KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

		L	T	P	C
17MMP211	**OPTIMIZATION TECHNIQUES - PRACTICAL**	**0**	**0**	**4**	**2**

List of Practical:

1. Solution for a system of equations- Simplex method.
2. Decision Making with minimax criteria.
3. Decision Making under risk.
4. Travelling salesman problem to find the shortest path.
5. Write a C program to calculate the minimum cost using North West Corner Rule.
6. To calculate the EOQ for purchasing model without shortage using C program.
7. To calculate the EOQ for manufacturing model without shortage using C program.
8. To calculate the EOQ for manufacturing model with shortage using C program.
9. To calculate the EOQ for purchasing model with shortage using C program.
10. Probabilistic Model-EOQ.

Content

EX.NO:1	FAMILY OF DIFFERENTIAL EQUATIONS

AIM:

To write a c program to find the solution for LPP using simplex method.

ALGORITHM:

STEP 1: Start the program.

STEP 2: Declare the variable required for the program.

STEP 3: Print the maximum and minimum choice.

STEP 4: Get the pivotal row and column and pivotal element to find the solution.

STEP 5: Find the new equation for s1 and s2.

STEP 6: The requesting is printout.

STEP 7: Display the result.

STEP 8: Stop the process.

PROGRAM:

```
#include<stdio.h>

#include<conio.h>

float a[10][10]={0},b[10],d[10],x[10][3]={3};

int m,n,s=1;

void main()

{

int i,j,m1,n1,c[10]={0};
```

```
float m2,d[10],s=0;
void table();
clrscr();
printf("\n \t \t ***SIMPLEX METHOD***\n ");
printf("\n 1.maximum \n 2.minimum \n choice");
scanf("%d",&m1);
printf("enter the coefficient in the main equation:");
scanf("%d",&n);
printf("enter the coefficient:");
for(i=1;i<=n;i++)
{
scanf("%d",&c[i]);
if(m1==2)
c[i]=-1*c[i];
}
printf("\n enter the number of constraints:");
scanf("%d",&m);
printf("\n enter the coefficient one by one:");
for(i=1;i<=m;i++)
{
printf("enter the coeffient of the constraints %d:",i);
for(j=1;j<=n;j++)
scanf("%f",&a[i][j]);
printf("enter the contant:");
```

```
scanf("%f",&a[i][0]);
if((d[i]!=0)&&(d[j]>0))
if(d[j]<m2)
{
m2=d[j];
n1=j;
}
}
m2=a[n1][i];
printf("\n pivotal column:y %d",i);
printf("\n pivotal row: %d",n1);
printf("\n pivotal element: %3.2f",m2);
getch();
for(j=0;j<=m+n;j++)
a[n1][j]=a[n1][j]/m2;
m2=i;
x[n1][0]=c[m2];
x[n1][1]=m2;
for(i=1;i<m;i++)
if(n1!=i)
{
s=a[i][m2];
for(j=0;j<=m+n;j++)
a[i][j]=a[i][j]-(s*a[i][j]);
```

```
}
goto line;
{
if(m1==m2)
b[0]=-1*b[0];
for(i=1;i<=m;i++)
x[i][2]=a[i][0];
printf("\n when");
for(i=1;i<=m;i++)
{
if(x[i][j]<=n)
printf("\n \t x%1.0f = 3.3f",x[j][i],x[1][2]);
}
printf("\n \n \n z=\t %5.3f",b[0]);
line;
getch();
}
int s1,s2;
clrscr();
printf("\n table %d \n ",s);
s++;
line();
printf("\n <b \t \t x");
for(s1=1;s1<=m;s1++)
```

```
{
printf(“\n \n %1.0f \t ”x[s1][0],x[s1][1]);
for(s2=0;s2<=m+n;s2++)
printf(“%2.1f”,a[s1][s2]);
}
printf(“\n”);
line();
printf(“\n \t z \t ”);
for(s1=0;s1<=m+n;s1++)
printf(“%2.1f”,b[s1]);
}
void line();
int s1;
for(s1=1;s1<=(m+n+3)*7;s1++)
printf(“*”);
}
```

OUTPUT:

EX.NO:2	DECISION MAKING WITH MINIMAX CRITERIA

QUESTION:

Find decision making under risk

AIM:

To Write a program to find decision making under risk.

ALGORITHM:

STEP 1: Start the process.

STEP 2: Include the necessary header file.

STEP 3: Declare the variable in intdatatype.

STEP 4: Print the row,column and matrix.

STEP5: Using the for loop statement,

for(i=1;i<=n;i++)

for(j=1;j<=m;j++)

STEP 6: Print the greatest value first and second rows.

STEP 7: Find the minimax value.

STEP 8: Stop the process.

PROGRAM:

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
void main()
{
int i,j,m,n,a[10][10],p,q;
clrscr();
printf("\n enter the number of rows and columns:");
scanf("%d %d",&m, &n);
printf("\n enter the matrix:");
{
for(i=1;i<=m;i++)
for(j=1;j<=n;j++)
scanf("%d",&a[i][j]);
}
printf("\n enter the greatest value of 1st row p:");
scanf("%d",^&p);
printf("\n enter the greatest value of 2nd row q:");
scanf("%d",&q);
if(p<q)
{
printf("\n the minimax value is:%d",p);
}
```

```
else
{
printf("the minimax value is:%d",q);
}
getch();
}
```

OUTPUT

EX.NO:3	DECISION MAKING UNDER RISK

QUESTION:

Find decision making under risk

AIM:

Write a C program to find decision making under risk.

ALGORITHM:

STEP 1: Start the process.

STEP 2: Declare the necessary header file.

STEP 3: Declare the variable.

STEP 4: Calculate the Ua,Pa,Pb,Ub,P1,P2 using this formula

P1=Ua*Pa;

P2=Ub*Pb;

STEP 5: Display the result.

STEP 6: Stop the process.

PROGRAM:

```
#include<stdio.h>

#include<conio.h>

#include<math.h>

void main()
```

```
{
float d,ed,c1,c3,q;
clrscr();
printf("\n ***INVENTORY CONTROL***\n");
printf("\n enter the setup cost c3=");
scanf("%f",&c3);
printf("\n enter the demand d=");
scanf("%f"&d);
printf("\n enter the carrying cost c1=");
scanf("%f",&c1);
printf("\n purchasing problem without shortage");
ed=(2*d*c3);
q=sqrt(ed/c1);
printf("\n the economic quantity= %f",q);
getch();
}
```

EX.NO:4	ASSIGNMENT PROBLEM

QUESTION:

Find maximum cost using assignment problem.

AIM:

To write a C program to find maximum cost using assignment problem.

ALGORITHM:

STEP 1: Start the process.

STEP 2: Declare the variables required for the program.

STEP 3: Get the value of last matrix and assign the matrix do other temporary matrix to noted.

STEP 4: Find the minimum value each row and column find the row minimum matrix displays it

STEP 5: Do the allocation in the result and matrix corresponding allocation in the cost matrix is added.

STEP 5: Stop the process.

PROGRAM:

```
clear;
L=1.0;
W=4.0;
T=10.;
k=200;
dt=T/k;
n=10.;
dx=L/n;
m=20.;
dy=W/m;
velx=.1;
vely=.4;
decay=.0;
for i=1:n+1
x(i)=(i-1)*dx;
for j=1:m+1
y(j)=(j-1)*dy;
u(i,j,1)=0.;
end
end
for k=1:k+1
time(k)=(k-1)*dt;
for j=1:m+1
```

```
u(1,j,k)=.0;
end
for i=1:n+1
u(i,1,k)=(i<=(n/2+1))*(k<26)*5.0*sin(pi*x(i)*2)+(i>(n/2+1))*.1;
end
end
for k=1:k
for i=2:n+1;
for j=2:m+1;
u(i,j,k+1)=(1-velx*dt/dx-vely*dt/dy-decay*dt)*u(i,j,k)+velx*dt/dx*u(i-1,j,k)+vely*dt/dy*u(i,j-1,k);
end
end
end
mesh(x,y,u(:,:,k)')
```

EX.NO:5	NORTH WEST CORNER RULE

QUESTION:

Use c program to find the solution of north west corner rule.

AIM:

To write a c program using north west corner rule.

ALGORITHM:

STEP 1: Start the process.

STEP 2: Declare the variable. Get the number of rows and columns of the matrix .

STEP 3: Using loop increment I and j values oneby one and get the matrix a[i][]j.

STEP 4: Using loop I values one by one and get the availability ofthe matrix ava[i].

STEP 5: Initialize n1,m1,m2 equal to zero.

STEP 6: Check whether req[j]>ava[i];

STEP 7: Initialize tp[m2][i]=ava[i];

Tp[m2][0]=a[i][j];

Calculate req[j]=req[i]-ava[i];

elseava[i]=ava[j]-req[j];

STEP 8: Terminate the program.

PROGRAM:

```
function V=elongation(t)
%Function elongation has input variable
 t and output variable V
%It gives the bacterium volume after
 time t: V=0.4+0.02*t
V=0.4+0.02*t;
elongation(4)
```

EX.NO:6	PURCHASING PROBLEM WITHOUT SHORTAGE

QUESTION:

Compute the purchasing problem without shortage.

AIM:

To write a C program to find purchasing problem without shortage.

ALGORITHIM:

STEP1: Start the process.

STEP2: Include necessary header file.

STEP3: Declare a float value q,d,c1,ed,c3.

STEP4: Find the value of ed,is calculate ed=(2*d*c3).

STEP5: Find the sqrt(ed/c1).

STEP6: Print the result.

STEP7: Stop the process.

PROGRAM:

```
#include<stdio.h>

#include<conio.h>

#include<math.h>

void main()

{

float d,c1,eq,c2,,k,q1,c3,q;

clrscr();

printf("\n ***INVENTORY CONTROL***\n");

printf("\n enter the setup cost c3=");

scanf("%f",&c3);

printf("\n enter the demand d=");

scanf("%f"&d);

printf("\n enter the carrying cost c1=");

scanf("%f",&c1);

printf("\n enter the production rate k=");

scanf("%f",&k);

printf("\n manufacturing problem without shortage");

q=(2*d*c3)/c1;

q1=(k/(k-d));

eq=sqrt(q*q1);

printf("\n the economic quantity= %f",eq);

getch();

}
```

EX.NO:7	**MANUFACTURING PROBLEM WITH SHORTAGE**

QUESTION:

Calculate EOQ manufacturing problem without shortage using C program

AIM:

To calculate EOQ manufacturing problem without shortage using C program..

ALGORITHIM:

STEP1: Start the process.
STEP2: Include necessary header file.
STEP3: Declare the variables.
STEP4:.Calculate q1,q,eq using the formula
q =(2*d*c3)/c1;
q1 =(k-/(c-d));
eq=srqt(q*q1);
STEP6: Print the eoq.
STEP7: Display the result.
STEP7: Stop the process.

PROGRAM:

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
void main()
{
float d,c1,eq,c2,,k,q1,c3,q,q2;
clrscr();
printf("\n ***INVENTORY CONTROL***\n");
printf("\n enter the setup cost c3=");
scanf("%f",&c3);
printf("\n enter the demand d=");
scanf("%f"&d);
printf("\n enter the carrying cost c1=");
scanf("%f",&c1);
printf("\n enter the production rate k=");
scanf("%f",&k);
printf("\n enter the shortage cost c2=");
scanf("%f",&c2);
printf("\n manufacturing problem with shortage");
q=(2*d*c3)/c1;
q1=(c1+c2)/c2;
```

```
q2=(k/(k-d));

eq=sqrt(q*q1*q2);

printf("\n the economic quantity= %f",eq);

getch();

}
```

OUTPUT:

EX.NO:8	PURCHASING PROBLEM WITH SHORTAGE

QUESTION:

Calculate EOQ for purchasing model with shortage using C program

AIM:

To calculate EOQ for purchasing model with shortage using C program.

ALGORITHIM:

STEP1: Start the process.

STEP2: Include necessary header file.

STEP3: Declare a float value q,d,c1,c2,q1,c3 in type of float.

STEP4: Print one setup cost,demand,carryingcost,shortage cost.

STEP5: Calculate q1,q and eoq using the formula

q =(2*d*c3)/c1.

q1 =(c1*c2)/c2.

STEP6: Print the eoq and display the result.

STEP7: Stop the process.

PROGRAM:

```
#include<stdio.h>

#include<conio.h>

#include<math.h>

void main()

{

float d,c1,eoq,c2,q1,c3,q;

clrscr();

printf("\n ***INVENTORY CONTROL***\n");

printf("\n enter the setup cost c3=");

scanf("%f",&c3);

printf("\n enter the demand d=");

scanf("%f"&d);

printf("\n enter the carrying cost c1=");

scanf("%f",&c1);

printf("\n enter the shortage cost c2=");

scanf("%f",&c2);

printf("\n purchasing problem with shortage");

q=(2*d*c3)/c1;

q1=(c1+c2)/c2;

eoq=sqrt(q*q1);

printf("\n the economic quantity= %f",eoq);

getch();

}
```

EX.NO:9	PROBABILISTIC MODEL

QUESTION:

Find a probabilistic model using C program .

AIM:

To write a C program to find a probabilistic model.

ALGORITHIM:

STEP1: Start the process.

STEP2: Include necessary header file.

STEP3: Declare the variables.

STEP4:.Calculate c1,c2,c3,p using the formula

c2=c3-c1;

p=c2/(c1+c2);

STEP5: Display the result.

STEP6: Stop the process.

```
#include<conio.h>
#include<math.h>
void main()
{
float c1,c2,c3,p;
clrscr();
printf("\n enter the holding cost c1=");
scanf("%f",&c1);
printf("\n enter the selling cost c3=");
scanf("%f",&c3);
c2=c3-c1;
printf("\n enter the carrying cost:%f",c2);
p=c2/(c1+c2);
printf("\n enter the probabilistic eoq is %f",p);
getch();
}
```

OUTPUT :

EX.NO:9	DECISION MAKING UNDER RISK

```
#include<stdio.h>
#include<conio.h>
#include<math.h>
void main()
{
float ua,ub,pa,pb,p1,p2;
clrscr();
printf("\n enter the utitlity value ua:");
scanf("%f",&ua);
printf("\n enter the probability value pa:");
scanf("%f",&pa);
p1=ua*pa;
printf("\n the expected utitlity is:%f",p1);
printf("\n enter the utitlity value is:%f:",p1);
scanf("%f",&ub);
printf("\n enter the probability value pb:");
scanf("%f",&pb);
printf("\n the expected utitlity is:%f",p2);
if(p1>p2)
{
```

```
printf("\n p1 is the best choice invested:");
}
Else
{
printf("\n p2 is the best choice invested:");
}
getch();
}
```

OUTPUT :

Reg. No ----------------

(17MMP206)

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE-21

DEPARTMENT OF MATHEMATICS

Second Semester

Fluid Dynamics

I Internal Test - Jan'2018

Date : 03 .02.2018 (FN) **Time : 2 Hours**

Class : I M. Sc Mathematics **Maximum: 50 Marks**

PART - A (20 x 1 = 20 Marks)

Answer all the questions:

1. The behaviour of fluid when it is in motion without considering the pressure force is called___________.
   a) fluid kinematics b) fluid mechanics
   c) fluid statics d) fluids
2. If any material deformation vanishes when a force applied withdrawn a material is said to be_________.
   a) elastic b) plastic
   c) deformation d) fluid
3. The _________ can be classified as liquids and gases.
   a) solid b) pressure
   c) fluid d) force
4. The density of fluids is defined as _________ volume.
   a) limit per unit b) solid per unit
   c) forces per unit d) mass per unit
5. A force per unit area is known as ____________.
   a) force b) pressure
   c) fluid d) density
6. The pressure changes in the fluid beings changes in the density of fluid is called__________.
   a) compressible fluid b) incompressible fluid
   c) body force d) surface force
7. The _________ are proportional to mass of the body.
   a) pressure b) body force
   c) surface force d) force
8. The tangential force per unit area is said to be ____________.
   a) normal stress b) stress
   c) shearing stress d) strain
9. The differential equation of the path line is__________.
   a) .u=dy/s b) v=dx/w
   c) q=s/r d) q=dr/dt
10. A flow in which each fluid particle posses different velocity at each section of the pipe are called_________.
    a) non uniform flow b) uniform flow
    c) barotropic flow d) rotational flow
11. A stream tube of an infinitesimal cross sectional area is called__________.
    a) stream line b) stream filament
    c) path line d) stream tube
12. When the flow is_______ the stream line changes from instant to instant.
    a) non uniform b) steady
    c) unsteady d) uniform
13. A force is said to be _________ if the force can be derivable from the potential.
    a) conservative b) non conservative
    c) acceleration d) surface
14. A flow is called a Beltrami's flow when_________.
    a) q.E=0 b) q+E=0
    c) q\E=0 d) q*E=0
15. Bernoulli's equation occurs when the motion is ___________.
    a) rotational b) irrotational
    c) steady d) unsteady
16. The __________ flow can occurs when the vertex and stream lines coincide.
    a) viscous flow b) beltrami's flow
    c) invisid flow d) normal flow

17. The product of the cross sectional area and magnitude of the vorticity is ________________ along a vortex filament

a) constant b) zero

c) parallel d) normal

18. When the forces are conservative and the pressure is a function of the density, then ______________.

a) $\nabla\backslash a = 0$ b) $\nabla * a = 0$

c) $\nabla + a = 0$ d) $\nabla . a = 0$

19. When a force is conservative, there exist a potential Ω such that f= ___.

a) $f = -\nabla\Omega$ b) $f = \nabla * \Omega$

c) $f = \nabla\Omega$ d) $f = \nabla + \Omega$

20. The pressure is function of density then the flow is said to be ______.

a) non uniform flow b) uniform flow

c) barotropic flow d) rotational flow

PART – B (3 x 2 = 6 Marks)

Answer all the questions:

21. Define Laminar flow.
22. Write about streak lines.
23. Derive the equation to the vortex line.

PART – C (3 x 8 =24 Marks)

Answer all the questions:

24. a) Obtain the differential equation of a stream line.

(OR)

b) Show that in a 2D incompressible steady flow fluid the equation of continuity is satisfied with the velocity components in a rectangular co-ordinates given by $u(x,y) = \frac{k(x^2-y^2)}{(x^2+y^2)^2}$, $v(x,y) = \frac{2kxy}{(x^2+y^2)^2}$ where k is an arbitrary constant.

25. a) Derive the equation of continuity.

(OR)

b) Derive the Euler's equation of motion.

26. a) State and prove the Euler's generalized momentum theorem.

(OR)

b) Derive the Energy equation.

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

LECTURE PLAN
DEPARTMENT OF MATHEMATICS

STAFF NAME: A.HENNA SHENOFER
SUBJECT NAME: FLUID DYNAMICS SUB.CODE:17MMP206
SEMESTER: II CLASS: I M.SC MATHEMATICS

S.No	Lecture Duration Period	Topics to be Covered	Support Material/Page Nos
		UNIT-I	
1	1	Introduction to fluid dynamics	T1:1-3
2	1	Basic concepts of fluid dynamics, viscosity, compressible and non compressible fluids	T1:3-8
3	1	Stream surface, tube filament, streak lines, path lines	R2:1.5-1.7
4	1	Problems on path lines	R2:1.7-1.9
5	1	Geometrical significance of velocity, problems on rotational and irrotational flow	T1:65-68
6	1	Theorem on equation of continuity	T1:68-73
7	1	Conservation of mass	T1:74-75
8	1	Boundary conditions	T1:75-76
9	1	Continuation of boundary conditions	T1:77
10	1	Theorems on rate of change of linear momentum	T1:77-79
11	1	Equation of motion of an inviscid fluid	T1:79-80
12	1	Recapitulation and discussion	

		on possible questions	
	Total No of Hours Planned For Unit 1=12		
		UNIT-II	
1	1	Euler's equation of motion interms of vorticity	T1:80-81
2	1	Euler's momentum theorem	T1:81-82
3	1	Equations of motion	T1:106-108
4	1	Theorem on equations of motion interms of vorticity	T1:108-110
5	1	Problems on Barotropic flow	T1:110-112
6	1	Bernoulli's theorem in steady motion	R1:181-182
7	1	Continuation of Bernoulli's theorem	R1:182-183
8	1	Theorem on energy equation for inviscid fluid	R1:184-185
9	1	Circulation	R1:185-187
10	1	Kelvins theorem	R1:187-189
11	1	Theorem on Helmholtz equation of vorticity	R1:190-192
12	1	Recapitulation and discussion on possible questions	
	Total No of Hours Planned For Unit II=12		
		UNIT-III	
1	1	Two dimensional motion	T2:42-43
2	1	Functions- problems	T2:43-44
3	1	Theorem on stream lines	T2:44-45
4	1	Potential lines	T2:45-46
5	1	Problems on the flow patterns	T2:46-47

6	1	Basic singularities	T2:47-50
7	1	Theorem on source and sink in 2D flow	T2:50-55
8	1	Theorem on complex potential for doublet and vortex	T2:56-60
9	1	Milne Thomson's circle theorem	T2:69-70
10	1	Blasius theorem and lift force	T2:70-71
11	1	Lift force	T2:71-72
12	1	Recapitulation and discussion on possible questions	
	Total No of Hours Planned For Unit III=12		
		UNIT-IV	
1	1	Dynamics of real fluid: Definition of plane coquette flow	T2:123-124
2	1	Theorem on Reynolds's number	T2:124-125
3	1	Theorem on Navier Stokes equation	T2:140-144
4	1	Theorem on energy equation	T2:145-147
5	1	Diffusion of vorticity	T2:147-150
6	1	Steady flow through an arbitrary cylinder under pressure	T2:150-151
7	1	Problems on steady flow	T2:151-152
8	1	Steady Couette flow between cylinders in relative motion	T2:153-155
9	1	Problems on steady couette flow	T2:155-157
10	1	Steady flow between parallel planes – problems	T2:157-158
11	1	Theorem on Poiseuille flow	T2:159-160

12	1	Recapitulation and discussion on possible questions	
	Total No of Hours Planned For Unit IV=12		
		UNIT-V	
1	1	Laminar boundary layer in incompressible fluid: Definition and problems on equation of boundary layer	T2:175-178
2	1	Theorems on displacement	T2:184-185
3	1	Theorems on momentum thickness	T2:186-187
4	1	Boundary layer separation: Theorem on integral equation of boundary layer	T2:179-180
5	1	Problems on momentum integral equation	T2:187-190
6	1	Theorems on boundary layer along a semi infinite flat plate	T2:192-192
7	1	Blasius equation and its solution in series	T2:193-195
8	1	Problems on flow near to the stagnation point of a cylinder	T2:197-198
9	1	Recapitulation and discussion on possible questions	
10	1	Discussion on previous ESE question papers	
11	1	Discussion on previous ESE question papers	
12	1	Discussion on previous ESE question papers	
	Total No of Hours Planned for unit V=12		
Total Planned Hours	**120**		

TEXT BOOK

1. Milne Thomson .L.M., (1968). Theoretical Hydrodynamics, Fifth edition, Dover Publications INC, NewYork.**(for unit I,II)**
2. Curle.N., and Davies H.J., (1971), Modern Fluid Dynamics Volume-I , D Van Nostrand Company Ltd., London. **(for unit III,IV,V)**

REFERENCES

1. Yuan, S.W, (1976). Foundations of Fluid Mechanics, Prentice- Hall, India.
2. Shanthi swarup, (2003), Fluid dynamics, Krishna Prakasan media Pvt Ltd, Meerut.

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Coimbatore –641 021

Subject: Fluid Dynamics **Subject Code: 17MMP206**
Class : I - M.Sc. Mathematics **Semester : II**

Unit I

Introductory Notions

Part A (20x1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
The behavior of fluid at rest gives the study of_______________.	fluid dynamics	fluid statics	elastic	plastic	fluid statics
The behavior of fluid when it is in motion without considering the pressure force is called____________.	fluid kinematics	fluid mechanics	fluid statics	fluids	fluid kinematics
____________ is a branch of science which deals with the behavior of fluid at rest as well as motion.	fluid mechanics	fluid statics	fluid kinematics	fluids	fluid mechanics
The behavior of fluid when it is in motion with considering the pressure force is called__________.	fluid kinematics	fluid dynamics	fluid statics	fluid mechanics	fluid dynamics
_____________ is the branch of science which deals with the study of fluids.	fluid kinematics	fluid dynamics	fluid statics	fluid mechanics	fluid dynamics
If any material deformation vanishes when a force applied withdrawn a material is said to be__________.	elastic	plastic	deformation	fluid	elastic
If deformation remains even after the force applied withdrawn the material is said to be____________	elastic	plastic	fluid	fluid statics	plastic

If the deformation remains even after the force applied withdrawn this property of material is_____________.	elastic	plasticity	fluid	deformation	plasticity
_________ can be classified as liquids and gases.	solids	pressure	fluids	forces	fluids
The density of fluids is defined as _________ volume.	limit per unit	solid per time	mass per unit	forces per unit	mass per unit
A force per unit area is known as _____________.	force	pressure	fluid	density.	pressure
ƏF is the ___________ force due to fluid on Əs	normal	constant	force	pressure	normal
The pressure changes in the fluid beings changes in the dencity of fluid is called__________.	compressible fluid	incompressible fluid	body force	surface force	compressible fluid
The change in pressure of fluid do not alter the density of the fluid is called_____________.	compressible fluid	incompressible fluid	body force	surface force	incompressible fluid
_________ are propotional to mass of the body.	pressure	body force	surface force	force	body force
_________ are propotional to the surface area.	body force	surface force	force	mass	surface force
The normal force per unit area is said to be _____________.	normal stress	shearing stress	stress	strain	normal stress
The tangential force per unit area is said to be _____________.	normal stress	shearing stress	stress	strain	shearing stress
In a high viscosity fluid there exist normal as well as shearing stress is called ___________.	viscous fluid	inviscid fluid	frictionless	ideal	viscous fluid
Rate of change of linear momentum equation is_______________.					
Which is the velocity of the equation.	q=dr/dt	.q=s/r	.v=dx/w	.u=dy/s	q=dr/dt
The differential equation of the path line is__________.	.u=dy/s	.v=dx/w	q=dr/dt	.q=s/r	q=dr/dt

A flow in which each fluid particle posses different velocity at each section of the pipe are called__________.	uniform flow	rotational floe	barotropic flow	non-uniform flow	non-uniform flow
A flow in which each fluid particle go on rotating about their own axis while flowing is said to be__________.	rotational floe	uniform flow	non-uniform flow	barotropic flow	uniform flow
The pressure is function of density then the flow is said to be ____________.	rotational floe	uniform flow	barotropic flow	non-uniform flow	barotropic flow
The direction of the fluid velocity at the point is called____________.	stream line	velocity	fluid	pressure	stream line
___________ is defined as the locus of different fluid particles passing through a fixed point.	stream filament	stream line	path line	stream tube	stream line
A stream tube of an infinitesimal cross sectional area is called___________.	stream line	stream filament	path line	stream tube	stream filament
The equation of volume is___________.	cross section area*speed	speed/cross section area	cross section area/speed	speed	cross section area*speed
The equation of speed is___________.	time/length	length/speed	length*time	time*speed	length/speed
When a fluid particle moves it changes in both ________________.	speed and time	time and frequency	speed and position	position and time	position and time
When the flow is __________ the strem line have same form at all times.	steady	unsteady	stream surface	stream tube	steady
When the flow is________ the stream line changes from instant to instant.	stream tube	steady	unsteady	steady	unsteady
If Δ.f=0 then f is said to be a _______________.	solenoid	rotation	irrotation	constant	solenoid

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Unit II

Conservative Forces

Part A (20x1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
A force is said to be -------- if the force can be derivable from the potential.	conservative	non conservative	acceleration	surface	conservative
A flow is called a Beltrami's flow when---	q.E=0	q*E=0	q/E=0	q+E=0	q*E=0
Bernoulli's equation occurs when the motion is--	unsteady	rotational	steady	irrotational	steady
The ---- ------ flow can occurs when the vertex and stream lines coincide	viscous flow	beltrami's flow	invisid flow	normal flow	beltrami's flow
When the motion is both steady and irrotational then---	∇ .E	∇*E	∇+E	∇-E	∇ .E
The product of the cross sectional area and magniyude of the vorticity is -------- along a vortex filament	parallel	zero	constant	normal	constant
When the forces are conservative and the pressure is a function of the density,then--------	∇.a =0	∇*a=0	∇+a =0	∇-a=0	∇.a =0
When a force is conservative,there exist a potentialΩ such that f=	f=∇Ω	**f=-∇ Ω**	f=- ∇*Ω	f= ∇*Ω	**f=-∇ Ω**
circulation around a closed circuit 'c' is defined as	∫q.rdr	∫q.dr	∫qx.rdr	∫qx+dr	∫q.dr

Euler's equation of motion is	dq/dt=F-∇P	dq/dt=F	dq/dt=F-∇p/P	qd/dt=-∇Ω	dq/dt=F-∇p/P
----------- from is called the acceleration potential	Ω-∫ ð P/ ρ	∇[∫ ð P/ ρ] +dp	∇[∫ ð P/ ρ]	Ω+∫ ð P /p	Ω+∫ ð P /p
Beltram's flow is ---------	ðq/ ðt=∇	ðq/ ðt=-∇	ðq/ ðt=-Ω∇	ðq/ ðt=-∇ ρ /p	ðq/ ðt=-∇
q*E=0 can become zero when E ≠0,but q*E can be to each other	parallel	non parallel	zero	normal	parallel
The motion is both steady and irrotational if	∇.ψ≠0	∇+ψ =0	∇ .ψ =0	∇*a=0	∇ .ψ =0
Which is the constant of kelvin's theorem	a	ρ	B	ψ	ρ
Circulation is always defined around a ---------- ciruit	open	parallel	closed	normal	closed
When a conservative force f a potentialΩ such that	F=∇Ω	F=-∇Ω	F≠∇*Ω	F≠∇.Ω	F=-∇Ω
The euler's equation of motion corresponding to a beltrami's flow is	ðq/ðt=-∇ ψ	ðq/ðt=-∇ ψ	ðq/ðt=-∇* ψ	ðq/ðt≠-∇ ψ	ðq/ðt=-∇ ψ
A force is said to be conservative if the force can be derivable from the ---------	potential	density	area	viscosity	potential
The euler's theory is confined only for ideal or inviscid fluid	viscid	stream	inviscid	fluid	inviscid
The rate of change of linear momentum is equal to the ------- of the forces acting on a body	sum	product	proportional	difference	sum
the inward normal is --------	ρ	q	n^	F	n^
The rate of change of momentum of the fluid body is given by---	d/dt(cir c)=∫B.n ds	d/dt(cir c)=∫n ds	d/dt(cir c)=∫B.n dc	d/dt(cir c)=∫n dc	d/dt(cir c)=∫B.n ds
The -------- is the motion the rate of change of linear momentum =the sum of the forces acting on the body	Kelvin's theorem	Energy equation	Newton's second law	Euler's theorem	Newton's second law
rate of change of circulation is	ð/ðt(cir c)= ∫b.nds	ð/ðt(cir c)= ∫q.dr	ð/ðt(cir c)= ∫dq/dt.dr	ð/ðt(cir c)= ∫a.dr	ð/ðt(cir c)= ∫b.nds
Accelaration is given by	a=dm/dt	a=dq/dt	a=dr/dt	a=dc/dt	a=dq/dt
The ------ is the internal energy per unit mass	E	F	r	a	E
Density of a fluid is denoted by	F	ρ	a	E	ρ

In Red wood viscometer	Absolute value of viscosity is detemiined	Part of the head of fluid is utilized inOvercoming friction	Fluid discharges through orifice with negligible velocity	Comparison of viscosity is done.	Comparison of viscosity is done.
Centre of buoyancy is	The point of intersection of buoyant force and centre line of the body	Centre of gravity of the body	Centric of displaced volume fluid	Midpoint between C.G. and metacentric.	Centric of displaced volume fluid
In isentropic flow; the temperature	Cannot exceed the reservoir temperature	Cannot drop and again increase downstream	Is independent of Match number	Is a function of Match number only	Cannot exceed the reservoir temperature
A stream line is	The line of equal velocity in a flow	The line along which the rate of pressure drop is uniform	The line along the geometrical centre of the flow	Fixed in space in steady flow.	Fixed in space in steady flow.
The flow of water in a pure of diameter 3000mm can be measured by	Venturimeter	Rotameter	Pilot tube	Orifice plate	Pilot tube
Apparent shear forces	Can never occur in frictionless fluid regardless of its motion	Can never occur when the fluid is at rest	Depend upon cohesive forces	All of the above	All of the above
Weber number is the ratio of	Inertial forces to surface tension	Inertial forces to viscous forces	Elastic forces to pressure forces	Viscous forces to gravity	Inertial forces to surface tension
A small plastic boat loaded with pieces of steel rods is floating in a bath tub. If the cargo is dumped into the water allowing the both to float empty, the water level in the tub will water level in the tub will	Rise	Fall	Remains same	Rise and then fall	Fall
A flow in which each liquid particle has a definite path and their paths do not cross each other, is called	Steady flow	Uniform flow	Streamline flow	Turbulent flow	Streamline flow

Buoyant force is	Resultant of up thrust and gravity forces acting on the body	Resultant force on the body due to the fluid surrounding it	Resultant of static weight of body and dynamic thrust of fluid	Equal to the volume of liquid displaced by the body	Equal to the volume of liquid displaced by the body

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Unit III

Two Dimensional Motion

Part A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
The stream function is constant along a ___________________	Stream line	Path line	Vortex line	Filament line	Stream line
If the stream function is _____________ along a stream line	equal to zero	zero	constant	not equal	constant
If the motion is steady, the stream line pattern is ____________	equal	fixed	not fixed	constant	fixed
When the motion is not steady the stream line pattern is _____________ fixed	not	equal	constant	zero	not
The velocity potential ϕ exits when the fluid is _____________	Rotational	Irrotational	Stream line	Path line	Irrotational
If the velocity potential function are ___________	Velocity	Density	Pressure	Force	Velocity
The necessary and sufficient condition for q = -Φ= - grad Φ is hold is______________	$\nabla . q \neq 0$	$\nabla x q=0$	$\nabla x q=0$	$\nabla . q \neq 0$	$\nabla x q=0$
The complex potential functions are satisfying _____________ equation	Laplace equation	Differential equation	C – R equation	Homogeneous equation	C – R equation
If the velocity potential function are velocity Φ is called	$q=\nabla \Phi$	$q= -\nabla \Phi$	$q=\nabla \times \Phi$	$q=-\nabla \times \Phi$	$q= -\nabla \Phi$
The irrotational flow of an incompressible in viscid fluid is in _____________	3 – D	1 – D	2 – D	Multi – Dimension	2 – D

When the incompressible in viscid 2 – D fluid flow Φ and ψ satisfy the ____________ equation.	C – R equation	Laplace equation	Linear equation	Differential equation	Laplace equation
The stream function ψ exist whether the motes is ______________	Stream line	Path line	Irrotational	Rotational	Irrotational
The ________ potential can exist only when the motion is irrotational	Velocity	Density	Pressure	Force	Velocity
Part of the fluid may be moving irrotationally and the other parts may be ____________	Irrotational	constant	Rotational	Density	Rotational
The points where the velocity is ____________ are called stagnation points	1	0	Constant	Variable	0
In a 2 – D flow field where the fluid is assumed to be created is called	Doublet	Vertex	Sink	Sources	Sources
The flow is radically inverse is called ____________	Vertex	Sink	Sources	Doublet	Sink
The amount of the fluid going in to the sink in a unit time is called ________	Strength of the sink	Strength of the doublet	Strength of the source	Strength of the Vertex	Strength of the sink
The amount of the fluid going in to the sink in a ______ is called strength of the sink	Certain Interval	Unit time	Mean time	average	Unit time
If a source, the velocity of the fluid is ______________	Finite	Equal	Infinite	Zero	Infinite
Complex potential of the flow due to sink of strength m at the origin is given by	w = m logz	w = -m logz	w=log z	W=-log z	w = -m logz
A combination of a source and a sink in a particular way is known as a ____________	Doublet	Source	sink	vortex	Doublet
The line joining the source and sink is called as ____________ of the doublet	X – axis	Access	Y – axis	Z-axis	Access
If any point in the 2 – D field where the fluid is assumed to be ____________ is called a sink	Created	Constant	Moving	Annihilated	Annihilated
In a 2 – D field where the fluid is assumed to be annihilated is called a ______________	Sink	Source	Strength of source	Strength of sink	Sink

When the motion of a fluid consists of symmetrical radial flow in all directions proceeding from a point, Then the point is known a _____________	Source	Simple source	Sink	vortex	Simple source
When the fluid particles have circular motion under steady condition such a circular motion is called ____________	vortex	Sink	Doublet	Source	vortex
The Complex potential for a stream flow when a _____________ is placed in that	Surface	uniform	Circular Cylinder	continuous	Circular Cylinder
The complex potential for the uniform flow is ____________	w = v Z	w = V Z	w ≠ u × Z	w = u . Z	w = V Z
The circular cylinder is an irrotational incompressible	3 – D	1 – D	Multi – Dimension	2 – D	2 – D
The complex potential for the __________ flow is w = u Z	Uniform	Continuous	Discontinuous	Equal	Uniform
The complex potential for a ____________ flow when a circular cylinder is placed in that	Straight	Stream	Rotational	irrotational	Stream
A steady two dimensional irrotational incompressible in viscid fluid flow under no ____________Forces	External	Internal	Heat	mass	External
When are remembered that as the fluid is assumed to be in viscid, the drag force is	1	Equal	Zero	Not Equal	Zero
Cavitations is caused by	High velocity	Low barometric pressure	High pressure	Low pressure	Low pressure
The general energy equation is applicable to	Unsteady flow	Steady flow	Non-uniform flow	Turbulent flow	Steady flow
The friction resistance in Pipe is proportional To Square of V , according to	Froudeaiumber	Reynolds-Weber	Darcy-Reynolds	Weber-Froude	Froudeaiumber
Pitot tube is used to measure the velocity head of	Still fluid	Laminar flow	Turbulent flow	Flowing fluid	Flowing fluid
In equilibrium condition, fluids are not able to sustain	Shear force	Resistance to viscosity	Surface tension	Geometric similitude	Surface tension

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Unit IV

Viscous Flow

Part A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
In the case of a real fluid frictionless resistance is known as ----------------	shearing stress	tangential stress	friction stress	ideal fluid	tangential stress
In the case of --------------frictionless resistance is known as tangential stress	perfect fluid	friction stress	real fluid	ideal fluid	real fluid
On real fluid ,tangential stresses are ---------------	large	small	very small	infinite	small
The property which causes the tangential stress is known as-------	inviscosity	real fluid	velocity	viscosity	viscosity
On plane coutte flow if the fluid is perfect the motion of the plates has--------on the fluid	no effect	viscous	effect	speed	no effect
Shearing stress will be proportional to the rate of change of -------	speed	pressure	force	velocity	velocity
The force will be proportional to the area upon which it acts and it is known as -------	shearing stress	tangential stress	viscosity	effect of viscosity	shearing stress
In the effect of viscosity the shearing stress is denoted by --------	ψ	μ	τ	Ω	τ
The coefficient of viscosity is denoted by--------	ψ	μ	Ω	τ	μ
A typical viscous stress is in the form τ=---------	$\partial u/\partial y$	μ	$\mu(\partial u/\partial y)$	$\partial\mu$	$\mu(\partial u/\partial y)$

The viscous force are of order ---- per unit area	U/L	μ (U/L)	μ /L	μU	μ (U/L)
The typical pressure force will be of order------ per unit area	U^2	ρU	ρU/L	ρU^2	ρU^2
In a Reynold's numbers, the kinematic viscosity is -----	γ=μ/ρ	γ=μ	γ=1/μ	γ=0	γ=μ/ρ
The non-dimensional parameter R=UL/γ is called -----	viscous force	pressure force	Reynold's number	kinematic viscosity	Reynold's number
The equation of continuity in a real fluid on a viscous flow is ------	$\partial\rho/\partial t + (\partial/\partial x_i)(\rho v_i)=0$	$\partial/\partial t + (\partial/\partial x_i)(\rho v_i)=0$	$\partial\rho/\partial t + (\partial^2/\partial t^2)(\rho v_i)=0$	$\partial\rho/\partial t + (\partial/\partial x_i)(\rho)=0$	$\partial\rho/\partial t + (\partial/\partial x_i)(\rho v_i)=0$
In the Navier stokes equation,when the fluid is incompressible,then ρ and μ are-----	equal	zero	not equal	constant	constant
The Navier stokes equation in vector form is ---------------	$dq/dt=F-\nabla p/\rho$	$dq/dt=F-\nabla p/\rho+\gamma\nabla^2 q$	$dq/dt=F+\gamma\nabla^2 q$	$dq/dt=F+\nabla p/\rho+\gamma\nabla^2 q$	$dq/dt=F-\nabla p/\rho+\gamma\nabla^2 q$
The equation of an Helmholtz equation of the viscous fluid is-------	$d\varepsilon/dt=(\varepsilon.\nabla)q+\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q$	$d\varepsilon/dt=\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q-\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q+\gamma\nabla^2\varepsilon$
On the 2-D motion the equation of vorticity is ------	$d\varepsilon/dt=(\varepsilon.\nabla)q+\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q$	$d\varepsilon/dt=\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q-\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=\gamma\nabla^2\varepsilon$
In a circulation on a viscous fluid the space derivative of the vorticity vector are--------------	small	constant	large	infinite	large
The steady flow through an arbitrary cylinder under pressure is known as ---------	Hagen –Poiseuille flow	viscous flow	inviscous flow	vorticity flow	Hagen –Poiseuille flow
In the Reynolds number is the principal parameter determining the ------	role of the flow	nature of the flow	order of the flow	type of the flow	nature of the flow
The constant of proportionality, μ depends entirely upon the physical properties of the fluid is called ________	typical viscous stress	effect of viscosity	coefficient of viscosity	viscosity of a flow	coefficient of viscosity
An arbitrary volume of a fluid,the momentum of the fluid contained within the volume is -----	$\int v_i dv$	$\int \rho v_i\, dv$	$\int \rho dv$	$\int \rho^2 v_i\, dv$	$\int \rho v_i\, dv$
The resultant value of an poiseuille's law is ------	$M=(\pi p\, a^3)/4\mu$	$M=(\pi\rho p\, a^3)/6\mu$	$M=(\pi\rho p\, a^4)/8\mu$	$M=(\pi p\, a^4)/6\mu$	$M=(\pi\rho p\, a^4)/8\mu$

If we consider two infinite parallel planes.Aflow with pressure gradient when both planes are at rest then they are called as --------	pressure flow	plane poiseuille flow	coutte flow	plane coutte flow	plane poiseuille flow
If we consider two infinite parallel planes.A flow without pressure gradient when one plane moves relative to the other such a flow is called----------	plane coutte flow	plane poiseuille flow	infinite plane flow	viscous plane flow	plane coutte flow
A flow is said to be ---------- if all fluid particles moving in one direction	parallel	perpendicular	nonparallel	zero	parallel
A flow is said to be parallel if only one velocity component is ----------	zero	non zero	constant	variable	non zero
A flow is said to be parallel if all fluid particles moving in---------- direction	two	three	one	four	one
A flow is said to be parallel if only---------- velocity component is non zero	two	four	three	one	one
Skin friction σ= ----------	μ/h	μU	μU/h	U/h	μU/h
Skin friction is also known as ---------per unit area	circle	sphere	square	drag	drag
In plane couette flow the -------------------is zero	temperature gradient	temperature	pressure gradient	pressure	pressure gradient
In------------------- the pressure gradient is zero	plane poiseuille flow	plane couette flow	couette flow	poiseuille flow	plane couette flow
In ---------------------------the plates are at rest	plane poiseuille flow	plane couette flow	couette flow	poiseuille flow	plane poiseuille flow
In plane poiseuille flow the plates are at--------	motion	rest	stable	nonstable	rest
The -------------------for the drag of a sphere is given by D= 6 $\pi\mu a U_0$	stokes formula	Greens formula	Gauss formula	Laplace formula	stokes formula
The stokes formula for the drag of a sphere is given by D= -----------------	6 U_0	6 $\pi\mu a U_0$	6 $\pi\mu a$	6 aU_0	6 $\pi\mu a U_0$
The stokes formula for the drag of a ---------------is given by D= 6 $\pi\mu a U_0$	circle	flux	sphere	square	sphere

In steady flow the flow past a circular cylinder then the stokes equation reduces to -------	parallel	perpendicular	nonzero	zero	zero

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Unit V

Laminar Boundary Layer in incompressible flow

Part A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
In a boundary layer characteristics which streamlines far from the wall are displaced then $\delta_1(x)$ is referred to as--------------	displacement thickness	momentum thickness	kinetic energy thicknesss	friction thifckness	displacement thickness
The value of displacement thickness $\delta_1(x)$=--------------------	$\int u(1-(u/u_1))$ dy	$\int 1-(1/u_1))$ dy	$\int 1-(u/u_1)$ dy	$\int(u/u_1)(1-(u/u_1))$ dy	$\int 1-(u/u_1)$ dy
When separation ocurrs in which circumstances the boundary layer approximation is suspect in such case is __________	displacement thickness	momentum thickness	kinetic energy thicknesss	friction thifckness	momentum thickness
A momentum thickness $\delta_2(x)$ is defined for incompressible flow as ---------	$\int u(1-(u/u_1))$ dy	$\int 1-(1/u_1))$ dy	$\int 1-(u/u_1)$ dy	$\int(u/u_1)(1-(u/u_1))$ dy	$\int(u/u_1)(1-(u/u_1))$ dy
A physically significant measure of boundary layer thickness is --------	displacement thickness	momentum thickness	kinetic energy thicknesss	friction thifckness	kinetic energy thicknesss
A measuresthe flux of kinetic energy defect within the boundary layer as compared with---------	viscous flow	steady flow	inviscid flow	incompressible flow	incompressible flow
The kinetic energy thickness is defined as $\delta_3(x)$=------	$\int u(1-(u/u_1))$ dy	$\int 1-(1/u_1))$ dy	$\int 1-(u/u_1)$ dy	$\int(u/u_1)(1-(u^2/u_1^2))$ dy	$\int(u/u_1)(1-(u^2/u_1^2))$ dy
The wall shearing stress is defined as -------	μ	δ	τ_w	ρ_w	τ_w
The skin friction τ_w=------	$(\partial u/\partial y)_w$	$\mu(\partial u/\partial y)_w$	$\delta(\partial u/\partial y)_w$	$(\partial^2 u/\partial y^2)_w$	$\mu(\partial u/\partial y)_w$

The onset of reversed flow near the wall takes place at the position of zero skin frction.such a position is called a position of ------	boundary layer friction	boundary layer characteristics	boundary layer separation	boundary layer flow	boundary layer separation
Kinematic viscosity is denoted by -----	$\mu=\gamma/\rho$	$\gamma= \mu/ \rho$	$\rho= \mu\gamma$	$\gamma= \rho \mu$	$\gamma= \mu/ \rho$
Enthalpy is defined as ----	I=E+P	I=E-(P/ ρ)	I=E+(P/ ρ)	I=E+(ρ / P)	I=E+(P/ ρ)
Thermal conductivity is denoted by ---------	p	I	ρ	K	K
Reynold’s number is defined as -------	$R=U/ \gamma$	$R=L/ \gamma$	$R=UL/ \gamma$	$R=U \gamma / L$	$R=UL/ \gamma$
Viscosity is a function of temperature and -----------	pressure	mass	density	viscosity	pressure
Kinematic viscosity is a function of -------and pressure	pressure	temperature	density	force	temperature
The rate of increases of the boundary layer thickness depends on ------	$\partial p/\partial x$	$\partial q/\partial x$	$\partial p/\partial y$	$\partial q/\partial y$	$\partial p/\partial x$
The rate of -------- of the boundary layer thickness depends on boundary gradient	change	not change	increase	decrease	increase
The layer in which -----is called boundary layer	$\partial u/\partial y$	$\partial v/\partial y$	$\partial u/\partial x$	$\partial v/\partial x$	$\partial u/\partial y$
Kinetic energy thickness is also known as kinetic energy ---------	linear equation	laplace equation	integral equation	definite equation	integral equation
------- is called the pressure coefficient	c_v	P_c	V_C	c_p	c_p
-------- have zero velocity at the walls	real fluids	ideal fluid	viscous fluid	inviscid fluid	real fluids
Real fluids have-------- velocity at the walls	negative	positive	zero	nonzero	zero
Real fluids have zero velocity --------	near to the wall	opposite to the wall	at the walls	befor the wall	at the walls
If the pressure has ----then the boundary layer thickness increases rapidly	decreases	change	nochange	increases	increases
.If the pressure increases then the---- increases rapidly	boundary	boundary layer thickness	boundary layer	boundary surface	boundary layer thickness
If the ---------------increases then the boundary layer thickness increases rapidly	pressure	density	mass	force	pressure
If the pressure increases then the boundary layer thickness ------------- rapidly	decreases	gradually increases	increases	gradually decreases	increases
.-------- has no slip conditions	real fluids	ideal fluid	viscous fluid	inviscid fluid	real fluids
Real fluids has --------	no slip conditions	slip conditions	maximum slip conditions	minimum slip conditions	no slip conditions

The velocity component is normal to the wall is small if ----- is small	$\delta/2$	$\delta/3$	$\delta/4$	$\delta/5$	$\delta/2$
The velocity component is normal to the wall is small if $\delta/2$ is -----	normal	small	parallel	perpendicular	small
In the equation of boundary layer-------------------- normal to the wall is small	temperature gradient	temperature	pressure	pressure gradient	pressure gradient
In the equation of boundary layer pressure gradient -------------------- to the wall is small	parallel	normal	tangent	perpendicular	normal
The relationship between the pressure and main stream velocity can be obtained by --------	beltramis equation	linear equation	indefinite equation	Bernoulli's equation	Bernoulli's equation
------ is the flux of defect of momentum in the boundary layer	$\rho\mu_1\delta_2$	$\rho\mu_1$	$\rho\mu_1^2\delta_2$	$\mu_1^2\delta_2$	$\rho\mu_1^2\delta_2$
$\rho\mu_1^2\delta_2$ is the flux of defect of-------------------- in the boundary layer	acceleration	velocity	mass	momentum	momentum
In the equation of boundary layer the velocity component is-----to the wall	parralel	perpendicular	normal	tangent	normal
In the equation of ----- the velocity component is normal to the wall	boundary	boundary layer thickness	boundary layer	boundary surface	boundary layer
In the equation of boundary layer the velocity component is normal to the wall is -----	normal	parallel	small	perpendicular	small

Reg. No..................................

[15MMP302]

KARPAGAM UNIVERSITY

Karpagam Academy of Higher Education
(Established Under Section 3 of UGC Act 1956)
COIMBATORE – 641 021
(For the candidates admitted from 2015 onwards)

M.Sc., DEGREE EXAMINATION, NOVEMBER 2016

Third Semester

MATHEMATICS

FLUID DYNAMICS

Time: 3 hours | Maximum : 60 marks

PART – A (20 x 1 = 20 Marks) (30 Minutes)
(Question Nos. 1 to 20 Online Examinations)

(Part - B & C 2 ½ Hours)

PART B (5 x 6 = 30 Marks)
Answer ALL the Questions

21. a. Derive the equation of continuity of an incompressible fluid.
Or
b. Obtain the equation of motion of an inviscid fluid.

22. a. State and prove Euler's momentum theorem.
Or
b. Derive the Helmholtz equation.

23. a. Obtain the complex potential describing the flow of a uniform stream past a circular cylinder having a circulation.
Or
b. State and prove the Blasius theorem.

24. a. Derive the Helmholtz equation for the vorticity.
Or
b. Discuss the steady flow through a circular cylinder of radius a under pressure.

25. a. Discuss displacement thickness, momentum thickness and kinetic energy thickness.
Or
b. Derive the Blasius equation.

PART C (1 x 10 = 10 Marks)
(Compulsory)

26. Show that $u = \frac{-2xyz}{(x^2+y^2)^2}$, $v = \frac{(x^2-y^2)^2}{(x^2+y^2)^2}$, $w = 0$ are the velocity components of a possible fluid motion. Also check the irrotationality of the fluid.

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

SYLLABUS

Semester – II

17MMP206	**FLUID DYNAMICS**	**L T P C** **4 0 0 4**

Scope: This course has been intended to identify and use key concepts and fundamental principles of fluid dynamics, together with the assumptions made in their development pertaining to fluid behavior, both in static and flowing conditions.

Objectives: To understand the fluids, their characteristics, Bernoulli's theorem in steady motion, Complex Potential Navier-Stokes equations and to be exposed with Laminar Boundary Layer in incompressible flow.

UNIT I

Introductory Notions – Velocity – Stream Lines and Path Lines – Stream Tubes and Filaments – Fluid Body – Density – Pressure.Differentiation following the Fluid – Equation of continuity – Boundary conditions – Kinematical and physical – Rate of change of linear momentum – Equation of motion of an in viscid fluid.

UNIT II

Euler's momentum Theorem – Conservative forces – Bernoulli's theorem in steady motion – energy equation for in viscid fluid – circulation – Kelvin's theorem – vortex motion – Helmholtz equation.

UNIT III

Two Dimensional Motion – Two Dimensional Functions – Complex Potential – basic singularities – source – sink – Vortex – doublet – Circle theorem. Flow past a circular cylinder with circulation – Blasius Theorem – Lift force. (Magnus effect)

UNIT IV

Viscous flows – Navier-Stokes equations – Vorticity and circulation in a viscous fluid – Steady flow through an arbitrary cylinder under pressure – StaedyCouettc flow between cylinders in relative motion – Steady flow between parallel planes.

UNIT V

Laminar Boundary Layer in incompressible flow: Boundary Layer concept – Boundary Layer equations – Displacement thickness, Momentum thickness – Kinetic energy thickness – integral equation of boundary layer – flow parallel to semi infinite flat plate – Blasius equation and its solution in series.

SUGGESTED READINGS

TEXT BOOKS

1. Milne Thomson .L.M., (1968). Theoretical Hydrodynamics, Fifth edition, Dover Publications INC, NewYork**.(for unit I,II)**
2. Curle.N., and Davies H.J., (1971), Modern Fluid Dynamics Volume-I , D Van Nostrand Company Ltd., London. **(for unit III,IV,V)**

REFERENCES

1. Yuan, S.W, (1976). Foundations of Fluid Mechanics, Prentice- Hall, India.
2. Shanthiswarup, (2003), Fluid dynamics, Krishna Prakasan media Pvt Ltd, Meerut.

UNIT-I

SYLLABUS

Introductory Notions – Velocity – Stream Lines and Path Lines – Stream Tubes and Filaments – Fluid Body – Density – Pressure. Differentiation following the Fluid – Equation of continuity – Boundary conditions – Kinematical and physical – Rate of change of linear momentum – Equation of motion of an in viscid fluid.

Basic Concepts and Definitions

(i) Let $\overline{q} = \hat{i}u + \hat{j}v + \hat{k}w$, then

$$|\overline{q}| = \sqrt{u^2 + v^2 + w^2} = q$$

D.C's are given by $l = \cos\alpha = \dfrac{u}{|\overline{q}|}$, $m = \cos\beta = \dfrac{v}{|\overline{q}|}$, $n = \cos\gamma = \dfrac{w}{|\overline{q}|}$

where l, m, n, are components of a unit vector i.e. $l^2 + m^2 + n^2 = 1$

(ii) $\overline{a}.\overline{b} = ab\cos\theta$, $\overline{a} \times \overline{b} = ab\sin\theta\,\hat{n}$

(iii) $\nabla\phi = \hat{i}\dfrac{\partial\phi}{\partial x} + \hat{j}\dfrac{\partial\phi}{\partial y} + \hat{k}\dfrac{\partial\phi}{\partial z}$, where ϕ is a scalar and

$$\nabla \equiv \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \text{ is a vector (operator)}$$

(iv) $\text{div } \overline{q} = \nabla.\overline{q} = \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}, \overline{q} = (u, v, w)$

If $\nabla \cdot \overline{q} = 0$, then $\overline{q}$ is said to be solenoidal vector.

(v) $d\overline{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz, d\phi = \dfrac{\partial\varphi}{\partial x}dx + \dfrac{\partial\varphi}{\partial y}dy + \dfrac{\partial\varphi}{\partial z}dz$

and

$$\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z},$$

Therefore,

$$d\phi = (\nabla\phi).\ d\overline{r}$$

(vi) $\text{Curl}\,\overline{q} = \nabla \times \overline{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$

$$= \hat{i}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) + \hat{j}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \hat{k}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

(vii) **(a)** Gradient of a scalar is a vector.
(b) Divergence of a scalar and curl of a scalar are meaningless.
(c) Divergence of a vector is a scalar and curl of a vector is a vector.

(viii) $\nabla \cdot \nabla\phi = \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$

where ∇^2 is Laplacian operator.

(ix) Curl grad $\phi = 0$, div curl $\overline{q} = 0$

(x) Curl curl $\overline{q} = \text{grad div}\,\overline{q} - \nabla^2\overline{q}$

i.e. $\nabla^2\overline{q} = \text{grad div}\,\overline{q} - \text{curl curl}\,\overline{q}$

(xi) **Gauss's divergence theorem**

(a) $\int_S \overline{q} \cdot d\overline{S} = \int_V \text{div}\,\overline{q}\, dv$

(b) $\int_S \hat{n} \times \overline{q}\, dS = \int_V \text{curl}\,\overline{q}\, dv$

(xii) **Green's theorem**

(a) $\int_V \nabla\phi \cdot \nabla\psi dV = \int_S \phi\nabla\psi \cdot d\overline{S} - \int_V \phi\nabla^2\psi dV$

$$= \int_S \psi\nabla\phi \cdot d\overline{S} - \int_V \psi\nabla^2\phi \cdot dV$$

(b) $\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)dV = \int_V \left(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\phi n}\right)dS$

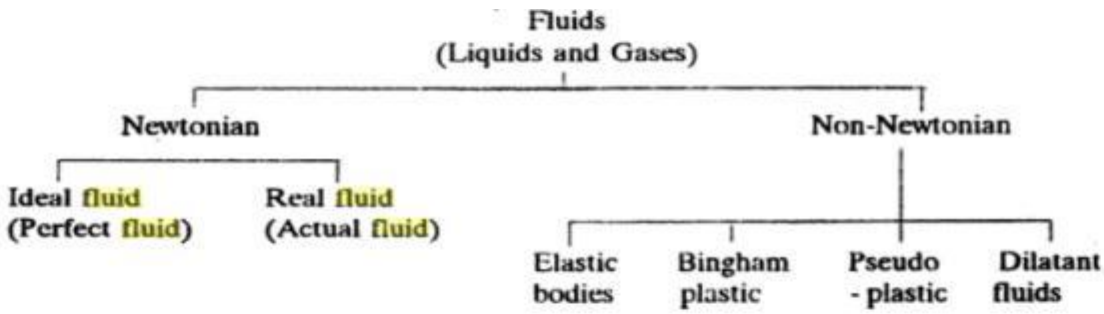
(xiii) **Stoke's theorem** $\int_C \overline{q} \cdot d\overline{r} = \int_S \text{curl}\,\overline{q} \cdot d\overline{S} = \int_S \text{curl}\,\overline{q} \cdot \hat{n}\, dS$

Fluid Dynamics

Fluid dynamics is the science treating the study of fluids in motion. By the term fluid, we mean a substance that flows i.e. which is not a solid. Fluids may be divided into two categories

(i) liquids which are incompressible i.e. their volumes do not change when the pressure changes

(ii) gases which are compressible i.e. they undergo change in volume whenever the pressure changes. The term hydrodynamics is often applied to the science of moving incompressible fluids. However, there is no sharp distinctions between the three states of matter i.e. solid, liquid and gases.



Fluid properties

Certain characteristics of a continuous fluid are independent of the motion of the fluid. These characteristics are known as the basic properties of the fluid. We shall discuss some of the properties of a fluid.

(i) Density : The *density* ρ represents a quantitative expression of the idea of mass. It is defined as the mass of the fluid contained within a unit volume. Consider δm is the mass of the fluid in a small volume δv surrounding that point, then, mathematically the density at a point is defined as

$$\rho = \lim_{\delta v \to 0} \frac{\delta m}{\delta v} .$$

(ii) Pressure : The *pressure* p at a point in the fluid is the limit of the ratio of normal force δF over an area δA by the surrounding fluid particles as the area approaches zero. It is defined as

$$p = \lim_{\delta A \to 0} \frac{\delta F}{\delta A} .$$

Definitions

Steady and unsteady flows : *Steady* flow occurs when at various points of the flow field the conditions and properties associated with the fluid flow remain unaltered with time *i.e.*, independent of time at all points. Mathematically, it can be expressed as

$$\frac{\partial A}{\partial t} = 0,$$

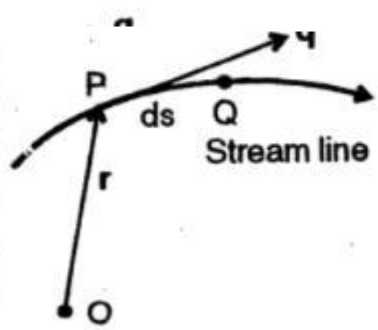
where A represents the characteristic of the fluid, *e.g.*, velocity, density, temperature and pressure etc. Thus in steady motion time drops out of the independent variables and the various field quantities become functions of the space coordinates. For example, *Water being pumped through a fixed system at a constant rate represents steady flow.*

The flow is said to be *unsteady* when conditions at any point change with regard to the time. For example, *Water being pumped through a fixed system at an increasing rate represents unsteady flow.*

Uniform and Non-uniform flows : If at every point the velocity vector is identical in magnitude and direction at any given instant, or, the conditions and properties are independent of the coordinate of the direction in which the fluid is moving then the motion is said to be *uniform*. If the flow characteristics, at any given time t, change with distance, it is said to be *non-uniform flow*.

Line of flow : A *line of flow* is a line whose direction coincides with the direction of the resultant velocity of the fluid.

Stream line : A *stream line* is a continuous line of flow drawn in the fluid so that the tangent at every point of it at any instant of time coincides with the direction of the motion of the fluid at that point. The component of velocity at right angles to the streamline is always zero. It follows that there is no flow across the streamline.



Consider ds be an element of the streamline passing through any point P (**r**) at an instant of time. Let **q** be the velocity at that point at the same instant. The direction of the tangent and direction of velocity are parallel.

i.e., $$d\mathbf{s} \times \mathbf{q} = 0$$

$$\Rightarrow \quad (\mathbf{i}dx + \mathbf{j}dy + \mathbf{k}dz) \times (\mathbf{i}u + \mathbf{j}v + \mathbf{k}w) = 0$$

$$\Rightarrow \quad (wdy - vdz)\,\mathbf{i} + (udz - wdx)\,\mathbf{j} + (vdx - udy)\,\mathbf{k} = 0$$

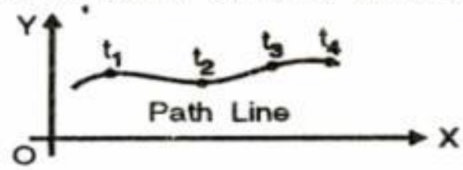
The vector equation is equivalent to three scalar equations

$$wdy - vdz = 0, udz - wdx = 0, vdx - udy = 0$$

which can be represented as

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w},$$

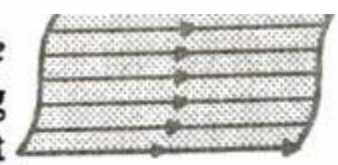
Pathline : The curve described in space by moving fluid element is known its *pathline* or trajectory *i.e.*, a *pathline* is a line traced by a particle in the fluid. The pathline shows the direction of the velocity of the fluid particle at any instant of time. Such a line is obtained by giving the position of an element as a function of time. The pathline are given by



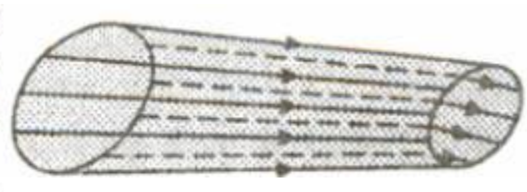
$$\mathbf{q} = \frac{d\mathbf{r}}{dt}$$

i.e., $$\frac{dx}{dt} = u(x, y, z, t), \frac{dy}{dt} = v(x, y, z, t), \frac{dz}{dt} = w(x, y, z, t).$$

(viii) Stream surface : A *stream surface* is a surface made by the streamlines passing through an arbitrary line in the fluid region at any instant of time.



(ix) Stream tube : A *stream tube* is obtained by drawing stream lines through every point of a closed curve in the fluid.

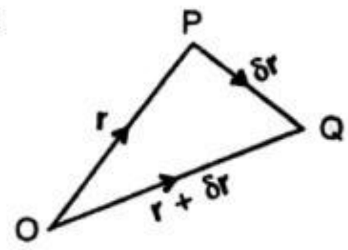


Velocity of a fluid particle at a point

Consider P and Q be the positions of the fluid particle at an instant of time t and $t + \delta t$ from the fixed point O such that

$$OP = \mathbf{r} \quad \text{and} \quad OQ = \mathbf{r} + \delta\mathbf{r}.$$

Let $\mathbf{q}$ be the velocity of the fluid particle at P, then



$$\mathbf{q} = \lim_{\delta t \to 0} \frac{(\mathbf{r} + \delta\mathbf{r}) - \mathbf{r}}{\delta t}$$

or
$$\mathbf{q} = \lim_{\delta t \to 0} \frac{\delta \mathbf{r}}{\delta t} = \frac{d\mathbf{r}}{dt}$$

Thus $\mathbf{q}$ is, in general, dependent on both $\mathbf{r}$ and t *i.e.*,

$$\mathbf{q} = \mathbf{q}(\mathbf{r}, t).$$

Example

The velocity $\mathbf{q}$ *in a three-dimensional flow field for an incompressible fluids is given by*

$$\mathbf{q} = 2x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Determine the equations of the streamlines passing through the point (1, 1, 1).

Solution. The equations of stream lines are given by

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \Rightarrow \underset{\text{(i)}}{\frac{dx}{2x}} = \underset{\text{(ii)}}{\frac{dy}{-y}} = \underset{\text{(iii)}}{\frac{dz}{-z}}$$

From (i) and (ii), we have

$$\frac{dx}{2x} = \frac{dy}{-y} \Rightarrow \frac{dx}{x} + \frac{2dy}{y} = 0$$

By integrating, we obtain

$$\log x + 2\log y = \log A$$

or $xy^2 = A$, where A is an integration constant.

From (i) and (iii), we have

$$\frac{dx}{2x} = \frac{dz}{-z} \Rightarrow \frac{dx}{x} + \frac{2dz}{z} = 0$$

By integrating, we have

$xz^2 = B$, where B is an integration constant.

At the point (1, 1, 1) $A = 1 = B$

Hence the required streamlines are

$$xy^2 = 1 \quad \text{and} \quad xz^2 = 1.$$

Equation of Continuity

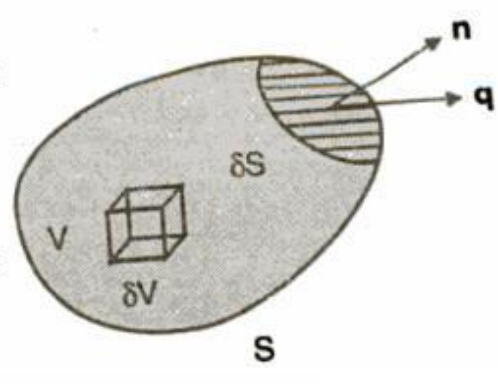
Physical quantities are said to be conserved when they do not change with regard to time during a process. The mathematical expression of the law of conservation of mass is known as the *equation of continuity*.

By continuity we mean physical continuity. The fluid always remain a continuum *i.e.*, as a continuously distributed matter. When a region of fluid contain neither sources nor sinks (*i.e.*, there is no creation or annihilation of the fluid) then the amount of fluid within the region is conserved in accordance with the principle of conservation of matter. The general conservation principle is defined as follows :

$$\text{In} - \text{Out} + \text{Source} - \text{Sink} = \text{Accumulation},$$

where each term represents a rate for a differential element of volume.

Consider a fluid element of infinitesimal volume δv and density ρ which is situated at a point $\mathbf{r}$ at any instant t. The mass of the element is $\rho\delta v$. Throughout the motion the mass of any element of fluid must be conserved, hence the mass of any fluid element remains unchanged as it moves about. This shows that the material derivative of $\rho\delta v$ vanishes, *i.e.*,



$$\frac{D}{Dt}(\rho v) = 0,$$

which is the equation of continuity in the simplest form.

Consider a closed surface S in a fluid medium containing a volume V fixed in space. Let $\mathbf{n}$ is the unit outward drawn normal at a surface element δS. If $\mathbf{q}$ be the fluid velocity at the element δS then the normal component of $\mathbf{q}$ measured outward from V will be $= \mathbf{n} \cdot \mathbf{q}$.

Rate of mass flow across δS per unit mass $= \rho(\mathbf{n} \cdot \mathbf{q})\, \delta S$

Total rate of mass flow out of V across $\delta S = \int_S \rho\, (\mathbf{n} \cdot \mathbf{q})\, dS$.

Total rate of mass flow into V

$$= -\int_S \mathbf{n} \cdot (\rho \mathbf{q})\, dS$$

$$= -\int_V \nabla \cdot (\rho \mathbf{q})\, dV \quad \text{(By Gauss theorem)}$$

Also rate of increase of mass within V

$$= \frac{\partial}{\partial t}\left[\int_V \rho\, dV\right] = \int_V \frac{\partial \rho}{\partial t} dV$$

By the principle of continuity, we have

$$\int_V \frac{\partial \rho}{\partial t} dV = -\int_V \nabla \cdot (\rho \mathbf{q})\, dV$$

or
$$\int_V \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{q})\right] dV = 0.$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{q}) = 0,$$

Equation (5) may be written in a different form as

$$\frac{\partial \rho}{\partial t} + \mathbf{q}\,(\nabla \rho) + \rho \nabla \cdot \mathbf{q} = 0 \Rightarrow \left(\frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla\right) \rho + \rho \nabla \cdot \mathbf{q} = 0$$

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{q} = 0, \quad \text{since} \quad \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{q} \cdot \nabla$$

$(D\rho/Dt)$ is the substantial derivative of density *i.e.*, the time derivative for a path following the fluid motion.

For a steady flow of fluid, the pattern of flow does not vary with regard to time then the relation (5) reduces to the form

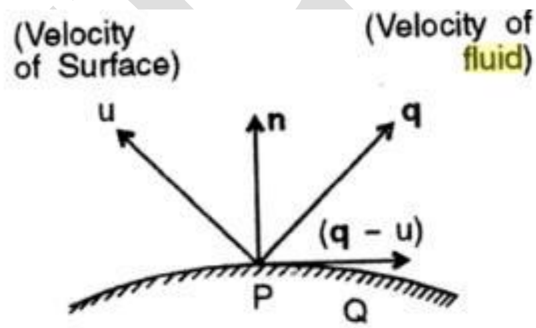
$\nabla \cdot (\rho \mathbf{q}) = 0.$

For a non-homogeneous incompressible fluid, the density of the fluid particle is invariable with time t *i.e.*, ρ remains constant throughout the entire region

$$\frac{D\rho}{Dt} = 0 \Rightarrow \nabla . \mathbf{q} = 0 \quad \Rightarrow \text{div}\, \mathbf{q} = 0$$

The quantity $\nabla . \mathbf{q}$ gives the rate of volume expansion of a fluid element. It may be called *dilatation* or *expansion*. A vector **q** having zero divergence is said to be *solenoidal*.

Boundary Surface



Moving boundary surface

Physical conditions that should be satisfied on given boundaries of the fluid are called as boundary condition. At the boundary of the fluid, the equation of continuity is replaced by a special surface condition. When the fluid is in contact with an impermeable (non-porous) bounding surface the velocity of a fluid particle at any point of the boundary relative to the surface must be tangential to the boundary. Thus at a fixed boundary, the velocity of the fluid perpendicular to the surface must vanish and the normal component of the velocity of the fluid must be equal to the normal component of the velocity of the surface.

Let **q** be the velocity of the fluid and **u** be the velocity of the surface at the point P. Let **n** be the unit normal vector drawn at the point P on the boundary surface $F(\mathbf{r}, t) = 0$. Since there must be no relative normal velocity at P between boundary and fluid so we must have the two normal components equal *i.e.*,

$$\mathbf{q.n} = \mathbf{u.n} \Rightarrow (\mathbf{q} - \mathbf{u}) . \mathbf{n} = 0.$$

For two fluids, in contact, a dynamical boundary is required; viz, the pressure must be continuous across the interface

$$(\mathbf{q} - \mathbf{u}) . \nabla F = 0, \qquad \mathbf{n} = \nabla F$$

The position of the point P on the moving surface at any instant of time $t + \delta t$ is given by

$$F(\mathbf{r} + \delta\mathbf{r}, t + \delta t) = 0$$

Expanding by Taylor's theorem, we have

$$F(\mathbf{r}, t) + \delta\mathbf{r}.\nabla F + \delta t.(\partial F/\partial t) = 0$$

or

$$\frac{\partial F}{\partial t} + \frac{\partial \mathbf{r}}{\partial t}.\nabla F = 0.$$

The above relation reduces to

$$\frac{\partial F}{\partial t} + \mathbf{u}.\nabla F = 0; \partial\mathbf{r} \to 0, \partial t \to 0; \mathbf{u} = d\mathbf{r}/dt$$

$$\frac{\partial F}{\partial t} + \mathbf{q}.\nabla F = 0 \Rightarrow \frac{\partial F}{\partial t} + u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z} = 0.$$

Thus the equation of every boundary surface must satisfy the differential equation (4).

If the surface is at rest then $\partial F/\partial t = 0$, the relation (4) reduces to

$$u(\partial F/\partial x) + v(\partial F/\partial y) + w(\partial F/\partial z) = 0,$$

which represents the condition when the liquid is in contact with a rigid surface. In order that contact is maintained, the fluid and the surface must have the same velocity normal to the surface.

Also, the normal velocity of the boundary is given by

$$\mathbf{u.n} = \frac{\mathbf{u}.\nabla F}{|\nabla F|} = -\frac{\partial F/\partial t}{\sqrt{\left\{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2 + \left(\frac{\partial F}{\partial z}\right)^2\right\}}}.$$

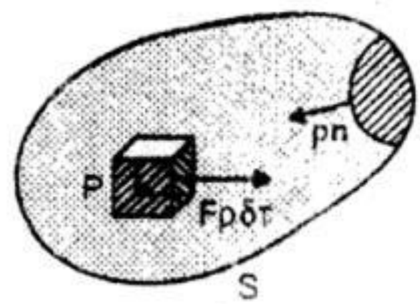
The momentum of a body is defined as the product of the mass of the body and its velocity *i.e.*, $\frac{m\mathbf{q}}{g_0}$, and has the dimensions of force-time. In the flow of fluids the momentum M per unit volume is given by

$$M = \frac{\sigma\mathbf{q}}{g_0} = \rho\mathbf{q}$$

Since velocity is a vector quantity so momentum is likewise, a vector quantity, having magnitude and direction both.

Equation of motion of an inviscid fluid

Consider any arbitrary closed surface S drawn in the region occupied by the incompressible fluid at an instant t. We know by Newton's second law of motion that the total force acting on this mass of fluid is equal to the rate of change of linear momentum. The forces are due to (i) the normal pressure thursts on the boundary, and (ii) the external force (*e.g.*, gravity) F per unit mass.



Let ρ be the density of the fluid particle P with in the closed surface and $d\tau$ be the volume enclosing P. The mass of the element $\rho\, d\tau$ will always remain constant. Consider $\mathbf{q}$ be the velocity of the fluid particle P then the momentum of the volume is

$$M = \int \mathbf{q}\rho\, d\tau. \qquad ...(1)$$

The time rate of change of momentum is given by differentiating (1) with regard to t, thus we have

$$\frac{dM}{dt} = \int \frac{d\mathbf{q}}{dt}(\rho\, d\tau) + \int \mathbf{q}\frac{d}{dt}(\rho\, d\tau) = \int \frac{d\mathbf{q}}{dt}.\rho\, d\tau \qquad ....(2)$$

The second integral vanishes as the mass $(\rho\, d\tau)$ remains constant for all time.

Let F be the external force per unit mass acting on fluid particle P then the total force on the volume is

$$= \int F\rho\, d\tau. \qquad ...(3)$$

Again, let p be the pressure at a point on the surface along the outward drawn unit normal $\hat{\mathbf{n}}$ then the force on the fluid particle due to the actions of the surrounding fluid is

$$= -\int p\hat{\mathbf{n}}\, dS = -\int \nabla p\, d\tau. \qquad ...(4)$$

The equation for the momentum balance is written as

Rate of momentum accumulation = Rate of momentum in − Rate of momentum out + Sum of forces acting on system.

$$\int \frac{d\mathbf{q}}{dt} . \rho \, d\tau = \int F\rho \, d\tau - \int \nabla p \, d\tau,$$

or

$$\int \left[\rho \frac{d\mathbf{q}}{dt} - \rho F + \nabla p\right] d\tau = 0.$$

Since the volume of integration enclosed in the surface is arbitrary, we can reduce this volume to a point. Therefore

$$\rho \frac{d\mathbf{q}}{dt} - \rho F + \nabla p = 0,$$

or

$$\frac{d\mathbf{q}}{dt} = F - \frac{1}{\rho} \nabla p, \quad \text{...(5)}$$

known as *Euler's equation of motion at all points of the fluid* which applies only to ideal fluids, the dissipative effects have not been considered.

The equation (5) may be expressed as

$$\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}.\nabla)\, \mathbf{q} = F - \frac{1}{\rho} \nabla p \qquad \left(\text{Since } \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{q}.\nabla\right)$$

or

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \left(\frac{1}{2} \mathbf{q}^2\right) - \mathbf{q} \times \text{curl}\, \mathbf{q} = F - \frac{1}{\rho} \nabla p, \quad \text{...(6)}$$

or

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \left(\frac{1}{2} \mathbf{q}^2\right) + \underline{\omega} \times \mathbf{q} = F - \frac{1}{\rho} \nabla p, \; (\text{Since } \underline{\omega} = \nabla \times \mathbf{q}) \quad \text{...(7)}$$

known as *Lamb's Hydrodynamical equations* which is a non-linear equation due to the convective term $(\mathbf{q}.\nabla)\,\mathbf{q}$ on the L.H.S. in (6).

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = X - \frac{1}{\rho} \frac{\partial p}{\partial x},$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = Y - \frac{1}{\rho} \frac{\partial p}{\partial y},$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = Z - \frac{1}{\rho} \frac{\partial p}{\partial z}$$

POSSIBLE QUESTIONS

PART – B (5 x 6 = 30 Marks)

Answer all the questions

1. Prove that the velocity field u=yzt,v=zxt,w=xyt is a possible case of irrotational flow.
2. Determine the stream lines and path lines of the particle u=x/(1+t) , v=y/(1+t) , w=z/(1+t).
3. Derive differential equation of a stream line.
4. Obtain the condition that the surface F(r ,t)=0.
5. Derive the differentiation following the motion of a fluid.
6. The velocity $\bar{q}$ in a three dimensional flow fluid for an incompressible fluid is given by $\bar{q}=2x\bar{i}-y\bar{j}-z\bar{k}$. Determine the equation of the stream line passing through the point(1,1,1).

PART – C (1 x 10 = 10 Marks)

Compulsory

1. The velocity components in a flow two dimensional flow fluid for an incompressible fluid is given by $u=e^x\cosh y, v=-e^x \sin hy$.
2. Show that the product of the speed and cross sectional area is constant along the stream filament of a liquid in steady motion.
3. Derive the equation of continuity.
4. The velocity field at a point in a fluid is given by $\bar{q}$ = (x/t,y,0).Obtain also a path line.
5. Determine the restriction on f_1,f_2,f_3 if $x^2/a^2.f_1(t) + y^2/b^2.f_2(t)+z^2/c^2.f_3(t)=1$ is a possible boundary surface of a liquid.

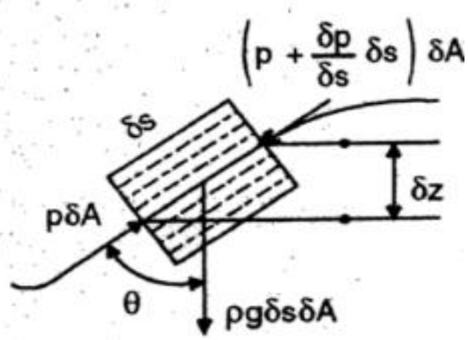
UNIT-II

SYLLABUS

Euler's momentum Theorem – Conservative forces – Bernoulli's theorem in steady motion – energy equation for in viscid fluid – circulation – Kelvin's theorem – vortex motion – Helmholtz equation.

Euler's equation of motion along a streamline

Consider an elementary section of a stream tube. Let δs be the length of the stream tube element. Mass of the fluid particle moving along a streamline in the positive direction is $\rho\,\delta A\,\delta s$. The force acting on the element are of two types : (i) Body forces and (ii) Surface forces exerted due to hydrostatic pressure on the end areas of the particle.



The body force is $\rho Fs\,\delta A\,\delta s$. On the upstream face the pressure force is $p\,\delta A$ in the $(+s)$ direction and on the downstream face it is $\left(p + \frac{\partial p}{\partial s}\delta s\right)\delta A$ acting in the $(-s)$ direction. The total force along the path δs with tangential unit vector is given by

$$= \rho F_S\,\delta s\,\delta A + \left[p\delta A - \left(p + \frac{\partial p}{\delta s}\delta s\right)\delta A\right]$$

$$= \rho F_S\,\delta s\,\delta A - \frac{\partial p}{\partial s}\delta s\,\delta A.$$

The acceleration of the fluid flowing along δs is $\frac{D\mathbf{q}}{Dt}$. By using Newton's second law of motion the equation of momentum along the path is given by

$$\frac{D\mathbf{q}}{Dt}\rho\,\delta s\,\delta A = \rho F_s\,\delta s\,\delta A - \frac{\partial p}{\partial s}\delta s\,\delta A$$

or $$\frac{D\mathbf{q}}{Dt} = F_S - \frac{1}{\rho}\frac{\partial p}{\partial s}$$

or $$\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q}\frac{\partial \mathbf{q}}{\partial s} = F_S - \frac{1}{\rho}\frac{\partial p}{\partial s}, \quad ...(1)$$

known as *the Euler's equation of motion for one-dimensional flow.*

Consider the body force due to the pull of gravity. The gravity force is $\rho g\, \delta s\, \delta A$, its component along the s direction are

$$\rho F_S\, \delta s\, \delta A = -\rho\, \delta s\, \delta A\, g \cos\theta \Rightarrow F_S = -g\cos\theta.$$

Since δz is the increase in elevation of the particle for a displacement δs then

$$F_S = -g\,(\partial z/\partial s), \text{ as } \cos\theta = (\partial z/\partial s) \quad ...(2)$$

From (1) and (2), we obtain

$$\frac{\partial \mathbf{q}}{\partial t} + \mathbf{q}\frac{\partial \mathbf{q}}{\partial s} = -g\frac{\partial z}{\partial s} - \frac{1}{\rho}\frac{\partial p}{\partial s}. \quad ...(3)$$

For steady flow $\partial \mathbf{q}/\partial t = 0$, the equation (3) reduces to

$$\mathbf{q}\frac{\partial \mathbf{q}}{\partial s} = -g\frac{\partial z}{\partial s} - \frac{1}{\rho}\frac{\partial p}{\partial s},$$

where $\mathbf{q}$, z and p are functions of s only. The partial derivatives may be replaced by the total derivatives

$$\frac{dp}{\rho} + g\,dz + \mathbf{q}\,d\mathbf{q} = 0$$

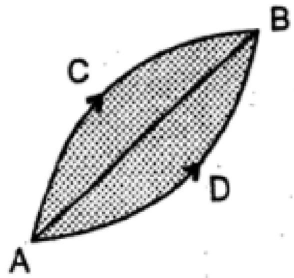
$$\int \frac{dp}{\rho} + qz + \frac{1}{2}\mathbf{q}^2 = \text{constant}. \quad ...(4)$$

which is an alternative form of *Euler's equation of motion along a streamline* for inviscid and steady flow. It may be integrated if ρ is known as a function of p or is a constant. The pressure along a stream line can be determined without assuming the existence of a velocity potential.

Conservative field of force

If the work done by the force F of the field in taking a unit mass from one point A to another point B independent of the path then it is termed as *conservative field of force*

$$\int_{ACB} F \cdot dr = \int_{ADB} F \cdot dr = -\Omega \text{ (say)},$$



where Ω is a scalar point function whose value depends on the initial and final position A and B. Thus

$$F = -\nabla\Omega,$$

where Ω is known as *force potential* which measures the potential energy of the field.

Bernoulli's Equation (Theorem)

For Steady Flow. We shall obtain a special form of Euler's dynamical equation in terms of pressure. The Euler's dynamical equation is

$$\frac{d\overline{q}}{dt} = \overline{F} - \frac{1}{\rho}\nabla p \qquad (1)$$

where $\overline{q}$ is velocity, $\overline{F}$ is the body force, p and ρ are pressure and density respectively.

$\overline{F}$ be conservative so that it can be expressed in terms of a body force potential function Ω as

$$\overline{F} = -\nabla\,\Omega \qquad (2)$$

When the flow is steady, then $\frac{\partial \overline{q}}{\partial t} = 0$ (3)

Therefore, in case of steady motion with a conservative body force equation (1), on using (2) and (3), gives

$$\nabla\left(\frac{1}{2}\overline{q}^2\right)-\overline{q}\times\overline{\xi}=-\nabla\Omega-\frac{1}{\rho}\nabla p$$

$$\Rightarrow \qquad \nabla\left(\frac{1}{2}\overline{q}^2+\Omega\right)+\frac{1}{\rho}\nabla p=\overline{q}\times\overline{\xi} \qquad (4)$$

Further, if we suppose that the liquid is barotropic i.e. density is a function of pressure p only, then we can write

$$\frac{1}{\rho}\nabla p=\nabla\int\frac{dp}{\rho}$$

Using this in (4), we get

$$\nabla\left[\frac{1}{2}\overline{q}^2+\Omega+\int\frac{dp}{\rho}\right]=\overline{q}\times\overline{\xi}. \qquad (5)$$

Multiplying (5) scalarly by $\overline{q}$ and noting that

$\overline{q}\cdot(\overline{q}\times\overline{\xi})=(\overline{q}\times\overline{q})\cdot\overline{\xi}=0$, we get

$$\overline{q}\cdot\nabla\left[\frac{1}{2}\overline{q}^2+\Omega+\int\frac{dp}{\rho}\right]=0 \qquad (6)$$

If $\hat{s}$ is a unit vector along the streamline through general point of the fluid and s measures distance along this stream line, then since $\hat{s}$ is parallel to $\overline{q}$, therefore equation (6) gives

$$\frac{\partial}{\partial s}\left[\frac{1}{2}\overline{q}^2+\Omega+\int\frac{dp}{\rho}\right]=0$$

Hence along any particular streamline, we have

$$\frac{1}{2}\overline{q}^2+\Omega+\int\frac{dp}{\rho}=C \qquad (7)$$

where C is constant which takes different values for different streamlines. Equation (7) is known as Bernaull's equation. This result applies to steady flow of ideal. barotropic fluids in which the body forces are conservative.
Now, if $\hat{s}$ is a unit vector taken along a vortexline, then, similarly, we get

$\frac{1}{2}\overline{q}^2 + \Omega + \int \frac{dp}{\rho} = C$ along any particular vortexline. (Here, we multiply scalarly by $\overline{\xi}$)

Remark. (i) If $\overline{q} \times \overline{\xi} = \overline{0}$ i.e. if $\overline{q}$ & $\overline{\xi}$ are parallel, then streamlines and vortex lines coincide and $\overline{q}$ is said to be **Beltrami vector.**

If $\overline{\xi} = \overline{0}$, the flow is irrotational.
For both of these flow patterns,

$$\frac{1}{2}\overline{q}^2 + \Omega + \int \frac{dp}{\rho} = C$$

where C is same at all points of the fluid.

(ii) For homogeneous incompressible fluids, ρ is constant and

$$\int \frac{dp}{\rho} = \frac{p}{\rho}.$$

The Bernoulli's equation becomes

$$\frac{p}{\rho} + \frac{1}{2}\overline{q}^2 + \Omega = C$$

so that if $\overline{q}$ is known, the pressure can be calculated.

Vorticity and the equations of motion.

Vortex lines and tubes.

We define a *vortex line* in analogy to a streamline as a line in the fluid that at each point on the line the vorticity vector is tangent to the line, i.e. the vortex line at each point is parallel to the vorticity vector.

It is important to note that the strength of the vector vorticity is not constant along a vortex line in the same way that the velocity is not (necessarily) constant along a streamline.

The circulation

The *circulation* of any vector field $\vec{J}$ around a closed curve C in the fluid is defined as:

$$\Gamma_J = \oint_C \vec{J} \cdot d\vec{x} = \oint_C J_i dx_i$$

where the contour is taken in the counter-clockwise sense.

The circulation involves the component of **J** tangent to the curve . If **J** is the velocity vector the resulting circulation is simply called the *circulation* and is denoted by Γ and is

$$\Gamma = \oint_C \vec{u} \cdot d\vec{x}$$

From Stokes theorem,

$$\Gamma = \oint_C \vec{u} \cdot d\vec{x} = \int_A [\nabla \times \vec{u}] \cdot \hat{n} dA = \int_A \vec{\omega} \cdot \hat{n} dA$$

so that the circulation is just vortex tube strength for the tube enclosed by C.

Kelvin's Circulation Theorem

The circulation Γ around a closed material contour $C(t)$ is defined by:

$$\Gamma(t) = \oint_C \boldsymbol{u} \cdot \mathrm{d}\boldsymbol{s}$$

where ***u*** is the velocity vector, and ***ds*** is an element along the closed contour.

The governing equation for an inviscid fluid with a conservative body force is

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\frac{1}{\rho}\boldsymbol{\nabla}p + \boldsymbol{\nabla}\Phi$$

where D/D*t* is the convective derivative, *ρ* is the fluid density, *p* is the pressure and *Φ* is the potential for the body force. These are the Euler equations with a body force.

The condition of barotropicity implies that the density is a function only of the pressure, i.e. $\rho = \rho(p)$.

Taking the convective derivative of circulation gives

$$\frac{\mathrm{D}\Gamma}{\mathrm{D}t} = \oint_C \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} \cdot \mathrm{d}\boldsymbol{s} + \oint_C \boldsymbol{u} \cdot \frac{\mathrm{D}\mathrm{d}\boldsymbol{s}}{\mathrm{D}t}.$$

For the first term, we substitute from the governing equation, and then apply Stokes' theorem, thus:

$$\oint_C \frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} \cdot \mathrm{d}\boldsymbol{s} = \int_A \boldsymbol{\nabla} \times \left(-\frac{1}{\rho}\boldsymbol{\nabla}p + \boldsymbol{\nabla}\Phi\right) \cdot \boldsymbol{n}\,\mathrm{d}S = \int_A \frac{1}{\rho^2}\left(\boldsymbol{\nabla}\rho \times \boldsymbol{\nabla}p\right) \cdot \boldsymbol{n}\,\mathrm{d}S = 0.$$

The final equality arises since $\boldsymbol{\nabla}\rho \times \boldsymbol{\nabla}p = 0$ owing to barotropicity. We have also made use

of the fact that the curl of any gradient is necessarily 0, or $\boldsymbol{\nabla} \times \boldsymbol{\nabla}f = 0$ for any

function f.

For the second term, we note that evolution of the material line element is given by

$$\frac{\mathrm{D}\mathrm{d}\boldsymbol{s}}{\mathrm{D}t} = (\mathrm{d}\boldsymbol{s} \cdot \boldsymbol{\nabla})\,\boldsymbol{u}.$$

Hence

$$\oint_C \boldsymbol{u} \cdot \frac{\mathrm{D}\mathrm{d}\boldsymbol{s}}{\mathrm{D}t} = \oint_C \boldsymbol{u} \cdot (\mathrm{d}\boldsymbol{s} \cdot \boldsymbol{\nabla})\,\boldsymbol{u} = \frac{1}{2}\oint_C \boldsymbol{\nabla}\left(|\boldsymbol{u}|^2\right) \cdot \mathrm{d}\boldsymbol{s} = 0.$$

The last equality is obtained by applying Stokes theorem.

Since both terms are zero, we obtain the result

$$\frac{D\Gamma}{Dt} = 0.$$

The theorem also applies to a rotating frame, with a rotation vector $\boldsymbol{\Omega}$, if the circulation is modified thus:

$$\Gamma(t) = \oint_C (\boldsymbol{u} + \boldsymbol{\Omega} \times \boldsymbol{r}) \cdot d\boldsymbol{s}$$

Here $\boldsymbol{r}$ is the position of the area of fluid. From Stokes' theorem, this is:

$$\Gamma(t) = \int_A \boldsymbol{\nabla} \times (\boldsymbol{u} + \boldsymbol{\Omega} \times \boldsymbol{r}) \cdot \boldsymbol{n}\, dS = \int_A (\boldsymbol{\nabla} \times \boldsymbol{u} + 2\boldsymbol{\Omega}) \cdot \boldsymbol{n}\, dS$$

Helmholtz Equations

Euler's equations of motion are given by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x},$$
$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y},$$
$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z}. \qquad ...(1, 2, 3)$$

Let V be the potential function of the external forces and the density ρ be the function of the pressure p. Equation (1) may be written as

$$\frac{\partial u}{\partial t} + \left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x} + w\frac{\partial w}{\partial x}\right) + v\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + w\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = -\frac{\partial V}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\Rightarrow \quad \frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}(\mathbf{q}^2) - 2v\zeta + 2w\eta = -\frac{\partial V}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x},$$

where $\underline{\Omega}\,(\xi, \eta, \zeta)$ are the spin components and $\mathbf{q}^2 = u^2 + v^2 + w^2$.

$$\Rightarrow \quad \frac{\partial u}{\partial t} - 2v\zeta + 2w\eta = -\frac{\partial}{\partial x}\left(V + \frac{1}{2}\mathbf{q}^2 + \int \frac{dp}{\rho}\right) = -\frac{\partial Q}{\partial x} \text{ (let)},$$

where $Q = V + \frac{1}{2}\mathbf{q}^2 + \int \frac{dp}{\rho}$.

Thus $\frac{\partial u}{\partial t} - 2v\zeta + 2w\eta = -\frac{\partial Q}{\partial x}$,

Similarly $\frac{\partial v}{\partial t} - 2w\xi + 2u\zeta = -\frac{\partial Q}{\partial y}$,

and $\frac{\partial w}{\partial t} - 2u\eta + 2v\xi = -\frac{\partial Q}{\partial z}$. ...(4, 5, 6)

Differentiating (5) and (6) partially with regard to z and y, we have

$$\frac{\partial^2 v}{\partial z\,\partial t} - 2w\frac{\partial \xi}{\partial z} - 2\xi\frac{\partial w}{\partial z} + 2u\frac{\partial \zeta}{\partial z} + 2\zeta\frac{\partial u}{\partial z} = \frac{\partial^2 w}{\partial y\,\partial t} - 2u\frac{\partial \eta}{\partial y} - 2\eta\frac{\partial u}{\partial y} + 2v\frac{\partial \xi}{\partial y} + 2\xi\frac{\partial v}{\partial y}$$

$$\Rightarrow \quad \frac{\partial}{\partial t}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - 2u\left(\frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z}\right) + 2v\left(\frac{\partial \xi}{\partial y}\right) + 2w\left(\frac{\partial \xi}{\partial z}\right) + 2\xi\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - 2\eta\left(\frac{\partial u}{\partial y}\right) - 2\zeta\frac{\partial u}{\partial z} = 0 \quad ...(7)$$

But $\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} = 0$ ...(8)

From (7) and (8), we have

$$2\frac{\partial \xi}{\partial t} + 2u\frac{\partial \xi}{\partial x} + 2v\frac{\partial \xi}{\partial y} + 2w\frac{\partial \xi}{\partial z} + 2\xi\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) - 2\xi\frac{\partial u}{\partial x} - 2\eta\frac{\partial u}{\partial y} - 2\zeta\frac{\partial u}{\partial z} = 0$$

$$\Rightarrow \quad \frac{D\xi}{Dt} + \xi\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = \xi\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z} \qquad \text{...(9)}$$

The equation of continuity is

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0. \qquad \text{...(10)}$$

From (9) and (10), we have

$$\frac{D\xi}{Dt} - \frac{\xi}{\rho}\frac{D\rho}{Dt} = \xi\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z}$$

$$\Rightarrow \quad \frac{1}{\rho}\frac{D\xi}{Dt} - \frac{\xi}{\rho^2}\frac{D\rho}{Dt} = \frac{\xi}{\rho}\frac{\partial u}{\partial x} + \frac{\eta}{\rho}\frac{\partial u}{\partial y} + \frac{\zeta}{\rho}\frac{\partial u}{\partial z}$$

$$\Rightarrow \quad \frac{D}{Dt}\left(\frac{\xi}{\rho}\right) = \frac{\xi}{\rho}\frac{\partial u}{\partial x} + \frac{\eta}{\rho}\frac{\partial u}{\partial y} + \frac{\zeta}{\rho}\frac{\partial u}{\partial z}, \qquad \text{...(11)}$$

Similarly $\displaystyle \frac{D}{Dt}\left(\frac{\eta}{\rho}\right) = \frac{\xi}{\rho}\frac{\partial v}{\partial x} + \frac{\eta}{\rho}\frac{\partial v}{\partial y} + \frac{\zeta}{\rho}\frac{\partial v}{\partial z}$,

and $$\frac{D}{Dt}\left(\frac{\zeta}{\rho}\right) = \frac{\xi}{\rho}\frac{\partial w}{\partial x} + \frac{\eta}{\rho}\frac{\partial w}{\partial y} + \frac{\zeta}{\rho}\frac{\partial w}{\partial z}. \qquad \text{...(12, 13)}$$

But $$\frac{\eta}{\rho}\frac{\partial u}{\partial y} + \frac{\zeta}{\rho}\frac{\partial u}{\partial z} = \frac{\eta}{\rho}\left\{\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) + \frac{\partial v}{\partial x}\right\} + \frac{\zeta}{\rho}\left\{\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \frac{\partial w}{\partial x}\right\}$$

$$= \frac{\eta}{\rho}(-2\zeta) + \frac{\eta}{\rho}\frac{\partial v}{\partial x} + \frac{\zeta}{\rho}(2\eta) + \frac{\zeta}{\rho}\frac{\partial w}{\partial x}$$

$$= \frac{\eta}{\rho}\frac{\partial v}{\partial x} + \frac{\zeta}{\rho}\frac{\partial w}{\partial x}. \qquad \text{...(14)}$$

Using the relation (14), the equations (11, 12, 13) reduce to

$$\frac{D}{Dt}\left(\frac{\xi}{\rho}\right) = \frac{\xi}{\rho}\frac{\partial u}{\partial x} + \frac{\eta}{\rho}\frac{\partial v}{\partial x} + \frac{\zeta}{\rho}\frac{\partial w}{\partial x},$$

$$\frac{D}{Dt}\left(\frac{\eta}{\rho}\right) = \frac{\xi}{\rho}\frac{\partial u}{\partial y} + \frac{\eta}{\rho}\frac{\partial v}{\partial y} + \frac{\zeta}{\rho}\frac{\partial w}{\partial y},$$

$$\frac{D}{Dt}\left(\frac{\zeta}{\rho}\right) = \frac{\xi}{\rho}\frac{\partial u}{\partial z} + \frac{\eta}{\rho}\frac{\partial v}{\partial z} + \frac{\zeta}{\rho}\frac{\partial w}{\partial z}. \qquad ...(15, 16, 17)$$

Equations (15), (16), (17) are known as *Helmholtz's equation.*

Let $\xi = \eta = \zeta = 0$ at an instant of time t then

$$\frac{D}{Dt}\left(\frac{\xi}{\rho}\right) = \frac{D}{Dt}\left(\frac{\eta}{\rho}\right) = \frac{D}{Dt}\left(\frac{\zeta}{\rho}\right) = 0$$

$\Rightarrow$ $$\frac{D\zeta}{Dt} = \frac{D\eta}{Dt} = \frac{D\zeta}{Dt} = 0, \rho = \text{const.}$$

$\Rightarrow$ ξ, η, ζ must be constant. Since they are all zero at an instant of time t and have to remain constant.

In general, let $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, ...$ are all finite and less than a quantity P then $\frac{\xi}{\rho}, \frac{\eta}{\rho}, \frac{\zeta}{\rho}$ can not increase faster than if they satisfy the equations.

$$\frac{D}{Dt}\left(\frac{\xi}{\rho}\right) = \frac{D}{Dt}\left(\frac{\eta}{\rho}\right) = \frac{D}{Dt}\left(\frac{\zeta}{\rho}\right) = \frac{P}{\rho}(\xi + \eta + \zeta)$$

Let $\xi + \eta + \zeta = PW$, then

$$\frac{D}{Dt}\left(\frac{\xi}{\rho} + \frac{\eta}{\rho} + \frac{\zeta}{\rho}\right) = \frac{D}{Dt}(W) = 3PW$$

$\Rightarrow$ $W = ke^{3Pt}, W \neq 0$

When $t = 0, W = 0 \Rightarrow k = 0$ *i.e.*, W be zero at time $t = 0$, it may be so for all time.

Since W is the sum of three quantities ξ, η, ζ which cannot be negative. Hence $W = 0$, it follows that each of these three quantities must be zero $\xi = 0 = \eta = \zeta$.

Hence if the motion is irrotational at any instant, it must be so for all time *i.e.*, if once, the velocity potential exists it exists for all time. This is known as the *principle of Permanance of irrotational motion*.

POSSIBLE QUESTIONS

PART – B (5 x 6 = 30 Marks)

Answer all the questions

1. Prove that the rate of change of total energy,kinetic energy,potential energy,intrinsic energy of any position of a compressible inviscid fluid as it moves about is equal to the rate at which work is being done by the pressure on the boundary Ω is constant w.r.t time.
2. State and prove Kelvin's theorem.
3. Explain Bernoulli's equation.
4. Obtain the Equation of motion interms of verticity vector when the force is conservative.
5. Derive Euler's generalised Momentum theorem.
6. Derive the Helmholtz equation of vorticity.

PART – C (1 x 10 = 10 Marks)

Compulsory

1. Explain Energy equation.
2. Explain Beltrami's flow.
3. Derive Equation of motion when the force is conservative.
4. Explain Circulation and rate of change of circulation.
5. Derive Euler's equation of motion.

UNIT-III

SYLLABUS

Two Dimensional Motion – Two Dimensional Functions – Complex Potential – basic singularities – source – sink – Vortex – doublet – Circle theorem. Flow past a circular cylinder with circulation – Blasius Theorem – Lift force. (Magnus effect)

Motion in two-dimensions.

Let a fluid move in such a way that at any given instant the flow pattern in a certain plane (say XOY) is the same as that in all other parallel planes within the fluid. Then the fluid is said to have two-dimensional motion. If (x, y, z) are coordinates of any point in the fluid, then all physical quantities (velocity, density, pressure etc.) associated with the fluid are independent of z. Thus u, are functions of x, y and t and $w = 0$ for such a motion.

Stream function or current function.

Let u and be the components of velocity in two-dimensional motion. Then the differential equation of lines of flow or streamline is

$$dx/u = dy/ \qquad \text{or} \qquad dx - udy = 0 \qquad ...(1)$$

and the equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial}{\partial y} = 0 \qquad \text{or} \qquad \frac{\partial}{\partial y} = \frac{\partial(-u)}{\partial x} \qquad ...(2)$$

(2) shows that $L.H.S.$ of (1) must be an exact differential, $d\psi$ (say). Thus, we have

$$dx - udy = d\psi = (d\psi/\partial x)dx + (\partial\psi/\partial y)dy \qquad ...(3)$$

so that $u = -\partial\psi/\partial y$ and $= \partial\psi/\partial x$ ...(4)

This function ψ is known as the *stream function*. Then using (1) and (3), the streamlines are given by $d\psi = 0$ *i.e.*, by the equation $\psi = c$, where c is an arbitrary constant. Thus the stream function is constant along a streamline. Clearly the current function exists by virtue of the equation of continuity and incompressibility of the fluid. Hence the current function exists in all types of two-dimensional motion whether rotational or irrotational.

Ex. 1. *To show that the curves of constant velocity potential and constant stream functions cut orthogonally at their points of intersection.*

OR

To shows that the family of curves $\phi(x, y) = c_1$ and $\psi(x, y) = c_2$, c_1, c_2 being constants, cut orthogonally at their points of intersection.

Proof. Let the curves of constant velocity potential and constant stream function be given by

$$\phi(x, y) = c_1 \qquad \ldots(1)$$

and

$$\psi(x, y) = c_2, \qquad \ldots(2)$$

where c_1 and c_2 are arbitrary constants. Let m_1 and m_2 be gradients of tangents PT_1 and PT_2 at point of intersection P of (1) and (2). Then, we have

$$m_1 = -\frac{\partial\phi/\partial x}{\partial\phi/\partial y} \qquad \text{and} \qquad m_2 = -\frac{\partial\psi/\partial x}{\partial\psi/\partial y} \qquad \ldots(3)$$

We know that ϕ and ψ satisfy the Cauchy-Riemann equations, namely,

$$\partial\phi/\partial x = \partial\psi/\partial y \qquad \text{and} \qquad \partial\phi/\partial y = -\partial\psi/\partial x. \qquad \ldots(4)$$

Now, from (3), $$m_1 m_2 = \frac{(\partial\phi/\partial x)(\partial\psi/\partial x)}{(\partial\phi/\partial y)(\partial\psi/\partial y)} = \frac{(\partial\psi/\partial y)(\partial\psi/\partial x)}{-(\partial\psi/\partial x)(\partial\psi/\partial y)}, \text{ by (4)}$$

Hence $m_1 m_2 = -1$, showing that the curves (1) and (2) cut each other orthogonally.

Source and sinks in two-dimensions.

In two-dimensions a source of strength m is such that the flow across any small curve surrounding is $2\pi m$. Sink is regarded as a source of strength $-m$.

Consider a circle of radius r with source at its centre. Then radial velocity q_r is given by

$$q_r = -\frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad \ldots(1)$$

or $$q_r = -\frac{\partial\phi}{\partial r}, \qquad \text{as} \qquad \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta} \qquad \ldots(2)$$

Then the flow across the circle is $2\pi r q_r$. Hence we have

$$2\pi r q_r = 2\pi m \qquad \text{or} \qquad r q_r = m \qquad \ldots(3)$$

or $$r\left(-\frac{1}{r}\frac{\partial\psi}{\partial\theta}\right) = m, \text{ by (1)}$$

Integrating and omitting constant of integration, we get

$$\psi = -m\theta \quad ...(4)$$

Using (2) and (3), we obtain as before

$$\phi = -m \log r \quad ...(5)$$

Equation (4) shows that the streamlines are θ = constant, *i.e.*, straight lines radiating from the source. Again (5) shows that the curves of equi-velocity potential are r = constant, *i.e.*, concentric circles with centre at the source.

Complex potential due to a source.

Let there be a source of strength m at origin. Then

$$w = \phi + i\psi = -m \log r - im\theta = -m(\log r + i \log e^{i\theta}) = -m \log(re^{i\theta}) = -m \log z.$$

If, however, the source is at z', then the complex potential is given by $w = -m \log(z - z')$

The relation between w and z for sources of strengths $m_1, m_2, m_3, \ldots$ situated at the points $z = z_1, z_2, z_3, \ldots$ is given by

$$w = -m_1 \log(z - z_1) - m_2 \log(z - z_2) - m_3 \log(z - z_3) - \ldots$$

leading to $\phi = -m_1 \log r_1 - m_2 \log r_2 - m_3 \log r_3 - \ldots$

and $\psi = -m_1\theta_1 - m_2\theta_2 - m_3\theta_3 - \ldots$

where $r_n = |z - z_n|$ and $\theta_n = \arg(z - z_n)$, $n = 1, 2, 3, \ldots$

Doublet (or dipole) in two dimensions

A combination of a source of strength m and a sink of strength $-m$ at a small distance δs apart, where in the limit m is taken infinitely great and δs infinitely small but so that the product $m\delta s$ remains finite and equal to μ, is called a *doublet of strength* μ, and the line δs taken in the sense from $-m$ to $+m$ is taken as the *axis of the doublet*.

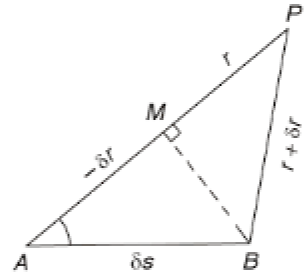
Complex potential due to a doublet in two-dimensions

Let A, B denote the positions of the sink and source and P be any point. Let $AP = r$, $BP = r + \delta r$ and $\angle PAB = \theta$. Let ϕ be the velocity potential due to this doublet.

Then $$\phi = m \log r - m \log(r + \delta r) = -m \log \frac{r + \delta r}{r}$$

or $$\phi = -m \log\left(1 + \frac{\delta r}{r}\right)$$

$\therefore$ $$\phi = -m\frac{\delta r}{r}, \text{ to first order of approximation.} \quad \ldots(1)$$



Let BM be perpendicular drawn from B on AP. Then,

$$AM = AP - MP = r - (r + \delta r) = -\delta r$$

$\therefore \quad \cos\theta = AM / AB = -\delta r / \delta s$ so that $\delta r = -\delta s \cos\theta$

$\therefore$ From (1), $$\phi = m\delta s \cdot \frac{\cos\theta}{r} = \frac{\mu\cos\theta}{r} \quad ...(2)$$

where $\mu = m\delta s$ = strength of the doublet.

From (2), $$\frac{\partial\phi}{\partial r} = -\frac{\mu\cos\theta}{r^2}$$

or $\frac{1}{r}\frac{\partial\psi}{\partial\theta} = -\frac{\mu\cos\theta}{r^2}$, as $\frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$

or $$\frac{\partial\psi}{\partial\theta} = -\frac{\mu\cos\theta}{r}$$

Integrating it with respect to θ, we get

$$\psi = -\frac{\mu\sin\theta}{r} + f(r) \quad ...(3)$$

Now, $$\frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r} \quad ...(4)$$

Using (2) and (3), (4) reduces to

$$\frac{1}{r}\left(-\frac{\mu\sin\theta}{r}\right) = -\left[\frac{\mu\sin\theta}{r^2} + f'(r)\right]$$

or $f'(r) = 0$ so that $f(r)$ = constant Hence omitting the additive constant, (3) reduces to

$$\psi = -\frac{\mu\sin\theta}{r} \quad ...(5)$$

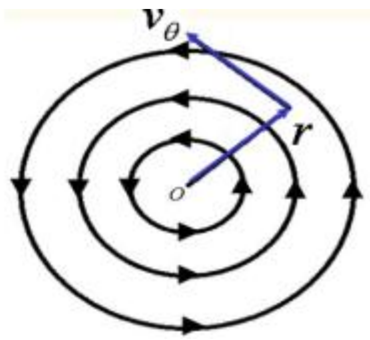
Using (2) and (5), the complex potential due to a doublet is given by

$$w = \phi + i\psi = \frac{\mu}{r}(\cos\theta - i\sin\theta) = \frac{\mu}{r}e^{-i\theta} = \frac{\mu}{re^{i\theta}} = \frac{\mu}{z}.$$

Vortex Flow

Three types of elementary flows (uniform flow, source/sink flow and doublet flow) have been discussed earlier. Now, the last elementary flow will be introduced called as *vortex flow* . Consider a flow field in which the streamlines are concentric circles about a given point which is exactly opposite case when the velocity potential and stream function for the source is interchanged. Here, the velocity along any given circular streamline is constant, while it can vary inversely with distance from one streamline to another from a common center. Referring to the Fig. 3.5.4, if v_r and v_θ are the components of velocities along radial and tangential direction respectively, then the flow field can be described as given below,

$$v_r = 0;\ v_\theta = \frac{c}{r}$$



Schematic representation of a vortex flow.

It may be easily shown that streamlines satisfy the continuity equation i.e. $\nabla \cdot \vec{V} = 0$ and the vortex flow is irrotational i.e. $\nabla \times \vec{V} = 0$ at every point except origin $(r = 0)$. In order to evaluate the constant appearing in Eq. (3.5.8), let us take the circulation around a given streamline of radius r:

$$\Gamma = \oint_C \vec{V} \cdot d\vec{s} = -v_\theta (2\pi r)$$

$$\Rightarrow v_\theta = -\frac{\Gamma}{2\pi r}$$

It may be seen by comparing Eqs. (3.5.8) and (3.5.9) that

$$c = -\frac{\Gamma}{2\pi} \Rightarrow \Gamma = -2\pi c$$

Thus, the circulation taken about all the streamlines is the same value. So, it is called as the strength of the vortex flow while the velocity field is given by Eq. (3.5.9). It may be noted that v_θ is negative when Γ is positive i.e. vortex of positive strength rotates in clockwise direction. Now, let us obtain the *velocity potential and stream function* for the vortex flow from the velocity field. By definition,

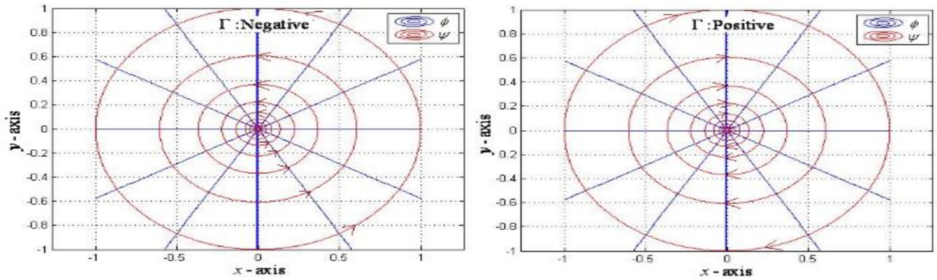
$$\frac{\partial \phi}{\partial r} = v_r = 0;\ \frac{1}{r}\frac{\partial \phi}{\partial \theta} = v_\theta = -\frac{\Gamma}{2\pi r}$$

$$\frac{1}{r}\frac{\partial \psi}{\partial \theta} = v_r = 0;\ -\frac{\partial \psi}{\partial r} = v_\theta = -\frac{\Gamma}{2\pi r}$$

Integrating the above equations, the *velocity potential and stream function* are obtained as,

$$\phi = -\frac{\Gamma}{2\pi}\theta; \quad \psi = \frac{\Gamma}{2\pi}\ln r$$

Once again it is clear from this equation that streamlines $(\psi = \text{constant})$ for a vortex flow is given by concentric circles with fixed radius while equipotential lines $(\phi = \text{constant})$ are the straight radial lines from the origin with constant θ. Both streamlines and equipotential lines are mutually perpendicular as shown in Fig. 3.5.5.



Flow nets drawn for of a free vortex flow.

MILNE–THOMSON CIRCLE THEOREM

Let $f(z)$ be the complex potential for a flow having no rigid boundaries and such that there are no singularities within the circle $|z| = a$. Then on introducing the solid cylinder $|z| = a$, with impermeable boundary, into the flow, the new complex potential for the fluid outside the cylinder is given by $W = f(z) + \bar{f}(a^2/z)$ for $|z| \geq a$.

Proof

All singularities of $f(z)$ occur in the region $|z| > a$. Hence the singularities of $f(a^2/z)$ occur in the region $a^2/|z| > a$, i.e., $|z| < a$. Thus the singularities of $\bar{f}(a^2/z)$ also lie in the region $|z| < a$.

It follows that in the region $|z| > a$, the functions $f(z)$ and $f(z) + \bar{f}(a^2/z)$ both have the same analytical singularities. Thus both functions considered as complex potentials represent the same hydrodynamical distributions in the region $|z| > a$.

The proof of the theorem is now completed by considering what happens on the circular boundary $|z| = a$. To this end, we write $z = ae^{i\theta}$ on the boundary where θ is real. Then $a^2 / z = ae^{-i\theta} = \bar{z}$ on the circular boundary. Thus, on the boundary $|z| = a$,

$$W = f(z) + \bar{f}(a^2 / z) = f(z) + \bar{f}(\bar{z}),$$

which is entirely real. Hence on the boundary,

$$\psi = \text{Im} W = 0.$$

This shows that the circular boundary is a streamline across which no fluid flows. Hence $|z| = a$ is a possible boundary for the new flow specified by the complex potential $W = f(z) + \bar{f}(a^2 / z)$.

Flow Around a Circular Cylinder

Flow around a circular cylinder can be approached from the previous example by bringing the source and the sink closer. Then we are considering a uniform flow in combination with a doublet. The stream function and the velocity potential for this flow are given by,

$$\psi = U_\infty r \sin\theta - \frac{K \sin\theta}{r}$$

$$\phi = U_\infty r \cos\theta + \frac{K \cos\theta}{r}$$

The velocity components are given by,

$$v_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \cos\theta\left(U_\infty - \frac{K}{r^2}\right)$$

$$v_\theta = -\frac{\partial\psi}{\partial r} = -\sin\theta\left(U_\infty + \frac{K}{r^2}\right)$$

It is seen that the radial velocity is zero when

$$\frac{K}{r^2} = U_\infty$$

If we recognise this particular streamline as the surface of the circular

cylinder then the radius of the cylinder a is given by,

$$a^2 = \frac{K}{U_\infty}$$

The equations for the streamline, velocity potential and the velocity components are replaced by,

$$\psi = U_\infty r \left(1 - \frac{a^2}{r^2}\right) \sin\theta$$

$$\phi = U_\infty r \left(1 + \frac{a^2}{r^2}\right) \cos\theta$$

$$v_r = U_\infty \left(1 - \frac{a^2}{r^2}\right) \cos\theta$$

$$v_\theta = -U_\infty \left(1 + \frac{a^2}{r^2}\right) \sin\theta$$

The velocity components on the surface of the cylinder are obtained by putting
r = a in the above expressions. Accordingly,

$$v_{rs} = 0 \quad and \quad v_{\theta_s} = -2U_\infty \sin\theta$$

$\sin\theta$ has a zero at 0 and 180^0 and a maximum of 1 at $\theta = 90^0$ and 270^0. The former set denotes the stagnation points of the flow and the later one
denotes the points of maximum surface velocity (of magnitude $2U_\infty$).
Thus the velocity decreases from a value of $2U_\infty$ at θ equals 90^0 to U_∞ as one moves away in a normal direction

The surface pressure distribution is calculated from Bernoulli equation.

If we denote the free stream speed and pressure as U_∞ and p_∞ we have

$$p_\infty + \frac{1}{2}\rho U_\infty^2 = p_s + \frac{1}{2}\rho v_{\theta s}^2$$

Substituting for $v_{\theta s} = -2U_\infty \sin\theta$, we have

$$p_s = p_\infty + \frac{1}{2}\rho U_\infty^2 (1 - 4\sin^2 \theta)$$

We can also express pressure in terms of pressure coefficient, C_p,

$$C_p = 1 - \left(\frac{v_s}{U_\infty}\right)^2$$

leading to

$$C_p = 1 - 4\sin^2\theta$$

A symmetry about y -axis is apparent. When compared to the experimentally observed C_p distribution we see that there is some agreement in the region between $\theta = 0^0$ and $\theta = 90^0$. But any agreement is lost in the other regions. The reasons for this are obvious. Viscous forces dominate the flow in the region to the right of the centreline giving rise to separation. The pressure tends to plateau out in a separated region, the level depending on whether it is a laminar separation or a turbulent one.

Symmetry in the theoretical C_p distribution about both y-axis and x-axis shows that drag and lift forces about the cylinder are each zero. This may also be proved by integrating pressure around the cylinder, thus,

Drag, $$D = -\int_0^{2\pi} p_s \cos\theta \; a \; d\theta$$

Lift, $$L = -\int_0^{2\pi} p_s \sin\theta \; a \; d\theta$$

By substituting for the surface pressure, p_s

$$D = -\int_0^{2\pi} p_\infty a \cos\theta d\theta - \frac{1}{2}\rho U_\infty^2 a \int_0^{2\pi} \left(\cos\theta - 4\sin^2\theta\cos\theta\right) d\theta$$

$$= \quad -p_\infty a \left[\sin\theta\right]_0^{2\pi} - \frac{1}{2}\rho U_\infty^2 a \left[\sin\theta\right]_0^{2\pi} + \frac{1}{2}\rho U_\infty^2 a \left[\frac{4}{3}\sin^3\theta\right]_0^{2\pi}$$

= -0 -0 + 0

$$L = -\int_0^{2\pi} p_\infty a \sin\theta d\theta - \frac{1}{2}\rho U_\infty^2 a \int_0^{2\pi} \left(\sin\theta - 4\sin^3\theta\right) d\theta$$

$$= \quad -p_\infty a \left[\cos\theta\right]_0^{2\pi} - \frac{1}{2}\rho U_\infty^2 a \left[\cos\theta\right]_0^{2\pi} + \frac{1}{2}\rho U_\infty^2 a \left[\frac{4}{3}\cos^3\theta - 4\cos\theta\right]_0^{2\pi}$$

= -0 -0 + 0

What we have just calculated is in contrast to the experimental results which do predict a significant drag for the flow about a circular cylinder.

Blasius Theorem

In a steady two dimensional irrotational flow given by the complex potential W = f(z), if the pressure forces on the fixed cylindrical surface C are represented by a force (X, Y) and a couple of moment M about the origin of co-ordinates, then neglecting the external forces,

$$X - iY = \frac{i\rho}{2}\int_C \left(\frac{dW}{dz}\right)^2 dz$$

$$M = \text{Real part of } \left[-\frac{\rho}{2}\int_C z\left(\frac{dW}{dz}\right)^2 dz\right]$$

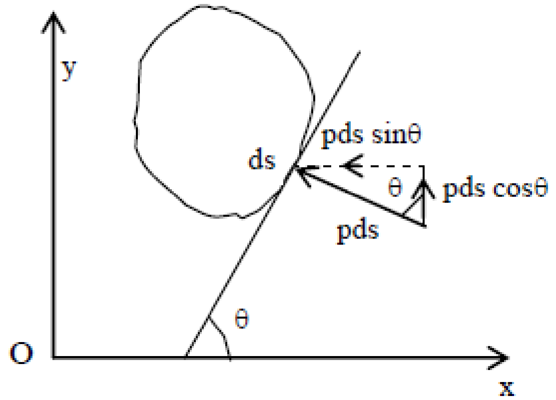
where ρ is the density of the fluid

Proof. Let ds be an element of arc at a point P(x, y) and the tangent at p makes an angle θ with the x-axis. The pressure at P(x, y) is pds, p is the pressure per unit length. pds acts along the inward normal to the cylindrical surface and its components along the co-ordinate axes are

pds cos (90 + θ), pds cosθ

i.e. $-pds\sin\theta$, $pds\cos\theta$

The pressure at the element ds is



$dF = dX + idY$

$= -p\sin\theta\, ds + ip\cos\theta\, ds$

$= ip(\cos\theta + i\sin\theta)\, ds$

| pds sinθ along negative x – axis
| ⇒ – pds sinθ along positive x – axis

$= ip\left(\frac{dx}{ds} + i\frac{dy}{ds}\right)ds \qquad \left|\ \cos\theta = \frac{dx}{ds},\quad \sin\theta = \frac{dy}{ds}\right.$

$= ip\,(dx + idy) = ip\, dz \qquad (1)$

The pressure equation, in the absence of external forces, is

$$\frac{p}{\rho} + \frac{1}{2}q^2 = \text{constant}$$

or $$p = -\frac{1}{2}\rho q^2 + k \qquad (2)$$

Further $$\frac{dW}{dz} = -u + iv = -q\cos\theta + iq\sin\theta$$

$$= -q\,(\cos\theta - i\sin\theta) = -q\,e^{-i\theta} \qquad (3)$$

and $$dz = dx + i dy = \left(\frac{dx}{ds} + i\frac{dy}{ds}\right) ds = (\cos\theta + i\sin\theta)\,ds = e^{i\theta}\,ds \qquad (4)$$

The pressure on the cylinder is obtained by integrating (1). Therefore,

$$F = X + iY = \int_C i p\,dz = \int_C i\,(k - 1/2\,\rho q^2)\,dz$$

$$= -\frac{i\rho}{2}\int_C q^2\,dz \qquad \Big|\ \because \int_C dz = 0$$

$$= -\frac{i\rho}{2}\int_C q^2\,e^{i\theta} ds$$

From here ;

$$X - iY = \frac{i\rho}{2}\int_C q^2\,e^{-i\theta}\,ds$$

$$= \frac{i\rho}{2}\int_C (q^2\,e^{-2i\theta})\,e^{i\theta}\,ds$$

$$= \frac{i\rho}{2}\int_C \left(\frac{dW}{dz}\right)^2 dz \qquad \Big|\ \text{using (3) \& (4)}$$

The moment M is given by

$$M = \int_C \left|\bar{r} \times d\bar{F}\right| = \int_C [(p ds \sin\theta)\,y + (p ds \cos\theta)\,x]$$

$$= \int_C \left[p\left(\frac{dy}{ds}\right) y\,ds + p\left(\frac{dx}{ds}\right) x\,ds\right]$$

$$= \int_C p(x\,dx + y dy]$$

$$= \int_C \left(k - \frac{1}{2}\rho q^2\right)(xdx + ydy)$$

$$= k\int_C d\left[\frac{1}{2}(x^2 + y^2)\right] - \frac{\rho}{2}\int_C q^2\,(xdx + ydy)$$

$$= -\frac{\rho}{2}\int_C q^2(xdx + ydy) \quad | \because \ 1^{st} \text{ integral}$$

vanishes.

$$= \frac{-\rho}{2}\int_C q^2 (x\cos\theta + y\sin\theta)\, ds \quad \begin{vmatrix} dx = \cos\theta ds \\ dy = \sin\theta ds \end{vmatrix}$$

$$= \text{R.P. of}\left[\frac{-\rho}{2}\int_C q^2(x+iy)(\cos\theta - i\sin\theta)ds\right]$$

$$= \text{R.P of}\left[\frac{-\rho}{2}\int_C q^2 z e^{-i\theta} ds\right]$$

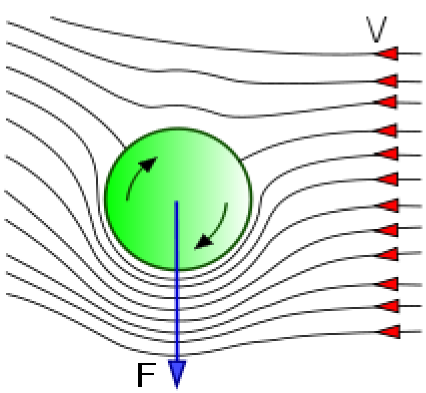
$$= \text{R.P of}\left[\frac{-e}{2}\int_C z(q^2 e^{-2i\theta})e^{i\theta} ds\right]$$

$$= \text{R.P. of}\left[-\frac{e}{2}\int_C z\left(\frac{dW}{dz}\right)^2 dz\right].$$

Hence the theorem.

The Magnus Effect

Spinning objects traveling through a viscus fluid act much like an airfoil (airplane wing)



http://schema-root.org/science/physics/effects/magnus/magnus_effect.png

First described in 1852 by Heinrich Magnus, the Magnus effect is a force generated by a spinning object traveling through a viscus fluid. The force is perpindicular to the velocity vector of the object. The direction of spin dictates the orientation of the Magnus force on the objecc. The

orientation of the force can change but it is important to remmeber that it is always perpindicualr to the direction of fluid.

Like an areofoil the rotation of the object forces some air to take a longer path around the spinning object. This air moves faster to cover the greater distance around the object in the same amount of time. The image above shows a ball rotating clockwise, we can see that the airstreams are pulled under the ball by its rotation. The resulting Magnus force is in the downward direction perpindicular to the direction of the air.

The force of the Magnus effect can be calculated with the following equation:

$$Fm = S\,(w \times v)$$

Where:
Fm =the Magnus force vector

w= angular velocity vector of the object

v=Velocity of the fluid (or velocity of object, depends on perspective)

S= air resistance coefficient across the surface of the object

Once Fm is found we can use the basic kinematic equations to predict the characteristics of spinning objects in flight.

POSSIBLE QUESTIONS

PART – B (5 x 6 = 30 Marks)

Answer all the questions

1. Discuss the flow for the complex potential $w=z^2$.
2. Explain Milne Thomson's circle theorem.
3. Show that in an irrotational incompressible inviscid 2-D fluid flow both φ & ψ satisfy the Laplace equation.
4. Explain Sink and its complex potential strength of the sink.
5. Discuss source in two dimensions.
6. Discuss the motion for the complex potential $w=iAz$.

PART – C (1 x 10 = 10 Marks)

Compulsory

1. In irrotational motions of 2-D, prove that $(\partial q/\partial x)^2+(\partial q/\partial y)^2=q.\Delta^2 q$.
2. Obtain the complex potential for the vortex.
3. Discuss on source and its complex potential.
4. A velocity field is given by $\bar{q} = -x\bar{i}+(y+t)\bar{j}$ find the stream function and the stream line for the field at t=2.
5. Show that $x^2/a^2.f(t) + y^2/b^2.\varphi(t)=1$ where $f(t)\varphi(t)$=constant is a possible form of the boundary surface.

UNIT-IV

SYLLABUS

Viscous flows – Navier-Stokes equations – Vorticity and circulation in a viscous fluid – Steady flow through an arbitrary cylinder under pressure – Staedy Couettc flow between cylinders in relative motion – Steady flow between parallel planes.

Viscous Flow

Viscous fluids are important in so many facets of everyday life that everyone has some intuition about the diverse flow phenomena that occur in practice. This course is distinctive in that it shows how quite advanced mathematical ideas such as asymptotics and partial differential equation theory can be used to analyse the underlying differential equations and hence give scientific understanding about flows of practical importance, such as air flow round wings, oil flow in a journal bearing and the flow of a large raindrop on a windscreen.

Naiver-Stokes equations.

For incompressible, viscous and Newtonian fluid, we then

obtain the Naiver-Stokes equation (plus suitable boundary and initial conditions).

$$\begin{aligned} -\mu\Delta \boldsymbol{u} + \rho(\boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}) + \nabla p &= \boldsymbol{f}, \\ \rho_t + \boldsymbol{u}\cdot\nabla\rho &= 0, \\ \operatorname{div}\boldsymbol{u} &= 0. \end{aligned}$$

In this note, we consider the constant mass density. Then the Navier-Stokes equations is simplified to

$$-\mu\Delta \boldsymbol{u} + \boldsymbol{u}_t + \boldsymbol{u}\cdot\nabla\boldsymbol{u} + \nabla p = \boldsymbol{f}, \tag{11}$$

$$\operatorname{div}\boldsymbol{u} = 0. \tag{12}$$

Note that in the momentum equation (11) the viscosity constant μ, the pressure, and the force is normalized by dividing the constant density ρ. The mass equation is equivalent to the incompressible equation.

Now let us consider the non-dimensionalization by the transformation

$$\bar{\boldsymbol{u}} = \boldsymbol{u}/U, \bar{p} = p/U^2, \bar{\boldsymbol{x}} = \boldsymbol{x}/L, \text{ and } \bar{t} = t/T.$$

Then the momentum equation (11) becomes

$$\bar{u}_{\bar{t}} + \bar{u} \cdot \nabla \bar{u} - \frac{1}{Re} \Delta \bar{u} + \nabla \bar{p} = f,$$

where

$$Re = LU/\mu.$$

Flow with the same Reynolds number (in domains with the same shape) will be similar. Therefore one can construct an experiment using practical size in lab to model flow in large scales. As Re increases, the equation becomes inviscid. From the definition of Re, for large scale problem (L or U big), the viscosity is tiny. In the limiting case, $Re = \infty, \mu = 0$, Navier-Stokes equation becomes the so-called Euler equation. The fluid is called ideal fluid. Note that N-S equation is second order while Euler is first order. The boundary conditions $u = 0$ should be changed accordingly to $u \cdot n = 0$. If there is a mismatch in the boundary condition, it cause problems near the boundary, known as boundary layer effect;

There are mainly three difficulties associated to the Navier-Stokes equations:

(1) First it is time dependent. Stability in time could be an issue for both PDE and numerical methods. For example, we still do not know wether solutions to N-S equation will below up in finite time or not (for a reasonable large class of initial conditions).

(2) Second, it is nonlinear. Efficient numerical methods can be developed for this special quadratic nonlinearity. But the convection $u \cdot \nabla$ derivative, especially when it is dominate ($\mu \ll 1$), will cause a serious trouble in the numerical computation.

(3) Third it is a coupled system of (u, p). The pressure p can be eliminate if we consider u in the exactly divergence free space. But the divergence free condition is hardly to impose in numerical methods.

We shall solve this tangle by focusing on one difficulty at a time. We first skip the time derivative and nonlinearity to get the steady-state Stokes equations

$$-\mu \Delta u + \nabla p = f, \quad (13)$$

$$\operatorname{div} u = 0. \quad (14)$$

Vorticity: The vorticity of a flow is defined as the curl of the velocity field:

$$\boxed{\textbf{vorticity}: \quad \vec{w} = \nabla \times \vec{u}}$$

It is a microscopic measure of rotation (vector) at a given point in the fluid, which can be envisioned by placing a paddle wheel into the flow. If it spins about its axis at a rate Ω, then $w = |\vec{w}| = 2\Omega$.

Circulation: The circulation around a closed contour C is defined as the line integral of the velocity along that contour:

$$\textbf{circulation}: \qquad \Gamma_C = \oint_C \vec{u} \cdot \mathrm{d}\vec{l} = \int_S \vec{w} \cdot \mathrm{d}\vec{S}$$

where S is an *arbitrary* surface bounded by C. The circulation is a macroscopic measure of rotation (scalar) for a finite area of the fluid.

Steady flow through an arbitrary cylinder under pressure

Now, we consider a steady-state, Navier with no external forces and take axes Ox_1, Ox_2, Ox_3, where Ox_1 is parallel to the generators of the cylinder and Ox_2, Ox_3 are perpendicular thereto. We look for a solution in which the flow is entirely parallel to the generators of the cylinder; thus

$$u_1 = u_1(x_1, x_2, x_3)\ , \quad u_2 = u_3 = 0\ , \tag{3.25}$$

and so the Navier equations become (with dimensions)

$$\frac{\partial u_1}{\partial x_1} = 0\ , \quad \frac{\partial p}{\partial x^2} = 0\ , \quad \frac{\partial p}{\partial x_3} = 0\ , \tag{3.26a}$$

$$\rho_0 u_1 \frac{\partial u_1}{\partial x_1} = -\frac{\partial p}{\partial x_1} + \mu_0 \nabla^2 u_1\ . \tag{3.26b}$$

Equations (3.26a) show that:

$$u_1 = u(x_2, x_3)\ , \quad p = p(x_1)\ ,$$

and in place of (3.26b), we obtain:

$$\frac{\mathrm{d}p}{\mathrm{d}x_1} = \mu_0 \left(\frac{\partial^2 u}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_3^2} \right)\ ,$$

where the left-hand side is a function only of x_1, and the right-hand side is a function only of x_2 and x_3. Consequently, we deduce that both are constants (denoted by Π_0), and write

$$\frac{1}{\mu_0}\frac{\mathrm{d}p}{\mathrm{d}x_1} = \frac{\partial^2 u}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_3^2} = -\frac{\Pi_0}{\mu_0} \ . \tag{3.27}$$

Equation (3.27) must be solved subject to the boundary conditions that $u = 0$ on the surface of the cylinder. The computation of full pipe flow for arbitrary cross-sectional shapes is based on solving the Poisson differential equation (3.27). Solutions for different cross-sections can be found in Shah and London (1978). We note also that an exact solution of the Navier equations can be given for a pipe of concentric circular cross-section (a so-called *annulus*); see Müller (1936).

The Case of a Circular Cylinder

For a circular cylinder of radius a, we transform into polar coordinates (r, θ, x), and note that the velocity $u(x_2, x_3)$ along the tube will be a function of r alone. Thus,

$$\frac{\partial^2 u}{\partial x_2^2} + \frac{\partial^2 u}{\partial x_3^2} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = -\frac{\Pi_0}{\mu_0} \ ,$$

which integrates to give

$$u(r) = A\ln r + B - \frac{1}{4\mu_0}\Pi_0 r^2 \ , \tag{3.28}$$

where A and B are arbitrary constants of integration. The constant A must be zero if the solution is to be physically acceptable along the axis $r = 0$, and B is then determined by the no-slip condition,

$$u = 0 \text{ on } r = a \quad \Rightarrow \quad B = \frac{1}{4\mu_0}\Pi_0 a^2 \ .$$

Thus, the solution is (with dimensions)

$$u(r) = \left(\frac{a^2}{4\mu_0}\right) \Pi_0 \left[1 - \left(\frac{r}{a}\right)^2\right] . \tag{3.29}$$

From this velocity profile, we may deduce the *mass flux* per unit time passing any cross-section of the tube:

$$M = \int_0^a \rho_0 u 2\pi r \, \mathrm{d}r = \rho_0 \pi \left(\frac{a^4}{8\mu_0}\right) \Pi_0 . \tag{3.30}$$

This result is known as *Poiseuille's law* – the basis of a method of measuring the viscosity of a fluid. Obviously, solution (3.29) is only valid "far" from the "entry flow" near the entrance to the tube, where the fully developed region with a velocity profile given by (3.29) has not been attained.

The Case of an Annular Region
Between Concentric Cylinders

If we consider an annular region between concentric cylinders of radii a and b ($b < a$), then the velocity profile for the flow through this annular region is

$$u(r) = \left(\frac{a^2}{4\mu_0}\right) \Pi_0 \left\{\left[1 - \left(\frac{r}{a}\right)^2\right] - \left[\frac{\ln(r/a)}{\ln(b/a)}\right]\left[1 - \left(\frac{b}{a}\right)^2\right]\right\} . \tag{3.31}$$

From (3.31), when b tends to zero, we derive (3.29) again. The resulting (3.31) is obtained from the solution (3.28), if we consider no-slip boundary conditions on the cylinders $r = a$ and $r = b$ (in this case the constant A is not zero because the singular axis $r = 0$ is outside the annular region).

Couette's Flow

It is the flow between two parallel planes (flat plates) one of which is at rest and other moving with velocity U parallel to the fixed plate. Here, the constants A and B in (7) are determined from the conditions

$$u = 0,\ y = 0 \ \}$$

and $$u = U,\ y = h \tag{8}$$

Using these conditions, we get

$$B = 0,\ U = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{h^2}{2} + Ah$$

$$\Rightarrow \qquad A = \frac{U}{h} - \frac{h}{2\mu}\left(\frac{dp}{dx}\right),\ B = 0 \qquad (9)$$

Therefore, the solution (7) becomes

$$u = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{y^2}{2} + y\left[\frac{U}{h} - \frac{h}{2\mu}\left(\frac{dp}{dx}\right)\right] \qquad (10)$$

$$= \frac{y^2 - hy}{2\mu}\left(\frac{dp}{dx}\right) + \frac{Uy}{h} \qquad (*)$$

$$= \frac{U}{h}y - \frac{h^2}{2\mu}\frac{dp}{dx}\frac{y}{h}\left(1 - \frac{y}{h}\right) \qquad (11)$$

We note that equation (10) represents a parabolic curve.

This equation is known as the equation of Couette's flow. Thus the velocity profile for Couetle's flow is parabolic. The flow Q per unit breadth is given by

$$Q = \int_0^h u\,dy = \int_0^h \left[\frac{1}{m}\frac{dp}{dx}\frac{y^2}{2} + y\left(\frac{U}{h} - \frac{h}{2\mu}\frac{dp}{dx}\right)\right]dy$$

$$= \frac{hU}{2} - \frac{h^3}{12\mu}\frac{dp}{dx} \qquad (**)$$

$$= \frac{hU}{2} + \frac{h^3}{12\mu}P, \quad P = -\frac{dp}{dx} \qquad (12)$$

In non-dimensional form (11) can be written as

$$\frac{u}{U} = \frac{y}{h} + \alpha\frac{y}{h}\left(1 - \frac{y}{h}\right) \qquad (13)$$

where $$\alpha = \frac{h^2}{2\mu U}\left(-\frac{dp}{dx}\right) \qquad (14)$$

α is the non-dimensional pressure gradient. If $\alpha > 0$, the pressure is decreasing in the direction of flow and the velocity is positive between the plates. If $\alpha < 0$, the equation (13) can be put as

$$\frac{u}{U} = \frac{y}{h}(1+\alpha) - \frac{\alpha y^2}{h^2} \qquad (15)$$

The pressure is increasing in the direction of flow and the reverse flow begins when $\alpha < -1$

| $\because$ y is small. i.e.

y^2 is neglected

If $\alpha = 0$ $\left(\text{i.e.} \frac{dp}{dx} = 0\right)$, then the particular case is known as simple Couette's flow and the velocity is given by

$$\frac{u}{U} = \frac{y}{h}$$

which gives u = 0 where y = 0 i.e. on the stationary plane.

(i) Average and Extreme Values of Velocity : The average velocity of a Couette's flow between two parallel straight plates is given by

$$u_0 = \frac{1}{h}\int_0^h u\, d_y \qquad (16) \qquad | \because u = u(y)$$

Using the value of u from (13), we get

$$u_0 = \frac{1}{h}\int_0^h \left[\frac{Uy}{h} + U\alpha \frac{y}{h}\left(1 - \frac{y}{h}\right)\right] dy$$

$$= \frac{Uh^2}{2h^2} + U\alpha\left(\frac{h^2}{2h^2} - \frac{h^3}{3h^3}\right)$$

$$= \frac{U}{2} + \frac{U\alpha}{6} = \left(\frac{1}{2} + \frac{\alpha}{6}\right)U \qquad (17)$$

$$= \frac{U}{2} - \frac{\mu^2}{12\mu}\frac{dp}{dx} = \frac{U}{2} + \frac{h^2}{12\mu}P,\ P = -\frac{dp}{dx} \qquad (18)$$

In the case of a simple Couette's flow, the velocity increases from zero on the stationary plate to U on the moving plate such that the average velocity is $\frac{U}{2}$.

When the non-dimensional pressure gradient is $\alpha = -3$, then from (17), we get $u_0 = 0$. This means that there is no flow because the pressure gradient is balanced by the viscous force.

For maximum & minimum values of u, we have

$$\frac{du}{dy} = 0 \Rightarrow \frac{U}{h} + U\alpha\left(\frac{1}{h} - \frac{2y}{h^2}\right) = 0$$

$$\Rightarrow y = \left(\frac{1+\alpha}{2\alpha}\right)h \qquad (19)$$

From here, $\frac{y}{h} = 1, \text{ when } \alpha = 1$

and $\frac{y}{h} = 0, \text{ when } \alpha = -1$

So, from (13), we get

$$u = \left[\frac{1+\alpha}{2\alpha} + \alpha\left(\frac{1+\alpha}{2\alpha}\right)\left(1 - \frac{1+\alpha}{2\alpha}\right)\right]U$$

$$= \frac{(1+\alpha)^2}{4\alpha} U$$

and thus u is maximum for $\alpha \geq 1$ and minimum for $\alpha \leq -1$.

(ii) Shearing Stress : The shearing stress (drag per unit area) in a Couette's flow is given by

$$\sigma_{yx} = \mu\frac{du}{dy} = \mu\frac{U}{h} + \frac{\mu\alpha U}{h}\left(1 - \frac{2y}{h}\right) \qquad (20)$$

$$= \frac{\mu U}{h}, \text{ for a simple Couette's flow } (\alpha = 0).$$

When $y = \frac{h}{2}$, then the second term in (20) vanishes. Thus the shearing stress is independent of α on the line midway between the flow. The shearing stress at the stationary plane is positive for $\alpha > -1$ and negative for $\alpha < -1$.
| y = 0 at stationary plate

The velocity gradient at the stationary plate is zero for $\alpha = -1$ and the shearing stress is zero for $\alpha = -1$.

Thus $\sigma_{yx} \gtrless 0$ when $\alpha \gtrless -1$.

Further, drag per unit area on the lower and the upper plates are obtained from (20) by putting y = 0 and y = h, as

$$\frac{\mu U}{h} + \frac{\mu\alpha U}{h} \text{ and } \frac{\mu U}{h} - \frac{\mu\alpha U}{h}$$

combining the two results, drag per unit area on the two plates is

$$\frac{\mu U}{h} \pm \frac{\mu\alpha U}{h} \text{ i.e. } \frac{\mu U}{h} \mp \frac{h}{2}\frac{dp}{dx} \qquad (***)$$

i.e. $$\frac{\mu U}{h} \pm \frac{Ph}{2}, \; P = -\frac{dp}{dx}$$

Plane Poiseuille Flow : A flow between two parallel stationary plates is said to be a plane Poiseuille Flow.

The origin is taken on the line midway between the plates which are placed at a distance h and x-axis is along this line.

The conditions to be used in this problem are

$$u = 0, \text{ when } y = \pm\frac{h}{2} \qquad (21)$$

Using these conditions in (7), we get

$$A = 0, \; B = \frac{1}{\mu}\left(-\frac{dp}{dx}\right)\frac{h^2}{8}$$

and thus the solution (7) is

$$u = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\left(\frac{y^2}{2} - \frac{h^2}{8}\right) \qquad (22)$$

This represents a parabola and thus the laminar flow in a Plane Poiseuille Flow is parabolic.

(i) Average and Maximum Velocity : For extreme values of u, we have $\frac{du}{dy} = 0$ and thus from (22), we get

$$\frac{1}{\mu}\left(\frac{dp}{dx}\right) y = 0 \quad \Rightarrow y = 0$$

Therefore ,
$$U_{max} = \frac{h^2}{8\mu}\left(-\frac{dp}{dx}\right) \qquad (23)$$

The average velocity in the plane Poiseuille flow is defined by

$$u_0 = \frac{1}{h}\int_{-h/2}^{h/2} u\, dy$$

Using the value of u from (22), we get

$$u_0 = \frac{1}{h}\int_{-h/2}^{h/2} \frac{-h^2}{8\mu}\frac{dp}{dx}\left(1 - \frac{4y^2}{h^2}\right)dy$$

$$= \frac{2}{3}\left(\frac{-h^2}{8\mu}\frac{dp}{d_x}\right) = \frac{2}{3}U_{max} \qquad (24)$$

From (23) & (24), decrease in the pressure is given by

$$\frac{dp}{dx} = -\frac{8\mu}{h^2}u_{max.} = \frac{-8\mu}{h^2}\frac{3}{2}u_0 = \frac{-12\mu}{h^2}u_0 \qquad (25)$$

This further shows that $\frac{dp}{dx}$ is a negative constant.

(ii) Shearing Stress : The shearing stress at a plate (lower plate) for a plane Poiseuille Flow is

$$(\sigma_{yx})_{y=\frac{-h}{2}} = \left(\mu\frac{du}{dy}\right)_{y=\frac{h}{2}} = -\mu\frac{1}{\mu}.\frac{dp}{dx}.\frac{h}{2}$$

$$= -\frac{h}{2}\frac{dp}{dx}$$

$$= \frac{4\mu}{h}u_{max}. \qquad (26)$$

The local frictional (skin) co-efficient C_f is defined by

$$C_f = \frac{(\sigma_{yx})_{-h/2}}{\rho u_{0/2}^2} = \frac{4\mu}{h}u_{max.} \Big/ \frac{\rho u_0^2}{2}$$

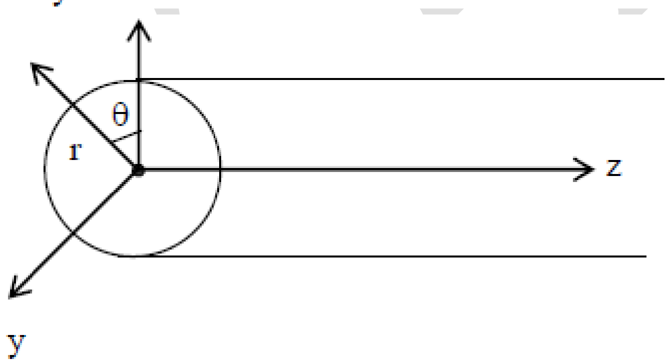
$$= \frac{4\mu}{\rho h}\left(\frac{3}{2}\frac{u_0}{u_0^2/2}\right) = \frac{12v}{h u_0} = \frac{12}{R_e}$$

Where $R_e = \frac{u_0 h}{v}$ is the Reynolds number of the flow based on the average velocity and the channel height.

Steady Flow Through Tube of Uniform Circular Cross-section (Poiseuille's Flow or Hagen-Poiseuill's Flow)

We consider a laminar flow, in the absence of body forces, through a long tube of uniform circular cross-section with axial symmetry.

Let z-axis be taken along the axis of the tube and the flow be in the direction of z-axis. Since the flow is along z-axis, the radial and transverse components of velocity are absent.



Thus $q_r = q_\theta = 0$ $\qquad \overline{q} = (q_r, q_\theta, q_z)$

The continuity equation for a viscous incompressible fluid gives.

$$\frac{\partial q_z}{\partial z} = 0 \Rightarrow q_z = q_z(r) \quad (1) \quad | \because \text{ axial symmetry i.e. independent of } \theta$$

The equations of motion in cylindrical co-ords are

$$\rho\left(\frac{dq_r}{dt}-\frac{{q_\theta}^2}{r}\right)=\rho\cdot X_r-\frac{\partial p}{\partial r}+\mu\left(\nabla^2 q_r-\frac{q_r}{r^2}-\frac{2}{r^2}\frac{\partial q_\theta}{\partial\theta}\right)$$

$$\rho\left(\frac{dq_\theta}{dt}+\frac{q_r q_\theta}{r}\right)=\rho\cdot X_\theta-\frac{1}{r}\frac{\partial p}{\partial\theta}+\mu\left(\nabla^2 q_\theta+\frac{2}{r^2}\frac{\partial q_r}{\partial\theta}-\frac{q_\theta}{r^2}\right)$$

$$\rho\frac{dq_z}{dt}=\rho X_z-\frac{\partial p}{\partial z}+\mu\nabla^2 q_z$$

where $\frac{d}{dt}=\frac{\partial}{\partial t}+q_r\frac{\partial}{\partial r}+q_\theta\frac{1}{r}\frac{\partial}{\partial\theta}+q_z\frac{\partial}{\partial z}$,

and $\overline{X}=(X_r,X_\theta,X_z)$

In the present case $\frac{\partial}{\partial t}\equiv 0$ and $q_r=q_\theta=0,\ \overline{X}=0$

Thus from the first two equations, we get

$$\frac{\partial p}{\partial r}=\frac{\partial p}{\partial\theta}=0 \quad\Rightarrow p=p(z) \qquad (2)$$

The third equation gives.

$$0=\frac{-\partial p}{\partial z}+\mu\nabla^2 q_z \qquad \Big|\ \because q_z=q_z(r)\ \text{and}\ r\ \text{is constant w.r.t. } t.$$

or
$$\frac{dp}{dz}=\mu\nabla^2 q_z=\mu\left(\frac{d^2q_z}{dr^2}+\frac{1}{r}\frac{dq_z}{dr}\right) \qquad (3)$$

(In cylindrical co-ordinates $\nabla^2\equiv\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2}$)

since q_z is a function of r only (from (1)) and p is a function of z only (from (2)).

Equation (3) can be put as

$$\mu\left(r\frac{d^2q_z}{dr^2}+\frac{dq_z}{dr}\right)=r\frac{dp}{dz}$$

i.e. $$\frac{d}{dr}\left(r\frac{dq_z}{dr}\right) = \frac{r}{\mu}\frac{dp}{dz}$$

Integrating, w.r.t. r, we get.

$$r\frac{dq_z}{dr} = \frac{1}{\mu}\left(\frac{dp}{dz}\right)\frac{r^2}{2} + A$$

i.e. $$\frac{dq_z}{dr} = \frac{1}{2\mu}\left(\frac{dp}{dz}\right)r + \frac{A}{r}$$

Integrating again, we get

$$q_z = \frac{1}{u\mu}\left(\frac{dp}{dz}\right)r^2 + A\log r + B \qquad (4)$$

where A and B are constants to be determined from the boundary conditions.

The first boundary condition is obtained from the symmetry of the flow such that

$$\frac{dq_z}{dr} = 0 \quad \text{on } r = 0 \qquad (5)$$

and the second boundary condition is

$$q_z = 0, \text{ when } r = a \qquad (6)$$

where a is the radius of the tube. Using these conditions, we get

$$A = 0,\ B = -\frac{1}{4\mu}\left(\frac{dp}{dz}\right)a^2 = \frac{1}{4\mu}\left(-\frac{dp}{dz}\right)a^2$$

Thus, the solution (4) becomes

$$q_z = \frac{1}{4\mu}\left(\frac{-dp}{dz}\right)(a^2 - r^2) \qquad (7)$$

This represents a paraboloid of revolution and thus the velocity profile is parabolic.

(i) The Max x Average Velocity : For extreme values of q_z, we have

$$\frac{dq_z}{dr} = 0 \qquad | \because q_z \text{ is a function of r only}$$

From (7), it implies that r = 0 and thus

$$q_{max.} = \frac{a^2}{4\mu}\left(-\frac{dp}{dz}\right) \tag{8}$$

where $\frac{dp}{dz}$ is a negative constant.

From (7) and (8), the velocity distribution, in non dimensional from, is given by

$$\frac{q_z}{q_{max}} = 1 - \left(\frac{r}{a}\right)^2$$

The average velocity is defined by

$$q_0 = \frac{1}{\pi a^2} \int_0^{2\pi} \int_0^a q_z \, r \, dr \, d\theta$$

Using the value of q_z, we get

$$q_0 = \frac{a^2}{8\mu}\left(-\frac{dp}{dz}\right) = \frac{1}{2} q_{max}.$$

The average velocity is therefore half of the maximum velocity

The volume of fluid discharged over any section per unit time (i.e. volumetric flow) is defined as

$$Q = \int_0^a q_z . 2\pi r \, dr$$

Using (7), it is obtained to be

$$Q = \frac{\pi a^4}{8\mu}\left(-\frac{dp}{dz}\right) = \frac{1}{2}\pi a^2\left[\frac{a^2}{4\mu}\left(\frac{-dp}{dz}\right)\right] = \frac{1}{2}\pi a^2 \, q_{max}. \qquad (9)$$

(ii) Shearing Stress : The shearing stress in Poiseuille's flow is given by

$$\sigma_{rz} = -\mu\frac{dq_z}{dr} = -\mu\frac{1}{4\mu}\left(\frac{dp}{dz}\right)(2r) = -\frac{r}{2}\left(\frac{dp}{dz}\right)$$

On the boundary of the tube, we have

$$(\sigma_{rz})_{r=a} = -\frac{a}{2}\left(\frac{dp}{dz}\right) = \frac{a}{2}\left(\frac{-dp}{dz}\right) = \frac{2\mu}{a}.q_{max}. \qquad (10)$$

The local frictional (skin) co-efficient C_f for laminar flow through a circular pipe is

$$C_f = \frac{(\sigma_{rz})_{r=a}}{\rho\, q_0^2/2} = \frac{2\mu}{a}\frac{q_{max}}{\rho\, q_0^2/2}$$

$$= \frac{4\mu}{\rho a}\frac{2q_0}{q_0^2} = \frac{8\mu}{\rho a}\frac{1}{q_0} = \frac{16}{R_e}$$

Where $R_e = 2aq_0/\nu$ is the Reynolds number. When R_e is less than the critical Reynolds number, which is 2300 in this flow problem, the flow is laminar but if $R_e > 2300$, the flow ceases to be laminar and becomes turbulent. Thus, in this problem, $R_e < 2300$.

POSSIBLE QUESTIONS

PART – B (5 x 6 = 30 Marks)

Answer all the questions

1. Explain Steady flow-through an arbitrary cylinder under pressure.
2. Obtain the Helmholtz equations for vorticity of viscous fluid.
3. Explain Vorticity of viscous fluid.
4. Explain Navier Strokes equation.
5. Discuss about Plane coutte flow.
6. Explain about Steady flow between parallel plane.

PART – C (1 x 10 = 10 Marks)

Compulsory

1. Discuss about Circulation in a viscous fluid.
2. Discuss about Energy equation.
3. Explain Reynold's numbers.
4. Explain about Steady coutte flow between cylinder in Relative motion.
5. Explain the Lift force.

UNIT-V

SYLLABUS

Laminar Boundary Layer in incompressible flow: Boundary Layer concept – Boundary Layer equations – Displacement thickness, Momentum thickness – Kinetic energy thickness – integral equation of boundary layer – flow parallel to semi infinite flat plate – Blasius equation and its solution in series.

Reynolds Number.

$$\frac{\nu}{UL}, \quad \frac{P}{\rho U^2}, \quad \frac{LX}{U^2} \qquad (6)$$

The first non-dimensional number in (6) ensures

dynamical similarity at corresponding points near the boundaries where viscous effects supervene. Its reciprocal is called the Reynolds number and is denoted by R_e so that

$$R_e = \frac{UL}{\nu}$$

Buckingham π-theorem.

The π-theorem makes use of the following

assumptions

(i) It is possible to select always m independent fundamental units in a physical phenomenon (in mechanics, m = 3 i.e. length, time, mass or force)

(ii) There exist quantities, say $Q_1, Q_2, \ldots, Q_n$ involved in a physical phenomenon whose dimensional formulae may be expressed in terms of m fundamental units

(iii) There exists a functional relationship between the n dimensional quantities $Q_1, Q_2, \ldots, Q_n$, say

$$\phi(Q_1, Q_2, \ldots, Q_n) = 0 \qquad (1)$$

(iv) Equation (1) is independent of the type of units chosen and is dimensionally homogeneous i.e. the quantities occurring on both sides of the equation must have the same dimensions.

Statement :- If $Q_1, Q_2, \ldots, Q_n$ be n physical quantities involved in a physical phenomenon and if there are $m(< n)$ independent fundamental units in this system, then a relation

$$\phi(Q_1, Q_2, \ldots, Q_n) = 0$$

is equivalent to the relation

$$f(\pi_1, \pi_2, \ldots, \pi_{n-r}) = 0,$$

$$f(\pi_1, \pi_2, \ldots, \pi_{n-r}) = 0,$$

where $\pi_1, \pi_2, \ldots, \pi_{n-r}$ are the dimensionless power products of $Q_1, Q_2, \ldots, Q_n$ taken $r + 1$ at a time, r being the rank of the dimensional matrix of the given physical quantities.

Proof. Let $Q_1, Q_2, \ldots, Q_n$ be n given physical quantities and let their dimensions be expressed in terms of m fundamental units $u_1, u_2, \ldots, u_m$ in the following manner

$$[Q_1] = \left[u_1^{a_{11}} u_2^{a_{21}} \ldots u_m^{a_{m1}}\right]$$

$$[Q_2] = \left[u_1^{a_{12}} u_2^{a_{22}} \ldots u_m^{a_{m2}}\right]$$

....

....

$$[Q_n] = \left[u_1^{a_{1n}} u_2^{a_{2n}} \ldots u_m^{a_{nm}}\right]$$

so that a_{ij} is the exponent of u_i in the dimension of Q_j . The matrix of dimensions i.e. the dimensional matrix of the given physical quantities is written as

$$\begin{array}{c} \\ u_1: \\ u_2: \\ \\ \\ u_m: \end{array} \begin{array}{ccc} Q_1: & Q_2: & Q_n: \end{array}$$

$$\begin{pmatrix} u_1: & a_{11} & a_{12}\ldots\ldots a_{1n} \\ u_2: & a_{21} & a_{22}\ldots\ldots a_{2n} \\ & \ldots\ldots\ldots\ldots & \\ & \ldots\ldots\ldots\ldots & \\ u_m: & a_{m_1} & a_{m_2}\ldots\ldots a_{mn} \end{pmatrix}$$

This m×n matrix is usually denoted by A.

Now, let us form a product π of powers of $Q_1, Q_2, \ldots\ldots, Q_n$, say

$$\pi = Q_1^{x_1}\ Q_2^{x_2} \ldots\ldots Q_n^{x_n}$$

then $[\pi] =$

$$\left[\left(u_1^{a_{11}}u_2^{a_{21}}\ldots u_m^{a_{m1}}\right)^{x_1}\left(u_1^{a_{12}}u_2^{a_{22}}\ldots u_m^{a_{m2}}\right)^{x_2}\ldots\ldots\left(u_1^{a_{1n}}u_2^{a_{2n}}\ldots u_m^{a_{mn}}\right)^{x_n}\right]$$

In order that the product π is dimensionless, the powers of $u_1, u_2, \ldots, u_m$ should be zero i.e. M^0, L^0, T^0 etc. Thus, we must have

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots + a_{1n}x_n = 0$$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots + a_{2n}x_n = 0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{m1} + x_1 + a_{m2}x_2 + \ldots\ldots + a_{mn}x_n = 0$$

This is a set of m homogeneous equations in n unknowns and in matrix form can be written as

$$AX = 0, X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

Now, from matrix algebra, we know the result that if there are m homogeneous equations in n unknowns, then the number of independent solutions will be n–r, where r is the rank of the matrix of co-efficients, and any other solution

can be expressed as a linear combination of these linearly independent solutions. Further there will be only r independent equations in the set of equations.

Thus if r is the rank of the dimensional matrix A, then the number of linearly independent solutions of the matrix equation AX = 0 are n–r. So, corresponding to each independent solution of X, we will have a dimensionless product π and therefore the number of dimensionless products in a complete set will be n–r

Therefore, $\phi(Q_1, Q_2, \ldots\ldots, Q_n) = 0$

$\Rightarrow$ $f(\pi_1, \pi_2, \ldots\ldots, \pi_{n-r}) = 0$

Hence the theorem.

Prandtl's Boundary Layer (case of small viscosity)

The simple problems of fluid motion which can be considered are divided into two classes according as the corresponding Reynolds number is small or large. In the case of small Reynolds number, viscosity is predominant and the inertia terms in the equations may be regarded as negligible. The case of large Reynolds number in which the frictional terms are small and inertia forces are predominant, was investigated by the German Scientist Ludwig Prandtl in 1904. He made an hypothesis that for fluids with very small viscosity i.e. large Reynolds number, the flow about a solid boundary can be divided into the following two regions.

(i) A thin layer in the neighbourhood of the body, known as the boundary layer, in which the viscous effect may be considered to be confined. The smaller the viscosity i.e. the larger the Reynolds number, the thinner is this layer. Its thickness is denoted by δ. In such layer, the velocity gradient normal to the wall of the body is very large.

(ii) The region outside this layer where the viscous effect may be considered as negligible and the fluid is regarded as non-viscous.

On the basis of this hypothesis, Prandtl simplified the Navier-Stokes equations to a mathematical tractable form which are termed as Prandtl boundary layer equations and thus he succeeded in giving a physically penetrating explanation of the importance of viscosity in the assessment of frictional drag. The theory was first developed for laminar flow of viscous incompressible fluids but was, later on, extended to include compressible fluids and turbulent flow. However, we shall consider only the case of incompressible fluids.

In the discussion of unsteady flow over a flat plate, we had obtained that

$$\delta \simeq 4\sqrt{\nu t}$$

i.e. the boundary layer thickness is proportional to the square root of kinematic viscosity. The thickness is very small compared with a linear dimension L of the body i.e. $\delta << L$.

Boundary Layer equation in Two-dimensions. The viscosity of water, air etc is very small. The Reynolds number for such fluids is large. This led Prandtl to introduce the concept of the boundary layer. We now discuss the mathematical procedure for reducing Navier-Stokes equations to boundary layer equations. The procedure is known as order of magnitude approach.

Let us consider a flow around a wedge submerged in a fluid of very small viscosity

At the stagnation point O, the thickness of the boundary layer is zero and it increases slowly towards the rear of the wedge. The velocity distribution and the pattern of streamlines deviate only slightly from those in the potential flow. We take the x-axis along the wall of the wedge and y-axis perpendicular to it, so that the flow is two-dimensional in the xy-plane. Within a very thin boundary layer of thickness δ, a very large velocity gradient exists i.e. the velocity u parallel to the wall in the boundary layer increases rapidly from a value zero at the wall to a value U of the main stream at the edge of the boundary layer.

The Navier–Stokes equations, in the absence of body forces, for two dimensional flow, are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \qquad (2)$$

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (3)$$

In studying the unsteady flow over a flat plate, we found that the thickness of the boundary layer δ is proportional to the square root of the kinematic viscosity *ν* which is indeed very small. For this reason $\delta << x$ except near the stagnation point 0 where the boundary layer begins. In order to compare the order of magnitude of the individual terms in the above equations, we put them in non-dimensional form by introducing the non-dimensional notations

$$x^* = \frac{x}{l}, y^* = \frac{y}{\delta}, u^* \frac{u}{U}, v^* = \frac{v}{V}\; t^* = \frac{t}{l/U}, p^* = \frac{p}{p_\infty} \qquad (4)$$

where l, δ, U, V and p_∞ are certain reference values of the corresponding quantities x, y, u , v and p respectively. The non-dimensional quantities are all of order unity. The continuity equation in non-dimensional form is

$$\frac{U}{l}\frac{\partial u^*}{\partial x^*}+\frac{V}{\delta}\frac{\partial v^*}{\partial y^*}=0 \qquad (5)$$

Integrating, we get

$$\frac{U}{l}\int_0^1 \frac{\partial u^*}{\partial x^*}dy^* + \frac{V}{\delta}\int_0^1 \frac{\partial v^*}{\partial y^*}dy^* = 0$$

or $$\frac{V}{U} = -\frac{\delta}{l}\int_0^1 \frac{\partial u^*}{\partial x^*}dy^*, \text{ where } (v^*)_{y^*=1} = 1 \qquad (6)$$

Since the integral in (6) is of the order of unity, the ratio $\frac{V}{U}$ is of order $\frac{\delta}{l}$.

Therefore $V << U$.

We now obtain the non-dimensional form of (1) using (4) such that

$$\frac{U^2}{l}\frac{\partial u^*}{\partial t^*}+\frac{U^2}{l}u^*\frac{\partial u^*}{\partial x^*}+\frac{UV}{\delta}v^*\frac{\partial u^*}{\partial y^*}=\frac{-p_\infty}{\rho l}\frac{\partial p^*}{\partial x^*}+\frac{\nu U}{l^2}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{l^2}{\delta^2}\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

or

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+\frac{V}{U}\;\frac{l}{\delta}v^*\frac{\partial u^*}{\partial y^*}=-\frac{p_\infty}{\rho U^2}\frac{\partial p^*}{\partial x^*}+\frac{l}{R_e}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{l^2}{\delta^2}\frac{\partial^2 u^*}{\partial y^{*2}}\right) \quad (7)$$

$$1 \qquad 1 \qquad \delta\;\frac{1}{\delta} \qquad 1 \qquad\qquad \delta^2 \qquad 1 \qquad \frac{1}{\delta^2}$$

The order of the terms involved are indicated.

Reynolds number, $R_e=\frac{lU}{\nu}\Rightarrow\frac{1}{R_e}=\frac{\nu}{lU}=0(\delta)^2$ as δ is proportional to $\nu^{1/2}$.

Similarly, the non-dimensional form of (2) is

$$\frac{UV}{l}\frac{\partial v^*}{\partial t^*}+\frac{UV}{l}u^*\frac{\partial v^*}{\partial x^*}+\frac{V^2}{\delta}v^*\frac{\partial v^*}{\partial y^*}$$

$$=-\frac{p_\infty}{\rho\delta}\frac{\partial p^*}{\partial y^*}+\nu\left(\frac{V}{l^2}\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{V}{\delta^2}\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

or
$$\frac{V}{U}\frac{\partial v^*}{\partial t^*}+\frac{V}{U}u^*\frac{\partial v^*}{\partial x^*}+\frac{V^2}{U^2}\frac{l}{\delta}v^*\frac{\partial v^*}{\partial y^*}$$

$$\delta \qquad \delta \qquad \delta^2\;\frac{1}{\delta}$$

$$=\frac{-p_\infty}{\rho U^2}\frac{l}{\delta}\frac{\partial p^*}{\partial y^*}+\frac{\nu V l}{l^2U^2}\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{l^2\partial^2 v^*}{\delta^2\partial y^{*2}}\right)$$

$$=\frac{-p_\infty}{\rho U^2}\frac{l}{\delta}\frac{\partial p^*}{\partial y^*}+\frac{1}{R_e}\frac{V}{U}\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{l^2}{\delta^2}\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

$$\delta^2 \quad \delta \qquad 1 \qquad \frac{1}{\delta^2} \qquad\qquad (8)$$

We neglect the terms of the order of δ and higher as δ is small. We then revert back to the dimensional variables to obtain

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (9)$$

$$\frac{\partial p}{\partial y}=0 \quad \Rightarrow p=p(x) \qquad (10)$$

and $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (11)$$

Equations (9–11) are known as Prandtl's boundary layer equations with boundary conditions

$$\left.\begin{array}{l} u=v=0, \quad y=0 \\ u=U(x,t), y\to\infty \end{array}\right\} \qquad (12)$$

Since p is independent of y, for given x, p has the same value through the boundary layer from y = 0 to y = δ. Thus, in boundary layer theory, there are only two variable terms u and v instead of three u, v and p in the Navier-Stokes equations. This is a great simplification in the solution of the differential equations.

Now, U is the velocity outside the boundary layer. The Euler's equation in the main stream (potential flow of non-viscous fluid) is obtained from (9) by taking $\nu= 0$ and

$$v=0,\ \frac{\partial u}{\partial y}=0 \text{ for } y\geq\delta$$

Thus, we get

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}=-\frac{1}{\rho}\frac{dp}{dx} \qquad (13)$$

From (9) and (13), we obtain

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (14)$$

and $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{15}$$

Although these equations are obtained for a rectilinear flow but they hold for curved flow if the curvature of the boundary is small in comparison to the boundary layer thickness.

The integration of (14) and (15) can be simplified if we can reduce the number of variables by introducing the stream function ψ.

where $$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{16}$$

The continuity equation is automatically satisfied. The boundary layer equation (14) in terms of ψ is

$$\frac{\partial^2 \psi}{\partial t \partial y} + \frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3} + U\frac{\partial U}{\partial x} + \frac{\partial U}{\partial t} \tag{17}$$

The boundary conditions (12) reduce to

$$\left.\begin{aligned} &\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial y} = 0, \; y = 0 \\ &\frac{\partial \psi}{\partial y} = U(x,t), \; y \to \infty \end{aligned}\right\} \tag{18}$$

The exact solution of (17) was given by H. Blasius in 1908, for the case of steady flow $(\partial/\partial t = 0)$ past a flat plate (U = constant).

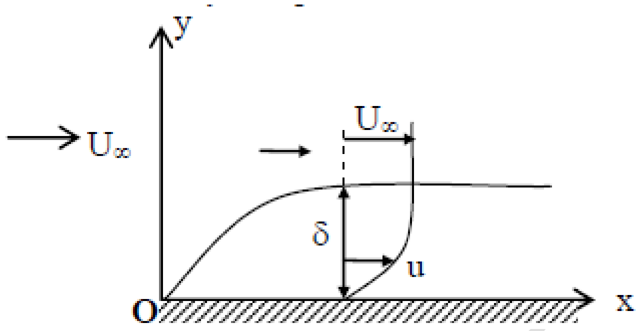
The Boundary Layer Along a Flat Plate (Blasius Solution or Blasius – Topfer for Solution)

Let us consider the steady flow of an incompressible viscous fluid past a thin semi-infinite flat plate which is placed in the direction of a uniform velocity U_∞. The motion is two-dimensional and can be analysed by using the Prandtl boundary layer equations. We choose the origin of the co-ordinates at the leading edge of the plate, x-axis along the direction of the uniformal stream and y-axis normal to the plate. The Prandtl boundary layer equations, for this case, are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

where u, v are the velocity components and ν is the kinematic viscosity.



The boundary conditions are

$$\left.\begin{array}{ll} u = v = 0 & \text{when } y = 0 \\ u = U_\infty & \text{when } y \to \infty \end{array}\right\} \qquad (3)$$

In this problem, the parameters in which the results are to be obtained, are U_∞, ν, x, y. So, we may take

$$\frac{u}{U_\infty} = F(x, y, \nu, U_\infty) = F(\eta) \qquad (4)$$

Further, according to the exact solution of the unsteady motion of a flat plate, we have

$$\delta \sim \sqrt{\nu t} \sim \sqrt{\frac{\nu x}{U_\infty}} \qquad (5)$$

where x is the distance travelled in time t with velocity U_∞. Hence the non-dimensional distance parameter may be expressed as

$$\eta = \frac{y}{\delta} = \frac{y}{\sqrt{\nu x/U_\infty}} = y\sqrt{\frac{U_\infty}{\nu x}} \qquad (6)$$

Thus, it can be seen that η in (4) is a function of x, y, ν, U_∞ as in (6)

The stream function ψ is given by

$$\psi = \int u \, dy \qquad \Big| \; u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

$$= \int U_\infty F(\eta) \frac{dy}{d\eta} d\eta$$

$$= U_\infty \sqrt{\frac{\nu x}{U_\infty}} \int F(\eta)\, d\eta = \sqrt{\nu x U_\infty}\; f(\eta) \qquad (7)$$

The velocity components in terms of η are (dash denotes derivative w.r.t. η)

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = \sqrt{\nu x U_\infty}\sqrt{\frac{U_\infty}{\nu x}} f'(\eta) = U_\infty f'(\eta) \quad (8)$$

$$-v = \frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\; f(\eta) + \sqrt{\nu x U_\infty}\; f'(\eta) y \sqrt{\frac{U_\infty}{\nu}}\left(-\frac{1}{2x^{3/2}}\right)$$

$$\Rightarrow \qquad v = -\frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\; f(\eta) + \frac{1}{2} y \frac{U_\infty}{x} f'(\eta)$$

$$= \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\left(\sqrt{\frac{U_\infty}{\nu x}}\; y\, f'(\eta) - f(\eta)\right)$$

$$= \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\; (\eta f'(\eta) - f(\eta)) \qquad (9)$$

Also,

$$\frac{\partial u}{\partial x} = \frac{\partial^2 \psi}{\partial x \partial y} = U_\infty\; f''(\eta)\frac{\partial \eta}{\partial x}$$

$$= -\frac{1}{2} U_\infty f''(\eta).\; y\sqrt{\frac{U_\infty}{\nu}}\; \frac{1}{x^{3/2}}$$

$$= -\frac{1}{2}\frac{U_\infty}{x}\eta\, f''(\eta) \qquad (10)$$

$$\frac{\partial u}{\partial y} = U_\infty \frac{\partial}{\partial y}(f'(\eta)) = U_\infty\sqrt{\frac{U_\infty}{\nu x}}\; f''(\eta) \qquad (11)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{U_\infty^2}{\nu x} f'''(\eta) \qquad (12)$$

Using these values of u, v and their derivatives in (1), we obtain

$$U_\infty f'(\eta)\left(-\frac{1}{2}\frac{U_\infty}{x}\eta f''(n)\right) + \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\,(\eta f'(\eta) - f(\eta))U_\infty\sqrt{\frac{U_\infty}{\nu x}}\, f''(\eta)$$

$$= \nu\,\frac{U_\infty^2}{\nu x} f'''(\eta)$$

or $$-\frac{U_\infty^2}{2x}\eta f' f'' + \frac{U_\infty^2}{2x}(\eta f' - f) f'' = \frac{U_\infty^2}{x} f'''$$

or $$-\eta f' f'' + \eta f' f'' - f f'' = 2f'''$$

or $$2 f''' + f f'' = 0$$

i.e. $$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \qquad (13)$$

The boundary conditions (3) in terms of f and η are obtained as follows

$$u = 0 \text{ when } y = 0 \text{ implies } f'(\eta) = 0 \text{ when } \eta = 0$$

and

$$v = 0 \;\Rightarrow \eta f'(\eta) - f(\eta) = 0 \;\Rightarrow f(\eta) = 0$$

Therefore,

$$f(\eta) = f'(\eta) = 0 \text{ when } \eta = 0 \qquad (14)$$

$u = U_\infty$ when $y \to \infty$ implies that $U_\infty f'(\eta) = U_\infty$ when $\eta \to \infty$

Therefore,

$$f'(\eta) = 1 \text{ when } \eta \to \infty \qquad (15)$$

Thus we have reduced the partial differential equation (1) to ordinary differential equation (13), known as Blasius equation, where η is the similarity parameter.

The third order non-linear differential equation (13) has no closed form solution, however, Blasius obtained the solution in the form of power series expansion about $\eta = 0$.

Let us consider

$$f(\eta) = c_0 + c_1\eta + \frac{c_2}{\underline{|2}}\eta^2 + \frac{c_3}{\underline{|3}}\eta^3 + \ldots\ldots \qquad (16)$$

$$f'(\eta) = c_1 + c_2\eta + \frac{c_3}{\underline{|2}}\eta^2 + \frac{c_4}{\underline{|3}}\eta^3 + \ldots\ldots \qquad (17)$$

$$f''(\eta) = c_2 + c_3\eta + \frac{c_4}{\underline{|2}}\eta^2 + \frac{c_5}{\underline{|3}}\eta^3 + \ldots\ldots \qquad (18)$$

$$f'''(\eta) = c_3 + c_4\eta + \frac{c_5}{\underline{|2}}\eta^2 + \frac{c_6}{\underline{|3}}\eta^3 + \ldots\ldots \qquad (19)$$

The constants c_i's are determined from the boundary conditions (14), (15) and the differential equation (13). From (14), we get

$$c_0 = c_1 = 0$$

From (13), we have

$$0 = (2c_3 + 2c_4\eta + c_5\eta^2 + \ldots..) + (c_0 + c_1\eta + \frac{c_2}{\underline{|2}}\ \eta^2 + \ldots.)\ (c_2 + c_3\ \eta + \frac{c_4}{\underline{|2}}\eta^2 + \ldots)$$

i.e.
$$(2c_3 + c_0\ c_2) + (2c_4 + c_0\ c_3 + c_1\ c_2)\eta + \left(c_5 + \frac{c_0c_4}{\underline{|2}} + c_1c_3 + \frac{c_2^2}{2}\right)\eta^2 + \ldots. = 0$$

i.e.
$$2c_3 + 2c_4\ \eta + \left(c_5 + \frac{c_2^2}{2}\right)\eta^2 + \ldots. = 0$$

Equating the co-efficients to zero, we get

$$c_3 = c_4 = c_6 = c_7 = c_9 = c_{10} = 0$$

$$c_5 = -\frac{c_2^2}{\lfloor 2}, \quad c_8 = \frac{11}{4} c_2^3, \quad c_{11} = -\frac{375}{8} c_2{}^4$$

The solution (16) is

$$f(\eta) = \frac{c_2}{2}\eta^2 - \frac{c_2^2}{2}\frac{\eta^5}{\lfloor 5} + \frac{11}{4}c_2^3\frac{\eta^8}{\lfloor 8} - \frac{375}{8}c_2^4\frac{\eta^{11}}{\lfloor 11} + \ldots \qquad (20)$$

The constant c_2 is determined by the condition (15) i.e.

$$\frac{df}{d\eta} = 1 \text{ as } n \to \infty$$

We write (20) as

$$f(\eta) = c_2^{1/3}\left[\frac{(c_2^{1/3}\eta)^2}{\lfloor 2} - \frac{1}{2}\frac{(c_2^{1/3}\eta)^5}{\lfloor 5} + \frac{11}{4}\frac{(c_2^{1/3}\eta)^8}{\lfloor 8} - \frac{375}{8}\frac{(c_2^{1/3}\eta)^{11}}{\lfloor 11} + \ldots\right]$$

$$= c_2^{1/3} F(c_2^{1/3}\eta) \qquad (21)$$

Therefore,

$$f'(\eta) = c_2^{2/3} F'(c_2^{1/3}\eta)$$

Thus, $\lim_{\eta\to\infty} c_2^{2/3} F'(c_2^{1/3}\eta) = \lim_{\eta\to\infty} f'(\eta) = 1$

Therefore,

$$c_2 = \left[\frac{1}{\lim_{\eta\to\infty} f'(c_2^{1/3}\eta)}\right]^{3/2} \qquad (22)$$

where c_2 is determined numerically by Howarth (1938) as 0.33206. Thus $f(\eta)$ in (20) is completely obtained which helps in finding u and v from (8) and (9). Hence the Blasius solution.

The shearing stress τ_0 on the surface of the plate can be calculated from the results of the Blasius solution. Thus, we have

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu U_\infty f''(0)}{\sqrt{\nu x/U_\infty}}$$

$$= \mu\frac{U_\infty C_2}{\sqrt{\nu x/U_\infty}} = \frac{0.332}{\sqrt{R_{e_x}}}\rho U_\infty^2 \qquad (23)$$

where $R_{e_x} = xU_\infty/\nu$ is the Reynolds number.

The frictional drag coefficients or local skin friction coefficients C_f is

$$C_f = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2} = \frac{0.664}{\sqrt{R_{e_x}}} \qquad (24)$$

The total frictional force F per unit width for one side of the plate of length l is given by

$$F = \int_0^l \tau_0 d_x = 0.664\,\rho U_\infty^2\sqrt{\frac{\nu l}{U_\infty}} \qquad (25)$$

Equation (25) shows that frictional force is proportional to the 3/2th power of the free stream velocity U_∞.

The average skin-friction co-efficient of the drag co-efficient is obtained as

$$C_F = \frac{F}{\frac{1}{2}\rho U_\infty^2 l} = \frac{0.664PU_\infty^2\sqrt{\nu l/U_\infty}}{\frac{1}{2}PU_\infty^2 l} = \frac{1.328}{\sqrt{R_{e_l}}} \qquad (26)$$

Where $R_{e_l} = \frac{lU_\infty}{\nu}$.

Characteristic Boundary Layer Parameters : (i) Boundary Layer Thickness. The boundary layer is the region adjacent to a solid surface in which viscous forces are important. According to the boundary conditions (3), the velocity u in the boundary layer does not reach the value U_∞ of the free stream until $y\to\infty$, because the influence of viscosity in the boundary layer decreases asymptotically outwards. Hence it is difficult to define an exact thickness of the boundary layer. However, at certain finite value of η, the velocity in the boundary layer asymptotically blends into the free stream velocity of the potential flow. If an arbitrary limit of the boundary layer at $u = 0.9975\, U_\infty$ is considered, the thickness of the boundary layer is found to be

$$\delta = 5.64\sqrt{\frac{\nu x}{U_\infty}} = \frac{5.64x}{\sqrt{R_{e_x}}} \qquad (27)$$

(ii) Displacement Thickness : The boundary layer thickness being somewhat arbitrary so more physically meaningful thickness is introduced. This thickness is known as displacement thickness, which is defined as

$$U_\infty \delta_1 = \int_{y=0}^{\infty} (U_\infty - u)dy \qquad (28)$$

where the right-hand size signifies the decrease in total flow caused by the influence of the friction and the left-hand side represents the potential flow that has been displaced from the wall. Hence the displacement thickness δ_1 is that distance by which the external potential field of flow is displaced outwards due to the decrease in velocity in the boundary layer.

i.e. $$\delta_1 = \int_0^{\infty}\left(1 - \frac{u}{U_\infty}\right)dy \qquad (29)$$

Using the expressions for $\frac{u}{U_\infty}$ and y from (8) and (6) respectively, we find δ_1 for the flow on a flat plate, as

$$\delta_1 = \sqrt{\frac{\nu x}{U_\infty}}\int_0^{\infty}(1 - f')d\eta$$

$$= \sqrt{\frac{Ux}{U_\infty}}\lim_{\eta\to\infty}[\eta - f(\eta)]$$

$$= 1.7208\sqrt{\frac{\nu x}{U_\infty}} = \frac{1.7208\,x}{\sqrt{R_{e_x}}} \qquad (30)$$

(iii) Momentum Thickness : Analogous to the displacement thickness, another thickness, known as momentum thickness (δ_2), may be defined in accordance with the momentum law. This is obtained by equating the loss of momentum flow as a consequence of the wall friction in the boundary layer to the momentum flow in the absence of the boundary layer. Thus

$$\rho\delta_2\, U_\infty^2 = \rho \int_{y=0}^{\infty} u(U_\infty - u)dy$$

or
$$\delta_2 = \int_0^\infty \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy \qquad (31)$$

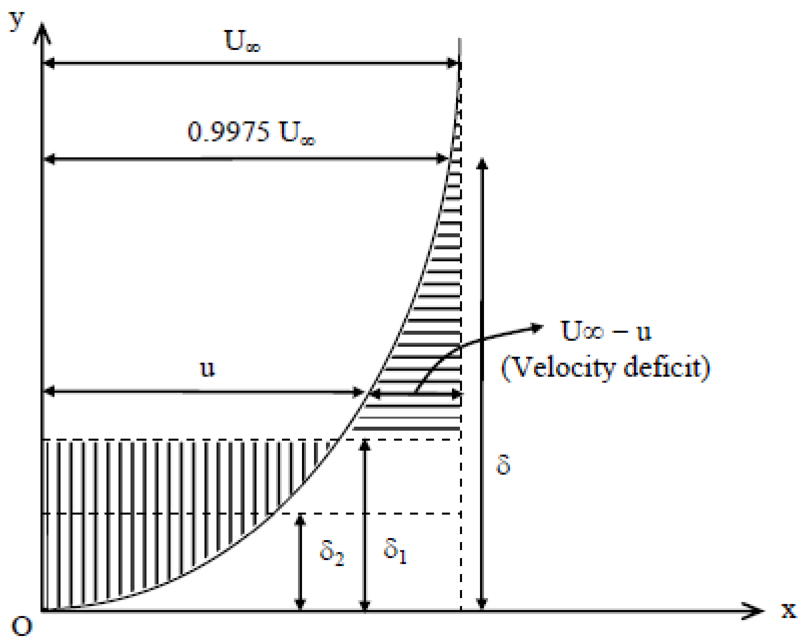
Again, using (8) and (6), we obtain δ_2 for the case of the flow on a flat plate, as

$$\delta_2 = \sqrt{\frac{\nu x}{U_\infty}}\int_0^\infty f'(1-f')\, d\eta$$

$$= 0.664\sqrt{\frac{\nu x}{U_\infty}} = \frac{0.664\, x}{\sqrt{R_{e_x}}} \qquad (32)$$

Comparison among various thicknesses of the boundary layer is shown in the figure. We note that

$\delta_2 < \delta_1 < \delta$.



POSSIBLE QUESTIONS

PART – B (5 x 6 = 30 Marks)

Answer all the questions

1. Explain the boundary layer characteristics.
2. Explain the momentum integral equation.
3. Explain the equation of boundary layer.
4. Derive the kinetic energy integral equation.
5. Explain the equation of boundary layer.

PART – C (1 x 10 = 10 Marks)

Compulsory

1. Explain the displacement and momentum thickness.
2. Explain the boundary layer separation.
3. Derive the kinetic energy thickness.
4. Explain the integral equations at boundary layer.
5. Explain the Steady Poisuille flow.
6.