Reg. No ------(17CCP103/17CMP103) KARPAGAM ACADEMY OF HIGHER EDUCATION COIMBATORE - 21 Department of Commerce First Semester First Internal Test – August – 2017 OPERATIONS RESEARCH Date : 30.08.2017 (FN) Class : I – M.Com / M.Com (CA) Time: 2 Hours

PART-A (20 x 1 = 20 Marks)

Answer All the Questions

1. Programming is another word for -----b) organizing c) managing d) decision making a) planning 2. Maximize $Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$ is called-----b) objective function a) constraints c)feasible function d)optimal function 3. LPP is a technique of finding the -----b) approximate solution a) optimal solution c) infeasible solution d) infeasible solution 4. More than two decision variables problem in LPP cannot be solved by ----a) simplex method b) Big-M method d) Dual simplex method c) Graphical method 5. Optimization means----a) maximization of profit b) minimization of constraints c) minimization of profit d) maximization of cost 6. A Feasible solution to a LPP which is also a basic solution to the problem is called -----a) basic solution b) basic feasible solution c) non basic feasible solution d) optimal solution 7. If Minimum (Z) = -5, then the maximum (Z) =-----a) –5 b) 5 c) 4 d) -4 8. The feasible region of an LPP is always-----b) upward c) downward d) a straight line a) convex

9. Linear programming problem involves ----- objective function. b) three a) four c) two d) one 10. Constraints appear as ------ when plotted in a graph b) straight line c) point d) circle a) curve 11. The optimum solution of transportation problem can be obtained by using ----a) VAM b) Modi method c) Hungarian method d) Simplex method 12. Degeneracy in a m x n transportation problem occurs when the number of occupied cells is -----a) < m+n-1 b) > m+n-1d) <m+n c) m+n-1 13. ----- is used to find IBFS of the transportation problem. b) Modi method a) Graphical method c) Least cost method d) Hungarian method 14. When the number of positive allocation in any basic feasible solution are less than m+n-1 then the solution is said to be -----b) degenerate c) balanced a) optimum d) independent 15. Transportation problem is said to be balanced if -----a) $\sum a_i = \sum b_i$ b) $\sum a_i < \sum b_i$ c) $\sum a_i > \sum b_j$ d) $\sum a_i \neq \sum b_j$ 16. The feasible solution of a transportation problem with 3 rows and 4 columns is said to be non-degenerate solution if the number of basic cells is equal to----a) 7 b) 8 c) 5 d) 6 17. The u-v method is also known as -----a) Modi method b) VAM c)LCM d) NVCR 18. A -----BFS is BFS in which none of the basic variables are zero. b) infeasible c) Non- degenerate d) feasible a) Degenerate 19. The total assignment value of ith machine is -----b) 2 c) 3 a) 1 d) 4 20. An assignment problem always a ----- form of a TP. a) independent b) dependent c) degenerate d) non degenerate

PART-B $(3 \times 2 = 6 \text{ Marks})$

Answer All the Questions

- 21. Define Slack Variable.
- 22. Mention the methods for finding IBFS for a transportation Problem.

23. Express the following LPP in Standard form.

Maximize $Z = 4x_1 + 2x_2 + 6x_3$ Subject to the constraints: $2x_1 + 3x_2 + 2x_3 \ge 6$ $3x_1 + 4x_2 = 8$ $6x_1 - 7x_2 + x_3 \le 10$ and $x_1, x_2, x_3 \ge 0$

PART-B (3 x 8 = 24 Marks)

Answer All the Questions

24. a) Solve the following LPP by the graphical method.

Maximize $Z = 3x_1 + 2x_2$ Subject to the constrains

 $-2x_1 + x_2 \le 1$ $x_1 \leq 2$

> $x_1 + x_2 \le 3$ and $x_1, x_2 \ge 0$

(**OR**)

b) Use simplex method to solve the following LPP

Maximize $Z = 4x_1 + 10x_2$ Subject to the constrains $2x_1 + x_2 \leq 50$ $2x_1 + 5x_2 \leq 100$ $2x_1 + 3x_2 \leq 90$ and $x_1, x_2 \ge 0$

25. a) Find the starting solution of the following transportation model



Using (i) North West Corner rule

(ii) Vogel's approximation method.

(**OR**)

b) Find the optimal solution to the following transportation problem. 1 2 3 Supply

4

					_
Ι	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	-

26. a) Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows:

			0			
			•	Job		
	1	2	3	4	5	
	ſ				7	
А	8	4	2	6	1	
В	0	9	5	5	4	
From C	3	8	9	2	6	
D	4	3	1	0	3	
E	9	5	8	9	5	
	C				J	

(**OR**)

b) Find an optimum solution to the following T.P

		Destina	tion		
	D	E	F	G	Available
А	11	20	7	8	50
Origin B	21	16	20	12	40
С	8	12	18	9	70
Requiremen	nt 30	25	35	40	

	Reg. No
	(17CCP103/17CMP103)
KARPAGAM ACADEMY OF HI	GHER EDUCATION
COIMBATORE	- 21
Department of Com	imerce
First Semester	r
Second Internal Test – Sept	tember – 2017
OPERATIONS RESI	EARCH
Date : 25.10.2017 (AN)	Time: 2 Hours
Class : I – M.Com / M.Com (CA)	Maximum: 50 Marks
Department of Com First Semester Second Internal Test – Sept OPERATIONS RESI Date : 25.10.2017 (AN) Class : I – M.Com / M.Com (CA)	- 21 imerce r tember – 2017 EARCH Time: 2 Hours Maximum: 50 Marks

PART-A (20 x 1 = 20 Marks)

Answer All the Questions

1. The arriving people in a queueing system are called -----. b) servers a) Input c) customers d) queue 2. Mean service time is denoted by -----. b) λ a) µ c) $1/\lambda$ d) $1 / \mu$ 3. The traffic intensity in queueing is defined by -----. b) μ / λ a) p / (p-1)c) λ / μ d) 1 / (1-p)4. A system is said to be in ----- state when its operating characteristics are dependent on time. a) Steady b) arrival c) service d) transient 5. Rate of consumption is different from -----. a) Rate of change b) Rate of production c) Rate of purchasing d) either b or c 6. ----- per unit item is affected by the quantity purchased due to quantity discounts or price breaks. a) Interested capital cost b) Handling cost d) Purchase price c) Holding cost 7. Storage cost is associated with -----a) Holding cost b) Shortage cost d) set up cost c) Carrying cost 8. If EOQ is 5000 units and Buffer stock is 500 units calculate minimum inventory. a) 5500 units b) 500 units c) 5000 units d) 5050 units

9. The penalty costs that are incurred as a result of running out of stock are known as---a) shortage cost b) set-up cost c) holding cost d) production cost 10. Reduction in procurement cost ----- EOQ a) increases b) decreases c) reduces d) neutral 11. If P is the purchase price of an item and I is the stock holding cost per unit time expressed as a fraction of stock value then the holding cost is -----. b) I + P d) IP a) I/P c) I - P12. Shortage cost is denoted by ----a) C_1 b) C_2 c) C_3 d) C_5 13. If total float in CPM is zero than the activity is called -----a) Critical activity b) Slack c) Free d) Normal 14. An event is also known as-----a) float b) slack c) arrow d) node 15. A dummy activity is denoted by -----in network diagram. b) arrow a) dotted line c) circle d) square 16. The expected time for each activity is computed by the formula --a) $(t_p + t_m + t_0) / 6$ b) $(4t_p + t_m + t_0) / 6$ c) $(t_p + 4t_m + t_0) / 6$ d) $(t_p + 4t_m + t_0) / 2$ 17. In network problem Total float = ----b) EST-LFT c) LST-EFT d) LFT – EFT a) LST-EST 18. ----- of an activity is the amount of time which the activity can be rescheduled without affecting the preceding or succeeding activities of that activity. a) Total float b) Free float c) Slack d) Independent float 19. An arrow in the network diagram represents -----a) starting point b) an activity c) finishing point d) node 20. A dummy activity in a project network always has ----- duration. a) two b) one c) zero d) four PART-B $(3 \times 2 = 6 \text{ Marks})$ **Answer All the Questions** 21. Define Steady State 22. Define holding cost.

23. Mention the formula for finding mean and SD in PERT method.

PART-B (3 x 8 = 24 Marks)

Answer All the Questions

24. a) In a railway marshalling yard, goods train arrive at a rate of 30 Trains per day. Assuming that inter arrival time follows an exponential distribution and the service time distribution is also exponential, with an average of 36 minutes. Calculate the following:

- (i) The mean Queue size (line length)
- (ii) The probability that Queue size exceeds 10
- (iii) If the input of the train increases to an average 33 per day, what will be the changes in (i), (ii)?

(**OR**)

b) A Project has the following characteristics

Activity :	А	В	С	D	Е	F
Duration:	6	8	4	9	2	7
Preceding						
activity :	_	А	А	В	С	D
Draw the network diagram and find the critical path.						

25. a) A company has a demand of 18,000 units per year for an item and it can produce 3000such items per month. The cost of one set up is Rs 500. and the holding cost per unit per month is Rs.0.15. The shortage cost of one unit is Rs.20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time between set- ups.

(**OR**)

b) Find the optimal order quantity for a product for which the pricebreak is as follows:

Quantity	Unit cost
$0 \le Q_1 < 50$	Rs. 10
$50 \le Q_2 < 100$	Rs. 9
$100 \leq Q_3$	Rs. 8

The monthly demand for the product is 200 units, the cost of the storage is 25% of the unit cost and ordering cost is Rs.20.00 per order.

26. a) Calculate the total float, free float and independent float for the project whose activities are given below:

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5
Duration	8	7	12	4	10	3	5
(in days)							
Activity	3-6	4-6	5-6				
Duration	10	7	4				
(in days)							

(**OR**)

b) The following table indicates the details of a project. The duration are in days. 'a' refers to optimistic time, 'm' refers to most likely time and 'b' refers to pessimistic time duration.

Activity	1 - 2	1 - 3	1 - 4	2 - 4	2 - 5	3 - 5	4 - 5
а	2	3	4	8	6	2	2
т	4	4	5	9	8	3	5
b	5	6	6	11	12	4	7

(a) Draw the network

(b) Find the critical path

(c) Determine the excepted standard deviation of the completion time.

Reg. No.

[16CCP103]

KARPAGAM UNIVERSITY

Karpagam Academy of Higher Education (Established Under Section 3 of UGC Act 1956) COIMBATORE - 641 021 (For the candidates admitted from 2016 onwards)

M.Com., DEGREE EXAMINATION, NOVEMBER 2016 First Semester

COMMERCE (COMPUTER APPLICATIONS)

OPERATIONS RESEARCH

Time: 3 hours

Maximum : 60 marks

PART - A (20 x 1 = 20 Marks) (30 Minutes) (Question Nos. 1 to 20 Online Examinations)

(Part - B & C 2 1/2 Hours)

PART B (5 x 6 = 30 Marks) Answer ALL the Questions

21. a. Explain the graphical method of solving a Linear Programming Problem involving two variables.

Or

b. A manufacturer can produce two types of Products A and B. Each of these products requires two different manufacturing operations namely grinding and finishing. The manufacturing requirements in hours are given below:

	A	B
Grinding	1	2
Finishing	3	2

The available capacities in hours are: Gridding 30 hours and Finishing. 90 hours. The contribution to profit is Rs. 2 per unit for A and Rs. 3 for B. Formulate the above problem as a linear programming problem.

22. a. Find initial solution for the following transportation problem using North West Corner Rule method.



- b. How the problem of degeneracy arises in a transportation problem? Explain the procedure for overcoming it.
- 23. a. Solve the following assignment problem (minimisation)

Job Worker	A	В	С	D
1	37	96	64	62
2	62	69	59	76
3	58	25	79	31
4	90	36	29	15
Or	-			

b. At a doctor's clinic patients arrive at an average rate of 5 patients per hour. It has been observed the doctor takes an average of 8 minutes per hour. Arrival time is based on poission and service is based on exponential distribution. Find The average number of patients in the doctor's clinic. The average number of patients waiting for their turn. The average time that a patient spends in the clinic.

24. a. List and explain the costs associated with inventories.

Or

Or

b. A shipbuilding firm uses rivets at a constant rate of 20,000 numbers per year. Ordering costs are Rs.30 per year. The rivets cost Rs.1.50 per number. The holding cost of rivets is estimated at 12.5% of unit cost per year. Determine the EOQ.

25. a. Write a brief note on PERT.

b. Find the critical path and the project duration for the following project.



2

PART C (1 x 10 = 10 Marks) (Compulsory)

26. Mechanics who work in a factory must get tools from a tool center. On an average of ten machinists per hour arrive asking for tools. At present, the tool center is staffed by a clerk who is paid \$6 per hour and who takes an average of 5 minutes to handle each request for tools. Since each machinist produces \$10 worth of goods per hour, each hour that a machinist spends at the tool center costs the company \$10. The company is deciding whether or not it is worthwhile to hire (at \$4 per hour) a helper for the clerk. If the helper is hired, the clerk will take an average of only 4 minutes to process requests for tools. Assume that service and interarrival times are exponential. Should the helper be hired?

And a Field initial solution of the company in ways to the boilt a At

Enable | Enlighten | Enrich (Deemed to be University) (Under Section 3 of UGC Act 1956)

KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore -641 021 LECTURE PLAN

Subject: OPERATIONS RESEARCH

Subject Code: 17CCP103

Class : I – M.Com (CA)

Semester : I

S.No.	Lecture Duration (Hr)	Topics to be covered	Support Materials			
		UNIT - I				
1.	1	Introduction to Operations Research	R2:Ch1;Pg:1.1-1.5			
2.	1	Application in Management Decision Making	T1:Ch 1;Pg:39-40			
3.	1	Linear Programming: Basic Concepts	T1:Ch 2;Pg:44-49			
4.	1	Formulation of LPP	T1:Ch 2;Pg:50-55			
5.	1	Graphical Solution to LPP	T1:Ch 2;Pg:56-59			
6.	1	Continuation of Graphical Solution of LPP	T1:Ch 2;Pg:60-62			
7.	1	Simplex Method (using slack variables only)	T1:Ch 3;Pg:74-80			
8.	1	Continuation of Simplex method	T1:Ch 3;Pg:80-86			
9.	1	Continuation of Simplex method	R2:Ch 3;Pg:3.9-3.15			
10.	1	Continuation of Simplex method	R2:Ch 3;Pg:3.16-3.21			
11.	11. 1 Recapitulation and discussion of possible question					
Total: 11 hrs						
T1:KanthiSwarup, Gupta P.K., Man Mohan., (2016) Operations Research, Sultan Chandand						
Sons, New Delhi.						
R2:Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., (2014), Resource						
Management Techniques, A. R. Publications, Nagapatinam.						
UNIT – II						

UNII – II							
1.	1	Transportation Model: Introduction	T1:Ch 6; Pg:146-148				
2.	1	Finding Initial Basic Feasible Solutions	T1:Ch 6; Pg:149-151				
3.	1	Continuation of finding IBFS	T1:Ch 6; Pg:152-156				
4.	1	Optimum Solution for Non-degeneracy Model	T1:Ch 6; Pg:158-160				
5.	1	Continuation on optimum solution for Non- degeneracy model	R2:Ch 7;Pg:7.26-7.28				
6.	1	Optimum Solution for Degeneracy Model	R2:Ch 7;Pg:7.29-7.31				
7.	1	Unbalanced Transportation Problems	R1:Ch 2;Pg:46-48				
8.	1	Continuation of Unbalanced Transportation Problems	R1:Ch 2;Pg:49-52				
9.	1	Maximization case in Transportation Problem	R2:Ch 7;Pg:7.40-7.47				
10.	1	Traveling Sales Man Problem.	R2:Ch 7;Pg:7.48-7.50				
11.	1	Continuation of Traveling Sales Man Problem.	R2:Ch 7;Pg:7.51-7.53				

		Recapitulation and discussion of possible										
12.	1 auestion											
Total •	Total: 12 hrs											
T1.Ka	1 Utal . 14 1113 T1-KanthiSwarun Cunta PK Man Mohan (2016) Operations Desearch Sultan Chandend											
Sons N	Sons New Delhi											
B1: Sharma I K (2008) Operations Research Theory Applications Macmillan India I td												
New Delhi												
R2.Su	ndaresan V	Gananathy Subramanian KS and Ganesa	n K (2014) Resource									
Manag	ement Techni	iques. A R Publications Naganatinam	in 13.9 (2011), 100500100									
Trianag		UNIT – III										
		The Assignment problem- Mathematical	R2:Ch 8:Po:8 1-8 5									
1.	1	Formulation of AP	R2.en 0,1 5.0.1 0.5									
2	1	Hungarian Method	R2.Ch 8.Pg.8 6-8 10									
2.	1	Continuation of using Hungarian Method	R2:Ch 8:Pg:8 11_8 1/									
<u>J.</u>	1	Unbalanced Assignment Problem	R2.Cli 8,1 g.8.11-8.14									
- + . 5	1	Maximization Case in AP	R2.Ch 4.Pg:// 17// 20									
5.	1	Travalling Salasman Brahlam	R3.Cli 4,1 g.4.17-4.20									
0.	1	Quanting Theory : Introduction	T1.Ch 17: Da:420 422									
1.	1	Queuing meory . Introduction	11.Cli 17, Fg.420-425									
8.	1	Problems in $(M/M/1):(\infty/FIFO)$ models	K2.Cli 15,Fg.15.5-									
		Continuation on Droblems in $(M/M/1)_{1}(\infty/EIEO)$	D2.Ch 12.Da.12 11									
9.	0. Continuation on Problems in $(M/M/1):(\infty/FIFO)$ R2:Ch 13;Pg:13											
		lilodels	13.10 D2.Ch 12.D $_{-12}24$									
10.	1	Problems in (M/M/1):(N/FIFO) models	K2.Cli 15,Fg.15.54-									
		Continuation on Problems in $(M/M/1)$: $(N/EIEO)$	13.30 P2.Ch 12.Dg.12.27									
11.	1	models	K2.Cli 15,Fg.15.57-									
		Recapitulation and discussion of possible	15.40									
12.	1	question										
Total	12 hm	question										
D2.Sm	12 IIIS ndoroson V	Canonathy Subramanian KS and Canasa	\mathbf{K} (2014) R esource									
Manag	ement Techni	iques A R Publications Naganatinam	\mathbf{M} N .,(2014), N esource									
R3.Sh	onthi Sonhia	Bharathi D (1900) Operations Research/Resource	ce management									
technia	ues Charulat	the Publications										
teening		INIT – IV										
1	1	Inventory Control: Introduction	T1.Ch 18. Pg./76-178									
1. 2	1	Costs involved in Inventory	T1.Ch 18, P_{g} :470-470 T1.Ch 18: P_{g} :470-480									
2.	1		R_{2} Ch 12.Pg 12.6									
3.	1	Deterministic EOQ Models	12.01 12,1 g.12.0-									
4	1	Purchasing Model without and with Shortage	T1:Ch 18: Pg:516-520									
	1	Continuation of problems on Purchasing Model	T1:Ch 18: Po:520-521									
5.	1	without and with Shortage	11.01110, 1 5.520 521									
		Continuation of problems on Purchasing Model	T1·Ch 18· Po·522-524									
6.	1	without and with Shortage	11.01110, 1 <i>5.322 32</i> 4									
		Manufacturing Model without and with	R3·Ch 6·Po·6 19-6 20									
7.	1	Shortage	1.0.01 0,1 5.0.17 0.20									

Lecture Plan 2017 Batch

8.	1	Continuation of Manufacturing Model without and with Shortage	R3:Ch 6;Pg:6.21-6.22						
9.	1	Manufacturing Model with Price Break R3:Ch 6;Pg:6.23-6.25							
10.	1	Continuation of Manufacturing Model with Price Break	R3:Ch 6;Pg:6.26-6.28						
		Recapitulation and discussion of possible							
11.	1	question							
Total :	11 hrs								
T1:Ka	nthiSwarup,	Gupta P.K., Man Mohan., (2016) Operations Res	search, Sultan Chandand						
Sons, N	lew Delhi.								
R2:Sur	ndaresan V.	, Ganapathy Subramanian K.S., and Ganesa	m K., (2014), Resource						
Manage	ement Techni	ques, A. R. Publications, Nagapatinam.							
R3:Sha	anthi Sophia	Bharathi D.,(1999), Operations Research/ Resource	ce management						
techniq	ues, Charulat	ha Publications.							
		UNIT – V							
1.	1	PERT and CPM: Network Representation	T1:Ch 21;Pg:611-613						
2	1	T1:Ch 21:Pg:614-616							
2.	1	11.01 21,1 5.01 1 010							
3.	1	T1:Ch 21;Pg:617-618							
4.	1	CPM - Various Floats for Activities	R4: Ch 2; Pg:907-909						
5.	1	CPM- Critical Path	R4: Ch 2; Pg:916-917						
6.	1	Continuation of Problems in CPM	R4: Ch 2; Pg:918-920						
7.	1	Time Estimates in PERT	T1:Ch 21;Pg:624-628						
8.	1	Continuation of Problems in PERT	T1:Ch 21;Pg:629-630						
9	1	Probability of Meeting scheduled date of	R5·Ch 22· Pg·234-240						
).	1	Completion of Projects.	K5.Ch 22, 1 g.23+ 2+0						
10	1	Continuation of Probability of Meeting	R5:Ch 22: Pg:241-250						
10.	-	scheduled date of Completion of Projects.	16.01 22, 1 8.2 11 200						
11.	1	Recapitulation and discussion of possible							
10		question							
12.	1	Discussion of pervious ESE question papers							
13.	1	Discussion of pervious ESE question papers							
14.	1	Discussion of pervious ESE question papers							
Total :	: 14hrs								
T1:Ka	nthiSwarup,	Gupta P.K., Man Mohan., (2016) Operations Res	search,Sultan Chandand						
Sons, N	lew Delhi.								
K4:Ha	mdyA.Taha	, Operations Research, (2007), Pearson education, Pr	rentice Hall.						
R5:Vit	tal– Operatio	ons Research – Margham Publications							

TEXT BOOK

T1:KanthiSwarup, **Gupta P.K., Man Mohan.,**(2016) Operations Research,Sultan Chandand Sons, New Delhi.

REFERENCES

R1: Sharma J.K., (2008), Operations Research Theory Applications, Macmillan India Ltd, New Delhi.

R2:Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K.,(2014), Resource Management Techniques, A. R. Publications, Nagapatinam.

R3:Shanthi Sophia Bharathi D.,(1999),Operations Research/ Resource management techniques, Charulatha Publications.

R4:HamdyA.Taha., Operations Research, (2007), Pearson education, Prentice Hall.

R5:Vittal– Operations Research – Margham Publications



Prepared by : R. Praveen Kumar, Department of Mathematics, KAHE

Linear Programming Problem / 2017

LPP is	optimal solution	approximate solution	both	infeasible solution	optimal solution
In an LPP maximization model	A constraint optimization model	A constraint decision making model	A mathematical programming model	A constraint inventory model	A constraint optimization model model
A Feasible solution to a LPP which is also a basic solution to the problem is called	the objective function is formed	the objective function is minimized	the objective function is maximized over	the constraints are formed	the objective function is maximized over
A Feasible solution to a LPP which is also a basic solution to the problem is called	basic solution	basic feasible solution	non basic feasible solution	optimal solution	basic feasible solution
The solution which optimizes the objective function are called	feasible solution	optimal solution	optional solution	arbitrary solution	optimal solution
A basic solution is said to be a if one of more of the basic variables are Zero	Non degenerate basic solution	infeasible solution	degenerate basic solution	unbounded solution	degenerate basic solution
More than two decision variables problem in LPP cannot be solved by	simplex method	Big-M method	Graphical method	Dual simplex method	Graphical method
More than two decision variables problem in LPP can be solved by	simplex method	Big-M method	Graphical method	Dual simplex method	simplex method
Another name for simplex method is	computational procedure	computational method	Big-M method	Dual simplex method	computational procedure
Constraints in the canonical LPP are of	>=	=	<=	<	<=
In standard form all constraints are expressed as	>=	=	<=	<	=
The minimization of the function f(x) is equivalent to the maximization of	-(-f(x))	f(x)	-f(-x)	1 / f(x)	-(-f(x))

In the entering variable is first calculated	Simplex method	Big-M method	graphical method	any one of these	Simplex method
Optimum solution of LPP in a Simplex procedure is always	un bounded	feasible	degenerate	basic feasible solution	basic feasible solution
A BFS is BFS in which none of the basic variables are Zero.	Degenerate	infeasible	non-degenerate	optimum	non-degenerate
The coefficient of slack variables in the constraints is	1	0	2	-1	1
In simplex method all the variables must be	negative	non-negative	have the same sign	initially zero	non-negative
The coefficient of slack variables in the objective function is	1	0	2	-1	0
Every equality constraint can be replaced equivalently by inequalities.	two	three	one	four	two
The standard form of the constraint 4x-5y < 20 is	4x-5y =20	4x-5y +S1=20	4x + 5y = 20	4x-5y - S ₁ =20	4x-5y+S1=20
The set of feasible solutions to an LPP is a	convex set	null set	concave	finite	convex set
The feasible region of an LPP is always	convex	upward	downward	a straight line	convex
A typical LPP must have at leastdecision alternatives.	three	two	one	many	two
The number of alternatives in a LPP is typically	finite	infinite	infeasible	feasible	finite

Linear Programming Problem / 2017

LPP deals with the problems involving only	one	two	more than one	more than two	one
Constraints appear as when plotted in a graph	curve	straight line	point	circle	straight line
An LPP is said to be infeasible if it has	no solution	infinite solution	Unbounded solution	infeasible solution	no solution
An LPP solution when permitted to be infinitely large is called	Unbounded	bounded	infeasible	large	Unbounded
The leaving variable row is called	key row	key column	pivot column	leaving row	key row
The entering variable column is called	-key row	pivot row	pivot row	entering row	pivot row
The intersection of the pivot column and pivot row is called the	pivot element	leaving element	unit element	first element	pivot element
For the optimal solution of an LPP, existence of an initial feasible solution is always	Assumed	given	does not exists	zero	Assumed
The solution $x_1 = 0, x_2 = 3$ is	feasible	in feasible	degenerate	non degenerate	degenerate
The solution $x_1 = 5$, $x_2 = 3$ is	feasible	in feasible	degenerate	non degenerate	non degenerate
If Minimum (Z) = -5, then the maximum (Z) =	5	5	4	-4	5
If the solution space is unbounded ,then the objective value will always be	bounded	unbounded	feasible	infeasible	unbounded

Prepared by : R. Praveen Kumar, Department of Mathematics, KAHE

In Simplex iteration ,the pivot element can be	-zero	one	negative	positive	one
Linear programming problem involves objective function.	four	three	two	one	one
Linear programming problems involving only variables can be effectively solved by a	-				
graphical method	four	three	two	one	two
Linear programming problems involving only					
two variables can be effectively solved by a					
method.	simplex	iteration	graphical	Big-M method	graphical
Pivot element is also called as	piyot row	key column	kev row	kev element	kev element
			key row		 Key clement
The element of intersection of the pivot column					
and pivot row is called the	pivot row	pivot column	keyrow	pivot element	pivot element
In maximization problem, the entering variable is					
the non-basic variable corresponding to the most					
negative value of	$ z_j + c_j $	z _j - c _j	$ \mathbf{z}_{j} / \mathbf{c}_{j} $	$z_j * c_j$	z _j - c _j



KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po),

Coimbatore -641 021

Subject: Operations Research

Subject Code:17CCP103

Semester : I

Class : I - M.Com (CA)

Unit II										
	Transportation Model									
Part A (20x1=20 Marks)										
	(Question Nos. 1 to 20 Online Examinations)									
Possible Questions										
Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer					
The Objective of the transportation problem is to be	Maximum	Minimum	Either maximum or minimum	Neither maximum nor minimum	Minimum					
When total supply is equal to total demand, the problem is called a	Feasible	Infeasible	Unbalanced	balanced	balanced					
Cells in the transportaion table having positive allocation will be called	occupied cells	unoccupied cells	empty cells	zero cells	occupied cells					
In a transportaion problem the various a's and b's are called	Supply	demand	rim requirement	destination	rim requirement					
The unit transportation cost from the ith source to jth destination is displayed in the of the (i j)th cell.	upper left side	upper right side	lower left side	lower right side	upper left side					
A balanced transportation problem will always have a	unique solution	infinite number of solution	infeasible solution	feasible solution	feasible solution					

A feasible solution to a (mxn) transportation problem that contains no more than m+n-1 non negative allocations is called	Feasible solution	optimul solution	basic feasible solution	infeasible solution	basic feasible solution
A feasible solution is said to be an optimul solution if it the total transportation cost	Minimize	Maximize	Either maximize or minimize	Neither maximize nor minimize	Minimize
The number of basic variables in an mxn balanced transportation problem is atmost	-m + n	m + n - 1	m + n +1	m - n -1	m + n -1
The number of non basic variables in an mxn balanced transportation problem is atleast	-mn - (m + n -1)	mn - (m - n -1)	mn - (m + n + 1)	mn - (m - n + 1)	mn - (m + n -1)
The non zero allocations imply that one cannot from a closed circuit by joining positive allocations by horizontal and vertical	independent	dependent	linear	non linear	independent
In a transportaion problem, the cost of transportation is	Linear	non linear	zero	one	Linear
For a feasible solution to exist, it is necessary that the total supply equal to total	cost	demand	cells	rows	demand
Least cost method is also called	Matrix method	Matrix maxima method	Matrix minima method	minima method	Matrix minima method
VAM method is also called	Penalty method	Matrix minima method	Lower cost method	Hungerian method	Penalty method
Vogel's approximation method is a	Penalty method	Matrix minima method	Hungerian method	Heuristic method	Heuristic method
In a VAM method allocations are made so that the penalty cost is	minimised	maximized	zero	non zero	minimised
The MODI method is based on the concept of	Linear Programming Problem	Heuristic method	Matrix maxima method	duality	duality

Transporatation Problem / 2017

Every loop has an number of cells	even	odd	equal	zero	even
Closed loops may or may not be in shape	circle	square	rectangle	triangle	square
When obtaining an initial basic feasible solution we may have m+n-1 allocations	non zero	zero	less than	greater than	less than
When total supply is the total demand, the dummy destination is added in the matrix with zero cost vectors	less than	greater than	greater than or equal to	less than or equal to	greater than
When total supply is the total demand, the dummy source is added in the matrix with zero cost vectors	less than	greater than	greater than or equal to	less than or equal to	less than
Penalty method is a	North - west corner rule	least cost method	VAM method	MODI method	VAM method
Matrix minima method is a	North - west corner rule	least cost method	VAM method	MODI method	least cost method
Every loop has an even number of cells and atleast	Two	Four	Six	Eight	Four
For a to exist, it is necessary that the total supply equal to total demand	unique solution	infinite number of solution	infeasible solution	feasible solution	feasible solution
The number of in an mxn balanced transportation problem is atmost m+n-1	basic variables	non basic variables	decision variables	non decision variables	basic variables
A transportation problem will always have a feasible solution	unique	unbalanced	balanced	distinct	balanced
In a transportation problem, the total transportation cost is	minimum	maximum	zero	unique	minimum

Transporatation Problem / 2017

In method, allocations are made so that the penalty cost is minimized.	North - west corner rule	least cost method	VAM method	MODI method	VAM method
The initial solution of a transportation problem can be obtained by applying any known method. However, the only condition is that	the solution be optimal	the rim conditions are satisfied	the solution not be degenerate	the solution be unique	the rim conditions are satisfied
The dummy source or destination in a transportation problem is added to	satisfy rim conditions	prevent solution from becoming degenerate	ensure that total cost does not exceed a limit	it is a balanced one	satisfy rim conditions
The occurrence of degeneracy while solving a transportation problem means that	total supply equals total demand	the solution so obtained is not feasible	the few allocations become negative	the solution so obtained is feasible	the solution so obtained is not feasible
An alternative optimul solution to a minimization transportation problem exists whenever opportunity cost corresponding to unused route of	positive and greater than zero	positive with atleast one equal to zero	negative with atleast one equal to zero	negative and lesser than zero	positive with atleast one equal to zero
One disadvantage of using North -west corner rule to find initial solution to the transportation problem is that	it is complicated to use	it does not take into account cost of transportation	it leads to a degenerate initial solution	it leads to non degenerate initial solution	it does not take into account cost of transportation
The solution to a transportation problem with m- rows (supplies) and n- columns (destination) is feasible if number os positive allocations are	m + n	m x n	m + n +1	m + n -1	m + n -1
The calculation of oppportunity cost in the MODI method is analogous to a	the net evaluation value for non basic variable	value of a variable in xB column of the simplex	variable in the B - column in the simplex method	variable in the Y - column in the simplex table	the net evaluation value for non basic variable columns in
If we were to use opportunity cost value for an unused cell to test optimality, it should be	equal to zero	most negative number	most positive number	any value	most negative number
A basic feasible solution to a (mxn) transportation problem is said to be a basic feasible solution if it contains exactly m+n-	degenerate	non degenerate	unique	infinite number of	non degenerate
A basic feasible solution to a (mxn) transportation problem is said to be a basic feasible solution if it contains less than m+n-	degenerate	non degenerate	unique	infinite number of	degenerate
In a transportation problem, least cost method gives a better solution to than	North - west corner rule	least cost method	VAM method	MODI method	North - west corner rule

Transporatation Problem / 2017

Ina transportation problem, gives a better starting solution than Least Cost method.	North - west corner rule	least cost method	VAM method	MODI method	VAM method
A transportation problem can always be represented by	balanced model	unbalanced model	simplex model	graphical model	balanced model
In a transportation model, north west corner rule starting solution is recommended because it ensure that there will be allocations	m + n	m x n	m + n +1	m + n -1	m + n -1
The transportation model is restricted to dealing with a	single	multiple	positive	negative	multiple
The transportation technique essentially uses the same steps of the	simplex method	graphical method	Big M method	dual simplex method	simplex method
For any transportation problem, the coefficients of all units in the constraints are	zero	any value	unity	unique	unity
A solution that satisfies all conditions of supply and demand but it may or may bot be optimal is called	feasible solution	infeasible solution	basic feasible solution	initial feasible solution	initial feasible solution
To solve degeneracy, an occupied cell with cost is converted into occupied cell by assigning infinitely small amount to it.	lowest	larger	unit	zero	lowest
In a north west corner rule, if the demand in the column is satisfied, one must move to the	left	right	middle	corner	right
Row wise and column wise difference between two minimum costs is calculated under method.	North - west corner rule	least cost method	VAM method	MODI method	VAM method
An optimum solution results when net costs change values of all unoccupied cells are	positive and greater than zero	negative	non negative	positive and lesser than zero	non negative
MODI method associated with transportation	Modified	Multiple	Matrix	Modified	Modified
	uisuibuuoli	uisuibuiloli	uisuibulloli	ustilicuoli	uisuiouuoli

Transporatation Problem / 2017

	Linear	Integer	Non Linear	Dynamic	Linear
Transportation problem is a sub class of	Programming	Programming	Programming	Programming	Programming
	Problem	Problem	Problem	Problem	Problem



KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po),

Coimbatore -641 021

Subject: Operations Research

Subject Code:17CCP103

Semester : I

Class : I - M.Com (CA)

Unit III									
Assignment Problem and Queuing System									
Part A (20x1=20 Marks)									
(Question Nos. 1 to 20 Online Examinations)									
	P	ossible Questions		Г					
Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer				
	Linear	Integer	Non Linear	Dynamic	Linear				
The assignment problem is a particular case of	Programming	Programming	Programming	Programming	Programming				
	Problem	Problem	Problem	Problem	Problem				
The objective of the assign a number of tasks to equal number of facilities at a cost.	Maximum	Minimum	Either maximum or minimum	Neither maximum nor minimum	Minimum				
The asssignment problem can be stated in the form of mxn matrix called a	cost matrix	unit matrix	zero matrix	scalar matrix	cost matrix				
In an assignment problem, source represents	- supply	jobs	facilities	origins	facilities				
Destination represents	supply	jobs	facilities	origins	jobs				
Each machine should be assigned tojob.	only one	two	three	four	only one				

Assignment Problem and Queuing System / 2017

The total assignment value of ith machine is	1	2	3	4	1
The assignment problem represents at problem with all demands and supplies equal to	1	2	3	4	1
An assignment problem always a form of a transportation problem.	independent	dependent	degenerate	non degenerate	degenerate
An assignment problem is said to be - if No.of rows = No.of columns.	balanced	unbalanced	equal	not equal	balanced
An assignment problem is said to be - if No.of rows \neq No.of columns.	balanced	unbalanced	equal	not equal	unbalanced
cannot be used to solve the assignment problem because of	independent	dependent	degeneracy	non degeneracy	degeneracy
Every basic solution in the assignment problem is	independent	dependent	degenerate	non degenerate	degenerate
The assignment problem can be solved by the	transportation problem	simplex problem	inventory problem	simulation problem	transportation problem
Every in the assignment problem is tansportation problem.	feasible solution	unique solution	basic solution	multiple solution	basic solution
An efficient method for solving an assignment problem is the	Penalty method	Matrix minima method	Hungerian method	Heuristic method	Hungerian method
Hungarian method is also known as	Matrix method	penalty method	Matrix minima method	reduced matrix method	reduced matrix method
Hungarian method is based on the concept of the -	optimal cost	opportunity cost	duality cost	lowest cost	opportunity cost

If the number of rows is not equal to the numbers					
of columns in the cost matrix of the given					
assignmnet problem than it is said to be	balanced	unbalanced	equal	not equal	unbalanced
If each row and each column contain exactly on				_	
encircled zero then the current assignment is					
	unique	distinct	optimal	not optimal	optimal
If at least one row/column is without an					
assignmnet then the current assignment is					
	unique	distinct	optimal	not optimal	not optimal
				Neither	
Assignment algorithm is only for			Either maximum	maximum nor	
problem	minimization	maximization	or minimum	minimum	minimization
The maximization assignment can be converted					
into a minimization assignmnet problem by					
from the highest element to all the elements	adding	subtracting	multiplying	dividing	subtracting
An optimal assignment requires that the					
maximum number of lines which cab be drawn			rows + colums -	rows - columns -	
through squares with zero opportuinty cost be	rows or columns	rows and columns	1	1	rows or columns
while solving an assignment problem, an activity	minimize the		reduce the cost		
is assigned to a resource through a square with	total cost of		of that particular		
zero opportunity cost because the objective is to	assignment	reduce the cost of	assignement to	maximize the	minimize the total
		assignemnt to zero	zero	total cost	cost of assignment
The purpose of dummy row or column in an	obtain balance	prevent a solution	provide a means	prevent a solution	obtain balance
assignment problem is to	between total	from becoming	of representing a	from becoming	between total
	activities and	degenerate	dummy problem	non degenerate	activities and total
				_	
If there were n workers and n jobs there would be					
	n solutions	(n-1)! Solutions	(n+1)! Solutions	n! solutions	n! solutions
For a salesmanwho has to visit n cities, following					
are the ways of his tour plan	n!	(n+1)!	(n-1)!	n	(n-1)!
	_				
	Departure		Arrival		
In $(a / b / c)$: (d / e) , a is called	distribution	Queue discipline	distribution	Number of units	Arrival distribution

Assignment Problem and Queuing System / 2017

	Departure		Arrival		
In $(a / b / c)$: (d / e) , c is called	distribution	Queue discipline	distribution	Number of units	Number of units
If the arrivals are completely random, then the					
probability distribution of a number of arrivals in					
a fixed-time interval follows a distribution	Poisson	normal	binomial	polynomial	Poisson
The describes the way in which the	service			customer	
customers arrive and join the system	mechanism	input	queue discipline	behaviour	input
The arriving people in a queueing system are					
called	Input	servers	customers	queue	customers
Mean service time is denoted by	m	1	1 / 1	1 / m	m
The traffic intensity in queueing is defined by					
	p / (p–1)	m / 1	l / m	1 / (1- p)	l / m
A system is said to be in state when its					
operating characteristics are dependent on time	Steady	arrival	service	transient	transient
A system is said to be in state when the					
behaviour of the system is independent on time.	Steady	arrival	service	transient	Steady
A customer who leaves the queue because the					•
queue is too long then his behaviour is said to be -					
	Reneging	balking	jockeying	priorities	balking
The Birth–death model is called	M / M / 1	M / M / N	M / M / ¥	M / M / 2	M / M / 1
Average queue length in $(M / M / 1) : (\infty / FCFS)$					
is	(1-r)/r	r / (1-r)	$r / (1 - r^2)$	$r^{2}/(1-r)$	$r^{2}/(1-r)$
The expected waiting time in the queue is					
calculated by the formula	1 / (m – l)	b) 1 / (m – 1)	l / m (m – l)	l / m	1 / m (m – l)

Prepared by : R. Praveen Kumar, Department of Mathematics, KAHE

Assignment Problem and Queuing System / 2017

In Birth–death model, the probability distribution					
of queue length is given by	$r^{n} / (1-r)$	b) $r^2 / (1-r)$	r / (1– r)	$(1-r) / r^{n}$	$r^{n}/(1-r)$
First In First Out (FIFO) is known as the	Input	service mechanism	customer behaviour	queue discipline	queue discipline
The probability of an empty system is given by	1 (1/m)	1/(m-1)	1/m(m-1)	1 / m	1 (1/m)
A Queuing system can be completely described	The input, the	The input the $\frac{1}{1}$	The input the	The input the	$\frac{1 - (17 \text{ m})}{\text{The input the}}$
by	service	service mechanism	service	service	service mechanism,
	mechanism	and the queue	mechanism, the	mechanism and	the queue discipline
The probability of Queue size ≥ N is	$r^{n} / (1-r)$	r ⁿ	$1-r^n$	$(1-r) / r^n$	r ⁿ
If $1 = 3$, $m = 2$ then $r =$	1.5	3	2	0.6	1.5
Average waiting time in the queue is given by					
	1/m - 1	1	m	1/m + 1	1

KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore -641 021									
Subject: Operations ResearchSubject Code:17CCP103									
Class : I - M.Com (CA)				Semester :	I				
		Unit IV							
	In	wentory Model							
Part A (20x1=20 Marks)									
(Question Nos. 1 to 20 Online Examinations)									
Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer				
may be defined as the stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business affairs	Inventory	Transportation	Queueing	Sequencing	Inventory				
Rate of consumption is different from	rate of change	rate of production	rate of purchasing	either b or c	either b or c				
Cost associated with carrying or holding the goods in stock is known as	interested capital cost	handling cost	holding cost	production cost	holding cost				
is the interest change over the capital invested.	interested capital cost	handling cost	holding cost	production cost	interested capital cost				
include costs associated with movement of stock, such as cost of labour etc.	interested capital cost	handling cost	holding cost	production cost	handling cost				
purchased deu to quantity discounts or price breaks.	interested capital cost	handling cost	holding cost	purchase price	purchase price				

If P is the purchase price of an item and I is the					
stock holding cost per unit time expressed as a					
fraction of stock value then the holding cost is	I/P	I + P	I – P	IP	IP
The penalty costs that are incurred as a result of					
running out of stock are known as	shortage cost	set-up cost	holding cost	production cost	shortage cost
Holding cost is denoted by	C ₁	C ₂	C ₃	C ₄	C ₁
		~		~	
Shortage cost is denoted by	C ₁	C ₂	C ₃	C ₅	C ₂
Set-up cost is denoted by	C ₁	C ₂	C ₃	C ₄	C ₃
Elapsed time between the placement of the order					
and its receipts in inventory is known as	lead time	recorder level	EOQ	variables	lead time
is the time when we should place an					
order by taking into consideration the interval					
between placing the order and receiving the	lead time	recorder level	EOQ	variables	recorder level
is that size of order which minimises total					
annual cost of carrying inventory and the cost of					
ordering under the assumed conditions of	lead time	recorder level	EOQ	variables	EOQ
	Economic Order	Economic Order	Economic Offer	Economic Offer	Economic Order
EOQ means	Quantity	Quality	Quality	Quantity	Quantity
	economic lot size	economic short	economic	economic	economic lot size
EOQ is also known as	formula	size formula	formula	variables	formula
include holding cost set up cost shortage		controlled	uncontrolled		uncontrolled
costs and demand	FOO	variables	variables	hasic variables	variables
costs and demand.					
Reduction in procurement cost EOQ	increases	decreases	reduces	neutral	reduces

An approximate percentage of 'C' items in a firm					
is around	60 - 65%	65 - 70%	70 - 75%	75 - 80%	70 - 75%
Economic order quantity results in equilization of cost and annual inventory cost.	annual procurement cost	procurement cost	inventory cost	shortage cost	annual procurement cost
Economic order quantity results in equilization of	annual inventory	progurament cost	inventory cost	shortaga agat	annual inventory
A company uses 10,000 units per year of an item. The purchase price is one rupee per item.	cost	procurement cost	Inventory cost	shortage cost	
Ordering cost = Rs. 25 per order. Carrying cost	2000 units	2083 units	2038 units	2050 units	2083 units
If the procurement cost per order increases 21%, the economic order quantity of the item shall increase by	10%	20%	30%	40%	10%
		2070			
If EOQ is 5000 units and Buffer stock is 500 units calculate max inventory.	5500 units	500 units	5000 units	5050 units	5500 units
If EOQ is 5000 units and Buffer stock is 500					7000
units calculate minimum inventory.	5500 units	500 units	normal lead time	5050 units	5000 units
Reorder level =	x monthly consumption	monthly consumption	- monthly consumption	/ monthly consumption	monthly consumption
An approximate percentage of A- items in a firm is around	5 - 10 %	10-20 %	20-25 %	70 – 75 %	5 - 10 %
	reduced stock –	increased stock –	equilisation of carrying cost	favourable procurement	equilisation of carrying cost and
Economic order quantity results in	outs	outs	and procurement	price	procurement costs
carrying cost is 1.5 % per month, the economic					
order quantity is	240 no's	200 no's	400 no's	500 no's	200 no's

In the ABC analysis, C items are those which					
have	low unit price	low cost price	low usage value	low consumption	low consumption
For an item with storage cost of each item Ks. 1,					
set up cost Rs.25, demand 200 units per month	D _a 100	D a 400	Da 500		D _a 100
C _{min} 18	KS.100	KS.400	KS.300	KS.800	K\$.100
Minimum inventory equals	EOQ	Reorder level	Safety stock	lead time	Safety stock
Given maximum lead time as 20 days and normal					•
lead time is 15 days with annual consumption					
12,000 units find the buffer stock.	176 units	167 units	157 units	186 units	167 units
Given $R = 1000$ units/year $I = 0.30$, $C =$					
Re.0.50/unit, $C_3 = Rs.10$ /order. Find minimum					
average cost.	54.77	55.77	53.77	50.77	54.77
The set up cost in inventory situation is of	1 1 /		1	11	• • • •
size of inventory.	dependent	independent	large	small	independent
	set up cost +	holding cost +	set up cost +	setup cost +	set up cost +
Total inventory cost =	purchasing cost	shortage cost	purchasing cost	shortage cost	purchasing cost +
			+ holding cost +	shore ge ease	holding cost +
Storage cost is associated with	holding cost	shortage cost	carrying cost	set up cost	carrying cost
	(FOO/2) + Safety	(EOO/2) - Safety	(FOO/2) / Safety	(FOO/2) * Safety	$(EOO/2) \pm Safety$
Average inventory -	(LOQ/2) + Safety	(LOQ/2) - Salety	(LOQ/2) / Salety	(LOQ/2) Salety	(LOQ/2) + Safety
	SIOCK	SIOCK	SIOCK	SIOCK	SIOCK
discounts reduce material cost and					
procurement costs	quantity	quality	carrying cost	set up cost	quantity
The ordering cost is independent of	ordering quantity	ordering quality	carrying cost	set up cost	ordering quantity

KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore –641 021								
Subject: Operations Research				Subject Code	e:17CCP103			
Class : I - M.Com (CA)				Semester : I				
		Unit V PERT and CPM						
	Part (Question Nos	A (20x1=20 Marks 1 to 20 Online Exa	s) aminations)					
	P	ossible Questions						
Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer			
A is defined as a combination of interrelated acitivities all of which be executed in a certain order to achieve a goal	Project	Acitivity	Event	Nodes	Project			
is a task or an item of work to be done in aproject.	Project	Acitivity	Event	Nodes	Acitivity			
node at the beginning and a node at the end indicating the start and finish of the activity.	Project	Nodes	Event	Acitivity	Acitivity			
Nodes are denoted by	dot	circle	arrow	square	circle			
The diagram in which arrow represents an activity is called	arrow diagram	network diagram	graph diagram	line diagram	arrow diagram			
The initial node are also called	head event	tail event	first event	last event	tail event			



The terminal node are called	head event	tail event	first event	last event	head event
An activity wgich must be completed before one					
or more other activities start is known as					
activity	Predecessor	successor	initial	final	Predecessor
An activity which started immediately after one					
or more of the other activities are completed is					
known as	Predecessor	successor	initial	final	successor
An activity which does not consumes either any					
resources and/or time is known as					
	Predecessor	successor	dummy	initial	dummy
If an activity B can start immediately after an	immediate	immediate			immediate
activity A, then A is called	predecessor	successor	Predecessor	successor	predecessor
If an activity B can start immediately after an	immediate	immediate			immediate
activity A then B is called	predecessor	successor	Predecessor	successor	successor
activity A, then B is called	predecessor	Successor	Tredecessor	successor	Successor
	A is a	B is a successor of	A is a successor	B is a	A is a predecessor
The notation ' $A < B$ ' is called	predecessor of B	A	of B	predecessor of A	of B
	<u>r</u>			F	
	A is a	B is a successor of	A is a successor	B is a	B is a successor of
The notation ' $B > A$ ' is called	predecessor of B	A	of B	predecessor of A	А
	1				
Activities which have no predecessors are called					start
activity	dummy	start	zero	terminal	
All the start activities can be made to have the	_				same
initial node.	same	different	multiple	zero	
Activities which have no successor are called					
activity	dummy	start	zero	terminal	terminal
The diagram denoting all the activities of a					
project by arrows taking into account the					
technological square of the activities is called	Project	nework	project network	Event	project network

There is another representation of a project					
network representing activities on nodes called					
	AON diagram	ANO diagram	NOA diagram	arrow diagram	AON diagram
only activity should connect any two					
nodes.	one	two	three	multiple	one
Path, connecting the first node to the very last					
terminal node of long duration in any project					
network is called	PERT	Critical path	Activity	network	Critical path
All the activities in any critical path are called					
	start activities	dummy activities	critical activities	terminal activities	critical activities
Critical path plays a very important role in project					
problems	schoduling	planning	controlling	notwork	schoduling
An activity is defined as the difference between	scheduning	praining	controlling	lictwork	scheduning
the latest start and the earliest start of the activity			independent		
is called	free float	total float	float	interfering float	total float
	nee noat		noat	Interfering float	
If the total float is then it may indicate					
that the resources for the activity are more that					
adaquata	positivo	nogotivo	7000	any valua	positivo
If the total float of an activity is it	positive	llegative	2010		positive
may indicate that the resources are just adequate					
for that activity	positivo	nogotivo	7000	any valua	70*0
	positive		Zelo		2010
If the total float is, it may indicate that					
the resources for that activity are inadequate.	positive	negative	zero	any value	negative
· · ·	-			-	
(L - E) of an aevent i-j is called the					
of the event j.	slack	surplus	dummy	total	slack
One of the portion of the total float is			independent		
	free float	total float	float	interfering float	free float
	nee noat	iotal noal	110al	menening noat	nee noat

rescheduling that activity without affecting the					
succeding activity we can use of an			independent		
activity	free float	total float	float	interfering float	free float
			1 .1		
Free float is the total float of an		greater than or	less than or		
activity	equal to	equal to	equal to	not equat to	less than or equal to
The amount of time by which the activity can be					
rescheduled with effecting the preceding or			independent		
succeeding of that activity is called	free float	total float	float	interfering float	independent float
The slack of the head event i is called the			independent		
of on activity i	free floot	total float	floot	interfering fleet	interfering fleat
	free float		noat	Interfering float	Interfering float
Interfering float of i i is the difference between			independent		
the total float and	free fleet	danandant float	floot	dummy floot	free fleet
	nee noai		110at	dummy moat	nee noai
All the critical activities have their total float as					
An the entited activities have then total hoat as	one	two	7 0r 0	any value	zero
	one		Zeit		2010
Critical path can also be defined as the path of					
entical path can also be defined as the path of	least total float	greatest total float	least free float	graatest free float	least total float
	least total moat	greatest total float	least filee filoat	greatest free froat	least total moat
The objective of network analysis is to	minimize total	minimize total	minimize	minimize the	minimize total
	project duration	project cost	prduction delay	interruption	project duration
			production delay	Interruption	project duration
The slack for an activity is equal to	LF - LS	EF - ES	LS - ES	LS - EF	LS - ES
PERT stands for	Project		Planning	Planning	
	Enumaration	Project Evaluation	Evaluation	Enumaration	Project Evaluation
	review	Review Technique	Review	review Technique	Review Technique
Generally PERT technique does not deals with		non repetitive	deterministic	non deterministic	non deterministic
the project of	repetitive nature	nature	nature	nature	nature
The technique of OR used for planning,					
scheduling and controlling large and complex					
projects are often referred as	network analysis	graphical analysis	critical activities	PERT	network analysis

A network is a	quality plan	control plan	graphical plan	inventory plan	graphical plan
Critical path method is used for commpletion of		1			
projects involving activities of	-	non repetitive	deterministic	non deterministic	
	repetitive nature	nature	nature	nature	repetitive nature
An event which represents the joint completion of					1
more than one activity is known as					
-	unique event	burst event	merge event	dummy event	merge event
An event which represents the initiation of more					
than one activity is known as	unique event	hurst event	merge event	dummy event	hurst event
than one activity is known as	unique event		merge event		
Events in the network diagram are identified by	_				
	numbers	variables	symbols	special characters	numbers
			symoons	special characters	
The negative value of the independent float is	-				
	one	zero	distinct	non zero	zero
In PERT the span of time between the optimistic					
and pesimistic time estimates of an acitivity is	-				
	3σ	6σ	12σ	4σ	6σ
			the project is		
If an activity has zero slack, it implies that	it lies on the	it is a dummy	progressing	the project is not	it lies on the critical
	critical path	activity	well.	progressing well	path
A dummy activity is used in the network diagram	two parallel	the chain of	two parallel	If the activities	two parallel
when	activities have	activities may have	activities have	have the tail and	activities have the
	the same tail and	a common event	the different tail	head events	same tail and head
The path of least cost float in a project is called					
	PERT	Critical path	unique path	network path	Critical path
The project dynation is offered if the dynatic soft					
The project duration is affected if the duration of	-11				-11
	changed	unchanged	same	exist	cnanged
The number of time estimates involved ina PERT	1		2		3
problem is	1	2	3	4	

PERT and CPM/ 2017

For a non critical activity, the total float is					non zero
	zero	non zero	unique	distinct	
The probability to complete a project in the					0.5
expected time is	1	1.5	0.5	1.15	0.5
In PERT ananlysis, the critical path is obtained by					
joining event having	positive slack	negative slack	non zero slack	unique slack	positive slack
In PERT network each activity time assumes a Beta - distribution because	it is a uni - model distribution that provides informing regarding the uncertainty of time estimates.	it has got finite non negative error	it need not be symmetrical about model value	it has infint negative error	it is a uni - model distribution that provides informing regarding the uncertainty of time estimates.
Float or slack analysis in useful for	projects behind the schedule only	projects ahead of the schedule only	projects behind the planning only	projects ahead of the planning only	projects behind the schedule only
In time cost trade off funtion analysis	cost decreases linearly as time increases	cost at normal time is zero	cost increases linearly as time increases	cost at normal time is unity	cost decreases linearly as time increases
The name of the proability distribution (used PERT) which estimates the expected duration and the expected variance of an activity is	Beta distribution	Gamma	poisson	normal	Beta distribution
PERT) which estimates the expected duration and the expected variance of an activity is	Beta distribution	Gamma distribution	poisson distribution	normal distribution	


KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore -641 021 **SYLLABUS**

17CCP103

OPERATIONS RESEARCH

Semester – I Т Р L С 4 0 0 4

Course Objective:

The course concentrates on Linear programming, transportation model, Queuing theory and Inventory

Course Outcome:

◆ This course emphasis to enhance learner knowledge in optimal use of performance measures of queues, optimal use of Inventory and Network scheduling with various applications in mathematics

Unit – I

Introduction to Operations Research – Application in Management Decision Making – Linear Programming: Formulation of LPP – Graphical Solution to LPP –Simplex Method (using slack variables only)

Unit - II

Transportation Model: Introduction – Mathematical Formulation –Finding Initial Basic Feasible Solutions - Optimum Solution for Nondegeneracy and Degeneracy Model - Unbalanced Transportation Problems and Maximization case in Transportation Problem- Traveling Sales Man Problem. Unit- III

The Assignment problem - Mathematical Formulation of the Problem - Hungarian Method -Unbalanced Assignment Problem - Maximization Case in Assignment Problem - Travelling Salesman Problem. Queuing Theory : Introduction - Characteristics of Queuing System. Problems in (M/M/1):(\omega/FIFO) and (M/M/1):(N/FIFO) models

Unit - IV

Inventory Control: Introduction - Costs involved in Inventory - Deterministic EOQ Models -Purchasing Model without and with Shortage, Manufacturing Model without and with Shortage -Price Break Unit – V

PERT and CPM: Network Representation - Calculation of Earliest expected time, latest allowable occurrence time. CPM - Various Floats for Activities - Critical Path- PERT - Time Estimates in PERT- Probability of Meeting scheduled date of Completion of Projects

TEXT BOOK

Kanthi Swarup, Gupta P.K., Man Mohan., (2014) Operations Research, Sultan Chand and Sons, New Delhi. REFERENCES

Sharma J.K., (2008), Operations Research Theory Applications, Macmillan India Ltd, New Delhi.

Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., (2005), Resource Management Techniques, A. R. Publications, Nagapatinam.

Shanthi Sophia Bharathi D., (1999). Operations Research/Resource management techniques, Charulatha Publications.

Hamdy A.Taha., Operations Research, (2007), Pearson education, Prentice Hall.

Vittal – Operations Research – Margham Publications



KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore -641 021. Department of Mathematics

Subject : Operation Research	Subject Code : 17CCP103	LTPC	
Class : I – M.Com(CA)	Semester : I	4 0 0 4	

UNIT – I

Introduction to Operations Research – Application in Management Decision Making – Linear Programming: Formulation of LPP – Graphical Solution to LPP – Simplex Method (using slack variables only)

TEXT BOOK

1. KanthiSwarup, Gupta P.K., Man Mohan., (2016) Operations Research, Sultan

Chandand Sons, New Delhi.

REFERENCES

1. Sharma J.K., (2008), Operations Research Theory Applications, Macmillan India Ltd, New Delhi.

2. Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K.,(2014), Resource Management Techniques, A. R. Publications, Nagapatinam.

3. Shanthi Sophia Bharathi D.,(1999),Operations Research/ Resource management

techniques, Charulatha Publications.

4. HamdyA.Taha., Operations Research,(2007), Pearson education, Prentice Hall.

5. Vittal- Operations Research - Margham Publications

UNIT – I

INTRODUCTION TO OPERATION RESEARCH

Definitions

To define anything non-trivial — like beauty or mathematics — is very difficult indeed. Here is a reasonably good definition of Operations Research:

Definition.

Operations Research (OR) is an interdisciplinary branch of applied mathematics and formal science that uses methods like mathematical modeling, statistics, and algorithms to arrive at optimal or near optimal solutions to complex problems. Definition is problematic: to grasp it we already have to know, e.g., what is formal science or near optimality.

From a practical point of view, OR can be defined as an art of optimization, i.e., an art of finding minima or maxima of some objective function, and — to some extend — an art of defining the objective functions. Typical objective functions are

- profit,
- assembly line performance,
- crop yield,
- bandwidth,
- loss,
- waiting time in queue,
- risk.

From an organizational point of view, OR is something that helps management achieve its goals using the scientific process.

The terms OR and Management Science (MS) are often used synonymously. When a distinction is drawn, management science generally implies a closer relationship to Business Management. OR also closely relates to Industrial Engineering. Industrial engineering takes more of an engineering point of view, and industrial engineers typically consider OR techniques to be a major part of their tool set. Recently, the term Decision Science (DS) has also be coined to OR.

OR Tools Some of the primary tools used in OR are

- statistics,
- optimization,
- probability theory,
- queuing theory,
- game theory,
- graph theory,

- decision analysis,
- simulation.

Because of the computational nature of these fields, OR also has ties to computer science, and operations researchers regularly use custom-written software. In this course we will concentrate on optimization, especially linear optimization.

Phases of Operations Research Study Seven Steps of OR Study An OR project can be split in the following seven steps:

Step 1: Formulate the problem The OR analyst first defines the organization's problem. This includes specifying the organization's objectives and the parts of the organization (or system) that must be studied before the problem can be solved.

Step 2: Observe the system Next, the OR analyst collects data to estimate the values of the parameters that affect the organization's problem. These estimates are used to develop (in Step 3) and to evaluate (in Step 4) a mathematical model of the organization's problem.

Step 3: Formulate a mathematical model of the problem The OR analyst develops an idealized representation — i.e. a mathematical model — of the problem.

Step 4: Verify the model and use it for prediction The OR analyst tries to determine if the mathematical model developed in Step 3 is an accurate representation of the reality. The verification typically includes observing the system to check if the parameters are correct. If the model does not represent the reality well enough then the OR analyst goes back either to Step 3 or Step 2.

Step 5: Select a suitable alternative Given a model and a set of alternatives, the analyst now chooses the alternative that best meets the organization's objectives. Sometimes there are many best alternatives, in which case the OR analyst should present them all to the organization's decision-makers, or ask for more objectives or restrictions.

Step 6: Present the results and conclusions The OR analyst presents the model and recommendations from Step 5 to the organization's decision-makers. At this point the OR analyst may find that the decisionmakers do not approve of the recommendations. This may result from incorrect definition of the organization's problems or decision-makers may disagree with the parameters or the mathematical model. The OR analyst goes back to Step 1, Step 2, or Step 3, depending on where the disagreement lies.

Step 7: Implement and evaluate recommendation Finally, when the organization has accepted the study, the OR analyst helps in implementing the recommendations. The system must be constantly monitored and updated dynamically as the environment changes. This means going back to Step 1, Step 2, or Step 3, from time to time.

Some areas of management where O.R techniques have been successfully utilized are as follow:

1. Allocation and Distribution in Projects:

- (i) Optimal allocation of resources such as men materials machines, time and money to projects.
- (ii) Determination and deployment of proper workforce.

(iii) Project scheduling, monitoring and control.

2. Production and Facilities Planning:

- (i) Factory size and location decision.
- (ii) Estimation of number of facilities required.

(iii) Preparation of forecasts for the various inventory items and computation of economic order quantities and reorder levels.

- (iv) Scheduling and sequencing of production runs by proper allocation of machines.
- (v) Transportation loading and unloading,
- (vi) Warehouse location decision.

(vii) Maintenance policy decisions.

3. Programmes Decisions:

- (i) What, when and how to purchase to minimize procurement cost.
- (ii) Bidding and replacement policies.

4. Marketing:

- (i) Advertising budget allocation.
- (ii) Product introduction timing.
- (iii) Selection of advertising media.
- (iv) Selection of product mix.

(v) Customer's preference of size, colour and packaging of various products.

5. Organization Behaviour:

- (i) Selection of personnel, determination of retirement age and skills.
- (ii) Recruitment policies and assignment of jobs.
- (iii) Recruitment of employees.
- (iv) Scheduling of training programs.

6. Finance:

- (i) Capital requirements, cash flow analysis.
- (ii) Credit policies, credit risks etc.
- (iii) Investment decision.
- (iv) Profit plan for the company.

7. Research and Development:

(i) Product introduction planning.

(ii) Control of R&D projects.

(iii) Determination of areas for research and development.

(iv) Selection of projects and preparation of their budgets.

(v) Reliability and control of development projects thus it may be concluded that operation research can be widely utilized in management decisions and can also be used as corrective measure.

LINEAR PROGRAMMING PROBLEM

Introduction:

LPP deals with determining optimal allocations of limited resources to meet given objectives. The resources may be in the form of men, raw materials, market demand, money and machines, etc. The objective is usually maximizing utility etc.

LPP deals with the optimization of a function of decision variables known as objective function. Subject to a set of simultaneous linear equation or inequality known as constraints.

The term linear means that all the variables occurring in the objective function and the constraints are of the 1st degree in the problem under consideration and the term programming means the process of determining the particular course of action.

Mathematical formulation of LPP:

If x_j (j = 1,2,....n) are n decision variables of the problem and if the system is subject to m constraints.

 \therefore The general model can be redundant the form,

Optimize $z = f(x_1, x_2, x_3, ..., x_n)$

Subject to the constraints are,

 $g_i(x_1, x_2, x_3, \dots, x_n) \le i \le b_i$ (i = 1, 2, ...,m) and

 $x_1, x_2, x_3, \dots, x_n \ge 0$ (non negativity constraints)

Procedure for forming a LPP model:

Step 1: Identify the unknown decision variables to be determined and assign symbols to them.

- **Step 2:** Identify all the restrictions or constraints in the problem and express them as linear equations or inequalities of decision variables.
- **Step 3:** Identify the objective or aim and represent it also as a linear function of decision variables.
- Step 4: Express the complete formulating of LPP as a general mathematical model.

Problems:

1. A firm manufactures two types of product A and B and sells them at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processed on two machines M_1 and M_2 . Type A requires 1 minute of processing time on M_1 and 2 minutes on M_2 . Type B requires 1 minute on M_1 and 1 minute on M_2 . Machine M_1 is available for not more than 6 hours 40 minutes, while machines M_2 is a available for 10 hours during any work hours. Formulate the problem as LPP so as to maximize the profit.

Solution:

Let us consider x_1 be the no. of units in Type A and x_2 be the no. of units in Type B.

To produce these units of Type A and Type B product it requires,

 $x_1 + x_2$ processing minutes on M_1

 $2x_1 + x_2$ processing minutes on M_2

Since M_1 is available for not more than 400 minutes and M_2 is available for not more then 600 minutes.

Therefore the constraints are:

$$x_1 + x_2 \le 400$$

 $2x_1 + x_2 \le 600$
and $x_1, x_2 \ge 0$

since the profit from Type A is Rs. 2 and profit from Type B is Rs. 3.

- \therefore The total Profit is $2x_1 + 3x_2$
- \therefore Here the objective is to maximize the profit
- \therefore The objective function is,

Maximize $z = 2x_1 + 3x_2$

The complete formulation of the LPP is

Maximize $z = 2x_1 + 3x_2$

Subject to the constraints,

$x_1 + x_2 \leq 400$	(i)
$2x_1 + x_2 \le 600$	(ii)
and $x_1, x_2 \ge 0$	(iii)

2.A person wants to decide the constituents of a diet which will fulfill his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from 4 different types of foods. The yields per unit of these foods are given in the following table.

Food Type		Cost / unit (Rs)		
	Proteins	Fats		
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum	800	200	700	
Requirement				

Formulate the LPP model for this problem.

Solution:

Let x_1, x_2, x_3, x_4 be the no. of units in the food type 1,2,3 and 4 respectively.

In this problem the main objective is to minimize the cost. ... The objective function is,

Minimize
$$Z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

The minimize requirement for proteins, fats and carbohydrates are 800, 200 and 700 respectively.

 \therefore The subject to the constraints are:

 $3x_1 + 4x_2 + 8x_3 + 6x_4 \ge 800$ $2x_1 + 2x_2 + 7x_3 + 5x_4 \ge 200$ $6x_1 + 4x_2 + 7x_3 + 4x_4 \ge 700$

And the complete formation of LPP is

Minimize $Z = 45x_1 + 40x_2 + 85x_3 + 65x_4$

 \therefore The subject to the constraints:

 $\begin{aligned} &3x_1 + 4x_2 + 8x_3 + 6x_4 \ge 800 \\ &2x_1 + 2x_2 + 7x_3 + 5x_4 \ge 200 \\ &6x_1 + 4x_2 + 7x_3 + 4x_4 \ge 700 \\ &and x_1, x_2, x_3, x_4 \ge 0. \end{aligned}$

3.A Television company operates 2 assembly section A and B. Each section is used assemble the components of 3 types of television (i.e.) color, standard and economy. The expected daily production on each section is as following.

T.V Model	Section A	Section B		
Color	3	1		
Standard	1	1		
Economy	2	6		

The daily running cost for two sections average Rs. 6000 for section A and Rs. 4000 for section B. It is given that the company must produce atleast 24 colors, 16 standard and 40 economy TV sets for which an order is pending. Formulate this as a LPP so as to minimize the total cost.

Solution:

Let x_1 and x_2 be the no. of units in section A and section B.

The objective function is, Minimize $Z = 6000x_1 + 4000x_2$

 \therefore The subject to the constraints,

$$3x_1 + x_2 \ge 24$$
$$x_1 + x_2 \ge 16$$
$$2x_1 + 6x_2 \ge 40$$

The complete formation of LPP is,

Minimize $Z = 6000x_1 + 4000x_2$

 \therefore The subject to the constraints,

$$3x_1 + x_2 \ge 24$$

 $x_1 + x_2 \ge 16$ $2x_1 + 6x_2 \ge 40$ and $x_1, x_2 \ge 0$

Graphical Method of the solution of the LPP:

Linear programming problems involving only 2 variables can be effective solved by a graphical method which provides a pictorial representation of the problems and its solutions. And which gives the basic concept used in solving general LPP which may involve any finite no. of variables.

Working procedure for graphical method:

Given a LPP optimize $Z = f(x_i)$,

Subject to the constraints,

$$g_i(x_i) \le =, or \ge b_i, (i=1,2,...,n), (j=1,2,...,m)$$

and $x_i \ge 0$ (non-negativity restrictions)

Step 1: Consider the inequality constraints as equalities. Draw the straight lines in the XOY plane corresponding to each equality and non-negativity restrictions.

Step 2: Find the permissible region (feasible region or solution space) for the values of the variable which is the region bound by the lines drawn in step1.

Step 3: Find the points of intersection of the bound lines by solving the equations of the corresponding lines.

Step 4: Find the values of Z at all vertices of the permissible region.

Step 5: (i) For minimization problem choose the vertex for which Z is maximum.

(ii) For minimization problem choose the vertex for which Z is minimum.

Problems:

1. Solve the following LPP by graphical method.

Max
$$Z = 3x_1 + 2x_2$$

Subject to the constraints are,

$$2x_1 + x_2 \le 1$$

$$x_1 \le 2$$

$$x_1 + x_2 \le 3 \text{ and } x_1, x_2 \ge 0$$

Solution:

Consider the inequality constraints as equality,

 $-2x_{1} + x_{2} = 1 \qquad \dots \dots (1)$ $x_{1} = 2 \qquad \dots \dots (2)$ $x_{1} + x_{2} = 3 \qquad \dots \dots (3)$ $x_{1} = 0 \qquad \dots \dots (4)$ $x_{2} = 0 \qquad \dots \dots (5)$

From equation (1), putting $x_1 = 0$.

We get $-2x_1 + x_2 = 1$ $x_2 = 1$

The point $(x_1, x_2) = (0, 1)$

Similarly, putting $x_2 = 0$, we get

 $-2x_1 + 0 = 1$ $x_1 = -1/2 = -0.5$

: The point $(x_1, x_2) = (-0.5, 0)$

: The point (0, 1) and (-0.5, 0) lies on the line $-2x_1 + x_2 = 1$

From equation (2) we get, the points (2,1) and (2,2) lies on the line $x_1 = 2$.

From equation (3) putting $x_1 = 0$,

we get $x_1 + x_2 = 3$

$$x_2 = 3$$

The point $(x_1, x_2) = (0, 3)$

Similarly putting $x_2 = 0$, we get $x_1 + 0 = 3$

 $x_1 = 3.$

The point $(x_1, x_2) = (3, 0)$

: The point (0, 3) and (3, 0) lies on the line $x_1 + x_2 = 3$



 \therefore From the graph the vertices of the solution space are,

O(0,0), A(2,0), B(2,1), C(0.7,2.3), D(0,1)

The values of the Z at these vertices are given by,

Vertex	$Z = 3x_1 + 2x_2$
O(0,0)	0
A(2,0)	6
B(2,1)	8
C(0.7,2.3)	6.7
D(0,1)	2

Since the problem is of maximization type.

 \therefore The optimum solution to the LPP is,

Max
$$Z = 8$$

$$\therefore \quad x_1 = 2 \text{ and } x_2 = 1$$

2.Maximize $Z = 5x_1 + 8x_2$

Subject to constraints, $15x_1 + 10x_2 \le 180$

$$10x_1 + 20x_2 \le 200$$

 $15x_1 + 20x_2 \le 210$
and $x_1, x_2 \ge 0$

Solution:

Consider the inequality constraints as equality,

 $15x_{1} + 10x_{2} = 180 \qquad \dots \qquad (1)$ $10x_{1} + 20x_{2} = 200 \qquad \dots \qquad (2)$ $15x_{1} + 20x_{2} = 210 \qquad \dots \qquad (3)$ $x_{1} = 0 \qquad \dots \qquad (4)$ $x_{2} = 0 \qquad \dots \qquad (5)$

From the (1) equation, putting $x_1 = 0$, we get

$$15(0) + 10x_2 = 180$$

 $10x_2 = 180$

 $x_2 = 18$

: The point $(x_1, x_2) = (0, 18)$

Similarly putting $x_2 = 0$, we get

 $15x_1 + 10(0) = 180$ $15x_1 = 180$ $x_1 = 12$

: The point $(x_1, x_2) = (12, 0)$

From the (2) equation putting $x_1 = 0$, we get

$$10(0) + 20x_2 = 200$$

 $20x_2 = 200$
 $x_2 = 10$

: The point $(x_1, x_2) = (0, 10)$

Similarly putting $x_2 = 0$, we get

$$10x_1 + 20(0) = 200$$

 $10x_1 = 200$
 $x_1 = 20$

: The point $(x_1, x_2) = (20, 0)$

From the 3^{rd} equation putting $x_1 = 0$, we get

 $15(0) + 20x_2 = 210$ $20x_2 = 210$ $x_2 = 10.5$

: The point $(x_1, x_2) = (0, 10.5)$

Similarly putting $x_2 = 0$, we get

$$15x_1 = 210$$

 $x_1 = 14$

: The point $(x_1, x_2) = (14, 0)$

Vertex	$Z = 5x_1 + 8x_2$
O(0,0)	0
A(12,0)	60
B(10,3)	74
C(2,9)	82
D(0,10)	80

 \therefore The solution Max Z = 82

 $\therefore x_1 = 2, \ x_2 = 9$

Some more cases in a LPP:

In general a LPP may have

- 1. A unique optimal solution.
- 2. An Infinite no. of optimal solutions.
- 3. An Unbounded solution.
- 4. No solution.

Problems:

1. Solve the following LPP graphically,

Maximize $Z = 100x_1 + 40x_2$

Subject to the constraints are:

 $5x_1 + 2x_2 \le 1000$

 $3x_1 + 2x_2 \le 900$ $x_1 + 2x_2 \le 500$ and $x_1, x_2 \ge 0$

Solution:

$5x_1 + 2x_2 = 1000$	(1)
$3x_1 + 2x_2 = 900$	(2)

$$x_1 + 2x_2 = 500$$
(3)

From (1) equation putting $x_2 = 0$, we get

$$5(0) + 2x_2 = 1000$$

 $2x_2 = 1000$

$$x_2 = 500$$

: The point $(x_1, x_2) = (0, 500)$

Similarly, $x_2 = 0$, we get $5x_1 + 0 = 1000$

 $x_1 = 200$

: The point $(x_1, x_2) = (200, 0)$

From equation (2) putting $x_1 = 0$, we get

$$3(0) + 2x_2 = 900$$

 $x_2 = 450$

: The point $(x_1, x_2) = (0, 450)$

Similarly $x_2 = 0$, we get, $3x_1 + 2(0) = 900$

$$3x_1 = 900$$

$$x_1 = 300$$

: The point $(x_1, x_2) = (300, 0)$

From equation (3) putting $x_1 = 0$, we get

$$0 + 2x_2 = 500$$
$$x_2 = 250$$

: The point $(x_1, x_2) = (0, 250)$

Similarly $x_2 = 0$, we get, $x_1 + 0 = 500$

 $x_1 = 500$

: The point $(x_1, x_2) = (500, 0)$

vertex	$\mathbf{Z} = 100x_1 + 40x_2$
O(0,0)	0
A(200,0)	20,000
B(125,187.5)	20,000
C(0,250)	10,000

Since the problem is of maximization type in the above table the two vertices have the same maximization Z value.

Hence the given LPP has an infinite no. of solution. For this problem the optimum solution is Maximize Z = 20,000

 $x_1 = 200 \text{ (or) } 125$ $x_2 = 187.5 \text{ (or) } 0$

- 2. Max $Z = 2x_1 + 3x_2$ Subject to the constraints:
 - $x_1 x_2 \le 2$

 $x_1 + x_2 \le 4$ and $x_1, x_2 \ge 0$

Solution:

 $x_1 - x_2 = 2$ (1) $x_1 + x_2 = 4$ (2)

From (1) equation $x_1 = 0$ we get,

 $0 - x_2 = 2$ $x_2 = -2$

: The point $(x_1, x_2) = (0, -2)$

Similarly putting $x_2 = 0$, we get

 $x_1 - 0 = 2$

 $x_1 = 2$

: The point $(x_1, x_2) = (2, 0)$

From (2) equation $x_1 = 0$ we get

 $0 + x_2 = 4$

 $x_2 = 4$

: The point $(x_1, x_2) = (0, 4)$

Similarly putting $x_2 = 0$, we get

 $x_1 + 0 = 4$ $x_1 = 4$

: The point $(x_1, x_2) = (4, 0)$

vertex	$\operatorname{Max} \mathbf{Z} = 2x_1 + 3x_2$
A(0,4)	12
B(3,1)	9

From the above graph, the maximization type with two vertex and have the unbounded solution.

 $3.\mathrm{Max}\ \mathrm{Z} = 4x_1 + 3x_2$

Subject to the constraints:

 $x_1 - x_2 \le -1$ $-x_1 + x_2 \le 0$ and $x_1, x_2 \ge 0$

Solution:

 $x_1 - x_2 = -1$ (1) $-x_1 + x_2 = 0$ (2)

From (1) equation $x_1 = 0$ we get,

 $0 - x_2 = -1$

- $x_2 = 1$
- : The point $(x_1, x_2) = (0, 1)$

Similarly putting $x_2 = 0$, we get

 $x_1 = -1$

 $x_1 - 0 = -1$

: The point $(x_1, x_2) = (-1, 0)$

From (2) equation $x_1 = 0$ we get

 $0+x_2=0$

 $x_2 = 0$

: The point $(x_1, x_2) = (0, 0)$

Similarly putting $x_2 = 0$, we get

 $-x_1 + 0 = 0$ $x_1 = 0$

: The point $(x_1, x_2) = (0, 0)$

These being no point (x_1, x_2) common to both the shaded regions. That is, we can not find a convex region for this problem. So the problem cannot be solved. Hence the problem have *no feasible solution*.

General Linear Programming Problem:

Simplex Methods:

and

General Linear Programming Problem:

The Linear Programming Problem involving more than 2 variables will be expressed as follows:

Maximize (or) Minimize $Z = C_1 x_1 + C_2 x_2 + \ldots + C_n x_n$

$$a_{11}x_{1} + a_{12}x_{2} + \dots + a_{1n}x_{n} \le =, \text{ (or) } \ge b_{1}$$

$$a_{21}x_{1} + a_{22}x_{2} + \dots + a_{2n}x_{n} \le =, \text{ (or) } \ge b_{2}$$

$$\dots$$

$$a_{m1}x_{1} + a_{m2}x_{2} + \dots + a_{mn}x_{n} \le =, \text{ (or) } \ge b_{m}$$
the non-negativity restrictions is, $x_{1}, x_{2}, \dots, x_{n} \ge 0$

Definition – 1:

A set of values x_1, x_2, \dots, x_n which satisfies the constraints of the LPP is called its *solution*.

Definition – 2:

Any solution to a LPP which satisfies the non-negativity restrictions of LPP is called its *Feasible Solution*.

Definition – 3:

Any feasible solution which optimizes the objective function of the LPP is called its *optimum or optimal solution*.

Definition – 4:

If a constraints of a general LPP, $\sum_{i=1}^{n} a_{ii} x_i \leq b_i$ (i=1,2,3,....,k)

Then the non – negative variables s_i which are introduced to convert inequalities to the equalities (i.e.) $\sum_{i=1}^{n} a_{ii} x_i + s_i = b_i$ are called *slack variables*.

Definition – 5:

If a constraints of a general LPP, $\sum_{i=1}^{n} a_{ij} x_i \ge b_i$ (i=1,2,3,....,k)

Then the non – negative variables s_i which are introduced to convert inequalities to the equalities (i.e.) $\sum_{j=1}^{n} a_{ij} x_j - s_i = b_i$ are called *surplus variables*.

Canonical and standard forms of LPP:

After the formulation of LPP the next step is to obtain its solution. But before any method is used to find its solution the problem must be presented in a suitable form.

There are 2 forms: (1) Canonical form

(2) Standard form

1. The Canonical Form:

The general LPP can always be expressed in the following form,

Maximize $Z = C_1 x_1 + C_2 x_2 + \ldots + C_n x_n$

```
a_{11}x_1 + a_{12}x_2 + \dots \dots + a_{1n}x_n \le b_1
```

```
a_{21}x_1 + a_{22}x_2 + \dots \dots + a_{2n}x_n \le b_2
```

.....

.....

 $a_{m1}x_1 + a_{m2}x_2 + \dots \dots + a_{mn}x_n \le b_m$

and the non-negativity restrictions is, $x_1, x_2, \dots, x_n \ge 0$

This form of LPP is called the *Canonical form of LPP*.

Characteristics of the Canonical Form:

- 1. The objective function is of Maximization type.
- 2. All constraints are of less or equal to type.
- 3. All variables x_i are non-negative.

1. The Standard Form:

The general LPP in the form,

Maximize $Z = C_1 x_1 + C_2 x_2 + ... + C_n x_n$

Subject to the constraints:

 $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$ $a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$ $a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$

and the non-negativity restrictions is, $x_1, x_2, \dots, x_n \ge 0$

is known as Standard from.

Characteristics of the Standard Form:

- 1. The objective function is of Maximization type.
- 2. All constraints are expressed as equation type.
- 3. Right hand sides of each constraint are non-negative.
- 4. All variables x_i are non-negative.

Problems:

1. Express the following LPP in Standard form.

Minimize $Z = 5x_1 + 7x_2$

Subject to the constraints:

 $x_1 + x_2 \le 8$ $3x_1 + 4x_2 \ge 3$ $6x_1 + 7x_2 \ge 5$ and $x_1, x_2 \ge 0$

Solution:

Since Minimize Z = -Max(-Z)

$$= -Max(Z^*)$$

= - (5x₁ + 7x₂)
= - 5x₁ - 7x₂

The given LPP becomes,

Maximize $Z^* = -5x_1 - 7x_2$

Subject to the constraints:

$$x_1 + x_2 \le 8$$

 $3x_1 + 4x_2 \ge 3$
 $6x_1 + 7x_2 \ge 5$ and $x_1, x_2 \ge 0$

By introducing slack variables S_1 surplus variables S_2 , S_3 then the standard form of the LPP is given by,

Maximize $Z^* = -5x_1 - 7x_2 + 0s_1 + 0s_2 + 0s_3$

```
x_1 + x_2 + s_1 = 8

3x_1 + 4x_2 - s_2 = 3

6x_1 + 7x_2 - s_3 = 5 \text{ and } x_1, x_2, s_1, s_2, s_3 \ge 0
```

2. Express the following LPP in Standard form.

Maximize $Z = 4x_1 + 2x_2 + 6x_3$

Subject to the constraints:

```
2x_1 + 3x_2 + 2x_3 \ge 6

3x_1 + 4x_2 = 8

6x_1 - 7x_2 + x_3 \le 10 \text{ and } x_1, x_2, x_3 \ge 0
```

Solution:

The given LPP becomes,

Max Z =
$$4x_1 + 2x_2 + 6x_3$$

By introducing slack variables S_2 surplus variables S_1 , then the standard form of the LPP is given by,

 $Max Z^* = 4x_1 + 2x_2 + 6x_3 + 0s_1 + 0s_2$

Subject to the constraints:

 $2x_1 + 3x_2 + 2x_3 - s_1 = 6$ $3x_1 + 4x_2 + 0x_3 = 8$ $6x_1 - 7x_2 + x_3 + s_2 = 10$ and $x_1, x_2, x_3, s_1, s_2 \ge 0$

3. Express the following LPP in Canonical form.

Minimize $Z = x_1 + 4x_2$

Subject to the constraints:

$$3x_1 + x_2 \le 5$$

 $-2x_1 + 4x_2 \ge -7$
and $x_1, x_2 \ge 0$

Solution:

The canonical form of the given LPP becomes,

Maximize $Z^* = -x_1 - 4x_2$

 $3x_1 + x_2 \le 5$ $2x_1 - 4x_2 \le 7$ and $x_1, x_2 \ge 0$

4. Maximize $Z = 3x_1 + x_2$

Subject to the constraints:

$$x_1 + 2x_2 \ge -5$$

$$3x_1 + 5x_2 \le 6$$

and $x_1, x_2 \ge 0$

Solution:

The canonical form of a given LPP is,

Max $Z = 3x_1 + x_2$

Subject to the constraints:

$$-x_1 - 2x_2 \le 5$$
$$3x_1 + 5x_2 \le 6$$
and $x_1, x_2 \ge 0$

5. Express the following LPP in the canonical form:

Maximize $Z = 2x_1 + 3x_2 + x_3$

Subject to the constraints:

 $4x_1 - 3x_2 + x_3 \le 6$ $x_1 + 5x_2 - 7x_3 \ge -4$

and $x_1, x_3 \ge 0$ and x_2 is unrestricted.

Solution:

Here x_2 is unrestricted, $\therefore x_2$ can be written as the difference of two non-negative variables, (i.e.) $x_2 = x_2' - x_2''$ where x'_2 , $x''_2 \ge 0$

 \therefore The given LPP becomes,

$$Max Z = 2x_1 + 3(x_2' - x_2'') + x_3$$

 $4x_1 - 3(x_2' - x_2'') + x_3 \le 6$ -x_1 - 5(x_2' - x_2'') + 7x_3 \le 4 and x_1, x_2', x_2'', x_3 \ge 0

The Simplex Method:

Definition – 1:

Given a system of m linear equations with n variables (m < n). The solution obtained by setting (n-m) variables = 0 and solving for the remaining m variables is called a *Basic Solution*.

The m variables are called Basic variables and they form Basic Solution. The (n-m) variables which are put to 0 are called as *Non- basic Variables*.

Definition – 2:

A basic solution is said to be a non-degenerate Basic Solution if none of the Basic variables is zero.

Definition – 3:

A basic solution is said to be a degenerate basic solution if one or more of the basic variables are zero.

Definition – 4:

A feasible solution which is also basic is called a *Basic feasible solution*.

The Simplex Algorithm:

Assuming the existence of an initial basic feasible solution, an optimum solution to any LPP by simplex method is found as follows:

Step 1: Check whether the objective function is to be maximized or minimized. If it is to be minimized, then convert it into a problem of maximization, by Minimize Z = -Maximize(-Z)

Step 2: Check whether all b_i 's are positive. If any of the b_i 's is negative, multiply both sides of that constraint by -1 so as to make its right hand positive.

Step 3: By introducing slack / surplus variables, convert the inequality constraints into equations and express the given LPP into its standard form.

Step 4: Find an initial basic feasible solution and express the above information conveniently in the following simplex table.

2017 Batch

		\mathbf{C}_{j}	(C ₁	C_2	C ₃	 0	0	0)
C _B	Y_B	X _B	<i>x</i> ₁	<i>x</i> ₂	<i>X</i> 3	 <i>S</i> ₁	<i>s</i> ₂	S 3	
C _{B1}	S_1	b_1	<i>a</i> ₁₁	<i>a</i> ₁₂	<i>a</i> ₁₃	 1	0	0	
C _{B2}	S_2	b_2	<i>a</i> ₂₁	<i>a</i> ₂₂	<i>a</i> ₂₃	 0	1	0	
C _{B3}	S_3	b_3	<i>a</i> ₃₁	<i>a</i> ₃₂	<i>a</i> ₃₃	 0	0	1	
				•					
•	•	•	•	•	•				
•					•				
•	•		Во	ody matrix			1	Unit m	atrix
(Z _j –	- C _j)	Z ₀	$(Z_1 - C_1)$	$\left(Z_2-C_2\right)$			•••••	•••••	•••••

 $\begin{array}{ll} (Where \ C_j - row \ denotes \ the \ coefficients \ of \ the \ variables \ in \ the \ objective \ function. \\ C_B \\ - \ column \ denotes \ the \ coefficients \ of \ the \ basic \ variables. \\ Y_B - \ column \ denotes \ the \ basic \ variables. \\ X_B - \ column \ denotes \ the \ basic \ variables. \\ The \ coefficients \ of \ the \ non-basic \ variables \ in \ the \ constraint \ equations \ constitute \ the \ body \ matrix \ while \ the \ coefficients \ of \ the \ basic \ variables \ constitute \ the \ unit \ matrix. \\ The \ row \ (Z_j - C_j) \ denotes \ the \ evaluations \ (or) \ index \ for \ each \ column). \end{array}$

Step 5: Compute the net evaluations $(Z_j - C_j)$ (j = 1, 2, ..., n) by using the relation $Z_j - C_j = C_B a_j - C_j$.

Examine the sign of $Z_j - C_j$

- (a) If all $(Z_j C_j) \ge 0$ then the current basic feasible solution X_B is optimal.
- (b) If atleast one $(Z_j C_j) < 0$, then the current basic feasible solution is not optimal, go to the next step.

Step 6: (To find the entering variable)

The entering variable is the non-basic variable corresponding to the most negative value of $(Z_j - C_j)$. Let it be x_r for some j = r. The entering variable column is known as the key column

Prepared by : R. Praveen Kumar, Department of Mathematics, KAHE.

(or) pivot column which is shown marked with an arrow at the bottom. If more than one variable has the same most negative $(Z_j - C_j)$, any of these variables may be selected arbitrarily as the entering variable.

Step 7: (To find the leaving variable)

Compute the ratio $\theta = \text{Min}\left\{\frac{x_{Bi}}{a_{ir}}, a_{ir} > 0\right\}$ (i.e., the ration between the solution column and the entering variable column by considering only the positive denominators)

- (a) If all $a_{ir} \leq 0$, then there is an unbounded solution to the given LPP.
- (b) If atleast one $a_{ir} > 0$, then the leaving variable is the basic variable corresponding to the minimum ratio θ . If $\theta = \frac{X_{Bk}}{a_{kr}}$, then the basic variable x_k leaves the basis. The leaving variable row is called the key row or pivot equation, and the element at the intersection of the pivot column and pivot row is called the pivot element (or) leading element.

Step 8: Drop the leaving variable and introduce the entering variable along with its associated value under C_B column. Convert the pivot element to unity by dividing the pivot equation by the pivot element and all other elements in its column to zero by making use of

- (i) New pivot equation = old pivot equation \div pivot element
- (ii) New equation (all other rows including $(Z_j C_j)$ row)

 $= \text{Old equation} - \begin{pmatrix} \text{Corresponding} \\ \text{Column} \\ \text{Coefficient} \end{pmatrix} x \begin{pmatrix} \text{New pivot} \\ \text{element} \end{pmatrix}$

Step 9: Go to step (5) and repeat the procedure until either an optimum solution is obtained or there is an indication of an unbounded solution.

Note(1): For maximization problems:

- (i) If all $(Z_j C_j) \ge 0$, then the current basic feasible solution is optimal.
- (ii) If at least one $(Z_j C_j) < 0$, then the current basic feasible solution is not optimal.
- (iii) The entering variable is the non-basic variable corresponding to the most negative value of $(Z_j C_j)$.

Note(2): For minimization problems:

- (i) If all $(Z_j C_j) \le 0$, then the current basic feasible solution is optimal.
- (ii) If at least one $(Z_j C_j) > 0$, then the current basic feasible solution is not optimal.
- (iii) The entering variable is the non-basic variable corresponding to the most positive value of $(Z_j C_j)$.

Note(3): For both maximization and minimization problems, the leaving variable is the basic variable corresponding to the minimum ratio θ .

Problems:

1. Use simplex method to solve following LPP.

Maximize $Z = 4x_1 + 10x_2$

Subject to the constraints:

$$2x_1 + x_2 \le 50$$

 $2x_1 + 5x_2 \le 100$
 $2x_1 + 3x_2 \le 90$
and $x_1, x_2 \ge 0$

Solution:

By introducing the slack variables s_1 , s_2 , s_3 . The standard form of the given LPP becomes,

Maximize $Z = 4x_1 + 10x_2 + 0s_1 + 0s_2 + 0s_3$

Subject to the constraints:

$$2x_1 + x_2 + s_1 + 0s_2 + 0s_3 \le 50$$

$$2x_1 + 5x_2 + 0s_1 + s_2 + 0s_3 \le 100$$

$$2x_1 + 3x_2 + 0s_1 + 0s_2 + s_3 \le 90$$

and $x_1, x_2, s_1, s_2, s_3 \ge 0$

since there are 3 equations with 5 variables. \therefore The initial basic Feasible Solution is obtained by equality, (5-3) = 2 to 0.

:. The initial basic Feasible Solution (IFBS), $s_1=50$, $s_2=100$, $s_3=90$ Initial iteration:

			Ci	4	10	0	0	0			
	CB	Y_B	X _B	<i>x</i> ₁	<i>x</i> ₂	<i>S</i> ₁	S ₂	<i>S</i> ₃	Ratio		
	0	<i>S</i> ₁	50	2	1	1	0	0	$\frac{50}{1^{2}} = 50$		
←	0	<i>s</i> ₂	100	2	5	0	1	0	100/5=20		
	0	<i>S</i> ₃	90	2	3	0	0	1	90/3=30		
		$Z_i - C_i$	0	-4	-10	0	0	0			
▲ · · · · · · · · · · · · · · · · · · ·											
x_2 is entering variable,											
	S_{2} is leaving variable										

5 is pivot element

I iteration:

		Cj	4	10	0	0	0
CB	Y_B	X_B	<i>x</i> ₁	<i>x</i> ₂	<i>S</i> ₁	<i>S</i> ₂	<i>S</i> ₃
0	<i>S</i> ₁	30	⁸ / ₅	0	1	$^{-1}/_{5}$	0
0	<i>x</i> ₂	20	$^{2}/_{5}$	1	0	¹ / ₅	0
0	<i>S</i> ₃	30	⁴ / ₅	0	0	$^{-3}/_{5}$	1
	$Z_i - C_i$	200	0	0	0	2	0

Since all $Z_j - C_j \ge 0$. \therefore The current Basic Feasible solution is optimal.

: The optimal solution is Max Z = 200, $x_1 = 0$, $x_2 = 20$.

2. Find the non-negative values of x_1 , x_2 , x_3 which Max $Z = 3x_1 + 2x_2 + 5x_3$ Subject to the constraints, $x_1 + 4x_2 \le 420$ $3x_1 + 2x_3 \le 460$ $x_1 + 2x_2 + x_3 \le 430$ and $x_1, x_2, x_3 \ge 0$

Solution:

By introducing slack variables s_1 , s_2 , s_3 . The standard form of the given

 $Max Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to the constraints,

 $x_1 + 4x_2 + 0x_3 + s_1 + 0s_2 + 0s_3 = 420$ $3x_1 + 0x_2 + 2x_3 + 0s_1 + s_2 + 0s_3 = 460$ $x_1 + 2x_2 + x_3 + 0s_1 + 0s_2 + s_3 = 430$

and
$$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$$

The IFBS is given by, $s_1 = 420, s_2 = 460, s_3 = 430$

Initial iteration:

			Cj	3	2	5	0	0	0	Ratio
	C _B	Y_B	X_B	<i>x</i> ₁	<i>x</i> ₂	<i>x</i> ₃	<i>S</i> ₁	<i>S</i> ₂	<i>S</i> ₃	θ
	0	<i>S</i> ₁	420	1	4	0	1	0	0	$420/0=\infty$
←	0	<i>S</i> ₂	460	3	0	2	0	1	0	$\frac{460}{2} = 230$
	0	<i>S</i> ₃	430	1	2	1	0	0	1	$\frac{430}{1} = 430$
		$Z_j - C_j$	0	-3	-2	-5	0	0	0	
						↑				

 s_2 is leaving variable, x_3 is entering variable, 2 is pivot element

I iteration:

			Cj	3	2	5	0	0	0	Ratio
	C_B	Y_B	X_B	<i>x</i> ₁	<i>x</i> ₂	<i>x</i> ₃	<i>s</i> ₁	<i>S</i> ₂	<i>S</i> ₃	θ
	0	<i>S</i> ₁	420	1	4	0	1	0	0	$\frac{420}{4} = 105$
	5	<i>x</i> ₃	230	$^{3}/_{2}$	0	1	0	$^{1}/_{2}$	0	$230/0=\infty$
←	0	<i>S</i> ₃	200	$^{-1}/_{2}$	2	0	0	$^{-1}/_{2}$	1	$\frac{200}{2^{2}}$
		$Z_j - C_j$	1150	⁹ / ₂	-2	0	0	⁵ /2	0	
					Ť					

 s_3 is leaving variable, x_2 is entering variable, 2 is pivot element

II iteration:

		c _j	3	2	5	0	0	0
CB	Y_B	X_B	<i>x</i> ₁	<i>x</i> ₂	<i>x</i> ₃	<i>s</i> ₁	<i>S</i> ₂	<i>S</i> ₃
0	<i>S</i> ₁	20	2	0	0	1	0	-2
5	<i>x</i> ₃	230	$^{3}/_{2}$	0	1	0	$^{1}/_{2}$	0
2	<i>x</i> ₂	100	$^{-1}/_{4}$	1	0	0	$^{-1}/_{4}$	$^{1}/_{2}$
	$Z_j - C_j$	1350	4	0	0	0	2	1

: Max Z = 1350, $x_1 = 0$, $x_2 = 100$, $x_3 = 230$

<u>PART – B (5 x 6 = 30)</u>

Possible Questions (6 Marks)

- 1. Explain the various models in operations research
- 2. Use graphical method to solve the following LPP

Max Z = $80x_1 + 55x_2$ Subject to $4x_1 + 2x_2 \le 40$ $2x_1 + 4x_2 \le 32$

$$x_1 \ge 0$$
, $x_2 \ge 0$

3. A manufacturer produces 3 models (I, II and III) of a certain product. He uses 2 raw materials A and B of which 4000 and 6000 units respectively are available. The raw materials per unit of 3 models are given below.

Raw materials	Ι	II	III
А	2	3	5
В	4	2	7

The labour time for each unit of model I is twice that of model II and thrice that of model III. The entire labour force of factory can produce an equivalent of 2500 units of model I. A model survey indicates that the minimum demand of 3 models is 500, 500 and 375 units respectively. However the ratio of number of units produced must be equal to 3:2:5. Assume that profits per unit of model are 60, 40 and 100 respectively. Formulate a LPP.

4. A pine apple firm produces two products canned pineapple and canned juice. Specific amounts of material, labour and equipment required to produce each product and the availability of each of these resources are shown in the table given below:

		<u> </u>	
	Canned Juice	Canned Pineapple	Available resources
Labour (Man hours)	3	2	12
Equipment (M/c hours)	1	2.3	6.9
Material (Unit)	1	1.4	4.9

Assuming one unit of canned juice and canned Pineapple has profit margins Rs.2 and Rs.1 respectively. Formulate this as a L.P.P and solve it graphically also.

5. Use simplex method to solve the following LPP

 $\begin{array}{l} \text{Maximize } Z=5x_1+8x_2\\ \text{Subject to the constrains}\\ -2\;x_1+\;\;x_2\;\leq 1\\ &x_1\;\leq 2\\ &x_1+\;x_2\;\leq 3\\ \text{and } x_1,x_2\;\leq 0 \end{array}$

6. Solve the following LPP by the graphical method.

 $\begin{array}{l} \text{Maximize } Z = 3x_1 + 2x_2\\ \text{Subject to the constrains}\\ -2x_1 + x_2 &\leq 1\\ x_1 &\leq 2\\ x_1 + x_2 &\leq 3\\ \text{and } x_1, x_2 \geq 0 \end{array}$

7. Use simplex method to solve the following LPP Maximize $Z = 4x_1 + 10x_2$ Subject to the constrains $2x_1 + x_2 \leq$ 50 $2x_1+5x_2 \hspace{.1in} \leq \hspace{.1in} 100$ $2x_1 + 3x_2 \leq$ 90 and $x_1, x_2 \ge 0$ 8. Use graphical method to solve the following LPP Maximize $Z = 4x_1 + 10x_2$ Subject to the constrains $2x_1 + x_2 \le 50$ $2x_1 + 5x_2 \le 100$ $2x_1 + 3x_2 \le 90$ and $x_1, x_2 \ge 0$ 9. Use simplex method to solve the following LPP Maximize $Z = 3x_1 + 2x_2$ Subject to the constrains $-2 x_1 + x_2 \leq 1$ $x_1 \leq 2$ $x_1 + x_2 \le 3$ and $x_1, x_2 \ge 0$ 10. A company manufactures 2 types of printed circuits. The requirements of transistors, resistors and capacitors for each type of printed circuits along with other data are given

below.

	Cir	cuit	Stock available
	A B		
Transistor	15	10	180
Resistor	10	20	200
Capacitor	15	20	210
Profit	Rs.5	Rs.8	

How many circuits of each type should the company produce from the stock to earn maximum profit. Formulate this as a LPP and solve it graphically also

11. Solve the following LPP by simplex method

 $\begin{array}{l} \text{Maximize } Z = 8x_1 - 2x_2 \\ \text{Subject to the constrains} \\ -4x_1 + 2 \ x_2 \ \leq \ 4 \\ 3x_1 + 8x_2 \ \leq \ 24 \\ 10x_1 + 7x_2 \ \leq \ 35 \\ \text{and } x_1, \ x_2, \ x_3 \ \geq \ 0 \end{array}$

PART – C $(1 \times 10 = 10)$

Possible Questions (10 Marks)

1. A manufacturer produces 3 models (I, II and III) of a certain product. He uses 2 raw materials A and B of which 4000 and 6000 units respectively are available. The raw materials per unit of 3 models are given below.

Raw materials	Ι	II	III
А	2	3	5
В	4	2	7

The labour time for each unit of model I is twice that of model II and thrice that of model III. The entire labour force of factory can produce an equivalent of 2500 units of model I. A model survey indicates that the minimum demand of 3 models is 500, 500 and 375 units respectively. However the ratio of number of units produced must be equal to 3:2:5. Assume that profits per unit of model are 60, 40 and 100 respectively. Formulate a LPP.

- 2. Explain the various models in operations research.
- 3. A company manufactures 2 types of printed circuits. The requirements of transistors, resistors and capacitors for each type of printed circuits along with other data are given below.

	Cir	cuit	Stock available
	A B		
Transistor	15	10	180
Resistor	10	20	200
Capacitor	15	20	210
Profit	Rs.5	Rs.8	

How many circuits of each type should the company produce from the stock to earn maximum profit. Formulate this as a LPP and solve it graphically also.

- 4. Discuss any three application of operations research
- 5. Use graphical method to solve the following LPP

Maximize $Z = 4x_1 + 10x_2$ Subject to the constrains $2x_1 + x_2 \le 50$ $2x_1 + 5x_2 \le 100$ $2x_1 + 3x_2 \le 90$ and $x_1, x_2 \ge 0$



KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore –641 021. Department of Mathematics

Subject : Operation Research	Subject Code : 17CCP103	LTPC		
Class : I – M.Com(CA)	Semester : I	4 0 0 4		

UNIT – II

Transportation Model: Introduction – Mathematical Formulation –Finding Initial Basic Feasible Solutions – Optimum Solution for Nondegeneracy and Degeneracy Model - Unbalanced Transportation Problems and Maximization case in Transportation Problem- Traveling Sales Man Problem.

TEXT BOOK

1. KanthiSwarup, Gupta P.K., Man Mohan., (2016) Operations Research, Sultan

Chandand Sons, New Delhi.

REFERENCES

1. Sharma J.K., (2008), Operations Research Theory Applications, Macmillan India Ltd, New Delhi.

2. Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K.,(2014), Resource Management Techniques, A. R. Publications, Nagapatinam.

3. Shanthi Sophia Bharathi D.,(1999),Operations Research/ Resource management techniques, Charulatha Publications.

4. HamdyA.Taha., Operations Research, (2007), Pearson education, Prentice Hall.

5. Vittal– Operations Research – Margham Publications

UNIT – II

TRANSPORTATION MODEL

Transportation Model :

Introduction

Transportation deals with the transportation of a commodity (single product) from 'm' sources (origins or supply or capacity centers) to 'n' destinations (sinks or demand or requirement centers). It is assumed that

- (i) Level of supply at each source and the amount of demand at each destination and
- (ii) The unit transportation cost of transportation is linear.

It is also assumed that the cost of transportation is linear.

The objective is to determine the amount to be shifted from each sources to each destination such that the total transportation cost is minimum.

Note: The transportation model also can be modified to Account for multiple commodities.

1. Mathematical Formulation of a Transportation problem:

Let us assume that there are m sources and n destinations.

Let a_i be the supply (capacity) at source i, b_j be the demand at destination j, c_{ij} be the unit transportation cost from source *i* to destination j and x_{ij} be the number of units shifted from sources *i* to destination *j*.

Then the transportation problems can be expressed mathematically as

Minimize
$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints

$\sum_{j=1}^n x_{ij} = a_i,$	$i = 1, 2, 3, \dots, m$
$\sum_{i=1}^m x_{ij} = b_j,$	$j = 1, 2, 3, \dots N.$

And $x_{ij} \ge 0$, for all *i* and *j*.

Note 1: The two sets of constraints will be consistent if

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

(total supply) (total demand)

Which is the necessary and sufficient condition for a transportation problems to have a feasible solution. Problems satisfying this condition are balanced transportation problems.

Note 2: If $\sum a_i \neq \sum b_i$

Note 3: For any transportation problems, the coefficient of all x_{ij} in the constraints are unity.

Note 4: The objective function and the constraints being all linear, the transportation problems is a special class of linear programming problem. Therefore it can be solved by simplex method. But the number of variables being large, there will be too many calculations. So we can look for some other technique which would be simpler that the usual simplex method.

Standard transportation table:

Transportation problem is explicitly represented by the following transportation table.

		D_1	D_1	D_1	 D_1	 D_1	supply
	S_1	<i>C</i> ₁₁	<i>C</i> ₁₂	C ₁₃	<i>C</i> _{1<i>j</i>}	<i>C</i> _{1<i>n</i>}	a_1
Source	<i>S</i> ₁	C ₂₁	C ₂₂	C ₂₃	C _{2j}	<i>C</i> _{2<i>n</i>}	<i>a</i> ₂
	<i>S</i> ₁	C _{i1}	<i>C</i> _{<i>i</i>2}		C _{ij}	C _{in}	
	<i>S</i> ₁	C_{m1}	<i>C</i> _{<i>m</i>2}		C _{mj}	C _{mn}	a_n
Demand		b_1	<i>b</i> ₂	<i>b</i> ₃	 	 b_n	$\sum a_i = \sum b_i$

Destination

The *mn* squares are called **cells.** The unit transportation cost c_{ij} from the ith source to the jth destination is displayed in the **upper left side of the** (i,j)th cell. Any feasible solution us shown in the table by entering the value of x_{ij} in the center of the (i,j)th cell. The various a's and b's are called **rim requirements.** The feasibility of a solution can be verified by summing the values if x_{ij} along the rows and down the columns.

Definition 1: A set of non-negative values x_{ij} , i=1,2.....m; j=1,2.....n. that satisfies the constraints (rim conditions and also the non-negativity restrictions) is called a **feasible solution** to the transportation problems.

Note: A balanced transportation problems will always have a feasible solution.

Definition 2: A feasible solution to a $(m \times n)$ transportation problems that contains no more than m + n-1 non-negative allocations is called a **basic feasible solution** (BFS) to the transportation problem.

Definition 3: A basic feasible solution to a $(m \times n)$ transportation problem is said to be a **non-degenerate basic feasible solution** if it contains exactly m + n-1 non-negative allocations in independent positions.

Definition 4: A basic feasible solution that contains less than m + n - 1 non-negative allocations is said to be a degenerate basic feasible solution.

Definition 5: A feasible solution (not necessarily basic) is said to be an **optimal** solution if it minimize is at most m + n - 1.

Note: The number of non-basic variables in an m x n balanced transportation problem is almost m + n - 1.

Note: The number of non-basic variables in an m x n balanced transportation problem is at least mn - (m + n - 1).

II. Methods for finding initial basic feasible solution

The transportation problems has a solution is and only if the problem is balanced. Therefore before starting to find the initial basic feasible solution, check whether the given transportation problem is balanced. If not once has to balance the transportation problems first. The way to doing this is discussed in section 7.4 page 7.40. In this section all the given transportation problems are balanced.

Method I: North west corner rule:

Step I: The first assignment is made in the cell occupying the upper left-hand (north-west) corner of the transportation table. The maximum possible amount is allocated there. That is $x_{11} = \min \{a_1, b1\}$.

Case (i): If min $\{a_1, b_1\} = a_1$, then put $x_{11} = a_1$, decrease b_1 by a_1 and move vertically to the 2^{nd} row (i.e.,) to the cell (2, 1) cross out the first row.

Case (ii): If min $\{a_1, b_1\} = b_1$, then put $x_{11} = b_1$, decrease a_1 by b_1 and move horizontally right (i.e.,) to the cell (2, 1) cross out the first column.

Case (iii): If min $\{a_1, b_1\} = a_1 = b_1$, the put $x_{11} = a_1 = b_1$ and move diagonally to the cell (2, 2) cross out the first row and the first column.

Step 2: Repeat the procedure until all the rim requirements are satisfied.
Method 2: Lest cost method (or) Matrix minima method (or) Lowest cost entry

method:

Step 1: Identify the cell with smallest cost and allocate $x_{ij} = Min \{a_i, b_j\}$

Case (i): If min $\{a_i, b_j\} = a_i$, then put $x_{ij} = a$, cross out the ith row and decrease b_j by a_i , go to step(2).

Case (ii): If min $\{a_i, b_j\} = b_j$, then put $x_{ij} = b_j$, cross out the jth column and decrease a_j by b_j , go to step(2).

Case (iii): If min $\{a_i, b_j\}=a_i=b_j$, then put $x_{ij}=a_i=b_j$, cross out either ith row and jth column but not both, go to step(2).

Step 2: Repeat step (1) for the resulting reduced transportation table until all the rim requirements are satisfied.

Method 3: Vogel's approximation method (VAM) (or) Unit cost penalty method:

Step 1: Find the difference (penalty) between the smallest and next smallest costs in each row (column) and write them in brackets against the corresponding row (column).

Step 2: Identify the row (or) column with large penalty. If a tie occurs, break the tie arbitrarily. Choose the sell with smallest cost in that selected row or column and allocate as much as possible to this cell and cross out the satisfied row or column and go to step (3).

Step 3: Again compute the column and row penalties for the reduced transportation table and then go to step (2). Repeat the procedure until all the rim requirements are satisfied.

Example 1: Determine basic feasible solution to the following transportation problems using North West Corner Rule:



Solution:

Since $a_i = b_j = 21$, the given problem is balanced. \therefore There exists a feasible solution to the transportation problem.

2	11	10	3	7	4
1	4	7	2	1	8
3	9	4	8	12	9
3	3	4	5	6	

Following North West Corner rule, the first allocation is made in the cell(1,1)

Here $x_{11} = \min \{a_1, b_1\} = \min \{4,3\} = 3$

Allocate 3 to the cell(1,1) and decrease 4 by 3 i.e., 4-3=1

As the first column is satisfied, we cross out the first column and the resulting reduced Transportation table is

11	10	3	7	1
1				
4	7	2	1	8
9	4	8	12	9
3	4	5	6	

Here the North West Corner cell is (1,2).

So allocate $x_{11} = \min \{1, 3\} = 1$ to the cell (1,2) and move vertically to cell (2, 2). The resulting transportation table is

4 2	7	2	1	8
9	4	8	12	9
2	4	5	6	I

Allocate $x_{22} = \min \{8, 2\} = 2$ to the cell (2, 2) and move horizontally to cell (2, 3). The resulting transportation table is

7 4	2	1	6
4	8	12	0
4	5	6	9

Allocate $x_{23} = \min \{6, 4\} = 4$ and move horizontally to cell (2, 4). The resulting reduced transportation table is



Allocate $x_{24} = \min \{2, 5\} = 2$ and move vertically to cell (3, 4). The resulting reduced transportation table is

Allocate $x_{34} = \min \{9, 3\} = 3$ and move horizontally to cell (3, 5).which is

Allocate $x_{35} = \min \{6, 6\} = 6$

Finally the initial basic feasible solution is as shown in the following table.

2	11	10	3	7
3	1			
1	4	7	2	1
	2	4	2	



From this table we see that the number of positive independent allocations is equal to

m + n - 1 = 3 + 5 - 1 = 7. This ensures that the solution is non degenerate basic feasible.

 $\therefore \text{The initial transportation} = \text{Rs. } 2 \times 3 + 11 \times 1 + 4 \times 2 + 7 \times 4 + 2 \times 2 + 8 \times 3$ $\text{cost} + 12 \times 6$ = Rs. 153/-

Example 2:

Find the initial basic feasible solution for the following transportation problem by Least Cost Method.



[MU. BE. Apr 95, BE. Nov 96]

Solution:

Since $\sum a_i = \sum b_j = 100$, the given TPP is balanced. There exists a feasible solution to the transportation problem.

1	2	1	4	
20				30
3	3	2	1	50
4	2	5	9	20
20	40	30	10	1

By least cost method, min $c_{ij} = c_{11} = c_{13} = c_{24} = 1$

Since more than one cell having the same minimum c_{ij} , break the tie.

Let us choose the cell (1,1) and allocate $x_{11} = \min \{a_1, b_1\} = \min \{30, 20\} = 20$ and cross out the satisfied column and decrease 30 by 20.

The resulting reduced transportation table is

2	1	4	
	10		10
3	2	1	50
2	5	9	20
40	30	10	

Here min $c_{ij} = c_{13} = c_{24} = 1$. Choose the cell (1,3) and allocate $x_{13} = \min \{a_1, b_3\} = \min \{10,30\} = 10$ and cross out the satisfied row.

The resulting reduced transportation table is

3	2	1	
		10	50
2	5	9	20
40	20	10	

Here min $c_{ij} = c_{24} = 1$

: Allocate $x_{24} = \min \{a_2, b_4\} = \min (50, 10) = 10$ and cross out the satisfied column.

The resulting transportation is

3	2	
	20	40
2	5	20
40	20	

Here $c_{ij} = c_{23} = c_{32} = 2$. Choose the cell (2,3) and allocate $x_{23} = \min \{a_2, b_3\} =$

min (40,20) = 10 and cross out the satisfied column.

The resulting reduced transportation table is



40

Here min = $c_{ij} = c_{32} = 2$. Choose the cell (3,2) and allocate $x_{32} = \min \{a_3, b_2\} =$

min (20,40) = 20 and cross out the satisfied row.

The resulting reduced transportation table is

Finally the initial basic feasible solution is as shown in the following table.

1	2	1	4
20		10	
3	3	2	1
	20	20	10
4	20	20	10 9

From this table we see that the number of positive independent allocations is equal to

m + n - 1 = 3 + 4 - 1 = 6. This ensures that the solution is non degenerate basic feasible.

 \therefore The initial transportation = Rs. 1 x 20 + 1 x 10 + 3 x 20 + 2 x 20

Cost $+1 \times 10 + 2 \times 20$ = Rs. 20 + 10 + 60 + 40 + 10 + 40 = Rs. 180/-

Example 3:

Find the initial basic feasible solution for the following transportation problem by VAM.

Distribution centres



S ₃	21	24	13	10	400
Requirements	200	225	275	250]

Solution:

Since $\sum a_i = \sum b_j = 100$, the given is balanced. \therefore There exists a feasible solution to this problem.

11	13	17	14	250	(2)
200					
16	18	14	10	300	(4)
- 21	24	10	10	100	$\langle \mathbf{O} \rangle$
21	24	13	10	400	(3)
200	225	075	250		
200	225	215	250		
(5)	(5)	(1)	(0)		

First let us find the difference (penalty) between the smallest and next smallest costs in each row and column and write them in brackets against the respective rows and columns.

The largest of these differences is (5) and is associated with the first two columns of the transportation table. We choose the first column arbitrarily.

In this selected column, the cell (1,1) has the minimum unit transportation cost $c_{11} = 11$.

: Allocate $x_{11} = \min (250,200) = 200$ to this cell (1,1) and decrease 250 by 200 and cross out the satisfied column.

The resulting reduced transportation table is

13 50	17	14	50	(1)
18	14	10	300	(4)
24	13	10	400	(3)
225 (5)	275 (1)	250 (0)	-	

The row and column differences are now computed for this reduced transportation table. The largest of these is (5) which is associated with the second column. Since $c_{12} = 13$ is the minimum cost, we allocate $x_{12} = \min (50,225) = 50$ to the cell (1,2) and decrease 225 by 50 and cross out the satisfied row.

Continuing in this manner, the subsequent reduced transportation tables and the differences for the surviving rows and columns are shown below:

18	14	10	300	(4)
175				
24	13	10	400	(3)
175	275	250		
(6)	(1)	(0)		
	<i>(i)</i>			





Finally the initial basic feasible solution is as shown in the following table.

11		13		17		14	
	200		50				
16		18		14		10	
			175				125
21		24		13		10	
					275		125

From this table we see that the number of positive independent allocation is equal to

m + n - 1 = 3 + 4 - 1 = 6. This ensures that the solution is non degenerate basic feasible.

 \therefore The initial transportation = Rs. 11 x 200 + 13 x 50 + 18 x 175 +

Cost =
$$+10 \times 125 + 13 \times 275 + 10 \times 125$$

= Rs. 12075/-

Example 4:

Find the starting solution of the following transportation model

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	1

Using (i). North West Corner rule

- (ii). Least Cost method
- (iii). Vogel's approximation method.

Solution:

Since $\sum a_i = \sum b_j = 100$, the given Transportation problem is balanced. \therefore There exists a basic feasible solution to this problem.

(i). North West Corner rule: Using this method, the allocation are shown in the tables below:



The starting solution is as shown in the following table

ſ	1		2		6	
		7				
I	0		4		2	
		3		9		
I	3		1		5	
				1		10

: The initial transportation cost = Rs. $1 \times 7 + 0 \times 3 + 4 \times 9 + 1 \times 1 + 5 \times 10$

= Rs. 94/-

(ii). Least Cost method: Using this method, the allocation are as shown in the table below:



(iv)

(v)

6

7

The starting solution is as shown in the following table:

	1		2		6	
						7
()		4		2	
		10				2
~	3		1		5	
				10		1

: The initial transportation cost = Rs. $6 \times 7 + 0 \times 10 + 2 \times 2 + 1 \times 10 + 5 \times 1$

= Rs. 61/-

(iii). Vogel's approximation Method: Using this method, the allocations are shown in the table below:





The starting solution is as shown in the following table:

1		2		6	
	7				
0		4		2	
	2				10
3		1		5	
	4		10		

: The initial transportation cost = Rs.1 x 7 + 0 x 2 + 2 x 10 + 3 x 1 + 1 x 10

$$=$$
 Rs. 40/-

Note: For the above problem, the number of positive allocation in independent positions is (m + n - 1) (i.e., m + n - 1 = 3 + 3 - 1 = 5). This ensures that the given problem has a non-

degenerate basic feasible solution by using all the three methods. This need not be the case in all the problems.

Transportation Algorithm (or) MODI Method (modified distribution method) (Test for optimal solution).

Step 1: Find the initial basic feasible solution of the given problems by Northwest Corner rule (or) Least Cost method or VAM.

Step 2: Check the number of occupied cells. If these are less than m + n - 1, there exists degeneracy and we introduce a very small positive assignment of $\in (\approx 0)$ in suitable independent positions, so that the number of occupied cells is exactly equal to m + n - 1.

Step 3: Find the set of values u_i , v_j (i=1,2,3,...m; j=1,2,3...n) from the relation $c_{ij} = u_i + v_j$ for each occupied cell (i,j), by starting initially with $u_i = 0$ or $v_j = 0$ preferably for which the corresponding row or column has maximum number of individual allocations.

Step 4: Find $u_i + v_j$ for each unoccupied cell (i,j) and enter at the upper right corner of the corresponding cell (i,j).

Step 5: Find the cell evaluations $d_{ij} = c_{ij} - (u_i + v_j) (d_{ij} = upper left - upper right)$ for each unoccupied cell (i,j) and enter at the lower right corner of the corresponding cell (i,j).

Step 6: Examine the cell evolutions d_{ij} for all unoccupied cells (i,j) and conclude that

- (i) If all $d_{ii}>0$, then the solution under the test is optimal and unique.
- (ii) If all $d_{ij}>0$, with atleast one $d_{ij}=0$, then the solution under the test is optimal and an alternative optimal solution exists.
- (iii) If at least one $d_{ij} < 0$, then the solution is not optimal. Go to the next step.

Step 7: Form a new B>F>S by giving maximum allocation to the cell for which d_{ij} is most negative by making an occupied cell empty. For that draw a closed path consisting of horizontal and vertical lines beginning and ending at the cell for which d_{ij} is most negative and having its **other corners at some allocated cells.** Along this closed loop indicate $+\theta$ and $-\theta$ alternatively at the corners. Choose minimum of the allocations from the cells having $-\theta$. Add this minimum allocation to the cells with $+\theta$ and subtract this minimum allocation from the allocation to the cells with $-\theta$.

Step 8: Repeat steps (2) to (6) to test the optimality of this new basic feasible solution.

Step 9: Continue the above procedure till an optimum solution is attained.

Note: The Vogels approximation method (VAM) takes into account not only the least $cost c_{ij}$ but also the costs that just exceed the least $cost c_{ij}$ and therefore yields better initial

solution than obtained from other methods in general. This can be justified by the above example (4). So to find the initial solution, give preference to VAM unless otherwise specified.

Example 1: Solve the transportation problem:

	1	2	3	4	Supply
Ι	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	

Solution: Since $\sum a_i = \sum b_j = 43$, the given transportation problem is balanced. \therefore There exists a basic feasible solution to this problem.By Vogel's approximation method, the initial solution is an shown in the following table.

			-						r						
	2	21		16		2	5		13	3		(3)	-	-	-
										1	1				
	1	7		18		1	4		23	3		(3)	(3)	(3)	(3)
			6		3					4	4				
	3	2	_	27		1	8		Δ	1		(9)	(9)	(9)	(9)
	5	-		21	7	1	1	12	т.	L		(\mathcal{I})	(\mathcal{I})	(\mathcal{I})	(\mathcal{I})
					'										
				(0)						(10)					
		(4)		(2)		(4)			(10)					
		(15)		(9)		(4)			(18)					
		(15)		(9)		(4)								
				(9)		(4)								
; [21		16	5		25		1	13		1				
				-						11					
	17		18	3		14		2	23						
		6		•	3					4					
	32		27	7		18		4	11						
				,	7		12								
]				

That is

From this table, we see that the number of non-negative independent allocations is (m + n - 1) = (3+4-1) = 6. Hence the solution is non-degenerate basic feasible.

 \therefore The initial transportation cost.

= Rs. 13 x 11 + 17 x 6 + 18 x 3 + 23 x 4 + 27 x 7 + 18 x 12

= Rs. 796/-

To find the optimal solution

Consider the above transportation table. Since m+n-1=6, we apply MODI method,

Now we determine a set of values u_i and v_j for each occupied cell (i,j) by using the

relation $c_{ij} = u_i + v_j$. As the 2nd row contains maximum number of allocations, we choose $u_2=0$.

Therefore

$$\begin{split} C_{21} &= u_2 + v_1 \Longrightarrow 17 = 0 + v_1 \Longrightarrow v_1 = 17 \\ C_{22} &= u_2 + v_2 \Longrightarrow 18 = 0 + v_2 \implies v_2 = 18 \\ C_{24} &= u_2 + v_4 \Longrightarrow 23 = 0 + v_4 \implies v_4 = 23 \\ C_{14} &= u_1 + v_4 \Longrightarrow 13 = u_1 + 23 \implies u_1 = -10 \\ C_{32} &= u_3 + v_2 \Longrightarrow 27 = u_3 + 18 \implies u_3 = 9 \\ C_{33} &= u_3 + v_3 \Longrightarrow 18 = 9 + v_3 \implies v_3 = 9 \end{split}$$

Thus we have the following transportation table:

21		16		25	13		$u_1 = -10$
						11	
17		18		14	23		$u_2 = 0$
	6		3			4	
32		27		18	41		$u_3 = 9$
			7	12			-
$v_1 =$	17	$v_2 =$	18	$v_3 = 9$	$v_4 = 2$	23	

Now we find $u_i + v_j$ for each unoccupied cell (i,j) and enter at the upper right corner of the corresponding unoccupied cell(i,j).

Then we find the cell evalutions $d_{ij} = c_{ij} - (u_i + v_j)$ (ie., upper left corner – upper right corner) for each unoccupied cell (i,j) and enter at the lower right corner of the corresponding unoccupied cell (i,j).

21	7	16	8	25	-1	13		u – 10
	14		8		26		11	$u_1 = -10$
17		18		14	9	23		
								$u_2 = 0$
	6		3		5		4	

32	26	27	18	41 32]
	6	7	12	0	$u_3 = 9$
	0	1	12	9	
$V_1 \equiv$	17	$v_2 = 18$	$v_2 - 0$	$v_4 - 23$	

Since all $d_{ij}>0$, with $d_{32}=0$, the current solution is optimal and unique.

:. The optimum allocation schedule is given by $x_{14} = 11$, $x_{21} = 6$, $x_{22} = 3$, $x_{24} = 4$, $x_{32} = 7$, $x_{33} = 12$, and the optimum (minimum) transportation cost = Rs. 13x 11 + 17 x 6 + 18 x 3 + 23 x 4 + 27 x 7 + 18 x 12 = Rs. 796/-

Example 2:

Obtain on optimum has feasible solution to the following transportation problem:

		То		Available
	7	3	2	2
From	2	1	3	3
	3	4	6	5
Demand	4	1	5	10

Solution:

Since $\sum a_i = \sum b_j = 43$, the given transportation problem is balanced. \therefore There exists a basic feasible solution to this problem.

By Vogel's approximation method, the initial solution is as shown in the following table:

ſ	7	3	2	(1)	(5)
			2		
ſ	2	1	3	(1)	(1) (1)
		1	2		
ľ	3	4	6	(1)	(3) (3)
	4		1		
L				1	
	(1)	(2)	(1)		
	(1)		(1)		
	(1)		(3)		

That is

7		3		2	
					2
2		1		3	
			1		2
3		4		6	
	4				1

From this table we see that the number of non-negative allocation is

m + n - 1 = (3 + 3 - 1) = 5.

Hence the solution is non-degenerate basic feasible

: The initial transportation cost = Rs. $2 \times 2 + 1 \times 1 + 3 \times 2 + 3 \times 4 + 6 \times 1$

= Rs. 29/-

For optimality: since the number of non – negative independent allocation is m + n - 1, we apply MODI method.

Since the third column contains maximum number of allocations, we choose $v_3 = 0$.

Now we determine a set of values u_i and v_j by using the occupied cells and the relation $c_{ij} = u_i + v_j$.

That is

7 -	-1	3	0	2		
					2	$u_1 = 2$
2		1		3		2
			1		2	$u_2 = 3$
3		4		6		
	4				1	$u_3 = 0$
$v_1 = -3$		$v_2 = -2$		$v_3 = 0$		-

Now we find $u_i + v_j$ for each unoccupied cell (i, j) and enter at the corresponding unoccupied cell (I,j).

Then we find the cell evaluations $d_{ij} = c_{ij} - (u_i + v_j)$ for each unoccupied cell (i, j) and enter at the lower right corner of the corresponding unoccupied cell (i, j).

Thus we get the following table

v ₁ =	= -3	$v_2 = -2$		$v_3 = 0$		
	4		0	1	l	$u_3 = 6$
3		4	4	6		
	2		1	2	2	$u_2 = 3$
2	0	1		3		$u_2 = 3$
	8		3	2	2	u1 – 2
7	-1	3	0	2		11 2

Since all $d_{ij}>0$, with $d_{32}=0$, the current solution is optimal and there exists an alternative optimal solution.

: The optimum allocation schedule is given by $x_{13} = 2$, $x_{32} = 1$, $x_{23} = 2$, $x_{31} = 4$, $x_{33} = 1$, and the optimum (minimum) transportation cost

$$= Rs.2x2 + 1x1 + 3x2 + 3x4 + 6x1 = Rs.29/-$$

Example 3: Find the optimal transportation cost of the following matrix using least cost

method for finding the critical solution.

		А	В	С	D	Е	Available
	Р	4	1	2	6	9	100
Factory	Q	6	4	3	5	7	120
	R	5	2	6	4	8	120
Demand		40	50	70	90	90	

Solution:

Since $\sum a_i = \sum b_j = 340$, the given transportation problem is balanced. \therefore There exists a basic feasible solution to this problem.

By using Least cost method, the initial solution is an shown in the following table:

4	1	2	6	9
	50	50		
6	4	3	5	7

10		20		90
5	2	б	4	8
30			90	

: The initial transportation cost = Rs. 1 x 50 + 2 x 50 + 6 x 10 + 3 x 20 + 7 x 90

For optimality: Since the number of non – negative independent allocations is (m + n -1), we apply MODI method:

] 1	6	4	6		2		1	5	4
$u_1 = -1$	3	2		50		50		-1	
	,	5	4		3	2	4		6
$u_2 = 0$	90	0		20		2		10	
] 1	6		4	2	6	1	2		5
$u_3 = -1$	2	90		4		1		30	
_	$r_5 = 7$	4 = 5	V	$_{3} = 3$	V 3	= 2	V	$_{1} = 6$	V

Since $d_{11} = -1 < 0$, the current solution is not optimal.

Now let us form a new basic feasible solution by giving maximum allocation to the cell (i,j) for which d_{ij} is most negative by making an occupied cell empty. Here the cell (1,1) having the negative value d_{11} =-1. We draw a closed loop consisting of horizontal and vertical lines beginning and ending at this cell (1,1) and having its other corners at some occupied cells. Along this closed loop indicate + θ and + θ alternatively at the corners. We have



6	4	3	5	7
10		20		90
<i>-</i> θ		$+\theta$		
5	2	6	4	8
30			90	

From the two cells (1,3), (2,1) having $+\theta$, we find that the minimum of the allocations 50,10 is 10. Add this cells with $+\theta$ and subtract this 10 to the cells with $+\theta$.

Hence the new basic feasible solution is displayed in the following table:

4		1		2		6		9	
	10		50		40				
6		4		3		5		7	
					30				90
5		2		6		4		8	
	30						90		

We see that the above table satisfies the rim conditions with (m + n - 1) non-negative allocations at independent position. So we apply MODI method.

	1		2		6	3	9	6	$u_1 = 0$
10		50		40					
						3		3	
5	4	2	3		5	4	7		
				20				00	$u_2 = 1$
1		2		30		1		90	
	10 5	1 10 5 4 1	1 50 50 50 51 2 2 1 2 2 1 2 2 1 2 2 1 2 2 1 2 2 1 2 2 1 2 2 1 2 2 1 2 2 1 2 2 1 2 2 1 2	1 2 10 50 5 4 2 1 2	1 2 10 50 40 5 4 2 3 1 2 30	1 2 6 10 50 40 5 4 2 3 5 1 2 30 1	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$

5		2	2	6	3	4		8	7	
,	20						00			$u_3 = 1$
•	30		0		3		90		1	
$v_1 = 4$	4	v ₂ =	1	v ₃ =	2	$v_4 =$	3	v ₅ =	6	

Since all $d_{ij}>0$, with $d_{32}=0$, the current solution is optimal and there exists an alternative optimal solution.

The optimum allocation schedule is given by $x_{11}=10$, $x_{12}=50$, $x_{13}=40$, $x_{23}=30$, $x_{25}=90$, $x_{31}=30$, $x_{34}=90$ and the optimum (minimum) transportation cost. = Rs. 4 x 10 + 1 x 50 + 2 x 40 + 3 x 30 + 7 x 90 + 5 x 30 + 4 x 90. = Rs. 1400/-

Degeneracy in Transportation Problems

In transportation problems, whenever the number of non-negative independent allocations in less than m + n - 1, the transportation problems is said to be **degenerate** one. Degeneracy may occur either at the initial stage or at an intermediate stage at some subsequent iteration.

To resolve degeneracy, we allocate an extremely small amount (close to zero)to one or more empty cells of the transportation table(generally minimum cost cells if possible), so that the total number of occupied cells becomes (m + n - 1) at independent positions. WE denote this small amount by \in (epsilon) satisfying the following conditions:

- (i) $0 < \epsilon < x_{ij}$, for all $x_{ij} > 0$
- (ii) $X_{ij} \pm \epsilon = x_{ij}$, for all $x_{ij} > 0$

The cells containing \in are then treated like other occupied cells and the problems is solved in the usual way. The \in 's are kept till the optimum solution is attained. Then we let each $\in \rightarrow 0$.

Example 1: find the non-degenerate basic feasible solution for the following transportation problems using

- (i) North west corner rule
- (ii) Least cost method
- (iii) Vogel's approximation method.



	13	9	12	8	20
From	4	5	7	9	30
	14	7	1	0	40
	3	12	5	19	50
Demand	60	60	20	10	

Solution: Since $\sum a_i = \sum b_i = 150$, the given transportation problems is balanced.

 \therefore There exists a basic feasible solution to this problem.

(i) The starting solution by NWC rule is an shown in the following table.

	10		20		5	7		
		10						
	13		9		12	8		
		20						
	4		5		7	9		
		30						
Since the	14		7		1	0		number of non-negative
allocations at				40				independent positions is 7 $(m + n - 1) = (5 + 4 - 1) = -$
8, this basic feasible	3		12		5	19		(m + n - 1) = (3 + 4 - 1) = solution is a degenerate one.
To resolve				20	20	1	10	this degeneracy, we allocate

a very small quantity \in to the unoccupied cell (5,1) so that the number of occupied cells becomes (m+n-1) (m + n - 1). Hence the non-degenerate basic feasible solution is an shown in the following table.

10	20	5	7
10			

	_		20		20		10
5	E	12		5		19	
3		12		5		10	
			40				
14		7		1		0	
	30						
4		5		7		9	
	20						
13		9		12		8	

The initial transportation cost = Rs. $10x10+13x20+4x30+7x40+3x \in +20x20+5x20+19x10$

$$= \operatorname{Rs.}(1290 = 3\varepsilon)$$
$$= \operatorname{Rs.} 1290/- \operatorname{as} \varepsilon \to 0.$$

(ii) Least cost method: Using this method the starting solution is an shown in the following table:

10	20		5	7
		10		
13	9		12	8
		20		
4	5		7	9
10		20		
10	7	20	1	0
10 14	7	20 10	1 20	0 10
10 14 3	7	20 10	1 20 5	0 10 19

UNIT – II

Since the number of non-negative allocations at independent positions is (m + n - 1) = 8, the solution is non-degenerate basic feasible.

The initial transportation cost = Rs.20x10+9x20+4x10+5x20+7x10+1x20+0x10+3x50

$$=$$
 Rs. 760/-

(iii) Vogel's approximation method: The starting solution by this method is an shown in the following table:

10	20		5		7	
10						
13	9		12		8	
		20				
4	5		7		9	
		30				
14	7		1		0	
		10		20		10
3	12		5		19	
50						

Since the number of non-negative allocations is 7 which is less than (m + n - 1) = (5+4-1)=8, this basic solution is a degenerate one.

To resolve this degeneracy, we allocate a very small quantity \in to the unoccupied cell(5,2) so that the number of occupied cells becomes (m + n - 1). Hence the non-degenerate basic feasible solution is an shown in the following table.

10		20	5	7
	10			
13		9	12	8
		20		

4	5		7		9	
		30				
14	7		1		0	
		10		20		10
3	12	10	5	20	19	10

 \therefore The initial transportation cost

= Rs. 10 x 10 + 9 x 20 + 5 x 30 + 7 x 10 + 1 x 20 + 0 x 10 + 3 x 50 + 12 x ∈

= Rs. (670 +12 \in)

= Rs.670/- = as $\in \rightarrow 0$.

Example 2: Solve the following transportation problems using vogel's method.

	Α	В	С	D	Ε	F	Available
1	9	12	9	6	9	10	5
Factory 2	7	3	7	7	5	5	6
3	6	5	9	11	3	11	2
4	6	8	11	2	2	10	9
Requirement	4	4	6	2	4	2	

Solution: Since $\sum a_i = \sum b_j = 22$, the given transportation problem is balanced. \therefore There exists a basic feasible solution to this problem. By Vogel's approximation method, the initial solution is as shown in the following table:

9	12	9	6	9	10
		5			
7	3	7	7	5	5
	4				2
6	5	9	11	3	11
1	E	1			

6	8	11	2	2	10
3			2	4	

Since the number of non-negative allocations is 8 which is less than (m + n - 1)=(4+6-1)=9, this basic solution is degenerate one.

To resolve degeneracy, we allocate a very small quantity \in to the cell (3,2), so that the number of occupied cells becomes (m + n - 1). Hence the non-degenerate basic feasible solution is as shown in the following table.

9	12	9	6	9	10
		5			
7	3 4	7	7	5	5 2
6	5	9	11	3	11
1	E	1			
6	8	11	2	2	10
3			2	4	

The initial transportation cost = Rs.9 x 5 + 3 x 4 + 5 x 2 + 6 x 1 + 5 x \in + 9 x 1

+ 6 x 3 + 2 x 2 + 2 x 4

$$=$$
Rs.(112+5 \in) $=$ Rs.112/-, $\in \rightarrow 0$.

To find the optimal solution

Now the number of non-negative allocations at independent positions is (m + n - 1). We apply the MODI method.

	9 6	12	5	9		6	2	9	2	10	7	
	2		_		5				_		2	$u_1 = 0$
	3		1				4		7		3	
ĺ	7 4	3		7	7	7	0	5	0	5		
			4								2	$u_2 = -2$
	3				0		7		5			

6	5	9	11 2	3 2	11 7	
1	E	1				$u_3 = 0$
			9	1	4	
6	8 5	11 9	2	2	10 7	
3			2	4		$u_4 = 0$
	3	2			3	
$v_1 = 6$	$v_2 = 5$	$v_3 = 9$	$v_4 = 2$	$v_5 = 2$	$v_6 = 7$	

Since all $d_{ij}>0$ with $d_{23}=0$, the solution under the test is optimal and an alternative optimal solution is also exists.

: The optimum allocation schedule is given by $x_{14}=5$, $x_{22}=4, x_{26}=2, x_{31}=1, x_{32}=\epsilon, x_{33}=1, x_{41}=3, x_{44}=2, x_{45}=4$ and the optimum(minimum) transportation cost is

 $= Rs.9x5+3x4+5x2+6x1+5x \in +9x1+6x3+2x2+2x4$ $= Rs. (112+5 \in)$ $= Rs. 112 \text{ as } \in \to 0.$

Example 3: Solve the following transportation problem to minimize the total cost of transportation.



Solution: Since $\sum a_i = \sum b_j = 24$, the given transportation problem is balanced. \therefore There exists a basic feasible solution to this problem.

By using Vogel's approximation method, the initial solution is as shown in the following table:

1	2	3	4
	6		
4	3	2	0
		2	6
0	2	2	1



Since the number of non-negative allocations is 5, which is less than (m + n - 1)=(3+4-1)=6, this basic feasible solution is degenerate.

To resolve degeneracy, we allocate a very small quantity \in to the cell (1,4), so that the number of occupied cells becomes (m + n - 1). Hence the non-degenerate basic feasible solution is given in the following table

	1	2		3		4	
			6				
	4	3		2		0	
					2		6
_	0	2		2	2	1	6

$$\therefore \text{ The initial transportation cost} = \text{Rs. } 2 \times 6 + 2 \times 2 + 0 \times 6 + 0 \times 4 + 2 \times 6 + 2 \times 6$$

$$= \text{Rs.} (28 + 2\varepsilon)$$
$$= \text{Rs.} 28/-, \text{ as } \varepsilon \longrightarrow 0.$$

To find the optimum solution:

Now the number of non-negative allocations at independent positions is (m + n - 1). We apply MODI method.

1	0	2		3	2	4	0	
			6					$u_1 = 0$
	1				1		4	
4	0	3	2	2		0		
					2		6	$u_2 = 0$
	4		1					

Since all $d_{ij} > 0$ the solution under the test is optimal and unique.

: The optimal allocation schedule is given by $x_{12} = 6$, $x_{23} = 2$, $x_{24} = 6$, $x_{31} = 4$, $x_{32} = 6$, $x_{33} = 6$ and the optimum (minimum) transportation cost

$$= \text{Rs. } 2 \text{ x } 6 + 2 \text{ x } 2 + 0 \text{ x } 6 + 0 \text{ x } 4 + 2 \text{ x } \in + 2 \text{ x } 6$$

$$=$$
 Rs. $(28 + 2\epsilon) =$ Rs. 28, as $\epsilon \longrightarrow 0$.

Example 5:

Solve the following transportation problem to minimize the total cost of transportation.

		Dest	tination		
	1	2	3	4	supply
1	14	56	48	27	70
Origin 2	82	35	21	81	47
3	99	31	71	63	93
Demand	70	35	45	60	210

Solution:

Since $\sum a_i = \sum b_j = 210$, the given transportation problem is balanced. \therefore There exists a basic feasible solution to this problem.

By using Vogel's approximation method, the initial solution is as shown in the following table:

14	-0	56	48	27
	70			
82		35	21	81
			45	2

99	31		71	63	
		35			58

Since the number of non-negative allocations is 5, which is less than (m + n - 1) = (3+4-1) = 6, this basic feasible solution is degenerate.

To resolve degeneracy, we allocate a very small quantity \in to the cell (1,4). So that the number of occupied cells becomes (m + n - 1). Hence the non-degenerate basic feasible solution is given in the following table.

14	56		48		27	
7	0				(Ξ
82	35		21		81	
				45		2
99	31		71		63	
		35				58

To find the optimum solution:

Now the number of non-negative allocations at independent positions is (m + n - 1) = 6. We apply MODI method.

14		56	-5	48	-33	27]
	70		61		81		E	$u_1 = 27$
02	60	25	40	21		01		_
82	08	33	49	21	45	01	2	$u_2 = 81$
	14		-14					_
99	50	31		71	3	63		
	49		35		68		58	$u_3 = 63$
$v_1 =$	-13	$v_2 =$	-32	V3 =	= 60	V4	= 0	

Since d_{22} =-14<0, the solution under the test is not optimal.

Now let us from a new basic feasible solution by giving maximum allocation to the cell (2,2) by making an occupied cell empty. We draw a closed loop consisting of horizontal and

vertical lines beginning and ending at this cell (2,2) and having its other corners at some occupied cells. Along this closed loop, indicate $+\theta$ and $-\theta$ alternatively at the corners.



From the two cells (2,4),(3,2) having $-\theta$ we find that the minimum of the allocations 2,35 is 2. Add this 2 to the cells with $+\theta$ and subtract this 2 to the cells with $+\theta$. Hence the new basic feasible solution is given by

14		56		48		27	
	70					e	Ξ
82		35		21		81	
			2		45		
99		31		71		63	
			33				60

We see that the above table satisfies the rim conditions with (m + n - 1) non-negative allocations at independent position. We apply MODI method for optimality.

14		56	-5	48	-19	27		
	70						E	$u_1 = -40$
			61		81			
82	54	35		21		81	67	
02	51	55	2	21	45	01	07	$u_2 = 0$
	28		-				14	
	28				-13		14	u ₂ – (

99	50 49	31	33	71	17 54	63	60	u ₃ = -4
$v_1 =$	54	$v_2 = 3$	5	V3	= 21	$v_4 =$	67	_

Since d_{ij} >0, the solution under the test is optimal.

: The optimal allocation schedule is given by $x_{11} = 70$, $x_{14} = \epsilon$, $x_{22} = 2$, $x_{23} = 45$, $x_{32} = 33$, $x_{34} = 60$ and the optimum (minimum) transportation cost

=Rs. 14 x 70 + 27 x ∈ + 35 x 2 + 21 x 45 + 31 x 33 + 63 x 60

=Rs.6798/- as $\in \rightarrow 0$.

Unbalanced Transportation Problems

If the given transportation problems is unbalanced one, i.e., if $\sum a_i \neq \sum b_j$, then convert this into a balanced one by introducing a dummy source or dummy destination with zero cost vector (zero unit transportation costs) as the case may be and then solve by usual method.

When the total supply if greater than the total demand, a dummy destination is included in the matrix with zero cost vectors. The excess supply is entered as a rim requirement for the dummy destination.

When the total demand is greater than the total supply, a dummy source is included in the matrix with zero cost vectors. The excess demand is entered as rim requirements for the dummy source.

Destination

Example 1: Solve the transportation problem

	А	В	С	D	supply
1	11	20	7	8	50
Source 2	21	16	20	12	40
3	8	12	18	9	70
Demand	30	25	35	40	_1

Solution: Since the total supply ($\sum a_i = 160$) is greater than the total demand ($\sum b_j = 130$), the given problem is an unbalanced transportation problem. To convert this i9nto a balanced one, we introduce a dummy destination E with zero unit transportation costs and having demand equal to 160-130=30 units.

 \therefore The given problem becomes

Destination										
	А	В	С	D	E	supply				
1	11	20	7	8	0	50				
Source 2	21	16	20	12	0	40				
3	8	12	18	9	0	70				
Demand	30	25	35	40	30	160				

By using VAM the initial solution is as shown in the following table

11	20	7	8	0
		35	15	
21	16	20	12 10	0 30
8	12	18	9	0
30	25		15	

 \therefore The initial transportation cost

=Rs. 7 x 35 + 8 x 15 + 12 x 10 + 0 x 30 + 8 x 30 + 12 x 25 + 9 x 15

=Rs. 1160/-

For Optimality: Since the number non-negative allocations at independent position is (m + n - 1), we apply the MODI method.

11	7	20	11	7		8	0	-4	
					35	15			$u_1 = 8$
	4		9					4	
21	11	16	15	20	11	12	0		
				_ •		10		30	$u_2 = 12$
	10		1		9				

8 30	12 25	18 8	9 15	0 -3	$u_3 = 9$
		10		3	
$v_1 = -1$	$v_2 = 3$	$v_3 = -1$	$v_4 = 0$	$v_5 = -12$	-

Since all d_{ij} >0, the solution under the test is optimum and unique.

: The optimum allocation schedule is $x_{13} = 35$, $x_{14} = 15$, $x_{24} = 10$, $x_{25} = 30$, $x_{31} = 30$, $x_{32} = 25$, $x_{34} = 15$

It can be noted that $x_{25}=30$ means that 30 units are dispatched from source 2 to the dummy destination E. In other words, 30 units are left undespatched from source 2.

The optimum (minimum) transportation cost

= Rs. 7 x 35 + 8 x 15 + 12 x 10 + 0 x 30 + 8 x 30 + 12 x 25 + 9 x 15

=Rs. 1160/-

Example 2: Solve the transportation problem with unit transportation costs, demands and supplies as given below:

Destination

	D_1	D_2	D_3	D_4	Supply
\mathbf{S}_1	6	1	9	3	70
Source S ₂	11	5	2	8	55
S_3	10	12	4	7	70
Demand	85	35	50	45	_

Solution: Since the total demand ($\sum b_j = 215$) is greater than the total supply ($\sum a_i = 195$), the given problem is unbalanced transportation problem. To convert this into a balanced one, we introduce a dummy source S_4 with zero unit transportation costs and having supply equal to 215-195=20 units. \therefore The given problems becomes

Destination





As this problem is balanced, there exists a basic feasible solution to this problem. By using Vogel's approximation method, the initial solution is as shown in the following table.

6		1		9		3	
	65		5				
11		5		2		8	
			30		25		
10		12		4		7	
					25		45
0		0		0		0	
U	20	0		0		U	
	20						

 \therefore The initial transportation cost

= Rs. 6x65+1x5+5x30+2x25+4x25+7x45+0x20

=Rs.1010/-

For optimality: Since number of non-negative allocations at independent positions is (m + n - 1), we apply the MODI method.

6		1		9	-2	3	1	
	65		5					$u_1 = 6$
					11		2	
11	10	5		2		8	5	
			30		25			$u_2 = 10$
	1						3	
10	12	12	7	4		7		
------------------	-----	-------------------------	------	----	--------	-------	------	------------
					25		45	$u_3 = 12$
	-2		5					
0		0	-5	0	-8	0	-5	
	20							$u_4 = 0$
			5		8		5	
v ₁ =	= 0	v ₂ =	= -5	V3	; = -8	v_4	= -5	

Since d_{31} =-2<0, the solution under the test is not optimal.

Now let us form a new basic feasible solution by giving maximum empty. For this, we draw a closed path consisting of horizontal and vertical lines beginning and ending at this cell (3,1) and having its other corners at some occupied cells. Along this closed loop, indicate $+\theta$ and $-\theta$ alternatively at the corners.

We have,



From the three cells (1,1), (2,2), (3,3) having $-\theta$, we find that the minimum of the allocations 65,30,25 is 25. Add this 25 to the cells with $+\theta$ and subtract this 25 to this cells with $-\theta$. Finally, the new feasible solution is displayed in the following table.

6	40	1	30	9		3	
11		5	5	2	50	8	
10	25	12		4		7	45
0	20	0		0		0	

We see that the above table satisfies the rim conditions with (m + n - 1) non-negative allocations at independent positions. Now we check for optimality.

6		1		9	-2	3	3
	40		30				
					11		0
11	10	5		2		8	7
			5		50		
	1						1
10		12	5	4	2	7	
	25						45
			7		2		
0		0	-5	0	-8	0	-3
	20						
			5		8		3

Since all $d_{ij}>0$ with $d_{14}=0$, the solution under the test is optimal and an alternative optimal solution exists.

: The optimum allocation schedule is given by $x_{13} = 35$, $x_{14} = 15$, $x_{24} = 10$, $x_{25} = 30$, $x_{31} = 30$, $x_{32} = 25$, $x_{34} = 15$, $x_{41} = 20$.

It can be noted that $x_{41}=20$ means that 20 units are dispatched from the dummy source S_4 to the destination D_1 . In other words, 20 units are not fulfilled for the destination D_1 .

The optimum (minimum) transportation cost

=Rs.960/-

Example 3:

Solve the transportation problem with unit transportation costs in rupees, demand and supplies as given below:



Solution: Since the total supply ($\sum a_i = 270$), the given transportation problem is unbalanced.

To convert this into a balanced one, we introduce a dummy source D_4 with zero unit transportation costs and having demand equal to 300-270=30 units. \therefore The given problem becomes

Destination

	D_1	D_2	D_3	D_4	Supply(units)
А	5	6	9	0	100
Origin B	3	5	10	0	75
С	6	7	6	0	50
D	6	4	10	0	75
Demand	70	80	120	30	300
(units)					

By using VAM the initial solution is given by

5	6	9 100	0
3 70	5 5	10	0
6	7	6 20	0 30
6	4 75	10	0

Since the number of non-negative allocations is 6, which is less than (m + n - 1) = 4+4-1 = 7, this basic feasible solution is degenerate.

To resolve this degeneracy, we allocate a very small quantity \in to the cell (2,4), so that the number of occupied cells becomes (m + n - 1). Hence the non-degenerate basic feasible solution is given in the following table.

5	6	9 100	0
3 70	5 5	10	0 ∈
6	7	6 20	0 30
6	4	10	0

Now the number of non-negative allocations at independent positions is (m + n - 1). We apply MODI method.

5	6	6	8	9		0	3	;	
				10)0				$u_1 = 3$
	-1		-2				-3	3	
3		5		10	6	0			
	70		5				E		$u_2 = 0$
					4				
6	3	7	5	6		0			
				2	0		30		$u_3 = 0$
	3		2						
6	2	4		10	5	0	-1	L	
			75						$u_4 = -1$
	4				5		1	l	
v	$r_1 = 3$	V	2 = 5	V 3	= 6	V	$v_4 = 0$		

Since there are some $d_{ij} < 0$, the current solution is not optimal.

Since d_{14} = -3 is the most negative, let is form a new basic feasible solution is giving maximum allocations to the corresponding cell(1,4) by making an occupied cell empty. We draw a closed loop consisting of horizontal and vertical lines beginning and ending at this cell (1,4) and having its other corners at some occupied cells. Along this closed loop indicate + θ and

 $-\theta$ Alternatively at the corners.

5	6	9 100		0	+ heta
		-θ			1
3	5	10		0	
70	5			E	
6	7	6		0	
		20	Ļ	30	
		$+\theta$			- θ
6	4	10		0	
	75				

From the two cells (1, 3), (3, 4) having $-\theta$, we find that the minimum of the allocations 100, 30 is 30. Add this 30 the cells with $+\theta$ and subtract this 30 to the cells with $-\theta$. Hence the new basic feasible solution is given in the following table.

Γ	5	6	9	0
			70	30
	3	5	10	0
	70	5		E
	6	7	6	0
			50	
ſ	6	4	10	0
		75		

We see that the above table satisfies the rim conditions with (m + n - 1) non-negative allocations at independent positions. We apply MODI method.

5	3	6	5	9		0		
				7	0		30	$u_1 \ = 0$
	2		1					
3		5		10	9	0		
	70		5				E	$u_2 = 0$
					1			
6	0	7	2	6		0		
				5	0			$u_3 = -3$
	6		5					
6	2	4		10	8	0	-1	
			75					$u_4 = -1$
	4				2		1	
v	$r_1 = 3$	\mathbf{v}_2	2 = 5	v ₃	= 9	V	$v_4 = 0$	-

Since all d_{ij} >0, the current solution is optimal and unique.

The optimum allocation schedule is given by $x_{13} = 70$, $x_{14} = 30$, $x_{21} = 70$, $x_{22} = 5$, $x_{24} = \epsilon$, $x_{33} = 50$, $x_{42} = 75$ and the optimum (minimum) transportation cost

=Rs. 1465/-

Maximization case in Transportation Problems

So far we have discussed the transportation problems in which the objectives has been to minimize the total transportation cost and algorithms have been designed accordingly.

If we have a transportation problems where the objective is to maximize the total profit, first we have to convert the maximization problem into a minimization problem by multiplying all the entries by -1 (or) by subtracting all the entries from the highest entry in the given transportation table. The modified minimization problem can be solved in the usual manner.

<u>PART – B (5 x 6 = 30)</u>

Possible Questions (6 Marks)

1. Determine basic feasible solution to the following transportation problem using i) North west corner rule ii) Matrix minima method

		Ware house					
Factory	W1	W2	W3	W4	Capacity		
F1	19	30	50	10	7		
F2	70	30	40	60	9		
F3	40	8	70	20	18		
Requirement	5	8	7	14	_		

2. Find the optimal solution to the following transportation problem.

	1	2	3	4	Supply
Ι	21	16	25	13	11
ΙΙ	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	-

3. Find the starting solution of the following transportation problem usingi) North west corner rule ii) Lowest cost entry method

	D	Е	F	G	Supply
А	1	2	1	4	30
В	3	3	2	1	50
С	4	2	5	9	20
Demand	20	40	30	10	

4. Solve the transportation problem.

		-	Го		Supply
Ī	1	2	3	4	6
From	4	3	2	0	8
	0	2	2	1	10
Demand	4	6	8	6	

5. Determine an initial basic feasible solution to the following transportation problem using Vogel's approximation method.

		Ι	II	III	IV	Supply
	А	13	11	15	20	2000
From	В	17	14	12	13	6000
	С	18	18	15	12	7000
	Demand	3000	3000	4000	5000	_

6. Find the initial basic feasible solution by using NWCR and Least cost entry method

$W \rightarrow$					
F	W	W.	W.	W	Factory
\downarrow	•• 1	vv ₂	•• 3	•• 4	Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F ₃	40	8	70	20	18
Warehouse	5	0	7	14	24
Requirement	5	0	7	14	54

7. Solve the transportation problem.

Distribution Centers

		D1	D2	D3	D4	Available
	A	- 11	13	17	14	250
Origin	В	16	18	14	10	300
	C	21	24	13	10	400
Demand		200	225	275	250	

8. Find an optimum solution to the following T.P Destination

	D	E	F	G	Available
А	11	20	7	8	50
Origin B	21	16	20	12	40
С	8	12	18	9	70

Requirement 30 25 35 40

 Find the Non- degenerate basic feasible solution for the following transportation problem using i) North west corner rule

ii) Lowest cost entry method

	A	В	С	D	Supply
Ι	10	20	5	7	10
II	13	9	12	8	20
III	4	5	7	9	30
IV	14	7	1	0	40
V	3	12	5	19	50
Demand	60	60	20	10	

10. Find the starting solution of the following transportation model

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	1

Using (i) North West Corner rule

(ii) Vogel's approximation method.

11. Find an optimum solution to the following T.P in which the cells contain the transportation cost in rupees.

	W_1	\mathbf{W}_2	W_3	\mathbf{W}_4	W_5	Available
А	7	6	4	5	9	40
В	8	5	6	7	8	30
С	6	8	9	6	5	20
D	5	7	7	8	6	10
Requiremen	t 30	30	15	20	5	-

PART – C $(1 \times 10 = 10)$

Possible Questions (10 Marks)

1. Determine an initial basic feasible solution to the following transportation problem using Vogel's approximation method.

			Ι	II		III	IV	Supply
	А		13	11		15	20	2000
From	В		17	14		12	13	6000
	С		18	18		15	12	7000
	Deman	d í	3000	3000	4	000	5000	
2. Solve the tran	nsportat	ion probl	em.					
		Distri	bution Ce	enters				
		D1	D2	D3	D4	Availa	ble	
	А	11	13	17	14	250		
Origin	В	16	18	14	10	300		
	С	21	24	13	10	400		
Demand		200	225	275	250			

3. Mention the procedure for finding IBFS.

4. Find the initial basic feasible solution by using North-West Corner Rule and Least cost entry method

$W \rightarrow$					
F	W/	XX 7	XX 7	XX 7	Factory
\downarrow	vv ₁	vv ₂	vv 3	vv 4	Capacity
F ₁	19	30	50	10	7
F ₂	70	30	40	60	9
F ₃	40	8	70	20	18
Warehouse	5	0	7	14	24
Requirement	3	0	/	14	34



KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore –641 021. Department of Mathematics

Subject : Operation Research	Subject Code : 17CCP103	LTPC
Class : I – M.Com(CA)	Semester : I	4 0 0 4

UNIT – III

The Assignment problem - Mathematical Formulation of the Problem – Hungarian Method –Unbalanced Assignment Problem- Maximization Case in Assignment Problem - Travelling Salesman Problem. Queuing Theory : Introduction – Characteristics of Queuing System. Problems in $(M/M/1):(\infty/FIFO)$ and (M/M/1):(N/FIFO) models

TEXT BOOK

1. KanthiSwarup, **Gupta P.K., Man Mohan.,**(2016) Operations Research,Sultan Chandand Sons, New Delhi.

REFERENCES

1. Sharma J.K., (2008), Operations Research Theory Applications, Macmillan India Ltd, New Delhi.

2. Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K.,(2014), Resource Management Techniques, A. R. Publications, Nagapatinam.

3. Shanthi Sophia Bharathi D.,(1999),Operations Research/ Resource management techniques, Charulatha Publications.

4. HamdyA.Taha., Operations Research, (2007), Pearson education, Prentice Hall.

5. Vittal– Operations Research – Margham Publications

UNIT – III

Assignment Problem

Introduction

The assignment problem is a particular case of the transportation problem in which the objective is to assign a number of tasks (Jobs or origins or sources) to an equal number of facilities (machines or persons or destinations) at a minimum cost (or maximum profit).

Suppose that we have 'n' jobs to be performed on 'm' machines (one Job to one machine) and our objective is to assign the jobs to the machines at the minimum cost (or maximum profit) under the assumption that each machine can perform each job but with varying degree of efficiencies.

The assignment problem can be stated in the form of $m \ge n$ matrix (c_{ij}) called a cost matrix (or) Effectiveness matrix where c_{ij} is the cost of assigning ith machine to the jth job.

	1	2	3	•••••	Ν
1	<i>c</i> ₁₁	<i>c</i> ₁₂	<i>c</i> ₁₃		C_{ln}
2	c_{21}	c_{22}	<i>C</i> ₂₃	•••••	C_{2n}
Machines 3	<i>C</i> ₃₁	<i>C</i> ₃₂	<i>C</i> ₃₃	•••••	C_{3n}
	•••••	•••••	•••••		•••••
•	•••••	•••••	•••••	•••••	•••••
•					
m	C_{m1}	C_{m2}	C_{m3}		C_{mn}

Mathematical formulation of an assignment problem.

Consider an assignment problem of assigning *n* jobs to *n* machines (one job to one machine). Let c_{ij} be the unit cost of assigning *i*th nachine to the *j*th job and

Let $x_{ij} = \begin{cases} 1, \text{ if } j^{\text{th}} \text{ job is assigned to } i^{\text{th}} \text{ machine} \\ 0, \text{ if } j^{\text{th}} \text{ job is not assigned to } i^{\text{th}} \text{ machine} \end{cases}$

The assignment model is then given by the following LPP

Minimize
$$Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints

 $\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \dots, n$ $\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \dots, n$ and $x_{ij} = 0$ (or) 1.

Difference between the transportation problem and the assignment problem.

	Transportation problem	Assignment problem
(a)	Supply at any source may be any positive quantity a _i	Supply at any source (machine) will be 1 i.e., $a_i = 1$.
(b)	Demand at any destination may be any positive b _j	Demand at any destination (job) will be 1 i.e., $b_j = 1$.
(c)	One or more source to any Number of destinations	One source (machine) to only one destination (job).

Assignment Algorithm (or) Hungarian Method.

First check whether the number of rows is equal to the number of columns. If it is so, the assignment problem is said to be **balanced**. Then proceed to step 1. If it is not balanced, then it should be balanced before applying the algorithm.

Step 1: Subtract the smallest cost element of each row from all the elements in the row of the row of the given cost matrix. See that each row contains atleast one zero.

Step 2: Subtract the smallest cost element of each column from all the elements in the column of the resulting cost matrix obtained by step 1.

Step 3: (Assigning the zeros)

- (a) Examine the rows successively until a row with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross all other zeros in the column of this enriched zero, as these will not be considered for any future assignment. Continues in this way until all the rows have been examined.
- (b) Examine the columns successively until a column with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it and cross any other zero in its row. Continue until all the columns have been examined.

Step 4: (Apply optimal Test)

UNIT – III Assignment Problem and Queuing System

- (a) If each row and each column contain exactly one encircled zero, then the current assignment is optimal.
- (b) It atleast one row/column is without an assignment (i.e., if there is atleast one row/column is without one encircled zero), then the current assignment is not optimal. Go to step 5.

Step 5: Cover all the zeros by drawing a minimum number of straight lines as follows.

- (a) Mark (\mathcal{V}) the rows that do not have assignment.
- (b) Mark (r) the columns (not already marked) that have zeros in marked columns.
- (c) Mark (\mathcal{V}) the rows (not already marked) that have assignments in marked columns.
- (d) Repeat (b) and (c) until no more marking is required.
- (e) Draw lines through all unmarked rows and columns. If the number of these lines is equal to the order of the matrix then it is an optimum solution otherwise not.
- **Step 6:** Determine the smallest cost element not covered by the straight lines. Subtract this smallest cost element from all the uncovered elements and add this to all those elements which are lying in the intersection of these straight lines and do not change the remaining elements which lie on the straight lines.
- Step 7: Repeat steps (1) to (6). Until an optimum assignment is attained.
 - **Note 1:** In case some rows or columns contain more than one zero, encircle any unmarked zero, encircle any unmarked zero arbitrarily and cross all other zeros in its column or row. Proceed in this way until no zero is left unmarked or encircled.
- Note 2: The above assignment algorithm is only for minimization problems.
- Note 3: If the given assignment problem is of maximization type, convert it to a minimization assignment problem by max Z = -min(-Z) and multiply all the given cost elements by -1 in the cost matrix and then solve by assignment algorithm.
- **Note 4:** Sometimes a final cost matrix contains more than required number of zeros at independent positions. This implies that there is more than one optimal solution (multiple optimal solutions) with the same optimum assignment cost.

Example 1:

Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows:

Job

	1	2	3	4	5
Α	8	4	2	6	1
В	0	9	5	5	4
From C	3	8	9	2	6
D	4	3	1	0	3
Ε	9	5	8	9	5
)

Determine the optimum assignment schedule.

Solution: The cost matrix of the given assignment problem is

/8	4	2	6	1\
0	9	5	5	4
3	8	9	2	6
4	3	1	0	3
\9	5	8	9	5/

Since the number of rows is equal to the number of columns in the cost matrix, the given assignment problem is balanced.

Step 1: Select the smallest cost element in each row and subtract this from all the elements of the corresponding row, we get the reduced matrix

$$\begin{pmatrix} 7 & 3 & 1 & 5 & 0 \\ 0 & 9 & 5 & 5 & 4 \\ 1 & 6 & 7 & 0 & 4 \\ 4 & 3 & 1 & 0 & 3 \\ 4 & 0 & 3 & 4 & 0 \end{pmatrix}$$

Step 2: select the smallest cost element in each column and subtract this from all the elements of the corresponding column, we get the reduced matrix.

/7	3	0	5	0\
0	9	4	5	4
1	6	6	0	4
4	3	0	0	3
$\setminus 4$	0	2	4	0/

Since each row and each column at least one zero, we shall make assignments in the reduced matrix.

Step 3: Examine the rows successively until a row with exactly one unmarked zero is found. Since the 2nd row contains a single zero, encircle this zero and cross all other zeros of its column. The 3rd row contains exactly one unmarked zero, so encircle this zero and cross all other zeros in its column. The 4th row contains exactly one unmarked zero, so encircle this zero and cross all other zeros in its column. The 1st row contains exactly one unmarked zero, so encircle this zero, so encircle this zero and cross all other zeros in its column. The 1st row contains exactly one unmarked zero, so encircle this zero, so encircle this zero and cross all other zeros in its column. The 1st row contains exactly one unmarked zero, so encircle this zero and cross all other zeros in its column. Finally the last row contains exactly one unmarked zero, so encircle this zero and cross all other zeros in its column. Likewise examine the columns successively. The assignments in rows and columns in the reduced matrix is given by

/ 7	3	0	5	(0)
(0)	9	4	5	4
1	6	6	(0)	4
4	3	(0)	0	3
$\setminus 4$	(0)	2	4	0 /

Step 4: Since each row and each column contains exactly one assignment (i.e., exactly one encircled zero) the current assignment is optimal.

: The optimal assignment schedule is given by A \rightarrow 5, B \rightarrow 1, C \rightarrow 4, D \rightarrow 3,

E → 2.

The optimum (minimum) assignment cost = (1 + 0 + 2 + 1 + 5) cost units = 9 units of cost.

Example 2:

The processing time in hours for the when allocated to the different machines are indicated below. Assign the machines for the jobs so that the total processing time is minimum.

	Machines				
	M_1	M_2	M_3	M_4	M_5
\mathbf{J}_1	9	22	58	11	19

J_2	43	78	72	50	63
Jobs J ₃	41	28	91	37	45
\mathbf{J}_4	74	42	27	49	39
J_5	36	11	57	22	25

Solution:

The cost matrix of the given problem is

9	22	58	11	19
43	78	72	50	63
41	28	91	37	45
74	42	27	49	39
36	11	57	22	25

Since the number of rows is equal to the number of columns in the cost matrix, the given assignment problem is balanced.

Step 1: select the smallest cost element in each row and subtract this from all the elements of the corresponding row, we get the reduced matrix.

1	´ 0	13	49	2	10 \	
1	0	35	29	7	20	
	13	0	63	9	17	
	47	15	0	22	12	
	25	0	46	11	14/	

Step 2: Select the smallest cost element in each column and subtract this from all the elements of the corresponding column, we get the following reduced matrix.

$$\begin{pmatrix} 0 & 13 & 49 & 0 & 0 \\ 0 & 35 & 29 & 5 & 10 \\ 13 & 0 & 63 & 7 & 7 \\ 47 & 15 & 0 & 20 & 2 \\ 25 & 0 & 46 & 9 & 4 \end{pmatrix}$$

Step 3: Now we shall examine the rows successively. Second row contains a single unmarked zero, encircle this zero and cross all other zeros in its column. Third row contains a single unmarked zero, encircle this zero and cross all other zeros in its column. Fourth row contains a single unmarked zero, encircle this zero and cross all other zero in its column. After this no row is with exactly one unmarked zero. So go for columns.

Examine the columns successively. Fourth column contains exactly one unmarked zero, encircle this zero and cross all other zeros in its row. After examining all the rows and columns.

We get

0	13	49	(0)	0
(0)	35	29	5	10
13	(0)	63	7	7
47	15	(0)	20	2
25	0	46	9	4
\ \				

Step 4: Since the 5th column do not have any assignment, the current assignment is not optimal. **Step 5:** Cover all the zeros by drawing a minimum number of straight lines as follows:

- (a) Mark (f) the rows that do not have assignment. The row 5 is marked.
- (b) Mark (\mathcal{J}) the columns (not already marked) that have zeros in marked rows. Thus column 2 is marked.
- (c) Mark the rows (not already marked) that have assignment in, marked columns. Thus row 3 is marked.
- (d) Repeat (b) and (c) until no more marking is required. In the present case this repetition is not necessary.
- (e) Draw lines through all unmarked rows (rows 1, 2 and 4). And marked columns (column 2). We get

$$\begin{pmatrix} 0 & 13 & 49 & 0 & 0 \\ 0 & 35 & 29 & 5 & 10 \\ 13 & 0 & 63 & 7 & 7 \\ 47 & 15 & 0 & 20 & 2 \\ 25 & 0 & 46 & 9 & (4) \end{pmatrix}$$

Step 6: Here 4 is the smallest element not covered by these straight lines. Subtract this 4 from all the uncovered element and add this 4 to all those elements which are lying in the intersections of these straight lines and do not change the remaining elements which lie on these straight lines. We get the following matrix.

0	17	49	0	0
0	39	29	5	10
9	0	59	3	3
47	19	0	20	2

21 0 42 5 0

Since each row and each column contains at least one zero, we examine the rows and columns successively, i.e., repeat step 3 above, we get

(0	17	49	(0)	0 \
	(0)	39	29	5	10
	9	(0)	59	3	3
	47	19	(0)	20	2
	21	0	42	5	(0)
	$\overline{}$				

In the above matrix, each row and each column contains exactly one assignment (i.e., exactly one encircled zero), therefore the current assignment is optimal.

: The optimum assignment schedule is $J_1 \rightarrow M_4$, $J_2 \rightarrow M_1$, $J_3 \rightarrow M_2$, $j_4 \rightarrow M_3$,

 $J_5 \longrightarrow M_5$ and the optimum (minimum) processing time

= 11+43+28+27+25 hours = 134 hours.

Unbalanced Assignment Models

If the number of rows is not equal to the number columns in the cost matrix of the given assignment problems, then the given assignment problems is said to be unbalanced.

First convert the unbalanced assignment problems in to a balanced one by adding dummy rows or dummy columns with zero cost element in the cost matrix depending upon whether m < n or m > n and then solve by the usual method.

Example 1: A company has four machines to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table.

	Machines				
	1 2 3 4				
Α	18	24	28	32	
В	8	13	17	19	
Jobs C	10	15	19	22	

What are job assignments which will minimize the cost?

Solution:

The cost matrix of the given assignment problems is

/18	24	28	32\
8	13	17	19
\10	15	19	22/

Since the number of rows is less than the number of columns in the cost matrix, the given assignment problems is unbalanced.

To make it a balanced one, add a dummy job D (row) with zero cost elements. The balanced cost matrix is given by

			\sim
(18	24	28	32
8	13	17	19
10	15	19	22
$\bigcup_{i=1}^{n}$	0	0	0)

Now select the smallest cost element in each row (column) and subtract this from all the elements of the corresponding row (columns), we get the reduced matrix

0	6	10	14
0	5	9	11
0	5	9	12
0	0	0	0)
\sim			

In this reduced matrix, we shall make the assignment in rows and columns having single zero. We have

(0)	6	10	14
0	5	9	11
0	5	9	12
0	(0)	0	0)

Since there are some rows and columns without assignment, the current assignment is not optimal.

Cover the all zeros by drawing a minimum number of straight lines. Choose the smallest cost element not covered by these straight lines.



Here 5 is the smallest cost element not covered by these straight lines. Subtract this 5 from all the uncovered element, add this 5 to those elements which lie in the intersections of these straight lines and do not change the remaining element which lie on the straight lines. We get

0	1	5	9
0	0	4	6
0	0	4	7
5	0	0	0)

Since each row and each column contains atleast one zero, we shall make assignment in the rows and columns having single zero. We get

(0)	1	5	9
0	(0)	4	6
0	0	4	7
5	0	(0)	0)

Since there are some rows and columns without assignment, the current assignment is not optimal.Cover all the zeros by drawing a minimum number of straight lines.

0	1	5	9
0	0	4	6
0	0	(4)	7
5	0	0	9

Choose the smallest cost element not covered by these straight line, subtract this from all the uncovered elements, add this to those elements which are in the intersection of the lines and do not change the remaining elements which lie on these straight lines. Thus we get

0	1	1	5
0	0	0	2
0	0	0	3
9	4	0	0

Since each row and each column contains atleast one zero, we shall make the assignment in the rows and columns having single zero. We get

 $\begin{pmatrix}
(0) & 1 & 1 & 5 \\
0 & (0) & 0 & 2 \\
0 & 0 & (0) & 3 \\
9 & 4 & 0 & (0)
\end{pmatrix}$

Since each row and each column contains exactly one assignment (i.e., exactly one encircled zero) the current assignment is optimal.

: The optimum assignment schedule is given by A \rightarrow 1, B \rightarrow 2, C \rightarrow 3, D \rightarrow 4 and the optimum (minimum) assignment cost

=(18+13+19+0) cost unit = 50/- units of cost

Note 1: For this problem, the alternative optimum schedule is $A \rightarrow 1, B \rightarrow 2, C \rightarrow 3, D \rightarrow 4$, with the same optimum assignment cost= Rs. (18+17+15+0) = 50/- units of cost.

Note 2: Here the assignment $D \longrightarrow 4$ means that the dummy Job D is assigned to the 4th Machine. It means that machine 4 is left without any assignment.

Maximization case in Assignment Problems

In an assignment problem, we may have to deal with maximization of an objective function. For example, we may have to assign persons to jobs in such a way that the total profit is maximized. The maximization problems has to be converted into an equivalent minimization problem and then solved by the usual Hungarian Method.

The conversion of the maximization problem into an equivalent minimization problems can be done by any of the following methods:

(i) Since max $Z = -\min(-Z)$, multiply all the cost element c_{ij} of the cost matrix by -1.

(ii) Subtract all the cost elements c_{ij} of the cost matrix from the highest cost element in that cost matrix.

Example:

Solve the assignment problem for maximization given the profit matrix (profit in rupees).

	Machines			
	Р	Q	R	S
А	51	53	54	50
В	47	50	48	50
Jobs C	49	50	60	61
D	63	64	60	60

Solution:

The profit matrix of the given assignment problem is

51	53	54	50
47	50	48	50
49	50	60	61
63	(64)	60	60

Since this is a maximization problem, it can be converted into an equivalent minimization problem by subtracting all the profit elements in the profit from the highest profit element 64 of this profit matrix. Thus the cost matrix of the equivalent minimization problem is

13	11	10	14
17	14	16	14
15	14	4	3
1	0	4	4

Select the smallest cost in each row and subtract this from all the cost elements of the corresponding row. We get

$\left(\right)$			
3	1	0	4
3	0	2	0
12	11	1	0
1	0	4	4)

Select the smallest cost element in each column and subtract this from all the cost elements of the corresponding column. We get

($\overline{2}$	1	0	4
	2	0	2	0
	11	11	1	0
	0	0	4	4

Since each row and each column contains atleast one zero, we shall make the assignment in rows and columns having single zero. We get

(2	1	(0)	4
	2	(0)	2	0
	11	11	1	(0)
	(0)	0	4	4)

Since each row and each column contains exactly one encircled zero, the current assignment is optimal.

:. The optimum assignment schedule is given by A \rightarrow R, B \rightarrow Q, C \rightarrow S, D \rightarrow P and the optimum (maximum) profit = Rs. (54 + 50 + 61 + 63)

= Rs. 228/-

Queuing Theory

Introduction

In everyday life it is seen that a number of people arrive at a cinema ticket window. If the people arrive too frequently they will have to wait for getting tickets or sometimes do without it. Such problems arise in Railways, Airline etc. Under such circumstances the only alternative is to form a Queue called the Waiting Line in order to get the service more effectively. If we have too many counters for service then expenditure may be high. On the other hand if we have only few countries then Queue may become longer resulting in the dissatisfaction or loss of customers. Queuing models are aids to determine the optimal number of counters so as to satisfy the customers keeping the total cost minimum. Here the arriving people are called customers and the person issuing the tickets is called a server.

Servers may be in parallel or in series. When in parallel, the arriving customers may form a single Queue or several Queues as is common in biog post offices. Service time may be constant or variable and customers may be served singly or in batches.

Queuing System

A queuing system can be completely described by

- (a) The input (or arrival pattern).
- (b) The service mechanism (or service pattern).
- (c) The Queue discipline.
- (d) Customer's behavior.

(a)The input (or arrival pattern)

The input describes the way in which the customers arrive and join the system. Generally the customers arrive in more or less random fashion which is not worth making the prediction. Thus the arrival pattern can best be described in term of probabilities and consequently the probability distribution for inter arrival times (the time between two successive arrivals) must be defined. We deal with those Queuing system in which the customers arrive in 'Poisson' fashion. Mean arrival rate is denoted by

(b) The service mechanism (or service pattern)

The service pattern is specified when it is known how many customers can be served at a time, what the statistical distribution of the service time is, and when the service is available. Service time may be constant or a random variable. Distribution of service time which is important in practice is the **negative exponential distribution**. The mean service rate is denoted by μ .

(c) The Queue discipline

The queue discipline is the rule determining the formation of the Queue, the manner of the customer's behavior while waiting, and the manner in which they are chosen for service. The simplest discipline id first come, First Served" according to which the customers are served in the order of their arrival. Such type of Queue discipline is observed at a ration shop. If the order is reversed, we have 'Last come, first served' discipline, as in the case of a big godown the items which come last are taken out first.

Some of the queue service disciplines are: FIFO – First in, First out or (FCFS) LIFO – Last in, First out, (LCFS) SIRO – Service in Random order.

(d)Customer's behavior:

The customer generally behaves in 4 ways:

- (i) Balking: A customer may leave the Queue, if there is no waiting space.
- (ii) Reneging: This occurs when the waiting customer leaves the Queue due to Impatience.
- (iii) Priorities: In certain applications some customers are served before others regardless of their order of arrival.
- (iv) Jockeying: Customers may jump from one, waiting line to another.

Transient and Steady States

A system is said to be in **Transient State** when its operating characteristics are dependent on time.

Steady State: A system is said to be in **Steady State** when the behavior of the system is independent of time. Let $P_n(t)$ denote the probability that there are 'n' units in the system at time t. Then in steady state

$$=> \lim_{t\to\infty} P'_n(t) = 0$$

Kendal'a Notation for representing Queuing models

Generally Queuing Model may be completely specified in the following symbol form :(a|b|c) :(d|e):

Where a=Probability law for the arrival

b=Probability law according to which customers are served.

c=Number of channels (or Service stations).

d=Capacity of the system.

e=Queue discipline.

Distribution of Arrivals "The Poisson Process" Arrival Distribution Theorem. (Pure Birth Process)

If the arrivals are completely random, then the probability distribution of a number of arrivals in a fixed – time interval follows a Poisson distribution.

Model 1: (M|M|I): (∞/FCFS) – Birth and Death Model

With usual notation, show that probability distribution of Queue length is given by ρ_n (1- ρ) where $\rho = \frac{\lambda}{u} < 1$ and $n \ge 0$.

Measure of Model I

1. To find the average (expected) number of units in the system, L. Solution:

By definition of excepted value

$$\begin{split} \mathrm{L}_{\mathrm{s}} &= \sum_{n=1}^{\infty} n P_{n} = \sum_{n=1}^{\infty} n \left(\frac{\lambda}{\mu}\right)^{n} \left(1 - \frac{\lambda}{\mu}\right) \\ &= \left(1 - \frac{\lambda}{\mu}\right) \left(\frac{\lambda}{\mu}\right) \sum_{n=1}^{\infty} n \left(\frac{\lambda}{\mu}\right)^{n-1} \\ &= \left(1 - \frac{\lambda}{\mu}\right) \left(\frac{\lambda}{\mu}\right) \left(1 + 2\frac{\lambda}{\mu} + 3\left(\frac{\lambda}{\mu}\right)^{2} + \dots + \right) \\ &= \left(1 - \frac{\lambda}{\mu}\right) \left(\frac{\lambda}{\mu}\right) \left(1 - \frac{\lambda}{\mu}\right)^{-2} \text{ using Bionomial series} \\ &= \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}} \\ \mathrm{L}_{\mathrm{s}} &= \frac{\rho}{1 - \rho} \text{ where } \rho = \frac{\lambda}{\mu} < 1 \end{split}$$

2. To find the average length of Queue, L_q

$$L_{q} = L_{s} - \frac{\lambda}{\mu}$$
$$= \frac{\rho^{2}}{1 - \rho}$$

3. Excepted waiting time in the system

$$W_s = \frac{L_s}{\lambda}$$

$$=\frac{1}{\mu-\lambda}$$

4. Waiting time in the Queue,

$$W_{q} = \frac{L_{q}}{\lambda}$$
$$= \frac{\lambda}{\mu(\mu - \lambda)}$$

5. Excepted waiting time of a customer who has to wait (W | W > 0)

$$=\frac{1}{\mu-\lambda}$$

- 6. Excepted length of the non empty Queue, (L | L > 0) = $\frac{\mu}{\mu - \lambda}$
- 7. Probability of Queue size $\geq N$ is ρ^N
- 8. Probability [Waiting time in the system $\geq t$] = $\int_{t}^{\infty} (\mu - \lambda) e^{-(\mu - \lambda)w} dw$
- 9. Probability [Waiting time in the queue $\geq t$] = $\int_t^{\infty} \rho(\mu - \lambda) e^{-(\mu - \lambda)w} dw$

10. Traffic Intensity $=\frac{\lambda}{\mu}$

Example 1:

In a railway Marshalling yard, goods train arrive at a rate of 30 Trains per day. Assuming that inter arrival time follows an exponential distribution and the service time distribution is also exponential, with an average of 36 minutes. Calculate the following:

- (i) The mean Queue size (line length)
- (ii) The probability that Queue size exceeds 10
- (iii) If the input of the train increases to an average 33 per dya, what will be the changes in (i), (ii)?

Solution:

$$\lambda = \frac{30}{60 \times 24} = \frac{1}{48}, \quad \mu = \frac{1}{36} \text{ trains per minute}$$

$$\therefore \rho = \frac{\lambda}{\mu} = \frac{36}{48} = 0.75$$

(i) $L_s = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = 3$ trains

(ii)
$$P (\ge 10) = (0.75)^{10} = 0.056$$

We have
$$\lambda = \frac{30}{60 \times 24} = \frac{1}{480}$$
 and $\mu = \frac{1}{36}$ trains per minute
Now, $L_s = \frac{\rho}{1-\rho}$ where $\rho = \frac{\lambda}{\mu}$; $\rho = 0.825$
 $\therefore L_s = \frac{0.825}{1-0.825} = 5$ trains (app)
Also P(≥ 10) = $\rho^{10} = (0.825)^{10}$
= 0.1460

Example 2:

In a super market, the average arrival rate of customer is 10 in every 30 minutes following Poisson process. The average time taken by the cashier to list and calculate the customer's purchases is 2.5 minutes, following exponential distribution. What is the probability that the Queue length exceeds 6? What is the excepted time spent by a customer in the system?

Solution:

Here the mean arrival rate

$$\lambda = \frac{10}{30}$$
 per minute

and mean service rate = $\frac{1}{2.5}$ per minute

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{3}}{\frac{1}{2.5}} = 0.8333$$

(i) (The probability of Queue size
$$>$$
 n) = ρ^n

When
$$n = 6 \implies (0.8333)^6 = 0.3348$$

(ii)
$$W_s = \frac{L_s}{\lambda} = \frac{\frac{\rho}{1-\rho}}{\lambda}, \ \rho = \frac{\lambda}{\mu}$$

= $\frac{0.8333}{1-0.8333} \ge 3 = \frac{2.499}{0.167}$
= 14.96 minutes

Example 3:

In a public telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone, find (i) Excepted number of callers in the booth at any time (ii) The proportion of the time the booth is excepted to be Idle?

Solution:

Mean arrival rate $\lambda = 15$ per hour

Mean service rate $\mu = \frac{1}{3} \ge 60 = 20$ per hr.

 \therefore (i). Excepted length of the non-empty

$$Queue = \frac{\mu}{\mu - \lambda} = \frac{20}{20 - 15} = 4$$

(ii). The service is busy means
$$=\frac{\lambda}{\mu}=\frac{15}{20}=\frac{3}{4}$$

 \therefore The booth excepted to Idle for $1 - \frac{3}{4} = \frac{1}{4}$ hrs

$$= 15$$
 minutes

Example 4:

A T.V repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is Poisson with an average rate of 10 per 8 hour day, what is his excepted Idle time day? How many hobs are ahead of the average set just brought in?

Solution:

Mean service rate
$$\mu = \frac{1}{30}$$
 per minute
= $\frac{1}{30} \ge 60 = 2$ sets per hour
Mean arrival rate = $\frac{10}{8}$ per hr

$$\rho = \frac{\lambda}{\mu}$$
 where $\mu = 2$ per hr.

$$\rho = \frac{5}{4}$$
 per hr

The utilization factor $\frac{\lambda}{\mu}$ is $\frac{5}{4 \ge 2} = \frac{5}{8}$

For 8 hr day, Repairman's busy time = 8 x $\frac{5}{8}$ = 5 hrs

: Idle time of repairman = 8 - 5 hrs = 3 hrs

The number of jobs ahead = No. of units in the system

$$=\frac{\rho}{1-\rho}=\frac{\frac{5}{8}}{1-\frac{5}{8}}=\frac{\frac{5}{8}}{\frac{3}{8}}=\frac{5}{3}$$

= 2 app, TV sets

Example 5:

At a public Telephone booth in a Post Office arrivals are considered to be Poisson with an average inter-arrival time of 12 minutes. The length of the phone call may be assumed to be distributed exponentially with an average of 4 minutes. Calculate the following:

- (a) What is the probability that a fresh arrival will not have to wait for the phone?
- (b) What is the probability than an arrival will have to wait more than 10 minutes before the phone is free?
- (c) What is the average length of Queues formed from time to time?

Solution:

Mean arrival rate, $\lambda = \frac{1}{12}$, $\mu = \frac{1}{4}$

Mean service rate, $\frac{\lambda}{\mu} = \frac{4}{12} = \frac{1}{3} = 0.33$

(a) Probability that a fresh arrival will not have to wait

$$=1-\frac{\lambda}{\mu}=1-0.33$$

= 0.67

(b) Probability that an arrival will have to wait for atleast 10 minutes

$$= \int_{t}^{\infty} \left(\frac{\lambda}{\mu}\right) (\mu - \lambda) e^{-(\mu - \lambda)t} dt$$

= $\int_{t}^{\infty} (0.33) (0.25 - 0.083) e^{-0.167t} dt$
= $0.05511 \left[\frac{e^{-0.167t}}{-0.167}\right]_{10}^{\infty}$

= 0.0621

(c) The average length of Queues from time to time

$$(L > 0) = \frac{\mu}{\mu - \lambda} = \frac{0.25}{0.25 - 0.085} = 1.5$$

Example 6:

People arrive at a Theatre ticket booth in Poisson distributed arrival rate of 25 per hour. Service time is constant at 2 minutes. Calculate

- (a) The mean number in the waiting line
- (b) The mean waiting time
- (c) The utilization factor.

Solution:

(i)

$$\lambda = \text{per hr};$$

$$\mu = \frac{1}{2} \ge 60 = 30 \text{ per hr}.$$

$$\therefore \rho = \frac{\lambda}{\mu} = \frac{25}{30} = \frac{5}{6} = 0.833$$
Length of the Queue
$$L_q = \frac{\rho^2}{1 - \rho}$$

$$= \frac{(0.833)^2}{1-0.833}$$

$$= \frac{0.693889}{0.167} = 4 \text{ (app)}$$
(ii) Mean waiting time
$$= \frac{L_q}{\lambda}$$

$$= \frac{4}{25}$$

$$= 9.6 \text{ minutes}$$
(iii) Utilization factor $\rho = \frac{\lambda}{\mu} = 0.833.$

Model II:

 $(\mathbf{M} \mid \mathbf{M} \mid \mathbf{I}) : (\mathbf{N} \mid \mathbf{FCFS})$

Here the capacity of the system is limited, say N. Infact arrivals will not exceed N in any case. The various measures of this Model are

1. $P_{0} = \frac{1-\rho}{1-\rho^{N+1}} \text{ where } \rho = \frac{\lambda}{\mu}, \left\{ \frac{\lambda}{\mu} > 1 \text{ is allowed} \right\}$ 2. $P_{n} = \frac{1-\rho}{1-\rho^{N+1}} \rho^{n} \text{ for } n = 0, 1, 2, \dots N$ 3. $L_{s} = P_{0} \sum_{n=0}^{N} n \rho^{n}$ 4. $L_{q} = L_{s} - \frac{\lambda}{\mu}$ 5. $W_{s} = \frac{L_{s}}{\lambda}$ 6. $W_{q} = \frac{L_{q}}{\lambda}$ Example 1: If for period of 2 hours in a day (8 - 10 AM) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period

(a) The probability that the yard is empty

(b) Average Queue length, assuming that capacity of the yard is 4 trains only.

Solution:

Here
$$\rho = \frac{36}{20} = 1.8$$
, N = 4
(a) $P_0 = \frac{\rho - 1}{\rho^5 - 1} = 0.04$
(b) Average Queue size
 $= P_0 \sum_{n=0}^{N} n\rho^n$
 $= 0.04 (\rho + 2\rho^2 - 3\rho^3 + 4\rho^4)$
 $= 2.9$
 $\approx 3 \text{ trains}$

Example 2:

In a railway marshalling yard, goods training trains arrive at a rate of 30 trains per day. Assume that the inter arrival – time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, calculate

(a) The probability that the yard is empty

(b) Average queue length assuming that the line capacity of the yard is 9 trains.

Solution:

Here
$$\rho = \frac{\lambda}{\mu} \longrightarrow \rho = 0.75$$

(a) The probability that the queue size is zero is given by

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} \text{ where } N = 9$$

$$P_0 = \frac{1 - 0.75}{1 - (0.75)^{10}} = \frac{0.25}{0.90} = 0.2649$$

(b) Average Queue length is given by the formula,

$$L_{s} = \frac{1-\rho}{1-\rho^{N+1}} \sum_{n=0}^{N} n\rho^{n}$$
$$L_{s} = \frac{1-0.75}{1-(0.75)^{10}} \sum_{n=0}^{9} n(0.75)^{n}$$
$$= 0.28 \ge 9.58 \approx 3 \text{ trains.}$$

Example 3:

A barbershop has space to accommodate only 10 customers. He can service only one person at a time. If a customer comes to his shop and finds it full, he goes to the next shop. Customers randomly, arrive at an average rate $\lambda = 10$ per hours and the barbers service time is negative exponential with an average of $\frac{1}{\mu} = 5$ minutes per customer. Find P₀, P_n.

Solution:

Here N = 10,
$$\lambda = \frac{10}{60}, \mu = \frac{1}{5}$$

 $\rho = \frac{\lambda}{\mu} = \frac{5}{6}$
 $P_0 = \frac{1-\rho}{1-\rho^{11}} = \frac{1-\frac{5}{6}}{1-(\frac{5}{6})^{11}}$
 $= \frac{0.1667}{0.8655} = 0.1926$
 $P_n = (\frac{1-\rho}{1-\rho^{N+1}}) \rho^n$
 $= (0.1926) \times (\frac{5}{6})^n, n = 0, 1, 2, \dots, 10$

Example 4:

A car park contains 5 cars. The arrival of cars is poisson at a mean rate of 10 per hour. The length of time each car spends in the car park is negative exponential distribution with mean of 2 hours. How many cars are in the car park on average?

Solution:

N = 5,
$$\lambda = \frac{10}{60}, \mu = \frac{1}{2 \times 60}, \rho = \frac{\lambda}{\mu} = 20$$

P₀ = $\left(\frac{1-\rho}{1-\rho^{N+1}}\right)$
= $\frac{1-20}{1-20^6} = \frac{-19}{-6399} = 2.962 \times 10^{-7}$

$$\mathbf{L}_{\mathrm{s}} = \frac{1-\rho}{1-\rho^{N+1}} \sum_{n=0}^{N} n\rho^{n}$$

= $(2.9692 \times 10^{-3}) \times \sum_{n=0}^{5} n (2.9692 \times 10^{-3})^n$

$$= (2.9692 \times 10^{-3}) \times [0 + (2.9692 \times 10^{-3}) + 2 \times (2.9692 \times 10^{-3})^{2} + 3 \times (2.9692 \times 10^{-3})^{3} + 4 \times (2.9692 \times 10^{-3})^{4} + 5 \times (2.9692 \times 10^{-3})^{5}]$$

= (2.9384 x 10⁻³) x [0 + (2.9692 x 10⁻³) + 2 x (2.9384 x 10⁻³) + 3 x (2.9692 x 10⁻³)^{2} + 4 x (2.9692 x 10⁻³)^{3} + 5 x (2.9692 x 10⁻³)^{4}]
= 5 (app)

Example 5:

At a one-man barber shop, the customers arrive following poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customers, find the average time a customers, find the system.

Solution:
2017 Batch

$W_{s} = \frac{P_{0}}{\lambda} \sum_{n=0}^{N} n\rho^{n}$ Here $\lambda = 5$ per hr $\mu = \frac{1}{10} \ge 60$
= 6 per hr and N = 5 $\therefore \frac{1}{\mu} = \frac{5}{6} = \rho$
$P_0 = \frac{1-\rho}{1-\rho^6} = \frac{1-\frac{5}{6}}{1-\left(\frac{5}{6}\right)^6}$
$=\frac{1-\frac{1}{6}}{1-\left(\frac{1}{6}\right)^{6}}=\frac{\frac{1}{6}}{1-1.07 \text{ x } 10^{-4}}$
$=\frac{0.1666}{1-0.0001}=\frac{0.1666}{1}$
= 0.1666

$$\frac{L_s}{\lambda} = W_s$$

Where Ls = 0.166 x $\sum_{n=0}^{N} n\rho^n$

$$= 0.166 \text{ x } \sum_{n=0}^{N} n \left(\frac{5}{6}\right)^{n}$$

$$= 0.166 \text{ x } \left[\rho + 2\rho^{2} + 3\rho^{3} + 4\rho^{4} + 5\rho^{5}\right]$$

$$= 0.166 \left[\frac{5}{6} + 2\left(\frac{5}{6}\right)^{2} + 3\left(\frac{5}{6}\right)^{3} + 4\left(\frac{5}{6}\right)^{4} + 5\left(\frac{5}{6}\right)^{5}\right]$$

$$= 0.166 \left[0.833 + (2 \text{ x } 0.694) + (3 \text{ x } 0.5782) + (4 \text{ x } 0.4816) + (5 \text{ x } 0.4012)\right]$$

$$W_{s} = \frac{0.1666}{5} \left[0.833 + 1.388 + 1.7346 + 1.9264 + 2.006\right]$$

$$=\frac{0.1666 \text{ x } 7.88}{5} = \frac{1.3094}{5} = 0.26 \text{ hrs } \approx 16 \text{ minutes}$$

<u>PART – B (5 x 6 = 30)</u>

Possible Questions (6 Marks)

- 1. Consider the problem of assigning five jobs to five persons. The assignment costs are given as follows: Job
- In a railway Marshalling yard, goods train arrive at a rate of 30 Trains per day. Assuming that inter arrival time follows an exponential distribution and the service time distribution is also exponential, with an average of 36 minutes. Calculate the following:

- (i) The mean Queue size (line length)
- (ii) The probability that Queue size exceeds 10
- (iii) If the input of the train increases to an average 33 per day, what will be the changes in (i), (ii)?
- 3. Solve the assignment problem.

	Ă	B	С	D
Ι	18	24	28	32
II	8	13	17	19
III	10	15	19	22

- 4. In a super market, the average arrival rate of customer is 10 in every 30 minutes following Poisson process. The average time taken by the cashier to list and calculate the customer's purchases is 2.5 minutes, following exponential distribution. What is the probability that the Queue length exceeds 6? What is the excepted time spent by a customer in the system?
- 5. The assignment cost of assigning any one operator to any one machine is given in the following table

	-	Operator			
		Ι	II	III	IV
Machine	1 2 3 4	10 3 10 5	5 9 7 11	13 18 3 9	15 3 2 7

- 6. In a public telephone booth the arrivals are on the average 15 per hour. A call on the average takes 3 minutes. If there is just one phone, find
 - (i) Excepted number of callers in the booth at any time
 - (ii) The proportion of the time the booth is excepted to be Idle?

7. At a one-man barber shop, the customers arrive following poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customer spends in the system.

8. Solve the assignment problem for maximization given the profit matrix (profit in rupees).

	Machines			
	Р	Q	R	S
Α	51	53	54	50
В	47	50	48	50
Jobs C	49	50	60	61
D	63	64	60	60

9. In a railway marshalling yard, goods training trains arrive at a rate of 30 trains per day. Assume that the inter arrival – time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes, calculate (i) The probability that the yard is empty

(ii)Average queue length assuming that the line capacity of the yard is 9 trains

- 10. Write algorithm for assignment problem (Hungarian Method)
- 11. A barbershop has space to accommodate only 10 customers. He can service only one person at a time. If a customer comes to his shop and finds it full, he goes to the next shop. Customers randomly, arrive at an average rate $\lambda = 10$ per hours and the barbers service time is negative exponential with an average of $\frac{1}{\mu} = 5$ minutes per customer. Find

 $P_{0,}P_{n}$.

<u>PART - C (1 x 10 = 10)</u>

Possible Questions (10 Marks)

1. The assignment cost of assigning any one operator to any one machine is given in the following table

			Oper	rator		
		Ι	II	III	IV	
Machine	1 2 3 4	10 3 10 5	5 9 7 11	13 18 3 9	15 3 2 7	

2. Solve the assignment problem.

	А	В	С	D
Ι	11	17	8	16
II	9	7	12	6
III	13	16	15	12
IV	14	10	12	11

3. At a one-man barber shop, the customers arrive following poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. Assuming that only 5 seats are available for waiting customers, find the average time a customers, find the average time a customer spends in the system.

4. Write algorithm for assignment problem (Hungarian Method)



KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore –641 021. Department of Mathematics

Subject : Operation Research	Subject Code : 17CCP103	LTPC
Class : I – M.Com(CA)	Semester : I	4 0 0 4

$\mathbf{UNIT} - \mathbf{IV}$

Inventory Control: Introduction – Costs involved in Inventory – Deterministic EOQ Models – Purchasing Model without and with Shortage, Manufacturing Model without and with Shortage -Price Break

TEXT BOOK

1. KanthiSwarup, Gupta P.K., Man Mohan., (2016) Operations Research, Sultan

Chandand Sons, New Delhi.

REFERENCES

1. Sharma J.K., (2008), Operations Research Theory Applications, Macmillan India Ltd, New Delhi.

2. Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K.,(2014), Resource Management Techniques, A. R. Publications, Nagapatinam.

3. Shanthi Sophia Bharathi D.,(1999),Operations Research/ Resource management techniques, Charulatha Publications.

4. HamdyA.Taha., Operations Research, (2007), Pearson education, Prentice Hall.

5. Vittal– Operations Research – Margham Publications

UNIT – IV

INVENTORY MODELS

Introduction

Inventory may be defined as the stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business affairs. The Inventory may be kept in any one of the following forms:

- Raw material Inventory.Raw materials which are kept in stock for using in production of goods.
- ii. Work in process Inventory.Semi finished goods which are stored during production process.
- iii. Finished goods Inventory.Finished goods awaiting shipments from the factory.

Type of Inventory

i. Fluctuation Inventorie

In real – life problems, there are fluctuations in the demand and lead time that affect the production of the items. Such types of safety stock are called *Fluctuation Inventories*.

ii. Anticipated Inventories

These are built up in advance for the season of large sales, a promotion programme or a plant shut down period. Anticipated Inventories keep men and machine hours for future participation.

iii. Lot – size Inventories

Generally rate of consumption is different from rate of production or purchasing. Therefore the items are produced in larger quantities, which result in Lot - size Inventories

Reasons for maintaining Inventory

- 1. Inventory helps in smooth and efficient running of business.
- 2. It provides service to the customers at short notice.
- 3. Because of long uninterrupted runs, production cost is less.
- 4. It acts as a buffer stock if shop rejections are too many.
- 5. It takes care of economic fluctuations.

Costs involved in Inventory Problems

1. Holding Cost (C_1)

The cost associated with carrying or holding the goods in stock is known as *holding cost (or) carrying cost* per unit of time. Holding cost is assumed to directly vary with the size of inventory as well as the time the item is held in stock. The following components constitute holding cost.

- (a) *Interested capital cost*: This is the interest charge over the capital invested.
- (b) Record keeping and Administrative costs.
- (c) *Handling cost:* These include costs associated with movement of stock, such as cost of labour etc.
- (d) Storage costs.
- (e) Depreciation costs.
- (f) Taxes and Insurance costs.
- (g) Purchase price or production costs.

Purchase price per unit item is affected by the quantity purchased due to quantity discounts or price breaks. If P is the purchase price of an item and 1 is the stock holding cost per unit time expressed as a fraction of stock value (in rupees), then the holding cost $C_1 = IP$.

2. Shortage Cost (C_2)

The penalty costs that are incurred as a result of running out of stock (i.e., shortage) are known as *shortage or stock – out costs*. These are denoted by C_2 . In case where the unfilled demand for the goods may be satisfied at a latter date,

these costs are assumed to vary directly with both the shortage quantity and the delaying time on the other hand if the unfilled demand is lost (no backlog case) shortage costs become proportional to shortage quantity only.

3. Set – up $cost(C_3)$

These costs are associated with obtaining goods through placing an order or purchasing or manufacturing or setting up a machinery before starting production. So they include costs of purchase, requisition, follow up receiving the goods, quality control etc. These are called *Ordering costs or replenishment costs*, or set-up cost usually denoted by C_3 per production run (cycle). They are assumed to be independent of the quantity ordered or produced.

Variables in Inventory Problem: The variables in inventory model are to two types.

- (a) Controlled Variables
- (b) Uncontrolled Variables

(a) Controlled Variables

- 1. How much quantity acquired.
- 2. The frequency or timing of acquisition.
- 3. The completion stage of stocked items.

(b) Uncontrolled Variables

These include holding costs, shortage costs, set-up cost and demand.

Lead time, Reorder Level (R.O.L)

Lead time: Elapsed time between the placement of the order and its receipts in inventory is known as Lead time.

Reorder Level: This is the time when we should place an order by taking into consideration the interval between placing the order and receiving the supply. For e.g., we would like to place a new order precisely at the time when Inventory Level reaches zero.

Definition: Economic Order Quantity (E.O.Q) or Economic lot size formula

Economic order quantity(EOQ) is that size or order which minimizes total annual cost of carrying inventory and the cost of ordering under the assumed conditions of certainty and that annual demands are known.

Deterministic Inventory Models

There are 4 types under this category which we shall study as follows:

Model I : Purchasing model with no shortages.

Model II : Manufacturing model with no shortages.

Model III: Purchasing model with shortages.

Model IV: Manufacturing model with shortages.

Model I : Purchasing model with no shortages.

(Demand rate uniform, Production rate infinite)

A manufacturer has order to supply goods at a uniform rate of R per unit time. No shortages are allowed, consequently the shortage cost is Infinity. He starts a production run every *t* time units, where *t* is fixed and the set up cost per production run is C_3 . Production time is negligible. (replacement Instantaneous) C_1 is the cost of holding one unit in inventory for a unit of time. The manufacturer's problem is to determine

- 1. How frequently he should make a production run
- How many units should be made per production run.
 Diagrammatic representation of this *model*.



If a production run is made at intervals t, a quantity q = Rt must be produced in each run. Since the stock in small time dt is Rtdt, the stock if period t will be

$$\int_{0}^{t} Rt \, dt = \frac{1}{2} Rt^{2}$$
$$= \frac{1}{2} qt = \text{Area of Inventory triangle OAP.}$$

Cost of holding inventory per production run $=\frac{1}{2}C_1Rt^2$

Set of cost per production run $= C_3$

Total cost per production run $=\frac{1}{2}C_1Rt^2+C_3$.

Average total cost per unit time $C(t) = \frac{1}{2}C_1Rt + \frac{C_3}{t}$ (1)

C will be minimum if
$$\frac{dC(t)}{dt} = 0$$
 and $\frac{d^2C(t)}{dt^2}$ is positive.

Differentiating (1) w.r.t t and equating to zero,

$$\frac{dC(t)}{dt} = \frac{1}{2}C_1Rt - \frac{C_3}{t^2} = 0 \qquad(2)$$

Which gives
$$t = \sqrt{\frac{2C_3}{C_1R}}$$

Differentiating (2) w.r.t. $t = \frac{d^2 C(t)}{dt^2} = \frac{2C_3}{t_3}$ which is positive for value of t given by the above equation.

Prepared by : R. Praveen Kumar, Department of Mathematics, KAHE

UNIT – IV

Thus C(*t*) is minimum for optimum time interval $t_0 = \sqrt{\frac{2C_3}{C_1R}}$.

Optimum quantity q_0 to be produced during each production run,

$$q_0 = Rt_0 = \sqrt{\frac{2C_3R}{C_1}}$$

which is known as the optimal lot – size formula due to R.H. Wilson.

The resulting minimum average cost per unit time,

$$C_{0}(q) = \frac{1}{2}C_{1}R\sqrt{\frac{2C_{3}}{C_{1}R}} + C_{3}\sqrt{\frac{C_{1}R}{2C_{3}}}$$
$$= \frac{1}{\sqrt{2}}\sqrt{c_{1}c_{3}R} + \frac{1}{\sqrt{2}}\sqrt{c_{1}c_{3}R}$$
$$= \sqrt{2c_{1}c_{3}R}$$

Remarks:

- 1. If the demand rate is not uniform, and if D is the total demand to be satisfied during the period T then $R = \frac{D}{T}$ in the above formula.
- 2. q_0, t_0, C_0 are sometimes referred as q^*, t^*, c^* .

Example

The annual demand for an item is 3200 units. The unit cost is Rs. 6/- and inventory carrying charges 25% per annum. If the cost of one procurement is Rs. 150/- determine

- i. Economic order quantity.
- ii. Time between two consecutive orders
- iii. Number of orders per year
- iv. The optimal total cost.

1:

Solution:

R = 3200 units,
$$C_1 = \frac{25}{100} X 6 = \frac{3}{2}$$

 $C_3 = 150 \text{ Rs}, \quad \therefore q^* = \sqrt{\frac{2C_3 R}{C_1}}$
 $= \sqrt{\frac{2X150X3200}{\frac{3}{2}}}$
= 800 units.

(ii)
$$t_0 = \{=t^*\} = \frac{q^*}{R}$$

$$= \frac{800}{3200} = \frac{1}{4}$$
 th of a year

(iii) Number of orders
$$=\frac{1}{t_0}$$
 $=\frac{1}{\frac{1}{4}}$ $=4$

(iv)
Total = (R x price per unit) +
$$C_0$$

(iv)
Optimal cost = (6 x 3200) + $\sqrt{2C_1C_3R}$
= Rs. 20,400 [Ans]
Otherwise optimal total cost = $\frac{R}{q}C_3 + (\frac{q}{2}XC_1) + RP$

Example 2:

A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs 20. The ordering cost per order is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer, and how much would it save the company per year?

Solution:

Hear
$$R = 9000$$
 parts per year

 $C_1 = 15\%$ unit cost

(Here 15% of average Inventory per year means that the carrying cost per unit per year is 15% of the unit cost)

$$= 20 \text{ x} \frac{15}{100} = \text{Rs. 3 each part per year}$$

 $C_3 = \text{Rs. 15 per order}$
 $\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$
 $= \sqrt{\frac{2X150X3200}{\frac{3}{2}}} = 300 \text{ units}$
 $t^* = \frac{q^*}{R} = \frac{300}{9000} = \frac{1}{30} \text{ year}$
 $= \frac{365}{30} = 12 \text{ days}$
 $C_{min} = \sqrt{2C_1C_3R}$
 $= \sqrt{2X3X15X9000}$

If the company follows the policy of ordering every month, then the annual ordering cost becomes $= 12 \times 15 = \text{Rs.} 180$

and lot – size of Inventory at any time $q = \frac{9000}{12} = 750$ parts.

Average Inventory at any time $=\frac{1}{2}q = 375$ parts.

Shortage cost at any time = $375 C_1$

$$= 375 \times 3 = \text{Rs. } 1125.$$

Total annual cost = 1125 + 180 = Rs. 1305

 \therefore The company purchases 300 parts at time intervals of 12 days instead of ordering 750 parts each month. So there will be a net saving of

Rs. 1305 - Rs. 900 = Rs. 405 per year

Example 3:

A certain item costs Rs. 235 per ton. The monthly requirements are 5 tons, and each item the stock is replenished, there is a setup cost of Rs. 1000. The cost of carrying inventory has been estimated at 10% of the average inventory per year. What is the optimum order quantity.

Solution:

R = 5 tons/month = 60 tons/year $C_3 = \text{Rs. 1000}$ $C_1 = 10\% \text{ of unit cost per year}$ $= \text{Rs. 235 x \frac{10}{100}}$ $= \text{Rs. 23.5 \text{ per item per year}}$ $\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$ $= \sqrt{\frac{2X1000X60}{23.5}} = 71.458 \text{ tons}$

Example 4:

A manufacturer has to supply his customer with 600 units of his products per year. Shortage are not allowed and storage cost amounts to 60 paise per unit per year. The set up cost is Rs. 80.00 find

- i. The economic order quantity
- ii. The minimum average yearly cost

- iii. The optimum number of orders per year
- iv. The optimum period of supply per optimum order

Solution:

- R = 600 units/year
- $C_1 = \text{Rs. 80}$

 $C_3 = 0.60$ per unit/year

i.
$$\therefore q^* = \sqrt{\frac{2C_3R}{C_1}}$$

$$=\sqrt{\frac{2X600X80}{0.60}} = 400 \text{ units/year}$$

ii. $C^* = \sqrt{2C_1C_3R} = \sqrt{2 X 0.60 X 80 X 600}$

$$=$$
 Rs. 240

iii.
$$N^* = \frac{demand}{EOQ} = \frac{600}{400} = \frac{3}{2}$$

iv.
$$t^* = \frac{1}{N^*} = \frac{2}{3}$$
 of a year

Model II : *Manufacturing model with no shortages.*(*Demand Rate uniform, production rate finite*)

It is assumed that run sizes are constant and that a new run will be started whenever Inventory is zero. Let

R = number of items required per unit time

- K = number of items produced per unit time
- $C_1 = \cos t$ of holding per item per unit time
- $C_3 = \cos t$ of setting up a production run
- q = number of items produced per run, q = Rt
- t = time interval between runs.

Here each production run of length *t* consists of two parts t_1 and t_2 ,

where (i) t_1 is the time during which the stock is building up at constant rate of K - R units per unit time. (ii) t_2 is the time during which there is no production (or supply) and inventory is decreasing at a constant rate R per unit time.

Let I_m be the maximum Inventory available at the end of time t_1 which is expected to be consumed during the remaining period t_2 at the demand rate R.

Then
$$I_m = (K - R) t_1$$
 (or)
 $t_1 = \frac{I_m}{K - R}$ (1)

Now the total quantity produced during time t_1 id q and quantity consumed during the same period is Rt, therefore the remaining quantity available at the end of time t_1 is

$$t_{1} = q - Rt_{1}$$

$$= q - \frac{R.I_{m}}{K - R} \qquad \text{from(1)}$$

$$\therefore I_{m} \left(1 + \frac{R}{k - R}\right) = q \text{ (or) } I_{m} = \frac{k - R}{k} q \dots \dots (2)$$

Now holding cost per production run for time period $t = \frac{1}{2}I_m tC_1$

And set up cost per production run $= C_3$

: Total average cost per unit time $C(I_m, t) = \frac{1}{2}I_mC_1 + \frac{C_3}{t}$

$$C(q, t) = \frac{1}{2} \left(\frac{k-R}{k} q\right) C_1 + \frac{C_3}{t}$$

$$C(q) = \frac{1}{2} \left(\frac{k-R}{k} q\right) C_1 + \frac{C_3}{\frac{q}{R}}$$

$$= \frac{1}{2} \frac{k-R}{k} C_1 q + \frac{C_3 R}{q}$$

For minimum value of C(q)

$$\frac{d}{dq} [c(q)] = \frac{1}{2} \frac{k-R}{k} C_1 - \frac{C_3 R}{q^2} = 0$$

Prepared by : R. Praveen Kumar, Department of Mathematics, KAHE

Which gives
$$q = \sqrt{\frac{2C_3}{c_1} \frac{RK}{K-R}}$$

 \therefore Optimum lot size $q_0 = \sqrt{\frac{K}{K-R}} \sqrt{\frac{2C_3R}{C_1}}$
 \therefore Optimum time interval $t_0 = \frac{q_0}{R}$
 $= \sqrt{\frac{K}{K-R}} \sqrt{\frac{2C_3}{C_1R}}$

Optimum average cost/unit time

$$C_{0} = \frac{1}{2} \frac{k-R}{k} C_{1} \sqrt{\frac{2C_{3}}{c_{1}} \frac{RK}{K-R}} + C_{3} R \sqrt{\frac{C_{1}(K-R)}{2C_{3}RK}}$$
$$= \sqrt{2C_{1}C_{3}R \frac{K-R}{K}}$$
$$= \sqrt{\frac{K-R}{K}} \sqrt{2C_{1}C_{3}R}$$

Note: (*i*) If K = R then $C_0 = 0$, (i.e.,) there will be no holding cost and set up cost

(*ii*) If $K = \infty$, (i.e.,) production rate is Infinite, this model reduces to model I.

Example 1:

A contractor has to supply 10,000 bearings per month to an automobile manufacturer. He finds that when he starts a production run he can produce 25,000 bearings per month. The cost of holding a bearing in stock for one year is Rs. 2 and the set up cost of a production run is Rs. 180. How frequently should the production run be made?

Solution:

R = 10,000/month x 12 = 1,20,000 per year

$$C_1 = \text{Rs. 2 per year}$$

$$C_{3} = \text{Rs. 180}$$

$$K = 25,000 \text{ x } 12 = 3,00,000 \text{ per year}$$

$$\therefore q^{*} = \sqrt{\frac{2C_{3}R}{C_{1}}} \sqrt{\frac{\kappa}{\kappa-R}}$$

$$= \sqrt{\frac{3,00,000}{30,000-120000}} X \sqrt{\frac{2 X 180 X 120000}{2}}$$

$$= 1.29 \text{ x } \sqrt{21600000} = 6000 \text{ units}$$

$$t^{*} = \frac{q^{*}}{R} = \frac{6000}{120000} = 0.05 \text{ years (i.e.,) 18 days.}$$

Example 2:

The annual demand for a product is 1,00,000 units. The rate or production is 2,00,000 units per year. The set - up cost per production run is Rs. 5000, and the variable production cost of each item is Rs. 10. The annual holding cost per unit is 20% of the value of the unit. Find the optimum production lot - size, and the length of production run.

Solution:

р

R = 1,00,000 per year

$$C_{1} = \frac{20}{100} \times 10 \text{ Rs. Per year}$$

$$C_{3} = \text{Rs. 5000}$$

$$K = 2,00,000$$

$$\therefore \text{EOQ} = \sqrt{\frac{\kappa}{\kappa - R}} \sqrt{\frac{2C_{3}R}{C_{1}}}$$

$$= \sqrt{\frac{2,00,000}{1,00,000}} \times \sqrt{\frac{2 \times 1,00,000 \times 5000}{\frac{20}{100} \times 10}}$$

$$= 1.4142 \times 22360.6$$

$$= 31622 \text{ units } (=q^*)$$
$$t^* = \frac{q^*}{R} = \frac{31622}{1,00,000} = 0.31622 \text{ years}$$
$$= 115 \text{ days}$$

Example 3:

An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set up cost is Rs. 100 per set up and holding cost is Rs. 0.01 per unit of item per day, find the economic lot size for one run, assuming that shortages are not permitted. Also find the time of cycle and minimum total cost for one run.

Solution:

R = 25 items per day
$C_1 = \text{Rs. 0.01}$ per unit per day
$C_3 = \text{Rs. 100 per set up}$
K = 50 items per day
$\therefore \text{EOQ} = \sqrt{\frac{K}{K-R}} \sqrt{\frac{2C_3R}{C_1}}$
$=\sqrt{\frac{2 X 100 X 25}{0.01}} \times \sqrt{\frac{50}{25}}$
= 1000 items
$t_0 = \frac{q_0}{R} = \frac{1000}{25} = 40$ days
Minimum daily cost = $\sqrt{2C_1C_3R} \sqrt{\frac{K-R}{K}}$
$= \text{Rs. } \sqrt{2 \ X \ 0.01 \ X \ 100 \ X \ 25 \ X \ \frac{25}{50}}$
= Rs. 5

Minimum total cost per run = 5×40

= Rs. 200

Example 4:

A company has a demand of 12,000 units/year for an item and it can produce 2000 such items per month. The cost of one setup is Rs. 400 and the holding cost/unit/month is Rs. 0.15. Find the optimum lot size, max inventory, manufacturing time, total time.

Solution:

R = 12,000 units/year $C_1 = \text{Rs. } 400/\text{ set up}$ $C_3 = \text{Rs. } 0.15 \text{ x } 12 = \text{Rs. } 1.80/\text{unit/year.}$ K = 2000 x 12 = 24,000 units/year $\therefore q_0 = \sqrt{\frac{K}{K-R}} \sqrt{\frac{2C_3R}{C_1}}$ $= \sqrt{\frac{2 X 400 X 12,000}{1.80}} \text{ x } \sqrt{\frac{24,000}{12,000}}$ = 3266 units/set upMax inventory $I_{m_0} = \frac{K-R}{K} q_0$ $= \frac{24,000-12,000}{24,000} \text{ x } 3266$ = 1632 units .

Manufacturing time $t_1 = \frac{I_{m_0}}{K-R} = \frac{1632}{12,000} = 0.136$ years.

Total time
$$t_0 = \frac{q_0}{R} = \frac{3264}{12,000} = 0.272$$
 years.

Example 5:

A certain item costs Rs. 250 per ton. The monthly requirements are 10 tons and each time the stock is replenished there is a setup cost of Rs. 1000. The cost of carrying inventory has been estimated as 12% of the value of the stock per year. What is the

optimal order quantity and how frequently should orders be placed? **Solution:**

$$C_{1} = \frac{12}{100} \ge 250$$

$$C_{3} = \text{Rs. 1000}$$

$$R = 10 \ge 12 = 120 \text{ tons/year}$$

$$\therefore \text{EOQ} = \sqrt{\frac{2C_{3}R}{C_{1}}}$$

$$= \sqrt{\frac{2 \times 1000 \times 120}{\frac{12}{100} \times 250}}$$

$$= \sqrt{\frac{24,000}{30}} = \sqrt{8000}$$

$$= 89.44 \text{ units}$$

$$t_{0} = \frac{q_{0}}{R} = \frac{89.44}{120} = 0.745 \text{ year} \approx 9 \text{ months.}$$

Model III : Purchasing model with shortages.

(Demand rate uniform, Production rate infinite, shortages allowed)

Assumptions are the same as model I, but shortages are allowed, consequently, a cost of shortage is incurred.

- C_1 Holding cost or carrying cost.
- C₃-Setup cost or Ordering cost.
- C_2 Shortage cost
- R Demand Rate.

The optimum quantities of this model are

a) The Economic order quantity
$$q^* = \sqrt{\frac{2C_3R}{C_1}} \sqrt{\frac{C_1 + C_2}{C_2}}$$

b) Time between two consecutive orders $t^* = \frac{q^*}{R}$

c) Number of orders per year $N^* = \frac{R}{a^*}$

Example 1:

The demand for an item is 18,000 units per year. The holding cost per unit time is Rs. 1.20 and the cost of shortage is Rs. 5.00, the production cost is Rs. 400. Assuming that replenishment rate is Instantaneous, determine the optimal order quantity.

Solution:

R = 18,000
C₁ = Rs. 1.20
C₂ = Rs. 5.00
C₃ = Rs. 400
∴
$$q^* = \sqrt{\frac{2C_3R}{C_1}} \sqrt{\frac{C_1 + C_2}{C_2}}$$

 $= \sqrt{\frac{2 X 400 X 18,000}{1.20}} \sqrt{\frac{1.20 + 5}{5}}$
= 1.113 x 3,464.10
= 3856 units (app)
 $t^* = \frac{q^*}{R} = \frac{3856}{18,000} = 0.214$ year
 $N^* = \frac{R}{q^*} = 4.67$ orders per year

Example 2:

A certain product has a demand of 25 units per month and the items are withdrawn uniformly. Each time a production run is made the setup cost is Rs. 15. The production cost is Rs. 1 per item and inventory holding cost is Rs. 0.30 per item per month. If

shortage cost is Rs. 1.50 per item per month, determine how often to make a production run and what size it should be ?

Solution:

Though the production cost is given, the cost equation remain the same.

$$\implies q^* = \sqrt{\frac{2C_3R}{C_1}} \sqrt{\frac{C_1 + C_2}{C_2}}$$

Given

R = 25 units/month $C_1 = 0.30$ per item per month $C_2 = \text{Rs. } 1.50$ per item per month $C_3 = \text{Rs. } 15$ $\therefore q^* = \sqrt{\frac{(1.50+0.30)2 X 15 X 25}{0.30 X 1.50}}$ $= \sqrt{\frac{54.77}{25}}$ units $t^* = \frac{q^*}{R} = \frac{54.77}{25} = 2.19$ month

Model IV : Manufacturing model with shortages.

(Demand rate uniform, Production rate finite, shortages allowed)

Assumptions are the same as model II, but shortages are allowed.

The optimum quantities of this model are

 C_1 – Holding cost or carrying cost.

C₃-Setup cost or Ordering cost.

C₂-Shortage cost

K – Production Rate.

a) The Economic order quantity $q^* = \sqrt{\frac{2C_3R}{C_1}} \sqrt{\frac{C_1 + C_2}{C_2}} \sqrt{\frac{\kappa}{\kappa - R}}$

b) Number of shortages $S = \frac{C_1}{C_1 + C_2} q^* \left(1 - \frac{R}{K} \right)$

c) Time between two consecutive orders $t^* = \frac{q^*}{R}$

- d) Number of orders per year $N^* = \frac{R}{q^*}$
- e) Manufacturing time $=\frac{q^*}{K}$

Example 1:

The demand for an item in a company is 18,000 units per year, and the company can produce the item at a rate of 3000 per month. The cost of one setup cost is Rs. 500.00 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs. 20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and time between set – ups.

Solution:

R = 18,000 units per year
= 1500 units per month
K = 3000 units per month
C₁ = Rs. 0.15 per month
C₂ = Rs. 20.00
C₃ = Rs. 500

$$q^* = \sqrt{\frac{2C_3R}{C_1}} \sqrt{\frac{C_1 + C_2}{C_2}} \sqrt{\frac{K}{K - R}}$$

$$= \sqrt{\frac{2 \times 500 \times 1500}{0.15}} \sqrt{\frac{0.15+20}{20}} \sqrt{\frac{3000}{3000-1500}}$$
$$= \frac{1224,744}{0.3872} \times 1.0037 \times 1.4142$$
$$= \frac{1738.458}{0.3872} = 4490 \text{ units (app)}$$
Number of shortages $S = \frac{C_1}{C_1 + C_2} q^* \left(1 - \frac{R}{K}\right)$
$$= \frac{0.15}{0.15+20} \times 4490 \left(1 - \frac{1500}{3000}\right)$$
$$= \frac{336.75}{20.15} = 16.71 \text{ units.}$$
Manufacturing time $= \frac{q^*}{K} = \frac{4490}{3000 \times 12} = 0.1247 \text{ years}$

Time between setup's $= \frac{q^*}{R} = \frac{4490}{18,000} = 0.2494$ years

Inventory Models with Price breaks:

In this section we shall consider a class of inventory problems in which the production (or) purchase cost per unit is a variable. This depends on the quality manufacture or purchased. This usually happen when discount are offered for the purchase of large quantities. These discounts take the form of Price-Breaks.

Consider the following three cases

Where
$$c_0(q) = \sqrt{2c_3 k_1 P R} + k_1 R + \frac{1}{2} c_3 p$$
(1)
and $q = \sqrt{\frac{2c_3 R}{k_1 P}}$,

Total expected cost per unit time

C (q) =
$$\frac{C_3 R}{q} + \frac{1}{2} q PI + PR$$
(2)

 K_1 = purchasing cost of care unit

	р	= holding cost/month expressed as a fraction of the values of the
unit		

Case (i): Purchase Inventory model with single price-break

Given:	Unit purchasing cost	Range of quality	
	K ₁₁	$0 < q_1 < b_1$	
	K ₁₂	$q_2 \ge b_1$	

- (i) If $b>q_2$ and $c_2(b) > c_0(q_1)$, the optimal lot size is q_1 and minimum values of $c(q) = c_0(q_1)$
- (ii) If $b>q_2$ and $c_2(b) < c_0(q)$, the optimal lot size is b and min $c(q) = c_2(b)$
- (iii) If b<q₂, the optimal lot size is q₂ and min $c(q) = c_0(q_2)$

Case (ii): Purchase Inventory Model with 2 prices - breaks

Unit purchasing cost	Range of quality
K ₁₁	$0 < q_1 < b_1$
K ₁₂	$b_1 \le q_2 < b_2$
K ₁₃	$b_2 \leq q_3$

The optimal purchase quality is determined in the following way

- (i) Calculate q_3 , If $q_3 > b_2$, optimal purchase quality is q_3
- (ii) If $q_3 \le b_2$, calculate q_2 since $q_3 < b_2$, the necessarily $q_2 < b_2$. As a consequence we have $q_2 < b_1$ or $q_2 > b_1$.
- (iii) If $q_3 < b_2$ and $b_1 < q_2 < b_2$, compare c_0 (q_2) with c_3 (b_2). The smaller of these qualities will be optimal purchase quantity.
- (iv) If $q_3 < b_2$ and $q_2 < b_1$. Calculated c_3 (q_1) which will necessarily satisfy the inequality $q_1 < b_1$. In this case compared c_0 (q_1), c_2 (b_1) and c_3 (b_2) to determine optimum purchase quantity.

Case (iii): Purchase inventory model with 'n' price breaks

When there are n price breaks, the situation can be represented as follows:

Unit purchasing cost	Range of quality
K ₁₁	$0 < q < b_1$
K ₁₂	$b_1 \leq q < b_2$
K_{1n}	$b_{n-1} \leq q$

- (i) Calculate q_n . If $q_n > b_{n-1}$, optimal purchase quality is q_n
- (ii) If q_n<b_{n-1}, calculate q_{n-1}. If q_{n-1}≥b_{n-2} proceed as in the case of one price break; (i.e.,) compare c₀ (q_{n-1}) with c(b_{n-1}) to determine optimum purchase quality.
- (iii) If q_{n-1}<b_{n-2}, compute q_{n-2}. If q_{n-2}≥b_{n-3}, proceed as in the case of 2 price breaks: (i.e.,) compare c₀(q_{n-2}) with c(b_{n-1}) and c(b_{n-2}) to determine optimal purchase quality.
- (iv) If $q_{n-2} < b_{n-3}$ compute q_{n-3} If $q_{n-3} \ge b_{n-4}$ compare $c_0(q_{n-3})$ with $C(b_{n-3})$, $C(b_{n-2})$ and $C(b_{n-1})$.
- (v) Continue in this number until $q_{n-j} \ge b_n (j+1)(0 \le j \le n-1)$ and then compare $C_0(q_{n-j})$ with $C(b_{n-j})$, $c(b_{n-j+1})$, $C(b_{n-j+2})$ $C(b_{n-1})$. This procedure involves only a finite number of steps.

Example 1:

Find the optimal order quantity for a product for which the price – break is as follows:

Quantity	unit cost
$0 \le Q_1 < 50$	Rs. 10
$50 \le Q_1 < 100$	Rs. 9
$100 \le Q_3$	Rs. 8

2017 Batch

The monthly demand for the product is 200 units, the cost of the storage is 25% of the unit cost and ordering cost is Rs. 20.00 per order.

Here R=200 units, P= $0.25, C_3 = Rs.20.00$

$$Q_3^0 = \sqrt{\frac{2 X 20 X 200}{8 X 0.25}}$$

Clearly 63 < 100(i.e.,)
$$Q_{3^0} < b_2$$

 \therefore We compute $Q_2^0 = \sqrt{\frac{2 X 20 X 200}{9 X 0.25}}$
= 60 units

Now since $Q_2^0 > b_1 (= 50)$ the optimum purchase quantity is determined by comparing $C_A(Q_{2^0})$ with $C_A(b_2)$

Now
$$C_A(Q_2^0) = 20 \ge x \frac{200}{60} + 200 \ge 9 + 9 \ge 0.25 \ge \frac{60}{2}$$

= Rs. 1934.16
 $C_A(b_2) = 20 \ge \frac{200}{100} + 200 \ge 8 + 8 \ge 0.25 \ge \frac{100}{2}$
= Rs.1740.00

Since $C_A(Q_2^0) > C_A(b_2)$, the optimum purchase quantity is $Q^0 = b_2 = 100$ units.

Example 2:

Find the optimal order quantity for which the price breaks are as follows:

Quantity	Unit cost
$0 \le q_1 < 500$	Rs.10
$500 \le q_2 < 750$	Rs.9.25
750≤q ₃	Rs.8.75

The monthly demand for the product is 200 units, shortage cost is 2% of the unit cost and the cost of ordering is Rs. 100.

Solution:

$$q_{3} = \sqrt{\frac{2c_{3}R}{k_{13}P}}$$

$$= \sqrt{\frac{2 X 100 X 200}{8.75 X 0.2}} = 478 \text{ Units}$$

$$b_{2} = 750$$

$$q_{3} < b_{2}, \text{ we calculate } q_{2}$$

$$q_{2} = \sqrt{\frac{2c_{3}R}{k_{12}P}}$$

$$= \sqrt{\frac{2 X 100 X 200}{9.25 X 0.02}} = 465 \text{ Units}$$

$$b_{1} = 500 \text{ Units}$$

$$q_{2} < b_{1}, \text{ we compute } q_{1}$$

$$q_{2} = \sqrt{\frac{2c_{3}R}{k_{11}P}}$$

$$= \sqrt{\frac{2 X 100 X 200}{10 X 0.02}} = 447 \text{ Units}$$

Next we compute

*Now C₀ (q₁) =
$$\sqrt{2c_3 k_{11}PR} + k_{11}R + \frac{1}{2}c_3P$$

=Rs. [$\sqrt{2 X 100 X 10 X 0.02 X 200} + 10 x 200 + \frac{1}{2} x 100 x 0.02$]
= Rs. 2090.42
C₂ (b₁) = $c_3 \frac{R}{q} + k_{12}R + \frac{1}{2}c_3P + \frac{1}{2}k_{12}pq$

= Rs. $[100 \ge \frac{200}{500} + 9.25 \ge 200 + \frac{1}{2} \ge 100 \ge 0.02 \ge \frac{1}{2} \ge 9.25 \ge 0.02 \ge 500]$ = 1937.25 $C_3(b_2) = c_3 \frac{R}{q} + k_{13} \ge R + \frac{1}{2} c_3 \ge R + \frac{1}{2} k_{13} \ge 100 \ge 0.02 \ge \frac{1}{2} \ge 0.02 \ge 750$ = Rs. $[100 \ge \frac{200}{750} + 8.75 \ge 200 + \frac{1}{2} \ge 100 \ge 0.02 \ge \frac{1}{2} \ge 8.75 \ge 0.02 \ge 750]$ = Rs. 1843.29

Since $C_3(b_2) < C_2(b_1) < C_0(q_1)$, the optimal order quantity is $b_2=750$ units. *we can use formula (2) under 12.9 also.

Example 3:

Find the optimum order quantity for a quantity for which the price breaks are as follows:

Quantity	Unit cost
$0 \le Q_1 < 100$	Rs.10
$500 < Q_2$	Rs.9.25

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering Rs.350.00

Solution:

$$Q_2^0 = \sqrt{\frac{2 X 350 X 200}{9.25 X 0.2}}$$

= 870 units

as
$$Q_2^0 > b_1, (870 > 500)$$

Optimum purchase quantity = 870 units.

Example 4:

Find the optimum order quantity for a quantity for which the price breaks are as follows:

Quantity

Unit cost

$0 \le Q_1 < 500$	Rs.20 per unit
$500 \le Q_2 < 750$	Rs.18 per unit
$750 \le Q_3$	Rs.16 per unit

The monthly demand for the product is 400 units. The shortage cost is 20% of the unit cost of the product and the cost of ordering is 25.00 per month.

Solution:

R = 100 units, I = Re. 0.20, $C_3 = Rs.25.00$

$$Q_3^0 = \sqrt{\frac{2c_3 R}{k_1 P}}$$
$$= \sqrt{\frac{2 X 25 X 400}{16 X 0.20}} = 79 \text{ units}$$

Since $Q_{3^0} < b_2$, we compute Q_2^0 ,

: We have
$$Q_2^0 = \sqrt{\frac{2 X 25 X 400}{20 X 0.20}} = 75$$
 units

Now since $Q_2^0 < b_1 (= 100)$ we next compute Q_1^0

Next we compute

Now
$$C_A(Q_1^0) = 25 \ge \frac{400}{70} + 400 \ge 20 + 20 \ge 0.20 \ge \frac{70}{2}$$
 = Rs. 8283.00

$$C_A(b_1) = 25 \ge \frac{400}{100} + 400 \ge 18 + 18 \ge 0.20 \ge \frac{100}{2} = \text{Rs.7480.00}$$

$$C_A(b_2) = 25 \text{ x} \frac{400}{200} + 400 \text{ x} 16 + 16 \text{ x} 0.20 \text{ x} \frac{200}{2} = \text{Rs.6770.00}$$

Since $C_A(b_2) < C_A(b_1) < C_A(Q_1^0)$, the optimal purchase quantity is $Q^0 = b_2 = 200$ units. Example 5: Find the optimal quantity for a product where the annual demand for the product is 500 units. The cost of storage per unit per year is 10% of the unit cost and the ordering cost per order is Rs.180.00. The unit costs are given below.

Quantity	Unit cost
$0 \le Q_1 < 500$	Rs. 25
$500 \le Q_2 < 1500$	Rs. 24.80
$1500 < Q_3 < 3000$	Rs. 24.60
$3000 < Q_4$	Rs. 24.40

Solution:

R = 500 units, P = Rs. 0.10, $C_3 = Rs.180.00$

$$Q_4^0 = \sqrt{\frac{2c_3 R}{k_4 P}}$$
$$= \sqrt{\frac{2 X 180 X 500}{(24.40) X 0.10}} = 272$$

Since $Q_4^0 < b_3$, we compute Q_3^0 ,

$$Q_3^0 = \sqrt{\frac{2c_3 R}{k_3 P}} = \sqrt{\frac{2 X 180 X 500}{(24.60) X 0.10}} = 270$$

Since $Q_3^0 < b_2 \ (= 1500) \therefore$ we calculate

$$Q_2^0 = \sqrt{\frac{2c_3 R}{k_2 P}} = \sqrt{\frac{2 X 180 X 500}{(24.80) X 0.10}} = 269$$

Since $Q_2^0 < b_1 \ (= 500) \therefore$ we calculate

$$Q_1^0 = \sqrt{\frac{2c_3 R}{k_1 P}} = \sqrt{\frac{2 X \, 180 \, X \, 500}{25 \, X \, 0.10}} = 268$$

Now
$$C_A(Q_1^0) = 180 \ge \frac{500}{268} + 500 \ge 25 + 25 \ge 0.10 \ge \frac{268}{2} = \text{Rs. } 13,170.82$$

2017 Batch

$$C_A (b_1) = 180 \text{ x} \frac{500}{500} + 500 \text{ x} 24.80 + (24.80) \text{ x} 0.10 \text{ x} \frac{500}{2}$$

= Rs.13,200.00
$$C_A (b_2) = 180 \text{ x} \frac{500}{1500} + 500 \text{ x} 24.60 + (24.60) \text{ x} 0.10 \text{ x} \frac{1500}{2}$$

= Rs.14,205.00
$$C_A (b_3) = 180 \text{ x} \frac{500}{3000} + 500 \text{ x} 24.40 + (24.40) \text{ x} 0.10 \text{ x} \frac{3000}{2}$$

= Rs.15,890.00

Since $C_A(b_3) > C_A(b_2) > C_A(b_1) > C_A(Q_1^0)$, the optimum purchase quantity is $Q^0 = Q_1^0 = 268$ units.

Example 6:

Find the optimal order quantity for the following annual demand = 3600 units, order cost = Rs. 50, cost of storage = 20% of the unit cost

Price break	$0 < Q_1 < 100$	Rs. 20
	$750 \le Q_2$	Rs. 18

Solution:

Given R = 3600 units per year,

I = Rs.
$$\frac{20}{100}$$
,
 $k_1 = \text{Rs.}200.00$
 $k_2 = \text{Rs.}18.00$
 $Q_2^0 = \sqrt{\frac{2 \times 50 \times 3600}{18 \times 0.20}} = 316.20$

Now b = 100 as $Q_2^0 > b$, Optimum purchase quantity = 316.20

<u>PART – B (5 x 6 = 30)</u>

Possible Questions (6 Marks)

1. (i) Explain various types of inventory.

(ii) A company has a demand of 12,000 units per year for an item and it can produce 2000 such items per month. The cost of one set up is Rs 400 and the holding cost unit/month is Rs.0.15. Find

1) The optimum lot size

- 2) Maximum inventory
- 3) Manufacturing time
- 4) Total time
- 2. Find the optimam order quantity for a quantity for which the price- break is as follows:

Quantity	Unit cost
$0 \le Q_1 < 500$	Rs. 10
$500 \leq Q_2$	Rs. 9.25

The monthly demand for the product is 200 units, the cost of the storage is 2% of the unit cost and ordering cost is Rs.350.00

- 3. A company has a demand of 18,000 units per year for an item and it can produce 3000 such items per month. The cost of one set up is Rs 500. and the holding cost per unit per month is Rs.0.15. The shortage cost of one unit is Rs.20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time between set- ups.
- 4. A manufacturer has to supply his customer with 600 units of his products per year. Shortages are not allowed and storage cost amounts to 60 paise per unit per year. The set up cost is Rs. 80.00. find
 - i) The economic order quantity
 - ii) The optimum number of orders per year
 - iii) The minimum average yearly cost

iv) The optimum period of supply per optimum order

- 5. The annual demand for an item is 3200 units. The unit cost is Rs.6 and inventory carrying charges 25 % per annum. If the cost of one procurement is Rs.150. Determine
 - i) Economic order quantity
 - ii) Time between two consecutive orders
 - iii) Number of orders per year
 - iv) The optimal cost.
- 6. Find the optimal order quantity for a product for which the price- break is as follows:

Quantity	Unit cost
$0 \le Q_1 < 50$	Rs. 10
$50 \le Q_2 < 100$	Rs. 9
$100 \leq Q_3$	Rs. 8

The monthly demand for the product is 200 units, the cost of the storage is 25% of the unit cost and ordering cost is Rs.20.00 per order.

- 7. A company has a demand of 12,000 units per year for an item and it can produce 2000 such items per month. The cost of one set up is Rs 400. and the holding cost per unit per month is Rs.0.15. Find
 - The optimum lot size
 Maximum inventory
 Manufacturing time
 Total time
- 8. Find the optimal order quantity for which the price break are as follows

Quantity	Unit cost
$0 \le Q_1 < 50$	Rs. 50
$50 \le Q_2 < 100$	Rs. 9
$100 \leq Q_3$	Rs. 8

The monthly demand for the product is 200 units. The cost of storage is 20% of the unit cost and the ordering per order is Rs.2

9. Find the optimum order quantity for a quantity for which the price breaks are as follows:

Quantity	Unit cost
$0 \leq Q_1 < 500$	Rs.10
$500 \le Q_2 < 750$	Rs.9.25
750≤Q ₃	Rs.8.75

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering Rs. 100

10. Find the optimal order quantity for a product where the annual demand for the product is 500 units. The cost of storage per unit per year is 10% of the unit cost and the ordering per order is Rs.180.00. The unit costs are given below.

Quantity	Unit cost
$0 \le Q_1 < 500$	Rs. 25
$500 \le Q_2 < 1500$	Rs. 24.80
$1500 \le Q_3 < 3000$	Rs.24.60
3000<Q₄

Rs.24.40

11. i) An item is produced at the rate of 50 items per day. The demand occurs at the rate of 25 items per day. If the set up cost is Rs.100 per set up and holding cost is Rs 0.01 per unit of item per day, find the economic lot size for one run, assuming that shortages are not permitted. Also find the time of cycle and minimum total cost for one run.
ii) Given R =800 units, cost of the item = Rs.40.Inventory carrying cost = 40%, Back order cost = Rs.10, Ordering cost = Rs.300. Find EOQ

PART – C $(1 \times 10 = 10)$

Possible Questions (10 Marks)

1. Find the optimum order quantity for a quantity for which the price breaks are as follows:

Unit cost
Rs.10
Rs.9.25
Rs.8.75

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering Rs. 100

2. Find the optimal order quantity for a product where the annual demand for the product is 500 units. The cost of storage per unit per year is 10% of the unit cost and the ordering per order is Rs.180.00. The unit costs are given below.

Quantity	Unit cost
$0 \le Q_1 < 500$	Rs. 25
$500 \le Q_2 < 1500$	Rs. 24.80
$1500 \le Q_3 < 3000$	Rs.24.60
$3000 < Q_4$	Rs.24.40

- 3. A company has a demand of 18,000 units per year for an item and it can produce 3000 such items per month. The cost of one set up is Rs 500. and the holding cost per unit per month is Rs.0.15. The shortage cost of one unit is Rs.20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time between set- ups.
- 4. A manufacturer has to supply his customer with 600 units of his products per year. Shortages are not allowed and storage cost amounts to 60 paise per unit per year. The set up cost is Rs. 80.00. find

i) The economic order quantity

ii) The optimum number of orders per year

- iii) The minimum average yearly costiv) The optimum period of supply per optimum order



KARPAGAM ACADEMY OF HIGHER EDUCATION (Deemed to be University Established Under Section 3 of UGC Act 1956) Pollachi Main Road, Eachanari (Po), Coimbatore –641 021. Department of Mathematics

Subject : Operation Research	Subject Code : 17CCP103	LTPC
Class : I – M.Com(CA)	Semester : I	4 0 0 4

UNIT – V

PERT and CPM: Network Representation – Calculation of Earliest expected time, latest allowable occurrence time. CPM - Various Floats for Activities – Critical Path- PERT –Time Estimates in PERT- Probability of Meeting scheduled date of Completion of Projects

TEXT BOOK

1. KanthiSwarup, Gupta P.K., Man Mohan., (2016) Operations Research, Sultan

Chandand Sons, New Delhi.

REFERENCES

1. Sharma J.K., (2008), Operations Research Theory Applications, Macmillan India Ltd, New Delhi.

2. Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K.,(2014), Resource Management Techniques, A. R. Publications, Nagapatinam.

3. Shanthi Sophia Bharathi D.,(1999),Operations Research/ Resource management techniques, Charulatha Publications.

4. HamdyA.Taha., Operations Research, (2007), Pearson education, Prentice Hall.

5. Vittal– Operations Research – Margham Publications

$\mathbf{UNIT} - \mathbf{V}$

PERT and CPM

Introduction

A **project** is defined as a combination of interrelated activities all of which must be executed in a certain order to achieve a set goal. A large and complex project involves usually a number of interrelated activities requiring men, machines and materials. It is impossible for the management to make and execute an optimum schedule for such a project just by intuition, based on the organizational capabilities and work experience. A systematic scientific approach has become a necessity for such project. So a number of methods applying networks scheduling techniques has been developed: **Programme Evaluation Review Technique** (PERT) and **Critical Path** method (CPM) are two of the many network techniques which are widely used for planning, scheduling and controlling large complex projects.

The main managerial functions for any project:

The main managerial functions for any project are

- 1. Planning
- 2. Scheduling
- 3. Control

Planning

This phase involves a listing of tasks or jobs that must be performed to complete a project under consideration. In this phase, men, machines and materials required for the project in addition to the estimates of costs and durations of various activities of the project are also determined

Scheduling

This phase involves the laying out of the actual activities of the project in a **logical sequence** of time in which they have to be performed.

Men and material requirements as well as the **expected completion time** of each activity at each stage of the project are also determined.

Control

This phase consists of reviewing the progress of the project whether the actual performance is according to the planned schedule and finding the reasons for difference, if any, between the schedule and performance. The basic aspect of control is to analyse and correct this difference by taking remedial action whether possible.

PERT and CPM are especially useful for scheduling and controlling

Basic Terminologies

Activity is a task or an item of work to be done in a project. An activity consumer resource like time, money, labour etc.

An activity is represented by an arrow with a node (event) at the beginning and a node (event) at the end indicating the start and termination (finish) of the activity. Nodes are denoted by circles. Since this is a logical diagram length or shape of the arrow has no meaning. The direction indicates the progress of the activity. Initial node and the terminal node are numbered as i-j(j<i) respectively. For example If A is the activity whose initial node is I and the terminal node is j then it is denoted diagrammatically by



The name of the activity is written over the arrow, **not inside the circle.** The diagram

In which arrow represents an activity is called **arrow diagram.** The Initial and terminal nodes of activities are also called tail and head events.

If an activity B can start immediately after an activity A then it is denoted by



A is called the **immediate predecessor** of B and B is called the **immediate successor** of A. If C can start only after completing activites A and B then it is diagrammatically represented as follows:



Notation: "A is a predecessor of B" is denoted as "A<B',"B is a successor of A" is denoted by "B>A".

If the project contains two or more activities which have some of their immediate predecessors in common then there is a need for introducing what is called **dummy activity**. Dummy activity is an imaginary activity which does not consume any resources and which serves the purpose of indicating the predecessor or successor relationship clearly in any activity on arrow diagram. The need for a dummy activity is illustrated by the following usual example.

Let P, Q be the predecessors of R and Q be the only predecessors of S.



Activities which have no *predecessors* are called *start activities* of the project. All the *start activities* can be made to have the *same initial node*. Activities which have *no successors* are called *terminal activities* of the project. These can be made to have the *same* terminal node (end node) of the project.

A project consists of a number of activities to be performed in some technological sequence. For example while constructing a building the activity of laying the foundation should be done before the activity of erecting the walls for the building. The diagram denoting all the activities of a project by arrows taking into account the technological sequence of the activities is called the project network represented by *activity on arrow diagram* or simply *arrow diagram*.

Note: There is another representation of a project network representing activites on nodes called AON diagram. To avoid confusion we use only activity on arrow diagram throughout the text.

Rules for constructing a project network

1. There must be no loops. For example, the activities F,D,E.



Obviously form a loop which is obviously not possible is any real project network.

2. Only one activity should connect any two nodes.

3. No dangling should appear in a project network i.e., no node of any activity except the terminal node of the project should be left without any activity emanating from it such a node can be joined to the terminal node of the project to avoid.

The Rules for numbering the Nodes:

Nodes may be numbered using the rule given below:

(Ford and Fulkerson's Rule)

- 1. Number the start node which has no predecessor activity, as 1.
- 2. Delete all the activites emanating from this node 1.
- 3. Number all the resulting start nodes without any predecessor as 2,3.....

- 4. Delete all the activities originating from the start nodes 2,3,...in step 3.
- 5. Number all the resulting new start nodes without any predecessor next to the last number used in step(3).
- 6. Repeat the process until the terminal node without any successor activity is reached and number this terminal node suitably.

Immediate predecessor (successor) will be simply called as predecessor (successor) unless otherwise stated.

Example 1: If there are five activities P, Q, R, S and T such that P, Q, R have no immediate predecessors but S and T have immediate predecessors P, Q, R respectively. Represent this situation by a network.

Solution:



 D_1 and D_2 are dummy activities.

Example 2:

Draw the network for the project whose activities and their precedence relationship are given below:

Activity	:	Р	Q	R	S	Т	U
Predecessor	:	-	-	-	P, Q	P, R	Q, R

Solution:



 D_1 , D_2 , D_3 , D_4 are dummy activities.

Example 3:

Draw the network for the project whose activities with their predecessor relationship are given below:

A, C, D can start simultaneously : E > B, C ; F, G > D ; H, I > E,F ; J > I,G ; K > H; B > A.

Solution:

Identify the start activities i.e., activities which have no predecessors. They are A,C and D as given. These three activities should start with the same start node. Also identify the terminal activities which have no successors. They are J and K. These two activities should end with the same end node, the last terminal node indicating the completion of the project. Taking into account the predecessors relationship given, the required network is as follows:



Example 4:

Construct the network for the project whose activities and their relationships are as given below:

Activities : A, D, E can start simultaneously.

Activities : B, C > A; G, F > D, C; H > E, F.

Solution:

Start activities are A, D, E. End activities are H, G, B. The required network is



Note : see how the nodes of the activity F are numbered. Can we number C as 2 - 4 and F as 4 - 3?

Example 5: Draw the network for the project whose activities and their precedence relationships are as given below:

Activities :	А	В	С	D	Е	F	G	Н	Ι
Immediate									
Predecessor:	-	А	А	-	D	B,C,E	F	Е	G,H
~ • •									

Solution:

Start activities : A,D, Terminal activities : I only. Activities B and C starting with the same node are both the predecessors of the activity F. Also the activity E has to be the predecessor of both F and H. Therefore dummy activities are necessary. Thus the required network is





 D_1 and D_2 are dummy activities.

Note: sometimes while constructing a network you may introduce more dummy activities than necessary. Redundant dummy activities can always be found out when one checks whether all the given precedence relationships given in the problem are satisfied exactly. (Nothing more, nothing less).

Example 6:

Construct the network for the project whose precedence relationships are as given below: B < E, F; C < G, L; E, G < H; L, H < I; L < M; H,M < N; A < J; I, J < P; P < Q. Solution:





Example 7:

Draw the event network for the following data:

 Event No
 :
 1
 2
 3
 4
 5
 6
 7

UNIT – V		PERT and CPM						
Immediate Predecessors:	_	1	1	2,3	3	4,5	5,6	
Solution:								



Network Computations and Critical Path

(Earliest Completion time of a Project and Critical path)

It is obvious that the completion time of the project is one of the very important things to be calculated knowing the durations of each activity. In real world situation the duration of any activity has an element of uncertainty because of sudden unexpected shortage of labour, machines, materials etc. Hence the completion time of the project also has an element of uncertainty. We first consider the situation where the duration of each activity is deterministic without taking the uncertainty into account.



The above diagram represents an activity whose direction is 10 time unit(hour or days or weeks or month etc)

The first network calculation one does is the computation of earliest start and earliest finish (completion) time of each activity given the duration of each activity. The method used is called forward pass calculation and it is best illustrated by means of the following example.

Example 1:

Compute the earliest start, earliest finish latest start and latest finish of each activity of the project given below:

Activity	1-2	1-3	2-4	2-5	3-4	4-5
Duration	8	4	10	2	5	3
(in days)						



Figure 1

To compute the Earliest start and Earliest finish of each activity:

We take the earliest time of all the start activities as zero.

So earliest starts of 1-2 and 1-3 are zero.

To find earliest start of 2-4.

The activity 2-4 can start only after finishing the only preceding activity 1-2 i.e., after 8 days.

: Earliest start of 2-4 is 8 days. Similarly earliest start of 2-5 is also 8 days.

Similarly earliest start of 3-4 is 4 days.

To find the earliest start of 4-5 we first notice that the activity 4-5 has more than one predecessor and also the activity 4-5 can start only after finishing all its preceding activities.

There are two paths leading to the activity 4-5: namely 1-2-4 which takes 18 days and 1-3-4 which takes 9 days. Obviously after 18 days all the activities 1-2, 1-3, 2-4, 3-4 can be finished but not earlier than that.

∴Earliest start of 4-5 is 18 days.

Note: Earliest start of an activity i-j can be denoted as ES_i or ES_{ij} . It can also be called the earliest occurrence of the event i.

Earliest finish of any activity i-j is got by adding the duration of the activity denoted by t_{ij} to the earliest start of i-j.

Hence the earliest finish of 1-2,1-3,2-4,2-5,3-4,4-5 are 8,4,18,10,9,21 respectively.



Figure 2

Obviously earliest completion time of the project is 21 days, the greater number among these since all the activities can be finished only after 21 days.

Formula for Earliest Start of an activity i-j in a project network is given by

 $ES_j = Max [ES_i+t_{ij}]$ where

 ES_j denoted the earliest start time of all the activities emanating from node i and t_{ij} is the estimated during of the activity i-j.

To compute the latest finish and latest start of each activity:

The method used here is called backward pass calculation since we start with the terminal activity and go back to the very first node.

We first calculate the latest finish of each activity as follows:

Latest finish of all the terminating (end) activities is taken as the earliest completion time of the project. Similarly latest finish of all the start activities is obviously taken as the same as the earliest start of these start activities.

Thus the latest finish of the terminal activities 2-5 and 4-5 are 21 days which is the earliest completion time of the project.

Latest finish of the activity 2-4 and 3-4 is 21-3 = 18 days.

Latest finish of 1-3 is 18-5 = 13 days

To find the latest finish of the activity 1-2, we observe that the activity 1-2 has more than one successor activity. Therefore the latest finish of the activity 1-2 is the smaller of the two numbers 21 - 2 = 19 and 18 - 10 = 8. i.e. 8 days.

Note : Latest finish of an activity can be denoted by LF_j or LF_{ij} . It can also be called the latest occurrence of the event j. Latest start of each activity is the latest finish of that activity minus the duration of that activity. The latest start of the activities 4 -5, 2 - 5, 2 - 4, 3 - 4, 1 - 3, 1 - 2 are 21, 21, 18, 18, 13, 8 respectively.



Prepared by :R. Praveen Kumar, Departent of Mathematics, KAHE.

5 L = 13 L = 18

4

Figure 3

Formula for the latest start time of all the activities emanating from, the event i of the activity i - j, $LS_i = Min [LS_j - t_{ij}]$ for all defined i-j activities where t_{ij} is the estimated duration of the activity i - j.

We can tabulate the results and represents these earliest and latest occurrences of the events in the network diagram as follows:



Figure 4

		Ea	arliest	Lat	test
Activity	Duration days	Start ES	Finish EF ES = ES+t _{ii}	Start LS LF – t _{ii}	Finish LF
			i ij	٦J	
1 - 2	8	0	8	0	8
1 – 3	4	0	4	9	13
2-4	10	8	18	8	18
2-5	2	8	10	19	21

3-4	5	4	9	13	18
4 – 5	3	18	21	18	21

Note: For small networks, it is not difficult to draw the network with E and L values calculated directly by looking at the diagram itself and constructing the table given above.

Critical path:

Path, connecting the first initial node to the very last terminal node, of longest duration in any project network is called the critical path.

All the activities in any critical path are called critical activities. Critical path is 1 - 2 - 4 - 5, usually denoted by doubt lines. (Ref fig.4)

Critical path plays a very important role in project scheduling problems.

Floats

Total float of an activity (T.F) is defined as the difference between the latest finish and the earliest finish of the activity or the difference between the latest start and the earliest start of the activity.

Total float of an activity $i - j = (LF)_{ij} - (EF)_{ij}$ Or $= (LS)_{ij} - (ES)_{ij}$.

Total float of an activity is the amount of time by which that particular activity may be delayed without affecting the duration of the project. If the total float is positive then it may indicate that the resources for the activity are more than adequate. If the total float of an activity is zero it may indicate that the resources are just adequate for that activity. If the total float is negative, it may indicate that the resources for that activity are inadequate.

Note: (L - E) of an event of I - j is called the slack of the event j.

There are three other types of floats for an activity, namely, Free float, Independent float and interference (interfering) float.

Free Float:

Free Float of an activity (F.F) is that portion of the total float which can be used for rescheduling that activity without affecting the succeeding activity. It can be calculated as follows:

Free float of an activity i - j = Total float of i - j - (L - E) of the event j

= Total float of i - j - slack of the head event j

= Total float of I - J – slack of the head event j

Where L = Latest occurrence, E = Earliest occurrence

Obviously Free Float \leq Total float for any activity.

Independent float (I.F):

Independent float (I.F) of an activity is the amount of time by which the activity can be rescheduled without affecting the preceding or succeeding activities of that activity.

Independent float of an activity i - j = Free float of i - j - (L - E) of event i.

= Free float of i - j – Slack of the tail event j.

Clearly, Independent float \leq Free float for any activity. Thus I.F \leq F.F \leq T.F.

Interfering Float or Interference Float of an activity i - j is nothing but the slack of the head event j.Obviously, Interfering Float of i - j = Total Float of i - j – Free Float of i - j.

Example 2:

Calculate the total float, free float and independent float for the project whose activities are given below:

Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration	8	7	12	4	10	3	5	10	7	4
(in days)										

The data is the same as given in example 2 above. The network with L and E of every event is given by



E = 17; L = 21

		Ea	rliest	La	Latest		Floats	
Activity	Duration (in weeks)	Start	Finish	Start	Finish	TF	FF	IF
1-2	8	0	8	0	8	0	0	0
1-3	7	0	7	8	15	8	5	5
1-5	12	0	12	9	21	9	5	5
2-3	4	8	12	11	15	3	0	0
2-4	10	8	18	8	18	0	0	0
3-4	3	12	15	15	18	3	3	0
3-5	5	12	17	16	21	4	0	-3
3-6	10	12	22	15	25	3	3	0
4-5	7	18	25	18	25	0	0	0
5-6	4	17	21	21	25	4	4	0

Explanation:

To find the total float of 2 - 3.

Total float of (2-3) = (LF - EF) of (2-3) = 15 - 12 = 3 from the table against the activity 2-3.

Free Float of (2-3) = Total float of (2-3) - (L - E) of event 3

 $= 3 - (15 \ 12)$ from the figure for event 3 = 0

Free Float of (1-5) = Total float of (1-5) - (L-F) of event 5

= (21 - 12) - (21 - 17) from the figure for event 5

= 9 - 4 = 5

Independent float of (1-5) = Free Float of (1-5) - (L-E) of event 1

= 5 - (0 - 0) = 5

Important Note:

Note that all the critical activities have their total float as zero. In fact the critical path can also be defined as the path of least (zero) total float. As we have noticed total float is 3 for the activity 2 - 3. This means that the activity 2 - 3 can be delayed by 3 weeks without delaying the duration (completion date) of the project.

Free float of 3 - 4 is 3. This means that the activity 3 - 4 can be delayed by 3 weeks without affecting its succeeding activity 4 - 6.

Independent float of 1 - 5 is 5 means that the activity 1 - 5 can be delayed by 5 weeks without affecting its preceding or succeeding activity. Of course 1 - 5 has no preceding activity. Uses of floats:

Floats are useful in resources leveling and recourse allocation problems which will be discussed in the last section of this chapter. Floats give some flexibility in rescheduling some activities so as to smoothen the level of resources or allocate the limited resources as best as possible.

Example 3:

Calculate the earliest start, earliest finish, latest start and latest finish of each activity of the project given below and determine the critical path of the project.

Activity 1-2 1-3 1-5 2-3 2-4

Duration



Program Evaluation Review Techniques: (PERT)

This technique, unlike CPM, take into account the uncertainly of project durations into account.

Optimistic (least) time estimate: (t_0 or a) is the duration of any activity when everything goes on very well during the project. i.e., labourers are available and come in time, machines are working properly, money is available whenever needed, there is no scarcity of raw material needed etc.

Pessimistic (greatest) time estimate: $(t_p \text{ or } b)$ is the duration of any activity when almost everything goes against our will and a lot of difficulties is faced while doing a project.

Most likely time estimate: $(t_m \text{ or } m)$ is the duration of any activity when sometimes things go on very well, sometimes things go on very bed while doing the project.

Two main assumption make in PERT calculations are

- (i) The activity durations are independent. Ie., the time required to complete an activity will have no bearing on the completion times of any other activity of the project.
- (ii) The activity durations follow β distribution. β Distribution is a probability distribution with density function $k(t-a)^{\alpha}$ $(b-t)^{\beta}$ with mean $t_e = \frac{1}{3}[2t_m + \frac{1}{2}(t_0 - t_p)]$ and the standard deviation $\sigma_t = \frac{t_p - t_0}{6}$

PERT procedure

(1) Draw the project network

(2) Compute the expected duration of each activity $t_e = \frac{t_0 + 4t_m + t_p}{6}$

- (3) Compute the expected variance $\sigma^2 = \left(\frac{t_p t_0}{6}\right)^2$ of each activity.
- (4) Compute the earliest start, earliest finish, latest start, latest finish and total float of each activity.
- (5) Determine the critical path and identity critical activites.
- (6) Compute the expected variance of the project length (also called the variance of the critical path) σ_c^2 which is the sum of the variance of all the critical activities.
- (7) Compute the expected standard deviation of the project length σ_c and calculate the standard normal deviate $\frac{T_s T_E}{\sigma_s}$ where
 - T_s = Specified or Schedule time to complete the project
 - T_E = Normal expected project duration
 - σ_c = Expected standard deviation of the project length.
- (8) Using (7) one can estimate the probability of completing the project within a specified time, using the normal curve (Area) tables.

Example 1:

Construct the network for the project whose activities and the three time estimates of these activities (in weeks) are given below. Compute

- (a) Excepted duration of each activity.
- (b) Excepted variance of each activity.

Activity	t_0	t_m	t_p
1-2	3	4	5
2-3	1	2	3
2-4	2	3	4
3 – 5	3	4	5
4-5	1	3	5
4-6	3	5	7
5-7	4	5	6
6-7	6	7	8
7-8	2	4	6
7 – 9	1	2	3
8-10	4	6	8
9 - 10	3	5	7

(c) Excepted variance of the project length.



Activity	t_0	t_m	t_p	Excepted duration $t_e = \frac{t_0 + 4t_m + t_p}{6}$	Excepted variance $\sigma^{2} = \left(\frac{t_{p} - t_{0}}{6}\right)^{2}$
1-2	3	4	5	4	$1/_9 = 0.11$ nearly
2-3	1	2	3	2	$\frac{1}{9} = 0.11$
2 - 4	2	3	4	3	$\frac{1}{9} = 0.11$

Prepared by :R. Praveen Kumar, Department of Mathematics, KAHE.

3 – 5	3	4	5	4	$\frac{1}{9} = 0.11$
4-5	1	3	5	3	$\frac{4}{9} = 0.44$
4-6	3	5	7	5	$\frac{4}{9} = 0.44$
5-7	4	5	6	5	$\frac{1}{9} = 0.11$
6-7	6	7	8	7	1/9 = 0.11
7 - 8	2	4	6	4	$\frac{4}{9} = 0.44$
7 – 9	1	2	3	2	$\frac{1}{9} = 0.11$
8 - 10	4	6	8	6	$\frac{4}{9} = 0.44$
9 - 10	3	5	7	5	$\frac{4}{9} = 0.44$

Critical path 1 - 2 - 4 - 6 - 7 - 8 - 10. Excepted project duration = 29 weeks.

(c) Excepted variance of the project length = Sum of the expected variances of all the critical activities

 $=\frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{15}{9} = \frac{15}{9} = \frac{5}{3} = 1.67$ or (0.11 + 0.11 + 0.44 + 0.11 + 0.44 + 0.44 = 1.32 + 0.33 = 1.65)

Example 2:

The following table indicates the details of a project. The duration are in days. 'a' refers to optimistic time, 'm' refers to most likely time and 'b' refers to pessimistic time duration.

Activity	1 - 2	1 - 3	1 - 4	2 - 4	2 - 5	3 - 5	4 - 5
a	2	3	4	8	6	2	2
т	4	4	5	9	8	3	5
b	5	6	6	11	12	4	7
(-) D							

(a) Draw the network

(b) Find the critical path

(c) Determine the excepted standard deviation of the completion time.

Solution:

Activity	a	т	b	Excepted	Excepted
				duration t _e	variance σ^2
1 – 2	2	4	5	3.83	1
					4
1-3	3	4	6	4.17	1
					$\overline{4}$

1-4	4	5	6	5	$\frac{1}{9}$
2-4	8	9	11	9.17	$\frac{1}{4}$
2-5	6	8	12	8.33	1
3-4	2	3	4	3	$\frac{1}{9}$
4 – 5	2	5	7	4.83	$\frac{25}{36}$



Critical path 1 - 2 - 4 - 5

Excepted project duration = 17.83 days

Excepted variance of the completion time = $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{43}{36}$

Excepted standard deviation of the completion time = $\sqrt{\frac{43}{36}} = 1.09$ nearly

Example 3:

A project consists of the following activities and time estimates:

Activity	Least time (days)	Greatest time (days)	Most likely time (days)
1 - 2	3	15	6
2 - 3	2	14	5
1 - 4	6	13	12
2 - 5	2	8	5
2 - 6	5	17	11

3-6	3	15	6
4 – 7	3	27	9
5-7	1	7	4
6-7	2	8	5

(a) Draw the network

(b) What is the probability that the project will be completed in 27 days?

Solution:

Obviously Greatest time = Pessimistic time = t_p

Least time = Optimistic time = t_0

Most Likely time = t_m



Activity	t ₀	t_p	t _m	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$
1 – 2	3	15	6	7	4
2-3	2	14	5	6	4
1-4	6	13	12	14	16
2-5	2	8	5	5	1
2-6	5	17	11	11	4
3-6	3	15	6	7	4
4 – 7	3	27	9	11	16
5 – 7	1	7	4	4	1
6-7	2	8	5	5	1

Critical path 1 - 4 - 7

Excepted project duration = 25 days

Sum of the excepted variance of Excepted variance of the project length = all the critical activities

$$= 16 + 16 = 32.$$

 σ_c = Standard deviation of the project length = $\sqrt{32}$ = $4\sqrt{2}$ = 5.656

 $Z = \frac{T_s - T_E}{\sigma_c} = \frac{27 - 25}{5.656} = 0.35$

Probability that the project will be completed in 27 days

$$= P(T_s \le 27) = P(Z \le 0.35)$$

= 0.6368 = 63.7%

Basic difference between PERT and CPM

PERT

- PERT was developed in a brand new R and D project it had to consider and deal with the uncertainties associated with such projects. Thus the project duration is regarded as a random variables and therefore probabilities are calculated so as to characteristics it.
- Emphasis is given to important stages of completion of task rather than the activities required to be performed to reach a particular event or task in the analysis of network.
 i.e., PERT network is essentially an event – oriented network.
- PERT is usually used for projects in which time estimates are uncertain. Example: R & D activities which are usually non-repetitive.
- 4. PERT helps in identifying critical areas in a project so that suitable necessary adjustments may be made to meet the scheduled completion date of the project.

CPM

 CPM was developed fir conventional projects like construction project which consists of well known routine tasks whose resources requirement and duration were known with certainty.

- 2. CPM is suited to establish a trade off for optimum balancing between schedule time and cost of the project.
- 3. CPM is used for projects involving well known activities of repetitive in nature, However the distinction between PERT and CPM is mostly historical.

<u>PART – B (5 x 6 = 30)</u>

Possible Questions (6 Marks)

- 1. Construct the network for the project whose activities and the three time estimates of these activities (in weeks) are given below. Compute
 - (d) Excepted duration of each activity.
 - (e) Excepted variance of each activity.
 - (f) Excepted variance of the project length.

Activity	t_0	t_m	t_p
1-2	3	4	5
2-3	1	2	3
2 - 4	2	3	4
3 – 5	3	4	5
4-5	1	3	5
4-6	3	5	7
5 – 7	4	5	6
6-7	6	7	8
7 - 8	2	4	6
7-9	1	2	3
8-10	4	6	8

UNIT – V

9 - 10	3	5	7

- 2. Describe the difference between PERT and CPM.
- 3. Calculate the total float, free float and independent float for the project whose activities are given below:

	10	1 0			a 4	a 4	~ -	<u> </u>		
Activity	1-2	1-3	1-5	2-3	2-4	3-4	3-5	3-6	4-6	5-6
Duration	8	7	12	4	10	3	5	10	7	4
(in days)										

4. The following table indicates the details of a project. The duration are in days. 'a' refers to optimistic time, 'm' refers to most likely time and 'b' refers to pessimistic time duration.

Activity	1 - 2	1 - 3	1 - 4	2 - 4	2 - 5	3 – 5	4 - 5
a	2	3	4	8	6	2	2
т	4	4	5	9	8	3	5
b	5	6	6	11	12	4	7

- (a) Draw the network
- (b) Find the critical path
- (c) Determine the excepted standard deviation of the completion time.

5. Calculate the earliest start, earliest finish, latest start and latest finish of each activity of the project given below and determine the critical path of the project.

Activity	0 - 1	1 - 2	1 - 3	2 - 4	2 - 5
Duration					
(in weeks)	3	8	12	6	3
Activity	3 - 4	3-6	4 - 7	5 - 7	6 - 7
Duration					
(in weeks)	3	8	5	3	8
	•	0 1 0 11 1		•	

6.	A project	consists o	of the	following	activities	and tim	e estimates:
	1 J			0			

Activity	Least time	Greatest	Most likely
	(days)	time (days)	time (days)
1 - 2	3	15	6
2-3	2	14	5
1-4	6	13	12
2-5	2	8	5
2 - 6	5	17	11
3-6	3	15	6
4-7	3	27	9
5-7	1	7	4
6-7	2	8	5

- (c) Draw the network
- (d) What is the probability that the project will be completed in 27 days?
- 7. A Project has the following characteristics

Activity :	А	В	С	D	Е	F
Duration:	6	8	4	9	2	7
Preceding						
activity :	_	А	А	В	С	D

Prepared by :R. Praveen Kumar, Department of Mathematics, KAHE.

Draw the network diagram and find the critical path.

8. The following table indicates the details of a project. The duration are in days. 'a' refers to optimistic time, 'm' refers to most likely time and 'b' refers to pessimistic time duration.

Activit	y 1-2	2 - 3	2 - 4	3 - 4	4 - 5	5-6
а	0.8	3.7	6.2	2.1	0.8	0.9
т	1.0	5.6	6.6	2.7	3.4	1.0
b	1.2	9.9	15.4	6.1	3.6	1.1
(a)	Drow the note	vork				

- (a) Draw the network
- (b) Find the critical path

(c) Determine the excepted standard deviation of the completion time

9. The following table given the characteristics of a project.

Job	Predecessors	Duration(days)
А	-	10
В	-	5
С	В	3
D	A,C	4
E	A,C	6
F	D	6
G	E	5
Н	F,G	5

Draw the network diagram and find the critical path.

10. The three estimates for the activities of a project are given below:

Activity	Estimated duration (days)					
	а	m	b			
1 - 2	5	6	7			
1-3	1	1	7			
1 - 4	2	4	12			
2 - 5	3	6	15			
3-5	1	1	1			
4 - 6	2	2	8			
5 - 6	1	4	7			

i) Draw the project network.

ii) What is the probability that the project will be completed on 22 days ?

$PART - C (1 \times 10 = 10)$

Possible Questions (10 Marks)

1. The following table lists the jobs of a network with their time estimate

Jobs	Optimistic	Duration days	Pessimistic
		Most likely	
1-2	3	6	15
1-6	2	5	14
2-3	6	12	30
2-4	2	5	8
3-5	5	11	17
4-5	3	6	15
6-7	3	9	27
5-8	1	4	7
7-8	4	19	28

Draw the project network and calculate the length and variance of the critical path

2. Construct the network for the project whose precedence relationships are as given below:

 $B < E, \ F \ ; \ C < G, \ L \ ; \ E, \ G < H \ ; \ L, \ H < I \ ; \ L < M \ ; \ H, M < N \ ; \ A < J \ ; \ I, \ J < P \ ; \ P < Q.$

3. A Project has the following characteristics

Activity :	А	В	С	D	E	F
Duration:	6	8	4	9	2	7
Preceding						

activity : _____A A B C D Draw the network diagram and find the critical path.

4. Draw the network for the project whose activities with their predecessor relationship are given below:

A, C, D can start simultaneously : E > B, C ; F, G > D ; H, I > E, F ; J > I, G ; K > H; B > A.