KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

FACULTY OF ENGINEERING

B.E (CIVIL ENGINEERING)

SYLLABUS

18BECE405	Introduction to Fluid Mechanics	4H-3C

Instruction Hours/week: L: 2 T: 0 P: 2 **Marks:** Internal**:40** External**:60** Total**:100**

End Semester Exam:3 Hours

Course Objective

- The course provides a first level exposure to the students to fluid statics, kinematics and dynamics.
- Measurement of pressure, computations of hydrostatic forces on structural components and the concepts of Buoyancy all find useful applications in many engineering problems.
- A training to analyze engineering problems involving fluids – such as those dealing with pipe flow, open channel flow, jets, turbines and pumps, dams and spillways, culverts, river and groundwater flow - with a mechanistic perspective is essential for the civil engineering students.

Course Outcome

1. Understand the broad principles of fluid statics, kinematics anddynamics
2. Understand definitions of the basic terms used in fluidmechanics
3. Understand classifications of fluidflow
4. Be able to apply the continuity, momentum and energyprinciples
5. Be able to apply dimensionalanalysis

UNIT-I: Basic Concepts and Definitions – Distinction between a fluid and a solid; Density, Specific weight, Specific gravity, Kinematic and dynamic viscosity; variation of viscosity with temperature, Newton law of viscosity; vapour pressure, boiling point, cavitation; surface tension, capillarity, Bulk modulus of elasticity,compressibility.

UNIT-II: Fluid Statics - Fluid Pressure: Pressure at a point, Pascals law, pressure variation with temperature, density and altitude. Piezometer, U-Tube Manometer, Single Column Manometer, U-Tube Differential Manometer, Micromanometers. pressure gauges, Hydrostatic pressure and force: horizontal, vertical and inclined surfaces. Buoyancy and stability of floatingbodies.

UNIT-III: Fluid Kinematics- Classification of fluid flow : steady and unsteady flow; uniform and non-uniform flow; laminar and turbulent flow; rotational and irrotational flow; compressible and incompressible flow; ideal and real fluid flow; one, two and three dimensional flows; Stream line, path line, streak line and stream tube; stream function, velocity potential function. One-, two- and three -dimensional continuity equations in Cartesian coordinates

UNIT-IV: Fluid Dynamics- Surface and body forces; Equations of motion - Euler's equation; Bernoulli's equation – derivation; Energy Principle; Practical applications of Bernoulli's equation : venturimeter, orifice meter and pitot tube; Momentum principle; Forces exerted by fluid flow on pipe bend; Vortex Flow – Free and Forced; Dimensional Analysis and Dynamic Similitude - Definitions of Reynolds Number, Froude Number, Mach Number, Weber *Number and Euler Number; Buckingham's* π-Theorem.

Lab Experiments

1. Calculation of viscosity
2. Study of Pressure MeasuringDevices
3. Verification of Bernoulli'sTheorem
4. Venturimeter
5. Orificemeter
6. Impacts ofjets
7. Velocity distribution inpipes
8. LaminarFlow

Text/Reference Books:

2. Fluid Mechanics and Machinery, C.S.P.Ojha, R. Berndtsson and P. N. Chadramouli, Oxford University Press,2010
3. Hydraulics and Fluid Mechanics, P M Modi and S M Seth, Standard BookHouse
4. Theory and Applications of Fluid Mechanics, K. Subramanya, Tata McGrawHill
5. Fluid Mechanics with Engineering Applications, R.L. Daugherty, J.B. Franzini and E.J. Finnemore, International Student Edition, Mc GrawHill.

LECTURE PLAN

Subject Name : Introduction to Fluid Mechanics
Subject Code : 18BECE442
Name of the Faculty : Ms. S. M. Leela Bharathi
Branch : B.E Civil Engineering

S. No.	Hours	Topics to be covered	Reference	Remarks
		UNIT I		
1	2	Basic Concepts and Definitions	T2	326
2	2	Density, Specific weight, Specific gravity, Kinematic and dynam viscosity	T2	297
3	1	Tutorial-1		
4	2	variation of viscosity with temperature, Newton law of viscosity	T2	326
5	1	vapour pressure, boiling point, cavitation	T2	331
6	1	surface tension, capillarity, Bulk modulus of elasticity,compressibility.	T2	331
7	1	Tutorial-2		
		TOTAL	10 Hrs	
		UNIT-II		
8	3	Fluid Statics - Fluid Pressure: Pressure at a point, Pascals la pressure variation with temperature, density and altitude.	T2	111
9	1	Tutorial-3		
10	3	Piezometer, U-Tube Manometer, Single Column Manometer, Tube Differential Manometer, Micromanometers. pressure gauge	T2	112
11	2	Hydrostatic pressure and force: horizontal, vertical and inclined surfaces. Buoyancy and stability of floatingbodies.	T2	113
12	1	Tutorial-4	T2	118
		TOTAL	10 Hrs	
		UNIT-III		
13	2	Fluid Kinematics- Classification of fluid flow : steady a unsteady flow; uniform and non-uniform flow; laminar a turbulent flow	T3	2,6,9
14	2	Rotational and irrotational flow; compressible and incompressib flow; ideal and real fluid flow; one, two and three dimensior flows	T3	48
15	1	Tutorial-5	T3	121
16	2	Stream line, path line, streak line and stream tube; stream functic velocity potential function.	T3	55
17	2	One-, two- and three -dimensional continuity equations in Cartesian coordinates	T3	55
18	1	Tutorial-6		
		TOTAL	10 Hrs	
		UNIT-IV		
19	2	Fluid Dynamics- Surface and body forces; Equations of motion - Euler's equation; Bernoulli's equation – derivation	T3	123-124
20	2	Energy Principle; Practical applications of Bernoulli's equation : venturimeter, orifice meter and pitot tube	T3	126
21	1	Tutorial-7	T3	128
22	2	Momentum principle; Forces exerted by fluid flow on pipe	T3	127

		bend; Vortex Flow – Free and Forced		
23	1	Dimensional Analysis and Dynamic Similitude	T3	168
24	1	Definitions of Reynolds Number, Froude Number, Mach Number, Weber Number and Euler Number; Buckingham's π-Theorem.	T3	128-140
25	1	Tutorial- 8		
		TOTAL	10 Hrs	
		Total Number of Hours	40 Hrs	

Text/Reference Books:

1. Fluid Mechanics and Machinery, C.S.P.Ojha, R. Berndtsson and P. N. Chadramouli, Oxford University Press,2010
2. Hydraulics and Fluid Mechanics, P M Modi and S M Seth, Standard BookHouse
3. Theory and Applications of Fluid Mechanics, K. Subramanya, Tata McGrawHill
4. Fluid Mechanics with Engineering Applications, R.L. Daugherty, J.B. Franzini and E.J. Finnemore, International Student Edition, Mc GrawHill.

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FLUID DYNAMICS

Objectives:

- Derive the Bernoulli (energy) equation
- Demonstrate practical uses of the Bernoulli and continuity equation in the analysis of flow
- Introduce the momentum equation for a fluid
- Demonstrate how the momentum equation and principle of conservation of momentum is used to predict forces induced by flowing fluids

After completing this chapter, you should be able to

- Apply the mass equation to balance the incoming and outgoing flow rates in a flow system.
- Recognize various forms of mechanical energy, and work with energy conversion efficiencies.
- Understand the use and limitations of the Bernoulli equation, and apply it to solve a variety of fluid flow problems.
- Work with the energy equation expressed in terms of heads, and use it to determine turbine power output and pumping power requirements.

- Introduction

This chapter deals with 3 equations commonly used in fluid mechanics

- ✓ *The mass equation* is an expression of the conservation of mass principle.
- ✓ *The Bernoulli equation* is concerned with the conservation of kinetic, potential, and flow energies of a fluid stream and their conversion to each other. *The energy equation* is a statement of the conservation of energy principle.
- ✓ Momentum Equation.

A Fluid in motion is subjected to several forces which results in the variation of acceleration and the energies involved in the flow phenomenon of the fluid. Hence the forces and the energies involved in the motion are to be considered in the study. This aspect of fluid motion is known as Dynamics of Fluid Flow.

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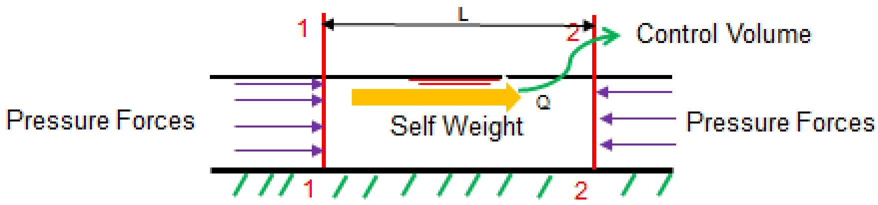
Forces acting on a fluid mass may be classified as (Ref. Fig)



- ✓ Self-Weight/ Gravity Force, Fg
- ✓ Pressure Forces, Fp
- ✓ Viscous Force, Fv
- ✓ Turbulent Force, Ft
- ✓ Surface Tension Force, Fs
- ✓ Compressibility Force, Fc

Dynamics of fluid is governed by Newton's Second law of motion, which states that the resultant force on any fluid element must be equal to the product of the mass and the acceleration of the element. ∑F= Ma

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- ✓ ∑F= Fg+Fp+Fv+Ft+Fs+Fc
- ✓ ∑F= Fg+Fp+Fv+Ft+Fs+Fc ………………. (1)
- ✓ Surface tension forces and Compressibility forces are not significant and may be neglected. Hence Eq. (1) becomes ∑F= Fg+Fp+Fv+Ft- Reynold's Equation of motion and used in the analysis of Turbulent flows. For laminar flows, turbulent force becomes less significant and hence Eq. (1) ∑F= Fg + Fp+ Fv – Navier - Stokes' Equation. If viscous forces are neglected then the Eq (1) reduces to ∑F= Fg + Fp = M*a – Euler's Equation of motion.

EULER'S EQUATION OF MOTION

Consider a stream lime in a flowing fluid in S direction as shown in the figure. On this stream line consider a cylindrical element having a cross sectional area dA and length ds.

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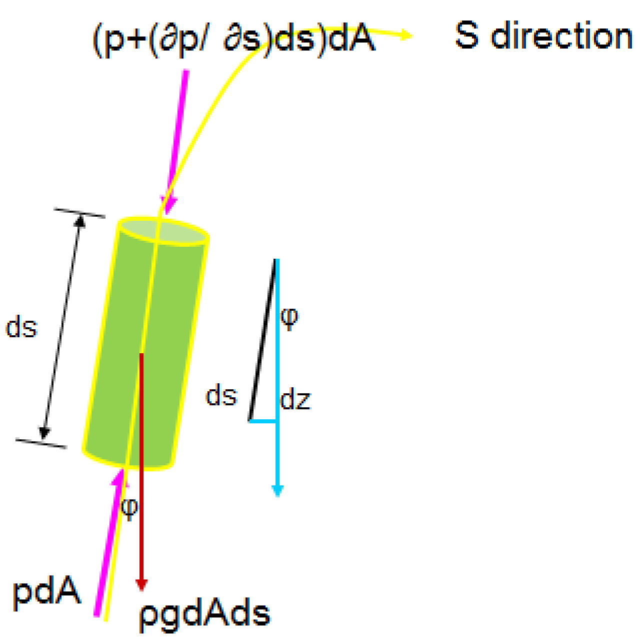
Forces acting on the fluid element are:

Pressure forces at both ends;

- ✓ Pressure force, pdA in the direction of flow



- ✓ Pressure force (p+(∂p/ ∂s)ds)dA in the direction opposite to the flow direction
- ✓ Gravity force, W acting vertically downwards

Let φ the angle between the direction of flow and the line of action of the weight of the element.

According to Newton's second law of motion, $\sum F_s = Ma_s$

$\sum F_s = pdA - (pdA + (\partial p/\partial s)ds)dA) - W\cos\Theta$

$\sum F_s = -(\partial p/\partial s)dsdA - W\cos\Theta$

$\sum F_s = -(\partial p/\partial s)dsdA - W\cos\Theta = Ma_s$

$\sum F_s = -(\partial p/\partial s)dV - W\cos\Theta = Ma_s$

$W/V = \gamma$;

$W = V\gamma$;

$\gamma = \rho g$;

$V = dV = dAds$;

$W = \rho gdAds$

$\rho = M/V$;

$M = \rho V$;

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$M = \rho dAds$

$= -(\partial p/\partial s)dsdA - WCos\Theta = Ma_s$

$= -(\partial p/\partial s)dsdA - \rho gdAdsCos\Theta = \rho dAdsa_s$

$= -(\partial p/\partial s) - \rho gCos\Theta = \rho a_s$

$= -(\partial p/\partial s)/\rho - gCos\Theta = a_s$

$a_s = (dv/dt)$;

v is a function of s & t;

$a_s = [(\partial v/\partial s)(ds/dt) + (\partial v/\partial t)]; [(\partial v/\partial s) v + (\partial v/\partial t)]$;

$\sum F_s = (-(\partial p/\partial s)/\rho - gCos\Theta) = [(\partial v/\partial s) v + (\partial v/\partial t)]$;

For steady flow, $\partial v/\partial t = 0$;

$\sum F_s = (-(\partial p/\partial s)/\rho - gCos\Theta) = (\partial v/\partial s) v$

$= (-(\partial p/\partial s)/\rho - gCos\Theta) - (\partial v/\partial s) v = 0$; $Cos\Theta = dz/ds$

$= (-(\partial p/\partial s)/\rho - g(dz/ds) - (\partial v/\partial s) v = 0$

$= (\partial p/\partial s)/\rho + g(dz/ds) + (\partial v/\partial s) v = 0$ --- Euler's equation of motion.

Bernoulli's Equation of Motion:

In the Euler's equation of motion, all quantities are functions of 's' only. Hence Integrating the Euler's equation with respect to 'ds', we have

$\int (\partial p/\partial s)/\rho + g(dz/ds) + (\partial v/\partial s) v = 0$

$\int (dp/ds)/\rho + g(dz/ds) + (dv/ds) v = 0$

$p/\rho + gz + v^2/2 = \text{Constant}$

$p/\rho g + z + v^2/2g = \text{Constant}$ – Bernoulli's Equation

$p/\rho g = p/\gamma$ = Pressure Energy per unit weight of the fluid

$v^2/2g$ = Kinetic Energy per Unit weight of the fluid

Z= Datum Energy per unit weight of the fluid.

Potential Energy/ Datum Energy: is the energy possessed by a fluid by virtue of its position or location in space.

Pressure Energy: It is the energy possessed by a fluid by virtue of the pressure at which it is maintained.

Kinetic Energy: It is the energy possessed by a fluid by virtue of its motion.

Statement: In a steady, incompressible fluid, the total energy remains same along a streamline throughout the reach.

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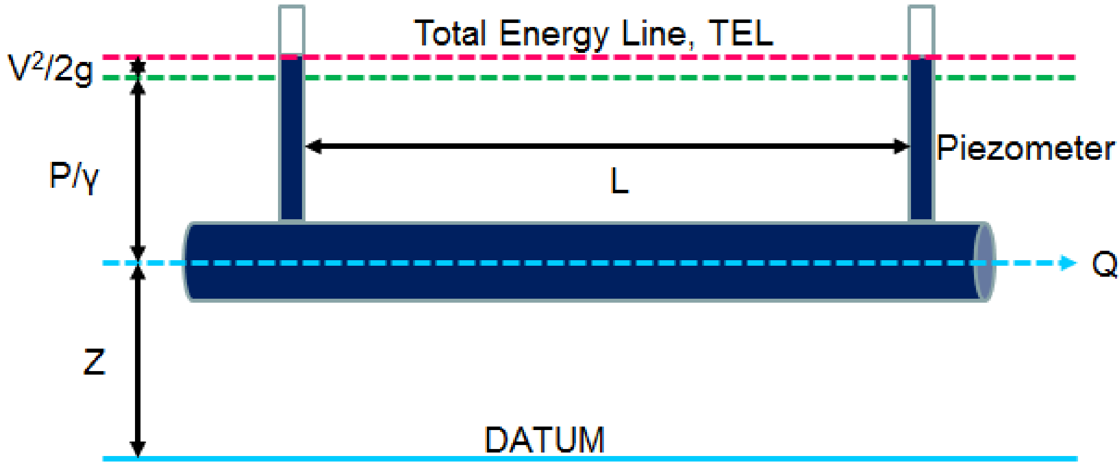
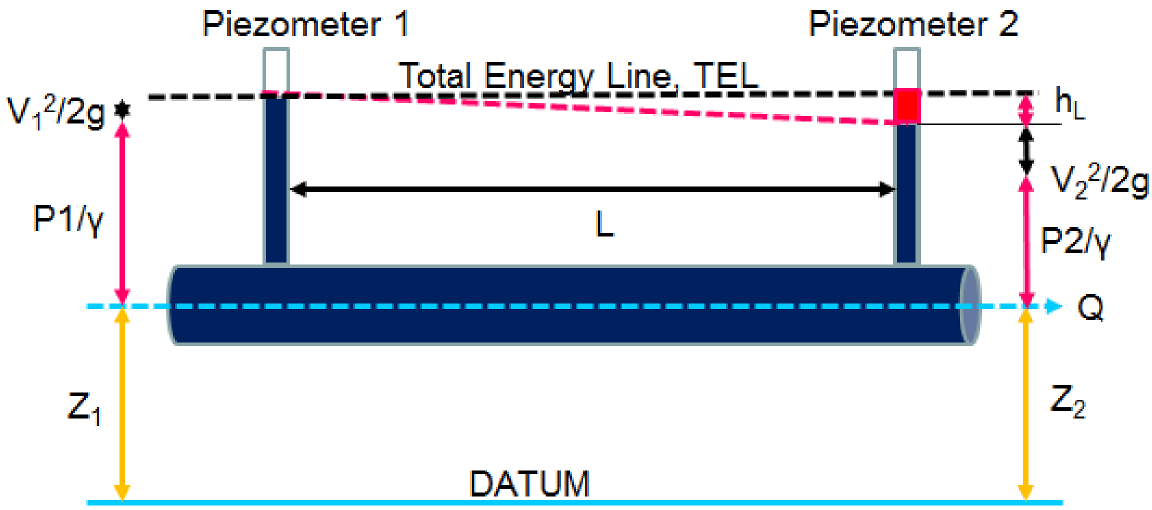
IDEAL FLUID:



$T = p_1/\rho + v_1^2/2g + z_1 = p_2/\rho + v_2^2/2g + z_2 = \text{Constant};$

REAL FLUID:



$T = p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L = \text{Constant}$

Limitations on the use of the Bernoulli Equation

Steady flow The first limitation on the Bernoulli equation is that it is applicable to *steady flow*.

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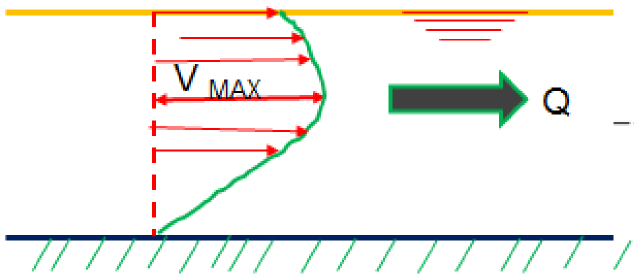
Frictionless flow Every flow involves some friction, no matter how small, and *frictional effects* may or may not be negligible.

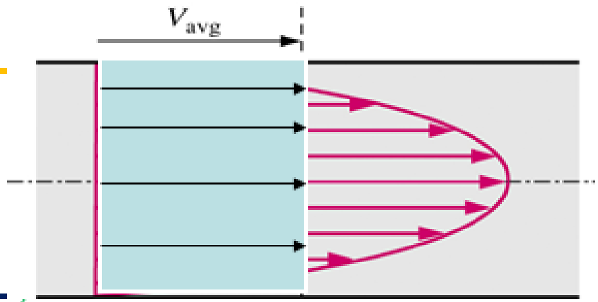
Incompressible flow One of the assumptions used in the derivation of the Bernoulli equation is that ρ= constant and thus the flow is incompressible.

Strictly speaking, the Bernoulli equation is applicable along a streamline, and the value of the constant *C*, in general, is different for different streamlines. But when a region of the flow is *irrational*, and thus there is no *vorticity* in the flow field, the value of the constant *C* remains the same for all streamlines, and, therefore, the Bernoulli equation becomes applicable *across* streamlines as well.

KINETIC ENERGY CORRECTION FACTOR:

In deriving the Bernoulli's Equation, the velocity head or the kinetic energy per unit weight of the fluid has been computed based on the assumption that the velocity is uniform over the entire cross section of the stream tube.





But in real fluids, the velocity distribution is not uniform. Therefore to obtain the kinetic energy possessed by the fluid at differenty sections is obtained by integrating the kinetic energies possessed by different fluid particles.

It is more convenient to express the kinetic energy in terms of the mean velocity of flow.

But the actual kinetic energy is greater than the computed using the mean velocity. Hence a correction factor called ' Kinetic Energy correction factor, α is introduced.

$T= p_1/\rho + \alpha_1(v_1^2/ 2g)+ z_1= p_2/\rho + \alpha_2 (v_2^2/ 2g) + z_2 +h_L=Constant$

In most of the problems of turbulent flow, the value of α = 1.

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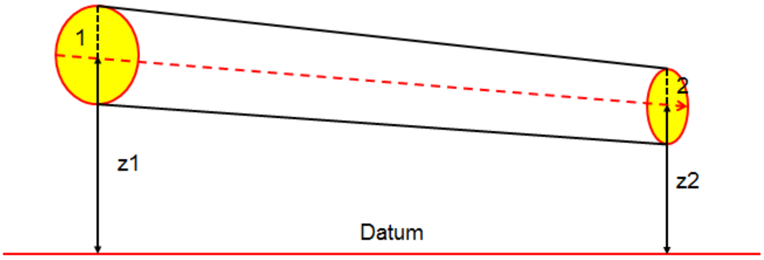
Problem 1: A Converging pipe 30 cm and 15 cm diameter carrying water is positioned inclined whose inlet and outlet are at 6m and 1m above the datum. The pressure at inlet is 1.5bar and velocity is 5m/s. Find the pressure of water at outlet of the pipe, neglecting the losses.



Given:

At Inlet: z_1=6m; d_1= 0.3m; v_1= 5m/s;

p_1= 1.5bar= 1.5X10^5 P_1= 1.5X10^5 N/m^2

Applying Continuity Equation between 1 and 2, we have,

Q_1= Q_2; A_1V_1= A_2V_2

At Inlet section: A_1= (3.14*0.3^2)/4 = 0.071m^2

V_1=5m/s;

At outlet section: A_2=(3.14*0.15^2)/4 = 0.018m^2

V_2= A_1V_1/ A_2 = 0.071*5/.018 = 19.625m/s

Applying Bernoulli's Equation between section 1 and 2, we have,

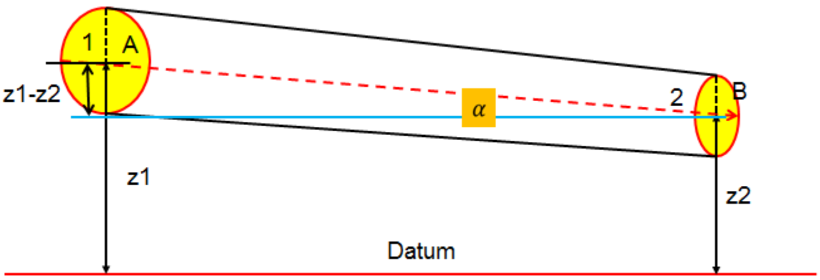
$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

= (1.5*10^5/9810)+5^2/2*9.81+6 =

(p_2 /9810)+19.63^2/2*9.81+1+0

p_2 = 0.118 bar

2. Water flows through an inclined pipe of 50 m long having a slope of 1in 20. The pipe tapers from 5m diameter to 1m dia from higher end to lower end. The water flow is 500lit/s. The pressure of water at the entry is 1.2 bar. Find the pressure of water at the exit.



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Given:

Tan α = 1/20 ; d_1= 5m;d_2=1m; Q= 0.5cum/s; p_1= 1.2bar; L= AB= 50m

Applying Continuity Equation between 1 and 2, we have,

$Q=Q_1= Q_2$; $A_1V_1= A_2V_2$

5= (3.14*5^2/4)V1 =(3.14*1^2/4) V_2

V_1=20/3.14*25 = 0.27m/s;

V_2= 0.27*25 = 6.7m/s

Tan α = 1/20; α = 2.86

Sin α = (z1-z2)/l; sin 2.86 = (z1-z2)/50; z1-z2= 2.5m

Applying Bernoulli's Equation between section 1 and 2, we have,

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$= p_2/\gamma = p_1/\gamma + (v_1^2 - v_2^2/2g) + (z_1 - z_2) + 0$

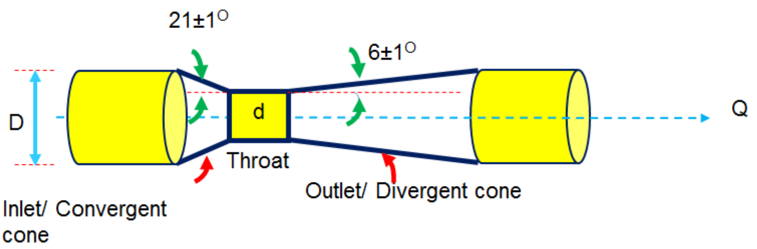
$= p_2/\gamma = 1.2*10^5/9810 + (0.27^2 - 6.7^2/2*9.81) + 2.5 + 0$

p_2= 1.22 bar

Applications of Bernoulli's Equation

1. VENTURIMETER

It is a device for measuring discharge in a pipe.



A Venturi meter consists of

1. Inlet/ Convergent cone
2. Throat
3. Outlet/ Divergent cone

The inlet section Venturi meter is same diameter as that type of the pipe to which it is connected, followed by the short convergent section with a converging cone angle of 21±1 and its length parallel to the axis is approximately equal to 2.7(D – d) , where D is the pipe diameter and d is the throat diameter.

The cylindrical throat – the section of constant cross-section with its length equal to diameter. The flow is minimum at the throat. Usually, diameter of throat is ½ the pipe diameter.

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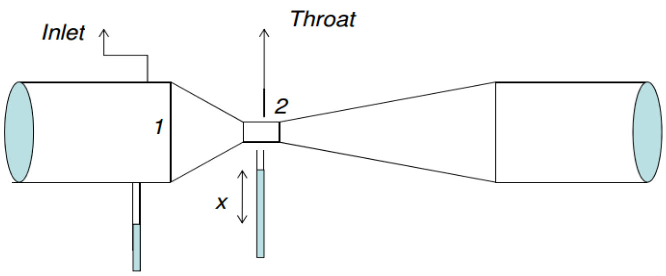
A long diverging section with a cone angle of about 5-7 where in the fluid is retarded and a large portion of the kinetic energy is converted back into the pressure energy.

- Principle of Venturi Meter: -

The basic principle on which a Venturi meter works is that by reducing the cross-sectional area of the flow passage, a pressure difference is created between the two sections, this pressure difference enables the estimation of the flow rate through the pipe.

EXPRESSION FOR DISCHARGE THROUGH VENTURIMETER



Let d_1 = diameter at the inlet (section 1)
p_1 = pressure at section 1
v_1 = velocity at section 1
A_1= area at section1

d_2, p_2, v_2, A_2 are the corresponding values at the throat (section 2)

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{w_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{w_2^2}{2g}+z_2.$$

As pipe is horizontal $z_1=z_2$

$$\Rightarrow \frac{p_1-p_2}{\rho g}=\frac{v_2^2-v_1^2}{2g}$$

$$\Rightarrow h=\frac{v_2^2-v_1^2}{2g}$$

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Where $h \equiv \frac{p_1 - p_2}{\rho g}$, difference of pressure heads at sections 1 and 2.

From the continuity equation at sections 1 and 2, we obtain

$$A_1 v_1 = A_2 v_2 \Rightarrow v_1 = \frac{A_2 v_2}{A_1}$$

Hence
$$h = \frac{v_2^{\;2}}{2g}\left[\frac{A_1^2 - A_2^2}{A_1^2}\right]$$

$$\Rightarrow v_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Actual discharge will be less than theoretical discharge.

$$Q_{actual} = C_d \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

C_d is the coefficient of venturimeter and its value is always less then 1.

Discharge
$$Q = A_1 v_1 = A_2 h$$

$$\Rightarrow Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

Note that the above expression is for ideal condition and is known as theoretical discharge.

Expression of 'h' given by differential U-tube manometer:

Case 1: The liquid in the manometer is heavier than the liquid flowing through the pipe

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

S_h: Specific gravity of the heavier liquid.
S_0: Specific gravity of the flowing liquid.

Case 2: The liquid in the manometer is lighter than the liquid flowing through the pipe

$$h = x\left[1 - \frac{S_L}{S_0}\right]$$

S_L: Specific gravity of the lighter liquid.
X: difference of the liquid columns in U-tube

Inclined Venturi Meter:

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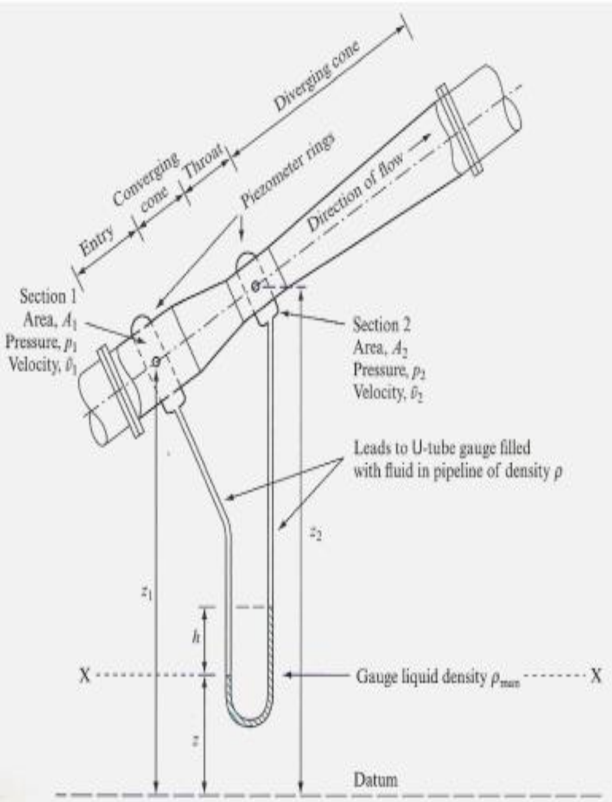
If the Manometric Fluid is heavier than the fluid flowing in the pipe

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

If the Manometric Fluid is Lighter than the fluid flowing in the pipe

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right]$$



P1. An oil of sp.gr. 0.8 is flowing through a venturimeter having inlet diameter 20cm and throat 10cm. The oil mercury differential manometer shows a reading of 25cm. Calculate the discharge of oil through the horizontal venturimeter. Take Cd= 0.98.

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Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$$\therefore \quad a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$$

$d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - 7a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$= 70465 \text{ cm}^3/\text{s}$ = **70.465 litres/s. Ans.**

P2. A horizontal venturimeter with inlet diameter 30cm and throat diameter 15cm is used to measure the flow of water. The differential manometer connected to the inlet and throat is 20cm. Calculate the discharge. Take Cd= 0.98.

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Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

$\therefore$ Difference of pressure head is given by (6.9)

or $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_0 = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

P3.A horizontal venturimeter with inlet diameter 20cm and throat diameter 10 cm is used to measure the flow of oil of specific gravity 0.8. The discharge of oil through venturimeter is 60li/s. Find the reading of the oil-mercury manometer. Take Cd= 0.98

Solution. Given : $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

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$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3/\text{s}$$

Using the equation (6.8), $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

or $$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h} = \frac{1071068.78\sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$$\therefore \quad h = (17.029)^2 = 289.98 \text{ cm of oil}$$

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$$\therefore \quad 289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$$\therefore \quad x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

∴ Reading of oil-mercury differential manometer = **18.12 cm. Ans.**

P4. A horizontal venturimeter with inlet diameter 20cm and throat diameter 10cm is used to measure the flow of water. The pressure at inlet is 17.658N/cm2 and vacuum pressure at throat is 30cm of Mercury. Find the discharge of water through venturimeter. Take Cd = 0.98.

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Solution. Given :

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$

$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$

ρ for water $= 1000 \frac{\text{kg}}{\text{m}^3}$ and $\therefore \frac{p_1}{\rho g} = \frac{17.658 \times 10^4}{9.81 \times 1000} = 18$ m of water

$\frac{p_2}{\rho g} = -30$ cm of mercury

$= -0.30$ m of mercury $= -0.30 \times 13.6 = -4.08$ m of water

$\therefore$ Differential head $= h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 18 - (-4.08)$

$= 18 + 4.08 = 22.08$ m of water $= 2208$ cm of water

The discharge Q is given by equation (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$$= \frac{50328837.21}{304} \times 165555 \text{ cm}^3/\text{s} = \mathbf{165.555 \text{ lit/s. Ans.}}$$

P5. The inlet and throat diameters of a horizontal venturimeter are 30cm and 10cm respectively. The liquid flowing through the venturimeter is water. The pressure intensity at inlet is 13.734N/cm^2 while the vacuum pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and the throat. Find also the values of C_d for the Venturimeter.

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Dia. at inlet, $d_1 = 30$ cm

$\therefore \quad a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore \quad a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Pressure, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

$\therefore$ Pressure head, $\frac{p_1}{\rho g} = \frac{13.734 \times 10^4}{1000 \times 9.81} = 14$ m of water

$$\frac{p_2}{\rho g} = -37 \text{ cm of mercury}$$

$$= \frac{-37 \times 13.6}{100} \text{ m of water} = -5.032 \text{ m of water}$$

Differential head,

$$h = p_1/\rho g - p_2/\rho g$$
$$= 14.0 - (-5.032) = 14.0 + 5.032$$
$$= 19.032 \text{ m of water} = 1903.2 \text{ cm}$$

Head lost, $h_f = 4\%$ of $h = \frac{4}{100} \times 19.032 = 0.7613$ m

$$\therefore \quad C_d = \sqrt{\frac{h - h_f}{h}} = \sqrt{\frac{19.032 - .7613}{19.032}} = 0.98$$

$\therefore$ Discharge

$$= C_d \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= \frac{0.98 \times 706.85 \times 78.54 \times \sqrt{2 \times 981 \times 1903.2}}{\sqrt{(706.85)^2 - (78.54)^2}}$$

$$= \frac{105132247.8}{\sqrt{499636.9 - 6168}} = 149692.8 \text{ cm}^3/\text{s} = \mathbf{0.14969 \text{ m}^3/\text{s}. \text{ Ans.}}$$

P6: A 30cmX15cm Venturimeter is inserted in a vertical pipe carrying water flowing in the upward direction. A differential mercury

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manometer connected to the inlet and throat gives a reading of 20cm. Find the discharge. Take Cd = 0.98

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore \quad a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore \quad a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

$$h = x\left[\frac{S_l}{S_o} - 1\right] = 20\left[\frac{13.6}{1.0} - 1.0\right] = 20 \times 12.6 = 252.0 \text{ cm of water}$$

$C_d = 0.98$

Discharge, $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.3 - 31222.9}} = \frac{86067593.36}{684.4}$$

$= 125756 \text{ cm}^3/\text{s} = $ **125.756 lit/s. Ans.**

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P7: A 20cmX10cm venturimeter is inserted in a vertical pipe carrying oil of sp.gr 0.8, the flow of oil is in the upward direction. The difference of levels between the throat and inlet section is 50cm. The oil mercury differential manometer gives a reading of 30cm of Mercury. Find the discharge of oil. Neglect the losses.

Solution. Dia. at inlet, $d_1 = 20$ cm

$\therefore \quad a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

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$\therefore \quad a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

Differential manometer reading, $x = 30$ cm

$$\therefore \quad h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_g}{S_o} - 1\right]$$

$$= 30\left[\frac{13.6}{0.8} - 1\right] = 30\,[17 - 1] = 30 \times 16 = 480 \text{ cm of oil}$$

$$C_d = 1.0$$

The discharge,
$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= \frac{1.0 \times 314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 480} \text{ cm}^3/s$$

$$= \frac{23932630.7}{304} = 78725.75 \text{ cm}^3/\text{s} = \textbf{78.725 litres/s. Ans.}$$

P.8:In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16cm and 8cm respectively. A is 2 meters above B. The pressure gauge readings have shown that the pressure at B is greater than at A by 0.981N/cm2. Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same fluid and connected to a U tube containing Mercury, Calculate the difference of level of Mercury in the two limbs of the U tube.

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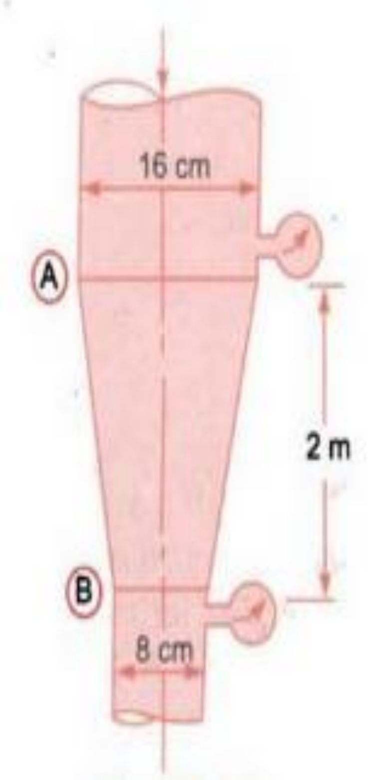

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

$\therefore$ Density, $\rho = 0.8 \times 1000 = 800 \frac{kg}{m^3}$

Dia. at A, $D_A = 16$ cm $= 0.16$ m

$\therefore$ Area at A, $A_1 = \frac{\pi}{4}(.16)^2 = 0.0201$ m^2

Dia. at B, $D_B = 8$ cm $= 0.08$ m

$\therefore$ Area at B, $A_2 = \frac{\pi}{4}(.08)^2 = 0.005026$ m^2

(*i*) Difference of pressures, $p_B - p_A = 0.981$ N/cm^2

$= 0.981 \times 10^4$ N/m^2 $= \frac{9810N}{m^2}$

Difference of pressure head

$$\therefore \quad \frac{p_B - p_A}{\rho g} = \frac{9810}{800 \times 9.81} = 1.25$$

Applying Bernoulli's equation between A and B, taking the reference line passing through B, we have,

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$

$(p_A/\gamma - p_B/\gamma) + z_A - z_B = (v_B^2/2g - v_A^2/2g)$

$(p_A/\gamma - p_B/\gamma) + 2.0 - 0.0 = (v_B^2/2g - v_A^2/2g)$

$-1.25 + 2.0 = (v_B^2/2g - v_A^2/2g)$

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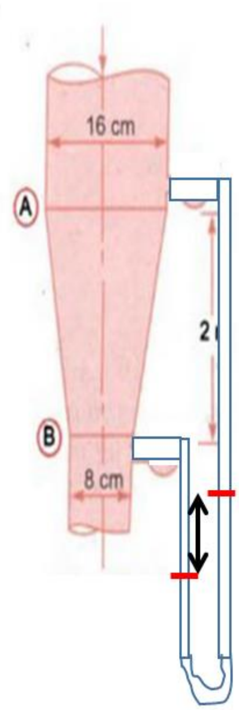

$0.75 = (v_B^2/ 2g - v_A^2/ 2g)$ ----------------- (1)

Now applying Continuity equation at A and B, we get,

$A_AV_A = A_BV_B$

$V_B = A_AV_A\ / A_B = 4V_A$

Substituting the value of V_B in equation (1), we get

$0.75 = 16\ v_A^2/ 2g - v_A^2/ 2g = 15\ v_A^2/ 2g$; Va= 0.99m/s

Rate of flow, Q= A_AV_A

Q= 0.99* 0.01989 Cum/s

Difference of level of mercury in the u – Tube:

Let x = Difference of Mercury level

Then h= x[(sm/so)- 1]

$h = (p_A/ \gamma + z_A) – (p_B/ \gamma + z_B)$

$= (p_A/ \gamma – p_B/ \gamma) + (z_A + z_B) = -1.25+2.00 = 0.75m$

0.75 = x[(13.6/0.8)-1] = 16x

X= 4.687 cm

P.9: Find the discharge of water flowing through a pipe 30cm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 15 cm. The difference of pressure between the main and the throat is measured by a liquid of sp.gr. 0.6 in an inverted U tube which gives a reading of 30cm. The loss of head between the main and the throat is 0.2 times the kinetic head of the pipe.

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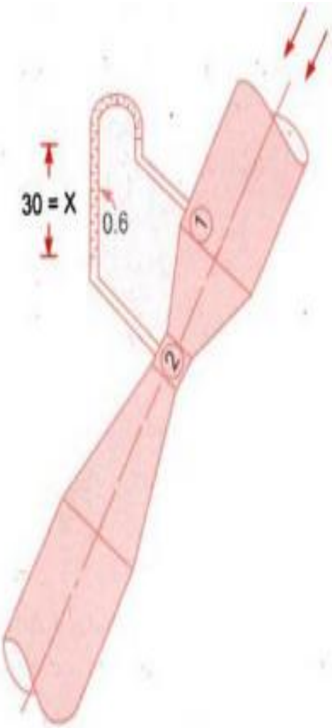

Solution. Dia. at inlet $= 30$ cm

$\therefore$ $d_1 = 30$ cm

$$a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$$

Dia. at throat, $d_2 = 15$ cm

$\therefore$
$$a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$$

Reading of differential manometer, $x = 30$ cm

Difference of pressure head, h is given by

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h$$

Also
$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where $S_l = 0.6$ and $S_o = 1.0$

$$= 30\left[1 - \frac{0.6}{1.0}\right] = 30 \times .4 = 12.0 \text{ cm of water}$$

Loss of head, $h_L = 0.2 \times$ kinetic head of pipe $= 0.2 \times \frac{v_1^2}{2g}$

Now applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_L$$

or
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = h_L$$

But
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = 12.0 \text{ cm of water}$$

and
$$h_L = 0.2 \times v_1^2/2g$$

$\therefore$
$$12.0 + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0.2 \times \frac{v_1^2}{2g}$$

$\therefore$
$$12.0 + 0.8\frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0$$

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Applying continuity equation at (1) and (2), we get

$$a_1v_1 = a_2v_2$$

$$\therefore \quad v_1 = \frac{a_2}{a_1}v_2 = \frac{\frac{\pi}{4}(15)^2 v_2}{\frac{\pi}{4}(30)^2} = \frac{v_2}{4}$$

Substituting this value of v_1 in equation (1), we get

$$12.0 + \frac{0.9}{2g}\left(\frac{v_2}{4}\right)^2 - \frac{v_2^2}{2g} = 0 \text{ or } 12.0 + \frac{v_2^2}{2g}\left[\frac{0.8}{16} - 1\right] = 0$$

$$\text{or} \quad \frac{v_2^2}{2g}[.05 - 1] = -12.0 \text{ or } \frac{0.95\,v_2^2}{2g} = 12.0$$

$$\therefore \quad v_2 = \sqrt{\frac{2 \times 981 \times 12.0}{0.95}} = 157.4 \text{ cm/s}$$

$$\therefore \text{ Discharge} = a_2v_2$$

$$= 176.7 \times 157.4 \text{ cm}^3\text{/s} = 27800 \text{ cm}^3\text{/s} = \mathbf{27.8\ litres/s.\ Ans.}$$

P.10: A 30cmX15cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.9, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 30cm. The differential U- tube mercury manometer shows a gauge deflection of 25cm. Calculate:

1. The discharge of the oil and
2. The pressure difference between the entrance section and the throat section. Take the coefficient of meter as 0.98 and the specific gravity of Mercury as 13.6.

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Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

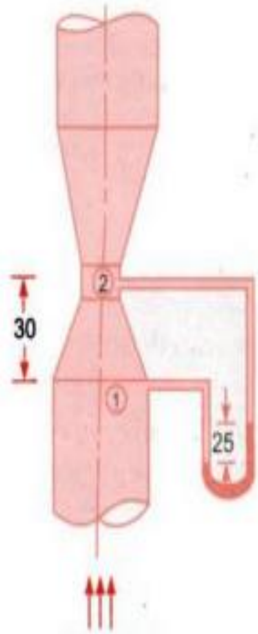
∴ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

∴ Area, $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Let section (1) represents inlet and section (2) represents throat. Then $z_2 - z_1 = 30$ cm

Sp. gr. of oil, $S_o = 0.9$



Sp. gr. of mercury, $S_g = 13.6$

Reading of diff. manometer, $x = 25$ cm

The differential head, h is given by

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$= x\left[\frac{S_g}{S_o} - 1\right] = 25\left[\frac{13.6}{0.9} - 1\right] = 352.77 \text{ cm of oil}$$

(*i*) The discharge, Q of oil $= C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= \frac{0.98 \times 706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} = \sqrt{2 \times 981 \times 352.77}$$

$$= \frac{101832219.9}{684.4} = 148790.5 \text{ cm}^3/\text{s}$$

= 148.79 litres/s. Ans.

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(*ii*) Pressure difference between entrance and throat section

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 352.77$$

or $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2 = 352.77$$

But $$z_2 - z_1 = 30 \text{ cm}$$

$\therefore$ $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) - 30 = 352.77$$

$\therefore$ $$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 352.77 + 30 = 382.77 \text{ cm of oil} = \mathbf{3.8277 \text{ m of oil. Ans.}}$$

or $$(p_1 - p_2) = 3.8277 \times \rho g$$

But density of oil $$= \text{Sp. gr. of oil} \times 1000 \text{ kg/m}^3$$
$$= 0.9 \times 1000 = 900 \text{ kg/cm}^3$$

$\therefore$ $$(p_1 - p_2) = 3.8277 \times 900 \times 9.81 \frac{\text{N}}{\text{m}^2}$$
$$= \frac{33795}{10^4} \text{ N/cm}^2 = \mathbf{3.3795 \text{ N/cm}^2\text{. Ans.}}$$

P.11: Crude oil of specific gravity 0.85 flows upwards at a volume rate of flow of 60 liter/sec through a vertical venturimeter with an inlet diameter of 200 mm and a throat diameter of 100mm. The coefficient of discharge of the venturimeter is 0.98. The vertical distance between the pressure tapings is 300mm.

1. If two pressure gauges are connected at the tapings such that they are positioned at the levels of their corresponding taping points, determine the difference of readings in N/cm2 of the two pressure gauges.
2. If a mercury differential manometer is connected in place of pressure gauge to the tapings such that the connecting tube upto mercury are filled with oil, determine the level of the mercury column.

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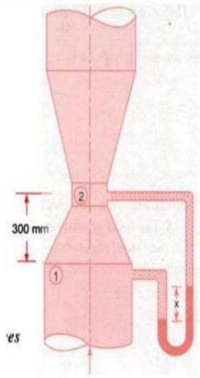
Solution. Given :

Specific gravity of oil, $S_o = 0.85$



∴ Density, $\rho = 0.85 \times 1000 = 850 \text{ kg/m}^3$

Discharge, $Q = 60$ litre/s

$$= \frac{60}{1000} = 0.06 \text{ m}^3/\text{s}$$

Inlet dia, $d_1 = 200 \text{ mm} = 0.2 \text{ m}$

∴ Area, $a_1 = \frac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

Throat dia., $d_2 = 100 \text{ mm} = 0.1 \text{ m}$

∴ Area, $a_2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$

Value of C_d $= 0.98$

Let section (1) represents inlet and section (2) represents throat. Then

$$z_2 - z_1 = 300 \text{ mm} = 0.3 \text{ m}$$

(i) Difference of readings in N/cm² of the two pressure gauges

The discharge Q is given by,

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or
$$0.06 = \frac{0.98 \times 0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times h}$$

$$= \frac{0.98 \times 0.00024649}{0.0304} \times 4.429\sqrt{h}$$

∴
$$\sqrt{h} = \frac{0.06 \times 0.0304}{0.98 \times 0.00024649 \times 4.429} = 1.705$$

∴
$$h = 1.705^2 = 2.908 \text{ m}$$

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But for a vertical venturimeter, $h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$

$$\therefore \quad 2.908 = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2$$

$$\frac{p_1 - p_2}{\rho g} = 2.908 + z_2 - z_1 = 2.908 + 0.3 \qquad (\because \; z_2 - z_1 = 0.3 \text{ m})$$

$$= 3.208 \text{ m of oil}$$

$$\therefore \quad p_1 - p_2 = \rho g \times 3.208$$

$$= 850 \times 9.81 \times 3.208 \text{ N/m}^2 = \frac{850 \times 9.81 \times 3.208}{10^4} \text{ N/cm}^2$$

$$= \mathbf{2.675\ N/cm^2.\ Ans.}$$

(ii) Difference in the levels of mercury columns (i.e., x)

The value of h is given by, $h = x\left[\frac{S_g}{S_o} - 1\right]$

$$\therefore \quad 2.908 = x\left[\frac{13.6}{0.85} - 1\right] = x\,[16 - 1] = 15\,x$$

$$\therefore \quad x = \frac{2.908}{15} = 0.1938 \text{ m} = \mathbf{19.38\ cm\ of\ oil.\ Ans.}$$

2. ORIFICE METER

- It is a device used for measuring the rate of flow through a pipe.

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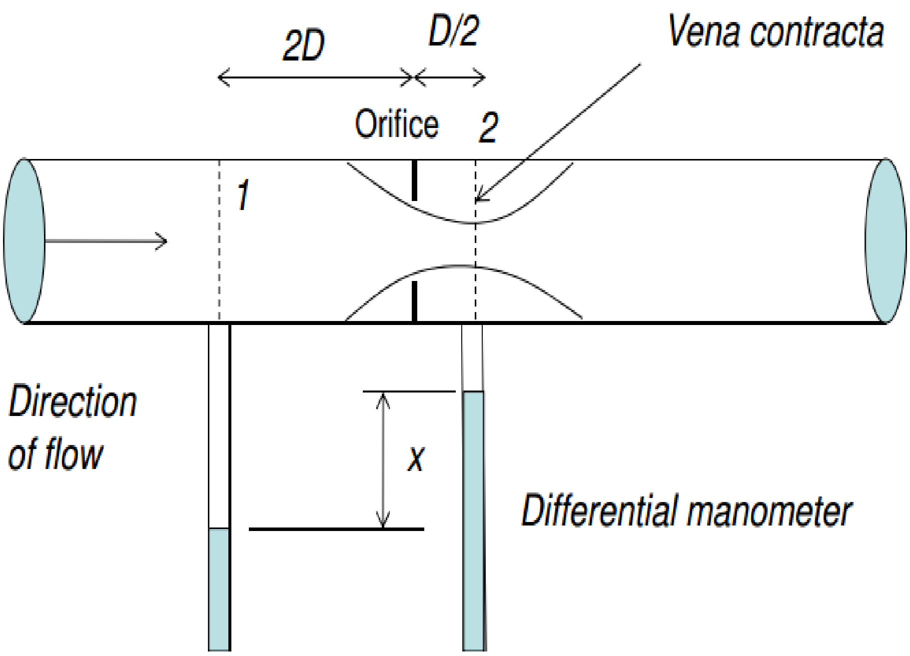
- It is a cheaper device as compared to venturimeter. The basic principle on which the Orifice meter works is same as that of Venturimeter.
- It consists of a circular plate with a circular opening at the center. This circular opening is called an Orifice.
- The diameter of the orifice is generally varies from 0.4 to 0.8 times the pipe diameter.



Let d_1 = diameter at section 1
p_1 = pressure at section 1
v_1 = velocity at section 1
A_1= area at section1

d_2, p_2, v_2, A_2 are the corresponding values at section 2.

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

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$$\Rightarrow \left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{v_2{}^2-v_1{}^2}{2g}$$

$$\Rightarrow h=\frac{v_2{}^2-v_1{}^2}{2g}$$

$$\Rightarrow v_2=\sqrt{2gh+v_1^2}$$

where h is the differential head.

Let A_0 is the area of the orifice.

Coefficient of contraction, $C_c=\frac{A_2}{A_0}$

By continuity equation, we have

$$A_1v_1=A_2v_2$$

$$\Rightarrow v_1=\frac{A_0C_c}{A_1}v_2$$

Hence,
$$v_2=\sqrt{2gh+\frac{A_0^2C_c^2v_2^2}{A_1^2}}$$

$$\Rightarrow v_2=\frac{\sqrt{2gh}}{\sqrt{1-\frac{A_0^2}{A_1^2}C_c^2}}$$

Thus, discharge,

$$Q=A_2v_2=v_2A_0C_c=\frac{A_0C_c\sqrt{2gh}}{\sqrt{1-\frac{A_0^2}{A_1^2}C_c^2}}$$

If C_d is the co-efficient of discharge for orifice meter, which is defined as

$$C_d=C_c\frac{\sqrt{1-\frac{A_0^2}{A_1^2}}}{\sqrt{1-\frac{A_0^2}{A_1^2}C_c^2}}$$

$$\Rightarrow C_c=C_d\frac{\sqrt{1-\frac{A_0^2}{A_1^2}C_c^2}}{\sqrt{1-\frac{A_0^2}{A_1^2}}}$$

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Hence,

$$Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

The coefficient of discharge of the orifice meter is much smaller than that of a venturimeter.

P1: An orifice meter with Orifice diameter 15cm is inserted in a pipe of 30cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50cm of Mercury. Find the rate of flow of oil of sp.gr. 0.9 when the co-efficient of discharge of the meter= 0.64.

Solution. Given :

Dia. of orifice, $d_0 = 15$ cm

∴ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Dia. of pipe, $d_1 = 30$ cm

∴ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Sp. gr. of oil, $S_o = 0.9$

Reading of diff. manometer, $x = 50$ cm of mercury

∴ Differential head, $h = x\left[\frac{S_g}{S_o} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right]$ cm of oil

$= 50 \times 14.11 = 705.5$ cm of oil

$C_d = 0.64$

∴ The rate of the flow, Q is given by equation (6.13)

$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= \frac{94046317.78}{684.4} = 137414.25 \text{ cm}^3\text{/s} = \textbf{137.414 litres/s.}$$ **Ans.**

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P2: An orifice meter with an orifice diameter 10cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62N/cm^2 and 9.81N/cm^2 respectively. Coefficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

∴ Area, $a_0 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. of pipe, $d_1 = 20$ cm

∴ Area, $a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

$p_1 = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$

∴ $\frac{p_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20$ m of water

Similarly $\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10$ m of water

∴ $h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10$ m of water $= 1000$ cm of water

$C_d = 0.6$

The discharge, Q is given by equation (6.13)

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3/\text{s} = \textbf{68.21 litres/s.}$$ **Ans.**

3. Pitot tube:

Pitot tube is a device used to measure the velocity of flow at any point in a pipe or a channel.

Principle: If the velocity at any point decreases, the pressure at that point increases due to the conversion of the Kinetic energy into pressure energy.

In Simplest form, the pitot tube consists of a glass tube, bent at right angles.

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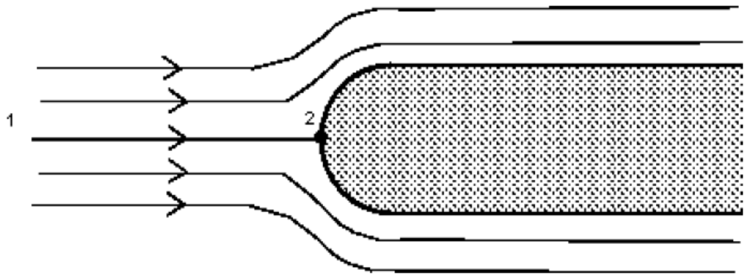
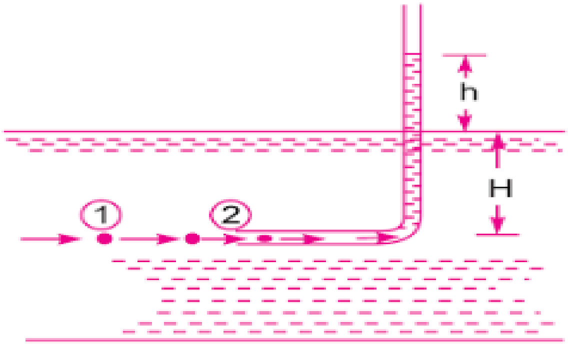
If a stream of uniform velocity flows into a blunt body, the stream lines take a pattern similar to this:



Streamlines around a blunt body

Note how some move to the left and some to the right. But one, in the centre, goes to the tip of the blunt body and stops. It stops because at this point the velocity is zero - the fluid does not move at this one point. This point is known as the *stagnation point*.



Let p_1 = pressure at section 1

p_2 = pressure at section 2

v_1 = velocity at section 1

v_2 = velocity at section 2 = 0

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface

Point 2 is just at the inlet of the Pitot-tube
Point 1 is far away from the tube

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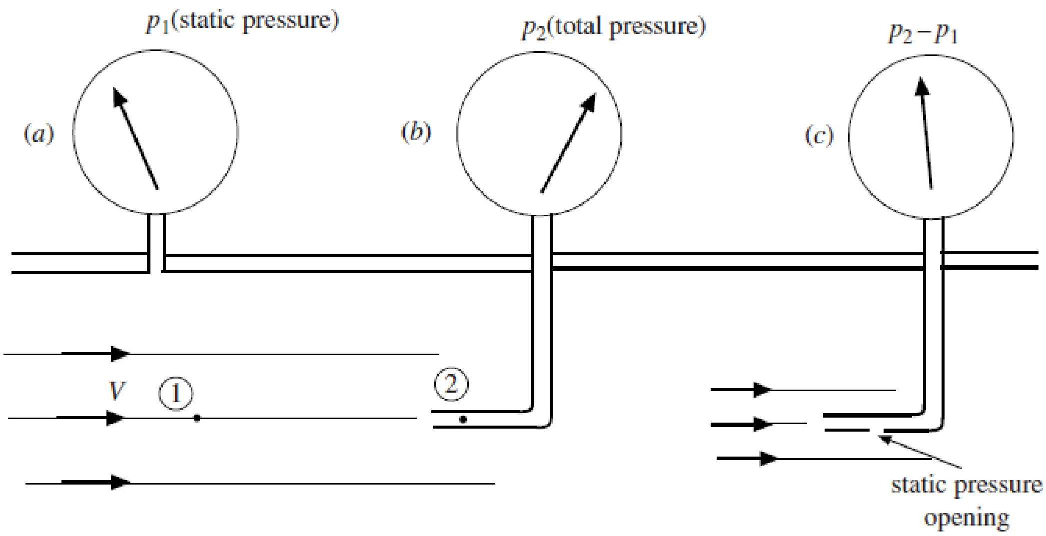

Pressure probes: (*a*) the piezometer, (*b*) a pitot tube, and (*c*) a pitot-static tube.

Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

But $z_1 = z_2$, and $v_2 = 0$.

$\frac{p_1}{\rho g} =$ Pressure head at 1=H

$\frac{p_2}{\rho g} =$ Pressure head at 2=h+H

Substituting these values, we get $H+\frac{v_1^2}{2g}=h+H$

$$\Rightarrow v_1=\sqrt{2gh}$$ Torricelli's Equation

This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act}=C_v\sqrt{2gh}$$

$C_v \equiv$ coefficient of pitot-tube

The pitot static tube, shown in the figure below, is one variation of the device which allows the static head (P/γ) and dynamic (total) head (P/γ + V^2/2g) to be separately measured

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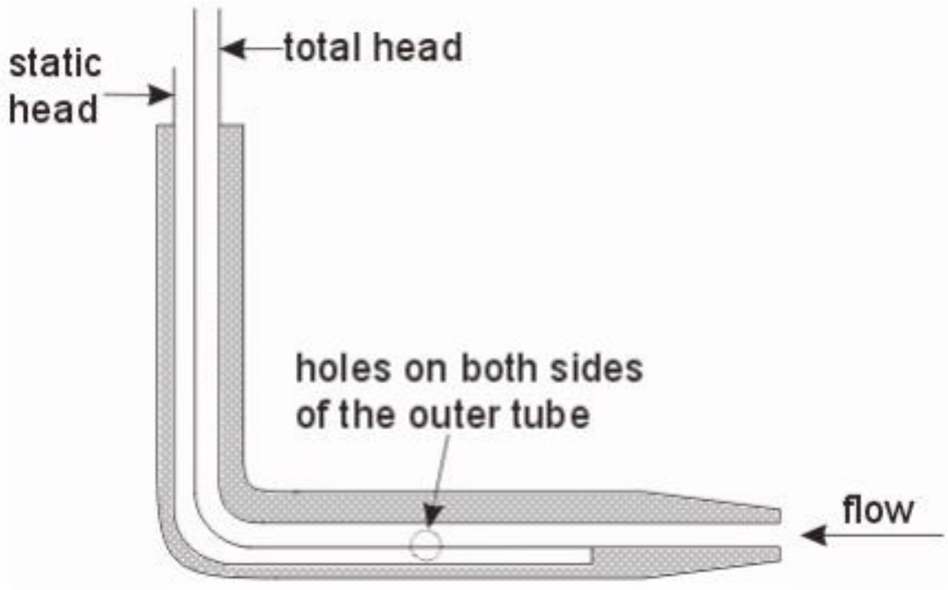

P1: A pitot static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6m and static pressure head is 5m. Calculate the velocity of flow assuming the coefficient of the tube = 0.98

Solution:

Stagnation Pressure head = 6m

Static Pressure Head = 5m KTUweb.com

Difference in head= Stagnation pressure head – static pressure head

= 6-5 = 1m

Velocity of flow , $V=\sqrt{2gH}$

$= 0.98\sqrt{2*9.81*1} = 4.34m/s$

MOMENTUM PRINCIPLE:

It is based on the principle of law of conservation of momentum which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

By Newton's second law of motion, the force acting on a fluid mass 'm' is given by F= Ma

Acceleration, a =

F= M

F= ----------- Momentum principle

F.dt = d(Mv) ---- Impulse momentum Equation – states that the impulse of the force F acting on a fluid mass M in a short interval of

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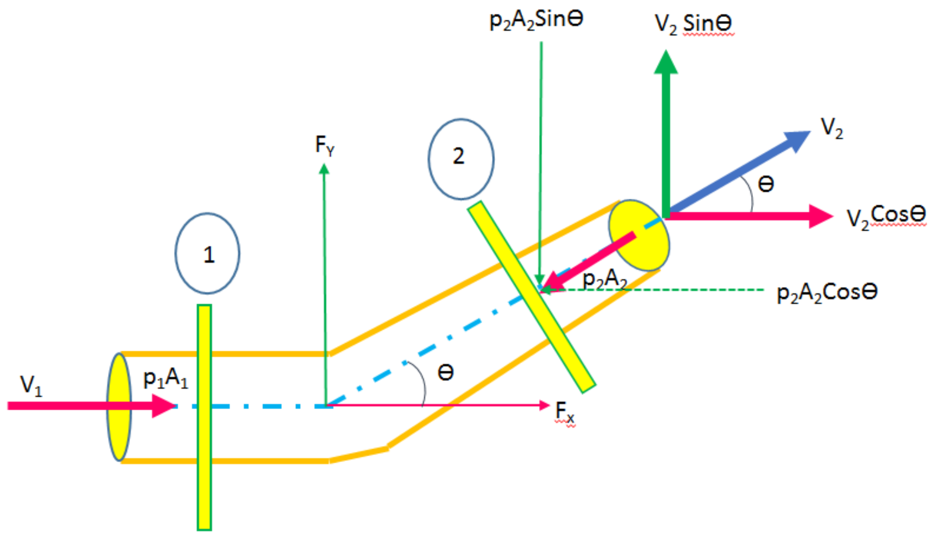
time dt is equal to the change in momentum d(Mv) in the direction of the force.

FORCE EXERTED BY A FLOWING FLUID ON A PIPE BEND



Consider 2 sections (1) and (2) as shown in the fig.

Let v_1 = Velocity of flow at section 1

P_1= Pressure intensity at section 1

A_1 = Cross sectional area of pipe at 1

Similarly, v_2, p_2 and A_2 the corresponding values at (2)

Let F_x, F_y be the components of the forces exerted by the flowing fluid on the bend in the x and y directions respectively.

The force exerted by the bend on the flowing fluid in the direction of x and y directions will be equal to F_x, F_y but in the opposite direction.

The other forces acting on the fluid are p_1A_1 and p_2A_2 on the sections 1 and 2 respectively.

Then the momentum equation in the x direction is given by : Net force acting on the fluid in the direction of x = Rate of change of momentum in the x- direction

$p_1A_1 - p_2A_2 \cos\Theta - F_x$ = mass per sec X change in velocity

= ρQ [Final velocity in the direction of x – Initial velocity in the direction of x]

$= \rho Q\ (v_2 \cos\Theta - v_1)$

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- $F_x = \rho Q\,(v_2 \cos\Theta - v_1)\; - p_1A_1 + p_2A_2 \cos\Theta$

$F_x = \rho Q\,(v_1 - v_2 \cos\Theta\,) + p_1A_1 - p_2A_2 \cos\Theta$

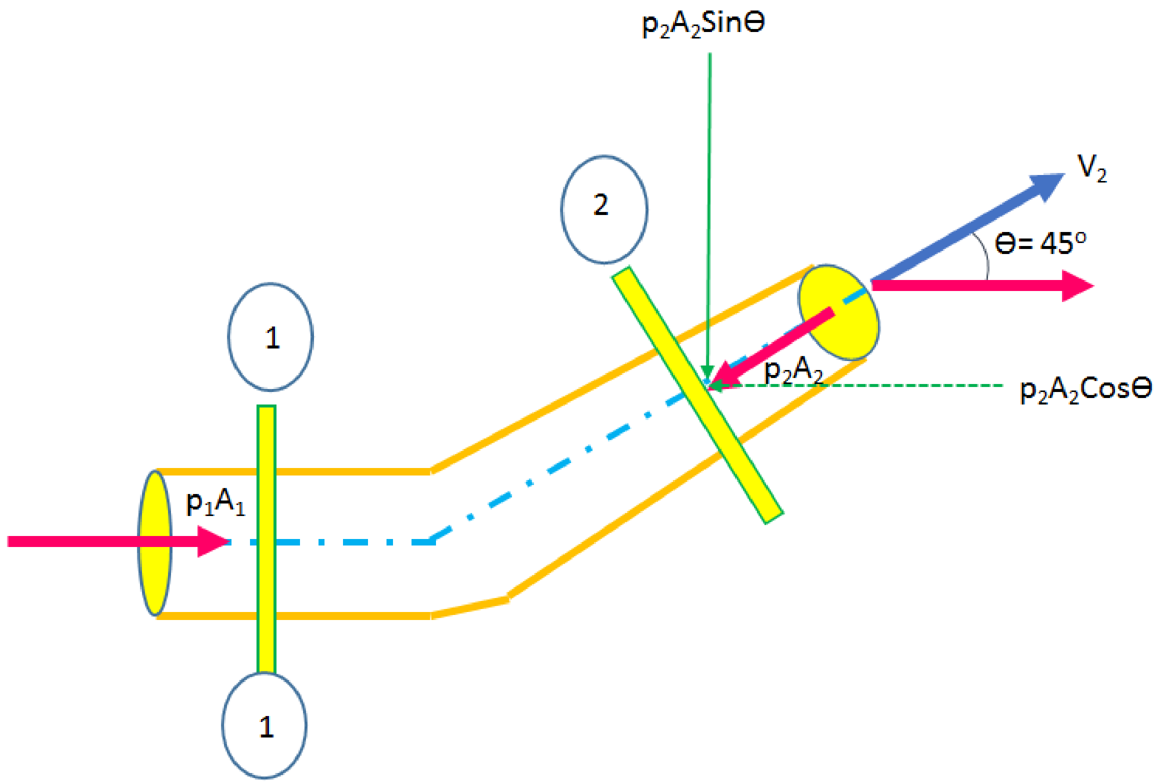
Similarly momentum in the y direction gives

$F_x = \rho Q\,(- v_2 \sin\Theta\,)\; - p_2A_2 \sin\Theta$

Now the resultant force (F_R) acting on the bend $=\sqrt{(F_x)^2+(F_Y)^2}$

The angle made by the resultant force with horizontal direction is given by $\tan\Theta = F_Y/F_x$

P1: A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 60cm and 30cm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 10N/cm^2 and the rate of flow is 0.6cum/s.



Solution:

Angle of the bend , $\Theta = 45°$

Dia. at Inlet, D_1 = 60cm = 0.6m

Area, A1 = = (3.14* 0.6*0.6)/4 = 0.2827 m^2

Dia. at Outlet, D_2 = 30cm = 0.3m

Area, A2= $\pi * D_2^2$ = (3.14* 0.3*0.3)/4 = 0.071 m^2

Pressure at Inlet, p_1 = 10N/cm^2 = 10*10^4N/m^2

Q= 0.6cum/s ; $v_1 = Q/A_1$ = 0.6/0.28 = 2.12m/s

$v_2 = Q/A_2$ = 0.6/0.071 = 8.49m/s

Applying Bernoulli's equation between sections 1 and 2, we have

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$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2$

But $z_1 = z_2$

$p_1/\gamma + v_1^2/2g = p_2/\gamma + v_2^2/2g$

$10*10^4/9810 + 2.122^2/(2*9.81) = p_2/\gamma + 8.488^2/(2*9.81)$

$p_2/\gamma = 10.19+0.23-3.67 = 6.75$m of water.

$p_2 = 66197$N/m^2.

Forces on the bend in the x and y directions are given by

$F_x = \rho Q\ (v_1 - v_2 \cos \Theta) + p_1A_1 - p_2A_2 \cos \Theta$

$F_y = \rho Q\ (-v_2 \sin \Theta) - p_2A_2 \sin \Theta$

$F_x = \rho Q\ (v_1 - v_2 \cos \Theta) + p_1A_1 - p_2A_2 \cos \Theta$

= 1000*0.6 (2.12 - 8.48 cos 45) + 10*10^4* 0.2827 -66197* 0.071 cos 45

= 24400N

$F_y = \rho Q\ (-v_2 \sin \Theta) - p_2A_2 \sin \Theta$

= 1000*0.6(-8.48sin45) – 10*10^4*0.2827sin 45

= -28384N

- Sign indicates F_y is acting downwards.

$F_R = \sqrt{(F_x^2 + F_y^2)}$

$= \sqrt{24400^2 + (-28384)^2}$

= 37,430N

$\tan\Theta = F_y/F_x = 28384/24400 = 1.16$

$\Theta = 0.85$;

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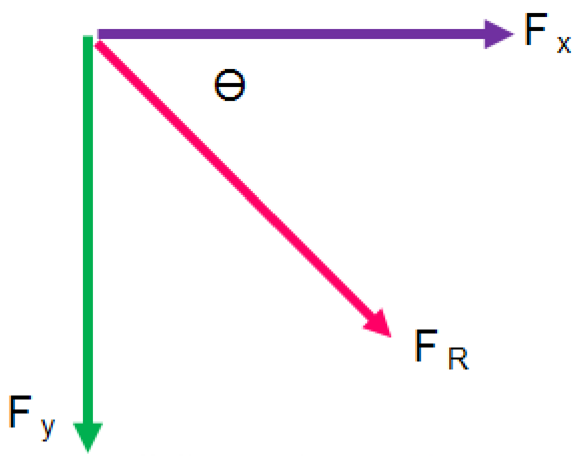

P2: 250litres/sec of water is flowing in a pipe having a diameter of 30cm. If the pipe is bent by 135 ° (that is change from initial to final direction is 135°), find the magnitude and direction of the resultant force on the bend. The pressure of water flowing is 4kgf/cm^2.

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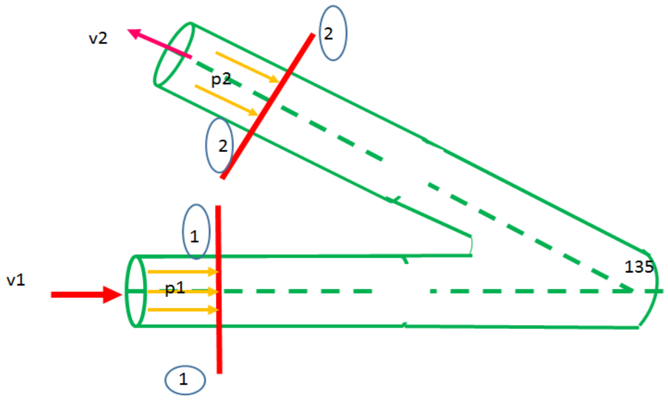


Given:

P1=p2= 4kgf/cm^2 = 4X10^4 m^2

Discharge, Q= 250l/s = 0.25m^3/s

Dia. of bend at inlet and outlet,

D1= D2= 30cm=0.3m

Area, A1= A2= 3.14*0.3^2/4 = 0.071m^2

Velocity of water at inlet and outlet, V=v1=v2= Q/A = 0.25/0.071 = 3.54m/s

Force acting along x-axis:

$F_x = \rho Q\,(v_1 - v_2 \cos \Theta) + p_1 A_1 - p_2 A_2 \cos \Theta$

= (1000/9.81)*0.25(3.54- (-3.54cos(180-135))+4X10^4*0.071-(-4X10^4*0.071cos45)

= 153.87+4826.3 = 4980.1kgf

$F_y = \rho Q\,(- v_2 \sin \Theta) - p_2 A_2 \sin \Theta$

= (1000/9.81)*0.25(-3.54sin 45) – 4X10^4*0.071Sin45

= - 2062.83kgf

Resultant Force:

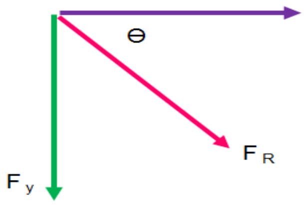
$F_R = \sqrt{(F_x^2 + F_y^2}$

$= \sqrt{4980.1^2 + (-2062.83)^2}$

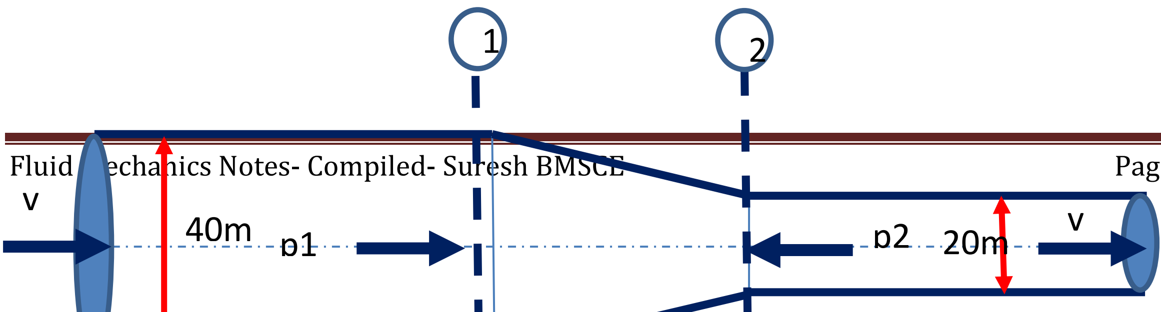
= 5390.4kgf

TanΘ = Fy/Fx = 2062.83/4980.1 = 0.4142

Θ = 22° 30';



P3: A nozzle of diameter 20mm is fitted to a pipe of diameter 40mm. Find the force exerted by the nozzle on the water which is flowing through the pipe at the rate of 1.2cum/minute.



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Dia of the pipe= D1= 40mm= 40X10^$^{-3}$ m = 0.04m

Area, A1 = 3.14*0.4^2/4 = 0.0013 m^2

Dia. of Nozzle, D2 = 20mm = 0.02m

Area, A2 = 3.14*0.02^2/4 = 0.000314m^2

Discharge, Q = 1.2m^3/minute = 1.2/60m^3/s = 0.02m^3/s

Applying continuity equation at (1) and (2),

A1V1 = A2V2= Q

V1 = Q/A1 = 0.2/(0.0013) = 15.92m/s

V2= Q/A2 = 0.2/ 0.000314 = 63.69m/s

Applying Bernoulli's Equation between sections (1) and (2)

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2$

$z_1 = z_2$; p_2/γ = atmospheric pressure =0

$p_1/\gamma + v_1^2/2g = v_2^2/2g$; $p_1/\gamma = v_2^2/2g - v_1^2/2g$ = 193.83m of water ;

p_1 = 193.83 *1000kgf/m^2

Let the force exerted by the nozzle on water = Fx

Net force in the direction of x = Rate of change of momentum in the direction of x

$F_x + p_1A_1 - p_2A_2 = \rho Q (v_2 - v_1)$; p2 = 0= atmospheric pressure

193.83X1000X0.001256-0+ F_x = (1000/9.81)X 0.02 (63.69-15.92)

243.45+ F_x = 93.39; F_x = -150.06kgf

- Sign indicates that the force exerted by the nozzle on water is acting from right to left.

P3: A discharge of 0.06m^3/s flows through a horizontal bend as shown in the Fig. Calculate the force on the bolts in section 1.

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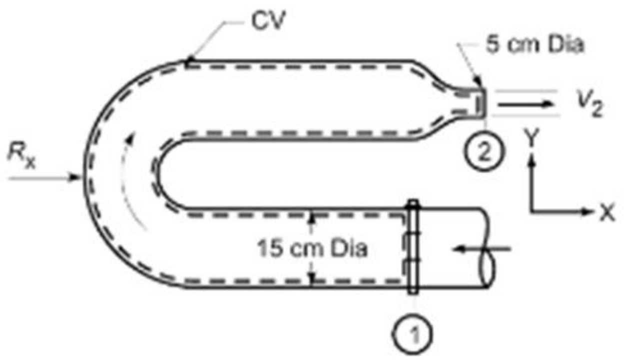

Solution: The control volume is shown in dotted lines. The reaction on the control volume fluid is shown as Rx in the positive X-direction

Discharge, $Q = 3.14* (D_2)^2 V_2/4 = 0.06 m^3/s$

$V_2 = 0.06/ 3.14(D_2)^2/4 = 30.56 m/s$

$Q= A_1 V_1 = A_2 V_2 = V_2 (D_2/D_1)^2 = 30.56*(5/15)^2 = 3.395 m/s$

Applying Bernoulli's equation to sections 2 and 1, by assuming the bend to be in horizontal plane,

$= p_2/\gamma + v_2^2/2g + z_2 = p_1/\gamma + v_1^2/2g + z_1; z_1 = z_2$

$= 0+ (30.56)^2/(2*9.81) = p_1/\gamma + (3.395)^2/(2*9.81)$

P1 = 460.2kPa.

By momentum equation in the x – direction

$-p_1 A_1 + R_x - 0 = \rho Q (v_2 - (-v_1))$

$= -(460.2*10^3)*3.14*(0.15)^2/4 + R_x = 998*0.06*(30.56+3.395)$

$R_x = 8132+2033 = 10165N$

The force F exerted by the fluid on the pipe and hence on the bolts in section 1, is equal and opposite to R_x

Thus F = 10165N and acts to the left, i.e., in the negative x- direction, as a pull (tension) on the joint.

ROTARY OR VORTEX MOTION

A mass of fluid in rotation about a fixed axis is called vortex. The rotary motion of fluid is also called vortex motion. In this case the rotating fluid particles have velocity in tangential direction.

Thus the vortex motion is defined as motion in which the whole fluid mass rotates about an axis.

The vortex motion is of two types

(i) Free vortex (ii) Forced vortex

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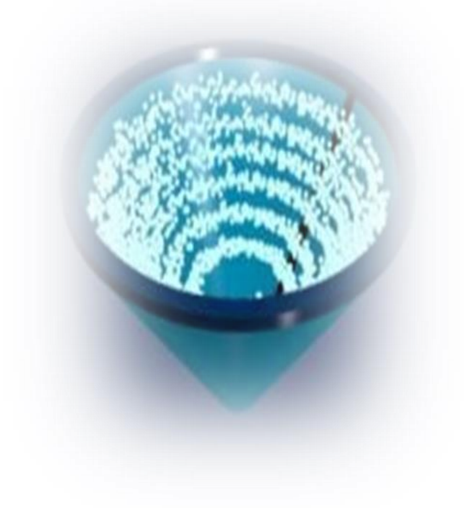

FREE VORTEX FLOW

- ✓ Free vortex flow is that type of flow in which the ***fluid mass rotates without any external impressed contact force.***
- ✓ The whole mass rotates either due to fluid pressure itself or the gravity or due to rotation previously imparted. ***Energy is not expended*** to any outside source.
- ✓ The free vortex motion is also called ***Potential vortex*** or ***Irrotational vortex***.

Examples of Free Vortex flow

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- Some instances of a vortex include a ring of smoke, drag from the wing of an aircraft, a tornado, dust devil or waterspout, hurricanes, sunspots on the Sun, black holes, and spiral galaxies, such as our own Milky Way.
- Rotary flow observed in wash basin while draining the liquid through the outlet at bottom.
- Flow of liquid through a hole provided at the bottom of the container

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- Flow through kitchen sink
- Draining the bath tub
- Flow of liquid around a circular bend in pipe
- A whirlpool in a river
- The flow fields due to a tornado.

The relation between velocity and radius in free vortex is obtained by putting the value of external torque equal to ZERO.OR the time rate of change of angular momentum, i.e., moment of the momentum must be Zero.

Consider a fluid particle of mass 'M' at a radial distance r from the axis of rotation, having a tangential velocity v. Then

Angular momentum= Mass X velocity = M*v

Moment of the Momentum = Momentum* radius® = Mvr

Time rate of change of angular momentum = δ/δt (Mvr)

But for free vortex δ/δt (Mvr)= 0

Integrating, we get

∫δ/δt (Mvr)= 0

Mvr= Constant = vr = constant

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FORCED VORTEX MOTION

Forced vortex motion is one in which the fluid mass is made to rotate by means of some external agencies. The external agency is generally the mechanical power which imparts the constant torque on the fluid mass. The forced vortex motion is also called ***flywheel vortex*** or ***rotational vortex***.

Examples:

Flow of water through the runner of a turbine

Flow of liquid through the passage of impeller of centrifugal pumps

Rotation of water in a washing machine

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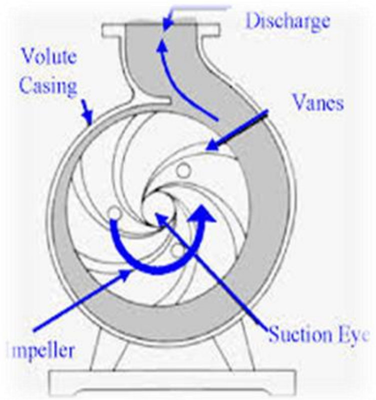
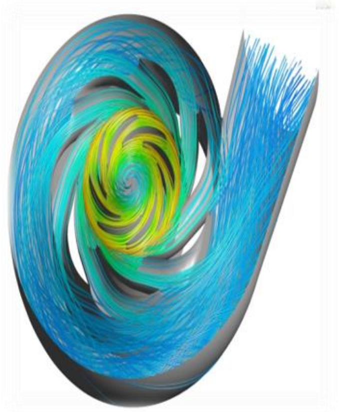
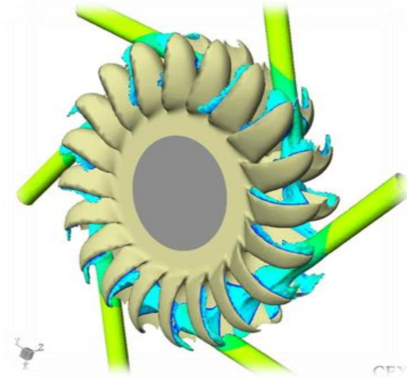






The fluid mass in this forced vortex flow rotates at constant angular velocity ω. The tangential velocity of any fluid particle is given by

$V = \omega * r$

Where r is the radius of the fluid particle from the axis of rotation.

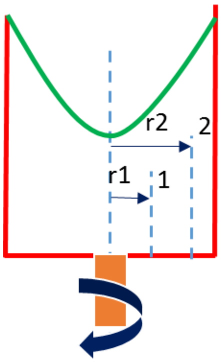
Hence angular velocity ω is given by, $\omega = v/r =$ constant.

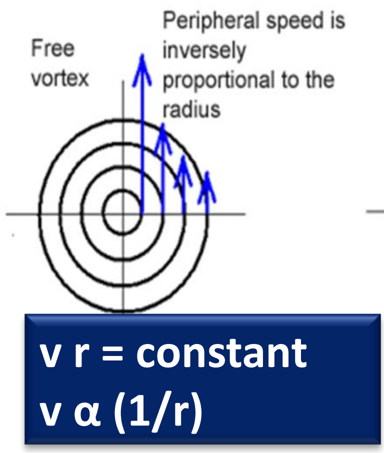
Variation of pressure of a rotating fluid in any plane is given by, $dp = \rho * (\omega^2 r^2/r)dr - \rho gdz$

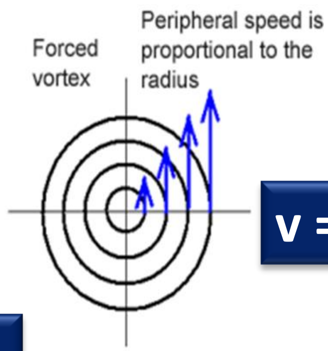
Integrating the above equation for points 1 and 2, we get

$\int dp = \int \rho * (\omega^2 r^2/r)dr - \int \rho gdz$

On simplification, $Z = z_2 - z_1 = v_2^2/2g = (\omega^2 r_2^2/2g)$







P3: Prove that in case of forced vortex, the rise of liquid level at the ends is equal to the fall of liquid level at the axis of rotation.

Solution:

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Let R- Radius of the cylinder

O-O – Initial; level of liquid in the cylinder when the cylinder is not rotating

Initial height of liquid = h+x

Volume of liquid in the cylinder = $\pi R^2 * (h+x)$………..(1)

Let the cylinder is rotated at constant angular velocity ω, the liquid will rise at the ends and will fall at the center.

Let y = Rise of liquid at the ends from O-O

X= Fall of liquid at the c entre O-O

Then the volume of Liquid = [Volume of cylinder up to level B-B] – [Volume of paraboloid]

= πR^2*(height of liquid up to level B-B)- [πR^2*(height of paraboloid]

$= \pi R^2 *(h+x+y) - (\pi R^2/2)*(x+y)$

$= \pi R^2 h + \pi R^2 (x+y) - (\pi R^2/2)*(x+y)$

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$= \pi R^2 h + (\pi R^2/2)*(x+y)$------ (2)

Equating (1) and (2), we get

$\pi R^2 *(h+x) = \pi R^2 h + (\pi R^2/2)*(x+y)$

$\pi R^2 h + \pi R^2 x = \pi R^2 h + (\pi R^2/2)*(x) + (\pi R^2/2)*(y)$

$= \pi R^2 x - (\pi R^2/2)*(x) = (\pi R^2/2)*(y)$

$(\pi R^2/2)*(x) = (\pi R^2/2)*(y)$

x=y; Fall of liquid = Rise of liquid at the ends.

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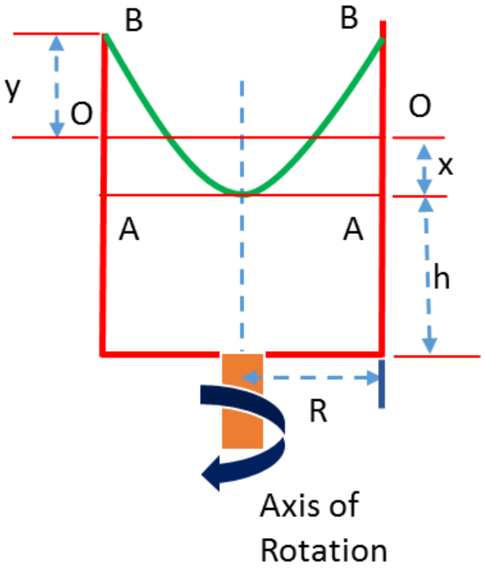

P2: An open circular tank of 20cm diameter and 100 cm long contains water up to a height of 60cm. The tank is rotated about its vertical axis at 300rpm. Find the depth of parabola formed at the free surface of water.

Solution:

Dia. Of the cylinder= 20cm

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Radius, R = 20/2 = 10cm

Height of liquid, H = 60cm

Speed, N = 300rpm

Angular velocity, ω= 2πN/60 = 2*3.14*300/60 = 31.41rad/s

Let the depth of parabola, Z

But Z= $(\omega^2 r_2^2/2g)$, r_2=R

Z= $(\omega^2 R_2^2/2g)$ = (31.41^2)*(10)^2/2*9.81 = 50.28cm

P3: An open circular cylinder of 15cm diameter and 100cm long contains water up to a height of 80cm. Find the maximum speed at which the cylinder is to be rotated about its vertical axis so that no water spills.

Given:

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Dia of the Cylinder = 15cm

Radius, R = 15/2 = 7.5cm

Length of cylinder, L = 100cm

Initial height of water= 80cm

Let the cylinder is rotated at an angular speed of ω rad/sec, when the water is about to spill. Then using,

Rise of liquid at ends= Fall of liquid at center

But rise of liquid at ends = length – initial length

= 100-80=20cm; Fall of liquid at center = 20cm;

Height of Parabola= 20+20=40cm = Z

But Z= $(\omega^2 R_2^2/2g)$, we get

40= ω^2 (7.5 ^{2}/2*981; ω^2 = 1395.2; ω = 37.35rad/s

Speed, N is given by

ω = 2πN/60 ; N= 60* ω/ 2 π = 356.66rpm

Orifices & Mouthpieces

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Orifice: An opening, in a vessel, through which the liquid flows out is known as orifice. This hole or opening is called an orifice, so long as the level of the liquid on the upstream side is above the top of the orifice.

The typical purpose of an orifice is the measurement of discharge. An orifice may be provided in the vertical side of a vessel or in the base. But the former one is more common.

Types of Orifices

Orifices can be of different types depending upon their size, shape, and nature of discharge. But the following are important from the subject point of view.

A. According to size:

- Small orifice
- Large orifice

B. According to shape:

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- Circular orifice
- Rectangular orifice
- Triangular orifice

C. According to shape of edge:

- Sharp-edged
- Bell-mouthed

D. According to nature of discharge:

- Discharging free Orifice
- Fully submerged Orifice
- Partially submerged Orifice

Venacontracta:

Consider an orifice is fitted with a tank. The liquid particles, in order to flow out through the orifice, move towards the orifice from all directions. A few of the particles first move downward, then take a turn to enter into the orifice and then finally flow through it.

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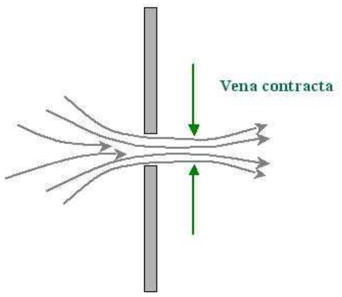
It may be noted, that the liquid particles loose some energy, while taking the turn to enter into the orifice. It can be observed that the jet, after leaving the orifice, gets contracted. **The maximum contraction takes place at a section slightly on the downstream side of the orifice, where the jet is more or less horizontal. Such a section is known as *Vena-Contracta*.**



Hydraulic Coefficients

The following four coefficients are known as *hydraulic coefficients* or *orifice coefficients*.

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- Coefficient of contraction
- Coefficient of velocity
- Coefficient of discharge
- Coefficient of resistance

Coefficient of Contraction

The ratio of the area of the jet, at vena-contracta, to the area of the orifice is known as *coefficient of contraction*.

$$C_c = \frac{Area\ of\ jet\ at\ vena\ contracta}{Area\ of\ the\ orifice}$$

The value of Coefficient of contraction varies slightly with the available head of the liquid, size and shape of the orifice. The average value of Cc is 0.64

Coefficient of Velocity

The ratio of actual velocity of the jet, at Venacontracta, to the theoretical velocity is known as *coefficient of velocity*.The theoretical velocity of jet at Venacontracta is given by the relation, V= √2gh

where *h* is the head of water at Venacontracta

$$C_v = \frac{Actual\ velocity\ of\ the\ jet\ at\ vena\ contracta}{Theoretical\ velocity\ of\ the\ jet}$$

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- The difference between the velocities is due to friction of the orifice.
- The value of Coefficient of velocity varies slightly with the different shapes of the edges of the orifice.
- This value is very small for sharp-edged orifices. For a sharp edged orifice, the value of Cv increases with the head of water.

Coefficient of Discharge

The ratio of a actual discharge through an orifice to the theoretical discharge is known as *coefficient of discharge*. Mathematically coefficient of discharge

$$C_d = \frac{Actual\ discharge}{Theoretical\ discharge} = \frac{Actual\ velocity \times Actual\ area}{Theoretical\ velocity \times Theoretical\ area}$$

$$= C_v \times C_c$$

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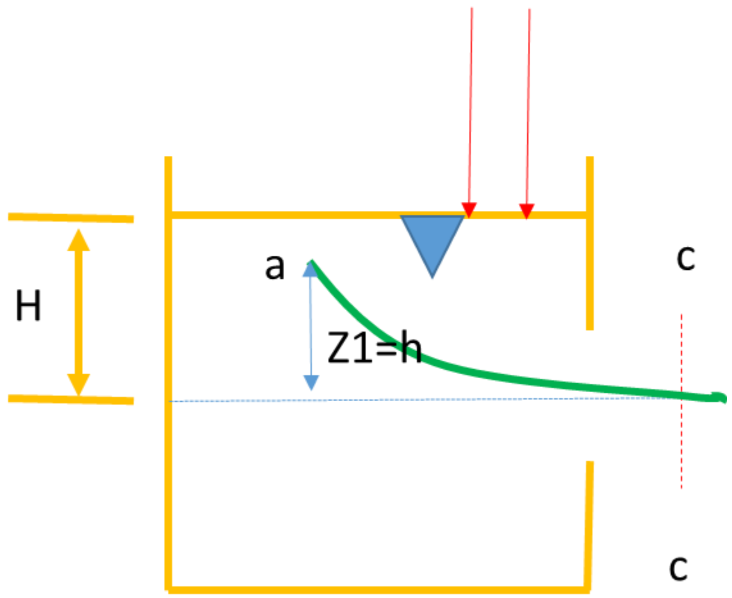
Thus the value of coefficient of discharge varies with the values of Cc and Cv . An average of coefficient of discharge varies from 0.60 to 0.64.

Discharge Through the orifice



The flow through the orifice from a tank is shown in the figure.

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Consider a stream line and on this stream line consider 2 points a and b. Point a is inside the tank and point b is at vena-contracta. Point a is at a depth of h from the center of the orifice and point b is on the center of the orifice.

Let p_1- Intensity of pressure at a

v_1- velocity of flow at a; v_1 is also called Velocity of Approach, v_a

Similarly p_2 and v_2 the corresponding values at b.

Wkt at vena-contracta, Pressure is atmospheric

Streamlines are parallel to each other. Jet has a minimum cross sectional area. Applying Bernoulli's equation between the points a and b, we have

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2$

$p_2/\gamma = p_{atm}$

$p_1/\gamma = p_{atm} + H-h$

$Z1 = h; z2=0$

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Substituting these values in the above equation, we have

$p_{atm}+ (H-h)+v_1^2/ 2g + h= p_{atm}+ v_2^2/ 2g + 0$

$(H)+v_1^2/ 2g = v_2^2/ 2g$

$(H) = v_2^2/ 2g- v_1^2/ 2g;$

$v_1^2= v_a^2$ – Velocity of approach- the velocity at which the fluid particle approaches the orifice and for a large reservoir, it is negligible.

Hence $v_2 = \sqrt{2gH}$; Discharge, $Q = A_2v_2$; A_2 = Area of the jet at vena-contracta

EXPERIMENTAL DETERMINATION OF HYDRAULIC COEFFICIENTS

DETERMINATION OF Cv

Consider an experimental set up as shown in the figure.

Let H the constant head of water in the tank supplied a constant supply as shown in the fig.

Let c- c represents the venacontracta.

Consider a point P (x,y) on the jet. Point P is at a distance of x from c-c and y from the center of the orifice.

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Let t time taken by a fluid particle to move from c-c to point P.

Then the horizontal distance travelled by the fluid particle (x) is given by $x= v_{act}*t$ --------(1)

Where v_{act} is the actual velocity.
Then the vertical distance is given by $S= ut+(1/2)at^2$
In this case S=y; u = 0 and a=g
$y= 0+(1/2)gt^2$; $2y = gt^2$; $t = \sqrt{2y/g}$
But from (1), $v_{act} = x/t$; Wkt $Cv = v_{act} / v_{the}$; $v_{the} = \sqrt{2gh}$;$Cv = (x/t) / \sqrt{2gh}$
$Cv = (x^* \sqrt{g/2y})/ \sqrt{2gh}$
$\mathbf{Cv = \sqrt{x^2/4yh}}$
Determination of Cd
Maintain a constant head in the tank. Note down the head, H in the tank. Note down the length and breadth of the measuring tank. At

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time t=0, note down the level of water in the tank. For a known time interval, t, note down the final water level in the tank.

Then volume of water collected in the tank = Area*depth= $(L*B*(h_2-h_1))$

Actual Discharge, Qa = Volume/time = $(L*B*(h_2-h_1))$ /time

Determination of Cc; Cc= Cd/Cv

Problem 7.2 *The head of water over the centre of an orifice of diameter 20 mm is 1 m. The actual discharge through the orifice is 0.85 litre/s. Find the co-efficient of discharge.*

Solution. Given :

Dia. of orifice, $d = 20 \text{ mm} = 0.02 \text{ m}$

∴ Area, $a = \frac{\pi}{4}(0.02)^2 = 0.000314 \text{ m}^2$

Head, $H = 1 \text{ m}$

Actual discharge, $Q = 0.85 \text{ litre/s} = 0.00085 \text{ m}^3\text{/s}$

Theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1} = 4.429 \text{ m/s}$

∴ Theoretical discharge, $Q_{th} = V_{th} \times \text{Area of orifice}$

$= 4.429 \times 0.000314 = 0.00139 \text{ m}^3\text{/s}$

∴ Co-efficient of discharge $= \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{0.00085}{0.00139} = \mathbf{0.61}$. **Ans.**

Problem 7.8 *A tank has two identical orifices in one of its vertical sides. The upper orifice is 3 m below the water surface and lower one is 5 m below the water surface. If the value of C_v for each orifice is 0.96, find the point of intersection of the two jets.*

Solution. Given :

Height of water from orifice (1), $H_1 = 3$ m

From orifice (2), $H_2 = 5$ m

C_v for both = 0.96

Let P is the point of intersection of the two jets coming from orifices (1) and (2), such that

x = horizontal distance of P

y_1 = vertical distance of P from orifice (1)

y_2 = vertical distance of P from orifice (2)

Then $y_1 = y_2 + (5 - 3) = y_2 + 2$ m

The value of C_v is given by equation (7.6) as

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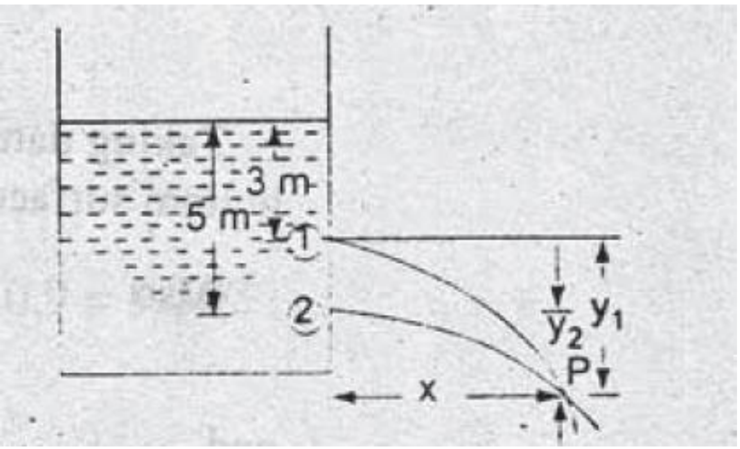

For orifice (1), $C_{v_1} = \frac{x}{\sqrt{4y_1 H_1}} = \frac{x}{\sqrt{4y_1 \times 3.0}}$

For orifice (2), $C_{v_2} = \frac{x}{\sqrt{4y_2 H_2}} = \frac{x}{\sqrt{4 \times y_2 \times 5.0}}$

As both the orifices are identical

$\therefore \quad C_{v_1} = C_{v_2}$

or $\frac{x}{\sqrt{4y_1 \times 3.0}} = \frac{x}{\sqrt{4y_2 \times 5.0}}$ or $3y_1 = 5y_2$

But $y_1 = y_2 + 2.0$

$\therefore \quad 3(y_2 + 2.0) = 5y_2$

$\therefore \quad 2y_2 = 6.0$

From (*ii*), $C_{v_2} = \frac{x}{\sqrt{4y_2 \times H_2}}$

or $0.96 = \frac{x}{\sqrt{4 \times 3.0 \times 5.0}}$

$\therefore \quad x = 0.96 \times \sqrt{4 \times 3.0 \times 5.0} = 7.436$ **m. Ans.**

Time of Emptying a Tank through an Orifice at its Bottom

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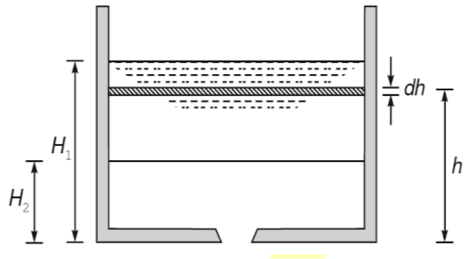
Let us consider a liquid tank of uniform cross section area, A, filled up to a height of H_1. Consider an orifice of cross section area a, fitted at the bottom of the tank. Let in the period of time T, liquid level in the tank falls from H_1 to H_2



Consider an elemental strip of thickness dh at a height h from the bottom. Let it takes time dT for falling the liquid level by small height dh. Therefore, volume of the liquid discharged from the tank in time dT is

$$dq = -A dh$$

Negative sign is used because h is decreasing with the increase in q.

Theoretical velocity through the orifice = $V = \sqrt{2gh}$

Discharge through the small time dT,

$$dq = \text{Coefficient of discharge} \times \text{Area of orifice} \times \text{Theoretical velocity} \times \text{Time}$$

$$dq = C_d a\sqrt{2gh}\, dT$$

Volume of liquid discharging through orifice will be equal to decreased volume of liquid in the tank.

$$\therefore \qquad -Adh = C_d a\sqrt{2gh}\, dT$$

$$dT = \frac{-Adh}{C_d a\sqrt{2gh}}$$

$$\Rightarrow \qquad dT = \frac{-A(h)^{-1/2}}{C_d a\sqrt{2g}}\, dh$$

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Now, total time T for emptying the tank can be calculated by integrating Eq. from H_1 to H_2.

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-A(h)^{-1/2}}{C_d a\sqrt{2g}}\, dh$$

$$T = \frac{-A}{C_d a\sqrt{2g}}\left[\frac{h^{-1/2+1}}{-\frac{1}{2}+1}\right]_{H_1}^{H_2}$$

$$T = \frac{-2A}{C_d a\sqrt{2g}}[\sqrt{H_2} - \sqrt{H_1}]$$

$$T = \frac{2A[\sqrt{H_1} - \sqrt{H_2}]}{C_d a\sqrt{2g}}$$

But, $H_2 = 0$ when tank is empty. So,

$$T = \frac{2A\sqrt{H_1}}{C_d a\sqrt{2g}}$$

Problem 7.17 *A circular tank of diameter 4 m contains water upto a height of 5 m. The tank is provided with an orifice of diameter 0.5 m at the bottom. Find the time taken by water (i) to fall from 5 m to 2 m (ii) for completely emptying to tank. Take $C_d = 0.6$.*

Solution. Given :

Dia. of tank, $D = 4$ m

$\therefore$ Area, $A = \frac{\pi}{4}(4)^2 = 12.566\ m^2$

Dia. of orifice, $d = 0.5$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.5)^2 = 0.1963\ m^2$

Initial height of water, $H_1 = 5$ m

Final height of water, (*i*) $H_2 = 2$ m (*ii*) $H_2 = 0$

First Case. When $H_2 = 2$ m

Using equation (7.11), we have $T = \frac{2A}{C_d \cdot a \cdot \sqrt{2g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$

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$$= \frac{2 \times 12.566}{0.6 \times .1963 \times \sqrt{2 \times 9.81}} \left[\sqrt{5} - \sqrt{2.0}\right] \text{ seconds}$$

$$= \frac{20.653}{0.521} = \textbf{39.58 seconds. Ans.}$$

Second Case. When $H_2 = 0$

$$T = \frac{2A}{C_d \cdot a \cdot \sqrt{2g}} \sqrt{H_1} = \frac{2 \times 12.566 \times \sqrt{5}}{0.6 \times .1963 \times \sqrt{2 \times 9.81}}$$

$$= \textbf{107.7 seconds. Ans.}$$

Problem 7.18 *A circular tank of diameter 1.25 m contains water upto a height of 5 m. An orifice of 50 mm diameter is provided at its bottom. If $C_d = 0.62$, find the height of water above the orifice after 1.5 minutes.*

Solution. Given :

Dia. of tank, $D = 1.25$ m

$\therefore$ Area, $A = \frac{\pi}{4}(1.25)^2 = 1.227 \text{ m}^2$

Dia. of orifice, $d = 50 \text{ mm} = .05$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.05)^2 = .001963 \text{ m}^2$

$C_d = 0.62$

Initial height of water, $H_1 = 5$ m

Time in seconds, $T = 1.5 \times 60 = 90$ seconds

Let the height of water after 90 seconds $= H_2$

Using equation (7.11), we have $T = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2}\right]}{C_d \cdot a \cdot \sqrt{2g}}$

or $$90 = \frac{2 \times 1.227\left[\sqrt{5} - \sqrt{H_2}\right]}{0.62 \times 0.001963 \times \sqrt{2 \times 9.81}} = 455.215\left[2.236 - \sqrt{H_2}\right]$$

$\therefore$ $$\sqrt{H_2} = 2.236 - \frac{90}{455.215} = 236 - 0.1977 = 2.0383$$

$\therefore$ $$H_2 = 2.0383 \times 2.0383 = \textbf{4.154 m. Ans.}$$

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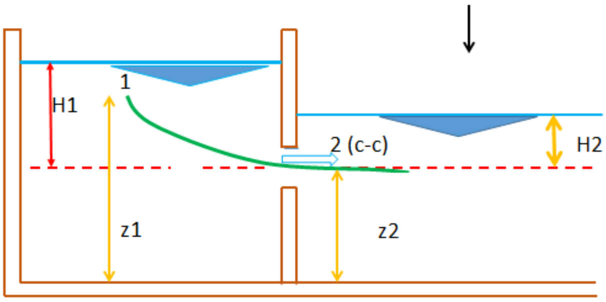
Discharge through Fully Submerged Orifice

In case of a totally submerged orifice, the orifice has a whole submerged under water so that it discharges a jet of water into a water on the downstream side. Let H1 the height of water on the U/S above the center of the orifice and H2 be the height of water on the D/S above the center of the Orifice.



Consider two points, 1 and 2 , point 1 being on the U/S and point 2 being on the D/s and at c-c.

Applying Bernoulli's Equation between 1 and 2, we have,

$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2$

But $p_1/\gamma = (H1+z2-z1)$

$p_2/\gamma = H2$

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Substitute these values in the above equation, we have

$H1+z2-z1+ v_1^2/2g + z_1 = H2+ v_2^2/2g + z_2$

$H1+ v_1^2/2g = H2+ v_2^2/2g$

$(H1- H2) = (v_2^2/2g - v_1^2/2g)$; v_1 = Velocity of approach and may be negligible

Hence $(H1- H2) = (v_2^2/2g)$;

$2g(H1- H2) = v_2^2$

$v_2 = \sqrt{2g(H1- H2)}$; Discharge through the orifice, $Q = Av = A\sqrt{2g(H1- H2)}$

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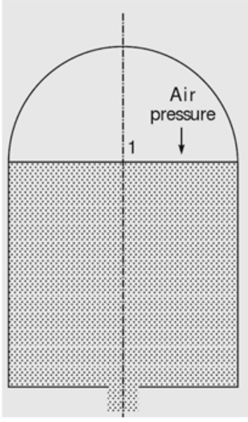
EXAMPLE 7.9 A vessel having air pressure above atmospheric pressure is 7.84 N/cm^2. It contains water up to a height of 2 m. An orifice is placed at the bottom having diameter 200 mm. Calculate the rate of flow of water from orifice if C_d is 0.51.



Solution Given

Diameter, $d = 200 \text{ mm} = 0.02 \text{ m}$

$C_d = 0.51$

Height of water, $H = 2 \text{ m}$

Air pressure, $p = 7.848 \text{ N/cm}^2 = 7.848 \times 10^4 \text{ N/m}^2$

Applying Bernoulli's equation at sections 1 and 2, as shown in Figure 7.5,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

Take datum at section 2 which is very closed to bottom surface. Then, $z_2 = 0$, $z_1 = 2$ m

Also $\frac{p_2}{\rho g} = 0$ (atmospheric pressure)

and $\frac{p_1}{\rho g} = \frac{7.848 \times 10^4}{1000 \times 9.81} = 8 \text{ m}$

$$8 + 0 + 2 = 0 + \frac{v_2^2}{2g} + 0$$

$$\frac{v_2^2}{2g} = 10$$

$$v_2 = \sqrt{2g \times 10}$$

$$v_2 = \sqrt{20 \times 9.81} = 14 \text{ m/s}$$

$$\text{Rate of flow} = C_d \times a_2 \times v_2$$

$$= 0.51 \times \frac{\pi}{4} \times (0.1)^2 \times 14$$

$$= 0.056 \text{ m/s}$$

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EXAMPLE 7.10 If the orifice is 2.5 m wide and 0.5 m deep, compute discharge through totally drowned orifice. Assume the difference of water level of an orifice is 5 m and $C_d = 0.6$.

Solution Given,

Width of orifice, $b = 2.5$ m

Depth of orifice, $d = 0.5$ m

Difference of water level, $H = 5$ m

$C_d = 0.6$

Discharge through orifice, $Q = C_d \times \text{Area} \times \sqrt{2gH}$

$$Q = 0.6 \times 2.5 \times 0.5 \times \sqrt{2 \times 9.81 \times 5}$$

$$Q = 7.4284 \text{ m}^3/\text{s}$$

EXAMPLE 7.11 Water head of an orifice is 3.5 m and 4 m from top and bottom, respectively. The difference of water level on both sides is 55 cm if the width of orifice is 2.5 m. Find the discharge through fully submerged orifice if $C_d = 0.51$.

Solution: Given,

Width of the orifice, $b = 2.5$ m

Difference of water level, $H = 55$ cm $= 0.55$ m

Height of water from top of orifice, $H_1 = 3.5$ m

Height of water from bottom of orifice, $H_2 = 4$ m

$C_d = 0.51$

Discharge through fully submerged orifice, $Q = C_d b(H_2 - H_1)\sqrt{2gH}$

$$Q = 0.51 \times 2.5 \times (4 - 3.5) \times \sqrt{2 \times 9.81 \times 0.55}$$

$$Q = 2.094 \text{ m}^3/\text{s}$$

MOUTHPIECE

Mouthpiece It is a short tube of length not more than two to three times its diameter, provided in a tank or a vessel containing fluid such that it is an extension of the orifice and through which also the fluid may be discharged.

Both orifices and mouthpieces are usually used for measuring the rate of flow of fluid.

Classification: • Cylindrical
• Convergent
• Divergent
• Convergent-divergent

- External mouthpieces
- Internal mouthpieces
- Running full mouthpieces
- Running free mouthpieces

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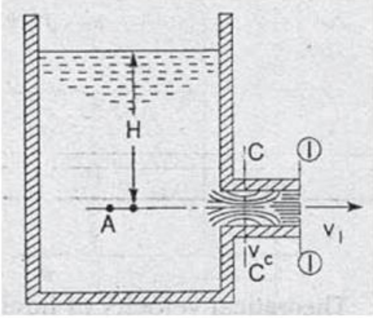
FLOW THROUGH AN EXTERNAL CYLINDRICAL MOUTHPIECE

Consider a tank having an external cylindrical mouthpiece of cross-sectional area a_1, attached to one of its sides as shown in Fig. ...3. The jet of liquid entering the mouthpiece contracts to form a vena-contracta at a section C-C. Beyond this section, the jet again expands and fill the mouthpiece completely.



Let H = Height of liquid above the centre of mouthpiece
v_c = Velocity of liquid at C-C section
a_c = Area of flow at vena-contracta
v_1 = Velocity of liquid at outlet
a_1 = Area of mouthpiece at outlet
C_c = Co-efficient of contraction.

Applying continuity equation at C-C and (1)-(1), we get

$$a_c \times v_c = a_1 v_1$$

$$\therefore \quad v_c = \frac{a_1 v_1}{a_1} = \frac{v_1}{a_c / a_1}$$

But $\dfrac{a_c}{a_1} = C_c$ = Co-efficient of contraction

Taking $C_d = 0.62$, we get $\dfrac{a_c}{a_1} = 0.62$

$$\therefore \quad v_c = \frac{v_1}{0.62}$$

The jet of liquid from section C-C suddenly enlarges at section (1)-(1). Due to sudden enlargement, there will be a loss of head, h_L* which is given as $h_L = \dfrac{(v_c - v_1)^2}{2g}$

But $$v_c = \frac{v_1}{0.62} = \frac{\left(\frac{v_1}{0.62} - v_1\right)^2}{2g} = \frac{v_1^2}{2g}\left[\frac{1}{0.62} - 1\right]^2 = \frac{0.375\, v_1^2}{2g}$$

Applying Bernoulli's equation to point A and (1)-(1)

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

where $z_A = z_1$, v_A is negligible,

$\dfrac{p_1}{\rho g}$ = atmospheric pressure = 0

$$\therefore \quad H + O = O + \frac{v_1^2}{2g} + .375\frac{v_1^2}{2g}$$

$$\therefore \quad H = 1.375\frac{v_1^2}{2g}$$

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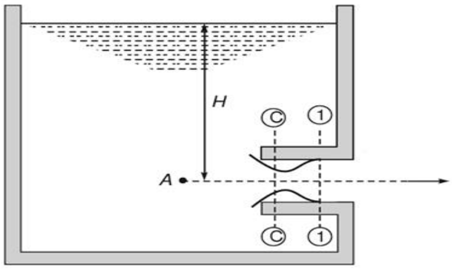
FLOW THROUGH INTERNAL OR RE-ENTRANT ON BORDA'S MOUTHPIECE

Mouthpiece Running Full



Consider a mouthpiece running full shown in Figure Let a_c be the area at vena contracta, a be the area of orifice or mouth piece, v_c be the velocity of liquid at vena contracta, v_1 be the velocity of liquid at section 1-1 and H be the head of the liquid at the centre of the mouthpiece.

Applying continuity equation between sections 1-1 and C-C,

Applying continuity equation between sections 1-1 and C-C,

$$a_c v_c = a_1 v_1$$

$$v_c = \frac{a_1 v_1}{a_c}$$

Coefficient of contraction for an internal mouthpiece is 0.5.

$$C_c = \frac{a_c}{a_1} = 0.5.$$

Therefore, substituting the value in continuity equation, we get,

$$v_c = 2v_1$$

$$\therefore \quad v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\sqrt{2gH}$$

Theoretical velocity of liquid at outlet is $v_{th} = \sqrt{2gH}$

$\therefore$ Co-efficient of velocity for mouthpiece

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855.$$

C_c for mouthpiece = 1 as the area of jet of liquid at outlet is equal to the area of mouthpiece at outlet.

Thus $$C_d = C_c \times C_v = 1.0 \times .855 = 0.855$$

Thus the value of C_d for mouthpiece is more than the value of C_d for orifice, and so discharge through mouthpiece will be more.

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Loss of head due to sudden enlargement after vena contracta will be

$$h_L = \frac{(v_c - v_1)^2}{2g} = \frac{(2v_1 - v_1)^2}{2g} = \frac{v_1^2}{2g}$$

Now, applying Bernoulli's equation between free water surface and section 1.1,

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

Let the datum line passes through the centre line of mouthpiece.

$$0 + 0 + H = 0 + \frac{v_1^2}{2g} + 0 + \frac{v_1^2}{2g}$$

$$H = \frac{v_1^2}{2g} + \frac{v_1^2}{2g} = \frac{v_1^2}{g}$$

$$v_1 = \sqrt{gH}$$

Therefore, the theoretical velocity will be

$$V_{th} = \sqrt{2gH}$$

Coefficient of the velocity will be

$$C_v = \frac{v_1}{V_{th}} = \frac{\sqrt{gH}}{\sqrt{2gH}} = \frac{1}{\sqrt{2}}$$

As the area of jet at outlet is same as the area of the mouthpiece, thus coefficient of contraction will be 1.

$$C_d = C_c C_v = 1 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707$$

Discharge, $Q = C_d a\sqrt{2gH} = 0.707 \times a \times \sqrt{2gH}$

NOTCHS & WEIRS

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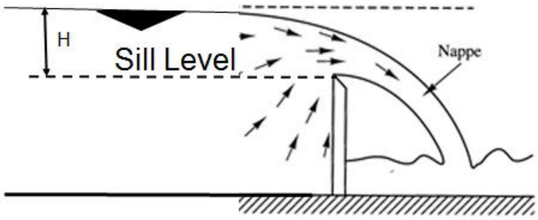
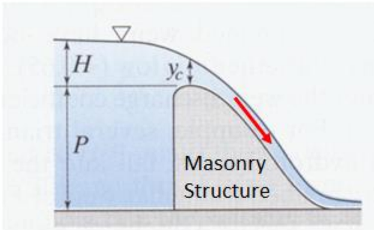
A notch is a device used for measuring the rate of flow of a liquid through a small channel or tank. It is an opening in the side of a measuring tank or reservoir such that the water level is always below the top edge of the opening.

A weir is a concrete or a masonry structure , placed in an open channel over which the flow occurs . It is generally in the form of vertical wall, with Bell mouthed edge.





- The sheet of water flowing over a notch or a weir is known as Nappe.
- The bottom edge of the opening is known as 'Sill' or Crest.

Classification of Notches

The notches are classified as:

- According to the shape of opening
1. Rectangular notch
2. Triangular notch
3. Trapezoidal notch
4. Stepped notch
- According to the effect of the sides on the nappe:
1. Notch with end contraction
2. Notch without end contraction or suppressed notch

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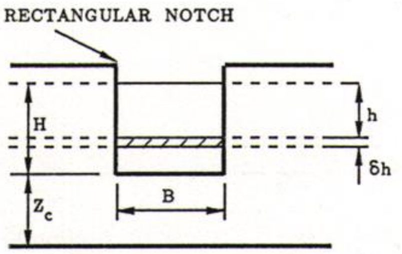
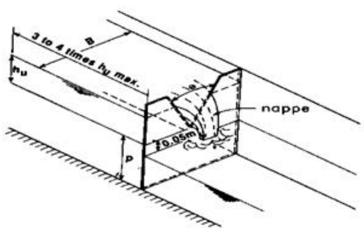
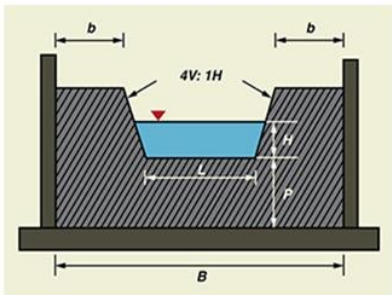
Rectangular notch



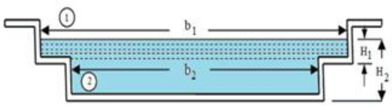
Triangular notch



Trapezoidal notch



Stepped notch



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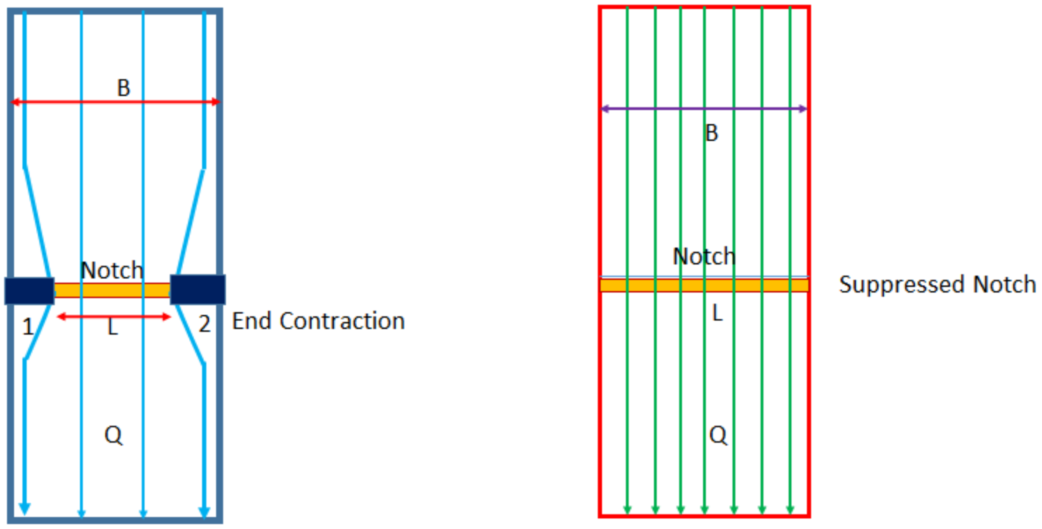
End Contractions



Discharge through a Rectangular Notch

Consider a sharp edge rectangular notch with crest horizontal and normal to direction of flow.

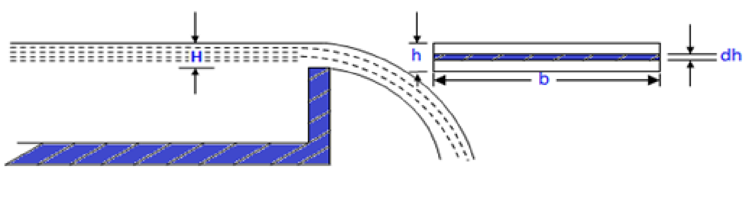
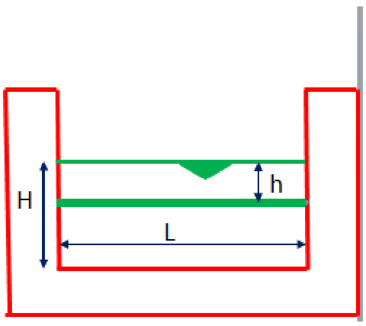


Fig : Rectangular Notch



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Let H = head of water over the crest

L = length of the notch

consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water.

Area of strip = $L \times dh$

Theoretical velocity of water flowing through strip = $\sqrt{2gh}$

The discharge through strip $dQ = C_d \times Ldh \times \sqrt{2gh}$

The total discharge, over the whole notch, may be found by integrating the above equation with in the limits 0 and H.

$$Q = \int_0^H C_d \times L \times \sqrt{2gh} \times dh$$

$$= C_d \times L \times \sqrt{2g} \int_0^H \sqrt{h} dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^H$$

$$= \frac{2}{3} C_d . L . \sqrt{2g} \, (H)^{\frac{3}{2}}$$

$$Q = \frac{2}{3} C_d . L . \sqrt{2g} \, (H)^{\frac{3}{2}}$$

The equation mentioned above is valid for a suppressed notch. But for notches with end contractions, the crest length is not equal to width of the channel. This decrease in crest length results in less discharge . According to J B Francis, the decrease in crest length depends only on head and for each end contraction, the crest length reduction is equal to H/10 or 0.1H

If there are 'n' end contractions, then
$Q = (2/3) C_d \sqrt{2g} \, (L - 0.1nH) \, H^{3/2}$

For a rectangular notch, n=2
$Q = (2/3) C_d \sqrt{2g} \, (L - 0.2H) \, H^{3/2}$ m^3/s

Ventilation of Notches/ Weirs:

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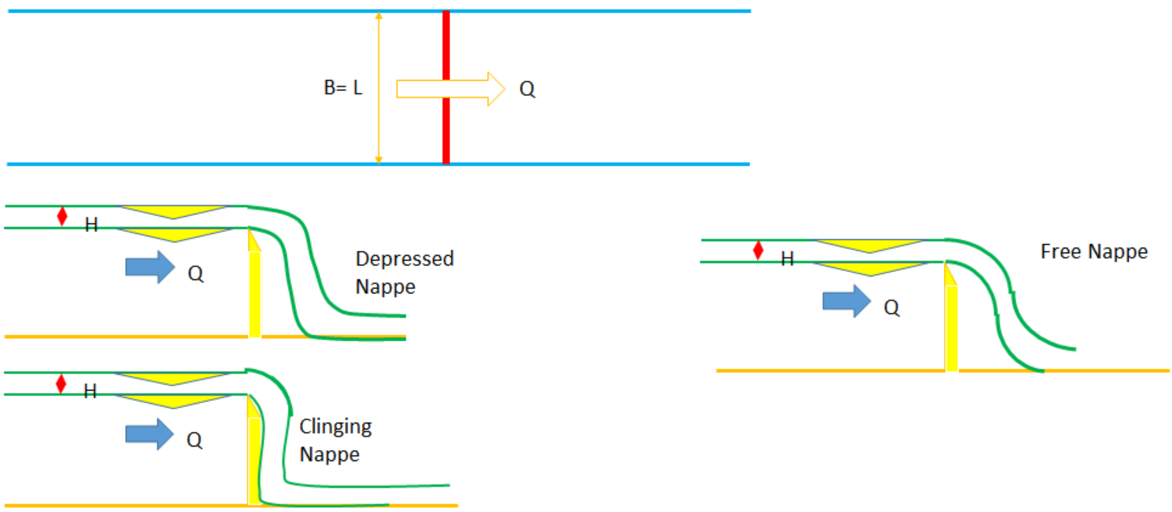

- In case of a suppressed notch, the free nappe emerging from the notch is in contact with the sides of the channel
- On account of this, air is trapped between the notch and the nappe.
- This air is gradually carried away by the flowing water.
- At the same time, the pressure of entrapped air reduces from atmospheric to suction pressure.
- Due this, the nappe is drawn towards the notch. This nappe is called ‘Depressed Nappe’
- If this process continues, at one stage, the nappe clings to the wall of the notch. This nappe is called ‘Clinging Nappe’.
- This is not desirable because it draws more water, thus more actual discharge for a given head over the notch.
- In order to avoid this, usually vents are provided in both the sides of the wall so that the entrapped air freely circulates with the atmospheric air.
- Another way of avoiding this is by providing a masonry or concrete structure between the notch and the nappe, then the structure behaves as a weir.

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A rectangular notch 0.5m wide has constant head of 400 mm. Find the discharge over the notch in liters per second, if the coefficient of discharge for the notch is 0.62.

Given,

- b = 0.5 m
- H = 400 mm = 0.4 m
- C_d = 0.62

We know that discharge over the rectangular notch,

$$Q = \frac{2}{3}C_d.b\sqrt{2g}(H)^{\frac{3}{2}}\ m^3/s$$

$$\Rightarrow Q = \frac{2}{3}\times 0.62\times 0.5\sqrt{2\times 9.81}(0.4)^{\frac{3}{2}}\ m^3/s$$

$$\Rightarrow Q = 0.915\times 0.253 = 0.231\ m^3/s = 231\ liters/s$$

Discharge through Triangular Notch

Triangular notch is also called V-notch .

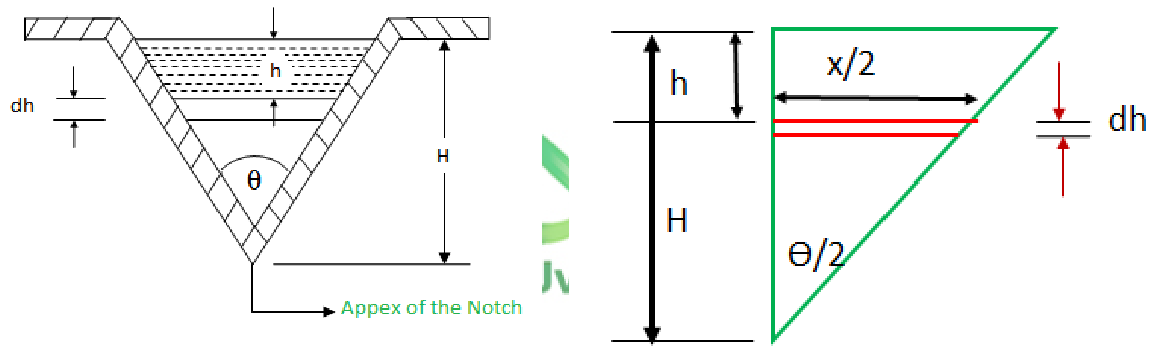

Fig : Triangular Notch

Let H =head of water above V-notch

=Angle of notch (Apex angle)

Consider a horizontal strip of the water having thickness dh at depth h below the free surface of water. If ‘x’ is the width of the strip, then we have,

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{H-h}$$

$$AC = (H-h)\tan\left(\frac{\theta}{2}\right)$$

AC= x/2 = (H-h)tan (Θ/2); x= 2(H-h)tan (Θ/2)

Area of the strip= dA = x*dh ;dA= (2(H-h)tan (Θ/2)) dh

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Area of strip, dA= $2(H-h)\tan\frac{\theta}{2}\times dh$

The theoretical velocity of water through strip = $\sqrt{2gh}$
Discharge through the strip, dQ:

$$dQ = C_d \times 2(H-h)\tan\frac{\theta}{2}\times dh \times \sqrt{2gh}$$

$$dQ = 2C_d\,(H-h)\tan\frac{\theta}{2}\times\sqrt{2gh}\times dh$$

Total discharge Q:

$$Q = \int_0^H 2C_d\,(H-h)\tan\frac{\theta}{2}\times\sqrt{2gh}\times dh$$

$$Q = 2C_d\tan\frac{\theta}{2}\times\sqrt{2g}\int_0^H (H-h)h^{\frac{1}{2}}dh$$

$$Q = 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{Hh^{\frac{3}{2}}}{\frac{3}{2}}-\frac{h^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^H$$

$$Q = 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{2}{3}H^{\frac{5}{2}}-\frac{2}{5}H^{\frac{5}{2}}\right]$$

$$Q = 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{4}{15}H^{\frac{5}{2}}\right]$$

$$Q = \frac{8}{15}C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\times H^{\frac{5}{2}}$$

For a right angle V-notch , if C_d =0.6

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$$\theta = 90, \tan\frac{\theta}{2} = 1$$

Discharge Q;

$$Q = \frac{8}{15}\times 0.6\times 1\times\sqrt{2\times 9.81}\times H^{\frac{5}{2}}$$

$$Q = 1.417H^{\frac{5}{2}}$$

- **Advantages of triangular notch over a rectangular notch:**

1. The expression for discharge for a right angled V-notch or weir is very simple.
2. For low discharges a triangular notch gives more accurate results than rectangular notch.
3. Reasonably stable value of discharge coefficient over a wide range of operating condition.

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4. Ventilation of triangular notch is not required.

A right-angled **V-notch** was used to measure the discharge of a centrifugal pump. If the depth of water at V-notch is 200mm, calculate the discharge over the notch in liters per minute. Assume coefficient of discharge as 0.62.

Given,

- $\theta = 90°$
- $H = 200\ mm = 0.2\ m$
- $C_d = 0.62$

We know that the discharge over the triangular notch,

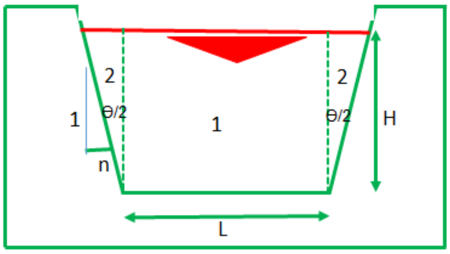
$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\frac{\theta}{2} \times H^{\frac{5}{2}}$$

$$\Rightarrow Q = \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \tan 45° \times (0.2)^{\frac{5}{2}}$$

$$\Rightarrow Q = 1.465 \times 0.018 = 0.026 m^3/s$$

$$\therefore Q = 26\ liters/s = 1560\ liters/min$$

Trapezoidal Notch



Consider a trapezoidal notch with side slopes 1:n

In this case, Discharge is given by Q= Q1+Q2

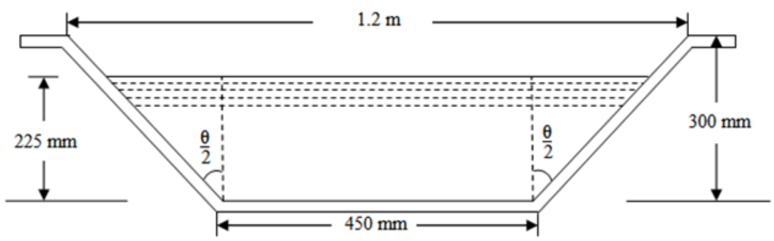
Discharge through the Rectangular notch (Q1)+

Discharge through V notch(Q2).

Q=[Q1= (2/3)Cd √2g (L-0.2H) $H^{3/2}$

+Q2= (8/15)Cd tan(Θ/2)√2g $H^{5/2}$]

A trapezoidal notch of 1.2m wide at the top and 450mm at the bottom is 300mm high. Find the discharge through the notch, if the head of water is 225mm. Take coefficient of discharge as 0.6.



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Given,

- Width of the notch = 1.2m
- b = 450mm = 0.45m
- Height of the notch = 300mm = 0.3m
- H = 225mm = 0.225m
- C_d = 0.6

From the geometry of the notch, we get,

$$\tan\frac{\theta}{2} = \frac{1200-450}{2} \times \frac{1}{300} = \frac{750}{600} = 1.25$$

and the discharge over trapezoidal notch,

$$Q = \frac{2}{3}C_d.b\sqrt{2g}(H)^{\frac{3}{2}} + \frac{8}{15}C_d\sqrt{2g}\tan\frac{\theta}{2} \times H^{\frac{5}{2}}$$

$$= \frac{2}{3} \times 0.6 \times 0.45\sqrt{2\times 9.81} \times (0.225)^{\frac{3}{2}} + \frac{8}{15} \times 0.6\sqrt{2\times 9.81} \times 1.25 \times (0.225)^{\frac{5}{2}}\ m^3/s$$

$$= 0.085 + 0.043 = 0.128\ m^3/s = 128\ liters/s$$

Cipolletti Notch

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- It is a trapezoidal notch that is often used .
- This is a fully contracted notch in which the notch ends (sides) are not vertical, as they are for a rectangular notch.
- The effects of end contraction are compensated by this trapezoidal notch type, meaning that mathematical corrections for end corrections are unnecessary and the equation is simpler.
- The side slopes of the notch are designed to correct for end contraction, splayed out at an angle of 14° with the vertical, or nearly 1 H to 4V (tan 14° ≈0.2493)

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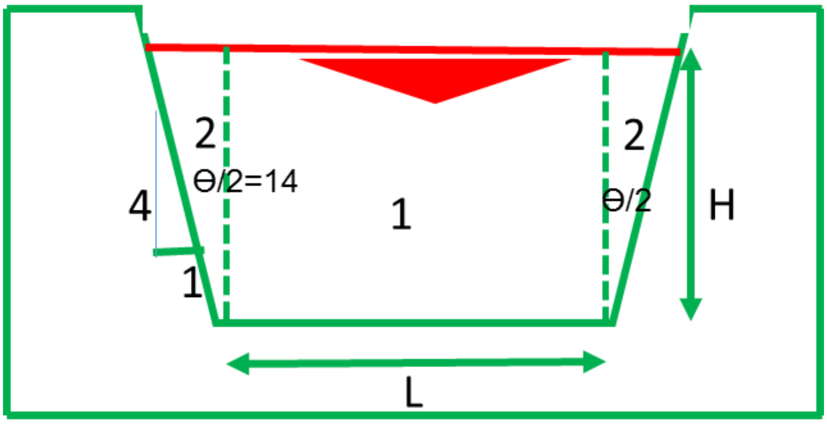

Cipolletti notch is made of Rectangular notch and a triangular notch.

Discharge through Cipolletti is given by Q= Q1+Q2

Discharge through the Rectangular notch (Q1)+ Discharge through V notch(Q2).

$Q=[(2/3)Cd \sqrt{2g} (L-0.2H) H^{3/2} +(8/15)Cd \sqrt{2g} \tan(\Theta/2) H^{5/2}]$

$Q=[(2/3)Cd \sqrt{2g} (L-0.2H) H^{3/2} +(8/15)Cd * \sqrt{2g} * 1/4 * H^{5/2}]$

$Q=[(2/3)Cd \sqrt{2g} (L-0.2H) H^{3/2} +(2/15)Cd * \sqrt{2g} * H^{5/2}]$

$Q= (2/3)Cd \sqrt{2g} [(LH^{3/2} -0.2H^{5/2} + H^{5/2}/5]$

$Q= (2/3)Cd \sqrt{2g} [(LH^{3/2} -0.8H^{5/2} /5]$

$Q= (2/3)Cd \sqrt{2g} LH^{3/2}$

Velocity of approach

The velocity with which the water approaches or reaches the weir or notch before it flows over it is known as velocity of approach.

The velocity of approach , Va is determined by finding the discharge over the notch or weir neglecting velocity of approach . Then dividing the discharge by the cross sectional area of the channel on the upstream side of the weir or notch , the velocity of approach is obtained.

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Mathematically,

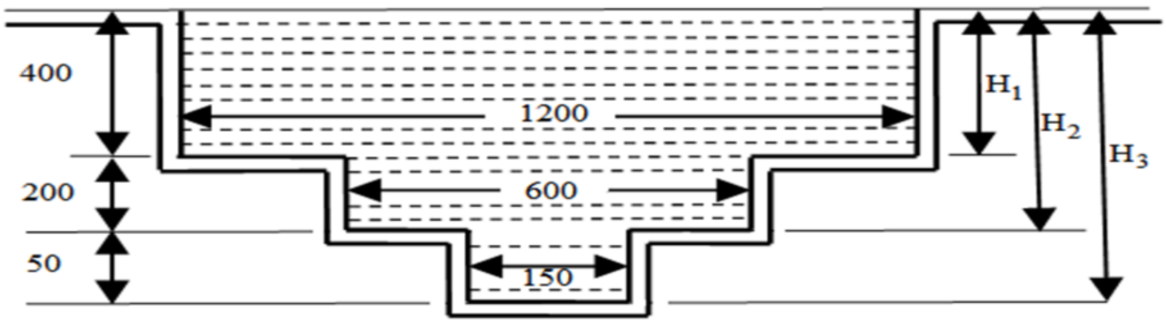
$$V_a = \frac{Q}{A}$$

Discharge over a rectangular weir , with velocity of approach,

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g} \left[(H_1 + H_a)^{\frac{3}{2}} - H_a^{\frac{3}{2}} \right]$$

Discharge through a stepped notch

A stepped notch is a combination of rectangular notches . The discharge through a stepped notch is equal to the sum of the discharge through the different rectangular notches.



Consider a stepped notch .

Let H=Height of water above the crest of notch 1,

L1=Length of notch 1,

H2,L2 and H3 , L3 are corresponding values for notches 2 and 3 respectively. Coefficient of discharge for all notches=Cd

Total discharge, Q=Q1+Q2+Q3

$$Q = \frac{2}{3} C_d L_1 \sqrt{2g} \left[H_1^{\frac{3}{2}} - H_2^{\frac{3}{2}} \right] \frac{2}{3} + C_d L_2 \sqrt{2g} \left[H_2^{\frac{3}{2}} - H_3^{\frac{3}{2}} \right] + \frac{2}{3} C_d L_3 \sqrt{2g} \times H_3^{\frac{3}{2}}$$

Time of emptying a reservoir or tank with rectangular notch:

Consider a tank or reservoir of uniform cross sectional area A and a rectangular notch provided in one of its side.

let L= length of crest of notch

H1 = initial height of liquid above the crest of notch

H2= final height of liquid above the crest of notch

T= time required form H1 to H2

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Let at any instant, the height of liquid surface above the crest of the notch be h and in a small time dT, let the liquid surface falls by 'dh'. Then

-Adh=Q*dT

- Ve sign is taken as with the increase of T, h decreases.

Q= (2/3)Cd $\sqrt{2g}$ $Lh^{3/2}$

= (2/3)Cd $\sqrt{2g}$ $Lh^{3/2}$

-Adh=Q*dT

-Adh = [(2/3)Cd $\sqrt{2g}$ $Lh^{3/2}$]*dT

dT = -Adh/ [(2/3)Cd $\sqrt{2g}$ $Lh^{3/2}$]

The total time is obtained by integrating the above equation between the limits H1 to H2

= / [(2/3)Cd $\sqrt{2g}$ $Lh^{3/2}$]

$$T = \frac{3A}{C_d \times L \times \sqrt{2g}}\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right]$$

Effect on Discharge over a notch or weir due to error in the measurement of head

Discharge over the notches is directly proportional to the head over the notches. Hence one must be careful in noting the head over the notches to get an accurate value of the discharge.

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A small error in the measurement of head will affect the discharge considerably.

1. For Rectangular Notch
2. For Triangular Notch

Rectangular Notch

The discharge for a rectangular notch is given by

Q= (2/3)Cd $\sqrt{2g}$ $LH^{3/2}$

Q= $kH^{3/2}$ .... (1)

where k = (2/3)Cd $\sqrt{2g}$ L

Differentiating equation (1), we get

dQ= k*(3/2)$H^{1/2}$ dh ....(2)

Dividing (2)by (1)

dQ/Q = k*(3/2)$H^{1/2}$ dh / $kH^{3/2}$

dQ/Q = 3/2 * (dH/H) --------- (3)

Above equation shows that an error of 1% in measuring H will produce 1.5% error in discharge over a rectangular notch.

Triangular Notch

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The discharge for a Triangular notch is given by

$Q = (8/15)C_d \sqrt{2g}\tan(\Theta/2)H^{5/2}$

$Q = kH^{5/2}$ ……… (1)

where $k = (8/15)C_d \sqrt{2g}\tan(\Theta/2)$

Differentiating equation (1), we get

$dQ = k*(5/2)H^{3/2}\,dh$ …………..(2)

Dividing (2)by (1)

$dQ/Q = k*(5/2)H^{3/2}\,dh / kH^{5/2}$

$dQ/Q = 5/2 * (dH/H)$ --------- (3)

Above equation shows that an error of 1% in measuring H will produce 2.5% error in discharge over a Triangular notch.

Weirs

- Generally, any flow obstruction that causes water to rise to flow over it, but used exclusively for intentional obstructions
- Uses include flow measurement (sharp-crested weirs) and control of water surface profile, e.g., by inducing super-critical flow (broad-crested weirs)
- According to the shape of crest :

1. Sharp – crested weir
2. Broad –crested weir
3. Narrow – crested weir
4. Ogee – shaped weir

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- According to the effect of sides on the emerging nappe:

1. Weir with end contraction
2. Weir without end contraction


Rectangular weir


V-notch weir


Broad-crested weir

References:

1. Fluid Mechanics- R .K.Bansal – e- book.