18MMU311 PDE AND SYSTEMS OF ODE - PRACTICAL 4H – 2C

Instruction Hours / week: L: 0 T: 0 P: 4 **Marks:** Internal: **40** External: **60** Total: **100**

End Semester Exam: 3 Hours

Course Objectives

This course enables the students to learn

- To demonstrate comprehension in relevant area of differential equations.
- Problem solving through programming.

Course Outcomes (COs)

On successful completion of this course, the student will be able to

1. Familiarize with the programming environment.
2. Acquire the problem solving skills through computer programming.
3. Understand to write diversified solutions using programming language.

List of Practical:

1. Solution of second order ordinary differential equations with initial conditions.
2. Solving Non Homogeneous Wave Equation.
3. Solving the Heat Conduction Problem.
4. Solving two dimensional Laplace equations.
5. Solving system of linear differential Equations.
6. Solution of differential equation using Euler method.
7. Solution of differential equation using Modified Euler method.
8. Solution of differential equation using 4^{th} order Runge-Kutta method.

Ex.No.1 **Solution of second order ordinary differential equations with initial conditions.**

Question:

Solve: $\frac{d^2y}{dt^2} = \frac{1}{t+1} + \sin(t)\sqrt{t}$ with the initial conditions $y(0) = 0$ and $y'(0) = -2$.

Aim:

To solve the given second order ODE with initial conditions and to plot the curves using Scilab.

Algorithm:

Step:1 Start the program.

Step:2 Define the given differential equation as custom function.

Step:3 Define the range of t, initial time t_0 and the given initial conditions.

Step:4 Define the ode( ) function with the given parameters.

Step:5 Plot the solution curve using the plot command.

Step:6 Analytical solution also be calculated in the console window using step 4.

Step:7 Stop the program.

Coding:

```
clc;
function dx=f(t, x)
dx(1) = x(2);
dx(2) = 1/(t+1) + sin(t)*sqrt(t);
endfunction
t = [0:1:5];
t0 = min(t);
y0 = [0; -2];
y = ode(y0, t0, t, f);
plot(t,y(1,:),'LineWidth',5)
plot(t,y(2,:),'r','LineWidth',5)
xgrid();
xlabel('$t \quad [s]$','FontSize',3)
ylabel('$f(t,x)$','FontSize',3)
title(['Integration of ' '$\frac{d^2 x}{dt^2} = \frac{1}{t+1} +
sint(t)\sqrt{t}$'],'FontSize',3)
legend(['$\Large{x}$' '$\Large{dx/dt}$'],2)
```

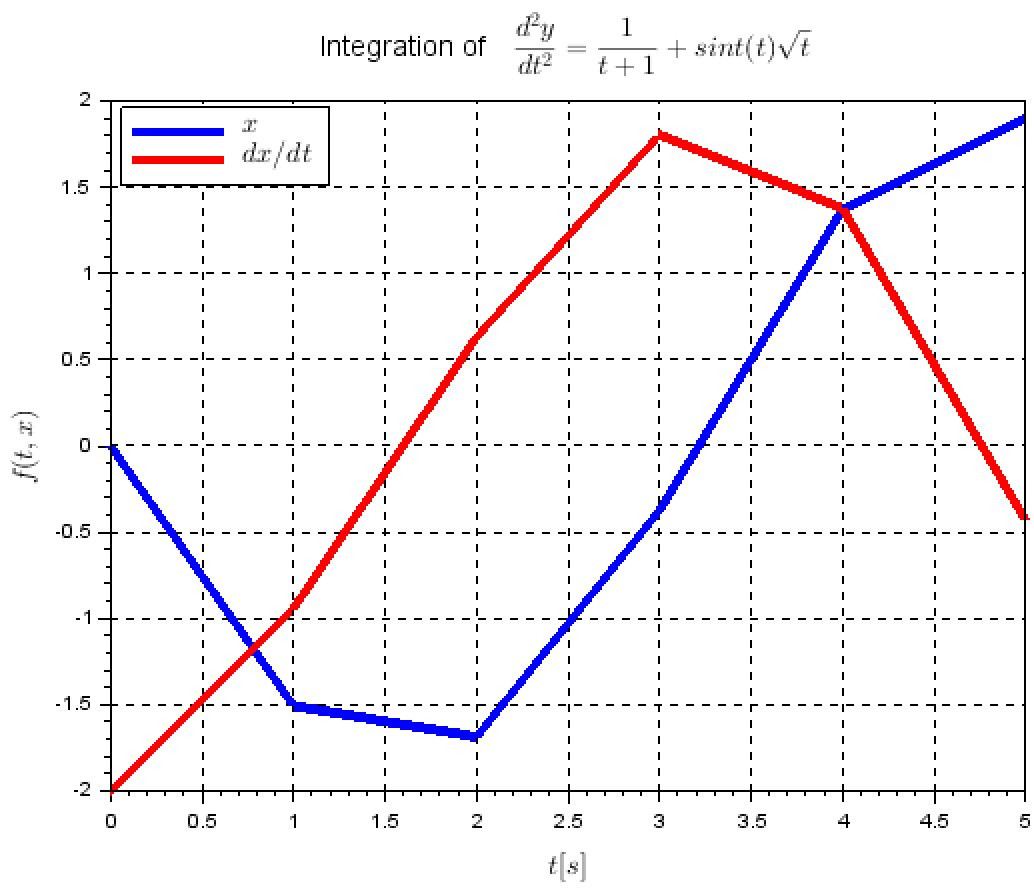
Output:

Numerical Solution

$y =$: 0 -1.5059853 -1.6866851 -0.3741111 1.3706081 1.9011403

-2 -0.9426308 0.6312574 1.8041464 1.3781866 -0.4308691

Graphical Solution:



Ex.No. 2 **Solving Non Homogeneous Wave Equation.**

Question:

Consider the case of a vibrating string with the initial displacement given by $f(x) = \frac{x}{L}\left(1-\frac{x}{L}\right)$ and the initial velocity given by $g(x) = \left(\frac{x}{L}\right)^2\left(1-\frac{x}{L}\right)$. The boundary conditions are $u(0,t) = 0; u(L,t) = 0$. Determine the solution $u(x,t)$ for this problem using components of the resulting Fourier series for $n = 1,2,3, \ldots ,20$ if $c = 1$ and $L = 1$.

Aim:

To solve the one dimensional wave equation and to plot the curves using Scilab.

Algorithm:

Step:1 Start the program.

Step:2 Define the given function $f(x)$ as embedded function.

Step:3 Again define an another embedded function to evaluate $a(n)$ using $f(x)$.

Step:4 Define the function $g(x)$ as embedded function.

Step:5 Once again define the embedded function to evaluate $b(n)$ using $g(x)$.

Step:6 Define an embedded function using for loop to find $u(x,t)$.

Step:7 Evaluate $u(x,t)$ using multiple evaluation function.

Step:7 Plot the solution curve using the plot command.

Step:8 Stop the program.

Coding:

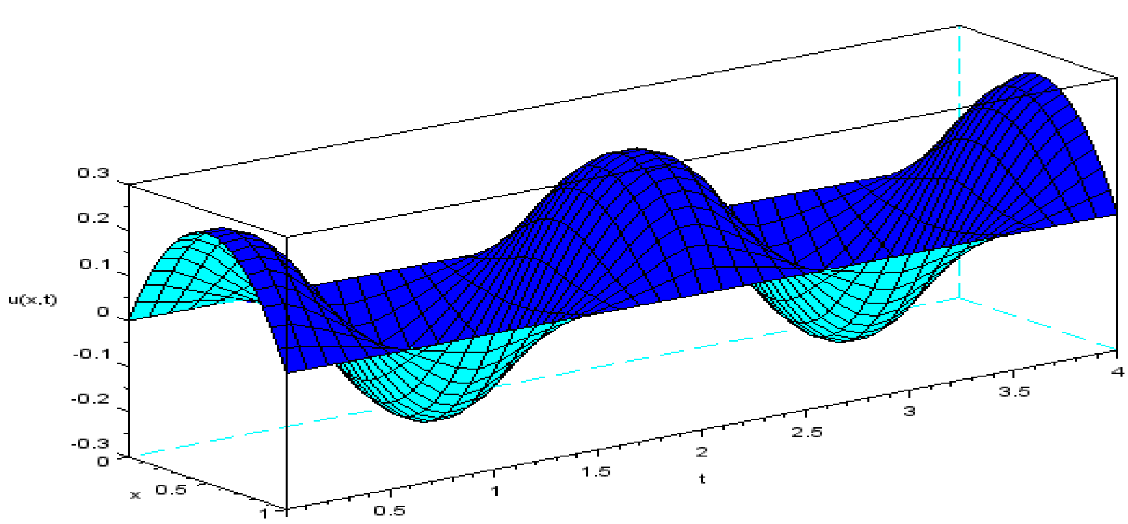
```
clc;
deff('[w]=f(x)','w=(x/L)*(1-x/L)')
deff('[w]=ff(x)','w=f(x)*sin(n*%pi*x/L)')
deff('[aa]=a(n)','aa=(2/L)*intg(0,L,ff,0.001)')
deff('[w]=g(x)','w=(x/L)^2*(1-x/L)')
deff('[w]=gg(x)','w=g(x)*sin(n*%pi*x/L)')
deff('[bb]=b(n)','bb=(2/(n*%pi*c))*intg(0,L,gg,0.001)')
L=1;c=1;
aa=[ ];for n=1:5, aa=[aa a(n)]; end;
bb=[ ];for n=1:5, bb=[bb b(n)]; end;
deff('[uu]=u(x,t)',['uu=0';'for n=1:20';
'uu=uu+sin(n*%pi*x/L)*(aa(n)*cos(n*%pi*c*t/L)+bb(n)*sin(n*%pi
*c*t/L))';'end'])
x=[0:0.05:1];t=[0:0.1:4];
uu=feval(x,t,u);
plot3d(x,t,uu,45,45,'x@t@u(x,t)')
```

Output:

--> uu

0.	0.	0.	0.	0.	0.
0.0475596	0.0404651	0.0316116	0.0231521	0.0147908	0.0062292
0.0899577	0.0811213	0.0633449	0.0463379	0.0295351	0.0123333
0.1275319	0.1196362	0.0952287	0.0695545	0.0441721	0.0181875
0.1599743	0.1533026	0.1274592	0.09288	0.0586712	0.0236666
0.1875219	0.1822956	0.1575792	0.1162487	0.0729512	0.0286459
0.2099806	0.2063122	0.1828377	0.1397925	0.0870116	0.0329999
0.2275177	0.2254648	0.2033156	0.1609758	0.1007225	0.0366043
0.2399833	0.2395157	0.2186455	0.1769693	0.1141212	0.0393332
0.2475161	0.2485377	0.2288615	0.1877894	0.1246289	0.0410629
0.2499841	0.2523167	0.2336473	0.1929742	0.1292909	0.0416658
0.2475161	0.2509127	0.2330115	0.1925145	0.1281297	0.0410629
0.2399833	0.2441157	0.2266454	0.1859689	0.1205188	0.0393332
0.2275177	0.2319899	0.2145658	0.1733518	0.1089485	0.0366043
0.2099806	0.2143121	0.1964373	0.15419	0.0960112	0.0329999
0.1875219	0.1911708	0.1723302	0.1309998	0.0818264	0.0286459
0.1599743	0.1623022	0.1418567	0.1064796	0.0666711	0.0236666
0.1275319	0.1278622	0.1076047	0.0808047	0.0506971	0.0181875
0.0899577	0.0875189	0.0723445	0.0543378	0.034135	0.0123333
0.0475596	0.0439659	0.0363368	0.0273021	0.0171659	0.0062292
0.	0.	0.	0.	0.	0.

Graphical Solution:



Ex.No.3 **Solving the Heat Conduction Problem**

Question:

Determine the solution for the one-dimensional heat equation subjected to $u(0,t) = u(L,t) = 0$, if the initial conditions are given by $u(x,0) = f(x) = 4\left(\frac{x}{L}\right)\left(1-\frac{x}{L}\right)$. Use $k = 1$ and $L = 1$.

Aim:

To solve the heat equation and to plot the curve using Scilab.

Algorithm:

Step:1 Start the program.

Step:2 Define the function $f(x)$ as custom function.

Step:3 Define the function $g(x)$ as custom function.

Step:4 Define the integral between the specific limits.

Step:5 Use for loop and feval function to find the value of u

Step:6 Analytical solution also be calculated in the console window using step 5.

Step:7 Plot the graph using plot3d function.

Step:7 Stop the program.

Coding:

```
Clc();
deff('[w]=f(x)','w=4*(x/L)*(1-x/L)')
deff('[w]=g(x)','w=f(x)*sin(n*%pi*x/L)')
deff('[bb]=b(n)','bb=intg(0,L,g,0.001)')
L=1;k=1;
bb=[ ];for n=1:40, bb=[bb b(n)]; end;
deff('[uu]=u(x,t)',['uu=0';'for j=1:40';
    'uu=uu+bb(j)*sin(j*%pi*x/L)*exp(-j^2*%pi^2*k*t/L^2)';'end'])
xx=[0:0.05:1];tt=[0:0.025:0.25];
uu=feval(xx,tt,u);
plot3d(xx,tt,uu,45,45,'x@t@u(x,t)')
```

Output:

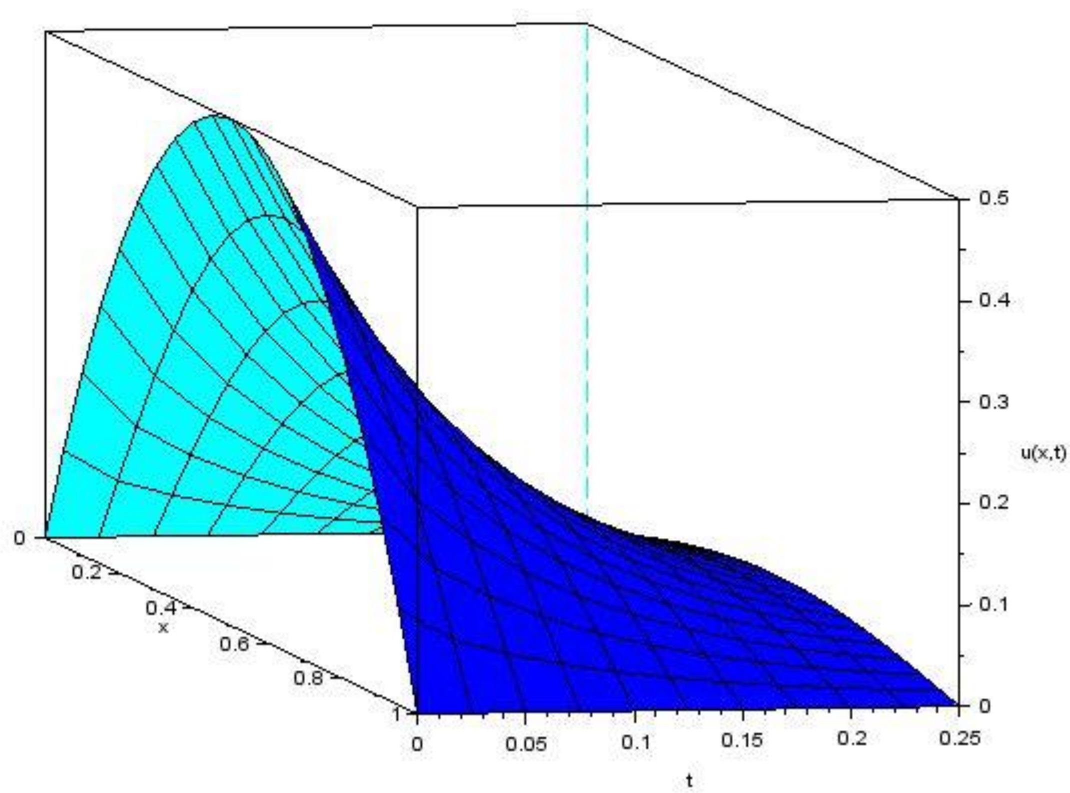
Numerical Solution

```
--> uu
 uu  =

       column 1 to 6
```

0.	0.	0.	0.	0.	0.
0.0949792	0.0640211	0.0493841	0.0385172	0.0300877	0.023508
0.1799878	0.1262802	0.0975324	0.0760838	0.0594343	0.0464372
0.2549914	0.1851005	0.1432439	0.1117731	0.087317	0.0682228
0.3199933	0.2389635	0.1853853	0.1447056	0.1130491	0.0883285
0.3749944	0.2865609	0.2229204	0.1740704	0.1359972	0.1062593
0.4199951	0.326822	0.2549358	0.1991457	0.1555961	0.1215735
0.4549955	0.3589166	0.2806603	0.2193159	0.1713637	0.1338942
0.4799958	0.3822396	0.2994809	0.2340865	0.1829117	0.142918
0.4949959	0.3963864	0.3109528	0.243096	0.1899562	0.1484227
0.499996	0.4011268	0.3148068	0.2461238	0.1923237	0.1502727
0.4949959	0.3963864	0.3109528	0.243096	0.1899562	0.1484227
0.4799958	0.3822396	0.2994809	0.2340865	0.1829117	0.142918
0.4549955	0.3589166	0.2806603	0.2193159	0.1713637	0.1338942
0.4199951	0.326822	0.2549358	0.1991457	0.1555961	0.1215735
0.3749944	0.2865609	0.2229204	0.1740704	0.1359972	0.1062593
0.3199933	0.2389635	0.1853853	0.1447056	0.1130491	0.0883285
0.2549914	0.1851005	0.1432439	0.1117731	0.087317	0.0682228
0.1799878	0.1262802	0.0975324	0.0760838	0.0594343	0.0464372
0.0949792	0.0640211	0.0493841	0.0385172	0.0300877	0.023508
0.	0.	0.	0.	0.	0.

Graphical Solution:



Ex.No.4 **Solving the Two dimensional Laplace equation**

Question:

Solve the two dimensional Laplace equation for contour plots in the solution domain $L = 2$ and $H = 1$, and the boundary condition at $y = H$ is given by $f(x) = 100\,x\,(L - x)^3$.

Aim:

To solve the two dimensional Laplace equation and plotting the curve using Scilab

Algorithm:

Step:1 Start the program.

Step:2 Define the function $g(x)$ as custom function $deff$.

Step:3 Define the function $gg(x)$ as custom function.

Step:4 Find the value of w using Fourier series technique.

Step:5 Use for loop and feval function to find the value of $u(x, y)$

Step:6 Analytical solution also be calculated in the console window using step 5.

Step:7 Plot the graph using contour function

Step:8 Stop the program.

Coding:

```
Clc
deff('[w]=g(x)','w=100*x*(L-x)^3')
deff('[w]=gg(x)','w=g(x)*sin(n*%pi*x/L)')
deff('[aa]=a(n)','aa=2*intg(0,L,gg,0.0001)/(L*sinh(n*%pi*H/L))')
L=2;H=1;
aa=[];for n=1:20, aa=[aa a(n)]; end;
deff('[uu]=u(x,y)',['uu=0';'for n=1:20';...
'uu=uu+aa(n)*sin(n*%pi*x/L)*sinh(n*%pi*y/L)';'end'])
x=[0:L/5:L];y=[0:H/5:H];
uu=feval(x,y,u);
contour(x,y,uu,15)
xtitle('Contour plots for Laplace equation solution','x','y')
```

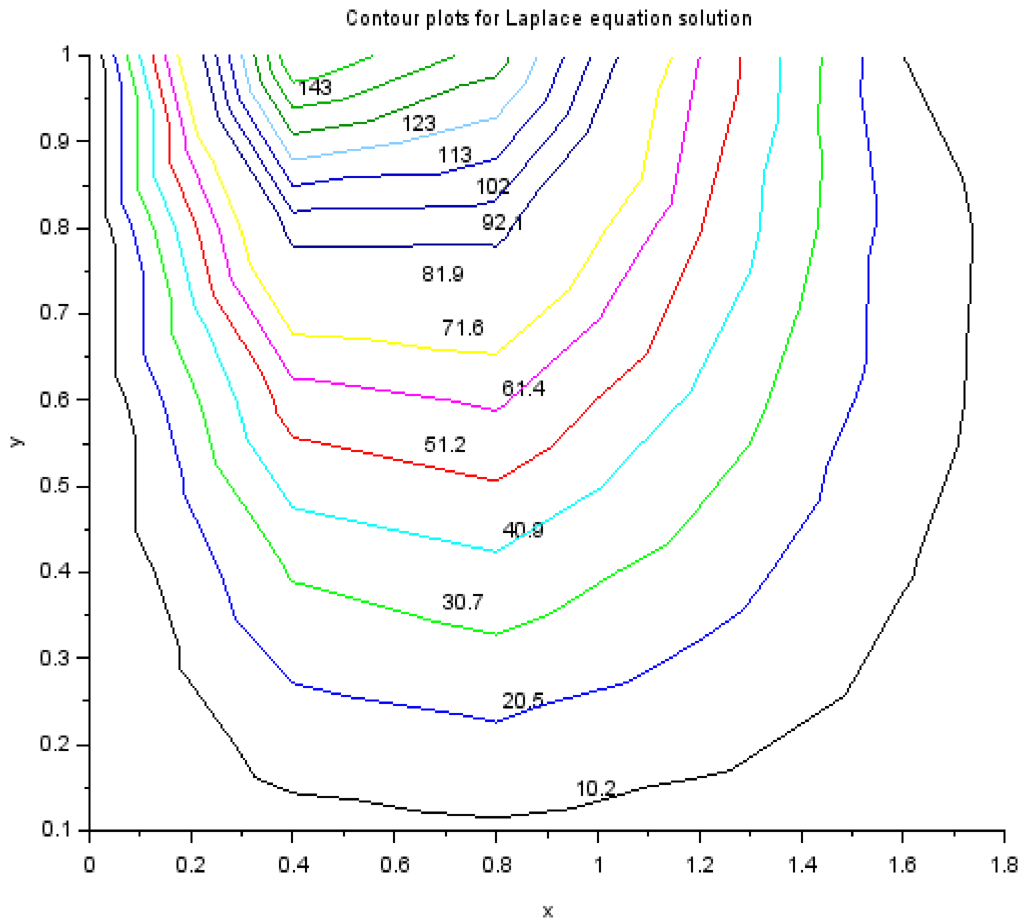
Output:

Numerical Solution

```
--> uu
 uu  =

   0.   0.          0.          0.          0.          0.
   0.   14.26303    31.647844   56.509153   96.263066   163.729
   0.   17.828795   37.953953   62.900703   95.465755   138.18792
   0.   12.654198   25.636441   38.926215   51.669127   61.412276
   0.   5.6485426   10.817843   14.666927   15.530574   10.227643
   0.   1.018D-15   1.890D-15   2.382D-15   2.085D-15   1.079D-15
```

Graphical Solution:



Ex.No.5 **Solving system of linear differential Equations.**

Question:

Solve the system of linear homogeneous ODE with constant coefficients given by

$$\frac{dy_1}{dx} = y_1 + y_3; \frac{dy_2}{dx} = y_1 + y_2 - y_3; \frac{dy_3}{dx} = 5y_1 + y_2 + y_3.$$

Aim:

To solve the given system of linear homogeneous ODE using Scilab.

Algorithm:

Step:1 Start the program.

Step:2 Define the function $spec()$ to find the eigen values of A

Step:3 Display the eigen vectors of A

Step:4 Solve for S using the values of X and B

Step:5 Display the value of S

Step:6 Stop the program.

Scilab Coding:

```
Clc;
A=[1 0 1;1 1 -1;5 1 1]
[c,d]=spec(A);
disp(spec(A),"The Eigenvalues of the Matrix A are:")
disp(c, "The corresponding Eigen vextor is:")
X=c;Y=[1;2;3]
B=linsolve(X,Y)
S=X.*[B B B]
```

Output:

```
 The Eigenvalues of Matrix A are:

   3.1149075
  -0.8608059
   0.7458983

 The corresponding Eigen vextor is:

  -0.4170021  -0.3827458   0.1983289
   0.2198294   0.588434   -0.9788391
  -0.8819208   0.7122156  -0.0503957

--> B
 B  =

   3.5181246
   0.3600066
   3.049763


--> S
 S  =

  -1.4670652  -1.3465474   0.6977459
   0.07914     0.2118401  -0.3523886
  -2.6896495   2.1720889  -0.153695
```

Ex.No.6. Solution of differential equation using Euler method

Question:

Solve $\frac{dy}{dx} = x + y, y(1) = 1$ with h=0.1 using Euler’s method.

Aim:

To solve the given differential equation using Euler’s method.

Algorithm:

Step:1 Start the program.

Step:2 Define the given function $f(x, y)$ as custom function.

Step:3 Define the size size h and initial value of y.

Step:4 Define the Euler formula using for loop.

Step:5 Display the value of $y(i + 1)$

Step:6 Stop the program.

Scilab Coding:

```
clc ;
deff ('y=f(x,y)' , 'y=x+y') y(1) =1;
h =0.1;
for i =1:6
printf( '\ny(%g)=%g\n' ,(i -1) /10 ,y(i))
y(i+1)=y(i)+h*f((i –1)/10,y(i))
end
```

Output:

```
Console

y(0)=1

y(0.1)=1.1

y(0.2)=1.22

y(0.3)=1.362

y(0.4)=1.5282

y(0.5)=1.72102
```

Ex.No.7 **Solution of differential equation using Modified Euler method**

Question:

Solve $\frac{dy}{dx} = x^2 + y^2, y(1) = 1$ with h=0.2 using Modified Euler's method.

Aim:

To solve the given differential equation using Modified Euler's method.

Algorithm:

Step:1 Start the program.

Step:2 Define the given function $f(x, y)$ as custom function. and initial value of y.

Step:3 Define the size size h

Step:4 Define the modified Euler formula using for loop.

Step:5 Print the value of $y(i + 1)$

Step:6 Stop the program.

Scilab Coding:

```
clc ;
deff ('y=f(x,y)','y=x^2+y^2')
y=1;
h=0.2;
for i =1:5
x =(i -1)*h
x1=x+h
ye=y+h*f(x,y)
y=y+h*(f(x,y)+f(x1 ,ye))/2
printf ( '\n y (%g) = %g\n' ,x1 ,y)
end
```

Output:

```
Console

y (0.2) = 1.248

y (0.4) = 1.66946

y (0.6) = 2.51042

y (0.8) = 4.71739

y (1) = 15.7486
```

Ex.No.8 Solution of differential equation using 4^{th} order Runge-Kutta method.

Question:

Solve $\frac{dy}{dx} = x + y, y(1) = 1$ with h=0.1 using RK method of fourth order.

Aim:

To solve the given differential equation using RK method.

Algorithm:

Step:1 Start the program.

Step:2 Define the given function $f(x, y)$ as custom function. and initial value of y.

Step:3 Define the size size h

Step:4 Find the values of K1,K2,K3 and K4 using the given function.

Step:5 Define the modified Euler formula using for loop.

Step:5 Print the value of y_1

Step:6 Stop the program.

Scilab Coding:

```
clc ;
deff ( 'y=f(x,y)','y=x+y')
y=1; x=1;h =0.1;
K1=h*f(x,y);
K2=h*f(x+h/2,y+K1/2) ;
K3=h*f(x+h/2,y+K2/2) ;
K4=h*f(x+h,y+K3);
disp("RK Method of Fourth Order")
disp(K4,'K4 =',K3,'K3=',K2,'K2=',K1,'K1=')
y1=y+(K1+2*K2+2*K3+K4)/6
```

Output:

```
Console

RK Method of Fourth Order

K1=

 0.2

K2=

 0.215

K3=

 0.21575

K4 =

 0.231575

--> y1
 y1  =

  1.2155125
```