KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

Semester – II

18MBAP204 QUANTITATIVE TECHNIQUES 5H -4C

Instruction Hours / week: L: 4 T: 1 P : 0 Marks: Internal: 40 External: 60 Total: 100

End Semester Exam: 3 Hours

COURSE OBJECTIVES:

To make the students

1. To understand the scientific approaches to decision-making through mathematical modeling and solving linear programming models.
2. To use variables for formulating complex mathematical models in management science, industrial engineering and transportation science.
3. To know the advanced methods for large-scale transportation, assignment problems and inventory models.
4. To formulate and solve problems as networks and graphs
5. To recognise the mathematical and computational modeling of real decision-making problems,

COURSE OUTCOMES :

Learners should be able to

1. Understand the principles and techniques of Operations Research and their applications in decision-making.
2. Realize and apply mathematical techniques for shortest path, maximum flow, minimal spanning tree, critical path, minimum cost flow, and transshipment problems.
3. Formulate linear programming (LP) models and understand the cost minimization and profit maximization concepts.
4. Select the best strategy on the basis of decision criteria under the uncertainty.
5. Demonstrate capabilities of problem-solving, critical thinking, and communication skills.

UNIT I

Operations Research and Linear Programming

Concepts and Scope of Operations Research (OR) – Phases of OR study – Models in OR – Advantages and limitations of OR – Role of computers in OR- Formulating Linear programming models, graphical solution of linear programming models, the simplex method-outline, and computing procedure, use of artificial variables, Big M- method and Two phase method.

UNIT II

Transportation Problems

Transportation Problems (TP) – Initial basic feasible solution to Transportation Cost – Northwest corner rule, Least cost method – Vogel's approximation method, Optimal solution using Modified Distribution (MODI) method, Degeneracy in TP, Unbalanced TP, Alternative optimal solutions, Maximization in TP – Assignment Problems – Hungarian method of solving assignment problem, Multiple optimum solutions, Maximisation in Assignment Problems, Unbalanced Assignment Problems, Restrictions in Assignment Problems.

UNIT III

Network Analysis

Network Analysis – Construction of networks, Components and Precedence relationships – Event – activities – rules of network construction, errors and dummies in network. PERT/CPM networks –project scheduling with uncertain activity times – Critical Path Analysis – Forward Pass method, Backward Pass method – Float (or slack) of an activity and event –Time – cost trade-offs – crashing activity times.

UNIT IV

Inventory Models

Inventory models – Economic order quantity models – Quantity discount models – Stochastic inventory models – Multi product models – Inventory control models in practice - Queueing models – Queueing systems and structures – Notation parameter – Single server and multi server models – Poisson input – Exponential service – Constant rate service – Infinite population.

UNIT V

Decision Models

Decision models – Anatomy of Decision Theory - Decision Models: Probabilistic Decision Models: Maximum Likelihood Rule- Expected Payoff Criterion- Competitive Decision Models: Maximin, Minimax, Savage, Hurwicz, Laplace Decision Models, Game theory – Two person zero sum games – Graphical solution- Algebraic solution– Linear Programming solution – Replacement models – Models based on service life – Economic life– Single / Multi variable search technique – Dynamic Programming. Simulation techniques: Introduction – Types of simulation- Monte Carlo Simulation

SUGGESTED READINGS:

1. Frederick S. Hillier , Gerald J. Lieberman , Bodhibrata Nag , Preetam Basu (2017), Introduction to Operations Research, 10th edition, McGraw Hill Education, New Delhi.
2. J.K. Sharma(2017). Operations Research - Theory and Applications., 6th edition, Laxmi Publications, New Delhi.
3. G. Srinivasan (2017) , Operations Research: Principles and Applications, PHI, New Delhi
4. Taha (2014), Operations Research: An Introduction, 9th edition, Pearson education, New Delhi.
5. PK Gupta , D.S Hira (1976), Operations Research, S Chand Publishing, New Delhi.

UNIT-I

SYLLABUS

Concepts and Scope of Operations Research (OR) – Phases of OR study – Models in OR – Advantages and limitations of OR – Role of computers in OR- Formulating Linear programming models, graphical solution of linear programming models, the simplex method-outline, and computing procedure, use of artificial variables, Big M- method and Two phase method.

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UNIT-I

LINEAR PROGRAMMING MODELS

CONCEPT

Linear programming (LP also called linear optimization) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming (mathematical optimization).

APPLICATIONS

1. Linear programming techniques are used in many industrial and economic problems such as planning, production, transportation, airlines railways, textiles industries, chemical industries, steel industries, food processing and other issues.

2. Linear programming is heavily used in microeconomics and company management.

3. The modern management issues are ever-changing, most companies would like to maximize profits or minimize costs with limited resources. Therefore, many issues can be characterized as linear programming problems.

4. Designing the layout of a factory for efficient flow of materials.

5. Managing the flow of raw materials and products in a supply chain based on uncertain demand for the finished products.

6. (a) Investment.
(b) Production planning and inventory control.
(c) Manpower planning.
(d) Urban development planning.
(e) Oil refining and blending.

Each model is detailed, and its optimum solution is interpreted.

7. Real life application-Optimization of heart valve production-Biological heart valves in different sizes are bioprostheses manufactured from porcine hearts for human implantation.

TECHNICAL TERMS

Linear-The term ‘Linear’ is used to describe the proportionate relationship of two or more variables in a model.

Programming-The word, ‘programming’ is used to specify a sort of planning that involves the economic allocation of limited resources by adopting a particular course of action.

Linear Programming -Linear Programming is a mathematical technique for optimum allocation of limited or scarce resources, such as labour, material, machine, money energy etc.

Decision Variables-The variables whose values determine the solution of a problem are called decision variables.

Objective Function- A linear programming problem may be defined as the problem of maximizing or minimizing a linear function subject to system of linear constraints.

Constraints- A system of linear inequalities.

Feasible region-A region in which all the constraints are satisfied simultaneously is called a feasible.

Feasible solution-Any solution to a LPP which satisfied the non-negative restrictions of the LPP is called is feasible solution.

Optimal solution- Any feasible solution which optimizes the objective function is called its optimal solution.

Basic solution-given a system of m linear equation with n variable ,any solution which is obtained by solving for m variables keeping the remaining n-m variable are called a basic solution.

Corner Point- A vertex of the feasible region. Not every intersection of lines is a corner point.

Bounded Region- A feasible region that can be enclosed in a circle. A bounded region will have both a maximum and minimum values.

Unbounded Region- A feasible region that cannot be enclosed in a circle.

Basic solution-given a system of m linear equation with n variable ,any solution which is obtained by solving for m variables keeping the remaining n-m variable are called a basic solution.

Slack variable-the non-negative variable which is added to LHS of the constraint to convert the inequality less than or equal to , into an equation is called slack variable.

Surplus variable-the non-negative variable which is removed from the LHS of the constraint to convert the inequality less than or equal to, into an equation is called surplus variable.

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1.1. MATHEMATICAL FORMULATION

General Mathematical Model of an LPP

Optimize (Maximize or Minimize) $Z=C_1 X_1 + C_2 X_2 +\ldots\ldots+C_nX_n$

Subject to constraints,

$a_{11}X_1 + a_{12}X_2+\ldots\ldots\ldots\ldots\ldots\ldots+ a_{1n}X_n$ (<,=,>) b_1

$a_{21}X_1 + a_{22}X_2+\ldots\ldots\ldots\ldots\ldots\ldots+ a_{2n}X_n$ (<,=,>) b_2

$a_{31}X_1 + a_{32}X_2+\ldots\ldots\ldots\ldots\ldots\ldots+ a_{3n}X_n$ (<,=,>) b_3

$a_{m1}X_1 + a_{m2}X_2+\ldots\ldots\ldots\ldots\ldots\ldots+ a_{mn}X_n$ (<,=,>) b_m

and $X_1, X_2 \ldots.X_n >0$

1.1.1. Example

A firm is engaged in producing two products. A and B. Each unit of product A requires 2 kg of raw material and 4 labour hours for processing, where as each unit of B requires 3 kg of raw materials and 3 labour hours for the same type. Every week, the firm has an availability of 60 kg of raw material and 96 labour hours. One unit of product A sold yields Rs.40 and one unit of product B sold gives Rs.35 as profit.

Solution:

Formulate this as an Linear Programming Problem to determine as to how many units of each of the products should be produced per week so that the firm can earn maximum profit.

i) Identify and define the decision variable of the problem. Let X_1 and X_2 be the number of units of product A and product B produced per week.

ii) Define the objective function Since the profits of both the products are given, the objective function is to maximize the profit.

$$\text{Max } Z = 40X_1 + 35X_2$$

i) State the constraints to which the objective function should be optimized (i.e. Maximization or Minimization)

ii) There are two constraints one is raw material constraint and the other one is labour constraint.. The raw material constraint is given by $2X_1 + 3X_2 < 60$ The labour hours constraint is given by

$$4X_1 + 3X_2 < 96$$

Finally we have,

Max $Z = 40X_1 + 35X_2$

Subject to constraints, $2X_1 + 3X_2 < 60$

$4X_1 + 3X_2 < 96$

$X_1,X_2 > 0$

Problems

1.1.2. A manufacturer produces two types of models M1 and M2.Each model of the type M1 requires 4 hours of grinding and 2 hours of polishing; where as each model of M2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works 60 hours a week. Profit on M1 model is Rs.3.00 and on model M2 is Rs.4.00.Whatever produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models, so that he makes maximum profit in a week?

Answer:

Max $Z = 40X_1 + 35X_2$

Subject to constraints,

$2X_1 + 3X_2 < 60$

$4X_1 + 3X_2 < 96$

$X_1,X_2 > 0$

1.1.3. The agricultural research institute suggested the farmer to spread out atleast 4800 kg of special phosphate fertilizer and not less than 7200 kg of a special nitrogen fertilizer to raise the productivity of crops in his fields. There are two sources for obtaining these – mixtures A and mixtures B. Both of these are available in bags weighing 100kg each and they cost Rs.40 and Rs.24 respectively. Mixture A contains phosphate and nitrogen equivalent of 20kg and 80 kg respectively, while mixture B contains these ingredients equivalent of 50 kg each. Write this as an LPP and determine how many bags of each type the farmer should buy in order to obtain the required fertilizer at minimum cost.

Answer:

Min $Z = 40X_1 + 24X_2$

is subjected to three constraints

$20X_1 + 50X_2 >4800$

$80X_1 + 50X_2 >7200$

$X_1, X_2 >0$

1.2. GRAPHICAL SOLUTION OF LINEAR PROGRAMMING MODELS

1.2.1. Example

Solve the following LPP by graphical method

Maximize $Z = 5X_1 + 3X_2$

Subject to constraints

$2X_1 + X_2 \leq 1000$

$X_1 \leq 400$

$X_1 \leq 700$

$X_1 , X_2 \geq 0$

Solution:

The first constraint $2X_1 + X_2 \leq 1000$ can be represented as follows. We get

$$2X_1 + X_2 = 1000$$

When $X_1 = 0$ in the above constraint, we get,

$$2 X_1 + X_2 = 1000$$

$$X_2 = 1000$$

Similarly when $X_2 = 0$ in the above constraint, we get,

$$2X_1 + 0 = 1000$$

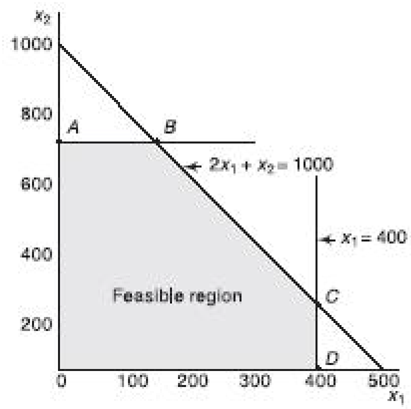
$X_1 = 1000/2 = 500$

The second constraint $X_1 \le 400$ can be represented as follows, We get

$X_1 = 400$

The third constraint $X_2 \le 700$ can be represented as follows, We get

$X_2 = 700$



The constraints are shown plotted in the above figure

Point	X1	X2	Z = 5X1 +3X2
0	0	0	0
A	0	700	Z = 5 x 0 + 3 x 700 = 2,100
B	150	700	Z = 5 x 150 + 3 x 700 = 2,850* Maximum
C	400	200	Z = 5 x 400 + 3 x 200 = 2,600
D	400	0	Z = 5 x 400 + 3 x 0 = 2,000

The Maximum profit is at point B

When $X_1 = 150$ and $X_2 = 700$

$Z = 2850$

Problems

1.2.2. Solve the following LPP by graphical method

Maximize $Z = 400X_1 + 200X_2$

Subject to constraints

$18X_1 + 3X_2 \leq 800$

$9X_1 + 4X_2 \leq 600$

$X_2 \leq 150$

$X_1, X_2 \geq 0.$

Answer:

The Maximum profit is at point A
When X1 = 150 and X2 = 0
Z = 30,000

1.2.3. Solve the following LPP by graphical method

Minimize $Z = 20X_1 + 40X_2$

Subject to constraints

$36X_1 + 6X_2 \geq 108$

$3X_1 + 12X_2 \geq 36$

$20X_1 + 10X_2 \geq 100$

$X_1, X_2 \geq 0$

Answer:

The Minimum cost is at point C
When X1 = 4 and X2 = 2
Z = 160

1.3. SIMPLEX METHOD

1.3.1. Example

Find the non negative values of X_1,X_2 and X_3 which Max Z= $3X_1 + 2X_2 + 5X_3$

Subject to $X_1 + 4X_2$ 420

$3X_1 + 2X_3$ 460

$X_1 + 2X_2 + X_3$ 430

And X_1 , X_2 , X_3

Solution:

Given

$\text{Max } Z = 3X_1 + 2X_2 + 5X_3$

Subject to $X_1 + 4X_2$ 420

$3X_1 + 2X_3$ 460

$X_1 + 2X_2 + X_3$ 430

X_1, X_2, X_3

By introducing non negative slack variables s1,s2 and s3 , the standard form of the LPP becomes

$\text{Max } Z = 3X_1 + 2X_2 + 5X_3 + 0S_1 + 0S_2 + 0S_3$

Subject to $X_1 + 4X_2$ 420

$3X_1 + 0X$ 460

$X_1 + 2X_2 + X_3$ 430

$X_1 , X_2 , X_3 , S_1 , S_2 , S_3$

Since there are 3 equations with 6 variables, the initial basic feasible solution is obtained by equating (6-3) variables to zero.

Therefore the initial basic feasible solution is $S_1 = 420$, $S_2 = 460$, $S_3 = 430$

$(X_1=X_2=X_3 =)$

The initial simplex table is given by

Initial iteration:

		C_j	3	2	5	0	0	0	
C_B	Y_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	

0	S_1	420	1	4	0	1	0	0	-
0	S_2	460	3	0	2	0	1	0	
0	S_3	430	1	2	1	0	0	1	
Z_j-Cj		0	-3	-2	-5	0	0	0	

Since there are some $(Z_j - C_j) < 0$, the current basic feasible solution is not optimal.

To find the entering variable :

Since $(Z_3\text{-}C_3) = -5$ is the most negative, the corresponding non basic variable X3 enters into the basis. The column corresponding to this X_3 is called the key column or pivot column.

To find the leaving variable:

Find the ratio = min = min =230

Therefore the leaving variable is the basic variable S_2 which corresponds to the minimum ratio = 230. The leaving variable row is called the key row or pivot equation and 2 is the pivot element.

First iteration:

		C_j	3	2	5	0	0	0	
C_B	Y_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	
0	S_1	420	1	4	0	1	0	0	
5	X_2	230	3/2	0	1	0	1/2	0	
0	S_3	200	-1/2	(2)	0	0	-1/2	1	
Zj-Cj		1150	9/2	-2	0	0	5/2	0	

Since there are some (Z_2-C_2) = -2, the current basic feasible solution is not optimal.

Here the non-basic variable X2 enters into the basis and the basic variable S_3 leaves the basis.

Second iteration:

		C_j	3	2	5	0	0	0	
C_B	Y_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	
0	S_1	420	1	4	0	1	0	0	
5	X_2	230	3/2	0	1	0	1/2	0	
0	S_3	200	-1/2	(2)	0	0	-1/2	1	
Zj-Cj		1150	9/2	-2	0	0	5/2	0	

Since all z_j- $c_j \geq 0$ the current basic feasible solution is optimal.

Therefore the optimal solution is Max Z = 1350, X_1 = 0, X_2 = 100, X_3 =230.

Problem

1.3.2. Use simplex method to solve the LPP

Max Z= $4X_1$ +$10X_2$

Subject to $2X_1 + X_2$ 50

$2X_1 + 5X_2$ 100

$2X_1 + 3X_2$ 90

And X_1 , X_2 .

Answer:

$X_1=0$, $X_2=20$, and Max $Z=200$.

1.4. ARTIFICIAL VARIABLE TECHNIQUES

1.4.1. Solve the following linear programming problem using Big M method

$$\text{Minimize } Z=10X_1+15X_2+20X_3$$

Subject to

$$2X_1+4X_2+6X_3\geq 24$$
$$3X_1+9X_2+6X_3\geq 24$$
$$X_1,X_2,X_3\geq 0$$

Solution

The standard form of this problem is as shown below. In this form S_1,S_2 are called as surplus variables which are introduced to balance the constraints.

$$\text{Minimize } Z=10X_1+15X_2+20X_3$$

Subject to

$$2X_1+4X_2+6X_3-S_1=24$$
$$3X_1+9X_2+6X_3-S_2=30$$
$$X_1,X_2,X_3,S_1,S_2\geq 0$$

The canonical form of the above standard form ,which consists of the artificial variables $R_1$and R_2 , is presented below

$$\text{Minimize } Z=10X_1+15X_2+20X_3+MR_1+MR_2$$

Subject to

$$2X_1+4X_2+6X_3-S_1+R_1=24$$
$$3X_1+9X_2+6X_3-S_2+R_2=30$$
$$X_1,X_2,X_3,S_1,S_2 \text{, } R_1 \text{ and } R_2\geq 0$$

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Table 1 : Initial table

		C_j	10	15	20	0	0	-M	-M	
C_B	Y_B	X_B						R_1	R_2	Ɵ
-M	R_1	24	2	4	6	-1	0	1	1	6
-M	R_2	30	3	9	6	0	-1	0	1	
Z_j-C_j		-54M	10-5M	15-13M*	20-12M	M	M	0	0	

Table 2: Iteration 1

		C_j	10	15	20	0	0	-M	-M	
C_B	Y_B	X_B						R_1	R_2	Ɵ
-M	R_1			0		-1		1		
-M	X_2			1		0		0		5
Z_j-C_j		M+50	M+5	0	M+10*	M	M+	0	M-	

Reached. With the entering variable as X_3 and the leaving variable as R_1 , the corresponding pivot operations are shown in Table

Table 3 : Iteration 2

		C_j	10	15	20	0	0	-M	-M
C_B	Y_B	X_B						R_1	R_2
20	X_3			0	1				

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15	X_2			1	0				
Z_j-C_j		82	3	0	0	3		M-3	

The optimality is reached and the corresponding optimal result is presented below

X_1=0 , X_2 = , X_3 = and Z(Optimum) =82.

1.3.3. Use Two phase simplex method to solve the LPP

Max Z= $5X_1 + 8X_2$

Subject to $3X_1 + 2X_2$ 3

$X_1 + 4X_2$ 4

$X_1 + X_2$ 5

And X_1 , X_2

Solution

Given

Max Z= $5X_1 + 8X_2$

Subject to $3X_1 + 2X_2$ 3

$X_1 + 4X_2$ 4

$X_1 + X_2$ 5

And X_1 , X_2

By introducing non negative slack variables ,surplus and artificial variables , the standard form of the LPP becomes

Max Z = $5X_1 + 8X_2 + 0S_1 + 0S_2 + 0S_3$

Subject to $3X_1 + 2X_2 - S_1 + 0S_2 + 0S_3 + R_1 = 3$

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$$X_1 + 4X_2 + 0S_1 - S_2 + 0S_3 + R_2 = 4$$

$$X_1 + X_2 + 0S_1 + 0S_2 + 0S_3 = 5$$

And $X_1, X_2, S_1, S_2, S_3, R_1, R_2$

Phase – 1

Max $Z^* = -R_1 - R_2$

Initial iteration:

		C_j	0	0	0	0	0	-1	-1	
C_B	Y_B	X_B	X_1	X_2	S1	S_2	S_3	R1	R2	
-1	R_1	3	3	2	-1	0	0	1	0	3/2
-1	R_2	4	1	(4)	0	-1	0	0	1	4/4
0	S_3	5	1	1	0	0	1	0	0	5/1
Z_j-Cj		-7	-4	-6	1	1	0	0	0	

Since there are some $(Z^*_j - C_j) < 0$, the current basic feasible solution is not optimal.

First iteration: Introduce x_2 and drop R_2

		C_j	0	0	0	0	0	-1	-1	
C_B	Y_B	X_B	X_1	X_2	S1	S_2	S_3	R1	R2	
-1	R_1	1	(5/2)	0	-1	1/2	0	1	1/2	2/5
0	X_2	1	1/4	1	0	-1/4	0	0	1/4	4
0	S_3	4	3/4	0	0	¼	1	0	-1/4	16/3
Z_j-Cj		-1	-5/2	0	1	-1/2	0	0	3/2	

Since there are some $(Z^*_j - C_j) < 0$, the current basic feasible solution is not optimal.

Second iteration: Introduce x_2 and drop R_1

		C_j	0	0	0	0	0	-1	-1
C_B	Y_B	X_B	X_1	X_2	S1	S_2	S_3	R1	R2
0	X_1	2/5	1	0	-2/5	1/5	0	2/5	-1/5
0	X_2	9/10	0	1	1/10	-3/10	0	-1/10	3/10
0	S_3	37/10	0	0	3/10	1/10	1	-3/10	-1/10
Z_j-Cj		0	0	0	0	0	0	1	1

Since there are some $(Z_j - C_j)$ 0, the current basic feasible solution is optimal. Furthermore no artificial variable appears in the optimum basis so we proceed to phase – II.

Phase – II:

Here, we consider the actual costs associated optimal variables. The new objective function then becomes

$$\text{Max } Z = 5X_1 + 8X_2 + 0S_1 + 0S_2 + 0S_3$$

Initial iteration:

		C_j	0	0	0	0	-1	
C_B	Y_B	X_B	X_1	X_2	S1	S_2	S_3	
5	X_1	2/5	2/5	1	0	-2/5	1/5	2
8	X_2	9/10	9/10	0	1	1/10	-3/10	-
0	S_3	37/10	37/10	0	0	3/10	1/10	37

Z_j-Cj	46/5	0	0	6/5	-7/5	0	

Since there are some $(Z_j - C_j) < 0$, the current basic feasible solution is not optimal.

First iteration:

		C_j	5	8	0	0	0	
C_B	Y_B	X_B	X_1	X_2	S1	S_2	S_3	
0	S_2	2	5	0	-2	1	0	-
8	X_2	3/2	3/2	1	-1/2	0	0	-
0	S_3	7/2	-1/2	0	(½)	0	1	7
Z_j-Cj		12	7	0	-4	0	0	

Since there are some $(Z_j - C_j) < 0$, the current basic feasible solution is not optimal.

Second iteration:

		C_j	5	8	0	0	0
C_B	Y_B	X_B	X_1	X_2	S1	S_2	S_3
0	S_2	16	3	0	0	1	4
8	X_2	5	1	1	0	0	1
0	S_1	7	-1	0	1	0	2

Z_j-Cj	40	3	0	0	0	8

Since there are some (Z_j -C_j) 0, the current basic feasible solution is optimal.

Therefore the optimal solution is Max Z = 40 X_1 = 0, X_2 = 5.

Problems

1.4.2. Minimize Z= $4X_1 + X_2$

Subject to $3X_1 + X_2$

$4X_1 + X_2$ 6

$X_1 + 2X_2$ 4

And X_1 , X_2 .

1.5. VARIANTS OF SIMPLEX METHOD

1.5.1. Example

Solve the following LPP by Dual simplex method:

Maximize Z=$x_1+2x_2+x_3$

Subject to

$2x_1+x_2- x_3 \leq 2$

$-2x_1+x_2-5x_3 \geq -6$

$4x_1+x_2+x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

Solution:

Given

Maximize Z= $x_1+2x_2+x_3$

Subject to

$2x_1+x_2- x_3 \leq 2$

$2x_1-x_2+5x_3 \leq 6$

$4x_1+x_2+x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

By introducing the non-negative slack variables s_1, s_2, s_3 the standard form of LPP becomes

Maximize $Z= x_1+2x_2+x_3$

Subject to

$2x_1+x_2- x_3+ s_1+0s_2+0s_3 =2$

$2x_1-x_2+5x_3+ 0s_1+ s_2+0 s_3 =6$

$4x_1+x_2+x_3+ 0s_1+0s_2+s_3=6$

And $x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

The initial basic feasible solution is given by

$s_1=2, s_2 =6, s_3 =6$(basic)($x_1= x_2 =x_3=0$, non basic)

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Initial iteration:

		C_j	1	2	1	0	0	0	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ
0	S_1	2	2	(1)	-1	1	0	0	
0	S_2	6	2	-1	5	0	1	0	_

0	S_3	6	4	1	1	0	0	1	
$(Z_j\text{-}C_j)$		0	-1	-2	-1	0	0	0	

Since there are some $(Z_j\text{-}C_j)<0$, the current basic feasible solution is not optimal.

First iteration: Introduce x_2 and drop s_1

		C_j	1	2	1	0	0	0	
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	θ
2	S_1	2	2	1	-1	1	0	0	–
0	S_2	8	4	0	4	1	1	0	
0	S_3	4	2	0	(2)	-1	0	1	
$(Z_j\text{-}C_j)$		4	3	0	-3	2	0	0	

Since there are some $(Z_j\text{-}C_j)<0$, the current basic feasible solution is not optimal.

Second iteration: Introduce x_3 and drop s_3

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		C_j	1	2	1	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
2	S_1	4	3	1	0		0	
0	S_2	0	0	0	0	3	1	-2
1	S_3	2	1	0	1		0	
(Z_j-C_j)		10	6	0	0		0	

Since all $(Z_j-C_j)\geq 0$, the current basic feasible solution is optimal.

Therefore, the optimal is Max Z=10, $x_1=0$, $x_2=4$, $x_3=2$.

Problems

1.5.2. Use Dual simplex method to solve the LPP

Max Z= $2X_1 + X_2$

Subject to $3X_1 + X_2$ 3

$4X_1 + 3X_2$ 6

Answer:

Min Z =12/5, X_1 =3/5, X_2 =6/5)

$X_1 + 2X_2$ 3

And X_1 , X_2 .

1.5.3. Use Dual simplex method to solve the LPP

Maximize Z= $6x_1+4x_2+4x_3$

Subject to $3x_1+ x_2+2x_3$ 2

$2x_1 + x_2 - x_3 \quad 1$

$- X_1 + X_2 + 2x_3 \quad 1$

Answer:

This method is fail .ie., we cannot solve this problem by this dual simplex method.

And $x_1, x_2, x_3 \geq 0$.

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UNIT-II

SYLLABUS

Transportation Problems (TP) – Initial basic feasible solution to Transportation Cost – Northwest corner rule, Least cost method – Vogel's approximation method, Optimal solution using Modified Distribution (MODI) method, Degeneracy in TP, Unbalanced TP, Alternative optimal solutions, Maximization in TP – Assignment Problems – Hungarian method of solving assignment problem, Multiple optimum solutions, Maximisation in Assignment Problems, Unbalanced Assignment Problems, Restrictions in Assignment Problems.

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UNIT II

TRANSPORTATION AND ASSIGNMENT MODELS

CONCEPT

The transportation problem deals with the transportation of a product manufactured at different plants or factories (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy the destination requirements within the Plants capacity constraints at the minimum transportation cost. Transportation problems thus typically arise in situations involving physical movement of goods from plants to warehouses, warehouses to wholesalers, wholesalers to retailers, retailers to customers. Solution of the transportation problems requires the determination of how many units should be transported from each supply origin to each demand destination in order to satisfy all the destination demands while minimizing the total associated cost of transportation.

APPLICATIONS

1. Minimize shipping costs from factories to warehouses (or from warehouses)

2. Determine lowest cost Location for new factory, warehouses, office, or other outlet facility.

3. Find minimum cost production schedule that satisfies firms demand and production limitations (called production smoothing).

4. The military distribution system usually called logistics systems lay due emphasis on the distribution of personnel and material to vessels, installation or troop locations using transportation models.

5. Example of transshipment nodes all the connecting airports between the starting point of a trip and the final destinations, satellites that act as relay station between a transmitted TV signal and the reception of that signal, etc,

6. In assignment problem, Assign sales people to sales territories, Assign vehicles to routes. ,Assign accountants to client accounts, Assign contracts to bidders through systematic evaluation of bids from competing suppliers, Assign naval vessels to petrol sectors, Assign development engineers to several constructions sites, Schedule teachers to classes, etc.,

7. In traveling salesman problems used for, In postal deliveries, Inspection, School bus rooting, Television relays, Assembly lines.

8. NORTH WEST- It is used to in case of transportation within the campus of an organization as costs are not significant. It is used for transportation to satisfy such obligations where cost is not the criteria. For example in case of Food Corporation of India Ltd.

9. VAM- It is used to compute transportation routes in such a way as to minimize transportation cost for finding out locations of warehouses. It is used to find out locations of transportation corportations depots where insignificant total cost difference may not matter.

TECHNICAL TERMS

BALANCED TRANSPORTATION PROBLEM: A transportation problem in which the total supply available (at all the origins) exactly satisfies the total demand required(at all the destinations).

CELL: the rectangle in a transportation tableau used to identify the route between an origin and destination.

DEGENERACY: a condition that occurs when the number of rows plus the number of columns minus 1 in a transportation table.

DESTINATION: The various customers which are supplied by the multiple facilities in the transportation method.

A location with a demand for material in a transportation problem.

DUMMY DESTINATION: An artificial destination added when total supply is greater than total demand. The demand at the dummy destination is set so that total supply and demand are equal.

DUMMY SOURCE: An artificial source added when total demand is greater than total supply. The supply at the dummy source is set so that total demand and supply are equal.

FACILITY LOCATION ANALYSIS: An application of the transportation method to help a firm decide where to locate a new factory or a warehouse.

IMPROVEMENT INTEX: The net cost of shipping one unit on a route not used in the current transportation problem solution.

OPPORTUNITY COST: The cost savings (if any) foregone by not using an empty cell of the transportation matrix.

ORIGINS: The multiple sites for facilities location which are analyzed by the transportation method.

MODIFIED DISTRIBUTION METHOD (MODI): Another algorithm for finding the optimal solution to a transportation problem. It can be used in place of the stepping stone method.

NORTH WEST CORNER METHOD: Systematic allocation to cells beginning in the upper left hand corner of the transportation table to obtain an initial feasible solution.

STEPPING STONE METHOD: An iterative technique for moving from an initial feasible solution to an optimal solution in transportation problems.

STEPPING STONE PATH: A series of adjustments in a feasible solution to a transportation problem that incorporates a new route and retains a feasible solution.

TRANSPORTATION PROBLEM: A specific case of linear programming concerned with scheduling shipments from sources to destinations so that total transportation costs are minimized.

TRANSPORTATION TABLEAU: A table used to display or summarize all transportation data to help keep track of all algorithm computations by storing information on demands , supplies, shipping costs , units shipped, origins and destinations.

TRANSPORTATION METHOD: A method for determining where to locate multiple facilities in order to minimize at the total cost of transportation.

TRANS-SHIPMENT PROBLEM: A transportation problem where shipment is possible from an origin to an origin or a destination as well as from a destination to an origin or a destination.

UNBALANCED TRANSPORTATION PROBLEM: A transportation problem where the total availability at the origins is different from the total requirement at the destinations.

VOGELS APPROXIMATION METHOD (VAM): An algorithm used to find a relatively efficient to initial feasible solution to a transportation problem.

SYMMETRICAL: Distance independent of the direction of travel.

ASYMMETRICAL: Distance varies with the direction of travel.

2.1. MATHEMATICAL FORMULATION OF TRANSPORTATION PROBLEM:

2.1.2. LP Formulation: The linear programming formulation in terms of the amounts shipped from the sources to the destinations, x_{ij} can be written as

Minimize (total cost) $Z= \sum \sum c_{ij}\, x_{ij}$ (total transportation cost)

Subject to the constraints

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$\sum x_{ij} \leq s_i$ for each source i (capacity constraints)

$\sum x_{ij} = d_j$ for each destination j (requirement constraints)

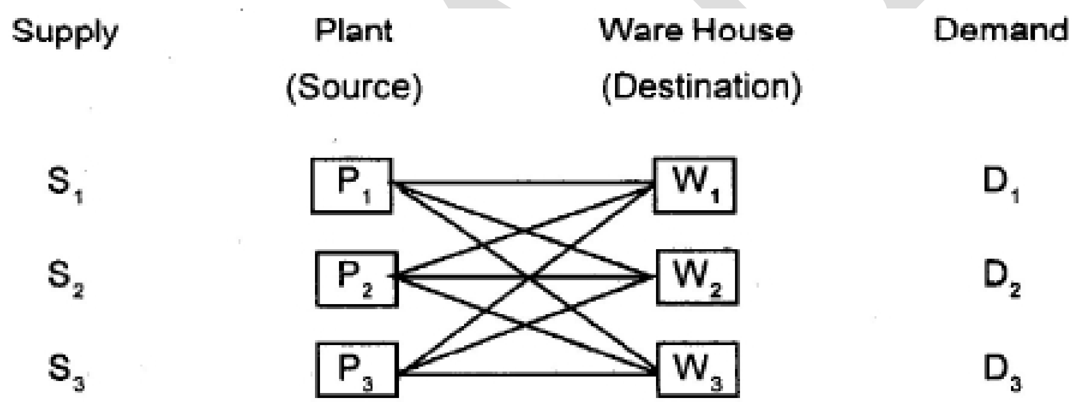
$x_{ij} \geq 0$ for all i and j (non-negativity constraints)

2.1.3. EXAMPLE

Transportation models deal can be formulated as a standard LP problem. Typical situation shown in the manufacturer example

- Manufacturer has three plants P_1, P_2, P_3 producing same products.
- From these plants, the product is transported to three warehouses W_1, W_2 and W_3.
- Each plant has a limited capacity, and each warehouse has specific demand. Each plant transport to each warehouse, but transportation cost vary for different combinations.

The problem is to determine the quantity each warehouse in order to minimize total transportation costs.



Solution Procedure for Transportation Problem:

- Conceptually, the transportation is similar to simplex method.
- Begin with an initial feasible solution.

This initial feasible solution may or may not be optimal. The only way you can find it out is to test it.

- If the solution is not optimal, it is revised and the test is repeated. Each iteration should bring you closer to the optimal solution.

2.2. FINDING AN INITIAL FEASIBLE SOLUTION:

There are a number of methods for generating an initial feasible solution for a transportation problem.

Consider three of the following

i) North West Corner Method

ii) Least Cost Method

(iii) Vogel's Approximation Method

2.2.1. NORTH WEST CORNER METHOD:

The simplest of the procedures used to generate an initial feasible solution is NWCM. It is so called because we begin with the North West or upper left corner cell of our transportation table. Various steps of this method can be summarized as under.

Step 1: Select the North West (upper left-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand requirement i.e., min (S_1, D_1).

Step 2: Adjust the supply and demand numbers in the respective rows And columns allocation.

Step 3: If the supply for the first row is exhausted, then move down to the first cell in t he second row and first column and go to step 2.

If the demand for the first column is satisfied, then move horizontally to the next cell in the second column and first row and go to step` 2.

Step 4 : If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column.

Step 5: Continue the procedure until the total available quantity id fully allocated to the cells as required.

PROBLEMS: Determine an initial basic feasible solution for the following Transportation Problem

		Ware houses W_1	W_2	W_3	Supply S_i
Plants	P_1	7	6	9	20
	P_2	5	7	3	28
	P_3	4	5	8	17
Demand	D_j	21	25	19	65

SOLUTION:

As stated in this method, we start with the cell (P_1 W_1) and Allocate the min (S_1, D_1) = min (20,21)=20. Therefore we allocate 20 Units this cell which completely exhausts the supply of Plant P_1 and leaves a balance of (21-20) =1 unit of demand at warehouse W_1.

Now, we move vertically downward to cell (P2 W1) At this stage the largest allocation possible is the (S_2, D_1 –20) =min (28, 1)= 1. This allocation of 1 unit to the cell (P1 W2). Since the demand of warehouse W2 is 25 units while supply available at plant P2 is 27 units, therefore, the min (27-5) = 25 units are located to the cell (P_1 W_2). The demand of warehouse W2 is now satisfied and a balance of (27-225) = 2 units of supply remain at plant P2. Moving again horizontally, we allocated two units to the cell (P2 W3) Which Completely exhaust the supply at plant P2 and leaves a balance of 17 units to this cell (P3, W3).. At this, 17 Units ae availbale at plant P3 and 17 units are required at warehouse W3. So we allocate 17 units to this cell (P3, W3). Hence we have made all the allocation. It may be noted here that there are 5 (+3+1) allocation which are necessary to proceed further. The initial feasible solution is shown below .

		WARE HOUSES W_1	W_2	W_3	SUPPLY S_i
	P_1	7 (20)	6	9	~~20~~ 0
	P_2	5 (1)	7 (25)	3 (2)	~~28~~ ~~27~~ ~~2~~ 0
PLANTS	P_3	4	5	8 (17)	17
	D_j	~~21~~ ~~1~~ 0	~~25~~ 0	~~19~~ ~~17~~ 0	65

The Total transportation cost for this initial solution is

Total Cost = 20x7 +1 X 5 + 25 x 7 + 2 X 3 + 17 + 8 = **Rs. 462**

2.2.2. LEAST COST METHOD:

The allocation according to this method is very useful as it takes into consideration the lowest cost and therefore, reduce the computation as well as the amount of time necessary to arrive at the optimal solution.

Step 1: Select the cell with the lowest transportation cost among all the rows or columns of the transportation table.

If the minimum cost is not unique, then select arbitrarily any cell with this minimum cost.

Step 2: Allocate as many units as possible to the cell determined in Step 1 and eliminate that row (column) in which either supply is exhausted or demand is satisfied.

Repeat Steps 1 and 2 for the reduced table until the entire supply at different plants is exhausted to satisfied the demand at different warehouses.

	Ware House			Supply
	W_1	W_2	W_3	S_i
P_1	7	6 (20)	9	20 0
P_2	5 (4)	7 (5)	3 (19)	~~28~~ ~~9~~ ~~5~~ 0
P_3	4 (17)	5	8	~~17~~ 0
Demand D_j	~~21~~ ~~4~~ 0	~~25~~ ~~5~~ 0	~~19~~ 0	65

Total cost = 20 x 6 + x 5 + 5 x 7 + 19 x 3 + 17 x 4 = Rs. 300

This total cost is less than the total cost obtained by NWCM.

2.2.3. VOGEL'S APPROXIMATION METHOD (VAM)

This method is preferred over the other two methods because the initial feasible solution obtained is either optimal or very close to the optimal solution.

Step 1: Compute a penalty for each row and column in the transportation table.

Step 2: Identify the row or column with the largest penalty.

Step 3: Repeat steps 1 and 2 for the reduced table until entire supply at plants are exhausted to satisfy the demand as different warehouses.

Warehouse

	W_1	W_2	W_3	S_i
P_1	$\boxed{7}$	$\boxed{6}$ 20	$\boxed{9}$	~~20~~ 0
P_2	$\boxed{5}$ 9	$\boxed{7}$	$\boxed{3}$ 19	~~28~~ ~~9~~ 0
P_3	$\boxed{4}$ 12	$\boxed{5}$ 5	$\boxed{8}$	~~17~~ ~~5~~ 0
D_j	~~21~~ ~~12~~ 0	~~25~~ ~~20~~ 0	~~19~~ 0	65

Column penalties

1	1	5
1	1	-
3	1	-

Row penalities

1	1	1
2	2	-
1	1	1

The total transportation cost associated with this method is

Total cost = 20 x 6 + 9 x 5 + 19 x 3 + 12 x 4 + 5 x 5 = Rs. 295.

PROBLEMS:

Determine an initial basic feasible solution by the following transportation problem by

1. Northwest corner method
2. Least cost method method

Warehouses

Factory	W 1	W 2	W 3	W4	Capacity (supply)
F1	21	16	25	13	11
F2	17	18	14	23	13
F3	32	27	18	41	19
Requirements (demand)	6	10	12	15	43

SOLUTION: INITIAL SOLUTION –NORTHWEST CORNER METHOD

Warehouses

Factory	W1	W2	W3	W4	Capacity (supply)
F1	21 **(6)**	16 **(5)**	25	13	11
F2	17	18 **(5)**	14 **(8)**	23	13
F3	32	27	18 **(4)**	41 **(15)**	19
Requirements (demand)	6	10	12	15	43

TOTAL COST=6(21)+5(16)+5(18)+8(14)+4(18)+15(41)=Rs.1055

INITIAL SOLUTION –LEAST COST METHOD

Warehouses

Factory	W1	W2	W3	W4	Capacity (supply)
F1	21	16	25	13 **(11)**	11
F2	17 **(1)**	18	14 **(12)**	23	13
F3	32	27	18	41	19

		(5)	**(10)**		**(4)**	
TOTAL	Requirements (demand)	6	10	12	15	43

Totalcost = 11(13)+1(17)+12(14)+5(32)+10(27)+4(41)=Rs.922

EXERCISES

1. Determine an initial basic feasible solution by the following transportation problem by

(i) Northwest corner method

(ii)Least cost method method

(iii)Vogel's approximation method

DESTINATION

SOURCES		D1	D2	D3	D4	SUPPLY
	A	11	13	17	14	250
	B	16	18	14	10	300
	C	21	24	13	10	400
	DEMAND	200	225	275	250	=950

ANSWER:

(i) $x_{11}=200$, $x_{12}=50$, $x_{22}=175$, $x_{23}=125$, $x_{33}=150$, $x_{34}=250$, Total cost=12200

(ii) $x_{11}=200$,$x_{12}=50$, $x_{22}=175$, $x_{23}=125$,$x_{33}=150$,$x_{34}=250$, Total cost=12200

(iii) $x_{11}=200$,$x_{12}=50$,$x_{22}=175$,$x_{23}=125$,$x_{33}=275$,$x_{34}=125$, Total cost=12075

2.3. FINDING THE OPTIMAL SOLUTION

Once an initial solution has been found, the next step is to test that solution for optimality. The following two methods are widely used for testing the solutions:

- **Stepping Stone Method**
- **Modified Distribution Method**

The two methods differ in their computational approach but give exactly the same results and use the same testing procedure.

2.3.1. STEPPING STONE METHOD:

Step 1: For each unoccupied cell, calculate the reduced cost by the MODI method described below. Select the unoccupied cell with the most negative reduced cost. (For maximization problems select the unoccupied cell with the largest reduced cost.) If none, STOP**.**

Step 2: For this unoccupied cell generate a stepping stone path by forming a closed loop with this cell and occupied cells by drawing connecting alternating horizontal and vertical lines between them.

Determine the minimum allocation where a subtraction is to be made along this path.

Step 3: Add this allocation to all cells where additions are to be made, and subtract this allocation to all cells where subtractions are to be made along the stepping stone path.

(**Note:** An occupied cell on the stepping stone path now becomes 0 (unoccupied). If more than one cell becomes 0, make only one unoccupied; make the others occupied with 0's.) GO TO STEP 1.

2.3.2. MODI Method (for obtaining reduced costs)

Associate a number, u_i, with each row and v_j with each column.

Step 1: Set $u_1 = 0$.

Step 2: Calculate the remaining u_i's and v_j's by solving the relationship $c_{ij} = u_i + v_j$ for occupied cells.

Step 3: For unoccupied cells (i,j), the reduced cost $= c_{ij} - u_i - v_j$.

EXAMPLE: Consider the following transportation problem solve this by the method of stepping stone

DESTINATIONS		D1	D2	D3	D4	CAPACITY

	F1	4	6	18	6	700
ORIGIN	F2	3	5	2	5	400
	F3	3	19	6	5	600
REQUIREMENTS		400	450	350	500	

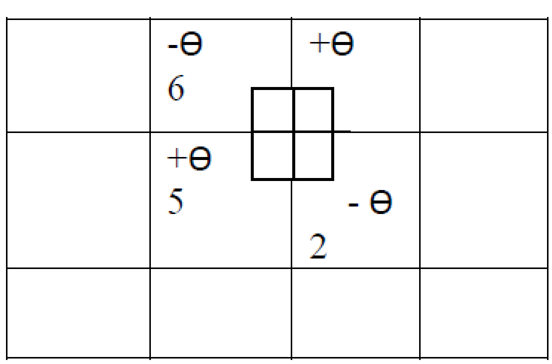
SOLUTION: Since ∑ai=∑bj=1700, problem is balanced, the IBFS got by NWC rule is

	D1	D2	D3	D4	CAPACITY
F1	4 **400**	6 **300**	18	6	700
F2	3	5 **150**	2 **250**	5	400
F3	3	19	6 **100**	5 **500**	600
	400	450	350	500	

Since we have m+n-1=6 allocation this problem is not degenerate.

Now let us check the change in cost if we give one unit allocation to empty cell in the position F1D3.

The net change in the total cost = +18-2+5-6 =15



Thus one unit allocation increases 15 units costs in the total cost. Then take F1D4 cell. Give one unit allocation

* 4	* -θ 6	18	+θ 6
3	* +θ 5	* -θ 2	5
3	19	* +θ 16	* -θ 5

Change in total cost =+6-5+6-2+5-6=4

Again this increases the total cost. the next empty cell is F2D1. Give allocation to F2D1.

* -θ 4	* -θ 6	18	+θ 6
-θ 3	* +θ 5	* 2	5
3	19	* 16	* 5

the change in total cost =+3-4+6-5=0

this makes no change in the total cost.so choose the next empty cell.

Give one unit allocation to the cellF2D4 change in

* 4	* 6	18	6
3	* 5	* -ϴ 2	+ϴ 5
3	19	* +ϴ 6	* -ϴ 5

Total cost=+5-5+6-2=4

The increases the total cost which is desirable. the next empty cell is F3D1. Allocation of one unit to F3D1 the change in the total cost = +3-6+2-5+6-4=-4.

* **400** -ϴ 4	* +ϴ 6	18	6
3	* **150** -ϴ 5	* +ϴ 2	5
+ϴ 3	19	* **100** -ϴ 6	* 5

Hence this allocation reduces the total cost. The maximum amount can be allocated F3D1. To find the maximum possible amount take the allocations in the cells with -ϴ, choose the minimum allocation. Here min(100,150,400)=100. The reallocated table is

4 **300**	6 **400**	18	6	700
3	5 **50**	2 **350**	5	400
3 **100**	19	6	5 **500**	600
400	450	350	500	

The next unallocated cell is F3D2.allocate one unit to F3D2. The change in the total cost=+19-3+4-6>0

4 * +ϴ	6 * -ϴ	18	6	700
3	5 *	2 *	5	400
3 * -ϴ	19 +ϴ	6	5 *	600
400	450	350	500	

This allocation increases the total cost. so we have checked all the empty cells. now the last table is the optimum one.

The optimum total cost=300x4+400x6+50x5+350x2+3x100+5x500=7350.

EXERCISES:

Consider the following transportation problem solve this by the method of stepping stone

		SINK		SUPPLY
SOURCE	1	2	0	30
	2	3	4	35

	1	5	6	45
DEMAND	30	40	30	

ANSWER: Total optimal cost=160 as ε→0.

2. Solve the transportation problem by MODI Method

	D1	D2	D3	D4	SUPPLY
S1	21	16	25	13	11
S2	17	18	14	23	13
S3	32	27	18	41	19
DEMAND	6	10	12	15	

SOLUTION:

Since $\sum a_i = \sum b_j$ =43, problem is balanced, the IBFS got by VAM rule is

	D1	D2	D3	D4	SUPPLY/PENEALTY
S1	21	16	25	**11** 13	11/(3)
S2	**6** 17	**3** 18	14	**4** 23	13/(3) (3) (3) (4)
S3	32	**7** 27	**12** 18	41	19/ (9) (9) (9) (9)
DEMAND/ PENEALTY	6 (4) (15)	10 (2) (9)	12 (4) (4)	15 (10) (18)	

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	(15) -	(9) (9)	(4) (4)	- -	

Initial Transportation Cost Is Rs.796

T0 FIND OPTIMUM SOLUTION BY MODI METHOD:

Since the number of basic cells =6=3+4-1.the solution is non degenerate

We calculate u_i and v_j using u_{i+} $v_{j=}$ c_{ij} basic cells. Here start with u2=0 as the second row has maximum number of allocations.

$$C_{21} = u_{2} + v_1 \quad 17 = 0 + v_1 \quad v_1=17$$

$$C_{22} = u_{2} + v_2 \quad 18 = 0 + v_2 \quad v_2=18$$

$$C_{24} = u_{2} + v_4 \quad 23= 0 + v_4 \quad v_4=23$$

$$C_{1\,4} = u_{1} + v_4 \quad 13 = u_1 + 23 \quad u_1=-10$$

$$C_{32} = u_{3} + v_2 \quad 27 = u_3+ 18 \quad u_3=9$$

$$C_{33} = u_{3} + v_3 \quad 18 = 9 + v_3 \quad v_3=9$$

We find the net evaluations dij = c_{ij} $_-$ $u_{i\,+}$ v_j for each non basic cells and enter at the upper right Horner of the cell

(14) 21	(8) 16	(26) 25	**11** 13	$u_{1=-10}$
6 17	**3** 18	(5) 14	**4** 23	$u_{2=0}$
(6) 32	**7** 27	**12** 18	(9) 41	$u_{3=9}$
$v_{1=17}$	$V_{2=18}$	$v_{3=9}$	$V_{4=23}$	

Since all dij>0 the solution under the test is optimal and unique.

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The optimum allocation schedule is given by

X_{14}=11, X_{21}=6, X_{22}=3, X_{24}=4 ,X_{32}=7, X_{33}=12 and minimum transportation cost =Rs.796.

EXERCISES:

2. Solve the transportation problem by MODI Method

	I	II	III	IV	SUPPLY
A	5	2	4	3	22
B	4	8	1	6	15
C	4	6	7	5	8
REQUIREMENTS	7	12	17	9	

ANSWER: The optimum allocation schedule is given by

X_{12}=12, X_{13}=2, X_{14}=8, $X_{23=}$15 ,X_{31}=7, X_{34}=1and minimum transportation cost = Rs.104.

2.4. UNBALANCED TRANSPORTATION PROBLEM:

To solve the transportation problem by its special purpose algorithm, the sum of the supplies at the origins must equal the sum of the demands at the destinations.

- If the total supply is greater than the total demand, a dummy destination is added with demand equal to the excess supply, and shipping costs from all origins are zero.
- Similarly, if total supply is less than total demand, a dummy origin is added.

- When solving a transportation problem by its special purpose algorithm, unacceptable shipping routes are given a cost of +M (a large number).

PROBLEMS:

Acme Block Company has orders for 80 tons of concrete blocks at three suburban locations as follows: Northwood -- 25 tons,Westwood -- 45 tons, and Eastwood -- 10 tons. Acme has two plants, each of which can produce 50 tons per week.Delivery cost per ton from each plant to each suburban location is shown below.How should end of week shipments be made to fill the above orders?

Delivery Cost Per Ton

	Northwood	Westwood	Eastwood
Plant 1	**24**	**30**	**40**
Plant 2	**30**	**40**	**42**

INITIAL TRANSPORTATION TABLEAU:

Since total supply = 100 and total demand = 80, a dummy destination is created with demand of 20 and 0 unit costs.

	Northwood	Westwood	Eastwood	Dummy	supply
Plant1	24	30	40	0	50
Plant2	30	40	42	0	50
demand	25	45	10	20	100

Least Cost Starting Procedure:

Iteration 1: Tie for least cost (0), arbitrarily select x_{14}. Allocate 20. Reduce s_1 by 20 to 30 and delete the Dummy column.

Iteration 2: Of the remaining cells the least cost is 24 for x_{11}. Allocate 25. Reduce s_1 by 25 to 5 and eliminate the Northwood column.

Iteration 3: Of the remaining cells the least cost is 30 for x_{12}. Allocate 5. Reduce the Westwood column to 40 and eliminate the Plant 1 row.

Iteration 4: Since there is only one row with two cells left, make the final allocations of 40 and 10 to x_{22} and x_{23}, respectively.

MODI METHOD

ITERATION 1

1. Set $u_1 = 0$

2. Since $u_1 + v_j = c_{1j}$ for occupied cells in row 1, then $v_1 = 24$, $v_2 = 30$, $v_4 = 0$.

3. Since $u_i + v_2 = c_{i2}$ for occupied cells in column 2, then $u_2 + 30 = 40$, hence

$u_2 = 10$.

4. Since $u_2 + v_j = c_{2j}$ for occupied cells in row 2, then $10 + v_3 = 42$, hence $v_3 = 32$.

Calculate the reduced costs (circled numbers on the next slide) by $c_{ij} - u_i + v_j$.

Unoccupied Cell	Reduced Cost
(1,3)	40 - 0 - 32 = 8
(2,1)	30 - 24 -10 = -4
(2,4)	0 - 10 - 0 = -10

	Northwood	Westwood	Eastwood	Dummy	ui
Plant1	24 **(25)**	30 **(8)**	40 **(8)**	0 **(20)**	0
Plant2	30 **(-4)**	40 **(40)**	42	0 **(-10)**	10
vj	24	32	30	0	100

ITERATION 2

The reduced costs are found by calculating the u_i's and v_j's for this tableau.

1. Set $u_1 = 0$.

2. Since $u_1 + v_j = c_{ij}$ for occupied cells in row 1, then $v_1 = 24$, $v_2 = 30$.

3. Since $u_i + v_2 = c_{i2}$ for occupied cells in column 2, then $u_2 + 30 = 40$, or $u_2 = 10$.

4. Since $u_2 + v_j = c_{2j}$ for occupied cells in row 2, then $10 + v_3 = 42$ or $v_3 = 32$; and, $10 + v_4 = 0$ or $v_4 = -10$.

Calculate the reduced costs (circled numbers on the next slide) by c_{ij} - u_i + v_j.

Unoccupied Cell	Reduced Cost
(1,3)	40 - 0 - 32 = 8
(1,4)	0 - 0 - (-10) = 10
(2,1)	30 - 10 - 24 = -4

	Northwood	Westwood	Eastwood	Dummy	ui
Plant1	**24** (25)	**30** (25)	**40** (8)	**0** (10)	**0**
Plant2	**30** (-4)	**40** (20)	**42** (10)	**20** (0)	**10**
vj	**24**	**30**	**36**	**-6**	**100**

ITERATION 3

The reduced costs are found by calculating the u_i's and v_j's for this tableau. 1. Set $u_1 = 0$

2. Since $u_1 + v_j = c_{1j}$ for occupied cells in row 1, then $v_1 = 24$ and $v_2 = 30$.

3. Since $u_i + v_1 = c_{i1}$ for occupied cells in column 2, then $u_2 + 24 = 30$ or $u_2 = 6$.

4. Since $u_2 + v_j = c_{2j}$ for occupied cells in row 2, then $6 + v_3 = 42$ or $v_3 = 36$, and $6 + v_4 = 0$ or $v_4 = -6$.

Calculate the reduced costs (circled numbers on thenext slide) by c_{ij} - u_i + v_j.

Unoccupied Cell	Reduced Cost
(1,3)	40 - 0 - 36 = 4
(1,4)	0 - 0 - (-6) = 6

(2,2) 40 - 6 - 30 = 4

	Northwood	Westwood	Eastwood	Dummy	ui
Plant1	**24** (25)	**30** (45)	**40** (+4)	**0** (+6)	**0**
Plant2	**30** (20)	**40** (+4)	**42** (10)	**20** (0)	**10**
vj	**24**	**30**	**36**	**-6**	**100**

Optimal Solution

From	To	Amount	Cost
Plant 1	Northwood	5	120
Plant 1	Westwood	45	1,350
Plant 2	Northwood	20	600
Plant 2	Eastwood	10	420

Total Cost = $2,490

Stepping Stone Method

Iteration 1

The stepping stone path for cell (2,4) is (2,4), (1,4), (1,2), (2,2). The allocations in the subtraction cells are 20 and 40, respectively. The minimum is 20, and hence reallocate 20 along this path. Thus for the next tableau:

x_{24} = 0 + 20 = 20 (0 is its current allocation)

x_{14} = 20 - 20 = 0 (blank for the next tableau)

x_{12} = 5 + 20 = 25

x_{22} = 40 - 20 = 20

The other occupied cells remain the same

Iteration 2

The most negative reduced cost is = -4 determined by x_{21}. The stepping stone path for this cell is (2,1),(1,1),(1,2),(2,2). The allocations in the subtraction cells are 25 and 20 respectively. Thus the new solution is obtained by reallocating 20 on the stepping stone path. Thus for the next tableau:

x_{21} = 0 + 20 = 20 (0 is its current allocation)

x_{11} = 25 - 20 = 5

x_{12} = 25 + 20 = 45

x_{22} = 20 - 20 = 0 (blank for the next tableau)

The other occupied cells remain the same.

Optimal Solution

From	To	Amount	Cost
Plant 1	Northwood	5	120
Plant 1	Westwood	45	1,350
Plant 2	Northwood	20	600
Plant 2	Eastwood	10	420

Total Cost = $2,490

2.5. MAXIMIZATION CASE IN TRANSPORTATION PROBLEMS:

So far we have discussed the transportation problems in which the objective has been to minimize the total transportation cost and algorithms have been designed accordingly.

If we have a transportation problem where the objective is to maximize the total profit ,first we have to convert the maximization problem into a minimization problem by multiplying all the entries by -1 (or) by subtracting all the entries from the highest entry in the

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given transportation table. The modified minimization problem can be solved in the usual manner.

EXAMPLE: Solve the **following** transportation problem to maximize profit

DESTINATION

SOURCES	A	B	C	D	SUPPLY
1	15	51	42	33	23
2	80	42	26	81	44
3	90	40	66	60	33
DEMAND	23	31	16	30	100

SOLUTION:

Since the given problem is of maximization type. We convert this into minimization problem by multiplying the profit cost C_{ij} by -1.

DESTINATION

SOURCES	A	B	C	D	SUPPLY
1	-15	-51	-42	-33	23
2	-80	-42	-26	-81	44
3	-90	-40	-66	-60	33
DEMAND	23	31	16	30	100

Since $\sum a_i = \sum b_j = 100$, there exists a basic feasible solution to this problem and is displayed in the following table by using VAM.

-15	-51 **23**	-42	-33
-80 **6**	-42 **8**	-26	-81 **30**
-90 **17**	-40	-66 **16**	-60

Since the number of non negative allocations at independent positions is (m+n-1) =6, we apply MODI method for optimal solution.

-15 -89 -4	-51 **23**	-42 -65 23	-33 -90 57	=-9
-80 **6**	-42 **8**	-26 -56 30	-81 **30**	0
-90 **17**	-40 -52 12	-66 **16**	-60 -91 31	-10

-80	-42	-56	-81	

Since all dij =0, the current solution is optimal and unique.

The optimum allocations are given by x_{12}=23,x_{21}=6,x22=8,x_{24}=30,x_{31}=17,x_{33}=16.

The optimum profit Rs.51x23+80x6+42x8+81x30+90x17+66x16=Rs.7005.

EXERCISES: Solve the following transportation problem to maximize profit

DESTINATION Profits(Rs)/unit

SOURCE	A	B	C	D	SUPPLY
1	40	25	22	33	100
2	44	35	30	30	30
3	38	38	28	30	70
DEMAND	40	20	60	30	200

ANSWER: The optimum allocations are given by
x_{11}=20,x_{14}=30,x_{22}=8,x_{15}=50,x_{21}=20,x_{23}=16,x_{32}=20,x_{33}=50.

The optimum profit Rs.40x20+33x30+0x50+44x20+30x10+38x20+28x50=Rs.5130.

2.6. DEGENERACY IN TRANSPORTATION PROBLEM:

In a transportation problem, whenever the number of non negative independent allocations is less than m+n-1, the transportation problem is said to be degenerate one. Degeneracy may occur either at the initial stage or at an intermediate stage at some subsequent iteration.

To resolve degeneracy, we allocate an extremely small amount (close to zero) to one or more empty cells of the transportation table (generally minimum cost cells if possible), so that total number of occupied cells becomes (m+n-1) at independent positions. We denote this small amount by ϵ (epsilon) satisfying the following conditions.

(i) $0 < \epsilon < x_{ij}$ for all $x_{ij} > o$

(ii) $X_{ij} \pm \epsilon = x_{ij}$ for all $x_{ij} > o$

The cells containing ϵ are than treated like other occupied cells and the problem is solved in the usual way. The ϵ,s are kept till the optimum solution is attained. Then we let each $\epsilon \rightarrow 0$.

EXAMPLE: Find the non-degenerate basic feasible solution for the following transportation problem using

(i) Northwest corner method

(ii)Least cost method method

(iii)Vogel's approximation method

10	20	5	7	SUPPLY
13	9	12	8	19
4	5	7	9	20
14	7	1	0	30
3	12	5	19	40
60	60	20	10	50

DEMAND

SOLUTION: Since $\sum a_i = \sum b_j = 150$, the given transportation problem is balanced'

There exists a basic feasible solution to this problem.

(i) The starting solution by NWC rule is shown in the following table.

10 **10**	20	5	7
13 **20**	9	12	8
4 **30**	5	7	9

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14	7 **40**	1	0
3	12 **20**	5 **20**	19 **10**

Since the number of non negative allocations at independent positions is 7 which is less than (m+n-1) = (5+4-1)=8, the basic feasible solution degenerate one.

To resolve this degeneracy , we allocate a very small quantity ϵ to the un occupied cell (5,1) so that the number of occupied cells becomes (m+n-1). Hence the non-degenerate basic feasible solution is shown in the following table.

10 **10**	20	5	7
13 **20**	9	12	8
4 **30**	5	7	9
14	7 **40**	1	0

3 ϵ	12 **20**	5 **20**	19 **10**

The initial transportation cost =Rs.10x10+13x20+4x30+7x40+3x ϵ+12x20+5x20+19x10

=Rs.(1290+3 ϵ)

=Rs.1290/-, as ϵ→0.

(ii)Least cost method method: using this method the starting solution is shown in the following table.

10	20 **10**	5	7
13	9 **20**	12	8
4 **10**	5 **20**	7	9
14	7 **10**	1 **20**	0 **10**
3 **50**	12	5	19

Since the number of non negative allocations at independent positions is (m+n-1) = (5+4-1)=8, the solution is basic non- degenerate basic feasible .

The initial transportation cost =Rs.20x10+9x20+4x10+5x20+7x 10+1x20+0x10+3x50

=Rs.760

(iii)Vogel's approximation method: the starting solution by this method as shown in the following table.

10 **10**	20	5	7
13	9 **20**	12	8
4	5 **30**	7	9
14	7 **10**	1 **20**	0 **10**
3 **50**	12	5	19

Since the number of non negative allocations at independent positions is 7 which is less than (m+n-1) = (5+4-1)=8, the basic feasible solution degenerate one.

To resolve this degeneracy , we allocate a very small quantity ϵ to the un occupied cell (5,2) so that the number of occupied cells becomes (m+n-1). Hence the non-degenerate basic feasible solution is shown in the following table.

10 **10**	20	5	7
13	9 **20**	12	8

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4	5 **30**	7	9
14	7 **10**	1 **20**	0 **10**
3 **50**	12 **ϵ**	5	19

The initial transportation cost =Rs.10x10+9x20+5x30+7x 10+1x20+0x10+3x50+12x ϵ

=Rs.(670+12 **ϵ)**

=Rs.670/-, as ϵ→0.

EXERCISES:

1. Solve the transportation problem using vogels method

WAREHOUSE

FACTORY	A	B	C	D	E	F	AVAILABLE
1	9	12	9	6	9	10	5
2	7	3	7	7	5	5	6
3	6	5	9	11	3	11	2
4	6	8	11	2	2	10	9
REQUIREMENT	4	4	6	2	4	2	

ANSWER: Rs.112/-, as ϵ→0.

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2. Solve the transportation problem

TO SUPPLY

FROM

1	2	3	4	6
4	3	2	0	8
0	2	2	1	10
4	6	8	6	

DEMAND

ANSWER: Rs.28/-, as $\epsilon \to 0$

2.7. MATHEMATICAL FORMULATION OF ASSIGNMENT MODELS

2.7.1. Linear Programming Formulation

Min $c_{ij}x_{ij}$
i j

Such that $x_{ij} = 1$ for each worker i
j
$x_{ij} = 1$ for each job j
i
$x_{ij} = 0$ or 1 for all i and j.

- Note: A modification to the right-hand side of the first constraint set can be made if a worker is permitted to work more than 1 job.

2.8. HUNGARIAN METHOD

- The Hungarian method solves minimization assignment problems with m workers and m jobs.

Special considerations can include:

- number of workers does not equal the number of jobs -- add dummy workers or jobs with 0 assignment costs as needed
- worker i cannot do job j -- assign $c_{ij} = +M$
- maximization objective -- create an opportunity loss matrix subtracting all profits for each job from the maximum profit for that job before beginning the Hungarian method

2.7.3. WORKING RULE FOR HUNGARIAN METHOD:

Step 1: For each row, subtract the minimum number in that row from all numbers in that row.

Step 2: For each column, subtract the minimum number in that column from all numbers in that column.

Step 3: Draw the minimum number of lines to cover all zeroes. If this number = m, STOP -- an assignment can be made.

Step 4: Determine the minimum uncovered number (call it d).

- Subtract d from uncovered numbers.
- Add d to numbers covered by two lines.
- Numbers covered by one line remain the same.
- Then, GO TO STEP 3.

Finding the Minimum Number of Lines and Determining the Optimal Solution

- Step 1: Find a row or column with only one unlined zero and circle it. (If all rows/columns have two or more unlined zeroes choose an arbitrary zero.)
- Step 2: If the circle is in a row with one zero, draw a line through its column. If the circle is in a column with one zero, draw a line through its row. One approach, when all rows and columns have two or more zeroes, is to draw a line through one with the most zeroes, breaking ties arbitrarily.

Step 3: Repeat step 2 until all circles are lined. If this minimum number of lines equals m, the circles provide the optimal assignment.

2.7.4. BALANCED METHOD PROBLEM:

➢ In the problem, the No of activities is exactly equal to the no.of.resources.

EXAMPLE:

A Company wishes to assign 3 jobs to 3 machines in such a way that each job is assigned to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix below(ij-th entry):

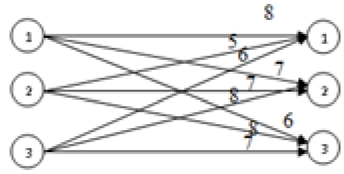
$$\text{Cost matrix: } \begin{pmatrix} 8 & 7 & 6 \\ 5 & 7 & 8 \\ 6 & 8 & 7 \end{pmatrix}$$

Draw the associated network. Formulate the network LPP and find the minimum cost of making the assignment.

SOLUTION:

(a) Network formulation of the given problem:

Job Machine



(b) Linear programming formulation:

Min z = $(8x_{11} + 7x_{12} + 6x_{13}) + (5x_{21} + 7x_{22} + 8x_{23}) + (6x_{31} + 8x_{32} + 7x_{33})$

Subject to the constraints:

$X_{i1} + X_{i2} + X_{i3} = 1;$ $\quad i = 1, 2, 3$

$X_{1j} + X_{2j} + X_{3j} = 1;$ $\quad j = 1, 2, 3$

$X_{ij} = 0$ or 1, for all i and j.

(c) Reduce the cost matrix by subtracting smallest element of each row (or) column from the corresponding row (or) column elements. In the reduced matrix, make assignments in rows (or) columns having single zeros.

Initial Iteration: Draw the minimum no.of.lines to cover all the zeros of the reduced matrix.

Final Iteration:

$$\begin{bmatrix} 2 & 1 & \boxed{0} \\ \boxed{0} & 2 & 3 \\ \cancel{0} & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & \boxed{0} \\ \boxed{0} & 1 & 2 \\ \cancel{0} & 1 & \cancel{0} \end{bmatrix}$$

$$\begin{bmatrix} 4 & \cancel{0} & \boxed{0} \\ \cancel{0} & \boxed{0} & 1 \\ \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & \boxed{0} \\ \boxed{0} & \cancel{0} & 1 \\ \cancel{0} & \boxed{0} & \cancel{0} \end{bmatrix}$$

$$\begin{array}{ccc} 4 & \boxed{0} & \cancel{0} \\ \boxed{0} & \cancel{0} & 1 \\ \cancel{0} & \cancel{0} & \boxed{0} \end{array}$$

Optimum assignment schedule:

Job 1 → Machine 3, Job 2 → Machine 2, Job 3 → Machine 3,

Job 1 → Machine 3, Job 2 → Machine 1, Job 3 → Machine 3,

Job 1 → Machine 2, Job 2 → Machine 1, Job 3 → Machine 3.

Total minimum cost in both the cases will be 19.

2.7.5. EXERCISE:

1. A construction company has four large bulldozers located at four different garages.

The bulldozers are to be moved to four different construction sites. The distances in miles

The bulldozers and the construction sites are given below.

BULLDOZERS\ SITE	A	B	C	D
1	90	75	75	80
2	75	85	55	65
3	125	95	90	105
4	45	110	95	115

How the bulldozers should be moved to the construction sites in order to minimize the total

Distance traveled?

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2. A Department head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below:

MEN

TASKS	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

2.7.6. UNBALANCED METHOD PROBLEM:

- In the problem, the Number of activities is not equal to the number of resources.

2.7.7. Example:
A contractor pays his subcontractors a fixed fee plus mileage for work performed. On a given day the contractor is faced with three electrical jobs associated with various projects. Given below are the distances between the subcontractors and the projects.

		Project		
		A	B	C
	Westside	50	36	16
Subcontractors:	Federated	28	30	18
	Goliath	35	32	20
	Universal	25	25	14

How should the contractors be assigned to minimize total distance (and total cost)?

Solution:

- Objective Function:

  Minimize total distance:

  $$\text{Min} = 50x_{11} + 36x_{12} + 16x_{13} + 28x_{21} + 30x_{22} + 18x_{23} + 35x_{31} + 32x_{32} + 20x_{33} + 25x_{41} + 25x_{42} + 14x_{43}$$

- Constraints:

  $x_{11} + x_{12} + x_{13} \leq 1$ (no more than one

  $x_{21} + x_{22} + x_{23} \leq 1$ project assigned

  $x_{31} + x_{32} + x_{33} \leq 1$ to any one

  $x_{41} + x_{42} + x_{43} \leq 1$ subcontractor)

  $x_{11} + x_{21} + x_{31} + x_{41} = 1$ (each project must

  $x_{12} + x_{22} + x_{32} + x_{42} = 1$ be assigned to just

  $x_{13} + x_{23} + x_{33} + x_{43} = 1$ one subcontractor)

  all $x_{ij} \geq 0$ (non-negativity)

- Optimal Assignment

Subcontractor	Project	Distance
Westside	C	16
Federated	A	28
Universal	B	25
Goliath	(unassigned)	

Total Distance = 69 miles.

2.7.8. Exercise

1. A Company has 4 machines on which to do 3 jobs. Each job can be assigned to one and only machine. The cost of each job on each machine is given in the following table:

Typist	Rate per hour (Rs.)	No.of.pages typed per hr.
A	5	12
B	6	14
C	3	8
D	4	10
E	4	11

2. A Manufacturer of garments plans to add four regional warehouses to meet increased demand. The following bids in lakhs of rupees have been for construction of the warehouses

Contractor	A	B	C	D
1	30	27	31	39
2	28	18	28	37
3	33	17	29	41
4	27	18	30	43
5	40	20	27	36

Explain why each warehouses contract cannot simply be given to the contractor who bids cheapest. How will you determine the optimal contract?

2.8. VARIANTS OF THE ASSIGNMENT PROBLEM:

2.8.1. Concept and Formulation:

Let k be the level number in the branching tree (for root node, it is 0), σ be an assignment make in the current node of a branching tree. $P^{K}{}_{\sigma}$ be an assignment at level k of the branching tree. A be the set of assigned cells (partial assignment) up to the node $P^{K}{}_{\sigma}$ from the root node (set of i

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and j) values with respect to the assigned cells up to the node P^{K}_{σ} from the root node, and V_0 be the lower bound of the partial assignment, A up to P^{K}_{σ}, such that,

$$V_0 = \underset{i,j\epsilon A}{C_{ij}} + \underset{i\epsilon x}{\ } \underset{j\epsilon y}{\min c_{ij}} \,)$$

where C_{ij} the cell entry of the cost matrix with respect to the ith row and jth column, X be the set of rows which are not deleted up to the node P^{K}_{σ} from the root node in the branching tree, and Y be the set of columns which are not deleted up to P^{K}_{σ}, from the root node in the branching tree.

2.8.2. Branching Guidelines:

1. At level k, the row marked as k of the assignment problem, will be assigned with the best column of the assignment problem.
2. If there is a tie on the lower bound, then the terminal node at the lower-most level is to be considered for the further branching.
3. Stopping rule: if the minimum lower bound happens to be at any one of the terminal notes at the (n-1) level, the optimality is reached. Then assignments on the path from the root node to the node along with the missing pair of row-column will form the optimum solution.

2.8.3. Example:

1. Solve the assignment problem using branch and bound algorithm. Cell entries represent the processing time in hours (C_{ij}) of the job i if it's assigned to the operator j.

Solution:

Initially, no job is assigned to any operator. so, the assignment (σ) at the root node (level 0) of the branching tree is a null set and the corresponding lower bound V_{σ} is also 0,as shown

Job i

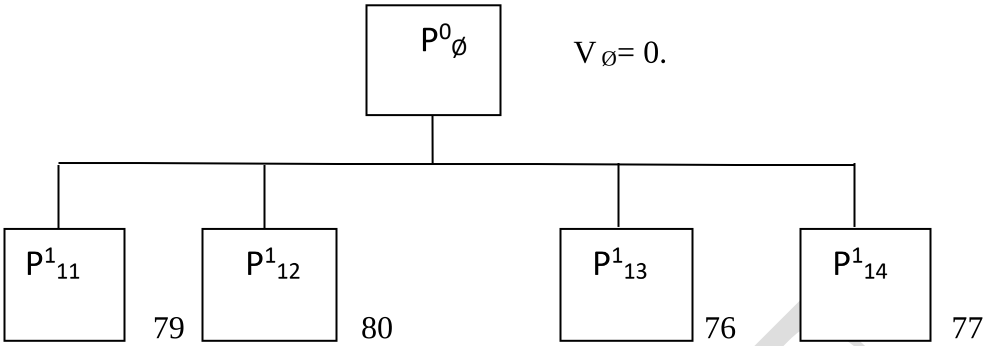
$\boxed{P^{0}_{\emptyset}}$

$\sigma = \emptyset$

$V_{\emptyset} = 0.$

(fig(a)- Branching tree at the root node)

Further branching: The four different sub-problems under the root node are shown by the fig:2. The lower bound for each of the sub-problem is shown on its right hand side.



(Tree with lower bounds after branching from ($P^0_\emptyset$)-fig(b)

Sample calculations to compute the lower bound for the first and the third sub-problems below.

Lower bound for P^1_{11}

$$V_0 = \underset{i,j\epsilon A}{C_{ij}} + \underset{i\epsilon x}{\min}\ \underset{j\epsilon y}{c_{ij}}\ )$$

where $\sigma = \{(11)\}$, $A = \{(11)\}$, $X = \{2,3,4\}$, $Y = \{2,3,4\}$

$$V_{(11)} = C_{11} + \underset{i\epsilon(2,3,4)}{\min}\ \underset{j\epsilon(2,3,4)}{c_{ij}}\ ) = 23+(20+18+18\ ) = 79.$$

Lower bound for P^1_{13}

$\sigma = \{(13)\}$, $A = \{(13)\}$, $X = \{2,3,4\}$, $Y = \{1,2,4\}$

Then

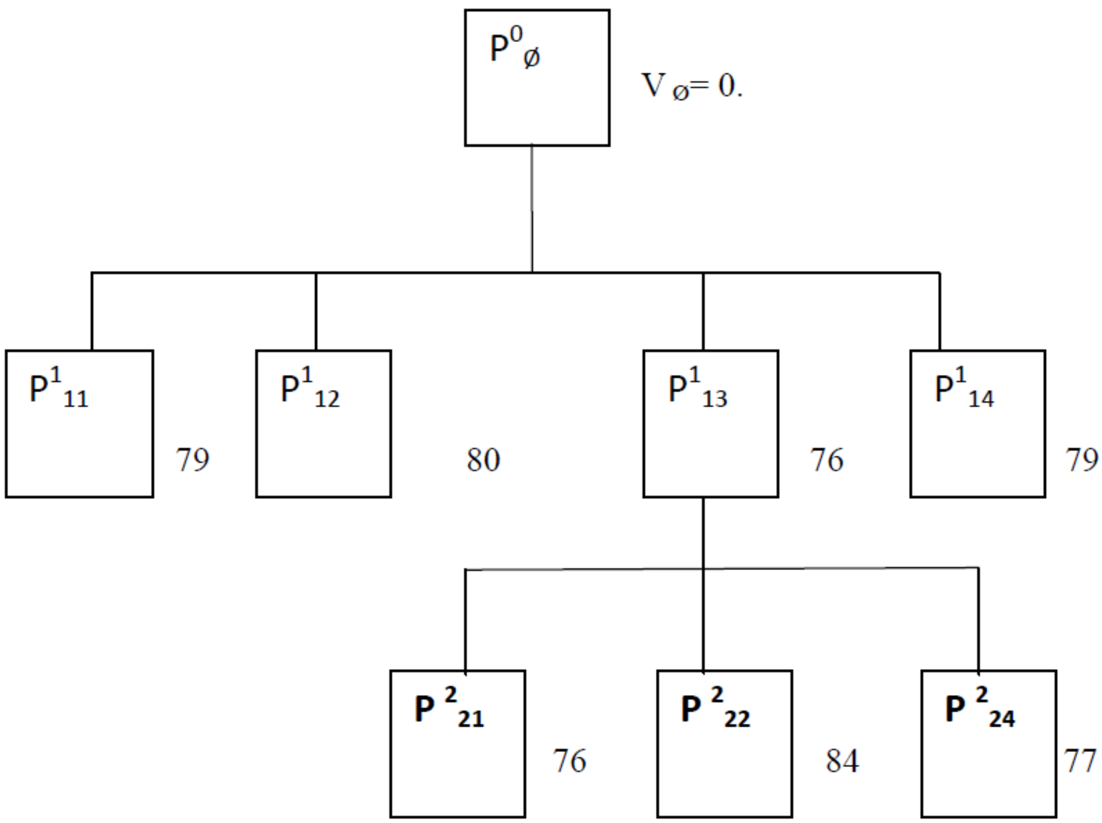
$$V_{(13)} = C_{13} + \underset{i\epsilon(2,3,4)}{\min}\ \underset{j\epsilon(1,2,4)}{c_{ij}}\ ) = 21+(19+18+18\ ) = 76.$$

Further branching: Further branching is done from the terminal node which has the least lower bound. At this stage, the nodes P^1_{11}, P^1_{12}, P^1_{13}, and P^1_{14} are the terminal nodes. Among these nodes, P^1_{13} has the least –lower bound. Hence, further branching from this node is shown as in the figure(c). the lower bound of each of the newly created nodes is shown by the side of it. As an example, the calculation pertaining to the lower bound of the node P^2_{22} is presented below.

$\sigma = \{(22)\}$, $A = \{(13), (22)\}$, $X = \{3, 4\}$, $Y = \{1,4\}$

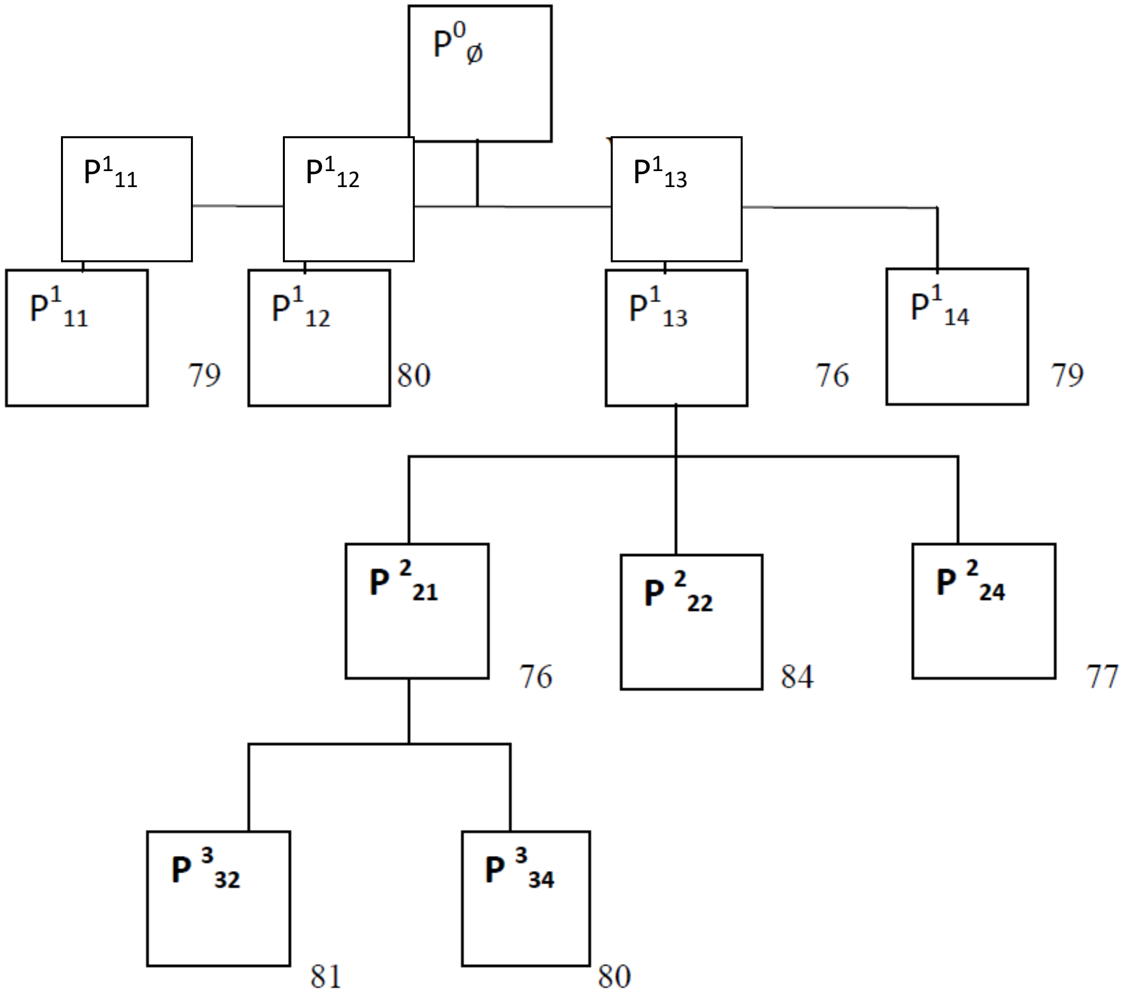
Then

$$V_{(22)} = C_{13} + C_{22} + \min_{i\epsilon(3,4)\; j\epsilon(1,4)} c_{ij}) = 21+21+ (20+22) = 84.$$



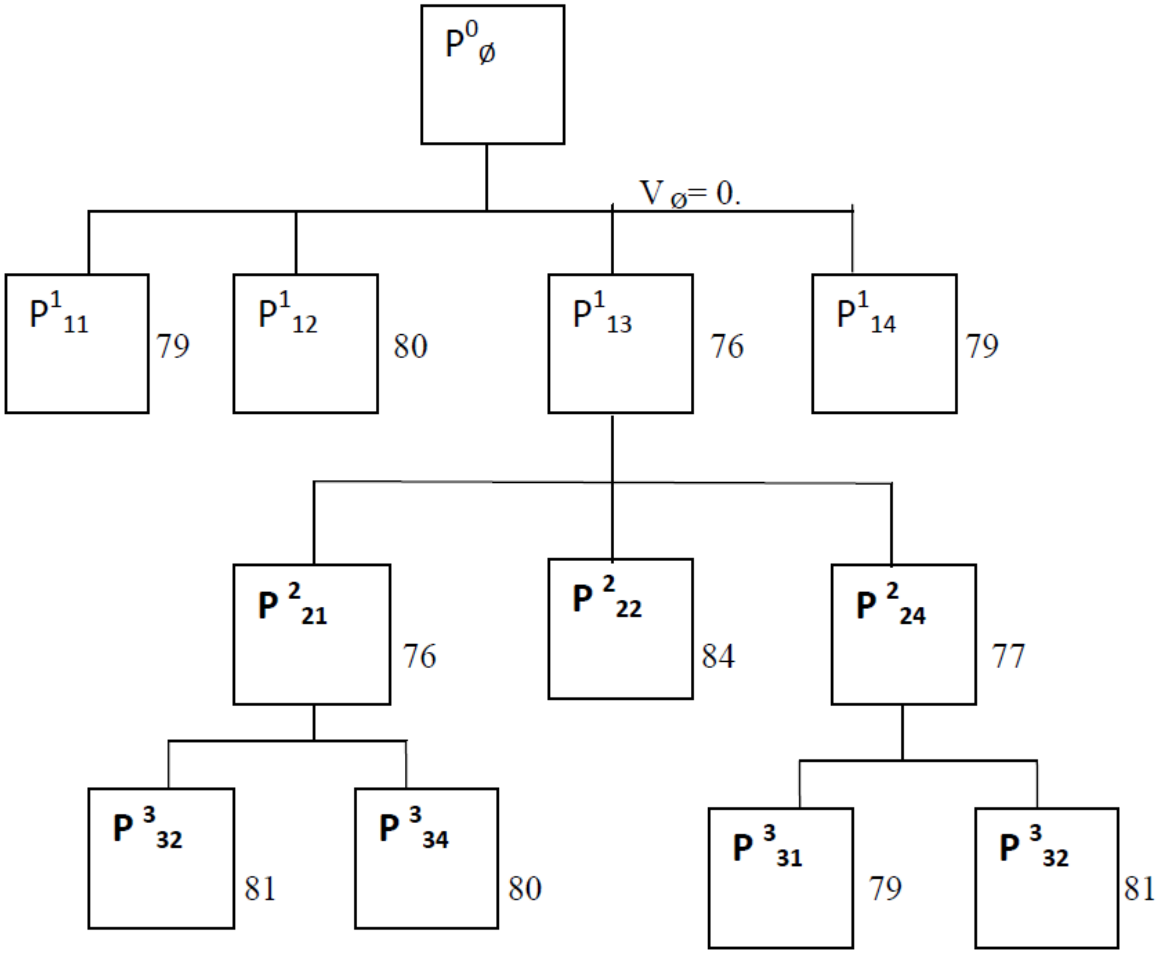
(Tree with lower bounds after branching from P^1_{13} – fig ©)

Further branching: At this stage, the nodes P^1_{11}, P^1_{12}, P^2_{21}, P^2_{22}, P^2_{24} and P^1_{14} are the terminal nodes. Among these nodes, P^2_{21} has the least –lower bound. Hence, further branching from this node is shown as in the figure (d). The lower bound of each of the newly created nodes is shown by the side of it.



(Tree with lower bounds after branching from P^2_{21}– fig (d))

Further branching: At this stage, the nodes P^1_{11}, P^1_{12}, P^3_{32}, P^3_{34}, P^2_{22}, P^3_{31}, P^3_{32} and P^1_{14} are the terminal nodes. Among these nodes, there are 3 nodes with the least lower bound of 79. So, the node P^3_{31} which is at the bottom-most level is considered for further branching tree. Since this node lies at (n-1) th level (k=3) of the branching tree, where n is the size of the problem, optimality is reached.



(Tree with lower bounds after branching from P^2_{24} – fig (e))

The corresponding solution is traced from the root node to the node P^3_{31} along with the missing pair of job and operator combination, (4,2) shown in the following table.

JOB	OPERATOR	TIME (in hours)
1	3	21
2	4	20
3	1	20

4	2	18

Hence total time = 79 hours.

2.8.4. TRAVELLING SALESMAN PROBLEM: (Shortest cyclic route models)

The problem is to find the routes shortest in distance (or time or cost) for salesman.

2.8.5. Example:

1. Salesman wants to visit cities 1, 2, 3 and 4. He does not want to visit any city twice before completing the tour of all the cities and wishes to return to his home city, the starting station. Cost of going from one city to another in rupees is given in table. Find the least cost route.

TO CITY

FROM CITY

1	0	30	80	50
2	40	0	140	30
3	40	50	0	20
	70	80	130	0

Solution:

If the optimum assignment table also satisfies the additional constraint that no city is to be visited twice before completing the tour of all the cities, it is also the optimal solution to the given travelling salesman problem. if it is not, it can be adjusted.

As going from the city 1 → 1, 2 → 2, etc.is not allowed, assign a large penalty $C_{ii} = \infty$ to these cells in the table.

	1	2	3	4
1	∞	0	50	20
2	10	∞	110	0
3	20	30	∞	0
	0	10	60	∞

→

	1	2	3	4
1	∞	0	0	20
2	10	∞	60	0
3	20	30	∞	0
	0	10	10	∞

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4

(Reduced matrix with zero in every row)

	1	2	3	4
1	∞	$\boxed{0}$	$\cancel{0}$	$\boxed{0}$
2	10	∞	60	$\times$
3	$\boxed{0}$	30	∞	0
4	0	10	10	∞

4

(Reduced matrix with zero in every row and column)

	1	2	3	4
	∞	$\boxed{0}$	$\cancel{0}$	30
	$\cancel{0}$	∞	50	$\cancel{0}$
	$\boxed{0}$	20	∞	$\boxed{0}$
	0	10	10	∞

(Second basic feasible solution)

1	∞	$\cancel{0}$	$\boxed{0}$	40
2	$\boxed{0}$	∞	40	$\cancel{0}$
3	10	10	∞	0
4	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	∞

1	∞	0	0	40
2	0	∞	40	0
3	10	10	∞	0
4	0	0	0	∞

(optimum solution)

The salesman should visit the city 1 → 3 → 4 → 2 → 1 (least cost route)

Cost is Rs.(80 + 20 + 80 + 40) = Rs.220

2.8.6. Exercise:

1. A Salesman wants to visit cities A, B, C, D and E. He does want to visit any city twice before completing his tour of all the cities and wish to return to the point of starting journey. Cost of going from one city to another (in rupees) is shown in the below table. Find the least cost route.

	A	B	C	D	E
A	0	2	5	7	1
B	6	0	3	8	2
C	8	7	0	4	7
D	12	4	6	0	5
E	1	3	2	8	0

2. What is the Travelling salesman problem? Which situations can be treated as the Travelling salesman problem? How does its solution differ from the solution of the assignment problem?

3. What are its practical applications? Describe the role of branch and bounding to solve the Travelling salesman model?

UNIT-III

SYLLABUS

Network Analysis – Construction of networks, Components and Precedence relationships – Event – activities – rules of network construction, errors and dummies in network. PERT/CPM networks –project scheduling with uncertain activity times – Critical Path Analysis – Forward Pass method, Backward Pass method – Float (or slack) of an activity and event –Time – cost trade-offs – crashing activity times.

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SCHEDULING BY PERT AND CPM

CONCEPT :

A project define a combination of inter-related activities which must be executed in a certain order to achieve a set goal before the entire task is completed.

APPLICATIONS

To plan, schedule, monitor and control projects such as :

1. Construction of buildings, bridges , factories, highways , stadiums, irrigation projects etc.,
2. Budget and auditing procedures.
3. Missile development programmes.
4. Installation of a complex new equipment such as computers or large machinery.
5. Advertising programmes and for development and launching of new products.
6. Planning of political campaigns.
7. Strategic and tactical military planning.
8. Research and development of new products.
9. Finding the best traffic flow patterns in a large city.
10. Maintenance and overhauling complicated equipment in the chemical power plants.
11. Long-range planning and developing staffing plans.
12. Organization of big conferences, public works, etc.
13. Shifting of manufacturing plant from one site to another.
14. Preparation of bids and proposals for projects of large size.
15. Launching space programmes.

TECHNICAL TERMS

ACTIVITY

An activity is a part of the project denoted by an arrow on the network. The tail of the arrow indicates the start of the activity whereas the head indicates the end of the activity. One and only one arrow is used to represent one activity of given duration. The arrows of the activities are not drawn to scale. The durations of the activities are written along their arrows.

DUMMY ACTIVITY

The activity which neither uses any resources nor any time for its completion is called dummy activity. It is represented by a dotted arrow or a solid arrow with zero time duration.

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EVENT

Event is the stage or point where all previous jobs merging in it are completed and the jobs bursting out are still to be completed. Event 1 is the starting point of the project.

See the figure shown below, where, event 6 is the point where activities B(3), F(3), H(2) and K(5) merge and activity L(4) bursts.

Events are generally represented by circles or nodes at the junctions of arrows. Events are serially numbered in their sequential order.

NETWORK

The diagrammatic representation of the activities of the entire project is called network of flow diagram. On this diagram various jobs of the project are shown in the order in which these are required to be performed.

EARLY START TIME (E.S.T.)

The earliest possible time at which an activity may start, is called early start time.

EARLY FINISH TIME (E.F.T.)

The sum of the earliest start time of an activity and the time required for its completion is called early finish time.

LATE START TIME (L.S.T.)

The latest possible time at which an activity may start without delaying the date of the project, is called late start time.

LATE FINISH TIME (L.F.T.)

The sum of the late start time of an activity and the time required for its completion is called late finish time.

TOTAL FLOAT

The difference between the maximum time allowed for an activity and its estimated duration is called total float. It is the duration of time by which the activity can be started late, without disturbing the project schedule. It is generally denoted by **S**.

FREE FLOAT

The duration of time by which the completion time of an activity can be delayed without affecting the start of succeeding activities is called free float. It is generally denoted by **S.F**.

CRITICAL ACTIVITIES

The event which has no float, are called critical activities. The critical events are required to be completed on schedule.

CRITICAL PATH

The path in the network joining the critical events is called the critical path of the work.

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4.1 Network Construction

A network is a graphical representation of the project activities arranged in a logical sequence and depicting all the interrelationships among them. A network consists of activities and events.

Rules for drawing network diagram

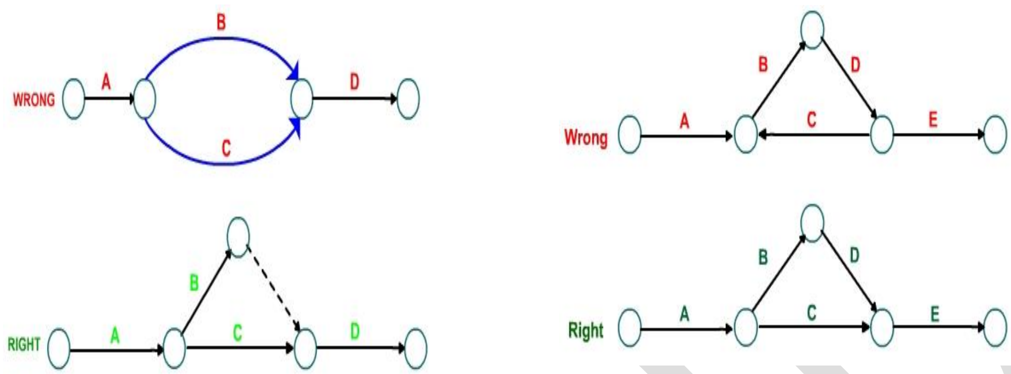
Rule 1: Each activity is represented by one and only one arrow in the network.

Rule 2: No two activities can be identified by the same end events.

Rule 3: Precedence relationships among all activities must always be maintained.

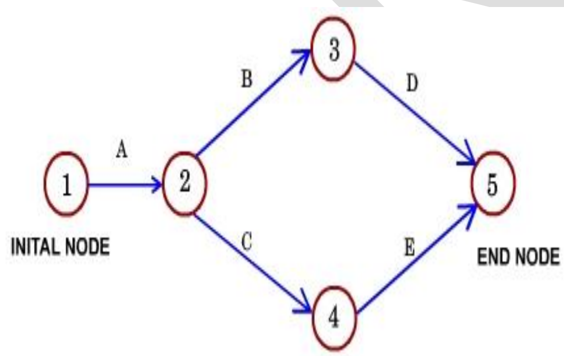
Rule 4: Dummy activities can be used to maintain precedence relationships only when actually required. Their use should be minimized in the network diagram.

Rule 5: Looping among the activities must be avoided



Rules for numbering the events is the no event can be numbered until all preceding events have been numbered. The number at the head of an arrow is always larger than that at its tail, i.e., events should be numbered such that for every (I,j) i<j. In order to conform this rule , the number of the events was done by a procedure given by Flukerson:

- Number of initial node with 1.
- Delete all arrows leaving the nodes which have been numbered
- Continue the numbering by identifying all nodes with no incoming arrows and by assigning them consecutive numbers in any order.
- Repeat (i) and (ii) until the terminal node is numbered.



Activity	Immediate Predecessor
A	–
B	A
C	A
D	B
E	C

Activity	Immediate Successor
A	B,C
B	D
C	E

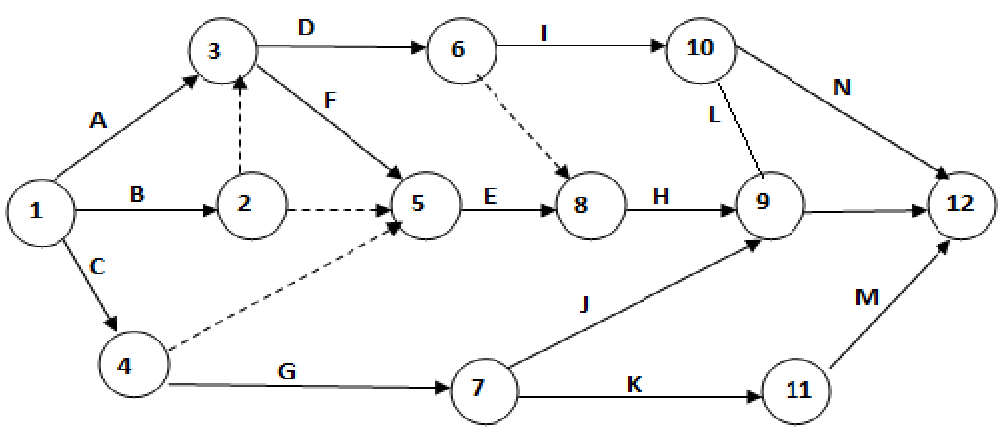
Example 3.4.1:

Draw an arrow diagram showing the following relationships:

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Immediate prede.	-	-	-	A,B	B,C	A,B	C	D,E,F	D	G	G	H,J	K	I,L

Solution:

The use of dummies may carefully be noted here. In particular , the dummy activity 2 – 5 is necessary here because f it is eliminated, node 5 becomes the ending node of activity b and the initial node of



activity E, implying that D and F require all A,B and C to be completed before their start , which is not the case. Inclusion o this activity thus enables us to present the precedence relationships in a correct manner.

CPM and PERT

The CPM (critical path method) system of networking is used, when the activity time estimates are deterministic in nature. For each activity, a single value of time, required for its execution, is estimated. Time estimates can easily be converted into cost data in this technique. CPM is an activity oriented technique.

The longest path through the network is called the critical path and its length determines the minimum

duration in which the said project can be completed.

The PERT (Project Evaluation and Review Technique) technique is used, when activity time estimates are stochastic in nature. For each activity, three values of time (optimistic, most likely, pessimistic) are estimated.

- **Optimistic time** (t_o) estimate is the shortest possible time required for the completion of activity.
- **Most likely time** (t_m) estimate is the time required for the completion of activity under normal circumstances.
- **Pessimistic time** (t_p) estimate is the longest possible time required for the completion of activity.

In PERT β-distribution is used to represent these three time estimates. As PERT activities are full of uncertainties, times estimates cannot easily be converted in to cost data.
PERT is an event oriented technique.
In PERT expected time of an activity is determined by using the below given formula:

$$t_e = \frac{(t_o + 4t_m + t_p)}{6}$$

The variance of an activity can be calculated as:

$$\sigma^2 = \left[\frac{(t_p - t_o)}{6}\right]^2$$

The basic objective of the time analysis is to get a planned schedule of the project for which the following factors should be known:

(i) Total completion time of the project.
(ii) Earliest time when each activity can begin.
(iii) Latest time when each activity can be started, without delaying the total project.
(iv) Float for each activity, i.e., amount of tie by which the completion of an activity can be delayed without delaying the total project completion time.
(v) Identification of critical activities and critical path.

Let us define some notations for the purpose of computing various times of events and activities.

(i,j) = Activity (i,j)with tail end event number I and head end event number j

E_{ij} = Earliest occurrence time event , i. It is the earliest time at which an event can occur without affecting the total project time.

L_i = Latest occurrence time of event, I, It is the latest time at which event can occur without affecting the total project time.

t_{ij} = Duration of activity (I , j).

ES_{ij} = Earliest start time for activity (I , j). It is the time at which the activity can start without affecting the total project time.

LS_{ij} = Latest start time for activity (I ,j). It is the earliest possible time by which an activity must start without affecting the total project time.

EF_{ij} = Earliest finish time for activity (I ,j) . It is the earliest possible time at which an activity can finish without affecting the total project time.

LF_{ij} = Latest finish time for activity (I ,j) . It is the latest time by which an activity must get completed without delaying the project completion.

4.2. CPM (critical path method)

4.2.1. Forward Pass Method (For Earliest Event Time)

In this method, calculations begin from the initial event 1, proceed through the events in an increasing order of event numbers and end at the final event, say N. At each event we calculate its earliest occurrence time (E) and earliest start and finish time for each activity that begins at that event. When calculations end at the final event N , its earliest occurrence time given the earliest possible completion time of the entire project. The method may be summarized as follows:

1. Set the earliest occurrence time of initial event 1 to zero. That is, $E_1 = 0$; i=1
2. Calculate earliest start time for eah activity that begins at event i(= 1). This is equal to earliest occurrence time of event, i(tail event). That is,

   $ES_{ij} = E_{i,}$, for all activities (I ,j) starting at event i.
3. Calculate the earliest finish time of each activity that begins at event i. This is equal to earliest occurrence of the activity plu7s the duration of the activity. That is,

$EF_{ij} = ES_{ij} + t_{ij} = E_i + t_{ij}$ for all activities (I ,j) beginning at event i.

4. Proceed to th next event, say j , j>1
5. Calculate the earliest occurrence time for the event j. This is the maximum of the earliest finish times of an activities ending into that event, that is,

   $E_j = \text{Max}\ \{EF_{ij}\} = \text{Max}\ \{E_i + t_{ij}\}$, for all immediate predecessor activities.
6. If j = N (final event), then earliest finish time for the project, that is , the earliest occurrence time E_N for the final event is given by

   $E_N = \text{Max}\ \{ EF_{ij}\} = \text{Max}\{E_{N-1}+t_{ij}\}$, for all terminal activities.

4.2.2. Backward Pass Method (For Latest Allowable Event Time)

In this method calculations begin from final event N. Proceed through the events in the decreasing order of event numbers and end at the initial event 1. At each event, we calculate its latest occurrence time (L) and latest finish and start time for each activity that is terminating at that event and the procedure continue till the initial event. The method may be summarized as follows:

1. Set the latest occurrence of last event, N equal to its earliest occurrence time Known from forward pass method)
2. Calculate latest start times of all activities ending at j. It is obtained by subtracting the duration of the activity from the latest finish time of the activity. That is,

   $LF_{ij} = L_i$; for all activities (I ,j) ending at event j.
3. Calculate the latest start times of all activities ending at j. It is obtained by subtracting the duration o the activity from the latest finish time of the activity. That is ,

   $LF_{ij} = L_j$ and $LS_{ij} = LF_{ij} - t_{ij}$, for all activity (i,j) ending at event j.
4. Proceed backward to the event in the sequence that decreases j by 1.
5. Calculate the latest occurrence time of event I (I < j). This is the minimum of the latest start times of all activities from the event. That is,

   $L_i = \text{Min}\ \{LS_{ij}\} = \text{Min}\ \{L_j - t_{ij}\}$, for all immediate successor activities.
6. If j=1 (initial event), then the latest finish time for project, i.e., latest occurrence time L_1 for the initial event is given by

   $L_1 = \text{Min}\ \{ LS_{ij}\} = \text{Min}\ \{L_{j-1} - t_{ij}\}$

4.2.3. Example

An established company has decided to add a new product to its line. It will buy the product from a manufacturing concern, package it, and sell it to a number of distributors selected on a geographical basis. Market research has indicated the column expected and the size of sales force required. The steps shown in the following table are to be planned.

Activity	Description	Predecessors	Duration
A	Organize sales office	--	6
B	Hire salesmen	A	4
C	Train Salesmen	B	7
D	Select advertising agency	A	2
E	Plan advertising agency	D	4
F	Conduct advertising campaign	E	10
G	Design package	--	2
H	Set-up packaging facilities	G	10
I	Package initial stocks	J, H	6
J	Order stock from manufacturer	--	13
K	Select distributors	A	9
L	Sell to distributors	C, K	3
M	Ship stocks to distributors	I, L	5

a) Draw an arrow diagram for this project and find the critical path.
b) For each non-critical activity, find the total and free float.

Solution :

a) The arrow diagram for the given project along with E-values and L-values are as follows

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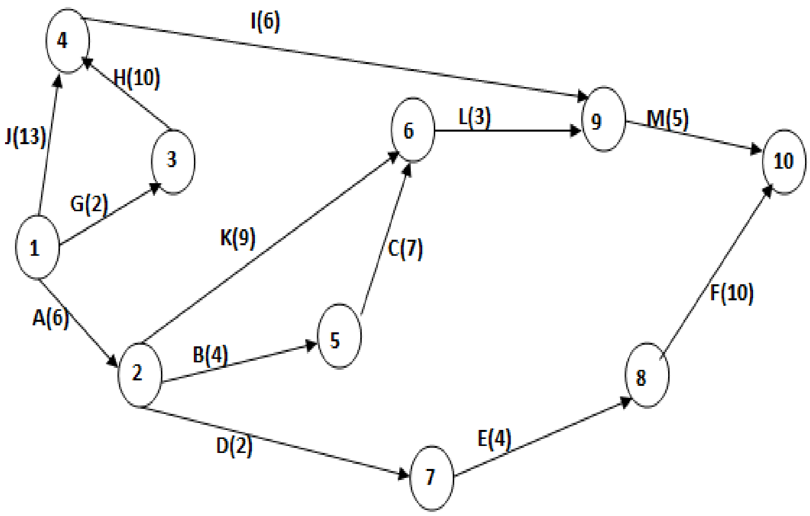


Forward pass calculation:

$E_1 = 0$ $\qquad$ $E_2 = E_1 + t_{1,2} = 0+6 = 6$

$E_3 = E_1 + t_{1,3} = 0 + 2 = 2$ $\qquad$ $E_6 = \underset{i=1,3}{Max}\{E_i + t_{i,4}\} = Max\{E_1 + t_{1,4}; E_3 + t_{3,4}\}$

$E_5 = E_2 + t_{2,5} = 6 + 4 = 10$ $\qquad$ $= Max\{0+13; 2+10\} = 13$

$E_6 = \underset{i=2,5}{Max}\{E_i + t_{i,6}\} = Max\{E_2 + t_{2,6}; E_5 + t_{5,6}\}$ $\qquad$ $E_8 = E_7 + t_{7,8} = 8+4 = 12$

$= Max\{6+9; 10+7\} = 17$ $\qquad$ $E_{10} = \underset{i=8,9}{Max}\{E_i + t_{i,10}\}$

$= Max\{E_8 + t_{8,10}; E_9 + t_{9,10}\}$

$E_7 = E_2 + t_{2,7} = 6 + 2 = 8$ $\qquad$ $= Max\{12+10; 20+5\} = 25$

$E_9 = \underset{i=4,6}{Max}\{E_i + t_{i,9}\} = Max\{E_4 + t_{4,9}; E_6 + t_{6,9}\}$

$= Max\{13+6; 17+3\} = 20$

Backward Pass Method :

$L_{10} = E_{10} = 25$ $\qquad$ $L_9 = L_{10} - t_{9,10} = 25-5 = 20$

$L_8 = L_{10} - t_{8,10} = 25 - 10 = 15$ $\qquad$ $L_7 = L_8 - t_{7,8} = 15-4 = 11$

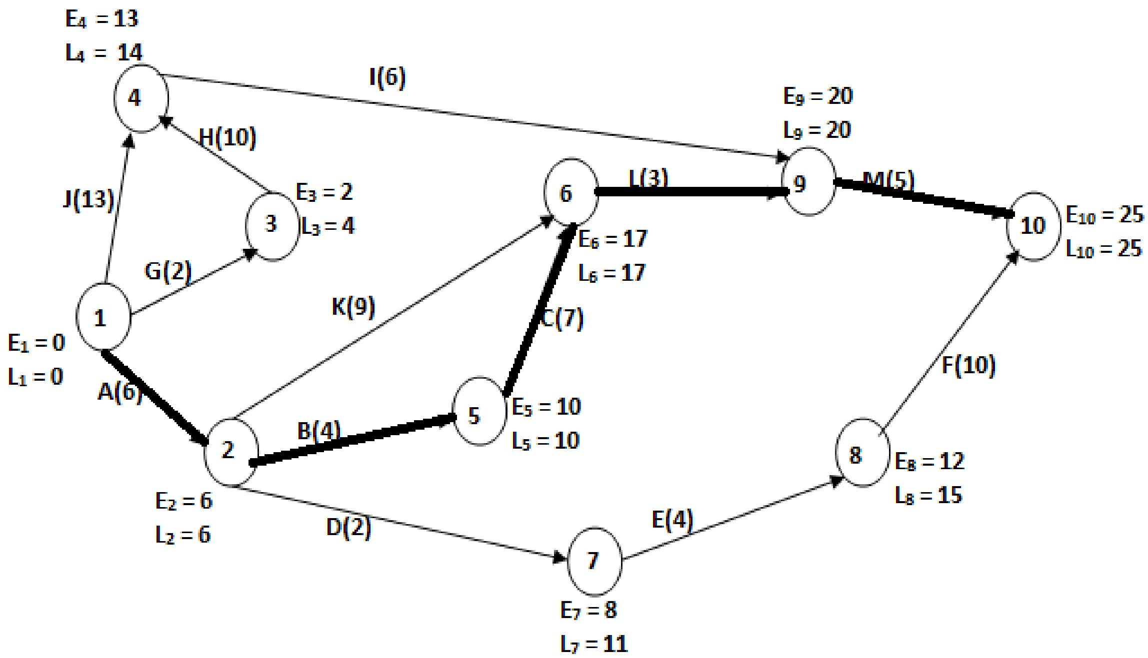
$L_6 = L_9 - t_{6,9} = 20 - 3 = 17$ $\qquad$ $L_5 = L_6 - t_{5,6} = 17-7 = 10$

$L_4 = L_9 - t_{4,9} = 20-6 = 14$ $\qquad$ $L_3 = L_4 - t_{3,4} = 14-10 = 4$

$L_2 = \underset{j=5,6,7}{Min}\{L_j - t_{2,j}\}$ $\qquad$ $L_2 = \underset{j=2,3,4}{Min}\{L_j - t_{1,j}\}$

$= Min\{L_5 - t_{2,5}; L_6 - t_{2,6}; L_7 - t_{2,7}\}$ $\qquad$ $= Min\{L_2 - t_{1,2}; L_3 - t_{1,3}; L_4 - t_{1,4}\}$

$= Min\{10 - 4; 17 - 9; 11 - 2\} = 6$ $\qquad$ $= Min\{6 - 6; 4 - 2; 4 - 13\} = 0$



The Critical path in the network diagram has been shown by double lines by joining all those events where E-values and L-values are equal. The critical path of the project is 1-2-5-6-9-10 and Critical activities are A , B ,C ,L and M. The total project time is 25 weeks.

b) For each non-critical activity, the total float and free float calculations are as follows:

Activity	**Duration**	**Earliest Time**		**Latest Time**		**Float**	
(i , j)	**(t_{ij})**	**Start**	**Finish**	**Start**	**Finish**	**Total**	**Free**
		(E_i)	**(E_i+t_{ij})**	**(L_j - t_{ij})**	**(L_j)**	**(L_j - t_{ij})- (E_i)**	**(E_j – E_i) - t_{ij}**
1 – 3	2	0	2	2	4	2	0
1 – 4	13	0	13	1	14	1	0
2 – 6	9	6	15	8	17	2	2
2 – 7	2	6	8	9	11	3	0
3 – 4	10	2	12	4	14	2	1
4 – 9	6	13	19	14	20	1	1
7 – 8	4	8	12	11	15	3	0
8 – 10	10	12	` 22	15	25	3	3

4.3. PERT (Project Evaluation and Review Technique)

Calculations

Estimation of Project Completion time :

If σ denotes the standard deviation , then $\sigma = t_p - t_o/6$ or $\sigma^2 = (t_p - t_o/6)^2$

The probability distribution of times for completing an event can be approximated by the normal distribution due to central limit theorem. Thus, the probability of completing the project by scheduled time (T_s) can be determined by using the standard normal variate value,

Standard normal variate value , $Z = T_s - T_e / \sigma_e$

Where, T_e = expected completion time of the project

σ_e = number of standard deviations the scheduled time lies from the expected (mean) time.

4.3.1. Example

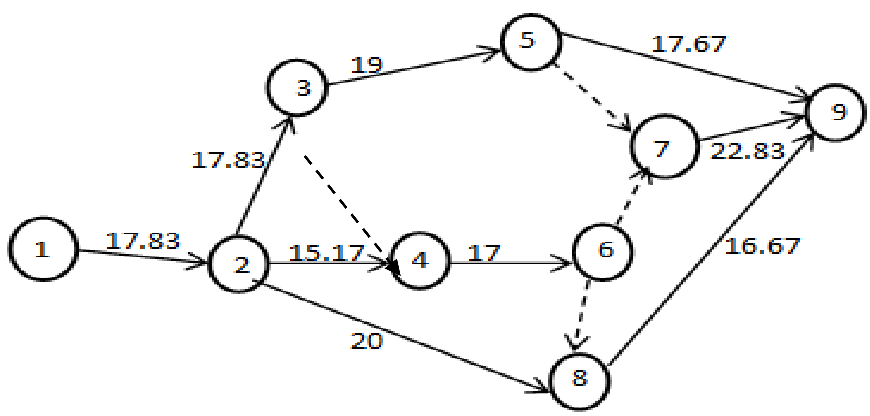
A civil engineering firm has to bid for the construction of a dam. The activities and time estimates are given below:

Activity	Estimated Duration (in Days)		
	Optimistic	Most likely	Pessimistic
1 – 2	14	17	25
2 – 3	14	18	21
2 – 4	13	15	18
2 – 8	16	19	28
3 – 4(dummy)	--	--	--
3 – 5	15	18	27
4 – 6	13	17	21
5 – 7(dummy)	--	--	--
5 – 9	14	18	20
6 – 7(dummy)	--	--	--
6 – 8(dummy)	--	--	--
7 – 9	16	20	41
8 - 9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide 95 percent of probability of at best breaking even. The fixed costs for the project are eight lakh and the variable costs are 9,000 everyday spent working on the project. The duration is in days and the costs are in terms of rupees.

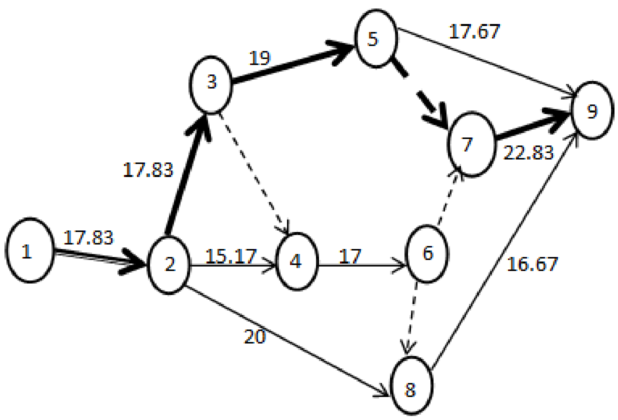
What amount should the firm bid under this policy?(You may perform the calculations on duration, etc., upto two decimal places.

Solution: The network diagram for the given activities is as follows



The calculations of expected duration and variance of each activity are given below:

Activity	Estimated Duration (in Days)			$t_e = (t_o + 4t_m + t_p)/6$	$\sigma^2 = (t_p - t_o/6)^2$
	Optimistic	Most likely	Pessimistic		
1 – 2	14	17	25	17.83	3.36
2 – 3	14	18	21	17.83	1.36
2 – 4	13	15	18	15.17	--
2 – 8	16	19	28	20.00	--
3 – 4(dummy)	--	--	--	--	--
3 – 5	15	18	27	19.00	4
4 – 6	13	17	21	17.00	--
5 – 7(dummy)	--	--	--	--	--
5 – 9	14	18	20	17.67	--
6 – 7(dummy)	--	--	--	--	--
6 – 8(dummy)	--	--	--	--	--
7 – 9	16	20	41	22.83	17.36
8 - 9	14	16	22	16.67	--



The various paths and their length are as follows:

	Path	*Duration*
(i)	*1 – 2 – 3 – 5 – 7 – 9*	*77.49*
(ii)	*1 – 2 – 3 – 5 – 9*	*72.33*
(iii)	*1 – 2 – 3 – 4 – 6 – 7 – 9*	*75.49*
(iv)	*1 – 2 – 3 – 4 – 6 – 8 – 9*	*69.30*
(v)	*1 – 2 – 8 – 9*	*54.50*
(vi)	*1 – 2 – 4 – 6 – 8 – 9*	*66.67*
(vii)	*1 – 2 – 4 – 6 – 7 – 9*	*72.83*

Thus the critical path is : 1 – 2 – 3 – 5 – 7 – 9 with maximum project duration of 77.49 days.
The variance of project duration is obtained by summing variances of critical activities,

Variance , $\sigma^2 = 3.36 + 1.36 + 4 + 17.36 = 26.08$

Standard deviation , $\sigma = \sqrt{26.08} = 5.12$

To calculate the project duration which will have 95 percent chances of its completion, we find the value of Z corresponding to 95 percent area under normal distribution curve which is 1.645.
Thus, $Z = T_s - T_e / \sigma$ or $1.645 = (T_s - 77.49)/5.12$

i.e., $T_s = 86$ days.

Since the fixed cost of the project is Rs . 8 lakhs and the variable cost is Rs.9,000 per day, amount to bid = Rs . 8 lakh + Rs . 9,000x86 = Rs . 15,74,000

Exercises

4.3.2. Determine the Critical Path for the activity data given below. All durations are in days.

Initial Node	Final Node	Duration (days)
1	2	5
1	3	6
2	3	3
2	4	8
3	5	2
3	6	11
4	5	0
4	6	1
5	6	12

2. Given the list of activities in a project and their time estimates (in days):

a)Draw the project network,

b)Determine the critical path(s) and the expected project duration

c)What is the probability that project will be completed in 35 days?

d)What due date has 90% chance of being met?

Activity	t_o	t_m	t_p
1-2	6	12	30
1-3	3	6	15
1-4	3	9	27
2-6	4	19	28
3-5	3	9	27
3-6	2	5	8
4-5	1	4	7
5-6	6	12	30

KAHE

UNIT-IV

SYLLABUS

Inventory models – Economic order quantity models – Quantity discount models – Stochastic inventory models – Multi product models – Inventory control models in practice - Queueing models – Queueing systems and structures – Notation parameter – Single server and multi server models – Poisson input – Exponential service – Constant rate service – Infinite population.

QUEUEING MODELS

CONCEPTS:

1. Queueing theory is a mathematics of waiting times.
2. Be exposed to basic characteristic features of a queuing system and acquire skills in analyzing queuing models.
3. To give insight view of the steady-state behavior of queuing processes and running the simulation experiments to obtain the required statistical results.
4. Queuing theory provides models for a number of situations that arise in real life.

APPLICATIONS:

- **Telecommunications**

1. Airport traffic, airline ticket sales.
2. Used at most airline ticket counters and in many post offices.
3. It is extremely useful in predicting and evaluating system performance.
4. Queueing theory has been used for operations research, manufacturing and systems analysis.
5. Traditional queueing theory problems refer to customers visiting a store, analogous to requires arriving at a device.
6. Running multiple programs on a computer.

7. A print queue is formed when many documents are sent to the printer.
8. Telephone calls on a switchboard and cell phone.
9. Vehicles waiting at a traffic light.
10. Reception desk.
11. Repair facility.

TECHNICAL TERMS:

ARRIVAL RATE (λ): Average rate at which customers arrive to the system. Has units of "customers / time unit".

SERVICE RATE (μ) : The average rate at which an individual server can serve customers. Has units of "customers / time unit" and is the reciprocal of the average time it takes to serve one customer.

FIFO: First in, first out (FIFO) queuing is the most basic queue scheduling discipline.

LIFO: Last-in, first-out (LIFO) queuing discipline.

SERVICE DISCIPLINE: Queue discipline or service discipline refers to the manner in which customers are selected for service when a queue has formed like FIFO/FCFS, LIFO etc.

SERVERS: The number of servers available to serve customers entering a queuing system. The number of servers must be a whole number that is greater than or equal to one.

STEADY STATE: The state of the system after it has been in operation for a long time.

SYSTEM CAPACITY: The total number of customers that can be in the system, either waiting or being served. Must be a whole number that is greater than or equal to one.

TRAFFIC INTENSITY: Traffic intensity is a measure of the average occupancy of a server or resource during a specified period of time.

UTILIZATION : The proportion of time a server is busy is the server utilization.

WAITING TIMES: Customer waiting time can be of two types, the time the customer spends in the queue and the total time a customer spends in the system(waiting time in queue + service).

- $\mu = \frac{1}{E[T]}$
- **P[system is busy OR a customer has to wait]** $= \rho = \frac{\lambda}{\mu}$
- **Average number of customers in the system** $L_s = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$
- **The average number of customers in the queue or average length of the queue** L_q:

$$L_q = L_s - \rho = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

- **Expected waiting time of customer in the system** $\boldsymbol{W_s = \frac{1}{\lambda} L_s = \frac{1}{\mu-\lambda}}$
- **Expected waiting time of customer in the queue** $\boldsymbol{W_q = \frac{1}{\lambda} L_q = \frac{\lambda}{\mu(\mu-\lambda)}}$
- $P[server\ is\ idle\ ]\ P_0 = 1-\rho$
- $P_n = (1-\rho)\rho^n\ ,\ n = 0,1,2,\dots,\infty$
- $P[N \geq n] = \rho^n$
- $P[N > n] = \rho^{n+1}$
- $\rho = \frac{\lambda}{\mu s}$
- $P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!\left(1-\frac{\lambda}{\mu s}\right)\left(\frac{\lambda}{\mu}\right)^s}\right]^{-1}$
- $L_q = \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu-\lambda)^2} P_0$
- $L_s = L_q + \frac{\lambda}{\mu}$
- $W_q = \frac{L_q}{\lambda}$
- $W_q = \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu}{(s\mu-\lambda)^2} P_0$
- $W_s = W_q + \frac{1}{\mu}$
- $P\ (n \geq s) = \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!\left(1-\frac{\lambda}{\mu s}\right)} P_0$

5.1 MARKOVIAN MODELS

5.1 (i) The queue Discipline

The queue discipline specifies the manner in which the customers from the queue or equivalently the manner in which they selected for service, when a queue has been formed. The most common discipline is the FCFS (First come first served) or FIFO (First in first out) as per which the customers are served in the strict order o their arrival. If the last arrival in the system is served first, we have the LCFS or LIFO (last in first out) discipline. If the service is given in random order, we have the SIRO discipline. In the queueing systems which we deal with, we shall assume that service is provided on the FCFS basis.

5.1 (ii) Symbolic representation of a queueing model:

Usually a queueing model is specified and represented symbolically in the form **(a/ b/ c) : (d / e)**

Where

a → the type of distribution of arrivals per unit time;

b → the type of distribution of the service time

c → the number of servers

d → the capacity of the system, via, the maximum queue size.

e → the queue discipline.

Accordingly, the four models we will deal with be denoted by the systems

- **(i)** **(M/M/I) : ($\infty/FIFO$)**
- **(ii)** **(M/M/C) : ($\infty/FIFO$)**
- **(iii)** **(M/M/I) : ($K/FIFO$) and**
- **(iv)** **(M/M/C) : ($K/FIFO$)**

5.1 (iii) Kendall's Notation for Representing Queueing Models

D.G. Kendall (1953) and later A. Lee (1966) introduced useful notation for queueing models. The complete notation can be expressed as **(a/b/c) : (d/e/f)**

Where

a = arrival (or inter arrival) distribution,

b = departure (or service time) distribution,

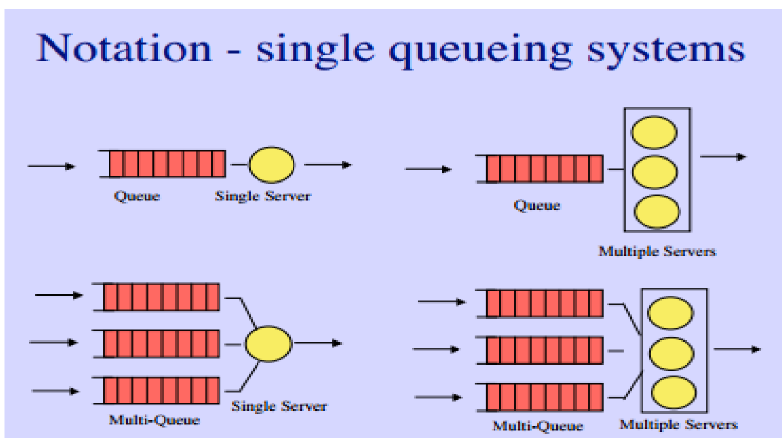
c = number of parallel service channels in the system,

d = service discipline,

e = maximum number of customers allowed in the system,

f = calling source or population

QUEUEING MODELS:



5.1 (a) Characteristics of infinite capacity, single server Poisson queue model I

(M/M/1): ($\infty/FIFO$),

When $\lambda_n = \lambda \; and \; \mu_n = \mu \; (\lambda = \mu)$.

1. Average number L_s of customers in the system:

Let 'N' denotes the number of customers in the queueing system (i.e., those in the queue and one who is being served).

'N' is a discrete random variable which can take the values 0, 1, 2…∞ such that

$$P\,(N = n) = P_n = \left(\frac{\lambda}{\mu}\right)^n . P_o \ , \text{ where } \frac{P_n}{\left(\frac{\lambda_0\lambda_1\lambda_2\ldots\lambda_{n-1}}{\mu_1\mu_2\mu_3\ldots\mu_n}\right)} = P_o.$$

$$P_o = \frac{1}{1+\sum_{n=1}^{\infty}\left(\frac{\lambda}{\mu}\right)^n}$$

$$= \frac{1}{\sum_{n=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^n}$$

$$= 1-\frac{\lambda}{\mu}$$

$$\therefore \qquad P_n = \left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right)$$

$$\text{Now } L_q = \mathrm{E(N)}$$

$$= \sum_{\mathrm{n=0}}^{\infty} \mathrm{nP_n}$$

$$= \left(\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right)\sum_{n=1}^{\infty} n\left(\frac{\lambda}{\mu}\right)^{n-1}$$

$$= \left(\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right)^2$$

$$= \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}}$$

$$= \frac{\lambda}{\mu-\lambda}$$

2. The average number L_q of customers in the queue or average length of the queue:

If 'N' is the number of customers in the system, then the number of customers in the queue is (N-1).

$$\therefore \qquad L_q = \mathrm{E(N-1)}$$

$$= \sum_{\mathrm{n=0}}^{\infty}\mathrm{(n-1)P_n}$$

$$= \left(1-\frac{\lambda}{\mu}\right)\sum_{n=1}^{\infty}(n-1)\left(\frac{\lambda}{\mu}\right)^n$$

$$= \left(\frac{\lambda}{\mu}\right)^2\left(1-\frac{\lambda}{\mu}\right)\sum_{n=2}^{\infty}(n-1)\left(\frac{\lambda}{\mu}\right)^{n-2}$$

$$= \left(\frac{\lambda}{\mu}\right)^2\left(1-\frac{\lambda}{\mu}\right)\left(1-\frac{\lambda}{\mu}\right)^{-2}$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^2}{1-\frac{\lambda}{\mu}}$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} \ .$$

3. Average number L_W of customers in non-empty queues:

$L_W = E\{(N-1)/(N-1) > 0\}$, Since the queue is non-empty.

$$= \frac{E(N-1)}{P(N-1)>0}$$

$$= \frac{\lambda^2}{\mu-\lambda} \text{X} \frac{1}{\sum_{n=2}^{\infty} \mathrm{P_n}}$$

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} \text{X} \frac{1}{\left(\frac{\lambda}{\mu}\right)^2 \left(1-\frac{\lambda}{\mu}\right) \sum_{n=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^n}$$

$$= \frac{\mu}{\mu-\lambda}$$

4. The probability that the number of customers in the systems in the system exceeds k:

$$P(N > K) = \sum_{n=k+1}^{\infty} \mathrm{P_n} = \sum_{n=k+1}^{\infty} \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right)$$

$$= \left(\frac{\lambda}{\mu}\right)^{k+1} \left(1-\frac{\lambda}{\mu}\right) \sum_{n=k+1}^{\infty} \left(\frac{\lambda}{\mu}\right)^{n-(k+1)}$$

$$= \left(\frac{\lambda}{\mu}\right)^{k+1} \left(1-\frac{\lambda}{\mu}\right) \sum_{n=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^{n}$$

$$= \left(\frac{\lambda}{\mu}\right)^{k+1} \left(1-\frac{\lambda}{\mu}\right) \left(1-\frac{\lambda}{\mu}\right)^{-1}$$

$$P(N > K) = \left(\frac{\lambda}{\mu}\right)^{k+1}.$$

5. The average waiting time of a customer in the system:

W_s Follows an exponential distribution with parameter$(\mu-\lambda)$.

$$\therefore E(W_s) = \frac{1}{\mu-\lambda}$$

$\because$ The mean of an exponential distribution is the reciprocal of its parameter.

6. Probability that the waiting time of a customer in the system exceeds't'

$$P(W_s > t) = \int_t^{\infty} f(w)dw$$

$$= \int_t^{\infty} (\mu - \lambda)e^{-(\mu-\lambda)w}dw$$

$$= \left[-e^{-(\mu-\lambda)w}\right]_t^{\infty}$$

$$= e^{-(\mu-\lambda)t}$$

7. Probability density Function of the waiting time W_q in the system:

W_q Represents the time between arrival and reach of service point. Let the probability density function of W_q *be* g(w) and let g(w/n) be the density function of W_q subject to the condition that there are n customers in the system or there (n-1) customers in the queue apart from one customer receiving service. Now g(w/n)= probability density function of sum of n service times [one residual service time + (n-1) full service times]

$$= \frac{\mu^n}{(n-1)!} e^{-\mu w} w^{n-1}; w > 0$$

$$g(w) = \sum_{n=1}^{\infty} \frac{\mu^n}{(n-1)!} e^{-\mu w} w^{n-1} \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

$$= \lambda\left(1 - \frac{\lambda}{\mu}\right) e^{-\mu w} \sum_{n=1}^{\infty} \frac{1}{(n-1)!} (\lambda w)^{n-1}$$

$$= \frac{\lambda}{\mu}(\mu - \lambda)e^{-\mu w}e^{\lambda w}$$

$$= \frac{\lambda}{\mu}(\mu - \lambda)e^{-(\mu-\lambda)w}; w > 0$$

and $$g(w) = 1 - \frac{\lambda}{\mu}; w > 0 .$$

8. The average waiting time of a customer in the queue:

$$E(W_q) = \frac{\lambda}{\mu}(\mu - \lambda)\int_0^{\infty} we^{-(\mu-\lambda)w}dw$$

$$u = w\, ; v = e^{-(\mu-\lambda)w}$$

$$u^1 = 1\ ;\ v_1 = \frac{-e^{-(\mu-\lambda)\mathrm{w}}}{\mu-\lambda}$$

$$u^{11} = 0\ ;\ v_2 = \frac{-e^{-(\mu-\lambda)\mathrm{w}}}{(\mu-\lambda)^2}$$

$$\therefore E(W_q) = \frac{\lambda}{\mu}(\mu-\lambda)\left[\frac{-we^{-(\mu-\lambda)\mathrm{w}}}{\mu-\lambda} - \frac{-e^{-(\mu-\lambda)\mathrm{w}}}{(\mu-\lambda)^2}\right]_0^\infty = \frac{\lambda}{\mu(\mu-\lambda)}$$

9. The average waiting time of a customer in the queue, if he has to wait,

$$E(W_q/W_q > 0) = \frac{E(W_q)}{P(W_q>0)}$$

$$= \frac{E(W_q)}{1-P(W_q=0)}$$

$$= \frac{E(W_q)}{1-P(no\ customer\ in\ the\ queue)}$$

$$= \frac{E(W_q)}{1-P_0}$$

$$= \frac{\lambda}{\mu(\mu-\lambda)} X \frac{\mu}{\lambda} \qquad \because P_0 = 1-\frac{\mu}{\lambda}$$

$$= \frac{\mathbf{1}}{\boldsymbol{\mu-\lambda}}$$

Relations among $\mathrm{E(N_s), E(N_q), E(W_q)}$ and $\mathrm{E(W_s)}$:

$$\text{(i) } \mathrm{E(N_s)} = \frac{\lambda}{\mu-\lambda} = \lambda\,\mathrm{E(W_s)}, \because\ \mathrm{E(N_s)} = L_s$$

$$(ii)\ \mathrm{E(N_q)} = \frac{\lambda^2}{\mu(\mu-\lambda)} = \lambda \mathrm{E(W_q)}, \because \mathrm{E(N_q)} = L_q$$

$$(iii)\ \mathrm{E(W_s)} = \mathrm{E(W_q)} + \frac{1}{\mu}$$

$$(iv)\ \mathrm{E(N_s)} = \mathrm{E(N_q)} + \frac{\lambda}{\mu}\ .$$

Example: 5.1a (1)

Arrivals of a telephone in a booth are considered to be Poisson with an average time of 12 minutes between one arrival and the text. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes.

(1) Find the average number of persons waiting in the system?

(2) What is the probability that a person arriving at the booth will have to wait in the queue?

(3) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?

(4) Estimate the fraction of the day when the phone will be in use.

(5) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for at least 3 minutes for phone. By how much the flow of the arrivals should increase in order to justify a second booth?

(6) What is the average length of the queue that forms from time to time?

Solution:

Given Telephone booth → single server

Arrivals at a telephone booth → infinite capacity

The given problem is (M/M/1): (∞/FIFO) model.

(or) (M/M/1): (∞/FCFS) [FIFO = FCFS]

Given $\frac{1}{\lambda}$[The mean inter-arrival time] = 12minutes

λ[mean arrival rate] = $\frac{1}{12}$ per minutes

E[T] = 4 minutes

$\mu = \frac{1}{\text{E[T]}} = \frac{1}{4}$ minutes

$\rho = \frac{\lambda}{\mu} = \frac{1/12}{1/4} = \frac{1}{3}$minutes.

(1) L_s [average number of persons waiting in the system] = $\frac{\rho}{1-\rho}$

$= \frac{1/3}{1-\frac{1}{3}}$

$= \frac{1/3}{2/3}$

$= \frac{1}{2}$

(2) Probability that the person arriving in the booth has to wait in the queue $= P[W > 0]$

= 1- $P[W = 0]$

= 1- P[no customer in the system]

= 1-P_0

= 1-[1-ρ]

= ρ

= $\frac{1}{3}$

(3) A persons takes more than 10 minutes to wait

and complete this call = $P[W > 0]$

$P[W > t]$ = $e^{-(\mu-\lambda)t}$

$P[W > 10]$ = $e^{-\left(\frac{1}{4}-\frac{1}{12}\right)10}$

= $e^{-(5/3)}$

= 0.1889

(4) P[Phone in use] = P [phone is busy]

= 1 – p [phone is idle]

= 1- P_0

= 1- [1-ρ]

= ρ

= $\frac{1}{3}$

(5) The second phone will be installed if E[W_q] > 3

$$\frac{\lambda}{\mu(\mu-\lambda)} > 3$$

(i.e) If $\frac{\lambda_R}{\left(\frac{1}{4}\right)\left(\frac{1}{4}-\lambda_R\right)} > 3$ where λ_R is the required arrival rate.

$$\text{If } \lambda_R > \frac{3}{4}\left(\frac{1}{4} - \lambda_R\right)$$

$$\lambda_R + \frac{3}{4}\lambda_R > \frac{3}{6}$$

$$\frac{7}{4}\lambda_R > \frac{3}{16}$$

$$\lambda_R > \frac{3}{16}\,\frac{4}{7}$$

$$\lambda_R > \frac{3}{28}$$

Hence the arrival rate should increase by $\frac{3}{28} - \frac{1}{12} = \frac{1}{42}$ pe minute.

(6) The average length of the queue $= \frac{\mu}{\mu-\lambda}$

$$= \frac{\frac{1}{4}}{\frac{1}{4}-\frac{1}{12}}$$

$$= \frac{\frac{1}{4}}{\frac{2}{12}}$$

$$= \frac{3}{2}$$

$$= 1.5$$

Example 5.1a (2)

Consider an M/M/1 Queuing system. If λ = 6 and μ = 8, find the probability of at least 10 customers in the system.

Solution:

The traffic intensity of the system is given by

$$\rho = \frac{\lambda}{\mu} = \frac{6}{8} = 0.75$$

Thus, the probability of at least 10 customers in the system is given by

$$P\ (N \geq 10) = \rho^{10}$$

$$= (0.75)^{10}$$

$$= 0.0563$$

Example 5.1a (3)

What is the probability that a customer has to wait more than 15 minutes to get his service completed in (M/M/1) : (∞/FIFO) queue system if λ = 6 per hour and μ = 10 per hour?

Solution:

Given λ = 6 per hour and μ = 10 per hour

The probability that the waiting time of a customer in the system exceeds t is given by P $(T_s > t)$ = $e^{-(\mu-\lambda)t}$ = e^{-4t}

Thus, the probability that a customer has to wait more than 15 minutes or one-quarter of an hour is given by

$$P\,(T_s > 0.25) = e^{-4(0.25)} = e^{-1} = \frac{1}{e}.$$

EXERCISE: 5.1 (a)

1. Suppose that customer arrive at a Poisson rate of one per every 12 minutes and that the service time is exponential at a rate of one service per 8 minutes. What is?
   (a) The average number of customers in the system. (b) The average time a customer spends in the system.
   [Ans: (a) L_s = 2 (b) W_s = 24 minutes]

2. Automatic car wash facility operates with only one boy. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the boy is busy. If the service time for all cars is constant and equal to 10 minutes, determine L_s, L_q, W_s *and* W_q.

[Ans : (i) Ls=2 cars (ii) Lq =1.3333 cars (iii) Ws=1/2 hours (iv) Wq=1/3hours]

3. Customers arrive at a one-man barber shop according to a Poisson process with a mean inter arrival time of 20 mins. Customers spend an average of 15 mins in the barber's chair. If an hour is used as a unit of time

(a)What is the probability that a customer need not wait for a hair cut?

(b) What is the expected number of customers in the barber shop and in the queue?

(c) How much time can a customer expect to spend in the barber's shop?

(d) What is the average time customer spends in the queue?

(e) What is the probability that there will be 6 or more customers waiting for service

[Ans: (a) P0= 1/4 (b) Ls=3, Lq=2.25 customers (c) Ws=1 hr (d) wq=0.75 hr (e) P[N≥ 6]=0.1779]

4. A T.V repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8 hour day.

i) what is the repairman's expected idle time each day?

ii) How many jobs are ahead of the average set just brought in?

iii)What is the average number of jobs in a non-empty queue?

[Ans: (i) P_0=3/8 (ii) Ls=5/3 (iii) Lw=2.667]

5.1. (b) MULTI-SERVER QUEUES

Characteristics of infinite capacity, multiple server Poisson queue model II

$(\mathrm{M/M/C}):(\infty/\mathrm{FIFO})$ Model, when $\lambda_n = \lambda, \forall\, n (\lambda < S\mu\,)$.

1. **Values of P_0 and P_n:**

   For the Poisson queue system, P_n is given by

$$P_n = \frac{\lambda_0\lambda_1\lambda_2 \dots \lambda_{n-1}}{\mu_1\mu_2\mu_3 \dots \mu_n} P_0\,, n \geq 1 \quad \dots (1)$$

Where $P_o = \left[1 + \sum_{n=1}^{\infty}\left(\frac{\lambda_0\lambda_1\lambda_2 \dots \lambda_{n-1}}{\mu_1\mu_2\mu_3 \dots \mu_n}\right)\right]$ .... (2)

Now P_o is given by $\sum_{n=0}^{\infty} P_0 = 1$

$$\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{\infty}\frac{1}{s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n\right] P_0 = 1\ .$$

$$\therefore\ P_0 = \left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!\left(1-\frac{\lambda}{\mu s}\right)\left(\frac{\lambda}{\mu}\right)^s}\right]^{-1}.$$

2. **The average number of customers in the queue or average queue length :**

$$L_q = E\left(N_q\right)$$

$$= E(N - s)$$

$$= \sum_{n=s}^{\infty}(n-s)\,P_n$$

$$= \sum_{x=0}^{\infty} x.P_{x+s}\,,\ put\ x = n - s$$

$$L_q = \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu-\lambda)^2} P_0$$

3. **The average number of customers in the system:**

   By little's Formula $L_s = L_q + \frac{\lambda}{\mu}$

$$= \frac{\lambda}{\mu} + \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu-\lambda)^2} P_0$$

4. **Average time a customer has to spend in the queue:**

$$W_q = \frac{L_q}{\lambda}$$

$$W_q = \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu}{(s\mu-\lambda)^2} P_0$$

5. **Average time a customer has to spend in the system:**

$$W_s = W_q + \frac{1}{\mu}$$

$$= \frac{1}{\mu} + \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu}{(s\mu-\lambda)^2} P_0 .$$

6. **Probability that an arrival has to wait:**

The probability that an arrival has to wait = The probability that there are s or more customers in the system.

(ie) $P(W_s > 0) = P\ (n \geq s) = \sum_{n=s}^{\infty} P_n$

$$= \sum_{n=s}^{\infty} \frac{1}{s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0$$

$$= \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \boldsymbol{P_0} \sum_{n=s}^{\infty}\left(\frac{\lambda}{\mu s}\right)^{n-s}$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!\left(1-\frac{\lambda}{\mu s}\right)} \boldsymbol{P_0}$$

$$= \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{\mu}{s\mu-\lambda}\right) \boldsymbol{P_0}$$

7. **Probability that an arrival enters the service without waiting:**

Required Probability = 1- P(an arrival has to wait)

$$= 1 - \frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \left(\frac{\mu}{s\mu-\lambda}\right) P_0$$

8. **Mean waiting time in the queue for those who actually wait:**

$$E(\mathrm{Wq/Ws} > 0) = \frac{E(W_q)}{P(W_s > 0)}$$

$$= \frac{\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu}{(s\mu-\lambda)^2} P_0}{\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu}{(s\mu-\lambda)^2} P_0 + \frac{1}{\mu}}$$

$$= \frac{1}{s\mu\left(1-\frac{\lambda}{\mu s}\right)}$$

$$= \frac{1}{\mu s-\lambda} \quad .$$

9. The probability that there will be same one waiting:

$$= P(N \geq s+1)$$

$$= \sum_{n=s+1}^{\infty} P_n$$

$$= \sum_{n=s}^{\infty} P_n - P(N = s)$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!\left(1-\frac{\lambda}{\mu s}\right)} P_0 - \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} P_0$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} P_0 \left[\frac{1}{1-\frac{\lambda}{\mu s}-1}\right]$$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} \frac{\frac{\lambda}{\mu s}}{1-\frac{\lambda}{\mu s}} P_0$$

10. The average number of customers (in non-empty queues), who have to actually wait,

$$L_w = E\left(N_q / N_q \geq 1\right)$$

$$= E\left((N_q)/P(N_q \geq 1)\right)$$

$$= \frac{1}{s\,.s!} \frac{\left(\frac{\lambda}{\mu}\right)^{s+1}}{1-\frac{\lambda}{\mu s}} P_0 \; \frac{s!\left(1-\frac{\lambda}{\mu s}\right)}{\left(\frac{\lambda}{\mu}\right)^s P_0}$$

$$= \frac{\frac{\lambda}{\mu s}}{1-\frac{\lambda}{\mu s}} \quad .$$

Example: 5.1 b (1)

There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour,

(i) What fraction of the time all the typists will the busy?

(ii) What is the average number of letters waiting to be typed?

(iii) What is the average time a letter has to spend for waiting and for being typed?

(iv) What is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed?

Solution:

$$\frac{\lambda}{\mu} = \frac{15}{6} = 2.5$$

$$\rho = \frac{\lambda}{\mu s} = \frac{1}{s}\left(\frac{\lambda}{\mu}\right) = \frac{1}{3}\left(\frac{5}{2}\right) = \frac{5}{6}$$

$$1 - \frac{\lambda}{\mu s} = 1 - \frac{5}{6} = \frac{1}{6}$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!(1-\rho)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{2} \frac{1}{n!}\left(\frac{5}{2}\right)^n + \frac{\left(\frac{5}{2}\right)^3}{3!\left(1-\frac{5}{6}\right)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{2} \frac{1}{n!}\left(\frac{5}{2}\right)^n + \frac{\frac{125}{8}}{6\left(\frac{1}{6}\right)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{2} \frac{1}{n!}\left(\frac{5}{2}\right)^n + \frac{125}{8}\right]^{-1}$$

$$= \left[\frac{1}{0!}\left(\frac{5}{2}\right)^0 + \frac{1}{1!}\left(\frac{5}{2}\right)^1 + \frac{1}{2!}\left(\frac{5}{2}\right)^2 + \frac{125}{8}\right]^{-1}$$

$$= \left[1 + \frac{5}{2} + \frac{1}{2}\left(\frac{25}{4}\right) + \frac{125}{8}\right]^{-1}$$

$$= \left[1 + \frac{5}{2} + \left(\frac{25}{8}\right) + \frac{125}{8}\right]^{-1}$$

$$= \left[\frac{8+20125}{8}\right]^{-1}$$

$$= \left[\frac{178}{8}\right]^{-1}$$

= 0.0449

(i) P(all the typists are busy) = P[N≥ 3]

$$= \frac{\left(\frac{\lambda}{\mu}\right)^3}{3!\left(1-\frac{\lambda}{3\mu}\right)}$$

$$= \frac{\left(\frac{5}{2}\right)^3}{3!\left(\frac{1}{6}\right)} P_0$$

$$= \frac{\frac{125}{8}}{6\left(\frac{1}{6}\right)} P_0$$

$$= \frac{125}{8} P_0$$

\ = 0.7016

Hence the fraction of the time all the typists will be busy=0.7016

(ii) $L_q = \frac{\lambda}{s\mu-\lambda}$ P[N≥ s]

$$= \frac{\lambda}{3\mu-\lambda} \text{ P[N} \geq 3]$$

$$= \frac{15}{3(6)-15} [0.7016]$$

$$= \frac{15}{18-15} [0.7016]$$

$$= \frac{15}{3} [0.7016]$$

= 3.508

(iii) $W_s = W_q + \frac{1}{\mu} = \frac{1}{\lambda} L_q + \frac{1}{\mu}$

$$= \frac{1}{15}[3.508] + \frac{1}{6}$$

= 0.2338 + 0.1667

= 0.4005 hours

= 0.4005 x 60 minutes

= 24.03 minutes

= 24 minutes (approximately)

(iv) $P\,(W > t) = e^{-\mu t}\left\{1 + \frac{\left(\frac{\lambda}{\mu}\right)^s [1-e^{-\mu t\left(s-1+\frac{\lambda}{\mu}\right)}]}{s!\left(1-\frac{\lambda}{\mu s}\right)\left(s-1-\frac{\lambda}{\mu}\right)} P_0\right\}$ [assumed formula]

$\therefore P\,(W > 1/3) = e^{-6\,X\,1/3}\left\{1 + \frac{(2.5)^3[1-e^{-2\,X\,(-0.5)}]}{6\left(1-\frac{2.5}{3}\right)(-0.5)}\,X\,0.0449\right\}$

= 0.4616.

Example: 5.1.b (2)

Consider an M/M/C queueing system. Find the probability that an arriving customer is forced to join in the queue.

Solution:

The probability that an arrival is forced to join in the queue

= the probability that an arrival has to wait

= probability that there are C or more customers in the system.

$= P\,(N \geq C) = \sum_{n=c}^{\infty} P_n$, where $P_n = \frac{1}{c!c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n P_0$

$\therefore P\,(N \geq C) = \sum_{n=c}^{\infty} \frac{1}{c!c^{n-c}}\left(\frac{\lambda}{\mu}\right)^n P_0$

$= \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^C P_0 \sum_{n=c}^{\infty}\left(\frac{\lambda}{\mu_c}\right)^{n-c}$

$= \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^C P_0\left[1 + \frac{\lambda}{\mu_c} + \left(\frac{\lambda}{\mu_c}\right)^2 + \cdots\right]$

$= \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^C P_0\left(1 - \frac{\lambda}{\mu_c}\right)^{-1}$

EXERCISE: 5.1(b)

1.A bank has two tellers working on savings accounts. The first teller handles with drawls only. The second teller handles deposits only. It has be found that the service time distribution for the deposits and withdrawals both are exponential with mean service time 3 minutes per customers. Depositors are found to arrive in a Poisson fashion throughout the day with mean arrival rate 16/hour. Withdrawers also arrive in a Poisson fashion with mean arrival rate 14/hour.

(i) What would be the effect on the average waiting time for depositors and withdrawers if each teller could handle both withdrawals and deposits?

(ii) What would be the effect if this could only be accomplished by increasing the service time to 3.5 minutes?

[Ans: (i)P_0=0.143,Lq=1.93,Wq=3.84min (ii)P_0=0.067,Lq=5.745, Wq=11.49min]

2. A supermarket has 2 girls serving at the counters. If people arrive in Poisson fashion at the rate of 10 per hour. The service time for each customer is exponential with mean 4 mins. Find

(i) The probability that an arriving customer has to wait for service?

(ii)What is expected % of idle time for each girl?

(iii)If the customer has to wait in queue, what is the expected length of his waiting time?

[Ans: (i)P[N≥2]=0.168 ,(ii) Expected %=67, (iii) E(Wq/Ws > 0)=3min]

5.1 (c) Characteristics of Finite Capacity, Single Server Poisson Queue Model

$(\mathbf{M/M/1}):(\mathbf{K/FIFO})$**Model:**

1. **Values of P_0 and P_n :**
For the Poisson queue system, $P_n = P(N = n)$ in the steady state is given by the difference equations.

$$\lambda_{n-1}P_{n-1} - (\lambda_n + \mu_n)P_n + \mu_{n+1}P_{n+1} = 0; n > 0$$

and $-\lambda_0 P_0 + \mu_1 P_1 = 0 ; n = 0$

This model represents the situation in which the system can accommodate only a finite number k of arrivals. If a customer arrives and the queue is full, the customer leaves without joining the queue.

Therefore for this model,

$$\mu_n = \mu, n = 1,2,3 \ldots$$

and $\lambda_n = \begin{cases} \lambda, \; for\; n = 0,1,2 \dots (k-1) \\ 0, \; for\; n = k, k+1, \dots \end{cases}$

Using these values in the difference equations given above, we have

$$\mu P_1 = \lambda P_0 \quad \dots (1)$$

$$\mu P_{n+1} = (\lambda + \mu) P_n - \lambda P_{n-1}, for\; 1 \le n \le k-1 \quad \dots (2)$$

And $$\mu P_k = \lambda P_{k-1}, for\; n = k \quad \dots (3)$$

$[\because P_{k+1}\; has\; no\; meaning]$

From (1), $P_1 = \frac{\lambda}{\mu} P_0$

From (2), $\mu P_2 = (\lambda + \mu) \frac{\lambda}{\mu} . P_0 - \lambda P_0$

$\therefore P_2 = \left(\frac{\lambda}{\mu}\right)^2 P_0\; and\; so\; on.$

In general, $P_n = \left(\frac{\lambda}{\mu}\right)^n P_0, true\; for\; 0 \le n \le k-1$

From (3), $P_k = \frac{\lambda}{\mu} \left(\frac{\lambda}{\mu}\right)^{k-1} P_0 = \left(\frac{\lambda}{\mu}\right)^k P_0$

Now $\sum_{n=0}^{k} P_n = 1$

i.e. $P_0 \sum_{n=0}^{k} \left(\frac{\lambda}{\mu}\right)^n = 1$

i.e., $P_0 \frac{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}{1-\frac{\lambda}{\mu}} = 1$

Which is valid even for $\lambda > \mu$

$$P_0 = \begin{cases} \dfrac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}, & if\; \lambda \ne \mu \quad (4) \\ \dfrac{1}{k+1}, & if\; \lambda \ne \mu, \quad since\; \lim_{\frac{\lambda}{\mu} \to 1} \left\{ \dfrac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} \right\} = \dfrac{1}{k+1} \end{cases}$$

$$P_n = \begin{cases} \left(\frac{\lambda}{\mu}\right)^n \left[\frac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}\right], \; if\; \lambda \ne \mu \; \dots \; (6) \\ \frac{1}{k+1}, \; if\; \lambda \ne \mu \dots \; (7) \end{cases}$$

2. Average number of customers in the system:

$$E(N_s) = \sum_{n=0}^{k} nP_n = \frac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} \sum_{n=0}^{k} n\left(\frac{\lambda}{\mu}\right)^n$$

$$= \frac{\left(1-\frac{\lambda}{\mu}\right).\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}.\sum_{n=0}^{k}\frac{d}{dx}(x^n) \qquad where\ x = \frac{\lambda}{\mu}$$

$$= \frac{\left(1-\frac{\lambda}{\mu}\right).\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}.\frac{d}{dx}\left(\frac{1-x^{k+1}}{1-x}\right)$$

$$= \frac{(1-x).x}{1-x^{k+1}}.\left[\frac{(1-x)\{-(k+1)x^k\}+1-x^{k+1}}{(1-x)^2}\right]$$

$$= \frac{x(1-x^{k+1})-(k+1)(1-x)x^{k+1}}{(1-x)(1-x^{k+1})}$$

$$= \frac{x}{1-x} - \frac{(k+1)x^{k+1}}{1-x^{k+1}}$$

$$= \frac{\lambda}{\mu-\lambda} - \frac{(k+1)\left(\frac{\lambda}{\mu}\right)^{k+1}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}, if\ \lambda \neq \mu \quad \dots (8)$$

and $$E(N_s) = \sum_{n=0}^{k}\frac{n}{k+1} = \frac{k}{2}, if\ \lambda = \mu \qquad \dots (9)$$

3. Average number of customers in the queue:

$$E(N_q) = E(N-1) = \sum_{n=1}^{k}(n-1)P_n$$

$$= \sum_{n=0}^{k} nP_n - \sum_{n=1}^{k} P_n$$

$$= E(N_s) - (1-P_0) \qquad \dots (10)$$

By little's formula,

$$E(N_q) = E(N_s) - \frac{\lambda}{\mu},$$

Which is true when the average arrival rate is λ through out. Now we see that, in step (10),

$1 - P_0 \neq \frac{\lambda}{\mu}$,because the average arrival rate is λ as long there is a vacancy in the queue and it is zero when the system is full.

Hence we define the overall effective arrival rate, denoted by $\lambda' or\ \lambda_{eff}$, by using step (8) & little's formula as

$$\frac{\lambda'}{\mu} = 1 - P_0 \quad (or)\lambda' = \mu(1 - P_0) \quad \dots(11)$$

Thus $E(N_q)$, can be rewritten as ,

$$E(N_q) = E(N_s) - \frac{\lambda'}{\mu}$$

Which is the modified little's formula for this model.

4. **Average waiting times in the system and in the queue**

By the modified Little's formula, $E(W_s) = \frac{1}{\lambda'} E(N_s)$

$$E(W_q) = \frac{1}{\lambda'} E(N_q)$$

Where λ' is the effective arrival rate, given by step (11).

Example: 5.1.c (1)

A one-person barber shop has 6 chairs to accommodate people waiting for hair cut. Assume that customers who arrive when all the 6 chairs are full leave without entering the barber shop. Customers arrive at the average rate of 3 per hour and spend an average of 15 min in the barber's chair.

(a) What is the probability that a customer can get directly into the barber's chair upon arrival?
(b) What is the expected number of customers waiting for a hair cut?
(c) How much time can a customer expect to spend in the barber shop?
(d) What fraction Potential customers are turned away?

Solution:

$\lambda = 3$ per hour

$\mu = \frac{1}{15}$ Minutes

$= \frac{1}{15}$ x 60 per hour

= 4 per hour

k= 6+1=7

$\rho = \frac{\lambda}{\mu} = \frac{3}{4}$

$P_0 = \frac{1-\rho}{1-\rho^{k+1}}$

$= \frac{1-\frac{3}{4}}{1-\left(\frac{3}{4}\right)^8}$

= 0.2778

(i) $L_s = \frac{\rho}{1-\rho} - \frac{(k+1)(\rho)^{k+1}}{1-\rho^{k+1}}$

$= \frac{\frac{3}{4}}{1-\frac{3}{4}} - \frac{8\left(\frac{3}{4}\right)^8}{1-\left(\frac{3}{4}\right)^8}$

$= \frac{\frac{3}{4}}{\frac{1}{4}} - \frac{(8)(3)^8}{(4)^8-(3)^8}$

= 2.11

$\frac{\lambda^1}{\mu} = 1 - P_0$ = 1-0.2778 = 0.7222

λ^1 = 4(0.7222) = 2.8888

$L_q = L_s - \frac{\lambda^1}{\mu}$

= 2.11 – 0.7222

=1.3878

$W_s = \frac{1}{\lambda^1} L_s$

$= \frac{1}{2.8888}$ (2.11)

= 0.7304 hours

= (0.7304) (60) minutes

= 43.8 minutes

P [N = 7] = $P_7 = \rho^7 P_0$

$= \left(\frac{3}{4}\right)^7 (0.2778)$

= 0.037

= 3.7%

EXERCISE: 5.1. (c)

1. Patients arrive at clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.
   (a) Find the effective arrival rate at the clinic.
   (b) What is the probability that an arriving patient will not wait.
   (c) What is the expected waiting time until a patient is discharged from the clinic?
   [Ans: (a) effective arrival rate=20/hr (b) P[arriving patient will not wait]=13 patients (c)Ws=39 min]

2. At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on the average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady- state probabilities for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard.

[Ans: (i) Ls=1.5 (ii) Ws=20 min]

5.1 (d) Characteristics of Finite Queue, Multiple Server Poisson Queue Model IV

$(M/M/S):(K/FIFO)$Model:

1. **Values of P_0 and P_n :**
   For the Poisson queue system, P_n is given by

$$P_n = \frac{\lambda_0\lambda_1\lambda_2 \ldots \lambda_{n-1}}{\mu_1\mu_2\mu_3 \ldots \mu_n} P_0 , n \geq 1 \quad \ldots (1)$$

Where $P_o = \left[1 + \sum_{n=1}^{\infty} \left(\frac{\lambda_0\lambda_1\lambda_2 \ldots \lambda_{n-1}}{\mu_1\mu_2\mu_3 \ldots \mu_n}\right)\right]^{-1}$ .... (2)

For this$(M/M/C):(K/FIFO)$Model:

$$\lambda_n = \begin{cases} \lambda, & for\ n = 0,1,2 \ldots (k-1) \\ 0, & for\ n = k, k+1 , \ldots \end{cases}$$

$$\mu_n = \begin{cases} n\mu, & for\ n = 0,1,2 \ldots (s-1) \\ s\mu, & for\ n = s, s+1 , \ldots \end{cases}$$

Using these values of λ_n and μ_n in (2) and noting that $1 < s < k$, we get

$$P_o{}^{-1} = \left\{1 + \frac{\lambda}{1!\,\mu} + \frac{\lambda^2}{2!\,\mu^2} + \cdots + \frac{\lambda^{s-1}}{(s-1)!\,\mu^{s-1}}\right\}$$
$$+ \left\{\frac{\lambda^s}{(s-1)!\,\mu^{s-1}.\mu s} + \frac{\lambda^{s+1}}{(s-1)!\,\mu^{s-1}.(\mu s)^2} + \ \dots \frac{\lambda^k}{(s-1)!\,\mu^{s-1}.(\mu s)^{k-s+1}}\right\}$$
$$= \sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{\lambda^s}{s!\,\mu^s}\left[1 + \frac{\lambda}{s\mu} + \left(\frac{\lambda}{s\mu}\right)^2 + \cdots + \left(\frac{\lambda}{s\mu}\right)^{k-s}\right]$$
$$= \sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{\lambda^s}{s!\,\mu^s}\sum_{n=s}^{k}\left(\frac{\lambda}{s\mu}\right)^{n-s} \qquad \dots(3)$$

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_{0,}\ for\ n \le s \\ \dfrac{1}{s!\,s^{n-s}}\left(\dfrac{\lambda}{\mu}\right)^n P_{0,}\ for\ s < n \le k \\ 0\,,\quad for\ n > k \end{cases}$$

2. Average queue length or average number of customers in the queue:

$$E(N_q) = E(N-s) = \sum_{n=s}^{k}(n-s)P_n$$

$$= \frac{P_0}{s!}\sum_{n=s}^{k}\frac{(n-s)\left(\frac{\lambda}{\mu}\right)^n}{s^{n-s}}\ [using\ 4]$$

$$= \frac{P_0\ \left(\frac{\lambda}{\mu}\right)^s}{s!}\sum_{x=0}^{k-s} x.\left(\frac{\lambda}{\mu s}\right)^s$$

$$= \frac{P_0\ \left(\frac{\lambda}{\mu}\right)^s}{s!}\sum_{x=0}^{k-s} x.\rho^{x-1} \quad \text{where } \rho = \frac{\lambda}{\mu s}$$

$$= \left(\frac{\lambda}{\mu}\right)^s \frac{P_0\ \rho}{s!}\sum_{x=0}^{k-s}\ \frac{d}{d\rho}(\rho^x)$$

$$= \left(\frac{\lambda}{\mu}\right)^s \frac{P_0\ \rho}{s!}\frac{d}{d\rho}\left[\frac{1-\rho^{k-s+1}}{1-\rho}\right]$$

$$= \left(\frac{\lambda}{\mu}\right)^s \frac{P_0\ \rho}{s!}\frac{d}{d\rho}\left[\frac{-(1-\rho)(k-s+1)\rho^{k-s}+(1-\rho)^{k-s+1}}{(1-\rho)^2}\right]$$

$$= \left(\frac{\lambda}{\mu}\right)^s \frac{P_0\ \rho}{s!}\left[\frac{-(k-s)(1-\rho)\rho^{k-s}+1-\rho^{k-s}(1-\rho+\rho)}{(1-\rho)^2}\right]$$

$$= P_0\ \left(\frac{\lambda}{\mu}\right)^s \frac{\rho}{s!(1-\rho)^2}\left[1-\rho^{k-s}-(k-s)(1-\rho)\rho^{k-s}\right]$$

3. **Average number of customers in the queue:**

$$E(N) = \sum_{n=0}^{k} nP_n = \sum_{n=0}^{s-1} nP_n + \sum_{n=s}^{k} nP_n$$
$$= \sum_{n=0}^{s-1} nP_n + \sum_{n=s}^{k} (n-s)P_n + \sum_{n=s}^{k} sP_n$$
$$= \sum_{n=0}^{s-1} nP_n + E(N_q) + s\{\sum_{n=0}^{k} P_n - \sum_{n=0}^{s-1} P_n\}$$
$$= E(N_q) + s - \sum_{n=0}^{s-1} (s-n)P_n \qquad \because \sum_{n=0}^{k} P_n = 1.$$

Obviously, $\{s - \sum_{n=0}^{s-1}(s-n)P_n\} \neq \frac{\lambda}{\mu}$, so that the above step represents Little's formula.

Now let us define overall effective arrival rate λ' or λ_{eff} as follows:

$$\frac{\lambda'}{\mu} = s - \sum_{n=0}^{s-1}(s-n)P_n$$
$$(i.e.,)\lambda' = \mu(s - \sum_{n=0}^{s-1}(s-n)P_n)$$
$$E(N) = E(N_q) + \frac{\lambda'}{\mu} ,$$

Which is the modified little's formula for this model.

4. **Average waiting time in the system and in the queue : By the modified Little's formula,**

$$E(W_s) = \frac{1}{\lambda'} E(N)$$

$$E(W_q) = \frac{1}{\lambda'} E(N_q)$$

Where λ' is the effective arrival rate.

Example: 5.1.d (1)

1. A car servicing station has 2 boys where service can be offered simultaneously. Because of space limitations, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the boys is exponentially distributed with μ = 8 cars per day. Find the average number of cars in the service station, the average number of cars waiting for service and the average time a car spends in the system.

   Solution:

   Given: s=2, k=4, λ=12 per day

   μ = 8 per day

   $\frac{\lambda}{\mu} = \frac{12}{8} = \frac{3}{2}$

Let $\rho = \frac{\lambda}{s\mu} = \frac{1}{s}\left(\frac{3}{2}\right) = \frac{1}{2}\left(\frac{3}{2}\right) = \frac{3}{4}$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \sum_{n=s}^{k} \rho^{n-s}\right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{1}{n!}\left(\frac{3}{2}\right)^n + \frac{1}{2!}\left(\frac{3}{2}\right)^2 \sum_{n=2}^{4} \left(\frac{3}{4}\right)^{n-2}\right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{1}{n!}\left(\frac{3}{2}\right)^n + \frac{1}{2}\left(\frac{9}{4}\right)\left[\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2\right]\right]^{-1}$$

$$= \left[\frac{1}{0!}\left(\frac{3}{2}\right)^0 + \frac{1}{1!}\left(\frac{3}{2}\right)^1 + \left(\frac{9}{8}\right)\left[\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2\right]\right]^{-1}$$

$$= \left[1 + \frac{3}{2} + \frac{9}{8}\left(\frac{37}{16}\right)\right]^{-1}$$

$$= \left[1 + \frac{3}{2} + \frac{333}{128}\right]^{-1}$$

$$= \left[\frac{653}{128}\right]^{-1} = \frac{128}{653} = 0.1960$$

$$\frac{\lambda^1}{\mu} = s - \sum_{n=0}^{s-1} (s-n)P_n$$

$$= 2 - \sum_{n=0}^{1} (2-n)P_n$$

$= 2 – [2P_0+P_1]$

$= 2 – 2P_0-P_1$

$= 2-2(0.1960)-0.294$ $\quad [\, P_1 = \frac{\lambda}{\mu}P_0 = \frac{3}{2}(0.1960) = 0.294\,]$

$= 1.314$

$\lambda^1 = \mu(0.314) = (8)\,(1.314) = 10.512$

$$L_q = \frac{P_0}{s!}\left(\frac{\lambda}{\mu}\right)^s \left[\frac{\rho(1-\rho^{k-s})}{(1-\rho)^2} - \frac{(k-s)\rho^{k-s+1}}{1-\rho}\right]$$

$$= \frac{0.1960}{2!}\left(\frac{3}{2}\right)^2 \left[\frac{\frac{3}{4}(1-\left(\frac{3}{4}\right)^{4-2})}{\left(1-\frac{3}{4}\right)^2} - \frac{(4-2)\left(\frac{3}{4}\right)^{4-2+1}}{1-\frac{3}{4}}\right]$$

$$= \frac{0.1960}{2}\left(\frac{3}{2}\right)^2\left[\frac{\frac{3}{4}(1-\left(\frac{3}{4}\right)^2)}{\left(\frac{1}{4}\right)^2} - \frac{(2)\left(\frac{3}{4}\right)^3}{\frac{1}{4}}\right]$$

$$= \frac{0.1960}{2}\left(\frac{3}{2}\right)^2\left[\frac{\frac{3}{4}(1-\left(\frac{3}{4}\right)^2)}{\left(\frac{1}{4}\right)^2} - \frac{(2)\left(\frac{3}{4}\right)^3}{\frac{1}{4}}\right]$$

$$= \frac{0.1960}{2}\left(\frac{9}{4}\right)\left[\frac{\frac{3}{4}\left[1-\frac{9}{16}\right]}{\frac{1}{16}} - \frac{(2)\frac{27}{64}}{\frac{1}{4}}\right]$$

$$= (0.2205)\left[\frac{21}{4} - \frac{27}{8}\right]$$

$$= (0.2205)\left[\frac{15}{8}\right] = 0.4134$$

∴ The average number of cars waiting for service in the queue is

L_q = 0.4134

$$L_s = L_q + \frac{\lambda^1}{\mu}$$

= (0.4134) +(1.314)

= 1.7274

∴ The number of cars in the service station is = 1.73 (approximately)

The average time spent in the system is

$$W_s = \frac{1}{\lambda^1} L_s$$

$$= \frac{1.7274}{10.512}$$

= 0.1643 day

= (0.1643) (24) hours

= 3.94 hours.

Exercise: 5.1(d)

1. At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20kms down the river. Tankers arrive according to a Poisson process with a mean of 1 every 2 hrs. It takes for an unloading crew, on the average 10 hour to unload a tanker, the unloading time following an exponential distribution.

(a) how many tankers are at the port on the average?

(b) How long does a tanker spend at the port on the average?

(c) What is the average arrival rate at the overflow facility?

[Ans: (a) 4 tankers (b)12.33 hrs (c) 0.1467 per hour]

2. A 2 person barber shop has 5 chairs to accommodate waiting customers.Potential customers, who arrive when all 5 chairs are full, leave without entering barber shop. Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber's chain compute P_0, P_1, P_7, $E[N_q]$ and $E[w]$.

[Ans; (i) P_0=0.429 (ii) P_1=0.343 (iii)P_7=0.0014 (iv)Lq=0.15 customer (v) Ws=14.20 min]

5.1(e) Finite Source Models

In previous models we have assumed that the population from which arrivals come (the calling population) is infinite.

We now treat a problem where the calling population is finite, say of size M, and future event occurrence probabilities are functions of system state.

A typical application of this model is that of machine repair, where the calling population is the machines, an arrival corresponds to a machine breakdown, and the repair crews are the servers.

Type I

(a) When s = 1 (only one server)

When there are n customers in the systems, then system is left with the capacity to accommodate M - n more customers.

Arrival rate of customers to the system will be $\lambda(M - n)$. That is for s = 1, we have

$$\lambda_n = \begin{cases} \lambda(M - n), & n = 0,1,2,\dots,M \\ 0, & n > M \end{cases}$$

$$\mu_n = \mu, \qquad n = 0,1,2,\dots,M$$

P_0 and P_n are the same as in the model $(M/M/S) : (\infty/FCFS)$

(i)Probability that the system is idle

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

(ii) Probability that there are n customers in the system

$$P_n = \frac{M!}{(M-n)!}\left(\frac{\lambda}{\mu}\right)^n P_0 \; , n = 0,1,2, \dots , M$$

(iii) Expected number of customers in the queue

$$L_q = \sum_{n=1}^{M}(n-1)\, P_n$$

$$= M - \left(\frac{\lambda+\mu}{\lambda}\right)(1-P_0)$$

(iv) Expected number of customers in the system

$$L_s = \sum_{n=0}^{M} n\, P_n = L_q + (1-P_0)$$

$$= M - \left(\frac{\mu}{\lambda}\right)(1-P_0)$$

(v)Expected waiting time of a customer in the queue

$$W_q = \frac{L_q}{\mu(1-P_0)} = \frac{1}{\mu}\left[\frac{M}{1-P_0} - \frac{\lambda+\mu}{\lambda}\right]$$

(vi)Expected waiting time of a customer in the system

$$W_s = W_q + \frac{1}{\mu} = \frac{1}{\mu}\left[\frac{M}{1-P_0} - \frac{\lambda+\mu}{\lambda} + 1\right]$$

Type II

When s >1: (more than one mechanic-multi-channel problem)

It is called Machine-Repair Problem, modeled as (M/M/S) : (M/GD) where M < s.

Here we are talking M machines (S > M). The breakdown (arrival) and service follow Poisson distribution with parameters λ and μ respectively.

We define $\boldsymbol{\lambda_n}$ and μ_n as the rate of breakdown per machine and service rate as follows

$$\lambda_n = \begin{cases} \lambda(M - n), & 0 \le n < M \\ 0, & n \ge M \end{cases}$$

$$\mu_n = \begin{cases} n\mu, & 0 \le n < s \\ s\mu, & n \ge s \end{cases}$$

The steady state equations are derived as $M\rho P_0 = P_1 \quad for\ n = 0$

$$\{(M-n)\rho + n\}P_n = (M-n+1)\rho P_{n-1} + (n+1)P_{n+1} \qquad for\ 0 < n < s$$

$$\{(M-n)\rho + s\}P_n = (M-n+1)\rho P_{n-1} + sP_{n+1} \qquad for\ s \le n < M$$

$$\rho P_{k-1} = sP_M \qquad for\ n = M$$

The expression for P_0 and P_n are obtained as

$$P_n = \begin{cases} \frac{1}{n!}\frac{M!}{(M-n)!}\left(\frac{\lambda}{\mu}\right)^n P_{0,}\ for\ 0 \le n \le s \\ \frac{M!}{(M-n)!}\frac{1}{s!\,s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_{0,}\ for\ s \le n \le M \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{M!}{n!(M-n)!}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} \frac{M!}{(M-n)!s!s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

Example: 5.1e (1)

In a machine repair station, the machine mechanic repairs four machines. Meantime between service requirements is 5 hours for each machine with exponential distribution and mean repair time is one hour with exponential distribution. Find

(a) Probability that the service will be idle.
(b) Average number of machine waiting to be repaired and being repaired.
(c) Expected time a machine will wait in queue to be repaired.

Solution:

Given arrival rate $\lambda = 1/5$ =0.2 machines/hour

Service rate $\mu = 1$ machines/hour
M=4 machines

$$\rho = \frac{\lambda}{\mu} = \left(\frac{1}{5}\right) X(1) = 0.2$$

(a)Probability that repair man will be idle:

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$
$$P_0 = \left[\sum_{n=0}^{4} \frac{4!}{(4-n)!}(0.2)^n\right]^{-1}$$
$$P_0 = \left[1 + \frac{4!}{3!}(0.2) + \frac{4!}{2!}(0.2)^2 + \frac{4!}{1!}(0.2)^3 + \frac{4!}{0!}(0.2)^4\right]^{-1}$$
$$P_0 = 1 + 0.8 + 0.48 + 0.192 + 0.000384$$

$$P_0 = [2.481]^{-1} = 0.4030$$

(b)Expected number of machines in the system:

$$L_s = M - \left(\frac{1}{P}\right)(1 - P_0)$$
$$L_s = 4 - \left(\frac{1}{0.2}\right)(1 - 0.4030)$$

$$L_s = 1.015 \text{ Machines}$$

(c)Expected time a machine will wait in the queue to be repaired

$$W_q = \frac{1}{\mu}\left[\frac{M}{1-P_0} - \frac{\lambda+\mu}{\lambda}\right] = \left[\frac{4}{0.597} - 6\right]$$

$$= 0.70 \text{ hour} = 42 \text{ minutes}$$

5.1. (f) Model D/D/1

This model is having single channel and determines inter-arrival time distribution along with deterministic service time distribution. In this model, the arrival rates and service rates are constant.

Example: 5.1 f (1)

A packing company has one labeling machine which labels packed box at the rate of 1 box each 2 seconds; The boxes arrive at the rate of 1 box every 4 seconds. If in the starting, there are 22 boxes to be labeled, and then find time which is required to label boxes that are waiting?

Solution:

From the data of problem, we have

a=4 seconds

b=2 seconds

n=22 boxes

Here b<a, therefore initial queue will diminish as proceeds and will eventually time required to label the waiting boxes:

$$T = \left(\frac{na-b}{a-b}\right) b$$

$$T = \left(\frac{(22)(4)-2}{4-2}\right)(2)$$

T =86 seconds.

5.2 QUEUES WITH IMPATIENT CUSTOMERS: BALKING AND RENEGING

The impatient customers generally behave in four ways,viz.

BALKING : A customer who leaves the queue because the queue is too long and he has no time to wait or has no sufficient waiting space.

RENEGING: This occurs when a waiting customer leaves the queue due to impatience**.**

JOCKEYING: Customers may jockey from one waiting line to another. This is most common in a "supermarket".

PRIORITY: In certain applications some people are served before other regardless of their order to arrival. These people have priority over others.

5.2 (a) Baling: M/M/1 Balking

$$\mathbf{P_n = P_0\left(\frac{\lambda}{\mu}\right)^n \prod_{i=1}^{n} bi - 1}$$

5.2 (b) M/M/1 Reneging

$$\mathbf{P_n = P_0\, \lambda^n \prod_{i=1}^{n} \frac{b_{i-1}}{\mu+r(i)},\ n} \geq 1$$

Where $\mathbf{P_0 = [1+\sum_{i=1}^{\infty} \lambda^n \prod_{i=1}^{n} \frac{b_{i-1}}{\mu+r(i)}\ ]^{-1}}$

5.3 PURE BIRTH PROCESS:

We have studied in the Poisson process in a interval of infinitesimal length h, the probability of exactly one occurrence is $\lambda h + O(h)$ and that of more than one occurrence is of $O(h)$. Here $O(h)$ is used as a symbol to denote a function of h which tends to O more rapidly than h i.e.,

As $h \to 0$, $\frac{O(h)}{h} \to 0$

Therefore in the interval $(t,t+h)$

$P[N(h) = 1] = \lambda h + O(h)$

$$\sum_{K=2}^{\infty} P[N(h) = K] = O(h)$$

(a)Birth and Death Process:

The birth and death process is a special type of continuous time discrete state space, stochastic process. W.K.T.

$$P_n(h) = P[N(h) = K/N(t) = n]$$

$$= \lambda_n h + O(h), \qquad K = 1$$

$$= O(h), \qquad K \geq 2$$

$$= 1 - \lambda_n h + O(h), \qquad K = 0$$

P (Number of deaths between *t* & *t+h* is K, number of individuals at time t is n)is given by,

$$q_n(h) = \mu_n h + O(h), \qquad K = 1$$

$$= O(h), \qquad K = 2$$

$$= 1 - \mu_n h + O(h), \qquad K = 0$$

Where $n \geq K$ and $\mu_0 = 0$.

(b)Equation of Birth And Death Process:

Let N(t)denotes the total number of individual at epoch'*t*' starting from *t=0*. Consider the interval 0 to *t+h*. Suppose that this is split up into 2 periods 0 to t and *t+h*. The events $\{N(t+h) = n, n \geq 1\}$ having the probability $P_n(t+h)$ can occur in a no. of mutually exclusive ways.

A_{ij} : $(n - i + j)$ Individuals by epoch *t* , *i*-birth and *j*-death between *t* & *t+h*, *i* , *j* =0,1.

Let $P_n(t) = P[N(t) = n]$, Then

$$P(A_{00}) = P_n(t)\big(1 - \lambda_n h + O(h)\big)(1 - \mu_n h + O(h))$$

$$= P_n(t)[1 - (\lambda_n + \mu_n)h + O(h)]$$

$$P(A_{10}) = P_{n-1}(t)\big(\lambda_{n-1} h + O(h)\big)(- \mu_{n-1} h + O(h))$$

$$= P_{n-1}(t)[\lambda_{n-1}h + O(h)]$$

$$P(A_{01}) = P_{n+1}(t)\big(1 - \lambda_{n-1}h + O(h)\big)(\mu_{n+1}h + O(h))$$

$$= P_{n+1}(t)[\mu_{n+1}h + O(h)]$$

& $$P(A_{11}) = P_n(t)\big(\lambda_n h + O(h)\big)(\mu_n h + O(h))$$

$$= O(h)$$

Therefore for $n \geq 1$, we have

$$P_n(t+h) = P_n(t)[1 - (\lambda_n + \mu_n)h] + P_{n-1}(t)[\lambda_{n-1}h] + P_{n-1}(t)[\mu_{n+1}h] + O(h)$$

$$\frac{P_n(t+h) - P_n(t)}{h} = -(\lambda_n + \mu_n)P_n(t) + P_{n-1}(t)[\lambda_{n-1}] + P_{n-1}(t)[\mu_{n+1}] + \frac{O(h)}{h}$$

As $h \to 0$, we have

$$P_n^1(t) = -(\lambda_n + \mu_n)P_n(t) + P_{n-1}(t)[\lambda_{n-1}] + P_{n-1}(t)[\mu_{n+1}] \leq 1 \ \ldots (i)$$

For $n=0$, we have

$$\frac{P_0(t+h) - P_0(t)}{h} = -(\lambda_0 + \mu_0)P_0(t) + P_1(t)(\mu_1) + \frac{O(h)}{h}$$

As $h \to 0$, we have

$$P_n^1(t) = -(\lambda_0)P_0(t) + P_1(t)(\mu_1)$$

The equations (i) and (ii) are known as equations of birth and death process.

Question	Option 1	Option 2	Option 3	Option 4	Answer
In Simplex iteration ,the pivot element can be------------------	zero	one	negative	positive	one
Linear programming problem involves ------ objective function.	four	three	two	one	one
Linear programming problems involving only ------- variables can be effectively solved by a graphical method	four	three	two	one	two
Linear programming problems involving only two variables can be effectively solved by a ------- method.	simplex	iteration	graphical	Big-M method	graphical
Pivot element is also called as -------	pivot row	key column	key row	key element	key element
An LPP is said to be infeasible if it has ---------------- that satisfies all the constraints	no solution	infinite solution	Unbounded solution	infeasible solution	no solution
An LPP solution when permitted to be infinitely large is called-------------------	Unbounded	bounded	infeasible	large	Unbounded
The leaving variable row is called -----------------	key row	key column	pivot column	leaving row	key row
The entering variable column is called -----------------	key row	pivot row	pivot row	entering row	pivot row
The intersection of the pivot column and pivot row is called the--------------------	pivot element	leaving element	unit element	first element	pivot element
The dual of the dual is the ---------------------	dual	primal	feasible	optimal	primal
If the primal is a maximization problem then the dual is a ---------------------- problem	maximization	linear programming	minimization	both maximization and minimization	minimization
The values of the dual variables are called----------------	shadow prices	cost prices	sales prices	zero prices	shadow prices
If either the primal or the dual problem has an unbounded solution, then the other problem has ---------------------------	feasible solution	no feasible solution	optimal solution	basic solution	no feasible solution
If a primal constraint is a strict inequality, then the corresponding dual variable is --------------------- at the optimum and vice versa.	positive	negative	zero	non zero	zero
A Feasible solution to a LPP which is also a basic solution to the problem is called ---	basic solution	basic feasible solution	non basic feasible solution	optimal solution	basic feasible solution
The solution which optimizes the objective function are called --------------	feasible solution	optimal solution	optional solution	arbitrary solution	optimal solution
A basic solution is said to be a ------------- if one of more of the basic variables are Zero	Non degenerate basic solution	infeasible solution	degenerate basic solution	unbounded solution	degenerate basic solution
More than two decision variables problem in LPP cannot be solved by -------	simplex method	Big-M method	Graphical method	Dual simplex method	Graphical method
More than two decision variables problem in LPP can be solved by -------	simplex method	Big-M method	Graphical method	Dual simplex method	simplex method
Another name for simplex method is -----------------------	computational procedure	computational method	Big-M method	Dual simplex method	computational procedure
Constraints in the canonical LPP are of ---------.	>=	=	<=	<	<=
In standard form all constraints are expressed as ----------------.	>=	=	<=	<	=
The minimization of the function f(x) is equivalent to the maximization of --------	-(-f(x))	f(x)	-f(-x)	1 / f(x)	-(-f(x))
In ------------- the entering variable is first calculated	Simplex method	Big-M method	graphical method	any one of these	Simplex method

Optimum solution of LPP in a Simplex procedure is always -------------.	un bounded	feasible	degenerate	basic feasible solution	basic feasible solution
A ------------- BFS is BFS in which none of the basic variables are Zero.	Degenerate	infeasible	non-degenerate	optimum	non-degenerate
The coefficient of slack variables in the constraints is -----------------	1	0	2	-1	1
In simplex method all the variables must be--------------------	negative	non-negative	have the same sign	initially zero	non-negative
The coefficient of slack variables in the objective function is -----------------	1	0	2	-1	0
Every equality constraint can be replaced equivalently by ---------------- inequalities.	two	three	one	four	two
The standard form of the constraint 4x-5y < 20 is ---------------------	4x-5y =20	4x-5y +S1=20	4x+5y = 20	4x-5y - S_1=20	4x-5y +S1=20
The set of feasible solutions to an LPP is a --------------	convex set	null set	concave	finite	convex set
The feasible region of an LPP is always------------------	convex	upward	downward	a straight line	convex
A typical LPP must have at least -------------- decision alternatives.	three	two	one	many	two
The number of alternatives in a LPP is typically---------------	finite	infinite	infeasible	feasible	finite
LPP deals with the problems involving only ------------- objective	one	two	more than one	more than two	one
Constraints appear as ---------------------- when plotted in a graph	curve	straight line	point	circle	straight line
The linear function is to be maximized or minimized is called-----------	objective function	subjective function	optional function	odd function	objective function
LPP is a technique of finding the ------------	optimal solution	approximate solution	both	infeasible solution	optimal solution
LPP is ---------------.	optimal solution	approximate solution	both	infeasible solution	optimal solution
In an LPP maximization model -------------------	A constraint optimization model	A constraint decision making model	A mathematical programming model	A constraint inventory model	A constraint optimization model model
A Feasible solution to a LPP which is also a basic solution to the problem is called ---	the objective function is formed	the objective function is minimized	the objective function is maximized over the allowable set of decisions	the constraints are formed	the objective function is maximized over the allowable set of decisions
For the optimal solution of an LPP, existence of an initial feasible solution is always	Assumed	given	does not exists	zero	Assumed
The solution x_1 =0, x_2 =3 is -------------------	feasible	in feasible	degenerate	non degenerate	degenerate
The solution x_1 =5, x_2 =3 is -------------------	feasible	in feasible	degenerate	non degenerate	non degenerate
If Minimum (Z) = -5, then the maximum (Z) =-----------------	–5	5	4	-4	5
If the solution space is unbounded ,then the objective value will always be-------------	bounded	unbounded	feasible	infeasible	unbounded
Programming is another word for --------------------	planning	organizing	managing	decision making	planning
Maximize Z= $c_1x_1+c_2x_2+\ldots\ldots\ldots\ldots+c_nx_n$ is called------------------	constraints	objective function	feasible function	optimal function	objective function
An example of objective function--------------	Max Z = $4x^2$ + 5y	Max Z = $4x^2$ + $5y^2$	Max Z = 4x+ 5y	4x+ 5y <= 30	Max Z = 4x+ 5y

Optimization means----------------------	maximization of profit	minimization of constraints	minimization of profit	maximization of cost	maximization of profit
The variables that appear in the objective function are called-----------------	decision variables	non decision variables	optimal variables	feasible variables	decision variables
The element of intersection of the pivot column and pivot row is called the ------	pivot row	pivot column	keyrow	pivot element	pivot element
In maximization problem, the entering variable is the non-basic variable corresponding to the most negative value of ------	$z_j + c_j$	$z_j - c_j$	z_j / c_j	$z_j * c_j$	$z_j - c_j$
The Big-M method is otherwise known as ---------------------------	method of penalties	the two phase method	simplex method	computational method	method of penalties
The purpose of introducing artificial variables is just to obtain an/a -------------------------	arbitrary solution	feasible solution	initial basic feasible solution	unbounded solution	initial basic feasible solution
The solution satisfies the constraints but does not optimize the objective function since it contains a very large penalty M and is called ------------	feasible solution	pseudo-optimal solution	non basic feasible solution	infeasible solution	pseudo-optimal solution
If no artificial variable remains in the basis and the optimality condition is satisfied, then the current solution is an/a -----------------------	non basic feasible solution	optimal basic feasible solution	degenerate basic solution	optional solution	optimal basic feasible solution
The variables in primal-dual problems are -------------------	non-negative	negative	zero	non zero	non-negative

Question	Option 1	Option 2	Option 3	Option 4	Answer
Cells in the transportaion table having positive allocation will be called --------------------------	occupied cells	unoccupied cells	empty cells	zero cells	occupied cells
In a transportaion problem the various a's and b's are called ---------------------	Supply	demand	rim requirement	destination	rim requirement
The unit transportation cost from the ith source to jth destination is displayed in the ------------- of the (i j)th cell.	upper left side	upper right side	lower left side	lower right side	upper left side
A balanced transportation problem will always have a ------------------	unique solution	infinite number of solution	infeasible solution	feasible solution	feasible solution
A feasible solution to a (mxn) transportation problem that contains no more than m+n-1 non negative allocations is called -------------	Feasible solution	optimul solution	basic feasible solution	infeasible solution	basic feasible solution
The Objective of the transportation problem is to be -----------------------	Maximum	Minimum	Either maximum or minimum	Neither maximum nor minimum	Minimum
When total supply is equal to total demand, the problem is called a ----------------------- transportation problem	Feasible	Infeasible	Unbalanced	balanced	balanced
When total supply is ----------- the total demand, the dummy source is added in the matrix with zero cost vectors	less than	greater than	greater than or equal to	less than or equal to	less than
Penalty method is a -----------------------	North - west corner rule	least cost method	VAM method	MODI method	VAM method
Matrix minima method is a ----------------------------	North - west corner rule	least cost method	VAM method	MODI method	least cost method
Every loop has an even number of cells and atleast ----------------	Two	Four	Six	Eight	Four
For a ---------------- to exist, it is necessary that the total supply equal to total demand	unique solution	infinite number of solution	infeasible solution	feasible solution	feasible solution
A feasible solution is said to be an optimul solution if it ------------------ the total transportation cost	Minimize	Maximize	Either maximize or minimize	Neither maximize nor minimize	Minimize
The number of basic variables in an mxn balanced transportation problem is atmost ----------------------	m + n	m + n -1	m + n +1	m - n -1	m + n -1
The number of non basic variables in an mxn balanced transportation problem is atleast -------------------	mn - (m + n -1)	mn - (m - n -1)	mn - (m + n + 1)	mn - (m - n + 1)	mn - (m + n -1)
The ------------- non zero allocations imply that one cannot from a closed circuit by joining positive allocations by horizontal and vertical lines only	independent	dependent	linear	non linear	independent
In a transportaion problem, the cost of transportation is ---------------------	Linear	non linear	zero	one	Linear
For a feasible solution to exist, it is necessary that the total supply equal to total ---------------------	cost	demand	cells	rows	demand
Least cost method is also called ---------------------	Matrix method	Matrix maxima method	Matrix minima method	minima method	Matrix minima method
VAM method is also called ---------------------	Penalty method	Matrix minima method	Lower cost method	Hungerian method	Penalty method
Vogel's approximation method is a -----------------------	Penalty method	Matrix minima method	Hungerian method	Heuristic method	Heuristic method
In a VAM method allocations are made so that the penalty cost is ----------------------	minimised	maximized	zero	non zero	minimised
The MODI method is based on the concept of ----------------------	Linear Programming Problem	Heuristic method	Matrix maxima method	duality	duality
Every loop has an ----------------- number of cells	even	odd	equal	zero	even
Closed loops may or may not be ---------------- in shape	circle	square	rectangle	triangle	square
When obtaining an initial basic feasible solution we may have ---------------- m+n-1 allocations	non zero	zero	less than	greater than	less than
When total supply is ----------- the total demand, the dummy destination is added in the matrix with zero cost vectors	less than	greater than	greater than or equal to	less than or equal to	greater than

In a north west corner rule, if the demand in the column is satisfied, one must move to the ---------- cell in the next column.	left	right	middle	corner	right
Row wise and column wise difference between two minimum costs is calculated under ---------------- method.	North - west corner rule	least cost method	VAM method	MODI method	VAM method
An optimum solution results when net costs change values of all unoccupied cells are ---------------	positive and greater than zero	negative	non negative	positive and lesser than zero	non negative
MODI method associated with transportation problem, MODI stands for -----------------	Modified distribution	Multiple distribution	Matrix distribution	Modified distinction	Modified distribution
Transportation problem is a sub class of ------------------------------	Linear Programming Problem	Integer Programming Problem	Non Linear Programming Problem	Dynamic Programming Problem	Linear Programming Problem
The calculation of oppportunity cost in the MODI method is analogous to a ------------------------	the net evaluation value for non basic variable columns in the simplex method	value of a variable in xB column of the simplex method	variable in the B - column in the simplex method	variable in the Y - column in the simplex table	the net evaluation value for non basic variable columns in the simplex method
If we were to use opportunity cost value for an unused cell to test optimality, it should be -----------------	equal to zero	most negative number	most positive number	any value	most negative number
A basic feasible solution to a (mxn) transportation problem is said to be a ----------- basic feasible solution if it contains exactly m+n-1 non negative allocations in independent positions.	degenerate	non degenerate	unique	infinite number of	non degenerate
A basic feasible solution to a (mxn) transportation problem is said to be a ----------- basic feasible solution if it contains less than m+n-1 non negative allocations in independent positions.	degenerate	non degenerate	unique	infinite number of	degenerate
In a transportation problem, least cost method gives a better solution to than ----------------------	North - west corner rule	least cost method	VAM method	MODI method	North - west corner rule
The dummy source or destination in a transportation problem is added to ----------------	satisfy rim conditions	prevent solution from becoming degenerate	ensure that total cost does not exceed a limit	it is a balanced one	satisfy rim conditions
The occurrence of degeneracy while solving a transportation problem means that ------------------	total supply equals total demand	the solution so obtained is not feasible	the few allocations become negative	the solution so obtained is feasible	the solution so obtained is not feasible
An alternative optimul solution to a minimization transportation problem exists whenever opportunity cost corresponding to unused route of transportation is ---------	positive and greater than zero	positive with atleast one equal to zero	negative with atleast one equal to zero	negative and lesser than zero	positive with atleast one equal to zero
One disadvantage of using North -west corner rule to find initial solution to the transportation problem is that --------------------	it is complicated to use	it does not take into account cost of transportation	it leads to a degenerate initial solution	it leads to non degenerate initial solution	it does not take into account cost of transportation
The solution to a transportation problem with m-rows (supplies) and n-columns (destination) is feasible if number os positive allocations are--------	m + n	m x n	m + n +1	m + n -1	m + n -1
Ina transportation problem, ------------- gives a better starting solution than Least Cost method.	North - west corner rule	least cost method	VAM method	MODI method	VAM method
A transportation problem can always be represented by --------------------	balanced model	unbalanced model	simplex model	graphical model	balanced model
In a transportation model, north west corner rule starting solution is recommended because it ensure that there will be -------------- allocations	m + n	m x n	m + n +1	m + n -1	m + n -1
The transportation model is restricted to dealing with a --------------------- commodity only.	single	multiple	positive	negative	multiple
The transportation technique essentially uses the same steps of the -------------------	simplex method	graphical method	Big M method	dual simplex method	simplex method
For any transportation problem, the coefficients of all units in the constraints are ------------	zero	any value	unity	unique	unity
A solution that satisfies all conditions of supply and demand but it may or may bot be optimal is called ------------------	feasible solution	infeasible solution	basic feasible solution	initial feasible solution	initial feasible solution
To solve degeneracy, an occupied cell with ------------cost is converted into occupied cell by assigning infinitely small amount to it.	lowest	larger	unit	zero	lowest
The number of ---------------- in an mxn balanced transportation problem is atmost m+n-1	basic variables	non basic variables	decision variables	non decision variables	basic variables
A ------------ transportation problem will always have a feasible solution	unique	unbalanced	balanced	distinct	balanced

In a transportation problem, the total transportation cost is -----------------------	minimum	maximum	zero	unique	minimum
In ------------- method, allocations are made so that the penalty cost is minimized.	North - west corner rule	least cost method	VAM method	MODI method	VAM method
The initial solution of a transportation problem can be obtained by applying any known method. However, the only condition is that------------	the solution be optimal	the rim conditions are satisfied	the solution not be degenerate	the solution be unique	the rim conditions are satisfied

Question	Option 1	Option 2	Option 3	Option 4	Answer
The penalty costs that are incurred as a result of running out of stock are known as----	shortage cost	set-up cost	holding cost	production cost	shortage cost
Holding cost is denoted by -------	C_1	C_2	C_3	C_4	C_1
Shortage cost is denoted by ------	C_1	C_2	C_3	C_5	C_2
Set-up cost is denoted by --------	C_1	C_2	C_3	C_4	C_3
Elapsed time between the placement of the order and its receipts in inventory is known as -------.	lead time	recorder level	EOQ	variables	lead time
---------- may be defined as the stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business affairs	Inventory	Transportation	Queueing	Sequencing	Inventory
Rate of consumption is different from ---------.	rate of change	rate of production	rate of purchasing	either b or c	either b or c
Cost associated with carrying or holding the goods in stock is known as -------.	interested capital cost	handling cost	holding cost	production cost	holding cost
-------- is the interest change over the capital invested.	interested capital cost	handling cost	holding cost	production cost	interested capital cost
-------- include costs associated with movement of stock, such as cost of labour etc.	interested capital cost	handling cost	holding cost	production cost	handling cost
-------- per unit item is affected by the quantity purchased deu to quantity discounts or price breaks.	interested capital cost	handling cost	holding cost	purchase price	purchase price
If P is the purchase price of an item and I is the stock holding cost per unit time expressed as a fraction of stock value then the holding cost is ----------.	I/P	I + P	I – P	IP	IP
Reduction in procurement cost ------- EOQ	increases	decreases	reduces	neutral	reduces
An approximate percentage of 'C' items in a firm is around --------	60 – 65%	65 – 70%	70 – 75%	75 – 80%	70 – 75%
Economic order quantity results in equilization of -------- cost and annual inventory cost.	annual procurement cost	procurement cost	inventory cost	shortage cost	annual procurement cost
Economic order quantity results in equilization of annual procurement cost cost and ------- cost.	annual inventory cost	procurement cost	inventory cost	shortage cost	annual inventory cost
A company uses 10,000 units per year of an item. The purchase price is one rupee per item. Ordering cost = Rs. 25 per order. Carrying cost 12% of the inventory value. Find the EOQ.	2000 units	2083 units	2038 units	2050 units	2083 units
---------- is the time when we should place an order by taking into consideration the interval between placing the order and receiving the supply.	lead time	recorder level	EOQ	variables	recorder level
--------- is that size of order which minimises total annual cost of carrying inventory and the cost of ordering under the assumed conditions of certainty and that annual demands are known.	lead time	recorder level	EOQ	variables	EOQ
EOQ means -------	Economic Order Quantity	Economic Order Quality	Economic Offer Quality	Economic Offer Quantity	Economic Order Quantity
EOQ is also known as -------	economic lot size formula	economic short size formula	economic formula	economic variables	economic lot size formula
------- include holding cost, set up cost, shortage costs and demand.	EOQ	controlled variables	uncontrolled variables	basic variables	uncontrolled variables
Economic order quantity results in -------	reduced stock – outs	increased stock – outs	equilisation of carrying cost and procurement costs	favourable procurement price	equilisation of carrying cost and procurement costs
The EOQ of an item which cost is Rs.36 and carrying cost is 1.5 % per month, the economic order quantity is ------	240 no's	200 no's	400 no's	500 no's	200 no's

In the ABC analysis, C items are those which have -------	low unit price	low cost price	low usage value	low consumption	low consumption
For an item with storage cost of each item Rs. 1, set up cost Rs.25, demand 200 units per month C_{min} is ------	Rs.100	Rs.400	Rs.500	Rs.800	Rs.100
Minimum inventory equals	EOQ	Reorder level	Safety stock	lead time	Safety stock
If the procurement cost per order increases 21%, the economic order quantity of the item shall increase by -------.	10%	20%	30%	40%	10%
If EOQ is 5000 units and Buffer stock is 500 units calculate max inventory.	5500 units	500 units	5000 units	5050 units	5500 units
If EOQ is 5000 units and Buffer stock is 500 units calculate minimum inventory.	5500 units	500 units		5050 units	5000 units
Reorder level = ---------	normal lead time x monthly	monthly consumption	normal lead time - monthly consumption	monthly consumption	normal lead time x monthly consumption
An approximate percentage of A- items in a firm is around ------	5 - 10 %	10 – 20 %	20 – 25 %	70 – 75 %	5 - 10 %
Total inventory cost = -------	set up cost + purchasing cost	holding cost + shortage cost	set up cost + purchasing cost + holding cost + shortage cost	setup cost + shortage cost	set up cost + purchasing cost + holding cost + shortage cost
Storage cost is associated with -------	holding cost	shortage cost	carrying cost	set up cost	carrying cost
Average inventory = -------	(EOQ/2) + Safety stock	(EOQ/2) - Safety stock	(EOQ/2) / Safety stock	(EOQ/2) * Safety stock	(EOQ/2) + Safety stock
------ discounts reduce material cost and procurement costs	quantity	quality	carrying cost	set up cost	quantity
The ordering cost is independent of ------	ordering quantity	ordering quality	carrying cost	set up cost	ordering quantity
Given maximum lead time as 20 days and normal lead time is 15 days with annual consumption 12,000 units find the buffer stock.	176 units	167 units	157 units	186 units	167 units
Given R = 1000 units/year I = 0.30, C = Re.0.50/unit, C_3 = Rs.10/order. Find minimum average cost.	54.77	55.77	53.77	50.77	54.77
The set up cost in inventory situation is ------- of size of inventory.	dependent	independent	large	small	independent

Question	Option 1	Option 2	Option 3	Option 4	Answer
Hungarian method is also known as -------------------	Matrix method	penalty method	Matrix minima method	reduced matrix method	reduced matrix method
Hungarian method is based on the concept of the ---------------------	optimal cost	opportunity cost	duality cost	lowest cost	opportunity cost
If the number of rows is not equal to the numbers of columns in the cost matrix of the given assignmnet problem than it is said to be ---------------	balanced	unbalanced	equal	not equal	unbalanced
If each row and each column contain exactly on encircled zero then the current assignment is ------------------------	unique	distinct	optimal	not optimal	optimal
If at least one row/column is without an assignmnet then the current assignment is ------------------	unique	distinct	optimal	not optimal	not optimal
Assignment algorithm is only for ----------------- problem	minimization	maximization	Either maximum or minimum	Neither maximum nor minimum	minimization
The maximization assignment can be converted into a minimization assignmnet problem by ------------ from the highest element to all the elements of the given table.	adding	subtracting	multiplying	dividing	subtracting
An optimal assignment requires that the maximum number of lines which cab be drawn through squares with zero opportuinty cost be equal to the number of --------------------	rows or columns	rows and columns	rows + colums -1	rows - columns -1	rows or columns
while solving an assignment problem, an activity is assigned to a resource through a square with zero opportunity cost because the objective is to ---------	minimize the total cost of assignment	reduce the cost of assignemnt to zero	reduce the cost of that particular assignement to zero	maximize the total cost	minimize the total cost of assignment
The purpose of dummy row or column in an assignment problem is to----------------	obtain balance between total activities and total resources	prevent a solution from becoming degenerate	provide a means of representing a dummy problem	prevent a solution from becoming non degenerate	obtain balance between total activities and total resources
If there were n workers and n jobs there would be ------------------------	n solutions	(n-1)! Solutions	(n+1)! Solutions	n! solutions	n! solutions
The total assignment value of ith machine is ---------------------	1	2	3	4	1
The assignment problem represents at problem with all demands and supplies equal to -----------------	1	2	3	4	1
An assignment problem always a ------------------ form of a transportation problem.	independent	dependent	degenerate	non degenerate	degenerate
An assignment problem is said to be ----------------- if No.of rows = No.of columns.	balanced	unbalanced	equal	not equal	balanced
An assignment problem is said to be ----------------- if No.of rows ≠ No.of columns.	balanced	unbalanced	equal	not equal	unbalanced
The transportation technique or simplex method cannot be used to solve the assignmnet problem because of --------	independent	dependent	degeneracy	non degeneracy	degeneracy
Every basic solution in the assignment problem is ---------------	independent	dependent	degenerate	non degenerate	degenerate
The assignment problem can be solved by the-------------------	transportation problem	simplex problem	inventory problem	simulation problem	transportation problem
Every ------------- in the assignment problem is tansportation problem.	feasible solution	unique solution	basic solution	multiple solution	basic solution
An efficient method for solving an assignment problem is the -------------------------	Penalty method	Matrix minima method	Hungerian method	Heuristic method	Hungerian method
The assignment problem is a particular case of ------------------------	Linear Programming Problem	Integer Programming Problem	Non Linear Programming Problem	Dynamic Programming Problem	Linear Programming Problem
The objective of the assign a number of tasks to equal number of facilities at a ---------------- cost.	Maximum	Minimum	Either maximum or minimum	Neither maximum nor minimum	Minimum
The asssignment problem can be stated in the form of mxn matrix called a ----------------------	cost matrix	unit matrix	zero matrix	scalar matrix	cost matrix
In an assignment problem, source represents ---------------------	supply	jobs	facilities	origins	facilities
Destination represents -------------------	supply	jobs	facilities	origins	jobs
The probability of an empty system is given by ---------------.	1 – (l / m)	l / (m – l)	l / m (m – l)	l / m	1 – (l / m)
A Queuing system can be completely described by --------------.	The input, the service mechanism	The input, the service mechanism and the queue discipline	The input, the service mechanism, the queue discipline and customer behaviour	The input, the service mechanism and customer behaviour	The input, the service mechanism, the queue discipline and customer behaviour

The probability of Queue size ≥ N is ------------.	$r^n / (1– r)$	r^n	$1– r^n$	$(1– r) / r^n$	r^n
If l = 3 , m = 2 then r = ------	1.5	3	2	0.6	1.5
Average waiting time in the queue is given by -----------	1/m - l	l	m	1/m + l	l
For a salesmanwho has to visit n cities, following are the ways of his tour plan -----------	n!	(n+1)!	(n-1)!	n	(n-1)!
In (a / b / c) : (d / e), a is called -------------.	Departure distribution	Queue discipline	Arrival distribution	Number of units	Arrival distribution
In (a / b / c) : (d / e), c is called -------------.	Departure distribution	Queue discipline	Arrival distribution	Number of units	Number of units
If the arrivals are completely random, then the probability distribution of a number of arrivals in a fixed-time interval follows a ------- distribution	Poisson	normal	binomial	polynomial	Poisson
The ---------- describes the way in which the customers arrive and join the system	service mechanism	input	queue discipline	customer behaviour	input
The arriving people in a queueing system are called ------------.	Input	servers	customers	queue	customers
Mean service time is denoted by ----------.	m	l	1 / l	1 / m	m
The traffic intensity in queueing is defined by ----------.	p / (p–1)	m / l	l / m	1 / (1– p)	l / m
A system is said to be in --------- state when its operating characteristics are dependent on time	Steady	arrival	service	transient	transient
A system is said to be in --------- state when the behaviour of the system is independent on time.	Steady	arrival	service	transient	Steady
A customer who leaves the queue because the queue is too long then his behaviour is said to be ----------.	Reneging	balking	jockeying	priorities	balking
The Birth–death model is called -----------.	M / M / 1	M / M / N	M / M / ¥	M / M / 2	M / M / 1
Average queue length in (M / M / 1) : (∞ / FCFS) is --------------.	(1– r) / r	r / (1– r)	$r / (1– r^2)$	$r^2 / (1– r)$	$r^2 / (1– r)$
The expected waiting time in the queue is calculated by the formula ------------.	1 / (m – l)	b) l / (m – l)	l / m (m – l)	l / m	l / m (m – l)
In Birth–death model, the probability distribution of queue length is given by --------.	$r^n / (1– r)$	b) $r^2 / (1– r)$	r / (1– r)	$(1– r) / r^n$	$r^n / (1– r)$
First In First Out (FIFO) is known as the --------------.	Input	service mechanism	customer behaviour	queue discipline	queue discipline

Question	Option 1	Option 2	Option 3	Option 4	Answer
An activity wgich must be completed before one or more other activities start is known as ------------------ activity	Predecessor	successor	initial	final	Predecessor
An activity which started immediately after one or more of the other activities are completed is known as -----------------	Predecessor	successor	initial	final	successor
An activity which does not consumes either any resources and/or time is known as -----------------------	Predecessor	successor	dummy	initial	dummy
If an activity B can start immediately after an activity A, then A is called ---------------------	immediate predecessor	immediate successor	Predecessor	successor	immediate predecessor
If an activity B can start immediately after an activity A, then B is called ---------------------	immediate predecessor	immediate successor	Predecessor	successor	immediate successor
A ----------- is defined as a combination of interrelated acitivities all of which be executed in a certain order to achieve a goal	Project	Acitivity	Event	Nodes	Project
----------is a task or an item of work to be done in aproject.	Project	Acitivity	Event	Nodes	Acitivity
An ------------ is represented by an arrow with a node at the beginning and a node at the end indicating the start and finish of the activity.	Project	Nodes	Event	Acitivity	Acitivity
Nodes are denoted by ------------------	dot	circle	arrow	square	circle
The diagram in which arrow represents an activity is called ------------------	arrow diagram	network diagram	graph diagram	line diagram	arrow diagram
The initial node are also called --------------	head event	tail event	first event	last event	tail event
The terminal node are called -------------	head event	tail event	first event	last event	head event
Critical path plays a very important role in project ------------- problems	scheduling	planning	controlling	network	scheduling
An activity is defined as the difference between the latest start and the earliest start of the activity is called ----------------	free float	total float	independent float	interfering float	total float
If the total float is ------------ then it may indicate that the resources for the activity are more that adequate.	positive	negative	zero	any value	positive
If the total float of an activity is -------------- it may indicate that the resources are just adequate for that activity.	positive	negative	zero	any value	zero
If the total float is ------------ , it may indicate that the resources for that activity are inadequate.	positive	negative	zero	any value	negative
The notation ' A < B ' is called -----------------	A is a predecessor of B	B is a successor of A	A is a successor of B	B is a predecessor of A	A is a predecessor of B
The notation ' B > A ' is called ----------------	A is a predecessor of B	B is a successor of A	A is a successor of B	B is a predecessor of A	B is a successor of A
Activities which have no predecessors are called---------------- activity	dummy	start	zero	terminal	start
All the start activities can be made to have the ---------------- initial node.	same	different	multiple	zero	same
Activities which have no successor are called---------------- activity	dummy	start	zero	terminal	terminal
The diagram denoting all the activities of a project by arrows taking into account the technological square of the activities is called -------------	Project	nework	project network	Event	project network
There is another representation of a project network representing activities on nodes called ---------------	AON diagram	ANO diagram	NOA diagram	arrow diagram	AON diagram
only ------------ activity should connect any two nodes.	one	two	three	multiple	one
Path, connecting the first node to the very last terminal node of long duration in any project network is called ---------------	PERT	Critical path	Activity	network	Critical path
All the activities in any critical path are called -----------------	start activities	dummy activities	critical activities	terminal activities	critical activities
In PERT ananlysis, the critical path is obtained by joining event having --------------------	positive slack	negative slack	non zero slack	unique slack	positive slack
In PERT network each activity time assumes a Beta - distribution because ----------------	it is a uni - model distribution that provides informing regarding the uncertainty of time estimates.	it has got finite non negative error	it need not be symmetrical about model value	it has infint negative error	it is a uni - model distribution that provides informing regarding the uncertainty of time estimates.
Float or slack analysis in useful for ------------------	projects behind the schedule only	projects ahead of the schedule only	projects behind the planning only	projects ahead of the planning only	projects behind the schedule only
In time cost trade off funtion analysis ----------------	cost decreases linearly as time increases	cost at normal time is zero	cost increases linearly as time increases	cost at normal time is unity	cost decreases linearly as time increases
A dummy activity is used in the network diagram when ------------------	two parallel activities have the same tail and head events	the chain of activities may have a common event not yet be independent by themselves	two parallel activities have the different tail and head events	If the activities have the tail and head events	two parallel activities have the same tail and head events

The path of least cost float in a project is called ----------------	PERT	Critical path	unique path	network path	Critical path
The project duration is affected if the duration of any activity is --------------	changed	unchanged	same	exist	changed
The number of time estimates involved ina PERT problem is ---------------	1	2	3	4	3
For a non critical activity, the total float is --------------------	zero	non zero	unique	distinct	non zero
The objective of network analysis is to ----------------	minimize total project duration	minimize total project cost	minimize prduction delay	minimize the interruption	minimize total project duration
The slack for an activity is equal to -----------------	LF - LS	EF - ES	LS - ES	LS - EF	LS - ES
PERT stands for -------------------	Project Enumaration review Technique	Project Evaluation Review Technique	Planning Evaluation Review Technique	Planning Enumaration review Technique	Project Evaluation Review Technique
Generally PERT technique does not deals with the project of	repetitive nature	non repetitive nature	deterministic nature	non deterministic nature	non deterministic nature
The technique of OR used for planning, scheduling and controlling large and complex projects are often referred as -------	network analysis	graphical analysis	critical activities	PERT	network analysis
A network is a -----------------	quality plan	control plan	graphical plan	inventory plan	graphical plan
Critical path method is used for commpletion of projects involving activities of --------------------------	repetitive nature	non repetitive nature	deterministic nature	non deterministic nature	repetitive nature
An event which represents the joint completion of more than one activity is known as -------------------	unique event	burst event	merge event	dummy event	merge event
An event which represents the initiation of more than one activity is known as ---------------	unique event	burst event	merge event	dummy event	burst event
Events in the network diagram are identified by ---------------------	numbers	variables	symbols	special characters	numbers
The negative value of the independent float is --------------	one	zero	distinct	non zero	zero
In PERT the span of time between the optimistic and pesimistic time estimates of an acitivity is ------------	3σ	6σ	12σ	4σ	6σ
If an activity has zero slack, it implies that --------------------	it lies on the critical path	it is a dummy activity	the project is progressing well.	the project is not progressing well	it lies on the critical path
The amount of time by which the activity can be rescheduled with effecting the preceding or succeeding of that activity is called ------------	free float	total float	independent float	interfering float	independent float
The slack of the head event j is called the ---------------------- of an activity i-j.	free float	total float	independent float	interfering float	interfering float
Interfering float of i-j is the difference between the total float and ------------------	free float	dependent float	independent float	dummy float	free float
All the critical activities have their total float as -----------------	one	two	zero	any value	zero
Critical path can also be defined as the path of ---------------------	least total float	greatest total float	least free float	greatest free float	least total float
The probability to complete a project in the expected time is ----------------	1	1.5	0.5	1.15	0.5
The name of the proability distribution (used PERT) which estimates the expected duration and the expected variance of an activity is -----------	Beta distribution	Gamma distribution	poisson distribution	normal distribution	Beta distribution
(L - E) of an aevent i-j is called the -------------- of the event j.	slack	surplus	dummy	total	slack
One of the portion of the total float is -----------------	free float	total float	independent float	interfering float	free float
rescheduling that activity without affecting the succeding activity we can use ----------------- of an activity	free float	total float	independent float	interfering float	free float
Free float is ------------------ the total float of an activity	equal to	greater than or equal to	less than or equal to	not equat to	less than or equal to