
Objectives: Electromagnetic theory is very important to understand the propagation of waves in different media, its transmission and reception. This paper is intended to give information about the theories of electromagnetic waves and their propagation through different media.

UNIT I

Maxwell Equations: Review of Maxwell's equations. Displacement Current. Vector and Scalar Potentials. Gauge Transformations: Lorentz and Coulomb Gauge. Boundary Conditions at Interface between Different Media. Wave Equations. Plane Waves in Dielectric Media. Poynting Theorem and Poynting Vector. Electromagnetic (EM) Energy Density. Physical Concept of Electromagnetic Field Energy Density, Momentum Density and Angular Momentum Density.

UNIT II

EM Wave Propagation in Unbounded Media: Plane EM waves through vacuum and isotropic dielectric medium, transverse nature of plane EM waves, refractive index and dielectric constant, wave impedance. Propagation through conducting media, relaxation time, skin depth. Wave propagation through dilute plasma, electrical conductivity of ionized gases, plasma frequency, refractive index, skin depth, application to propagation through ionosphere.

UNIT III

EM Wave in Bounded Media: Boundary conditions at a plane interface between two media. Reflection & Refraction of plane waves at plane interface between two dielectric media-Laws of Reflection & Refraction. Fresnel's Formulae for perpendicular & parallel polarization cases, Brewster's law. Reflection & Transmission coefficients. Total internal reflection, evanescent waves. Metallic reflection (normal Incidence).

UNIT IV

Polarization of Electromagnetic Waves: Description of Linear, Circular and Elliptical Polarization. Propagation of E.M. Waves in Anisotropic Media. Symmetric Nature of Dielectric Tensor. Fresnel's Formula. Uniaxial and Biaxial Crystals. Light Propagation in Uniaxial Crystal. Double Refraction. Polarization by Double Refraction. Nicol Prism. Ordinary & extraordinary refractive indices. Production & detection of Plane, Circularly and Elliptically Polarized Light. Phase Retardation Plates: Quarter-Wave and Half-Wave Plates. Babinet Compensator and its Uses. Analysis of Polarized Light

UNIT V

Wave Guides: Planar optical wave guides. Planar dielectric wave guide. Condition of continuity at interface. Phase shift on total reflection. Eigenvalue equations. Phase and group velocity of guided waves. Field energy and Power transmission.

Optical Fibres:- Numerical Aperture. Step and Graded Indices (Definitions Only). Single and Multiple Mode Fibres (Concept and Definition Only).

Reference Books:

1. Introduction to Electrodynamics, D.J. Griffiths, 3rd Ed., 1998, Benjamin Cummings.
 2. Elements of Electromagnetics, M.N.O. Sadiku, 2001, Oxford University Press.
 3. Introduction to Electromagnetic Theory, T.L. Chow, 2006, Jones & Bartlett Learning
 4. Fundamentals of Electromagnetics, M.A.W. Miah, 1982, Tata McGraw Hill
 5. Electromagnetic field Theory, R.S. Kshetrimayun, 2012, Cengage Learning
 6. Engineering Electromagnetic, William H. Hayt, 8th Edition, 2012, McGraw Hill.
- Electromagnetic Field Theory for Engineers & Physicists, G. Lehner, 2010, Springer

UNIT-I

S No	Lecture	Topics to be covered	Support materials
1.	1Hr	Review of Maxwell's equations. Displacement Current. Vector and Scalar	T1-326-330
2.	1Hr	Gauge Transformations: Lorentz and	T1-416-417
3.	1Hr	Boundary Conditions at Interface between Different Media. Wave	T1-419-422
4.	1Hr	Poynting Theorem and Poynting	T1-331-333
5.	1Hr	Electromagnetic (EM) Energy Density.	T1-346-348
6.	1Hr	Physical Concept of Electromagnetic Field Energy	T1-355-359
7.	1Hr	Revision	
8.	1Hr	Discussion of Possible 2 marks	
9.	1Hr	Discussion of Possible 8 marks	
10.	1Hr	Class test in 2 marks	
11.	1Hr	Class test in 8 marks	
	Total no of hours		11

Text Book

T1: Introduction to Electrodynamics, David Jeffery Griffiths, 3rd Ed., 1998, Benjamin Cummings

Reference book

R2: Elements of Electromagnetics, M.N.O.Sadiku, 2001, Oxford University Press.

UNIT-II

S No	Lecture Duration (Hr)	Topics to be covered	Support materials
1.	1Hr	Plane EM waves through vacuum and isotropic dielectric medium	T2- 235
2.	1Hr	Transverse nature of plane EM waves, refractive index and dielectric constant, wave impedance	T2- 236
3.	1Hr	Propagation through conducting media, relaxation time, skin depth	T2- 243
4.	1Hr	Wave propagation through dilute plasma, electrical conductivity of ionized gases	T2- 254
5.	1Hr	plasma frequency, refractive index, skin depth	T2- 246, 253
6.	1Hr	application to propagation through ionosphere	T2– 257
7.	1Hr	Revision	
8.	1Hr	Discussion of Possible 2 marks	
9.	1Hr	Discussion of Possible 8 marks	
10.	1Hr	Class test in 2 marks	
11.	1Hr	Class test in 8 marks	
Total no of hours			11

Text Book

T2: Electromagnetic Theory, Chopra & Agarwal, 6th Ed., 2004 Nath & Co, Meerut.

Reference Book

R1: Introduction to the Electrodynamics, David Jeffery Griffiths, 3rd Ed., 1998, Benjamin Cummings

UNIT-III

S No	Lecture	Topics to be covered	Support materials
1.	1Hr	Boundary conditions at a plane interface between two media	T1-370-373
2.	1Hr	Reflection & Refraction of plane waves at plane interface between two dielectric media-Laws of Reflection &	T1-388
3.	1Hr	Fresnel's Formulae for perpendicular & parallel polarization cases	T1-386-390
4.	1Hr	Brewster's law	T1-390
5.	1Hr	Reflection & Transmission coefficients	T1-391-392
6.	1Hr	. Total internal reflection, evanescent waves. Metallic reflection (normal Incidence).	T1- 384-386
7.	1Hr	Revision	
8.	1Hr	Discussion of Possible 2 marks	
9.	1Hr	Discussion of Possible 8 marks	
10.	1Hr	Class test in 2 marks	
11.	1Hr	Class test in 8 marks	
Total no of hours			11

Text Book

T1:Introduction to the Electrodynamics, David Jeffery Griffiths, 3rd Ed., 1998, Benjamin Cummings

Reference Book

R2: Elements of Electromagnetics, M.N.O.Sadiku, 2001, Oxford University Press.

UNIT-IV

S No	Lecture Duration (Hr)	Topics to be covered	Support materials
1.	1Hr	Description of Linear, Circular and Elliptical Polarization.	T2- 1.19-1.21
2.	1Hr	Propagation of E.M. Waves in Anisotropic Media. Symmetric Nature of Dielectric Tensor.	T4-22.26
3.	1Hr	Fresnel's Formula. Uniaxial and Biaxial Crystals	T4- 20.6
4.	1Hr	Light Propagation in Uniaxial Crystal. Double Refraction. Polarization by Double Refraction.	T4-22.14
5.	1Hr	Nicol Prism. Ordinary & extraordinary refractive indices.	T4-22.8
6.	1Hr	Production & detection of Plane, Circularly and Elliptically Polarized Light.	T4-22.6
7.	1 Hr	Phase Retardation Plates: Quarter-Wave and Half-Wave Plates	T4-22.18
8.	1Hr	Babinet Compensator and its Uses. Analysis of Polarized Light	T4-22.20
9.	1 Hr	Revision	
10.	1Hr	Discussion of Possible 2 marks	
11.	1Hr	Discussion of Possible 8 marks	
12.	1Hr	Class test in 2 marks	
13.	1Hr	Class test in 8 marks	
Total no of hours			13

Text Book

T3: Electromagnetic waves & wave guides, Dr.P.Dananjayan, I Ed., T4: Optics, Ajoy Ghatak, 5th Ed., 2012, Tata Mc Graw Hills

Reference Book

R1: Introduction to Electrodynamics, David Jeffery Griffiths, 3rd Ed., 1998,

Benjamin Cummings

UNIT-V

S No	Lecture Duration (Hr)	Topics to be covered	Support materials
1.	1Hr	Planar optical wave guides. Planar dielectric wave guide.	T1-533-535
2.	1Hr	Condition of continuity at interface. Phase shift on total reflection.	T1-468
3.	1Hr	Eigenvalue equations. Phase and group velocity of guided waves.	T1-468
4.	1Hr	Field energy and Power transmission.	T1-500
5.	1Hr	Numerical Aperture. Step and Graded Indices (Definitions Only).	T4-27.8
6.	1Hr	Single and Multiple Mode Fibres (Concept and Definition Only).	T4-29.6
7.	1Hr	Revision	
8.	1Hr	Old question paper discussion	
9.	1Hr	Old question paper discussion	
10.	1Hr	Old question paper discussion	
11.	1Hr	Test in Unit 1 & 2	
12.	1Hr	Test in Unit 2 & 3	
13.	2Hr	Full portion Test	
Total no of hours			14

Text Book

T1: Introduction to Electrodynamics, David Jeffery Griffiths, 3rd Ed., 1998, Benjamin Cummings

T4: Optics, Ajoy Ghatak, 5th Ed., 2012, Tata Mc Graw- Hills

Reference Book

R2: Elements of Electromagnetics, M.N.O.Sadiku, 2001, Oxford University Press.

Maxwell Equations:

Review of Maxwell's equations. Displacement Current. Vector and Scalar Potentials. Gauge Transformations: Lorentz and Coulomb Gauge. Boundary Conditions at Interface between Different Media. Wave Equations. Plane Waves in Dielectric Media. Poynting Theorem and Poynting Vector. Electromagnetic (EM) Energy Density. Physical Concept of Electromagnetic Field Energy Density, Momentum Density and Angular Momentum Density.

REVIEW OF MAXWELLS EQUATION :

The four fundamental equation of electromagnetism and corresponds to a generalisation of certain experimental observations-regarding electricity and magnetism. The following four laws of electricity and magnetism constitutes the so called differential form of Maxwell's equation.

1. Guass law for the electric field of charge yields

$$\text{div } \mathbf{D} = \nabla \cdot \mathbf{D} = \rho \quad \text{-----(A)}$$

\mathbf{D} – electric displacement in coulombs / m²

ρ – free charge density in coulombs / m³

2. Guass law for magnetic field yields

$$\text{div } \mathbf{B} = \nabla \cdot \mathbf{B} = 0 \quad \text{-----(B)}$$

\mathbf{B} – magnetic induction in web / m³

3. Ampere's Law in circuital form for the magnetic field accompanying a current when modified by Maxwell yields

$$\text{Curl } \mathbf{H} = \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{----- (C)}$$

\mathbf{H} – magnetic field intensity in amperes / m

\mathbf{J} – current density amperes / m²

4. Faradays law in circuital form for the induced electromotive force produced by the rate of change of magnetic flux linked with the path yields.

$$\text{Curl } \mathbf{E} = \nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \quad \text{----- (D)}$$

\mathbf{E} – electric field intensity in Volts / m

DERIVATIONS :

1. $\text{div } \mathbf{D} = \nabla \cdot \mathbf{D} = \rho$

Let us consider a surface S bounding a volume τ within a dielectric. The volume τ contains no net charge but we allow the dielectric to be polarised say by placing it in an electric field. Some charge on the dielectric body are placed. Thus we have two types of charges

a) real charge of density ρ

b) bound charge density ρ' , Gauss law then can be written as,

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} \int_{\tau} (\rho + \rho') d\tau$$

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} \int_{\tau} \rho d\tau + \int_{\tau} \rho' d\tau \text{-----(1)}$$

But as the bound charge density ρ' is defined as

$$\rho' = -\text{div } \mathbf{P}$$

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \int_{\tau} \text{div } \mathbf{E} d\tau$$

Equation (1) can be written as ,

$$\oint_S \text{div } \mathbf{E} d\tau = \int_{\tau} \rho d\tau - \int_{\tau} \text{div } \mathbf{P} d\tau$$

$$\int_{\tau} \text{div } (\epsilon_0 \mathbf{E} + \mathbf{P}) d\tau = \int_{\tau} \rho d\tau \quad [\epsilon_0 \mathbf{E} + \mathbf{P} = \mathbf{D}]$$

$$\int_{\tau} (\text{div } \mathbf{D} - \rho) d\tau = 0$$

This equation is true for all volumes, the integration must vanish.

$$\text{div } \mathbf{D} = \rho, \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

2. $\text{div } \mathbf{B} = \nabla \cdot \mathbf{B} = 0$

Experiments to date have shown that magnetic poles do not exist. This in turn implies that the magnetic lines of force are either closed loops or go off to infinity. Hence the no. of magnetic lines of force entering any arbitrary closed surface is exactly the same as leaving it.

The flux of magnetic induction \mathbf{B} across any closed surface is always zero.

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

Transforming this surface integral to volume integral by Gauss theorem, we get,

$$\int_{\tau} \text{div } \mathbf{B} d\tau = 0$$

But as the surface bounding the volume is quite arbitrary the above equation will be true only when the integrand vanishes .

$$\text{div } \mathbf{B} = \nabla \cdot \mathbf{B} = 0$$

3. $\text{Curl } \mathbf{H} = \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$

From Ampere's circuital law the work done in carrying unit magnetic pole once round a closed arbitrary path linked with the current I is expressed by,

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} \quad [I = \int_S \mathbf{J} \cdot d\mathbf{s}]$$

Where S is the surface bounded by the closed path C .

Now changing the line integral into surface integral by Stokes theorem, we get

$$\int_S \text{curl } \mathbf{H} \cdot d\mathbf{s} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

$$\text{Curl } \mathbf{H} = \mathbf{J} \quad \text{-----(2)}$$

But Maxwell found it to be incomplete for changing electric fields and assumed that a quantity,

$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$ is called displacement current must also be included in it so that it may satisfy the continuity equation, \mathbf{J} must be replaced by $\mathbf{J} + \mathbf{J}_d$, so the law becomes,

$$\text{Curl } \mathbf{H} = \mathbf{J} + \mathbf{J}_d$$

$$\text{Curl } \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

4. $\text{Curl } \mathbf{E} = \nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$

According to Faradays law of electromagnetic induction, we know that the induced e.m.f is proportional to the rate of change flux

$$\epsilon = -d\Phi_B / dt \quad \text{----- (3)}$$

Now if E be the electric intensity at a point the work done in moving a unit charge through a small distance $d\mathbf{l}$ is

$E \cdot d\mathbf{l}$. So the work done in moving the unit charge once round the circuit is $\oint_C \mathbf{E} \cdot d\mathbf{l}$. Now as e.m.f is defined as the amount of work done in moving a unit charge once round the electric circuit.

$$\epsilon = \oint_C \mathbf{E} \cdot d\mathbf{l} \quad \text{----- (4)}$$

Comparing equation (3) and (4), we get,

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = d\Phi_B / dt \quad \text{----- (5)}$$

$$\mathbf{B} = \int_S \mathbf{B} \cdot d\mathbf{s}$$

$$\text{So, } \oint_C \mathbf{E} \cdot d\mathbf{l} = - \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s}$$

Now changing the line integral into surface integral by Stokes theorem, we get

$$\int_S \text{curl } \mathbf{E} \cdot d\mathbf{s} = - \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{s}$$

The surface S is fixed in space and only B changes with time, above equation yields,

$$\int_S (\text{curl} + \frac{\partial \mathbf{B}}{\partial t}) \cdot d\mathbf{s} = 0$$

Integrated vanish is the integral is true for arbitrary,

$$\text{Curl } \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

Special Cases :

1. In a conducting medium of relative permittivity ϵ_r and permeability μ_r as

$$\mathbf{D} = \epsilon_r \epsilon_0 \mathbf{E}$$

$$\mathbf{B} = \mu_r \mu_0 \mathbf{H}$$

And Maxwell equation reduced to

$$(i) \quad \nabla \cdot \mathbf{E} = \rho / \epsilon_r \epsilon_0$$

$$(ii) \quad \nabla \cdot \mathbf{H} = 0$$

$$(iii) \quad \nabla \times \mathbf{H} = \mathbf{J} + \epsilon_r \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$(iv) \quad \nabla \times \mathbf{E} = - \mu_r \mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

2. In a non-conducting media of relative permittivity ϵ_r and permeability μ_r as

$$\rho = \sigma = 0$$

$$\mathbf{J} = \sigma \mathbf{E} = 0$$

And Maxwell equation reduced to

$$(i) \quad \nabla \cdot \mathbf{E} = 0$$

$$(ii) \quad \nabla \cdot \mathbf{H} = 0$$

$$(iii) \quad \nabla \times \mathbf{H} = \epsilon_r \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$(iv) \quad \nabla \times \mathbf{E} = - \mu_r \mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

3. In free space as

$$\epsilon_r = \mu_r = 1$$

$$\rho = \sigma = 0$$

And Maxwell equation reduced to

$$(i) \quad \nabla \cdot \mathbf{E} = 0$$

$$(ii) \quad \nabla \cdot \mathbf{H} = 0$$

$$(iii) \quad \nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$(iv) \quad \nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

Discussion :

(i) The equations are based on experimental observations the equation (A) and (C) correspond to electricity and (B) and (D) to magnetism.

(ii) These equations are general and apply to all electromagnetic phenomena in media, which are at rest with respect to the co-ordinate system.

(iii) These equations are not independent of each other as from equation (D) we can derive (B) and from (C), (A). The equation (B) and (D) are called first pair of Maxwell's equation while (A) and (C) are called the second pair.

(iv) The equation A represents Coulomb's law while C the law of conservation of charge (i.e.) continuity equation.

DISPLACEMENT CURRENT :

We know that Ampere's circuital law in its most general form is given by

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} \quad \mathbf{J} - \text{Current density}$$

$$\int_S \text{curl } \mathbf{H} \cdot d\mathbf{s} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

$$\text{Curl } \mathbf{H} = \mathbf{J} \quad \text{----- (1)}$$

Let us now examine the validity of this equation in the event that the fields are allowed to vary with time. If we take the divergence of both sides of equation (1) then,

$$\text{div}(\text{curl}) = \text{div } \mathbf{J} \quad \text{----- (2)}$$

Now as div of curl of any vector is zero, we get from equation (2),

$$\text{div } \mathbf{J} = 0 \quad \text{----- (3)}$$

Now the continuity equation in general state

$$\text{div } \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad \text{----- (4)}$$

and will therefore vanish only in the special case that the charge density is static. We must conclude that Ampere's law as stated in equation (1) is valid only for steady state conditions and is insufficient for the case of time-dependent fields. Because of this Maxwell assumed

that equation (1) is not complete but should have something be denoted be J_d , then equation (1) can be written as

$$\text{curl } H = J + J_d \quad \text{----- (5)}$$

In order to identify J_d , we calculate the divergence of equation (2) again and get

$$\text{div curl } H = \text{div } (J + J_d) \quad [\text{div curl } H = 0]$$

$$\text{div } (J + J_d) = 0$$

$$\text{div } J + \text{div } J_d = 0$$

$$\text{div } J_d = - \text{div } J$$

$$\text{div } J_d = \frac{\partial \rho}{\partial t} \quad \{ \text{from a equation (4)} \}$$

$$\text{div } J_d = \frac{\partial}{\partial t} \text{div } D$$

$$\text{div } J_d - \frac{\partial}{\partial t} \text{div } D = 0 \quad \{ \text{div } D = 0 \}$$

$$\text{div } (J_d - \frac{\partial D}{\partial t}) = 0 \quad \text{----- (6)}$$

As equation (6) is true for any arbitrary volume $J_d = \frac{\partial D}{\partial t}$

And so the modified form of Ampere's circuital law becomes,

$$\text{Curl } H = J + \frac{\partial D}{\partial t}$$

D / t called the displacement current to distinguish it from J , the conduction current. By adding this term to Ampere's law, Maxwell assumed that the time rate of change of displacement produce a magnetic field just as a conduction current.

SCALAR AND VECTOR POTENTIALS :

The analysis of an electromagnetic field is often facilitated by the use of auxiliary functions know as electromagnetic potentials. At every point of space the field vectors satisfy the equations,

$$\text{div } D = \quad \text{----- (A)}$$

$$\text{div } B = 0 \quad \text{----- (B)}$$

$$\text{Curl } H = J + D / t \quad \text{----- (C)}$$

$$\text{Curl } E = - B / t \quad \text{----- (D)}$$

According to equation (B) , the field of vector B is always solenoidal, B can be represented as the curl of another vector say A .

$$B = \text{curl } A \quad \text{----- (1)}$$

Where A is a vector which is function of space (x, y, z) and time (t) both. Now sub the value B in equation (1) we get,

$$\begin{aligned}\text{Curl } E &= -\frac{\partial}{\partial t} \text{curl } A \\ \text{Curl } (E + \frac{\partial A}{\partial t}) &= 0 \quad \text{----- (2)}\end{aligned}$$

(i.e.) $E + \frac{\partial A}{\partial t}$ is irrotational and must be equal to the gradient of some scalar function.

$$\begin{aligned}E + \frac{\partial A}{\partial t} &= -\text{grad } \phi \\ E &= -\text{grad } \phi - \frac{\partial A}{\partial t} \quad \text{----- (3)}\end{aligned}$$

Thus we have introduced a vector A and a scalar ϕ both being functions of position and time. These are called electromagnetic potentials. The scalar ϕ is called the scalar potential and vector A , vector potential.

Properties of scalar and vector potential :

- (i) These are mathematical function, which are not physically measurable.
- (ii) They are not independent of each other.
- (iii) They play an important role in relativistic electrodynamics.

NON-UNIQUENESS OF ELECTROMAGNETIC POTENTIALS AND CONCEPT OF GAUGE :

In terms of electromagnetic potentials field vectors are given by ,

$$B = \text{curl } A \quad \text{----- (1)}$$

And
$$E = -\text{grad } \phi - \frac{\partial A}{\partial t} \quad \text{----- (2)}$$

From equations (1) and (2) it is clear that for a given A and ϕ , each of the field vectors B and E has only value i.e. A and ϕ determine B and E uniquely. However the converse is not true i.e. field vectors do not determine the potentials A and ϕ completely. This in turn implies that for a given A and ϕ there will be only one E and B while for a given E and B there can be infinite number of A 's and ϕ 's. This is because the curl of the gradient of any scalar vanishes identically and hence we may add to A the gradient of a scalar Λ without affecting B . That is A may be replaced by,

$$A' = A + \text{grad } \Lambda \quad \text{----- (3)}$$

But if this is done equation (2) becomes,

$$E = -\text{grad } \phi - \frac{\partial}{\partial t} (A' - \text{grad } \Lambda)$$

$$\mathbf{E} = -\text{grad} \left(-\frac{\partial A'}{\partial t} \right) - \frac{\partial A'}{\partial t}$$

So if we make the transformation given by equation (3). We must also replace by

$$\mathbf{A}' = -\frac{\partial \Lambda}{\partial t} \quad (4)$$

The expressions for field vectors \mathbf{E} and \mathbf{B} remain unchanged under transformations equations (3) and (4).

$$\mathbf{B} = \text{curl } \mathbf{A} = \text{curl} \left(\mathbf{A}' - \text{grad } \Lambda \right) = \text{curl } \mathbf{A}'$$

And
$$\mathbf{E} = -\text{grad} \left(-\frac{\partial A}{\partial t} \right) = -\text{grad} \left(\mathbf{A}' + \frac{\partial \Lambda}{\partial t} \right) - \frac{\partial}{\partial t} \left(\mathbf{A}' - \text{grad } \Lambda \right)$$

$$\mathbf{E} = \text{grad} \left(\mathbf{A}' - \frac{\partial A'}{\partial t} \right)$$

We get the same field vectors whether we use the set (\mathbf{A}, Λ) or (\mathbf{A}', Λ') . So electromagnetic potentials define the field vectors uniquely though they themselves are non-unique.

The transformations given by equations (3) and (4) are called gauge transformations and the arbitrary scalar Λ gauge function. From the above it is also clear that even though we add the gradient of a scalar function, the field vectors remain unchanged. Now it is the field quantities and not the potentials that possess physical meaningfulness. We therefore say that the field vectors are invariant to gauge transformations i.e. they are gauge invariant.

LORENTZ GAUGE :

The Maxwell's field equations in terms of electromagnetic potentials are,

$$\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} - \text{grad} \left(\text{div } \mathbf{A} + \mu\epsilon \frac{\partial \phi}{\partial t} \right) = -\mu \mathbf{J} \quad (1)$$

$$\nabla^2 \phi - \mu\epsilon \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial t} \left(\text{div } \mathbf{A} + \mu\epsilon \frac{\partial \phi}{\partial t} \right) = -\frac{\rho}{\epsilon} \quad (2)$$

A casual glance at equations (1) and (2) reveals that these equations will be much more simplified (i.e. will become identical and uncoupled) if we choose

$$\text{div } \mathbf{A} + \mu\epsilon \frac{\partial \phi}{\partial t} = 0 \quad (3)$$

This requirement is called the Lorentz condition when the vector and scalar potential satisfy it, the gauge is called is known as Lorentz gauge.

So with Lorentz condition field equation reduce to

$$\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\mu \mathbf{J} \quad (4)$$

$$\nabla^2 \phi - \mu\epsilon \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon} \quad (5)$$

But as $\mu\epsilon = 1/v^2$

So equations (4) and (5) can be written as

$$\nabla^2 A = -\mu J \quad \text{----- (6)}$$

$$\nabla^2 \phi = -\frac{\rho}{\epsilon} \quad \text{----- (7)}$$

With $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Equations (6) and (7) are inhomogeneous wave equations and are known as D'Alembertian equations and can be solved in general by a method similar to that we use to solve Poisson's equation. The potentials obtained by solving these equations are called retarded potentials.

In order to determine the requirement that Lorentz condition places on A' and ϕ' from equations (3) and (4).

$$\text{div}(A' - \text{grad } \Lambda) + \mu\epsilon \frac{\partial}{\partial t}(\phi' + \frac{\partial \Lambda}{\partial t}) = 0$$

$$\text{div } A' + \mu\epsilon \frac{\partial \phi'}{\partial t} = \nabla^2 \Lambda - \mu\epsilon \frac{\partial^2 \Lambda}{\partial t^2}$$

So A' and ϕ' will also satisfy equation (3) i.e. Lorentz condition provided that

$$\nabla^2 \Lambda - \mu\epsilon \frac{\partial^2 \Lambda}{\partial t^2} = 0$$

i.e. $\nabla^2 \Lambda = 0 \quad \text{----- (8)}$

Lorentz condition is invariant under those gauge transformations for which the gauge functions are solutions of the homogeneous wave equations.

The advantages of this particular gauge are :

- (i) It makes the equations for A and ϕ independent of each other.
- (ii) It leads to the wave equations which treat ϕ and A on equivalent footings.
- (iii) It is a concept which is independent of the co-ordinate system chosen and so fits naturally into the considerations of special theory of relativity.

COULOMB GAUGE :

An inspection of field equations in terms of electromagnetic potentials,

$$\nabla^2 A - \mu\epsilon \frac{\partial^2 A}{\partial t^2} - \text{grad}(\text{div } A + \mu\epsilon \frac{\partial \phi}{\partial t}) = -\mu J \quad \text{----- (1)}$$

$$\nabla^2 \phi - \mu\epsilon \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial t}(\text{div } A + \mu\epsilon \frac{\partial \phi}{\partial t}) = -\frac{\rho}{\epsilon}$$

i.e. $\nabla^2 \phi + \frac{\partial}{\partial t}(\text{div } A) = -\frac{\rho}{\epsilon} \quad \text{----- (2)}$

Shows that if we assume ,

$$\text{div } A = 0 \quad \text{----- (3)}$$

equation (2) reduces to Poisson's equation

$$\nabla^2 \phi(r, t) = -\frac{\rho(r', t)}{\epsilon} \quad \text{----- (4)}$$

Whose solution is ,

$$\phi(r, t) = \frac{1}{4\pi\epsilon} \int \frac{\rho(r', t)}{R} d\tau' \quad \text{----- (5)}$$

i.e. the scalar potential is just the instantaneous Coulombian potential due to charge (x', y', z', t) . This is the origin of the name Coulomb gauge. Equation (1) in the light of (3) reduced to

$$\nabla^2 A - \frac{1}{V^2} \frac{\partial^2 A}{\partial t^2} = -\mu J + \mu\epsilon \nabla \cdot \frac{\partial \phi}{\partial t} \quad \text{----- (6)}$$

Now to express equation (6) in more convenient way we use Poisson's equation (4) which with the help of (5) can be written as

$$\nabla^2 \left\{ \frac{1}{4\pi\epsilon} \int \frac{\rho(r', t)}{R} d\tau' \right\} = -\frac{\rho(r', t)}{\epsilon} \quad \text{----- (7)}$$

Now as Poisson's equation holds good for scalar and vectors both, replacing (r', t) by J' we get,

$$\nabla^2 \left\{ \frac{1}{4\pi\epsilon} \int \frac{J'}{R} d\tau' \right\} = -\frac{J'}{\epsilon} \quad \text{----- (8)}$$

Now from the vector identity

$$\nabla \times \nabla \times G = (\nabla \cdot G) - \nabla^2 G$$

$$\nabla^2 G = (\nabla \cdot G) - \nabla \times \nabla \times G$$

Taking $G = \int \left(\frac{J'}{R} \right) d\tau'$, we get

$$\nabla^2 \left(\frac{J'}{R} \right) d\tau' = \nabla \cdot \left(\frac{J'}{R} d\tau' \right) - \nabla \times \nabla \times \int \frac{J'}{R} d\tau'$$

Which in the light of equation (8) reduces to

$$-4\pi J' = \nabla \cdot \left(\frac{J'}{R} d\tau' \right) - \nabla \times \nabla \times \int \frac{J'}{R} d\tau'$$

$$\text{i.e. } J' = -\frac{1}{4\pi} \nabla \cdot \left(\nabla \cdot \int \frac{J'}{R} d\tau' \right) + \frac{1}{4\pi} \nabla \times \nabla \times \int \frac{J'}{R} d\tau' \quad \text{----- (9)}$$

Now as $\nabla \cdot \int \frac{J'}{R} d\tau'$

$$= \left[\frac{1}{R} \nabla \cdot J' + J' \cdot \nabla \left(\frac{1}{R} \right) \right] \{ \text{as } (\nabla \cdot s) = \nabla \cdot s + s \cdot \nabla \}$$

$$= J' \cdot \nabla \left(\frac{1}{R} \right) \{ \text{as } J' \text{ is not a function of } x, y \text{ and } z \}$$

$$= -J' \cdot \nabla \left(\frac{1}{R} \right) \{ \text{as } \nabla \left(\frac{1}{R} \right) = -\nabla' \left(\frac{1}{R} \right) \}$$

$$= \left[\nabla' \cdot \frac{J'}{R} - \nabla' \cdot \left(\frac{J'}{R} \right) \right] d\tau' \{ \text{as } \nabla' \cdot \frac{J'}{R} = \left(\frac{1}{R} \right) \nabla' \cdot J' + J' \cdot \nabla' \left(\frac{1}{R} \right) \}$$

$$= \nabla' \cdot \frac{J'}{R} d\tau' - \oint_s \left(\frac{J'}{R} \right) \cdot ds \{ \text{as } \oint \nabla' \left(\frac{J'}{R} \right) d\tau' = \oint_s \left(\frac{J'}{R} \right) \cdot ds \}$$

As J' is confined to the vol τ' , the surface contribution will vanish so

$$\cdot \left(\frac{J'}{R} \right) d\tau' = \nabla' \cdot \frac{J'}{R} d\tau' \quad \text{----- (10)}$$

And $\times \left(\frac{J'}{R} \right) d\tau'$

$$\begin{aligned} &= \left[\nabla' \times \frac{J'}{R} - J' \times \nabla' \left(\frac{1}{R} \right) \right] d\tau' \{ \text{as } \text{curl } SV = S \text{ curl } V - V \times \text{grad } S \} \\ &= - \left[J' \times \nabla' \left(\frac{1}{R} \right) \right] d\tau' \{ \text{as } J' \text{ is not a function of } x, y, \text{ and } z \} \\ &= J' \times \nabla' \left(\frac{1}{R} \right) d\tau' \{ \text{as } \nabla' \left(\frac{1}{R} \right) = - \nabla' \left(\frac{1}{R} \right) \} \\ &= \left[\nabla' \times \frac{J'}{R} - \nabla' \times \left(\frac{J'}{R} \right) \right] d\tau' \{ \text{as } \nabla' \times (J'/R) = (1/R) \nabla' \times J' - J' \times \nabla' (1/R) \} \\ &= \left[\nabla' \times \frac{J'}{R} + \oint \frac{J'}{R} \times ds \right] d\tau' \{ \text{as } \oint \nabla' \times V d\tau' = - \oint V \times ds \} \end{aligned}$$

As J' is confined to vol τ' , surface contribution will vanish so

$$\times \left(\frac{J'}{R} \right) d\tau' = \int \nabla' \times \frac{J'}{R} d\tau' \quad \text{----- (11)}$$

So equation (9) becomes

$$J' = - \frac{1}{4\pi} \nabla' \cdot \left(\frac{J'}{R} \right) d\tau' + \frac{1}{4\pi} \nabla' \times \left(\frac{J'}{R} \right) d\tau'$$

$$\text{i.e. } J' = J'_1 + J'_T \quad \text{----- (12)}$$

$$\text{With } J'_1 = - \frac{1}{4\pi} \nabla' \cdot \left(\frac{J'}{R} \right) d\tau' \text{ and } J'_T = \frac{1}{4\pi} \nabla' \times \left(\frac{J'}{R} \right) d\tau' \quad \text{----- (13)}$$

Now as

$$\begin{aligned} \times J'_1 &= \times \left[- \frac{1}{4\pi} \nabla' \cdot \left(\frac{J'}{R} \right) d\tau' \right] \\ \times J'_1 &= 0 \quad \{ \text{as } \text{curl grad } \phi = 0 \} \quad \text{----- (14)} \end{aligned}$$

$$\text{And } \nabla' \cdot J'_T = \nabla' \cdot \left[\frac{1}{4\pi} \nabla' \times \left(\frac{J'}{R} \right) d\tau' \right]$$

$$\cdot J'_T = 0 \quad \{ \text{as } \text{div curl } V = 0 \} \quad \text{----- (15)}$$

The first term on R.H.S of equation (12) is irrotational and second is solenoidal. The first term is called longitudinal current and the other transverse current.

So in the light of equation (12), (6) can be written as

$$\nabla^2 A = \frac{1}{\epsilon_0} \frac{\partial^2 \rho}{\partial t^2} = - \mu (J_1 + J_T) + \mu \epsilon \nabla \cdot \frac{\partial \phi}{\partial t}$$

$$\nabla^2 A = \frac{1}{\epsilon_0} \frac{\partial^2 \rho}{\partial t^2} = - \mu J_T - \mu J_1 + \mu \epsilon \nabla \cdot \left[\frac{1}{4\pi \epsilon} \int \frac{\rho(r', t)}{R} d\tau' \right] \quad \{ \text{Substituting from equation (5)} \}$$

$$\nabla^2 A - \frac{1}{v^2} \frac{\partial^2 A}{\partial t^2} = -\mu J_T - \mu J_L + \mu \frac{1}{4\pi} \nabla \cdot \left(-\frac{\nabla \cdot \mathbf{J}}{R} d\tau' \right) \quad \left\{ \text{as from continuity equation } \frac{\rho(r',t)}{R} = -\frac{1}{R} \cdot \mathbf{J} \right\}$$

$$\text{Or } \nabla^2 A - \frac{1}{v^2} \frac{\partial^2 A}{\partial t^2} = -\mu J_T - \mu J_L + \mu J_L \quad \left\{ \text{from equation (13)} \right\}$$

$$\nabla^2 A - \frac{1}{v^2} \frac{\partial^2 A}{\partial t^2} = -\mu J_T$$

$$\boxed{\nabla^2 A = -\mu J_T} \quad \text{----- (16)}$$

The equation for A can be expressed entirely in terms of the transverse current. So this gauge sometimes is also called as transverse gauge.

The Coulomb gauge has a entire advantage. In it the scalar potential is exactly the electrostatic potential and electric field,

$$\mathbf{E} = -\text{grad } \phi - \frac{\partial A}{\partial t}$$

Is separable into an electrostatic field $\mathbf{V} = -\text{grad } \phi$ and a wave field given by $-\frac{\partial A}{\partial t}$.

This gauge is often used when no sources are present. Then according to equation 5, $\nabla \cdot \mathbf{E} = 0$ and A satisfies the homogeneous wave equation 16. The fields are given by,

$$\mathbf{E} = -\frac{\partial A}{\partial t} \text{ and } \mathbf{B} = \nabla \times \mathbf{A}$$

POYNTING THEOREM (OR) ENERGY IN ELECTROMAGNETIC FIELDS :

“ The rate of decrease of energy in the electrodynamic fields in a specific region is equal to the sum of rate of work done on charges and rate of escape of energy through the surface in the form of electromagnetic radiation.”

According to Lorentz law, the force acting in a electromagnetic field is given by

$$\vec{F} = [\vec{E} + (\vec{v} \times \vec{B})] \quad \text{----- (1)}$$

For an elementary volume $d\tau'$, the force experienced in an electromagnetic field is given by

$$\vec{F} = \oint_v [\vec{E} + (\vec{v} \times \vec{B})] \rho d\tau \quad \text{or } \oint_v \rho d\tau$$

The work done in causing a displacement $d\mathbf{l}$ in the electromagnetic field is given by

$$W = \oint_v [\vec{E} + (\vec{v} \times \vec{B})] \rho d\tau \cdot d\mathbf{l} \quad \text{----- (2)}$$

$$W = \oint_v [\vec{E} + (\vec{v} \times \vec{B})] \rho d\tau \cdot \vec{v} dt$$

Rate of work done in an electromagnetic field is given by

$$\frac{dW}{dt} = \oint_v [\vec{E} + (\vec{v} \times \vec{B})] \rho d\tau \cdot \vec{v} \quad \text{----- (3)}$$

Assuming the rate of work done in the electric field only, we get

$$\begin{aligned}\frac{dW}{dt} &= \oint_V \vec{E} \cdot \rho d\tau, \vec{v} \times \vec{B} = 0 \\ &= \oint_V \vec{E} \cdot \rho \vec{v} d\tau \\ P &= \frac{dW}{dt} = \oint_V (\vec{E} \cdot \vec{j}) d\tau \quad \text{----- (4)}\end{aligned}$$

We know that the modified Ampere's law is applicable to electrodynamics.

$$\frac{\nabla \times \vec{B}}{\mu_0} - \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{j} \quad \text{----- (5)}$$

Putting the value of \vec{j} from equation (5) in equation (4), we get

$$\begin{aligned}\oint_V (\vec{E} \cdot \vec{j}) d\tau &= \oint_V \vec{E} \cdot \left[\frac{\nabla \times \vec{B}}{\mu_0} - \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right] d\tau \\ &= \oint_V \vec{E} \cdot \frac{\nabla \times \vec{B}}{\mu_0} d\tau - \oint_V \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} d\tau \quad \text{----- (6)}\end{aligned}$$

$$\text{We know that } \nabla \cdot \left[\vec{E} \times \frac{\vec{B}}{\mu_0} \right] = \left[\frac{\vec{B}}{\mu_0} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot \frac{\nabla \times \vec{B}}{\mu_0} \right]$$

$$\begin{aligned}\text{Now } \oint_V \vec{E} \cdot \vec{j} d\tau &= \oint_V \frac{1}{\mu_0} (\vec{E} \cdot \nabla \times \vec{B}) d\tau - \oint_V \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} d\tau \\ &= \oint_V \frac{\vec{B}}{\mu_0} \cdot (\nabla \times \vec{E}) d\tau - \oint_V \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} d\tau\end{aligned}$$

According to Maxwell's third equation in the differential form,

$$\begin{aligned}\vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \oint_V \vec{E} \cdot \vec{j} d\tau &= \frac{1}{\mu_0} \oint_V \left[\vec{B} \cdot \left(-\frac{\partial \vec{B}}{\partial t} \right) \right] d\tau - \oint_V \nabla \cdot (\vec{E} \times \vec{H}) d\tau - \frac{\epsilon_0}{2} \oint_V \frac{dE^2}{dt} d\tau \\ \oint_V \vec{E} \cdot \vec{j} d\tau &= -\frac{1}{2\mu_0} \oint_V \frac{dB^2}{dt} d\tau - \frac{\epsilon_0}{2} \oint_V \frac{dE^2}{dt} d\tau - \oint_V \nabla \cdot (\vec{E} \times \vec{H}) d\tau \\ &= -\frac{\partial}{\partial t} \oint_V \left[\frac{B^2}{2\mu_0} + \frac{1}{2} \epsilon_0 E^2 \right] d\tau - \oint_V \nabla \cdot (\vec{E} \times \vec{H}) d\tau \\ &= -\frac{\partial}{\partial t} \oint_V \left[\frac{B^2}{2\mu_0} + \frac{1}{2} \epsilon_0 E^2 \right] d\tau - \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{a} \quad \text{----- (8)}\end{aligned}$$

$$\frac{\partial}{\partial t} \oint_V \left[\frac{B^2}{2\mu_0} + \frac{1}{2} \epsilon_0 E^2 \right] d\tau = \oint_V (\vec{E} \cdot \vec{j}) d\tau + \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{a} \quad \text{----- (9)}$$

$\vec{E} \times \vec{H}$ is called the pointing vector or power density. It is denoted by symbol \vec{S} .

$$\vec{S} = \vec{E} \times \vec{H}$$

The unit of pointing vector is Watts/m². The pointing theorem,

$$-\frac{\partial}{\partial t} \oint_V \left[\frac{B^2}{2\mu_0} + \frac{1}{2} \epsilon_0 E^2 \right] d\tau = \oint_V (\vec{E} \cdot \vec{j}) d\tau + \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{a} \quad \text{is called integral form.}$$

POYNTING VECTOR (OR) POWER DENSITY :

According to the law of conservation of energy in an electromagnetic fields,

$$S = E \times H$$

E – the electric field H – magnetic field

The amount of the field energy passing through unit area of the surface in a direction perpendicular to the plane containing E and H per unit time.

MOMENTUM DENSITY :

The momentum density of an electromagnetic wave is given by ,

$$= \frac{1}{4\pi c} E \times B$$

Where ,

C – speed of light , E – electric field , B – magnetic field , S – pointing vector

$$= \frac{S}{4\pi c} = \frac{S}{c^2}$$

ANGULAR MOMENTUM DENSITY :

For an electromagnetic waves the angular momentum is defined as

$$\begin{aligned} L &= r \times p \\ &= \frac{1}{4\pi c} r \times (E \times B) \\ &= (E \times B) \times r^2 / c^2 \end{aligned}$$

$$L = \frac{S}{c^2} \times r$$

Where

r- position , p – momentum density , E – electric field, B – magnetic field , C – speed of light

BOUNDARY CONDITIONS :

In general the field E,D,B and H will be discontinuous at a boundary between two different media or at a surface that carries charge density σ or current density k. The discontinuous can be deduced from Maxwell's equations as,

$$\begin{aligned} (1) \oint_S D \cdot da &= 0_{\text{fenc}} \text{ Over any closed surface } S \\ (2) \oint_S B \cdot da &= 0 \\ (3) \oint_p E \cdot dl &= - \frac{d}{dt} \int_S B \cdot da \\ (4) \oint_p H \cdot dl &= I_{\text{fenc}} + \frac{d}{dt} \int_S D \cdot da \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{ for any surface } S \text{ bounded by the closed loop } p.$$

Apply equation (1) to tiny water thin Gaussian pill box extending just slightly into the material on either side of the boundary, we obtain

$$D_1 a - D_2 a = \sigma_f a \quad \text{----- (5)}$$

The component of D_1 that is perpendicular to the interface is discontinuous in the amount

$$D_1 - D_2 = \sigma_f \quad \text{----- (6)}$$

Identical reasoning applied to equation (2) yields,

$$B_1 - B_2 = 0 \quad \text{----- (7)}$$

Turning to equation (3), a very thin Amperian loop straddling the surface

$$E_1 l - E_2 l = - \frac{d}{dt} \int_s B \cdot da$$

But in the limit as the width of the loop goes to zero, the flux vanishes,

$$E_1^{\parallel} - E_2^{\parallel} = 0 \quad \text{----- (8)}$$

The component of E parallel to the interface are continuous across the boundary, By the same equation (4) implies,

$$H_1 l - H_2 l = I_{\text{fenc}}$$

Where I_{fenc} is the free current passing through the Amperian loop. No volume current density will contribute but a surface current can, in fact if \hat{n} is a unit vector \perp to the interface, so that $(\hat{n} \times 1)$ is normal to the Amperian loop, then

$$I_{\text{fenc}} = k_f \cdot (\hat{n} \times 1) = K_f (\hat{n} \times 1) l$$

And hence

$$H_1^{\parallel} - H_2^{\parallel} = K_f \times \hat{n} \quad \text{----- (9)}$$

So the parallel components of H are discontinuous by an amount perpendicular to the free surface density. Equation (6) & (9) are the general boundary conditions for electrodynamics.

In case of linear media, they can be expressed in terms of E and B alone.

$$(i) \quad \epsilon_1 E_1 - \epsilon_2 E_2 = \sigma_f$$

$$(ii) \quad B_1 - B_2 = 0$$

$$(iii) \quad E_1^{\parallel} - E_2^{\parallel} = 0$$

$$(iv) \quad \frac{1}{\mu_1} B_1^{\parallel} - \frac{1}{\mu_2} B_2^{\parallel} = 0$$

Part B

Possible 2 Marks

1. Write short notes on displacement current.
2. Define coulomb gauge transformation.
3. Write about scalar and vector potentials.
4. State ampere circuital law.
5. What are the different types of current densities?
6. Write short notes on pointing vector.
7. State pointing theorem.

Part C

Possible 6 Marks

1. Deduce Faraday's law of electromagnetic induction.
2. Establish the non uniqueness of electromagnetic potential.
3. Deduce Maxwell equation of electromagnetic field and discuss their empirical basis.
4. Explain the gauge transformation.
5. State and prove $\text{curl } \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$.
6. How are the scalar and vector potentials related to the electric and magnetic fields?
7. State ampere circuital law and discuss how it was modified to displacement current.
8. State and prove pointing theorem.

**Karpagam academy of higher education
Coimbatore -21**

Department of Physics

II B.Sc Physics

Electromagnetic Theory [16PHU303]

QUESTIONS	OPTION 1	OPTION 2	OPTION 3	OPTION 4	ANSWER
UNIT-I					
charge enclosed by it.	ϵ_0	m_0	$1/\epsilon_0$	$1/m_0$	$1/\epsilon_0$
The differential form of Gauss's law is _____.	$\text{div } E = r/\epsilon_0$	$-\text{div } E = r/\epsilon_0$	$\text{div } E = 0$	$\text{div } E = r$	$\text{div } E = r/\epsilon_0$
The insulators whose behaviour gets modified in an electric field are called _____.	semiconductor	superconductor	p-type semiconductor	dielectrics	dielectrics
The force between two magnetic poles varies with the distance between them. The variation is _____ to the square of that distance.	Equal	Greater than	Directly proportional	Inversely proportional	Inversely proportional
The property of magnetic materials of retaining magnetism after withdrawal of the magnetizing force is known as	Retentivity	Reluctivity	Resistivity	Conductivity	Retentivity
Permeability means	The conductivity of the material for magnetic lines of force	The magnetization test in the material after exciting field has been removed	the strength of an electromagnet	The strength of the permanent magnet	The conductivity of the material for magnetic lines of force
The magnetic field inside a solenoid	is constant	is uniform	increases with distance from the axis	decreases with distance from the axis	is uniform
Paramagnetic substance has a relative permeability of	Slightly less than one	Equal to one	Slightly greater than one	Very much greater than one	Slightly greater than one
The point in a magnet where the intensity of magnetic lines of force is	Magnetic pole	South pole	North pole	Unit pole	Unit pole
For which of the following substance, the magnetic susceptibility is independent of temperature _____	Dia	Para	Ferro	None of these	Dia
Atoms may or may not have an _____ dipole moments.	Extrinsic	Intrinsic	Intermediate	Extreme	Intrinsic
The group of parallel oriented atomic dipoles is called a _____.	domain	co-domain	N-domain	V-domain	domain
All magnetic moments within a domain will point in the _____	Different	Same	Positive	Negative	Same
The unit of magnetization M is _____.	Am^{-1}	Am^{-2}	Am	A/m	Am^{-1}
The unit of magnetic induction (B) is _____.	Weber	Weber/m	Weber/m^2	Weber.meter	Weber/m^2
The field vectors are invariant to _____	gauge transformations	Hertz potential	Maxwell's equation	ampere's law	gauge transformations
Gauge functions are solutions of _____ wave equations.	homogenous	non homogenous	Independent	dependent	homogenous
The magnetic lines of force are always closed may be expressed by _____.	$\Delta \times B = 0$	$\Delta \cdot B = 0$	$\Delta \times B = \mu_0 J$	$\Delta \times A = 0$	$\Delta \cdot B = 0$
A magnetic dipole of moment m placed in a nonuniform magnetic field B experience a force	$\Delta(m \cdot B)$	$m \times B$	$m \cdot B$	none of these	$\Delta(m \cdot B)$
A moving charge produces _____.	electric field only	magnetic field only	both electric and magnetic fields	none of these	both electric and magnetic fields

Differential form of Ampere's law for a steady current is _____.	$\Delta \times H = J + \partial D / \partial t$	$\Delta \times B = \mu_0 J$	$\Delta \cdot B = 0$	$\int B \cdot dl = \mu_0 I$	$\Delta \times B = \mu_0 J$
The magnetic dipole moment induced per unit volume of the material is _____.	magnetization	polarization	magnetic induction	magnetic intensity	magnetization
Magnetic intensity is a _____.	Phasor quantity	Physical quantity	Scalar quantity	Vector quantity	Vector quantity
The time dependent electromagnetic field equation are _____.	Maxwell's	Ampere's law	Faraday's law	Gauss law	Maxwells equation
div of curl of any vector is _____.	0	infinity	1	J	0
Dielectric polarization is proportional to _____.	applied electric field	applied magnetic field	applied electromagnetic field	applied electrostatic field	applied electric field
The unit of polarization is _____.	coul/m	coul/m ²	coul/m ³	coul ² /m ²	coul/m ²
The addition of _____ to Ampere's law results in the unification of electric and magnetic phenomena.	displacement current	current density	scalar potential	vector potential	displacement current
The unit for electric displacement D is _____.	coul ² /m	coul/m ²	coul/m ³	coul ² /m ²	coul/m ²
The continuity equation is _____.	$\text{div } J - \partial r / \partial t = 0$	$-\text{div } J + \partial r / \partial t = 0$	$\text{div } J + \partial r / \partial t = 0$	$-\text{div } J - \partial r / \partial t = 0$	$\text{div } J + \partial r / \partial t = 0$
The flux of magnetic induction B across any closed surface is always _____.	0	unity	varying	constant	0
Electric and magnetic phenomena are _____.	symmetric	asymmetric	same	converse	asymmetric
Electric and magnetic phenomena are asymmetry arises due to the non-_____.	dipoles	electric field	magnetic field	monopole	monopole
Maxwell's first equation signifies that the total flux of electric displacement _____.	equal to	less than	greater than	inversely	equal to
Maxwell's second equation signifies that the total flux of magnetic induction _____.	unity	same	zero	constant	zero
Maxwell's third equation signifies that _____ force around a _____.	magnetomotive	electromotive	restoring	none of the above	magnetomotive
Maxwell's fourth equation signifies that _____ is equal to the negative _____.	magnetomotive	electromotive	electric	magnetic	electromotive
_____ is the amount of field energy passing through unit area of the _____.	electric energy	magnetic energy	poynting vector	mechanical energy	poynting vector
Poynting vector at any arbitrary point in the field varies _____ as _____.	inversely	directly	sinusoidally	abnormally	inversely
The definition of a poynting vector is not a _____.	vector	scalar	mandatory	none of the above	mandatory
If the poynting vector is _____ then no electromagnetic energy _____.	unity	finite	infinite	zero	zero
In case of time varying fields $S = E \times H$ gives the _____ value of the _____.	instantaneous	total	random	half the	instantaneous
According to Ampere's law of force, the force between current carrying _____.	inversely	independently	directly	infinitely	independently
According to Ampere's law of force, the force between current carrying _____.	colour	nature	property	length	nature
According to Ampere's law of force, the force is _____ if the _____.	repulsive	infinite	attractive	finite	attractive
According to ampere's law of force, the force is _____ if the current _____.	attractive	infinite	finite	repulsive	4
The vector B is called _____ vector.	magnetic flux	magnetic intensity	magnetic induction	magnetic force	magnetic induction
The unit of B is _____.	Tesla	Web	Web/m	Web ² /m	Tesla
equal to _____ times the total current crossing any surface _____.	ϵ_0	m_0	$1/\epsilon_0$	$1/m_0$	m_0
The div of curl of any vector is _____.	1	-1	Grad	0	0
The magnitude of displacement current is equal to the time rate of change of _____.	electric	current density J	charge density r	none of the above	electric
The displacement current in a good conductor is _____.	constant	proportional to	current density	negligible	negligible
$J_d =$ _____.	$-\partial D / \partial t$	$J + \partial D / \partial t$	$\partial D / \partial t$	$J - \partial r / \partial t$	$\partial D / \partial t$
Maxwell's fourth equation signifies that _____ is equal to the negative _____.	magnetomotive	electromotive	electric	magnetic	electromotive

EM Wave Propagation in Unbounded Media:

Plane EM waves through vacuum and isotropic dielectric medium, transverse nature of plane EM waves, refractive index and dielectric constant, wave impedance. Propagation through conducting media, relaxation time, skin depth. Wave propagation through dilute plasma, electrical conductivity of ionized gases, plasma frequency, refractive index, skin depth, application to propagation through ionosphere.

PLANE ELECTROMAGNETIC WAVE IN FREE SPACE:

Maxwell's equations in derivative form for empty space.

$$\nabla \cdot \mathbf{D} = \rho \quad \text{_____ (1)}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \text{_____ (2)}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{_____ (3)}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{_____ (4)}$$

where,

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\mathbf{B} = \mu_0 \mathbf{H}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E}$$

and for free space (i.e.) vacuum

$$\rho = 0$$

$$\mathbf{J} = 0$$

$$\epsilon_r = 1$$

$$\mu_r = 1$$

and the Maxwell's equation reduces to,

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$

taking curl for third equation

$$\nabla \times (\nabla \times \mathbf{H}) = \epsilon_0 \nabla \times \left(\frac{\partial \mathbf{E}}{\partial t} \right)$$

$$\nabla \cdot (\nabla \cdot H) - \nabla^2 H = \varepsilon_0 \frac{\partial}{\partial t} (\nabla \times E)$$

since $\nabla \cdot H = 0$ and $\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$

$$\nabla^2 H - \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} = 0$$

same is repeated for fourth equation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\mu_0 \frac{\partial H}{\partial t} \right)$$

solving,

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0$$

these two equations satisfies the wave equation which is,

$$\nabla^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

the last equation is a standard wave equation representing unattenuated wave travelling at a speed of light. so we conclude that field vector E and H are propagated in free space at a velocity equal to the speed of light.

PROPAGATION OF ELECTRO-MAGNETIC WAVES IN FREE SPACE:-

1. The wave propagates with a speed equal to that of light in free space.
2. The electromagnetic waves are transverse in nature.
3. The electromagnetic energy is transmitted in the direction of wave propagation at speed "C".
4. The wave vector E and H are mutually perpendicular to each other.
5. The vector E and H are in phase with each other.

ELECTRO-MAGNETIC WAVES IN ISO-TROPIC DI-ELECTRIC MEDIUM:-

with respect to the Maxwell's equation

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{E} = \frac{-\partial B}{\partial t}$$

for isotropic medium,

$$J = \sigma E$$

$$B = \mu H$$

$$D = \epsilon E$$

here,

$$\sigma = 0$$

$$\rho = 0$$

then the Maxwell's equation reduces to,

$$\nabla \cdot E = 0$$

$$\nabla \cdot H = 0$$

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

taking curl for third and fourth equation,

for third equation,

$$\nabla^2 H - \frac{1}{v^2} \frac{\partial^2 H}{\partial t^2} = 0$$

for fourth equation,

$$\nabla^2 E - \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2} = 0$$

these two waves satisfies the wave equation,

$$\nabla^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

the solution for the wave equation is,

$$\psi = \psi_0 e^{-i(\omega t - k \cdot r)}$$

the solution of equations are will be of the given in the form,

$$E = E_0 e^{-i(\omega t - k \cdot r)}$$

$$H = H_0 e^{-i(\omega t - k \cdot r)}$$

where k is the wave vector

$$k = k_n = \frac{2\pi}{\lambda} n = \frac{2\pi f_n}{c} = \frac{\omega_n}{c}$$

with n as the unit vector in the direction of wave propagation

Dr.A.Saranya

Department of Physics

Karpagam Academy of Higher Education

Coimbatore -21

The equation can be written as

$$k \cdot E = 0$$

$$k \cdot H = 0$$

$$-k \times H = \omega \epsilon_0 E$$

$$k \times E = \mu_0 H$$

propagation of electromagnetic waves in dielectric:-

1. The waves E and H are orthogonal.
2. The electromagnetic wave is transverse in nature.
3. The electric and magnetic vectors are also mutually orthogonal.

TRANSVERSE NATURE OF ELECTROMAGNETIC WAVES:-

consider an electromagnetic wave propagating in free space along z-direction. Then the E and H vary only in z-direction.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \text{ and } \frac{\partial}{\partial z} \neq 0$$

such wave is called as planar wave, since its vector are functions of (z, t) only then we write,

$$E = E(z, t)$$

$$H = H(z, t)$$

from first Maxwell's equation,

$$\nabla \cdot D = 0 \text{ or } \epsilon_0 (\nabla \cdot E) = 0$$

$$\nabla \cdot E = 0 \text{ or } \frac{\partial E_z}{\partial z} = 0$$

$$E_z = \text{constant in time}$$

from second Maxwell's equation,

$$\nabla \cdot B = 0 \text{ or } \mu_0 (\nabla \cdot H) = 0$$

$$\nabla \cdot H = 0 \text{ or } \frac{\partial H_z}{\partial z} = 0$$

$$H_z = \text{constant in time}$$

from third Maxwell's equation,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$(\nabla \times E)_z = -\mu_0 \frac{\partial H_z}{\partial t}$$

$$K \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = -\mu_0 \frac{\partial H_z}{\partial t}$$

$$\frac{\partial H_z}{\partial t} = 0$$

$$H_z = \text{constant in time}$$

similarly by using fourth equation,

$$E_z = \text{constant in time}$$

Thus we have concluded that E_z and H_z are constant as regards for time and space. they represent the static components and consequently, no part of wave motion. we can therefore write,

$$E_z = H_z = 0$$

$$E = iE_x + jE_y$$

$$H = iH_x + jH_y$$

the electric E and magnetic H vector don't have any Z-component, the Z-direction being the direction of propagation ,

both these vectors are perpendicular to the direction of propagation ,Maxwell's electromagnetic waves are purely transverse in nature.

PROPAGATION OF ELECTROMAGNETIC WAVES IN CONDUCTING MEDIUM:-

Considering the Maxwell's equations,

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

here,

$$J = \sigma E$$

$$D = \epsilon E$$

$$B = \mu H$$

$$\sigma = 0$$

$$\rho = 0$$

Then,

$$\nabla \cdot D = 0$$

$$\nabla \cdot H = 0$$

$$\nabla \times H = \sigma E + \frac{\partial H}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

taking curl for the third and fourth equation,

for third equation,

$$\nabla \times (\nabla \times H) = \nabla \times \left(\sigma E + \frac{\partial H}{\partial t} \right)$$

solving we get,

$$\nabla^2 H - \sigma \mu \frac{\partial H}{\partial t} - \mu \varepsilon \frac{\partial^2 H}{\partial t^2} = 0$$

for fourth equation,

$$\nabla \times (\nabla \times E) = -\mu \nabla \times \left(\frac{\partial H}{\partial t} \right)$$

solving we get,

$$\nabla^2 E - \sigma \mu \frac{\partial E}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = 0$$

These two equations are called as the equation of telegraphy,

EQUATION OF TELEGRAPHY:-

$$\nabla^2 H - \sigma \mu \frac{\partial H}{\partial t} - \mu \varepsilon \frac{\partial^2 H}{\partial t^2} = 0$$

$$\nabla^2 E - \sigma \mu \frac{\partial E}{\partial t} - \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = 0$$

IMPEDANCE IN PHASE:-

$$\left| \frac{E}{H} \right| = \frac{E_0}{H_0} = \sqrt{\left(\frac{\mu_r}{\varepsilon_r} \right) z_0} = z$$

PROPAGATION OF ELECTROMAGNETIC WAVES IN IONIZED GASES:-

In certain situations such as the ionosphere or a tenuous plasma there is so little air that the electrons may vibrate without colliding

with the molecules. so the force on a charged particle is an electromagnetic field, neglecting the earth's magnetic field will be

$$\mathbf{F} = e[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

now as in a plane wave

$$\mathbf{B} = \frac{n \times \mathbf{E}}{c}$$

$$|\mathbf{v} \times \mathbf{B}| = vB = \frac{v}{c} E$$

and also,

$$E = E_0 e^{-i(\omega t - k \cdot r)}$$

$$E = E_0 e^{-i(\omega t - (2\pi/\lambda)n \cdot r)}$$

$$E = E_0 e^{-i(\omega t)}$$

so equation reduces to,

$$F = eE_0 e^{-i\omega t}$$

$$m \frac{d^2 r}{dt^2} = eE_0 e^{-i\omega t}$$

$$\frac{d^2 r}{dt^2} = \frac{e}{m} E_0 e^{-i\omega t}$$

$$\frac{dr}{dt} = \frac{eE_0 e^{-i\omega t}}{m(-i\omega)}$$

$$v = i \frac{e}{m\omega} E$$

now if there are N electrons per unit volume then as

$$J = Nev$$

substituting the value of v from equation we get,

$$J = i \frac{Ne^2}{m\omega} E$$

$$J = \sigma E$$

we find that the conductivity is purely imaginary,

$$\sigma = i \frac{Ne^2}{m\omega}$$

various shortcuts are possible in deriving equations of wave propagation in an ionized medium but it seems worthwhile to go all the way both Maxwell's equation.

$$\begin{aligned}\nabla \cdot D &= \rho \\ \nabla \cdot B &= 0 \\ \nabla \times H &= J + \frac{\partial D}{\partial t} \\ \nabla \times E &= -\frac{\partial B}{\partial t}\end{aligned}$$

which for the present situation reduces to

$$\begin{aligned}\nabla \cdot E &= 0 \\ \nabla \cdot H &= 0 \\ \nabla \times H &= \sigma E + \epsilon_0 \frac{\partial E}{\partial t} \\ \nabla \times E &= -\mu_0 \frac{\partial H}{\partial t}\end{aligned}$$

in case of ionized medium $\rho = 0$, $\epsilon_r = 1$ and $\mu_r = 1$

now taking curl for fourth equation,

$$\nabla \times (\nabla \times E) = -\mu_0 \frac{\partial}{\partial t} (\nabla \times H)$$

solving this we get,

$$\nabla^2 E - \sigma \mu_0 \frac{\partial E}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$

similarly taking curl for third equation,

$$\nabla^2 H - \sigma \mu_0 \frac{\partial H}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} = 0$$

the solution of these two equations be,

$$\Psi = \Psi_0 e^{-i(\omega t - k \cdot r)}$$

then,

$$\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} E_0 \\ H_0 \end{pmatrix} e^{-i(\omega t - k \cdot r)}$$

so that field equation reduces,

$$(K^2 - i\mu_0\omega\sigma - \mu_0\epsilon_0\omega^2) \begin{pmatrix} E \\ H \end{pmatrix} = 0$$

as vector E or H is not zero,

$$K^2 = \mu_0 \epsilon_0 \omega^2 \left[1 + \frac{i\sigma}{\epsilon_0 \omega} \right]$$

$$K^2 = \frac{\omega^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega^2} \right]$$

$$m = \frac{c}{v} = \frac{c}{\omega/k} = \frac{kc}{\omega}$$

so the index of refraction in this case will be given by,

$$n = \sqrt{\left(1 - \frac{\omega_p^2}{\omega^2} \right)}$$

from this equation it is clear that for frequencies $\omega^2 > \omega_p^2$

in region of vanishing small ionization and high frequency range index of refraction is real and so waves propagate freely as in dielectric, however if the plasma frequency increases with distance, the index of refractive will decreases according. this is turn means that the beam will bends in a direction away from the normal as it moving from a region of higher index of refraction to that of lower index of refraction. This bending of high frequency or short wavelength electromagnetic wave by earth's ionosphere is used in long distance radio transmission .

in the limit $\omega^2 \gg \omega_p^2$ as $n \rightarrow 1 = \text{constant}$, the transmission is unaffected by the presences of ionosphere this is why the radar signals that have been received after reflection from the moon had to be rather higher frequency waves to pass through the ionized part of earth's atmosphere.

for frequency $\omega^2 < \omega_p^2$ in heavily ionized region and for low frequencies ranges the index of refraction is purely imaginary. so if we write $n \rightarrow in$ then from equation

$$k = \frac{\omega}{c} (in) = i \frac{\omega}{c} \sqrt{\left(\frac{\omega_p^2}{\omega^2} - 1 \right)}$$

so that

$$\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} E_0 \\ H_0 \end{pmatrix} e^{-\beta(n.r)} e^{-i(\omega.t)}$$

with $\beta = \frac{\omega n}{c}$

SKIN DEPTH:-

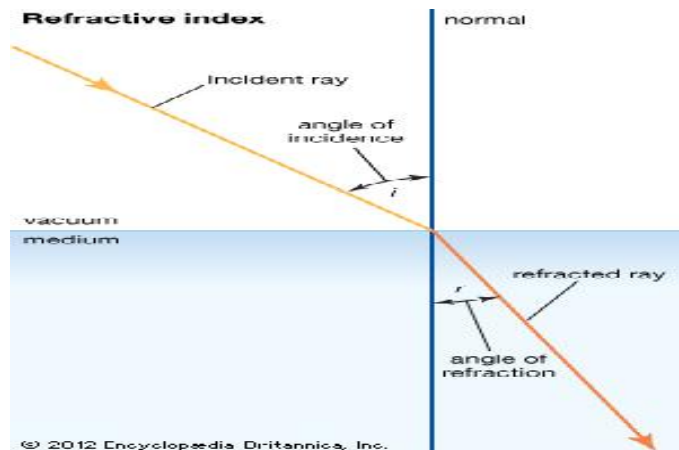
The attenuation of wave will result in the medium and there will be no propagation in the medium. therefore electromagnetic waves with frequency below the plasma frequency ω_p will be reflected at the plasma boundary in the plasma the field will fall off exponentially with distance from surface depth for plasma will be,

$$\delta_{\text{plasma}} = \frac{1}{\beta} = \frac{1}{\frac{\omega_p}{c}} = \frac{1}{\frac{\omega}{c} \sqrt{\left(\frac{\omega_p}{\omega}\right)^2 - 1}}$$

$$\delta_{\text{plasma}} = \frac{c}{\sqrt{\omega_p^2 - \omega^2}} \propto \frac{c}{\omega_p} \text{ when } \omega \ll \omega_p$$

the field from within a plasma is a well know effect in process and is exploited in attempts at hot plasma.

REFRACTIVE INDEX:



Refractive index, also called index of refraction, measure of the bending of a ray of light when passing from one medium into another. If i is the angle of incidence of a ray in vacuum (angle between the incoming ray and the perpendicular to the surface of a medium, called the normal) and r is the angle of refraction (angle between the ray in the medium and the normal), the refractive index n is defined as the ratio of the sine of the angle of incidence to the sine of the angle of refraction; i.e., $n = \sin i / \sin r$. Refractive index is also equal to the velocity of light c of a given wavelength in empty space divided by its velocity v in a substance, or $n = c/v$

Some typical refractive indices for yellow light (wavelength equal to 589 nanometres [10⁻⁹ metre]) are the following: air, 1.0003; water, 1.333; crown glass, 1.517; dense flint glass, 1.655; and diamond, 2.417. The variation of refractive index with wavelength is the source of chromatic aberration in lenses. The refractive index of X-rays is slightly less than 1.0, which means that an X-ray entering a piece of glass from air will be bent away from the normal, unlike a ray of light, which will be bent toward the normal. The equation $n = c/v$ in this case indicates, correctly, that the velocity of X-rays in glass and in other materials is greater than its velocity in empty space.

DIELECTRIC CONSTANT:

A quantity of measuring the ability of a substance to store electrical energy in an electric field.

Part B

Possible 2 Marks

1. What are electromagnetic waves?
2. Explain Cerenkov – radiation.
3. Write the condition of E.M. W. propagating in free space.
4. Write about the equation of telegraphy.
5. What are called skin depth ?
6. Write about the relaxation time.
7. What is momentum density ?

Part C

Possible 6 Marks

1. How the electromagnetic wave propagate in free space.
2. Explain the propagation of E. M. W in isotropic dielectrics.
3. Explain the propagation of electromagnetic waves in dielectric medium.
4. Discuss the propagation of e.m. waves in plasma for $\omega > \omega_p$.
5. Discuss the propagation of electromagnetic waves in a conducting media.
6. Show that the propagation of electromagnetic waves in an ionosphere.
7. Show that inside the conducting medium the wave is damped and obtain an expression for skin depth δ .
8. Discuss the propagation of electromagnetic waves in a ionized gases.

Karpagam academy of higher education
Coimbatore -21
Department of Physics
II B.Sc Physics
Electromagnetic Theory [16PHU303]

The ratio ϵ/ϵ_0 is a dimensionless quantity known as _____	relative permeability	relative permittivity	absolute permittivity	permeability	relative permittivity
Gauss law is _____	$\epsilon_0 \oint E \cdot ds = q$	$\Delta \cdot D = \rho/\epsilon_0$	$\Delta \cdot D = q/\epsilon_0$	$\oint E \cdot ds = \rho$	$\epsilon_0 \oint E \cdot ds = q$
Gauss's law in a dielectric medium takes the form $\oint D \cdot ds = q$, where q is _____.	total free charge enclosed	polarization charges	total of both free and polarization charges	zero	total free charge enclosed
In gauss's law the electric flux E through a closed surface (s) depends on the value of net charge _____.	Inside the surface	outside the surface	on the surface	None of the above	Inside the surface
The Flux of electric field is _____	scalar	vector	zero	infinity	scalar
The unit of permittivity is _____	$C^2 N^{-1} M^{-2}$	$C^2 N^1 M^2$	$C^2 N^1 M^2$	$C^2 N^{-1} M^{-2}$	$C^2 N^{-1} M^{-2}$
Dielectric constant of any medium is always _____ than permittivity.	greater	lesser	neither greater not lesser	neither lesser nor greater	greater
Gauss law for magnetic field yields, $\text{div } \mathbf{B} =$	0	1	ρ	σ	0
The unit of electric field intensity is	Volts/m	amp/m	weber/m	volts/m ²	Volts/m
The equation of poynting vector is	$S = E \times H$	$S = E/H$	$S = E + H$	$S = \text{curl}(E \times H)$	$S = E \times H$
The unit watt/m ² is a unit of	Gauss law	Ampere's circuital law	Faraday's law	Poynting vector	Poynting vector
$\text{div } \mathbf{B} = 0$, the field of vector B is always _____	scleronomic	rheonomic	unilateral	solenoidal	solenoidal
The electromagnetic energy is transmitted in the direction of the wave propagation at speed of	light	time	position	momentum	light
The unit of capacitance is _____.	farad	coulomb	volt	henry	farad
Gaussian surface is a _____.	real surface	open surface	Imaginary surface	Smooth surface.	Imaginary surface
Which of the following is a ferromagnetic material	Tungsten	Aluminum	Copper	Nickel	Nickel
The symbol of relative permittivity of a medium is	ϵ_0	ϵ_r	$\epsilon_r \epsilon_0$	μ	$\epsilon_r \epsilon_0$
The field vector operator ∇ is equivalent to _____.	-iw	iw	iw^2	$i^2 w^2$	-iw

The path followed by a unit positive charge in an electric field called as	The line of force	coulombs forces	Electric force	Electromagnetic field	The line of force
$\Delta \cdot D =$	ρ	0	1	μ	ρ
$\Delta \cdot B =$	π	0	1	μ	0
$\Delta \times H =$	$\text{div } J + \frac{\partial r}{\partial t} = 0$	$J + \frac{\partial D}{\partial t}$	$J - \frac{\partial D}{\partial t}$	0	$J + \frac{\partial D}{\partial t}$
$\Delta \times E =$	ρ	π	$J - \frac{\partial D}{\partial t}$	$-\frac{\partial B}{\partial t}$	$-\frac{\partial B}{\partial t}$
Electromagnetic waves propagates in free space with the velocity of _____.	light	sound	electron	proton	light
The velocity of electromagnetic waves in free space is _____.	$30 \times 10^8 \text{ m/s}$	$356 \times 10^8 \text{ m/s}$	330 m/s	$3 \times 10^8 \text{ m/s}$	$3 \times 10^8 \text{ m/s}$
The electromagnetic energy density is equal to _____	magnetostatic energy density	charge density	energy density	space energy	magnetostatic energy density
The flow of energy in a electromagnetic wave in free space is in the direction of _____.	electric field	magnetic field	electrons	wave propagation	wave propagation
The electromagnetic energy density is equal to the _____ energy density.	magnetostatic	electrostatic	magnetic flux	electric flux	magnetostatic
The electromagnetic field vectors E and H are in _____.	out of phase	phase	proportional	none of the above	phase
In plane electromagnetic wave, the wave vectors E, H and K are _____.	parallel	rotational	irrotational	orthogonal	orthogonal
The field vector operator $\nabla \cdot \nabla$ is equivalent to _____.	-iw	iw	iw^2	$i^2 w^2$	-iw
The speed of electromagnetic wave in isotropic dielectrics is _____ than the speed of electromagnetic waves in free space.	greater	lesser	absolute	none of the above	lesser
In an anisotropic medium, the energy is _____ in the wave propagation.	not propagated	propagated	orthogonal	parallel	not propagated
In case of propagation of EMW in conducting medium the wave gets _____ with penetration.	reflected	refracted	attenuated	scattered	attenuated
In case of propagation of EMW in conducting medium, the wave is _____ with respect to the E and H.	longitudinal	parallel	transverse	none of the above	transverse
In case of propagation of EMW in conducting medium, magnetic energy is _____ electric energy density.	greater than	lesser than	equal to	inversely proportional to	greater than
The wave vectors E and H are mutually _____	perpendicular	parallel	equal	greater	perpendicular

When high energy particles having velocities greater than c passes through a dielectric a light known as cerenkov radiation is emitted.	greenish	bluish	reddish	greenish-blue	bluish
The flow of energy in a electromagnetic wave in free space is in the direction of _____.	electric field	magnetic field	electrons	wave propagation	wave propagation
_____ radiation is emitted due to the interaction of uniformly moving charged particles with the dielectric medium.	X-rays	Gamma	Alpha	Cerenkov	Cerenkov
The total energy density in case of electromagnetic waves in isotropic dielectrics is _____ times of the energy density if the wave propagates through free space.	μ_r	$-\mu_r$	ϵ_r	$-\epsilon_r$	ϵ_r
When electromagnetic waves crosses a boundary surface, then the normal component of the electric displacement is _____ by an amount equal to the free density of charge.	equals	continuous	proportional	discontinuous	discontinuous
The poynting vector in case of propagation of electromagnetic waves in isotropic dielectric is _____ times of the poynting vector if the same wave propagates through free space.	n/ϵ_r	n/μ_r	$-n/\epsilon_r$	$-n/\mu_r$	n/μ_r
In plane electromagnetic wave, the wave vectors D , H and K are _____.	parallel	rotational	irrotational	orthogonal	orthogonal
In plane electromagnetic wave, the wave vectors D , E and K are _____.	parallel	rotational	co-planer	orthogonal	co-planer
The field vectors are spatially _____.	attenuated	unattenuated	rotated	conducted	attenuated
The vectors E and H are both zero inside _____.	super conductors	conductors	insulators	semi conductors	super conductors
Poynting vector is _____ as the wave progress.	attenuated	unattenuated	rotated	conducted	attenuated
the wave gets attenuated with _____.	penetration	transmission	absorption	refraction	penetration
The current flows through the surface and the effect is called _____.	strong skin effect	sound effect	light effect	wave effect	strong skin effect
$J =$ _____.	NeV	0	1	H	NeV
The earth's magnetic field will be _____.	$e[E + (v \times B)]$	$E \times H$	D	J	$F = e[E + (v \times B)]$
Light waves consists of _____.	protons	electrons	photons	neutrons	photons
White light is incident on the interface of glass and air medium. If green light is just totally internally reflected, then the emerging ray in air contains _____.	yellow, orange, red	violet, indigo, blue	all colours	all colours except green	yellow, orange, red
If the EMW incident on a system of charged particles, the electric and magnetic fields of the wave exert a _____ force on the charges.	Lorentz	Mechanical	Electrical	No	Lorentz
If the energies of the incident and scattered radiations are equal the scattering is called _____.	inelastic	coherent	elastic	incoherent	elastic
If the energies of the incident and scattered radiations are not equal, the scattering is called _____.	inelastic	coherent	elastic	incoherent	inelastic

EM Wave in Bounded Media:

Boundary conditions at a plane interface between two media. Reflection & Refraction of plane waves at plane interface between two dielectric media-Laws of Reflection & Refraction. Fresnel's Formulae for perpendicular & parallel polarization cases, Brewster's law. Reflection & Transmission coefficients. Total internal reflection, evanescent waves. Metallic reflection (normal Incidence).

REFLECTION AND REFRACTION OF ELECTROMAGNETIC WAVE :

We now need to consider that what happens when plane electromagnetic waves which are travelling in one medium are incident upon an infinite plane surface separation this medium from another, with different electromagnetic properties. When an electric waves is travelling through space there is an exact balance between the electric and magnetic field. Half of the energy of wave as a matter of fact is in electric field and half in the magnetic. If the wave enters some different medium, there must be a new distribution of energy whether the new medium is a dielectric a magnetic a conducting or an ionised region, there will have to be a readjustment of energy relation as the wave reaches its surface. Since no energy can be added to the wave as it only way that a new balance can be achieved is for some of the incident energy to be reflected.

This is what actually happens, the transmitted energy constitutes the refracted wave and the reflected one the reflected wave. The reflection and refraction of light at a plane surface between two media of different dielectric properties is a familiar, example of reflection and refraction of electromagnetic waves. The various aspects of the phenomenon divide themselves into two classes.

Kinematic properties :

Following are the kinematic properties of reflection and refraction.

(i) Law of frequency :

The frequency of the wave remains unchanged by reflection or refraction.

(ii) The reflected and refracted waves are in the same plane as the incident wave and the normal to the boundary surface.

(iii) Law of reflection :

In case of reflection the angle of reflection is equal to the angel of incident.

$$\theta_1 = \theta_R$$

(IV) Snell's Law :

In case of reflection the ratio of the sin of the angle of refraction to the sin of angle of incident is equal to the ratio of the refractive indices of two media.

$$n_1 \sin \theta_i = n_2 \sin \theta_R$$

Dynamic properties :

These properties are concerned with the

- (i) intensities of reflected and refracted waves
- (ii) Phase changes and polarisation of waves

The kinematic properties follow immediately from the wave nature of phenomenon and the fact that these are boundary condition to be satisfied. But they do not depend on the nature of the wave or the boundary conditions.

FRESNEL FORMULAE : (DYNAMIC PROPERTIES)

The formulae relating the amplitude of the reflected and transmitted electromagnetic waves with that of incident one when the boundary is between two dielectrics are called Fresnel formulae. These are contained in the boundary condition.

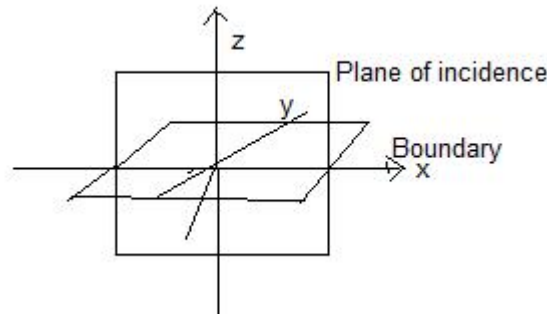
$$(D_i)_n + (D_R)_n = (D_T)_n \quad \text{----- (1)}$$

$$(B_i)_n + (B_R)_n = (B_T)_n \quad \text{----- (2)}$$

$$(E_i)_t + (E_R)_t = (E_T)_t \quad \text{----- (3)}$$

$$(H_i)_t + (H_R)_t = (H_T)_t \quad \text{----- (4)}$$

The condition (1) and (2) when coupled with Snell's law yield no information not included in the (3) and (4) conditions. So it is necessary to consider only condition (3) and (4). Now to derive the desired formulae we consider a plane EMW in x-z plane incident on a plane boundary and consider it as a superposition of two waves one with the electric vector perpendicular to the plane of incidence. Therefore it is sufficient to consider these two cases separately. The general result may be obtained from the appropriate linear combination of the two cases.



CASE I : E parallel to the plane of incidence

The situation is shown in figure. The electric and propagation vectors in two media are indicated. The directions of H vector are chosen as to give a positive flow of energy in the direction of wave vectors. In this situation the magnetic vectors are all parallel to the boundary surface.

$$(H_i)_t = H_i$$

$$(H_r)_t = H_r$$

$$(H_t)_t = H_t$$

And $(E_i)_t = E_i \cos \theta_i$

$$(E_r)_t = -E_r \cos \theta_r$$

$$(E_t)_t = E_t \cos \theta_t$$

So the boundary condition (3) and (4) reduce to,

$$E_i \cos \theta_i - E_r \cos \theta_r = E_t \cos \theta_t \quad \text{----- (5)}$$

$$H_i - H_r = H_t \quad \text{----- (6)}$$

In equation (5) and (6) we have omitted the zero subscript on E and H, it being understood that the phases now cancel and equations are relations between amplitudes.

$$E_i = E_r \text{ and } H = (E/Z) = (n/\mu_r Z_0) = E$$

$$H = (n/Z_0)E$$

So equation (5) and (6) reduce to

$$E_i \cos \theta_i - E_r \cos \theta_r = E_t \cos \theta_t \quad \text{----- (7)}$$

$$n_1 E_i + n_2 E_r = n_2 E_t \quad \text{----- (8)}$$

The interest lies in the fraction of incident amplitudes which are reflected and transmitted. So eliminating E_t from equation (7) with the help of (8) we get

$$(E_i - E_r) \cos \theta_i = \frac{n_1}{n_2} (E_i + E_r) \cos \theta_r$$

$$\left(\frac{E_r}{E_i}\right) = \frac{\frac{n_2}{n_1} \cos \theta_i - \cos \theta_r}{\frac{n_2}{n_1} \cos \theta_i + \cos \theta_r}$$

$$\left(\frac{E_R}{E_i}\right) = \frac{\left(\frac{\sin \theta_i}{\sin \theta_T}\right) \cos \theta_i - \cos \theta_T}{\left(\frac{\sin \theta_i}{\sin \theta_T}\right) \cos \theta_i + \cos \theta_T}$$

$$\left(\frac{E_R}{E_i}\right) = \frac{\sin \theta_i \cos \theta_i - \sin \theta_T \cos \theta_T}{\sin \theta_i \cos \theta_i + \sin \theta_T \cos \theta_T} = \frac{\sin 2\theta_i - \sin 2\theta_T}{\sin 2\theta_i + \sin 2\theta_T} \quad \text{----- (A)}$$

Similarly eliminating E_R from equation (7) with the help of (8)

$$E_i \cos \theta_i - \left(\frac{n_2^2}{n_1^2} E_T - E_i\right) \cos \theta = E_T \cos \theta_T$$

$$\left(\frac{E_T}{E_i}\right)_{II} = \frac{2 \cos \theta_i}{\frac{n_2^2}{n_1^2} \cos \theta_i + \cos \theta_T}$$

$$\left(\frac{E_T}{E_i}\right)_{II} = \frac{2 \cos \theta_i \sin \theta_T}{\sin \theta_i \cos \theta_i + \sin \theta_T \cos \theta_T}$$

$$\left(\frac{E_T}{E_i}\right)_{II} = \frac{2 \cos \theta_i \sin \theta_T}{\sin(\theta_i + \theta_T) \cos(\theta_i - \theta_T)} \quad \text{----- (B)}$$

CASE II : E perpendicular to the plane of incidence

The situation is shown the magnetic field vectors and the propagation vectors are indicated.

The electric vectors all directed into the plane of the figure.

Since the electric vectors are all parallel to the boundary surface,

$$(E_i)_t = E_i$$

$$(E_R)_t = E_R$$

$$(E_T)_t = E_T$$

And

$$(H_i)_t = -H_i \cos \theta_i$$

$$(H_R)_t = H_R \cos \theta_R$$

$$(H_T)_t = -H_T \cos \theta_T$$

So boundary condition (3) and (4) reduce to

$$E_i - E_R = E_T$$

$$H_i \cos \theta_i - H_R \cos \theta_R = H_T \cos \theta_T \quad \text{----- (9)}$$

$$\theta_i = \theta_R \text{ and } H = (E/Z) = (n/Z_0) \quad \text{----- (10)}$$

So equation (10) reduce to

$$n_1 E_i \cos \theta_i - n_2 E_R \cos \theta_R = n_2 E_T \cos \theta_T \quad \text{----- (11)}$$

Now eliminating E_R from equation (11) with help of (9) we get,

$$(E_i - E_R) n_1 \cos \theta_i = n_2 \cos \theta_T (E_i + E_R)$$

$$\left(\frac{E_R}{E_i}\right)_\perp = \frac{\cos \theta_i - \frac{n_2}{n_1} \cos \theta_T}{\cos \theta_i + \frac{n_2}{n_1} \cos \theta_T}$$

$$\left(\frac{E_R}{E_i}\right)_\parallel = \frac{\cos \theta_i - \frac{\sin \theta_i}{\sin \theta_T} \cos \theta_T}{\cos \theta_i + \frac{\sin \theta_i}{\sin \theta_T} \cos \theta_T}$$

$$\left(\frac{E_R}{E_i}\right)_\perp = \frac{\sin \theta_r \cos \theta_i - \cos \theta_T \sin \theta_i}{\sin \theta_r \cos \theta_i + \cos \theta_T \sin \theta_i}$$

$$\left(\frac{E_R}{E_i}\right)_\perp = \frac{\sin(\theta_i - \theta_T)}{\sin(\theta_i + \theta_T)} \quad \text{----- (C)}$$

Similarly eliminating E_R from equation (11) with the help of (9) we get,

$$n_1 E_i \cos \theta_i - n_1 (E_T - E_R) \cos \theta_i = n_2 E_T \cos \theta_T$$

$$\left(\frac{E_T}{E_i}\right)_\perp \approx \frac{2 \cos \theta_i}{\cos \theta_i + \frac{n_2}{n_1} \cos \theta_T}$$

$$\left(\frac{E_T}{E_i}\right)_\parallel \approx \frac{2 \cos \theta_i}{\cos \theta_i + \frac{\sin \theta_i}{\sin \theta_T} \cos \theta_T}$$

$$\left(\frac{E_T}{E_i}\right)_\perp \approx \frac{2 \cos \theta_i \sin \theta_T}{\cos \theta_i \sin \theta_T + \sin \theta_i \cos \theta_T}$$

$$\left(\frac{E_T}{E_i}\right)_\perp \approx \frac{2 \cos \theta_i \sin \theta_T}{\sin(\theta_i + \theta_T)} \quad \text{----- (D)}$$

Equation (A), (B), (C), (D) are the desired result known as Fresnel formulae.

REFLECTION FROM A METALLIC SURFACE:

We now consider the case in which the boundary surface separates a dielectric from a conducting medium, for simplicity we shall treat only the case of normal incidence here. The boundary condition for the continuity of the tangential components of electric and magnetic vectors E and H for the situation depicted in figure.

$$E_i - E_R = E_T \quad \text{----- (1)}$$

And $H_i + H_R = H_T \quad \text{----- (2)}$

But as $\mu_r \mu_0 H = \frac{n}{\mu_r \mu_0} E$ & $E = \frac{n}{\mu_r \mu_0} H$ as

$$\mu_r \approx 1$$

Equation (2) reduces to

$$n_1 (E_i + E_R) = n_2 E_T \quad \text{----- (3)}$$

So substituting the value of E_T from (3) in (1) we get,

$$(E_i - E_R) = \frac{n_1}{n_2} (E_i + E_R)$$

$$\frac{E_R}{E_i} = \frac{n_2 - n_1}{n_2 + n_1} \quad \text{----- (4)}$$

And substituting the value of E_R from

Sub equation (3) in equation (1) we get,

$$E_i = \left[\frac{n_2}{n_1} E_R - E_I \right] + E_T$$

$$\left[\frac{E_T}{E_i} \right] = \frac{2n_1}{n_2 + n_1} \quad \text{----- (5)}$$

Now as index of refraction is related to propagation vector by the relation

$$K = n \frac{\omega}{c} \quad \text{I.e. } n = \frac{c}{\omega} K$$

Interaction of EMW with matter on microscopic scale and as in case of good conductor.

$$K \rightarrow K^* = \alpha + i\beta \quad \text{with } \alpha \simeq \beta = \frac{1}{\delta} = \sqrt{\frac{\omega \mu_0 \sigma}{2}}$$

$$n_2 \rightarrow n^* = \frac{c}{\omega} (K^*) = \frac{c}{\omega} (\alpha + i\beta)$$

$$n_2 \rightarrow n^* = \left[\frac{c}{\omega} \alpha \right] + i \left[\frac{c}{\omega} \beta \right]$$

$$n_2 \rightarrow n^* = n + i\alpha \quad \text{with } n = k = \sqrt{\frac{\alpha}{2\omega\epsilon_0}} \quad \text{----- (6)}$$

So equation (4) and (5) reduce

$$\left(\frac{E_R}{E_i} \right) = \frac{(n + ik) - n_1}{n + ik + n_1} = \frac{n - n_1 + ik}{n + n_1 + ik} \quad \text{----- (7)}$$

$$\text{And } \left(\frac{E_T}{E_i} \right) = \frac{2n_1}{n + ik + n_1} = \frac{2n_1}{(n + n_1) + ik} \quad \text{----- (8)}$$

Now if, $(n - n_1) + ik = a e^{i\psi_1}$

$$(n - n_1) + ik = b e^{i\psi_2}$$

$$a = \sqrt{(n - n_1)^2 + k^2} \quad ; \quad \psi_1 = \tan^{-1} \frac{k}{n - n_1} \quad \text{----- (9)}$$

$$b = \sqrt{(n + n_1)^2 + k^2} \quad ; \quad \psi_2 = \tan^{-1} \frac{k}{n + n_1} \quad \text{----- (10)}$$

So equation (7) and (8) reduce to ,

$$\left[\frac{E_T}{E_i} \right] = a e^{i\psi_1} / b e^{i\psi_2} = \left[(n - n_1)^2 + k^2 / (n + n_1)^2 + k^2 \right]^{1/2} e^{i(\psi_2 - \psi_1)} \quad \text{----- (11)}$$

$$\left[\frac{E_T}{E_i} \right] = 2n_1 / b e^{i\psi_2} = 2n_1 / [(n + n_1)^2 + k^2] e^{i\psi_2} \quad \text{----- (12)}$$

Equation (11) and (12) are our final equation representing the reflected and transmitted waves ψ_1 and ψ_2 are given by equation (9) and (10) while n and k by equation (6). From these it is clear that both reflected and transmitted waves exist and they are not in phase with the incident wave. Further the amplitude of the transmitted wave is very small due to large values of n and k in the denominator, waves which are most strongly absorbed are very strongly reflected. i.e. All good conductors are good absorbers and good reflectors. Light are

complementary. A good example is offered by the optical properties of thin sheets of gold. They appear yellowish by reflection. This means that of the white light is incident on thin gold foils. Then the transmitted light will be devoid of yellow component. As a result the transmitted light appears greenish or bluish.

TOTAL INTERNAL REFLECTION :

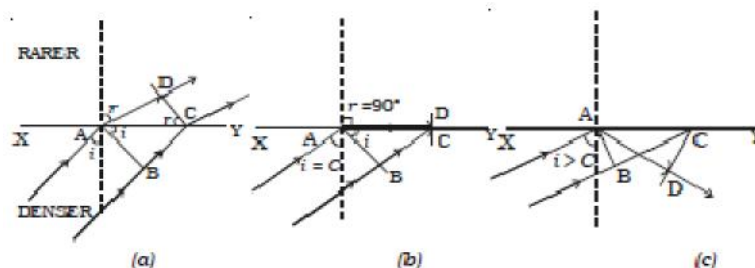
Let XY be a plane surface which separates a rarer medium (air) and a denser medium. Let the velocity of the wavefront in these media be C_a and C_m respectively. A plane wavefront AB passes from denser medium to rarer medium. It is incident on the surface with angle of incidence i . Let r be the angle of refraction.

$$\frac{\sin i}{\sin r} = \frac{(BC/AC)}{(AD/AC)} = \frac{BC}{AD} = \frac{c_m t}{c_a t} = \frac{c_m}{c_a}$$

Since, $\frac{c_m}{c_a} < 1$, i is less than r . This means that the refracted wavefront is deflected away from the surface XY.

In right angled triangle ADC, there are three possibilities

(i) $AD < AC$ (ii) $AD = AC$ and (iii) $AD > AC$



(i) **$AD < AC$** : For small values of i , BC will be small and so $AD > BC$ but less than AC (Fig.a)

$\sin r = AD/AC$, which is less than unity

i.e. $r < 90^\circ$

For each value of i , for which $r < 90^\circ$, a refracted wavefront is possible

(ii) **$AD = AC$** : As i increases r also increases. When $AD = AC$, $\sin r = 1$ (or) $r = 90^\circ$. i.e. a refracted wavefront is just possible (Fig.b). Now the refracted ray grazes the surface of separation of the two media. The angle of incidence at which the angle of refraction is 90° is called the critical angle C .

(iii) **$AD > AC$** : When $AD > AC$, $\sin r > 1$. This is not possible (Fig. c). Therefore no refracted wave front is possible, when the angle of incidence increases beyond the critical angle. The

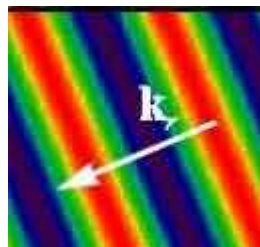
incident wavefront is totally reflected into the denser medium itself. This is called total internal reflection.

Hence for total internal reflection to take place (i) light must travel from a denser medium to a rarer medium and (ii) the angle of incidence inside the denser medium must be greater than the critical angle. i.e. $i > C$.

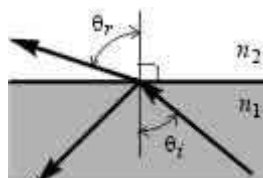
EVANESCENT WAVES :

"Evanescent" means "tending to vanish", which is appropriate because the intensity of evanescent waves decays exponentially (rather than sinusoidally) with distance from the interface at which they are formed. Evanescent waves are formed when sinusoidal waves are (internally) reflected off an interface at an angle greater than the critical angle so that total internal reflection occurs.

The colors in the image at right indicate the instantaneous electric field magnitude of the incident light. In this view, the plane of the page is the plane of incidence (contains the wave vector \mathbf{k}_i and the normal to the interface, the latter indicated by the black line). Surfaces on which the electric field magnitude is uniform are planes normal to the wave vector \mathbf{k}_i . Hence the incident light is a linearly polarized plane wave (LPPW). As time progresses, these planes move at the speed of light in a direction given by the wave vector \mathbf{k}_i . A LPPW is the type of wave produced by a laser.



The next image at right shows the reflected wave, which is also a LPPW. The direction of the wave vector \mathbf{k}_r is determined such that the angle of incidence equals the angle of reflection.



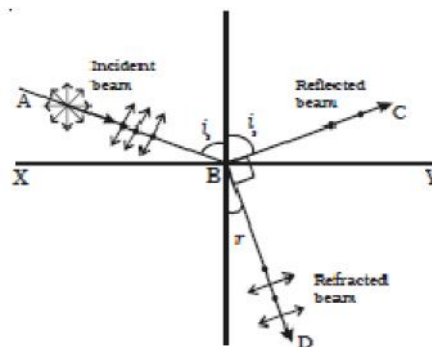
A wave (called the refracted wave) also arises on the other side of the interface where the reflection occurs. The three arrows in the sketch at left represent the 3 wave vectors for the

incident, reflected and refracted waves. All 3 wave vectors lie in the same plane (the plane of incidence). The angle of incidence θ_i and the angle of refraction θ_r are related by Snell's law:

$$n_1 \sin \theta_i = n_2 \sin \theta_r$$

where n_1 and n_2 are indexes of refraction on either side of the interface. When $n_1 < n_2$ Snell's law predicts that the refracted wave vector will be bent toward to the normal. This is called an external reflection because it often occurs when the incident light strikes the outside surface of a solid object. External reflections of LPPW's always produce a refracted wave which is also a LPPW. Our interest is in internal reflections.

Brewster's law :



Sir David Brewster conducted a series of experiments with different reflectors and found a simple relation between the angle of polarisation and the refractive index of the medium. It has been observed experimentally that the reflected and refracted rays are at right angles to each other, when the light is incident at polarising angle.

$$\text{From } i_p + 90^\circ + r = 180^\circ$$

$$r = 90^\circ - i_p$$

$$\text{From Snell's law, } \sin i_p / \sin r = \mu$$

where μ is the refractive index of the medium (glass) Substituting for r , we get

$$\frac{\sin i_p}{\sin(90^\circ - i_p)} = \mu \quad ; \quad \frac{\sin i_p}{\cos i_p} = \mu$$

$$\therefore \tan i_p = \mu$$

The tangent of the polarising angle is numerically equal to the refractive index of the medium.

TRANSMISSION AND REFLECTION CO-EFFICIENT :

The transmission coefficient is used in physics and electrical engineering when wave propagation in a medium containing discontinuities is considered. A transmission coefficient

describes the amplitude, intensity, or total power of a transmitted wave relative to an incident wave.

In physics and electrical engineering the reflection coefficient is a parameter that describes how much of an electromagnetic wave is reflected by an impedance discontinuity in the transmission medium. It is equal to the ratio of the amplitude of the reflected wave to the incident wave, with each expressed as phasors. For example, it is used in optics to calculate the amount of light that is reflected from a surface with a different index of refraction, such as a glass surface, or in an electrical transmission line to calculate how much of the electromagnetic wave is reflected by an impedance. The reflection coefficient is closely related to the transmission coefficient. The reflectance of a system is also sometimes called a "reflection coefficient".

Part B

Possible 2 Marks

1. What are called wave guides?
2. What are kinematic properties?
3. State Snell's law.
4. Define Brewster's law.
5. What are called evanescent waves?
6. Write about the boundary conditions .
7. State Fresnel's formula.

Part C

Possible 6 Marks

1. Determine the boundary conditions satisfied by electromagnetic fields at the interface between two dielectric media.
2. Discuss the phenomenon of total internal reflection on the basis of electromagnetic theory.
3. Discuss the boundary condition at the interface between two different media.
4. Explain the significance of brewster angle.
5. Write in detail about the reflection and refraction of electromagnetic waves.
6. What are the peculiarities of metallic reflection? Describe a method for its verification.
7. Discuss about the fresnel formulae for perpendicular polarization condition.
8. Obtain the laws of reflection and refraction and write about the dynamic properties.

_____ is defined as the ratio of the energy scattered by the system per unit time per unit solid angle to the energy flux density of the incident radiation.	surface cross-section	area cross-section	cross-section	differential scattering cross-section	differential scattering cross-section
_____ is defined as the ratio of the power scattered to the intensity of the incident radiation.	cross section	area cross section	total scattering cross-section	differential scattering cross section	total scattering cross-section
The factor _____ is called degree of polarization.	$-1/2 (1 + \cos^2 \theta)$	$1/2 (1 - \cos^2 \theta)$	$-1/2 (1 - \cos^2 \theta)$	$1/2 (1 + \cos^2 \theta)$	$1/2 (1 + \cos^2 \theta)$
Scattering depends on the nature of the _____ particles.	charged	uncharged	elementary	none of the above	charged
Thomson formula for scattering is appropriate for the scattering for _____.	alpha	neutrons	cosmic	electrons	electrons
Scattering occurs in all directions and is maximum when $\theta =$ _____.	0°	270°	90°	45°	0°
Scattering occurs in all directions and is minimum when $\theta =$ _____.	180°	0	90° or 270°	360°	90° or 270°
The total scattering cross-section according to Thomson's scattering is $\sigma_T =$ _____.	$-8\pi/3 r_0^2$	$8\pi/3 r_0^2$	$8\pi/3 r_0$	$-8\pi/3 r_0$	$8\pi/3 r_0^2$
In general the scattered radiation is more concentrated in the _____ direction.	forward	backward	random	none of the above	forward
Scattering of electromagnetic waves is _____ of the nature of the incident wave.	dependent	independent	infinite	finite	independent
An oscillating charge behave like an oscillating dipole with dipole moment $p =$ _____.	$-qx$	qx^2	qx^2	$-qx^2$	qx^2
The relation between the wavelength of the scattered radiation at an angle θ and the incident radiation is _____.	$\lambda_s = \lambda_i + (h/mc) (1 - \cos \theta)$	$\lambda_s = \lambda_i - (h/mc) (1 - \cos \theta)$	$\lambda_s = \lambda_i + (h/mc) (1 + \cos \theta)$	$\lambda_s = -\lambda_i + (h/mc) (1 + \cos \theta)$	$\lambda_s = \lambda_i + (h/mc) (1 - \cos \theta)$
If the amount of scattered light is proportional to $1/\lambda^4$ where λ is the wavelength of the incident radiation, then scattering is known as _____ scattering.	Thomson	Compton	Inelastic	Rayleigh	Rayleigh
The blue color of the sky is due to _____ scattering.	Rayleigh	Compton	Thomson	None of the above	Rayleigh
_____ light has longest wavelength in the visible region.	blue	violet	red	green	red

In a medium, the index of refraction varies with frequency, then the medium is said to be _____.	rarer	dispersive	denser	none of the above	dispersive
The rate of change of refractive index with wavelength is known as _____.	dispersion	refractive index	dispersive power	none of the above	dispersive power
The index of refraction _____ as the frequency increases.	equals	decreases	varies	increases	increases
As an electromagnetic wave passes through a gas the electric field induces _____ in the gas molecules.	electrostatic energy	dipole moment	magnetostatic energy	none of the above	dipole moment
The electrons are bound to the nucleus in an atom by _____.	covalent bond	ionic bond	linear restoring force	none of the above	linear restoring force
In polarization the positions of the electrons are altered from their equilibrium value while _____ remains stationary.	nuclei	neutron	proton	muon	nuclei
The classical radius of the electron $r_0 =$ _____.	$-q^2/4\pi\epsilon_0 mc^2$	$q^2/4\pi\epsilon_0 mc$	$q^2/4\pi\epsilon_0 mc^2$	$q/4\pi\epsilon_0 mc^2$	$q^2/4\pi\epsilon_0 mc^2$
In a plane wave $B =$ _____.	$-(n \times E)$	$(n \times E)$	$-(n \times E)/c$	$(n \times E)/c$	$(n \times E)/c$
If the index of refraction decreases with the increase in frequency over small frequency range, then it is called _____ dispersion.	normal	abnormal	finite	anomalous	anomalous
In dispersion in gases, there is a damping proportional to the velocity of the _____.	proton	electron	neutron	muon	electron
The dipole moment results from the displacement of the electron is $p =$ _____.	er	$-er$	er^2	$-er^2$	er
In case of gases, $e_r \propto$ _____.	-1	0	1	2	1
If $\omega \gg 0$, the frequency of the incident wave is _____ in comparison to the natural frequency of the electron.	very large	very small	zero	none of the above	very small
Thomson scattering is known as _____ scattering.	resonance	abnormal	normal	anomalous	resonance
Oscillating charge is equivalent to an induced _____ of moment $p = qx$.	electric dipole	magnetic dipole	electromagnetic dipole	none of the above	electric dipole
Thomson result become significant for incident photon energy $h\nu$ which is comparable with or larger than rest energies of the scattering electron.	mc	mv	mc^2	mv^2	mc^2
According to quantum mechanical calculations, the frequency of the scattered radiation is _____ that of the incoming waves.	greater than	lesser than	equal to	none of the above	lesser than
According to quantum mechanical calculations, the frequency of the scattered radiation depends on _____ of the scattering.	angle	nature	energy	momentum	angle
The restoring force is _____.	$m\omega_0^2 x$	$-m\omega_0^2 x$	$-m\omega_0 x$	$-m^2 \omega_0 x$	$-m\omega_0^2 x$
Example of resonance scattering is _____.	neon lamp	mercury vapour lamp	fluorescent lamp	sodium vapour lamp	sodium vapour lamp

The color of the sky during sunset or sunrise is _____.	red	blue	yellow	yellowish red	red
_____ light has shorter wavelength in visible	red	blue	violet	green	violet
According to normal dispersion, the refractive index is _____.	Real	imaginary	complex	rational	Real
According to normal dispersion, the refractive index _____ with frequency of the incident waves.	proportional	equals	increases	decreases	increases
For a given medium _____ light has the lowest index of refraction in the optical range of frequencies.	blue	green	yellow	red	red
For a given medium _____ light has the largest index of refraction in the optical range of frequency.	red	violet	blue	green	violet
For anomalous dispersion, there is _____ natural frequency.	one	two	four	three	one
The index of refraction is a _____ function of frequency of the electromagnetic waves propagating through the gas.	linear	logarithmic	complex	exponential	complex
At very low frequencies, the index of refraction is slightly _____ unity.	lesser than	greater than	equal to	proportional to	greater than
The imaginary part of index of refraction corresponds to the _____ of electromagnetic waves propagating through gases.	emission	reflection	absorption	refraction	absorption
For any real gas, there exists _____ resonant frequencies.	many	two	three	four	many
The tangential component of E is _____ across a surface of discontinuity.	discontinuous	Proportional	continuous	equals	continuous
According to law of frequency, the frequency of the wave remains _____ by reflection or refraction.	multiplied	changed	decreased	unchanged	unchanged
According to law of reflection, the angle of reflection is _____ angle of incidence.	equal to the	greater than	lesser than	none of the above	equal to the
In case of refraction, the ratio of the sine of the angle of refraction to the sine of the angle of incidence is _____ ratio of the refractive index of the two media.	greater than	equal to the	lesser than	none of the above	equal to the
For all angles of incidence there is a phase change of _____ on reflection for EMW whose vibrations are perpendicular to the plane of incidence.	2π	$\pi/2$	π	$\pi/3$	π
_____ angle is also called as polarizing angle.	brewster's angle	Snell's angel	Fresnel's angle	None of the above	brewster's angle
If light is incident on a glass plate at 56° , the reflected light will _____.	circularly polarized	plane polarized	spherically polarized	elliptically polarized	plane polarized

Water _____ reflect radiowaves which are polarized with vibration in the plane of incidence and are incident on it at 84° .	cannot	can	multi reflection	none of the above	cannot
All the light is reflected as the angle of incidence approaches 90° , the angle is called _____.	Snell's angle	Brewster's angle	Fresnel's angle	Grazing angle	Grazing angle
_____ glasses transmit only one direction of vibration.	light	dark	colour	crown	dark
The value of angle of incidence for which θ_T becomes 90° is called _____.	critical angle	Brewster's angle	Fresnel's angle	Snell's angle	critical angle
The _____ velocity is a function of angle of incidence.	group	angular	phase	linear	phase
The waves which do not have energy are called _____ waves.	cerenkov	Radio	Microwaves	Evanescent	Evanescent
If a linearly polarized wave is reflected from the boundary at an incident angle greater than the critical angle, the reflected wave will be _____.	circularly	elliptically	spherically	plane	elliptically
The phenomena of total internal reflection is used to produce _____ polarized lights.	elliptically	spherically	plane	none of the above	elliptically
All good conductors are good _____ and good _____.	absorber and scatterer	absorbers and reflectors	absorber and refractor	none of the above	absorbers and reflectors
Good conductor of electricity are _____ to light.	opaque	absorbers	reflectors	scatterers	opaque
If white light is incident on thin gold foils, then the transmitted light appears _____.	yellowish or reddish	brownish or greenish	brownish or bluish	greenish or bluish	greenish or bluish
The reflection coefficient of substance of high conductivity at low frequency will _____.	not be unity	be infinite	will be unity	be finite	will be unity
Transmission of electromagnetic waves by successive reflections from inner walls of the tube is called _____.	transmission tube	wave guide	reflection tube	total-internal reflection	wave guide
If the cross-section of the waveguide is rectangular, it is called _____ waveguide.	cylindrical	circular	square	rectangular	rectangular
If the cross-section of waveguide is circular, it is called _____ waveguide.	cylindrical	circular	elliptical	none of the above	cylindrical
The walls of the waveguide are perfectly _____.	non-conducting	semi-conducting	conducting	none of the above	conducting
The tangential component of E and normal component of B _____ at the surface of the walls of the wave guide.	multiplies	vanishes	coincides	none of the above	vanishes
The electromagnetic fields E and B are propagated as waves in the waveguides at a speed equal to _____.	c	0.9 c	0.3 c	0.2 c	c
The phase velocity becomes _____ exactly at cutoff frequency.	finite	zero	infinite	unity	infinite
In the waveguide the Maxwell's first equation for the propagation of EMW is _____.	$\text{div } E = \rho$	$\text{div } E = -\rho/\epsilon_0$	$\text{div } E = -\rho$	$\text{div } E = 0$	$\text{div } E = 0$
If the EMW propagates in a waveguide, the Maxwell's second equation is _____.	$H = 0$	$\text{Div } B = 0$	$H = 1$	$\text{Div } B = H$	$\text{Div } B = 0$

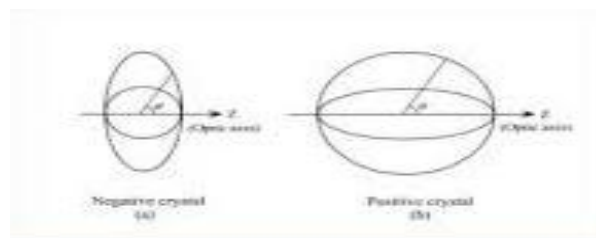
_____ waves cannot be propagated along the axis of a waveguide.	stationary	TE	TEM	TM	TEM
In _____ mode all the transverse components of E and B can be expressed in terms of longitudinal component of magnetic vector B_z .	TM	TE	TEM	Transverse	TE
The _____ mode is called the principal or dominant mode.	TE_{11}	TE_{10}	TE_{01}	TE_{00}	TE_{01}
In _____ mode, all the transverse components of E and B can be expressed in terms of longitudinal component of the electric field E_z .	TM	TE	TEM	Longitudinal	TM
In _____ waveguide TE and TM have the same set of cutoff frequencies.	circular	all	square	rectangular	rectangular
The reflection of the electromagnetic waves at the conducting plane involves no change in _____.	frequency	amplitude	phase	energy	amplitude
If the magnetic field H of electromagnetic wave has a component along the assumed axis of propagation then the wave is called _____.	H-wave	E-wave	EH-wave	Transverse wave	H-wave

Polarization of Electromagnetic Waves:

Description of Linear, Circular and Elliptical Polarization. Propagation of E.M. Waves in Anisotropic Media. Symmetric Nature of Dielectric Tensor. Fresnel's Formula. Uniaxial and Biaxial Crystals. Light Propagation in Uniaxial Crystal. Double Refraction. Polarization by Double Refraction. Nicol Prism. Ordinary & extraordinary refractive indices. Production & detection of Plane, Circularly and Elliptically Polarized Light. Phase Retardation Plates: Quarter-Wave and Half-Wave Plates. Babinet Compensator and its Uses. Analysis of Polarized Light

HUYGENS EXPLANATION OF DOUBLE REFRACTION IN UNIAXIAL CRYSTALS :

Huygens explained the phenomenon of double refraction with the help of the principal of secondary wavelets. A point source of light in a double refracting medium is the origin of two wave fronts. For the ordinary ray, for which the velocity of light is same in all directions. The wavefront is spherical. For extraordinary ray the velocity varies with the directions and wavefront is ellipsoid. The velocities of ordinary and extraordinary rays are the same along the optic axis.



Consider a point source of light S in a calcite crystal, the sphere is the wave surface of the ordinary ray and ellipsoid is the wave surface of the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface. Such crystals are known as negative crystals. For crystals like quartz, which are known as positive crystals, the extraordinary wave surface lies within the ordinary wave surface.

1) For the negative uniaxial crystals $\mu_o > \mu_E$:

The velocity of the extraordinary ray varies as the radius vector of the ellipsoid. It is least and equal to the velocity of the ordinary ray along the optic axis but it is maximum at right angle to the direction of the optic axis.

2) For the positive uniaxial crystals $\mu_E > \mu_o$:

The velocity of the extraordinary ray is least in the direction at the right angles to the optic axis. It is maximum and equal to the velocity of the ordinary ray along the optic axis. Hence from the Huygens theory, The wavefronts or surfaces in uniaxial crystals are a sphere and an ellipsoid and there are two points where these two wavefronts touch each other. The direction of the line joining these two points is the optic axis.

QUARTER WAVE PLATE:

It is a plate of doubly refracting uniaxial crystal of calcite (or) quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis. The incident plane-polarized light is perpendicular to its surface and the ordinary and the extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays are μ_o and μ_E respectively, Then the path difference introduced between the two rays is given by,

For negative crystals, path difference = $(\mu_o - \mu_E)t$

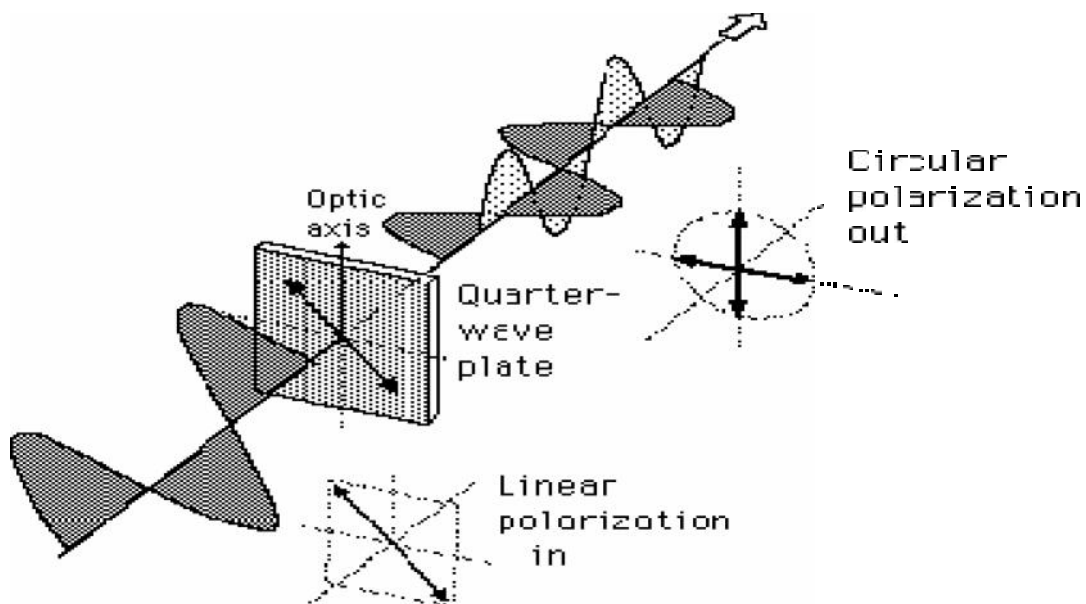
For positive crystals, path difference = $(\mu_E - \mu_o)t$

To produce a path difference of $\lambda/4$, in calcite

$$(\mu_o - \mu_E)t = \lambda/4$$

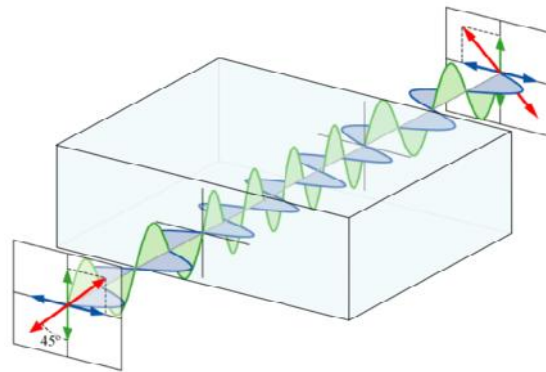
$$t = \lambda/4 (\mu_o - \mu_E) \text{----- (1)}$$

and in case of quartz, If the plane-polarized light, whose plane of vibration is inclined at an angle of 45° to the optic axis, is incident on a quartz wave plate, the emergent light is circularly polarized.



HALF WAVE PLATE:

This plate is also made from a doubly refracting uniaxial crystal of calcite (or) quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis. the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays have a path difference $= \lambda/2$ after passing through the crystals



For negative crystal, path difference $= (\mu_0 - \mu_E)t$

For positive crystals, path difference $= (\mu_E - \mu_0)t$

To produce a path difference of $\lambda/2$, in calcite

$$(\mu_0 - \mu_E)t = \lambda/2$$

$$t = \lambda/2 (\mu_0 - \mu_E) \text{----- (1)}$$

and in case of quartz,

$$t = \lambda/2 (\mu_E - \mu_0) \text{----- (2)}$$

When plane polarised light is incident on a half waveplate, such that it makes an angle of 45° with the optic axis a path difference of $\lambda/2$ is introduced between the extraordinary and the ordinary rays. The emergent light is plane-polarized and the direction of polarization of the linear incident light is rotated through 90° . Thus a half waveplate rotates the azimuth of a beam of plane-polarized light by 90° , provided the incident light makes an angle 45° with the optic axis of the half wave plate.

PRODUCTION OF PLANE, CIRCULAR AND ELLIPTICALLY POLARIZED LIGHT:

Production of plane polarized light:

A beam of monochromatic light is passed through a nicol prism. While passing through the nicol prism, the beam is split up into extraordinary ray and ordinary ray. The

ordinary ray is totally internally reflected back at the Canada balsam layer, while the extraordinary ray passes through the nicol prism. The emergent beam is plane polarized

Production of circularly polarized light:

To produce circularly polarized light, the two waves vibrating at right angle to each other having the same amplitude and time period should have a phase difference of $\pi/2$ (or) a path difference of $\lambda/4$. For this purpose a parallel beam of monochromatic light is allowed to fall on a nicol prism N_1 . The beam after passing through the prism N_1 is plane polarized.

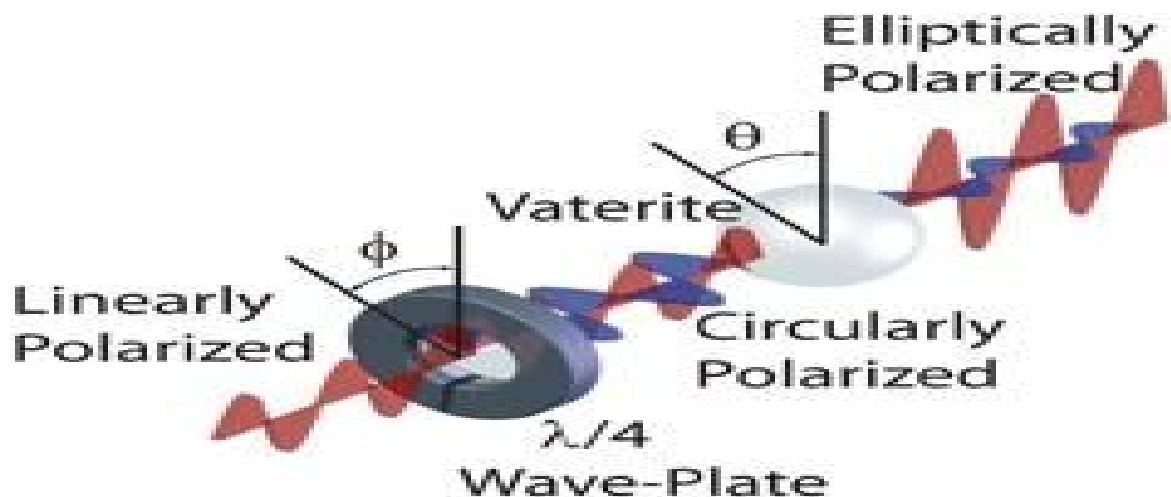
The nicol prism N_2 is placed at some distance from N_1 so that N_1 and N_2 are crossed. The field of view will be dark as viewed by the eye in this position. A quarter waveplate P is mounted on a tube A . The tube A can rotate about the outer fixed tube B introduced between the nicol prism N_1 and N_2 . The plane polarised light from N_1 falls normally on P and the field of view may be bright. The quarter waveplate is rotated until the field of view may be dark keeping P fixed, A is rotated such that the mark S on P coincides with 0 mark on A . Afterwards, By rotating the quarter waveplate P , the mark S is made to coincide with the 45° mark on A .

The quarter waveplate is in the desired position. In this case, the vibration of plane polarised light falling on the quarter waveplate makes an angle 45° with the direction of optic axis of the quarter wave plate. The polarised light is split up into two rectangular components having equal amplitude and time period and on coming out of the quarter waveplate, the beam is circular polarised if the nicol prism N_2 is rotated at this stage, the field of view is uniform in intensity similar to the ordinary light passing through the nicol prism

Elliptically polarised light:

To produce elliptically polarised light, the two waves vibrating at right angle to each other having unequal amplitudes should have a phase difference of $\pi/2$ or a path difference of $\lambda/4$. The arrangement of figure can be used for this purpose. A parallel beam of monochromatic light is allowed to fall on the nicol prism N_1 . The prisms N_1 and N_2 are crossed and the field of view is dark. A quarter wave plate is introduced between N_1 and N_2 . The plane polarised light from the nicol prism N_1 falls normally on the quarter wave plate. The field of view is illuminated and the light coming out of the quarter wave plate is elliptically polarised. The only precaution in the case is that the vibrations of the plane polarised light falling on the quarter plate should not make an angle of 45° with the optic

axis, in which case, the light will be circularly polarised. When the nicol prism N_2 is rotated, it is observed that the intensity of illumination of the field of view varies between a maximum and a minimum. This is just similar to the case when a beam consists of a mixture of plane-polarised light and ordinary light is examined by a nicol prism.



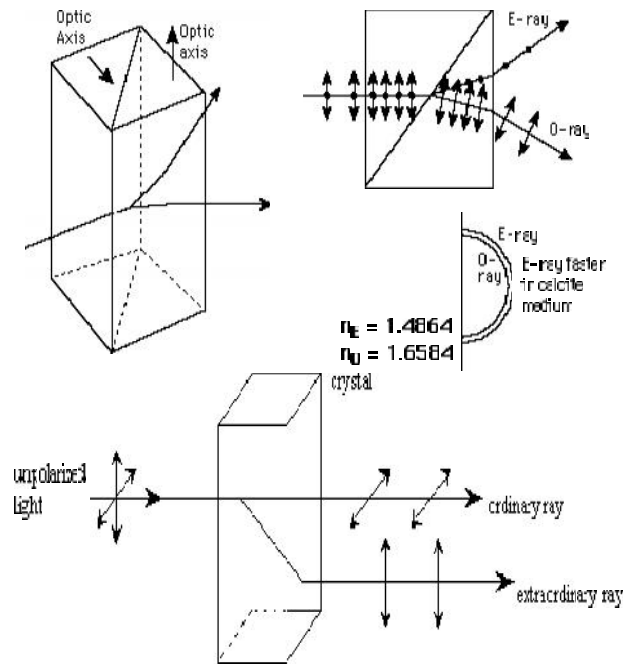
DOUBLE REFRACTION :

Erasmus Bratholinus discovered in 1669, that when a ray of light is refracted by a crystal of calcite it gives two refracted rays. The phenomenon is called double refraction. Calcite or Iceland spar is crystallised calcium carbonate (CaCO_3) and was found in large quantities in Iceland as very large transparent crystals. Due to this reason calcite is known as Iceland spar. It crystallises in many forms and can be reduced by cleavage or breakage into a rhombohedron, bounded by six parallelograms with angle equal to 102° and 78° . (more accurately $101^\circ 55'$ and $78^\circ 5'$).

Optic Axis:

At two opposite corners A and H, of the rhombohedron all the angles of the faces are obtuse. These corners A and H are known as the blunt corners of the crystal. A line drawn through A making equal angles with each of the three edges gives the direction of the optic axis. In fact any line parallel to this line is also an optic axis. Therefore optic axis is not a line but it is a direction. Moreover, it is not defined by joining the two blunt corners. Only in a spherical case, when the three edges of the crystal are equal, the line joining the two blunt corners A and H coincides with the crystallographic axis of the crystal and it gives the direction of the optic axis. If a ray of light is incident along the optic axis or in a direction parallel to the optic axis, then it will not break into two rays. Thus the phenomenon of double refraction is

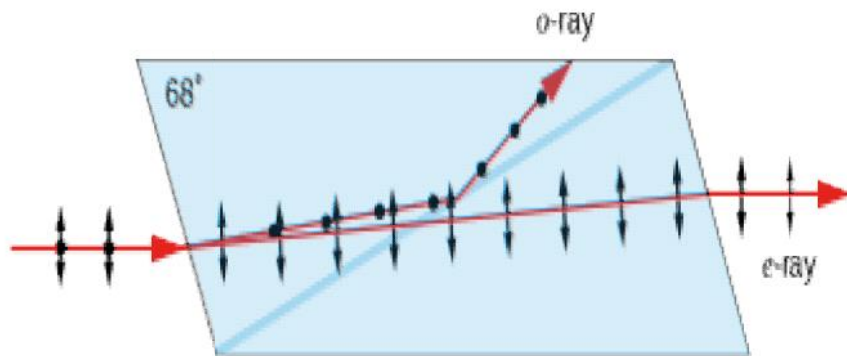
absent . When a light is allowed to enter the crystal along the optic axis.



NICOL PRISM :

It is an optical device for producing and analysing plane polarised light. It was invented by William Nicol, in 1828 who was expert in cutting and polishing gems and crystals . we have discussed that when a beam of light is transmitted through a calcite crystal, it breaks up into two rays (1) the ordinary ray which has its vibrations perpendicular to the principal section of the crystal and (2) the extraordinary ray which has its vibrations parallel to the principal section.

The nicol prism is made in such a way that it eliminates one of the two rays by total internal reflection. It is generally found that the ordinary ray is eliminated and only the extraordinary ray is transmitted through the prism. A calcite crystal whose length is three times its breadth is taken. Let A'BCDEFG'H represent such a crystal having A' and G' as its blunt corners and A'CG'E is one of the principle sections with $\angle A'CG'E = 70^\circ$.



The faces A'BCD and EFG'H are grounded such a way that the angle ACG becomes 68° instead of 71° . The crystal is then cut along the plane AKGL. The two out surfaces are grounded and polished optically flat and then cemented together by Canada balsam whose refractive index lies between the refractive indices for the ordinary and the extraordinary rays for calcite.

Refractive index for the ordinary $\mu_o = 1.658$

Refractive index for Canada balsam $\mu_B = 1.55$

Refractive index for the extraordinary $= 1.486$

The diagonal AC represents the Canada balsam layer in the plane ALKG.

It is clear that Canada balsam act as a rarer medium for an ordinary ray and it act as a denser medium for the extraordinary ray. Therefore, when the ordinary ray passes from a portion of the crystal into the layer of Canada balsam it passes from a denser to a rarer medium. When the angle of incidence is greater than the critical angle, the ray is totally internally reflected and is not transmitted. The extraordinary ray is not affected and is therefore transmitted through the prism.

Nicol prism



Calcite

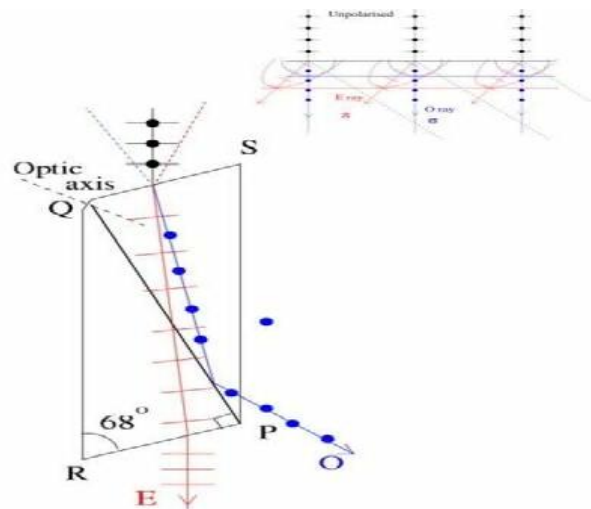
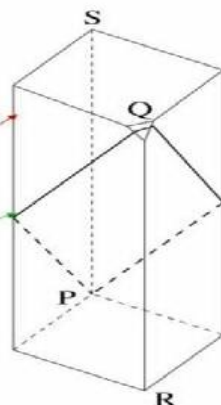
$$n_o = 1.6584$$

$$n_e = 1.4864$$

Canada

balsam

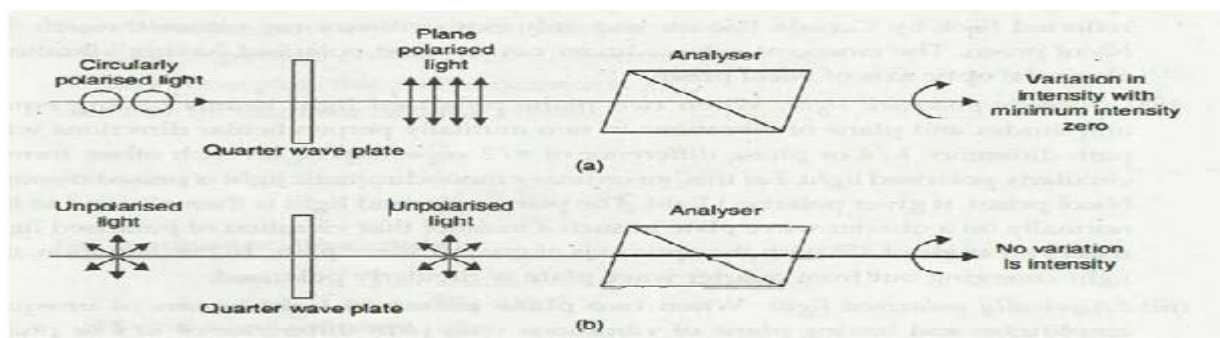
$$n = 1.55$$



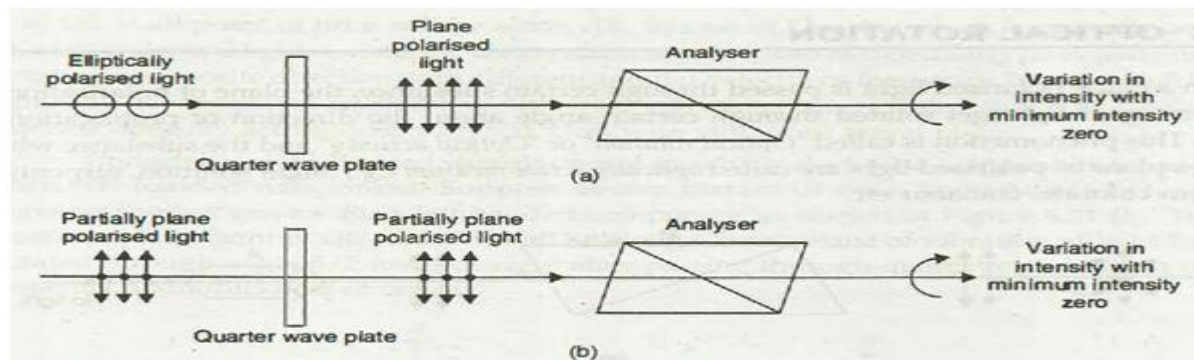
DETECTION OF PLANE, CIRCULAR AND ELLIPTICALLY POLARIZED LIGHT:

Plane Polarised Light: The light beam is allowed to fall on Nicol prism. If on rotation of Nicol prism, intensity of emitted light can be completely extinguished at two places in each rotation, then light is plane polarised.

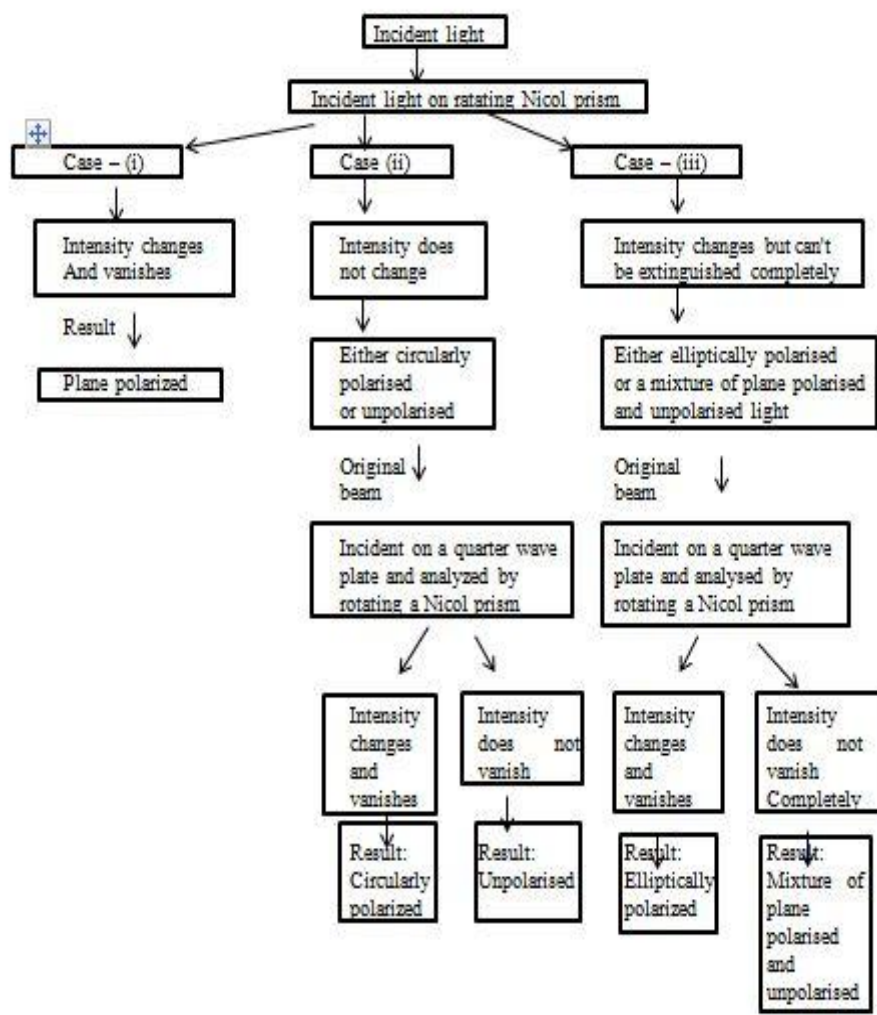
Circularly Polarised Light: The light beam is allowed to fall on a Nicol prism. If on rotation of Nicol prism the intensity of emitted light remains same, then light is either circularly polarised or unpolarised. To differentiate between unpolarised and circularly polarised light, the light is first passed through quarter wave plate and then through Nicol prism. Because if beam is circularly polarised then after passing through quarter wave-plate an extra difference of $\pi/4$ is introduced between ordinary and extraordinary component and gets converted into plane polarised. Thus on rotating the Nicol, the light can be extinguished at two places. If, on the other hand, the beam is unpolarised, it remains unpolarised after passing through quarter wave plate and on rotating the Nicol, there is no change in intensity of emitted light (Figure).



Elliptically Polarised Light: The light beam is allowed to fall on Nicol prism. If on rotation of Nicol prism, the intensity of emitted light varies from maximum to minimum, then light is either elliptically polarised or a mixture of plane polarized and unpolarised. To differentiate between the two, the light is first passed through quarter wave plate and then through Nicol prism. Because, if beam is elliptically polarised, then after passing through quarter wave plate, an extra path difference of $\pi/4$ is introduced between O-ray and E-ray and get converted into plane polarized. Thus, on rotating the Nicol, the light can be extinguished at two places. If, on the other hand, beam is mixture of polarised and unpolarised it remains mixture after passing through quarter wave plate and on rotating the Nicol intensity of emitted light varies from maximum to minimum (Figure 6.19).



Summary of detection of plane, circular and elliptical polarized light :



BABINET COMPENSATOR :

A quarter wave plate or a half wave plate produces only a fixed path difference between the ordinary and the extraordinary rays and can be used only for light of a particular wavelength. For different wavelengths, different quarter wave plates are to be used. To avoid this difficulty, Babinet designed a compensator by means of which a desired path difference can be introduced.

It consists of two wedge-shaped sections A and B of quartz. The optic axis is lengthwise in A and transverse in B. The outer faces of the compensator are parallel to the optic axis. Therefore, the ordinary and the extraordinary rays travel with different velocities along the

same direction inside the compensator. Moreover, the extraordinary ray in A behaves as ordinary in B while the ordinary in A behaves as extraordinary in B. Suppose a plane polarized parallel beam of light is incident normally at the point C of the Babinet's compensator. The beam is split up into extraordinary and ordinary rays. The path difference introduced between them after they have travelled a distance CD in A is .

$$(\mu_E - \mu_O) t_1$$

In B, the path difference introduced by B is,

$$(\mu_O - \mu_E) t_2$$

Therefore, the resultant path difference

$$(\mu_E - \mu_O) (t_1 - t_2)$$

The crystals A and B are mounted such that A is fixed and B can slide along the surface of A with the help of a rack and pinion arrangement. In this way $(t_1 - t_2)$ can be made to have any desired value. Hence any path difference can be introduced with the help of the Babinet's Compensator and it can be used for light of any wavelength.

DESCRIPTION OF PLANE, CIRCULAR AND ELLIPTICALLY POLARIZATION :

Light in the form of a plane wave in space is said to be linearly polarized. Light is a transverse electromagnetic wave, but natural light is generally unpolarized, all planes of propagation being equally probable. If light is composed of two plane waves of equal amplitude by differing in phase by 90° , then the light is said to be circularly polarized. If two plane waves of differing amplitude are related in phase by 90° , or if the relative phase is other than 90° then the light is said to be elliptically polarized.

Plane Polarization :

A plane electromagnetic wave is said to be linearly polarized. The transverse electric field wave is accompanied by a magnetic field wave as illustrated.

Circular Polarization :

Circularly polarized light consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase. The light illustrated is right- circularly polarized. If light is composed of two plane waves of equal amplitude but differing in phase by 90° , then the light is said to be circularly polarized. If you could see the tip of the electric field vector, it would appear to be moving in a circle as it approached you. If while looking at the source, the electric vector of the light coming toward you appears to be rotating counterclockwise,

the light is said to be right-circularly polarized. If clockwise, then left-circularly polarized light. The electric field vector makes one complete revolution as the light advances one wavelength toward you. Another way of saying it is that if the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.

Circularly polarized light may be produced by passing linearly polarized light through a quarter-wave plate at an angle of 45° to the optic axis of the plate.

Elliptical Polarization :

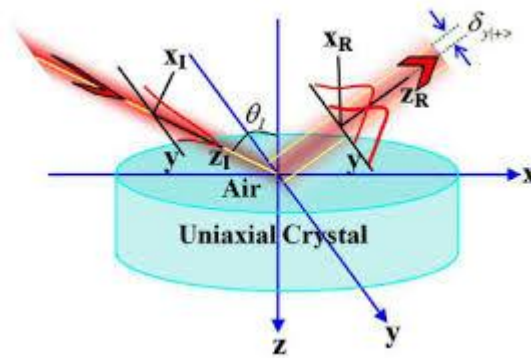
Elliptically polarized light consists of two perpendicular waves of unequal amplitude which differ in phase by 90° . The illustration shows right- elliptically polarized light.

If the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.



UNIAXIAL CRYSTAL :

Uniaxial crystals are transmissive optical elements in which the refractive index of one crystal axis is different from the other two crystal axes (i.e. $n_i \neq n_j = n_k$). This unique axis is called the extraordinary axis and is also referred to as the optic axis. Light travels with a higher phase velocity through an axis that has the smallest refractive index and this axis is called the fast axis. Similarly, an axis which has the highest refractive index is called a slow axis since the phase velocity of light is the lowest along this axis. The optic axis can be the fast or the slow axis for the crystal depending upon the material. Negative uniaxial crystals (e.g. calcite CaCO_3 , ruby Al_2O_3) have $n_e < n_o$ so for these crystals, the extraordinary axis (optic axis) is the fast axis whereas for positive uniaxial crystals (e.g. quartz SiO_2 , sapphire (magnesium fluoride) MgF_2 , rutile TiO_2), $n_e > n_o$ and thus the extraordinary axis (optic axis) is the slow axis. These crystals show birefringent property.



BIAXIAL CRYSTAL :

Biaxial Crystal - A birefringent crystal which has two axes along which the polarized vectors of a monochromatic light beam will travel with the same speed, or along which no double refraction occurs. This type of crystal may be monoclinic, orthorhombic, or triclinic. Sulfur, mica, and turquoise form biaxial crystals.

Part B

Possible 2 Marks

1. What is meant by polarization of light?
2. Define fresnel's formula.
3. Write about circular polarization.
4. What is mean by quarter wave plate?
5. Difference between polarized and unpolarized light
6. What are babinet compensator ?
7. What are double refraction ?

Part B

Possible 6 Marks

1. Describe the construction of nicol prism.
2. Give the construction and theory of quarter – wave plate.
3. Explain the phenomenon of double refraction in a calcite crystal.
4. What is babinets compensator? Explain how this can be used to analyze fully elliptically polarized light.
5. What is meant by 1) plane polarized light 2) circularly polarized light 3) elliptically polarized light. How they are produced.
6. Explain in detail about the double refraction in uniaxial crystal.
7. Discuss the detection of plane, circularly and elliptically polarized light.
8. what is known as principal refractive index and write in detail about the refractive index for extraordinary ray.

Karpagam academy of higher education
Coimbatore -21
Department of Physics
II B.Sc Physics
Electromagnetic Theory [16PHU303]

The experiments on interference and diffraction have shown that light is form of _____	transverse waves	particle motion	light waves	wave motion	wave motion
The light waves are _____	circular	longitudinal	elliptical	transverse	transverse
The light is not propagated as _____	circular	longitudinal	elliptical	transverse	longitudinal
After passing through a crystal light waves vibrate in _____ direction	all	only in one	perpendicular	parallel	only in one
Light coming from a crystal is known as _____	polarized	non polarized	refraction	transverse waves	polarized
Its experimentally proved that light waves are _____ in nature	circular	longitudinal	elliptical	transverse	transverse
When light is passed through a _____ crystal, the light is polarized to only one direction	quartz	diamond	ruby	tourmaline	tourmaline
The plane of polarization is that plane in which no _____ occur	disturbance	attenuation	polarization	vibration	vibration
The plane of vibration occurs at _____ angle	acute	obtuse	straight	right	right
Polarization of light by _____ from the surface of glass was discovered by Malus	reflection	refraction	polarization	diffraction	reflection
Polarized light is obtained when ordinary light is reflected by a plane sheet of _____	glass	tourmaline	mica	quartz	glass
Brewster performed no of experiments to study the polarization of light by _____	reflection	refraction	polarization	diffraction	reflection
Reflection from a transparent medium at a particular angle is known as _____	angle of polarization	angle of vibration	diffraction	vibration	angle of polarization
Snell's law _____	$\sin i / \sin r$	$\sin r / \sin i$	$\tan i$	$\tan r$	$\sin i / \sin r$
Brewster law _____	$\sin i / \sin r$	$\sin r / \sin i$	$\tan i$	$\tan r$	$\tan i$
The refractive index of glass is _____	1.52	0	1.98	2	1.52
_____ window type of window is used in laser	Brewster	Snell's	Langivian	Malus	Brewster
The pile of plates consists of _____ plates	glass	ceramic	FTO	Si	glass
A beam of _____ light is allowed to fall on the pile of plates at the polarizing angle	sodium	mercury	dichromatic	monochromatic	monochromatic
Who discovered the double refraction phenomenon ?	Malus	Brewster	Snell	Erasmus Bartholinus	Erasmus Bartholinus
calcite is also known as _____	ice	iceland	calcium	iceland spar	iceland spar
The phenomenon of double refraction is absent when _____ is allowed to enter the crystal along the optic axis	light	sound	wave	particle	light
The stationary image is known as _____	ordinary image	extraordinary image	imaginary image	standing image	ordinary image
The image which rotates with the rotation of the crystal is known as _____	ordinary image	extraordinary image	imaginary image	standing image	extraordinary image
The velocity of light for the ordinary ray inside the crystal will be _____	less	high	zero	one	less
The ordinary and the extraordinary rays are _____ polarized	plane	elliptically	circularly	optically	plane
_____ is a device used for producing and analysing plane polarised light	prism	grating	lens	Nicol prism	Nicol prism
William Nicol is expert in cutting and polishing gems and _____	prism	quartz	crystal	glass	crystal
In calcite crystal the ordinary ray has its vibration _____ to its direction	all	only in one	perpendicular	parallel	perpendicular

In calcite crystal the extraordinary ray has its vibration _____ to its direction	all	only in one	perpendicular	parallel	parallel
_____ can be used in the detection of plane polarizer light	nicol prism	brewster law	Snells law	calcite crystal	nicol prism
Nicol prism are coated with _____ paint to absorb ordinary ray	blue	red	green	black	black
Canada balsam acts as _____ medium for ordinary ray	rarer	denser	thinner	thicker	rarer
Canada balsam acts as _____ medium for extraordinary ray	denser	thinner	rarer	thicker	denser
Huygens explained the phenomenon of double refraction with the help of his principle of _____ wavelets	primary	secondary	tetra	zero	secondary
The velocities of ordinary and extraordinary rays are same along the _____	wave axis	particle axis	optic axis	sound axis	optic axis
The _____ is the wave surface for ordinary ray	sphere	ellipsoid	triangle	circular	sphere
The _____ is the wave surface for extraordinary ray	sphere	ellipsoid	triangle	circular	ellipsoid
The ordinary wave surface lies within the extraordinary wave surface and the crystal is known as _____ crystal	negative	positive	quartz	calcite	negative
_____ crystal are known as positive crystal	quartz	calcite	tourmaline	calcium	quartz
The direction of the line joining the two points is the _____	wave axis	particle axis	optic axis	sound axis	optic axis
The _____ of the extraordinary ray through a uniaxial crystal depends upon the direction of the ray	velocity	time	speed	distance	velocity
The refractive index of extraordinary ray is known as principle of _____	refractive index	angle of vibration	angle of refraction	refractive media	refractive index
The plane of vibration inclined at an angle of _____ to the optic axis, is incident on a quarter wave	45°	90°	75°	180°	45°
The ordinary and extraordinary rays have a path difference of _____ after passing through the crystal	$\lambda/2$	$\lambda/4$	λ	0	$\lambda/2$
A half wave plate rotates the azimuth of a beam of plane polarized light by _____	45°	90°	75°	180°	90°
The ordinary and extraordinary rays travel with different velocities along the same direction inside the _____	crystal	media	glass	compensator	compensator
Polaroids are widely used as polarizing _____	windows	sun glasses	gems	crystal	sunglasses

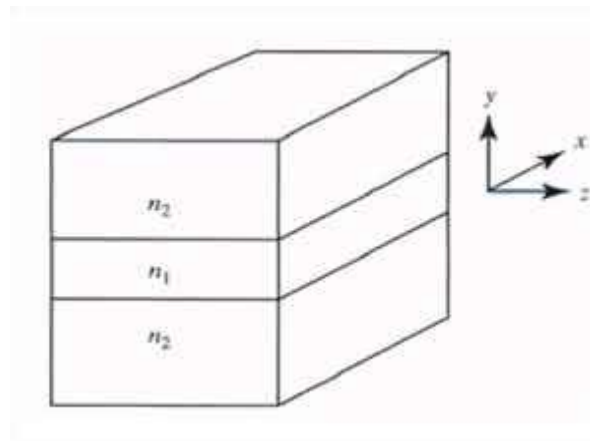
Wave Guides:

Planar optical wave guides. Planar dielectric wave guide. Condition of continuity at interface. Phase shift on total reflection. Eigenvalue equations. Phase and group velocity of guided waves. Field energy and Power transmission.

Optical Fibres:- Numerical Aperture. Step and Graded Indices (Definitions Only).

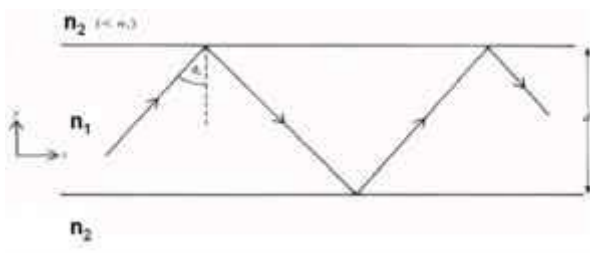
Single and Multiple Mode Fibres (Concept and Definition Only).

PLANAR DIELECTRIC WAVE GUIDE :



Planar (slab) waveguides are the basis of waveguides used in integrated optoelectronics. The same mathematical ideas can be applied (with minor modifications) to circular waveguides.

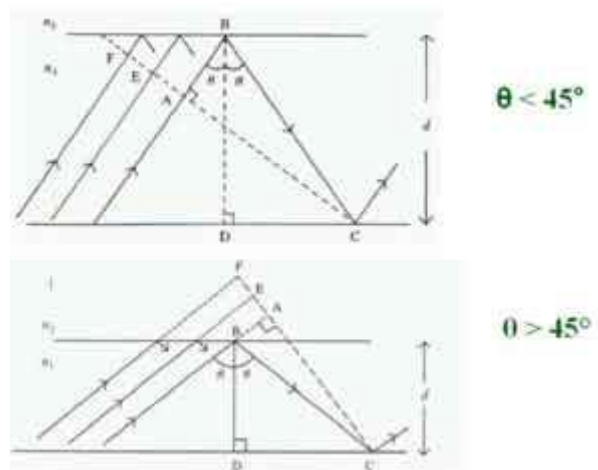
The waveguide consists of a semi-infinite slab of dielectric materials with thickness d and refractive index n_1 (the core) that is sandwiched between two regions (the cladding) both of refractive index n_2 , and where $n_1 > n_2$.



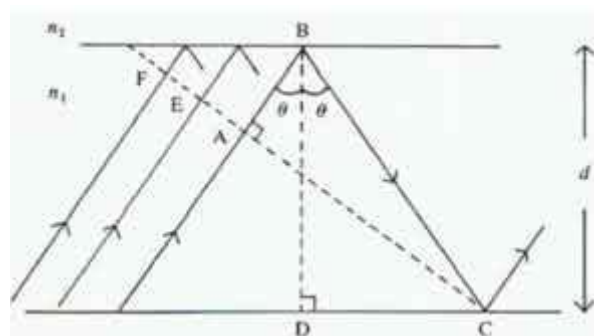
A beam propagating down a waveguide within the core layer

A ray of light may propagate down the core provided that total internal reflection occurs at the core/cladding interface, this requires that: $90^\circ > \theta_i > \theta_c$

Where θ_i is the internal ray angle (from now on written as θ)



In fact there are ‘infinite’ number of rays, all slightly displaced from each other, also propagating down the guide. The dotted line that is perpendicular to the wave lines is the wave front of the propagating beam. The rays represent lines drawn normally to the plane wave fronts.



The wave front FC intersects two the upwardly traveling portions of the same ray at points A and C. Therefore the phase at C and A must be the same or differ by a multiple of 2π .

Otherwise there would be destructive interference between out-of-phase waves and the light will not propagate. It also requires very specific angles above the critical angle. Consider the phase difference between A and C.

There are two factors -the path length of $AB + BC$ -the phase change due to reflection at B and C. We write the phase change resulting from reflection simply as ϕ . For perpendicular radiation ϕ is 2π , for parallel radiation $\phi = \pi$. The total phase change is equivalent to:

$$(AB + BC) \frac{2\pi}{\lambda_0} - 2\phi$$

Where λ_0 is the wavelength of light in the medium.

To determine the path of the light from a to b to c using trigonometry: $AB = BC \cos 2\theta$

Thus $AB + BC = BC (1 + \cos 2\theta)$

Since $\cos 2\theta = 2 \cos^2 \theta - 1$

$AB + BC = 2 BC \cos^2 \theta$ that is the thickness of the slab

So that $AB + BC = 2d \cos \theta$

The thickness of the slab determines the number of modes or angles that light will propagate at. In order for the mode to propagate the total phase change must be a multiple of 2π :

$$\frac{4\pi n_1 d \cos \theta}{\lambda_0} - 2\alpha = 2m\pi$$

$$\frac{2\pi n_1 d \cos \theta}{\lambda_0} - \phi = m\pi$$

Where m is an integer, S_0 for each value of m there will be an angle θ_m that satisfies the equation.

Each value of θ_m (those $> \theta_c$) has a distinct distribution of electric field across the guide. This distribution is known as a mode. Depending on the mode there may a distribution that is centered in the core or may have 2 spots, 4 spots etc when view in cross section.

When : θ_m is $= \theta_c$ the mode is at cut-off

If $\theta_m < \theta_c$: the mode is below cut off resulting in rapid attenuation and light will not be propagated.

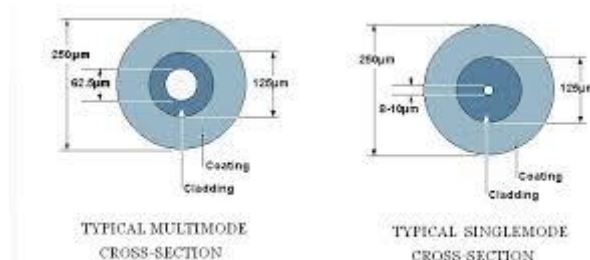
If $\theta_m > \theta_c$: the mode is above cut-off which can propagate.

SINGLE AND MULTI MODE FIBRE:

Fibre Optics is sending signals down hair-thin strands of glass or plastic fibre. The light is “guided” down the center of the fibre called the “core”. The core is surrounded by an optical material called the “cladding” that traps the light in the core using an optical technique called “total internal reflection.”

The core and cladding are usually made of ultra-pure glass. The fibre is coated with a protective plastic covering called the “primary buffer coating” that protects it from moisture and other damage. More protection is provided by the “cable” which has the fibres and strength members inside an outer covering called a “jacket”.

Multicom’s Fibre Optic Product Line and services also includes stocking and same day shipment of a large quantity and variety of custom-cut fibre optic cable (including loose tube, ADSS, Armored, etc), Corning fibre optics-based products and a wide selection of fibre. optic Transmitters, EDFAs, Receivers, Nodes, accessories, splitters, jumpers, pigtails, and media converters designed to meet the demanding requirements of data, video, and voice networks.



Single Mode Fibre Optic Cable :

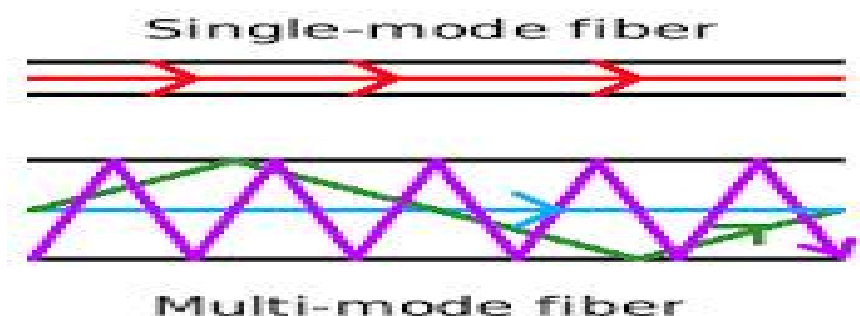
Single Mode fibre optic cable has a small diametral core that allows only one mode of light to propagate. Because of this, the number of light reflections created as the light passes through the core decreases, lowering attenuation and creating the ability for the signal to travel further. This application is typically used in long distance, higher bandwidth runs by Telcos, CATV companies, and Colleges and Universities.

Left: Single Mode fibre is usually 9/125 in construction. This means that the core to cladding diameter ratio is 9 microns to 125 microns.

Multimode Fibre Optic Cable :

Multimode fibre optic cable has a large diametral core that allows multiple modes of light to propagate. Because of this, the number of light reflections created as the light passes through the core increases, creating the ability for more data to pass through at a given time. Because of the high dispersion and attenuation rate with this type of fibre, the quality of the signal is reduced over long distances. This application is typically used for short distance, data and audio/video applications in LANs. RF broadband signals, such as what cable companies commonly use, cannot be transmitted over multimode fibre.

Above: Multimode fiber is usually 50/125 and 62.5/125 in construction. This means that the core to cladding diameter ratio is 50 microns to 125 microns and 62.5 microns to 125 microns. Due to its large core, some of the light rays that make up the digital pulse may travel a direct route, whereas others zigzag as they bounce off the cladding. These alternate paths cause the different groups of light rays, referred to as modes, to arrive separately at the receiving point. The pulse, an aggregate of different modes, begins to spread out, losing its well-defined shape. The need to leave spacing between pulses to prevent overlapping limits the amount of information that can be sent. This type of fibre is best suited for transmission over short distances.



NUMERICAL APERTURE :

Numerical Aperture (also termed Object-Side Aperture) is a value (often symbolized by the abbreviation NA) originally defined by Abbe for microscope objectives and condensers. It is given by the simple expression:

Numerical Aperture (NA) = $n \times \sin(\mu)$ or $n \times \sin(\theta)$ Numerical Aperture (NA) μ or θ

Note: Many authors use the variable μ to designate the one-half angular aperture while others employ the more common term θ , and in some instances, α .

In the numerical aperture equation, n represents the refractive index of the medium between the objective front lens and the specimen, and μ or θ is the one-half angular aperture of the objective. The numerical

aperture of a microscope objective is a measure of its ability to gather light and resolve fine specimen detail at a fixed object distance. Image-forming light waves pass through the specimen and enter the objective in an inverted cone as illustrated in Figure 1 (above). A longitudinal slice of this cone of light reveals the angular aperture, a value that is determined by the focal length of the objective.

Numerical aperture is a measure of the highly diffracted light rays captured by the objective. In practice, it is difficult to achieve numerical aperture values above 0.95 with dry objectives. Figure 1 illustrates a series of light cones derived from objectives of varying focal length and numerical aperture. As the light cones grow larger, the angular aperture (θ) increases from 7° to 60° , with a resulting increase in the numerical aperture from 0.12 to 0.87, nearing the limit when air is utilized as the imaging medium. Higher numerical apertures can be obtained by increasing the imaging medium refractive index (n) between the specimen and the objective front lens. Microscope objectives are now available that allow imaging in alternative media such as water (refractive index = 1.33), glycerin (refractive index = 1.47), and immersion oil (refractive index = 1.51). The numerical aperture of an objective is also dependent, to a certain degree, upon the amount of correction for optical aberration.

CONDITION OF CONTINUITY AT INTERFACE:

Maxwell's equations describe the behaviour of electromagnetic fields; electric field, electric displacement field, and the magnetic field. The differential forms of these equations require that there is always an open neighbourhood around the point to which they are applied, otherwise the vector fields E , D , B and H are not differentiable. In other words, the medium must be continuous.

On the interface of two different media with different values for electrical permittivity and magnetic permeability, that condition does not apply. However the interface conditions for the electromagnetic field vectors can be derived from the integral forms of Maxwell's equations.

PHASE SHIFT ON TOTAL REFLECTION :

The absolute, average, and differential phase shifts that p- and s-polarized light experience in total internal reflection (TIR) at the planar interface between two transparent media are considered as functions of the angle of incidence θ . Special angles at which quarter-wave phase shifts are achieved are determined as functions of the relative refractive index N . When the average phase shift equals $\pi/2$, the differential reflection phase shift ϕ is maximum, and the reflection Jones matrix assumes a simple form. For $N > 3$, the average and differential phase shifts are equal (hence $\phi = 3\pi/4$) at a certain angle θ that is determined as a function of N . All phase shifts rise with infinite slope at the critical angle. The limiting slope of the ϕ -versus- θ curve at grazing incidence ($\theta \rightarrow 90^\circ$) is $\phi' = -(2/N)(N^2 - 1)^{1/2} = -2 \cos \theta_c$, where θ_c is the critical angle and $(\theta^2 / 2) \rightarrow 90^\circ = 0$. Therefore ϕ is proportional to the grazing incidence angle $\theta \rightarrow 90^\circ$ (for small θ) with a slope that depends on N . The largest separation between the angle of maximum

and the critical angle is 9.88° and occurs when $N=1.55377$. Finally, several techniques are presented for determining the relative refractive index N by using TIR ellipsometry.

PLANER OPTICAL WAVE GUIDE:

An optical waveguide is a physical structure that guides electromagnetic waves in the optical spectrum. Common types of optical waveguides include optical fibre and rectangular waveguides.

Optical waveguides are used as components in integrated optical circuits or as the transmission medium in local and long haul optical communication systems.

Optical waveguides can be classified according to their geometry (planar, strip, or fibre waveguides), mode structure (single-mode, multi-mode), refractive index distribution (step or gradient index) and material (glass, polymer, semiconductor).

STEP AND GRANDED INDICES FIBRE :

Fibre optics, a graded index is an optical fibre whose core has a refractive index that decreases with increasing radial distance from the optical axis of the fibre.

Because parts of the core closer to the fibre axis have a higher refractive index than the parts near the cladding, light rays follow sinusoidal paths down the fibre. The most common refractive index profile for a graded-index fiber is very nearly parabolic. The parabolic profile results in continual refocusing of the rays in the core, and minimizes modal dispersion.

Multi-mode optical fibre can be built with either graded index or step index. The advantage of the multi-mode graded index compared to the multi-mode step index is the considerable decrease in modal dispersion. Modal dispersion can be further decreased by selecting a smaller core size (less than $5\text{-}10\mu\text{m}$) and forming a single mode step index fibre.

An optical fibre, a step-index profile is a refractive index profile characterized by a uniform refractive index within the core and a sharp decrease in refractive index at the core-cladding interface so that the cladding is of a lower refractive index. The step-index profile corresponds to a power-law index profile with the profile parameter approaching infinity. The step-index profile is used in most single-mode fibres and some multimode fibres.

A step-index fibre is characterized by the core and cladding refractive indices n_1 and n_2 and the core and cladding radii a and b . Examples of standard core and cladding diameters $2a/2b$ are $8/125$, $50/125$, $62.5/125$, $85/125$, or $100/140$ (units of μm). Step-index optical fibre is generally made by doping high-purity fused silica glass (SiO_2) with different concentrations of materials like titanium, germanium, or boron.

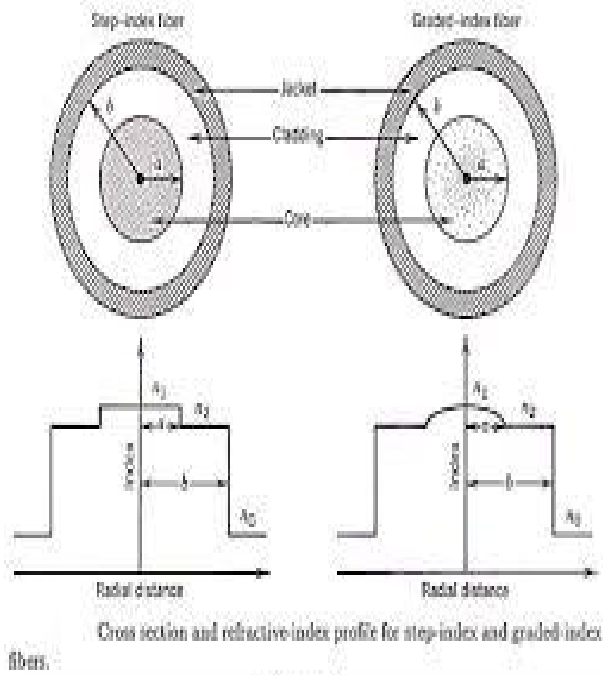


Figure 1.30

Part B

Possible 2 Marks

1. Write short notes on fibre optic communication.
2. Define Numerical aperture.
3. Define step indices.
4. Write short notes on single mode fiber.
5. Write about the condition of continuity.
6. What are the types of communications ?
7. Write short notes on multimode fibres.

Part C

Possible 6 marks

1. Explain briefly about plane optical wave guide
2. Define and explain the concept of single and multiple mode fibers.
3. Writ in detail about phase and group velocity of guide waves
4. Write short notes on numerical aperture and define step and graded indices
5. Explain briefly about planer dielectric wave guide.
6. What is mean by optical fibre? Write short notes on single and multiple mode fibres.
7. Write about the condition of continuity of interference and phase shift on total reflection.
8. Define optical fibres. Write in detail about step and graded indices.

Karpagam academy of higher education
Coimbatore -21
Department of Physics
II B.Sc Physics
Electromagnetic Theory [16PHU303]

The principle of fibre is same as that of _____	light	waves	particle	sound	light
Snells law states that refraction cannot take place when angle of incidence is not	large	small	same	unity	large
The core fibre is typically made of _____ doped with impurities	impurities	silica	glass	copper	silica
The _____ which surrounds the fibre core is made from pure silica and has lower refractive index than core	cladding	gratting	prism	silica	cladding
The first type of fibre optics put to use was called	step index fibre	gradded index fibre	bending	core	step index fibre
The change in index also has the effect of _____ the light back towards the centre of the core	bending	tagging	covariant	invareint	bending
The property of rotating the plane of vibration by certain crystals is known as _____	wave axis	particle axis	optic axis	optical activity	optical activity
The beams are treated with a quarter wave plate and a nicol prism, both are found to be _____ polarized	ellipitically	circurlarly	plane	optically	circurlarly
The multimode step index fibre has a core of _____ in diameter surrounded by cladding	50-200 μm	20-50 μm	50-200 cm	20-50 cm	50-200 μm
The light ray bends continuously and travel in a _____ pattern	helical	sperical	circular	ellipsoid	helical
Complex intrference pattern on screen are called	hologram	telegram	bibilogram	interference	hologram
Waveguides are used in _____ region.	MUF	VHF	OHF	OWF	OHF
Waveguides are used in _____ region.	microwave	ultra-violet	infra-red	radio-wave	microwave
Wave guide dispersion occurs only in fibers with a _____ mode	single	double	multi	tetra	single
The most important application of optical fibres in the field of _____	communication	transmission	modulation	propagation	communication

The source can be a _____ laser	semiconductor	conductor	signal	receiver	semiconductor
The fibre optical system is widely used in _____ services	defence	computer	signal	none	defence
The loss of optical fibre is measured in terms of the	decibel	ampere	hertz	intensity	decibel
_____ is used as a joint in connecting 2 fibres	fiber slices	fiber splices	connctetors	coupler	fibre splices
_____ is a passive devices	optical fiber coupler	fibre splices	connectors	jointer	optical fiber coupler
Fiber connectors are _____ joints	removable	fixed	connecting	jointer	removable
The loss of _____ in a fibre occurs because of mechanism	light	sound	energy	particle	light
_____ of light in a fibre is also wavelength dependent	scattering	dispersion	attenuation	absorbtion	scattering
Modal dispersion occurs fibres that have morethan mode	one	double	multi	none	one
When data is sent in fibre as it cointains pulses in the intensity of _____	light	sound	particle	wave	light
The variation in optical fibrre are known as	microwave	microbend	nanobend	millibend	microbend
Light is lost when it is first launched into any _____ along the fibre	splices	slices	couplers	fibres	splices
Scattering of light in a fibre is also wavelength	dependent	independent	greater	smaller	dependent
scattering loss is inversely proportional to the power of wavelength	fourth	third	second	zeroth	fourth
Light beams are generally used in _____	laser	optics	fiber	satellite	laser
The scattered wavefront are _____ with the centre	spherical	circular	plane	ellptically	spherical
When a phottograph is illuminated by laser light the object is _____	constructed	reconstructed	varient	covariant	reconstructed
The photographic record is called _____	hologram	intreferece pattern	telegram	wavefront	hologram
Light travels in a _____ line	straight	brnd	loop	ninety	straight
Total internal reflection is the theory for	optical fiber coupler	optical fibre	hologram	silica	optical fibre
Optical fibre is made of _____	glass	mica	silica	iron	glass

Only fundamental mode is used to transmit _____ in fibre	energy	particle	quanta	packets	energy
The size of the step index fibre are _____ μm	125	25	500	350	125
Marginal ray travels more distance than _____ ray	axial	coaxial	parabolic	helical	axial
The refractive index of fibre _____ for parabolic	increases	decreases	remains constant	zero	decreases

Reg No.....

(16PHU303) KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE-21

(Under Section 3 of UGC Act 1956) DEPARTMENT OF PHYSICS

II B.Sc PHYSICS Third Semester

I-Internal Examination (July 2017)

Electromagnetic Theory

Time: 2 hours

Maximum: 50 marks **PART-A (20x1=20 Marks) Answer all questions**

1. The time dependent electromagnetic field equation are called
 - a. Maxwell's equation
 - b. Ampere's law
 - c. Faraday's law
 - d. Gauss law
2. *div of curl* of any vector is
 - a. Zero
 - b. infinity
 - c. one
 - d. J.
3. The addition of $\epsilon_0 \mu_0$ to Ampere's law results in
 - a. light
 - b. time
 - c. position
 - d. momentum
4. Gauss law for magnetic field yields, $\text{div } \mathbf{B} =$
 - a. 0
 - b. 1
 - c. μ_0
 - d. ϵ_0
5. The unit of electric field intensity is
 - a. Volts/m
 - b. amp/m
 - c. weber/m
 - d. volts/m²
6. The equation of Poynting vector is
 - a. $\mathbf{S} = \mathbf{E} \times \mathbf{H}$
 - b. $\mathbf{S} = \mathbf{E}/\mathbf{H}$
 - c. $\mathbf{S} = \mathbf{E} + \mathbf{H}$
 - d. $\mathbf{S} = \text{curl}(\mathbf{E} \times \mathbf{H})$

7. The unit watt/m^2 is a unit of
a. Gauss law b. Ampere's circuital law c. Faraday's law
d. Poynting vector
8. $\text{div } \mathbf{B} = 0$, the field of vector \mathbf{B} is always _____
a. scleronomic b. rheonomic c. unilateral d. solenoidal
9. The field vectors are invariant to _____
a. gauge transformations b. Hertz potential c.
Maxwell's equation d. ampere's law
10. Gauge functions are solutions of _____ wave equations.
a. homogenous b. non homogenous c. Independent d. dependent
11. Electromagnetic waves are _____ in nature.
a. transverse b. longitudinal c. circular d. sperical
12. The vector \mathbf{E} and \mathbf{H} are mutually _____
a. perpendicular b. parallel c. variant d. invariant
13. The electromagnetic energy is transmitted in the direction of the wave propagation at speed of _____

14. Equation of motion in Poisson bracket from depends on
a. b. c. time d. all the three
15. The unit of capacitance is _____.

- a. farad b. coulomb c. volt d. henry
16. Gaussian surface is a _____
- a. Real surface b. Open surface c. Imaginary surface d. Smooth surface.
17. Which of the following is a ferromagnetic material
- a. Tungsten b. Aluminum c. Copper d. Nickel
18. The symbol of relative permittivity of a medium is _____
- a. ϵ_r b. 0 c. ϵ_0 d. μ
19. The path followed by a unit positive charge in an electric field called as
- a. The line of force b. coulombs forces c. Electric force d. Electromagnetic field
20. The point in a magnet where the intensity of magnetic lines of force is maximum
- a. Magnetic pole b. South pole c. North pole d. Unit pole

PART-A (20x1=20Marks) Answer all questions

Answer key's

- 1) maxwell's equation
- 2) zero
- 3) displacement current
- 4) 0
- 5) volts/m
- 6) $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

- 7) poynting vector
- 8) solenoidal
- 9) gauge transformations
- 10) non homogenous
- 11) transverse
- 12) perpendicular
- 13) light
- 14) all the three
- 15) farad
- 16) imaginary surface
- 17) nickel
- 18) $\epsilon_r \epsilon_0$
- 19) line of force
- 20) unit pole

PART-B (3x2=6 Marks) Answer all the questions

21. What is Displacement current?

$$\frac{\partial D}{\partial t}$$

– called the displacement current to distinguish it from J, the conduction current. By adding this term to ampere's law

Maxwell assumed that the time rate of change of displacement produces a magnetic field just as a conduction current

22. State poynting vector.

The amount of field energy passing through the unit area of the surface in a direction perpendicular to the plane containing E and H per unit time

$$S = E \times H$$

23. When E.M.W. propagate in free space?

- * The wave propagate with a speed equal to that of light in free space
- *the electromagnetic waves are transverse in nature
- * The wave vectors E and H are, mutually perpendicular
- *the wave vectors E and H are in phase

PART-C (3x8=24 Marks) Answer all the questions

24. a. Obtain maxwells's equation of electromagnetic field and discuss their empirical basis.

The four fundamental equations of electromagnetism correspond to a generalisation of certain experimental observations regarding electricity and magnetism. The following four laws of electricity and magnetism constitute the so-called differential form of Maxwell's equations.

1. Gauss law for the electric field of charge yields

$$\text{div } \mathbf{D} = \nabla \cdot \mathbf{D} = \rho \quad \text{----- (A)}$$

\mathbf{D} – electric displacement in coulombs / m²
 ρ – free charge density in coulombs / m³

2. Gauss law for magnetic field yields

$$\text{div } \mathbf{B} = \nabla \cdot \mathbf{B} = 0 \quad \text{----- (B)}$$

\mathbf{B} – magnetic induction in weber / m²

3. Ampere's Law in circuital form for the magnetic field accompanying a current when modified by Maxwell yields

$$\text{Curl } \mathbf{H} = \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{----- (C)}$$

\mathbf{H} – magnetic field intensity in amperes / m
 \mathbf{J} – current density amperes / m²

4. Faraday's law in circuital form for the induced electromotive force produced by the rate of change of magnetic flux linked with the path yields.

$$\text{Curl } \mathbf{E} = \nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t} \quad \text{----- (D)}$$

\mathbf{E} – electric field intensity in Volts / m

DERIVATIONS :

1. $\text{div } \mathbf{D} = \nabla \cdot \mathbf{D} =$

Let us consider a surface S bounding a volume τ with in a dielectric. The volume τ contains no net charge but we allow the dielectric to be polarised say by placing it in an electric field. Some charge on the dielectric body are placed. Thus we have two types charges

a) real charge of density

b) bound charge density ρ' , Gauss law then can be written as,

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} \int_{\tau} (\rho + \rho') d\tau$$

$$\epsilon_0 \oint_S \mathbf{E} \cdot d\mathbf{s} = \int_{\tau} \rho d\tau + \int_{\tau} \rho' d\tau \text{-----(1)}$$

But as the bound charge density ρ' is defined as

$$\rho' = -\text{div } \mathbf{P}$$

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = \int_{\tau} \text{div } \mathbf{E} d\tau$$

Equation (1) can be written as ,

$$\epsilon_0 \int_{\tau} \text{div } \mathbf{E} d\tau = \int_{\tau} \rho d\tau - \int_{\tau} \text{div } \mathbf{P} d\tau$$

$$\int_{\tau} \text{div } (\epsilon_0 \mathbf{E} + \mathbf{P}) d\tau = \int_{\tau} \rho d\tau \quad [\epsilon_0 \mathbf{E} + \mathbf{P} = \mathbf{D}]$$

$$\int_{\tau} (\text{div } \mathbf{D} - \rho) d\tau = 0$$

This equation is true for all volumes, the integration must vanish.

$$\text{div } \mathbf{D} = \nabla \cdot \mathbf{D} =$$

2. $\text{div } \mathbf{B} = \nabla \cdot \mathbf{B} = 0$

Experiments to data have shown that magnetic poles do not exist. This in turn implies that the magnetic lines of force are either closed group or go off to infinity. Hence the no of magnetic lines of force entering any arbitrary closed surface is exactly the same as leaving it.

The flux of magnetic induction \mathbf{B} across any closed surface is always zero.

$$\oint_s \mathbf{B} \cdot d\mathbf{s} = 0$$

Transforming this surface integral to volume integral by Guass theorem, we get,

$$\int_\tau \text{div } \mathbf{B} d\tau = 0$$

But as the surface bounding the volume is quite arbitrary the above equation will be true only when the integrated vanishes .

$$\text{div } \mathbf{B} = \nabla \cdot \mathbf{B} = 0$$

b. Obtain Poynting theorem for the conservation of energy in an electromagnetic field.

“ The rate of decrease of energy in the electrodynamic fields in a specific region is equal to the sum of rate of work done on charges and rate of escape of energy through the surface in the form of electromagnetic radiation.”

According to Lorentz law, the force acting in a electromagnetic field is given by

$$\vec{F} = [\vec{E} + (\vec{v} \times \vec{B})] \quad \text{----- (1)}$$

For an elementary volume $d\tau$, the force experienced in an electromagnetic field is given by

$$\vec{F} = \oint_v [\vec{E} + (\vec{v} \times \vec{B})] \rho d\tau \quad [q = \oint_v \rho d\tau]$$

The work done in causing a displacement $d\vec{l}$ in the electromagnetic field is given by

$$W = \oint_v [\vec{E} + (\vec{v} \times \vec{B})] \rho d\tau \cdot d\vec{l} \quad \text{----- (2)}$$

$$W = \oint_v [\vec{E} + (\vec{v} \times \vec{B})] \rho d\tau \cdot \vec{v} dt$$

Rate of work done in an electromagnetic field is given by

$$\frac{dW}{dt} = \oint_v [\vec{E} + (\vec{v} \times \vec{B})] \rho d\tau \cdot \vec{v} \quad \text{----- (3)}$$

Assuming the rate of work done in the electric field only, we get

$$\begin{aligned} \frac{dW}{dt} &= \oint_v \vec{E} \cdot \rho d\tau \cdot \vec{v} \quad \{ \vec{v} \times \vec{B} = 0 \} \\ &= \oint_v \vec{E} \cdot \rho \vec{v} d\tau \\ P &= \frac{dW}{dt} = \oint_v (\vec{E} \cdot \vec{J}) d\tau \quad \text{----- (4)} \end{aligned}$$

We know that the modified Ampere's law is applicable to electrodynamics.

$$\frac{\nabla \times \vec{B}}{\mu_0} - \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{J} \quad \text{----- (5)}$$

Putting the value of \vec{J} from equation (5) in equation (4), we get

$$\begin{aligned} \oint_v (\vec{E} \cdot \vec{J}) d\tau &= \oint_v \vec{E} \cdot \left[\frac{\nabla \times \vec{B}}{\mu_0} - \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right] d\tau \\ &= \oint_v \left[\vec{E} \cdot \frac{\nabla \times \vec{B}}{\mu_0} \right] d\tau - \oint_v \epsilon_0 \vec{E} \cdot \frac{d\vec{E}}{dt} d\tau \quad \text{----- (6)} \end{aligned}$$

We know that $\nabla \cdot [\vec{E} \times \frac{\vec{B}}{\mu_0}] = [\frac{\vec{B}}{\mu_0} \cdot (\nabla \times \vec{E})] - \vec{E} \cdot \frac{\nabla \times \vec{B}}{\mu_0}$

$$\begin{aligned} \text{Now } \oint_V \vec{E} \cdot \vec{j} \, d\tau &= \oint_V \frac{\vec{B}}{\mu_0} \cdot (\nabla \times \vec{E}) \, d\tau - \oint_V \nabla \cdot [\vec{E} \times \frac{\vec{B}}{\mu_0}] \, d\tau - \oint_V \epsilon_0 \vec{E} \cdot \frac{d\vec{E}}{dt} \, d\tau \\ &= \oint_V \frac{\vec{B}}{\mu_0} \cdot (\nabla \times \vec{E}) \, d\tau - \oint_V \nabla \cdot [\vec{E} \times \frac{\vec{B}}{\mu_0}] \, d\tau - \frac{\epsilon_0}{2} \oint_V \frac{dE^2}{dt} \, d\tau \end{aligned}$$

According to Maxwell's third equation in the differential form,

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\oint_V \vec{E} \cdot \vec{j} \, d\tau = \frac{1}{\mu_0} \oint_V [\vec{B} \cdot (\frac{\partial \vec{B}}{\partial t})] \, d\tau - \oint_V \nabla \cdot (\vec{E} \times \vec{H}) \, d\tau - \frac{\epsilon_0}{2} \oint_V \frac{dE^2}{dt} \, d\tau$$

$$\begin{aligned} \oint_V \vec{E} \cdot \vec{j} \, d\tau &= - \frac{1}{2\mu_0} \oint_V \frac{dB^2}{dt} \, d\tau - \frac{\epsilon_0}{2} \oint_V \frac{dE^2}{dt} \, d\tau - \oint_V \nabla \cdot (\vec{E} \times \vec{H}) \, d\tau \\ &= - \frac{\partial}{\partial t} \oint_V [\frac{B^2}{2\mu_0} + \frac{1}{2} \epsilon_0 E^2] \, d\tau - \oint_V \nabla \cdot (\vec{E} \times \vec{H}) \, d\tau \\ &= - \frac{\partial}{\partial t} \oint_V [\frac{B^2}{2\mu_0} + \frac{1}{2} \epsilon_0 E^2] \, d\tau - \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{a} \quad \text{----- (8)} \end{aligned}$$

$$- \frac{\partial}{\partial t} \oint_V [\frac{B^2}{2\mu_0} + \frac{1}{2} \epsilon_0 E^2] \, d\tau = \oint_V (\vec{E} \cdot \vec{j}) \, d\tau + \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{a} \quad \text{----- (9)}$$

$\vec{E} \times \vec{H}$ is called the pointing vector or power density. It is denoted by symbol S.

$$\vec{S} = \vec{E} \times \vec{H}$$

The unit of pointing vector is Watts/m². The pointing theorem,

$$- \frac{\partial}{\partial t} \oint_V [\frac{B^2}{2\mu_0} + \frac{1}{2} \epsilon_0 E^2] \, d\tau = \oint_V (\vec{E} \cdot \vec{j}) \, d\tau + \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{a} \quad \text{is called integral form.}$$

25. a. Discuss in detail about coulomb gauge.

An inspection of field equations in terms of electromagnetic potentials,

$$\nabla^2 A - \mu\epsilon \frac{\partial^2 A}{\partial t^2} - \text{grad} \left(\text{div} A + \mu\epsilon \frac{\partial \phi}{\partial t} \right) = -\mu J \quad \text{----- (1)}$$

$$\nabla^2 \phi - \mu\epsilon \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial t} \left(\text{div} A + \mu\epsilon \frac{\partial \phi}{\partial t} \right) = -\frac{\rho}{\epsilon}$$

$$\text{i.e.} \quad \nabla^2 \phi + \frac{\partial}{\partial t} (\text{div} A) = -\frac{\rho}{\epsilon} \quad \text{----- (2)}$$

Shows that if we assume ,

$$\text{div} A = 0 \quad \text{----- (3)}$$

equation (2) reduces to Poisson's equation

$$\nabla^2 \phi(r, t) = -\frac{\rho(r', t)}{\epsilon} \quad \text{----- (4)}$$

Whose solution is ,

$$\phi(r, t) = \frac{1}{4\pi\epsilon} \int \frac{\rho(r', t)}{R} d\tau' \quad \text{----- (5)}$$

i.e. the scalar potential is just the instantaneous Coulombian potential due to charge (x', y', z', t) . This is the origin of the name Coulomb gauge.

Equation (1) in the light of (3) reduced to

$$\nabla^2 A - \frac{1}{\epsilon} \frac{\partial^2 A}{\partial t^2} = -\mu J + \mu\epsilon \nabla \frac{\partial \phi}{\partial t} \quad \text{----- (6)}$$

Now to express equation (6) in more convenient way we use Poisson's

equation (4) which with the help of (5) can be written as

$$\nabla^2 \left\{ \frac{1}{4\pi\epsilon} \int \frac{\rho(r',t)}{R} d\tau' \right\} = - \frac{\rho}{\epsilon}(r',t) \quad \text{----- (7)}$$

Now as Poisson's equation holds good for scalar and vectors both, replacing

(r', t) by J' we get,

$$\nabla^2 \left\{ \frac{1}{4\pi\epsilon} \int \frac{J'}{R} d\tau' \right\} = - \frac{J'}{\epsilon} \quad \text{----- (8)}$$

Now from the vector identity

$$\nabla \times \nabla \times G = \nabla(\nabla \cdot G) - \nabla^2 G$$

$$\nabla^2 G = \nabla(\nabla \cdot G) - \nabla \times \nabla \times G$$

Taking $G = \int \left(\frac{J'}{R} \right) d\tau'$, we get

$$\nabla^2 \int \left(\frac{J'}{R} \right) d\tau' = \nabla(\nabla \cdot \int \frac{J'}{R} d\tau') - \nabla \times \nabla \times \int \frac{J'}{R} d\tau'$$

Which in the light of equation (8) reduces to

$$- 4\pi J' = \nabla(\nabla \cdot \int \frac{J'}{R} d\tau') - \nabla \times \nabla \times \int \frac{J'}{R} d\tau'$$

$$\text{i.e. } J' = - \frac{1}{4\pi} \nabla(\nabla \cdot \int \frac{J'}{R} d\tau') + \frac{1}{4\pi} \nabla \times \nabla \times \int \frac{J'}{R} d\tau' \quad \text{----- (9)}$$

Now as $\nabla \cdot \int \frac{J'}{R} d\tau'$

$$= \int \left[\frac{1}{R} \nabla \cdot J' + J' \cdot \nabla \left(\frac{1}{R} \right) \right] \quad \{ \text{as } \nabla(sv) = s\nabla \cdot v + v \cdot \nabla s \}$$

$$= \int J' \cdot \nabla \left(\frac{1}{R} \right) d\tau' \quad \{ \text{as } J' \text{ is not a function } x, y \text{ and } z \}$$

$$\begin{aligned}
 &= - \int J' \cdot \nabla' \frac{1}{R} d\tau' \left\{ \text{as } \nabla \left(\frac{1}{R} \right) = - \nabla' \left(\frac{1}{R} \right) \right\} \\
 &= \int \left[\nabla' \cdot \frac{J'}{R} - \nabla' \cdot \left(\frac{J'}{R} \right) \right] d\tau' \left\{ \text{as } \nabla' \cdot \frac{J'}{R} = \left(\frac{1}{R} \right) \nabla' \cdot J' + J' \cdot \nabla' \left(\frac{1}{R} \right) \right\} \\
 &= \int \nabla' \cdot \frac{J'}{R} d\tau' - \oint_s \left(\frac{J'}{R} \right) \cdot ds \left\{ \text{as } \int \nabla' \left(\frac{J'}{R} \right) d\tau' = \oint_s \left(\frac{J'}{R} \right) \cdot ds \right\}
 \end{aligned}$$

As J' is confined to the vol τ' , the surface contribution will vanish so

$$\nabla \cdot \int \left(\frac{J'}{R} \right) d\tau' = \nabla' \cdot \frac{J'}{R} d\tau' \text{ ----- (10)}$$

And $\nabla \times \int \left(\frac{J'}{R} \right) d\tau'$

$$= \int \left[\nabla \times \frac{J'}{R} - J' \times \nabla \left(\frac{1}{R} \right) \right] d\tau' \left\{ \text{as } \text{curl } SV = S \text{ curl } V - V \times \text{grad } S \right\}$$

grad S }

$$= - \int J' \times \nabla \left(\frac{1}{R} \right) d\tau' \left\{ \text{as } J' \text{ is not a function of } x, y, \text{ and } z \right\}$$

$$= \int J' \times \nabla' \left(\frac{1}{R} \right) d\tau' \left\{ \text{as } \nabla \left(\frac{1}{R} \right) = - \nabla' \left(\frac{1}{R} \right) \right\}$$

$$= \int \left[\nabla' \times \frac{J'}{R} - \nabla' \times \left(\frac{J'}{R} \right) \right] d\tau' \left\{ \right.$$

$$\text{as } \nabla \times (J'/R) = (1/R) \nabla' \times J' - J' \times \nabla' (1/R) \left. \right\}$$

$$= \int \nabla' \times \frac{J'}{R} d\tau' + \oint_s \frac{J'}{R} \times ds \left\{ \text{as } \int \nabla \times V d\tau' = - \oint_s V \times ds \right\}$$

As J' is confined to vol τ' , surface contribution will vanish so

$$\nabla \times \int \left(\frac{\mathbf{J}'}{R} \right) d\tau' = \int \nabla' \times \frac{\mathbf{J}'}{R} d\tau' \quad \text{----- (11)}$$

So equation (9) becomes

$$\mathbf{J}' = -\frac{1}{4\pi} \nabla \int \nabla' \cdot \frac{\mathbf{J}'}{R} d\tau' + \frac{1}{4\pi} \nabla \times \int \nabla' \times \frac{\mathbf{J}'}{R} d\tau'$$

$$\text{i.e. } \mathbf{J}' = \mathbf{J}'_1 + \mathbf{J}'_T \quad \text{----- (12)}$$

$$\text{With } \mathbf{J}'_1 = -\frac{1}{4\pi} \nabla \int \nabla' \cdot \frac{\mathbf{J}'}{R} d\tau' \text{ and } \mathbf{J}'_T = \frac{1}{4\pi} \nabla \times \int \nabla' \times \frac{\mathbf{J}'}{R} d\tau' \quad \text{-----}$$

----- (13)

Now as

$$\nabla \times \mathbf{J}'_1 = \nabla \times \left[-\frac{1}{4\pi} \nabla \int \nabla' \cdot \frac{\mathbf{J}'}{R} d\tau' \right]$$

$$\nabla \times \mathbf{J}'_1 = 0 \quad \{ \text{as curl grad} = 0 \} \quad \text{----- (14)}$$

$$\text{And } \nabla \cdot \mathbf{J}'_T = \nabla \cdot \left[\nabla \times \int \nabla' \times \frac{\mathbf{J}'}{R} d\tau' \right]$$

$$\nabla \cdot \mathbf{J}'_T = 0 \quad \{ \text{as div curl } \mathbf{V} = 0 \} \quad \text{----- (15)}$$

The first term on R.H.S of equation (12) is irrotational and second is solenoidal. The first term is called longitudinal current and the other transverse current.

So in the light of equation (12),(6) can be written as

$$\nabla^2 \mathbf{A} = -\mu (\mathbf{J}_1 + \mathbf{J}_T) + \mu \epsilon \nabla \frac{\partial \phi}{\partial t}$$

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}_T - \mu \mathbf{J}_1 + \mu \epsilon \nabla \frac{\partial}{\partial t} \left[\frac{1}{4\pi \epsilon} \int \frac{\rho(\mathbf{r}', t)}{R} d\tau' \right]$$

{ Substituting from $\nabla^2 A - \frac{1}{v^2} \frac{\partial^2 A}{\partial t^2} = -\mu J_T - \mu J_L + \mu \frac{1}{4\pi} \nabla \cdot \int -\frac{\nabla \cdot \mathbf{J}}{R} d\mathbf{r}'$ { as from continuity equation $\frac{\rho(r',t)}{R} = -\nabla \cdot \mathbf{J}$ }

Or $\nabla^2 A - \frac{1}{v^2} \frac{\partial^2 A}{\partial t^2} = -\mu J_T - \mu J_L + \mu J_L$ { from equation (13) }

$\nabla^2 A - \frac{1}{v^2} \frac{\partial^2 A}{\partial t^2} = -\mu J_T$

$\square^2 A = -\mu J_T$ ----- (16)

The equation for A can be expressed entirely in terms of the transverse current. So this gauge sometimes is also called as transverse gauge.

The Coulomb gauge has a entire advantage. In it the scalar potential is exactly the electrostatic potential and electric field,

$$\mathbf{E} = -\text{grad } \phi - \frac{\partial \mathbf{A}}{\partial t}$$

Is separable into an electrostatic field $\mathbf{V} = -\text{grad } \phi$ and a wave field given by $-\frac{\partial \mathbf{A}}{\partial t}$.

This gauge is often used when no sources are present. Then according to equation 5, $\rho = 0$ and A satisfies the homogeneous wave equation 16. The fields are given by,

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \text{ and } \mathbf{B} = \nabla \times \mathbf{A}$$

b. Establish the non uniqueness of electromagnetic potentials and concept of gauge.

In terms of electromagnetic potentials field vectors are given by ,

$$\mathbf{B} = \text{curl } \mathbf{A} \quad \text{----- (1)}$$

And
$$\mathbf{E} = - \text{grad } \left(- \frac{\partial \mathbf{A}}{\partial t} \right) \quad \text{----- (2)}$$

From equations (1) and (2) it is clear that for a given \mathbf{A} and $\frac{\partial \mathbf{A}}{\partial t}$, each of the field vectors \mathbf{B} and \mathbf{E} has only value i.e. \mathbf{A} and $\frac{\partial \mathbf{A}}{\partial t}$ determine \mathbf{B} and \mathbf{E} uniquely. However the converse is not true i.e. field vectors do not determine the potentials \mathbf{A} and $\frac{\partial \mathbf{A}}{\partial t}$ completely. This in turn implies that for a given \mathbf{A} and $\frac{\partial \mathbf{A}}{\partial t}$ there will be only one \mathbf{E} and \mathbf{B} while for a given \mathbf{E} and \mathbf{B} there can be infinite number of \mathbf{A}' S and $\frac{\partial \mathbf{A}'}{\partial t}$ S. This is because the curl of the gradient of any scalar vanishes identically and hence we may add to \mathbf{A} the gradient of a scalar Λ without affecting \mathbf{B} . That is \mathbf{A} may be replaced by,

$$\mathbf{A}' = \mathbf{A} + \text{grad } \Lambda \quad \text{----- (3)}$$

But if this is done equation (2) becomes,

$$\mathbf{E} = - \text{grad } \left(- \frac{\partial}{\partial t} (\mathbf{A}' - \text{grad } \Lambda) \right)$$

$$\mathbf{E} = - \text{grad } \left(- \frac{\partial \mathbf{A}'}{\partial t} \right) - \frac{\partial \Lambda}{\partial t}$$

So if we make the transformation given by equation (3). We must also replace $\frac{\partial \mathbf{A}}{\partial t}$ by

$$\frac{\partial \mathbf{A}'}{\partial t} = \frac{\partial \mathbf{A}}{\partial t} - \frac{\partial \Lambda}{\partial t} \quad \text{----- (4)}$$

The expressions for field vectors \mathbf{E} and \mathbf{B} remain unchanged under transformations equations (3) and (4).

$$\mathbf{B} = \text{curl } \mathbf{A} = \text{curl } (\mathbf{A}' - \text{grad } \Lambda) = \text{curl } \mathbf{A}'$$

And
$$\mathbf{E} = - \text{grad } \left(- \frac{\partial \mathbf{A}}{\partial t} \right) = - \text{grad } \left(- \frac{\partial \mathbf{A}'}{\partial t} + \frac{\partial \Lambda}{\partial t} \right) - \frac{\partial \Lambda}{\partial t} = - \text{grad } \left(- \frac{\partial \mathbf{A}'}{\partial t} \right) - \frac{\partial \Lambda}{\partial t} + \frac{\partial \Lambda}{\partial t} = - \text{grad } \left(- \frac{\partial \mathbf{A}'}{\partial t} \right)$$

$$\mathbf{E} = \text{grad } \phi - \frac{\partial \mathbf{A}}{\partial t}$$

We get the same field vectors whether we use the set (\mathbf{A}, ϕ) or (\mathbf{A}', ϕ') . So electromagnetic potentials define the field vectors uniquely though they themselves are non-unique.

The transformations given by equations (3) and (4) are called gauge transformations and the arbitrary scalar Λ gauge function. From the above it is also clear that even though we add the gradient of a scalar function, the field vectors remain unchanged. Now it is the field quantities and not the potentials that possess physical meaningfulness. We therefore say that the field vectors are invariant to gauge transformations i.e. they are gauge invariant.

26. a. Making use of Maxwell's field equations derive the equation for plane electromagnetic waves in free space.

Let's start with Maxwell's equations in derivative form for empty space.

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

where,

$$\mathbf{J} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{B} = \mu_0 \mathbf{H}$$

$$\mathbf{D} = \epsilon_0 \mathbf{E}$$

and for free space (i.e.) vacuum

$$=0$$

$$=0$$

$$\epsilon_r=1$$

$$\mu_r=1$$

and the Maxwell's equation reduces to,

$$\nabla \cdot E = 0$$

$$\nabla \cdot H = 0$$

$$\nabla \times H = \epsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$$

taking curl for third equation

$$\nabla \times (\nabla \times H) = \epsilon_0 \nabla \times \left(\frac{\partial E}{\partial t} \right)$$

$$\nabla \cdot (\nabla \cdot H) - \nabla^2 H = \epsilon_0 \frac{\partial}{\partial t} (\nabla \times E)$$

$$\text{since } \nabla \cdot H = 0 \text{ and } \nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$$

$$\nabla^2 H - \frac{1}{C^2} \frac{\partial^2 H}{\partial t^2} = 0$$

same is repeated for fourth equation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\mu_0 \frac{\partial H}{\partial t} \right)$$

solving,

$$\nabla^2 E - \frac{1}{C^2} \frac{\partial^2 E}{\partial t^2} = 0$$

these two equations satisfies the wave equation which is,

$$\nabla^2 \Psi - \frac{1}{v^2} \frac{\partial^2 \Psi}{\partial t^2} = 0$$

the last equation is a standard wave equation representing unattenuated wave travelling at a speed of light. so we conclude that field vector E and H are propagated in free space at a velocity equal to the speed of light.

PROPAGATION OF ELECTRO-MAGNETIC WAVES IN FREE SPACE:-

1. The wave propagates with a speed equal to that of light in free space.
2. The electromagnetic waves are transverse in nature.
3. The electromagnetic energy is transmitted in the direction of wave propagation at speed "C".
4. The wave vector E and H are mutually perpendicular to each other.
5. The vector E and H are in phase with each other.

b. Show that inside the conducting medium the wave is damped and obtain an expression for the skin depth.

considering the Maxwell's equations,

$$\nabla \cdot D = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

here,

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\mathbf{D} = \epsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

$$\sigma = 0$$

$$\rho = 0$$

Then,

$$\nabla \cdot \mathbf{D} = 0$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

taking curl for the third and fourth equation,

for third equation,

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla \times \left(\sigma \mathbf{E} + \frac{\partial \mathbf{H}}{\partial t} \right)$$

solving we get,

$$\nabla^2 H - \sigma\mu \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0$$

for fourth equation,

$$\nabla \times (\nabla \times E) = -\mu \nabla \times \left(\frac{\partial H}{\partial t} \right)$$

solving we get,

$$\nabla^2 E - \sigma\mu \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0$$

These two equations are called as the equation of telegraphy,

EQUATION OF TELEGRAPHY:-

$$\nabla^2 H - \sigma\mu \frac{\partial H}{\partial t} - \mu\varepsilon \frac{\partial^2 H}{\partial t^2} = 0$$

$$\nabla^2 E - \sigma\mu \frac{\partial E}{\partial t} - \mu\varepsilon \frac{\partial^2 E}{\partial t^2} = 0$$

IMPEDANCE IN PHASE:-

$$\left| \frac{E}{H} \right| = \frac{E_0}{H_0} = \sqrt{\left(\frac{\mu_r}{\epsilon_r} \right)} z_0 = z$$

SKIN DEPTH:-

thus we see that attenuation of wave will result in the medium and there will be no propagation in the medium. therefore

electromagnetic waves with frequency below the plasma frequency ω_p will be reflected at the plasma boundary in the plasma the field will fall off exponentially with distance from surface depth for plasma will be,

$$\delta_{plasma} = \frac{1}{\beta} = \frac{1}{\frac{\omega_p}{c}} = \frac{1}{\frac{\omega}{c} \sqrt{\left[\left(\frac{\omega_p}{\omega} \right)^2 - 1 \right]}}$$

$$\delta_{plasma} = \frac{c}{\sqrt{\omega_p^2 - \omega^2}} \propto \frac{c}{\omega_p} \text{ when } \omega \ll \omega_p$$

the field from within a plasma is a well know effect in process and is exploited in attempts at hot plasma.

Reg No.....
(16PHU303)

KARPAGAM ACADEMY OF HIGHER EDUCATION
COIMBATORE-21
DEPARTMENT OF PHYSICS
II B.Sc PHYSICS
Third Semester
II-Internal Examination
Electromagnetic Theory

Time:2 hours

Maximum:50 marks

PART-A(20x1=20Marks)

Answer all questions

1. The velocity of electromagnetic waves in free space is _____.
a) 30×10^8 m/s b) 356×10^8 m/s c) 330 m/s d) 3×10^8 m/s
2. The wave enters some different medium, there must be a new distribution of _____.
a. Velocity b. Time c. Energy d. wave
3. Light waves consists of _____.
a) protons b) electrons c) photons d) neutrons
4. The reflection of EMW by metallic boundaries is subjected to _____.
a. wave guides b. waves c. unilateral d. None of these
5. The wave gets attenuated with _____

a) penetration b) transmission c) absorption d) refraction

6. Dielectric polarization is proportional to _____.

a) applied electric field b) applied magnetic field c) applied electromagnetic field d) applied electrostatic field

7. When electromagnetic waves crosses a boundary surface, then the normal component of the electric displacement is _____ by an amount equal to the free density of charge.

a) equals b) continuous c) proportional d) discontinuous

8. In plane electromagnetic wave, the wave vectors D, H and K are _____.

a) parallel b) rotational c) irrotational d) orthogonal

9. The field vectors are spatially _____

a) attenuated b) unattenuated c) rotated d) conducted

10. Electric and magnetic phenomena are asymmetry arises due to the non-existence of _____.

a) Dipoles b) electric field c) magnetic field d) monopoles

11. The unit watt/m^2 is a unit of

a) Gauss law b) Ampere's circuital law c) Faraday's law d) Poynting vector

12. The unit of capacitance is _____.

a) farad b) coulomb c) volt d) Henry

13. When an _____ wave is traveling through a space there is an exact balance between the electric and magnetic fields.
 a) convergent b) divergent c) electric d) magnetic
14. The frequency of the waves remains unchanged by _____
 a. polarization b. diffraction c. absorption d. refraction
15. Dynamic properties are concerned with _____
 a. position b. momentum c. time d. intensity
16. In case of reflection the angle of reflection is equal to angle of _____
 a. incidence b. diffraction c. absorption d. refraction
17. The ratio of refractive index of two media is _____
 a. $n_1 \sin i = n_2 \sin T$ b. $\sin i / \sin T$ c. n_2 / n_1 d. refraction
18. The transmitted wave to incident on the boundary between two dielectrics are called _____ formulae
 a). Snell's b) Lens c) Fresnel d) Rayleigh
19. Angle of incidence holds good in all _____
 a) statistical b) dielectrics c) dynamic d) electronics
20. _____ is sometimes called as the polarization angle
 a) Brewster angle b) right angle c) Snell's law d) incidence angle

PART-B (2x3=6 Marks)

Answer all the questions

21. Define the law of reflection.
22. What are wave guides ?

23. State snells law.

PART-C (3x8=24 Marks)

Answer all the questions

24. a. Making use of the Maxwell's field equation derives the equation for plane e.m.waves in ionosphere.

OR

b. Discuss the propagation of electromagnetic waves in a ionized gases.

25. a. Discuss about the reflection and refraction of EMW.

OR

b. Discuss about the reflection from a metallic surface

26. a . Obtain the laws of reflection and refraction and write about the dynamic properties.

OR

b. Discuss about the fresnel formulae for perpendicular polarization condition.

Answer keys

PART-A(20x1=20Marks)

1) $3 \times 10^8 \text{ m/s}$

2) Velocity

3) Photons

4) Wave guides

5) Penetration

6) Applied electric field

7) Discontinuous

8) Orthogonal

9) Attenuated

10) Monopoles

11) Pointing vector

12) Farad

13) Convergent

14) Absorption

15) Time

16) Incidence

17) $n_i \sin \theta_i = n_t \sin \theta_t$

18) Snell's

19) Dynamic

20) Brewster's angle

PART-B (2x3=6 Marks)

Answer all the questions

21. Define the law of reflection.

Laws of reflection states that the angle of reflection is equal to the angle of incidence and that the incident ray, normal ray and the reflected ray all lie in the same plane incidence

22. What are wave guides ?

Wave guide is an electromagnetic feed line used in microwave communication, broadcasting, and radar installations

23. State Snell's law.

In case of reflection the ratio of sin of angle of refraction to the sin of angle of incident is equal to the ratio of the refractive indices of the two media i.e.

$$n_1 \sin \theta_i = n_2 \sin \theta_i$$

PART-C (3x8=24 Marks)

Answer all the questions

24 a. Making use of the Maxwell's field equation derives the equation for plane e.m.waves in ionosphere.

In certain situations such as the ionosphere or a tenuous plasma there is so little air that the electrons may vibrate without colliding with the molecules. so the force on a charged particle is an electromagnetic field, neglecting the earth's magnetic field will be

$$\mathbf{F} = e[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

now as in a plane wave

$$\mathbf{B} = \frac{\mathbf{n} \times \mathbf{E}}{c}$$

$$|\mathbf{v} \times \mathbf{B}| = vB = \frac{v}{c} E$$

and also,

$$\mathbf{E} = E_0 e^{-i(\omega t - k \cdot r)}$$

$$\mathbf{E} = E_0 e^{-i(\omega t - (2\pi/\lambda) \cdot \mathbf{n} \cdot \mathbf{r})}$$

$$\mathbf{E} = E_0 e^{-i(\omega t)}$$

so equation reduces to,

$$\mathbf{F} = eE_0 e^{-i\omega t}$$

$$m \frac{d^2 \mathbf{r}}{dt^2} = eE_0 e^{-i\omega t}$$

$$\frac{d^2 \mathbf{r}}{dt^2} = \frac{e}{m} E_0 e^{-i\omega t}$$

$$\frac{dr}{dt} = \frac{eE_0 e^{-i\omega t}}{m(-i\omega)}$$

$$v = i \frac{e}{m\omega} E$$

now if there are N electrons per unit volume then as

$$J = Nev$$

substituting the value of v from equation we get,

$$J = i \frac{Ne^2}{m\omega} E$$

$$J = \sigma E$$

we find that the conductivity is purely imaginary,

$$\sigma = i \frac{Ne^2}{m\omega}$$

various shortcuts are possible in deriving equations of wave propagation in an ionized medium but it seems worthwhile to go all the way both Maxwell's equation.

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

which for the present situation reduces to

$$\nabla \cdot E = 0$$

$$\nabla \cdot H = 0$$

$$\nabla \times H = \sigma E + \varepsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$$

in case of ionized medium $\rho = 0, \epsilon_r = 1$ and $\mu_r = 1$

now taking curl for fourth equation,

$$\nabla \times (\nabla \times E) = -\mu_0 \frac{\partial}{\partial t} (\nabla \times H)$$

solving this we get,

$$\nabla^2 E - \sigma \mu_0 \frac{\partial E}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$

similarly taking curl for third equation,

$$\nabla^2 H - \sigma \mu_0 \frac{\partial H}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2} = 0$$

the solution of these two equations be,

$$\Psi = \Psi_0 e^{-i(\omega t - kr)}$$

then,

$$\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} E_0 \\ H_0 \end{pmatrix} e^{-i(\omega t - kr)}$$

so that field equation reduces,

$$\nabla (K^2 - i\mu_0 \omega \sigma - \mu_0 \epsilon_0 \omega^2) \begin{pmatrix} E \\ H \end{pmatrix} = 0$$

as vector E or H is not zero,

$$K^2 = \mu_0 \epsilon_0 \omega^2 \left[1 + \frac{i\sigma}{\epsilon_0 \omega} \right]$$

$$K^2 = \frac{\omega^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega^2} \right]$$

$$m = \frac{c}{v} = \frac{c}{\omega/k} = \frac{kc}{\omega}$$

so the index of refraction in this case will be given by,

$$n = \sqrt{\left(1 - \frac{\omega_p^2}{\omega^2}\right)}$$

from this equation it is clear that for frequencies $\omega^2 > \omega_p^2$

in region of vanishing small ionization and high frequency range index of refraction is real and so waves propagate freely as in dielectric, however if the plasma frequency increases with distance, the index of refractive will decreases according. this is turn means that the beam will bends in a direction away from the normal as it moving from a region of higher index of refraction to that of lower index of refraction. this bending of high frequency or short wavelength electromagnetic wave by earth's ionosphere is used in long distance radio transmission .

in the limit $\omega^2 \gg \omega_p^2$ as $n \rightarrow 1 = \text{constant}$, the transmission is unaffected by the presences of ionosphere this is why the radar signals that have been received after reflection from the moon had to be rather higher frequency waves to pass through the ionized part of earth's atmosphere.

for frequency $\omega^2 < \omega_p^2$ in heavily ionized region and for low frequencies ranges the index of refraction is purely imaginary. so if we write $n \rightarrow in$ then from equation

$$k = \frac{\omega}{c}(in) = i \frac{\omega}{c} \sqrt{\left(\frac{\omega_p^2}{\omega^2} - 1\right)}$$

so that

$$\begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} E_0 \\ H_0 \end{pmatrix} e^{-\beta(n.r)} e^{-i(\omega.t)}$$

With $\beta = \frac{\omega n}{c}$

b. Discuss the propagation of electromagnetic waves in a ionized gases.

consider an electromagnetic wave propagating in free space along z-direction. Then the E and H vary only in z-direction.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \text{ and } \frac{\partial}{\partial z} \neq 0$$

such wave is called as planar wave, since it's vector are functions of (z, t) only then we write,

$$E = E(z, t)$$

$$H = H(z, t)$$

from first Maxwell's equation,

$$\nabla \cdot D = 0 \text{ or } \epsilon_0 (\nabla \cdot E) = 0$$

$$\nabla \cdot E = 0 \text{ or } \frac{\partial E_z}{\partial z} = 0$$

$$E_z = \text{constant in time}$$

from second Maxwell's equation,

$$\nabla \cdot B = 0 \text{ or } \mu_0 (\nabla \cdot H) = 0$$

$$\nabla \cdot H = 0 \text{ or } \frac{\partial H_z}{\partial z} = 0$$

$$H_z = \text{constant in time}$$

from third Maxwell's equation,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$(\nabla \times E)_z = -\mu_0 \frac{\partial H_z}{\partial t}$$

$$K \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) = -\mu_0 \frac{\partial H_z}{\partial t}$$

$$\frac{\partial H_z}{\partial t} = 0$$

$$H_z = \text{constant in time}$$

similarly by using fourth equation,

$$E_z = \text{constant in time}$$

Thus we have concluded that E_z and H_z are constant as regards for time and space. they represent the static components and consequently, no part of wave motion. we can therefore write,

$$E_z = H_z = 0$$

$$\mathbf{E} = iE_x + jE_y$$

$$\mathbf{H} = iH_x + jH_y$$

the electric E and magnetic H vector don't have any Z-component, the Z-direction being the direction of propagation , both these vectors are perpendicular to the direction of propagation ,Maxwell's electromagnetic waves are purely transverse in nature

25 a. Discuss about the reflection and refraction of EMW.

when plane electromagnetic waves which are travelling in one medium are incident upon an infinite plane surface separation this medium from another, with different electromagnetic properties. When an electric waves is travelling through space there is an exact balance between the electric and magnetic field. Half of the energy of wave as a matter of fact is in electric field and half in the magnetic. If the wave enters some different medium, there must be a new distribution of energy whether the new medium is a dielectric a magnetic a conducting or an ionised region, there will have to be a readjustment of energy relation as the wave reaches

its surface. Since no energy can be added to the wave as it only way that a new balance can be achieved is for some of the incident energy to be reflected.

This is what actually happens, the transmitted energy constitutes the refracted wave and the reflected one the reflected wave. The reflection and refraction of light at a plane surface between two media of different dielectric properties is a familiar, example of reflection and refraction of electromagnetic waves. The various aspects of the phenomenon divide themselves into two classes.

Kinematic properties :

Following are the kinematic properties of reflection and refraction.

(i) Law of frequency :

The frequency of the wave remains unchanged by reflection or refraction.

(ii) The reflected and refracted waves are in the same plane as the incident wave and the normal to the boundary surface.

(iii) Law of reflection :

In case of reflection the angle of reflection is equal to the angel of incident.

$$\theta_i = \theta_r$$

(IV) Snell's Law :

In case of reflection the ration of the sin of the angle of refraction to the sin of angle of incident is equal to the ratio of the refractive indices of two media.

$$n_1 \sin \theta_i = n_2 \sin \theta_r$$

b. Discuss about the reflection from a metallic surface.

We now consider the case in which the boundary surface separates a dielectric from a conducting medium, for simplicity we shall treat only the case of normal incidence here. The boundary condition for the continuity of the tangential components of electric and magnetic vectors E and H for the situation depicted in figure.

$$E_i - E_R = E_T \quad \text{----- (1)}$$

And $H_i + H_R = H_T \quad \text{----- (2)}$

But as $H_i = \frac{n}{\mu_0} E_i$ & $H_R = \frac{n}{\mu_0} E_R$ as

$$\mu_r \simeq 1$$

Equation (2) reduces to

$$n_1 (E_i + E_R) = n_2 E_T \quad \text{----- (3)}$$

So substituting the value of E_T from (3) in (1) we get,

$$(E_i - E_R) = \frac{n_1}{n_2} (E_i + E_R)$$

$$\frac{E_R}{E_i} = \frac{n_2 - n_1}{n_2 + n_1} \quad \text{----- (4)}$$

And substituting the value of E_R from

Sub equation (3) in equation (1) we get,

$$E_i = \left[\frac{n_2}{n_1} E_R - E_i \right] + E_T$$

$$\left[\frac{E_T}{E_i} \right] = \frac{2n_1}{n_2 + n_1} \quad \text{----- (5)}$$

Now as index of refraction is related to propagation vector by the relation

$$K = n \frac{\omega}{c} \quad \text{i.e. } n = \frac{c}{\omega}$$

Interaction of EMW with matter on microscopic scale and as in case of good conductor.

$$K \rightarrow k^* = \alpha + i\beta \quad \text{with } \alpha \simeq \beta = \frac{1}{\delta} = \sqrt{\frac{\omega \mu_0 \sigma}{2}}$$

$$n_2 \rightarrow n^* = \frac{c}{\omega} (K^*) = \frac{c}{\omega} (\alpha + i\beta)$$

$$n_2 \rightarrow n^* = \left[\frac{c}{\omega} \alpha \right] + i \left[\frac{c}{\omega} \beta \right]$$

$$n_2 \rightarrow n^* = n + i\alpha \quad \text{with} \quad n = k = \sqrt{\frac{\alpha}{2\omega\epsilon_0}} \quad \text{----- (6)}$$

So equation (4) and (5) reduce

$$\left(\frac{ER}{Ei}\right) = \frac{(n+ik)-n_1}{n+ik+n_1} = \frac{n-n_1+ik}{n+n_1+ik} \quad \text{----- (7)}$$

$$\text{And} \quad \left(\frac{ET}{Ei}\right) = \frac{2n_1}{n+ik+n_1} = \frac{2n_1}{(n+n_1)+ik} \quad \text{----- (8)}$$

Now if, $(n - n_1) + ik = a e^{i\theta_1}$

$(n - n_1) + ik = b e^{i\theta_2}$

$$a = [\sqrt{(n - n_1)^2 + k^2}] \quad ; \quad \theta_1 = \tan^{-1} \frac{k}{n - n_1} \quad \text{----- (9)}$$

$$b = [\sqrt{(n - n_1)^2 + k^2}] \quad ; \quad \theta_2 = \tan^{-1} \frac{k}{n + n_1} \quad \text{----- (10)}$$

So equation (7) and (8) reduce to ,

$$\left[\frac{ER}{Ei}\right] = a e^{i\theta_1} / b e^{i\theta_2} = [(n - n_1)^2 + k^2 / (n + n_1)^2 + k^2]^{1/2} e^{i(\theta_2 - \theta_1)} \quad \text{----- (11)}$$

$$\left[\frac{ET}{Ei}\right] = 2n_1 / b e^{i\theta_2} = 2n_1 / [(n + n_1)^2 + k^2] e^{i\theta_2} \quad \text{----- (12)}$$

Equation (11) and (12) are our final equation representing the reflected and transmitted waves r_1 and r_2 are given by equation (9) and (10) while n and k by equation (6). From these it is clear that both reflected and transmitted waves exist and they are not in phase with the incident wave. Further the amplitude of the transmitted wave is very small due to large values of n and k in the denominator, waves which are most strongly absorbed are very strongly reflected. i.e. All good conductors are good absorbers and good reflectors. Light are complementary. A good example is offered by the optical properties of thin sheets of gold. They appears yellowish by reflection. This mean that of the white light is incident on thin gold foils. Then the transmitted light will be devoid of yellow component. As a result the transmitted light appears greenish or bluish.

26 a .Obtain the laws of reflection and refraction and write about the dynamic properties.

(i) Law of frequency :

The frequency of the wave remains unchanged by reflection or refraction.

(ii) The reflected and refracted waves are in the same plane as the incident wave and the normal to the boundary surface.

(iii) Law of reflection :

In case of reflection the angle of reflection is equal to the angel of incident.

$$\theta_i = \theta_r$$

(IV) Snell's Law :

In case of reflection the ration of the sin of the angle of refraction to the sin of angle of incident is equal to the ratio of the refractive indices of two media.

$$n_1 \sin \theta_i = n_2 \sin \theta_r$$

Dynamic properties :

These properties are concerned with the

(i) intensities of reflected and refracted waves

(ii) Phase changes and polarisation of waves

The kinematic properties follow immediately from the wave nature of phenomenon and the fact that these are boundary condition to be satisfied. But they do not depends on the nature of the wave or the boundary conditions.

26 b. Discuss about the fresnel formulae for perpendicular polarization condition.

The formulae relating the amplitude of the reflected and transmitted electromagnetic waves with that of incident one when the boundary is between two dielectrics are called Fresnel formulae. These are contained in the boundary condition.

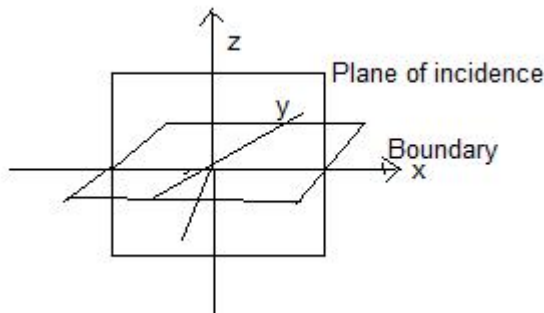
$$(D_i)_n + (D_R)_n = (D_T)_n \quad \text{----- (1)}$$

$$(B_i)_n + (B_R)_n = (B_T)_n \quad \text{----- (2)}$$

$$(E_i)_t + (E_R)_t = (E_T)_t \quad \text{----- (3)}$$

$$(H_i)_t + (H_R)_t = (H_T)_t \quad \text{----- (4)}$$

The condition (1) and (2) when coupled with Snell's law yield no information not included in the (3) and (4) conditions. So it is necessary to consider only condition (3) and (4). Now to derive the desired formulae we consider a plane EMW in x-z plane incident on a plane boundary and consider it as a superposition of two waves one with the electric vector perpendicular to the plane of incidence. Therefore it is sufficient to consider these two cases separately. The general result may be obtained from the appropriate linear combination of the two cases.



The situation is shown the magnetic field vectors and the propagation vectors are indicated. The electric vectors all directed into the plane of the figure.

Since the electric vectors are all parallel to the boundary surface,

$$(E_i)_t = E_i$$

$$(E_R)_t = E_R$$

$$(E_T)_t = E_T$$

And

$$(H_i)_t = -H_I \cos \theta_i$$

$$(H_R)_t = H_R \cos \theta_p$$

$$(H_T)_t = -H_T \cos \theta_T$$

So boundary condition (3) and (4) reduce to

$$E_I - E_R = E_T$$

$$H_i \cos \theta_i - H_R \cos \theta_R = H_T \cos \theta_T \quad \text{----- (9)}$$

$$i = R \text{ and } H = (E/Z) = (n / Z_0) \quad \text{----- (10)}$$

So equation (10) reduce to

$$n_1 E_I \cos \theta_i - n_2 E_R \cos \theta_R = n_2 E_T \cos \theta_T \quad \text{----- (11)}$$

Now eliminating E_R from equation (11) with help of (9) we get,

$$(E_i - E_R) n_1 \cos \theta_i = n_2 \cos \theta_T (E_i + E_R)$$

$$\left(\frac{E_R}{E_I} \right)^+ = \frac{\cos \theta_i - \frac{n_2}{n_1} \cos \theta_T}{\cos \theta_i + \frac{n_2}{n_1} \cos \theta_T}$$

$$\left(\frac{E_R}{E_I} \right)^+ = \frac{\cos \theta_i - \frac{\sin \theta_i}{\sin \theta_T} \cos \theta_T}{\cos \theta_i + \frac{\sin \theta_i}{\sin \theta_T} \cos \theta_T}$$

$$\left(\frac{E_R}{E_I} \right)^+ = \frac{\sin \theta_T \cos \theta_i - \cos \theta_T \sin \theta_i}{\sin \theta_T \cos \theta_i + \cos \theta_T \sin \theta_i}$$

$$\left(\frac{E_R}{E_I} \right)^+ = \frac{\sin(\theta_i - \theta_T)}{\sin(\theta_i + \theta_T)} \quad \text{----- (C)}$$

Similarly eliminating E_R from equation (11) with the help of (9) we get,

$$n_1 E_I \cos \theta_i - n_1 (E_T - E_I) \cos \theta_i = n_2 E_T \cos \theta_T$$

$$\left(\frac{E}{E_i}\right)^{\perp} = \frac{2 \cos \theta_i}{\cos \theta_i + \frac{n_2}{n_1} \cos \theta_T}$$

$$\left(\frac{E}{E_i}\right)^{\perp} = \frac{2 \cos \theta_i}{\cos \theta_i + \frac{\sin \theta_i}{\sin \theta_T} \cos \theta_T}$$

$$\left(\frac{E}{E_i}\right)^{\perp} = \frac{2 \cos \theta_i \sin \theta_T}{\cos \theta_i \sin \theta_T + \sin \theta_i \cos \theta_T}$$

$$\left(\frac{E}{E_i}\right)^{\perp} = \frac{2 \cos \theta_i \sin \theta_T}{\sin(\theta_i + \theta_T)} \quad \text{----- (D)}$$

Equation (A), (B), (C), (D) are the desired result known as Fresnel formulae.

Reg No.....

(16PHU303)

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE-21

DEPARTMENT OF PHYSICS

II B.Sc PHYSICS

Third Semester

III-Internal Examination

Electromagnetic Theory

Time:2 hours

Maximum:50 marks

PART-A(20x1=20Marks)

Answer all questions

1. Polarized light is obtained when ordinary light is reflected by a plane sheet of _____
a)glass b)tourmaline c)mica d)Erasmus Bartholinus
2. calcite is also known as _____
a)ice b)land c)calcium d)Iceland
3. Canada balsam acts as _____ medium for ordinary ray
a) rarer b) denser c) thinner d) thicker
4. A half wave plate rotates the azimuth of a beam of plane polarized light by _____

Dr.A.Saranya

Karpagam Academy of Higher Education

Page

1 of 18

Department of Physics

2017-2018 Odd

a) 45° b) 90° c) 75° d) 180°

5. The light waves are _____

a) Circular b) Longitudinal c) Elliptical d) Transverse

2017-2018 Odd

6. Its experimentally proved that light waves are _____ in nature
a) Circular b) Longitudinal c) Elliptical d) Transverse
7. When light is passed through a _____ crystal, the light is polarized to only one direction
a) Quartz b) Diamond c) Ruby d) Tourmaline
8. The plane of polarization is that plane in which no _____ occur
a) Disturbance b) Attenuation c) Polarization d) Vibration
9. The plane of vibration occurs at _____ angle
a) Acute b) Obtuse c) Straight d) Right
10. Light waves consists of
a) Protons b) Electrons c) Photons d) Neutrons
11. White light is incident on the interface of glass and air medium. If green light is just totally internally reflected, then the emerging ray in air contains
a) Yellow, Orange, Red b) Violet, Indigo, Blue c) All colours d) All colours except Green
12. The principle of optical fiber its same as that of _____
a) Light b) Wave c) Sound d) Energy.
13. Polaroids are widely used as polarizing _____
a) sunglasses b) Wave c) windows d) crystal
14. The core fibre is typically made of _____ doped with impurities.
a) silica b) copper c) glass d) crystal

2017-2018 Odd

15. The first type of fibre put in use is _____ .

a)step index fibre b)graded index c)single mode d) multi mode

16. The current flows through the surface and the effect is called _____

2017-2018 Odd

a) strong skin effect b) sound effect c) light effect d) wave effect

17. Light waves consists of _____

a) protons b) electrons c) photons d) neutrons

18. White light is incident on the interface of glass and air medium. If green light is just totally internally reflected, then the emerging ray in air contains _____

a) yellow, orange, red b) violet, indigo, blue c) all colours d) all colours except green

19. Wave guides are used in _____

a) microwave b) infra red c) ultra violet d) radiowave

20. The refractive index _____ parabolic

a) decreases b) increases c) constant d) 180°

Answer Key

1) Glass

2) Iceland

3) Rarer

4) 90°

5) Transverse

2017-2018 Odd

- 6) Transverse
- 7) Tourmaline
- 8) Vibration
- 9) Right
- 10) Photons
- 11) Yellow, orange, red,
- 12) Light
- 13) Sunglasses
- 14) Silica
- 15) Step index fibre
- 16) Strong skin effect
- 17) Photons
- 18) Yellow, orange , red
- 19) Microwave
- 20) Increases

PART-B (2x3=6 Marks)

2017-2018 Odd

Answer all the questions

21. Define optic axis.

The phenomena of double refraction is absent when the light is allowed to enter the crystal and this is called as optic axis

22. What are the types of polarization ?

- i) Plane polarisation
- ii) Elliptically polarisation
- iii) Circularly polarisation

23. Define step indices fibre.

A multi-mode or single mode optical fibre with a uniform refractive index throughout the core. The step is the shift between core and the cladding, which has the lower refractive index

PART-C (3x8=24 Marks)

Answer all the questions

24. a) What is meant by 1) plane polarized light 2) circularly polarized light 3) elliptically polarized light. How they are produced.

Production of plane polarized light:

A beam of monochromatic light is passed through a nicol prism. While passing through the nicol prism, the beam is split up into extraordinary ray and ordinary ray. The ordinary ray is totally internally reflected back at the Canada balsam layer, while the

extra ordinary ray passes through the nicol prism. The emergent beam is plane polarized

Production of circularly polarized light:

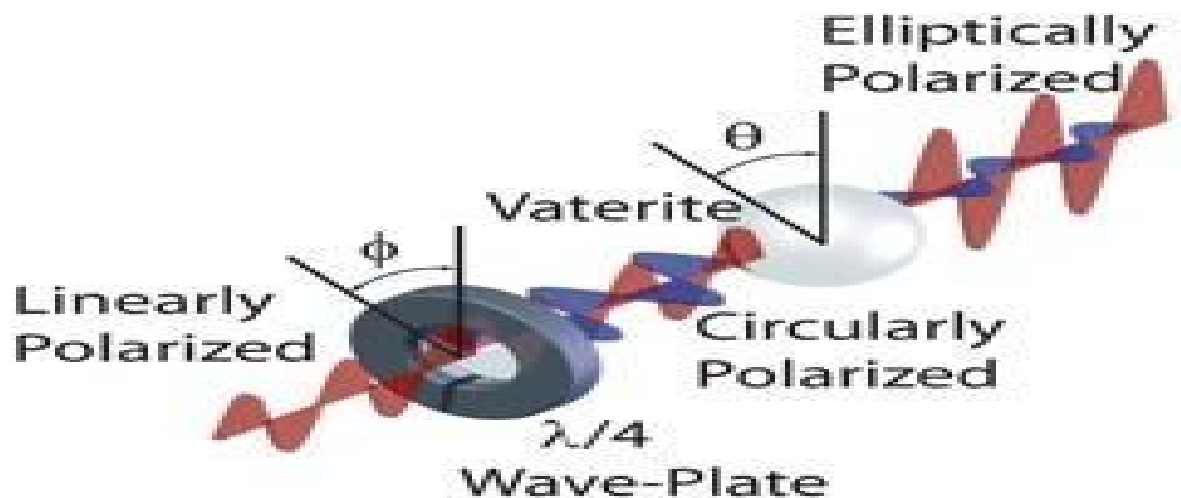
To produce circularly polarized light, the two waves vibrating at right angle to each other having the same amplitude and time period should have a phase difference of $\pi/2$ (or) a path difference of $\lambda/4$. For this purpose a parallel beam of monochromatic light is allowed to fall on a nicol prism N_1 . The beam after passing through the prism N_1 is plane polarized.

The nicol prism N_2 is placed at some distance from N_1 so that N_1 and N_2 are crossed. The field of view will be dark as viewed by the eye in this position. A quarter waveplate P is mounted on a tube A . The tube A can rotate about the outer fixed tube B introduced between the nicol prism N_1 and N_2 . The plane polarised light from N_1 falls normally on P and the field of view may be bright. The quarter waveplate is rotated until the field of view may be dark keeping P fixed, A is rotated such that the mark S on P coincides with 0 mark on A . Afterwards, By rotating the quarter waveplate P , the mark S is made to coincide with the 45° mark on A .

The quarter waveplate is in the desired position. In this case, the vibration of plane polarised light falling on the quarter waveplate makes an angle 45° with the direction of optic axis of the quarter wave plate. The polarised light is split up into two rectangular components having equal amplitude and time period and on coming out of the quarter waveplate, the beam is circular polarised. If the nicol prism N_2 is rotated at this stage, the field of view is uniform in intensity similar to the ordinary light passing through the nicol prism.

Elliptically polarised light:

To produce elliptically polarised light, the two waves vibrating at right angle to each other having unequal amplitudes should have a phase difference of $\pi/2$ or a path difference of $\lambda/4$. The arrangement of figure can be used for this purpose. A parallel beam of monochromatic light is allowed to fall on the nicol prism N_1 . The prisms N_1 and N_2 are crossed and the field of view is dark. A quarter wave plate is introduced between N_1 and N_2 . The plane polarised light from the nicol prism N_1 falls normally on the quarter wave plate. The field of view is illuminated and the light coming out of the quarter wave plate is elliptically polarised. The only precaution in the case is that the vibrations of the plane polarised light falling on the quarter plate should not make an angle of 45° with the optic axis, in which case, the light will be circularly polarised. When the nicol prism N_2 is rotated, it is observed that the intensity of illumination of the field of view varies between a maximum and a minimum. This is just similar to the case when a beam consists of mixture of plane-polarised light and ordinary light is examined by a nicol prism.



b) Explain in detail about the double refraction in uniaxial crystal.

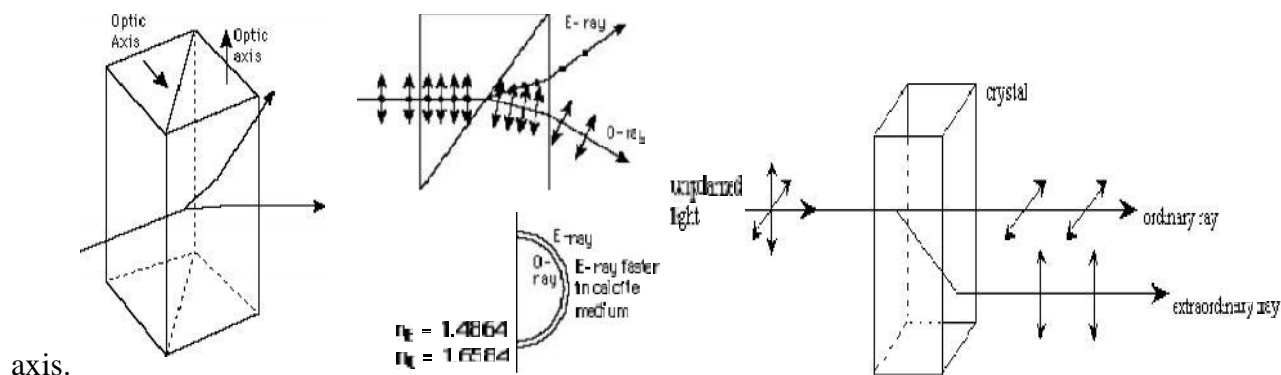
Erasmus Bratholinus discovered in 1669, that when a ray of light is refracted by a crystal of calcite it gives two refracted rays. The phenomenon is called double refraction. Calcite or Iceland spar is crystallised calcium carbonate (CaCO_3) and was found in large quantities in Iceland as very large transparent crystals. Due to this reason calcite is known as Iceland spar. It crystallises in many forms and can be reduced by cleavage or breakage into a rhombohedron, bounded by six parallelograms with angle equal to 102° and 78° . (more accurately $101^\circ 55'$ and $78^\circ 5'$).

Optic Axis:

At two opposite corners A and H, of the rhombohedron all the angles of the faces are obtuse. These corners A and H are known as the blunt corners of the crystal. A line drawn through A making equal angles with each of the three edges gives the

2017-2018 Odd

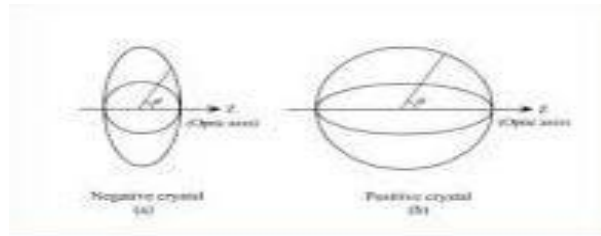
direction of the optic axis. In fact any line parallel to this line is also an optic axis. Therefore optic axis is not a line but it is a direction. Moreover, it is not defined by joining the two blunt corners. Only in a spherical case, when the three edges of the crystal are equal, the line joining the two blunt corners A and H coincides with the crystallographic axis of the crystal and it gives the direction of the optic axis. If a ray of light is incident along the optic axis or in a direction parallel to the optic axis, then it will not break into two rays. Thus the phenomenon of double refraction is absent. When a light is allowed to enter the crystal along the optic



25. Explain Huygens theory of double refraction.

Huygens explained the phenomenon of double refraction with the help of the principle of secondary wavelets. A point source of light in a double refracting medium is the origin of two wave fronts. For the ordinary ray, for which the velocity of light is same in all directions, the wavefront is spherical. For the extraordinary ray, the velocity varies with the directions and the wavefront is ellipsoidal. The velocities of ordinary and extraordinary rays are the same along the optic axis.

2017-2018 Odd



Consider a point source of light S in a calcite crystal, the sphere is the wave surface of the ordinary ray and ellipsoid is the wave surface of the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface. Such crystals are known as negative crystals. For crystals like quartz, which are known as positive crystals, the extraordinary wave surface lies within the ordinary wave surface.

1) For the negative uniaxial crystals $\mu_o > \mu_E$:

The velocity of the extraordinary ray varies as the radius vector of the ellipsoid. It is least and equal to the velocity of the ordinary ray along the optic axis but it is maximum at right angle to the direction of the optic axis.

2) For the positive uniaxial crystals $\mu_E > \mu_o$:

The velocity of the extraordinary ray is least in the direction at the right angles to the optic axis. It is maximum and equal to the velocity of the ordinary ray along the optic axis. Hence from the Huygens theory, the wavefronts or surfaces in uniaxial crystals are a sphere and an ellipsoid and there are two points where these two wavefronts touch each other. The direction of the line joining these two points is the optic axis.

b) Write short notes on half wave plate and describe its construction in detail.

This plate is also made from a doubly refracting uniaxial crystal of calcite (or) quartz of suitable thickness whose refracting faces

2017-2018 Odd

are cut parallel to the direction of the optic axis. the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays have a path difference $= \lambda/2$ after passing through the crystals

For negative crystal, path difference $= (\mu_o - \mu_e)t$

For positive crystals, path difference $= (\mu_e - \mu_o)t$

To produce a path difference of $\lambda/2$, in calcite

$$(\mu_o - \mu_e)t = \lambda/2$$

$$t = \lambda/2 (\mu_o - \mu_e) \text{----- (1)}$$

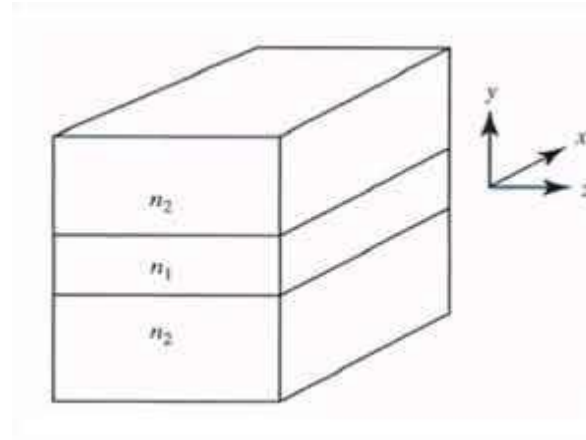
and in case of quartz,

$$t = \lambda/2 (\mu_e - \mu_o) \text{----- (2)}$$

When plane polarised light is incident on a half waveplate, such that it makes an angle of 45° with the optic axis a path difference of $\lambda/2$ is introduced between the extraordinary and the ordinary rays. The emergent light is plane-polarized and the direction of polarization of the linear incident light is rotated through 90° . Thus a half waveplate rotates the azimuth of a beam of plane-polarized light by 90° , provided the incident light makes an angle 45° with the optic axis of the half wave plate.

26. a) Explain briefly about planer dielectric wave guide.

2017-2018 Odd



Planar (slab) waveguides are the basis of waveguides used in integrated optoelectronics. The same mathematical ideas can be applied (with minor modifications) to circular waveguides.

The waveguide consists of a semi-infinite slab of dielectric materials with thickness d and refractive index n_1 (the core) that is sandwiched between two regions (the cladding) both of refractive index n_2 , and where $n_1 > n_2$.

A beam propagating down a waveguide within the core layer

A ray of light may propagate down the core provided that total internal reflection occurs at the core/cladding interface.

this requires that: $90^\circ > \theta > \theta_c$

Where θ is the internal ray angle (from now on written as θ)

In fact there are ‘infinite’ number of rays, all slightly displaced from each other, also propagating down the guide. The dotted line that is perpendicular to the wave lines is the wave front of the propagating beam. The rays represent lines drawn normally to the plane wave fronts.

2017-2018 Odd

The wave front FC intersects two the upwardly traveling portions of the same ray at points A and C. Therefore the phase at C and A must be the same or differ by a multiple of 2π .

Otherwise there would be destructive interference between out-of-phase waves and the light will not propagate. It also requires very specific angles above the critical angle. Consider the phase difference between A and C.

There are two factors -the path length of $AB + BC$ -the phase change due to reflection at B and C. We write the phase change resulting from reflection simply as ϕ . For perpendicular radiation ϕ is 2π , for parallel radiation $\phi = 0$. The total phase change is equivalent to:

$$(AB + BC) \frac{2\pi}{\lambda} + n\pi - \phi$$

Where λ is the wavelength of light in the medium.

To determine the path of the light from a to b to c using trigonometry: $AB = BC \cos 2\theta$

Thus $AB + BC = BC (1 + \cos 2\theta)$

Since $\cos 2\theta = 2 \cos^2 \theta - 1$

$$AB + BC = 2 BC \cos^2 \theta \quad \text{that is the thickness of the slab}$$

So that $AB + BC = 2d \cos^2 \theta$

The thickness of the slab determines the number of modes or angles that light will propagate at. In order for the mode to propagate the total phase change must be a multiple of 2π :

$$(4 \pi n d \cos^2 \theta) - \phi = 2\pi m$$

$$(2 \pi n d \cos^2 \theta) - \phi = m\lambda$$

Where m is an integer, So for each value of m there will be an angle θ_m that satisfies the equation.

Each value of θ_m (those $> \theta_c$) has a distinct distribution of electric field across the guide. This distribution is known as a mode.

Depending on the mode there may a distribution that is centered in the core or may have 2 spots, 4 spots etc when view in cross section.

When : m is = c the mode is at cut-off

If $m < c$: the mode is below cut off resulting in rapid attenuation and light will not be propagated.

If $m > c$: the mode is above cut-off which can propagate.

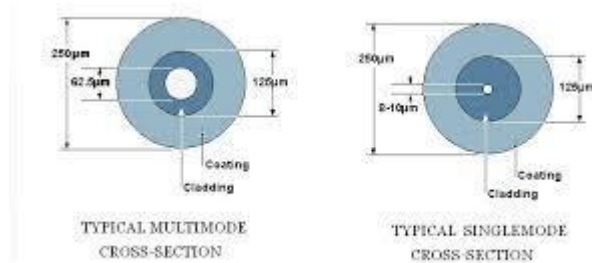
b) What is mean by optical fibre? Write short notes on single and multiple mode fibres.

Fibre Optics is sending signals down hair-thin strands of glass or plastic fibre. The light is “guided” down the center of the fibre called the “core”. The core is surrounded by a optical material called the “cladding” that traps the light in the core using an optical technique called “total internal reflection.”

The core and cladding are usually made of ultra-pure glass. The fibre is coated with a protective plastic covering called the “primary buffer coating” that protects it from moisture and other damage. More protection is provided by the “cable” which has the fibres and strength members inside an outer covering called a “jacket”.

Multicom’s Fibre Optic Product Line and services also includes stocking and same day shipment of a large quantity and variety of custom-cut fibre optic cable (including loose tube, ADSS, Armored, etc), Corning fibre optics-based products and a wide selection of fibre. optic Transmitters, EDFAs, Receivers, Nodes, accessories, splitters, jumpers, pigtails, and media converters designed to meet the demanding requirements of data, video, and voice networks.

2017-2018 Odd

**Single Mode Fibre Optic Cable :**

Single Mode fibre optic cable has a small diametral core that allows only one mode of light to propagate. Because of this, the number of light reflections created as the light passes through the core decreases, lowering attenuation and creating the ability for the signal to travel further. This application is typically used in long distance, higher bandwidth runs by Telcos, CATV companies, and Colleges and Universities.

Left: Single Mode fibre is usually 9/125 in construction. This means that the core to cladding diameter ratio is 9 microns to 125 microns.

Multimode Fibre Optic Cable

Multimode fibre optic cable has a large diametral core that allows multiple modes of light to propagate. Because of this, the number of light reflections created as the light passes through the core increases, creating the ability for more data to pass through at a given time. Because of the high dispersion and attenuation rate with this type of fibre, the quality of the signal is reduced over long distances. This application is typically used for short distance, data and audio/video applications in LANs. RF broadband signals, such as what cable companies commonly use, cannot be transmitted over multimode fibre.

Above: Multimode fiber is usually 50/125 and 62.5/125 in construction. This means that the core to cladding diameter ratio is 50 microns to 125 microns and 62.5 microns to 125 microns. What's Happening Inside The Multimode Fibre Step-Index Multimode Fibre.

Due to its large core, some of the light rays that make up the digital pulse may travel a direct route, whereas others zigzag as they bounce off the cladding. These alternate paths cause the different groups of light rays, referred to as modes, to arrive separately at the receiving point. The pulse, an aggregate of different modes, begins to spread out, losing its well-defined shape. The need to leave spacing between pulses to prevent overlapping limits the amount of information that can be sent. This type of fibre is best suited for transmission over short distances.

