KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

Semester – II

18MMP206 **FLUID DYNAMICS** **4H – 4C**

Instruction Hours / week: L: 4 T: 0 P: 0 **Marks:** Internal: **40** External: **60** Total: **100**

End Semester Exam: 3 Hours

Course Objectives

This course enables the students to learn

- The concepts of fluid, its properties and behavior under various conditions of internal and external flows.
- The fundamentals of Fluid Dynamics, which is used in the applications of Aerodynamics, Hydraulics, Marine Engineering, Gas dynamics etc.
- To imbibe basic laws and equations used for analysis of static and dynamic fluids

Course Outcomes (COs)

On successful completion of this course, students will be able to

1. Classify and exploit fluids based on the physical properties of a fluid.
2. Compute correctly the kinematical properties of a fluid element.
3. Apply correctly the conservation principles of mass, linear momentum, and energy to fluid flow systems.
4. Understand both flow physics and mathematical properties of governing Navier-Stokes equations and define proper boundary conditions for solution.
5. Provide the student with the basic mathematical background and tools to model fluid motion.
6. Calculate the flow of an ideal fluid in a variety of situations.
7. Develop a physical understanding of the important aspects that govern fluid flows that can be observed in a variety of situations in everyday life.

UNIT I

INTRODUCTORY NOTIONS

Velocity – Stream Lines and Path Lines – Stream Tubes and Filaments – Fluid Body – Density – Pressure. Differentiation following the Fluid – Equation of continuity – Boundary conditions – Kinematical and physical – Rate of change of linear momentum – Equation of motion of an in viscid fluid.

UNIT II

EQUATION OF MOTION OF A FLUID

Euler's momentum Theorem – Conservative forces – Bernoulli's theorem in steady motion – energy equation for in viscid fluid – circulation – Kelvin's theorem – vortex motion – Helmholtz equation.

UNIT III

TWO DIMENSIONAL FLOW

Two Dimensional Motion – Two Dimensional Functions – Complex Potential – basic singularities – source – sink – Vortex – doublet – Circle theorem. Flow past a circular cylinder with circulation – Blasius Theorem – Lift force. (Magnus effect)

UNIT IV

VISCOUS FLOWS

Viscous flows – Navier-Stokes equations – Vorticity and circulation in a viscous fluid – Steady flow through an arbitrary cylinder under pressure – Steady Couettc flow between cylinders in relative motion – Steady flow between parallel planes.

UNIT V

LAMINAR BOUNDARY LAYER IN INCOMPRESSIBLE FLOW

Boundary Layer concept – Boundary Layer equations – Displacement thickness, Momentum thickness – Kinetic energy thickness – integral equation of boundary layer – flow parallel to semi infinite flat plate – Blasius equation and its solution in series.

SUGGESTED READINGS

1. Milne Thomson .L.M., (1968). Theoretical Hydrodynamics, Fifth edition, Dover Publications INC, NewYork.
2. Curle.N., and Davies H.J., (1971), Modern Fluid Dynamics Volume-I , D Van Nostrand Company Ltd., London.
3. Yuan, S.W, (1976). Foundations of Fluid Mechanics, Prentice- Hall, India.
4. Shanthi swarup, (2003), Fluid dynamics, Krishna Prakasan media Pvt Ltd, Meerut.

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LECTURE PLAN
DEPARTMENT OF MATHEMATICS

STAFF NAME: V.KUPPUSAMY
SUBJECT NAME: FLUID DYNAMICS SUB.CODE:18MMP206
SEMESTER: II CLASS: I M.SC MATHEMATICS

S.No	Lecture Duration Period	Topics to be Covered	Support Material/Page Nos
		UNIT-I	
1	1	Introduction to fluid dynamics	R1:Ch:1; Pg.No:1-3
2	1	Basic concepts of fluid dynamics, viscosity, compressible and non compressible fluids	R1:Ch:1; Pg.No:3-8
3	1	Stream surface, tube filament,streak lines, path lines, problems on path lines	R1:Ch:1; Pg.No:8-9
4	1	Geometrical significance of velocity, problems on rotational and irrotational flow	R1:Ch:1; Pg.No:65-68
5	1	Theorem on equation of Continuity	R1:Ch:3; Pg.No:68-76
6	1	Conservation of mass, Boundary conditions	R1:Ch:3; Pg.No:76-77
7	1	Theorems on rate of change of linear momentum, equation of motion of an inviscid fluid	R1:Ch:3; Pg.No:78-80
8	1	Recapitulation and discussion on possible questions	
	Total No of Hours Planned For Unit 1=8		

		UNIT-II	
1	1	Euler's equation of motion interms of vorticity	R1: Ch:3; Pg.No:80-81
2	1	Euler's momentum theorem	R1: Ch:3; Pg.No:81-82
3	1	Equations of motion	R1: Ch:3; Pg.No:106-108
4	1	Theorem on equations of motion interms of vorticity, Problems on Barotropic flow	R1: Ch:3; Pg.No:108-112
5	1	Bernoulli's theorem in steady motion, Theorem on energy equation for inviscid fluid	R3: Ch:3; Pg.No:181-185
6	1	Circulation, Kelvins theorem	R4: Ch:4; Pg.No:146-147
7	1	Theorem on Helmholtz equation of vorticity	R4: Ch:4; Pg.No:148-155
8	1	Recapitulation and discussion on possible questions	
	Total No of Hours Planned For Unit II=8		
		UNIT-III	
1	1	Two dimensional motion, Functions- problems	R2:Ch:3; Pg.No:42-44
2	1	Theorem on stream lines, Potential lines	R2: Ch:3; Pg.No:44-46
3	1	Problems on the flow patterns, Basic singularities	R2: Ch:3; Pg.No:46-50
4	1	Theorem on source and sink in 2D flow	R2: Ch:3; Pg.No:50-55
5	1	Theorem on complex potential for doublet and vortex	R2: Ch:3; Pg.No:56-60
6	1	Milne Thomson's circle theorem	R2: Ch:3; Pg.No:69-70
7	1	Blasius theorem and lift force	R2: Ch:3; Pg.No:70-72
8	1	Recapitulation and discussion on possible questions	
	Total No of Hours Planned For Unit III=8		

		UNIT-IV	
1	1	Introduction and definition of plane coquette flow	R2:Ch:5; Pg.No:123-124
2	1	Theorem on Reynolds's number	R2: Ch:5; Pg.No:124-125
3	1	Theorem on Navier Stokes equation	R2: Ch:5; Pg.No:140-144
4	1	Theorem on energy equation, Diffusion of vorticity	R2: Ch:5; Pg.No:145-150
5	1	Steady flow through an arbitrary cylinder under pressure	R2: Ch:5; Pg.No:150-152
6	1	Steady Couette flow between cylinders in relative motion	R3: Ch:5; Pg.No:80-85
7	1	Steady flow between parallel planes – problems, Theorem on Poiseuille flow	R3: Ch:5; Pg.No:86-88
8	1	Recapitulation and discussion on possible questions	
	Total No of Hours Planned For Unit IV=8		
		UNIT-V	
1	1	Laminar boundary layer in incompressible fluid: Definition and problems on equation of boundary layer	R2: Ch:6; Pg.No:175-178
2	1	Theorems on displacement	R2: Ch:5; Pg.No:184-185
3	1	Theorems on momentum thickness	R2: Ch:5; Pg.No:186-187
4	1	Boundary layer separation: Theorem on integral equation of boundary layer	R2: Ch:5; Pg.No:187-188
5	1	Problems on momentum integral equation	R2: Ch:5; Pg.No:188-190
6	1	Theorems on boundary layer along a semi-infinite flat plate	R2: Ch:5; Pg.No:191-192
7	1	Blasius equation and its solution in series	R2: Ch:5; Pg.No:193-195
8	1	Problems on flow near to the stagnation point of a cylinder	R2: Ch:5; Pg.No:197-198
9	1	Recapitulation and discussion on possible questions	

10	1	Discussion on previous ESE question papers	
11	1	Discussion on previous ESE question papers	
12	1	Discussion on previous ESE question papers	
	Total No of Hours Planned for unit V=12		
Total Planned Hours			**44**

REFERENCES

1. Milne Thomson .L.M., (1968). Theoretical Hydrodynamics, Fifth edition, Dover Publications INC, New York**.**
2. Curle.N., and Davies H.J., (1971), Modern Fluid Dynamics Volume-I , D Van Nostrand Company Ltd., London.
3. Yuan, S.W, (1976). Foundations of Fluid Mechanics, Prentice- Hall, India.
4. Shanti swarup, (2003), Fluid dynamics, Krishna Prakasan media Pvt Ltd, Meerut.

UNIT-I

Velocity – Stream Lines and Path Lines – Stream Tubes and Filaments – Fluid Body – Density – Pressure. Differentiation following the Fluid – Equation of continuity – Boundary conditions – Kinematical and physical – Rate of change of linear momentum – Equation of motion of an in viscid fluid.

INTRODUCTION

Fluid dynamics is the science of treating of fluids in motion. Fluid may be divided into two kinds

Liquids

Gases

A liquids are incompressible and gases are compressible fluids

COMPRESSIBLE

It means changes in volume whenever the pressure changes.

INCOMPRESSIBLE

It means changes in volume donot change when the pressure changes.

NOTE I

The term hydro dynamics is often applied to the science of measuring.

Incompressible fluid

NOTE II

Matter classified into three types

Elasticity

Plasticity

Flow

VISCOUS AND INVISCID FLUID

Suppose that the fluid element is enclosed by the surface S. Let ds be the surface element around a point p. Then a surface force acting on the surface. It may be resolved into normal direction and tangential direction.

Normal forces per unit area is said to be normal stress.(pressure)

The tangential forces per unit area is called shearing stress.

A fluid is said to be viscous (real fluid) when normal stress as well as shearing stress exists .

Eg: oil for viscous fluid dam water for inviscid fluid.

Velocity of the fluid at a point

At a time 't' a fluid particle is at the point p.

Here $\overline{OP}$ = r and at a time $t+\delta t$ the same particle has reached P'

$$\overline{OP}' = r + \delta r$$

And $\overline{PP}' = \delta r$

The particle velocity q at p is

$$q = \lim_{8t \to 0} \frac{\delta r}{\delta t} \quad ; \quad q = \frac{dr}{dt}$$

Clearly q is displacement on both r and t

So q=q(r,t)

It p has Cartesian coordinates (x,y,z) relative to the fixed point O

$\therefore$ We get q=q(x,y,z,t)

Ie further suppose u,v,w are the Cartesian components of q in their direction $q = qi + u\vec{i} + v\vec{j} + w\vec{k}$

In general r is represented by $r = x\vec{i} + y\vec{j} + z\vec{k}$ then $q = \frac{dr}{dt}$

$$= \frac{d}{dt}\left(x\vec{i} + y\vec{j} + z\vec{k}\right)$$

$$= \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Let u= $\frac{dx}{dt}$

V= $\frac{dy}{dt}$

W= $\frac{dz}{dt}$

$$q = u\vec{i} + v\vec{j} + w\vec{k}$$

DEFINITION

Fluid dynamics is a branch of science treating the study of fluid in motion

The term fluid is a substance that flows is called solid.

The fluid is divided into two kinds.

Liquids => which are in compression
Gases => which are in compression

LAMINAR FLOW

A flow in which the fluid particles trace out a definite curve and a curve traced by any two fluid particles do not intersect is said to be laminar flow

TURBULENTFLOW

A flow in which the fluid particles do not trace out a definite curve and curve traced by any two fluids will intersect is said to be turbulent flow

STEADY FLOW

A flow in which the flow pattern remains unchanged with time is said to be steady flow

Ie $\frac{\delta p}{\delta t} = 0$

Here p may be velocity, density, pressure, temperature etc.

UNSTEADY FLOW

A flow in which the flow pattern changes with time is said to be unsteady

UNIFORM FLOW

The flow in which the fluid particles possesses equal velocity at each section of the channel or pipe is called uniform flow

NON – UNIFORM FLOW

The flow in which the fluid particles possesses different velocity at each section of the channel or pipe called non-uniform flow

ROTATIONAL OR IRROTATIONAL FLOW

A flow in which the fluid particles go on rotating about their own axes while flowing is called rotational

The fluid particles does not rotate about their own axes while flowing called irrotational flow

BAROTROPHIC FLOW

A flow is said to be Barotrophic when the pressure is the function of density

PRESSURE

When a fluid is contained in a vessel. It exaerts a force at each point of the linear side of the vessel such a force per unit area is called pressure.

VELOCITY OF A FLUID PARTICLE:

Let a fluid particle at a point P at any time t. let it be at Q at the time $t + \delta t$ such that OP=r.

Then the moment of the particle PQ is δr

Hence the velocity $q = \lim_{\partial t \to 0} \frac{\partial r}{\partial t}$

$$q = \frac{dr}{dt}$$

Here q is a function of r and t or q=f(r,t)

u,v,w are the components of then we have $\bar{q} = u\vec{i} + v\vec{j} + w\vec{k}$

STREAM LINES:

A stream line is a curve drawn in the fluid. Such that the tangent to the curve gives the direction of the fluid velocity at a particular point

Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ be the position vector of point P and $q = u\vec{i} + v\vec{j} + w\vec{k}$ be the fluid velocity at the point P. then the equation of the stream line is given by

$$q \times dr = 0$$

$$\left(u\vec{i} + v\vec{j} + w\vec{k}\right) \times d\left(x\vec{i} + y\vec{j} + z\vec{k}\right) = 0$$

$$\begin{vmatrix} i & j & k \\ u & v & w \\ dx & dy & dz \end{vmatrix} = 0$$

$$i(vdz - wdy) - j(udz - wdx) + k(udy - vdx) = 0$$

$$i(vdz - wdy) = 0$$

$$j(udz - wdx) = 0$$

$$k(udy - vdx) = 0$$

$$i(vdz - wdy) = 0$$

$$vdz = wdy$$

$$\frac{dz}{w} = \frac{dy}{v} \text{...............(1)}$$

$$udz = wdx$$

$$\frac{dz}{w} = \frac{dx}{u} \text{...............(2)}$$

$$udy = vdx$$

$$\frac{dx}{u} = \frac{dy}{v} \text{...............(3)}$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \text{.......................(4)}$$

This is the equation 4 of the stream line thus stream line shows how each particle is moving at a given instant

If the velocity vanishes at a given point such a point is known as critical point stagnation.

PATH LINE:

The path traced out by the fluid particle as it moves with evaluation of time is called path line

THE VELOCITY VECTOR

$$\bar{q} = (u, v, w) = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right)$$

$$\frac{dx}{dt} = u(x, y, z, t)$$

$$\frac{dy}{dt} = v(x, y, z, t)$$

$$\frac{dz}{dt} = w(x, y, z, t)$$

STREAK LINES:

The locus of all fluid particles which has crossed a particular point at an earlier instant is called as streak lines.

EXAMPLE:

The powder line formed in the river water when we pour pouder by standing in a particular place a particular point.

STREAM TUBE:

The stream tube is the collection of number of stream lines forming an imaginary tube.

STREAM FILAMENT:

A stream tube of infinite estimal cross section is known as stream filament.

Problem 1:

Given the velocity vector $q = x\vec{i} + y\vec{j}$ determine the equation of stream line.

Solution:

The equation of steam line

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\vec{q} = x\vec{i} + y\vec{j}$$

$$\frac{dx}{x} = \frac{dy}{y}$$

Integrate

$$\int \frac{dx}{x} = \int \frac{dy}{y}$$

$$\log x = \log y + \log c$$

$$\log x - \log y = \log c$$

$$\frac{x}{y} = c$$

Problem 2:

The velocity component in three dimension flow fluid for a incompressible fluid (2x,-y,-z) determine the equation of steam line passing through (1,1,1)

Solution:

The equation stream line

$$\frac{dx}{u}=\frac{dy}{v}=\frac{dz}{w}$$

$$\frac{dx}{2x}=\frac{dy}{-y}$$

$$\int\frac{dx}{2x}=\int\frac{dy}{-y}$$

$$\frac{1}{2}\log x=-\log y+\log c$$

$$\frac{1}{2}\log x+\log y=\log c$$

$$\log x^{\frac{1}{2}}+\log y=\log c_1$$

$$\frac{dy}{-y}=\frac{dw}{-z}$$

$$\int\frac{dy}{-y}=\int\frac{dz}{-z}$$

$$-\log y=-\log z+\log c_2$$

$$-\log y+\log z=\log c_2$$

$$\log z-\log y=\log c_2$$

$$\frac{z}{y}=c_2$$

$$\frac{dz}{w}=\frac{dx}{u}$$

$$\frac{dz}{-z}=\frac{dx}{2x}$$

$$\int \frac{dz}{-z} = \int \frac{dx}{2x}$$

$$-\log z = \frac{1}{2}\log x + \log c_3$$

$$c_3 = x^{\frac{1}{2}}.z$$

$$c_1 = x^{\frac{1}{2}}.y$$

$$c_2 = \frac{z}{y}$$

$$c_3 = x^{\frac{1}{2}}.z$$

Apply points (x,y,z)=(1,1,1)

$c_1 = 1$

$c_2 = 1$

$c_3 = 1$

Problem 3:

find the equation of stream line for the flow $q = -i(3y^2) - j(6x)$

at the point (1,1)

VISCOSITY:

A Fluid which has viscosity is called viscosity fluid.

A Fluid which has no viscosity is called non – viscous fluid or inciscid fluid.

It is a property of exerting internal resistance to the change in shape is form is called viscousity

Example:

Honey is more viscous than water

It is clear that there exist a property in the fluid which controls the rate of flow. This property of flow is called viscousity or internal friction.

DIFFERENCE BETWEEN STREAM LINE AND PATH LINE:

Stream line:

1. A tangent to the stream line gives the direction of velocity of fluid particles at various point at a given time
2. Stream line shows how each fluid particle is moving at the given instant
3. In steady flow stream lines do not vary with time and co inside with path lines.

Path line:

1. A tangent to path line gives the direction of velocity given fluid particles at various time.
2. The path shows how the given fluid particle is moving at each instant.

THEOREM:

Show that the product of speed and cross sectional area is constant along the stream filament of a liquid in a steady motion

(or)

Show that the stream filament widest at place where the speed is narrowest and the speed is greatest.

Solution

Consider the stream filament of a liquid in steady motion.

Let q_1 and q_2 be the speeds of the flow at places where the cross section area $\sigma_1 and \sigma_2$

The liquid is incompressible in a given time the same volume of fluid must flow out at one end as flow in at other end

$$\sigma_1 q_1 = \sigma_2 q_2$$

The product of speed and cross section area is constant along the stream filament of the liquid in steady motion.

VELOCITY POTENTIAL OR VELOCITY FUNCTION:

Let the velocity of the fluid the time t be $q = u\vec{i} + v\vec{j} + w\vec{k}$ at any point p further suppose that at a particular instant t there exists a scalar function $\phi(x, y, z, t)$ which is uniform throughout the entire field of flow and such that

$$d\phi = -\left(\frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy + \frac{\partial \phi}{\partial z}dz + \frac{\partial \phi}{\partial t}dt\right)$$

$$= -(udx + vdy + wdz)$$

Since $\frac{\partial \phi}{\partial t} = 0$

Let the expression on the right hand side is exact differential they we have

$u = -\frac{\partial \phi}{\partial x}$ $v = -\frac{\partial \phi}{\partial y}$ $w = -\frac{\partial \phi}{\partial z}$ and $\frac{\partial \phi}{\partial t} = 0$

Hence $q = u\vec{i} + v\vec{j} + w\vec{k}$

$$= -\left(i\frac{\partial \phi}{\partial x} + j\frac{\partial \phi}{\partial y}dy + k\frac{\partial \phi}{\partial z}\right)$$

$= -\nabla\phi$

$\bar{q} = -\nabla\phi \Rightarrow -grad\phi$

ϕ is called the velocity potential.

Here the negative sign indicates the flow taking place flow the higher to lower potential.

VORTEX LINE:

Vortex line is a curve drawn in the fluid such that the tangent to the curve gives the direction of the vorticity vector.

VORTEX TUBE:

A vortex line drawn through each point of a closed curve enclosed by the tubular space in the fluid known as vortex tube

VORTEX FILAMENT:

The vortex tube of infinitesimal cross section is called as vortex filament.

BELTRANIC FLOW:

A fluid motion is said to be Beltranic flow if q is parallel to w

i.e $q \times w = 0$

Here q is called Beltranic vector

ROTATIONAL AND IRROTATIONAL MOTION:

A motion of a fluid is said to be irrotational when the velocity vector of the every fluid particle is zero.

When the vorticity vector is different from zero then the motion is said to be rotational.

THEOREM:

Show that the pressure at a point in an inviscid fluid is a scalar quantity

Proof:

Let P,Q,R,S be the tetrahedral of the small of the small dimension with common centroid o in the fluid.

Let p_1 and p_2 be the average pressure on the phase pRS+qRS

Whose areas are σ_1 and σ_2. Let σ be the common area of projection of σ_1 and σ_2 on pq

The component of the pressure stress in the direction of PQ of all phase of tetrahedran

$$= p_1\sigma_1 - p_2\sigma_2 + 0 + 0$$

The volume of the fluid within PQRS= $l\sigma$

Where l is the small length

Let f be the component of external force per unit mass in PQ and f be component of acceleration of the fluid per unit mass in PQ

By the second law of motion $F = ma$

We have $p_1\sigma_1 - p_2\sigma_2 + Fl\sigma\rho = fl\sigma\rho$

Where ρ is the density

$$(p_1 - p_2)\sigma = l(F - f)\sigma\rho$$

Here the area $\sigma_1 = \sigma_2 = \sigma$ is infinite estimal cross section

$l \to 0$ Is a point

$$(p_1 - p_2)\sigma = 0$$

$$p_1 = p_2$$

The pressure is a scalar quantity which is independent of direction

The orientation of phase is arbitrary

We conclude that the pressure at o is same for all orientation

THEOREM:

DIFFERENTIATION OF FLUID:

Fluid particles moves from $p(x,y,z)$ at time t to $p'(x+\delta x, y+\delta y, z+\delta z)$ at the time $t+\delta t$

Let $f(x,y,z,t)$ be a scalar function associated with some property of the fluid then motion

$$\delta F = \frac{\partial F}{\partial x}\delta x + \frac{\partial F}{\partial y}\delta y + \frac{\partial F}{\partial z}\delta z + \frac{\partial F}{\partial t}\delta t$$

The total of changes of f at p at the time t is the motion

$$\lim_{\delta t\to 0}\left(\frac{\delta F}{\delta t}\right) = \frac{dF}{dt} = \frac{\partial F}{\partial x}\frac{dx}{dt} + \frac{\partial F}{\partial y}\frac{dy}{dt} + \frac{\partial F}{\partial z}\frac{dz}{dt} + \frac{\partial F}{\partial t}$$

$$\lim_{\delta t\to 0}\left(\frac{\delta F}{\delta t}\right) = \frac{dF}{dt} = u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + z\frac{\partial F}{\partial z} + \frac{\partial F}{\partial t}\ldots\ldots\ldots\ldots\ldots(1)$$

Here $q = [u,v,w]$ is the velocity of the fluid particle at p

$$\frac{dF}{dt} = q.\nabla + \frac{\partial f}{\partial t}\ldots\ldots\ldots\ldots(2)$$

Similarly for a vector function $f(x,y,z,t)$ associated with same property of fluid

We get the differential equation of motion

$$\frac{df}{dt} = q.\nabla f + \frac{\partial f}{\partial t}\ldots\ldots\ldots\ldots(3)$$

From equation (2) and (3) we get operation equivalence

$$\frac{d}{dt} = q.\nabla + \frac{\partial}{\partial t}............(4)$$

Hence the equation (4) is called differential for the fluid.

Note 1:

In equation (3) and (4) $\frac{dF}{dt}, \frac{df}{dt}$ are called particle rate of change

$\frac{\partial F}{\partial t}, \frac{\partial f}{\partial t}$ are called local rate of change.

Note 2:

In equation (3) replace $F = \overline{q}$

$$\frac{d\overline{q}}{dt} = q.\nabla\overline{q} + \frac{\partial\overline{q}}{\partial t}............(5)$$

This is known as the analytic expression for acceleration

Note 3:

If the fluid is incompressible then $\frac{d\overline{q}}{dt} = 0$

Equation (5) becomes $q.\nabla\overline{q} + \frac{\partial\overline{q}}{\partial t} = 0$

EQUATION OF CONTINUITY:

If is based on the law of conservation of energy which states that energy can neither created nor destroyed. In this case the conservation of mass is interpreted in the following form it express the fact that the rate of generation of mass within the given volume is entirely due to net flow volume is enterly due to net flow of mass through the surface enclosing the given volume

Let us consider the closed surface s enclosing the volume v in the region occupied by the moving fluid.

Let the $\hat{n}$ be the unit outward drawn normal vector.

Let ds be any elementary surface enclosing the volume dv

Then the elementary mass dm is given by $dm = \rho dv$

Where ρ is the density of the fluid. Now the mass of the fluid within the whole surface s is $\int_v \rho dv$

Now the rate at which the mass is generated as $\frac{\partial}{\partial t}\int_v \rho dv$............(1)

This is because the rate refers to the time and $\frac{d}{dt}$ is the total derivative its takes care of changes in both time and position

Now equation (1) becomes $\int_v \frac{\partial \rho}{\partial t} dv$

Since the differentiation under the integral sign is allowed

But according to the conservative of mass this should be equal to the mass of the fluid entering per unit time across the surface S.

The mass of the fluid entering per unit time through the element ds is give by $ds = \rho \times$ length

$= \rho \times ds \times$ Velocity

$= \rho \times ds \times$ Velocity component $\times$ time

$= \rho \times ds \times -q\hat{n} \times t$

But time is unity

$$= -\int_S \rho \times q \times \hat{n} \times ds ....(2)$$

The mass of the fluid entering inside the surface is

$$S = -\int_S \rho \times (q \times \hat{n}) \times ds ....(3)$$

But the conservation of mass claim's that (1)=(2)

$$\int_v \frac{\partial \rho}{\partial t} dv = -\int_S \rho \times (q \times \hat{n}) \times ds$$

Now the L.H.S is given in volume integral and R.H.S is given is surface integral.

We should change surface integral and this is done by guass divergent theorem

If s is the closed surface enclosed surface in volume v and n is the unit normal vector outward to S

$$\int_S \hat{n} F ds = \int_v \nabla . F dv$$

$$\int_v \frac{\partial \rho}{\partial t} dv = -\int_v \nabla (\rho . q) \times dv$$

$$\int_v \frac{\partial \rho}{\partial t} dv + \int_v \nabla (\rho . q) \times dv = 0$$

Since v is an arbitrary choosen volume then we get

$$\frac{\partial \rho}{\partial t} + \nabla (\rho . q) = 0$$

This is called a continuity equation

Note 1:

Equation of continuity for a steady compressible flow:

Since the flow is steady $\frac{\partial \rho}{\partial t} = 0$ and hence the equation of continuity for a steady compressible flow is $\nabla(\rho.q) = 0$

Note 2:

Equation of continuity for a incompressible flow:

Since ρ is constant foe any incompressible fluid we get $\nabla.u = 0$

In other words to check any fluid velocity or to find the velocity of a liquid then we check $\nabla.q = 0$

Note 3:

Derive the equation of continuity for incompressible fluid

Proof:

The fluid is incompressible fluid so ρ is constant

By the equation of continuity we have

$$\frac{\partial \rho}{\partial t} + \nabla(\rho.q) = 0$$

$$\frac{\partial \rho}{\partial t} = 0$$

$$\nabla(\rho.q) = 0$$

$$\rho(\nabla.q) = 0$$

$$\nabla.q = 0$$

$div q = 0$

THEOREM:

DERIVE EULER'S EQUATION OF MOTION FOR INVISCID FLUID:

OR

DERIVE EQUATION OF MOTION OF AN INVISCID FLUID IN THE FORM $\frac{dq}{dt} = \bar{F} - \frac{1}{\rho}\nabla p$

PROOF:

Consider a fluid of volume v inside a closed surface.

ρ be the density of the fluid

ds elementary surface area

$\hat{n}$ unit outward vector

$\bar{q}$ velocity of the fluid particle

Elementary mass of the fluid $= \rho dv$

Linear momentum of elementary mass $= \bar{q}\rho dv$

Linear momentum of entire mass $= \int_v \bar{q}\rho dv$

The rate of change of linear momentum $= \frac{d}{dt}\int_v \bar{q}\rho dv$

$$= \int_v \frac{dq}{dt}\rho dv \ldots\ldots\ldots\ldots\ldots(1)$$

By Newton's second law of motion the total force on the body is equal to the rate of change of linear momentum

The force acting on this area

(i) External force $= \int_v \overline{F} \rho dv$

Normal pressure= stress of the body

$$= -\int_S p\hat{n}ds$$

Here p indicates the pressure

F indicates the force

The total force acting on the body $= \int_v \overline{F}\rho dv - \int_S p\hat{n}ds$

$$= \int_v \overline{F}\rho dv - \int_v \nabla p dv \text{...........}(2)$$

Equate (1) and (2)

$$\int_v \frac{dq}{dt}\rho dv = \int_v \overline{F}\rho dv - \int_v \nabla p dv$$

$$\int_v \frac{dq}{dt} dv = \int_v \overline{F} dv - \frac{1}{\rho}\int_v \nabla p dv$$

By vanishing the integral over the volume we get

$$\frac{dq}{dt} = \overline{F} - \frac{1}{\rho}\nabla p$$

This is the given equation of motion for an inviscid fluid

UNIT 1

POSSIBLE QUESTIONS

PART-B (6MARKS)

1. Show that the surface will be a surface of discontinuity of direction of the velocity not of speed
2. Derive the equation of motion of an inviscid fluid
3. Show that the velocity q is a function of r and t
4. Show that the path lines coincide with the stream lines when the motion is steady.
5. Discuss about the concept of kinematical boundary condition.
6. Explain briefly about adherence condition.
7. Derive equation of motion of an inviscid fluid.
8. Explain compressible and incompressible fluid.
9. Given the velocity vector $q = x\vec{i} + y\vec{j}$ determine the equation of stream line.
10. Derive equation of continuity for a incompressible flow
11. Fluid particles moves from $p(x, y, z)$ at time t to $p'(x+\delta x, y+\delta y, z+\delta z)$ at the time $t+\delta t$
12. Show that the pressure at a point in an inviscid fluid is a scalar quantity
13. Explain rotational and Irrotational terms.
14. Difference between path lines and steam lines.
15. Explain briefly about the viscous flow with examples.

PART-C (10 MARKS)

1. Show that the surface will be a surface of discontinuity of direction of the velocity not of speed
2. Find the rate of change of the momentum as S moves about with the fluid
3. Prove that the pressure at a point in an inviscid fluid is independent of direction

4. Show that the product of the speed and cross sectional area is constant along a stream filament of a liquid in steady motion.
5. Derive equation of motion of an inviscid fluid in the form $\frac{dq}{dt} = \overline{F} - \frac{1}{\rho}\nabla p$
6. Explain (i) Compressible (ii) incompressible (i)turbulent Flow with examples.
7. Explain the concept of viscous and invicous flow.
8. Define path line and steam line with application.

KARPAGAM ACADEMY OF HIGHER EDUCATION
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Subject: Fluid Dynamics **Subject Code: 18MMP206**
Class : I - M.Sc. Mathematics **Semester : II**

Unit I

Part A (20x1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)
Possible Questions

Questions	Opt1	Opt2	Opt3	Opt4	Answer
The behavior of fluid at rest gives the study of_______________.	fluid dynamics	fluid statics	elastic	plastic	fluid statics
The behavior of fluid when it is in motion without considering the pressure force is called____________.	fluid kinematics	fluid mechanics	fluid statics	fluids	fluid kinematics
____________ is a branch of science which deals with the behavior of fluid at rest as well as motion.	fluid mechanics	fluid statics	fluid kinematics	fluids	fluid mechanics
The behavior of fluid when it is in motion with considering the pressure force is called_________.	fluid kinematics	fluid dynamics	fluid statics	fluid mechanics	fluid dynamics
_____________ is the branch of science which deals with the study of fluids.	fluid kinematics	fluid dynamics	fluid statics	fluid mechanics	fluid dynamics
If any material deformation vanishes when a force applied withdrawn a material is said to be_________.	elastic	plastic	deformation	fluid	elastic
If deformation remains even after the force applied withdrawn the material is said to be____________	elastic	plastic	fluid	fluid statics	plastic
If the deformation remains even after the force applied withdrawn this property of material is_____________.	elastic	plasticity	fluid	deformation	plasticity
__________ can be classified as liquids and gases.	solids	pressure	fluids	forces	fluids
The density of fluids is defined as __________ volume.	limit per unit	solid per time	mass per unit	forces per unit	mass per unit
A force per unit area is known as _____________.	force	pressure	fluid	density.	pressure
ƏF is the ____________ force due to fluid on Əs	normal	constant	force	pressure	normal
The pressure changes in the fluid beings changes in the dencity of fluid is called___________.	compressible fluid	incompressible fluid	body force	surface force	compressible fluid
The change in pressure of fluid do not alter the density of the fluid is called_____________.	compressible fluid	incompressible fluid	body force	surface force	incompressible fluid
__________ are propotional to mass of the body.	pressure	body force	surface force	force	body force
__________ are propotional to the surface area.	body force	surface force	force	mass	surface force
The normal force per unit area is said to be _____________.	normal stress	shearing stress	stress	strain	normal stress
The tangential force per unit area is said to be _____________.	normal stress	shearing stress	stress	strain	shearing stress
In a high viscosity fluid there exist normal as well as shearing stress is called ___________.	viscous fluid	inviscid fluid	frictionless	ideal	viscous fluid

Which is the velocity of the equation.	q=dr/dt	.q=s/r	.v=dx/w	.u=dy/s	q=dr/dt
The differential equation of the path line is___________.	.u=dy/s	.v=dx/w	q=dr/dt	.q=s/r	q=dr/dt
A flow in which each fluid particle posses different velocity at each section of the pipe are called__________.	uniform flow	rotational floe	barotropic flow	non-uniform flow	non-uniform flow
A flow in which each fluid particle go on rotating about their own axis while flowing is said to be__________.	rotational floe	uniform flow	non-uniform flow	barotropic flow	uniform flow
The pressure is function of density then the flow is said to be ____________.	rotational floe	uniform flow	barotropic flow	non-uniform flow	barotropic flow
The direction of the fluid velocity at the point is called____________.	stream line	velocity	fluid	pressure	stream line
___________ is defined as the locus of different fluid particles passing through a fixed point.	stream filament	stream line	path line	stream tube	stream line
A stream tube of an infinitesimal cross sectional area is called___________.	stream line	stream filament	path line	stream tube	stream filament
The equation of volume is___________.	cross section area*speed	speed/cross section area	cross section area/speed	speed	cross section area*speed
The equation of speed is___________.	time/length	length/speed	length*time	time*speed	length/speed
When a fluid particle moves it changes in both _______________.	speed and time	time and frequency	speed and position	position and time	position and time
When the flow is __________ the strem line have same form at all times.	steady	unsteady	stream surface	stream tube	steady
When the flow is________ the stream line changes from instant to instant.	stream tube	steady	unsteady	steady	unsteady
If Δ.f=0 then f is said to be a ______________.	solenoid	rotation	irrotation	constant	solenoid

UNIT II

Euler's momentum Theorem – Conservative forces – Bernoulli's theorem in steady motion – energy equation for in viscid fluid – circulation – Kelvin's theorem – vortex motion – Helmholtz equation.

EULER'S MOMENTUM THEOREM:

STATEMENT:

A resultant thrust on the fluid enclosed with a closed surface S is equal to the reserve resultant of the boundary force enclosed the fluid and rate of flow of momentum outwards across the boundary S.

PROOF:

Consider a fluid of volume V enclosed with the surface S. let dv be an elementary volume enclosing the fluid particle p at time t.

dv =elementary volume

p = one point of the fluid particle

q = velocity of the fluid particle at time t

ρ = density of the fluid

$\hat{n}$ =unit outward normal vector

Elementary mass of the fluid= $\rho.dv$

Linear momentum of the elementary mass= $\bar{q}\rho dv$

Rate of change of linear momentum of entire fluid= $\frac{d}{dt}\int \bar{q}\rho dv$

We know that $\frac{d}{dt}=\frac{\partial}{\partial t}+\bar{q}\nabla$

$$\frac{d}{dt}\int \bar{q}\rho dv=\frac{\partial}{\partial t}\int \bar{q}\rho dv+\int(\bar{q}.\nabla)\bar{q}\rho dv.............(1)$$

Using Gauss divergence theorem

$$\int_S \bar{F}.n.ds=\int_v(\nabla.\bar{F})dv$$

The equation (1) becomes

$$\frac{d}{dt}\int \bar{q}\rho dv=\frac{\partial}{\partial t}\int \bar{q}\rho dv+\int(\bar{q}.n)\,p\bar{q}ds.............(2)$$

The minus symbol indicates the opposite direction of surface.

The force acting on the fluid body

(1) Normal pressure on the surface $\int_S p.\bar{n}.ds$

(2) External force (gravity) $\bar{F}$ per unit mass= $\int_v \bar{F}.\rho.dv$

The total force acting on the fluid = $\int_S p.\bar{n}.ds+\int_v \bar{F}.\rho.dv................(3)$

By Newton's second law the total force acting on the particle = Rate of change linear momentum

(2)=(3)

$$\int_S p.\bar{n}.ds=-\int_v \bar{F}.\rho.dv+\frac{\partial}{\partial t}\int \bar{q}\rho dv-\int(\bar{q}.n)\,p\bar{q}ds.............(4)$$

NOTE:

When the fluid is at rest the Euler momentum theorem is nothing but the principle of Archimedes.

PROOF:

When the particle is at rest then $\bar{q} = 0$

Equation (4)

$$\int_S p.\bar{n}.ds = -\int_v \bar{F}.\rho.dv - \int (\bar{q}.n)\, p\bar{q}\, ds$$

$$\int_S p.\bar{n}.ds = -\int_v \bar{F}.\rho.dv$$

This is the principle of Archimedes

CONSERVATIVE FORCE:

The force $\bar{F}$ is conservative iff there exists a potential function Ω such that $\bar{F} = -\nabla\Omega$

BOOK WORK:

Derive the equation of motion in the form $\frac{d\bar{q}}{dt} = -\nabla\left[\int \frac{dp}{p} + \Omega\right]$ where the force is conservative and derived from potential Ω and the pressure is the function of density.

PROOF:

From unit 1 Euler equation of in viscid fluid is $\frac{d\bar{q}}{dt} = \bar{F} - \frac{1}{\rho}\nabla p$................(1)

The force $\bar{F}$ is conservative

$\bar{F} = -\nabla\Omega$............(2)

Now $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$

Divide ρ by above equation

$$\frac{dp}{\rho} = \frac{d\vec{r} - \nabla p}{\rho}$$

$$\frac{1}{\rho}(d\vec{r}.\nabla p) = d\int\frac{dp}{\rho}$$

$$\frac{1}{\rho}(d\vec{r}.\nabla p) = dr\nabla\int\frac{dp}{\rho}$$

Use equation (2) and (3) in (1)

$$\frac{d\overline{q}}{dt} = -\nabla\Omega - \nabla\int\frac{dp}{\rho}\ \frac{\nabla p}{\rho} = \nabla\int\frac{dp}{\rho}\text{.............}(3)$$

$$\frac{d\overline{q}}{dt} = -\nabla\left[\Omega + \int\frac{dp}{\rho}\right]$$

$$d\bar{r}\nabla\phi = d\left(x\vec{i} + y\vec{j} + z\vec{k}\right)\left(i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}\right)$$

$$= \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$$

$$d\bar{r}\nabla\phi = d\phi$$

$$d\bar{r}\nabla = d$$

STATE AND PROVE BERNOULLI'S THEOREM

OR

DERIVE BERNOULLI'S EQUATION OF STEADY MOTION IN THE FORM:

$$\frac{d\overline{q}}{dt} - \overline{q}\times\overline{\zeta} = -\nabla\Psi \text{ where}$$

$$\Psi = \int \frac{dp}{\rho} + \Omega + \frac{1}{2}q^2$$

PROOF:

Equation of motion for inviscid fluid is

$$\frac{d\bar{q}}{dt} = \bar{F} - \frac{1}{\rho}\nabla p................(1)$$

The force $\bar{F}$ is conservative

$$\bar{F} = -\nabla\Omega.............(2)$$

Then we know that

$$\frac{d\bar{q}}{dt} = \frac{d\bar{q}}{dt} - \bar{q}\times\bar{\zeta} + \frac{1}{2}q^2..............(3)$$

When the pressure is the function of density

$$\frac{\nabla p}{\rho} = \nabla\int\frac{dp}{\rho}.............(4)$$

Use equation (2)(3)(4)in(1)

$$\frac{d\bar{q}}{dt} - \bar{q}\times\bar{\zeta} + \frac{1}{2}\nabla q^2 = -\nabla\Omega - \nabla\int\frac{dp}{\rho}.$$

$$\frac{d\bar{q}}{dt} - \bar{q}\times\bar{\zeta} = -\nabla\Omega - \nabla\int\frac{dp}{\rho}. - \frac{1}{2}\nabla q^2$$

$$\frac{d\bar{q}}{dt} - \bar{q}\times\bar{\zeta} = -\nabla\left(\Omega - \int\frac{dp}{\rho}. - \frac{1}{2}q^2\right)$$

$$\frac{d\bar{q}}{dt} - \bar{q}\times\bar{\zeta} = -\nabla\Psi$$

Where $\Psi = \int\frac{dp}{\rho} + \Omega + \frac{1}{2}q^2$

When the motion is steady $\frac{\partial\bar{q}}{\partial t} = 0$

$$\bar{q}\times\bar{\zeta} = \nabla\Psi$$

$\bar{q}\times\bar{\zeta}$ is normal to the surface Ψ

In this surface Ψ is constant

$$\int\frac{dp}{\rho} + \Omega + \frac{1}{2}q^2 = constant$$

This is known as Bernoulli's equation for fluid in steady motion.

NOTE 2:

Derive the Bernoulli's equation of motion for an incompressible fluid

PROOF:

Bernoulli's equation for fluid in steady motion is $\Psi = \int\frac{dp}{\rho} + \Omega + \frac{1}{2}q^2$ =constant

The given fluid is incompressible

$$\int\frac{dp}{\rho} = \frac{p}{\rho}$$

Bernoulli's equation for incompressible fluid is

$\frac{p}{\rho}+\Omega+\frac{1}{2}q^2$ is constant

CIRCULATION:

The line integral of the fluid velocity around of the fluid velocity the closed curve c is called the circulation.

$$\Gamma=\oint_C \bar{q}.dr$$

KELVIN'S THEOREM:

If fluid is inviscid and the force are conservative then circulation on any closed curve moving with the fluid is constant for all the time.

PROOF:

First we want to prove the following lemma.

LEMMA:

The necessary and sufficient condition for the constant c of circulation in a closed curve moving with the fluid is $\nabla\times a=0$.

PROOF:

We know that $\bar{a}=\frac{d\bar{q}}{dt}$........(1) and the circulation is

$$\Gamma=\oint_C \bar{q}.dr..............(2)$$

$$\frac{d\Gamma}{dt}=\frac{d}{dt}\oint_C \bar{q}.dr$$

$$\frac{d\Gamma}{dt}=\oint_C \frac{d\bar{q}}{dt}.d\bar{r}+\oint_C \bar{q}\,\frac{d}{dt}.d\bar{r}$$

$$=\oint_C \frac{d\bar{q}}{dt}.d\bar{r}$$

Using stroke's theorem

$$\oint_C \bar{F}.d\bar{r}=\oint_S curl\bar{F}\bar{n}ds$$

$$\oint_C \bar{F}.d\bar{r}=\oint_S curl\,\frac{d\bar{q}}{dt}\,\bar{n}ds$$

$$\frac{d\Gamma}{dt}=\oint_S curl\bar{a}\bar{n}ds.................(3)$$

From equation (3) it follows that necessary and sufficient condition for constant c of circular in a closed for constant C of circular in a closed curve moving with the velocity is

$$curl\bar{a}=0$$

$$\nabla\times a=0$$

Hence the lemma

PROOF OF THE THEOREM:

Equation of motion for an inviscid fluid is

$$\frac{d\bar{q}}{dt}=\bar{F}-\frac{1}{\rho}\nabla p$$

Here the forces are conservative

$$\bar{F}=-\nabla\Omega$$

Sub this value in the above equation

$$\frac{d\bar{q}}{dt} = -\nabla\Omega - \frac{1}{\rho}\nabla p$$

$$\bar{a} = -\nabla\Omega - \frac{1}{\rho}\nabla p$$

Here $\bar{a}$ is acceleration value.

Taking curl on both sides

$$\nabla \times a = \nabla \times \left(-\nabla\Omega - \frac{1}{\rho}\nabla p \right)$$

$$= \nabla \times \frac{1}{\rho}\nabla p$$

$$= -\left[\nabla \times \frac{1}{\rho}\nabla p + \frac{1}{\rho}\nabla \times \nabla p \right]$$

$$= -\left[\nabla \times \frac{1}{\rho} \times \nabla p \right]$$

$$\nabla \times a = \nabla p \times \frac{1}{\rho}\nabla p \text{..........(4)}$$

CASE 1:

For an incompressible fluid ρ is constant then equation 4 becomes $\nabla \times a = 0$

CASE 2:

For compressible fluid ρ is a function of p.

Let $\frac{1}{\rho} = f(P)$

$$\nabla \frac{1}{\rho} = \nabla\left[f(P)\right]$$

$$= i\frac{\partial}{\partial x} f(P) + j\frac{\partial}{\partial y} f(P) + k\frac{\partial}{\partial z} f(P)$$

$$= i\frac{\partial f}{\partial p}\frac{\partial p}{\partial x} + j\frac{\partial f}{\partial p}\frac{\partial p}{\partial y} + k\frac{\partial f}{\partial p}\frac{\partial p}{\partial z}$$

$$= f'(p)\left[i\frac{\partial p}{\partial x} + j\frac{\partial p}{\partial y} + k\frac{\partial p}{\partial z}\right]$$

$$\nabla \frac{1}{\rho} = f'(p)\nabla p \text{.........}(5)$$

Use (5) in (4)

$$\nabla \times a = \nabla p \times f'(p)\nabla p$$

$$\nabla \times a = 0$$

If either ρ is a constant or ρ is the function of p

We have $\nabla \times a = 0$

From the lemma we can say

$\frac{d\Gamma}{dt} = 0$ Γ is a constant

Hence the fluid is inviscid and the forces are conservative then circulation on any closed curve moving with fluid is constant for all the time.

Hence proved

BOOK WORK:

Derive the equation of motion in Cartesian co-ordination when the force are conservative

PROOF:

The equation of motion for an inviscid fluid is

$$\frac{d\bar{q}}{dt} = \bar{F} - \frac{1}{\rho}\nabla p \text{..........(1)}$$

Here the forces are conservative

$$\bar{F} = -\nabla\Omega \text{...........(2)}$$

And we know that

$$\frac{d\bar{q}}{dt} = \frac{d\bar{q}}{dt} + (\bar{q}.\nabla\bar{q}) \text{.........(3)}$$

By (1)(2) and (3)

$$\frac{d\bar{q}}{dt} + (\bar{q}.\nabla\bar{q}) = \bar{F} - \frac{1}{\rho}\nabla p$$

$$= -\nabla\Omega - \frac{1}{\rho}\nabla p$$

Let $\bar{q} = u\vec{i} + v\vec{j} + w\vec{k}$

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\frac{\partial(u\vec{i} + v\vec{j} + w\vec{k})}{\partial t} + \left((u\vec{i} + v\vec{j} + w\vec{k})\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\right)(u\vec{i} + v\vec{j} + w\vec{k})$$

$$= -\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\Omega - \frac{1}{\rho}\nabla p$$

$$\frac{\partial(u\vec{i} + v\vec{j} + w\vec{k})}{\partial t} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)(u\vec{i} + v\vec{j} + w\vec{k})$$

$$= -\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\Omega - \frac{1}{\rho}\nabla p$$

By equation the co-efficient I,j,k

We get

$$\frac{\partial u}{\partial t} + \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial \Omega}{\partial x} - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial \Omega}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} + \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial \Omega}{\partial w} - \frac{1}{\rho}\frac{\partial p}{\partial w}$$

ENERGY EQUATION:

STATEMENT:

The rate of change of total energy of any portion of a inviscid fluid as it moves about is equal to the rate of at which working is being done by a pressure on the boundary.

PROOF:

Consider any arbitrary closed surface S drawn in the region occupied by the inviscid fluid and let v be the volume of the fluid with in s.

Let ρ be the density of the fluid particle p and dv be the volume element surrounding p.

Let $q(r,t)$ be the velocity of p then the Euler equation of motion.

$$\frac{dq}{dt} = F - \frac{\nabla p}{\rho}$$

The force is conservative

$F = -\nabla\Omega$

Sub $F = -\nabla\Omega$ in

$$\frac{dq}{dt} = -\nabla\Omega - \frac{\nabla p}{\rho} ....(1)$$

Multiplying both sides ρq

$$\rho q.\frac{dq}{dt} = -\rho q\nabla\Omega - \rho q\frac{\nabla p}{\rho}$$

$$\rho q.\frac{dq}{dt} = -\rho q\nabla\Omega - q\nabla p.............(2)$$

$$\frac{d}{dt}(q.q) = q\frac{dq}{dt} + q\frac{dq}{dt}$$

$$= 2q\frac{dq}{dt}$$

$$\frac{1}{2}\frac{d}{dt}(q^2) = q\frac{dq}{dt}..............(3)$$

Sub (3) in (2)

$$\frac{1}{2}\rho\frac{d}{dt}(q^2) = -\rho(q.\nabla)\Omega - q\nabla p$$

$$\frac{d\Omega}{dt} = \frac{d\Omega}{dt} + (q.\nabla)\Omega$$

$$\frac{1}{2}\rho\frac{d}{dt}(q^2) = -\rho\frac{d\Omega}{dt} - q\nabla p$$

$$\frac{1}{2}\rho\frac{d}{dt}(q^2) + \rho\frac{d\Omega}{dt} = -q\nabla p$$

$$\rho \frac{d}{dt}\left(\frac{1}{2}q^2 + \Omega\right) = -q\nabla p$$

$$\int_v \rho \frac{d}{dt}\left(\frac{1}{2}q^2 + \Omega\right) dv = -\int_v q\nabla p dv$$

$$\int_v \rho \frac{d}{dt}\left(\frac{1}{2}q^2\right) dv + \int_v \rho \frac{d}{dt}\Omega dv = -\int_v q\nabla p dv$$

Let $T = \int_v \rho \frac{d}{dt}\left(\frac{1}{2}q^2\right) dv$

$$v = \int_v \rho \frac{d}{dt}\Omega dv$$

$$I = \int_v E\rho dv$$

$$\nabla(pq) = p\nabla q + q\nabla p$$

$$(q.\nabla)p = \nabla(pq) - p\nabla q \ldots\ldots\ldots\ldots\ldots(5)$$

Use (5) in (4)

$$\int_v \rho \frac{d}{dt}\left(\frac{1}{2}q^2\right) dv + \int_v \rho \frac{d}{dt}\Omega dv = -\int_v \left(\nabla(pq) - p\nabla q\right) dv$$

$$\int_v \rho \frac{d}{dt}\left(\frac{1}{2}q^2\right) dv + \int_v \rho \frac{d}{dt}\Omega dv = -\int_v \nabla(pq) dv + \int_v \left(p\nabla q\right) dv$$

$$\frac{d}{dt}(T+V) = \int_S p(q\hat{n})dS + \int_v \left(p\nabla q\right) dv$$

To prove: $\int_v \left(p\nabla q\right) dv = \frac{dI}{dt}$

Suppose e is defined as the work done by the unit mass of the fluid against external pressure p in which p_0 and p_0 are the values of the pressure and density respectively.

$$E = \int_{v_0}^{v} p dv$$

$$= \int_{p_0}^{p} p d\left(\frac{1}{p}\right)$$

$$= -\int_{p}^{p_0} p\left(-\frac{1}{p^2}\right) dp$$

$$= -\int_{p}^{p_0} \frac{p}{p^2} dp$$

$$\frac{dE}{dt} = \frac{dE}{dp} \times \frac{dp}{dt}$$

$$\frac{dE}{dt} = \frac{p}{p^2} \times \frac{dp}{dt}$$

Multiplying both sides by pdv

$$\frac{dE}{dt} pdv = \frac{p}{p^2} \frac{dp}{dt} pdv$$

$$\frac{dE}{dt} pdv = \frac{p}{p} \frac{dp}{dt} dv$$

Integrating

$$\int_{v} \frac{dE}{dt} pdv = \int_{v} \frac{p}{p} \frac{dp}{dt} dv$$

$$\int_v \frac{d}{dt}(Epdv) = \int_v \frac{p}{p}\frac{dp}{dt}dv \ldots\ldots\ldots\ldots(6)$$

From the equation of continuity

$\frac{dp}{dt} + p(\nabla.q) = 0$ we have

$$\frac{dp}{dt} = -p(\nabla.q)$$

Use (7) in (6)

$$\int_v \frac{d}{dt}(Epdv) = \int_v \frac{p}{p}\left(-p(\nabla.q)\right)dv$$

$$= -\int_v p(\nabla.q)dv$$

$$\int_v \frac{d}{dt}(Epdv) = -\int_v p(\nabla.q)dv$$

$$= -\frac{dI}{dt}$$

$$\frac{d}{dt}(T+V) = \int_S p(q\hat{n})dS - \frac{dI}{dt}$$

$$\frac{d}{dt}(T+V) + \frac{dI}{dt} = \int_S p(q\hat{n})dS$$

$$\frac{d}{dt}(T+V+I) = \int_S p(q\hat{n})dS$$

This shoes that rate of change of total energy of position of the fluid as it moves about is equal to the rate of working done by the pressure on the boundary.

BOOK WORK:

Show that magnitude of the vorticity multiplied by the cross sectional area along the filament is a constant.

Or

Show that vertex filaments if cannot terminate at a point within the fluid.

Or

Show that vertex filament must be either closed or terminated at the boundary.

PROOF:

Consider the volume of the fluid enclosed between two cross sectional area $d\sigma_1$ and $d\sigma_2$ of the vertex filament

Consider $\int_S \bar{\zeta}.\bar{n}ds = \int_v \nabla\bar{\zeta}dv$

$\int_S \bar{\zeta}.\bar{n}ds = \int_v \nabla(\nabla\times\bar{q})dv$

$\int_S \bar{\zeta}.\bar{n}ds = 0$

$\bar{\zeta}.\bar{n} = 0$ on the walls of the filament

Then we have $\bar{\zeta}_1.\bar{n}_1 = 0$

$\bar{\zeta}_2.\bar{n}_2 = 0$

At the place of the cross sectional areas the above equation becomes

$\bar{\zeta}_1.\bar{n}_1 d\sigma_1 = 0$

$\bar{\zeta}_2.\bar{n}_2 d\sigma_2 = 0$

Where $\bar{\zeta}_1$ and $\bar{\zeta}_2$ are the vertices at the end of the filaments whose cross sectional areas are $d\sigma_1$ and $d\sigma_2$.

$\bar{n}_1$ and $\bar{n}_2$ be the unit normal vectors

Then the magnitude value is

$$\left|\bar{\zeta}_1\right|.\left|\bar{n}_1\right|\left|d\sigma_1\right| = \left|\bar{\zeta}_2\right|.\left|\bar{n}_2\right|\left|d\sigma_2\right|$$

$$\Rightarrow \left|\bar{\zeta}_1\right|d\sigma_1 = \left|\bar{\zeta}_2\right|d\sigma_2$$

HELMHOLTZ THEOREM:

Derive Helmholtz equation in the form

$$\frac{d}{dt}\left(\frac{\bar{\zeta}}{p}\right) = \left(\frac{\bar{\zeta}}{p}\nabla\right)\bar{q}$$

PROOF:

We know that $\bar{a} = \frac{d\bar{q}}{dt}$

$$\bar{a} = \frac{d\bar{q}}{dt} - \bar{q}\times\bar{\zeta} + \frac{1}{2}\nabla\bar{q}^2$$

$$\nabla\times\bar{a} = \nabla\left[\frac{d\bar{q}}{dt} - \bar{q}\times\bar{\zeta} + \frac{1}{2}\nabla\bar{q}^2\right]$$

$$= \nabla\frac{d\bar{q}}{dt} - \nabla\left(\bar{q}\times\bar{\zeta}\right) + \nabla\frac{1}{2}\nabla\bar{q}^2$$

$$= \frac{\partial}{\partial t}(\nabla\times\bar{q}) - \nabla\left(\bar{q}\times\bar{\zeta}\right)$$

$$= \frac{\partial}{\partial t}(\nabla \times \bar{q}) - \left[(\bar{\zeta}.\nabla)\bar{q} - (\bar{q}.\nabla)\bar{\zeta} - \bar{\zeta}(\nabla \bar{q}) + \bar{q}(\nabla \bar{\zeta})\right]$$

$$= \frac{\partial}{\partial t}(\nabla \times \bar{q}) - (\bar{\zeta}.\nabla)\bar{q} + (\bar{q}.\nabla)\bar{\zeta} + \bar{\zeta}(\nabla \bar{q}) - \bar{q}(\nabla \bar{\zeta})$$

$$= \frac{\partial}{\partial t}(\nabla \times \bar{q}) - (\bar{\zeta}.\nabla)\bar{q} + (\bar{q}.\nabla)\bar{\zeta} + \bar{\zeta}(\nabla \bar{q}) - 0$$

$$= \frac{d\bar{\zeta}}{dt} - (\bar{\zeta}.\nabla)\bar{q} + \bar{\zeta}(\nabla \bar{q})$$

We know that $\nabla \bar{q} = -\frac{1}{\rho}\frac{d\rho}{dt}$

$$\nabla \times a = \frac{d\bar{\zeta}}{dt} - (\bar{\zeta}.\nabla)\bar{q} + \bar{\zeta}\left(-\frac{1}{\rho}\frac{d\rho}{dt}\right)$$

$$\nabla \times a = \frac{d\bar{\zeta}}{dt} - (\bar{\zeta}.\nabla)\bar{q} - \bar{\zeta}\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$$

$$\nabla \times a = \rho\frac{d}{dt}\left(\frac{\bar{\zeta}}{\rho}\right) - (\bar{\zeta}.\nabla)\bar{q}$$

$$\rho\frac{d}{dt}\left(\frac{\bar{\zeta}}{\rho}\right) = (\nabla \times a) + (\bar{\zeta}.\nabla)\bar{q}$$

$$\frac{d}{dt}\left(\frac{\bar{\zeta}}{\rho}\right) = \frac{1}{\rho}\left[(\nabla \times a) + (\bar{\zeta}.\nabla)\bar{q}\right] \text{..........(1)}$$

Hence the equation (1) indicates the rate of change of $\frac{\bar{\zeta}}{\rho}$

If the force are consecutive and pressure is a function of density

$$\bar{a} = \frac{d\bar{q}}{dt} = -\nabla\left[\int \frac{dp}{\rho} + \Omega\right]$$

Taking curl on both sides

$$\nabla \times \bar{a} = \nabla \times \frac{d\bar{q}}{dt}$$

$$= \nabla \times -\nabla$$

$$= 0$$

$\nabla \times \bar{a} = 0$ ……..(2)

Sub (2) in (1)

$$\frac{d}{dt}\left(\frac{\bar{\zeta}}{\rho}\right) = \frac{1}{\rho}\left[(\nabla \times a) + (\bar{\zeta}.\nabla)\bar{q}\right]$$

$$\frac{d}{dt}\left(\frac{\bar{\zeta}}{\rho}\right) = \frac{1}{\rho}\left[0 + (\bar{\zeta}.\nabla)\bar{q}\right]$$

$$\frac{d}{dt}\left(\frac{\bar{\zeta}}{\rho}\right) = \frac{1}{\rho}\left[(\bar{\zeta}.\nabla)\bar{q}\right]$$

Hence the proof

NOTE:

In the case of liquid $\nabla.\bar{q} = 0$ and so $\bar{q}$ becomes solenoidal we also know that $\overline{\Omega}$ is also a solenoidal.

$$\nabla \times \overline{\Omega} = 0$$

UNIT 2

POSSIBLE QUESTIONS

PART- B (6 MARKS)

1. Show that the vortex filaments must be either closed or terminate at the boundary
2. If w is the area of the cross- section of a stream filament prove that the equation of continuity is $\frac{\partial}{\partial t}(\rho w)+\frac{\partial}{\partial s}(\rho wq)=0$ where ∂s an element of arc of the filament in is the direction of the flow and q is the speed.
3. State and prove the Euler's momentum theorem
4. Show that the mass of the particle remain unaltered as it moves.
5. Derive the equation of motion in the form $\frac{d\bar{q}}{dt}=-\nabla\left[\int\frac{dp}{p}+\Omega\right]$ where the force is conservative and derived from potential Ω and the pressure is the function of density.
6. Derive Helmholtz equation in the form

$$\frac{d}{dt}\left(\frac{\bar{\zeta}}{p}\right)=\left(\frac{\bar{\zeta}}{p}\nabla\right)\bar{q}$$

7. Show that magnitude of the vorticity multiplied by the cross sectional area along the filament is a constant.
8. Derive the equation of motion in Cartesian co-ordination when the force are conservative
9. State and prove energy equation.
10. Show that vertex filaments if cannot terminate at a point within the fluid.
11. Show that vertex filament must be either closed or terminated at the boundary.
12. Derive the necessary and sufficient condition for the constant c of circulation in a closed curve moving with the fluid is $\nabla\times a=0$.

13. State and prove if fluid is inviscid and the force are conservative Then circulation on any closed curve moving with the fluid is constant for all the time.
14. Derive the equation of motion.
15. Explain the concept of rate of change of circulation.

PART-C (10 MARKS)

1. Find the equation of motion of an inviscid fluid
2. Find the rate of change of circulation
3. Show that the rate of change of total energy of any portion of the fluid as it moves about is equal to the rate of working of the pressures on the boundary
4. Show that the rotational motion permanent and so is irrotational motion
5. Show that he equation of motion in the form $\frac{d\bar{q}}{dt} = -\nabla\left[\int \frac{dp}{p} + \Omega\right]$ is the function of density.
6. Show that vertex filament must be either closed or terminated at the boundary.
7. Show that the rate of change of total energy of any portion of a inviscid fluid as it moves about is equal to the rate of at which working is being done by a pressure on the boundary.
8. State and prove Kelvin's theorem

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Subject: Fluid Dynamics **Subject Code: 18MMP206**
Class : I - M.Sc. Mathematics **Semester : II**

Unit II

Part A (20x1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)
Possible Questions

Questions	Opt 1	Opt 2	Opt 3	Opt 4	Answer
A force is said to be -------- if the force can be derivable from the potential.	conservative	non conservative	acceleration	surface	conservative
A flow is called a Beltrami's flow when---	q.E=0	q*E=0	q/E=0	q+E=0	q*E=0
Bernoulli's equation occurs when the motion is--	unsteady	rotational	steady	irrotational	steady
The ---- ------ flow can occurs when the vertex and stream lines coincide	viscous flow	beltrami's flow	invisid flow	normal flow	beltrami's flow
When the motion is both steady and irrotational then---	$\nabla .E$	$\nabla *E$	$\nabla +E$	$\nabla -E$	$\nabla .E$
The product of the cross sectional area and magniyude of the vorticity is -------- along a vortex filament	parallel	zero	constant	normal	constant
When the forces are conservative and the pressure is a function of the density,then--------	$\nabla .a =0$	$\nabla *a=0$	$\nabla +a =0$	$\nabla -a=0$	$\nabla .a =0$
When a force is conservative,there exist a potentialΩ such that f=	$f=\nabla\Omega$	**$f=-\nabla \Omega$**	$f=- \nabla *\Omega$	$f= \nabla *\Omega$	**$f=-\nabla \Omega$**
circulation around a closed circuit 'c' is defined as	$\int q.rdr$	$\int q.dr$	$\int qx.rdr$	$\int qx+dr$	$\int q.dr$
Euler's equation of motion is	$dq/dt=F-\nabla P$	$dq/dt=F$	$dq/dt=F-\nabla p/P$	$qd/dt=-\nabla\Omega$	$dq/dt=F-\nabla p/P$
----------- from is called the acceleration potential	$\Omega-\int \partial P/ \rho$	$\nabla[\int \partial P/ \rho] +dp$	$\nabla[\int \partial P/ \rho]$	$\Omega+\int \partial P /p$	$\Omega+\int \partial P /p$
Beltram's flow is ---------	$\partial q/ \partial t=\nabla$	$\partial q/ \partial t=-\nabla$	$\partial q/ \partial t=-\Omega\nabla$	$\partial q/ \partial t=-\nabla \rho /p$	$\partial q/ \partial t=-\nabla$
q*E=0 can become zero when $E \neq 0$,but q*E can be to each other	parallel	non parallel	zero	normal	parallel
The motion is both steady and irrotational if	$\nabla.\psi\neq0$	$\nabla+\psi =0$	$\nabla .\psi =0$	$\nabla *a=0$	$\nabla .\psi =0$
Which is the constant of kelvin's theorem	a	ρ	B	ψ	ρ
Circulation is always defined around a ---------- ciruit	open	parallel	closed	normal	closed
When a conservative force f a potentialΩ such that	$F=\nabla\Omega$	$F=-\nabla\Omega$	$F\neq\nabla *\Omega$	$F\neq\nabla .\Omega$	$F=-\nabla\Omega$
The euler's equation of motion corresponding to a beltrami's flow is	$\partial q/\partial t=-\nabla \psi$	$\partial q/\partial t=-\nabla \psi$	$\partial q/\partial t=-\nabla * \psi$	$\partial q/\partial t\neq-\nabla \psi$	$\partial q/\partial t=-\nabla \psi$
A force is said to be conservative if the force can be derivable from the ---------	potential	density	area	viscosity	potential
The euler's theory is confined only for ideal or inviscid fluid	viscid	stream	inviscid	fluid	inviscid

The rate of change of linear momentum is equal to the ------- of the forces acting on a body	sum	product	proportional	difference	sum
the inward normal is --------	ρ	q	n^	F	n^
The rate of change of momentum of the fluid body is given by---	d/dt(cir c)=∫B.n ds	d/dt(cir c)=∫n ds	d/dt(cir c)=∫B.n dc	d/dt(cir c)=∫n dc	d/dt(cir c)=∫B.n ds
The -------- is the motion the rate of change of linear momentum =the sum of the forces acting on the body	Kelvin's theorem	Energy equation	Newton's second law	Euler's theorem	Newton's second law
rate of change of circulation is	ð/ðt(cir c)= ∫b.nds	ð/ðt(cir c)= ∫q.dr	ð/ðt(cir c)= ∫dq/dt.dr	ð/ðt(cir c)= ∫a.dr	ð/ðt(cir c)= ∫b.nds
Accelaration is given by	a=dm/dt	a=dq/dt	a=dr/dt	a=dc/dt	a=dq/dt
The ------ is the internal energy per unit mass	E	F	r	a	E
Density of a fluid is denoted by	F	ρ	a	E	ρ
In Red wood viscometer	Absolute value of viscosity is detemiined	Part of the head of fluid is utilized inOvercoming	Fluid discharges through orifice with negligible velocity	Comparison of viscosity is done.	Comparison of viscosity is done.
Centre of buoyancy is	The point of intersection of buoyant force and centre line of the body	Centre of gravity of the body	Centric of displaced volume fluid	Midpoint between C.G. and metacentric.	Centric of displaced volume fluid
In isentropic flow; the temperature	Cannot exceed the reservoir temperature	Cannot drop and again increase downstream	Is independent of Match number	Is a function of Match number only	Cannot exceed the reservoir temperature
A stream line is	The line of equal velocity in a flow	The line along which the rate of pressure drop is uniform	The line along the geometrical centre of the flow	Fixed in space in steady flow.	Fixed in space in steady flow.
The flow of water in a pure of diameter 3000mm can be measured by	Venturimeter	Rotameter	Pilot tube	Orifice plate	Pilot tube
Apparent shear forces	Can never occur in frictionless fluid regardless of its motion	Can never occur when the fluid is at rest	Depend upon cohesive forces	All of the above	All of the above
Weber number is the ratio of	Inertial forces to surface tension	Inertial forces to viscous forces	Elastic forces to pressure forces	Viscous forces to gravity	Inertial forces to surface tension
A small plastic boat loaded with pieces of steel rods is floating in a bath tub. If the cargo is dumped into the water allowing the both to float empty, the water level in the tub will water level in the tub will	Rise	Fall	Remains same	Rise and then fall	Fall
A flow in which each liquid particle has a definite path and their paths do not cross each other, is called	Steady flow	Uniform flow	Streamline flow	Turbulent flow	Streamline flow
Buoyant force is	Resultant of up thrust and gravity forces acting on the body	Resultant force on the body due to the fluid surrounding it	Resultant of static weight of body and dynamic thrust of fluid	Equal to the volume of liquid displaced by the body	Equal to the volume of liquid displaced by the body

UNIT III

Two Dimensional Motion – Two Dimensional Functions – Complex Potential – basic singularities – source – sink – Vortex – doublet – Circle theorem. Flow past a circular cylinderwith circulation – Blasius Theorem – Lift force. (Magnus effect)

TWO DIMENSIONAL MOTIONS

When the lines of motion are parallel to the fixes plane z=0 and the velocity at the corresponding points of all planes are parallel to z=0 has same magnitude and direction. Then the motion is said to be two dimensional motion.

EXAMPLE:

Stream function or of current function potential function or velocity function.

Two dimensional function:

STREAM FUNCTION OR CURRENT FUNCTION:

In 2D motion velocity vector q is the function of (x,y,t).

Hence the differential equation of stream line is given by $\frac{dx}{u}=\frac{dy}{v}$

$$udy-vdx=0..............(1)$$

Equation of continuity for an incompressible fluid in 2D motion is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.........(2)$$

In differential equation

We know that $Mdx+Ndy=0$

$$\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0$$

Equation (2) is condition for equation (1) to be exact

Thus there exists a function $\Psi(x,y)$ such that

$$d\Psi = udy - vdx$$

$$\frac{\partial \psi}{\partial x}+\frac{\partial \psi}{\partial y}=udy-vdx$$

$$u=\frac{\partial \psi}{\partial x};v=-\frac{\partial \psi}{\partial y}$$

Here the function $\Psi(x,y)$ is called stream function or current function.

POTENTIAL FUNCTION:

In case of irrotational curl q=0

$$\nabla\times q=\begin{vmatrix} i & j & k \\ \frac{\partial}{dx} & \frac{\partial}{dy} & \frac{\partial}{dz} \\ u & v & 0 \end{vmatrix}$$

$$\Rightarrow i\left(0-v\frac{\partial}{dz}\right)-j\left(0-u\frac{\partial}{dz}\right)+k\left(v\frac{\partial}{dx}-u\frac{\partial}{dy}\right)$$

$$\Rightarrow i\left(\frac{\partial v}{dz}\right)-j\left(\frac{\partial u}{dz}\right)+k\left(\frac{\partial v}{dx}-\frac{\partial u}{dy}\right)$$

$$=k\left(\frac{\partial v}{dx}-\frac{\partial u}{dy}\right)$$

$$\frac{\partial v}{dx}=\frac{\partial u}{dy}\text{...............}(1)$$

Equation (1) is the condition for the differential equation

$udy + vdx = 0$ to be exact there exists a function $\phi(x, y)$ such that

$$d\phi = udy + vdx$$

$$\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = udx + vdy$$

$$\frac{\partial \phi}{\partial x} = u$$

$$\frac{\partial \phi}{\partial y} = v$$

Hence the function $\phi(x, y)$ is called potential function or velocity potential.

NOTE:

Write down the relationship connecting velocity components of the stream function or potential function.

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

BOOK WORK:

Explain physical interpretation of stream function.

Let us consider the portion of the curve between A and B through the fluid.

Let p be any point on the curve at which the tangent makes an angle θ with OX.

Let u,v be the components of velocity at the point p(x,y) and let $q(x+\Delta x, y+\Delta y)$ be its neighbourhood points so that pq=ds

Component of u along the normal = $u\cos(90-\theta)$

$$=u\sin\theta$$

Components of v along the normal = $u\cos(90+90-\theta)$

$$=-v\cos\theta$$

Components of velocity along the normal $=u\sin\theta - v\cos\theta$

The amounts of flow access the elementary all.

$$ds=(u\sin\theta - v\cos\theta)ds$$

$$ds=u\sin\theta ds - v\cos\theta ds$$

$$=udy-vdx$$

The amount of flow access $AB=\int_A^B (udy-vdx)$

$$=\int_A^B d\psi$$

$$=[\psi]_A^B$$

$$=\psi_B-\psi_A$$

The difference is the value of stream function at difference point gives the element of flow access. The curve joining the two points.

PROBLEMS:

1. Express velocity components in terms of stream function in polar co-ordinates

Or

Show that in the polar co-ordinates $V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ and $V_\theta = -\frac{\partial \psi}{\partial r}$

Solution:

Hence $x = r\cos\theta$

$y = r\sin\theta$

$$\left.\begin{aligned}\frac{\partial x}{\partial r} &= \cos\theta \\ \frac{\partial y}{\partial r} &= \sin\theta\end{aligned}\right\}\text{.................}(1)$$

$$\left.\begin{aligned}\frac{\partial x}{\partial \theta} &= -r\sin\theta \\ \frac{\partial y}{\partial \theta} &= r\cos\theta\end{aligned}\right\}\text{.................}(2)$$

Also we know that the relation connecting velocity components of stream function and potential function

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}\text{...............}(3)$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}\text{...........}(4)$$

We know from dynamics. The radial components of velocity

$$V_r = \frac{\partial \phi}{\partial r}$$

$$V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

To find V_r

$$V_r = \frac{\partial\phi}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial\phi}{\partial y}\frac{\partial y}{\partial r}$$

$$= \frac{\partial\phi}{\partial x}\cos\theta + \frac{\partial\phi}{\partial y}\sin\theta$$

$$= \frac{\partial\phi}{\partial x}\frac{1}{r}\frac{\partial y}{\partial\theta} + \frac{\partial\phi}{\partial y}\left(-\frac{1}{r}\frac{\partial x}{\partial\theta}\right)$$

$$= \frac{1}{r}\frac{\partial\phi}{\partial x}\frac{\partial y}{\partial\theta} + \frac{1}{r}\frac{\partial\phi}{\partial y}\frac{\partial x}{\partial\theta}$$

Using eq (3) and (4)

$$= \frac{1}{r}\left(\frac{\partial\psi}{\partial y}\frac{\partial y}{\partial\theta}\right) + \frac{1}{r}\left(\frac{\partial\psi}{\partial x}\frac{\partial x}{\partial\theta}\right)$$

$$V_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

To find V_θ

$$V_r = \frac{\partial\phi}{\partial r}$$

$$V_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta}$$

$$= \frac{1}{r}\left[\frac{\partial\phi}{\partial x}\frac{\partial y}{\partial\theta} + \frac{\partial\phi}{\partial y}\frac{\partial x}{\partial\theta}\right]$$

$$= \frac{1}{r}\left[\frac{\partial\phi}{\partial x}(-r\sin\theta) + \frac{\partial\phi}{\partial y}(r\cos\theta)\right]$$

$$= -\frac{\partial \phi}{\partial x}(\sin\theta) + \frac{\partial \phi}{\partial y}(\cos\theta)$$

$$V_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$$

2. Show that stream function ψ and the potential function ϕ satisfies the Laplace equation.

Solution:

The relation correction velocity components of stream function and potential function

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}\text{................(1)}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{\partial \phi}{\partial y}\text{...........(2)}$$

The equation continuity for an incompressible fluid in 2D motion is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\text{................(3)}$$

Use eqn (1) and (2) in (3)

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right) = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

$$\nabla^2 \phi = 0$$

The velocity potential ϕ satisfies the Laplace Eqn (1) and (2)

$$\frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial^2 \psi}{\partial y^2}\text{............(4)}$$

$$\frac{\partial^2 \phi}{\partial x \partial y} = -\frac{\partial^2 \psi}{\partial x^2} \text{............(5)}$$

$$\frac{\partial^2 \phi}{\partial y^2} = -\frac{\partial^2 \psi}{\partial x^2}$$

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2} = 0$$

STREAM FUNCTION SATISFIES THE LAPLACE EQUATION.

PROBLEMS:

1. Investigate the orthogonality of two families of curves $\phi(x, y)$ =constant $\psi(x, y)$ =constant

Solution:

$\phi(x, y)$ =constant

$\psi(x, y)$ =constant

$\phi(x, y)$ =constant

Differential $\frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = 0$

$$\frac{\partial \phi}{\partial x} dx = -\frac{\partial \phi}{\partial y} dy$$

$$\frac{dy}{dx} = -\frac{\frac{\partial \phi}{\partial x}}{\frac{\partial \phi}{\partial y}}$$

$$\frac{dy}{dx} = -\frac{u}{v}$$

$\psi(x, y)$ =constant

Differential $\frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = 0$

$$\frac{\partial \psi}{\partial x}dx = -\frac{\partial \psi}{\partial y}dy$$

$$\frac{dy}{dx} = -\frac{\frac{\partial \psi}{\partial x}}{\frac{\partial \psi}{\partial y}}$$

$$\frac{dy}{dx} = \frac{v}{u}$$

$$= -\frac{u}{v} \times \frac{v}{u}$$

$$= -1$$

They intersect orthogonally.

C-R equations from stream function and potential function we have

$$\frac{\partial \phi}{\partial x} = u$$

$$\frac{\partial \phi}{\partial y} = v$$

$$\frac{\partial \psi}{\partial x} = u$$

$$-\frac{\partial \psi}{\partial y} = v$$

Equation u

$$\frac{\partial \phi}{\partial y} = v$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\phi_x = \psi_y$$

Equating v we get

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$\phi_x = -\psi_y$$

This is known as C-R equation

2. The velocity filed is given by $q = -x\hat{i} + (y+t)\,j$ find the stream function and stream line for this field at t=2.

Solution:

$$q = -x\hat{i} + (y+t)\,j$$

To find stream function

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$$

From the equation

$$u = -x$$

$$v = y + t$$

$$d\psi = -vdx + udy$$

$$d\psi = -(y+t)dx - xdy$$

Integrate the equation

$$\int d\psi = \int -(y+t)dx - xdy$$

$$= -y$$

Put t=2

$$\int d\psi = \int -y - tdx - \int xdy$$

$$= -yx - tx - xy$$

$$= -2xy - tx$$

$$= -2xy - 2x$$

$$\psi = -2x(y+1)$$

Stream line

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{-x} = \frac{dy}{(y+t)}$$

Integrate

$$\int \frac{dx}{-x} = \int \frac{dy}{(y+t)}$$

$$-\log x = \log(y+t) + \log c$$

$$-x = (y+t) + c$$

Put t=2

$$-x = (y+2)c$$

$$-x = 2y + 2c$$

$$x + 2y + 2c = 0$$

3. A two dimensional flow is given by $\psi = xy$ then show that is irrotational.

Solution:

To prove this flow is irrotational then $\nabla \times q = 0$

$$\nabla \times q = \begin{vmatrix} i & j & k \\ \frac{\partial}{dx} & \frac{\partial}{dy} & \frac{\partial}{dz} \\ u & v & 0 \end{vmatrix}$$

$$\Rightarrow i\left(0 - v\frac{\partial}{dz}\right) - j\left(0 - u\frac{\partial}{dz}\right) + k\left(v\frac{\partial}{dx} - u\frac{\partial}{dy}\right)$$

$$\Rightarrow i\left(\frac{\partial v}{dz}\right) - j\left(\frac{\partial u}{dz}\right) + k\left(\frac{\partial v}{dx} - \frac{\partial u}{dy}\right)$$

$$= k\left(\frac{\partial v}{dx} - \frac{\partial u}{dy}\right)$$

$$\frac{\partial v}{dx} = \frac{\partial u}{dy} \ldots\ldots\ldots\ldots\ldots(1)$$

$$u = \frac{\partial \psi}{\partial y}$$

$$u = \frac{\partial}{\partial y}(xy)$$

$$u = x$$

$$v = -\frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial}{\partial x}(xy)$$

$$v = -y$$

Sub u and v in

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

$$-\frac{\partial y}{\partial x} = \frac{\partial x}{\partial y}$$

$$= 0$$

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial(-y)}{\partial x} - \frac{\partial x}{\partial y} = 0$$

$$0 = 0$$

It is irrotational

4. Obtain C-R equation of complex potential in polar form

Solution:

Let w be the analytic function of z

W=f(z)

$\phi + i\psi = f(re^{i\theta})$.........(1)

$$\frac{\partial \phi}{\partial r} + i\frac{\partial \psi}{\partial r} = f'(re^{i\theta}).e^{i\theta} \text{.............(2)}$$

$$\frac{\partial \phi}{\partial \theta} + i\frac{\partial \psi}{\partial \theta} = f'(re^{i\theta})ri.e^{i\theta}$$

$$\frac{\partial \phi}{\partial \theta} + i\frac{\partial \psi}{\partial \theta} = r\left[\frac{\partial \phi}{\partial r} + i\frac{\partial \psi}{\partial r}\right]$$

$$\frac{\partial \phi}{\partial \theta} = -r\frac{\partial \psi}{\partial r}$$

$$\frac{\partial \psi}{\partial \theta} = -r\frac{\partial \phi}{\partial r}$$

$$\frac{\partial \phi}{\partial \theta} = \frac{1}{r}\frac{\partial \psi}{\partial r}$$

COMPLEX FUNCTION:

A complex function $w = \phi + i\psi$ where real and imaginary parts are velocity potential and stream function this w is the complex potential of the fluid.

NOTE 1:

Since velocity potential ϕ and the stream function ψ satisfies the C-R equation.

$w = \phi + i\psi$ is an analytic function of $z(x+iy)$

NOTE 2:

Since $w = \phi + i\psi$

$$\frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x} = \frac{\partial w}{\partial x} = \frac{\partial w}{\partial z}.\frac{\partial z}{\partial x} = \frac{\partial w}{\partial z}$$

$$\frac{\partial w}{\partial z} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x}$$

$$\frac{\partial w}{\partial z} = u - iv$$

$$= qe^{i\theta}$$

Hence θ is the velocity direction relation to the real axis given by

$$\theta = -\arg\left(\frac{dw}{dz}\right)$$

NOTE 3:

Discuss the motion for the inverse function $z = c\cosh w$ where c is real positive number and w is the complex potential.

Solution:

We know that $z = x + iy$

$$z = c\cosh w$$

$$= \cosh(u + iv)$$

$$= c\cosh(\phi + i\psi)$$

$$= c(\cosh\phi \cosh i\psi + \sinh\phi \sinh i\psi)$$

$$= c(\cosh\phi \cosh\psi) + ci(\sinh\phi \sinh\psi)$$

$$x = c(\cosh\phi \cosh\psi)$$

$$y = c(\sinh\phi \sinh\psi)$$

$$x^2 = c^2(\cosh^2\phi \cosh^2\psi)$$

$$y^2 = c^2(\sinh^2\phi \sinh^2\psi)$$

$$\frac{x^2}{c^2\cosh^2\psi} = \cosh^2\phi$$

$$\frac{y^2}{c^2\sinh^2\psi} = \sinh^2\phi$$

$$\frac{x^2}{c^2\cosh^2\psi} - \frac{y^2}{c^2\sinh^2\psi} = \cosh^2\phi - \sinh^2\phi$$

For the different values of ψ we get the same focus

Hence the streamlines are confocal hyperbola.

BASIC SINGULARITY:

SOURCE:

Any point in the two dimensional motion where the fluid is assumed to be created is called source.

SINK:

Any point in the two dimensional motion where the fluid is assumed to be ignored is called sink.

DOUBLETS OR DOUBLE SOURCE OR DIPOLE:

The combination of the source of strength m and the sink of strength M at the distance δ apart such that $M\delta$ is finite is called doublets.

NOTE:

If the total flux act outwards as a small surface closed surrounding a point $2\pi m$. Here m is called the strength of the source.

BOOK WORK:

Obtain the complex potential of the flow due to a source of strength of origin.

Proof:

Let m be the strength of source placed at the origin.

Then the flex across the circle of unit radius

$r = 2\pi m$

For a source the flow is purely radial and symmetrical.

The flex across the circle of radius for satisfying the condition of continuity we have

$$2\pi m = 2\pi \frac{\partial \phi}{\partial r}$$

$$m = r\frac{\partial \phi}{\partial r}$$

$$\frac{\partial \phi}{\partial r} = \frac{m}{r}$$

Integrate w r t r we get

$$\int \frac{\partial \phi}{\partial r} = \int \frac{m}{r} dr$$

$$\phi = m \int \frac{1}{r} dr$$

$$\phi = m \log r$$

Also we know that

$$V_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

Here $\frac{\partial \phi}{\partial r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$

$$\frac{m}{r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$$

$$m = \frac{\partial \psi}{\partial \theta}$$

Integrate w r t to θ we get

$$\int m d\theta = \int \frac{\partial \psi}{\partial \theta}$$

$$\psi = m\theta$$

The complex potential of the flow due to a source of strength m is place at the origin is

$$w = \phi + i\psi$$

$$w = m \log r + im\theta$$

$$w = m(\log(r + i\theta))$$

$$w = m \log re^{i\theta}$$

$w = m \log z$

NOTE:

The complex potential of the flow due to a source of strength m placed at $a = \alpha + i\beta$ is $w = m\log(z - a)$

PROBLEMS:

1. Obtain the complex potential of the flow due to the doublet of strength m.

Proof:

We know that the complex potential of the flow due to the source of strength n placed at origin is $w = m\log z$

Hence sink of strength –m is placed at the origin

Its complex potential is

$w_1 = -m\log z$

Here source of strength n is placed at $\delta e^{i\alpha}$

Its complex potential is

$w_2 = m\log(z - \delta e^{i\alpha})$

The complex potential of the doublet is

$w = w_1 + w_2$

$w = -m\log z + m\log(z - \delta e^{i\alpha})$

$= m(\log z + \log(z - \delta e^{i\alpha}))$

$= m(\log z - \delta e^{i\alpha}) - \log(z))$

$$= m\left(\frac{(\log z - \delta e^{i\alpha})}{\log z}\right)$$

$$= m\frac{(\log(1-\delta e^{i\alpha})}{z}$$

$$w = \frac{-\mu e^{i\alpha}}{z}$$

BOOK WORK 4:

Obtain the complex potential of the flow due to rectilinear vertex filament or point vertex in two dimensional motion or symmetrical circulation flow about the point.

Proof:

If the stream line of the flow are concentric circle.

The radial component of the velocity will be zero and from the symmetrical conservation of the transverse component is $\frac{1}{r}\frac{\partial\phi}{\partial\theta}$

Let us suppose that the circulation around any stream line is constant say k

Circulation around the stream line of radius $r = 2\pi r\frac{1}{r}\frac{\partial\phi}{\partial\theta}$

$$= 2\pi\frac{\partial\phi}{\partial\theta}$$

$$k = 2\pi\frac{\partial\phi}{\partial\theta}$$

$$\frac{\partial\phi}{\partial\theta} = \frac{k}{2\pi}$$

Integrating on both sides

$$\int \frac{\partial \phi}{\partial \theta} = \int \frac{k}{2\pi} d\theta$$

$$\phi = \frac{k}{2\pi}\theta$$

We know that

$$\frac{\partial \phi}{\partial \theta} = -r\frac{\partial \psi}{\partial r}$$

$$\frac{k}{2\pi} = -r\frac{\partial \psi}{\partial r}$$

$$\frac{-k}{2\pi r} = \frac{\partial \psi}{\partial r}$$

Integrating w r t r

$$\int \frac{-k}{2\pi r} dr = \int \frac{\partial \psi}{\partial r} dr$$

$$\psi = \frac{-k}{2\pi}(\log r)$$

The complex potential of the flow due to vector linear vertex filament is

$$w = \phi + i\psi$$

$$= \frac{k}{2\pi}\theta + i\left(\frac{-k}{2\pi}(\log r)\right)$$

$$= \frac{k}{2\pi}(\theta - i\log r)$$

$$= -\frac{k}{2\pi}(i\log r + \theta)$$

$$= -\frac{ik}{2\pi}(\log r + i\theta)$$

$$= -\frac{ik}{2\pi}(\log re^{i\theta})$$

$$= -\frac{ik}{2\pi}(\log z)$$

THEOREM:

STATE AND PROVE CIRCLE THEOREM

OR

STATE AND PROVE MILNE THOMSON CIRCLE THEOREM:

STATEMENT:

Let f(z) be a complex potential for a flow having no rigid boundary and such that they we have no singularities of the flow within the circle $|z| = a$

Then on introducing the solid cylinder $|z| = a$ into the flow the new complex potential given by

$$w = f(z) + \bar{f}\left(\frac{a^2}{z}\right)$$

Proof:

Let c be the cross section of the circular cylinder $|z| = a$

$$z\bar{z} = a^2$$

$$\bar{z} = \frac{a^2}{z}$$

Hence for the points of the circle $|z| = a$ we have

$$w = f(z) + \bar{f}\left(\frac{a^2}{z}\right)$$

$$\phi + i\psi = f(z) + \bar{f}\left(\frac{a^2}{z}\right) \ldots\ldots\ldots\ldots\ldots(1)$$

Hence the quantity R.H.S equation (1) is purely real part $\psi = 0$

Hence c is the stream line in the net flow by the hypothesis of singularities of f(z) at which source sink doublet may be present lie outside the circle $|z| = a$ and so the singularities of $\bar{f}\left(\frac{a^2}{z}\right)$ lies inside the circle $|z| = a$

Here z and $\frac{a^2}{z}$ are inverse points with respect to the circle $|z| = a$

Thus we find the additional form $\bar{f}\left(\frac{a^2}{z}\right)$ introduce no singularities into the flow inside the circle $|z| = a$

Hence $|z| = a$ is the possible boundary for the new flow and $w = f(z) + \bar{f}\left(\frac{a^2}{z}\right)$ for $|z| \geq a$ is the approximate complex potential for the net flow.

Blasius theorem:

Statement:

Consider the steady two dimensional irrotational incompressible inviscid fluids under no external force. Let w=f(z) be the complex potential for the flow.

Consider the cylinder of any cross section c placed in the flow

Let (x,y) be the component of the force and M be the pitching movement (moment about the origin for the cylinder)

Then $X - iY = \frac{1}{2} i\rho \oint_C \left(\frac{dw}{dz}\right)^2 dz$

M= real part of- $\frac{1}{2} i\rho \oint_C \left(\frac{dw}{dz}\right)^2 dz$

Where ρ is the density of the fluid and the integrals are taken around the contour c of the cylinder

Proof:

Let p be the point on the curve c

Consider any element ds of R surrounding the point p of (x,y)

C denotes the boundary of the cylinder of any shape and size

Let the tangent at p makes an angle θ with x-axis so that invert normal at p will make an angle 90+θ with x-axis.

Now the force exerted on the cylinder is only the pressure force because given that there is no external force.

Pressure is the normal stress and the stress is nothing but force per unit area.

Hence force due to pressure of the length=pds

Let x and y are the component of force exerted on the cylinder in xy direction.

$$X = \oint_C p \cos\left(\frac{\pi}{2} + \theta\right) ds$$

$$= -\oint_C p \sin\theta ds$$

$$= -\oint_C p \frac{dy}{ds} ds$$

$$= -\oint_C p dy$$

$$Y = \oint_C p \cos\theta ds$$

$$= \oint_C p \frac{dx}{ds} ds$$

$$= \oint_C p dx$$

$$X - iY = -\oint_C p dy - i\oint_C p dx$$

$$= -\oint_C p(dy - idx)$$

$$= -i\oint_C p(dx - idy)$$

$$= -i\oint_C p d\bar{z}$$

We have given that the motion is steady 2 dimensional irrotational and incompressible.

Hence by the Bernoulli's equation

$\Omega + \frac{p}{\rho} + \frac{1}{2} q^2$ =constant

Here Ω =0

$$0 + \frac{p}{\rho} + \frac{1}{2} q^2 = c$$

$$\frac{p}{\rho}+\frac{1}{2}q^2=c$$

$$\frac{p}{\rho}=c-\frac{1}{2}q^2$$

$$p=\left(c-\frac{1}{2}q^2\right)\rho$$

$$p=\left(B-\frac{1}{2}q^2\rho\right)$$

Where B is a constant

Then $X-iY=-i\int\left(B-\frac{1}{2}q^2\rho\right)dz$

$$=-i\int Bdz+\frac{i\rho}{2}\int q^2dz$$

$$=0+\frac{i\rho}{2}\int q^2dz$$

$$=\frac{i\rho}{2}\int q^2dz$$

$$=\frac{i\rho}{2}\int\left|\frac{dw}{dz}\right|^2dz$$

$$=\frac{i\rho}{2}\int\frac{dw}{dz}.\frac{dw}{dz}dz$$

$$=\frac{i\rho}{2}\int\frac{dw}{dz}.dw$$

Given c is the circuit of cross section of the cylinder also therefore c is also one of the streamline of the flow and there won't be any flow across this line.

On c ψ remains constant

$d\psi = 0$

We know that $w = \phi + i\psi$

$dw = d\phi + id\psi$

$dw = d\phi + i.0$

$dw = d\phi$

$d\overline{w} = d\phi - id\psi$

$d\overline{w} = d\phi - i.0$

$d\overline{w} = d\phi$

$d\overline{w} = dw$

$$X - iY = \frac{i\rho}{2}\int \frac{dw}{dz}.\frac{dw}{dz}dz$$

$$X - iY = \frac{i\rho}{2}\int \left(\frac{dw}{dz}\right)^2 dz$$

If dM is the elementary moment then the entire moment is given by $\oint_c dM$

$dM = pdyy + pdxx$

$$M = \int_c dM$$

$$M = p\int_c ydy + xdx$$

$$M = realpartofP\int_c z d\bar{z}$$

Since we need to obtained the result in terms of complex potential. We express p in terms of q by using Bernoulli's equation

$$\frac{p}{\rho} + \frac{1}{2}q^2 = c$$

$$\frac{p}{\rho} = c - \frac{1}{2}q^2$$

$$p = \left(c - \frac{1}{2}q^2\right)\rho$$

$$p = \left(B - \frac{1}{2}q^2\rho\right)$$

M= real part of $\left(B - \frac{1}{2}q^2\rho\int_c z d\bar{z}\right)$

=real part of $B\int_c z d\bar{z} - \frac{1}{2}q^2\rho\int_c z d\bar{z}$

=real part of $0 - \frac{1}{2}q^2\rho\int_c z d\bar{z}$

=real part of $\frac{-\rho}{2}\int_c q^2 z d\bar{z}$

=real part of $\frac{-\rho}{2}\int\left(\frac{dw}{dz}\right)^2 dz$

=real part of $\frac{-\rho}{2}\int\frac{dw}{dz}.\frac{dw}{dz} dz$

=real part of $\frac{-\rho}{2}\int \frac{dw}{dz}.dw$

CONFORMAL TRANSFORMATION:

In studying the two dimensional irrotational motion of the incompressible inviscid fluid. We are able to study the flow about the circular cylinder but the cylinder is not a circular say aerofoil (aerofoil is a fish like profile with the sharp tailing edge) then there is no method by which it is possible to obtain the exact solution of the quantities of the flow. In such case we use the conformal transformation.

Suppose it where possible to establish the relationship between the complex variable z and ξ

Say $\xi = f(z)$

Then the mapping is said to be conformal transformation.

It satisfies the following 3 conditions

1. Geometrical condition
2. Dynamical condition
3. Physical condition

By uniform transformation the mapping $\xi = f(z)$ is said to be uniform transformation if it satisfies

1. **GEOMETRICAL CONDITION:**

For every point in z plane there must exists only one point in ξ plane and conversely corresponding to one point in ξ plane there must exists one and only point in z- plane.

2. **DYNAMICAL CONDITION:**

There must be a relationship between the velocity of the corresponding point of infinitesimal length must be similar and the angles are preserved.

3. **PHYSICAL CONDITION:**

The corresponding masses or areas must be similar.

TRANSFORMATION OF FLOW FIELD:

Let the complex potential describing the flow in the z plane is given by $w = f_1(z)$ then by the conformal mapping $\xi = f(z)$, the corresponding complex potential in ξ plane will be given by $w = f_2(z)$ then

$$\frac{dw}{dz} = \frac{dw}{d\xi} \times \frac{d\xi}{dz}$$

$$\left|\frac{dw}{dz}\right| = \left|\frac{dw}{d\xi} \times \frac{d\xi}{dz}\right|$$

Velocity of a point in z plane is equal to the velocity at the corresponding point in ξ plane.

STATE THE THEORY OF KUTTA AND JOUKOWSKI :

STATEMENT:

Suppose $\frac{dw}{dz}$ has no singularities outside the contour c.

Let their be a fixed aerofoil in an uniform stream with velocity u and the direction of motion makes n angle α with real axis.

Let k be the circulation around the aerofoil

Let x and y be the components of the force acting on the surface c of the aerofoil then by the Blasius theorem $X - iY = \frac{i\rho}{2}\int\left(\frac{dw}{dz}\right)^2 dz$

We take Γ as the contour at ∞ then we expand $\frac{dw}{dz}$ in inverse powers of z

$$\frac{dw}{dz} = a_0 + \frac{a_1}{z} + \frac{a_2}{z^2} + ...$$

At ∞ the fluid velocity must tend to the constant which is the velocity of uniform stream but uniform stream velocity u makes an angle α with real axis then $|z| \to \infty$

$$\frac{dw}{dz} = a_0 = ue^{-i\alpha}$$

$$\frac{dw}{dz} = ue^{-i\alpha} + \frac{a_1}{z} + \frac{a_2}{z^2} + ...$$

Integrate with respect to z

$$w = ue^{-i\alpha}z - a_1 \log z - \frac{a_2}{z} + ...$$

The 2nd term in this equation being a multiple valued function and represent the circulation $2\pi k$ around the aerofoil

$$\frac{dw}{dz} = ue^{-i\alpha} + \frac{ik}{z} + \frac{a_2}{z} + ...$$

By Blasius theorem

$$X - iY = \frac{i\rho}{2}\oint_C \left(\frac{dw}{dz}\right)^2 dz$$

$$\left(\frac{dw}{dz}\right)^2 = \left(ue^{-i\alpha} + \frac{ik}{z} + 0\left(\frac{1}{z^2}\right)\right)\left(ue^{-i\alpha} + \frac{ik}{z} + 0\left(\frac{1}{z^2}\right)\right)$$

$$= \left(u^2 e^{-2i\alpha} + \frac{ikue^{-i\alpha}}{z} + \frac{ikue^{-i\alpha}}{z}\right) + 0\left(\frac{1}{z^2}\right)$$

$$\left(\frac{dw}{dz}\right)^2 = u^2 e^{-2i\alpha} + \frac{2ikue^{-i\alpha}}{z} + 0\left(\frac{1}{z^2}\right)$$

The function $\left(\frac{dw}{dz}\right)^2$ as z=0 which is essential singularities

Then by Cauchy residue theorem

$$\oint \left(\frac{dw}{dz}\right)^2 dz = 2\pi i \text{ (sum of residue)}$$

$$= 2\pi i \text{ (co-efficient of 1/z in } \left(\frac{dw}{dz}\right)^2$$

$$= 2\pi i (2iuke^{-i\alpha})$$

$$= 4\pi uke^{-i\alpha}$$

$$X - iY = \frac{i\rho}{2} \oint_C \left(\frac{dw}{dz}\right)^2 dz$$

$$X - iY = \frac{i\rho}{2} (4\pi uke^{-i\alpha})$$

$$= -2i\rho\pi uke^{-i\alpha}$$

$$= -2i\rho\pi uk(\cos\alpha - i\sin\alpha)$$

$$X = -2i\rho\pi uk\sin\alpha$$

$Y = 2i\rho\pi uk\cos\alpha$

Lift force $\sqrt{X^2 + Y^2}$

$= \sqrt{(-2i\rho\pi uk\sin\alpha)^2 + (2i\rho\pi uk\cos\alpha)^2}$

$= 2\rho\pi uk$

POSSIBLE QUESTION

UNIT 3

PART-B (6 MARKS)

1. Show that ϕ and ψ satisfy Laplace's equation
2. Describe the transformation of flow field
3. Obtain the velocity potential
4. Obtain the transformation of doublet.
5. Explain physical interpretation of stream function.
6. Explain the concept of conformal transformation.
7. State and prove circle theorem.
8. Obtain the complex potential of the flow due to rectilinear vertex filament or point vertex in two dimensional motion or symmetrical circulation flow about the point.
9. Obtain the complex potential of the flow due to the doublet of strength m.
10. Express velocity components in terms of stream function in polar co-ordinates .
11. Show that stream function ψ and the potential function ϕ satisfies the Laplace equation.
12. Investigate the orthogonality of two families of curves $\phi(x, y)$ =constant $\psi(x, y)$ =constant
13. The velocity filed is given by $q = -x\hat{i} + (y + t)j$ find the stream function and stream line for this field at t=2.
14. A two dimensional flow is given by $\psi = xy$ then show that is irrotational.
15. Obtain the complex potential of the flow due to the doublet of strength m.

PART C (10 MARKS)

1. Show that a source is an abstract in introduced to describe the flow in a domain which excludes the source
2. State and prove the circle theorem
3. Describe the flow due to a rectilinear vortex filament a point vortex in a two dimensional plane
4. State and prove Blasius theorem
5. State the theory of kutta and joukowski.
6. State and prove milne Thomson circle theorem.
7. Prove that f(z) be a complex potential for a flow having no rigid boundary and such that they we have no singularities of the flow within the circle $|z| = a$ into the flow the new complex potential given by $w = f(z) + \bar{f}\left(\frac{a^2}{z}\right)$
8. Show that in the polar co-ordinates $V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ and $V_\theta = -\frac{\partial \psi}{\partial r}$

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Subject: Fluid Dynamics **Subject Code: 18MMP206**
Class : I - M.Sc. Mathematics **Semester : II**

Unit III

Part A (20x1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)
Possible Questions

Questions	Opt 1	Opt 2	Opt 3	Opt 4	Answer
The stream function is constant along a ___________________	Stream line	Path line	Vortex line	Filament line	Stream line
If the stream function is _____________ along a stream line	equal to zero	zero	constant	not equal	constant
If the motion is steady, the stream line pattern is ___________	equal	fixed	not fixed	constant	fixed
When the motion is not steady the stream line pattern is ____________ fixed	not	equal	constant	zero	not
The velocity potential ϕ exits when the fluid is _____________	Rotational	Irrotational	Stream line	Path line	Irrotational
If the velocity potential function are ___________	Velocity	Density	Pressure	Force	Velocity
The necessary and sufficient condition for q = - Φ= - grad Φ is hold is______________	∇.q≠0	∇xq=0	∇xq=0	∇.q≠0	∇xq=0
The complex potential functions are satisfying _____________ equation	Laplace equation	Differential equation	C – R equation	Homogeneous equation	C – R equation
If the velocity potential function are velocity Φ is called	q=∇ Φ	q= -∇ Φ	q=∇x Φ	q=-∇x Φ	q= -∇ Φ
The irrotational flow of an incompressible in viscid fluid is in ____________	3 – D	1 – D	2 – D	Multi – Dimension	2 – D
When the incompressible in viscid 2 – D fluid flow Φ and ψ satisfy the ____________ equation.	C – R equation	Laplace equation	Linear equation	Differential equation	Laplace equation
The stream function ψ exist whether the motes is ______________	Stream line	Path line	Irrotational	Rotational	Irrotational
The ________ potential can exist only when the motion is irrotational	Velocity	Density	Pressure	Force	Velocity
Part of the fluid may be moving irrotationally and the other parts may be ____________	Irrotational	constant	Rotational	Density	Rotational
The points where the velocity is ____________ are called stagnation points	1	0	Constant	Variable	0
In a 2 – D flow field where the fluid is assumed to be created is called	Doublet	Vertex	Sink	Sources	Sources
The flow is radically inverse is called ____________	Vertex	Sink	Sources	Doublet	Sink
The amount of the fluid going in to the sink in a unit time is called ________	Strength of the sink	Strength of the doublet	Strength of the source	Strength of the Vertex	Strength of the sink

The amount of the fluid going in to the sink in a ______ is called strength of the sink	Certain Interval	Unit time	Mean time	average	Unit time
If a source, the velocity of the fluid is _______________	Finite	Equal	Infinite	Zero	Infinite
Complex potential of the flow due to sink of strength m at the origin is given by	w = m logz	w = -m logz	w=log z	W=-log z	w = -m logz
A combination of a source and a sink in a particular way is known as a _____________	Doublet	Source	sink	vortex	Doublet
The line joining the source and sink is called as _____________ of the doublet	X – axis	Access	Y – axis	Z-axis	Access
If any point in the 2 – D field where the fluid is assumed to be _____________ is called a sink	Created	Constant	Moving	Annihilated	Annihilated
In a 2 – D field where the fluid is assumed to be annihilated is called a _______________	Sink	Source	Strength of source	Strength of sink	Sink
When the motion of a fluid consists of symmetrical radial flow in all directions proceeding from a point, Then the point is known a _____________	Source	Simple source	Sink	vortex	Simple source
When the fluid particles have circular motion under steady condition such a circular motion is called ____________	vortex	Sink	Doublet	Source	vortex
The Complex potential for a stream flow when a _____________ is placed in that	Surface	uniform	Circular Cylinder	continuous	Circular Cylinder
The complex potential for the uniform flow is ____________	w = v Z	w = V Z	w ≠ u × Z	w = u . Z	w = V Z
The circular cylinder is an irrotational incompressible	3 – D	1 – D	Multi – Dimension	2 – D	2 – D
The complex potential for the _________ flow is w = u Z	Uniform	Continuous	Discontinuous	Equal	Uniform
The complex potential for a ____________ flow when a circular cylinder is placed in that	Straight	Stream	Rotational	irrotational	Stream
A steady two dimensional irrotational incompressible in viscid fluid flow under no ____________Forces	External	Internal	Heat	mass	External
When are remembered that as the fluid is assumed to be in viscid, the drag force is	1	Equal	Zero	Not Equal	Zero
Cavitations is caused by	High velocity	Low barometric pressure	High pressure	Low pressure	Low pressure
The general energy equation is applicable to	Unsteady flow	Steady flow	Non-uniform flow	Turbulent flow	Steady flow
The friction resistance in Pipe is proportional To Square of V , according to	Froudeaiumber	Reynolds-Weber	Darcy-Reynolds	Weber-Froude	Froudeaiumber
Pitot tube is used to measure the velocity head of	Still fluid	Laminar flow	Turbulent flow	Flowing fluid	Flowing fluid
In equilibrium condition, fluids are not able to sustain	Shear force	Resistance to viscosity	Surface tension	Geometric similitude	Surface tension

UNIT IV

Viscous flows – Navier-Stokes equations – Vorticity and circulation in a viscous fluid – Steady flow through an arbitrary cylinder under pressure – Steady Couettc flow between cylinders in relative motion – Steady flow between parallel planes.

VISCOSITY AND REYNOLDS NUMBER

Consider the simple type of flow in which a streamline are parallel.

The velocity field is one dimensional and hence the velocity is U

H is the distance between the stream lines and y denotes the normal line of the stream lines

The velocity profile for this flow is a straight line

$$u(y) = \frac{U}{h}Y.........(1)$$

In the view of the linear nature of velocity profit the stresses will be determined by the velocity gradient $\frac{du}{dy}$ and all higher derivatives of velocity will be zero

From (1)

$$\frac{du}{dy} = \frac{U}{h}Y$$

By varying u and h the measures of force experienced by upper plain. It is found that the tangential stress τ is direct proportional to the velocity gradients.

$$\tau \alpha \frac{du}{dy}$$

$$\tau = \frac{u}{y}$$

$$\tau = \mu.\frac{u}{y} \text{..............(2)}$$

The constant proportionality μ depends upon the physical properties of the fluid and it is called the co-efficient of viscosity

In many fluids the co-efficient of viscosity μ is very small.

Because of the reason the viscosity stress is neglect able in ideal fluid.

In practice the relative magnitude of viscous flow in the form equ (2) is varied

If U typical velocity and l is typical length in the flow under consideration then

typical pressure force / typical viscous force $= \dfrac{\rho u^2}{\mu \frac{u}{h}}$

$$= \frac{\rho u^2}{\mu \frac{u}{L}}$$

$$= \frac{UL}{\frac{\mu}{\rho}}$$

$$= \frac{UL}{\gamma} \text{ where } \gamma = \frac{\mu}{\rho}$$

Where $\gamma = \dfrac{\mu}{\rho}$ is called kinematical viscosity

The non-dimensional parameter $R = \dfrac{UL}{\gamma}$ is called the Reynolds number.

NAVIER-STROKES EQUATION:

Navier strokes equation are the set of equations which expresses the basic physical concept of flow of the real fluid they are,

1. Equation of mass continuity
2. Momentum equation
3. Equation of energy conservation

BOOK WORK1:

Derive the equation of continuity for a real or viscous fluid in cartisian co-ordinates

PROOF:

Consider a fluid of volume v inside a closed surface s

Let ρ be the density of the fluid consider an elementary surface ds and $\hat{n}$ be the unit outward vector.

Let $\hat{q}$ be the velocity of the fluid particle at ρ on the elementary surface ds.

The rate at which the mass of fluid flows out of the surface ds is $\rho(\vec{q}.\vec{n}).ds$

The rate at which the mass of the fluid flows in the surface ds is $-\rho(\vec{q}.\vec{n}).ds$

The rate of which the mass of the fluid flow into the surface s

$$\iint_s \rho(\vec{q}.\vec{n}).ds = -\iiint_v (\nabla.\rho\vec{q}).\vec{n}dv................(1)$$

Let us consider the elementary volume dv

The elementary mass = ρdv

The mass of the fluid inside the volume v= $\iiint_v \rho dv$

The rate of change of mass= $\frac{\partial}{\partial t}\iiint_v \rho dv$.............(2)

If we assume that the motion of the fluid is created or destroyed inside the volume v the equation (1) and (2) are same

$$-\iiint_v (\nabla.\rho\vec{q}).\vec{n}ds = \frac{\partial}{\partial t}\iiint_v \rho dv$$

$$'\frac{\partial}{\partial t}\iiint_v \rho dv + \iiint_v (\nabla.\rho\vec{q}).\vec{n}ds = 0$$

$$'\iiint_v \left(\frac{\partial \rho}{\partial t} + \nabla.\rho\vec{q}\vec{n}\right)dv = 0$$

Since the volume under consideration is arbitrary an d hence the integral must vanish

$$\frac{\partial \rho}{\partial t} + \nabla.\rho\vec{q}\vec{n} = 0$$

Which is known as the equation of continuity

Hence the proof

BOOK WORK 2:

In usually notation derive the momentum equation for viscous fluid

PROOF;

Consider the orbitray volume v bounded by the surface s

Let l_j be the direction cosines of the outward normal from the fluid surface

Let dv be an elementary volume enclosing a fluid particle at p, where the velocity components along x_i direction at time t is v_i

Let ρ be the density of the fluid

Elementary mass of the fluid= ρdv

Linear momentum of the elementary mass = $v_i \rho dv$

The momentum of the fluid containing within the volume v= $\int_v v_i \rho dv$

Rate of change of momentum= $\frac{D}{Dt}\int_v v_i \rho dv$

$$= \int_v \frac{Dv_i}{Dt} \rho dv \ldots\ldots\ldots\ldots(1)$$

By Newton's second law of motion the rate of change of momentum is must be equal to the total force acting upon the fluid within the volume v.

The force acting on the fluid are

(i) External force = $\int_v F_i \rho dv \ldots\ldots\ldots\ldots\ldots(2)$

Where F_i is external force per unit mass

(ii) The resultant of the fluid stress at the surface $S = -\int_s p_{ij} l_j ds \ldots\ldots\ldots\ldots\ldots(3)$

Where p_{ij} is a stress component in x_i direction

By Newton second law

$$\int_v \frac{Dv_i}{Dt} \rho dv = \int_v F_i \rho dv - \int_s p_{ij} l_j ds$$

$$= \int_v F_i \rho dv - \int_s \frac{\partial}{\partial x_j} p_{ij} l_j ds$$

Since the volume is orbitray

$$\rho \frac{Dv_i}{Dt} = \rho F_i - \frac{\partial}{\partial x_j}(p_{ij})$$

Divide by ρ

$$\frac{Dv_i}{Dt} = F_i - \frac{1}{\rho}\frac{\partial}{\partial x_j}(p_{ij}) \text{............(4)}$$

In case of rectangle cartesian co-ordinates

$$\frac{Dv_i}{Dt} = \frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \text{............(5)}$$

Also,

$$P_{ij} = P\delta_{ij} - \mu\left(\frac{\partial v_i}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right) + \frac{2}{3}\mu\left(\frac{\partial v_k}{\partial x_k}\right)\delta_{ij} \text{..............(6)}$$

Sub (5) and (6) in (4)

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} = F_i - \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(P\delta_{ij} - \mu\left(\frac{\partial v_i}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right) + \frac{2}{3}\mu\left(\frac{\partial v_k}{\partial x_k}\right)\delta_{ij}\right)$$

$$= F_i - \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\left[P + \frac{2}{3}\mu\left(\frac{\partial v_k}{\partial x_k}\right)\right]\delta_{ij} + v\frac{\partial}{\partial x_i}\left(\frac{\partial v_i}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right)\right)$$

It is known as equation for momentum of viscous

DERIVE THERMAL ENERGY EQUATION FOR VISCOUS FLUID:

Consider an orbitray volume B enclosed by the surface S

Let l_i be the direction cosines of the outward from the fluid surface

Let dv be the elementary volume enclosing a fluid particle at p. where the velocity components along x_j direction at time t is v_i

Let ρ be the density of the fluid

The elementary mass of the fluid $= \rho dv$

The total energy of the volume is = kinetic energy + potential energy

$$= \frac{1}{2}\rho dv \times v_i^{\ 2}$$

Here potential energy $= \rho dvgh$

$$= \rho E dv$$

Where E is the internal energy

The total energy of the entire volume $= \int_v \left(\frac{1}{2} v_i^{\ 2} + E \right) \rho dv$

Rate of change of total energy $= \frac{D}{Dt} \int_v \left(\frac{1}{2} v_i^{\ 2} + E \right) \rho dv$............(1)

From thermo dynamics we know that the rate of change of total energy is determined by the following fact.

(i) Q_i - heat conduction

(ii) v_{ij} -pressure thrust

(iii) F_i -external force

1. If Q_i is the heat conduction per unit area in x_i direction then $-l_i Q_i ds$ is the heat conduction into the elementary surface ds.

   The total heat conducted within the volume enclosed by the surface $S = -\int_s l_i Q_i ds$...........(2)

2. The stress in x_i direction upon the fluid on the elementary surface ds is $-l_i P_{ij} ds$

   The rate at which the elementary surface works upon the fluid is $-l_i P_{ij} dsv_i$

   The total rate of work upon the entire fluid $= -\int_s l_i P_{ij} dsv_i$ ....(3)

3. The external force in the x_i th direction is F_i per unit mass

   The external force on the mass= $F_i \rho dv$

   The rate of change of work is done by the external force= $F_i \rho dvv_i$

   The total rate of change of work done apart the entire fluid = $\int_v F_i \rho dvv_i$ ....(4)

Now equation (1)=(2)+(3)+(4)

$$\frac{D}{Dt}\int_v \left(\frac{1}{2}v_i^2 + E\right)\rho dv = -\int_s l_i Q_i ds - \int_s l_i P_{ij} dsv_i + \int_v F_i \rho dvv_i$$

$$\frac{D}{Dt}\int_v \left(\frac{1}{2}v_i^2 + E\right)\rho dv = \int_v F_i \rho dvv_i - \int_v \frac{\partial Q_i}{\partial x_i} dv - \int_v \frac{\partial (p_{ij} v_i)}{\partial x_i} dv$$

The volume under the consideration is orbitray

$$\frac{D}{Dt}\int_v \left(\frac{1}{2}v_i^2 + E\right)\rho = (F_i \rho v_i) - \frac{\partial Q_i}{\partial x_i} - \frac{\partial (p_{ij})}{\partial x_i} v_i$$

$$\frac{D}{Dt}\int_v \left(\frac{1}{2}v_i^{\ 2} + \frac{DE}{Dt}\right)\rho = (F_i \rho v_i) - \frac{\partial Q_i}{\partial x_i} - \frac{\partial (p_{ij})}{\partial x_i} v_i$$

$$\rho \frac{1}{2} 2v_i \frac{Dv_i}{Dt} + \frac{DE}{Dt}\rho = (F_i \rho v_i) - \frac{\partial Q_i}{\partial x_i} - \frac{\partial (p_{ij})}{\partial x_i} v_i \text{...........}(5)$$

$$\rho v_i \frac{Dv_i}{Dt} + \frac{DE}{Dt}\rho = (F_i \rho v_i) - \frac{\partial Q_i}{\partial x_i} - \frac{\partial (p_{ij})}{\partial x_i} v_i$$

From the previous bookwork we know that

$$\frac{Dv_i}{Dt} = F_i - \frac{1}{\rho}\frac{\partial}{\partial x_i} p_{ij}$$

Multiply fully by ρv_i

$$\left(\frac{Dv_i}{Dt}\right)\rho v_i = F_i \rho v_i - v_i \frac{\partial}{\partial x_i} p_{ij} \text{...........}(6)$$

Equation (5)-(6)

$$\frac{DE}{Dt}\rho = -\frac{\partial Q_i}{\partial x_i} - \frac{\partial (p_{ij})}{\partial x_i} v_i + v_i \frac{\partial}{\partial x_i} p_{ij}$$

$$= -\frac{\partial Q_i}{\partial x_i} - p_{ij}\frac{\partial v_i}{\partial x_i}$$

$$\frac{DE}{Dt}\rho = -\frac{\partial Q_i}{\partial x_i} - \frac{1}{2} p_{ij} \ell_{ij} \text{.....................}(7)$$

Introducing enthalpy which is defined as

$$I = E + \frac{P}{\rho}$$

Differentiating with respect to t we get

$$\frac{DI}{Dt} = \frac{DE}{Dt} + \frac{D}{Dt}\left(\frac{P}{\rho}\right)$$

Multiply by ρ we get

$$\rho\frac{DI}{Dt} = \rho\frac{DE}{Dt} + \rho\frac{D}{Dt}\left(\frac{P}{\rho}\right)$$

Use equation (7) we get

$$\rho\frac{DI}{Dt} = -\frac{\partial Q_i}{\partial x_i} - \frac{1}{2}p_{ij}\ell_{ij} + \frac{DP}{Dt} - \frac{P}{\rho}\frac{DP}{Dt} \quad \ldots\ldots\ldots\ldots\ldots(8)$$

Using equation of continuity

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i) = 0$$

$$\frac{\partial v_i}{\partial t} = \frac{\partial v_i}{\partial x_i}\delta_{ij} = \frac{1}{2}\ell_{ij}P_{ij}$$

$$\rho\frac{DI}{Dt} = -\frac{\partial Q_i}{\partial x_i} + \frac{DP}{Dt} - \frac{1}{2}\ell_{ij}P_{ij} + P\frac{\partial v_i}{\partial x_i}$$

(8) $\Rightarrow$

$$\rho\frac{DI}{Dt} = -\frac{\partial Q_i}{\partial x_i} - \frac{1}{2}\ell_{ij}P_{ij} + \frac{DP}{Dt} - \frac{1}{2}\delta_{ij}P_{ij}$$

$$\rho\frac{DI}{Dt} = \frac{DP}{Dt} - \frac{\partial Q_i}{\partial x_i} + \phi \quad \ldots\ldots\ldots\ldots(9)$$

Where $\phi = \frac{1}{2}\ell_{ij}(P\delta_{ij} - P_{ij})$ is the rate of description of energy per unit of volume due to viscousity.

If we assume that conduction Q_i is propositional to temperature gradients then $Q_i = -k\frac{\partial T}{\partial x_i}$

Where k is called thermal conductivity

From equation (9)

$$\rho \frac{DI}{Dt} = \frac{DP}{Dt} - \frac{\partial}{\partial x_i}\left(-k\frac{\partial T}{\partial x_i}\right) + \phi \ldots\ldots\ldots\ldots\ldots(10)$$

For perfect gas contains specific

$$I = C_p T$$

Equation (10)

$$\rho \frac{D}{Dt}\left(C_p T\right) = \frac{DP}{Dt} - \frac{\partial}{\partial x_i}\left(-k\frac{\partial T}{\partial x_i}\right) + \phi$$ is called thermal energy equation.

BOOK WORK 4:

Derive the Navier stoke equation for incompressible fluids.

PROOF:

We know for an incompressible fluid ρ is an constant

i. Equation of continuity:

Equation of continuity for a viscid fluid is

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_i)}{\partial x_i} = 0$$

Equation of continuity for an incompressible viscous fluid is

$$0 + \rho\frac{\partial v_i}{\partial x_i} = 0$$

$$\frac{\partial v_i}{\partial x_i} = 0$$

ii. Momentum equation:

The momentum equation for a viscous fluid is

$$\frac{Dv_i}{Dt} = F_i - \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[P + \frac{2}{3}\mu\frac{\partial v_k}{\partial x_k}\right]\delta_{ij} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right)\right]$$

Here $\frac{\partial^2 v_k}{\partial x_j \partial x_k} = \frac{\partial^2 v_j}{\partial x_j \partial x_j} = 0$

$$\frac{\partial^2 v_k}{\partial^2 x_k} = 0$$

$$\frac{Dv_i}{Dt} = F_i - \frac{1}{\rho}\left[\frac{\partial}{\partial x_j}P + \frac{2}{3}\mu\frac{\partial^2 v_k}{\partial^2 x_k}\right]\delta_{ij} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right)\right]$$

$$\frac{Dv_i}{Dt} = F_i - \frac{1}{\rho}\left[\frac{\partial}{\partial x_j}P + 0\right]\delta_{ij} + \frac{\mu}{\rho}\frac{\partial^2 v_i}{\partial^2 x_j}$$

$$\frac{Dv_i}{Dt} = F_i - \frac{1}{\rho}\left[\frac{\partial}{\partial x_j}P\right]\delta_{ij} + \frac{\mu}{\rho}\frac{\partial^2 v_i}{\partial^2 x_j}$$

iii. Thermal energy equation:

We know the thermal energy equation for viscous fluid

$$\rho\frac{D}{Dt}(C_p T) = \frac{DP}{Dt} + \frac{\partial}{\partial x_i}\left(k.\frac{\partial T}{\partial x_i}\right) + \phi$$

Here $\frac{\partial \rho}{\partial t} = 0 \Rightarrow$ the pressure value

$\frac{DP}{Dt} = 0$ and $\phi = 0$ because the rate of description value is zero

The above equation becomes

$$\rho \frac{D}{Dt}(C_p T) = k.\frac{\partial^2 T}{\partial {x_i}^2}$$

$$\rho C_p \frac{DT}{Dt} = k.\frac{\partial^2 T}{\partial {x_i}^2}$$

$$\frac{DT}{Dt} = \frac{k}{\rho C_p}.\frac{\partial^2 T}{\partial {x_i}^2}$$

$$\frac{DT}{Dt} = K.\frac{\partial^2 T}{\partial {x_i}^2}$$

(Where $K = \frac{k}{\rho C_p}$ is called thermo metric conduction)

BOOK WORK 5:

Derive the momentum equation in the form

$$\frac{\partial \vec{v}}{\partial t} - \vec{v} \times \vec{w} = \vec{F} - grad\left(\frac{P}{\rho} + \frac{1}{2}\vec{v}.\vec{v}\right) - \gamma curl \vec{w}$$

PROOF:

The momentum equation for incompressible viscous fluid is

$$\frac{D\overrightarrow{v_i}}{Dt} = F_i - \frac{1}{\rho}\left[\frac{\partial P}{\partial x_j}\right]\delta_{ij} + \gamma \frac{\partial^2 v_i}{\partial^2 x_j} \text{.................}(1)$$

Where $\gamma = \frac{\mu}{\rho}$

We know that

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + (\vec{v}.\nabla)\vec{v}$$

We know that from the vector identities

$$\nabla(\vec{u}.\vec{v}) = (u.\nabla)\vec{v} + (\vec{v}.\nabla)\vec{u} + \vec{u} \times curl\vec{v} + \vec{v} \times curl\vec{u}$$

Take u=v

$$\nabla(\vec{v}.\vec{v}) = 2 \times \left((\vec{v}.\nabla)\vec{v} + \vec{v} \times curl\vec{v}\right)$$

$$\frac{1}{2}\nabla(\vec{v}.\vec{v}) = \left((\vec{v}.\nabla)\vec{v} + \vec{v} \times curl\vec{v}\right)$$

$$\frac{1}{2}\nabla(\vec{v}.\vec{v}) - +\vec{v} \times curl\vec{v} = \left((\vec{v}.\nabla)\vec{v}\right)$$

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \nabla \frac{1}{2}(\vec{v}.\vec{v}) - \vec{v} \times \vec{w} \ldots\ldots\ldots\ldots\ldots(2)$$

We consider

$$\nabla(\nabla.\vec{v}) - \nabla \times \vec{w} = \nabla^2 v_i \ldots\ldots\ldots\ldots\ldots(3)$$

For an incompressible fluid

$$\nabla.\vec{q} = 0$$

$$\nabla.\nabla = 0$$

(3) becomes

$$-\nabla.\vec{w} = \nabla^2 v_i$$

Here $\nabla^2 v_i$ is the component of –curl w

From equation (1)

$$\frac{Dv_i}{Dt} = F_i - \frac{1}{\rho}\left[\frac{\partial P}{\partial x_j}\right]\delta_{ij} + \gamma$$

In general I th +j th +k th components of momentum equation

$$\frac{D\vec{v}}{Dt} = F_i - grad\frac{P}{\rho} - \gamma curlw..............(4)$$

Using (2) and (4)

$$\frac{\partial \vec{v}}{\partial t} - \vec{v}\times\vec{w} = \vec{F} - grad\left(\frac{P}{\rho} + \frac{1}{2}\vec{v}.\vec{v}\right) - \gamma curl\vec{w}$$

THE BOUNDARY LAYER ALONG A FLAT PLATE:

Let us consider the steady flow of an incompressible viscous fluid past a thin semi infinite flate which is placed in direction of a uniform velocity u. the motion is two dimensional and can be analyzed by using the prandt boundary layer equations. We choose the origin of the co-ordinates at the leading edge of the plate x-axis along the direction of uniformly stream and y-axis normal to the plate. The prandt boundary layer equations for this case are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial y^2}.................(1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0................(2)$$

Where u,v are the velocity components and v is the kinematic viscosity

The boundary conditions are

U=v=0 when y=0

U=u∞ when $y \to \infty$ …………(3)

In this problem the parameter in which the result are to be obtained are u∞,v,x

So we may take

$$\frac{u}{u_0} = F(x, y, v, u_\infty) = F(\eta)\text{..........}(4)$$

Further according to the exact solution of the unsteady motion of a flat plate we have

$$\delta = \sqrt{vt} = \sqrt{\frac{vx}{u\infty}}\text{..........}(5)$$

Where x is the distance travelled in time with velocity u∞. hence the non-dimensional distance parameter may be expressed as

$$\eta = \frac{y}{\delta} = \frac{y}{\sqrt{\frac{vx}{u\infty}}} = y\sqrt{\frac{u\infty}{vx}}\text{..........}(6)$$

Thus it can be seen that η is (4) is a function of x,y,v, $u\infty$ in (6)

The stream function ψ is given by

$$\psi = \int u dy$$

$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$\psi = \int u_\infty F(\eta)\frac{dy}{d\eta}d\eta$$

$$= u_\infty\sqrt{\frac{vx}{u\infty}}\int F(\eta)d\eta = \sqrt{vxu_\infty F(\eta)}\text{.........}(7)$$

The velocity components in term of η are

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = \sqrt{vxu_\infty}\sqrt{\frac{u\infty}{vx}}F'(\eta)$$

$$= u_\infty F'(\eta)..............(8)$$

$$-v = \frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial x}$$

$$= \frac{1}{2}\sqrt{v\frac{u\infty}{x}}F'(\eta) + \sqrt{vxu_\infty}F'(\eta)y\sqrt{\frac{u\infty}{vx}}$$

$$v = -\frac{1}{2}\sqrt{v\frac{u\infty}{x}}F(\eta) + \frac{1}{2}y\frac{u\infty}{x}F'(\eta)$$

$$= -\frac{1}{2}\sqrt{v\frac{u\infty}{x}}\left(\sqrt{\frac{u\infty}{vx}}yF'(\eta) - F(\eta)\right)$$

$$= -\frac{1}{2}\sqrt{v\frac{u\infty}{x}}\left(\eta F'(\eta) - F(\eta)\right)...............(9)$$

Also $\frac{\partial u}{\partial x} = \frac{\partial^2 \psi}{\partial x \partial y} = u_\infty f''(\eta)\frac{\partial \eta}{\partial x}$

$$= -\frac{1}{2}u_\infty f''(\eta).y\sqrt{\frac{u_\infty}{v}}\frac{1}{x^{3/2}}$$

$$= -\frac{1}{2}\frac{u_\infty}{x}\eta f''(\eta)..................(10)$$

$$\frac{\partial u}{\partial y} = u_\infty \frac{\partial}{\partial y} f''(\eta) = u_\infty\sqrt{\frac{u_\infty}{vx}} f''(\eta)............(11)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_\infty}{vx} f''(\eta)..............(12)$$

Using these values of u,v and their derivatives in (1) we obtain

$u_\infty f'(\eta)$

$$\left(\frac{-1}{2}\frac{u_\infty}{vx}\eta f''(\eta)\right)+\frac{1}{2}\sqrt{\frac{vu_\infty}{x}}\left(\eta f'(\eta)-f(\eta)\right)u_\infty\sqrt{\frac{u_\infty}{vx}}f''(\eta)=v\frac{u_\infty{}^2}{vx}f'''(\eta)$$

$$-\frac{u_\infty{}^2}{2x}\eta f f''+\frac{u_\infty{}^2}{2x}(\eta f'-f)f''=\frac{u_\infty{}^2}{x}f''$$

Or

$$-\eta f f''+\eta f f''-f f''=2\eta f''$$

Or

$$2f'''+ff''=0$$

$$2\frac{d^3f}{d\eta^3}+f\frac{d^2f}{d\eta^2}=0\ldots\ldots\ldots\ldots\ldots(13)$$

Thus we have reduced the partial differential equation (1) to ordinary differential equation (13) known as Blasius equation.

POSSIBLE QUESTIONS

UNIT 4

PART-B (6 MARKS)

1. Explain inviscid flow past of a circular cylinder
2. Explain steady flow between parallel planes
3. Show that the rate of change of momentum must equal the total force acting upon the fluid within the volume
4. Deduce the equation for incompressible
5. Explain the concept of boundary layer of a flat plane.
6. Derive the momentum equation .
7. Derive the Navier stoke equation for incompressible fluids.
8. Define thermal equation for viscous fluid.
9. In usually notation derive the momentum equation for viscous fluid.
10. Derive the equation of continuity for a real or viscous fluid.
11. What are the basic physical concept of flow of the real fluid
12. Brief the concept Equation of mass continuity
13. Show that the Derivation of Momentum equation
14. Expain Equation of energy conservation
15. Derive Navier stoke equation.

PART-C (10 MARKS)

1. Explain stokes's flow foe very slow motion
2. Obtain Helmholtz's equation for the vorticity

3. Derive Helmholtz's equation for vorticity

4. Deduce the thermal energy equation $\rho \frac{D}{Dt}(C_p T) = \frac{Dp}{Dt} + \frac{\partial}{\partial x_i}\left(k \frac{\partial T}{\partial x_i}\right) + \phi$

5. Derive the equation of continuity for a real or viscous fluid in Cartesian equation.

6. Derive the momentum equation in the form $\frac{\partial \vec{v}}{\partial t} - \vec{v} \times \vec{w} = \vec{F} - grad\left(\frac{P}{\rho} + \frac{1}{2}\vec{v}.\vec{v}\right) - \gamma curl \vec{w}$

7. Define (i) inviscid flow and (ii) Reynolds number with examples.

8. Derive the momentum equation for viscous fluid.

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Subject: Fluid Dynamics **Subject Code: 18MMP206**
Class : I - M.Sc. Mathematics **Semester : II**

Unit IV

Part A (20x1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)
Possible Questions

Questions	Opt 1	Opt 2	Opt 3	Opt 4	Answer
In the case of a real fluid frictionless resistance is known as ---------------	shearing stress	tangential stress	friction stress	ideal fluid	tangential stress
In the case of --------------frictionless resistance is known as tangential stress	perfect fluid	friction stress	real fluid	ideal fluid	real fluid
On real fluid ,tangential stresses are ---------------	large	small	very small	infinite	small
The property which causes the tangential stress is known as-------	inviscosity	real fluid	velocity	viscosity	viscosity
On plane coutte flow if the fluid is perfect the motion of the plates has--------on the fluid	no effect	viscous	effect	speed	no effect
Shearing stress will be proportional to the rate of change of -------	speed	pressure	force	velocity	velocity
The force will be proportional to the area upon which it acts and it is known as -------	shearing stress	tangential stress	viscosity	effect of viscosity	shearing stress
In the effect of viscosity the shearing stress is denoted by --------	ψ	μ	τ	Ω	τ
The coefficient of viscosity is denoted by--------	ψ	μ	Ω	τ	μ
A typical viscous stress is in the form τ=---------	$\partial u/\partial y$	μ	$\mu(\partial u/\partial y)$	$\partial\mu$	$\mu(\partial u/\partial y)$
The viscous force are of order ---- per unit area	U/L	μ (U/L)	μ /L	μU	μ (U/L)
The typical pressure force will be of order------ per unit area	U^2	ρU	$\rho U/L$	ρU^2	ρU^2
In a Reynold's numbers, the kinematic viscosity is ------	$\gamma=\mu/\rho$	$\gamma=\mu$	$\gamma=1/\mu$	$\gamma=0$	$\gamma=\mu/\rho$
The non-dimensional parameter R=UL/γ is called -------	viscous force	pressure force	Reynold's number	kinematic viscosity	Reynold's number
The equation of continuity in a real fluid on a viscous flow is ------	$\partial\rho/\partial t + (\partial/\partial x_i)(\rho v_i)=0$	$\partial/\partial t + (\partial/\partial x_i)(\rho v_i)=0$	$\partial\rho/\partial t + (\partial^2/\partial t^2)(\rho v_i)=0$	$\partial\rho/\partial t + (\partial/\partial x_i)(\rho)=0$	$\partial\rho/\partial t + (\partial/\partial x_i)(\rho v_i)=0$
In the Navier stokes equation,when the fluid is incompressible,then ρ and μ are-----	equal	zero	not equal	constant	constant
The Navier stokes equation in vector form is ---------------	$dq/dt=F-\nabla p/\rho$	$dq/dt=F-\nabla p/\rho+\gamma\nabla^2 q$	$dq/dt=F+\gamma\nabla^2 q$	$dq/dt=F+\nabla p/\rho+\gamma\nabla^2 q$	$dq/dt=F-\nabla p/\rho+\gamma\nabla^2 q$
The equation of an Helmholtz equation of the viscous fluid is-------	$d\varepsilon/dt=(\varepsilon.\nabla)q+\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q$	$d\varepsilon/dt=\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q-\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q+\gamma\nabla^2\varepsilon$
On the 2-D motion the equation of vorticity is ------	$d\varepsilon/dt=(\varepsilon.\nabla)q+\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q$	$d\varepsilon/dt=\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=(\varepsilon.\nabla)q-\gamma\nabla^2\varepsilon$	$d\varepsilon/dt=\gamma\nabla^2\varepsilon$

In a circulation on a viscous fluid the space derivative of the vorticity vector are--------------	small	constant	large	infinite	large
The steady flow through an arbitrary cylinder under pressure is known as ---------	Hagen –Poiseuille flow	viscous flow	inviscous flow	vorticity flow	Hagen –Poiseuille flow
In the Reynolds number is the principal parameter determining the ------	role of the flow	nature of the flow	order of the flow	type of the flow	nature of the flow
The constant of proportionality, μ depends entirely upon the physical properties of the fluid is called _________	typical viscous stress	effect of viscosity	coefficient of viscosity	viscosity of a flow	coefficient of viscosity
An arbitrary volume of a fluid,the momentum of the fluid contained within the volume is -----	$\int v_i dv$	$\int \rho v_i \, dv$	$\int \rho dv$	$\int \rho^2 v_i \, dv$	$\int \rho v_i \, dv$
The resultant value of an poiseuille's law is ------	$M=(\pi p\ a^3)/4\mu$	$M=(\pi\rho p\ a^3)/6\mu$	$M=(\pi\rho p\ a^4)/8\mu$	$M=(\pi p\ a^4)/6\mu$	$M=(\pi\rho p\ a^4)/8\mu$
If we consider two infinite parallel planes.Aflow with pressure gradient when both planes are at rest then they are called as --------	pressure flow	plane poiseuille flow	coutte flow	plane coutte flow	plane poiseuille flow
If we consider two infinite parallel planes.A flow without pressure gradient when one plane moves relative to the other such a flow is called----------	plane coutte flow	plane poiseuille flow	infinite plane flow	viscous plane flow	plane coutte flow
A flow is said to be ---------- if all fluid particles moving in one direction	parallel	perpendicular	nonparallel	zero	parallel
A flow is said to be parallel if only one velocity component is ----------	zero	non zero	constant	variable	non zero
A flow is said to be parallel if all fluid particles moving in---------- direction	two	three	one	four	one
A flow is said to be parallel if only---------- velocity component is non zero	two	four	three	one	one
Skin friction σ= ----------	μ/h	μU	μU/h	U/h	μU/h
Skin friction is also known as ---------per unit area	circle	sphere	square	drag	drag
In plane couette flow the -------------------is zero	temperature gradient	temperature	pressure gradient	pressure	pressure gradient
In------------------- the pressure gradient is zero	plane poiseuille flow	plane couette flow	couette flow	poiseuille flow	plane couette flow
In ---------------------------the plates are at rest	plane poiseuille flow	plane couette flow	couette flow	poiseuille flow	plane poiseuille flow
In plane poiseuille flow the plates are at--------	motion	rest	stable	nonstable	rest
The -------------------for the drag of a sphere is given by D= 6 $\pi\mu a U_0$	stokes formula	Greens formula	Gauss formula	Laplace formula	stokes formula
The stokes formula for the drag of a sphere is given by D= -----------------	$6\ U_0$	$6\ \pi\mu a U_0$	$6\ \pi\mu a$	$6\ aU_0$	$6\ \pi\mu a U_0$
The stokes formula for the drag of a ---------------is given by D= 6 $\pi\mu a U_0$	circle	flux	sphere	square	sphere
In steady flow the flow past a circular cylinder then the stokes equation reduces to -------	parallel	perpendicular	nonzero	zero	zero

UNIT V

Boundary Layer concept – Boundary Layer equations – Displacement thickness, Momentum thickness – Kinetic energy thickness – integral equation of boundary layer – flow parallel to semi infinite flat plate – Blasius equation and its solution in series.

BOOK WORK 1

Derive the boundary layer equation for the two dimensional flow along a plane all.

PROOF:

Let us take a rectangle Cartesian co-ordinates (x,y) with x measure on the surface in the direction of flow and y measured normal to the surface.

Let (u,v) be the velocity components then the equation of motion are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0........(1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\gamma\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right).......(2)$$

$$u=v$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\gamma\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right).......(3)$$

The approximate boundary layer equation may be obtained either physically or mathematically.

Physically we have u is order of U and typical length scale parallel and normal to the wall are L and δ respectively.

Then v is the order of $\frac{u\delta}{L}$ where $\frac{\delta}{L}$ is the order of Reynolds's number.

The terms in equation (2) are of the order $\left(\frac{u}{L}\right)^2$ expect the term $\frac{\partial^2 u}{\partial x^2}$

The term may be neglected

Then equation (2) becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\gamma\left(\frac{\partial^2 v}{\partial y^2}\right).........(4)$$

And also from equation (3) except the term $-\frac{1}{\rho}\frac{\partial p}{\partial x}$ the remaining terms are of order $\left(\frac{u^2}{L^2}\right)\delta$

Then equation (3) becomes

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=o\left(\left(\frac{u^2}{L^2}\right)\delta\right).........(5)$$

The pressure gradient normal to the wall is small and the total pressure changes a cross the boundary layer.

The pressure is the function of x only.

P=p(x)

Equation (4)becomes

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\gamma\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right).........(6)$$

The equation (1) and (6) are approximate boundary layer equation for u and v

By the continuity equation (1) we may introduce the stream layer function ψ such that

$$\left.\begin{aligned} u &= \frac{\partial \psi}{\partial x} \\ v &= -\frac{\partial \psi}{\partial x} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

And equation (6) becomes the equation of 3rd order of ψ

The boundary conditions are u=v=0 when y=0

In addition to the velocity u(x,y) we join smoothly onto the main stream velocity for some suitable value of y

It is found that $u = u_1(x)$ atleast the boundary layer solution is concerned

The 3rd boundary condition is $u = u_1(x)$ when $y = \infty$

At $y \to \infty$ $\frac{\partial u}{\partial y} \to 0$ and $\frac{\partial^2 u}{\partial y^2} \to 0$

Then equation (6) becomes

$$\int u_1 \frac{du_1}{dx} = -\int \frac{1}{\rho} \frac{dp}{dx}$$

$$\frac{u_1^{\,2}}{2} = \frac{-p}{\rho} + c$$

$$p + \frac{\rho u_1^{\,2}}{2} = c = p_0 + \frac{1}{2}\rho u_0^{\,2}$$

The thermal pressure co-efficient

$$cp = 1 - \frac{u_1^{\,2}}{u_0^{\,2}}$$

BOOK WORK 2:

Some important boundary layer characteristics are

1. Displacement thickness δ_1
2. Momentum thickness δ_2
3. Kinetic energy thickness δ_3
4. Skin friction or wall shearing stress τ_w
5. Discipation of energy within a boundary layer.

Displacement thickness δ_1

Let us consider a particular stream line which is at a distance $h(x,\psi_0)$ from the wall.

In this case inviscid flow the stream would have be a distance $h_i(x,\psi_0)$ from the wall.

We know that mass of the fluid flowing in unit time between y=0 and y=h is equal to the mass of the fluid per unit time between y=0 and $y = h_i$

In inviscid flow $u = u_1(x)$ for every y

We have $\int_0^h \rho u dy = \int_0^{hi} \rho u_1 dy = \rho u_1 [y]_0^{hi}$

$$\int_0^h \rho u dy = \rho u_1 h_i$$

$$h_i = \int_0^h \frac{u}{u_1} dy$$

The amount by which the stream is displaced outwards under the influence of viscosity

$$h - h_i = h - \int_0^h \frac{u}{u_1} dy$$

$$= \int_0^h \left(1 - \frac{u}{u_1}\right) dy$$

It follows that the amount by which the stream line for from the wall is displaced is

$$\lim_{n \to \infty} (h - h_i) = \delta_1(x) = \int_0^h \left(1 - \frac{u}{u_1}\right) dy$$

Hence $\delta_1(x)$ is called as displacement thickness.

Momentum thickness δ_2:

It is defined by comparing the loss of momentum due to the way function in the boundary to the momentum in the free flow region the momentum thickness δ_2 can be calculated as

$$\rho {u_1}^2 \delta_2 = \int_0^\infty u(\rho u_1 - \rho u) dy$$

$$\delta_2 = \int_0^\infty \frac{u}{u_1}\left(1 - \frac{u}{u_1}\right) dy$$

$\rho {u_1}^2 \delta_2$ is equal to the flux of defect of momentum in the boundary layer

$\delta_2(x)$ is called momentum thickness of a boundary layer.

Kinetic energy thickness δ_3:

There is always loss in kinetic energy because of viscosity now the loss of kinetic energy in the boundary layer at a distance y from the fluid is

$$\int_0^\infty \frac{1}{2} \rho (u_1 - u^2) u dy$$

If this integral is equaled to the quantity $\frac{1}{2}\rho u_1{}^3\delta_3, \delta_3$ can be considered as kinetic energy flux as the rate of which the kinetic energy loss of a boundary layer

$$\frac{1}{2}\rho u_1{}^3\delta_3 = \int_0^\infty \frac{1}{2}\rho(u_1{}^2 - u^2)u\,dy$$

$$\delta_3 = \int_0^\infty \frac{u}{u_1{}^3}(u_1{}^2 - u^2)dy$$

$$\delta_3 = \int_0^\infty \frac{u}{u_1}\left(1 - \frac{u^2}{u_1{}^2}\right)dy$$

Skin friction or wall shearing stress τ_w:

We considered the stress expectation upon the wall by the fluid in the boundary in 2D flow the components of the stress are

$$p_{ij} = p\delta_{ij} - \mu\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j}\right) + \frac{2\mu}{3}\left(\frac{\partial v_k}{\partial x_k}\delta_{ij}\right)$$

$$p_{11} = p - 2\mu\frac{\partial v_1}{\partial x_1}$$

$$= p - 2\mu\frac{\partial u}{\partial x}$$

$$p_{12} = -\mu\left(\frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2}\right)$$

$$= -\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = p_{21}$$

$$p_{22} = p - 2\mu \frac{\partial v_2}{\partial x_2}$$

$$= p - 2\mu \frac{\partial v}{\partial y}$$

Within the boundary layer $\frac{\partial u}{\partial y}$ is of order $\frac{u}{\delta}$ and $\frac{\partial v}{\partial x}$ is of order $\frac{\delta u}{L^2}$ so the ratios of these terms is $1:\left(\frac{\delta}{L}\right)^2$

$1:R^{-1}$ and $\frac{\partial v}{\partial x}$ may be neglated by comparison with $\frac{\partial v}{\partial y}$

Also by using 2n from of continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

Now from equation (1)

$$p_{11} = p - 2\mu \frac{\partial u}{\partial x}$$

$$p_{11} = p_{21} = -\mu \frac{\partial u}{\partial y}$$

$$p_{22} = p - 2\mu \frac{\partial v}{\partial y}$$

$$= p + 2\mu \frac{\partial u}{\partial y}$$

At the wall itself the stress acting on the wall in the direction is simply $-p_{21}$

$$\tau_w = -p_{21} = \mu \frac{\partial u}{\partial y}$$

Here τ_w is the skin friction or wall shearing stress.

The rate of energy destination per unit volume by viscosity or Discipation of energy within a boundary layer:

We know $\phi = \frac{1}{2}\mu(\zeta_{ij})^2$

Where $\zeta_{ij} = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}$

From the equation of continuity for the compressible flow

$\zeta_{kk} = 0$ and $\zeta_{11} = -\zeta_{22} = 2.\frac{\partial u}{\partial x}$

$\zeta_{12} = \zeta_{21} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

Sub these values in (1)

$$\phi = \frac{1}{2}\mu\left[4\left(\frac{\partial u}{\partial x}\right)^2 + 4\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right)^2\right]$$

$$= \frac{1}{2}\mu\left[8\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right)^2\right]$$

$$= 4\mu\left(\frac{\partial u}{\partial x}\right)^2 + \mu\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial x}\right)^2$$

The magnitude of varies terms in this expression is found that is

$\mu\left(\frac{\partial u}{\partial x}\right)^2 = o\left(\frac{\mu u^2}{\delta^2}\right)$ And the remaining terms are almost the order of R^{-1}

This expression may be neglated

The boundary layer approximation to the equation (1) is $\phi = \mu\left(\frac{\partial u}{\partial x}\right)^2$

This is the rate of Discipation per unit volume by viscosity

BOOK WORK 3:

Derive the integral equation for the boundary layer

PROOF:

Here there are two types of integral layer

1. Momentum integral
2. Kinetic energy integral equation

Momentum integral:

For 2D flow the momentum equation of the boundary layer is

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\gamma\left(\frac{\partial^2 u}{\partial y^2}\right)\text{..........(1)}$$

$$u_1\frac{\partial u_1}{\partial x}=-\frac{1}{\rho}\frac{dp}{dx}\text{..................(2)}$$

Sub (2) in (1)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_1\frac{\partial u_1}{\partial x}+\gamma\left(\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-u_1\frac{\partial u_1}{\partial x}=\gamma\left(\frac{\partial^2 u}{\partial y^2}\right)\text{...........(3)}$$

On integrating w r t y from 0 to ∞

$$\int_0^\infty\left(u\frac{\partial u}{\partial x}-u_1\frac{\partial u_1}{\partial x}\right)dy+\int_0^\infty v\frac{\partial u}{\partial y}\,dy=\int_0^\infty\gamma\left(\frac{\partial^2 u}{\partial y^2}\right)dy$$

$$\int_0^\infty \left(u\frac{\partial u}{\partial x} - u_1\frac{\partial u_1}{\partial x} \right) dy + \int_0^\infty v\frac{\partial u}{\partial y}\, dy = \gamma \left(\frac{\partial u}{\partial y} \right)_0^\infty$$

$$= \gamma \left(0 - \frac{\partial u}{\partial y} \right)_w$$

$$= -\gamma \left(\frac{\partial u}{\partial y} \right)_w$$

$$= -\frac{\mu}{\rho} \left(\frac{\partial u}{\partial y} \right)_w$$

$$= -\frac{\tau_w}{\rho} \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Where $\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_w$

Consider the 2nd integral

Since by integral parts first subtraction the zero quantity $v\frac{\partial u}{\partial y}$ from the integral

$$\int_0^\infty v\frac{\partial u}{\partial y}\, dy = \int_0^\infty \left(v\frac{\partial u}{\partial y} - v\frac{\partial u_1}{\partial y} \right) dy$$

$$= \int_0^\infty v\frac{\partial}{\partial y}(u - u_1)\, dy$$

$$= [v(u - u_1)]_0^\infty - \int_0^\infty (u - u_1)\, dy$$

Since u=v=0 when y=0 and $u = u_1$ when $y \to \infty$. In above equation the first term becomes zero

$$\int_0^\infty v\frac{\partial u}{\partial y}\,dy=\int_0^\infty (u-u_1)\frac{\partial u}{\partial y}\,dy$$

$$=\int_0^\infty (u-u_1)\frac{\partial u}{\partial y}\,dy$$

Equation (4) becomes

$$\frac{\tau_w}{\rho}==\int_0^\infty\left(u_1\frac{\partial u_1}{\partial x}-u\frac{\partial u}{\partial x}\right)dy-\int_0^\infty (u-u_1)\frac{\partial u}{\partial y}\,dy$$

But $\int_0^\infty (u-u_1)\frac{\partial u}{\partial y}\,dy=\frac{d}{dx}\left[\int_0^\infty u(u-u_1)dy\right]-\int_0^\infty u\left(\frac{\partial u}{\partial x}-\frac{\partial u_1}{\partial x}\right)dy$

Where $\left(\frac{d}{dx}=\frac{\partial}{\partial x}+q\nabla\right)$

$$\frac{\tau_w}{\rho}=\int_0^\infty\left(u_1\frac{\partial u_1}{\partial x}-u\frac{\partial u}{\partial x}\right)dy-\frac{d}{dx}\int_0^\infty u(u-u_1)dy+\int_0^\infty u\left(\frac{\partial u}{\partial x}-\frac{\partial u_1}{\partial x}\right)dy$$

$$\frac{\tau_w}{\rho}=\int_0^\infty\left(u_1\frac{\partial u_1}{\partial x}-u\frac{\partial u}{\partial x}\right)dy-\frac{d}{dx}\int_0^\infty u(u-u_1)dy-\int_0^\infty\left(u_1\frac{\partial u_1}{\partial x}-u\frac{\partial u}{\partial x}\right)dy$$

$$\frac{\tau_w}{\rho}=-\frac{d}{dx}\int_0^\infty u(u-u_1)dy$$

Here u and u_1 are functions of x along

$$\frac{\partial u_1}{\partial x}=\frac{du}{dx}$$

$$\frac{\tau_w}{\rho}=-\frac{d}{dx}u\int_0^\infty (u-u_1)dy+\frac{du_1}{dx}\int_0^\infty (u_1-u)dy$$

$$= -\frac{d}{dx}uu_1\int_0^\infty\left(\frac{u}{u_1}-1\right)dy + \frac{du_1}{dx}\int_0^\infty u_1\left(1-\frac{u}{u_1}\right)dy$$

$$= -\frac{d}{dx}uu_1\int_0^\infty\left(\frac{u}{u_1}-1\right)dy + \frac{du_1}{dx}u_1\int_0^\infty\left(1-\frac{u}{u_1}\right)dy$$

$$= -\frac{d}{dx}u_1{}^2\int_0^\infty\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)dy + \frac{du_1}{dx}u_1\int_0^\infty\left(1-\frac{u}{u_1}\right)dy$$

$$= -\frac{d}{dx}\left[u_1{}^2\delta_2\right] + \frac{du_1}{dx}u_1\delta_1 \text{...............(*)}$$

Here δ_1 is displacement thickness and δ_2 is momentum thickness.

Equation (*) is called momentum integral equation.

ii. kinetic energy integral equation:

for a 2d flow the momentum equation of the boundary layer is

$$u\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \gamma\left(\frac{\partial^2 u}{\partial y^2}\right) \text{..........(1)}$$

We know that

$$u_1\frac{du_1}{dx} = -\frac{1}{\rho}\frac{dp}{dx} \text{..................(2).}$$

$$u\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial y} = u_1\frac{du_1}{dx} + \gamma\left(\frac{\partial^2 u}{\partial y^2}\right).$$

$$u\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial y} - u_1\frac{du_1}{dx} = \gamma\left(\frac{\partial^2 u}{\partial y^2}\right) \text{............(3)}$$

Multiply equation (3) by u and integrate w r t y with the limit 0 to ∞ we have

$$\int_0^\infty u\left(u\frac{\partial u}{\partial y}-u_1\frac{du_1}{dx}\right)dy+\int_0^\infty uv\frac{\partial u}{\partial y}dy=\int_0^\infty \gamma u\left(\frac{\partial^2 u}{\partial y^2}\right)dy.............(4)$$

Using integration by parts

$$\int_0^\infty uv\frac{\partial u}{\partial y}dy=\int_0^\infty v\frac{\partial}{\partial y}\left(\frac{1}{2}u^2\right)dy$$

$$\int_0^\infty uv\frac{\partial u}{\partial y}dy=\frac{1}{2}\int_0^\infty v\frac{\partial}{\partial y}\left(u^2-u_1{}^2\right)dy$$

$$=\frac{1}{2}\left[v\left(u^2-u_1{}^2\right)\right]_0^\infty-\frac{1}{2}\int_0^\infty\left(u^2-u_1{}^2\right)\frac{\partial v}{\partial y}dy$$

Since u=v=0 when y=0

U= u_1 when y=0

$$=\frac{1}{2}\int_0^\infty\left(u^2-u_1{}^2\right)\frac{\partial v}{\partial y}dy.............(5)$$

Consider from equation (4) R.H.S

$$\int_0^\infty u\left(\frac{\partial^2 u}{\partial y^2}\right)dy=\left[u\frac{\partial u}{\partial y}\right]_0^\infty-\int_0^\infty\left(\frac{\partial u}{\partial y}\right)^2dy$$

$$=-\int_0^\infty\left(\frac{\partial u}{\partial y}\right)^2dy...............(6)$$

Sub equation (5) and (6) in (4)

$$\int_0^\infty u\left(u\frac{\partial u}{\partial y}-u_1\frac{du_1}{dx}\right)dy+\frac{1}{2}\int_0^\infty\left(u^2-u_1{}^2\right)\frac{\partial u}{\partial y}dy=-\gamma\int_0^\infty\left(\frac{\partial u}{\partial y}\right)^2dy$$

Using the equation of continuity

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$$

$$\int_0^\infty u\left(u\frac{\partial u}{\partial y} - u_1\frac{du_1}{dx}\right)dy + \frac{1}{2}\int_0^\infty \left(u^2 - u_1^{\,2}\right)\frac{\partial u}{\partial y}\,dy = -\gamma\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

Multiply throughout by -2

$$-2\int_0^\infty u\left(u\frac{\partial u}{\partial y} - u_1\frac{du_1}{dx}\right)dy - \int_0^\infty \left(u^2 - u_1^{\,2}\right)\frac{\partial u}{\partial y}\,dy = 2\gamma\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$2\int_0^\infty u\left(u_1\frac{du_1}{dx} - u\frac{\partial u}{\partial y}\right)dy - \int_0^\infty \left(u_1^{\,2} - u^2\right)\frac{\partial u}{\partial y}\,dy = 2\gamma\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\int_0^\infty \left(2uu_1\frac{du_1}{dx} - 2u^2\frac{\partial u}{\partial y} + u_1^{\,2}\frac{\partial u}{\partial y} - u^2\frac{\partial u}{\partial y}\right)dy = 2\gamma\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\int_0^\infty \frac{\partial}{\partial x}\left(uu_1^{\,2} - u^3\right)dy = 2\gamma\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\int_0^\infty \frac{\partial}{\partial x}\frac{u}{u_1}\left(u_1^{\,3} - u_1^{\,2}u\right)dy = 2\gamma\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\int_0^\infty \frac{\partial}{\partial x}\frac{u}{u_1}\left(u_1^{\,3} - u_1^{\,2}u\right)dy = \frac{2\mu}{\rho}\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\frac{\partial}{\partial x}\int_0^\infty u_1^{\,3}\frac{u}{u_1}\left(1 - \frac{u^2}{u_1^{\,2}}\right)dy = \frac{2\mu}{\rho}\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho u_1^{\,3}\delta_3\right) = \mu\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Where δ_3 is the kinetic energy thickness

$$u,\delta_3 = \int_0^\infty \frac{u}{u_1}\left(1-\frac{u^2}{{u_1}^2}\right)dy$$

Equation (1) is called kinetic energy integral equation. The rate of change of flux of kinetic energy defeat with the boundary layer is equal to the rate at which the kinetic energy is discipated by viscousity

BOOK WORK 4:

Derive Blasius equation at boundary layer

Or

Flow parallel to a semi infinite plate

Or

Boundary layer along a semi infinite plate

Let us consider a semi infinite plate with thickness zero, with velocity u in the stream study motion along x-axis

The plane is at y=0 and leading edge at x=0

We assume that the stream is neglibility effected by the pressure of the plane expect at the boundary layer

$$\frac{\partial P}{\partial x} = 0$$

Then the boundary layer equation becomes

$$\left.\begin{array}{l} u\dfrac{\partial u}{\partial y} + v\dfrac{\partial u}{\partial y} = \gamma\left(\dfrac{\partial^2 u}{\partial y^2}\right) \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial y} = 0 \end{array}\right\}\text{.....................(1)}$$

With boundary conditions $u = u_0$ at $y = 0$ at $u = u_1(x)$ at $y \to \infty$

We show the stream function has the relationship

$$\left. \begin{aligned} u &= \frac{\partial \psi}{\partial y} \\ v &= -\frac{\partial \psi}{\partial y} \end{aligned} \right\} \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

By using the idea of Blasius we introduce a function ψ

$$\psi = (2u_0\gamma_x)^{1/2} f(\eta) \ldots\ldots\ldots\ldots(3)$$

Her f is a function of η

And $\eta = \left(\frac{u_0}{2\gamma_x}\right)^{1/2} y$

From the equation (2)

$$u = \frac{\partial \psi}{\partial y}$$

$$u = \frac{\partial}{\partial y}(2u_0\gamma_x)^{1/2} f\left(\frac{u_0}{2\gamma_x}\right)^{1/2} y$$

$$u = (2u_0\gamma_x)^{1/2} f'\left(\frac{u_0}{2\gamma_x}\right)^{1/2} y$$

$$u = u_0 f'$$

$$v = -\frac{\partial \psi}{\partial y}$$

$$-v=\frac{\partial\psi}{\partial y}=(2u_0\gamma_x)^{1/2} f'\frac{\partial\eta}{\partial x}+f(\eta)\frac{1}{2}(2u_0\gamma_x)^{-1/2}\,2u_0\gamma$$

$$=(2u_0\gamma_x)^{1/2} f'\left(\frac{u_0}{2\gamma_x}\right)^{1/2}\left(-\frac{1}{2}x^{-3/2}\right)y+\frac{f'(2u_0\gamma)^{1/2}}{2x^{1/2}}$$

$$=\left(\frac{u_0 v}{2x}\right)^{1/2} f-\left(\frac{u_0\gamma}{2x}\right)^{1/2} f'(\eta)$$

$$v=\left(\frac{u_0 v}{2x}\right)^{1/2}(f'(\eta)-f)$$

Now $\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}(u_0 f')$

$$=u_0 f''\frac{\partial\eta}{\partial x}$$

$$=u_0 f''\left(\frac{u_0}{2\gamma}\right)^{1/2}\left(\frac{1}{x-x^{1/2}}\right)y$$

$$=-\frac{u_0}{2x} f''\left(\frac{u_0}{2\gamma x}\right)^{1/2} y$$

$$\frac{\partial u}{\partial x}=-\frac{u_0}{2x} f''(\eta)$$

$$\frac{\partial u}{\partial y}=\frac{\partial}{\partial y}(u_0 f')$$

$$=u_0 f''\frac{\partial\eta}{\partial y}$$

$$=u_0 f''\left(\frac{u_0}{2\gamma x}\right)^{1/2}$$

$$\frac{\partial^2 u}{\partial y^2} = u_0 f'''\left(\frac{u_0}{2\gamma x}\right)^{1/2} \frac{\partial \eta}{\partial y}$$

$$= u_0 f'''\left(\frac{u_0}{2\gamma x}\right)$$

Sub all these in equation (1)

$$-u_0 f' \frac{u_0}{2x}\eta f'' + \left(\frac{u_0 \gamma}{2x}\right)^{1/2} (\eta f' - f) u_0 f''\left(\frac{u_0}{2\gamma x}\right)^{1/2} = v u_0 \left(\frac{u_0}{2\gamma x}\right) f'''$$

$$\div \frac{u_0{}^2}{2x} \Rightarrow$$

$$-f f''\eta + f f''\eta - f f'' = f'''$$

$$f''' + f f'' = 0 \text{................(4)}$$

$$u\frac{\partial^3 f}{\partial y^3} + f\frac{\partial^2 f}{\partial y^2} = 0$$

With boundary condition

$$f = f' = 0 (or) \eta = 0 \quad f'' \to 1 \text{for } n \to \infty$$

This equation is known as Blasius equation for boundary layer along semi infinite plate.

BOOK WORK 5:

Show that the Blasius equation to the boundary layer along flate is a profile f(y) such that

$$\int_0^\infty (f' - f'^2) d\eta = f''(0)$$

SOLUTION:

The Blasius equation gives

$$f''' + ff'' = 0$$

Adding f'^2 on both sides we get

$$f''' + ff'' + f'^2 = f'^2$$

Integrate w r t η between the limit 0 to ∞ we get

$$\int_0^\infty \left(f''' + ff'' + f'^2 \right) d\eta = \int_0^\infty f'^2 d\eta$$

$$\int_0^\infty d\left(f'' + ff' \right) = \int_0^\infty f'^2 d\eta$$

$$\left[f'' + ff' \right]_0^\infty = \int_0^\infty f'^2 d\eta$$

Using the boundary condition $f = f'$ as $n \to \infty$ $f' = 0$ & $f' = 1$ $n \to \infty$

$$\left[f''(\infty) + ff'(\infty) \right] - \left[f''(0) + ff'(0) \right] = \int_0^\infty f'^2 d\eta$$

$$\left[0 + f(\infty) - f''(0) + 0 \right] = \int_0^\infty f'^2 d\eta$$

$$f(\infty) - \int_0^\infty f'^2 d\eta = f''(0)$$

$$\int_0^\infty f' d\eta = f(\infty) - f(0) = f(\infty)$$

$$\int_0^\infty f' d\eta - \int_0^\infty f'^2 d\eta = f''(0)$$

$$\int_0^\infty \left(f' - f'^2\right) d\eta = f''(0)$$

POSSIBLE QUESTIONS

UNIT 5

PART-B (6 MARKS)

1. Explain boundary layer separation
2. Obtain von mises transformation
3. Derive the equation that hold for curved if the radius of curvature is large compared to the boundary layer thickness
4. Explain the concept of the boundary layer
5. Define integral layer and its types.
6. Show that the Blasius equation to the boundary layer along flate is a profile f(y) such that

   $$\int_0^\infty (f' - f'^2) d\eta = f''(0)$$

7. Derive Blasius equation at boundary layer
8. Show that the Flow parallel to a semi infinite plate
9. Derive the concept of Boundary layer along a semi infinite plate

10. Explain characteristics of Some important boundary layer

11. Define (i) Displacement thickness δ_1 (ii)Momentum thickness δ_2

12. Explain (i) Kinetic energy thickness δ_3 (ii)Skin friction or wall shearing stress τ_w

13. Explain the concept Discipation of energy within a boundary layer.

14. Derive the boundary layer equation for the two dimensional flow along a plane.

15. State blasius equation and prandfl's boundary layer with application.

PART-C (10 MARKS)

1. Obtain the Blasius equation
2. Obtain the momentum integral equation
3. Derive Prandtl's boundary layer equation
4. Find the displacement thickness of boundary layer
5. Derive the integral equation for the boundary layer
6. Explain the applications of boundary layer.
7. Define (i) Displacement thickness δ_1 (ii)Momentum thickness δ_2 (iii) Kinetic energy thickness δ_3 (iv)Skin friction or wall shearing stress τ_w
8. Explain briefly about thickness of boundary layers.

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Established Under Section 3 of UGC Act 1956)
Pollachi Main Road, Eachanari (Po),
Coimbatore –641 021

Subject: Fluid Dynamics **Subject Code: 18MMP206**

Class : I - M.Sc. Mathematics **Semester : II**

Unit V

Part A (20x1=20 Marks)

(Question Nos. 1 to 20 Online Examinations)

Possible Questions

Questions	Opt 1	Opt 2	Opt 3	Opt 4	Answer
In a boundary layer characteristics which streamlines far from the wall are displaced then $\delta_1(x)$ is referred to as--------------	displacement thickness	momentum thickness	kinetic energy thicknesss	friction thifckness	displacement thickness
The value of displacement thickness $\delta_1(x)$=-------------------	$\int u(1-(u/u_1))$ dy	$\int 1-(1/u_1))$ dy	$\int 1-(u/u_1)$ dy	$\int (u/u_1)(1-(u/u_1))$ dy	$\int 1-(u/u_1)$ dy
When separation ocurrs in which circumstances the boundary layer approximation is suspect in such case is __________	displacement thickness	momentum thickness	kinetic energy thicknesss	friction thifckness	momentum thickness
A momentum thickness $\delta_2(x)$ is defined for incompressible flow as ---------	$\int u(1-(u/u_1))$ dy	$\int 1-(1/u_1))$ dy	$\int 1-(u/u_1)$ dy	$\int (u/u_1)(1-(u/u_1))$ dy	$\int (u/u_1)(1-(u/u_1))$ dy
A physically significant measure of boundary layer thickness is --------	displacement thickness	momentum thickness	kinetic energy thicknesss	friction thifckness	kinetic energy thicknesss
A measuresthe flux of kinetic energy defect within the boundary layer as compared with---------	viscous flow	steady flow	inviscid flow	incompressible flow	incompressible flow
The kinetic energy thickness is defined as $\delta_3(x)$=------	$\int u(1-(u/u_1))$ dy	$\int 1-(1/u_1))$ dy	$\int 1-(u/u_1)$ dy	$\int (u/u_1)(1-(u^2/u_1^2))$ dy	$\int (u/u_1)(1-(u^2/u_1^2))$ dy
The wall shearing stress is defined as -------	μ	δ	τ_w	ρ_w	τ_w
The skin friction τ_w=------	$(\partial u/\partial y)_w$	$\mu(\partial u/\partial y)_w$	$\delta(\partial u/\partial y)_w$	$(\partial^2 u/\partial y^2)_w$	$\mu(\partial u/\partial y)_w$
The onset of reversed flow near the wall takes place at the position of zero skin frction.such a position is called a position of ------	boundary layer friction	boundary layer characteristics	boundary layer separation	boundary layer flow	boundary layer separation
Kinematic viscosity is denoted by -----	$\mu=\gamma/\rho$	$\gamma=\mu/\rho$	$\rho=\mu\gamma$	$\gamma=\rho\mu$	$\gamma=\mu/\rho$
Enthalpy is defined as ----	I=E+P	I=E-(P/ ρ)	I=E+(P/ ρ)	I=E+(ρ / P)	I=E+(P/ ρ)
Thermal conductivity is denoted by ---------	p	I	ρ	K	K
Reynold's number is defined as -------	R=U/ γ	R=L/ γ	R=UL/ γ	R=U γ / L	R=UL/ γ
Viscosity is a function of temperature and -----------	pressure	mass	density	viscosity	pressure
Kinematic viscosity is a function of -------and pressure	pressure	temperature	density	force	temperature
The rate of increases of the boundary layer thickness depends on ------	$\partial p/\partial x$	$\partial q/\partial x$	$\partial p/\partial y$	$\partial q/\partial y$	$\partial p/\partial x$
The rate of -------- of the boundary layer thickness depends on boundary gradient	change	not change	increase	decrease	increase
The layer in which -----is called boundary layer	$\partial u/\partial y$	$\partial v/\partial y$	$\partial u/\partial x$	$\partial v/\partial x$	$\partial u/\partial y$
Kinetic energy thickness is also known as kinetic energy ---------	linear equation	laplace equation	integral equation	definite equation	integral equation
------- is called the pressure coefficient	c_v	P_c	V_C	c_p	c_p

-------- have zero velocity at the walls	real fluids	ideal fluid	viscous fluid	inviscid fluid	real fluids
Real fluids have-------- velocity at the walls	negative	positive	zero	nonzero	zero
Real fluids have zero velocity --------	near to the wall	opposite to the wall	at the walls	befor the wall	at the walls
If the pressure has ----then the boundary layer thickness increases rapidly	decreases	change	nochange	increases	increases
.If the pressure increases then the---- increases rapidly	boundary	boundary layer thickness	boundary layer	boundary surface	boundary layer thickness
If the ---------------increases then the boundary layer thickness increases rapidly	pressure	density	mass	force	pressure
If the pressure increases then the boundary layer thickness ------------- rapidly	decreases	gradually increases	increases	gradually decreases	increases
.-------- has no slip conditions	real fluids	ideal fluid	viscous fluid	inviscid fluid	real fluids
Real fluids has --------	no slip conditions	slip conditions	maximum slip conditions	minimum slip conditions	no slip conditions
The velocity component is normal to the wall is small if ----- is small	δ/2	δ/3	δ/4	δ/5	δ/2
The velocity component is normal to the wall is small if δ/2 is -----	normal	small	parallel	perpendicular	small
In the equation of boundary layer-------------------- normal to the wall is small	temperature gradient	temperature	pressure	pressure gradient	pressure gradient
In the equation of boundary layer pressure gradient -------------------- to the wall is small	parallel	normal	tangent	perpendicular	normal
The relationship between the pressure and main stream velocity can be obtained by --------	beltramis equation	linear equation	indefinite equation	Bernoulli's equation	Bernoulli's equation
------ is the flux of defect of momentum in the boundary layer	$\rho\mu_1\delta_2$	$\rho\mu_1$	$\rho\mu_1^2\delta_2$	$\mu_1^2\delta_2$	$\rho\mu_1^2\delta_2$
$\rho\mu_1^2\delta_2$ is the flux of defect of-------------------- in the boundary layer	acceleration	velocity	mass	momentum	momentum
In the equation of boundary layer the velocity component is-----to the wall	parralel	perpendicular	normal	tangent	normal
In the equation of ----- the velocity component is normal to the wall	boundary	boundary layer thickness	boundary layer	boundary surface	boundary layer
In the equation of boundary layer the velocity component is normal to the wall is -----	normal	parallel	small	perpendicular	small

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(18MMP206)

KARPAGAM ACADEMY OF HIGHER EDUCATION
COIMBATORE –21
DEPARTMENT OF MATHEMATICS
Second Semester
II Internal Test- February'19
Fluid Dynamics

Date : 06.02.2019(AN) **Time: 2 Hours**
Class : I-M.Sc. Mathematics **Maximum: 50 Marks**

PART - A (20 × 1 =20 Marks)
Answer All the Questions

1. The __________ can be classified as liquids and gases.
   a) solids b) fluids
   c) pressure d) forces
2. The density of fluids is defined as __________ volume.
   a) mass per time b) solid per time
   c) mass per unit d) limit per unit
3. A force per unit area is known as _____________.
   a) force b) pressure
   c) fluid d) density.
4. ∂F is the ____________ force due to fluid on ∂S.
   a) normal b) constant
   c) force d) pressure
5. The normal force per unit area is said to be _____________.
   a) normal stress b) shearing stress
   c) stress d) strain
6. The tangential force per unit area is said to be _____________.
   a) normal stress b) shearing stress
   c) stress d) strain
6. In a high viscosity fluid there exist normal as well as shearing stress is called ___________.
   a) viscous fluid b) inviscid fluid
   c) frictionless d) ideal
7. Which is the velocity of the equation.
   a) q=dr/dt b) q=s/r
   c) v=dx/w d) u=dy/s
8. The____________ is a branch of science which deals with the behavior of fluid at rest as well as motion.
   a) fluid mechanics b) fluid statics
   c) fluid kinematics d) fluids
9. The behavior of fluid at rest gives the study of_______________.
   a) fluid dynamics b) fluid statics
   c) elastic d) plastic
10. The behavior of fluid when it is in motion without considering the pressure force is called____________.
   a) fluid kinematics b) fluid mechanics
   c) fluid statics d) fluids
11. Circulation around a closed circuit 'c' is defined as___________
   a) ∫q.dr b) ∫q.rdr
   c) ∫qx.rdr d) ∫qx+dr
12. Euler's equation of motion is_______________
   a) dq/dt=F-∇p/P b) dq/dt=F-∇P
   c) dq/dt=F d) qd/dt=-∇ Ω
13. The ___________form is called the acceleration potential
   a) Ω+∫ ð P /p b) Ω-∫ ð P/ ρ
   c) -∇Ω-∇[∫ ð P/ ρ] d) ∇[∫ ð P/ ρ]
14. The irrotational flow of an incompressible in viscid fluid is in _____________
   a) 3 – D b) 1 – D
   c) 2 – D d) multi – Dimension

15. Circulation is always defined around a__________ circuit

a) open b) closed

c) parallel d) normal

16. When a conservative force F a potential Ω such that

a) F=∇Ω b) F=-∇Ω

c) F≠∇*Ω d) F≠∇.Ω

17. The Euler's equation of motion corresponding to a Beltrami's flow is

a) $\frac{\partial q}{\partial t} = \nabla$ b) $\frac{\partial q}{\partial t}$=-∇*

c) $\frac{\partial q}{\partial t}$=-∇ d) $\frac{\partial q}{\partial t}$≠-∇

18. A force is said to be conservative if the force can be derivable from the

a) density b) potential

c) area d) viscosity

19. If the motion is both steady and irrotational then

a) $\nabla . E = 0$ b) $\nabla \times E = 0$

c) $\nabla E = 0$ d) $\nabla - E = 0$

20. The product of the cross sectional area and magnitude of the vorticity is _______ along a vortex filament.

a) constant b) zero

c) parallel d) normal

PART-B (3 × 2 = 6 Marks)

Answer All the Questions

21. Define Laminar flow.

22. What are the two method of describing fluid motion?

23. Explain about the circulation flow around the closed curve.

PART-C (3 × 8 = 24 Marks)

Answer All the Questions

24. a) Prove that the velocity field u=yzt,v=zxt,w=xyt is a possible case of irrotational flow.

(OR)

b) Derive differential equation of a stream line.

25. a) The velocity components in a flow two dimensional flow fluid for an incompressible fluid is given by u=e^xcoshy, v=-e^xsinhy. Determine the equation of the streamline for this flow.

(OR)

b) Derive Euler's Generalized Momentum theorem.

26. a) Explain Beltrami's flow.

(OR)

b) Show that in an irrotational incompressible inviscid 2-D fluid flow both φ & ψ satisfy the Laplace equation.

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE-21

Department of Mathematics

Answer key for CIA-I

Subject : Fluid Dynamics.

Subject code : 18MMP206

Class : II. M.Sc. Mathematics.

Part - A

1) b) fluids

2) c) mass per unit.

3) b) pressure

4) a) normal.

5) a) normal stress

6) b) shearing stress.

7) a) $q = dr/dt$

8) a) fluid mechanics.

9) b) fluid statics.

10) b) fluid mechanics.

11) a) $\int q.dr$

12) a) $\frac{dq}{dt} = F - \nabla p/\rho$

13) a) $\Omega + \int \frac{\partial p}{\rho}$

14) c) 2-D

15) b) closed

16) b) $F = -\nabla \Omega$

17) c) $\frac{\partial q}{\partial t} = -\nabla.$

18) b) potential

19) a) $\nabla . E = 0$

20) a) constant.

Part - B

21) A flow in which each fluid particle out a definite curve and the curve traced any two different fluid particle donot intersect is said to be Laminar flow.

22) 1. Lagrangian Method

2. Eulerian method.

23) circulation is always defined around a closed circuit is nothing but the curve threating as a portion of surface

when the circulation around a closed circuit Γ is defined as.

$$\Gamma = \int_C \vec{q} \cdot d\vec{r}.$$

Part - C

24) a) $\nabla \times \vec{q} = 0$

$$\nabla \times \vec{q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yzt & zxt & xyt \end{vmatrix}$$

$$= \vec{i}\left[\frac{\partial}{\partial y}(xyt) - \frac{\partial}{\partial z}(zxt)\right] - \vec{j}\left[\frac{\partial}{\partial x}(xyt) - \frac{\partial}{\partial z}(yzt)\right]$$

$$+ \vec{k}\left[\frac{\partial}{\partial x}(zxt) - \frac{\partial}{\partial y}(yzt)\right]$$

$$= \vec{i}(0) + \vec{j}(0) + \vec{k}(0)$$

$$= 0.$$

24) or) b)

$$\vec{q} \times \vec{ds} = 0$$

$$\vec{ds} \times q = 0$$

$$(dx\vec{i} + dy\vec{j} + dz\vec{k}) \times (u\vec{i} + v\vec{j} + w\vec{k}) = 0$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0$$

$$\vec{i}(w\,dy - v\,dz) - \vec{j}(w\,dx - u\,dz) + \vec{k}(v\,dx - u\,dy) = 0$$

$$\Rightarrow \frac{dy}{v} = \frac{dz}{w}, \quad \frac{dx}{u} = \frac{dz}{w}, \quad \frac{dx}{u} = \frac{dy}{v}$$

$$\Rightarrow \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}.$$

25) a)

$$\frac{dx}{e^x \cosh y} = \frac{dy}{-e^x \sinh y}.$$

$$\Rightarrow dx = -\coth y\,dy.$$

$$\Rightarrow x = \log\left(c/\sinh y\right)$$

$$\Rightarrow e^x = c/\sinh y.$$

$$\Rightarrow c = e^x \sinh y.$$

25) OR) b)

$$\frac{d\vec{M}}{dt} = \frac{d}{dt}\int_V \vec{q}\cdot\rho\, d\tau$$

$$= \left(\frac{\partial}{\partial t} + \vec{q}\nabla\right)\int_V \vec{q}\cdot\rho\, d\tau$$

$$= \frac{\partial}{\partial t}\int_V \vec{q}\cdot\rho\, d\tau + (\vec{q}\cdot\nabla)\rho\,\vec{q}\, d\tau$$

$$= \frac{\partial}{\partial t}\int_V \vec{q}\cdot\rho\, d\tau + \int_V (\vec{q}\cdot\nabla)\rho\cdot\vec{q}\, d\tau$$

$$\frac{d\vec{M}}{dt} = \frac{\partial}{\partial t}\int_V \vec{q}\cdot\rho\, d\tau - \int_S (\vec{q}\cdot\hat{n})\,\vec{q}\cdot\rho\, ds.$$

$\frac{d\vec{M}}{dt}$ = The pressure force per unit mass.

$$\Rightarrow \frac{d\vec{M}}{dt} = \int_S p\,\hat{n}\, ds + \int_V \rho\,\vec{F}\, d\tau$$

$$\int_S p\,\hat{n}\, ds = -\int_V \rho\,\vec{F}\, d\tau + \frac{\partial}{\partial t}\vec{q}\cdot\rho\, d\tau - \int_S (\vec{q}\cdot\hat{n})\,\vec{q}\,\rho\, ds.$$

26) a) $\vec{q}\times\vec{\xi} = 0.$

$$\frac{\partial\vec{q}}{\partial t} = -\nabla\phi$$

$$\phi = \Omega + \int\frac{dp}{\rho} + \frac{q^2}{2}$$

The Beltrami's flow is

i) If $\vec{\xi}\neq 0$, $\vec{q}\times\vec{\xi} = 0$

ii) $\vec{q} \times \vec{q} = 0$

iii) In the case of a homogeneous liquid $\int \frac{dp}{\rho}$ is replaced throughout in all the equation by p/ρ.

All the previous or forgoing equations are known as Eulerian equation of motion.

2b) b)

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \psi}{\partial x \partial y}.$$

$$\frac{\partial^2 \phi}{\partial y^2} = -\frac{\partial^2 \psi}{\partial x \partial y}.$$

$$\Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0.$$

ϕ satisfied the Laplace equation,

Reg. No ----------------

(18MMP206)

KARPAGAM ACADEMY OF HIGHER EDUCATION
COIMBATORE –21
DEPARTMENT OF MATHEMATICS
Second Semester
II Internal Test- March'19
Fluid Dynamics

Date : 13.03.2019 (AN) **Time: 2 Hours**
Class : I-M.Sc. Mathematics **Maximum: 50 Marks**

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PART - A (20 × 1 =20 Marks)
Answer All the Questions

1. The irrotational flow of an incompressible in viscid fluid is in _____________
   a) 3 – D b) 1 – D
   c) 2 – D d) multi – Dimension
2. When the incompressible in viscid 2 – D fluid flow Φ and ψ satisfy the __________.
   a) C – R equation b) laplace equation
   c) linear equation d) differential equation
3. The stream function ψ exist whether the motes is_______________
   a) stream line b) path line
   c) rotational d) irrotational
4. The amount of the fluid going in to the sink in a unit time is called _________
   a) strength of the sink b) strength of the doublet
   c) strength of the source d) strength of the Vertex
5. The velocity potential Ø exits when the fluid is ____________
   a) rotational b) irrotational
   c) stream line d) path line
6. If a source, the velocity of the fluid is _______________
   a) finite b) equal
   c) infinite d) zero
7. If any point in the 2 – D field where the fluid is assumed to be _____________ is called a sink.
   a) created b) constant
   c) moving d) annihilated
8. In a 2 – D field where the fluid is assumed to be annihilated is called a _______________
   a) sink b) source
   c) strength of source d) strength of sink
9. The stream function is constant along a _________________
   a) stream line b) path line
   c) vortex line d) filament line
10. The amount of the fluid going in to the sink in a ______ is called strength of the sink
    a) certain Interval b) unit time
    c) mean time d) average
11. If the motion is steady, the stream line pattern is ____________
    a) fixed b) not fixed
    c) equal d) constant
12. When the motion is not steady the stream line pattern is _____________ fixed.
    a) not b) equal
    c) constant d) zero
13. In a circulation on a viscous fluid the space derivative of the vorticity vector are__________
    a) small b) constant
    c) large d) infinite
14. The steady flow through an arbitrary cylinder under pressure is known as_____________
    a) Hagen –Poiseuille flow b) viscous flow
    c) inviscous flow d) vorticity flow

15. In the Reynolds number is the principal parameter determining the _____________

a) role of the flow b) nature of the flow

c) order of the flow d) type of the flow

16. The constant of proportionality, μ depends entirely upon the physical properties of the fluid is called__________

a) typical viscous stress b) effect of viscosity

c) coefficient of viscosity d) viscosity of a flow

17. In the case of a real fluid frictionless resistance is known as ________________

a) shearing stress b) tangential stress

c) friction stress d) ideal fluid

18. In the case of___________frictionless resistance is known as tangential stress.

a) perfect fluid b) real fluid

c) friction stress d) ideal fluid

19. On real fluid, tangential stresses are _______________

a) large b) very small

c) small d) infinite

20. The property which causes the tangential stress is known as____________

a) inviscosity b) viscosity

c) real fluid d) velocity

PART-B (3 × 2 = 6 Marks)
Answer All the Questions

21. Define Sink and Dublet.

22. Show that in an irrotational incompressible inviscid 2-D fluid flow both φ & ψ satisfy the Laplace equation.

23. Write the Cauchy's Riemann equation in both cartesian and polar Coordinates.

PART-C (3 × 8 = 24 Marks)
Answer All the Questions

24. a) Explain Sink and its complex potential strength of the sink.

(OR)

b) Obtain the complex potential for the vortex.

25. a) In irrotational motions of 2-D, prove that $(\partial q/\partial x)^2+((\partial q/\partial y)^2=q.\Delta^2 q$.

(OR)

b) A velocity field is given by $\bar{q} = -x\bar{i}+(y+t)\bar{j}$ find the stream function and the stream line for the field at t=2.

26. a) Obtain the Helmholtz equations for vorticity of viscous fluid.

(OR)

b) Explain Milne Thomson's circle theorem.

KARPAGAM ACADEMY OF HIGHER EDUCATIONS

COIMBATORE - 21

Department of Mathematics.

Answer key for CIA-II

Subject : Fluid Dynamic

Subject code : 18MMP206

class : I - M.Sc. Mathematics.

Part - A

1) c) 2-D

2) b) laplace equation

3) d) irrotational

4) a) strength of the sink

5) b) irrotational.

6) c) infinite

7) a) created

8) a) sink

9) a) stream line

10) b) unit time

11) a) fixed

12) a) not

13) c) large

14) a) Hagen-poiseuille flow

15) b) nature of the flow

16) c) coefficient of viscosity

17) b) tangential stress

18) b) real fluid

19) c) small

20) b) viscosity.

~~(4)~~

part - B

21) Sink: Any two dimensional fluid field where the fluid is assumed to be annihilated is called a sink.

Doublet:- A combination of sink of source in particular is called doublet.

22)

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}, \quad \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{\partial^2 \phi}{\partial y^2} = -\frac{\partial^2 \psi}{\partial y \partial x}, \quad \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \psi}{\partial x \partial y}.$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

23) C-R equation, $\frac{\partial \phi}{\partial y} = -\frac{\partial \phi}{\partial x}, \quad \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}.$

Polar, $\frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad \frac{\partial \psi}{\partial r} = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}.$

Part - C

24) a) The equation of continuity,

$$-2\pi r q_r = 2\pi m$$

$$q_r = -m/r$$

Since the flow is symmetric radially so

$q_\theta = 0$

$$-2\pi r \frac{\partial \phi}{\partial r} = 2\pi m$$

$$\frac{\partial \phi}{\partial r} = -m/r.$$

$$\frac{\partial \psi}{\partial \theta} = -m$$

$$\psi = -m\theta$$

$$w = \phi + i\psi = -m\log r - m\theta$$

$$= -m(\log r + \log e^{i\theta})$$

$$w = -m\log z.$$

24) or) b) $q_r = 0$

$$q_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad — ①$$

$$q_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r} \quad — ②.$$

Equ ①, $k = \frac{1}{r}\frac{\partial \phi}{\partial \theta}(2\pi r)$

$$k/2\pi \, \partial\theta = \partial\phi$$

$$\phi = k/2\pi \, \theta$$

Equ ②, $-\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta}$

$$-\frac{\partial \psi}{\partial r} = \frac{1}{r}\frac{\partial}{\partial \theta}(k/2\pi)$$

$$\psi = -k/2\pi \log r$$

$$w = \phi + i\psi$$

$$w = k/2\pi \, \theta + i(-k/2\pi \log r)$$

$$w = \frac{-ik}{2\pi}\log z$$

Q5) b) $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$

$\vec{q}=u\vec{i}+v\vec{j}$

$\vec{q}^2=\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2$

Diff Partially w.r.to 'x'

$$q.\frac{\partial\phi}{\partial x}=\frac{\partial\phi}{\partial x}.\frac{\partial^2\phi}{\partial x^2}+\frac{\partial\phi}{\partial y}.\frac{\partial^2\phi}{\partial x\partial y} \quad - (1)$$

Diff Partially w.r.to 'y'

$$q.\frac{\partial q}{\partial y}=\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial y\partial x}+\frac{\partial\phi}{\partial y}.\frac{\partial^2\phi}{\partial y^2} \quad - (2)$$

Differentiation (1) w.rto x.

$$q.\frac{\partial^2 q}{\partial x^2}+\frac{\partial q}{\partial x}\frac{\partial q}{\partial x}=\frac{\partial\phi}{\partial x}.\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^2\phi}{\partial x^2}.\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial x\partial y}.\frac{\partial^2\phi}{\partial x\partial y}$$
$$+\frac{\partial\phi}{\partial y}.\frac{\partial^3\phi}{\partial x^2\partial y}.$$

Differentiation (2) w.r.to 'y'.

$$q.\frac{\partial^2 q}{\partial y^2}+\left(\frac{\partial q}{\partial y}\right)^2=\frac{\partial\phi}{\partial x}.\frac{\partial^3\phi}{\partial y^2\partial x}+\left(\frac{\partial^2\phi}{\partial x\partial y}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)$$
$$\left(\frac{\partial^3\phi}{\partial y^3}\right)+\frac{\partial^2\phi}{\partial y^2}.\frac{\partial^2\phi}{\partial y^2}$$

$$\Rightarrow q^2\left[\left(\frac{\partial^2\phi}{\partial x^2}\right)^2+\left(\frac{\partial^2\phi}{\partial x\partial y}\right)^2\right]=q^2\left[\left(\frac{\partial q}{\partial x}\right)^2+\left(\frac{\partial q}{\partial y}\right)^2\right]$$
$$=\left(\frac{\partial^2\phi}{\partial x^2}\right)^2+\left(\frac{\partial^2\phi}{\partial x\partial y}\right)^2$$

(OR)
25) b) $\vec{q} = -x\vec{i} + (y+t)\vec{j}$

$u = -x, \quad v = (y+t)$

$u = \frac{\partial\psi}{\partial y}, \quad v = \frac{\partial\psi}{\partial x}.$

$\psi = -xy + f(x,t)$

$\frac{\partial\psi}{\partial x} = (y+t)$

Integrate w.r.to x

$\psi = yx + tx + f(y,t)$

Put $t = 2$.

$\psi = yx + 2x + f(y,2).$

26) a). W.K.T, $\vec{a} = \frac{d\vec{q}}{dt} = \frac{\partial\vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}$

$\nabla(\vec{q}\cdot\vec{q}) = \vec{q}\times(\nabla\times\vec{q}) + (\vec{q}\times\nabla)\vec{q} + (\vec{q}\cdot\nabla)\vec{q}$

$\nabla(q^2/2) = \vec{q}\times(\nabla\times\vec{q}) + (\vec{q}\cdot\nabla)\vec{q}$

$(\vec{q}\cdot\nabla)\vec{q} = \nabla(q^2/2) - \vec{q}\times(\nabla\times\vec{q}).$

$\frac{d\vec{q}}{dt} = \frac{\partial\vec{q}}{\partial t} + \nabla(q^2/2) - \vec{q}\times\vec{q}$

$\vec{a} = \nabla\times\frac{\partial\vec{q}}{\partial t} - \frac{\partial}{\partial t}(\nabla\times\vec{q}).$

$$\nabla \times \vec{q} = \frac{\partial \vec{q}}{\partial t} - (\nabla \cdot \vec{q})\vec{q} + (\vec{q} \cdot \nabla)\vec{q} - (\vec{q} \cdot \nabla)\vec{q}.$$

equ of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \cdot \vec{q}) = 0.$$

$$\nabla \cdot \vec{q} = 1/\rho \frac{d\rho}{dt}$$

$$\nabla \times \vec{q} = \nabla \times \left(\frac{d\vec{q}}{dt}\right) = R.H.S.$$

$$\nabla \times \vec{q} = \nabla \times \frac{d\vec{q}}{dt} = R.H.S.$$

$$\therefore \frac{d\vec{q}}{dt} = \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q} - (\vec{q} \cdot \nabla)\vec{q}.$$

$$\nabla \times \vec{q} = \frac{d\vec{q}}{dt} = \frac{1}{\rho}\frac{d\rho}{dt}\vec{q} - (\vec{q} \cdot \nabla)\vec{q}'$$

2). $\frac{\nabla p}{\rho} = \nabla \int \frac{dp}{\rho}$

$$\nabla \times \vec{q} = -\nabla \times \nabla \left[\int \frac{dp}{\rho} + \Omega\right] = 0$$

$$\nabla \times \vec{q} = 0$$

$$\frac{d}{dt}\left(\frac{\vec{q}}{\rho}\right) = \left(\frac{\vec{q}}{\rho} \cdot \nabla\right)\vec{q}$$

2b) (OR) b) Let C be the cross section of the circle $|z|=a$

$$\phi + i\psi = f(z) + \bar{f}\left(a^2/z\right)$$

$$z\bar{z} = a^2$$

$$\bar{z} = a^2/z$$

$$|\bar{z}| = a^2/z.$$

∴ z is lies outsides of the circle

and (a^2/z) is lies inside of the circle.

∴ the sigularities of z is lies outsides.

the circle and $f(a^2/z)$ is lies insides.

Then the new possible flow is,

$w = f(z) + \bar{f}\left(a^2/z\right)$ is the complex

potential flow for the plane.

Reg. No................................

[18MMP206]

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University)
(Established Under Section 3 of UGC Act, 1956)
Pollachi Main Road, Eachanari Post, Coimbatore – 641 021
(For the candidates admitted from 2018 onwards)

M.Sc., DEGREE EXAMINATION, APRIL 2019

Second Semester

MATHEMATICS

FLUID DYNAMICS

Time: 3 hours Maximum : 60 marks

PART – A (20 x 1 = 20 Marks) (30 Minutes)
(Question Nos. 1 to 20 Online Examinations)

(Part - B & C 2 ½ Hours)

PART B (5 x 6 = 30 Marks)
Answer ALL the Questions

21. a. Derive differential equation of a stream line.

Or

b. Obtain the condition that the surface F(r,t)=0.

22. a. Explain Bernoulli's equation.

Or

b. Obtain the Equation of motion in terms of vorticity vector when the force is conservative.

23. a. Show that in an irrotational incompressible inviscid 2-D fluid flow both φ & ψ satisfy the Laplace equation.

Or

b. Explain Sink and its complex potential strength of the sink.

24. a. Explain Vorticity of viscous fluid.

Or

b. Explain Navier Strokes equation.

25. a. Explain the equation of boundary layer.

Or

b. Derive the kinetic energy integral equation.

PART C (1 x 10 = 10 Marks)
COMPULSORY

26. Derive Euler's equation of motion.

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE - 21

DEPARTMENT OF MATHEMATICS

Answer key for ESE

subject : Fluid Dynamics

subject code } : 18MMP206

class : I. M.Sc. Mathematics.

Part - B

21. a) Let p be any point and q be the velocity that point at the some instant.

Since line is the stream line; — (2)

$\vec{q} \times ds = 0$

$\vec{ds} \times q = 0$

$(dx\vec{i} + dy\vec{j} + dz\vec{k}) \times (u\vec{i} + v\vec{j} + w\vec{k}) = 0$ — (2)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ dx & dy & dz \\ u & v & w \end{vmatrix} = 0.$$

$\Rightarrow i[w dy - v dz] - \vec{j}[w dx - u dz] + \vec{k}[v dx - u dy] = \vec{0}.$

$w dy - v dz = 0, \quad w dx - u dz = 0, \quad v dx - u dy = 0$ — (2)

$\Rightarrow \frac{dy}{v} = \frac{dz}{w}, \quad \frac{dx}{u} = \frac{dz}{w}, \quad \frac{dx}{u} = \frac{dy}{v}$

$\Rightarrow \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

21) or) b) Let P be a point ... surface, $F(r,t)=0$ —— ①

Let $\vec{q}$ be a fluid velocity and $\vec{u}$ be the velocity of the surface then by the boundary condition, } —— ①

$\vec{q}\cdot\hat{n} = \vec{u}\cdot\hat{n}$ —— ②

Since the direction ratio of $\hat{n}$ are $\left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)$ replace $\hat{n} = \nabla F$ —— ③

From ①, $(\vec{q}-\vec{u})\cdot\nabla\vec{F} = 0$ } —— ②

$\vec{q}\cdot\nabla F - \nabla F\cdot\vec{u} = 0$

Let $P(\vec{r},t)$ moves to the point $(\vec{r}+\delta\vec{r}, \vec{t}+\delta\vec{t})$ in time ∇t.

Then $\vec{q}$ must satisfies the equ of the boundary surface, $f(\vec{r}+\delta\vec{r}, \vec{t}+\delta\vec{t})=0$ —— ①

By tailor's theorem,

$F(\vec{r},t) + \delta\vec{r}\,\frac{\partial F}{\partial\vec{r}}(\vec{r},t) + \delta t\,\frac{\partial F}{\partial t}(\vec{r},t) = 0$

$\div$ by ∇v and $\delta\vec{r}\to 0$, $\delta t\to 0$, we get } —— ②

$\frac{\partial\vec{F}}{\partial t} + \vec{q}\cdot\nabla F = 0.$

2) a) The Equation of motion is given by

$$\frac{\partial \vec{q}}{\partial t} - \vec{q} \times \vec{\xi} = -\nabla\left(-\Omega + \int \frac{dp}{\rho} + \frac{q^2}{2}\right) \quad \text{—(1)}$$

when the motion is steady. This equation

reduces to $-\vec{q} \times \vec{\xi} = -\nabla\psi$

$\Rightarrow \; {}^{1}\!/_{2} = \vec{\xi} = \nabla\psi$ —(1) } —(1)

The L.H.S is a vector $\perp^r$ to the both $\vec{q}$

and $\vec{\xi}$. The R.H.S is normal to surfaces,

$$\psi = q^2/2 + \Omega + \int \frac{dp}{\rho} = \text{constant} \quad \text{—(1)}$$

—(2)

Hence $\vec{q} \times \vec{\xi}$ is perpendicular to both

$\vec{q} \times \vec{\xi}$ It follows that any particular

surface of the system. (2) contains

both stream lines and vertex lines. —(1)

$\therefore -\Omega + q^2/2 + \int dp/\rho = \psi$ has the same constant

→ (1)

value on a stream line. This is the general

form of Bernoulli's equation. For a liquid.

$\int \frac{dp}{\rho} = p/\rho$ and hence for a liquid Bernoulli's

equation becomes $q^2/2 + \Omega + \frac{p}{\rho} = \text{constant}.$

$\Rightarrow \vec{q} \times \vec{\xi} = 0$ and $\frac{\partial \vec{q}}{\partial t} = 0.$ } —(1)

$\therefore \nabla\psi = 0.$

22) OR) b) To Find the "Holmholtz equation of V... and when the force is conservative an... $\rightarrow$ (2)

and, if the pressure is the function of density, then.

$$\vec{a} = \frac{d\vec{q}}{dt} = \vec{F} - \frac{\nabla p}{\rho}$$

$$\vec{a} = -\nabla\Omega - \frac{\nabla p}{\rho}$$

$$\nabla \times \vec{a} = -\nabla \times \left(\nabla\Omega + \frac{\nabla p}{\rho}\right)$$

$$= -\nabla \times \nabla\Omega - \nabla \times \frac{\nabla p}{\rho}$$

— (2)

$\nabla \times \vec{a} = 0$

Then the Helmholtz equation reduces to the form $\frac{d}{dt}\left(\frac{\vec{\xi}}{\rho}\right) = \frac{1}{\rho}(\vec{\xi}\cdot\nabla)\vec{q}$. — (2)

23) a) By C-R equation

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \quad \text{— (1)}, \qquad \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \quad \text{— (2)}$$

— (1)

$$(1) \Rightarrow \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right) = \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial x}\right) = -\frac{\partial^2 \psi}{\partial x \partial y} \quad \text{— (3)}$$

$$(2) \Rightarrow \frac{\partial^2 \phi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) = \frac{\partial^2 \psi}{\partial y \partial x} \quad \text{— (4)}$$

$\longrightarrow$ (2)

∴ Laplace equation is,

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \frac{\partial^2\phi}{\partial x \partial y} - \frac{\partial^2\phi}{\partial y \partial x} = 0. \quad \text{—— (2)}$$

$$\left.\begin{aligned} \nabla^2\phi &= 0. \\ \text{llly } \nabla^2\psi &= 0. \end{aligned}\right\} \quad \text{—— (1)}$$

23) OR) b) The strength "m" of the sink is the measure of the flux acrosses any curve substaining unit radian at the sink, A sink of the strength "m" is the source of the stregth m. Any point in the two dimensional field where the the fluid is assured to be anhilated is called sink.

To satisfies the containing of continuity

we have, $-2\pi r q_r = 2\pi m,$

$$\left.\begin{aligned} q_r &= -m/r \end{aligned}\right\} \quad \text{—— (1)}$$

Since the flow is radial and symmetrical we have only the rth component of the velocity and $q_\theta = 0$

$$\left.\begin{aligned} -2\pi r \frac{\partial\phi}{\partial r} &= 2\pi m \end{aligned}\right\} \quad \text{—— (1)}$$

ie) $\frac{\partial \phi}{\partial r} = -m/r$

Since $q_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = 0$

we have ϕ is end of θ and hence on

integrating equation (1)

$\phi = -m \log r$

$\frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ and $\frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r}$

$\frac{\partial \psi}{\partial \theta} = -r\frac{\partial \phi}{\partial r}$ — (2)

$\Rightarrow -r\frac{m}{r} = -m$

$\Rightarrow \frac{\partial \psi}{\partial \theta} = -m$ — (2)

we have $\frac{\partial \phi}{\partial r} = 0$,

integrating $\psi = -m\theta$

$\Rightarrow w = \phi + i\psi = -m\log r + i(-m\theta)$

$= -m\{\log r + \log e^{i\theta})$ — (2)

$= -m \log re^{i\theta}$.

$w = -m \log z$.

1) a) The Navier Stokes equation for an incompressible fluid is given by,

$$\frac{d\vec{q}}{dt} = \vec{F} - \frac{\nabla p}{\rho} + \nu \nabla^2 \vec{q} \quad \text{—} (1)$$

$$\frac{d\vec{q}}{dt} = \frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q}$$

$$= \frac{\partial \vec{q}}{\partial t} + \tfrac{1}{2}\nabla q^2 - \vec{q}\times(\nabla\times\vec{q})$$

$$= \frac{\partial \vec{q}}{\partial t} + \tfrac{1}{2}\nabla q^2 - \vec{q}\times\vec{\xi}$$

— (2)

Since the fluid is incompressible, $\nabla\cdot\vec{q} = 0$

$$\therefore \nabla\times(\nabla\times\vec{q}) = -\nabla^2\vec{q} \quad \text{—} (2)$$

$$\nabla^2\vec{q} = -(\nabla\times\vec{\xi}) \quad \text{—} (3)$$

— (1)

If the force are compressible.

$$F = -\nabla\phi$$

$$\frac{\partial \vec{q}}{\partial t} - \vec{q}\times\vec{\xi} = -\nabla\phi - \frac{\nabla p}{\rho} - \nu[\nabla\times\vec{\xi}) - \tfrac{1}{2}\nabla q^2$$

$$= -\nabla\left(\phi + \frac{p}{\rho} + \tfrac{1}{2}q^2\right) - \nu(\nabla\times\vec{\xi})$$

$\longrightarrow$ (1)

Taking curl on both sides,

$$\frac{\partial}{\partial t}(\nabla\times\vec{\xi}) - \nabla(\vec{q}\times\vec{\xi}) = -\nabla\times\nabla\left(\phi + \frac{p}{\rho} + \tfrac{1}{2}q^2\right) - \nu\nabla\times(\nabla\times\vec{\xi})$$

$$\Rightarrow \frac{\partial \vec{\xi}}{\partial t} + (\vec{q}\cdot\nabla)\vec{\xi} = (\vec{\xi}\cdot\nabla)\vec{q} + \nu\nabla^2\vec{\xi}$$

$$\frac{d\vec{\xi}}{dt} = (\vec{\xi}\cdot\nabla)\vec{q} + \nu\nabla^2\vec{\xi}$$

— (2)

This is required equation.

24)(OR) b) consider an arbitrary volume V bounded by a closed surface S and let the outward drawn normal of this surface have direction cosine l_s. If v_i be the velocity then the momentum

$$= \rho \, \delta v \times v_i$$

$$= \int_V \rho v_i \, dv. \quad \Big\} \text{ --- (1)}$$

$$\Rightarrow \frac{d}{dt}\int_V \rho v_i \, dv = \int_V \frac{dv_i}{dt}(\rho . dv) + \int_V v_i \frac{d}{dt}(\rho dv)$$

$$\therefore \frac{d}{dt}\int_V v_i \rho \, dv = \int_V \frac{dv_i}{dt}(\rho dv) \text{ --- (1)} \quad \Big\} \text{ --- (2)}$$

If F_i be the external force,

$$\int_V F_i \rho \, ds \text{ --- (2)}$$

$$\therefore = -P_{ij} l_j \delta s \quad (\text{Since } P_i = -P_{ij} l_j)$$

x_i direction $= -\int_S P_{ij} . l_j \, ds.$

$$= -\int_V \frac{\partial}{\partial x_i} P_{ij} \, dv \text{ --- (3)} \quad \Big\} \text{ (1)}$$

using (1), (2), (3).

$$\int_V \rho \frac{dv_i}{dt} dv = \int_V F_i \rho \, dv - \int_V \frac{\partial}{\partial x_i} P_{ij} \, dv \quad \Big\} \text{ --- (1)}$$

L

The volume is arbitrary we get.

$$\frac{dv_i}{dt} = F_i - \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[p\delta_{ij} - \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \frac{\partial \mu}{\partial x}\left(\frac{\partial v_k}{\partial x_k}\right)\delta_{ij}\right] \quad \longrightarrow (4)$$

$$\frac{dv_i}{dt} = F_i + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] - \frac{1}{\rho}\frac{\partial}{\partial x_i}\left(p + \frac{\partial \mu}{\partial x}\left(\frac{\partial v_k}{\partial x_k}\right)\right) \quad \longrightarrow (2)$$

This is Navier Stokes equation.

25) a) Let us consider the 2-D flow of the fluid in the xy-plane where x is measured along the direction of the flow and y is normal to the surface.

The velocity increase rapidly from zero at the velocity u, within a short distance δ from the wall. Accordingly, $\frac{\partial u}{\partial y}$ is large and the viscous stress $\mu \frac{\partial u}{\partial y}$, a because importan[t] when μ is small. This layer in which $\frac{\partial u}{\partial y}$ is a dominant factor is called boundary layer.

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad \text{—— (1)} \quad \text{—— (1)}$$

The typical length normal the velocity s.

$$\left.\begin{aligned} \frac{\partial v}{\partial y} &= -\frac{\partial u}{\partial x} \\ \text{order of } v &= o\left[\frac{U}{L}\cdot\delta\right] \end{aligned}\right\} \text{---} (1)$$

the velocity component normal to the wall is small if δ/L is small. By Navier Stokes equ,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \text{---} (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] \text{---} (3)$$

There are in the proprotion of $\longrightarrow$ (2)

$$\left(1, 1, 1, 1/R, 1/R, \frac{L}{\delta^2}\right)$$

the order of $\nu.\frac{\partial^2 u}{\partial x^2}$ is small,

$$(2) \Rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \text{---} (4)$$

$$\Rightarrow -\frac{1}{\rho}\frac{\partial p}{\partial y} = o\left[\frac{u^2}{L^2}\cdot\delta\right]$$

(1)

Thus the pressure gradient normal to the wall is small and so it is neglated.

The boundary layer equation are -

$$\left.\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\ &u.\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 v}{\partial y^2} \text{---} (5) \end{aligned}\right\} \text{---} (1)$$

In such case the boundary conditions are

$u=0, v=0$ for $y=0$.

i.e) x-axis, $u=u(x)$, $P=P(x)$ for $y=\infty$.

(OR)

25) b) consider the boundary layer equ,

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=u_1\frac{\partial u_1}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \quad \text{---} (1)$$

multiply u throught and integrate between $y=0$ and $y=\infty$.

$$\int_0^\infty u\left(u.\frac{\partial u}{\partial x}-u_1\frac{\partial u_1}{\partial x}\right)dy+\int_0^\infty u.v\frac{\partial v}{\partial y}dy=\nu\int_0^\infty u.\frac{\partial^2 u}{\partial y^2}dy \quad \text{---} (1)$$

consider, $\nu\int_0^\infty u\frac{\partial^2 u}{\partial y^2}dy=\nu\int_0^\infty u\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy$

$$=\nu\left[\left(u.\frac{\partial u}{\partial y}\right)_0^\infty-\int_0^\infty\frac{\partial u}{\partial y}.\frac{\partial u}{\partial y}dy\right]$$

$$=-\nu\int_0^\infty\left(\frac{\partial u}{\partial y}\right)^2dy \quad \text{---} (2)$$

consider, $\int_0^\infty uv\frac{\partial v}{\partial y}dy=\int_0^\infty v\frac{\partial}{\partial y}\left(\tfrac{1}{2}u^2\right)dy$

$$=\tfrac{1}{2}\int_0^\infty v\frac{\partial}{\partial y}(u^2-u_1^2)dy$$

$$= \frac{1}{2}\int_0^\infty (u^2 - u_1^2)\frac{\partial u}{\partial x}\, dy.$$

$$① \Rightarrow \int_0^\infty u\left(u\frac{\partial v}{\partial x} - u_1\frac{\partial u_1}{\partial x}\right) dy = \nu\int_0^\infty u\frac{\partial^2 u}{\partial y^2}\, dy$$

$$\Rightarrow \int_0^\infty u\left(u\frac{\partial v}{\partial x} - u_1\frac{\partial u}{\partial x}\right) dy + \frac{1}{2}\int_0^\infty u^2 - u_1^2\frac{\partial u}{\partial x}\, dy = -\nu\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy.$$ ②

(×) by (−2).

$$\int_0^\infty -2u\left[u\frac{\partial v}{\partial x} - u_1\frac{\partial u_1}{\partial x}\right] dy - \int_0^\infty (u^2 - u_1^2)\frac{\partial u}{\partial x}\, dy = 2\nu\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\Rightarrow \int_0^\infty \frac{\partial}{\partial x}\left[u_1^2 u - u^3\right] dy = 2\nu\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

$$\frac{d}{dx}\left[u_1^3\int_0^\infty \frac{u_0}{u_1}\left(1 - \frac{u^2}{u_1^2}\right) dy\right] = 2\nu\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$ — ①

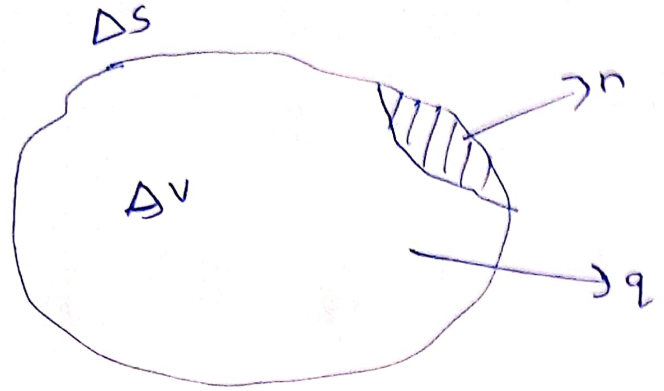
$$\therefore \frac{d}{dx}\left[\frac{1}{2}u_1^2\delta_3\right] = \nu\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy.$$

$$\text{i.e.) } \frac{d}{dx}\left[\frac{1}{2}u\, u_1^2\delta_3\right] = \nu\int_0^\infty \left(\frac{\partial u}{\partial y}\right)^2 dy$$

∴ This is required equation.

Part – C

6)



} — ①

i) The normal pressures on the boundary on the boundary — ①

ii) External force such gravity, say $\vec{F}$ be unit mass body force.

The total force is,

$$\int_S p.\hat{n}\, ds + \int_V \vec{F}\cdot \rho\, d\tau = -\int_V \nabla .p\, d\tau + \int_V \vec{F}\, \rho\, d\tau \quad — ②$$

where $\hat{n}$ is unit inward drawn normal vector (using Gauss divergence theorem). equating this to the rate of change of linear momentum, we get

$$\int_V \rho.\frac{d\vec{q}}{dt}\, d\tau = -\int_V \nabla.p\, d\tau + \int_V \vec{F}.\rho\, d\tau$$

$$\Rightarrow \int_V \left(\vec{F}\rho - \nabla p - \rho\frac{d\vec{q}}{dt}\right). d\tau = 0.$$

} — ②

Since the shape of the fluid body and therefore the volume of integration is arbitrary, we must have.

$$\Rightarrow \vec{F}\rho - \nabla p = \rho . \frac{d\vec{q}}{dt} = 0. \Bigg\} \longrightarrow (2)$$

dived by ρ

$$\frac{d\vec{q}}{dt} = \vec{F} - \frac{1}{\rho} \nabla . p \quad \Bigg\} \text{——— } (2)$$

∴ This is the Euler equation of motion.

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Established Under Section 3 of UGC Act 1956)
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Subject: Fluid Dynamics **Subject Code: 18MMP206**
Class : I - M.Sc. Mathematics **Semester : II**

Unit I

Part A (20x1=20 Marks)
(Question Nos. 1 to 20 Online Examinations)
Possible Questions

Questions	Opt1	Opt2	Opt3	Opt4	Answer
The behavior of fluid at rest gives the study of_______________.	fluid dynamics	fluid statics	elastic	plastic	fluid statics
The behavior of fluid when it is in motion without considering the pressure force is called____________.	fluid kinematics	fluid mechanics	fluid statics	fluids	fluid kinematics
____________ is a branch of science which deals with the behavior of fluid at rest as well as motion.	fluid mechanics	fluid statics	fluid kinematics	fluids	fluid mechanics
The behavior of fluid when it is in motion with considering the pressure force is called__________.	fluid kinematics	fluid dynamics	fluid statics	fluid mechanics	fluid dynamics
_____________ is the branch of science which deals with the study of fluids.	fluid kinematics	fluid dynamics	fluid statics	fluid mechanics	fluid dynamics
If any material deformation vanishes when a force applied withdrawn a material is said to be__________.	elastic	plastic	deformation	fluid	elastic
If deformation remains even after the force applied withdrawn the material is said to be____________	elastic	plastic	fluid	fluid statics	plastic
If the deformation remains even after the force applied withdrawn this property of material is_____________.	elastic	plasticity	fluid	deformation	plasticity
__________ can be classified as liquids and gases.	solids	pressure	fluids	forces	fluids
The density of fluids is defined as __________ volume.	limit per unit	solid per time	mass per unit	forces per unit	mass per unit
A force per unit area is known as _____________.	force	pressure	fluid	density.	pressure
ƏF is the ____________ force due to fluid on Əs	normal	constant	force	pressure	normal
The pressure changes in the fluid beings changes in the dencity of fluid is called___________.	compressible fluid	incompressible fluid	body force	surface force	compressible fluid
The change in pressure of fluid do not alter the density of the fluid is called_____________.	compressible fluid	incompressible fluid	body force	surface force	incompressible fluid
__________ are propotional to mass of the body.	pressure	body force	surface force	force	body force
__________ are propotional to the surface area.	body force	surface force	force	mass	surface force
The normal force per unit area is said to be _____________.	normal stress	shearing stress	stress	strain	normal stress
The tangential force per unit area is said to be _____________.	normal stress	shearing stress	stress	strain	shearing stress
In a high viscosity fluid there exist normal as well as shearing stress is called ___________.	viscous fluid	inviscid fluid	frictionless	ideal	viscous fluid

Which is the velocity of the equation.	q=dr/dt	.q=s/r	.v=dx/w	.u=dy/s	q=dr/dt
The differential equation of the path line is___________.	.u=dy/s	.v=dx/w	q=dr/dt	.q=s/r	q=dr/dt
A flow in which each fluid particle posses different velocity at each section of the pipe are called__________.	uniform flow	rotational floe	barotropic flow	non-uniform flow	non-uniform flow
A flow in which each fluid particle go on rotating about their own axis while flowing is said to be__________.	rotational floe	uniform flow	non-uniform flow	barotropic flow	uniform flow
The pressure is function of density then the flow is said to be ____________.	rotational floe	uniform flow	barotropic flow	non-uniform flow	barotropic flow
The direction of the fluid velocity at the point is called____________.	stream line	velocity	fluid	pressure	stream line
___________ is defined as the locus of different fluid particles passing through a fixed point.	stream filament	stream line	path line	stream tube	stream line
A stream tube of an infinitesimal cross sectional area is called___________.	stream line	stream filament	path line	stream tube	stream filament
The equation of volume is___________.	cross section area*speed	speed/cross section area	cross section area/speed	speed	cross section area*speed
The equation of speed is___________.	time/length	length/speed	length*time	time*speed	length/speed
When a fluid particle moves it changes in both _______________.	speed and time	time and frequency	speed and position	position and time	position and time
When the flow is __________ the strem line have same form at all times.	steady	unsteady	stream surface	stream tube	steady
When the flow is________ the stream line changes from instant to instant.	stream tube	steady	unsteady	steady	unsteady
If $\Delta .f=0$ then f is said to be a ______________.	solenoid	rotation	irrotation	constant	solenoid