Reg. No----------------

(17MMP203)

KARPAGAM ACADEMY OF HIGHER EDUCATION
COIMBATORE – 641021
DEPARTMENT OF MATHEMATICS
SECOND SEMESTER
I INTERNAL TEST- JAN'18
OPTIMIZATION TECHNIQUES

Time: 2 Hours **Maximum: 50 Marks**
Date: .01.18( ) **Class: I M.Sc Mathematics**

PART – A(20X1=20 Marks)

1. A LPP in which all or some of the decision variables are constrained to assume non negative integer values is called an -------------
   a) **Integer programming problem**
   b) b) Dynamic programming problem
   c) Non linear programming problem
   d) d) Decision analysis
2. In a LPP, if all the variables in the optimal solution are restricted to assume non negative integer values, then it is called the ------------------
   a) Zero - one IPP b) **Pure IPP**
   b) c) Mixed IPP d) Non IPP
3. In cutting algorithm, each cut involves the introduction of _______
   a) an equality constraint
   ~~b~~) **less than or equal to constraint**
   c) greater than or equal to constraint
   d)an artificial variable.

4. In the context of Branch and bound method, which of the following is not correct?
   a) **It can be used to solve any kind of programming problem**
   b) It divides the feasible region into smaller parts by the process of branching
   c) It is very usefull employed in problems where there a finite number of solutions
   d) It is not a standardised method and is applied differently for different problems
5. A systematic procedure for solving pure I.P.P is ----------
   a) Hungarian method b) **Cutting method**
   c) Revised simplex method d)Modi method
6. The fractional part of the negative number -7 / 3 is _____
   a) 7/3 b) 1/3 **c) 2/3** d) 5/3
7. __________ can be solved using Branch and Bound method
   a) **A travelling salesman problem**
   b) assignment problem
   c) geometric problem
   d) simulation problem
8. The part of the feasible solution space eliminated by plotting a cut constraints _______
   a) **Onlynon integer solutions**
   b) only integer solutions
   c) both integer and non integer solutions only
   d) rational solutions

9. Which of the following is correct?

a) A linear programming problem with only one decision variable restricted to integer value is not an integer programming problem

b) An integer programming problem is an LPP with decision variables restricted to integer values.

c) A mixed IPP is one where mixed constraints are involved.

d) A pure IPP is one where all the decision variables are either zero or unity

10. ______ can be considered as a zero - one programming problem.

a) A travelling salesman problem

b) assignment problem

c) geometric problem

d) simulation problem

11. While solving IP problem any non - integer variable in the solution is picked up to ________

a) obtain the cut constraint b) enter the solution

c) leave the solution d) no solution

12. Which of the following is not an integer linear programming problem?

a) Zero - one IPP b) Pure IPP

c) Mixed IPP **d)continuous IPP**

13. The mathematical Techniques of optimizing such a sequence of interrelated decisions over a period of time is called------------------.

a) Integer programming problem

b) **Dynamic programming problem**

c) Non linear programming problem

d) Decision analysis

14. A stage in a dynamic programming problem represents

a) **number of decision alternatives**

b) status of the system at a particular state

c) same time periods in the planning period

d) different time periods in the planning period

15. The variables which specify the condition of the decision process or describe the status of the system at a particular stage are called ---------------------

a) **State variables** b) decision variables

c) dependent variables d) independent variables

16. A decision making rule that at any stage permits a feasible sequence of decisions is called______

a) State b) Stage **c)Policy** d) Optimal policy

17. Dynamic Programming Problem deals with the ______

a) multi stage decision making problems

b) single stage decision making problems

c) time independent decision making problems

d) problems which fix the levels of different decision variables so as to maximize profit or minimize cost

18. Which of the following is not correct?

a) DP approach helps in reducing the computational efforts in sequential decision making

b) DPP can be divided into a sequence of smaller sub problems called stages of the original problem.

c) DP cannot be dealt with non linear constraints

d) The concept of dynamic programming is based upon the principle of optimality due to Bellman.

19. DP divides the problem into a number of --------

a) conflicting objective function **b) decision stages**

c) unrelated constraints d) policies

20. Which of the following is not correct?

a) DPP is solved starting from the initial stage to the next till the final stage is reached

b) **DPP can be solved by simplex method**

c) Optimum solution is DPP depends on the initial solution.

d) Computation in DPP are done recursively, in the sense that the optimum solution of one sub problem is used as an input to the next sub problem.

PART –B(3X2=6 MARKS)

ANSWER ALL THE QUESTIONS

21 . Define Pure Integer Programming Problem.

22. Define Zero – one Programming Problem.

23. What is dynamic programming.

PART –B(3X8=24 MARKS)

ANSWER ALL THE QUESTIONS

24 . a) Find the optimum integer solution to the following LPP

Maximize $Z = x_1 + 4x_2$

Subject to the constraints

$2x_1 + 4x_2 \leq 7$

$5x_1 + 3x_2 \leq 15$

and $x_1, x_2 \geq 0$ and are integers

(OR)

b) Solve the following ILPP using Branch and Bound method

Maximize $Z = 7x_1 + 9x_2$

Subject to the constraints

$-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

$x_2 \leq 7$

and $x_1, x_2 \geq 0$ and are integers

25. a) Solve the following mixed integer programming problem.

Minimize $Z = x_1 - 3x_2$

Subject to

$x_1 + x_2 \leq 5$

$-2x_1 + 4x_2 \leq 11$

and x_1 , $x_2 \geq 0$ and x_2 be an integers.

(OR)

b) Use dynamic programming to solve the following LPP

Maximize $Z = 3x_1 + 5x_2$

Subject to the constraints

$x_1 \leq 4$

$3x_1 + 2x_2 \leq 18$

and $x_1 , x_2 \geq 0$

26. a) Divide a positive quantity c into n-parts in such a way that their product is maximum.

(OR)

b) Solve the following mixed integer programming problem.

Maximize $Z = x_1 + x_2$

Subject to the constraints

$2x_1 + 5x_2 \leq 16$

$6x_1 + 5x_2 \leq 30$

and $x_2 \geq 0$ and x_1 is non negative integers.

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SECOND SEMESTER
II INTERNAL TEST- MAR'18
OPTIMIZATION TECHNIQUES

Time: 2 Hours **Maximum: 50 Marks**
Date: .03.18( ) **Class: I M.Sc Mathematics**

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1. In DPP, the number of stage variable is ----------- the number of state variable
   a) equal to b) not equal to
   c) greater than d) lesser than
2. ---------- may be defined as the stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business affairs
   a) Inventory b) Transportation c) Queueing d) Sequencing
3. Holding cost is denoted by -------
   a) C_1 b) C_2 c) C_3 d) C_4
4. Elapsed time between the placement of the order and its receipts in inventory is known as---------
   a) lead time b) recorder level
   c) EOQ d) variables
5. Total inventory cost = -------
   a) set up cost + purchasing cost
   - b) holding cost + shortage cost
   c) set up cost + purchasing cost + holding cost + shortage cost
   d) setup cost + shortage cost
6. The several alternatives available are called --------
   a) acts b) payoff
   c) state of nature d) outcome
7. A situation in which a decision maker knows all of the possible outcomes of a decision andalso knows the probability associated with each outcome is referred to as ---
   a) certainty b) risk
   c) uncertainty d) strategy
8. Which one of the following does measure risk?
   a) Coefficient of variance b) Standard deviation
   c) Expected value d) Expected variance
9. States of nature _____________
   a) can describe uncontrollable natural events such as floods or freezing temperatures
   b) can be selected by the decision maker
   c) cannot be enumerated by the decision maker
   d) can not describe uncontrollable natural events
10. A NLP problem with non linear objective function and linear constraints such as NLP is called -----------
    a) Dynamic Programming b) Quadratic programming
    c) Geometric Programming d) Separable programming
11. In case of maximization of NLPP all the constraints must be converted into -------- type
    a) strictly less than b) less than or equal to
    c) strictly greater d) greater than or equal to
12. If the principle minor of bordered ---------- is positive, the objective function is convex
    a) Hessian matrix b) Hermitan matrix

c) diagonal matrix d) identity matrix

13. A NLPP is solved using __________
    a) Lagrange multipliers b) Simplex method
    c) Dual simplex method d) Degeneracy multipliers
14. A matrix that, for each state of nature and strategy, shows the difference between a strategy's payoff and the best strategy's payoff is called-----------
    a) a maximin matrix b) a minimax regret matrix
    c) a payoff matrix d) an expected utility matrix
15. A graphical method of representing events and course of action may be referred to as --------
    a) decision tree b) decision criterion
    c) payoff table d) strategy
16. In constructing a tree diagram, decision point is denoted by ----
    a) Circle b) Square c) Triangle d) Dot
17. In case of minimization of NLPP all the constraints must be converted into -------- type
    a) strictly less than b) less than or equal to
    c) strictly greater than d) greater than or equal to
18. A ________of a convex function on a convex set is a unique global minimum of that function.
    a) Local minimum b) Local maximum
    c) weak minimum d) weak maximum
19. The method of ______multipliers is a systematic way of generating the necessary condition for a stationary point
    a)Lagrange b) Cauchy's c) Euler's d) Fourier
20. X^TQX is said to be _______ if $X^TQX > 0$ for $x \neq 0$
    a) Positive definite b) negative definite
    c) Positive semi definite d) negative semi definite

PART –B(3X2=6 MARKS)

ANSWER ALL THE QUESTIONS

21 . Define EOQ.

22. Define payoff matrix.

23. Define NLPP.

PART –B(3X8=24 MARKS)

ANSWER ALL THE QUESTIONS

24. a) The probability distribution of monthly sale of a certain item is as follows:

Monthly sales:	0	1	2	3	4	5	6
Probability :	0.01	0.06	0.25	0.35	0.20	0.03	0.10

The cost of carrying inventory is Rs.30 per unit per month and the cost of unit shortage is Rs.70 per month. Determine the optimum stock level that minimizes the total expected cost.

(OR)

b) Electro uses resin in its manufacturing process at the rate of 1000 gallons per month. It cost electro 100 to place an order. The following cost per gallon per month is 2, and the shortage cost per gallon is 10 .historical data show that the demand during lead time is uniform over the range(0,50) gallons. Determine the optimal ordering policy for Electro.

25. a) Explain about decision making under uncertainty.

(OR)

b) A businessman has two independent investment portfolios A and B, available to him, but he lacks the capital to undertake both of them simultaneously. He can either choose A first and then stop, or if A is not successful, then take, B or vice versa. The probability of success of A is 0.6, while for B it is 0.4. Both investment schemes require an initial capital outlay of Rs.10,000 and both return nothing if the venture proves to be unsuccessful. Successful completion of A will return Rs.20, 000 (over cost) and successful completion of B will return Rs.24,000(over cost). Draw a decision tree in order to determine the best strategy.

26. a) A manufacturing company produces two products: Radios and TV sets. Sales-price relationships for these two products are given below:

Product	Quantity Demanded	Unit Price
Radios	$1500 - 5p_1$	p_1
TV sets	$3800 - 10p_2$	P_2

The total cost functions for these two products are given by $200x_1 + 0.1x_2^2$ and $300x_2 + 0.1x_2^2$respectively. The production takes place on two assembly lines. Radio sets are assembled on assembly line I and TV sets are assembled on assembly line II. Because of the limitations of the assembly-line capacities, the daily production is limited to no more than 80 radio sets and 60 TV sets. The production of both types of products requires electronic components. The production of each of these sets requires five units and six units of electronic equipment components respectively. The electronic components are supplied by another manufacturer, and the supply is limited to 600 units per day. The company has 160 employees, i.e., the labour supply amounts to 160 man-days. The production of one unit of radio set requires 1 man-day of labour, whereas 2 man-days of labour are required for a TV set. How many units of radio and TV sets should the company produce in order to maximize the total profit? Formulate the problem as a non-linear programming problem.

(OR)

b) Solve the following LPP by using dynamic programming approach

$$\text{Max } Z = 3x_1+5x_2$$

Subject to

$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$3x_1+5x_2 \leq 18$$

and $x_1, x_2 \geq 0$

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Established Under Section 3 of UGC Act 1956)
Coimbatore – 641 021.

LECTURE PLAN
DEPARTMENT OF MATHEMATICS

STAFF NAME: A.NEERAJAH
SUBJECT NAME: Optimization Techniques SUB.CODE: 17MMP203

SEMESTER: II CLASS: I M.SC MATHEMATICS

S.No	Lecture Hours (Hr)	Topics to be covered	Support Materials
Unit-I			
1	1	Integer Linear Programming Problem: Introduction and Types of Integer Linear Programming	R3: Chapter 3 : Pg.no:281
2	1	Concept of Cutting Plane	R4: Chapter10:Pg.no: 670-671
3	1	Gomory's All Integer Cutting Plane Method-Problems	R1: Chapter 7:Pg.no:126-128
4	1	Continuation on problems on Gomory's All Integer Cutting Plane Method	R4: Chapter10:Pg.no: 672-676
5	1	Gomory's mixed Integer Cutting Plane method-Problems	R1:Chapter 7 : Pg.no:129-130
6	1	Continuation on problems on Gomory's mixed Integer Cutting Plane method	R1:Chapter 7 : Pg.no:130-131
7	1	Branch and Bound Method - problems	R2: Chapter 6:Pg.no: 209-212
8	1	Continuation on problems on Branch and Bound Method	R2: Chapter 6:Pg.no: 213-218
9	1	Zero-One Integer Programming-problems	R3: Chapter 3:Pg.no: 282-283

10	1	Real life application in Integer Linear Programming.	R1:Chapter7: Pg.no:134-136
11	1	Recapitulation and discussion of possible questions	
Total	**11Hrs**		
		Unit-II	
1	1	Dynamic Programming problem: Introduction and Definitions	R3: Chapter 20: Pg.no:893-894
2	1	Characteristics of Dynamic Programming	R1:Chapter 13: Pg.no :236
3	1	Developing Optimal Decision Policy-problems	R3: Chapter20: Pg.no : 895-896
4	1	Continuation on problems on Developing Optimal Decision Policy	R4: Chapter13: Pg.no : 249-252
5	1	Dynamic Programming under Certainty-Model I-Shortest Route Problem.	R3: Chapter20: Pg.no: 896-897
6	1	Continuation on problems on Dynamic Programming under Certainty- Model I-Shortest Route Problem.	R3: Chapter20: Pg.no: 898-899
7	1	Model II- Multiplicative Separable Return Function& single additive constraint	R3: Chapter20: Pg.no: 899-901
8	1	Continuation on problems on Model II-Multiplicative Separable Return Function& single additive constraint	R3: Chapter20: Pg.no: 902-904
9	1	Model III-Additive Separable Return Function& single additive constraint	R3: Chapter20: Pg.no: 909-911
10	1	Continuation on problems on Model III-Additive Separable Return Function& single additive constraint	R3: Chapter20: Pg.no: 911-914
11	1	Model IV-Additivity Separable Return	R3: Chapter20: Pg.no: 920-922

		Function& single multiplicative constraint	
12	1	Dynamic Programming approach to solve LPP	R3: Chapter20: Pg.no: 922-924
13	1	Recapitulation and discussion of possible questions	
Total	**13Hrs**		

		Unit-III	
1	1	Probabilistic Inventory Model: Introduction and Real life application	T1: Chapter 14: Pg.no:551
2	1	Continuous review models	T1: Chapter 14: Pg.no:551-554
3	1	Probabilistic Economic order quantity (EOQ) Model-problems	T1: Chapter 14: Pg.no:554-555
4	1	Continuation on problems on Probabilistic Economic order quantity (EOQ) Model	T1: Chapter 14: Pg.no:556-558
5	1	Single-period with no setup model-problems	T1: Chapter 14: Pg.no:559-560
6	1	Continuation on problems on Single-period with no setup model	T1: Chapter 14: Pg.no:560-561
7	1	Single-period with setup model-problems	T1: Chapter 14: Pg.no:563-564
8	1	Continuation on problems on Single-period with setup model-problems	T1: Chapter 14: Pg.no:564-565
9	1	Multi period model-problems	T1: Chapter 14: Pg.no:565-567
10	1	Recapitulation and discussion of possible questions	
Total	**10Hrs**		
		Unit-IV	

1	1	Decision Analysis: Introduction and Real life application	T1: Chapter 13:Pg.no:509-510
2	1	Decision making under certainty –problems	R5: Chapter 13:Pg.no:492-493
3	1	Analytic hierarchy process-problems	T1: Chapter 13:Pg.no:510-511
4	1	Continuation on problems on Analytic hierarchy process	T1: Chapter 13:Pg.no:511-513
5	1	Decisions under Risk- problems	R1: Chapter 16:Pg.no:289-290
6	1	Decision Trees-based expected value criterion	T1: Chapter 13:Pg.no:520-522
7	1	variations of the expected value criterion-problems	T1: Chapter 13:Pg.no:526-527
8	1	Continuation on variations of the expected value criterion-problems	T1: Chapter 13:Pg.no:527-528
9	1	Decisions under Uncertainty- problems	T1: Chapter 13:Pg.no:535-538
10	1	Continuation on problems on Decisions under Uncertainty	R1: Chapter 16:Pg.no:296-297
11	1	Real life application in Decision Analysis	R1: Chapter 16:Pg.no:297-299
12	1	Recapitulation and discussion of possible questions	
Total	**12Hrs**		
		Unit-V	
1	1	Non-linear Programming Methods: Introduction and Examples of NLPP	R1: Chapter 24:Pg.no:522-525
2	1	General NLPP -problem	R1: Chapter 24:Pg.no:525-526
3	1	Constrained optimization with equality constraints	R1: Chapter 24:Pg.no:526-528
4	1	Continuation on Constrained optimization	R1: Chapter 24:Pg.no:529-531

		with equality constraints	
5	1	Constrained optimization with inequality constraints	R1: Chapter 24:Pg.no:532-533
6	1	Graphical solution and Quadratic Programming	R1: Chapter 25:Pg.no:541-543
7	1	Continuation on problems on Graphical solution and Quadratic Programming	R1: Chapter 25:Pg.no:543-546
8	1	Wolfe's modified Simplex Methods	R1: Chapter 25:Pg.no:547-548
9	1	Continuation on problems on Wolfe's modified Simplex Methods	R1: Chapter 25:Pg.no:548-550
10	1	Beale's Method	R1: Chapter 25:Pg.no:550-553
11	1	Recapitulation and discussion of possible questions	
12	1	Discussion of previous ESE question papers	
13	1	Discussion of previous ESE question papers	
14	1	Discussion of previous ESE question papers	
Total	**14Hrs**		

SUGGESTED READINGS

TEXT BOOK

T1: Handy, A. Taha.(2007). Operations Research, Seventh edition, Prentice Hall of India Pvt Ltd, New Delhi.

REFERENCES

R1:Kanti swarup., Gupta, P. K. and Manmohan., (2006). Operations Research, Twelfth edition, Sultan Chand & Sons Educational Publishers, New Delhi.

R2:Panneerselvam, R., (2007). Operations Research, Second edition, Prentice Hall of India Private Ltd, New Delhi.

R3:Sharma, J. K., (2008). Operations Research Theory and Practice, Third edition, Macmillan India Ltd.

R4**:**Singiresu, S. Rao., (2006). Engineering Optimization Theory and Practice, Third edition New Age International Pvt Ltd, New Delhi.

R5:Sivarethina Mohan. R., (2005). Operations Research, First edition, Tata Mc Graw Hill Publishing Company Ltd, New Delhi.

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

SYLLABUS

		Semester – II
17MMP203	**OPTIMIZATION TECHNIQUES**	**L T P C**
		4 0 0 4

Scope : This course has been intended to provide the knowledge in understanding the need and origin of the optimization methods which plays an essential role in present , future in the applications of Mathematics.

Objectives: To apply Mathematical techniques to model and analyze decision problems which play an essential role in the solution real life problems.

UNIT I

Integer Linear Programming: Types of Integer Linear Programming Problems - Concept of Cutting Plane - Gomory's All Integer Cutting Plane Method - Gomory's mixed Integer Cutting Plane method.Branch and Bound Method. - Zero-One Integer Programming – Real life application in Integer Linear Programming.

UNIT II

Dynamic Programming: Characteristics of Dynamic Programming Problem - Developing Optimal Decision Policy - Dynamic Programming under Certainty - DP approach to solve LPP.

UNIT III

Probabilistic Inventory Model: Real life application -Continuous review models- Probabilistic Economic order quantity (EOQ) Model. Single-period models – No setup model – setup model. Multi period model.

UNIT IV

Decision Analysis: Real life application - Decision making under certainty- Analytic hierarchy process. Decisions under Risk- Decision Trees-based expected value criterion, variations of the expected value criterion. Decisions under Uncertainty Real life application in Decision Analysis

UNIT V

Non-linear Programming Methods: Examples of NLPP - General NLPP - Graphical solution - Quadratic Programming - Wolfe's modified Simplex Methods - Beale's Method.

SUGGESTED READINGS

TEXT BOOK

T1:Handy, A. Taha.(2007). Operations Research, Seventh edition, Prentice Hall of India Pvt Ltd,New Delhi.

REFERENCES

R1. Kanti swarup., Gupta, P. K. and Manmohan., (2006). Operations Research, Twelfth edition, Sultan Chand & Sons Educational Publishers, New Delhi.

R2. Panneerselvam, R., (2007). Operations Research, Second edition, Prentice Hall of India Private Ltd, New Delhi.

R3. Sharma, J. K., (2008). Operations Research Theory and Practice, Third edition, Macmillan India Ltd.

R4. Singiresu, S. Rao., (2006). Engineering Optimization Theory and Practice, Third edition New Age International Pvt Ltd, New Delhi.

R5. Sivarethina Mohan. R., (2005). Operations Research, First edition, Tata Mc Graw Hill Publishing Company Ltd, New Delhi.

UNIT-I

SYLLABUS

Types of Integer Linear Programming Problems - Concept of Cutting Plane - Gomory's All Integer Cutting Plane Method - Gomory's mixed Integer Cutting Plane method Branch and Bound Method. - Zero-One Integer Programming – Real life application in Integer Linear Programming

INTEGER PROGRAMMING

In this section, we focus on a class of problems that are modeled as linear programs with the additional requirement that some requirement that some or all of the decision variable must be integer. Such problems are called **Integer Linear Programming Problems.** The use of integer variables provides additional modeling flexibility. As a result, the number of practical applications that can be addressed with LP methodology is enlarged and includes capital budgeting, distribution system Design, Location problems etc.

Types of integer Linear Programming Models

Formally, the general integer programming problem is

Maximize $f(x)$

s.t

$g_j(x) = 0, \quad j = 1, 2, \ldots, m$

$h_i(x) \leq 0, \quad i = 1, 2, \ldots, k$

$X = (x_1, x_2, \ldots, x_q, x_{q+1}, \ldots, x_n)$

Where $x_1, x_2, \ldots, x_q$ are integers for a given q. As problem remains essentially unsolved in the general case we confine our attentive to a useful simplification. We assume f and the hi's are **linear,** these are no gj's and all the variables in X must be non-negative. Then the formulation can be expressed in matrix notation as

Maximize CX … (1)

s.t

$AX \leq b$ … (2)

$X \geq 0$ … (3)

$x_1, x_2, \ldots, x_q$, integers … (4)

Where $X = (x_1, x_2, \ldots, x_{q+1}, \ldots, x_n)^T$, C is a 1 x n real vector, b is an m x 1 real vector, A is an m x n real matrix.

If q = n, the problem is termed an **all- integer linear programming problem**

If $1 < q < n$, the problem is termed a **mixed- integer linear programming problem.** If $(x_1, x_2, \ldots, x_q)$ is replaced by $xj = 0$ or 1, i =1, 2…..n problem is termed a **zero-one programmed problem.**

Rounding (Graphical Solution)

One obvious approach to (1) – (4) is to neglect (4) and solve the resulting problem graphically. If the solution produced satisfies (4) then it must be optimal. If it does not then there are a number of options available. One straight forward strategy is to round the values of non integer values either up or down to achieve n integer solution.

Let us explore this idea on the following integer – programming problem:

(1) Max $z = 2x_1 + 3x_2$

s.t

$$195x_1 + 273x_2 \le 1365$$

$$4x_1 + 40x_2 \le 140$$

$$x_1 \le 4$$

$$x_1\ x_2 \ge 0 \text{ and integer}$$

The Linear programming version of this problem has been solved graphically in (fig 1.0) it can be been that the optimal solution is

$$(x_1, x_2) = (2.44, 3.26) \qquad Z = 14.66$$

Rounding the decision variable to the nearest integer value yields a solution of $x_1 =2$ and $x_2 = 3$ for an objective function value of 13 or #13,000 annual cash flow. In fig 1, we shows the feasible solution points that provide integer values for x_1 and x_2. Is the rounded solution $(x_1, x_2) = (2, 3)$ the optimal solution? The answer is no! as can be seen that the optimal integer solution is $x_1 = 4$ and $x_2 = 2$, with object function value of 14.00 or #14,000 annual cash flow.

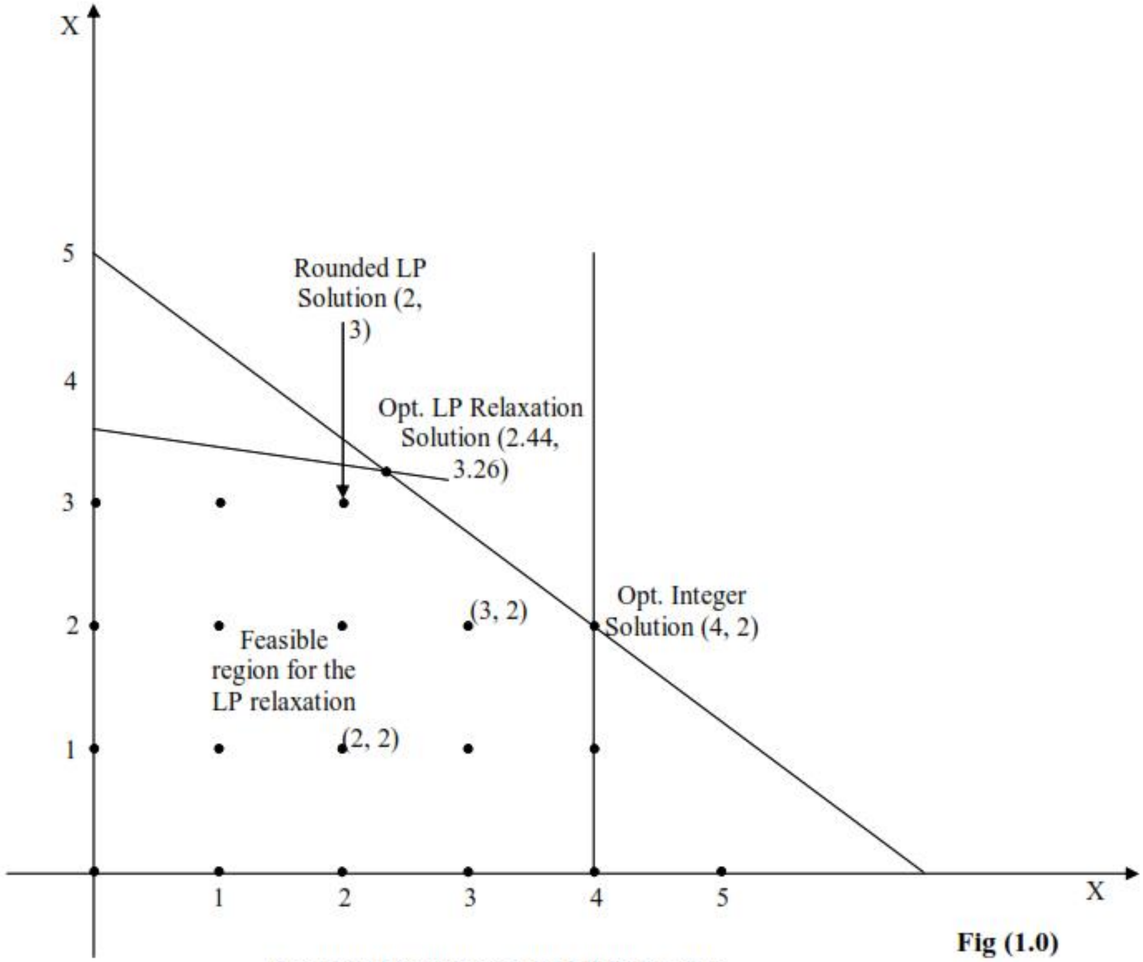


Fig (1.0)

Graphical Solution to the LP Relaxation

Methods of Integer Programming

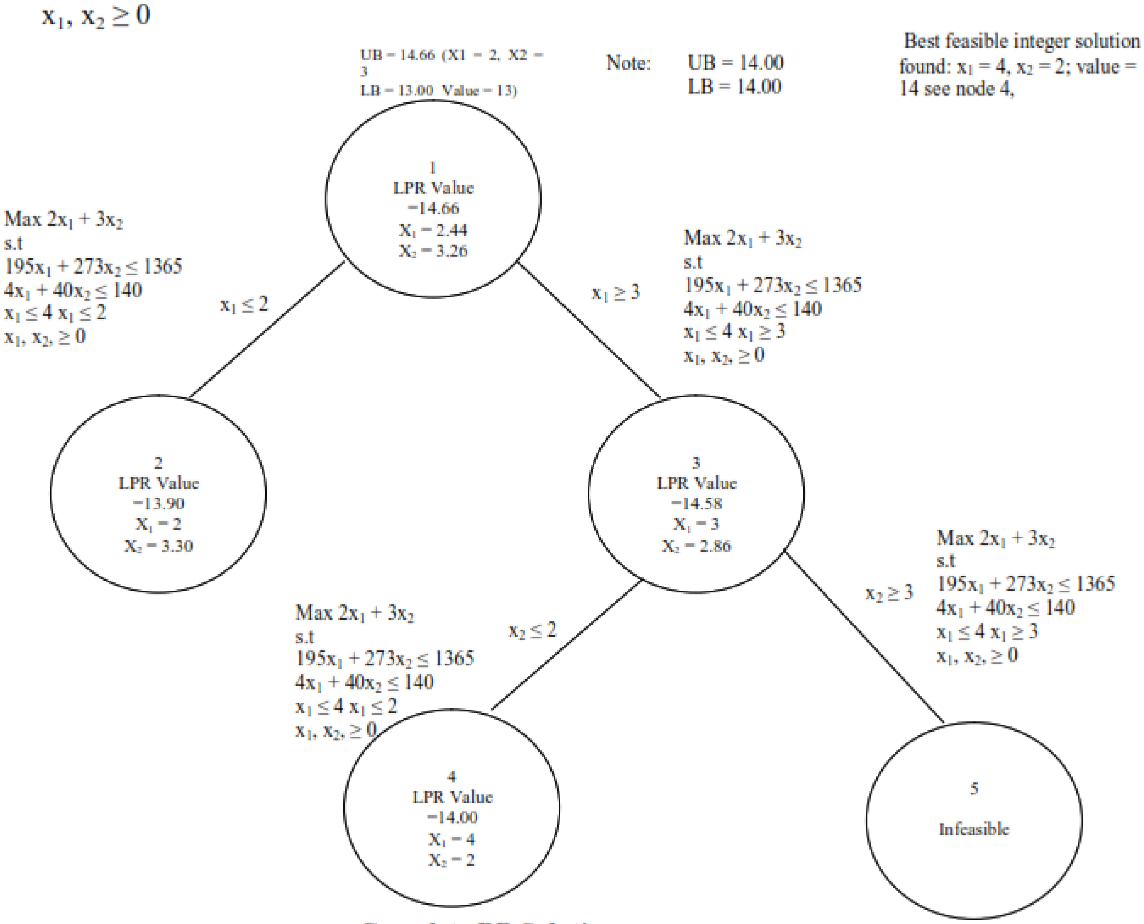
The section unveils the methods that guarantees to find an optimal solution (if one exists) to any integer – programming problem. The two broad approaches **Branch and Bound Technique** and **Cutting plane method**. The earlier technique starts with the continuous optimum; but systematically "partitions" the solution space into sub problems by deleting parts that contain no feasible integer points. The cutting methods systematically adding special "secondary" constraints, which essentially represent necessary conditions for integrality, the continuous solution space is gradually modified until its continuous optimum extreme points satisfies the integer conditions.

The branch and bound algorithm was originally developed by A.H. Larid and A. G. Doig. However, R.J. Dakin's modification offers greater computational advantage and his version will be presented here.

Branch and Bond Solution

BB is currently the most efficient general-purpose solution procedure or integer linear programs. The BB procedure begins by solving the LP Relaxation of the integer linear program. The LP Relaxation of the above problem is stated below,

Max $2x_1 + 3x_2$	x_1 = blocks of town houses purchased
s.t	x_2 = apartment building
$195x_1 + 273x_2 \leq 1365$	funds available
$4x_1 + 40x_2 \leq 140$	Manager's time
$x_1 \leq 4$	Town house availability
$x_1, x_2 \geq 0$	



Complete BB Solution

The Branch and bound solution procedure could be summarized as follows

1. Solve the LP Relaxation of the IP at node 1 set UB value equal to the value of the LP Solution
2. Find n feasible integer solution. Set LB equal to the valueo f the feasible integer solution
3. Is UB = LB? If yes, the optimal solution is the feasible solution with value = LB
4. Otherwise branch from the node with the greatest LP value. Find the variable (call it x_j) that is furthest from being integral. Create two branches and two descendant nodes; one with $x_j \leq (x_j)$ and one with $x_j \geq (k_j) + 1$.
5. Solve the LP Relaxation at each of the descendant nodes, and record its LP value
6. Re compute the upper bound by finding the maximum over all nod from which there are no branches
7. Re compute the lower bound as the max value of all feasible integer solution found to date. Test for 93) and treat the decision arrive at accordingly.

Extension to Mixed-Integer Integer Programs

One of the advantage of BB Solution procedure for integer programming is that it is applicable to both all-integer and mixed-integer linear programs. To see how the BB solution approach can be applied to a mixed-integer linear program, let us return to problem (1) and suppose that x_2 was not required to be integer i.e.

Max $2x_1 + 3x_2$

s.t

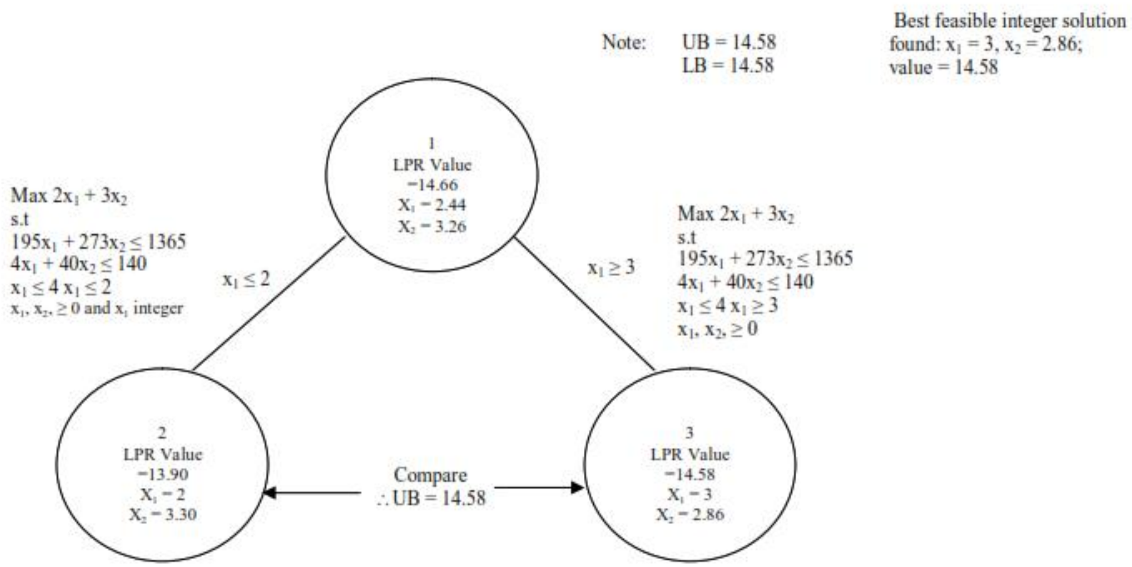
$195x_1 + 273x_2 \leq 1365$

$4x_1 + 40x_2 \leq 140$

$x_1 \leq 4$

$x_1, x_2, \geq 0$ and x_1 integer

The BB solution procedure is illustrated using the decision tree below:



From the above, since the upper and lower bound are equal, the optimal solution to the problem (original) with only x_1 required to be integer has been found. It is given by $x_1 = 3$ and $x_2 = 2.86$, with an objective function value of 14.58

Gomory Cuts: Solution

First of all, we reformulate the problem from canonical to standard form:

$$\left.\begin{array}{lrcl} \min & x_1 - 2x_2 & & \\ t.c. & -4x_1 + 6x_2 + x_3 & = & 9 \\ & x_1 + x_2 + x_4 & = & 4 \\ & x \geq 0 & , & x_1, x_2 \in \mathbb{Z} \end{array}\right\}$$

where x_3, x_4 are slack variables.

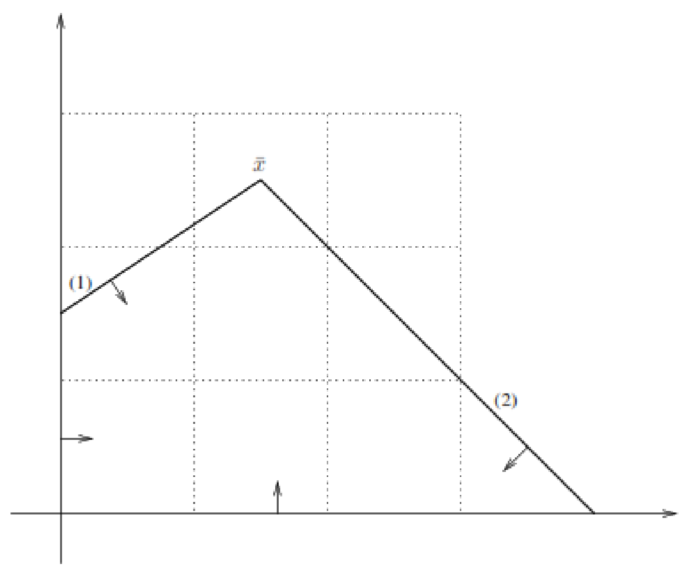
Using the simplex method applied to the feasible basis $x_B = (x_3, x_4)$, we obtain the following tableaux sequence (the pivot element is emphasized as $\boxed{\text{p}}$):

	x_1	x_2	x_3	x_4
0	1	-2	0	0
9	-4	$\boxed{6}$	1	0
4	1	1	0	1

	x_1	x_2	x_3	x_4
3	$-\frac{1}{3}$	0	$\frac{1}{3}$	0
$\frac{3}{2}$	$-\frac{2}{3}$	1	$\frac{1}{6}$	0
$\frac{5}{2}$	$\boxed{\frac{5}{3}}$	0	$-\frac{1}{6}$	1

	x_1	x_2	x_3	x_4
$\frac{7}{2}$	0	0	$\frac{3}{10}$	$\frac{1}{5}$
$\frac{5}{2}$	0	1	$\frac{1}{10}$	$\frac{2}{5}$
$\frac{3}{2}$	1	0	$-\frac{1}{10}$	$\frac{3}{5}$

Therefore the optimal solution of the relaxation is $\bar{x} = (\frac{3}{2}, \frac{5}{2})$, with slack variables $x_3 = x_4 = 0$ (also see the figure below).



From the optimal tableau, we derive a Gomory cut from the first row $x_2 + \frac{1}{10}x_3 + \frac{2}{5}x_4 = \frac{5}{2}$. The Gomory cut is expressed as

$$x_i + \sum_{j \in N} \lfloor \bar{a}_{ij} \rfloor x_j \leq \lfloor \bar{b}_i \rfloor, \tag{10.1}$$

where N is the set of indices of the nonbasic variables and i is the index of the chosen row. In this case, we obtain $x_2 \leq 2$.

We now have to add the Gomory cut $x_2 \leq 2$ in the current (optimal) simplex tableau. By definition, a valid cut "cuts off" the current optimal relaxed solution from the feasible region, the currently optimal basis ceases to be feasible after the Gomory cut is inserted. The primal simplex algorithm relies on having a feasible basis at all times, so it cannot be used. The dual simplex algorithm, on the other hand, relies on having a basis which is always optimal (or super-optimal) and works towards reaching feasibility. The currently optimal basis becomes "super-optimal" after the insertion of the Gomory cut in the sense that the new optimal solution will surely have a higher objective function value (recall we are minimizing the objective) since it is constrained by one more inequality.

In order to insert the constraint $x_2 \leq 2$ in the tableau it is necessary to express it in function of the nonbasic variables x_3, x_4. If from (10.1) we subtract the i-th row of the optimal tableau

$$x_i + \sum_{j \in N} \bar{a}_{ij} x_j \leq \bar{b}_i$$

we obtain the Gomory cut in fractional form:

$$\sum_{j \in N} (\lfloor \bar{a}_{ij} \rfloor - \bar{a}_{ij}) x_j \leq (\lfloor \bar{b}_i \rfloor - \bar{b}_i).$$

Applied to our instance this is:

$$-\frac{1}{10} x_3 - \frac{2}{5} x_4 \leq -\frac{1}{2}.$$

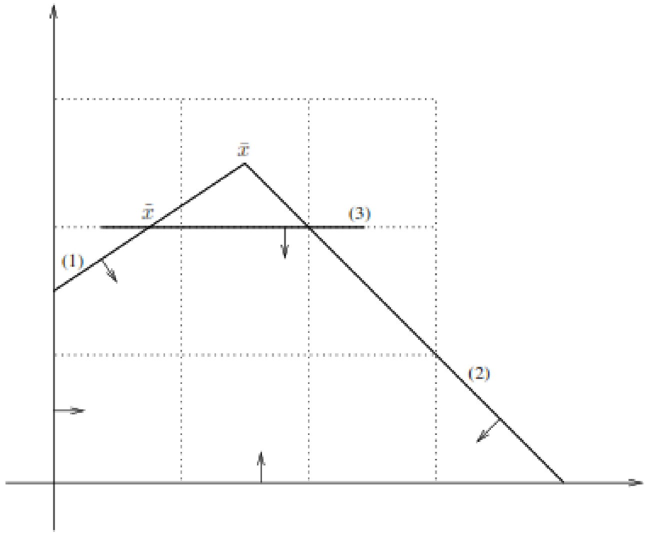
Since the simplex algorithm in tableau form requires all inequalities to be in equation form, we need to add a slack variable $x_5 \geq 0$ to the problem:

$$-\frac{1}{10} x_3 - \frac{2}{5} x_4 + x_5 = -\frac{1}{2}.$$

In this form, the equation can be added to the currently optimal tableau, which gains a new row and column (corresponding respectively to the new cut and the new slack variable, which is inserted in the basis):

	x_1	x_2	x_3	x_4	x_5
$\frac{7}{2}$	0	0	$\frac{3}{10}$	$\frac{1}{5}$	0
$\frac{5}{2}$	0	1	$\frac{1}{10}$	$\frac{2}{5}$	0
$\frac{3}{2}$	1	0	$-\frac{1}{10}$	$\frac{3}{5}$	0
$-\frac{1}{2}$	0	0	$-\frac{1}{10}$	$\boxed{-\frac{2}{5}}$	1

The new row corresponds to the Gomory cut $x_2 \leq 2$, depicted in the figure below as constraint (3).



We carry out an iteration of the dual simplex algorithm using the new tableau. The reduced costs are all non-negative, but $\bar{b}_3 = -\frac{1}{2} < 0$ implies that $x_5 = \bar{b}_3$ has nagative value and so is not primal feasible (recall $x_5 \geq 0$ is a constraint in the problem). We pick x_5 to exit the basis. The entering variable is given by the index j such that

$$j = \text{argmin}\{\frac{\bar{c}_j}{|\bar{a}_{ij}|} \mid j \leq n \wedge \bar{a}_{ij} < 0\},$$

which in this case is the minimum index in $\{3, \frac{1}{2}\}$, i.e. $j = 4$. Therefore x_4 enters the basis instead of x_5, and the pivot element is that indicated in the tableau above. We get the new tableau

	x_1	x_2	x_3	x_4	x_5
$\frac{13}{4}$	0	0	$\frac{1}{4}$	0	$\frac{1}{2}$
2	0	1	0	0	1
$\frac{3}{4}$	1	0	$-\frac{1}{4}$	0	$\frac{3}{2}$
$\frac{5}{4}$	0	0	$\frac{1}{4}$	1	$-\frac{5}{2}$

with optimal solution $\bar{x} = (\frac{3}{4}, 2)$. The solution is not yet integer, so we choose the second row:

$$x_1 - \frac{1}{4}x_3 + \frac{3}{2}x_5 = \frac{3}{4},$$

whence we obtain the Gomory cut

$$x_1 - x_3 + x_5 \leq 0,$$

which expressed in the original problem variable is

$$-3x_1 + 5x_2 \leq 7.$$

The fractional form of this Gomory cut is

$$-\frac{3}{4}x_3 - \frac{1}{2}x_5 \leq -\frac{3}{4}.$$

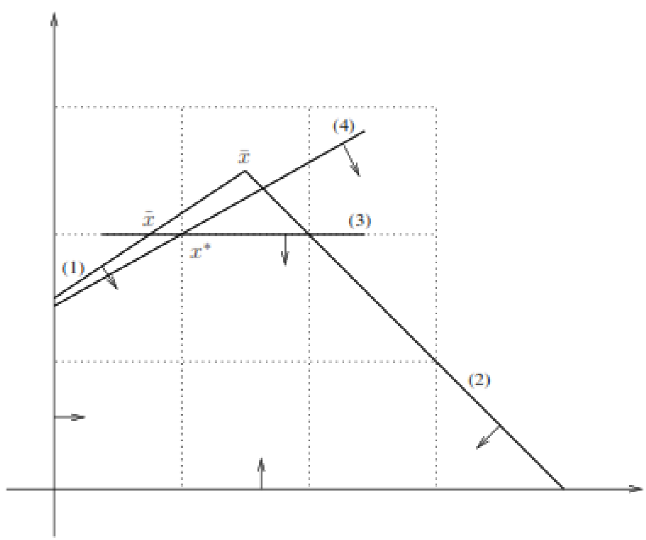
We insert it in the tableau as before and we obtain:

	x_1	x_2	x_3	x_4	x_5	x_6
$\frac{13}{4}$	0	0	$\frac{1}{4}$	0	$\frac{1}{2}$	0
2	0	1	0	0	1	0
$\frac{3}{4}$	1	0	$-\frac{1}{4}$	0	$\frac{3}{2}$	0
$\frac{5}{4}$	0	0	$\frac{1}{4}$	1	$-\frac{5}{2}$	0
$-\frac{3}{4}$	0	0	$\boxed{-\frac{3}{4}}$	0	$-\frac{1}{2}$	1

where the pivot element is emphasized (row 4 was selected because $\bar{b}_4 < 0$, column 5 because $\frac{\bar{c}_3}{|\bar{a}_{43}|} = \frac{1}{3} < 1 = \frac{\bar{c}_5}{|\bar{a}_{45}|}$). Pivoting yields

	x_1	x_2	x_3	x_4	x_5	x_6
3	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$
2	0	1	0	0	1	0
1	1	0	0	0	$\frac{5}{3}$	$-\frac{1}{3}$
1	0	0	0	1	$-\frac{8}{3}$	$\frac{1}{3}$
1	0	0	1	0	$\frac{2}{3}$	$-\frac{4}{3}$

which corresponds to the optimal integer solution $x^* = (1, 2)$ depicted below.



POSSIBLE QUESTIONS

ANSWER ALL THE QUESTIONS (5X6=30)

1. Find the optimum integer solution to the following LPP

Maximize $Z = x_1 + 4x_2$

Subject to the constraints

$2x_1 + 4x_2 \leq 7$, $5x_1 + 3x_2 \leq 15$ and $x_1, x_2 \geq 0$ and are integers

2. Solve the following ILPP using Branch and Bound method

Maximize $Z = 7x_1 + 9x_2$

Subject to the constraints

$-x_1 + 3x_2 \leq 6$, $7x_1 + x_2 \leq 35$, $x_2 \leq 7$ and $x_1, x_2 \geq 0$ and are integers

3. Solve the following IPP.

Minimize $Z = -2x_1 - 3x_2$

Subject to

$2x_1 + 2x_2 \leq 7$, $x_1 \leq 2$, $x_2 \leq 2$ and $x_1 \geq 0$, $x_2 \geq 0$ and are integers.

4. Solve the following mixed integer programming problem.

Minimize $Z = x_1 - 3x_2$

Subject to

$x_1 + x_2 \leq 5$, $-2x_1 + 4x_2 \leq 11$ and $x_1, x_2 \geq 0$ and x_2 be an integers.

5. Find the optimum integer solution to the following IPP.

Maximize $Z = x_1 + x_2$

Subject to

$2x_1 + 5x_2 \leq 16$, $6x_1 + 5x_2 \leq 30$ and $x_1 \geq 0$, $x_2 \geq 0$ and are all integers.

6. Use Branch and Bound method to solve the following:

Maximize $Z = 2x_1 + 2x_2$

Subject to

$5x_1 + 3x_2 \leq 8$, $x_1 + 2x_2 \leq 4$ and $x_1 \geq 0$, $x_2 \geq 0$ and are all integers.

7. Find the optimum integer solution to the following LPP.

Maximize $Z = x_1 + 2x_2$

Subject to the Constraints

$2x_2 \leq 7$, $x_1 + x_2 \leq 7$, $2x_1 \leq 11$ and $x_1 \geq 0$, $x_2 \geq 0$ and are all integers.

COMPULSORY (1X10=10)

1. Solve the following mixed integer programming problem.

   Minimize $Z = 10x_1 + 9x_2$

   Subject to the Constraints

   $x_1 \leq 8$, $x_2 \leq 10$, $5x_1 + 3x_2 \geq 45$ and x_1 , $x_2 \geq 0$, x_1 be an integers.

2. Find the optimum integer solution to the following LPP.

   Maximize $Z = 2x_1 + 2x_2$

   Subject to the constraints

   $5x_1 + 3x_2 \leq 8$, $2x_1 + 4x_2 \leq 8$ and $x_1 \geq 0$, $x_2 \geq 0$ and are integers.

3. Solve the following mixed integer programming problem.

   Maximize $Z = x_1 + x_2$

   Subject to the constraints

   $2x_1 + 5x_2 \leq 16$, $6x_1 + 5x_2 \leq 30$ and $x_2 \geq 0$ and x_1 is non negative integers.

UNIT-I

SYLLABUS

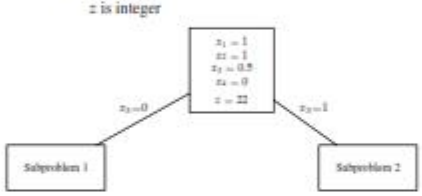
Characteristics of Dynamic Programming Problem - Developing Optimal Decision Policy - Dynamic Programming under Certainty - DP approach to solve LPP.

Dynamic Programming

Knapsack

Consider the Knapsack problem from the previous lecture.

$$\begin{aligned} \text{Max } z = \ & 8x_1 + 11x_2 + 6x_3 + 4x_4 \\ & 5x_1 + 7x_2 + 4x_3 + 3x_4 \le 14 \\ & x_1, x_2, x_3, x_4 \in \{0,1\} \\ & z \text{ is integer} \end{aligned}$$



Subproblem 1: $x_3 = 0$

$$\begin{aligned} \text{Max } & 8x_1 + 11x_2 + 4x_4 \\ & 5x_1 + 7x_2 + 3x_4 \le 14 \end{aligned}$$

Subproblem 2: $x_3 = 1$

$$\begin{aligned} \text{Max } & 6 + 8x_1 + 11x_2 + 4x_4 \\ & 5x_1 + 7x_2 + 3x_4 \le 10 \end{aligned}$$

Note that every subproblem is characterized by the set of remaining objects (variables) and the total **budget** (the right-hand side). Observe that the above two subproblems only differ in the value of budget (as we can ignore the absolute constant in the objective function). We should exploit this symmetry.

Instead of solving just for 10 and 14, we solve the subproblem for **all** meaningful values of the right-hand side (here for values $1, 2, \ldots, 14$). Having done that, we pick the best solution. This may seem wasteful but it can actually be faster. To make this work efficiently, we branch systematically on variables $x_1, x_2, \ldots$

Let us describe it first more generally. Consider the problem (all coefficients are integers)

$$\begin{aligned} \max \ & c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ & d_1x_1 + d_2x_2 + \ldots + d_nx_n \le B \\ & x_1, \ldots, x_n \in \{0,1\} \end{aligned}$$

For every $i \in \{1, \ldots, n\}$ and $j \in \{1, \ldots, B\}$, we solve a subproblem with variables $x_1, \ldots, x_i$ and budget j.

$$\begin{aligned} \max\ & c_1x_1 + c_2x_2 + \ldots + c_ix_i \\ & d_1x_1 + d_2x_2 + \ldots + d_ix_i \leq j \\ & x_1, \ldots, x_i \in \{0,1\} \end{aligned}$$

Let $f_i(j)$ denote the value of this solution. We want the value of $f_n(B)$.

How can we calculate $f_i(j)$**?** We observe that the optimal solution consisting of first i items either contains the i-th item or it does not. If it does not contain the i-th item, then the value of $f_i(j)$ is the same as that of $f_{i-1}(j)$; the best solution using just the first $i-1$ items. If, on the other hand, an optimal solution using the first i items contains the i-th item, then removing this item from the solution gives an **optimal solution** for **first** $i-1$ items **with budget** $j-d_i$. (Convince yourself of this fact.) So the value of $f_i(j)$ is obtained by taking $f_{i-1}(j-d_i)$ and adding the value c_i of item i. We don't know which of the two situations happens, but by taking the better of the two, we always choose correctly.

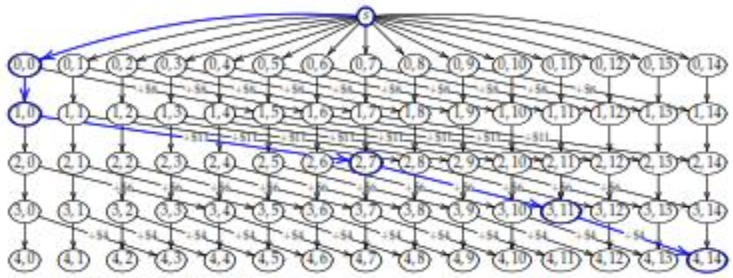
> What this shows is that from optimal solutions to smaller subproblems we can build an optimal solution to a larger subproblem. We say that the problem has **optimal substructure**.

As we just described, the function $f_i(j)$ satisfies the following **recursion**:

$$f_i(j) = \begin{cases} 0 & i = 0 \\ \max\{f_{i-1}(j), c_i + f_{i-1}(j-d_i)\} & i \geq 1 \end{cases}$$

We can calculate it by filling the table of all possible values.

$f_i(j)$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	8	8	8	8	8	8	8	8	8	8
2	0	0	0	0	0	8	8	11	11	11	11	11	19	19	19
3	0	0	0	0	6	8	8	11	11	14	14	17	19	19	19
4	0	0	0	4	6	8	8	11	12	14	15	17	19	19	21



(edges with no cost indicated have \$0 cost)

The longest path (in **bold**) from s to $t = (4, 14)$ gives the optimal solution.

Dynamic programming characteristics

- Problem can be divided into stages
- Each stage has a (finite) number of possible **states**, each associated a value.
- Next stage only depends on the values of states of the previous stage

Resource allocation

Knapsack with fixed costs.

Investment 1: investing $x_1 > 0$ dollars yields $c_1(x_1) = 7x_1 + 2$ dollars, where $c_1(0) = 0$
Investment 2: investing $x_2 > 0$ dollars yields $c_2(x_2) = 3x_2 + 7$ dollars, where $c_2(0) = 0$
Investment 3: investing $x_3 > 0$ dollars yields $c_3(x_3) = 4x_3 + 5$ dollars, where $c_3(0) = 0$

(Note that if no money is invested, then there is no yield; the yield is non-linear.)

Suppose that we can invest \$6,000 and each investment must be a multiple of \$1,000. We identify stages

stages: in stage i, we consider only investments $1, \ldots, i$.

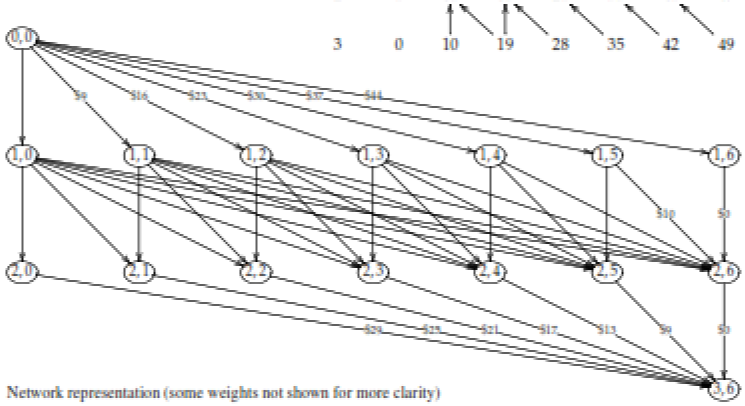
states: the available budget as a multiple of \$1,000, up to \$6,000.

values: $f_i(j)$ = maximum yield we can get by investing j thousands of \$ into investments #1,…,#i.

recursion: $f_i(j) = \begin{cases} 0 & i = 0 \\ \max\limits_{k \in \{0,1,\ldots,j\}} \left\{ c_i(k) + f_{i-1}(j-k) \right\} & i \geq 1 \end{cases}$

Note: the arrow indicates from which subproblem was an optimal solution obtained

stage	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	9	16	23	30	37	44
2	0	10	19	26	33	40	47
3	0	10	19	28	35	42	49



Network representation (some weights not shown for more clarity)

Inventory problem

A company must meet demand in the next four months as follows: month 1: $d_1 = 1$ unit, month 2: $d_2 = 3$ units, month 3: $d_3 = 2$ units, month 4: $d_4 = 4$ units. At the beginning of each month it has to be determined how many units to produce. Production has a setup cost of \$3 and then \$1 for each unit. At the end of the month there is holding cost \$0.50 for each unit at hand. The company's warehouse can store up to 4 units from month to month. The capacity

of the production line allows at most 5 units to be produced each month. Initially no products at hand. Determine the minimum cost of production that meets the demand.

stages: production until (and including) month i

states: number of units at hand at the end of month i

values: $f_i(j)$ = the minimum cost of production that ends in month i with j units in the inventory

For example, if we have 4 units at the end of month 2, then we meet month 3 demand of 3 units if we either

- do not produce and are left with 1 unit, held for \$0.50 **total cost \$0.50**
- pay setup cost \$3, produce 1 units for \$1, and are left with 2 units, held for \$1 **total cost \$5.00**
- pay setup cost \$3, produce 2 units for \$2, and are left with 3 units, held for \$1.50 **total cost \$6.50**
- pay setup cost \$3, produce 3 units for \$3, and are left with 4 units, held for \$2 **total cost \$8.00**

Note that in our calculation with already include the holding cost.

$$f_i(j) = \underbrace{j \times \$0.50}_{\text{holding cost}} + \min \begin{cases} f_{i-1}(j+d_i) & \text{no production } k=0 \\ \underbrace{\$3 + k \times \$1}_{\text{production cost}} + f_{i-1}(j+d_i-k) & \text{production } k \in \{1,2,\ldots,j+d_i\} \end{cases}$$

$f_i(j)$	0	1	2	3	4
	inventory				
1	0	–	–	–	–
2	4	5.5	7	8.5	10
3	8.5	10.5	13	15	17
4	13	15	16.5	18	20.5
5	20	–	–	–	–

Shortest paths

All-pairs Shortest Paths: (Floyd-Warshall) given a network $G = (V, E)$ where each edge $(u, v) \in E$ has length/cost c_{uv}, determine the distance between every pair of nodes.

We can solve this problem using dynamic programming as follows. We label the vertices $v_1, v_2, \ldots, v_n$.

stages: in stage i consider only shortest paths going through intermediate nodes $v_1, \ldots, v_i$

values: $f_i(u, v)$ = minimum length of path from u to v whose all intermediate nodes are among $v_1, \ldots, v_i$.

$$f_0(u, w) = \begin{cases} 0 & u = w \\ c_{uw} & uw \in E \\ \infty & uw \notin E \end{cases}$$

$$f_i(u, w) = \min \left\{ f_{i-1}(u, w),\ f_{i-1}(u, v_i) + f_{i-1}(v_i, w) \right\} \quad \text{for } i \geq 1$$

Single-source Shortest Paths: (Bellman-Ford) find distances from a fixed source s to all other vertices

stages: in stage i consider only shortest paths using at most i edges

values: $f_i(u)$ = minimum length of a path going from s to u and having at most i edges

We have $f_0(s) = 0$ and $f_0(u) = \infty$ for all $u \neq s$, since we cannot use any edge at this stage. For later stages:

$$f_i(u) = \min\left\{ f_{i-1}(u),\ \min_{\substack{v \in V \\ vu \in E}} \left\{ f_{i-1}(v) + c_{vu} \right\} \right\}$$

Knapsack revisited

$$\begin{aligned} \max\ & c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ & d_1x_1 + d_2x_2 + \ldots + d_nx_n \leq B \\ & x_1, \ldots, x_n \in \{0, 1\} \end{aligned}$$

$f_i(j)$ = optimal solution using first i items and bag of size j

$$f_i(j) = \begin{cases} -\infty & j < 0 \\ 0 & i = 0 \\ \max\left\{ f_{i-1}(j),\ c_i + f_{i-1}(j - d_i) \right\} & i \geq 1 \end{cases}$$

Alternative recursion (only for unbounded Knapsack)

Item	Weight (lbs)	Price ($)
1	4	11
2	3	7
3	5	12

→ fill a 10 pound bag

$$\begin{aligned} \max\ & 11x_1 + 7x_2 + 12x_3 \\ & 4x_1 + 3x_2 + 5x_3 \le 10 \\ & x_1, x_2, x_3 \ge 0 \text{ and integer} \end{aligned}$$

$g(j)$ = the maximum total price obtained by filling a j pound bag

recursion formula: $g(j) = \max\limits_{\substack{i \in \{1,2,3\} \\ j \ge d_i}} \left\{ c_i + g(j - d_i) \right\}$

We try to put item i into the bag and fill the rest as best as possible (note that here we need that we have unlimited number of each item rather than just one; it would not work for binary Knapsack).

j	0	1	2	3	4	5	6	7	8	9	10
$g(j)$	0	0	0	7	11	12	14	18	22	23	25

$g(0) = 0$
$g(1) = 0$
$g(2) = 0$
$g(3) = \max\{c_2 + g(3 - d_2)\} = c_2 + g(0) = 7 + 0 = 7$

$$g(4) = \max \begin{cases} c_1 + g(0) = 11 + 0 = 11^* \\ c_2 + g(1) = 7 + 0 = 7 \end{cases}$$

$$g(5) = \max \begin{cases} c_1 + g(1) = 11 + 0 = 11 \\ c_2 + g(2) = 7 + 0 = 7 \\ c_3 + g(0) = 12 + 0 = 12^* \end{cases}$$

$$g(6) = \max \begin{cases} c_1 + g(2) = 11 + 0 = 11 \\ c_2 + g(3) = 7 + 7 = 14^* \\ c_3 + g(1) = 12 + 0 = 12 \end{cases}$$

$$g(7) = \max \begin{cases} c_1 + g(3) = 11 + 7 = 18^* \\ c_2 + g(4) = 7 + 11 = 18^* \\ c_3 + g(2) = 12 + 0 = 12 \end{cases}$$

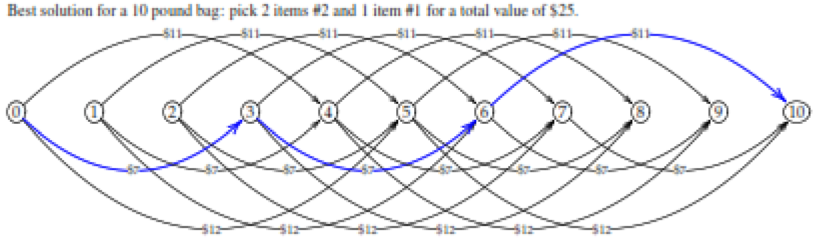
$$g(8) = \max \begin{cases} c_1 + g(4) = 11 + 11 = 22^* \\ c_2 + g(5) = 7 + 12 = 19 \\ c_3 + g(3) = 12 + 7 = 19 \end{cases}$$

$$g(9) = \max \begin{cases} c_1 + g(5) = 11 + 12 = 23^* \\ c_2 + g(6) = 7 + 14 = 21 \\ c_3 + g(4) = 12 + 11 = 23^* \end{cases}$$

$$g(10) = \max \begin{cases} c_1 + g(6) = 11 + 14 = 25^* \\ c_2 + g(7) = 7 + 18 = 25^* \\ c_3 + g(5) = 12 + 12 = 24 \end{cases}$$

Optimal solution: $g(10) = c_1 + g(6) = c_1 + (c_2 + g(3)) = c_1 + c_2 + (c_2 + g(0)) = c_1 + c_2 + c_2$

Best solution for a 10 pound bag: pick 2 items #2 and 1 item #1 for a total value of $25.



Optimal solution (longest path) shown in **blue** color.

Scheduling

We have 7 jobs $j_1, j_2, \ldots, j_7$ with processing times $p_1, p_2, \ldots, p_7$ given as follows:

job j_i	j_1	j_2	j_3	j_4	j_5	j_6	j_7
processing time p_i	10	8	6	5	4	4	3

We wish to schedule all 7 jobs on 2 machines. The goal is to minimize the completion time of all jobs.

n jobs $j_1, \ldots, j_n$
processing times $p_1, \ldots, p_n$
m machines

$$x_{ij} = \begin{cases} 1 & \text{if job } i \text{ scheduled on machine } j \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \min \quad & z \\ & \sum_j x_{ij} = 1 \qquad i = 1, \ldots, n \\ & \sum_i p_i x_{ij} \le z \qquad j = 1, \ldots, m \\ & x_{ij} \in \{0, 1\} \end{aligned}$$

stages: at stage i we schedule first i jobs $j_1, \ldots, j_i$

state: pair (t_1, t_2) denoting used up time t_1 on machine #1 and time t_2 on machine #2

value: $f_i(t_1, t_2) = 1$ if it is possible to schedule first i jobs on the two machines using t_1 time on machine #1 and t_2 time on machine #2;

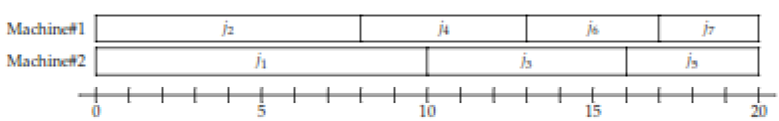
$f_i(t_1, t_2) = 0$ if otherwise

(Note that the order of the jobs does not matter here; only on which machine each job is executed.)

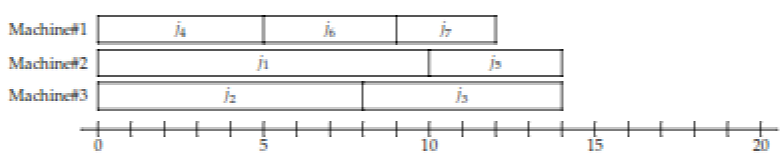
recursion:
$$f_i(t_1, t_2) = \begin{cases} 0 & t_1 < 0 \text{ or } t_2 < 0 \\ 1 & i = 0 \\ \max\left\{ f_{i-1}(t_1 - p_i, t_2), f_{i-1}(t_1, t_2 - p_i) \right\} & i \ge 1 \end{cases}$$

We either execute job j_i on machine #1 or on machine #2. We pick better of the two options.

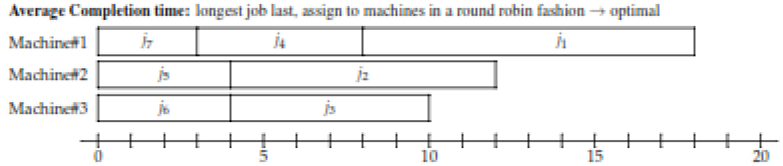
answer: is given by taking smallest t such that $f_7(t, t) = 1$



(How would you formulate the problem for 3 machines instead of 2?)



Average Completion time: longest job last, assign to machines in a round robin fashion → optimal



Average completion time = $8\frac{3}{7} \sim 8.43$ (note that moving j_7 to any other machine does not change the average)

POSSIBLE QUESTIONS

ANSWER ALL THE QUESTIONS (5X6=30)

1. Solve the following LPP by using dynamic Programming approach

   Max $Z = 3x_1+5x_2$

   Subject to

   $x_1 \leq 4$, $x_2 \leq 6$, $3x_1+5x_2 \leq 18$ and $x_1, x_2 \geq 0$

2. By dynamic programming technique, solve the problem.

   Min $Z = x_1^2 + x_2^2 + x_3^2$

   Subject to

   $x_1+x_2+x_3 \geq 15$ and $x_1,x_2,x_3 \geq 0$.

3. Factorize a positive quantity b into n factors in such a way that the sum of their squares is a minimum.

4. Use dynamic programming to solve the following LPP

   Maximize $Z = 3x_1 + 5x_2$

   Subject to the constraints

   $x_1 \leq 4$, $3x_1 + 2x_2 \leq 18$ and x_1, $x_2 \geq 0$

5. Divide unity n parts so as to minimize the quantity $\sum p_i \log p_i$.

6. Solve the following LPP using dynamic programming principles:

   Maximize $Z = 2x_1 + 5x_2$

   Subject to the constraints

   $2x_1 + x_2 \leq 43$, $2x_2 \leq 46$ and $x_1, x_2 \geq 0$.

7. Solve the following by dynamic programming.

   Minimize $Z = y_1^2+y_2^2+\ldots..y_n^2$

   Subject to the Constraints

   $y_1\ y_2\ldots..y_n = b$ and $y_1, y_2, \ldots.. y_n \geq 0$

COMPULSORY (1X10=10)

1. Use dynamic programming, Solve

   Maximum $Z = y_1 . y_2 . y_3$

   Subject to the Constraints

   $y_1 + y_2 + y_3 = 5$ and $y_1, y_2, y_3 \geq 0$

2. Divide a positive quantity c into n-parts in such a way that their product is maximum.

3. Solve the following LPP using dynamic programming approach:

   Maximize $Z = 8x_1 + 7x_2$

   Subject to the constraints

   $2x_1 + x_2 \leq 8$, $5x_1 + 2x_2 \leq 15$ and $x_1, x_2 \geq 0$

UNIT-III

SYLLABUS

Real life application -Continuous review models- Probabilistic Economic order quantity (EOQ) Model. Single-period models – No setup model – setup model. Multi period model.

INTRODUCTION

An inventory can be defined as any idle resource of an enterprise. An inventory problem exists when it is necessary to stock physical goods or commodities for the purpose of satisfying demand over a specified time horizon (finite or infinite). Almost every business must stock goods to ensure smooth and efficient running of its operation. Decisions regarding how much and when to order are typical of every inventory problem. The required demand maybe satisfied by stocking once for the entire time horizon or by stocking separately for every time unit of the horizon. The two extreme situations (overstocking and under-stocking) are costly. Decisions may thus be based on the minimization of an appropriate cost function that balances the total costs resulting from over-stocking and under-stocking.

A GENERALIZED INVENTORY MODEL

The ultimate objective of an inventory model is to answer two questions.

1. How much to order?
2. When to order

The answer to the first question is expressed in terms of what we call the order quantity and the when-to-order decision is the inventory level at which a new order should be placed usually expressed in terms of re-order point.

The order quantity and re-order pint are normally determined by minimizing the total inventory cost that can be expressed as a function of these two variables. We can summarize the total cost of a general inventory model as a function of its principal components in the following manner:

(Total inventory cost) = (purchasing cost) + (setup cost) + (holding cost)
+ (shortage cost)

TYPES OF INVENTORY MODELS

In general, inventory models are classified into two categories:

1. Deterministic model and
2. Stochastic model

Definitions:

1. Delivery lags or lead times is the time between the placement of an order and its receipt and may be deterministic or stochastic.
2. Time Horizon defined the period over which the inventory level will be controlled. This horizon may be finite or infinite depending on the time period which demand can be forecast reliably.
3. Stock replacement: Although an inventory system may operate with delivery lags. The actual replenishment of stock may occur instantaneously or uniformly. Instantaneous replenishment can occur when the stock is purchased from outside sources. Uniform replenishment may occur when the product is manufactured locally within the organization.

DETERMINISTIC MODEL (Single item static model)

The simplest type of inventory model occurs when demand is constant over time with instantaneous replenishment and no shortages. Typical situations to which this model may apply are:

1. The use of light bulbs in a building
2. The use of clerical supplies, such as paper, pads and pencil in a large company
3. The consumption of staple food items, such as bread and milk

Fig (1) illustrates the variation of the inventory level. It is assumed that demand occurs at the rate β (per unit time). The highest level of inventory occurs when the inventory level reaches zero level y/β time units after the order quantity y is received.

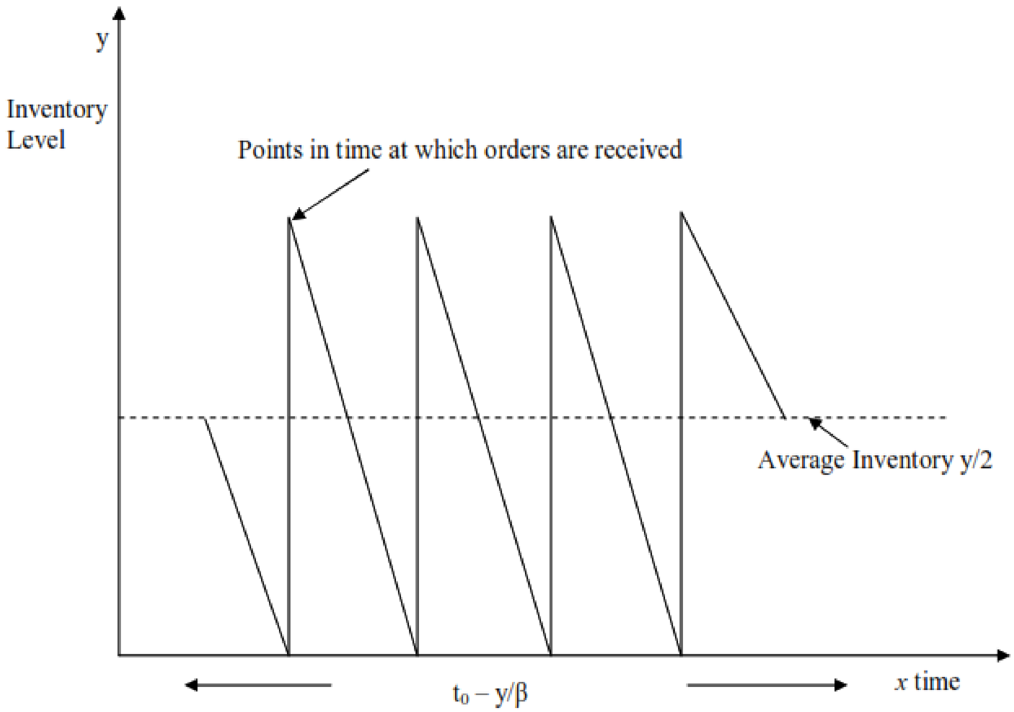


Fig. 1

The smaller the order quantity y, the more frequent will be the placement of new orders. However, the average level of inventory hold in stock will be reduced. On the other hand, larger order quantities indicate larger inventory level but less frequent placement of order (see fig. 2).

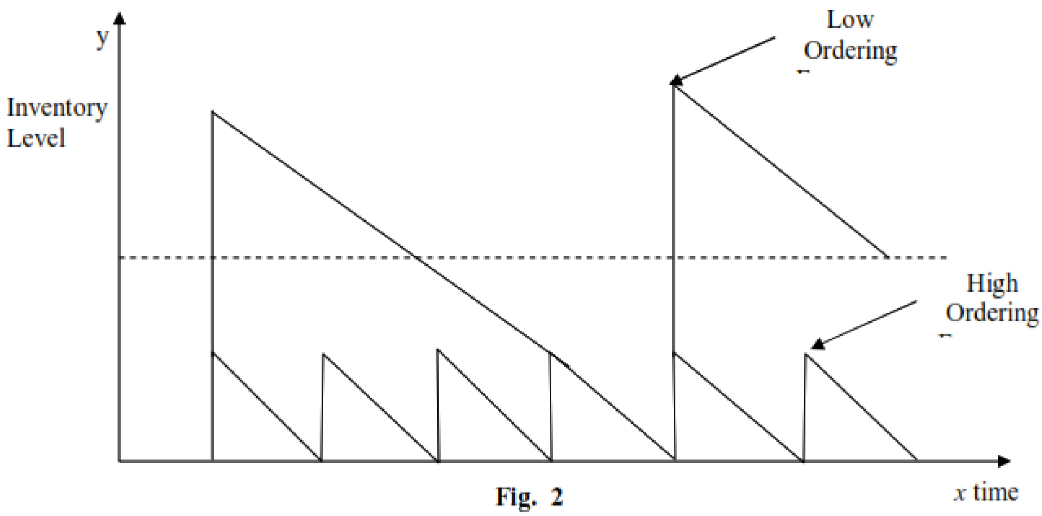


Fig. 2

Because there are costs associated with placing orders and holding inventory in stock, the quantity y is selected to allow a compromise between the two types of costs. This is the basis for formulating the inventory model.

Let K be the set up cost incurred every time an order is placed and assume that the holding cost per unit inventory per unit time is k. Hence, the total cost per unit time TCU as a function of y may be written as

TCU (y) = Setup cost/unit time + holding cost/unit time

$$TCU\ (y) = \frac{k}{y/\beta} + h\ (y/2)$$

As seen from fig. (1), the length of each inventory cycle is to = y / β and the average inventory in stock is y/2

The optimum value of y is obtained by minimizing TCU (y) with respect to y. Thus, assuming that y is a continuous variable, we have

$$\frac{dTCU\ (y)}{dy} = k\beta/y + h/2 = 0$$

Which yields the optimum order quantity as

$$y = \sqrt{\frac{2k\beta}{h}}$$

(It can be proved that y minimizes TCU(y) by showing that the 2^{nd} derivates at y is strictly positive). The order quantity above is usually referred to as Wilson's economic lot size.

The optimum policy of the model calls for ordering units every to time units. The optimum cost TCU (y) obtained by direct substitution is √2k/β.

EXAMPLE 1:

The daily demand for a commodity is approximately 100 units. Every time an order dared, fixed cost is N100 is incurred. The daily holding cost per unit inventory if N0.02. If the lead time is 12 days. Determine the economic lot size and the re-order point

SOLUTION:

From the earlier formula the economic lot size is

$$y = \sqrt{\frac{2k\beta}{h}}$$

$= \sqrt{2* 100*/0.02} = 1000$ units

The associated optimum cycle length is this given as

$b_0* = \frac{y_s}{\beta} = 1000/100 = 10$ days

Since the lead time is 12 days add the cycle length is 10 days re-ordering occurs when the level of inventory is sufficient to satisfy the demand for two (- 12, 10) days. Thus the quantity y* = 1000 is ordered when the level of inventory reaches 2 * 100 = 200 units.

Notice that the "effective" lead time is taken equal to 2 days rather than 12 days. This result occurs because the lead time is longer than t_0*.

EXAMPLE 12:

A manufacturer has to supply his customer' with 600 units of his product per year. Shortages are not allowed and the shortage cost amounts to N0.60 per and per year. The setup cost per run is N80.00. Find the optimum run size and minimum average yearly cost.

Solution:

Since β = 600 units/year

K = N80.00

h = N0.06

$y = \sqrt{\frac{2k\beta}{h}}$

$\sqrt{2 * 80 * 600 / 0.06} = 400$ units opt. run time

And the minimum average yearly cost = $\sqrt{2k\beta h}$

$= \sqrt{2 * 80 * 60 * 0.60}$

= N240.00

EXERCISES

1. XYZ Company purchases a component used in the manufacturing automobile generators directly from the suppliers. XYZ's generator production which is operated at a constant rate will required 1000 components per month throughout the year (12,000 units annually). If ordering cost is N25 per order, unit cost is

N200 per component and annual inventory holding costs are charged at 20%. Answer the following inventory policy question for XYZ

a. What is the economic order quantity (EOQ) for this component?

b. What is the length of cycle time in months?

c. What are the total annual inventory building and ordering cost associated with your recommended EOQ?

2. The demand for a particular item is 18,000 units per year. The holding cost per unit is N1.20 per year, and the cost of the replenishment rate is instantaneous. Determine

a. Optimum order quantity

b. Number of orders per year

c. Time between orders and

d. Total cost per year when the cost of 1 unit is N1.00

- **Economic Order Quantity (EOQ)**
- **Graphical Metod**

Concept of Economic Ordering Quantity (EOQ)

The concept of *economic ordering quantity* was first developed by *F. Harris* in 1916. The concept is that management is confronted with a set of opposing costs-as the lot size *(q)* increases, the carrying charges *(C)* will increase while the ordering costs *(C3)* will decrease. On the other hand, as the lot size *(q)* decreases, the carrying cost *(C)* will decrease but the ordering costs will increase (assuming that only minor deviations from these trends may occur). Thus, *economic ordering quantity (EOQ) is that size of order which minimizes total annual (or other time period as determined by individual firms) cost of carrying inventory and cost of ordering under the assumed conditions of certainty and that annual demands are known.* The concept of EOQ applies to items, which are replenished periodically into inventory in lots covering several periods' need. The EOQ concept is applicable under the following conditions:

a. The item is replenished in lots or batches, either by purchasing or by manufacturing.

b. Consumption of items (or sales or usage rate) is uniform and continuous.

EOQ is that order quantity or optimal order size which minimises the total cost. The model is described under the following situations:

i. Planning period is one year.

ii. Demand is deterministic and indicated by parameter D units per year.

iii. Cost of purchases, or of one unit is C.

iv. Cost of ordering (or procurement cost of replenishment cost) is C3or *Co* For manufacturing goods, it is known as set-up cost.

v. Cost of holding stock (also known as inventory carrying cost) is C1or *Cb* per unit per year expressed either in items of cost per unit per period or in terms of percentage charge of the purchase price.

vi. Shortage cost (mostly it is back order cost) is C2or *Cs* per unit per year.

vii. Lead time is L, expressed in unit of time.

viii. Cycle period in replenishment is *t*.

ix. Order size is Q.

Determination of EOQ by Trial and Error Method (or Tabular Method)

This method involves the following steps:

Step 1. Select a number of possible lot sizes to purchase.

Step 2. Determine total cost for each lot size chosen.

Step 3. Finally, select the ordering quantity, which minimizes total cost.

Following illustrative example will make the procedure clear.

For example, suppose annual demand *(D)* equals 8000 units, ordering cost *(C3)* per order is Rs. 12.50, the carrying cost of average inventory is 20% per year, and the cost per unit is Re. 1.00. The following table is computed.

No. of Orders per Year	Lot Size	Average Inventory	Carrying Charges 20% per Year	Ordering Costs Rs. 12.50 per Order	Total Cost per Year
(1)	(2)	(3)	(4)	(5)	(6) = (4) + (5)
			(Rs.)	(Rs.)	(Rs.)
1	8,000	4,000	800·00	12·50	812·50
2	4,000	2,000	400·00	25·00	425·00
4	2,000	1,000	200·00	50·00	250·00
→ 8	1,000	500	100·00	100·00	200·00
12	667	333	66·00	150·00	$\boxed{216·00}$
16	500	250	50·00	200·00	250·00
32	50	125	25·00	400·00	425·00

This table indicates that an order size of 1000 units will give us the lowest total cost among all the seven alternatives calculated in the table. Also, it is important to note that this minimum total cost occurs when:

Carrying Costs =Ordering Costs.

Example 1. *Novelty Ltd. carries a wide assortment of items for its customers. One item, Gay look, is very popular. Desirous of keeping its inventory under control, a decision is taken to order only the optimal economic quantity, for this item, each time. You have the following information. Make your recommendations: Annual Demand:* 1,60,000 *units; Price per unit:* Rs. 20; *Carrying Cost:* Re. 1*per unit or* 5% *per rupee of inventory value; Cost per order:* Rs. 50. *Determine the optimal economic quantity by developing the following table.*

No. of Orders	Size of Orders	Average Inventory	Carrying Costs	Ordering Costs	Total Cost
1	...	...	...	...	...
10	...	...	...	...	...
20	...	...	...	...	...
40	...	...	...	...	...
80	...	...	...	...	...
100	...	...	...	...	...

Solution:

Tabular Method to Find EOQ

Orders per Year	Lot Size	Average Inventory	Carrying Cost (Re. 1) (Rs.)	Ordering Cost (Rs. 50 per order) (Rs.)	Total Cost per Year (Rs.)
1	1,60,000	80,000	80,000	50	80,050
10	16,000	8,000	8,000	500	8,500
20	8,000	4,000	4,000	1,000	5,000
40	4,000	2,000	2,000	2,000	4,000
80	2,000	1,000	1,000	4,000	5,000
100	1,600	800	800	5,000	5,800

Disadvantage of trial & error (or tabular) method.

In above example, we were fortunate enough in finding the lowest possible cost. But, suppose the computation for 8 orders per year had not been made.

Then, we could choose only among the six remaining alternatives for the lowest cost solution. This imposes a serious limitation of this method. So, a relatively large number of alternatives must be computed before the best possible least cost solution is obtained. In this situation, the following *graphical method* may be advantageous.

Graphical Method

The data calculated in the above table can be graphed as below to demonstrate the nature of the opposing costs involved in an EOQ model. This graph shows that annual total costs of inventory, carrying costs and ordering costs first decrease, then hit a lowest point where *inventory carrying costs* equal *ordering costs*, and finally increases as the ordering quantity increases. Our main objective is to find a numerical value for EOQ that will minimize the total variable costs on the graph.

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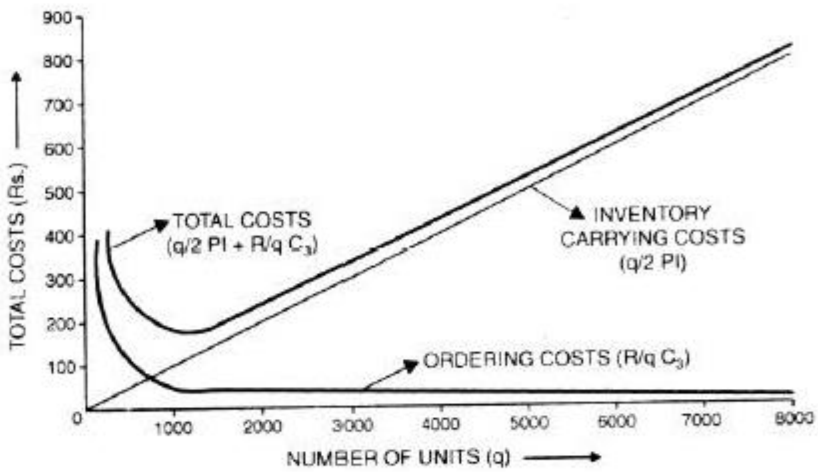


Fig. Economic ordering quantity graph

Disadvantage of graphical Method.

Without specific costs and values an accurate plotting of the carrying costs, ordering costs, and Iota I costs is not feasible, we can solve the EOQ models by the following two more accurate methods.

.*Algebraic method.* This method is based on the fact that the most economical point in terms of total Inventory cost is where the *inventory carrying cost* equals *ordering cost.*

Calculus method. This method is based on the technique of finding the minimum total cost by utilizing the differentiation. This is the best method since it does not suffer from the limitations like previous methods.

1. Stationary Economic Order Interval. Suppose that there is a constant demand rate $r > 0$ per unit time for a single product. There is a fixed setup cost $K > 0$ for placing an order, a holding cost $h > 0$ per unit stored per unit time and a penalty cost $p > 0$ per unit backordered demand per unit time. Each time the net inventory level (i.e., inventories less backorders) reaches s (< 0), an order for D ($> -s$) units is placed that brings the net inventory on hand immediately up to $S \equiv s + D$.

(a) Optimal Ordering Policy. Show that the values of s, S and D that minimize the long-run average cost per unit time are given by

$$D = \sqrt{\frac{2Kr}{h} \cdot \frac{h+p}{p}},$$

$$S = \frac{p}{h+p} D \text{ and}$$

$$s = -\frac{h}{h+p} D.$$

(b) Limiting Optimal Ordering Policy as Penalty Cost Increases to Infinity. Find the limiting values of s, S and D as $p \to \infty$. Show that the limiting value of D agrees with the Harris square-root formula given in §1.2 of *Lectures on Supply-Chain Optimization.*

(c) Independence of Optimal Fraction Backordered and the Demand Rate and Setup Cost. What fraction f of the time will there exist no backorders under the policy in part (a)? Show that f does not depend on the demand rate or setup cost. Explain this fact by showing that for any fixed order interval, the same fraction f is optimal.

2. Extreme Flows in Single-Source Networks. Consider the minimum-additive-concave-cost network-flow problem on the graph $(\mathcal{N}, \mathcal{A})$ with demand vector $r = (r_i)$ in which the flows are required to be nonnegative and there is a single source $\sigma \in \mathcal{N}$, i.e., $r_\sigma < 0$ and $r_i \geq 0$ for $i \in \mathcal{N} \setminus \{\sigma\}$. Use a graph-theoretic argument to establish the equivalence of 1°-3° below about a nonnegative flow $x = (x_{ij})$. Also show that 1° implies the second assertion of 3° using Leontief substitution theory from problem 1(a) of Homework 1.

1° x is an extreme flow.

2° The subgraph induced by x is a tree with all arcs directed away from σ.

3° The subgraph induced by x is connected and contains no arc whose head is σ, and $x_{ij}x_{kj} = 0$ for all arcs $(i, j), (k, j) \in \mathcal{A}$ for which $i \neq k$.

3. Cyclic Economic Order Interval. Consider the problem of scheduling orders, inventories and backorders of a single product over periods $1, 2, \ldots$ so as to minimize the (long-run) average-cost per period of satisfying given demands $r_1, r_2, \ldots$ for the product in periods $1, 2, \ldots$. There is a real-valued concave cost $c_i(x_i)$ of ordering $x_i \geq 0$, $h_i(y_i)$ of storing $y_i \geq 0$ and $b_i(z_i)$ of backordering $z_i \geq 0$ in each period $i \geq 1$. Assume that $c_i(0) = h_i(0) = b_i(0) = 0$ for each $i \geq 1$ and that the data are *n-periodic*, i.e.,

$$(c_i, h_i, b_i, r_i) = (c_{i+n}, h_{i+n}, b_{i+n}, r_{i+n}) \text{ for } i = 1, 2, \ldots.$$

Clearly, there is a feasible schedule if and only if $\sum_1^n r_i \geq 0$. Assume this is so in the sequel.

(a) Reduction to Network-Flow Problem on a Wheel. Show that the problem of finding an ordering, inventory and backorder schedule that minimizes the average-cost in the class of n-periodic schedules is a minimum-additive-concave-cost uncapacitated nearly-1-planar network-flow problem. Show also that apart from duplicate arcs between pairs of nodes, the graph is a wheel with the hub node labeled 0 and the nodes associated with demands in periods $1, \ldots, n$ labeled by those periods cyclically around the hub. Thus, the node that immediately follows n in the cyclic order is, of course, node 1. Denote by $(i, k]$ the interval of periods in the cyclic order that begins with the node following i and ends with k for $1 \leq i, k \leq n$.

(b) Existence of Optimal Periodic Schedules. Give explicit necessary and sufficient conditions for the existence of a minimum-average-cost n-periodic schedule in terms of the derivatives at infinity of the cost functions (c_i, h_i, b_i) for $1 \leq i \leq n$.

(c) Extreme Periodic Schedules. Show that a feasible n-periodic schedule $(x_1, y_1, z_1, \ldots, x_n, y_n, z_n)$ is an extreme point of the set of such schedules if and only if

- inventories and backorders do not occur in the same period, i.e., $y_i z_i = 0$ for $i = 1, \ldots, n$,
- there is a period $1 \leq l \leq n$ with no inventories or backorders, i.e., $y_l = z_l = 0$, and
- between any two distinct periods $1 \leq i, k \leq n$ of positive production, there is a period j in the interval $[i, k)$ with no inventories or backorders, i.e., $y_j = z_j = 0$.

Show that if also the demands are all nonnegative, then $y_{i-1}x_i = z_i x_i = y_{i-1} z_i = 0$, for $1 \leq i \leq n$ where $y_0 \equiv y_n$.

(d) An $O(n^3)$ Running-Time Algorithm. Use the second condition of (c) to show that if there is a minimum-average-cost n-periodic schedule, then such a schedule can be found with at most $\frac{3}{2}n^3 + O(n^2)$ additions and $n^3 + O(n^2)$ comparisons by solving n n-period economic-order-interval problems. This implementation improves upon the $O(n^4)$ running time of straightforward application of the send-and-split method. [*Hint*: First show how to compute the minimum costs c_{ik} incurred in the interval $(i, k]$ with zero inventories and backorders in periods i and k, and with at most one order placed in the interval $(i, k]$ for all $1 \leq i, k \leq n$ in $O(n^3)$ time.]

(e) Constant-Factor Reduction in Running Time with Nonnegative Demands and No Backorders. Show that if also the demands are nonnegative and no backorders are allowed, then the running time in (d) can be reduced to $\frac{1}{2}n^3 + O(n^2)$ additions and a like number of comparisons.

(f) An $O(n)$ Running-Time Algorithm with Nonnegative Demands and Linear Costs. Suppose that the demands are nonnegative and the cost functions are linear, i.e., there are constants c_i, h_i and b_i such that $c_i(z) = c_i z$, $h_i(z) = h_i z$ and $b_i(z) = b_i z$ for all $z \geq 0$ and $1 \leq i \leq n$. Show that a minimum-average-cost n-periodic schedule can be found with $4n$ additions and $4n$ comparisons. [*Hint*: Consider the following two-step algorithm. The first step is to determine an optimal period i in which to produce to satisfy the demand in any fixed period j. This can be done by choosing the cheapest of the solutions to two n-period problems. One finds an optimal period in which to order and store until period j, and the other finds an optimal period in which to order after backordering since period j. The second step is to solve the problem under the assumption that one orders in period i, in which case one can delete the inventory and backorder arcs whose head is node i.]

POSSIBLE QUESTIONS

ANSWER ALL THE QUESTIONS (5X6=30)

1. The probability distribution of monthly sale of a certain item is as follows:

Monthly sales:	0	1	2	3	4	5	6
Probability :	0.01	0.06	0.25	0.35	0.20	0.03	0.10

The cost of carrying inventory is Rs.30 per unit per month and the cost of unit shortage is Rs.70 per month. Determine the optimum stock level that minimizes the total expected cost.

2. Electro uses resin in its manufacturing process at the rate of 1000 gallons per month. It cost electro 100 to place an order. The following cost per gallon per month is 2, and the shortage cost per gallon is 10 .historical data show that the demand during lead time is uniform over the range(0,50) gallons. Determine the optimal ordering policy for Electro.

3.Determine optimal order level of Z.

4.The owner of a newsstand wants to determine the number of USA now newspapers that must be stocked at the start of each day. It costs 30 cents to buy a copy, and the owner sells it for 75 cents . The sale of the newspaper typically occurs between 7:00 and 8:00 am newspapers left at the end of the day are recycled for an in come of 5 cents a copy .How many copies should the owner stock every morning , assuming that the demand for the day can be described as

i) a normal distribution with mean 300 copies and standard deviation 20 copies

ii) A discrete probability distribution function f(D), defined as

D	200	220	300	320	340
f(D)	0.1	0.1	0.4	0.2	0.1

5. If the demand for a certain product has a rectangular distribution between 4000 and 5000, find the optimal order quantity if shortage cost is Rs. 1 per unit and shortage cost is Rs.7 per unit.

6.Explain the optimum order level of single period problem without setup cost.

COMPULSORY (1X10=10)

1.The probability distribution of monthly sales of a certain item as follows:

Monthly Sales:	0	1	2	3	4	5	6
Probability :	0.02	0.05	0.30	0.27	0.20	0.10	0.06

The cost of carrying inventory is Rs.10 per unit per month. The current policy is to maintain a stock of four items at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the imputed cost of a shortage of one time unit.

2.The daily demand form an item during a single period occurs instantaneously at the start of the period. The p. d. f. of the demand is uniform between 0 and 10 units.The unit holding cost of the item during the period is 50 ,and the unit penalty cost for running out of stock is 4.50 . The unit purchase cost is 50. A fixed cost of 25 is incurred each time an order is placed. Determine the optimal inventory policy for the item.

3. A shop is about to order some heaters for a forecast spell of cold weather. The shop pays Rs.1,000 for each heater, and during the cold spell they sell for Rs.2,000 each.The demand for the heater declines after the cold spell is over, and any unsold units are sold at Rs.500. Previous experience suggests the likely demand for heaters is as follows:

Demand :	10	20	30	40	50
Probability:	0.20	0.30	0.30	0.10	0.10

How many heaters should the shop buy?

10. An ice cream company sells one of its types of ice creams by weights. If the products is not sold on the day it is prepared, it can be sold for a loss of 50 paisa per pound but there is an unlimited market for one day old ice – creams. On the other hand, the company makes a profit of Rs. 3.20 on every pound of ice – cream sold on the day it is prepares. If daily orders form a distribution with $f(x) = 0.02- 0.0002x$, $0 \leq x \leq 100$ how many pounds of ice – cream should the company prepare every day?.

UNIT-IV

SYLLABUS

Real life application - Decision making under certainty- Analytic hierarchy process. Decisions under Risk- Decision Trees-based expected value criterion, variations of the expected value criterion. Decisions under Uncertainty Real life application in Decision Analysis

DECISION THEORY

The ultimate purpose of any operations research analysis is to enable operations to be run efficiently and effectively, and this in turn involves selecting the best of the alternative ways and means of conducting operations. Fundamental, then, to any operations research exercise is the final step of making a decision between alternatives, and the principles underlying such decision making are referred to as **Decision Theory**

Structuring the Decision Problem

To illustrate the decision analysis approach, let us consider the case of Political Systems, Inc (PSI), a newly formed computer service from specializing in information services such as surveys and data analysis for individuals running for political office. The firm is in the final stages of selecting a computer system for its Midwest branch, located in Lagos. While PSI has decided on a computer manufacturer, it is currently attempting to determine the size of the computer system that would be most economical. We will use decision theory to help PSI make its computer decision.

The first step is to identify the alternatives considered by the decision maker. For PSI, the final decision will be to select one of the three computer systems, which differ in size and capacity. The three decision alternatives denoted by D_1, D_2, and D_3 are as follows:

D_1 – large computer system

D_2 – Medium computer system

D_3 – Small computer system

The second step is to identify the future events that might occur. There events, which are not under the control of the decision maker, are referred to as the **States of nature**. Thus, the PSI states of nature denoted S_1 and S_2 are as follows:

S_1 – high customer acceptance of PSI services

S_2 – low customer acceptance of PSI services

Given the three decision alternatives and the two states of nature, **which computer system should PSI select?** To answer this question, we will need information on the profit associated with each combination of a decision alternative and a state of nature.

Payoff Tables

We denote the decision alternatives by D_1, D_2, …, D_m, the states of nature by S_1, S_2, … S_n; and the return associated with decision Di and state Sj by Vij (I = 1, 2, …, m i j = 1, 2, …, n). A process requiring the implementation of just one decision is defined completely by Table 1. A table of this form is referred as a payoff table. In general, entries in (a) can be stated in terms of profits, costs etc. Using the best information available, management has estimated the payoffs or profits for the PSI problem. There estimates are presented in Table 2

Table 1: States of Nature

	S_1	S_2	…	S_n
D_1	V_{11}	V_{12}	…	V_{1n}
D_2	V_{21}	V_{22}	…	V_{2n}
…	….	…	…	…
D_m	V_{m1}	V_{m2}	…	V_{mn}

Table 2:

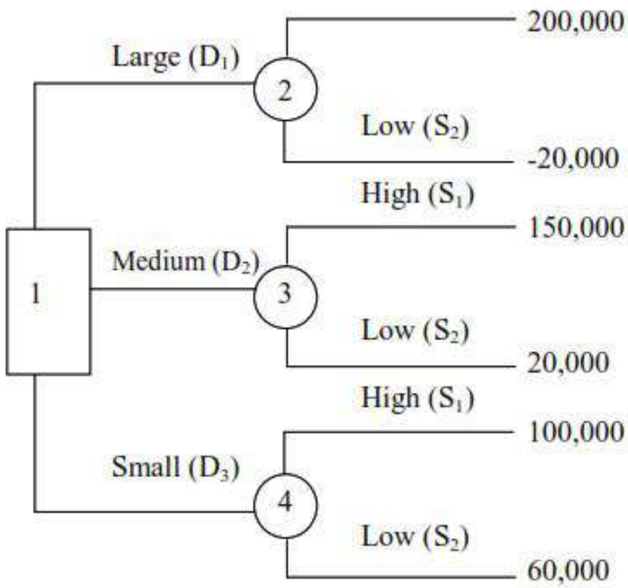
Decision alternatives		High Acceptance S_1	Low Acceptance S_2
Large system	D_1	200,000	20,000
Medium system	D_2	150,000	20,000
Small system	D_3	100,000	60,000

Decision Trees

A decision tree provides a graphical presentation of the decision-making process. Figure 1 shows a decision tree for the PSI problem. Note that the three shows the natural or logical progress that will occur overtime.

Figure 1

High (S_2)



Using the general terminology associated with decision trees, we will refer to the intersection or junction points of the tree as **nodes** and the arcs or connectors between the nodes as **branches**. Fig 1 shows the PSI decision tree with the nodes numbered 1 to 4. When the branches leaving a given node are decision branches, we refer to the nodes as decision node. Decision nodes are denoted by squares. Similarly, when the branches leaving a given node are state-of-nature branches, we refer to the node as a state-of-nature node. State-of-nature nodes are denoted by circles. Using the node-labelling procedure, node 1 is a decision node, where as nodes 2, 3 and 4 are states of nature nodes.

Decision Making without Probabilities

This section consider approaches to decision making that do not require knowledge of the probabilities of the states of nature

Optimistic Approach

The (~) evaluates each decision alternative in terms of the best payoff that can occur. The decision alternative that is recommended is the one that provides the best possible payoff. For a problem in which maximum profit is desired, as it is in the PSI problem, the optimistic approach would lead the decision maker to choose the alternative corresponding to the largest profit. For problems involving minimization, this approach leads to choosing the alternative with the smallest payoff. To illustrate the use of the ~, we will show how it can be used to develop a recommendation for the PSI problem.

Table 3

Decision alternatives		Maximum Payoff	
Large system	D_1	200,000	←Maximum of the maximum
Medium system	D_2	150,000	payoff values
Small system	D_3	100,000	

Conservative Approach

The conservative approach evaluates each decision alternatives in terms of the most payoff that can occur. The decision alternative recommended is the one that provides the best of the worst possible payoffs. For a problem in which the output measure is profit, as it is in PSI problems. The (~) would lead the decision maker to chose the alternative that maximizes the minimum possible profit that could be obtained. For problems involving minimization, this approach identifies the alternative that will minimize the maximum payoff.

Table 4

Decision alternatives		Maximum Payoff	
Large system	D_1	-20,000	
Medium system	D_2	20,000	
Small system	D_3	60,000 ←	Maximum of the maximum payoff values

Minimax Regret Approach

(~) is another approach to decision making with certainty. This approach is neither purely optimistic nor purely conservative. We illustrate the (~) for the PSI problem. In maximization problem, the general expression for opportunity loss or regret is given by the formula:

Opportunity loss or Regret

$R_{ij} = V_j - V_{ij}$ --- (*)

Where

Rij = reject associated with decision alternative Di and state of nature Sj

Vj = best payoff value under state of nature Sj

Vij = payoff associated with decision alternative D1 and state of nature Sj

Using eq (*) and the payoff in Table 2, we can compute the regret associated with all combinations of decision alternatives Di and States of nature Sj

Table 5

Regret or opportunity loss for the PSI problem		States of Nature	
Decision Alternatives		High Acceptance S_1	Low Acceptance S_2
Large System	D_1	0	80,000
Medium System	D_2	50,000	40,000
Small System	D_3	100,000	0

Table 6

Decision Alternatives		Maximum Regret or Opportunity Loss
Large system	D_1	80,000
Medium system	D_2	50,000 ← Minimum of the maximum
Small system	D_3	100,000 regret

For the PSI problem, the decision to select a medium-computer system, with a corresponding regret of N50,000, is the recommended minimax regret decision.

Rank: In cost minimization problems, Vj will be the smallest entry in column j, and equation (*) must be changed to

$$Rij = Vij - Vj$$

Decision Making with Probabilities

In many decision-making situations, it is possible to obtain probability estimates for each of the states of nature. When such probabilities are available, the **expected value**

approach can be used to identify the best decision alternative. The expect value approach evaluates each decision alternative in terms of its expected value. The recommended decision alternative is the one that provides the best expected value.

Let

N = the number of states of nature

P (Sj) = the probability of state of nature Sj

P (Sj) $\geq$ 0 for all states of nature

P (Sj) = P (S_1) + P (S_2) + … + P (S_N) = 1

Expected Value of Decision Alternative Di

$$EV\ (Di) = \sum_{j=1}^{N} P\ (Sj)\ Vij \qquad **$$

Using the payoff values Vij shown in Tale 1 and supposes that PSI management believes that S_1, the high acceptance state of nature, has a 0.3 probability of occurrence and that S2, the low-acceptance state of nature, has a 0.7 probability. Thus, P (S_1) = 0.3 and P (S_2) = 0.7 and equation (**), expected values for the three decision alternatives can be calculated:

EV (D_1) = 0.3 (200,000) + 0.7 (-20,000) = N46,000

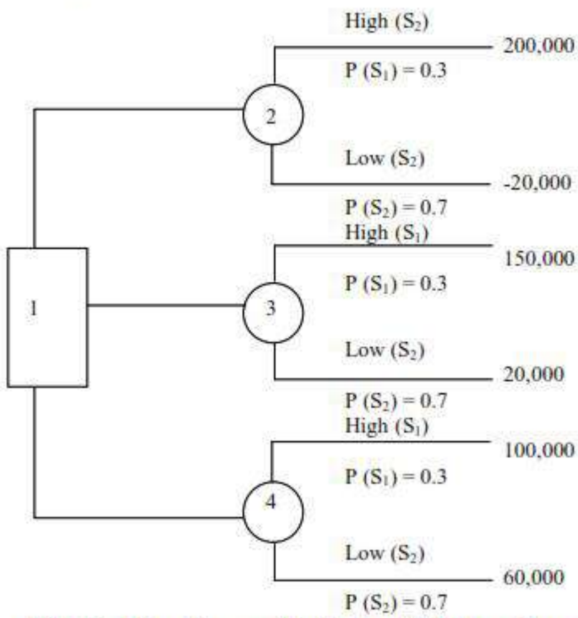
EV (D_2) = 0.3 (150,000) + 0.7 (20,000) = N59,000

EV (D_3) = 0.3 (100,000) + 0.7 (60,000) = N72,000

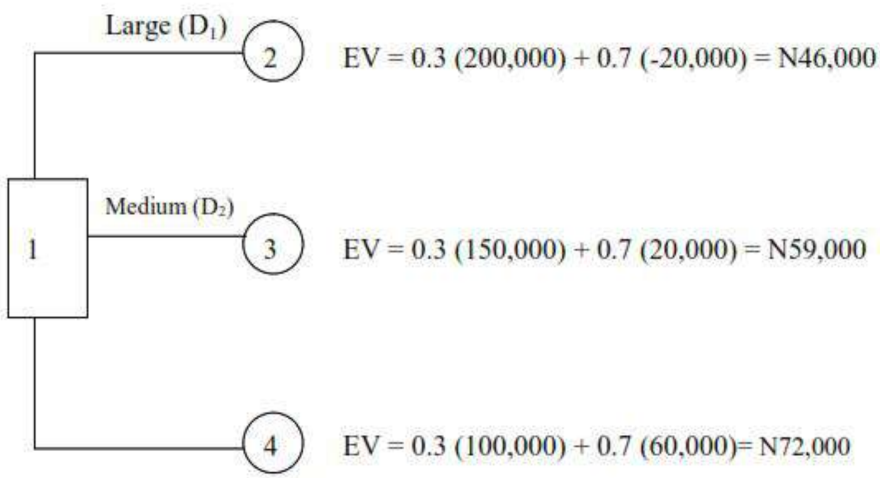
Thus, according to the expected value approach, D3 is the recommended decision since D_3 has the highest expected value (N72,000)

Using Decision Tree

Figure 2



PSI Decision Tree with State-of-Nature Branch Probabilities



Applying the Expected Value Approach Using Decision Trees

Sensitivity Analysis

In this section, we consider how changes in the probability estimates for the states of nature affect or utter the recommended decision. The study of the effect of such changes is referred to as ~. One approach to ~ is to consider different probabilities for the states of nature and then recompute the expected value for each decision alternative. Repeating this several times, we can begin to learn how changes in the probabilities for the states of nature affect the recommended decision. For example, suppose that we consider a change in the probabilities for the states of nature such that P (S_1) = 0.6 and P (S_2) = 0.4. Using these probabilities and repeating the expected value computations, we find the following:

EV (D_1) = 0.6 (200,000) + 0.4 (-20,000) = N112,000

EV (D_2) = 0.6 (150,000) + 0.4 (20,000) = N98,000

EV (D_3) = 0.6 (100,000) + 0.4 (60,000) = N84,000

Thus, with these probabilities, the recommended decision alternative is D_1, with an expected value of N112,000.

The only drawback to this approach is the numerous calculations required to evaluate the effect of several possible changes in the state-of-nature probabilities.

POSSIBLE QUESTIONS

ANSWER ALL THE QUESTIONS (5X6=30)

1. Explain about decision making under uncertainty.

2. A businessman has two independent investment portfolios A and B, available to him, but he lacks the capital to undertake both of them simultaneously. He can either choose A first and then stop, or if A is not successful, then take, B or vice versa. The probability of success of A is 0.6, while for B it is 0.4. Both investment schemes require an initial capital outlay of Rs.10,000 and both return nothing if the venture proves to be unsuccessful. Successful completion of A will return Rs.20, 000 (over cost) and successful completion of B will return Rs.24,000(over cost). Draw a decision tree in order to determine the best strategy.

3. A businessman has three alternative open to him each of which can be followed by any of the four possible event. The conditional payoffs (in Rs) for each action event combination are given below.

Alternative	Payoffs conditional on event			
	A	B	C	D
X	8	0	-10	6
Y	-4	12	18	-2
Z	14	6	0	8

Determine which alternative should the businessman choose, if he adopts the
i) maximin criterion ii) maximax criterion iii) savage criterion

4. A glass factory that specializes in crystal is developing a substantial backlog and for this the firm's management is considering three courses of action: To arrange for subcontracting (S_1), to begin overtime production(S_2), and to construct new facilities(S_3). The correct choice depends largely upon the future demand , which may be low, medium, or high. By consensus, management ranks the respective probabilities as 0.10, 0.50 and 0.40. A cost analysis reveals the effect upon the profits. This is shown in the table below.

Demand	Probability	Course of Action		
		S_1	S_2	S_3
Low(L)	0.10	10	-20	-150
Medium(M)	0.50	50	60	20

High(H)	0.40	50	100	200

Show this decision situation in the form of a decision tree and indicate the most preferred decision and its corresponding expected values.

5. A food products company is contemplating the introduction of a revolutionary new product with new packaging or replacing the existing product at much higher price(S_1). It may even make a moderate change in the composition of the existing product, with new packaging at a small increase in price (S_2), or may mall a small change in the composition of the existing product, backing it with the word 'New" and a negligible increase in price(S_3). The three possible states of nature or events are: (i) high increase in sales (N1), (ii) no change in sales(N_2) and (iii) decrease in sales(N_3). The marketing department of the company worked out the payoffs in terms of yearly net profits for each of the strategies of three events(expected sales).
This is represented in the following table:

Strategies	States of Nature		
	N_1	N_2	N_3
S_1	7,00,000	3,00,000	1,50,000
S_2	5,00,000	4,50,000	0
S_3	3,00,000	3,00,000	3,00,000

Which strategy should the concerned executive choose on the basis of
i) Maximin criterion ii) Maximax criterion
iii) Minimax regret criterion iv) Laplace criterion

6.A retailer purchases cherries every morning at Rs.50 a case and sells them for Rs.80 a case. Any case that remains unsold at the end of the day can be disposed of the next day at a salvage value of Rs.20 per case(thereafter they have no value). Past sales have ranged from 15 to 18 cases per day. The following is the record of sales for the past 120 days.

Cases sold :	15	16	17	18
Number of days:	12	24	48	36

Find out how many cases should the retailer purchase per day in order to maximize his profit.

COMPLUSOURY (1X10=10)

7.Explain about decision making under risk.

8.A manager has a choice between (i) risky contract promising Rs. 7 lakhs with probability 0.6 and Rs. 4 lakhs with probability0.4, and (ii) a diversified portfolio consisting of two contracts with independent outcomes each promising Rs. 3.5 lakhs with probability 0.6 and Rs. 2 lakhs with probability 0.4. Construct a decision tree for using EMV criteria.

9. A firm manufactures three types of products. The fixed and variable costs are given below.

	Fixed cost (Rs)	Variable Cost Per Unit(Rs)

Product A	25000	12
Product B	35000	9
Product C	53000	7

The likely demand (units) of the products is given below:

Poor demand : 3000
Moderate demand : 7000
High demand : 11000

If the sale price of each type of product is Rs.25, then prepare the payoff matrix.

10.A super bazaar must decide on the level of supplies it must stock to meet the needs of its customers during Diwali days. The exact number of customers is not know, but it is expected to be in one of the 4 categories: 300, 350, 400 (or) 450 customers. Four levels of supplies are thus suggested with level j being ideal, if the number of customers falls in category j deviations from the ideal levels results in additional costs either because extra supplies are stocked needlessly or because demand cannot be satisfied. The table below provides these costs in thousands of rupees.

Customers category	Supplies level			
	A1	A2	A3	A4
E1	7	12	20	27
E2	10	9	10	25
E3	23	20	14	23
E4	32	24	21	17

Find i) maximin criterion ii) maximax criterion iii) savage criterion.

INTRODUCTION

An inventory can be defined as any idle resource of an enterprise. An inventory problem exists when it is necessary to stock physical goods or commodities for the purpose of satisfying demand over a specified time horizon (finite or infinite). Almost every business must stock goods to ensure smooth and efficient running of its operation. Decisions regarding how much and when to order are typical of every inventory problem. The required demand maybe satisfied by stocking once for the entire time horizon or by stocking separately for every time unit of the horizon. The two extreme situations (overstocking and under-stocking) are costly. Decisions may thus be based on the minimization of an appropriate cost function that balances the total costs resulting from over-stocking and under-stocking.

A GENERALIZED INVENTORY MODEL

The ultimate objective of an inventory model is to answer two questions.

1. How much to order?
2. When to order

The answer to the first question is expressed in terms of what we call the order quantity and the when-to-order decision is the inventory level at which a new order should be placed usually expressed in terms of re-order point.

The order quantity and re-order pint are normally determined by minimizing the total inventory cost that can be expressed as a function of these two variables. We can summarize the total cost of a general inventory model as a function of its principal components in the following manner:

(Total inventory cost) = (purchasing cost) + (setup cost) + (holding cost)
+ (shortage cost)

TYPES OF INVENTORY MODELS

In general, inventory models are classified into two categories:

1. Deterministic model and
2. Stochastic model

Definitions:

1. Delivery lags or lead times is the time between the placement of an order and its receipt and may be deterministic or stochastic.
2. Time Horizon defined the period over which the inventory level will be controlled. This horizon may be finite or infinite depending on the time period which demand can be forecast reliably.
3. Stock replacement: Although an inventory system may operate with delivery lags. The actual replenishment of stock may occur instantaneously or uniformly. Instantaneous replenishment can occur when the stock is purchased from outside sources. Uniform replenishment may occur when the product is manufactured locally within the organization.

DETERMINISTIC MODEL (Single item static model)

The simplest type of inventory model occurs when demand is constant over time with instantaneous replenishment and no shortages. Typical situations to which this model may apply are:

1. The use of light bulbs in a building
2. The use of clerical supplies, such as paper, pads and pencil in a large company
3. The consumption of staple food items, such as bread and milk

Fig (1) illustrates the variation of the inventory level. It is assumed that demand occurs at the rate β (per unit time). The highest level of inventory occurs when the inventory level reaches zero level y/β time units after the order quantity y is received.

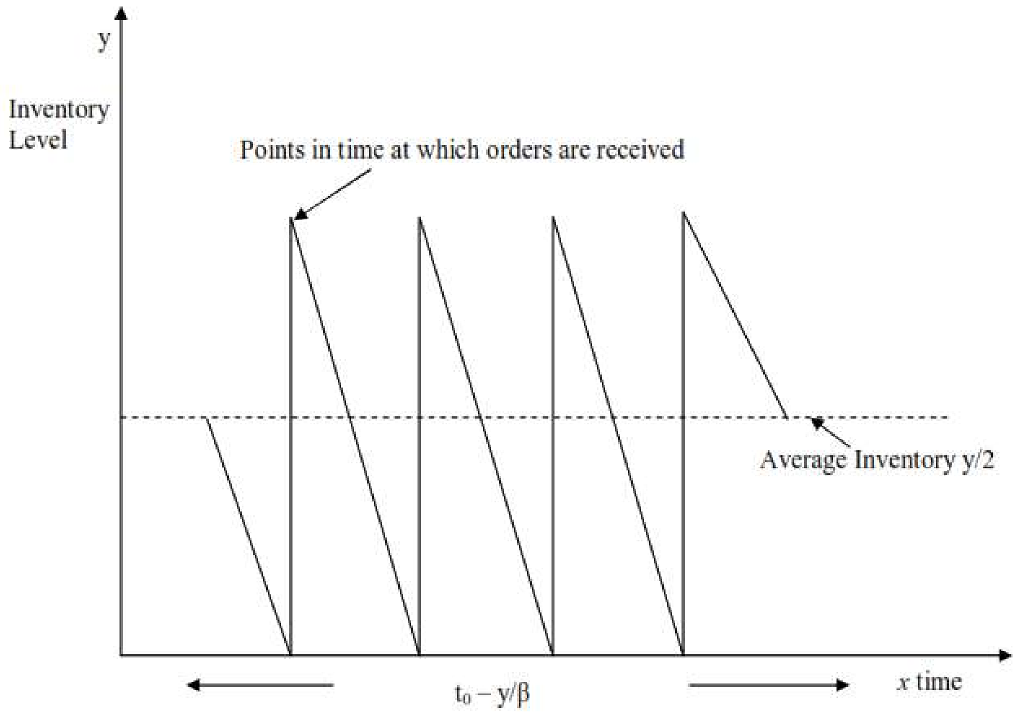


Fig. 1

The smaller the order quantity y, the more frequent will be the placement of new orders. However, the average level of inventory hold in stock will be reduced. On the other hand, larger order quantities indicate larger inventory level but less frequent placement of order (see fig. 2).

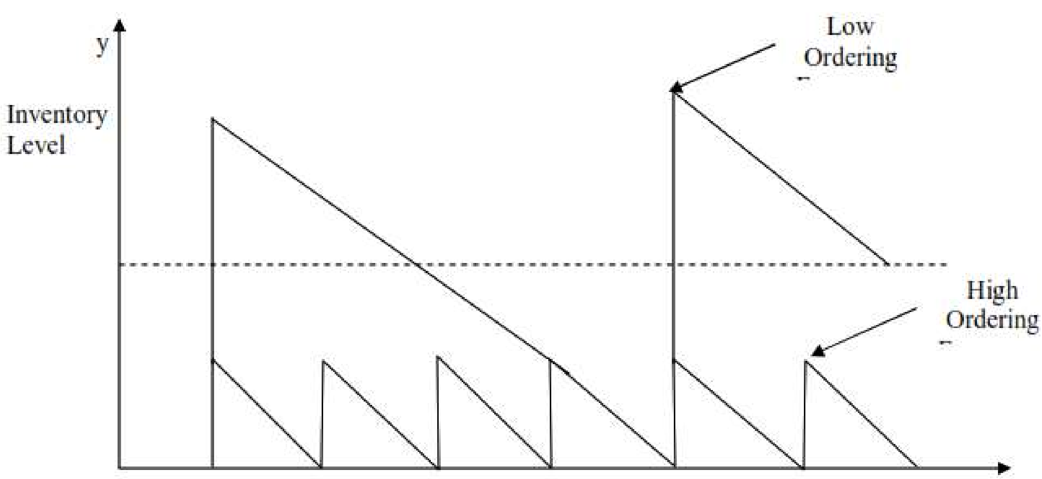


Fig. 2

Because there are costs associated with placing orders and holding inventory in stock, the quantity y is selected to allow a compromise between the two types of costs. This is the basis for formulating the inventory model.

Let K be the set up cost incurred every time an order is placed and assume that the holding cost per unit inventory per unit time is k. Hence, the total cost per unit time TCU as a function of y may be written as

TCU (y) = Setup cost/unit time + holding cost/unit time

$$TCU\,(y) = \frac{k}{y/\beta} + h\,(y/2)$$

As seen from fig. (1), the length of each inventory cycle is to = y / β and the average inventory in stock is y/2

The optimum value of y is obtained by minimizing TCU (y) with respect to y. Thus, assuming that y is a continuous variable, we have

$$\frac{dTCU\,(y)}{dy} = k\beta/y + h/2 = 0$$

Which yields the optimum order quantity as

$$y = \sqrt{\frac{2k\beta}{h}}$$

(It can be proved that y minimizes TCU(y) by showing that the 2^{nd} derivates at y is strictly positive). The order quantity above is usually referred to as Wilson's economic lot size.

The optimum policy of the model calls for ordering units every to time units. The optimum cost TCU (y) obtained by direct substitution is $\sqrt{2k/\beta}$.

EXAMPLE 1:

The daily demand for a commodity is approximately 100 units. Every time an order dared, fixed cost is N100 is incurred. The daily holding cost per unit inventory if N0.02. If the lead time is 12 days. Determine the economic lot size and the re-order point

SOLUTION:

From the earlier formula the economic lot size is

$$y = \sqrt{\frac{2k\beta}{h}}$$

$= \sqrt{2} * 100*/0.02 = 1000$ units

The associated optimum cycle length is this given as

$b_0* = \frac{y_*}{\beta} = 1000/100 = 10$ days

Since the lead time is 12 days add the cycle length is 10 days re-ordering occurs when the level of inventory is sufficient to satisfy the demand for two (- 12, 10) days. Thus the quantity y* = 1000 is ordered when the level of inventory reaches 2 * 100 = 200 units.

Notice that the "effective" lead time is taken equal to 2 days rather than 12 days. This result occurs because the lead time is longer than t_0*.

EXAMPLE 12:

A manufacturer has to supply his customer' with 600 units of his product per year. Shortages are not allowed and the shortage cost amounts to N0.60 per and per year. The setup cost per run is N80.00. Find the optimum run size and minimum average yearly cost.

Solution:

Since β = 600 units/year

K = N80.00

h = N0.06

$y = \sqrt{\frac{2k\beta}{h}}$

$\sqrt{2} * 80 * 600 / 0.06 = 400$ units opt. run time

And the minimum average yearly cost = $\sqrt{2k\beta h}$

$= \sqrt{2} * 80 * 60 * 0.60$

= N240.00

EXERCISES

1. XYZ Company purchases a component used in the manufacturing automobile generators directly from the suppliers. XYZ's generator production which is operated at a constant rate will required 1000 components per month throughout the year (12,000 units annually). If ordering cost is N25 per order, unit cost is

N200 per component and annual inventory holding costs are charged at 20%. Answer the following inventory policy question for XYZ

a. What is the economic order quantity (EOQ) for this component?

b. What is the length of cycle time in months?

c. What are the total annual inventory building and ordering cost associated with your recommended EOQ?

2. The demand for a particular item is 18,000 units per year. The holding cost per unit is N1.20 per year, and the cost of the replenishment rate is instantaneous. Determine

a. Optimum order quantity

b. Number of orders per year

c. Time between orders and

d. Total cost per year when the cost of 1 unit is N1.00

- **Economic Order Quantity (EOQ)**
- **Graphical Metod**

Concept of Economic Ordering Quantity (EOQ)

The concept of *economic ordering quantity* was first developed by *F. Harris* in 1916. The concept is that management is confronted with a set of opposing costs-as the lot size *(q)* increases, the carrying charges *(C)* will increase while the ordering costs *(C3)* will decrease. On the other hand, as the lot size *(q)* decreases, the carrying cost *(C)* will decrease but the ordering costs will increase (assuming that only minor deviations from these trends may occur). Thus, *economic ordering quantity (EOQ) is that size of order which minimizes total annual (or other time period as determined by individual firms) cost of carrying inventory and cost of ordering under the assumed conditions of certainty and that annual demands are known*. The concept of EOQ applies to items, which are replenished periodically into inventory in lots covering several periods' need. The EOQ concept is applicable under the following conditions:

a. The item is replenished in lots or batches, either by purchasing or by manufacturing.

b. Consumption of items (or sales or usage rate) is uniform and continuous.

EOQ is that order quantity or optimal order size which minimises the total cost. The model is described under the following situations:

i. Planning period is one year.

ii. Demand is deterministic and indicated by parameter D units per year.

iii. Cost of purchases, or of one unit is C.

iv. Cost of ordering (or procurement cost of replenishment cost) is C3or *Co* For manufacturing goods, it is known as set-up cost.

v. Cost of holding stock (also known as inventory carrying cost) is C1or *Cb* per unit per year expressed either in items of cost per unit per period or in terms of percentage charge of the purchase price.

vi. Shortage cost (mostly it is back order cost) is C2or *Cs* per unit per year.

vii. Lead time is L, expressed in unit of time.

viii. Cycle period in replenishment is *t*.

ix. Order size is Q.

Determination of EOQ by Trial and Error Method (or Tabular Method)

This method involves the following steps:

Step 1. Select a number of possible lot sizes to purchase.

Step 2. Determine total cost for each lot size chosen.

Step 3. Finally, select the ordering quantity, which minimizes total cost.

Following illustrative example will make the procedure clear.

For example, suppose annual demand *(D)* equals 8000 units, ordering cost *(C3)* per order is Rs. 12.50, the carrying cost of average inventory is 20% per year, and the cost per unit is Re. 1.00. The following table is computed.

No. of Orders per Year	Lot Size	Average Inventory	Carrying Charges 20% per Year	Ordering Costs Rs. 12.50 per Order	Total Cost per Year
(1)	(2)	(3)	(4)	(5)	(6) = (4) + (5)
			(Rs.)	(Rs.)	(Rs.)
1	8,000	4,000	800·00	12·50	812·50
2	4,000	2,000	400·00	25·00	425·00
4	2,000	1,000	200·00	50·00	250·00
→ 8	1,000	500	100·00	100·00	200·00
12	667	333	66·00	150·00	$\boxed{216·00}$
16	500	250	50·00	200·00	250·00
32	50	125	25·00	400·00	425·00

This table indicates that an order size of 1000 units will give us the lowest total cost among all the seven alternatives calculated in the table. Also, it is important to note that this minimum total cost occurs when:

Carrying Costs =Ordering Costs.

Example 1. *Novelty Ltd. carries a wide assortment of items for its customers. One item, Gay look, is very popular. Desirous of keeping its inventory under control, a decision is taken to order only the optimal economic quantity, for this item, each time. You have the following information. Make your recommendations: Annual Demand:* 1,60,000 *units; Price per unit: Rs.* 20; *Carrying Cost: Re.* 1*per unit or* 5% *per rupee of inventory value; Cost per order: Rs.* 50. *Determine the optimal economic quantity by developing the following table.*

No. of Orders	Size of Orders	Average Inventory	Carrying Costs	Ordering Costs	Total Cost
1	....	...	...	...	...
10	....	...	...	...	...
20	....	...	...	...	...
40	....	...	...	...	...
80	....	...	...	...	...
100	....	...	...	...	...

Solution:

Tabular Method to Find EOQ

Orders per Year	Lot Size	Average Inventory	Carrying Cost (Re. 1) (Rs.)	Ordering Cost (Rs. 50 per order) (Rs.)	Total Cost per Year (Rs.)
1	1,60,000	80,000	80,000	50	80,050
10	16,000	8,000	8,000	500	8,500
20	8,000	4,000	4,000	1,000	5,000
40	4,000	2,000	2,000	2,000	4,000
80	2,000	1,000	1,000	4,000	5,000
100	1,600	800	800	5,000	5,800

Disadvantage of trial & error (or tabular) method.

In above example, we were fortunate enough in finding the lowest possible cost. But, suppose the computation for 8 orders per year had not been made.

Then, we could choose only among the six remaining alternatives for the lowest cost solution. This imposes a serious limitation of this method. So, a relatively large number of alternatives must be computed before the best possible least cost solution is obtained. In this situation, the following *graphical method* may be advantageous.

Graphical Method

The data calculated in the above table can be graphed as below to demonstrate the nature of the opposing costs involved in an EOQ model. This graph shows that annual total costs of inventory, carrying costs and ordering costs first decrease, then hit a lowest point where *inventory carrying costs* equal *ordering costs*, and finally increases as the ordering quantity increases. Our main objective is to find a numerical value for EOQ that will minimize the total variable costs on the graph.

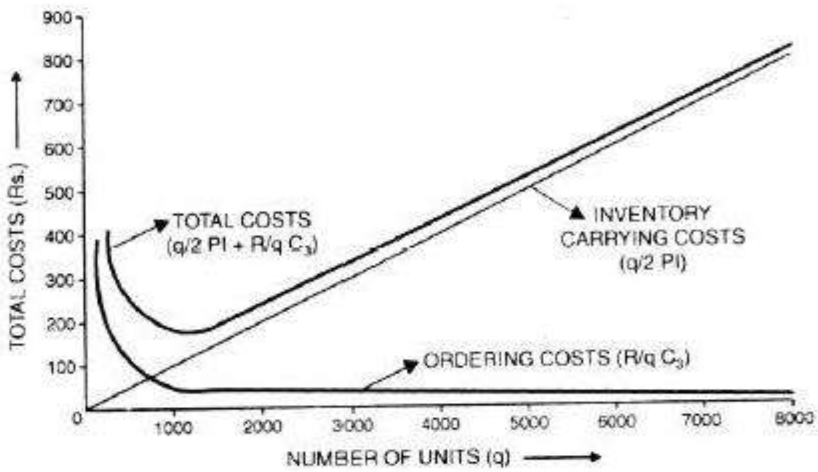


Fig. Economic ordering quantity graph

Disadvantage of graphical Method.

Without specific costs and values an accurate plotting of the carrying costs, ordering costs, and Iota I costs is not feasible, we can solve the EOQ models by the following two more accurate methods.

. *Algebraic method.* This method is based on the fact that the most economical point in terms of total Inventory cost is where the *inventory carrying cost* equals *ordering cost.*

Calculus method. This method is based on the technique of finding the minimum total cost by utilizing the differentiation. This is the best method since it does not suffer from the limitations like previous methods.

1. Stationary Economic Order Interval. Suppose that there is a constant demand rate $r > 0$ per unit time for a single product. There is a fixed setup cost $K > 0$ for placing an order, a holding cost $h > 0$ per unit stored per unit time and a penalty cost $p > 0$ per unit backordered demand per unit time. Each time the net inventory level (i.e., inventories less backorders) reaches s (< 0), an order for D ($> -s$) units is placed that brings the net inventory on hand immediately up to $S \equiv s + D$.

(a) Optimal Ordering Policy. Show that the values of s, S and D that minimize the long-run average cost per unit time are given by

$$D = \sqrt{\frac{2Kr}{h} \cdot \frac{h+p}{p}},$$
$$S = \frac{p}{h+p} D \text{ and}$$
$$s = -\frac{h}{h+p} D.$$

(b) Limiting Optimal Ordering Policy as Penalty Cost Increases to Infinity. Find the limiting values of s, S and D as $p \to \infty$. Show that the limiting value of D agrees with the Harris square-root formula given in §1.2 of *Lectures on Supply-Chain Optimization*.

(c) Independence of Optimal Fraction Backordered and the Demand Rate and Setup Cost. What fraction f of the time will there exist no backorders under the policy in part (a)? Show that f does not depend on the demand rate or setup cost. Explain this fact by showing that for any fixed order interval, the same fraction f is optimal.

2. Extreme Flows in Single-Source Networks. Consider the minimum-additive-concave-cost network-flow problem on the graph $(\mathcal{N}, \mathcal{A})$ with demand vector $r = (r_i)$ in which the flows are required to be nonnegative and there is a single source $\sigma \in \mathcal{N}$, i.e., $r_\sigma < 0$ and $r_i \geq 0$ for $i \in \mathcal{N} \setminus \{\sigma\}$. Use a graph-theoretic argument to establish the equivalence of 1°-3° below about a nonnegative flow $x = (x_{ij})$. Also show that 1° implies the second assertion of 3° using Leontief substitution theory from problem 1(a) of Homework 1.

1° x is an extreme flow.

2° The subgraph induced by x is a tree with all arcs directed away from σ.

3° The subgraph induced by x is connected and contains no arc whose head is σ, and $x_{ij}x_{kj} = 0$ for all arcs $(i, j), (k, j) \in \mathcal{A}$ for which $i \neq k$.

3. Cyclic Economic Order Interval. Consider the problem of scheduling orders, inventories and backorders of a single product over periods $1, 2, \ldots$ so as to minimize the (long-run) average-cost per period of satisfying given demands $r_1, r_2, \ldots$ for the product in periods $1, 2, \ldots$. There is a real-valued concave cost $c_i(x_i)$ of ordering $x_i \geq 0$, $h_i(y_i)$ of storing $y_i \geq 0$ and $b_i(z_i)$ of backordering $z_i \geq 0$ in each period $i \geq 1$. Assume that $c_i(0) = h_i(0) = b_i(0) = 0$ for each $i \geq 1$ and that the data are *n-periodic*, i.e.,

$$(c_i, h_i, b_i, r_i) = (c_{i+n}, h_{i+n}, b_{i+n}, r_{i+n}) \text{ for } i = 1, 2, \ldots.$$

Clearly, there is a feasible schedule if and only if $\sum_1^n r_i \geq 0$. Assume this is so in the sequel.

(a) Reduction to Network-Flow Problem on a Wheel. Show that the problem of finding an ordering, inventory and backorder schedule that minimizes the average-cost in the class of n-periodic schedules is a minimum-additive-concave-cost uncapacitated nearly-1-planar network-flow problem. Show also that apart from duplicate arcs between pairs of nodes, the graph is a wheel with the hub node labeled 0 and the nodes associated with demands in periods $1, \ldots, n$ labeled by those periods cyclically around the hub. Thus, the node that immediately follows n in the cyclic order is, of course, node 1. Denote by $(i, k]$ the interval of periods in the cyclic order that begins with the node following i and ends with k for $1 \leq i, k \leq n$.

(b) Existence of Optimal Periodic Schedules. Give explicit necessary and sufficient conditions for the existence of a minimum-average-cost n-periodic schedule in terms of the derivatives at infinity of the cost functions (c_i, h_i, b_i) for $1 \leq i \leq n$.

(c) Extreme Periodic Schedules. Show that a feasible n-periodic schedule $(x_1, y_1, z_1, \ldots, x_n, y_n, z_n)$ is an extreme point of the set of such schedules if and only if

- inventories and backorders do not occur in the same period, i.e., $y_i z_i = 0$ for $i = 1, \ldots, n$,
- there is a period $1 \leq l \leq n$ with no inventories or backorders, i.e., $y_l = z_l = 0$, and
- between any two distinct periods $1 \leq i, k \leq n$ of positive production, there is a period j in the interval $[i, k)$ with no inventories or backorders, i.e., $y_j = z_j = 0$.

Show that if also the demands are all nonnegative, then $y_{i-1} x_i = z_i x_i = y_{i-1} z_i = 0$, for $1 \leq i \leq n$ where $y_0 \equiv y_n$.

(d) An $O(n^3)$ Running-Time Algorithm. Use the second condition of (c) to show that if there is a minimum-average-cost n-periodic schedule, then such a schedule can be found with at most $\frac{3}{2}n^3 + O(n^2)$ additions and $n^3 + O(n^2)$ comparisons by solving n n-period economic-order-interval problems. This implementation improves upon the $O(n^4)$ running time of straightforward application of the send-and-split method. [*Hint:* First show how to compute the minimum costs c_{ik} incurred in the interval $(i, k]$ with zero inventories and backorders in periods i and k, and with at most one order placed in the interval $(i, k]$ for all $1 \leq i, k \leq n$ in $O(n^3)$ time.]

(e) Constant-Factor Reduction in Running Time with Nonnegative Demands and No Backorders. Show that if also the demands are nonnegative and no backorders are allowed, then the running time in (d) can be reduced to $\frac{1}{2}n^3 + O(n^2)$ additions and a like number of comparisons.

(f) An $O(n)$ Running-Time Algorithm with Nonnegative Demands and Linear Costs. Suppose that the demands are nonnegative and the cost functions are linear, i.e., there are constants c_i, h_i and b_i such that $c_i(z) = c_i z$, $h_i(z) = h_i z$ and $b_i(z) = b_i z$ for all $z \geq 0$ and $1 \leq i \leq n$. Show that a minimum-average-cost n-periodic schedule can be found with $4n$ additions and $4n$ comparisons. [*Hint*: Consider the following two-step algorithm. The first step is to determine an optimal period i in which to produce to satisfy the demand in any fixed period j. This can be done by choosing the cheapest of the solutions to two n-period problems. One finds an optimal period in which to order and store until period j, and the other finds an optimal period in which to order after backordering since period j. The second step is to solve the problem under the assumption that one orders in period i, in which case one can delete the inventory and backorder arcs whose head is node i.]

POSSIBLE QUESTIONS

ANSWER ALL THE QUESTIONS (5X6=30)

1. The probability distribution of monthly sale of a certain item is as follows:

Monthly sales:	0	1	2	3	4	5	6
Probability :	0.01	0.06	0.25	0.35	0.20	0.03	0.10

The cost of carrying inventory is Rs.30 per unit per month and the cost of unit shortage is Rs.70 per month. Determine the optimum stock level that minimizes the total expected cost.

2. Electro uses resin in its manufacturing process at the rate of 1000 gallons per month. It cost electro 100 to place an order. The following cost per gallon per month is 2, and the shortage cost per gallon is 10 .historical data show that the demand during lead time is uniform over the range(0,50) gallons. Determine the optimal ordering policy for Electro.

3.Determine optimal order level of Z.

4.The owner of a newsstand wants to determine the number of USA now newspapers that must be stocked at the start of each day. It costs 30 cents to buy a copy, and the owner sells it for 75 cents . The sale of the newspaper typically occurs between 7:00 and 8:00 am newspapers left at the end of the day are recycled for an in come of 5 cents a copy .How many copies should the owner stock every morning , assuming that the demand for the day can be described as

i) a normal distribution with mean 300 copies and standard deviation 20 copies

ii) A discrete probability distribution function f(D), defined as

D	200	220	300	320	340
f(D)	0.1	0.1	0.4	0.2	0.1

5. If the demand for a certain product has a rectangular distribution between 4000 and 5000, find the optimal order quantity if shortage cost is Rs. 1 per unit and shortage cost is Rs.7 per unit.

6.Explain the optimum order level of single period problem without setup cost.

COMPULSORY (1X10=10)

1.The probability distribution of monthly sales of a certain item as follows:

Monthly Sales:	0	1	2	3	4	5	6
Probability :	0.02	0.05	0.30	0.27	0.20	0.10	0.06

The cost of carrying inventory is Rs.10 per unit per month. The current policy is to maintain a stock of four items at the beginning of each month. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the imputed cost of a shortage of one time unit.

2.The daily demand form an item during a single period occurs instantaneously at the start of the period. The p. d. f. of the demand is uniform between 0 and 10 units.The unit holding cost of the item during the period is 50 ,and the unit penalty cost for running out of stock is 4.50 . The unit purchase cost is 50. A fixed cost of 25 is incurred each time an order is placed. Determine the optimal inventory policy for the item.

3. A shop is about to order some heaters for a forecast spell of cold weather. The shop pays Rs.1,000 for each heater, and during the cold spell they sell for Rs.2,000 each.The demand for the heater declines after the cold spell is over, and any unsold units are sold at Rs.500. Previous experience suggests the likely demand for heaters is as follows:

Demand :	10	20 30	40	50
Probability:	0.20	0.30	0.30	0.10	0.10

How many heaters should the shop buy?

4. An ice cream company sells one of its types of ice creams by weights. If the products is not sold on the day it is prepared, it can be sold for a loss of 50 paisa per pound but there is an unlimited market for one day old ice – creams. On the other hand, the company makes a profit of Rs. 3.20 on every pound of ice – cream sold on the day it is prepares. If

daily orders form a distribution with $f(x) = 0.02- 0.0002x$, $0 \le x \le 100$ how many pounds of ice – cream should the company prepare every day?.

UNIT-V

SYLLABUS

Examples of NLPP - General NLPP - Graphical solution - Quadratic Programming - Wolfe's modified Simplex Methods - Beale's Method.

Non-Linear Programming Problem

Introduction

The Linear Programming Problem which can be review as to

$$Maximize\ \ Z = \sum_{j=1}^{n} c_j x_j$$

$$subject\ to \sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad for \quad i = 1, 2, ..., m$$

$$and\ \ x_j \ge 0 \qquad for \quad j = 1, 2, ..., m$$

The term 'non linear programming' usually refers to the problem in which the objective function (1) becomes non-linear, or one or more of the constraint inequalities (2) have non-linear or both.

Ex. Consider the following problem

$$Maximize(\ Minimize\ )\ Z = x_1^2 + x_2^2 + x_3^3$$

$$subject\ to \qquad x_1 + x_2 + x_3 \ = \ 4 \quad and \quad x_1\ , x_2\ , x_3 \geqslant 0$$

Graphical Solution

In a linear programming, the optimal solution was usually obtained at one of the extreme points of the convex region generated by the constraints and the objective function of the problem. But, it is not necessary to find the solution at extreme points of the feasible region of non-linear programming problem. Here, we take an example below :-

Example 1. Solve graphically the following problem:

$$Maximize \quad Z = 2x_1 + 3x_2 \tag{1}$$

$$subject\ to \quad x_1^2 + x_2^2 \leqslant 20, \tag{2}$$

$$x_1x_2 \leqslant 8 \quad and \quad x_1,\ x_2 \geqslant 0 \tag{3}$$

Solution:

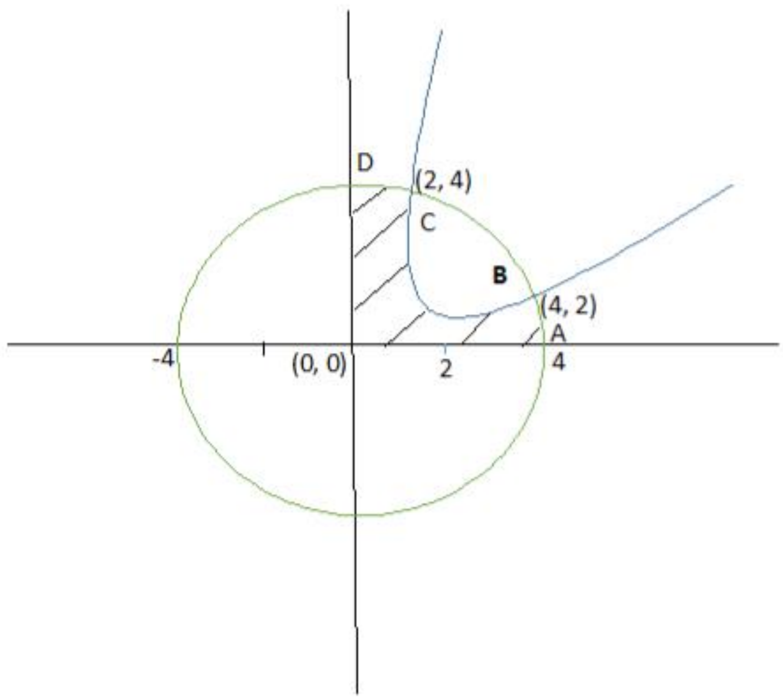
In this problem objective function is linear and the constraints are non-linear.

$x_1^2 + x_2^2 = 20$ represents circle and $x_1 x_2 = 0$ represents hyperbola. Asymptotes are represented by $X - axis$ and $Y - axis$.

Solving eq^n (2) *and* (3) , we get $x_1 = -2, -4, 2, 4$. But $x_1 = -2, -4$ are impossible $(x_1 \geqslant 0)$

Take $x_1 = 2$ *and* 4 in eq^n (2) *and* (3) , then we get $x_2 = 4$ *and* 2 respectively. So, the points are $(2, 4)$ *or* $(4, 2)$. Shaded non-convex region of OABCD is called the feasible region. Now, we maximize the objective function *i.e* $2x_1 + 3x_2 = K$ lines for different constant values of K and stop the process when a line touches the extreme boundary point of the feasible region for some value of K .

At $(2, 4)$, $K = 16$ which touches the extreme boundary point. We have boundary point of like $(0,0)$, $(0,4)$, $(2,4)$, $(4,2)$, $(4,0)$. Where the value of Z is maximum at point $(2,4)$.



$\therefore$ *Max.* $Z = 16$

General Non-linear Programming Problem

Let Z be a real valued function of n variables defined by:

(a) $Z = f(x_1, x_2, ..., x_n) \longrightarrow$ Objective function.

Let $(b_1, b_2, ..., b_m)$ be a set of constraints, such that:

$$(b) \quad g_1\ (x_1, x_2, ..., x_n)\ [\leqslant\ or\ \geqslant\ or\ =]\ b_1$$

$$g_2\ (x_1, x_2, ..., x_n)\ [\leqslant\ or\ \geqslant\ or\ =]\ b_2$$

$$g_3\ (x_1, x_2, ..., x_n)\ [\leqslant\ or\ \geqslant\ or\ =]\ b_3$$

$$g_m\ (x_1, x_2, ..., x_n)\ [\leqslant\ or\ \geqslant\ or\ =]\ b_m$$

Where g_1 are real valued functions of n variables, $x_1, x_2, ..., x_n$.

Finally, let (c) $x_j \geqslant 0$ where $j = 1, 2, ..., n$. $\longrightarrow$Non-negativity constraint.

If either $f(x_1, x_2, ..., x_n)$ or some $g_1(x_1, x_2, ..., x_n)$ or both are non-linear, then the problem of determining the n-type $(x_1, x_2, ..., x_n)$ which makes z a minimum or maximum and satisfies both (b) and (c), above is called a general non-linear programming problem.

Global Minima and Local Minima of a Function

It gives optimal solution for the objective function at the point but also optimize the function over the complete solution space.

Global Minimum: A function $f(x)$ has a global minimum at a point x_0 of a set of points K if an only if $f(x_0) \leqslant f(x)$ for all x in K.

Local Minimum: A function $f(x)$ has the local minimum point x_0 of a set of points K if and only if there exists a positive number such that $f(x_0) \leqslant f(x)$ for all x in K at which $||x_0 - x|| < \delta$

There is no general procedure to determine whether the local minimum is really a global minimum in a non-linear optimization problem.

The simplex procedure of an LPP gives a local minimum, which is also a global minimum. This is the reason why we have to develop some new mathematical concepts to deal with NLPP.

Quadratic Programming Problem

Quadratic programming deals with the non-linear programming problem of maximizing(or minimizing) the quadratic objective function subject to a set of linear inequality constraints.

The general *quadratic programming problem* can be defined as follows:

$$Maximize\ Z = CX + \frac{1}{2}X^TQX$$

subject to

$$AX \leqslant B \quad and \quad X \geqslant 0$$

where

$$X = (x_1, x_2, ..., x_n)^T$$

$$C = (c_1, c_2, ..., c_n) \quad , \quad B = (b_1, b_2, ..., b_m)^T$$

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix}$$

$$Q = \begin{pmatrix} q_{11} & \dots & q_{1n} \\ \vdots & \vdots & \vdots \\ q_{n1} & \dots & q_{nn} \end{pmatrix}$$

The function X^TQX is said to be negative-definite in the maximization case, and positive definite in the minimization case. The constraints are to be linear which ensures a convex solution space.

In this algorithm, the objective function is convex (minimization) or concave(maximization) and all the constraints are linear.

Wolfe's modified simplex method

Let the quadratic programming problem be :

$$Maximize \quad Z = f(X) = \sum_{j=1}^{n} c_j x_j + \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} c_{jk} x_j x_k$$

subject to the constraints :

$$\sum_{j=1}^{n} a_{ij} x_j \leqslant b_i, \ x_j \geqslant 0 (i = 1, ..., m, j = 1, ..., n)$$

Where $c_{jk} = c_{kj}$ for all j and k, $b_i \geqslant 0$ for all i = 1,2,...,m.

Also, assume that the quadratic form

$$\sum_{j=1}^{n}\sum_{k=1}^{n} c_{jk} x_j x_k$$

be negative semi-definite.

Then, the Wolfe's iterative procedure may be outlined in the following steps:

Step 1. First, convert the inequality constraints into equation by introducing slack-variable q_i^2 in the ith constraint $(i = 1, ..., m)$ and the slack variable r_j^2 the jth non-negative constraint $(j = 1, 2, ..., n)$.

Step 2. Then, construct the Lagrangian function

$$L(X, \mathbf{q}, \mathbf{r}, \lambda, \mu) = f(X) - \sum_{i=1}^{m} \lambda_i [\sum_{j=1}^{n} a_{ij} x_j - b_i + q_i^2] - \sum_{j=1}^{n} \mu_j [-x_j + r_j^2]$$

Where $X = (x_1, x_2, ..., x_n)$, $\mathbf{q} = (q_1^2, q_2^2, ..., q_m^2)$, $\mathbf{r} = (r_1^2, r_2^2, ..., r_n^2)$, and $\mu = (\mu_1, \mu_2, ..., \mu_n)$, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_m)$,

Differentiating the above function '**L**' partially with respect to the components of X,**q**,**r**,λ, μ and equating the first order partial derivatives to zero, we derive Kuhn-Tucker conditions from the resulting equations.

Step 3. Now introduce the non-negative artificial variable v_j, $j = 1, 2, ..., n$ in the Kuhn-Tucker conditions

$$c_j + \sum_{k=1}^{n} c_{jk} x_k - \sum_{i=1}^{m} \lambda_i a_{ij} + \mu_j = 0$$

for $j = 1, 2, ..., n$ and to construct an objective function

$$Z_v = v_1 + v_2 + ... + v_n$$

Step 4. In this step, obtain the initial basic feasible solution to the following linear programming problem :

$$Minimize\ Z_v = v_1 + v_2 + ... + v_n.$$

Subject to the constraints :

$$\sum_{k=1}^{n} c_{jk}x_k - \sum_{i=1}^{m} \lambda_i a_{ij} + \mu_j + v_j = -c_j \quad (j = 1, ...n)$$

$$\sum_{j=1}^{n} a_{ij}x_j + q_i^2 = b_i \quad (i = 1, ..., m)$$

$$v_j, \lambda_j \mu_j, x_j \geqslant 0 \quad (i = 1, ..., m; j = 1, ..., n)$$

and satisfying the complementary slackness condition:

$$\sum_{j=1}^{n} \mu_j x_j + \sum_{i=1}^{m} \lambda_i s_i = 0, \quad (where\ s_i = q_i^2)$$

or

$$\lambda_i s_i = 0 \quad and \quad \mu_j x_j \ = \ 0 \quad (for\ i = 1, ..., m; j = 1, ..., n).$$

Step 5. Now, apply two-phase simplex method in the usual manner to find an optimum solution to the LP problem constructed in Step 4. The solution must satisfy the above complementary slackness condition.

Step 6. The optimum solution thus obtained in Step 5 gives the optimum solution of given QPP also.

Important Remarks:

1.If the QPP is given in the minimization form, then convert it into maximization one by suitable modifications in $f(x)$ and the '$\geqslant$' constraints.

2. The solution of the above system is obtained by using *Phase I* of simplex method. The solution does not require the consideration of *Phase II*. Only maintain the condition $\lambda_i s_i = 0 = \mu_j x_j$ all the time.

3. It should be observed that *Phase I* will end in the usual manner with the sum of all artificial variables *equal to zero* only if the feasible solution to the problem exists.

problem.2

$Maximize\;\; 2x_1 + x_2 - {x_1}^2$

subject to

$$2x_1 + 3x_2 \leq 6$$

$$2x_1 + x_2 \leq 4 \qquad and \quad x_1,\; x_2 \;\geq\; 0$$

solution:

Since the given objective function is convex and each constraint is convex therefore the given NLPP is a CNLPP.

Now $L(X, \overline{\lambda}) = (-2x_1 - x_2 + {x_1}^2) + \lambda_1(2x_1 + 3x_2 - 6) + \lambda_2(2x_1 + x_2 - 4)$

Therefore the khun-tucker condition are

$$\therefore\;\; \frac{\partial L}{\partial x_j} \geq 0 \Rightarrow -2 + 2x_1 + 2\lambda_1 + 2\lambda_2 \geq 0$$

$$\Rightarrow -2 + 2x_1 + 2\lambda_1 + 2\lambda_2 - \mu_1 = 0$$

$$-1 + 3\lambda_1 + \lambda_2 \geq 0$$

$$\Rightarrow -1 + 3\lambda_1 + \lambda_2 - \mu_2 = 0$$

$$\therefore\;\; \frac{\partial L}{\partial \lambda_i} \leq 0 \Rightarrow 2x_1 + 3x_2 - 6 \leq 0$$

$$\Rightarrow 2x_1 + 3x_2 - 6 + S_1 = 0$$

$$2x_1 + x_2 - 4 \leq 0$$

$$\Rightarrow 2x_1 + x_2 - 4 + S_2 = 0$$

$$\therefore \quad x_j \frac{\partial L}{\partial x_j} = 0 \Rightarrow x_1\mu_1 = 0, x_2\mu_2 = 0 \qquad (1)$$

$$\therefore \quad \lambda_i \frac{\partial L}{\partial \lambda} = 0 \Rightarrow \lambda_1 S_1 = 0, \lambda_2 S_2 = 0 \qquad (2)$$

The above system of equation can be written as

$2x_1 + 2\lambda_1 + 2\lambda_2 - \mu_1 = 2$

$3\lambda_1 + \lambda_2 - \mu_2 = 1$

$2x_1 + 3x_2 + S_1 = 6$

$2x_1 + x_2 + S_2 = 4 \qquad (3)$

$x_1, x_2, \lambda_1, \lambda_2, S_1, S_2, \mu_1, \mu_2 \geq 0$

$x_1\mu_1 = 0, x_2\mu_2 = 0, \lambda_1 S_1 = 0, \lambda_2 S_2 = 0.$

This equation (3) is a LPP with out an objective function. To find the solution we can write (3) as the following LPP.

$max. Z = -R_1 - R_2$

subject to

$2x_1 + 2\lambda_1 + 2\lambda_2 - \mu_1 = 2$

$3\lambda_1 + \lambda_2 - \mu_2 = 1$

$2x_1 + 3x_2 + S_1 = 6$

$2x_1 + x_2 + S_2 = 4$

Now solve this by the two phase simplex method. The end of the phase (1) gives the feasible solution of the problem

The optimal solution of the QPP is

$x_1 = \frac{2}{3}, x_2 = \frac{14}{9}, \lambda_1 = \frac{1}{3}, S_2 = \frac{10}{9}.$ □

Beale's Method

In Beale's method we solve Quadratic Programming problem and in this method we does not use the Kuhn-Tucker condition. At each iteration the objective function is expressed in terms of non basic variables only.

Let the QPP be given in the form

$$Maximize\ f(X) = CX + \frac{1}{2}X^T QX$$

subject to $AX = b,\ X \geq 0.$

Where

$$X = (x_1, x_2, ..., x_{n+m})$$

$$c \quad is \quad 1 \times n$$

$$A \quad is \quad m \times (n+m)$$

and Q is symmetric and every QPP with linear constraints.

Algorithm

Step 1

First express the given QPP with Linear constraints in the above form by introducing slack and surplus variable.

Step 2

Now select arbitrary m variables as basic and remaining as non-basic.

Now the constraints equation $AX = b$ can be written as

$$BX_B + RX_{NB} = b \Rightarrow X_B = B^{-1}b - B^{-1}RX_{NB}$$

where

X_B-basic vector X_{NB}-non-basic vector

and the matrix A is partitioned to submatrices B and R corresponding to X_B and X_{NB} respectively.

Step 3

Express the basis X_B in terms of non-basic X_{NB} only, using the given additional constraint equations, if any.

Step 4

Express the objective function $f(x)$ in terms of X_{NB} only using the given and additional constraint , if any . Thus we observe that by increasing the value of any of the non-basic variables, the value of the objective function can be improved. Now the constraints on

the new problem become

$$B^{-1}RX_NB \leqslant B^{-1}b \qquad (since \ \ X_B \geqslant 0)$$

Thus, any component of X_{NB} can increase only until $\dfrac{\partial f}{\partial x_{NB}}$ becomes zero or one or more components of X_B are reduced to zero.

Step 5

Now we have $m+1$ non-zero variables and $m+1$ constraints which is a basic solution to the extended set of constraints.

Step 6

We go on repeating the above procedure until no further improvement in the objective function may be obtain by increasing one of the non-basic variables.

problem.1:

Use Beale's Method to solve following problem

$$Maximize\ Z = 4x_1 + 6x_2 - 2{x_1}^2 - 2x_1x_2 - 2{x_2}^2$$

$$subject\ to\ x_1 + 2x_2 \le 2\ and\ x_1, x_2 \ge 0$$

Solution:

Step:1

$$Max.\ Z = 4x_1 + 6x_2 - 2{x_1}^2 - 2x_1x_2 - 2{x_2}^2 \quad (1)$$

subject to

$$x_1 + 2x_2 + x_3 = 2 \quad (2)$$

and $x_1, x_2, x_3 \ge 0$

taking $X_B = (x_1); X_{NB} = \binom{x_2}{x_3}$

and $x_1 = 2 - 2x_2 - x_3 \quad (3)$

Step:2

put (3) in (1), we get

$Max.\ f(x_2, x_3) = 4(2 - 2x_2 - x_3) + 6x_2 - 2(2 - 2x_2 - x_3)^2 - 2(2 - 2x_2 - x_3)x_2 - 2{x_2}^2$

$\dfrac{\partial f}{\partial x_2} = -2 + 8(2 - 2x_2 - x_3) + 8x_2 - 4x_2 - 2(2 - x_3)$

$\dfrac{\partial f}{\partial x_3} = -4 + 4(2 - 2x_2 - x_3) + 2x_2$

Now $\dfrac{\partial f}{\partial x_2}_{(0,0)} = 10$

$\dfrac{\partial f}{\partial x_3}_{(0,0)} = 4$

Here '+ve' value of $\dfrac{\partial f}{\partial x_i}$ indicates that the objective function will increase if x_i increased . Similarly '-ve' value of $\dfrac{\partial f}{\partial x_i}$ indicates that the objective function will decrease if x_i is decrease . Thus, increase in x_2 will give better improvement in the objective function.

Step:3

$f(x)$ will increase if x_2 increased .

If x_2 is increased to a value greater then 1, x_1 will be negative.

Since $x_1 = 2 - 2x_2 - x_3$

$x_3 = 0; \dfrac{\partial f}{\partial x_2} = 0$

$$\Rightarrow 10 - 12x_2 = 0$$

$$\Rightarrow x_2 = \frac{5}{6}$$

$Min.\ (1, \frac{5}{6}) = \frac{5}{6}$

The new basic variable is x_2.

Second Iteration:

Step:1

let $X_B = (x_2)$, $X_{NB} = \binom{x_1}{x_3}$

$$x_2 = 1 - \frac{1}{2}(x_1 + x_3)$$

Step:2

Substitute (4) in (1)

$Max.\ f(x_1, x_3) = 4x_1 + 6(1 - \frac{1}{2}(x_1 + x_3)) - 2x_1{}^2 - 2x_1(1 - \frac{1}{2}(x_1 + x_3)) - 2(1 - \frac{1}{2}(x_1 + x_3))^2$

$\dfrac{\partial f}{\partial x_1} = 1 - 3x_1$, $\dfrac{\partial f}{\partial x_2} = -1 - x_3$

$\dfrac{\partial f}{\partial x_1}_{(0.0)} = 1$

$\dfrac{\partial f}{\partial x_3}_{(0.0)} = -1$

This indicates that x_1 can be introduce to increased objective function.

Step:3

$x_2 = 1 - \frac{1}{2}(x_1 + x_3)$ *and* $x_3 = 0$

If x_1 is increased to a value greater then 2, x_2 will become negative.

$$\frac{\partial f}{\partial x_1} = 0$$
$$\Rightarrow 1 - 3x_1 = 0$$
$$\Rightarrow x_1 = \frac{1}{3}$$
$$Min.\ (2, \frac{1}{3}) = \frac{1}{3}$$
Therefore $x_1 = \frac{1}{3}$

Hence $x_1 = \frac{1}{3}, \qquad x_2 = \frac{5}{6}, \qquad x_3 = 0$

and $\qquad Max.\ f(x) = \frac{25}{6}$ □

Kuhn-Tucker Conditions

Here we developing the necessary and sufficient conditions for identifying the stationary points of the general inequality constrained optimization problems. These conditions are called the Kuhn-Tucker Conditions. The development is mainly based on Lagrangian method. These conditions are sufficient under certain limitations which will be stated in the following .

Kuhn-Tucker Necessary Conditions

$Maximize\ \ f(X)\ ,\ X = (x_1, x_2, ..., x_n)$ subject to constraints

$$g_i(X) \leqslant b_i\ ,\ i = 1, 2, ..., m,$$

including the non-negativity constraints $X \geqslant 0$, the necessary conditions for a local maxima at $\bar{X}$ are

$$(i)\ \ \frac{\partial L(\bar{X}, \bar{\lambda}, \bar{s})}{\partial x_j} = 0,\ j = 1, 2, ..., n,$$

$$(ii)\ \ \bar{\lambda}_i\ [\ g_i(\bar{X})\ -\ b_i\ ]\ =\ 0,$$

$$(iii)\ g_i(\bar{X})\ \leqslant\ b_i\ ,\quad (iv)\ \bar{\lambda}_i \geqslant 0\ ,\ i\ =\ 1, 2, ..., m.$$

Kuhn-Tucker Sufficient Conditions

The Kuhn-Tucker conditions which are necessary conditions are also sufficient if $f(x)$ is concave and the feasible space is convex, i.e. if $f(x)$ is strictly concave and $g_i(x)$, $i = 1, ..., m$ are convex.

Problem.1

$Max.Z = 10x_1 + 4x_2 - 2x_1{}^2 - 3x_2{}^2,$

subject to

$$2x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

.

Solution:

We have,

$$f(X) = 10x_1 + 4x_2 - 2{x_1}^2 - {x_2}^2$$

$$h(X) = 2x_1 + x_2 - 5$$

The Kuhn-Tucker condition are

$$\frac{\partial f}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0$$

$$\frac{\partial f}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} = 0$$

$\lambda h(X) = 0,$

$h(X) \leq 0, \quad \lambda \geq 0.$

Applying these condition , we get

$$10 - 4x_1 - 2\lambda = 0 \qquad (i)$$

$$4 - 2x_2 - \lambda = 0 \qquad (ii)$$

$$\lambda(2x_1 + x_2 - 5) = 0 \qquad (iii)$$

$$2x_1 + x_2 - 5 \leq 0 \qquad (iv)$$

$$\lambda \geq 0 \qquad (v)$$

From (iii) either $\lambda = 0$ or $2x_1 + x_2 - 5 = 0$

When $\lambda = 0$, the solution of (i) and (ii) gives $x_1 = 2.5$ and $x_2 = 2$ which does not satisfy the equation (iv). Hence $\lambda = 0$ does not yield a feasible solution.

When $2x_1 + x_2 - 5 = 0$ and $\lambda \neq 0$, the solution of (i),(ii) and (iii) yields, $x_1 = \frac{11}{6}, x_2 = \frac{4}{3}, \lambda = \frac{4}{3}$, which satisfy all the necessary conditions.

It can be verified that the objective function is concave in X, while the constraint is convex in X. Thus these necessary conditions are also the sufficient conditions of maximization of $f(X)$.

Therefore the optimal solution is $x_1{}^* = \frac{11}{6}, x_2{}^* = \frac{4}{3}$, which gives $Z_{max} = \frac{91}{6}$ $\square$.

POSSIBLE QUESTIONS

ANSWER ALL THE QUESTIONS (5X6=30)

1.A manufacturing company produces two products: Radios and TV sets. Sales-price relationships for these two products are given below:

Product	Quantity Demanded	Unit Price
Radios	$1500 - 5p_1$	p_1
TV sets	$3800 - 10p_2$	P_2

The total cost functions for these two products are given by $200x_1 + 0.1x_i^2$ and $300x_2 + 0.1x_2^2$respectively. The production takes place on two assembly lines. Radio sets are assembled on assembly line I and TV sets are assembled on assembly line II. Because of the limitations of the assembly-line capacities, the daily production is limited to no more than 80 radio sets and 60 TV sets. The production of both types of products requires electronic components. The production of each of these sets requires five units and six units of electronic equipment components respectively. The electronic components are supplied by another manufacturer, and the supply is limited to 600 units per day. The company has 160 employees, i.e., the labour supply amounts to 160 man-days. The production of one unit of radio set requires 1 man-day of labour, whereas 2 man-days of labour are required for a TV set. How many units of radio and TV sets should the company produce in order to maximize the total profit? Formulate the problem as a non-linear programming problem.

2. Solve graphically the following NLPP

Maximize $Z = 2x_1 + 3x_2$
Subject to the constraints:
$x_1 + x_2 \leq 8$
$x_1^2 + x_2^2 \leq 20$, and
$x_1, x_2 \geq 0$

Verify that the Kuhn-Tucker conditions hold for the maxima you obtain.

3. Solve the non-linear programming problem:

Minimize $z = 2x_1^2 - 24x_1 + 2x_2^2 - 8x_2 + 2x_3^2 - 12x_3 + 200$
Subject to the constraints:
$x_1 + x_2 + x_3 = 11$
and $x_1, x_2, x_3 \geq 0$

4. Use Beale's method to solve the following NLPP:

Minimize $z = 2x_1 + 3x_2 - x_1^2$
Subject to the constraints:
$x_1 + 2x_2 \leq 4$, and $x_1, x_2 \geq 0$

5. Solve the non-linear programming problem:
Optimize $z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$
Subject to the constraints:
$x_1 + x_2 + x_3 = 15$
$2x_1 - x_2 + 2x_3 = 20$
and $x_1, x_2, x_3 \geq 0$.

6.Solve graphically the following NLPP

Minimize $Z = x_1^2 + x_2^2$
Subject to the constraints:
$x_1 + x_2 \geq 4$
$2x_1 + x_2 \geq 5$
and $x_1 , x_2 \geq 0$.

7. Use Beale's method to solve the following NLPP:
Minimize $z = 6 - 6x_1 + 2x_1^2 - 2 x_1 x_2 + 2x_2^2$
Subject to the constraints:
$x_1 + x_2 \leq 2$, and $x_1, x_2 \geq 0$.

COMPULSORY (1X10=10)

1. Solve the non-linear programming problem:
Maximum $z = 8x_1 - x_1^2 + 8x_2 - x_2^2$
Subject to the constraints:
$x_1 + x_2 \leq 12$
$x_1 - x_2 \geq 4$
and $x_1, x_2 \geq 0$
2. Explain about Kuhn – Tucker condition with non negative constraints.
3.Solve the quadratic programming problem by using Beal's method
Maximize $z = 4x_1 + 6x_2 - x_1^2 - 3x_2^2$
Subject to the constraints:
$x_1 + 2x_2 \leq 4$ and $x_1, x_2 \geq 0$

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Class : I - M.Sc. Mathematics **Semester : II**

Unit I Integer Linear Programming

Part A (20x1=20 Marks) (Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
A LPP in which all or some of the decision variables are constrained to assume non negative integer values is called an -------------	Integer programming problem	Dynamic programming problem	Non linear programming problem	Decision Analysis	Integer programming problem
In a LPP, if all the variables in the optimal solution are restricted to assume non negative integer values, then it is called the ------------------	Zero - one IPP	Pure IPP	Mixed IPP	Non IPP	Pure IPP
A systematic procedure for solving pure I.P.P is ----------	Hungarian method	Cutting method	Revised simplex method	Modi method	Cutting method
The fractional part of the negative number -7 / 3 is _____	5/3	1/3	2/3	7/3	2/3
Which of the following is correct?	A linear programming problem with only one decision variable restricted to integer value is not an integer programming problem	An integer programming problem is an LPP with decision variables restricted to integer values.	A mixed IPP is one where mixed constraints are involved.	A pure IPP is one where all the decision variables are either zero or unity	An integer programming problem is an LPP with decision variables restricted to integer values.
Which of the following is not correct?	An IPP where all the variables must be equal to zero or one is called a 0-1 IPP.	An LPP in which all the decision variables are non negative integers is called a pure IPP.	Variables in an IPP that are not integer constrainted are called discontinuous variables.	Variables in an IPP that are not integer constrained are called continuous variables.	Variables in an IPP that are not integer constrainted are called discontinuous variables.
In cutting algorithm, each cut involves the introduction of _______	an equality constraint	less than or equal to constraint	greater than or equal to constraint	an artificial variable.	less than or equal to constraint
In the context of Branch and bound method, which of the following is not correct?	It can be used to solve any kind of programming problem	It divides the feasible region into smaller parts by the process of branching	It is very usefull employed in problems where ther a finite number of solutions	It is not a standardised method and is applied differently for different problems	It can be used to solve any kind of programming problem
Which of the following is correct?	If the optimum solution to an LPP has all integer values, it may or may not be an optimum integer solution	the solution to the IPP can always be obtainted by rounding off the fractional values in the solution obtainted by simplex method	A cut may or may not eliminate any points that are feasible for the IPP	a cut does not eliminate any points which are feasible for the IPP.	a cut does not eliminate any points which are feasible for the IPP.
Maximize z = 3x +5y, subject to : x + 2y ≤ 4, 2x + y ≥ 6 and x ≥0, y ≥ 0. This problem represents a _________	Zero - one IPP	Pure IPP	Mixed IPP	Non IPP	Non IPP
Which of the following is not correct?	The optimum solution to LPP satisfies the cut that is introduced on the basis of it.	A cut is formed be choosing a row in the optimum simples table that corresponds to a non integer variable.	the cutting plane algorithm assures optimal integer solution in a finite number of iterations	The cutting plane algorithm requires all RHS values as well as all the coefficients in the constraints to be integers.	The optimum solution to LPP satisfies the cut that is introduced on the basis of it.
Which of the following is not an integer linear programming problem?	Zero - one IPP	Pure IPP	Mixed IPP	continuous IPP	continuous IPP
__________ can be solved using Branch and Bound method	A travelling salesman problem	assignment problem	geometric problem	simulation problem	A travelling salesman problem
______ can be considered as a zero - one programming problem.	A travelling salesman problem	assignment problem	geometric problem	simulation problem	assignment problem
The part of the feasible solution space eliminated by plotting a cut constraints__________	only non integer solutions	only integer solutions	both integer and non integer solutions	only rational solutions	only non integer solutions
While solving IP problem any non integer variable in the solution is picked up to ________	obtain the cut constraint	enter the solution	leave the solution	no solution	obtain the cut constraint
In a Branch and Bound minimization tree, the lower bounds on objective function value________	do not decrease in value	do not increase in value	remains constant	no constant	remains constant
A travelling salesman problem can be solved using __________	Cutting method	Branch and Bound method	Simplex method	Critical path method	Branch and Bound method
An assignment problem can be considered as a ________	Pure IPP	Mixed IPP	0 - 1 programmign problem	linear programming problem	0 - 1 programmign problem
______ can be considered as a zero - one programming problem.	Transportation problem	assignment problem	inventory problem	simulation problem	assignment problem
In a mixed integer programming problem we have ------	all of the decision v	few of the decision va	different objective functio	mixed solution to the problem	few of the decision variables require integer solutions
Branch and Bound method divides the feasible solution space into smaller parts by --------	branching	bounding	enumering	parts	branching
In a mixed-integer programming problem--------- requries integer solutions	all of the decision variables	few of the decision variables	different objective functions	mixed variables	few of the decision variables
A non integer variable is chosen in the optimal simplex table of the integer LP problem to ---------	leave the basis	enter the basis	to construct a gomory cut	Branch and Bound method	to construct a gomory cut
The corners of the reduced feasible region of an integer LP problem contains	only integer solution	optimal integer solution	only non-integer solution	not a solution	only integer solution
An------------ programming was used for capital budgeting in hospital	integer	Hungerian method	Dynamic programming problem	Decision Analysis	integer
An integer programming was used for capital budgeting in --------------	school	railway	transport	hospital	hospital
A travelling salesman problem can be solved using -----	Simplex method	Hungerian method	cutting method	Branch and Bound method	Branch and Bound method

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Class : I - M.Sc. Mathematics **Semester : II**

Unit II Dynamic Programming

Part A (20x1=20 Marks) (Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
The mathematical Techniques of optimizing such a sequence of interrelated decisions over a period of time is called------------------.	Integer programming problem	Dynamic programming problem	Non linear programming problem	Decision Analysis	Dynamic programming problem
The DP problem can be decomposed or divided into a sequence of smaller sub problems called-------------	State	Stage	Policy	Optimal policy	Stage
The variables which specify the condition of the decision process or describe the status of the system at a particular stage are called ---------------------	State variables	decision variables	independent variables	dependent variables	State variables
A decision making rule that at any stage permits a feasible sequence of decisions is called_____	State	Stage	Policy	Optimal policy	Policy
A transition function is expressed as -----------	$s_{n-1} = t_n (s_n , d_n)$	$s_n = t_n (s_{n-1} , d_{n-1})$	$s_{n+1} = t_n (s_{n+2} , d_{n+2})$	$s_{n-1} = t_n (s_{n-1} , d_{n-1})$	$s_{n-1} = t_n (s_n , d_n)$
A stage in a dynamic programming problem represents	number of decision alternatives	status of the system at a particular state	same time periods in the planning period	status of the system at a same state	number of decision alternatives
The return function in a DP model depends on --------	State	Stage	Policy	Optimal policy	State
Dynamic Programming Problem deals with the _____	multi stage decision making problems	single stage decision making problems	time independent decision making problems	problems which fix the levels of different decision variables so as to maximize profit or minimize cost	multi stage decision making problems
which of the following is not correct?	DP approach helps in reducing the computational efforts in sequential decision making	DPP can be divided into a sequence of smaller sub problems called stages of the original problem.	DP cannot be dealt with non linear constraints	The concept of dynamic programming is based upon the principle of optimality due to Bellman.	DP cannot be dealt with non linear constraints
When a positive quantity C is divided into five parts, the maximum value of their products is _______	$(C/5)^5$	$(5C)^5$	5x5C	5(C/5)	$(C/5)^5$
which of the following is not correct?	DPP is solved starting from the initial stage to the next till the final stage is reached	DPP can be solved by simplex method	Optimum solution is DPP depends on the initial solution.	Computation in DPP are done recursively, in the sense that the optimum solution of one sub problem is used as an input to the next sub problem.	DPP can be solved by simplex method
which of the following is correct?	DPP can be solved only by recursive equation approach	Monte - Carlo method is useful in solving DPP	LPP cannot be solved by using DP approach	In DP, when the current state is known, an optimum policy for the remaining stage is independent of the policy of the previous ones.	In DP, when the current state is known, an optimum policy for the remaining stage is independent of the policy of the previous ones.
The solution of recursive equation in DP does not involve the___________	backward computational procedure or forward computational procedure	two types of computations, according as the system is continuous or discrete	classical methods of optimization or tabular computational scheme to achieve the optimum solution	number os stages that provides an optimum solution or there is an indication of an unbounded solution.	number os stages that provides an optimum solution or there is an indication of an unbounded solution.
In DPP, the number of stage variable is ----------- the number of state variable	equal to	not equal to	greater than	lesser than	not equal to
The number of ----------- may change from the stage to stage	decision varaible	dependent variable	independent variables	state variables	state variables
DP divides the problem into a number of --------	confilcting objective function	decision stages	unrelated constraints	policies	decision stages
The relationship between stages of a DPP is called ----	State	random variable	node	transformation	transformation
In DP the output to stage n become the input to -----	stage n-1	stage n itself	stage n+1	stage n - 2	stage n-1
The decision criterion for the shortest route problem is ________.	minimization of the total number of cities visited	minimization of the distance traveled	minimization of the total cost	minimization of the route itself	minimization of the distance traveled
The decsion variables in a dynamic programming problem are ___________	the stages of the DPP	the returns at the stages of a DPP	the alternatives that exist at each stage of the DPP	the possible beginning situations of a stage	the alternatives that exist at each stage of the DPP
In dynamic programming, the output from a stage is called a ________	return	state variable	decision variable	transformation variable	state variable
The knapsack problem is an example of a __________problem that can be solved by dynamic programming	network	Monte Carlo	Infeasible	Non network	Non network
The equation describing the relationship between stages is called a __________	transformation function	relationship function	input - output function	inter stage function	transformation function
state variables, decision varibales, the dicision criterion and the optimal policy can be determined for ___________ of a dynamic programming problem	only stage 1	stages 1 and 2	stage n-1	any stage	any stage
_______ is an example of a problem to which dynamic programming might provide a solution	Determining the best route to travel through a city	Routing of emergency services	finding the optimal product mix	Finding the optimal adverting mix	Routing of emergency services

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Unit III Probabilistic Inventory Model

Part A (20x1=20 Marks) (Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
---------- may be defined as the stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business affairs	Inventory	Transportation	Queueing	Sequencing	Inventory
Rate of consumption is different from ---------.	rate of change	rate of production	rate of purchasing	either b or c	either b or c
Cost associated with carrying or holding the goods in stock is known as -------.	interested capital cost	handling cost	holding cost	production cost	holding cost
-------- is the interest change over the capital invested.	interested capital cost	handling cost	holding cost	production cost	interested capital cost
-------- include costs associated with movement of stock, such as cost of labour etc.	interested capital cost	handling cost	holding cost	production cost	handling cost
-------- per unit item is affected by the quantity purchased deu to quantity discounts or price breaks.	interested capital cost	handling cost	holding cost	purchase price	purchase price
If P is the purchase price of an item and I is the stock holding cost per unit time expressed as a fraction of stock value then the holding cost is ----------.	I/P	I + P	I – P	IP	IP
The penalty costs that are incurred as a result of running out of stock are known as----	shortage cost	set-up cost	holding cost	production cost	shortage cost
Holding cost is denoted by -------	C_1	C_2	C_3	C_4	C_1
Shortage cost is denoted by ------	C_1	C_2	C_3	C_5	C_2
Set-up cost is denoted by --------	C_1	C_2	C_3	C_4	C_3
Elapsed time between the placement of the order and its receipts in inventory is known as -------.	lead time	recorder level	EOQ	variables	lead time
---------- is the time when we should place an order by taking into consideration the interval between placing the order and receiving the supply.	lead time	recorder level	EOQ	variables	recorder level
--------- is that size of order which minimises total annual cost of carrying inventory and the cost of ordering under the assumed conditions of certainty and that annual demands are known.	lead time	recorder level	EOQ	variables	EOQ
EOQ means -------	Economic Order Quantity	Economic Order Quality	Economic Offer Quality	Economic Offer Quantity	Economic Order Quantity
EOQ is also known as -------	economic lot size formula	economic short size formula	economic formula	economic variables	economic lot size formula
------- include holding cost, set up cost, shortage costs and demand.	EOQ	controlled variables	uncontrolled variables	basic variables	uncontrolled variables
Reduction in procurement cost ------- EOQ	increases	decreases	reduces	neutral	reduces
Economic order quantity results in equilization of -------- cost and annual inventory cost.	annual procurement cost	procurement cost	inventory cost	shortage cost	annual procurement cost
Economic order quantity results in equilization of annual procurement cost cost and ------- cost.	annual inventory cost	procurement cost	inventory cost	shortage cost	annual inventory cost
Reorder level = ---------	normal lead time x monthly consumption	normal lead time + monthly consumption	normal lead time - monthly consumption	normal lead time / monthly consumption	normal lead time x monthly consumption
Economic order quantity results in -------	reduced stock – outs	increased stock – outs	equilisation of carrying cost and procurement costs	favourable procurement price	equilisation of carrying cost and procurement costs
Minimum inventory equals	EOQ	Reorder level	Safety stock	lead time	Safety stock
The set up cost in inventory situation is ------- of size of inventory.	dependent	independent	large	small	independent
Total inventory cost = -------	set up cost + purchasing cost	holding cost + shortage cost	set up cost + purchasing cost + holding cost + shortage cost	setup cost + shortage cost	set up cost + purchasing cost + holding cost + shortage cost
Storage cost is associated with -------	holding cost	shortage cost	carrying cost	set up cost	carrying cost
Average inventory = -------	(EOQ/2) + Safety stock	(EOQ/2) - Safety stock	(EOQ/2) / Safety stock	(EOQ/2) * Safety stock	(EOQ/2) + Safety stock
------ discounts reduce material cost and procurement costs	quantity	quality	carrying cost	set up cost	quantity
The ordering cost is independent of ------	ordering quantity	ordering quality	carrying cost	set up cost	ordering quantity
Inventory used for decoupling is used to ________	Make businesses wait longer for products	make one part of a businessless directly dependent on other parts of the businedd	Reduce overall inventory levels	Provide JIT deliveries	make one part of a businessless directly dependent on other parts of the businedd
An example of dependent demand inventory would be _______	Milk in a grocery store	Finished products in a toy factory's warehouse	Silos full of gaint at a brewery	Lubricants used to maintain a manufacturer's machinery	Silos full of gaint at a brewery
A key difference between independent and dependent demand inventory is ___________	Independent demand is probabilistic, while dependent demand can be calculated precisely	Dependent demand is probilistic, while independent demand can be calculated precisely.	dependent demand is always lower than independent demand.	Dependent demand is more likely to result in forecast error	Independent demand is probabilistic, while dependent demand can be calculated precisely

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Unit IV Decision Analysis

Part A (20x1=20 Marks) (Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
Once a decision tree has been drawn it should be ______	Compared to a payoff table	Analyzed from right to left	Compared to sensitivity analysis	Analyzed from left to right	Analyzed from right to left
A decision tree is used when ________	Non-sequential dec	Only with monetary v	The decision is a sensitiv	Sequential decisions are involved	Sequential decisions are involved
A graph is used to depict ________	Sensitivity analysis	Payoff tables	Decision trees	The expected monetary value	Sensitivity analysis
The several alternatives available are called --------	Acts	payoff	state of nature	outcome	Acts
Conditionsoutside the control of the decision maker which will determine the consequence of a particular act is -----------	Acts	payoff	state of nature	outcome	state of nature
The consequence of a decision taken against a state of nature is defined as an -----------	Acts	payoff	state of nature	outcome	outcome
If the outcome is measured interms of money it is called a ----------------	Acts	payoff	state of nature	outcome	payoff
___________provides a way of incorporating the decision makers attitude towards risk into the analysis	Utility theory	Game theory	decision theory	queuing theory	Utility theory
A utility function is said to be --------, if there exist a measure of utility for all alternatives.	Complete	compact	certinity	uncertainty	Complete
_____________is indicated interms of propabilities assigned to events	certainty	uncertainty	risk	strategy	uncertainty
A situation in which a decision maker knows all of the possible outcomes of a decision and also knows the probability associated with each outcome is referred to as _______	certainty	risk	uncertainty	strategy	risk
Which of the following methods of selecting a strategy is consistent with risk averting behavior?	If two strategies have the same expected profit, select the one with the smaller standard deviation.	If two strategies have the same standard deviation, select the one with the smaller expected profit	Select the strategy with the larger coefficient of variation	All of the above are correct	If two strategies have the same expected profit, select the one with the smaller standard deviation.
Which one of the following does measure risk?	Coefficient of varianc	Standard deviation	Expected value	Expected variance	Expected value
If a person's utility doubles when their income doubles, then that person is risk ________	averse	neutral	seeking	There is not enough information	neutral
The coefficient of variation measures ________	the risk per unit of expected payoff	the risk-adjusted expected value	the payoff per unit of risk	a decision maker's risk-return tradeoff	the risk per unit of expected payoff
A situation in which a decision maker must choose between strategies that have more than one possible outcome when the probability of each outcome is unknown is referred to as ________	diversification	certainty	risk	uncertainty	uncertainty
If a decision maker is risk averse, then the best strategy to select is the one that yields the ___________	highest expected payoff	lowest coefficient of variation	highest expected utility	lowest standard deviation	highest expected utility
Circumstances that influence the profitability of a decision are referred to as ________	strategies	a payoff matrix	states of nature	the marginal utility of money	states of nature
The marginal utility of money diminishes for a decision maker who is ________	a risk seeker	risk neutral	a risk averter	ina situation of uncertainty.	a risk averter
A payoff matrix presents all the information required to determine the optimal strategy using the ________	expected value criterion	the maximin criterion	the utility maximization criterion	simulation criterion	the maximin criterion
A matrix that, for each state of nature and strategy, shows the difference between a strategy's payoff and the best strategy's payoff is called _______	a maximin matrix	a minimax regret matrix	a payoff matrix	an ecpected utility matrix	a minimax regret matrix
The sequence of possible managerial decisions and their expected outcome under each set of circumstances can be represented and analyzed by using _________	the minimax regret criterion	a decision tree	a payoff matrix	simulation criterion	a decision tree
Which one of these refers to decision making under risk with additional information?	EVPI	EMV	Maximax	Maximin	EVPI
The expected monetary value approach is most appropriate when the decision maker is _______	risk averse	risk seeking	risk neutral	risk free	risk neutral
States of nature ____________	can describe uncontrollable natural events such as floods or freezing temperatures	can be selected by the decision maker	cannot be enumerated by the decision maker	can not describe uncontrollable natural events	can describe uncontrollable natural events such as floods or freezing temperatures
A payoff ________	is always measured in profit	is always measured in cost	exists for each pair of decision alternative and state of nature	exists for each state of nature	exists for each pair of decision alternative and state of nature
A decision tree ___________	presents all decision alternatives first and follows them with all states of nature	presents all states of nature first and follows them with all decision alternatives	alternates the decision alternatives and states of nature	arranges decision alternatives and states of nature in their natural chronological order	arranges decision alternatives and states of nature in their natural chronological order
. Which of the methods for decision making without probabilities best protects the decision maker from undesirable results?	the optimistic approach	the conservative approach	minimum regret	minimax regret	the conservative approach
Which of the following criteria is not used for decision making under uncertainty?	Maxmin	Maximax	Minimax	Minimize expected loss	Minimize expected loss
Which of the following criteria is not applicable to decision making under risk?	Maximize expected r	Maximize return	Minimize expected regret	knowledge of likelihood occurance of each state of nature	knowledge of likelihood occurance of each state of nature
The minimum expected opportunity loss (EOL) is _________	Maximum regret	Minimum regret	equal to EMV	Minimum and maximum regret	equal to EMV
The value of the coefficient of optimism is needed while using the criterion of _________	equally likely	Maximin	realism	minimax	realism
The decision maker's knowledge and experience may influence the decision making process when using the criterion of _________	Maximax	Minimax	Maximin	realism	realism
Which of the following refers to the random(chance) occurance that can affect the outcome of an individual's decision?	payoff	states	nature	decision criteria	states
A graphical method of representing events and course of action may be referred to as _________	decision tree	decision criterion	payoff table	strategy	decision tree
What will you obtain when you sum the profit for each combination of a state and action times the probability of occurance of the state?	Expected monetary value	Expected opportunity loss	Expected value of perfect information	Expected value strategy	Expected monetary value
The utility curve that shows greater utility for larger dollar amounts would best represents a _________	risk averter	risk netural person	risk seeker	There is not enough information	risk seeker
The utility curve that shows each additional dollar of profit yielding the samr value as the previous dollar would best represents a _________	risk averter	risk netural person	risk seeker	There is not enough information	risk netural person
Which of the following is correct?	Every decision making situation involves an equal number of course of actions and events	Opportunity loss in the payoff matrix is also called conditional payoff	Regret is the amount of money suffered for not adopting the optimal course	Events are also called states of nature	Regret is the amount of money suffered for not adopting the optimal course
Which of the following is correct?	A conditional payoff table always involves orifits	Different criteria for making decision under risk would always lead to the same optimal choice	Laplace principle is based on the premise ofequally likely occurance of possible events	Minimax is an optimist's choice while minimum is a pessimist criterion	Laplace principle is based on the premise ofequally likely occurance of possible events
In _____ approach to decision making , the optimum strategy is determined using posterion probabilities	Laplace	Bayesian	Dymnamic	Simplex	Bayesian
__________ is used when sequential decision making is involved.	Laplace	Bayesian	Dymnamic	Decision tree	Decision tree
The EVSI is the _________	Expected value of strategic information	Expected value of static information	Expected value of state information	Expected value of stage information	Expected value of strategic information
In constructing a tree diagram, decision point is denoted by _________	Circle	Square	Triangle	Dot	Square
In constructing a tree diagram, event is denoted by _________	Circle	Square	Triangle	Dot	Circle

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Subject: OPTIMIZATION TECHINIQUES **Subject Code: 17MMP203**
Class : I - M.Sc. Mathematics **Semester : II**

Unit V Non-linear Programming Methods

Part A (20x1=20 Marks) (Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
A NLP problem with non linear objective function ans linear constraints such as NLP is called -----------	Dynamic Programming	Quadratic programming	Geometric Programming	Separable programming	Quadratic programming
In case of maximization of NLPP all the constraints must be converted into -------- type	strictly less than	less than or equal to	strictly greater	greater than or equal to	less than or equal to
In case of minimization of NLPP all the constraints must be converted into -------- type	strictly less than	less than or equal to	strictly greater	greater than or equal to	greater than or equal to
If the principle minor of bordered ---------- is positive, the objective function is convex	Hessian matrix	Hermitan matrix	diagonal matrix	identity matrix	Hessian matrix
The objective function is _______, if the principle minors of borded Hessian matrix alternate in sign, beginning with the negative sign.	Convex	Concave	both convex and concave	neither convex nor concave	Concave
A NLPP is solved using _________	Lagrange multipliers	Simplex method	Dual simplex method	Degeneracy multipliers	Lagrange multipliers
A ________of a cenvex function on a convex set is a unique global minimum of that function.	Local minimum	Local maximum	weak minimum	weak maximum	Local minimum
A local minimum of a cenvex function on a convex set is a unique _______ of that function.	Local minimum	Local maximum	global minimum	global maximum	global minimum
The method of ______multipliers is a systematic way of generating the necessary condition for a stationary point	Lagrange	Cauchy's	Euler's	Fourier	Lagrange
The necessary conditions become sufficient condition for a maximum if the objective function is _________	Convex	Concave	both convex and concave	neither convex nor concave	Concave
The necessary conditions become sufficient condition for a minimum if the objective function is _________	Convex	Concave	both convex and concave	neither convex nor concave	Convex
The sign pattern being alternate the stationary point is a _________	Local minimum	Local maximum	global minimum	global maximum	Local maximum
The sign pattern being always negative, the stationary point is _________	Local minimum	Local maximum	global minimum	global maximum	Local minimum
X^TQX represents a ___________	Quadratic form	Normal form	standard form	canonical form	Quadratic form
X^TQX is said to be _______ if $X^TQX > 0$ for $x \neq 0$	Positive definite	negative definite	Positive semi definite	negative semi definite	Positive definite
X^TQX is said to be _______ if $X^TQX < 0$ for $x \neq 0$	Positive definite	negative definite	Positive semi definite	negative semi definite	negative definite
X^TQX is said to be _______ if $X^TQX \geq 0$ for $x \neq 0$	Positive definite	negative definite	Positive semi definite	negative semi definite	Positive semi definite
X^TQX is said to be _______ if $X^TQX \leq 0$ for $x \neq 0$	Positive definite	negative definite	Positive semi definite	negative semi definite	negative semi definite
If X^TQX is positive semi definite then it is ________	Convex	Concave	Strictly convex	Strictly concave	Convex
If X^TQX is negative semi definite then it is ________	Convex	Concave	Strictly convex	Strictly concave	Concave
If X^TQX is positive definite then it is ________	Convex	Concave	Strictly convex	Strictly concave	Strictly convex
If X^TQX is negative definite then it is ________	Convex	Concave	Strictly convex	Strictly concave	Strictly concave
The iterative procedure for the solution of a quadratic programming problem by ________	Beale's method	Fletcher's method	Wolfe's modified simplex method	Frank - wolfe's method	Wolfe's modified simplex method
In _________ method, at each iteration the objective function is expressed interms of only the non basic variables	Beale's method	Fletcher's method	Wolfe's modified simplex method	Frank - wolfe's method	Beale's method
In Beale's method, at each iteration the objective function is expressed interms of only the _________ variables	basic	non basic	decision	free	non basic
In Beale's method, we introduce an additional non basic variables is called_______	basic variables	state variables	decision variables	free variables	free variables
In an NLPP, if the objective function is _______, the Kuhn - Tucker conditions are sufficient conditions for a absolute maximum	Convex	Concave	Strictly convex	Strictly concave	Strictly concave
In an NLPP, if the objective function is Strictly concave, the Kuhn - Tucker conditions are sufficient conditions for a __________	local maximum	weak maximum	global maximum	absolute maximum	absolute maximum
The general NLPP with inequality constraints are usually solved using _____	Lagrange condition	Kuhn - Tucker condition			Kuhn - Tucker condition
which of the following methods of solving a QPP is based on modified simplex method?	Beale's method	Fletcher's method	Wolfe's method	Frank - wolfe's method	Wolfe's method
In _______ method , the technique involves partitioning of the variables into basic and non basic pools and then using results from calculus	Beale's method	Fletcher's method	Wolfe's method	Frank - wolfe's method	Beale's method
An optimum solution to an NLPP involving only two decision variables can conveniently be determined by ________	Simplex method	dual simplex method	Big - M method	graphical method	graphical method
Quadratic programming concerned with the NLPP of optimizing the quadratic objective function subject to _____	Linear inequality constraints	Non linear inequality constraints	Non linear equality constraints	No constraints	Linear inequality constraints
If X^TQX is ___________ then it is convex	Positive definite	negative definite	Positive semi definite	negative semi definite	Positive semi definite
If X^TQX is __________ then it is concave	Positive definite	negative definite	Positive semi definite	negative semi definite	negative semi definite
If X^TQX is __________ then it is strictly convex	Positive definite	negative definite	Positive semi definite	negative semi definite	Positive definite
If X^TQX is _________ then it is strictly concave	Positive definite	negative definite	Positive semi definite	negative semi definite	negative definite
X^TQX is said to be positive definite if ________ for $x \neq 0$	$X^TQX > 0$	$X^TQX < 0$	$X^TQX \geq 0$	$X^TQX \leq 0$	$X^TQX > 0$
X^TQX is said to be negative definite if _______for $x \neq 0$	$X^TQX > 0$	$X^TQX < 0$	$X^TQX \geq 0$	$X^TQX \leq 0$	$X^TQX < 0$
X^TQX is said to be positive semi definite, if ________for $x \neq 0$	$X^TQX > 0$	$X^TQX < 0$	$X^TQX \geq 0$	$X^TQX \leq 0$	$X^TQX \geq 0$
X^TQX is said to be negative semi definite if _________ for $x \neq 0$	$X^TQX > 0$	$X^TQX < 0$	$X^TQX \geq 0$	$X^TQX \leq 0$	$X^TQX \leq 0$
The stationary point is a local maximum, the sign pattern being ________	Positive	negative	alternate	zero	alternate
The stationary point is a local minimum, the sign pattern being always ________	Positive	negative	alternate	zero	negative
__________ is concerned with the NLPP of optimiing the objective function subject to linear inequality constriants	Quadratic programming	Dynamic Programming	separable programming	Geometric programming	Quadratic programming
The necessary conditions become sufficient condition for a ________ if the objective function is concave	Maximum	Minimum	both maximum and minimum	neither maximum nor minimum	Maximum
The necessary conditions become sufficient condition for a ________ if the objective function is convex	Maximum	Minimum	both maximum and minimum	neither maximum nor minimum	Minimum
If the principle minors of borded Hessian matrix is ________, the objective function is convex	Positive	negative	zero	non zero	Positive
If the principle minors of borded Hessian matrix is positive, the objective function is ________	Convex	Concave	Strictly convex	Strictly concave	Convex
In ________ we introduce an additional non basic variables is called free variables	Beale's method	Fletcher's method	Wolfe's method	Frank - wolfe's method	Beale's method