Semester – II

18CCU202 BUSINESS MATHEMATICS AND STATISTICS 8H – 6C

Instruction Hours / week L: 6 T: 2 P : 0 Marks: Internal: 40 External: 60 Total: 100

End Semester Exam: 3 Hours

COURSE OBJECTIVES

To make the students

1. To understand the concept of matrices
2. To acquire the knowledge of differential calculus
3. To know the concepts of central tendency and dispersion
4. To understand the correlation and regression concepts
5. To be aware of the index numbers and trend analysis

COURSE OUTCOMES:

Learners should be able to

1. Utilize the concept of matrices, differential calculus to solve business problems
2. Calculate and apply the measure of central tendency and dispersion in decision making.
3. Evaluate the relationship and association between variables to formulate the strategy in business.
4. Apply the concept of index numbers and trend analysis in business decisions.
5. Demonstrate capabilities as problem-solving, critical thinking, and communication skills related to the discipline of statistics.

UNIT I Matrices & Basic Mathematics of Finance

Definition of a matrix. Types of matrices; Algebra of matrices. Calculation of values of determinants up to third order; Adjoint of a matrix; Finding inverse of a matrix through ad joint; Applications of Matrices to solution of simple business and economic problems- Simple and compound interest Rates of interest; Compounding and discounting of a sum using different types of rates

UNIT II Differential Calculus

Mathematical functions and their types – linear, quadratic, polynomial; Concepts of limit and continuity of a function; Concept of differentiation; Rules of differentiation – simple standard forms. Applications of differentiation – elasticity of demand and supply; Maxima and Minima of functions (involving second or third order derivatives) relating to cost, revenue and profit.

UNIT III Uni-variate Analysis

Measures of Central Tendency including arithmetic mean, geometric mean and harmonic mean: properties and applications; mode and median. Partition values - quartiles, deciles, and percentiles. Measures of Variation: absolute and relative. Range, quartile deviation and mean deviation; Variance and Standard deviation: calculation and properties.

UNIT IV Bi-variate Analysis

Simple Linear Correlation Analysis: Meaning, and measurement. Karl Pearson's co-efficient and Spearman's rank correlation Simple Linear Regression Analysis: Regression equations and estimation. Relationship between correlation and regression coefficients

UNIT V Time-based Data: Index Numbers and Time-Series Analysis

Meaning and uses of index numbers; Construction of index numbers: Aggregative and average of relatives – simple and weighte Tests of adequacy of index numbers, Construction of consumer price indices. Components of time series; additive and multiplicativ models; Trend analysis: Finding trend by moving average method and Fitting of linear trend line using principle of least squares

SUGGESTED READINGS :

1. Sreyashi Ghosh and Sujata Sinha (2018), Business Mathematics and Statistics, 1st edition, Oxford University Press; New Delhi.
2. Asim Kumar Manna (2018), Business Mathematics and Statistics, 1st edition, McGraw Hill Education, New Delhi.
3. S.P. Gupta and P.K. Gupta (2013), Business Statistics and Business Mathematics, S Chand Publishing, New Delhi.
4. Mariappan (2015), Business Mathematics, 1st edition, Pearson Education, New Delhi.
5. J.K.Sharma,)2014) Business statistics, 4th edition, Vikas Publishing House, New Delhi

Semester – II

18BPU202 BUSINESS MATHEMATICS AND STATISTICS 8H – 6C

Instruction Hours / week L: 6 T: 2 P : 0 Marks: Internal: 40 External: 60 Total: 100

End Semester Exam: 3 Hours

COURSE OBJECTIVES

To make the students

6. To understand the concept of matrices
7. To acquire the knowledge of differential calculus
8. To know the concepts of central tendency and dispersion
9. To understand the correlation and regression concepts
10. To be aware of the index numbers and trend analysis

COURSE OUTCOMES:

Learners should be able to

6. Utilize the concept of matrices, differential calculus to solve business problems
7. Calculate and apply the measure of central tendency and dispersion in decision making.
8. Evaluate the relationship and association between variables to formulate the strategy in business.
9. Apply the concept of index numbers and trend analysis in business decisions.
10. Demonstrate capabilities as problem-solving, critical thinking, and communication skills related to the discipline of statistics.

UNIT I Matrices & Basic Mathematics of Finance

Definition of a matrix. Types of matrices; Algebra of matrices. Calculation of values of determinants up to third order; Adjoint of a matrix; Finding inverse of a matrix through ad joint; Applications of Matrices to solution of simple business and economic problems- Simple and compound interest Rates of interest; Compounding and discounting of a sum using different types of rates

UNIT II Differential Calculus

Mathematical functions and their types – linear, quadratic, polynomial; Concepts of limit and continuity of a function; Concept of differentiation; Rules of differentiation – simple standard forms. Applications of differentiation – elasticity of demand and supply; Maxima and Minima of functions (involving second or third order derivatives) relating to cost, revenue and profit.

UNIT III Uni-variate Analysis

Measures of Central Tendency including arithmetic mean, geometric mean and harmonic mean: properties and applications; mode and median. Partition values - quartiles, deciles, and percentiles. Measures of Variation: absolute and relative. Range, quartile deviation and mean deviation; Variance and Standard deviation: calculation and properties.

UNIT IV Bi-variate Analysis

Simple Linear Correlation Analysis: Meaning, and measurement. Karl Pearson's co-efficient and Spearman’s rank correlation Simple Linear Regression Analysis: Regression equations and estimation. Relationship between correlation and regression coefficients

UNIT V Time-based Data: Index Numbers and Time-Series Analysis

Meaning and uses of index numbers; Construction of index numbers: Aggregative and average of relatives – simple and weighted, Tests of adequacy of index numbers, Construction of consumer price indices. Components of time series; additive and multiplicative models; Trend analysis: Finding trend by moving average method and Fitting of linear trend line using principle of least squares

SUGGESTED READINGS :

6. Sreyashi Ghosh and Sujata Sinha (2018), Business Mathematics and Statistics, 1st edition, Oxford University Press; New Delhi.
7. Asim Kumar Manna (2018), Business Mathematics and Statistics, 1st edition, McGraw Hill Education, New Delhi.
8. S.P. Gupta and P.K. Gupta (2013), Business Statistics and Business Mathematics, S Chand Publishing, New Delhi.
9. Mariappan (2015), Business Mathematics, 1st edition, Pearson Education, New Delhi.
10. J.K.Sharma,)2014) Business statistics, 4th edition, Vikas Publishing House, New Delhi

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

LECTURE PLAN

DEPARTMENT OF MATHEMATICS

STAFF NAME: M.Jannath Begam

SUBJECT NAME: Business Mathematics and Statistics **SUB.CODE: 19CCU202**

SEMESTER: II **CLASS: I B.Com CA-B**

UNIT I			
1	1	Introduction and definition of a matrix.	S4:Chap-15 Pg.No:280-281
2	1	Types of matrices	S4:Chap-15 Pg.No:281-284
3	1	**Tutorial-I**	
4	1	Algebra of matrices	S4:Chap-15 Pg.No:285-286
5	1	Calculation of values of determinants up to third order;	S4:Chap-15 Pg.No:297-301
6	1	**Tutorial-II**	
7	1	Adjoint of a matrix	S4:Chap-15 Pg.No:306-307
8	1	Problems on finding inverse of a matrix through ad joint	S4:Chap-15 Pg.No:307-313
9	1	**Tutorial-III**	
10	1	Applications of matrices to solution of simple business and economic problems	S4:Chap-16 Pg.No:355-361
11	1	Continuations on applications of matrices to solution of simple business and economic problems	S4:Chap-16 Pg.No:355-361
12	1	**Tutorial-IV**	
13	1	Simple and compound interest Rates of interest	S1:Chap:6:Pg.No:111-143
14	1	Continuation onSimple and compound interest Rates of interest	S1:Chap:6:Pg.No:111-143
15	1	**Tutorial-V**	
16	1	Problems on Compounding and discounting of a sum using different types of rates	S1:Chap:6:Pg.No:143-144
17	1	**Tutorial-VI**	

18	1	Recapitulation & discussion of possible questions	
	Total No of Hours Planned For Unit I=18		
		UNIT II	
1	1	Introduction of Mathematical Functions	S4:Chap:17:Pg.No:363-364
2	1	Types of Mathematical Functions: linear,quadratic,polynomial formula and its problems	S4:Chap:17:Pg.No:363-364
3	1	**Tutorial-I**	
4	1	Concept of limit of a function	S4:Chap:17:Pg.No:363-364
5	1	Concept of Continuity of a function	S4:Chap:17:Pg.No:364-365
6	1	**Tutorial-II**	
7	1	Concept of differentiation	S4:Chap:17:Pg.No:366-371
8	1	Problems on differentiation	S4:Chap:17:Pg.No:372-377
9	1	**Tutorial-III**	
10	1	Rules of differentiation- simple standard forms	S4:Chap:17:Pg.No:380-384
11	1	Applications of Differentiation in elasticity of demand and supply	S4:Chap:17:Pg.No: 384-387
12	1	**Tutorial-IV**	
13	1	Maxima and Minima of functions	S4:Chap:17:Pg.No:390-394
14	1	Maxima and Minima of functions (involving second order derivatives) relating to cost, revenue and profit	S4:Chap:17:Pg.No:395-410
15	1	**Tutorial-V**	
16	1	Maxima and Minima of functions (involving third order derivatives) relating to cost, revenue and profit	S4:Chap:17:Pg.No: 410-415
17	1	**Tutorial-VI**	
18	1	Recapitulation & discussion of possible questions	

	Total No of Hours Planned For Unit II=18		
		UNIT III	
1	1	Introduction on Measures of Central Tendency including arithmetic mean, geometric mean and its problems	S2:Chap:5:Pg.No:5.1-5.3
2	1	Measures of Central Tendency including harmonic mean and its problems	S5:Chap:3:Pg.No:101-102
3	1	**Tutorial-I**	
4	1	Properties and applications of mean	S2:Chap:5:Pg.No:5.3-5.5
5	1	Mode and median and its problems	S2:Chap:5:Pg.No:5.6-5.8
6	1	Partition values – quartiles and its problems	S5:Chap:3:Pg.No:105-106
7	1	**Tutorial-II**	
8	1	Partition values –deciles and its problems-percentiles and its problems	S5:Chap:3:Pg.No:107-111
9	1	Measures of Variation: absolute and relative.	S5:Chap:3:Pg.No:128-129
10	1	**Tutorial-III**	
11	1	Range and quartile deviation definition and its problems	S5:Chap:4:Pg.No:131-137
12	1	Problems on Range and Problems on quartile deviation and Mean deviation	S5:Chap:4:Pg.No:131-137
13	1	**Tutorial-IV**	
14	1	Variance and Standard deviation: problems and properties	S3:Chap:5:Pg.No:134-140
15	1	**Tutorial-V**	
16	1	Continuation on Variance and Standard deviation: problems and properties	S3:Chap:5:Pg.No:140-143
17	1	**Tutorial-VI**	
18	1	Recapitulation & discussion of possible questions	
	Total No of Hours Planned For Unit III=18		
		UNIT IV	
1	1	Introduction on Simple Linear Correlation Analysis, Meaning and measurement of Correlation	S5:Chap:13:Pg.No:452-455
2	1	Karl Pearson's co-efficient problems	S5:Chap:13:Pg.No:458-459
3	1	**Tutorial-I**	

4	1	Problems on Karl Pearson's co-efficient	S5:Chap:13:Pg.No:459-460
5	1	**Tutorial-II**	
6	1	Spearman's rank correlation problems	S5:Chap:13:Pg.No:466-467
7	1	Continuation of problems on Spearman's rank correlation	S5:Chap:13:Pg.No:468-471
8	1	**Tutorial-III**	
9	1	Simple Linear Regression Analysis: Regression equations	S2:Chap:7:Pg.No:7.1-7.3
10	1	Problems on Simple Linear Regression Analysis: Regression equations	S2:Chap:7:Pg.No:7.4-7.5
11	1	**Tutorial-IV**	
12	1	Problems on Simple Linear Regression Analysis: Regression equations and estimation	S2:Chap:7:Pg.No:7.6-7.7
13	1	Relationship between correlation and regression coefficients	S5:Chap:13:Pg.No:610-611
14	1	**Tutorial-V**	
15	1	Continuation on problems on correlation and regression coefficients	S1:Chap:8:Pg.No:163-212
16	1	Continuation on problems on correlation and regression coefficients	S1:Chap:8:Pg.No:163-212
17	1	**Tutorial-VI**	
18	1	Recapitulation & discussion of possible questions	
	Total No of Hours Planned For Unit IV=18		
		UNIT V	
1	1	Introduction on index numbers Meaning and its uses	S5:Chap:17:Pg.No:587-590
2	1	Construction of index numbers	S5:Chap:17:Pg.No:598-599
3	1	**Tutorial-I**	
4	1	Problems on unweighted price index and weighted price index	S5:Chap:17:Pg.No:599-607
5	1	**Tutorial-II**	
6	1	Tests of adequacy of index numbers	S5:Chap:17:Pg.No:617-618
7	1	**Tutorial-III**	
8	1	Construction of consumer price indices.	S5:Chap:17:Pg.No:632-634

9	1	Components of time series; additive and multiplicative models	S5:Chap:16:Pg.No:550-552
10	1	**Tutorial-IV**	
11	1	Trend analysis: Finding trend by moving average method	S5:Chap:16:Pg.No:565-566
12	1	**Tutorial-V**	
13	1	Fitting of linear trend line using principle of least squares	S5:Chap:16:Pg.No:566-567
14	1	**Tutorial-VI**	
15	1	Recapitulation & discussion of possible questions	
16	1	Discussion of previous ESE question papers	
17	1	Discussion of previous ESE question papers	
18	1	Discussion of previous ESE question papers	
	Total No of Hours Planned For Unit I=18		

Suggested Reading

S1. Sreyashi Ghosh and Sujata Sinha (2018), Business Mathematics and Statistics, 1st edition,Oxford University Press; New Delhi.
S2. Asim Kumar Manna (2018), Business Mathematics and Statistics, 1st edition, McGraw Hill Education, New Delhi.
S3. S.P. Gupta and P.K. Gupta (2013), Business Statistics and Business Mathematics, S Chand Publishing, New Delhi.
S4. Mariappan (2015), Business Mathematics, 1st edition, Pearson Education, NewDelhi.
S5. J.K.Sharma, (2014) Business statistics, 4th edition, Vikas Publishing House, New Delhi

Signature student Representative Signature of the Course Faculty

Signature of the Class Tutor Signature of Coordinator

Head of the Department

UNIT – I

SYLLABUS

Matrices & Basic Mathematics of Finance
Definition of a matrix.Types of matrices; Algebra of matrices. Calculation of values of determinants up to third order; Adjoint of a matrix; Finding inverse of a matrix through ad joint; Applications of matrices to solution of simple business and economic problems- Simple and compound interest Rates of interest – nominal, effective and continuous – their interrelationships; Compounding and discounting of a sum using different types of rates

Matrix

An array of **mn** numbers arranged in **m** rows and **n** columns and bounded by square bracket [ ] brackets ( ) or || || is called **m** by **n** matrix and is represented as

$$A = \begin{bmatrix} a_{11} & a_{12} & .... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ a_{31} & a_{32} & ... & a_{3n} \\ .... & .... & .... & .... \\ a_{i1} & a_{i2} & ... & a_{in} \\ .... & .... & .... & .... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ .... & .... & .... & .... \\ a_{i1} & a_{i2} & .... & a_{in} \\ .... & .... & .... & .... \\ a_{m1} & a_{m2} & .... & a_{mn} \end{pmatrix} \quad ...(1)$$

(1) is known as m × n matrix in which there are m rows and n columns.Each member of m × n matrix is known as an element of the matrix.

Note:

1. In general, we denote a Matrix by capital letter $A = [a_{ij}]$, where a_{ij} are elements of Matrix in which its position is in i^{th} row and j^{th} column i.e. first suffix denote row number and second suffix denote column number.
2. The elements $a_{11}, a_{22},...,a_{nn}$ in which both suffix are same called diagonal elements, all other elements in which suffix are not same are called non diagonal elements.
3. The line along which the diagonal element lie is called the Principal Diagonal.

Different types of Matrices

Zero Matrix or Null Matrix. A Matrix in which each elements is equal to zero is called a zero matrix or null matrix.

e.g., $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

are zero matrix respectively of order 2 × 3; 3×2 and 3×3.

In general we denote zero matrix of order m× n by $O_{m \times n}$. Matrix other than Zero Matrix are called Non- Zero Matrix.

Square matrix. Matrix in which number of row becomes equal to number of column is called square matrix i.e.

If matrix A is of type m × n, where m = n then the matrix is called square matrix otherwise it is called rectangular matrix.

Row Matrix. A matrix of type 1 × n, having only one row is called row matrix. For example $\begin{pmatrix} 1 & -2 & 3 \end{pmatrix}$ is a row matrix.

Column Matrix. A matrix of type m × 1, having only one column is called column matrix. For example $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ is a column matrix.

Diagonal Matrix. A square matrix in which all non diagonal elements are equal to zero is called diagonal matrix i.e.
A square matrix $A = [a_{ij}]$, is diagonal matrix if $a_{ij} = 0$ for $i \neq j$. Thus

$\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$ or $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ are diagonal matrices.

Scalar Matrix. Diagonal matrices in which all diagonal elements are equal are called scalar matrix i.e.
A square matrix $A = [a_{ij}]$, is scalar matrix if $a_{ij} = 0$ for $i \neq j$ and $a_{ij} = k$, for $i = j$ .Thus
$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ is scalar matrix.

Unit Matrix or Identity Matrix. A scalar matrix in which all diagonal elements are unity are called Unit matrix or Identity matrix generally denoted by I_n.
A square matrix $A = [a_{ij}]$, is Identity matrix if $a_{ij} = 0$ for $i \neq j$ and $a_{ij} = 1$, for $i = j$ .Thus
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, is identity matrix of order 3 × 3.

Triangular matrix are of two types:
(a) Upper Triangular Matrix. It is a matrix in which all elements below the principal diagonal are zero

e.g. $\begin{pmatrix} 2 & 4 & 8 \\ 0 & -7 & -3 \\ 0 & 0 & 3 \end{pmatrix}$

(b) Lower Triangular Matrix. It is a Matrix in which all elements above the principal diagonal are zero

e.g. $\begin{pmatrix} 2 & 0 & 0 \\ 4 & -7 & 0 \\ -6 & 5 & 3 \end{pmatrix}$

Sub Matrix: A matrix B obtained by deleting some rows or some column or both of a matrix A, is called a sub matrix of A.

For example.If $A=\begin{pmatrix} 2 & 1 & 5 \\ 0 & -7 & 9 \\ -3 & 4 & 3 \end{pmatrix}$ then the matrices $\begin{pmatrix} 2 & 1 \\ 0 & -7 \end{pmatrix}$, $\begin{pmatrix} 2 & 1 & 5 \\ 0 & -7 & 9 \end{pmatrix}$ etc. are sub matrix of A.

Transpose of a matrix

If matrix A is of type m × n, then the matrix obtained by interchanging the rows and the columns of A is known as Transpose of Matrix A, denoted by A' or A^T i.e.

A= $[a_{ij}]$ of m × n order then

A' or $A^T=[a_{ji}]$ of n × m order Matrix,

Now if A', B' be the transpose of matrix A and B respectively, then

(i) A = (A′)′ i.e. the transpose of transpose of a matrix A is matrixA itself.

(ii) (A + B)′ = A′ + B′ i.e. the transpose of the sum of two matrices is equal to the sum of their transposes.

(iii) (kA)′=k A′, where k is a scalar.

(iv) (AB)′ = B′A′ i.e. the transpose of the product of two matrices is equal to the product of their transposes, taken in reversed order.

Conjugate of a Matrix

If A be a matrix of order m × n, over complex number system, then the matrix obtained from A by replacing each of its elements by their corresponding complex conjugates is called the conjugate of A and is denoted by $\overline{A}$, where $\overline{A}$ is also of same order m × n. If $\overline{A}$, $\overline{A}$ be the conjugate matrices of A, B respectively, then

(i) $\overline{\left(\overline{A}\right)} = A$.

(ii) $\overline{\left(A + B\right)} = \overline{A} + \overline{B}$, where A and B are conformable for addition.

(iii) $\overline{\left(kA\right)} = \overline{k}.\overline{A}$, where k is any complex number.

(iv) $\overline{AB} = \overline{A}.\overline{B}$, where A and B are conformable for multiplication.

Transpose Conjugate of a Matrix

The transpose of the conjugate or conjugate of the transpose of a matrix A is called Transpose conjugate of A and is denoted by A^θ. Thus

$A^\theta = (\overline{A})' = \overline{(A')}$.

If A^θ, B^θ denote the transposed conjugate of A, B respectively, then

(i) $(A^\theta)^\theta = A$.

(ii) $(A + B)^\theta = A^\theta + B^\theta$, where A and B are conformable for addition.

(iii) $(kA)^\theta = \overline{k}.A^\theta$, where k is any complex number.

(iv) $(AB)^\theta = B^\theta.A^\theta$, where A and B are conformable for multiplication.

Adjoint of a Square Matrix

If A is an n- rowed square matrix, then adjoint of A is defined as a transpose of matrix obtained by replacing each of its elements by its cofactors.

Theorem 1.1: If A be an n-square matrix, then A(adj. A)= (adj.A)A=$|A|I_n$, where I_n denotes the unit matrix of order n.

Theorem 1.2: If A and B are square matrix of the same order n, then adj. (AB)=(adj.B)(adj.A) .

Inverse of Square Matrix

Let A be n-square matrix, if there exist an n-square matrix B such that

$AB = BA = I_n$, then the matrix A is called invertible and the matrix B is called inverse of A. Inverse of a square matrix is denoted by A^{-1}.

Note. 1. From definition it is clear that A is the inverse of B.

2. A non-square matrix does not have any inverse.

Singular and Non Singular Matrices

A square matrix A is said to be singular or non singular according as $|A| = 0$ or $|A| \neq 0$.

Orthogonal Matrix

Any square matrix A is said to be orthogonal if $AA' = A'A = I$, this indicates that the row vectors (column vectors) of an orthogonal matrix A are mutually orthogonal unit vectors.

Unitary Matrix

Any square A with complex elements is said to be unitary if $A.A^{\theta} = A^{\theta}A = I$.

Symmetric And Skew Symmetric Matrices

Symmetric Matrix

A square matrix A = $[a_{ij}]$ is said to be symmetric if $a_{ij} = a_{ji}$ for all i and j.

Examples $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$, $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 7 \end{bmatrix}$

Skew Symmetric Matrix

If a square matrix A has its elements such that $a_{ij} = -a_{ji}$ for i and j and the leading diagonal elements are zeros, then matrix A is known as skew matrix. For example $\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & h & g \\ -h & 0 & -f \\ -g & f & 0 \end{bmatrix}$ are skew symmetric matrices.

Example

Some examples of matrices are:

$$A = \begin{pmatrix} -4 & 3 & -6 & 9 \end{pmatrix} \quad B = \begin{pmatrix} 5 & -2 \\ 3 & -7 \end{pmatrix} \quad C = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 7 & -4 \\ 0 & 1 & -3 \end{pmatrix}$$

1. $(A)+(B)$ Given: $A = \begin{pmatrix} 4 & -3 & 6 \\ -8 & 5 & -9 \end{pmatrix} B = \begin{pmatrix} -5 & 6 & -2 \\ 3 & 7 & -4 \end{pmatrix}$

Solution:
$$A+B = \begin{pmatrix} 4+-5 & -3+6 & 6+-2 \\ -8+3 & 5+7 & -9+-4 \end{pmatrix} \Rightarrow A+B = \begin{pmatrix} -1 & 3 & 4 \\ -5 & 12 & -13 \end{pmatrix}$$

2. $(A)-(B)$ Given: $A = \begin{pmatrix} 6 & -7 \\ -4 & 5 \\ -3 & 2 \end{pmatrix} B = \begin{pmatrix} -8 & 3 \\ 3 & -1 \\ 2 & -8 \end{pmatrix}$

Solution:
$$A-B = \begin{pmatrix} 6--8 & -7-3 \\ -4-3 & 5--1 \\ -3-2 & 2--8 \end{pmatrix} \Rightarrow A-B = \begin{pmatrix} 14 & -10 \\ -7 & 6 \\ -5 & 10 \end{pmatrix}$$

3. $5\begin{pmatrix} -4 & 3 \\ 6 & -2 \end{pmatrix} \Rightarrow$ Solution: $\begin{pmatrix} -20 & 15 \\ 30 & -10 \end{pmatrix}$

4. $(A)(B)$ Given: $A = \begin{pmatrix} 6 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -3 \\ 4 & -5 \\ 1 & -6 \end{pmatrix}$

2. Given: $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 3 & -5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -2 & 1 \\ 4 & -3 \end{bmatrix}$

Find:

(a) $\mathbf{A} + \mathbf{B}$ $\quad \mathbf{A} + \mathbf{B} = \begin{bmatrix} -1 & 1 \\ 7 & -8 \end{bmatrix}$

(b) $2\mathbf{A} - 3\mathbf{B}$ $\quad 2\mathbf{A} - 3\mathbf{B} = \begin{bmatrix} 8 & -3 \\ -6 & -1 \end{bmatrix}$

(c) $\mathbf{AB}$ $\quad \mathbf{AB} = \begin{bmatrix} -2 & 1 \\ -26 & 18 \end{bmatrix}$

(d) $\mathbf{AB} + \mathbf{BA}$ $\quad \mathbf{AB} + \mathbf{BA} = \begin{bmatrix} -1 & -4 \\ -31 & 33 \end{bmatrix}$

(e) $\mathbf{A}^{\mathrm{T}}\mathbf{B}$ (where $\mathbf{A}^{\mathrm{T}}$ denotes the transpose of $\mathbf{A}$)

$$\mathbf{A}^{T}\mathbf{B} = \begin{bmatrix} 10 & -8 \\ -20 & 15 \end{bmatrix}$$

(f) $\mathbf{B}^{\mathrm{T}}\mathbf{A}$ $\quad \mathbf{B}^{\mathbf{T}}\mathbf{A} = \begin{bmatrix} 10 & -20 \\ -8 & 15 \end{bmatrix}$

(g) $\mathbf{A}^{2}\mathbf{B}$ $\quad \mathbf{A}^{\mathbf{2}}\mathbf{B} = \begin{bmatrix} -2 & 1 \\ 124 & -87 \end{bmatrix}$

3. Given: $\mathbf{A} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}$

Find **AB** and **BA**

$$\mathbf{AB} = \begin{bmatrix} a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & a_3b_3 \end{bmatrix}$$

$\mathbf{BA} = b_1a_1 + b_2a_2 + b_3a_3$ (Note, a scalar)

4. Find the determinant, all minors and cofactors, and the inverse of each of the following matrices:

(a) $\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \\ 4 & 0 & 0 \end{bmatrix}$

Determinant = 20

Matrix of minors: $\begin{bmatrix} 0 & -12 & -8 \\ 0 & 4 & -4 \\ 5 & 5 & 0 \end{bmatrix}$

Matrix of cofactors (= signed minors): $\begin{bmatrix} 0 & 12 & -8 \\ 0 & 4 & 4 \\ 5 & -5 & 0 \end{bmatrix}$

Inverse matrix: $\begin{bmatrix} 0 & 0 & 0.25 \\ 0.6 & 0.2 & -0.25 \\ -0.4 & 0.2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & -1 & -1 \\ 2 & 0 & 3 \\ 4 & 0 & 4 \end{bmatrix}$

Determinant: –4

Matrix of minors: $\begin{bmatrix} 0 & -4 & 0 \\ -4 & 12 & 4 \\ -3 & 8 & 2 \end{bmatrix}$

Matrix of cofactors (=signed minors) $\begin{bmatrix} 0 & 4 & 0 \\ 4 & 12 & -4 \\ -3 & -8 & 2 \end{bmatrix}$

Inverse matrix: $\begin{bmatrix} 0 & -1 & 0.75 \\ -1 & -3 & 2 \\ 0 & 1 & -0.5 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 2 & -1 \\ 2 & -1 & 4 \\ 2 & 0 & 2 \end{bmatrix}$

Determinant: –4

Minors $\begin{bmatrix} -2 & -4 & 2 \\ 4 & 12 & -4 \\ 7 & 22 & -9 \end{bmatrix}$

Cofactors (= signed minors): $\begin{bmatrix} -2 & 4 & 2 \\ -4 & 12 & 4 \\ 7 & -22 & -9 \end{bmatrix}$

Inverse matrix: $\begin{bmatrix} 0.5 & 1 & -1.75 \\ -1 & -3 & 5.5 \\ -0.5 & -1 & 2.25 \end{bmatrix}$

(d) $\begin{bmatrix} 10 & 5 & 1 \\ 2 & -1 & 4 \\ 4 & 0 & 4 \end{bmatrix}$

Determinant: 4

Minors: $\begin{bmatrix} -4 & -8 & 4 \\ 20 & 36 & -20 \\ 21 & 38 & -20 \end{bmatrix}$

Cofactors: $\begin{bmatrix} -4 & 8 & 4 \\ -20 & 36 & 20 \\ 21 & -38 & -20 \end{bmatrix}$

inverse matrix: $\begin{bmatrix} -1 & -5 & 5.25 \\ 2 & 9 & -9.5 \\ 1 & 5 & -5 \end{bmatrix}$

The Cofactor Expansion for Determinants

Every square matrix has a determinant. All matrices with zero determinant are singular.
All matrices with non-zero determinant are invertible.

The determinant of a (1×1) matrix $A = [\ a\ ]$ is just $\det A = a$.

From section 2.3, the determinant of a (2×2) matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\det A = ad - bc$.

The determinants of all higher-order matrices can be expressed in terms of lower-order determinants.

Determinants and Inverse Matrices

For any set of square matrices of the same dimensions, the determinant of a product is the product of the determinants:

$\det(AB) = (\det A)(\det B)$, $\det(ABC) = (\det A)(\det B)(\det C)$, etc.

It then follows that

$$\boxed{\det\left(A^k\right) = \left(\det A\right)^k}$$

$$AA^{-1} = I \ \Rightarrow\ \det\left(AA^{-1}\right) = 1 \ \Rightarrow\ \det A \det\left(A^{-1}\right) = 1 \ \Rightarrow$$

$$\boxed{\det\left(A^{-1}\right) = \frac{1}{\det A}}$$

The **adjugate** (or **adjoint**) of any square matrix A is the transpose of the matrix of cofactors of A:

$\text{adj}(A) = \left[c_{ij}(A)\right]^{\mathrm{T}}$ [The 2×2 case is $\text{adj}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.]

Example 1

Compute adj (A), Aadj (A) and det (A) for $A = \begin{bmatrix} 1 & 9 & 1 \\ 0 & 2 & 0 \\ 1 & -4 & 2 \end{bmatrix}$.

The matrix of cofactors is

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} +\begin{vmatrix} 2 & 0 \\ -4 & 2 \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} & +\begin{vmatrix} 0 & 2 \\ 1 & -4 \end{vmatrix} \\ -\begin{vmatrix} 9 & 1 \\ -4 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 9 \\ 1 & -4 \end{vmatrix} \\ +\begin{vmatrix} 9 & 1 \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} & +\begin{vmatrix} 1 & 9 \\ 0 & 2 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 4 & 0 & -2 \\ -22 & 1 & 13 \\ -2 & 0 & 2 \end{bmatrix}$$

$$\text{adj}\, A = C^{\mathrm{T}} = \begin{bmatrix} 4 & -22 & -2 \\ 0 & 1 & 0 \\ -2 & 13 & 2 \end{bmatrix}$$

$$A\,\text{adj}\, A = \begin{bmatrix} 1 & 9 & 1 \\ 0 & 2 & 0 \\ 1 & -4 & 2 \end{bmatrix}\begin{bmatrix} 4 & -22 & -2 \\ 0 & 1 & 0 \\ -2 & 13 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2I$$

Expand along the middle row to find $\det A$:
$\det A = 0 + 2 \times c_{22} + 0 = 2$

Note that $A\text{adj}(A) = (\det(A))\, I \quad \Rightarrow \quad A^{-1} = \dfrac{\text{adj}\, A}{\det A}$

Associated with every square matrix is a value called the determinant. This value for a 2×2 matrix is the number that results from the difference between the products of the numbers in each diagonal of the matrix. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ represents any 2×2 matrix, then the determinant is the result of **ad – bc.** The determinant of **A** can be represented as detA or as $|A|$.

Notation: $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Examples: Find the determinant of each of the following matrices:

1. $\begin{pmatrix} 5 & 3 \\ 7 & 2 \end{pmatrix}$ 2. $\begin{pmatrix} 6 & -4 \\ 2 & -3 \end{pmatrix}$ 3. $\begin{pmatrix} 4 & -2 \\ 3 & 5 \end{pmatrix}$ 4. $\begin{pmatrix} -3 & 8 \\ -2 & -4 \end{pmatrix}$

$(5)(2)-(7)(3)$	$(6)(-3)-(2)(-4)$	$(4)(5)-(3)(-2)$	$(-3)(-4)-(-2)(8)$
$10-21$	$-18--8$	$20--6$	$12--16$
-11	-10	26	28

The inverse of a 2×2 matrix can also be determined. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ represents any 2×2 matrix, then the inverse of **A**, written as $\mathbf{A^{-1}}$, is found by:

$$A^{-1} = \begin{pmatrix} \frac{d}{\det A} & \frac{-b}{\det A} \\ \frac{-c}{\det A} & \frac{a}{\det A} \end{pmatrix}$$

When a 2×2 matrix is multiplied by its inverse, the result is the identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Examples: Find the inverse of each of the following matrices:

1. $A = \begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}$

Solution: The first step is to find the determinant of (A).

$|A| = -12 - -10$

$|A| = -2$

$$A^{-1} = \begin{pmatrix} \frac{-4}{-2} & \frac{2}{-2} \\ \frac{-5}{-2} & \frac{3}{-2} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{4}{2} & \frac{-2}{2} \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix} or \begin{pmatrix} 2 & -1 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix}$$

Do not leave a negative sign in the denominator of any fraction. Apply the rule for division of integers and place the resulting sign in the numerator of the fraction.
When using the inverse matrix to solve a system of linear equations, it is best to leave all the elements in fraction form. To confirm that the answer is correct, multiply the matrix by its inverse.

Check: $$\begin{pmatrix} 3 & -2 \\ 5 & -4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ \frac{5}{2} & \frac{-3}{2} \end{pmatrix} \Rightarrow \begin{pmatrix} 6+\frac{-10}{2} & -3+\frac{6}{2} \\ 10+\frac{-20}{2} & -5+\frac{12}{2} \end{pmatrix} \Rightarrow \begin{pmatrix} 6-5 & -3+3 \\ 10-10 & -5+6 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of $(A)\times(A)^{-1}$ is the identity matrix. Therefore the inverse matrix is correct.

2. $A = \begin{pmatrix} 2 & -6 \\ 1 & 3 \end{pmatrix}$

Solution: The first step is to find the determinant of (A).

$$|A| = 6 - -6$$
$$|A| = 12$$

$$A^{-1} = \begin{pmatrix} \frac{3}{12} & \frac{6}{12} \\ \frac{-1}{12} & \frac{2}{12} \end{pmatrix} \quad or A^{-1} = \begin{pmatrix} \frac{1}{4} & \frac{1}{2} \\ \frac{-1}{12} & \frac{1}{6} \end{pmatrix}$$

When using the inverse matrix to solve a system of linear equations, it is best to leave all of the elements in fraction form with the same common denominator as shown in the first inverse rather than as reduced fractions as shown in the second inverse.

Check: $$\begin{pmatrix} 2 & -6 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} \frac{3}{12} & \frac{6}{12} \\ \frac{-1}{12} & \frac{2}{12} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{6}{12}+\frac{6}{12} & \frac{12}{12}-\frac{12}{12} \\ \frac{3}{12}-\frac{3}{12} & \frac{6}{12}+\frac{6}{12} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{12}{12} & \frac{0}{12} \\ \frac{0}{12} & \frac{12}{12} \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of $(A)\times(A)^{-1}$ is the identity matrix. Therefore the inverse matrix is correct.

The multiplication above was done using A^{-1} in which all the elements had the common denominator 12. Therefore, the resulting products could be manipulated in order to produce the identity matrix.

3. $A=\begin{pmatrix}-10 & 4\\ 5 & 2\end{pmatrix}$

Solution: The first step is to find the determinant of (A).

$$|A|=-20-20$$
$$|A|=-40$$
$$A^{-1}=\begin{pmatrix}\frac{2}{-40} & \frac{-4}{-40}\\ \frac{-5}{-40} & \frac{-10}{-40}\end{pmatrix}$$
$$A^{-1}=\begin{pmatrix}\frac{-2}{40} & \frac{4}{40}\\ \frac{5}{40} & \frac{10}{40}\end{pmatrix} \quad or A^{-1}=\begin{pmatrix}\frac{-1}{20} & \frac{1}{10}\\ \frac{1}{8} & \frac{1}{4}\end{pmatrix}$$

Check: $$\begin{pmatrix}-10 & 4\\ 5 & 2\end{pmatrix}\begin{pmatrix}\frac{-2}{40} & \frac{4}{40}\\ \frac{5}{40} & \frac{10}{40}\end{pmatrix}\Rightarrow\begin{pmatrix}\frac{20}{40}+\frac{20}{40} & \frac{-40}{40}+\frac{40}{40}\\ \frac{-10}{40}+\frac{10}{40} & \frac{20}{40}+\frac{20}{40}\end{pmatrix}\Rightarrow\begin{pmatrix}\frac{40}{40} & \frac{0}{40}\\ \frac{0}{40} & \frac{40}{40}\end{pmatrix}\Rightarrow\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

The product of $(A)\times(A)^{-1}$ is the identity matrix. Therefore the inverse matrix is correct.

4. $A=\begin{pmatrix}6 & -3\\ 5 & -2\end{pmatrix}$

Solution: The first step is to find the determinant of (A).

$$|A|=-12--15$$
$$|A|=3$$

$$A^{-1} = \begin{pmatrix} \frac{-2}{3} & \frac{3}{3} \\ \frac{-5}{3} & \frac{6}{3} \end{pmatrix} \quad or A^{-1} = \begin{pmatrix} \frac{-2}{3} & 1 \\ \frac{-5}{3} & 2 \end{pmatrix}$$

Check:

$$\begin{pmatrix} 6 & -3 \\ 5 & -2 \end{pmatrix} \begin{pmatrix} \frac{-2}{3} & \frac{3}{3} \\ \frac{-5}{3} & \frac{6}{3} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{-12}{3} + \frac{15}{3} & \frac{18}{3} - \frac{18}{3} \\ \frac{-10}{3} + \frac{10}{3} & \frac{15}{3} - \frac{12}{3} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{3}{3} & \frac{0}{3} \\ \frac{0}{3} & \frac{3}{3} \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The product of $(A)\times(A)^{-1}$ is the identity matrix. Therefore the inverse matrix is correct.

1. Find the determinant of each of the following matrices:

a) $\begin{pmatrix} 2 & -3 \\ -4 & -10 \end{pmatrix}$ b) $\begin{pmatrix} 7 & 3 \\ -4 & -2 \end{pmatrix}$ c) $\begin{pmatrix} 5 & -2 \\ -3 & 2 \end{pmatrix}$ d) $\begin{pmatrix} -8 & 5 \\ -4 & 3 \end{pmatrix}$

Solution :

a) $\begin{pmatrix} 2 & -3 \\ -4 & -10 \end{pmatrix}$ b) $\begin{pmatrix} 7 & 3 \\ -4 & -2 \end{pmatrix}$ c) $\begin{pmatrix} 5 & -2 \\ -3 & 2 \end{pmatrix}$ d) $\begin{pmatrix} -8 & 5 \\ -4 & 3 \end{pmatrix}$

det = -20- + 12 det = -14- -12 det = 10- + 6 det -24- - 20
det = -32 det = -2 det = 4 det = -4

2. Find the inverse of each of the following 2×2 matrices. Check your solution by performing the operation of $(A)\times(A)^{-1}$.

a) $A = \begin{pmatrix} 4 & 3 \\ -1 & -2 \end{pmatrix}$ b) $A = \begin{pmatrix} 6 & -3 \\ 7 & -4 \end{pmatrix}$ c) $A = \begin{pmatrix} 5 & 0 \\ -4 & 2 \end{pmatrix}$ d) $A = \begin{pmatrix} -9 & 3 \\ 5 & -2 \end{pmatrix}$

Solution :

a) $A = \begin{pmatrix} 4 & 3 \\ -1 & -2 \end{pmatrix}$ b) $A = \begin{pmatrix} 6 & -3 \\ 7 & -4 \end{pmatrix}$ c) $A = \begin{pmatrix} 5 & 0 \\ -4 & 2 \end{pmatrix}$ d) $A = \begin{pmatrix} -9 & 3 \\ 5 & -2 \end{pmatrix}$

$|A| = -8 - -3$ $|A| = -24 - -21$ $|A| = 10 - 0$ $|A| = 18 - 15$

$|A| = -5$ $|A| = -3$ $|A| = 10$ $|A| = 3$

$A^{-1} = \begin{pmatrix} \frac{-2}{-5} & \frac{-3}{-5} \\ \frac{1}{-5} & \frac{4}{-5} \end{pmatrix}$ $A^{-1} = \begin{pmatrix} \frac{-4}{-3} & \frac{3}{-3} \\ \frac{-7}{-3} & \frac{6}{-3} \end{pmatrix}$ $A^{-1} = \begin{pmatrix} \frac{2}{10} & \frac{0}{10} \\ \frac{4}{10} & \frac{5}{10} \end{pmatrix}$ $A^{-1} = \begin{pmatrix} \frac{-2}{3} & \frac{-3}{3} \\ \frac{-5}{3} & \frac{-9}{3} \end{pmatrix}$

$A^{-1} = \begin{pmatrix} \frac{2}{5} & \frac{3}{5} \\ \frac{-1}{5} & \frac{-4}{5} \end{pmatrix}$ $A^{-1} = \begin{pmatrix} \frac{4}{3} & \frac{-3}{3} \\ \frac{7}{3} & \frac{-6}{3} \end{pmatrix}$ $A^{-1} = \begin{pmatrix} \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{2} \end{pmatrix}$ $A^{-1} = \begin{pmatrix} \frac{-2}{3} & -1 \\ \frac{-5}{3} & -3 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 1 & -1 \\ \frac{7}{3} & -2 \end{pmatrix}$

Minors and cofactors of a Matrix

Let A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Minor of $a_{ij} \equiv M_{ij}$ **,**

is determinant obtained by deleting ith row and jth column.

$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$ is determinant obtained by deleting 1st row and 1st column

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}, M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}, M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}, M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}$$

$$M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}, M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}, M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Cofactor of $a_{ij} = C_{ij} = (-1)^{i+j} M_{ij}$

Signs of Cofactors

For 2x2 – matrix $\begin{bmatrix} + & - \\ - & + \end{bmatrix}$

For 3x3 – matrix $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

For 4x4 – matrix $\begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$

Example:1. Find all minors and cofactors of the matrix

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$$

Solution:

$$M_{11} = \begin{vmatrix} 0 & 3 \\ 5 & -4 \end{vmatrix} = -15, M_{12} = \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} = -10, M_{13} = \begin{vmatrix} 1 & 0 \\ 2 & -5 \end{vmatrix} = 5$$

$$M_{21} = \begin{vmatrix} 4 & -1 \\ 5 & -4 \end{vmatrix} = -11, M_{22} = \begin{vmatrix} 3 & -1 \\ 2 & -4 \end{vmatrix} = -10, M_{23} = \begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix} = 7$$

$$M_{31} = \begin{vmatrix} 4 & -1 \\ 0 & 3 \end{vmatrix} = 12, \quad M_{32} = \begin{vmatrix} 3 & -1 \\ 1 & 0 \end{vmatrix} = 10, \quad M_{33} = \begin{vmatrix} 3 & 4 \\ 1 & 0 \end{vmatrix} = -4$$

Cofactor of $a_{ij} = C_{ij} = (-1)^{i+j} M_{ij}$

$$C_{11} = -15, \quad C_{12} = 10, \quad C_{13} = 5$$
$$C_{21} = 11, \quad C_{22} = -10, \quad C_{23} = -7$$
$$C_{31} = 12, \quad C_{32} = -10, \quad C_{33} = -4$$

NOTE: Matrix of cofactors , $C = \begin{bmatrix} -15 & 10 & 5 \\ 11 & -10 & -7 \\ 12 & -10 & -4 \end{bmatrix}$

NOTE: Determinant of matrix of Cofactors by the method of Cofactors

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$
$$\det(A) = a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$$
$$\det(A) = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$$

The above equations can be used to check that the cofactors are found correctly as the values of determinants found must be equal, we open matrix from any row or column.

Example: 2 .Find the determinant of the matrix A by method of cofactors,

$$A = \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix}$$

Solution: **:Using the cofactors found in the last example**

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$
$$= 3(-15)+4(10)+(-1)(5)$$
$$=-45 + 40 - 5 = -10$$

NOTE: 3.We can find determinant by opening matrix from second or third row or first column, the value of the determinant will be same

$$\det(A) = a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$$
$$=(1)(11)+0(-10)+3(-7)=11 - 21 = -10$$

$$\det(A) = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$$
$$= 2(12) +5(-10)+(-4)(-4) =24 - 50 +16 = -10$$

NOTE : 4. Determinant of A can be obtained by multiplying any row or any column of matrix A with the corresponding row or column of the matrix of cofactors.

NOTE: 5. **Determinant of matrix A** = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

det A = $a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Inverse by method of Cofactors:

A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ **det A ≠ 0.**

Step:1. **Find Matrix of cofactors**

C = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

Step : 2. **Find Adjoint of matrix A , adj(A)**

Adj(A) = $\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$

Step: 3.

If A is an invertible matrix, det(A)≠ 0, then

$$A^{-1} = \frac{1}{\det A}[adj(A)]$$

Example: 3 . Find A^{-1} of matrix A

$$A = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 2 \\ -2 & 0 & -4 \end{bmatrix}$$ by the method of cofactors.

Solution: Cofactors of the matrix A are

$$C_{11} = \begin{vmatrix} 3 & 2 \\ 0 & -4 \end{vmatrix} = -12, C_{12} = -\begin{vmatrix} 0 & 2 \\ -2 & -4 \end{vmatrix} = -4, C_{13} = \begin{vmatrix} 0 & 3 \\ -2 & 0 \end{vmatrix} = 6$$

$$C_{21} = -\begin{vmatrix} 0 & 3 \\ 0 & 4 \end{vmatrix} = 0, \quad C_{22} = \begin{vmatrix} 2 & 3 \\ -2 & -4 \end{vmatrix} = -2, \; C_{23} = -\begin{vmatrix} 2 & 0 \\ -2 & 0 \end{vmatrix} = 0,$$

$$C_{31} = \begin{vmatrix} 0 & 3 \\ 3 & 2 \end{vmatrix} = -9, \quad C_{32} = -\begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = -4, \quad C_{33} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6$$

Matrix of cofactors, $C = \begin{bmatrix} -12 & -4 & 6 \\ 0 & -2 & 0 \\ -9 & -4 & 6 \end{bmatrix}$

Adjoint of matrix A, $\text{adj}(A) = \begin{bmatrix} -12 & 0 & -9 \\ -4 & -2 & -4 \\ 6 & 0 & 6 \end{bmatrix}$

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$
$$= 2(-12)+0(-4)+3(6)$$
$$=-24 +18 = - 6 \neq 0$$

Inverse of matrix A is

$$A^{-1} = \frac{1}{\det A}[adj(A)] = \frac{1}{-6}\begin{bmatrix} -12 & 0 & -9 \\ -4 & -2 & -4 \\ 6 & 0 & 6 \end{bmatrix}$$

NOTE :

If we can find A^{-1} , then solution of linear system

AX = B is $X = A^{-1}B$

Adjoint:
The adjoint is the transpose of the cofactors (c^T)

To find the cofactors you find the '**minor**' of each element (and use the alternate signs as shown above).

If: $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

The minors of each element

Matrix of cofactors = $\begin{bmatrix} +\begin{vmatrix} e & f \\ h & i \end{vmatrix} - \begin{vmatrix} d & f \\ g & i \end{vmatrix} + \begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ -\begin{vmatrix} b & c \\ h & i \end{vmatrix} + \begin{vmatrix} a & c \\ g & i \end{vmatrix} - \begin{vmatrix} a & b \\ g & h \end{vmatrix} \\ +\begin{vmatrix} b & c \\ e & f \end{vmatrix} - \begin{vmatrix} a & c \\ d & f \end{vmatrix} + \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix}$

Nb. To find the adjoint we then need to <u>transpose</u> this matrix.

1. Find the inverse of the following matrix: A = $\begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{5} \\ \mathbf{3} & \mathbf{3} & \mathbf{2} \\ \mathbf{4} & \mathbf{1} & \mathbf{1} \end{bmatrix}$

Determinant: $|A| \quad = +2\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} + 5\begin{vmatrix} 3 & 3 \\ 4 & 1 \end{vmatrix}$

$$= 2(1) - 1(-5) + 5(-9) \qquad = -38$$

Cofactors = $\begin{bmatrix} +\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} & +\begin{vmatrix} 3 & 3 \\ 4 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 5 \\ 1 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 5 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 1 & 5 & -9 \\ 4 & -18 & 2 \\ -13 & 11 & 3 \end{bmatrix}$

Adjoint(C^T): = $\begin{bmatrix} 1 & 4 & -13 \\ 5 & -18 & 11 \\ -9 & 2 & 3 \end{bmatrix}$

Inverse: $A^{-1} = \frac{1}{|A|}$ x adj $= -\frac{1}{38}\begin{bmatrix} \mathbf{1} & \mathbf{4} & \mathbf{-13} \\ \mathbf{5} & \mathbf{-18} & \mathbf{11} \\ \mathbf{-9} & \mathbf{2} & \mathbf{3} \end{bmatrix}$

Simple Interest

The price to be paid for the use of a certain amount of money (called principal) for a certain period is known as *Interest*. The interest is payable yearly, half-yearly, quarterly or monthly.

The sum of the principal and interest due at any time, is called the *Amount* at that time.

The rate of interest is the interest charged on one unit of principal for one year and is denoted by **i.** If the principal is ₹ 100 then the interest charged for one year is usually called the amount of interest per annum, and is denoted by r (= Pi).

e.g. if the principal is ₹ 100 and the interest ₹ 3, then we say usually that the rate of interest is 3 percent per annum (or r = 3 %)

Here $i = \frac{3}{100} = 0.03$(i.e. interest for 1 rupee for one year).

Simple interest is calculated always on the original principal for the total period for which the sum (principal) is used.

Let P be the principal (original)

n be the number of years for which the principal is used

r be the rate of interest p.a.

I be the amount of interest

i be the rate of interest per unit (i.e. interest on Re. 1 for one year)]

Now I = P.i.n, where $i = \frac{r}{100}$

Amount A = P + I = P + P. i. n = P (1+ i.n) i.e. **A = P (1 + n .i)**

SOLVED EXAMPLES :

1. **Amit deposited ₹ 1200 to a bank at 9% interest p.a. find the total interest that he will get at the end of 3 years.**

Here P = 1200, $i = \frac{9}{100} = 0.09$, n = 3, I = ?

I = P. i.n = 1200 x 0.09 x 3 = 324.

Amit will get ₹ 324 as interest.

2. **Sumit borrowed ₹ 7500 at 14.5% p.a. for $2\frac{1}{2}$ years. Find the amount he had to pay after that period.**

$P = 7500, \; i = \frac{14.5}{100} = 0.145, \; n = 2\frac{1}{2} = 2.5, A = ?$

$A = P(1 + in) = 7500(1 + 0.145 \times 2.5) = 7500(1 + 0.3625)$

$= 7500 \times 1.3625 = 10218.75$

Reqd. amount = ₹ 10218.75.

3. **Find the simple interest on ₹ 5600 at 12% p.a. from July 15 to September 26, 2013.**

Time = number of days from July 15 to Sept. 26

= 16 (July) + 31 (Aug.) + 26 (Sept.)= 73 days.

$P = 5600, \; i = \frac{12}{100} = 0.12, \; n = \frac{73}{365} \text{yr.} = \frac{1}{5} \text{yr.}$

$S.I. = P. \; i.n. = 5600 \times 0.12 \times \frac{1}{5} = 134.4$

$\therefore$ Reqd. S.I. = ₹ 134.40.

To find Principle :

4. **What sum of money will amount to ₹ 1380 in 3 years at 5% p.a. simple interest?**

Here $A = 1380, n = 3, \; i = \frac{5}{100} = 0.05, P = ?$

From $A = P(1 + 0.05 \times 3)$ or, $1380 = P(1 + 0.15)$

Or, $1380 = P(1.15)$ or, $P = \frac{1380}{1.15} = 1200$

$\therefore$ Reqd. sum = ₹ 1200

Problems to find rate % :

5. **At what rate percent will a sum, become double of itself in $5\frac{1}{2}$ years at simple interest?**

$A = 2P, P =$ Principal, $n = 5\frac{1}{2}, i = ?$

$A = P(1 + ni)$ or, $2P = P(1 + \frac{11}{2} i)$

or, $2 = 1 + \frac{11}{2} i$ or, $i = \frac{2}{11}$

or, $n = \frac{2}{11} \times 100 = 18.18$ (approx); Reqd. rate = 18.18%.

Problems to find time :

6. **In how many years will a sum be double of itself at 10% p.a. simple interest.**

A = 2P, P = Principal, $i = \frac{10}{100} = 0.10$, n= ?

A = P (1 + ni) or, 2P = P [1 + n(.10)] or, 2 = 1 + n (.10)

or, n (.10) = 1 or, $n = \frac{1}{.10} = 10$ ∴ Reqd. time = 10 years.

COMPOUND INTEREST

Interest as soon as it is due after a certain period, is added to the principal and the interest for the succeeding period is based upon the principal and interest added together. Hence the principal does not remain same, but increases at the end of each interest period.

A year is generally taken as the interest-period, but in most cases it may be half-year or quarter-year.

Symbols :

Let P be the Principal (original)

A be the amount

i be the Interest on Re. 1 for 1 year

n be the Number of years (interest period).

Formula : $\mathbf{A = P(1 + i)^n}$ .... (i)

Cor.1. In formula (i) since P amount to A in years, P may be said to be present value of the sum A due in n years.

$$P = \frac{A}{(1+i)^n} = A(1+i)^{-n}$$

Cor.2. Formula (1) may be written as follows by using logarithm :

log A = log P + n log (1 + i)

FEW FORMULAE :

Compound Interest may be paid half-yearly, quarterly, monthly instead of a year. In these cases difference in formulae are shown below :

(Taken P = principal, A = amount, T = total interest, i = interest on Re. 1 for 1 year, n = number of years.)

	Time	Amount	I = A – P
(i)	Annual	$A = P(1 + I)^n$	$I = P\{(1 + i)^n - 1\}$
(ii)	Half-Yearly	$A = P\left(1+\frac{i}{2}\right)^{2n}$	$I = P\left[\left(1+\frac{i}{2}\right)^{2n} - 1\right]$
(iii)	Quarterly	$A = P\left(1+\frac{i}{4}\right)^{4n}$	$I = P\left[\left(1+\frac{i}{4}\right)^{4n} - 1\right]$

In general if C.I. is paid p times in a year, then $A = P\left(1+\frac{i}{p}\right)^{pn}$.

i.e. : Let P = ₹ 1000, r = 5% i.e., i = 0.05, n = 24 yrs.

If interest is payable yearly the A = 1000 $(1 + 0.5)^{24}$

If int. is payable half-yearly the A = $1000\left(1+\frac{0.05}{2}\right)^{2\times24}$

If int. is payable quarterly then A = $1000\left(1+\frac{0.05}{4}\right)^{4\times24}$

Note. or r = 100 I = interest per hundred.

If r = 6% then If, however i = 0.02 then, r = 100 × 0.02 = 2%.

SOLVED EXAMPLES..

1. Find the compound interest on ₹ 1,000 for 4 years at 5% p.a.

Here P = ₹ 1000, n = 4, i = 0.05, A = ?

We have A = P $(1 + i)^n$

A = 1000 $(1+0.05)^4$

Or log A = log 1000 + 4log (1 + 0.05) = 3 + 4 log (1. 05) = 3 + 4 (0.0212) = 3 + 0.0848 = 3.0848

A = antilog 3.0848 = 1215

C.I. = ₹ 1215 – ₹ 1000 = ₹ 215

2. In what time will a sum of money double itself at 5% p.a. C.I.

Here, P = P, A = 2P, i = 0.05, n = ?

A = P $(1 + i)^n$ or, 2P = P$(1 + 0.05)^n$ = P $(1.05)^n$

or, 2 = $(1.05)^n$ or log 2 = n log 1.05

$$\therefore n = \frac{\log 2}{\log 1.05} = \frac{0.3010}{0.0212} = 14.2 \text{ years (Approx)}$$

3. The difference between simple and compound interest on a sum put out for 5 years at 3% was ₹ 46.80. Find the sum.

Let P = 100, i = .03, n = 5. From $A = P(1+i)^n$,

$A = 100(1+.03)^5 = 100(1.03)^5$

log A = log 100 + 5 log (1.03) = 2 + 5 (.0128) = 2 + .0640 = 2.0640

∴ A = antilog 2.0640 = 115.9 ∴ C.I. = 115.9 – 100 = 15.9

Again S.I. = 3 × 5 = 15. ∴ difference 15.9 – 15 = 0.9

Diff.	Capital
0.9	100
46.80	x

$$x = 100 \times \frac{46.80}{0.9} = 5,200$$

∴ original sum = ₹ 5,200.

4. What is the present value of ₹ 1,000 due in 2 years at 5% compound interest, according as the interest is paid (a) yearly, (b) half-yearly ?

(a) Here A = ₹ 1,000, $i = \frac{5}{100} = 0.05$, n = 2, P = ?

$A = P(1+i)^n$ or $1000 = P(1+.05)^2 = P(1.05)^2$

$$\therefore P = \frac{1000}{(1.05)^2} = \frac{1000}{1.1025} = 907.03$$

∴ Present value = ₹ 907.03

(b) Interest per unit per half-year $\frac{1}{2} \times 0.05 = 0.025$

From $A = P\left(1+\frac{i}{2}\right)^{2n}$ we find.

$$1,000 = P\left(1+\frac{0.05}{2}\right)^{2\times 2} = P(1+.025)^4 = P(1.025)^4$$

or, $P = \frac{1000}{(1.025)^4}$

∴ log P = log 1000 – 4log (1.025) = 3 – 4 (0.0107) = 3 – 0.0428 = 2.9572

∴ P = antilog 2.9572 = 906.1.

Hence the present amount = ₹ 906.10

Present value of an annuity :

Definition : Present value of an annuity is the sum of the present values of all payments (or instalments) made at successive annuity periods.

Formula :

(i) The present value V of an annuity P to continue for n years is given by

$$V = \frac{P}{i}\left\{1 - (1+i)^{-n}\right\}$$ Where i = interest per rupee per annum.

(ii) The Present value V of an annuity P payable half-yearly, then

$$V = \frac{2P}{i} \quad 1 - \quad 1 + \frac{i}{2}^{\,-2n}$$

(ii) The Present value V of an annuity P payable quarterly, then

$$V = \frac{4P}{i} \quad 1 - \quad 1 + \frac{i}{4}^{\,-4n}$$

SOLVED EXAMPLES :

A man decides to deposit ₹ 20,000 at the end of each year in a bank which pays 10% p.a. compound interest. If the instalments are allowed to accumulate, what will be the total accumulation at the end of 9 years?

Solution :

Let ₹ A be the total accumulation at the end of 9 years. Then we have

$$A = \frac{P}{i}\left\{(1+i)^n - 1\right\}$$

Here P = ₹ 20,000, $i = \frac{10}{100} = 0.1$, n = 9 years.

$$\therefore A = \frac{20,000}{0.1}\left\{(1+.1)^9 - 1\right\} = 2,00,000\left\{(1.1)^9 - 1\right\} = 2,00,000\,(2.3579 - 1)$$

$$= 2,00,000 \times 1.3579 = ₹\,2,71,590$$

∴ The required total accumulation = ₹ 2,71,590.

Nominal and Effective Interest

An interest rate takes two forms: nominal interest rate and effective interest rate. The nominal interest rate does not take into account the compounding period. The effective interest rate does take the compounding period into account and thus is a more accurate measure of interest charges.

A statement that the "interest rate is 10%" means that interest is 10% per year, compounded annually. In this case, the nominal annual interest rate is 10%, and the effective annual interest rate is also 10%. However, if compounding is more frequent than once per year, then the effective interest rate will be greater than 10%. The more often compounding occurs, the higher the effective interest rate.

The relationship between nominal annual and effective annual interest rates is:

$i_a = [\ 1 + (r / m)\ ]^m - 1$

where "i_a" is the effective annual interest rate, "r" is the nominal annual interest rate, and "m" is the number of compounding periods per year.

Example: A credit card company charges 21% interest per year, compounded monthly. What effective annual interest rate does the company charge?
r = 0.21 per year
m = 12 months per year
$i_a = [\ 1 + (.21\ /\ 12)\ ]^{12} - 1$
$= [1 + 0.0175\ ]^{12} - 1$
$= (1.0175)^{12} - 1 = 1.2314 - 1$
$= 0.2314 = 23.14\%$
It may be desired to find the effective interest rate for a period other than annual. In this case, adjust the period for "r" and "m" as needed. For example, if the effective interest rate per semi annual period (every 6 months) is desired, then
r = nominal interest rate per 6 months
m = number of compounding periods per 6 months
and the effective interest rate, i_{sa}, per semi-annual period, is:
$i_{sa} = [\ 1 + (r\ /\ m)\ ]^{m} - 1$

2. If a lender charges 12% interest, compounded monthly, what is the effective interest rate per quarter?

Hint: m = number of compounding periods per quarter

Let i = effective interest rate per quarter.

Choose an answer by clicking on one of the letters below, or click on "Review topic" if needed.

A $i = [\ 1 + (0.12\ /\ 3)\ ]^{3} - 1 = (1.04)^{3} - 1 = 0.1249 = 12.49\%$

B $i = [\ 1 + 0.03\ ]^{12} - 1 = (1.03)^{12} - 1 = 0.4258 = 42.58\%$

C $i = [\ 1 + (0.03\ /\ 3)\ ]^{3} - 1 = (1.01)^{3} - 1 = 0.0303 = 3.03\%$

D $i = [\ 1 + (0.03\ /\ 12)\ ]^{3} - 1 = (1.0025)^{3} - 1 = 0.0075 = 0.75\%$

Continuous Compounding

Single payment formulas for continuous compounding are determined by taking the limit of compound interest formulas as m approaches infinity, where m is the number of compounding periods per year. Here "e" is the exponential constant (sometimes called Euler's number).
With continuous compounding at nominal annual interest rate r (time-unit, e.g. year) and n is the number of time units we have:
$F = P\ e^{r\,n}$ F/P
$P = F\ e^{-r\,n}$ P/F
$i_a = e^{r} - 1$ Actual interest rate for the time unit
Example 1: If \$100 is invested at 8% interest per year, compounded continuously, how much will be in the account after 5 years?
P = \$100
r = 8%

n = 5 years
F = P $e^{r\,n}$ = ($100) $e^{(.08)(5)}$
= ($100) $e^{0.4}$ = ($100)(1.4918) = $149.18
Example 2: If $100 is invested at 0.667% interest per month, compounded continuously, how much will be in the account after 5 years?
P = $100
r = 0.666667%
n = 5 years * 12 months
F = P $e^{r\,n}$ = ($100) $e^{(.00666667)(60)}$
= ($100) $e^{0.4}$ = ($100)(1.4918) = $149.18

2 MARK QUESTIONS

1. What is an upper triangular matrix? Give example.
2. Write the formula for calculates Simple interest and compound interest?
3. Find the simple interest on the sum of Rs.6,000 at 10% p.a. for 3 years?
4. Find the inverse of A= $\begin{pmatrix} 5 & 5 \\ 8 & 6 \end{pmatrix}$
5. What is singular matrix and non singular matrix?
6. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 9 \end{bmatrix}$ then show that $(A')' = A$
7. Evaluate $A = \begin{vmatrix} 6 & 4 & 2 \\ 7 & 1 & 3 \\ 0 & -1 & 6 \end{vmatrix}$
8. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 4 \\ 3 & 4 \end{bmatrix}$, then find (A+B), (A–B).

8 MARK QUESTIONS

1. If A = $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$, Find the inverse of A.

2. Mr. Ravi borrows Rs.20000 at 4% compound interest and agrees to pay both the principal and the interest in 10 equal installments at the end of each year. Find the amount of these installments.

3. If $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 9 \\ 1 & 6 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 2 & 5 \\ 6 & -2 & 7 \end{bmatrix}$, Show that $5(A+B) = 5A + 5B$.

4. Mr. Puthiyanayagam has two daughters A and B aged 10 ½ and 16 years. He has Rs.180, 000 with him now but he wants that both of them should get an equal amount when they are 18 years old. How he should divide the money if it were to be deposited with his friend who gives only 5 % p.a.

5. Verify that $B^T A^T = (AB)^T$, when $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$

6. i) If a term deposit of Rs.4000 earns an interest of Rs.2500 in 50 months find the rate of interest.

ii) Find the present value of Rs.695 due 2 years from now receiving simple interest at 5% per annum.

7. If A= $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$, find the inverse of the given matrix A.

8.i) Find the effective rate of interest percent per annum equivalent to a nominal rate 12% per annum the interest being payable half yearly.

ii). Find the simple interest on the sum of Rs.3000 at 5% p.a. for 2 years?

9. Show that $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5I = 0$.hence deduce the inverse of A.

10. A person has two daughters A and b aged 13 and 16 years. He has Rs.40, 000 with him now but wants that both of them should get an equal amount when they are 20 years old. How he should divide the money if it were to be deposited in a bank giving 9% compound interest p.

11. Find the effective rate of interest equivalent nominal rate of 12% p.a., compounded monthly. Further, find the effective rate when interest is compounded continuously.

UNIT – II

SYLLABUS

Differential Calculus Mathematical functions and their types – linear, quadratic, polynomial; Concepts of limit and continuity of a function; Concept of differentiation; Rules of differentiation – simple standard forms. Applications of differentiation – elasticity of demand and supply; Maxima and Minima of functions (involving second or third order derivatives) relating to cost, revenue and profit.

Concept of Functions

Let A and B be any two non–empty sets. Then a function ‘ff’ is a rule or law which associates each element of ‘A’ to a unique element of set ‘B’.

Notation:

(i) A function is usually denoted by small letters, i.e. f,g,hf,g,h etc. and Greek letters, i.e. $\alpha,\beta,\gamma,\phi,\psi$α,β,γ,ϕ,ψetc.

(ii) If ‘ff’ is a function from ‘A’ to ‘B’ then we write $f:A\rightarrow B$f:A→B.

Ordered Pair:

Let ‘a’ and ‘b’ be any two elements then an element(a,b)(a,b) is called an ordered pair.

Constant Function:

Let ‘A’ and ‘B’ be any two non–empty sets, then a function ‘ff’ from ‘A’ to ‘B’ is called a constant function if and only if the range of ‘ff’ is a singleton.

Algebraic Function:

A function defined by an algebraic expression is called an algebraic function.

e.g. $f(x)=x^2+3x+6$f(x)=x2+3x+6

Polynomial Function:

A function of the form $P(x)=a_mx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$P(x)=amxn+an−1xn−1+⋯+a1x+a0 where ‘n’ is a positive integer and $a_n,a_{n-1},\cdots,a_1,a_0$an,an−1,⋯,a1,a0 are real numbers is called a polynomial function of degree ‘n’.

Linear Function:

A polynomial function with degree ‘tt’ is called a linear function. The most general form of a linear function is

$f(x)=ax+b$f(x)=ax+b

Quadratic Function:

A polynomial function with degree ‘2’ is called a quadratic function. The most general form of a quadratic equation is $f(x)=ax^2+bx+c$f(x)=ax2+bx+c

Cubic Function:

A polynomial function with degree '3' is called a cubic function. The most general form of a cubic function is $f(x)=ax^3+bx^2+cx+d$ f(x)=ax3+bx2+cx+d

Identity Function:

Let $f:A\rightarrow B$ f:A→B be a function then 'ff' is called an identity function if $f(x)=x, \forall x \in A$ f(x)=x,∀x∈A.

Rational Function:

A function $R(x)$ R(x) defined by $R(x)=\frac{P(x)}{Q(x)}$ R(x)=P(x)Q(x), where both $P(x)$ P(x)and $Q(x)$ Q(x) are polynomial functions is called a rational function.

Trigonometric Function:

A function $f(x)=\sin x$ f(x)=sin x, $f(x)=\cos x$ f(x)=cos x etc., then $f(x)$ f(x) is called a trigonometric function.

Exponential Function:

A function in which the variable appears as an exponent (power) is called an exponential function e.g. (i) $f(x)=a^x$ f(x)=ax (ii) $f(x)=3^x$ f(x)=3x.

Logarithmic Function:

A function in which the variable appears as an argument of a logarithm is called a logarithmic function.

e.g. $f(x)=\log_a(x)$ f(x)=loga(x).

LIMIT AND CONTINUITY

Limit of a Function at Infinity

Defintion Let f(x) be a function defined on R. $\lim_{x\to\infty} f(x) = \ell$ means that for any ε> 0 , there exists X > 0 such that when x > X , $|f(x)-\ell| < \varepsilon$.

Theorem **UNIQUENESS of Limit Value**

If $\lim_{x\to\infty} f(x) = a$ and $\lim_{x\to\infty} f(x) = b$, then $a = b$.

Theorem **Rules of Operations on Limits**

If $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} g(x)$ exist , then

(a) $\lim_{x\to\infty}[f(x) \pm g(x)] = \lim_{x\to\infty} f(x) \pm \lim_{x\to\infty} g(x)$

(b) $\lim_{x\to\infty} f(x)g(x) = \lim_{x\to\infty} f(x) \cdot \lim_{x\to\infty} g(x)$

(c) $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\frac{\lim_{x\to\infty}f(x)}{\lim_{x\to\infty}g(x)}$ if $\lim_{x\to\infty}g(x)\neq 0$.

(d) For any constant k, $\lim_{x\to\infty}[kf(x)]=k\lim_{x\to\infty}f(x)$.

(e) For any positive integer n,

(i) $\lim_{x\to\infty}[f(x)]^n=[\lim_{x\to\infty}f(x)]^n$

(ii) $\lim_{x\to\infty}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to\infty}f(x)}$

DERIVATIVES

Definition: The derivative of a function f at a point a, denoted by $f'(a)$, is

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

provided that the limit exists.

If we denote $y = f(x)$, then $f'(a)$ is called the derivative of f, with respect to (the independent variable) x, at the point $x = a$.

Recall that the value of this limit is, if it exists, is the slope of the line tangent to the curve $y = f(x)$ at the point $x = a$. As well, it also represents the instantaneous rate of change, with respect to x, of the function f at a. Therefore, a positive $f'(a)$ means that the function f is *increasing* at a, while a negative $f'(a)$ means that f is *decreasing* at a. If $f'(a) = 0$, then f is neither increasing nor decreasing at a.

Equivalently, the derivative can be stated as

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

ex. Let $f(t) = t^5 + 6t$, find $f'(a)$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{[(a+h)^5 + 6(a+h)] - [a^5 + 6a]}{h}$$

$$= \lim_{h\to 0} \frac{[a^5 + 5a^4h + 10a^3h^2 + 10a^2h^3 + 5ah^4 + h^5 + 6a + 6h] - [a^5 + 6a]}{h}$$

$$= \lim_{h\to 0} \frac{5a^4h + 10a^3h^2 + 10a^2h^3 + 5ah^4 + h^5 + 6h}{h}$$

$$= \lim_{h\to 0} \frac{h(5a^4 + 10a^3h + 10a^2h^2 + 5ah^3 + h^4 + 6)}{h}$$

$$= \lim_{h\to 0} (5a^4 + 10a^3h + 10a^2h^2 + 5ah^3 + h^4 + 6) = 5a^4 + 6$$

ex. Let $f(x) = \sqrt{4x^2 + 5}$, find $f'(a)$. Write an equation of the line tangent to $y = f(x)$ when $a = 1$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{\sqrt{4(a+h)^2 + 5} - \sqrt{4a^2 + 5}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{4(a+h)^2 + 5} - \sqrt{4a^2 + 5}}{h} \cdot \frac{\sqrt{4(a+h)^2 + 5} + \sqrt{4a^2 + 5}}{\sqrt{4(a+h)^2 + 5} + \sqrt{4a^2 + 5}}$$

$$= \lim_{h\to 0} \frac{4(a+h)^2 + 5 - (4a^2 + 5)}{h\left(\sqrt{4(a+h)^2 + 5} + \sqrt{4a^2 + 5}\right)} = \lim_{h\to 0} \frac{4a^2 + 8ah + 4h^2 + 5 - 4a^2 - 5}{h\left(\sqrt{4(a+h)^2 + 5} + \sqrt{4a^2 + 5}\right)}$$

$$= \lim_{h\to 0} \frac{8ah + 4h^2}{h\left(\sqrt{4(a+h)^2 + 5} + \sqrt{4a^2 + 5}\right)} = \lim_{h\to 0} \frac{8a + 4h}{\sqrt{4(a+h)^2 + 5} + \sqrt{4a^2 + 5}}$$

$$=\frac{8a+4(0)}{\sqrt{4(a+0)^2+5}+\sqrt{4a^2+5}}=\frac{8a}{2\sqrt{4a^2+5}}=\frac{4a}{\sqrt{4a^2+5}}$$

At $a = 1$, the point on the curve, $(a, f(a))$ is (1, 3), and the slope of the tangent line is $f'(1)=\frac{4}{3}$.

The equation of the line is, in point-slope form, therefore

$$y-3=\frac{4}{3}(x-1)$$

or, in slope-intercept form,

$$y=\frac{4}{3}x+\frac{5}{3}.$$

ex. Let $f(x)=\frac{1}{\sqrt{x^2+3}}$, find $f'(a)$.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{h\to 0}\frac{\frac{1}{\sqrt{(a+h)^2+3}}-\frac{1}{\sqrt{a^2+3}}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{\sqrt{(a+h)^2+3}}-\frac{1}{\sqrt{a^2+3}}}{h}\cdot\frac{\sqrt{(a+h)^2+3}\sqrt{a^2+3}}{\sqrt{(a+h)^2+3}\sqrt{a^2+3}}$$

$$=\lim_{h\to 0}\frac{\sqrt{a^2+3}-\sqrt{(a+h)^2+3}}{h(\sqrt{(a+h)^2+3})(\sqrt{a^2+3})}$$

$$=\lim_{h\to 0}\frac{\sqrt{a^2+3}-\sqrt{(a+h)^2+3}}{h(\sqrt{(a+h)^2+3})(\sqrt{a^2+3})}\cdot\frac{\sqrt{a^2+3}+\sqrt{(a+h)^2+3}}{\sqrt{a^2+3}+\sqrt{(a+h)^2+3}}$$

$$= \lim_{h \to 0} \frac{(a^2+3)-(a^2+2ah+h^2+3)}{h(\sqrt{(a+h)^2+3})(\sqrt{a^2+3})(\sqrt{a^2+3}+\sqrt{(a+h)^2+3})}$$

$$= \lim_{h \to 0} \frac{-2ah-h^2}{h(\sqrt{(a+h)^2+3})(\sqrt{a^2+3})(\sqrt{a^2+3}+\sqrt{(a+h)^2+3})}$$

$$= \lim_{h \to 0} \frac{-2a-h}{(\sqrt{(a+h)^2+3})(\sqrt{a^2+3})(\sqrt{a^2+3}+\sqrt{(a+h)^2+3})} = \frac{-2a}{(\sqrt{a^2+3})(\sqrt{a^2+3})(2\sqrt{a^2+3})}$$

$$= \frac{-a}{(a^2+3)^{3/2}}$$

Basic Differentiation Formulas

Suppose f and$g$ are differentiable functions, c is any real number, then

1. $\frac{d}{dx}(c) = 0$

2. $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

3. $\frac{d}{dx}[f(x)-g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

4. $\frac{d}{dx}[c\,f(x)] = c \cdot \frac{d}{dx}f(x)$

5. $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$

Or, in short-hand notations:

1. $(c)' = 0$
2. $[f(x) + g(x)]' = f'(x) + g'(x)$
3. $[f(x) - g(x)]' = f'(x) - g'(x)$
4. $[cf(x)]' = cf'(x)$
5. $[f(x)g(x)]' = f(x)g'(x) + g(x)f'(x)$
6. $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

The Power Rule: For any real number n,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

For $n = 1$, this means that $\frac{d}{dx}(x) = 1 \cdot x^{1-1} = x^0 = 1$.

And if n = 0, then $\frac{d}{dx}(x^0) = \frac{d}{dx}(1) = 0 \cdot x^{0-1} = 0$, which is consistent with the constant rule of differentiation (rule #1 above).

ex. The instantaneous rate of change of a line

Suppose $f(x) = mx + b$, where m and b are constants, then

$$f'(x) = (mx)' + (b)' = m(x)' + (b)' = m(1) + 0 = m$$

Therefore, any linear function has a constant derivative equals to the slope of its graph, which is a line of slope m. It says that the instantaneous rate of change of a linear function is constant, and that the tangent line to the graph of a line is always the line itself (because the tangent line has the same slope as the line, and they obviously contain one common point, therefore they have the same equation and are therefore the same line).

ex. Differentiate $y = 2t^3 - t^{\pi} + t^{-2} + 9$

$y' = 2(t^3)' - (t^{\pi})' + (t^{-2})' + (9)' = 2(3t^2) - \pi t^{\pi-1} + (-2t^{-3}) + 0$

$= 6t^2 - \pi t^{\pi-1} - 2t^{-3}$

ex. Differentiate $s(t) = 5\sqrt{t} - \dfrac{2}{\sqrt{t}} + 4\sqrt[3]{t}$

This would be easier to do if we first rewrite $s(t)$ in terms of powers of x.

$s(t) = 5t^{1/2} - 2t^{-1/2} + 4t^{1/3}$, then

$$s'(t) = 5\left(\frac{1}{2}t^{(\frac{1}{2}-1)}\right) - 2\left(\frac{-1}{2}t^{(\frac{-1}{2}-1)}\right) + 4\left(\frac{1}{3}t^{(\frac{1}{3}-1)}\right) = \frac{5}{2}t^{\frac{-1}{2}} + t^{\frac{-3}{2}} + \frac{4}{3}t^{\frac{-2}{3}}$$

ex. Differentiate $f(x) = \dfrac{x^2 + 3x + 2}{x^2 - 3x + 2}$

$$f'(x) = \frac{(x^2 - 3x + 2)(x^2 + 3x + 2)' - (x^2 + 3x + 2)(x^2 - 3x + 2)'}{(x^2 - 3x + 2)^2}$$

$$= \frac{(x^2 - 3x + 2)(2x + 3) - (x^2 + 3x + 2)(2x - 3)}{(x^2 - 3x + 2)^2}$$

ex. Differentiate $g(x) = \dfrac{2x^3 + 3x^2 - x + 5}{x^2}$

The easiest way to do this is to rewrite $g(x)$ as

$g(x) = 2x + 3 - \dfrac{1}{x} + \dfrac{5}{x^2}$, then

$$g'(x) = 2(x)' + (3)' - (x^{-1})' + 5(x^{-2})' = 2 + 0 - (-x^{-2}) + 5(-2x^{-3})$$

$$= 2 + x^{-2} - 10x^{-3} = 2 + \frac{1}{x^2} - \frac{10}{x^3}$$

ex. Differentiate $y = \sqrt{x}\,(x^2 - 5x + 2)$

Simplify first: $y = x^{5/2} - 5x^{3/2} + 2x^{1/2}$.

$$y' = \frac{5}{2}x^{3/2} - 5\left(\frac{3}{2}x^{1/2}\right) + 2\left(\frac{1}{2}x^{-1/2}\right) = \frac{5}{2}x^{3/2} - \frac{15}{2}x^{1/2} + x^{-1/2}$$

The longer way to do this is by using the product rule:

$$y' = \sqrt{x}\,(x^2 - 5x + 2)' + (x^2 - 5x + 2)(\sqrt{x})' = \sqrt{x}\,(2x - 5) + (x^2 - 5x + 2)(\frac{1}{2}x^{-1/2})$$

$$= x^{1/2}(2x - 5) + (x^2 - 5x + 2)(\frac{1}{2}x^{-1/2}) = 2x^{3/2} - 5x^{1/2} + \frac{1}{2}x^{3/2} - \frac{5}{2}x^{1/2} + x^{-1/2}$$

$$= \frac{5}{2}x^{3/2} - \frac{15}{2}x^{1/2} + x^{-1/2}$$

Derivatives of logarithmic and exponential functions

Formula. *The exponential rule. For the function of the type* $f(x) = a^x$ *, where a is a constant, then* $f'(x) = a^x \ln a$ *. Derivative of* $f(x) = e^x$ *is* $f'(x) = e^x$ *.*

Example 1. *Find* $\frac{dy}{dx}$.

a. $f(x) = 3x^{20} + 10^x$ *b.* $f(x) = e^x + e^{5x}$

Solution.a. $\frac{df}{dx} = 60x^{19} + 10^x \ln 10$, *b.* $\frac{df}{dx} = e^x + 5e^{5x}$

Formula. *For the logarithmic function* $f(x) = \ln x$ *,* $f'(x) = \frac{1}{x}$.

Derivative of $f(x) = \log_a x$ *is* $f'(x) = \frac{1}{x \ln a}$.

Example 2. *Find* $\frac{dy}{dx}$.

a. $f(x) = 3\ln x + 10$ *b.* $f(x) = \log x + 3$

Solution.a. $\frac{df}{dx} = \frac{3}{x}$, *b.* $\frac{df}{dx} = \frac{1}{x \ln 10}$

Example 1. *Find* $\frac{dy}{dx}$ *or* y'.

a. $3y^2 - 5xy + 9x - 2 = 0$ *b.* $y^2 = \log x + 3$

Solution.a. $6yy' - 5y - 5xy' + 9 = 0 \Rightarrow y' = \frac{5y-9}{6y-5x}$

b. $2yy' = \frac{1}{x \ln 10} \Rightarrow y' = \frac{1}{2xy \ln 10}$

Example 2. *Find the slope of the tangent line to the curve* $3xy - 2x^2 = 7$ *at (1, 3). Also compute the second derivative.*

Solution. $3xy' + 3y - 4x = 0 \Rightarrow y' = \frac{4x-3y}{3x} = \frac{4(1)-3(3)}{3(1)} = -\frac{5}{3}$

For the second derivative we consider again $3xy' + 3y - 4x = 0$

Taking derivative we find $3xy'' + 3y' + 3y' - 4 = 0 \Rightarrow y'' = \frac{4-6y'}{3x} = \frac{6y-4x}{3x^2} = \frac{14}{3}$

DEMAND AND SUPPLY

The market and its economic agents

Purpose of this lesson: to study the behaviour of people as they interact with one another in markets.

Market: a group of buyers and sellers of a particular good (or service).

Demand: represents the behaviour of buyers.

Supply: represents the behaviour of sellers.

Main assumption that we will use for the time being: markets are competitive.

Characteristics of a competitive market:

a) Goods offered for sale in one market are all the same.
b) Buyers and sellers are so numerous, that they individually cannot influence the market price. We say that both buyers and sellers are price takers.

Not all markets are competitive. If sellers are few and individually can influence the market price, then we say that markets are not competitive (oligopoly, monopoly).

The concepts of demand and supply

To be specific let us concentrate on one particular market: for example, the market for butter.

Demand

- Factors that determine the quantity demanded of butter:

a) The price of the good. The higher the price, the lower the quantity demanded. This is the Law of demand.
b) Income. Normally, the richer people are, the more of a good they will buy. Let us call income m.
c) Prices of related goods.

x and y substitutes. Think of butter (x) and margarine (y).

$$p_y \downarrow \;\Rightarrow\; y\uparrow \;\Rightarrow\; x\downarrow; \quad \text{Therefore } p_y \text{ and } x \text{ positively related.}$$

x and y complements. Think of butter (x)and bread (y)

$$p_y \downarrow \;\Rightarrow\; y\uparrow \;\Rightarrow\; x\uparrow; \quad \text{Therefore } p_y \text{ and } x \text{ negatively related.}$$

d) Tastes. Some people like butter, others like olive oil.

- The demand curve

Suppose that we give a mathematical form to the relationship between butter (*x*) and the factors that determine the quantity demanded of butter.

$$x = 10 - 4p_x + 0.005m + 2p_y$$

Discuss signs.

Normally we do not want to work with so many variables. The ones we are interested in depend on the problem at hand. Suppose we are only interested in the price and quantity of butter, and that we are given the values of the other variables $m = 3{,}000€;\ p_y = 0.5€$. Then,

$$x = 10 - 4p_x + 0.005(3{,}000) + 2(0.5)$$

$$x = (10 + 15 + 1) - 4p_x$$

$$\boxed{x = 26 - 4p_x}$$

The expression in the box is the demand curve (or the demand function).
Suppose we have a market with three buyers: Ana, Víctor and Pilar. Each has the following demand curve:

Ana: $x_A = 10 - 2p_x$
Víctor: $x_V = 20 - 4p_x$
Pilar: $x_P = 15 - 3p_x$

We obtain the aggregate demand curve by adding horizontally (adding the x's).

$$\begin{aligned} x &= x_A + x_V + x_P \\ &= (10 - 2p_x) + (20 - 4p_x) + (15 - 3p_x) \\ &= 45 - 9p_x \end{aligned}$$

Therefore, the aggregate demand curve is

$$\boxed{x = 45 - 9p_x}$$

2. A consumer buys 100 units of a product when the price is €1. When the price is increased to €1.25 the consumer buys 80 units. Calculate the Price Elasticity of demand for this consumer.

$$\frac{P1 + P2}{Q1 + Q2} \times \frac{\Delta Q}{\Delta P}$$

$$\frac{1 + 1.25}{100 + 80} \quad x \quad \frac{-20}{+0.25}$$

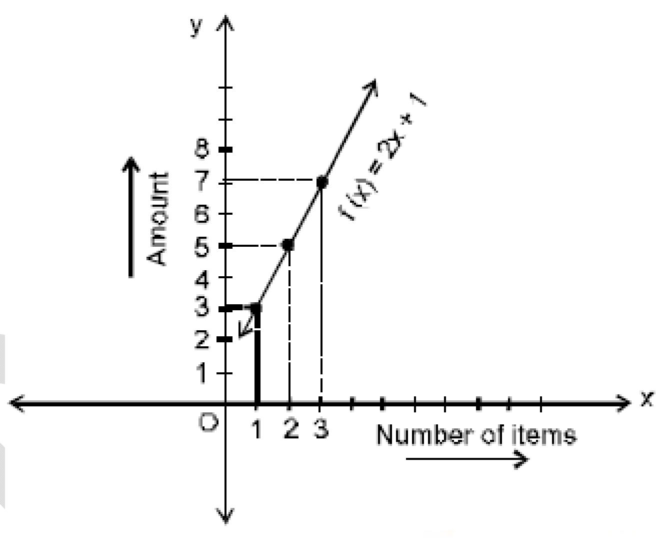
= -1

Demand for this good is unitary elastic as the answer is equal to 1

MAXIMA AND MINIMA

You are aware that in any transaction the total amount paid increases with the number of items purchased. Consider a function as $f(x) = 2x + 1, x > 0$. Let the function f(x) represent the amount required for purchasing 'x' number of items.

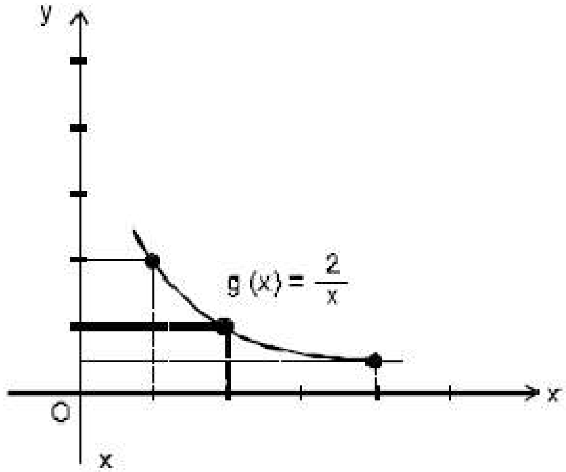
The graph of the function $y = f(x) = 2x + 1, x > 0$ for different values of x is shown in



A look at the graph of the above function prompts us to believe that the function is increasing for positive values of x i.e., for x>0 Can you think of another example in which value of the function decreases when x increases ? Any such relation would be relationship between time and manpower/person(s) involved. You have learnt that they are inversely propotional . In other words we can say that time required to complete a certain work increases when number of persons (manpower) invovled decreases and vice-versa. Consider such similar function as

$$g(x)=\frac{2}{x}, x>0$$

The values of the function g (x) for different values of x are plotted



All these examples, despite the diversity of the variables involved, have one thing in common : function is either increasing or decreasing.

INCREASING AND DECREASING FUNCTIONS

Let a function $f(x)$ be defined over the closed interval [a,b].

Let $x_1, x_2 \in [a,b]$, then the function f(x) is said to be an increasing function in the given interval if $f(x_2) \geq f(x_1)$ whenever $x_2 > x_1$. It is said to be strictly increasing if $f(x_2) > f(x_1)$ for all $x_2 > x_1$, $x_1, x_2 \in [a,b]$.

In Fig. 25.3, sin x increases from -1 to +1 as x increases from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$.

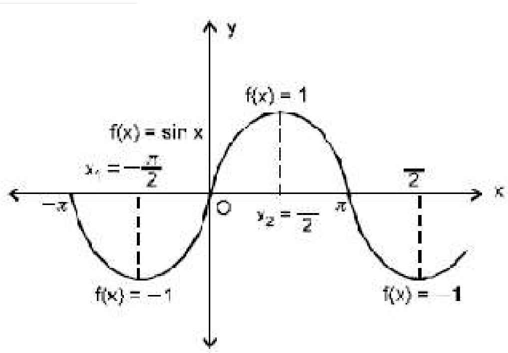


Fig. 25. 3

Note : A function is said to be an increasing function in an interval if $f(x+h) > f(x)$ for all x belonging to the interval when h is positive.

A function f(x) defined over the closed interval [a, b] is said to be a decreasing function in the given interval, if $f(x_2) \le f(x_1)$, whenever $x_2 > x_1$, $x_1, x_2 \in [a, b]$. It is said to be strictly decreasing if $f(x_1) > f(x_2)$ for all $x_2 > x_1$, $x_1, x_2 \in [a, b]$.

In Fig. 25.4, cos x decreases from 1 to −1 as x increases from 0 to π.

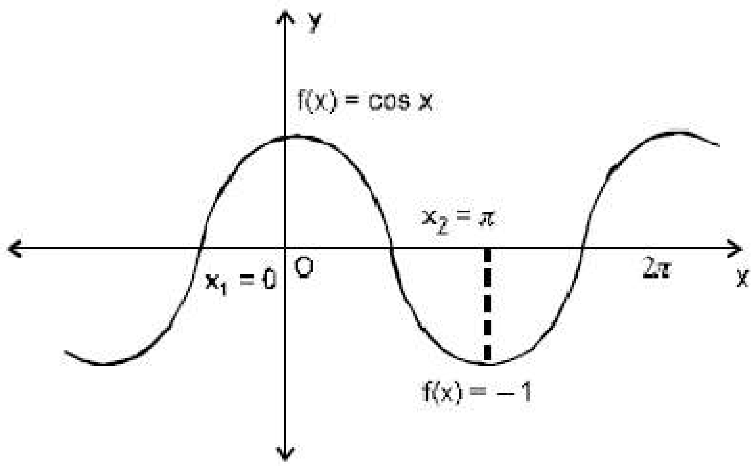


Fig. 25.4

Note : A function is said to be a decreasing in an internal if $f(x+h) < f(x)$ for all x belonging to the interval when h is positive.

Example :

Prove that the function $f(x) = 4x + 7$ is monotonic for all values of $x \in R$.

Solution : Consider two values of x say $x_1, x_2 \in R$

such that $x_2 > x_1$ .....(1)

Multiplying both sides of (1) by **4**, we have $4x_2 > 4x_1$ .....(2)

Adding **7** to both sides of (2), to get

$$4x_2 + 7 > 4x_1 + 7$$

We have $f(x_2) > f(x_1)$

Thus, we find $f(x_2) > f(x_1)$ whenever $x_2 > x_1$.

Hence the given function $f(x) = 4x + 7$ is monotonic function. (monotonically increasing).

Example

Show that

$$f(x) = x^2$$

is a strictly decreasing function for all $x < 0$.

Solution : Consider any two values of x say x_1, x_2 such that

$$x_2 > x_1, \qquad x_1, x_2 < 0 \qquad \text{.......(i)}$$

Order of the inequality reverses when it is multiplied by a negative number. Now multiplying (i) by x_2, we have

$$x_2 \cdot x_2 < x_1 \cdot x_2$$

or, $$x_2^2 < x_1 x_2 \qquad \text{......(ii)}$$

Now multiplying (i) by x_1, we have

$$x_1 \cdot x_2 < x_1 \cdot x_1$$

or, $$x_1 x_2 < x_1^2 \qquad \text{.....(iii)}$$

From (ii) and (iii), we have

$$x_2^2 < x_1 x_2 < x_1^2$$

or, $$x_2^2 < x_1^2$$

or, $$f(x_2) < f(x_1)$$

Thus, from (i) and (iv), we have for

$$x_2 > x_1,$$

$$f(x_2) < f(x_1)$$

Hence, the given function is strictly decreasing for all $x < 0$.

Example

Find the interval in which $f(x) = 2x^3 - 3x^2 - 12x + 6$ is increasing or decreasing.

Solution : $f(x) = 2x^3 - 3x^2 - 12x + 6$

$$f'(x) = 6x^2 - 6x - 12$$

$$= 6\left(x^2 - x - 2\right)$$

$$= 6(x - 2)(x + 1)$$

For $\mathbf{f}(x)$ to be increasing function of x,

$$f'(x) > 0$$

i.e. $6(x-2)(x+1) > 0$ or, $(x-2)(x+1) > 0$

Since the product of two factors is positive, this implies either both are positive or both are negative.

Either $x - 2 > 0$ and $x + 1 > 0$ or $x - 2 < 0$ and $x + 1 < 0$

i.e.	$x > 2$ and $x > -1$	i.e.	$x < 2$ and $x < -1$
	$x > 2$ implies $x > -1$		$x < -1$ implies $x < 2$
$\therefore$	$x > 2$	$\therefore$	$x < -1$

Hence f (x) is increasing for $x > 2$ or $x < -1$.

Now, for f (x) to be decreasing,

$$f'(x) < 0$$

or, $6(x-2)(x+1) < 0$ or, $(x-2)(x+1) < 0$

Since the product of two factors is negative, only one of them can be negative, the other positive.

Therefore,

Either	or
$x - 2 > 0$ and $x + 1 < 0$	$x - 2 < 0$ and $x + 1 > 0$
i.e. $x > 2$ and $x < -1$	i.e. $x < 2$ and $x > -1$
There is no such possibility that $x > 2$ and at the same time $x < -1$	This can be put in this form $-1 < x < 2$

$\therefore$ The function is decreasing in $-1 < x < 2$.

Example

Determine the intervals for which the function $f(x) = \dfrac{x}{x^2+1}$ is increasing or decreasing.

Solution : $$f'(x) = \frac{\left(x^2+1\right)\frac{dx}{dx} - x\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2}$$

$$= \frac{\left(x^2+1\right) - x\cdot(2x)}{\left(x^2+1\right)^2}$$

$$= \frac{1-x^2}{\left(x^2+1\right)^2}$$

$$\therefore \quad f'x = \frac{(1-x)(1+x)}{\left(x^2+1\right)^2}$$

As $\left(x^2+1\right)^2$ is positive for all real x.

Therefore, if $-1 < x < 0$, $(1-x)$ is positive and $(1+x)$ is positive, so $f'(x) > 0$;

$\therefore$ If $0 < x < 1$, $(1-x)$ is positive and $(1+x)$ is positive, so f $f'(x) > 0$;

2 MARK QUESTIONS

1. What are the types of Functions?
2. Define derivative.
3. If $f(x)=(x^2-7)^2$, find f ' '(x).
4. Find the derivative of x^3-3x^2+4x+3
5. Define limit of a function.
6. Find $\lim_{x\to 1} \frac{x^3+1}{2x^2+5x=3}$
7. Define elasticity of supply.
8. Define elasticity of demand.
9. Find $\frac{dy}{dx}$,if $y = (2x+5)^3$

8 MARK QUESTIONS

1.A firm sells a product at Rs.3 per unit. The total cost of the firm for producing x units is given by $C=20+0.6x+0.001x^2$. How many units should be made to achieve maximum profit? Verify that the condition for a maximum is satisfied.

2. If the demand law is $x = \frac{20}{p+1}$ find the elasticity of demand at the point when p=3.

3. Find the elasticity of supply from the function p = -2+5x.

4. Find $\frac{dy}{dx}$ i) $x^2 + y^2 = 1$ ii) $xy = c^2$

5.If $f(x) = \frac{x^3-2x^2+50}{x^2}$ find $f'(5)$ and $f'(10)$.

6.Differentiate the following with respect to x

(i) x^3-3x^2+4x+3 (ii) $x^5 + 3\log x - 4e^x$

7.Differentiate the following with respect to x

(i) y = $(x^2+5)(3x+1)$ (ii)$y = \frac{3x^2}{4x-1}$

8.Find$\lim_{x\to 0} \frac{(1+x)^4-1}{(1+x)^2-1}$

9. Find $\lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x^2}$

10. Find $\lim_{x\to 3} \frac{x^2+x-12}{x^2-x-6}$

11. If the demand law is $x = \frac{20}{p+1}$, find the elasticity of demand at the point when p=3.

12. Find for what values of x, the following expression is maximum and minimum respectively. $2x^3 - 21x^2 + 36x - 20$.find also the maximum and the minimum values.

UNIT – III

SYLLABUS

Uni-variate Analysis
Measures of Central Tendency including arithmetic mean, geometric mean and harmonic mean: properties and applications; mode and median. Partition values - quartiles, deciles, and percentiles. Measures of Variation: absolute and relative. Range, quartile deviation and mean deviation; Variance and Standard deviation: calculation and properties.

Measures of Central Tendency:

In the study of a population with respect to one in which we are interested we may get a large number of observations. It is not possible to grasp any idea about the characteristic when we look at all the observations. So it is better to get one number for one group. That number must be a good representative one for all the observations to give a clear picture of that characteristic. Such representative number can be a central value for all these observations. This central value is called a measure of central tendency or an average or a measure of locations. There are five averages. Among them mean, median and mode are called simple averages and the other two averages geometric mean and harmonic mean are called special averages.

Characteristics for a good or an ideal average :

The following properties should possess for an ideal average.

1. It should be rigidly defined.
2. It should be easy to understand and compute.
3. It should be based on all items in the data.
4. Its definition shall be in the form of a mathematical formula.
5. It should be capable of further algebraic treatment.
6. It should have sampling stability.
7. It should be capable of being used in further statistical computations or processing.

Arithmetic mean or mean :

Arithmetic mean or simply the mean of a variable is defined as the sum of the observations divided by the number of observations. If the variable x assumes n values $x_1, x_2 .. x_n$ then the mean, $\bar{x}$, is given by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i$$

This formula is for the ungrouped or raw data.

Example 1 :

Calculate the mean for 2, 4, 6, 8, 10

Solution:

$$\bar{x} = \frac{2+4+6+8+10}{5}$$

$$= \frac{30}{5} = 6$$

Short-Cut method :

Under this method an assumed or an arbitrary average (indicated by A) is used as the basis of calculation of deviations from individual values. The formula is

$$\bar{x} = A + \frac{\sum d}{n}$$

where, A – the assumed mean or any value in x
d – the deviation of each value from the assumed mean

Example 2 :

A student' s marks in 5 subjects are 75, 68, 80, 92, 56. Find his average mark.

Solution:

	X	d=x-A
	75	7
A	68	0
	80	12
	92	24
	56	-12
	Total	31

$$\bar{x} = A + \frac{\sum d}{n}$$

$$= 68 + \frac{31}{5}$$

$$= 68 + 6.2$$

$$= 74.2$$

Grouped Data :

The mean for grouped data is obtained from the following formula:

$$\bar{x} = \frac{\sum fx}{N}$$

where x = the mid-point of individual class

f = the frequency of individual class

N = the sum of the frequencies or total frequencies.

Short-cut method :

$$\bar{x} = A + \frac{\sum fd}{N} \times c$$

where $d = \frac{x - A}{c}$

A = any value in x

N = total frequency

c = width of the class interval

Example 3:

Given the following frequency distribution, calculate the arithmetic mean

Marks	: 64	63	62	61	60	59
Number of Students	: 8	18	12	9	7	6

Solution:

X	F	fx	d=x-A	fd
64	8	512	2	16
63	18	1134	1	18
62	12	744	0	0
61	9	549	–1	–9
60	7	420	–2	–14
59	6	354	–3	–18
	60	3713		- 7

Direct method

$$\bar{x} = \frac{\sum fx}{N} - \frac{3713}{60} = 61.88$$

Short-cut method

$$\bar{x} = A + \frac{\sum fd}{N} - 62 - \frac{7}{60} - 61.88$$

Example 4 :

Following is the distribution of persons according to different income groups. Calculate arithmetic mean.

Income Rs(100)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of persons	6	8	10	12	7	4	3

Solution:

Income C.I	Number of Persons (f)	Mid X	$d = \frac{x - A}{c}$	Fd
0-10	6	5	-3	-18
10-20	8	15	-2	-16
20-30	10	25	-1	-10
30-40	12	A 35	0	0
40-50	7	45	1	7
50-60	4	55	2	8
60-70	3	65	3	9
	50			-20

$$\text{Mean} = \bar{x} = A + \frac{\sum fd}{N}$$

$$= 35 - \frac{20}{50} \times 10$$

$$= 35 - 4$$
$$= 31$$

Merits and demerits of Arithmetic mean :

Merits:

1. It is rigidly defined.
2. It is easy to understand and easy to calculate.
3. If the number of items is sufficiently large, it is more accurate and more reliable.
4. It is a calculated value and is not based on its position in the series.
5. It is possible to calculate even if some of the details of the data are lacking.
6. Of all averages, it is affected least by fluctuations of sampling.
7. It provides a good basis for comparison.

Demerits:

1. It cannot be obtained by inspection nor located through a frequency graph.
2. It cannot be in the study of qualitative phenomena not capable of numerical measurement i.e. Intelligence, beauty, honesty etc.,
3. It can ignore any single item only at the risk of losing its accuracy.
4. It is affected very much by extreme values.
5. It cannot be calculated for open-end classes.
6. It may lead to fallacious conclusions, if the details of the data from which it is computed are not given.

Median :

The median is that value of the variate which divides the group into two equal parts, one part comprising all values greater, and the other, all values less than median.

Ungrouped or Raw data :

Arrange the given values in the increasing or decreasing order. If the number of values are odd, median is the middle value .If the number of values are even, median is the mean of middle two values.

By formula

$$\text{Median} = \text{Md} = \left(\frac{n+1}{2}\right)^{\text{th}} \text{item.}$$

Example 11:

When odd number of values are given. Find median for the following data

25, 18, 27, 10, 8, 30, 42, 20, 53

Solution:

Arranging the data in the increasing order 8, 10, 18, 20, 25, 27, 30, 42, 53

The middle value is the 5^{th} item i.e., 25 is the median

Using formula

$$Md = \left(\frac{n+1}{2}\right)^{th} \text{item.}$$

$$= \left(\frac{9+1}{2}\right)^{th} \text{item.}$$

$$= \left(\frac{10}{2}\right)^{th} \text{item}$$

$$= 5^{th} \text{ item}$$

$$= 25$$

Example 12 :

When even number of values are given. Find median for the following data

5, 8, 12, 30, 18, 10, 2, 22

Solution:

Arranging the data in the increasing order 2, 5, 8, 10, 12, 18, 22, 30

Here median is the mean of the middle two items (ie) mean of (10,12) ie

$$= \left(\frac{10+12}{2}\right) = 11$$

$$\therefore \text{median} = 11.$$

Using the formula

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{item.}$$

$$= \left(\frac{8+1}{2}\right)^{th} \text{ item.}$$

$$= \left(\frac{9}{2}\right)^{th} \text{ item } = 4.5^{th} \text{ item}$$

$$= 4^{th} \text{ item} + \left(\frac{1}{2}\right)(5^{th} \text{ item} - 4^{th} \text{ item})$$

$$= 10 + \left(\frac{1}{2}\right)[12-10]$$

$$= 10 + \left(\frac{1}{2}\right) \times 2$$

$$= 10 + 1$$

$$= 11$$

Example 13:

The following table represents the marks obtained by a batch of 10 students in certain class tests in statistics and Accountancy.

Serial No	1	2	3	4	5	6	7	8	9	10
Marks (Statistics)	53	55	52	32	30	60	47	46	35	28
Marks (Accountancy)	57	45	24	31	25	84	43	80	32	72

Indicate in which subject is the level of knowledge higher ?

Solution:

For such question, median is the most suitable measure of central tendency. The mark in the two subjects are first arranged in increasing order as follows:

Serial No	1	2	3	4	5	6	7	8	9	10
Marks in Statistics	28	30	32	35	46	47	52	53	55	60
Marks in Accountancy	24	25	31	32	43	45	57	72	80	84

Median = $\left(\frac{n+1}{2}\right)^{th}$ item = $\left(\frac{10+1}{2}\right)^{th}$ item $=5.5^{th}$ item

$$= \frac{Value\ of\ 5^{th}\ item + value\ of\ 6^{th}\ item}{2}$$

$$\text{Md (Statistics)} = \frac{46+47}{2} = 46.5$$

$$\text{Md (Accountancy)} = \frac{43+45}{2} = 44$$

There fore the level of knowledge in Statistics is higher than that in Accountancy.

Grouped Data:

In a grouped distribution, values are associated with frequencies. Grouping can be in the form of a discrete frequency distribution or a continuous frequency distribution. Whatever may be the type of distribution , cumulative frequencies have to be calculated to know the total number of items.

Cumulative frequency : (cf)

Cumulative frequency of each class is the sum of the frequency of the class and the frequencies of the pervious classes, ie adding the frequencies successively, so that the last cumulative frequency gives the total number of items.

Discrete Series:

Step1: Find cumulative frequencies.

Step2: Find $\left(\frac{N+1}{2}\right)$

Step3: See in the cumulative frequencies the value just greater than $\left(\frac{N+1}{2}\right)$

Step4: Then the corresponding value of x is median.

Example 14:

The following data pertaining to the number of members in a family. Find median size of the family.

Number of members x	1	2	3	4	5	6	7	8	9	10	11	12
Frequency F	1	3	5	6	10	13	9	5	3	2	2	1

Solution:

X	f	cf
1	1	1
2	3	4
3	5	9
4	6	15
5	10	25
6	13	38
7	9	47
8	5	52
9	3	55
10	2	57
11	2	59
12	1	60
	60	

Median = size of $\left(\frac{N+1}{2}\right)^{th}$ item

$= \text{size of} \left(\frac{60+1}{2} \right)^{\text{th}} \text{item}$

$= 30.5^{\text{th}}$ item

The cumulative frequencies just greater than 30.5 is 38.and the value of x corresponding to 38 is 6.Hence the median size is 6 members per family.

Continuous Series:

The steps given below are followed for the calculation of median in continuous series.

Step1: Find cumulative frequencies.

Step2: Find $\left(\frac{N}{2} \right)$

Step3: See in the cumulative frequency the value first greater than $\left(\frac{N}{2} \right)$, Then the corresponding class interval is called the Median class. Then apply the formula

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times c$$

Where l = Lower limit of the median class
m = cumulative frequency preceding the median
c = width of the median class
f =frequency in the median class.
N=Total frequency.

Example 15:

The following table gives the frequency distribution of 325 workers of a factory, according to their average monthly income in a certain year.

Income group (in Rs)	Number of workers
Below 100	1
100-150	20
150-200	42
200-250	55
250-300	62
300-350	45
350-400	30
400-450	25
450-500	15
500-550	18
550-600	10
600 and above	2
	325

Calculate median income

Solution:

Income group (Class-interval)	Number of workers (Frequency)	Cumulative frequency c.f
Below 100	1	1
100-150	20	21
150-200	42	63
200-250	55	118
250-300	62	180
300-350	45	225
350-400	30	255
400-450	25	280
450-500	15	295
500-550	18	313
550-600	10	323
600 and above	2	325
	325	

$$\frac{N}{2} = \frac{325}{2} = 162.5$$

Here $l = 250$, N = 325, f = 62, c = 50, m = 118

$$Md = 250 + \left(\frac{162.5 - 118}{62}\right) \times 50$$

$$= 250 + 35.89$$

$$= 285.89$$

Example 16:

Following are the daily wages of workers in a textile. Find the median.

Wages (in Rs.)	Number of workers
less than 100	5
less than 200	12
less than 300	20
less than 400	32
less than 500	40
less than 600	45
less than 700	52
less than 800	60
less than 900	68
less than 1000	75

Solution :

We are given upper limit and less than cumulative frequencies. First find the class-intervals and the frequencies. Since the values are increasing by 100, hence the width of the class interval equal to 100.

Class interval	f	c.f
0-100	5	5
100-200	7	12
200-300	8	20
300- 400	12	32
400-500	8	40
500-600	5	45
600-700	7	52
700-800	8	60
800-900	8	68
900-1000	7	75
	75	

$$\left(\frac{N}{2}\right) = \left(\frac{75}{2}\right) = 37.5$$

$$\text{Md} = l + \left(\frac{\frac{N}{2} - m}{f}\right) \times \text{c}$$

$$= 400 + \left(\frac{37.5 - 32}{8}\right) \times 100 = 400 + 68.75 = 468.75$$

Merits of Median :

1. Median is not influenced by extreme values because it is a positional average.
2. Median can be calculated in case of distribution with open-end intervals.
3. Median can be located even if the data are incomplete.
4. Median can be located even for qualitative factors such as ability, honesty etc.

Demerits of Median :

1. A slight change in the series may bring drastic change in median value.
2. In case of even number of items or continuous series, median is an estimated value other than any value in the series.
3. It is not suitable for further mathematical treatment except its use in mean deviation.
4. It is not taken into account all the observations.

Mode :

The mode refers to that value in a distribution, which occur most frequently. It is an actual value, which has the highest concentration of items in and around it.

Computation of the mode:

Ungrouped or Raw Data:

For ungrouped data or a series of individual observations, mode is often found by mere inspection.

Example 29:

2 , 7, 10, 15, 10, 17, 8, 10, 2

$\therefore$ Mode = M_0 = 10

In some cases the mode may be absent while in some cases there may be more than one mode.

Example 30:

1. 12, 10, 15, 24, 30 (no mode)
2. 7, 10, 15, 12, 7, 14, 24, 10, 7, 20, 10

$\therefore$ the modes are 7 and 10

Grouped Data:

For Discrete distribution, see the highest frequency and corresponding value of X is mode.

Continuous distribution :

See the highest frequency then the corresponding value of class interval is called the modal class. Then apply the formula.

$$\text{Mode} = M_0 = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C$$

l = Lower limit of the model class

$$\triangle_1 = f_1 - f_0$$
$$\triangle_2 = f_1 - f_2$$

f_1 = frequency of the modal class
f_0 = frequency of the class preceding the modal class
f_2 = frequency of the class succeeding the modal class

The above formula can also be written as

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

Example 31:

Calculate mode for the following :

C- I	f
0-50	5
50-100	14
100-150	40
150-200	91
200-250	150
250-300	87
300-350	60
350-400	38
400 and above	15

Solution:

The highest frequency is 150 and corresponding class interval is 200 – 250, which is the modal class.

Here $l=200, f_1=150, f_0=91, f_2=87, C=50$

$$\text{Mode} = M_0 = 1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$= 200 + \frac{150-91}{2 \times 150 - 91 - 87} \times 50$$

$$= 200 + \frac{2950}{122}$$

$$= 200 + 24.18 = 224.18$$

Determination of Modal class :

For a frequency distribution modal class corresponds to the maximum frequency. But in any one (or more) of the following cases

i. If the maximum frequency is repeated
ii. If the maximum frequency occurs in the beginning or at the end of the distribution
iii. If there are irregularities in the distribution, the modal class is determined by the method of grouping.

Steps for Calculation :

We prepare a grouping table with 6 columns

1. In column I, we write down the given frequencies.
2. Column II is obtained by combining the frequencies two by two.
3. Leave the 1st frequency and combine the remaining frequencies two by two and write in column III
4. Column IV is obtained by combining the frequencies three by three.
5. Leave the 1st frequency and combine the remaining frequencies three by three and write in column V
6. Leave the 1st and 2nd frequencies and combine the remaining frequencies three by three and write in column VI

Mark the highest frequency in each column. Then form an analysis table to find the modal class. After finding the modal class use the formula to calculate the modal value.

Example 32:

Calculate mode for the following frequency distribution.

Class interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	9	12	15	16	17	15	10	13

Grouping Table

C I	f	2	3	4	5	6
0- 5	9	21				
5-10	12		27	36		
10-15	15	31			**43**	
15-20	16		**33**			**48**
20-25	**17**	**32**		**48**		
25-30	15		25		42	38
30-35	10	23				
35-40	13					

Analysis Table

Columns	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
1					1			
2					1	1		
3				1	1			
4				1	1	1		
5		1	1	1				
6			1	1	1			
Total		1	2	4	5	2		

The maximum occurred corresponding to 20-25, and hence it is the modal class.

$$\text{Mode} = M_o = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C$$

Here $l = 20$; $\Delta_1 = f_1 - f_0 = 17 - 16 = 1$

$\Delta_2 = f_1 - f_2 = 17 - 15 = 2$

$$\therefore M_0 = 20 + \frac{1}{1+2} \times 5$$

$$= 20 + 1.67 = 21.67$$

MEASURES OF DISPERSION

Characteristics of a good measure of dispersion:

An ideal measure of dispersion is expected to possess the following properties

1. It should be rigidly defined
2. It should be based on all the items.
3. It should not be unduly affected by extreme items.

4. It should lend itself for algebraic manipulation.
5. It should be simple to understand and easy to calculate

Absolute and Relative Measures :

There are two kinds of measures of dispersion, namely

1. Absolute measure of dispersion
2. Relative measure of dispersion.

The various absolute and relative measures of dispersion are listed below.

Absolute measure	**Relative measure**
1. Range	1. Co-efficient of Range
2. Quartile deviation	2. Co-efficient of Quartile deviation
3. Mean deviation	3. Co-efficient of Mean deviation
4. Standard deviation	4. Co-efficient of variation

7.3 Range and coefficient of Range:

7.3.1 Range:

This is the simplest possible measure of dispersion and is defined as the difference between the largest and smallest values of the variable.

In symbols, Range $= L - S$.
Where L = Largest value.
S = Smallest value.

In individual observations and discrete series, L and S are easily identified. In continuous series, the following two methods are followed.

Method 1:

L = Upper boundary of the highest class
S = Lower boundary of the lowest class.

Method 2:

L = Mid value of the highest class.
S = Mid value of the lowest class.

7.3.2 Co-efficient of Range :

$$\text{Co-efficient of Range} = \frac{L-S}{L+S}$$

Example1:

Find the value of range and its co-efficient for the following data.

7, 9, 6, 8, 11, 10, 4

Solution:

L=11, S = 4.

Range = L – S = 11- 4 = 7

$$\text{Co-efficient of Range} = \frac{L-S}{L+S}$$

$$= \frac{11-4}{11+4}$$

$$= \frac{7}{15} = 0.4667$$

Example 2:

Calculate range and its co efficient from the following distribution.

Size:	60-63	63-66	66-69	69-72	72-75
Number:	5	18	42	27	8

Solution:

L = Upper boundary of the highest class.
= 75

S = Lower boundary of the lowest class.
= 60

Range = L – S = 75 – 60 = 15

$$\text{Co-efficient of Range} = \frac{L-S}{L+S} = \frac{75-60}{75+60} = \frac{15}{135} = 0.1111$$

7.3.3 Merits and Demerits of Range :

Merits:

1. It is simple to understand.
2. It is easy to calculate.
3. In certain types of problems like quality control, weather forecasts, share price analysis, et c., range is most widely used.

Demerits:

1. It is very much affected by the extreme items.
2. It is based on only two extreme observations.
3. It cannot be calculated from open-end class intervals.
4. It is not suitable for mathematical treatment.
5. It is a very rarely used measure.

7.6 Standard Deviation and Coefficient of variation:

7.6.1 Standard Deviation :

Karl Pearson introduced the concept of standard deviation in 1893. It is the most important measure of dispersion and is widely used in many statistical formulae. Standard deviation is also called Root-Mean Square Deviation. The reason is that it is the square–root of the mean of the squared deviation from the arithmetic mean. It provides accurate result. Square of standard deviation is called Variance.

Definition:

It is defined as the positive square-root of the arithmetic mean of the Square of the deviations of the given observation from their arithmetic mean.

The standard deviation is denoted by the Greek letter σ (sigma)

7.6.2 Calculation of Standard deviation-Individual Series :

There are two methods of calculating Standard deviation in an individual series.

a) Deviations taken from Actual mean
b) Deviation taken from Assumed mean

a) Deviation taken from Actual mean:

This method is adopted when the mean is a whole number.

Steps:

1. Find out the actual mean of the series ($\bar{x}$)
2. Find out the deviation of each value from the mean ($x = X - \bar{X}$)
3. Square the deviations and take the total of squared deviations $\sum x^2$

4. Divide the total ($\sum x^2$) by the number of observation $\left(\frac{\sum x^2}{n}\right)$

The square root of $\left(\frac{\sum x^2}{n}\right)$ is standard deviation.

Thus $\sigma = \sqrt{\left(\frac{\sum x^2}{n}\right)}$ or $\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

b) Deviations taken from assumed mean:

This method is adopted when the arithmetic mean is fractional value.

Taking deviations from fractional value would be a very difficult and tedious task. To save time and labour, We apply short –cut method; deviations are taken from an assumed mean. The formula is:

$$\sigma = \sqrt{\frac{\sum d^2}{N} - \left(\frac{\sum d}{N}\right)^2}$$

Where d-stands for the deviation from assumed mean = (X-A)

Steps:

1. Assume any one of the item in the series as an average (A)
2. Find out the deviations from the assumed mean; i.e., X-A denoted by d and also the total of the deviations $\sum d$
3. Square the deviations; i.e., d^2 and add up the squares of deviations, i.e, $\sum d^2$
4. Then substitute the values in the following formula:

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

Note: We can also use the simplified formula for standard deviation.

$$\sigma = \frac{1}{n}\sqrt{n\sum d^2 - \left(\sum d\right)^2}$$

For the frequency distribution

$$\sigma = \frac{c}{N}\sqrt{N\sum fd^2 - \left(\sum fd\right)^2}$$

Example 9:

Calculate the standard deviation from the following data.

14, 22, 9, 15, 20, 17, 12, 11

Solution:

Deviations from actual mean.

Values (X)	$X - \overline{X}$	$(X - \overline{X})^2$
14	-1	1
22	7	49
9	-6	36
15	0	0
20	5	25
17	2	4
12	-3	9
11	-4	16
120		140

$$\overline{X} = \frac{120}{8} = 15$$

$$\sigma = \sqrt{\frac{\Sigma(x-\overline{x})^2}{n}}$$

$$= \sqrt{\frac{140}{8}}$$

$$= \sqrt{17.5} = 4.18$$

Example 10:

The table below gives the marks obtained by 10 students in statistics. Calculate standard deviation.

Student Nos :	1	2	3	4	5	6	7	8	9	10
Marks :	43	48	65	57	31	60	37	48	78	59

Solution: (Deviations from assumed mean)

Nos.	Marks (x)	d=X-A (A=57)	d^2
1	43	-14	196
2	48	-9	81
3	65	8	64
4	57	0	0
5	31	-26	676
6	60	3	9
7	37	-20	400
8	48	-9	81
9	78	21	441
10	59	2	4
n = 10		$\sum d=-44$	$\sum d^2=1952$

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{1952}{10} - \left(\frac{-44}{10}\right)^2}$$

$$= \sqrt{195.2 - 19.36}$$

$$= \sqrt{175.84} = 13.26$$

7.6.3 Calculation of standard deviation:

Discrete Series:

There are three methods for calculating standard deviation in discrete series:

(a) Actual mean methods
(b) Assumed mean method
(c) Step-deviation method.

(a) Actual mean method:

Steps:

1. Calculate the mean of the series.
2. Find deviations for various items from the means i.e.,
   $x - \bar{x} = d$.
3. Square the deviations ($= d^2$) and multiply by the respective frequencies(f) we get fd^2
4. Total to product ($\sum fd^2$) Then apply the formula:

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f}}$$

(b) Assumed mean method:

Here deviation are taken not from an actual mean but from an assumed mean. Also this method is used, if the given variable values are not in equal intervals.

Steps:

1. Assume any one of the items in the series as an assumed mean and denoted by A.
2. Find out the deviations from assumed mean, i.e, X-A and denote it by d.
3. Multiply these deviations by the respective frequencies and get the $\sum fd$
4. Square the deviations (d^2).
5. Multiply the squared deviations ($d^{2)}$ by the respective frequencies (f) and get $\sum fd^2$.
6. Substitute the values in the following formula:

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

Where $d = X - A$, $N = \sum f$.

Example 11:

Calculate Standard deviation from the following data.

X :	20	22	25	31	35	40	42	45
f :	5	12	15	20	25	14	10	6

Solution:

Deviations from assumed mean

x	f	d = x – A (A = 31)	d^2	fd	fd^2
20	5	-11	121	-55	605
22	12	-9	81	-108	972
25	15	-6	36	-90	540
31	20	0	0	0	0
35	25	4	16	100	400
40	14	9	81	126	1134
42	10	11	121	110	1210
45	6	14	196	84	1176
	N=107			$\sum fd=167$	$\sum fd^2 =6037$

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$$

$$= \sqrt{\frac{6037}{107} - \left(\frac{167}{107}\right)^2}$$

$$= \sqrt{56.42 - 2.44}$$

$$= \sqrt{53.98} = 7.35$$

7.6.4 Calculation of Standard Deviation –Continuous series:

In the continuous series the method of calculating standard deviation is almost the same as in a discrete series. But in a continuous series, mid-values of the class intervals are to be found out. The step- deviation method is widely used.

The formula is,

$$\sigma = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times C$$

$$d = \frac{m - A}{C}$$, C- Class interval.

Steps:

1. Find out the mid-value of each class.
2. Assume the center value as an assumed mean and denote it by A
3. Find out $d = \frac{m - A}{C}$
4. Multiply the deviations d by the respective frequencies and get Σfd
5. Square the deviations and get d^2
6. Multiply the squared deviations (d^2) by the respective frequencies and get Σfd^2
7. Substituting the values in the following formula to get the standard deviation

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times C$$

Example 13:

The daily temperature recorded in a city in Russia in a year is given below.

Temperature C^0	No. of days
-40 to –30	10
-30 to –20	18
-20 to –10	30
-10 to 0	42
0 to 10	65
10 to 20	180
20 to 30	20
	365

Calculate Standard Deviation.

Solution:

Temperature	Mid value (m)	No. of days f	d = $\frac{m-(-5^{n})}{10^{n}}$	fd	fd ^{2}
-40 to -30	-35	10	-3	-30	90
-30 to -20	-25	18	-2	-36	72
-20 to -10	-15	30	-1	-30	30
-10 to - 0	-5	42	0	0	0
0 to 10	5	65	1	65	65
10 to 20	15	180	2	360	720
20 to 30	25	20	3	60	180
		N=365		$\sum fd$ = 389	$\sum fd^2$ =1157

$$\sigma = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times C$$

$$= \sqrt{\frac{1157}{365} - \left(\frac{389}{365}\right)^2} \times 10$$

$$= \sqrt{3.1699 - 1.1358} \times 10$$

$$= \sqrt{2.0341} \times 10$$

$$= 1.4262 \times 10$$

$$= 14.26^\circ c$$

7.6.6 Merits and Demerits of Standard Deviation:

Merits:

1. It is rigidly defined and its value is always definite and based on all the observations and the actual signs of deviations are used.
2. As it is based on arithmetic mean, it has all the merits of arithmetic mean.
3. It is the most important and widely used measure of dispersion.
4. It is possible for further algebraic treatment.
5. It is less affected by the fluctuations of sampling and hence stable.
6. It is the basis for measuring the coefficient of correlation and sampling.

Demerits:

1. It is not easy to understand and it is difficult to calculate.
2. It gives more weight to extreme values because the values are squared up.
3. As it is an absolute measure of variability, it cannot be used for the purpose of comparison.

7.6.7 Coefficient of Variation :

The Standard deviation is an absolute measure of dispersion. It is expressed in terms of units in which the original figures are collected and stated. The standard deviation of heights of students cannot be compared with the standard deviation of weights of students, as both are expressed in different units, i.e heights in centimeter and weights in kilograms. Therefore the standard deviation must be converted into a relative measure of dispersion for the purpose of comparison. The relative measure is known as the coefficient of variation.

The coefficient of variation is obtained by dividing the standard deviation by the mean and multiply it by 100. symbolically,

$$\text{Coefficient of variation (C.V)} = \frac{\sigma}{\overline{X}} \times 100$$

If we want to compare the variability of two or more series, we can use C.V. The series or groups of data for which the C.V. is greater indicate that the group is more variable, less stable, less uniform, less consistent or less homogeneous. If the C.V. is less, it indicates that the group is less variable, more stable, more uniform, more consistent or more homogeneous.

Example 15:

In two factories A and B located in the same industrial area, the average weekly wages (in rupees) and the standard deviations are as follows:

Factory	Average	Standard Deviation	No. of workers
A	34.5	5	476
B	28.5	4.5	524

1. Which factory A or B pays out a larger amount as weekly wages?
2. Which factory A or B has greater variability in individual wages?

Solution:

Given $N_1 = 476$, $\overline{X}_1 = 34.5$, $\sigma_1 = 5$

$N_2 = 524,\ \overline{X}_2 = 28.5,\ \sigma_2 = 4.5$

1. Total wages paid by factory A

$= 34.5 \times 476$

$=$ Rs.16.422

Total wages paid by factory B

$= 28.5 \times 524$

$-$ Rs.14,934.

Therefore factory A pays out larger amount as weekly wages.

2. C.V. of distribution of weekly wages of factory A and B are

$$C.V.(A) = \frac{\sigma_1}{\overline{\overline{X}}_1} \times 100$$

$$= \frac{5}{34.5} \times 100$$

$$= 14.49$$

$$C.V\ (B) = \frac{\sigma_2}{\overline{\overline{X}}_2} \times 100$$

$$= \frac{4.5}{28.5} \times 100$$

$$= 15.79$$

Factory B has greater variability in individual wages, since C.V. of factory B is greater than C.V of factory A

Example 16:

Prices of a particular commodity in five years in two cities are given below:

Price in city A	Price in city B
20	10
22	20
19	18
23	12
16	15

Which city has more stable prices?

Solution:

Actual mean method

City A			City B		
Prices (X)	Deviations from $\overline{X}$=20 dx	dx^2	Prices (Y)	Deviations from $\overline{Y}$ =15 dy	dy^2
20	0	0	10	-5	25
22	2	4	20	5	25
19	-1	1	18	3	9
23	3	9	12	-3	9
16	-4	16	15	0	0
$\sum x=100$	$\sum dx=0$	$\sum dx^2=30$	$\sum y=75$	$\sum dy=0$	$\sum dy^2$ =68

City A: $\overline{X} = \frac{\Sigma x}{n} = \frac{100}{5} = 20$

$$\sigma_x = \sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\sum dx^2}{n}}$$

$$= \sqrt{\frac{30}{5}} = \sqrt{6} = 2.45$$

$$C.V(x) = \frac{\sigma_x}{\overline{x}} \times 100$$

$$= \frac{2.45}{20} \times 100$$

$$= 12.25\ \%$$

City B: $\overline{Y} = \frac{\Sigma y}{n} = \frac{75}{5} = 15$

$$\sigma_y = \sqrt{\frac{\Sigma(y-\overline{y})^2}{n}} = \sqrt{\frac{\sum dy^2}{n}}$$

$$= \sqrt{\frac{68}{5}} = \sqrt{13.6} = 3.69$$

$$C.V.(y) = \frac{\sigma_y}{\overline{y}} \text{ x } 100$$

$$= \frac{3.69}{15} \times 100$$

$$= 24.6\ \%$$

City A had more stable prices than City B, because the coefficient of variation is less in City A.

2 MARK QUESTIONS

1.Find the Arithmetic Mean for the following data.
60 75 50 42 95 46

2.Find Median and Mode for the following data.
13 16 17 15 18 14 19 15 12

3.Write the relation between Standard Deviation and Variance.

4. Calculate the Range and its Coefficient for the following data.

X :	12	14	16	18	20
f :	1	3	5	3	1

5. Define Quartile Deviation?

6. Find the Standard Deviation for the following data:
240 260 290 245 255 288 272 263 277 251

8 MARK QUESTIONS

1.Find median ,quartiles,9thdecile and 56th percentile on the basis of the following data.

Marks:	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of students:	7	11	24	32	9	14	2	1

2. Calculate the Standard Deviation and Coefficient of Variance (CV) for the following data.

X	0 – 10	10 - 20	20 - 30	30 – 40	40 – 50
f	2	5	9	3	1

3.Calculate the G.M and H.M are shown below.

value	0 - 10	10-20	20-30	30-40	40-50
frequency	8	12	20	6	4

4.Find the Mean deviations , Median of the following data.

Marks	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 – 89	90 - 99
No. of students	7	11	24	32	9	14	2	1

5. Calculate the Median for the following Continuous Frequency Distribution.

Wages (in Rs.) :	0 - 19	20 - 39	40 - 59	60 – 79	80 - 99
No. of Workers:	5	20	35	20	12

6.The income distribution of 102 persons is given.Calculate the quartile deviation and its coefficient.

Income (Rs.)	200	250	300	350	400	700
No.of persons	10	16	30	21	15	10

7.Calculate the Arithmetic Mean ,median and mode for the following data.

Height (cms):	160	161	162	163	164	165	166
No. of Persons :	27	36	43	78	65	48	28

8. Calculate the Coefficient of Variance for the following data.

77 73 75 70 72 76 75 72 74 76

9.Calculate the Median for the following.

Hourly Wages (in Rs.)	40 - 50	50 – 60	60 - 70	70 - 80	80 - 90	90 - 100
Number of Employees	10	20	15	30	15	10

10. The following data give the details about salaries (in thousands of rupees) of seven employees randomly selected from a Pharmaceutical Company.

Serial No.	1	2	3	4	5	6	7
Salary per Annum ('000)	89	57	104	73	26	121	81

Calculate the Standard Deviation and Coefficient of variance of the given data.

11. i) Calculate the range and its coefficient for the following data.

7 4 10 9 15 12 7 9 7

ii) Calculate the standard deviation for the following data.

X : 20 22 25 31 35 40 42 45

f : 5 12 15 20 25 14 10 6

12. Calculate the Coefficient of Variation for the following data.

X	12	15	15	18	20
F	5	7	10	4	2

UNIT – IV

SYLLABUS

Bi-variate Analysis
Simple Linear Correlation Analysis: Meaning, and measurement. Karl Pearson's co-efficient and Spearman's rank correlation Simple Linear Regression Analysis: Regression equations and estimation. Relationship between correlation and regression coefficients

CORRELATION

Introduction:

The term correlation is used by a common man without knowing that he is making use of the term correlation. For example when parents advice their children to work hard so that they may get good marks, they are correlating good marks with hard work.

Thus Correlation refers to the relationship of two variables or more. (e-g) relation between height of father and son, yield and rainfall, wage and price index, share and debentures etc.

Definitions:

1. Correlation Analysis attempts to determine the degree of relationship between variables- Ya-Kun-Chou.
2. Correlation is an analysis of the covariation between two or more variables.- A.M.Tuttle.

Correlation expresses the inter-dependence of two sets of variables upon each other. One variable may be called as (subject) independent and the other relative variable (dependent). Relative variable is measured in terms of subject.

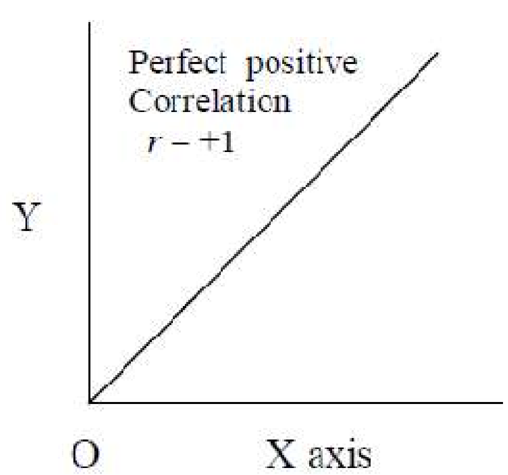
Uses of correlation:

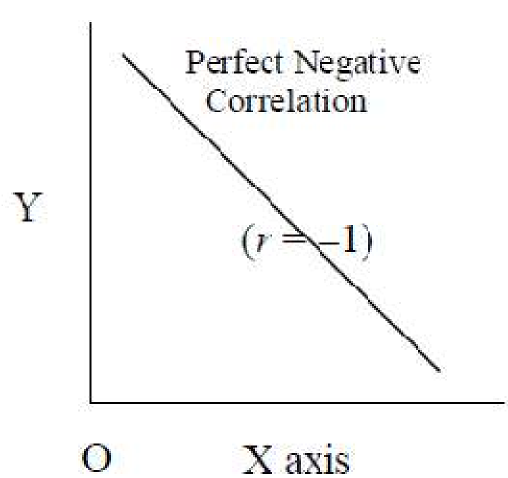
1. It is used in physical and social sciences.
2. It is useful for economists to study the relationship between variables like price, quantity etc. Businessmen estimates costs, sales, price etc. using correlation.
3. It is helpful in measuring the degree of relationship between the variables like income and expenditure, price and supply, supply and demand etc.
4. Sampling error can be calculated.
5. It is the basis for the concept of regression.

Scatter Diagram:

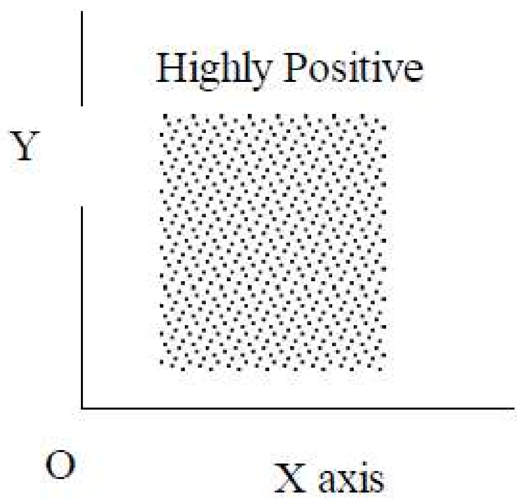
It is the simplest method of studying the relationship between two variables diagrammatically. One variable is represented along the horizontal axis and the second variable along the vertical axis. For each pair of observations of two variables, we put a dot in the plane. There are as many dots in the plane as the number of paired observations of two variables. The direction of dots shows the scatter or concentration of various points. This will show the type of correlation.

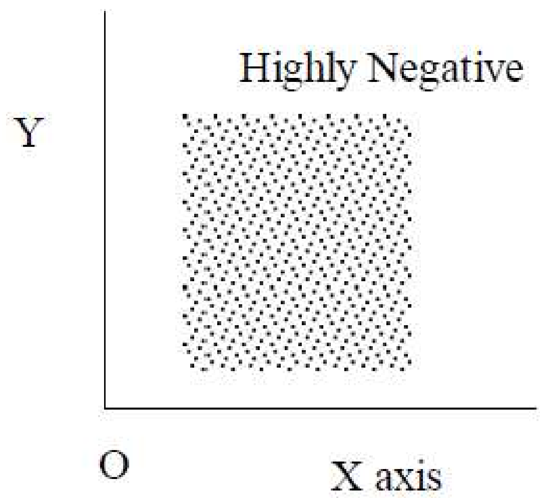
1. If all the plotted points form a straight line from lower left hand corner to the upper right hand corner then there is
Perfect positive correlation. We denote this as $r = +1$



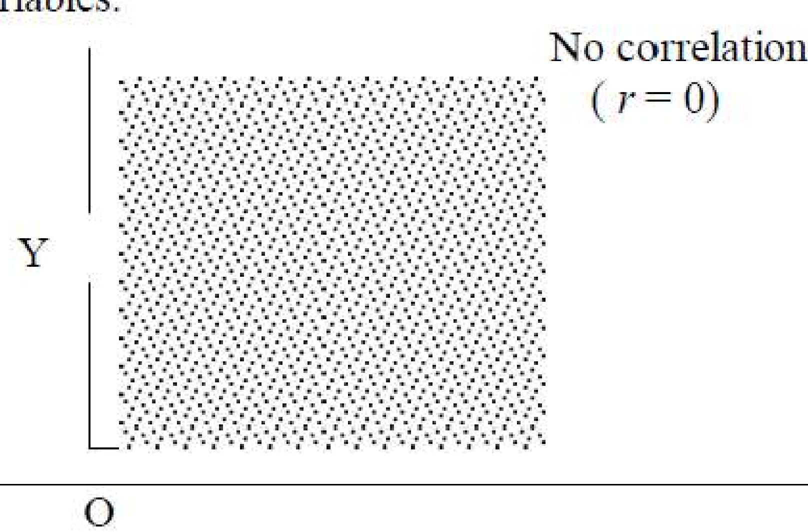


1. If all the plotted dots lie on a straight line falling from upper left hand corner to lower right hand corner, there is a perfect negative correlation between the two variables. In this case the coefficient of correlation takes the value $r = -1$.
2. If the plotted points in the plane form a band and they show a rising trend from the lower left hand corner to the upper right hand corner the two variables are highly positively correlated.





1. If the points fall in a narrow band from the upper left hand corner to the lower right hand corner, there will be a high degree of negative correlation.
2. If the plotted points in the plane are spread all over the diagram there is no correlation between the two variables.



Merits:

1. It is a simplest and attractive method of finding the nature of correlation between the two variables.
2. It is a non-mathematical method of studying correlation. It is easy to understand.
3. It is not affected by extreme items.
4. It is the first step in finding out the relation between the two variables.
5. We can have a rough idea at a glance whether it is a positive correlation or negative correlation.

Demerits:

By this method we cannot get the exact degree or correlation between the two variables.

Types of Correlation:

Correlation is classified into various types. The most important ones are

i) Positive and negative.
ii) Linear and non-linear.
iii) Partial and total.
iv) Simple and Multiple.

Positive and Negative Correlation:

It depends upon the direction of change of the variables. If the two variables tend to move together in the same direction (ie) an increase in the value of one variable is accompanied by an increase in the value of the other, (or) a decrease in the value of one variable is accompanied by a decrease in the value of other, then the correlation is called positive or direct correlation. Price and supply, height and weight, yield and rainfall, are some examples of positive correlation.

If the two variables tend to move together in opposite directions so that increase (or) decrease in the value of one variable is accompanied by a decrease or increase in the value of the other variable, then the correlation is called negative (or) inverse correlation. Price and demand, yield of crop and price, are examples of negative correlation.

Linear and Non-linear correlation:

If the ratio of change between the two variables is a constant then there will be linear correlation between them.

Consider the following.

X	2	4	6	8	10	12
Y	3	6	9	12	15	18

Here the ratio of change between the two variables is the same. If we plot these points on a graph we get a straight line.

If the amount of change in one variable does not bear a constant ratio of the amount of change in the other. Then the relation is called Curvi-linear (or) non-linear correlation. The graph will be a curve.

Simple and Multiple correlation:

When we study only two variables, the relationship is simple correlation. For example, quantity of money and price level, demand and price. But in a multiple correlation we study more than two variables simultaneously. The relationship of price, demand and supply of a commodity are an example for multiple correlation.

Partial and total correlation:

The study of two variables excluding some other variable is called **Partial correlation**. For example, we study price and demand eliminating supply side. In total correlation all facts are taken into account.

Computation of correlation:

When there exists some relationship between two variables, we have to measure the degree of relationship. This measure is called the measure of correlation (or) correlation coefficient and it is denoted by ' r' .

Co-variation:

The covariation between the variables x and y is defined as

Cov(x,y) = $\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$ where $\bar{x}, \bar{y}$ are respectively means of x and y and ' n' is the number of pairs of observations.

Karl pearson' s coefficient of correlation:

Karl pearson, a great biometrician and statistician, suggested a mathematical method for measuring the magnitude of linear relationship between the two variables. It is most widely used method in practice and it is known as pearsonian coefficient of correlation. It is denoted by 'r'. The formula for calculating 'r' is

(i) $r = \frac{Cov(x, y)}{\sigma_x . \sigma_y}$ where σ_x, σ_y are S.D of x and y respectively.

(ii) $r = \frac{\sum xy}{n \ \sigma_x \ \sigma_y}$

(iii) $r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 . \Sigma Y^2}}$, $X = x - \bar{x}$, $Y = y - \bar{y}$

when the deviations are taken from the actual mean we can apply any one of these methods. Simple formula is the third one.

The third formula is easy to calculate, and it is not necessary to calculate the standard deviations of x and y series respectively.

Steps:

1. Find the mean of the two series x and y.
2. Take deviations of the two series from x and y.
   $X = x - \bar{x}$, $Y = y - \bar{y}$
3. Square the deviations and get the total, of the respective squares of deviations of x and y and denote by ΣX^2 , ΣY^2 respectively.
4. Multiply the deviations of x and y and get the total and Divide by n. This is covariance.
5. Substitute the values in the formula.

$$r = \frac{\text{cov}(x, y)}{\sigma x . \sigma y} = \frac{\Sigma(x - \bar{x}) \ (y - \bar{y}) / n}{\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}} . \sqrt{\frac{\Sigma(y - \bar{y})^2}{n}}}$$

The above formula is simplified as follows

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 . \Sigma Y^2}}, \quad X = x - \bar{x}\ , Y = y - \bar{y}$$

Example 1:

Find Karl Pearson's coefficient of correlation from the following data between height of father (x) and son (y).

X	64	65	66	67	68	69	70
Y	66	67	65	68	70	68	72

Comment on the result.

Solution:

x	Y	$X = x - \bar{x}$ $X = x - 67$	X^2	$Y = y - \bar{y}$ $Y = y - 68$	Y^2	XY
64	66	-3	9	-2	4	6
65	67	-2	4	-1	1	2
66	65	-1	1	-3	9	3
67	68	0	0	0	0	0
68	70	1	1	2	4	2
69	68	2	4	0	0	0
70	72	3	9	4	16	12
469	476	0	28	0	34	25

$$\bar{x} = \frac{469}{7} = 67\ ;\ \bar{y} = \frac{476}{7} = 68$$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 . \Sigma Y^2}} = \frac{25}{\sqrt{28 \times 34}} = \frac{25}{\sqrt{952}} = \frac{25}{30.85} = 0.81$$

Since r = + 0.81, the variables are highly positively correlated. (ie) Tall fathers have tall sons.

Working rule (i)

We can also find r with the following formula

We have $r = \frac{Cov(x,y)}{\sigma_x . \sigma_y}$

$$Cov(x,y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} = \frac{\Sigma(xy + \bar{x}\bar{y} - \bar{y}x - \bar{x}y)}{n}$$

$$= \frac{\Sigma xy}{n} - \frac{\bar{y}\Sigma x}{n} - \frac{\bar{x}\Sigma y}{n} + \frac{\Sigma \bar{x}\bar{y}}{n}$$

$$\text{Cov(x,y)} = \frac{\Sigma xy}{n} - \bar{y}\bar{x} - \bar{x}\bar{y} + \bar{x}\bar{y} = \frac{\Sigma xy}{n} - \bar{x}\bar{y}$$

$$\sigma^2 x^2 = \frac{\Sigma x^2}{n} - \bar{x}^2, \quad \sigma^2 y^2 = \frac{\Sigma y^2}{n} - \bar{y}^2$$

$$\text{Now } r = \frac{Cov(x,y)}{\sigma_x . \sigma_y}$$

$$r = \frac{\frac{\Sigma xy}{n} - \bar{x}\bar{y}}{\sqrt{\left(\frac{\Sigma x^2}{n} - \bar{x}^2\right)} . \sqrt{\left(\frac{\Sigma y^2}{n} - \bar{y}^2\right)}}$$

$$r = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

Note: In the above method we need not find mean or standard deviation of variables separately.

Example 2:

Calculate coefficient of correlation from the following data.

X	1	2	3	4	5	6	7	8	9
Y	9	8	10	12	11	13	14	16	15

x	y	x^2	y^2	xy
1	9	1	81	9
2	8	4	64	16
3	10	9	100	30
4	12	16	144	48
5	11	25	121	55
6	13	36	169	78
7	14	49	196	98
8	16	64	256	128
9	15	81	225	135
45	108	285	1356	597

$$r = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$r = \frac{9 \times 597 - 45 \times 108}{\sqrt{(9\times 285 - (45)^2).(9\times 1356 - (108)^2)}}$$

$$r = \frac{5373 - 4860}{\sqrt{(2565-2025)(12204-11664)}}$$

$$= \frac{513}{\sqrt{540 \times 540}} = \frac{513}{540} = 0.95$$

Working rule (ii) (shortcut method)

We have $r = \frac{Cov(x,y)}{\sigma_x.\sigma_y}$

where Cov(x,y) = $\frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$

Take the deviation from x as x – A and the deviation from y as y – B

$$Cov(x,y) = \frac{\Sigma\ [(x-A)-(\bar{x}-A)]\ [(y-B)-(\bar{y}-B)]}{n}$$

$$= \frac{1}{n}\Sigma\ [(x-A)\ (y-B)\ -\ (x-A)\ (\bar{y}-B)$$

$$-\ (\bar{x}-A)(y-B)+(\bar{x}-A)(\bar{y}-B)]$$

$$= \frac{1}{n}\Sigma\ [(x-A)\ (y-B)\ -\ (\bar{y}-B)\ \frac{\Sigma(x-A)}{n}$$

$$-\ (\bar{x}-A)\frac{\Sigma(y-B)}{n}+\frac{\Sigma(\bar{x}-A)(\bar{y}-B)}{n}$$

$$= \frac{\Sigma(x-A)(y-B)}{n} - (\bar{y}-B)\ (\bar{x}-\frac{nA}{n})$$

$$-\ (\bar{x}-A)\ (\bar{y}-\frac{nB}{n})+(\bar{x}-A)\ (\bar{y}-B)$$

$$= \frac{\Sigma(x-A)(y-B)}{n} - (\bar{y}-B)(\bar{x}-A)$$

$$- (\bar{x}-A)(\bar{y}-B) + (\bar{x}-A)(\bar{y}-B)$$

$$= \frac{\Sigma(x-A)(y-B)}{n} - (\bar{x}-A)(\bar{y}-B)$$

Let $x-A = u$; $y-B = v$; $\bar{x}-A=\bar{u}$; $\bar{y}-B=\bar{v}$

$$\therefore \text{Cov}(x,y) = \frac{\Sigma uv}{n} - \bar{u}\bar{v}$$

$$\sigma\sigma_x^2 = \frac{\Sigma u^2}{n} - \bar{u}^2 = \sigma u^2$$

$$\sigma\sigma_y^2 = \frac{\Sigma v^2}{n} - \bar{v}^2 = \sigma v^2$$

$$\therefore r = \frac{n\Sigma uv - (\Sigma u)(\Sigma v)}{\sqrt{\left[n\Sigma u^2 - (\Sigma u)^2\right].\left[(n\Sigma v^2) - (\Sigma v)^2\right]}}$$

Example 3:

Calculate Pearson' s Coefficient of correlation.

X	45	55	56	58	60	65	68	70	75	80	85
Y	56	50	48	60	62	64	65	70	74	82	90

X	Y	u – x-A	v – y-B	u^2	v^2	uv
45	56	-20	-14	400	196	280
55	50	-10	-20	100	400	200
56	48	-9	-22	81	484	198
58	60	-7	-10	49	100	70
60	62	-5	-8	25	64	40
65	64	0	-6	0	36	0
68	65	3	-5	9	25	-15
70	70	5	0	25	0	0
75	74	10	4	100	16	40
80	82	15	12	225	144	180
85	90	20	20	400	400	400
		2	-49	1414	1865	1393

$$r = \frac{n\Sigma uv - (\Sigma u)\ (\Sigma v)}{\sqrt{[n\Sigma u^2 - (\Sigma u^2)]\ [n\Sigma v^2 - (\Sigma v)^2]}}$$

$$r = \frac{11 \times 1393 - 2 \times (-49)}{\sqrt{(1414 \times 11 - (2)^2) \times (1865 \times 11 - (-49)^2)}}$$

$$= \frac{15421}{\sqrt{15550 \times 18114}} = \frac{15421}{16783.11} = +0.92$$

Limitations:

1. Correlation coefficient assumes linear relationship regardless of the assumption is correct or not.
2. Extreme items of variables are being unduly operated on correlation coefficient.
3. Existence of correlation does not necessarily indicate cause-effect relation.

Interpretation:

The following rules helps in interpreting the value of 'r'.

1. When r = 1, there is perfect +ve relationship between the variables.
2. When r = -1, there is perfect –ve relationship between the variables.
3. When r = 0, there is no relationship between the variables.
4. If the correlation is +1 or –1, it signifies that there is a high degree of correlation. (+ve or –ve) between the two variables.

If r is near to zero (ie) 0.1,-0.1, (or) 0.2 there is less correlation.

Rank Correlation:

It is studied when no assumption about the parameters of the population is made. This method is based on ranks. It is useful to study the qualitative measure of attributes like honesty, colour, beauty, intelligence, character, morality etc.The individuals in the group can be arranged in order and there on, obtaining for each individual a number showing his/her rank in the group. This method was developed by Edward Spearman in 1904. It is defined

as $r = 1 - \frac{6\Sigma D^2}{n^3 - n}$ r = rank correlation coefficient.

Note: Some authors use the symbol ρ for rank correlation.
ΣD^2 = sum of squares of differences between the pairs of ranks.
n = number of pairs of observations.

The value of r lies between –1 and +1. If r = +1, there is complete agreement in order of ranks and the direction of ranks is also same. If r = -1, then there is complete disagreement in order of ranks and they are in opposite directions.

Computation for tied observations: There may be two or more items having equal values. In such case the same rank is to be given. The ranking is said to be tied. In such circumstances an average rank is to be given to each individual item. For example if the value so is repeated twice at the 5^{th} rank, the common rank to be assigned to each item is $\frac{5+6}{2}$ = 5.5 which is the average of 5 and 6 given as 5.5, appeared twice.

If the ranks are tied, it is required to apply a correction factor which is $\frac{1}{12}$ (m^3-m). A slightly different formula is used when there is more than one item having the same value.

The formula is

$$r = 1 - \frac{6[\Sigma D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + ...]}{n^3 - n}$$

Where m is the number of items whose ranks are common and should be repeated as many times as there are tied observations.

Example 6:

In a marketing survey the price of tea and coffee in a town based on quality was found as shown below. Could you find any relation between and tea and coffee price.

Price of tea	88	90	95	70	60	75	50
Price of coffee	120	134	150	115	110	140	100

Price of tea	Rank	Price of coffee	Rank	D	D^2
88	3	120	4	1	1
90	2	134	3	1	1
95	1	150	1	0	0
70	5	115	5	0	0
60	6	110	6	0	0
75	4	140	2	2	4
50	7	100	7	0	0
					$\Sigma D^2 = 6$

$$r = 1 - \frac{6\Sigma D^2}{n^3 - n} = 1 - \frac{6 \times 6}{7^3 - 7}$$

$$= 1 - \frac{36}{336} = 1 - 0.1071$$

$$= 0.8929$$

The relation between price of tea and coffee is positive at 0.89. Based on quality the association between price of tea and price of coffee is highly positive.

Example 7:

In an evaluation of answer script the following marks are awarded by the examiners.

1st	88	95	70	960	50	80	75	85
2nd	84	90	88	55	48	85	82	72

Do you agree the evaluation by the two examiners is fair?

x	R1	y	R2	D	D^2
88	2	84	4	2	4
95	1	90	1	0	0
70	6	88	2	4	16
60	7	55	7	0	0
50	8	48	8	0	0
80	4	85	3	1	1
85	3	75	6	3	9
					30

$$r = 1 - \frac{6\Sigma D^2}{n^3 - n} = 1 - \frac{6 \times 30}{8^3 - 8}$$

$$-1 \quad \frac{180}{504} \quad -1 \quad 0.357 - 0.643$$

r = 0.643 shows fair in awarding marks in the sense that uniformity has arisen in evaluating the answer scripts between the two examiners.

Example 8:

Rank Correlation for tied observations. Following are the marks obtained by 10 students in a class in two tests.

Students	A	B	C	D	E	F	G	H	I	J
Test 1	70	68	67	55	60	60	75	63	60	72
Test 2	65	65	80	60	68	58	75	63	60	70

Calculate the rank correlation coefficient between the marks of two tests.

Student	Test 1	R1	Test 2	R2	D	D^2
A	70	3	65	5.5	-2.5	6.25
B	68	4	65	5.5	-1.5	2.25
C	67	5	80	1.0	4.0	16.00
D	55	10	60	8.5	1.5	2.25
E	60	8	68	4.0	4.0	16.00
F	60	8	58	10.0	-2.0	4.00
G	75	1	75	2.0	-1.0	1.00
H	63	6	62	7.0	-1.0	1.00
I	60	8	60	8.5	0.5	0.25
J	72	2	70	3.0	-1.0	1.00
						50.00

60 is repeated 3 times in test 1.
60,65 is repeated twice in test 2.
m = 3; m = 2; m = 2

$$r = 1 - \frac{6[\Sigma D^2 + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m) + \frac{1}{12}(m^3 - m)}{n^3 - n}$$

$$= 1 - \frac{6[50 + \frac{1}{12}(3^3 - 3) + \frac{1}{12}(2^3 - 2) + \frac{1}{12}(2^3 - 2)]}{10^3 - 10}$$

$$= 1 - \frac{6[50 + 2 + 0.5 + 0.5]}{990}$$

$$= 1 - \frac{6 \times 53}{990} = \frac{672}{990} = 0.68$$

REGRESSION

9.1 Introduction:

After knowing the relationship between two variables we may be interested in estimating (predicting) the value of one variable given the value of another. The variable predicted on the basis of other variables is called the "dependent" or the ' explained' variable and the other the ' independent' or the ' predicting' variable. The prediction is based on average relationship derived statistically by regression analysis. The equation, linear or otherwise, is called the regression equation or the explaining equation.

For example, if we know that advertising and sales are correlated we may find out expected amount of sales for a given advertising expenditure or the required amount of expenditure for attaining a given amount of sales.

The relationship between two variables can be considered between, say, rainfall and agricultural production, price of an input and the overall cost of product, consumer expenditure and disposable income. Thus, regression analysis reveals average relationship between two variables and this makes possible estimation or prediction.

9.1.1 Definition:

Regression is the measure of the average relationship between two or more variables in terms of the original units of the data.

9.2 Types Of Regression:

The regression analysis can be classified into:

a) Simple and Multiple
b) Linear and Non –Linear
c) Total and Partial

a) Simple and Multiple:

In case of simple relationship only two variables are considered, for example, the influence of advertising expenditure on sales turnover. In the case of multiple relationship, more than two variables are involved. On this while one variable is a dependent variable the remaining variables are independent ones.

For example, the turnover (y) may depend on advertising expenditure (x) and the income of the people (z). Then the functional relationship can be expressed as $y = f(x,z)$.

b) Linear and Non-linear:

The linear relationships are based on straight-line trend, the equation of which has no-power higher than one. But, remember a linear relationship can be both simple and multiple. Normally a linear relationship is taken into account because besides its simplicity, it has a better predective value, a linear trend can be easily projected into the future. In the case of non-linear relationship curved trend lines are derived. The equations of these are parabolic.

c) Total and Partial:

In the case of total relationships all the important variables are considered. Normally, they take the form of a multiple relationships because most economic and business phenomena are affected by multiplicity of cases. In the case of partial relationship one or more variables are considered, but not all, thus excluding the influence of those not found relevant for a given purpose.

9.3 Linear Regression Equation:

If two variables have linear relationship then as the independent variable (X) changes, the dependent variable (Y) also changes. If the different values of X and Y are plotted, then the two straight lines of best fit can be made to pass through the plotted points. These two lines are known as regression lines. Again, these regression lines are based on two equations known as regression equations. These equations show best estimate of one variable for the known value of the other. The equations are linear.

Linear regression equation of Y on X is

$$Y = a + bX \text{ ......(1)}$$

And X on Y is

$$X = a + bY \text{ ......(2)}$$

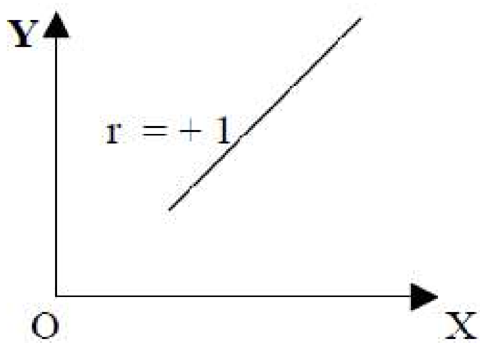
a, b are constants.

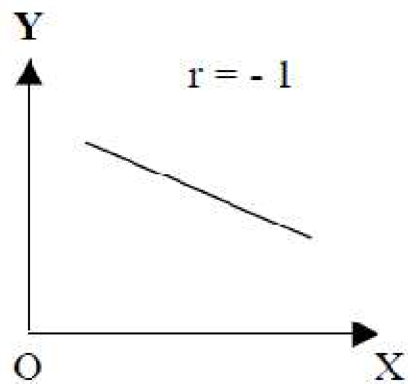
From (1) We can estimate Y for known value of X.
(2) We can estimate X for known value of Y.

9.3.1 Regression Lines:

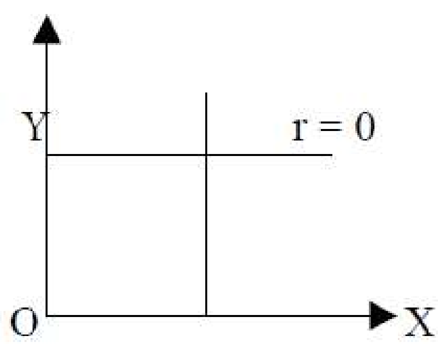
For regression analysis of two variables there are two regression lines, namely Y on X and X on Y. The two regression lines show the average relationship between the two variables.

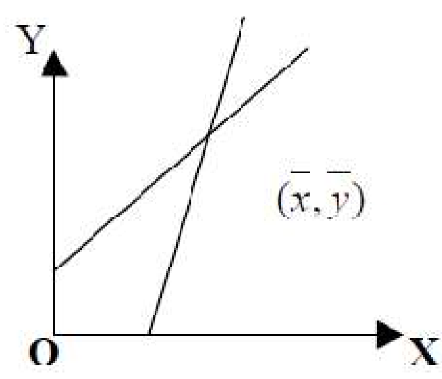
For perfect correlation, positive or negative i.e., $r = \pm 1$, the two lines coincide i.e., we will find only one straight line. If $r = 0$, i.e., both the variables are independent then the two lines will cut each other at right angle. In this case the two lines will be parallel to X and Y-axes.





Lastly the two lines intersect at the point of means of X and Y. From this point of intersection, if a straight line is drawn on X-axis, it will touch at the mean value of x. Similarly, a perpendicular drawn from the point of intersection of two regression lines on Y-axis will touch the mean value of Y.





9.3.2 Principle of 'Least Squares' :

Regression shows an average relationship between two variables, which is expressed by a line of regression drawn by the method of "least squares". This line of regression can be derived graphically or algebraically. Before we discuss the various methods let us understand the meaning of least squares.

A line fitted by the method of least squares is known as the line of best fit. The line adapts to the following rules:

(i) The algebraic sum of deviation in the individual observations with reference to the regression line may be equal to zero. i.e.,

$\Sigma(X - Xc) = 0$ or $\Sigma(Y - Yc) = 0$

Where Xc and Yc are the values obtained by regression analysis.

(ii) The sum of the squares of these deviations is less than the sum of squares of deviations from any other line. i.e.,

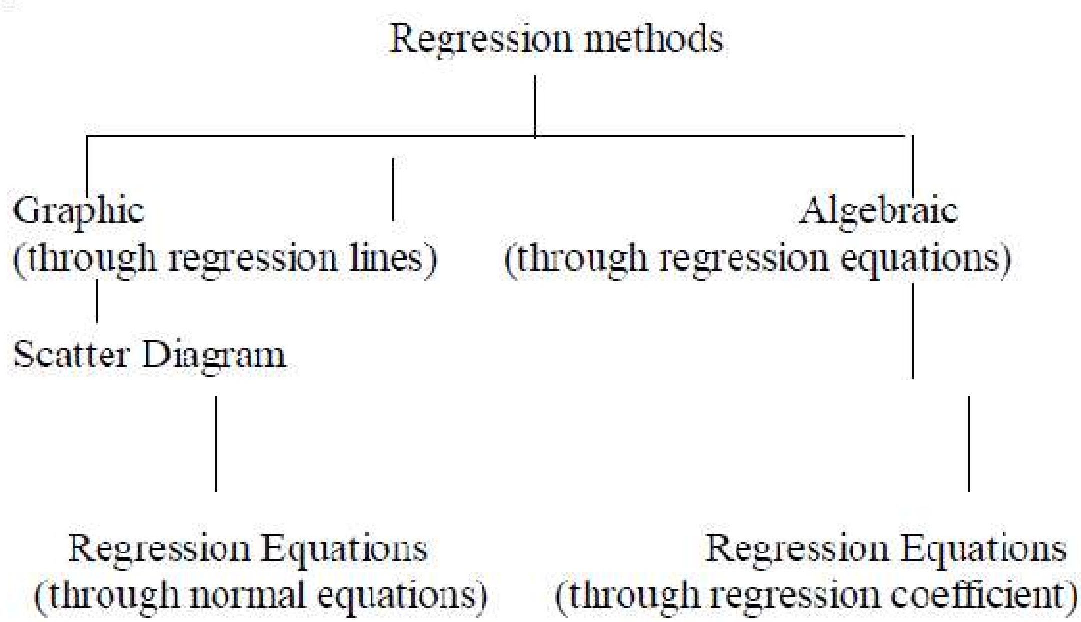
$\Sigma(Y - Yc)^2 < \Sigma(Y - Ai)^2$

Where Ai = corresponding values of any other straight line.

(iii) The lines of regression (best fit) intersect at the mean values of the variables X and Y, i.e., intersecting point is $\bar{x}, \bar{y}$.

9.4 Methods of Regression Analysis:

The various methods can be represented in the form of chart given below:



9.4.1 Graphic Method:

Scatter Diagram:

Under this method the points are plotted on a graph paper representing various parts of values of the concerned variables. These points give a picture of a scatter diagram with several points spread over. A regression line may be drawn in between these points either by free hand or by a scale rule in such a way that the squares of the vertical or the horizontal distances (as the case may be) between the points and the line of regression so drawn is the least. In other words, it should be drawn faithfully as the line of best fit leaving equal number of points on both sides in such a manner that the sum of the squares of the distances is the best.

9.4.2 Algebraic Methods:

(i) Regression Equation.

The two regression equations

for X on Y; $X = a + bY$

And for Y on X; $Y = a + bX$

Where X, Y are variables, and a,b are constants whose values are to be determined

For the equation, $X = a + bY$

The normal equations are

$$\sum X = na + b\sum Y \text{ and}$$

$$\sum XY = a\sum Y + b\sum Y^2$$

For the equation, $Y = a + bX$, the normal equations are

$$\sum Y = na + b\sum X \text{ and}$$

$$\sum XY = a\sum X + b\sum X^2$$

From these normal equations the values of a and b can be determined.

Example 1:

Find the two regression equations from the following data:

X:	6	2	10	4	8
Y:	9	11	5	8	7

Solution:

X	Y	X^2	Y^2	XY
6	9	36	81	54
2	11	4	121	22
10	5	100	25	50
4	8	16	64	32
8	7	64	49	56
30	40	220	340	214

Regression equation of Y on X is $Y = a + bX$ and the normal equations are

$$\sum Y = na + b\sum X$$
$$\sum XY = a\sum X + b\sum X^2$$

Substituting the values, we get

$$40 = 5a + 30b \;\ldots\ldots\; (1)$$
$$214 = 30a + 220b \;\ldots\ldots (2)$$

Multiplying (1) by 6

$$240 = 30a + 180b. \ldots\ldots (3)$$

(2) – (3) $\quad -26 = 40b$

or $b = -\dfrac{26}{40} = -0.65$

Now, substituting the value of 'b' in equation (1)

$$40 = 5a - 19.5$$
$$5a = 59.5$$
$$a = \frac{59.5}{5} = 11.9$$

Hence, required regression line Y on X is $Y = 11.9 - 0.65X$.

Again, regression equation of X on Y is

$X = a + bY$ and

The normal equations are

$$\sum X = na + b\sum Y \text{ and}$$
$$\sum XY = a\sum Y + b\sum Y^2$$

Now, substituting the corresponding values from the above table, we get

$$30 = 5a + 40b \quad ....(3)$$
$$214 = 40a + 340b \quad ....(4)$$

Multiplying (3) by 8, we get

$$240 = 40a + 320\,b \quad ...(5)$$

(4) – (5) gives

$$-26 = 20b$$
$$b = -\frac{26}{20} = -1.3$$

Substituting $b = -1.3$ in equation (3) gives

$$30 = 5a - 52$$
$$5a = 82$$
$$a = \frac{82}{5} = 16.4$$

Hence, Required regression line of X on Y is

X = 16.4 – 1.3Y

(ii) Regression Co-efficents:

The regression equation of Y on X is $y_e = \overline{y} + r\frac{\sigma_y}{\sigma_x}(x - \overline{x})$

Here, the regression Co.efficient of Y on X is

$$b_1 = b_{yx} = r\frac{\sigma_y}{\sigma_x}$$
$$y_e = \overline{y} + b_1(x - \overline{x})$$

The regression equation of X on Y is

$$X_e = \overline{x} + r\frac{\sigma_x}{\sigma_y}(y - \overline{y})$$

Here, the regression Co-efficient of X on Y

$$b_2 = b_{xy} = r\frac{\sigma_x}{\sigma_y}$$
$$X_e = \overline{X} + b_2(y - \overline{y})$$

If the deviation are taken from respective means of x and y

$$b_1 = b_{yx} = \frac{\sum (X-\overline{X})(Y-\overline{Y})}{\sum (X-\overline{X})^2} = \frac{\sum xy}{\sum x^2} \quad \text{and}$$

$$b_2 = b_{xy} = \frac{\sum (X-\overline{X})(Y-\overline{Y})}{\sum (Y-\overline{Y})^2} = \frac{\sum xy}{\sum y^2}$$

where $x = X-\overline{X}, y = Y-\overline{Y}$

If the deviations are taken from any arbitrary values of x and y (short – cut method)

$$b_1 = b_{yx} = \frac{n\sum uv - \sum u \sum v}{n\sum u^2 - \left(\sum u\right)^2}$$

$$b_2 = b_{xy} = \frac{n\sum uv - \sum u \sum v}{n\sum v^2 - \left(\sum v\right)^2}$$

where u = x – A : v = Y-B
A = any value in X
B = any value in Y

9.5 Properties of Regression Co-efficient:

1. Both regression coefficients must have the same sign, ie either they will be positive or negative.
2. correlation coefficient is the geometric mean of the regression coefficients ie, $r = \pm\sqrt{b_1 b_2}$
3. The correlation coefficient will have the same sign as that of the regression coefficients.
4. If one regression coefficient is greater than unity, then other regression coefficient must be less than unity.
5. Regression coefficients are independent of origin but not of scale.
6. Arithmetic mean of b_1 and b_2 is equal to or greater than the coefficient of correlation. Symbolically $\frac{b_1 + b_2}{2} \geq r$

7. If r=0, the variables are uncorrelated , the lines of regression become perpendicular to each other.
8. If r= $\pm$1, the two lines of regression either coincide or parallel to each other
9. Angle between the two regression lines is $\theta = \tan^{-1}\left[\frac{m_1 - m_2}{1 + m_1 m_2}\right]$

   where m_1 and,m_2 are the slopes of the regression lines X on Y and Y on X respectively.
10. The angle between the regression lines indicates the degree of dependence between the variables.

Example 2:

If 2 regression coefficients are $b_1 = \frac{4}{5}$ and $b_2 = \frac{9}{20}$ .What would be the value of r?

Solution:

The correlation coefficient , $r = \pm\sqrt{b_1 b_2}$

$$= \sqrt{\frac{4}{5} \text{x} \frac{9}{20}}$$

$$= \sqrt{\frac{36}{100}} = \frac{6}{10} = 0.6$$

Example 3:

Given $b_1 = \frac{15}{8}$ and $b_2 = \frac{3}{5}$, Find r

Solution:

$r = \pm\sqrt{b_1 b_2}$

$$= \sqrt{\frac{15}{8} \text{x} \frac{3}{5}}$$

$$= \sqrt{\frac{9}{8}} = 1.06$$

It is not possible since r, cannot be greater than one. So the given values are wrong

Example 4:

Compute the two regression equations from the following data.

X	1	2	3	4	5
Y	2	3	5	4	6

If x =2.5, what will be the value of y?

Solution:

X	Y	$x = X - \overline{X}$	$y = Y - \overline{Y}$	x^2	y^2	xy
1	2	-2	-2	4	4	4
2	3	-1	-1	1	1	-1
3	5	0	1	0	1	0
4	4	1	0	1	0	0
5	6	2	2	4	4	4
15	20	20		10	10	9

$$\overline{X} = \frac{\sum X}{n} = \frac{15}{5} = 3$$

$$\overline{Y} = \frac{\sum Y}{n} = \frac{20}{5} = 4$$

Regression Co efficient of Y on X

$$b_{yx} = \frac{\sum xy}{\sum x^2} = \frac{9}{10} = 0.9$$

Hence regression equation of Y on X is

$$Y = \overline{Y} + b_{yx}(X - \overline{X})$$

$= 4 + 0.9\ (X - 3)$

$- 4 + 0.9X - 2.7$

$=1.3 + 0.9X$

when X = 2.5

$Y - 1.3 + 0.9 \times 2.5$

$= 3.55$

Regression co efficient of X on Y

$$b_{xy} = \frac{\sum xy}{\sum y^2} = \frac{9}{10} = 0.9$$

So, regression equation of X on Y is

$$X = \overline{X} + b_{xy}(Y - \overline{Y})$$

$$= 3 + 0.9\,(Y - 4)$$

$$= 3 + 0.9Y - 3.6$$

$$= 0.9Y - 0.6$$

Example 6:

In a correlation study, the following values are obtained

	X	Y
Mean	65	67
S.D	2.5	3.5

Co-efficient of correlation = 0.8
Find the two regression equations that are associated with the above values.

Solution:

Given,

$\overline{X} = 65,\ \overline{Y} = 67,\ \sigma_x = 2.5,\ \sigma_y = 3.5,\ r = 0.8$

The regression co-efficient of Y on X is

$$b_{yx} = b_1 = r\frac{\sigma_y}{\sigma_x}$$

$$= 0.8 \times \frac{3.5}{2.5} = 1.12$$

The regression coefficient of X on Y is

$$b_{xy} = b_2 = r\frac{\sigma_x}{\sigma_y}$$

$$= 0.8 \times \frac{2.5}{3.5} = 0.57$$

Hence, the regression equation of Y on X is

$$Y_e = \overline{Y} + b_1(X - \overline{X})$$

$= 67 + 1.12\ (X-65)$

$= 67 + 1.12\ X - 72.8$

$= 1.12X - 5.8$

The regression equation of X on Y is

$$X_e = \overline{X} + b_2(Y - \overline{Y})$$

$$= 65 + 0.57\ (Y-67)$$

$$= 65 + 0.57Y - 38.19$$

$$= 26.81 + 0.57Y$$

9.7 Uses of Regression Analysis:

1. Regression analysis helps in establishing a functional relationship between two or more variables.
2. Since most of the problems of economic analysis are based on cause and effect relationships, the regression analysis is a highly valuable tool in economic and business research.
3. Regression analysis predicts the values of dependent variables from the values of independent variables.
4. We can calculate coefficient of correlation (r) and coefficient of determination (r^2) with the help of regression coefficients.
5. In statistical analysis of demand curves, supply curves, production function, cost function, consumption function etc., regression analysis is widely used.

9.8 Difference between Correlation and Regression:

S.No	Correlation	Regression
1.	Correlation is the relationship between two or more variables, which vary in sympathy with the other in the same or the opposite direction.	Regression means going back and it is a mathematical measure showing the average relationship between two variables
2.	Both the variables X and Y are random variables	Here X is a random variable and Y is a fixed variable. Sometimes both the variables may be random variables.
3.	It finds out the degree of relationship between two variables and not the cause and effect of the variables.	It indicates the causes and effect relationship between the variables and establishes functional relationship.

4.	It is used for testing and verifying the relation between two variables and gives limited information.	Besides verification it is used for the prediction of one value, in relationship to the other given value.
5.	The coefficient of correlation is a relative measure. The range of relationship lies between –1 and +1	Regression coefficient is an absolute figure. If we know the value of the independent variable, we can find the value of the dependent variable.
6.	There may be spurious correlation between two variables.	In regression there is no such spurious regression.
7.	It has limited application, because it is confined only to linear relationship between the variables.	It has wider application, as it studies linear and non-linear relationship between the variables.
8.	It is not very useful for further mathematical treatment.	It is widely used for further mathematical treatment.
9.	If the coefficient of correlation is positive, then the two variables are positively correlated and vice-versa.	The regression coefficient explains that the decrease in one variable is associated with the increase in the other variable.

2 MARK QUESTIONS

1. Define Correlation.
2. Define Standard Error and Probable Error.
3. Give any two points on merits and Demerits of Spearman's Rank correlation and Regression
4. Distinguish between correlation and Regression
5. Mention the different type of correlation
6. write the formula for correlation and Regression equstions.
7. If $r = 0.78, \sigma_x = 2.5, \sigma_y = 3.5$ then find b_{xy}
8. write the formula for Rank correlation

8 MARK QUESTIONS

1. Distinguish between correlation and Regression (minimum 8 Points)
2. Marks obtained by 8 students in Accountancy (X) and Statistics (Y) are given below. Compute Rank Correlation Coefficient.

X	25	20	28	22	40	60	20
Y	40	30	50	30	20	10	30

3. You are given the following data:

	X	Y
Arithmetic mean	36	85
Standard deviation	11	8

Correlation coefficient between X and Y = 0.66

i)Find the two regression equations.

ii) Find r.

4. Find Karl Pearson s coefficient of correlation from the following data .

X :	100	101	102	102	100	99	97	98	96	95
Y :	98	99	99	97	95	92	95	94	90	91

5. From the data given below find the two regression lines.

X:	10	12	13	12	16	15
Y:	40	38	43	45	37	43.

i) Estimate Y when X = 20.

ii) Estimate X when Y = 25.

6. Find the Karl Pearson's coefficient of correlation from the marks secured by 10 students in Accountancy and statistics.

Marks in Accountancy (X): 45 70 65 30 90 40 50 75 85 60

Marks in statistics (Y): 35 90 70 40 95 40 60 80 80 50.

7. In a correlation study, the following values are obtained

	X	Y
Mean	65	67
S.D	2.5	3.5

Coefficient of correlation = 0.8

Find the two regression equations.

8. Find the Spearman's rank correlation coefficient for the following data.

X: 68	64	75	50	64	80	75	40	55	64
Y: 62	58	68	45	81	60	68	48	50	70

9. Following are the rank obtained by 10 students in two subjects. Find the Spearman's rank correlation coefficient.

Statistics:	1	2	3	4	5	6	7	8	9	10
Maths:	2	4	1	5	3	9	7	10	6	8

10. Develop the Regression Equation that best fit the data given below using annual income as an independent variable and amount of life insurance as dependent variable.

Annual Income (Rs. in 000's)	62	78	41	53	85	34
Amount of Life Insurance (Rs. in 00's)	25	30	10	15	50	7

UNIT – V

SYLLABUS

Time-based Data: Index Numbers and Time-Series Analysis
Meaning and uses of index numbers; Construction of index numbers: Aggregative and average of relatives – simple and weighted, Tests of adequacy of index numbers, Construction of consumer price indices. Components of time series; additive and multiplicative models; Trend analysis: Finding trend by moving average method and Fitting of linear trend line using principle of least squares

Index Numbers

Introduction:

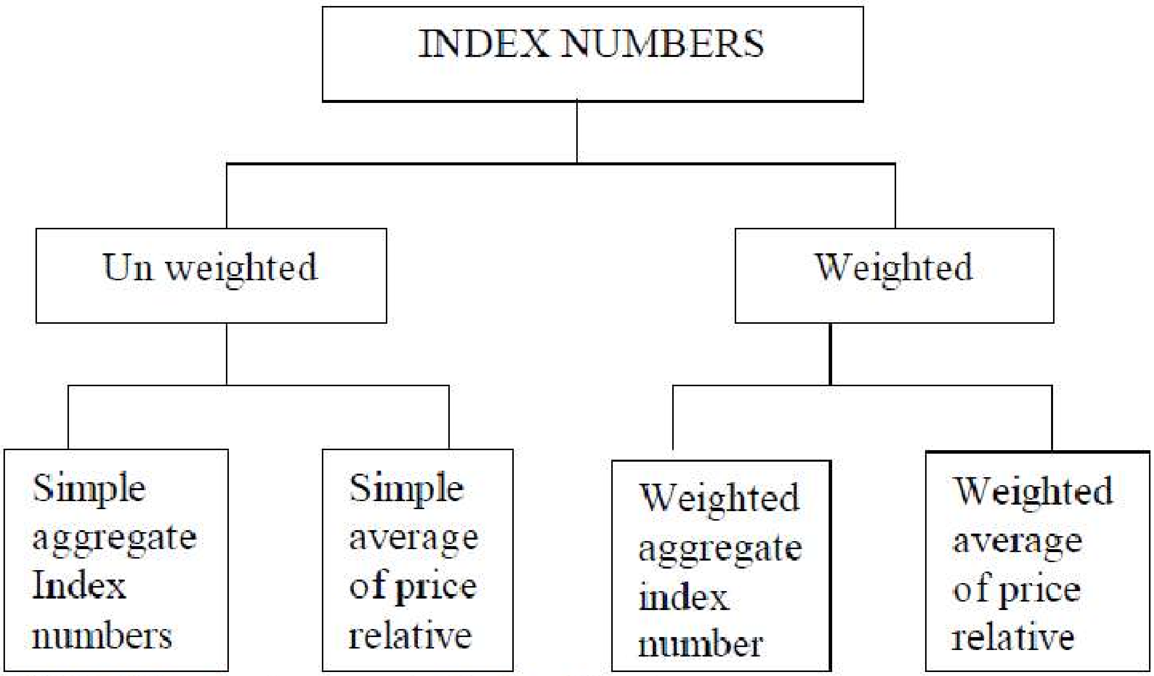
An index number is a statistical device for comparing the general level of magnitude of a group of related variables in two or more situation. If we want to compare the price level of 2000 with what it was in 1990, we shall have to consider a group of variables such as price of wheat, rice, vegetables, cloth, house rent etc., If the changes are in the same ratio and the same direction, we face no difficulty to find out the general price level. But practically, if we think changes in different variables are different and that too, upward or downward, then the price is quoted in different units i.e milk for litre, rice or wheat for kilogram, rent for square feet, etc

Notation: For any index number, two time periods are needed for comparison. These are called the Base period and the Current period. The period of the year which is used as a basis for comparison is called the base year and the other is the current year. The various notations used are as given below:

P_1 = Price of current year P_0 = Price of base year
q_1 = Quantity of current year q_0 = Quantity of base year

10.5 Method of construction of index numbers:

Index numbers may be constructed by various methods as shown below:



10.5.1 Simple Aggregate Index Number

This is the simplest method of construction of index numbers. The price of the different commodities of the current year are added and the sum is divided by the sum of the prices of those commodities by 100. Symbolically,

$$\text{Simple aggregate price index} = P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

Where , Σp_1 = total prices for the current year

Σp_0 = Total prices for the base year

Example 1:

Calculate index numbers from the following data by simple aggregate method taking prices of 2000 as base.

Commodity	Price per unit (in Rupees)	
	2000	2004
A	80	95
B	50	60
C	90	100
D	30	45

Solution:

Commodity	Price per unit (in Rupees)	
	2000 (P_0)	2004 (P_1)
A	80	95
B	50	60
C	90	100
D	30	45
Total	250	300

$$\text{Simple aggregate Price index} = P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$$

$$= \frac{300}{250} \times 100 = 120$$

10.5.2 Simple Average Price Relative index:

In this method, first calculate the price relative for the various commodities and then average of these relative is obtained by using arithmetic mean and geometric mean. When arithmetic mean is used for average of price relative, the formula for computing the index is

Simple average of price relative by arithmetic mean

$$P_{01} = \frac{\Sigma\left(\frac{p_1}{p_0}\times 100\right)}{n}$$

P_1 = Prices of current year
P_0 = Prices of base year
n = Number of items or commodities
when geometric mean is used for average of price relative, the formula for obtaining the index is
Simple average of price relative by geometric Mean

$$P_{01} = \text{Antilog}\left(\frac{\Sigma \log(\frac{p_1}{p_0}\times 100)}{n}\right)$$

Example 2:
From the following data, construct an index for 1998 taking 1997 as base by the average of price relative using (a) arithmetic mean and (b) Geometric mean

Commodity	Price in 1997	Price in 1998
A	50	70
B	40	60
C	80	100
D	20	30

Solution:
(a) Price relative index number using arithmetic mean

Commodity	Price in 1997 (P_0)	Price in 1998 (P_1)	$\frac{p_1}{p_0}\times 100$
A	50	70	140
B	40	60	150
C	80	100	125
D	20	30	150
		Total	565

Simple average of price relative by arithmetic mean

$$P_{01} = \frac{\sum\left(\frac{p_1}{p_0}\times 100\right)}{n}$$

P_1 = Prices of current year
P_0 = Prices of base year
n = Number of items or commodities
when geometric mean is used for average of price relative, the formula for obtaining the index is
Simple average of price relative by geometric Mean

$$P_{01} = \text{Antilog}\left(\frac{\sum \log\left(\frac{p_1}{p_0}\times 100\right)}{n}\right)$$

Example 2:
From the following data, construct an index for 1998 taking 1997 as base by the average of price relative using (a) arithmetic mean and (b) Geometric mean

Commodity	Price in 1997	Price in 1998
A	50	70
B	40	60
C	80	100
D	20	30

Solution:
(a) Price relative index number using arithmetic mean

Commodity	Price in 1997 (P_0)	Price in 1998 (P_1)	$\frac{p_1}{p_0}\times 100$
A	50	70	140
B	40	60	150
C	80	100	125
D	20	30	150
		Total	565

Simple average of price relative by arithmetic mean

$$P_{01} = \frac{\Sigma\left(\frac{p_1}{p_0}\times 100\right)}{n}$$

P_1 = Prices of current year
P_0 = Prices of base year
n = Number of items or commodities
when geometric mean is used for average of price relative, the formula for obtaining the index is
Simple average of price relative by geometric Mean

$$P_{01} = \text{Antilog}\left(\frac{\Sigma \log(\frac{P_1}{P_0}\times 100)}{n}\right)$$

Example 2:
From the following data, construct an index for 1998 taking 1997 as base by the average of price relative using (a) arithmetic mean and (b) Geometric mean

Commodity	Price in 1997	Price in 1998
A	50	70
B	40	60
C	80	100
D	20	30

Solution:
(a) Price relative index number using arithmetic mean

Commodity	Price in 1997 (P_0)	Price in 1998 (P_1)	$\frac{P_1}{P_0}\times 100$
A	50	70	140
B	40	60	150
C	80	100	125
D	20	30	150
		Total	565

$$\text{Simple average of price relative index} = (P_{01}) = \frac{\Sigma\left(\frac{p_1}{p_0}\times 100\right)}{4}$$

$$= \frac{565}{4} = 141.25$$

(b) Price relative index number using Geometric Mean

Commodity	Price in 1997 (P_0)	Price in 1998 (P_1)	$\frac{p_1}{p_0}\times 100$	$\log(\frac{p_1}{p_0}\times 100)$
A	50	70	140	2.1461
B	40	60	150	2.1761
C	80	100	125	2.0969
D	20	30	150	2.1761
			Total	8.5952

Simple average of price Relative index

$$(P_{01}) = \text{Antilog}\ \frac{\Sigma\log\left[\frac{p_1}{p_o}x100\right]}{n}$$

$$= \text{Antilog}\ \frac{8.5952}{4}$$

$$= \text{Antilog}\ [2.1488] = 140.9$$

10.5.3 Weighted aggregate index numbers

In order to attribute appropriate importance to each of the items used in an aggregate index number some reasonable weights must be used. There are various methods of assigning weights and consequently a large number of formulae for constructing index numbers have been devised of which some of the most important ones are

1. **Laspeyre's method**
2. **Paasche's method**
3. **Fisher's ideal Method**
4. **Bowley's Method**
5. **Marshall- Edgeworth method**
6. **Kelly's Method**

1. Laspeyre's method:

The Laspeyres price index is a weighted aggregate price index, where the weights are determined by quantities in the based period and is given by

$$\text{Laspeyre's price index} = P_{01}^{L} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

2. Paasche's method

The Paasche's price index is a weighted aggregate price index in which the weight are determined by the quantities in the current year. The formulae for constructing the index is

$$\text{Paasche's price index number} = P_{01}^{P} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

Where

P_0 = Price for the base year P_1 = Price for the current year

q_0 = Quantity for the base year q_1 = Quantity for the current year

3. Fisher's ideal Method

Fisher's Price index number is the geometric mean of the Laspeyres and Paasche indices Symbolically

$$\text{Fisher's ideal index number} = P_{01}^{F} = \sqrt{L \times P}$$

$$= \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

It is known as ideal index number because

(a) It is based on the geometric mean
(b) It is based on the current year as well as the base year
(c) It conform certain tests of consistency
(d) It is free from bias.

4. Bowley's Method:

Bowley's price index number is the arithmetic mean of Laspeyre's and Paasche's method. Symbolically

$$\text{Bowley's price index number} = P_{01}^{B} = \frac{L + P}{2}$$

$$= \frac{1}{2}\left[\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}\right] \times 100$$

5. Marshall- Edgeworth method

This method also both the current year as well as base year prices and quantities are considered. The formula for constructing the index is

$$\text{Marshall Edgeworth price index} = P_{01}^{ME} = \frac{\Sigma(q_0 + q_1)p_1}{\Sigma(q_0 + q_1)p_0} \times 100$$

$$= \frac{\Sigma p_1 q_0 + \Sigma p_1 q_1}{\Sigma p_0 q_0 + \Sigma p_0 q_1} \times 100$$

6. Kelly' s Method

Kelly has suggested the following formula for constructing the index number

$$\text{Kelly's Price index number} = P_{01}^{k} = \frac{\Sigma p_1 q}{\Sigma p_0 q} \times 100$$

$$\text{Where } = q = \frac{q_0 + q_1}{2}$$

Here the average of the quantities of two years is used as weights

Example 3:

Construct price index number from the following data by applying

1. Laspeyere' s Method
2. Paasche' s Method
3. Fisher' s ideal Method

Commodity	2000		2001	
	Price	Qty	Price	Qty
A	2	8	4	5
B	5	12	6	10
C	4	15	5	12
D	2	18	4	20

Solution:

Commodity	p_0	q_0	p_1	q_1	p_0q_0	p_0q_1	p_1q_0	p_1q_1
A	2	8	4	5	16	10	32	20
B	5	12	6	10	60	50	72	60
C	4	15	5	12	60	48	75	60
D	2	18	4	20	36	40	72	80
					172	148	251	220

$$\text{Laspeyre's price index} = P_{01}^{L} = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

$$= \frac{251}{172} \times 100 = 145.93$$

$$\text{Paasche price index number} = P_{01}^{P} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100$$

$$= \frac{220}{148} \times 100$$

$$= 148.7$$

$$\text{Fisher's ideal index number} = \sqrt{L \times P}$$

$$= \sqrt{(145.9) \times (148.7)}$$

$$= \sqrt{21695.33}$$

$$= 147.3$$

Or

$$\text{Fisher's ideal index number} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$$

$$= \sqrt{\frac{251}{172} \times \frac{220}{148}} \times 100$$

$$= \sqrt{(1.459) \times (1.487)} \times 100$$

$$= \sqrt{2.170} \times 100$$

$$= 1.473 \times 100 = 147.3$$

Interpretation:

The results can be interpreted as follows:

If 100 rupees were used in the base year to buy the given commodities, we have to use Rs 145.90 in the current year to buy the same amount of the commodities as per the Laspeyre's formula. Other values give similar meaning .

IV. Weighted Average of Price Relative index.

When the specific weights are given for each commodity, the weighted index number is calculated by the formula.

Weighted Average of Price Relative index = $\frac{\Sigma pw}{\Sigma w}$

Where w = the weight of the commodity

P = the price relative index

Example 6:

Compute the weighted index number for the following data.

Commodity	Price		Weight
	Current year	Base year	
A	5	4	60
B	3	2	50
C	2	1	30

10.7 Tests of Consistency of index numbers:

Several formulae have been studied for the construction of index number. The question arises as to which formula is appropriate to a given problems. A number of tests been developed and the important among these are

1. Unit test
2. Time Reversal test
3. Factor Reversal test

1. Unit test:

The unit test requires that the formula for constructing an index should be independent of the units in which prices and quantities are quoted. Except for the simple aggregate index (unweighted) , all other formulae discussed in this chapter satisfy this test.

2. Time Reversal test:

Time Reversal test is a test to determine whether a given method will work both ways in time, forward and backward. In the words of Fisher, "the formula for calculating the index number should be such that it gives the same ratio between one point of comparison and the other, no matter which of the two is taken as base". Symbolically, the following relation should be satisfied.

$$P_{01} \times P_{10} = 1$$

Where P_{01} is the index for time ' 1' as time ' 0' as base and P_{10} is the index for time ' 0' as time ' 1' as base. If the product is not unity, there is said to be a time bias is the method. Fisher' s ideal index satisfies the time reversal test.

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

$$P_{10} = \sqrt{\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$\text{Then } P_{01} \times P_{10} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$= \sqrt{1} = 1$$

Therefore Fisher ideal index satisfies the time reversal test.

3. Factor Reversal test:

Another test suggested by Fisher is known s factor reversal test. It holds that the product of a price index and the quantity index should be equal to the corresponding value index. In the words of Fisher, "Just as each formula should permit the interchange of the two times without giving inconsistent results, so it ought to permit interchanging the prices and quantities without giving inconsistent result, ie, the two results multiplied together should give the true value ratio.

In other word, if P_{01} represent the changes in price in the current year and Q_{01} represent the changes in quantity in the current year, then

$$P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

ie. $$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

Then $$P_{01} \times Q_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times \frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$= \sqrt{\left(\frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}\right)^2}$$

$$= \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Since $P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$, the factor reversal test is satisfied by the Fisher's ideal index.

Example 8:

Construct Fisher's ideal index for the Following data. Test whether it satisfies time reversal test and factor reversal test.

	Base year		Current year	
Commodity	Quantity	Price	Quantity	Price
A	12	10	15	12
B	15	7	20	5
C	5	5	8	9

Solution:

Commodity	q_0	p_0	q_1	p_1	P_0q_0	p_0q_1	p_1q_0	p_1q_1
A	12	10	15	12	120	150	144	180
B	15	7	20	5	105	140	75	100
C	5	5	8	9	25	40	45	72
					250	330	264	352

Fisher ideal index number $P_{01}^{F} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}} \times 100$

$$= \sqrt{\frac{264}{250} \times \frac{352}{330}} \times 100$$

$$= \sqrt{(1.056) \times (1.067)} \times 100$$

$$= \sqrt{1.127} \times 100$$

$$= 1.062 \times 100 = 106.2$$

Time Reversal test:

Time Reversal test is satisfied when $P_{01} \times P_{10} = 1$

$$P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$$

$$= \sqrt{\frac{264}{250} \times \frac{352}{330}}$$

$$P_{10} = \sqrt{\frac{\Sigma p_0 q_1}{\Sigma p_1 q_1} \times \frac{\Sigma p_0 q_0}{\Sigma p_1 q_0}}$$

$$= \sqrt{\frac{330}{352} \times \frac{250}{264}}$$

Now $P_{01} \times P_{10} = \sqrt{\frac{264}{250} \times \frac{352}{330} \times \frac{330}{352} \times \frac{250}{264}}$

$$= \sqrt{1}$$

$$= 1$$

Hence Fisher ideal index satisfy the time reversal test.

Factor Reversal test:

Factor Reversal test is satisfied when $P_{01} \times Q_{01} = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$

Now $P_{01} = \sqrt{\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}$

$$= \sqrt{\frac{264}{250} \times \frac{352}{330}}$$

$$Q_{01} = \sqrt{\frac{\Sigma q_1 p_0}{\Sigma q_0 p_0} \times \frac{\Sigma q_1 p_1}{\Sigma q_0 p_1}}$$

$$= \sqrt{\frac{330}{250} \times \frac{352}{264}}$$

Then $P_{01} \times Q_{01} = \sqrt{\frac{264}{250} \times \frac{352}{330} \times \frac{330}{250} \times \frac{352}{264}}$

$$= \sqrt{\left(\frac{352}{250}\right)^2}$$

$$= \frac{352}{250}$$

$$= \frac{\Sigma p_1 q_1}{\Sigma p_0 q_0}$$

Hence Fisher ideal index number satisfy the factor reversal test.

10.8 Consumer Price Index

Consumer Price index is also called the cost of living index. It represent the average change over time in the prices paid by the ultimate consumer of a specified basket of goods and services. A change in the price level affects the costs of living of different classes of people differently. The general index number fails to reveal this. So there is the need to construct consumer price index. People consume different types of commodities. People's consumption habit is also different from man to man, place to place and class to class i.e richer class, middle class and poor class.

The scope of consumer price is necessary, to specify the population group covered. For example, working class, poor class, middle class, richer class, etc and the geographical areas must be covered as urban, rural, town, city etc.

Use of Consumer Price index

The consumer price indices are of great significance and is given below.

1. This is very useful in wage negotiations, wage contracts and dearness allowance adjustment in many countries.
2. At government level, the index numbers are used for wage policy, price policy, rent control, taxation and general economic policies.
3. Change in the purchasing power of money and real income can be measured.
4. Index numbers are also used for analysing market price for particular kinds of goods and services.

Method of Constructing Consumer price index:

There are two methods of constructing consumer price index. They are

1. Aggregate Expenditure method (or) Aggregate method.
2. Family Budget method (or) Method of Weighted Relative method.

1. Aggregate Expenditure method:

This method is based upon the Laspeyre's method. It is widely used. The quantities of commodities consumed by a particular group in the base year are the weight.

The formula is Consumer Price Index number = $\frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$

2. Family Budget method or Method of Weighted Relatives:

This method is estimated an aggregate expenditure of an average family on various items and it is weighted. The formula is

Consumer Price index number = $\frac{\Sigma pw}{\Sigma w}$

Where P = $\frac{p_1}{p_0} \times 100$ for each item. w = value weight (i.e) p_0q_0

"Weighted average price relative method" which we have studied before and "Family Budget method" are the same for finding out consumer price index.

Example 9:

Construct the consumer price index number for 1996 on the basis of 1993 from the following data using Aggregate expenditure method.

		Price in	
Commodity	Quantity consumed	1993	1996
A	100	8	12
B	25	6	7
C	10	5	8
D	20	15	18

Solution:

Commodity	q_0	p_0	p_1	p_0q_0	p_1q_0
A	100	8	12	800	1200
B	25	6	7	150	175
C	10	5	8	50	80
D	20	15	18	300	360
			Total	1300	1815

Consumer price index by Aggregate expenditure method

$$= \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100$$

$$= \frac{1815}{1300} \times 100 = 139.6$$

Example 10:

Calculate consumer price index by using Family Budget method for year 1993 with 1990 as base year from the following data.

		Price in	
Items	Weights	1990 (Rs.)	1993 (Rs.)
Food	35	150	140
Rent	20	75	90
Clothing	10	25	30
Fuel and lighting	15	50	60
Miscellaneous	20	60	80

Solution:

Items	W	P_0	P_1	$P = \frac{p_1}{p_0} \times 100$	PW
Food	35	150	140	93.33	3266.55
Rent	20	75	90	120.00	2400.00
Clothing	10	25	30	120.00	1200.00
Fuel and lighting	15	50	60	120.00	1800.00
Miscellaneous	20	60	80	133.33	2666.60
	100				11333.15

Consumer price index by Family Budget method = $\frac{\Sigma pw}{\Sigma w}$

$$= \frac{11333.15}{100}$$

$$= 113.33$$

Introduction:

Arrangement of statistical data in chronological order ie., in accordance with occurrence of time, is known as "Time Series". Such series have a unique important place in the field of Economic and Business statistics. An economist is interested in estimating the likely population in the coming year so that proper planning can be carried out with regard to food supply, job for the people etc. Similarly, a business man is interested in finding out his likely sales in the near future, so that the businessman could adjust his production accordingly and avoid the possibility of inadequate production to meet the demand. In this connection one usually deal with statistical data, which are collected, observed or recorded at successive intervals of time. Such data are generally referred to as ' time series' .

Definition:

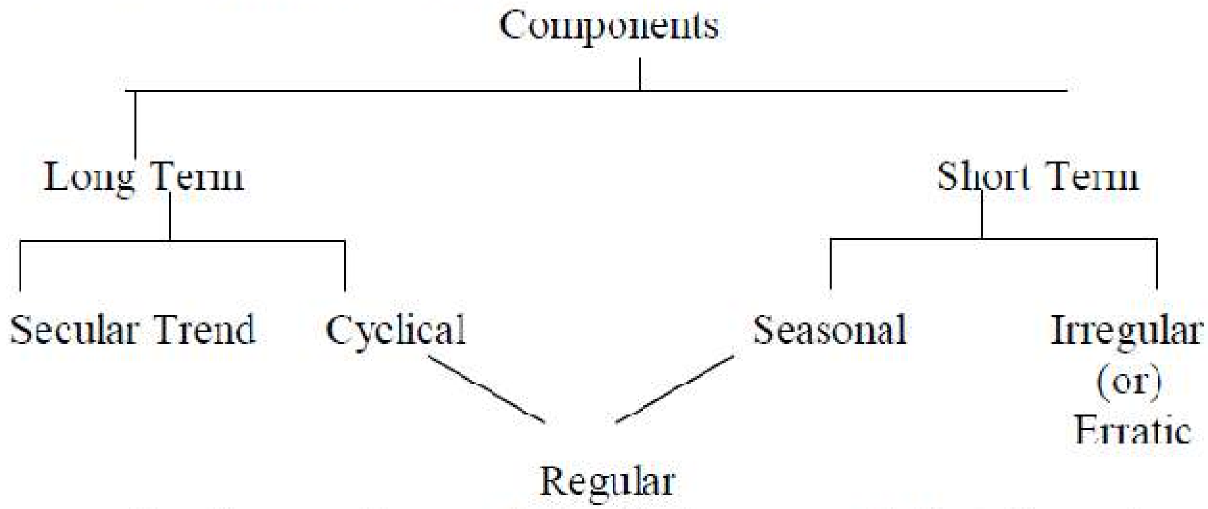
According to Mooris Hamburg "A time series is a set of statistical observations arranged in chronological order"

Ya-Lun- chou defining the time series as "A time series may be defined as a collection of readings belonging to different time periods, of some economic variable or composite of variables. A time series is a set of observations of a variable usually at equal intervals of time. Here time may be yearly, monthly, weekly, daily or even hourly usually at equal intervals of time.

Hourly temperature reading, daily sales, monthly production are examples of time series. Number of factors affect the observations of time series continuously, some with equal intervals of time and others are erratic studying, interpreting analyzing the factors is called Analysis of Time Series.

8.2 Components of Time series:

The components of a time series are the various elements which can be segregated from the observed data. The following are the broad classification of these components.



In time series analysis, it is assumed that there is a multiplicative relationship between these four components. Symbolically,

$$Y = T \times S \times C \times I$$

Where Y denotes the result of the four elements; T = Trend ; S = Seasonal component; C = Cyclical components; I = Irregular component

In the multiplicative model it is assumed that the four components are due to different causes but they are not necessarily independent and they can affect one another.

Another approach is to treat each observation of a time series as the sum of these four components. Symbolically

$$Y = T + S + C + I$$

The additive model assumes that all the components of the time series are independent of one another.

1) Secular Trend or Long - Term movement or simply Trend
2) Seasonal Variation
3) Cyclical Variations
4) Irregular or erratic or random movements(fluctuations)

8.2.2 Methods of Measuring Trend:

Trend is measured by the following mathematical methods.

1. Graphical method
2. Method of Semi-averages
3. Method of moving averages
4. Method of Least Squares

Method of Moving Averages:

This method is very simple. It is based on Arithmetic mean. Theses means are calculated from overlapping groups of successive time series data. Each moving average is based on values covering a fixed time interval, called "period of moving average" and is shown against the center of the interval.

The method of ' odd period of moving average is as follows. (3 or 5) . The moving averages for three years is $\frac{a+b+c}{3}$, $\frac{b+c+d}{3}$, $\frac{c+d+e}{3}$ etc

The formula for five yearly moving average is $\frac{a+b+c+d+e}{5}$, $\frac{b+c+d+e+f}{5}$, $\frac{c+d+e+f+g}{5}$ etc.

Steps for calculating odd number of years.

1. Find the value of three years total, place the value against the second year.
2. Leave the first value and add the next three years value (ie 2^{nd}, 3^{rd} and 4^{th} years value) and put it against 3^{rd} year.
3. Continue this process until the last year' s value taken.
4. Each total is divided by three and placed in the next column.

These are the trend values by the method of moving averages

Example 4 :

Calculate the three yearly average of the following data.

Year	1975	1976	1977	1978	1979	1980
Production in (tones)	50	36	43	45	39	38

Year	1981	1982	1983	1984
Production in (tones)	33	42	41	34

Solution:

Year	Production (in tones)	3 years moving total	3 years moving average as Trend values
1975	50	-	-
1976	36	129	43.0
1977	43	124	41.3
1978	45	127	42.3
1979	39	122	40.7
1980	38	110	36.7
1981	33	113	37.7
1982	42	116	38.7
1983	41	117	39.0
1984	34	-	-

Even Period of Moving Averages:

When the moving period is even, the middle period of each set of values lies between the two time points. So we must center the moving averages.

The steps are

1. Find the total for first 4 years and place it against the middle of the 2nd and 3rd year in the third column.
2. Leave the first year value, and find the total of next four-year and place it between the 3rd and 4th year.
3. Continue this process until the last value is taken.
4. Next, compute the total of the first two four year totals and place it against the 3rd year in the fourth column.
5. Leave the first four years total and find the total of the next two four years' totals and place it against the fourth year.
6. This process is continued till the last two four years' total is taken into account.
7. Divide this total by 8 (Since it is the total of 8 years) and put it in the fifth column.

These are the trend values.

Example 5 :

The production of Tea in India is given as follows. Calculate the Four-yearly moving averages

Year	1993	1994	1995	1996	1997	1998
Production (tones)	464	515	518	467	502	540

Year	1999	2000	2001	2002
Production (tones)	557	571	586	612

Solution:

Year	Production (in tones)	4 years Moving total	Total of Two four years	Trend Values
1993	464		-	-
		-		
1994	515			
		1964		
1995	518		3966	495.8
		2002		
1996	467		4029	503.6
		2027		
1997	502		4093	511.6
		2066		
1998	540		4236	529.5
		2170		
1999	557		4424	553.0
		2254		
2000	571		4580	572.5
		2326		
2001	586			
		-		
2002	612			

8.3 Method of Least Square:

This method is widely used. It plays an important role in finding the trend values of economic and business time series. It helps for forecasting and predicting the future values. The trend line by this method is called the line of best fit.

The equation of the trend line is $y = a + bx$, where the constants a and b are to be estimated so as to minimize the sum of the squares of the difference between the given values of y and the estimate values of y *by* using the equation. The constants can be obtained by solving two normal equations.

$$\Sigma y - na + b\Sigma x \quad \text{.........} \; (1)$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{.......} \; (2)$$

Here x represent time point and y are observed values. 'n' is the number of pair- values.

When odd number of years are given

Step 1: Writing given years in column 1 and the corresponding sales or production etc in column 2.

Step 2: Write in column 3 start with 0, 1, 2 .. against column 1 and denote it as X

Step 3: Take the middle value of X as A

Step 4: Find the deviations $u = X - A$ and write in column 4

Step 5: Find u^2 values and write in column 5.

Step 6: Column 6 gives the product uy

Now the normal equations become

$$\Sigma y = na + b\Sigma u \qquad (1) \qquad \text{where } u = X-A$$

$$\Sigma uy = a\Sigma u + b\Sigma u^2 \qquad (2)$$

Since $\Sigma u = 0$, From equation (1)

$$a = \frac{\Sigma y}{n}$$

From equation (2)

$$\Sigma uy = b\Sigma u^2$$

$$\therefore b = \frac{\Sigma uy}{\Sigma u^2}$$

$\therefore$ The fitted straight line is

$$y = a + bu = a + b(X - A)$$

Example 6:

For the following data, find the trend values by using the method of Least squares

Year	1990	1991	1992	1993	1994
Production (in tones)	50	55	45	52	54

Estimate the production for the year 1996

Solution:

Year (x)	Production (y)	X= x -1990	u = X-A = X-2	u^2	uy	Trend values
1990	50	0	-2	4	-100	50.2
1991	55	1	-1	1	-55	50.7
1992	45	**2 A**	0	0	0	51.2
1993	52	3	1	1	52	51.7
1994	54	4	2	4	108	52.2
Total	256			10	5	

Where A is an assumed value

The equation of straight line is

$Y = a + bX$

$= a + bu$, where $u = X - 2$

the normal equations are

$$\Sigma y = na + b\Sigma u \text{ ....(1)}$$

$$\Sigma uy - a\Sigma u + b\Sigma u^2 \text{ ....(2)}$$

since $\Sigma u = 0$ from(1) $\Sigma y = na$

$$a = \frac{\Sigma y}{n} = \frac{256}{5} = 51.2$$

From equation (2)

$$\Sigma uy = b\Sigma u^2$$

$$5 = 10b$$

$$b - \frac{5}{10} = 0.5$$

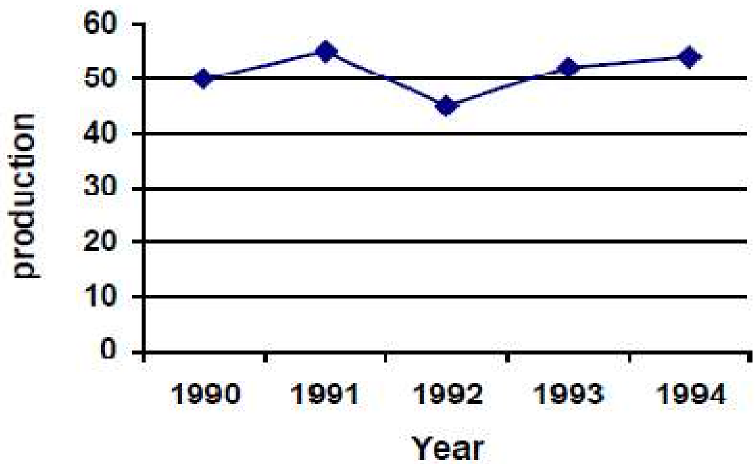
The fitted straight line is

$$y = a + bu$$
$$y = 51.2 + 0.5\,(X-2)$$
$$y = 51.2 + 0.5X - 1.0$$
$$y = 50.2 + 0.5X$$

Trend values are, 50.2, 50.7, 51.2, 51.7, 52.2

The estimate production in 1996 is put $X = x - 1990$

$$X = 1996 - 1990 = 6$$
$$Y = 50.2 + 0.5X = 50.2 + 0.5(6)$$
$$= 50.2 + 3.0 = 53.2 \text{ tonnes.}$$



When **even number of years** are given

Here we take the mean of middle two values of X as A

Then $u = \frac{X - A}{1/2} = 2\,(X-A)$. The other steps are as given in the odd number of years.

Example 7:

Fit a straight line trend by the method of least squares for the following data.

Year	1983	1984	1985	1986	1987	1988
Sales (Rs. in lakhs)	3	8	7	9	11	14

Also estimate the sales for the year 1991

Solution:

Year (x)	Sales (y)	X = x-1983	u =2X-5	u2	*uy*	Trend values
1983	3	0	-5	25	-15	3.97
1984	8	1	-3	9	-24	5.85
1985	7	2	-1	1	-7	7.73
1986	9	3	1	1	9	9.61
1987	11	4	3	9	33	11.49
1988	14	5	5	25	70	13.37
Total	52		0	70	66	

$$u = \frac{X - A}{1/2}$$

$$= 2(X - 2.5) = 2X - 5$$

The straight line equation is

$$y = a + bX = a + bu$$

The normal equations are

$$\Sigma y = na \;.....(1)$$

$$\Sigma uy = b\Sigma u^2 \;....(2)$$

From (1) $52 = 6a$

$$a = \frac{52}{6}$$
$$= 8.67$$

From (2) $66 = 70\,b$

$$b = \frac{66}{70}$$
$$= 0.94$$

The fitted straight line equation is

$y = a + bu$

$y = 8.67+0.94(2X-5)$

$y = 8.67 + 1.88X - 4.7$

$y = 3.97 + 1.88X$ -----------(3)

The trend values are

Put $X = 0,\ y = 3.97$ $\quad X = 1,\ y = 5.85$

$X = 2,\ y = 7.73$ $\quad X = 3,\ y = 9.61$

$X = 4,\ y = 11.49$ $\quad X = 5,\ y = 13.37$

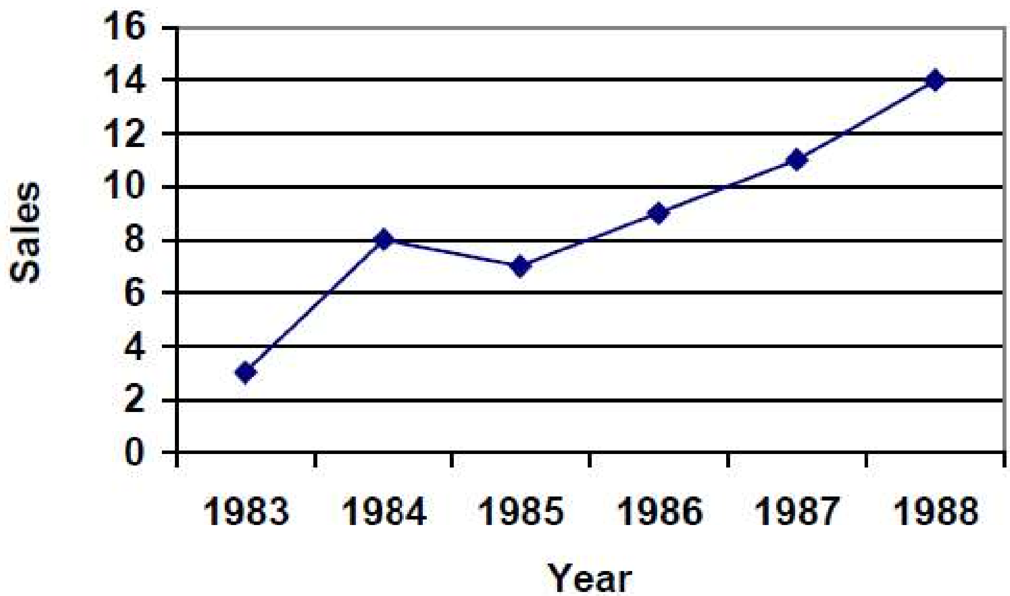
The estimated sale for the year 1991 is; put $X = x - 1983$

$$= 1991 - 1983 = 8$$

$$y = 3.97 + 1.88 \times 8$$

$$= 19.01 \text{ lakhs}$$

The following graph will show clearly the trend line.



Parabolic Trend Model

The curvilinear relationship for estimating the value of a dependent variable *y*from an independent variable *x* might take the form

$\hat{y} = a + bx + cx^2$.This trend line is called the *parabola*.

For a non-linear equation $\hat{y} = a + bx + cx^2$, the values of constants *a, b*, and *c* can be determined by solving three normal equations.

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

When the data can be coded so that $\Sigma x = 0$ and $\Sigma x^3 = 0$, two term in the above expressions drop out and we have

$\Sigma y = na + c\Sigma x^2$

Σxy = *b*Σx2
Σx2y = aΣx2 + *c*Σ*x4*

To find the exact estimated value of the variable *y*, the values of constants *a, b,* and *c* need to be calculated. The values of these constants can becalculated by using the following shortest method:

$$a = \frac{\sum y - c\sum x^2}{n}; b = \frac{\sum xy}{\sum x^2} and c = \frac{n\sum x^2 y - \sum x^2 \sum y}{n\sum x^4 - (\sum x^2)^2}$$

Example : The prices of a commodity during 1999-2004 are given below. Fit a parabola to these data. Estimate
the price of the commodity for the year 2005.

Year	Price	Year	Price
1999	100	2002	140
2000	107	2003	181
2001	128	2004	192

Solution: To fit a parabola *y*ˆ = a + *bx*+ *cx*2, the calculations to determine the values of constants *a*, *b,* and *c* are shown in table

Calculations for Parabola Trend Line

Year	Time Scale (x)	Price (y)	x^2	x^3	x^4	xy	x^2y	Trend Values (*y*ˆ)
1999	-2	100	4	-8	16	-200	400	97.72
2000	-1	107	1	-1	1	-107	107	110.34
2001	0	128	0	0	0	0	0	126.68

2002	1	140	1	1	1	140	140	146.50
2003	2	181	4	8	16	362	724	169.88
2004	3	192	9	27	81	576	1728	196.82
	3	**848**	**19**	**27**	**115**	**771**	**3099**	**847.94**

(i) $\Sigma y= na- b\Sigma x+ c\Sigma x2$

$848 = 6a + 3b + 19c$

(ii) $\Sigma xy= a\Sigma x+ b\Sigma x2 + c\Sigma x3$

$771 = 3a + 19b + 27c$

(iii) $\Sigma x2y = a\Sigma x2 + b\Sigma x2 + c\Sigma x4$

$3099 = 19a + 27b + 115c$

Eliminating a from eqns. (i) and (ii), we get

(iv) $694 = 35b + 35c$

Eliminating a from eqns. (ii) and (iii), we get

(v) $5352 = 280b + 168c$

Solving eqns. (iv) and (v) for b and c we get b =18.04 and c = 1.78.

Substituting values of b and c in eqn. (i), we get a = 126.68.

Hence, the required non-linear trend line becomes

$y = 126.68 +18.04x + 1.78x^2$

Conversion of Trend Equation

Trend equation depends on:

1. The origin of time reference
2. The units of time viz., weekly, monthly, yearly etc.
3. The time series value relate to annual figures or monthly averages.

Trend values can be computed by the straight line method. For simplicity and convenience, the following methods are also used. They are

1. Shifting of origin
2. Conversion of annual trend equation to monthly trend equation when the y values are in annual totals and are given as monthly averages

Conversion of annual trend values to monthly trend equation

We calculate monthly tend equation from annual trend equation. When Y unit are annual total and X units are in year then the annual trend equation can be converted into monthly trend equation i.e. ; we divide 'a' by 12 and 'b' by 144. $Y = \frac{a}{12} + \frac{b}{144}X$. If the time unit of X in the trend equation represents only 6 months, then 'b' is divided by 72 i.e.,

$$Y = \frac{a}{12} + \frac{b}{72}X$$

The annual trend equation trend equation is Y = a + b X

The monthly trend equation is $Y = \frac{a}{12} + \frac{b}{144}X$

Example:

The trend of annual production of a company described by the following equation

$Y_c = 18 + 0.6X$

Origin 1988; X-unit = 1 Year; Y-unit = annual production. Convert the equation to a monthly trend equation.

Solution:

Monthly trend equation:

$$Y_c = \frac{18}{12} + \frac{0.6}{144}X$$

$$= 1.50 + 0.0041\ X$$

Exponential Trend Model

When the given values of dependent variable *y* from approximately a geometric progression while the corresponding independent variable *x* values form an arithmetic progression, the relationship between variables *x* and *y* is given by an exponential function, and the best fitting curve is said to describe the *exponential trend.* Data from the fields of biology, banking, and

economics frequently exhibit such a trend. For example, growth of bacteria, money accumulating at compound interest, sales or earnings over a short period, and so on, follow exponential growth.

The characteristics property of this law is that the rate of growth, that is, the rate of change of *y* with respect to *x* is proportional to the values of the function. The following function has this property.

y = abcx, a > 0

The letter *b* is a fixed constant, usually either 10 or *e,* where *a* is a constant to be determined from the data.

To assume that the law of growth will continue is usually unwarranted, so only short range predictions can be made with any considerable degree or reliability.

If we take logarithms (with base 10) of both sides of the above equation, we obtain

Log $y = \log a + (c \log b)\, x$ (7.2)

For b =10, log b =1, but for b=e, log b =0.4343 (approx.). In either case, this equation is of the form $y' = c + dx$

Where $y' = \log y$, $c = \log a$, and $d = c \log b$.

Equation (7.2) represents a straight line. A method of fitting an exponential trend line to a set of observed values of y is to fit a straight trend line to the logarithms of the y-values.

In order to find out the values of constants a and b in the exponential function, the two normal equations to be solved are

$\Sigma \log y = n \log a + \log b \Sigma x$

$\Sigma x \log y = \log a \Sigma x + \log b \Sigma x2$

When the data is coded so that $\Sigma x = 0$, the two normal equations become

$\Sigma \log y = n \log a$ or $\log a = \frac{1}{n} \Sigma \log y$

and $\Sigma x \log y = \log b\ \Sigma x^2$ or $\log b = \frac{\Sigma x \log y}{\Sigma x2}$

Coding is easily done with time-series data by simply designating the center of the time period as x =0, and have equal number of plus and minus period on each side which sum to zero.

Example :

The sales (Rs. In million) of a company for the years 1995 to 1999 are:

Year :	1995	1996	1997	1998	1999
Sales :	1.6	4.5	13.8	40.2	125.0

Find the exponential trend for the given data and estimate the sales for 2002.

Solution:

computational time can be reduced by coding the data. For this consider $u = x$-3. The necessary computations are shown in table

Fitting the Exponential Trend Line

Year	Time Period x	$u=x-3$	u^2	Sales y	Log y	u log y
1995	1	-2	4	1.60	0.2041	-0.4082
1996	2	-1	1	4.50	0.6532	-0.6532
1997	3	0	0	13.80	1.1390	0
1998	4	1	1	40.20	1.6042	1.6042
1999	5	2	4	125.00	2.0969	4.1938
			10		**5.6983**	**4.7366**

$\log a = \frac{1}{n}\Sigma \log y = \frac{1}{5}(5.6983) = 1.1397$

$\log b = \frac{\sum u \log y}{\sum u^2} = \frac{4.73\ 66}{10} = 0.4737x$

Therefore log y= log a + (x+3) log b = 1.1397 + 0.4737x

For sales during 2002, x =3, and we obtain

log$y$= 1.1397 + 0.4737 (3) = 2.5608

y = antilog (2.5608) = 363.80

Seasonal Variations:

Seasonal Variations are fluctuations within a year during the season. The factors that cause seasonal variation are

i) Climate and weather condition.

ii) Customs and traditional habits.

For example the sale of ice-creams increase in summer, the umbrella sales increase in rainy season, sales of woolen clothes increase in winter season and agricultural production depends upon the monsoon etc.,

Method of simple averages

(i)applying the method of least squares, we can obtain the trend values.

(ii)We must divide the original data of time series for each season (month quarter) by the corresponding trend values and multiply these ratio by 10, i.e. $\frac{\text{T x S x C x I}}{\text{T}}\text{x}100 = \text{S x C x I x } 100$. Thus the trend eliminated values are obtained. This percentage will include seasonal, cyclical and irregular fluctuation.

(iii)In order to eliminate the irregular and cyclical movement, the seasonal figures are averaged with any one of the measures of central tendency, mean or median. Thu we obtain the indices for seasonal variation for different season.

(iv)Thee indices are adjusted to a total of ,200 for monthly data or 400 for quarterly data by multiplying each index by a suitable factor $\left(\frac{1200}{\text{thesumofthe 12 monthlyvalues}} \text{or} \frac{400}{\text{thesumofthe 4 quarterlyvalues}}\right)$ Thus we get Seasonal Index.

Ratio to Moving Average Method

The steps for calculation:

(i)Calculate12 month moving average , which eliminates seasonal and irregular fluctuations and represents trend and cycle. i.e., T.C.

(ii)Express the original values a percentage of centres moving average values for all month, i.e.,

$$\frac{\text{Original Value}}{\text{Moving Average vlue}}\text{x } 100 = \frac{\text{TSCI}}{\text{TC}}\text{x}100$$
$$= \text{SI x } 100$$

(iii)As in the above said two methods, arrange these percentages according to year and months. By averaging these percentages for each month, we can eliminate irregular factors. Mean or Median can be used for averaging.

(iv)The sum of thee indices should be 1,200 (400) for monthly or quarterly data. If it is not so, an adjustment is made to eliminate the discrepancy; i.e.,

2 MARK QUESTIONS

1.Write any two characteristics of index numbers.
2..Find the trend, where $Y_T = 140 + 20(X - 1981)$,if x = 1979,1980.
3.What are the types of secular trend?
4.Write the formula for laspeyre's and pasche's method.
5.What is moving average?

8 MARK QUESTIONS

1.Compute Price Index based on the simple average of price relatives by using Arithmetic Mean.

Commodity	A	B	C	D	E	F	G	H
Price 1997 (in Rs.):	40	120	140	130	60	70	65	75
Price 1998 (in Rs.)	60	140	170	135	100	80	75	80

2.Draw a trend line by the method of semi averages.
Year :1987 1988 1989 1990 1991 1992 1993
Production : 90 110 130 150 100 150 200

3. Construct the Cost of Living Index number for the following data:

Item	Base Year Price	Current Year Price	Weight
Food	39	47	4
Fuel	8	12	1
Clothing	14	18	3
House rent	12	15	2
Miscellaneous	5	30	1

4. Fit a linear trend equation by the method of least squares and estimate the net profit in 2003.
Year : 199519961997 1998 1999 2000 2001
Net profit : 32 36 44 37 71 72 109
(Rs.)

5.Calculate the Price Index by Weighted G.M method for the following data.

Item	A	B	C	D	E
Weight	40	25	15	10	10
Price 2001 (in Rs.):	25	50	40	10	20
Price 2011 (in Rs.)	40	80	60	20	40

6. Using the four year moving averages determine the trend and short term fluctuation.
Year: 1981 1982 1983 1984 1985 1986 1987 1988 1989 1990
Production : 464 515 518 467 502 540 557 571 586 612

7. Fit a linear trend equation by the method of least squares and estimate the net profit in 2003.
Year : 1995 1996 1997 1998 1999 2000 2001
Net profit : 32 36 44 37 71 72 109
(Rs.)

8.FindLaspeyre's, Paasche's and Fisher's Price Index for the following data:

Commodity	**2003**		**2004**	
	Price (Rs)	**Quantity ('00)**	**Price (Rs)**	**Quantity ('00)**
Erasers	05	25	06	30
Pencils	10	05	15	04
Markers	03	40	02	50
Ball Pens	06	30	08	35

9. Calculate the price index numbers by (i) Simple G.M method (ii) Weighted G.M method for the following data.

Item	A	B	C	D	E
Weight	40	25	15	10	10
Price in 1991(Rs.)	25	50	40	10	20
Price in 2001(Rs.)	40	80	60	20	40

10. Fit a straight line trend equation to the following data by the method of Least squares .

Year:	1979	1980	1981	1982	1983
Sales (Rs.):	100	120	140	160	180

Questions	opt1	opt2	opt3	opt4	Answer
A rectangular arrangement of numbers in rows	set	matrix	order	sub matrix	matrix
A scalar matrix in which all the leading diagonal	square matrix	unit matrix	null matrix	Identity matrix	unit matrix
A diagonal matrix in which all the diagonal elements	diagonal matrix	scalar matrix	unit matrix	null matrix	scalar matrix
[3 8 9 -2] is a row matrix of order------------------	4 x 1	1x 4	1x 1	4 x 4	1x 4
A square matrix such that A' = A is called------------------	symmetric	skew symmetric	hermitiIan	scalar	symmetric
An association of a real numbers with a matrix is known	real matrix	complex matrix	null matrix	unit matrix	real matrix
The number of elements in an m x n matrix is -------	mn	n	m^2	n^2	mn
If in a square matrix A, $a_{ij} = 0$ for i>j, then it is called	lower triangular	upper triangular	diagonal	scalar matrix	upper triangular
$(A^T)^T$ = -----------.	A^{-1}	A^T	A	1 / A	A
A matrix A is said to be Orthogonal matrix if --------------	$AA^T = I$	$AA^T = 0$	$A = A^T$	A = 1/ A	$AA^T = I$

The addition of two matrices are possible only when	of same order	of any order	scalar matrices	unit matrices	of same order
A square matrix is said to be singular if its determinant is	0	1	2	-1	0
If A is non singular ,its inverse is ----------------	Adj A / |A|	|A| / Adj A	1/Adj A	1/|A|	Adj A / |A|
The condition for existing of A^{-1} is --------------	A is non singular	A is singular	A^T is singular	A square matrix	A is non singular
in a square matrix A, $a_{ij} = 0$ for i <j, then it is called a ---	lower triangular	upper triangular	diagonal	triangular	lower triangular
A Square matrix such that A' = – A is called --------------	symmetric	skew symmetric	hermit ion	scalar	skew symmetric
The sum of the diagonal elements in a matrix A is the	Trace of A	unit of A	Transpose of A	inverse of A	Trace of A
If a square matrix A of order n is of the form $A^n = 0$.	Identity	Idempotent	Nilpotent	Orthogonal	Nilpotent
If a square matrix A of order n is of the form $A^n = A$.	Identity	Idempotent	Nilpotent	Orthogonal	Idempotent
$(AB)^{-1}$ = ---------------.	A^{-1}	B^{-1}	$A^{-1}B^{-1}$	$B^{-1}A^{-1}$	$B^{-1}A^{-1}$
I*A = ---------------.	A	0	Identity matrix	Zero matrix	A

An identity matrix is also found as a ------------- matrix	Scalar	Diagonal	triangular	scalar and diagonal	scalar and diagonal
In a 3x3 square matrix the minor and the cofactor of	Same sign and same value	same sign and different values	Opposite sign and same value	opposite sign and different values	Opposite sign and same value
Every matrix A and its transpose A^T have --------------	Different	same	one as the negative of other	one as the multiple of other	same
The determinant value of the unit matrix of order 2	1	0	-1	2	1
If A is singular ,its inverse is ----------------	null matrix	does not exists	1/Adj A	1/|A|	does not exists
A rectangular matrix will not possesses-------------	inverse	cofactor	determinant	transpose	inverse
For any square matrix (adj A).A = A. (adj A) is ---------	|A| .I	|A |	1 / |A|	A^T	|A| .I
For any two square matrices A and B (adj AB) = ---	(adj A). (adj B)	(adj B). (adj A)	(adj BA)	(adj B +adj A)	(adj B). (adj A)
The formula for computing simple Interest is ------------	pnr / 100	p+n+r /100	p-n-r /100	p+n+r	pnr / 100
Simple interest will be an income for ------------------.	lender	borrower	customer	creditor	lender
The simple interest on Rs. 5000 at 10 % for 3 years is -----	Rs.500	Rs.1000	Rs.1500	Rs.2000	Rs.1500

If the simple interest for 2 years at 12 % is Rs. 3000	20,000	3,000	12,500	10,000	12,500
The formula for compound interest is ------------------	PNR /100	$P(1+r/100)^n$ –P	A/(1+i)n	CI /(I+I)n	$P(1+r/100)^n$ –P
The compound interest on Rs. 1000 at 12 % for	Rs.150	Rs.175	Rs.120	Rs.126	Rs.120
The formula for calculating principal under	A $/(1+i)^n$	$P(1+i)^n$	Pni	CI $/(1+i)^n$	A $/(1+i)^n$
If the principal isRs.5000 ,Interest is Rs.100 then	Rs.5100	Rs.4900	Rs.5000	Rs.6000	Rs.5100
If Rs.6,000 amounts to Rs.8,940 then the	2940	Rs.3450	Rs.4560	Rs.3540	2940
The simple interest on Rs.6,000 at 10% for 2 years is--------	Rs.1200	Rs.1000	Rs.2000	Rs.1250	Rs.1200
The formula for simple interest is -------	Pni	P-n-r/100	P+n+r/100	Pn	Pni
The formula for calculating amount under compound	$P(1+r/100)^n$	P(1+r/100)n-1	Pni	P	$P(1+r/100)^n$
Compound interest is more than ---------	Principal	amount	simple interest	compound interest	simple interest
Formula for rate of interest under compound interest	$100[(A/P)^{1/n}-1]$	P(1+A /100)n-1	P(1+I)n	P(1+I)	$100[(A/P)^{1/n}-1]$

In compound interest the formula for period is--------	log A-log P/log(1+r/100)	log A-log P/(1+r/100)	log A+log P/(1+r/100)	A+P	log A-log P/log(1+r/100)
Compound interest for Rs.2,500 for 4 years at 8% per	Rs.801.32	Rs.701.22	Rs.601.42	Rs.901.22	Rs.901.22
The effective rate of compound interest is -------	$100[(1-r/100m)^m - 1]$	$100[(1+r/100m)^m - 1]$	$P(1+i)^n$	P	$100[(1+r/100m)^m - 1]$
In the effective rate of interest, a unit of time m is	4	2	3	1	4
In the effective rate of interest, a unit of time m is half	4	2	3	1	2
Interest is compounded continuously	large	small	middle	first	large
When interest is compounded continuously A is	Pe^{in}	Pe-in	$- Pe^{in}$	P	Pe^{in}
When interest is compounded continuously P is	Pe^{in}	Pe-in	$- Pe^{in}$	P	Pe-in
When interest is compounded continuously,	Pe^{in}	A/ e^{in}	$- Pe^{in}$	$100(e^i - 1)$	$100(e^i - 1)$
In what time a sum of Rs.1234 amount to Rs.5678 at 8%	18.72 years	19.27 years	19.72 years	17.95 years	19.27 years
Under compound interest , interest earns-------	interest	principal	amount	discount	interest

The simple interest on a sum of Rs.2000 for two	Rs.800	Rs.700	Rs.1000	Rs.1200	Rs.1200
If the simple interest is Rs 150 ,number of years is	Rs. 500	Rs. 300	Rs.1000	Rs. 500	Rs. 500
If the simple interest is Rs 72 , the rate if interest	3	4	5	1	3
Compound interest is-------------------	Amount - principal	Amount + principal	principal –Amount	Amount x principal	Amount - principal
The amount of money deposited or borrowed is	principal	interest	amount	discount	principal
The extra money which we get or which we pay is	principal	interest	amount	discount	interest
The simple interest on a sum of Rs.1225 for 4	Rs.539	Rs.700	Rs.100	Rs.120	Rs.539

Questions	opt1	opt2	opt3	opt4	Answer
Quantities which take the same values throughout particular investigation are called_________	Constants	Variables	Functions	Integers	Constants
Quantities which change that is different values in an investigation are called_________	Constants	Variables	Functions	Integers	Variables
The gross profit ddepends on the sales and the volume of a sphere depends on its_________	Circle	Square	Radius	Diameter	Radius
Two variables x and y are said to be function of each other when the values of one of them ______ on those of ther. other	Independent	propotional	depend	perpendicular	depend
If corresponding to each value of x, there is only one value of y then y is called_________Valued	Single	many	two	three	Single
If corresponding to each value of x, there existsmore than one value of y then y is called_________Valued	Single	many	two	three	many
The relation between two variables can be expressed in the form y=f(x), y is_________	implicit	explicit	odd	even	explicit
The relation between two variables can be expressed in the formf(x,y)=0, y is_________	implicit	explicit	odd	even	implicit
If f(-x)=f(x) then f(x) is called___________	odd	even	implicit	explicit	even
If f(-x)=-f(x) then f(x) is called___________	odd	even	implicit	explicit	odd

y=a0+a1x+……….anxn is called________ function.	odd	even	implicit	polynomial	polynomial
Limit x tends to a f(x)=1 is _______ function.	limit	even	implicit	polynomial	limit
Limit h tends to 0 f(a-h)=l1 is __________ limit	Right hand	Left hand	finite	infinite	Left hand
Limit h tends to 0 f(a+h)=l1 is __________ limit	Right hand	Left hand	finite	infinite	Right hand
If left hand limit = right hand limit then limit _______	does not exists	zero	exists	one	exists
If the limit is in indeterminant form then apply the method of _______	Induction	partial fraction	Integration	factorization	factorization
If the limit is in indeterminant form then apply the method of _______	rationalisation	Induction	partial fraction	Integration	rationalisation
If the limit is in indeterminant form then apply the method of _______	Induction	partial fraction	Substitution	Integration	Substitution
If the limit is in indeterminant form then apply the method of _______	Induction	partial fraction	Integration	L'Hospitals	L'Hospitals
If the limit is in indeterminant form then apply the method of _______	Induction	partial fraction	Integration	Infinite Limit	Infinite Limit
The rate of change of the object is called________	Derivative	Induction	Integration	Velocity	Derivative

The process of finding derivatives are called___________	Integration	Differentiation	Induction	Acceleration	Differentiation
Derivative of exponential is _________	algebra	exponential	logerthms	function	exponential
Derivative of constant is _________	one	two	three	zero	zero
One of the rule of differentiation is __________	addition	commutative	assiciative	closure	addition
One of the rule of differentiation is __________	commutative	assiciative	closure	Difference	Difference
One of the rule of differentiation is __________	commutative	Product	closure	assiciative	Product
One of the rule of differentiation is __________	closure	assiciative	Quotient	commutative	Quotient
If x is function of t and y is function of t then it is in ________ form.	closure	assiciative	Quotient	Parametric	Parametric
Process of finding second derivative ,third derivative etc is called _________differentiation.	Initial	final	Successive	Single	Successive
One of the application of Differentiation is ____________	Boundary	Elasticity	Simple Interest	Compound Interest	Elasticity
If the graph of the function rises then the function is __________ function	Decreasing	Increasing	rising	falling	Increasing

If the graph of the function fallss then the function is __________ function	Decreasing	Increasing	rising	falling	Decreasing
If the first dereivative zero and the second derivative less than zero then function is ________	Minima	Maxima	Optima	Extrima	Maxima
If the first dereivative zero and the second derivative greater than zero then function is ________	Minima	Maxima	Optima	Extrima	Minima

Questions	opt1	opt2	opt3	opt4	Answer
Which one of the following is a measure of central tendency?	Median	range	variation	correlation	Median
The total of the values of the items divided by their number of items is known as	Median	Arithmetic mean	mode	range	Arithmetic mean
In the short-cut method of arithmetic mean, the deviation is taken as	x – A	A – x	(x – A) / c	(A – x) / c	x – A
The sum of the deviations of the values from their arithmetic mean is	– 1	one	two	zero	zero
The formula for the weighted arithmetic mean is	∑wx / ∑w	∑w / ∑wx	∑x / n	∑x / ∑f	∑wx / ∑w
Find the Mean of the following values. 5, 15, 20, 10, 40	5	18	41	20	18
Which of the followings represents median?	First quartile	Third quartile	Second quartile	Q.D	Second quartile
Which of the measure of central tendency is not affected by extreme values?	Mode	Median	sixth deciles	Mean	Median
Which one of the following is relative measure of dispersion?	Range	Q.D	S.D	coefficient of variation	coefficient of variation
Quartile deviation is half of the difference between the --------------- quartiles	Q_3 and Q_1	Q_2 and Q_1	Q_4 and Q_1	Q3 and Q2	Q_3 and Q_1
The coefficient of Quartile deviation is given by	(Q3 – Q1)/(Q3+Q1)	(Q3 + Q1)/(Q3-Q1)	(Q3 – Q1)/(Q3-Q1)	(Q3 – Q1)	(Q3 – Q1)/(Q3+Q1)

Coefficient of variation is defined as	(AM * 100)/S.D	(S.D* 100)/A.M	S.D/A.M	(1/S.D)*100	(S.D* 100)/A.M
In a symmetrical distribution	A.M =G.M=H.M	A.M >H.M >G.M	H.M >G.M >A.M	A.M <H.M <G.M	A.M =G.M=H.M
If the values of median and mean are 72 and 78 respectively, then find the mode.	16	60	70	76	60
If variance is 64, then find S.D.	8	13	14	11	8
Find Mean for the following 3, 4, 5.	4.25	2.25	3	2.28	3
The coefficient of range	L-S /L+S	L+S /L-S	L-S	L+S	L-S /L+S
Second quartile is also called as	Mode	mean	median	G.M	median
If S.D = 6, then find variance.	6	36	42	12	36
The mean of age of 5 men is 40 years. Three of them are of some age and they are excluded. The mean of the remaining two is 25. Age of one of the excluded person in years is:	20	25	40	50	50
If the mean of 50 observations is 50 and one observation 94 is wrongly recorded there as 49 then correct mean will be	49.1	50	50.9	58	50.9

Median is	Average point	Midpoint	Most likely point	Most remote point	Midpoint
Mode is the value which	Is a mid point	Occur the most	Average of all	Most remote Likely	Occur the most
. ……………. Is known as positional average	Median	Mean	Mode	Range	Median
The median of marks 55, 60, 50, 40, 57, 45, 58, 65, 57, 48 of 10 students is	55	57	52.5	56	56
In a group of 150 observations the arithmetic mean is 60 and arithmetic mean of first 100 observations of the group is 50. Then arithmetic mean of the remaining observations of the group is	80	60	50	70	80
The middle most value of a frequency distribution table is known as	Mean	Median	Mode	Range.	Median
Quartiles are values dividing a given set of data into….. equal parts	4	6	3	2	4
The median value for the series 3, 5, 5, 2, 6, 2, 9, 5, 8, 6 is …	6	5	5.5	6.5	5
The mode for the series 3, 5, 6, 2, 6, 2, 9, 5, 8, 6 is	5	6	5.5	6.5	6
The Arithmetic mean for the series 51.6, 50.3, 48.9, 48.7, 48.5 is……….	49.8	50	48.9	49.6	49.8
The Median for the series 51.6, 50.3, 48.9, 48.7, 49.5, is…………	49.8	50	48.9	49.6	49.6

The Arithmetic mean for the series 51.6, 50.3, 48.9, 48.7, 49.5 is…………..	49.8	50	48.9	49.6	48.9
The Mode for the series 51.6, 50.3, 48.9, 48.7, 49.5 is………….	49.8	50	48.9	49.6	48.9
Mathematical ……………..is a positional average	Mean	median	mode	Standard deviation	median
The sum of deviations taken from arithmetic mean is ……………	minimum	zero	maximum	one	zero
The value of the variable which occurs most frequently in a distribution is called	Mean	median	mode	Standard deviation	mode
The formula of bimodal series is ……………..	Mode=2Median-3Mean	Mode= 3Median-2Mean	Mode= Median-Mean	Mode= Median-2Mean	Mode=3Median-2Mean
Deciles are the values dividing a given set of observations into	10	5	6	4	10
Percentiles divides a set of observations into	100	80	60	10	100
The middle most value of a frequency distribution table is known as	Mean	Median	Mode	Range	Median
Which of the following measures of averages divide the observation into two parts	Mean	Median	Mode	Range	Median
Which of the following measures of averages divide the observation into four equal parts	Mean	Median	Mode	Quartile	Quartile

The first quarter is known as	Lower quarter	Middle quarter	Upper quarter	Median	Lower quarter
The third quarter is known as	Lower quarter	Middle quarter	Upper quarter	Mode	Upper quarter
Arithmetic mean of the series 1, 3, 5, 7, 9 is	5	6	5.5	6.5	5
Arithmetic mean of the series 3, 4, 5, 6, 7 is ………	5.5	6	5	6.5	5
The Arithmetic mean for the series 3, 5, 5, 2, 6, 2, 9, 5, 8, 6, is…………….	5	6	5.5	6.5	5
Extreme values in a series affects the …………………….	Mean	median	mode	Standard deviation	Mean
Dispersion is also known as………..	Scatter	not scatter	line	nor line	Scatter
The simple Range is …………..	R=L*S	R=L+S	R=L/S	R=L-S	R=L-S
Variance cannot be …………..	positive	negative	zero	one	negative
If A.M = 8, N=12, then find ∑X.	76	80	86	96	96
If the value of mode and mean is 60 and 66 then, find the value of median.	64	46	54	44	64
If standard deviation is 5, then the variance is	5	625	25	2.23068	25

Standard deviation is also called as	Root mean square deviation	mean square deviation	Root deviation	Root median square	Root mean square deviation
Measures of central tendency is also known as	Dispersion	averages	correlation	tendency	correlation
$Q_1 = 40$, $Q_3 = 60$ then coefficient of Q.D is	0.3	0.4	0.2	0.1	0.4
From the given data 35,40,43,32,27 the coefficient of range is	23	0.23	13	0.13	13
Sum of square of the deviations about mean is	Maximum	one	zero	Minimum	Minimum
Median is the value of -------------- item when all the items are in order of magnitude	First	second	Middle most	last	Middle most
Find the Median of the following data 160, 180, 175, 179, 164, 178, 171, 164, 176	160	175	176	180	175
The position of the median for an individual series is taken as	(N + 1) / 2	(N + 2) / 2	N/2	N/4	(N + 1) / 2
Mode is the value, which has	Average frequency density	less frequency density	greatest frequency density	graetest frequency	greatest frequency density
A frequency distribution having two modes is said to be	unimodal	bimodal	trimodal	modal	bimodal
Mode has --------------- stable than mean.	less	more	same	most	less
Which of the following is not a measure of dispersion?	Range	quartile deviation	standard deviation	median	median

Which one of the following shows the relation between variance and standard deviation?	var = square root of S.D	S.D = square root of variance	variance = S.D	variance / S.D = 1	S.D = square root of variance
Range of the given values is given by	L- S	L+S	S+L	LS	L- S

Questions	opt1	opt2	opt3	opt4	Answer
A hypothesis may be classified as ----------------- .	simple and composite	composite only	null only	total population	simple and composite
Student's t-test is applicable in case of -------------------.	Small samples	for sample of size between 5	Large samples	for sample of size of more	Small samples
Degree of freedom for statistic chi-square incase of contingency	4	3	2	1	1
The term STATISTIC refers to the statistical measures	Population	Hypothesis	Sample	Parameter	Sample
A good way to get a small standard error is to use a _________.	Repeated sampling	Small sample	Large Sample	Large Population	Large Sample
The mean of Chi - distribution with *n* degrees of freedom	n	n+1	2n	0	n
If the calculated value is less than the table value, then we accept the ---------------	Alternative	Null	Statistics	Sample	Null
Rejecting null hypothesis when it is true leads to	Type I error	Type II error	Type III error	Correct decision	Type I error
Estimation is possible only in case of a:	Parameter	Random sample	Population	Sample	Random sample
If T is the estimator of parameter t, then T is called	$E(T) < t$	$E(T) > t$	$E(T) \neq t$	$E(T) = t$	$E(T) = t$
If point estimate is 8 and margin of error is 5 then confidence	5 to 15	4 to 14	6 to 16	3 to 13	3 to 13
Distance between true value of population parameter and	Error of central	Error of confidence	Error of estimatio	Error of hypothesis	Error of estimation
Small sample test is also known as	Z-test	t-test	Exact test	Normal test	t-test
In F – test, the variance of population from which samples	Equal	Different	Large	Small	Equal
Estimation is of two types	One sided and two sided	Type I and Type II	Biased and unbiased	Point estimation and interval estimation	Point estimation and interval estimation
The value of Z test at 5% level of significance is	0.96	3.95	1.96	2.56	1.96

Which of the following is a non-parametric test	Chi square	F	t	Z	Chi square
An estimator is a random variable because it varies from	Population to	Population to sample	Sample to populatio	Sample to sample	Sample to sample
Considering sample statistic, if mean of sampling distribution is	Unbiased estimator	Point estimator	Biased estimator	Interval estimator	Unbiased estimator
In confidence interval estimation, formula of calculating	Point estimate ± margin	Point estimate -margin of	Point estimate x margin	Point estimate + margin of	Point estimate ± margin of
In chi – square test, if the values of expected frequency	Goodness of fit	Degrees of freedom	Level of significan	Pooling	Pooling
Interval estimate is associated with	Probability	Non-probability	Range of values	Number of parameters	Range of values
Student's t-test is applicable in case of	Small samples	Large samples	For sample of	For sample size	Small samples
Z – test is applicable only when the sample size is ……….	Zero	2	Small	Large	Large
Which one of the following refers the term Correlation?	Perfectly positive	Perfectly negative	No correlatio	Low Positive	Perfectly negative
If r = +1, then the relationship between the given two variables is…………	Perfectly positive	Perfectly negative	No correlatio	Both positive	No correlation
If r = - 1, then the relationship between the given two variables is……….	1 and –1	0 and 1	0 and infinity	0 and –1.	1 and –1
If r = 0, then the relationship between the given two variables is……..	Perfect positive	Simple positive	Perfect negative	No correlation	Perfect negative
Coefficient of correlation value lies between ……………..	0 to 1	–1 to 1	0 to ∞	– ∞ to ∞	–1 to 1
While drawing a scatter diagram if all points appear to form a straight line getting Downward	Zero degree	Sixty degree	Ninety	Thirty degree	Ninety degree
The range of the rank correlation coefficient is……….	Origin	Scale	Both origin	Neither origin nor	Origin
If r =0, then the angle between two lines of regression is……..	Positive	Negative	Not certain	Zero	Negative
Regression coefficient is independent of……………	Positive	Negative	Not certain	Zero	Positive
If the correlation coefficient between two variables X and Y is negative, then the	Average of X only	Average of Y only	Average of X and	the median of X on Y	Average of X and Y

If the correlation coefficient between two variables X and Y is positive, then the	Less than one	Greater than one	Equal to one	Equal to zero	Greater than one
The regression line cut each other at the point of…………..	R.A.Fisher	Sir Francis Galton	Karl Pearson	Spearman	Spearman
If b_{xy} and b_{yx} represent regression coefficients and if $b_{yx} > 1$ then b_{xy} is…..	$1- (6\Sigma d^2 /(n(n^2-1)))$	$1- (6\Sigma d^2 /(n(n^2+1)))$	$1+ (6\Sigma d^2$ /(	$1 /(n(n^2-1))$	$1- (6\Sigma d^2 /(n(n^2-1)))$
Rank correlation was discovered by………..	6.4	2.5	10	25.6	2.5
Formula for Rank correlation is ……..	$(b_{xy.} b_{yx})^{1/4}$	$(b_{xy\,.}b_{yx})^{-1/2}$	$(b_{xy}$ b $)^{1/3}$	$(b_{xy.} b_{yx})^{1/2}$	$(b_{xy.} b_{yx})^{1/2}$
With b_{xy}=0.5, r = 0.8 and the variance of Y=16, the standard deviation of X=	Zero	Negative	Positive	One	Positive
The coefficient of correlation r =……………….	Positive	Negative	Zero	One	Negative
If two regression coefficients are positive then the coefficient of correlation must be	X = a + bY	X = a + bX	X= a - bY	Y= a + bX	X = a + bY
If two-regression coefficients are negative then the coefficient of correlation must be………….	Perfect positive	simple positive	Perfect negative	no correlation	Perfect negative
The regression equation of X on Y is ………………	Zero degree	Sixty degree	Ninety	Thirty degree	Zero degree
While drawing a scatter diagram if all points appear to form a straight line getting downward	Origin	Scale	Both origin	Neither origin nor	Origin
If r =1, the angle between two lines of regression is----------	r =0	r = 2	r = -2	r is either +1 or –1	r is either +1 or –1
Regression coefficient is independent of--------	Average of X only	Average of Y only	Average of X and	The median of X on Y	Average of X and Y
There will be only one regression line in case of two variables if------------	2	1.5	1	0	1
The regression line cut each other at the point of----------	-1	1.5	1	0	-1
Maximum value of correlation is………	Graphic correlation	Scatter diagrams	Both Graphic	Either graphic	Both Graphic
Minimum value of correlation is…………….	Regression	Skewness	Correlati on	Quartile	Correlation
Which is a method of measuring correlation?	Ordial	Interval	Ratio	Nomial	Interval

If there exists any relation between the sets of variables, it is called…………	0 and 1	-1 and 1	-1 and 0	1 and 2	-1 and 1
Which of the following measurement scales is required for the valid calculation of Karl	0	-1	1	2	1
Which of the following is the range of r ?	Perfect positive	simple positive	Perfect negative	no correlation	Perfect negative
When the two regression lines coincide, then r is…………..	Zero degree	Sixty degree	Ninety	Thirty degree	Zero degree

Questions	opt1	opt2	opt3	opt4	Answer
A ------- is an arrangement of statistical data in a chronological order.	forecasting	evaluation	comparison	all the above	all the above
Time series helps in ----------.	3	4	2	5	4
There are ------- types of components of a time series.	multiplicative	secular	additive	cyclical	additive
The ------------ model assumes that the observed value is the sum of four component of time series	multiplicative	secular	additive	cyclical	multiplicative
The ---------- model assumes that the observed value is obtained by multiplying the trend by the rates of three other	5	1	7	3	1
Seasonal variations repeat during a period of----- years.	depression in business	growth of population	weather and social customs	none of these	weather and social customs
The most important factor causing seasonal variations is-----	ratio to trend method	simple average method	ratio to moving average method	none of these	simple average method
If the trend is absent, the seasonal indices are known by--------	trend is not clear	the trend is linear	trend is not linear	none of these	trend is not linear
The trend can be found by the method of least squares if the --------	rate of growth is positive	growth rate is constant	growth is not constant	none of these\	growth rate is constant
The trend is linear if ---------	ratio to trend method	link relative	ratio to moving average	none of these	ratio to moving average

The most widely used method of measuring seasonal variations is ---------	graphic method	method of least squares	semi average method	moving average method	method of least squares
The ----------------may be used either to fit a straight line trend or a parabolic trend.	which year was selected as the origin?	what is the unit of time represented by	In what kind of units is Y being measured?	all the above	all the above
Whenever we fit any straight line trend by the least squares method,which things should be specified?	linear trend	secular trend	non-linear trend	none of the above	non-linear trend
The simplest example of the --------- is the second degree parabola.	1	2	3	0	0
In second deree parabola when time origin is taken between two middle years ∑X would be --------.	straight line	non-linear curve	either a or b	none of these	either a or b
Trends may also be plotted on a semi-loarithmic chart in the form of a-------	1	2	3	4	2
How many types of trend are usually computed by logarithms?	exponential tends	growth curves	both a and b	none of these	both a and b
The types of trend usually computed by logarithms are ----------	seasonal	secular	cyclical	irregular	seasonal
A ----------- variations repeat during a period of 1 year.	time series	data	correlation	index number	time series
The------ helps in forescating,evaluation and comparison.	non-linear	linear	clear	none of these	linear
The trend is ----------- if growth rate is constant.	seasonal	secular	cyclical	irregular	seasonal

The most important factor causing -------- variations is weather and social customs.	ellipse	parabola	hyperbola	circle	parabola
The simplest example of the non-linear trend is the second degree --------	averages	percentages	economic activity	time series	time series
Index numbers are special type of ----------	current	past	future	arbitrary	current
The base period should not be too distant from the -----	unit test	time reversal test	factor test and circular test	all the above	all the above
A good index number is one that satisfies-----------------------------	Index numbers	averages	time series	trend	Index numbers
-------------- help to calculate the real wages	A.M	G.M	H.M	both A.M and G.M	G.M
The best average to calculate index number is --------------	simple A.M method	Kelly's method	Laspeyre's method	Fisher's method	Fisher's method
Current year quantity is used in --------------	Base year quartiles	Current year quartiles	Both of them	Average of current and base year	Base year quartiles
Laspeyre's index is based on--------------------------	Laspeyre's method	Paasche's method	Fisher's method	Bow ley's method	Fisher's method
Time Reversal test is satisfied by-------------------------------	8	90	111.11	110	90
If the price of a commodity is Rs.80 in the base year and Rs.72 in the current year, the Price index number is-------	current year	base year	future year	arbitrary year	base year

In Laspeyre's index number, importance is given to the quantity of---------	Bow ley's formula	Laspeyre's formula	Paasche's formula	Fisher's formula	Paasche's formula
The current year quantities are taken as weights in------	P01 xP10 = 0	P_{01} xP_{10} < 1	P_{01} xP_{10} = 1	P_{01} xP_{10} > 1	P_{01} xP_{10} = 1
Time reversal test condition is--------	$P_{01}Q_{01}$ = åp_1q_1/ åp_0q_0	$P_{01}Q_{01}$< åp_1q_1/ åp_0q_0	$P_{01}Q_{01}$> åp_1q_1/ åp_0q_0	$P_{01}Q_{01}$ = åp_1q_1	$P_{01}Q_{01}$ = åp_1q_1/ åp_0q_0
Factor reversal test condition is = ----------	an upward bias	a downward bias	either upward or downward bias	no bias	an upward bias
Paasche's index number is generally expected to have--------	å log P/N	åP/N	N/åP	åP/ log N	åP/N
The formula for unweighted averages of relatives' method by using A.M.,	arithmetic mean of Laspeyre's and Paasche's	geometric mean of Laspeyre's and Paasche's	median of Lasoeyre's and Paasche's	all of the above	geometric mean of Laspeyre's and Paasche's
Fisher's ideal index is--------	151	150	151.61	125.2	151.61
If åp_0 is 3100 and åp_1 is 4700 then P_{01} =--------------	Laspeyre's formula	Paasche's formula	Fisher's formula	both a) and b)	Fisher's formula
$P = \sqrt{P_{01}^{L} \times P_{01}^{P}}$ is -------------------	consumer	Laspeyre's	Paasche's	Fisher	consumer
Family budget method is a method to calculate ----- price index.	median	geometric mean	arithmetic mean	mode	geometric mean
The best average in the construction of index number is	value index	price index	quantity index	quality index	quantity index

Commodities which show considerable price fluctuation could be best measured by a	factor reversal test	time reversal test	t-test	f-test	time reversal test
The circular test is an extension of the	weighted	fixed weighted	un weighted	fixed un weighted	weighted
Most frequently used index number formulae are	base year quantities	current year quantities	base year qualities	current year qualities	base year quantities
Laspeyre's index is based on	Kelly's	Walsch's	Fisher's price	Marshall-Edgeworth's	Walsch's
------- index number uses the geometric mean of the base year and current year quantities as weights.	Value index	Laspeayre's index	Paasche's index	Fisher ideal index	Value index
----- is the sum of the values of a given year divided by the sum of the values of the base year.	$(p_1 / p\text{-}_0) * 100$	$(p_1 / q\text{-}_0) * 100$	$(q_1 / p\text{-}_1) * 100$	$(p_1 / q_1) * 100$	$(p_1 / p\text{-}_0) * 100$
Formula for price relative or price index number of a commodity P is-----	Ideal	economic	special	commercial	Ideal
Fisher's formula is called --------- index number formula	Laspeyre's method	Paasche's method	Fisher's method	both a) and b)	Fisher's method
Factor reversal test is satisfied by------------	current	base	average	calculated	current
The year for which index number is calculated is called ---------year.	p_1	p_0	q_0	q_1	p_1
Notation of price of a commodity in the current year is-------	Time series	Mean	Mode	Index number	Index number

---------- are the pulse of an economy	100/Price index	Price index /100	Money wage/Price index *100	Price index *100	100/Price index
Purchasing power = ------------	p_1	p_0	q_0	q_1	p_0
Notation of price of a commodity in the base year is-------	p_1	p_0	q_0	q_1	q_1
Notation of quantity of a commodity in the current year is-------	25%	10%	125%	35%	25%
If the price of a commodity is Rs.40 in the base year and Rs.50 in the current year, the Price has increased by-------	$P_{01} \times P_{12} \times P_{20} = 1$	$P_{12} \times P_{20} = 1$	$P_{01} \times P_{12} = 1$	$P_{01} \times P_{12} \times P_{20} = 0$	$P_{01} \times P_{12} \times P_{20} = 1$
By circular test ---------------	current year	base year	arbitrary year	previous year	base year
Link relative is a price or quantity relative with the condition that ------------- is the preceding year.	Consumer price	Consumer price index number	Consumer price index number	price index number	Consumer price index number
Cost of living index number is also known as -----------	price	quantity	both	neither price nor quantity	price
In Factor reversal test P_{01} gives the relative change in -----------	price	quantity	both	neither price nor quantity	quantity
In Factor reversal test Q_{01} gives the relative change in -----------	Time reversal test	Factor reversal test	Fisher's test	both a and b	Time reversal test
----------- satisfies the Kelly's test.	Time reversal test	Factor reversal test	Fisher's test	both a and b	both a and b

Fisher’s index satisfy ------------	price multiplied by quantity	quantity	both	price	price multiplied by quantity
In Factor reversal test P_{01} x Q_{01} gives the relative change in -----------	ratio	percentage	fraction	mean	percentage
Index number are expressed in ------	base year quantities	current year quantities	base year qualities	current year qualities	current year quantities
Paasche index is based on	forecasting	evaluation	comparison	index numbers	Index numbers