SEMESTER-V

17CAP503 **Optimization Techniques** **4H - 4C**

Instruction Hours / week:L: 4 T: 0 P: 0 **Marks: Internal: 40 External: 60 Total: 100**
End Semester Exam: 3Hours

SCOPE

Optimization Techniques is used to design construction and maintenance of engineering systems involved in decision making both at the managerial and technological level

Course Objective:

- To understand the need and origin of the optimization methods.
- To get a broad picture of the various applications of optimization methods used in engineering
- The process of finding the conditions that give the minimum or maximum value of the function where the function represents the effort required or the desired benefits.

Course Outcomes:

- Understanding the concept of transportation and assignment problems.
- Finding the optimal solution of unconstrained optimization problems.
- Understanding the importance of optimization of industrial process management
- Skill to analyze a variety of performance measures for various optimization problems
- Understanding the role of uncertainty in decision-making

UNITI

Optimization Techniques Linear Programming: Graphical method for two dimensional problems - Central problem of linear programming various definitions - statements of basic theorems and properties - Phase I and Phase II of the simplex method - revised simplex method - primal and dual - dual simplex method - sensitivity analysis transportation problem and its solution - assignment problem and its solution by Hungarian method.

UNITII

Integer Programming: Gomory cutting plane methods - Branch and Bound method.**QueueingTheory:**Characteristics of queueing systems - steady state MIMIt, MlMit/K and MIMIC queueing models.

UNITIII

Replacement Theory: replacement of items that deteriorate - Replacement of items that fail Group replacement and individual replacement.

UNITIV

Inventory theory: Costs involved in inventory problems - single item deterministic models-economic lot size models without shortages and with shortages having production rate infinite and finite.

UNITV

PERT and CPM: Arrow networks - time estimates- earliest expected time, latest allowable occurrence time and slack - critical path - probability of meeting scheduled date of completion of project calculations on CPM network - various floats for activities - critical path - updating project - operation time cost trade off curve - project time cost trade off curve - selection of schedule based on cost analysis.
(Remarks: No mathematical derivations included).

SUGGESTED READINGS

1. Gillet, B.E.,(1990), "Introduction to Operations Research : A Computer Oriented Algorithmic Approach". Tata McGraw Hill, New York.
2. Gross D., and Harris. C.M (1980), "Fundamentals of Queueing Theory", John Wiley and Sons, New York.
3. Hillier F., and Lieberman. GJ.(1985), "Introduction to Operations Research", Holden Day, New York.
4. Karnbo, N.S.,.(1985) "Mathematical Programming Techniques", McGraw Hill, New York.
5. KantiSwarup, Gupta, P.K., and Man Mohan (1980), "Operations Research", Sultan Chand & Sons. New Delhi.
6. Mital K. V.(1992), "Optimization Methods In Operations Research and System Analysis", New Age International (P) Ltd., New Delhi.
7. Saffer, L.R., Fitter J.B., and MeyerW.L.,(1990), "The Critical Path Method". McGraw Hill. New York.

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

LESSON PLAN

DEPARTMENT OF MATHEMATICS

Name of the faculty : M.Jannath Begam

Class : III M.C.A

Subject : Optimization Techniques

Subject Code : 17CAP503

S.No	Lecture Duration	Topics to be covered	Support Materials
Unit – I			
1.	1	Introduction to Linear Programming Problem	R3:Ch:3:Pg:31-34
2.	1	Problems on Mathematical Formulation of Linear Programming Problem	R5:Ch:2:Pg:44-49
3.	1	Graphical solution method – Problems	R5:Ch:2:Pg:56-59
4.	1	Continuation of problems on Graphical method	R5:Ch:2:Pg:60-62
5.	1	Statements of basic theorems and properties	R5:Ch:2:Pg:78-85
6.	1	Phase I and Phase II of the simplex method	R5:Ch:2:Pg:60-62
7.	1	Revised simplex method	R6:Ch:3:Pg:77-80
8.	1	Primal and dual problem	R6:Ch:8:Pg:228-230
9.	1	Dual simplex method	R6:Ch:3:Pg:89-90
10.	1	Sensitivity analysis transportation problem and its solution	R5:Ch:6:Pg:146-182
11.	1	Assignment problem and its solution by Hungarian method	R5:Ch:7:Pg:184-189
12.	1	Recapitulation and discussion of possible question	
Total No of Hours Planned For Unit I– 12 hours			

Unit – II			
1.	1	Introduction to Integer Programming problem	R1:Ch:4:Pg:129
2.	1	Gomory cutting plane methods	R1:Ch:4:Pg:164-169
3.	1	Branch and Bound method	R3:Ch:11:Pg:505-510
4.	1	Introduction to Queueing Theory	R3:Ch:17:Pg:766-768
5.	1	Characteristics of queueing systems	R5:Ch:17:Pg:421
6.	1	Steady state M/M/1, queueing model.Steady state M/M/1/K queueing model	R5:Ch:17:Pg:427-430 R5:Ch:17:Pg:439-442
7.	1	Steady state M/M/C queueing model.	R5:Ch:17:Pg:446-456
8.	1	Recapitulation and discussion of possible questions	
Total No of Hours Planned For Unit II – 8 hours			
Unit – III			
1.	1	Introduction to Replacement Theory	R5:Ch:19:Pg:535-536
2.	1	Replacement of items that deteriorate	R5:Ch:19:Pg:537-539
3.	1	Continuation on replacement of items that deteriorate	R5:Ch:19:Pg:540-545
4.	1	Replacement of items that fail Group replacement	R5:Ch:19:Pg:546-549
5.	1	Replacement of items that fail individual replacement.	R5:Ch:19:Pg:555-558
6.	1	Continuation on Replacement of items that fail individual replacement.	R5:Ch:19:Pg:559-563
7.	1	Recapitulation and discussion of possible questions	
Total No of Hours Planned For Unit III – 7 hours			
Unit – IV			
1.	1	Introduction to Inventory theory	R5:Ch:18:Pg:476-477
2.	1	Costs involved in inventory problems	R5:Ch:18:Pg:478-479

3.	1	Single item deterministic models	R5:Ch:18:Pg:480-485
4.	1	Economic lot size models without shortages	R5:Ch:18:Pg:486
5.		Continuatuion on Economic lot size models without shortages	R5:Ch:18:Pg:487
6.	1	Economic lot size models with shortages having production rate infinite and finite.	R5:Ch:18:Pg:488
7.	1	Economic lot size models with shortages	R5:Ch:18:Pg:489-490
8.	1	Continuatuion on Economic lot size models with shortages	R5:Ch:18:Pg:491-493
9.	1	Economic lot size models without shortages having production rate infinite and finite.	R5:Ch:18:Pg:494-496
10.	1	Recapitulation and discussion of possible question	
Total No of Hours Planned For Unit IV– 10 hours			
Unit – V			
1.	1	Introduction to CPM, Arrow networks , Time estimates-problems	R1:Ch:12:Pg:434-439
2.	1	Earliest expected time, latest allowable occurrence time and slack-problems	R1:Ch:12:Pg:439-442
3.	1	Various floats for activities ,critical path-problems	R1:Ch:12:Pg:442-443
4.	1	Continuatuion of problems on Various floats for activities ,critical path	R1:Ch:12:Pg:443-444
5.	1	Introduction to PERT:Probability of meeting scheduled date of completion of project	R1:Ch:12:Pg:445-446
6.	1	Operation time cost trade off curve, Project time cost trade off curve-problems	R5:Ch:22:Pg:635-640
7.	1	Selection of schedule based on cost analysis	R5:Ch:22:Pg:648-657
8.	1	Recapitulation and discussion of possible questions	R5:Ch:22:Pg:635-637 R5:Ch:22:Pg:638-640
9.	1	Discussion of previous ESE question papers	
10.	1	Discussion of previous ESE question papers	

11	1	Discussion of previous ESE question papers	

11.	1Discussion of previous ESE question papers		
Total No of Hours Planned For Unit V -11 hours			

SUGGESTED READINGS

1. Gillet, B.E.,(1990), "Introduction to Operations Research : A Computer Oriented Algorithmic Approach". Tata McGraw Hill, New York.
2. Gross D., and Harris. C.M (1980), "Fundamentals of Queueing Theory", John Wiley and Sons, New York.
3. Hillier F., and Lieberman. GJ.(1985), "Introduction to Operations Research", Holden Day, New York.
4. Karnbo, N.S.,.(1985) "Mathematical Programming Techniques", McGraw Hill, New York.
5. KantiSwarup, Gupta, P.K., and Man Mohan (1980), "Operations Research", Sultan Chand & Sons. New Delhi.
6. Mital K. V.(1992), "Optimization Methods In Operations Research and System Analysis", New Age International (P) Ltd., New Delhi.
7. Saffer, L.R., Fitter J.B., and MeyerW.L.,(1990), "The Critical Path Method". McGraw Hill. New York.

UNIT-I

SYLLABUS

Optimization Techniques Linear Programming: Graphical method for two dimensional problems - Central problem of linear programming various definitions - statements of basic theorems and properties - Phase I and Phase II of the simplex method - revised simplex method - primal and dual - dual simplex method - sensitivity analysis transportation problem and its solution - assignment problem and its solution by Hungarian method

LINEAR PROGRAMMING PROBLEM

Introduction:

LPP deals with determining optimal allocations of limited resources to meet given objectives. The resources may be in the form of men, raw materials, market demand, money and machines, etc. The objective is usually maximizing utility etc.

LPP deals with the optimization of a function of decision variables known as objective function. Subject to a set of simultaneous linear equation or inequality known as constraints.

The term linear means that all the variables occurring in the objective function and the constraints are of the 1^{st} degree in the problem under consideration and the term programming means the process of determining the particular course of action.

Mathematical formulation of LPP:

If (j = 1,2,…..n) are n decision variables of the problem and if the system is subject to m constraints.

∴ The general model can be redundant the form, Optimize z = f(1, 2, 3,………)

Subject to the constraints are,

(1, 2, 3,………) ≤, =, ≥, (i = 1, 2, …..,m) and

1, 2, 3,……… >= 0 (non negativity constraints)

Procedure for forming a LPP model:

Step 1: Identify the unknown decision variables to be determined and assign symbols to them.

Step 2: Identify all the restrictions or constraints in the problem and express them as linear equations or inequalities of decision variables.

Step 3: Identify the objective or aim and represent it also as a linear function of decision variables.

Step 4: Express the complete formulating of LPP as a general mathematical model.

Problems:

1. A firm manufactures two types of product A and B and sells them at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processed on two machines $_1$and $_2$. Type A requires 1 minute of processing time on $_1$ and 2 minutes on $_2$. Type B requires 1 minute on $_1$ and 1 minute on $_2$. Machine $_1$ is available for not more than 6 hours 40 minutes, while machines $_2$ is a available for 10 hours during any work hours. Formulate the problem as LPP so as to maximize the profit.

Solution:

Let us consider $_1$ be the no. of units in Type A and $_2$ be the no. of units in Type B. To produce these units of Type A and Type B product it requires,

$_1 +$ $_2$ processing minutes on $_1$

$2\,_1 +$ $_2$ processing minutes on $_2$

Since $_1$ is available for not more than 400 minutes and $_2$ is available for not more then 600 minutes.

Therefore the constraints are:

$$_1 + \quad _2 \leq 400$$
$$2\,_1 + \quad _2 \leq 600$$
and $_{1,}\ _2 \geq 0$

since the profit from Type A is Rs. 2 and profit from Type B is Rs. 3.

∴ The total Profit is $2\,{}_1 + 3\,{}_2$
∴ Here the objective is to maximize the profit
∴ The objective function is,

Maximize z = $2\,{}_1 + 3\,{}_2$

The complete formulation of the LPP is

Maximize z = $2\,{}_1 + 3\,{}_2$

Subject to the constraints,

${}_1 + {}_2 \le 400$ ………… (i)

$2\,{}_1 + {}_2 \le 600$ ………… (ii)

and ${}_{1},{}_{2} \ge 0$ ………… (iii)

2.A person wants to decide the constituents of a diet which will fulfill his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from 4 different types of foods. The yields per unit of these foods are given in the following table.

Food Type	Yield/unit			Cost / unit (Rs)
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum Requirement	800	200	700	

Formulate the LPP model for this problem.

Solution:

Let ${}_1, {}_2, {}_3, {}_4$ be the no. of units in the food type 1,2,3 and 4 respectively.
In this problem the main objective is to minimize the cost. ∴ The objective function is,

Minimize Z = $45\,{}_1 + 40\,{}_2 + 85\,{}_3 + 65\,{}_4$

The minimize requirement for proteins, fats and carbohydrates are 800, 200 and 700 respectively.

∴ The subject to the constraints are:

$$3\,{}_1 + 4\,{}_2 + 8\,{}_3 + 6\,{}_4 \geq 800$$
$$2\,{}_1 + 2\,{}_2 + 7\,{}_3 + 5\,{}_4 \geq 200$$
$$6\,{}_1 + 4\,{}_2 + 7\,{}_3 + 4\,{}_4 \geq 700$$

And the complete formation of LPP is

Minimize $Z = 45\,{}_1 + 40\,{}_2 + 85\,{}_3 + 65\,{}_4$

∴ The subject to the constraints:

$$3\,{}_1 + 4\,{}_2 + 8\,{}_3 + 6\,{}_4 \geq 800$$
$$2\,{}_1 + 2\,{}_2 + 7\,{}_3 + 5\,{}_4 \geq 200$$
$$6\,{}_1 + 4\,{}_2 + 7\,{}_3 + 4\,{}_4 \geq 700$$

and $\,{}_1,\ {}_2,\ {}_3,\ {}_4 \geq 0.$

3.A Television company operates 2 assembly section A and B. Each section is used assemble the components of 3 types of television (i.e.) color, standard and economy. The expected daily production on each section is as following.

T.V Model	Section A	Section B
Color	3	1
Standard	1	1
Economy	2	6

The daily running cost for two sections average Rs. 6000 for section A and Rs. 4000 for section B. It is given that the company must produce atleast 24 colors, 16 standard and 40 economy TV sets for which an order is pending. Formulate this as a LPP so as to minimize the total cost.

Solution:

Let ${}_1$ and ${}_2$ be the no. of units in section A and section B.

The objective function is, Minimize Z = 6000 $_1$ + 4000 $_2$

∴ The subject to the constraints,

3 $_1$ + $_2 \geq 24$

$_1$ + $_2 \geq 16$

2 $_1$ + 6 $_2 \geq 40$

The complete formation of LPP is,

Minimize Z = 6000 $_1$ + 4000 $_2$

∴The subject to the constraints,

3 $_1$ + $_2 \geq 24$

$_1$ + $_2 \geq 16$

2 $_1$ + 6 $_2 \geq 40$

and $_1$, $_2 \geq 0$

Graphical Method of the solution of the LPP:

Linear programming problems involving only 2 variables can be effective solved by a graphical method which provides a pictorial representation of the problems and its solutions. And which gives the basic concept used in solving general LPP which may involve any finite no. of variables.

Working procedure for graphical method:

Given a LPP optimize Z = f( ),

Subject to the constraints,

( ) ≤, =, ≥ , (i=1,2,…..,n), (j=1,2,…..,m)

and ≥ 0 (non-negativity restrictions)

Step 1: Consider the inequality constraints as equalities. Draw the straight lines in the XOY plane corresponding to each equality and non-negativity restrictions.

Step 2: Find the permissible region (feasible region or solution space) for the values of the variable which is the region bound by the lines drawn in step1.

Step 3: Find the points of intersection of the bound lines by solving the equations of the corresponding lines.

Step 4: Find the values of Z at all vertices of the permissible region.

Step 5:

(i) For minimization problem choose the vertex for which Z is maximum.

(ii) For minimization problem choose the vertex for which Z is minimum.

Problems:

1. Solve the following LPP by graphical method. Max Z = 3 $_1$ + 2 $_2$

Subject to the constraints are,

-2 $_1$ + $_2 \leq 1$

$_1 \leq 2$

$_1$ + $_2 \leq 3$ and $_1$, $_2 \geq 0$

Solution:

Consider the inequality constraints as equality,

-2 $_1$ + $_2$ = 1 ……….. (1)

$_1$ = 2 ……….. (2)

$_1$ + $_2$ = 3 ……….. (3)

$_1$ = 0 ……….. (4)

$_2$ = 0 ……….. (5)

From equation (1), putting $_1$ = 0.

We get -2 $_1$ + $_2$ = 1

$_2$ = 1

The point ($_1$, $_2$) = (0,1)

Similarly, putting $_2$ = 0, we get

-2 $_1$ + 0 = 1

$_1 = {}^{-1}/_2 = -0.5$

∴ The point ($_1$, $_2$) = (-0.5, 0)

∴ The point (0, 1) and (-0.5, 0) lies on the line -2 $_1$ + $_2$ = 1 From equation (2) we get, the points (2,1) and (2,2) lies on the line $_1$ = 2. From equation (3) putting $_1$ = 0,

we get $_1$ + $_2$ = 3

$_2$ = 3

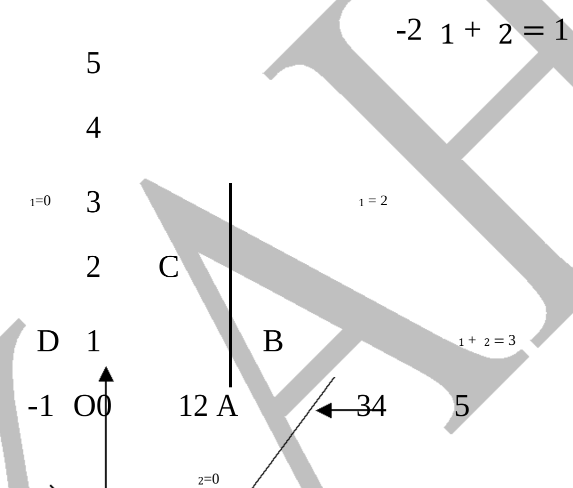
The point ($_1$, $_2$) = (0, 3)

Similarly putting $_2$ = 0, we get $_1$ + 0 = 3

$_1$ = 3.

The point ($_1$, $_2$) = (3, 0)

∴ The point (0, 3) and (3, 0) lies on the line $_1$ + $_2$ = 3



∴ From the graph the vertices of the solution space are, O(0,0), A(2,0), B(2,1), C(0.7,2.3), D(0,1)

The values of the Z at these vertices are given by,

Vertex	$Z = 3{}_1 + 2{}_2$
O(0,0)	0
A(2,0)	6
B(2,1)	8
C(0.7,2.3)	6.7
D(0,1)	2

Since the problem is of maximization type.

∴ The optimum solution to the LPP is, Max Z = 8

∴ ${}_1 = 2$ and ${}_2 = 1$

2.Maximize $Z = 5{}_1 + 8{}_2$

Subject to constraints, $15{}_1 + 10{}_2 \leq 180$

$10{}_1 + 20{}_2 \leq 200$

$15{}_1 + 20{}_2 \leq 210$

and ${}_{1,}{}_2 \geq 0$

Solution:

Consider the inequality constraints as equality,

$15{}_1 + 10{}_2 = 180$ ……….. (1)

$10{}_1 + 20{}_2 = 200$ ……….. (2)

$15{}_1 + 20{}_2 = 210$ ……….. (3)

${}_1 = 0$ ……….. (4)

${}_2 = 0$ ……….. (5)

From the (1) equation, putting ${}_1 = 0$, we get

$15(0) + 10{}_2 = 180$

$$10\ {}_{2} = 180$$
$${}_{2} = 18$$

∴ The point $({}_{1}, {}_{2}) = (0, 18)$ Similarly putting ${}_{2} = 0$, we get

$$15\ {}_{1} + 10(0) = 180$$
$$15\ {}_{1} = 180$$
$${}_{1} = 12$$

∴ The point $({}_{1}, {}_{2}) = (12, 0)$

From the (2) equation putting ${}_{1} = 0$, we get

$$10(0) + 20\ {}_{2} = 200$$
$$20\ {}_{2} = 200$$
$${}_{2} = 10$$

∴ The point $({}_{1}, {}_{2}) = (0, 10)$ Similarly putting ${}_{2} = 0$, we get

$$10\ {}_{1} + 20(0) = 200$$
$$10\ {}_{1} = 200$$
$${}_{1} = 20$$

∴ The point $({}_{1}, {}_{2}) = (20, 0)$

From the 3rd equation putting ${}_{1} = 0$, we get

$$15(0) + 20\ {}_{2} = 210$$
$$20\ {}_{2} = 210$$
$${}_{2} = 10.5$$

∴ The point $({}_{1}, {}_{2}) = (0, 10.5)$ Similarly putting ${}_{2} = 0$, we get

$15\ {}_1 = 210$

${}_1 = 14$

∴ The point $({}_1, {}_2) = (14, 0)$

Vertex	$Z = 5\ {}_1 + 8\ {}_2$
O(0,0)	0
A(12,0)	60
B(10,3)	74
C(2,9)	82
D(0,10)	80

∴ The solution Max Z = 82

∴ ${}_1 = 2, \quad {}_2 = 9$

Some more cases in a LPP:

In general a LPP may have

1. A unique optimal solution.
2. An Infinite no. of optimal solutions.
3. An Unbounded solution.
4. No solution.

Problems:

1. Solve the following LPP graphically, Maximize $Z = 100\ {}_1 + 40\ {}_2$

Subject to the constraints are:

$5\ {}_1 + 2\ {}_2 \leq 1000$

$3\ {}_1 + 2\ {}_2 \leq 900$

${}_1 + 2\ {}_2 \leq 500$

and ${}_{1,}\ {}_2 \geq 0$

Solution:

$$5\,{}_1 + 2\,{}_2 = 1000 \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

$$3\,{}_1 + 2\,{}_2 = 900 \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

$$ {}_1 + 2\,{}_2 = 500 \qquad \ldots\ldots\ldots\ldots\ldots (3)$$

From (1) equation putting ${}_2 = 0$, we get

$5(0) + 2\,{}_2 = 1000$

$2\,{}_2 = 1000$

${}_2 = 500$

$\therefore$ The point $({}_1, {}_2) = (0, 500)$

Similarly, ${}_2 = 0$, we get $5\,{}_1 + 0 = 1000$

${}_1 = 200$

$\therefore$ The point $({}_1, {}_2) = (200, 0)$

From equation (2) putting ${}_1 = 0$, we get

$3(0) + 2\,{}_2 = 900$

${}_2 = 450$

$\therefore$ The point $({}_1, {}_2) = (0, 450)$

Similarly ${}_2 = 0$, we get, $3\,{}_1 + 2(0) = 900$

$3\,{}_1 = 900$

${}_1 = 300$

$\therefore$ The point $({}_1, {}_2) = (300, 0)$

From equation (3) putting ${}_1 = 0$, we get

$0 + 2\,{}_2 = 500$

${}_2 = 250$

∴ The point ($_1$, $_2$) = (0, 250)

Similarly $_2$ = 0, we get, $_1$ + 0 = 500

$_1$ = 500

∴ The point ($_1$, $_2$) = (500, 0)

vertex	Z = 100 $_1$ + 40 $_2$
O(0,0)	0
A(200,0)	20,000
B(125,187.5)	20,000
C(0,250)	10,000

Since the problem is of maximization type in the above table the two vertices have the same maximization Z value.

Hence the given LPP has an infinite no. of solution. For this problem the optimum solution is Maximize Z = 20,000

$_1$ = 200 (or) 125
$_2$ = 187.5 (or) 0

2. Max Z = 2 $_1$ + 3 $_2$ Subject to the constraints:

$_1$ - $_2$ ≤ 2

$_1$ + $_2$ ≤ 4 and $_1$, $_2$ ≥ 0

Solution:

$_1$ - $_2$ = 2 ……………. (1)

$_1$ + $_2$ = 4 ……………. (2)

From (1) equation $_1$ = 0 we get,

0 - $_2$ = 2

$_2 = -2$

∴ The point ($_1$, $_2$) = (0, -2)

Similarly putting $_2 = 0$, we get

$_1 - 0 = 2$

$_1 = 2$

∴ The point ($_1$, $_2$) = (2, 0)

From (2) equation $_1 = 0$ we get

$0 + {}_2 = 4$

$_2 = 4$

∴ The point ($_1$, $_2$) = (0, 4)

Similarly putting $_2 = 0$, we get

$_1 + 0 = 4$

$_1 = 4$

∴ The point ($_1$, $_2$) = (4, 0)

vertex	Max Z = 2 $_1$ + 3 $_2$
A(0,4)	12
B(3,1)	9

From the above graph, the maximization type with two vertex and have the unbounded solution.

3.Max Z = 4 $_1$ + 3 $_2$

Subject to the constraints:

$_1 - {}_2 \leq -1$

$-{}_1 + {}_2 \leq 0$ and $_1, {}_2 \geq 0$

Solution:

$_1$ - $_2$ = -1 ……………. (1)

- $_1$ + $_2$ = 0 ……………. (2)

From (1) equation $_1$ = 0 we get,

0 - $_2$ = -1

$_2$ = 1

∴ The point ($_1$, $_2$) = (0, 1) Similarly putting $_2$ = 0, we get

$_1$ - 0 = -1

$_1$ = -1

∴ The point ($_1$, $_2$) = (-1, 0) From (2) equation $_1$ = 0 we get

0 + $_2$ = 0

$_2$ = 0

∴ The point ($_1$, $_2$) = (0, 0) Similarly putting $_2$ = 0, we get

− $_1$ + 0 = 0

$_1$ = 0

∴ The point ($_1$, $_2$) = (0, 0)

These being no point ($_1$, $_2$) common to both the shaded regions. That is, we can not find a convex region for this problem. So the problem cannot be solved. Hence the problem have ***no feasible solution***.

General Linear Programming Problem:

Simplex Methods:

General Linear Programming Problem:

The Linear Programming Problem involving more than 2 variables will be expressed as follows:

Maximize (or) Minimize Z = $_{1}{}_{1}+{}_{2}{}_{2}+\ldots\ldots+$

Subject to the constraints:

$11\ 1 + 12\ 2 + \ldots\ldots\ldots + 1$ $\leq, =,$ (or) $\geq$ $1\ 21\ 1 + 22\ 2 + \ldots\ldots$
$\ldots + 2$ $\leq, =,$ (or) ≥ 2

…………………………………………………..

…………………………………………………..
$1\ 1 + \ 2\ 2 + \ldots\ldots\ldots +$ $\leq, =,$ (or) $\geq$

and the non-negativity restrictions is, $1, 2, \ldots\ldots\ldots\ldots,$ ≥ 0

Definition – 1:

A set of values $1, 2, \ldots\ldots\ldots\ldots,$ which satisfies the constraints of the LPP is called its ***solution***.

Definition – 2:

Any solution to a LPP which satisfies the non-negativity restrictions of LPP is called its ***Feasible Solution***.

Definition – 3:

Any feasible solution which optimizes the objective function of the LPP is called its ***optimum or optimal solution.***

Definition – 4:

If a constraints of a general LPP, $\sum_{=1}$ $\leq$ (i=1,2,3,…..,k)

Then the non – negative variables which are introduced to convert inequalities to the equalities (i.e.) $\sum_{=1} + =$ are called ***slack variables***.

Definition – 5:

If a constraints of a general LPP, $\sum_{=1}$ $\geq$ (i=1,2,3,…..,k)

Then the non – negative variables which are introduced to convert inequalities to the equalities (i.e.) $\sum_{=1} - =$ are called ***surplus variables***.

Canonical and standard forms of LPP:

After the formulation of LPP the next step is to obtain its solution. But before any method is used to find its solution the problem must be presented in a suitable form.

There are 2 forms: (1) Canonical form

(2) Standard form

1. The Canonical Form:

The general LPP can always be expressed in the following form,

Maximize Z = $_{1\,1} + _{2\,2} + \ldots + $

Subject to the constraints:

$_{11\,1} + _{12\,2} + \ldots\ldots + _{1} \leq _{1}$

$_{21\,1} + _{22\,2} + \ldots\ldots + _{2} \leq _{2}$

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……………………………………………………..

$_{1\,1} + _{2\,2} + \ldots\ldots + \leq$

and the non-negativity restrictions is, $_{1}, _{2}, \ldots\ldots, \geq 0$

This form of LPP is called the ***Canonical form of LPP.***

Characteristics of the Canonical Form:

1. The objective function is of Maximization type.
2. All constraints are of less or equal to type.
3. All variables are non-negative.

1. The Standard Form:

The general LPP in the form,

Maximize Z = $_{1\,1} + _{2\,2} + \ldots \ldots +$

Subject to the constraints:

$_{11\,1} + _{12\,2} + \ldots\ldots\ldots + _{1} \quad = \quad _{1}$

$_{21\,1} + _{22\,2} + \ldots\ldots\ldots + _{2} \quad = \quad _{2}$

……………………………………………

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$_{1\,1} + _{2\,2} + \ldots\ldots\ldots + \quad =$

and the non-negativity restrictions is, $_{1}, _{2}, \ldots\ldots\ldots .., \geq 0$ is known as ***Standard from.***

Characteristics of the Standard Form:

1. The objective function is of Maximization type.
2. All constraints are expressed as equation type.
3. Right hand sides of each constraint are non-negative.
4. All variables are non-negative.

Problems:

1. Express the following LPP in Standard form.

Minimize Z = $5\,_{1} + 7\,_{2}$

Subject to the constraints:

$_{1} + \quad _{2} \leq 8$

$3\,_{1} + 4\,_{2} \geq 3$

$6\,_{1} + 7\,_{2} \geq 5$ and $\quad _{1}, _{2} \geq 0$

Solution:

Since Minimize Z = - Max(-Z)

$= \text{-Max}(\ ^{*})$

$= -5\ _{1} - 7\ _{2}$

The given LPP becomes,

Maximize $^{*} = -5\ \ -7$

$_{1} \qquad _{2}$

Subject to the constraints:

$_{1} + \ _{2} \leq 8$

$3\ _{1} + 4\ _{2} \geq 3$

$6\ _{1} + 7\ _{2} \geq 5$ and $_{1},\ _{2} \geq 0$

By introducing slack variables $_{1}$ surplus variables $_{2},\ _{3}$ then the standard form of the LPP is given by,

Maximize $^{*} = -5\ \ -7\ \ +0\ \ +0\ \ +0$

$_{1} \quad _{2} \quad _{1} \quad _{2} \quad _{3}$

Subject to the constraints:

$_{1} + \ _{2} + \ _{1} = 8$

$3\ _{1} + 4\ _{2} - \ _{2} = 3$

$6\ _{1} + 7\ _{2} - \ _{3} = 5$ and $_{1},\ _{2},\ _{1},\ _{2},\ _{3} \geq 0$

2. Express the following LPP in Standard form.

Maximize $Z = 4\ _{1} + 2\ _{2} + 6\ _{3}$

Subject to the constraints:

$2\ _{1} + 3\ _{2} + 2\ _{3} \geq 6$

$3\ _{1} + 4\ _{2} = 8$

$6\ _{1} - 7\ _{2} + \ _{3} \leq 10$ and $_{1},\ _{2},\ _{3} \geq 0$

Solution:

The given LPP becomes,

Max Z = $4\ _1 + 2\ _2 + 6\ _3$

By introducing slack variables $_2$ surplus variables $_1$, then the standard form of the LPP is given by,

Max $^* = 4\ _1 + 2\ _2 + 6\ _3 + 0\ _1 + 0\ _2$

Subject to the constraints:

$2\ _1 + 3\ _2 + 2\ _3 - \ _1 = 6$

$3\ _1 + 4\ _2 + 0\ _3 = 8$

$6\ _1 - 7\ _2 + \ _3 + \ _2 = 10$

and $_1, _2, _3, _1, _2 \geq 0$

3. Express the following LPP in Canonical form.

Minimize Z = $_1 + 4\ _2$

Subject to the constraints:

$3\ _1 + \ _2 \leq 5$

$-2\ _1 + 4\ _2 \geq -7$

and $_1, _2 \geq 0$

Solution:

The canonical form of the given LPP becomes,

Maximize $^* = -\ _1 - 4\ _2$

Subject to the constraints:

$3\ _1 + \ _2 \leq 5$

$2\ _1 - 4\ _2 \leq 7$

and $_1, _2 \geq 0$

4. Maximize Z = $3_{1} + {}_{2}$ Subject to the constraints:

$${}_{1} + 2_{2} \geq -5$$
$$3_{1} + 5_{2} \leq 6$$
and $\quad {}_{1,2} \geq 0$

Solution:

The canonical form of a given LPP is,

Max Z = $3_{1} + {}_{2}$

Subject to the constraints:

$$-_{1} - 2_{2} \leq 5$$
$$3_{1} + 5_{2} \leq 6$$
and $\quad {}_{1,2} \geq 0$

5. Express the following LPP in the canonical form: Maximize Z = $2_{1} + 3_{2} + {}_{3}$

Subject to the constraints:

$$4_{1} - 3_{2} + {}_{3} \leq 6$$
$${}_{1} + 5_{2} - 7_{3} \geq -4$$
and $\quad {}_{1,3} \geq 0$ and ${}_{2}$ is unrestricted.

Solution:

Here ${}_{2}$ is unrestricted, $\therefore$ ${}_{2}$ can be written as the difference of two non-negative variables, (i.e.) ${}_{2} = {}_{2}' - {}_{2}''$ where ${}_{2}', {}_{2}'' \geq 0$

$\therefore$ The given LPP becomes,

Max Z = $2_{1} + 3({}_{2}' - {}_{2}'') + {}_{3}$

Subject to the constraints:

$$4\,{}_1 - 3({}_2{}' - {}_2{}'') + \qquad {}_3 \leq 6$$
$$-\,{}_1 - 5({}_2{}' - {}_2{}'') + 7\,{}_3 \leq 4$$
and $${}_1, {}_2{}', {}_2{}'', {}_3 \geq 0$$

The Simplex Method:

Definition – 1:

Given a system of m linear equations with n variables (m<n). The solution obtained by setting (n-m) variables = 0 and solving for the remaining m variables is called a ***Basic Solution.***

The m variables are called Basic variables and they form Basic Solution. The (n-m) variables which are put to 0 are called as ***Non- basic Variables***.

Definition – 2:

A basic solution is said to be a non-degenerate Basic Solution if none of the Basic variables is zero.

Definition – 3:

A basic solution is said to be a degenerate basic solution if one or more of the basic variables are zero.

Definition – 4:

A feasible solution which is also basic is called a ***Basic feasible solution***.

The Simplex Algorithm:

Assuming the existence of an initial basic feasible solution, an optimum solution to any LPP by simplex method is found as follows:

Step 1: Check whether the objective function is to be maximized or minimized. If it is to be minimized, then convert it into a problem of maximization, by Minimize Z = -Maximize(-Z)

Step 2: Check whether all b_i's are positive. If any of the b_i's is negative, multiply both sides of that constraint by -1 so as to make its right hand positive.

Step 3: By introducing slack / surplus variables, convert the inequality constraints into equations and express the given LPP into its standard form.

Step 4: Find an initial basic feasible solution and express the above information conveniently in the following simplex table.

		C_j	(C_1	C_2	C_3	…….	0	0	0	…….)
C_B	Y_B	X_B	x_1	x_2	x_3	…….	s_1	s_2	s_3	…….
C_{B1}	S_1	b_1	a_{11}	a_{12}	a_{13}	…….	1	0	0	…….
C_{B2}	S_2	b_2	a_{21}	a_{22}	a_{23}	…….	0	1	0	…….
C_{B3}	S_3	b_3	a_{31}	a_{32}	a_{33}	…….	0	0	1	…….
.	.	.	.	.	.	…….				…….
.	.	.	.	.	.					
.	.	.	.	.	.	…….				…….
.	.	.	.	.	.					
.	.	.	.	.	.	…….				…….
.	.	.	.	.	.					
.	.	.	Body matrix			…….		Unit matrix		
.	.	.								
(Z_j – C_j)		Z_0	(Z_1 – C_1)	(Z_2 – C_2)		…….	…………………….			

(Where C_j – row denotes the coefficients of the variables in the objective function.
C_B – column denotes the coefficients of the basic variables in the objective function.
Y_B – column denotes the basic variables. X_B – column denotes the values of the basic variables. The coefficients of the non-basic variables in the constraint equations constitute the body matrix while the coefficients of the basic variables constitute the unit matrix. The row (Z_j – C_j) denotes the evaluations (or) index for each column).

Step 5: Compute the net evaluations ($Z_j - C_j$) (j = 1,2,…..,n) by using the relation
$Z_j - C_j = C_B\, a_j - C_j$.

Examine the sign of Z_j - C_j

(a) If all $(Z_j – C_j) \geq 0$ then the current basic feasible solution X_B is optimal.

(b) If atleast one $(Z_j – C_j) < 0$, then the current basic feasible solution is not optimal, go to the next step.

Step 6: (To find the entering variable)

The entering variable is the non-basic variable corresponding to the most negative value of $(Z_j – C_j)$. Let it be x_r for some j = r. The entering variable column is known as the key column (or) pivot column which is shown marked with an arrow at the bottom. If more than one variable has the same most negative $(Z_j – C_j)$, any of these variables may be selected arbitrarily as the entering variable.

Step 7: (To find the leaving variable)

Compute the ratio = Min { , > 0} (i.e., the ration between the solution column and the entering variable column by considering only the positive denominators)

(a) If all $a_{ir} \leq 0$, then there is an unbounded solution to the given LPP.

(b) If atleast one $a_{ir} > 0$, then the leaving variable is the basic variable corresponding to the minimum ratio . If = , then the basic variable x_k leaves the basis. The leaving variable row is called the key row or pivot equation, and the element at the intersection of the pivot column and pivot row is called the pivot element (or) leading element.

Step 8: Drop the leaving variable and introduce the entering variable along with its associated value under C_B column. Convert the pivot element to unity by dividing the pivot equation by the pivot element and all other elements in its column to zero by making use of

(i) New pivot equation = old pivot equation ÷ pivot element

(ii) New equation (all other rows including $(Z_j – C_j)$ row)

= Old equation - (Corresponding Column Coefficient) x (New pivot element)

Step 9: Go to step (5) and repeat the procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

Note(1): For maximization problems:

(i) If all $(Z_j - C_j) \geq 0$, then the current basic feasible solution is optimal.
(ii) If atleast one $(Z_j - C_j) < 0$, then the current basic feasible solution is not optimal.
(iii) The entering variable is the non-basic variable corresponding to the most negative value of $(Z_j - C_j)$.

Note(2): For minimization problems:

(i) If all $(Z_j - C_j) \leq 0$, then the current basic feasible solution is optimal.
(ii) If atleast one $(Z_j - C_j) > 0$, then the current basic feasible solution is not optimal.
(iii) The entering variable is the non-basic variable corresponding to the most positive value of $(Z_j - C_j)$.

Note(3): For both maximization and minimization problems, the leaving variable is the basic variable corresponding to the minimum ratio .

Problems:

1. Use simplex method to solve following LPP. Maximize Z = $4\,_1 + 10\,_2$

Subject to the constraints:

$$2\,_1 + \,_2 \leq 50$$
$$2\,_1 + 5\,_2 \leq 100$$
$$2\,_1 + 3\,_2 \leq 90$$
and $\,_1, \,_2 \geq 0$

Solution:

By introducing the slack variables $_1, _2, _3$. The standard form of the given LPP becomes,
Maximize Z = $4\,_1 + 10\,_2 + 0\,_1 + 0\,_2 + 0\,_3$

Subject to the constraints:

$$2\,_1 + \,_2 + \,_1 + 0\,_2 + 0\,_3 \leq 50$$
$$2\,_1 + 5\,_2 + 0\,_1 + \,_2 + 0\,_3 \leq 100$$

$2\ _1 + 3\ _2 + 0\ _1 + 0\ _2 + \ _3 \leq 90$

and $_1, _2, _1, _2, _3 \geq 0$

since there are 3 equations with 5 variables. ∴ The initial basic Feasible Solution is obtained by equality, (5-3) = 2 to 0.

∴ The initial basic Feasible Solution (IFBS) , $_1$=50, $_2$=100, $_3$=90

Initial iteration:

			4	10	0	0	0	
			$_1$	$_2$				Ratio
0	$_1$	50	2	1	1	0	0	$^{50}/_{1}$= 50
0	$_2$	100	2	$\boxed{5}$	0	1	0	$^{100}/_{5}$= 20
0	$_3$	90	2	3	0	0	1	$^{90}/_{3}$= 30
	–	0	-4	-10	0	0	0	

2 is entering variable,

2 is leaving variable,

5 is pivot element

I iteration:

			4	10	0	0	0
			$_1$	$_2$			
0	$_1$	30	$^{8}/_{5}$	0	1	$^{-1}/_{5}$	0
0	$_1$	20	$^{2}/_{}$	1	0	$^{1}/_{}$	0
0	$_3$	30	$^{4}/_{5}$	0	0	$^{-3}/_{5}$	1
	–	200	0	0	0	2	0

Since all – ≥ 0. ∴ The current Basic Feasible solution is optimal.

∴ The optimal solution is Max Z = 200, $_1$ = 0, $_2$ = 20.

2. Find the non-negative values of $_1, _2, _3$ which

Max Z = $3\ _1 + 2\ _2 + 5\ _3$

Subject to the constraints,

$_1 + 4\ _2 \le 420$
$3\ _1 + 2\ _3 \le 460$
$_1 + 2\ _2 + \quad _3 \le 430$
and $_{1,\ 2,\ 3} \ge 0$

Solution:

By introducing slack variables $_1,\ _2,\ _3$. The standard form of the given
Max Z = $3\ _1 + 2\ _2 + 5\ _3 + 0\ _1 + 0\ _2 + 0\ _3$

Subject to the constraints,

$_1 + 4\ _2 + 0\ _3 + \quad _1 + 0\ _2 + 0\ _3 = 420$
$3\ _1 + 0\ _2 + 2\ _3 + 0\ _1 + \quad _2 + 0\ _3 = 460$
$_1 + 2\ _2 + \quad _3 + 0\ _1 + 0\ _2 + \ _3 = 430$
and $_{1,\ 2,\ 3,\ 1,\ 2,\ 3} \ge 0$

The IFBS is given by, $_1 = 420,\ _2 = 460,\ _3 = 430$

Initial iteration:

			3	2	5	0	0	0	Ratio
			$_1$	$_2$	$_3$				
0	$_1$	420	1	4	0	1	0	0	$420/0=\infty$
0	$_2$	460	3	0	$\boxed{2}$	0	1	0	$460/2=230$
0	$_3$	430	1	2	1	0	0	1	$430/1=430$
	$_-$	0	-3	-2	-5	0	0	0	

$_2$ is leaving variable, $_3$ is entering variable, 2 is pivot element

I iteration:

			3	2	5	0	0	0	Ratio
			1	2	3				
0	1	420	1	4	0	1	0	0	$420/4=105$
5		230	3/2	0	1	0	1/2	0	230/
0	3	200	$-1/2$	$\boxed{2}$	0	0	$-1/2$	1	$200/2=100$
		1150	9/2	-2	0	0	5/2	0	

3 is leaving variable, 2 is entering variable, 2 is pivot element

II iteration:

			3	2	5	0	0	0
			1	2	3			
0	1	20	2	0	0	1	0	-2
5		230	3/2	0	1	0	1/2	0
2		100	−1/4	1	0	0	−1/4	1/2
		1350	4	0	0	0	2	1

∴ Max Z = 1350, 1 = 0, 2 = 100, 3= 230

Artificial Variable Techniques:

To solve a LPP by a simplex method. We have to start with initial Basic feasible solution and construct the initial simplex table. In the previous problems the slack variable provided the IFBS. However in some problems the slack variables cannot provide the IFBS. In these problems atleast one of the constraints is of equal to or greater than or equal to type. To solve such a LPP there are 2 methods available:

(1) Big-M method (or) M-Technique (or) The Method of penalties
(2) Two Phase method.

Problems:

1. Use Big- M \method to solve, Min Z = 4 1 + 3 2

Subject to the constraints,

$2\,{}_{1} + {}_{2} \geq 10$
$-3\,{}_{1} + 2\,{}_{2} \leq 6$
${}_{1} + {}_{2} \leq 6$ and ${}_{1},\ {}_{2} \geq 0$

Solution:

The standard form of the given LPP is

Max $^{*} = -4\,{}_{1} - 3\,{}_{2} + 0\,{}_{1} + 0\,{}_{2} + 0\,{}_{3} - M\,{}_{1} - M\,{}_{2}$ Subject to the constraints,

$2\,{}_{1} + {}_{2} - {}_{1} + {}_{1} = 10$
$-3\,{}_{1} + 2\,{}_{2} + {}_{2} = 6$
${}_{1} + {}_{2} - {}_{3} + {}_{2} = 6$
and ${}_{1},\ {}_{2},\ {}_{1},\ {}_{2},\ {}_{3},\ {}_{1},\ {}_{2} \geq 0$

$\therefore$ The IFBS is given by, ${}_{1} = 10,\ {}_{2} = 6,\ {}_{2} = 6$

Initial Iteration:

			-4	-3	0	0	0	-M	-M	Ratio
			${}_{1}$	${}_{2}$	${}_{1}$	${}_{2}$	${}_{3}$	${}_{1}$	${}_{2}$	
-M	${}_{1}$	10	$\boxed{2}$	1	-1	0	0	1	0	$^{10}/_{2} = 5$
0	${}_{2}$	6	-3	2	0	1	0	0	0	$^{-6}/_{3} = -2$
-M	${}_{2}$	6	1	1	0	0	-1	0	1	$^{6}/_{1} = 6$
		-16M	-3M+4	-2M+3	M	0	M	0	0	

${}_{1}$ is leaving variable, ${}_{1}$ is entering variable, 2 is pivot element

I iteration:

			-4	-3	0	0	0	-M	Ratio
			$_1$	$_2$	$_1$	$_2$	$_3$	$_2$	
-4	$_1$	5	1	$1/2$	$-1/2$	0	0	0	10
0	$_2$	21	0	$7/2$	$-3/2$	1	0	0	6
-M	$_2$	1	0	$\boxed{1/_2}$	$1/2$	0	-1	1	2
	–	-20-M	0	$2-/_2$	$4-/_2$	0	M	0	

$_2$ is leaving variable, $_2$ is entering variable, $1/_2$ is pivot element

II iteration:

			-4	-3	0	0	0
			$_1$	$_2$			
-4	$_1$	4	1	0	-1	0	1
0	$_2$	14	0	0	-5	1	7
-3	$_2$	2	0	1	1	0	-2
	–	-22	0	0	1	0	2

Since all – ≥ 0, $\therefore$ the current solution is optimal.

The optimal solution is, Max $^* = -22$, $_1 = 4$, $_2 = 2$

But, Min Z = - (Max (-Z))

= - (- 22)

Min Z = 22, $_1 = 4$, $_2 = 2$

2. Solve the following LPP by Simplex Method: Max Z = $3\,_1 + 2\,_2$

Subject to the constraints,

$2\,_1 + \,_2 \leq 2$

$3\,_1 + 4\,_2 \geq 12$ and $_1, _2 \geq 0$

Solution:

By introducing the slack variable $_1$ and surplus variable $_2$, artificial variable $_1$. The standard form of the LPP becomes, Max Z = 3 $_1$ + 2 $_2$ + 0 $_1$+ 0 $_2$ + 0 $_3$ – M $_1$

Subject to the constraints,

2 $_1$ + $_2$+ $_1$ = 2

3 $_1$ + 4 $_2$ – $_2$+ $_1$ = 12

and $_1$, $_2$, $_1$, $_2$, $_3$, $_1$ ≥0

∴ The IBFS, $_1$ = 2, $_1$= 12

Initial Iteration:

			3	2	0	0	-M	Ratio
			$_1$	$_2$	$_1$	$_2$	$_1$	
← 0	$_1$	2	2	$\boxed{1}$	1	0	0	2
-M	$_1$	12	3	4	0	-1	1	3
	–	-12M	-3M-3	-4M-2	0	M	0	

$_1$ is leaving variable, $_2$ is entering variable, 1 is pivot element

↑

I Iteration:

			3	2	0	0	-M
			$_1$	$_2$			
2	$_2$	2	2	1	1	0	0
-M	$_1$	4	-5	0	-4	-1	1
	–	4-4M	1+5M	0	4M+2	M	0

∴ Since all – ≥0 and artificial variable $_1$ appears in the basis at the non – zero level. ∴ The given LPP does not possess any feasible solution. But the given LPP Possess a pseudo optimal solution.

3. Use penalty method to solve,

Max Z = 2 $_1$ + $_2$+ $_3$

Subject to the constraints,

$4_1 + 6_2 + 3_3 \le 8$
$3_1 - 6_2 - 4_3 \le 1$
$2_1 + 3_2 - 5_3 \ge 4$
and $_1, _2, _3 \ge 0$

Solution:

By introducing the slack variable $_1, _2$ and surplus variable $_3$. Artificial variable $_1$. The standard form of the LPP becomes,

Max Z = $2_1 + _2 + _3 + 0_1 + 0_2 + 0_3 - M_1$

Subject to the constraints,

$4_1 + 6_2 + 3_3 + _1 = 8$
$3_1 - 6_2 - 4_3 + _2 = 1$
$2_1 + 3_2 - 5_3 - _3 + _1 = 4$
and $_1, _2, _3, _1, _2, _3, _1 \ge 0$

∴ The IBFS is $_1 = 8$, $_2 = 1$, $_1 = 4$

Initial Iteration:

			2	1	1	0	0	0	-M	Ratio
			$_1$	$_2$	$_3$					
0	$_1$	8	4	6	3	1	0	0	0	$8/6 = 4/3$
0	$_2$	1	3	-6	-4	0	1	0	0	---
-M	$_1$	4	2	$\boxed{3}$	-5	0	0	-1	1	$4/3$
			-4M	-2M-2	-3M-1	5M-10	0	M	0	

$_1$ is leaving variable, $_2$ is entering variable, 3 is pivot element

I Iteration:

			2	1	1	0	0	0	Ratio
			$_1$	$_2$	$_3$	$_1$	$_2$	$_3$	
0	$_1$	0	0	0	$\boxed{13}$	1	0	2	0
0	$_2$	9	7	0	-14	0	1	-2	----
1	$_2$	$^4/_3$	$^2/_3$	1	$^{-5}/_3$	0	0	$^{-1}/_3$	----
	–	$^4/_3$	$^{-4}/_3$	0	$^{-8}/_3$	0	0	$^{-1}/_3$	

$_1$ is leaving variable, $_3$ is entering variable, 13 is pivot element

II iteration:

			2	1	1	0	0	0	Ratio
			$_1$	$_2$	$_3$	$_1$	$_2$	$_3$	
1	$_3$	0	0	0	1	$^1/_{13}$	0	$^2/_{13}$	----
0	$_2$	9	$\boxed{7}$	0	0	$^{14}/_{13}$	1	$^2/_{13}$	$^9/_7$
1	$_2$	$^4/_3$	$^2/_3$	1	0	$^5/_{39}$	0	$^{-1}/_{13}$	2
	–	$^4/_3$	$^{-4}/_3$	0	0	$^8/_{39}$	0	$^1/_{13}$	

$_2$ is leaving variable, $_1$ is entering variable, 7 is pivot element

III Iteration:

			2	1	1	0	0	0
			$_1$	$_2$	$_3$			
1	$_3$	0	0	0	0	$^1/_{13}$	0	$^2/_{13}$
2	$_1$	$^9/_7$	1	0	0	$^2/_{13}$	$^1/_7$	$^2/_{91}$
1	$_2$	$^{10}/_{21}$	0	1	0	$^1/_{39}$	$^2/_{21}$	$^{-17}/_{273}$
	–	$^{64}/_{21}$	0	0	0	$^{16}/_{39}$	$^8/_{21}$	$^{37}/_{273}$

∴ The optimal solution is Max Z = $^{64}/_{21}$, 1= $^{9}/_{7}$, 2 = $^{10}/_{21}$, 3 = 0

Duality in LPP:

1. For every LPP there is a unique LPP associated with it, involving the same data and closely related optimal solutions.
2. The original problem is then called ***Primal Problem***. While other is called it***s*** ***Dual Problem***.
3. But in General, the two problems are said to be Dual of each other.

Formulation of Dual Problems:

There are 2 important forms of primal – Dual pairs namely,

1. Symmetric Form
2. Unsymmetrical Form

I.Symmetric Form:

Consider the following LPP

Max Z = 1 1+ 2 2+. . ... + ≤ 1 ≤ 2

Subject to the constraints:

11 1 + 12 2 + ... + 1 21 1 + 22 2 + ... + 2

……………………………………………………….

……………………………………………………….

1 1 + 2 2 + ... + ≤

and the non-negativity restrictions is, 1, 2, ... , ≥ 0 This form of LPP is called the ***Symmetric Form of LPP.***

Procedure for constructing the Dual Problem:

Rule 1:

The Maximization problem in the primal becomes the minimization problem in the Dual and vice versa.

Rule 2:

The Maximization problem has less or equal to constrain while minimization problem has greater than or equal.

Rule 3:

If the primal contains m contains and n variables then the dual will contain n constraints and m variables. (i.e) Transpose of the Body Matrix of the primal problem gives the Body Matrix of the Dual and vice versa.

Rule 4:

The constants $_1, \ _2, \ldots,$ in the objective function of the primal appear in the constraints of the dual.

Rule 5:

The constants $_1, \ _2, \ldots,$ in the constraints of the primal appear in the objective function of the dual.

Rule 6:

The variables in both problems are non-negative.

Problems:

1. Write the dual of the primal LPP, Max Z = $_1 + 2\ _2 + \ _3$

Subject to the constraints,

$$2\ _1 + \ _2 - \ _3 \leq 2$$
$$-2\ _1 + \ _2 - 5\ _3 \geq -6$$
$$4\ _1 + \ _2 + \ _3 \leq 6$$

and $_1, \ _2, \ _3 \geq 0$

Solution:

The given primal problem is,

Max Z = $_1 + 2\ _2 + \ _3$

Subject to the constraints,

$2\ _1 + \quad _2 - \ _3 \leq 2$
$2\ _1 - \quad _2 + 5\ _3 \leq 6$
$4\ _1 + \quad _2 + \ _3 \leq 6$

and $_1, \ _2, \ _3 \geq 0$

since the given problem is Maximization type with less than or equal to constraints and also it has the three variables $_1$, $_2$, $_3$ and three constraints.

∴ The dual problem may be Minimization type with ≥ to constraints and also it will have the three variables $_1$, $_2$, $_3$and three constraints.
∴ The dual problem will become,

Min W = $2\ _1 + 6\ _2 + 6\ _3$

Subject to the constraints,

$2\ _1 + \quad _2 + 4\ _3 \geq 1$
$_1 - \ _2 + \ _3 \geq 2$
$-\ _1 + 5\ _2 + \ _3 \geq 1$

and $_1, \ _2, \ _3 \geq 0$

2. Min Z = $4\ _1 + 6\ _2 + 18\ _3$ Subject to the constraints,

$_1 + 3\ _2 \geq 3\ _2 + 2\ _3 \geq 5$

and $_1, \ _2, \ _3 \geq 0$

Solution:

The given primal problem is

$\text{Min } Z = 4\,_1 + 6\,_2 + 18\,_3$

Subject to the constraints,

$_1 + 3\,_2 + 0\,_3 \geq 3$
$_2 + 0\,_1 + 2\,_3 \geq 5$
and $_1, _2, _3 \geq 0$

since the given problem is minimization type with ≥ to constraints and has the three variables and 2 constraints.

∴ The Dual problem will be Maximization type ≤ constraints and then 2 variables and 3 constraints.
∴ The Dual problems will becomes,

$\text{Max } W = 3\,_1 + 5\,_2$

Subject to the constraints,

$_1 + 0\,_2 \leq 4$
$3\,_1 + _2 \leq 6$
$0\,_1 + 2\,_2 \leq 18$
and $_1, _2 \geq 0$

II. Unsymmetrical Form:

1. Write the dual of the given primal LPP,

$\text{Max } Z = 3\,_1 + 10\,_2 + 2\,_3$

Subject to the constraints,

$2\,_1 + 3\,_2 + 2\,_3 \leq 7$
$3\,_1 - 2\,_2 + 4\,_3 = 5$
and $_1, _2, _3 \geq 0$

Solution:

Since the primal problem is maximization type with ≤ constraints and equality constraints. It has 3 variables and 2 constraints.

∴ The Dual problem is maximization type with ≥ type inequality. It has 2 variables and 3 constraints. If the given primal is an equality constraints.
∴ In Dual problem, the corresponding dual variable $_2$ is unrestricted.
∴ The Dual problem will become,

Min W = $7\,_1 + 3\,_2$

Subject to the constraints,

$2\,_1 + 3\,_2 \geq 4$
$3\,_1 - 2\,_2 \geq 6$
$2\,_1 + 4\,_2 \geq 18$
and $_1 \geq 0$, $_2$is unrestricted.

2. Min Z = $_2 + 3\,_3$

Subject to the constraints,

$2\,_1 + \;_2 \leq 3 \Rightarrow -2\,_1 - \,_2 \geq -3$
$_1 + 2\,_2 + 6\,_3 \geq 5$
$-\,_1 + \,_2 + \,_3 = 2$
and $_1,\ _2 \geq 0$, $_3$ is unrestricted.

Solution:

Since the primal problem is minimization type with ≥ constraints and equality constraints. It has 3 variables and 3 constraints.

∴ The Dual problem is Maximization type with ≤ constraints. It will have 3 variables and 3 constraints.
∴ In the primal problem, the 3rd constraint is equality constraint.

∴ The corresponding dual variable ${}_3$is unrestricted and also. In the primal problem ${}_3$ is unrestricted.
∴ The corresponding dual 3rd constraint is an equality constraint.
∴ The dual problem will becomes,

$$\text{Max W} = -3\,{}_1 + 5\,{}_2 + 2\,{}_3$$

Subject to the constraints,

$$-2\,{}_1 + {}_2 - {}_3 \le 0$$
$$-\,{}_1 + 2\,{}_2 + {}_3 \le 1$$
$$0\,{}_1 + 6\,{}_2 + {}_3 = 3$$

and ${}_1, {}_2 \ge 0$, ${}_3$is unrestricted.

Use duality to solve the following LPP:

1. Min Z = $2\,{}_1 + 2\,{}_2$ Subject to the constraints,

$$2\,{}_1 + 4\,{}_2 \ge 1$$
$$-\,{}_1 - 2\,{}_2 \le -1 \Rightarrow {}_1 + 2\,{}_2 \ge 1$$
$$2\,{}_1 + {}_2 \ge 1$$

and ${}_1, {}_2, {}_3 \ge 0$

Solution:

The Dual problem of the given primal problem is

$$\text{Max W} = {}_1 + {}_2 + {}_3$$

Subject to the constraints,

$$2\,{}_1 + {}_2 + 2\,{}_3 \le 2$$

$4_1 + 2_2 + _3 \leq 2$

and $_1, _2, _3 \geq 0$

By introducing the slack variables $_1$ and $_2$ the standard form of the dual problem is,

Max W = $_1 + _2 + _3 + 0_1 + 0_2$

Subject to the constraints,

$2_1 + _2 + 2_3 + _1 + 0_2 = 2$

$4_1 + 2_2 + _3 + 0_1 + _2 = 2$

and $_1, _2, _3, _1, _2 \geq 0$

∴ The IBFS is given by $_1 = 2$, $_2 = 2$

Initial Iteration:

			1	1	1	0	0	Ratio
			1	2	3	1	2	
0	1	2	2	1	2	1	0	$^2/_2$=1
0	2	2	$\boxed{4}$	2	1	0	1	$^2/_4$= 0.5
	–	0	-1	-1	-1	0	0	

2 is leaving variable, 1 is entering variable, 4 is pivot element

I Iteration:

			1	1	1	0	0	Ratio
			1	2	3	1	2	
0	1	1	0	0	$\boxed{3/2}$	1	$-^1/_2$	$^2/_3$
1	1	$1/2$	1	$1/2$	$1/4$	0	$1/4$	4/=2 2
	–	$1/2$	0	$-1/2$	$-3/4$	0	$1/4$	

1 is leaving variable, 3 is entering variable, 3/2 is pivot element

II Iteration:

			1	1	1	0	0	Ratio
			1	2	3	1	2	
1	3	2/3	0	0	1	2/3	-1/3	----
1	1	1/3	1	$\boxed{1/2}$	0	-1/6	1/3	2/3
	-	1	0	-1/2	0	1/2	0	

$_1$ is leaving variable, $_2$ is entering variable, $^1/_2$ is pivot element

III Iteration:

			1	1	1	0	0
			1	2	3	1	2
1	3	2/3	0	0	1	2/3	-1/3
1	2	2/3	2	1	0	-1/3	2/3
	-	4/3	1	0	0	1/3	1/3

The Solution of Dual problem is,

Max W = $^4/_3$, $_1 = 0$, $_2 = {}^2/_3$, $_3 = {}^2/_3$

The Solution of Primal problem is,

Min Z = $^4/_3$, $_1 = {}^1/_3$, $_2 = {}^1/_3$

2. Apply the principal of Duality to solve the LPP:

Max Z = 3 $_1$ + 2 $_2$

Subject to the constraints,

$_1 +$ $_2 \geq 1$ $\Rightarrow$ $-_1 -$ $_2 \leq -1$

$_1 +$ $_2 \leq 7$

$_1 + 2\ _2 \leq 10$
$_2 \leq 3$

and $_1,\ _2 \geq 0$

Solution:

The Dual problem of the given primal problem is given by,

Min W = $-\ _1 + 7\ _2 + 10\ _3 + 3\ _4$

Subject to the constraints,

$-\ _1 + \ _2 + \ _3 \geq 3$
$-\ _1 + \ _2 + 2\ _3 + \ _4 \geq 2$
and $_1, _2, _3, _4 \geq 0$

By introducing the 2 surplus variables $_1$ and $_2$ and Artificial variables $_1$, $_2$. The standard form of the dual problem is,

Max $^* = -\ 7\ -10\ -3\ +0\ +0\ -M\ -M$

Subject to the constraints,

$-\ _1 + \ _2 + \ _3 - \ _1 + 0\ _2 + \ _1 = 3$

$-\ _1 + \ _2 + 2\ _3 + \ _4 + 0\ _1 - \ _2 + \ _2 = 2$

and $_1, _2, _3, _4, _1, _2, _1, _2 \geq 0$

The IBFS is given by, $_1 = 3,\ _2 = 2$

Initial Iteration:

			1	-7	-10	-3	0	0	-M	-M	Ratio
			$_1$	$_2$	$_3$	$_4$	$_1$	$_2$	$_1$	$_2$	
-M	$_1$	3	-1	1	1	0	-1	0	1	0	
← -M	$_2$	2	-1	1	$\boxed{2}$	1	0	-1	0	0	
		-3M-2M	2M-1	-2M+7	-3M+10 ↑	-M+3	M	M	0	0	

2 is leaving variable, 3 is entering variable,2 is pivot element

I Iteration:

			1	-7	-10	-3	0	0	-M	Ratio
-M		2			0		-1		1	4
-10		1		$\boxed{1}$	1		0		0	2
		-2M-10			0		M	+5	0	

3 is leaving variable, 2 is entering variable, $^1/_2$ is pivot element

II Iteration:

			1	-7	-10	-3	0	0	-M	Ratio
-M		1	0	0	-1	-1	-1	$\boxed{1}$	1	1
-7		2	-1	1	2	1	0	-1	0	---
		-M-14	6	0	M-4	M-4	M	-M+7	0	

1 is leaving variable, 2 is entering variable,1 is pivot element

III Iteration:

			1	-7	-10	-3	0	0
0		1	0	0	-1	-1	-1	1
-7		3	-1	1	1	0	-1	0
		-21	6	0	3	3	7	0

Max $^{*}= -21, \ _1= 0, \ _2= 3, \ _3= 0, \ _4= 0.$

Dual Solution: Max W = 21
Primal Solution: Min Z = 21, $_1= 7, \ _1= 0.$

3. Write down the dual of the following LPP and solve it.

Max Z $=4 \ _1 + 2 \ _2$

Subject to the constraints,

$- \ _1 - \ _2 \leq -3$

$- \ _1 + \ _2 \geq -2 \quad \Longrightarrow \quad _1 - \ _2 \leq 2$

and $_1, \ _2 \geq 0$

$\therefore$ The Dual problem of the given primal problem is,

Min W $= -3 \ _1 + 2 \ _2$

Subject to the constraints,

$- \ _1 + \ _2 \geq 4$, $- \ _1 - \ _2 \geq 2$, and $_1, \ _2 \geq 0$ To solve the Dual problem,

Consider Max $^{*}= 3 \ _1 - 2 \ _2$

Subject to the constraints,

$- \ _1 + \ _2 \geq 4$

$- \ _1 - \ _2 \geq 2$

and $_1, \ _2 \geq 0$

$\therefore$ By introducing the surplus variables $_1$ and $_2$ and Artificial variable $_1, \ _2$. The standard form is given by,

Max $^{*}= 3 \ _1 - 2 \ _2 + 0 \ _1 + 0 \ _2 - M \ _1 - M \ _2$

Subject to the constraints,

$- \ _1 + \ _2 + \ _1 + 0 \ _2 + \ _1 = 4$

$- _1 - _2 + 0 _1 - _2 + _2 = 2$

and $_1, _2, _1, _2, _1, _2 \geq 0$

$\therefore$ The IBFS is given by, $_1 = 4, _2 = 2$

Initial Iteration:

			3	-2	0	0	-M	-M	Ratio
			$_1$	$_2$			$_1$	$_2$	
-M	$_1$	4	-1	1	-1	0	1	0	4
-M	$_2$	2	-1	-1	0	-1	0	1	----
		-6M	2M-3	2	M	M	0	0	

Since all $^* - \geq 0$ and the artificial variable $_1, _2$ appears in the basis at non-zero level.
$\therefore$ The Dual problem does not possess any optimum Basic Feasible Solution.

4. Use duality to solve the LPP:

Min $Z = 50 _1 - 80 _2 - 140 _3$

Subject to the constraints,

$_1 - _2 - 3 _3 \geq 4$

$_1 - 2 _2 - 2 _3 \geq 3$

and $_1, _2, _3 \geq 0$

Solution:

The Dual Problem of the given primal is,

Max $W = 4 _1 + 3 _2$

Subject to the constraints,

$_1 + _2 \leq 50$

$- _1 - 2 _2 \leq -80 \Rightarrow _1 + 2 _2 \geq 80$

$-3 _1 - 2 _2 \leq -140 \Rightarrow 3 _1 + 2 _2 \geq 140$

and $_1, \ _2 \geq 0$

By introducing the 2 surplus variables $_2, _3$ and Artificial variable $_1, \ _2$.
The slack variable is $_1$. The Standard form of the given LPP is:

Max W = 4 $_1$ + 3 $_2$ + 0 $_1$ + 0 $_2$ + 0 $_3$ − $_1$ − $_2$

Subject to the constraints:

$_1 + \ _2 + \ _1 = 50$
$_1 + 2 \ _2 - \ _2 + \ _1 = 80$
$3 \ _1 + 2 \ _2 - \ _3 + \ _2 = 140$

and $_1, \ _2, \ _1, \ _2, \ _3, \ _1, \ _2 \geq 0$

∴ The IBFS is given by, $_1 = 50$, $_1 = 80$, $_2 = 140$

Initial Iteration:

			4	3	0	0	0	-M	-M	Ratio
			1	2	1	2	3	1	2	
0	1	50	1	1	1	0	0	0	0	$^{50}/_1 = 50$
-M	1	80	1	2	0	-1	0	1	0	80/1 = 80
-M	2	140	$\boxed{3}$	2	0	0	-1	0	1	140/3
		-220M	-4M-4	-4M-3	0	M	M	0	0	

$_2$ is leaving variable, $_1$ is entering variable, 3 is pivot element

I Iteration:

			4	3	0	0	0	-M	Ratio
			1	2	1	2	3	1	
0	1	$^{10}/_3$	0	$\boxed{1/3}$	1	0	$^1/_3$	0	10
-M	1	100/3	0	4/3	0	-1	2/3	1	25
4	1	140/3	1	2/3	0	0	-1/3	0	70
		−100/3 + 560/3	0	−4/3 − 1/3	0	M	− − 4/3	0	

1 is leaving variable, 2 is entering variable, 1/3 is pivot element

II Iteration:

			4	3	0	0	0	-M
3		10	0	1	3	0	1	0
-M		20	0	0	-4	-1	-1	1
4		40	1	0	-2	0	-1	0
		190-20M	0	0	4M+1	M	M-1	0

Since all * – ≥0 and the artificial variable 1 appears in the basis at non-zero level.

∴ The Dual Problem does not possess any optimum basic feasible solution.

TRANSPORTATION MODEL AND ASSIGNMENT PROBLEM

Transportation Model :

Introduction

Transportation deals with the transportation of a commodity (single product) from 'm' sources (origins or supply or capacity centers) to 'n' destinations (sinks or demand or requirement centers). It is assumed that

(i) Level of supply at each source and the amount of demand at each destination and
(ii) The unit transportation cost of transportation is linear.

It is also assumed that the cost of transportation is linear.

The objective is to determine the amount to be shifted from each sources to each destination such that the total transportation cost is minimum.

Note: The transportation model also can be modified to Account for multiple commodities.

1. **Mathematical Formulation of a Transportation problem:**
Let us assume that there are m sources and n destinations.

Let a_i be the supply (capacity) at source i, b_j be the demand at destination j, c_{ij} be the unit transportation cost from source *i* to destination j and x_{ij} be the number of units shifted from sources *i* to destination *j*.

Then the transportation problems can be expressed mathematically as

Minimize Z = $\sum_{=1} \sum_{=1}$

Subject to the constraints

$\sum_{=1}$ = , $i = 1, 2, 3, \ldots\ldots m$

$\sum_{=1}$ = , $j = 1, 2, 3, \ldots\ldots N.$

And ≥ 0, for all *i* and *j*.

Note 1: The two sets of constraints will be consistent if

$\sum_{=1} = \sum_{=1}$

(total supply) *(total demand)*

Which is the necessary and sufficient condition for a transportation problems to have a feasible solution. Problems satisfying this condition are **balanced transportation problems.**

Note 2: If $\sum$ i $\neq \sum$ j

Note 3: For any transportation problems, the coefficient of all x_{ij} in the constraints are unity.

Note 4: The objective function and the constraints being all linear, the transportation problems is a special class of linear programming problem. Therefore it can be solved by simplex method. But the number of variables being large, there will be too many calculations. So we can look for some other technique which would be simpler that the usual simplex method.

Standard transportation table:

Transportation problem is explicitly represented by the following transportation table.

Destination

	1	1	1	.....	1	.....	1	supply
Source								
								.
								.
								.
Demand				.....	.....	.....		

The *mn* squares are called **cells.** The unit transportation cost c_{ij} from the i^{th} source to the j^{th} destination is displayed in the **upper left side of the (i,j)th cell.** Any feasible solution us shown in the table by entering the value of x_{ij} **in the center of the** $(i,j)^{th}$ cell. The various a's and b's are called **rim requirements.** The feasibility of a solution can be verified by summing the values if x_{ij} along the rows and down the columns.

Definition 1: A set of non-negative values x_{ij}, i=1,2……………m; j=1,2………..n. that satisfies the constrains (rim conditions and also the non-negativity restrictions) is called a **feasible solution** to the transportation problems.

Note: A balanced transportation problems will always have a feasible solution.

Definition 2: A feasible solution to a (m x n) transportation problems that contains no more than m + n-1 non-negative allocations is called a **basic feasible solution** (BFS) to the transportation problem.

Definition 3: A basic feasible solution to a (m x n) transportation problem is said to be a **non-degenerate basic feasible solution** if it contains exactly m + n-1 non-negative allocations in independent positions.

Definition 4: A basic feasible solution that contains less than m + n -1 non-negative allocations is said to be a degenerate basic feasible solution.

Definition 5: A feasible solution (not necessarily basic) is said to be an **optimal solution** if it minimize is atmost m + n -1.

Note: The number of non-basic variables in an m x n balanced transportation problem is almost m + n -1.

Note: The number of non-basic variables in an m x n balanced transportation problem is atleast mn – (m + n -1).

II. Methods for finding initial basic feasible solution

The transportation problems has a solution is and only if the problem is balanced. Therefore before starting to find the initial basic feasible solution, check whether the given transportation problem is balanced. If not once has to balance the transportation problems first. The way to doing this is discussed in section 7.4 page 7.40. In this section all the given transportation problems are balanced.

Method I: North west corner rule:

Step I: The first assignment is made in the cell occupying the upper left-hand (north-west) corner of the transportation table. The maximum possible amount is allocated there. That is x_{11} = min {a_1, b1}.

Case (i): If min {a_1, b1} =a_1, then put x_{11}=a_1, decrease b_1 by a_1 and move vertically to the 2nd row (i.e.,) to the cell (2, 1) cross out the first row.

Case (ii): If min {a_1, b1} =b_1, then put x_{11}=b_1, decrease a_1 by b_1 and move horizontally right (i.e.,) to the cell (2, 1) cross out the first column.

Case (iii): If min {a_1, b1} = a_1 =b_1, the put x_{11} = a_1=b_1 and move diagonally to the cell (2, 2) cross out the first row and the first column.

Step 2: Repeat the procedure until all the rim requirements are satisfied.

Method 2: Lest cost method (or) Matrix minima method (or) Lowest cost entry method:

Step 1: Identify the cell with smallest cost and allocate x_{ij} = Min {a_i,b_j}

Case (i): If min {a_i,b_j}= a_i, then put x_{ij} = a, cross out the ith row and decrease b_j by a_i, go to step(2).

Case (ii): If min {a_i,b_j}= b_j, then put x_{ij} = b_j, cross out the jth column and decrease a_j by b_j, go to step(2).

Case (iii): If min {a_i,b_j}=a_i= b_j, then put x_{ij}=a_i = b_j, cross out either ith row and jth column but not both, go to step(2).

Step 2: Repeat step (1) for the resulting reduced transportation table until all the rim requirements are satisfied.

Method 3: Vogel's approximation method (VAM) (or) Unit cost penalty method:

Step 1: Find the difference (penalty) between the smallest and next smallest costs in each row (column) and write them in brackets against the corresponding row (column).

Step 2: Identify the row (or) column with large penalty. If a tie occurs, break the tie arbitrarily. Choose the sell with smallest cost in that selected row or column and allocate as much as possible to this cell and cross out the satisfied row or column and go to step (3).

Step 3: Again compute the column and row penalties for the reduced transportation table and then go to step (2). Repeat the procedure until all the rim requirements are satisfied.

Example 1: Determine basic feasible solution to the following transportation problems using North West Corner Rule:

Sin k

	A	B	C	D	E	Supply
OriginP	**2**	**11**	**10**	**3**	**7**	**4**
Q	**1**	**4**	**7**	**2**	**1**	**8**
R	**3**	**9**	**4**	**8**	**12**	**9**
Demand	**3**	**3**	**4**	**5**	**6**	

[MU. BE. Apr 94]

Solution:

Since = = 21, the given problem is balanced. ∴ There exists a feasible solution to the transportation problem.

2 **3**	11	10	3	7	4
1	4	7	2	1	8
3	9	4	8	12	9
3	3	4	5	6	

Following North West Corner rule, the first allocation is made in the cell(1,1)
Here $_{11}$ = min { $_1$, $_1$} = min {4,3} = 3

Allocate 3 to the cell(1,1) and decrease 4 by 3 i.e., 4 – 3 = 1

As the first column is satisfied, we cross out the first column and the resulting reduced Transportation table is

11 **1**	**10**	3	7	1
4	7	2	1	8
9	4	8	12	9
3	4	5	6	

Here the North West Corner cell is (1,2).

So allocate $_{11}$= min {1, 3} = 1 to the cell (1,2) and move vertically to cell (2, 2). The resulting transportation table is

4 **2**	7	2	1	8
9	4	8	12	9
2	4	5	6	

Allocate $_{22}$= min {8, 2} = 2 to the cell (2, 2) and move horizontally to cell (2, 3). The resulting transportation table is

7 **4**	2	1	6
4	8	12	9
4	5	6	

Allocate $_{23}$= min {6, 4} = 4 and move horizontally to cell (2, 4). The resulting reduced transportation table is

2 **2**	1	2
8	12	9
5	6	

Allocate $_{24}$= min {2, 5} = 2 and move vertically to cell (3, 4). The resulting reduced transportation table is

8 **3**	12	9
3	6	

Allocate 34= min {9, 3} = 3 and move horizontally to cell (3, 5).which is

12 **6**	6
6	

Allocate $_{35}$= min {6, 6} = 6

Finally the initial basic feasible solution is as shown in the following table.

2 **3**	11 **1**	10	3	7
1	4 **2**	7 **4**	2 **2**	1
3	9	4	8 **3**	12 **6**

From this table we see that the number of positive independent allocations is equal to

$m + n - 1 = 3 + 5 - 1 = 7$. This ensures that the solution is non degenerate basic feasible.

∴The initial transportation cost = Rs. 2 x 3 + 11 x 1 + 4 x 2 + 7 x 4 + 2 x 2 + 8 x 3 + 12 x 6

= Rs. 153/-

Example 2:

Find the initial basic feasible solution for the following transportation problem by Least Cost Method.

	To				Supply
	1	2	1	4	30
From	3	3	2	1	50
	4	2	5	9	20
Demand	20	40	30	10	

Solution:

Since ∑ = ∑ = 100, the given TPP is balanced. There exists a feasible solution to the transportation problem.

1 **20**	2	1	4	30
3	3	2	1	50
4	2	5	9	20
20	40	30	10	

By least cost method, min = 11 = 13 = 24 = 1

Since more than one cell having the same minimum , break the tie.

Let us choose the cell (1,1) and allocate 11= min { 1, 1} = min {30,20} = 20 and cross out the satisfied column and decrease 30 by 20.

The resulting reduced transportation table is

2	1 **10**	4	10
3	2	1	50
2	5	9	20
40	30	10	

Here min = $_{13}$ = $_{24}$ = 1.Choose the cell (1,3) and allocate $_{13}$= min { $_1$, $_3$} = min {10,30} = 10 and cross out the satisfied row.

The resulting reduced transportation table is

3	2	1 **10**	50
2	5	9	20
40	20	10	

Here min = $_{24}$ = 1

∴ Allocate $_{24}$= min { $_2$, $_4$} = min (50,10) = 10 and cross out the satisfied column. The resulting transportation is

3	**2** **20**	40
2	**5**	20
40	20	

Here = $_{23}$ = $_{32}$ = 2. Choose the cell (2,3) and allocate $_{23}$= min { $_2$, $_3$}= min (40,20) = 10 and cross out the satisfied column.

The resulting reduced transportation table is

3 20	20
2 20	20
40	

Here min = = $_{32}$ = 2. Choose the cell (3,2) and allocate $_{32}$= min { $_3$, $_2$}= min (20,40) = 20 and cross out the satisfied row.

The resulting reduced transportation table is

3 **20**	20
20	

Finally the initial basic feasible solution is as shown in the following table.

1 **20**	2	1 **10**	4
3	3 **20**	2 **20**	1 **10**
4	2 **20**	5	9

From this table we see that the number of positive independent allocations is equal to

$m + n - 1 = 3 + 4 - 1 = 6$. This ensures that the solution is non degenerate basic feasible.

∴ The initial transportation = Rs. 1 x 20 + 1 x 10 + 3 x 20 + 2 x 20

Cost +1 x 10 + 2 x 20

= Rs. 20 + 10 + 60 + 40 + 10 + 40

= Rs. 180/-

Example 3:

Find the initial basic feasible solution for the following transportation problem by VAM. Distribution centres

						Availability
	1	11	13	17	14	250
Origin	2	16	18	14	10	300
	3	21	24	13	10	400
Requirements		200	225	275	250	

Solution:

Since ∑ = ∑ = 100, the given is balanced. ∴ There exists a feasible solution to this problem.

11 **200**	13	17	14	250	(2)
16	18	14	10	300	(4)
21	24	13	10	400	(3)
200	225	275	250		
(5)	(5)	(1)	(0)		

First let us find the difference (penalty) between the smallest and next smallest costs in each row and column and write them in brackets against the respective rows and columns.

The largest of these differences is (5) and is associated with the first two columns of the transportation table. We choose the first column arbitrarily.
In this selected column, the cell (1,1) has the minimum unit transportation cost $_{11}$ = 11.

∴ Allocate $_{11}$= min (250,200) = 200 to this cell (1,1) and decrease 250 by 200 and cross out the satisfied column.

The resulting reduced transportation table is

13 50	17	14	50	(1)
18	14	10	300	(4)
24	13	10	400	(3)
225	275	250		
(5)	(1)	(0)		

The row and column differences are now computed for this reduced transportation table. The largest of these is (5) which is associated with the second column. Since $_{12}$ = 13 is the minimum cost, we allocate $_{12}$= min (50,225) = 50 to the cell (1,2) and decrease 225 by 50 and cross out the satisfied row.

Continuing in this manner, the subsequent reduced transportation tables and the differences for the surviving rows and columns are shown below:

18 175	14	10	300	(4)
24	13	10	400	(3)
175	275	250		
(6)	(1)	(0)		

(i)

14	10 125	125	**(4)**
13	10	400	(3)
	250		
(1)	(0)		

(ii)

14	10 125	400
275	125	

(iii)

13 275	275
275	

(iv)

Finally the initial basic feasible solution is as shown in the following table.

11 **200**	13 **50**	17	14
16	18 **175**	14	10 **125**
21	24	13 **275**	10 **125**

From this table we see that the number of positive independent allocation is equal to

$m + n - 1 = 3 + 4 - 1 = 6$. This ensures that the solution is non degenerate basic feasible.

∴ The initial transportation Cost = Rs. 11 x 200 + 13 x 50 + 18 x 175 ++ 10 x 125 + 13 x 275 + 10 x 125

= Rs. 12075/-

Example 4:

Find the starting solution of the following transportation model

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

Using ***(i). North West Corner rule***

(ii). Least Cost method

(iii). Vogel's approximation method.

Solution:

Since ∑ = ∑ = 100, the given Transportation problem is balanced. ∴ There exists a basic feasible solution to this problem.

(i). North West Corner rule: Using this method, the allocation are shown in the tables below:

1 **7**	**2**	**6**	7
0	4	2	12
3	1	5	11
10	10	10	

(i)

0 **3**	4	2	12
3	1	5	11
3	10	10	

(ii)

4 **8**	**2**	9
1	5	11
10	10	

(iii)

1 **1**	5	11
1	10	

(iv)

5 10	10
10	

(v)

The starting solution is as shown in the following table

1 **7**	2	6
0 **3**	4 **9**	2
3	1 **1**	5 **10**

∴ The initial transportation cost = Rs. 1 x 7 + 0 x 3 + 4 x 9 + 1 x 1 + 5 x 10

= Rs. 94/-

(ii). **Least Cost method:** Using this method, the allocation are as shown in the table below:

1	2	6 7	7
0 10	4	2	12
3	1	5	11
10	10	10	

(i)

2	6	7
4	2	2
1 10	5	11
10	10	

(ii)

6	7
2 2	2
5	1
10	

(iii)

6	7
5 1	1
8	

(iv)

6 7	7
7	

(v)

The starting solution is as shown in the following table:

1	2	6 **7**
0 **10**	4	2 **2**
3	1 **10**	5 **1**

∴ The initial transportation cost = Rs. 6 x 7 + 0 x 10 + 2 x 2 + 1 x 10 + 5 x 1

= Rs. 61/-

(iii). Vogel's approximation Method: Using this method, the allocations are shown in the table below:

1	2	6	7 (1)
0	4	2 10	12 (2)
3	1	5	11 (2)
10 (1)	10 (1)	10 **(3)**	

(i)

1	2	7 (1)
0 2	4	2 **(4)**
3	1	11 (2)
10 (1)	10 (1)	

(ii)

1	2	7 (1)
3	1 10	11 **(2)**
8 (2)	10 (1)	

(iii)

1 **7**	7
3	1
8	

(iv)

3 **1**	1
1	

(v)

The starting solution is as shown in the following table:

1 **7**	2	6
0 **2**	4	2 **10**
3 **1**	1 **10**	5

∴ The initial transportation cost = Rs.1 x 7 + 0 x 2 + 2 x 10 + 3 x 1 + 1 x 10

= Rs. 40/-

Note: For the above problem, the number of positive allocation in independent positions is (m + n -1) (i.e., m + n – 1 = 3 + 3 – 1 = 5). This ensures that the given problem has a non-degenerate basic feasible solution by using all the three methods. This need not be the case in all the problems.

Transportation Algorithm (or) MODI Method (modified distribution method) (Test for optimal solution).

Step 1: Find the initial basic feasible solution of the given problems by Northwest Corner rule (or) Least Cost method or VAM.

Step 2: Check the number of occupied cells. If these are less than m + n – 1, there exists degeneracy and we introduce a very small positive assignment of ∈ (≈ 0) in suitable independent positions, so that the number of occupied cells is exactly equal to m + n – 1.

Step 3: Find the set of values u_i, v_j (i= 1,2,3,….m; j=1,2,3…n) from the relation $c_{ij} = u_i + v_j$ for each occupied cell (i,j), by starting initially with $u_i = 0$ or $v_j = 0$ preferably for which the corresponding row or column has maximum number of individual allocations.

Step 4: Find $u_i + v_j$ for each unoccupied cell (i,j) and enter at the upper right corner of the corresponding cell (i,j).

Step 5: Find the cell evaluations $d_{ij} = c_{ij} – (u_i + v_j)$ (d_{ij} = upper left – upper right) for each unoccupied cell (i,j) and enter at the lower right corner of the corresponding cell (i,j).

Step 6: Examine the cell evolutions d_{ij} for all unoccupied cells (i,j) and conclude that

(i) If all $d_{ij}>0$, then the solution under the test is optimal and unique.

(ii) If all $d_{ij}>0$, with atleast one $d_{ij}=0$, then the solution under the test is optimal and an alternative optimal solution exists.

(iii) If atleast one $d_{ij}<0$, then the solution is not optimal. Go to the next step.

Step 7: Form a new B>F>S by giving maximum allocation to the cell for which $d_{ij}$is most negative by making an occupied cell empty. For that draw a closed path consisting of horizontal and vertical lines beginning and ending at the cell for which d_{ij} is most negative and having its **other corners at some allocated cells.** Along this closed loop indicate + and – alternatively at the corners. Choose minimum of the allocations from the cells having – . Add this minimum allocation to the cells with + and subtract this minimum allocation from the allocation to the cells with – .

Step 8: Repeat steps (2) to (6) to test the optimality of this new basic feasible solution.

Step 9: Continue the above procedure till an optimum solution is attained.

Note: The Vogels approximation method (VAM) takes into account not only the least cost c_{ij} but also the costs that just exceed the least cost c_{ij} and therefore yields better initial solution than obtained from other methods in general. This can be justified by the above example (4). So to find the initial solution, give preference to VAM unless otherwise specified.

Example 1: Solve the transportation problem:

	1	**2**	**3**	**4**	**Supply**
I	**21**	**16**	**25**	**13**	**11**
II	**17**	**18**	**14**	**23**	**13**
III	**32**	**27**	**18**	**41**	**19**
Demand	**6**	**10**	**12**	**15**	

Solution: Since $\sum = \sum = 43$, the given transportation problem is balanced. ∴ There exists a basic feasible solution to this problem.

By Vogel's approximation method, the initial solution is an shown in the following table.

21	16	25	13 **11**	(3) - - -
17 **6**	18 **3**	14	23 **4**	(3) (3) (3) (3)
32	27 **7**	18 **12**	41	(9) (9) (9) (9)
(4) (15) **(15)**	(2) (9) (9) **(9)**	(4) (4) (4) (4)	**(10)** **(18)**	

That is

21	16	25	13 **11**
17 **6**	18 **3**	14	23 **4**
32	27 **7**	18 **12**	41

From this table, we see that the number of non-negative independent allocations is (m + n – 1) = (3+4-1) =6. Hence the solution is non-degenerate basic feasible.

∴ The initial transportation cost.

= Rs. 13 x 11 + 17 x 6 +18 x 3 +23 x 4 + 27 x 7 + 18 x 12

= Rs. 796/-

To find the optimal solution

Consider the above transportation table. Since m+n-1=6, we apply MODI method,

Now we determine a set of values u_i and v_j for each occupied cell (i,j) by using the relation $c_{ij} = u_i + v_j$. As the 2nd row contains maximum number of allocations, we choose $u_2=0$. Therefore

$C_{21} = u_2 + v_1 \Rightarrow 17 = 0 + v_1 \Rightarrow v_1 = 17$

$C_{22} = u_2 + v_2 \Rightarrow 18 = 0 + v_2 \Rightarrow v_2 = 18$

$C_{24} = u_2 + v_4 \Rightarrow 23 = 0 + v_4 \Rightarrow v_4 = 23$

$C_{14} = u_1 + v_4 \Rightarrow 13 = u_1 + 23 \Rightarrow u_1 = -10$

$C_{32} = u_3 + v_2 \Rightarrow 27 = u_3 + 18 \Rightarrow u_3 = 9$

$C_{33} = u_3 + v_3 \Rightarrow 18 = 9 + v_3 \Rightarrow v_3 = 9$

Thus we have the following transportation table:

21	16	25	13 **11**	$u_1 = -10$
17 **6**	18 **3**	14	23 **4**	$u_2 = 0$
32	27 **7**	18 **12**	41	$u_3 = 9$
$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$	

Now we find $u_i + v_j$ for each unoccupied cell (i,j) and enter at the upper right corner of the corresponding unoccupied cell(i,j).

Then we find the cell evalutions $d_{ij} = c_{ij} - (u_i + v_j)$ (ie., upper left corner – upper right corner) for each unoccupied cell (i,j) and enter at the lower right corner of the corresponding unoccupied cell (i,j).

21 7 14	16 8 8	25 -1 26	13 **11**	$u_1 = -10$
17 **6**	18 **3**	14 9 5	23 **4**	$u_2 = 0$
32 26 6	27 **7**	18 **12**	41 32 9	$u_3 = 9$
$v_1 = 17$	$v_2 = 18$	$v_3 = 9$	$v_4 = 23$	

Since all $d_{ij}>0$, with $d_{32} = 0$, the current solution is optimal and unique.

∴ The optimum allocation schedule is given by $x_{14} = 11$, $x_{21} = 6$, $x_{22} = 3$, $x_{24} = 4$, $x_{32} = 7$, $x_{33} = 12$, and the optimum (minimum) transportation cost

= Rs. 13x 11 + 17 x 6 + 18 x 3 +23 x 4 + 27 x 7 + 18 x 12 = Rs. 796/-

Example 2:

Obtain on optimum has feasible solution to the following transportation problem:

	To			Available
	7	3	2	2
From	2	1	3	3
	3	4	6	5
Demand	4	1	5	10

Solution:

Since ∑ = ∑ =43, the given transportation problem is balanced. ∴ There exists a basic feasible solution to this problem.

By Vogel's approximation method, the initial solution is as shown in the following table:

7	3	2 **2**	**(1)** **(5)**
2	1 **1**	3 **2**	(1) (1) (1)
3 **4**	4	6 **1**	(1) (3) **(3)**
(1)	**(2)**	(1)	
(1)		(1)	
(1)		(3)	

That is

7	3	2 **2**
2	1 **1**	3 **2**
3 **4**	4	6 **1**

From this table we see that the number of non-negative allocation is $m + n - 1 = (3 + 3 - 1) = 5$.

Hence the solution is non-degenerate basic feasible

∴ The initial transportation cost = Rs. 2 x 2 + 1 x 1 + 3 x 2 + 3 x 4 + 6 x 1

= Rs. 29/-

For optimality: since the number of non – negative independent allocation is m + n -1, we apply MODI method.

Since the third column contains maximum number of allocations, we choose $v_3 = 0$.

Now we determine a set of values u_i and v_j by using the occupied cells and the relation $c_{ij} = u_i + v_j$.

That is

7 -1	3 0	2 **2**	$u_1 = 2$
2	1 **1**	3 **2**	$u_2 = 3$
3 **4**	4	6 **1**	$u_3 = 6$
$v_1 = -3$	$v_2 = -2$	$v_3 = 0$	

Now we find $u_i + v_j$ for each unoccupied cell (i, j) and enter at the corresponding unoccupied cell (I,j).

Then we find the cell evaluations $d_{ij} = c_{ij} - (u_i + v_j)$ for each unoccupied cell (i, j) and enter at the lower right corner of the corresponding unoccupied cell (i, j).

Thus we get the following table

7 -1 8	3 0 3	2 **2**	$u_1 = 2$
2 0 2	1 **1**	3 **2**	$u_2 = 3$
3 **4**	4 4 0	6 **1**	$u_3 = 6$
$v_1 = -3$	$v_2 = -2$	$v_3 = 0$	

Since all $d_{ij}>0$, with $d_{32} = 0$, the current solution is optimal and there exists an alternative optimal solution.

∴ The optimum allocation schedule is given by $x_{13} = 2$, $x_{32} = 1$, $x_{23} =2$, $x_{31} =4$, $x_{33} =1$, and the optimum (minimum) transportation cost

= Rs.2x2+1x1+3x2+3x4+6x1 = Rs.29/-

Example 3: Find the optimal transportation cost of the following matrix using least cost method for finding the critical solution.

		A	B	C	D	E	Available
	P	4	1	2	6	9	100
Factory	Q	6	4	3	5	7	120
	R	5	2	6	4	8	120
Demand		40	50	70	90	90	

Solution:

Since ∑ = ∑ = 340, the given transportation problem is balanced.∴ There exists a basic feasible solution to this problem.

By using Least cost method, the initial solution is an shown in the following table:

4	1 **50**	2 **50**	6	9
6 **10**	4	3 **20**	5	7 **90**
5 **30**	2	6	4 **90**	8

∴ The initial transportation cost = Rs. 1 x 50 + 2 x 50 + 6 x 10 + 3 x 20 + 7 x 90 + 5 x 30 + 4 x 90

= Rs. 1410/-

For optimality: Since the number of non – negative independent allocations is (m + n -1), we apply MODI method:

4 5 -1	1 **50**	2 **50**	6 4 2	9 6 3	$u_1 = -1$
6 **10**	4 2 2	3 **20**	5 5 0	7 **90**	$u_2 = 0$
5 **30**	2 1 1	6 2 4	4 **90**	8 6 2	$u_3 = -1$
$v_1 = 6$	$v_2 = 2$	$v_3 = 3$	$v_4 = 5$	$v_5 = 7$	

Since $d_{11} = -1 < 0$, the current solution is not optimal.

Now let us form a new basic feasible solution by giving maximum allocation to the cell (i,j) for which d_{ij} is most negative by making an occupied cell empty. Here the cell (1,1) having the negative value d_{11}=-1. We draw a closed loop consisting of horizontal and vertical lines beginning and ending at this cell (1,1) and having its other corners at some occupied cells. Along this closed loop indicate + and + alternatively at the corners. We have

4 +	1 **50**	2 **50** -	6	9
6 **10** -	4	3 **20** +	5	7 **90**
5 **30**	2	6	4 **90**	8

From the two cells (1,3), (2,1) having + , we find that the minimum of the allocations 50,10 is 10. Add this cells with + and subtract this 10 to the cells with+ .

Hence the new basic feasible solution is displayed in the following table:

4 **10**	1 **50**	2 **40**	6	9
6	4	3 **30**	5	7 **90**
5 **30**	2	6	4 **90**	8

We see that the above table satisfies the rim conditions with (m + n – 1) non-negative allocations at independent position. So we apply MODI method.

4 **10**	1 **50**	2 **40**	6 3 3	9 6 3	$u_1 = 0$
6 5 1	4 2 2	3 **30**	5 4 1	7 **90**	$u_2 = 1$
5 **30**	2 2 0	6 3 3	4 **90**	8 7 1	$u_3 = 1$
$v_1 = 4$	$v_2 = 1$	$v_3 = 2$	$v_4 = 3$	$v_5 = 6$	

Since all $d_{ij}>0$, with $d_{32}=0$, the current solution is optimal and there exists an alternative optimal solution.

The optimum allocation schedule is given by $x_{11}=10$, $x_{12}=50$, $x_{13}=40$, $x_{23}=30$, $x_{25}=90$, $x_{31}=30$, $x_{34}=90$ and the optimum (minimum) transportation cost.

= Rs. 4 x 10 + 1 x 50 + 2 x 40 + 3 x 30 + 7 x 90 + 5 x 30 + 4 x 90.

= Rs. 1400/-

Degeneracy in Transportation Problems

In transportation problems, whenever the number of non-negative independent allocations in less than m + n – 1, the transportation problems is said to be **degenerate** one. Degeneracy may occur either at the initial stage or at an intermediate stage at some subsequent iteration.

To resolve degeneracy, we allocate an extremely small amount (close to zero)to one or more empty cells of the transportation table(generally minimum cost cells if possible), so that the total number of occupied cells becomes (m + n – 1) at independent positions. WE denote this small amount by ∈ (epsilon) satisfying the following conditions:

(i) $0 < \in < x_{ij}$, for all $x_{ij} > 0$
(ii) $X_{ij} \pm \in = x_{ij}$, for all $x_{ij} > 0$

The cells containing ∈ are then treated like other occupied cells and the problems is solved in the usual way. The∈'s are kept till the optimum solution is attained. Then we let each $\in \to 0$.

Example 1: find the non-degenerate basic feasible solution for the following transportation problems using

(i) North west corner rule
(ii) Least cost method
(iii) Vogel's approximation method.

	To				supply
	10	**20**	**5**	**7**	**10**
	13	**9**	**12**	**8**	**20**
From	**4**	**5**	**7**	**9**	**30**
	14	**7**	**1**	**0**	**40**
	3	**12**	**5**	**19**	**50**
Demand	**60**	**60**	**20**	**10**	

Solution: Since ∑ = ∑ = 150, the given transportation problems is balanced.

∴ There exists a basic feasible solution to this problem.

The starting solution by NWC rule is an shown in the following table.

10 **10**	20	5	7
13 **20**	9	12	8
4 **30**	5	7	9
14	7 **40**	1	0
3	12 **20**	5 **20**	19 **10**

Since the number of non-negative allocations at independent positions is 7 which is less than (m + n – 1) = (5 + 4 – 1) = 8, this basic feasible solution is a degenerate one.

To resolve this degeneracy, we allocate a very small quantity ∈ to the unoccupied cell (5,1) so that the number of occupied cells becomes (m+n-1) (m + n – 1). Hence the non-degenerate basic feasible solution is an shown in the following table.

10 **10**	20	5	7
13 **20**	9	12	8
4 **30**	5	7	9
14	7 **40**	1	0
3 ∈	12 **20**	5 **20**	19 **10**

The initial transportation cost = Rs. 10x10+13x20+4x30+7x40+3x∈+20x20+5x20+19x10
= Rs.(1290 = 3∈)
= Rs. 1290/- as ∈→ 0.

(ii) Least cost method: Using this method the starting solution is an shown in the following table:

10	20 **10**	5	7
13	9 **20**	12	8
4 **10**	5 **20**	7	9
14	7 **10**	1 **20**	0 **10**
3 **50**	12	5	19

Since the number of non-negative allocations at independent positions is (m + n – 1) =8, the solution is non-degenerate basic feasible.

The initial transportation cost = Rs.20x10+9x20+4x10+5x20+7x10+1x20+0x10+3x50

= Rs. 760/-

(iii) Vogel's approximation method: The starting solution by this method is an shown in the following table:

10 **10**	20	5	7
13	9 **20**	12	8
4	5 **30**	7	9
14	7 **10**	1 **20**	0 **10**
3 **50**	12	5	19

Since the number of non-negative allocations is 7 which is less than (m + n – 1) = (5+4-1)=8., this basic solution is a degenerate one.

To resolve this degeneracy, we allocate a very small quantity ∈ to the unoccupied cell(5,2) so that the number of occupied cells becomes (m + n – 1). Hence the non-degenerate basic feasible solution is an shown in the following table.

10 **10**	20	5	7
13	9 **20**	12	8
4	5 **30**	7	9
14	7 **10**	1 **20**	0 **10**
3 **50**	12 ∈	5	19

∴ The initial transportation cost

= Rs. 10 x 10 + 9 x 20 + 5 x 30 + 7 x 10 + 1 x 20 + 0 x 10 + 3 x 50 + 12 x ∈
= Rs. (670 +12∈)
= Rs.670/- = as ∈→0.

Example 2: Solve the following transportation problems using vogel's method.

		A	B	C	D	E	F	Available
	1	9	12	9	6	9	10	5
Factory	2	7	3	7	7	5	5	6
	3	6	5	9	11	3	11	2
	4	6	8	11	2	2	10	9
Requirement		4	4	6	2	4	2	

Solution: Since ∑ = ∑ = 22, the given transportation problem is balanced. ∴ There exists a basic feasible solution to this problem. By Vogel's approximation method, the initial solution is as shown in the following table:

9	12	9 **5**	6	9	10
7	3 **4**	7	7	5	5 **2**
6 **1**	5 ∈	9 **1**	11	3	11
6 **3**	8	11	2 **2**	2 **4**	10

Since the number of non-negative allocations is 8 which is less than (m + n – 1)=(4+6-1)=9, this basic solution is degenerate one.

To resolve degeneracy, we allocate a very small quantity ∈ to the cell (3,2), so that the number of occupied cells becomes (m + n – 1). Hence the non-degenerate basic feasible solution is as shown in the following table.

9	12	9 **5**	6	9	10
7	3 **4**	7	7	5	5 **2**
6 **1**	5 ∈	9 **1**	11	3	11
6 **3**	8	11	2 **2**	2 **4**	10

The initial transportation cost = Rs.9 x 5 + 3 x 4 + 5 x 2 + 6 x 1 + 5 x ∈ + 9 x 1

+ 6 x 3 + 2 x 2 + 2 x 4

=Rs.(112+5∈) = Rs.112/-, ∈→0.

To find the optimal solution

Now the number of non-negative allocations at independent positions is (m + n – 1). We apply the MODI method.

9 6 3	12 5 7	9 **5**	6 2 4	9 2 7	10 7 3	$u_1 = 0$
7 4 3	3 **4**	7 7 0	7 0 7	5 0 5	5 **2**	$u_2 = -2$
6 **1**	5 ∈	9 **1**	11 2 9	3 2 1	11 7 4	$u_3 = 0$
6 **3**	8 5 3	11 9 2	2 **2**	2 **4**	10 7 3	$u_4 = 0$
$v_1 = 6$	$v_2 = 5$	$v_3 = 9$	$v_4 = 2$	$v_5 = 2$	$v_6 = 7$	

Since all $d_{ij}>0$ with $d_{23}=0$, the solution under the test is optimal and an alternative optimal solution is also exists.

∴ The optimum allocation schedule is given by $x_{14}=5$, $x_{22}=4, x_{26}=2, x_{31}=1, x_{32}=\in, x_{33}=1, x_{41}=3, x_{44}=2, x_{45}=4$ and the optimum(minimum) transportation cost is

$=Rs. 9x5+3x4+5x2+6x1+5x\in+9x1+6x3+2x2+2x4$

$=Rs. (112+5\in)$

$=Rs. 112$ as $\in \rightarrow 0$.

Example 3: Solve the following transportation problem to minimize the total cost of transportation.

			To		Supply
	1	**2**	**3**	**4**	**6**
From	**4**	**3**	**2**	**0**	**8**
	0	**2**	**2**	**1**	**10**
Demand	**4**	**6**	**8**	**6**	

Solution: Since $\sum = \sum = 24$, the given transportation problem is balanced. ∴ There exists a basic feasible solution to this problem.

By using Vogel's approximation method, the initial solution is as shown in the following table:

1	2 **6**	3	4
4	3	2 **2**	0 **6**
0 **4**	2	2 **6**	1
	4		

Since the number of non-negative allocations is 5, which is less than $(m + n - 1)=(3+4-1)=6$, this basic feasible solution is degenerate.

To resolve degeneracy, we allocate a very small quantity ∈ to the cell (1,4), so that the number of occupied cells becomes (m + n – 1). Hence the non-degenerate basic feasible solution is given in the following table

1	2 **6**	3	4
4	3	2 **2**	0 **6**
0 **4**	2 ∈	2 **6**	1

∴ The initial transportation cost = Rs. 2 x 6 + 2 x 2 + 0 x 6 + 0 x 4 + 2 x ∈ + 2 x 6

= Rs. (28 + 2∈)

= Rs. 28/-, as ∈ ⟶ 0.

To find the optimum solution:

Now the number of non-negative allocations at independent positions is (m + n – 1). We apply MODI method.

1 0 1	2 **6**	3 2 1	4 0 4	$u_1 = 0$
4 0 4	3 2 1	2 **2**	0 **6**	$u_2 = 0$
0 **4**	2 ∈	2 **6**	1 0 1	$u_3 = 0$
$v_1 = 0$	$v_2 = 2$	$v_3 = 2$	$v_4 = 0$	

Since all $d_{ij} > 0$ the solution under the test is optimal and unique.

∴ The optimal allocation schedule is given by $x_{12} = 6$, $x_{23} = 2$, $x_{24} = 6$, $x_{31} = 4$, $x_{32} = \in$, $x_{33} = 6$ and the optimum (minimum) transportation cost = Rs. 2 x 6 + 2 x 2 + 0 x 6 + 0 x 4 + 2 x ∈ + 2 x 6

= Rs. (28 + 2∈) = Rs. 28, as $\in \to 0$.

Example 5:

Solve the following transportation problem to minimize the total cost of transportation.

Destination

	1	2	3	4	supply
1	14	56	48	27	70
Origin 2	82	35	21	81	47
3	99	31	71	63	93
Demand	70	35	45	60	210

Solution:

Since ∑ = ∑ = 210, the given transportation problem is balanced. ∴ There exists a basic feasible solution to this problem.

By using Vogel's approximation method, the initial solution is as shown in the following table:

14 **70**	56	48	27
82	35	21 **45**	81 **2**
99	31 **35**	71	63 **58**

Since the number of non-negative allocations is 5, which is less than (m + n – 1) = (3+4-1) = 6, this basic feasible solution is degenerate.

To resolve degeneracy, we allocate a very small quantity ∈ to the cell (1,4). So that the number of occupied cells becomes ($m + n - 1$). Hence the non-degenerate basic feasible solution is given in the following table.

14 70	56	48	27 ∈
82	35	21 **45**	81 **2**
99	31 **35**	71	63 **58**

To find the optimum solution:

Now the number of non-negative allocations at independent positions is (m + n – 1) =6. We apply MODI method.

14 **70**	56 -5 61	48 -33 81	27 ∈	$u_1 = 27$
82 68 14	35 49 -14	21 **45**	81 **2**	$u_2 = 81$
99 50 49	31 **35**	71 3 68	63 **58**	$u_3 = 63$
$v_1 = -13$	$v_2 = -32$	$v_3 = 60$	$v_4 = 0$	

Since $d_{22}=-14<0$, the solution under the test is not optimal.

Now let us from a new basic feasible solution by giving maximum allocation to the cell (2,2) by making an occupied cell empty. We draw a closed loop consisting of horizontal and vertical lines beginning and ending at this cell (2,2) and having its other corners at some occupied cells. Along this closed loop, indicate + and - alternatively at the corners.

14 **70**	56	48	27 є
82	35 +	21 **45**	81 **2** -
99	31 - **35**	71	63 + **58**

From the two cells (2,4),(3,2) having – we find that the minimum of the allocations 2,35 is 2. Add this 2 to the cells with + and subtract this 2 to the cells with + . Hence the new basic feasible solution is given by

14 **70**	56	48	27 є
82	35 **2**	21 **45**	81
99	31 **33**	71	63 **60**

We see that the above table satisfies the rim conditions with (m + n – 1) non-negative allocations at independent position. We apply MODI method for optimality.

14 **70**	56 -5 61	48 -19 81	27 є	u_1 = -40
82 54 28	35 **2**	21 **45**	81 67 14	u_2 = 0
99 50 49	31 **33**	71 17 54	63 **60**	u_3 = -4
v_1 = 54	v_2 = 35	v_3 = 21	v_4 = 67	

Since $d_{ij}>0$, the solution under the test is optimal.

∴ The optimal allocation schedule is given by $x_{11} = 70$, $x_{14} = \in$, $x_{22} = 2$, $x_{23} = 45$, $x_{32} = 33$, $x_{34} = 60$ and the optimum (minimum) transportation cost
=Rs. 14 x 70 + 27 x ∈ + 35 x 2 + 21 x 45 + 31 x 33 + 63 x 60
=Rs.6798/- as $\in \to 0$.

Unbalanced Transportation Problems

If the given transportation problems is unbalanced one, i.e., if$\sum \neq \sum$, then convert this into a balanced one by introducing a dummy source or dummy destination with zero cost vector (zero unit transportation costs) as the case may be and then solve by usual method.

When the total supply if greater than the total demand, a dummy destination is included in the matrix with zero cost vectors. The excess supply is entered as a rim requirement for the dummy destination.

When the total demand is greater than the total supply, a dummy source is included in the matrix with zero cost vectors. The excess demand is entered as rim requirements for the dummy source.

Example 1: Solve the transportation problem

Destination

	A	B	C	D	supply
1	11	20	7	8	50
Source 2	21	16	20	12	40
3	8	12	18	9	70
Demand	30	25	35	40	

Solution: Since the total supply ($\sum$ =160) is greater than the total demand ($\sum$ =130), the given problem is an unbalanced transportation problem. To convert this i9nto a balanced one, we introduce a dummy destination E with zero unit transportation costs and having demand equal to 160-130=30 units.

∴ The given problem becomes

Destination

	A	B	C	D	E	supply
1	11	20	7	8	0	50
Source 2	21	16	20	12	0	40
3	8	12	18	9	0	70
Demand	30	25	35	40	30	160

By using VAM the initial solution is as shown in the following table

11	20	7 **35**	8 **15**	0
21	16	20	12 **10**	0 **30**
8 **30**	12 **25**	18	9 **15**	0

The initial transportation cost

=Rs. 7 x 35 + 8 x 15 + 12 x 10 + 0 x 30 + 8 x 30 + 12 x 25 + 9 x

15 =Rs. 1160/-

For Optimality: Since the number non-negative allocations at independent position is (m + n – 1), we apply the MODI method.

11 7 4	20 11 9	7 **35**	8 **15**	0 -4 4	u_1 = 8
21 11 10	16 15 1	20 11 9	12 **10**	0 **30**	u_2 = 12
8 **30**	12 **25**	18 8 10	9 **15**	0 -3 3	u_3 = 9
v_1 = -1	v_2 = 3	v_3 = -1	v_4 = 0	v_5 = -12	

Since all $d_{ij}>0$, the solution under the test is optimum and unique.

∴ The optimum allocation schedule is $x_{13} = 35$, $x_{14} = 15$, $x_{24} = 10$, $x_{25} = 30$, $x_{31} = 30$, $x_{32} = 25$, $x_{34} = 15$

It can be noted that $x_{25}=30$ means that 30 units are dispatched from source 2 to the dummy destination E. In other words, 30 units are left undespatched from source 2.

The optimum (minimum) transportation cost

= Rs. 7 x 35 + 8 x 15 + 12 x 10 + 0 x 30 + 8 x 30 + 12 x 25 + 9 x 15

=Rs. 1160/-

Example 2: Solve the transportation problem with unit transportation costs, demands and supplies as given below:

Destination

	D_1	D_2	D_3	D_4	Supply
S_1	6	1	9	3	70
Source S_2	11	5	2	8	55
S_3	10	12	4	7	70
Demand	85	35	50	45	

Solution: Since the total demand (∑ =215) is greater than the total supply (∑ =195), the given problem is unbalanced transportation problem. To convert this into a balanced one, we introduce a dummy source 4 with zero unit transportation costs and having supply equal to 215-195=20 units. ∴ The given problems becomes

Destination

	D_1	D_2	D_3	D_4	Supply
S_1	6	1	9	3	70
Source S_2	11	5	2	8	55
S_3	10	12	4	7	70
S_4	0	0	0	0	20
Demand	85	35	50	45	215

As this problem is balanced, there exists a basic feasible solution to this problem. By using Vogel's approximation method, the initial solution is as shown in the following table.

6 **65**	1 **5**	9	3
11	5 **30**	2 **25**	8
10	12	4 **25**	7 **45**
0 **20**	0	0	0

∴ The initial transportation cost

= Rs. 6x65+1x5+5x30+2x25+4x25+7x45+0x20

=Rs.1010/-

For optimality: Since number of non-negative allocations at independent positions is (m + n – 1), we apply the MODI method.

6 **65**	1 **5**	9 -2 11	3 1 2	$u_1 = 6$
11 10 1	5 **30**	2 **25**	8 5 3	$u_2 = 10$
10 12 -2	12 7 5	4 **25**	7 **45**	$u_3 = 12$
0 **20**	0 - 5 5	0 - 8 8	0 - 5 5	$u_4 = 0$
$v_1 = 0$	$v_2 = -5$	$v_3 = -8$	$v_4 = -5$	

Since $d_{31}=-2<0$, the solution under the test is not optimal.

Now let us form a new basic feasible solution by giving maximum empty. For this, we draw a closed path consisting of horizontal and vertical lines beginning and ending at this cell

(3,1) and having its other corners at some occupied cells. Along this closed loop, indicate + and - alternatively at the corners.

We have,

6 **65** -	1 **5** +	9	3
11	5 **30** -	2 **25** +	8
10 +	12	4 **25** -	7 **45**
0 **20**	0	0	0

From the three cells (1,1), (2,2), (3,3) having – , we find that the minimum of the allocations 65,30,25 is 25. Add this 25 to the cells with + and subtract this 25 to this cells with– . Finally, the new feasible solution is displayed in the following table.

6 **40**	1 **30**	9	3
11	5 **5**	2 **50**	8
10 **25**	12	4	7 **45**
0 **20**	0	0	0

We see that the above table satisfies the rim conditions with (m + n – 1) non-negative allocations at independent positions. Now we check for optimality.

6 **40**	1 **30**	9 -2 11	3 3 0
11 10 1	5 **5**	2 **50**	8 7 1
10 **25**	12 5 7	4 2 2	7 **45**
0 **20**	0 -5 5	0 -8 8	0 -3 3

Since all $d_{ij}>0$ with $d_{14}=0$, the solution under the test is optimal and an alternative optimal solution exists.

∴ The optimum allocation schedule is given by $x_{13} = 35$, $x_{14} = 15$, $x_{24} = 10$, $x_{25} = 30$, $x_{31} = 30$, $x_{32} = 25$, $x_{34} = 15$, $x_{41} = 20$.

It can be noted that $x_{41}=20$ means that 20 units are dispatched from the dummy source 4 to the destination 1. In other words, 20 units are not fulfilled for the destination 1.

The optimum (minimum) transportation cost

= Rs. 6 x 40 + 1 x 30 + 5 x 5 + 2 x 50 + 10 x 25 + 7 x 45 + 0 x 20

=Rs.960/-

Example 3:

Solve the transportation problem with unit transportation costs in rupees, demand and supplies as given below:

Destination

	D_1	D_2	D_3	Supply(units)
A	5	6	9	100
Origin B	3	5	10	75
C	6	7	6	50
D	6	4	10	75
Demand (units)	70	80	120	

Solution: Since the total supply (∑ =270), the given transportation problem is unbalanced.

To convert this into a balanced one, we introduce a dummy source 4 with zero unit transportation costs and having demand equal to 300-270=30 units. ∴ The given problem becomes

Destination

	D_1	D_2	D_3	D_4	Supply(units)
A	5	6	9	0	100
Origin B	3	5	10	0	75
C	6	7	6	0	50
D	6	4	10	0	75
Demand (units)	70	80	120	30	300

By using VAM the initial solution is given by

5	6	9 **100**	0
3 **70**	5 **5**	10	0
6	7	6 **20**	0 **30**
6	4 **75**	10	0

Since the number of non-negative allocations is 6, which is less than (m + n – 1) = 4+4-1 =7, this basic feasible solution is degenerate.

To resolve this degeneracy, we allocate a very small quantity ∈ to the cell (2,4), so that the number of occupied cells becomes (m + n – 1). Hence the non-degenerate basic feasible solution is given in the following table.

5	6	9 **100**	0
3 **70**	5 **5**	10	0 ε
6	7	6 **20**	0 **30**
6	4 **75**	10	0

Now the number of non-negative allocations at independent positions is (m + n – 1). We apply MODI method.

5 6 -1	6 8 -2	9 **100**	0 3 -3	u_1 = 3
3 **70**	5 **5**	10 6 4	0 ε	u_2 = 0
6 3 3	7 5 2	6 **20**	0 **30**	u_3 = 0
6 2 4	4 **75**	10 5 5	0 -1 1	u_4 = -1
v_1 = 3	v_2 = 5	v_3 = 6	v_4 = 0	

Since there are some d_{ij} < 0, the current solution is not optimal.

Since d_{14}= -3 is the most negative, let is form a new basic feasible solution is giving maximum allocations to the corresponding cell(1,4) by making an occupied cell empty. We draw a closed loop consisting of horizontal and vertical lines beginning and ending at this cell (1,4) and having its other corners at some occupied cells. Along this closed loop indicate + and

– Alternatively at the corners.

5	6	9 **100** -	0 +
3 **70**	5 **5**	10	0 ε
6	7	6 **20** +	0 **30** -
6	4 **75**	10	0

From the two cells (1, 3), (3, 4) having – , we find that the minimum of the allocations 100, 30 is 30. Add this 30 the cells with + and subtract this 30 to the cells with – . Hence the new basic feasible solution is given in the following table.

5	6	9 **70**	0 **30**
3 **70**	5 **5**	10	0 ε
6	7	6 **50**	0
6	4 **75**	10	0

We see that the above table satisfies the rim conditions with (m + n – 1) non-negative allocations at independent positions. We apply MODI method.

5 3 2	6 5 1	9 **70**	0 **30**	$u_1 = 0$
3 **70**	5 **5**	10 9 1	0 ε	$u_2 = 0$
6 0 6	7 2 5	6 **50**	0	$u_3 = -3$
6 2 4	4 **75**	10 8 2	0 -1 1	$u_4 = -1$
$v_1 = 3$	$v_2 = 5$	$v_3 = 9$	$v_4 = 0$	

Since all $d_{ij}>0$, the current solution is optimal and unique.

The optimum allocation schedule is given by $x_{13} = 70$, $x_{14} = 30$, $x_{21} = 70$, $x_{22} = 5$, $x_{24} = \in$, $x_{33} = 50$, $x_{42} = 75$ and the optimum (minimum) transportation cost

= Rs. $9 \times 70 + 0 \times 30 + 3 \times 70 + 5 \times 5 + 0 \times \in + 6 \times 50 + 4 \times 75$ =Rs. 1465/-

Maximization case in Transportation Problems

So far we have discussed the transportation problems in which the objectives has been to minimize the total transportation cost and algorithms have been designed accordingly.

If we have a transportation problems where the objective is to maximize the total profit, first we have to convert the maximization problem into a minimization problem by multiplying all the entries by -1 (or) by subtracting all the entries from the highest entry in the given transportation table. The modified minimization problem can be solved in the usual manner.

Assignment Problem :Introduction

The assignment problem is a particular case of the transportation problem in which the objective is to assign a number of tasks (Jobs or origins or sources) to an equal number of facilities (machines or persons or destinations) at a minimum cost (or maximum profit).

Suppose that we have '*n*' jobs to be performed on '*m*' machines (one Job to one machine) and our objective is to assign the jobs to the machines at the minimum cost (or maximum profit) under the assumption that each machine can perform each job but with varying degree of efficiencies.

The assignment problem can be stated in the form of *m* x *n* matrix (c_{ij}) called a cost matrix (or) Effectiveness matrix where c_{ij} is the cost of assigning i^{th} machine to the j^{th} job.

		1	2	3	……………	n
	1	c_{11}	c_{12}	c_{13}	……………	c_{1n}
	2	c_{21}	c_{22}	c_{23}	……………	c_{2n}
Machines	3	c_{31}	c_{32}	c_{33}	……………	c_{3n}
	.	……….		…..	……………	…..
	.					
	.	…..	…..	…..	……………	…..
	.					
	.	…..	…..	…..	……………	…..
	.					
	m	c_{m1}	c_{m2}	c_{m3}	……………	c_{mn}

Mathematical formulation of an assignment problem.

Consider an assignment problem of assigning n jobs to n machines (one job to one machine). Let c_{ij} be the unit cost of assigning i^{th} nachine to the j^{th} job and

$$\text{Let } x_{ij} = \begin{cases} 1, \text{ if } j^{th} \text{ job is assigned to } i^{th} \text{ machine} \\ 0, \text{ if } j^{th} \text{ job is not assigned to } i^{th} \text{ machine} \end{cases}$$

The assignment model is then given by the following LPP

Minimize Z = $\sum =1 \sum =1$

Subject to the constraints

$\sum = 1$ $= 1, \quad j = 1,2,\ldots.,n$

$\sum = 1$ $= 1, \quad i = 1,2,\ldots.,n$

and $= 0$ (or) 1.

Difference between the transportation problem and the assignment problem.

	Transportation problem	*Assignment problem*
(a)	Supply at any source may be any positive quantity a_i	Supply at any source (machine) will be 1 i.e., $a_i = 1$.
(b)	Demand at any destination may be any positive b_j	Demand at any destination (job) will be 1 i.e., $b_j = 1$.
(c)	One or more source to any Number of destinations	One source (machine) to only one destination (job).

Assignment Algorithm (or) Hungarian Method.

First check whether the number of rows is equal to the number of columns. If it is so, the assignment problem is said to be **balanced**. Then proceed to step 1. If it is not balanced, then it should be balanced before applying the algorithm.

Step 1: Subtract the smallest cost element of each row from all the elements in the row of the row of the given cost matrix. See that each row contains atleast one zero.

Step 2: Subtract the smallest cost element of each column from all the elements in the column of the resulting cost matrix obtained by step 1.

Step 3: (Assigning the zeros)

(a) Examine the rows successively until a row with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross all other zeros in the column of this enriched zero, as these will not be considered for any future assignment. Continues in this way until all the rows have been examined.
(b) Examine the columns successively until a column with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it and cross any other zero in its row. Continue until all the columns have been examined.

Step 4: (Apply optimal Test)

(a) If each row and each column contain exactly one encircled zero, then the current assignment is optimal.
(b) It atleast one row/column is without an assignment (i.e., if there is atleast one row/column is without one encircled zero), then the current assignment is not optimal. Go to step 5.

Step 5: Cover all the zeros by drawing a minimum number of straight lines as follows.

(a) Mark ▌) the rows that do not have assignment.
(b) Mark ▌) the columns (not already marked) that have zeros in marked columns.
(c) Mark ▌) the rows (not already marked) that have assignments in marked columns.
(d) Repeat (b) and (c) until no more marking is required.
(e) Draw lines through all unmarked rows and columns. If the number of these lines is equal to the order of the matrix then it is an optimum solution otherwise not.

Step 6: Determine the smallest cost element not covered by the straight lines. Subtract this smallest cost element from all the uncovered elements and add this to all those elements which are lying in the intersection of these straight lines and do not change the remaining elements which lie on the straight lines.

Step 7: Repeat steps (1) to (6). Until an optimum assignment is attained.

Note 1: In case some rows or columns contain more than one zero, encircle any unmarked zero, encircle any unmarked zero arbitrarily and cross all other zeros in its column or row. Proceed in this way until no zero is left unmarked or encircled.

Note 2: The above assignment algorithm is only for minimization problems.

Note 3: If the given assignment problem is of maximization type, convert it to a minimization assignment problem by max Z = - min (-Z) and multiply all the given cost elements by -1 in the cost matrix and then solve by assignment algorithm.

Note 4: Sometimes a final cost matrix contains more than required number of zeros at independent positions. This implies that there is more than one optimal solution (multiple optimal solutions) with the same optimum assignment cost.

Example 1:

Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows:

$$\begin{array}{cc} & \textbf{Job} \\ & \begin{array}{ccccc} \textbf{1} & \textbf{2} & \textbf{3} & \textbf{4} & \textbf{5} \end{array} \\ \begin{array}{r} \textbf{A} \\ \textbf{B} \\ \textbf{From C} \\ \textbf{D} \\ \textbf{E} \end{array} & \begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix} \end{array}$$

Determine the optimum assignment schedule.

Solution: The cost matrix of the given assignment problem is

$$\begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix}$$

Since the number of rows is equal to the number of columns in the cost matrix, the given assignment problem is balanced.

Step 1: Select the smallest cost element in each row and subtract this from all the elements of the corresponding row, we get the reduced matrix

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

Step 2: select the smallest cost element in each column and subtract this from all the elements of the corresponding column, we get the reduced matrix.

$$\begin{pmatrix} 7 & 3 & 0 & 5 & 0 \\ 0 & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & 0 & 4 \\ 4 & 3 & 0 & 0 & 3 \\ 4 & 0 & 2 & 4 & 0 \end{pmatrix}$$

Since each row and each column at least one zero, we shall make assignments in the reduced matrix.

Step 3: Examine the rows successively until a row with exactly one unmarked zero is found. Since the 2nd row contains a single zero, encircle this zero and cross all other zeros of its column. The 3rd row contains exactly one unmarked zero, so encircle this zero and cross all other zeros in its column. The 4th row contains exactly one unmarked zero, so encircle this zero and cross all other zeros in its column. The 1st row contains exactly one unmarked zero, so encircle this zero and cross all other zeros in its column. Finally the last row contains exactly one unmarked zero, so encircle this zero and cross all other zeros in its column. Likewise examine the columns successively. The assignments in rows and columns in the reduced matrix is given by

$$\begin{pmatrix} 7 & 3 & 0 & 5 & (0) \\ (0) & 9 & 4 & 5 & 4 \\ 1 & 6 & 6 & (0) & 4 \\ 4 & 3 & (0) & 0 & 3 \\ 4 & (0) & 2 & 4 & 0 \end{pmatrix}$$

Step 4: Since each row and each column contains exactly one assignment (i.e., exactly one encircled zero) the current assignment is optimal.

$\therefore$ The optimal assignment schedule is given by A $\longrightarrow$ 5, B $\longrightarrow$ 1, C $\longrightarrow$ 4, D $\longrightarrow$ 3, E $\longrightarrow$ 2.

The optimum (minimum) assignment cost = (1 + 0 + 2 + 1 + 5) cost units = 9 units of cost.

Example 2:

The processing time in hours for the when allocated to the different machines are indicated below. Assign the machines for the jobs so that the total processing time is minimum.

		Machines				
		M_1	M_2	M_3	M_4	M_5
	J_1	9	22	58	11	19
	J_2	43	78	72	50	63
Jobs	J_3	41	28	91	37	45
	J_4	74	42	27	49	39
	J_5	36	11	57	22	25

Solution:

The cost matrix of the given problem is

9	22	58	11	19
43	78	72	50	63
41	28	91	37	45
74	42	27	49	39
36	11	57	22	25

Since the number of rows is equal to the number of columns in the cost matrix, the given assignment problem is balanced.

Step 1: select the smallest cost element in each row and subtract this from all the elements of the corresponding row, we get the reduced matrix.

$$\begin{pmatrix} 0 & 13 & 49 & 2 & 10 \\ 0 & 35 & 29 & 7 & 20 \\ 13 & 0 & 63 & 9 & 17 \\ 47 & 15 & 0 & 22 & 12 \\ 25 & 0 & 46 & 11 & 14 \end{pmatrix}$$

Step 2: Select the smallest cost element in each column and subtract this from all the elements of the corresponding column, we get the following reduced matrix.

$$\begin{pmatrix} 0 & 13 & 49 & 0 & 0 \\ 0 & 35 & 29 & 5 & 10 \\ 13 & 0 & 63 & 7 & 7 \\ 47 & 15 & 0 & 20 & 2 \\ 25 & 0 & 46 & 9 & 4 \end{pmatrix}$$

Step 3: Now we shall examine the rows successively. Second row contains a single unmarked zero, encircle this zero and cross all other zeros in its column. Third row contains a single unmarked zero, encircle this zero and cross all other zeros in its column. Fourth row contains a single unmarked zero, encircle this zero and cross all other zero in its column. After this no row is with exactly one unmarked zero. So go for columns.

Examine the columns successively. Fourth column contains exactly one unmarked zero, encircle this zero and cross all other zeros in its row. After examining all the rows and columns. We get

$$\begin{pmatrix} 0 & 13 & 49 & (0) & 0 \\ (0) & 35 & 29 & 5 & 10 \\ 13 & (0) & 63 & 7 & 7 \\ 47 & 15 & (0) & 20 & 2 \\ 25 & 0 & 46 & 9 & 4 \end{pmatrix}$$

Step 4: Since the 5^{th} column do not have any assignment, the current assignment is not optimal.

Step 5: Cover all the zeros by drawing a minimum number of straight lines as follows:

(a) Mark (✓) the rows that do not have assignment. The row 5 is marked.
(b) Mark (✓) the columns (not already marked) that have zeros in marked rows. Thus column 2 is marked.
(c) Mark the rows (not already marked) that have assignment in, marked columns. Thus row 3 is marked.
(d) Repeat (b) and (c) until no more marking is required. In the present case this repetition is not necessary.
(e) Draw lines through all unmarked rows (rows 1, 2 and 4). And marked columns (column 2). We get

$$\begin{pmatrix} 0 & 13 & 49 & 0 & 0 \\ 0 & 35 & 29 & 5 & 10 \\ 13 & 0 & 63 & 7 & 7 \\ 47 & 15 & 0 & 20 & 2 \\ 25 & 0 & 46 & 9 & (4) \end{pmatrix}$$

Step 6: Here 4 is the smallest element not covered by these straight lines. Subtract this 4 from all the uncovered element and add this 4 to all those elements which are lying in the intersections of these straight lines and do not change the remaining elements which lie on these straight lines. We get the following matrix.

$$\begin{pmatrix} 0 & 17 & 49 & 0 & 0 \\ 0 & 39 & 29 & 5 & 10 \\ 9 & 0 & 59 & 3 & 3 \\ 47 & 19 & 0 & 20 & 2 \\ 21 & 0 & 42 & 5 & 0 \end{pmatrix}$$

Since each row and each column contains at least one zero, we examine the rows and columns successively, i.e., repeat step 3 above, we get

$$\begin{pmatrix} 0 & 17 & 49 & (0) & 0 \\ (0) & 39 & 29 & 5 & 10 \\ 9 & (0) & 59 & 3 & 3 \\ 47 & 19 & (0) & 20 & 2 \\ 21 & 0 & 42 & 5 & (0) \end{pmatrix}$$

In the above matrix, each row and each column contains exactly one assignment (i.e., exactly one encircled zero), therefore the current assignment is optimal.

∴ The optimum assignment schedule is $1 \rightarrow 4$, $2 \rightarrow 1$, $3 \rightarrow 2$, $4 \rightarrow 3$, $5 \rightarrow 5$ and the optimum (minimum) processing time

= 11+43+28+27+25 hours = 134 hours.

Unbalanced Assignment Models

If the number of rows is not equal to the number columns in the cost matrix of the given assignment problems, then the given assignment problems is said to be unbalanced.

First convert the unbalanced assignment problems in to a balanced one by adding dummy rows or dummy columns with zero cost element in the cost matrix depending upon whether m<n or m>n and then solve by the usual method.

Example 1: A company has four machines to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table.

	Machines			
	1	**2**	**3**	**4**
A	**18**	**24**	**28**	**32**
B	**8**	**13**	**17**	**19**
Jobs C	**10**	**15**	**19**	**22**

What are job assignments which will minimize the cost?

Solution:

The cost matrix of the given assignment problems is

$$\begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \end{pmatrix}$$

Since the number of rows is less than the number of columns in the cost matrix, the given assignment problems is unbalanced.

To make it a balanced one, add a dummy job D (row) with zero cost elements. The balanced cost matrix is given by

$$\begin{pmatrix} 18 & 24 & 28 & 32 \\ 8 & 13 & 17 & 19 \\ 10 & 15 & 19 & 22 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Now select the smallest cost element in each row (column) and subtract this from all the elements of the corresponding row (columns), we get the reduced matrix

$$\begin{pmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

In this reduced matrix, we shall make the assignment in rows and columns having single zero. We have

$$\begin{pmatrix} (0) & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & 5 & 9 & 12 \\ 0 & (0) & 0 & 0 \end{pmatrix}$$

Since there are some rows and columns without assignment, the current assignment is not optimal.

Cover the all zeros by drawing a minimum number of straight lines. Choose the smallest cost element not covered by these straight lines.

$$\begin{pmatrix} 0 & 6 & 10 & 14 \\ 0 & 5 & 9 & 11 \\ 0 & (5) & 9 & 12 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Here 5 is the smallest cost element not covered by these straight lines. Subtract this 5 from all the uncovered element, add this 5 to those elements which lie in the intersections of these straight lines and do not change the remaining element which lie on the straight lines. We get

$$\begin{pmatrix} 0 & 1 & 5 & 9 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 4 & 7 \\ 5 & 0 & 0 & 0 \end{pmatrix}$$

Since each row and each column contains atleast one zero, we shall make assignment in the rows and columns having single zero. We get

$$\begin{pmatrix} (0) & 1 & 5 & 9 \\ 0 & (0) & 4 & 6 \\ 0 & 0 & 4 & 7 \\ 5 & 0 & (0) & 0 \end{pmatrix}$$

Since there are some rows and columns without assignment, the current assignment is not optimal.

Cover all the zeros by drawing a minimum number of straight lines.

$$\begin{pmatrix} 0 & 1 & 5 & 9 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & (4) & 7 \\ 5 & 0 & 0 & 0 \end{pmatrix}$$

Choose the smallest cost element not covered by these straight line, subtract this from all the uncovered elements, add this to those elements which are in the intersection of the lines and do not change the remaining elements which lie on these straight lines. Thus we get

$$\begin{pmatrix} 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 9 & 4 & 0 & 0 \end{pmatrix}$$

Since each row and each column contains atleast one zero, we shall make the assignment in the rows and columns having single zero. We get

$$\begin{pmatrix} (0) & 1 & 1 & 5 \\ 0 & (0) & 0 & 2 \\ 0 & 0 & (0) & 3 \\ 9 & 4 & 0 & (0) \end{pmatrix}$$

Since each row and each column contains exactly one assignment (i.e., exactly one encircled zero) the current assignment is optimal.

∴ The optimum assignment schedule is given by A $\longrightarrow$ 1, B $\longrightarrow$ 2, C $\longrightarrow$ 3, D $\longrightarrow$ 4 and the optimum (minimum) assignment cost

= (18+13+19+0) cost unit = 50/- units of cost

Note 1: For this problem, the alternative optimum schedule is A $\longrightarrow$ 1, B $\longrightarrow$ 2, C $\longrightarrow$ 3, D $\longrightarrow$ 4, with the same optimum assignment cost= Rs. (18+17+15+0) = 50/- units of cost.

Note 2: Here the assignment D $\longrightarrow$ 4 means that the dummy Job D is assigned to the 4th Machine. It means that machine 4 is left without any assignment.

Maximization case in Assignment Problems

In an assignment problem, we may have to deal with maximization of an objective function. For example, we may have to assign persons to jobs in such a way that the total profit is maximized. The maximization problems has to be converted into an equivalent minimization problem and then solved by the usual Hungarian Method.

The conversion of the maximization problem into an equivalent minimization problems can be done by any of the following methods:

(i) Since max Z = - min (-Z), multiply all the cost element c_{ij} of the cost matrix by -1.

(ii) Subtract all the cost elements c_{ij} of the cost matrix from the highest cost element in that cost matrix.

Example:

Solve the assignment problem for maximization given the profit matrix (profit in rupees).

$$\begin{array}{cc} & \text{Machines} \\ & \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{array}{r} A \\ B \\ \text{Jobs } C \\ D \end{array} & \begin{array}{|cccc|} \hline 51 & 53 & 54 & 50 \\ 47 & 50 & 48 & 50 \\ 49 & 50 & 60 & 61 \\ 63 & 64 & 60 & 60 \\ \hline \end{array} \end{array}$$

Solution:

The profit matrix of the given assignment problem is

$$\begin{pmatrix} 51 & 53 & 54 & 50 \\ 47 & 50 & 48 & 50 \\ 49 & 50 & 60 & 61 \\ 63 & (64) & 60 & 60 \end{pmatrix}$$

Since this is a maximization problem, it can be converted into an equivalent minimization problem by subtracting all the profit elements in the profit from the highest profit element 64 of this profit matrix. Thus the cost matrix of the equivalent minimization problem is

$$\begin{pmatrix} 13 & 11 & 10 & 14 \\ 17 & 14 & 16 & 14 \\ 15 & 14 & 4 & 3 \\ 1 & 0 & 4 & 4 \end{pmatrix}$$

Select the smallest cost in each row and subtract this from all the cost elements of the corresponding row. We get

$$\begin{pmatrix} 3 & 1 & 0 & 4 \\ 3 & 0 & 2 & 0 \\ 12 & 11 & 1 & 0 \\ 1 & 0 & 4 & 4 \end{pmatrix}$$

Select the smallest cost element in each column and subtract this from all the cost elements of the corresponding column. We get

$$\begin{pmatrix} 2 & 1 & 0 & 4 \\ 2 & 0 & 2 & 0 \\ 11 & 11 & 1 & 0 \\ 0 & 0 & 4 & 4 \end{pmatrix}$$

Since each row and each column contains atleast one zero, we shall make the assignment in rows and columns having single zero. We get

$$\begin{pmatrix} 2 & 1 & (0) & 4 \\ 2 & (0) & 2 & 0 \\ 11 & 11 & 1 & (0) \\ (0) & 0 & 4 & 4 \end{pmatrix}$$

Since each row and each column contains exactly one encircled zero, the current assignment is optimal.

$\therefore$ The optimum assignment schedule is given by $A \longrightarrow R$, $B \longrightarrow Q$, $C \longrightarrow S$, $D \longrightarrow P$ and the optimum (maximum) profit = Rs. (54 + 50 + 61 + 63)

= Rs. 228/-

Possible Questions

Part-B(6 mark)

1. Use two phase simplex method to solve

   Max $Z=5x_1 + 3x_2$

   Subject to

   $2x_1+ x_2 \leq 1$

   $x_1 + 4x_2 \geq 6$

   $x_1 \geq 0$, $x_2 \geq 0$

2.Use graphical method to solve the following LPP

   Max $Z = 80x_1 + 55x_2$Type equation here.

   Subject to

   $4x_1+ 2x_2 \leq 40$

   $2x_1 + 4x_2 \leq 32$

   $x_1 \geq 0$, $x_2 \geq 0$

3. Use simplex method to solve the following

   LPP Maximize $Z = 5x_1 + 8x_2$

   Subject to the constrains

$-2x_1 + x_2 \leq 1$

$x_1 \leq 2$

$x_1+ x_2 \leq 3$

and $x_1, x_2 \geq 0$

4. The assignment cost of assigning any one operator to any one machine is given in the following table

		Operator			
		I	II	III	IV
Machine	1	10	5	13	15
	2	3	9	18	3
	3	10	7	3	2
	4	5	11	9	7

   Find the optimal assignment by Hungarian method.

5. Consider the problem of assigning five jobs to five persons. The assignment costs are given asfollows:

Job

$$\begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{From} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 8 & 4 & 2 & 6 & 1 \\ 0 & 9 & 5 & 5 & 4 \\ 3 & 8 & 9 & 2 & 6 \\ 4 & 3 & 1 & 0 & 3 \\ 9 & 5 & 8 & 9 & 5 \end{pmatrix} \end{array}$$

Determine the optimum assignment schedule

6. Solve the transportation problem.

Distribution Centres

		D1	D2	D3	D4	Available
	A	11	13	17	14	250
Origin	B	16	18	14	10	300
	C	21	24	13	10	400
Demand		200	225	275	250	

7. Solve the assignment problem.

	A	B	C	D
I	18	24	28	32
II	8	13	17	19
III	10	15	19	22

8. Find the Non- degenerate basic feasible solution for the following transportation problem using

i) North west corner rule

ii) Lowest cost entry method

	A	B	C	D	Supply
I	10	20	5	7	10
II	13	9	12	8	20
III	4	5	7	9	30
IV	14	7	1	0	40
V	3	12	5	19	50
Demand	60	60	20	10	

9. Solve the following LPP by simplex method Maximize $Z = 8x_1 - 2x_2$
   Subject to the constrains
   $-4x_1 + 2x_2 \leq 4$
   $3x_1 + 8x_2 \leq 24$
   $10x_1 + 7x_2 \leq 35$
   and $x_1, x_2, x_3 \geq 0$

10. Find the starting solution of the following transportation model

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

Using (i) North West Corner rule
(ii) Vogel's approximation method.

Part-C(10 mark)

1. Find the optimal solution to the following transportation problem.

	1	2	3	4	Supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
	6	10	12	15	

Demand

Question	Opt 1	Opt 2	Opt 3	Opt 4	Opt 5	Opt 6	Answer
Programming is another word for ---------------------	planning	organizing	managing	decision making			planning
The linear function is to be maximized or minimized is called------------	objective function	subjective function	optiomal function	odd function			objective function
LPP is a technique of finding the -------------	optimal solution	approximate solution	both	infeasible solution			optimal solution
A Feasible solution to a LPP which is also a basic solution to the problem is called ---	basic solution	basic feasible solution	non basic feasible solution	optimal solution			basic feasible solution
The solution which optimizes the objective function are called ---------------	feasible solution	optimal solution	optional solution	arbitrary solution			optimal solution
A basic solution is said to be a -------------- if one of more of the basic variables are Zero	Non degenerate basic solution	infeasible solution	degenerate basic solution	unbounded solution			degenerate basic solution
More than two decision variables problem in LPP can be solved by -------	simplex method	Big-M method	Graphical method	Dual simplex method			simplex method
Another name for simplex method is ------------------------	computational procedure	computational method	Big-M method	Dual simplex method			computational procedure
The minimization of the function f(x) is equivalent to the maximization of --------	-(-f(x))	f(x)	-f(-x)	1 / f(x)			-(-f(x))
The coefficient of slack variables in the constraints is ------------------	1	0	2	-1			1
In simplex method all the variables must be--------------------	negative	non-negative	have the same sign	initially zero			non-negative
The coefficient of slack variables in the objective function is ------------------	1	0	2	-1			0
The set of feasible solutions to an LPP is a ---------------	convex set	null set	concave	finite			convex set
The number of alternatives in a LPP is typically----------------	finite	infinite	infeasible	feasible			finite
LPP deals with the problems involving only ------------- objective	one	two	more than one	more than two			one
Constraints appear as ----------------------- when plotted in a graph	curve	straight line	point	circle			straight line
An LPP is said to be infeasible if it has ---------------- that satisfies all the constraints	no solution	infinite solution	Unbounded solution	infeasible solution			no solution
The leaving variable row is called -----------------	key row	key column	pivot column	leaving row			key row
The entering variable column is called -----------------	key row	pivot row	pivot row	entering row			pivot row
The intersection of the pivot column and pivot row is called the---------------------	pivot element	leaving element	unit element	first element			pivot element
For the optimal solution of an LPP, existence of an initial feasible solution is always	Assumed	given	does not exists	zero			Assumed
If Minimum (Z) = -5, then the maximum (Z) =------------------	–5	5	4	-4			5
If the solution space is unbounded ,then the objective value will always be-------------	bounded	unbounded	feasible	infeasible			unbounded
Linear programming problems involving only two variables can be effectively solved by a ------- method.	simplex	iteration	graphical	Big-M method			graphical
The element of intersection of the pivot column and pivot row is called the ------	pivot row	pivot column	keyrow	pivot element			pivot element
The Objective of the transportation problem is to be -----------------------	Maximum	Minimum	Either maximum or minimum	Neither maximum nor minimum			Minimum
When total supply is equal to total demand, the problem is called a ------------------------ transportation problem	Feasible	Infeasible	Unbalanced	balanced			balanced
Cells in the transportaion table having positive allocation will be called ---------------------------	occupied cells	unoccupied cells	empty cells	zero cells			occupied cells
In a transportaion problem the various a's and b's are called ----------------------	Supply	demand	rim requirement	destination			rim requirement
The unit transportation cost from the ith source to jth destination is displayed in the ------------- of the (i, j)th cell.	upper left side	upper right side	lower left side	lower right side			upper left side
A balanced transportation problem will always have a ------------------	unique solution	infinite number of solution	infeasible solution	feasible solution			feasible solution
A feasible solution to a (mxn) transportation problem that contains no more than m+n-1 non negative allocations is called -------------	Feasible solution	optimul solution	basic feasible solution	infeasible solution			basic feasible solution
A feasible solution is said to be an optimul solution if it ------------------- the total transportation cost	Minimize	Maximize	Either maximize or minimize	Neither maximize nor minimize			Minimize
The number of basic variables in an mxn balanced transportation problem is atmost -----------------------	m + n	m + n -1	m + n +1	m - n -1			m + n -1
The number of non basic variables in an mxn balanced transportation problem is atleast --------------------	mn - (m + n -1)	mn - (m - n -1)	mn - (m + n + 1)	mn - (m - n + 1)			mn - (m + n -1)
In a transportaion problem, the cost of transportation is ----------------------	Linear	non linear	zero	one			Linear
For a feasible solution to exist, it is necessary that the total supply equal to total ----------------------	cost	demand	cells	rows			demand
Least cost method is also called ----------------------	Matrix method	Matrix maxima method	Matrix minima method	minima method			Matrix minima method
VAM method is also called ----------------------	Penalty method	Matrix minima method	Lower cost method	Hungerian method			Penalty method
Vogel's approximation method is a ------------------------	Penalty method	Matrix minima method	Hungerian method	Heuristic method			Heuristic method
In a VAM method allocations are made so that the penalty cost is -----------------------	minimised	maximized	zero	non zero			minimised
The MODI method is based on the concept of -----------------------	Linear Programming Problem	Heuristic method	Matrix maxima method	duality			duality

Every loop has an ------------------ number of cells	even	odd	equal	zero	even
Closed loops may or may not be ----------------- in shape	circle	square	rectangle	triangle	square
Penalty method is a ------------------------	North - west corner rule	least cost method	VAM method	MODI method	VAM method
Matrix minima method is a -----------------------------	North - west corner rule	least cost method	VAM method	MODI method	least cost method
Every loop has an even number of cells and atleast ----------------	Two	Four	Six	Eight	Four
The number of ----------------- in an mxn balanced transportation problem is atmost m+n-1	basic variables	non basic variables	decision variables	non decision variables	basic variables
In a transportation problem, the total transportation cost is -----------------------	minimum	maximum	zero	unique	minimum
The dummy source or destination in a transportation problem is added to -----------------	satisfy rim conditions	prevent solution from becoming degenerate	ensure that total cost does not exceed a limit	it is a balanced one	satisfy rim conditions
The occurrence of degeneracy while solving a transportation problem means that -------------------	total supply equals total demand	the solution so obtained is not feasible	the few allocations become negative	the solution so obtained is feasible	the solution so obtained is not feasible
One disadvantage of using North -west corner rule to find initial solution to the transportation problem is that --------------------	it is complicated to use	it does not take into account cost of transportation	it leads to a degenerate initial solution	it leads to non degenerate initial solution	it does not take into account cost of transportation
The solution to a transportation problem with m-rows (supplies) and n- columns (destination) is feasible if number of positive allocations are---------	m + n	m x n	m + n +1	m + n -1	m + n -1
If we were to use opportunity cost value for an unused cell to test optimality, it should be ------------------	equal to zero	most negative number	most positive number	any value	most negative number
A basic feasible solution to a (mxn) transportation problem is said to be a ----------- basic feasible solution if it contains exactly m+n-1 non negative allocations in independent positions.	degenerate	non degenerate	unique	infinite number of	non degenerate
A basic feasible solution to a (mxn) transportation problem is said to be a ----------- basic feasible solution if it contains less than m+n-1 non negative allocations in independent positions.	degenerate	non degenerate	unique	infinite number of	degenerate
In a transportation problem, least cost method gives a better solution to than ----------------------	North - west corner rule	least cost method	VAM method	MODI method	North - west corner rule
Ina transportation problem, -------------- gives a better starting solution than Least Cost method.	North - west corner rule	least cost method	VAM method	MODI method	VAM method
A transportation problem can always be represented by ---------------------	balanced model	unbalanced model	simplex model	graphical model	balanced model
In a transportation model, north west corner rule starting solution is recommended because it ensure that there will be --------------- allocations	m + n	m x n	m + n +1	m + n -1	m + n -1
The transportation model is restricted to dealing with a ---------------------- commodity only.	single	multiple	positive	negative	multiple
For any transportation problem, the coefficients of all units in the constraints are -------------	zero	any value	unity	unique	unity
A solution that satisfies all conditions of supply and demand but it may or may bot be optimal is called -------------------	feasible solution	infeasible solution	basic feasible solution	initial feasible solution	initial feasible solution
To solve degeneracy, an occupied cell with -------------cost is converted into occupied cell by assigning infinitely small amount to it.	lowest	larger	unit	zero	lowest
In a north west corner rule, if the demand in the column is satisfied, one must move to the ----------- cell in the next column.	left	right	middle	corner	right
Row wise and column wise difference between two minimum costs is calculated under ----------------- method.	North - west corner rule	least cost method	VAM method	MODI method	VAM method
An optimum solution results when net costs change values of all unoccupied cells are ----------------	positive and greater than zero	negative	non negative	positive and lesser than zero	non negative
MODI method associated with transportation problem, MODI stands for -----------------	Modified distribution	Multiple distribution	Matrix distribution	Modified distinction	Modified distribution
Transportation problem is a sub class of ------------------------------	Linear Programming Problem	Integer Programming Problem	Non Linear Programming Problem	Dynamic Programming Problem	Linear Programming Problem

UNIT-II

SYLLABUS

Integer Programming: Gomory cutting plane methods - Branch and Bound method. **Queueing Theory:**Characteristics of queueing systems - steady state MIMIt, MlMit/K and MIMIC queueing models.

Queuing Theory

Introduction

In everyday life it is seen that a number of people arrive at a cinema ticket window. If the people arrive too frequently they will have to wait for getting tickets or sometimes do without it. Such problems arise in Railways, Airline etc. Under such circumstances the only alternative is to form a Queue called the Waiting Line in order to get the service more effectively. If we have too many counters for service then expenditure may be high. On the other hand if we have only few countries then Queue may become longer resulting in the dissatisfaction or loss of customers. Queuing models are aids to determine the optimal number of counters so as to satisfy the customers keeping the total cost minimum. Here the arriving people are called customers and the person issuing the tickets is called a server.

Servers may be in parallel or in series. When in parallel, the arriving customers may form a single Queue or several Queues as is common in biog post offices. Service time may be constant or variable and customers may be served singly or in batches.

Queuing System

A queuing system can be completely described by

(a) The input (or arrival pattern).
(b) The service mechanism (or service pattern).
(c) The Queue discipline.
(d) Customer’s behavior.

(a) The input (or arrival pattern)

The input describes the way in which the customers arrive and join the system. Generally the customers arrive in more or less random fashion which is not worth making the prediction. Thus the arrival pattern can best be described in term of probabilities and consequently the probability distribution for inter arrival times (the time between two successive arrivals) must be defined. We deal with those Queuing system in which the customers arrive in 'Poisson' fashion. Mean arrival rate is denoted by

(b) The service mechanism (or service pattern)

The service pattern is specified when it is known how many customers can be served at a time, what the statistical distribution of the service time is, and when the service is available. Service time may be constant or a random variable. Distribution of service time which is important in practice is the **negative exponential distribution.** The mean service rate is denoted by μ.

(c) The Queue discipline

The queue discipline is the rule determining the formation of the Queue, the manner of the customer's behavior while waiting, and the manner in which they are chosen for service. The simplest discipline id first come, First Served" according to which the customers are served in the order of their arrival. Such type of Queue discipline is observed at a ration shop. If the order is reversed, we have 'Last come, first served' discipline, as in the case of a big godown the items which come last are taken out first.

Some of the queue service disciplines are:

FIFO – First in, First out or (FCFS)

LIFO – Last in, First out, (LCFS)

SIRO – Service in Random order.

(d) Customer's behavior:

The customer generally behaves in 4 ways:

(i) Balking: A customer may leave the Queue, if there is no waiting space.
(ii) Reneging: This occurs when the waiting customer leaves the Queue due to Impatience.
(iii) Priorities: In certain applications some customers are served before others regardless of their order of arrival.
(iv) Jockeying: Customers may jump from one, waiting line to another.

Transient and Steady States

A system is said to be in **Transient State** when its operating characteristics are dependent on time.

Steady State: A system is said to be in **Steady State** when the behavior of the system is independent of time. Let $P_n(t)$ denote the probability that there are 'n' units in the system at time t. Then in steady state

$$\Rightarrow \lim_{t \to \infty} P_n'(t) = 0$$

Kendal'a Notation for representing Queuing models

Generally Queuing Model may be completely specified in the following symbol form :(a|b|c) :(d|e):

Where a=Probability law for the arrival

b=Probability law according to which customers are served.

c=Number of channels (or Service stations).

d=Capacity of the system.

e=Queue discipline.

Distribution of Arrivals "The Poisson Process" Arrival Distribution Theorem. (Pure Birth Process)

If the arrivals are completely random, then the probability distribution of a number of arrivals in a fixed – time interval follows a Poisson distribution.

Model 1: (M|M|I): ∞/FCFS) – Birth and Death Model

With usual notation, show that probability distribution of Queue length is given by ρ_n (1- ρ) where $\rho = \frac{\lambda}{\mu} < 1$ and n≥0.

Measure of Model I

1. **To find the average (expected) number of units in the system, L.**
   Solution:

   By definition of excepted value

$$L_s = \sum_{n=1}^{\infty} nP_n = \sum_{n=1}^{\infty} n \left(\frac{\lambda}{\mu}\right)_n \left(1 - \frac{\lambda}{\mu}\right)$$

$$= \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right) \sum_{n=1}^{\infty} n \left(\frac{\lambda}{\mu}\right)^{n-1}$$

$$= \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right) \left[1 + \frac{\lambda}{\mu} + 3\left(\frac{\lambda}{\mu}\right)_2 + \ldots\ldots\ldots + \right.$$

$$= \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)\left(1 - \frac{\lambda}{\mu}\right)^{-2} \text{ using Bionomial series}$$

$$= \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}}$$

$$L_s = \frac{\rho}{1-\rho} \text{ where } \rho = \frac{\lambda}{\mu} < 1$$

2. **To find the average length of Queue, L_q**

$$L_q = L_s - \frac{\lambda}{\mu}$$

$$= \frac{\rho^2}{1-\rho}$$

3. **Excepted waiting time in the system**

$$W_s = \frac{L_s}{\lambda}$$

$$= \frac{1}{\mu-\lambda}$$

4. **Waiting time in the Queue,**

$$W_q = \frac{L_q}{\lambda}$$

$$= \frac{\lambda}{\mu(\mu-\lambda)}$$

5. **Excepted waiting time of a customer who has to wait (W | W > 0)**

$$= \frac{1}{\mu-\lambda}$$

6. Excepted length of the non – empty Queue, (L | L > 0)

$$= \frac{\mu}{\mu - \lambda}$$

7. Probability of Queue size ≥ N is ρ^N

8. Probability [Waiting time in the system ≥ *t*]

$$= \int_t^{\infty} (\mu - \lambda) e^{-(\mu-\lambda)w} dw$$

9. Probability [Waiting time in the queue ≥ *t*]

$$= \int_t^{\infty} \rho(\mu - \lambda) e^{-(\mu-\lambda)w} dw$$

10. Traffic Intensity = $\frac{\lambda}{\mu}$

Example 1:

In a railway Marshalling yard, goods train arrive at a rate of 30 Trains per day. Assuming that inter arrival time follows an exponential distribution and the service time distribution is also exponential, with an average of 36 minutes. Calculate the following:

(i) The mean Queue size (line length)
(ii) The probability that Queue size exceeds 10
(iii) If the input of the train increases to an average 33 per dya, what will be the changes in (i), (ii)?

Solution:

$\lambda = \frac{30}{60 \times 24} = \frac{1}{48}$, $\mu = \frac{1}{36}$ trains per minute

$\therefore \rho = \frac{\lambda}{\mu} = \frac{36}{48} = 0.75$

(i) $L_s = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = 3$ trains

(ii) P (≥ 10) = $(0.75)^{10} = 0.056$

(iii)When the input increases to 33 trains per day,

We have $\lambda = \frac{30}{60 \times 24} = \frac{1}{480}$ and $\mu = \frac{1}{36}$ trains per minute

Now, $L_s = \frac{\rho}{1-\rho}$ where $\rho = \frac{\lambda}{\mu}$; $\rho = 0.825$

$\therefore L_s = \frac{0.825}{1-0.825} = 5$ trains (app)

Also $P(\geq 10) = \rho^{10} = (0.825)^{10}$

$= 0.1460$

Example 2:

In a super market, the average arrival rate of customer is 10 in every 30 minutes following Poisson process. The average time taken by the cashier to list and calculate the customer's purchases is 2.5 minutes, following exponential distribution. What is the probability that the Queue length exceeds 6? What is the excepted time spent by a customer in the system?

Solution:

Here the mean arrival rate

$\lambda = \frac{10}{30}$ per minute

and mean service rate $= \frac{1}{2.5}$ per minute

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{3}}{\frac{1}{2.5}} = 0.8333$$

(i) (The probability of Queue size > n) $= \rho^n$

When $n = 6 \Longrightarrow (0.8333)^6 = 0.3348$

(ii) $W_s = \frac{L_s}{\lambda} = \frac{\frac{\rho}{1-\rho}}{\lambda}, \rho = \frac{\lambda}{\mu}$

$= \frac{0.8333}{1-0.8333} \times 3 = \frac{2.499}{0.167}$

= 14.96 minutes

Example 3:

In a public telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone, find (i) Excepted number of callers in the booth at any time (ii) The proportion of the time the booth is excepted to be Idle?

Solution:

Mean arrival rate λ = 15 per hour

Mean service rate $\mu = \frac{1}{3}$ x 60 = 20 per hr.

∴ (i). Excepted length of the non-empty

Queue = $\frac{\mu}{\mu-\lambda} = \frac{20}{20-15} = 4$

(ii). The service is busy means = $\frac{\lambda}{\mu} = \frac{15}{20} = \frac{3}{4}$

∴ The booth excepted to Idle for 1 - $\frac{3}{4} = \frac{1}{4}$ hrs

= 15 minutes

Example 4:

A T.V repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is Poisson with an average rate of 10 per 8 hour day, what is his excepted Idle time day? How many hobs are ahead of the average set just brought in?

Solution:

Mean service rate $\mu = \frac{1}{30}$ per minute

$= \frac{1}{30}$ x 60 = 2 sets per hour

Mean arrival rate = $\frac{10}{8}$ per hr

$\rho = \frac{\lambda}{\mu}$ where μ = 2 per hr.

$\rho = \frac{5}{4}$ per hr

The utilization factor $\frac{\lambda}{\mu}$ is $\frac{5}{4 \times 2} = \frac{5}{8}$

For 8 hr day, Repairman's busy time = 8 x $\frac{5}{8}$ = 5 hrs

∴ Idle time of repairman = 8 – 5 hrs = 3 hrs

The number of jobs ahead = No. of units in the system

$$= \frac{\rho}{1-\rho} = \frac{\frac{5}{8}}{1-\frac{5}{8}} = \frac{\frac{5}{8}}{\frac{3}{8}} = \frac{5}{3}$$

= 2 app, TV sets

Example 5:

At a public Telephone booth in a Post Office arrivals are considered to be Poisson with an average inter-arrival time of 12 minutes. The length of the phone call may be assumed to be distributed exponentially with an average of 4 minutes. Calculate the following:

(a) What is the probability that a fresh arrival will not have to wait for the phone?
(b) What is the probability than an arrival will have to wait more than 10 minutes before the phone is free?
(c) What is the average length of Queues formed from time to time?

Solution:

Mean arrival rate, $\lambda = \frac{1}{12}$, $\mu = \frac{1}{4}$

Mean service rate, $\frac{\lambda}{\mu} = \frac{4}{12} = \frac{1}{3}$ = 0.33

(a) Probability that a fresh arrival will not have to wait

$$= 1 - \frac{\lambda}{\mu} = 1 - 0.33$$

$$= 0.67$$

(b) Probability that an arrival will have to wait for atleast 10 minutes

$$= \int_t^{\infty} \left(\frac{\lambda}{\mu}\right)(\mu - \lambda)e^{-(\mu-\lambda)t}\,dt$$

$$= \int_t^{\infty} (0.33)(0.25 - 0.083)e^{-0.167t}\,dt$$

$$= 0.05511\left[\frac{e^{-0.167t}}{-0.167}\right]_{10}^{\infty}$$

$$= 0.0621$$

(c) The average length of Queues from time to time

$$(L > 0) = \frac{\mu}{\mu-\lambda}$$

$$= \frac{0.25}{0.25-0.085}$$

$$= 1.5$$

Example 6:

People arrive at a Theatre ticket booth in Poisson distributed arrival rate of 25 per hour. Service time is constant at 2 minutes. Calculate

(a) The mean number in the waiting line
(b) The mean waiting time
(c) The utilization factor.

Solution:

λ = per hr;

$\mu = \frac{1}{2}$ x 60 = 30 per hr.

$\therefore \rho = \frac{\lambda}{\mu} = \frac{25}{30} = \frac{5}{6} = 0.833$

(i) Length of the Queue

$$L_q = \frac{\rho^2}{1-\rho}$$

$$= \frac{(0.833)^2}{1-0.833}$$

$$= \frac{0.693889}{0.167} = 4 \text{ (app)}$$

(ii) Mean waiting time = $\frac{L_q}{\lambda}$

$$= \frac{4}{25}$$

= 9.6 minutes

(iii) Utilization factor $\rho = \frac{\lambda}{\mu}$ = 0.833.

Model II:

(M | M | I) : (N | FCFS)

Here the capacity of the system is limited, say N. Infact arrivals will not exceed N in any case. The various measures of this Model are

1. $P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ where $\rho = \frac{\lambda}{\mu}$, $\left\{\frac{\lambda}{\mu} > 1\right.$ is allowed

2. $P_n = \frac{1-\rho}{1-\rho^{N+1}}\ \rho^n$ for n = 0, 1, 2,….N

3. $L_s = P_0 \sum_{n=0}^{N} n\rho^n$

4. $L_q = L_s - \frac{\lambda}{\mu}$

5. $W_s = \frac{L_s}{\lambda}$

6. $W_q = \frac{L_q}{\lambda}$

Example 1:

If for period of 2 hours in a day (8 – 10 AM) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period

(a) The probability that the yard is empty
(b) Average Queue length, assuming that capacity of the yard is 4 trains only.

Solution:

Here $\rho = \frac{36}{20} = 1.8$, N = 4

(a) $P_0 = \frac{\rho - 1}{\rho^5 - 1} = 0.04$

(b) Average Queue size

$= P_0 \sum_{n=0}^{N} n\rho^n$

$= 0.04\, \rho + \rho^2 - \rho^3 + \rho^4)$

$= 2.9$

$\approx$ 3 trains

Example 2:

In a railway marshalling yard, goods training trains arrive at a rate of 30 trains per day. Assume that the inter arrival – time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, calculate

(a) The probability that the yard is empty
(b) Average queue length assuming that the line capacity of the yard is 9 trains.

Solution:

Here N = 10, $\lambda = \frac{10}{60}, \mu = \frac{1}{5}$

$\rho = \frac{\lambda}{\mu} = \frac{5}{6}$

$P_0 = \frac{1-\rho}{1-\rho^{11}} = \frac{1-\frac{5}{6}}{1-\left(\frac{5}{6}\right)^{11}}$

$= \frac{0.1667}{0.8655} = 0.1926$

$P_n = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)\rho^n$

$= (0.1926) \times \left(\frac{5}{6}\right)^n$, n = 0, 1, 2, …..,10

Example 4:

A car park contains 5 cars. The arrival of cars is poisson at a mean rate of 10 per hour. The length of time each car spends in the car park is negative exponential distribution with mean of 2 hours. How many cars are in the car park on average?

Solution:

N = 5, $\lambda = \frac{10}{60}, \mu = \frac{1}{2 \times 60}, \rho = \frac{\lambda}{\mu} = 20$

$P_0 = \left(\frac{1-\rho}{1-\rho^{N+1}}\right)$

$= \frac{1-20}{1-20^6} = \frac{-19}{-6399} = 2.962 \times 10^{-7}$

$L_s = \frac{1-\rho}{1-\rho^{N+1}} \sum_{n=0}^{N} n\rho^n$

$= (2.9692 \times 10^{-3}) \times \sum_{n=0}^{5} n(2.9692 \times 10^{-3})^{n}$

$= (2.9692 \times 10^{-3}) \times [0 + (2.9692 \times 10^{-3}) + 2 \times (2.9692 \times 10^{-3})^{2} + 3 \times (2.9692 \times 10^{-3})^{3} + 4 \times (2.9692 \times 10^{-3})^{4} + 5 \times (2.9692 \times 10^{-3})^{5}]$

$= (2.9384 \times 10^{-3}) \times [0 + (2.9692 \times 10^{-3}) + 2 \times (2.9384 \times 10^{-3}) + 3 \times (2.9692 \times 10^{-3})^{2} + 4 \times (2.9692 \times 10^{-3})^{3} + 5 \times (2.9692 \times 10^{-3})^{4}]$

$= 5$ (app)

Example 5:

At a one-man barber shop, the customers arrive following poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customers, find the average time a customer spends in the system.

Solution:

$$W_s = \frac{P_0}{\lambda} \sum_{n=0}^{N} n\rho^{n}$$

Here $\lambda = 5$ per hr

$\mu = \frac{1}{10} \times 60$

$= 6$ per hr and $N = 5$

$\therefore \frac{1}{\mu} = \frac{5}{6} = \rho$

$$P_0 = \frac{1-\rho}{1-\rho^6} = \frac{1-\frac{5}{6}}{1-\left(\frac{5}{6}\right)^6}$$

$$= \frac{1-\frac{1}{6}}{1-\left(\frac{1}{6}\right)^6} = \frac{\frac{1}{6}}{1-1.07 \text{ x } 10^{-4}}$$

$$= \frac{0.1666}{1-0.0001} = \frac{0.1666}{1}$$

$$= 0.1666$$

$$\frac{L_s}{\lambda} = W_s$$

Where $Ls = 0.166 \text{ x } \sum_{n=0}^{N} n\rho^n$

$$= 0.166 \text{ x } \sum_{n=0}^{N} n\left(\frac{5}{6}\right)^n$$

$$= 0.166 \text{ x } \rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + 5\rho^5]$$

$$= 0.166\left(\frac{5}{6} + 2\left(\frac{5}{6}\right)^2 + 3\left(\frac{5}{6}\right)^3 + 4\left(\frac{5}{6}\right)^4 + 5\left(\frac{5}{6}\right)^5\right)$$

$$= 0.166\ [0.833 + (2 \text{ x } 0.694) + (3 \text{ x } 0.5782) + (4 \text{ x } 0.4816) + (5 \text{ x } 0.4012)]$$

$$W_s = \frac{0.1666}{5}[0.833 + 1.388 + 1.7346 + 1.9264 + 2.006]$$

$$= \frac{0.1666 \text{ x } 7.88}{5} = \frac{1.3094}{5} = 0.26 \text{ hrs} \approx 16 \text{ minutes}$$

Possible Questions

Part-B(6 marks)

1. Find the optimum integer solution to the following LPP

   Maximize $Z = x_1 + 4x_2$

   Subject to the constraints

   $2x_1 + 4x_2 \leq 7$

   $5x_1 + 3x_2 \leq 15$

   and $x_1, x_2 \geq 0$ and are integers

2. Solve the following ILPP using Branch and Bound method

   Maximize $Z = 7x_1 + 9x_2$

   Subject to the constraints

   $-x_1 + 3x_2 \leq 6$

   $7x_1 + x_2 \leq 35$

   $x_2 \leq 7$

   and $x_1, x_2 \geq 0$ and are integers

3. Solve the following mixed integer programming problem.

   Minimize $Z = x_1 - 3x_2$

   Subject to

   $x_1 + x_2 \leq 5$

   $-2x_1 + 4x_2 \leq 11$

   and $x_1, x_2 \geq 0$ and x_2 be an integer

4. In a super market, the average arrival rate of customer is 10 in every 30 minutes following Poisson process. The average time taken by the cashier to list and calculate the customer's purchases is 2.5 minutes, following exponential distribution. What is the probability that the Queue length exceeds 6? What is the excepted time spent by a customer in the system?

5. Find the optimum integer solution to the following IPP.

   Maximize $Z = x_1 + x_2$

   Subject to

   $2x_1 + 5x_2 \leq 16$

   $6x_1 + 5x_2 \leq 30$

   and $x_1 \geq 0$, $x_2 \geq 0$ and are all integers

6. At a one-man barber shop, the customers arrive following poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average

service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customers, find the average time a customer spends in the system.

7. Find the optimum integer solution to the following LPP.

Maximize $Z = x_1 + 2x_2$

Subject to the constraints

$2x_2 \leq 7$

$x_1 + x_2 \leq 7$

$2x_1 \leq 11$

and $x_1 \geq 0$, $x_2 \geq 0$ and are all integers.

8. Solve the following mixed integer programming problem.

Minimize $Z = 10x_1 + 9x_2$

Subject to the constraints

$x_1 \leq 8$

$x_2 \leq 10$

$5x_1 + 3x_2 \geq 45$

and x_1 , $x_2 \geq 0$, x_1 be an integer.

9. Find the optimum integer solution to the following LPP.

Maximize $Z = 2x_1 + 2x_2$

Subject to the constraints

$5x_1 + 3x_2 \leq 8$

$2x_1 + 4x_2 \leq 8$

and $x_1 \geq 0$, $x_2 \geq 0$ and are integers.

10. Solve the following mixed integer programming problem.

Maximize $Z = x_1 + x_2$

Subject to the constraints

$2x_1 + 5x_2 \leq 16$

$6x_1 + 5x_2 \leq 30$

and $x_2 \geq 0$ and x_1 is non negative integers.

Part-C(10 marks)

1. At a one-man barber shop, the customers arrive following poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customers, find the average time a customer spends in the system.

Question	Opt 1	Opt 2	Opt 3	Opt 4	Opt 5	Opt 6	Answer
A LPP in which all or some of the decision variables are constrained to assume non negative integer values is called an -------------	Integer programming problem	Dynamic programming problem	Non linear programming problem	Decision Analysis			Integer programming problem
In a LPP, if all the variables in the optimal solution are restricted to assume non negative integer values, then it is called the -----------------	Zero - one IPP	Pure IPP	Mixed IPP	Non IPP			Pure IPP
A systematic procedure for solving pure I.P.P is ----------	Hungarian method	Cutting method	Revised simplex method	Modi method			Cutting method
In cutting algorithm, each cut involves the introduction of _______	an equality constraint	less than or equal to constraint	greater than or equal to constraint	an artificial variable.			less than or equal to constraint
In the context of Branch and bound method, which of the following is not correct?	It can be used to solve any kind of programming problem	It divides the feasible region into smaller parts by the process of branching	It is very usefull employed in problems where ther a finite number of solutions	It is not a standardised method and is applied differently for different problems			It can be used to solve any kind of programming problem
Which of the following is correct?	If the optimum solution to an LPP has all integer values, it may or may not be an optimum integer solution	the solution to the IPP can always be obtainted by rounding off the fractional values in the solution obtainted by simplex method	A cut may or may not eliminate any points that are feasible for the IPP	a cut does not eliminate any points which are feasible for the IPP.			a cut does not eliminate any points which are feasible for the IPP.
Which of the following is not correct?	The optimum solution to LPP satisfies the cut that is introduced on the basis of it.	A cut is formed be choosing a row in the optimum simples table that corresponds to a non integer variable.	the cutting plane algorithm assures optimal integer solution in a finite number of iterations	The cutting plane algorithm requires all RHS values as well as all the coefficients in the constraints to be integers.			The optimum solution to LPP satisfies the cut that is introduced on the basis of it.
Which of the following is not an integer linear programming problem?	Zero - one IPP	Pure IPP	Mixed IPP	continuous IPP			continuous IPP
The part of the feasible solution space eliminated by plotting a cut constraints _________	only non integer solutions	only integer solutions	both integer and non integer solutions	only rational solutions			only non integer solutions
While solving IP problem any non integer variable in the solution is picked up to _______	obtain the cut constraint	enter the solution	leave the solution	no solution			obtain the cut constraint
In a Branch and Bound minimization tree, the lower bounds on objective function value________	do not decrease in value	do not increase in value	remains constant	no constant			remains constant
In a mixed integer programming problem we have ------	all of the decision variables require integer solutions	few of the decision variables require integer solutions	different objective functions are mixed together	mixed solution to the problem			few of the decision variables require integer solutions
Branch and Bound method divides the feasible solution space into smaller parts by --------	branching	bounding	enumering	parts			branching
In a mixed-integer programming problem--------- requries integer solutions	all of the decision variables	few of the decision variables	different objective functions	mixed variables			few of the decision variables
A non integer variable is chosen in the optimal simplex table of the integer LP problem to ---------	leave the basis	enter the basis	to construct a gomory cut	Branch and Bound method			to construct a gomory cut
The corners of the reduced feasible region of an integer LP problem contains	only integer solution	optimal integer solution	only non-integer solution	not a solution			only integer solution
An------------ programming was used for capital budgeting in hospital	integer	Hungerian method	Dynamic programming problem	Decision Analysis			integer
An integer programming was used for capital budgeting in --------------	school	railway	transport	hospital			hospital
If the arrivals are completely random, then the probability distribution of a number of arrivals in a fixed-time interval follows a ------- distribution	Poisson	normal	binomial	polynomial			Poisson
The ---------- describes the way in which the customers arrive and join the system	service mechanism	input	queue discipline	customer behaviour			input

The arriving people in a queueing system are called ------------.	Input	servers	customers	queue	customers
Mean service time is denoted by ----------.	m	l	1 / l	1 / m	m
The traffic intensity in queueing is defined by ----------.	p / (p–1)	m / l	l / m	1 / (1– p)	l / m
A system is said to be in --------- state when its operating characteristics are dependent on time	Steady	arrival	service	transient	transient
A system is said to be in --------- state when the behaviour of the system is independent on time.	Steady	arrival	service	transient	Steady
A customer who leaves the queue because the queue is too long then his behaviour is said to be ----------.	Reneging	balking	jockeying	priorities	balking
The Birth–death model is called -----------.	M / M / 1	M / M / N	M / M / ¥	M / M / 2	M / M / 1
Average queue length in (M / M / 1) : (∞ / FCFS) is --------------.	(1– r) / r	r / (1– r)	$r / (1– r^2)$	$r^2 / (1– r)$	$r^2 / (1– r)$
The expected waiting time in the queue is calculated by the formula ------------.	1 / (m – l)	b) l / (m – l)	l / m (m – l)	l / m	l / m (m – l)
In Birth–death model, the probability distribution of queue length is given by --------.	$r^n / (1– r)$	b) $r^2 / (1– r)$	r / (1– r)	$(1– r) / r^n$	$r^n / (1– r)$
First In First Out (FIFO) is known as the --------------.	Input	service mechanism	customer behaviour	queue discipline	queue discipline
The probability of an empty system is given by --------------.	1 – (l / m)	l / (m – l)	l / m (m – l)	l / m	1 – (l / m)
A Queuing system can be completely described by --------------.	The input, the service mechanism	The input, the service mechanism and the queue discipline	The input, the service mechanism, the queue discipline and customer behaviour	The input, the service mechanism and customer behaviour	The input, the service mechanism, the queue discipline and customer behaviour
The probability of Queue size ≥ N is ------------.	$r^n / (1– r)$	r^n	$1– r^n$	$(1– r) / r^n$	r^n
If l = 3 , m = 2 then r = ------	1.5	3	2	0.6	1.5
Average waiting time in the queue is given by ----------	1/m - l	l	m	1/m + l	l

UNIT-III

SYLLABUS .

Replacement Theory: replacement of items that deteriorate - Replacement of items that fail Group replacement and individual replacement.

Replacement Theory:

The Replacement Theory in Operations Research is used in the decision making process of replacing a used equipment with a substitute; mostly a new equipment of better usage. The replacement might be necessary due to the deteriorating property or failure or breakdown of particular equipment. The 'Replacement Theory' is used in the cases like; existing items have out-lived, or it may not be economical anymore to continue with them, or the items might have been destroyed either by accident or otherwise. The above discussed situations can be solved mathematically and categorized on some basis like:

- Items that deteriorate with time e.g. machine tools, vehicles, equipment buildings etc.
- Items becoming out-of-date due to new developments like ordinary weaving looms by automatic, manual accounting by tally etc.
- Items which do not deteriorate but fail completely after certain amount of use like electronic parts, street lights etc *(Group Replacement)* and
- The existing working staff in an organization gradually diminishing due to death, retirement, retrenchment & otherwise *(Staff Replacement)*.

Replacement Policy for Equipment which Deteriorate Gradually

Let us see the fiUSDt case of gradual failure of items with time. Consider the example of a Motor Vehicle; the pattern of failure here is progressive in nature i.e. as the life of vehicle increases; its efficiency decreases. This results in additional expenditure in running or maintaining this vehicle and at the same time its resale value (also called as scrap value) also keeps on decreasing. The above case makes this situation a typical case for applying 'Replacement Theory'.

Group Replacement theory:

Replacement of items that fail suddenly

There are certain items which do not deteriorate but fail completely after certain amount of use. These kinds of failures are analyzed by the method called as group replacement theory. Here, large numbers of items are failing at their average life expectancy. This kind of items may not have maintenance costs as such but they fail suddenly without any prior warning. Also, in case of sudden breakdowns immediate replacement may not be available. Few examples are fluorescent tubes, light bulbs, electronic chips, fuse etc.

Let's consider the example of street lights. We often see street-lights being repaired by the corporation staff using extendable ladders. If a particular light is beyond repairs, then it is replaced. This kind of policy of replacement is called as 'replacement of items as-and-when they fail' or *'Individual Replacement'*. On the other hand, if all the street lights in a particular cluster are replaced as and when they fail and also simultaneously in groups, then the policy is called as *'Group Replacement'*. It should be noted that, **group replacement does involve periodic simultaneous replacements along with individual replacements in between**.

It is found that replacing these random failing items simultaneously at specific intervals is economical as compared to replacing them only when an item fails. A long period between group replacements results in increase in cost of individual replacements, while frequent group replacements are definitely costly. There lies the need to balance this and find an optimum replacement time for optimum cost of replacement.

- Replacement of the equipment that fails completely.
  - Individual Replacement Policy : Mortality Theorem
  - Group Replacement of items that fail completely

OBJECTIVES OF REPLACEMENT

The primary objective of replacement is to direct the organization towards profit maximization or cost minimization. Deciding the replacement policy that determines the optimal replacement age of equipment, instead of using with higher maintenance costs for long time, is the main objective of replacement problem. For instance, in order to replace an:

- item whether to wait till its failure or replacing at an early age with higher cost.
- equipment whether to replace the inefficient equipment with a similar type of equipment or with a modern one.

FAILURE MECHANISMS OF EQUIPMENTS

The term 'failure' has a wider meaning in *business* than what it has in our daily life. Failures can be discussed under two categories viz., Gradual Failures, and Sudden Failures.

Costs to be considered: Various costs that are to be included in this model are all those costs that depend upon the choice or age of the equipment. In some special cases, certain costs need to be included in the calculations. For example, in considering the optimum decisions of replacement for a particular machine, the costs that do not change with the age of the equipment need not be considered. The costs to be considered while calculating the optimum replacement period are:

(i) Capital Recovery Cost =Average First Cost, if rate of interest is zero percent.

(ii) Running cost = Average operating and maintenance cost (O&M cost).

The above associated costs can be expressed as average cost per period and the sum of the above two costs can be considered as total cost.

GRADUAL FAILURE

The mechanism under this category is progressive. That is, as the life of an item increases, its efficiency deteriorates, causing:

- Increased expenditure for operating costs
- Decreased equipments' productivity
- Decrease in the value of the equipment

Example: bearings, pistons, piston rings, 'Automobile Tyres', mechanical systems like machines, machine tools, flexible manufacturing equipment etc. fall under this category.

A machine loses efficiency with time and we have to determine the best time at which we have to go for a new one. In case of a vehicle, the maintenance cost is increasing as it is getting aged. These costs increase day by day if we postpone the replacement.

INDIVIDUAL REPLACEMENT POLICY (MORTALITY THEOREM)

In this policy, 'an equipment or item is to be replaced as soon as it fails. Mortality tables will be referred to determine the life span of any equipment or the probability distribution of failure. To discuss such type of replacement policy, the problem of human population is considered. No group of people ever existed under the conditions.

a) that all deaths are immediately replaced by births, and

b) that there are no other entries or exits.

Individual Replacement:

The mean age of items $= 1\times p_1 + 2\times p_2 + 3\times p_3 + 4\times p_4 + 5\times p_5 + \ldots + k.P_k = \sum_{i=1}^{n} i.p_i$

The number of failures in each period in steady state $= N/\sum_{i=1}^{n} i.p_i$

The cost of replacing items individually on failure $= = \left[(N/\sum_{i-1}^{n}(i.p_i)\right]C_1$

M(t) = Number of items surviving at time t
M(t - 1) = Number of items surviving at time (t - 1)
N = Total number of items in the system

The probability of failure of items during the interval t and (t - 1) is given by

$$\frac{M(t-1) - M(t)}{N}$$

The conditional probability that any item survived upto age (t - 1) and will fail in the next period is given by

$$\frac{M(t-1) - M(t)}{M(t-1)}$$

Following mortality rates have been observed for certain type of light bulbs.

Time (weeks)	0	1	2	3	4	5	6	7	8	9	10
Number of bulbs still operating	100	94	82	58	40	28	19	13	7	3	0

Calculate the probability of failure.

Solution.

Here, t is the time (weeks) and M (t) is the number of bulbs still operating. The probability of failure can be calculated as shown in the following table.

Table

Time (t)	M (t)	Probability of failure $p_i = [M(t-1) - M(t)] / N$
0	100	----
1	94	(100 - 94)/100 = 0.06
2	82	(94 - 82)/100 = 0.12
3	58	(82 - 58)/100 = 0.24
4	40	(58 - 40)/100 = 0.18
5	28	(40 - 28)/100 = 0.12
6	19	(28 - 19)/100 = 0.09
7	13	(19 - 13)/100 = 0.06
8	7	(13 - 7)/100 = 0.06
9	3	(7 - 3)/100 = 0.04
10	0	(3 - 0)/100 = 0.03

Following mortality rates have been observed for a certain type of electronic component.

Month	0	1	2	3	4	5	6
% surviving at the end of the month	100	97	90	70	30	15	0

There are 10000 items in operation. It costs Re 1 to replace an individual item and 35 paise per item, if all items are replaced simultaneously. It is decided to replace all items at fixed intervals & to continue replacing individual items as and when they fail. At what intervals should all items be replaced? Assume that the items failing during a month are replaced at the end of the month.

Solution.

Month	% surviving at the end of the month	Probability of failure p_i
0	100	----
1	97	(100 - 97)/100 = 0.03
2	90	(97 - 90)/100 = 0.07
3	70	(90 - 70)/100 = 0.20
4	30	(70 - 30)/100 = 0.40
5	15	(30 - 15)/100 = 0.15
6	0	(15 - 0)/100 = 0.15

The given problem can be divided into two parts.

I. Individual replacement.
II. Group replacement.

Case I

It should be noted that no item survives for more than 6 months. Thus, an item which has survived for 5 months is sure to fail during sixth month.

The expected life of each item is given by

= $\Sigma x_i p_i$, where x_i is the month and p_i is the corresponding probability of failure.
= (1 X 0.03) + (2 X 0.07) + (3 X 0.20) + (4 X 0.40) + (5 X 0.15) + (6 X 0.15)
= 4.02 months.

∴ Average number of replacement every month = N/(average expected life) = 10000/4.02 = 2487.5
= 2488 items (approx.).
Here average cost of monthly individual replacement policy = 2488 X 1 = Rs. 2488/-, (the cost being Re 1/- per item).

Case II

Let N_i denote the number of items replaced at the end of ith month.

Calculating values for N_i

N_0 = Number of items in the beginning = 10,000
N_1 = Number of items during the 1st month X probability that an item fails during 1st month of installation
= 10000 X 0.03 = 300

N_2 = Number of items replaced by the end of second month
=(Number of items in beginning X probability that these items will fail in 2nd month) + (Number of items replaced in first month X probability that these items will fail during second month)

$=N_0P_2 + N_1P_1$
=(10000 X 0.07) + (300 X 0.03) = 709

$N_3 = N_0P_3 + N_1P_2 + N_2P_1$
= (10000 X 0.20) + (300 X 0.07)+ (709 X 0.03) = 2042

$N_4 = N_0P_4 + N_1P_3 + N_2P_2 + N_3P_1$
= (10000 X 0.40) + (300 X 0.20)+ (709 X 0.07) + (2042 X 0.03) = 4171

$N_5 = N_0P_5 + N_1P_4 + N_2P_3 + N_3P_2 + N_4P_1$
= (10000 X 0.15) + (300 X 0.40)+ (709 X 0.20) + (2042 X 0.07) + (4171 X 0.03) = 2030

$N_6 = N_0P_6 + N_1P_5 + N_2P_4 + N_3P_3 + N_4P_2 + N_5P_1$
= (10000 X 0.15) + (300 X 0.15)+ (709 X 0.40) + (2042 X 0.20) + (4171 X 0.07) + (2030 X 0.03) = 2590.

From the above calculations, it is observed that N_i increases upto fourth month and then decreases. It can also be seen that N_i will later tend to increase and the value of N_i will oscillate till the system acquires a steady state.

The optimum replacement cycle under group replacement is given in the following table.

End of month	Total no. of items failed	Cumulative no. of failure	Cost of replacement after failure (Re 1/ item)	Cost of all replacement (Rs. 0.35/ item)	Total cost (Rs.)	Average cost per month (Rs.)
1	300	300	300	3500	3800	3800
2	709	1009	1009	3500	4509	2254.50
3	2042	3051	3051	3500	6551	2183.66
4	4171	7222	7222	3500	10722	2680.50
5	2030	9252	9252	3500	12752	2550.40
6	2590	11842	11842	3500	15342	2557.00

Possible Questions:

Part- B(6 marks)

1. A firm is considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value, Rs. 200. The running (maintenance and operating) costs in rupees are found from experience to be as follows :

Year :	1	2	3	4	5	6	7	8
Running cost :	200	500	800	1200	1800	2500	3200	4000

When should the machine be replaced ?

2. Machine A costs Rs. 9000. Annual operating costs are Rs. 200 for the first year, and then increase by Rs. 2000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine ?
3. Machine A costs Rs. 10,000. Annual operating costs are Rs. 400 for the first year, and then increase by Rs. 800 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine ?
4. The data collected in running a machine, the cost of which is Rs. 60,000 are given below:

Year:	1	2	3	4	5
Resale value (Rs.)	42,000	30,000	20,400	14,400	9,650
Cost of spares (Rs.)	4,000	4,270	4,880	5,700	6,800
Cost of labour (Rs.)	14,000	16,000	18,000	21,000	25,000

Determine the optimum period for replacement of the machine.

5. The cost of a machine is Rs. 6,100 and its scrap value is Rs. 100. The maintenance costs found from experience are as follows :

Year	1	2	3	4	5	6	7	8
Maintenance cost (Rs.)	100	250	400	600	900	1,200	1,600	2,000

When should the machine be replaced?

6. A firm is considering replacement of a machine whose cost price is Rs. 17,500 and the scrap value is Rs. 500. The maintenance costs (in Rs.) are found from experience to be as follows:

Year:	1	2	3	4	5	6	7	8
Maintenance cost (Rs.)	200	300	3,500	1,200	1,800	2,400	3,300	4,500

When should the machine be replaced?

7. Let the value of money be assumed to be 10% per year and suppose that machine A is replaced after every 3 years whereas machine B is replaced after every six years. The yearly costs of both the machines are given below:

Year:	1	2	3	4	5	6
Machine A :	1000	200	400	1000	200	400
Machine B :	1700	100	200	300	400	500

Determine which machine should be replaced.

8. The initial cost of an item is Rs. 15,500 and maintenance or running costs for different years are given below :

Year :	1	2	3	4	5	6	7
Running cost (in Rs.)	2,500	3,000	4,000	5,000	6,500	8,000	10,000

Part C(10 marks)

1. Machine A costs Rs. 9000. Annual operating costs are Rs. 200 for the first year, and then increase by Rs. 2000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine ?
(b) Machine B costs Rs. 10,000. Annual operating costs are Rs. 400 for the first year, and then increase by Rs.800 every year. You now have a machine of type A which is one year old. Should you replace it with B. if so when?

Question	Opt 1	Opt 2	Opt 3	Opt 4	Opt 5	Opt 6	Answer
The study of replacement is concerned with situations that arise when items need ______________ due to their deteriorating efficiency,failure or breakdown.	replacement	shortage	inventory	expensive			replacement
Total cost = Capital cost - scrap value + _______________	setup cost	maintenance cost	replacement cost	initial cost			maintenance cost
Equipment should be replacement when maintenance cost ________ the average annual total cost.	greater than	less than	equals	greater than or equal to			equals
The rate of interest becomes _________ and hence v approaches unity when the time value of money is taken into consideration.	three	one	two	zero			zero
If w(n1) _________ w(n2), choose equipment A	less than	greater than	equals	not equal			less than
If w(n1) _________ w(n2), choose equipment B	less than	greater than	equals	not equal			greater than
If w(n1) _________ w(n2), both equipments are equally good.	less than	greater than	equal	not equal			equal
In ____________ policy an item is replaced immediately after its failure.	individual replacement	group replacement	order replacement	removable			individual replacement
In ____________ policy all the items must be replaced	individual replacement	group replacement	order replacement	removable			group replacement
The probability of failure during time period 't' is given by	p(t)=[M(t-1)+M(t)]/N	p(t)=[M(t-1)+M(t)]	p(t)=[M(t-1)-M(t)]	p(t)=[M(t-1)-M(t)]/N			p(t)=[M(t-1)-M(t)]/N
The conditional probability of failure is given by ___________________	p(t)=[M(t-1)+M(t)]/M(t-1)	p(t)=[M(t-1)-M(t)]/M(t-1)	p(t)=[M(t-1)-M(t)]	p(t)=[M(t-1)+M(t)]			p(t)=[M(t-1)-M(t)]/M(t-1)
The probability of survival till age t is given by	p(t)=M(t)	p(t)=N.	p(t)=M(t)/N+1	p(t)=M(t)/N.			p(t)=M(t)/N.
Group replacement must be made at the end of _____________ if the cost of individual replacement for the period is greater than the average cost per unit time period through the end of 't'periods.	t+1' period	t -1' period	't' period	t+2' period			't' period
The quantity (1+r) to the power -n is called the _______________	worth factor	present factor	factor	present worth factor			present worth factor
The quantity (1+r) to the power n is called the _______________	compound amount factor	factor	payment	payment compound amount factor			payment compound amount factor
The expression which lies between Rn-1 and Rn is called the __________ of all the previous n years.	weighted average cost	average cost	weighted cost	cost			weighted average cost
If the next period's __________ is less than the weighted average of previous costs, donot replace the equipments.	average cost	operating cost	cost	weighted cost			operating cost
Replace the equipments if the next period's operating cost is_____ the weighted average of previous costs.	less than	greater than	greater than or equal to	equal			greater than

UNIT-IV

SYLLABUS

Inventory theory: Costs involved in inventory problems - single item deterministic models-economic lot size models without shortages and with shortages having production rate infinite and finite.

INVENTORY MODELS

Introduction

Inventory may be defined as the stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business affairs. The Inventory may be kept in any one of the following forms:

i. Raw material Inventory.

Raw materials which are kept in stock for using in production of goods.

ii. Work – in process Inventory.

Semi finished goods which are stored during production process.

iii. Finished goods Inventory.

Finished goods awaiting shipments from the factory.

Type of Inventory

i. Fluctuation Inventories

In real – life problems, there are fluctuations in the demand and lead time that affect the production of the items. Such types of safety stock are called ***Fluctuation Inventories.***

ii. Anticipated Inventories

These are built up in advance for the season of large sales, a promotion programme or a plant shut down period. Anticipated Inventories keep men and machine hours for future participation.

iii. Lot – size Inventories

Generally rate of consumption is different from rate of production or purchasing. Therefore the items are produced in larger quantities, which result in ***Lot – size Inventories***

Reasons for maintaining Inventory

1. Inventory helps in smooth and efficient running of business.
2. It provides service to the customers at short notice.
3. Because of long – uninterrupted runs, production cost is less.
4. It acts as a buffer stock if shop rejections are too many.
5. It takes care of economic fluctuations.

Costs involved in Inventory Problems

1. **Holding Cost (C_1)**

The cost associated with carrying or holding the goods in stock is known as ***holding cost (or) carrying cost*** per unit of time. Holding cost is assumed to directly vary with the size of inventory as well as the time the item is held in stock. The following components constitute holding cost.

(a) ***Interested capital cost***: This is the interest charge over the capital invested.
(b) Record keeping and Administrative costs.
(c) ***Handling cost:*** These include costs associated with movement of stock, such as cost of labour etc.
(d) Storage costs.
(e) Depreciation costs.
(f) Taxes and Insurance costs.
(g) Purchase price or production costs.

Purchase price per unit item is affected by the quantity purchased due to quantity discounts or price breaks. If P is the purchase price of an item and 1 is the stock holding cost per unit time expressed as a fraction of stock value (in rupees), then the holding cost C_1 = IP.

2. Shortage Cost (C_2)

The penalty costs that are incurred as a result of running out of stock (i.e., shortage) are known as ***shortage or stock – out costs***. These are denoted by C_2. In case where the unfilled demand for the goods may be satisfied at a latter date, these costs are assumed to vary directly with both the shortage quantity and the delaying time on the other hand if the unfilled demand is lost (no backlog case) shortage costs become proportional to shortage quantity only.

3. Set – up cost(C_3)

These costs are associated with obtaining goods through placing an order or purchasing or manufacturing or setting up a machinery before starting production. So they include costs of purchase, requisition, follow up receiving the goods, quality control etc. These are called ***Ordering costs or replenishment costs***, or set-up cost usually denoted by C_3 per production run (cycle). They are assumed to be independent of the quantity ordered or produced.

Variables in Inventory Problem: The variables in inventory model are to two types.

(a) Controlled Variables
(b) Uncontrolled Variables

(a) Controlled Variables

1. How much quantity acquired.
2. The frequency or timing of acquisition.
3. The completion stage of stocked items.

(b) Uncontrolled Variables

These include holding costs, shortage costs, set-up cost and demand.

Lead time, Reorder Level (R.O.L)

Lead time: Elapsed time between the placement of the order and its receipts in inventory is known as Lead time.

Reorder Level: This is the time when we should place an order by taking into consideration the interval between placing the order and receiving the supply. For e.g., we would like to place a new order precisely at the time when Inventory Level reaches zero.

Definition: Economic Order Quantity (E.O.Q) or Economic lot size formula

Economic order quantity(EOQ) is that size or order which minimizes total annual cost of carrying inventory and the cost of ordering under the assumed conditions of certainty and that annual demands are known.

Deterministic Inventory Models

There are 4 types under this category which we shall study as follows:

Model I : Purchasing model with no shortages.

Model II : Manufacturing model with no shortages.

Model III : Purchasing model with shortages.

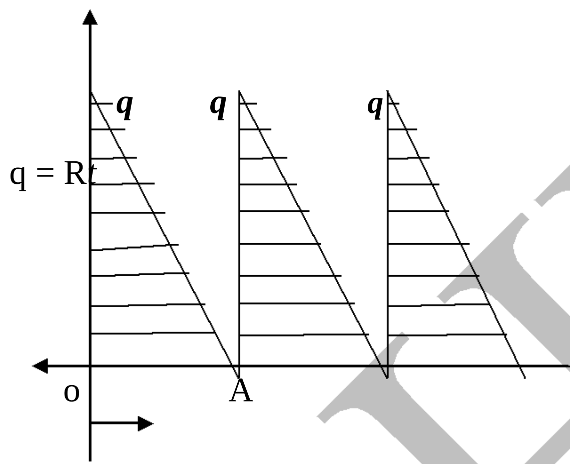
Model IV : Manufacturing model with shortages.

Model I : Purchasing model with no shortages.

(*Demand rate uniform, Production rate infinite***)**

A manufacturer has order to supply goods at a uniform rate of R per unit time. No shortages are allowed, consequently the shortage cost is Infinity. He starts a production run every *t* time units, where *t* is fixed and the set up cost per production run is C_3. Production time is negligible. (replacement Instantaneous) $C_1$is the cost of holding one unit in inventory for a unit of time. The manufacturer's problem is to determine

1. How frequently he should make a production run
2. How many units should be made per production run. Diagrammatic representation of this *model*.



If a production run is made at intervals t, a quantity q = Rt must be produced in each run. Since the stock in small time dt is Rtdt, the stock if period t will be

$$\int_0^t Rt\ dt = \frac{1}{2}Rt^2$$

$$= \frac{1}{2}qt = \text{Area of Inventory triangle OAP.}$$

Cost of holding inventory per production run $= \frac{1}{2}C_1Rt^2$

Set of cost per production run $= C_3$

Total cost per production run $= \frac{1}{2}C_1Rt^2 + C_3$.

Average total cost per unit time C(t) = $\frac{1}{2}C_1Rt + \frac{C_3}{t}$ ………(1)

C will be minimum if $\frac{dC(t)}{dt} = 0$ and $\frac{d^2C(t)}{dt^2}$ is positive.

Differentiating (1) w.r.t t and equating to zero,

$$\frac{dC(t)}{dt} = \frac{1}{2}C_1Rt - \frac{C_3}{t^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Which gives $t = \sqrt{\frac{2C_3}{C_1R}}$

Differentiating (2) w.r.t. t $\frac{d^2C(t)}{dt^2} = \frac{2C_3}{t_3}$ which is positive for value of t given by the above equation.

Thus C(t) is minimum for optimum time interval $t_0 = \sqrt{\frac{2C_3}{C_1R}}$

Optimum quantity q_0 to be produced during each production run,

$$q_0 = Rt_0 = \sqrt{\frac{2C_3R}{C_1}}$$

which is known as the **optimal lot – size formula due to R.H. Wilson.**

The resulting minimum average cost per unit time,

$$C_0(q) = \frac{1}{2}C_1R\sqrt{\frac{2C_3}{C_1R}} + C_3\sqrt{\frac{C_1R}{2C_3}}$$

$$= \frac{1}{\sqrt{2}}\sqrt{c_1c_3R} + \frac{1}{\sqrt{2}}\sqrt{c_1c_3R}$$

$$= \sqrt{2c_1c_3R}$$

Remarks:

1. If the demand rate is not uniform, and if D is the total demand to be satisfied during the period T then R = $\frac{D}{T}$ in the above formula.
2. q_0, t_0, C_0 are sometimes referred as q^*, t^*, c^*.

Example 1:

The annual demand for an item is 3200 units. The unit cost is Rs. 6/- and inventory carrying charges 25% per annum. If the cost of one procurement is Rs. 150/-determine

i. Economic order quantity.
ii. Time between two consecutive orders
iii. Number of orders per year
iv. The optimal total cost.

Solution:

R = 3200 units, $C_1 = \frac{25}{100} X \; 6 = \frac{3}{2}$

C_3 = 150 Rs, $\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$

$= \sqrt{\frac{2X150X3200}{\frac{3}{2}}}$

= 800 units.

(ii) = $\{= t^*\} = \frac{q^*}{R}$

= $= \frac{1}{4}$ th of a year

(iii) Number of orders = $\frac{1}{t_0}$ $= \frac{1}{\frac{1}{4}} = 4$

Total = (R x price per unit) + C_0

(iv) Optimal cost = (6 x 3200) +

= Rs. 20,400 ***[Ans]***

Otherwise optimal total cost = $\frac{R}{q} C_3 + \left(\frac{q}{2} X C_1\right)$ + RP

Example 2:

A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer, and how much would it save the company per year?

Solution:

Here R = 9000 parts per year

C_1 = 15% unit cost

(Here 15% of average Inventory per year means that the carrying cost per unit per year is 15% of the unit cost)

= 20 x $\frac{15}{100}$ = Rs. 3 each part per year

C_3 = Rs. 15 per order

$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$

$= \sqrt{\frac{2X150X3200}{\frac{3}{2}}}$ = 300 units

$t^* = \frac{q^*}{R} = \frac{300}{9000} = \frac{1}{30}$ year

= = 12 days

$C_{min} = \sqrt{2C_1C_3R}$

$= \sqrt{2\ X\ 3\ X\ 15\ X\ 9000}$

If the company follows the policy of ordering every month, then the annual ordering cost becomes = 12 x 15 = Rs. 180

and lot – size of Inventory at any time $q = \frac{9000}{12}$ = 750 parts.

Average Inventory at any time = $\frac{1}{2}q$ = 375 parts.

Shortage cost at any time = 375 C_1

= 375 x 3 = Rs. 1125.

Total annual cost = 1125 + 180 = Rs. 1305

$\therefore$ The company purchases 300 parts at time intervals of 12 days instead of ordering 750 parts each month. So there will be a net saving of

Rs. 1305 – Rs. 900 = Rs. 405 per year

Example 3:

A certain item costs Rs. 235 per ton. The monthly requirements are 5 tons, and each item the stock is replenished, there is a setup cost of Rs. 1000. The cost of carrying inventory has been estimated at 10% of the average inventory per year. What is the optimum order quantity.

Solution:

R = 5 tons/month

= 60 tons/year

C_3 = Rs. 1000

C_1 = 10% of unit cost per year

= Rs. 235 x $\frac{10}{100}$

= Rs. 23.5 per item per year

$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$

$$= \sqrt{\frac{2X1000X60}{23.5}} = 71.458 \text{ tons}$$

Example 4:

A manufacturer has to supply his customer with 600 units of his products per year. Shortage are not allowed and storage cost amounts to 60 paise per unit per year. The set up cost is Rs. 80.00 find

i. The economic order quantity
ii. The minimum average yearly cost
iii. The optimum number of orders per year
iv. The optimum period of supply per optimum order

Solution:

R = 600 units/year

C_1 = Rs. 80

C_3 = 0.60 per unit/year

i. $\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$

$$= \sqrt{\frac{2X600X80}{0.60}} = 400 \text{ units/year}$$

ii. $C^* = \sqrt{2C_1C_3R} = \sqrt{2\ X\ 0.60\ X\ 80\ X\ 600}$ = Rs. 240

iii. $N^* = \frac{demand}{EOQ} = \frac{600}{400} = \frac{3}{2}$

iv. $t^* = \frac{1}{N^*} = \frac{2}{3}$ of a year

Model II : Manufacturing model with no shortages.*(Demand Rate uniform, production rate finite)*

It is assumed that run sizes are constant and that a new run will be started whenever Inventory is zero. Let

R = number of items required per unit time

K = number of items produced per unit time

= cost of holding per item per unit time

= cost of setting up a production run

q = number of items produced per run, q = Rt

t = time interval between runs.

Here each production run of length t consists of two parts $t_1$and t_2,

where (i) t_1 is the time during which the stock is building up at constant rate of K – R units per unit time. (ii) t_2 is the time during which there is no production (or supply) and inventory is decreasing at a constant rate R per unit time.

Let $I_m$be the maximum Inventory available at the end of time t_1 which is expected to be consumed during the remaining period t_2 at the demand rate R.

Then I_m = (K – R)t_1 (or)

$$t_1 = \frac{I_m}{\mathrm{K - R}} \quad \ldots\ldots\ldots\ldots\ldots(1)$$

Now the total quantity produced during time t_1 id q and quantity consumed during the same period is Rt, therefore the remaining quantity available at the end of time t_1 is

$$t_1 = q - \mathrm{R}t_1$$

$$= q - \frac{R.I_m}{\mathrm{K - R}} \qquad \text{from(1)}$$

$$\therefore I_m\left(1 + \frac{R}{k-R}\right) = q \text{ (or)} = I_m \quad \frac{k-R}{k} q \ \ldots\ldots(2)$$

Now holding cost per production run for time period t =

And set up cost per production run =

∴ Total average cost per unit time C(I_m, t) $= \frac{1}{2} I_m C_1 + \frac{C_3}{t}$

C(q, t) $= \frac{1}{2}\left(\frac{k-R}{k} q\right) C_1 +$

C(q) = +

= +

For minimum value of C(q)

$\frac{d}{dq}$ [c(q)] = = 0

Which gives $q = \sqrt{\frac{2C_3}{c_1} \frac{RK}{K-R}}$

∴ Optimum lot size q_0 =

∴ Optimum time interval $t_0 = \frac{q_0}{R}$

=

Optimum average cost/unit time

$$C_0 = \frac{1}{2} \frac{k-R}{k} C_1 \sqrt{\frac{2C_3}{c_1} \frac{RK}{K-R}} + C_3 R \sqrt{\frac{C_1(K-R)}{2C_3 RK}}$$

$$= \sqrt{2C_1 C_3 R \frac{K-R}{K}}$$

$$= \sqrt{\frac{K-R}{K}} \sqrt{2C_1 C_3 R}$$

Note: *(i)* If K = R then C_0= 0, (i.e.,) there will be no holding cost and set up cost

(ii) If K = ∞, (i.e.,) production rate is Infinite, this model reduces to model I.

Example 1:

A contractor has to supply 10,000 bearings per month to an automobile manufacturer. He finds that when he starts a production run he can produce 25,000 bearings per month. The cost of holding a bearing in stock for one year is Rs. 2 and the set up cost of a production run is Rs. 180. How frequently should the production run be made?

Solution:

R = 10,000/month x 12 = 1,20,000 per year

C_1 = Rs. 2 per year

C_3 = Rs. 180

K = 25,000 x 12 = 3,00,000 per

year $\therefore q^* = \sqrt{\frac{2C_3 R}{C_1}}\sqrt{\frac{K}{K-R}}$

$= \sqrt{\frac{3,00,000}{30,000-120000}}\ X\ \sqrt{\frac{2\ X\ 180\ X\ 120000}{2}}$

= 1.29 x $\sqrt{21600000}$ = 6000 units

$t^* = \frac{q^*}{R} = \frac{6000}{120000}$ = 0.05 years (i.e.,) 18 days.

Example 2:

The annual demand for a product is 1,00,000 units. The rate or production is 2,00,000 units per year. The set – up cost per production run is Rs. 5000, and the variable production cost of each item is Rs. 10. The annual holding cost per unit is

20% of the value of the unit. Find the optimum production lot – size, and the length of production run.

Solution:

R = 1,00,000 per year

$C_1 = \frac{20}{100}$ x 10 Rs. Per year

C_3 = Rs. 5000

K = 2,00,000

$\therefore$ EOQ = $\sqrt{\frac{K}{K-R}}\sqrt{\frac{2C_3R}{C_1}}$

$= \sqrt{\frac{2,00,000}{1,00,000}} \text{ x } \sqrt{\frac{2 X 1,00,000 X 5000}{\frac{20}{100} X 10}}$

= 1.4142 x 22360.6

= 31622 units (=q^*)

$t^* = \frac{q^*}{R} = \frac{31622}{1,00,000}$ = 0.31622 years

= 115 days

Example 3:

An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set up cost is Rs. 100 per set up and holding cost is Rs. 0.01 per unit of item per day, find the economic lot size for one run, assuming that shortages are not permitted. Also find the time of cycle and minimum total cost for one run.

Solution:

R = 25 items per day

C_1 = Rs. 0.01 per unit per day

C_3 = Rs. 100 per set up

K = 50 items per day

$\therefore$ EOQ = $\sqrt{\frac{K}{K-R}}\sqrt{\frac{2C_3R}{C_1}}$

$= \sqrt{\frac{2 X 100 X 25}{0.01}} \text{ x } \sqrt{\frac{50}{25}}$

= 1000 items

$t_0 = \frac{q_0}{R} = \frac{1000}{25}$ = 40 days

Minimum daily cost = $\sqrt{2C_1C_3R}\sqrt{\frac{K-R}{K}}$

= Rs. $\sqrt{2 X 0.01 X 100 X 25 X \frac{25}{50}}$

= Rs. 5

Minimum total cost per run = 5 x 40

= Rs. 200

Example 4:

A company has a demand of 12,000 units/year for an item and it can produce 2000 such items per month. The cost of one setup is Rs. 400 and the holding cost/unit/month is Rs. 0.15. Find the optimum lot size, max inventory, manufacturing time, total time.

Solution:

R = 12,000 units/year

C_1 = Rs. 400/ set up

C_3 = Rs. 0.15 x 12 = Rs. 1.80/unit/year.

K = 2000 x 12 = 24,000 units/year

$\therefore q_0 = \sqrt{\frac{K}{K-R}}\sqrt{\frac{2C_3R}{C_1}}$

$= \sqrt{\frac{2 X 400 X 12{,}000}{1.80}} \text{ x } \sqrt{\frac{24{,}000}{12{,}000}}$

= 3266 units/set up

Max inventory $I_{m_o} = \frac{K-R}{K} q_0$

= x 3266 = 1632 units .

Manufacturing time $t_1 = \frac{I_{m_o}}{K-R} = \frac{1632}{12{,}000}$ = 0.136 years.

Total time $= \frac{q_0}{R} = \frac{3264}{12{,}000}$ = 0.272 years.

Example 5:

A certain item costs Rs. 250 per ton. The monthly requirements are 10 tons and each time the stock is replenished there is a setup cost of Rs. 1000. The cost of carrying inventory has been estimated as 12% of the value of the stock per year. What is the optimal order quantity and how frequently should orders be placed? **Solution:**

$C_1 = \frac{12}{100}$ x 250

C_3 = Rs. 1000

R = 10 x 12 = 120 tons/year

$\therefore$ EOQ $= \sqrt{\frac{2C_3R}{C_1}}$

$= \sqrt{\frac{2 X 1000 X 120}{\frac{12}{100} X 250}}$

$= \sqrt{\frac{24,000}{30}} = \sqrt{8000}$

$= 89.44$ units

$t_0 = \frac{q_0}{R} = \frac{89.44}{120} = 0.745$ year $\approx$ 9 months.

Model III : Purchasing model with shortages.

(*Demand rate uniform, Production rate infinite, shortages allowed)*

Assumptions are the same as model I, but shortages are allowed, consequently, a cost of shortage is incurred.

C_1 – Holding cost or carrying cost.

C_3 – Setup cost or Ordering cost.

C_2 **–** Shortage cost

R – Demand Rate.

The optimum quantities of this model are

a) The Economic order quantity $q^* = \sqrt{\frac{2C_3R}{C_1}}\sqrt{\frac{C_1+C_2}{C_2}}$

b) Time between two consecutive orders $t^* = \frac{q^*}{R}$

c) Number of orders per year $N^* = \frac{R}{q^*}$

Example 1:

The demand for an item is 18,000 units per year. The holding cost per unit time is Rs. 1.20 and the cost of shortage is Rs. 5.00, the production cost is Rs. 400. Assuming that replenishment rate is Instantaneous, determine the optimal order quantity.

Solution:

R = 18,000

C_1 = Rs. 1.20

C_2 = Rs. 5.00

C_3 = Rs. 400

$$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}\sqrt{\frac{C_1+C_2}{C_2}}$$

$$= \sqrt{\frac{2\ X\ 400\ X\ 18{,}000}{1.20}}\sqrt{\frac{1.20+5}{5}}$$

= 1.113 x 3,464.10

= 3856 units (app)

$$t^* = \frac{q^*}{R} = \frac{3856}{18{,}000} = 0.214 \text{ year}$$

$$N^* = \frac{R}{q^*} = 4.67 \text{ orders per year}$$

Example 2:

A certain product has a demand of 25 units per month and the items are withdrawn uniformly. Each time a production run is made the setup cost is Rs. 15. The production cost is Rs. 1 per item and inventory holding cost is Rs. 0.30 per item per month. If shortage cost is Rs. 1.50 per item per month, determine how often to make a production run and what size it should be ?

Solution:

Though the production cost is given, the cost equation remain the same.

$$q^* = \sqrt{\frac{2C_3R}{C_1}}\sqrt{\frac{C_1+C_2}{C_2}}$$

Given

R = 25 units/month

C_1 = 0.30 per item per month

C_2 = Rs. 1.50 per item per month

C_3 = Rs. 15

$$\therefore q^* = \sqrt{\frac{(1.50+0.30)2\ X\ 15\ X\ 25}{0.30\ X\ 1.50}}$$

$$= \sqrt{\frac{54.77}{25}} \text{ units}$$

$$t^* = \frac{q^*}{R} = \frac{54.77}{25} = 2.19 \text{ month}$$

Model IV : Manufacturing model with shortages.

(Demand rate uniform, Production rate finite, shortages allowed)

Assumptions are the same as model II, but shortages are allowed. The optimum quantities of this model are

C_1 – Holding cost or carrying cost.

C_3 – Setup cost or Ordering cost.

C_2 – Shortage cost

R – Demand Rate.

K – Production Rate.

a) The Economic order quantity $q^* = \sqrt{\frac{2C_3R}{C_1}}\sqrt{\frac{C_1+C_2}{C_2}}\sqrt{\frac{K}{K-R}}$

b) Number of shortages S = $\frac{C_1}{C_1+C_2}q^*\left(1-\frac{R}{K}\right)$

c) Time between two consecutive orders $t^* = \frac{q^*}{R}$

d) Number of orders per year $N^* = \frac{R}{q^*}$

e) Manufacturing time $= \frac{q^*}{K}$

Example 1:

The demand for an item in a company is 18,000 units per year, and the company can produce the item at a rate of 3000 per month. The cost of one setup cost is Rs. 500.00 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs. 20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and time between set – ups.

Solution:

R = 18,000 units per year

= 1500 units per month

K = 3000 units per month

C_1 = Rs. 0.15 per month

C_2 = Rs. 20.00

C_3 = Rs. 500

$$q^* = \sqrt{\frac{2C_3R}{C_1}}\sqrt{\frac{C_1+C_2}{C_2}}\sqrt{\frac{K}{K-R}}$$

$$= \sqrt{\frac{2 X 500 X 1500}{0.15}}\sqrt{\frac{0.15+20}{20}}\sqrt{\frac{3000}{3000-1500}}$$

$$= \frac{1224.744}{0.3872} \text{ x } 1.0037 \text{ x } 1.4142$$

$$= \frac{1738.458}{0.3872} = 4490 \text{ units (app)}$$

Number of shortages S = $\frac{C_1}{C_1 + C_2} q^* \left(1 - \frac{R}{K}\right)$

$$= \frac{0.15}{0.15+20} \times 4490 \left(1 - \frac{1500}{3000}\right)$$

$$= \frac{336.75}{20.15} = 16.71 \text{ units.}$$

Manufacturing time $= \frac{q^*}{K} = \frac{4490}{3000 \, X \, 12} = 0.1247$ years

Time between setup's $= \frac{q^*}{R} = \frac{4490}{18{,}000} = 0.2494$ years

Inventory Models with Price breaks:

In this section we shall consider a class of inventory problems in which the production (or) purchase cost per unit is a variable. This depends on the quality manufacture or purchased. This usually happen when discount are offered for the purchase of large quantities. These discounts take the form of Price-Breaks.

Consider the following three cases

Where $c_0(q) = \quad + k_1 R + \frac{1}{2} c_3 p$ ……(1)

and $q = \sqrt{\frac{2 c_3 R}{k_1 P}}$,

Total expected cost per unit time

$$C(q) = \frac{C_3 R}{q} + \frac{1}{2} q \, PI + PR \qquad \text{……(2)}$$

K_1 = purchasing cost of care unit

p = holding cost/month expressed as a fraction of the values of the unit

Case (i): Purchase Inventory model with single price-break

Given:

Unit purchasing cost	Range of quality
K_{11}	$0<q_1<b_1$
K_{12}	$q_2 \geq b_1$

(i) If $b>q_2$ and $c_2(b) > c_0(q_1)$, the optimal lot size is q_1 and minimum values of $c(q) = c_0(q_1)$

(ii) If $b>q_2$ and $c_2(b) < c_0(q)$, the optimal lot size is b and min $c(q) = c_2(b)$

(iii) If $b<q_2$, the optimal lot size is q_2 and min $c(q) = c_0(q_2)$

Case (ii): Purchase Inventory Model with 2 prices – breaks

Unit purchasing cost	Range of quality
K_{11}	$0<q_1<b_1$
K_{12}	$b_1\leq q_2<b_2$
K_{13}	$b_2\leq q_3$

(i) Calculate q_3, If $q_3>b_2$, optimal purchase quality is q_3

(ii) If $q_3\leq b_2$, calculate q_2 since $q_3<b_2$, the necessarily $q_2<b_2$. As a consequence we have $q_2<b_1$ or $q_2>b_1$.

(iii) If $q_3<b_2$and $b_1<q_2<b_2$, compare c_0 (q_2) with c_3 (b_2). The smaller of these qualities will be optimal purchase quantity.

(iv) If $q_3<b_2$ and $q_2<b_1$. Calculated c_3 (q_1) which will necessarily satisfy the inequality $q_1<b_1$. In this case compared c_0 (q_1), c_2 (b_1) and c_3 (b_2) to determine optimum purchase quantity.

When there are n price breaks, the situation can be represented as follows:

Unit purchasing cost Range of quality

K_{11}	$0<q<b_1$
K_{12}	$b_1 \leq q<b_2$
…………	………..
K_{1n}	$b_{n-1} \leq q$

(i) Calculate q_n. If $q_n>b_{n-1}$, optimal purchase quality is q_n

(ii) If $q_n<b_{n-1}$, calculate q_{n-1}. If $q_{n-1} \geq b_{n-2}$ proceed as in the case of one price break; (i.e.,) compare c_0 (q_{n-1}) with $c(b_{n-1})$ to determine optimum purchase quality.

(iii) If $q_{n-1} < b_{n-2}$, compute q_{n-2}. If $q_{n-2} \geq b_{n-3}$, proceed as in the case of 2 price breaks: (i.e.,) compare $c_0(q_{n-2})$ with $c(b_{n-1})$ and $c(b_{n-2})$ to determine optimal purchase quality.

(iv) If $q_{n-2} < b_{n-3}$ compute q_{n-3}

If $q_{n-3} \geq b_{n-4}$ compare $c_0(q_{n-3})$ with $C(b_{n-3})$, $C(b_{n-2})$ and $C(b_{n-1})$.

(v) Continue in this number until $q_{n-j} \geq b_n - (j+1)(0 \leq j \leq n-1)$ and then compare $C_0(q_{n-j})$ with $C(b_{n-j})$, $c(b_{n-j+1})$, $C(b_{n-j+2})$……….$C(b_{n-1})$. This procedure involves only a finite number of steps.

Example 1:

Find the optimal order quantity for a product for which the price – break is as follows:

Quantity	unit cost
$0 \leq Q_1 < 50$	Rs. 10
$50 \leq Q_1 < 100$	Rs. 9
$100 \leq Q_3$	Rs. 8

The monthly demand for the product is 200 units, the cost of the storage is 25% of the unit cost and ordering cost is Rs. 20.00 per order.

Here R=200 units, P=0.25, C_3= Rs.20.00

$$Q_3^0 = \sqrt{\frac{2 X 20 X 200}{8 X 0.25}}$$

= 63 units

Clearly 63 < 100(i.e.,) $Q_3{}^0 < b_2$

∴ We compute $Q_2^0 = \sqrt{\frac{2 X 20 X 200}{9 X 0.25}}$

= 60 units

Now since $Q_2^0 > b_1$(= 50) the optimum purchase quantity is determined by comparing $C_A(Q_2{}^0)$ with $C_A(b_2)$

Now $C_A(Q_2^0)$ = 20 x $\frac{200}{60}$ + 200 x 9 + 9 x 0.25 x $\frac{60}{2}$

= Rs. 1934.16

$C_A(b_2)$ = 20 x $\frac{200}{100}$ + 200 x 8 + 8 x 0.25 x $\frac{100}{2}$

= Rs.1740.00

Since $C_A(Q_2^0) > C_A(b_2)$, the optimum purchase quantity is Q^0 = b_2 = 100 units.

Example 2:

Find the optimal order quantity for which the price breaks are as follows:

Quantity	Unit cost
$0 \le q_1 < 500$	Rs.10
$500 \le q_2 < 750$	Rs.9.25
$750 \le q_3$	Rs.8.75

The monthly demand for the product is 200 units, shortage cost is 2% of the unit cost and the cost of ordering is Rs. 100.

Solution:

$$q_3 = \sqrt{\frac{2c_3 R}{k_{13} P}}$$

$$= \sqrt{\frac{2 X 100 X 200}{8.75 X 0.2}} = 478 \text{ Units}$$

$b_2 = 750$

$q_3 < b_2$, we calculate q_2

$$q_2 = \sqrt{\frac{2c_3 R}{k_{12} P}}$$

$$= \sqrt{\frac{2 X 100 X 200}{9.25 X 0.02}} = 465 \text{ Units}$$

$b_1 = 500$ Units

$q_2 < b_1$, we compute q_1

$$q_2 = \sqrt{\frac{2c_3 R}{k_{11} P}}$$

$$= \sqrt{\frac{2 X 100 X 200}{10 X 0.02}} = 447 \text{ Units}$$

Next we compute

*Now $C_0 (q_1) = \sqrt{2c_3 k_{11} PR} + k_{11}R + \frac{1}{2} c_3 P$

$= \text{Rs. } [\sqrt{2 X 100 X 10 X 0.02 X 200} + 10 \text{ x } 200 + \frac{1}{2} \text{ x } 100 \text{ x } 0.02]$

$= \text{Rs. } 2090.42$

$C_2 (b_1) = c_3 \frac{R}{q} + k_{12} R + \frac{1}{2} c_3 P + \frac{1}{2} k_{12}\, pq$

$= \text{Rs.} [100 \times \frac{200}{500} + 9.25 \times 200 + \frac{1}{2} \times 100 \times 0.02 \times \frac{1}{2} \times 9.25 \times 0.02 \times 500]$

$= 1937.25$

$C_3 (b_2) = c_3 \frac{R}{q} + k_{13} R + \frac{1}{2} c_3 P + \frac{1}{2} k_{13} pq$

$= \text{Rs.} [100 \times \frac{200}{750} + 8.75 \times 200 + \frac{1}{2} \times 100 \times 0.02 \times \frac{1}{2} \times 8.75 \times 0.02 \times 750]$

$= \text{Rs. } 1843.29$

Since $C_3 (b_2) < C_2 (b_1) < C_0 (q_1)$, the optimal order quantity is b_2=750 units.

*we can use formula (2) under 12.9 also.

Example 3:

Find the optimum order quantity for a quantity for which the price breaks are as follows:

Quantity	Unit cost
$0 \le Q_1 < 100$	Rs.10
$500 < Q_2$	Rs.9.25

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering Rs.350.00

Solution:

$$Q_2^0 = \sqrt{\frac{2 X 350 X 200}{9.25 X 0.2}}$$

= 870 units

as $Q_2^0 > b_1$,(870 > 500)

Optimum purchase quantity = 870 units.

Example 4:

Find the optimum order quantity for a quantity for which the price breaks are as follows:

Quantity	Unit cost
$0 \le Q_1 < 500$	Rs.20 per unit
$500 \le Q_2 < 750$	Rs.18 per unit
$750 \le Q_3$	Rs.16 per unit

The monthly demand for the product is 400 units. The shortage cost is 20% of the unit cost of the product and the cost of ordering is 25.00 per month.

Solution:

R = 100 units, I = Re. 0.20, C_3 = Rs.25.00

$$Q_3^0 = \sqrt{\frac{2c_3 R}{k_1 P}}$$

$$= \sqrt{\frac{2 X 25 X 400}{16 X 0.20}} \qquad = 79 \text{ units}$$

Since $Q_3^0 < b_2$, we compute Q_2^0

$\therefore$ We have $Q_2^0 = \sqrt{\frac{2 X 25 X 400}{20 X 0.20}}$ = 75 units

Now since $Q_2^0 < b_1 (= 100)$ we next compute Q_1^0

Next we compute

Now $C_A(Q_1^0) = 25 \text{ x} \frac{400}{70} + 400 \text{ x } 20 + 20 \text{ x } 0.20 \text{ x} \frac{70}{2}$ = Rs. 8283.00

$C_A(b_1) = 25 \text{ x} \frac{400}{100} + 400 \text{ x } 18 + 18 \text{ x } 0.20 \text{ x} \frac{100}{2}$ = Rs.7480.00

$C_A(b_2) = 25 \text{ x} \frac{400}{200} + 400 \text{ x } 16 + 16 \text{ x } 0.20 \text{ x} \frac{200}{2}$ = Rs.6770.00

Since $C_A(b_2) < C_A(b_1) < C_A(Q_1^0)$, the optimal purchase quantity is= b_2 = 200 units.

Example 5:

Find the optimal quantity for a product where the annual demand for the product is 500 units. The cost of storage per unit per year is 10% of the unit cost and the ordering cost per order is Rs.180.00. The unit costs are given below.

Quantity	Unit cost
$0 \leq Q_1 < 500$	Rs. 25
$500 \leq Q_2 < 1500$	Rs. 24.80
$1500 < Q_3 < 3000$	Rs. 24.60
$3000 < Q_4$	Rs. 24.40

Solution:

R = 500 units, P = Rs. 0.10, C_3 = Rs.180.00

$$Q_4^0 = \sqrt{\frac{2c_3 R}{k_4 P}}$$

$$= \sqrt{\frac{2 X 180 X 500}{(24.40) X 0.10}} = 272$$

Since $Q_4^0 < b_3$, we compute Q_3^0,

$$Q_3^0 = \sqrt{\frac{2c_3 R}{k_3 P}} = \sqrt{\frac{2 X 180 X 500}{(24.60) X 0.10}} = 270$$

Since $Q_3^0 < b_2$ (= 1500) $\therefore$ we calculate

$$Q_2^0 = \sqrt{\frac{2c_3 R}{k_2 P}} = \sqrt{\frac{2 X 180 X 500}{(24.80) X 0.10}} = 269$$

Since $Q_2^0 < b_1$ (= 500) $\therefore$ we calculate

$$Q_1^0 = \sqrt{\frac{2c_3 R}{k_1 P}} = \sqrt{\frac{2 X 180 X 500}{25 X 0.10}} = 268$$

Now $C_A(Q_1^0) = 180 \text{ x} \frac{500}{268} + 500 \text{ x } 25 + 25 \text{ x } 0.10 \text{ x} \frac{268}{2}$ = Rs. 13,170.82

$C_A(b_1) = 180 \text{ x} \frac{500}{500} + 500 \text{ x } 24.80 + (24.80) \text{ x } 0.10 \text{ x} \frac{500}{2}$

= Rs.13,200.00

$C_A(b_2) = 180 \text{ x} \frac{500}{1500} + 500 \text{ x } 24.60 + (24.60) \text{ x } 0.10 \text{ x} \frac{1500}{2}$

= Rs.14,205.00

$C_A(b_3) = 180 \text{ x} \frac{500}{3000} + 500 \text{ x } 24.40 + (24.40) \text{ x } 0.10 \text{ x} \frac{3000}{2}$

= Rs.15,890.00

Since $C_A(b_3) > C_A(b_2) > C_A(b_1) > C_A(Q_1^0)$, the optimum purchase quantity is $Q^0 = Q_1^0$ = 268 units.

Example 6:

Find the optimal order quantity for the following annual demand = 3600 units, order cost = Rs. 50, cost of storage = 20% of the unit cost

Price break	$0 < Q_1 < 100$	Rs. 20
	$750 \leq Q_2$	Rs. 18

Solution:

Given R = 3600 units per year,

I = Rs.$\frac{20}{100}$,

k_1 = Rs.200.00

k_2 = Rs.18.00

$$Q_2^0 = \sqrt{\frac{2 X 50 X 3600}{18 X 0.20}} \quad = 316.20$$

Now b = 100 as $Q_2^0 > b$, Optimum purchase quantity = 316.20

POSSIBLE QUESTIONS:

PART-B(5X6 = 30 MARKS)

1. A contractor has to supply 10,000 bearings per month to an automobile manufacturer. He finds that when he starts a production run he can produce 25,000 bearings per month. The cost of holding a bearing in stock for one year is Rs 2 and the set up cost of a production run is Rs 180.How frequently the production run be made?

2. A company has a demand of 18,000 units per year for an item and it can produce 3000 such items per month. The cost of one set up is Rs 500. and the holding cost /unit/month is Rs.0.15. The shortage cost of one unit is Rs.20 per month.Determinethe optimum manufacturing quantity and the number of shortages.Also determine the manufacturing time between set- ups.

3. i) Explain various types of inventory.
   ii) A company has a demand of 12,000 units per year for an item and it can produce 2000 such items per month. The cost of one set up is Rs 400 and the holding cost unit/month is Rs.0.15. Find
      1) The optimum lot size
      2) Maximum inventory
      3) Manufacturing time
      4) Total time

4. A manufacturer has to supply his customer with 600 units of his products per year. Shortages are not allowed and storage cost amounts to 60 paise per unit per year. The set up cost is Rs. 80.00. find
   i) the economic order quantity
   ii) the optimum number of orders per year
   iii) the minimum average yearly cost
   iv) the optimum period of supply per optimum order

5. The annual demand for an item is 3200 units. The unit cost is Rs.6 and inventory carrying charges 25 % per annum. If the cost of one procurement is Rs.150.Determine

   i) Economic order quantity ii) Time between two consecutive orders iii) Number of orders per year iv) The optimal cost.

6. Find the optimal order quantity for which the price break are as follows

Quantity	Unit cost
$0 \leq Q_1 < 50$	Rs. 50
$50 \leq Q_2 < 100$	Rs. 9
$100 \leq Q_3$	Rs. 8

The monthly demand for the product is 200 units. The cost of storage is 20% of the unit cost and the ordering per order is Rs.2

7. Find the optimal order quantity for a product where the annual demand for the product is 500 units. The cost of storage per unit per year is 10% of the unit cost and the ordering per order is Rs.180.00. The unit costs are given below.

Quantity	Unit cost
$0 \le Q_1 < 500$	Rs. 25
$500 \le Q_2 < 1500$	Rs. 24.80
$1500 \le Q_3 < 3000$	Rs.24.60
$3000 < Q_4$	Rs.24.40

8. Find the optimam order quantity for a quantity for which the price- break is as follows:

Quantity	Unit cost
$0 \le Q_1 < 500$	Rs. 10
$500 \le Q_2$	Rs. 9.25

The monthly demand for the product is 200 units, the cost of the storage is 2% of the unit cost and ordering cost is Rs.350.00

9. Find the optimal order quantity for a product for which the price- break is as follows:

Quantity	Unit cost
$0 \le Q_1 < 100$	Rs. 20
$100 \le Q_2 < 200$	Rs. 18
$200 \le Q_3$	Rs. 16

The monthly demand for the product is 400 units, the cost of the storage is 20% of the unit cost and ordering cost is Rs.25.00 per month.

10. Find the optimal order quantity for a product for which the price- break is as follows:

Quantity	Unit cost
$0 \le Q_1 < 50$	Rs. 10
$50 \le Q_2 < 100$	Rs. 9
$100 \le Q_3$	Rs. 8

The monthly demand for the product is 200 units, the cost of the storage is 25% of the unit cost and ordering cost is Rs.20.00 per order.

PART-C (1X10=10 MARKS)

COMPULSORY:

1. An item is produced at the rate of 50 items per day.The demand ocurs at the rate of 25 items per day.If the set up cost is Rs.100 per set up and holding cost is Rs 0.01 per unit of item per day, find the economic lot size for one run,assuming that shortages are not permitted.Also find the time of cycle and minimum total cost for one run.

2. A company has a demand of 18,000 units per year for an item and it can produce 3000 such items per month. The cost of one set up is Rs 500. and the holding cost /unit/month is Rs.0.15. The shortage cost of one unit is Rs.20 per month.Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time between set- ups.

3. Find the optimum order quantity for a quantity for which the price breaks are as follows:

Quantity	Unit cost
$0 \leq Q_1 < 500$	Rs.10
$500 \leq Q_2 < 750$	Rs.9.25
$750 \leq Q_3$	Rs.8.75

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering Rs. 100

Question	Opt 1	Opt 2	Opt 3	Opt 4	Opt 5	Opt 6	Answer
---------- may be defined as the stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business affairs	Inventory	Transportation	Queueing	Sequencing			Inventory
Rate of consumption is different from ---------.	rate of change	rate of production	rate of purchasing	either b or c			either b or c
Cost associated with carrying or holding the goods in stock is known as -------.	interested capital cost	handling cost	holding cost	production cost			holding cost
-------- is the interest change over the capital invested.	interested capital cost	handling cost	holding cost	production cost			interested capital cost
-------- include costs associated with movement of stock, such as cost of labour etc.	interested capital cost	handling cost	holding cost	production cost			handling cost
-------- per unit item is affected by the quantity purchased due to quantity discounts or price breaks.	interested capital cost	handling cost	holding cost	purchase price			purchase price
If P is the purchase price of an item and I is the stock holding cost per unit time expressed as a fraction of stock value then the holding cost is ----------.	I/P	I + P	I – P	IP			IP
The penalty costs that are incurred as a result of running out of stock are known as----	shortage cost	set-up cost	holding cost	production cost			shortage cost
Holding cost is denoted by -------	C_1	C_2	C_3	C_4			C_1
Shortage cost is denoted by ------	C_1	C_2	C_3	C_5			C_2
Set-up cost is denoted by --------	C_1	C_2	C_3	C_4			C_3
Elapsed time between the placement of the order and its receipts in inventory is known as -------.	lead time	recorder level	EOQ	variables			lead time
---------- is the time when we should place an order by taking into consideration the interval between placing the order and receiving the supply.	lead time	recorder level	EOQ	variables			recorder level
--------- is that size of order which minimises total annual cost of carrying inventory and the cost of ordering under the assumed conditions of certainty and that annual demands are known.	lead time	recorder level	EOQ	variables			EOQ
EOQ means -------	Economic Order Quantity	Economic Order Quality	Economic Offer Quality	Economic Offer Quantity			Economic Order Quantity
EOQ is also known as -------	economic lot size formula	economic short size formula	economic formula	economic variables			economic lot size formula
------- include holding cost, set up cost, shortage costs and demand.	EOQ	controlled variables	uncontrolled variables	basic variables			uncontrolled variables
Reduction in procurement cost ------- EOQ	increases	decreases	reduces	neutral			reduces
An approximate percentage of 'C' items in a firm is around --------	60 – 65%	65 – 70%	70 – 75%	75 – 80%			70 – 75%
Economic order quantity results in equilization of -------- cost and annual inventory cost.	annual procurement cost	procurement cost	inventory cost	shortage cost			annual procurement cost
Economic order quantity results in equilization of annual procurement cost cost and ------- cost.	annual inventory cost	procurement cost	inventory cost	shortage cost			annual inventory cost
A company uses 10,000 units per year of an item. The purchase price is one rupee per item. Ordering cost = Rs. 25 per order. Carrying cost 12% of the inventory value. Find the EOQ.	2000 units	2083 units	2038 units	2050 units			2083 units
If the procurement cost per order increases 21%, the economic order quantity of the item shall increase by -------.	10%	20%	30%	40%			10%
If EOQ is 5000 units and Buffer stock is 500 units calculate max inventory.	5500 units	500 units	5000 units	5050 units			5500 units
If EOQ is 5000 units and Buffer stock is 500 units calculate minimum inventory.	5500 units	500 units		5050 units			5000 units
Reorder level = ---------	normal lead time x monthly consumption	normal lead time + monthly consumption	normal lead time - monthly consumption	normal lead time / monthly consumption			normal lead time x monthly
An approximate percentage of A- items in a firm is around ------	5 - 10 %	10 – 20 %	20 – 25 %	70 – 75 %			5 - 10 %
Economic order quantity results in -------	reduced stock – outs	increased stock – outs	equilisation of carrying cost and procurement costs	favourable procurement price			equilisation of carrying cost and procurement costs

The EOQ of an item which cost is Rs.36 and carrying cost is 1.5 % per month, the economic order quantity is ------	240 no’s	200 no’s	400 no’s	500 no’s	200 no’s
In the ABC analysis, C items are those which have -------	low unit price	low cost price	low usage value	low consumption	low consumption
For an item with storage cost of each item Rs. 1, set up cost Rs.25, demand 200 units per month C_{min} is ------	Rs.100	Rs.400	Rs.500	Rs.800	Rs.100
Minimum inventory equals	EOQ	Reorder level	Safety stock	lead time	Safety stock
Given maximum lead time as 20 days and normal lead time is 15 days with annual consumption 12,000 units find the buffer stock.	176 units	167 units	157 units	186 units	167 units
Given R = 1000 units/year I = 0.30, C = Re.0.50/unit, C_3 = Rs.10/order. Find minimum average cost.	54.77	55.77	53.77	50.77	54.77
The set up cost in inventory situation is ------- of size of inventory.	dependent	independent	large	small	independent
Total inventory cost = -------	set up cost + purchasing cost	holding cost + shortage cost	set up cost + purchasing cost + holding cost + shortage cost	setup cost + shortage cost	set up cost + purchasing cost + holding cost + shortage cost
Storage cost is associated with -------	holding cost	shortage cost	carrying cost	set up cost	carrying cost
Average inventory = -------	(EOQ/2) + Safety stock	(EOQ/2) - Safety stock	(EOQ/2) / Safety stock	(EOQ/2) * Safety stock	(EOQ/2) + Safety stock
------ discounts reduce material cost and procurement costs	quantity	quality	carrying cost	set up cost	quantity
The ordering cost is independent of ------	ordering quantity	ordering quality	carrying cost	set up cost	ordering quantity

UNIT-V

SYLLABUS

PERT and CPM: Arrow networks - time estimates- earliest expected time, latest allowable occurrence time and slack - critical path - probability of meeting scheduled date of completion of project calculations on CPM network - various floats for activities - critical path - updating project - operation time cost trade off curve - project time cost trade off curve - selection of schedule based on cost analysis.

PERT and CPM

Introduction

A **project**is defined as a combination of interrelated activities all of which must be executed in a certain order to achieve a set goal. A large and complex project involves usually a number of interrelated activities requiring men, machines and materials. It is impossible for the management to make and execute an optimum schedule for such a project just by intuition, based on the organizational capabilities and work experience. A systematic scientific approach has become a necessity for such project. So a number of methods applying networks scheduling techniques has been developed: **Programme Evaluation Review Technique** (PERT) and **Critical Path** method (CPM) are two of the many network techniques which are widely used for planning, scheduling and controlling large complex projects.

The main managerial functions for any project:

The main managerial functions for any project are

1. Planning
2. Scheduling
3. Control

Planning

This phase involves a listing of tasks or jobs that must be performed to complete a project under consideration. In this phase, men, machines and materials required for the project in addition to the estimates of costs and durations of various activities of the project are also determined

Scheduling

This phase involves the laying out of the actual activities of the project in a **logical sequence** of time in which they have to be performed.

Men and material requirements as well as the **expected completion time** of each activity at each stage of the project are also determined.

Control

This phase consists of reviewing the progress of the project whether the actual performance is according to the planned schedule and finding the reasons for difference, if any, between the schedule and performance. The basic aspect of control is to analyse and correct this difference by taking remedial action whether possible.

PERT and CPM are especially useful for scheduling and controlling

Basic Terminologies

Activity is a task or an item of work to be done in a project. An activity consumer resource like time, money, labour etc.

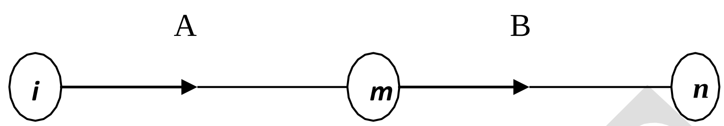
An activity is represented by an arrow with a node (event) at the beginning and a node (event) at the end indicating the start and termination (finish) of the activity. Nodes are denoted by circles. Since this is a logical diagram length or shape of the arrow has no meaning. The direction indicates the progress of the activity. Initial node and the terminal node are numbered as i-j(j<i) respectively. For example If A is the activity whose initial node is I and the terminal node is j then it is denoted diagrammatically by

$$\textcircled{i} \xrightarrow{\quad A \quad} \textcircled{j}$$

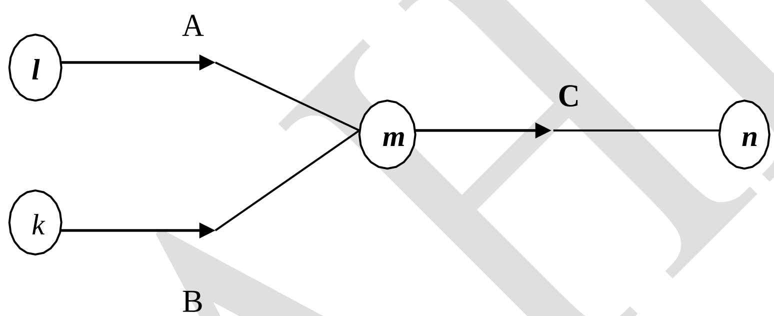
The name of the activity is written over the arrow, **not inside the circle.** The diagram

In which arrow represents an activity is called **arrow diagram.** The Initial and terminal nodes of activities are also called tail and head events.

If an activity B can start immediately after an activity A then it is denoted by



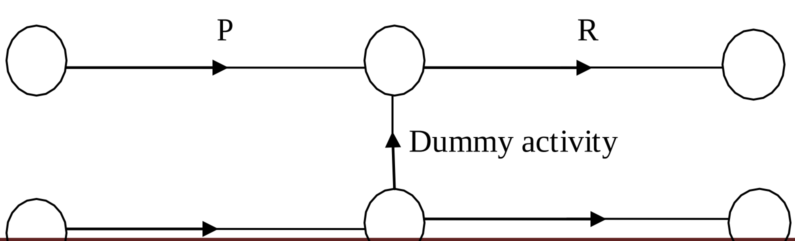
A is called the **immediate predecessor** of B and B is called the **immediate successor** of A. If C can start only after completing activites A and B then it is diagrammatically represented as follows:



Notation: “A is a predecessor of B” is denoted as “A<B“,”B is a successor of A” is denoted by “B>A”.

If the project contains two or more activities which have some of their immediate predecessors in common then there is a need for introducing what is called **dummy activity.** Dummy activity is an imaginary activity which does not consume any resources and which serves the purpose of indicating the predecessor or successor relationship clearly in any activity on arrow diagram. The need for a dummy activity is illustrated by the following usual example.

Let P, Q be the predecessors of R and Q be the only predecessors of S.



Q S

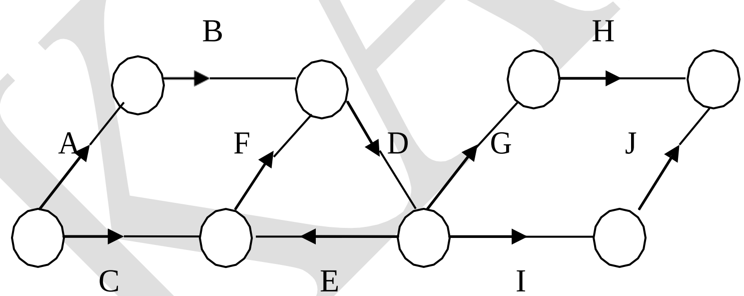
Activities which have no ***predecessors*** are called ***start activities***of the project. All the ***start activities*** can be made to have the ***same initial node***. Activities which have ***no successors*** are called ***terminal activities*** of the project. These can be made to have the ***same*** terminal node (end node) of the project.

A project consists of a number of activities to be performed in some technological sequence. For example while constructing a building the activity of laying the foundation should be done before the activity of erecting the walls for the building. The diagram denoting all the activities of a project by arrows taking into account the technological sequence of the activities is called the project network represented by ***activity on arrow diagram*** or simply ***arrow diagram***.

Note: There is another representation of a project network representing activites on nodes called AON diagram. To avoid confusion we use only activity on arrow diagram throughout the text.

Rules for constructing a project network

1. There must be no loops. For example, the activities F,D,E.



Obviously form a loop which is obviously not possible is any real project network.

2. Only one activity should connect any two nodes.

3. No dangling should appear in a project network i.e., no node of any activity except the terminal node of the project should be left without any activity emanating from it such a node can be joined to the terminal node of the project to avoid.

The Rules for numbering the Nodes:

Nodes may be numbered using the rule given below:

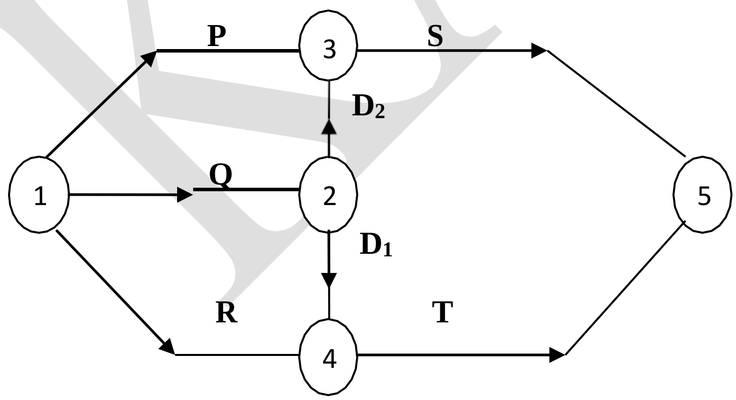
(Ford and Fulkerson's Rule)

1. Number the start node which has no predecessor activity, as 1.
2. Delete all the activites emanating from this node 1.
3. Number all the resulting start nodes without any predecessor as 2,3………
4. Delete all the activities originating from the start nodes 2,3,….in step 3.
5. Number all the resulting new start nodes without any predecessor next to the last number used in step(3).
6. Repeat the process until the terminal node without any successor activity is reached and number this terminal node suitably.

Immediate predecessor (successor) will be simply called as predecessor (successor) unless otherwise stated.

Example 1: If there are five activities P, Q, R, S and T such that P, Q, R have no immediate predecessors but S and T have immediate predecessors P, Q, R respectively. Represent this situation by a network.

Solution:



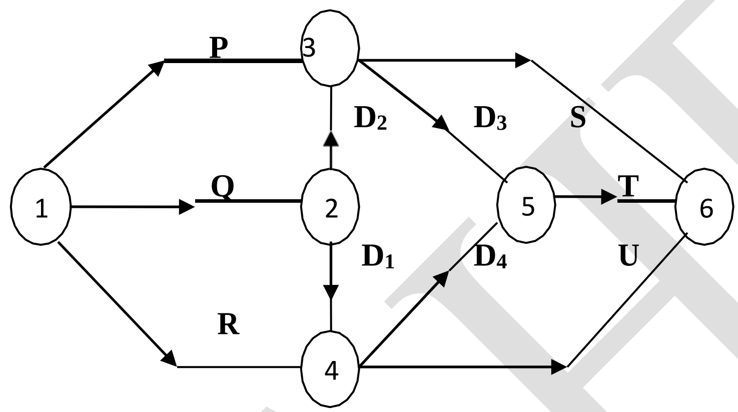
D_1 and D_2 are dummy activities.

Example 2:

Draw the network for the project whose activities and their precedence relationship are given below:

Activity	P	Q	R	S	T	U
Predecessor:	-	-	-	P, Q	P, R	Q, R

Solution:



D_1, D_2, D_3, D_4 are dummy activities.

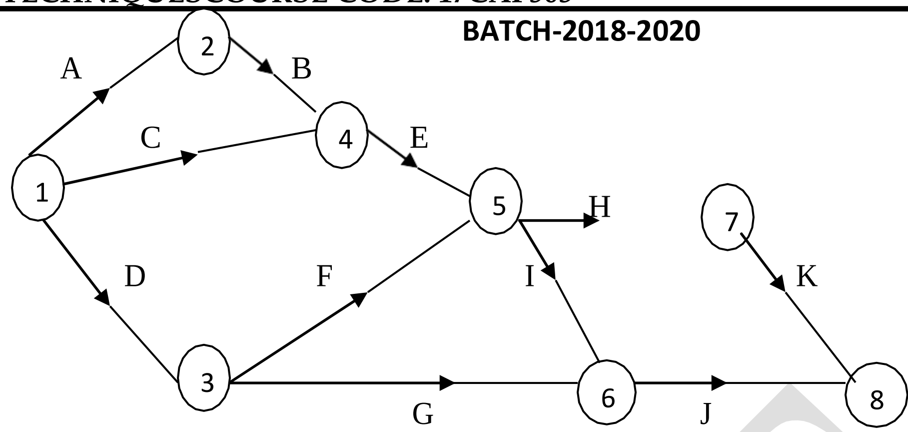
Example 3:

Draw the network for the project whose activities with their predecessor relationship are given below:

A, C, D can start simultaneously : E > B, C ; F, G > D ; H, I > E,F ; J > I,G ; K > H; B> A.

Solution:

Identify the start activities i.e., activities which have no predecessors. They are A,C and D as given. These three activities should start with the same start node. Also identify the terminal activities which have no successors. They are J and K. These two activities should end with the same end node, the last terminal node indicating the completion of the project. Taking into account the predecessors relationship given, the required network is as follows:



Example 4:

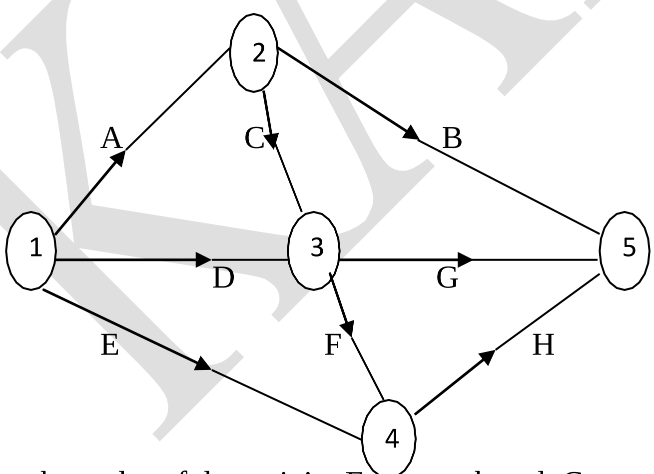
Construct the network for the project whose activities and their relationships are as given below:

Activities : A, D, E can start simultaneously.

Activities : B, C > A; G, F > D, C ; H > E, F.

Solution:

Start activities are A, D, E.
End activities are H, G, B.
The required network is



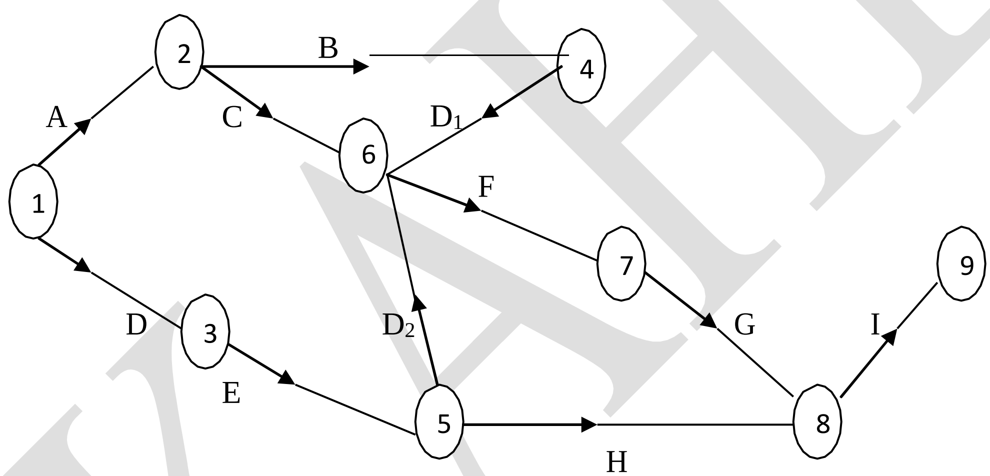
Note : see how the nodes of the activity F are numbered. Can we number C as 2 – 4 and F as 4 – 3?

Example 5: Draw the network for the project whose activities and their precedence relationships are as given below:

Activities :	A	B	C	D	E	F	G	H	I
Immediate Predecessor:	-	A	A	-	D	B,C,E	F	E	G,H

Solution:

Start activities : A,D, Terminal activities : I only. Activities B and C starting with the same node are both the predecessors of the activity F. Also the activity E has to be the predecessor of both F and H. Therefore dummy activities are necessary.Thus the required network is



D_1 and D_2 are dummy activities.

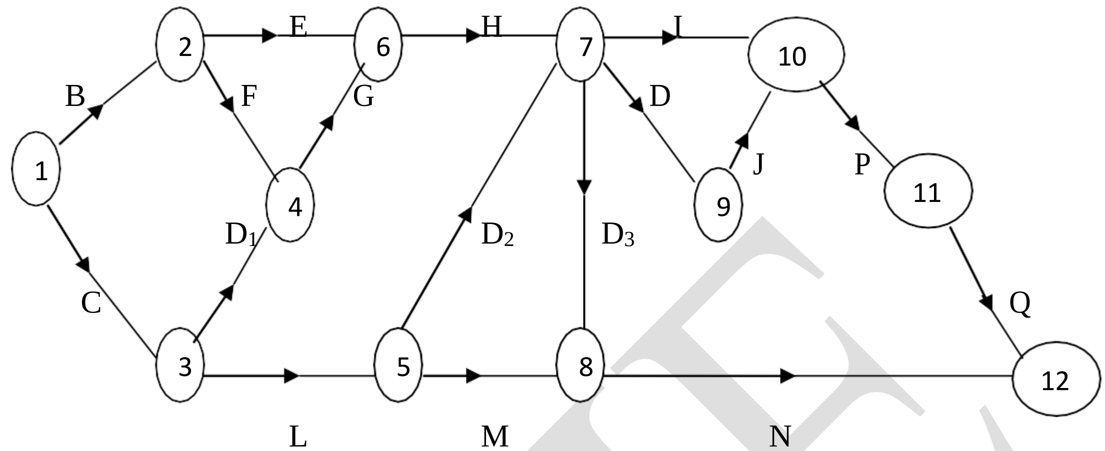
Note: Sometimes while constructing a network you may introduce more dummy activities than necessary. Redundant dummy activities can always be found out when one checks whether all the given precedence relationships given in the problem are satisfied exactly. (Nothing more, nothing less).

Example 6:

Construct the network for the project whose precedence relationships are as given below: B < E, F ; C < G, L ; E, G < H ; L, H < I ; L < M ; H,M < N ; A < J ; I, J < P ; P < Q.

Solution:

Start activities: B,C End activities : N, Q



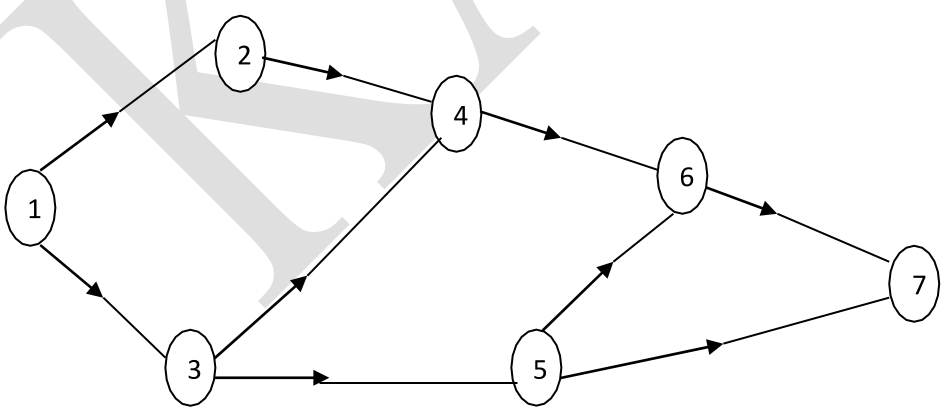
D_1 , D_2, D_3 and D_4 are dummy activities.

Example 7:

Draw the event network for the following data:

Event No :	1	2	3	4	5	6	7
Immediate Predecessors:	-	1	1	2,3	3	4,5	5,6

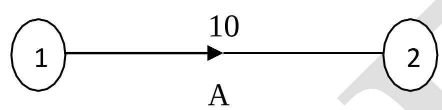
Solution:



Network Computations and Critical Path

(Earliest Completion time of a Project and Critical path)

It is obvious that the completion time of the project is one of the very important things to be calculated knowing the durations of each activity. In real world situation the duration of any activity has an element of uncertainty because of sudden unexpected shortage of labour, machines, materials etc. Hence the completion time of the project also has an element of uncertainty. We first consider the situation where the duration of each activity is deterministic without taking the uncertainty into account.



The above diagram represents an activity whose direction is 10 time unit(hour or days or weeks or month etc)

The first network calculation one does is the computation of earliest start and earliest finish (completion) time of each activity given the duration of each activity. The method used is called forward pass calculation and it is best illustrated by means of the following example.

Example 1:

Compute the earliest start, earliest finish latest start and latest finish of each activity of the project given below:

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration (in days)	8	4	10	2	5	3

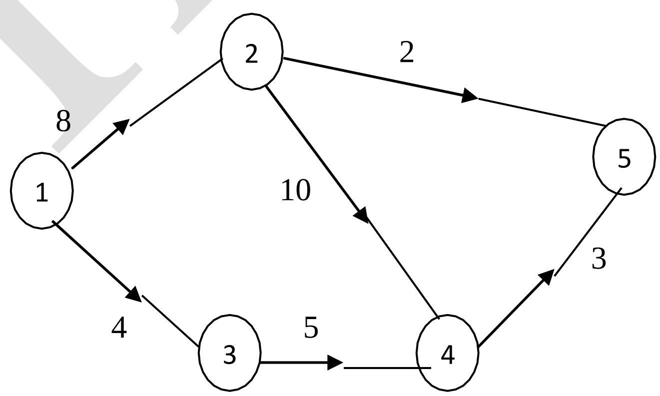


Figure 1

To compute the Earliest start andEarliest finish of each activity:

We take the earliest time of all the start activities as zero.

So earliest starts of 1-2 and 1-3 are zero.

To find earliest start of 2-4.

The activity 2-4 can start only after finishing the only preceding activity 1-2 i.e., after 8 days.

∴ Earliest start of 2-4 is 8 days. Similarly earliest start of 2-5 is also 8 days.

Similarly earliest start of 3-4 is 4 days.

To find the earliest start of 4-5 we first notice that the activity 4-5 has more than one predecessor and also the activity 4-5 can start only after finishing all its preceding activities.

There are two paths leading to the activity 4-5: namely 1-2-4 which takes 18 days and 1-3-4 which takes 9 days. Obviously after 18 days all the activities 1-2, 1-3, 2-4, 3-4 can be finished but not earlier than that.

∴Earliest start of 4-5 is 18 days.

Note: Earliest start of an activity i-j can be denoted as ES_i or ES_{ij}. It can also be called the earliest occurrence of the event i.

Earliest finish of any activity i-j is got by adding the duration of the activity denoted by t_{ij} to the earliest start of i-j.

Hence the earliest finish of 1-2,1-3,2-4,2-5,3-4,4-5 are 8,4,18,10,9,21 respectively.

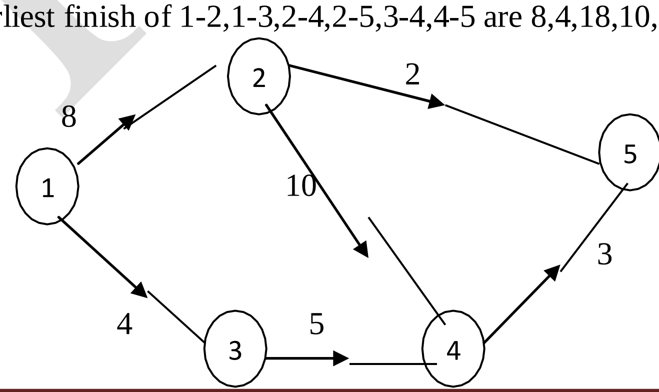


Figure 2

Obviously earliest completion time of the project is 21 days, the greater number among these since all the activities can be finished only after 21 days.

Formula for Earliest Start of an activity i-j in a project network is given by

$$ES_j = \text{Max}\ [ES_i+t_{ij}] \text{ where}$$

ES_j denoted the earliest start time of all the activities emanating from node i and t_{ij} is the estimated during of the activity i-j.

To compute the latest finish and latest start of each activity:

The method used here is called backward pass calculation since we start with the terminal activity and go back to the very first node.

We first calculate the latest finish of each activity as follows:

Latest finish of all the terminating (end) activities is taken as the earliest completion time of the project. Similarly latest finish of all the start activities is obviously taken as the same as the earliest start of these start activities.

Thus the latest finish of the terminal activities 2-5 and 4-5 are 21 days which is the earliest completion time of the project.

Latest finish of the activity 2-4 and 3-4 is 21-3 = 18 days.

Latest finish of 1-3 is 18-5 = 13 days

To find the latest finish of the activity 1-2, we observe that the activity 1-2 has more than one successor activity. Therefore the latest finish of the activity 1-2 is the smaller of the two numbers 21 - 2 = 19 and 18 – 10 = 8. i.e. 8 days.

Note : Latest finish of an activity can be denoted by LF_j or LF_{ij}. It can also be called the latest occurrence of the event j.Latest start of each activity is the latest finish of that activity minus the duration of that activity. The latest start of the activities 4 -5, 2 – 5,2 – 4, 3 – 4,1 – 3, 1 – 2 are 21, 21, 18,18,13,8 respectively.

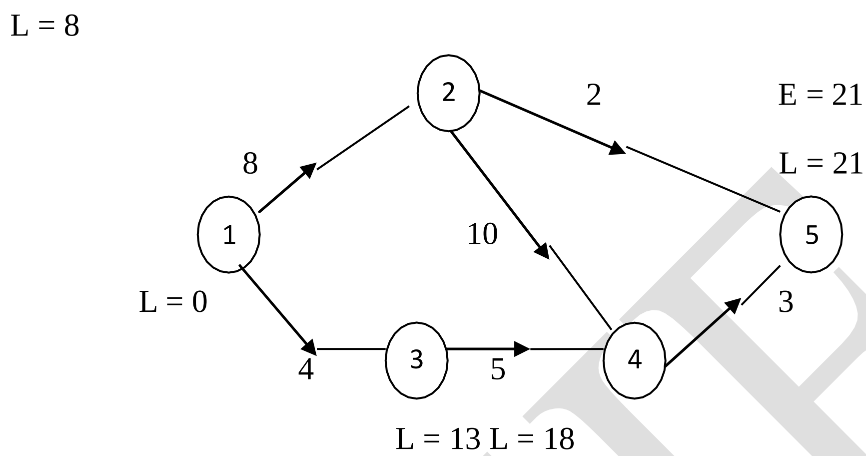


Figure 3

Formula for the latest start time of all the activities emanating from, the event i of the activity i – j, LS_i = Min [LS_j – t_{ij}] for all defined i-j activities where t_{ij} is the estimated duration of the activity i – j.

We can tabulate the results and represents these earliest and latest occurrences of the events in the network diagram as follows:

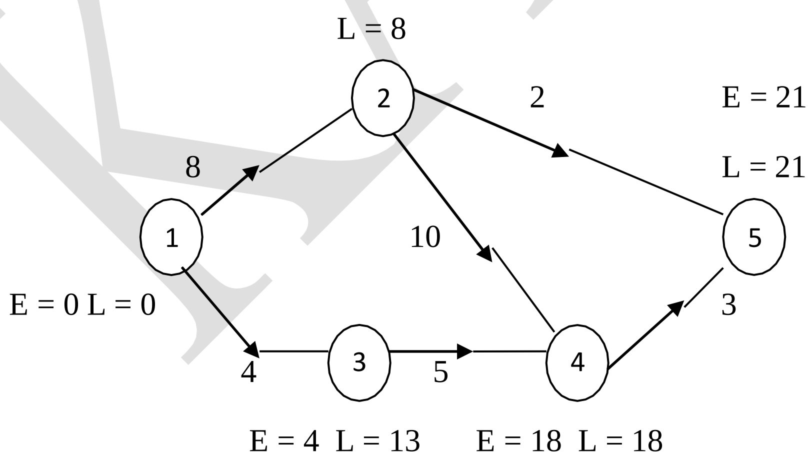


Figure 4

		Earliest		Latest	
Activity	Duration days	Start ES	Finish EF ES = ES+t_{ij}	Start LS LF – t_{ij}	Finish LF
1 – 2	8	0	8	0	8
1 – 3	4	0	4	9	13
2 – 4	10	8	18	8	18
2 – 5	2	8	10	19	21
3 – 4	5	4	9	13	18
4 – 5	3	18	21	18	21

Note: For small networks, it is not difficult to draw the network with E and L values calculated directly by looking at the diagram itself and constructing the table given above.

Critical path:

Path, connecting the first initial node to the very last terminal node, of longest duration in any project network is called the critical path.

All the activities in any critical path are called critical activities. Critical path is 1 – 2 – 4 – 5, usually denoted by doubt lines. (Ref fig.4)

Critical path plays a very important role in project scheduling problems.

Floats

Total float of an activity (T.F) is defined as the difference between the latest finish and the earliest finish of the activity or the difference between the latest start and the earliest start of the activity.

Total float of an activity i - j = $(LF)_{ij} – (EF)_{ij}$ Or = $(LS)_{ij} – (ES)_{ij}$.

Total float of an activity is the amount of time by which that particular activity may be delayed without affecting the duration of the project. If the total float is positive then it may indicate that the resources for the activity are more than adequate. If the total float of an activity is zero it may indicate that the resources are just adequate for that activity. If the total float is negative, it may indicate that the resources for that activity are inadequate.

Note: (L – E) of an event of I – j is called the slack of the event j.

There are three other types of floats for an activity, namely, Free float, Independent float and interference (interfering) float.

Free Float:

Free Float of an activity (F.F) is that portion of the total float which can be used for rescheduling that activity without affecting the succeeding activity. It can be calculated as follows:

Free float of an activity i – j = Total float of i – j – (L – E) of the event j

= Total float of i – j – slack of the head event j

= Total float of I – J – slack of the head event j

Where L = Latest occurrence , E= Earliest occurrence

Obviously Free Float ≤ Total float for any activity.

Independent float (I.F):

Independent float (I.F) of an activity is the amount of time by which the activity can be rescheduled without affecting the preceding or succeeding activities of that activity.

Independent float of an activity i – j = Free float of i- j – (L – E) of event i.

= Free float of i – j – Slack of the tail event j.

Clearly, Independent float ≤ Free float for any activity. Thus I.F ≤ F.F ≤ T.F.

Interfering Float or Interference Float of an activity i – j is nothing but the slack of the head event j. Obviously, Interfering Float of i – j = Total Float of i – j – Free Float of i – j.

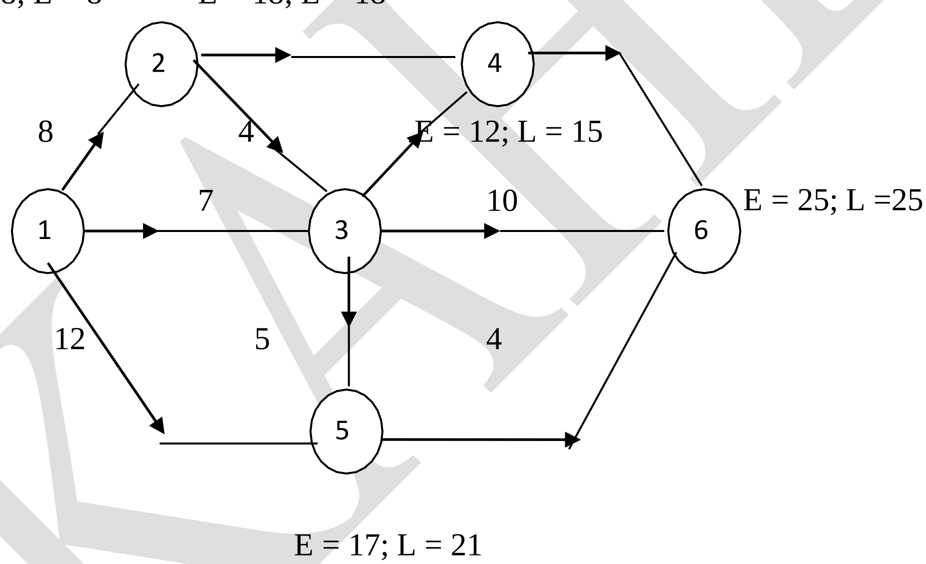
Example 2:

Calculate the total float, free float and independent float for the project whose activities are given below:

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration (in days)	8	7	12	4	10	3	5	10	7	4

The data is the same as given in example 2 above. The network with L and E of every event is given by

Solution: E = 8; L = 8 E = 18; L = 18



		Earliest		Latest		Floats		
Activity	Duration (in weeks)	Start	Finish	Start	Finish	TF	FF	IF
1 – 2	8	0	8	0	8	0	0	0
1 – 3	7	0	7	8	15	8	5	5
1 – 5	12	0	12	9	21	9	5	5
2 – 3	4	8	12	11	15	3	0	0

2 – 4	10	8	18	8	18	0	0	0
3 – 4	3	12	15	15	18	3	3	0
3 – 5	5	12	17	16	21	4	0	-3
3 – 6	10	12	22	15	25	3	3	0
4 – 5	7	18	25	18	25	0	0	0
5 – 6	4	17	21	21	25	4	4	0

Explanation:

To find the total float of 2 – 3.

Total float of (2 – 3) = (LF – EF) of (2 – 3) = 15 – 12 = 3 from the table against the activity 2 – 3.

Free Float of (2- 3) = Total float of (2 – 3) – (L – E) of event 3

= 3 – (15 12) from the figure for event 3 = 0

Free Float of (1 – 5) = Total float of (1 – 5) – (L – F) of event 5

= (21 – 12) – (21 – 17) from the figure for event 5

= 9 – 4 = 5

Independent float of (1 – 5) = Free Float of (1 – 5) – (L – E) of event 1

= 5 – (0 – 0) = 5

Important Note:

Note that all the critical activities have their total float as zero. In fact the critical path can also be defined as the path of least (zero) total float. As we have noticed total float is 3 for the activity 2 – 3. This means that the activity 2 – 3 can be delayed by 3 weeks without delaying the duration (completion date) of the project.

Free float of 3 – 4 is 3. This means that the activity 3 – 4 can be delayed by 3 weeks without affecting its succeeding activity 4 – 6.

Independent float of 1 – 5 is 5 means that the activity 1 – 5 can be delayed by 5 weeks without affecting its preceding or succeeding activity. Of course 1 – 5 has no preceding activity.

Uses of floats:

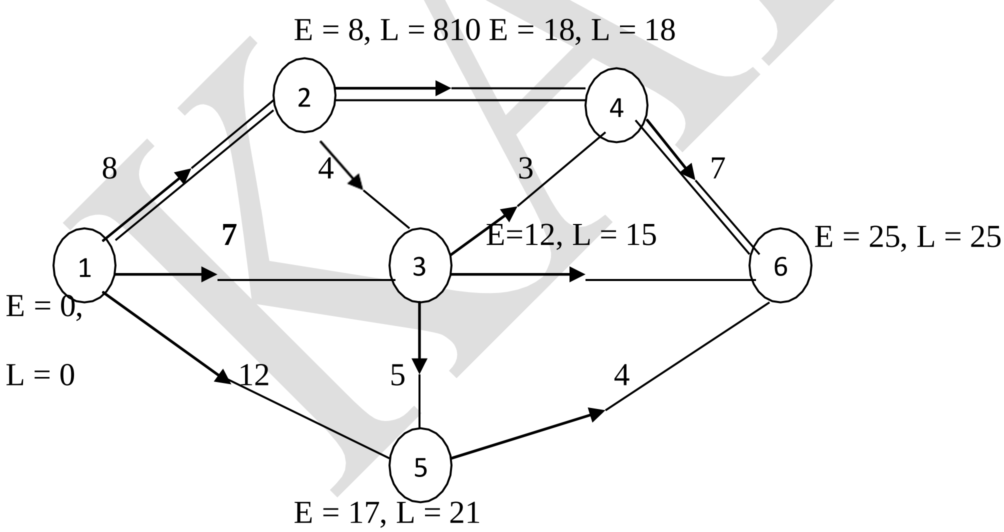
Floats are useful in resources leveling and recourse allocation problems which will be discussed in the last section of this chapter. Floats give some flexibility in rescheduling some activities so as to smoothen the level of resources or allocate the limited resources as best as possible.

Example 3:

Calculate the earliest start, earliest finish, latest start and latest finish of each activity of the project given below and determine the critical path of the project.

Activity	1 – 2	1 – 3	1 – 5	2 – 3	2 – 4
Duration (in weeks)	8	7	12	4	10
Activity	3 – 4	3 – 5	3 – 6	4 – 6	5 – 6
Duration (in weeks)	3	5	10	7	4

Solution:



Program Evaluation Review Techniques: (PERT)

This technique, unlike CPM, take into account the uncertainly of project durations into account.

Optimistic (least) time estimate: (t_0 or a)is the duration of any activity when everything goes on very well during the project. i.e., labourers are available and come in time, machines are working properly, money is available whenever needed, there is no scarcity of raw material needed etc.

Pessimistic (greatest) time estimate: (t_p or b)is the duration of any activity when almost everything goes against our will and a lot of difficulties is faced while doing a project.

Most likely time estimate: (t_m or m) is the duration of any activity when sometimes things go on very well, sometimes things go on very bed while doing the project.

Two main assumption make in PERT calculations are

(i) The activity durations are independent. Ie., the time required to complete an activity will have no bearing on the completion times of any other activity of the project.

(ii) The activity durations follow β – distribution.

β Distribution is a probability distribution with density function $k(t-a)^{\alpha}(b-t)^{\beta}$ with mean $t_e = \frac{1}{3}[2t_m+\frac{1}{2}(t_0 - t_p)]$ and the standard deviation $\sigma_t=\frac{t_p-t_0}{6}$

PERT procedure

(1) Draw the project network

(2) Compute the expected duration of each activity $t_e = \frac{t_0+4t_m+t_p}{6}$

(3) Compute the expected variance $\sigma^2= \left(\frac{t_p-t_0}{6}\right)^2$ of each activity.

(4) Compute the earliest start, earliest finish, latest start, latest finish and total float of each activity.

(5) Determine the critical path and identity critical activites.

(6) Compute the expected variance of the project length (also called the variance of the critical path) $\sigma_c{}^2$ which is the sum of the variance of all the critical activities.

(7) Compute the expected standard deviation of the project length σ_c and calculate the standard normal deviate $\frac{T_S - T_E}{\sigma_C}$ where

T_s = Specified or Schedule time to complete the project

T_E = Normal expected project duration

σ_c= Expected standard deviation of the project length.

(8) Using (7) one can estimate the probability of completing the project within a specified time, using the normal curve (Area) tables.

Example 1:

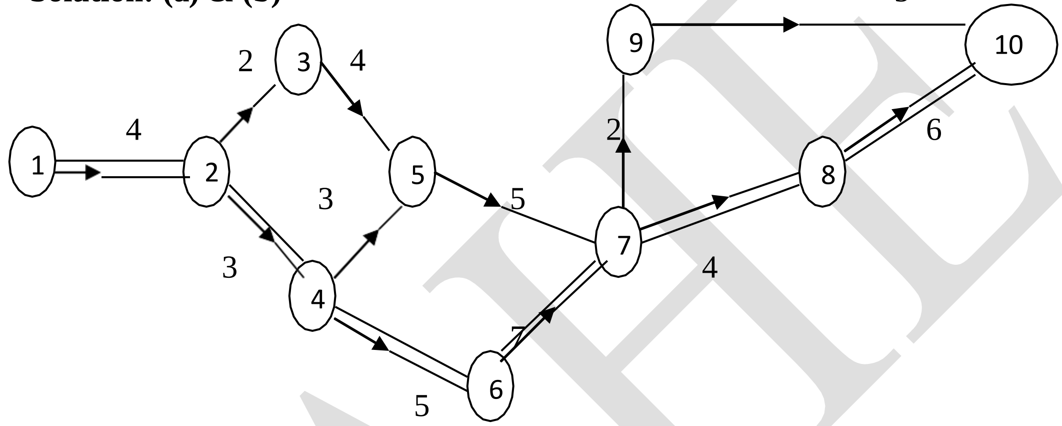
Construct the network for the project whose activities and the three time estimates of these activities (in weeks) are given below. Compute

(a) Excepted duration of each activity.
(b) Excepted variance of each activity.
(c) Excepted variance of the project length.

Activity	t_0	t_m	t_p
1 – 2	3	4	5
2 – 3	1	2	3
2 – 4	2	3	4
3 – 5	3	4	5
4 – 5	1	3	5
4 – 6	3	5	7
5 – 7	4	5	6

6 – 7	6	7	8
7 – 8	2	4	6
7 – 9	1	2	3
8 – 10	4	6	8
9 – 10	3	5	7

Solution: (a) & (b)



Activity	t_0	t_m	t_p	Excepted duration $t_e = \dfrac{t_0 + 4t_m + t_p}{6}$	Excepted variance $\sigma^2 = \left(\dfrac{t_p - t_0}{6}\right)^2$
1 – 2	3	4	5	4	$\mathbf{{}^1/_9}$ **= 0.11** nearly
2 – 3	1	2	3	2	${}^1/_9$ = 0.11
2 – 4	2	3	4	3	$\mathbf{{}^1/_9}$ **= 0.11**
3 – 5	3	4	5	4	${}^1/_9$ = 0.11
4 – 5	1	3	5	3	${}^4/_9$ = 0.44
4 – 6	3	5	7	5	$\mathbf{{}^4/_9}$ **= 0.44**
5 – 7	4	5	6	5	${}^1/_9$ = 0.11
6 – 7	6	7	8	7	$\mathbf{{}^1/_9}$ **= 0.11**
7 – 8	2	4	6	4	$\mathbf{{}^4/_9}$ **= 0.44**
7 – 9	1	2	3	2	${}^1/_9$ = 0.11
8 – 10	4	6	8	6	$\mathbf{{}^4/_9}$ **= 0.44**
9 – 10	3	5	7	5	${}^4/_9$ = 0.44

Critical path 1 – 2 – 4 – 6 – 7 – 8 – 10. Excepted project duration = 29 weeks.
(c) Excepted variance of the project length = Sum of the expected variances of all the critical activities

$$= \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{15}{9} = \frac{15}{9} = \frac{5}{3} = 1.67$$

or (0.11 + 0.11 + 0.44 + 0.11 + 0.44 + 0.44 = 1.32 + 0.33 = 1.65)

Example 2:

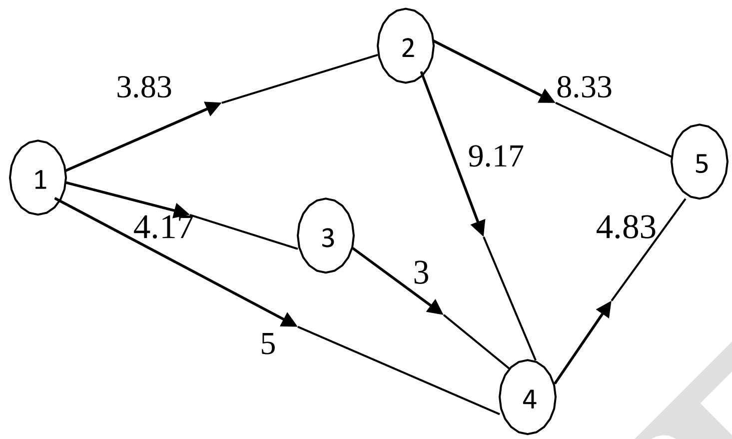
The following table indicates the details of a project. The duration are in days. „a" refers to optimistic time, „m" refers to most likely time and „b" refers to pessimistic time duration.

Activity	1 – 2	1 – 3	1 – 4	2 – 4	2 – 5	3 – 5	4 – 5
a	2	3	4	8	6	2	2
m	4	4	5	9	8	3	5
b	5	6	6	11	12	4	7

(a) Draw the network
(b) Find the critical path
(c) Determine the excepted standard deviation of the completion time.

Solution:

Activity	*a*	*m*	*b*	Excepted duration t_e	Excepted variance σ^2
1 – 2	2	4	5	3.83	$\mathbf{\frac{1}{4}}$
1 – 3	3	4	6	4.17	$\frac{1}{4}$
1 – 4	4	5	6	5	$\frac{1}{9}$
2 – 4	8	9	11	9.17	$\mathbf{\frac{1}{4}}$
2 – 5	6	8	12	8.33	1
3 – 4	2	3	4	3	$\frac{1}{9}$
4 – 5	2	5	7	4.83	$\mathbf{\frac{25}{36}}$



Critical path 1 – 2 – 4 – 5

Excepted project duration = 17.83 days

Excepted variance of the completion time = $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{43}{36}$

Excepted standard deviation of the completion time = $\sqrt{\frac{43}{36}}$ = 1.09 nearly

Example 3:

A project consists of the following activities and time estimates:

Activity	*Least time (days)*	*Greatest time (days)*	*Most likely time (days)*
1 – 2	**3**	**15**	**6**
2 – 3	**2**	**14**	**5**
1 – 4	**6**	**13**	**12**
2 – 5	**2**	**8**	**5**
2 – 6	**5**	**17**	**11**
3 – 6	**3**	**15**	**6**
4 – 7	**3**	**27**	**9**
5 – 7	**1**	**7**	**4**
6 – 7	**2**	**8**	**5**

(a) Draw the network
(b) What is the probability that the project will be completed in 27 days?

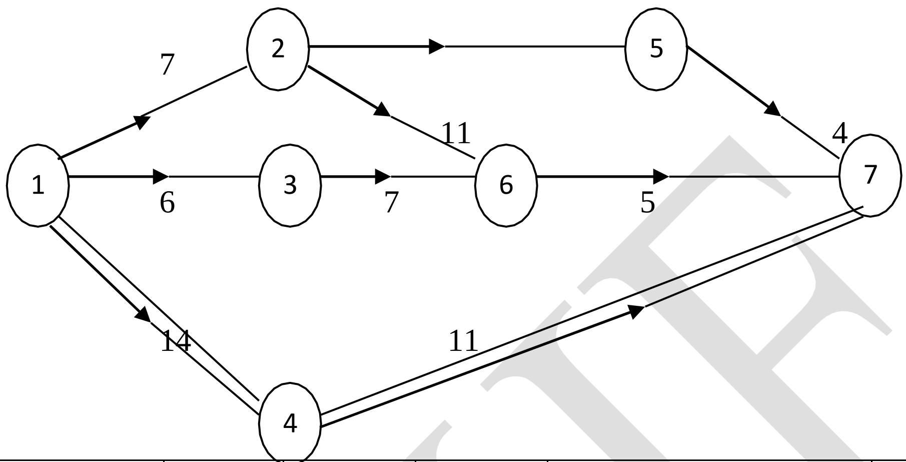
Solution:

Obviously Greatest time = Pessimistic time = t_p

Least time = Optimistic time = t_0

Most Likely time = t_m

(a) 5



Activity	t_0	t_p	t_m	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
1 – 2	3	15	6	7	4
2 – 3	2	14	5	6	4
1 – 4	6	13	12	14	16
2 – 5	2	8	5	5	1
2 – 6	5	17	11	11	4
3 – 6	3	15	6	7	4
4 – 7	3	27	9	11	16
5 – 7	1	7	4	4	1
6 – 7	2	8	5	5	1

Critical path 1 – 4 – 7

Excepted project duration = 25 days

Excepted variance of the project length = Sum of the excepted variance of all the critical activities

= 16 + 16 = 32.

σ_c = Standard deviation of the project length = $\sqrt{32}$ = $4\sqrt{2}$ = 5.656

$Z = \frac{T_S - T_E}{\sigma_C} = \frac{27-25}{5.656} = 0.35$

Probability that the project will be completed in 27 days

$= P(T_s \leq 27) = P(Z \leq 0.35)$

$= 0.6368 = 63.7\%$

Basic difference between PERT and CPM

PERT

1. PERT was developed in a brand new R and D project it had to consider and deal with the uncertainties associated with such projects. Thus the project duration is regarded as a random variables and therefore probabilities are calculated so as to characteristics it.
2. Emphasis is given to important stages of completion of task rather than the activities required to be performed to reach a particular event or task in the analysis of network. i.e., PERT network is essentially an event – oriented network.
3. PERT is usually used for projects in which time estimates are uncertain. Example: R & D activities which are usually non-repetitive.
4. PERT helps in identifying critical areas in a project so that suitable necessary adjustments may be made to meet the scheduled completion date of the project.

CPM

1. CPM was developed fir conventional projects like construction project which consists of well known routine tasks whose resources requirement and duration were known with certainty.
2. CPM is suited to establish a trade off for optimum balancing between schedule time and cost of the project.
3. CPM is used for projects involving well known activities of repetitive in nature, However the distinction between PERT and CPM is mostly historical.

POSSIBLE QUESTIONS:

PART-B(5X6 = 30 MARKS)

1. A Project has the following characteristics

Activity :	A	B	C	D	E	F
Duration:	6	8	4	9	2	7
Preceding activity :	_	A	A	B	C	D

Draw the network diagram and find the critical path.

2. Calculate the total float, free float and independent float for theproject whose activities are given below.

Activity:	1-2	1-3	2-4	3-4	3-5	4-5	4-6	5-6
Duration:	6	5	10	3	4	6	2	9

3. Draw the network and find the critical path of the project given below.

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration	8	4	10	2	5	3

4. Calculate the total float, free float and independent float for the project whose activities are given below:

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration (in days)	8	7	12	4	10	3	5	10	7	4

5. Draw the network diagram and find the critical path.

Job	Predecessors	Duration(days)

A	-	10
B	-	5
C	B	3
D	A,C	4
E	A,C	6
F	D	6
G	E	5
H	F,G	5

6. Write the difference between CPM and PERT.

7. A Project consists of the following activities and time estimates:

Activity	Least time(days)	Greatest time(days)	Most likely Time (days)
1-2	2	4	5
1-3	3	4	6
1-4	4	5	6
2-4	8	9	11
2-5	6	8	12
3-4	2	3	4
4-5	2	5	7

i) Draw the network and find the critical path.

ii) Find the expected standard deviation of completion of time.

8. A Project consists of the following activities and time estimates:

Activity	t_o	t_m	t_p
1-2	0.8	1.0	1.2
2-3	3.7	5.6	9.9
2-4	6.2	6.6	15.4
3-4	2.1	2.7	6.1
4-5	0.8	3.4	3.0
5-6	0.9	3.4	2.7

i) Find the critical path.

ii) Determine the expected duration and standard deviation of each activity

iii) Find the probability that the project will be completed in 2 months earlier than expected.

9. A project consists of the following activities and time estimates:

Activity	Least time (days)	Greatest time (days)	Most likely time (days)
1 – 2	3	15	6
2 – 3	2	14	5
1 – 4	6	13	12
2 – 5	2	8	5
2 – 6	5	17	11
3 – 6	3	15	6
4 – 7	3	27	9
5 – 7	1	7	4
6 – 7	2	8	5

(c) Draw the network
(d) What is the probability that the project will be completed in 27 days?

10. The following table indicates the details of a project. The duration are in days. „a" refers to optimistic time, „m" refers to most likely time and „b" refers to pessimistic time duration.

Activity	1 – 2	1 – 3	1 – 4	2 – 4	2 – 5	3 – 5	4 – 5
a	2	3	4	8	6	2	2
m	4	4	5	9	8	3	5
b	5	6	6	11	12	4	7

(a) Draw the network
(b) Find the critical path
(c) Determine the excepted standard deviation of the completion time.

PART-C (1X10=10 MARKS)

COMPULSORY:

1. The three estimates for the activities of a project are given below:

Activity	Estimated duration (days)		
	a	m	b
1 – 2	5	6	7
1 – 3	1	1	7
1 – 4	2	4	12
2 – 5	3	6	15
3- 5	1	1	1
4 – 6	2	2	8
5 – 6	1	4	7

i) Draw the project network.
ii) What is the probability that the project will be completed on 22 days ?

2. The following table lists the jobs of a network with their time estimate

Jobs	Optimistic	Duration days Most likely	Pessimistic

1-2	3	6	15
1-6	2	5	14
2-3	6	12	30
2-4	2	5	8
3-5	5	11	17
4-5	3	6	15

6-7	3	9	27
5-8	1	4	7
7-8	4	19	28

Draw the project network and calculate the length and variance of the critical path.

3. A Project consists of the following activities and time estimates:

Activity	t_o	t_m	t_p
1-2	3	4	5
2-3	2	3	10
2-4	4	4	10
3-5	3	5	7
4-5	1	7	7
4-6	2	9	10
5-6	2	3	4

i) Find the critical path.
ii) Determine the expected project completion time and its variance

Question	Opt 1	Opt 2	Opt 3	Opt 4	Opt 5	Opt 6	Answer
A ----------- is defined as a combination of interrelated acitivities all of which be executed in a certain order to achieve a goal	Project	Acitivity	Event	Nodes			Project
----------is a task or an item of work to be done in aproject.	Project	Acitivity	Event	Nodes			Acitivity
An ------------ is represented by an arrow with a node at the beginning and a node at the end indicating the start and finish of the activity.	Project	Nodes	Event	Acitivity			Acitivity
Nodes are denoted by -----------------	dot	circle	arrow	square			circle
The diagram in which arrow represents an activity is called ------------------	arrow diagram	network diagram	graph diagram	line diagram			arrow diagram
The initial node are also called --------------	head event	tail event	first event	last event			tail event
The terminal node are called -------------	head event	tail event	first event	last event			head event
An activity wgich must be completed before one or more other activities start is known as ----------------- activity	Predecessor	successor	initial	final			Predecessor
An activity which started immediately after one or more of the other activities are completed is known as -----------------	Predecessor	successor	initial	final			successor
An activity which does not consumes either any resources and/or time is known as -----------------------	Predecessor	successor	dummy	initial			dummy
If an activity B can start immediately after an activity A, then A is called --------------------	immediate predecessor	immediate successor	Predecessor	successor			immediate predecessor
If an activity B can start immediately after an activity A, then B is called --------------------	immediate predecessor	immediate successor	Predecessor	successor			immediate successor
The notation ' A < B ' is called -----------------	A is a predecessor of B	B is a successor of A	A is a successor of B	B is a predecessor of A			A is a predecessor of B
The notation ' B > A ' is called ----------------	A is a predecessor of B	B is a successor of A	A is a successor of B	B is a predecessor of A			B is a successor of A
Activities which have no predecessors are called---------------- activity	dummy	start	zero	terminal			start
All the start activities can be made to have the ---------------- initial node.	same	different	multiple	zero			same
Activities which have no successor are called---------------- activity	dummy	start	zero	terminal			terminal
The diagram denoting all the activities of a project by arrows taking into account the technological square of the activities is called -------------	Project	nework	project network	Event			project network
There is another representation of a project network representing activities on nodes called ---------------	AON diagram	ANO diagram	NOA diagram	arrow diagram			AON diagram
only ------------ activity should connect any two nodes.	one	two	three	multiple			one
Path, connecting the first node to the very last terminal node of long duration in any project network is called ---------------	PERT	Critical path	Activity	network			Critical path
All the activities in any critical path are called -----------------	start activities	dummy activities	critical activities	terminal activities			critical activities
Critical path plays a very important role in project ------------ problems	scheduling	planning	controlling	network			scheduling
An activity is defined as the difference between the latest start and the earliest start of the activity is called ----------------	free float	total float	independent float	interfering float			total float
If the total float is ------------ then it may indicate that the resources for the activity are more that adequate.	positive	negative	zero	any value			positive

If the total float of an activity is -------------- it may indicate that the resources are just adequate for that activity.	positive	negative	zero	any value	zero
If the total float is ------------ , it may indicate that the resources for that activity are inadequate.	positive	negative	zero	any value	negative
(L - E) of an aevent i-j is called the -------------- of the event j.	slack	surplus	dummy	total	slack
One of the portion of the total float is -----------------	free float	total float	independent float	interfering float	free float
rescheduling that activity without affecting the succeding activity we can use ----------------- of an activity	free float	total float	independent float	interfering float	free float
Free float is ------------------ the total float of an activity	equal to	greater than or equal to	less than or equal to	not equat to	less than or equal to
The amount of time by which the activity can be rescheduled with effecting the preceding or succeeding of that activity is called ------------	free float	total float	independent float	interfering float	independent float
The slack of the head event j is called the --------------------- of an activity i-j.	free float	total float	independent float	interfering float	interfering float
Interfering float of i-j is the difference between the total float and -------------------	free float	dependent float	independent float	dummy float	free float
All the critical activities have their total float as -----------------	one	two	zero	any value	zero
Critical path can also be defined as the path of ---------------------	least total float	greatest total float	least free float	greatest free float	least total float
The objective of network analysis is to ----------------	minimize total project duration	minimize total project cost	minimize prduction delay	minimize the interruption	minimize total project duration
The slack for an activity is equal to -----------------	LF - LS	EF - ES	LS - ES	LS - EF	LS - ES
PERT stands for -------------------	Project Enumaration review Technique	Project Evaluation Review Technique	Planning Evaluation Review Technique	Planning Enumaration review Technique	Project Evaluation Review Technique
Generally PERT technique does not deals with the project of	repetitive nature	non repetitive nature	deterministic nature	non deterministic nature	non deterministic nature
The technique of OR used for planning, scheduling and controlling large and complex projects are often referred as -------	network analysis	graphical analysis	critical activities	PERT	network analysis
A network is a -----------------	quality plan	control plan	graphical plan	inventory plan	graphical plan
Critical path method is used for commpletion of projects involving activities of -------------------------	repetitive nature	non repetitive nature	deterministic nature	non deterministic nature	repetitive nature
An event which represents the joint completion of more than one activity is known as -------------------	unique event	burst event	merge event	dummy event	merge event
An event which represents the initiation of more than one activity is known as ---------------	unique event	burst event	merge event	dummy event	burst event
Events in the network diagram are identified by ---------------------	numbers	variables	symbols	special characters	numbers
The negative value of the independent float is --------------	one	zero	distinct	non zero	zero
In PERT the span of time between the optimistic and pesimistic time estimates of an acitivity is ------------	3σ	6σ	12σ	4σ	6σ
If an activity has zero slack, it implies that --------------------	it lies on the critical path	it is a dummy activity	the project is progressing well.	the project is not progressing well	it lies on the critical path

A dummy activity is used in the network diagram when ------------------	two parallel activities have the same tail and head events	the chain of activities may have a common event not yet be independent by themselves	two parallel activities have the different tail and head events	If the activities have the tail and head events	two parallel activities have the same tail and head events
The path of least cost float in a project is called ----------------	PERT	Critical path	unique path	network path	Critical path
The project duration is affected if the duration of any activity is --------------	changed	unchanged	same	exist	changed
The number of time estimates involved ina PERT problem is ---------------	1	2	3	4	3
For a non critical activity, the total float is ---------------------	zero	non zero	unique	distinct	non zero
The probability to complete a project in the expected time is ----------------	1	1.5	0.5	1.15	0.5
In PERT ananlysis, the critical path is obtained by joining event having --------------------	positive slack	negative slack	non zero slack	unique slack	positive slack
In PERT network each activity time assumes a Beta - distribution because ----------------	it is a uni - model distribution that provides informing regarding the uncertainty of time estimates.	it has got finite non negative error	it need not be symmetrical about model value	it has infint negative error	it is a uni - model distribution that provides informing regarding the uncertainty of time estimates.
Float or slack analysis in useful for ------------------	projects behind the schedule only	projects ahead of the schedule only	projects behind the planning only	projects ahead of the planning only	projects behind the schedule only
In time cost trade off funtion analysis ----------------	cost decreases linearly as time increases	cost at normal time is zero	cost increases linearly as time increases	cost at normal time is unity	cost decreases linearly as time increases
The name of the proability distribution (used PERT) which estimates the expected duration and the expected variance of an activity is -----------	Beta distribution	Gamma distribution	poisson distribution	normal distribution	Beta distribution