# DEPARTMENT OF MATHEMATICS FACULTY OF ARTS, SCIENCE AND HUMANITIES PG PROGRAM (CBCS) – M.Sc. Mathematics

Course code	Name of the course	Objectives and Out Comes		Instruction Hours / Week			it(s)	Maximum Marks			- P.No.
	Traine of the course	PEOs	POs	L	Т	P	Credit(s)	CIA	ESE	Total	
								40	60	100	
		SEMEST	LEK-I	T		1	1		,		
20MMP101	Algebra	III	a, c, e	5	0	0	4	40	60	100	5
20MMP102	Real Analysis	I	a, g, e	5	0	0	4	40	60	100	7
20MMP103	Numerical Analysis	I	b, d, g	5	0	0	4	40	60	100	9
20MMP104	Theory of Ordinary Differential Equations	II	b, d, e	5	0	0	4	40	60	100	11
20MMP105A	Calculus of variation	III	e					4 40	60	100	13
20MMP105B	Number Theory	I	a, g	4	0	0	4				15
20MMP105C	Combinatorics	II	e								17
20MMP111	Numerical Computing – Practical	I	a	0	0	4	2	40	60	100	19
Journal Paper analysis & Presentation				2	-	-	-	-	-	-	
Semester Total				26	0	4	22	240	360	600	
	S	EMEST	ER – II								
20MMP201	Linear Algebra	III	c, e	5	0	0	4	40	60	100	20
20MMP202	Complex Analysis	I	a, c	5	0	0	4	40	60	100	22
20MMP203	Optimization Techniques	III	f	5	0	0	4	40 60 10		100	24
20MMP204	Theory of Partial Differential Equations	II	d, e	5	0	0	4	40			26
20MMP205A	Advanced Graph Theory	I	a							100	28
20MMP205B	Mechanics	I	a, g	4	0	0	4	40	60		30
20MMP205C	Fundamentals of Actuarial Mathematics	III	b, g	'		0	'	10		100	32
20MMP211	Optimization Techniques  - Practical	II	g	0	0	4	2	40	60	100	34
Journal Paper analysis & Presentation				2	_	-	-	-	-	-	
Semester Total				26	0	4	22	240	360	600	
	EMEST	ER – III									
20MMP301	Topology	III	c, e	5	0	0	5	40	60	100	35
20MMP302	Fluid Dynamics	II	c, e	5	0	0	4	40	60	100	37

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20MMP303	Measure Theory	III	f,g	5	0	0	4	40	60	100	39
20MMP304	Mathematical Statistics	I	i,j	5	0	0	4	40	60	100	41
20MMP305A	Fuzzy Sets and Fuzzy Logic	I	e,i							43	
20MMP305B	Matlab Programming	II	e,j	4	$\begin{vmatrix} 0 & 0 \end{vmatrix}$		4	40	60	100	45
20MMP305C	Neural Networks	III	b, e								48
20MMP311	Mathematical Statistics -		4	2	40	60	100	50			
Journal Paper and	Journal Paper analysis & Presentation			2	-	-	-	-	-	-	
Semester total				26	0	4	23	240	360	600	
SEMESTER – I	SEMESTER – IV				<b>!</b>						
20MMP401	Functional Analysis	III	c, e	5	0	0	5	40	60	100	52
20MMP402	Mathematical Methods	I	g,e,j	5	0	0	5	40	60	100	54
20MMP403	Stochastic Processes	II	j,g	5	0	0	5	40	60	100	56
20MMP411	Project	III	e	-		-	8	80	120	200	58
Semester total	Semester total			15	0	0	23	200	300	500	
Grand Total				93	0	12	90	920	1380	2300	

# **Electives Courses\***

Elective I		Elective II		Elective III			
Course code Name of the course		Course code Name of the course		Course code	Name of the course		
20MMP105A	Calculus of Variation	20MMP205A	Advanced Graph Theory	20MMP305A	Fuzzy Sets and Fuzzy Logic		
20MMP105B	Number theory	20MMP205B	Mechanics	20MMP305B	Matlab Programming		
20MMP105C	Combinatorics	20MMP205C	Fundamentals of Actuarial Mathematics	20MMP305C	Neural Networks		

# DEPARTMENT OF MATHEMATICS FACULTY OF ARTS, SCIENCE AND HUMANITIES PG PROGRAM (CBCS) – M.Sc. Mathematics (2020–2021 Batch and onwards)

## **PROGRAMME OUTCOMES (POs)**

- a. Solve intricate mathematical problems using the knowledge of pure and applied Mathematics.
- b. Explain the knowledge of modern issues in the field of mathematics.
- c. Proficiency in all lectureship exams approved by UGC.
- d. Solve differential equations governing real life issues.
- e. Pursue further studies and conduct research.
- f. Mathematical lifelong learning through continuous professional development.
- g. Employ technology in solving and understanding mathematical problems.

# PROGRAM SPECIFIC OUTCOMES (PSOs)

- h. Acquire knowledge of mathematics and its applications in all the fields.
- i. Acquaint with the recent advances in applied mathematical sciences such as numerical computations and mathematical modeling.
- j. Capable of formulating and analyzing mathematical models of real life applications.

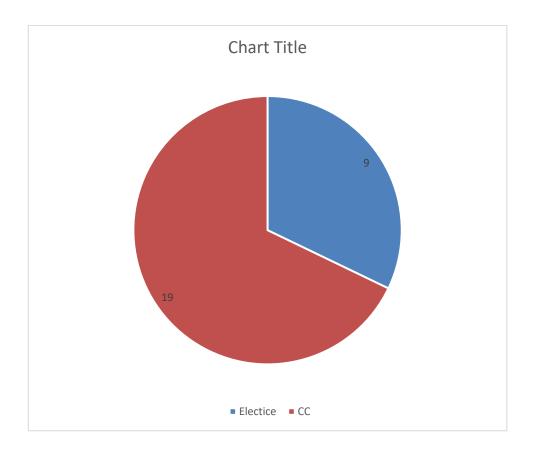
## PROGRAMME EDUCATIONAL OBJECTIVES (PEOs)

- **PEO I**: To engender problem-solving skills and apply them to the problems of pure and applied Mathematics.
- **PEO II :** To assimilate complicated mathematical concepts and arguments.
- **PEO III**: To enhance your own learning and create mathematical thinking

#### MAPPING OF POS AND PEOS

Pos	a	b	C	d	e	f	g	h	i	j
PEO I	X		X		X			X		X
PEO II	X			X			X			X
PEO III		X				X			X	

# COURSE DETAIL USING PIE CHART



Semester – I

**ALGEBRA** 

5H - 4C

#### 20MMP101

Instruction Hours / week: L: 5 T: 0 P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

This course enables the students to learn

- Sylow's theorems, Cauchy's theorem and Index theorem.
- The basic central ideas of Polynomial ring.
- The various algebraic structures
- The elementary concepts and basic ideas involved in homomorphism and isomorphism.
- The fundamental concepts of algebraic ring theory and fields.
- How to test if a polynomial is irreducible Finite Field (Galois Fields).

## **Course Outcomes (COs)**

After successful completion of this course the students will be able to

- 1. Recognize some advances of the theory of groups.
- 2. Define conjugate and conjugate classes.
- 3. Use Sylow's theorems in the study of finite groups.
- 4. Formulate some special types of rings and their properties.
- 5. Recognize the interplay between fields and vector spaces.
- 6. Apply the algebraic methods for solving problems.

#### **UNIT I**

#### SYLOW'S THEOREMS

Conjugate—Normalizer — Conjugate classes — application — Cauchy's theorem —Sylow's theorems—p-Sylow's subgroup—second and third proof of Sylow's theorem.

## UNIT II

# FINITE ABELIAN GROUP

Double co-set—Sylow's in doubt co-set. internal and external direct product – isomorphism –Finite abelian groups– invariant.

#### **UNIT III**

## POLYNOMIAL RINGS

Polynomial rings – Degree of polynomial – Polynomials over the rational field– Primitive – Content of the polynomial – Gauss lemma – integer monic – Eisenstein Criterion – Polynomial rings over commutative rings– unique factorization domain.

#### **UNIT IV**

## **GALOIS THEORY**

More about roots – derivative –simple extension–fixed field–Normal extension– Splitting field – Galois group–fundamental theorem of Galois theory.

## **UNIT V**

#### FINITE FIELD

Solvable group—the commutator subgroup—solvability by radicals—Finite fields—Wedderburn's theorem on finite division rings.

#### SUGGESTED READINGS

- 1. Herstein.I.N.,(2006). Topics in Algebra, Second edition, Wiley and sons Pvt. Ltd, Singapore.
- 2. Michiel Hazewinkel, Nadiya Gubareni and Kirichenko. V.V, (2011). Algebras, Rings and Modules, Vol.1, Springer International Edition, (Indian Print).
- 3. Artin. M., (2015). Algebra, Pearson Prentice-Hall of India, New Delhi.
- 4. Fraleigh. J. B., (2013). A First Course in Abstract Algebra, Seventh edition, Pearson Education Ltd. New Delhi.
- 5. Kenneth Hoffman., Ray Kunze., (2015). Linear Algebra, Second edition, Prentice Hall of India Pvt Ltd, New Delhi.
- 6. Vashista.A.R., (2014). Modern Algebra, KrishnaPrakashan Media Pvt Ltd, Meerut.

- 1. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25
- 2. https://ocw.mit.edu/courses/mathematics/18-702-algebra-ii-spring-2011/index.htm
- 3. <a href="https://www.youtube.com/watch?v=PN-cro0J\_v8&list=PLEAYkSg4uSQ1Yhxu2U-BxtRiZElrfVVcO">https://www.youtube.com/watch?v=PN-cro0J\_v8&list=PLEAYkSg4uSQ1Yhxu2U-BxtRiZElrfVVcO</a>
- 4. http://172.16.25.76/course/view.php?id=1646

Semester – I
REAL ANALYSIS
5H – 4C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

20MMP102

This course enables the students to learn

- The basic principles of Riemann Stieltjes Integral.
- The basic principles of real analysis.
- The concept of sequence and series of functions.
- Apply mathematical concepts and principles to power series.
- The concept inverse and implicit function theorem.
- How to identify sets with various properties such as finiteness, countability, infiniteness.

## **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Get specific skill in Riemann Stieltjes integral and Lebesgue integral.
- 2. Apply the concept of man value theorem for differentiable functions.
- 3. Apply implicit and inverse function theorem moving towards calculus on manifolds.
- 4. Solve given problems at a high level of abstraction based on logical and structured reasoning.
- 5. Attain knowledge in infinite series.
- 6. Describe the fundamental properties of the real numbers that underpin the formal development of real analysis.

#### **UNIT I**

#### DIFFERENTIATION

The derivative of real function – mean value theorems-The continuity of derivatives L'Hospital's rule – Derivatives of higher order – Taylor's theorem. Differentiation of vector-valued functions.

#### **UNIT-II**

## THE RIEMANN STIELTJES INTEGRAL

The Riemann-Stieltjes integral: Definition and existence of the integral – Properties of the integral - Integration and differentiation - Integration of vector valued functions - Rectifiable curves.

#### **UNIT III**

#### SEQUENCES OF FUNCTIONS

Sequences and series of functions: Discussion of Main problem – Uniform Convergence - Uniform convergence and Integration - Uniform convergence and

differentiation - Equicontinuous families of functions - The Stone-Weierstrass theorem.

#### **UNIT IV**

#### SOME SPECIAL FUNCTIONS

Some special functions: Power series - The exponential and Logarithmic functions - The trigonometric functions - The algebraic completeness of the complex field – Fourier Series - The Gamma functions.

#### **UNIT V**

#### IMPLICIT FUNCTIONS AND EXTREMUM PROBLEMS

Introduction – Functions with non zero Jacobian determinant – Inverse function theorem – Implicit function theorem – Extrema of real valued functions of one variable and several variables

#### SUGGESTED READINGS

- 1. Rudin. W., (2013). Principles of Mathematical Analysis, Tata McGraw Hill, New York.
- 2. Balli. N.P., (2017). Real Analysis, Laxmi Publication Pvt Ltd, New Delhi.
- 3. Bartle R.G. and Sherbert D. R. (2015). Introduction to Real Analysis, John Wiley and Sons (Asia) Pvt. Ltd.
- 4. Gupta. S.L. and Gupta. N. R., (2003). Principles of Real Analysis, Second edition, Pearson Education Pvt. Ltd, Singapore.
- 5. Royden. H.L., (2002). Real Analysis, Third edition, Prentice hall of India, New Delhi.
- 6. Sterling. K. Berberian., (2004). A First Course in Real Analysis, Springer Pvt Ltd, New Delhi.
- 7. Tom. M. Apostol., (2002). Mathematical Analysis, Second edition, Narosa Publishing House, New Delhi.

- 1. <a href="https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25">https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25</a>
- 2. https://ocw.mit.edu/courses/mathematics/18-100c-real-analysis-fall-2012/index.htm
- 3. http://172.16.25.76/course/view.php?id=1647

Semester – I
NUMERICAL ANALYSIS
5H – 4C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

20MMP103

This course enables the students

• To develop the working knowledge on different numerical techniques.

- To solve algebraic and transcendental equations.
- Appropriate numerical methods to solve differential equations.
- To provide suitable and effective methods for obtaining approximate representative numerical results of the problems.
- To solve complex mathematical problems using only simple arithmetic operations. The approach involves formulation of mathematical models of physical situations that can be solved with arithmetic operations.
- Provide a basic understanding of the derivation, analysis, and use of these numerical methods, along
  with a rudimentary understanding of finite precision arithmetic and the conditioning and stability of
  the various problems and methods.

## **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Identify the concept of numerical differentiation and integration.
- 2. Provide information on methods of iteration.
- 3. Solve ordinary differential equations by using Euler and modified Euler method.
- 4. Study in detail the concept of boundary value problems.
- 5. Attain mastery in the numerical solution of partial differential equations.
- 6. Apply numerical methods to obtain approximate solutions to mathematical problems.

#### **UNIT I**

## **SOLUTIONS OF NON-LINEAR EQUATIONS**

Newton's method- Convergence of Newton's method-Barstow's method for quadratic factors. Numerical Differentiation and Integration: Derivatives from difference tables – Higher order derivatives – divided difference. Trapezoidal rule– Romberg integration – Simpson's rules.

## **UNIT II**

## **SOLUTIONS OF SYSTEM OF EQUATIONS**

The Elimination method: Gauss Elimination and Gauss Jordan Methods – LU decomposition method. Methods of Iteration: Gauss Jacobi and Gauss Seidal iteration-Relaxation method.

#### UNIT III

## SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS

One step method: Euler and Modified Euler methods–Runge kutta methods. Multistep methods: Adams Moulton method – Milne's method.

#### **UNIT IV**

# BOUNDARY VALUE PROBLEMS AND CHARACTERISTIC VALUE PROBLEMS

The shooting method: The linear shooting method – The shooting method for non-linear systems. Characteristic value problems –Eigen values of a matrix by Iteration-The power method.

#### **UNIT V**

## NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

Classification of Partial Differential Equation of the second order – Elliptic Equations. Parabolic equations: Explicit method – The Crank Nicolson difference method. Hyperbolic equations – solving wave equation by Explicit Formula.

#### SUGGESTED READINGS

- 1. Gerald, C. F., and Wheatley. P. O., (2009). Applied Numerical Analysis, Seventh edition, Dorling Kindersley (India) Pvt. Ltd. New Delhi.
- 2. Jain. M. K., Iyengar. S. R. K. and R. K. Jain., (2012). Numerical Methods for Scientific and Engineering Computation, New Age International Publishers, New Delhi.
- 3. Burden R. L., and Douglas Faires.J., (2014). Numerical Analysis, Seventh edition, P. W. S. Kent Publishing Company, Boston.
- 4. Sastry S.S., (2009). Introductory methods of Numerical Analysis, Fourth edition, Prentice Hall of India. New Delhi.

- 1. <a href="https://ocw.mit.edu/courses/mathematics/18-330-introduction-to-numerical-analysis-spring-2012/index.htm">https://ocw.mit.edu/courses/mathematics/18-330-introduction-to-numerical-analysis-spring-2012/index.htm</a>
- 2. <a href="https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25">https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25</a>
- 3. <a href="http://vidyamitra.inflibnet.ac.in/index.php/search?subject%5B%5D=&course%5B%5D=Numerical+analysis&domain%5B%5D=Physical+%26+Basic+Sciences">http://vidyamitra.inflibnet.ac.in/index.php/search?subject%5B%5D=&course%5B%5D=Numerical+analysis&domain%5B%5D=Physical+%26+Basic+Sciences</a>
- 4. <a href="http://172.16.25.76/course/view.php?id">http://172.16.25.76/course/view.php?id</a>=1648

Semester - I

## 20MMP104 THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

5H-4C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

This course enables the students to learn

• The formulation and solutions of second order ordinary differential equations and get exposed to physical problems with applications.

- The concept of solve the system of first order equations.
- Linear homogeneous and non homogeneous equations with constant coefficients.
- Understanding the elementary linear oscillations.
- Understand all of the concepts relating to the order and linearity of ordinary differential equations, analytic and computational solution methods for ordinary differential equations, and the real-world applications of ordinary differential equations.
- Apply your understanding of the concepts, formulas, and problem solving procedures to thoroughly investigate relevant physical models.

# **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Model a simple physical system to obtain a first and second order differential equation.
- 2. Understand the basic notions of linearity, superposition, existence and uniqueness of solution to differential equations and use these concepts in solving linear differential equations.
- 3. Identify homogeneous equations, homogeneous equations with constant coefficients and exact linear differential equations.
- 4. Solve higher order and system of differential equations of Successive approximation.
- 5. Understand the difficulty of solving problems for elementary linear oscillations.
- 6. Identify, analyze and subsequently solve physical situations whose behavior can be described by ordinary differential equations.

### **UNIT I**

## SECOND ORDER LINEAR EQUATIONS

Second order linear equations with ordinary points – Legendre equation and Legendre polynomial – Second order equations with regular singular points – Bessel equation.

#### **UNIT II**

# **EXISTENCE AND UNIQUENESS SOLUTIONS**

System of first order equations – existence and uniqueness theorems – fundamental matrix.

#### **UNIT III**

## NON-HOMOGENEOUS EQUATIONS

Non homogeneous linear system – linear systems with constant coefficient – Linear systems with periodic coefficients.

#### **UNIT IV**

## SUCCESSIVE APPROXIMATIONANDNON UNIQUENESS SOLUTIONS

Successive approximation – Picard's theorem – Non uniqueness of solution – Continuation and dependence on initial conditions – Existence of solution in the large existence and uniqueness of solution in the system.

#### **UNIT V**

#### OSCILLATION THEORY

Fundamental results – Sturms comparison theorem – Elementary linear oscillations – Comparison theorem of Hille winter – Oscillations of x'' + a(t)x = 0 elementary non linear oscillations.

#### SUGGESTED READINGS

- 1. Earl A. Coddington, (2004). An introduction to Ordinary differential Equations, Prentice Hall of India Private limited, New Delhi.
- 2. Deo. S. G, Lakshmikantham, V. and Raghavendra, V. (2005). Ordinary Differential Equations and Stability Theory, Second edition, Tata McGraw Hill Publishing Company limited, New Delhi.
- 3. Rai. B, Choudhury, D. P. and Freedman, H. I. (2004). A course of Ordinary differential Equations, Narosa Publishing House, New Delhi.
- 4. George F. Simmons, (2017). Differential Equations with application and historical notes, 3<sup>rd</sup> edition by Taylor & Francis Group, LLC.
- 5. Sanchez. D.A,(1968).Ordinary Differential Equations and Stability Theory, W.H.Freeman & Co., San Francisco.

- 1. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25
- 2. <a href="https://www.youtube.com/watch?v=4QllSn2v7p4&list=PLbMVogVj5nJSGlf9sluucwobyr\_zz6glD&index=14">https://www.youtube.com/watch?v=4QllSn2v7p4&list=PLbMVogVj5nJSGlf9sluucwobyr\_zz6glD&index=14</a>
- 3. http://172.16.25.76/course/view.php?id=1649

Semester – I 20MMP105A CALCULUS OF VARIATION 4H –4C

Instruction Hours/week: L:4T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

This course enables the students to learn

• Formulate variational problems and analyse them to deduce key properties of system behaviour.

- The calculus of variations concerns problems in which one wishes to find the minima or extrema of some quantity over a system that has functional degrees of freedom.
- Many important problems arise in this way across pure and applied mathematics.
- Finding the configuration of a piece of elastic that minimises its energy.
- Perhaps most importantly, the principle of least action is now the standard way to formulate the laws of mechanics and basic physics.
- The minimizing principle that underlies these equations leads to direct methods for analysing the solutions to these equations.

## **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Understand what functionals are, and have some appreciation of their applications
- 2. Apply the formula that determines stationary paths of a functional to deduce the differential equations for stationary paths in simple cases
- 3. Use the Euler-Lagrange equation or its first integral to find differential equations for stationary paths
- 4. Solve differential equations for stationary paths, subject to boundary conditions, in straightforward cases.
- 5. Describe the brachistochrone problem mathematically and solve it;
- 6. Solve isoperimetric problems of standard type

#### **UNIT I**

#### **ELEMENTS OF THE THEORY**

Functionals. Some Simple Variational Problems – Function Spaces – The Variation of a Functional- A Necessary Condition for an Extremum – The Simplest Variational Problem. Euler's Equation—The Case of Several Variables—A Simple Variable End Point Problem—The Variational Derivative—Invariance of Euler's Equation.

FURTHER GENERALIZATIONS: The Fixed End Point Problem for n Unknown Functions, Variational Problems in Parametric Form, Functionals Depending on Higher – Order Derivatives, Variational Problems with Subsidiary Conditions.

THE GENERAL VARIATION OF A FUNCTIONAL: Derivation of the Basic Formula – End Points Lying on Two Given Curves or Surfaces–Broken Extremals – The Weierstrass Erdmann Conditions.

#### UNIT II

# THE CANONICAL FORM OF THE EULER EQUATIONS AND RELATED TOPICS The Canonical Form of the Euler Equations – First Integrals of the Euler Equations, The Legendre Transformation – Canonical Transformations – Noether's Theorem – The Principle of Least Action – Conservation Laws – The Hamilton-Jacobi Equation – Jacobi's Theorem.

#### **UNIT III**

#### THE SECOND VARIATION. SUFFICIENT CONDITIONS FOR A WEAK EXTREMUM

Quadratic Functionals – The Second Variation of a Functional – The Formula for the Second Variation – Legendre's Condition – Analysis of the Quadratic Functionals  $\int_a^b (ph' + qh') dx$  Jacobi's Necessary Condition. More on Conjugate Points – Sufficient Conditions for a Weak Extremum – Generalization to n unknown functions – Connection Between Jacobi's Condition and the Theory of Quadratic Forms – Sufficient conditions for a strong extremum: Consistent Boundary Conditions – General Definition of a Field, The Field of a Functional – Hilbert's Invariant Integral – The Weierstrass E-Function – Sufficient Conditions for a Strong Extremum.

# **UNIT IV**

#### DIRECT METHODS IN THE CALCULUS OF VARIATIONS

Minimizing Sequences – The Ritz Method and the Method of Finite Differences – The Sturm-Liouville Problem – Integral Equations

#### **UNIT V**

## INTEGRAL EQUATIONS

Introduction and basic examples, Classification – Conversion of Volterra Equation to ODE, Conversion of IVP and BVP to Integral Equation – The Green's function. 26 Decomposition – direct computation – Successive approximation, Successive substitution methods for Fredholm Integral Equation series solution – successive approximation – successive substitution method for Volterra Integral Equations – Volterra Integral Equation of first kind, Integral Equations with separable Kernel – Fredholm's first, second and third theorem – Integral Equations with symmetric kernel – Eigen function expansion – Hilbert Schmidt theorem, Fredholm and Volterra Integro – Differential equation – Singular and nonlinear Integral Equation.

#### SUGGESTED READINGS

- 1. Gelfand. I.M. and Francis. S. V.,(1991). Calculus of Variation, Prentice Hall, (All the chapters except chapter 7 are included).
- 2. Tricomi.F. G.,(1985). Integral equations, Dover Publications.
- 3. Gupta. A. S., (2006). Calculus of Variations with Applications, PHI.
- 4. Weinstock, Robert.,(1974). Calculus of Variations with Applications to Physics and Engineering, Dover.
- 5. Cordumeanu, C.,(1991). Integral Equations and Applications, Cambridge University Press.

- 1. http://vidyamitra.inflibnet.ac.in/index.php/content/index/5a5dcd198007be95cebc3043
- 2. https://www.youtube.com/watch?v=urfILCPQYYY

Semester – I 20MMP105B NUMBER THEORY 4H –4C

Instruction Hours/week: L: 4T: 0P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

This course enables the students to learn

• Mathematical concepts and principles to perform numerical and symbolic computations.

- Investigate and solve mathematical problems.
- Write clear and precise proofs.
- Communicate effectively in both written and oral form.
- Some foundational ideas in number theory without the technical baggage often associated with a more advanced courses.
- The opportunity to develop an appreciation of pure mathematics while engaged in the study of number theoretic results.

## **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Identify and apply various properties of and relating to the integers including the Well-Ordering Principle, primes, unique factorization, the division algorithm, and greatest common divisors.
- 2. Identify certain number theoretic functions and their properties.
- 3. Understand the concept of congruence and use various results related to congruence including the Chinese Remainder Theorem.
- 4. Solve certain types of Diophantine equations.
- 5. Identify how number theory is related to and used in cryptography.
- 6. Acquire a broad knowledge in Greatest Integer Function.

#### **UNIT I**

#### DIVISIBILTY

Introduction-Divisibility-definition - Properties greatest common divisor-Least common multiple-Primes -Euclid Lemma-The Binomial theorem.

#### **UNIT II**

#### **CONGRUNCES**

Congruences, solutions of congruences, Congruences of Degree one. The functions of n, Congruences of higher degree, Prime power moduli, Prime modulus.

#### **UNIT III**

## **CONGRUENCES (CONTINUITY)**

Congruences degree 2, prime modulus, Power Residues, Number theory from an algebraic view point, Multiplicative groups, Rings and fields, quadratic residues.

## **UNIT IV**

#### **OUADRATIC RECIPROCITY**

Quadratic reciprocity – The Jacobi Symbol – Greatest integer function.

#### **UNIT V**

#### SOME FUNCTIONS OF NUMBER THEORY

Arithmetic functions – The Moebius Inversion formula – The multiplication of arithmetic functions – Recurrence functions.

#### SUGGESTED READINGS

- 1. Ivan Nivan and Herberts Zucherman., (2008). An Introduction to Theory of Numbers third Edition, Wiley Eastern Limited, New Delhi.
- 2. Apostol T.M., (1998). Introduction to Analytic Number Theory, Springer Verlag,.
- 3. Kennath and Rosan, (1986). Elementary Number Theory and its Applications, Addison Wesley Publishing Company.
- 4. George E. Andrews., (2012). Number Theory, Hindustan Publishing, New Delhi.

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- 2. http://vidyamitra.inflibnet.ac.in/index.php/content/index/5704c5378ae36c6ab9b0b9d0
- 3. http://172.16.25.76/course/view.php?id=1653

Semester – I 20MMP105C COMBINATORICS 4H –4C

Instruction Hours/week: L: 4T: 0P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

This course enables the students to learn

- Improve mathematical proof writing skills.
- Cater mathematical verbal communication skills.
- Afford problem-solving skills.
- Combinatorial proofs of identities and inequalities.
- Model and analyze computational processes using analytic and combinatorial methods.
- Structures to represent mathematical and applied questions, and they will become comfortable with the combinatorial tools commonly used to analyze such structures.

## **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Cognition in various combinatorial methods.
- 2. Solve real-life problems through computational skills.
- 3. Develop different combinatorial techniques.
- 4. Apply the inclusion/exclusion principle.
- 5. Analyze combinatorial objects satisfying certain properties and answer questions related to Necklace problem.
- 6. Know the concept of Burnside's lemma.

#### UNIT I

#### PERMUTATIONS AND COMBINATIONS:

The rules of sum and product - Distributions of distinct objects - Distributions of Non distinct objects - Stirling's formula.

#### **UNIT II**

#### **GENERATING FUNCTIONS**

Generating functions for combinations - Enumerators for permutations- Distributions of distinct objects into non distinct cells - Partitions of integers - Ferrers graph - Elementary relations.

### **UNIT III**

#### RECURRENCE RELATIONS

Linear recurrence relations with constant coefficients - Solutions by the technique of generating functions - A special class of nonlinear difference equations - Recurrence relations with two indices.

#### **UNIT IV**

#### THE PRINCIPLE OF INCLUSION AND EXCLUSION

General formula - Permutations with restrictions on relative positions - The Rook polynomials - Permutations with forbidden positions.

#### **UNIT V**

## **EQUIVANCE CLASS OF FUNCTIONS**

Polya's Theory of Counting - Equivalence classes under a permutation group - Equivalence classes of functions - Weights and inventories of functions - Polya's fundamental theorem.

#### SUGGESTED READINGS

- 1. Liu C. L., (2000). Introduction of Combinatorial Mathematics, McGraw Hill, Singapore.
- 2. Marshall Hall J.R., (1998). Combinatorial Theory, John Wiley & Sons, New York.
- 3. Ryser H.J.,(1979). Combinatorial Mathematics, The Mathematical Association of America, John Wiley & Sons, Inc, New York.

- 1. https://freevideolectures.com/course/3542/combinatorics
- **2.** http://172.16.25.76/course/view.php?id=642

Semester-I

20MMP111 NUMERICAL COMPUTING – PRACTICAL

4H-2C

Instruction Hours/week: L: 0 T: 0 P: 4 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam: 3** Hours

## **Course Objectives**

This course enables the students to learn

• In-depth understanding of functional, logic, and programming paradigms.

- How to implement several programs in languages other than the one emphasized in the core curriculum.
- This course provides an introduction to the basic concepts and techniques of numerical solution of algebraic equation.
- This course is to provide students with an introduction to the field of numerical analysis.
- Understand the concept of Gauss elimination method.
- How to find the differential equation in numerical method.

## **Course Outcomes (COs)**

On successful completion of this course, the student will be able to

- 1. Know the concepts for problem solving.
- 2. Acquire new knowledge in computing, including the ability to learn about new ideas and advances, techniques, tools, and languages, and to use them effectively; and to be motivated to engage in life-long learning
- 3. Comprehend important issues related to the development of computer-based systems in a professional context using a well-defined process.
- 4. Be familiar with programming with numerical packages.
- 5. Be aware of the use of numerical methods in modern scientific computing.
- 6. To develop the mathematical skills of the students in the areas of numerical methods.

#### List of Practical

- 1. Solution of non-linear equation-Bairstow's method for quadratic factors.
- 2. Solution of simultaneous equations-Gauss Elimination.
- 3. Solution of simultaneous equations-Gauss Jordan.
- 4. Solution of simultaneous equations-Gauss Jacobi.
- 5. Solution of simultaneous equations-Gauss Seidal.
- 6. Solution of simultaneous equations-Triangularisation.
- 7. Numerical integration-Trapezoidal rule.
- 8. Numerical integration-Simpson's rules.
- 9. Solution for ordinary differential equation-Euler method.
- 10. Solution for ordinary differential equation- Runge Kutta Second order.
- 11. Solution for parabolic equation Explicit method.
- 12. Solution for parabolic equation The Crank Nicolon method.

Semester – II LINEAR ALGEBRA 5H – 4C

Instruction Hours/week: L:5 T:0 P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam: 3 Hours** 

# **Course Objectives**

20MMP201

This course enables the students to learn

• To develop student's mathematical maturity and enables to build mathematical thinking and use results from canonical forms and inner product spaces to solve contemporary problems.

- Be able to use and understand matrix and vector notation, addition, scalar multiplication, the dot product, matrix multiplication, and matrix transposition.
- Construct visualizations of matrices related to vector fields. Explain how eigenvalues and eigenvectors relate to vector fields.
- Gaussian elimination to solve systems of linear equations and write the solution in parametric and vector form.
- Normalize vectors, obtain vectors of a given length in a given direction, and explain how to tell if two vectors are orthogonal.
- Concept of cyclic decomposition.

## **Course Outcomes (COs)**

On successful completion of this course, the student will be able to

- 1. Recognize some advances of vector spaces and linear transformations.
- 2. Understand the concepts of linear algebra in geometric point of view.
- 3. Visualize linear transformations as a matrix form.
- 4. Decompose a given vector space in to certain canonical forms.
- 5. Formulate several classes of linear transformations and their properties.
- 6. Know the concept of rational form.

#### **UNIT I**

#### LINEAR TRANSFORMATIONS

Linear transformation – null space – rank – nullity – Algebra of linear transformation. The algebra of linear transformations-Isomorphism – Representation of transformations by matrices – Linear functional-The double dual – The transpose of a linear transformation.

#### **UNIT II**

#### **POLYNOMIALS**

Linear algebras over the field – Algebra of polynomials – Lagrange Interpolation – isomorphic – Polynomial ideals – root – Taylor's Formula – ideal – greatest common divisor – relatively prime – the prime factorization of a polynomial.

#### **UNIT III**

#### **DETERMINANTS**

Commutative Ring- Determinant Functions- n-linear - Permutations- degree of permutation - Signature of permutation - the Uniqueness of Determinants- Additional Properties of Determinants- Cramer's rule.

#### **UNIT IV**

#### ELEMENTARY CANONICAL FORM

Introduction— Characteristic Values— Characteristic vector — Characteristic space — example — Diagonalizable — Annihilating Polynomials— Invariant Subspaces— Simultaneous Triangulation— Simultaneous Diagonalization.

#### **UNIT V**

## THE RATIONAL AND JORDAN FORM

Cyclic Subspaces and Annihilators— Cyclic Decompositions theorem — generalized Cayley Hamilton theorem — The Jordon Form.

# **SUGGESTED READINGS**

- 1. Kennath M. Hoffman and Ray Kunze, (2015). Linear Algebra, 2 Edition, Pearson India Publishing, New Delhi.
- 2. Herstein.I.N, (2006). Topics in Algebra, 2nd Edition, John Wiley & Sons, Singapore.
- 3. Vivek Sahai & Vikas Bist (2013). Linear Algebra, Narosa Publishing House.
- 4. Rao.A.R. & Bhimashankaram. P, (2000). Linear Algebra, Tata McGraw Hill.
- 5. Golan.J.S,(2010). Foundations of linear Algebra, Kluwer Academic publisher.
- 6. Kumaresan. S, (2006). Linear Algebra A Geometric Approach, Prentice Hall of India.

- 1. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25
- 2. http://vidyamitra.inflibnet.ac.in/index.php/content/index/5704c5378ae36c6ab9b0b0fa
- 3. https://ocw.mit.edu/courses/mathematics/18-06sc-linear-algebra-fall-2011/
- 4. <a href="https://ocw.mit.edu/courses/mathematics/18-s096-topics-in-mathematics-with-applications-in-finance-fall-2013/video-lectures/lecture-2-linear-algebra/">https://ocw.mit.edu/courses/mathematics/18-s096-topics-in-mathematics-with-applications-in-finance-fall-2013/video-lectures/lecture-2-linear-algebra/</a>

Semester – II
COMPLEX ANALYSIS
5H – 4C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

20MMP202

This course enables the students to learn

• To learn the concepts of Oriented circles and level curves.

- Fundamental concepts of complex integration.
- To know the concepts of harmonic function.
- To develop the skill of contour integration to evaluate complicated real integrals via residue calculus.
- The development of the complex variable in boundary behaviour.
- Contour integral using parametrization, fundamental theorem of calculus and Cauchy's integral formula.

## **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Explain the role of the Conformal mapping.
- 2. Evaluate complex contour integrals and some of their consequences.
- 3. Determine the Taylor series or the Laurent series of an analytic function in a given region
- 4. Describe the convergence properties of a power series.
- 5. Know the basic properties of singularities of analytic functions.
- 6. Demonstrate familiarity with a range of examples of these concepts of conformal mapping.

#### UNIT I

#### CONFORMALITY

Conformal mapping-Linear transformations- cross ratio- symmetry- Oriented circles-families of circles-level curves.

## **UNIT II**

#### **FUNDAMENTAL THEOREMS**

Complex integration - rectifiable Arcs- Cauchy's theorem for Rectangle and disc - Cauchy's integral formula - higher derivatives.

#### UNIT III

#### HARMONIC FUNCTIONS

Harmonic functions - mean value property-Poisson's formula-Schwarz theorem, Reflection principle-Weierstrass theorem- Taylor series and Laurent series.

## **UNIT IV**

#### **PARTIAL FRACTIONS**

Partial Fractions- Infinite products – Canonical products-The gamma function – Stirling's Formula – Entire functions – Jensen's formula.

#### **UNIT V**

#### THE RIEMANN MAPPING THEOREM

Riemann Mapping Theorem – Boundary behavior – Use of Reflection Principle – Analytical arcs – Conformal mapping of polygons- The Schwartz - Christoffel formula.

#### SUGGESTED READINGS

- 1. Lars V .Ahlfors., (2015). Complex Analysis, Third edition, Mc-Graw Hill Book Company, New Delhi.
- 2. Ponnusamy, S.,(2019). Foundation of Complex Analysis, Second edition, Narosapublishing house, New Delhi.
- 3. Choudhary, B., (2005). The Elements of Complex Analysis, New Age International Pvt. Ltd ,New Delhi.
- 4. Vasishtha, A. R., (2014). Complex Analysis, Krishna Prakashan Media Pvt. Ltd., Meerut.
- 5. Walter Rudin., (2017) .Real and Complex Analysis,3<sup>rd</sup> edition, McGraw Hill Book Company, New York.

- 1. https://nptel.ac.in/courses/111107056/
- 2. https://nptel.ac.in/courses/111103070/
- 3. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25

Semester – II

## 20MMP203 OPTIMIZATION TECHNIQUES

5H-4C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

This course enables the students to learn

- The basic concepts of integer linear programming.
- How to solve quadratic programming problems, dynamic programming problems and non-linear programming problems.
- Classical optimization techniques and numerical methods of optimization.
- Know the basics of different evolutionary algorithms.
- Explain Integer programming techniques and apply different optimization techniques to solve various models.
- Enumerate the fundamental knowledge of Linear Programming and Dynamic Programming problems.

# **Course Outcomes (COs)**

On successful completion of this course, the students will be able to

- 1. Understand the concept of linear programming and integer programming.
- 2. Develop optimal decision policy skill.
- 3. Familiarize with real life applications of inventory models.
- 4. Skill in decision analysis.
- 5. Mastery in Beale's method and simplex method.
- 6. Use classical optimization techniques and numerical methods of optimization.

## **UNIT I**

#### INTEGER LINEAR PROGRAMMING

Types of Integer Linear Programming Problems - Concept of Cutting Plane - Gomory's All Integer Cutting Plane Method - Gomory's mixed Integer Cutting Plane method Branch and Bound Method.

- Zero-One Integer Programming – Real life application in Integer Linear Programming.

# **UNIT II**

## DYNAMIC PROGRAMMING

Characteristics of Dynamic Programming Problem - Developing Optimal Decision Policy - Dynamic Programming under Certainty - DP approach to solve LPP.

#### **UNIT III**

#### PROBABILISTIC INVENTORY MODEL

Real life application -Continuous review models- Probabilistic Economic order quantity (EOQ) Model. Single-period models – No setup model – setup model. Multi Period model.

## **UNIT IV**

## **DECISION ANALYSIS**

Real life application - Decision making under certainty- Analytic hierarchy process. Decisions under Risk- Decision Trees-based expected value criterion, variations of the expected value criterion. Decisions under Uncertainty Real life application in Decision Analysis

#### **UNIT V**

#### NON-LINEAR PROGRAMMING METHODS

Examples of NLPP - General NLPP - Graphical solution - Quadratic Programming - Wolfe's modified Simplex Methods - Beale's Method.

#### **SUGGESTED READINGS**

- 1. Sharma, J. K., (2017). Operations Research Theory and Practice, Third edition, Macmillan India Ltd.
- 2. Handy, A. Taha.(2007). Operations Research, Eighth edition, Prentice Hall of India Pvt Ltd, New Delhi.
- 3. Kantiswarup., Gupta, P. K. and Manmohan., (2006). Operations Research, Twelfth edition, Sultan Chand & Sons Educational Publishers, New Delhi.
- 4. Panneerselvam, R., (2007). Operations Research, Second edition, Prentice Hall of India Private Ltd, New Delhi.
- 5. Singiresu, S. Rao., (2006). Engineering Optimization Theory and Practice, Third edition New Age International Pvt Ltd, New Delhi.
- 6. Sivarethina Mohan. R., (2008). Operations Research, First edition, Tata McGraw Hill Publishing Company Ltd, New Delhi.

- 1. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25
- 2. http://172.16.25.76/course/view.php?id=2082

Semester – II 5H – 4C

## 20MMP204 THEORY OF PARTIAL DIFFERENTIAL EQUATIONS

Instruction Hours/week: L:5 T:0 P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

This course enables the students to learn

• To solve Laplace equations, diffusion equations and wave equations arising in real life problems.

- About initial and boundary value problems for PDEs of first and second order which includes Laplace Equation, Diffusion Equation and Wave Equation.
- Introduce students to how to solve linear Partial Differential with methods.
- Technique of separation of variables to solve PDEs and analyze the behavior of solutions in terms of eigen function expansions.
- Solutions of PDEs are determined by conditions at the boundary of the spatial domain and initial conditions at time zero.
- Basic questions concerning the existence and uniqueness of solutions, and continuous dependence of initial and boundary data.

# **Course Outcomes (COs)**

On successful completion of this course the students will be able to

- 1. Classify linear and Nonlinear first order differential equations with constant coefficients.
- 2. Solve the linear partial differential equations with constant coefficient equations.
- 3. Describe the method of separable variables and integral transforms.
- 4. Solve the elementary Laplace equation with symmetry.
- 5. Acquire the knowledge of wave equation and vibrating membranes.
- 6. Enrich their knowledge about diffusion equations with sources.

## UNIT I

## FIRST ORDER PARTIAL DIFFERENTIAL EQUATIONS

Non linear partial differential equation of first order –Compatible systems of first order equations – Special type of first order equations – Partial differential equations of second order – The origin of second order equations – Linear partial differential equations with constant coefficient equations with variable coefficients.

## UNIT II

#### METHOD OF SEPARATION OF VARIABLES

Separation of variables —The vibrating string problem-Existence and Uniqueness of solution of the vibrating string problem. The heat conduction problem-existence and uniqueness of solution of the heat conduction problem —The Laplace and beam Equations.

#### **UNIT III**

#### **BOUNDARY VALUE PROBLEMS**

Elementary solutions of Laplace equations- Families of Equi-potential surfaces – Interior Dirichlet and Neumann problems- Seperation of variables--Dirichlet problems for a circular annulus-Neumann problem for a circle Dirichlets problems for a rectangle – Neumann problems for a rectangle – problems with axial symmetry.

#### **UNIT IV**

#### **GREEN FUNCTION**

The Delta Function-Green' function – Method of green's function-Dirichlet problem for laplace operator – Method of image – Method of eigen functions.

#### **UNIT V**

# **WAVE EQUATION**

Elementary solutions of one dimensional wave equation-Vibrating membrane – Applications of calculus of variations – Diffusion equation-Elementary solution of Diffusion equation – Separation of variables.

#### SUGGESTED READINGS

- 1. Sharma, J. N, Keharsingh, (2009). Partial Differential Equations for Engineering and Scientists, Narosa Publishing House, New Delhi.
- 2. Ian. N. Sneedon, (2006). Elementary Partial differential equations, Tata Mcgraw Hill Ltd.
- 3. Geraold. B. Folland, (2001). Introduction to Partial Differential Equations, Prentice Hall of India Private limited, New Delhi.
- 4. Sankara Rao. K, (2010). Introduction to Partial Differential Equations, Third edition, Prentice Hall of India Private limited, New Delhi.
- 5. Veerarajan, T, (2004). Partial Differential Equations and Integral Transforms, Tata McGraw-Hill Publishing Company limited, New Delhi.
- 6. John, F, (1991). Partial Differential equations, Third edition, Narosa publication co, New Delhi
- 7. Tyn-Myint-U and Lokenath Debnath(2008). Linear Partial Differential Equations for Scientists and Engineers, Fourth Edition, Birkhauser, Berlin.
- 8. Evans. L.C,(2003). Partial Differential Equations, AMS, Providence RI.

- 1. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25
- 2. <a href="http://vidyamitra.inflibnet.ac.in/index.php/search?subject%5B%5D=&course%5B%5D=Partial+differential+equations&domain%5B%5D=Physical+%26+Basic+Sciences">http://vidyamitra.inflibnet.ac.in/index.php/search?subject%5B%5D=&course%5B%5D=Physical+%26+Basic+Sciences</a>

Semester – II

20MMP205A

# ADVANCED GRAPH THEORY

4H - 4C

Instruction Hours/week: L: 4T: 0P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

This course enables the students to learn

- The fundamental concepts in Graph Theory and some of its modern applications.
- The use of these methods in subsequent courses in the design and analysis of algorithms, computability theory, software engineering, and computer systems.
- Apply graph-theoretic terminology and notation.
- Analyze new networks using the main concepts of graph theory.
- Central theorems about trees, matching, connectivity, colouring and planar graphs.
- Know the concept of domination in graphs.

## **Course Outcomes (COs)**

On successful completion of this course, the students will be able to

- 1. Applied the knowledge of graphs to solve the real-life problem.
- 2. Overview of properties of trees and a minimal spanning tree for a given weighted graph.
- 3. Understanding the basic concepts of graphs, directed graphs, and weighted graphs and able to present a graph by matrices.
- 4. Understand Eulerian and Hamiltonian graphs.
- 5. Determine, whether graphs are Planer and/or non planer.
- 6. Identify induced subgraphs, cliques, matchings, covers in graphs.

#### **UNIT I**

#### **GRAPHS AND TREES**

Graphs – Introduction – Isomorphism – Sub graphs – Walks, Paths, Circuits – Connectedness – Components – Euler Graphs – Hamiltonian Paths and Circuits – Trees – Properties of trees – Distance and Centers in Tree – Rooted and Binary Trees - Spanning trees – Fundamental Circuits.

#### UNIT II

## **CUT SETS AND CONNECTIVITY**

Spanning Trees in a Weighted Graph – Cut Sets – Properties of Cut Set – All Cut Sets – Fundamental Circuits and Cut Sets – Connectivity and separability – Network flows – 1-Isomorphism – 2-Isomorphism – Planer Graphs – Different Representation of a Planer Graph.

#### **UNIT III**

#### MATRIX AND GRAPH COLORING

Incidence matrix – Sub matrices – Circuit Matrix – Path Matrix – Adjacency Matrix – Chromatic Number – Chromatic partitioning – Chromatic polynomial - Matching - Covering – Four Color Problem.

#### **UNIT IV**

#### **COUNTING TREE**

Directed Graphs – Types of Directed Graphs - Types of enumeration, counting labeled trees, counting unlabelled trees, Polya's counting theorem, graph enumeration with Polya's theorem.

#### **UNIT V**

#### DOMINATION IN GRAPHS

Introduction – Terminology and concepts – Applications – Dominating set and domination number – Independent set and independence number.

#### SUGGESTED READINGS

- 1. Deo N, (2007). Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India Pvt Ltd, New Delhi.
- 2. Balakrishnan R, Ranganathan K (2008) A Textbook of Graph Theory, Springer, International Edition, New Delhi.
- 3. Teresa W. Haynes, Stephen T. Hedetniemi and Peter J.Slater, (1998). Fundamentals of Domination in Graphs, Marcel Dekker, New York.
- 4. Jonathan L Gross, Jay Yellen, (2014). Handbook of Graph Theory, CRC Press LLC. Taylor & Francis Group, Boca Rotan.
- 5. Diestel. R Springer-Verlag, (2012). Graph Theory. Springer-Verleg, New York.
- 6. Flouds C. R., (2009). Graph Theory Applications, Narosa Publishing House. New Delhi, India.
- 7. Arumugam. S, Ramachandran. S, (2006). Invitation to graph theory, Scitech publications, Chennai.
- 8. Harary F, (2001). Graph Theory, Addison-Wesley Publishing Company Inc USA.

- 1. http://vidyamitra.inflibnet.ac.in/index.php/content/index/5704c5378ae36c6ab9b0b602
- 2. http://172.16.25.76/course/view.php?id=2085

Semester – II 20MMP205B MECHANICS 4H –4C

Instruction Hours/week: L: 4T: 0P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

This course enables the students to learn

• How to use Newton's laws of motion to solve advanced problems involving the dynamic motion of classical mechanical systems.

- Applications of differential equations in advanced mathematical problems.
- To solve dynamics problems such as conservation of energy and linear and angular momentum.
- Parameters defining the motion of mechanical systems and their degrees of freedom.
- The components of a force in rectangular or nonrectangular coordinates. ÿ Determine the resultant of a system of forces.
- Complete and correct free-body diagrams and write the appropriate equilibrium equations from the free-body diagram.

# **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Understand the concept of the D'Alembert's principle.
- 2. Derive the Lagrange's equation for holomonic and non holomonic constraints.
- 3. Classify Scleronomic and Rheonomic systems.
- 4. Solve the problems of Hamilton equations of motion.
- 5. Study of the canonical transformations.
- 6. Know the concept of Hamilton Jacobi Theory.

#### **UNIT I**

### SURVEY OF ELEMENTARY PRINCIPLES

Constraints - Generalized coordinates, Holonomic and non-holonomic systems, Scleronomic and Rheonomic systems. D'Alembert's principle and Lagrange's equations – Velocity – dependent potentials and the dissipation function – some applications of the Lagrange formulation.

#### **UNIT II**

## VARIATION PRINCIPLES AND LAGRANGE'S EQUATIONS

Hamilton's principle – Some techniques of calculus of variations – Derivation of Lagrange's Equations from Hamilton's principle – Extension of Hamilton's principle to non-holonomic systems – Conservation theorems and symmetry properties.

#### **UNIT III**

## HAMILTON EQUATIONS OF MOTION

Legendre Transformations and the Hamilton Equations of motion-canonical equations of Hamilton – Cyclic coordinates and conservation theorems – Routh's procedure - Derivation of Hamilton's equations from a variational principle – The principle of least action.

#### **UNIT IV**

## **CANONICAL TRANSFORMATIONS**

The equations of canonical transformation – Examples of Canonical transformations – Poission Brackets and other Canonical invariants – integral invariants of Poincare, Lagrange brackets.

#### **UNIT V**

## **HAMILTON JACOBI THEORY**

Hamilton Jacobi equations for Hamilton's principle function – Harmonic oscillator problem - Hamilton Jacobi equation for Hamilton's characteristic function – Separation of variables in the Hamilton-Jacobi equation.

#### SUGGESTED READINGS

- 1. Goldstein. H. (2011). Classical Mechanics Third Edition, Narosa Publishing House, New Delhi.
- 2. Gantmacher, F., (2013). Lectures in Analytic Mechanics, MIR Publishers, Moscow.
- 3. Gelfand, I. M., and Fomin, S. V., (2003). Calculus of Variations, Prentice Hall, New Delhi.
- 4. Loney, S. L., (2015). An elementary treatise on Statics, Kalyani Publishers, New Delhi.

- 1. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25
- 2. <a href="https://www.youtube.com/playlist?list=PLq-Gm0yRYwTjpY9BlDxFGNXIaQJIOQRdo">https://www.youtube.com/playlist?list=PLq-Gm0yRYwTjpY9BlDxFGNXIaQJIOQRdo</a>
- 3. http://172.16.25.76/course/view.php?id=1650

Semester -II

#### 20MMP205C FUNDAMENTALS OF ACTUARIAL MATHEMATICS

4H-4C

Instruction Hours/week: L: 4T: 0P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam: 3 Hours** 

# **Course Objectives**

This course enables the students to learn

• The fundamental theories of actuarial science as they apply in life insurance, general insurance and superannuation.

- How to assess the suitability of actuarial, financial and economic models in solving actuarial problems
- Interpretation and critically evaluating the articles in the actuarial research literature.
- About the concept of educational annuity plan.
- Understand the Premium Conversion tables for calculation of Policy values.
- The concept of Premiums for Annuity Plans.

## **Course Outcomes (COs)**

On successful completion of this course the student will be able to

- 1. Explain the basic concepts of accounts and calculations of interest rates in banking / financial institution system.
- 2. Define Annuity and Summarize / calculate different values Annuities.
- 3. Leant about how to read Mortality Table and from that how to calculate the Probability of Survival and Death.
- 4. Describe about Premiums of Life Insurance and Endowment Assurance (Pure, Double and Marriage) and Educational Annuity plan.
- 5. Find the Annuity values for various Annuities.
- 6. Calculation of Net Premiums for Assurance Plans.

#### UNIT I

#### BASIC CONCEPTS OF ACTUARIAL MATHEMATICS

Accumulated Value – Present Value – Formula for present value- Annuities Certain- present Values-Amounts - Deferred Annuities –Perpetuities - Present Value of an Immediate Annuity Certain – Accumulated Value of Annuity – Relation between Sn and  $a_n$  – Present Value of Deferred Annuity Certain – Accumulated Value of a term of n years – Perpetuity – Present Value of an Immediate Perpetuity of 1p.a. – Present Value of a Perpetuity due of 1 p.a. – Deferred Perpetuity with Deferment Period of m years – Mortality Table – The Probabilities of Survival and Death.

#### **UNIT II**

#### CALCULATION OF DIFFERENT INSURANCE PREMIUMS

Life Insurance Premiums – General considerations - Assurance Benefits – Pure Endowment Assurance – Endowment Assurance – Temporary Assurance or Term Assurance - Whole Life Assurance – Pure Endowment Assurance – Endowment Assurance – Double Endowment Assurance Increasing Temporary Assurance – Increasing Whole Life Assurance – Fixed Term (Marriage) Endowment – Educational Annuity Plan.

#### **UNIT III**

#### VARIOUS VALUES OF ANNUITIES

Life Annuities and Temporary Annuities – Commutation Functions Nx – To Find the Present Value of an Annuity Due of Re.1 p.a. for Life – Temporary Immediate Life Annuity – Expression for ax : n – Deferred Temporary Life Annuity – Variable Life Annuity – Increasing Life Annuity – Variations in the Present Values of Annuities – Life Annuities Payable at Frequent Intervals.

#### **UNIT IV**

#### ANNUAL PREMIUMS AND ANNUITY PLANS

Net Premiums for Assurance Plans – Natural Premiums – Level Annual Premium – Symbols for Level Annual Premium under Various Assurance Plans – Mathematical Expressions for level Annual Premium under Level Annual Premium under Various Plans for Sum Assure of Re. 1 – Net Premiums – Consequences of charging level Premium – Consequences of withdrawals – Net Premiums for Annuity Plans – Immediate Annuities – Deferred Annuities.

## **UNIT V**

## POLICYVALUE AND ITS CALCULATION

Premium Conversion tables – Single Premium Conversion tables – Annual Premium Conversion Tables – Policy Values – Two kinds of Policy values – Policy value in symbols – Calculation of Policy Value for Unit Sum Assure – Other Expressions for Policy Value – Surrender Values – Paid up Policies – Alteration of Policy Contracts.

#### SUGGESTED READING

1. Mathematical Basis of Life Insurance - Insurance Institute of India

- 1. https://www.youtube.com/watch?v=Z95I07ZauOo
- 2. https://www.youtube.com/watch?v=Uun217imHhs

Semester -II

## 20MMP211 OPTIMIZATION TECHNIQUES – PRACTICAL

4H-2C

Instruction Hours/week: L: 0 T: 0 P: 4 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam: 3 Hours** 

## **Course Objectives**

This course enables the students to learn

- To provide the students an exposure to develop well-structured optimization techniques knowledge arising process in various level of science.
- The course aims at building capabilities in the students for analyzing different situations in the industrial/ business scenario involving limited resources and finding the optimal solution within constraints.
- This module aims to introduce students to use Probabilistic Model and techniques.
- The course aims at providing fundamental knowledge and exposure of the concepts, theories and practices in the field of management.
- Study the basic components of an optimization problem.
- Formulation of design problems as mathematical programming problems.

#### **Course Outcomes**

On successful completion of this course, the student will be able to

- 1. Use the object oriented concepts for implementation of Optimization Techniques.
- 2. Implement the data structure concepts for Optimization Techniques problems.
- 3. Acquire skills to solve various multivariable optimization problems
- 4. Solve of different optimization problems.
- 5. Identify and develop operational research models from the verbal description of the real system. Understand the mathematical tools that are needed to solve optimization problems.
- 6. Use mathematical software to solve the proposed models.

#### **List of Practical:**

- 1. Solution for a system of equations- Simplex method.
- 2. EOQ for purchasing model without shortage
- 3. EOQ for manufacturing model without shortage
- 4. EOQ for manufacturing model with shortage
- 5. EOQ for purchasing model with shortage
- 6. Probabilistic Model-EOQ.
- 7. Decision making with minimax criterion.
- 8. Decision making with maximin criterion.
- 9. Decision making under risk.
- 10. Decision making with Hurwicz criterion.

Semester – III 5H –5C

## 20MMP301 TOPOLOGY

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

## **Course Objectives**

This course enables the students to learn

- Topological properties of sets.
- The properties of compact spaces and connected spaces.
- To explore the foundations of linear subspace.
- The concepts of metric spaces and topological spaces.
- Metric spaces and metrizability of topological spaces; separation axioms.
- Interior, closure and boundary: applications to geographic information systems

## **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Develop their abstract thinking skills in topological concepts.
- 2. Know concept of metric spaces.
- 3. Acquire knowledge about various types of topological spaces and their properties.
- 4. Know the result of Compactness problems and theorems.
- 5. Admire the deep mathematical results like Urysohn's lemma.
- 6. Create examples and counterexamples in the fundamental concepts of separation space.

#### UNIT I

# TOPOLOGY OF METRIC SPACES

Topological spaces-Basis for a topologies-The order topology-The product topology X x Y-The subspace topology.

#### **UNIT II**

#### TOPOLOGICAL PROPERTIES

Closed set and limit points-Continuous functions-The product topologies-The metric topologies.

#### **UNIT III**

#### **CONNECTEDNESS**

Connected spaces-Connected subspaces of the real line-Components and local connectedness.

#### **UNIT IV**

#### **COMPACTNESS**

Compact spaces-Compact subspaces of the Real line-Limit point compactness-Local compactness.

#### **UNIT V**

## **COUNTABILITY AND SEPARATION AXIOMS**

The countability axioms-The separation axioms-Normal spaces-The Urysohn lemma, The Urysohn metrization theorem-The Tietze Extension theorem.

#### SUGGESTED READINGS

- 1. James R.Munkres., (2008). Topology, Second edition, Pearson Prentice Hall, New Delhi.
- 2. Simmons, G. F., (2017). Introduction to Topology and Modern Analysis, Tata McGraw Hill, New Delhi.
- 3. Deshpande, J. V., (1990). Introduction to topology, Tata McGraw Hill, New Delhi.
- 4. James Dugundji., (2002). Topology, Universal Book Stall, New Delhi.
- 5. Joshi, K. D.(2017). Introduction to General Topology, New Age International Pvt Ltd, New Delhi.

- 1. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25
- 2. <a href="https://ocw.mit.edu/courses/mathematics/18-901-introduction-to-topology-fall-2004/index.htm">https://ocw.mit.edu/courses/mathematics/18-901-introduction-to-topology-fall-2004/index.htm</a>
- 3. http://vidyamitra.inflibnet.ac.in/index.php/content/index/5a5dcf758007be2bd1bc30b1
- 4. http://172.16.25.76/course/view.php?id=1170

Semester – III 5H –4C

### 20MMP302

### **FLUID DYNAMICS**

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Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

This course enables the students to learn

• The concepts of fluid, its properties and behavior under various conditions of internal and external flows.

- The fundamentals of Fluid Dynamics, which is used in the applications of Aerodynamics, Hydraulics, Marine Engineering, Gas dynamics etc.
- To imbibe basic laws and equations used for analysis of static and dynamic fluids
- About the Two-Dimensional Motion of the fluid.
- Identify the fundamental kinematics of a fluid element.
- State the conservation principles of mass, linear momentum, and energy for fluid flow.

# **Course Outcomes (COs)**

On successful completion of this course, students will be able to

- 1. Classify and exploit fluids based on the physical properties of a fluid.
- 2. Compute correctly the kinematical properties of a fluid element.
- 3. Apply correctly the conservation principles of mass, linear momentum, and energy to fluid flow systems.
- 4. Understand both flow physics and mathematical properties of governing Navier-Stokes equations and define proper boundary conditions for solution.
- 5. Provide the basic mathematical background and tools to model fluid motion.
- 6. Calculate the flow of an ideal fluid in a variety of situations.

# UNIT I

## INTRODUCTORY NOTIONS

Velocity – Stream Lines and Path Lines – Stream Tubes and Filaments – Fluid Body – Density – Pressure. Differentiation following the Fluid – Equation of continuity – Boundary conditions – Kinematical and physical – Rate of change of linear momentum – Equation of motion of an in viscid fluid.

# **UNIT II**

### **EOUATION OF MOTION OF A FLUID**

Euler's momentum Theorem – Conservative forces – Bernoulli's theorem in steady motion – Energy equation for in viscid fluid – Circulation – Kelvin's theorem – Vortex motion – Helmholtz equation.

#### **UNIT III**

## TWO DIMENSIONAL FLOW

Two Dimensional Motion – Two Dimensional Functions – Complex Potential – Basic singularities

- Source Sink Vortex Doublet Circle theorem. Flow past a circular cylinder with circulation
- Blasius Theorem Lift force. (Magnus effect)

## **UNIT IV**

### VISCOUS FLOWS

Viscous flows – Navier-Stokes equations – Vorticity and circulation in a viscous fluid – Steady flow through an arbitrary cylinder under pressure – Steady Couettc flow between cylinders in relative motion – Steady flow between parallel planes.

### **UNIT V**

### LAMINAR BOUNDARY LAYER IN INCOMPRESSIBLE FLOW

Boundary Layer concept – Boundary Layer equations – Displacement thickness, Momentum thickness – Kinetic energy thickness – Integral equation of boundary layer – Flow parallel to semi infinite flat plate – Blasius equation and its solution in series.

### SUGGESTED READINGS

- 1. Milne Thomson .L.M., (2011). Theoretical Hydrodynamics, Fifth edition, Dover Publications INC, NewYork.
- 2. Curle.N., and Davies H.J., (1971). Modern Fluid Dynamics Volume-I, D Van Nostrand Company Ltd., London.
- 3. Yuan, S.W. (1976). Foundations of Fluid Mechanics, Prentice-Hall, India.
- 4. Shanthiswarup, (2014). Fluid dynamics, Krishna Prakasan media Pvt Ltd, Meerut.

- 1. <a href="https://ocw.mit.edu/courses/mechanical-engineering/2-06-fluid-dynamics-spring-2013/">https://ocw.mit.edu/courses/mechanical-engineering/2-06-fluid-dynamics-spring-2013/</a>
- 2. http://172.16.25.76/course/view.php?id=2086

Semester – III 5H –4C

20MMP303

### **MEASURE THEORY**

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Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

This course enables the students to learn

- Perspective on the broader impact of measure theory in ergodic theory.
- To apply the general principles of measure theory and integration.
- About the concept of Measurable spaces.
- To understand the basic concepts Riemann integral and Lebesgue integral.
- Basic knowledge of measure theory needed to understand probability theory, statistics and functional analysis.
- Develop the ideas of Lebesgue integration and its properties.

# **Course Outcomes (COs)**

After successful completion of this course the students will be able to:

- 1. Get a clear view of the fundamentals of measure theory.
- 2. Acquaint with the proofs of the fundamental theorems underlying the theory of Lebesgue integration.
- 3. Identify the broader impact of measure theory in ergodic theory and ability to pursue further studies in this area.
- 4. Mastery in the measure spaces and its properties.
- 5. Apply the theorems of monotone and dominated convergence and Fatou's lemma.
- 6. Apply Lebesgue decomposition and the Radon-Nikodym theorem.

# **UNIT I**

### **MEASURES**

Introduction – Outer measure – Measurable sets and Lebesgue Measure – A non measurable set – Measurable functions – Littlewoods's three principles.

## **UNIT II**

## **FUNCTIONS AND INTEGRALS**

The Riemann integral – The Lebesgue integral of a bounded function over a set finite measure – The integral of a non negative function – The general Lebesgue integral – Convergence in measure.

## **UNIT III**

## **DIFFERENTIATION**

Differentiation of monotone function—Functions of bounded variation—differentiation of an integral-Absolute continuity.

### **UNIT IV**

## **MEASURE SPACES**

Measure spaces-Measurable functions-Integration-General convergence Theorems.

### **UNIT V**

# **SIGNED MEASURES**

Signed measures-The Radon-Nikodym theorem-the L<sup>P</sup> spaces.

## SUGGESTED READINGS

- 1. Royden, H. L, (2008). Real Analysis, Third Edition, Prentice Hall of India Pvt.Ltd, New Delhi.
- 2. Keshwa Prasad Gupta, (2014). Measure Theory, Krishna Prakashan Ltd, Meerut.
- 3. Donald L. Cohn, (2013). Measure Theory, United States.
- 4. Paul R. Halmos, (2008). Measure Theory, Princeton University Press Dover Publications, New York.
- 5. Rudin W, (2017). Real and Complex Analysis, 3 <sup>rd</sup> Edition, Mcgraw Hill Education India PvtLtd, New Delhi.
- 6. G.de Barra(2014), Measure Theory and Integration, Ist Edition, New Age International Publishers, India

- 1. https://ocw.mit.edu/courses/mathematics/18-125-measure-and-integration-fall-2003/
- 2. <a href="https://www.youtube.com/playlist?list=PLgMDNELGJ1CYKDzKdGcM1-kuH a1NCfQA">https://www.youtube.com/playlist?list=PLgMDNELGJ1CYKDzKdGcM1-kuH a1NCfQA</a>
- 3. http://172.16.25.76/course/view.php?id=2069

Semester – III

### 20MMP304

## **MATHEMATICAL STATISTICS**

5H -4C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

This course enables the students to learn

- To understand the basic concepts in probability generating functions, sample moments and their functions, sampling, significance tests and statistical measures
- Probability distributions, significance of testing hypothesis and its interpretation,
- Estimation, ANOVA and their applications in various disciplines.
- Understand the concept of distribution functions.
- The knowledge of fixed-sample and large-sample statistical properties of point and interval estimators.
- Understanding of how to design experiments and surveys for efficiency.

# **Course Outcomes (COs)**

On successful completion of this course, the students will be able to

- 1. Explain the concepts of probability, including conditional probability.
- 2. Explain the concepts of random variable, probability distribution, distribution function, expected value, variance and higher moments, and calculate expected values and probabilities associated with the distributions of random variables.
- 3. Summarize the main features of a data set and test statistical hypotheses.
- 4. Define basic discrete and continuous distributions, be able to apply them and simulate them in simple cases.
- 5. Explain the concepts of analysis of variance and use them to investigate factorial dependence.
- 6. Describe the main methods of estimation and the main properties of estimators, and apply them.

# **UNIT I**

## INTRODUCTION TO PROBABILITY AND HYPOTHESIS TESTING

Random Events – Preliminary remarks – random events and operations performed on them – the system of axioms of the theory of probability – conditional probability – Bayes theorem – Independent Events – functions of random variables –Introduction to hypothesis testing, Population and Sample - Parameter and Statistic, Sampling and its methods, type-I and type-II errors, standard error, confidence interval, confidence limits. Level of Significance and degrees of freedom.

### **UNIT II**

## **PARAMETRIC TESTS**

Concept of a statistical test, Small and Large sample tests - t- test, two sample t- test, Z- test - one tailed test about population mean when sigma known, two tailed test about population mean when sigma unknown, two tailed test about population means.

### **UNIT III**

### NON-PARAMETRIC TESTS

Difference between Parametric and Non-parametric tests. Non-parametric tests - Chi-square test - Goodness of fit test, Independence Tests by contingency tables. Kolmogorov Theorem-Smirnov Theorem-Tests of Kolmogorov and Smirnov test for comparing two populations, The Wald-Wolfovitz and Wilcoxon-Mann-Whitney tests.

### **UNIT IV**

# ESTIMATION AND ANALYSIS OF VARIANCE (ANOVA)

Preliminary notion-Consistency estimation-Unbiased estimates -Sufficiency- Efficiency-Asymptotically most efficient estimates -methods of finding estimates -confidence Interval. Analysis of Variance: One way classification and two-way classification.

### **UNIT V**

### STATISTICAL LEARNING AND DATA ANALYTICS

Introduction to Big data concepts, Artificial intelligence and Machine Learning, Classification of Data Analytics and Popular Software used for Data Analytics - EXCEL, SPSS, Python, R-Programming.

## SUGGESTED READINGS

- 1. MarekFisz, (1980). Probability Theory and Mathematical Statistics, John Wiley and Sons, NewYork.
- 2. Meyer, (2006). Introduction to Probability and Statistical applications, Oxford and IBH Publishing Co. Pvt Ltd. New Delhi.
- 3. Dinesh Kumar.D, (2017). Business Analytics: The Science of Data Driven Decision Making, Wiley, New Delhi.
- 4. Srivastava T.N., and ShailajaRego., (2012). 2e, Statistics for Management, McGraw Hill Education, New Delhi.
- 5. Sheldon M. Ross, (2009). Introduction to probability and statistics for engineers and scientists, Third edition, Academic press.
- 6. Parimal Mukhopadhyay, (2012). Theory of Probability, New central book agency, Calcutta.

- 1. <a href="https://ocw.mit.edu/courses/mathematics/18-175-theory-of-probability-spring-2014/index.htm">https://ocw.mit.edu/courses/mathematics/18-175-theory-of-probability-spring-2014/index.htm</a>
- 2. https://www.youtube.com/playlist?list=PLRw1YtKsDxwqIE8WVFtL2Q s8Sk7LkC8e
- 3. http://172.16.25.76/course/view.php?id=1655

Semester - III

## 20MMP305A FUZZY SETS AND FUZZY LOGIC

4H-4C

Instruction Hours/week: L: 4T: 0P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam: 3 Hours** 

# **Course Objectives**

This course enables the students to learn

• Understand the basic knowledge of fuzzy sets and fuzzy logic.

- To gain knowledge in fuzzy relations and fuzzy measures.
- Be exposed to basic fuzzy system applications.
- The basic concepts of modeling in systems using fuzzy sets.
- The concepts of fuzzy sets, knowledge representation using fuzzy rules, approximate reasoning, fuzzy inference systems, and fuzzy logic control and other machine intelligence applications of fuzzy logic.
- The importance of tolerance of imprecision and uncertainty for design of robust & low cost intelligent machines.

# **Course Outcomes (COs)**

On successful completion of this course, the student will be able to

- 1. Comprehend the fuzzy logic and the concept of fuzziness involved in various systems and fuzzy set theory.
- 2. Understand the concepts of fuzzy sets, knowledge representation using fuzzy rules, approximate reasoning, fuzzy inference systems, and fuzzy logic.
- 3. Understand basic knowledge of the fuzzy sets, operations and their properties.
- 4. Understand the fundamental concepts of Fuzzy functions and Fuzzy logic
- 5. Apply the concepts of Fuzzy sets in image processing, Pattern reorganization and Decision making.
- 6. Understand the concept of Fuzzy measures.

#### **UNIT I**

### CRISP SETS AND FUZZY SETS

Introduction-Crisp sets: An over view-The Notion of Fuzzy Sets-basic concepts of Fuzzy sets. Classical Logic: complement-Fuzzy Union-Fuzzy interaction – Combination of operations—general aggregation of operations.

#### UNIT II

#### **FUZZY RELATIONS**

Crisp and Fuzzy relations – Binary relations – Binary relations on a single set – Equivalence and similarity relations – Compatibility on Tolerance Relations-Orderings – Morphism – fuzzy relations Equations.

## **UNIT III**

### **FUZZY MEASURES**

General discussion – Belief and plausibility Measures –Probability measures – Possibility and Necessity measures.

## **UNIT IV**

## **FUZZY MEASURES**

Relationship among Classes of Fuzzy Measures - Types of Uncertainity- Measures of Fuzziness - Classical measures of Uncertainity.

# **UNIT V**

## **UNCERTAINTY AND INFORMATION**

Measures of Dissonance-Measures of Confusion – Measures of Non-Specificity – Uncertainty and Information – Information and Complexity – Principles of Uncertainty and information.

# **SUGGESTED READINGS**

- 1. George J. Klir and Bo Yuan, (1988). Fuzzy Sets and Fuzzy Logic, Prentice Hall of India.
- 2. George J. Klir and Tina A. Folger, (2015). Fuzzy Sets, Uncertainty and Information, pearson publications.
- 3. Zimmerman. H.J, (2006). Fuzzy Set Theory and Its Applications, Kluwer Academic publishers.
- 4. DuBois. D and Prade. H. M,(1994). Fuzzy Sets and Systems: Theory and Applications, AcademicPress.
- 5. Ross. T. J,(2016). Fuzzy Logic with Engineering Applications, 4<sup>th</sup>edition,Willey Publications.

- 1. https://youtu.be/BFEwuFatM4Q.
- 2. <a href="http://videolectures.net/acai05\_berthold\_fl/">http://videolectures.net/acai05\_berthold\_fl/</a>

Semester – III MATLAB PROGRAMMING 4H –4C

Instruction Hours/week: L: 4T: 0P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam: 3** Hours

# **Course Objectives**

20MMP305B

This course enables the students to learn

• To provide basic fundamentals on MATLAB, primarily for numerical computing.

- To learn the characteristics of script files, functions and function files, two-dimensional plots and three-dimensional plots.
- Develop enhance the programming skills with the help of MATLAB and its features which allow to learn and apply specialized technologies.
- Understanding the MATLAB environment
- Do simple calculations using MATLAB.
- Carry out simple numerical computations and analyses using MATLAB.

# **Course Outcomes (COs)**

On successful completion of the course, students will be able to:

- 1. Formulate matrix manipulations, plotting of functions and data, implementation of algorithms, and creation of user interfaces.
- 2. Understanding integrating computation, visualization and programming in an easy to use environment where problems and solutions are expressed in familiar mathematical notations.
- 3. Use this software is a more flexible programming tool for users in order to create large and complex application programs.
- 4. Know it consists of set of tools that facilitates for developing, managing, debugging and profiling M files, and MATLAB's applications.
- 5. Understand the main features of the MATLAB development environment.
- 6. Design simple algorithms to solve problems.

# **UNIT I**

### **STARTING WITH MATLAB:**

Starting MATLAB, MATLAB Windows - Working in the Command Window - Arithmetic Operations with Scalars - Display Formats - Elementary Math Built-In Functions - Defining Scalar Variables - Useful Commands for Managing Variables - Script Files - Examples of MATLAB Applications.

## **CREATING ARRAYS:**

Creating a One-Dimensional Array (Vector) - Creating a Two Dimensional Array (Matrix) - Notes about Variables n MATLAB - The Transpose Operator-Array Addressing - Using a Colon : In Addressing Arrays - Adding Elements to Existing Variables - Deleting Elements - Built-In Functions for Handling Arrays - Strings and Strings as Variables.

# **UNIT II**

## **MATHEMATICAL OPERATIONS WITH ARRAYS:**

Addition and Subtraction - Array Multiplication - Array Division - Element-By-Element Operations - Using Arrays In MATLAB Built-In Math Functions - Built-In Functions For Analyzing Arrays - Generation Of Random Numbers - Examples Of MATLAB Applications.

## USING SCRIPT FILES AND MANAGING DATA:

The MATLAB Workspace and the-Workspace Window - Input To A Script File - Output Commands - The Save And Load Commands-Importing And Exporting Data - Examples Of MATLAB Applications.

## **UNIT III**

## **TWO-DIMENSIONAL PLOTS:**

The plot Command - The fplot Command - Plotting Multiple Graphs in the Same Plot - Formatting a Plot - Plots With Logarithmic Axes - Plots With Error Bars-Plots With Special Graphics - Histograms - Polar Plots - Putting Multiple Plots on the Same Page-Multiple Figure Windows - Examples of MATLAB Applications.

## **THREE-DIMENSIONAL PLOTS:**

Line Plots - Mesh and Surface Plots - Plots With Special Graphics - The View Command - Examples on MATLAB Applications.

### **UNIT IV**

### **PROGRAMMING IN MATLAB:**

Relational and Logical Operators - Conditional Statements - The Switch-Case Statement - Loops - Nested Loops and Nested Conditional Statements - The Break and Continue Commands - Examples of MATLAB Applications.

## **USER-DEFINED FUNCTIONS AND FUNCTION FILES:**

Creating a Function File -Structure of a Function File - Local And Global Variables - Saving A Function File - Using A User Defined Function - Examples of Simple User-Defined Functions - Comparison Between Script Files and Function Files - Anonymous And Inline Functions - Function Functions - Subfunctions-Nested Functions - Examples Of MATLAB Applications.

# **UNIT V**

# POLYNOMIALS, CURVE FITTING, AND INTERPOLATION:

Polynomials - Curve Fitting-Interpolation - The Basic Fitting Interface - Examples of MATLAB Applications.

# **APPLICATIONS IN NUMERICAL ANALYSIS:**

Solving an Equation with One Variable - Finding a Minimum or a Maximum of a Function - Numerical Integration - Ordinary Differential Equations - Examples of MATLAB Applications.

# **SUGGESTED READINGS**

1. Rudra Pratap, Getting Started with MATLAB – A Quick Introduction for Scientists and Engineers, Oxford University Press.

- 2. William John Palm,(2005). Introduction to MATLAB 7 for Engineers, McGraw-Hill Professional. s
- 3. Dolores M. Etter, David C. Kuncicky, (2004) Introduction to MATLAB 7, Printice Hall.
- 4. A. Gilat, John Wiley & Sons, (2011)."MATLAB An Introduction with Application", Singapore.

- 1. <a href="https://ocw.mit.edu/courses/mathematics/18-s997-introduction-to-matlab-programming-fall-2011/">https://ocw.mit.edu/courses/mathematics/18-s997-introduction-to-matlab-programming-fall-2011/</a>
- 2. https://www.youtube.com/playlist?list=PLRWKj4sFG7-6 Xr9yqg6SMr F80KdFVhN

Semester – III

### 20MMP305C

### **NEURAL NETWORKS**

4H-4C

Instruction Hours/week: L: 4T: 0P: 0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives:**

This course enables the students to learn

• To introduce the neural networks for classification and regression

- To give design methodologies for neural networks
- Understand the differences between networks for supervised and unsupervised learning.
- The introduction and different architectures of neural networks.
- The applications of neural networks.
- The fundamental concepts artificial neural networks.

# **Course Outcomes (COs)**

On successful completion of this course the student will be able to

- 1. Comprehend the concepts of feed forward neural networks
- 2. Analyze the various Linear Associator.
- 3. Analyze the various Back Propagation Algorithm
- 4. Supervised learning and unsupervised learning.
- 5. Design single and multi-layer feed-forward neural networks
- 6. Analyze the performance of neural networks in directional derivatives.

### UNIT-I

# **EVOLUTION OF NEURAL NETWORKS**

Mathematical Neuron Model- Network Architectures- Perceptron-Hamming Network-Hopfield Network-Learning Rules.

# UNIT - II

### PERCEPTRON LEARNING RULE

Perceptron Architectures and Learning Rule with Proof of Convergence. Supervised Hebbian Learning -Linear Associator.

## **UNIT I- III**

### MULTI-LAYER PERCEPTRONS

The Hebb Rule-Pseudo inverse Rule-Variations of Hebbian Learning-Back Propagation - Multilayer Perceptrons.

### UNIT - IV

# **BACK PROPAGATION**

Back propagation Algorithm-Convergence and Generalization –Performances Surfaces and Optimum Points-Taylor series.

### **UNIT V**

# **OPTIMITATION IN NURAL NETWORK**

Directional Derivatives - Minima-Necessary Conditions for Optimality-Quadratic Functions-Performance Optimizations-Steepest Descent-Newton's Method-Conjugate Gradient.

## SUGGESTED READINGS

- 1. Martin T.Hagan, Howard B. Demuth and Mark Beale, (2014). Neural Network Design, Vikas, Publishing House, New Delhi,
- 2. James A. Freeman, David M. Skapura, (2011). Neural Networks Algorithms, Applications and Programming Techniques, Pearson Education.
- 3. Robert J. Schalkoff, (2000). Artificial Neural Network, McGraw-Hill International Edition.

- 1. <a href="https://ocw.mit.edu/courses/brain-and-cognitive-sciences/9-641j-introduction-to-neural-networks-spring-2005/">https://ocw.mit.edu/courses/brain-and-cognitive-sciences/9-641j-introduction-to-neural-networks-spring-2005/</a>
- 2. <a href="https://www.youtube.com/watch?v=xbYgKoG4x2g&list=PL53BE265CE4A6C05">https://www.youtube.com/watch?v=xbYgKoG4x2g&list=PL53BE265CE4A6C05</a>

Semester - III

## 20MMP311 MATHEMATICAL STATISTICS-PRACTICAL

4H-2C

Instruction Hours/week: L: 0 T: 0 P: 4 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

This course enables the students to learn

- In-depth understanding of SPSS Software Package.
- It is well recognized nowadays the importance of Statistics as an indispensable tool for obtaining and spreading information.
- Importance has been enhanced by the use of computational resources and particularly the software SPSS, that showed, during the last decades, to be an effective tool for treating and analyzing statistical data.
- Ability to use SPSS procedures in handling data files and performing statistical analysis, and to interpret the outputs provided by the program.
- Acquiring sensitivity and critical thinking towards arguments and conclusions based on statistical studies.
- Understanding the fundamental principles underlying descriptive and inferential statistical reasoning.

# **Course Outcomes (COs)**

On successful completion of this course, the student will be able to

- 1. Describe and classify data using statistical terminology.
- 2. Use SPSS to conduct basic descriptive analyses and graphical presentations.
- 3. Define the null hypothesis and the alternative hypothesis and Interpret P values and confidence intervals.
- 4. Understand different measures of effect (e.g. mean difference).
- 5. Know when to use basic statistical hypothesis tests (t-tests, chi-squared tests, correlation) and how to carry out these tests using SPSS.
- 6. Appreciate how to present and interpret these results in scientific reports.

# **List of Practical**

- 1. Various Software Package available for Data Analytics
- 2. Introduction to SPSS Software Package for Data Analytics
- 3. Drawing of graphs and diagrams using software
- 4. Calculation of Standard deviation for individual and discrete series.
- 5. Calculation of Standard deviation continuous series.
- 6. Calculation of Coefficient of Variation.
- 7. Calculation of Karl Pearson's Correlation
- 8. Calculation of Rank Correlation Coefficient
- 9. Fitting of Linear Regression

- 10. Hypothesis Testing for small sample test (t test)
- 11. Hypothesis Testing for two sample t test
- 12. Hypothesis Testing for Large sample (Z-test)
- 13. Testing Hypothesis using chi-square test (for Goodness of fit)
- 14. Testing Hypothesis using chi-square test (for Contingency Table)

## **SUGGESTED READINGS**

- 1. SPSS Lab Manual
- 2. U Dinesh Kumar (2017), Business Analytics: The Science of Data Driven Decision Making, Wiley, New Delhi.
- 3. Daniel Y Chen (2017), Pandas for everyone Python data Analysis,
- 4. R. Evans James (2017), Business Analytics, 2nd edition, Pearson Education, New Delhi.

Semester – IV FUNCTIONAL ANALYSIS 5H – 5C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

20MMP401

This course enables the students to learn

- The concept of Banach spaces and related properties.
- Pure concepts on open mapping and closed graph theorem.
- The specific techniques for bounded operators over normed and Hilbert spaces.
- The demonstrate significant applications of the theory of operators.
- Understand how to use the main properties of compact operators.
- Apply the spectral analysis of compact self-adjoint operators to the resolution of integral equations.

# **Course Outcomes (COs)**

On successful completion of this course the students will be able to:

- 1. Develop Banach spaces from vector spaces.
- 2. Describe the open mapping theorem.
- 3. Discuss Hilbert spaces and its properties.
- 4. Study in detail about the adjoint of an operator.
- 5. Handle complex problems concerning topics within the area of Functional Analysis.
- 6. Understand and apply fundamental theorems from the theory of normed and Banach spaces.

### **UNIT I**

## **BANACH SPACES**

Banach Spaces—Normed linear space — Definitions and Examples-Theorems. Continuous Linear Transformations—Some theorems- Problems. The Hahn-Banach Theorem—Lemma and Theorems. The Natural imbedding of N in N\*\*—Definitions and Theorems.

## **UNIT II**

## **OPEN MAPPING THEOREM**

The Open Mapping Theorem – Theorem and Examples – Problems. The closed graph theorem. The conjugate of an operation – The uniform boundedness theorem – Problems.

## **UNIT III**

# **HILBERT SPACES**

The Definition and Some Simple Properties – Examples and Problems. Orthogonal Complements –Some theorems. Ortho-normal sets– Definitions and Examples-Bessel's inequality–The conjugate space H\*.

# **UNIT IV**

### THE ADJOINT OF AN OPERATOR

Definitions and Some Properties-Problems. Self-adjoint operators – Some Theorems and Problems. Normal and Unitary operators –theorems and problems. Projections –Theorems and Problems.

## **UNIT V**

### **BANACH ALGEBRAS**

Matrices—Determinant and Spectrum of bounded operator—The spectral Theorem—The definition and some examples of Banach algebra — Regular and singular elements — Topological divisors of zero — The spectrum — The formula for the spectral radius.

### SUGGESTED READINGS

- 1. Simmons. G. F., (2015). Introduction to Topology & Modern Analysis, Tata McGraw-Hill Publishing Company Ltd, New Delhi.
- 2. Balmohan V. and Limaye., (2004). Functional Analysis, Second edition, New Age International Pvt.Ltd, Chennai.
- 3. ChandrasekharaRao, K., (2004). Functional Analysis, Narosa Publishing House, Chennai.
- 4. Choudhary, .B and Sundarsan Nanda. (2003). Functional Analysis with Applications, New Age International Pvt. Ltd, Chennai.
- 5. Ponnusamy, S., (2002). Foundations of functional analysis, Narosa Publishing House, Chennai.

- 1. <a href="https://ocw.mit.edu/courses/mathematics/18-102-introduction-to-functional-analysis-spring-2009/">https://ocw.mit.edu/courses/mathematics/18-102-introduction-to-functional-analysis-spring-2009/</a>
- 2. https://epgp.inflibnet.ac.in/Home/ViewSubject?catid=25
- 3. http://vidyamitra.inflibnet.ac.in/index.php/content/index/5704c5378ae36c6ab9b0b5bf
- 4. <a href="http://vidyamitra.inflibnet.ac.in/index.php/content/index/5704c5378ae36c6ab9b0b5a9">http://vidyamitra.inflibnet.ac.in/index.php/content/index/5704c5378ae36c6ab9b0b5a9</a>
- 5. http://172.16.25.76/course/view.php?id=1652

Semester – IV
MATHEMATICAL METHODS
5H – 5C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

20MMP402

This course enables the students to learn

• Range of mathematics tools with emphasis on engineering applications.

- To think quantitatively and analyze problems critically.
- Understand the concept of Volterra integral and Fredholm theory.
- The concepts of Functional dependent on higher order derivatives.
- Understand and be able to use the language, symbols and notation of mathematics.
- Develop mathematical curiosity and use inductive and deductive reasoning when solving problems.

# **Course Outcomes (COs)**

On successful completion of this course, the students will be able to

- 1. Calculate the Fourier transform of elementary functions from the definition.
- 2. Find the Fourier transforms of functions of one variable.
- 3. Calculate the Laplace equation in half plane of standard functions both from the definition and by using tables.
- 4. Equation with separable kernel and Fredholm alternative approximation Method.
- 5. Select and combine the necessary Laplace transform techniques to solve second-order ordinary differential equations.
- 6. Understand the concept of Functionals of the integral forms.

# **UNIT I**

## FOURIER TRANSFORM

Fourier Transforms – Definition of Inversion theorem –Fourier cosine transforms - Fourier sine transforms – Fourier transforms of derivatives -Fourier transforms of some simple functions - Fourier transforms of rational function.

### **UNIT II**

## PARTIAL DIFFERENTIAL EQUATION OF FOURIER TRANSFORM

The convolution integral – convolution theorem – Parseval's relation for Fourier transforms – solution of PDE by Fourier transform – Laplace's Equation in Half plane – Laplace's Equation in an infinite strip - The Linear diffusion equation on a semi-infinite line - The two-dimensional diffusion equation.

### UNIT III

### INTEGRAL EQUATIONS

Types of Integral equations—Equation with separable kernel- Fredholm Alternative Approximate method – Volterra integral equations—Classical Fredholm theory – Fredholm's First, Second, Third

theorems-Conversion of ordinary differential equation into integral equation - Method of converting initial value problem into a Volterra integral equation - Boundary value problem - Method of converting a boundary value problem into a Fredholm integral equation.

## **UNIT IV**

# INTEGRAL EQUATIONS FOR ORDINARY DIFFERENTIAL EQUATIONS

Application of Integral equation to ordinary differential equation – initial value problems – Boundary value problems – singular integral equations – Abel Integral equation – The solution of Abel's integral equation – Some general form of Abel's singular integral equation – Problem-Applications of integral equation and Green's functions to ordinary differential equation.

# **UNIT V**

### CALCULUS OF VARIATION

Variation and its properties – Euler's equation – Functionals of the integral forms - Functional dependent on higher order derivatives – functionals dependent on the functions of several independent variables – variational problems in parametric form.

### SUGGESTED READINGS

- 1. Sneedon. I. N, (1974). The Use of Integral Transforms, Tata McGraw Hill, New Delhi.
- 2. Kanwal, R. P, (2013). Linear integral Equations Theory and Technique, Academic press, New York.
- 3. Elsgots, L., (2003). Differential Equations and Calculus of Variation, Mir Publication Moscow.
- 4. Gelfand, I. M and Francis, S.V. (2000). Calculus of Variation, Prentice Hall, India.
- 5. Tricomi.F.G. (2012). Integral Equations, Dover, New York.
- 6. Srivastava.A.N. and Mohammad Ahmad, (2012). Integral Transforms and Fourier Series, Alpha Science International, Limited.

- 1. <a href="http://vidyamitra.inflibnet.ac.in/index.php/content/index/570b7b068ae36c5432d66dbb">http://vidyamitra.inflibnet.ac.in/index.php/content/index/570b7b068ae36c5432d66dbb</a>
- 2. http://vidyamitra.inflibnet.ac.in/index.php/content/index/570b7b068ae36c5432d66dcd
- 3. http://172.16.25.76/course/view.php?id=1657

Semester – IV 20MMP403 STOCHASTIC PROCESSES 5H – 5C

Instruction Hours/week: L:5T:0P:0 Marks: Internal: 40 External: 60 Total: 100

**End Semester Exam:** 3 Hours

# **Course Objectives**

This course enables the students to learn

- The mathematical theory of random variables and random processes
- How queueing theory are used as tools and mathematical models in the study of networks.
- The theoretical concepts and techniques for solving problems that arises in practice
- Markov processes in discrete and continuous time.
- The essential mathematical tools for handling random processes.
- The familiarize the students with the stochastic simulation techniques.

# **Course Outcomes (COs)**

On successful completion of the course, students will be able to:

- 1. Capable to expose the students to different types mathematical models with a view of random processes.
- 2. Understanding in the concept of birth and death process.
- 3. Solve the Kolmogrov equations problems.
- 4. Compute probabilities of transition between states and return to the initial state after long time intervals in Markov chains.
- 5. Identify classes of states in Markov chains and characterize the classes.
- 6. Stochastic Processes in Queuing Systems.

# **UNIT I**

## STOCHASTIC PROCESSES

Definition of Stochastic Processes – Markov chains: definition, order of a Markov Chain – Higher transition probabilities – Classification of states and chains.

### **UNIT II**

## MARKOV PROCESS WITH DISCRETE STATE SPACE

Poisson process – and related distributions – Properties of Poisson process, Generalizations of Poisson Processes – Birth and death Processes – continuous time Markov Chains.

### **UNIT III**

# MARKOV PROCESSES WITH CONTINUOUS STATE SPACE

Introduction, Brownian motion – Weiner Process and differential equations for Weiner process, Kolmogrov equations – First passage time distribution for Weiner process – Ornstein – Uhlenbech process.

## **UNIT IV**

### **BRANCHING PROCESSES**

Introduction – properties of generating functions of Branching process– Distribution of the total number of progeny, Continuous- Time Markov Branching Process, Age dependent branching process: Bellman-Harris process.

### **UNIT V**

# STOCHASTIC PROCESSES IN OUEUING SYSTEMS

Concepts – Queuing model M/M/1 – transient behavior of M/M/1 model – Birth and death process in Queuing theory: M/M/1 – Model related distributions – M/M/1 - M/M/S/S – loss system - M/M/S/M – Non birth and death Queuing process: Bulk queues – M(x)/M/1.

## SUGGESTED READINGS

- 1. Medhi, J., (2006). Stochastic Processes, 2nd Edition, New age international Private limited, New Delhi.
- 2. Basu, K., (2003). Introduction to Stochastic Process, Narosa Publishing House, New Delhi.
- 3. Goswami and Rao, B. V., (2006). A Course in Applied Stochastic Processes, Hindustan Book Agency, New Delhi.
- 4. Grimmett, G. and Stirzaker D., (2001). Probability and Random Processes, 3rd Ed., Oxford University Press, New York.
- 5. Papoulis.A and UnnikrishnaPillai, (2002). Probability, Random variables and Stochastic Processes, Fourth Edition, McGraw-Hill, New Delhi.
- 6. V.Sundarapandian.,(2009).Probability statistics and Queuing theory, PHI learning private limited, New Delhi.

- 1. <a href="https://ocw.mit.edu/courses/mathematics/18-445-introduction-to-stochastic-processes-spring-2015/">https://ocw.mit.edu/courses/mathematics/18-445-introduction-to-stochastic-processes-spring-2015/</a>
- 2. http://172.16.25.76/course/view.php?id=2070

20MMP411	PROJECT	Semester – IV –8C
Instruction Hours/week: L: 0 T: 0 P: 0	Marks: Internal: 80	External: 120 Total: 200 End Semester Exam: -

# LIST OF VALUE ADDED COURSES

- **❖** Data Analysis using SPSS
- **❖** LATEX
- Vedic Mathematics
- **❖** MATLAB
- **❖** MAPLE
- **❖** GEOGEBRA
- DIA
- ❖ Data Analysis Using Advanced Excel
- Mathematical Modelling with Excel
- Productivity Analysis and Audit
- ❖ Data Analysis Using R Programming
- Statistical Quality Control
- ❖ Six Sigma Analysis-