

**(i)Theory****Course Objectives**

To enhance the fundamental knowledge in Physics and its applications relevant to various streams of Engineering and Technology.

**Course Outcomes**

Upon completion of this course,

- 1 The students will gain knowledge on the basics of properties of matter and its applications, The students will acquire knowledge on the concepts of Laser and optical devices, The students will have adequate knowledge on the concepts of thermal properties of materials and their applications in expansion joints and heat exchanges, The students will get knowledge on advanced physics concepts of quantum theory and its applications in tunneling microscopes, and The students will understand the basics of crystals, their structures and different crystal growth techniques.

**UNIT I – PROPERTIES OF MATTER**

Elasticity –Three types of modulus of elasticity – basic definitions, relation connecting the moduli (Derivation)-factors affecting elastic modulus and tensile strength–Poisson’s ratio- Torsional pendulum- bending of beams – bending moment – uniform and non-uniform bending – I-shaped girders - stress due to bending in beams.

**UNIT II – LASER AND FIBER OPTICS**

Introduction – emission and absorption process- Einstein’s coefficients derivation. Types of LASER – CO<sub>2</sub>, Semiconductor LASER- Applications of LASER in industry and medicine.

Total internal reflection – modes of propagation of light in optical fibers – numerical aperture and acceptance angle –derivations, types of optical fibers (Material, refractive index and mode) – fiber optical communication system (block diagram).

**UNIT III – THERMAL PHYSICS**

Transfer of heat energy – thermal expansion of solids and liquids – expansion joints – bimetallic strips - thermal conduction, convection and radiation – heat conductions in solids – thermal conductivity – Forbe’s and Lee’s disc method: theory and experiment – conduction through compound media (series and parallel) – thermal insulation – applications: heat exchangers, refrigerators, ovens and solar water heaters.

## **UNIT IV – QUANTUM PHYSICS**

Introduction to quantum theory – Black body radiation – dual nature of matter and radiation – de Broglie wavelength, uncertainty principle – Schrödinger's wave equation – time dependent and time independent equations – particle in one dimensional box- physical significance of wave function, scanning electron microscope.

## **UNIT V – CRYSTAL PHYSICS**

Single crystalline, polycrystalline and amorphous materials – single crystals: unit cell, crystal systems, Bravais lattices, directions and planes in a crystal, Miller indices – inter-planar distances – Coordination number and packing factor for SC, BCC, FCC, HCP – crystal Imperfections: point defects, line defects

## **SUGGESTED READINGS**

Bhattacharya D.K. & Poonam T., (2015) Engineering Physics, Oxford University Press.  
Gaur R.K. and Gupta S.L, (2012), Engineering Physics, Dhanpat Rai Publications  
Pandey .B.K. & Chaturvedi .S, (2012), Engineering Physics, Cengage Learning India  
Halliday.D., Resnick R. & Walker. J, (2015), Principles of Physics, Wiley  
Serway R.A and Jewett J.W., (2010), Physics for Scientists and Engineers with Modern Physics, Thomson Brooks/Cole Publishing Co  
Tipler P.A. and Mosca G.P, (2007)., Physics for Scientists and Engineers with Modern Physics, W.H. Freeman

**KARPAGAM ACADEMY OF HIGHER EDUCATION**  
(Deemed to be University Established under Section 3 of UGC Act 1956)  
**COIMBATORE – 641021**  
**FACULTY OF ENGINEERING**  
**DEPARTMENT OF SCIENCE AND HUMANITIES**  
**LECTURE PLAN**

**Subject : ENGINEERING PHYSICS**

**Code : 19BTBT241**

Unit No.	List of Topics	No. of Hours
<b>UNIT I</b>	<b>Properties of matter</b>	
	Elasticity , Three types of modulus of elasticity	1
	Basic definitions ,relation connecting the moduli (Derivation)	1
	Factors affecting elastic modulus and tensile strength	1
	Tutorial	1
	Poisson's ratio , Torsional pendulum	1
	Bending of beams, bending moment	1
	Uniform and non-uniform bending	2
	I- shaped girders, stress due to bending in beams	1
	Tutorial	1
	<b>TOTAL</b>	<b>10</b>
<b>UNIT – II</b>	<b>Laser and fiber optics</b>	
	Introduction , emission and absorption process	1
	Einstein's coefficients derivation	1
	Types of LASER - CO <sub>2</sub> , Semiconductor LASER	1
	Applications of LASER in industry and medicine	1
	Tutorial	1
	Total internal reflection , modes of propagation of light in optical fibers	1
	numerical aperture and acceptance angle –derivations	1
	types of optical fibers (Material, refractive index and mode)	1
	fiber optical communication system (block diagram)	1
	Tutorial	1
	<b>TOTAL</b>	<b>10</b>
<b>UNIT – III</b>	<b>Thermal Physics</b>	
	Transfer of heat energy	1
	Thermal expansion of solids and liquids	1
	Expansion joints , bimetallic strips	1
	Thermal conduction, convection and radiation	1
	Tutorial	1
	Heat conduction in solids, thermal conductivity	1
	Forbe's and Lee's disc method :theory and experiment	1
	conduction through compound media (series and parallel)–thermal insulation	1
	Applications : heat exchangers, Refrigerators , ovens and solar water heaters	1

	Tutorial	1
	<b>TOTAL</b>	<b>10</b>
<b>UNIT – IV</b>	<b>Quantum physics</b>	
	Introduction to quantum theory	1
	Blackbody radiation	1
	dual nature of matter and radiation de Broglie wavelength	1
	Uncertainty principle	1
	Tutorial	1
	Schrödinger's wave equation – time dependent and time independent equations	2
	particle in one dimensional box- physical significance of wave function	1
	Scanning electron microscope	1
	Tutorial	1
	<b>TOTAL</b>	<b>10</b>
<b>UNIT – V</b>	<b>Crystal Physics</b>	
	Single crystalline, poly crystalline and amorphous materials	1
	Single crystals: unit cell, crystal systems	1
	Bravais lattices	1
	Directions and planes in a crystal	1
	Tutorial	1
	Miller indices	1
	inter-planar distances - Coordination number and packing factor for SC ,BCC crystal	1
	inter-planar distances - Coordination number and packing factor for FCC, HCP – crystal	1
	Imperfections: pointdefects, line defects	1
	Tutorial	1
	<b>TOTAL</b>	<b>10</b>
	<b>TOTAL NO OF HOURS</b>	<b>50</b>

**TEXT BOOK& REFERENCES:**

S.no	Author(s) Name	Title of the Book	Publisher	Year of Publicatio
1	Bhattacharya D.K. & Poonam T.	Engineering Physics	Oxford University Press	2015
2	Gaur R.K. and Gupta S.L	Engineering Physics	Dhanpat Rai Publications	2012
3	Pandey. B.K. & Chaturvedi .S	Engineering Physics	Cengage Learning India	2012
4	Halliday. D., Resnick R. & Walker. J	Principles of Physics	Wiley	2015
5	Serway R. A and Jewett J. W	Physics for Scientists and Engineers with Modern Physics	Thomson Brooks/Cole Publishing Co.	2010
6	Tipler P.A. and Mosca G.P	Physics for Scientists and Engineers with Modern Physics	W.H. Freeman	2007

**WEBSITES:**

- |  |
|--|
| 1. <a href="http://www.nptel.ac.in">www.nptel.ac.in</a><br>2. <a href="http://www.ocw.mit.edu">www.ocw.mit.edu</a> |
|--|

**STAFF IN-CHARGE****HOD**

**UNIT I****PROPERTIES OF MATTER****INTRODUCTION - ELASTICITY**

Elasticity is the property by virtue of which a body offers resistance to any deforming force and regains its original condition when the deforming force is removed. All bodies can be deformed by the action of external forces. Bodies which can completely regain their original condition of shape and size on removal of deforming forces are said to be perfectly elastic. Bodies which retain their deformed nature even after the removal of the deforming forces are said to be perfectly plastic. If external forces fail to produce any deformation or relative displacements of the particles of the body, the body is said to be perfectly rigid. It is defined as the distance between any two points in a body is unaltered due to application of force.

**STRESS AND STRAIN****Stress:**

Stress is defined as the restoring force per unit area which brings back the body to its original state from the deformed state. As long as no permanent change is produced in the body, the restoring force is equal to the force applied. Unit of stress is  $\text{N/m}^2$

**Types of Stresses:****i. Normal Stress:**

When the force is applied perpendicular to the surface of the body, then the stress applied is called as normal stress.

**ii. Tangential stress:**

When the force is applied along the surface of the body, then the stress applied is called as tangential stress or shearing stress.

**Strain:**

The change produced in the body due to change in dimension of a body under a system of forces of Equilibrium is called Strain. It has no unit.

$$\text{Strain} = \frac{\text{Change in Dimension}}{\text{Original Dimension}}$$

**Types of Strain****i. Longitudinal (or) Tensile Strain:**

It is defined as the ratio between the changes in length to the original length, without any change in its shape, after the removal of the external forces.

**ii. Shearing strain:**

It is defined as the angular deformation produced on the body due to the application of external tangential forces on it.

**iii. Volumetric strain:**

It is defined as the ratio between the changes in volume to the original volume, without any change in its shape.

**HOOKE'S LAW**

Robert Hooke proposed a relation between stress and strain and is named as Hooke's law by his name. According to this law, Stress is directly proportional to the strain produced, within the elastic limit.

$$\text{i.e.,} \quad \text{Stress} \propto \text{Strain}, \quad \text{Stress} = E \times \text{Strain}, \quad E = \frac{\text{Stress}}{\text{Strain}} \text{ Nm}^{-2}$$

Where, E - modulus of Elasticity and the value of E depends upon the nature of the material.

**CLASSIFICATION OF ELASTIC MODULUS**

Depending on the three types of strain, there are three types of modulus they are,

- i. Young's modulus(Y) (or) modulus corresponding to longitudinal (or) tensile strength
- ii. Bulk modulus(K) (or) modulus corresponding to the volume strain
- iii. Rigidity modulus(n) (or) modulus corresponding to the shearing strain

**i. Young's modulus (Y)**

It is defined as the ratio between the longitudinal stress to longitudinal strain, within the elastic limits.

$$\text{Young's modulus} = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} \quad \text{----- (1)}$$

**Explanation:**

Let us consider a wire of length L with an area of cross section A. Let one end of the wire is fixed and the other end is loaded (or) stretched as shown in Fig 1.1.

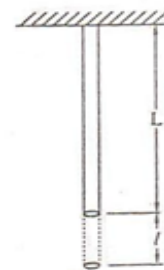


Fig 1.1

Let l be change in length due to the action of force, then

$$\text{The longitudinal stress} = \frac{\text{Longitudinal force}}{\text{Area}} = \frac{F}{A}$$

$$\text{and} \quad \text{The longitudinal strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{l}{L}$$

$$\text{Young's modulus, } Y = \frac{F/A}{l/L} = \frac{FL}{Al} \text{ Nm}^{-2} \text{ (or) pascal}$$

**ii. Bulk modulus (K)**

It is defined as the ratio between the volume stress (or) bulk stress to the volume strain (or) bulk strain within the elastic limit.

$$\text{Bulk's modulus} = \frac{\text{Volume stress}}{\text{Volume strain}} \text{----- (1)}$$

**Explanation:**

Let us consider a body of volume  $V$  with an area of cross section  $A$ . Let three equal forces act on the body in mutually perpendicular directions as shown in Fig 1.2. Let  $v$  be the change in volume due to the action of forces, then,

$$\text{The volume stress (or) bulk stress} = \frac{\text{Normal force}}{\text{Area}} = \frac{F}{A}$$

$$\text{The volume strain (or) bulk strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{v}{V}$$

$$\text{Bulk modulus (K)} = \frac{F/A}{v/V}$$

$$\text{Bulk's modulus, } K = \frac{Fv}{Va} = \frac{pv}{v} \text{NM}^{-2} \text{ (or) pascal}$$

Where  $P$  is the pressure  $= F/A$ , The reciprocal of bulk modulus of a material is known as compressibility of that material.

**iii. Rigidity modulus (G)**

It is defined as the ratio between the tangential stress to the shearing strain, within the elastic limit.

$$\text{Rigidity's modulus} = \frac{\text{Tangential stress}}{\text{Tangential strain}} \text{----- (1)}$$

**Explanation:**

Let us consider a solid cube ABCDEFGH as in Fig 1.3 Whose lower face CDHG is fixed.

A tangential force 'F' is applied over the upper face ABEF. The result is that the cube gets deformed into a rhombus shape A'B'CDE'F'GH. The lines joining the two faces are shifted to an angle  $\phi$ . If 'L' is the original length and 'l' is the relative displacement of the upper face of the cube with respect to the lower fixed face, then Tangential stress  $= \frac{F}{A}$ . The shearing strain ( $\phi$ ) can be defined as the ratio of the relative displacement between the two layers in the direction of the stress, to the distance measured perpendicular to the layers,

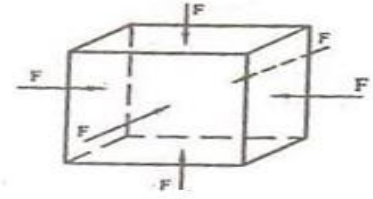


Fig 1.2

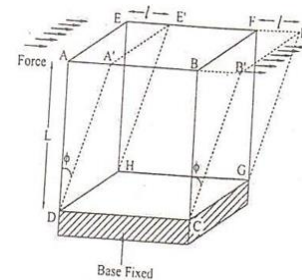


Fig 1.3



$$\text{Rigidity modulus, } G = \frac{F/A}{l/L}$$

$$\text{Rigidity modulus} = \frac{FL}{Al} \text{ Nm}^{-2} \text{ (or) pascal}$$

### POISSON'S RATIO ( $\sigma$ )

It is defined as the ratio between the lateral strain per unit stress ( $\beta$ ) to the longitudinal strain per unit stress ( $\alpha$ ), within the elastic limits.

#### Explanation:

Let us consider a wire, fixed at one end and is stretched along the other end as shown in Fig 1.4. Due to the force applied the wire becomes longer but it also becomes thinner (i.e) although there is an increase in its length, there is a decrease in diameter. Therefore the wire elongates freely in the direction of tensile force and contracts laterally in the direction perpendicular to the force. Let 'L' be the original length and 'D' be the original diameter of the wire. After the application of force, let the length increases from L to L+l and the diameter decreases from D to d, then

$$\text{Longitudinal strain} = l/L$$

$$\text{Lateral strain} = \frac{(d-D)}{D}$$

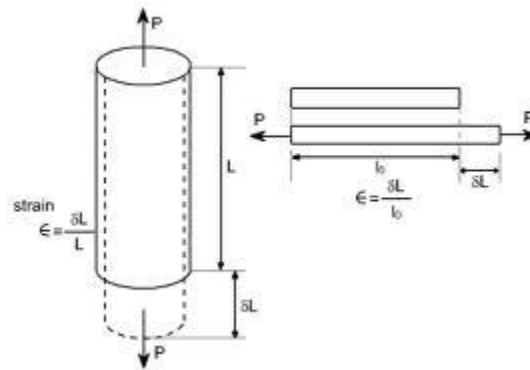


Fig 1.4

The negative sign indicates that longitudinal strain and lateral strain are opposite to each other.

$$\text{Poisson's ratio } (\sigma) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = - \frac{(d-D)L}{DL}$$

Strains along the direction of the applied force are referred to as primary strains. Hence longitudinal strains are primary strains. The strains acting at right angles to the applied force are referred to as secondary strains. Hence lateral strains are secondary. The negative sign in the above equation indicates opposite nature of the two strains, one is tensile while the other is compressive.

$$\begin{aligned}\text{Hence } \sigma &= \frac{1}{m} \\ &= \frac{\text{secondary strain}}{\text{primary strain}} \\ &= \frac{\text{compressive strain}}{\text{tensile strain}}\end{aligned}$$

It ( $\sigma$ ) has no units and dimensions. Theoretically  $\sigma$  lies between -1 and  $+\frac{1}{2}$ .

Practically no substance has been found with negative value of  $\sigma$ . i.e., practically  $\sigma$  lies between zero and  $+\frac{1}{2}$ .

### ELASTIC LIMIT

The maximum stress up to which a body can recover its original shape and size, after removing the external forces is called as Elastic limit. After elastic limit the body will be in a limit called as plastic limit.

### YIELD POINT

The point at which the body loses its elasticity is called as Yield point.

### ELASTIC FATIGUE

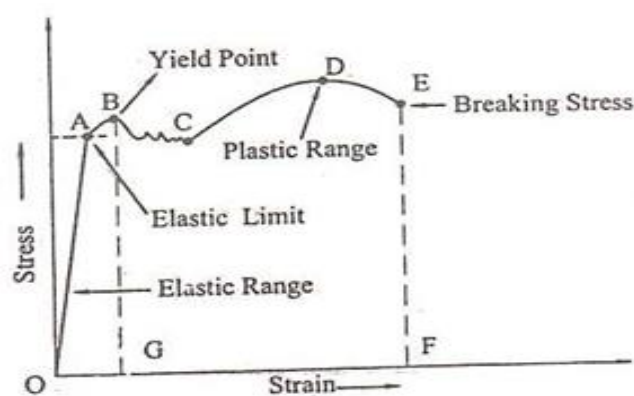
If a body is continuously subjected to stress or strain, it gets fatigued (weak) called as elastic fatigue.

### STRESS – STRAIN DIAGRAM AND ITS USES

Let us consider a body which is subjected to uniformly increasing stress. Due to the application of the stress, the change in dimension of the body take place; the strain is developed. If we plot a graph between stress and strain we get a curve as shown in Fig 1.5 and is called as stress-strain Diagram.

- i. From the fig (1.5) it is found that the body obeys Hooke's law upto the region **OA** called as **elastic range**.
- ii. As soon as the maximum elastic limit, yield point B is crossed, the strain increases rapidly than the stress.
- iii. At this stage the body remains **partly elastic** and **partly plastic** which is represented by the curve **BC**.
- iv. Now, even if a small external force is applied, the body will take a new path **CD** and remains as plastic called as **Plastic range**, where D is called as **ultimate strength**.

- v. After this, the body will not come to its original state and the body acquires a permanent residual strain and it breaks down at a point called as **breaking stress**, indicated by dotted line **EF**.



### Uses of stress – strain diagram

1. It is used to categorize the materials into ductile (or) Brittle (or) plastic in nature.
2. For ductile material the portion of curve between C to E will be very large.  
Examples: Annealed copper, low carbon steel, brass, aluminum etc.,
3. For a brittle material, the yield point coincides with the breaking point.  
Examples: Glass, high carbon steel, cast iron, brick, stone etc.
4. For a plastic material the stress – strain diagram runs parallel to the strain axis beyond the yield point.

### FACTORS AFFECTING ELASTIC MODULUS AND TENSILE STRENGTH

Some bodies lose their elastic property even within the elastic limit, due to elastic fatigue. Apart from elastic fatigue some materials will have change in their elastic property because of the following factors:

- i. Effect of stress
- ii. Effect of annealing
- iii. Change in Temperature
- iv. Presence of impurities
- v. Due to the nature of cycles

#### (i) Effect of Stress

When a material is subjected to large number of cycles of stresses, it loses its elastic property even within the elastic limit. Therefore, the working stress on the material should be kept lower than the ultimate tensile strengthening and the safety factor.

### **(ii) Effect of Annealing**

Annealing is a process by which the material is heated to a very high temperature and then it is slowly cooled. Usually this process is adopted for the materials to increase the softness and ductility in the materials. But if annealing is made to a material it results in the formation of large crystal grains, which ultimately reduces the elastic property of the material.

### **(iii) Effect of Temperature**

The elastic property of the materials changes with the temperature. Normally the elasticity increases with the decrease in temperature and vice-versa.

#### **Examples:**

- a) The elastic property of Lead increases when the temperature is decreased.
- b) The carbon filament becomes plastic at higher temperature.

### **(iv) Effect of impurities**

The addition of impurities produces variation in the elastic property of the materials.

The increase and decrease of elasticity depends on the type of impurity added to it.

#### **Examples:**

- a) When potassium is added to gold, the elastic property of gold increases.
- b) When carbon is added to molten iron, the elastic property of iron decreases provided the carbon content should be more than 1% in iron.

### **(v) Effect of nature of Crystals**

The elasticity also depends on the types of the crystals, whether it is a single crystal or poly crystal. For a single crystal the elasticity is more and for a poly-crystal the elasticity is less.

## **MOMENT, COUPLE AND TORQUE**

### **(i) Moment of a Force**

The moment of a force about a point is defined as the product of the magnitude of the force and perpendicular distance from the point to the line of action of force.

### **(ii) Couple**

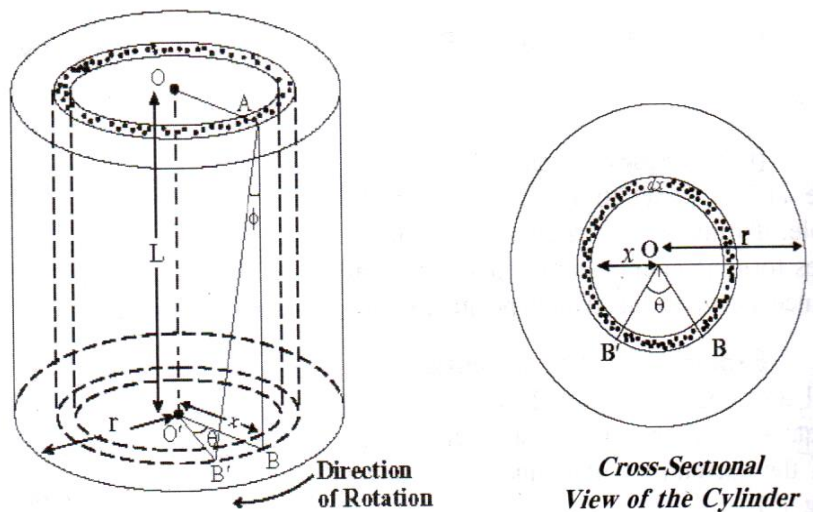
A couple constitutes a pair of two equal and opposite forces acting on a body, in such a way that the lines of action of the two forces are not in the same straight line.

### **(iii) Torque**

Torque is the rotating force and is equal to the moment of the couple. Torque is the product of one of the forces forming couple and the perpendicular distance between the two opposite forces.

### TWISTING COUPLE ON A WIRE – Expression for Torque per unit twist

Let us consider a cylindrical wire of length 'L' and radius 'r'. The wire is fixed at its upper end and twisted through an angle 'θ' by applying a torque at lower end. The wire can be assumed to be made up of a number of hollow cylindrical tubes (co-axial) whose radii varies from 0 to r.



Due to twisting torque the line AB which is initially parallel to the axis  $OO'$  of the cylinder is displaced to a position  $AB'$  through an angle  $\phi$  as shown in figure 1.6.

The result of twisting the cylinder is a shearing strain.

The angle of shear = angle  $BAB' = \phi$

Here  $BB' = x\theta = L\phi$

$$\phi = \frac{x\theta}{L} \text{ ----- (1)}$$

$$\text{Rigidity modulus } n = \frac{\text{Tangential stress}}{\text{Shearing strain}} \quad (\text{or}) \quad \frac{\text{Shearing strain}}{\text{Angle of shear}}$$

$$\text{Shearing stress} = n\phi \text{ -----(2)}$$

Substituting for  $\phi$  from equation (1) in equation (2), we have

$$\text{Shearing stress} = \frac{nx\theta}{L} \text{ -----(3)}$$

We know shearing stress =  $\frac{\text{Shearing force}}{\text{Area}}$

Shearing Force = Shearing stress x Area on which the shearing Force is acting

$$F = \frac{nx\theta}{L} \cdot 2\pi x dx \text{ -----(4)}$$

Where  $2\pi x dx$  is the area over which the shearing Force acts, as shown in figure 1.6.

Moment of force about the oo' axis of the cylinder = Shearing Force x Distance

$$\begin{aligned} &= \frac{nx\theta}{L} \cdot 2\pi x dx \cdot x \\ &= \frac{2\pi n\theta}{L} \cdot x^3 \cdot dx \text{ ----- (5)} \end{aligned}$$

Twisting couple of the whole wire can be derived by integrating equation (5) with in the limits 0 to r (since the radii varies from 0 to r).

$$\text{Twisting couple on the wire} = C = \int_0^r \frac{2\pi n\theta}{L} \cdot x^3 \cdot dx$$

$$\text{(or)} \quad C = \frac{2\pi n\theta}{L} \cdot \frac{r^4}{4}$$

$$\text{(or)} \quad C = \frac{\pi n\theta r^4}{2L}$$

If twist  $\theta$  is unity ie.  $\Theta = 1$  radian.

Then, we can write

$$\text{The torque per unit twist } C = \frac{\pi n r^4}{2L} \text{ ----- (6)}$$

### TORSIONAL STRESS & DEFORMATIONS (THEORY)

When a body is fixed at one end and twisted about its axis by means of a torque at the other end, then the body is said to be under torsion. The torsion involves shearing strain and hence the modulus involved is the “Rigidity Modulus”.

### TORSION PENDULUM

#### Principle

When a disc (torsion pendulum) is rotated in a horizontal plane, the disc executes simple harmonic oscillation due to the restoring couple produced in the wire.

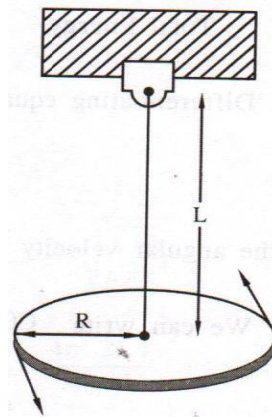
#### Description

A torsion pendulum consists of a wire with one end fixed to a split chuck and other end fixed to the centre of the circular disc of radius  $R$  as shown in figure 1.7.

Let ' $L$ ' be the distance between the chuck end to the disc and ' $r$ ' be the radius of suspended wire.

### Working

The circular disc is rotated in horizontal plane so that the wire is twisted through an angle ' $\theta$ '. The various elements of the wire will undergo shearing strain and a restoring couple is produced. Now if the disc is released, the disc will produce torsion oscillations.



The couple acting on the disc produces an angular acceleration in it, which is proportional to the angular displacement and is always directed towards its mean position.

Therefore from the law of conservation of energy the total energy of the system is conserved.

**Total energy of the torsion pendulum = Potential Energy + Kinetic Energy --- (1)**

The potential energy confined to the wire is equal to the work done in twisting the disc, thereby creating a restoring couple ( $c$ ).

Restoring couple (P.E) through an angle  $\theta = \int_0^\theta \text{Moment of couple} \times d\theta$

$$\text{P.E} = \int_0^\theta C\theta \cdot d\theta$$

$$\text{P.E} = \frac{C\theta^2}{2} \quad \text{---- (2)}$$

Let ' $\omega$ ' the angular velocity with which the disc oscillates, due to the restoring couple, then

The kinetic energy confined to the rotating disc (Deflecting couple) =  $\frac{1}{2} I\omega^2$

$$\text{i.e. } K.E = \frac{1}{2} I \omega^2 \quad \text{--- (3)}$$

Here I is the moment of inertia of the circular disc

$$\text{Total Energy } T = \frac{C\theta^2}{2} + \frac{I\omega^2}{2} = \text{constant} \quad \text{--- (4)}$$

Differentiating equation (4) with respect to time 't' we get,

$$C\theta \frac{d\theta}{dt} + I\omega \frac{d\omega}{dt} = 0$$

Since the angular velocity  $\omega = \frac{d\theta}{dt}$  and angular acceleration  $\frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

We can write

$$C\theta \frac{d\theta}{dt} + I \frac{d\theta}{dt} \cdot \frac{d^2\theta}{dt^2} = 0$$

$$(\text{or}) \frac{d\theta}{dt} \left[ C\theta + I \frac{d^2\theta}{dt^2} \right] = 0$$

$$\text{Here, } \frac{d\theta}{dt} \neq 0 \quad C\theta + I \frac{d^2\theta}{dt^2} = 0$$

$$\text{Angular acceleration} = \frac{d^2\theta}{dt^2} = -\frac{C\theta}{I} \quad \text{--- (5)}$$

The negative sign indicates that the couple tends to decrease the twist on the wire.

### Period of Oscillation

$$\text{We know, the time period of oscillation } T = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$\text{Substituting from equation (5), we have } T = 2\pi \sqrt{\frac{\theta}{\frac{C\theta}{I}}}$$

$$(\text{or}) \text{ Time period of torsion oscillation } T = 2\pi \sqrt{\frac{I}{C}} \quad \text{--- (6)}$$

$$\text{Frequency of oscillation } f = \frac{1}{2\pi} \sqrt{\frac{C}{I}} \quad \left( f = \frac{1}{T} \right)$$

### Rigidity modulus of the wire

If 'r' is the radius of the wire and 'L' is the length of the wire suspended, then we know

$$\text{The torque per unit twist } C = \frac{n\pi r^4}{2L} \quad \text{--- (7)}$$

Substituting equation 7 in equation 6 we get,

$$T = 2\pi \sqrt{\frac{I2L}{n\pi r^4}}$$

$$T^2 = \frac{4\pi^2 2LI}{n\pi r^4}$$

$$(\text{or}) \text{ Rigidity modulus of the wire } (n) = \frac{8\pi IL}{T^2 r^4} \text{ Nm}^{-2}$$

Thus the torsion pendulum is used to find the rigidity modulus for various materials.



## EXPERIMENTS USING TORSION PENDULUM – MOMENT OF INERTIA OF A REGULAR AND IRREGULAR BODY

### i) Determination of the moment of inertia of the disc and rigidity modulus of the wire using torsion pendulum with mass:

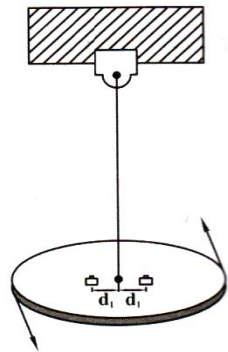
To determine the moment of inertia of the disc and the rigidity modulus of the wire, the disc is set into torsional oscillations without any mass over it and the time period of oscillations (T) is measured.

i.e. Time period of oscillation without mass  $T = 2\pi \sqrt{\frac{I}{C}}$  (or)

$$T^2 = \frac{4\pi^2}{C} \quad \text{---(1)}$$

Where, I is the moment of inertia of the disc about the axis of rotation and C is the restoring couple.

Now two equal cylindrical masses are placed over the disc at equal distances say  $d_1$ , from the center of the disc as shown in figure 1.8 and are set into torsional oscillations. Time period of oscillation ( $T_1$ ) is measured.



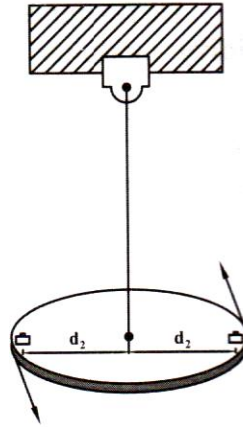
Time period of oscillation when the masses are at a distance

$$(d_1) = T_1 = 2\pi \sqrt{\frac{I_1}{C}} \quad \text{(or)}$$

$$T_1^2 = \frac{4\pi^2 I_1}{C} \quad \text{--- (2)}$$

Where  $I_1$  is moment of inertia of the disc along with the cylindrical masses placed over the disc at a distance  $d_1$ .

Now the cylindrical masses are placed at the edges of the disc at equal distances say  $d_2$  from the center of the disc as shown in figure 1.9 and the Time period of oscillation ( $T_2$ ) is measured.



$$(d_2) = T_2 = 2\pi \sqrt{\frac{I_2}{C}} \quad (\text{or})$$

$$T_2^2 = \frac{4\pi^2 I_2}{C} \quad \text{--- (3)}$$

Where  $I_2$  is moment of inertia of the disc along with the two equal cylindrical masses placed over the disc at a distance  $d_2$ .

a) **Moment of inertia of the disc**

From equations (1), (2) and (3), we can write

$$\begin{aligned} \frac{T^2}{T_2^2 - T_1^2} &= \frac{\frac{4\pi^2 I}{C}}{\frac{4\pi^2 I}{C}(I_2 - I_1)} \\ \frac{T^2}{T_2^2 - T_1^2} &= \frac{I}{I_2 - I_1} \\ I &= \frac{T^2(I_2 - I_1)}{T_2^2 - T_1^2} \quad \text{--- (4)} \end{aligned}$$

From the parallel axis theorem we can write the moment of inertia  $I_1$  as

$$I_1 = I + 2I_m + 2md_1^2 \quad \text{--- (5)}$$

Where,  $I_m \rightarrow$  moment of inertia of each Cylindrical mass passing through its center.

$m \rightarrow$  mass of the cylindrical weights placed over the disc.

Similarly from the parallel axis theorem we can write the moment of inertia  $I_2$  as

$$I_2 = I + 2I_m + 2md_2^2 \quad \text{----- (6)}$$

From equation (5) and (6), we can write,

$$I_2 - I_1 = 2m(d_2^2 - d_1^2) \quad \text{----- (7)}$$

Substituting equation (7) in equation (4)

Moment of inertia of the disc about the axis of rotation

$$I = \frac{2m(d_2^2 - d_1^2)T^2}{T_2^2 - T_1^2} \quad \text{----- (8)}$$

b) Rigidity modulus of the wire

From the theory of torsion pendulum, we know

$$\text{The rigidity modulus of the wire } \eta = \frac{8\pi L}{T^2 r^4} \quad \text{----- (9)}$$

Substituting equation (8) in equation (9) we have

$$\eta = \frac{8\pi L}{T^2 r^4} \frac{2m(d_2^2 - d_1^2)T^2}{T_2^2 - T_1^2}$$

$$\eta = \frac{16m\pi L}{r^4} \frac{(d_2^2 - d_1^2)}{T_2^2 - T_1^2} \quad \text{----- (10)}$$

## BENDING OF BEAMS

### Beam

A beam is a rod or bar of uniform cross-section (either circular or rectangular) of a homogeneous, isotropic elastic material whose length is large compared to its thickness.

When such a beam is fixed at one end and loaded at the other, within the limit of elasticity a bending is produced due to the moment of the load. The deformation produced by the load brings about restoring forces due to elasticity tending to bring the strip back to its original position. In equilibrium position,

$$\text{Restoring couple} = \text{Bending couple}$$

These two couples act in the opposite directions. The plane in which these couples act is called the plane of bending. The moment of the couple due to the elastic reactions (restoring couple) which balances the external couple due to the applied load is called the bending moment.

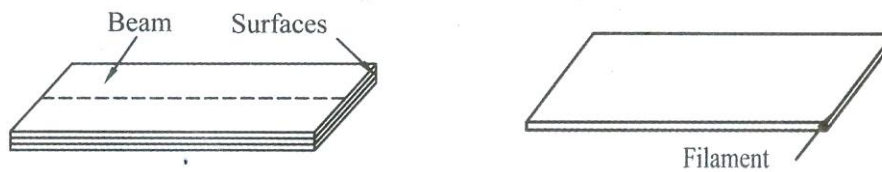
### Assumptions

While studying about the bending of beams, the following assumptions have to be made.

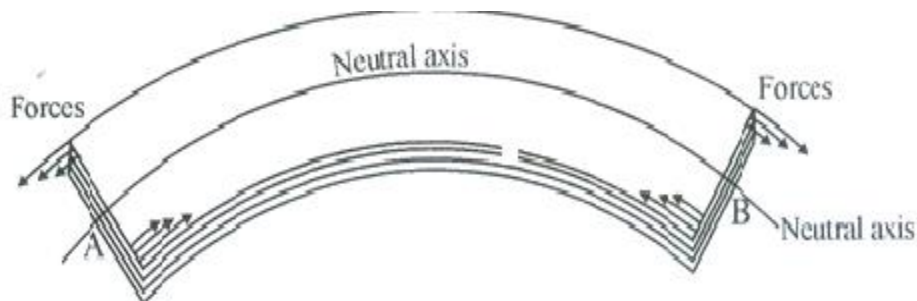
- i. The length of the beam should be large compared to other dimensions.
- ii. The load(forces) applied should be large compared to the weight of the beam.
- iii. The cross section of the beam remains constant and hence the geometrical moment of inertia  $I_g$  also remains constant.
- iv. The shearing stresses are negligible.
- v. The curvature of the beam is very small.

**Bending of a beam and neutral axis:**

Let us consider a beam of uniform rectangular cross section Fig.1.10. A beam may be assumed to consist of a number of parallel longitudinal metallic fibres placed one over the other and are called as filaments as shown in Fig.

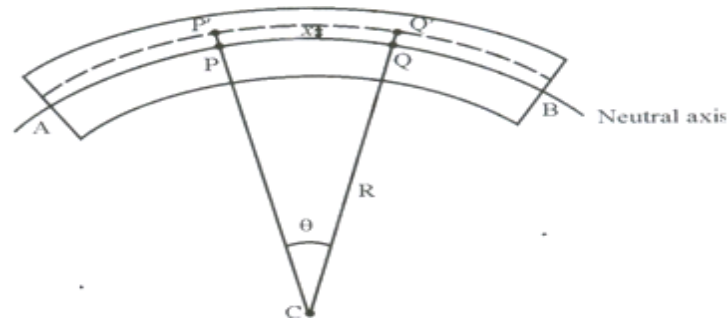


Let the beam be subjected to deforming forces at its ends as shown in Fig 1.12. Due to the deforming forces the beam bends. We know the beam consists of many filaments. Let us consider a filament AB at the centre of the beam. It is found that the filaments (layers) lying above AB gets elongated, while the filaments (layers) lying below AB gets compressed. Therefore the filament (i.e) layer AB which remains unaltered is taken as the reference axis called as NEUTRAL AXIS and the plane is called as neutral plane. Further, the deformation of any filament can be measured with reference to the neutral axis.

**EXPRESSION FOR THE BENDING MOMENT:**

Let us consider a beam under the action of deforming forces. The beam bends into a circular arc as shown in Fig 1.13. Let AB be the neutral axis of the beam. Here the filaments above AB are elongated and the filaments below AB are compressed. The filament AB remains unchanged.

Let PQ be the arc chosen from the neutral axis. If R is the radius of curvature of the neutral axis and  $\theta$  is the angle subtended by it at its centre of curvature 'C'.



Then we can write original length  $PQ = R\theta$  -----(1)

Let us consider a filament P'Q' at a distance 'x' from the neutral axis.

We can write the extended length  $= P'Q' = (R+x)\theta$  -----(2)

From eqn.(1) and eqn.(2), we have

$$\text{Increase in its length} = P'Q' - PQ$$

$$\text{Increase in its length} = (R+x)\theta - R\theta$$

$$\text{Increase in its length} = x\theta \text{ -----(3)}$$

$$\text{We know Linear Strain} = \frac{\text{Increase in length}}{\text{Original length}}$$

$$(\text{or}) \text{ Linear Strain} = \frac{x\theta}{R\theta}$$

$$\text{Linear Strain} = \frac{x}{R} \text{ -----(4)}$$

We know,

$$\text{The Young's modulus of the material } Y = \frac{\text{Stress}}{\text{Linear Strain}}$$

$$(\text{or}) \text{ Stress} = Y \times \text{Linear strain} \text{ -----(5)}$$

Substituting eqn.(4) in (5), we have

$$\text{Stress} = \frac{Yx}{R}$$

If  $\delta A$  is the area of cross section of the filament P'Q'. Then,

$$\text{The tensile force on the area } (\delta A) = \text{Stress} \times \text{Area}$$

$$(\text{i.e}) \text{ Tensile Force} = \frac{Yx}{R} \cdot \delta A$$

We know,

$$\text{Moment of force} = \text{Force} \times \text{Perpendicular Distance}$$

Moment of the tensile force about the neutral axis AB (or)

$$PQ = \frac{Yx}{R} \cdot \delta Ax$$

$$PQ = \frac{Y}{R} \cdot \delta Ax^2$$

The moment of force acting on both the upper and lower halves of the neutral axis can be got by summing all the moments of tensile and compressive forces about the neutral axis.

The moment of all the forces about the neutral axis  $= \frac{Y}{R} \cdot \sum x^2 \delta A$

Here  $I_g = \sum x^2 \delta A = AK^2$  is called the geometrical moment of Inertia.

Where, A is the total area of the beam and K is the radius of the Gyration.

Total moment of all the forces (or) Internal bending moment  $= \frac{Y I_g}{R}$  -----(6)

### Special cases:

#### 1. Rectangular cross section:

If 'b' is the breadth and 'd' is the thickness of the beam, then

$$\text{Area } A = bd \text{ and } K^2 = \frac{d^2}{12}$$

$$I_g = AK^2 = \frac{bd \cdot d^2}{12} = \frac{bd^3}{12}$$

Substituting the value of in eqn.(6), we can write

Bending moment for a rectangular cross section  $= \frac{Y b d^3}{12 R}$  -----(7)

#### 2. Circular cross section

For a circular cross section if 'r' is the radius, then Area  $A = \pi r^2$

$$\text{and } K^2 = \frac{r^2}{4}$$

$$I_g = AK^2 = \frac{\pi r^2 \times r^2}{4}$$

$$I_g = \frac{\pi r^4}{4}$$

Substituting the value of  $I_g$  in eqn.(6), we can write

The Bending moment of a circular cross section  $= \frac{\pi Y r^4}{4 R}$  -----(8)

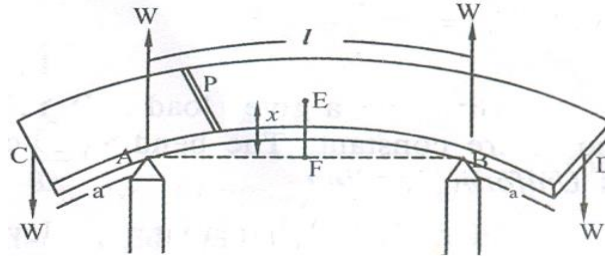
### UNIFORM BENDING – ELEVATION AT THE CENTRE OF THE BEAM LOADED AT BOTH ENDS

#### Theory

Let us consider a beam of negligible mass, supported symmetrically on the two knife edges A and B as shown. Let the length between A and B be 'l'. let equal weights W, be added to either end of the beam C and D.

Let the distance CA = BD = a.

Due to the load applied the beam bends from position F to E into an arc of a circle and produces an elevation 'x' from position F to E. Let 'W' be the reaction produced at the points A and B which acts vertically upwards as shown in fig 1.18.



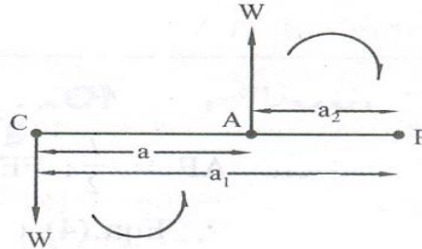
Consider a point 'P' on the cross section of the beam. Then the forces acting on the part PC of the beam are

- i. Force W at 'C'
- ii. Reaction W at A as shown in fig 1.19.

Let the distance PC =  $a_1$  and PA =  $a_2$ , then

The external bending moment about 'P' is

$$M_p = W \times a_1 - W \times a_2$$



Here, the clockwise moment is taken as negative and anticlockwise moment is taken as positive.

External bending moment about P can be written as

$$M_p = W (a_1 - a_2)$$

$$M_p = Wa \quad \text{-----(1)}$$

$$\text{We know the internal bending moment} = \frac{YIg}{R} \quad \text{-----(2)}$$

Under equilibrium condition, External bending moment = Internal bending moment

Since for a given load (W), Y, Ig, a and R are constant. The bending is called as Uniform Bending.

Here it is found that the elevation 'x' forms an arc of the circle of radius 'R' as shown in fig 1.20.

From the  $\Delta AFO$  we can write

Since  $OF = FE$ , therefore we can write

$$OA^2 = AF^2 + FE^2$$

Rearranging we can write

$$AF^2 = FE \left[ \frac{OA^2}{FE} - FE \right] \text{ -----(4)}$$

Here  $AF = \frac{l}{2}$ ;  $FE = x = \frac{R}{2}$ ;  $OA = R$

Eqn. (4) can be written as  $\left(\frac{l}{2}\right)^2 = x \left[ \frac{R^2}{\frac{R}{2}} - x \right]$

$$\frac{l^2}{4} = x[2R - x]$$

$$\frac{l^2}{4} = [2xR - x^2]$$

If the elevation 'x' is very small, then the term  $x^2$  can be neglected.

We can write  $\frac{l^2}{4} = 2xR$

$$(\text{or}) x = \frac{l^2}{8R}$$

$$\text{Radius of curvature } R = \frac{l^2}{8x} \text{ -----(5)}$$

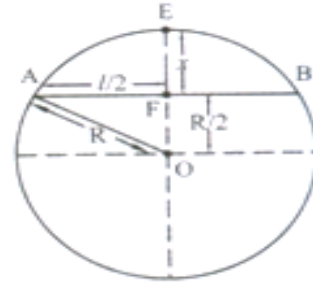
Substituting the value of 'R' value in eqn. (3) we have

$$(\text{or}) W.a = \frac{Y I_g}{\left(\frac{l^2}{8x}\right)}$$

$$(\text{or}) W.a = \frac{8Y I_g x}{l^2}$$

Rearranging eqn. (6)

$$\text{The Elevation of point 'E' above 'A' is } x = \frac{W a l^2}{8Y I_g}$$



## EXPERIMENTAL DETERMINATION OF YOUNG'S MODULUS BY UNIFORM BENDING

### Statistical method - Description

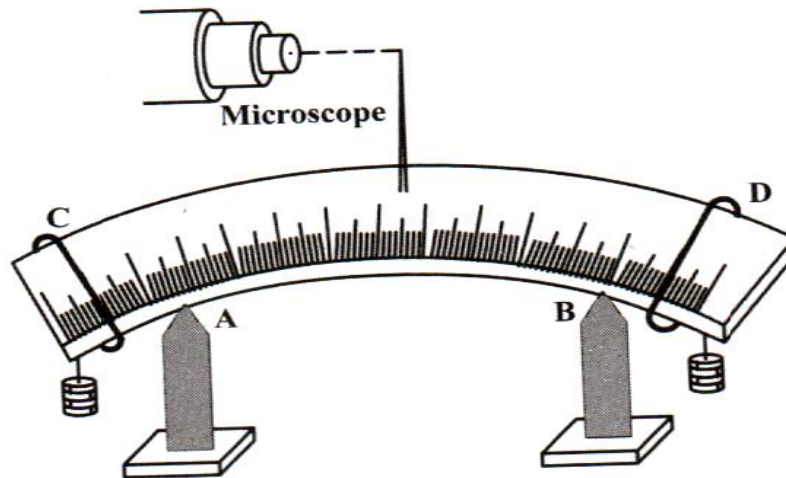
It consists of a beam, symmetrically supported on the two knife edges A and B. Two weight hangers are suspended on both side of the beam at the position 'C' and 'D'. The distance between AC and BD are adjusted to be equal. A pin is fixed vertically at the centre of the beam



as shown in fig. A travelling microscope is placed in front of the whole set up for finding the position of the pin.

**Procedure:**

Taking the weight hanger as the dead load ( $W$ ), the microscope is adjusted and the tip of the pin is made to coincide with the vertical cross wire. The reading is noted from the vertical scale of the microscope.



Now the load on each hanger is increased in equal steps of ' $m$ ', ' $2m$ ',  $3m$  etc. kilogram and the corresponding readings are noted from the vertical scale of the microscope. The same procedure is repeated during unloading. The readings are noted from the vertical scale of the microscope. The readings are tabulated in the tabular column.

Sl.No.	Load (M)	Microscope readings			Elevation (x)	(M/x)
		Increasing Load	Decreasing Load	Mean		
Unit	Kg	$\times 10^{-2}m$	$\times 10^{-2}m$	$10^{-2}m$	metre	Kg $m^{-1}$
1.	W			$x_0$		
2.	W + m			$x_1$		
3.	W + 2m			$x_2$		
4.	W + 3m			$x_3$		
5.	W + 4m			$x_4$	$x_4 - x_0$	
6.	W + 5m			$x_5$	$x_5 - x_1$	
7.	W + 6m			$x_6$	$x_6 - x_2$	
8.	W + 7m			$x_7$	$x_7 - x_3$	

The mean elevation ' $x$ ' of the centre for  $M$  kg is found. The distance between the two knife edges is measured as ' $l$ ' and the distance from the point of suspension of the load to the knife edge is measured as ' $a$ '.

Then we know the elevation produced is

$$x = \frac{Wal^2}{8YI_g} \text{-----(1)}$$

If 'b' is the breadth of the beam and 'd' is the thickness of the beam, then

$$\text{For a rectangular bar geometrical moment of Inertia } I_g = \frac{bd^3}{12} \text{-----(2)}$$

$$\text{Also we know, Weight } W = Mg \text{-----(3)}$$

Substituting eqn.(2) and eqn.(3) in eqn.(1), we have

$$x = \frac{Mgal^2}{8Y\left(\frac{bd^3}{12}\right)} \text{-----(4)}$$

Rearranging eqn.(4) we can write, the Young's Modulus

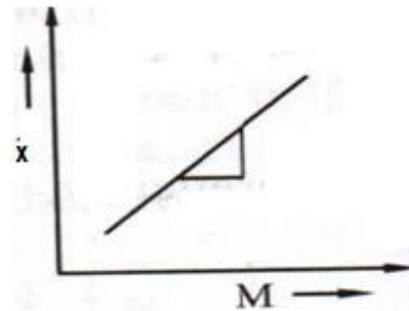
$$Y = \frac{3Mgal^2}{2xbd^3}$$

$$\text{(or) The Young's modulus } Y = \frac{3gal^2}{2bd^3} \left(\frac{M}{x}\right) \text{Nm}^{-2} \text{-----(5)}$$

Substituting the mean value of  $\frac{M}{x}$  from the tabular column the Young's modulus Y of the material of the given beam can be calculated.

#### Graphical method (or) Dynamical method

A graph is drawn between load (M) along x axis and elevation (x) along y axis. It is found to be a straight line as shown in fig. The slope of the straight line gives the value of  $\frac{x}{M}$ . Eqn.(5) can be written as

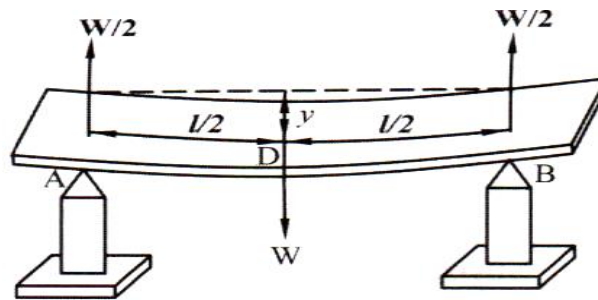


$$\text{Young's modulus } Y = \frac{3gal^3}{2bd^3} \frac{1}{\text{slope}}$$

#### YOUNG'S MODULUS BY NON UNIFORM BENDING –DEPRESSION AT THE MID POINT OF A BEAM LOADED AT THE MIDDLE.

##### THEORY

Let us consider a beam of length 'l' (distance between two knife edges) supported on the two knife edges A and B as shown in figure 1. The load of weight 'w' is suspended at the center 'c'. It is found that the beam bends and the maximum displacement is at the point 'D', where the load is given.



Due to the load (w) applied, at the middle of the beam the reaction  $W/2$  is acted vertically upwards at each knife edges. The bending is called as non-uniform bending.

The beam may be considered as two cantilever, whose free end carries a load  $W/2$  each of length  $l/2$  and fixed at the point 'D'.

Hence we can say the elevation of A above D as the depression of D below A. we know depression of a cantilever  $y = \frac{wl^3}{3YI_g}$

Therefore substituting the value of l as  $l/2$  and w as  $w/2$  in the expression for the depression of a cantilever, ie.,  $y = \frac{wl^3}{3YI_g}$

We have, **Depression of D below A**  $y = \frac{\frac{w}{2} \left(\frac{l}{2}\right)^3}{3YI_g}$  (or)  $y = \frac{wl^3}{48YI_g}$

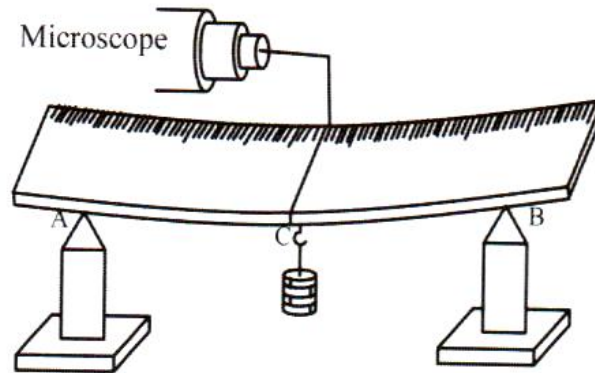
## EXPERIMENTAL DETERMINATION OF YOUNGS MODULUS – NON UNIFORM BENDING

### Statical Method - Description

It consists of a beam, symmetrically supported on the two knife edges A and B. A weight hanger is suspended at the center (C) of the beam by means of a loop (or) thread. A pin is fixed vertically at 'c' by some wax as shown in fig. In order to focus the tip of the pin a travelling microscope (M) is placed in front of this arrangement.

### Procedure

Taking the weight hanger as the dead load (W) the microscope is adjusted and the tip of the pin is made to coincide with the horizontal cross wire. The reading is noted from the vertical scale the microscope.



The weights are added (loaded) in steps of  $m$ ,  $2m$ ,  $3m$  kilograms and the corresponding readings are noted from the vertical scale of the microscope. The same procedure is repeated while unloading and the readings are tabulated in the tabular column as shown. The mean depression ' $y$ ' is found for a load of  $M$  kg.

Sl.No.	Load (M)	Microscope readings			Depression (y)	(M/y)
		Increasing Load	Decreasing Load	Mean		
Unit	Kg	$\times 10^{-2}m$	$\times 10^{-2}m$	$10^{-2}m$	metre	Kg $m^{-1}$
1.	W			$y_0$		
2.	W + m			$y_1$		
3.	W + 2m			$y_2$		
4.	W + 3m			$y_3$		
5.	W + 4m			$y_4$	$y_4 - y_0$	
6.	W + 5m			$y_5$	$y_5 - y_1$	
7.	W + 6m			$y_6$	$y_6 - y_2$	
8.	W + 7m			$y_7$	$y_7 - y_3$	

Theoretically, we know the depression produced is  $y = \frac{Wl^3}{48YI_g}$  ----- (1)

Where ' $l$ ' be the length of the beam (ie.) the distance between the knife edge. If ' $b$ ' is the breadth of the beam and ' $d$ ' is the thickness of the beam, then

The geometrical of moment of inertia  $I_g = \frac{bd^3}{12}$  ----- (2)

Also the weight  $W = Mg$  ----- (3)

Substituting equation (2) and (3) in equation (1), we get

$$y = \frac{Mgl^3}{48Y \frac{bd^3}{12}}$$

$$y = \frac{Mgl^3}{4Ybd^3} \text{ ----- (4)}$$

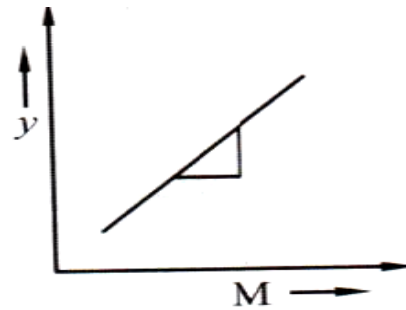
Rearranging equation (4) we get

$$\text{Young's modulus } Y = \frac{gl^3}{4bd^3} \left( \frac{M}{y} \right) \text{ Nm}^{-2} \text{ ----- (5)}$$

Substituting the mean value of  $\left( \frac{M}{y} \right)$ , from the tabular column, the Youngs modulus Y can be determined.

### Graphical method (or) Dynamical method

A graph is drawn between load (M) along x axis and depression (y) along y axis. It is found to be a straight line as shown in fig. The slope of the straight line gives the value of  $\frac{y}{M}$ . Eqn.(5) can be written as



$$\text{Young's modulus } Y = \frac{gl^3}{4bd^3} \frac{1}{\text{slope}}$$

Substituting the slope value in the given formula the Young's modulus can be calculated.

## I SHAPE GIRDERS

### Definition

The girders with upper and lower section broadened and the middle section tapered, so that it can withstand heavy loads over it is called as I shaped girders. Since the girder look like letter I as shown in fig. they are named as I shape girders.

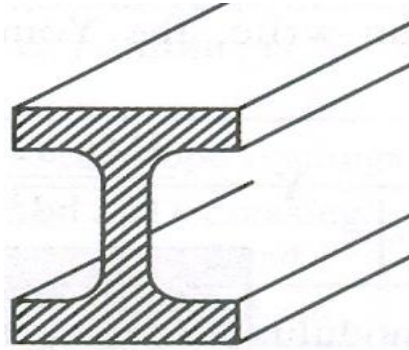
### Explanation:

Girder is supported at its two ends as on the opposite walls of a room, bends under its own weight and a small depression is produced at the middle portion. This may also be caused when loads are applied to the beams. Due to the depression produced, the upper parts of the girder above the neutral axis contracts, while the lower parts below the neutral axis expands. The stresses have a maximum value at the top and bottom. The stresses progressively decrease as it approaches towards the neutral axis. Therefore, the upper and lower surfaces of the girder must be stronger than the intervening part. Thus the girders are made of I shape and are called as I-Shape girders.

**Minimization of the Depression:** We know the depression produced in the case of a rectangular section.

The depression can be minimized by either decreasing the load (W) or the length of the girder (l) or by increasing the Young's modulus or the breadth or the thickness of the girder. Since the

length of the beam  $l$  is the fixed quantity, it cannot be decreased. Therefore, the breadth and thickness may be adjusted by making the girder of large depth and small breadth. Thus the volume of the girder is increased and hence the depression produced is reduced. Therefore for stability, the upper part and the lower part are made broader than the centre part and hence forming an 'I' shape called as I shape girders. The depression can also be reduced by properly choosing the materials of high Young's modulus.

**Applications of I- shape Girders:**

- 1) They are used in the construction of bridges over the rivers
- 2) They are very much useful in the production of iron rails which are employed in railway tracks
- 3) They are used as supporting beams for the ceiling in the construction of buildings.
- 4) They are used in the construction of iron beams to support the bridges for the heavy vehicles and also in the construction of dams.

**Advantages of I- Shape Girders:**

- 1) More stability
- 2) More stronger
- 3) High durability

**PROBLEMS**

1. A wire of length 1 metre and diameter 1mm is clamped at one of its ends. Calculate the couple required to twist the other end by 90°. (Given rigidity modulus = 298 gpa)

**Given data**

$$n = 298 \times 10^9 \text{ Pa}$$

$$\theta = 90^\circ = \frac{\pi}{2} \text{ radians}$$

$$r = 0.5 \times 10^{-3} \text{ m}$$

$$L = 1 \text{ m}$$

**Solution**

$$\text{Twisting couple } C = \frac{\pi n \theta r^4}{2L}$$

$$C = \frac{\pi \times 298 \times 10^9 \times \pi \times (0.5 \times 10^{-3})^4}{2 \times 1}$$

$$= \frac{0.18363}{4}$$

$$= 4.59 \times 10^{-2} \text{ NM.}$$

**Couple required to twist the other end by 90° = 4.59 X 10<sup>-2</sup> NM.**

2. Calculate the Poisson's ratio for the material, given  $Y = 12.25 \times 10^{10} \text{ Nm}^{-2}$  and  $n = 4.55 \times 10^{10} \text{ Nm}^{-2}$ .

**Given data:**

$$Y = 12.25 \times 10^{10} \text{ Nm}^{-2}$$

$$n = 4.55 \times 10^{10} \text{ Nm}^{-2}$$

**Solution:**

$$\text{We have } \sigma = \frac{Y}{2n} - 1$$

$$= \left( \frac{12.25 \times 10^{10}}{2(4.55 \times 10^{10})} \right) - 1$$

$$\sigma = 1.34615 - 1$$

$$\sigma = 0.34615$$

**Poisson's ratio  $\sigma = 0.34615$**

3. Calculate the Young's modulus in the cantilever depression method used. The length of cantilever beam is 1 m which is suspended with a load of 150 gm. The depression is found to be 4 cm. The thickness of the beam is 5 mm and breadth of the beam is 3 cm.

**Given data:**

Length of cantilever beam = 1 m

Depression = 4 cm

Thickness of the beam = 5 mm

Breadth of the beam = 3 cm

Load = 150 gm

**Solution:**

$$\begin{aligned}
 Y &= \frac{4gl^3}{bd^3} \left( \frac{M}{y} \right) \\
 &= \frac{4 \times 9.8 \times 1^3 \times 150 \times 10^{-3}}{3 \times 10^{-2} \times (5 \times 10^{-3}) \times 4 \times 10^{-2}} \\
 &= \frac{5.88}{1.5 \times 10^{-10}} \text{ N m}^{-2}
 \end{aligned}$$

**Young's modulus =  $3.92 \times 10^{10} \text{ N m}^{-2}$**

4. An elastic wire is cut into half its original length. How will it affect the maximum load the wire can support?

**Solution**

$$\text{We know i. } E = \left( \frac{\text{Load}}{\text{Area}} \right) \times \left( \frac{\text{Original length}}{\text{change in length}} \right)$$

$$\text{ii. Tensile strength} = \frac{\text{Maximum tensile load}}{\text{Original crosssectional area}}$$

When original length is halved, strain is changed so that the maximum load that the wire can support will remain the same as Elastic modulus is constant.



## UNIT – I

## PART A

**1. Define Elasticity.**

Elasticity is the property by virtue of which a body offers resistance to any deforming force and regains its original condition when the deforming force is removed.

**2. Define Perfectly Elastic Body and Perfectly Plastic Body.****Perfectly Elastic Body:**

All bodies can be deformed by the action of external forces. Bodies which can completely regain their original condition of shape and size on removal of deforming forces are said to be perfectly elastic.

**Perfectly Plastic Body:**

Bodies which retain their deformed nature even after the removal of the deforming forces are said to be perfectly plastic.

**3. Define a perfectly Rigid Body.**

If external forces fail to produce any deformation or relative displacements of the particles of the body, the body is said to be perfectly rigid.

**4. Define Stress and Strain and write down their units.****Stress:**

Stress is defined as the restoring force per unit area which brings back the body to its original state from the deformed state. As long as no permanent change is produced in the body, the restoring force is equal to the force applied.

Unit of stress is  $\text{N/m}^2$

**Strain**

The change produced in the body due to change in dimension of a body under a system of forces of Equilibrium is called Strain.

$$\text{Strain} = \frac{\text{Change in Dimension}}{\text{Original Dimension}}$$

It has no unit.

**5. What is meant by Normal Stress & Tangential Stress?****Normal Stress:**

When the force is applied perpendicular to the surface of the body, then the stress applied is called as normal stress.

**Tangential stress:**

When the force is applied along the surface of the body, then the stress applied is called as tangential stress. The tangential stress is called as Shearing stress.

**6. Define Longitudinal (or) Tensile Strain.**

It is defined as the ratio between the changes in length to the original length, without any change in its shape, after the removal of the external forces.

**7. What is meant by Shearing strain?**

It is defined as the angular deformation produced on the body due to the application of external tangential forces on it.

**8. What is volumetric strain?**

It is defined as the ratio between the changes in volume to the original volume, without any change in its shape.

**9. What do you understand from Hooke's Law (or) State Hooke's Law.**

Robert Hooke proposed a relation between stress and strain and is named as Hooke's law by his name. According to this law, Stress is directly proportional to the strain produced, within the elastic limit.

$$(i.e) \text{ Stress} \propto \text{Strain}$$

$$\text{Stress} = E \times \text{Strain}$$

$$E = \frac{\text{Stress}}{\text{Strain}} \text{ Nm}^{-2}$$

Where, E - modulus of Elasticity. The value of E depends upon the nature of the material.

**10. List out the different types of moduli.**

Depending on the three types of strain, there are three types of modulus they are,

- Young's modulus(Y) (or) modulus corresponding to longitudinal (or) tensile strength
- Bulk modulus(K) (or) modulus corresponding to the volume strain
- Rigidity modulus(n) (or) modulus corresponding to the shearing strain

**11. Define Young's modulus.**

It is defined as the ratio between the longitudinal stress to longitudinal strain, within the elastic limits. Unit of Young's modulus is  $\text{Nm}^{-2}$

$$\text{Young's modulus} = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$\text{Young's modulus} = \frac{FL}{al}$$

**12. Define Bulk modulus.**

It is defined as the ratio between the volume stress (or) bulk stress to the volume strain (or) bulk strain within the elastic limit.

$$\text{Bulk's modulus} = \frac{\text{Volume stress}}{\text{Volume strain}}$$
$$\text{Bulk's modulus, } K = FV/Va = pV/V \text{ NM}^{-2} \text{ (or) pascal}$$

**13. Define Rigidity modulus.**

It is defined as the ratio between the tangential stress to the shearing strain, within the elastic limit.

$$\text{Rigidity's modulus} = \frac{\text{Tangential stress}}{\text{Tangential strain}}$$
$$\text{Rigidity modulus} = \frac{FL}{Al} \text{ Nm}^{-2} \text{ (or) pascal}$$

**14. Mention the formula for the calculation of Young's modulus by uniform bending expression.**

$$Y = \frac{3mgal^2}{2bd^3y}$$

By knowing the mass(m), the distance between the point of suspension of the load and the nearest knife edge (a), length (l), breadth(b) and thickness(d) of the beam and depression (y), the young's modulus (Y) is determined.

**15. Define Poisson's ratio.**

It is defined as the ratio between the lateral strain per unit stress ( $\beta$ ) to the longitudinal strain per unit stress ( $\alpha$ ), within the elastic limits.

**16. What is meant by Yield point.**

The point at which the body loses its elasticity is called as Yield point.

**17. Define Elastic limit and Plastic limit.****Elastic limit.**

The maximum stress up to which a body can recover its original shape and size, after removing the external forces is called as Elastic limit.

**Plastic limit.**

After the elastic limit the body will not recover its original shape and size when the deforming force is removed i.e., the body will be in its plastic limits.

**18. Define Elastic Fatigue.**

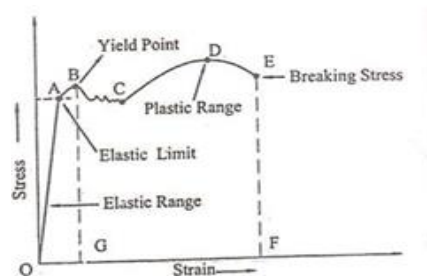
If a body is continuously subjected to stress or strain, it gets fatigued (weak) called as elastic fatigue.

**19. What do you infer from Stress – Strain diagram?**

- The stress is directly proportional to the strain within elastic limit.
- It determines the ultimate strength of the material.
- It distinguishes the elastic and plastic limit of a material.
- This diagram also helps us to distinguish the material based on the properties such as ductility and brittleness.

**20. What is the use of Stress – Strain diagram?**

The elastic behavior of solid materials is studied by using this stress-strain diagram.

**21. Draw the stress-strain diagram.****22. What are the Factors affecting Elasticity?**

- Effect of stress
- Effect of annealing
- Change in Temperature
- Presence of impurities
- Due to the nature of cycles

**23. What is meant by Annealing?**

Annealing is a process by which the material is heated to a very high temperature and then it is slowly cooled. Usually this process is adopted for the materials to increase the softness and ductility in the materials. But if annealing is made to a material it results in the formation of large crystal grains, which ultimately reduces the elastic property of the material.

**24. How do temperature and impurity in a material affect the elasticity of the materials?****Effect of Temperature:**

The elastic property of the materials changes with the temperature . Normally the elasticity increases with the decrease in temperature and vice-versa.

**Examples:**

- The elastic property of Lead increases when the temperature is decreased.
- The carbon filament becomes plastic at higher temperature.

**Effect of impurities:**

The addition of impurities produces variation in the elastic property of the materials. The increase and decrease of elasticity depends on the type of impurity added to it.

**Examples:**

- When potassium is added to gold, the elastic property of gold increases.
- When carbon is added to molten iron, the elastic property of iron decreases, provided the carbon content should be more than 1% in iron.

**26. What are the effects of hammering and annealing on elasticity of a material?**

While being hammered or rolled, crystal grains break into smaller units resulting in increase of their elastic properties. While annealing i.e heating and then cooling gradually, constituent crystals are uniformly oriented and form larger crystal grains, which results in decrease in their elastic properties.

**27. Define Moment, Couple and Torque.****(i) Moment of a Force:**

The moment of a force about a point is defined as the product of the magnitude of the force and perpendicular distance from the point to the line of action of force.

**(ii) Couple:**

A couple constitutes a pair of two equal and opposite forces acting on a body, in such a way that the lines of action of the two forces are not in the same straight line.

**(iii) Torque:**

Torque is the rotating force and is equal to the moment of the couple. Torque is the product of one of the forces forming couple and the perpendicular distance between the two opposite forces.

**28. Define Beam.**

A beam is a rod or bar of uniform cross-section (either circular or rectangular) of a homogeneous, isotropic elastic material whose length is large compared to its thickness.

**29. Explain Bending moment of Beam.**

The moment of the couple due to the elastic reactions (restoring couple) which balances the external couple due to the applied load is called the bending moment.

**30. What is Uniform Bending?**

The beam is loaded uniformly on its both ends, the bent beam forms an arc of a circle.

The radius of curvature of the bent beam is constant for given load. This type of bending is called Uniform Bending.

**31. What are the assumptions to study the bending of the beams?**

While studying about the bending of beams, the following assumptions have to be made.

1. The length of the beam should be large compared to other dimensions.
2. The load (forces) applied should be large compared to the weight of the beam.
3. The cross section of the beam remains constant and hence the geometrical moment of inertia  $I_g$  also remains constant.
4. The shearing stresses are negligible.
5. The curvature of the beam is very small.

**32. Explain Neutral axis or Define Neutral axis.**

The middle layer or filament of a beam which remains unaltered even with the presence of load on the beam is called Neutral axis. Filament which are lying above it are elongated and those are lying below it are compressed.

**33. Define a Cantilever.**

A cantilever is a beam fixed horizontally at one end and loaded at the other end.

**34. Define I Shape Girders:**

The girders with upper and lower section broadened and the middle section tapered, so that it can withstand heavy loads over it is called as I shaped girders. Since the girder look like I and show it is named as I shape girders.

**35. Explain the Advantages of I- Shape Girders.**

- More stability
- More stronger
- High durability

**36. Give the applications of I-Shape Girders.**

- Since they reduce the area of neutral axis, they have higher strength and more stability.
- Minimum amount of raw material is enough to make I-Shape Girders.
- They have high durability.

**PART B**

1. Derive an expression for the period of oscillation of a torsion pendulum. How can it be used to determine the torsional rigidity of a wire?
2. Describe with necessary theory, the method to determine the Young's modulus of the material of a rectangular bar by uniform bending.
3. Describe with necessary theory, the method to determine the Young's modulus of the material of a rectangular bar by Non uniform bending.
4. Describe an experiment to determine the Young's modulus of a beam using bending of beams?
5. Derive an expression for the internal bending moment of a beam in terms of radius of curvature?
6. Describe an experiment to determine Young's modulus of a beam by uniform bending.
7. Write short notes on I-shape Girders.
8. Derive an relation between three modulus of elasticity.

S.No	Questions	opt1	opt2	opt3	opt4	answer
1	The modulus of elasticity is dimensionally equivalent to	Strain	Stress	Surface tension	Poisson's ratio	Stress
2	If by applying a force, the shape of a body is changed, then the corresponding stress is known as	Tensile stress	Bulk stress	Shearing stress	Compressive stress	Shearing stress
3	According to Hooke's law of elasticity, within elastic limits, if the stress is increased, the ratio of stress to strain	Increases	Decreases	Becomes zero	Remains constant	Remains constant
4	Which one of the following does not affect the elasticity of a substance?	Hammering	Adding impurity in the substance	Changing the dimensions	Change of temperature	Changing the dimensions
5	The bulk modulus of a fluid is inversely proportional to the	Change in pressure	Volume of the fluid	Density of the fluid	Change in its volume	Change in its volume
6	Shearing strain is given by	Deforming force	Shape of shear	Angle of shear	Change in volume of body	Angle of shear
7	The ratio of the change in dimension at right angles to the applied force to the initial dimension is known as	Young's modulus	Poisson's ratio	Lateral strain	Shearing strain	Lateral strain
8	Which of the following is dimensionless quantity?	Stress	Young's modulus	Pressure	Strain	Strain
9	The energy per unit volume of a stretched wire is	$1/2 \times \text{stress} \times \text{strain}$	$\text{strain} \times \text{stress}$	$1/2 \times \text{load} \times \text{extension}$	$\text{load} \times \text{Extension}$	$1/2 \times \text{stress} \times \text{strain}$
10	Out of the following materials, whose elasticity is independent of temperature?	Copper	Invar steel	gold	Silver	Invar steel
11	Theoretical value of Poisson's ratio lies between	- 1 to 0.5	-1 to -2	0.5 to 1	1 to 2	- 1 to 0.5
12	Strain has	No units but only dimensions	Only units but no dimensions	No units, no dimensions but a constant value	No units, no dimensions but a variable value	No units, no dimensions but a variable value
13	When impurities are added to an elastic substance, its elasticity	Increases	Decreases	Becomes zero	May increase or decrease	May increase or decrease
14	Longitudinal strain is possible in the case of	Gases	Liquid	only solids	Only gases & liquids	only solids
15	When the intermolecular distance increases due to tensile force, then	There is no force between the molecules	There is a repulsive force between the molecules	There is an attractive force between the	There is zero resultant force between the	There is an attractive force between the molecules
16	If a material is heated and annealed, then its elasticity is	Increased	Decreased	Not change	Becomes zero	Decreased
17	Hooke's law essentially defines	stress	strain	yield point	elastic limit	elastic limit
18	The Young's modulus for a plastic body is	one	zero	infinity	less than one	zero
19	Which of the following have highest elasticity?	steel	copper	rubber	aluminium	steel
20	Energy in a stretched wire is	Half of load x strain	Half of stress / strain	Stress x strain	Load / strain	Half of load x strain
21	The property due to which then sheets can be prepared from a material is called	Elasticity	Brittleness	Malleability	Ductility	Malleability
22	The substance which shows practically no elastic effect is	Quartz	Copper	silk	Rubber	Quartz
23	In uniform bending experiment ,which property can be calculated	Rigidity modulus	youngs modulus	bulk modulus	none	youngs modulus
24	In torsional pendulum experiment ,which property can be calculated	Rigidity modulus	youngs modulus	bulk modulus	none	Rigidity modulus
25	The property by virtue of which a deformed body tends to regain its original shape after the removal of deforming forces is called -----	Elasticity	Plasticity	Rigidity	Moldity	Elasticity
26	Restoring force per unit area is given by	Stress	Strain	Modulus	Shearing strain	Stress
27	Unit of stress is -----	$\text{Nm}^{-2}$	$\text{Nm}^2$	$\text{Nm}^3$	$\text{Nm}^{-1}$	$\text{Nm}^{-2}$
28	Unit of strain is -----	Metre	$\text{Metre}^3$	No unit	Stress	No unit
29	Based on Hookes law stress is directly proportional to-----	Modulus	Strain	Stress	Elasticity	Strain
30	Young's modulus = Longitudinal stress/-----	Longitudinal strain	Lateral strain	Shearing strain	Bulk strain	Longitudinal strain
31	The ratio of tangential stress to shearing strain is -----	Young's modulus	Rigidity modulus	Bulk modulus	Poisson's ratio	Rigidity modulus
32	Volume stress/volume strain is known as -----	Poisson's ratio	Bulk modulus	Rigidity modulus	Young's modulus	Rigidity modulus
33	Relationship between three modulli of elasticity is -----	$3K+G/ 9Gk$	$9GK/ 3k+G$	$9Gk/9G+K$	$3K-2G/6K+2G$	$9GK/ 3k+G$
34	A rod of uniform cross section with greatest length is known as ----	Bar	Beam	Cantilever	support	Beam
35	If a beam fixed horizontally at one end and loaded at the other end is known as-----	Bar	Cantilever	Support	Beam	Cantilever
36	Poisson's ratio = -----	lateral strain/ longitudinal strain	lateral stress /lateral strain	longitudinal stress/strain	stress/strain	lateral strain/ longitudinal strain
37	What is the ratio of stress to tensile strain called?	Poisson's ratio	Pascal law	Hooke's law	Youngs modulus	Youngs modulus
38	What is the SI unit of pressure?	Nm	$\text{Nm}^{-1}$	$\text{Nm}^{-2}$	$\text{Nm}^{-3}$	$\text{Nm}^{-2}$
39	What is the density of air?	1.3	13.6	9.23	123.32	1.3



## UNIT-II

### LASER AND FIBRE OPTICS

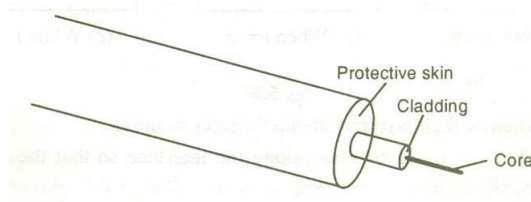
#### INTRODUCTION TO FIBRE OPTICS

The development of lasers and optical fiber has brought about a revolution in the field of communication systems. Experiments on the propagation of information – carrying light waves through an open atmosphere were conducted. The atmospheric conditions like rain, fog etc affected the efficiency of communication through light waves. To have efficient communication systems, the information carried by light waves should need a guiding medium through which it can be transmitted safely. This guiding mechanism is optical fiber. The communication through optical fiber is known as light wave communication or optical communication. A light beam acting as a carrier wave is capable of carrying more information than that of radio waves and microwaves due to its larger bandwidth. Currently in most part of the world, fiber optics is used to transmit voice, video and digital data signals using light waves from one place to other place.

#### OPTICAL FIBER

It is made up of transparent dielectrics ( $\text{SiO}_2$ ), (glass or dielectrics).

An optical fibre of a central core glass ( $50\mu\text{m}$  dia) surrounded by a cladding ( $125\text{--}200\mu\text{m}$ ) which is of slightly lower refractive index than core as shown in figure.



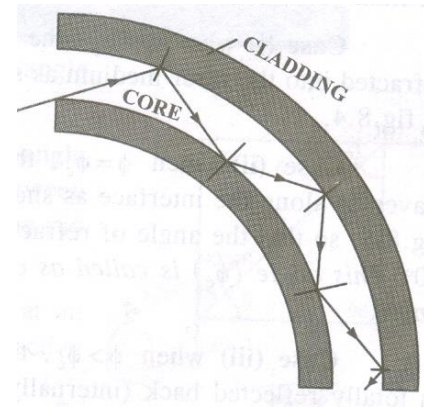
The cladding is enclosed by strength members and polyurethane jacket, which act as protective skin for core and cladding as shown in figure. The protective layer is used so has to make the optical cable to withstand for hard pulling, bending, sketching, rolling etc.. the layer also traps the escaping light from the core.

#### Features of Optical fibers

- (i) It is light in weight
- (ii) It is smaller in size
- (iii) It is flexible
- (iv) It is non – conductive, non – radiative and non – inductive
- (v) It has high bandwidth and low loss
- (vi) There is no cross talk / internal noise
- (vii) It can withstand to any range of temperature and moisture condition.
- (viii) No voltage problem occurs

## PRINCIPLE AND PROPAGATION OF LIGHT IN OPTICAL FIBERS

For optical fibers, the process of propagation of light (optical signal) is simple, because once the light enters the fiber, the rays do not encounter any new surfaces, but repeatedly they hit the same surface. The reason of confining the light beam inside the fibers is the total internal reflection. Even for a bent fiber, the light guidance takes place by multiple total internal reflections all over the length of the fiber as shown in figure.



### Principle

The principle of optical fiber communication is Total Internal Reflection.

### Total Internal Reflection

The phenomenon of Total Internal Reflection takes place when it satisfies the following two conditions.

#### Condition 1:

Light should travel from denser medium to rarer medium i.e  $n_1 > n_2$

Where  $n_1$  = refractive index of core

$n_2$  = refractive index of cladding

#### Condition 2:

The angle of incidence on core should be greater than the critical angle.

$$\text{i.e. } \phi > \phi_c$$

Where,

$\phi$  - angle of incidence

$\phi_c$  – critical angle

### Propagation Phenomenon

Let the light rays traverses from denser medium to rarer medium.

### Case i.

When  $\phi < \phi_c$  the ray is refracted into the rarer medium as shown in figure.

### Case ii.

When  $\phi = \phi_c$ , the ray traverses along the interface as shown in figure. So that the angle of refraction is  $90^\circ$ . This angle is called as critical angle.

### Case iii.

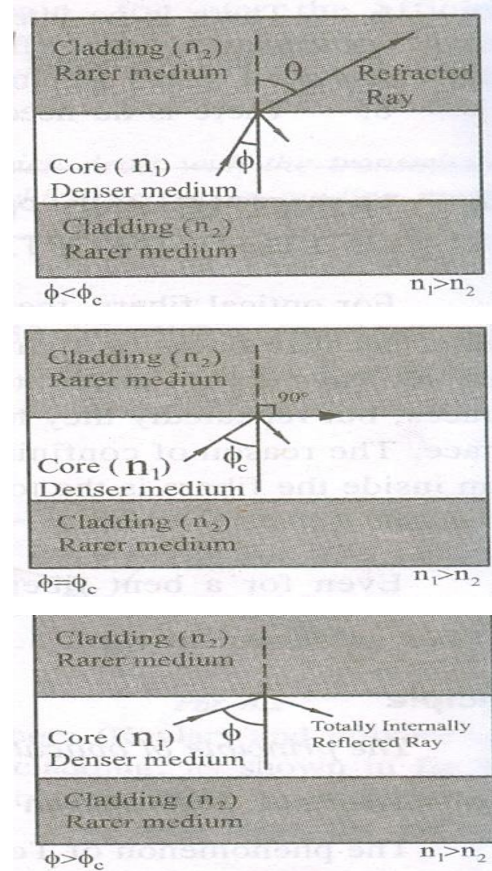
When  $\phi > \phi_c$  the ray is totally reflected back (internally) into the denser medium itself as shown in figure. From Snell's law (the maximum angle for Total Internal Reflection  $\phi_c$ ).

$$n_1 \sin \phi_c = n_2 \sin 90^\circ$$

$$\sin \phi_c = n_2/n_1$$

since  $\sin 90^\circ = 1$ , we have

$$\phi_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$



## ACCEPTANCE ANGLE AND NUMERICAL APERTURE

Let us consider a cylindrical fiber. it consists of core of refractive index  $n_1$ , and cladding of refractive index  $n_2$  and let  $n_0$  be the refractive .

The incident ray travels along AO and enters the core at an angle 'i' to the fiber axis.

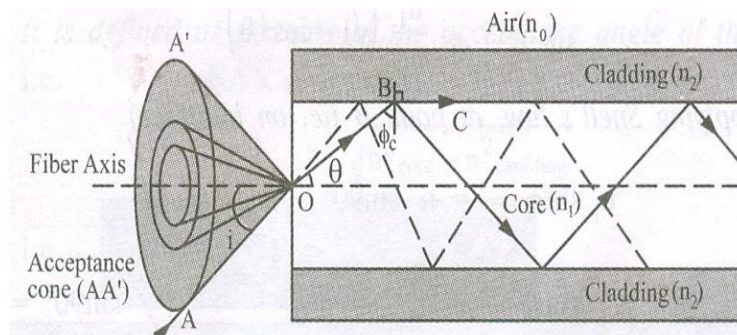
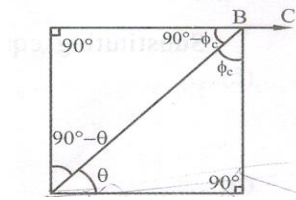


Fig.5.24

The ray is refracted along OB at an angle  $\theta$  in the core as shown in figure 5.24.

It further proceeds to fall at critical angle of incidence  $(\phi_c) = 90 - \theta$  on the interface between core and cladding. At this angle the ray just moves along BC.

Any ray which enters in to the core at an angle of incidence less than  $i_m$  will have refractive angle less than  $\theta_c$ .



Hence, the angle of incidence ( $\phi = 90 - \theta$ ) at the interface of core and cladding will be more than the critical angle. Hence the ray is totally internally reflected ray. Thus, only those ray which passes within the acceptance angle (cone) will be totally internally reflected. Therefore, the light incident on the core within this maximum external incident angle ( $i_m$ ) can be coupled into the fiber to propagate. This angle ( $i_m$ ) is called as wave guide acceptance angle.

### Mathematical Relation

- i. Applying snell's law, at point entry of ray (AO) we have

$$n_o \sin i = n_1 \sin \theta$$

$$\sin i = \frac{n_1}{n_o} \sin \theta$$

$$\sin i = \frac{n_1}{n_o} \sqrt{1 - \cos^2 \theta} \quad \text{----- (1)}$$

- ii. Applying snell's law, at point B (ie.on surface)

$$n_1 \sin \phi_c = n_2 \sin 90^\circ.$$

$$\sin \phi_c = \frac{n_2}{n_1} \quad (\sin 90^\circ = 1)$$

$$\sin (90 - \theta) = \frac{n_2}{n_1}$$

$$\cos \theta = \frac{n_2}{n_1} \quad \text{----- (2)}$$

substituting equation (2) in equation (1) we get

$$\sin i = \frac{n_1}{n_o} \left[ \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} \right]$$

$$\sin i = \frac{n_1}{n_o} \cdot \frac{1}{n_1} \sqrt{n_1^2 - n_2^2}$$

$$i = \sin^{-1} \left( \frac{\sqrt{n_1^2 - n_2^2}}{n_o} \right)$$

If the refractive index of air,  $n_o = 1$ , then the maximum value of  $\sin i$  is given as

$$\sin i_{\max} = \sqrt{n_1^2 - n_2^2} \quad \text{----- (3)}$$

Where,  $n_1$  and  $n_2$  are refractive indices of core and cladding respectively.

## Acceptance angle

Thus the maximum angle at or below which the light can suffer Total Internal Reflection is called acceptance angle. The cone is referred as acceptance cone.

## Numerical Aperture [NA]

It is defined as the sine of the acceptance angle of the fiber.

$$\begin{aligned} \text{NA} &= \sin i_m \\ &= \sqrt{n_{\text{core}}^2 - n_{\text{cladding}}^2} \end{aligned}$$

$$\text{NA} = \sin i_m = \sqrt{n_1^2 - n_2^2} \quad \text{-----(4)}$$

## Fractional index change ( $\Delta$ )

It is the ratio of refractive index difference in core and cladding to the refractive index of core.

$$\Delta = \frac{n_1 - n_2}{n_1} \quad \text{-----(5)}$$

Relation between NA and  $\Delta$

$$n_1 \Delta = n_1 - n_2 \quad \text{-----(6)}$$

We know  $\text{NA} = \sqrt{n_1^2 - n_2^2}$

$$\text{(Or )} \quad \text{NA} = \sqrt{(n_1 + n_2)(n_1 - n_2)} \quad \text{-----(7)}$$

Substituting equation (6) in equation (7) we have

$$\text{or } \text{NA} = \sqrt{(n_1 + n_2)(n_1 \Delta)}$$

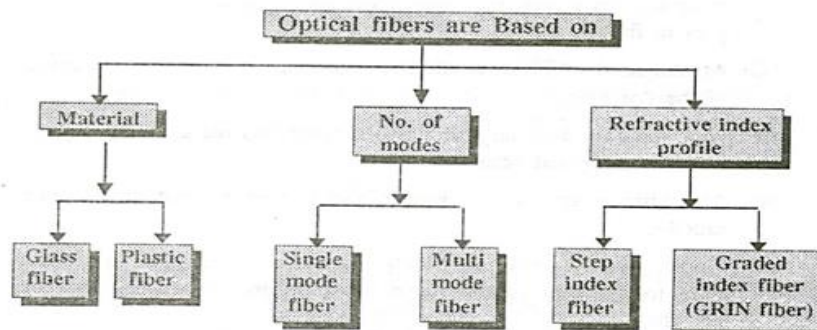
$$\text{if } n_1 \approx n_2, \text{ then } \text{NA} = \sqrt{2n_1^2 \Delta}$$

$$\text{NA} = n_1 \sqrt{2\Delta}$$

## TYPES OF OPTICAL FIBERS

Optical fibers are classified into three major categories

- i. The type of material used
- ii. The number of modes
- iii. The refractive index profile



## GLASS AND PLASTIC FIBERS

Based on the type of the material used, they are classified into two types.

### Glass fibers:

The glass fibres are made up of mixture of metal metal oxides and silica glasses.

**Example:** The glass fibres are made by the following combinations of core and cladding.

Core:  $\text{SiO}_2$ , Cladding:  $\text{SiO}_2$

Core:  $\text{GeO}_2$ -  $\text{SiO}_2$ , Cladding:  $\text{SiO}_2$

### Plastic fibers:

The fibers which are made up of plastics can be handled without any care due to its toughness and durability are called plastic fiber.

**Example:** The plastic fibres are made by the following combinations of core and cladding.

Core: polymethyl methacrylate , Cladding: Co- Polymer

Core: Polystyrene, Cladding: Methyl methacrylate

## SINGLE AND MULTIMODE FIBER

Light propagates as electromagnetic waves through an optical fiber. Based on the modes of propagation the fibers are classified into two types.

1. Single mode fiber
2. Multimode fiber

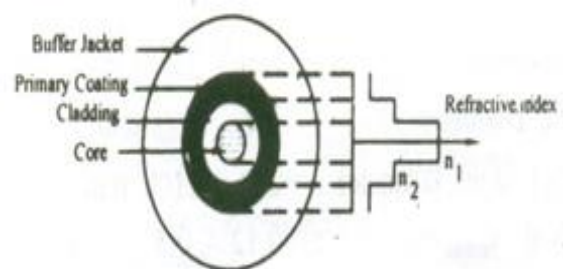
### 1. Single mode fiber

In general, the single mode fibers are step – index fibers. These types of fibers are made from doped silica. It has a very small core diameter so that it can allow only one mode of propagation and hence called single mode fibers.

The cladding diameter must be very large compared to the core diameter. Thus in the case of single mode fiber, the optical loss is very much reduced. The structure of a single mode fiber as shown in figure.

#### Structure

Core diameter	: 5-10 $\mu\text{m}$
Cladding diameter	: Around 125 $\mu\text{m}$
Protective layer	: 250 to 1000 $\mu\text{m}$
Numerical aperture	: 0.08 to 0.10
Band width	: More than 50MHz km.



### Application:

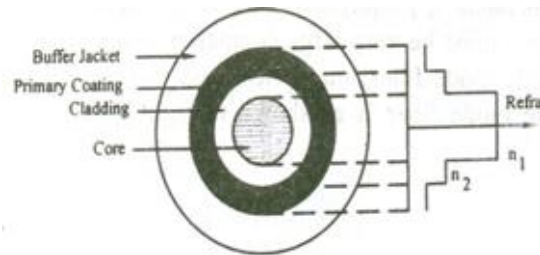
Because of high bandwidth, they are used in long haul communication systems.

## 2. Multimode fiber

The multi modes fibers are useful in manufacturing both for step – index and graded index fibers. The multi-mode fibers are made by multi-component glass compounds such as Glass – Clad Glass, Silica – Clad – Silica, doped silica etc. Here the core diameter is very large compared to single mode fibers, so that it can allow many modes to propagate through it and hence called as Multi mode fibers. The cladding diameter is also larger than the diameter of the single mode fibers. The structure of the multimode fiber is as shown in the figure.

### Structure

Core diameter : 50-350 $\mu$ m  
Cladding diameter : 125 $\mu$ m - 500 $\mu$ m  
Protective layer : 250 to 1100 $\mu$ m  
Numerical aperture : 0.12 to 0.5  
Band width : Less than 50MHz km.



### Application:

Because of its less band width it is very useful in short haul communication systems.

### DIFFERENCE BETWEEN SINGLE AND MULTIMODE FIBER

S. No.	SINGLE MODE FIBER	MULTI MODE FIBER
1.	In single mode fiber only one mode can propagate through the fiber	In multimode it allows a large number of paths or modes for the light rays travelling through it.
2.	It has smaller core diameter and the difference between the refractive index of the core and cladding is very small.	It has larger core diameter and refractive index difference is larger than the single mode fiber.
3.	<b>Advantages:</b> No dispersion(i.e. there is no degradation of signal during propagation)	<b>Disadvantages:</b> Dispersion is more due to degradation of signal owing to multimode.
4.	The fiber can carry information to longer distances.	Information can be carried to shorter distances only.
5.	<b>Disadvantages:</b> Launching of light and connecting of two fibers difficult.	<b>Advantages:</b> Launching of light and also connecting of two fibers is easy.
6.	Installation (fabrication) is difficult as it is more costly	Fabrication is easy and the installation cost is low.

### STEP INDEX AND GRADED INDEX FIBERS

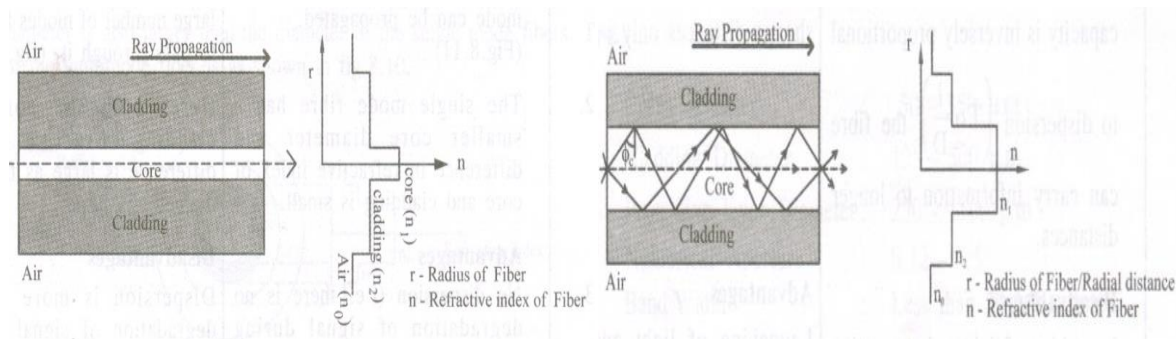
Based on the variation in the refractive index of the core and the cladding, the fibers are classified into two types

- Step index fiber
- Graded index fiber



## Step index fiber

The refractive indices of air, cladding and core vary by step by step and hence it is called as step index fiber. In step index fiber we have both single mode and multimode fibers as shown in figure 5.28.

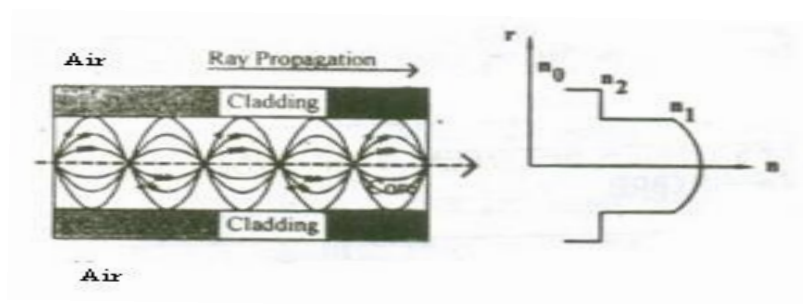


Single mode step index fiber

Multi mode step index fiber

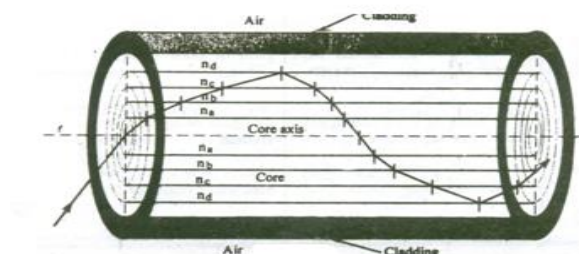
## Graded index fiber

The refractive index of the core varies radially from the axis of the fiber. The refractive index of the core is maximum along the fiber axis and it gradually decreases. Thus it is called as graded index fiber. Here the refractive index becomes minimum at the core-cladding interface. In general the graded index fibers will be of multi mode system. The multi mode graded index fiber has very less intermodal dispersion compared to multi mode step index fiber. A typical multi mode graded index fiber is as shown in figure.



## Propagation of light in GRIN fiber

Let  $n_a, n_b, n_c, n_d$  etc be the refractive index of different layers in graded index fiber with  $n_a > n_b > n_c > n_d$  etc. then the propagation of light through the graded index fiber is as shown in the figure .





Here, since  $n_a > n_b$  the ray gets refracted. Similarly since  $n_b > n_c$ , the ray gets refracted and so on. In a similar manner, due to decrease in refractive index the ray gets gradually curved towards the upward direction and at one place, where in it satisfies the condition for total internal reflection, ( $\phi > \phi_c$ ) it is totally internally reflected. The reflected rays travels back towards the core axis and without crossing the fiber axis, it is refracted towards downwards direction and again gets totally internally reflected and passes towards upward direction. In this manner the ray propagates inside the fiber in a helical or spiral manner.

### DIFFERENCE BETWEEN STEP INDEX FIBER & GRADED INDEX FIBER

S. No.	STEP INDEX FIBER	GRADED INDEX (GRIN) FIBER
1.	The difference in refractive indices is obtained in single step and hence called as step-index fiber.	Due to non-uniform refractive indices, the difference in refraction index is obtained gradually from centre towards interface and hence called graded index fiber.
2.	The light ray propagation is in the form of meridional rays and it passes through the fiber axis.	The light ray propagation is in the form of skew rays and it will not cross the fiber axis.
3.	The path of light propagation is in zig-zig manner.	The path of light is <b>helical</b> in manner
4.	This fiber has <b>lower bandwidth</b>	This fiber has <b>higher bandwidth</b>
5.	<b>Attenuation is more</b> for multimode step index fiber but for single mode it is very less. Explanation: When a ray travels through the longer distances there will be some difference in reflected angles. Hence high angle rays arrive later than low angle rays causing dispersion resulting in distorted output.	<b>Attenuation is less.</b> Explanation: Here the light rays travel with different velocity in different paths because of their variation in their refractive indices. At the outer edge it travels faster than near the center. But almost all the rays reach the exit at the same time due to helical path. Thus, there is no dispersion.
6.	<b>No of modes of Propagation</b> $N_{\text{step}} = 4.9 \left( \frac{d \times NA}{\lambda} \right)^2 = \frac{V^2}{2}$ Where d= diameter of the fiber core $\lambda$ = wavelength NA = Numerical Aperture V- V-number is less than or equal to 2.405 for single mode fibers & greater than 2.405 for multimode fibers.	<b>No of modes of Propagation</b> $N_{\text{step}} = 4.9 \left( \frac{d \times NA}{\lambda} \right)^2 = \frac{V^2}{2}$ Or $N_{\text{graded}} = \frac{N_{\text{step}}}{2}$

## LOSSES IN OPTICAL FIBERS- ATTENUATION

When light propagates through an optical fiber, a small percentage of light is lost through different mechanisms. The loss of optical power is measured in terms of decibels per kilometer for attenuation losses.

### Attenuation

It is defined as the ratio of the optical power output ( $P_{out}$ ) from a fiber of length 'L' to the power input ( $p_{in}$ )

$$\text{ie. Attenuation } (\alpha) = \frac{-10}{L} \log \frac{P_{in}}{P_{out}} \text{ dB/km}$$

Since attenuation plays a major role in determining the transmission distance, the following attenuation mechanisms are to be considered in designing an optical fiber.

- (1) Absorption
- (2) Scattering
- (3) Radiative losses.

### 1. Absorption

Usually absorption of light occurs due to imperfections of the atomic structure such as missing molecules, (OH<sup>-</sup>) hydroxyl ions, high density cluster of atoms etc., which absorbs light. Absorption also depends on the wavelength of the light used. The three bands of wavelength at which the absorption increases drastically is 950 nm, 1250 nm and 1380 nm. For example, at the wavelength say 850 nm the absorption is 1.5 dB/Km and for 1500 nm, it is 0.5 dB/Km.

### 2. Scattering

Scattering is also a wavelength dependent loss, which occurs inside the fibers. Since the glass is used in fabrication of fibers, the disordered structure of glass will make some variations in the refractive index inside the fiber. As a result, if it is passed through the atoms in the fiber, a portion of the light is scattered (elastic scattering). This type of scattering is called Rayleigh scattering. i.e., Rayleigh scattering loss  $\propto \frac{1}{\lambda^4}$

### 3. Radiative losses

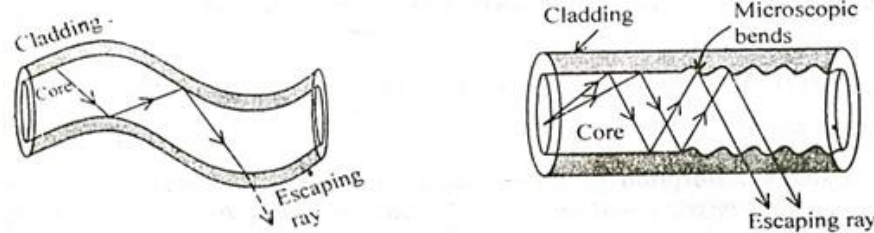
Radiative loss occurs in fibers, due to bending of finite radius of curvature in optical fibers. The types of bends are

- (a) Macroscopic bend and
- (b) Microscopic bend

#### (a) Macroscopic bends

If the radius of core is large compared to fiber diameter as shown in fig. it may cause large-curvature at the position where the fiber cable turns at the corner. At these corners the light

will not satisfy the condition for total internal reflection and hence it escapes out from the fiber. This is called as macroscopic/macro bending losses. Also note that this loss is negligible for small bends.



### (b) Microscopic bends

Micro-bends losses are caused due to non-uniformities (or) micro-bends inside the fiber as shown in fig. This micro bends in fiber appears due to non uniform pressures created during the cabling of the fiber (or) even during the manufacturing itself. This lead to loss of light by leakage through the fiber.

### Remedy

Micro-bend losses can be minimized by extruding (squeezing out)a compressible jacket over the fiber. In such cases even when the external forces are applied, the jacket will be deformed but the fiber will tend to stay relatively straight and safe, without causing more loss.

Fiber optic cables find many uses in a wide variety of industries and applications. Some uses of fiber optic cables include:

## APPLICATION OF OPTICAL FIBER

- **Medical**

Used as light guides, imaging tools and also as lasers for surgeries

- **Defense/Government**

Used as hydrophones for seismic waves and SONAR , as wiring in aircraft, submarines and other vehicles and also for field networking

- **Data Storage**

Used for data transmission

- **Telecommunications**

Fiber is laid and used for transmitting and receiving purposes

- **Networking**

Used to connect users and servers in a variety of network settings and help increase the speed and accuracy of data transmission

- **Industrial/Commercial**

Used for imaging in hard to reach areas, as wiring where EMI is an issue, as sensory devices to make temperature, pressure and other measurements, and as wiring in automobiles and in industrial settings

- **Broadcast/CATV**

Broadcast/cable companies are using fiber optic cables for wiring CATV, HDTV, internet, video on-demand and other applications

## **INTRODUCTION**

The word "**laser**" is an acronym for Light Amplification by Stimulated Emission of Radiation. Lasers have many important applications. They are used in common consumer devices such as DVD players, laser printers, and barcode scanners.

They are used in medicine for laser surgery and various skin treatments, and in industries for cutting and welding materials.

## **CHARACTERISTICS OF LASER**

### **(i) Directionality**

Ordinary light spreads in all directions and its angular spread is 1 metre/metre. But it is found that laser is highly directional and its angular spread is 1mm/metre.

### **(ii) Intensity**

An ordinary light spreads in all directions; the intensity reaching the target is very less. But in the case of laser, due to high directionality the intensity of laser beam reaching the target is of high intense beam.

For example, 1 milli watt power of He-Ne laser appears to be brighter than the sunlight.

### **(iii) Monochromaticity**

Laser beam is highly monochromatic i.e. the wavelength is single, whereas in ordinary light like mercury vapour lamp, many wavelengths of light are emitted.

### **(iv) Coherence**

The light from a laser is said to be highly coherent, which means that the waves of laser light are in same amplitude and phase. There are two types of coherence,

- temporal coherence
- spatial coherence.

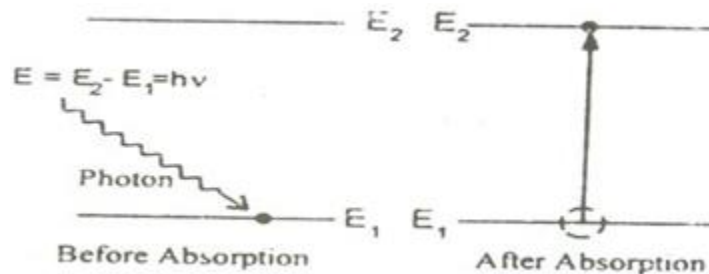
# PRINCIPLE OF SPONTANEOUS AND STIMULATED EMISSION EINSTEIN'S QUANTUM THEORY OF RADIATION

When light is absorbed by the atoms or molecules, then it goes from the lower energy level ( $E_1$ ) to the higher energy level ( $E_2$ ) and during the transition from higher energy level ( $E_2$ ) to lower energy level ( $E_1$ ), the light is emitted from the atoms or molecules. Let us consider an atom exposed to (light) photons of energy  $E_2 - E_1 = h\nu$ , three distinct processes takes place.

- i. Absorption
- ii. Spontaneous emission
- iii. Stimulated emission

## i. Absorption

An atom in the lower energy level or ground state energy level  $E_1$  absorbs the incident photon radiation of energy ( $h\nu$ ) and goes to the higher energy level or excited energy state  $E_2$  as shown in fig 5.1. This process is called as absorption.



If there are many number of atoms in the ground state then each atom will absorb the energy from the incident photon and goes to the excited state then,

The rate of absorption ( $R_{12}$ ) is proportional to the following factors.

- i.e.  $R_{12} \propto$  Energy density of incident radiation ( $\rho_\nu$ )  
 $\propto$  No of atoms in the ground state ( $N_1$ )

i.e.  $R_{12} \propto \rho_\nu N_1$

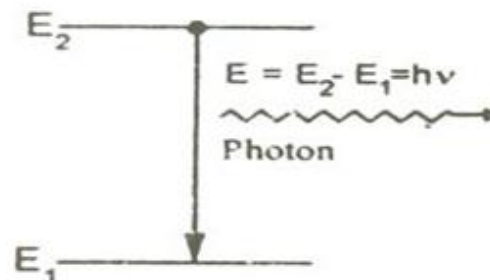
or  $R_{12} = B_{12} \rho_\nu N_1$  ----(1)

Where,  $B_{12}$  is a constant which gives the probability of absorption transition per unit time.

## ii. Spontaneous emission

The atom in the excited state returns to the ground state by emitting a photon of energy  $E = (E_2 - E_1) = h\nu$ , spontaneously without any external triggering as shown in figure.

This process is known as spontaneous emission. Such an emission is random and is



independent of incident radiation.

If  $N_1$  and  $N_2$  are the numbers of atoms in the ground state ( $E_1$ ) and excited state ( $E_2$ ) respectively, then

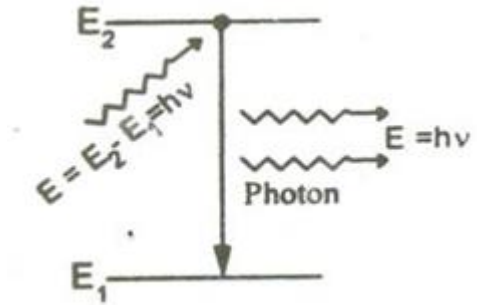
The rate of spontaneous emission is  $R_{21}(\text{Sp}) \propto N_2$

$$\text{(or)} R_{21}(\text{Sp}) = A_{21}N_2 \quad \text{-----(2)}$$

Where,  $A_{21}$  is a constant which gives the probability of spontaneous emission transition per unit time.

### iii. Stimulated emission

The atom in the excited state can also return to the ground state by external triggering (or) inducement of photon there by emitting a photon of energy equal to the energy of the incident photon, known as stimulated emission. Thus results in two photons of same energy, phase difference and of same directionality as shown in figure.



The rate of stimulated emission is  $R_{21}(\text{St}) \propto \rho_\nu N_2$

$$\text{(or)} R_{21}(\text{St}) = B_{21} \rho_\nu N_2 \quad \text{-----(3)}$$

Where,  $B_{21}$  is a constant which gives the probability of stimulated emission transition per unit time.

### Einstein's theory

Einstein's theory of absorption and emission of light by an atom is based on Planck's theory of radiation. Also under thermal equilibrium, the population of energy levels obeys the Boltzmann's distribution law.

i.e. under thermal equilibrium

The rate of absorption = The rate of emission

$$B_{12} \rho_\nu N_1 = A_{21}N_2 + B_{21} \rho_\nu N_2$$

$$\rho_\nu [B_{12} N_1 - B_{21} N_2] = A_{21}N_2$$

$$\therefore \rho_\nu = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$\therefore \rho_\nu = \frac{A_{21}}{B_{12}(N_1/N_2) - B_{21}} \quad \text{-----(4)}$$

We know from Boltzmann distribution law

$$N_1 = N_0 e^{-E_1/K_B T}$$

Similarly

$$N_2 = N_0 e^{-E_2/K_B T}$$

Where

$K_B$  - Boltzmann Constant

$N_0$  - Number of atoms at absolute zero

T- Absolute temperature

At equilibrium, we can write the ratio of population levels as follows,

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/K_B T}$$

Since  $E_2 - E_1 = h\nu$ , we have

$$\frac{N_1}{N_2} = e^{h\nu/K_B T} \quad \text{----(5)}$$

Sub eqn (5) in (4), we get

$$\begin{aligned} \rho_v &= \frac{A_{21}}{B_{12}(e^{h\nu/K_B T}) - B_{21}} \\ \rho_v &= \frac{A_{21}}{B_{21}} \frac{1}{(B_{12}/B_{21})e^{h\nu/K_B T} - 1} \end{aligned} \quad \text{----(6)}$$

This equation has a very good agreement with plank's energy distribution radiation law

$$\rho_v = \frac{8\pi h\nu^3}{C^3} \frac{1}{e^{h\nu/K_B T} - 1} \quad \text{---(7)}$$

Therefore comparing (6) and (7), we have

$$B_{12} = B_{21} = B$$

and

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{C^3} \quad \text{----(8)}$$

Taking  $A_{21} = A$

The constants A and B are called as Einstein Coefficients, which accounts for spontaneous and stimulated emission probabilities. Ratio of magnitudes of stimulated and spontaneous emission rates are as follows,

From eqn (2) and (3) we have

$$\begin{aligned} \frac{R_{21}(\text{st})}{R_{21}(\text{sp})} &= \frac{B_{21}\rho_v N_2}{A_{21} N_2} \\ \frac{R_{21}(\text{st})}{R_{21}(\text{sp})} &= \frac{B_{21}\rho_v}{A_{21}} \end{aligned} \quad \text{---(9)}$$

Rearranging eqn (6) we can write

$$\frac{B_{21}\rho_v}{A_{21}} = \frac{1}{(B_{12}/B_{21})e^{h\nu/K_B T} - 1}$$

Since  $B_{12} = B_{21}$ , we have

$$\frac{1}{e^{\frac{h\nu}{k_B T} - 1}} = \frac{B_{21} \rho_\nu}{A_{21}} \quad \text{--- (10)}$$

Comparing (9) and (10) we get

$$\frac{R_{21}(\text{st})}{R_{21}(\text{sp})} = \frac{1}{e^{\frac{h\nu}{k_B T} - 1}} = \frac{B_{21} \rho_\nu}{A_{21}}$$

In simpler way the ratio can be written as

$$R = \frac{B_{21} \rho_\nu}{A_{21}}$$

Generally spontaneous emission is more predominant in the optical region (ordinary light). To increase the number of coherent photons stimulated emission should dominate over spontaneous emission.

## DIFFERENCE BETWEEN SPONTANEOUS AND STIMULATED EMISSION OF RADIATION

S. No	Stimulated emission	spontaneous emission
1.	An atom in the excited state is induced to return to ground state, thereby resulting in two photons of same frequency and energy is called stimulated emission.	The atom in the excited state returns to ground state thereby emitting a photon, without any external inducement is called spontaneous emission.
2.	The emitted photons move in same direction and is highly directional	The emitted photons move in all directions and are random.
3.	The radiation is high intense, monochromatic and coherent.	The radiation is less intense and is incoherent.
4.	The photons are in phase (i.e.) there is a constant phase difference.	The photons are not in phase (ie.) there is no phase relationship between them.
5.	The rate of transition is given by $R_{21}(\text{St}) = B_{21} \rho_\nu N_2$	The rate of transition is given by $R_{21}(\text{Sp}) = A_{21} N_2$

## POPULATION INVERSION

Consider two energy level systems  $E_1$  and  $E_2$ . Suppose a photon of energy equal to the energy difference between the two energy levels, incident on the system, then there is equal chances for stimulated emission and absorption to occur. At this situation the chance for emission or absorption depends only on the number of atoms in the ground state and in the excited state.

Let  $N_1$  be the number of atoms in ground state and  $N_2$  be the number of atoms in excited state. Then,

If  $N_1 > N_2$  there is more chance for absorption takes place.

If  $N_2 > N_1$  there is more chance for stimulated emission takes place.



Therefore, the number of atoms in the excited state should be increased by some means. Thus the state of achieving more number of atoms in the excited state compared to the ground state atoms is called population inversion.

We know from Boltzmann distribution law  $N_1/N_2 = e^{(E_2-E_1)/K_B T}$

**Case (i):** If T is +ve

$$N_1 = N_2 e^{+ve}$$

For example if  $N_2 = 5$  and if  $(E_2 - E_1) / k_B T \approx 2$ ,

$$\text{Then, } N_1 = 5 \cdot e^{+2} = 36.9$$

$$N_1 > N_2 \text{ since } 36.9 > 5$$

**Case (ii)** If T is -ve

$$N_1 = N_2 e^{-ve}$$

For example If  $N_2 = 5$  and if  $(E_2 - E_1) / k_B T \approx 2$ ,

$$N_1 = 5 \cdot e^{-2} = 0.6766$$

$$N_2 > N_1 \text{ since } 5 > 0.6766$$

This shows that number of atoms in excited state can be made more than number of atoms in the ground state only under negative temperature. But, the negative temperature is practically not possible. Therefore population inversion can be achieved by some other artificial process known as pumping process.

### **Active medium**

The medium in which the population inversion takes place is called as active medium.

### **Active centre**

The material in which the atoms are raised to excited state to achieve population inversion is called as active centre.

## **PUMPING METHODS**

### **Pumping**

The process of raising more number of atoms to excited state by artificial means is called as pumping process. There are several methods by which the population inversion (pumping) can be achieved. Some of the most commonly used methods are as follows,

- (i) Optical pumping
- (ii) Electric discharge
- (iii) Inelastic atom – atom collision.
- (iv) Direct conversion
- (v) Chemical process

#### **(i) Optical pumping**

The atoms are excited with help of photons emitted by an external optical source. The atoms absorb energy from the photons and raises to excited state.

Ex. Ruby laser, Nd-YAG laser.

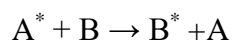
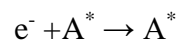
### **(ii) Electric discharge**

The electrons are accelerated to very high velocity by strong electric field and they collide with gas atoms and these atoms are raised to excited state (e.g) argon laser, Helium-Neon laser, CO<sub>2</sub> Laser etc...

### **(iii) Inelastic atom-atom collision**

In this method a combination of two types of gases are used. Say A and B, either having same (or) nearly coinciding excited states A\* and B\*.

During electric discharge 'A' atoms get excited due to collision with electrons. The excited A\* atoms now collide with 'B' atoms so that B goes to excited state B\* (e.g), Helium-Neon laser, CO<sub>2</sub> Laser.



### **(iv) Direct conversion**

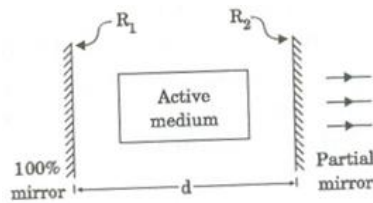
Due to electrical energy applied in direct band gap semiconductor like GaAs etc., the combination of electrons and holes take place and electrical energy is converted into light energy directly. (e.g) Semiconductor laser.

### **(v) Chemical process**

Due to some chemical reactions, the atoms may be raised to excited state. (e.g) Dye laser

## **OPTICAL RESONATOR**

The Optical resonator constitutes an active medium kept in between a 100% reflecting mirror and a partially reflecting mirror as shown in figure 5.4. This optical resonator acts as a feedback system in amplifying the light emitted from the active medium, by making it to undergo multiple reflections between the 100% mirror and partial mirror. Here the light bounces back and forth between the two mirrors and hence the intensity of the light is increased enormously. Finally the intense, amplified beam called LASER is allowed to come out through the partial mirror as shown in figure 5.4.



## TYPES OF LASERS

Based on the type of active medium, Laser systems are broadly classified into the following categories.

S.No	TYPES OF LASER	EXAMPLES
1.	Solid State Laser	Ruby Laser Nd:YAG laser
2.	Gas laser	He-Ne Laser, CO <sub>2</sub> Laser, Argon – ion Laser
3.	Liquid Laser	SeOCL <sub>2</sub> Laser, Europium Chelate Laser
4.	Dye Laser	Rhodamine 6G laser, Coumarin dye laser
5.	Semiconductor Laser	GaAs laser, GaAsP laser

## CARBON – DI - OXIDE LASER

### Characteristics of CO<sub>2</sub> laser

Type – Molecular Gas laser

Active medium – Mixture of CO<sub>2</sub> , N<sub>2</sub> and helium or Water vapour

Active centre – Co<sub>2</sub>

Pumping method – Electric discharge method

Optical resonator – Metallic mirror of gold or silicon mirrors coated with aluminium

Power output – 10Kw

Nature of output – continuous or pulsed

Wavelength emitted – 9.6μm & 10.6 μm

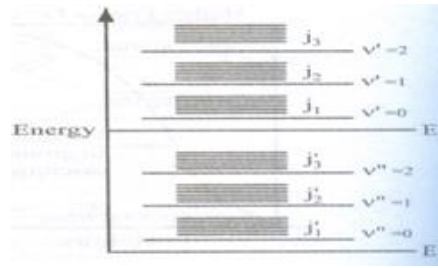
### Introduction

An Indian engineer C.K.N designed the co<sub>2</sub> laser we know I n the case if atoms, electrons can be excited to higher energy levels of the molecule eg. He - Ne laser. Besides these electronic energy levels, the molecule can have other energy levels also due to rotation and vibration of the molecule (co<sub>2</sub>) they give rise to various vibrational and rotational energy levels as shown in figure 5.6.

Where,  $E_1 E_2$  – electronic energy levels

$v', v''$  – vibrational energy levels

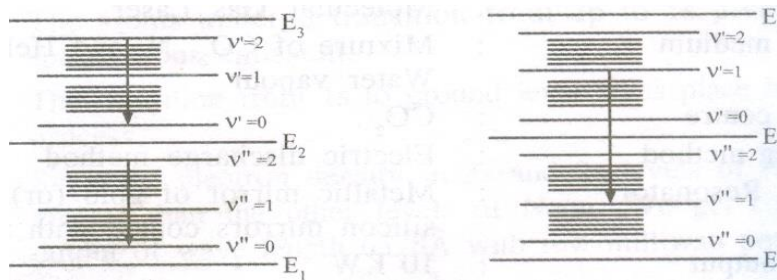
$j, j'$  – rotational energy levels.



## Principle

The transition between these vibrational and rotational energy levels leads to the construction of molecular gas laser. Here the nitrogen atoms are initially raised to excited state. The nitrogen atoms deliver the energy to  $\text{CO}_2$  atoms which has closest energy level to it. Then, the transition takes place between the vibrational energy 7 levels of the  $\text{CO}_2$  atoms and hence laser beam is emitted. The molecular gas laser can have two types of transitions such as,

- Transition between vibrational levels of same electronic state as shown in figure.
- Transition between vibrational levels of different electronic state as shown in figure.



$\text{CO}_2$  laser satisfies them first condition. i.e. here the laser transition occurs between vibrational levels of same electronic state.

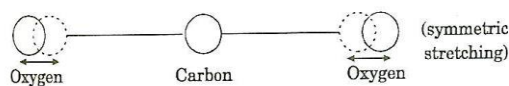
## Fundamental modes of vibration of the $\text{CO}_2$ molecule.

**There are three fundamental modes of vibration.**

- Symmetric stretching mode ( $10^0$ )
- Bending mode ( $01^0$ ,  $02^0$ )
- Asymmetric stretching mode ( $00^1$ ,  $00^2$ )

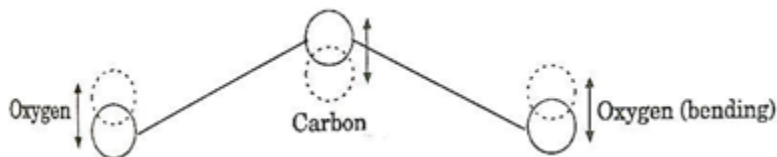
### Symmetric stretching mode ( $10^0$ )

The carbon atom is stationary and the oxygen atoms oscillate or vibrate along the axis of the molecule as shown in figure. The state of vibration is given by 3 integers ( $mn^lq$ ) here ( $10^0$ ), which corresponds, to the degree of excitation.



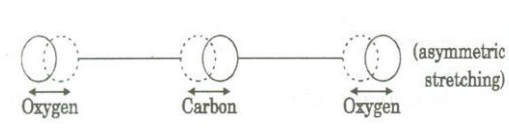
### Bending mode ( $01^{\circ}0$ , $02^{\circ}0$ )

Here the atoms will not be linear, rather the atoms will vibrate perpendicular to the molecular axis as shown in figure. This gives rise to two quanta of frequency represented by ( $01^{\circ}0$ ,  $02^{\circ}0$ ).



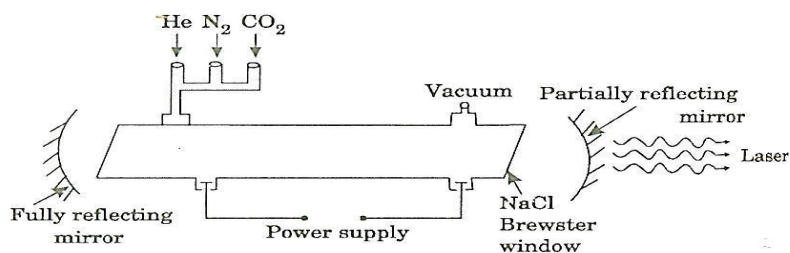
### Asymmetric stretching mode ( $00^{\circ}1$ , $00^{\circ}2$ )

Here all the atoms will vibrate. Here the oxygen atoms vibrate in the opposite direction to the vibration direction of carbon atom as shown in figure 5.10. This gives the quanta of frequency ( $00^{\circ}1$ ,  $00^{\circ}2$ ).



### Construction

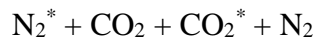
It consists of a discharge tube in which  $\text{CO}_2$  is taken along with nitrogen and helium gases with their pressure level of 0.33:1.2:7 mm of Hg for  $\text{CO}_2$ , nitrogen and the He respectively. Nitrogen helps to increase the population of atoms in the upper level of  $\text{CO}_2$ , while helium helps to depopulate the atoms in the lower level of  $\text{CO}_2$  and also to cool the discharge tube.



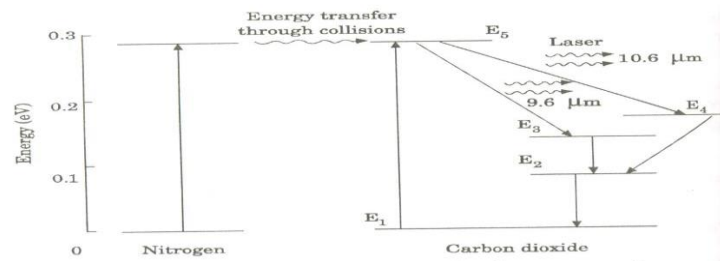
The discharge is produced by DC excitation. At the ends of the tube sodium chloride/Brewster windows are placed as shown in figure. Confocal silicon mirrors coated with aluminium or metallic mirror of gold is employed for proper reflection, which form the resonant cavity. The output power can be increased by increasing the diameter of the tube.

### Working

- (i) The discharge is passed through the tube first, the nitrogen atoms are raised to excited state
$$e^- + \text{N}_2 \rightarrow \text{N}_2^*$$
- (ii) The excited  $\text{N}_2$  atoms undergo resonant energy transfer with  $\text{CO}_2$  atom and raises  $\text{CO}_2$  ( $00^{\circ}1$ ) to excited state due to closer energy level of  $\text{CO}_2$  ( $00^{\circ}1$ ) and nitrogen.



- (iii) When transition takes place between  $00^{\circ}1$  to  $10^{\circ}0$ , the laser of wavelength  $10.6 \mu\text{m}$  is emitted as shown in figure.
- (iv) Similarly when transition takes place between  $00^{\circ}1$  and  $02^{\circ}0$  laser beam of wavelength  $9.6 \mu\text{m}$  is emitted as shown in figure.
- (v) Since  $00^{\circ}1$  to  $10^{\circ}0$  has higher gain than  $00^{\circ}1$  to  $02^{\circ}0$  transition, usually the laser beam of wavelength  $10.6 \mu\text{m}$  is produced more.
- (vi) When the gas flow is longitudinal power output is 50 to 60 watts but if the gas flow is perpendicular to the discharge tube the output power may be raised to 10 kilo watt/m.



- (vii) This type of CO<sub>2</sub> laser is known as TEA laser.  
(i.e.), (Transversely Excited Atmospheric Pressure laser).
- (viii) The contamination of carbon monoxide and oxygen will also have some effect on the laser action. To avoid these unused gases can be pumped out and fresh CO<sub>2</sub> must be inside the discharge tube.

### Application of CO<sub>2</sub> laser

- (i) This laser has applications in medical field such as neurosurgery. Microsurgery, treatment of liver, lungs and also in bloodless operations.
- (ii) It is widely used in open air communication.
- (iii) This laser also has wide applications over military field.

## HOMOJUNCTION SEMICONDUCTOR LASER

### Characteristics of Homojunction laser

Type – Homojunction semiconductor laser

Active medium – PN junction diode

Active centre – Recombination of electrons and holes

Pumping method – Direct pumping

Optical resonator – junctions of diodes- polished

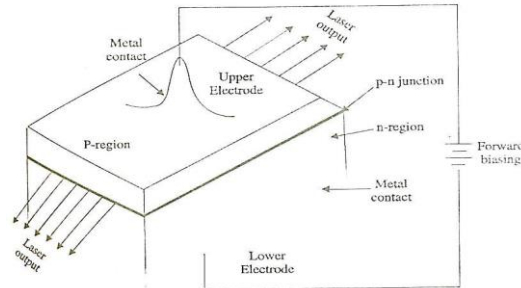
Power output – The power output from this laser is 1mW.

Nature of output – The nature of output is continuous wave or pulsed output

Wavelength emitted – 8400 – 8600 Å°

## Principle

The electron in conduction band combines with a hole in the valence band and hence the recombination of electron and hole produces energy in the form of light. This photon, in turn may induce another electron in the conduction band to valence band and thereby stimulate the emission of another photon.

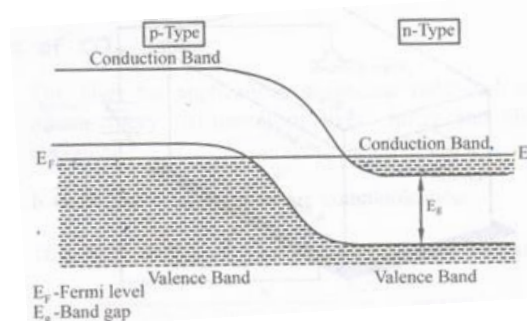


## Construction

The active medium is a p-n-junction diode made from a single crystalline material i.e. Gallium Arsenide, in which p-region is doped with germanium and n-region with tellurium. The thickness of the p-n-junction layer is very narrow so that the emitted laser radiation has large divergence. The junctions of the p and n are well polished and are parallel to each other as shown in figure. Since the refractive index of GaAs is high, it acts as optical resonator so that the external mirrors are not needed. The upper and lower electrodes fixed in the p and n region helps for the flow of current to the diode while biasing.

## Working

- (i) The population inversion in a p-n-junction is achieved by heavily doping p and n materials, so that the Fermi level lies within the conduction band of n type and within the valence band of p type as shown in figure.

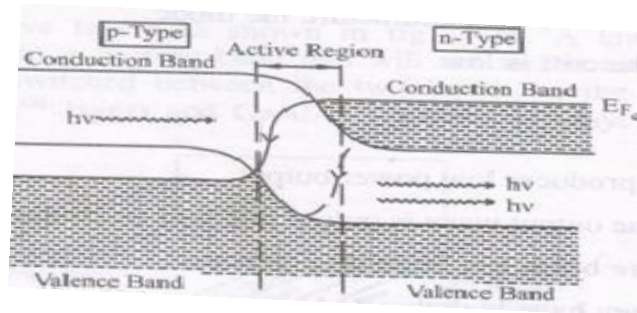


- (ii) If, the junction is forward biased with an applied voltage nearly equal to the band gap voltage, direct conduction takes place. Due to high current density, active region is generated near the depletion region.

(iii) At this junction, if a radiation having frequency ( $\gamma$ ) is made to incident on the p-n-junction then the photon emission is produced as shown in figure.

Thus the frequency of the incident radiation should be in the range

$$E_g < h\gamma < \frac{(E_{FC} - E_{FV})}{h}$$



(iv) Further the emitted photon increases the rate of recombination of injected electrons from the n region and holes in p region by inducing more recombination. Hence the emitted photons have the same phase and frequency as that of original inducing photons and will be amplified to get intense beam of laser.

(v) The wavelength of emitted radiation depends on i. the band gap and ii. The concentration of donor and acceptor atoms in GaAs,

### 1. Calculation of wavelength

Band gap of GaAs = 1.44 eV

$$\begin{aligned} E_g &= h\gamma = h\frac{c}{\lambda} \\ \lambda &= \frac{hc}{E_g} \\ &= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.44 \times 1.6 \times 10^{-19}} \\ &= 8626 \text{ \AA} \end{aligned}$$

The wavelength is near IR region.

### Advantages

- (i) It is easy to manufacture the diode.
- (ii) The cost is low.

### Disadvantages

- (i) It produces low power output.
- (ii) The output wave is pulsed and will be continuous only for some time.
- (iii) The beam has large divergence.
- (iv) They have high threshold current density.

## HETEROJUNCTION SEMICONDUCTOR LASER



## Characteristics of Hetero junction semiconductor laser

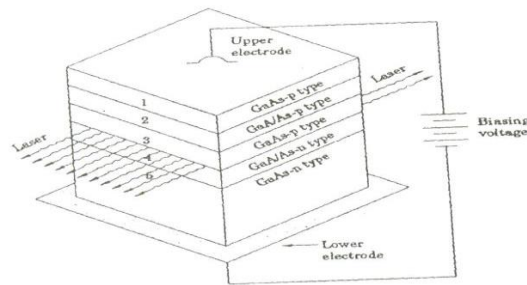
Type	-	Hetero junction semiconductor laser
Active medium	-	p-n-junctions with various layers
Active centre	-	Recombination of electrons and holes
Pumping method	-	Direct pumping
Optical resonator	-	Junctions of diodes- polished
Power output	-	The power output from this laser is 10mW.
Nature of output	-	continuous wave form
Band gap	-	1.55ev

## Principle

The electron in conduction band combines with a hole in the valence band and hence the recombination of electron and hole produces energy in the form of light. This photon, in turn may induce another electron in the conduction band to valence band and thereby stimulate the emission of another photon.

## Construction

It consists of five layers as shown in figure. A layer of GaAs- p-type (3<sup>rd</sup> layer) which has narrow band gap will act as the active region. This layer (3<sup>rd</sup> layer) is sandwiched between the two layers having wider band gap viz. GaAlAs – p-type (2<sup>nd</sup> layer) and GaAlAs – n-type (4<sup>th</sup> layer).



A contact layer made of GaAs – p-type (1<sup>st</sup> layer) is made to form at the top of the 2<sup>nd</sup> layer for necessary biasing. All these four layers are grown over the substrate (5<sup>th</sup> layer) made of GaAs-n-type. The junctions of GaAs – p-type (3<sup>rd</sup> layer) and GaAlAs – n-type (4<sup>th</sup> layer) are well polished and hence it acts as an optical resonator. The upper and lower electrodes help in forward biasing the diode.

## Working

Working of a heterojunction laser is similar to that of the working of a homojunction laser.

- The diode is forward biased with the help of upper and lower electrodes.

- (ii) Due to forward biasing the charge carriers are produced in the wide band gap layers (2 and 4).
- (iii) These charge carriers are injected into the active region (layer 3).
- (iv) The charge carriers are continuously injected from 2<sup>nd</sup> and 4<sup>th</sup> layer to the 3<sup>rd</sup> layer, until the population inversion is achieved.
- (v) At this state some of the injected charge carriers recombine and produce spontaneously emitted photons.
- (vi) These spontaneously emitted photons stimulate the injected charge carriers to emit photons.
- (vii) As a result more number of stimulated emissions arise and thus a large number of photons is produced.
- (viii) These photons are reflected back and forth at the junction and hence an intense, coherent beam of LASER emerges out from the p-n junctions of the active region i.e. between layer-3 and layer-4 as shown in figure.

(ix) The wavelength of the emitted radiation is given by  $\lambda = \frac{hc}{E_g}$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.55 \times 1.6 \times 10^{-19}}$$

$$= 8014 \text{ \AA}$$

The wavelength lies in the IR region.

### **Advantages**

- i. Power output is high.
- ii. It produces continuous wave output.
- iii. It has high directionality and high coherence.
- iv. It has low threshold current density compared to homojunction laser.
- v. These diodes are highly stable and have longer life time.

### **Disadvantages**

- i. Cost is higher than homojunction laser.
- ii. Practical difficulties arise while growing the different layers of p-n junction.

## **INDUSTRIAL APPLICATIONS [LASERS IN WELDING, HEAT TREATMENT AND CUTTING]**

### **Laser heat treatment**

Laser is a light beam of high intensity, directionality and coherence. So, when laser light is focused on a particular area, even of micrometer size, for a very longer time, then that

particular area alone will be heated and the other area will remain as such. This is called thermal effect or laser heat treatment. In this process the light energy is converted into heat energy.

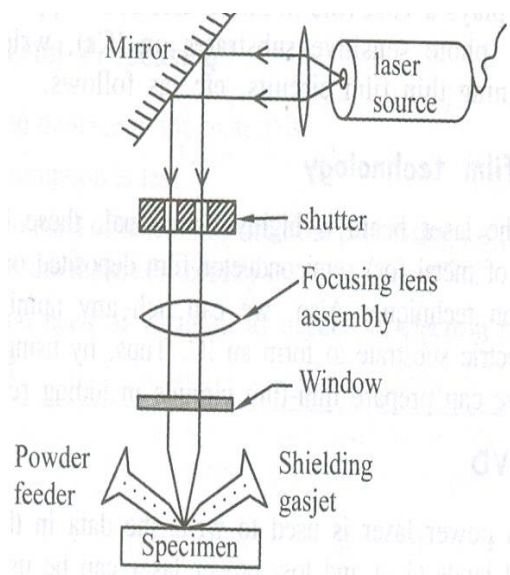
## **Instrumentation technique**

### **Principle**

The technique of laser heat treatment is used in engineering applications like surface hardening, coating, glazing, alloying, cutting, welding, drilling and perforating holes in the materials and hence this process is called material processing. In general ruby laser, Nd-YAG laser and CO<sub>2</sub> laser are used for this purpose.

### **Instrumentation**

The Instrumentation for materials processing consists of a laser source to produce laser beam, shutter to control the intensity of the laser beam and an assembly of lenses to effectively focus the laser onto the specimen as shown figure.



Apart from these Instrumentation, separate control arrangements are made for removing the molten materials, smokes, fumes etc.. with the help of a shielding gas jet, which consist of the assisting gases such as air, N<sub>2</sub>, O<sub>2</sub>, Ar etc. the powder feeder is used feed the metal powder, wherever necessary.

### **Processing**

Laser source is switched on. The light by the plane mirror is made to pass through the shutter. The intensity of the laser beam is controlled by the shutter and the controlled laser beam is allowed to fall on the focusing lens assembly. This lens assembly focuses the light effectively onto the window and is made to incident on the specimen.

Now the specimen gets heated, giving rise to smokes, fumes and molten materials. These smokes, fumes and molten materials are removed immediately by blowing the assisting gas from

the shielding gas jet and this in turn makes the laser beam to continuously fall on the specimen, thereby increasing the cutting rate. Thus the materials can be drilled, cut, put holes etc. using this technique effectively and easily. In case of alloying, cladding, molding, welding etc. the power feeder will be used to spray the metal power over the specimen, during the focusing of laser beam on to the specimen.

## **Applications**

### **Laser in Microelectronics**

Laser plays a vital role in micro-electronics applications, such as making photos masks, writing/reading CDs and DVDs, designing thin film circuits, etc as follows.

#### **(i) Thin film technology**

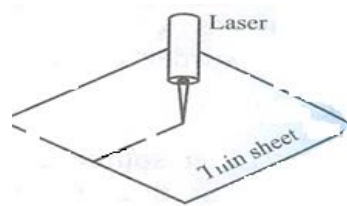
As the laser beam is highly directional, these beams are used to trim off a portion of metal or semiconductor film deposited on the dielectric substrate, by evaporation technique. Also, we can etch any number of micro components over the dielectric substrate to form an IC. Thus, by using as accurately controlled laser beam we can prepare thin film circuits including resistors, capacitors etc...

#### **(ii) CD/DVD**

High power laser is used to write the data in the CD/DVD by creating pits (0's) and lands (1's) and low power laser can be used to read the data.

### **Laser cutting**

Laser is used as a tool to cut thin metal sheets by properly focusing the laser onto any particular area to be cut, for a longer time. Thus due to thermal effect the sheet is cut as shown in figure.



### **Laser drilling and perforating holes**

The same technique as used for cutting will be adopted for drilling and perforating holes, even upto 0.2 to 0.5  $\mu\text{m}$  of thickness.

### **Laser welding**

In ordinary welding process heat will be made to fall on the area to be welded, so that the material in that area will go to molten state. This on cooling will join the material. In this process the heat will spread all over the surroundings and will affect the other areas of the material and

hence the material gets damaged. To avoid this difficulty, laser is used for welding. Due to its high directionality, it is focused on to that particular area alone, even of very small size and the other area remains unaffected. Thus due to thermal effect the parts can be welded. This process is also called Micro-Welding.

Laser and Fiber optics						
S.No	questions	opt1	opt2	opt3	opt4	answer
1	The laser beam from laser source has frequency upto _____ leads to many scientific applications.	109HZ	109KHZ	1014KHZ	1014HZ	1014HZ
2	The angular spread (or) divergence of a laser beam is _____	1m/m	1 $\mu$ m/m	1mm/m	1cm/m	1mm/m
3	The laser source emitting radiation are _____	polychromatic in nature	not in phase	monochromatic in nature	monochromatic,coherent in nature	monochromatic,coherent in nature
4	In a given light beam containing wave trains of same frequency are in phase means that light beam is _____	ordinary light beam	extradinary light beam	laser light beam	monochromatic light beam	laser light beam
5	The spontaneously emitted photons are _____	high intense & incoherent	less intense & coherent	less intense & incoherent	monochromatic & coherent	less intense & incoherent
6	During population inversion, the number of atoms in excited state can be made more than ground state only under _____ condition.	high pressure	negative temperature	low pressure	high temperature	negative temperature
7	The method of pumping used in Carbon dioxide laser is _____	optical pumping	direct electron excitation	direct conversion	chemical method	direct electron excitation
8	Active centre of Carbon dioxide laser is _____	nitrogen & helium	water vapour	Carbon dioxide	mixture of Carbon dioxide and nitrogen	Carbon dioxide
9	Active medium of homo junction semiconductor laser is _____	silicon	P-N junction diode	P-type semi conductor	N-type semiconductor	P-N junction diode
10	Active centre of homojunction semi conductor laser is _____	neon	P-N junction diode	silicon	recombination of electrons and holes	recombination of electrons and holes
11	The wavelength of laser beam emitted by homojunction semiconductor laser source is _____	8400 $\mu$ m - 8600 $\mu$ m	8400A-8600A	6382 A	6485 A	8400A-8600A
12	Which of the following act as a optical resonator of homojunction semiconductor laser?	metallic mirror	silver coated mirrors	P-N junction	polished-junction of diodes	polished-junction of diodes
13	Gallium Arsenide acts as optical resonator in homojunction semiconductor laser due to its _____	high refractive index	low refractive index	high conductivity	high reflecting property	high refractive index
14	The method of pumping used in heterojunction semiconductor laser is _____	optical pumping	chemical process	inelastic collision	direct pumping	optical pumping
15	The wavelength of laser beam emitted by heterojunction semiconductor laser is _____	6382 A	5892 A	1.06 A	8000 A	8000 A
16	semiconductor laser is _____, its band gap is 1.44 eV	5892 A	1.06 $\mu$ m	5892 A	8626 A	8626 A
17	The wavelength of laser beam emitted by the GaAlAs laser is _____, its band gap is 1.55 eV.	5892 A	1.08 $\mu$ m	8626 A	8014 A	8014 A
18	The method of pumping used in homojunction semiconductor laser is _____	direct pumping	optical pumping	chemical pumping	elastic collision	direct pumping
19	The wavelength of the laser beam emitted by a semiconductor laser depends on _____	biasing voltage across the junctions	band gap	the concentration of donar and acceptor atoms in an active medium	band gap and the concentration of donor and acceptor atoms in an active medium	band gap and the concentration of donor and acceptor atoms in an active medium
20	Due to _____ property laser is used to destroy tumors.	highly intense	directionality	high penetrating power	not absorbed by water	high penetrating power
21	Due to _____ property laser is used to do micro surgery.	highly intense	directionality	high penetrating power	not absorbed by water	directionality
22	Due to _____ property, laser is used in endoscopic applications.	directionality	high penetrating power	highly intense	narrow band width	highly intense
23	The term "monochromatic" means _____	inphase	single wavelength	directional	intense	single wavelength
24	The term "coherence" means _____	inphase with same frequency	single wavelength	intense	less angular spread	inphase with same frequency
25	Which of following parameter induces the lasing action?	spontaneous emission	stimulated emission	stimulated absorption	spontaneous absorption	stimulated emission
26	The angular spread at ordinary light is _____	1mm/m	1m/m	1cm/m	10 <sup>-1</sup> m/m	1m/m
27	Spontaneous emission is random and _____	dependent of incident radiation	independent of incident radiation	frequency of incident radiation	intensity of incident radiation	independent of incident radiation
28	Under _____ the population of energy levels obey the Boltzmann's distribution law.	low temperature	high pressure	high temperature	thermal equilibrium	thermal equilibrium
29	Spontaneously emitted photons are _____	inphase	not inphase	incoherent	not inphase and incoherent	not inphase and incoherent
30	In stimulated emission, the emitted photons are _____	not inphase	random in direction	highly coherent and monochromatic	polychromatic	highly coherent and monochromatic
31	The medium in which the population inversion takes place.	active centre	active medium	optical resonator	vacuum	active medium
32	A photon incident on the excited atoms in the active medium and initiates the _____	absorption	stimulated emission	spontaneous emission	stimulated and spontaneous emission	stimulated emission
33	The atoms in the active medium are allowed to go from ground state to excited state, the _____ is achieved.	laser action	normal population	population inversion	pumping	population inversion
34	The more number of atoms stimulated from excited state to the ground state _____ is achieved.	laser action	population inversion	spontaneous emission	pumping	laser action
35	Brewster windows produces the _____ by reflection, of perpendicularly polarized light.	population inversion	plane polarised light	un polarised light	pumping	plane polarised light
36	The output nature of Carbon dioxide laser is _____	continuous	pulsed	continuous & pulsed	continuous(or)pulsed	continuous(or)pulsed
37	The output power of the Carbon dioxide laser is increased by _____	increasing the diameter of the discharge tube	decreasing the diameter of the discharge tube	increasing the pressure of active medium	increasing the pressure of active centre	increasing the diameter of the discharge tube
38	The band gap energy of the homojunction semiconductor laser is _____	1.55eV	1.44MeV	1.44eV	1.55MeV	1.44eV
39	The band gap energy of the heterojunction semiconductor laser is _____	1.55eV	1.44MeV	1.44eV	1.55MeV	1.55eV
40	Due to _____ property the laser is used in drilling process.	directionality	highly intense	narrow band width	high penetrating power	high penetrating power
41	The output power of the Ga-As laser is _____	1.8W	1.8mW	1mW	1W	1mW
42	Which carrier wave is capable of carrying more information's?	Radio waves	microwaves	light waves	sound waves	light waves
43	The central portion of optical fiber consists of _____	Cladding	core	strengthening material	outer jacket	core
44	The refractive index of the core is _____ than cladding surface of optical fiber.	Greater	below	similar	dissimilar	Greater
45	Polyurethane is used as _____	strengthening material	strengthening material	outer jacket	core	outer jacket
46	_____ layer traps the escaping light from the core.	Outer jacket	strengthening material	cladding	core	Outer jacket
47	The optical fibers has _____ band width.	High	low	zero	very narrow	High
48	The optical fibers has _____ loss.	Maximum	minimum	zero	1	minimum
49	The principle of optical fiber communication is _____	Refraction	diffraction	total internal reflection	reflection	total internal reflection
50	The diameter of the core in single mode fibers are multi mode fibers.	Smaller than	larger than	same as	non-linear with respect to	Smaller than
51	The band width of single mode fiber is _____	more than 50 MHz Km	less than 50MHz Km	less than 50KHz Km	Less than 50 KHz Km.	more than 50 MHz Km
52	In general the single mode fibers are _____ fibers.	Step index	graded index	Step & graded index	either step (or) graded index	Step index
53	The path of light propagation in graded index fiber is _____	Zigzag	helical	circle	straight line	helical
54	In photodiode, the electron hole pair is created due to the _____ of photon.	Emission	absorption	both emission and absorption	interference	absorption
55	The unit of Numerical Aperture is _____	No unit	decibel	meter	Hz	No unit
56	_____ communication can be made even in the absence of power.	Radio wave	microwave	optical wave	matter wave	optical wave
57	In the fiber communication, the optical signals are _____ by electric signals, (lightening).	Affected	not affected	slightly affected	transmitted	not affected

**UNIT II****THERMAL PHYSICS****INTRODUCTION TO THERMAL PHYSICS**

Heat or Thermal energy is the physical cause of the sensation of the hotness and coldness of a body and temperature is the degree of the hotness and coldness.

**TRANSFER OF HEAT ENERGY:****CONDUCTION, CONVECTION & RADIATION**

Normally there are three modes by which transfer of heat take from one place to another viz. conduction, convection and radiation.

**Conduction:** It is the process in which heat is transferred from hotter end to colder end without the actual motion of the particles.

**Example:** When one end of the metal rod is heated, the molecules at the hot end vibrate with higher amplitude (kinetic energy) and transmit the heat energy from one particle to the other adjacent particle and so on. Thus heat is transferred from one end to the other, at the same time each individual particle remains in their mean position of equilibrium. Even though solids, liquids and gases conduct heat, it is prominent in solids since atoms are well bound in their positions.

**Convection:** It is the process in which heat is transmitted from hotter end to colder end by the actual movement of heated particles.

**Example:** When some potassium permanganate is added to a beaker of water and heated, we can find that the lower region become warm first and becomes less dense. Hence it moves up and the denser cold water comes down. Now this cold water is heated and becomes less dense so that it moves towards upward direction. Thus the convection current is maintained and the actual motion of the grains of potassium permanganate can be seen, through the naked eye. Convection takes place in liquids and gases in which molecules are relatively free.

**Radiation:** It is the process in which heat is transmitted from one place to another without the necessity of the intervening medium.

**Example:** Though there is no material medium between the Earth and the Sun, the radiation from the Sun reaches us. Here the energy is sent out in the form of radiant heat waves. These waves when fall on our body induce the molecules to vibrate and hence the body is heated up. Thus the radiation can take place even through vacuum. The properties of heat radiations are similar to light radiations.

## THERMAL EXPANSION OF SOLIDS AND LIQUIDS

### Thermal Expansion in solids

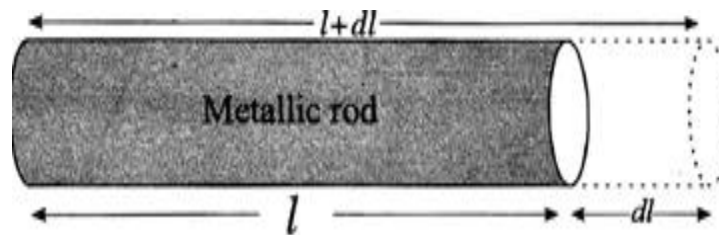
When a metal is heated, due to increase in temperature it will expand and when it is cooled it contracts. However, an internal force will always act so as to keep the metal to regain its original length/position.

#### Definition

**Thermal expansion:** The expansion of a metal, when subjected to heat is called thermal expansion.

**Thermal stress:** The force (or) the stress developed inside the metal so as to regain its original position is called thermal stress.

#### Explanation



Let us consider a metal rod of length 'l' at a temperature T. When the rod is heated to the temperature from T to T +  $\theta$ , then the length of the rod increases, linearly from l to l + dl as shown in Fig. 3.1

The co-efficient of thermal expansion  $\alpha = \frac{dl}{l\theta}$

If  $\theta = 1$ , then  $\alpha = \frac{dl}{l}$

#### Co-efficient of thermal expansion ( $\alpha$ )

Thus the co-efficient of thermal expansion can be defined as the ratio between the change in length to the original length per unit rise of temperature.

### Thermal expansion in Liquids-Volume Expansion

We know when an ordinary alcohol-in-glass thermometer (or) mercury in glass thermometer is kept in the temperature bath, the alcohol (or) mercury rises, due to thermal expansion.

In this case it should be noted that the temperature rises not because of the expansion of liquid, it is only due to the volume expansion of the liquid.



**Co-efficient of volume expansion ( $\beta$ )**

The co-efficient of volume expansion is defined as the ratio between the fractional change ( $\Delta V$ ) in volume to the original volume ( $V$ ) per unit rise of temperature ( $\Delta T$ ).

$$\text{i.e., } \frac{\Delta V}{V} \alpha \Delta T$$

$$(\text{or}) \quad \frac{\Delta V}{V} = \beta \Delta T$$

**Expansion Joints**

$$\text{We know, the Young's Modulus } E = \frac{\text{Longitudinal Stress}}{\text{Longitudinal strain}} \text{ ----(1)}$$

$$\text{Here } \quad \text{Longitudinal stress} = \frac{\text{Force}}{\text{Area}} \quad \text{....(2)}$$

$$\text{We know Longitudinal strain} = \alpha \theta = \frac{dl}{l} \quad \text{....(3)}$$

Substituting equations (2) & (3) in equation (1) we get,

$$E = \frac{F/A}{\alpha \theta}$$

$$F = E \alpha \theta A$$

From equation (4) we can see that if the Area is less, then the force required to restore the material to its Original position is less. Suppose, if the large then the restoring force should also be more, which is quite impossible. Hence to avoid this problem, while constructing a large area of beams, gaps are provided and these gaps are called expansion joints.

Examples:

- (1) Expansion joints are provided while laying the railway lines.
- (2) Expansion joints are provided even in the construction of buildings. However the joints area well packed and are not visible.

**BIMETALLIC STRIPS****Definition:**

As the name itself implies that Bimetallic strips are made up of two thin metal strips with different co-efficient of thermal expansion.

**Fabrication:**

Let us consider two metals viz. Brass of higher co-efficient of thermal expansion, Steel of lower co-efficient of thermal expansion these two metals are welded as shown in Fig.3.2. When the bimetallic strip is heated then the strip will start expanding and therefore the brass, which has

large co-efficient of thermal expansion expands more than the steel and hence the bimetallic strip bends like an arc as shown in Fig.

While cooling

Now, when the bimetallic strip is cooled then the strip will start contracting and therefore the brass which has large co-efficient of thermal expansion contracts more than the steel and bends like an arc as shown in Fig.

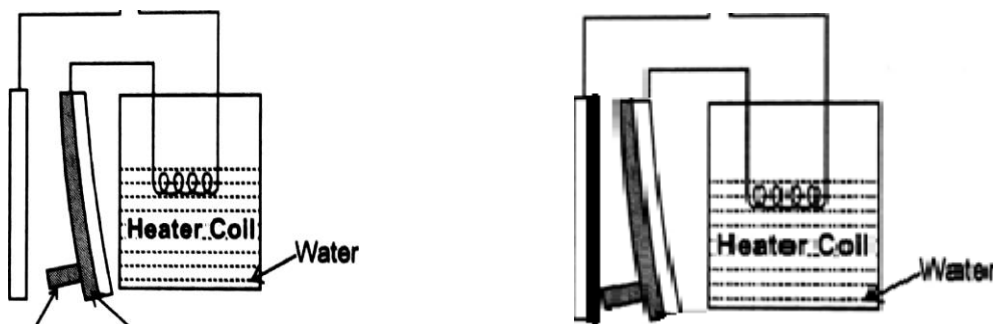


#### Applications:

Bimetallic strips are commonly used in water heaters as temperature controller as detailed below.

A water heater consists of a heater coil connected to the bimetallic strip as shown in Fig. At room temperature the bimetallic strip remains straight and the knob attached to the bimetallic strip helps the circuit to be in the closed condition as shown in Fig.

When the power supply is switched ON, then the current starts flowing through the bimetallic strip to the heater coil as shown in Fig. and the water is heated. Due to increase in temperature of the water, the heater coil becomes hot and in turn the bimetallic strip also becomes hot.



Now, due to thermal expansion, the bimetallic strip starts bending and at a particular temperature the knobs get detached and hence, the circuit becomes open circuit as shown in Fig.

Again, when the water becomes cool, the temperature of the bimetallic strip decreases, and in turn the strip contracts and the circuit is closed as shown in Fig. This process continues and hence the bimetallic strip act as a temperature controller (or) as a switch to maintain the temperature of the water.

## HEAT CONDUCTION IN SOLIDS-THERMAL CONDUCTIVITY

The conducting property varies from material to material, so it is essential to know about the conducting property of a material.

Let us consider a solid material slab having an area of cross section 'A', thickness 'x' as shown in fig 3.7. Let the temperature at the hot end be  $\theta_1$  and that of cold end be  $\theta_2$ . Now the amount of heat (Q) flowing from hot end to the cold end is directly proportional to

- The area of cross section 'A' (i.e.,)  $Q \propto A$
- The temperature difference (i.e.,)  $Q \propto (\theta_1 - \theta_2)$
- The time of conduction (i.e.,)  $Q \propto t$
- The amount of heat flowing from hot end to cold end is inversely proportional to the thickness of the slab.

$$(i.e.,) Q \propto \frac{1}{x}$$

Combining all the factors we can write that the amount of heat conducted

$$Q \propto \frac{A(\theta_1 - \theta_2)t}{x}$$

$$(or) \quad Q = K \frac{A(\theta_1 - \theta_2)t}{x} \quad \text{-----(1)}$$

Where K is the proportionality constant, known as coefficient of thermal conductivity, which depends on the materials

From eqn. (1) we can write,

$$K = \frac{Q \cdot x}{A(\theta_1 - \theta_2)t} \quad \text{Wm}^{-1}\text{K}^{-1}$$

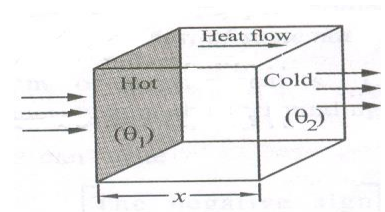
If  $A = 1 \text{ cm}^2$ ;  $(\theta_1 - \theta_2) = 1^\circ\text{C}$ ;  $t = 1 \text{ sec}$ ;  $x = 1 \text{ cm}$  then  $\boxed{Q = K}$

### Definition

The coefficient of thermal conductivity is defined as the amount of heat conducted per second, normally across unit area of cross section, maintained at unit temperature gradient.

### Temperature Gradient:

The quantity  $\frac{(\theta_1 - \theta_2)}{x}$  represents the rate of fall of temperature with respect to the distance or thickness which is known as temperature gradient.



For smaller values  $\frac{(\theta_1 - \theta_2)}{x}$  is written as  $\frac{-d\theta}{dx}$

Eqn(1) can be written as  $Q = -\frac{KA d\theta}{dx} t$

The negative sign indicates the decrease in temperature as the distance increases.

### Thermal Diffusivity (h):

Before the steady state is reached, the rate of heat flow is determined by a quantity called thermal diffusivity (h) or thermometric conductivity. It is defined as the ratio of thermal conductivity to the thermal capacity per unit volume. [The product of specific heat capacity (S) and the density ( $\rho$ )] of the material.

$$(i.e) h = \frac{\text{Thermal Conductivity}}{\text{Thermal Capacity}} \quad (or) \quad \frac{K}{\rho S}$$

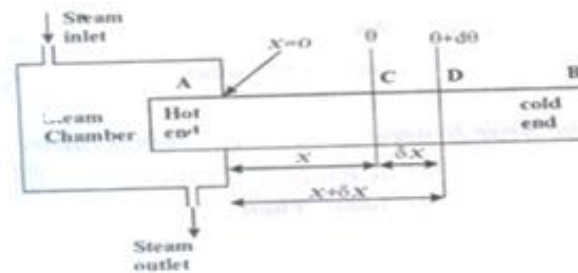
### Specific Heat Capacity:

It is defined as the amount of heat required to raise the temperature of unit mass of the substance through one Kelvin.

$$(i.e) S = \frac{Q}{m\theta} \text{ J Kg}^{-1} \text{ K}^{-1}$$

## RECTILINEAR FLOW OF HEAT ALONG AN UNIFORM BAR ONE DIMENSIONAL FLOW OF HEAT

Let us consider the bar AB of uniform area of cross section 'A' exposed to air, lying along the X axis (one dimensional). Let one end of the bar be heated with the help of steam chamber as shown in the Fig.



Let us consider the plane C and D at a distance  $x$  and  $x + \delta x$  from the hot end respectively. Let  $\theta$  and  $\theta + d\theta$  be the temperature at C and D respectively.

Then the temperature gradient at Plane 'C' =  $\frac{d\theta}{dx}$

Here the excess of temperature at Plane 'D' =  $\left(\theta + \frac{d\theta}{dx} \delta x\right)$

Temperature gradient at Plane D =  $\frac{d}{dx} \left(\theta + \frac{d\theta}{dx} \delta x\right)$

The amount of heat conducted per second at

$$C = Q_1 = -KA \frac{d\theta}{dx} \quad \text{----- (1)}$$

The amount of heat conducted per second at

$$D = Q_2 = -KA \frac{d}{dx} \left( \theta + \frac{d\theta}{dx} \delta x \right) \quad \text{----- (2)}$$

The amount of heat gained by the rod per second between C and D is

$$Q = Q_1 - Q_2$$

Substituting from equations (1) and (2) we get

$$Q = -KA \frac{d\theta}{dx} - \left( KA \frac{d}{dx} \left( \theta + \frac{d\theta}{dx} \delta x \right) \right)$$

$$Q = KA \frac{d^2\theta}{dx^2} \cdot \delta x \quad \text{----- (3)}$$

**Before steady state is reached:**

Before the steady state is reached the amount of heat 'Q' is used in two ways.

**(i) Part of the heat is used to raise the temperature of the rod**

(i.e) If  $\frac{d\theta}{dt}$  is the rise in temperature per second,  $\rho$  is the density of the rod and 's' is the Specific heat. Then

Amount of heat used per second to raise the temperature of rod

$$Q_3 = \text{mass} \times \text{specific heat} \times \frac{d\theta}{dt}$$

$$= A\delta x \rho s \frac{d\theta}{dt} \quad \text{----- (4)}$$

Where mass = Volume x density

**(ii) Rest of the heat is radiated from the surface of the rod.**

(i.e.,) If E is the emissive power and P is the perimeter then we can write

The amount of heat lost per second due to radiation =  $EP\delta x\theta$  --- (5)

Before steady state is reached, the amount of heat used to raise the temperature.

$$Q = Q_3 + Q_4$$

Substituting from equation (4) and equation (5), we get

$$Q = A\delta x \rho s \frac{d\theta}{dt} + EP\delta x\theta \quad \text{----- (6)}$$

Substituting equation (3) in (6) we have

$$KA \frac{d^2\theta}{dx^2} \cdot \delta x = A\delta x \rho s \frac{d\theta}{dt} + EP\delta x\theta$$

$$\frac{d^2\theta}{dx^2} = \frac{\rho s}{K} \frac{d\theta}{dt} + \frac{EP}{KA} \theta \quad \text{----- (7)}$$

This is the general equation for the flow of heat in one dimension. Here  $\frac{K}{\rho s}$  is called as thermal diffusivity ( $h$ ) of the bar.

**b) After Steady state is reached and if the bar is of infinite length:**

After the steady state is reached, the rod does not require further heat to raise to temperature will become constant at this stage.

$$\text{i.e } \frac{d\theta}{dt} = 0$$

$$\text{Equation (7) becomes } \frac{d^2\theta}{dx^2} = \frac{EP}{KA}\theta$$

$$\text{Assuming } \frac{EP}{KA} = \mu^2$$

$$\text{We can write } \frac{d^2\theta}{dx^2} = \mu^2\theta \quad \text{-----(8)}$$

The general solution for equation (8) is

$$\theta = Ae^{\mu x} + Be^{-\mu x} \quad \text{----- (9)}$$

Where, A and B are the arbitrary constants, which can be determined by applying boundary conditions.

If the Bar is assumed to be infinite length then the boundary condition are

$$\text{i) At } x=0; \theta = \theta_0$$

$$\text{Equation (9) becomes } \theta_0 = A + B \quad \text{----- (10)}$$

$$\text{ii) At } x=\infty; \theta = 0 \text{ (Since it is assumed that the bar is of infinite length, the excess of temperature at the other end is zero)}$$

$$\text{Equation (9) becomes } 0 = Ae^{\infty}$$

$$\text{Here } e^{\infty} \neq 0 \text{ 'A' should be equal to zero (i.e.) } A = 0$$

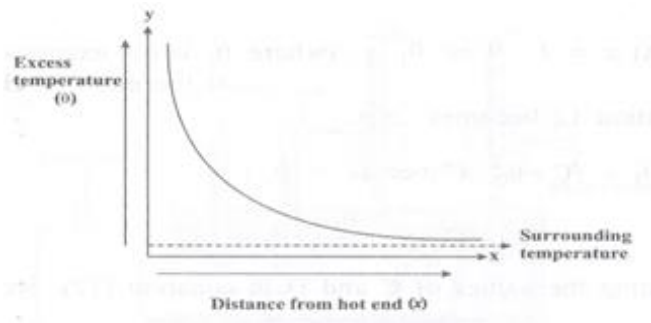
Substituting  $A = 0$  in equation (10) we have

$$\theta_0 = 0 + B \quad \text{(or) } B = \theta_0$$

Substituting the values of A and B in equation (9)

$$\theta = \theta_0 e^{-\mu x} \quad \text{----- (11)}$$

Equation (11) represents the excess of temperature of any point / plane at a distance  $x$  from the hot end, after steady state is reached, which is an exponential function. Therefore a graph is plotted between  $x$  and  $\theta$ , which gives rise to exponential form as shown in Fig.3.9.



**After steady state is reached and if the bar is of finite length (l) covered with insulating materials)**

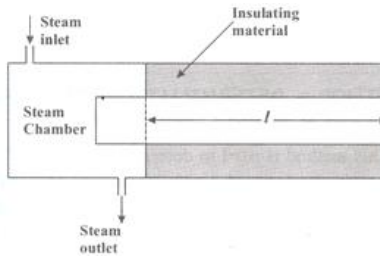
When the bar of finite length (l) is covered by insulating materials as shown in Fig 3.10, then the heat lost due to radiation will be very small.

Emissive power (E) = 0 and After steady state,  $\frac{d\theta}{dt} = 0$

Therefore equation (7) becomes  $\frac{d^2\theta}{dx^2} = 0$

Integrating twice, we can write  $\theta = Cx + D$  ----- (12)

Where, C and D are the arbitrary constants which can be evaluated by applying the boundary conditions.



i) At  $x = 0$ ;  $\theta = \theta_0$

Equation (12) becomes  $\theta = D$

ii) At  $x = l$ ;  $\theta = \theta_l$  (where  $\theta_l$  is the excess of temperature at the end of bar)

Equation (12) becomes

$$\theta = lx + \theta_0 \quad (\text{Since } D = \theta_0)$$

$$(\text{or}) \quad C = \frac{\theta_l - \theta_0}{l}$$

Substituting the values of C and D in equation (12), we get

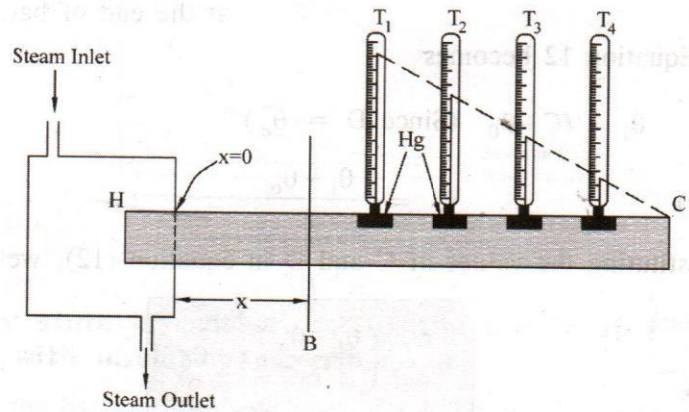
$$\theta = \left( \frac{\theta_l - \theta_0}{l} \right)x + \theta_0 \quad \text{----- (13)}$$

Equation (13) represents the excess of temperature of any point / plane at a distance x from the hot end, when it is covered by insulator (or) insulating material.

## FORBE'S METHOD -DETERMINATION OF THERMAL CONDUCTIVITY FOR GOOD CONDUCTORS

### Description:

This method is used to determine the absolute thermal conductivity of metals. It consists of a long rod (HC) of uniform area of cross section. One end of the rod is enclosed by a steam chamber and the other end is let free as shown in fig. A number of provision are made in the rod at equal distances in which thermometers are inserted. A small quantity of mercury is poured in each provision to have good contact between the rod and the thermometers.



### Working:

The rod is heated till the steady state is reached, i.e. all the thermometers indicate constant values (but different). Then,

The amount of heat flowing per second across 'B' at a distance 'x' from the hot end

$$= K A \left( \frac{d\theta}{dx} \right)_B \dots\dots\dots (1)$$

The heat conducted at 'B' is somehow should be lost between B to end 'C', therefore,

The amount of heat lost per second by radiation by the rod beyond the section 'B' =  $\int_B^C ms \left( \frac{d\theta}{dt} \right)$

Since mass = volume x density, we get mass =  $A dx \rho$

$$\text{Heat lost per second} = \int_B^C A dx \rho s \left( \frac{d\theta}{dt} \right) \dots\dots\dots (2)$$

At steady state,

$$\left( \begin{array}{c} \text{The amount of heat} \\ \text{flowing per second across 'B'} \end{array} \right) = \left( \begin{array}{c} \text{The amount of heat lost per} \\ \text{second by radiation between B to end 'C'} \end{array} \right)$$

$$K A \left( \frac{d\theta}{dx} \right)_B = \int_B^C A dx \rho s \left( \frac{d\theta}{dt} \right)$$



Equating equations (1) and (2) we have

$$K = \frac{\rho s \int_B^C \left(\frac{d\theta}{dt}\right) dx}{\left(\frac{d\theta}{dx}\right)_B} \text{ Wm}^{-1} \text{ K}^{-1} \quad \dots\dots\dots (3)$$

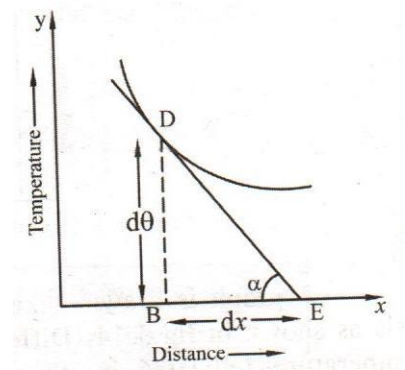
In order to find K, we have to find  $\int_B^C \left(\frac{d\theta}{dt}\right) dx$  and  $\left(\frac{d\theta}{dx}\right)_B$

The experiment is divided into parts

- i) Static experiment to find  $\int_B^C \left(\frac{d\theta}{dt}\right) dx$
- ii) Dynamic experiment to find  $\left(\frac{d\theta}{dx}\right)_B$

### i) Static experiment:

The experimental set up is the same as shown in the fig. The rod is heated up to say six or seven house until the steady state is reached. The temperatures indicated by the thermometers  $T_1, T_2, T_3, T_4$  are noted as  $\theta_1, \theta_2, \theta_3, \theta_4$  respectively. The distance of the thermometers from the hot end is also noted. A graph is plotted taking distance along x-axis and the temperature along y-axis as shown in fig. A tangent is drawn to the corresponding distance (B) i.e. on point D in the curve.

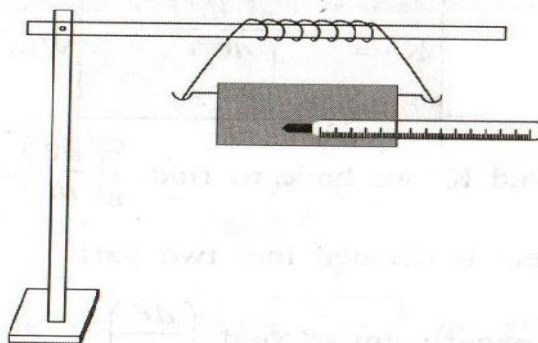


From the graph we can write  $\left(\frac{d\theta}{dx}\right)_B = \tan \alpha = \frac{BD}{BE}$

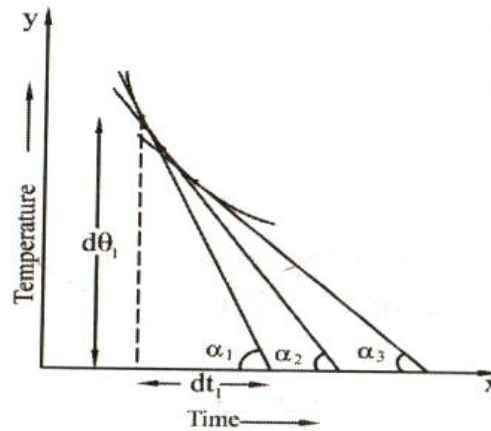
Hence  $\left(\frac{d\theta}{dx}\right)_B$  has been found.

### ii) Dynamic Experiment:

A sample piece of the original rod of same area of cross section is heated until it reaches the temperatures of the hot end (H). Then the sample rod fixed with a thermometer at the centre of the rod is suspended in the open atmosphere and is allowed to cool as shown in the fig.



The fall of temperature is noted at regular intervals of time until it reaches the temperature below the temperature of the section 'B' chosen in static part.

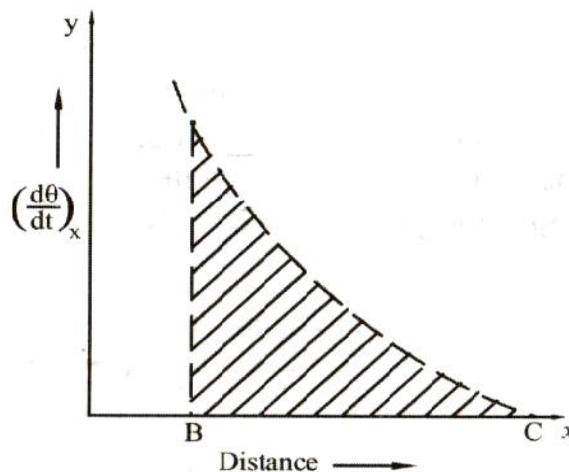


A graph is drawn, taking time along x axis and temperature along y axis as shown in fig. Different tangents are drawn for the corresponding temperature indicated by thermometer  $T_1$ ,  $T_2$ ,  $T_3$ ,  $T_4$  in the static experiment.

Now we get  $(\frac{d\theta}{dt})_{x1} = \tan \alpha_1$ ,  $(\frac{d\theta}{dt})_{x2} = \tan \alpha_2$  .....etc.

The rate of cooling at that corresponding distances beyond the section 'B' in static experiment is obtained.

With the above data, a third curve is plotted taking distance 'x' from the hot end beyond the section 'B' along x axis and  $(\frac{d\theta}{dt})_{x1}$ ,  $(\frac{d\theta}{dt})_{x2}$  etc., along y axis as shown in the fig. 3.15.



Then the area of the shade portion of the curve will correspond to  $\int_B^C (\frac{d\theta}{dt}) dx$ , where 'B' is point chosen in the static experiment,

$$\text{Area of the Shaded portion} = \int_B^C (\frac{d\theta}{dt}) dx \dots\dots\dots (5)$$

Substituting eqn. (4) and eqn. (5) in eqn. (3) we have

$$\text{The Thermal Conductivity } K = \frac{\rho s \times \text{Area of the shaded portion}}{BD/BE}$$

The experiment is repeated by choosing the point 'B' at different distances from the hot end and the average value 'K' is determined.

**Limitations:**

1. It is tedious to draw three graphs.
2. It takes a long time to complete the experiment.
3. Also, distribution of the heat is not the same all over the bar in static and dynamic experiments.

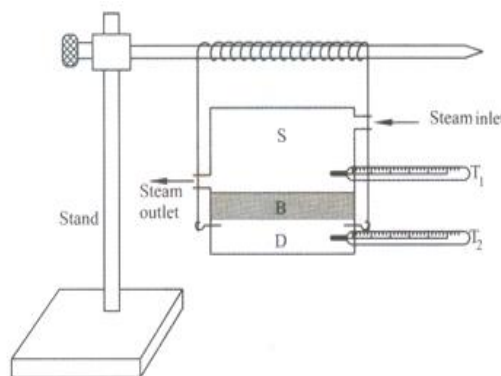
**LEE'S DISC METHOD – DETERMINATION OF THERMAL CONDUCTIVITY OF A BAD CONDUCTOR**

**Description:**

The given bad conductor (B) is shaped with the diameter as that of the circular slab (or) disc 'D'. The bad conductor is placed in between the steam chamber (S) and the disc (or) slab (D), provided the bad conductor steam chamber and the slab should be of same diameter. Holes are provided in the steam chamber (S) and the disc (or) slab (D) in which thermometer is inserted to measure the temperatures. The total arrangement is hanged over the stand in Fig.

**Working:**

Steam is passed through the steam chamber till the steady state is reached (i.e) the thermometer show constant temperature. Let the temperature of the steam chamber (hot end) and the disc or slab (cold end) be  $\theta_1$  &  $\theta_2$  respectively.



**Calculation:**

Let 'x' be the thickness of the bad conductor (B), 'm' is the mass of the slab, 's' be the specific heat capacity of the slab, 'r' is the radius of the slab and 'h' be the height of the slab, then, Amount of heat conducted by the bad conductor per second =  $\frac{KA(\theta_1 - \theta_2)}{x}$

Since the Area of cross section is  $= \pi r^2$

$$\text{Amount of heat conducted per second} = \frac{K\pi r^2(\theta_1 - \theta_2)}{x} \text{-----(1)}$$

$$\begin{aligned} \text{The amount of heat lost by the slab per second} &= m \times s \times \text{Rate of cooling} \\ &= msR_c \text{-----(2)} \end{aligned}$$

### Under steady state

The amount of heat conducted by the bad conductor (B) per second = Amount of heat lost by the slab (D) per second

Therefore, we can write

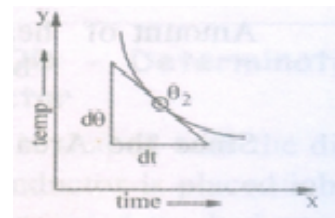
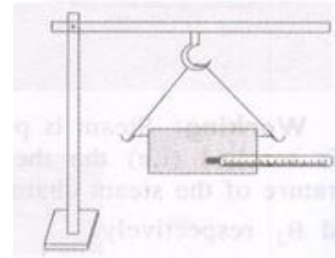
$$\text{eqn.(1)} = \text{eqn.(2)}$$

$$\frac{K\pi r^2(\theta_1 - \theta_2)}{x} = msR_c$$

$$K = \frac{msxR_c}{\pi r^2(\theta_1 - \theta_2)} \text{-----(3)}$$

### To find the rate of cooling ( $R_c$ ):

In Eqn.(3)  $R_c$  represents the rate of cooling of the slab along with the steam chamber. To find the rate of cooling for the slab alone, the bad conductor is removed and the steam chamber is directly placed over the slab and heated. When the temperature of the slab attains  $5^\circ\text{C}$  higher than  $\theta_2$ , the steam chamber is removed. The slab is allowed to cool, as shown in Fig. A graph is plotted taking time along 'x' axis and temperature along 'y' axis, the rate of cooling for the slab alone (i.e)  $\left(\frac{d\theta}{dt}\right)$  is found from graph, as shown Fig.



The rate of cooling is directly proportional to the surface area exposed.

### Case (i)

Steam chamber and bad conductor are placed over slab, in which radiation takes place from the bottom surface of area  $(\pi r^2)$  of the slab and the sides of the slab of area  $(2\pi rh)$ .

$$R_c = \pi r^2 + 2\pi rh$$

$$\text{(or)} \quad R_c = \pi r(r + 2h) \text{-----(4)}$$

**Case (ii)**

The heat is radiated by the slab alone, (i.e.,) from the bottom of area ( $\pi r^2$ ), top surface of the slab of area  $\pi r^2$  and also through the sides of the slab of area  $2\pi rh$ .

$$\begin{aligned} \left(\frac{d\theta}{dt}\right)_{\theta_2} &= \pi r^2 + \pi r^2 + 2\pi rh \\ \text{(or)} \quad \left(\frac{d\theta}{dt}\right)_{\theta_2} &= 2\pi r^2 + 2\pi rh \\ \left(\frac{d\theta}{dt}\right)_{\theta_2} &= 2\pi r(r+h) \quad \text{-----(5)} \\ \frac{Rc}{\left(\frac{d\theta}{dt}\right)_{\theta_2}} &= \frac{\pi r(r+2h)}{2\pi r(r+h)} \\ Rc &= \frac{(r+2h)}{2(r+h)} \left(\frac{d\theta}{dt}\right)_{\theta_2} \quad \text{----- (6)} \end{aligned}$$

Substitute eqn. (6) in eqn. (3) we have

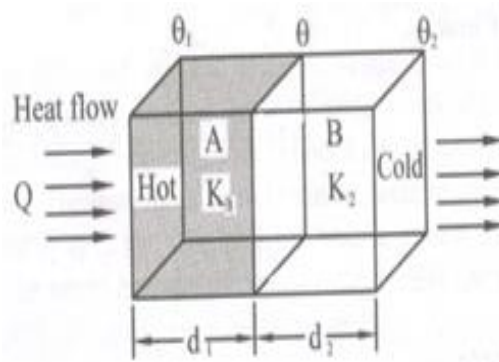
$$K = \frac{msx \left(\frac{d\theta}{dt}\right)_{\theta_2} \cdot (r+2h)}{\pi r^2 (\theta_1 - \theta_2) 2(r+h)} \text{ Wm}^{-1} \text{ K}^{-1}$$

Hence, thermal conductivity of the given bad conductor can be determined from the above formula.

## HEAT CONDUCTION THROUGH A COMPOUND MEDIA (SERIES AND PARALLEL)

### i) Bodies in Series:

Consider a composite slab or compound wall of two different (heterogeneous) materials, A and B of thermal conductivity  $K_1$  and  $K_2$  respectively. Let the thickness of these two layers A and B be  $d_1$  and  $d_2$  respectively as shown in fig. 3.19. Let the temperature of the end faces be  $\theta_1$  and  $\theta_2$  and temperature  $\theta_1$  and  $\theta_2$  at the contact surface be, which is unknown. Heat will flow from face-A and face-B through surface of contact only if  $\theta_1 > \theta_2$ . After steady state is reached, heat flowing per second (Q) through ever layer is same.



$$\text{Amount of heat flowing per second through A} \quad Q = \frac{K_1 A (\theta_1 - \theta)}{d_1} \quad \text{----- (1)}$$

$$\text{Amount of heat flowing per second through B} \quad Q = \frac{K_2 A (\theta - \theta_2)}{d_2} \quad \text{----- (2)}$$

Where, A is Area of cross-section of both layers, which is same.

Since the amount of heat flowing through A and B are equal, we can write

Equation (1) = Equation (2)

$$\frac{K_1 A (\theta_1 - \theta)}{d_1} = \frac{K_2 A (\theta - \theta_2)}{d_2}$$

$$\frac{K_1 \theta_1}{d_1} - \frac{K_1 \theta}{d_1} = \frac{K_2 \theta}{d_2} - \frac{K_2 \theta_2}{d_2}$$

Rearranging we get

$$\frac{K_1 \theta_1}{d_1} + \frac{K_2 \theta_2}{d_2} = \theta \left[ \frac{K_1}{d_1} + \frac{K_2}{d_2} \right]$$

$$(\text{or}) \quad \theta = \frac{\frac{K_1 \theta_1}{d_1} + \frac{K_2 \theta_2}{d_2}}{\frac{K_1}{d_1} + \frac{K_2}{d_2}}$$

Equation (3) gives us the expression for temperature at the interface of the compound media which is having two layers.

By substituting the value of  $\theta$  in equation (1) we get

$$Q = \frac{K_1 A}{d_1} \left[ \theta_1 - \frac{\frac{K_1 \theta_1}{d_1} + \frac{K_2 \theta_2}{d_2}}{\frac{K_1}{d_1} + \frac{K_2}{d_2}} \right]$$

$$Q = \frac{K_1 A}{d_1} \left[ \theta_1 - \frac{(d_2 K_1 \theta_1 + d_1 K_2 \theta_2) / d_1 d_2}{(K_1 d_2 + K_2 d_1) / d_1 d_2} \right]$$

$$Q = \frac{K_1 A}{d_1} \left[ \frac{(K_1 d_2 + K_2 d_1) \theta_1 - d_2 K_1 \theta_1 - d_1 K_2 \theta_2}{(K_1 d_2 + K_2 d_1)} \right]$$

$$Q = \frac{K_1 A}{d_1} \left[ \frac{K_1 d_2 \theta_1 + K_2 d_1 \theta_1 - K_1 d_2 \theta_1 - K_2 d_1 \theta_2}{(K_1 d_2 + K_2 d_1)} \right]$$

$$Q = \frac{K_1 A [K_2 d_1 (\theta_1 - \theta_2)]}{d_1 (K_1 d_2 + K_2 d_1)}$$

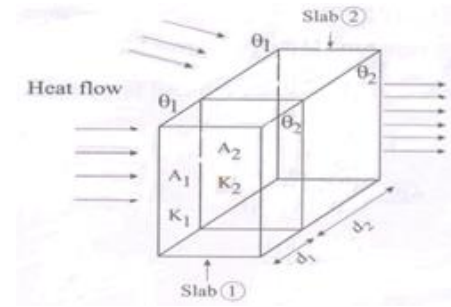
$$Q = \frac{A (\theta_1 - \theta_2)}{\frac{K_1 d_2}{K_1 K_2} + \frac{K_2 d_1}{K_1 K_2}}$$

$$Q = \frac{A (\theta_1 - \theta_2)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \quad \text{----- (4)}$$

Equation (4) gives the amount of heat conducted by the two layers in series.

**ii) Bodies in Parallel**

Consider a composite slab or compound wall of two different (heterogeneous) materials A and B of thermal conductivities  $K_1$  and  $K_2$  and of thickness  $d_1$  and  $d_2$  respectively. These two material layers are arranged in parallel as shown in fig. The opposite faces of the material are kept at temperatures  $\theta_1$  and  $\theta_2$  respectively. Let  $A_1$  and  $A_2$  be the areas of cross-section of the materials.



Then,

The amount of heat flowing through the first slab  $Q_1 = \frac{K_1 A_1 (\theta_1 - \theta_2)}{d_1}$  -----(1)

The amount of heat flowing through the first slab  $Q_2 = \frac{K_2 A_2 (\theta_1 - \theta_2)}{d_2}$  -----(2)

The total heat flowing through these two slabs per second  $Q = Q_1 + Q_2$

Substituting equation (1) and (2) in equation (3), we get

$$Q = \frac{K_1 A_1}{d_1} (\theta_1 - \theta_2) + \frac{K_2 A_2}{d_2} (\theta_1 - \theta_2) \text{-----(3)}$$

$$Q = \left[ \frac{K_1 A_1}{d_1} + \frac{K_2 A_2}{d_2} \right] (\theta_1 - \theta_2) \text{-----(4)}$$

Equation (4) gives us the amount of heat flowing through compound wall of two layers in parallel.

**BLACK BODY RADIATION****Perfect black body:**

A perfect black body is one which absorbs and emits in all the radiations (corresponding to all wavelengths) that fall on it. The radiation given out by a perfect black body is called Black body radiation.

**Kirchoff's law:**

Ratio of emissive power to the coefficient of absorption of any given wavelength is the same for all bodies at a given temperature and is equal to the emissive power of the black body at that temperature.

$$E_\lambda = \frac{e_\lambda}{a_\lambda}$$

**LAWS FOR EXPLAINING THE ENERGY DISTRIBUTION:****1. Stefan- Boltzmann Law**

According to this law the radiant energy (E) of the body is directly proportional to the fourth power of the temperature (T) of the body.

$$(i.e) \quad E = \sigma T^4$$

Where,  $\sigma$  - Stefan constant, given by  $\sigma = \frac{2\pi^5 K_B^4}{15h^3 c^2}$

**2. Wien's Displacement Law**

When the temperature of a blackbody radiator increases, the overall radiated energy increases and the peak of the radiation curve moves to shorter wavelengths. When the maximum is evaluated from the Planck radiation formula, the product of the peak wavelength and the temperature is found to be a constant.

$$(i.e) \quad \lambda_m T = \text{Constant}$$

This law shows that, as the temperature increases, the wavelength corresponding to maximum energy decreases.

Wien also showed that the maximum energy ( $E_{\max}$ ) is directly proportional to the fifth power of the absolute temperature.

$$E_{\max} \propto T^5$$

By deducing this law he obtained a law called Wien's law of distribution of energy ( $E_\lambda$ ), given by

$$E_\lambda = C_1 \lambda^{-5} e^{C_2/\lambda T}$$

Where  $C_1$  and  $C_2$  are constants given by  $C_1 = 8\pi hc$  and  $C_2 = \frac{hc}{K_B}$

This law holds good only for shorter wavelengths and not for longer wavelengths.

**Rayleigh – Jeans Law**

The energy distribution is directly proportional to the absolute temperature and is inversely proportional to the fourth power of the wavelength.

$$(i.e.,) \quad E_\lambda = \frac{8\pi K_B T}{\lambda^4}$$

Where,  $K_B$  is the Boltzmann constant.



This law holds good only for longer wavelengths and not for shorter wavelengths.

It is found that, both Wien's and Raleigh Jeans law don't agree with the experimental results. Therefore we can conclude that the classical theory was not able to explain the emission of black body radiation. Thus Max Planck used Quantum theory to explain Black body radiation.

### **The four laws of thermodynamics**

#### **Zeroth Law**

It states that if two bodies are in thermal equilibrium with some third body, then they are also in equilibrium with each other. This establishes temperature as a fundamental and measurable property of matter.

#### **First Law**

It states that the total increase in the energy of a system is equal to the increase in thermal energy plus the work done on the system. This states that heat is a form of energy and is therefore subject to the principle of conservation.

#### **Second Law**

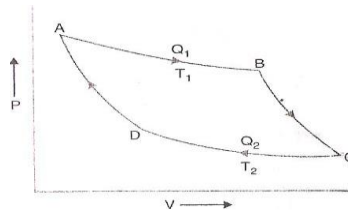
It states that heat energy cannot be transferred from a body at a lower temperature to a body at a higher temperature without the addition of energy. This is why it costs money to run an air conditioner.

#### **Third Law**

It states that the entropy of a pure crystal at absolute zero is zero. As explained above, entropy is sometimes called "waste energy," i.e., energy that is unable to do work, and since there is no heat energy whatsoever at absolute zero, there can be no waste energy. Entropy is also a measure of the disorder in a system, and while a perfect crystal is by definition perfectly ordered, any positive value of temperature means there is motion within the crystal, which causes disorder. For these reasons, there can be no physical system with lower entropy, so entropy always has a positive value.

### **Change of Entropy in Reversible Cycle**

Consider a complete reversible Carnot's cycle ABCD as shown in Fig. for an ideal gas formed by two isothermals i.e., AB at a temperature and CD at a temperature and the two adiabatics BC and DA.

**(i) Isothermal expansion AB:**

Let be the quantity of heat absorbed by the working Substance in going from state A to state B during isothermal expansion AB at a constant Temperature. The increase in entropy of the working substance is given by

$$\int_A^B ds = + Q_1 / Q_2$$

**(ii) Adiabatic Expansion BC:**

In going from state B to C along the adiabatic BC, there is no Change in entropy of the working substance, but the temperature falls from to due to expansion.

$$\int_A^B ds = 0$$

**(iii) Isothermal compression CD:**

In going from state C to D along the isothermal CD, the Working substance rejects heat to the sink at temperature. The entropy of the working substance decreases and change in entropy is given by

$$\int_C^D ds = -Q_1 / Q_2$$

**(iv) Adiabatic Compression DA:**

In going from D to A along the adiabatic DA, there is no Change in entropy but temperature rises from to.

$$\int_D^A ds = 0$$

Thus the net gain in entropy of the working substance in the whole cycle ABCDA

$$= \int_A^B ds + \int_B^C ds + \int_C^D ds + \int_D^A ds$$

$$\oint dS = Q_1/Q_2 - Q_1/Q_2$$

But for a reversible Carnot's cycle

$$Q_1/Q_2 = Q_1/Q_2$$

$$(\text{or}) \quad Q_1/Q_2 - Q_1/Q_2 = 0$$

Substituting, we get

$$\oint dS = Q_1/Q_2 - Q_1/Q_2 = 0$$

Where the integral sign with a circle refers to a complete cycle. Thus in a cycle of reversible process, the entropy of the system remains unchanged or remains constant. This means that the total change in entropy is also zero.

### **Change of Entropy in Irreversible Process:**

The thermo dynamical state of the system is defined with a help of the thermo dynamical coordinates of the system. The state of the system can be changed by altering the thermo dynamical coordinates. Changing from one state to other by changing the thermo dynamical coordinates is called a process.

Consider two states of a system i.e., state A and state B. change of the state from A to B or vice versa is a process and the direction of the process will depend upon a new thermo dynamical coordinate called entropy. All process are not possible in the universe.

### **Consider the following process:**

1. Let two blocks A and B at different temperatures and ( $>$ ) be kept in contact but the system as a whole is insulated from the surroundings. Conductions of heat takes place between the blocks, the temperature of A falls and the temperature of B rises and thermo dynamical equilibrium will be reached.

2. Consider a fly wheel rotating with an angular velocity. Its kinetic energy is  $I$ . After sometime the wheel comes to rest and kinetic energy is utilised in overcoming friction at the bearings. The temperature of the wheel and the bearings rises and the increase in their internal energy is equal to the original kinetic energy of the fly wheel.

3. Consider two flasks A and B connected by a glass tube provided with a soap cock. Let A contain air at high pressure and B is evacuated. The system is isolated from the surroundings.

If the stop cock is opened, air rushes from A to B, the pressure in A decreases and the volume of air increases.

All the above three examples are thermo dynamical process involving change in thermo dynamical coordinates. Also, in accordance with the first law of thermodynamics, the principle of conservation of energy is not violated because the total energy of the system is conserved. It is also clear that, with the initial conditions described above, the three process will takes place.

But, it is a matter of common experience, that none of the above conditions for the reversed process is reached. It means that the direction of the process cannot be determined by knowing the thermo dynamical coordinates in the two end states. To determine the direction of the process a thermo dynamical coordinate has been devised by Clasius and this is called the entropy of the system. Similar to internal energy, entropy is also a function of the state of the system. For any possible process, the entropy of an isolated system should increase or remain constant. The process in which there is a possibility of decrease in entropy cannot take place. If the entropy of an isolated system is maximum, any change of state will mean decrease in entropy and hence that change of state will not take place. For concluding, process in which the entropy of an isolated system decreases do not take place or for all process taking place in an isolated system the entropy of an system should

Increase or remain constant.

It means a process is irreversible if the entropy decreases when the direction of the process is reversed. A process is said to be irreversible if it cannot be retraced back exactly in the opposite direction. During an irreversible process, heat energy is always used to overcome friction. Energy is also dissipated in the form of conduction and radiation. This loss of energy is always take place whether the engines works in one direction or the reverse direction. Such energy cannot be regained. In actual practice all the engines are irreversible. If electric current is passed through a wire, heat is produced. If the direction of the current is reversed, heat is again is produced. This is also an example of an irreversible process. All chemical reactions are irreversible. In general, all natural processes are irreversible.

## **REFRIGERATORS**

### **Introduction**

Generally he cannot flow from a cold body (i.e., a body at lower temperature) to hot body i.e., (a body at higher temperature). But, it is possible to do so, if some external work (or) pressure is done on the working substance. This concept is used in refrigerators.

**General Terminologies****1. Refrigeration:**

Refrigeration is the process of reducing and maintaining the temperature of a body below the normal temperature or it is the process of removing heat from a substance under controlled conditions.

**2. Refrigerator:**

Refrigerator is equipment used to reduce and maintain the temperature below atmospheric temperature. It is obtained by removing the heat from the space continuously.

**3. Refrigerant:**

Refrigerant is a fluid which absorbs the heat from the body and rejects the heat at high temperature.

Examples: Ammonia, carbon-di-oxide, Freon, methyl chloride, Chloro Fluro carbons (CFC).

**4. Capacity of Refrigerator (or) Refrigerating effect :**

It is the amount of heat extracted from the cold body per unit mass per second. (or) The rate at which refrigeration produced is called the capacity of refrigerator. It is expressed in tonne of refrigeration.

**5. Tonne of refrigeration**

A tonne of refrigeration is defined as the amount of refrigeration effect produced (amount of heat extracted) by uniform melting of one tonne (1000 kg) of ice at 0°C to water in 24 hours.

1 Tonne of refrigeration = 210 KJ/min (or) 3.5 KT/sec,

**6. Performance coefficient:**

Coefficient of performance (COP) is the ratio of heat extracted and the work done.

$$COP = \frac{\text{Heat extracted}}{\text{Work done}}$$

**Types of refrigerators**

There are two types of refrigerators, viz.

1. Vapour Compression refrigerator

Example: Domestic refrigerators

2. Vapour absorption refrigerator.

Example: Commercial refrigerators.

## DOMESTIC REFRIGERATOR

### Principle

The second law of thermodynamics as given by clausius, i.e., "without doing external work it is impossible to transfer heat from a cold body to a hot body" is the principle used in refrigerator.

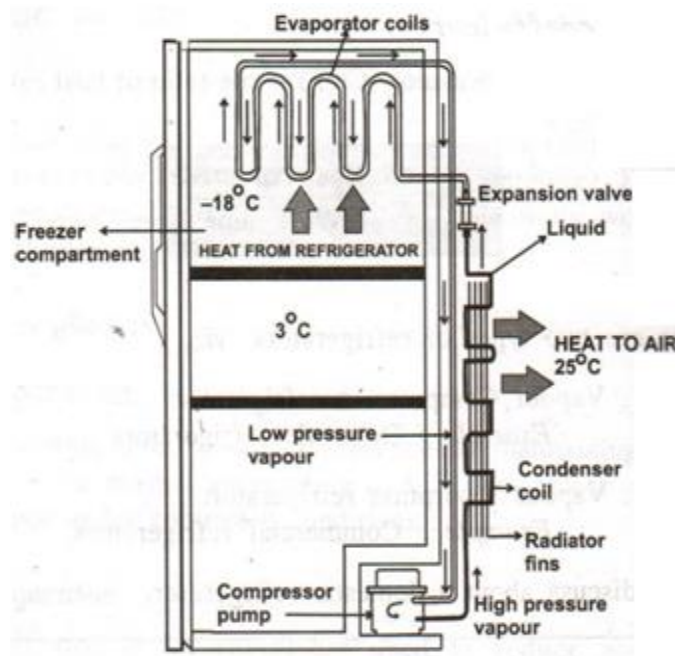
Here, the ammonia (working substance) takes heat from the refrigerator (at lower temperature) and due to external work done on ammonia, it give heat to atmospheric air (at normal (or) high temperature) and keep the refrigerator continuously cool.

### Design

It consists of two coils viz.

1. Evaporator coil to convert liquid ammonia to vapour and
2. Condenser coil to convert vapour to liquid ammonia, as shown in Fig.

The compressor in the refrigerator is used to compress the ammonia vapour using a piston to a very high pressure and it helps in doing the external work on the ammonia. The whole setup is kept in well air circulation area for better performance.



**Working**

1. In domestic refrigerator, liquid ammonia is used as the working substance for cooling the refrigerator.
2. Here, Liquid ammonia at low pressure is passed through the evaporator coils, wherein it expands and absorbs the heat from the refrigerator.
3. This liquid ammonia takes up the heat from the refrigerator and is converted into low pressure vapour.
4. Now the compressor is used to compress the ammonia vapour externally using a piston, to a very high pressure.
5. This ammonia at high pressure is allowed to pass through the condenser coils.
6. While passing, the ammonia vapour gives heat to the atmospheric air at room temperature and becomes liquid ammonia again due to cooling.
7. This cool liquid ammonia in turn acts as a primary refrigerant and keeps the refrigerator cool.
8. This cycle of process continues and makes the refrigerator to be in cool condition always.

**Applications**

- ◆ It is used for preserving the Food, Fruits and drinks for a long duration.
- ◆ It is used to preserve flowers, medicines and medical drugs.
- ◆ Refrigerator is used to manufacture ice in ice plants.
- ◆ Refrigerator is used in refineries for removing wax.
- ◆ In industries they are used for processing lubricants, rubber, steel etc.,
- ◆ It is used for producing frozen foods, ice creams, chemicals and other products.

**Advantages**

- ◆ It is used to store food for a long time.
- ◆ It protects the food from microbes, insects and rodents.
- ◆ It protects the food from direct sunlight and heat.
- ◆ Cost of refrigerant is low.

**Disadvantages**

- ◆ It consumes large amount of electricity.
- ◆ It causes global warming.
- ◆ Harmful pollutant gas like CFC (chloro fluoro carbon) is used in refrigerators.
- ◆ Preserving food in refrigerator for a long duration is not good for health.

## PROBLEMS

1. A copper rod of length 50 cm and cross sectional area  $6 \times 10^{-2} \text{ cm}^2$  is connected in series with an iron rod of same area of cross section and length 25 cm. One end of copper is immersed in boiling water. The far end of the iron rod is in an ice bath of  $0^\circ\text{C}$ . Find the rate of transfer of heat from boiling water to ice bath. (Thermal conductivity of copper and iron are  $401 \text{ Wm}^{-1}\text{K}^{-1}$  and  $80 \text{ Wm}^{-1}\text{K}^{-1}$  respectively) (Dec 2005)

$$d_1 = 50 \times 10^{-2} \text{ m} \quad \theta_1 = 100^\circ\text{C}$$

$$K_1 = 401 \text{ Wm}^{-1}\text{K}^{-1}$$

$$d_2 = 25 \times 10^{-2} \text{ m}$$

$$\theta_2 = 0^\circ\text{C}$$

$$K_2 = 80 \text{ Wm}^{-1}\text{K}^{-1} \quad \text{and}$$

$$\text{area } A = 0.06 \times 10^{-4} \text{ m}^2$$

## Solution

$$\text{The amount of heat conducted per second } Q = \frac{A (\theta_1 - \theta_2)}{\left[\frac{d_1}{K_1} + \frac{d_2}{K_2}\right]}$$

$$Q = \frac{0.06 \times 10^{-4} \times (373 - 273)}{\left[\frac{0.5}{401} + \frac{0.25}{80}\right]}$$

$$\text{Rate of transfer of heat } Q = 0.1372 \text{ Watts}$$

2. The total area of the glass window pane is  $0.9 \text{ m}^2$ . Calculate how much heat is conducted per hour through the glass window pane if thickness of the glass 4mm, the temperature inside is  $29^\circ\text{C}$  and of the outside surface is  $3^\circ\text{C}$ . Thermal conductivity of glass is  $1.1 \text{ Wm}^{-1}\text{K}^{-1}$

## Given data

$$\text{Area } A = 0.9 \text{ m}^2$$

$$\text{Thickness } x = 4\text{mm}$$

$$\theta_1 = 29^\circ\text{C}$$

$$\theta_2 = 3^\circ\text{C}$$

## Solution

## Amount of heat conducted

$$Q = \frac{KA (\theta_1 - \theta_2)t}{x}$$

$$Q = \frac{1.1 \times 0.9 \times 26 \times 3600}{0.004}$$

$$\text{Heat conducted per hour } Q = 2.316 \times 10^7 \text{ Joules.}$$



3. A metal pipe having an external diameter 20 cm carries steam at 100 °C. This is covered by a layer 2.0 cm thick of insulating material with co-efficient of thermal conductivity 0.20 Wm<sup>-1</sup>K<sup>-1</sup>. If the outer surface is 30 °C, Calculate the heat lost by the pipe of 2m length per hour. Neglect the temperature drop across the pipe.

**Given data:**

Diameter = 20 cm,

$\theta_1 = 100^\circ\text{C}$ ,  $\theta_2 = 30^\circ\text{C}$ ,

time 't'=1 hour = 3600

**Solution:**

Quantity of heat flowing out across the pipe of length 'l' m in time 't' seconds.

$$Q = \frac{2\pi l K (\theta_1 - \theta_2)}{2.303 \log_{10} \left( \frac{r_2}{r_1} \right)} \times t \text{ joules}$$

$$\begin{aligned} Q \text{ per hour} &= \frac{2\pi \times 2 \times 0.2 (373 - 303)}{2.303 \log_{10} \left( \frac{12}{10} \right)} \times 3600 \\ &= \frac{6.33 \times 10^5}{0.1823} \end{aligned}$$

The quantity of heat lost by the piper per hour  $Q = 3.472 \times 10^6$  joules

4. A 30 cm length of iron rod is heated at one end to 100 °C, while the other end is kept at a temperature of 35 °C. The area of cross section of the iron rod is 0.725 cm<sup>2</sup>. Assume that the iron rod is thermally insulated. Calculate the amount of heat conducted through the rod in 8 minutes along the way.

Given the thermal conductivity of iron  $K = 62 \text{ Wm}^{-1} \text{ K}^{-1}$ .

**Given data:**

$K = 62 \text{ Wm}^{-1} \text{ K}^{-1}$ ,

$A = 0.725 \text{ cm}^2$ ,

$\theta_1 = 100^\circ\text{C}$ ,

$\theta_2 = 35^\circ\text{C}$ ,  $t = 8\text{min}$ ,  $x = 30\text{cm}$

**Solution:**

The quantity of heat conducted

$$\begin{aligned} Q &= \frac{KA (\theta_1 - \theta_2) t}{x} \\ &= \frac{62 \times 0.725 \times 10^{-4} \times (373 - 308) \times 480}{0.3} \\ &= 467.48 \text{ J} \end{aligned}$$

The iron rod conducts 467.48 joules in 8 minutes.

5. Two bars of copper and steel of length 1.0m and 0.5m respectively and of co-efficient of thermal conductivity 400 W/m-K and 50 W/m-K respectively are jointed end to end. The free ends of copper and steel are maintained at 100° C and 0° C respectively. Calculate the temperature of copper-steel junction if both bars have the same area of cross-section.

**Given data:**

$$K_1 = 400 \text{ W/m-K} \qquad d_1 = 1 \text{ m} \qquad \theta_1 = 100^\circ \text{ C}$$

$$K_2 = 50 \text{ W/m-K} \qquad d_2 = 0.5 \text{ m} \qquad \theta_2 = 0^\circ \text{ C}$$

**Solution:**

$$\frac{K_1}{K_2} = \frac{d_1}{d_2} \left( \frac{\theta_2 - \theta_3}{\theta_1 - \theta_2} \right)$$

$$\frac{400}{50} = \frac{1}{0.5} \left( \frac{\theta_2 - 0}{100 - \theta_2} \right)$$

$$\frac{\theta_2}{100 - \theta_2} = 4$$

$$400 - 4\theta_2 = \theta_2$$

$$5\theta_2 = 400$$

$$\theta_2 = 80^\circ \text{ C}$$

**The temperature of copper – Steel junction = 80° C**

**Part A –TWO MARKS****1. Define Heat energy.**

Heat or Thermal energy is the physical cause of the sensation of the hotness and coldness of a body and temperature is the degree of the hotness and coldness.

**2. What are the three modes of transfer of heat:**

Heat flow from one point to another by three distinct processes they are:

- Conduction
- Convection
- Radiation

**3. Explain the term heat Conduction. (Jan. 2011)**

Conduction is the process in which heat is transmitted from the hotter to the colder part of a body without the motion of the particles of the body.

**4. Define Convection process.**

Convection is the process by which heat is transmitted from the hotter to the colder part of a fluid with actual motion of the particles of the medium.

**5. Define the process Radiation.**

Radiation is the process by which heat energy is transmitted from one place to another without the aid of any material medium.

**5. Define Co-efficient of Thermal conductivity. (Dec. 1997)**

The coefficient of thermal conductivity is defined as the amount of heat conducted per second, normally across unit area of cross section, maintained at unit temperature gradient.

$$K = \frac{Q \cdot x}{A(\theta_1 - \theta_2)tx} \quad \text{Wm}^{-1}\text{K}^{-1}$$

**6. What are the basic entities responsible for thermal conduction of a solid?**

- Area of Cross section (A)
- Temperature difference between the hot and cold layers of the solid ( $\theta_1 - \theta_2$ )
- Time of Conduction (t)
- Thickness of the solid (x)

**7. What is meant by Temperature Gradient?**

The quantity  $\frac{(\theta_1 - \theta_2)}{x}$  represents the rate of fall of temperature with respect to the distance or thickness which is known as temperature gradient.

For smaller values  $\frac{(\theta_1 - \theta_2)}{x}$  is written as  $\frac{-d\theta}{dx}$

The negative sign indicates the decrease in temperature as the distance increases.

**8. How are heat conduction and electrical conduction analogous to each other? (Dec. 1998) (Any 4 points)**

S.No.	HEAT CONDUCTION	ELECTRICAL CONDUCTION
1.	Heat is conducted from a point of higher temperature to a point of lower temperature.	Electricity is conducted from a point at higher potential to a point at lower potential.
2.	In metals heat conduction is mainly due to free electrons.	In metals electrical conduction is due to free charge carriers namely electrons. In semi-conductors both electrons and holes contribute for the electrical contribution.
3.	In non metals heat conduction is only due to lattice vibrations.	In insulators, at high voltages electric breakdown occurs.
4.	The ability to conduct heat is measured by thermal conductivity which is defined as the quantity of heat conducted per second through unit area of the material when unit temperature gradient is measured.	The ability to conduct electricity is measured by electrical conductivity which is defined in the total electrical charge flowing per second per unit area of the conduction when unit potential gradient is maintained.
5.	Unit for thermal conductivity is $\text{Wm}^{-1}\text{K}^{-1}$	Unit for electrical conductivity is $\text{mho m}^{-1}$
6.	Thermal resistance of the conductor $= \frac{\text{Temperature gradient}}{\text{Thermal current flow}} = \frac{\Delta T}{Q}$	Electrical resistance of the conductor $= \frac{\text{potential gradient}}{\text{electrical current flow}} = \frac{\Delta V}{i}$

**9. Define Thermal Diffusivity (h):**

It is defined as the ratio of thermal conductivity to the thermal capacity per unit volume. [Since thermal capacity is the product of specific heat capacity (S) and the density ( $\rho$ )] of the material, we have,

$$\text{(i.e.,) } h = \frac{\text{thermal conductivity}}{\text{thermal capacity}} \quad (\text{or}) \quad \frac{K}{\rho S} \text{ m}^2\text{s}^{-1}$$

**10. What is meant by Specific Heat Capacity?**

It is defined as the amount of heat required to raise the temperature of unit mass of the substance through one Kelvin.

$$\text{(i.e) } S = \frac{Q}{m\theta} \text{ J Kg}^{-1} \text{ K}^{-1}$$

**11. Why the specimen used to determine thermal conductivity of a bad conductor should have a larger area and smaller thickness?**

For a bad conductor with a smaller thickness and larger area of cross section, the amount of heat conducted will be more.

**12. Give the methods of determining the thermal conductivity of good and bad conductors.**

The methods used for determining the thermal conductivity of good and bad conductor are:

- Searle's method: Good conductors like metallic rod.
- Forbe's method: For determining absolute conductivity of metals.
- Lee's disc method: For bad conductors.
- Radial flow method: For bad conductors.

**13. What is the basic principle employed in Lee's disc method for bad conductors? (Jan. 2012)**

The given bad conductor is taken in the form of disc and is placed in between the disc and steam chamber. The steam is passed through bad conductor. Heat conducted through the bad conductor per second is calculated. Amount of heat lost per second by the disc is also calculated. When steady state is reached, the amount of heat conducted through the bad conductor per second = Amount of heat lost per second by the disc

**14. What is meant by thermal resistance? (Jan. 2012)**

The thermal resistance of a body is a measure of its opposition to the flow of heat through it. (i.e) everybody posses some resistive power when it is subjected to heat. This resistive power is termed as thermal resistance.

**15. Distinguish between conduction and convection.**

**Conduction:** It is the process in which the heat is transferred from hot end to cold end without the actual movement of the particles.

**Convection:** It is the process in which the heat is transmitted from hot end to cold end by the actual movement of the particles.

**16. What is meant by thermal expansion in solids?**

When a metal is heated, due to increase in temperature it will expand and when it is cooled it contracts. However, an internal force will always act so as to keep the metal to regain its original length/position.

**Thermal expansion:** The expansion of a metal, when subjected to heat is called thermal expansion.

**17. Define Co-efficient of Thermal expansion.**

Co-efficient of thermal expansion ( $\alpha$ ) : The co-efficient of thermal expansion can be defined as the ratio between the change in length to the original length per unit rise of temperature.

**18. What is meant by Thermal expansion in liquid?**

We know when an ordinary alcohol-in-glass thermometer (or) mercury in-glass thermometer is kept in the temperature bath, the alcohol (or) mercury rises, due to thermal expansion.

In this case it should be noted that the temperature rises not because of the expansion of liquid, it is only due to the volume expansion of the liquid, it is only due to the volume expansion of the liquid.

**19. What do you understand by the term Bimetallic Strip? Give its use.****Definition**

As the name itself implies that Bimetallic strips are made up of two thin metal strips with different co-efficient of thermal expansion.

Bimetallic strips are commonly used in water heaters as temperature controller.

**20. What is meant by thermal insulation?**

Thermal insulation is made for reducing the heat transfer between the objects in thermal contact (or) in range of radiative influence.

It also provides a region of insulation in which thermal conduction is reduced. In other way we can say that the thermal radiation is reflected rather than absorbed by the body at lower temperature.

**21. What are the important properties of thermal insulating materials?**

The following are the important properties of thermal insulating materials.

- (i) The material should be fire proof.
- (ii) It should have volumetric specific heat.
- (iii) It should have low thermal conductivity.
- (iv) It should be a poor absorber of moisture.
- (v) It should have a fibrous, granular and porous structure

**21. What is meant by Heat Exchanger? How the heat is measured using it?****Definition**

A heat exchanger is a device that is used to transfer the heat between a solid and a liquid (or) between two (or) more liquid, without mixing and is used to reduce the heat produced by a device (or) machine.

Measurement: In a heat exchanger, the driving temperature across the heat transfer surface varies with position. Therefore the temperature difference is measured only in terms of log mean temperature difference (LMTD).

**22. Mention any two applications of heat exchanger?**

Heat exchangers have a very wide range of applications. Some of them are detailed below.

1. They are used in Refrigerators, air conditioners etc.
2. Heat exchangers are often used in power plants (or) Engines to cool the exhaust hot gases.
3. They are widely used in petroleum refineries, Petro-chemical Plants etc.
4. They are also used in natural-gas processing and sewage water treatment plants.

**23. What is meant by a refrigerator? Give its principle.**

Refrigerator is equipment used to 'reduce and maintain the temperature below atmospheric temperature. It is obtained by removing the heat from the space continuously.

Generally heat cannot flow from a cold body (i.e, a body at lower temperature) to hot body (i.e, a body at higher temperature). But, it is possible to do so, if some external work (or) pressure is done on the working substance. This concept is used in refrigerators.

**24. What do you understand by the term refrigerant? Give examples.**

Refrigerant is a fluid which absorbs the heat from the body and rejects the heat at high temperature.

Examples: Ammonia, carbon-di-oxide, Freon, methyl chloride, Chloro Fluro carbons (CFC).

**25. Define tonne of refrigeration.**

A tonne of refrigeration is defined as the amount of refrigeration effect produced (amount of heat extracted) by uniform melting of one tonne (1000 kg) of ice at 0°C to water in 24 hours.

$$1 \text{ Tonne of refrigeration} = 210 \text{ KJ/ms (or) } 3.5 \text{ KJ/sec.}$$

**26. What is the principle used in domestic refrigerator?**

The second law of thermodynamics as given by clausius, i.e, "without doing external work it is impossible to transfer heat from a cold body to a hot body". is the principle used in refrigerator.

Here, the ammonia (working substance) takes heat from the refrigerator (at lower temperature) and due to external work done on ammonia, it gives heat to atmospheric air (at normal (or) high temperature) and keeps the refrigerator continuously cool.

**27. Name any four applications of a refrigerator.**

1. It is used for preserving the Food, Fruits and drinks for a long duration.
2. It is used to preserve flowers, medicines and medical drugs.
4. Refrigerator is used to manufacture ice in ice plants. Refrigerator is used in refineries for removing wax.
5. In industries they are used for processing lubricants, rubber, steel etc.,
6. It is used for producing frozen foods, ice creams, chemicals and other products.

**28. What is meant by oven?**

Oven is a cylindrical shape material made up of heavy cast-Iron, which is used to roast (or) cook food in ancient days. In modern days ovens have become more high - Tech, in which electricity is used as the source for the heating coil. More recently micro-wave ovens become very popular for cooking in which micro wave radiations are used to excite the molecules in the food causing friction and they in turn produce heat, which is utilized for cooking the food.

**29. What are the types of oven?**

There are different types of oven as given below.

1. Earth oven
2. Ceramic oven
3. Gas oven
4. Masonry oven
5. Toaster oven
6. Microwave oven

**30. What is the principle used in microwave oven? Mention any two advantages & disadvantages of it.**

**Principle**

High powered microwaves are generated and are allowed to fall on the food stuff. These waves heat the molecules in the food particles evenly and cook the food.



**Advantages**

1. It is portable, small in size and low of cost.
2. Easy and faster to cook, with high efficiency.

**Disadvantages**

1. Microwaves are dangerous and so there should not be any leakage.
2. Uneven heating (or) cooking of food is not good for health.

**31. What do you meant by solar power? How will you estimate it?**

Solar Power is the process of converting (or) utilizing the abundantly available solar energy either directly as heat (or) indirectly by converting it into electrical power using photo voltaic cells.

**PART-B QUESTIONS**

1. Derive a differential equation (Second order) to describe the heat conduction along a uniform bar. Hence obtain the steady solution of it. ( DEC 1997)
2. Derive an expression for the rectilinear flow of heat along an uniform bar (One dimensional flow of heat). ( MAY/JUNE 2014, DEC 1998, NOV 2001)
3. Explain the Forbe's method of determining absolute thermal conductivity of good conductors.
4. How will you determine the thermal conductivity of a poor conductor using Lee's Disc method?
5. Derive an expression for the quantity of heat flow through a metal slab whose faces are kept at two different temperatures. Use this expression to determine the thermal conductivity of a bad conductor by lee's disc method. ( NOV 2002)
6. Explain in detail the conduction of heat through a compound media (Series and Parallel).
7. What is Heat exchanger? Detail the different modes in which the heat exchanged through it.
8. With a neat sketch describe the design and working of a refrigerator. Mention its advantages and disadvantages.
9. Describe the principal, construction and working of a Microwave oven.
10. Describe the principal, construction and working of a Solar water heater.

Thermal Physics						
S.No	questions	opt1	opt2	opt3	opt4	answer
1	If there is no transfer of energy between two objects then their temperature	same	different	zero	infinite	same
2	Heat is transferred in solids by	radiation	convection	conduction	fusion	conduction
3	Bad conductors are also called	insulators	convectors	radiators	termoids	insulators
4	Heat is measured in	joule	kelvin	celsius	joule second	joule
5	In a refrigeration system, the expansion device is connected between the	Compressor and condenser	Condenser and receiver	Receiver and evaporator	Evaporator and compressor	Receiver and evaporator
6	Quantity of thermal energy absorbed by a body for 1 kelvin increase in its temperature is known as	heat capacity	pressure capacity	Kinetic energy	potential energy	heat capacity
7	Gaps are left in railway tracks to compensate thermal expansion during	rainy season	winter	hot season	wind	hot season
8	An increase in breadth, length and thickness of a substance is due to	fusion	thermal expansion	stress	boiling	thermal expansion
9	The coefficient of thermal conductivity of a rubber can be determined by the principle of	Rectilinear	Cylindrical	Radial	Axial	Cylindrical
10	Lee's method for bad conductors a steady current passed through	heater coil	thermo couples	thin disk	copper plates	heater coil
11	Thermal diffusivity is defined as	ratio of thermal conductivity to thermal capacity per unit volume	directly proportional to the thermal conductivity	inversely proportional to thermal conductivity	directly proportional to the square of the thermal conductivity	ratio of thermal conductivity to thermal capacity per unit volume
12	Thermal capacity of a good conductor is determined by	Lee's method	Forbe's method	Searle's apparatus	Charlton's method	Lee's method
13	Which of the following has highest heat capacity?	metal	soil	water	air	water
14	Which law gives the relation between the work done and the heat produced?	Dulong and peti's law	Newton's law	Kirchoff's law	Joule's law	Joule's law
15	Molecules of a solid vibrate with larger amplitude at	zero temperature	lower temperature	higher temperature	pressure	higher temperature
16	Heat is transferred in gases by	radiation	convection	conduction	fusion	convection
17	Which of the following is not a common microwave application?	radar	mobile radio	telephone	satellite communication	mobile radio
18	Microwave oven operates at a frequency of	1.37 GHz	1.45 GHz	2.45 GHz	3.94 GHz	2.45 GHz
19	During a refrigeration cycle,heat is rejected by the refrigerant in a	Condenser	Compressure	Evaporator	Expansion valve	Condenser
20	One tonne of refrigeration is equal to	210 kJ/min	210 kJ/min	420 kJ/min	620 kJ/min	210kJ/min
21	Air refrigeration cycle is used in	Commercial refrigerators	Domestic refrigerators	Air-conditioning	Gas liquefaction	Gas liquefaction
22	Which of the following refrigerant is highly toxic and flammable	Ammonia	Carbon dioxide	Sulphur dioxide	Freon-12	Ammonia
23	Which of the following refrigerant has the lowest boiling point?	Ammonia	Carbon dioxide	Sulphur dioxide	Freon-12	Carbon dioxide
24	What is the principle of domestic refrigerator?	zeroth law	First law	Second law	third law	Second law
25	The ceramic oven is constructed using	Clay	Glass	Plastics	Rubber	Clay
26	For the same heat transfer Q and same overall heat transfer coefficient Uo, surface area required for cross flow operation is always	less than LMTD for parallel flow	more than LMTD for parallel flow	same as LMTD for parallel flow	unpredictable	less than LMTD for parallel flow
27	In parallel flow heat exchangers,	the exit temperature of hot fluid is always equal to the exit temperature of cold fluid	the exit temperature of hot fluid is always less than the exit temperature of cold fluid	the exit temperature of hot fluid is always more than the exit temperature of cold fluid	we cannot predict comparison between exit temperatures of hot fluid and cold fluid	the exit temperature of hot fluid is always more than the exit temperature of cold fluid
28	For the same heat transfer Q and same overall heat transfer coefficient Uo, surface area required for parallel flow operation is always	less than LMTD for counter flow	more than LMTD for counter flow	same as LMTD for counter flow	unpredictable	more than LMTD for counter flow
29	For the same inlet and exit temperatures of two fluids, the LMTD for counterflow is always	smaller than LMTD for parallel flow	greater than LMTD for parallel flow	same as LMTD for parallel flow	unpredictable	greater than LMTD for parallel flow
30	Which of the following temperature difference is safer than other to consider in designing of heat exchangers?	Arithmetic Mean Temperature Difference	Logarithmic Mean Temperature Difference (LMTD)	Both have nothing to do with safety	Other	Logarithmic Mean Temperature Difference (LMTD)
31	When is the arithmetic mean temperature difference of heat exchanger used instead of LMTD?	when the temperature profiles of two fluids of heat exchanger are sloping downward with curve	when the temperature profiles of two fluids of heat exchanger are sloping upward with curve	when the temperature profiles of two fluids of heat exchanger are straight	when the temperature profiles of two fluids of heat exchanger are quadratic	when the temperature profiles of two fluids of heat exchanger are straight
32	How can the arithmetic mean temperature difference and LMTD of a same heat exchanger be compared?	the arithmetic mean temperature difference is less than LMTD of a same heat exchanger	the arithmetic mean temperature difference is more than LMTD of a same heat exchanger	the arithmetic mean temperature difference and LMTD of a same heat exchanger are equal	the arithmetic mean temperature difference is double than LMTD of a same heat exchanger	the arithmetic mean temperature difference is more than LMTD of a same heat exchanger
33	The response of a material due to the function of heat is known as	Mechanical property	Electrical property	Chemical property	Thermal property	Thermal property
34	Tendency of matter to change in shape, area, and volume in response to a change in a particular temperature is called	non linear expansion	electrical expansion	thermal expansion	mechanical expansion	thermal expansion
35	Internal energy comprises of two types of energies, those are	mechanical and electrical energy	magnetic and electrical energy	kinetic and potential energy	kinetic and magnetic energy	kinetic and potential energy
36	A pure substance would freeze or solidify at its	boiling point	condensation point	melting point	sublimation point	melting point
37	Which of following does not belong to list of factors that affect rate of transfer of energy by radiation?	Color and texture of the surface	Temperature of the surface	Movement of air above the surface	Surface area	Movement of air above the surface
38	Rate of transfer of energy by radiation can be increased by	increasing the surface temperature	decreasing the surface area	using shiny white surfaces instead of dull and black surfaces	decreasing the atmospheric pressure	increasing the surface temperature
39	Vacuum in a vacuum flask prevents heat transfer through process of	Conduction only	Convection only	Conduction and Convection	Radiation only	Conduction and Convection
40	With respect to heat transfer through conduction, which of following inequalities place solids, 'liquids' and 'gases' in right order?	Solids < liquids < gases	Solids < gases < liquids	Solids > gases > liquids	Solids > liquids > gases	Solids > liquids > gases

# UNIT I

## QUANTUM MECHANICS

### INTRODUCTION TO QUANTUM THEORY

Laws of thermodynamics and classical laws of electricity and magnetism provide the basis for explanation of all phenomena in classical physics. It was general belief of the scientists that these laws would suffice to account for any subsequent discovered phenomena. Classical mechanics successfully explained the motion of the objects, which are directly observable. When the objects are not observable, then the concept of classical mechanics cannot be applied.

The phenomena in the realm of the atoms, nuclei and elementary particles are commonly referred to as quantum phenomena and subject matter containing all these phenomena constitutes what is known as Quantum Physics.

#### **Inadequacy of classical mechanics:**

According to the classical mechanics, if we consider the case of an electron moving round the nucleus, its energy should decrease (because the accelerated charged particle loses energy in the form of electromagnetic waves) and therefore its velocity should decrease continuously. The ultimate result is that the electron comes closer and closer to the nucleus until it collapses. This shows the instability of the atom; it is in contradiction to the observed fact of the stability of an atom. Thus the classical mechanics fails to explain the stability of an atom.

The classical mechanics also failed to explain the spectrum of the hydrogen atom. According to the classical theory, the excited atoms of hydrogen emit electromagnetic radiations of all wavelengths continuously, while it is observed that they emit the radiation of certain wavelengths only.

#### **Difficulties with classical theories of Black Body Radiation and Origin of Quantum Theory of Radiation:**

We know that when bodies radiate energy, their temperature falls until the loss of energy is compensated by an external source. In case of heat radiation, we can obtain the thermal equilibrium by maintaining the body at a fixed temperature with the help of some heat-giving source. In this case the body gives as much radiation as it receives. If the body absorbs all the incident radiation, then it is called Black Body radiation. In actual practice, it is not possible to realize a perfectly black body, but an enclosure provided with a small opening serves the purpose because the radiation entering the enclosure will be reflected many times inside the enclosure and ultimately absorbed.

## BLACK BODY RADIATION

### Perfect black body:

A perfect black body is one which absorbs and emits in all the radiations (corresponding to all wavelengths) that fall on it. The radiation given out by a perfect black body is called Black body radiation.

### Kirchoff's law:

Ratio of emissive power to the coefficient of absorption of any given wavelength is the same for all bodies at a given temperature and is equal to the emissive power of the black body at that temperature.

$$E_{\lambda} = \frac{e_{\lambda}}{a_{\lambda}}$$

### Experiment:

In practice a perfect black body is not available. Therefore let us consider a hollow sphere coated with lamp black on its inner surface.

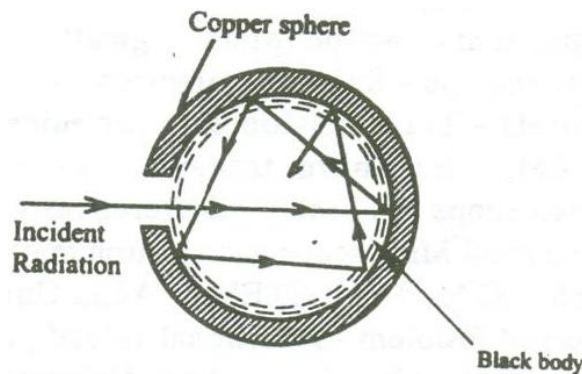


Fig 3.1

A fine hole is made for radiations to enter into the sphere as shown in the fig 3.1.

Now when the radiations are made to pass through the hole it undergoes multiple reflections and are completely absorbed. Thus the black body acts as a perfect absorber. Now when the black body is placed in a temperature bath of fixed temperature, the heat radiations will come out only through the hole in the sphere and not through the walls of the sphere.

Therefore, we can conclude that the radiations are emitted from the inner surface of the sphere and not from the outer surface of the sphere. Thus a perfect black body is a perfect absorber and also a perfect radiator of all wavelengths.

### Energy spectrum:

When a perfect black body is allowed to emit radiations at different temperatures, then the distribution of the energy for different wavelengths at various temperatures is obtained as shown in the fig 3.2.

From figure the following results are formulated.

- i. The energy distribution is not uniform for a given temperature.
- ii. The intensity of radiation ( $E$ ) increases with respect to the increase in wavelength at particular wavelength in becomes maximum ( $\lambda_m$ ) and after this it starts decreasing with respect to the increase in wavelength.
- iii. When the temperature is increased, the maximum wavelength ( $\lambda_m$ ) decreases.
- iv. For all the wavelengths an increase in its temperature causes increase in energy.

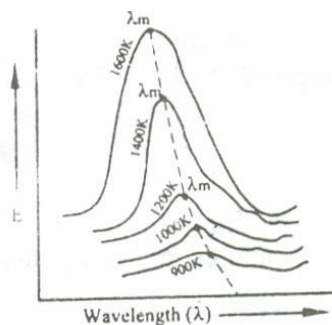


Fig 3.2

- v. The total energy emitted at any particular temperature can be calculated from the area under that particular curve.

### PHOTON AND ITS PROPERTIES:

According to the Quantum theory of radiation, we know that the exchange of energy values between the light radiation and particles have discrete energy values.

#### Photon

The discrete energy values in the form of small packets (or) bundles (or) quantas of definite frequency or wavelength are called photon. These photons are propagates like a particle like a particle but with the speed of light ( $3 \times 10^8 \text{ m/s}$ ).

#### Properties of photon:

- Photons are similar to that of electrons.
- We know for electrons the definite quantities are 'e' and 'm'. Similarly for photons the definite quantities are 'h' and 'c'.
- Photons will not have any charge. They are neutral and hence they are not affected by magnetic (or) electric fields.
- They do not ionize gases.
- The energy of the photon is given by  $E = h\nu$ , which varies with respect to the type of radiation frequencies.
- The momentum of photon is given by  $p = mc$ , where 'm' is the mass of the photon and 'c' is the velocity of light.
- The relation between energy and the momentum of the photon is given by  $E = pc$ .

$$[\text{i.e., } E = mc^2 = mc(c) = pc].$$

## DUAL NATURE OF RADIATION (LIGHT) AND MATTER (PARTICLES) – MATTER WAVES

### De- Broglie concept of dual nature:

The universe is made of radiation (light) and matter (particles). The light exhibits the dual nature (ie.) it can behave both a wave (interference, diffraction, phenomenon) and as a particle (Compton Effect, photo electric effect etc.)

Since the nature loves symmetry, in 1923 Louis de Broglie suggested that an electron or any other material particle must exhibit wave like properties in addition to particle nature.

The waves associated with a material particle are called as matter waves.

### De-Broglie wavelength:

From the theory of light, considering a photon as a particle the total energy of the photon is given by

$$E = mc^2 \quad \text{---(1)}$$

Where, 'm' is the mass of the particle and 'c' is the velocity of light.

Considering the photon as a wave, the total energy is given by

$$E = h\nu \quad \text{--- (2)}$$

Where, 'h' is the Planck's constant and 'ν' is the frequency of the radiation.

From equations (1) and (2)

$$E = mc^2 = h\nu \quad \text{--- (3)}$$

We know Momentum = Mass × velocity

$$p = mc$$

∴ Equation (3) becomes  $h\nu = pc$

$$p = \frac{h\nu}{c}$$

Since  $\lambda = \frac{c}{\nu}$  we can write  $p = \frac{h}{\lambda}$

(or) The wavelength of a photon  $\lambda = \frac{h}{p} \quad \text{--- (4)}$

de-Broglie suggested that the equation 3 can be applied both for photons and material particles. If m is the mass of the particle and v is the velocity the particle, then

$$\text{Momentum } p = mv$$

∴ de-Broglie wavelength  $\lambda = \frac{h}{mv} \quad \text{--- (5)}$

### De-Broglie wavelength in terms of energy:

We know kinetic energy  $E = \frac{1}{2}mv^2$

Multiplying by 'm' on both sides we get

$$\begin{aligned} Em &= \frac{1}{2}m^2v^2 \\ 2Em &= m^2v^2 \end{aligned}$$

$$(or) \quad mv = \sqrt{2mE}$$

$$\therefore \text{de-Broglie wavelength} \quad \lambda = \frac{h}{\sqrt{2mE}} \quad \text{--- (6)}$$

### De-Broglie wavelength in terms of Voltage

If a charged particle of charge 'e' is accelerated through a potential difference 'v', Then the Kinetic Energy of the particle =  $\frac{1}{2}mv^2$  --- (7)

Also we know energy = eV --- (8)

Equating (7) and (8), we get,

$$\frac{1}{2}mv^2 = eV$$

Multiplying by 'm' on both sides we get

$$\frac{1}{2}m^2v^2 = meV$$

$$m^2v^2 = 2meV$$

$$(or) \quad mv = \sqrt{2meV} \quad \text{--- (9)}$$

Substituting equation (9) in equation (5), we get

$$\therefore \text{de-Broglie wavelength} \quad \lambda = \frac{h}{\sqrt{2meV}} \quad \text{--- (10)}$$

### De-Broglie wavelength in terms of Temperature:

When a particle like neutron is in thermal equilibrium at temperature T, then they possess Maxwell distribution of velocities.

$$\therefore \text{Their kinetic energy} \quad E_k = \frac{1}{2}mv_{rms}^2 \quad \text{--- (11)}$$

Where, ' $v_{rms}$ ' is the Root mean square velocity of the particle.

$$\text{Also, we know} \quad \text{energy} = \frac{3}{2}K_B T \quad \text{--- (12)}$$

Where, ' $K_B$ ' is the Boltzmann constant.

Equating (11) and (12) we get

$$\frac{1}{2}mv^2 = \frac{3}{2}K_B T$$

Multiplying by 'm' on both sides we get

$$\frac{1}{2}m^2v^2 = \frac{3}{2}mK_B T$$

$$m^2v^2 = 3mK_B T$$

$$(or) \quad mv = \sqrt{3mK_B T}$$

$$\therefore \text{De-Broglie wavelength} \quad \lambda = \frac{h}{\sqrt{3mK_B T}}$$

### PROPERTIES OF MATTER WAVES:

- Matter waves are not electromagnetic waves.
- Matter waves are new kind of waves in which due to the motion of the charged particles, electromagnetic waves are produced.
- The wave and particle aspects cannot appear together.
- Locating the exact position of the particle in the wave is uncertain.
- Lighter particles will have high wavelength.

- Particles moving with less velocity will have high wavelength.
- The velocity of matter wave is not a constant; it depends on the velocity of the particle.
- The velocity of matter wave is greater than the velocity of light.

## UNCERTAINTY PRINCIPLE

According to classical ideas, it is possible for a particle to occupy a fixed position and have a definite momentum. Hence we can predict exactly its position and momentum, at any time.

But according to quantum mechanics, there is an inherent uncertainty in the determination of the position and momentum of the particle. According to Heisenberg's principle the position and momentum of a particle cannot be determined simultaneously to any degree of accuracy.

### Statement

It is impossible to determine precisely and simultaneously the values of both the position and momentum of a particle.

### Example

Considering the position and momentum as a pair of physical variables. These quantities are related as

$$\Delta x \Delta p \approx \frac{h}{2\pi}$$

where  $\Delta x$  is the error in determining position

$\Delta p$  is error in determining momentum of the particle.

Similarly we have

$$\Delta E \Delta t \approx \frac{h}{2\pi}$$

$$\Delta J \Delta \theta \approx \frac{h}{2\pi}$$

where  $\Delta E$  and  $\Delta t$  is the error in determining energy and time respectively and  $\Delta J$  and  $\Delta \theta$  are the error in determining the angular momentum and angle respectively.

## SCHROEDINGER WAVE EQUATION:

Schroedinger wave equation describes the wave nature of a particle in the mathematical form. It is the basic equation of motion of matter waves.

If the particle has wave properties, then there should be some sort of wave equation to describe the behavior of that particle.



Schrodinger connected the expression of de-Broglie's wavelength with the classical wave equation for a moving particle and he obtained a new wave equation

## FORMS OF SCHROEDINGER WAVE EQUATION

There are two forms of Schroedinger wave equation. They are

- a. Time independent wave equation
- b. Time dependent wave equation

### SCHROEDINGER TIME INDEPENDENT WAVE EQUATION:

Consider a wave associated with a moving particle.

Let x, y, z be the coordinates of the particle and  $\psi$  wave function for de – Broglie's waves at any given instant of time 't'.

The classical differential equation for wave motion is given by

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (1)}$$

Here, 'v' is wave velocity.

The eqn (1) is written as

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (2)}$$

Where,  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$  is a Laplacian's operator.

The solution of eqn (2) gives  $\psi$  as a periodic variations in terms of 't',

$$\psi(x, y, z, t) = \psi_0(x, y, z) e^{-i\omega t} \quad \text{--- (3)}$$

Here,  $\psi_0(x, y, z)$  is a function of x, y, z only, which is the amplitude at the point considered.

' $\omega$ ' is angular velocity of the wave.

Differentiating eqn (3) with respect to 't', we get

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t}$$

Again differentiating with respect to 't', we have

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)(-i\omega) \psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = i^2 \omega^2 \psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \quad \text{--- (4)}$$

Where,  $i^2 = -1$  and  $\psi = \psi_0 e^{-i\omega t}$

Substituting eqn (4) in eqn (2), we have

$$\nabla^2 \psi = -\frac{\omega^2}{v^2} \psi$$

$$\nabla^2 \psi + \frac{\omega^2}{v^2} \psi = 0 \quad \text{--- (5)}$$

We know that angular frequency  $\omega = 2\pi\nu$

$$\omega = 2\pi \frac{v}{\lambda}$$

Where,  $\nu = \frac{v}{\lambda}$ ,  $\nu$  is the frequency and  $v$  is the wave velocity

$$\frac{\omega}{v} = \frac{2\pi}{\lambda} \quad \text{--- (6)}$$

Squaring the eqn (6) on both sides, we get

$$\begin{aligned} \frac{\omega^2}{v^2} &= \frac{2^2 \pi^2}{\lambda^2} \\ \frac{\omega^2}{v^2} &= \frac{4\pi^2}{\lambda^2} \end{aligned} \quad \text{--- (7)}$$

Substituting eqn (7) in eqn (5), we get

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0 \quad \text{--- (8)}$$

On substituting,  $\lambda = \frac{h}{mv}$  in eqn (8), we get

$$\begin{aligned} \nabla^2 \psi + \frac{4\pi^2}{\left(\frac{h}{mv}\right)^2} \psi &= 0 \\ \nabla^2 \psi + \frac{4\pi^2}{\frac{h^2}{m^2 v^2}} \psi &= 0 \\ \nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2} \psi &= 0 \end{aligned} \quad \text{--- (9)}$$

If 'E' is the total energy of the particle, 'V' is potential energy and  $\frac{1}{2}mv^2$  is kinetic energy, then

Total energy = Potential energy + Kinetic energy

$$\begin{aligned} E &= V + \frac{1}{2}mv^2 \\ E - V &= \frac{1}{2}mv^2 \\ 2(E - V) &= mv^2 \end{aligned}$$

Multiplying by 'm' on both sides, we have

$$2m(E - V) = m^2 v^2 \quad \text{--- (10)}$$

Substituting eqn (10) in eqn (9), we get

$$\begin{aligned} \nabla^2 \psi + \frac{4\pi^2 2m(E - V)}{h^2} \psi &= 0 \\ \nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi &= 0 \end{aligned} \quad \text{--- (11)}$$

The eqn (11) is known as Schroedinger time independent wave equation.

Let us now introduce  $\hbar = \frac{h}{2\pi}$  in eqn (11),

$$\begin{aligned} \hbar^2 &= \frac{h^2}{2^2 \pi^2} \\ \hbar^2 &= \frac{h^2}{4\pi^2} \end{aligned} \quad \text{--- (12)}$$

where, ' $\hbar$ ' is a reduced Planck's constant.

The eqn (11) is modified by substituting  $\hbar$ ,

$$\nabla^2 \psi + \frac{m(E - V)}{\frac{h^2}{8\pi^2}} \psi = 0$$

$$\nabla^2 \psi + \frac{m(E-V)}{\frac{h^2}{2 \times 4\pi^2}} \psi = 0$$

$$\nabla^2 \psi + \frac{2m(E-V)}{\frac{h^2}{4\pi^2}} \psi = 0 \quad \text{--- (13)}$$

On substituting eqn (12) in eqn (13), Schroedinger time independent wave equation is written as,

$$\nabla^2 \psi + \frac{2m}{h^2} (E - V) \psi = 0 \quad \text{--- (14)}$$

### Special case:

If we consider one dimensional motion i.e., particle moving along only X-direction, then Schroedinger time independent wave equation (14) reduces to

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{h^2} (E - V) \psi = 0 \quad \text{--- (15)}$$

### SCHROEDINGER TIME DEPENDENT WAVE EQUATION:

Schrodinger time independent wave equation is derived from Schroedinger time independent wave equation.

The solution of classical differential equation of the wave motion is given by

$$\psi(x, y, z, t) = \psi_0(x, y, z) e^{-i\omega t} \quad \text{--- (1)}$$

Differentiating eqn (1) with respect to 't', we get

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t} \quad \text{--- (2)}$$

$$\frac{\partial \psi}{\partial t} = -i(2\pi\nu) \psi_0 e^{-i\omega t}$$

Where,  $\omega = 2\pi \nu$

$$\frac{\partial \psi}{\partial t} = -i(2\pi\nu) \psi \quad \text{--- (3)}$$

Where,  $\psi = \psi_0 e^{-i\omega t}$

$$\frac{\partial \psi}{\partial t} = -2\pi i \frac{E}{h} \psi$$

Where,  $E = h\nu$  (or)  $\nu = \frac{E}{h}$

$$\frac{\partial \psi}{\partial t} = -i \frac{E}{h} \psi$$

We know that  $\hbar = \frac{h}{2\pi}$

$$\therefore \frac{\partial \psi}{\partial t} = -i \frac{E}{h} \psi \quad \text{--- (4)}$$

Multiplying 'i' on both sides in eqn (4), we get

$$i \frac{\partial \psi}{\partial t} = -i^2 \frac{E}{h} \psi$$

We know that  $i^2 = -1$

$$\therefore i \frac{\partial \psi}{\partial t} = -(-1) \frac{E}{h} \psi$$

$$i \frac{\partial \psi}{\partial t} = \frac{E}{h} \psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi \quad \text{--- (5)}$$

Schroedinger time independent wave equation is given by

$$\begin{aligned}\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi &= 0 \\ \nabla^2\psi + \frac{2m}{\hbar^2}(E\psi - V\psi) &= 0\end{aligned}\quad \text{--- (6)}$$

Substituting eqn (5) in eqn (6), we get

$$\begin{aligned}\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) &= 0 \\ \nabla^2\psi &= -\frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) \\ -\frac{\hbar^2}{2m}\nabla^2\psi &= i\hbar\frac{\partial\psi}{\partial t} - V\psi \\ -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi &= i\hbar\frac{\partial\psi}{\partial t} \\ \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi &= i\hbar\frac{\partial\psi}{\partial t}\end{aligned}\quad \text{--- (7)}$$

$$(or) H\psi = E\psi \quad \text{--- (8)}$$

Where,  $H = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$  is Hamiltonian operator and  $E = i\hbar\frac{\partial}{\partial t}$  is energy operator

The eqn (8) is known as Schroedinger time dependent wave equation

### PHYSICAL SIGNIFICANCE OF WAVE FUNCTION:

#### Wave function:

It is the variable quantity that is associated with a moving particle at any position (x, y, z) and at any time 't' and it relates the probability of finding the particle at that point and at that time.

- It relates the particle and the wave statistically (i.e.,)  $\psi = \psi_0 e^{-i\omega t}$
- Wave function gives the information about the particle behavior.
- $\Psi$  is a complex quantity and individually it does not have any meaning.
- $|\psi|^2 = \psi^*\psi$  is real and positive, it has physical meaning. This concept is similar to light. In light, amplitude may be positive (or) negative but the intensity, which is square amplitude, is real and is measurable.
- $|\psi|^2$  represents the probability density (or) probability of finding the particle per unit volume.
- For a given volume  $d\tau$ , the probability of finding the particle is given by  
Probability (P) =  $\iiint |\psi|^2 d\tau$   
Where,  $d\tau = dx.dy.dz$
- The probability will have any value between zero and one. (i.e.,)
  - i. If  $P = 0$  then there is no chance for finding the particle (i.e.,) there is no particle, within the given limits.
  - ii. If  $P = 1$  then there is 100% chance for finding the particle (i.e.,) the particle is definitely present, within the given limits.

- iii. If  $P = 0.7$  then there is 70% chance for finding the particle and 30% there is no chance for finding the particle, within the given limits.

**Example:**

If a particle is definitely present within a one dimensional box (x-direction) of length 'l', then the probability of finding the particle can be written as

$$P = \int_0^l |\psi|^2 dx = 1$$

**PARTICLE IN A ONE DIMENSIONAL BOX:**

Let us consider particle (electron) of mass 'm' moving along the x - axis, enclosed in a one dimensional potential box as shown in the figure 3.10.

Since the walls are of infinite potential the particle does not penetrate out from the box.

Also, the particle is confined between the length 'l' of the box and has elastic collisions with the walls. Therefore, the potential energy of the electron inside the box is constant and can be taken as zero for simplicity.

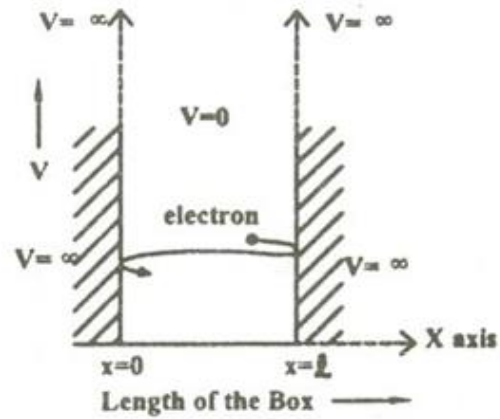


Fig 3.10

∴ We can say that the Outside the box and on the wall of the box, the potential energy V of the electron is 'α'.

Inside the box the potential energy (V) of the electron is zero.

In other words we can write the boundary condition as

$$V(x) = 0 \text{ when } 0 < x < l$$

$$V(x) = \alpha \text{ when } 0 \geq x \geq l$$

Since the particle cannot exist outside the box the wave function  $\psi = 0$  when  $0 \geq x \geq l$ .

To find the wave function of the particle within the box of length 'l', let us consider the Schroedinger one dimensional time independent wave equation(i.e.,)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

Since the potential energy inside the box is zero [i.e.  $V = 0$ ], the particle has kinetic energy alone and thus it is named as a free particle (or) free electron

∴ For a free particle (electron), the Schroedinger wave equation is given by

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0$$

$$\text{Or} \quad \frac{\partial^2 \psi}{\partial x^2} + k^2 E \psi = 0 \quad \text{--- (1)}$$

$$\text{Where,} \quad k^2 = \frac{2mE}{\hbar^2} \quad \text{--- (2)}$$

Equation (1) is a second order differential equation; therefore, it should have solution with two arbitrary constants.

∴ The solution for equation (1) is given by

$$\Psi(x) = A \sin kx + B \cos kx \quad \text{--- (3)}$$

Where, A and B are called Arbitrary constants, which can be found by applying the boundary conditions.

$$\text{(i.e.,) } V(x) = \alpha \text{ when } x = 0 \text{ and } x = l$$

**Boundary condition (i)** at  $x=0$  Potential energy  $V = \alpha$ . ∴ There is no chance for finding the particle at the walls of the box, ∴  $\psi(x) = 0$

∴ Equation (3) becomes

$$0 = A \sin 0 + B \cos 0$$

$$0 = 0 + B \quad (1)$$

$$\therefore \mathbf{B = 0}$$

**Boundary condition (ii)** at  $x = l$  Potential energy  $V = \alpha$ . ∴ There is no chance for finding the particle at the walls of the box, ∴  $\psi(x) = 0$

∴ Equation (3) becomes

$$0 = A \sin kl + B \cos kl$$

Since  $B = 0$  (from the first boundary condition), we have

$$0 = A \sin kl$$

$$\text{Since } A \neq 0; \quad \sin kl = 0$$

$$\text{We know} \quad \sin n\pi = 0$$

Comparing these two equations,

$$\text{We can write} \quad kl = n\pi$$

Where, n is an integer.

$$\text{(or) } k = \frac{n\pi}{l} \quad \text{--- (4)}$$

Substituting the value of B and k in equation 3 we can write the wave function associated with the free electron confined in a one dimensional box as

$$\Psi_n(x) = A \sin \frac{n\pi}{l} \quad \text{--- (5)}$$

### Energy of the particle (Electron)

We know from equation (2),

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\text{Where, } \hbar^2 = \frac{h^2}{4\pi^2}$$

$$k^2 = \frac{2mE}{\left(\frac{h^2}{4\pi^2}\right)}$$

$$\text{(or) } k^2 = \frac{8\pi^2 mE}{h^2} \quad \text{--- (6)}$$

Squaring eqn (4), we get

$$k^2 = \frac{n^2 \pi^2}{l^2} \quad \text{--- (7)}$$

Equating (6) and (7), we can write

$$\frac{8\pi^2 m E}{h^2} = \frac{n^2 \pi^2}{l^2}$$

$$E = \frac{n^2 \pi^2 h^2}{8\pi^2 m l^2}$$

$$\therefore \text{Energy of the particle (electron) } E_n = \frac{n^2 h^2}{8ml^2} \quad \text{--- (8)}$$

$\therefore$  From equations (8) and (5) we can say that, for each value of 'n', there is an energy level and the corresponding wave function.

Thus we can say that, each value of  $E_n$  is known as Eigen value and the corresponding value of  $\psi_n$  is called Eigen function.

### Energy levels of an electron

For various values of 'n' we get various energy values of the electron. The lowest energy value or ground state energy value can be got by substituting  $n = 1$  in equation (8)

$$\therefore \text{When } n=1 \text{ we get } E_1 = \frac{1^2 h^2}{8ml^2} = \frac{h^2}{8ml^2}$$

Similarly we can get the other energy values

$$\text{When } n=2, \text{ we get } E_2 = \frac{2^2 h^2}{8ml^2} = \frac{4h^2}{8ml^2} = 4E_1$$

$$\text{When } n=3, \text{ we get } E_3 = \frac{3^2 h^2}{8ml^2} = \frac{9h^2}{8ml^2} = 9E_1$$

$$\text{When } n=4, \text{ we get } E_4 = \frac{4^2 h^2}{8ml^2} = \frac{16h^2}{8ml^2} = 16E_1$$

$\therefore$  In general we can write the energy Eigen function as

$$E_n = n^2 E_1 \quad \text{--- (9)}$$

It is found that from the energy levels  $E_1, E_2, E_3$  etc the energy levels of an electron are discrete.

This is the great success which is achieved in Quantum Mechanics than classical mechanics, in which the energy levels are found to be continuous.

The various energy Eigen values and their corresponding Eigen functions of an electron enclosed in a one dimensional box is as shown in the fig 3.11.

Thus we have discrete energy values.

### Normalization of the wave function:

#### Normalization:

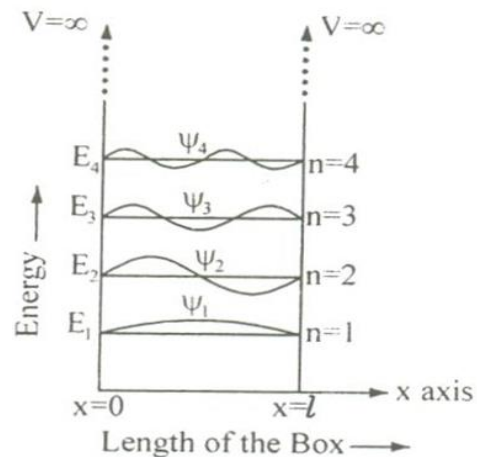


Fig 3.11

It is the process by which the probability (P) of finding the particle (electron) inside the

$$A = \sqrt{\frac{2}{l}}$$

Substituting the Value of A in equation (5),

The normalized wave function can be written as

$$\Psi_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l}$$

The normalized wave function and their energy values are as shown in the fig 3.12

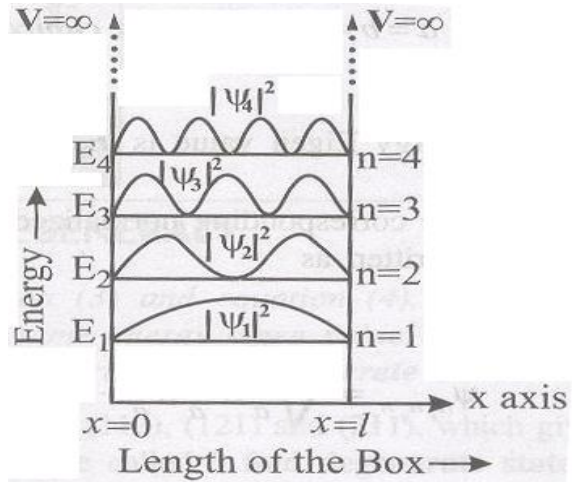


Fig 3.12

box can be done.

We know that the total probability (P) is equal to 1 means then there is a particle inside the box. Therefore, for a one dimensional potential box of length 'l' the probability

$$P = \int_0^l |\psi|^2 dx = 1 \quad \text{--- (10)}$$

(Since the particle is present inside the well between the length 0 to 'l' the limits are chosen between 0 to l)

Substituting equation (5) in equation (10), we get

$$P = \int_0^l A^2 \sin^2 \frac{n\pi}{l} dx = 1$$

$$(\text{or}) A^2 \int_0^l \left[ \frac{1 - \cos 2\left(\frac{n\pi x}{l}\right)}{2} \right] dx = 1$$

$$A^2 \left[ \frac{x}{2} - \frac{1}{2} \frac{\sin\left(\frac{2n\pi x}{l}\right)}{\left(\frac{2n\pi}{l}\right)} \right]_0^l = 1$$

$$A^2 \left[ \frac{l}{2} - \frac{1}{2} \frac{\sin\left(\frac{2n\pi l}{l}\right)}{\left(\frac{2n\pi}{l}\right)} \right] = 1$$

$$A^2 \left[ \frac{l}{2} - \frac{1}{2} \frac{\sin(2n\pi)}{\left(\frac{2n\pi}{l}\right)} \right] = 1$$

We know  $\sin n\pi = 0 \therefore \sin 2n\pi$  is also = 0

$\therefore$  Equation 11 can be written as

$$A^2 \left[ \frac{l}{2} \right] = 1$$

$$A^2 = \frac{2}{l}$$

## DEGENERACY AND NON-DEGENERACY:

**Degeneracy:**



It is seen from equation (3) and equation (3), for several combination of quantum numbers we have same energy Eigen value but different Eigen functions. Such states and energy levels are called Degenerate state.

The three combinations of quantum numbers (112), (121) and (211) which gives same Eigen value but different Eigen functions are 3 fold degenerate state.

#### **Non –degeneracy:**

For various combinations of quantum number if we have same energy value and same (one) Eigen function then such states and energy levels are called Non – degenerate state.

#### **BASICS OF A MICROSCOPE:**

A microscope is a device which is used to view the magnified image of a smaller object, which cannot be clearly seen through a naked eye.

In general we can classify the microscope as simple and compound microscope. A simple microscope is made up of a single biconvex magnifying lens held in a simple frame. A compound microscope is made up of two lenses (or) system of lenses for better magnification.

Depending on the field of application, many other microscopes such as phase contrast microscope, UV microscope, metallurgical microscope, electron microscope, etc are designed. These types of microscopes give a stereoscopic vision and reduce the strain of our eyes.

#### **Magnifying power:**

The magnifying power (M) of a microscope is defined as the ratio between the angle subtended by the final image at the eye ( $\beta$ ) to the angle subtended by the object at the eye ( $\alpha$ ), placed at the near point.

$$M = \frac{\beta}{\alpha}$$

#### **Resolving power:**

It is the ability of an optical instrument to form a distinct and separable image of the two point objects which are close to each other.

If 'd' is the least distance between two close point objects, then we can write

$$d = \frac{\lambda_0}{2 NA}$$

$$\therefore \text{Resolving power} = \frac{1}{d} = \frac{2NA}{\lambda_0}$$

Where, NA be the numerical aperture of the objective of the microscope and  $\lambda_0$  be the wavelength of light through vacuum.

Therefore the resolving power of a microscope can be increased by decreasing the value of  $\lambda_0$ . Thus, by using UV light and quartz lenses, the resolving power can be increased.

#### **ELECTRON MICROSCOPE**

It is a type of microscope in which instead of light beam, a beam of electrons are used to form a large image of very small object. These microscopes are widely used in the field of engineering and medicine.

**Principle:**

A stream of electrons is passed through the object and the electron which carries the information about the object are focused by electric and magnetic fields.

Since the resolving power is inversely proportional to the wavelength, the electron microscope has high resolving power because of its shorter wavelength.

**Construction:**

An electron microscope is similar to that of an optical microscope. Here the focusing of electrons can be done either by magnetic lens or by electrostatic lens. Normally in electron microscope magnetic lenses are used for focusing.

In general, the magnetic lenses are made of two coils  $C_1$  and  $C_2$  enclosed inside the iron cases which have one hole as shown in fig 3.13. When the holes face each other, the magnetic field in space between the two coils focuses the electrons emerging out from the electron gun. Similarly the divergence of the electrons can also be made by adjusting the position of the holes in the iron cases.

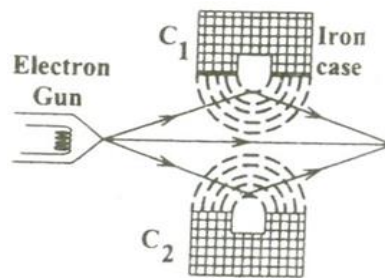


Fig 3.13

The essential parts of an electron microscope are as shown in the fig 3.14 and for comparison an optical microscope is also shown in fig 3.15.

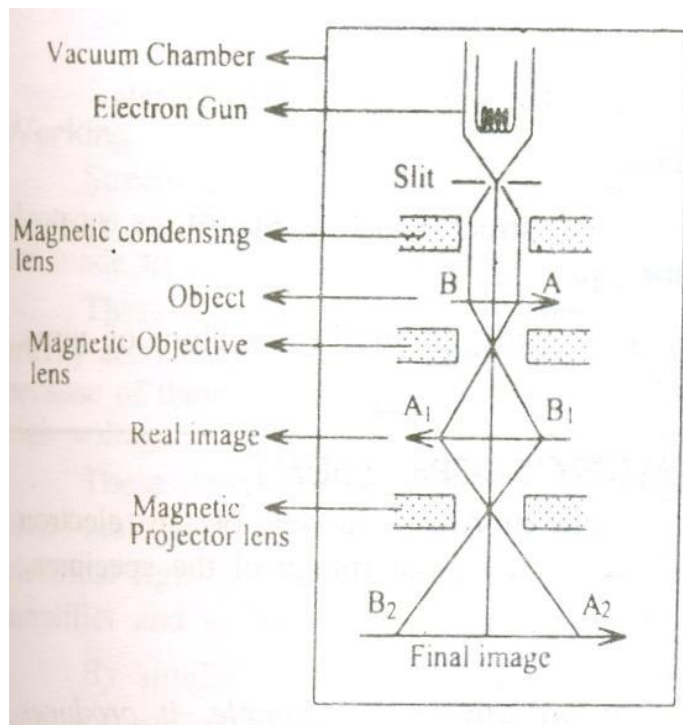


Fig 3.14

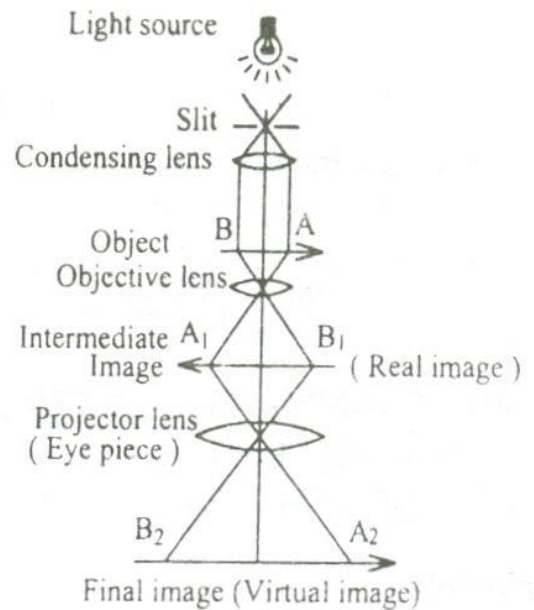


Fig 3.15

The electron microscope consists of an electron gun to produce the stream of electrons. Similar to the condensing lens, objective and eye piece in an optical microscope here three magnetic lenses are used.

- Magnetic condensing lens
- Magnetic objective lens
- Magnetic projector lens

The whole arrangement is kept inside a vacuum chamber to allow the passage of electron beam.

### Working:

Stream of electrons are produced and accelerated by the electron gun. The electron beam is made to pass through the center of the doughnut shaped magnetic condensing lens. These electrons are made as parallel beam and are focused on to the object AB (fig 3.14). The electrons are transmitted more in the less dense region of the object and is transmitted less (i.e.,) absorbed by the denser region of the object. Thus the transmitted electron beam on the falling over the magnetic objective lens, resolves the structure of the object to form a magnified real image of the object. Further the image can be magnified by the magnetic projector lens and the final image is obtained on the fluorescent screen.

In order to make a permanent record of the image of the object, the final image can also be obtained on a photographic plate.

### Advantages:

- It can produce magnification as high as 1, 00,000 times as that of the size of the object.

- The focal length of the microscopic system can be varied.

### Applications:

It has a very wide area of applications (Eg.) in biology, metallurgy, physics, chemistry, medicine, engineering etc.

- It is used to determine the complicated structure of the crystals.
- It is used in the study of the colloids.
- In industries it is used to study the structure of textile fibers, surface of metals, composition of paper, paints etc.
- In the medical field it is used to study about the structure of virus, bacterial etc which are of smaller size.

### SCANNING ELECTRON MICROSCOPE

Scanning electron microscope is an improved model of an electron microscope. SEM is used to study the three dimensional image of the specimen.

#### Principle:

When the accelerated primary electron strikes the sample, it produces secondary electrons. These secondary electrons are collected by a positive charged electron detector which in turn gives a 3- dimensional image of the sample.

#### Construction:

It consists of an electron gun to produce high energy electron beam. A magnetic condensing lens is used to condense the electron beam and a scanning coil is arranged in-between magnetic condensing lens and the sample.

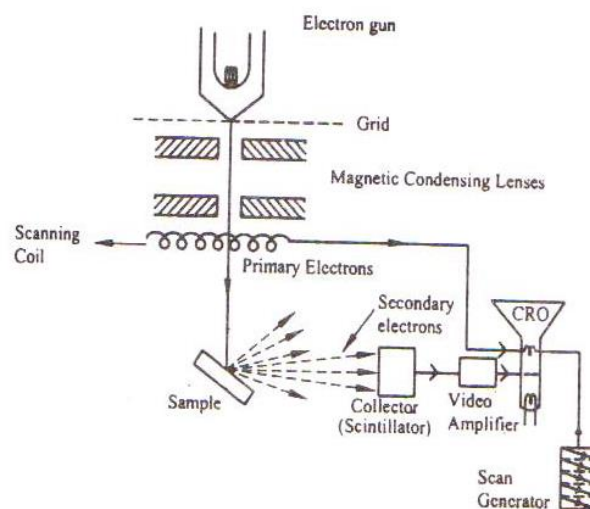


Fig 3.16

The electron detector (Scintillator) is used to collect the secondary electrons and can be converted into electrical signal. These signals can be fed into CRO through video amplifier as shown in fig 3.16.

#### Working:

Stream of electrons are produced by the electron gun and these primary electrons are accelerated by the grid and anode. These accelerated primary electrons are made to be incident on the sample through condensing lenses and scanning coil.

These high speed primary electrons on falling over the sample produce low energy secondary electrons. The collections of secondary electrons are very difficult and hence a high voltage is applied to the collector.

These collected electrons produce scintillations on the photo multiplier tube are converted into electrical signals. These signals are amplified by the video amplifier and are fed to the CRO.

By similar procedure the electron beam scans from left to right and again right to left etc., similar to we read a book (fig 3.17) and the whole picture of the sample is obtained in the CRO screen.

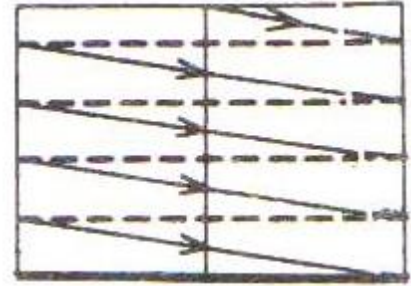


Fig 3.17

**Advantages:**

- It can be used to examine specimens of large thickness.
- It has large depth of focus.
- It can be used to get a three dimensional image of the object.
- Since the image can be directly viewed in the screen, structural details can be resolved in the precise manner.
- The magnification may be upto 3, 00,000 times greater than that of the size of the object.

**Disadvantages:**

The resolution of the image is limited to about 10-20 nm, hence it is very poor.

**Applications:**

- It is used to examine the structure of very large specimens in a three dimensional view.
- Similar to the application of electron microscope this SEM also has applications over various fields such as Biology, Industries, Engineering, Physics, Chemistry, etc.

## PROBLEMS

1. Calculate de-Broglie wavelength associated with a proton moving with a velocity equal to  $\frac{1}{20}$  of the velocity of light.

Mass of proton =  $1.675 \times 10^{-27}$  kg

**Given data:**

Mass of proton  $m = 1.675 \times 10^{-27}$  kg

Velocity of proton  $v = \frac{1}{20}$  x velocity of light

$$v = \frac{1}{20} \times 3 \times 10^8$$

$$v = 15 \times 10^6 \text{ m/s}$$

**Solution:**

de-Broglie wavelength  $\lambda = \frac{h}{mv}$

$$\lambda = \frac{6.63 \times 10^{-34}}{1.675 \times 10^{-27} \times 15 \times 10^6}$$

$$\lambda = 2.64 \times 10^{-14} \text{ m}$$

2. Calculate the de-Broglie wavelength of an electron of energy 100 eV.

**Given data:**

Energy of electron (E) = 100 eV =  $100 \times 1.6 \times 10^{-19}$  Joules

$$E = 1.6 \times 10^{-17} \text{ Joules}$$

**Solution:**

de-Broglie wavelength  $\lambda = \frac{h}{\sqrt{2mE}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-17}}}$$

$$\lambda = 1.227 \times 10^{-10} \text{ m}$$

$$\lambda = 1.227 \text{ \AA}$$

3. An electron is accelerated by a potential of 150 V. what is the wavelength of that electron wave?

**Given data:**

Accelerated potential of an electron (V) = 150 V

**Solution:**

de-Broglie wavelength  $\lambda = \frac{h}{\sqrt{2meV}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 150}}$$

$$\lambda = 1.0018 \times 10^{-10} \text{ m}$$

$$\lambda = 1 \text{ \AA}$$

4. Calculate the de-Broglie wavelength corresponding to the root mean square velocity of hydrogen molecules at 27°C.

**Given data:**

Temperature  $T = 27^\circ\text{C} = 300\text{K}$

Mass of hydrogen = mass of proton =  $1.678 \times 10^{-27}\text{kg}$

**Solution:**

de-Broglie wavelength  $\lambda = \frac{h}{\sqrt{3mK_B T}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.678 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$$\lambda = 1.451 \text{ \AA}$$

5. An electron is confined to a one dimensional box of side  $10^{-10}\text{m}$ . Obtain the first two Eigen values of the electron.

**Given data:**

Length of one dimensional box 'l' =  $10^{-10}\text{m}$

**Solution:**

1<sup>st</sup> Eigen value,

$$\text{Eigen Energy of the particle (electron)} E_1 = \frac{1^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$E_1 = 6.022 \times 10^{-18}\text{J}$$

(or) 
$$E_1 = \frac{6.022 \times 10^{-18}}{1.6 \times 10^{-19}} = 37.63 \text{ eV}$$

2<sup>nd</sup> Eigen value,

$$\text{Eigen Energy of the particle (electron)} E_2 = 2^2 E_1 = 2.408 \times 10^{-17}\text{J} = 150 \text{ eV}$$

(or) 
$$E' = \frac{7.8002 \times 10^{-14}}{1.6 \times 10^{-19}}$$

$$E' = 0.4875 \times 10^6 \text{ eV}$$

6. Calculate the magnifying power of a microscope. Give that the angle subtended by the final image ( $\beta$ ) is  $40^\circ$  at eye and the angle subtended by the object at the eye kept at the near point ( $\alpha$ ) is  $10^\circ$ .

**Given data:**

$$\beta = 40^\circ; \alpha = 10^\circ$$

**Solution:**

$$\text{Magnifying power } M = \frac{\beta}{\alpha}$$

Or 
$$M = \frac{\tan \beta}{\tan \alpha}$$

$$= \frac{\tan 40^\circ}{\tan 10^\circ}$$

$$\therefore \text{Magnifying power } M = 4.758$$

questions

A perfect black body is one which \_\_\_\_\_ all the radiations.

The classical theory was not able to explain the \_\_\_\_\_.

The discrete energy values in the form of quantum of definite frequency are called \_\_\_\_\_.

Which of the following are not affected by the magnetic field?

The relation between energy and the momentum of the photon is \_\_\_\_\_.

The wave associated with a material particle are called as \_\_\_\_\_.

De-Broglie wavelength in terms of energy is \_\_\_\_\_.

De-Broglie wavelength in terms of voltage is \_\_\_\_\_.

According to \_\_\_\_\_ theory the hydrogen spectrum is a continuous spectrum.

According to \_\_\_\_\_ theory the hydrogen spectrum is a discrete spectrum.

The de-Broglie wavelength of an electron of energy 100 eV is \_\_\_\_\_.

In one dimensional potential box,the potential energy of the electron inside the box is \_\_\_\_\_.

According to quantum mechanics, the energy levels of an electron are \_\_\_\_\_.

Source used in the SEM is \_\_\_\_\_.

energy and other characteristics to be calculated. In classical physics, what is analogue of the wave function?

The speed of propagation of an electromagnetic wave in vacuum is:

The expression of the momentum of a photon is :

If the uncertainty of a proton accelerated in a laboratory is 400 m/s, that of its position is:

Planck's constant has the same units as \_\_\_\_\_.

Which of the following is known as the Schrodinger equation ?

The principle that all microscopic physical entities have both wave and particle properties is called the wave-particle:

In the probabilistic interpretation of wave function  $\Psi$ , the quantity  $|\Psi|^2$  is:

Quantum Mechanics is also called as \_\_\_\_\_.

Classical Mechanics failed to explain \_\_\_\_\_.

Quantum Theory was introduced by \_\_\_\_\_.

As temperature increase, the peak wavelength emitted by the black body \_\_\_\_\_.

As temperature increase, the total energy emitted by black body \_\_\_\_\_.

According to de-braglie wave equation, when mass of the particle decreases wavelength will be \_\_\_\_\_.

Speed of light \_\_\_\_\_.

A particle in one dimensional box at the walls of the box potential Energy will be \_\_\_\_\_.

Photons propagate with \_\_\_\_\_ speed of light.

According to de-braglie wave equation, when mass of the particle increases wavelength will be \_\_\_\_\_.

Black body emits radiation in \_\_\_\_\_.

According to plancks quantum theory,electrons in the black body radiator are assumed to be \_\_\_\_\_.

The black body radiator emits energy in \_\_\_\_\_.

The term  $k_B$  represents \_\_\_\_\_.

Expression for momentum is \_\_\_\_\_.

$(\Delta E)^2$  represents \_\_\_\_\_.

Relation between angular and linear frequencies is \_\_\_\_\_.

Which one of the following is true?

For a free particle, potential energy is \_\_\_\_\_.

According to de-broglie wave equation, when velocity of the particle increases wavelength will be \_\_\_\_\_.

A particle in one dimensional box at the walls of the box wave function will be \_\_\_\_\_.

Energy of photon is directly related to the \_\_\_\_\_.

Calculate the minimum uncertainty in velocity of an electron trapped in 30.3nm wide?

Frequency below which no electrons are emitted from metal surface is \_\_\_\_\_.

Loss of energy of an electron results in \_\_\_\_\_.

Electrons show diffraction effects because their de Broglie wavelength is similar to \_\_\_\_\_.

Wavelength of ultraviolet region of electromagnetic spectrum is \_\_\_\_\_.

A perfect black body is a perfect absorber and radiator of \_\_\_\_\_ radiation.

opt1

Absorbs

diffraction

photons

electrons

$E=EC$

standing wave

$h/2mE$

$h/2mV$

classical

12.27 Angstrom

maximum

continuous

electrical enrgy

particle's momentum.

$3 \times (10)^{-6}$  m/s

$p = h \cdot \lambda$

7.88 nm

angular momentum

$E = h \cdot \nu$

singularity

a probability density

Classical Mechanics

Thermodynamics of maths

Max Planck

Increases

No Change

Decreases

$3 \times (10)^{-8}$  m/s

Increases

smaller

Decreases

visible region

electric oscillators

discrete

Bohr radius

$p = 2 \text{ m} \cdot v$

Hermition operator

$\omega = \nu/2\pi$

$\nu = p/\lambda$

0

Doubles

Increases

wavelength

$1.20 \times 10^{-4}$  m/s

minimum frequency

absorption of photon

spacing between atomic layers

121 nm

monochromatic

opt2

emits

interference

photons

protons

$E=P/C$

progressive wave

$h/\sqrt{2mE}$

$h/2mE$

quantum

classical

122.7 Angstrom

minimum

discrete

chemical source

particle's energy

$3 \times (10)^{-8}$  m/s

$p = h / \lambda$

9.70 nm

the Hamiltonian

$E = m^* (\dot{c})^2$

duality

a probability amplitude.

Wave mechanics

Electromagnetic Theory

De-Broglie

No Change

Decreases

Increases

$(2.5) \times (10)^{-8}$  m/s

Decreases

greater

Increases

single wavelengths

harmonic oscillators

continuos

Plancks constant

$p = m \cdot v$

Laplacian operator

$\omega = 2 \cdot \pi \cdot \nu$

$\nu = c/\lambda$

1

Increases

Decreases

wave number

$2.87 \times 10^{-4}$  m/s

angular frequency

emission of photon

no. of atomic layers

120 nm

all wavelengths of

opt3

absorbs and emits

emission of black body radiation

neutrons

photons

$C=EP$

transverse wave

$h/2mEe$

$h/2m$

Electro magnetic

electro magnetic

1.227 Angstrom

zero

scattering

electron gun

particle's mass

$3 \times (10)^{-8}$  km/s

$p = c/\lambda$

112 nm

frequency

$H^* \Psi = E^* \Psi$

triality

1

Statistical mechanics

Atomic Spectra

Newton

Decreases

Saturates

Doubles

$(3.5) \times (10)^{-8}$  m/s

Zero

equal

Doubles

Discrete wavelengths

simple pendulum

pulse

Boltzmann constant

$p = h \cdot \nu$

Energy operator

$\omega = 2 \cdot \pi \cdot \nu$

$\lambda = \nu \cdot c$

2

Decreases

Zero

frequency

$1.20 \times 10^{-6}$  m/s

maximum frequency

destruction of photon

nature of atomic layers

119 nm

coherent

opt4

reflects

scattering

scattering

positive ions

$E=PC$

matter wave

$h/2m$

$h/\sqrt{2meV}$

wave

electro magnetic

0.1227 Angstrom

infinity

diffraction

neutron gun

The sum of the forces on the particle

$3 \times (10)^{-8}$  km/s

$p = c/\lambda$

115nm

de Broglie wavelength

$\lambda = h/p$

infinitality

0

Newtonian mechanics

Wave mechanics

Velocity of Particle

Einstein

Saturates

Increases

Zero

$2 \times (10)^{-8}$  m/s

infinity

same

Decreases

All wavelengths

non-linear oscillators

wave

Stefans constant

$p = h \cdot (\nu)/c$

Hamiltonian operator

$\nu = 2 \cdot \pi \cdot \omega$

$\lambda = 1/\nu$

3

Zero

infinity

amplitude

$1.20 \times 10^{-5}$  m/s

threshold frequency

formation of photon

positioning of atomic layers

130 nm

polychromatic

answer

absorbs and emits

emission of black body radiation

photons

photons

$E=PC$

matter wave

$h/\sqrt{2mE}$

$h/\sqrt{2meV}$

classical

quantum

1.227 Angstrom

zero

discrete

electron gun

particle's mass

$3 \times (10)^{-8}$  km/s

$p = c/\lambda$

7.88 nm

angular momentum

$H^* \Psi = E^* \Psi$

duality

a probability density

Wave mechanics

Atomic Spectra

Max Planck

Decreases

Increases

Increases

$3 \times (10)^{-8}$  m/s

infinity

same

Decreases

All wavelengths

harmonic oscillators

discrete

Boltzmann constant

$p = m \cdot v$

Laplacian operator

$\omega = 2 \cdot \pi \cdot \nu$

$\nu = c/\lambda$

0

Decreases

Zero

frequency

$1.20 \times 10^{-4}$  m/s

threshold frequency

emission of photon

spacing between atomic layers

121 nm

all wavelengths of



## UNIT V

### CRYSTAL PHYSICS

#### 5.1 INTRODUCTION

Materials differ from one another in their properties. Some solids are brittle, some are ductile, some are weak, some are malleable, some are good conductors of electricity and heat, some are magnetic and so on. But all these materials are composed of atoms and molecules. These atoms are held together by the forces of attraction. The attractive forces which hold the particles of substance together are “bonds”. The differences in the properties of the solids are due to their structure.

#### 5.2 CLASSIFICATION OF SOLIDS

Most of the materials do not have any characteristic difference in their outward appearance. But if we examine them under a microscope we shall find these materials to have different internal atomic structures. Based on internal structures, the solids can be classified into two categories namely

- i) Crystalline solids
- ii) Non – crystalline solids or amorphous materials

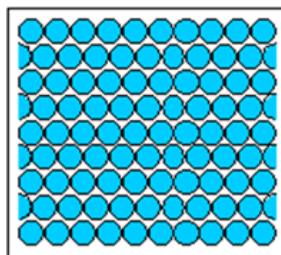
#### 5.3 CRYSTALLINE SOLIDS (OR) CRYSTALS

Crystals are those in which the atoms are arranged in an orderly fashion throughout in a three dimensional pattern. Each atom is fixed at a definite point in space, at a definite distance from each other and in a definite angular orientation to all other atoms surrounding it. Therefore crystalline solids have **well defined geometrical form**. Further when crystal breaks, all the broken pieces will have a regular shape. It is called as **anisotropic substances**. These crystalline solids are classified into two such as,

- i) Single crystal
- ii) Poly crystal

##### Single crystal:

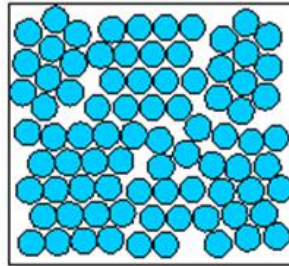
The crystalline solid which contains only one crystal, it is called as **single crystal**. Fig (5.1) represents the schematic structure of single crystal.



**Fig 5.1 Schematic structure of single crystal**

**Poly crystal:**

The polycrystalline materials are aggregate of many grains separated by well defined grain boundary. Fig (5.2) represents the schematic structure of poly crystal.



**Fig 5.2 Schematic structure of poly crystal**

**Examples for crystalline solids:** Diamond, Copper, Platinum, Silver, Polonium, Gold, Molybdenum, Nickel, Cadmium, Iron, etc.

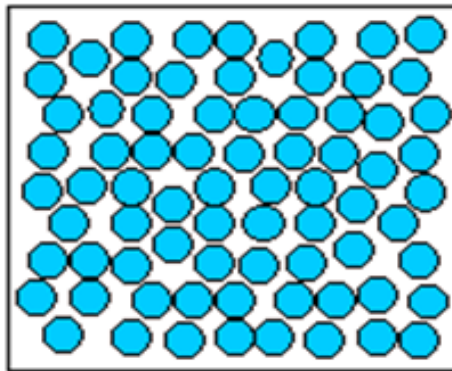
**Crystallography:**

The study of the geometric form and other physical properties of crystalline solids, using X-rays (or) electron beam (or) neutron beam etc., is termed as the science of crystallography.

**5.4 NON – CRYSTALLINE SOLIDS (AMORPHOUS MATERIALS)**

**Amorphous** means **without form**. The materials in which atoms are arranged in an irregular fashion are known as amorphous materials. **Example:** rubber, glass and plastics.

The schematic representation of amorphous materials as shown in fig 5.3



**Fig 5.3 Structure of amorphous materials**

**5.5 DIFFERENCES BETWEEN CRYSTALLINE AND NON – CRYSTALLINE MATERIAL**

S. No	Crystalline Material	Non – Crystalline Material
1.	They have a definite and regular geometrical shapes which extend throughout the crystal	They do not have definite and regular geometrical shape

S. No	Crystalline Material	Non – Crystalline Material
2.	They are anisotropic	They are isotropic
3.	They are most stable	They are less stable
4.	They have sharp melting point	They do not have sharp melting point
5.	Examples: NaCl, KCl	Examples: Glasses, Rubber

## 5.6 FUNDAMENTALS OF CRYSTALS AND ITS STRUCTURES

### Crystal:

A crystal is a three dimensional solid which consists of periodic arrangement of atoms. Crystal is regular polyhedral form bounded by smooth surfaces, which is formed by chemical compound under the action of its inter atomic forces, when passing from the state of liquid to that of a solid, under suitable condition.

X –rays are most widely used to study the crystal structures, because the wavelength of X –rays are almost equal to that of the inter atomic distances.

### Crystallographic terms:

A crystal is a collection of atoms in three dimensions. As a matter of convenience, these atoms are considered as **points** to study the crystal structure. The representation of atoms in the crystals as points in three dimensions is known as **space lattice**.

### Lattice:

Lattice is a geometrical concept. It is defined as an array of points which are imaginarily kept to represent the position of atoms in the crystal such that every lattice point has got the same environment as that of the other and hence one lattice point cannot be distinguished from the other point.

### Lattice plane:

A set of parallel and equally spaced plane in space lattice is defined as lattice plane and is as shown in figure (5.4).

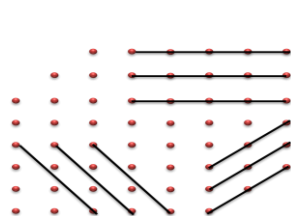


Fig 5.4

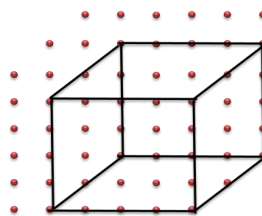


Fig 5.5

→ Lattice point  
→ Lattice line

### Lattice point:

The atom in the crystal is replaced by the point is called as lattice point and is as shown in figure (5.5).

**Lattice line:**

The lattice points are joined with the lines as shown in figure (5.5). These lines are known as lattice line.

**Basis (motif):**

The crystal structure is obtained by adding a unit assembly of atoms to each lattice point. This unit assembly is called as motif (or) basis. The number of atoms in the basis may be 1 or 2 or 3.etc and it may be go even above 1000 which are identical in composition, arrangement and orientation. Example, Aluminum and Barium has the basis of the one atom, NaCl and KCl has the basis of the two atoms and  $\text{CaF}_2$  has the basis of the three atoms.

**Crystal structure:**

When the basis is repeated in a space lattice with correct periodicity in all directions, then it gives the actual crystal structure.

Therefore, a space lattice combines with a basis gives a crystal structure.

(i.e.,) Space lattice + Basis = Crystal Structure.

**Unit cell:**

It is defined as the smallest volume of a solid from which the entire crystal structure is constructed by translation repetition in the three dimensions. The unit cell fully represents the characteristics of entire crystal. A unit cell in three dimensions is shown in fig. 5.6.

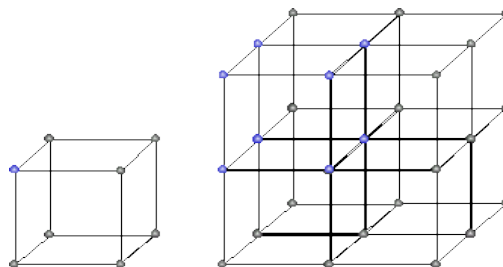


Fig. 5.6

**Lattice parameters of the unit cell:**

A unit cell is constructed if the distance between two neighbouring lattice points along three dimensions and angle between them are known.

The distance between two neighbouring lattice point is nothing but the edges of the unit cell. The lengths OA, OB, OC in three axes OX, OY and OZ are the **axial lengths or intercepts**.(fig 5.7). In fig 1.7 the axial lengths  $OA = a$ ,  $OB = b$  and  $OC = c$  are known as intercepts a, b and c along three axes. The angles between three intercepts ( $\alpha$ ,  $\beta$  and  $\gamma$ ) are

called **interfacial angles**. Therefore, the both intercepts and interfacial angles are the lattice parameters of the unit cell. They determine the actual shape and size of the unit cell.

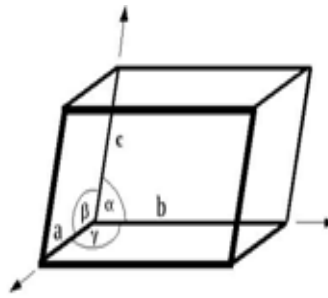


Fig 5.7

### Primitive cell & Non - Primitive cell:

A primitive cell is the simplest type of unit cell which contains only one lattice points or atom per unit cell. Example: simple cubic.

Non - primitive cell:

If there are more than one lattice points in a unit cell, it is called a non - primitive cell. Example: BCC and FCC.

## 5.7 THE CRYSTAL SYSTEMS

On the basis of lattice parameters such as intercepts or axial lengths (a, b & c) and interfacial angles (α, β and γ) the crystals are classified into 7 crystal system.

The 7 basic crystal systems are

1. Triclinic, 2. Monoclinic, 3. Orthorhombic, 4. Tetragonal, 5. Hexagonal
6. Trigonal (or) Rhombohedral, 7. Cubic

### 1. Triclinic crystal system:

In this crystal system, all three axial lengths of unit cell are not equal and all the axes are inclined obliquely to each other (fig 5.8 a).

Example: Copper Sulphate ( $\text{CuSO}_4$ ),

Potassium dichromate ( $\text{K}_2\text{Cr}_2\text{O}_7$ )

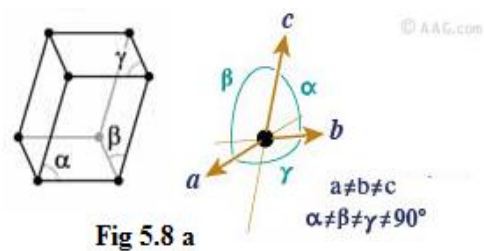


Fig 5.8 a

### 2. Monoclinic crystal system:

In this crystal system, all three axial lengths of unit cell are not equal. Two axes are perpendicular to each other and third is obliquely inclined (fig 5.8 b).

Example: Sodium Sulphate ( $\text{Na}_2\text{SO}_3$ )

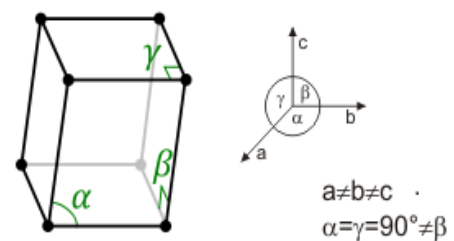


Fig 5.8 b

**3. Orthorhombic crystal system:**

In this crystal system, all three axial lengths of unit cell are not equal but they are perpendicular to each other (fig 5.8 c).

Example: Sulphur, Topaz

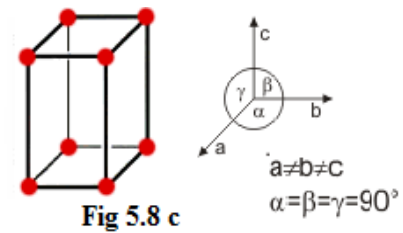


Fig 5.8 c

In this crystal system, two axial lengths of the unit cell are equal and third axial length is either longer or shorter (fig 5.8 d). but they are perpendicular to each other

Example: Ordinary white tin, Indium

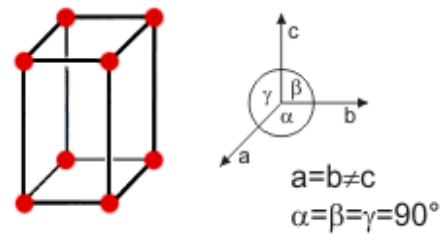


Fig 5.8 d

**4. Tetragonal crystal system:****5. Hexagonal crystal system:**

In this crystal system, two axial lengths of the unit cell are equal and lying in one plane at  $120^\circ$  with each other. The third axial length is either longer or shorter than the other two and it is perpendicular to this plane (fig 5.8 e).

Example: Quartz, Tourmaline.

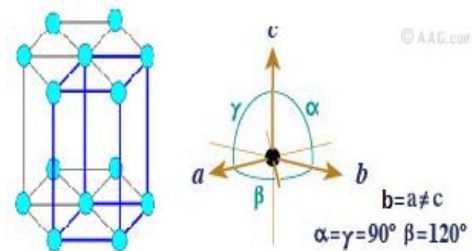


Fig 5.8 e

**6. Trigonal (or rhombohedral) crystal system:**

In this crystal system, all three axial lengths of unit cell are equal. And also they are equally inclined to each other at an angle other than  $90^\circ$  (fig 5.8 f).

Example: Calcite

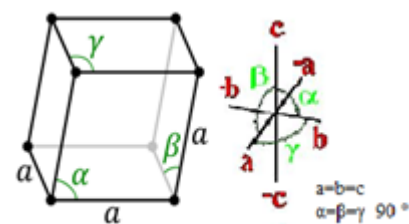


Fig 5.8 f

**7. Cubic crystal system:**

In this crystal system, all three axial lengths of unit cell are equal and they are perpendicular to each other (fig 5.8 g).

Example: Sodium Chloride (NaCl),

Calcium Fluoride (CaF<sub>2</sub>)

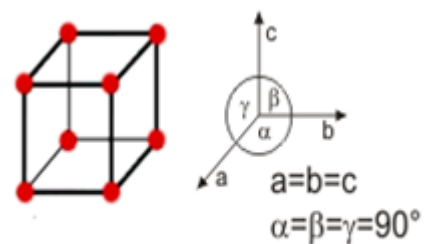


Fig 5.8 g

Table 5.1  
Seven crystal systems

S.No	Crystal systems	Axial lengths (a, b, c)	Interfacial angles ( $\alpha$ , $\beta$ , $\gamma$ )	Example
1	Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$	$\text{CuSO}_4$ , $\text{K}_2\text{Cr}_2\text{O}_7$
2	Monoclinic	$a \neq b \neq c$	$\alpha = \beta = 90^\circ$ , $\gamma \neq 90^\circ$	$\text{Na}_2\text{SO}_3$ , $\text{FeSO}_4$
3	Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	Sulphur, Topaz
4	Tetrahedral	$a = b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	Ordinary white tin, Indium
5	Hexagonal	$a = b \neq c$	$\alpha = \beta = 90^\circ$ , $\gamma = 120^\circ$	Quartz, Tourmaline
6	Trigonal (or) Rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \neq 90^\circ$	Calcite
7	Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$	$\text{NaCl}$ , $\text{CaF}_2$

### 5.8 BRAVAIS LATTICE

In 1880 Bravais introduced the concept of space lattice. He showed that there are only 14 ways of arranging points in space such that the environment looks same from each point.

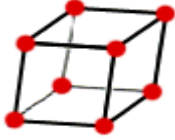
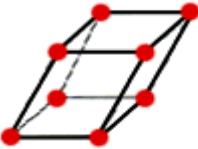
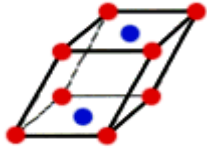

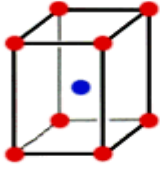
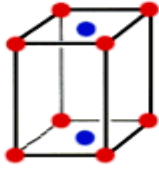
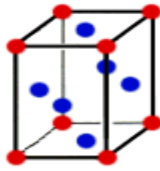
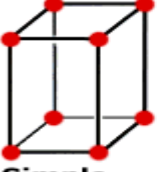
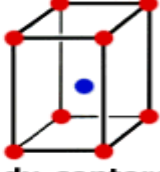
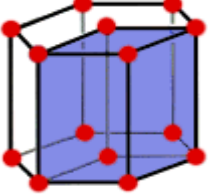

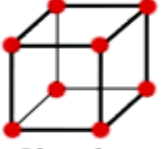
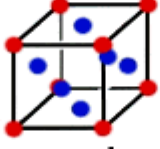
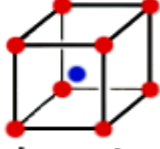
Hence, there are only 14 types of space lattices which can be possibly developed from 7 crystal systems as shown in table (5.2). These 14 types of space lattices are known as Bravais lattice.

Table 5.2  
Possible Bravais Lattice

S.No	Crystal systems	No.of possible Bravais Lattices
1	Triclinic	1
2	Monoclinic	2
3	Orthorhombic	4
4	Tetrahedral	2
5	Hexagonal	1
6	Trigonal (or) Rhombohedral	1
7	Cubic	3
Total		14

The 14 possible Bravais lattices drawn from the crystal systems are shown in table 5.3.

Table 5.3  
Bravais lattices of seven crystal systems

S.No	Crystal systems	Bravais Lattices
1	<b>Triclinic</b> ( $a \neq b \neq c$ , $\alpha \neq \beta \neq \gamma \neq 90^\circ$ )	 <b>Triclinic</b>
2	<b>Monoclinic</b> ( $a \neq b \neq c$ , $\alpha = \beta = 90^\circ$ , $\gamma \neq 90^\circ$ )	 <b>Simple Monoclinic</b>  <b>Base-centered monoclinic</b>
3	<b>Orthorhombic</b> ( $a \neq b \neq c$ , $\alpha = \beta = \gamma = 90^\circ$ )	 <b>Simple orthorhombic</b>  <b>Body-centered orthorhombic</b>  <b>Base-centered orthorhombic</b>  <b>Face-centered orthorhombic</b>
4	<b>Tetragonal</b> ( $a = b \neq c$ , $\alpha = \beta = \gamma = 90^\circ$ )	 <b>Simple tetragonal</b>  <b>Body-centered tetragonal</b>
5	<b>Hexagonal</b> ( $a = b \neq c$ , $\alpha = \beta = 90^\circ$ , $\gamma \neq 120^\circ$ )	 <b>Hexagonal</b>
6	<b>Trigonal (or) Rhombohedral</b> ( $a = b = c$ , $\alpha = \beta = \gamma \neq 90^\circ$ )	 <b>simple Rhombohedral</b>
7	<b>Cubic</b> ( $a = b = c$ , $\alpha = \beta = \gamma = 90^\circ$ )	 <b>Simple cubic</b>  <b>Face-centered cubic</b>  <b>Body-centered cubic</b>



### 5.9 MILLER INDICES

We imagine certain planes in the crystal which contains large concentration of atoms. The physical properties are dependent on concentration of atoms at planes. The orientation of the plane can be described by the intercepts of the plane with three crystallographic axes. In figure 5.9, a plane is drawn which cuts the three crystallographic directions. It has one unit in the x-axis, three units in the y axis and one unit in the z axis. The intercept values are (1,3,1).

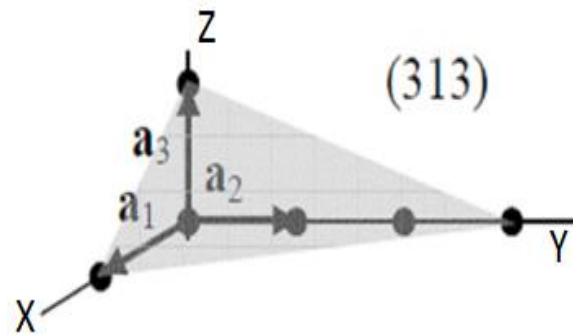


Fig 5.9

Miller suggested a better way for the representation of a plane in a crystal, and the indices are called as Miller indices.

The procedure for finding the Miller indices for a plane is illustrated as follows:

1. Determine the intercept of the plane on the three coordinate axes in terms of lattice constant. In the above example the intercepts are (1,3,1)
2. Take a reciprocal of these intercepts. In the above example the reciprocals are (1/1, 1/3, 1/1).
3. Reduce the reciprocals by whole numbers by multiplying each with LCM which gives the Miller indices of concerned plane. In the example 3,1,3
4. Enclosed these numbers in square brackets. The miller indices of the plane in the figure 1.10 are [3,1,3]

#### Example:

Find the miller indices of the following planes given in figure 5.10 (a, b, c).

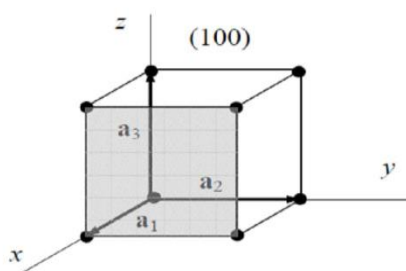


Fig 5.10 (a)

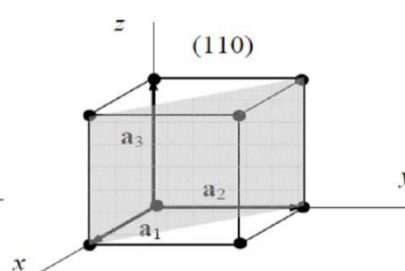


Fig 5.10(b)

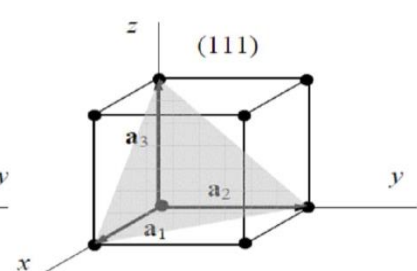


Fig 5.10(c)

1. The plane is at a distance of one unit to x- axis,  $\infty$  distance to both y and z-axes. The

intercepts are  $1, \infty, \infty$ .

2. The reciprocals are  $1/1, 1/\infty, 1/\infty$ . Hence, Miller indices are  $[1\ 0\ 0]$  shown in fig 5.10 (a).
3. The plane touches the x-axis at one unit distance-axis at one unit distance and does not touch to the z-axis. The intercepts are  $1, 1, \infty$ . The reciprocals of the intercepts are  $1/1, 1/1, 1/\infty$ . The miller indices are  $[1\ 1\ 0]$  shown in fig 5.10 (b).
4. The plane intercepts are  $1, 1, 1$  reciprocals of the intercepts are  $1, 1, 1$ . Hence the Miller indices are  $[1\ 1\ 1]$  shown in fig 5.10 (c).
5. If the intercept of a plane to certain axes are in negative direction, then a bar should be kept on the indices.

#### **Important features of Miller indices:**

1. Miller indices describe the angular position of a plane with respect to the crystallographic axes but not their actual distances from the origin.
2. All equally spaced parallel planes have the same miller indices. Thus the Miller indices do not describe the angular position of a single plane but a set of parallel planes.
3. When the plane is parallel to one of the coordinate axis, it is said to meet that axis at infinity, i.e. the length of the intercept is infinity. The reciprocal of this parameter is zero and the Miller index for that intercept is zero.
4. It is only the ratio of indices that is important, i.e.  $[8, 4, 2]$ ,  $[4, 2, 1]$  represents the same set of planes.
5. A negative index shows that the plane cuts that axes on the negative side of the origin.
6. A normal to the plane with index number  $[h, k, l]$  is the direction of  $[h, k, l]$ .

#### **Procedure for drawing the plane for given Miller indices:**

It is the reverse procedure for the finding of Miller indices when intercepts are given.

1. Take the reciprocal of Miller indices. If Miller indices are  $[h, k, l]$ , the reciprocals are  $1/h, 1/k, 1/l$ .
2. These reciprocals will give the intercepts of the plane on the three axes.
3. These intercepts are marked in the unit cell and a plane is drawn.
4. If intercept value is infinity on any axis, then the plane drawn is parallel to that axis.

### **5.10 EXPRESSION FOR DISTANCE BETWEEN SUCCESSIVE PLANES**

Studying the distance between two successive crystals planes are very imported for the structure determination and to find certain crystal properties by x-ray diffraction methods.

Let us consider a rectangular Cartesian coordinate system and O is the origin. Let

$[h,k,l]$  are the Miller indices of plane ABC, which makes intercepts OA, OB and OC on x,y,z axes respectively as shown in the figure 5.11

Let ON is the normal to this plane such that ON=  $d_1$  the distance of the plane from the origin, makes angles  $\alpha$ ,  $\beta$  and  $\gamma$  with the three axes.

$OA = a/h$ ,  $\cos \alpha = d_1 / OA$ , substitution the value of OA,  $\cos \alpha = \frac{d_1}{a/h} = \frac{hd_1}{a}$

$OB = \frac{a}{k}$ ,  $\cos \beta = \frac{d_1}{OB}$ , on substituting the value of OB,  $\cos \beta = \frac{d_1}{a/k} = \frac{kd_1}{a}$ ,  $OC = \frac{a}{l}$ ,

$\cos \gamma = \frac{d_1}{OC}$ , On substituting the value of OC

$$\cos \gamma = \frac{d_1}{a/l} = \frac{ld_1}{a}$$

As  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$  and substituting the values

$$\left(\frac{hd_1}{a}\right)^2 + \left(\frac{kd_1}{a}\right)^2 + \left(\frac{ld_1}{a}\right)^2 = 1$$

$$\left(\frac{d_1}{a}\right)^2 (h^2 + k^2 + l^2) = 1 \text{ Square rooting both}$$

$$\text{the sides } \frac{d_1}{a} \sqrt{h^2 + k^2 + l^2} = 1$$

$$\text{Or } d_1 = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad \dots \quad (4)$$

Now let us consider another plane A' B' C' which is parallel to the plane ABC which is at a distance of  $d_2$  from the origin O. Let the intercepts  $OA' = OB' = OC' = 2a$  and the normal to the plane passing through origin is OM such that  $d_2 = OM$ .

$$OA' = \frac{2a}{h}, \cos \alpha = \frac{d_2}{OA'}$$

Substituting the value of OA',

$$\cos \alpha = \frac{d_2}{2a/h} = \frac{d_2 h}{2a}$$

Similarly

$$\cos \beta = \frac{d_2 k}{2a} \text{ and } \cos \gamma = \frac{d_2 l}{2a}$$

As  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$ , substituting the values in the equation and on simplification

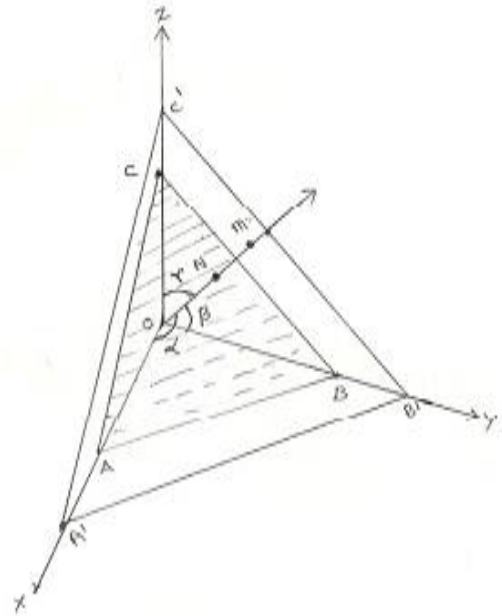


Fig5.11

$$\frac{d_2}{2a} \sqrt{h^2 + k^2 + l^2} = 1 \quad \text{or}$$

$$d_2 = \frac{2a}{\sqrt{h^2 + k^2 + l^2}} \quad \dots \quad (5)$$

The distance between two successive planes,  $d = d_2 - d_1$

From equation (4) and (5)

$$d = \frac{2a}{\sqrt{h^2 + k^2 + l^2}} - \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad \dots \quad (6)$$

### 5.11 CHARACTERISTIC OF A UNIT CELL

A unit cell is characterized by the following properties

- i) Number of atoms per unit cell
- ii) Co-ordination number
- iii) Nearest neighboring distance
- iv) Atomic radius and
- v) Packing factor or density factor

#### (i) Number of atoms per unit cell:

**It is the number of atoms possessed by a unit cell.** This can be determined if the arrangement of atoms inside the unit cell is known.

#### (ii) Co-ordination number:

**It is the number of nearest atoms directly surrounding a particular atom in a crystal.** The co-ordination number gives the information about the packing of atoms in the structure. i.e., whether the crystal structure is closely packed structure or loosely packed structure. If the co-ordination number is high, then the structure is more closely packed. If it is low, then the structure is loosely packed.

#### (iii) Nearest neighboring distance (2r):

**The distance between the centers of two nearest atoms is called nearest neighboring distance.**

It is expressed in terms of the length of edge of the unit cell 'a' and it is 2r.

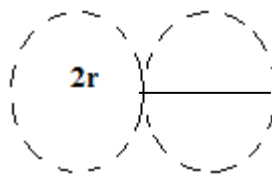


Fig 5.12

**(iv) Atomic radius (r):**

**It is half of the nearest neighbouring distance in a crystal.** It is denoted by 'r'. It is usually expressed in terms of cube edge 'a'(lattice parameter ).(Fig 5.12).

$$r = \frac{a}{2}$$

**(v) Packing factor (PF):**

**It is defined as the ratio of total volume occupied by the atoms in a unit cell to the total volume of a unit cell.**

$$\text{packing factor} = \frac{\text{Total volume occupied by the atoms in a unit cell (v)}}{\text{Total volume of the unit cell (V)}}$$

$$PF = \frac{\text{Number of atoms present in a unit cell} \times \text{Volume of one atom}}{\text{Total volume of the unit cell}}$$

A high packing factor indicates that the atoms are very closely packed and therefore there is very little unoccupied space. On the other hand, a low packing factor indicates loose packing of atoms and hence there is relatively more unoccupied space. It also called as **density of packing**.

**5.12 CRYSTAL STRUCTURES - SIMPLE CUBIC (SC) STRUCTURE**

This is one of the most common and simplest shapes found in crystals. The simple cubic unit cell is a cube (all sides of the same length and all face perpendicular to each other) with an atom at each corner of the unit cell. These corner atoms touch each other along cube edges. The figure 5.13 represents the arrangement of atoms in simple cubic crystal.

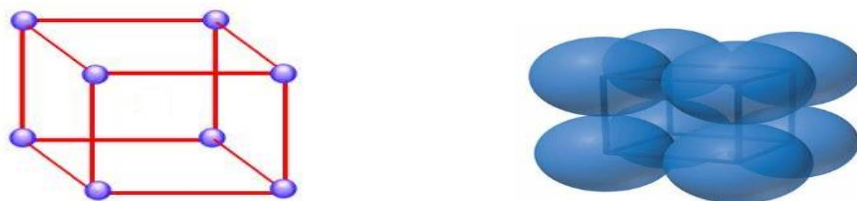


Fig 5.13 simple cubic system

**Number atoms per unit cell:**

The unit of a simple cubic structure is shown in fig 5.14. There are 8 atoms, one atom at each corner of the unit cell. Each corner atom is shared by 8 surrounding unit cell.

Therefore, Share of each unit cell =  $\frac{1}{8}$  of corner atom

$$\begin{aligned} \text{Total number of atom in unit cell} &= \frac{1}{8} \times 8 \\ &= 1 \text{ atom} \end{aligned}$$

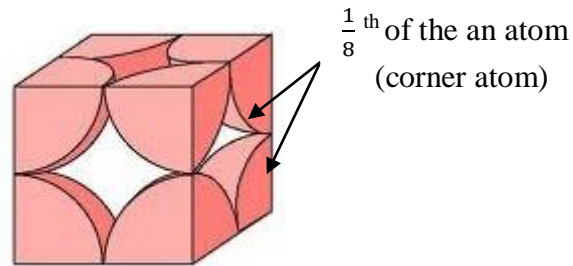


Fig 5.14 simple cubic unit cell

**Co-ordination number:**

Simple cubic unit cell has 8 corner atoms. Let us consider one of the corner atoms (say X). It is shared by 8 adjacent unit cells as shown in fig 5.15.

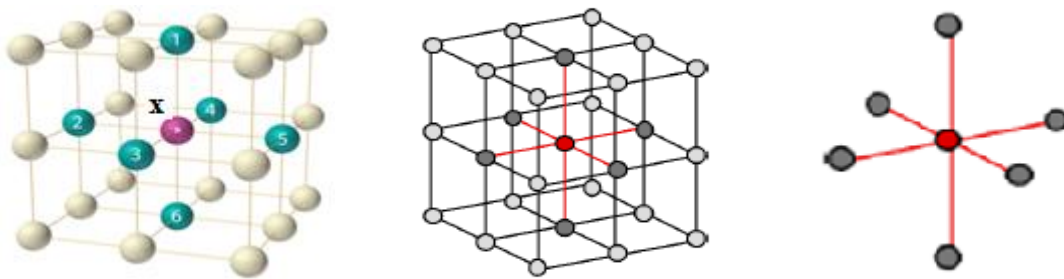


Fig 5.15

There are 4 nearest neighbouring atoms to this particular atom 'X' which are shown by 2, 3, 4 and 5 in a plane (horizontal plane). Further, there are 2 more nearest atoms, one directly above (atom 1) and the other one directly below (atom 6) to the atom X. Thus, there are only six (4+2) nearest neighbours to the atom X.

Hence, the co-ordination number for simple cubic is 6.

**Atomic radius:**

Consider a face of the unit cell of a simple cubic structure (fig 5.16). The atoms touch each other along the edges of the cube. It is clear that the distance between the centers of two nearest atoms is just equal to the cube edge 'a'.

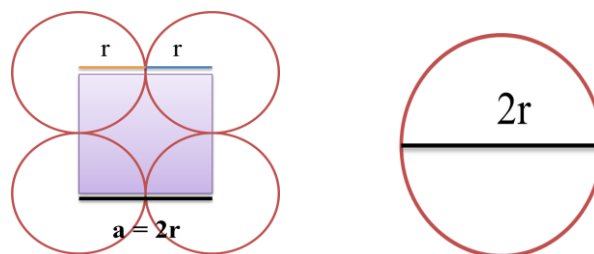


Fig 5.16

If 'a' is the side of the unit cell and 'r' its radius, then, from 5.16,

$$2r = a$$

$$r = \frac{a}{2}$$

**Atomic Packing factor:**

Number of atoms per unit cell = 1

Volume of one unit cell,  $v = \frac{4}{3}\pi r^3$

Where  $r$  is the atomic radius

Atomic radius,  $r = \frac{a}{2}$  and  $a = 2r$

Total volume of the unit cell,  $V = a^3$

We know that, Packing factor =  $\frac{v}{V}$

Substituting the values of  $v$  and  $V$  in above equation we get,

$$PF = \frac{\frac{4}{3}\pi r^3}{a^3}$$

Substituting the value of 'a' in above equation we get,

$$\begin{aligned} PF &= \frac{\frac{4}{3}\pi r^3}{(2r)^3} = \frac{\frac{4}{3}\pi r^3}{2^3 r^3} \\ &= \frac{\frac{4}{3}\pi}{8} = \frac{4\pi}{24} \\ &= \frac{\pi}{6} = 0.5236 \end{aligned}$$

$$PF = 52\%$$

Thus, 52% of the volume is occupied by the atoms and remaining 48% volume is vacant.

Example: Only one element Polonium (Po) at a certain temperature range exhibits this crystal structure.

**5.13 BODY CENTERED CUBIC (BCC) CRYSTAL SYSTEM**

The body-centered cubic unit cell is a cube (all sides of the same length and all face perpendicular to each other) with an atom at each corner of the unit cell and an atom in the center of the unit cell. These corner atoms do not touch each other but all these corner atoms touch the body center atom. The figure 5.17 represents the arrangement of atoms in BCC crystal.



Fig 5.17

**Number atoms per unit cell:**

The unit of a Body-centered cubic structure is shown in fig 5.18.

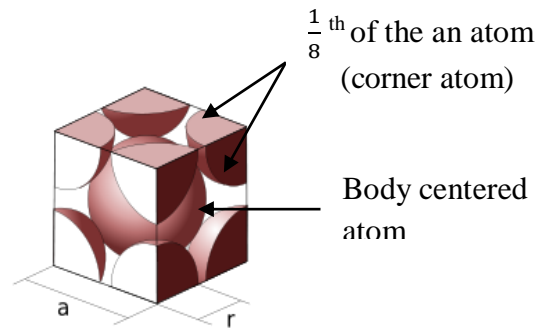


Fig 5.18 Body-centered cubic unit cell

There are 8 atoms, one atom at each corner of the unit cell. Each corner atom is shared by 8 surrounding unit cell. Therefore,

$$\text{Share of each unit cell} = \frac{1}{8} \text{ of corner atom}$$

$$\text{Total number of corner atom in unit cell} = \frac{1}{8} \times 8 = 1 \text{ atom}$$

There is one atom at the body center of every unit cell.

Therefore,

$$\begin{aligned} \text{Total number of atoms per unit cell in BCC} &= \text{Total number of corner atom in unit cell} + \text{Total number of body centered atom} \\ &= 1 + 1 \\ &= 2 \text{ atoms} \end{aligned}$$

### Co-ordination number:

In the unit cell of BCC structure, there is one atom (say atom X) at the body center of the unit cell. Further there are '8' atoms at the 8 corners of the unit cell as shown in fig 5.19.

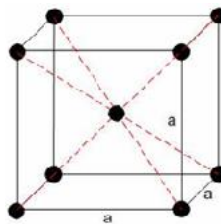


Fig 5.19

The corner atoms do not touch each other but all the '8' corner atoms touch the body center atom along the body diagonal. Thus, for body center atom 'X', there are 8 nearest neighbours (ie., 8 corner atoms). Hence, the co-ordination number of body centered cubic structure is 8.

### Atomic radius:

The corner atoms do not touch each other. However, each corner atoms touches the body centered atom along the body diagonal of the cube. The unit cell for BCC is shown in fig 5.20.



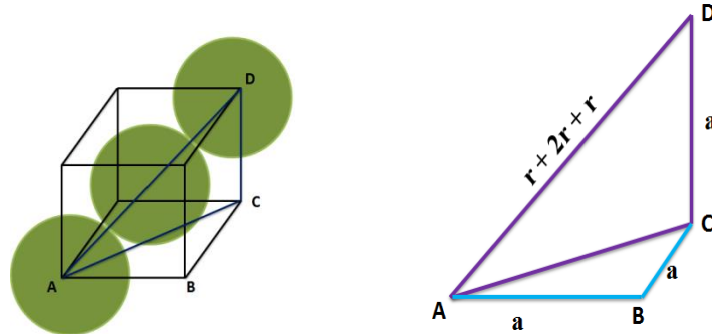


Fig 5.20

It is clear from fig 5.17 that the nearest neighbouring atoms are corner atoms A and D and the body center atom. Consider the atoms at A, D and the body center atom. These atoms lie in one straight line along the body diagonal AD of the cube.

From the right angled  $\triangle ABC$ ,

$$AC^2 = AB^2 + BC^2$$

Substituting the values for AB and BC from the fig 5.18, we have

$$AC^2 = a^2 + a^2$$

$$AC^2 = 2a^2$$

From the geometry of figure 5.18,

$$AD = r + 2r + r = 4r$$

On squaring on both sides, we get

$$AD^2 = (4r)^2$$

From the right angled triangle ACD,

$$\begin{aligned} AD^2 &= AC^2 + CD^2 \\ &= AB^2 + BC^2 + CD^2 \end{aligned}$$

Substituting the values for  $AD^2$ ,  $AB^2$ ,  $BC^2$  and  $CD^2$ , we get

$$(4r)^2 = a^2 + a^2 + a^2$$

$$16r^2 = 3a^2$$

$$r^2 = 3a^2/16$$

Taking square root on both sides, we get

$$r = \frac{a\sqrt{3}}{4}$$

**Atomic packing factor:**

Number of atoms per unit cell = 2

$$\text{Volume of 2 atoms, } v = 2 \times \frac{4}{3}\pi r^3 = \frac{8}{3}\pi r^3$$

Where r is the atomic radius

$$\text{Atomic radius, } r = \frac{a\sqrt{3}}{4} \text{ and } a = \frac{4r}{\sqrt{3}}$$

Total volume of the unit cell,  $V = a^3$

We know that, Packing factor =  $\frac{v}{V}$

Substituting the values of  $v$  and  $V$  in above equation we get,

$$PF = \frac{\frac{8}{3}\pi r^3}{a^3}$$

Substituting the value of 'a' in above equation we get,

$$\begin{aligned} PF &= \frac{\frac{8}{3}\pi r^3}{\left(\frac{4r}{\sqrt{3}}\right)^3} = \frac{\frac{8}{3}\pi r^3}{\frac{64r^3}{3\sqrt{3}}} \quad (\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{3}) \\ &= \frac{\frac{8}{3}\pi r^3}{\frac{64r^3}{3\sqrt{3}}} = \frac{8}{3}\pi r^3 \times \frac{3\sqrt{3}}{64r^3} \\ &= \frac{\sqrt{3}\pi}{8} = 0.6798 \end{aligned}$$

$$PF = 68 \%$$

Thus, 68% of the volume is occupied by the atoms and remaining 32% volume is vacant.

Example: Tungsten, Chromium and Molybdenum.

#### 5.14 FACE CENTERED CUBIC (FCC) CRYSTAL STRUCTURE

Face Center Cubic Structure consists of an atom at each cube corner and an atom in the center of each cube face. The FCC structure is shown in fig 5.21. These corner atoms do not touch each other but all these corner atoms touch the face center atom.

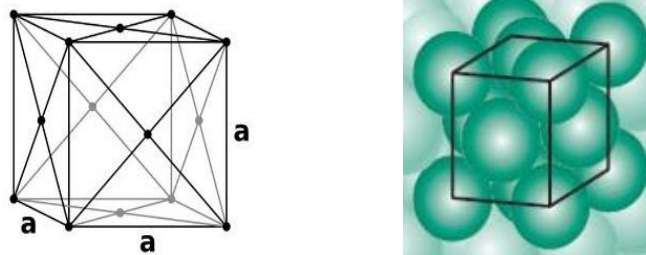


Fig 5.21

##### Number atoms per unit cell:

The unit of a Face-centered cubic structure is shown in fig 5.22. It consists of two types of atoms such as corner atoms and face centered atoms.

There are 8 atoms, one atom at each corner of the unit cell. Each corner atom is shared by 8 surrounding unit cell. Therefore,

$$\text{Share of each unit cell} = \frac{1}{8} \text{ of corner atom}$$

$$\text{Total number of corner atom in unit cell} = \frac{1}{8} \times 8 = 1 \text{ atom}$$

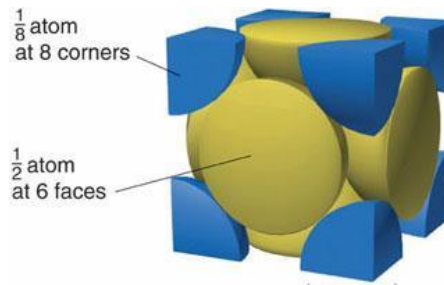


Fig 5.22 Face-centered cubic unit cell

Each face centered atom is shared by only two unit cell, which lie on either side of the atom. Similarly we have six face centered atoms in a unit cell.

Therefore,

$$\text{Total number of face centered atom in unit cell} = \frac{1}{2} \times 6 = 3 \text{ atoms}$$

Therefore,

Total number of atoms per unit cell in FCC	=	Total number of corner atom in unit cell	+	Total number of face centered atom
	=	1	+	3
		= 4 atoms		

### Co-ordination number:

To calculate the co-ordination number, for FCC, let us consider a corner atom (X) as shown in fig 5.23. In its own plane it has 4 face centered atoms (5, 6, 7, 8) as nearest neighbours. In a plane which is lie just above this corner atom, it has 4 more face centered atoms (1, 2, 3, 4) as nearest neighbours. In a plane which is lie just below this corner atom, it has 4 more face centered atoms (9, 10, 11, 12) as nearest neighbours. Therefore, the total number of nearest atoms to any corner atom is  $4+4+4=12$ . Hence, the co-ordination number for FCC is 12.

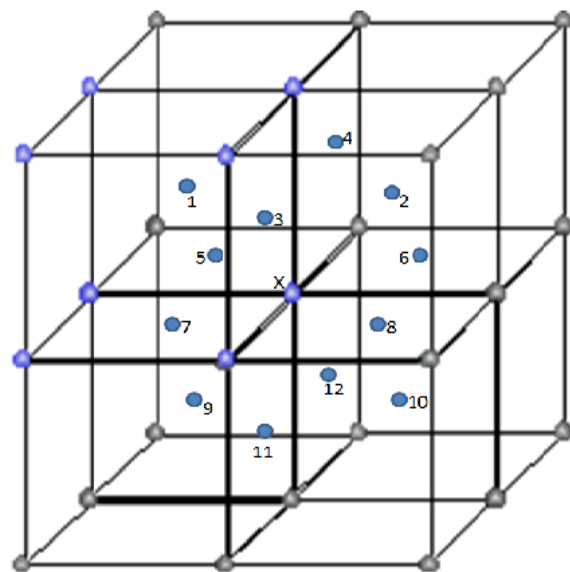


Fig 5.23

### Atomic radius:

The corner atoms do not touch each other. However, each corner atoms touches the face centered atoms along the diagonal of the face of the cube as shown in fig 5.24 (a). It is clear from fig 5.23 that the nearest neighbouring atoms are corner atoms A and C and the face

center atom. Consider the atoms at A, C and the face center atom. These atoms lie in one straight line along the face diagonal AC of the cube.

From the right angled  $\triangle ABC$ ,

$$AC^2 = AB^2 + BC^2$$

Substituting the values for AB and BC from the fig

5.24 (b), we have

$$AC^2 = a^2 + a^2$$

$$AC^2 = 2a^2$$

Taking square root on both sides, we get

$$AC = \sqrt{2}a$$

From the geometry of figure 1.20,

$$AC = r + 2r + r = 4r$$

Substituting the values for AC, we get

$$4r = \sqrt{2}a$$

$$r = \frac{\sqrt{2}a}{4} = \frac{\sqrt{2}a}{2\sqrt{2}\sqrt{2}}$$

$$r = \frac{a}{2\sqrt{2}}$$

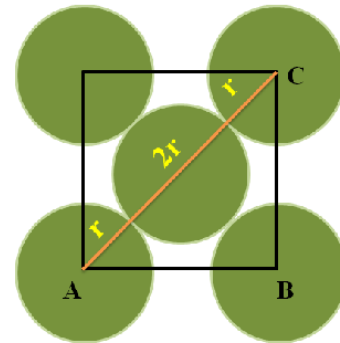


Fig 5.24 (a)

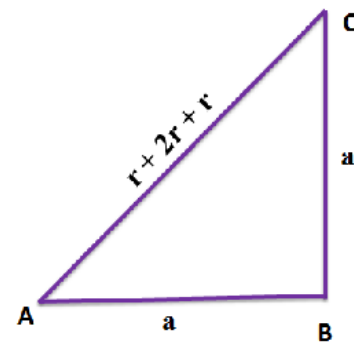


Fig 5.24 (b)

### Atomic packing factor:

Number of atoms per unit cell = 4

$$\text{Volume of 4 atoms, } v = 4 \times \frac{4}{3}\pi r^3 = \frac{16}{3}\pi r^3$$

Where r is the atomic radius

$$\text{Atomic radius, } r = \frac{a}{2\sqrt{2}} \text{ and } a = 2\sqrt{2}r$$

$$\text{Total volume of the unit cell, } V = a^3$$

We know that,

$$\text{Packing factor} = \frac{v}{V}$$

Substituting the values of v and V in above equation we get,

$$PF = \frac{\frac{16}{3}\pi r^3}{a^3}$$

Substituting the value of 'a' in above equation we get,

$$PF = \frac{\frac{16}{3}\pi r^3}{(2\sqrt{2}r)^3} = \frac{\frac{16}{3}\pi r^3}{2^3 r^3 \sqrt{2}^3} \quad (\sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2})$$

$$= \frac{\frac{16}{3}\pi r^3}{16\sqrt{2}r^3} = \frac{1}{3\sqrt{2}}\pi$$

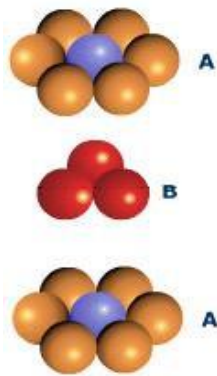
$$= 0.7402$$

$$PF = 74 \%$$

Thus, 74% of the volume is occupied by the atoms and remaining 26% volume is vacant.

### 5.15 HEXAGONAL CLOSELY PACKED STRUCTURE (HCP)

The unit cell of HCP system has 12 corner atoms, one at each and every corner of the hexagon with two base centered atoms, one at the top face of the hexagon and another at the bottom face of the hexagon as shown in figure 5.25. In addition to the corner and base atoms, there are three symmetrically arranged atoms in between the top and bottom face of the hexagon. Three additional atoms are located between top and bottom planes, which are illustrated in hard sphere model as shown in figure 5.25(a).



5.25 (a) Hard sphere model



5.25 (b) Lattice point representation

#### Number of atoms per unit cell:

The unit of HCP structure is shown in fig 5.26. In HCP, there are three types of atoms viz., (i) corner atoms (ii) base center atoms and (iii) middle layer atoms.

##### (i) Number of corner atoms per unit cell:

Each corner atom is shared by six surrounding hexagon unit cell. Similarly we have 12 corner atoms in a unit cell.

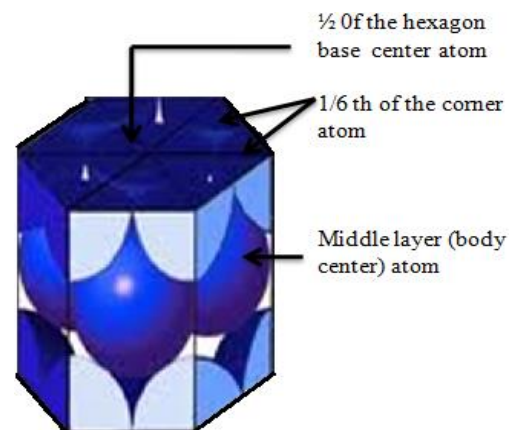


Fig 5.26

Therefore,

The number of corner atoms per unit cell

$$= \frac{1}{6} \times 12 = 2 \text{ atoms.}$$

##### (ii) Number of base centered atoms per unit cell:

Each base atom is shared by two unit cells. Similarly we have two base centered atoms in

a unit cell. Therefore, the number of base centered atoms per unit cell  $= \frac{1}{2} \times 2 = 1$  atom.

(iii) Number of middle layer atoms per unit cell:

The 3 atoms situated at the middle layer, within the body of the unit cell are fully contributing to that of the unit cell alone i.e., they are not shared by any other unit cells.

Therefore,

The number of middle layer atoms per unit cell = 3

Therefore,

The total number of atoms per unit cell in HCP structure	=	Number of corner atoms per unit cell	+	Number of base centered atoms per unit cell	+	Number of middle layer atoms per unit cell
	=	2	+	1	+	3
	=	6 atoms				

### Co-ordination number:

The HCP structure consists of three layer viz., top layer, bottom layer and middle layer as shown in fig 5.27. In the top and bottom layer, the base centered atom is surrounded by six corner atoms.

In the middle layer we have 3 atoms stacked inside the unit cell as shown in fig 5.27. Let us consider two unit cells as shown in fig 5.27. Let 'X' be the reference atom taken in the bottom layer of unit cell-1.

This atom has 6 neighbouring atom in its own plane. Further at a distance of  $c/2$  it has 3 atoms in the middle layer of unit cell-1 and 3 more atoms in the middle layer of unit cell-2. Therefore, the total number of neighbouring atoms are  $6 + 3 + 3 = 12$ .

Thus, the co-ordination number is 12.

### Atomic radius:

To find the atomic radius of the HCP structure, consider any two corner atoms. It has to be noted that, each and every corner atoms touches each other, therefore they are the nearest neighbours.

From fig 5.28, we can write,  $a = 2r$

i.e., the atomic radius is  $r = \frac{a}{2}$

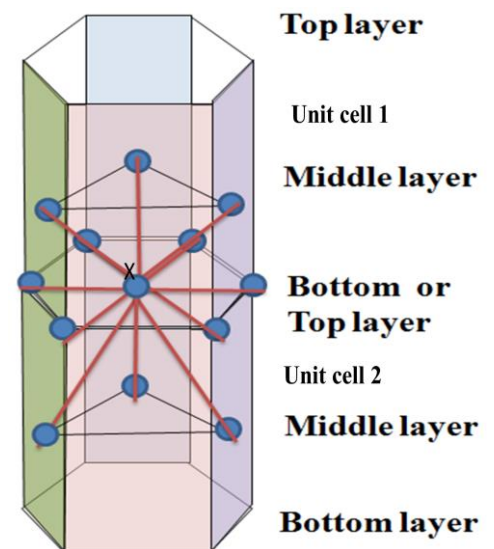


Fig 5.27

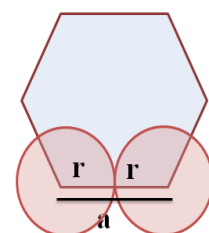


Fig 5.28

**Relation between 'c' and 'a' [c/a ratio]:**

To calculate c/a ratio, the triangle in the bottom layer of the HCP unit cell. Let 'c' be the height of the unit cell and 'a' be the distance between two nearest neighboring atoms. Consider the triangle AOC which is shown in the figure. Here, A, O and C are the atoms of the bottom layer and y is the next layer atom which lies exactly above this plane at a distance c/2.

From triangle ABC,  $\cos 30^\circ = \frac{AB}{AC}$

$$AB = AC \cos 30^\circ$$

From the fig 5.29,  $AC = a$

Substituting AC and  $\cos 30^\circ$  values, we get

$$AB = a \frac{\sqrt{3}}{2}$$

But, from the fig 5.29 (b),  $AX = (2/3) AB$

Substituting for AB, we get

$$AX = \frac{2}{3} \times \frac{a\sqrt{3}}{2} = \frac{a\sqrt{3}}{3} = \frac{a\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{a}{\sqrt{3}}$$

In the triangle AXY,  $AC^2 = AX^2 + XY^2$

Substituting AX, AC & XY from fig 5.29(a), we get

$$a^2 = \left(\frac{a}{\sqrt{3}}\right)^2 + \left(\frac{c}{2}\right)^2 = (a^2/3) + (c^2/4)$$

$$\frac{c^2}{4} = a^2 - \frac{a^2}{3}$$

$$\frac{c^2}{4} = \frac{3a^2 - a^2}{3} = \frac{2a^2}{3}$$

$$\frac{c^2}{a^2} = \frac{4 \times 2}{3} = \frac{8}{3} = \frac{2\sqrt{2}}{\sqrt{3}}$$

Taking square root on both sides,

$$\frac{c}{a} = 1.633$$

**Atomic packing factor:**

To find Volume of all atoms in a unit cell (v):

Number of atoms per unit cell = 6

$$\text{Volume of 6 atoms, } v = 6 \times \frac{4}{3}\pi r^3 = \frac{24}{3}\pi r^3 = 8\pi r^3$$

Where r is the atomic radius

$$\text{Atomic radius, } r = \frac{a}{2}$$

Substituting 'r' values, we get

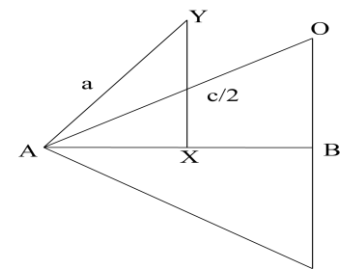


Fig 5.29(a)

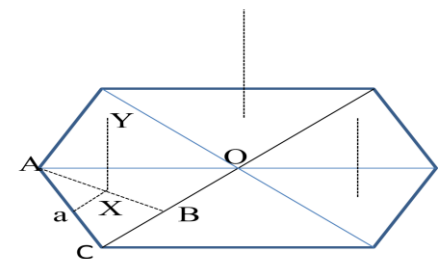


Fig 5.29(b)

$$\text{Volume of 6 atoms (v)} = 8\pi(a/2)^3 = 8\pi a^3/8$$

$$v = \pi a^3$$

Volume of the unit cell (V):

$$\text{Volume of a HCP unit cell} = \left( \text{Area of hexagonal face} \right) \times \left( \text{height of the hexagon} \right)$$

To find area of hexagon:

$$\text{Area of the base} = 6 \times \text{area of triangle ABC}$$

$$\text{Area of triangle ABC} = \frac{1}{2} \times AB \times BO$$

Substituting for  $BO = a$  and  $AB = a \frac{\sqrt{3}}{2}$ ,

$$\begin{aligned} \text{Area of triangle ABC} &= \frac{1}{2} \times a \times a \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3}a^2}{4} \end{aligned}$$

$$\therefore \text{Area of the base} = 6 \times \frac{\sqrt{3}a^2}{4} = \frac{3\sqrt{3}a^2}{2}$$

$$\text{Therefore, Volume of a HCP unit cell} = \frac{3\sqrt{3}a^2}{2} \times c$$

We know that,

$$\text{Packing factor} = \frac{v}{V}$$

Substituting the values of  $v$  and  $V$  in above equation we get,

$$\text{PF} = \frac{\pi a^3}{\frac{3\sqrt{3}a^2}{2} \times c}$$

$$\begin{aligned} \text{PF} &= \frac{\pi}{3\sqrt{3}} \times \frac{a}{c} \\ &= \frac{\pi}{3\sqrt{3}} \times \frac{1}{1.633} \\ &= 0.7402 \end{aligned}$$

$$\text{PF} = 74 \%$$

Therefore, we can say that 74% volume of the unit cell of HCP is occupied by atoms and remaining 26% volume is vacant which implies that HCP structure can be termed as tightly or closely packed structure.

**Examples:** Zinc, Titanium, and cobalt

Comparison between the simple cubic, body centered cubic, face centered cubic and hexagonal closed packed structure is given in table 5.4



Table 5.4

Sr. No.	Properties	sc	bcc	fcc	hcp
1	Volume of unit cell	$a^3$	$a^3$	$a^3$	$\frac{3}{2}\sqrt{3} a^2 c$
2	Number of atoms per unit cell	1	2	4	6
3	Number of atoms per unit volume	$\frac{1}{a^3}$	$\frac{2}{a^3}$	$\frac{4}{a^3}$	$\frac{4}{\sqrt{3} a^2 c}$
4	Number of nearest neighbour (Coordination number)	6	8	12	12
5	Nearest neighbour distance ( $2r$ )	$a$	$\frac{a\sqrt{3}}{2}$	$\frac{a\sqrt{2}}{2}$	$a$
6	Atomic radius	$\frac{a}{2}$	$\frac{a\sqrt{3}}{4}$	$\frac{a\sqrt{2}}{4}$	$\frac{a}{2}$
7	Atomic Packing Fraction (APF)	$\frac{\pi}{6} = 0.52$	$\frac{\sqrt{3}\pi}{8} = 0.68$	$\frac{\sqrt{2}\pi}{6} = 0.74$	$\frac{\sqrt{2}\pi}{6} = 0.74$

### 5.16. DIAMOND CUBIC STRUCTURE

The diamond structure is an FCC with the basis of two carbon atoms, viz., 'X' and 'Y'. The 'X' atom is located with origin of (0, 0, 0) and the 'Y' atom is located with origin of  $(\frac{a}{4}, \frac{a}{4}, \frac{a}{4})$  where 'a' is the lattice constant of the FCC lattice. The figure 5.30 illustrates the diamond structure.

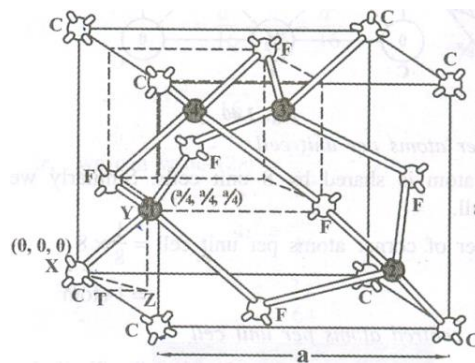


Fig 5.30 diamond structure

Thus, from fig 5.30, we can say that the diamond structure is formed due to the combination of two interpenetrating FCC sub lattices, having the origin (0, 0, 0) and  $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ , along the body diagonal.

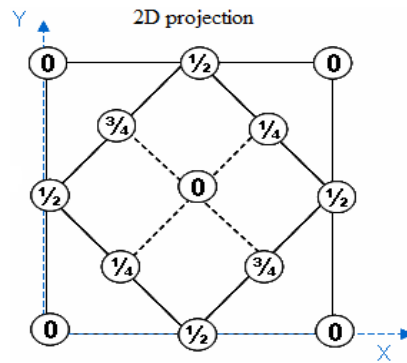


Fig 5.31 The diamond unit cell

The figure 5.31 shows a the unit cell of the diamond structure with co-ordinates of each point and the tetrahedral bonds of the nearest neighbours. The diamond unit cell has 4 tetrahedral bonds as shown in figure 5.30.

#### Number of atoms per unit cell:

The unit cell has 8 corner atoms, 6 face-centered atoms and 4 atoms inside the cube. Since each corner atom is shared by 8 adjacent unit cells and each face-centered atom is shared by 2 unit cells. Therefore,

$$\text{The number of atoms /unit cell} = \left(\frac{1}{8}\right) \times 8 + \left(\frac{1}{2}\right) \times 6 + 4 = 8$$

(Corner atoms)      (FC atoms)

#### Co-ordination number:

The co-ordination number of an atom in diamond structure is obviously 4 as can be seen from the unit cell of diamond and is loosely packed.

#### Atomic radius:

To determine the distance between nearest neighbours one needs to estimate the distance AJ as shown in the figure 5.32.

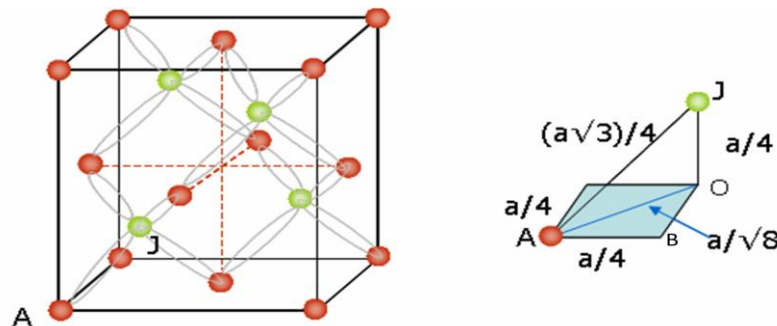


Fig 5.32

The atom at J is displaced from that at A by  $a/4$  along x-axis and  $a/4$  along Y axis, therefore

$$AO^2 = AB^2 + BO^2 = \left(\frac{a}{4}\right)^2 + \left(\frac{a}{4}\right)^2 = 2a^2/16 = a^2/8$$

Then,

$$AJ^2 = AO^2 + OJ^2 = \frac{a^2}{8} + \frac{a^2}{16} = \frac{3a^2}{16}$$

Therefore,

$$AJ = \frac{a\sqrt{3}}{4}$$

Substituting the values for AJ from the fig 5.32, we get

$$2r = \frac{a\sqrt{3}}{4}$$

Therefore,

$$r = \frac{a\sqrt{3}}{8}$$

From the atomic radius, the lattice constant becomes,

$$a = \frac{8r}{\sqrt{3}}$$

**Packing factor:**

Number of atoms per unit cell = 8

Volume of one unit cell,  $v = 8 \times \frac{4}{3}\pi r^3$

Where r is the atomic radius

$$\text{Atomic radius, } r = \frac{a\sqrt{3}}{8}$$

Total volume of the unit cell,  $V = a^3$

We know that,  $\text{Packing factor} = \frac{v}{V}$

Substituting the values of v and V in above equation we get,

$$PF = \frac{8 \times \frac{4}{3}\pi r^3}{a^3}$$

Substituting the value of 'r' in above equation we get,

$$\begin{aligned} PF &= \frac{\frac{32}{3}\pi\left(\frac{a\sqrt{3}}{8}\right)^3}{a^3} = \frac{\frac{32}{3}\pi a^3 \sqrt{3}^3}{8^3 a^3} \\ &= \frac{\frac{32}{3}\pi 3\sqrt{3}}{8^3} = \frac{32\sqrt{3}\pi}{512} \\ &= \frac{\pi\sqrt{3}}{16} = 0.34 \end{aligned}$$

$$PF = 34 \%$$

Thus, 34% of the volume is occupied by the atoms and remaining 66% volume is vacant which implies that diamond structure can be termed as loosely packed structure.

### 5.17 CRYSTAL IMPERFECTIONS

If atoms in the solid are not arranged in a perfectly regular manner, it is called defects (or) imperfections in crystals. It is observed that the crystals are rarely found to be perfect.

The effect of imperfections is found to be very important in understanding the properties of crystals. This is because, structure insensitive properties like stiffness, density

and electrical conductivity are not affected much by the presence of imperfections or defects in the crystals.

While, structure sensitive properties such as mechanical strength, ductility, crystal growth, magnetic hysteresis, dielectric strength which are of greatest technical importance are found to be greatly affected by the presence of imperfections or defects. The various types of structural imperfections or defects in crystals are classified as

1. Lattice vibrations or phonons
2. Point defects or zero dimensional defects
  - Vacancies
  - Interstitial defects
  - Self-interstitial defects
  - Schottky defect
  - Frenkel defect
3. Line defects or one dimensional defects or dislocations.
  - Edge dislocation
  - Screw dislocation
  - Dislocation climb.
  - Dislocation slip.
4. Surface defects or plane defects or two dimensional defects
  - Grain boundaries
  - Twin boundaries
  - Tilt boundaries
  - Stacking faults
5. Volume defects or three dimensional defects or bulk defects.

#### **5.17.1 LATTICE VIBRATIONS OR PHONONS**

Atoms are not stationary in their designated lattice positions, but are vibrating about their mean position. This happens as the atoms interact with one another and, the frequency of vibration depends on the temperature. These vibrations of atoms play a fundamental role in affecting electrical, specific heat and magnetic properties of materials to a greater extent.

#### **5.17.2 POINT DEFECTS**

The defects which take place due to imperfect packing of atoms during crystallization are known as point defects. As the name implies, they are imperfect point like regions in the

crystal and, therefore, they are also referred as zero dimensional imperfections. The most common point defects in a crystal are described below and are shown in fig 5.33.

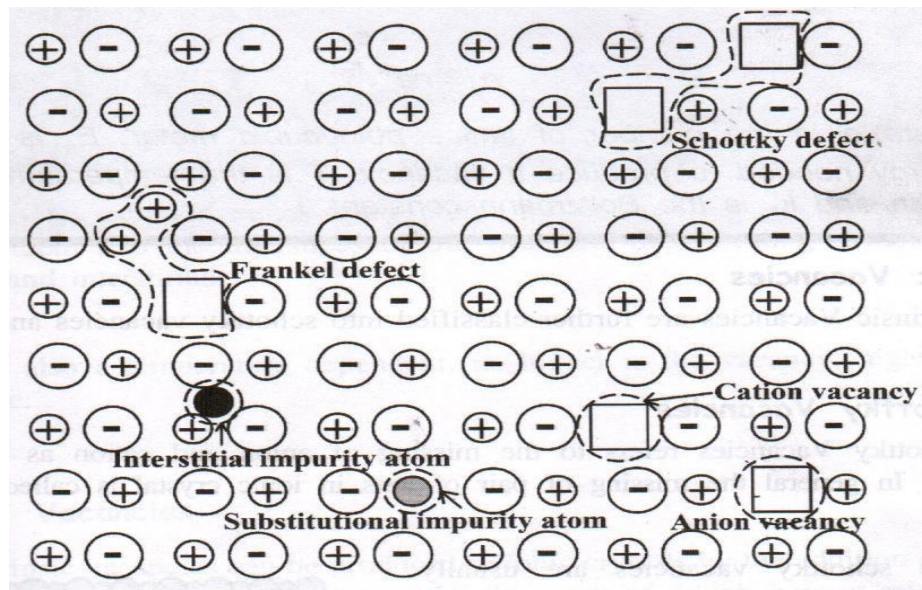


Fig 5.33

Point imperfections may be classified into two forms viz., a) Vacancies b) Impurities

#### (a) Vacancies

A vacancy is the simplest point defect in a crystal. This refers to a missing atom (or) a vacant atomic site as shown in fig. (5.34). such defects may arise either from imperfect packing density crystallization process or from thermal vibration of atoms at high temperature. When the thermal energy due to vibration is increased among atoms, it can result in a particular atom acquiring sufficient energy to escape, thereby creating a vacancy. The vacancies may be single (one atom missing) or divacancies (two atoms missing) or trivacancies (three atoms missing) and so on.

Vacancy may also occur if an atom leaves its own site and dissolved interstitially into the structure. Generally vacancies are classified into intrinsic and extrinsic vacancies. They are discussed as follows.

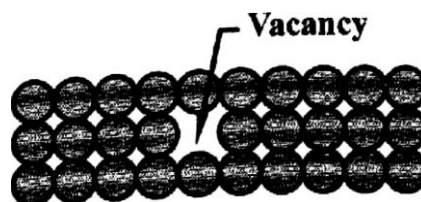


Fig 5.34

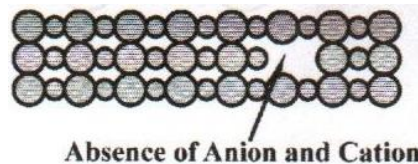
#### Intrinsic Vacancies

Intrinsic Vacancies are further classified into schottky vacancies and Frankel Vacancies.

**(i) Schottky Vacancies**

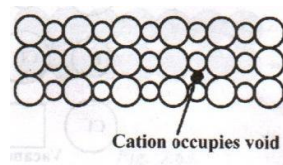
Schottky Vacancies refers to the missing of anion and cation as shown in fig 5.35. In general the missing of pair of ions in ionic crystal is called schottky defect.

The schottky vacancies are usually produced as a result of the thermal incorporation of the unoccupied lattice sites from the exterior of the crystal. The lattice undergoes thermal vibration and thermal expansion when temperature is raised above 0K. This shows that the vacancies may be increased by increasing temperature.

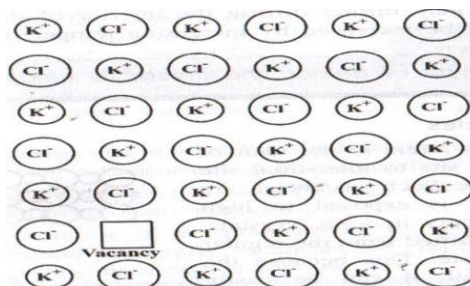
**Fig 5.35****(ii) Frankel Vacancies**

Frankel vacancies refer to the shift of cation from the regular site to interstitial site as shown in the fig 5.36. Such vacancies are produced when crystal is exposed to high radiation. It always occurs in ionic crystals. Normally the cation is shifted from the regular site to the interstitial site. This because the cation being very small, it can be easily accommodated in the void space.

It is also temperature dependent i.e., higher is the vacancy, higher is the temperature.

**Fig 5.36****Extrinsic Vacancies**

Extrinsic vacancies can be produced in an ionic crystal by the addition of proper type of impurities, if we add these molecules each of which have a missing ion in the crystal environment.

**Fig 5.37**

For example when  $\text{CaCl}_2$  is added with  $\text{KCl}$ , it has a missing positive ion since there are two  $\text{Cl}^{-1}$  ion for every  $\text{Ca}^{2+}$  ion as shown in fig 5.37.

### (b) Impurities

Point defects may also occur due to the addition of impurity atom. Impurity atom may fit into the crystal structure by two ways, viz.,

- It may occupy a position normally occupied by the crystal atom [substitutional impurity]
- It may stay in the normally unfilled volume [interstitial impurity]

#### Substitutional impurities

When impurity atom replaces the parent atom in the atomic lattice, substitutional impurities are formed. Here the impurity atoms have almost the same size and valency as that of host atoms. Substitutional impurity cannot exist in ionic and covalent crystals. Zn is a substitution for the copper atom in case of brass formation as shown in fig (5.38)

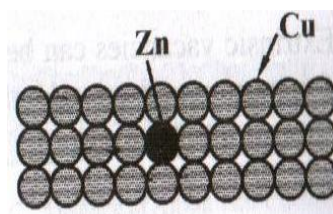


Fig 5.38

#### (ii) Interstitial impurities

Interstitial impurity is created when impurity atom occupies the interstitial position (void) without replacing the parent atom in the crystal. These can exist only ionic and covalent crystals. Here the impurity atoms have very small size as that of the host atoms. The presence of carbon in iron is an example for this defect as shown in fig (5.39).

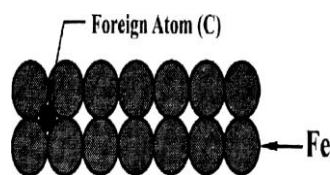


Fig 5.39

#### Applications of point defects

1. If there is any point defect in a solid, through which some impurities can be added so that to increase the hardness of a solid. For example, surface hardness of the low carbon steels and alloy steels is increased by addition of carbon and nitrogen atoms which act as point defects.



2. Similarly, Ductility of a solid can be increased. For example, copper atoms are added with gold so that gold can be drawn into ornaments.
3. Bearing properties of copper can be increased. For example, in a copper lattice, low melting point tin atoms behave as a substitutional impurity so that to increase the bearing properties of copper.

### 5.17.3 LINE DEFECTS

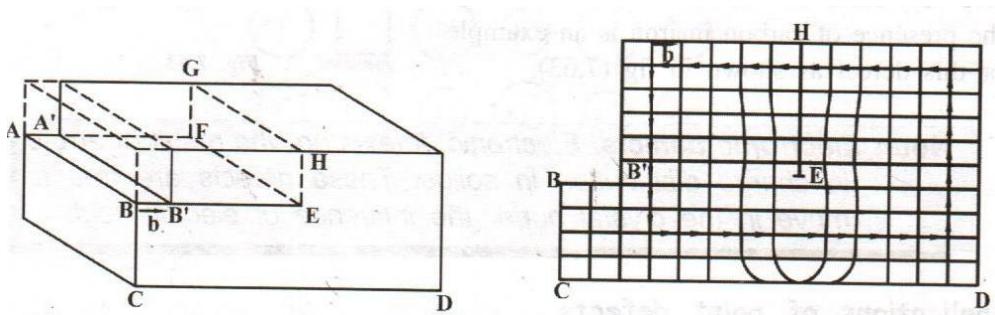
The defects which take place due to dislocations or distortion of atoms along a line in some direction is called line defect.

The dislocation is a boundary between the slipped region and the unslipped region and lies in the slip plane. The structure and behaviour of dislocations affect many of the properties of engineering materials. This is nothing but one part of the crystal shifts or slips relative to the rest of the crystal. The imperfections are formed due to solidification of metals. Dislocations are generally classified into four types.

They are (a) Edge dislocations (b) Screw dislocations (c) Dislocation slip and (d) Dislocation climb.

#### (a) Edge Dislocation

The geometry of edge dislocation may be understood by slip process as shown in fig 5.40.



**Fig 5.40**

Let the crystal be cut across the area ABEF, so that the "lower and upper parts are disconnected. Let the upper half is pushed sideways, i.e., line A' B' coincides with AB and is shifted by an amount  $\vec{b}$ . Finally the two halves are glued (joined quickly) together in this position. This operation is compared with realistic operation which might actually occur in crystals, as discussed below.

The upper half of the left side is subjected to a shearing stress. The result of this operation is also to shift the upper half of the left side with respect to the rest of the crystal. The



slip process introduces an extra line of atoms in the upper half of the network which corresponds to the extra plane of atoms in the crystal.

The extra plane of atoms terminates along the edge in the plane between the slipped and unslipped regions. This edge is the boundary along with the displacement of the slipped region over the unslipped region is terminated and constitutes a dislocation in the crystal.

This edge dislocation is centered along the edge and the edge is a dislocation line. The dislocation line runs indefinitely in the slip plane in the direction normal to it. It is perpendicular to the paper through the point E, i.e., EF is the dislocation line.

The displacement of atoms around the dislocation is called the slip or Burger's vector  $\vec{b}$ .

The extra plane of atoms lying above the slip plane is called positive edge dislocations and those lying below the slip plane are called negative edge dislocations. Symbols T and I are used for positive and negative edge dislocations.

#### (b) Screw dislocation

Screw dislocation results from a displacement of the atoms in one part of a crystal relative to the rest of crystal forming a spiral ramp around the dislocation line.

The geometry of the screw dislocation may be understood by slip process as shown in Fig. 5.41 and 5.42. Let a sharp cut be made partway through a perfect crystal. Let the crystal on one side of the cut displaced relative to the other side by an amount  $b$  parallel to the cut. Finally let the rows of atoms are placed back into contact and the structure is glued in this position.

When the atoms are displaced in two separate planes perpendicular to each other, the imperfection produced is called screw dislocation.

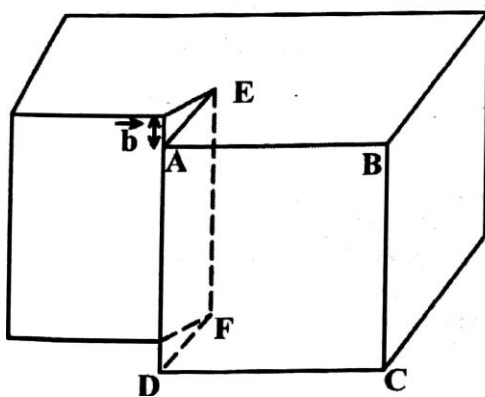


Fig. 5.41

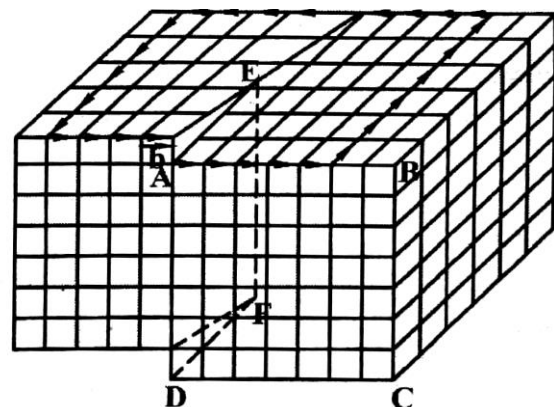


Fig. 5.42

A more realistic operation which might actually occurs in crystals consists in subjecting the front half of the right side of the crystal to a shearing stress but keeping the

left side stationary. Fig. 5.42 shows that the network of lines drawn inside the structure Fig. 5.41 only after the operation.

From the figures, the displacement in this case terminates in the crystal along a low EF of atoms, and consequently constitutes a dislocation in the crystal. This is screw dislocation and EF is the dislocation line. It should be noted that, this dislocation line runs along the cut and is parallel to the slip direction. The name "screw" follows from the new character of the atomic planes which are transferred. Such that as one moves around the dislocation line in a circuit. This is as shown in Fig. 5.42.

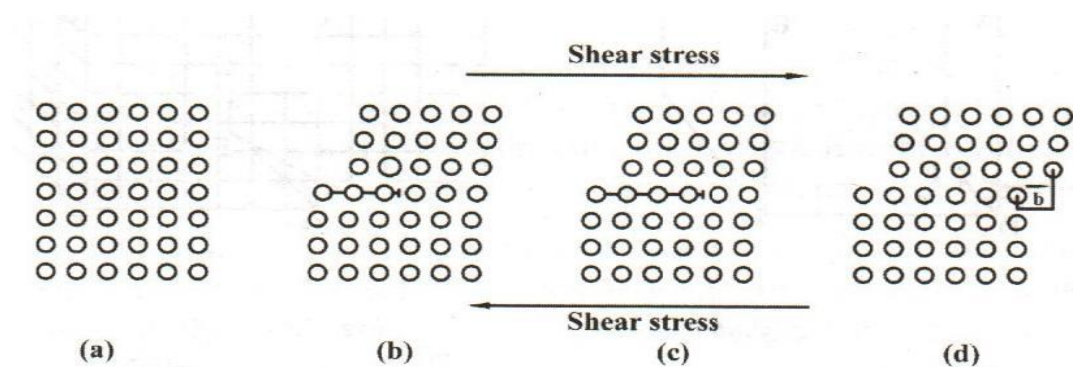
It is clear that, from the Fig. 5.41 and 5.42, the vector  $b$  describes the direction as well (as the magnitude of the slip). The vector  $b$  which is closely related to the dislocations in the crystal is called the "**Burger vector**", named after Burger's whose work on dislocations.

The burger vector and the screw dislocation line are parallel to each other. It is represented by the symbol  $\pm$  and  $\mp$  when the burger vector and dislocation line are in the same direction or opposite direction.

### (c) Dislocation slip

Slip is the relative shift of one part of the crystal with respect to the rest of crystal under the influence of applied stress.

Fig. 5.43 (a) represents a piece of perfect crystal free from dislocations. Fig. 5.43 (b) represents the same piece when it contains one edge dislocation which is under the action of stress. Fig. 7.68 (c) shows that the edge dislocation moves from one lattice site to another under the influence of shear stress.

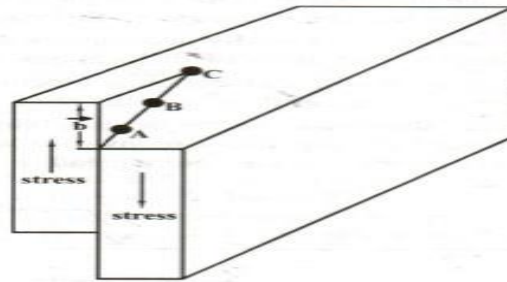


**Fig. 5.43**

It should be noted that the extra plane of atoms in this process does not physically move from one site to another. Suppose the dislocation is over from one

surface to another, we can get new structure as shown in Fig. 5.43(d) this figure represents the top of the crystal slips with respect to the bottom.

Slip is caused due to the motion of screw dislocation. From the Fig. 5.44, the dislocation moves from A to B to C, the slip of one part of the crystal over the other proceeds and finally result, in the complete displacement of one portion with respect to the rest by an amount equal to the Burger vector when the dislocation moves from one surface to other.



**Fig. 5.44**

It can be seen that the screw dislocation alters the shape of the crystal, it is quite different when compared edge dislocation. This indicates that the slip may occur due to screw dislocation.

For example, let a crystal be deformed due to the application stress.

If the crystal regains its original state upon the removal of stresses, the deformation is called "elastic deformation".

If the crystal does not regain its original state even after the removal of stresses, if it retains a certain amount of deformation, it is called "**plastic deformation**".

Plastic deformations are somewhat related with dislocation, which may cause slip. Dislocations also promote the growth of crystals. The screw dislocations Provide steps of atoms and the stacking of atoms, the crystal will grow around these steps. The strength of single crystals increases when the dislocation motion is impeded.

#### **(d) Dislocation climb**

Atomic movements permit the dislocation to climb at high temperatures.

Simple climb of edge dislocation depending on the diffusion of the vacancies either toward edge dislocation or away from edge dislocation.

Obstacles are used to stop dislocations in the glide plane to form piled-up groups. The rate of creep is governed by the rate of escape of dislocations past the obstacles.

The deformations in the metal is initially accomplished by slip, but the factor controlling the amount of slip is the climb of dislocations over obstacles.

Fig. 5.45 (a) shows that the sessile dislocation is blocking the motion of other dislocations in its slip plane and the result is a dislocation pile-up.

Fig. 5.45 (b) shows that the climb of the leading slip dislocation over the sessile dislocation has permitted all slip dislocations to advance and deformation by slip to continue. Here the dislocations are spread more evenly along the boundary after climb is over.

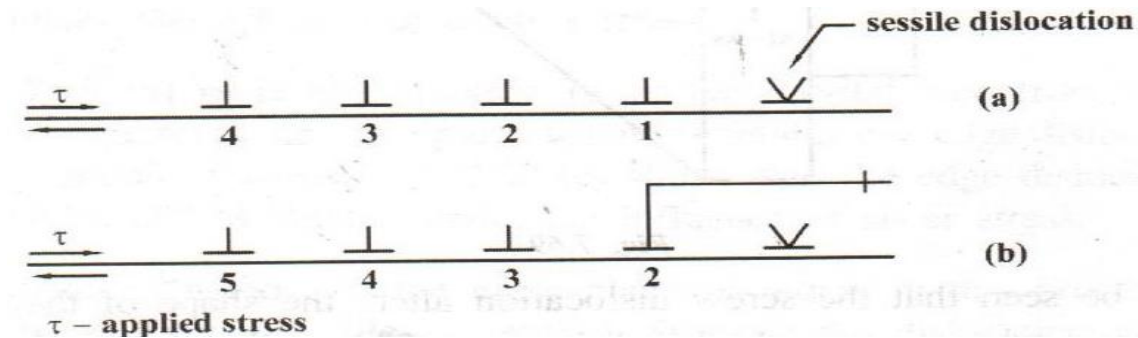


Fig. 5.45

### PROBLEMS

1. A crystal plane cut at 3a, 4b and 2c distances along the crystallographic axes. Find the Miller Indices of the plane.

**Given data :**

$$\text{Intercepts} = 3a : 4b : 2c$$

**Solution**

$$\text{Step (i) : Co-efficients of intercepts} = 3 : 4 : 2$$

$$\text{Step (ii) : Reciprocal of intercepts} = \frac{1}{3} : \frac{1}{4} : \frac{1}{2}$$

$$\text{Step (iii) : LCM} = 12$$

Step (iv) : Multiplying by LCM with the reciprocals

$$12 \times \frac{1}{3} : 12 \times \frac{1}{4} : 12 \times \frac{1}{2}$$

We have 4 3 6

Miller Indices = (4 3 6)
--------------------------

2. Calculate the value of d-spacing for (100) planes in a rock salt crystal of  $a = 2.814 \text{ \AA}$ .

**Givendata:**

$$a = 2.814 \text{ \AA}$$

$$h = 1$$

$$k = 0$$

$$l = 0$$

**Solution:**

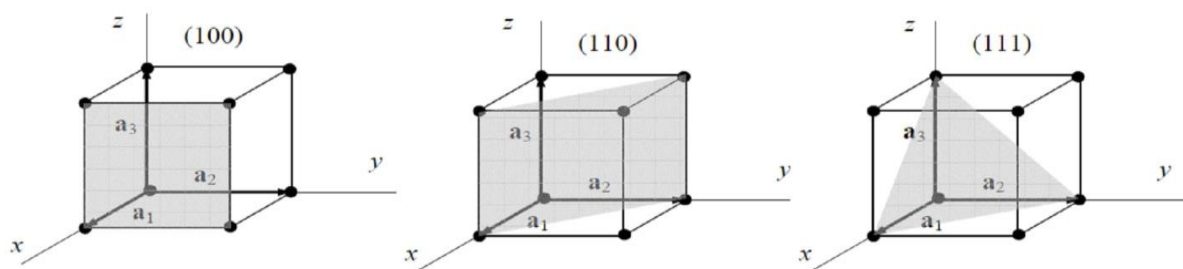
$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$d = \frac{2.814 \times 10^{-10}}{\sqrt{(1^2 + 0^2 + 0^2)}}$$

$$d = 2.814 \text{ \AA}$$

3. Draw the following planes in a cubic structure.

(100), (110) and (111)



4. The interplanar spacing is 1.3 Å. The first order of Bragg's reflection is located at 23°. Calculate the wavelength of X-ray.

**Given data:**

$$d = 1.3 \text{ \AA}$$

$$\theta = 23^\circ$$

$$n = 1$$

$$\lambda = ?$$

**Solution:**

$$n\lambda = 2d\sin\theta$$

$$\lambda = \frac{2d\sin\theta}{n}$$

$$\lambda = \frac{2 \times 1.3 \times 10^{-10} \times \sin 23^\circ}{1}$$

$$\lambda = 1.015 \times 10^{-10} \text{m}$$

(or)

$$\lambda = 1.015 \text{ \AA}$$

5. Sodium is a BCC crystal. Its density is  $9.6 \times 10^2 \text{ Kg/m}^3$  and atomic weight is 23.

Calculate the lattice constant for sodium crystal.

**Given data:**

$$\text{Atomic weight (A)} = 23$$

$$\text{Density } (\rho) = 9.6 \times 10^2 \text{ Kg/m}^3$$

$$\text{For BCC, (n)} = 2$$

$$\text{Lattice constant (a)} = ?$$

$$\text{Avagadro Number (N)} = 6.023 \times 10^{26}$$

**Solution:**

$$a^3 = \frac{nA}{N\rho}$$

$$a^3 = \frac{2 \times 23}{6.023 \times 10^{26} \times 9.6 \times 10^2}$$

$$a^3 = (7.955 \times 10^{-30})$$

$$a = \sqrt[3]{(7.955 \times 10^{-30})}$$

$$\mathbf{a = 1.996 \text{ \AA}}$$

6. A crystal of BCC structure has atomic radius  $1.2 \text{ \AA}$ . Find the volume of its unit cell.

**Given data:**

$$\text{Radius (r)} = 1.2 \times 10^{-10} \text{m}$$

**Solution:**

$$\text{Lattice constant of BCC structure, } a = \frac{4r}{\sqrt{3}}$$

$$a = \frac{4 \times 1.2 \times 10^{-10}}{\sqrt{3}}$$

$$\mathbf{a = 2.771 \times 10^{-10} \text{m}}$$

$$\text{Volume of the cell, } a^3 = (2.771 \times 10^{-10})^3 \text{ m}^3$$

$$a^3 = 2.128 \times 10^{-29} \text{ m}^3$$

- 7. Copper has FCC structure and its lattice parameter is 3.6 Å. Find the atomic radius.**  
**Given data:**

$$\text{Lattice parameter of copper (a) = 3.6 Å}$$

**Solution:**

$$\text{Atomic radius of copper, } r = \frac{a\sqrt{2}}{4}$$

$$r = \frac{3.6 \times 10^{-10} \times \sqrt{2}}{4} \text{ m}$$

$$r = 1.273 \times 10^{-10} \text{ m}$$

$$r = 1.273 \text{ Å}$$

- 8. Magnesium has HCP structure. The radius of magnesium atom is 0.1605 nm. Calculate the volume of the unit cell of magnesium.**

**Given data:**

$$\text{Radius of magnesium, } r = 0.1605 \text{ nm}$$

**Solution:**

$$\text{For HCP, the lattice constant, } a = 2r$$

$$a = 2 \times 0.1605 \times 10^{-9} \text{ m}$$

$$a = 0.321 \times 10^{-9} \text{ m}$$

Also,

$$c/a = \sqrt{8/3}$$

$$c = 0.321 \times 10^{-9} \sqrt{8/3}$$

$$c = 0.5242 \times 10^{-9} \text{ m}$$

$$\text{Volume of unit cell, } V = \frac{3\sqrt{3} a^2 c}{2}$$

$$V = \frac{3\sqrt{3} (0.321 \times 10^{-9})^2 (0.5242 \times 10^{-9})}{2}$$

$$V = 1.409 \times 10^{-28} \text{ m}^3$$

**PART A****1. What is a crystal? (or) What are crystalline materials? Give examples.**

Crystals (or) crystalline materials are those in which the constituent atoms (or) molecules are arranged in an orderly fashion throughout, in a three dimensional pattern.

Example: copper, silver.

**2. List out the differences between Crystalline and Non – Crystalline material.**

S. No	Crystalline Material	Non – Crystalline Material (or) Amorphous material
1.	They have a definite and regular geometrical shapes which extend throughout the crystal	They do not have definite and regular geometrical shape
2.	They are anisotropic	They are isotropic
3.	They are most stable	They are less stable
4.	They have sharp melting point	They do not have sharp melting point
5.	Examples: NaCl, KCl	Examples: Glasses, Rubber

**3. Define single crystal and poly crystal.****Single crystal:**

The crystalline solid which contains only one crystal, it is called single crystal.

**Poly crystal:**



The polycrystalline materials are collection of many crystals (or) grains separated by well defined grain boundary.

**4. Define Crystallography.**

The study of geometric form and other physical properties of crystalline solids, using X- rays (or) electron beam (or) neutron beam etc., is termed as the science of crystallography.

**5. Define Lattice.**

Lattice is defined as an array of points which are imaginarily kept to represent the position of atoms in the crystal such that every lattice point has got the same environment as that of the other.

**6. Define Space lattice (or) Crystal lattice?(May 2004, June 2005)**

A three dimensional collection of points in space is called a space lattice or crystal lattice.

**7. Define the following terms.**

**(i) Lattice point (ii) Lattice line**

**Lattice point:**

The atom in the crystal is replaced by the point is called lattice point and is as shown in figure 1.

**Lattice line:**

The lattice points are joined with the lines and these lines are known as lattice lines as shown in figure 1.

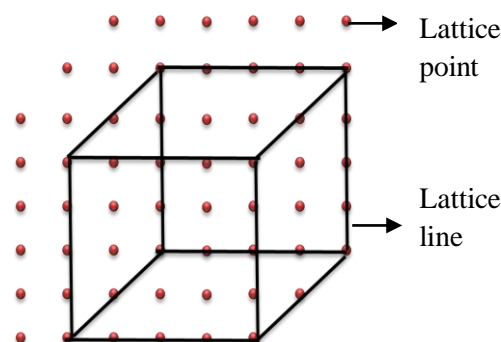


Fig. 1

**8. What is a Lattice plane?**

A set of parallel and equally spaced plane in space lattice is defined as lattice plane.

**9. What is a Basis (or) Motif?**

A unit assembly of atoms or molecules identical in composition, arrangement and orientation is called Basis or Motif.

**10. What is a crystal structure?**

When the basis is repeated in a space lattice with correct periodicity in all directions, then it gives the actual crystal structure. Therefore, a space lattice combined with a basis gives a crystal structure.

(i.e.,) Space lattice + Basis = Crystal Structure.

**11. Define Unit cell? (Jan. 2011, Jan. 2012)**

Unit cell is defined as the smallest volume of a solid from which the entire crystal structure is constructed by translational repetition in three dimensions. The unit cell fully represents the characteristics of the entire crystal.

**12. What is a Primitive cell? Give example. (Jan. 2010, Jan. 2012)**

A primitive cell is the simplest type of unit cell which contains one lattice point per unit cell.

**Example:** SC

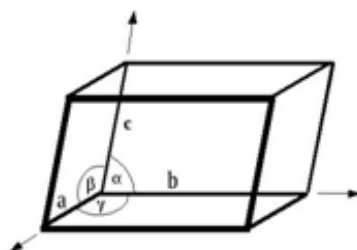
**13. What is Non-Primitive cell? Give examples. (Jan. 2010)**

The unit cell which contains more than one lattice point per unit cell is called Non-Primitive cell.

**Example:** BCC, FCC

**14. What are lattice parameters (or) unit cell parameters? (Jan. 2013)**

The intercepts (or) axial lengths  $a$ ,  $b$  and  $c$  and the interfacial angles  $\alpha$ ,  $\beta$  and  $\gamma$  along the three axes of a unit cell are called lattice parameters (or) unit cell parameters.



**15. Name the seven crystal systems. (Jan. 2010)**

On the basis of lattice parameters such as intercepts or axial lengths ( $a$ ,  $b$  &  $c$ ) and interfacial angles ( $\alpha$ ,  $\beta$  and  $\gamma$ ) the crystals are classified into seven crystal systems. The seven basic crystal systems are Triclinic, Monoclinic, orthorhombic, tetragonal, hexagonal, trigonal and cubic crystal systems.

**16. What are Bravais lattices? (Jan. 2009, Jan. 2013)**

There are 14 possible ways of arranging points in space lattice such that, all the lattice points have exactly the same surroundings. These 14 lattices are called Bravais lattices.

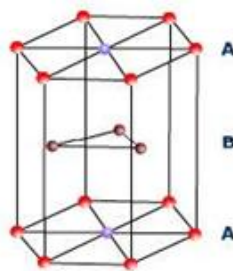
**17. What are the different types of cubic crystal system?**

Cubic crystal system:

- Simple cubic system (SC)
- Body centered cubic crystal system (BCC)
- Face centered cubic crystal system (FCC)

**18. What is meant by closed packed structure? ( Jan. 2010)**

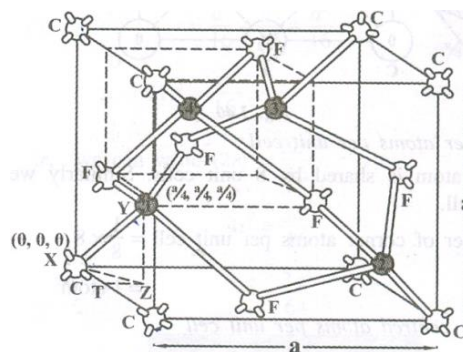
**HCP structure:**



**Explanation:**

The unit cell of HCP system has 12 corner atoms, one at each and every corner of the hexagon with two base centered atoms, one at the top face of the hexagon and another at the bottom face of the hexagon as shown in figure. In addition to the corner and base atoms, there are three symmetrically arranged atoms in between the top and bottom face of the hexagon and they are called middle layer atoms.

**19. Draw the structure of Diamond.**



**20. What are the characteristic of a unit cell? (or) What are the parameters used to describe a crystal structure?**

A unit cell is characterized by the following parameters

- vi) Number of atoms per unit cell
- vii) Co-ordination number

- viii) Nearest neighboring distance
- ix) Atomic radius and
- x) Packing factor or density factor

**21. Define Atomic Radius. (Jan. 2011)**

Atomic Radius is defined as half of the distance between any two nearest neighbouring atoms which have direct contact with each other, in a crystal of pure element. It is denoted by 'r'.

**22. Define Co-ordination number. (Dec. 2010)**

Co-ordination number is the number of nearest neighbouring atoms to a particular atom in a crystal system. The co-ordination number gives the information about the packing of atoms in the structure. i.e., whether the crystal structure is closely packed structure or loosely packed structure. If the co-ordination number is high, then the structure is more closely packed. If it is low, then the structure is loosely packed.

**23. Define Atomic Packing Factor (or) packing density. (Jan 2011)**

Atomic Packing Factor (or) packing density is defined as the ratio of total volume occupied by the atoms in a unit cell to the total volume of a unit cell.

$$\text{packing factor} = \frac{\text{Total volume occupied by the atoms in a unit cell } (v)}{\text{Total volume of the unit cell } (V)}$$

$$APF = \frac{\text{Number of atoms present in a unit cell} \times \text{Volume of one atom}}{\text{Total volume of the unit cell}}$$

**24. Define Number of atoms per unit cell (or) Effective number.**

The total number of atoms present in (or) shared by an unit cell is known as number of atoms per unit cell (or) Effective number. This can be determined if the arrangement of atoms inside the unit cell is known.

**25. State the values of Number of atoms/unit cell, atomic radius, co-ordination number and APF for SC, BCC, FCC and HCP crystal systems.**

Sr. No.	Properties	sc	bcc	fcc	hcp
1	Number of atoms per unit cell	1	2	4	6
2	Number of nearest neighbour (Coordination number)	6	8	12	12
3	Atomic radius	$\frac{a}{2}$	$\frac{a\sqrt{3}}{4}$	$\frac{a\sqrt{2}}{4}$	$\frac{a}{2}$
4	Atomic Packing Fraction (APF)	$\frac{\pi}{6} = 0.52$	$\frac{\sqrt{3}\pi}{8} = 0.68$	$\frac{\sqrt{2}\pi}{6} = 0.74$	$\frac{\sqrt{2}\pi}{6} = 0.74$

**26. Write the atomic radius, co-ordination number for diamond structure? (Jan. 2009)**

- Atomic radius for diamond structure is  $(a\sqrt{3})/8$
- Co-ordination number is 4

**27. What are miller indices?(Jan. 2009, Jan. 2010, Jan. 2011 )**

Miller indices are the smallest possible integers which have the same ratios as the reciprocals of the intercepts of the plane concerned on the three axes.

It is represented as (h k l).

**28. Give any four important features of Miller indices.**

1. Miller indices describe the angular position of plane with respect to crystallographic axes.
2. All equally spaced parallel planes have same miller indices.
3. When the plane is parallel to one of the coordinate axis, it is said to meet that axis at infinity, i.e. the length of the intercept is infinity. The reciprocal of this parameter is zero and the Miller index for that intercept is zero.
4. A negative index shows that the plane cuts that axes on the negative side of the origin.

**29. Define inter planar distance (or) d spacing? (Jan. 2012)**

The distance between any two successive planes is called inter planar distance.

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

**30. Give the methods of crystal growth.**

Single crystals may be produced by the transport of crystal constituents in the solid, liquid or vapour phase. On the basis of this, crystal growth may be classified into three categories as follows,

- Growth from solution
- Growth from melt

- Growth from vapour

**31. What are the methods that are employed in solution growth method?**

Materials, which have high solubility and have variation in solubility with temperature, can be grown easily by solution method. There are two methods in solution growth depending on the solvents and the solubility of the solute.

They are

- High temperature solution growth and
- Low temperature solution growth

**32. Define Gel.**

A gel is defined as a semi solid having high viscosity. It is a porous material comprised of a semi rigid network.

**33. What are types of various gel growth methods?**

The various gel methods to grow single crystals may be classified into three categories.

- Reaction method
- Complex dilution method
- Reduction of solubility method

**34. What is meant by hydrothermal growth?**

Crystals of materials which don't have sufficient solubility in water or other solvents at normal temperatures and pressures may be grown under hydrothermal conditions i.e, at high temperatures and pressures are called hydrothermal growth.

**35. Define Flux growth.**

The crystal growth, carried out by using high temperature solvent, usually molten salts also called flux is called flux growth.

**36. What are the various types of melt growth techniques?**

Various melt growth techniques are as follows,

- Normal Freezing
- Crystal Pulling
- Zone Melting
- Flame Fusion

**37. Define Flame fusion method.**

Flame fusion method involves the process of melting a finely powdered substance using an oxy hydrogen flame and the melted droplets crystallizes into an ingot (large crystal).

**38. What is the basic principle of Bridgman technique?**

In Bridgman technique, a boat with the molten charge is moved across a temperature gradient so as to allow the molten charge contained in the boat to solidify starting from an oriented seed.

**39. What are the advantages of Bridgman technique?**

- Relatively cheaper when compare to other pulling techniques and simpler technology.
- Melt compensation can be controlled during the growth.
- The thermal gradient can be easily minimized with a consequent reduction of the dislocation density
- It gives cylindrical crystals without the necessity of sophisticated diameter control devices.

**40. What are the disadvantages of Bridgman technique?**

- Growth rate is very low
- The material is in contact with the walls of the container for long period and it leads to dislocations of the nucleus.
- Sometimes instead of single crystal, poly crystals may be grow
- It cannot be used for the materials which decompose before melting

**41. What is the method used to produce bulk crystals?**

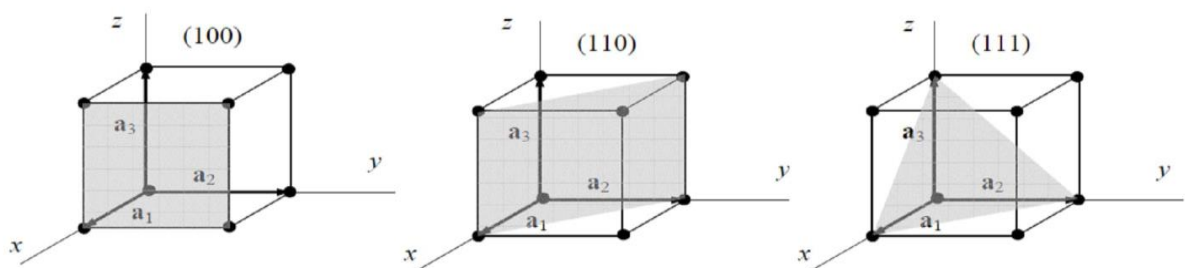
Czochralski method or crystal pulling method from the melt is the principle for the production of bulk single crystals of silicon.

**42. State the basic principle of Czochralski method.**

Czochralski method is a crystal pulling technique of growth of crystal by a gradual layer by layer condensation of melt. It is based on liquid – solid phase transition initiated by a seed crystal.

**43. Draw the following planes in a cubic structure. (Dec. 2011)**

(100), (110) and (111)

**44. What is crystal imperfection?**

If atoms in the solid are not arranged in a perfectly regular manner, it is called defects (or) imperfections in crystals.

**45. What are the types of crystal imperfections?**

- Lattice vibrations or phonons
- Point defects or zero dimensional defects
- Line defects or one dimensional defects or dislocations.
- Surface defects or plane defects or two dimensional defects
- Volume defects or three dimensional defects or bulk defects.

**46. What are the types of point defects?**

- Vacancies
- Interstitial defects
- Self-interstitial defects
- Schottky defect
- Frenkel defect

**47. What are the types of line defects?**

- Edge dislocation
- Screw dislocation
- Dislocation climb.
- Dislocation slip

**48. What are the types of surface defects?**

- Grain boundaries
- Twin boundaries
- Tilt boundaries
- Stacking faults

**49. What are Frenkel and Schottky imperfections?**

Frenkel imperfection refers to the shift of cation from the regular site to interstitial site.

The missing of pair of ions in ionic crystal is called Schottky imperfection.

**50. What are point defects?**

The defect which takes place due to imperfect packing of atoms during crystallization is known as point defects.

**51. What is a line defect?**



The defects which take place due to dislocations or distortion of atoms along a line in some direction is called line defect.

**52. What is meant by stacking fault?**

Stacking faults are planar surface imperfections, and, are caused by fault in the stacking sequence of atomic planes in crystals of FCC and HCP materials.

**53. Define Burger vector.**

The vector which indicates the direction and magnitude of the shift of the lattice on the slip plane is called a Burger vector.

**54. What are the differences between screw and edge dislocation?**

S.No	Edge dislocation	Screw dislocation
1	These dislocations arise due to introduction or elimination of an extra plane of atom.	These are results from a displacement of the atoms in one part of the crystal relative to the rest of crystal forming a spiral ramp around the dislocation line.
2	Region of lattice disturbance along an edge inside a crystal.	Region of lattice disturbance extends in two separate planes at right angles to each other.
3	An edge dislocation can glide and climb.	An edge dislocation can glide only.
4	Burger vector is always perpendicular to the dislocation line.	Burger vector is parallel to the dislocation line.

**55. What are the differences between spin and twinning in plastic deformation?**

S.No	Slip	Twinning
1	The shear deformation which moves atoms in one crystal plane over the atoms of another crystal plane by many inter atomic distances relative to their initial positions is called slip.	The shear deformation in a solid crystalline material through a process by which a portion of the crystal takes up an orientation which makes that position as a mirror image of the parent crystal is called twinning.
2	Lower stress is enough to produce slip	Higher stress is required
3	It occurs due to the movement of dislocations	This can be produced by mechanical deformation.
4	The orientation of the crystal above and below the slip plane is same before and after deformation.	Twinning results in an orientation difference across the twin plane.
5	Slip band is formed after several milliseconds.	Twins can form in a time as short as few microseconds.

**PART B**

1. (i) Explain the terms: atomic radius, Co-ordination number and packing factor.  
(ii) Show that the packing factor for Face centered Cubic and Hexagonal Close packed Structures are equal. (JAN. 2010)
2. (i) What are Miller indices? Explain.  
(ii) Derive an expression for the interplanar spacing for (hkl) planes of a cubic structure. (JAN. 2010)
3. (i) What is packing factor? Obtain packing factor for SC, BCC and FCC lattices  
(ii) Describe the crystal structure of diamond. (JAN 2011)
4. Calculate atomic radius and packing factor for diamond structure. Explain what type of bond is present in diamond. (JAN 2012)
5. (i) What are Bravais lattice?  
(ii) What is primitive cell?

- (iii) Derive an expression for the interplanar spacing for (hkl) planes of a cubic structure. (DEC. 2008)
6. (i). Define atomic radius and Packing Factor.
- (ii). Describe a HCP structure. Show that for an HCP structure  $c/a = \sqrt{8/3}$  and hence calculate the packing fraction for the HCP structure. (DEC. 2008)
7. Explain the Bridgman and Czochralski techniques for growing crystal. (JAN 2014)
8. What is meant by crystal defects? Describe in details point, line and surface defects.
9. Write a note on types of crystal system.

questions	opt1	opt2	opt3	opt4	answer
The boundary separating the two adjacent grains is called _____	crystal	x-rays	grain boundary	crystallography	grain boundary
The example of amorphous solids is _____	plastic	nickel	platinum	silver	plastic
The example of crystalline solids is _____	gold	rubber	glass	plastic	gold
Crystalline material is _____	anisotropic	isotropic	sharp melting point	definite geometrical shape	anisotropic
Non crystalline material is _____	anisotropic	isotropic	regular geometrical shapes	isochoric	isotropic
The lattice means _____	imaginary concept	real concept	regular concept	lattice concept	imaginary concept
Space lattice is _____	3-dimensional	2-dimensional	1-dimensional	4-dimensional	3-dimensional
The example of basis is _____	aluminium lattice+basis	platinum basis+molecule	aluminium lattice+atoms	Nacl basis+atoms	aluminium lattice+basis
The crystal structure is _____	smallest geometric figure	largest geometric figure	middle geometric figure	very largest geometric figure	smallest geometric figure
The unit cell is _____.	one lattice point	two lattice point	three lattice point	four lattice point	one lattice point
The primitive cell is _____	SC	BCC	FCC	triclinic	SC
The crystal system is _____	7	8	6	5	7
The crystal parameters of triclinic crystal system is _____	$a \neq b \neq c$	$a \neq b = c$	$a = b \neq c$	$a = b = c$	$a \neq b \neq c$
The lattice parameters of monoclinic crystal system is _____	$a \neq b \neq c$	$a \neq b \neq c$	$a = b = c$	$a = b \neq c$	$a \neq b \neq c$
The lattice parameters of orthorhombic crystal system is _____	$\alpha = \beta = \gamma = 90^\circ$	$\alpha \neq \beta = \gamma = 90^\circ$	$\alpha = \beta \neq \gamma = 90^\circ$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$	$\alpha = \beta = \gamma = 90^\circ$
The lattice parameters of tetragonal crystal system is _____	$a = b \neq c$	$a \neq b \neq c$	$a = b = c$	$a \neq b = c$	$a = b \neq c$
The lattice parameters of hexagonal crystal system is _____	$\alpha = \beta = 90^\circ$	$\alpha \neq \beta \neq 90^\circ$	$\alpha \neq \beta = 90^\circ$	$\alpha = \beta \neq 90^\circ$	$\alpha = \beta = 90^\circ$
The number of Bravais lattice of triclinic system is _____	1	2	3	4	1
The number of Bravais lattice of monoclinic system is _____	1	2	3	4	2
The number of Bravais lattice of tetragonal system is _____	2	1	3	4	2
The number of Bravais lattice of hexagonal system is _____	1	2	3	4	1
The number of Bravais lattice of trigonal system is _____	1	2	3	4	1
The number of Bravais lattice of cubic system is _____	3	2	4	1	3
The number of Bravais lattice is _____	4	7	14	3	14
The total number of atoms present in (or) shared by an atoms is called _____	effective number	atomic radius	co-ordinations number	cubic system	effective number
_____ is the number of nearest neighbouring atoms to a particular atom.	atomic number	atomic mass	atomic weight	co-ordination number	co-ordination number
Atomic packing factor is _____	$V/V$	$V/v$	$v/p$	$V/p$	$v/V$
The total number of atoms per unit cell in SC is _____	2	1	3	4	1
The total number of atoms per unit cell in BCC is _____	1	3	4	2	2
The total number of face centre atoms per unit cell in FCC is _____	2	1	3	6	3
The total number of atoms per unit cell in FCC is _____	1	2	3	4	4
The number of body centre atoms per unit cell is _____	4	3	2	1	1
The atomic radius of SC structure is _____	$a/2$	$a/4$	$a/6$	$a/8$	$a/2$
The atomic radius of BCC structure is _____	$a/2$	$a/4$	$a \cdot \sqrt{3}/4$	$a/8$	$a \cdot \sqrt{3}/4$
The atomic radius of FCC structure is _____	$a/2$	$a/4$	$a \cdot \sqrt{2}/4$	$a/8$	$a \cdot \sqrt{2}/4$
The co-ordination of SC structure is _____	4	2	3	6	6
The co-ordination number of BCC structure is _____	4	6	8	10	8
The co-ordination number of the FCC structure is _____	10	12	14	8	12
Atomic packing factor for BCC structure is _____	0.68	0.78	0.74	0.52	0.68
The APF for SC structure is _____	0.68	0.78	0.74	0.52	0.52
The APF of FCC structure is _____	0.68	0.78	0.74	0.52	0.74
The volume of 1 atom is _____	$4/3\pi r^2$	$4/3\pi r^3$	$4/\pi r^2$	$4/\pi r^3$	$4/3\pi r^3$
The number of atoms per unit cell in hexagonal _____	12	8	6	4	6
The co-ordination number of hexagonal structure is _____	12	8	6	4	12
The atomic radius of HCP structure is _____	$a/2$	$a/4$	$a/6$	$a/8$	$a/2$
The APF of HCP structure is _____	0.68	0.74	0.72	0.52	0.74
The total number of diamond cubic structure is _____	4	6	8	10	8
The total number of atoms present in (or) shared by an atoms is called _____	effective number	atomic radius	co-ordinations number	cubic system	effective number
An element that can exist in 2 or more forms in the same state is called _____	polymorphism	allotrophy	crystal	SC	allotrophy
If the atoms in the solid are not arranged in a perfectly regular manner, it is called _____	crystal point	crystal defect	BCC	FCC	crystal defect
The point defect is _____	schottky defect	grain boundaries	edge dislocation	screw dislocation	schottky defect
The line defect is _____	schottky defect	grain boundaries	edge dislocation	frenkel defect	edge dislocation
The surface defect is _____	schottky defect	edge dislocation	grain boundaries	frenkel defect	grain boundaries
The defect which take place due to imperfect packing of atoms during crystallization are known as _____	line defect	crystal defect	point defect	surface defect	point defect