M.Sc.Microbiology 2018-2019

Semester - II

18MBP203 BIOSTATISTICS ANDRESEARCHMETHODOLOGY 4H –4C

Instruction Hours / week:L: 4 T: 0P: 0 Marks: Internal: 40 External: 60 Total:100

End Semester Exam: 3Hours

COURSE OBJECTIVE

- This course has been intended to provide the learner insights into helpful areas of Statistics which plays an essential role in present, future use and applications ofBiology.
- Students get an idea about collection, interpretation and presentation of statisticaldata.

COURSE OUTCOMES(CO'S)

On successful completion of this course the learners will be able to

1. apply basic statistical concepts commonly used in health and medicalsciences
2. use basic analytical techniques to generateresults
3. interpret results of commonly used statistical analyses in writtensummaries
4. demonstrate statistical reasoning skills correctly andcontextually

UNIT I -Scope of Biostatistics

Definitions-Scopeof Biostatistics- Variables in biology, collection, classification and tabulation of data- Graphical and diagrammaticrepresentation.

Measures of central tendency – Arithmetic mean, median and mode.Measures of dispersion-Range, standard deviation, Coefficient of variation.

UNIT II - Correlation

Correlation – Meaning and definition - Scatter diagram –Karl pearson's correlation coefficient. Rank correlation.Regression: Regression in two variables – Regression coefficient problems – uses of regression.

UNIT III - Test of significance

Test of significance: Tests based on Means only-Both Large sample and Small sample tests - Chi square test - goodness of fit. Analysis of variance – one way and two way classification.CRD, RBD Designs.

UNIT IV-Research

Research: Scope and significance – Types of Research – Research Process – Characteristics of good research – Problems in Research – Identifying research problems. Research Designs – Features of good designs.

UNIT V - Sampling Design

Sampling Design: Meaning – Concepts – Steps in sampling – Criteria for good sample design.Scaling measurements – Techniques – Types of scale.

SUGGESTED READINGS

1. Jerrold H. Zar. (2003). *Biostatistical Analysis.* (4th ed.). Pearson Education (P) Ltd, New Delhi.
2. Kothari. C.R. (2004). *Research Methodology – Methods and Techniques.* (2nd ed.). New Age International Pvt. Ltd, NewDelhi.

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Pollachi Main Road, Coimbatore – 641 021, Tamil Nadu

Department of Mathematics

Subject : Mathematical Statistics **Semester VI** **L T P C**
Subject Code : 16MMU603A **Class : III B.Sc Mathematics** **4 2 0 4**

Glossary of Statistical Terms

2 X 5 factorial design	A factorial design with one variable having two levels and the other having five levels.
Alpha	The probability of a Type I error.
Abscissa	Horizontal axis.
Additive law of probability	The rule giving the probability of the occurrence of one or more mutually exclusive events.
Adjacent values	Actual data points that are no more extreme than the inner fences.
Alternative hypothesis (H_1)	The hypothesis that is adopted when H_0 is rejected. Usually the same as the research hypothesis.
ß (Beta)	The probability of a Type II error.
Categorical data	Data representing counts or number of observations in each category.
Cell	The combination of a particular row and column ‹the set of observations obtained under identical treatment conditions.
Central limit theorem	The theorem that specifies the nature of the sampling distribution of the mean.
Chi-square test	A statistical test often used for analyzing categorical data.
Conditional probability	The probability of one event *given* the occurrence of some other event.
Confidence interval	An interval, with limits at either end, with a specified probability of including the parameter being estimated.
Confidence limits	An interval, with limits at either end, with a specified probability of including the parameter being estimated.
Constant	A number that does not change in value in a given situation.
Continuous variables	Variables that take on *any* value.
Correlation	Relationship between variables.
Correlation coefficient	A measure of the relationship between variables.
Count data	Data representing counts or number of observations in each category.
Covariance	A statistic representing the degree to which two variables vary together.
Criterion variable	The variable to be predicted.
Critical value	The value of a test statistic at or beyond which we will reject H0 .
Decision making	A procedure for making logical decisions on the basis of sample data.
Degrees of	The number of independent pieces of information remaining after

freedom (*df*)	estimating one or more parameters.
Density	Height of the curve for a given value of X- closely related to the probability of an observation in an interval around X.
Dependent variables	The variable being measured. The data or score.
Discrete variables	Variables that take on a small set of possible values.
Dispersion	The degree to which individual data points are distributed around the mean.
Distribution free tests	Statistical tests that do not rely on parameter estimation or precise distributional assumptions.
Effect size	The difference between two population means divided by the standard deviation of either population.
Efficiency	The degree to which repeated values for a statistic cluster around the parameter.
Event	The outcome of a trial.
Exhaustive	A set of events that represents all possible outcomes.
Expected value	The average value calculated for a statistic over an infinite number of samples.
Expected frequencies	The expected value for the number of observations in a cell if H_0 is true.
Experimental hypothesis	Another name for the research hypothesis.
Exploratory data analysis (EDA)	A set of techniques developed by Tukey for presenting data in visually meaningful ways.
External Validity	The ability to generalize the results from this experiment to a larger population.
Frequency distribution	A distribution in which the values of the dependent variable are tabled or plotted against their frequency of occurrence.
Frequency data	Data representing counts or number of observations in each category.
Goodness of fit test	A test for comparing observed frequencies with theoretically predicted frequencies.
Grand total ($\sum X$)	The sum of all of the observations.
Hypothesis testing	A process by which decisions are made concerning the values of parameters.
Independent variables	Those variables controlled by the experimenter.
Independent events	Events are independent when the occurrence of one has no effect on the probability of the occurrence of the other.
Interaction	A situation in a factorial design in which the effects of one independent variable depend upon the level of another independent variable.
Intercept	The value of Y when X is 0.
Interval scale	Scale on which equal intervals between objects represent equal differences ‹differences are meaningful.
Interval estimate	A range of values estimated to include the parameter.
Joint probability	The probability of the co-occurrence of two or more events.
Kurtosis	A measure of the peakedness of a distribution.
Leptokurtic	A distribution that has relatively more scores in the center and in the tails.

Linear relationship	A situation in which the best-fitting regression line is a straight line.
Linear regression	Regression in which the relationship is linear.
Marginal totals	Totals for the levels of one variable summed across the levels of the other variable.
Matched samples	An experimental design in which the same subject is observed under more than one treatment.
Mean absolute deviation (m.a.d.)	Mean of the absolute deviations about the mean.
Mean	The sum of the scores divided by the number of scores.
Measurement	The assignment of numbers to objects.
Measurement data	Data obtained by measuring objects or events.
Measures of central tendency	Numerical values referring to the center of the distribution.
Median location	The location of the median in an ordered series.
Median (Med)	The score corresponding to the point having 50% of the observations below it when observations are arranged in numerical order.
Mesokurtic	A distribution with a neutral degree of kurtosis.
Midpoints	Center of interval -- average of upper and lower limits.
Mode (Mo)	The most commonly occurring score.
Monotonic relationship	A relationship represented by a regression line that is continually increasing (or decreasing), but perhaps not in a straight line.
Multiplicative law of probability	The rule giving the probability of the joint occurrence of independent events.
Mutually exclusive	Two events are mutually exclusive when the occurrence of one precludes the occurrence of the other.
Negative relationship	A relationship in which increases in one variable are associated with decreases in the other.
Negatively skewed	A distribution that trails off to the left.
Nominal scale	Numbers used only to distinguish among objects.
normal distribution	A specific distribution having a characteristic bell-shaped form.
Ordinal scale	Numbers used only to place objects in order.
Ordinate	Vertical axis.
Outlier	An extreme point that stands out from the rest of the distribution.
p level	The probability that a particular result would occur by chance if H_0 is true. The exact probability of a Type I error.
Parameters	Numerical values summarizing population data.
Parametric tests	Statistical tests that involve assumptions about, or estimation of, population parameters.
Pearson product-moment correlation coefficient (*r*)	The most common correlation coefficient.

Percentile	The point below which a specified percentage of the observations fall.
Phi	The correlation coefficient when both of the variables are measured as dichotomies.
Platykurtic	A distribution that is relatively thick in the "shoulders."
Pooled variance	A weighted average of the separate sample variances.
Population variance	Variance of the population ‹usually estimated, rarely computed.
Population	Complete set of events in which you are interested.
Positively skewed	A distribution that trails off to the right.
Power	The probability of correctly rejecting a false H_0 .
Predictor variable	The variable from which a prediction is made.
Protected *t*	A technique in which we run *t* tests between pairs of means only if the analysis of variance was significant.
Quantitative data	Data obtained by measuring objects or events.
Random sample	A sample in which each member of the population has an equal chance of inclusion.
Random Assignment	Assigning participants to groups or cells on a random basis.
Range	The distance from the lowest to the highest score.
Range restrictions	Refers to cases in which the range over which X or Y varies is artificially limited.
Ranked data	Data for which the observations have been replaced by their numerical ranks from lowest to highest.
Rank - randomization tests	A class of nonparametric tests based on the theoretical distribution of randomly assigned ranks.
Ratio scale	A scale with a true zero point -- ratios are meaningful.
Real lower limit	The points halfway between the top of one interval and the bottom of the next.
Real upper limit	The points halfway between the top of one interval and the bottom of the next.
Rectangular distribution	A distribution in which all outcomes are equally likely.
Regression	The prediction of one variable from knowledge of one or more other variables.
Regression equation	The equation that predicts Y from X.
Regression coefficients	The general name given to the slope and the intercept ‹most often refers just to the slope.
Rejection region	The set of outcomes of an experiment that will lead to rejection of H_0 .
Rejection level	The probability with which we are willing to reject H0 when it is in fact correct.
Related samples	An experimental design in which the same subject is observed under more than one treatment.
Relative frequency view	Definition of probability in terms of past performance.

Research hypothesis	The hypothesis that the experiment was designed to investigate.
Sample	Set of actual observations. Subset of the population.
Sample statistics	Statistics calculated from a sample and used primarily to describe the sample.
Sample variance (s^2)	Sum of the squared deviations about the mean divided by N - 1.
Sample with replacement	Sampling in which the item drawn on trial N is replaced before the drawing on trial N + 1.
Sampling distribution of differences between means	The distribution of the differences between means over repeated sampling from the same population(s).
Sampling distribution of the mean	The distribution of sample means over repeated sampling from one population.
Sampling distributions	The distribution of a statistic over repeated sampling from a specified population.
Sampling error	Variability of a statistic from sample to sample due to chance.
Scales of measurement	Characteristics of relations among numbers assigned to objects.
Scatter plot	A figure in which the individual data points are plotted in two-dimensional space.
Scatter diagram	A figure in which the individual data points are plotted in two-dimensional space.
Scattergram	A figure in which the individual data points are plotted in two-dimensional space.
Sigma	Symbol indicating summation.
Significance level	The probability with which we are willing to reject H_0 when it is in fact correct.
Simple effect	The effect of one independent variable at one level of another independent variable.
Skewness	A measure of the degree to which a distribution is asymmetrical.
Slope	The amount of change in Y for a one unit change in X.
Spearman's correlation coefficient for ranked data (r_s)	A correlation coefficient on ranked data.
Standard deviation	Square root of the variance.
Standard error	The standard deviation of a sampling distribution.
Standard error of differences between means	The standard deviation of the sampling distribution of the differences between means.
Standard error of estimate	The average of the squared deviations about the regression line.
Standard scores	Scores with a predetermined mean and standard deviation.
Standard normal	A normal distribution with a mean equal to 0 and variance equal to 1.

distribution	Denoted *N* (0, 1).
Statistics	Numerical values summarizing sample data.
Student's *t* distribution	The sampling distribution of the *t* statistic.
Subjective probability	Definition of probability in terms of personal subjective belief in the likelihood of an outcome.
Sufficient statistic	A statistic that uses all of the information in a sample.
Sums of squares	The sum of the squared deviations around some point (usually a mean or predicted value).
Symmetric	Having the same shape on both sides of the center.
T scores	A set of scores with a mean of 50 and a standard deviation of 10.
Test statistics	The results of a statistical test.
Type I error	The error of rejecting H_0 when it is true.
Type II error	The error of not rejecting H_0 when it is false.
Unconditional probability	The probability of one event *ignoring* the occurrence or nonoccurrence of some other event.
Unimodal	A distribution having one distinct peak.
Variables	Properties of objects that can take on different values.
Weighted average	The mean of the form: $(a_1X_1 + a_2X_2)/(a_1 + a_2)$ where a_1 and a_2 are weighting factors and X_1 and X_2 are the values to be average.

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UNIT I

Definitions scope of Biostatistics, Variables in Biology, Collection, Classification and Tabulation of data – Graphical and Diagrammatic representation.

Measures of central tendency: Arithmetic Mean, Median, Mode. Measures of dispersion – Range, Standard deviation and Coefficient of variation.

SUGGESTED READINGS

1. Jerrold H Zat (2003) Bio-statistical Analysis (4^{th} ed.) Pearson Education(P) Ltd, New Delhi.
2. Kothari C.R (2004). Research Methodology – Methods and Techniques (2^{nd} Ed.) New age International (P) Ltd, New Delhi.

Unit – I

Introduction

Statistical tools are found useful in progressively increasing of disciplines. In ancient times the statistics or the data regarding the human force and wealth available in their land had been collected by the rulers. Now-a-days the fundamental concepts of statistics are considered by many to be essential part of their knowledge.

Origin and Growth

The origin of the word 'statistics' has been traced to the Latin word 'status', the Italian word 'statista' , the French word 'statistique' and the German word 'statistik'. All these words mean political state.

Meaning

The word 'statistics' is used in two different meanings. As a plural word it means data or numerical statements. As a singular word it means the science of statistics and statistical methods. The word 'statistics' is also used currently as singular to mean data.

Definitions

Statistics is "the science of collection, organization, presentation, analysis and interpretation of numerical data". – Dr S.P.Gupta.

Statistics are numerical statement of facts in any department of enquiry, placed in relation to each other". – Dr.A.L.Bowley.

Functions

The following are the important functions of statistics.

- σ Collection
- σ Numerical Presentation
- σ Diagrammatic Presentation
- σ Condensation

- σ Comparison
- σ Forecasting
- σ Policy Making
- σ Effect Measuring
- σ Estimation
- σ Tests of significance.

Characteristics

* Statistics is a Quantitative Science.

* It never considers a single item.

* The values should be different.

* Inductive logic is applied.

* Statistical results are true on the average.

* Statistics is liable to be misused.

Samples vs. Populations

Population: A complete set of observations or measurements about which conclusions are to be drawn.

Sample: A subset or part of a population.

Not necessarily random

Statistics vs. Parameters

Parameter: A summary characteristic of a population.

Summary of Central tendency, variability, shape, correlation

E.g., Population mean, Population Standard Deviation, Population Median, Proportion of population of registered voters voting for Bush, Population correlation between Systolic & Diastolic BP

Statistic: A summary characteristic of a sample. Any of the above computed from a sample taken from the population.

E.g., Sample mean, Sample Standard Deviation, median, correlation coefficient

MEASURES OF CENTRAL TENDENCY

Introduction

In this chapter we are going to deal with Measures of central tendency and about the measures of dispersion. The measures of central tendency concentrate about the values in the central part of the distribution. Plainly speaking an average of a statistical series is the value of the variable which is the representative of the entire distribution. If we know the average alone we cannot form a complete idea about the distribution so for the completeness of the idea we use Measures of dispersion.

Measures of Central Tendency

According to Professor Bowley the measures of central tendency are "statistical constants which enable us to comprehend in a single effort the significance of the whole "

The following three are the basic measures of central tendency in this chapter we deal with

- Arithmetic Mean or simply Mean
- Median
- Mode

Arithmetic Mean or Mean

Arithmetic Mean or simply Mean is the total values of the item divided by their number of the items. It is usually denoted by $\bar{X}$

Individual series

$\bar{X} = \Sigma X / N$

Example 1

The expenditure of ten families are given below .Calculate arithmetic mean.

30, 70, 10, 75, 500, 8, 42, 250, 40, 36

Solution

Here N=10

$\Sigma X = 30 + 70 + 10 + 75 + 500 + 8 + 42 + 250 + 40 + 36 = 1061$

$\bar{X} = 1061 / 10 = 106.1$

Discrete series

$\bar{X} = \Sigma f X / \Sigma f$

Example 2

Calculate the mean number of person per house.

No. of person : 2 3 4 5 6

No. of house : 10 25 30 25 10

Solution

X	f	f X
2	10	20
3	25	75

4	30	120
5	25	125
6	10	60

$\Sigma f = 100 \quad \Sigma f X = 400$

$\bar{X} = 400 / 100 = 4$.

Continuous series

$\bar{X} = \Sigma f m / \Sigma f$ where m represents the mid value

Mid-value = (upper boundary + lower boundary) / 2.

Example 3

Calculate the mean for the following.

Marks :	20-30	30-40	40-50	50-60	60-70	70-80
No. of student :	5	8	12	15	6	4

Solution

C.I	f	m	f m
20-30	5	25	125
30-40	8	35	280
40-50	12	45	540
50-60	15	55	825
60-70	6	65	390
70-80	4	75	300

$\Sigma f = 50 \quad \Sigma f m = 2460$

X = 2460 / 50 = 49.2.

Median

The median is the value for the middle most items when all the items are in the order of magnitude. It is denoted by M or Me.

Individual series

For odd number of items Median Position = (N+1) / 2

For even number of item

Position of the Median = [(N / 2)+((N/2)+1)] / 2

Example 1

Calculate median for the following.

22 10 6 7 12 8 5

Solution

Here N =7

Arrange in ascending order or descending order.

5 6 7 8 10 12 22

(N+1) / 2= (7+1) /2 = 4 th item = 8

Discrete series

Position of the median = (N+1) / 2 th item.

Example 2

Find the median for the following.

X : 10 15 17 18 21

F: 4 16 12 5 3

Solution

X f c.f

10	4	4
15	16	20
17	12	32
18	5	37
21	3	40

N= 40

(N+1) /2 = (40+1) / 2 = 20.5th item

= (20 th item +21st item) /2 =(15+17) /2

= 16.

Continuous series

$$M = L+\frac{[((N/2) - c.f) \times i]}{f}$$

Where L - lower boundary, f - frequency, I - size of class interval and c.f - cumulative frequency.

Example 3

Calculate the median height (Ht) for the No .of Students (NoS) given below.

Ht: 145-150 150-155 155-160 160-165 165-170 170-175

NoS: 2 5 10 8 4 1

Solution

Height	No. of student	c.f
145-150	2	2
150-155	5	7
155-160	10	17
160-165	8	25

165-170	4	29
170-175	1	30
	$\Sigma f = 30$	

Position of the median = $N/2^{th}$ item = 30 / 2 =15.

$$M = L+\frac{[((N/2) - c.f) \times i]}{f.}$$

$$= 155+ \frac{[(15-7)x5]}{10} = 155+(40/10) = 159.$$

Mode

Mode is the value which has the greatest frequency density. Mode is usually denoted by Z.

Individual series

In a set of observations the value which occur more number of time is known as Mode. In other way the most frequented value in a set of value is Mode.

Example 1

Determine the mode for the set of Individual observations given as follows 32, 35, 42, 32, 42, 32.

Solution

Mode = 32 Uni-model

Discrete series

Determine the Mode

Size of dress	**No. of set**
18	55
20	120
22	108

24 45

Here the mode represents highest frequency ie. I 120.

So, Mode = 20

Continuous series

$Z = L + [i(f_1-f_0) /(2f_1 –f_0 –f_2)]$

Where L- lower boundary , f_1-frequency of the modal class, f_0 – frequency of the preceding modal class, f_2- frequency of the succeeding modal class, i-size of class interval , c.f- cumulative frequency.

Example

Determine the mode

Marks : 0-10 10-20 20-30 30-40 40-50

No.of student : 5 20 35 20 12

Solution

Marks	No. of student
0-10	5
10-20	20
20-30	35
30-40	20
40-50	12

$Z = L + [i(f_1-f_0) /(2f_1 –f_0 –f_2)]$

$= 20+[10(35-20)/(2(35)-20-20)] = 20+5 = 25.$

Empirical relation

- Mode = 3 median -2 mean.

MEASURES OF DISPERSION

Measure of dispersion deals mainly with the following three measures

- Range
- Standard deviation
- Coefficient of variation

Range

Range is the difference between the greatest and the smallest value.

- Range = L – S , where L-largest value & S-Smallest value
- Coefficient of range = (L-S) /(L+S)

Individual series

Example

Find the value of range and its coefficient of range for the following data.

8 ,10, 5, 9,12,11

Solution

Range = L – S

= 12- 5 =7

Coefficient of range = (L-S) /(L+S)

= (12-5) / (12+5)

= 7 /17

= 0.4118

Continuous series

Range = L – S, where L-Mid-value of largest boundary & S-Mid-value of smallest boundary

Calculate the range for the following continuous frequency distribution

Marks : 20-30 30-40 40-50 50-60 60-70 70-80

No.of student : 5 8 12 15 6 4

Solution

C.I	f	m
20-30	5	25
30-40	8	35
40-50	12	45
50-60	15	55
60-70	6	65
70-80	4	75

Here L=75 & S=25

Range = L – S = 75-25 = 50

Quartile Deviation

Quartile Deviation is half of the difference between the first and the third quartiles. Hence it is called Semi Inter Quartile Range.

Coefficient of Quartile Deviation

A relative measure of dispersion based on the quartile deviation is called the coefficient of quartile deviation. It is defined as

$$= \frac{\frac{Q_3 - Q_1}{2}}{\frac{Q_3 + Q_1}{2}} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

It is pure number free of any units of measurement. It can be used for comparing the dispersion in two or more than two sets of data.

Example

The wheat production (in Kg) of 20 acres is given as: 1120, 1240, 1320, 1040, 1080, 1200, 1440, 1360, 1680, 1730, 1785, 1342, 1960, 1880, 1755, 1720, 1600, 1470, 1750, and 1885. Find the quartile deviation and coefficient of quartile deviation.

Solution

After arranging the observations in ascending order, we get

1040, 1080, 1120, 1200, 1240, 1320, 1342, 1360, 1440, 1470, 1600, 1680, 1720, 1730, 1750, 1755, 1785, 1880, 1885, 1960.

$$Q_1 = \text{Value of } \left(\frac{n+1}{4}\right)th \text{ item}$$

$$= \text{Value of } \left(\frac{20+1}{4}\right)th \text{ item}$$

$$= \text{Value of } (5.25)th \text{ item}$$

$$= 5th \text{ item} + 0.25(6th \text{ item} - 5th \text{ item}) = 1240 + 0.25(1320 - 1240)$$

$$Q_1 = 1240 + 20 = 1260$$

$$Q_3 = \text{Value of } \frac{3(n+1)}{4}th \text{ item}$$

$$= \text{Value of } \frac{3(20+1)}{4}th \text{ item}$$

$$= \text{Value of } (15.75)th \text{ item}$$

$$= 15th \text{ item} + 0.75(16th \text{ item} - 15th \text{ item}) = 1750 + 0.75(1755 - 1750)$$

$$Q_3 = 1750 + 3.75 = 1753.75$$

Quartile Deviation (Q.D)

$$= \frac{Q_3 - Q_1}{2} = \frac{1753.75 - 1260}{2} = \frac{492.75}{2} = 246.875$$

Coefficient of Quartile Deviation

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{1753.75 - 1260}{1753.75 + 1260} = 0.164$$

Standard deviation

The standard deviation is the root mean square deviation of the values from the arithmetic mean .It is a positive square root of variants. It is also called root mean square deviation. This is usually denoted by σ .

Individual series

$$\sigma = \sqrt{(\Sigma x^2 / N) - (\Sigma x / N)^2}$$

Example 1

Calculate standard deviation for the following data.

40,41,45,49,50,51,55,59,60,60.

Solution

X	X^2
40	1600
41	1681
45	2025
49	2401
50	2500
51	2601
55	3025
59	3481
60	3600
60	3600
510	$\Sigma x^2 = 26504$

$$\sigma = \sqrt{(\Sigma x^2 / N) - (\Sigma x / N)^2}$$

$= \sqrt{(26514/10) - (510/10)^2}$

$= 7.09$

Discrete series

$\sigma = \sqrt{(\Sigma fx^2 / \Sigma f) - (\Sigma fx / \Sigma f)^2}$

Example 2

Calculate standard deviation for the following data.

X : 0 1 2 3 4 5

F : 1 2 4 3 0 2

Solution

X	**f**	**fx**	**x^2**	**fx^2**
0	1	0	0	0
1	2	2	1	2
2	4	8	4	16
3	3	9	9	27
4	0	0	16	0
5	2	10	25	50

$\Sigma f = 12$ $\Sigma fx = 29$ $\Sigma fx^2 = 95$

$\sigma = \sqrt{(\Sigma fx^2 / \Sigma f) - (\Sigma fx / \Sigma f)^2}$

$= \sqrt{}\ (95/12\ ) - (\ 29\ /12\ )^2 \qquad = 1.44$

Continuous series

$$\sigma = \sqrt{}\ (\Sigma\ fm^2 / \Sigma\ f) - (\ \Sigma\ fm / \Sigma\ f)^2$$

Example 3

C.I : 0-10 10-20 20-30 30-40 40-50

F : 2 5 9 3 1

Solution

C.I	f	m	fm	m^2	fm^2
0-10	2	5	10	25	50
10-20	5	15	75	225	1125
20-30	9	25	225	625	5625
30-40	3	35	105	1225	3675
40-50	1	45	5	2025	2025
	20		460		12500

$$\sigma = \sqrt{}\ (\Sigma\ fm^2 / \Sigma\ f) - (\ \Sigma\ fm / \Sigma\ f)^2$$

$= \sqrt{}\ (12500/20\ ) - (\ 460\ /20\ )^2$

$= 9.79$

Coefficient of variation

Coefficient of variation = [standard deviation / arithmetic mean] x100

Example 1

Calculate the coefficient of variation.

Mean= 51, standard deviation = 7.09

Solution

Coefficient of variation = [standard deviation / arithmetic mean] x100

= (7.09 /51) x100

= 13.9

Questions

1) Calculate the Mean for the following.

X	20	30	35	15	10
f	2	3	4	3	2

2) Define Median and give Example.

3) Calculate the Range and its Coefficient for the following data.

X	:	12	14	16	18	20
f	:	1	3	5	3	1

4) What do you mean by Bimodal?

5) Calculate the Median for the following data.

80 100 50 90 120 110

6) Write the relation between Standard Deviation and Variance.

7) Calculate the Average number of students per class for the following data.

26 46 33 25 36 27 34 29

8) Find Median and Mode for the following data.

13 16 17 15 18 14 19 15 12

9) Define the term Range.

10) Find the Arithmetic Mean for the following data.

70 60 75 50 42 95 46

11) Calculate the Range and its Coefficient for the following data.

17 10 56 19 12 11 18 14

12) Define the term Quartile Deviation.

13) Find the median for 57, 58, 61, 42, 38, 65, 72, and 66

14) Write the empirical relation for Mode.

Exercise

1. Calculate the Mode for the following Continuous Frequency Distribution.

Salary (in Rs. 1000s)	0 -19	20-39	40 - 59	60-79	80-99
No. of Employees	5	20	35	20	12

2. Find the Mean and the Standard Deviation for the given below data set.

10	14	20	12	21	16	19	17	14	25

3. Calculate the Standard Deviation and Coefficient of Variance (CV) for the following data.

X	0 – 10	10 - 20	20 - 30	30 – 40	40 - 50
f	2	5	9	3	1

4. Calculate the Median for the following Continuous Frequency Distribution.

Wages (in Rs.)	0 - 19	20 - 39	40 - 59	60 – 79	80 - 99
No. of Workers	5	20	35	20	12

5. Calculate the Coefficient of Variation for the following data.

X	6	9	12	15	18
f	7	12	13	10	8

6. Calculate the Median for the following.

Hourly Wages (in Rs.)	40 - 50	50 – 60	60 - 70	70 - 80	80 - 90	90 - 100
Number of Employees	10	20	15	30	15	10

7. The following data give the details about salaries (in thousands of rupees) of seven employees randomly selected from a Pharmaceutical Company.

Serial No.	1	2	3	4	5	6	7
Salary per Annum ('000)	89	57	104	73	26	121	81

Calculate the Standard Deviation and Coefficient of variance of the given data.

8. Calculate the Arithmetic Mean for the following data.

Height (cms) : 160 161 162 163 164 165 166

No. of Persons : 27 36 43 78 65 48 28

9. Calculate the Coefficient of Variance for the following data. 77 73 75 70 72 76 75 72 74 7

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
The word statistics is used as----------	singular word	a plural word	both singular and plural words	neither singular nor plural word	both singular and plural words
Classification is a process of arranging data in----------	grouping of related facts in different classes	different rows	different columns and rows	different columns grouping of related facts in different classes	grouping of related facts in different classes
To represent two or more sets of interrelated data , we use ----------	bar diagram	pie diagram	histogram	multiple bar diagram	multiple bar diagram
Histogram is a graph of ----------	Time series	frequency distribution	cumulative frequency distribution	normal distribution	frequency distribution
Univariate data consists of ----------	one variable	two variables	three variable	four	one variable
Data are generally obtained from----------	Primary sources	Secondary sources	Both primary and secondary sources	neither primary nor secondary sources	Both primary and secondary sources
In geographical classification data are classified on the basis of----------	area	attributes	time	location	area
In qualitative classification data are classified on the basis of----------	area	attributes	time	location	attributes
In quantitative classification data are classified on the basis of----------	area	attributes	time	magnitude	magnitude
Number of source of data is----------	2	3	4	1	2
Squares and rectangles are----------	Two dimensional diagram	One dimensional diagram	Three dimensional diagram	Multi dimensional diagram	Two dimensional diagram
Data originally collected for an investigation is known as----------	Tabulation	Primary data	Secondary data	Published data	Primary data
The heading of a row in a statistical table is known as ----------	stub	caption	title	heading	stub

Statistics can ----------	prove anything	disprove anything	neither prove nor disprove anything but it is just a tool	none of these	neither prove nor disprove anything but it is just a tool
Statistics is also a science of----------	estimates	both a and b	probabilities	neither a nor b	both a and b
Statistics is---------	quantitative science	a qualitative science	both quantitative and qualitative science	neither quantitative nor qualitative	both quantitative and qualitative science
Statistics considers----------	a single item	a set of item	either a single item or a set of item	neither a single item or a set of item	a set of item
Statistics can be considered as----------	an art	a science	both an art and science	neither an art nor a science	both an art and science
The other name of cumulative frequency curve is ----	Ogive	Bars	Histogram	Pie diagram	Ogive
Number of methods of collection of primary data is----------	2	3	4	5	5
Number of questions in a questionnaire should be----------	5	10	maximum	minimum	minimum
Sources of secondary data are----------	Published sources	Unpublished sources	Either Published sources or Unpublished sources	primary source	Either Published sources or Unpublished sources
Compared with primary data , secondary data are----------	more reliable	less reliable	equally reliable	uniformly reliable	less reliable
---------- are column headings	stub	heading	bar	captions	captions
Mid value= ------	lower boundary/2	upper boundary/2	lower boundary+ upper boundary)/2	lower boundary+ upper boundary	lower boundary+ upper boundary)/2
The origin of the word statistics has been traced to the Latin word -------	statista	status	statistik	statistique	status
Graphs of frequency distribution are----------	histogram	pie diagram	bar chart	circle	histogram

cubes are----------	Two dimensional diagram	One dimensional diagram	Three dimensional diagram	Multi dimensional diagram	Three dimensional diagram
---------- is the difference between the value of the smallest item and the valueof the largest item.	class interval	frequency	number of items	range	range
---------- is one which is used by the individual or agency which collect it.	primary data	secondary data	both		primary data
Exclusive class intervals suit ----------	discrete variables	continuous variables	both	neither	continuous variables
A table is a systematic arrangement of statistical data in----------	columns	rows	both columns and rows	stubs	both columns and rows
The collected data in any statistical investigation are known as ----------	raw data	arranged data	classified data	tabulated data	raw data
The emitting form of a frequency polygon is called ----------	histogram	ogive	bar diagram	frequency curve	frequency curve
In chronological classification data are classified on the basis of----------	time	attributes	class intervals	location	time
Bar diagrams are ---------- dimensional diagrams	two	three	one	multi	one
Diagrams and graphs are tools of ----------	collection of data	presentation	analysis	summarization	presentation
In a two dimensional diagram ----------	only height is considered	only width is considered	height,width and thickness are considered	Both height and width are considered	only height is considered
Which one of the following is a measure of central	Median	range	variation	correlation	Median
The total of the values of the items divided by their number of items is known as	Median	Arithmetic mean	mode	range	Arithmetic mean
In the short-cut method of arithmetic mean, the deviation is taken as	$x - A$	$A - x$	$(x - A) / c$	$(A - x) / c$	$x - A$
The sum of the deviations of the values from their arithmetic mean is	– 1	one	two	zero	zero
The formula for the weighted arithmetic mean is	$\sum wx / \sum w$	$\sum w / \sum wx$	$\sum x / n$	$\sum x / \sum f$	$\sum wx / \sum w$
Find the Mean of the following values. 5, 15, 20, 10,	5	18	41	20	18
Which of the followings represents median?	First quartile	Third quartile	Second quartile	Q.D	Second quartile

Which of the measure of central tendency is not affected by extreme values?	Mode	Median	sixth deciles	Mean	Median
Sum of square of the deviations about mean is	Maximum	one	zero	Minimum	Minimum
Median is the value of ---------- item when all the items are in order of magnitude.	First	second	Middle most	last	Middle most
Find the Median of the following data 160, 180, 175, 179, 164, 178, 171, 164, 176.	160	175	176	180	175
The position of the median for an individual series is	(N + 1) / 2	(N + 2) / 2	N/2	N/4	(N + 1) / 2
Mode is the value, which has	Average frequency density	less frequency density	greatest frequency density	graetest frequency	greatest frequency density
A frequency distribution having two modes is said to	unimodal	bimodal	trimodal	modal	bimodal
Mode has ---------- stable than mean.	less	more	same	most	less
Which of the following is not a measure of dispersion?	Range	quartile deviation	standard deviation	median	median
Range of the given values is given by	L- S	L+S	S+L	LS	L- S
Which one of the following is relative measure of dispersion?	Range	Q.D	S.D	coefficient of variation	coefficient of variation
Coefficient of variation is defined as	(AM * 100)/S.D	(S.D* 100)/A.M	S.D/A.M	(1/S.D)*100	(S.D* 100)/A.M
If the values of median and mean are 72 and 78 respectively, then find the mode.	16	60	70	76	60
Find Mean for the following 3, 4, 5.	4	2.25	3	2.28	4
The coefficient of range	L-S /L+S	L+S /L-S	L-S	L+S	L-S /L+S
Second quartile is also called as	Mode	mean	median	G.M	median
If A.M = 8, N=12, then find $\sum X$.	76	80	86	96	96
If the value of mode and mean is 60 and 66 then, find the value of median.	64	46	54	44	64
The formula for median for continuous series is	M = (N+1) / 2	M = L + [(N/2 + cf) / f] * i	M =L - (N/2+cf)/f* i	M = L + [(N/2 - cf) / f] * i	M = L + [(N/2 - cf) / f] * i

Median is	Average point	Midpoint	Most likely point	Most remote point	Midpoint
Mode is the value which	Is a mid point	Occur the most	Average of all	Most remote Likely	Occur the most
. ……. Is known as positional average	Median	Mean	Mode	Range	Median
The median of marks 55, 60, 50, 40, 57, 45, 58, 65, 57, 48 of 10 students is	55	57	52.5	56	56
The middle most value of a frequency distribution table is known as	Mean	Median	Mode	Range.	Median
The middle most value of a frequency distribution table is known as	Mean	Median	Mode	Range	Median
Measures of central tendency is also known as	Dispersion	averages	correlation	tendency	correlation
From the given data 35,40,43,32,27 the coefficient	23	0.23	13	0.13	13
If S.D = 6, then find variance.	6	36	42	12	36
Which one of the following shows the relation between variance and standard deviation?	var = square root of S.D	S.D = square root of variance	variance = S.D	variance / S.D = 1	S.D = square root of variance
If variance is 64, then find S.D.	8	13	14	11	8
Which of the following measures of averages divide the observation into two parts	Mean	Median	Mode	Range	Median
Which of the following measures of averages divide the observation into four equal parts	Mean	Median	Mode	Quartile	Quartile
Arithmetic mean of the series 1, 3, 5, 7, 9 is	5	6	5.5	6.5	5
Arithmetic mean of the series 3, 4, 5, 6, 7 is ……	5.5	6	5	6.5	5
The Arithmetic mean for the series 3, 5, 5, 2, 6, 2, 9, 5, 8, 6, is…….	5	6	5.5	6.5	5
The median value for the series 3, 5, 5, 2, 6, 2, 9, 5,	6	5	5.5	6.5	5
The mode for the series 3, 5, 6, 2, 6, 2, 9, 5, 8, 6 is	5	6	5.5	6.5	6
The Arithmetic mean for the series 51.6, 50.3, 48.9, 48.7, 48.5 is…….	49.8	50	48.9	49.6	49.8
The Median for the series 51.6, 50.3, 48.9, 48.7, 49.5, is……	49.8	50	48.9	49.6	49.6

The Mode for the series 51.6, 50.3, 48.9, 48.7, 49.5 is……	49.8	50	48.9	49.6	48.9
If standard deviation is 5, then the variance is	5	625	25	2.23068	25
Standard deviation is also called as	Root mean square deviation	mean square deviation	Root deviation	Root median square deviation	Root mean square deviation

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Department of Mathematics

Subject : Biostatistics and Research Methodology **Semester II** **L T P C**
Subject Code : 18MBP203 **Class : I M.Sc Microbiology** **4 2 0 4**

UNIT II

Correlation meaning and definition – scatter diagram – Karl Pearson correlation coefficient, Rank Correlation. Regression – Regression in two variables – Regression coefficient problem – uses of Regression.

SUGGESTED READINGS

1. Jerrold H Zat (2003) Bio-statistical Analysis (4^{th} ed.) Pearson Education(P) Ltd, New Delhi.
2. Kothari C.R (2004). Research Methodology – Methods and Techniques (2^{nd} Ed.) New age International (P) Ltd, New Delhi.

CHAPTER – II

CORRELATION AND REGRESSION ANALYSIS

3.1 Simple Linear Correlation

The term Correlation refers to the relationship between the variables. Simple correlation refers to the relationship between two variables. Various types of correlation are considered.

3.2 Type of Correlation

Positive or Negative when the values of two variables change in the same direction, their positive correlation between the two variables.

Example

X:	50	60	70	95	100	105	34	25	18	10	7
Y:	23	32	37	41	46	50	51	49	42	33	19

Simple or Partial or Multiple

When only two variables are considered as under positive or negative correlation above the correlation between them is called Simple correlation. When more than two variables as considered the correlation between two of them when all other variables are held constant, i.e., when the linear effects of all other variables on them are removed is called partial correlation. When more than two variables are considered the correlation between one of them and its estimate based on the group consisting of the other variables is called multiple correlation.

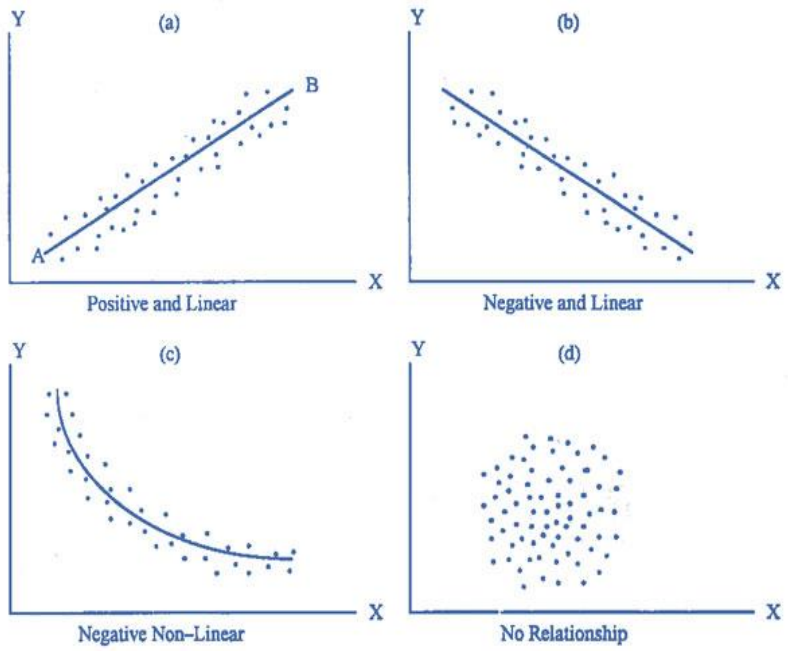
3.3 Methods of Finding Correlation Coefficient

The following four methods are available under simple linear correlation and among them; product moment method is the best one.

- Scatter Diagram
- Karl Pearson's correlation coefficient or product moment correlation coefficient (r)
- Spearman's rank correlation coefficient (ρ)
- Correlation coefficient by concurrent deviation method (r_c).

3.3.1 Scatter Diagram

Scatter diagram is a graphic picture of the sample data. Suppose a random sample of n pairs of observations has the values $(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), \ldots, (X_n, Y_n)$. These points are plotted on a rectangular co-ordinate system taking independent variable on *X*-axis and the dependent variable on *Y*-axis. Whatever be the name of the independent variable, it is to be taken on *X*-axis. Suppose the plotted points are as shown in figure (a). Such a diagram is called scatter diagram. In this figure, we see that when *X* has a small value, *Y* is also small and when *X* takes a large value, *Y* also takes a large value. This is called direct or positive relationship between *X* and *Y*. The plotted points cluster around a straight line. It appears that if a straight line is drawn passing through the points, the line will be a good approximation for representing the original data. Suppose we draw a line ***AB*** to represent the scattered points. The line ***AB*** rises from left to the right and has positive slope. This line can be used to establish an approximate relation between the random variable *Y* and the independent variable *X*. It is nonmathematical method in the sense that different persons may draw different lines. This line is called the regression line obtained by inspection or judgment.



Making a scatter diagram and drawing a line or curve is the primary investigation to assess the type of relationship between the variables. The knowledge gained from the scatter diagram can be used for further analysis of the data. In most of the cases the diagrams are not as simple as in figure (a). There are quite complicated diagrams and it is difficult to choose a proper

mathematical model for representing the original data. The scatter diagram gives an indication of the appropriate model which should be used for further analysis with the help of method of least squares. Figure (b) shows that the points in the scatter diagram are falling from the top left corner to the right. This is a relation called inverse or indirect. The points are in the neighborhood of a certain line called the regression line.

As long as the scattered points show closeness to a straight line of some direction, we draw a straight line to represent the sample data. But when the points do not lie around a straight line, we do not draw the regression line. Figure (c) shows that the plotted points have a tendency to fall from left to right in the form of a curve. This is a relation called non-linear or curvilinear. Figure (d) shows the points which apparently do not follow any pattern. If X takes a small value, Y may take a small or large value. There seems to be no sympathy between X and Y. Such a diagram suggests that there is no relationship between the two variables.

3.3.2 Karl Pearson's Correlation Coefficient

Karl Pearson's Product-Moment Correlation Coefficient or simply Pearson's Correlation Coefficient for short, is one of the important methods used in Statistics to measure Correlation between two variables.

A few words about Karl Pearson: Karl Pearson was a British mathematician, statistician, lawyer and a eugenicist. He established the discipline of mathematical statistics. He founded the world's first statistics department In the University of London in the year 1911. He along with his colleagues Weldon and Galton founded the journal "Biometrika" whose object was the development of statistical theory.

The Correlation between two variables X and Y, which are measured using Pearson's Coefficient, give the values between +1 and -1. When measured in population the Pearson's Coefficient is designated the value of Greek letter rho (ρ). But, when studying a sample, it is designated the letter r. It is therefore sometimes called Pearson's r. Pearson's coefficient reflects the linear relationship between two variables. As mentioned above if the correlation coefficient is +1 then there is a perfect positive linear relationship between variables, and if it is -1 then there is a perfect negative linear relationship between the variables. And 0 denotes that there is no relationship between the two variables.

The degrees -1, +1 and 0 are theoretical results and are not generally found in normal circumstances. That means the results cannot be more than -1, +1. These are the upper and the lower limits.

Pearson's Coefficient computational formula

$$r = \frac{\sum XY - \frac{\sum X \sum Y}{N}}{\sqrt{(\sum X^2 - \frac{(\sum X)^2}{N})(\sum Y^2 - \frac{(\sum Y)^2}{N})}}$$

Sample question: compute the value of the correlation coefficient from the following table:

Subject	Age x	Weight Level y
1	43	99
2	21	65
3	25	79
4	42	75
5	57	87
6	59	81

Step 1: Make a chart. Use the given data, and add three more columns: xy, x2, and y2.

Subject	Age x	Weight Level y	xy	x^2	y^2
1	43	99			
2	21	65			
3	25	79			
4	42	75			
5	57	87			
6	59	81			

Step 2: Multiply x and y together to fill the xy column. For example, row 1 would be 43 × 99 = 4,257

Step 3: Take the square of the numbers in the x column, and put the result in the x^2 column.

Subject	Age x	Weight Level y	xy	x^2	y^2

1	43	99	4257	1849	
2	21	65	1365	441	
3	25	79	1975	625	
4	42	75	3150	1764	
5	57	87	4959	3249	
6	59	81	4779	3481	

Step 4: Take the square of the numbers in the y column, and put the result in the y^2 column.

Step 5: Add up all of the numbers in the columns and put the result at the bottom.2 column. The Greek letter sigma (Σ) is a short way of saying "sum of."

Subject	**Age X**	**Weight Y**	**XY**	$\mathbf{X^2}$	$\mathbf{Y^2}$
1	43	99	4257	1849	9801
2	21	65	1365	441	4225
3	25	79	1975	625	6241
4	42	75	3150	1764	5625
5	57	87	4959	3249	7569
6	59	81	4779	3481	6561
Σ	247	486	20485	11409	40022

Step 6: Use the following formula to work out the correlation coefficient.

The answer is: 1.3787 × 10-4 the range of the correlation coefficient is from -1 to 1. Since our result is 1.3787 × 10-4, a tiny positive amount, we can't draw any conclusions one way or another.

3.3.3 Spearman's Rank Correlation Coefficient

The Spearman correlation coefficient is often thought of as being the Pearson correlation coefficient between the ranked variables. In practice, however, a simpler procedure is normally used to calculate ρ. The n raw scores X_i, Y_i are converted to ranks x_i, y_i, and the differences $d_i = x_i - y_i$ between the ranks of each observation on the two variables are calculated.

If there are no tied ranks, then ρ is given by

$$\rho = 1 - \left(\frac{6 \sum d^2}{N(N^2 - 1)} \right)$$

If tied ranks exist, Pearson's correlation coefficients between ranks should be used for the calculation:

One has to assign the same rank to each of the equal values. It is an average of their positions in the ascending order of the values.

Example 1

X : 21 36 42 37 25

Y : 47 40 37 42 43

For the data given above, calculate the rank correlation coefficient.

Solution

		Rank of X and Y			
X	Y	R(X)	R(Y)	d	d^2
21	47	5	1	4	16
36	40	3	4	-1	1
42	37	1	5	-4	16
37	42	2	3	-1	1
25	43	4	2	2	4
			Total	$\sum d = 0$	$\sum d^2 = 38$

$$\rho = 1 - \left(\frac{6 \sum d^2}{N(N^2 - 1)} \right)$$

$$= 1 - \left(\frac{6 \times 38}{5 (5^2 - 1)} \right)$$

$$= 1 - 1.9 = -0.9$$

Tied Ranks

When one or more values are repeated the two aspects- ranks of the repeated values and changes in the formula are to be considered.

Example 2

Find the rank correlation coefficient for the percentage of marks secured by a group of 8 students in Economics and Statistics.

Marks in Eco:	50	60	65	70	75	40	70	80
Marks in Stat:	80	71	60	75	90	82	70	50

Solution

Let X be Marks in Economics and Y be Marks in Statistics

		Rank of X and Y			
X	Y	X	Y	d	d^2
50	80	7	3	4	16
60	71	6	5	1	1
65	60	5	7	-2	4
70	75	3.5	4	-0.5	0.25
75	90	2	1	1	1
40	82	8	2	6	36
70	70	3.5	6	-2.5	6.25
80	50	1	8	-7	49
			Total	$\sum d = 0$	$\sum d^2$ =113.5

$$\rho = 1 - \left(\frac{6\{ \sum d^2 + m(m^2-1)/12\}}{N(N^2-1)} \right)$$

When $m=2$, $m(m^2-1)/12 = 0.5$

Therefore $\rho = 1-\left(6\{113.5+0.5\}/8(8^2-1)\} \right)$

$= 1- 1.3571 = -0.3571$

3.4 Simple Linear Regression

The line which gives the average relationship between the two variables is known as the regression equation. The regression equation is also called estimating equation.

Uses

1. Regression analysis is used in statistics and other displines.
2. Regression analysis is of practical use in determining demand curve, supply curve, consumption function, etc from market survey.

3. In Economics and Business, there are many groups of interrelated variables.
4. In social resarch, the relation between variables may not known; the relation may differ from place to place.
5. The value of dependent variable is estimated corresponding to any value of the independent variable using the appropriate regression equation.

3.4.1 Method of Least Squares

From a scatter diagram, there is virtually no limit as to the number of lines that can be drawn to make a linear relationship between the 2 variables

- the objective is to create a BEST FIT line to the data concerned
- the criterion is the called the method of least squares
- i.e. the *sum of squares* of the *vertical deviations* from the points to the line be a minimum (based on the fact that the dependent variable is drawn on the vertical axis)
- the linear relationship between the dependent variable (Y) and the independent variable(x) can be written as $Y = a + bX$, where a and b are parameters describing the vertical intercept and the slope of the regression.
- Similarly the linear relationship between the dependent variable (XY) and the independent variable(Y) can be written as $X = a' + b'Y$, where a and b are parameters describing the vertical intercept and the slope of the regression.

Calculating the coefficients a and b

The values of a and b for the given pairs of values of (xi,yi) I = 1, 2, 3 ….. are determined

Using the normal equations as

$\sum y = Na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$

Similarly, the values of a' and b' for the given pairs of values of (xi,yi) I = 1, 2, 3 ….. are determined,

Using the normal equations as,

$\sum x = Na' + b'\sum y$

$\sum xy = a'\sum y + b'\sum y^2$

3.4.2 Methods of forming the regression equations

- Regression equations based on normal equations.
- Regression equations based on X and Y and b_{YX} and b_{XY}.

Example 1

From the following data, obtain the two regression equations.

X	6	2	10	4	8
Y	9	11	5	8	7

use normal equations

Solution

X	Y	XY	X^2	Y^2
6	9	54	36	81
2	11	22	4	121
10	5	50	100	25
4	8	32	16	64
8	7	56	64	49
$\sum x=0$	$\sum y=0$	$\sum xy=214$	$\sum x^2=220$	$\sum y^2=340$

Let the regression equation Y on X is $Y = a + bX$

The normal equations are,

$\sum y = Na + b\sum x$

$\sum xy = a\sum x + b\sum x^2$

By substituting the values from the table, we get

$5a+30b = 40$ ------- 1

$30a + 180b = 214$ -------- 2

Solving these two equations we get,

$a=11.90$ and $b= -0.65$

Therefore the regression Y on X is $Y = 11.90-0.65X$.

Let the regression equation X on Y is $X = a' + b'Y$

The normal equations are,

$\sum x = Na + b\sum y$

$\sum xy = a\sum y + b\sum y^2$

By substituting the values from the table, we get

5a'+40'b = 30 ------- 3

40a' + 340b' = 214 -------- 4

Solving these two equations we get,

a' = 16.40 and b= -1.30

Therefore the regression equation X on Y is X = 16.40-1.30Y

Example 2

From the data given below, find

(i) the two regression equations

(ii) The correlation coefficient between the variables X and Y

(iii) The value of Y when X= 30

X : 25 28 35 32 31 36 29 38 34 32

Y : 43 46 49 41 36 32 31 30 33 39

Solution

X	Y	x= X- X`	Y= Y-Y`	xy	x^2	y^2
25	43	-7	5	-35	49	25
28	46	-4	8	-32	16	64
35	49	3	11	33	9	121
32	41	0	3	0	0	9
31	36	-1	-2	2	1	4
36	32	4	-6	-24	16	36
29	31	-3	-7	21	9	49
38	30	6	-8	-48	36	64
34	33	2	-5	-10	4	25
32	39	0	1	0	0	1
320	380	0	0	-93	140	398

X` = 32, Y`= 38, $b_{xy} = \sum xy / \sum y^2 = -0.2337$, $b_{yx} = \sum xy / x^2 = -0.6643$

iv) Regression equation of Y on X ,(Y - Y`)

$= b_{yx} (X-X`) (Y – 38) = -0.6643(X-32) \Rightarrow$

$Y = 59.26-0.6643X$

(ii) Regression equation of X on Y , $(X - X`)$

$= b_{xy} (Y-Y`) (X – 32) = -0.2337Y +8.88 \Rightarrow$

$X = 40.88 - 0.233 Y$

(iii) $r = + \sqrt{b_{yx} b_{xy}} = -0.3940$

(iv) $Y = 59.26-0.6643x30= 39$

3.4.3 Properties of Regression coefficients

1. The two regression equations are generally different and are not to be interchanged in their usage. ` `
2. The two regression lines intersect at (X, Y).
3. Correlation coefficient is the geometric mean of two regression coefficients.
4. The two regression coefficients and the correlation coefficient have the same sign.
5. Both the regression coefficients and the correlation coefficient cannot be greater than one numerically and simultaneously.
6. Regression coefficients are independent of change of origin but are affected by the change of scale.
7. Each regression coefficient is in the unit of the measurement of the dependent variable.
8. Each regression coefficient indicates the quantum of change in the dependent variable corresponding to unit increase in the independent variable.

Questions

1) What are the types of Correlation?
2) Write any two properties of Correlation.
3) What is the range of Correlation Coefficient?
4) Define Positive Correlation.
5) What is meant by Regression?
6) What are the formulae for Regression co-efficients?
7) Distinguish between Correlation and Regression.
8) Write the formula for Rank Correlation, when more than one rank is repeated.
9) If $b_{xy} = -0.2337$ and $b_{yx} = -0.6643$ then find the Correlation Coefficient.
10) What is Negative Correlation? Give an example?
11) Write down the formula for Karl Pearson's Coefficient of Correlation.
12) Define Scatter Diagram.
13) What is Simple Correlation?
14) Define Regression Equation.
15) When $X = 40$, $Y = 60$, $\sigma_x = 10$, $\sigma_Y = 15$ and $r = 0.7$ find the Regression Equation of Y on X.

Exercise

1) Calculate the Correlation Coefficient from the following variables.

Sales in ('0000)	57	58	59	59	60	61	62	64
Advertisement Expenditure ('000)	17	16	15	18	12	14	19	11

2) Marks obtained by 8 students in Accountancy (X) and Statistics (Y) are given below. Compute Rank Correlation Coefficient.

X	25	20	28	22	40	60	20
Y	40	30	50	30	20	10	30

3) Calculate the two Regression Equations from the following data.

X	10	12	13	12	16	15
Y	40	38	43	45	37	43

4) Calculate Karl Pearson's Coefficient of Correlation from the following data.

Wages	100	101	102	102	100	99	97	98
Cost of Living	98	99	99	97	95	92	95	94

5) From the data given find two Regression Equations.

X	10	12	13	12	16	15
Y	20	28	23	25	27	30.

i) Estimate Y when X = 20.

ii) Estimate X when Y = 35.

6) A comparison of the undergraduate Grade Point Averages of 10 corporate employees with their scores in a managerial trainee examination produced the results shown *in* the following table.

Exam Score	89	83	79	91	95	82	69	66	75	80
GPA	2.4	3.1	2.5	3.5	3.6	2.5	2.0	2.2	2.6	2.7

Measure the Correlation Coefficient between Exam scores and GPA by using Rank Method and also interpret the data given with the help of Scatter Diagram.

7) Develop the Regression Equation that best fit the data given below using annual income as an independent variable and amount of life insurance as dependent variable.

Annual Income (Rs. in 000's)	62	78	41	53	85	34
Amount of Life Insurance (Rs. in 00's)	25	30	10	15	50	7

8) The ranks of ten students in Economics and Statistics subjects are as follows.

Economics	3	5	8	4	7	10	2	1	6	9
Statistics	6	4	9	8	1	2	3	10	5	7

Calculate Spearman's Rank Correlation Coefficient.

9) You are given the following data:

	X	Y
Arithmetic Mean	36	85
Standard Deviation	11	8

Correlation coefficient between X and Y = 0.66

Find the two Regression Equations. And also find Correlation Coefficient.

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
The regression line cut each other at the point of-------	Average of X only	Average of Y only	Average of X and Y	the median of X on Y	Average of X and Y
Given the coefficient of correlation being 0.8, the coefficient of determination will be	0.98	0.64	0.66	0.54	0.64
Given the coefficient of correlation being 0.9, the coefficient of determination will be	0.98	0.81	0.66	0.54	0.81
If the coefficient of determination being 0.49, what is the coefficient of correlation	0.7	0.8	0.9	0.6	0.7
Given the coefficient of determination being 0.36, the coefficient of correlation will be	0.3	0.4	0.6	0.5	0.6
Which one of the following refers the term Correlation?	Relationship between two values	Relationship between two variables	Average relationship between two variables	Relationship between two things	Relationship between two variables
If r = +1, then the relationship between the given two variables is	perfectly positive	perfectly negative	no correlation	high positive	perfectly positive
If r = - 1, then the relationship between the given two variables is	perfectly positive	perfectly negative	no correlation	low Positive	perfectly negative
If r = 0, then the relationship between the given two variables is	Perfectly positive	perfectly negative	no correlation	both positive and negative	no correlation
Coefficient of correlation value lies between	1 and –1	0 and 1	0 and ∞	0 and –1.	1 and –1
While drawing a scatter diagram if all points appear to form a straight line getting Downward from left to right, then it is inferred that there is	Perfect positive correlation	simple positive correlation	Perfect negative correlation	no correlation	Perfect negative correlation
The range of the rank correlation coefficient is	0 to 1	–1 to 1	0 to ∞	– ∞ to ∞	–1 to 1
If r =1, then the angle between two lines of regression is	Zero degree	sixty degree	ninety degree	thirty degree	ninety degree

Regression coefficient is independent of	Origin	scale	both origin and scale	neither origin nor scale.	Origin
If the correlation coefficient between two variables X and Y is negative, then the Regression coefficient of Y on X is	Positive	negative	not certain	zero	negative
If the correlation coefficient between two variables X and Y is positive, then the Regression coefficient of X on Y is	Positive	negative	not certain	zero	Positive
There will be only one regression line in case of two variables if	r =0	r = +1	r = –1	r is either +1 or –1	r =0
The regression line cut each other at the point of	Average of X only	Average of Y only	Average of X and Y	the median of X on Y	Average of X and Y
If b_{xy} and b_{yx} represent regression coefficients and if $b_{yx} > 1$ then b_{xy} is	Less than one	greater than one	equal to one	equal to zero	Less than one
Rank correlation was discovered by	R.A.Fisher	Sir Francis Galton	Karl Pearson	Spearman	Spearman
Formula for Rank correlation is	$1- (6\Sigma d^2 /(n(n2-1)))$	$1- (6\Sigma d^2 /(n(n2+1)))$	$1+ (6\Sigma d^2 /(n(n2+1)))$	$1 /(n(n2-1))$	$1- (6\Sigma d^2 /(n(n2-1)))$
With b_{xy}=0.5, r = 0.8 and the variance of Y=16, the standard deviation of X=	6.4	2.5	10	25.6	2.5
The coefficient of correlation r =	$(b_{xy.} b_{yx})^{1/4}$	$(b_{xy .} b_{yx})^{-1/2}$	$(b_{xy .} b_{yx})^{1/3}$	$(b_{xy.} b_{yx})^{1/2}$	$(b_{xy.} b_{yx})^{1/2}$
If two regression coefficients are positive then the coefficient of correlation must be	Zero	negative	positive	one	positive
If two-regression coefficients are negative then the coefficient of correlation must be	Positive	negative	zero	one	Positive
The regression equation of X on Y is	X = a + bY	X = a + bX	X= a - bY	Y= a + bX	X = a + bY
The regression equation of Y on X is	X = a + bY	X = a + bX	X= a - bY	Y= a + bX	Y= a + bX
The given two variables are perfectly positive, if	r = +1	r = -1	r = 0	$r \neq +1$	r = +1
The relationship between two variables by plotting the values on a chart, known as-	coefficient of correlation	Scatter diagram	Correlogram	rank correlation	Scatter diagram

If x and y are independent variables then,	cov(x,y)≠ 0	cov(x,y)= 1	cov(x,y)= 0	cov(x,y) >1	cov(x,y)= 0
Correlation coefficient is the ---------- of the two regression coefficients.	Mode	Geometric mean	Arithmetic mean	median	Geometric mean
b_{xy} = 0.4, b_{yx} = 0.9 then r =	0.6	0.3	0.1	-0.6	0.6
b_{xy}=1/5, r=8/15, s_x = 5 then s_y=	40/13	13/40	40/3	3	40/3
The geometric mean of the two regression coefficients.	Correlation coefficient	regression coefficients	coefficient of range	coefficient of variation	Correlation coefficient
If two variables are uncorrelated, then the lines of regression	Do not exist	coincide	Parallel to each other	perpendicular to each other	perpendicular to each other
If the given two variables are correlated perfectly negative, then	r = +1	r = -1	r = 0	r ≠ +1	r = -1
If the given two variables have no correlation, then	r = +1	r = -1	r = 0	r ≠ +1	r = 0
If the correlation coefficient between two variables X and Y is ---------, the Regression coefficient of Y on X is positive	Negative	positive	not certain	zero	positive
If the correlation coefficient between two variables X and Y is ---------, the Regression coefficient of Y on X is negative	Negative	positive	not certain	zero	Negative
---------- is independent of origin and scale.	Correlation coefficient	regression coefficients	coefficient of range	coefficient of variation	Correlation coefficient
The angle between two lines of regression is ninety degree, if ----------	r = 2	r = 0	r = 1	r = -1	r = 1
--------- is used to measure closeness of relationship between variables.	Regression	mean	Rank correlation	correlation	correlation
If r is either +1 or –1, then there will be only one -------- line in case of two variables	Correlation	regression	rank correlation	mean	regression
When b_{xy}=0.85 and b_{yx}= 0.89, then correlation coefficient r =	0.98	0.5	0.68	0.87	0.87

If b_{xy} and b_{yx} represent regression coefficients and if $b_{xy} < 1$, then b_{yx} is	less than 1	greater than one	equal to one	equal to zero	greater than one
While drawing a scatter diagram if all points appear to form a straight line getting Downward from left to right, then it is inferred that there is--------	Perfect positive correlation	simple positive correlation	Perfect negative correlation	no correlation	Perfect negative correlation
If r =1, the angle between two lines of regression is----------	Zero degree	sixty degree	ninety degree	thirty degree	ninety degree
Regression coefficient is independent of----------	Origin	scale	both origin and scale	neither origin nor scale.	Origin
There will be only one regression line in case of two variables if----------	r =0	r = +1	r = –1	r is either +1 or –1	r =0

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Department of Mathematics

Subject : Biostatistics and Research Methodology **Semester II** **L T P C**
Subject Code : 18MBP203 **Class : I M.Sc Microbiology** **4 2 0 4**

UNIT III

Tests of Significance – Tests based on Means only – both large samples and small sample tests – Chi square tests - goodness of fit Analysis of variance – one way and two way classification CRD and RBD design.

SUGGESTED READINGS

1. Jerrold H Zat (2003) Bio-statistical Analysis (4^{th} ed.) Pearson Education(P) Ltd, New Delhi.
2. Kothari C.R (2004). Research Methodology – Methods and Techniques (2^{nd} Ed.) New age International (P) Ltd, New Delhi.

Hypothesis:

A statistical hypothesis is an assumption that we make about a population parameter, which may or may not be true concerning one or more variables.

According to Prof. Morris Hamburg "A hypothesis in statistics is simply a quantitative statement about a population".

Hypothesis testing:

Hypothesis testing is to test some hypothesis about parent population from which the sample is drawn.

Example:

A coin may be tossed 200 times and we may get heads 80 times and tails 120 times, we may now be interested in testing the hypothesis that the coin is unbiased.

To take another example we may study the average weight of the 100 students of a particular college and may get the result as 110lb. We may now be interested in testing the hypothesis that the sample has been drawn from a population with average weight 115lb.

Hypotheses are two types

1. Null Hypothesis
2. Alternative hypothesis

Null hypothesis:

The hypothesis under verification is known as *null hypothesis* and is denoted by H_0 and is always set up for possible rejection under the assumption that it is true.

For example, if we want to find out whether extra coaching has benefited the students or not, we shall set up a null hypothesis that *"extra coaching has not benefited the students".* Similarly, if we want to find out whether a particular drug is effective in curing malaria we will take the null hypothesis that *"the drug is not effective in curing malaria*".

Alternative hypothesis:

The rival hypothesis or hypothesis which is likely to be accepted in the event of rejection of the null hypothesis H_0 is called alternative hypothesis and is denoted by H_1 or H_a.

For example, if a psychologist who wishes to test whether or not a certain class of people have a mean I.Q. 100, then the following null and alternative hypothesis can be established.

The null hypothesis would be

$$H_0 : \mu = 100$$

Then the alternative hypothesis could be any one of the statements.

$$H_1 : \mu \neq 100$$
$$(or)\, H_1 : \mu > 100$$
$$(or)\, H_1 : \mu < 100$$

Errors in testing of hypothesis:

After applying a test, a decision is taken about the acceptance or rejection of null hypothesis against an alternative hypothesis. The decisions may be four types.

1) The hypothesis is true but our test rejects it.(type-I error)
2) The hypothesis is false but our test accepts it. .(type-II error)
3) The hypothesis is true and our test accepts it.(correct)
4) The hypothesis is false and our test rejects it.(correct)

The first two decisions are called errors in testing of hypothesis.

i.e.1) Type-I error

2) Type-II error

1) Type-I error: The type-I error is said to be committed if the null hypothesis (H_0) is true but our test rejects it.

2) Type-II error: The type-II error is said to be committed if the null hypothesis (H_0) is false but our test accepts it.

Level of significance:

The maximum probability of committing type-I error is called level of significance and is denoted by α .

α = P (Committing Type-I error)

= P (H_0 is rejected when it is true)

This can be measured in terms of percentage i.e. 5%, 1%, 10% etc…….

Power of the test:

The probability of rejecting a false hypothesis is called power of the test and is denoted by $1-\beta$.

Power of the test =P (H_0 is rejected when it is false)

= 1- P (H_0 is accepted when it is false)

= 1- P (Committing Type-II error)

= $1-\beta$

- A test for which both α and β are small and kept at minimum level is considered desirable.
- The only way to reduce both α and β simultaneously is by increasing sample size.
- The type-II error is more dangerous than type-I error.

Critical region:

A statistic is used to test the hypothesis H_0. The test statistic follows a known distribution. In a test, the area under the probability density curve is divided into two regions i.e. the region of acceptance and the region of rejection. The region of rejection is the region in which H_0 is rejected. It indicates that if the value of test statistic lies in this region, H_0 will be rejected. This region is called critical region. The area of the critical region is equal to the level of significance α. The critical region is always on the tail of the distribution curve. It may be on both sides or on one side depending upon the alternative hypothesis.

One tailed and two tailed tests:

A test with the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta \neq \theta_0$, it is called a two tailed test. In this case the critical region is located on both the tails of the distribution.

A test with the null hypothesis $H_0 : \theta = \theta_0$ against the alternative hypothesis $H_1 : \theta > \theta_0$ (right tailed alternative) or $H_1 : \theta < \theta_0$ (left tailed alternative) is called one tailed test. In this case the critical region is located on one tail of the distribution.

$H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ ------- right tailed test

$H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$ ------- left tailed test

Sampling distribution:

Suppose we have a population of size 'N' and we are interested to draw a sample of size 'n' from the population. In different time if we draw the sample of size n, we get different samples of different observations i.e. we can get ${}^{N}c_{n}$ possible samples. If we calculate some particular statistic from each of the ${}^{N}c_{n}$ samples, the distribution of sample statistic is called sampling distribution of the statistic. For example if we consider the mean as the statistic, then the distribution of all possible means of the samples is a distribution of the sample mean and it is called sampling distribution of the mean.

Standard error:

Standard deviation of the sampling distribution of the statistic t is called standard error of t.

$$\text{i.e.} \quad \text{S.E (t)} = \sqrt{Var(t)}$$

Utility of standard error:

1. It is a useful instrument in the testing of hypothesis. If we are testing a hypothesis at 5% l.o.s and if the test statistic i.e. $|Z| = \left|\frac{t - E(t)}{S.E(t)}\right| > 1.96$ then the null hypothesis is rejected at 5% l.o.s otherwise it is accepted.
2. With the help of the S.E we can determine the limits with in which the parameter value expected to lie.
3. S.E provides an idea about the precision of the sample. If S.E increases the precision decreases and vice-versa. The reciprocal of the S.E i.e. $\frac{1}{S.E}$ is a measure of precision of a sample.
4. It is used to determine the size of the sample.

Test statistic:

The test statistic is defined as the difference between the sample statistic value and the hypothetical value, divided by the standard error of the statistic.

$$\text{i.e. test statistic } Z = \frac{t - E(t)}{S.E(t)}$$

Procedure for testing of hypothesis:

1. Set up a null hypothesis i.e. $H_0 : \theta = \theta_0$.
2. Set up a alternative hypothesis i.e. $H_1 : \theta \neq \theta_0$ or $H_1 : \theta > \theta_0$ or $H_1 : \theta < \theta_0$
3. Choose the level of significance i.e. α.
4. Select appropriate test statistic Z.
5. Select a random sample and compute the test statistic.
6. Calculate the tabulated value of Z at α % l.o.s i.e. Z_α.
7. Compare the test statistic value with the tabulated value at α % l.o.s. and make a decision whether to accept or to reject the null hypothesis.

Large sample tests:

The sample size which is greater than or equal to 30 is called as large sample and the test depending on large sample is called large sample test.

The assumption made while dealing with the problems relating to large samples are

Assumption-1: The random sampling distribution of the statistic is approximately normal.

Assumption-2: Values given by the sample are sufficiently closed to the population value and can be used on its place for calculating the standard error of the statistic.

Large sample test for single mean (or) test for significance of single mean:

For this test

The null hypothesis is $H_0 : \mu = \mu_0$

against the two sided alternative $H_1 : \mu \neq \mu_0$

where μ is population mean

μ_0 is the value of μ

Let $x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n$ be a random sample from a normal population with mean μ and variance σ^2

i.e. if $X \sim N(\mu, \sigma^2)$ then $\bar{x} \sim N\left(\mu, \sigma^2/n\right)$, Where $\bar{x}$ be the sample mean

Now the test statistic $Z = \dfrac{t - E(t)}{S.E(t)} \sim N(0,1)$

$$= \frac{\bar{x} - E(\bar{x})}{S.E(\bar{x})} \sim N(0,1)$$

$$\Rightarrow Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

Note: if the population standard deviation is unknown then we can use its estimate s, which will be calculated from the sample. $s = \sqrt{\dfrac{1}{n-1}\sum (x - \bar{x})^2}$.

Large sample test for difference between two means:

If two random samples of size n_1 and n_2 are drawn from two normal populations with means μ_1 and μ_2, variances σ_1^2 and σ_2^2 respectively

Let $\bar{x}_1$ and $\bar{x}_2$ be the sample means for the first and second populations respectively

Then $\bar{x}_1 \sim N\left(\mu_1, {\sigma_1}^2 / n_1\right)$ and $\bar{x}_2 \sim N\left(\mu_2, {\sigma_2}^2 / n_2\right)$

Therefore $\bar{x}_1 - \bar{x}_2 \sim N\left(\mu_1 - \mu_2, \frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}\right)$

For this test

The null hypothesis is $H_0 : \mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = 0$

against the two sided alternative $H_1 : \mu_1 \neq \mu_2$

Now the test statistic $Z = \frac{t - E(t)}{S.E(t)} \sim N(0,1)$

$$= \frac{(\bar{x}_1 - \bar{x}_2) - E(\bar{x}_1 - \bar{x}_2)}{S.E(\bar{x}_1 - \bar{x}_2)} \sim N(0,1)$$

$$\Rightarrow Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{S.E(\bar{x}_1 - \bar{x}_2)} \sim N(0,1)$$

$$\Rightarrow Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}} \sim N(0,1) \qquad [\text{since } \mu_1 - \mu_2 = 0 \text{ from } H_0]$$

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

Note: If ${\sigma_1}^2$ and ${\sigma_2}^2$ are unknown then we can consider ${S_1}^2$ and ${S_2}^2$ as the estimate value of ${\sigma_1}^2$ and ${\sigma_2}^2$ respectively..

Large sample test for single standard deviation (or) test for significance of standard deviation:

Let $x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n$ be a random sample of size n drawn from a normal population with mean μ and variance σ^2,

for large sample, sample standard deviation s follows a normal distribution with mean σ and variance $\sigma^2/2n$ i.e. $s \sim N\left(\sigma, \sigma^2/2n\right)$

For this test

The null hypothesis is $H_0 : \sigma = \sigma_0$

against the two sided alternative $H_1 : \sigma \neq \sigma_0$

Now the test statistic $Z = \dfrac{t - E(t)}{S.E(t)} \sim N(0,1)$

$$= \frac{s - E(s)}{S.E(s)} \sim N(0,1)$$

$$\Rightarrow Z = \frac{s - \sigma}{\sigma/\sqrt{2n}} \sim N(0,1)$$

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

Large sample test for difference between two standard deviations:

If two random samples of size n_1 and n_2 are drawn from two normal populations with means μ_1 and μ_2, variances σ_1^2 and σ_2^2 respectively

Let s_1 and s_2 be the sample standard deviations for the first and second populations respectively

Then $s_1 \sim N\left(\sigma_1, {\sigma_1}^2 / 2n_1\right)$ and $\bar{x}_2 \sim N\left(\sigma_2, {\sigma_2}^2 / 2n_2\right)$

Therefore $s_1 - s_2 \sim N\left(\sigma_1 - \sigma_2, \frac{{\sigma_1}^2}{2n_1} + \frac{{\sigma_2}^2}{2n_2}\right)$

For this test

The null hypothesis is $H_0 : \sigma_1 = \sigma_2 \Rightarrow \sigma_1 - \sigma_2 = 0$

against the two sided alternative $H_1 : \sigma_1 \neq \sigma_2$

Now the test statistic $Z = \frac{t - E(t)}{S.E(t)} \sim N(0,1)$

$$= \frac{(s_1 - s_2) - E(s_1 - s_2)}{S.E(s_1 - s_2)} \sim N(0,1)$$

$$\Rightarrow Z = \frac{(s_1 - s_2) - (\sigma_1 - \sigma_2)}{S.E(s_1 - s_2)} \sim N(0,1)$$

$$\Rightarrow Z = \frac{(s_1 - s_2)}{\sqrt{\frac{{\sigma_1}^2}{2n_1} + \frac{{\sigma_2}^2}{2n_2}}} \sim N(0,1) \qquad [\text{since } \sigma_1 - \sigma_2 = 0 \text{ from } H_0]$$

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

Large sample test for single proportion (or) test for significance of proportion:

Let x is number of success in n independent trails with constant probability p, then x follows a binomial distribution with mean np and variance npq.

In a sample of size n let x be the number of persons processing a given attribute then the sample proportion is given by $\hat{p} = \frac{x}{n}$

Then $E(\hat{p}) = E\left(\frac{x}{n}\right) = \frac{1}{n}E(x) = \frac{1}{n}np = p$

And $V(\hat{p}) = V\left(\frac{x}{n}\right) = \frac{1}{n^2}V(x) = \frac{1}{n^2}npq = \frac{pq}{n}$

$S.E(\hat{p}) = \sqrt{\frac{pq}{n}}$

For this test

The null hypothesis is $H_0 : p = p_0$

against the two sided alternative $H_1 : p \neq p_0$

Now the test statistic $Z = \frac{t - E(t)}{S.E(t)} \sim N(0,1)$

$$= \frac{\hat{p} - E(\hat{p})}{S.E(\hat{p})} \sim N(0,1)$$

$$\Rightarrow Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \sim N(0,1)$$

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

Large sample test for single proportion (or) test for significance of proportion:

let x_1 and x_2 be the number of persons processing a given attribute in a random sample of size n_1 and n_2 then the sample proportions are given by $\hat{p}_1 = \frac{x_1}{n_1}$ and $\hat{p}_2 = \frac{x_2}{n_2}$

Then $E(\hat{p}_1) = p_1$ and $E(\hat{p}_2) = p_2 \Rightarrow E(\hat{p}_1 - \hat{p}_2) = p_1 - p_2$

And $V(\hat{p}_1) = \frac{p_1 q_1}{n_1}$ and $V(\hat{p}_2) = \frac{p_2 q_2}{n_2} \Rightarrow V(\hat{p}_1 - \hat{p}_2) = \frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}$

$$S.E(\hat{p}_1) = \sqrt{\frac{p_1 q_1}{n_1}} \text{ and } S.E(\hat{p}_2) = \sqrt{\frac{p_2 q_2}{n_2}} \Rightarrow S.E(\hat{p}_1 - \hat{p}_2) = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$$

For this test

The null hypothesis is $H_0 : p_1 = p_2$

against the two sided alternative $H_1 : p_1 \neq p_2$

Now the test statistic $Z = \frac{t - E(t)}{S.E(t)} \sim N(0,1)$

$$= \frac{\hat{p}_1 - \hat{p}_2 - E(\hat{p}_1 - \hat{p}_2)}{S.E(\hat{p}_1 - \hat{p}_2)} \sim N(0,1)$$

$$\Rightarrow Z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{S.E(\hat{p}_1 - \hat{p}_2)} \sim N(0,1)$$

$$\Rightarrow Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}} \sim N(0,1)$$

$$\Rightarrow Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{pq}{n_1} + \frac{pq}{n_2}}} \sim N(0,1) \quad \text{Since } p_1 = p_2 \text{ from } H_0$$

$$\Rightarrow Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$$

When p is not known p can be calculated by $p = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$ and $q = 1 - p$

Now calculate $|Z|$

Find out the tabulated value of Z at α % l.o.s i.e. Z_α

If $|Z| > Z_\alpha$, reject the null hypothesis H_0

If $|Z| < Z_\alpha$, accept the null hypothesis H_0

- **As σ is unknown,**

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} = \left[\bar{x} - z_{\alpha/2} \frac{s}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{s}{\sqrt{n}} \right]$$

Step 2: If μ_0 falls into the above confidence intervals, then

do *not* reject H_0. Otherwise, reject H_0.

Example 1:

The average starting salary of a college graduate is $19000 according to government's report. The average salary of a random sample of 100 graduates is $18800. The standard error is 800.

(a) Is the government's report reliable as the level of significance is 0.05.
(b) Find the p-value and test the hypothesis in (a) with the level of significance $\alpha = 0.01$.
(c) The other report by some institute indicates that the average salary is $18900. Construct a 95% confidence interval and test if this report is reliable.

[solutions:]

(a)

$$H_0 : \mu = \mu_0 = 19000 \text{ vs. } H_a : \mu \neq \mu_0 = 19000,$$
$$n = 100, \bar{x} = 18800, s = 800, \alpha = 0.05$$

Then,

$$|z| = \left| \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \right| = \left| \frac{18800 - 19000}{800/\sqrt{100}} \right| = |-2.5| = 2.5 > z_{\alpha/2} = z_{0.025} = 1.96.$$

Therefore, reject H_0.

(b)

$$\text{p-value} = P(|Z| > |z|) = P(|Z| > 2.5) = 2 \cdot P(Z > 2.5) = 0.0124 > 0.01$$

Therefore, ***not*** reject H_0.

(c)

$$H_0 : \mu = \mu_0 = 18900 \text{ vs } H_a : \mu \neq \mu_0 = 18900,$$

A 95% confidence interval is

$$\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}} = 18800 \pm z_{0.025} \frac{800}{\sqrt{100}} = 18800 \pm 1.96 \cdot 80 = [18643.2, 18956.8].$$

Since $\mu_0 = 18900 \in [18643.2, 18956.8]$, Therefore, ***not*** reject H_0 **.**

Example 2:

A sample of 49 provides a sample mean of 38 and a sample standard deviation of 7. Let $\alpha = 0.05$. Please test the hypothesis

$$H_0 : u = 40 \text{ vs. } H_a : u \neq 40.$$

based on

(a) classical hypothesis test
(b) p-value
(c) confidence interval.
[solution:]

$$\bar{x} = 38,\ s = 7,\ u_0 = 40,\ n = 49,\ z = \frac{\bar{x} - u_0}{s/\sqrt{n}} = \frac{38 - 40}{7/\sqrt{49}} = -2.$$

(a)

$$|z| = 2 > 1.96 = z_{0.025}$$

we reject H_0.

(b)

$$p - value = P(|Z| > |z|) = P(|Z| > 2) = 2 * (1 - 0.9772) = 0.0456 < 0.05 = \alpha$$

we reject H_0.

(c)

$100\times(1-\alpha)\%=95\%$ confidence interval is

$$\bar{x}\pm z_{\alpha/2}\frac{s}{\sqrt{n}}=38\pm z_{0.025}\frac{7}{\sqrt{49}}=38\pm 1.96=\left[36.04,39.96\right].$$

Since $40\notin\left[36.04,39.96\right]$, we reject H_0.

Hypothesis Testing for the Mean (Small Samples)

For samples of size less than 30 and when σ is unknown, if the population has a normal, or nearly normal, distribution, the *t*-distribution is used to test for the mean μ.

Using the t-Test for a Mean μ when the sample is small		
Procedure	**Equations**	**Example 4**
State the claim mathematically and verbally. Identify the null and alternative hypotheses	State H_0 and H_a	$H_0:\mu\geq 16500$ $H_a:\mu<16500$ $n=14,\bar{x}=15700,s=1250$
Specify the level of significance	Specify α	$\alpha=0.05$
Identify the degrees of freedom and sketch the sampling distribution	$d.f=n-1$	$d.f.=13$
Determine any critical values. If test is left tailed, use One tail, α column with a negative sign. If test is right tailed, use One tail, α column with a positive sign. If test is two tailed, use Two tails, α column with a negative and positive sign.	Table 5 (*t*-distribution) in appendix B	The test is left-tailed. Since test is left tailed and $d.f=13$, the critical value is $t_0=-1.771$
Determine the rejection regions.	The rejection region is $t<t_0$	The rejection region is $t<-1.771$

Find the standardized test statistic	$t = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \approx \frac{\bar{x} - \mu}{s/\sqrt{n}}$	$t = \frac{15700 - 16500}{1250/\sqrt{14}} \approx -2.39$
Make a decision to reject or fail to reject the null hypothesis	If t is in the rejection region, reject H_0, Otherwise do not reject H_0	Since $-2.39 < -1.771$, reject H_0
Interpret the decision in the context of the original claim.		Reject claim that mean is at least 16500.

Chi-Square Tests and the F-Distribution

Goodness of Fit

DEFINITION A **chi-square goodness-of-fit test is** used to test whether a frequency distribution fits an expected distribution.

The test is used in a multinomial experiment to determine whether the number of results in each category fits the null hypothesis:

H_0 : The distribution fits the proposed proportions

H_1 : The distribution differs from the claimed distribution.

To calculate the test statistic for the chi-square goodness-of-fit test, you can use observed frequencies and expected frequencies.

DEFINITION The **observed frequency O** of a category is the frequency for the category observed in the sample data.

The **expected frequency E** of a category is the calculated frequency for the category. Expected frequencies are obtained assuming the specified (or hypothesized) distribution. The expected frequency for the *i*th category is

$E_i = np_i$

where *n* is the number of trials (the sample size) and p_i is the assumed probability of the *i*th category.

The Chi-square Goodness of Fit Test: The sampling distribution for the goodness-of-fit test is a chi-square distribution with $k-1$ degrees of freedom where k is the number of categories. The test statistic is

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where *O* represents the observed frequency of each category and *E* represents the expected frequency of each category. To use the chi-square goodness of fit test, *the following must be true* .

1. The observed frequencies must be obtained using a random sample.
2. The expected frequencies must be ≥ 5.

Performing the Chi-Square Goodness-of-Fit Test (p 496)		
Procedure	Equations	Example **(p 497)**
Identify the claim. State the null and alternative hypothesis.	State H_0 and H_1	H_0: Classical 4% Country 36% Gospel 11% Oldies 2% Pop 18% Rock 29%
Specify the significance level	Specify α	$\alpha = 0.01$
Determine the degrees of freedom	d.f. = #categories - 1	$d.f. = 6-1 = 5$
Find the critical value	χ^2_α: Obtain from Table 6 Appendix B	$\varphi^2_{0.01}(d.f = 5) = 15.086$
Identify the rejection region	$\chi^2 \geq \chi^2_\alpha$	$\chi^2 \geq 15.086$
Calculate the test statistic	$\chi^2 = \sum \frac{(O-E)^2}{E}$	Survey results, n = 500 Classical O= 8 E = .04*500 = 20 Country O = 210 E = .36*500 = 180 Gospel O = 7 E = .11*500 = 55 Oldies O = 10 E = .02*500 = 10 Pop O = 75 E = .18*500 = 90 Rock O= 125 E = .29*500 = 145 Substituting $\chi^2 = 22.713$
Make the decision to reject or fail to reject the null hypothesis	Reject if χ^2 is in the rejection region Equivalently, we reject if the P-value (the	Since 22.713 > 15.086 we reject the null hypothesis Equivalently $P(X \geq 22.713) < 0.01$so reject

	probability of getting as extreme a value or more extreme) is $\leq \alpha$	the null hypothesis. (Note Table 6 of Appendix B doesn't have a value less than 0.005.)
Interpret the decision in the context of the original claim		Music preferences differ from the radio station's claim.

Using Minitab to perform the Chi-Square Goodness-of-Fit Test (Manual p 237)

The data from the example above (Example 2 p 497) will be used.

Enter Three columns: Music Type: Classical, etc, Observed: 8 etc, Distribution 0.04, etc. (Note the names of the columns 'Music Types', 'Observed' and 'Distribution' are entered in the gray row at the top.)

Select **Calc->Calculator**, **Store the results in** C4, and calculate the **Expression** C3*500, click **OK**, Name C4 'Expected' since it now contains the expected frequencies

Music Type	Observed	Distribution	Expected
Classical	8	0.04	20
Country	210	0.36	180
Gospel	72	0.11	55
Oldies	10	0.02	10
Pop	75	0.18	90
Rock	125	0.29	145

Next calculate the chi-square statistic, $(O-E)^2/E$ as follows: Click **Calc->Calculator**. **Store the results in** C5 and calculate the **Expression** (C2-C4)**2/C4. Click on **OK** and C5 should contain the calculated values.

7.2000
5.0000
41.8909
0.0000
2.5000

2.7586

Next add up the values in C5 and the sum is the test statistic as follows: Click on **Calc->Column Statistics**. Select **Sum** and enter C5 for the **Input Variable**. Click OK. The chi-square statistic is displayed in the session window as follows:

Sum of C5

```
Sum of C5 = 22.7132
```

Next calculate the P-value: Click on **Calc->Probability Distributions->Chi-square**. Select **Cumulative Probability** and enter 5 **Degrees of Freedom** Enter the value of the test statistic `22.7132` for the **Input Constant**. Click **OK**.

The following is displayed on the Session Window.

Cumulative Distribution Function

```
Chi-Square with 5 DF

      x  P( X <= x )
22.7132     0.999617
```

P(X≤22.7132) = 0.999617 So the P-value = 1 – 0.999617 = 0.000383. This is less that α = 0.01 so we reject the null hypothesis.

Instead of calculating the P-value, we could have found the critical value from the Chi-Square table (Table 6 Appendix B) for 5 degrees of freedom as we did above. The value is 15.086, and since our test statistic is 22.7132, we reject the null hypothesis.

Chi-Square with M&M's

H_0: Brown: 13%, Yellow: 14%, Red: 13%, Orange: 20%, Green 16%, Blue 24%
Significance level: α = 0.05
Degrees of freedom: number of categories – 1 = 5
Critical Value: $\chi^2_{0.05}(d.f.=5)=11.071$
Rejection Region: $\chi^2 \geq 11.071$
Test Statistic: $\chi^2 = \sum \frac{(O-E)^2}{E}$, where *O* is the actual number of M&M's of each color in the bag and *E* is the proportions specified under H_0 times the total number.

Reject H_0 if the test statistic is greater than the critical value (1.145)

Section 10.2 Independence

This section describes the chi-square test for independence which tests whether two random variables are independent of each other.

DEFINTION An ***r x c* contingency table** shows the observed frequencies for the two variables. The observed frequencies are arranged in r rows and c columns. The intersection of a row and a column is called a **cell.**

The following is a contingency table for two variables A and B where f_{ij} is the frequency that A equals A_i and B equals B_j.

	A_1	A_2	A_3	A_4	A
B_1	f_{11}	f_{12}	f_{13}	f_{14}	$f_{1.}$
B_2	f_{21}	f_{22}	f_{23}	f_{24}	$f_{2.}$
B_3	f_{31}	f_{32}	f_{33}	f_{34}	$f_{3.}$
B	$f_{.1}$	$f_{.2}$	$f_{.3}$	$f_{.4}$	f

If A and B are independent, we'd expect

$$f_{ij} = prob(A = A_i) * prob(B = B_j) * f = \left(\frac{f_{i.}}{f}\right)\left(\frac{f_{.j}}{f}\right) f = \frac{(f_{i.})(f_{.j})}{f}$$

$$\frac{(sum of\ row\ i) * (sum of\ column\ j)}{sample\ size}$$ (

Example 1 Determining the expected frequencies of CEO's ages as a function of company size under the assumption that age is independent of company size.

	<= 39	40 - 49	50 - 59	60 - 69	>= 70	Total
Small/midsize	42	69	108	60	21	300
Large	5	18	85	120	22	250
Total	47	87	193	180	43	550

	<= 39	40 - 49	50 - 59	60 - 69	>= 70	Total
Small/midsize	$\frac{300*47}{550}$ ≈ 25.64	$\frac{300*87}{550}$ ≈ 47.45	$\frac{300*193}{550}$ ≈ 105.27	$\frac{300*180}{550}$ ≈ 98.18	$\frac{300*43}{550}$ ≈ 23.45	300
Large	$\frac{250*47}{550}$ ≈ 21.36	$\frac{250*87}{550}$ ≈ 39.55	$\frac{250*193}{550}$ ≈ 87.73	$\frac{250*180}{550}$ ≈ 81.82	$\frac{250*43}{550}$ ≈ 19.55	250
Total	47	87	193	180	43	550

After finding the expected frequencies under the assumption that the variables are independent, you can test whether they are independent using the chi-square independence test.

DEFINITION A **chi-square independence test** is used to test the independence of two random variables. Using a chi-square test, you can determine whether the occurrence of one variable affects the probability of occurrence of the other variable.

To use the test,

1. The observed frequencies must be obtained from a random sample
2. Each expected frequency must be ≥ 5

The sampling distribution for the test is a chi-square distribution with

$(r-1)(c-1)$

degrees of freedom, where r and c are the number of rows and columns, respectively, of the contingency table. The test statistic for the chi-square independence test is

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where O represents the observed frequencies and E represents the expected frequencies.

To begin the test we state the null hypothesis that the variables are independent and the alternative hypothesis that they are dependent.

Performing a Chi-Square Test for Independence (p 507)		
Procedure	Equations	Example2 **(p 507)**
Identify the claim. State the null and alternative	State H_0 and H_1	H_0: CEO's ages are independent of company

hypotheses.		size H_1: CEO's ages are dependent on company size.
Specify the level of significance	Specify α	$\alpha = 0.01$
Determine the degrees of freedom	$d.f. = (r-1)(c-1)$	$d.f. = (2-1)(5-1) = 4$
Find the critical value.	χ^2_α :Obtain from Table 6, Appendix B	$\chi^2_\alpha \geq 13.277$
Identify the rejection region	$\chi^2 \geq \chi^2_\alpha$	$\chi^2 \geq 13.277$
Calculate the test statistic	$\chi^2 = \sum \frac{(O-E)^2}{E}$	$\sum \frac{(O-E)^2}{E} \approx 77.9$ Note that O is in the table of actual CEO's ages above, and E is in the table of Expected CEO's ages (if independent of size) above
Make a decision to reject or fail to reject the null hypothesis	Reject if χ^2 is in the rejection region. Equivalently, we reject if the P-value (the probability of getting as extreme a value or more extreme) is $\leq \alpha$	Since 77.9 > 13.277 we reject the null hypothesis Equivalently $P(X \geq 77.0) < \alpha$ so reject the null hypothesis. (Note Table 6 of Appendix B doesn't have a value less than 0.005.)
Interpret the decision in the context of the original claim		CEO's ages and company size are dependent.

The test statistic (77.887) is greater than the critical value obtained from Table 6, Appendix B (13.277) so the null hypothesis is rejected. (Alternatively the P-Value (0.000) is less than the level of significance, α (0.01) so the null hypothesis is rejected.)

3. An urban geographer randomly samples 20 new residents of a neighborhood to determine their ratings of local bus service. The scale used is as follows: 0–very dissatisfied, 1–dissatisfied, 2– neutral, 3–satisfied, 4–very satisfied. The 20 responses are 0,4,3, 2,2,1,1,2,1,0,01,2,1,3,4,2,0,4,1. Use the sign test to see whether the population median is 2.

   Solution:

There are 5 observations above the hypothesized median. Because the sample size is larger than 10, we test using the sample proportion $p = 5/20 = 0.25$. Using the PROB-VALUE method the steps in this test are:

1) H_0: $\pi = 0.5$ and H_A: $\pi \neq 0.5$
2) We will use the *Z*-distribution
3) We will use the 5%-level, thus $\alpha = 0.05$
4) The test statistic is $z = (0.25 - 0.5)/\sqrt{0.25/20} = -2.24$
5) Table A-4 shows that $P(|Z| > 2.24) \approx 0.025$.
6) Because PROB-VALUE < α, we reject H_0. We conclude π is different than 0.5, and thus the median is different than 2.

4. A course in statistical methods was team-taught by two instructors, Professor Jovita Fontanez and Professor Clarence Old. Professor Fontanzez used many active learning techniques, whereas Old employed traditional methods. As part of the course evaluation, students were asked to indicate their instructor preference. There was reason to think students would prefer Fontanez, and the sample obtained was consistent with that idea: of the 12 students surveyed, 8 preferred Professor Fontanez and 2 preferred Professor Old. The remaining students were unable to express a preference. Test the hypothesis that the students prefer Fontanez. (*Hint:* Use the sign test.)

Solution:

Although the sample is large enough for a normal approximation, we will use the binomial distribution to illustrate its application. Of the 12 observations, 8 preferred Prof. Fontanez, thus we need the probability of observing 8 or more successes in 12 trials of a Bernoulli process with the probability of success equal to 0.5. From Table A-1, we get

$$P(X \geq 8) = P(X = 8) + P(X = 9) + P(X = 10) + P(X = 11) + P(X = 12)$$

$$P(X \geq 8) = 0.1208 + 0.0537 + 0.0161 + 0.0029 + 0.002 = 0.1937$$

Adopting the 5% uncertainty level, we see that PROB-VALUE > α. Thus we fail to reject H_0. We cannot conclude students prefer Fontanez.

5. Use the data in Table 10-8 to perform two Mann–Whitney tests: (a) compare uncontrolled intersections and intersections with yield signs, and (b) compare uncontrolled intersections and intersections with stop signs.

   Solution:

   (a) The rank sums are 119.5 and 90.5 for the yield-signed and uncontrolled intersections respectively. Given the small sample size, we use an exact test rather than the normal approximation. The associated PROB-VALUE is 0.272. Adopting a 5% level of uncertainty, we fail to reject the hypothesis of no difference. We cannot conclude the samples were drawn from different populations.

   (b) The rank sums are 130.5 and 59.5 for the stop-signed and uncontrolled intersections respectively. Given the small sample size, we use an exact test rather than the normal approximation. The associated PROB-VALUE is 0.013. Adopting a 5% level of uncertainty, we reject the hypothesis of no difference. We conclude the samples were drawn from different populations.

6. Solid-waste generation rates measured in metric tons per household per year are collected randomly in selected areas of a township. The areas are classified as high-density, low density, or sparsely settled. It is thought that generation rates probably differ because of differences in waste collection and opportunities for on-site storage. Do the following data support this hypothesis?

High Density	Low Density	Sparsely Settled
1.84	2.04	1.07
3.06	2.28	2.31
3.62	4.01	0.91
4.91	1.86	3.28
3.49	1.42	1.31

   Solution:

   We will use the multi-sample Kruskal-Wallis test with an uncertainly level $\alpha = 0.1$. The null hypothesis is that all samples have been drawn from the same population. The rank sums are 55, 39 and 26 for the high density, low density, and sparsely settled samples respectively. The Kruskal-Wallis statistic is

$$H = \frac{12}{15(15+1)}\left(\frac{55^2}{5}+\frac{39^2}{5}+\frac{26^2}{5}\right) - 3(15+1) = 4.22$$

Using the $\chi 2$ distribution with $3-1=2$ degrees of freedom, the associated PROB-VALUE is 0.121. We fail to reject the null hypothesis. The sample does not support the hypothesis of differing waste generation rates.

7. The distances travelled to work by a random sample of 12 people to their places of work in 1996 and again in 2006 are shown in the following table.

	Distance (km)			Distance (km)	
Person	1996	2006	Person	1996	2006
1	8.6	8.8	7	7.7	6.5
2	7.7	7.1	8	9.1	9
3	7.7	7.6	9	8	7.1
4	6.8	6.4	10	8.1	8.8
5	9.6	9.1	11	8.7	7.2
6	7.2	7.2	12	7.3	6.4

Has the length of the journey to work changed over the decade?

Solution:

The sample can be considered as twelve paired observations. By taking differences between paired values, we get measures of the change for each individual. If the median change for the population is zero, we expect a sample to have a median difference near zero. Thus we will do a sign test for the median difference with a hypothesized value of zero. In other words, the hypotheses are $H_0 : \eta = 0$ and $H_A : \eta \neq 0$. We denote samples values whose distance decreased with a minus sign. Sample values with a positive difference get a plus sign. The sample becomes

S = {-,+,+,+,+,0,+,-,+,-,+,+}

Ignoring the tie, this is a sample of size 11 with 8 values above the hypothesized median. We are using Format (C) of Table 10-1, thus the PROB-VALUE is

$2P(X \geq 8)$ where X is a binomial variable with $\pi = 0.5$. From the equation for the binomial, the PROB-VALUE is found to be 0.113. At the $\alpha = 10\%$ level, we fail the reject the null hypothesis. We cannot conclude there has been a change in distance.

8. One hundred randomly sampled residents of a city subject to periodic flooding are classified according to whether they are on the floodplain of the major river bisecting the city or off the floodplain. These households are then surveyed to determine whether they currently have flood insurance of any kind. The survey results are as follows:

	On the Floodplain	Off the Floodplain
Insured	50	10
No Insurance	15	25

Test a relevant hypothesis.

Solution:

We will do a χ^2 test for a relationship between insurance and house location. The null hypothesis is no relationship (independence). Augmenting the data with expected frequencies, we have:

	On the Floodplain	Off the Floodplain
Insured	50 (39)	10 (21)
No Insurance	15 (26)	25 (14)

The corresponding χ^2 value is 22.16. Table A-8 shows that with 1 degree of freedom, $P(\chi^2 > 20)$ is zero to 3 decimal places. Thus for any reasonable level of uncertainty (any $\alpha < 0.0005$), we can reject the null hypothesis.

9. The occurrence of sunshine over a 30-day period was calculated as the percentage of time the sun was visible (i.e., not obscured by clouds). The daily percentages were:

Day	Percentage of sunshine	Day	Percentage of sunshine	Day	Percentage of sunshine
1	75	11	21	21	77
2	95	12	96	22	100
3	89	13	90	23	90
4	80	14	10	24	98
5	7	15	100	25	60
6	84	16	90	26	90
7	90	17	6	27	100
8	18	18	0	28	90
9	90	19	22	29	58
10	100	20	44	30	0

If we define a sunny day as one with over 50% sunshine, determine whether the pattern of occurrence of sunny days is random.

Solution:

For this we can use the number-of-runs test. Rather than calculate runs across two samples, here we will simply note if a day has 50% or more sunshine. The sample becomes

S={+,+,+,+,-,+,+,-,+,+,-,+,+,-,+,+,-,-,-,-,+,+,+,+,+,+,+,+,-}

We see that the sample consists of 12 runs. There are $n_x = 21$ sunny days, and $n_y = 9$ cloudy days. Because $n_x < 20$, we cannot use the normal approximation given in Table 10-5. Instead the probability is computed using combinatorial rules, and is approximately 0.4. This is far too large for rejection of the randomness hypothesis. We cannot conclude the pattern is non-random.

10. Test the normality of the DO data (a) using the Kolmogorov–Smirnov test with the ungrouped data of Table 2-4 and (b) using the χ^2 test with $k = 6$ classes of Table 2-6.

Solution:

(a) We will take the mean and standard deviation as known rather than estimated from the sample. Doing so results in calculated PROB-VALUES that are smaller than the true values (i.e., we are more likely to reject the null hypothesis). For the

DO data the mean and standard deviation are 5.58 and 0.39 respectively. We sort the data, and then find the differences between the observed and expected cumulative distributions. The table below shows the results for a few of the 50 observations:

x_i	$S(x_i)$	$F(x_i)$	$\lvert S(x_i)-F(x_i)\rvert$
4.2	0.020	0.015	0.005
4.3	0.040	0.023	0.017
4.4	0.060	0.032	0.028
…	…	…	…
5.9	0.780	0.692	0.088
…	…	...	…
6.7	0.960	0.960	0.000
6.8	0.980	0.972	0.008
6.9	1.000	0.981	0.019

The maximum difference is 0.088. Table A-9 shows that with 50 degrees of freedom, the corresponding PROB-VALUE is about 0.6. We obviously cannot reject the hypothesis of normality.

(b) Here we will take the mean and standard deviation as unknown, to be estimated from the sample. In other words, we estimate two parameters from the sample. In building the χ^2 table, we combine the first two and the last two categories in Table 2-6 to ensure at least 2 expected frequencies per cell. This reduces the number of categories 4, as seen in the table below:

Group	Minimum	Maximum	O_j	E_j	$(O_j-E_j)^2/E_j$
1	4.000	4.990	9	3.3	10.13
2	5.000	5.490	10	17.0	2.89
3	5.500	5.990	20	21.7	0.14
4	6.000	6.990	11	7.0	2.24

The observed Chi-square value is 15.4. With $k - p - 1 = 4 - 2 - 1 = 1$ degrees of freedom, Table A-8 shows that the PROB-VALUE is less than 0.0005. We therefore reject the null hypothesis.

Note that with only 4 classes, we can obtain only a rough idea of the distribution of DO. The 4 classes given in Table 2-6 do not yield a distribution that is at all similar to the normal distribution. In practice one would need many classes (and observations) for the χ^2 test to be reliable.

Possible Questions

PART-B

1. How Z-test is used for testing significance of proportions?
   In a referendum submitted to the student body and 850 men and 550 women voted. Out of these, 530 of men and 310 of women voted 'yes'. Does this indicate a significant difference in opinion on matter between men and women students? (Use $\alpha = 5\%$ and $Z_{(0.05)} = 1.96$)
2. Test Median class size for Math is larger than the median class size for English for the following data using Mann – Whitney U test.

Class size (Math, M)	23	45	34	78	34	66	62	95	81
Class size (English, E)	30	47	18	34	44	61	54	28	40

3. According to the IQ level and the economic conditions of their homes 1000 students at a college were graded. Use χ^2 test to find out whether there is any association between economic condition at home and IQ.

Economic Conditions	IQ		Total
	High	Low	
Rich	**460**	**140**	600
Poor	**240**	**160**	400
Total	700	300	1000

(Note: The level of significance is 0.05 and table value is 3.84).

4. Test Median class size for Math is larger than the median class size for English for the following data using Mann – Whitney U test.

Class size (Math, M)	23	45	34	78	34	66	62	95	81
Class size (English, E)	30	47	18	34	44	61	54	28	40

5. Mr. Gowtham, Personal Manager is concerned about absenteeism. He decides to sample the records to determine if absenteeism is distributed evenly throughout the six-day work-week. The null hypothesis to be tested is: absenteeism is distributed evenly throughout the week. The sample results are as follows:

Day	**Number of Absentees**
Monday	12
Tuesday	9
Wednesday	11
Thursday	10
Friday	9
Saturday	9

   i. Using χ^2 test of significance, compute χ^2 value.
   ii. Is the null hypothesis rejected?
   iii. Specifically, what does this indicate to the Personal Manager?

   (Note: The level of significance is 0.01 and table value is 15.086).

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
Completely randomized design is similar to ----------	three way	one way	two way	t test	one way
Randomized block design is similar to ----------	two way	three way	one way	many	two way
ANOVA is the technique of analysis of ----------	standard deviation	variance	mean	range	variance
Under one way classification , the influence of only ----- attribute or factor is considered	two	three	one	many	one
Under two way classification , the influence of only ----- attribute or factor is considered	four	two	three	one	two
The word ---------- is used to indicate various statistical measures like mean, standard deviation, correlation etc, in the universe.	Statistic	parameter	hypothesis	none of these	parameter
The term STATISTIC refers to the statistical measures relating to the ----------.	Population	hypothesis	sample	universe	sample
A hypothesis may be classified as ----------.	Simple	Composite	null	all the above	all the above
Level of significance is the probability of ----------.	Type I error	Type II error	Not committing error	any of the above	any of the above
Degrees of freedom are related to ----------.	No. of observations in a set	hypothesis under test	No. of independent observations in a set	No. of dependent observations in a set	No. of independent observations in a set
A critical function provides the basis for ----------.	Accepting H_0	rejecting H_0	no decision about H_0	all the above	all the above

Student's t-test is applicable in case of ----------.	Small samples	for sample of size between 5 and 30	Large samples	none of the above	Small samples
Student's t-test is applicable only when ----------.	The variate values are independent	the variable is distributed normally	The sample is not large	all the above	all the above
If the calculated value is less than the table value then we accept the ---------- hypothesis.	Alternative	null	both	sample	null
Small sample test is also known as ----------.	Exact test	t – test	normal test	F-test	t – test
The formula for c^2 is ----------	å(O–E) ^{2}/E	å(E+O) ^{2}/E	å(O-E) /E	å(O-E) ^{2}/O	å(O–E) ^{2}/E
If a statistic 't' follows student's t distribution with n degrees of freedom then t^2 follows ------	c^2distribution with (n-1) degrees of freedom	c^2distribution with n degrees of freedom	c^2distribution with n^2 degrees of freedom	c^2distribution with (n+1) degrees of freedom	c^2distribution with (n-1) degrees of freedom
The distribution used to test goodness of fit is----------	F distribution	c^2distribution	t distribution	Z distribution	c^2distribution
Degree of freedom for statistic chi-square incase of contingency table of order 2x2 is----------	3	4	2	1	1
Larger group from which the sample is drawn is called ----------.	Sample	sampling	universe	parameter	universe
Any hypothesis concerning a population is called a ----------.	Sample	population	statistical measure	statistical hypothesis	statistical hypothesis
Rejecting Ho when it is true leads ----------.	Type I error	Type II error	correct decision	either (a) or (b)	Type I error
Accept Ho when it is true leads ----------.	Type I error	Type II error	correct decision	either (a) or (b)	correct decision
Type II error occurs only if ----------.	Reject Ho when it is true	Accept Ho when it is false	Accept Ho when it is true	reject Ho when it is false	Accept Ho when it is false

The correct decision is ----------.	Reject Ho when it is true	Accept Ho when it is false	Reject Ho when it is false	none of these	Reject Ho when it is false
The maximum probability of committing type I error, which we specified in a test is known as ----------.	Null hypothesis	alternative hypothesis	DOF	level of significance	level of significance
If the computed value is less than the critical value, then ----------.	Null hypothesis is accepted	Null hypothesis is rejected	Alternative hypothesis is accepted	population	Null hypothesis is accepted
If the computed value is greater than the critical value, then ----------.	Null hypothesis is accepted	Null hypothesis is rejected	Alternative hypothesis is accepted	small sample	Null hypothesis is rejected
In sampling distribution the standard error is ----------.	np	pq	npq	sqrt(npq)	sqrt(npq)
If the sample size is greater than 30, then the sample is called ----------.	Large sample	small sample	population	Null hypothesis	Large sample
If the sample size is less than 30, then the sample is called ----------.	Large sample	small sample	population	alternative hypothesis	small sample
Z – test is applicable only when the sample size is ----------.	zero	one	small	large	large
The degrees of freedom for two samples in t – test is ---------.	$n_1 + n_2 + 1$	$n_1 + n_2 – 2$	$n_1 + n_2 + 2$	$n_1 + n_2 – 1$	$n_1 + n_2 – 2$
An assumption of t – test is population of the sample is ----------.	Binomial	Poisson	normal	exponential	normal
The degrees of freedom of chi – square test is ----------.	(r – 1)(c – 1)	(r + 1)(c + 1)	(r + 1)(c – 1)	(r – 1)(c + 1)	(r – 1)(c – 1)
In chi – square test, if the values of expected frequency are less than 5, then they are combined together with the neighbouring frequencies. This is known as ----------.	Goodness of fit	DOF	LOS	pooling	pooling

The expected frequency of chi – square test can be calculated as ----------.	(RT + CT) / GT	(RT – CT) / GT	(RT * CT) / GT	(RT*CT)	(RT * CT) / GT
In F – test, the variance of population from which samples are drawn are ----------.	equal	not equal	small	large	equal
If the data is given in the form of a series of variables, then the DOF is ---------.	n	n-1	n+1	(r – 1)(c – 1)	n-1
The characteristic of the chi–square test is ----------.	DOF	LOS	ANOVA	independence of attributes	independence of attributes
If $S_1^2 > S_2^2$, then the F – statistic is ----------.	S_1 / S_2	S_2 / S_1	S_1^2 / S_2^2	S_1^3 / S_2^3	S_1^2 / S_2^2
The value of Z test at 5% level of significance is ----------.	3.96	2.96	1.96	0.96	1.96
In -------, the variance of population from which samples are drawn are equal	t-test	Chi-Square test	Z-test	F-test	F-test
F – statistics is ----------.	Variance between the samples / variance within the samples	Variance within the samples / variance between the samples	Variance between the rows / variance between the columns	Variance within the rows / variance within the columns	Variance between the samples / variance within the samples
Analysis of variance utilizes:	t-test	Chi-Square test	Z-test	F-test	F-test
F – test whish is also known as ------	Chi-Square test	Z-test	varience ratio test	t-test	varience ratio test
The technique of analysis of variance refered to as ------	ANOVA	F – test	Z – test	Chi- square test	ANOVA
The two variations, variation within the samples and variations between the samples are tested for their significance by -------	Chi- square test	F – test	t-test	Z – test	F – test

Under ------- classification , the influence of only one attribute or factor is considered.	two way	three way	one way	many	one way
Under --------- classification , the influence of two attribute or factors is considered	two way	three way	one way	many	two way

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
Study to portray accurately characteristics of a particular individual, situation or a group is called ---------- research	Exploratory	Diagnostic	Descriptive	Hypothesis testing	Descriptive
Critical evaluation made by the researcher with the facts and information already available is called ---------- research.	Analytical	Exploratory	Diagnostic	Hypothesis testing	Analytical
Research to find reason, why people think or do certain things is an example of ----------	Quantitative Research	Applied Research	Qualitative research	Fundamental research	Qualitative research
Which one is considered a major component of the research study	Interpretation	research report	finding	draft	research report
Research task remains incomplete till the _____ has been presented.	Report	objective	finding	objective and finding	Report
What is the last step in a research study	Writing report	writing finding	limitations	research report	Writing report
Which is the final step in report writing……..	Writing report	writing finding	writing drafts	writing limitations	writing drafts
What is usually appended to the research work	Editing	bibliography	coding	research report	bibliography
The _____ is one which gives emphasis on simplicity and attractiveness	popular report	research report	article report	writing limitations	popular report
_____ should slow originality and should necessarily be on attempt to solve some intellectual problem	Interpretation	research report	finding	draft	research report
The researcher must remain caution about the _____ that can possibly arise in the process of interpreting results	Analysis	conclusions	findings	error	error
Which one should be considered while interpreting a given data	Validity	reliability	both validity and reliability	technical jargon	reliability

_____ is asking questions face to face	Indirect method	mailed questionnaire	through post	personal interview	personal interview
Journals, books, magazines etc are useful sources of collecting----------	Primary data	secondary data	both primary and secondary data	objective	secondary data
The collected raw data to detect errors and are called,	Editing	coding	classification	all the above	Editing
The formal, systematic and intensive process of carrying on a scientific method of analysis is _____	Research Design	research	interpretation	research analysis	research
------ Refers to the process of assigning numerals or symbols to answers of response	Coding	editing	classification	all the above	Coding
The research study, which is based on describing the characteristic of a particular individual or group	Experience survey	Descriptive	Diagnostic	Exploratory	Descriptive
Research is a_____	Finding	assumption	statement	all the above	all the above
The research, which has the purpose of improving a product or a process testing theoretical concepts in actual problem situations is-------research.	Statistical	Applied	Domestic	Biological	Applied
The chart of research process indicates that the process consists of a number of ---.	Closely related activities	unrelated activities	Closely unrelated activities	moderately related activities	Closely related activities

The objective of a good design is ----------.	Maximize the bias andmaximize the reliability of data	Minimize the bias and minimize the reliability of data	Minimize the bias and maximize the reliability of data	Maximize the bias and maximize the reliability of data	Maximize the bias and maximize the reliability of data
A ---------- is used whenever a full written report of the study is required.	Popular report	Technical report	article	monograph	Technical report
The ---------- is one which gives emphasis on simplicity and attractiveness.	Popular report	Technical report	article	monograph	Popular report
Study to portray accurately characteristics of a particular individual, situation or a group is called ---------- research	Exploratory	Diagnostic	Descriptive	Hypothesis testing	Descriptive
Critical evaluation made by the researcher with the facts and information already available is called ---------- research.	Analytical	Exploratory	Diagnostic	Hypothesis testing	Analytical
Research to find reason, why people think or do certain things is an example of ----------	Quantitative Research	Applied Research	Qualitative research	Fundamental research	Qualitative research
Which one is considered a major component of the research study	Interpretation	research report	finding	draft	research report
Research task remains incomplete till the _____ has been presented.	Report	objective	finding	objective and finding	Report
What is the last step in a research study	Writing report	writing finding	limitations	research report	Writing report
Which is the final step in report writing……..	Writing report	writing finding	writing drafts	writing limitations	writing drafts
What is usually appended to the research work	Editing	bibliography	coding	research report	bibliography
The _____ is one which gives emphasis on simplicity and attractiveness	popular report	research report	article report	writing limitations	popular report

_____ should slow originality and should necessarily be on attempt to solve some intellectual problem	Interpretation	research report	finding	draft	research report
The researcher must remain caution about the _____ that can possibly arise in the process of interpreting results	Analysis	conclusions	findings	error	error
Which one should be considered while interpreting a given data	Validity	reliability	both validity and reliability	technical jargon	reliability
_____ is asking questions face to face	Indirect method	mailed questionnaire	through post	personal interview	personal interview
Journals, books, magazines etc are useful sources of collecting----------	Primary data	secondary data	both primary and secondary data	objective	secondary data
The collected raw data to detect errors and are called,	Editing	coding	classification	all the above	Editing
The formal, systematic and intensive process of carrying on a scientific method of analysis is _____	Research Design	research	interpretation	research analysis	research
------ Refers to the process of assigning numerals or symbols to answers of response	Coding	editing	classification	all the above	Coding
The research study, which is based on describing the characteristic of a particular individual or group	Experience survey	Descriptive	Diagnostic	Exploratory	Descriptive
Research is a_____	Finding	assumption	statement	all the above	all the above
The research, which has the purpose of improving a product or a process testing theoretical concepts in actual problem situations is-------research.	Statistical	Applied	Domestic	Biological	Applied

The chart of research process indicates that the process consists of a number of ---.	Closely related activities	unrelated activities	Closely unrelated activities	moderately related activities	Closely related activities
The objective of a good design is ----------.	Maximize the bias andmaximize the reliability of data	Minimize the bias and minimize the reliability of data	Minimize the bias and maximize the reliability of data	Maximize the bias and maximize the reliability of data	Maximize the bias and maximize the reliability of data
A ---------- is used whenever a full written report of the study is required.	Popular report	Technical report	article	monograph	Technical report
The ---------- is one which gives emphasis on simplicity and attractiveness.	Popular report	Technical report	article	monograph	Popular report
The square of the S.D is …….	Variance	Coefficient of variation	Square of variance	Square of coefficient of variation	Variance
Analysis of variance is a statistical method of comparing the _____ of several populations.	Standard deviations	Means	Variances	Proportions	Means
The analysis of variance is a statistical test that is used to compare how many group means?	Three	More than three	Three or more	Two or more	Two or more
Analysis of variance utilizes:	F-test	Chi-Square test	Z-test	t-test	F-test
What is two-way ANOVA?	An ANOVA with two variables and one factor	An ANOVA with one variable and two factors	An ANOVA with one variable and three factors	An ANOVA with both categorical and scale variables	An ANOVA with one variable and two factors
Which of the following is the correct F ratio in the one-way ANOVA?	MSA/MSE	MSBL/MSE	MST/MSE	MSE/MST	MST/MSE
For validity of F-test in Anova, parent population should be…….	Binomial	Poisson	Normal	Exponential	Normal

_____ sum of squares measures the variability of the observed values around their respective tabulated values	Treatment	Error	Interaction	Total	Error
The _____ sum of squares measures the variability of the sample treatment means around the overall mean.	Total	Treatment	Error	Interaction	Treatment
If the true means of the k populations are equal, then MST/MSE should be:	more than 1.00	Close to 1.00	Close to -1.00	A negative value between 0 and - 1	Close to 1.00
If MSE of ANOVA for six treatment groups is known, you can compute……	Degree of freedom	The standard deviation of each treatment group	Variance	The pooled standard deviation	The pooled standard deviation
To determine whether the test statistic of ANOVA is statistically significant,to determine critical value we need	Sample size, number of groups	Mean, sample standard deviation	Expected frequency, obtained frequency	MSTR, MSE	Sample size, number of groups
Which of the following is an assumption of one-way ANOVA comparing samples from 3 or more experimental treatments?	Variables follow F-distribution	Variables follow normal distribution	Samples are dependent each other	Variables have different variances	Variables follow normal distribution
The error deviations within the SSE statistic measure distances:	Within groups	Between groups	Between each value and the grand mean	Betweeen samples	Within groups
In one-way ANOVA, which of the following is used within the F-ratio as a measurement of the variance of individual observations?	SSTR	MSTR	SSE	MSE	SSE
When conducting a one-way ANOVA, the _____ the between-treatment variability is when compared to the within-treatment variability	More random larger	Smalller	Larger	More random smaller	Smaller

When conducting a one-way ANOVA,the value of *F* DATA will be tend to be ……..	More random larger	Smalller	More random smaller	Larger	Smaller
When conducting an ANOVA, *F* DATA will always fall within what range?	Between negative infinity and infinity	Between 0 and 1	Between 0 and infinity	Between 1 and infinity	Between 0 and infinity
If F DATA = 5, the result is statistically significant	Always	Sometimes	Never	Is impossible	Sometimes
If F DATA= 0.9, the result is statistically significant	Always	Sometimes	Never	Is impossible	Never
When comparing three treatments in a one-way ANOVA ,the alternate hypothesis is…..	All three treatments have different effect on the mean response.	Exactly two of the three treatments have the same effect on the mean response.	At least two treatments are different from each other in terms of their effect on the mean response	All the treatments have same effect	At least two treatments are different from each other in terms of their effect on the mean response
If the sample means for each of *k* treatment groups were identical,the observed value of the ANOVA test statistic?	1	0	A value between 0.0 and 1.0	A negative value	0
If the null hypothesis is rejected, the probability of obtaining a *F* - ratio > the value in the *F* table as the 95th % is:	0.5	>0.5	<0.5	1	<0.5
ANOVA was used to test the outcomes of three drug treatments. Each drug was given to 20 individuals. If MSE =16, What is the standard deviation for all 60 individuals sampled for this study?	6.928	48	16	4	4
Analysis of variance technique originated in the field of……	Agriculture	Industry	Biology	Genetics	Agriculture

With 90, 35, 25 as TSS, SSR and SSC , in case of two way classification, SSE is…..	50	40	30	20	30
Variation between classes or variation due to different basis of classification is commonly known as ……...	Treatments	Total sum of squares	Sum of squares	Sum of squares due to error	Treatments
The total variation in observations in Anova is classified as:	Treatments and inherent variation	SSE and SST	MSE and MST	TSS and SSE	Treatments and inherent variation
In Anova, variance ratio is given by…….	MST/MSE	MSE/MST	SSE/SST	TSS/SSE	MST/MSE
Degree of freedom for TSS is……..	N-1	k-1	h-1	(k-1)(h-1)	N-1
For Anova, MST stands for……...	Mean sum of squares of treatment	Mean sum of squares of varieties	Mean sum of squares of tables	Mean sum of sources of treatment	Mean sum of squares of treatment
An ANOVA procedure is applied to data of 4 samples,where each sample contains 10 observations.Then degree of freedom for critical value of F are	4 numerator and 9 denominator	3 numerator and 40 denominator	3 numerator and 36 denominator	4 numerator and 10 denominator	3 numerator and 36 denominator
The power function of a test is denoted by…….	M(w,Q)	M(Q,Qo)	P(w,Q)	P(w,Qo)	M(w,Q)
Sum of power function and operation characteristic is…….	Unity	Zero	two	Negative	Unity
Operation characteristic is denoted by……	L(w,Q)	M(w,Q)	L(w,Qo)	M(w,Qo)	L(w,Q)
Operation characteristic is also known as ……	Test characteristic	Power function	best characteristic	unique characteristic	Test characteristic
The formula to find OC is L(w,Q)=	1-Power Function	2xPower Function	Power Funtion -1	2xConfidance Interval	1-Power Function
Operation Characteristic is of a test is related to…	Power Function	Best Test	Unique Test	Uniformally Best Test	Power Function

If the Hypothesis is correct the operation charectristics will be……	1	0	-1	0.5	1
If the Hypothesis is wrong the operation charectristics will be…….	0	1	0.5	0.333333	0
In which test we verify a null hypothesis against any other definite alternate hypothesis?	Best Test	Unique Test	Uniformally Best Test	Unbiased Test	Best Test
A Best Test is a Test such that the critical region for which _____ attains least value for a given α.	Beta	1-Beta	Alpha	1-Alpha	1-Beta
A Test whose power funtion attains its mean at point Q = Qo is called ____ Test	Unique	Unbiased	Power	Operation Characteristic	Unique
A Best Unique Test exist _____	Always	Never	Sometimes	When Q not = to Qo	Sometimes
Operation Characteristic is related to	Power Function	Unique Test	Best Test	Uniformally Best Test	Power Function
Power is the ability to detect:	A statistically significant effect where one exists	A psychologically important effect where one exists	Both (a) and (b) above	Design flaws	A statistically significant effect where one exists
Calculating how much of the total variance is due to error and the experimental manipulation is called:	Calculating the variance	Partitioning the variance	Producing the variance	Summarizing the variance	Partitioning the variance
ANOVA is useful for:	Teasing out the individual effects of factors on an Independent Variables	Analyzing data from research with more than one Independent Variable and one Dependent Variable	Analyzing correlational data	Individual effects of factors on an Dependent Variables	Analyzing data from research with more than one Independent Variable and one Dependent Variable

What is the definition of a simple effect?	The effect of one variable on another	The difference between two conditions of one Independent Variable at one level of another Independent Variable	The easiest way to get a significant result	Difference between two Dependent Variables	The difference between two conditions of one Independent Variable at one level of another Independent Variable
In a study with gender as the manipulated variable, the Independent Variable is:	Within participants	Correlational	Between participants	Regressional	Between participants
Which of the following statements are true of experiments?	The Independent Variable is manipulated by the experimenter	The Dependent Variable is assumed to be dependent upon the IV	They are difficult to conduct	both (a) and (b)	both (a) and (b)
All other things being equal, repeated-measures designs:	Have exactly the same power as independent designs	Are often less powerful than independent designs	Are often more powerful than independent designs	Are rarely less powerful when compare to than independent designs	Are often more powerful than independent designs
Professor P. Nutt is examining the differences between the scores of three groups of participants. If the groups show homogeneity of variance, this means that the variances for the groups:	Are similar	Are dissimilar	Are exactly the same	Are enormously different	Are similar
Differences between groups, which result from our experimental manipulation, are called:	Individual differences	Treatment effects	Experiment error	Within-participants effects	Treatment effects

Herr Hazelnuss is thinking about whether he should use a related or unrelated design for one of his studies. As usual, there are advantages and disadvantages to both. He has four conditions. If, in a related design, he uses 10 participants, how many would he need for an unrelated design?	40	20	10	100	40
Individual differences within each group of participants are called:	Treatment effects	Between-participants error	Within-participants error	Individual biases	Within-participants error
Calculating how much of the total variance is due to error and the experimental manipulation is called:	Calculating the variance	Partitioning the variance	Producing the variance	Summarizing the variance	Partitioning the variance
The decision on how many factors to keep is decided on:	Statistical criteria	Theoretical criteria	Both (a) and (b)	Neither (a) nor (b)	Both (a) and (b)
It is possible to extract:	As many factors as variables	More factors than variables	More variables than factors	Correlation between the actual and predicted variables	As many factors as variables
Four groups have the following means on the covariate: 35, 42, 28, 65. What is the grand mean?	43.5	42.5	56.7	58.9	42.5
You can perform ANCOVA on:	Two groups	Three groups	Four groups	All of the above	All of the above
When carrying out a pretest--posttest study, researchers often wish to:	Partial out the effect of the dependent variable	Partial out the effect of the pretest	Reduce the correlation between the pretest and posttest scores	Correlation between the two tests scores	Partial out the effect of the pretest

Using difference scores in a pretest--posttest design does not partial out the effect of the pretest for the following reason:	The pretest scores are not normally correlated with the posttest scores	The pretest scores are normally correlated with the different scores	The posttest scores are normally correlated with the different scores	Up normal relationship with the different scores	The pretest scores are normally correlated with the different scores
Experimental designs are characterized by:	Two conditions	No control condition	Random allocation of participants to conditions	More than two conditions	Random allocation of participants to conditions
Between-participants designs can be:	Either quasi-experimental or experimental	Only experimental	Only quasi-experimental	Only correlational	Either quasi-experimental or experimental
A continuous variable can be described as:	Able to take only certain discrete values within a range of scores	Able to take any value within a range of scores	Being made up of categories	Being made up of variables	Able to take any value within a range of scores
In a within-participants design with two conditions, if you do not use counterbalancing of the conditions then your study is likely to suffer from:	Order effects	Effects of time of day	Lack of participants	Effects of participants	Order effects
Demand effects are possible confounding variables where:	Participants behave in the way they think the experimenter wants them to behave	Participants perform poorly because they are tired or bored	Participants perform well because they have practiced the experimental task	Participants perform strongly	Participants behave in the way they think the experimenter wants them to behave

Power can be calculated by a knowledge of:	The statistical test, the type of design and the effect size	The statistical test, the criterion significance level and the effect size	The criterion significance level, the effect size and the type of design	The criterion significance level, the effect size and the sample size	The criterion significance level, the effect size and the sample size
Relative to large effect sizes, small effect sizes are:	Easier to detect	Harder to detect	As easy to detect	As difficult to detect	As difficult to detect
Differences between groups, which result from our experimental manipulation, are called:	Individual differences	Treatment effects	Experiment error	Within-participants effects	Treatment effects
Completely randomized design is similar to ----------	three way	one way	two way	t test	one way
Randomized block design is similar to ----------	two way	three way	one way	many	two way
ANOVA is the technique of analysis of ------	standard deviation	variance	mean	range	variance
Under one way classification , the influence of only ----- attribute or factor is considered	two	three	one	many	one
Under two way classification , the influence of only ----- attribute or factor is considered	four	two	three	one	two

Question	Opt 1	Opt 2	Opt 3	Opt 4	Answer
Psychometric Methods book is written by_____	J.P.Guilford	Likert	L.L.Thurstone	Louis Guttman	J.P.Guilford
Respondents are asked to rank their choices in_____	Comparative scaling	arbitrary scaling	rating scale	differential scale	Comparative scaling
_____ is developed on ad-hoc basis	Differential scale	arbitrary scale	rating scale	ranking scale	arbitrary scale
_____scale is developed by utilizing item analysis approach	Comparative scale	likert scale	differential scale	rating scale	likert scale
Scalogram analysis is developed by_____	J.P.Guilford	Likert	L.L.Thurstone	Louis Guttman	Louis Guttman
A complete enumeration of all items in the population is known as _____	sampling unit	sample design	census inquiry	all the above	census inquiry
The selected respondents constitute_____	population	sample	sample size	population size	sample
The selection process of respondents is called_____	survey	sampling technique	sample survey	census inquiry	sampling technique
The survey conducted to select the respondents is called_____	sampling technique	sample survey	census inquiry	population size	sample survey
A sample design is a definite plan for obtaining a sample from a given_____	universe	sample design	population	sample survey	population
The number of items in universe can be_____	finite	infinite	both	zero	both
The population of a city, number of workers in a company is_____	infinite	finite	both	zero	finite
Source list is also known as_____	sampling size	sampling size	sampling frame	population size	sampling frame
The size of the sample should be _____	large	optimum	small	all the above	optimum

Inappropriateness in sampling frame will result in _____	systematic bias	optimum	problems	sampling error	systematic bias
Sampling error _____ with increase in size of sample	decrease	increase	both	optimum	decrease
Sampling error can be measured from_____	sample design	sample size	population	sample design and sample size	sample design and sample size
On the representation basis samples may be_____	probability sampling	non-probability sampling	both	restricted	both
On element selection basis the samples may be_____	restricted	unrestricted	both	probability sampling	both
Non-probability sampling is also known as_____	quota sampling	purposive sampling	deliberate sampling	all the three	all the three
Quota sampling is an example of_____	probability sampling	non-probability sampling	both	purposive sampling	non-probability sampling
Probability sampling is also known as_____	random sampling	choice sampling	random and choice sampling	multistage sampling	random and choice sampling
Lottery method of selecting data is an example of_____	random sampling	choice sampling	purposive sampling	quota sampling	random sampling
Systematic sampling is an improved version of_____	quota sampling	simple random sampling	choice sampling	purposive sampling	simple random sampling
If population is not drawn from homogeneous group_____ technique is applied	simple random sampling	quota sampling	choice sampling	stratified sampling	stratified sampling
In_____ total population is divided into number of relatively small sub divisions	cluster sampling	choice sampling	stratified sampling	quota sampling	cluster sampling
When a particular lot is to be accepted or rejected on the basis of single sampling it is known as_____	double sampling	single sampling	area sampling	purposive sampling	single sampling

Survey designed to determine attitude of students toward new teaching plan is known as_____-	cross stratification sampling	stratification sampling	cluster sampling	multi stage sampling	cross stratification sampling
Sample design is determined_____ datas are collected	before	after	both	based on the survey	before
Indeterminary principle step comes in_____	step in sample design	criteria to select sample procedure	both	step doesnot occur	criteria to select sample procedure
The measurement of sampling error is called as_____	precision of sampling plan	sampling survey	sampling plan	representation basis	precision of sampling plan
The different sub populations divided to constitute a sample is known as_____	stratified sampling	survey	population	strata	strata
Every nth item is selected in_____	stratified sampling	systematic sampling	judgement sampling	all the above	systematic sampling
_____is conducted for determining a more appropriate and efficient stratification plan	survey	sample	pilot study	sample plan	pilot study
_____is considered more appropriate when universe happens to be small	purposive sampling	area sampling	cluster sampling	simple random sampling	purposive sampling
When we use rating scales we judge an object in _____terms against some specified criteria.	real	absolute	imaginary	perfect	absolute
Rating scale is also known as_____	Categorical scale	arbitrary scale	cumulative scales	all the above	Categorical scale
The graphical scale is _____and is commonly used in practice.	Problematic	critical	simple	real	simple
_____is also known as numerical scale	Itemized rating scale	graphical rating scale	cumulative scale	likert scale	Itemized rating scale

The chief merit of itemized rating scale is it provides_____ information	more	deep	critical	all the above	more
_____occurs when the respondents are either easy raters or hard raters	error of hallo effect	error of leniency	error of central tendency	cumulative scales	error of leniency
_____ occurs when the rater carries a generalized impression of the subject from one rating to another.	error of hallo effect	error of leniency	error of central tendency	graphical rating scale	error of hallo effect
When the raters are reluctant to give extreme judgments, the result is_____	error of hallo effect	error of leniency	error of central tendency	cluster sampling	error of central tendency
Systematic bias is also known as_____	Error of hallo effect	error of leniency	error of central tendency	cumulative scales	Error of hallo effect
_____occurs when the rater is asked to rate more factors, which has no evidence for judgment.	error of hallo effect	error of leniency	error of central tendency	cluster sampling	error of hallo effect
_____ is also known as ranking scale	rating scale	comparative scale	likert scale	graphical rating scale	comparative scale
We make relative judgments against similar objects in _____	comparative scale	likert scale	differential scale	rating scale	comparative scale
Paired comparisions provide_____ data.	nominal	ordinal	ratios	interval	ordinal
Ordinal data can be converted to _____ data through Law of comparative judgment.	nominal	ordinal	ratio	interval	interval
Law of comparative judgment is developed by_____	J.P.Guilford	Likert	L.L.Thurstone	all the three	L.L.Thurstone
--------- Scales have an absolute or true zero of measurement	Ordinal	Nominal	interval	ratio	ratio

The section of ---------- constitutes the main body of the report where in the results of the study are presented in clear.	Appendix	results	methods	Ordinal	results
Study to portray accurately characteristics of a particular individual, situation or a group is called ---------- research	Exploratory	Diagnostic	Descriptive	Hypothesis testing	Descriptive
Critical evaluation made by the researcher with the facts and information already available is called ---------- research.	Analytical	Exploratory	Diagnostic	Hypothesis testing	Analytical
Research to find reason, why people think or do certain things is an example of ----------	Quantitative Research	Applied Research	Qualitative research	Fundamental research	Qualitative research
Which one is considered a major component of the research study	Interpretation	research report	finding	draft	research report
Research task remains incomplete till the _____ has been presented.	Report	objective	finding	objective and finding	Report
What is the last step in a research study	Writing report	writing finding	writing limitations	research report	Writing report
Which is the final step in report writing……..	Writing report	writing finding	writing drafts	writing limitations	writing drafts
What is usually appended to the research work	Editing	bibliography	coding	research report	bibliography
The _____ is one which gives emphasis on simplicity and attractiveness	popular report	research report	article report	writing limitations	popular report
_____ should slow originality and should necessarily be on attempt to solve some intellectual problem	Interpretation	research report	finding	draft	research report

The researcher must remain caution about the _____ that can possibly arise in the process of interpreting results	Analysis	conclusions	findings	error	error
Which one should be considered while interpreting a given data	Validity	reliability	both validity and reliability	technical jargon	reliability
_____ is asking questions face to face	Indirect method	mailed questionnaire	through post	personal interview	personal interview
Journals, books, magazines etc are useful sources of collecting----------	Primary data	secondary data	both primary and secondary data	objective	secondary data
The collected raw data to detect errors and are called,	Editing	coding	classification	all the above	Editing
The formal, systematic and intensive process of carrying on a scientific method of analysis is _____	Research Design	research	interpretation	research analysis	research
------ Refers to the process of assigning numerals or symbols to answers of response	Coding	editing	classification	all the above	Coding
The research study, which is based on describing the characteristic of a particular individual or group	Experience survey	Descriptive	Diagnostic	Exploratory	Descriptive
Research is a_____	Finding	assumption	statement	all the above	all the above
The research, which has the purpose of improving a product or a process testing theoretical concepts in actual problem situations is-------research.	Statistical	Applied	Domestic	Biological	Applied

The chart of research process indicates that the process consists of a number of ----.	Closely related activities	unrelated activities	Closely unrelated activities	moderately related activities	Closely related activities
The objective of a good design is ----------.	Maximize the bias andmaximize the reliability of data	Minimize the bias and minimize the reliability of data	Minimize the bias and maximize the reliability of data	Maximize the bias and maximize the reliability of data	Maximize the bias and maximize the reliability of data
A ---------- is used whenever a full written report of the study is required.	Popular report	Technical report	article	monograph	Technical report
The ---------- is one which gives emphasis on simplicity and attractiveness.	Popular report	Technical report	article	monograph	Popular report
Which of the following are measurements of scale?	Nominal	ordinal	interval	all the above	all the above
____Scale is a system of assigning numbers, symbols to events in order to label them.	Interval	ordinal	Nominal	ratio	Nominal
The qualitative phenomena are considered in the ---------- scale.	Ordinal	Nominal	interval	ratio	Ordinal
---------- Scales can have an arbitrary zero, but it is not possible to determine the absolute zero.	Ordinal	Nominal	interval	ratio	interval