KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University

Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

LECTURE PLAN
DEPARTMENT OF MATHEMATICS

STAFF NAME: Dr. K.KALIDASS COURSE CODE: 18MMP402

COURSE NAME: STOCHASTIC PROCESSES0. CLASS: II M. Sc. MATHEMATICS

SEMESTER: IV

S. No	Lecture Duration Hour	Topics To Be Covered	Support Materials
UNIT-I			
1	1	Definitions and examples on stochastic processes	R2 Ch 1, 50-51
2	1	Definitions and Examples on Markov chains	R1 Ch 2, 63-65
3	1	Theorems on higher transition probabilities	R1 Ch 2, 70-74
4	1	Theorems on classification of states and chains	R1 Ch 2, 78-80
5	1	Recapitulation and Discussion of possible questions	
Total number of hours planed for unit I			**5 hours**
UNIT-II			
1	1	Markov process with discrete state space	R3 Ch 2, 101-103
2	1	Theorems on Poisson process	R1 Ch 3, 138-155
3	1	Theorems on birth and death processes	R6 Ch 5, 217-225
4	1	Theorems on continuous time Markov Chains	R1 Ch 3, 176-180
5	1	Recapitulation and Discussion of possible questions	
Total number of hours planed for unit II			**5 hours**
UNIT-III			
1	1	Markov processes with continuous state space	R1 Ch 2, 122-124
2	1	Some simple properties of Weiner process	R1 Ch 4, 198-200
3	1	Kolmogrov equations	R1 Ch 4, 201-202
4	1	First passage time distribution for Weiner process	R1 Ch 4, 202-203
5	1	Examples and Problems on Ornstein and Uhlenbech process	R1 Ch 4, 203-205
6	1	Recapitulation and Discussion of possible questions	
Total number of hours planed for unit III			**6 hours**
UNIT-IV			
1	1	Introduction to Branching Processes	R4 Ch 11, 302-310
2	1	Distribution of the total number of progeny	R1 Ch 9, 359-361

3	1	Continuous- Time Markov Branching Process	R1 Ch 9, 371-377
4	1	Age dependent branching process	R1 Ch 9, 377
5	1	Bellman-Harris process	R1 Ch 9, 378
6	1	Recapitulation and Discussion of possible questions	
Total number of hours planed for unit IV			**6 hours**
UNIT-V			
1	1	Queuing model M/M1	R5 Ch 9, 267
2	1	Transient behavior of M/M/1 model	R1 Ch 10,392
3	1	Birth and death process in Queuing theory	R1 Ch 10,392
4	1	Non birth and death Queuing process	R1 Ch 10,395
5	1	Recapitulation and Discussion of possible questions	
6	1	Discussion on previous year ESE questions	
7	1	Discussion on previous year ESE questions	
8	1	Discussion on previous year ESE questions	
Total number of hours planed for unit V			**8 Hours**

SUGGESTED READINGS

R1 Medhi, J., (2006). Stochastic Processes, 2nd Edition, New age international Private limited, New Delhi

R2 Basu, K., (2003). Introduction to Stochastic Process, Narosa Publishing House, New Delhi

R3 Goswami and Rao, B. V., (2006). A Course in Applied Stochastic Processes, Hindustan Book Agency, New Delhi

R4 Grimmett, G. and Stirzaker D., (2001). Probability and Random Processes, 3rd Ed., Oxford University Press, New York

R5 Papoulis.A and Unnikrishna Pillai,(2002). Probability, Random variables and Stochastic Processes, Fourth Edition, McGraw-Hill, New Delhi.

R6 V. Sundarapandian, (2009), Probability, Statistics and Queueing theory, PH1 learning private limited, New Delhi.

Possible questions

1. Describe about Polya's urn model
2. Derive exponential distribution from geometric distribution
3. State and prove Chapman Kolmogrov equation
4. Let two gamblers, A and B, initially have k dollars and $\boldsymbol{m}$ dollars, respectively. Suppose that at each round of their game, A wins one dollar from B with probability $\boldsymbol{p}$ and loses one dollar to B with probability $q = 1 - p$. Assume that A and B play until one of them has no money left. Let X_n, be A's capital after round n, where $n = 0, 1, 2, \ldots$ and X_0 = k.
   (i) Show that X_n is a Markov chain with absorbing states.
   (ii) Find its transition probability matrix P.
5. Consider a Markov chain with state space $S = \{0,1,2,\ldots\}$ and having
$$P = \begin{pmatrix} p_1 & p_2 & p_3 & p_4 & \cdots \\ p_1 & p_2 & p_3 & p_4 & \cdots \\ 0 & p_1 & p_2 & p_3 & \cdots \\ 0 & 0 & p_1 & p_2 & \cdots \\ 0 & 0 & 0 & p_1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$ Prove that $V(s) = \frac{\left(1-P'(1)\right)(1-s)P(s)}{P(s)-s}$
6. Show that exponential distribution has Markov property
7. Show that geometric distribution has Markov property
8. Write the transition matrices for the walks on the two connected graphs of order 3.
9. Write the transition matrices for the walks on the six connected graphs of order 4.
10. Draw graphs corresponding to the following transition matrices.

$$a. \begin{bmatrix} 0 & 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 \end{bmatrix} \qquad b. \begin{bmatrix} 0 & 1/3 & 1/3 & 0 & 1/3 \\ 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \end{bmatrix}$$

Question	Choice1	Choice 2	Choice3	Choice 4	Answer
Let X(t) be the number of customers waiting for service in a service facility. Then S is ----------- state space	continuous	discrete	neither continuous nor discrete	both continuous and discrete	discrete
Let X(t) be the number of customers waiting for service in a service facility. Then S =	{0,1,2,…,100}	{1,2,…,100}	{0,1,2,…}	{1,2,…}	{0,1,2,…}
Let X(t) be the number of customers waiting for service in a service facility. Then {X(t), t>=0} is a ------ time stochastic process with discrete state space	continuous	discrete	neither continuous nor discrete	both continuous and discrete	continuous
Let X(t) be the number of customers waiting for service in a service facility. Then {X(t), t>=0} is a continuous time stochastic process with ------ state space	continuous	discrete	neither continuous nor discrete	both continuous and discrete	discrete
The one step transition probability is -------------	stochastic	random	neither stochastic nor random	both stochastic and random	both stochastic and random
The ------ step transition probability is stochastic	1	2	3	all the above	all the above
Consider a game where a coin tossed repeatedly. If the coin turns head, the player wins a dollar which happens with p, otherwise the player losses all his winnings. Let X_n denote the player's fortune after the nth toss. Then X_(n+1)=0 with probability	p	1-p	0	1	1-p
Consider a game where a coin tossed repeatedly. If the coin turns head, the player wins a dollar which happens with p, otherwise the player losses all his winnings. Let X_n denote the player's fortune after the nth toss. Then X_(n+1)=X_n+1 with probability	p	1-p	0	1	p
Consider a game where a coin tossed repeatedly. If the coin turns head, the player wins a dollar which happens with p, otherwise the player losses all his winnings. Let X_n denote the player's fortune after the nth toss. Then {X_n:n≥0} is a ------- process	Markov	stochastic	random	all the above	all the above
Consider a game where a coin tossed repeatedly. If the coin turns head, the player wins a dollar which happens with p, otherwise the player losses all his winnings. Let X_n denote the player's fortune after the nth toss. Then {X_n:n≥0} is a ------- process	Markov	non Markov	neither Markov nor non Markov	both Markov and non Markov	Markov
At least one of the eigen value of tpm is	0	1	2	3	1
Which of the following cannot be an eigen value of tpm P?	0	1	-1	2	2
Eigen values of tpm lies between	0 and 1	1 and 2	-1 and 1	-1 and 0	-1 and 1
Absolute value of eigen values of a tpm is	1	0	less than or equal 1	less than or equal 0	less than or equal 1
A state j is said to be accessible from a state i if p_ij^((n))----0 for some n	less than	less than or equal	greater than	greater than or equal	greater than
A state j is said to be --------from a state i if p_ij^((n))>0 for some n	accessible	not accessible	neither accessible nor accessible	both accessible and not accessible	accessible
A state j is said to be accessible from a state i if p_ij^((n))>0 for ------- n	all	finite	infinite	some	some
States i and j are said to communicate if	j is accessible from i	i is accessible from j	both of them	neither of them	both of them
If i and j are communicate and i is positive recurrent then j is	positive	recurrent	positive recurrent	not positive recurrent	positive recurrent
If i and j are communicate and i is ------- then j is positive recurrent	positive	recurrent	positive recurrent	not positive recurrent	positive recurrent
A continuous non negative random variable has memory less property iff it is ---------------- random variable	Poisson	exponential	geometric	binomial	exponential
A continuous non negative random variable has --------- property iff it is exponential random variable	Markov	memory less	neither Markov nor memory less	either Markov or memory less	either Markov or memory less
A------ non-negative random variable has has memory less property iff it is exponential random variable	continuous	discrete	both continuous and discrete	neither continuous nor discrete	continuous
A continuous random variable has constant hazard rate iff it is an ------ random variable	Poisson	exponential	geometric	binomial	exponential

Possible questions

1. Find the Laplace transform of Poisson distribution
2. Prove that if P is t.p.m then $\boldsymbol{P^n}$ is also t.p.m
3. Find the eigen values of P where P is a t. p. m
4. Consider a Markov chain with two states and transition probability matrix

$$P = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

   (i) Find the stationary distribution of the chain.

   (ii) Find $lim_{n\to\infty}\ P^n$

5. Consider a Markov chain with two states and transition probability matrix

$$P = \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}, 0 \le p \le 1$$

   Find the stationary distribution of the chain.

6. Prove that $p_{jk}^{(m+n)} = \sum_r p_{jr}^{(m)} p_{rk}^{(n)}$ for all $r \in S$
7. Give examples of two stochastic matrices that cannot be transition matrices of graph walks
8. Consider transitions from A, referring to the first row of Display 5.4 as needed. Explain why $P(A \to G) = 0$ for the new chain whose states are A, C, G, T, AG, AGC, and AGCT. What other transitions from A have probability 0? Write the first row of the new transition matrix.
9. Consider transitions from AG. Which three of the seven states cannot be reached from AG? For which destination states are the transition probabilities from AG the same as from G? What is the transition probability from AG to AGC? Write the row of the transition matrix for AG.
10. The limiting probabilities for this Markov chain are:

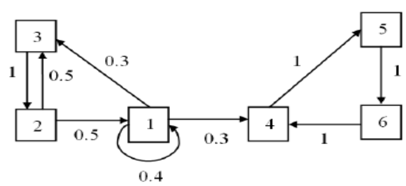
A	C	G	T	AG	AGC	AGCT
0.301	0.185	0.130	0.294	0.070	0.015	0.005

What does this tell you about the average distance between “restriction sites”, that is, between occurrences of AGCT?

Question	Choice1	Choice 2	Choice3	Choice 4	Answer
A -------- random variable has constant hazard rate iff it is an exponential random variable	continuous	discrete	both continuous and discrete	neither continuous nor discrete	continuous
A continuous random variable has ------- hazard rate iff it is an exponential	constant	non zero	zero	1	constant
A continuous random variable has constant --------rate iff it is an exponential	hazard rate	mean	variance	SD	hazard rate
Hazard rate of exponential random variable is	constant	non zero	zero	1	constant
Minimum of k independent exponential random variables is ----- random variable	Poisson	exponential	geometric	binomial	exponential
------ of k independent exponential random variables is an exponential random variable	Maximum	Minimum	sum	difference	Minimum
Minimum of k independent -------random variables is an exponential random variable	Poisson	exponential	geometric	binomial	exponential
Sum of iid exponential random variables is ------- random variable	Poisson	exponential	geometric	Erlang	Erlang
Sum of iid --------- random variables is an Erlang random variable	Poisson	exponential	geometric	binomial	exponential
Sum of ----- exponential random variables is an Erlang random variable	independent	identical	neither independent nor identical	both independent and identical	both independent and identical
Counting process is called ------- process	Poisson	exponential	geometric	binomial	Poisson
---------process is called Poisson process	Counting	sum	Minimum	difference	Counting
Shifted Poisson process is a -------- process	Poisson	exponential	geometric	binomial	Poisson
Shifted ---------- process is a Poisson process	Poisson	exponential	geometric	binomial	Poisson
A Poisson process has ------- increments	stationary	transient	neither stationary nor transient	both stationary and transient	stationary
A ------- process has stationary increments	Poisson	exponential	geometric	binomial	Poisson
A Poisson process has ------- increments	independent	dependent	neither independent nor dependent	both independent and dependent	independent
A ------- process has independent increments	Poisson	exponential	geometric	binomial	Poisson
A Poisson process has ------- increments	independent	stationary	neither stationary nor independent	both stationary and independent	both stationary and independent
Counting process has ------- increments	stationary	transient	neither stationary nor transient	both stationary and transient	stationary
A ------- process has stationary increments	Counting	sum	Minimum	difference	Counting
Counting process has ------- increments	independent	dependent	neither independent nor dependent	both independent and dependent	independent
------- process has independent increments	Counting	sum	Minimum	difference	Counting
Counting process has ------- increments	independent	stationary	neither stationary nor independent	both stationary and independent	both stationary and independent
Poisson process is a	DTDS	DTCS	CTDS	CTCS	CTCS
Counting process is a	DTDS	DTCS	CTDS	CTCS	CTCS
Rate –Equality principle is _____________	Rate down=rate up	Rate up =rate down	Rate up < rate down	Rate up > rate down	Rate up =rate down
The rate enters state n _____the rate leaves the state n	greater	lesser	equals	Not equal	equals
Rate in = Rate out is known as _____principle	Rate-equality	Rate-inequality	Inequality-rate	Equality-rate	Rate-equality

Possible questions

1. Classify states of a Markov chain.
2. Describe about ergodic state with examples
3. Classify states of a Markov chain with t.p. m $P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
4. Draw the transition graph of $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}$ and examine whether the chains are irreducible
5. Construct a t.p.m form given transition graph and classify the states



6. Suppose **P** is a 5x5 stochastic matrix with elements p_{ij} with i, j = 1, 2, ..., 5. Write an expression in terms of the p_{ij}, for $p^{(2)}[3, 5] = p^{(2)}{}_{3,5}$.
7. Limiting distributions. Find enough examples to support answers to the following questions:
   a. Does every graph walk have a limiting distribution?
   b. Does any graph walk have more than one limiting distribution?
   c. Does the limiting distribution ever depend on the starting vertex? Always depend on the starting vertex?
8. Use the relationship between $\mathbf{p}^{(n)}$ and $\mathbf{p}^{(n+1)}$ to find an equation that the equilibrium vector $\boldsymbol{x}$ must satisfy.
9. If the transition matrix **P** of a Markov chain is symmetric ($\mathbf{P} = \mathbf{P}^T$) what can you say about the stationary distribution of the chain?
10. Suppose $\mathbf{u} = (1/k, 1/k, \ldots, 1/k)$ is a stationary distribution for a Markov chain with transition matrix **P**. What must be true of the column sums of **P**?

Question	Choice1	Choice 2	Choice3	Choice 4	Answer
In M/M/1 queue the arrival from _____ source	Finite	infinite	open	none	infinite
Arrival time is ______distribution	Poisson	gamma	f	exponential	Poisson
The inter arrival time is _____distribution	Poisson	gamma	f	exponential	exponential
In M/M/1 queue number of servers______	1	n	n+1	C	1
Service time is ______distribution	gamma	exponential	poisson	F	exponential
The queue discipline is __________	FCFS	SIRO	random	none	FCFS
A stochastic process is {Xn,n>0}is known as_____chain	Non-markov	markov	Random	none	Markov
Markov chain is ________on future	independent	dependent	same	none	independent
P is known as _________	Traffic intensity	arrival	service	independent	Traffic intensity
The transition from state n to______	n+1	n	1-n	N^2	n+1
Steady state is ________ property	memorable	memoryless	none	system	memoryless
Steady state of M/M/1 is ________distribution	Poisson	geometric	erlang	gamma	Geometric
Number of customers in the system_____	E{N}	E{N-1}	E{N+1}	E{n}	E{N}
TPM stands for_______	Transient probability matrix	Transient poisson matrix	Transition probability matrix	none	Transition probability matrix

Possible questions

1. Prove that $P_j(t) = \frac{e^{-\lambda t}(\lambda t)^j}{j\,!}, j = 0,1,2 \cdots$.
2. Consider the Markov chain having state space $S = \{0,1,2\}$ and T.P.M $P = \begin{pmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{4} & \frac{1}{4} & 0 \end{pmatrix}$. Find the invariant measure.

3. Show that in a finite-state Markov chain, not all states can be transient

4. Find the p.g.f of branching processes.

5. Find the distribution of branching process

6. Find the pgf of branching process

7. Find the mean and variance of branching process

8. If Y follows branching distribution, show that $E[Y] = \mu$

9. Find the recurrence formula for branching distribution

10. Describe about branching distribution.

Question	Choice1	Choice 2	Choice3	Choice 4	Answer
P is a square matrix with row sum=__________	0	2	1	≠1	1
A nonnegative square matrix with unit row sum is____	Stochastic matrix	Square matrix	matrix	Non-negative matrix	Stochastic matrix
Markov chain $P_{jk}(n)$ depend on n is known as	Non-homogeneous	singular	random	Homogeneous	Homogeneous
P^2 is ______ step probability	4	0	2	1	2
Poisson process is a ____ process with constant birth rate	death	renewal	birth	none	birth
Poisson process also introduced as ______ process	Renewal	Non renewal	stable	numerable	Renewal
Pure birth process also known as_______process	Yule-furry	Erlang	Finite Erlang	none	Yule-furry
Memory loss property is also known as_____	Yule-furry	Erlang	Markov	Randomness	Markov
Poisson process is purely a _____event	uniform	straight	random	none	random
λ is a parameter of _____ process	output	Inter arrival	input	Inter service	input
The output process is Poisson with same parameter as______ process	output	input	Inter arrival	none	input
A queue with limited waiting space is known as______	Infinite buffer	buffer	Finite buffer	none	Finite buffer
In M/M/1/K queueing model K denotes____	Number of servers	Queue discpline	Infinite source	Limited waiting space	Limited waiting space
When the system contains K customer ____ is not allowed	departure	service	arrival	Rate out	arrival
In M/M/1/K the system cannot exceed___	n	K+1	K	K-1	K
Limited waiting space queueing model contain _____customers in system	K-1	K	K+1	1+k	K

Possible questions

1. Find the expected number of customers in $M/M/1$ queue
2. Show that $P_0 = 1 - \rho$ where $\rho = \frac{\lambda}{\mu}$ for $M/M/1$ queue
3. Find the expected waiting time of customers in $M/M/1$ queue
4. Find the pgf of Poisson process
5. Derive Erlong loss formula
6. Prove that $\rho < 1$ is a necessary condition for existence of steady states in M/M/1 queue
7. In an M/M/1 queue, prove that $p_n^*(s) = \frac{(1-z_2)(z_2)^n}{s}, n \geq 0$
8. Find the probability that the server to be idle in $M^X/\ M\ /1$ queue
9. Find the expected waiting customers in M/M/1/N
10. Find the expected waiting time of customers in the system of M/E_k/1 queue

Question	Choice1	Choice 2	Choice3	Choice 4	Answer
If system contains K customers then only ____ is possible	departure	arrival	interarrival	none	departure
In M/M/1/K model rate out _____rate in	≠	+	-	=	=
The probability sum =_____	0	2	1	none	1
If a< 1 the waiting space model turns to___	M/M/C	M/M/1/n	M/M/1	none	M/M/1
The c denotes in M/M/c model is_____	Arrival	service	Service channels	intensity	Servicechannel
P_n (h)=o(h) if	n=1	n=0	n=-1	n>=2	n>=2
P_n (h)= ----- if n>=2	0	1	-1	o(h)	o(h)
P_n (h)=o(h) if n	2	>2	neither =2 nor >2	either =2 or >2	either =2 or >2
P_100 (h)=	0	1	-1	o(h)	o(h)
P_1000 (h)=	0	1	-1	o(h)	o(h)
P_(-1) (h)=	0	1	-1	o(h)	0
P(z) converges for	z=1	z<1	either z=1 or z<1	neither z=1 nor z<1	either z=1 or z<1
P(z)	converges	absolutely converges	neither converges nor absolutely converges	diverges	absolutely converges
|P(z)|=	0	1	-1	∞	1
Which of the following distribution has memoryless property?	geometric	exponential	neither geometric nor exponential	both geometric and exponential	both geometric and exponential
E[X(X - 1)] =	P’(1)	P’’(1)	0	1	P’’(1)
The Poisson distribution is a —- probability distribution	continuous	discrete	neither continuous nor discrete	both continuous and discrete	discrete
--------- is a discrete distribution	Poisson	geometric	neither Poisson nor geometric	both Poisson and geometric	both Poisson and geometric
The geometric distribution is a —- probability distribution	continuous	discrete	neither continuous nor discrete	both continuous and discrete	discrete
The binomial distribution is a —- probability distribution	continuous	discrete	neither continuous nor discrete	both continuous and discrete	discrete
The exponential distribution is a —- probability distribution	continuous	discrete	neither continuous nor discrete	both continuous and discrete	continuous
--------- is a discrete distribution	Poisson	binomial	neither Poisson nor binomial	both Poisson and binomial	both Poisson and binomial
--------- is a discrete distribution	binomial	geometric	neither binomial nor geometric	both binomial and geometric	both binomial and geometric
--------- is a discrete distribution	Poisson	exponential	neither Poisson nor exponential	both Poisson and exponential	Poisson