Ph.D. MATHEMATICS

Curriculum and Syllabus
Regular (2022-2023)

Enable | Enlighten | Enrich
KARPAGAM
ACADEMY OF HIGHER EDUCATION
(Deemed to be University)
(Established Under Section 3 of UGC Act, 1956)

DEPARTMENT OF MATHEMATICS
FACULTY OF ARTS, SCIENCE, COMMERCE AND MANAGEMENT

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University)
(Established under section 3 of UGC Act, 1956)
(Accredited with A+ Grade by NAAC in the Second Cycle)
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DEPARTMENT OF MATHEMATICS
FACULTY OF ARTS, SCIENCE, COMMERCE AND MANAGEMENT
RESEARCH PROGRAM – Ph.D. in Mathematics
(2022–2023 Batch and onwards)

Course code	Name of the course	Instruction hours / week	Credits	Maximum Marks (100)	Page No.
Paper-I					
22RMAT101	Research Methodology and Pedagogy	4	4	100	02
Paper-II					
22RMAT201	Research Publication Ethics	4	4	100	04
Paper-III					
22RMAT301	Fuzzy Mathematics	4	4	100	06
22RMAT302	Advanced Topics in Fluid Dynamics				08
22RMAT303	Hydrodynamic and Hydromagnetic Stability				10
22RMAT304	Queueing Theory				12
22RMAT305	Advanced Graph Theory				14
22RMAT306	Oscillation Theory of Differential Equations				16
Program Total		**12**	**12**	**300**	

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Paper-I

22RMAT101 RESEARCH METHODOLOGY AND PEDAGOGY 4H – 4C

Instruction Hours / week: L: 4 **Total: 100**
End Semester Exam: 3 Hours

Course Objectives:

To make the learners to

- Understand the fundamentals of research terminology.
- Enrich the concepts of Implicit function theorem.
- Know about the linear oscillations and comparison theorem approaches to research.
- Get the basic working knowledge in LATEX programming.
- Critically analyse methods of evaluation.
- Know about e-learning researches and web-based learning.

Course Outcomes (Cos):

Learners able to:

1. Understand the basic framework of research process.
2. Understand the various research concepts of Implicit functions and extremum problems.
3. Attain mastery in the concept of oscillations of second order equations.
4. Develop LATEX coding for mathematical documentation.
5. Study about the Quality teaching and learning.
6. Acquiring the knowledge of e-learning researches and web-based learning.

UNIT – I

Research Methodology – Meaning of research, Objectives of Research, Motivation in Research – Types of Research – Research approaches – Research methods, Versus Research Methodology – Research process – Scientific method – Criteria for good research, Defining the research problem – Necessity of defining the problem – Techniques involved in defining the problem, Research Design – Meaning and need for Research Design – Features of good design – Important concepts relating to research design.

UNIT – II

Implicit functions and extremum problems: Introduction – Functions with non-zero Jacobian determinant – Inverse function theorem – Implicit function theorem – Extrema of real valued functions of one variable and several variables. Rank Theorem – Determinants – Derivatives of Higher order-Differentiation of Integrals.

UNIT – III

Oscillations of second order equation-Fundamental results – Sturms comparison theorem – elementary linear oscillations – comparison theorem of Hille winter – Oscillations of $x'' + a(t)x = 0$ elementary non linear oscillations – stability of linear and non linear systems – elementary critical points – system of equations with constant coefficient – the linear equations with constant co efficient – Lyabunov stability – Stability of quasi linear systems.

UNIT- IV

LATEX: The Basics - The Document -Bibliography - Bibliographic Databases - Table of contents, Index and Glossary - Displayed Text - Rows and Columns -Typesetting Mathematics - Typesetting - Several Kinds of Boxes - The figure environment -Cross References in LATEX - Footnotes, Marginpars, and Endnotes.

UNIT-V

Objectives and roll of higher education – Important characteristics of an effective Lecture – Quality teaching and learning – Lecture preparation – Characteristics of instructional design – Method of teaching and learning: Large group – Technique – Lecture, Seminar, Symposium, Team Teaching, Project, Small group Technique – Simulation, role playing Demonstration, Brain Storing, case discussion and assignment, Methods of evaluation – Self-evaluation, Student evaluation, Diagnostic testing and remedial teaching – Question banking – Electronic media in education: e-learning researches – web based learning.

SUGGESTED READINGS

1. Kothari C.R., (2014). Research Methodology, Method and Techniques, Second Edition, New age International publishers, New Delhi.
2. Rudin W., (2016). Principles of Mathematical Analysis, McGraw Hill, New York.
3. Earl Coddington A., (2002). An introduction to Ordinary Differential Equations, Prentice Hall of India Private limited, New Delhi. (For Unit III)
4. Krishnan E., (2003). Latex Tutorials – A primer, Indian TEX users group, Trivandrum, India.
5. Panneerselvam R, (2013). Research Methodology, Second Edition, Prentice Hall of India, New Delhi.
6. Gupta S. P, (2011). Statistical Methods, Fourth Edition, Sultan Chand & Sons, New Delhi.
7. Vedanayagam E. G., (2019). Teaching Technology for College teachers, New Delhi.
8. Kumar K. L., (2004). Educational Technologies, New age International, New Delhi.
9. Winkler Anthony C., and Jo Roy McCuen., (2015). Writing a research paper: A Handbook, 2nd edition, Harcourt, New York.

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Paper-II

22RMAT201 RESEARCH PUBLICATION ETHICS 4H – 4C

Instruction Hours / week: L: 4 **Total: 100**
End Semester Exam: 3 Hours

Course Objective:

To make the learners to

- Know about the basic of philosophy of science and ethics, research integrity, publication ethics
- Know about the hands on sessions are designed to identify misconduct and predatory publications.
- Learn indexing and citation databases, open access publication, research and p metrics and plagiarism tools introduced in the course.
- Understand the concept of Group discussions.
- Learn the impact factor of journal as per Journal Citation Report.
- Identify the concept development of e- content.

Course Outcomes (Cos):

Learners able to:

1. Attain mastery in basic concept and moral philosophy of research process.
2. Understand selective reporting and misrepresentation of date.
3. Identify violation of publication ethics, authorship and contributing and appeals.
4. Summarize the philosopher who developed the terms of ethics and their arguments about on ethics.
5. Know about the indexing database and impact factor of journals.
6. Understand the concepts of Learning Management system.

UNIT I: Philosophy and Ethics

Introduction to Philosophy: Definition, nature and scope, concept, branches – Ethics: Definition, moral philosophy, nature of moral judgments and reaction.

UNIT II: Scientific Conduct

Ethics with respect to science and research – Intellectual honesty and research integrity – scientific misconduct: Falsification – Fabrication – Fabrication and Plagiarism (FFP) – Redundant publications: duplicate and over lapping publication-salami slicing- selective reporting and misrepresentation of date.

UNIT III: Publication Ethics

Publication Ethics: Definition, introduction and importance- Best practices/ standards setting initiatives and guidelines: COPE, WAME, etc. – Conflicts of interest – publication misconduct: definition, concept , problems that lead to unethical behavior and vice versa, type- violation of publication ethics, authorship and contributing and appeals- predatory publishers and journals.

UNIT IV: Publication misconduct

Group discussions: Subject specific ethical issues, FFP, authorship – conflicts of interest- complaints and appeals: examples and fraud from India and abroad.

Software Tools: Use of Plagiarism software tools like Turnitin, Urkund and other open source software tools.

UNIT V: Database and Research Metrics

Database: indexing database- citation database: web of science, scopus, etc.

Research Metrics: impact factor of journal as per Journal Citation Report, SNIP, SJR, IPP, Cite Score – Metrics: h-index, g index, g index, I 10 indices, altmetrics.

UNIT VI: Development of e- content & IPR

Integrated Library Management System (ILMS): e-journals – e-books – e-shodhsindu – shodhganga – database – e-content development – Learning Management system (LMS) – e-PG – Pathshala – CEC (UG) SWAYAM – MOOCs – NPTEL – NMEICT.

IPR: Patent – Copyrights- trademark – Geographical Indication.

PRACTICE:
Open access publishing
Open access publications and initiatives-SHERPA/RoMEO online resource to check polisher copyright & self -archiving policies-software tool to identify predatory publications developed by SPPU-Journal finder / journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.

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Paper-III

22RMAT301 **FUZZY MATHEMATICS** **4H – 4C**

Instruction Hours / week: L: 4 **Total: 100**
End Semester Exam: 3 Hours

Course Objectives:

To make the learners to

- Study in detail the basic mathematical elements of the theory of fuzzy sets.
- Understand the concepts of operations on fuzzy sets.
- Know about the concepts fuzzy relations and fuzzy graphs.
- Know about the momentous of fuzzy measures and probability measure.
- Solve problems by using the concept decision making in fuzzy environments.
- Know the importance of fuzzy logic in real life problems.

Course Outcomes (Cos):

Learners able to:

1. Understand about the concepts of fuzzy sets and fuzzy logic.
2. Acquire the knowledge on general aggregation operations.
3. Know about the fuzzy relation equation and fuzzy graphs.
4. Describe the probability measures and fuzzy measures of fuzziness.
5. Import the knowledge on the decision making in fuzzy environments.
6. Understand decision making in fuzzy environments**.**

UNIT - I

Crisps sets and Fuzzy sets: Introduction –Crisp Sets: An overview-The notion of fuzzy sets – Basics concepts of fuzzy sets –Classical logic: An overview-Fuzzy logic.

UNIT- II

Operations on Fuzzy sets: Fuzzy complement - fuzzy union – fuzzy Intersection – combinations of operation – General Aggregation operations.

UNIT - III

Fuzzy relations and Fuzzy graphs: Crisp and fuzzy relations – Binary relations-Binary relations on a single set – Equivalence and similarity relations-Compatibility or Tolerance relations – ordering- Morphisms – Fuzzy relation equations – Fuzzy graphs.

UNIT- IV

Fuzzy Measures: Belief and Plausibility Measures – Probability Measures – Possibility and necessity measures-Relationship among classes of Fuzzy measures of Fuzziness.

UNIT- V

Decision making in fuzzy environments: Fuzzy Decisions – Fuzzy Linear programming – symmetric Fuzzy LP – Fuzzy LP with crisp objective function – Fuzzy Dynamic Programming-Fuzzy Dynamic with Crisp state Transformation Function- fuzzy multi criteria Analysis– Multi objective Decision Making (MODM) – Multi Attributive Decision making (MADM).

SUGGESTED READINGS

1. George Klir .J and Tina Folger. A., (2015). Fuzzy sets – Uncertainty and information, Prentice – Hall of India Pvt. Ltd. Chapters: I, II, III & IV.
2. Zimmermann H.J., (2007). Fuzzy set theory and its applications, Fourth Edition Springer. Chapter XIV.

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Paper-III

22RMAT302 **ADVANCED TOPICS IN FLUID DYNAMICS** **4H – 4C**

Instruction Hours / week: L: 4 **Total: 100**

End Semester Exam: 3 Hours

Course Objectives:

To make the learners to

- Acquire the knowledge on the properties of two dimensional flow.
- Import knowledge in thermal boundary layer in laminar flow.
- Understand the concept of equation of motion in rotating co-ordinate system.
- Describe the main properties of the electrodynamics systems.
- Introduce the system of magnetohydrodynamics equations and main theorems that follow from the magnetohydrodynamics system.
- Understand the importance of fluid dynamics in diverse real life applications.

Course Outcomes (Cos):

Learners able to:

1. Solve and classify the fluids based on the physical properties of a fluid.
2. Compute correctly the theory of similarity in heat transfer.
3. Understand non dimensional parameters and Ross by number.
4. Study the magnetic energy and its properties.
5. Analyze magnetohydro dynamic waves and its applications.
6. Assess the various equations for the incompressible MHD.

UNIT – I

Steady unidirectional flow – Poiseuille flow – Two dimensional flow – Paint-Brush model – unsteady unidirectional flow – Flow with circular stream lines – Flow fields in which inertia forces are negligible – Lubrication theory.

UNIT – II

Thermal boundary layer in laminar flow: Derivation of the energy equation – Temperature increase through adiabatic compression – Stagnation temperature – Theory of similarity in heat transfer – Exact solutions for the problem of temperature distribution in a viscous flow – Boundary layer simplifications.

UNIT – III

Equation of motion in rotating co-ordinate system – Potential vorticity – vorticity equation – Ertel's theorem – Non dimensional parameters – Rossby number – Ekman number – Geostrophic flow – Taylor – Proudman theorem – Taylor coloumn.

UNIT – IV

Magnetohydrodynamics: Electrodynamics of moving media – The electromagnetic effects and the magnetic Reynolds number – Alfen's theorem – The magnetic energy – The mechanical equations – Basic equations for the incompressible MHD – Steady Laminar motion – Hartmann flow.

UNIT – V

Magnetohydrodynamic waves – waves in an infinite fluid of infinite electrical conductivity – Alfen's waves – Magnetohydrodynamic waves in a compressible fluid – Magneto acoustic waves – Slow and Fast waves – Stability – Physical concepts – Linear-Pinch –Kink – Sausage and Flute types of instability – Method of small oscillations – Jeans criterion for gravitational stability.

SUGGESTED READINGS

1. Batchelor G.K., (2002). An Introduction to Fluid Dynamics, Cambridge University Press.
2. Schlichting H., (2003). Boundary – Layer Theory, Springer.
3. Friedlander S., (1980). An Introduction to the Mathematical Theory of Geophysical Fluid Dynamics, Elsevier.
4. Ferraro V.C.A and Plumpton C., (1972). An Introduction to Magneto Fluid Dynamics, Oxford University.

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Paper-III

22RMAT303 HYDRODYNAMIC AND HYDROMAGNETIC STABILITY 4H – 4C

Instruction Hours / week: L: 4 **Total: 100**
End Semester Exam: 3 Hours

Course Objectives:

To make the learners to

- Understand the concept of hydrodynamics systems.
- Impart the basic knowledge of benard problems.
- Attain mastery in the applications of rotation of fluid in stability analysis.
- Get the knowledge of the effect of magnetic field.
- Know about the perturbation techniques for determining the stability of superposed fluids.
- Understand the concept of important instabilities like Rayleigh-Taylor, Kelvin-Helmholtz instability.

Course Outcomes (Cos):

Learners able to:

1. Describe the fundamental principles of the motion of ideal (inviscid) and real (viscous) fluid flows.
2. Apply analytical concepts to analyze a range of two-dimensional engineering fluid flows, with appropriate choice of simplifying assumptions and boundary conditions.
3. Provide the details of the effect of rotations.
4. Study the magnetic field and its properties.
5. Impart knowledge in concepts of instability.
6. Analyze the analytical technique to characterize the hydrodynamic stability.

UNIT – I: Introduction:

Basic Concepts - Analysis in terms of normal modes - Non-dimensional number.

UNIT – II: Benard Problem:

Basic hydrodynamic equations. Boussinesq approximation. Perturbation equations. Analysis into normal modes. Principle of exchange of stabilities. Equations governing the marginal state. Exact solution when instability sets in as stationary convection for two free boundaries.

UNIT – III

The effect of rotation: The Perturbation equations. Analysis in terms of normal modes. Variation Principle for stationary convection. Solutions when instability sets in as stationary convection for two free boundaries. On the onset of convection as over stability; the solution for the case of two free boundaries.

UNIT – IV

The effect of magnetic field: The Perturbation equations. The case when instability sets in as stationary convection; A variational principle. Solutions for stationary convection and for over stability for the case of two free boundaries. The stability of superposed fluids.

UNIT – V

(i) **Rayleigh-Taylor instability:** The Perturbation equations. Inviscid case(the case of two uniform fluids of constant density separated by a horizontal boundary, the case of exponentially varying density). Effect of rotation. Effect of vertical magnetic field.

(ii) **The Kelvin-Helmholtz instability:** The perturbation equations, the case of two uniform fluids in relative horizontal motion separated by a horizontal boundary, the effect of rotation, the effect of horizontal magnetic field.

SUGGESTED READINGS

1. Chandrasekhar S., (2081). Hydrodynamic and Hydromagnetic Stability, Dover Publications.
2. Drazin P.G and Reid W.H., (2004). Hydrodynamic Stability, Cambridge University Press.

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Paper-III

22RMAT304 **QUEUEING THEORY** **4H – 4C**

Instruction Hours / week: L: 4 **Total: 100**

End Semester Exam: 3 Hours

Course Objectives:

To make the learners to

- Attain mastery in the basics of Markov Chains.
- Classify queueing models.
- Know about various Markovian queuing systems.
- Analyze multi server queueing models.
- Solve finite input source queues.
- Develop queueing models to analyze computer networks.

Course Outcomes (Cos):

Learners able to:

1. Mastery in concepts of discrete and continuous time Markov Chains
2. Explain single server queues
3. Examine steady state solution of important queues.
4. Investigate multi sever queues solution.
5. Understand input source models.
6. Model real life queueing scenarios into mathematically.

UNIT I

Introduction-Markov Chains- Basic ideas-Classification of states and chains-Sojourn time - Transition density matrix or infinitesimal generator - Limiting behavior: ergodicity - Transient solution -Alternative definition.

UNIT II

Birth-and-Death Processes: Special case: M/M/1 queue -Pure birth process-Yule-Furry process. Queueing Systems: General Concepts: Basic characteristics -The input or arrival pattern of customers -The pattern of service -The number of servers -The capacity of the system - The queue discipline. The Simple M/M/1 Queue: Steady-state solution of M/M/l - Waiting-time distributions - The output process -Semi-Markov process analysis.

UNIT III

System with Limited Waiting Space: The M/M/1/K Model: Steady-state solution - Expected number in the system L_K - Equivalence of an M/M/l//K model with a two-stage cyclic model - Birth-and-Death Processes: Exponential Models - The M/M/∞ Model: Exponential Model with an Infinite Number of Servers.

UNIT IV

The Model M/M/c: Steady-state distribution - Expected number of busy and idle servers - Waiting-time distributions - The output process .The M/M/c/c System: Erlang Loss Model: Erlang loss (blocking) formula: Recursive algorithm -Relation between Erlang's B and C formulas .

UNIT V

Model with Finite Input Source: Steady-state distribution: M/M/c//m (m>c). Engset delay model- Engset loss model M/M/c//m/(m > c) - The model M/M/c//m(m ≤ c).

SUGGESTED READINGS

1. Medhi J., (2003). Stochastic models in Queueing theory, 2e, Academic press.
2. Donald Gross, John Shortle F, James Thompson M , Carl M., and Harris ., (2008). Fundamentals of Queueing theory, Wiley.
3. Narayan Bhat U., (2008). An introduction to Queueing theory: Modelling and Analysis in Applications, Birkhauser Basel.

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Paper-III

22RMAT305 ADVANCED GRAPH THEORY 4H – 4C

Instruction Hours / week: L: 4 Total**: 100**

End Semester Exam: 3 Hours

Course Objectives:

To make the learners to

- Know about the basic concepts and definitions of graph colouring.
- Understand the dual graphs.
- Attain mastery in factorizations and decompositions of graphs.
- Acquire theoretical knowledge in graph domination.
- About the Ramsey numbers.
- Gain applicability of theoretical concepts to address network design problems.

Course Outcomes (Cos):

Learners able to:

1. Express vertex and edge coloring of graphs and apply in real life situations
2. Describe embeddings, dual graphs and Kuratowski's theorem, five color theorem.
3. Illustrate decomposition and labelling of any type of graphs.
4. Calculate domination and independent domination number of a graph.
5. Explain classical Ramsey numbers and generalize the theory of Ramsey.
6. Apply the concept of domination and labeling in recent research areas.

UNIT-I: COLORING OF GRAPHS

Vertex coloring and upper bounds – Brook's Theorem – Graphs with large chromatic number – Turan's Theorem – Counting proper coloring – Edge coloring – Characterization of line graph

UNIT-II: DUAL GRAPHS

Embeddings and Euler's Formula – Dual graphs – Kuratowski's Theorem – Five color theorem – Crossing number – Surface of higher genus

UNIT-III: DECOMPOSITION AND LABELING

Factorizations and Decompositions of graphs- Labeling of Graphs.

UNIT-IV:DOMINATION

Domination Theory -The Domination number of a graph- Bounds in terms of degree- The Independent Domination number of a graph

UNIT-V: RAMSEY THEORY

Classical Ramsey numbers- Generalized Ramsey Theory.

SUGGESTED READINGS

1. Chartrand G and Lesniak L., (2016). Graphs and Digraphs, Chapman and Hall/CRC, New York.
2. Douglas B. West., (2002). Introduction to Graph Theory, Prentice Hall of India.
3. Bondy J. A. and Murty U. S. R.,(2008). Graph Theory, Springer.
4. Harary F., (1972). Graph Theory, Addison-Wesley Publication.
5. Deo N., (2016). Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India.

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Paper-III

22RMAT306 OSCILLATION THEORY OF DIFFERENTIAL EQUATIONS 4H – 4C

Instruction Hours / week: L: 4 Total**: 100**

End Semester Exam: 3 Hours

Course Objectives:

To make the learners to

- Know about the basic concepts and brief knowledge of standard inequalities.
- Understand the concept of well-posed problems and show their existence and uniqueness of solutions.
- Acquire theoretical knowledge in oscillation theory of differential equations.
- Gain applicability of theoretical concepts to delay and neutral differential equations.
- Study comparison theorems for second-order ordinary differential equations.
- Study different kinds of oscillation criteria and asymptotic behavior of ordinary, delay and neutral differential equations.

Course Outcomes (Cos):

Learners able to:

1. To develop an understanding of the basic concepts of ordinary differential and functional differential equations (ODE and FDE).
2. To apply the knowledge of the ordinary differential and functional differential equations which will enable them to analyze the dynamics of the processes.
3. Explore some of the basic theories of functional differential equations, recognize basic types of FDEs for which exact solutions may be obtained, and apply the corresponding methods of solution.
4. Recognize ODEs and FDEs concepts that are encountered in the real-world problems, understand and be able to communicate the underlying mathematics involved in order to solve the problems using multiple approaches.
5. Import the knowledge of oscillation theory and apply it to the functional differential equations.
6. Students are introduced to modern concepts and methodologies in differential equations, with particular emphasis on the methods that can be used to solve largescale problems.

Unit-I: Algebraic and Integral Inequalities

The Theorem of the Arithmetic and Geometric Means - Holder's Inequality – Minkowski's Inequality – The Inequalities of Gronwall and Bellman – Some Generalization of Gronwall and Bellman inequalities.

Unit-II: Existence and Uniqueness of Solutions

Introduction – Lipschitz Continuity - Successive Approximations – Picard's Theorem – Continuation and Dependence on Initial Conditions – Existence of Solutions in the Large.

Unit-III: Oscillation of Second-Order Equations

Oscillation of Second-Order Equations – Fundamental Results – Sturm's Comparison Theorem – Elementary Linear Oscillations – Comparison Theorem of Hille-Wintner – Oscillations of $x''(t)+a(t)x=0$.

Unit-IV: Oscillation of Delay Differential Equations

Introduction – Equations with Single Delay - Equations with Variable Delay - Equations with Constant Delay - Equations with Several Delays - Equations with Piecewise Delay Constant Argument.

Unit-V: Oscillation of Neutral Differential Equations

Introduction - Comparison Theorem and Oscillation – Oscillation of Equations with Variable Coefficients – Existence of Non-Oscillating Solutions.

SUGGESTED READINGS:

1. Deo S. G., Lakshmikantham V. and Ragavendra V., (1997). Text Book of Ordinary Differential Equations, Tata McGraw Hill Publ. Co. New Delhi.
2. Agarwal R. P., Bohner M. and W-T Li., (2004). Nonoscillation and Oscillation: Theory for Functional Differential Equations, Marcel Dekker Inc., New York, 2004.
3. Gyori I. and Ladas. G., (1991). Oscillation Theory of Delay Differential Equations with Applications, Clarendon Press, Oxford.
4. Bainov D. D. and Mishev D. P., (1991). Oscillation Theory for Neutral Differential Equations with Delay, IOP Publishing Ltd, New York.
5. Erbe L. H., Kong Q. and Zang B. G., (1995). Oscillation Theory for Functional Differential Equations, Marcel Dekker Inc., New York.
6. Hardy G. H., Littlewood J. E. and Polya G., (1934). Inequalities, Cambridge University Press, London.
7. Pachpatte B.G., (1998). Inequalities for Differential and Integral Equations, Academic Press, London.