KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

SYLLABUS

Semester - III

		L	T	P	C
17MMU301	**NUMERICAL METHODS**	**4**	**0**	**0**	**4**

Scope: This course provides a deep knowledge to the learners to understand the basic concepts of Numerical Methods which utilize computers to solve Engineering Problems that are not easily solved or even impossible to solve by analytical means.

Objectives: To enable the students to study numerical techniques as powerful tool in scientific computing.

UNIT I

Convergence, Errors: Relative, Absolute, Round off, Truncation. Transcendental and Polynomial equations: Bisection method - Newton's method - False Position method - Secant method - Rate of convergence of these methods.

UNIT II

System of linear algebraic equations: Gaussian Elimination - Gauss Jordan methods - Gauss Jacobi method - Gauss Seidel method and their convergence analysis – LU decomposition - Power method.

UNIT III

Interpolation: Lagrange and Newton's methods. Error bounds - Finite difference operators. Gregory forward and backward difference interpolation – Newton's divided difference – Central difference – Lagrange and inverse Lagrange interpolation formula.

UNIT IV

Numerical Differentiation and Integration: Gregory's Newton's forward and backward differentiation- Trapezoidal rule, Simpson's rule, Simpsons 3/8th rule, Boole's Rule. Midpoint rule, Composite Trapezoidal rule, Composite Simpson's rule.

UNIT V

Ordinary Differential Equations: Taylor's series - Euler's method – modified Euler's method - Runge-Kutta methods of orders two and four.

SUGGESTED READINGS

TEXT BOOK

1. Jain. M.K., Iyengar. S.R.K.,and Jain R.K., (2012). Numerical Methods for Scientific and Engineering Computation, New Age International Publishers, New Delhi .

REFERNCES

1. Bradie B., (2007).A Friendly Introduction to Numerical Analysis, Pearson Education, India,

2.Gerald C.F., and Wheatley P.O., (2006). Applied Numerical Analysis, Sixth Edition, Dorling Kindersley (India) Pvt. Ltd., New Delhi.

3. Uri M. Ascher and Chen Greif., (2013). A First Course in Numerical Methods, Seventh Edition., PHI Learning Private Limited.

4. John H., Mathews and Kurtis D. Fink., (2012). Numerical Methods using Matlab, Fourth Edition., PHI Learning Private Limited.

5. Sastry S.S., (2008). Introductory methods of Numerical Analysis, Fourth edition, Prentice Hall of India, New Delhi.

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LECTURE PLAN
DEPARTMENT OF MATHEMATICS

STAFF NAME: K.AARTHIYA
SUBJECT NAME: NUMERICAL METHODS SUB.CODE:16MMU301
SEMESTER: III CLASS: I B.SC MATHEMATICS

S.No	Lecture Duration Period	Topics to be Covered	Support Material/Page Nos
		UNIT-I	
1.	1	Introduction to Convergence	T1:ch-1, Pg.No:12-15
2.	1	Convergence, Errors: Relative, Absolute, Round off, Truncation	T1:ch -1,Pg.No:7-8
3.	1	Solution of Algebraic and Transcendental Equation -Bisection Method	T1: ch -2,Pg.No:20-22
4.	1	Newton's method and its rate of convergence- problems	R2:Ch 1,Pg.No:48-49
5.	1	Continuous on Newton's method and rate of convergence problems	R2:Ch 1,Pg.No:50-51
6.	1	False Position method and its rate of convergence related examples	T1: ch -2,Pg.No:23-24
7.	1	Continuous on False Position method and its rate of convergence related examples	T1: ch -2,Pg.No:25-26
8.	1	Secant method related problems and its rate of convergence	R5:ch-2,Pg.No:43-44
9.	1	Recapitulation and Discussion of possible questions	
	Total No of Hours Planned For Unit I=9		
		UNIT-II	
1.	1	Introduction to Solution of Simultaneous Linear algebraic Equations	T1:ch -3,Pg.No:114-115
2.	1	Gauss Elimination Method: Procedure	T1:ch -3,Pg.No:116-117
3.	1	Gauss Jordan Method and their convergence related examples	T1:ch -3,Pg.No:119-120

4.	1	Gauss Jordan Method and its convergence related examples	R1:chater-3,Pg.No:216-224
5.	1	Gauss Jacobic Method and its convergence related examples	T1:ch -3,Pg.No:146-149
6.	1	Gauss Seidal Method and its convergence problems	T1:ch -3,Pg.No:150-152
7.	1	Continuation of Problems on Gauss Seidal Method	R2:ch-2,Pg.No:129-134
8.	1	LU Decomposition related problems	R3: ch -5 Pg.No:100-105
9.	1	Power Method with examples	T1: ch -3,Pg.No:192-194
10.	1	Recapitulation and Discussion of possible questions	
	Total No of Hours Planned For Unit II=10		
		UNIT-III	
1.	1	Introduction on Interpolation and its formulas	T1: ch -4,Pg.No: 212-214
2.	1	Lagrange and Newton's Methods related problmes	T1: ch -4, Pg.No: 215-216
3.	1	Continuous on Lagrange and Newton's Methods related problmes	T1: ch -4, Pg.No: 216-217
4.	1	Error bounds - Finite difference operators related examples	T1: ch -4,Pg.No: 218-220
5.	1	Continuous on Error bounds - Finite difference operators related examples	T1: ch -4,Pg.No: 221-224
6.	1	Gregory Forward and backward difference Interpolation related examples	T1: ch -4, Pg.No: 230-236
7.	1	Newton's Divided difference and its problems	T1: ch -4,Pg.No: 226-229
8.	1	Central difference	R3:ch -10,Pg.No:306-310
9.	1	Lagrange and Inverse Interpolation formula	R4: ch -6,Pg.No:334-335
10.	1	Recapitulation and Discussion of possible questions	
	Total No of Hours Planned For Unit III=10		
		UNIT-IV	
1.	1	Introduction to Numerical Differentiation and Integration	T1: ch -5,Pg.No: 320-322
2.	1	Gregory 's Newton's Forward and Backward differentiation	T1: ch -5,Pg.No: 323-324
3.	1	Continuous on Gregory 's Newton's Forward and Backward differentiation	T1: ch -5, Pg.No: 325-326
4.	1	Trapezoidal rule and its examples	T1: ch -5,Pg.No:350-352
5.	1	Simpson's 1/3 rule and Simpson's	T1: ch -5,Pg.No:353-355

		3/8 rule-Problems	
6.	1	Boole’s Rule & Midpoint rule related problems	R5:ch-5,Pg.No:200-202
7.	1	Composite Trapezoidal rule and its problems	T1: ch 5,Pg.No:386-387
8.	1	Composite Simpson’ rule related examples	T1: ch 5,Pg.No:388-390
9.	1	Recapitulation and Discussion of possible questions	
	Total No of Hours Planned For Unit IV=9		
		UNIT-V	
1.	1	Introduction to Ordinary Differential Equations	R4:ch 9,Pg.No:451-453
2.		Taylor’s series with examples	R4:ch 9,Pg.No:454-456
3.	1	Euler’s method and modified Euler’s method with problems	T1: ch -6, Pg.No:425-430
4.	1	Continuous on Euler’s method and modified Euler’s method with problems	R2:ch:6,Pg.No:455-458
5.	1	Runge-Kutta methods of orders two and four with problems	T1: ch -6, Pg.No:451-456
6.	1	Milne’s predictor – corrector method & Adam’s Bashforth predictor – corrector method and its examples	R2:ch:6,Pg.No:467-468 T1: ch -6,Pg.No:487-492
7.	1	Recapitulation and Discussion of possible questions	
8.	1	Discuss on Previous ESE Question Papers	
9.	1	Discuss on Previous ESE Question Papers	
10.	1	Discuss on Previous ESE Question Papers	
	Total No of Hours Planned for unit V=10		
Total Planned Hours	**48**		

SUGGESTED READINGS

TEXT BOOK

T1. Jain. M.K., Iyengar. S.R.K.,and Jain R.K., (2012). Numerical Methods for Scientific and Engineering Computation, New Age International Publishers, New Delhi .

REFERNCES

R1. Bradie B., (2007).A Friendly Introduction to Numerical Analysis, Pearson Education, India,

R2. Gerald C.F. and Wheatley P.O., (2006). Applied Numerical Analysis, Sixth Edition, Dorling Kindersley (India) Pvt. Ltd., New Delhi.

R3. Uri M. Ascher and Chen Greif., (2013). A First Course in Numerical Methods, Seventh Edition., PHI Learning Private Limited.

R4. John H., Mathews and Kurtis D. Fink., (2012). Numerical Methods using Matlab, Fourth Edition., PHI Learning Private Limited.

R5. Sastry S.S., (2008). Introductory methods of Numerical Analysis, Fourth edition, Prentice Hall of India, New Delhi.

UNIT I
SYLLABUS

Convergence, Errors: Relative, Absolute, Round off, Truncation. Transcendental and Polynomial equations: Bisection method - Newton's method - False Position method - Secant method - Rate of convergence of these methods.

SOLUTION OF ALGEBRAIC AND TRANSCENDENTAL EQUATIONS

Introduction

The solution of the equation of the form $f(x) = 0$ occurs in the field of science, engineering and other applications. If $f(x)$ is a polynomial of degree two or more ,we have formulae to find solution. But, if $f(x)$ is a transcendental function, we do not have formulae to obtain solutions.

When such type of equations are there, we have some methods like Bisection method, Newton-Raphson Method and The method of false position. Those methods are solved by using a theorem in theory of equations, *i.e., If* $f(x)$ is continuous in the interval (a,b) and if f(a) and f(b) are of opposite signs, then the equation f(x) = 0 will have atleast one real root between a and b.

Bisection Method

Let us suppose we have an equation of the form $f(x) = 0$ in which solution lies between in the range (a,b). Also $f(x)$ is continuous and it can be algebraic or transcendental. If $f(a)$ and $f(b)$ are opposite signs, then there exist atleast one real root between a and b.Let $f(a)$ be positive and $f(b)$ negative. Which implies atleast one root exits between a and b. We assume that root to be $x_o = (a+b)/2$. Check the sign of $f(x_o)$. *If* $f(x_0)$ is negative , the root lies between a and x_o. If $f(x_0)$ is positive , the root lies between x_o and b. Subsequently any one of this case occur.

$$X_1 = \frac{X_0+a}{2} \quad \text{(or)} \quad \frac{x_0+b}{2}$$

When $f(x_1)$ is negative , the root lies between xo and x1 and let the root be $x_2=(x_0 +x_1) / 2$.

Again $f(x_2)$ negative then the root lies between x_o and x_2, let $x_3 = (x_0+x_2)/2$ and so on. Repeat the process $x_o, x_1, x_2, \ldots.$ Whose limit of convergence is the exact root.

Steps:

1. Find a and b in which f(a) and f(b) are opposite signs for the given equation using trial and error method.
2. Assume initial root as $x_o = (a+b)/2$.
3. If $f(x_0)$ is negative , the root lies between a and x_o and take the root as $x_1 = (x_o+a)/2$.
4. If $f(x_o)$ is positive , then the root lies between x_o and b and take the root as $x_1 = (x_o + b)/ 2$.
5. If $f(x_1)$ is negative , the root lies between x_o and x_1 and let the root be $x_2 = (x_0 + x_1) / 2$.
6. If $f(x_2)$ is negative , the root lies between x_o and x_1 and let the root be $x_3 = (x_0 + x_2) / 2$.
7. Repeat the process until any two consecutive values are equal and hence the root.

Example:

Find the positive root of $x^3 - x = 1$ correct to four decimal places by bisection method.

Solution:

Let $f(x) = x^3 - x - 1$

$f(0) = 0^3 - 0 - 1 = -1 = -ve$

$f(1) = 1^3 - 1 - 1 = -1 = -ve$

$f(2) = 2^3 - 2 - 1 = 5 = +ve$

So root lies between 1 and 2 , we can take (1+2) /2 as initial root and proceed.

i.e., f(1.5) = 0.8750 = +ve

and f(1) = -1 = -ve

So root lies between 1 and 1.5 ,

Let x_o = (1+1.5) /2 as initial root and proceed.

f(1.25) = - 0.2969

So root lies between x_1 between 1.25 and 1.5

Now x_1 = (1.25 + 1.5) /2 = *1.3750*

f(1.375) = 0.2246 = +ve

So root lies between x_2 between 1.25 and 1.375

Now $x_2 = (1.25 + 1.375)/2 = 1.3125$

$f(1.3125) = -0.051514 = -ve$

Therefore, root lies between 1.375and 1.3125

Now $x_3 = (1.375 + 1.3125) / 2 = 1.3438$

$f(1.3438) = 0.082832 = +ve$

So root lies between 1.3125 and 1.3438

Now $x_4 = (1.3125 + 1.3438) / 2 = 1.3282$

$f(1.3282) = 0.014898 = +ve$

So root lies between 1.3125 and 1.3282

Now $x_5 = (1.3125 + 1.3282) / 2 = 1.3204$

$f(1.3204) = - 0.018340 = -ve$

So root lies between 1.3204 and 1.3282

Now $x_6 = (1.3204 + 1.3282) / 2 = 1.3243$

$f(1.3243) = -ve$

So root lies between 1.3243 and 1.3282

Now $x_7 = (1.3243 + 1.3282) / 2 = 1.3263$

$f(1.3263) = +ve$

So root lies between 1.3243 and 1.3263

Now $x_8 = (1.3243 + 1.3263) / 2 = 1.3253$

$f(1.3253) = +ve$

So root lies between 1.3243 and 1.3253

Now $x_9 = (1.3243 + 1.3253) / 2 = 1.3248$

$f(1.3248) = +ve$

So root lies between 1.3243 and 1.3248

Now x_{10} = (1.3243 + 1.3248) / 2 = *1.3246*

f(1.3246) = -ve

So root lies between 1.3248 and 1.3246

Now x_{11} = (1.3248 + 1.3246) / 2 = *1.3247*

f(1.3247) = -ve

So root lies between 1.3247 and 1.3248

Now x_{12} = (1.3247 + 1.3247) / 2 = *1.32475*

Therefore , the approximate root is 1.32475

Example

Find the positive root of x – cos x = 0 by bisection method.

Solution :

Let f(x) = x – cos x

f(0) = 0 - cos (0) = 0 -1 = -1 = -ve

f(0.5) = 0.5 – cos (0.5) = -0.37758 = -ve

f(1) = 1 – cos (1) = 0.42970 = +ve

So root lies between 0.5 and 1

Let x_0 = (0.5 +1) /2 as initial root and proceed.

f(0.75) = *0.75 – cos (0.75) = 0.018311 = +ve*

So root lies between 0.5 and 0.75

x_1 = (0.5 +0.75) /2 =0.625

f(0.625) = *0.625 – cos (0.625) = - 0.18596*

So root lies between 0.625 and 0.750

x_2 = (0.625 +0.750) /2 = 0.6875

f(0.6875) = - *0.085335*

So root lies between 0.6875 and 0.750

$x3$ = (0.6875 +0.750) /2 = 0.71875

f(0.71875) = 0.71875-cos(0.71875) = - *0.033879*

So root lies between 0.71875 and 0.750

$x4$ = (0.71875 +0.750) /2 = 0.73438

f(0.73438) = -0.0078664 = - ve

So root lies between 0.73438 and 0.750

$x5$ = 0.742190

f(0.742190) = 0.0051999 = + ve

$x6$ = (0.73438 +0.742190) /2 = 0.73829

f(0.73829) = -0.0013305

So root lies between 0.73829 and 0.74219

$x7$ = (0.73829+0.74219) = 0.7402

f(0.7402) = 0.7402-cos(0.7402) = 0.0018663

So root lies between 0.73829 and 0.7402

$x8$ = 0.73925

f(0.73925) = 0.00027593

$x9$ = 0.7388

The root is 0.7388.

Newton-Raphson method (or Newton's method)

Let us suppose we have an equation of the form $f(x) = 0$ in which solution is lies between in the range (a,b).

Also $f(x)$ is continuous and it can be algebraic or transcendental. If $f(a)$ and $f(b)$ are opposite signs, then there exist

atleast one real root between a and b.

Let $f(a)$ be positive and $f(b)$ negative. Which implies atleast one root exits between *a* and *b*. We assume that

root to be either a or b, in which the value of f(a) or f(b) is very close to zero. That number is assumed to be

initial root. Then we iterate the process by using the following formula until the value is converges.

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$

Steps:

1. Find *a* and *b* in which *f(a)* and *f(b)* are opposite signs for the given equation using trial and error method.

2. Assume initial root as $X_o = a$ i.e., *if* $f(a)$ is very close to zero or $X_o = b$ *if* $f(a)$ is very close to zero

3. Find X1 by using the formula

$$X_1 = X_o - \frac{f(X_o)}{f'(X_o)}$$

4. *Find* X_2 *by using the following formula*

$$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$$

5. *Find* $X_3, X_4, \ldots X_n$ *until any two successive values are equal.*

Example:

Find the positive root of f(x) = 2x3 - 3x-6 =0 by Newton – Raphson method correct to

five decimal places.

Solution:

Let $f(x) = 2x^3 - 3x - 6$; $f'(x) = 6x^2 - 3$

f(1) = 2-3-6 = -7 = -ve

f(2) = 16 – 6-6 = 4 = +ve

So, a root between 1 and 2 . In which 4 is closer to 0 Hence we assume initial root as 2.

Consider $x_0 =2$

So $X_1 = X_0 – f(X_0)/f'(X_0)$

$= X_0- ((2X_03 - 3X_0 - 6) / 6\alpha_0 -3) = (4X_03 +6)/(6X_02-3)$

$X_{i+1}= (4X_i3 + 6)/(6X_i2-3)$

$X_1 = (4(2)^2+6)/(6(2)^2- 3) = 38/21 = 1.809524$

$X_2= (4(1.809524)^3+6)/(6(1.809524)^2- 3) = 29.700256/16.646263 = 1.784200$

$X_3= (4(1.784200)^3+6)/(6(1.784200)^2- 3) = 28.719072/16.100218 = 1.783769$

$X_4= (4(1.783769)^3+6)/(6(1.783769)^2- 3) = 28.702612/16.090991 = 1.783769$

Example:

Using Newton's method, find the root between 0 and 1 of $x^3 = 6x – 4$ correct to 5 decimal places.

Solution :

Let $f(x) = x^3 -6x+4$; $f(0) = 4$ = +ve; $f(1) = -1$ = -ve

So a root lies between 0 and 1

f(1) is nearer to 0. Therefore we take initial root as $X_0= 1$

$f'(x) = 3x^2- 6$

$$= x - \frac{f(x)}{f'(x)}$$

$= x - (3x^3 -6x+4)/(3x^2-6)$

$= (2x^3 -4)/(3x^2 -6)$

$X_1 = (2X_0 3 – 4)/(3X_0 2-6) = (2-4)/(3-6) = 2/3 = 0.66666$

$X_2 = (2(2/3)^3 – 4)/(3(2/3)^2-6) = 0.73016$

$X_3 = (2(0.73015873)^3 – 4)/(3(0.73015873)^2-6)$

$= (3.22145837/ 4.40060469)$

$= 0.73205$

$X_4 = (2(0.73204903)^3 – 4)/(3(0.73204903)^2-6)$

$= (3.21539602/ 4.439231265)$

$= 0.73205$

The root is 0.73205 correct to 5 decimal places.

Method of False Position (or Regula Falsi Method)

Consider the equation $f(x) = 0$ and $f(a)$ and $f(b)$ *are* of opposite signs. Also let $a< b$.

The graph $y = f(x)$ *will* Meet the x-axis at some point between A(a, f(a)) and B (b,f(b)). The equation of the chord joining the two points A(a, f(a)) and B (b,f(b)) is

$$\frac{y - f(a)}{x - a} = \frac{f(a) - f(b)}{a - b}$$

The x- Coordinate of the point of intersection of this chord with the x-axis gives an approximate value for the of f(x) = 0. Taking y = 0 in the chord equation, we get

$$\frac{-f(a)}{x - a} = \frac{f(a) - f(b)}{a - b}$$

$x[f(a) - f(b)] - a f(a) + a f(b) = -a f(a) + b f(b)$

$x[f(a) - f(b)] = b f(a) - a f(b)$

This x_1 gives an approximate value of the root f(x) = 0. ($a < x_1 < b$)

Now $f(x_1)$ and f(a) are of opposite signs or $f(x_1)$ and f(b) are opposite signs.

If $f(x_1), f(a) < 0$. then x2 lies between x_1 and a.

Therefore

$$x2 = \frac{a f(x_1) - x_1 f(b)}{f(x_1) - f(a)}$$

This process of calculation of (x_3, x_4, x_5, ….) is continued till any two successive values are equal and subsequently we get the solution of the given equation.

Steps:

1. Find *a* and *b* in which *f(a)* and *f(b)* are opposite signs for the given equation using trial and error method.

2. Therefore root lies between *a* and *b if f(a)* is very close to zero select and compute x_1 by using the following formula:

$$x1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

3. If $f(x_1), f(a) < 0$. then root lies between x_1 and *a*. Compute x_2 by using the following formula:

$$x2 = \frac{a f(x_1) - x_1 f(b)}{f(x_1) - f(a)}$$

4. Calculate the values of (x_3, x_4, x_5, ….) by using the above formula until any

two successive values are equal and subsequently we get the solution of the given

equation.

. **Example:**

Solve for a positive root of x^3-4x+1=0 by and Regula Falsi method

Solution :

Let f(x) = $x^3-4x+1 = 0$

$f(0) = 0^3-4(0)+1 = 1 = +ve$

$f(1) = 1^3-4(1)+1 = -2 = -ve$

So a root lies between 0 and 1

We shall find the root that lies between 0 and 1.

Here a=0, b=1

$$x1 = \frac{a\,f(b) - b\,f(a)}{f(b) - f(a)}$$

$$= \frac{(0 \times f(1) - 1 \times f(0))}{(f(1) - f(0))}$$

$$= \frac{-1}{(-2 - 1)}$$

$$= 0.333333$$

$f(x_1) = f(1/3) = (1/27)-(4/3) +1 = -0.2963$

Now f(0) and f(1/3) are opposite in sign.

Hence the root lies between 0 and 1/3.

$$x_2 = \frac{(0 \times f(1/3) - 1/3 \times f(0))}{(f(1/3) - f(0))}$$

$x_2 = (-1/3)/(-1.2963) = 0.25714$

Now $f(x_2) = f(0.25714) = - 0.011558 = -ve$

So the root lies between 0 and 0.25714

$x_3 = (0 \times f(0.25714) - 0.25714 \times f(0)) / (f(0.25714) – f(0))$

$= -0.25714/-1.011558 = 0.25420$

$f(x_3) = f(0.25420) = -0.0003742$

So the root lies between 0 and 0.25420

$x_4 = (0 \times f(0.25420)- 0.25420 \times f(0)) / (f(0.25420) – f(0))$

$= -0.25420 / -1.0003742 = 0.25410$

$f(x_4) = f(0.25410) = - 0.000012936$

The root lies between 0 and 0.25410

$x_5 = (0 \times f(0.25410) - 0.25410 \times f(0)) / (f(0.25410) – f(0))$

$= -0.25410 / -1.000012936 = 0.25410$

Hence the root is 0.25410.

Example:

Find an approximate root of $x \log_{10}x – 1.2 = 0$ by False position method.

Solution :

Let $f(x) = x \log_{10}x – 1.2$

$f(1) = -1.2 = -ve$; $f(2) = 2 \times 0.30103 -1.2 = - 0.597940$

$f(3) = 3 \times 0.47712 – 1.2 = 0.231364 = +ve$

So, the root lies between 2 and 3.

$= 2.721014$

$f(x_1) = f(2.7210) = - 0.017104$

The root lies between x_1 and 3.

$_1 xf(3) – 3 \times f(x_1)$ $\quad$ $2.721014 \times 0.231364 – 3 \times (-0.017104)$

$= 2.740211$

$f(x_2) = f(2.7402) = 2.7402 \text{ x } \log(2.7402) – 1.2$

$= - 0.00038905$

So the root lies between 2.740211 and 3

$$x_3 = \frac{2.7402 \text{ x } f(3) – 3 \text{ x } f(2.7402)}{f(3) – f(2.7402)} = \frac{2.7402 \text{ x } 0.231336 + 3 \text{ x } (0.00038905)}{0.23136 + 0.00038905}$$

$$= \frac{0.63514}{0.23175} = 2.740627$$

$f(2.7406) = 0.00011998$

So the root lies between 2.740211 and 2.740627

$$x_4 = \frac{2.7402 \text{ x } f(2.7406) – 2.7406 \text{ x } f(2.7402)}{f(2.7406) – f(2.7402)}$$

$$= \frac{2.7402 \text{ x } 0.00011998 + 2.7406 \text{ x } 0.00038905}{0.00011998 + 0.00038905}$$

$$= \frac{0.0013950}{0.00050903}$$

$= 2.7405$

Hence the root is 2.7405

POSSIBLE QUESTIONS

1. Define round-off error with example.
2. Define relative error with example.
3. Write the rate of convergence of the Regula falsi method.
4. Define absolute error with example
5. If x=2.536,find the absolute error and relative error.
6. Using secant method find the real root of the equation $f(x)=x^3 – 5x + 1=0$ lies in the interval (0,1)
7. Find all the roots of the equation $x^3- 4x^2+5x-2=0$ by method of false position.
8. Find the positive roots of the equation 3x – cosx -1=0 by Newton’s method.
9. Solve the equation x log x – 1.2=0 by Regula Falsi method.
10. Find the positive root of the equation $x^3 –x=1$ by bisection method.

11 Solve the following by Secant method $2x - log_{10} x = 7$.

12 Find the real positive root of 3x - cos x -1 =0 by Newton's method correct to 3 decimal place.

13 Solve for the positive root of x^3-4x+1=0 by method of false position.

14 Assuming that a root of x^3-9x+1 = 0 lies in the interval (2,4) ,find that root by bisection method.

15 Find the positive root of f(x)=2x^3-3x-6=0 by using Newton Raphson method correct to five decimal places.

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University)
(Established Under Section 3 of UGC Act 1956)
Pollachi Main Road, Eachanari (Po),
Coimbatore –641 021
Subject: Numerical Methods
Class : II - B.Sc. Mathematics

Unit I

Part A (20x1=20 Marks) **(Question Nos.**

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
-------- Method is based on the repeated application of the intermediate value theorem.	Gauss Seidal	Bisection	Regula Falsi	Newton Raphson	Bisection
The order of convergence of Newton Raphson method is ----------	4	2	1	0	2
Graeffe's root squaring method is useful to find -----------------	complex roots	single roots	unequal roots	polynomial roots	polynomial roots
The approximate value of the root of f(x) given by the bisection method is ----	x = a + b	x = f(a) + f(b)	x = (a + b)/ 2	x = (f(a) + f(b))/2	x= (a + b)/ 2
In Newton Raphson method, the error at any stage is proportional to the ------- of the error in the previous stage.	cube	square	square root	equal	square
The convergence of bisection method is -------.	linear	quadratic	slow	fast	slow
The order of convergence of Regula falsi method may be assumed to ----------.	1	1.618	0	0.5	1.618
----------------- Method is also called method of tangents.	Gauss Seidal	Secant	Bisection	Newton Raphson	Newton Raphson
If f (x) contains some functions like exponential, trigonometric, logarithmic etc., then f (x) is called ------------- equation.	Algebraic	transcendental	numerical	polynomial	transcendental
A polynomial in x of degree n is called an algebraic equation of degree n if -----	f (x) = 0	f (x) = 1	f (x) <1	f (x) >1	f (x) = 0
The method of false position is also known as ------------- method.	Gauss Seidal	Secant	Bisection	Regula falsi	Regula falsi
The Newton Rapson method fails if ---------------.	f ' (x) = 0	f (x) = 0	f (x) =1	f ' (x)=1	f ' (x) = 0
The bisection method is simple but ------------------.	slowly divergent	fast convergent	slowly convergent	divergent	slowly convergent
________Method is also called as Bolzano method or interval having method.	Bisection	false position	Newton raphson	Horner's	Bisection
The another name of Bisection method is ___________	Bozano	Regula falsi	Newtons	Giraffes	Bozano
In Regula-Falsi method, to reduce the number of iterations we start with ________ interval	Small	large	equal	none	Small
The rate of convergence in Newton-Raphson method is of order ________	1	2	3	4	2
Newton's method is useful when the graph of the function crosses the x-axis is nearly ______.	vertical	horizontal	close to zero	none	vertical
If the initial approximation to the root is not given we can find any two values of x say a and bsuch that f (a) and f(b) are of _________signs.	opposite	same	positive	negative	opposite
The Newton – Raphson method is also known as method of __________	secant	tangent	iteration	interpolation	tangent
If the derivative of f(x) = 0, then _________ method should be used.	Newton – Raphson	Regula-Falsi	iteration	interpolation	Regula-Falsi
The rate of convergence of Newton – Raphson method is _________	quadratic	cubic	4	5	quadratic
If f (a) and f (b) are of opposite signs the actual root lies between _________	(a, b)	(0, a)	(0, b)	(0, 0)	(a, b)
The convergence of root in Regula-Falsi method is slower than ________	Gauss – Elimination	Gauss – Jordan	Newton – Raphson	Power method	Newton – Raphson
Regula-Falsi method is known as method of _________	secant	tangent	chords	elimination	chords
______________method converges faster than Regula-Falsi method.	Newton – Raphson	Power method	elimination	interpolation	Newton – Raphson
If f(x) is continuous in the interval (a, b) and if f (a) and f (b) are of opposite signs the equation f(x) = 0 has at least one _________ lying between a and b.	equation	function	root	polynomial	root
Rounding errors arise during_________	Solving	Algorithm	Truncation	Computation	Computation
The other name for truncation error is _________ error.	Absolute	Rounding	Inherent	Algorithm	Algorithm
Rounding errors arise from the process of _________ the numbers.	Truncating	Rounding off	Approximating	Solving	Rounding off
Absolute error is denoted by________	Ea	Er	Ep	Ex	Ea
Truncation errors are caused by using _________ results.	Exact	True	Approximate	Real	Approximate
Truncation errors are caused on replacing an infinite process by ________ one.	Approximate	True	Finite	Exact	Finite
If a word length is 4 digits, then rounding off of 15.758 is	15.75	15.76	15.758	16	15.76
The actual root of the equation lies between a and b when f (a) and f (b) are of ____ signs.	Opposite	same	negative	positive	Opposite

UNIT-II

SOLUTIONS OF SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS

System of linear algebraic equations: Gaussian Elimination - Gauss Jordan methods – Gauss Jacobi method - Gauss Seidel method and their convergence analysis – LU decomposition - Power method

INTRODUCTION

We will study here a few methods below deals with the solution of simultaneous Linear Algebraic Equations

GAUSS ELIMINATION METHOD (DIRECT METHOD).

This is a direct method based on the elimination of the unknowns by combining equations such that the *n* unknowns are reduced to an equation upper triangular system which could be solved by back substitution.

Consider the *n* linear equations in *n* unknowns, viz.

$$a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1$$

$$a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n=b_n \quad \ldots(1)$$

Where a_{ij} and b_i are known constants and x_i's are unknowns.

The system (1) is equivalent to $AX=B$ .....(2)

Where $A = \begin{pmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \ldots\ldots & \ldots\ldots & & \ldots\ldots \\ a_{n1} & a_{n2} & \ldots\ldots & a_{nn} \end{pmatrix}$ $X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$ and $B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$

Now our aim is to reduce the augmented matrix (A,B) to upper triangular matrix.

$$(A,B) = \left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \cdots & \cdots & \cdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & b_n \end{array}\right) \quad ..(3)$$

Now, multiply the first row of (3) (if $a_{11} \neq 0$) by $-\frac{a_{i1}}{a_{11}}$ and add to the ith row of (A,B), where i=2,3,…,n. By thia, all elements in the first column of (A,B) except a_{11} are made to zero. Now (3) is of the form

$$\left(\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ 0 & b_{22} & \cdots & b_{2n} & c_2 \\ \cdots & \cdots & \cdots & & \vdots \\ 0 & b_{n2} & \cdots & b_{nn} & c_n \end{array}\right) \quad …….(4)$$

Now take the pivot b_{22}. Now, considering b_{22} as the pivot, we will make all elements below b_{22} in the second column of (4) as zeros. That is, multiply second row of (4) by $-\frac{b_{i2}}{b_{22}}$ and add to the corresponding elements of the ith row (i=3,4,…,n). Now all elements below b_{22} are reduced to zero. Now (4) reduces to

$$\left(\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ 0 & b_{22} & b_{23} & \cdots & b_{2n} & c_2 \\ 0 & 0 & c_{23} & \cdots & c_{3n} & d_3 \\ \cdots & \cdots & \cdots & & \cdots & \vdots \\ 0 & 0 & c_{n3} & \cdots & c_{nn} & d_n \end{array}\right) \quad ……(5)$$

Now taking c_{33} as the pivot, using elementary operations, we make all elements below c_{33} as zeros. Continuing the process, all elements below the leading diagonal elements of A are made to zero.

Hence, we get (A,B) after all these operations as

$$\left(\begin{array}{cccccc|c} a_{11} & a_{12} & a_{13} & \cdots\cdots & & a_{1n} & b_1 \\ 0 & b_{22} & b_{23} & \cdots\cdots & & b_{2n} & c_2 \\ 0 & 0 & c_{23} & c_{34} & \cdots\cdots & c_{3n} & d_3 \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots & \cdots\cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots\cdots & c_{nn} & d_n \end{array}\right) \quad \ldots(6)$$

From, (6) the given system of linear equations is equivalent to

$$a_{11}x_1+a_{12}x_2+a_{13}x_3+\ldots.+a_{1n}x_n=b_1$$

$$b_{22}x_2+b_{23}x_3+\ldots.+b_{2n}x_n=c_2$$

$$c_{33}x_3+\ldots.+c_{3n}x_n=d_3$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.$$

$$\alpha_{nn}x_n=k_n$$

Going from the bottom of these equation, we solve for $x_n=\frac{k_n}{\alpha_{nn}}$. Using this in the penultimate equation, we get x_{n-1} and so. By this back substitution method for we solve x_n , x_{n-1}, x_{n-2}, ……x_2, x_1.

GAUSS – JORDAN ELIMINATION METHOD (DIRECT METHOD)

This method is a modification of the above Gauss elimination method. In this method, the coefficient matrix A of the system $AX=B$ is brought to a diagonal matrix or unit matrix by making the matrix A not only upper triangular but also lower triangular by making the matrix A not above the leading diagonal of A also as zeros. By this way, the system $AX=B$ will reduce to the form.

$$\left(\begin{array}{cccccc|c} a_{11} & 0 & 0 & 0 & \ldots\ldots\ldots\ldots & a_{1n} & b_1 \\ 0 & b_{22} & 0 & 0 & \ldots\ldots\ldots\ldots & b_{2n} & c_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots\ldots\ldots & \ldots & d_3 \\ 0 & 0 & 0 & 0 & \ldots\ldots\ldots & \alpha_{nn} & k_n \end{array}\right) \ldots(7)$$

From (7)

$$x_n = \frac{k_n}{\alpha_{nn}}, \ldots\ldots, \; x_2 = \frac{c_2}{b_{22}}, \; x_n = \frac{b_1}{a_{11}}$$

Note: By this method, the values of $x_1, x_2, \ldots. x_n$ are got immediately without using the process of back substitution.

Example 1. *Solve the system of equations by (i) Gauss elimination method (ii) Gauss – Jordan method.*

$x+2y+z=3$, $2x+3y+3z=10$, $3x-y+2z=13$.

Solution. (By Gauss method)

This given system is equivalent to

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ 13 \end{pmatrix}$$

$$A\,X \quad = \quad B$$

$$(A,B) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right) \ldots\ldots\ldots\ldots\ldots (1)$$

Now, we will make the matrix A upper triangluar.

$$(A,B) = \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{array}\right)$$

$$\sim \begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{array} \quad R_2+(-2)R_1,\ R_3+(-3)R_1$$

Now, take b_{22}=-1 as the pivot and make b_{32} as zero.

$$(A,B) \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right] R_{32}(-7) \ldots\ldots\ldots (2)$$

From this, we get

$x+2y+z = 3$, $-y+z = 4$, $-8z = -24$

∴ $z = 3$, $y = -1$, $x = 2$ by back substitution.

$x = 2$, $y = -1$, $z = 3$

Solution. (Gauss – Jordan method)

In stage 2, make the element, in the position (1,2), also zero.

$$(A,B) \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 11 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{array}\right] R_{12}(2)$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 3 & 11 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -1 & -3 \end{array}\right] \quad R_3\left(\frac{1}{8}\right)$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & -3 \end{array}\right] \quad R_{13}(3),\ R_{23}(1)$$

i.e., $x = 2,\ y = -1,\quad z = 3$

METHOD OF TRIANGULARIZATION (OR METHOD OF FACTORIZATION) (*DIRECT METHOD*)

This method is also called as *decomposition* method. In this method, the coefficient matrix A of the system $AX = B$, decomposed or factorized into the product of a lower triangular matrix L and an upper triangular matrix U. we will explain this method in the case of three equations in three unknowns.

Consider the system of equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

This system is equivalent to $AX = B$

Where $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Now we will factorize A as the product of lower triangular matrix

$$L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}$$

And an upper triangular matrix

$$U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} \text{ so that}$$

$LUX = B$ Let $UX = Y$ And hence $LY = B$

That is, $$\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$\therefore$ $y_1 = b$, $l_{21}y_1+y_2 = b_2$, $l_{31}y_1+l_{32}y_2+y_3 = b_3$

By forward substitution, y_1, y_2, y_3 can be found out if L is known.

From (4), $$\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$u_{11}x_1 + u_{12}x_2 + u_{13}x_3 = y_1$, $u_{22}x_2 + u_{23}x_3 = y_2$ and $u_{33}x_3 = y_3$

From these, x_1, x_2, x_3 can be solved by back substitution, since y_1, y_2, y_3 are known if U is known.Now L and U can be found from $LU = A$

i.e., $$\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

i.e.,

$$\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{31}u_{12}+l_{32}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Equating corresponding coefficients we get nine equations in nine unknowns. From these 9 equations, we can solve for 3 l's and 6 u's.

That is, L and U re known. Hence X is found out. Going into details, we get $u_{11} = a_{11}$, $u_{12} = a_{12}$, $u_{13} = a_{13}$. That is the elements in the first rows of U are same as the elements in the first of A.

Also, $l_{21}u_{11} = a_{21}$ $\quad l_{21}u_{12}+u_{22} = a_{22}$ $\quad l_{21}u_{13}+u_{23} = a_{23}$

$$l_{21} = \frac{a_{21}}{a_{11}},\quad u_{22} = a_{22} - \frac{a_{21}}{a_{11}}.a_{12} \quad \text{and} \quad u_{23} = a_{23} - \frac{a_{21}}{a_{11}}.\ a_{13}$$

again, $l_{31}u_{11} = a_{31}$, $l_{31}u_{12}+l_{32}u_{22} = a_{32}$ and $l_{31}u_{13}+l_{32}u_{23}+u_{33} = a_{33}$

solving, $l_{31} = \dfrac{a_{31}}{a_{11}}$, $l_{32} = \dfrac{a_{32} - \frac{a_{31}}{a_{11}}.a_{12}}{a_{22} - \frac{a_{21}}{a_{11}}.a_{12}}$

$$u_{33} = \left[a_{33} - \frac{a_{31}}{a_{11}}.a_{13}\right] - \left[\frac{a_{32} - \frac{a_{31}}{a_{11}}.a_{12}}{a_{22} - \frac{a_{21}}{a_{11}}.a_{12}}\right] a_{33} - \frac{a_{31}}{a_{11}}.a_{13}$$

Therefore L and U are known.

Example 2 *By the method of triangularization, solve the following system.*

$5x - 2y + z = 4$, $7x + y - 5z = 8$, $3x + 7y + 4z = 10$.

Solution. The system is equivalent to

$$\begin{pmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$$

$$A \quad X = B$$

Now, let $LU = A$

That is, $$\begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} = \begin{pmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{pmatrix}$$

Multiplying and equating coefficients,

$$u_{11} = 5,\quad u_{12} = -2,\quad u_{13} = 1$$

$$l_{21}u_{11} = 7 \quad l_{21}u_{12}+u_{22} = 1 \quad l_{21}u_{13}+u_{23} = -5$$

$l_{21} = \frac{7}{5}$, $u_{22} = 1 - \frac{7}{5}.(-2) = \frac{19}{5}$ and

$u_{23} = -5 - \frac{7}{5}.(1) = -\frac{32}{5}$

Again equating elements in the third row,

$l_{31}u_{11} = 3$, $l_{31}u_{12}+l_{32}u_{22} = 7$ and $l_{31}u_{13}+l_{32}u_{23}+u_{33} = 4$

$$\therefore \quad l_{31} = \frac{3}{5}, \quad l_{32} = \frac{7 - \frac{3}{5}.(-2)}{\frac{19}{5}} = \frac{41}{19}$$

$$u_{33} = 4 - \frac{3}{5}.(1) - \frac{41}{19}\left(-\frac{32}{5}\right) = 4 - \frac{3}{5} + \frac{1312}{95}$$

$$= \frac{1635}{95} = \frac{327}{19}$$

Now L and U are known.Since $LUX = B$, $LY = B$ where $UX = Y$.

From $LY = B$,

$$\begin{pmatrix} 1 & 0 & 0 \\ \frac{7}{5} & 1 & 0 \\ \frac{3}{5} & \frac{41}{19} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$$

$y_1 = 4$, $\frac{7}{5} y_1 + y_2 = 8$, $\frac{3}{5} y_1 + \frac{41}{19} y_2 + y_3 = 10$

$$y_2 = 8 - \frac{28}{5} = \frac{12}{5}$$

$$y_3 = 10 - \frac{12}{5} - \frac{41}{19} \times \frac{12}{5} = 10 - \frac{12}{5} - \frac{492}{95} = \frac{46}{19}$$

$UX = Y$ gives $\begin{pmatrix} 5 & -2 & 1 \\ 0 & \frac{19}{5} & -\frac{32}{5} \\ 0 & 0 & \frac{327}{19} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ \frac{12}{5} \\ \frac{46}{19} \end{pmatrix}$

$5x - 2y + z = 4$

$\frac{19}{5} y - \frac{32}{5} z = \frac{12}{5}$

$\frac{327}{19} z = \frac{46}{19}$

$z = \frac{46}{327}$

$\frac{19}{5} y = \frac{12}{5} + \frac{32}{5}\left(\frac{46}{327}\right)$

$y = \frac{284}{327}$

$5x = 4 + 2y - z = 4 + 2\left(\frac{568}{327}\right) - \frac{46}{327}$

$\therefore \quad x = \frac{366}{327}$

$\therefore \quad x = \frac{366}{327}, \; y = \frac{284}{327}, \; z = \frac{46}{327}$

Crout's Method

Crout's Method is a root-finding algorithm used in LU decomposition (see Foundation). Also known as Crout Matrix Decomposition and Crout Factorization, the method decomposes a matrix into a lower triangular matrix (L), an upper triangular matrix (U), and a permutation matrix (P). The last matrix is optional and not always needed.

Crout's Method solves the N^2 equations

$$i < j \quad l_i1u_1j + l_i2u_2j + \ldots + l_{ii}u_{ij} = a_{ij}$$

$$i = j \quad l_i1^u1j^+ l_i2^u2j^+ \ldots + l_{ii}u_{jj} = a_{ij}$$

$$i > j \quad l_i1u_1j + l_i2u_2j + \ldots + l_{ij}u_{jj} = a_{ij}$$

for the $N^2 + N$ unknowns l_{ij} and u_{ij}.

ITERATIVE METHODS

This iterative methods is not always successful to all systems of equations. If this method is to succeed, each equation of the system must possess one large coefficient and the large coefficient must be attached to a different unknown in that equation. This condition will be satisfied if the large coefficients are along the leading diagonal of the coefficient matrix. When this condition is satisfied, the system will be solvable by the iterative method. The system,

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

will be solvable by this method if

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$|a_{22}| > |a_{21}| + |a_{23}|$$

$$|a_{33}| > |a_{31}| + |a_{32}|$$

In other words, the solution will exist (iteration will converge) if the absolute values of the leading diagonal elements of the coefficient matrix A of the system $AX=B$ are greater than the sum of absolute values of the other coefficients of that row. The condition is *sufficient* but not *necessary*.

JACOBI METHOD OF ITERATION OR GAUSS – JACOBI METHOD

Let us explain this method in the case of three equations in three unknowns.

Consider the system of equations,

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x+b_2y+c_2z = d_2$$

$$a_3x+b_3y+c_3z = d_3 \quad \ldots\ldots\ldots\ldots (1)$$

Let us assume $|a_1| > |b_1| + |c_1|$

$$|b_2| > |a_2| + |c_2|$$

$$|c_3| > |a_3| + |b_3|$$

Then, iterative method can be used for the system (1). Solve for x, y, z (whose coefficients are the larger values) in terms of the other variables. That is,

$$x = \frac{1}{a_1}(d_1 - b_1y - c_1z)$$

$$y = \frac{1}{b_2}(d_2 - a_2x - c_2z)$$

$$z = \frac{1}{c_3}(d_3 - a_3x - b_3y) \quad \ldots\ldots\ldots\ldots (2)$$

If x^0, y^0, z^0 are the initial values of x, y, z respectively, then

$$x^{(1)} = \frac{1}{a_1}(d_1 - b_1y^{(0)} - c_1z^{(0)})$$

$$y^{(1)} = \frac{1}{b_2}(d_2 - a_2x^{(0)} - c_2z^{(0)})$$

$$z^{(1)} = \frac{1}{c_3}(d_3 - a_3x^{(0)} - b_3y^{(0)}) \quad \ldots\ldots\ldots (3)$$

Again using these values $x^{(2)}, y^{(2)}, z^{(2)}$ in (2), we get

$$x^{(2)} = \frac{1}{a_1}(d_1 - b_1y^{(1)} - c_1z^{(1)})$$

$$y^{(2)} = \frac{1}{b_2}(d_2 - a_2x^{(1)} - c_2z^{(1)})$$

$$z^{(2)} = \frac{1}{c_3}(d_3 - a_3x^{(1)} - b_3y^{(1)}) \quad \ldots.(4)$$

Proceeding in the same way, if the rth iterates are $x^{(r)}$, $y^{(r)}$, $z^{(r)}$, the iteration scheme reduces to

$$x^{(r+1)} = \frac{1}{a_1}(d_1 - b_1 y^{(r)} - c_1 z^{(r)})$$

$$y^{(r+1)} = \frac{1}{b_2}(d_2 - a_2 x^{(r)} - c_2 z^{(r)})$$

$$z^{(r+1)} = \frac{1}{c_3}(d_3 - a_3 x^{(r)} - b_3 y^{(r)}) \quad \text{…..(5)}$$

The procedure is continued till the convergence is assured (correct to required decimals).

GAUSS – SEIDEL METHOD OF ITERATION:

This is only a refinement of Guass – Jacobi method. As before,

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$

$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$

$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

We start with the initial values y^0, z^0 for y and z and get $x^{(1)}$ from the first equation. That is,

$$x^{(1)} = \frac{1}{a_1}(d_1 - b_1 y^{(0)} - c_1 z^{(0)})$$

While using the second equation, we use $z^{(0)}$ for z and $x^{(1)}$ for x instead of x^0 as in Jacobi's method, we get

$$y^{(1)} = \frac{1}{b_2}(d_2 - a_2 x^{(1)} - c_2 z^{(0)})$$

Now, having known $x^{(1)} and\ y^{(1)}$, use $x^{(1)}$ for x $and\ y^{(1)}$ for y in the third equation, we get

$$z^{(1)} = \frac{1}{c_3}(d_3 - a_3 x^{(1)} - b_3 y^{(1)})$$

In finding the values of the unknowns, we use the latest available values on the right hand side. If $x^{(r)}$, $y^{(r)}$, $z^{(r)}$ are the rth iterates, then the iteration scheme will be

$$x^{(r+1)} = \frac{1}{a_1}(d_1 - b_1 y^{(r)} - c_1 z^{(r)})$$

$$y^{(r+1)} = \frac{1}{b_2}(d_2 - a_2 x^{(r+1)} - c_2 z^{(r)})$$

$$z^{(r+1)} = \frac{1}{c_3}(d_3 - a_3 x^{(r+1)} - b_3 y^{(r+1)})$$

This process of iteration is continued until the convergence assured. As the current values of the unknowns at each stage of iteration are used in getting the values of unknowns, the convergence in Gauss – seidel method is very fast when compared to Gauss – Jacobi method. The rate of convergence in Gauss – Seidel method is roughly two times than that of Gauss – Jacobi method. As we saw the sufficient condition already, the sufficient condition for the convergence of this method is also the same as we stated earlier. That is, *the method of iteration will converge if in each equation of the given system, the absolute value of the largest* coefficient is greater than the sum of the absolute values of all the remaining coefficients. (The largest coefficients must be the coefficients for different unknowns).

Example 3 Solve the following system by Gauss – Jacobi and Gauss – Seidel methods:

10x-5y-2z = 3 ; 4x-10y+3z = -3 ; x+6y+10z = -3.

Solution: Here, we see that the diagonal elements are dominant. Hence, the iteration process can be applied.

That is, the coefficient matrix $\begin{bmatrix} 10 & -5 & -2 \\ 4 & -10 & 3 \\ 1 & 6 & 10 \end{bmatrix}$ is diagonally dominant, since $|10| > |-5| + |-2|$,

$|-10| > |4| + |3|$,

$|10| > |1| + |6|$

Gauss – Jacobi method, solving for *x*, *y*, *z* we have

$x = \frac{1}{10}(3+5y+2z)$ ……………… (1)

$y = \frac{1}{10}(3+4x+3z)$ ……………… (2)

$z = \frac{1}{10}(-3-x-6y)$ ……………… (3)

First iteration: Let the initial values be (0, 0, 0).

Using these initial values in (1), (2), (3), we get

$$x^{(1)} = \frac{1}{10}(3+5(0)+2(0)) = 0.3$$

$$y^{(1)} = \frac{1}{10}(3+4(0)+3(0)) = 0.3$$

$$z^{(1)} = \frac{1}{10}(-3-(0)-6(0)) = -0.3$$

Second iteration: using these values in (1), (2), (3), we get

$$x^{(2)} = \frac{1}{10}(3+5(0.3)+2(-0.3)) = 0.39$$

$$y^{(2)} = \frac{1}{10}(3+4(0.3)+3(-0.3)) = 0.33 \qquad z^{(2)} = \frac{1}{10}(-3-(0.3)-6(0.3)) = -0.51$$

Third iteration: using these values of $x^{(2)}$, $y^{(2)}$, $z^{(2)}$ in (1), (2), (3), we get,

$$x^{(3)} = \frac{1}{10}(3+5(0.33)+2(-0.51)) = 0.363$$

$$y^{(3)} = \frac{1}{10}(3+4(0.39)+3(-0.51)) = 0.303$$

$$z^{(3)} = \frac{1}{10}(-3-(0.39)-6(0.33)) = -0.537$$

Fourth iteration:

$$x^{(4)} = \frac{1}{10}(3+5(0.303)+2(-0.537)) = 0.3441$$

$y^{(4)} = \frac{1}{10}(3 + 4(0.363) + 3(-0.537)) = 0.2841$

$z^{(4)} = \frac{1}{10}(-3- (0.363) – 6(0.303)) = -0.5181$

Fifth iteration:

$x^{(5)} = \frac{1}{10}(3 + 5(0.2841) + 2 (-0.5181)) = 0.33843$

$y^{(5)} = \frac{1}{10}(3 + 4(0.3441) + 3(-0.5181)) = 0.2822$

$z^{(5)} = \frac{1}{10}(-3- (0.3441) – 6(0.2841)) = - 0.50487$

Sixth iteration:

$x^{(6)} = \frac{1}{10}(3 + 5(0.2822) + 2 (-0.50487)) = 0.340126$

$y^{(6)} = \frac{1}{10}(3 + 4(0.33843) + 3(-0.50487)) = 0.283911$

$z^{(6)} = \frac{1}{10}(-3- (0.33843) – 6(0.2822)) = - 0.503163$

Seventh iteration:

$x^{(7)} = \frac{1}{10}(3 + 5(0.283911) +2(-0.503163)) =0.3413229$

$y^{(7)} = \frac{1}{10}(3 + 4(0.340126) +3(-0.503163)) = 0.2851015$

$z^{(7)} = \frac{1}{10}(-3- (0.340126) – 6(0.283911)) = - 0.5043592$

Eighth iteration:

$x^{(8)} = \frac{1}{10}(3 + 5(0.2851015) +2 (-0.5043592))$

=0.34167891

$y^{(8)} = \frac{1}{10}(3 + 4(0.3413229) + 3(-0.5043592))$

= 0.2852214

$z^{(8)} = \frac{1}{10}(-3-(0.3413229)– 6(0.2851015))$

= - 0.50519319

Ninth iteration:

$x^{(9)} = \frac{1}{10}(3 + 5(0.2852214) + 2(-0.50519319))$

= 0.341572062

$y^{(9)} = \frac{1}{10}(3 + 4(0.34167891) + 3(-0.50519319))$

= 0.285113607

$z^{(9)} = \frac{1}{10}(-3- (0.34167891) – 6(0.2852214)) = - 0.505300731$

Hence, correct to 3 decimal places, the values are

$x = 0.342, \quad y = 0.285, \quad z = -0.505$

Gauss – seidel method: Initial values : $y = 0, z = 0$.

First iteration: $x^{(1)} = \frac{1}{10}(3 + 5(0) + 2(0)) = 0.3$

$y^{(1)} = \frac{1}{10}(3 + 4(0.3) + 3(0)) = 0.42$

$z^{(1)} = \frac{1}{10}(-3- (0.3) – 6(0.42)) = - 0.582$

Second iteration:

$x^{(2)} = \frac{1}{10}(3 + 5(0.42) + 2(-0.582)) = 0.3936$

$y^{(2)} = \frac{1}{10}$ (3 + 4(0.3936) + 3(- 0.582)) = 0.28284

$z^{(2)} = \frac{1}{10}$ (-3- (0.3936) – 6(0.28284)) = - 0.509064

Third iteration:

$x^{(3)} = \frac{1}{10}$ (3 + 5(0.28284) + 2(-0.509064)) = 0.3396072 $y^{(3)} = \frac{1}{10}$(3 +4(0.3396072)+3(-0.509064))= 0.28312368

$z^{(3)} = \frac{1}{10}$ (-3- (0.3396072) – 6(0.28312368))

= - 0.503834928

Fourth iteration:

$x^{(4)} = \frac{1}{10}$ (3 + 5(0.28312368) + 2(-0.503834928))

= 0.34079485

$y^{(4)} = \frac{1}{10}$ (3 + 4(0.34079485) + 3(-0.503834928))

= 0.285167464

$z^{(4)} = \frac{1}{10}$ (-3- (0.34079485) – 6(0.285167464))

= - 0.50517996

Fifth iteration:

$x^{(5)} = \frac{1}{10}$ (3 + 5(0.285167464) + 2(-0.50517996))

= 0.34155477

$y^{(5)} = \frac{1}{10}$ (3 + 4(0.34155477) + 3(-0.50517996))

= 0.28506792

$z^{(5)} = \frac{1}{10}$ (-3- (0.34155477) – 6(0.28506792))

= - 0.505196229

Sixth iteration:

$x^{(6)} = \frac{1}{10}(3 + 5(0.28506792) + 2(-0.505196229))$

= 0.341494714

$y^{(6)} = \frac{1}{10}(3 + 4(0.341494714) + 3(-0.505196229))$

= 0.285039017

$z^{(6)} = \frac{1}{10}(-3- (0.341494714) – 6(0.28506792))$

= - 0.5051728

Seventh iteration:

$x^{(7)} = \frac{1}{10}(3 + 5(0.285039017) + 2(-0.5051728))$

= 0.3414849

$y^{(7)} = \frac{1}{10}(3 + 4(0.3414849) + 3(- 0.5051728))$

= 0.28504212

$z^{(7)} = \frac{1}{10}(-3- (0.3414849) – 6(0.28504212))$

= - 0.5051737

The values at each iteration by both methods are tabulated below:

Iteration	Gauss - jacobi method			Gauss – seidel method		
	x	y	z	x	y	z
1	0.3	0.3	-0.3	0.3	0.42	-0.582
2	0.39	0.33	-0.51	0.3936	0.2828	-0.5090
3	0.363	0.303	-0.537	0.3396	0.2831	-0.5038
4	0.3441	0.2841	-0.5181	0.3407	0.2851	-0.5051
5	0.3384	0.2822	-0.5048	0.3415	0.2850	-0.5051
6	0.3401	0.2839	-0.5031	0.3414	0.2850	-0.5051
7	0.3413	0.2851	-0.5043	0.3414	0.2850	-0.5051
8	0.3416	0.2852	-0.5051			
9	0.3411	0.2851	-0.5053			

The values correct to 3 decimal places are

$$x = 0.342,\ \ y = 0.285,\ \ z = -0.505$$

POSSIBLE QUESTIONS

1. Write the formulae for method of triangularization.
2. Define iterative method.
3. Define power method.
4. Write the difference between the direct method and iterative method.
5. Explain the power method.
6. Solve the following system by Gauss elimination method.

$$3x + y - z = 3$$
$$2x - 8y + z = -5$$
$$x - 2y + 9z = 8$$

7. Solve the following system by Gauss Jacobi method.

$$8x + y + z = 8$$
$$2x + 4y + z = 4$$
$$x + 3y + 3z = 5$$

8. Solve the following system by Gauss Jordan method.

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

9. Solve the following system of equations by Gauss-Jacobi method

$$10x - 5y - 2z = 3$$
$$4x - 10y + 3z = -3$$
$$x + 6y + 10z = -3$$

10. Solve the following system by triangularization method.

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

11. Solve the following system of equations by Gauss-Seidal method.

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

12. Solve the following system by Gauss Jordan method.

$$x + 2y + z = 3$$
$$2x + 3y + 3z = 10$$
$$3x - y + 2z = 13$$

13. Find the numerically largest Eigen value of $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ and the corresponding Eigen vector.

14. Solve the following system by triangularization method.

x + y + 5z =16
2x + 3y + z = 4
4 x + y – z = 4

15. Solve the following system of equations by Gauss-Jacobi method

4x+2y +z = 14
x +5y -z = 10
x +y +8z = 20

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Pollachi Main Road, Eachanari (Po),
Coimbatore –641 021

Subject: Numerical Methods **Subject**

Class : II - B.Sc. Mathematics

Unit II

Part A (20x1=20 Marks) **(Question Nos. 1 to 20**

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
Iterative method is a ------------ method	Direct method	InDirect method	both 1st & 2nd	either 1st &2nd	InDirect method
-------------- is also a self-correction method.	Iteration method	Direct method	Interpolation	none	Iteration method
The condition for convergence of Gauss Seidal method is that the ------ should be diagonally dominant	Constant matrix	unknown matrix	Coefficient matrix	Unit matrix	Coefficient matrix
In ----------- method, the coefficient matrix is transformed into diagonal matrix	Gauss elimination	Gauss jordan	Gauss jacobi	Gauss seidal	Gauss jordan
-------------- Method takes less time to solve a system of equations comparatively than ' iterative method'	Direct method	Indirect method	Regula falsi	Bisection	Direct method
The iterative process continues till ------------- is secured.	convergency	divergency	oscillation	none	convergency
In Gauss elimination method, the solution is getting by means of ----------- from which the unknowns are found by back substitution.	Elementary operations	Elementary column operations	Elementary diagonal operations	Elementary row operations	Elementary row operations
The --------------- is reduced to an upper triangular matrix or a diagonal matrix in direct methods.	Coefficient matrix	Constant matrix	unknown matrix	Augment matrix	Augment matrix
The augment matrix is the combination of ------------------.	Coefficient matrix and constant matrix	Unknown matrix and constant matrix	Coefficient matrix and Unknown matrix	Coefficient matrix, constant matrix and Unknown matrix	Coefficient matrix and constant matrix
The given system of equations can be taken as in the form of -----------	A = B	BX= A	AX= B	AB = X	AX= B
Which is the condition to apply Gauss Seidal method to solve a system of equations?	1st row is dominant	1st column is dominant	diagonally dominant	last row dominant	diagonally dominant
Crout's method and triangularisation method are ------ method.	Direct	Indirect	Iterative	Interpolation	Direct
The solution of simultaneous linear algebraic equations are found by using-----	Direct method	Indirect method	both 1st & 2nd	Bisection	InDirect method
The matrix is ____ if the numerical value of the leading diagonal element in each row is greater than or equal to the sum of the numerical value of other element in that row.	orthogonal	symmetric	diagonally dominant	singular	diagonally dominant
If the Eigen values of A are -6, 2, 4 then ______ is dominant.	2	-6	4	-2	2
The Gauss – Jordan method is the modification of ________ method.	Gauss –Elimination	Gauss – Jacobi	Gauss – Seidal	interpolation	Gauss –Elimination
$x^2 + 5x + 4 = 0$ is a ________ equation.	algebraic	transcendental	wave	heat	algebraic
$a + b \log x + c \sin x + d = 0$ is a _________ equation.	algebraic	transcendental	wave	heat	transcendental
In Gauss – Jordan method, the augmented matrix is reduced into _____matrix	upper triangular	lower triangular	diagonal	scalar	diagonal
The 1st equation in Gauss – Jordan method, is called __________ equation.	pivotal	dominant	reduced	normal	pivotal
The element a_{11} in Gauss – Jordan method is called _________ element.	Eigen value	Eigen vector	pivot	root	pivot
The system of simultaneous linear equation in n unknowns AX = B if A is diagonally dominant then the system is said to be __________ system	dominant	diagonal	scalar	singular	diagonal
The convergence of Gauss – Seidal method is roughly _____ that of Gauss – Jacobi method	twice	thrice	once	4 times	twice
Jacobi's method is used only when the matrix is ____	symmetric	skew-symmetric	singular	non-singular	symmetric
Gauss Seidal method always --------- for a special type of systems.	Converges	diverges	oscillates	equal	Converges
Condition for convergence of Gauss Seidal method is ---------------.	Coefficient matrix is diagonally dominant	pivot element is Zero	Coefficient matrix is not diagonally dominant	pivot element is non Zero	Coefficient matrix is diagonally dominant
Modified form of Gauss Jacobi method is -------------------- method.	Gauss Jordan	Gauss Siedal	Gauss Jacobbi	Gauss Elimination	Gauss Siedal
In Gauss elimination method by means of elementary row operations, from which the unknowns are found by ---------------- method	Forward substitution	Backward substitution	random	Gauss Elimination	Backward substitution
In iterative methods, the solution to a system of linear equations will exist if the absolute value of the largest coefficient is ------------ the sum of the absolute values of all remaining coefficients in each equation.	less than	greater than or equal to	equal to	not equal	greater than or equal to

In -------------- iterative method, the current values of the unknowns at each stage of iteration are used in proceeding to the next stage of iteration.	Gauss Siedal	Gauss Jacobi	Gauss Jordan	Gauss Elimination	Gauss Siedal
The direct method fails if any one of the pivot elements become ----.	Zero	one	two	negative	Zero
In Gauss elimination method the given matrix is transformed into ------------- .	Unit matrix	diagonal matrix	Upper triangular matrix	lower triangular matrix	Upper triangular matrix
If the coefficient matrix is not diagonally dominant, then by ----------- that diagonally dominant coefficient matrix is formed.	Interchanging rows	Interchanging Columns	adding zeros	Interchangingrow and Columns	Interchangingrow and Columns
Gauss Jordan method is a -----------------.	Direct method	InDirect method	iterative method	convergent	Direct method
Gauss Jacobi method is a -----------------.	Direct method	InDirect method	iterative method	convergent	InDirect method
The modification of Gauss – Jordan method is called ---------------------.	Gauss Jordan	Gauss Siedal	Gauss Jacobbi	gauss elemination	Gauss Siedal
Gauss Seidal method always converges for ----------- of systems	Only the special type	all types	quadratic types	first type	Only the special type
In solving the system of linear equations, the system can be written as ---	BX = B	AX = A	AX = B	AB = X	AX = B
In solving the system of linear equations, the augment matrix is --------	(A, A)	(B, B)	(A, X)	(A, B)	(A, B)
In the direct methods of solving a system of linear equations, at first the given system is written as ------------- form.	An augment matrix	a triangular matrix	constant matrix	Coefficient matrix	An augment matrix
All the row operations in the direct methods can be carried out on the basis of --	all elements	pivot element	negative element	positiveelement	pivot element
The direct method fails if -----------.	1st row elements 0	1st column elements 0	Either 1st or 2nd	2 nd row is dominant	Either 1st or 2nd
The elimination of the unknowns is done not only in the equations below, but also in the equations above the leading diagonal is called ------------	Gauss elimination	Gauss jordan	Gauss jacobi	Gauss siedal	Gauss jordan
In Gauss Jordan method, we get the solution ---------------------	without using back substitution method	By using back substitution method	by using forward substitution method	Without using forward substitution method	By using back substitution method
If the coefficient matrix is diagonally dominant, then ---------- method converges quickly.	Gauss elimination	Gauss jordan	Direct	Gauss siedal	Gauss siedal
Which is the condition to apply Jocobi's method to solve a system of equations	1st row is dominant	1st column is dominant	diagonally dominant	2 nd row is dominant	diagonally dominant
Iterative method is a ------------ method	Direct method	InDirect method	Interpolation	extrapolation	InDirect method
As soon as a new value for a variable is found by iteration it is used immediately in the equations is called --------------.	Iteration method	Direct method	Interpolation	extrapolation	Iteration method
-------------- is also a self-correction method.	Iteration method	Direct method	Interpolation	extrapolation	Iteration method
The condition for convergence of Gauss Seidal method is that the ------ should be diagonally dominant	Constant matrix	unknown matrix	Coefficient matrix	extrapolation	Coefficient matrix
In ----------- method, the coefficient matrix is transformed into diagonal matrix	Gauss elimination	Gauss jordan	Gauss jacobi	Gauss seidal	Gauss jordan
We get the approximate solution from the ---------------.	Direct method	InDirect method	fast method	Bisection	InDirect method
The iterative process continues till ------------- is secured.	convergency	divergency	oscillation	point	convergency
In Gauss elimination method, the solution is getting by means of ----------- from which the unknowns are found by back substitution.	Elementary operations	Elementary column operations	Elementary diagonal operations	Elementary row operations	Elementary row operations
The method of iteration is applicable only if all equation must contain one coefficient of different unknowns as ---------- than other coefficients.	smaller	larger	equal	non zero	larger
The --------------- is reduced to an upper triangular matrix or a diagonal matrix in direct methods.	Coefficient matrix	Constant matrix	unknown matrix	Augment matrix	Augment matrix
The augment matrix is the combination of ------------------.	Coefficient matrix and constant matrix	Unknown matrix and constant matrix	Coefficient matrix and Unknown matrix	Coefficient matrix, constant matrix and Unknown matrix	Coefficient matrix and constant matrix
The sufficient condition of iterative methods will be satisfied if the large coefficients are along the ---------------- of the coefficient matrix.	Rows	Coloumns	Leading Diagonal	elements	Leading Diagonal
Which is the condition to apply Gauss Seidal method to solve a system of equations.	1st row is dominant	1st column is dominant	diagonally dominant	Leading Diagonal	diagonally dominant
In the absence of any better estimates, the -------------of the function are taken as x = 0, y = 0, z = 0.	initialapproximations	roots	points	final value	initialapproximations
The solution of simultaneous linear algebraic equations are found by using-	Direct method	InDirect method	fast method	Bisection	InDirect method

UNIT-III

INTERPOLATION

Interpolation: Lagrange and Newton's methods. Error bounds - Finite difference operators.Gregory forward and backward difference interpolation – Newton's divided difference – Central difference – Lagrange and inverse Lagrange interpolation formula.

Introduction

Interpolation means the process of computing intermediate values of a function a given set of tabular values of a function. Suppose the following table represents a set of values of x and y.

x : x_1 x_2 x_3............x_n

y : y_1 y_2 y_3...........y_n

We may require the value of $y = y_i$ for the given $x = x_i$, where x lies between x_0 *to* x_n

Let $y = f(x)$ be a function taking the values $y_0, y_1, y_2, \ldots\ y_n$ corresponding to the *values* $x_0,\ x_1, x2,\ \ldots\ldots\ldots\ x_n$. Now we are trying to find $y = y_i$ for the given $x = x_i$ under assumption that the function f(x) is not known. In such cases , we replace f(x) by simple fan arbitrary function and let Φ(x) denotes an arbitrary function which satisfies the set of values given in the table above . The function Φ(x) is called interpolating function or smoothing function or interpolation formula.

Newton's forward interpolation formula (or) Gregory-Newton forward interpolation formula (for equal intervals)

Let $y = f(x)$ denote a function which takes the values $y_0, y_1, y_2\ \ldots\ldots\ldots, y_n$ corresponding to the values $x_0, x_1, x_2\ \ldots\ldots\ldots, x_n$.

Let suppose that the values of x i.e., x_0, x_1, x_2 ………., x_n.are equidistant .

$$x_1 = x_0 + h\ ;\ \ x_2 = x_1 + h\ ;\ \ \text{and so on}\qquad x_n = x_{n-1} + h\ ;$$

Therefore $xi = x0 + i\,h$, *where* $i = 1,2,\ \ldots, n$

Let $P_n(x)$ be a polynomial of the n^{th} degree in which x is such that

$$y_I = f(x_i) = P_n(x_i), \quad I = 0,1,2, \ldots n$$

Let us assume Pn(x) in the form given below

$$P_n(x) = a_0 + a_1(x - x_0)^{(1)} + a_2(x - x_0)^{(2)} + \ldots\ldots + a_r(x - x_0)^{(r)} + \ldots\ldots + \ldots\ldots + a_n(x - x_0)^{(n)} \quad \ldots\ldots(1)$$

This polynomial contains the n + 1 constants $a_0, a_1, a_2, \ldots\ldots a_n$ can be found as follows :

$P_n(x_0) = y_0 = a_0$ (setting x = x0, in (1))

Similarly $y_1 = a_0 + a_1(x_1 - x_0)$

$y_2 = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)$

From these, we get the values of $a_0, a_1, a_2, \ldots\ldots a_n$

i.e.,

Therefore, $a_0 = y_0$

$\Delta y_0 = y_1 - y0 = a_1(x_1 - x_0)$

$= a_1 h$

$\Rightarrow a_1 = \Delta y_0 / h$

lly $\Rightarrow a_2 = (\Delta y_1 - \Delta y_0) / 2h^2 = \Delta^2 y_0 / 2!\, h^2$

lly $\Rightarrow a_3 = \Delta^3 y_0 / 3!\, h^3$

Putting these values in (1), we get

$$P_n(x) = = y_0 + (x - x_0)^{(1)}\Delta y_0 /h + (x - x_0)^{(2)} \Delta^2 y_0 /(2!\, h^2) + \ldots\ldots + (x - x_0)^{(r)} \Delta^r y_0 /(r!\, h^r) + \ldots\ldots + (x - x_0)^{(n)} \Delta^r y_0 /(n!\, h^n)$$

By substituting $\frac{x - x_0}{h} = u$ *, the above equation becomes*

$$y(x_0 + uh) = y_u = y_0 + u\,\Delta y_0 + \frac{u\,(u-1)}{2!}\Delta^2 y_0 + \frac{u\,(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots \ldots$$

By substituting $u = u^{(1)}$,

$u\,(u-1) = u^{(2)}$,

$u(u-1)(u-2) = u^{(3)}$, *… in the above equation, we get*

$$P_n\,(x) = P_n y(x_0 + uh) = y_0 + u^{(1)}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \frac{u^{(3)}}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{u^{(r)}}{r!}\Delta^r y0 + \ldots. + \frac{u^{(n)}}{n!}\Delta^n y_0$$

The above equation is known as **Gregory-Newton forward formula or Newton's forward interpolation formula.**

Note : 1. This formula is applicable only when the interval of difference is uniform.

2. This formula apply forward differences of y_0, hence this is used to interpolate the values of y nearer to beginning value of the table (i.e., x lies between x0 to ^{x}1 or x1 to x_2)

Example.

Find the values of y at x = 21 from the following data.

x: 20 23 26

x: 0.3420 0.3907

0.4384

29

0.4848

Solution.

Step 1. Since x = 21 is nearer to beginning of the table. Hence we apply Newton's forward formula.

Step 2. Construct the difference table

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$
20	0.3420	(0.3420-0.3907)		
		0.0487	(0.0477-0.0487)	
23	0.3907		-0.001	
		0.0477		-0.0003
26	0.4384		-0.0013	
		0.0464		
29	0.4848			

Step 3. Write down the formula and put the various values :

$$P_n(x) = P_n y(x_0 + uh) = y_0 + u^{(1)}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \frac{u^{(3)}}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{u^{(r)}}{r!}\Delta^r y0 + \ldots + \frac{u^{(n)}}{n!}\Delta^n y_0$$

Where $u^{(1)} = (x - x_0) / h = (21 - 20) / 3 = 0.3333$

$\underline{u(2) \;\; = u(u\text{-}1) = \;\; (0.3333)(0.6666)}$

$P_n\,(x=21) = y(21) = 0.3420 + (0.3333)(\,0.0487) + (0.3333)\,(-0.6666)\,(\,-0.001)$

$+ (0.3333)\,(-0.6666)(-1.6666)\,(\,-0.0003)$

$= \mathbf{0.3583}$

Example: . From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46.

Age	x:	45	50	55	60	65
Premium	y:	114.84	96.16	83.32	74.48	68.48

Solution.

Step 1. Since $x = 46$ is nearer to beginning of the table and the values of x is equidistant i.e., $h = 5$.. Hence we apply Newton's forward formula.

Step 2. Construct the difference table

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
45	114.84				
		-18.68			
50	96.16		5.84		
		-12.84		-1.84	
55	83.12		4.00		0.68
		-8.84		-1.16	
60	74.48		2.84		
		-6.00			
65	68.48				

Step 3. Write down the formula and put the various values :

$$P_n(x) = P_n y(x_0 + uh) = y_0 + u^{(1)}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \frac{u^{(3)}}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{u^{(r)}}{r!}\Delta^r y0 + \ldots + \frac{u^{(n)}}{n!}\Delta^n y_0$$

Where $u = (x - x_0) / h$ = (46 – 45) / 5 = 01/5 = 0.2

$$P_n(x=46) = y(46) = 114.84 + [0.2(-18.68)] + [0.2(-0.8)(5.84)/3]$$
$$+ [0.2(-0.8)(-1.8)(-1.84)/6]$$
$$+ [0.2(-0.8)(-1.8)(-2.8)(0.68)]$$
$$= 114.84 - 3.7360 - 0.4672 - 0.08832 - 0.228$$

$$= \mathbf{110.5257}$$

Example . From the following table , find the value of tan 45^0 15’

x^0:	45	46	47	48	49	50
tan x^0:	1.0	1.03553	1.07237	1.11061	1.15037	1.19175

Solution.

Step 1. Since $x = 45^o\ 15'$ is nearer to beginning of the table and the values of x is equidistant i.e., $h = 1$. Hence we apply Newton’s forward formula.

Step 2. Construct the difference table to find various Δ’s

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
45^0	1.0000					
		0.03553				
460	1.03553		0.00131			
		0.03684		0.00009		
47^0	1.07237		0.00140		0.00003	
		0.03824		0.00012		-0.00005
48^0	1.11061		0.00152		-0.00002	
		0.03976		0.00010		
49^0	1.15037		0.00162			
		0.04138				
50^0	1.19175					

Step 3. Write down the formula and substitute the various values :

$$P_n(x) = P_n y(x_0 + uh) = y_0 + u^{(1)}\Delta y_0 + \frac{u^{(2)}}{2!}\Delta^2 y_0 + \frac{u^{(3)}}{3!}\Delta^3 y_0 + \dots\dots + \frac{u^{(r)}}{r!}\Delta^r y0 + \dots. + \frac{u^{(n)}}{n!}\Delta^n y_0$$

Where $u = (45^o\, 15' - 45^0) / 1^0$

$= 15' / 1^0$

$= 0.25$ ……………(since $1^0 = 60$ ‘)

$$y\,(x=45^o\, 15') = P_5\,(45^o\, 15') = 1.00 + (0.25)(0.03553) + (0.25)(-0.75)(0.00131)/2$$
$$+(0.25)(-0.75)(-1.75)(0.00009)/6$$
$$+(0.25)(-0.75)(-1.75)(-2.75)(0.0003)/24$$
$$+(0.25)(-0.75)(-1.75)(-2.75)(-3.75)(-0.00005)/120$$

$= 1.000 + 0.0088825 - 0.0001228 + 0.0000049$

$= \mathbf{1.00876}$

Newton's backward interpolation formula (or) Gregory-Newton backward interpolation formula (for equal intervals)

Let $y = f(x)$ denote a function which takes the values $y_0, y_1, y_2 \ldots\ldots\ldots, y_n$ corresponding to the values $x_0, x_1, x_2 \ldots\ldots\ldots, x_n$.

Let suppose that the values of x i.e., $x_0, x_1, x_2 \ldots\ldots\ldots, x_n$ are equidistant .

$x_1 = x_0 + h$; $x_2 = x_1 + h$; and so on $x_n = x_{n-1} + h$;

Therefore $xi = x0 + i\,h$, where $i = 1,2, \ldots, n$

Let $P_n(x)$ be a polynomial of the n^{th} degree in which x is such that

$y_I = f(x_i) = P_n(x_i), \ I = 0,1,2, \ldots. n$

$$P_n(x) = a_0 + a_1(x - x_n)^{(1)} + a_2(x - x_n)(x - x_{n-1})^{)} + \ldots\ldots\ldots + a_n(x - x_n)(x - x_{n-1}) \ldots(x - x_1) \ \ldots\ldots(1)$$

Let us assume Pn(x) in the form given below

$$P_n(x) = a_0 + a_1(x - x_n)^{(1)} + a_2(x - x_n)^{(2)} + \ldots\ldots\ldots + a_r(x - x_n)^{(r)} + \ldots\ldots + \ldots\ldots + a_n(x - x_n)^{(n)} \ \ldots\ldots(1.1)$$

This polynomial contains the n + 1 constants $a_0, a_1, a_2, \ldots\ldots\ldots a_n$ can be found as follows :

$P_n(x_n) = \ y_n = a_0$ (setting x = xn, in (1))

Similarly $y_{n-1} = a_0 + a_1(x_{n-1} - x_n)$

$y_{n-2} = a_0 + a_1(x_{n-2} - x_n) + a_2(x_{n-2} - x_n)$

From these, we get the values of $a_0, a_1, a_2, \ldots\ldots\ a_n$

Therefore, $a_0 = \ y_n$

˘$yn = y_n - yn\text{-}1 = a_1(x_{n-1} - x_n)$

$= a_1 h$

$=> a_1 = y_n / h$

lly $=> a_2 = (˘y_1 - ˘y_n) / 2h^2 = ˘^2 y_n / 2!\, h^2$

lly $=> a_3 = ˘^3 / 3!\, h^3$

Putting these values in (1), we get

$$P_n(x) = = y_n + (x - x_n)^{(n)} \nabla y_n / h + (x - x_n)^{(2)} \nabla^2 y_n / (2!\, h^2) + (x - x_n)^{(r)} \nabla^r y_n / (r!\, h^r) + \ldots\ldots + (x - x_n)^{(n)} \nabla^n y_n / (n!\, h^n)$$

By substituting $\frac{x - x_n}{h} = v$ *, the above equation becomes*

$$y(x_n + vh) = y_n + v \nabla y_n + \frac{v(v+1)}{2!} \nabla^2 y_n + \frac{v(v+1)(v+2)}{3!} \nabla^3 y_n + \ldots\ldots$$

By substituting $v = v^{(1)}$,

$v(v+1) = v^{(2)}$,

$v(v+1)(v+2) = v^{(3)}$, … *in the above equation, we get*

$$P_n(x) = P_n y(x_n + vh) = y_n + v^{(1)} \nabla y_n + \frac{v^{(2)}}{2!} \nabla^2 y_n + \frac{v^{(3)}}{3!} \nabla^3 y_n + \ldots + \frac{v^{(r)}}{r!} \nabla^r y_n + \ldots + \frac{v^{(n)}}{n!} \Delta^n y_n$$

The above equation is known as **Gregory-Newton backward formula or Newton's backward interpolation formula.**

Note : 1. This formula is applicable only when the interval of difference is uniform.

2. This formula apply backward differences of y_n, hence this is used to interpolate the values of y nearer to the end of a set tabular values. (i.e., x lies between xn to xn- 1 and xn-1 to x_{n-2})

Example: Find the values of y at x = 28 from the following data.

x:	20	23	26	29
y	0.3420	0.3907	0.4384	0.4848

Solution.

Step 1. Since x = 28 is nearer to beginning of the table. Hence we apply Newton's backward formula.

Step 2. Construct the difference table

x	y	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$
20	0.3420	(0.3420-0.3907)		
		0.0487	(0.0477-0.0487)	
23	0.3907		-0.001	
		0.0477		
26	0.4384			-0.0003
			-0.0013	
29	0.4848	0.0464		

Step 3. Write down the formula and put the various values :

$$P_3(x) = P_3 y(x_n + vh) = y_n + v^{(1)}\nabla y_n + \frac{v^{(2)}}{2!}\nabla^2 y_n + \frac{v^{(3)}}{3!}\nabla^3 y_n$$

Where $v^{(1)} = (x - x_n) / h = (28 - 29) / 3 = -0.3333$

$v^{(2)} = v(v+1) = (-0.333)(0.6666)$

$v^{(3)} = v(v+1)(v+2) = (-0.333)(0.6666)(1.6666)$

$$P_n(x=28) = y(28) = 0.4848 + (-0.3333)(0.0464) + (-0.3333)(0.6666)(-0.0013)/2$$

$$+(-0.3333)\,(0.6666)(1.6666)\ \ (-0.0003)/6$$

$$= 0.4848 - 0.015465 + 0.0001444 + 0.0000185$$

$$= \mathbf{0.4695}$$

Example: From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 63.

Age	x:	45	50	55	60	65
Premium	y:	114.84	96.16	83.32	74.48	68.48

Solution.

Step 1. Since $x = 63$ is nearer to beginning of the table and the values of x is equidistant i.e., $h = 5$.. Hence we apply Newton's backward formula.

Step 2. Construct the difference table

x	y	Δy_0	$\Delta^2 y0$	$\Delta^3 y_0$	$\Delta^4 y_0$
45	114.84				
		-18.68			
50	96.16		5.84		
		-12.84		-1.84	
55	83.12		4.00		-
		-8.84		1.16	
60	74.48		2.84		
		-6.00			
65	68.48				
					0.68

Step 3. Write down the formula and put the various values :

$$P_3\,(x) = P_3 y(x_n + vh) = y_n + v^{(1)}\Delta y_n + \frac{v^{(2)}}{2!}\Delta^2 y_n + \frac{v^{(3)}}{3!}\Delta^3 yn + \frac{v^{(4)}}{4!}\Delta^4 y_n$$

Where $v^{(1)} = (x - x_n) / h = (63 - 65) / 5 = -2/5 = -0.4$

$v(2) = v(v+1)$ $v(3) = v(v+1)\,(v+2)$

$= (-0.4)(1.6)$ $= (-0.4)(1.6)(2.6)$

$v(4) = v(v+1)(v+2))(v+3) = (-0.4)(1.6)(2.6)(3.6)$

$P_4(x=63) = y(63) = 68.48 + [(-0.4)(-6.0)] + [(-0.4)(1.6)(2.84)/2]$

$+ [(-0.4)(1.6)(2.6)(-1.16)/6]$

$+ [(-0.4)(1.6)(2.6)(3.6)(0.68)/24]$

$= 68.48 + 2.40 - 0.3408 + 0.07424 – 0.028288$

= **70.5852**

Example: From the following table , find the value of tan $49^0\,15'$

x^0:	45	46	47	48	49	50
tan x^0:	1.0	1.03553	1.07237	1.11061	1.15037	1.19175

Solution.

Step 1. Since $x = 49^o\,45'$ is nearer to beginning of the table and the values of x is equidistant i.e., $h = 1$. Hence we apply Newton's backward formula.

Step 2. Construct the difference table to find various Δ's

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$
45^0	1.0000					
		0.03553				
46	1.03553		0.00131			
		0.03684		0.00009		
47^0	1.07237		0.00140		0.00003	
		0.03824		0.00012		-0.00005
48^0	1.11061		0.00152		-0.00002	
		0.03976		0.00010		
49^0	1.15037		0.00162			
		0.04138				
50^0	1.19175					

Step 3. Write down the formula and substitute the various values :

$$P_5(x) = P_5 y(x_n + vh) = y_n + v^{(1)}\Delta y_n + \frac{v^{(2)}}{2!}\Delta^2 y_n + \frac{v^{(3)}}{3!}\Delta^3 y_n + \frac{v^{(4)}}{4!}\Delta^4 y_n + \frac{v^{(5)}}{5!}\Delta^5 y_n$$

Where $v = (49^o 45' - 50^0) / 1^0$

$= -15' / 1^0$

$= -0.25$ ……………(since $1^0 = 60'$)

$v(2) = v(v+1) \qquad = (-0.25))(0.75)$

$\qquad\qquad\qquad\qquad = (-0.25)(0.75)(1.75)$

$v(3) = v(v+1)(v+2)$

$v(4) = v(v+1)(v+2))(v+3) = (-0.25)(0.75)(1.75)(2.75)$

$y(x=49^o 15') = P_5(49^o 15') = 1.19175 + (-0.25)(0.04138) + (-0.25)(0.75)(0.00162)/2$
$+(-0.25)(0.75)(1.75)(0.0001)/6$

$+(-0.25)(0.75)(1.75)(2.75)(-0.0002)/24$
$+(-0.25)(0.75)(1.75)(2.75)(3.75)(-0.00005)/120$

$= 1.19175 - 0.010345 - 0.000151875 + 0.000005 + \ldots$

$=$ ***1.18126***

Lagrange's Interpolation Formula

Interpolation means the process of computing intermediate values of a function a given set of tabular values of a function. Suppose the following table represents a set of values of x and y.

x:	x_0	x_1	x_2	$x3$	………..	x_n
y:	y_0	y_1	y_2	$y3$	………..	y_n

We may require the value of $y = y_i$ for the given $x = x_i$, where x lies between x_0 *to* x_n

Let y = $f(x)$ be a function taking the values $y_0, y_1, y_2, \ldots y_n$ corresponding to the *values* $x_0, x_1, x2, \ldots\ldots x_n$. Now we are trying to find $y = y_i$ for the given $x = x_i$ under assumption that the function f(x) is not known. In such cases, x_i *'s* are not equally spaced we use *Lagrange's interpolation formula.*

Newton's Divided Difference Formula:

The divided difference $f[x_0, x_1, x_2, \ldots, x_n]$, sometimes also denoted $[x_0, x_1, x_2, \ldots, x_n]$, on $n+1$ points

$X_0, x_1, \ldots, x_n$ of a function $f(x)$ is defined by $f[x_0] \equiv f(x_0)$ and

$$f[x_0, x_1, \ldots, x_n] = \frac{f[x_0, \ldots, x_{n-1}] - f[x_1, \ldots, x_n]}{x_0 - x_n}$$

for $n \geq 1$. The first few differences are

$$f[x_0, x_1] = \frac{f_0 - f_1}{x_0 - x_1}$$

$$f[x_0, x_1, x_2] = \frac{f[x_0, x_1] - f[x_1, x_2]}{x_0 - x_2}$$

$$f[x_0, x_1, \ldots, x_n] = \frac{f[x_0, \ldots, x_{n-1}] - f[x_1, \ldots, x_n]}{x_0 - x_n}.$$

Defining

$\pi_n(x) \equiv (x - x_0)(x - x_1)\cdots(x - x_n)$ and taking the derivative

$\pi'_n(x_k) = (x_k - x_0)\cdots(x_k - x_{k-1})(x_k - x_{k+1})\cdots(x_k - x_n)$ gives the identity

$$f[x_0, x_1, \ldots, x_n] = \sum_{k=0}^{n} \frac{f_k}{\pi'_n(x_k)}.$$

Lagrange's interpolation formula (for unequal intervals)

Let $y = f(x)$ denote a function which takes the values $y_0, y_1, y_2 \ldots\ldots\ldots, y_n$ corresponding to the values $x_0, x_1, x_2 \ldots\ldots\ldots, x_n$.

Let suppose that the values of x *i.e.*, $x_0, x_1, x_2 \ldots\ldots\ldots, x_n$. are not equidistant .

$$y_I = f(x_i) \quad I = 0,1,2, \ldots. N$$

Now, there are (n+1) paired values(x_i, y_i), $I = 0,1,2, \ldots n$ *and hence f(x) can be represented by a polynomial function of degree n in x.*

Let us consider f(x) as follows

$$f(x) = a_0(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n)$$
$$+ a_1(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_n)$$
$$+ a_2(x-x_0)(x-x_3)(x-x_4)\ldots(x-x_n)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+a_n(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_{n-1}) \quad \ldots\ldots\ldots(1)$$

Substituting $x= x_0, y=y_0$, *in the above equation*

$$y_0 = a_0(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n)$$

which implies $a_0 = y_0 /(x_0-x_1)(x_0-x_2)(x_0-x_3)\ldots(x_0-x_n)$

Similarly $a_1= y_1 /(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)$

$$a_2 = y_2 /(x_2-x_0)(x_2-x_1)(x_2-x_3)\ldots(x_2-x_n)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_n = y_n(x_n-x_0)(x_n-x_2)(x_n-x_3)\ldots(x_n-x_{n-1})$$

Putting these values in (1), we get

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\ldots(x_0-x_n)} y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)} y_1$$

$$(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n)$$

$$+ \frac{\qquad}{(x_2 - x_0)(x_2 - x_2)(x_1 - x_3) \ldots (x_1 - x_n)} y_2$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+ \frac{(x - x_0)(x - x_2)(x - x_3) \ldots (x - x_{n-1})}{(x_n - x_0)(x_n - x_2)(x_n - x_3) \ldots (x_n - x_{n-1})} y_n$$

The above equation is called ***Lagrange's interpolation formula*** for unequal intervals.

Note : 1. This formula is will be more useful when the interval of difference is not uniform.

POSSIBLE QUESTIONS

1. Prove that $E\Delta = \Delta = \nabla E$.
2. Write Gregory Newton backward interpolation formulae.
3. Define Inverse Lagrange's interpolation
4. Prove that $\mu = (1 + \frac{\delta^2}{4})^{\frac{1}{2}}$
5. Prove that $\Delta\nabla = \Delta - \nabla = \delta^2$.
6. From the following table, find the value of tan 45°15′

x° :	45	46	47	48	49	50
tan x° :	1.0000	1.0355	1.072	1.1106	1.1503	1.1917

7. Using inverse interpolation formula, find the value of x when y=13.5.

x:	93.0	96.2	100.0	104.2	108.7
y:	11.38	12.80	14.70	17.07	19.91

8. From the following table find f(x) and hence f(6) using Newton interpolation formula.

x :	1	2	7	8
f(x) :	1	5	5	4

9. Find the values of y at X=21 and X=28 from the following data.

X:	20	23	26	29
Y:	0.3420	0.3907	0.4384	0.4848

10. Using Newton's divided difference formula. Find the values of f(2),f(8) and f(15) given the following table

x:	4	5	7	10	11	13
f(x):	48	100	294	900	1210	2028

11. Using Lagrange's interpolation formula find the value corresponding to x = 10 from the following table

x :	5	6	9	11
y :	12	13	14	16

12. From the following table of half-yearly premium for policies maturing at different ages. Estimate the premium for policies maturing at age 46 & 63.

Age x :	45	50	55	60	65
Premium y :	114.84	96.16	83.32	74.48	68.48

13. Find the value of y at x = 1.05 from the table given below.

x :	1.0	1.1	1.2	1.3	1.4	1.5
y :	0.841	0.891	0.932	0.964	0.985	1.015

14. Using inverse interpolation formula, find the value of x when y=13.5.

x:	93.0	96.2	100.0	104.2	108.7
y:	11.38	12.80	14.70	17.07	19.91

15. Find the age corresponding to the annuity value 13.6 given the table

Age(x) :	30	35	40	45	50
Annuity Value(y):	15.9	14.9	14.1	13.3	12.5

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Coimbatore –641 021

Subject: Numerical Methods **Subject**

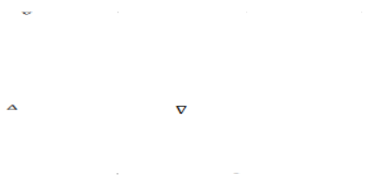
Class : II - B.Sc. Mathematics

Unit III

Part A (20x1=20 Marks) **(Question Nos. 1 to 20**

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
The x values of Interpolating polynomial of newton -Gregory has_________	even space	equal space	odd space	unequal	equal space
The value of E is __________	delta -1	1-delta	delta+1	delta+2	delta+1
We use the central difference formula such as ___________	lagrange's	Newton	Euler	bessel's	bessel's
--------------- Formula can be used for unequal intervals.	Newton’s forward	Newton’s backward	Lagrange	stirling	Lagrange
By putting n = 3 in Newton cote’s formula we get ----------- rule.	Simpson’s 1/3 rule	Simpson’s 3/8 rule	Trapezoidal rule	Simpson’s rule	Simpson’s 3/8 rule
The process of computing the value of a function outside the range is called -----	interpolation	extrapolation	triangularisation	integration	extrapolation
The process of computing the value of a function inside the range is called ------	interpolation	extrapolation	triangularisation	integration	interpolation
_________ Formula can be used for interpolating the value of f(x) near the end of the tabular values.	Newton’s forward	Newton’s backward	Lagrange	stirling	Newton’s backward
The technique of estimating the value of a function for any intermediate value is	interpolation	extrapolation	forward method	backward method	interpolation
The $(n+1)^{th}$ and higher differences of a polynomial of the nth degree are ---------	zero	one	two	three	zero
Numerical evaluation of a definite integral is called --------	integration	differentiation	interpolation	triangularisation	integration
The values of the independent variable are not given at equidistance intervals, we use --------------- formula.	Newton’s forward	Newton’s backward	Lagrange	stirling	Lagrange
To find the unknown values of y for some x which lies at the --------- of the table, we use Newton’s Backward formula.	beginning	end	center	outside	end
To find the unknown values of y for some x which lies at the --------- of the table, we use Newton’s Forward formula.	beginning	end	center	outside	beginning
To find the unknown value of x for some y, which lies at the unequal intervals we use ------------------- formula.	Newton’s forward	Newton’s backward	Lagrange	inverse interpolation	Lagrange
If the values of the variable y are given, then the method of finding the unknown variable x is called ----------------	Newton’s forward	Newton’s backward	interpolation	inverse interpolation	inverse interpolation
In Newton’s backward difference formula, the value of n is calculated by -------.	$n = (x-x_n) / h$	$n = (x_n-x) / h$	$n = (x-x_0) / h$	$n = (x_0-x) / h$	$n = (x-x_n) / h$
------------- Interpolation formula can be used for equal and unequal intervals.	Newton’s forward	Newton’s backward	Lagrange	none	Lagrange
The fourth differences of a polynomial of degree four are ----------.	zero	one	two	three	zero
If the values $x_0 = 0$, $y_0 = 0$ and h = 1 are given for Newton’s forward method, then the value of x is ------------.	0	1	n	X	n
The differences of constant functions are -----	Not equal to zero	zero	one	two	zero
In Newton’s forward interpolation formula, the first two terms will give the -----	extrapolation	linear interpolation	parabolic interpolation	interpolation	linear interpolation
In Newton’s forward interpolation formula, the three terms will give the ------	extrapolation	linear interpolation	parabolic interpolation	interpolation	parabolic interpolation
The difference $D^3f(x)$ is called -------differences f(x).	first	fourth	second	third	third
n th difference of a polynomial of n th degree are constant and all higher order difference are	constant	variable	zero	negative	zero
In divided difference the value of any difference is ----- of the order of their argument	Independent	dependent	Inverse	direct	Independent
The differences Dy are called -------differences f(x).	first	fourth	second	third	first
The value (delta +1)is ____________	E	h	h2	h4	E



UNIT-IV

NUMERICAL DIFFERENTIATION AND INTEGRATION

Numerical Differentiation and Integration: Gregory's Newton's forward and backward differentiation- Trapezoidal rule, Simpson's rule, Simpsons 3/8th rule, Boole's Rule. Midpoint rule, Composite Trapezoidal rule, Composite Simpson's rule.

Numerical differentiation

The problem of Interpolation is finding the value of y for the given value of x among (x_i , y_j) for i= 1 to n. Now we find the derivatives of the corresponding arguments . If the required value of y lies in the first half of the interval then we call it as Forward interpolation .If the required value of y (derivative value) lies in the second half of the interval we call it as Backward interpolation also if the derivative of y lies in the middle of of class interval then we solve by central difference.

Newton's forward formula for Interpolation :

$Y= y_0 +u\, \Delta y_0+u(u-1)/2!\, \Delta^2 Y_0 +u\, (u-1)(u-2) / 3!\, \Delta^3 Y_0+\ldots\ldots$

Where $u = (x-x_0)/h$

Differentiating with respect to x ,

$dy/dx = (dy/du). (du/dx) = (1/h) (dy / du)$

$(dy / dx)_{x \neq x_0} = (1 / h) [\Delta y_0 +(2u-1)/2\ \Delta^2 y_0 +(3u^2 -6u+2)/ 6\ \Delta^3 y_0 +\ldots\ldots\ldots]$

$(dy / dx)_{x = x_0} = (1 / h) [\Delta y_0 -(1/2)\ \Delta^2 y_0 +(1/ 3)\ \Delta^3 y_0 +\ldots\ldots\ldots]$

$(d^2y / dx^2)_{x \neq x_0}= d/dx (dy / dx) = d/dx(dy / du. du / dx)$

$= (1/h^2) [\Delta^2 y_0 + 6(u-1) / 6\, \Delta^3 y_0 + (12u^2 - 36 u +22) / 2\, \Delta^4 y_0 +\ldots\ldots.]$

$(d^2y / dx^2)_{x= x_0}=(1/h^2) [\Delta^2 y_0 - \Delta^3 y_0 +(11/12)\, \Delta^4 y_0 +\ldots\ldots.]$

Similarly,

$(d^3y / dx^3)_{x \neq x_0}= (1/h^3) [\Delta^3 y_0 + (2u - 3) / 2\, \Delta^4 y_0 +\ldots\ldots.]$

$(d^2y / dx^2)_{x= x_0}=(1/h^3) [\Delta^3 y_0 -(3/2)\Delta^4 y_0 +\ldots\ldots.]$.

In a similar manner the derivatives using backward interpolation an also be found out.

Using backward interpolation .

$(dy / dx)\ x \neq x_n = (1 / h) [\nabla y_n + (2u+1)/2\ \nabla^2 y_n + (3u^2 + 6u+2)/ 6\ \nabla^3 y_n + \ldots\ldots\ldots]$

$(dy / dx)\ x = x_n = (1 / h) [\nabla y_n - (1/2)\ \nabla^2 y_n + (1/ 3)\ \nabla^3 y_n + \ldots\ldots\ldots]$

$(d^2y / dx^2)\ x \neq x_0 = (1/h^2) [\nabla^2 y_0 + 6(u-1) / 6\ \nabla^3 y_0 + (12u^2 - 36 u +22) / 2\ \nabla^4 y_0 + \ldots\ldots.]$

$(d^2y / dx^2)\ x = x_0 = (1/h^2) [\nabla^2 y_0 - \nabla^3 y_0 + (11/12)\ \nabla^4 y_0 +.]$

Example

Find the first two derivatives of $x^{(1/3)}$ at x= 50 and x= 56, given the table below.

X: 50 51 52 53 54 55 56

Y: 3.6840 3.7084 3.7325 3.7563 3.7798 3.8030 3.8259

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
50	3.6840				
51	3.7084	0.0244			
52	3.7325	0.0241	-0.0003	0	
53	3.7563	0.0238	-0.0003	0	0
54	3.7798	0.0235	-0.0003	0	0
55	3.8030	0.0232	-0.0003	0	0
56	3.8259	0.0229	-0.0003		

At x= 50,

$(dy/dx)_{x = x0} = (1 /h)[\Delta y_0 - (1/2)\ \Delta^2 y_0 + (1/3)\Delta^3 y_0 + \ldots\ldots\ldots]$

$= (1/1)[0.024-(1/2)(-0.0003)+0] = 0.02455$

$(d^2y/dx^2)_{x= x0} = (1/h^2) [\Delta^2 y_0 - \Delta^3 y_0 + (11/12)\ \Delta^4 y_0 + \ldots\ldots.]$

$= (1/1)[-0.003-0]= -.0003$

At x=56,

$(dy/dx)_{x = xn} = (1/h)[\ \nabla y_n + (1/2)\ \nabla^2 y_n + (1/3)\ \nabla^3 y_n + \ldots\ldots\ldots]$

$= (1/1) [0.0229+(1/2)(-0.0003)+0] = 0.02275.$

$(d^2y/dx^2)_{x= xn} = (1/h^2)[\ \nabla^2 y_n + \nabla^3 y_n + (11/12)\ \nabla^4 y_n + \ldots]$

$= (1/1) [-0.003-0] = -0.0003.$

For the above ptroblem let us find the first two derivatives of x when x= 52 and x= 55.

When x=52, From Newton's forward formula

$(dy / dx)\ x \neq x_0 = (1 / h)\ [\Delta y_0 + (2u-1)/2\ \Delta^2 y_0 + (3u^2 - 6u+2)/ 6\ \Delta^3 y_0 + \ldots\ldots\ldots],$

$= (1/1)\ [\ 0.0244+(3/2)(-0.0003)+0] = 0.02395,$

Since here $u= (x-x_0) / h = (52-50)/1 = 2$.

$(d^2y / dx^2)\ x \neq x_0 = (1/h^2)\ [\Delta^2 y_0 + 6(u-1) / 6\ \Delta^3 y_0 + (\ 12u^2 - 36 u +22) / 2\ \Delta^4 y_0 + \ldots\ldots.]$

$= (1/)m\ [\ -0.0003+0] = -0.0003.$

When x= 55,from backward interpolation

$(dy / dx)\ x \neq x_n = (1 / h)\ [\nabla y_n + (2v+1)/2\ \nabla^2 y_n + (3v^2 + 6v+2)/ 6\ \nabla^3 y_n + \ldots\ldots\ldots]$

$= (1/1)\ [\ 0.0229+(-1/2)(-0.0003)+0] = 0.02305,$

Since here $v= (x-x_n) / h = (55-56)/1 = -1$.

$(d^2y / dx^2)\ x \neq x_n = (1/h^2)\ [\nabla^2 y_n + 6(v+1) / 6\ \nabla^3 y_n + (\ 12v^2 + 36 v +22) / 2\ \nabla^4 y_n + \ldots\ldots.]$

$= (1/1)\ [\ 0.0229+(-1/2)(-0.0003)+0] = 0.02305.$

Numerical Integration:

We know that $\int_a^b f(x)dx$ represents the area between y = f(x), x – axis and the ordinates x = a and x = b. This integration is possible only if the f(x) is explicitly given and if it is integrable. The problem of numerical integration can be stated as follows: Given as set of (n+1) paired values (x_i, y_i), i = 0,1,2,…,n of the function y=f(x), where f(x) is not known explicitly, it is required to compute $\int_{x_0}^{x_n} y\,dx$.

A general quadrature formula for equidistant ordinates (or Newton – cote's formula)

For equally spaced intervals, we have Newton's forward difference formula as

$$y(x)=y(x_0+uh)=y_0+u\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\ldots \qquad \ldots\ldots(1)$$

Now, instead of *f(x)*, we will replace it by this interpolating formula of Newton.

Here, $u = \frac{x-x_0}{h}$ where h is interval of differencing.

Since $x_n = x_0 + nh$, and $u = \frac{x-x_0}{h}$ we have $\frac{x-x_0}{h} = n = u$.

$$\int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_n+nh} f(x)dx$$

$$= \int_{x_0}^{x_n+nh} P_n(x)\, dx$$ where $P_n(x)$ is interpolating polynomial

$$=\int_0^n \left(y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \ldots\ldots\right)(hdu)$$

Since $dx = hdu$, and when $x = x_0$, $u = 0$ and when $x = x_0+nh$, $u = n$.

$$= h\left[y_0(u) + \frac{u^2}{2}\Delta y_0 + \frac{\left(\frac{u^3}{3}-\frac{u^2}{2}\right)}{2}\Delta^2 y_0 + \frac{1}{6}\left(\frac{u^4}{4} - u^3 + u^2\right)\Delta^3 y_0 + \cdots\right]_0^n$$

$$\int_{x_0}^{x_n} f(x)dx = h\,n\left[y_0 + \frac{n^2}{2}\Delta y_0 + \frac{1}{2}\left[\frac{n^3}{3} - \frac{n^2}{2}\right]\Delta^2 y_0 + \frac{1}{6}\left(\frac{n^4}{4} - n^3 + n^2\right)\Delta^3 y_0 + \ldots\right] \quad (2)$$

The equation (2), called Newton-cote's quadrature formula is a general quadrature formula. Giving various values for n, we get a number of special formula.

Trapezoidal rule:

By putting n = 1, in the quadrature formula (i.e there are only two paired values and interpolating polynomial is linear).

$\int_{x_0}^{x_n+nh} f(x)dx = h\left[1.y_0 + \frac{1}{2}\Delta y_0\right]$ since other differences do not exist if n = 1.

$$=\int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_n+nh} f(x)dx$$

$$= \int_{x_0}^{x_0+h} f(x)dx + \int_{x_0+h}^{x_n+2h} f(x)dx + \ldots\ldots\ldots + \int_{x_0+(n-1)h}^{x_n+nh} f(x)dx$$

$$= \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+\cdots.+y_{n-1})\right]$$

$= \frac{h}{2}$ [(sum of the first and the last ordinates) + 2(sum of the remaining ordinates)]

This is known as Trapezoidal Rule and the error in the trapezoidal rule is of the order h^2.

Romberg's method

For an interval of size h, let the error in the trapezoidal rule be kh^2 where k is a constant. Suppose we evaluate $I = \int_{x_0}^{x_n} y\,dx$, taking two different values of h, say h_1 and h_2, then

$$I = I_1 + E_1 = I_1 + kh_1^2 \quad I = I_2 + E_2 = I_2 + kh_2^2$$

Where I_1, I_2 are the values of I got by two different values of h, by trapezoidal rule and E_1, E_2 are the corresponding errors.

$$I_1 + kh_1^2 = I_2 + kh_2^2$$

$$k = \frac{I_1 - I_2}{h_2^2 - h_1^2}$$

substituting in (1), $I = I_1 + \frac{I_1 - I_2}{h_2^2 - h_1^2} h_1^2$ & $I = \frac{I_1 h_2^2 - I_2 h_1^2}{h_2^2 - h_1^2}$

This I is a better result than either I_1, I_2.

If $h_1 = h$ and $h_2 = \frac{1}{2}h$, then we get

$$I = \frac{I_1\left(\frac{1}{4}h^2\right) - I_2 h^2}{\frac{1}{4}h^2 - h^2} \qquad = \frac{4I_2 - I_1}{3} = I_2 + \frac{1}{2}(I_2 - I_1), \quad I = I_2 + \frac{1}{2}(I_2 - I_1)$$

We got this result by applying trapezoidal rule twice. By applying the trapezoidal rule many times, every time halving h, we get a sequence of results A_1, A_2, A_3,…… we apply the formula given by (3), to each of adjacent pairs and get the resultants B_1, B_2, B_3 ….. (which are improved values). Again applying the formula given by (3), to each of pairs B_1, B_2, B_3 …..we get another sequence of better results C_1, C_2, C_3 ….continuing in this way, we proceed until we get two successive values which are very close to each other. This systematic improvement of Richardson's method is called Romberg method or Romberg integration.

Simpson's one-third rule:

Setting n = 2 in Newton- cote's quadrature formula, we have $\int_{x_0}^{x_n} f(x)dx = h \quad 2y_0 + \frac{4}{2}\Delta y_0 + \frac{1}{2} \quad \frac{8}{3} - \frac{4}{2} \quad \Delta^2 y_0$ (since other terms vanish)

$= \frac{h}{3}(y_2+y_1+y_0)$

Similarly, $\int_{x_2}^{x_4} f(x)dx = \frac{h}{3}(y_2+4y_3+y_4)$

$\int_{x_2}^{x_4} f(x)dx = \frac{h}{3}(y_i+4y_{i+1}+y_{i+2})$

If n is an even integer, last integral will be

$\int_{x_{n-2}}^{x_n} f(x)dx = \frac{h}{3}(y_{n-2}+4y_{n-1}+y_n)$

Adding all the integrals, if n is an even positive integer, that is, the number of ordinates y_0, y_1, $y_2 \dots y_n$ is odd, we have

$\int_{x_0}^{x_n} f(x)dx = \int_{x_0}^{x_2} f(x)dx + \int_{x_2}^{x_4} f(x)dx + \dots\dots\dots +$

$\int_{x_{n-2}}^{x_n} f(x)dx$

$= \frac{h}{3}\left[(y_0+y_n) + 2(y_2+y_4+\dots) + \dots\dots + 4(y_1+y_3+\dots\dots)\right]$

$= \frac{h}{3}$ [(sum of the first and the last ordinates) + 2(sum of remaining odd ordinates) + 2(sum of even ordinates)]

Simpson's three-eighths rule:

Putting n = 3 in Newton – cotes formula

$= \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+\dots+y_{n-1})+2(y_3+y_6+y_9+\dots+y_n)\right] \dots(2)$

Equation (2) is called *Simpson's three – eighths rule* which is applicable only when n is a multiple of 3.Truncation error in simpson's rule is of the order h

Example

Evaluate $\int_{-3}^{3} x^4 dx$ by using (1) trapezoidal rule (2)simpson's rule. Verify your results by actual integration.

Solution.

Here $y(x) = x^4$. Interval length$(b - a) = 6$. So, we divide 6 equal intervals with $h = \frac{6}{6} = 1$.

We form below the table

x	-3	-2	-1	0	1	2	3
y	81	16	1	0	1	16	81

(i) By trapezoidal rule:

$\int_{-3}^{3} y\,dx = \frac{h}{2}$ [(sum of the first and the last ordinates) +

2(sum of the remaining ordinates)]

$= \frac{1}{2}$ [(81+81)+2(16+1+0+1+16)]

=115

(ii) By simpson's one - third rule (since number of ordinates is odd)**:**

$\int_{-3}^{3} y\,dx = \frac{1}{3}$ [(81+81) + 2(1+1) + 4(16+0+16)]

= 98.

(iii) Since $n = 6$, (multiple of three), we can also use **simpson's three - eighths rule.** By this rule,

$\int_{-3}^{3} y\,dx = \frac{1}{3}$ [(81+81) + 3(16+1+1+16) + 2(0)]

= 99

(iv) By actual integration,

$$\int_{-3}^{3} x^4 dx = 2 * \left[\frac{x^5}{5}\right]_0^3 = \frac{2*243}{5} = 97.2$$

From the results obtained by various methods, we see that simpson's rule gives better result than trapezoidal rule.

POSSIBLE QUESTIONS

1. Write the formulae for Newton forward difference formula for derivatives.
2. Write the formula for Newton backward difference formula for derivatives.
3. Write the Simpson's 3/8th rule formula.
4. Write Boole's rule formula.
5. Write the Simpson's 3/8th rule formula.
6. Given the following data, find $y'(6)$ and the maximum value of y.

X	:	0	2	3	4	7	9
Y	:	4	26	58	112	466	922

7. Evaluate I= $\int_0^6 dx \,/\, (1+\ x)$ using both of the Simpson's rule.
8. Find the first and second derivative of the function tabulated below atx = 0.6

X :	0.4	0.5	0.6	0.7	0.8
Y :	1.5836	1.7974	2.0442	2.3275	2.6511

9. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by (i) Trapezoidal rule (ii) Simpson's rule. Also check up the result by actual integration.
10. Find the gradient of the road at the middle point of the elevation above a datum line of seven point of road which are given below:

X	:	0	300	600	900	1200	1500	1800
Y	:	135	149	157	183	201	205	193

11. By dividing the range into the ten equal parts .Evaluate $\int_0^{\pi} \sin x dx$ by Trapezoidal rule and Simpson's rule.
12. The population of a certain town is given below, Find the rate of growth of population in 1931, 1941, 1961 and 1971.

Year	: 1931	1941	1951	1961	1971
Population in thousands	: 40.62	60.80	79.95	103.56	132.65

13. b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule with h = 0.2. Hence obtain the approximate value of π .

14. Find the first two derivatives of $x^{\frac{1}{3}}$ at x=50 and x=56 given the table below:

X	:	50	51	52	53	54	55	56
Y	:	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

15. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by (i) Trapezoidal rule (ii) Simpson's rule. Also check up the result by actual integration.

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University)
(Established Under Section 3 of UGC Act 1956)
Pollachi Main Road, Eachanari (Po),
Coimbatore –641 021
Subject: Numerical Methods
Class : II - B.Sc. Mathematics

Unit IV

Part A (20x1=20 Marks) **(Question Nos. 1**

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
If the given integral is approximated by the sum of 'n' trapezoids, then the rule	Newton's method	Trapezoidal rule	simpson's rule	power	Trapezoidal rule
The order of error in Trapezoidal rule is -------------.	h	h^3	h^2	h^4	h^2
The general quadratic formula for equidistant ordinates is ________________	raphson	Newton-cote's	interpolation	divide difference	Newton-cote's
h/2[(sum of the first and last ordinates)+2(sum of the remaining ordinates)] is_____________	simphson's 3/8	simphson's 1/3	trapezoidal	taylor series	trapezoidal
Use trapezoidal rule for y(x) _______________	linear	second degree	third degree	degree n	linear
Simpson's rule is exact for a ----------------- even though it was derived for a	cubic	less than cubic	linear	quadratic	linear
What is the order of the error in Simpson's formula?	Four	three	two	one	Four
Simpson's 1/3 is findind y(x) upto ____________	linear	second degree	degree n	third degree	second degree
In simpson's 1/3, the number of intervels must be ___________	any integer	odd	even	prime	even
In simpson's 1/3, the number of ordinates must be _______________	any integer	odd	prime	even	odd
Simpson's one-third rule on numerical integration is called a ---------	closed	open	semi closed	semi opened	closed
In simphson's 3/8 rule, we calculate the polynomial of degree ________	degree n	linear	second degree	third degree	third degree
The number of interval is multiple of three the use_________	simpson's 1/3	trapezoidal	simpson's 3/8	taylor series	simpson's 3/8
The number of interval is multiple of six _____________	simpson's 1/3	simphson's 3/8	weddle	trapezoidal	weddle
The error in Simpson's 1/3 is -------------.	h	h^3	h^2	h^4	h^4
The order of error is h^2 for_____________	lagrange's	trapezoidal	weddle	simpson's 1/3	trapezoidal
h^4 is the error of ________	simphson's 3/8	simphson's 1/3	trapezoidal	taylor series	simphson's
The value of integral e^x is evaluated from 0 to 0.4 by the following formula. Which method will give the least error ?	Trapezoidal rule with h = 0.2	Trapezoidal rule with h = 0.1	Simpson's 1/3 rule with h = 0.1.	weddle	Simpson's 1/3 rule with h = 0.1.
Using Simpson's rule the area in square meters included between the chain line, irregular boundary and the first and the last offset will be	7.33.28 sq-m	744.18 sq-m	880.48 sq-m.	820.38 sq-m	820.38 sq-m
By putting n = 1 in Newton cote's formula we get ----------- rule.	Simpson's 1/3 rule	Simpson's 3/8 rule	Trapezoidal rule	Simpson's rule	Trapezoidal rule
$I = (3h / 8) \{ (y_0 + y_n) + 3 (y_1 + y_2 + y_4 + y_5 + \ldots..)+2(y_3 + y_6 + y_9 + \ldots..) \}$	Simpson's 1/3 rule	Simpson's 3/8 rule	Trapezoidal rule	Simpson's rule	Simpson's 3/8 rule
$I = (h / 3) \{ (y_0 + y_n) + 2 (y_2+ y_4 + y_6 + y_8 + \ldots..)+ 4(y_1 + y_3 + y_5 + \ldots..) \}$ is	Simpson's 1/3 rule	Simpson's 3/8 rule	Trapezoidal rule	Simpson's rule	Simpson's 1/3 rule
The differentiation of logx is ________	1/x	e(x)	sinx	cosx	1/x
h/3[(sum of first and last ordinates)+2(sum of even ordinates)+4(sum of odd ordinates)] is the formula for ___________	trapezoidal	simphson's 1/3	simphson's 3/8	taylor series	simphson's 1/3
Differentiation of sinx is ________________	cosx	tanx	sinx	logx	cosx
Integration of cosx _____________	cosx	tanx	sinx	logx	sinx
If y(x) is linear then use ___________________	simphson's 3/8	simphson's 1/3	trapezoidal	taylor series	trapezoidal
The differentiation of secx is ____________	secx tanx	cotx	cosecx	tanx	secx tanx
The notation h is __________	differece of ordinates	sum of ordinates	number of ordinates	product of ordinates	differece of ordinates
While evaluating the definite integral by Trapezoidal rule, the accuracy can be increased by taking----------	Large number of sub-intervals	even number of sub-intervals	multipleof6	has multiple of 3	Large number of sub-
Numerical integration when applied to a function of a single variable, it is known as--------	maxima	minima	quadrature	quadrant	quadrature

UNIT-V

ORDINARY DIFFERENTIAL EQUATIONS

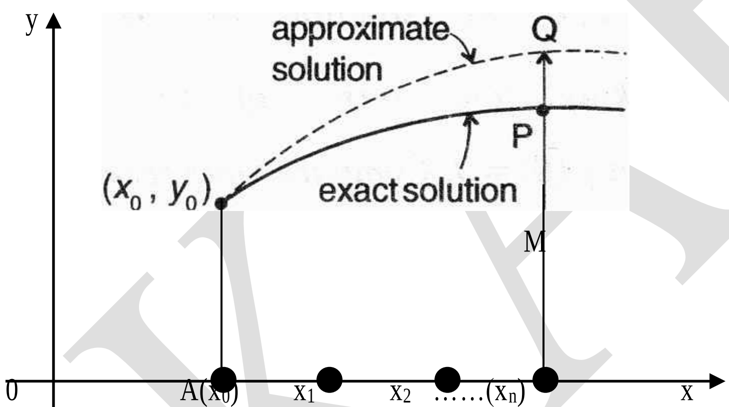
Ordinary Differential Equations: Taylor's series - Euler's method – modified Euler's method - Runge-Kutta methods of orders two and four.

INTRODUCTION

Suppose we require to solve dy/dx=f(x,y) with the initial condition $y(x_0)=y_0$. By numerical solution of y at $x=x_0, x_1, x_2$,.. let y=y(x)be the exact solution. If we plot and draw the graph of y=y(x), (exact curve) and also draw the approximate curve by plotting $(x_0, y_0), (x_1,y_1), (x_2, y_2)$,… we get two curves.

PM= exact value, QM=approximate value at $x=x_i$. Then

$QP=MQ-MP=y_i-y(x_i) = \varepsilon$ is called the truncation error at $x= x_i$



$QP=MQ-MP=y_i-y(x_i)= \varepsilon_i$ is called return error at $x=x_i$

RUNGE- KUTTA METHOD

Second order Runge-Kutta method (for first order O.D.E)

AIM : To solve dy / dx = f(x,y) given $y(x_0)=y_0$ ….(1)

Proof. By Taylor series, we have,

$y (x+h) = y(x)+ hy'(x)+ h^2/2!\, y''(x)+O(h^3)$ ….….(2)

Differentiating the equation (1) w.r.t.x,

$$y'' = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dx} = f_x + y' f_y = f_x + f f_y$$

Using the values of y' and y'' got from (1) and (3), in (2), we get,

$$Y(x + h) - y(x) = hf + \tfrac{1}{2} h^2 [f_x + ff_y] + O(h^3)$$

$$\Delta y = hf + \tfrac{1}{2} h^2 (f_x + ff_y) + O(h^3)$$

Let $\Delta_1 y = k_1 = f(x,y)$. $\Delta x = hf(x,y)$, $\Delta_2 y = k_2 = hf(x+mh, y+mk_1)$

and $\Delta y = ak_1 + bk_2$, Where a, b and m are constants to be determined to get the better accuracy of Δy. Expand k_2 and Δy in powers of h.

Expanding k_2, by Taylor series for two variables, we have

$K_2 = hf(x+mh, y+mk_1)$

$= h[f + mhf_x + mhff_y + \{(mh\partial/\partial x + mk_1 \partial/\partial y)^2 f / 2!\} + \ldots]$ …(8)

$= hf + mh_2(f_x + ff_y) + \ldots$ Higher powers of h ……….. (9)

Substituting k_1, k_2 in (7),

$\Delta y = ahf + b[hf + mh^2(f_x + ff_y) + O(h^3)]$

$= (a+b)hf + bmh^2(f_x + ff_y) + O(h^3)$ …… 10)

Equating Δy from (4) and (10), we get

$= hf + mh^2(f_x + ff_y) + \ldots$ higher powers of h………. (9)

Substituting k_1, k_2 in (7),

$\Delta y = ahf + b[hf + mh^2(f_x + ff_y) + O(h^3)]$ $= (a+b)hf + bmh^2(f_x + ff_y) + O(h^3)$ …………. (10)

Equating Δy from (4) and (10), we get

$a+b=1$ and $bm = \tfrac{1}{2}$ ………… (11)

Now we have only two equations given by (1) to solve for three unknowns a,b,m.

From a+b=1, a=1-b and also m= 1/2b using (7),

$\Delta y=(1-b)k_1+bk_2,$ Where $k_1=hf(x,y)$

$K_2=hf(x+h/2b, y+hf/2b)$ Now $\Delta y=y(x+h)-y(x)$

$Y(x+h)=y(x)+(1-b)hf+bhf(x+h/2b,y+hf/2b)$

i.e., $y_{n+1}=y_n+(1-b)hf(x_n,y_n) +bhf(x_n+h/2b,y_n+h/2bf(x_n,y_n))+O(h^3)$

from this general second order Runge kutta formula, setting a=0, b=1, m=1/2, we get the second order Runge kutta algorithm as

$k_1=hf(x,y)$ & $k_2=hf(x+ ½ h, y + ½ k_1)$ and $\Delta y=k_2$ where $h=\Delta x$

Since the derivation of third and fourth order Runge Kutta algorithm are tedious, we state them below for use.

The third order Runge Kutta method algorithm is given below :

$K_1=hf(x,y)$

$K_2=hf(x+1/2h, y+1/2k_1)$

$K_3=hf(x+h,y+2k_2-k_1)$

and $\Delta y=1/6 (k_1+4k_2+k_3)$

The fourth order Runge Kutta method algorithm is mostly used in problems unless otherwise mentioned. It is

$K_1=hf(x,y)$

$K_2=hf(x+1/2h, y+1/2k_1)$

$K_3=hf(x+1/2h, y+1/2k_2)$

$K_4=hf(x+h, y+k_3)$

and $\Delta y= 1/6 (k_1+2k_2+2k_3+k_4)$

$y(x+h)=y(x)+ \Delta y$

Working Rule :

To solve $dy/dx = f(x,y)$, $y(x_0)=y_0$

Calculate $k_1=hf(x_0,y_0)$

$K_2=hf(x_0+1/2h,y_0+1/2k_1)$

K_3= hf(x_0+1/2h,y_0+1/2k_2)

K_4=hf(x_0+h, y_0+k_3)

and Δy= 1/6 (k_1+2k_2+2k_3+k_4)

where Δx=h

Now y_1=y_0+ Δy

Now starting from (x_1,y_1) and repeating the process, we get (x_2,y_2) etc.,

Example

Obtain the values of y at x=0.1, 0.2 using R.K method of (i) second order (ii) third order and (iii) fourth order for the differential equation y'=-y, given y(0)=1.

Solution :Here f(x,y)=-y,x_0=0, y_0=1, x_1=0.1, x_2=0.2

(i) Second Order:

k_1=hf(x_0,y_0)=(0.1)(-y_0)= - 0.1

k_2=hf(x_0+ ½ h, y_0+ ½ k_1) = (0.1) f(0.05,0.95)

= -0.1(x0.95)= - 0.095= Δy

y_1=y_0+Δy=1-0.095=0.905

y_1=y(0.1)=0.905

Again starting from (0.1, 0.905) replacing (x_0,y_0) by (x_1,y_1)we get

k_1=(0.1) f(x_1,y_1)=(0.1) (-0.905)= - 0.0905

k_2=hf(x_1+ ½ h, y_1+ ½ k_1)

=(0.1)[f(0.15,0.85975)]=(0.1)(-0.85975)=-0.085975

Δy=k_2 y_2=y(0.2)=y_1+Δy=0.819025

(ii) Third Order:

k_1= hf(x_0, y_0) = -0.1

k_2= hf(x_0+ ½ h, y_0+ ½ k_1) = - 0.095

k_3=hf(x_0+h,y_0+2k_2-k_1)

= (0.1)f(0.1,0.9)=(0.1)(-0.9)= -0.0.9

Δy=1/6 (k_1+4k_2+k_3)

y(0.1)=y_1=y_0+Δy=1-0.09=0.91

Again taking (x_1,y_1) has (x_0,y_0) repeat the process

k_1=hf(x_1,y_1)=(0.1) (-0.91)=-0.091

k_2=hf(x_1+ ½ h, y_1+ ½ k_1)

= (0.1)f(0.15,0.865)=(0.1) (-0.865)= - 0.0865

k_3=hf(x_1+h, y_1+2k_2-k_1)

= (0.1)f(0.2,0.828) = -0.0828

y_2=y_1+Δy=0.91+1/6 (k_1+4k_2+k_3)

= 0.91+1/6 (- 0.091-0.3460 – 0.0828)

y(0.2)=0.823366

(iii) Fourth order:

k_1=hf(x_0,y_0)=(0.1)f(0.1)=-0.1

k_2=hf(x_0+ ½ h, y_0+1/2k_1)=(0.1)f(0.05, 0.95) = - 0.095

k_3=hf(x_0+ ½ h, y_0+ ½ k_2) = (0.1) f(0.05, 0.9525) = - 0.09525

k_4=hf(x_0+h,y_0+k_3)=(0.1)f(0.1,0.90475) = -0.090475

Δy=1/6 (k_1+2k_2+2k_3+k_4)

y_1=y_0+Δy=1+1/6 (k_1+2k_2+2k_3+k_4)

y_1=y(0.1)=0.9048375

Again start from this (x_1,y_1) and replace (x_0,y_0) and repeat

k_1=hf(x_1,y_1)=(0.1)(-y1)= - 0.09048375

k_2=hf(x_1+1/2h,y_1+1/2k_1)

= (0.1)f(0.15,0.8595956) = -0.08595956

$k_3=hf(x_1+ ½ h, y_1+ ½ k_2)$

$= (0.1)f(0.15,0.8618577)= - 0.08618577$

$k_4=hf(x_1+h, y_1+k_3)$

$= (0.1)f(0.2,0.8186517)= - 0.08186517$

$\Delta y=1/6(-0.09048375-2x\ 0.08595956- 2x\ 0.08618577 – 0.08186517) = -0.0861066067$

$y_2=y(0.2)=y_1+\Delta y=0.81873089$

Tabular values are:

x	Second order	Third order	Fourth order	Exact value Y=e-x
0.1	0.905	0.91	0.9048375	0.904837418
0.2	0.819025	0.823366	0.81873089	0.818730753

POSSIBLE QUESTIONS

1. Write the difference between Euler and modified Euler Method.
2. Define Euler method with formula.
3. Write the formula for Milne's predictor – corrector method.
4. Write the formula for Adam's Bash forth predictor – corrector method.
5. Write the modified Euler method formula.
6. Solve dy/dx = x + y, given y(1)=0 and get y(1.1),y(1.2) by Taylor's series method. Compare your result with the explicit method
7. Find y (1.5) taking h=0.5 given $y' = y - 1,\ y(0) = 1.1$ by using Euler method.
8. Using Adam's method for y (0.4) given $\frac{dy}{dx} = \frac{1}{2}xy,$ y(0)=1,y(0.1)=1.01,y(0.2)=1.022, y(0.3)=1.023.
9. Apply fourth order Runge-Kutta method to find y(0.2) given that $y' = x + y, y(0) = 1.$
10. Using Taylor method compute y(0.2) and y(0.4) correct to four decimal places given by $\frac{dy}{dx}$ = 1-2xy and y(0)=0
11. Compute y at x=0.25 by modified Euler method. Given y^1=2xy, y(0)=1
12. Solve the equation dy/dx=1-y, given y(0)=0 using modified Euler method and tabulate the values at x=0.1, 0.2, 0.3 compare your results with the exact solutions
13. Determine the value of y (0.4) using Milne's methods given $y' = xy + y^2$ use Taylor series to get the values of y(0.1),y(0.2) and y(0.3).
14. Find y (1.1) given y^1=2x-y, y (1) =3 by using Taylor series method
15. Obtain the values of y at x=0.1, 0.2 using R-K method of
    (i) Second order
    (ii) Fourth order

    For the differential equation y^1= -y given y(0)=1.

KARPAGAM ACADEMY OF HIGHER EDUCATION
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Subject: Numerical Methods
Class : II - B.Sc. Mathematics

Unit V

Part A (20x1=20 Marks) **(Question Nos. 1 to 20**

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
The order of error in Trapezoidal rule is -------------.	h	h^3	h^2	h^4	h^2
The order of error in Simpson's rule is -------------.	h	h^3	h^2	h^4	h^4
Numerical evaluation of a definite integral is called --------.	Integration	Differentiation	Interpolation	Triangularization	Integration
Simpson's ⅜ rule can be applied only if the number of sub interval is in -	Equal	even	multiple of three	unequal	multiple of
By putting n = 2 in Newton cote's formula we get ----------- rule.	Simpson's 1/3	Simpson's 3/8	Trapezoidal	Romberg	Simpson's 1/3
The Newton Cote's formula is also known as --------------- formula.	Simpson's 1/3	Simpson's 3/8	Trapezoidal	quadrature	quadrature
By putting n = 3 in Newton cote's formula we get ----------- rule.	Simpson's 1/3	Simpson's 3/8	Trapezoidal	Romberg	Simpson's 3/8
By putting n = 1 in Newton cote's formula we get ----------- rule.	Simpson's 1/3	Simpson's 3/8	Trapezoidal	newton's	Trapezoidal
The systematic improvement of Richardon's method is called--------- method	Simpson's 1/3	Simpson's 3/8	Trapezoidal	Romberg	Romberg
Simpson's 1/3 rule can be applied only when the number of interval is --	Equal	even	multiple of three	unequal	even
Simpson's rule is exact for a ----------------- even though it was derived for a	cubic	less than cubic	linear	quadratic	linear
The accuracy of the result using the Trapezoidal rule can be improved by --------	Increasing the interval h	Decreasing the interval h	Increasing the number of iterations	altering the given function	Increasing the number of iterations
A particular case of Runge Kutta method of second order is ------------------.	Milne's method	Picard's method	Modified Euler method	Runge's method	Modified Euler method
Runge Kutta of first order is nothing but the --------------.	modified Euler method	Euler method	Taylor series	none of these	Euler method
In Runge Kutta second and fourth order methods, the values of k1 and k2 are ----	same	differ	always positive	always negative	same
_________values are calculated in Runge Kutta fourth order method.	k_1, k_2, k_3, k_4 and Dy	k_1, k_2 and Dy	k_1, k_2, k_3 and Dy	none of these	k_1, k_2, k_3, k_4 and Dy
The use of Runge kutta method gives ------------- to the solutions of the differential equation than Taylor's series method.	Slow convergence	quick convergence	oscillation	divergence	quick convergence
In Runge – kutta method the value x is taken as ----------.	$x = x_0 + h$	$x_0 = x + h$	$h = x_0 + x$	$h = x_0 - x$	$x = x_0 + h$
In Runge – kutta method the value y is taken as ----------.	$y = y_0 + h$	$y_0 = x_0 + h$	$y = y_0 – Dy$	$y = y_0 + Dy$	$y = y_0 + Dy$
___________is nothing but the modified Euler method.	Runge kutta method of second order	Runge kutta method of third order	Runge kutta method of fourth order	Taylor series method	Runge kutta method of second order
In all the three methods of Rungekutta methods, the values ---------- are	k_4 & k_3	k_3 & k_2	k_2 & k_1	k_1, k_2, k_3 & k_4	k_2 & k_1
The formula of Dy in second order Runge Kutta method is given by ------	k_1	k_2	k_3	k_4	k_2
The Runge – Kutta methods are designed to give -------------- and they posses the advantage of requiring only the function values at some selected points on the sub intervals	greater accuracy	lesser accuracy	average accuracy	equal	greater accuracy
If dy/dx is a function x alone, then fourth order Runge – Kutta method reduces to --------------.	Trapezoidal rule	Taylor series	Euler method	Simpson method	Simpson method
In Runge Kutta methods, the derivatives of ----------- are not require and we require only the given function values at different points.	higher order	lower order	middle order	zero	higher order
The use of -------------- method gives quick convergence to the solutions of the differential equation than Taylor's series method.	Taylor series	Euler	Runge – Kutta	Simpson method	Runge – Kutta
If dy/dx is a function x alone, then ---------- Runge – Kutta method reduces to Simpson method	fourth order	third order	second order	first order	fourth order
If dy/dx is a function of ----------- then fourth order Runge – Kutta method reduces to Simpson method.	x alone	y alone	both x and y	none	x alone

Reg. No................................

[16MMU301]

KARPAGAM UNIVERSITY

Karpagam Academy of Higher Education
(Established Under Section 3 of UGC Act 1956)
COIMBATORE – 641 021
(For the candidates admitted from 2016 onwards)

B.Sc., DEGREE EXAMINATION, NOVEMBER 2017

Third Semester

MATHEMATICS

NUMERICAL METHODS

Time: 3 hours Maximum : 60 marks

PART – A (20 x 1 = 20 Marks) (30 Minutes)
(Question Nos. 1 to 20 Online Examinations)

PART B (5 x 2 = 10 Marks) (2 ½ Hours)
Answer ALL the Questions

21. Find the initial approximation of $x^3 - x = 1$ by bisection method.
22. List out the methods to find the solution of simultaneous linear algebraic Equations.
23. Write down the Newton's Gregory forward interpolation formula.
24. What are the methods available to find the value of the integration?
25. Write down the modified Euler's formula.

PART C (5 x 6 = 30 Marks)
Answer ALL the Questions

26. a. Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method.

Or

b. Derive the order of convergence of Newton's method.

27. a. Solve the system of equations by the Gauss Elimination method.
$x + 2y + z = 3;\ 2x + 3y + 3z = 10;\ 3x - y + 2z = 13$

Or

b. Solve the following system of equations by Gauss-Seidel method
$8x - 3y + 2z = 20$
$4x + 11y - z = 33$
$6x + 3y + 12z = 35$

28. a. Find a polynomial of degree four which takes the value

X:	2	4	6	8	10
Y:	0	0	1	0	0

Or

b. Using Lagrange's formula of interpolation find y (0.5) given

X:	7	8	9	10
Y:	3	1	1	9

29. a. Evaluate $I = \int_0^6 \frac{1}{1+x}\,dx$ using simpson's rule.

Or

b. The population of a certain town is given below. Find the rate of growth of the population in 1931

year	x :1931	1941	1951	1961	1971
population in thousands	y : 40.62	60.80	79.95	103.56	132.65

30. a. Using Taylor series method, find correct to four decimal places , the value of y (0.1), given $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$

Or

b. Compute (0.3) given $\frac{dy}{dx} + y + xy^2 = 0$; $y(0) = 1$ by taking h=0.1using R.K method of IVth order

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