SEMESTER VI

17BECE601 STRUCTURAL ANALYSIS II 3 2 0 4100

OBJECTIVES:

- To understand the influence line concepts for indeterminate structures
- To understand the methods of analysis of intermediate trusses for external loads, lack of fit and thermal effect
- To study behaviour of arches and their methods of analysis
- To know the concept and analysis of cable stayed bridge
- To study the multi storey frames subjected to gravity loads and lateral loads

UNIT I **12**

Indeterminate structures - Slope deflection method - Continuous beams and fixedbeam– Simplification of hinged end – support settlement - Simple frames - Portal frames Consistent-deformation method-continuous beams.

UNIT II **12**

Strain energy method- Castigliano's theorem- Deflection by strain energy method – evaluation of strain energy in member under different loading – Application of strain energy method for Beams and frames - Beams curved in plan.

UNIT III **12**

Flexibility method -Equilibrium and Compatibility – Determinate vs Indeterminate structures – Indeterminacy – Primary Structure – Compatibility conditions – Analysis of indeterminate pin – jointed plane frames, continuous beams, rigid jointed plane frames (with redundancy restricted to two).

UNIT IV **12**

Stiffness method-Beams-Trusses-Simple frames-Portal frames-Grids-Lack of fit-Temperature stresses-Support settlements-Elastic supports.(Direct approach)- Introduction to Finite element.

UNIT V **12**

Plastic Analysis of Structures : Statically indeterminate axial problems – Beams in pure bending – Plastic moment of resistance – Plastic modulus – Shape factor – Load factor – Plastic hinge and mechanism – Plastic analysis of indeterminate beams and frames.

TOTAL:60HRSTEXT BOOKS:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Strength of materials and Theory of Structures Vol.I, II	Dr.B.C.Punmia	Laxmi Publication, New Delhi	2012

REFERENCE BOOKS:

Sl.No	Title of the Book	Author of the Book	Publisher	Year of Publishing
1	Intermediate Structural Analysis	C.K. Wang	McGraw-Hill, New Delhi	2002
2	Matrix Analysis of Framed structures	W.Weaver and J.M Gere	Van NostrandReinhold,New York	2003
3	Structural analysis, a matrix approach	G.S.Pandit and S.P.Gupta	Tata McGraw Hill	2004
4	Theory of structures	S.Ramamrutham&R.Narayan	DhanpatRai Publishing Co, New Delhi	2013
5	Analysis of Structures-Vol.II	Prof.V.N. Vazirani, Dr.M.M.Ratwani, Dr.S.K.Duggal	Khanna Publishers, Chennai	2012

WEBSITES:

- http://www.icivilengineer.com
- http://www.engineeringcivil.com/
- http://www.aboutcivil.com/
- http://www.engineersdaily.com
- http://www.asce.org/
- http://www.cif.org/
- http://icevirtuallibrary.com/
- http://www.ice.org.uk/
- http://www.engineering-software.com/ce/

COURSE OUTCOMES

On completion of the course, the students will be able to:

- Demonstrate the concepts of qualitative influence line diagram for continuous beams and frames
- Apply the methods of indeterminate truss analysis.
- Demonstrate the behavior of arches and their methods of analysis.
- Analyse cable suspension bridges.
- Analyse multistory frames subjected to gravity loads and lateral loads.

CE6501 STRUCTURAL ANALYSIS I
UNIT IV
SLOPE DEFLECTION METHOD

4.1 Continuous beams without sway

4.2 Continuous beams with sway

4.3 Rigid frames without sway

4.4 Rigid frames with sway

4.5 Symmetry and antisymmetry

4.6 Simplification for hinged end

4.7 Support displacements

INTRODUCTION

- The slope deflection method is a structural analysis method for beams & frames introduced in 1914 by George A. Maney.
- This method considers the deflection as the primary unknowns, while the redundant forces were used in the force method . Hence this method is the displacement method.
- In the slope-deflection method, the relationship is established between moments at the ends of the members and the corresponding rotations and displacements.
- An important characteristic of the slope-deflection method is that it does not become increasingly complicated to apply as the number of unknowns in the problem increases. In the slope-deflection method the individual equations are relatively easy to construct regardless of the number of unknowns

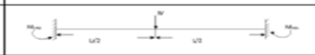
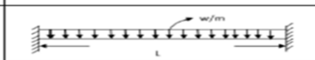
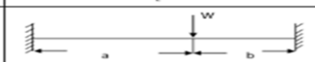
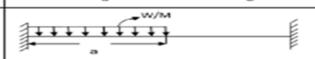
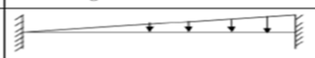
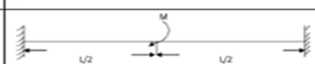
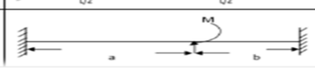
Assumptions In The Slope- Deflection Method

- The material of the structure is linearly elastic.
- The structure is loaded with in elastic limit.
- Axial displacements ,Shear displacements are neglected.
- Only flexural deformations are considered.
- All joints are considered rigid.

SIGN CONVENTION

- Clockwise moment and clockwise rotation are taken as negative ones.
- The down ward displacements of the right end with respect to the left end of horizontal member is considered as positive.
- The right ward displacement of upper end with respect to lower end of a vertical member is taken as positive.

Fixed end moments

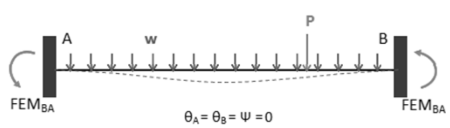
S.No	CASES	BOTH ENDS FIXED	
		M_{FAB}	M_{FBA}
1		$-\frac{wl}{8}$	$+\frac{wl}{8}$
2		$-\frac{wl^2}{12}$	$+\frac{wl^2}{12}$
3		$-\frac{wab^2}{l^2}$	$+\frac{wab^2}{l^2}$
4		$-\frac{wa}{12l^2}(6l^2-8al+3a^2)$	$+\frac{wa^3}{12l^2}(4l^2-3a)$
5		$-\frac{wl^2}{30}$	$+\frac{wl^2}{20}$
6		$+\frac{m}{4}$	$+\frac{m}{4}$
7		$\frac{Mb}{l^2}(3a-l)$	$\frac{Ma}{l^2}(3b-l)$

General Procedure OF Slope-Deflection Method

- Find the fixed end moments of each span (both ends left & right).
- Apply the slope deflection equation on each span & identify the unknowns.
- Write down the joint equilibrium equations.
- Solve the equilibrium equations to get the unknown rotation & deflections.
- Determine the end moments and then treat each span as simply supported beam subjected to given load & end moments so we can work out the reactions & draw the bending moment & shear force diagram.

Slope deflection equation

Slope deflection equation for a beam AB,



- The moments that would develop at the ends of such a fixed beam are referred to as fixed-end moments and their expression can be obtained by setting $\theta_A = \theta_B = \Psi = 0$; that is,

- We find that the second terms on the right sides of Eqs. 6 are equal to the fixed-end moments.

$$M_{AB} = \frac{2EI}{L}(2\ _A + \ _B - \quad + FEM_{AB} \qquad (1)$$

$$M_{BA} = \frac{2EI}{L}(\ _A + 2_B - 3\quad) + \quad _{BA} \qquad (2)$$

- Equations (1&2), which express the moments at the ends of a member in terms of its end rotations and translations for a specified external loading, are called slope-deflections equations.
- These equations are valid for prismatic members, composed of linearly elastic material and subjected to small deformations.
- The deformations due to axial and shear forces are neglected.

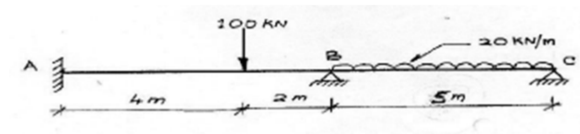
- The two slope-deflection equations have the same form and either end of equations can be obtained from the other simply by switching the subscript A and B.

$$M_{nf} = \frac{2EI}{L}\left(2 \;_{n} + \;_{f} \quad - \qquad\qquad _{nf}\right. \qquad (9)$$

in which the subscript n refers to the near end of the member where moment M_{nf} acts and the subscript f identifies the far (other) end of the member.

4.1 Continuous beams without sway
Problem: 1

Analyze two span continuous beam ABC by slope deflection method. Then draw Bending moment & Shear force diagram. Take EI constant.



Step: 1

Fixed end moments are

$$M_{AB=}\ -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44 \text{ KNm}$$

$$M_{BA=}\ \frac{Wa^2b}{L^2} = \frac{100 \times 4^2 \times 2}{6^2} = 88.89\text{KNm}$$

$$M_{BC=}\ -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67\text{KNm}$$

$$M_{CB=}\ \frac{wL^2}{12} = \frac{20 \times 5^2}{12} = 41.67 \text{ KNm}$$

Since A is fixed $\Theta_A = 0 \;\&\; \Theta_B \;\&\; \Theta_c \neq 0$

Step: 2

Slope deflection equations are

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -44.44 + \frac{2EI}{6}\theta_B = -44.44 + \frac{EI}{3}\theta_B \quad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 88.89 + \frac{4EI}{6}\theta_B = 88.89 + \frac{2EI}{3}\theta_B \quad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C \quad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B \quad \cdots\cdots(4)$$

Step: 3 Equilibrium equations:

$$M_{BA} + M_{BC} = 0$$

$M_{CB} = 0$ as end C is simply supported.

$$M_{BA} + M_{BC} = 88.89 + \frac{2EI}{3}\theta_B - 41.67 + \frac{4EI}{5}\theta_B + \frac{2EI}{5}\theta_C = 47.22 + \frac{22}{15}EI\theta_B + \frac{2}{5}EI\theta_C = 0 \quad \cdots\cdots(5)$$

$$M_{CB} = 41.67 + \frac{4EI}{5}\theta_C + \frac{2EI}{5}\theta_B = 0 \quad \cdots\cdots(6)$$

Solving the equations (5) & (6), we get $\theta_B = -\frac{20.83}{EI}$

$\theta_C = -\frac{41.67}{EI}$

Step: 4 Final moments

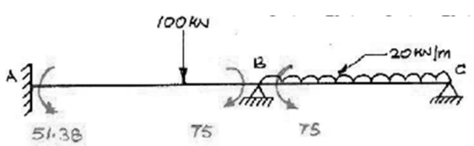
$$M_{AB} = -44.44 + \frac{EI}{3}\left(-\frac{20.83}{EI}\right) = -51.38 \text{ KNm}$$

$$M_{BA} = 88.89 + \frac{2EI}{3}\left(-\frac{20.83}{EI}\right) = 75 \text{ KNm}$$

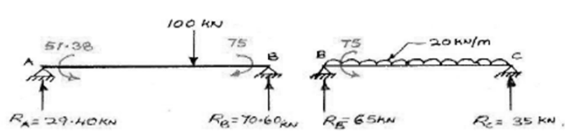
$$M_{BC} = -41.67 + \frac{4EI}{5}\left(-\frac{20.83}{EI}\right) + \frac{2EI}{5}\left(-\frac{41.67}{EI}\right) = -75 \text{ KNm}$$

$$M_{CB} = 41.67 + \frac{4EI}{5}\left(-\frac{41.67}{EI}\right) + \frac{2EI}{5}\left(-\frac{20.83}{EI}\right) = 0$$

Step: 5 BMD & SFD



Reactions: Consider the free body diagram of the beam



Find reactions using equations of equilibrium.

Span AB: $\Sigma M_A = 0$, $R_B \times 6 = 100\times4+75-51.38$

$\therefore R_B = 70.60$ KN

$\Sigma V = 0$, $R_A + R_B = 100$KN

$\therefore R_A = 100-70.60 = 29.40$ KN

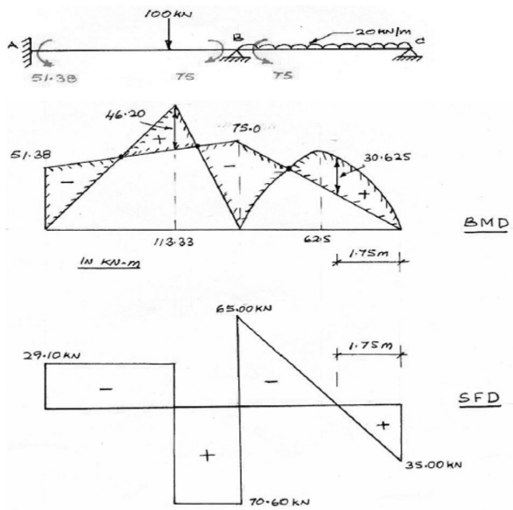
Span BC: $\Sigma M_C = 0$, $R_B \times 5 = 20\times5\times\frac{5}{2}+75$

$\therefore R_B = 65$ KN

$\Sigma V = 0$ $R_B + R_C = 20\times5 = 100$KN

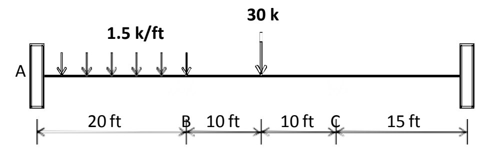
$R_C = 100-65 = 35$ KN

Using these data BM and SF diagram can be drawn



PROBLEM: 2

- Toillustrate the basic concept of the slope-deflection method, consider the three-span continuous beam shown in Figure below.



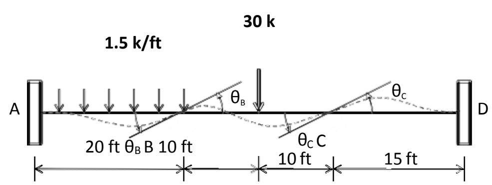
EI = constant
E = 29,000 ksi
I = 500 in4

Although the structure actually consists of a single continuous beam between the fixed supports A and D, for the purpose of analysis it is considered to be composed of three members, AB, BC, and CD, rigidly connected at joints A, B, C, and D located at the supports of the structure.

Degrees of Freedom

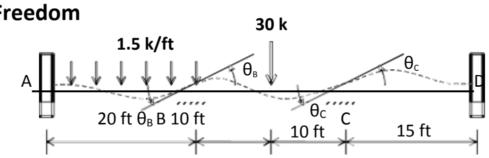
Identify the unknown independent displacements (translations and rotations) of the joints of the structure. These unknown joint displacements are referred to as the degrees of freedom of the structure.

From the qualitative deflected shape of the continuous beam shown in Figure below, we can see that none of its joints can translate.



The fixed joints A and D cannot rotate, whereas joints B and C are free to rotate.

Degrees of Freedom



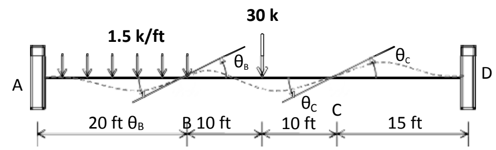
This beam has two degrees of freedom, θ_B and θ_C, which represent the unknown rotationsof joints B and C, respectively.

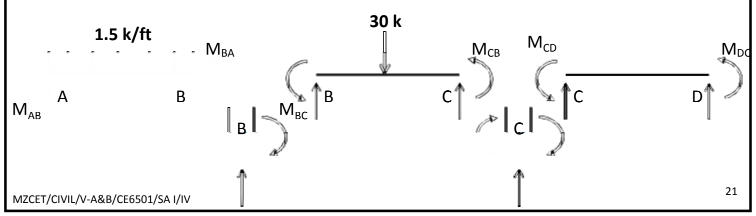
The number of degrees of freedom is sometimes called the degree of kinematic indeterminacy of the structure. This beam is kinematically indeterminate to the second degree.

A structure without any degrees of freedom is termed kinematically determinate.

Equations of Equilibrium

The unknown joint rotations are determined by solving the equations of equilibrium of the joints that are free to rotate. The free body diagrams of the members and joints B and C of the continuous beam are shown.



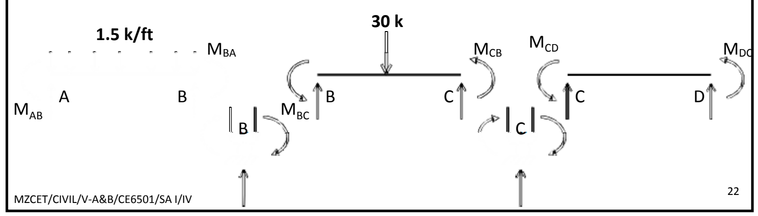


Equations of Equilibrium

In addition to the external loads, each member is subjected to an internal moment at each of its ends.

The correct senses of the member end moments are not yet known, it is assumed that the moments at the ends of all the members are positive (counterclockwise).

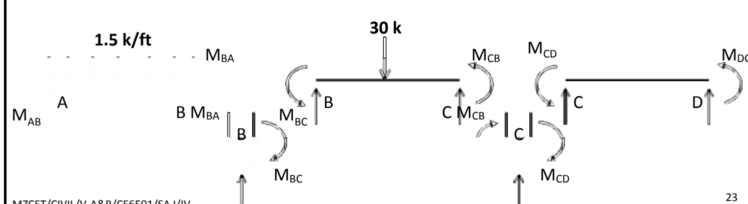
The free body diagrams of the joints show the member end moments acting in an opposite (clockwise) direction in accordance with Newton's law of action and reaction.



Equations of Equilibrium

Because the entire structure is in equilibrium, each of its members and joints must also be in equilibrium. By applying the moment equilibrium equations $\sum M_B = 0$ and $\sum M_C = 0$, respectively, to the free bodies of joints B and C, we obtain the equilibrium equations

$$M_{BA} + M_{BC} = 0 \quad (17a)$$
$$M_{CB} + M_{CD} = 0 \quad (17b)$$



Slope-Deflection Equations

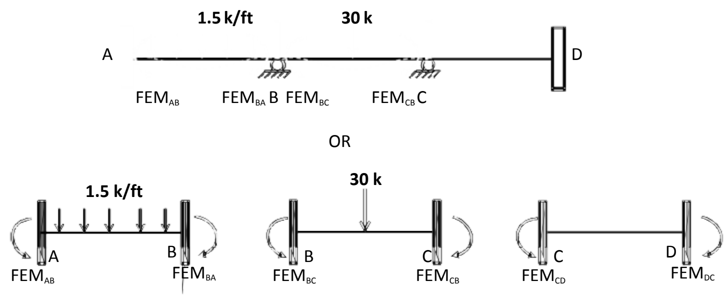
The equilibrium equations Eqs. (17) can be expressed in terms of the unknown joint rotations, θ_B and θ_C, by using slope-deflection equations that relate member end moments to the unknown joint rotations.

Before we can write the slope-deflection equations, we need to compute the fixed-end moments due to the external loads acting on the members of the continuous beam.

To calculate the fixed-end moments, we apply imaginary clamps at joints B and C to prevent them from rotating.

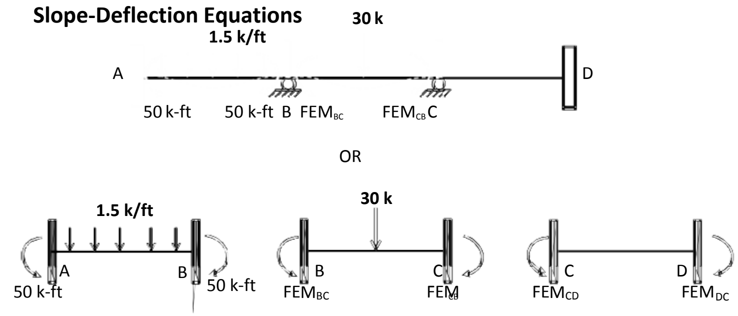
Or we generally provide fixed-supports at the ends of each member to prevent the joint rotations as shown.

Slope-Deflection Equations



The fixed-end moments that develop at the ends of the members of this fully restrained or kinematically determinate structure can easily be evaluated by using the fixed-end moment expressions given inside the back cover of book.

Slope-Deflection Equations

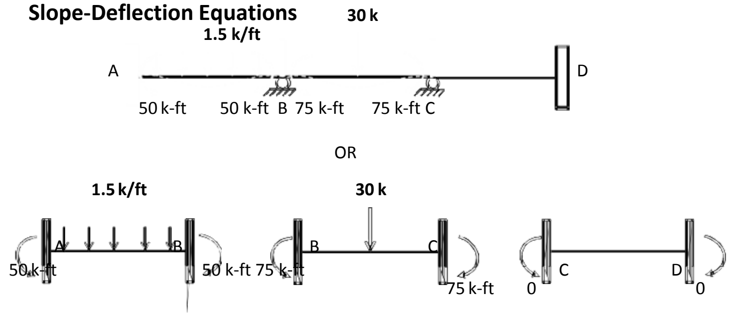


For member AB:

$$FEM_{AB} = \frac{wL^2}{12} = \frac{1.5(20)^2}{12} = 50k - ft$$

$$FEM_{BA} = \frac{wL^2}{12} = \frac{1.5(20)^2}{12} = 50k - ft$$

Slope-Deflection Equations



For member BC:

$$FEM_{BC} = \frac{PL}{8} = \frac{30(20)}{8} = 75k - ft$$

$$FEM_{CB} = \frac{PL}{8} = \frac{30(20)}{8} = 75k - ft$$

Slope-Deflection Equations

The slope-deflection equations for the three members of the continuous beam can now be written by using Eq. (9).

Since none of the supports of the continuous beam translates, the chord rotations of the three members are zero ($\Psi_{AB} = \Psi_{BC} = \Psi_{CD} = 0$).

Also, supports A and D are fixed, the rotations $\theta_A = \theta_D = 0$. By applying Eq. (9) for member AB, with A as the near end and B as the far end, we obtain the slope-deflection equation

$$M_{AB} = \frac{2EI}{20}\left(0 + {}_{B} - 0\right) + 50 = 0.1EI_B \quad + \qquad (18a)$$

Next, by considering B as the near end and A as the far end, we write

$$M_{BA} = \frac{2EI}{20}\left(2{}_{B} + 0 - 0\right) - 50 = 0.2EI_B \quad - \qquad (18b)$$

Slope-Deflection Equations

Similarly, by applying Eq. (9) for member BC, we obtain

$$M_{BC} = \frac{2EI}{20}(2\ _{B} + \ _{C}) + 75 = 0.2EI\ _{B} + 0.1EI\ _{C} + 75 \qquad (18c)$$

$$M_{CB} = \frac{2EI}{20}(2\ _{C} + \ _{B}) - 75 = 0.2EI\ _{C} + 0.1EI\ _{B} - 75 \qquad (18d)$$

and for member CD,

$$M_{CD} = \frac{2EI}{15}(2\ _{C}) = 0.267EI\ _{C} \qquad (18e)$$

$$M_{DC} = \frac{2EI}{15}(\ _{C}) = 0.133EI\ _{C} \qquad (18f)$$

Joint Rotations

To determine the unknown joint rotations θ_B & θ_C, we substitute the slope-deflection equations Eqs. (18) into the joint equilibrium equations Eqs. (17) and solve the resulting systems of equations simultaneously for θ_B & θ_C. By substituting Eqs. (18b) and (18c) into Eq. (17a), we obtain

$$(0.2EI\theta_B - 50) + (0.2EI\theta_B + 0.1EI\theta_C + 75) = 0$$

$$\text{or} \qquad 0.4EI\theta_B + 0.1EI\theta_C = -25 \qquad (19a)$$

and by substituting Eqs. (18d) and (18e) into Eq. (17b), we get

$$(0.2EI\theta_C + 0.1EI\theta_B - 75) + 0.267EI\theta_C = 0$$

$$\text{or} \qquad 0.1EI\theta_B + 0.467EI\theta_C = 75 \qquad (19b)$$

Joint Rotations

Solving Eqs. (19a) & (19b) simultaneously for $EI\theta_B$ and $EI\theta_C$, we obtain

$$EI\theta_B = -108.46\ k\ \ ft^2$$

$$EI\theta_C = 183.82\ k - ft_2$$

By substituting the numerical values of E = 29,000 ksi = 29,000(12)$_2$ ksf and I = 500 in.$_4$, we determine the rotations of joints B and C to be

$$\theta_B = -0.011\,rad \qquad \text{or} \qquad 0.011\,rad$$

$$\theta_C = 0.0018\ rad$$

Member End Moments

The moments at the ends of the three members of the continuous beam can now be determined by substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations (Eqs. 18).

$$M_{AB} = 0.1(-108.46) + 50 = 39.2\ k\text{-}ft$$

$$M_{BA} = 0.2(-108.46) - 50 = -71.7\ k\text{-}ft \qquad \text{or } 71.7\ k\text{-}ft$$

$$M_{BC} = 0.2(-108.46) + 0.1(183.82) + 75 = 71.7\,k\text{-}ft$$

$$M_{CB} = 0.2(183.82) + 0.1(-108.46) - 75 = -49.1\,k\text{-}ft \qquad \text{or } 49.1\ k\text{-}ft$$

$$M_{CD} = 0.267(183.82) = 49.1\,k\text{-}ft$$

$$M_{DC} = 0.133(183.82) = 24.4\ k\text{-}ft$$

Member End Moments

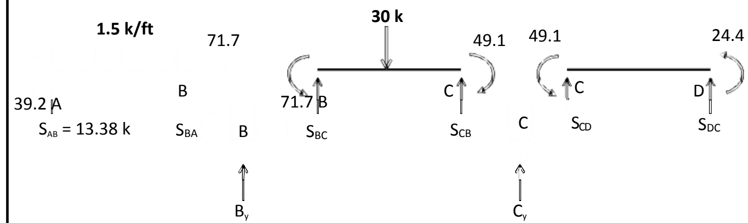
To check that the solution of simultaneous equations (Eqs. 19) has been carried out correctly, the numerical values of member end moments should be substituted into the joint equilibrium equations (Eqs. 17). If the solution is correct, then the equilibrium equations should be satisfied.

$$M_{BA} + M_{BC} = -71.7 + 71.7 = 0 \qquad \textit{Checks}$$

$$M_{CB} + M_{CD} = -49.1 + 49.1 = 0 \qquad \textit{Checks}$$

The member end moments just computed are shown on the free body diagrams of the members and joints in Figure on next slide.

Member End Moments



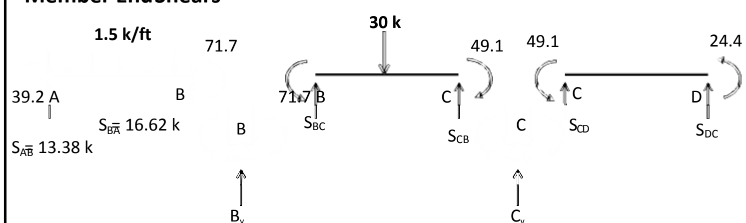
Member End Shears

The shear forces at the ends of members can now be determined by applying the equations of equilibrium to the free bodies of members. For member AB,

$$+\circlearrowleft \sum M_B = 0 \quad 39.2 - S_{AB}(20) + 1.5(20)(10) - 71.7 = 0$$

$$S_{AB} = 13.38\ k \uparrow$$

Member End Shears

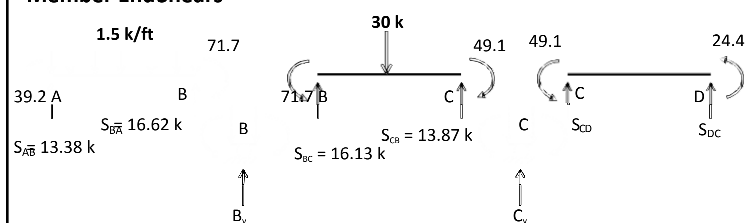


For member AB,

$$+\uparrow \sum F_y = 0 \qquad 13.38 - 1.5(20) + S_{BA} = 0$$

$$S_{BA} = 16.62k \uparrow$$

Member End Shears



For member BC,

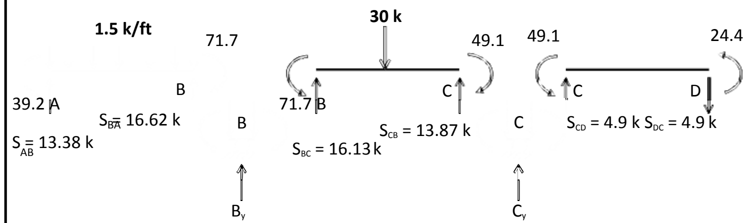
$$+\circlearrowleft \sum M_C = 0 \qquad 71.7 - S_{BC}(20) + 30(10) - 49.1 = 0$$

$$S_{BC} = 16.13\ k \uparrow$$

$$+\uparrow \sum F_y = 0 \qquad 16.13 - 30 + S_{CB} = 0$$

$$S_{CB} = 13.87k \uparrow$$

Member End Shears



For member CD,

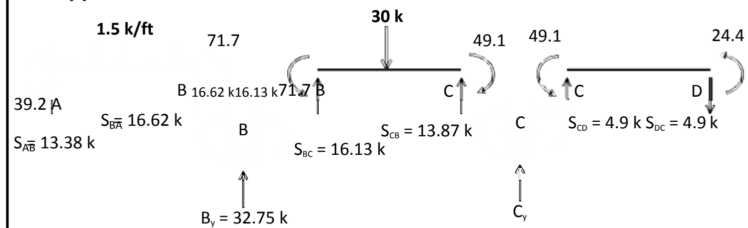
$$+\circlearrowleft \sum M_D = 0 \qquad 49.1 - S_{CD}(15) + 24.4 = 0$$

$$S_{CD} = 4.9\ k \uparrow$$

$$+\uparrow \sum F_y = 0 \qquad 4.9 + S_{DC} = 0$$

$$S_{DC} = 4.9\ k \downarrow$$

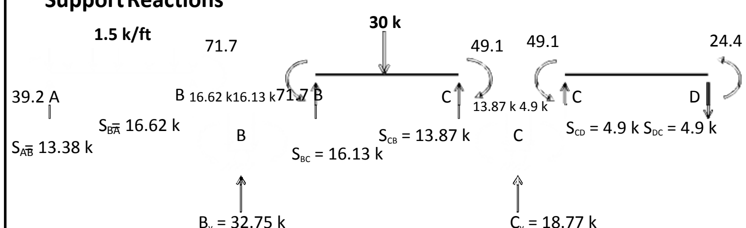
Support Reactions



From the free body diagram of joint B, we can see that the vertical reaction at the roller support B is equal to the sum of the shears at ends B of member AB and BC; that is

$$B_y = S_{BA} + S_{BC} = 16.62 + 16.13 = 32.75\ k \uparrow$$

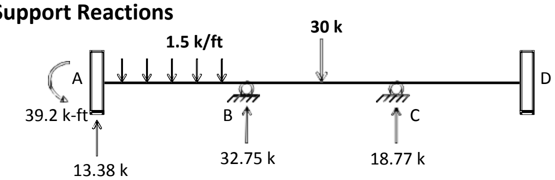
Support Reactions



The vertical reaction at the roller support C equals the sum of shears at ends C of members BC and CD.

$$C_y = S_{CB} + S_{CD} = 13.87 + 4.9 = 18.77\ k \uparrow$$

Support Reactions

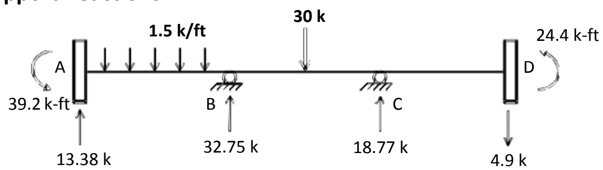


The reactions at the fixed support A are equal to the shear and moment at the end A of member AB.

$$A_y = S_{AB} = 13.38\ k \uparrow$$

$$M_A = M_{AB} = 39.2\ k-ft \circlearrowright$$

Support Reactions

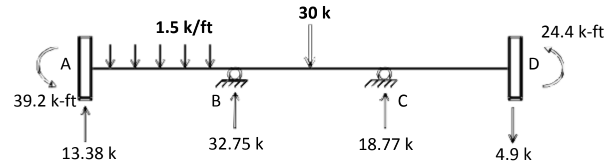


The reactions at the fixed support D equal the shear and moment at end D of the member CD.

$$D_y = S_{DC} = 4.9\ k \downarrow$$

$$M_D = M_{DC} = 24.4\ k-ft$$

Equilibrium Check



To check out computations of member end shears and support reactions, we apply the equations of equilibrium to the free body of the entire structure.

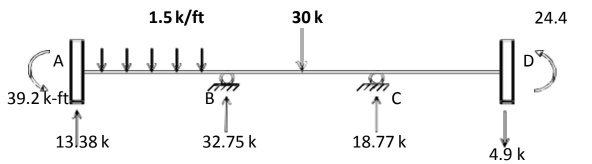
$$+\uparrow \sum F_y = 0$$

$$13.38 - 1.5(20) + 32.75 - 30 + 18.77 - 4.9 = 0 \qquad \text{Checks}$$

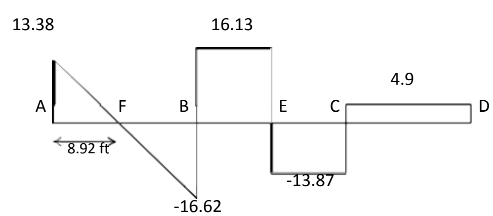
$$+\sum M_D = 0$$

$$39.2 - 13.38(55) + 1.5(20)(45) - 32.75(35) + 30(25)$$

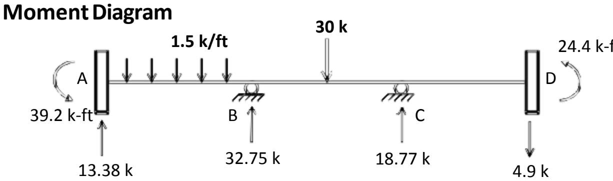
$$-18.77(15) + 24.4 = -0.1 \approx 0 \qquad \text{Checks}$$



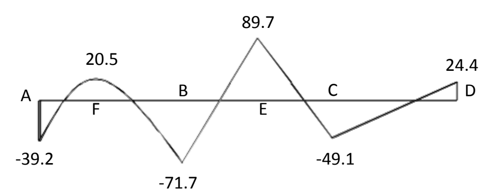
Using General sign conventions

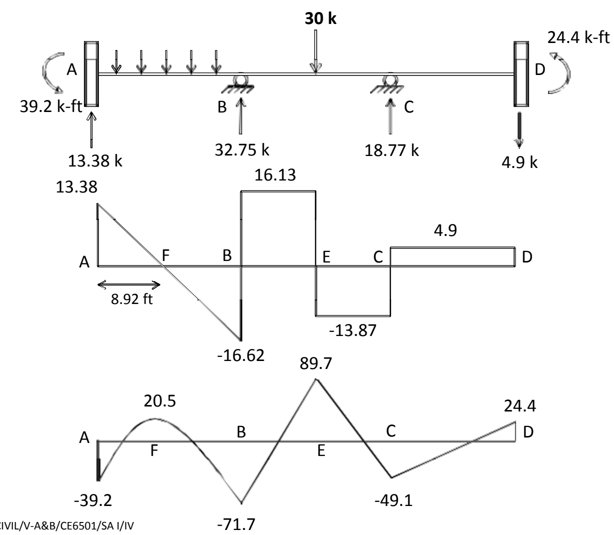


Moment Diagram



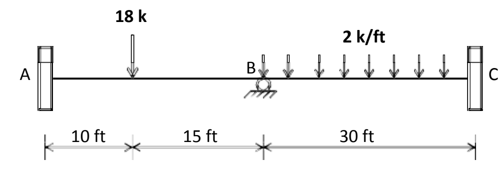
Using General sign conventions





PROBLEM: 3

- Determine the reactions and draw the shear and bending moment diagrams for the two-span continuous beam shown in Figure.

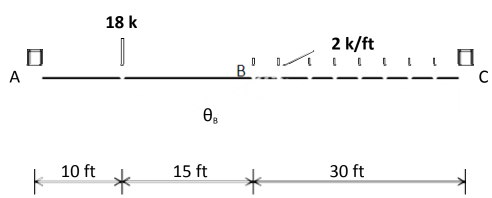


EI = constant

Solution

1. Degree of Freedom

We can see that only joint B of the beam is free to rotate. Thus, the structure has only one degree of freedom, which is the unknown joint rotation, θ_B.



EI = constant

2. Fixed-End Moments

By using the fixed-end moment expressions given inside the back cover of the book, we evaluate the fixed-end moments due to the external loads for each member.

$$FEM_{AB} = \frac{wab^2}{L^2} = \frac{18(10)(15)^2}{25^2} = 64.8\ k-ft \qquad or + 64.8\ k\text{-}ft$$

$$FEM_{BA} = \frac{wa^2b}{L^2} = \frac{18(10)^2(15)}{25^2} = 43.2\ k-ft \qquad or - 64.8 k\text{-}ft$$

$$FEM_{BC} = \frac{wL^2}{12} = \frac{2(30)^2}{12} = 150\ k-ft \qquad or +150\ k\text{-}ft$$

$$FEM_{CB} = 150\ k-ft \qquad or -150\ k\text{-}ft$$

Counterclockwise FEM are positive, whereas clockwise FEM are negative.

3. Chord Rotations

Since no support settlements occur, the chord rotations of both members are zero; that is, $\Psi_{AB} = \Psi_{BC} = 0$.

4. Slope-Deflection Equations

To relate the member end moments to the unknown joint rotation, θ_B, we write the slope deflection equation for the two members of the structure by applying Eq. (9).

$$M_{nf} = \frac{2EI}{L}\big(2\,{}_{n} + {}_{f} \quad - \qquad {}_{nf} \tag{9}$$

since the supports A and C are fixed, the rotations $\theta_A = \theta_C = 0$.

4. Slope-Deflection Equations

Slope-Deflection Equation for Member AB

$$M_{AB} = \frac{2EI}{25}\left(\ {}_{B}\right) + 64.8 = 0.08EI\ {}_{B} \quad + \tag{1}$$

$$M_{BA} = \frac{2EI}{25}\left(2\ {}_{B}\right) - 43.2 = 0.16EI\ {}_{B} \quad - \tag{2}$$

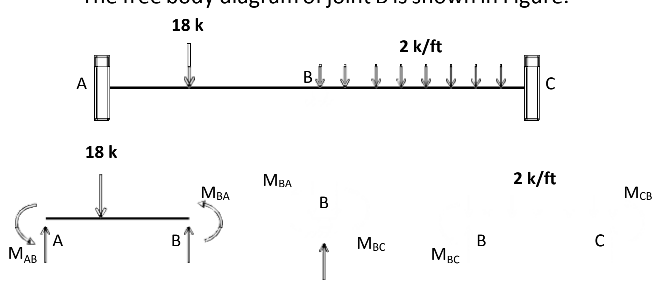
Slope-Deflection Equation for Member BC

$$M_{BC} = \frac{2EI}{30}\left(2\ {}_{B}\right) + 150 = 0.133EI\ \ {}_{B} \tag{3}$$

$$M_{CB} = \frac{2EI}{30}\left(\ {}_{B}\right) - 150 = 0.0667EI\ \ {}_{B} - 150 \tag{4}$$

5. Equilibrium Equations

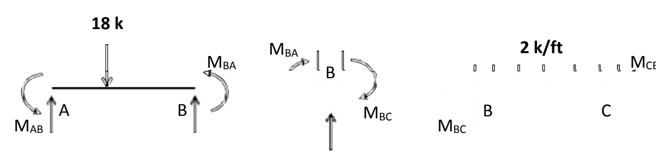
The free body diagram of joint B is shown in Figure.



Member end moments, which are assumed to be in counterclockwise direction on the ends of members, must be applied in (opposite) clockwise direction on the free body of the joint in accordance with Newton's Third Law.

5. Equilibrium Equations

The free body diagram of joint B is shown in Figure.



By applying the moment equilibrium equation $\sum M_B = 0$ to the free body of the joint B, we obtain

$$M_{BA} + M_{BC} = 0 \tag{5}$$

6. Joint Rotations

To determine the unknown joint rotations, θ_B, substitute the slope deflection equations (Eqs. 2 & 3) into the equilibrium equation (Eq. 5).

$$\left(0.16EI\ _{B}-43.2\right)+\left(0.133EI\ _{B}+150\right)=0$$

or

$$0.293EI\ _{B}=-106.8$$

fromwhich

$$EI\ _{B}=-364.5\ k-ft_{2}$$

7. Member End Moments

The member end moments can now be computed by substituting the numerical value of $EI\theta_B$ back into the slope-deflection equation (Eqs. 1 to 4).

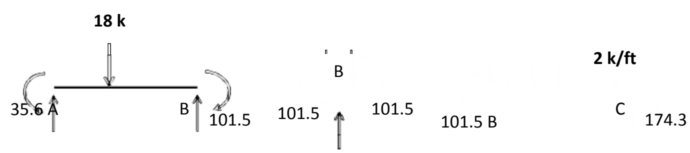
$$M_{AB}=0.08(-364.5)+64.8=35.6\ k-ft$$

$$M_{BA}=0.16(-364.5)-43.2=-101.5\,k-ft \qquad or \qquad 101.5\,k\text{-}ft$$

$$M_{BC}=0.133(-364.5)+150=101.5\,k-ft$$

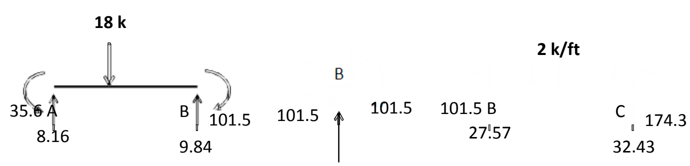
$$M_{CB}=0.0667(-364.5)-150=-174.3k-ft \qquad or \qquad 174.3\,k\text{-}ft$$

Positive answer for an end moment indicates that its sense is counterclockwise, whereas a negative answer implies a clockwise sense. As M_{BA} and M_{BC} are equal in magnitude but oppositein sense, the equilibrium equation $M_{BA}+M_{BC}=0$ is satisfied.



8. Member End Shears

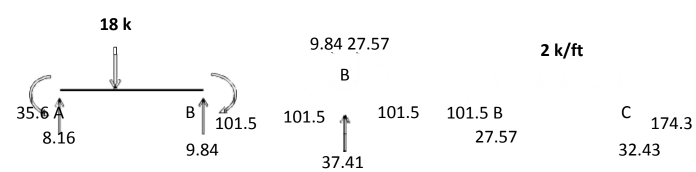
The member end shears, obtained by considering the equilibrium of each member, are shown in figure below



9. Support Reactions

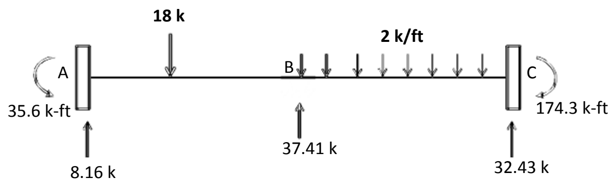
The reactions at the fixed support A and C are equal to the forces and moments at the ends of the members connected to these joints. To determine the reaction at roller support B, consider the equilibrium of the free body of joint B in the vertical direction.

$$B_y=S_{BA}+S_{BC}=9.84+27.57=37.41\ k\uparrow \qquad \text{ANS}$$



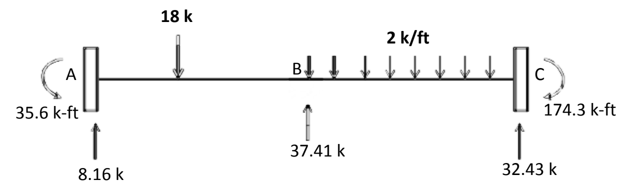
9. Support Reactions

The support reactions are shown in figure below.



10. Equilibrium Check

To check our calculations of member end shears and support reactions, we apply the equations of equilibrium to the free body of the entire structure.

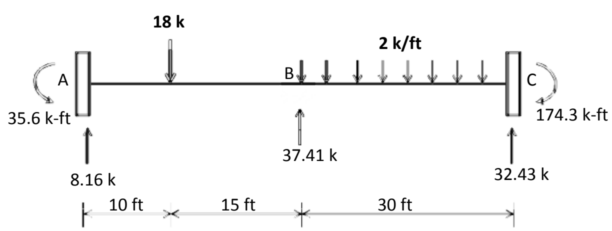


$+\uparrow \sum F_y = 0$

$8.16 - 18 + 37.41 - 2(30) + 32.43 = 0$ Checks

10. Equilibrium Check

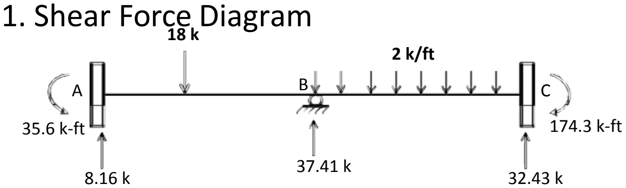
To check our calculations of member end shears and support reactions, we apply the equations of equilibrium to the free body of the entire structure.

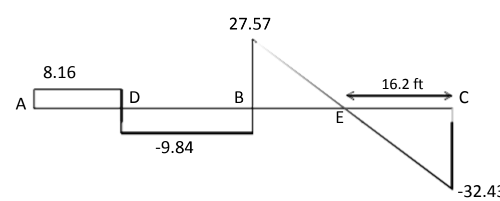


$+ \sum M_C = 0$

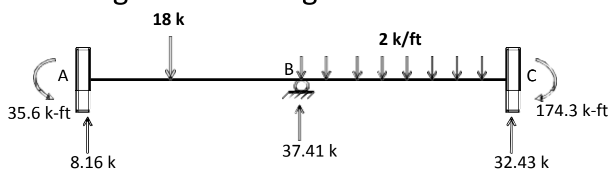
$35.6 - 8.16(55) + 18(45) - 37.41(30) + 2(30)(15) - 174.3 = 0.2 \approx 0$ Checks

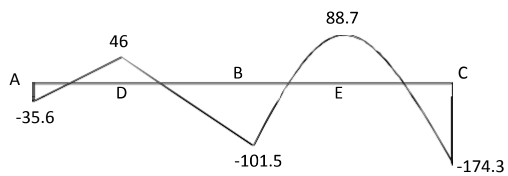
11. Shear Force Diagram





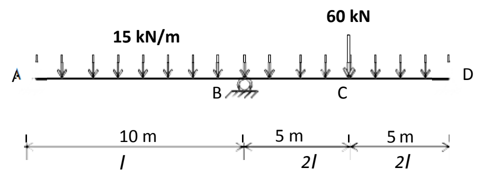
11. Bending Moment Diagram





PROBLEM: 4

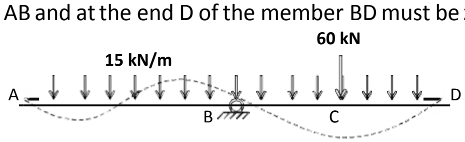
- Determine the reactions and draw the shear and bending moment diagrams for the continuous beam shown in Figure.

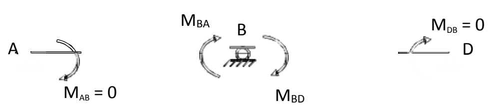


EI = 200 GPa
I = 700 (10⁶) mm⁴

Solution

- From figure we can see that all three joints of the beam are free to rotate. Thus the beam have 3 degrees of freedom, θ_A, θ_B, θ_D.
- The end supports A and D of the beam are simple supports at which no external moment is applied, the moments at the end A of the member AB and at the end D of the member BD must be zero.





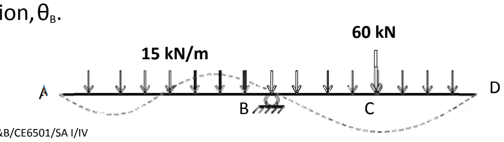
Solution

- The ends A and D can be considered as hinged ends and the modified slope-deflection equations can be used.

$$M_{rh} = \frac{3EI}{L}(\theta_r - \psi) + \left(FEM_{rh} - \frac{FEM_{hr}}{2}\right) \qquad (15a)$$

$$M_{hr} = 0 \qquad (15b)$$

- The modified SDE do not contain the rotations of the hinged ends, by using these equations the rotations θ_A, and θ_D of the simple supports can be eliminated, which will then involve only one unknown joint rotation, θ_B.



1. Degree of Freedom

θ_B

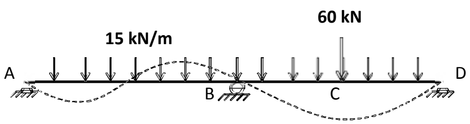
2. Fixed-End Moments

$$FEM_{AB} = \frac{15(10)^2}{12} = 125\ kN-m \qquad or\ +125\ kN\text{-}m$$

$$FEM_{BA} = 125\ kN-m \qquad or\ -125\ kN\text{-}m$$

$$FEM_{BD} = \frac{60(10)}{8} + \frac{15(10)^2}{12} = 200\ kN-m \qquad or\ +200\ kN\text{-}m$$

$$FEM_{DB} = 200\ kN-m \qquad or\ -200\ kN\text{-}m$$



3. Slope-Deflection Equations

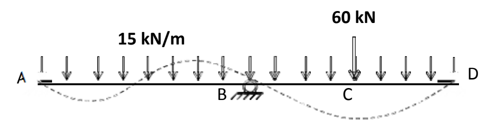
Since both members of the beam have one end hinged, we use Eqs. 15 to obtain the slope-deflection equations for both members.

$$M_{AB} = 0 \qquad \text{ANS}$$

$$M_{BA} = \frac{3EI}{10}(\theta_B) + \left(-125 - \frac{125}{2}\right) = 0.3EI\theta_B - 187.5 \qquad (1)$$

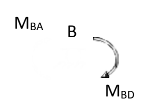
$$M_{BD} = \frac{3E(2I)}{10}(\theta_B) + \left(200 + \frac{200}{2}\right) = 0.6EI\theta_B + 300 \qquad (2)$$

$$M_{DB} = 0 \qquad \text{ANS}$$



4. Equilibrium Equations

By considering the moment equilibrium of the free body of joint B, we obtain the equilibrium equation



$$M_{BA} + M_{BD} = 0 \qquad (3)$$

5. Joint Rotation

To determine the unknown joint rotation θ_B we substitute the SDE (Eqs. 1 &2) into the equilibrium equations Eq. 3 to obtain

6. Joint Rotation

$$(0.3EI\theta_B - 187.5) + (0.6EI\theta_B + 300) = 0$$

or

$$0.9EI\theta_B = -112.5$$

from which

$$EI\theta_B = -125\ kN-m^2$$

7. Member End Moments

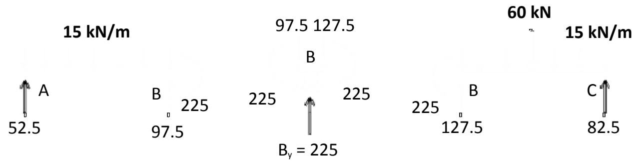
The member end moments can now be computed by substituting the numerical value of $EI\theta_B$ into the slope-deflection equations (Eqs. 1 & 2).

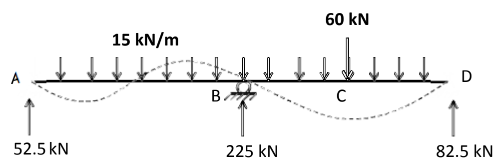
8. Member End Moments

$M_{BA} = 0.3(-125) - 187.5 = -225\ kN - m$ *or* $225\ kN\text{-}m$ ANS

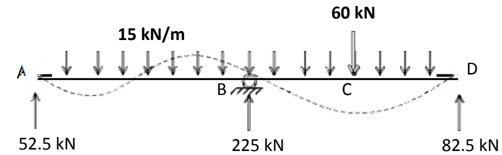
$M_{BD} = 0.6(-125) + 300 = 225\ kN - m$ ANS

9. Member End Shears and Support reactions





10. Equilibrium Checks



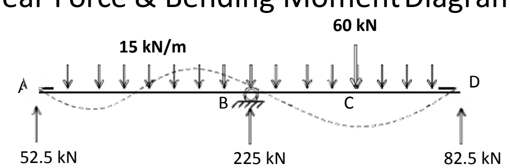
$+\uparrow \sum F_y = 0$

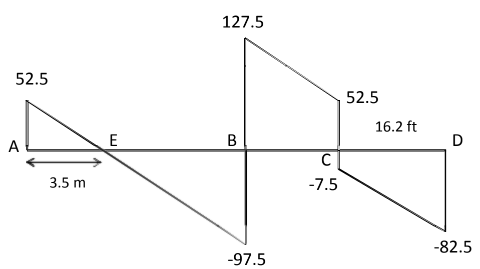
$52.5 - 15(20) + 225 - 60 + 82.5 = 0$ Checks

$+\sum M_C = 0$

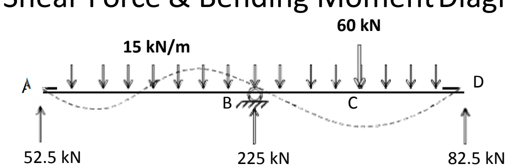
$-52.5(20) + 15(20)(10) - 225(10) + 60(5) = 0$ Checks

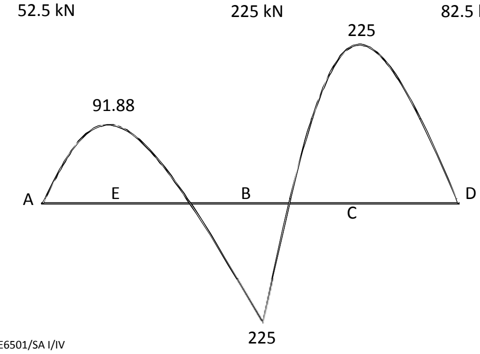
11. Shear Force & Bending Moment Diagrams





11. Shear Force & Bending Moment Diagrams

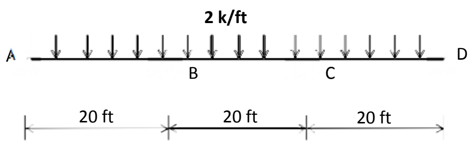




4.2 Continuous beams with sway

PROBLEM: 1

- Determine the member end moments and reactions for the three-span continuous beam shown, due to the uniformly distributed load and due to the support settlements of 5/8 in. at B, and 1.5 in. at C, and ¾ in. atD.



EI = 29,000 ksi

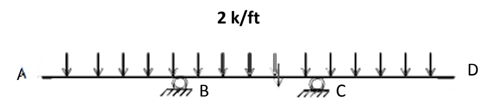
I = 7,800 in.4

Solution

1. Degree of Freedom

Four joints of the beam are free to rotate, we will eliminate the rotations of simple supports at ends A and D and use the modified SDE for member AB and CD respectively.

The analysis will involve only two unknown joint rotations, θ_B and θ_C.



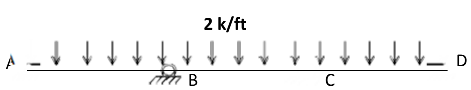
2. Fixed End Moments

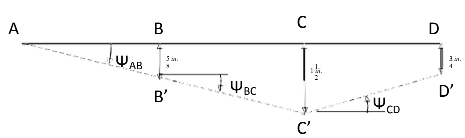
$$FEM_{AB} = FEM_{BC} = FEM_{CD} = \frac{2(20)^2}{12} = 66.7\ k-ft \qquad or + 66.7\ k\text{-}ft$$

$$FEM_{BA} = FEM_{CB} = FEM_{DC} = 66.7\ k-ft \qquad or - 66.7\ k\text{-}ft$$

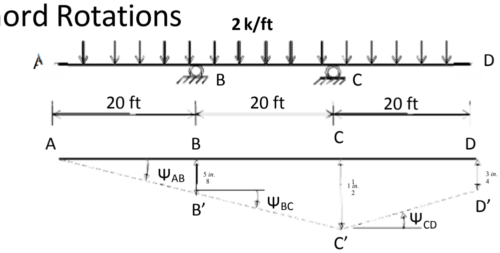
3. Chord Rotations

The specified support settlements are shown on a exaggerated scale.





3. Chord Rotations



$$_{AB} = -\frac{0.0521}{20} = -0.0026$$

$$_{BC} = -\frac{0.0729}{20} = -0.00365$$

$$_{CD} = \frac{1.5 - 0.75}{(12)20} = 0.00313$$

4. Slope-deflection Equations

$$M_{AB} = 0 \qquad \text{ANS}$$

$$M_{BA} = \frac{3EI}{10}\left(_{B} + 0.0026\right) - 100 = 0.15EI_{B} + 0.00039EI - 100 \qquad (1)$$

$$M_{BC} = \frac{2EI}{20}\left[2_{B} + _{C} - 3(-0.00365)\right] +$$

$$= 0.2EI_{B} + 0.1EI_{C} + 0.0011EI + 66.7 \qquad (2)$$

$$M_{CB} = \frac{2EI}{20}\left[2_{C} + _{B} - 3(-0.00365)\right] - 66.7$$

$$= 0.1EI_{B} + 0.2EI_{C} + 0.0011EI - \qquad (3)$$

$$M_{CD} = \frac{3EI}{20}\left(_{C} - 0.00313\right) + 100 = 0.15EI_{C} - 0.00047EI \qquad (4)$$

$$M_{DC} = 0 \qquad \text{ANS}$$

5. Equilibrium Equations

M_{BA} B M_{BC} $\qquad M_{BA} + M_{BC} = 0 \qquad (5)$

M_{CB} C M_{CD} $\qquad M_{CB} + M_{CD} = 0 \qquad (6)$

6. Joint Rotations

By substituting the slope-deflection equations (Eqs. 1 – 4) into the equilibrium equations (Eqs. 5 & 6), we obtain

$$0.35EI_{B} + 0.1EI_{C} = -0.00149EI + 33.3$$

$$0.1EI_{B} + 0.35EI_{C} = -0.00063EI - 33.3$$

substituting EI = (29,000)(7,800)/(12)$_2$k-ft$_2$ into the right sides of the above equations yields

6. Joint Rotations

$$0.35EI_{B} + 0.1EI_{C} = -2{,}307.24 \qquad (7)$$

- $0.1EI_{B} + 0.35EI_{C} = -1{,}022.93 \qquad (8)$

- By solving Eqs. (7) and (8) simultaneously, we determine the values of EIθ$_B$ and EIθ$_B$ to be

- $EI_{B} = -6{,}268.81\,k-ft_2$
- $EI_{C} = 1{,}131.57.81\,k-ft_2$

7. Member End Moments

To compute the member end moments, substitute the numerical values of EIθ$_B$ and EIθ$_C$ back into the slope-deflection equations (Eqs. 1 – 4) to obtain

7. Member End Moments

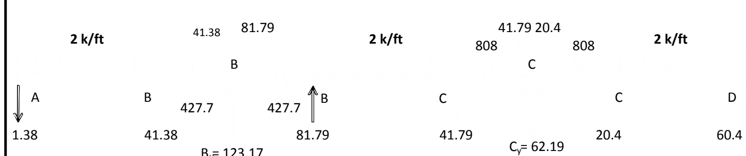
$M_{BA} = -427.7\ k-ft$ *or* $427\ k\text{-}ft$ ANS

$M_{BC} = 427\ k-ft$ ANS

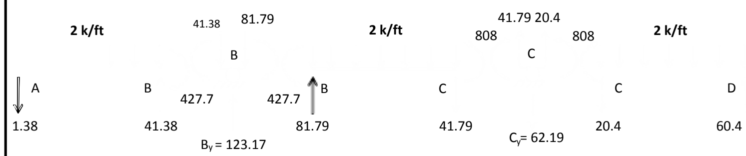
$M_{CB} = 808\ k-ft$ ANS

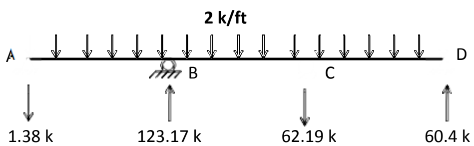
$M_{CD} = -808\ k-ft$ *or* $808\ k\text{-}ft$ ANS

8. Member End Shears and Support Reactions

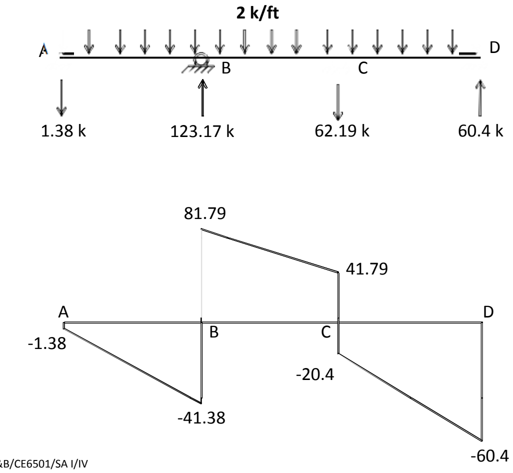


8. Member End Shears and Support Reactions

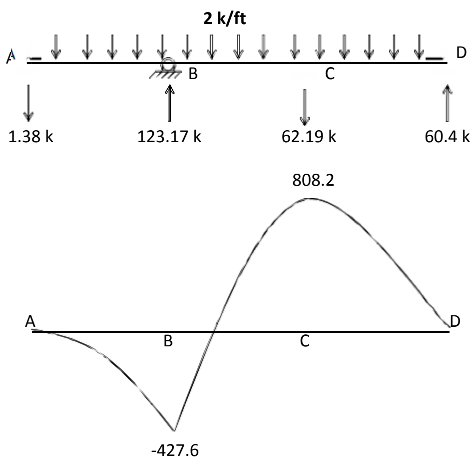




9. Shear and Bending Moment Diagrams



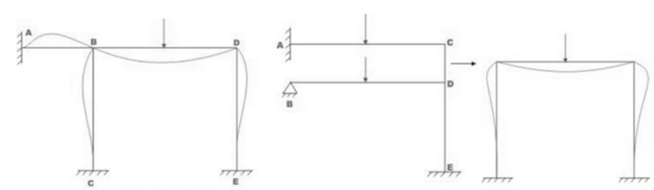
9. Shear and Bending Moment Diagrams



4.3 Rigid frames without sway

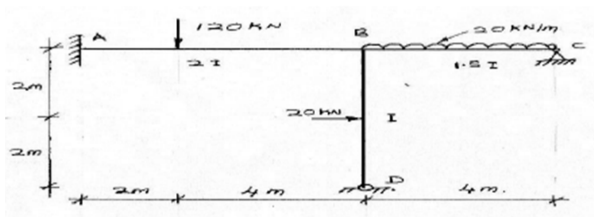
However the frames are symmetrical in geometry and in loading and hence these will not sidesway. In general, frames do not side sway if,

1) They are restrained against side sway.
2) The frame geometry and loading is symmetrical



PROBLEM:1

Analyse the simple frame shown in figure. End A is fixed and ends B & C are hinged. Draw the bending moment diagram.



Step: 1 Kinematic indeterminacy

$$\Theta_A = 0 \;\&\; \Theta_B, \Theta_c \;\&\; \Theta_D \neq 0$$

Step: 2 Fixed end moment

$$M_{AB=} -\frac{Wab^2}{L^2} = -\frac{120 \times 2 \times 4^2}{6^2} = -106.67\ KNm$$

$$M_{BA=} \frac{Wa^2b}{L^2} = \frac{120 \times 2^2 \times 4}{6^2} = 53.33KNm$$

$$M_{BC=} -\frac{wL^2}{12} = -\frac{20 \times 4^2}{12} = -26.67KNm$$

$$M_{CB=} \frac{wL^2}{12} = \frac{20 \times 4^2}{12} = 26.67\ KNm$$

$$M_{DB} = -\frac{wL}{8} = -\frac{20 \times 4}{8} = -10\ KNm$$

$$M_{BD} = \frac{wL}{8} = \frac{20 \times 4}{8} = 10\ KNm$$

Step: 3 Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}[2\theta_A + \theta_B] = -106.67 + \frac{2E(2I)}{6}\theta_B$$

$$= -106.67 + \frac{2EI}{3}\theta_B \qquad \cdots\cdots(1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}[2\theta_B + \theta_A] = 53.33 + \frac{2E(2I)}{6}2\theta_B = 53.33 + \frac{4EI}{3}\theta_B \qquad \cdots\cdots(2)$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}[2\theta_B + \theta_C] = -26.67 + \frac{3EI}{2}\theta_B + \frac{3EI}{4}\theta_C \qquad \cdots\cdots(3)$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}[2\theta_C + \theta_B] = 26.67 + \frac{3EI}{2}\theta_C + \frac{3EI}{4}\theta_B \qquad \cdots\cdots(4)$$

$$M_{BD} = M_{FBD} + \frac{2EI}{L}[2\theta_B + \theta_D] = 10 + \frac{2EI}{4}2\theta_B + \frac{2EI}{4}\theta_D$$

$$= 10 + EI\theta_B + \frac{EI}{2}\theta_D \qquad \cdots\cdots(5)$$

$$M_{DB} = M_{FDB} + \frac{2EI}{L}[2\theta_D + \theta_B] = -10 + \frac{2EI}{4}2\theta_D + \frac{2EI}{4}\theta_B$$

$$= -10 + EI\theta_D + \frac{EI}{2}\theta_B \qquad \cdots\cdots(6)$$

Step: 4 Equilibrium equation

In all the above equations there are only 3 unknowns $\theta_B \& \theta_C \& \theta_D$ and accordingly the boundary conditions are

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$M_{CB} = 0$$

$$M_{DB} = 0$$

$$M_{BA} + M_{BC} + M_{BD} = 53.33 + \frac{4EI}{3}\theta_B - 26.67 + \frac{3EI}{2}\theta_B + \frac{3EI}{4}\theta_C + 10 + EI\theta_B + \frac{EI}{2}\theta_D =$$

$$36.66 + \frac{23}{6}EI\theta_B + \frac{3}{4}EI\theta_C + \frac{EI}{2}\theta_D = 0 \qquad \cdots\cdots \quad (7)$$

$$M_{CB} = 26.67 + \frac{3EI}{2}\theta_C + \frac{3EI}{4}\theta_B = 0 \qquad \cdots\cdots \quad (8)$$

$$M_{DB} == -10 + EI\theta_D + \frac{EI}{2}\theta_B = 0 \qquad \cdots\cdots \quad (9)$$

Solving equations (7) & (8) & (9),

$$\theta_B = -\frac{8.83}{EI}$$

$$\theta_C = -\frac{13.36}{EI}$$

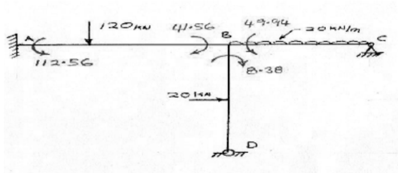
$$\theta_D = \frac{14.414}{EI}$$

Step: 5 Final end moments

Substituting the values in the slope deflections we have,

$$M_{AB} = -106.67 + \frac{2}{3}(-8.83) = -112.56KNm$$
$$M_{BA} = 53.33 + \frac{4}{3}(-8.83) = 41.56KNm$$
$$M_{BC} = -26.67 + \frac{3}{2}(-8.83) + \frac{3}{4}(-13.36) = -49.94KNm$$
$$M_{CB} = 26.67 + \frac{3}{2}(-13.36) + \frac{3}{4}(-8.83) = 0$$

$$M_{BD} = 10 - 8.83 + \frac{1}{2}(14.414) = 8.38KNm$$
$$M_{DB} = -10 + \frac{1}{2}(-8.83) + (14.414) = 0$$



Step: 6 BMD & SFD

REACTIONS:
SPAN AB:

$$R_B = \frac{41.56 - 112.56 + 120 \times 2}{6} = 28.17KN$$
$$R_A = 120 - R_B = 91.83KN$$

SPAN BC:

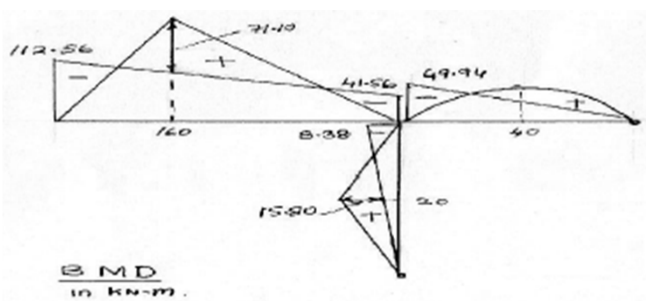
$$R_B = \frac{49.94 + 20 \times 4 \times 2}{4} = 52.485KN$$

$$R_C = 20 \times 4 - R_B = 27.515KN$$

Column BD:

$$H_D = \frac{20 \times 2 - 8.38}{4} = 7.92KN$$

$$H_B = 12.78KN$$



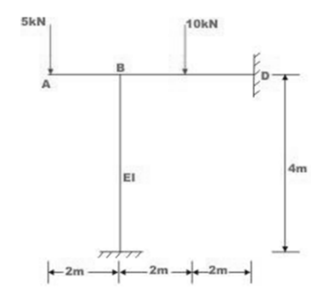
PROBLEM: 2

Analyse the rigid frame shown. Assume *EI* to be constant for all the members. Draw bending moment diagram and also sketch the elastic curve.

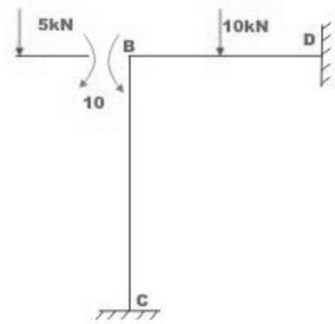
Solution:

Step: 1

Kinematic indeterminacy of frame is one.



Step: 2 Fixed end moment



$$M_{BD}^{F} = +5\,\text{kNm}$$

$$M_{DB}^{F} = -5\,\text{kNm}$$

$$M_{BC}^{F} = 0\,\text{kNm}$$

$$M_{BC}^{F} = 0\,\text{kNm}$$

Step:3 slope deflection equation

For writing slope–deflection equations two spans must be considered, BC and BD. Since supports C and D are fixed $\theta_C = \theta_D = 0$. Also the frame is restrained against sidesway.

$$M_{BD} = 5 + \frac{2EI}{4}\left[2\theta_B\right] = 5 + EI\theta_B$$

$$M_{DB} = 5 + \frac{2EI}{4}\left[\theta_B\right] = -5 + 0.5EI\theta_B$$

$$M_{BC} = EI\theta_B$$

$$M_{CB} = 0.5EI\theta_B$$

Step:4 Equilibrium equation

$$\sum M_B = 0 \quad \Rightarrow \quad M_{BD} + M_{BC} - 10 = 0$$

Substituting the value of M_{BD} and M_{BC}

$$5 + EI\theta_B + EI\theta_B - 10 = 0$$

$$\theta_B = \frac{2.5}{EI}$$

Step: 5 Final end moments

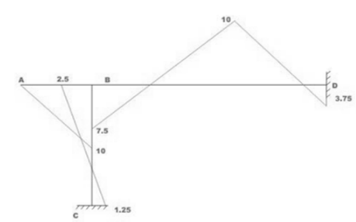
Substituting the values of θ_B

$$M_{BD} = +7.5\,\text{kN}\cdot\text{m}$$

$$M_{DB} = -3.75\,\text{kN}\cdot\text{m}$$

$$M_{BC} = +2.5\,\text{kN}\cdot\text{m}$$

$$M_{CB} = +1.25\,\text{kN}\cdot\text{m}$$



PROBLEM: 3

- **Determine support moments using slope deflection method for the frame shown in figure. Also draw bending moment diagram.**

Solution:

(a)Fixed end moments (FEM):

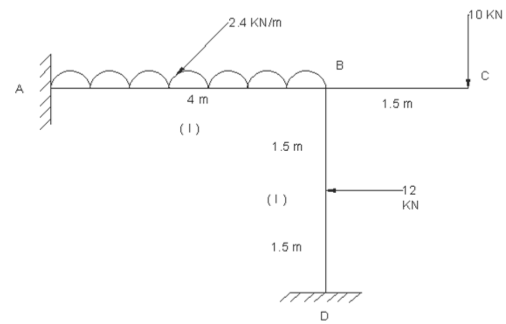
$$M_fAB = -\frac{wl^2}{12} = -\frac{2.4 \times 4^2}{12} = -3.20\ \mathrm{kN}$$

$$M_fBA = +\frac{wl^2}{12} = +3.20\ \mathrm{kN}$$

$$M_fBD = -\frac{wl^2}{8} = -\frac{12 \times 3}{8} = -4.5\ \mathrm{kN}$$

$$M_fDB = +\frac{wl}{8} = +4.5\ \mathrm{kN}$$

$$M_fBC = M_{BC} = -10 \times 1.5 = -15\ \mathrm{kN}$$



(b) Slope – Deflection equation :

$$M_{AB} = M_FAB + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$= -3.20 + \frac{2EI}{4}(0 + \theta_B - 0) \qquad [\because \theta_A = 0\,, \delta = 0\,]$$

$$= -3.20 + 0.5EI\theta_B \quad \cdots (1)$$

$$M_{BA} = M_FBA + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= 3.20 + \frac{2EI}{4}(2\theta_B + 0 - 0)$$

$$= 3.20 + EI\theta_B \quad \cdots (2)$$

$$M_{BD} = M_FBD + \frac{2EI}{l}\left(2\theta_B + \theta_D - \frac{3\delta}{l}\right)$$

$$= -4.5 + \frac{2EI}{3}(2\theta_B + 0 - 0)$$

$$= -4.5 + 1.333EI\theta_B \quad \cdots (3)$$

$$M_{DB} = M_FDB + \frac{2EI}{l}\left(2\theta_D + \theta_B - \frac{3\delta}{l}\right)$$

$$= 4.5 + \frac{2EI}{3}(0 + \theta_B - 0)$$

$$= 4.5 + 0.67EI\theta_B \quad \cdots (4)$$

(c) Equilibrium equation :

At joint B,

$$M_{BA} + M_{BC} + M_{BD} = 0$$

$$\therefore (3.20 + EI \cdot \theta_B) - 15 + (-4.5 + 1.333EI \cdot \theta_B) = 0$$

$$\therefore 2.333EI\theta_B = 16.3$$

$$\boxed{\therefore \theta_B = \frac{6.986}{EI} \quad \cdots \text{clockwise}}$$

(d) Final Moments :

$$M_{AB} = -3.20 + 0.5EI \cdot \theta_B$$

$$= -3.20 + 0.5EI\left(\frac{6.986}{EI}\right)$$

$$= 0.293 \text{ kN} \cdot \text{m}$$

$$\cong 0.30 \text{ kN} \cdot \text{m}$$

$$M_{BA} = 3.20 + EI \cdot \theta_B$$

$$= 3.20 + EI\left(\frac{6.986}{EI}\right)$$

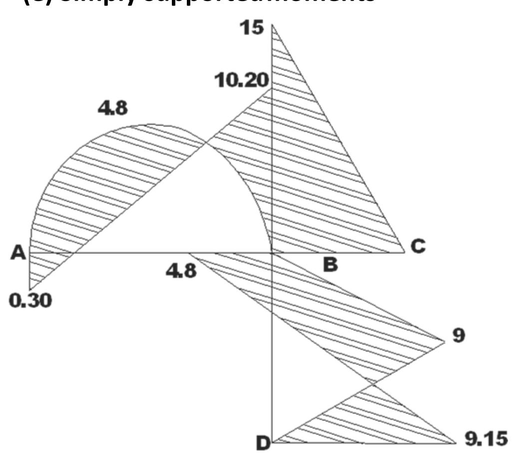
$$= 10.20 \text{ kN} \cdot \text{m}$$

(d) <u>Simply supported moments :</u>

$$\text{span AB}, M = \frac{wl^2}{8} = \frac{2.4 \times 4^2}{8} = 4.8 \text{ kN} \cdot \text{m}$$

$$\text{span BD}, M = \frac{wl}{4} = \frac{12 \times 3}{4} = 9 \text{ kN} \cdot \text{m}$$

(e) Simply supported moments



B.M Diagram

4.4 Rigid frames with sway

Causes of side sway :

1 Unsymmetrical loading (eccentric loading) 2
Unsymmetrical out-line of portal frame
3 Different end conditions of the columns of the portal frame . 4
Non-uniform sections (M.I.) of the members of the frame.
5 Horizontal loading on the columns of the frame.
6 settlement of the supports of the frame .

$$H_a = \frac{M_{AB} + M_{BA} - Ph}{l_1} \rightarrow$$

$$H_d = \frac{M_{CD} + M_{DC} + \frac{wl_2^2}{2}}{l_2} \rightarrow$$

for the equilibrium of the frame,

$$\sum H = 0$$

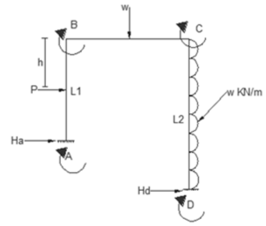
$$Ha + Hd + P - w \cdot l^2 = 0$$

$$\frac{M_{AB} + M_{BA} - Ph}{l_1} + \frac{M_{CD} + M_{DC} + \frac{wl_2^2}{2}}{l_2} + P - wl_2 = 0$$

if $P = 0$, $w = 0$, the above equation becomes

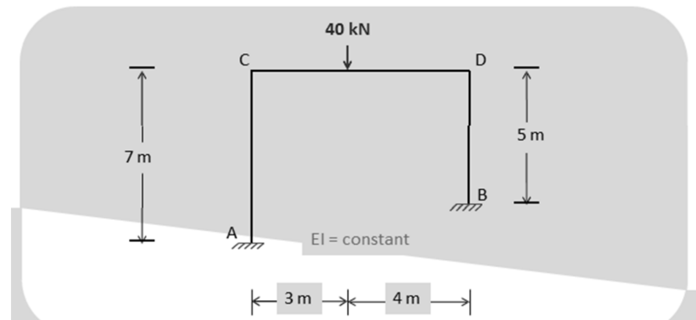
$$\frac{M_{AB} + M_{BA}}{l_1} + \frac{M_{CD} + M_{DC}}{l_2} = 0$$

Equation (4) and (5) *are* called shear equations.



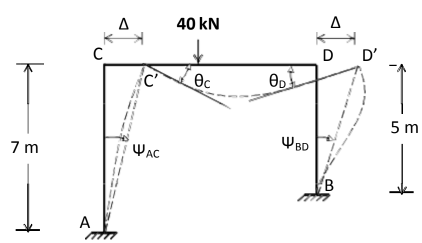
PROBLEM: 1

Determine the member end moments and reactions for the frame shown by the slope-deflection method.



1. Degrees of Freedom

- The degrees of freedom are θ_C, θ_D. and Δ.



2. Fixed-End Moments

- By using the fixed end moments expressions given inside the back coverof the book, we obtain

$$FEM_{CD} = \frac{40(3)(4)^2}{(7)^2} = 39.2 \text{ kN.m} \qquad \text{or} \qquad +39.2 \text{ kN.m}$$

$$FEM_{DC} = -\frac{40(3)^2(4)}{(7)^2} = 29.4 \text{ kN.m} \qquad \text{or} \qquad -29.4 \text{kN.m}$$

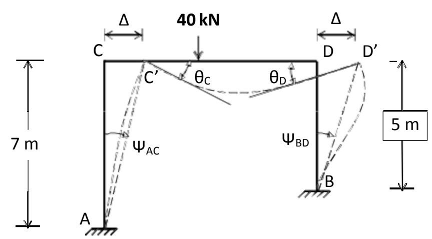
$$FEM_{CD} = FEM_{CA} = FEM_{BD} = FEM_{DB} = 0$$

3. Chord Rotations

$$_{AC} = -\frac{\Delta}{7}$$

$$_{BD} = -\frac{\Delta}{5}$$

$$_{CD} = 0$$



4. Slope-Deflection Equations

$$M_{AC} = \frac{2EI}{7}\left(_C - 3\left(-\frac{\Delta}{7}\right)\right) = 0.286EI _C + 0.122EI\Delta \qquad (1)$$

$$M_{CA} = \frac{2EI}{7}\left(2 _C - 3\left(\frac{\Delta}{7}\right)\right) = 0.571EI _C + 0.122EI\Delta \qquad (2)$$

$$M_{BD} = \frac{2EI}{5}\left(_D - 3\left(-\frac{\Delta}{5}\right)\right) = 0.4EI _D + 0.24EI\Delta \qquad (3)$$

$$M_{DB} = \frac{2EI}{5}\left(2 _D - 3\left(-\frac{\Delta}{5}\right)\right) = 0.8EI _D + 0.24EI\Delta \qquad (4)$$

$$M_{CD} = \frac{2EI}{7}\left(2 _C + _D\right) + 39.2 = 0.571EI _C + 0.286EI _D + \qquad (5)$$

$$M_{DC} = \frac{2EI}{7}\left(_C + 2 _D\right) - 29.4 = 0.286EI _C + 0.571EI _D - \qquad (6)$$

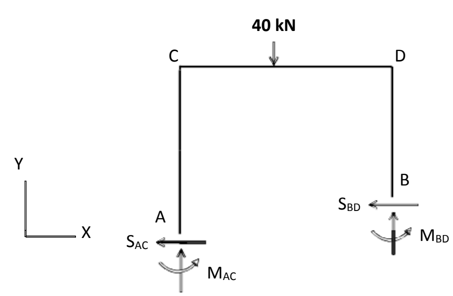
5. Equilibrium Equations

$$M_{CA} + M_{CD} = 0 \qquad (7)$$

$$M_{DB} + M_{DC} = 0 \qquad (8)$$

To establish the third equilibrium equation, we apply the force equilibrium equation $\sum F_x = 0$ to the free body of the entire frame, to obtain

$$S_{AC} + S_{BD} = 0$$



5. Equilibrium Equations

To express the column end shears in terms of column end moments, we draw the free body diagram of the two columns and sum the moments about the top of each column:

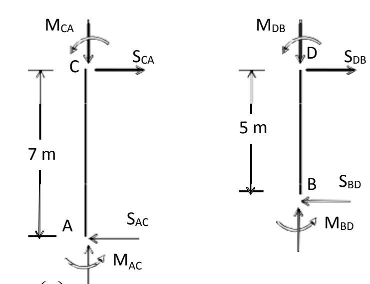
$$S_{AC} + S_{BD} = 0$$

$$S_{AC} = \frac{M_{AC} + M_{CA}}{7}$$

$$S_{BD} = +\frac{M_{BD} \; M_{DB}}{5}$$

$$\frac{M_{AC} + M_{CA}}{7} + \frac{M_{BD} + M_{DB}}{5} = 0$$

$$5(M_{AC} + M_{CA}) + 7(M_{BD} + M_{DB}) = 0 \qquad (9)$$



6. Joints Displacements

To determine the unknown joint displacements θ_c, θ_D and Δ, we substitute the SDE (1 to 6) into the equilibrium equations (7 to 9) to obtain

$$1.142EI\;_C + 0.286EI\;_D + 0.122EI\Delta = -39.2 \qquad (10)$$

$$0.286EI\;_C + 1.371EI\;_D + 0.24EI\Delta = 29.4 \qquad (11)$$

$$4.285EI\;_C + 8.4EI\;_D + 4.58EI\Delta = 0 \qquad (12)$$

Solving Eqs. 10 to 12 simultaneously yields

$$EI\;_C = -40.211 \text{ kN.m}^2$$

$$EI\;_D = 34.24 \text{ kN.m}^2$$

$$EI\Delta = -25.177 \text{ kN.m}^3$$

7. Member End Moments

By substituting the numerical values of $EI\theta_c$, $EI\theta_D$, and $EI\Delta$ into the slope-deflection equations (Eqs. 1 through 6), we obtain

$M_{CA} = -26\text{kN.m}$ or 26 kN.m ANS

$M_{BD} = 7.7\text{ kN.m}$ ANS

$M_{DB} = 21.3\text{ kN.m}$ ANS

$M_{CD} = 26\text{ kN.m}$ ANS

$M_{DC} = -21.3\text{kN.m}$ or 21.3 kN.m ANS

$M_{AC} = -14.6\text{ kN.m}$ or 14.6kN.m ANS

8. Equilibrium check

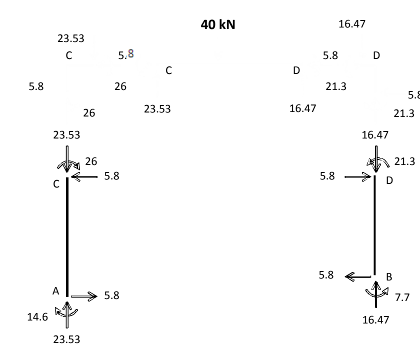
By substituting the numerical values of $EI\theta_c$, $EI\theta_D$, and $EI\Delta$ into the slope-deflection equations (Eqs. 1 through 6), we obtain

$$M_{CA} + M_{CD} = -26 + 26 = 0 \qquad \text{Checks}$$

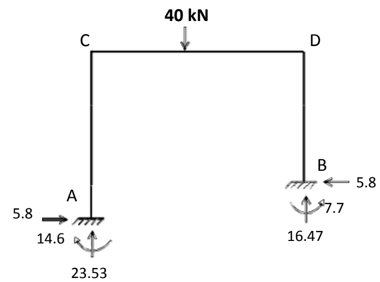
$$M_{DB} + M_{DC} = 21.3 - 21.3 = 0 \qquad \text{Checks}$$

$$5(M_{AC} + M_{CA}) + 7(M_{BD} + M_{DB}) = 5(-14.6 - 26) + 7(7.7 + 21.3) = 0 \qquad \text{Checks}$$

9. Member end shears and axial forces



10. Support Reactions

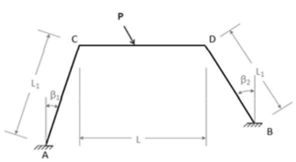


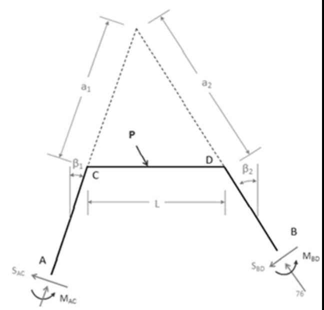
Frames with Inclined Legs

- The analysis of frames with inclined legs is similar to that of the rectangular frames considered previously.
- But when frames with inclined legs are subjected to sidesway, their horizontal members also undergo chord rotations, which must be included in the analysis.
- Recall that the chord rotations of the horizontal members of rectangular frames, subjected to sideway, are zero.

$$a_1 = \frac{L}{\cos\beta_1(\tan\beta_1 + \tan\beta_2)}$$

$$a_2 = \frac{L}{\cos\beta_2(\tan\beta_1 + \tan\beta_2)}$$

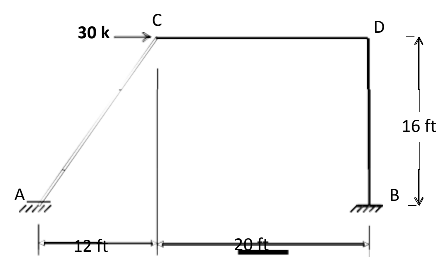




Once the equilibrium equations have been established, the analysis can be completed in the usual manner, as discussed previously.

PROBLEM: 2

- Determine the member end moments and reactions for the frame shown by the slope-deflection method.

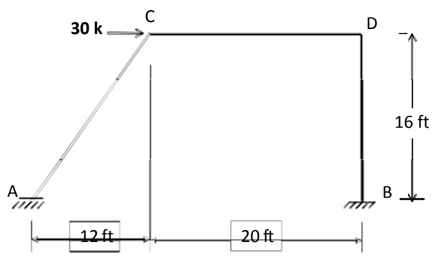


EI = constant

Solution

1. Degree of Freedom

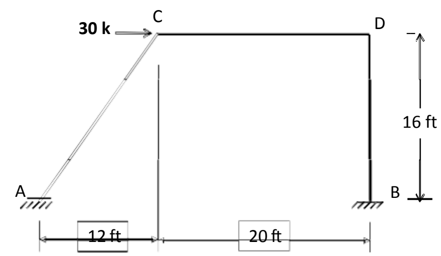
θ_C, θ_D and Δ



EI = constant

2. Fixed End Moments

Since no external loads are applied to the members, the fixed-end moments are zero.

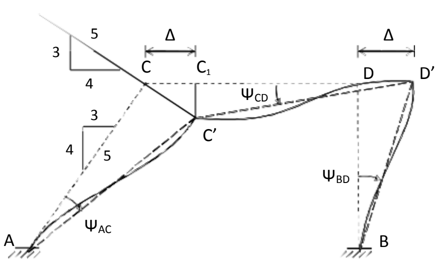


EI = constant

3. Chord Rotations

$$_{AC} = -\frac{CC'}{20} = -\frac{\left(\frac{5}{4}\right)\Delta}{20} = -0.0625\Delta \qquad _{BD} = -\frac{DD'}{16} = -\frac{\Delta}{16} = -0.0625\Delta$$

$$_{CD} = \frac{C'C_1}{20} = \frac{\left(\frac{3}{4}\right)\Delta}{20} = 0.0375\Delta$$



4. Slope-Deflection Equations

$$M_{AC} = \frac{2EI}{20}\left(_C - 3(-0.0625\Delta)\right) = 0.1EI _C + \qquad (1)$$

$$M_{CA} = \frac{2EI}{20}\left(2 _C - 3(-0.0625\Delta)\right) = 0.2EI _C + 0.0188EI\Delta \qquad (2)$$

$$M_{BD} = \frac{2EI}{16}\left(_D - 3(-0.0625\Delta)\right) = 0.125EI _D + \qquad (3)$$

$$M_{DB} = \frac{2EI}{16}\left(2 _D - 3(-0.0625\Delta)\right) = 0.25EI _D + \qquad (4)$$

$$M_{CD} = \frac{2EI}{20}\left(2 _C + _D (0.0375\Delta)\right) = 0.2EI _C + 0.1EI _D - 0.0113EI\Delta \qquad (5)$$

$$M_{DC} = \frac{2EI}{20}\left(2 _D + _C - 3(0.0375\Delta)\right) = 0.2EI_D + 0.1EI_C - \qquad (6)$$

5. Equilibrium Equations

By considering the moment equilibrium of joints C and D, we obtain the equilibrium equations

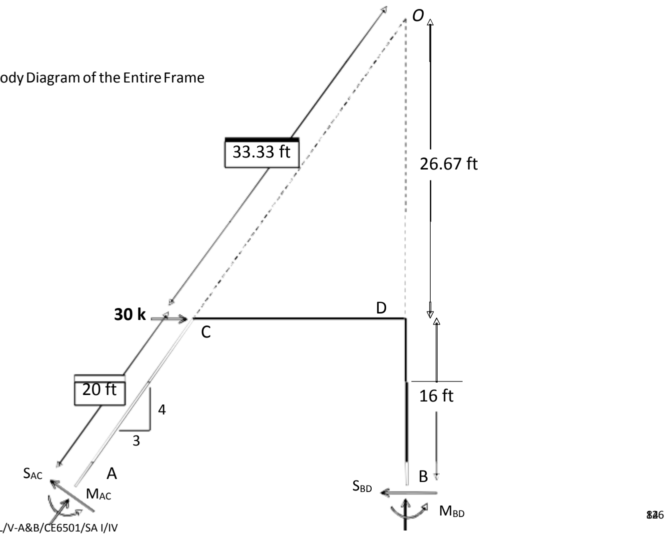
$$M_{CA} + M_{CD} = 0 \qquad (7)$$

$$M_{DB} + M_{DC} = 0 \qquad (8)$$

The third equilibrium equation is established by summing the moments of all the forces and couples acting on the free body of the entire frame about point *O*, which is located at the intersection of the longitudinal axes of the two columns as shown.

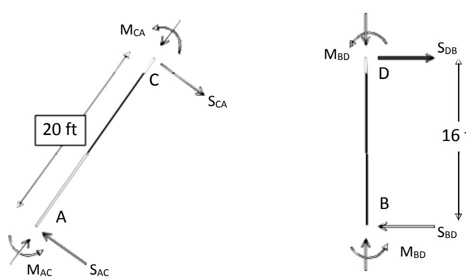
$$+\circlearrowleft \sum M_O = 0 \qquad M_{AC} - S_{AC}(53.33) + M_{BD} - S_{BD}(42.67) + 30(26.67) = 0$$

Free-body Diagram of the Entire Frame



$$+\circlearrowleft \sum M_O = 0 \qquad M_{AC} - S_{AC}(53.33) + M_{BD} - S_{BD}(42.67) + 30(26.67) = 0$$

in which the shears at the lower ends of the columns can be expressed in terms of column end moments as shown below.



$$S_{AC} = \frac{M_{AC} + M_{CA}}{20} \qquad S_{BD} = \frac{M_{BD} + M_{DB}}{16}$$

by substituting these expressions into the third equilibrium equation, we obtain

$$1.67M_{AC} + 2.67M_{CA} + 1.67M_{BD} + 2.67M_{DB} = 800 \qquad (9)$$

6. Joint Displacements

Substitution of the slope-deflection equations Eqs. 1 to 6 into the equilibrium equations Eqs. 7 to 9 yields

$$0.4EI_C + 0.1EI_D + 0.0075EI\Delta \qquad (10)$$

$$0.1EI_C + 0.45EI_D + 0.0121EI\Delta = 0 \qquad (11)$$

$$0.71EI_C + 0.877EI_D + 0.183EI\Delta = 800 \qquad (12)$$

$$EI\theta_C = -66.648 \text{ k - ft}^2$$

$$EI\theta_D = -125.912 \text{ k - ft}^2$$

$$EI\Delta = 5{,}233.6 \text{ k - ft}^3$$

7. Member End Moments

By substituting the numerical values of $EI\theta_C$, $EI\theta_D$, and $EI\Delta$ into the slope-deflection equations (Eqs. 1 through 6), we obtain

$M_{AC} = 91.7$ k - ft ANS

$M_{CA} = 85.1$ k - ft ANS

$M_{BD} = 106.7$ k - ft ANS

$M_{DB} = 91$ k - ft ANS

$M_{CD} = -85.1$ k - ft or 85.1 k - ft ANS

$M_{DC} = -91$ k - ft or 91 k - ft ANS

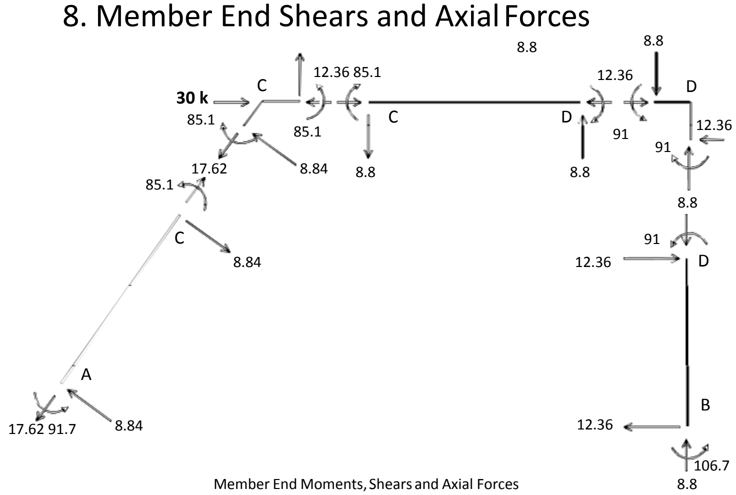
Back substitution of the numerical values of member end moments into the equilibrium equations yields

$M_{CA} + M_{CD} = 85.1 - 85.1 = 0$ Checks

$M_{DB} + M_{DC} = 91 - 91 = 0$ Checks

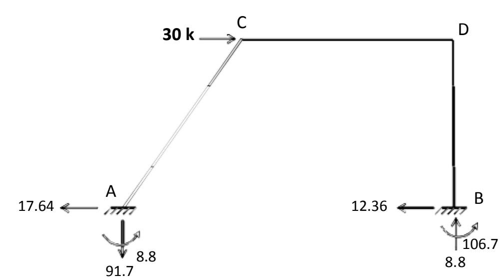
$$1.67M_{AC} + 2.67M_{CA} + 1.67M_{BD} + 2.67M_{DB} = 1.67(91.7) + 2.67(85.1) + 1.67(106.7) + 2.67(91) = 801.5 \cong 800$$ Checks

8. Member End Shears and Axial Forces



Member End Moments, Shears and Axial Forces

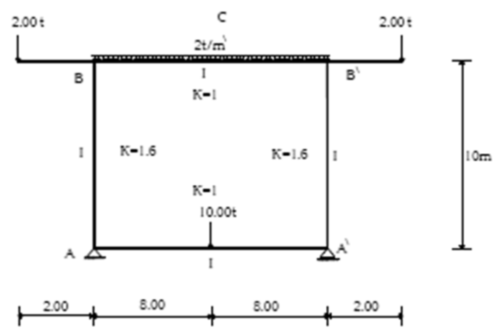
9. Support Reactions



Member End Moments, Shears and Axial Forces

4.5 Symmetry and antisymmetry PROBLEM: 1

Draw B.M.D for the shown frame.



Two equilibrium eqns.

$M_{AB} + M_{AA} = 0$ .... (1)

$M_{BB} + M_{BA} + 4 = 0$ ....(2)

Slope deflection eqns.

$M_{AB} = o + 1.6\,(2\theta_A + \theta_B)$

$M_{AA} = \frac{-10\times16}{8} + (2\theta_A + \theta_A)$

$M_{AA} = -20 + \theta_A$

$M_{BA} = o + 1.6\,(2\theta_B + \theta_A)$

$M_{BB} = -42.67 + (2\theta_B + \theta_B)$

$= -42.67 + \theta_B$

Hence:

$3.2\theta_A + 1.6\theta_B + \theta_A - 20 = o$

$4.2\theta_A + 1.6\theta_B = 20$ .... (1)

$-42.67 + 4.2\,\theta_B + 1.6\theta_A + 4 = 0$

$1.6\theta_A + 4.2\theta_B = 38.67$ ....(2)

$1.6\theta_A + 0.61\theta_B = 7.62$ ....(1)

$3.59\theta_B = 31.05$

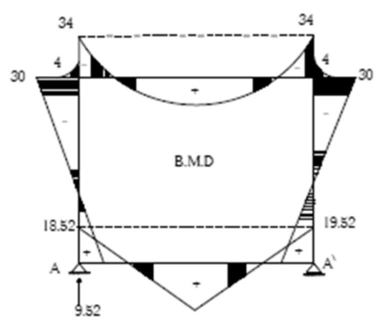
$\theta_B = 8.65$

$\theta_A = 1.46$

$M_{AB} = -18.52$

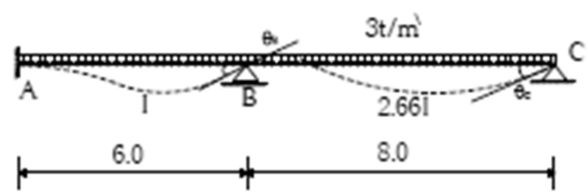
$M_{BA} = 30$

$M_{BB} = -34$



4.6 Simplification for hinged end PROBLEM: 1

Draw B.M.D for the shown beam.



Fixed and Moment:-

$$MF_{BA} = \frac{3 \times 6^2}{12} = -9 \ t.m.$$

$$MF_{BA} = + \frac{3 \times 6^2}{12} = +9 \ , \quad MF_{BC} = + \frac{3 \times 8^2}{12} = -18$$

$$MF_{CB} = + \frac{3 \times 8^2}{12} = +18$$

Two unknown $\theta_B + \theta_C$ then two static equations are

1) $\sum M_B = 0$

2) $M_C = 0$

$M_{BA} + M_{BC} = 0$ .... (1)

$M_{BC} = 0$.................. (2)

But:

$M_{AB} = -9 + \theta_B$

$M_{BA} = 9 + 1\,(2\theta_B)$

$M_{BC} = -16 + 2\,(2\theta_B + \theta_C)$

$M_{CB} = +16 + 2\,(2\theta_C + \theta_B)$

From eqns. (1&2)

$9 + 2\theta_B + (-16 + 2\,(2\theta_B + \theta_C) = 0$

$6\theta_B + 2\theta_C = 7$ ....(3)

and $4\theta_C + 2\theta_B = -16$

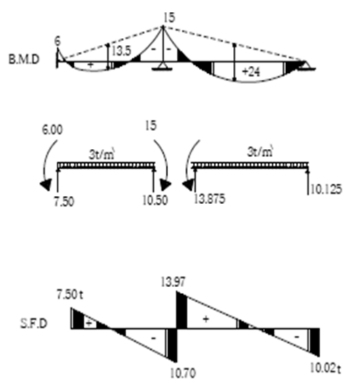
$2\theta_C + \theta_B = -8$ .... (4)

from 3 & 4

$5\,\theta_B = 15$

$\theta_B = \frac{15}{5} = 3.0$

$\theta_C = -5.5$



$M_{AB} = -9 + 3.4 = 5.6$ t.m

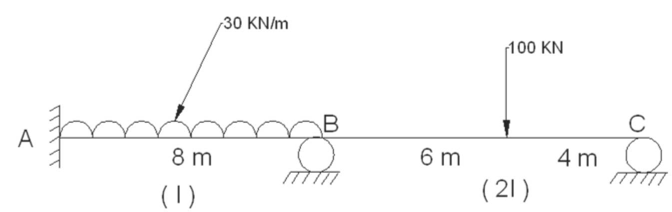
$M_{BA} = 9 + 2{\times}3.4 = 15.8$ t.m

$M_{BC} = -18 + 2\,(2{\times}3.4) + (-5.5) = -15.0$ t.m

$M_{CB} = 16 + 2\,(2.3 - 5.7 + 3.4) = 0.0$ (0.k

PROBLEM: 2

Using slope deflection method analyse the continuous beam shown in figure.
Draw the bending moment diagram.



(a) Fixed end Moments

$$M_{f}AB = -\frac{wl^2}{12} = -\frac{30 \times 8^2}{12} = -160\ kN.m$$

$$M_{f}BA = +\frac{wl^2}{12} = +160\ kN.m$$

$$M_{f}BC = -\frac{Wab^2}{l^2} = -\frac{100 \times 6 \times 4^2}{10^2} = -96\ kN.m$$

$$M_{f}CB = +\frac{Wba^2}{l^2} = +\frac{100 \times 4 \times 6^2}{10^2} = 144\ kN.m$$

(b) Slope Deflection equations

$$M_{AB} = M_{f}AB + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

$$= -160 + \frac{2EI}{8}(0 + \theta_B - 0)$$

$$= -160 + 0.25\,EI\,\theta_B \ldots\ldots(1)$$

$$M_{BA} = M_{f}BA + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\delta}{l}\right)$$

$$= 160 + \frac{2EI}{l}(2\theta_B + 0 - 0)$$

$$= 160 + 0.5\,EI\,\theta_B \ldots\ldots\ldots(2)$$

$$M_{BC} = M_{f}BC + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\delta}{l}\right)$$

$$= -96 + \frac{2E(2I)}{10}(2\theta_B + \theta_C - 0)$$

$$= -96 + 0.8EI\,\theta_B + 0.4EI\,\theta_C \ldots\ldots(3)$$

$$M_{CB} = M_{f}CB + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\delta}{l}\right)$$

$$= 144 + 0.4EI\,\theta_B + 0.8EI\,\theta_C \ldots\ldots\ldots(4)$$

(C) Equilibrium equations :

$M_{BA} + M_{BC} = 0 \ldots\ldots(A)$

$M_{CB} = 0 \ldots\ldots(B)$

$M_{BA} + M_{BC} = 0$

$(160 + 0.5EI\,\theta_B) + (-96 + 0.8EI\,\theta_B + 0.4EI\,\theta_C) = 0$

$\therefore\ 1.3EI\,\theta_B + 0.4EI\,\theta_C = -64 \ldots\ldots(A)$

$M_{CB} = 0,$

$\therefore\ 0.4EI\,\theta_B + 0.8EI\,\theta_C + -144 \ldots\ldots(B)$

Solving equation (A) and (B) by calculator.

Mode $\rightarrow$ equations $\rightarrow$ 2unknowns

$\therefore\ \theta_B = \frac{7.272}{EI}$ ..... clockwise

$\therefore\ \theta_C = -\frac{183.63}{EI}$ ..... anticlockwise

(d) Final Moments :

$$M_{AB} = -160 + 0.25\,EI\,\theta_B$$

$$= -160 + 0.25\,EI\left(\frac{7.272}{EI}\right)$$

$$= -158.18\ kN.m$$

$$M_{BA} = 160 + 0.5\,EI\left(\frac{7.272}{EI}\right)$$

$$= 163.64\ kN.m$$

$$M_{BC} = -96 + 0.8\,EI\,\theta_B + 0.4\,EI\,\theta_C$$

$$= -96 + 0.8\,EI\left(\frac{7.272}{EI}\right) + 0.4\,EI\left(-\frac{183.63}{EI}\right)$$

$$= -96 + 5.82 - 73.45$$

$$= -163.64\ kN.m$$

$$M_{CB} = 144 + 0.4\,EI\,\theta_B + 0.8\,EI\,\theta_C$$

$$= 144 + 0.4\,EI\left(\frac{7.272}{EI}\right) + 0.8\,EI\left(-\frac{183.63}{EI}\right)$$

$$= 144 + 2.91 - 146.90$$

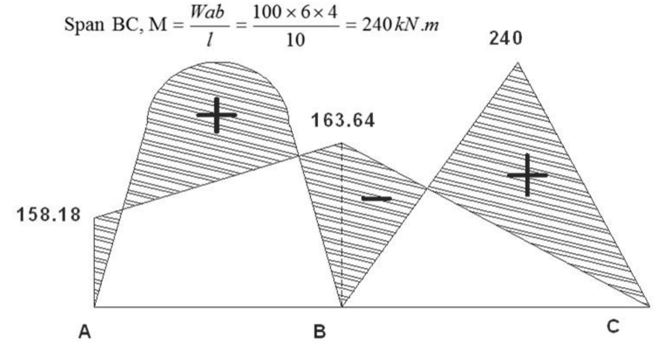
$$= 0$$

(e) B.M Diagram

Simply supported moments.

Span AB, $M = \frac{wl^2}{8} = \frac{30 \times 8^2}{8} = 240\,kN.m$

Span BC, $M = \frac{Wab}{l} = \frac{100 \times 6 \times 4}{10} = 240\,kN.m$

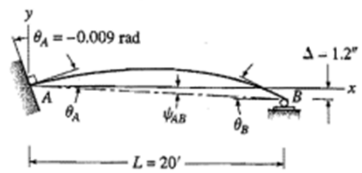


4.7 Support displacements
PROBLEM:1

Determine the reactions and draw the shear and moment curves for the beam. The support at A has been accidentally constructed with a slope that makes an angle of 0.009 rad with the vertical y-axis through support A, and B has been constructed 1.2 in below its intended position. Given: EI is constant, 1= 360 in_4, and E = 29,000 $kips/in_2$.

Solution:

$$\psi_{AB} = \frac{\Delta}{L} = \frac{1.2}{20(12)} = 0.005 \text{ radians}$$



$$M_{AB} = \frac{2EI_{AB}}{L_{AB}}(2\theta_A + \theta_B - 3\psi_{AB}) + \text{FEM}_{AB}$$

$$M_{AB} = \frac{2E(360)}{20(12)}[2(-0.009) + \theta_B - 3(0.005)] \qquad (1)$$

$$M_{BA} = \frac{2E(360)}{20(12)}[2\theta_B + (-0.009) - 3(0.005)] \qquad (2)$$

Writing the equilibrium equation at joint B yields

$$\circlearrowleft^{+} \quad \Sigma M_B = 0$$

$$M_{BA} = 0 \qquad (3)$$

Substituting Equation 2 into Equation 3 and solving for θ_B yield

$$3E(2\theta_B - 0.009 - 0.015) = 0$$

$$\theta_B = 0.012 \text{ radians}$$

To evaluate M_{AB}, substitute θ_B into Equation 1:

$$M_{AB} = 3(29{,}000)[2(-0.009) + 0.012 - 3(0.005)]$$

$$= -1827 \text{ kip·in} = -152.25 \text{ kip·ft}$$

Complete the analysis by using the equations of statics to compute the reaction at B and the shear at A (see Fig. 12.12b).

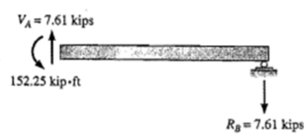
$$\circlearrowright^{+} \quad \Sigma M_A = 0$$

$$0 = R_B(20) - 152.25$$

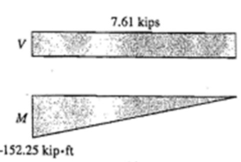
$$R_B = 7.61 \text{ kips} \qquad \textbf{Ans.}$$

$$\uparrow^{+} \quad \Sigma F_y = 0$$

$$V_A = 7.61 \text{ kips}$$



(b)



(c)

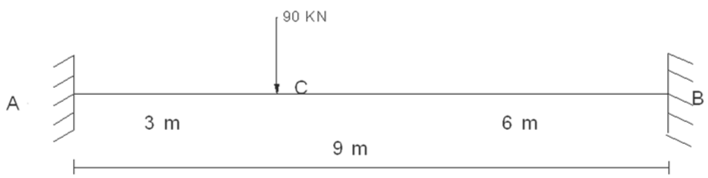
PROBLEM: 2

A beam AB of uniform section of span 9m and constant EI=3.6×10_4 Nm² is partially fixed at ends when the beam carries a point load 90 kN at distance 3m from the left end A. The following displecements were observed.

i) rotation at A =0.001 rad (clockwise) and settlement at A=20mm

ii) rotation at B=0.0075rad (anticlockwise) and settlement at B=15mm.

Analyse using slope deflection method.



Fixed end moments(FEM):

$$M_f AB = -\frac{wab^2}{l^2} = -\frac{90 \times 3 \times 6^2}{9^2} = -120kN \cdot m$$

$$M_f BA = -\frac{wba^2}{l^2} = -\frac{90 \times 6 \times 3^2}{9^2} = 60kN \cdot m$$

Slope deflection equations:

$$M_{AB} = M_f AB + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

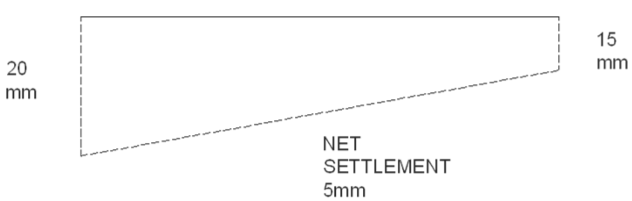
left support sinks more

$\therefore \delta$ *is taken* $-V_e$

$Net\ \delta = 20 - 15 = 5mm = 0.005m$

$$EI = 3.6 \times 10^4\ N \cdot m^2$$
$$= 36\ kN \cdot m^2$$

Slope deflection



$$M_{AB} = M_f AB + \frac{2EI}{l}\left[2\theta_a + \theta_b - \frac{3\delta}{l}\right]$$

$$= -120 + \frac{2 \times 36}{9}\left[2 \times 0.001 - 0.0075 - 3 \times \left(\frac{-0.005}{9}\right)\right]$$

$$= -120 + 8[-0.00383]$$

$$= -120{,}03\ kN \cdot m$$

$\theta_a = +0.001\ rad(clockwise)$

$\theta_b = -0.0075\ rad(anticlockwise)$

$$M_{BA} = M_f BA + \frac{2EI}{l}\left[2\theta_b + \theta_a - \frac{3\delta}{l}\right]$$

$$= 60 + \frac{2 \times 36}{9}\left|2 \times (-0.0075) + 0.001 - \frac{3 \times (-0.005)}{9}\right|$$

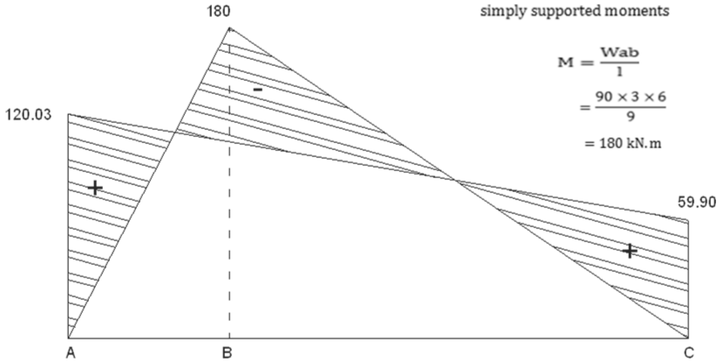
$$= 60 + 8[-0.0123]$$

$$= 59.90\ kN.m$$

B.M. diagram

simply supported moments

$$M = \frac{Wab}{l}$$
$$= \frac{90 \times 3 \times 6}{9}$$
$$= 180 \text{ kN.m}$$



B.M Diagram

PART A

1. **What are the assumptions made in slope-deflection method?**

(AUC Apr/May 2012, Nov/Dec 2013)

   i) Between each pair of the supports the beam section is constant.

   ii) The joint in structure may rotate or deflect as a whole, but the angles between the members meeting at that joint remain the same.

2. **What is the limitation of slope-deflection equations applied in structural analysis?**

(AUC Apr/May 2012)

   i) It is not easy to account for varying member sections.

   ii) It becomes very cumbersome when the unknown displacements are large in number.

3. **Mention the causes for sway in portal frames.** **(AUC Nov/Dec 2012, May/June 2014)**

   Because of sway, there will be rotations in the vertical members of a frame. This causes moments in the vertical members. To account for this, besides the equilibrium, one more equation namely shear equation connecting the joint-moments is used.

4. **Explain the use of slope deflection method.** **(AUC Nov/Dec 2012)**

   i) It can be used to analyze statically determinate and indeterminate beams and frames.

   ii) In this method it is assumed that all deformations are due to bending only.

   iii) In other words deformations due to axial forces are neglected.

   iv) The slope-deflection equations are not that lengthy in comparison.

5. **Compute the rotation at middle support of a two equal span continuous beam fixed at the ends and carrying UDL of 10 kN/m over the entire beam span 5 m. Take EI = 60000 kNm^2.**

(AUC Apr/May 2011)

   θ_B = 0.00155 mm and

   θ_C = 0.0062 mm

6. **Write down the slope deflection equation for a beam AB fixed at A and B subjected to a settlement δ at B.** **(AUC Apr/May 2011, May/June 2014)**

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B + \frac{3\delta}{L}\right)$$

7. **Mention two assumptions made in slope deflection method.** **(AUC Nov/Dec 2010)**

   i) Between each pair of the supports of the beam is constant.

   ii) The joint in a structure may rotate or deflect as a whole, but the angles between the members meeting at that joint remain the same.

8. **Write down the fundamental equation of slope deflection method. (AUC Nov/Dec 2010, 2013)**

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B + \frac{3\delta}{L}\right)$$

9. **How many slope deflection equations are available for a two span continuous beam?**

   There will be 4 Nos. of slope deflection equations, two for each span.

10. **What is the moment at a hinged end of a simple beam?**

   Moment at the hinged ends of a simple beam is zero.

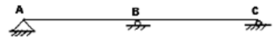
11. **What are the quantities in terms of which the unknown moments are expressed in slope deflection method?**

   In slope-deflection method, unknown moments are expressed in terms of

   (i) Slopes (θ) and

   (ii) Deflections (Δ)

12. The beam shown in figure is to be analyzed by slope deflection method. What are the unknowns and to determine them, what are the conditions used?



Unknowns are θ_A, θ_B and θ_C

Equilibrium equations used:

(i) $M_{AB} = 0$

(ii) $M_{BA} + M_{BC} = 0$

(iii) $M_{CB} = 0$

13. Mention any three reasons due to which sway may occur in portal frames.

Sway in portal frames may occur due to

i) Unsymmetry in geometry of the frame

ii) Unsymmetry in loading or

iii) Settlement of one end of a frame.

14. Write down the general slope deflection equations and state what each term represents.

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B + \frac{3\delta}{L}\right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(\theta_A + 2\theta_B + \frac{3\delta}{L}\right)$$

Where, M_{AB}, M_{BA} = fixed end moments at A and B due to given loading.

θ_A, θ_B = slopes at A and B.

Δ = Sinking of support A with respect to B.

15. A rigid frame is having totally 10 joints including support joints. Out of slope deflection and moment distribution methods, which method would you prefer for analysis? Why?

Moment distribution method is preferable.

If we use slope-deflection method, there would be 10 (or more) unknown displacements and an equal number of equilibrium equations. In addition, there would be 2 unknown support moments per span and the same number of slope-deflection equations. Solving them is difficult.