KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Under section 3 of UGC act 1956)
COIMBATORE-641021

FACULTY OF ENGINEERING
DEPARMENT OF CIVIL ENGINEERING

B.E Civil Engineering **2018-2019**

Semester-V

17BECE503	**Solid Mechanics-II**	**2H-2C**

Instruction Hours/week: L: 2 T: 0 P: 0 **Marks:** Internal:**40** External:**60**

Total**: 100**

End Semester Exam:3 Hours

Course Objective

- To understand the concept of Principle of virtual work
- To study the different methods of finding deflection of beam
- To analyze the Indeterminate beams subjected to various loading
- To study the different methods to find the deflection of truss
- To analyze the column with different end conditions and stress in thick cylinders

Course Outcome

On completion of the course, the students will be able to:

- Apply the principle of virtual work.
- Determine deflection of a beam for various loading conditions.
- Apply unit load method to find the deflection of truss.
- Determine different stresses developed in thick cylinders.
- Visualize the behavior of column for combined bending and axial loading.

UNIT-I: FORCES IN STATICALLY DETERMINATE FRAMES: Method of joints-Method of sections-Graphical method –Deflection -Unit load method-Graphical method-Forces in redundant frames-Castigliano‟s theorem-Maxwell‟s method-Tension Co-efficient method. **9**

UNIT-II: UNSYMMETRICAL BENDING: Stresses in beams subjected to unsymmetrical bending- Deflection of beams –simply supported beams – fixed end beams – Over hanging beams – different load conditions (Point load, UDL,UVL) - under unsymmetrical bending-shear centre. **9**

UNIT-III: COMBINED BENDING AND DIRECT STRESSES: Columns and struts-types-failure modes- Euler‟s formula-Rankin‟s formula-Johnson‟s - IS code formula-practical end conditions and effective length factors- Eccentric loading-Middle third rule-Core of a section. **9**

UNIT-IV: THIN CYLINDRICAL AND SPHERICAL SHELLS: Assumptions-Internal pressure-Change in volume-Minimum thickness of wall plates. **9**

UNIT-V: ELEMENTARY THEORY OF VIBRATIONS: Simple harmonic motion - Longitudinal vibration - Helical and Compound springs -Transverse vibrations of beams with point loads and UDL - Torsional vibrations of shafts. **9**

SUPPORTING MATERIALS

TEXT BOOKS:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Strength of materials and Theory of Structures Vol.I	Dr. B.C.Punmia	Laxmi Publications, Chennai	2011

REFERENCE BOOKS:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Applied mechanics	V.Devarajan	Padma Publications, New Delhi.	2012
2	Applied Mechanics and Strength of Materials	R.S.Khurmi	Niraja Construction and Development Limited, Tenth Edition, New Delhi,	2012

STAFF INCHARGE

(**Mr.S.Sanjay**)

HOD (Department of Civil Engineering **DEAN** (FOE)

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17BECE503/ SOLID MECHANICS-II
LECTURE PLAN

Number of credits : 3
Contact hours : 3 hours per week
Lecturer : Mr.S.Sanjay
Semester : V– (2018-2019)
Course Type : Core

Lecture	Hours	Topics to be Covered	Text / Reference	Page No
1	1	Method of joints	**T1,R1**	**125, 172**
2	1	Method of sections	**T1,R2**	**128, 182**
3	1	Graphical method	**T1,R1**	**130, 196**
4	1	Deflection , Unit load method	**T2,R2**	**131, 206**
5	1	Graphical method	**T1, R2**	**134, 215**
6	1	Forces in redundant frames	**T1,R2**	**139, 217**
7	1	Castigliano's heorem	**T1,R2**	**141, 220**
8	1	Maxwell's method	**T1,R1**	**147, 221**
9	1	Tension Co-efficient method	**T1,R1**	**151, 227**
Total	**9 Hrs**			
10	1	Stresses in beams subjected to unsymmetrical bending	**T1,R1**	**157, 228**
11	1	Deflection of beams	**T1,R1**	**159, 232**
12	1	Simply supported beams	**T1,R1**	**161, 233**
13	1	Fixed end beams	**T1,R1**	**167, 236**
14	1	Over hanging beams	**T1,R1**	**178, 237**
15	1	Different load conditions f point load and UDL	**T1,R2**	**177, 239**
16	1	UVL loading conditions	**T1,R2**	**177, 240**
17	1	Unsymmetrical bending	**T1,R2**	**183, 242**
18	1	shear centre	**T1,R1**	**186, 244**
Total	**9 Hrs**			

19	1	Columns and struts and their types	**T1,R1**	**199, 260**
20	1	Failure modes	**T1,R2**	**203, 263**
21	1	Euler’s formula	**T1,R2**	**206, 265**
22	1	Rankin’s and Johnson’s formula	**T2,R2**	**218, 266**
23	1	IS code formula	**T1,R1**	**236, 271**
24	1	Practical end conditions	**T1,R1**	**238, 277**
25	1	Effective length factors	**T1,R2**	**247, 282**
26	1	Eccentric loading	**T1,R2**	**249, 297**
27	1	Middle third rule, Core of a section	**T1,R2**	**264, 298**
Total	**9 Hrs**			
28	1	Intrduction to thin cylindrical	**T1,R1**	**266, 302**
29	1	Intrduction to spherical shells	**T2,R1**	**268, 313**
30	1	Assumptions in thin cylindrical	**T2,R1**	**269, 318**
31	1	Assumptions in spherical shells	**T1,R1**	**271, 321**
32	1	Design concepts of thin cylindrical	**T1,R1**	**272, 322**
33	1	Design concepts of spherical shells	**T1,R1**	**277, 326**
34	1	Internal pressure	**T2,R1**	**275, 329**
35	1	Change in volume	**T1,R1**	**281, 335**
36	1	Minimum thickness of wall plates	**T1,R1**	**283, 337**
Total	**9 Hrs**			
37	1	Intrduction to elementary theory of vibrations	**T1,R2**	**286, 339**
38	1	Concepts of harmonic motion	**T1,R1**	**299, 342**
39	1	Simple harmonic motion	**T2,R1**	**301, 345**
40	1	Longitudinal vibration	**T1,R1**	**315, 348**
41	1	Helical springs	**T1,R2**	**322, 349**
42	1	Compound springs	**T1,R1**	**323, 351**
43	1	Transverse vibrations of beams with point loads	**T1,R2**	**334, 372**
44	1	Transverse vibrations of beams with UDL	**T1 ,R2**	**353, 373**
45	1	Torsional vibrations of shafts	**T1,R1**	**359, 377**
Total	**9 Hrs**			

SUPPORTING MATERIALS

TEXT BOOKS:

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1	Strength of materials and Theory of Structures Vol.I	Dr. B.C.Punmia	Laxmi Publications, Chennai	2011

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I. SIMPLE STRESS, STRAIN AND THERMAL STRESSES :-

Technical Terms:-

1. Strength of material

The internal resistence developed (or) offered by any Material which is loaded, is known as strength of Material. Generally the external load is applied upto "limit of proportionality" where Hooke's law is valid (or) applicable. The Analysis of loaded Material is based on "Elastic Zone" only.

2. Engineering Stress :- (or) Conventional Stress :-

It is defined as The internal resistence offered by any material per unit of its original Cross-Sectional area. Its symbol is "σ or f"

$$\boxed{\sigma = f = \frac{P}{A_0}}$$

Its unit is "N/m^2" in S.I system.

kgf/m^2 in MKS System

$Dyne/cm^2$ - in CGS

$$1\,kgf = 1\,msl \times 1\,m/sec^2$$

$$= 9.81\,kg\text{-}m/sec^2$$

$$\boxed{1\,kgf = 9.81\,N}$$

In S.I system

$$1\,N/m^2 = (1\text{ pascal}) = 1\,pa.$$

1kg = 9.81N

1KN = 100Kg

$$1\,Pascal = 1\,N/m^2$$

$$= 1\,\frac{N}{(1000mm)^2}$$

$$10^6\,Pa = 1\,N/mm^2$$

$$\boxed{1\,Mpa = 1\,N/mm^2}$$

Dimension of stress :-

$$\sigma = f = \frac{\overset{force}{P}}{A} = \frac{\text{mass} \times \text{acceleration}}{\text{Area}}$$

$$= \frac{M \times LT^{-2}}{L^2}$$

$$\boxed{\text{SI unit of stress} = ML^{-1}T^{-2}}$$

3. **True stress :-**

It is defined as the internal resistence (force) developed by any loaded material per unit of its instantaneous cross sectional area

$$\boxed{\sigma_{true} = \frac{P}{A}}$$ A – instantanous area

$$\sigma_{true} = \frac{P}{A} \times \frac{A_0}{A_0}$$

$$\sigma_{true} = \left(\frac{P}{A_0}\right) \times \left(\frac{A_0}{A}\right)$$

$$\sigma_{true} = \sigma_{Engg.} \times \left(\frac{A_0}{A}\right)$$

Volume of material before loading = Volume of material after loading

$$\boxed{V_0 = V}$$

$$A_0 \times l_0 = A \times l.$$

$$\frac{A_0}{A} = \frac{l}{l_0} = \frac{l_0 \pm \Delta l}{l_0}$$

$$= \frac{l_0}{l_0} \pm \frac{\Delta l}{l_0}$$

$A_c = \frac{A_0}{1 \pm \epsilon}$

$$\boxed{\frac{A_0}{A} = 1 \pm \epsilon}$$

ϵ → linear strain

$$\boxed{\sigma_{True} = \sigma_{Engg.} \times (1 \pm \epsilon)}$$

$+$ → Tensile force

$-$ → compressive force.

4. Conventional strain (or) Engg. strain:

It is defined as the change in length per unit of original length of a loaded material. Its symbol is (ϵ or e)

$$\epsilon = e = \int_{l_0}^{l} \frac{\delta l}{l_0} = \int_{l_0}^{l} \frac{dl}{l_0}$$

$$= \frac{1}{l_0} [l]_{l_0}^{l}$$

$$= \frac{1}{l_0} \times [l - l_0]$$

$$\boxed{e = \epsilon = \frac{\Delta l}{l_0}}$$

Unit → MM/mm → $M^0 L^0 T^0$

→ No dimension.

5. Natural strain :- (ϵ_{nat})

It is defined as the change in length per unit instantaneous length of a loaded material.

$$\epsilon_{nat} = \frac{\delta l}{l} = \frac{dl}{l}$$

$$= \int_{l_0}^{l} \frac{dl}{l}$$

$$= [\log_e l]_{l_0}^{l}$$

$$= \log_e l - \log_e l_0$$

$$= \log_e \left[\frac{l}{l_0}\right]$$

$$= \log_e \left[\frac{l_0 \pm \Delta l}{l_0}\right]$$

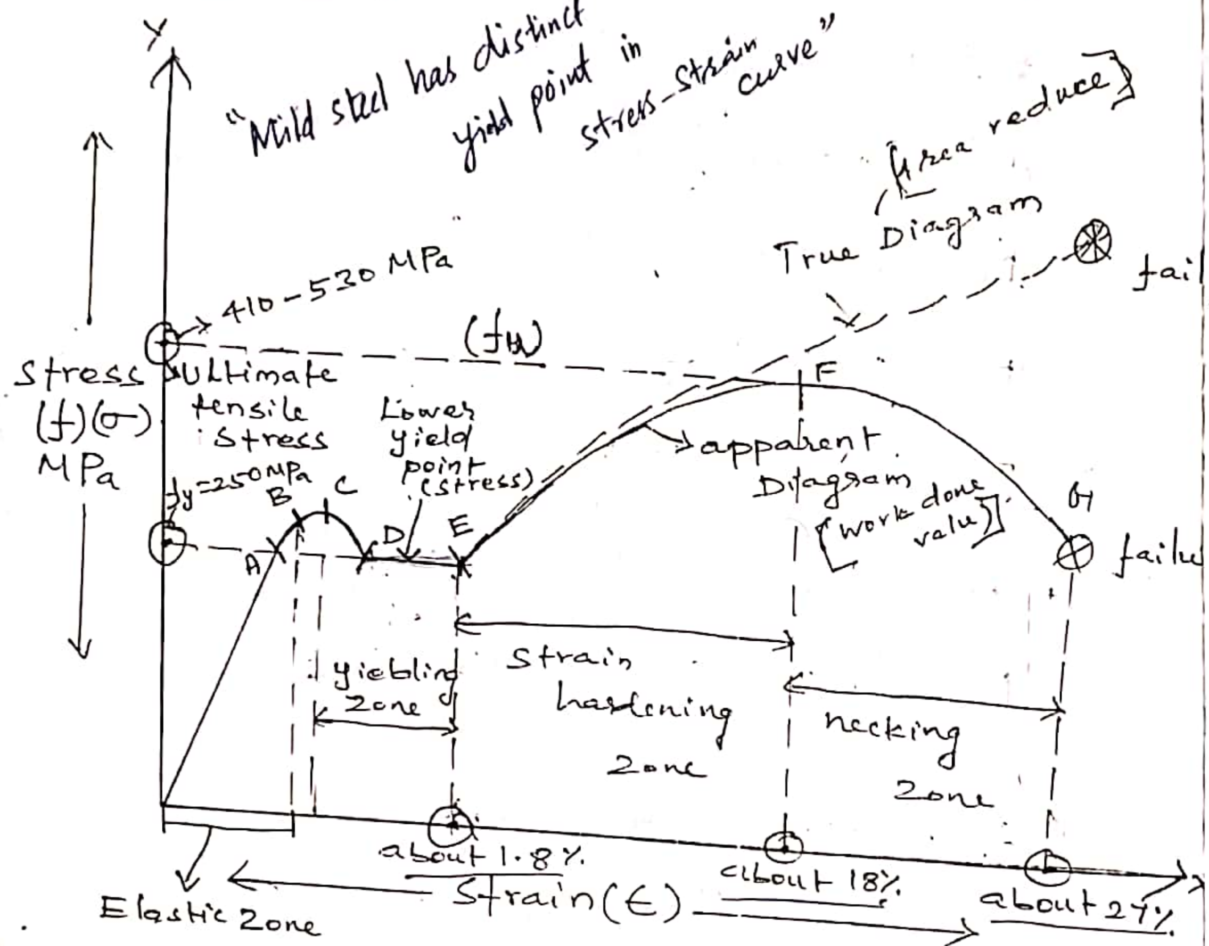
$$= \log_e \left[\frac{l_0}{l_0} \pm \frac{\Delta l}{l_0}\right]$$

$$\boxed{\epsilon_{nat} = \log_e [1 \pm \epsilon]}$$

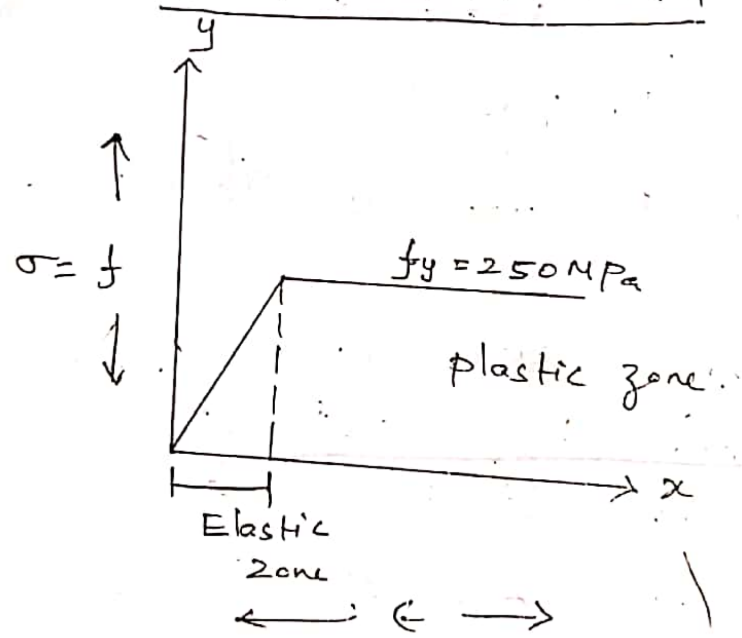
$$= \ln [1 \pm \epsilon]$$

Conventional strain

6. Stress - Strain curve of Mild steel :-
***



IDEALIZED DIAGRAM



A- Limit of proportionality.

B- Elastic limit

C- Upper yield point

D- Lower yield point

E- End of lower yield point

DE- Yield stress $[f_y = 250 \text{ Mpa} = \sigma_y]$

EF- Strain Hardening.

F- Ultimate stress point
(Tensile stress)

[f_u = 410 to 530 Mpa
(Tensile stress)
(Ultimate tensile stress)]

FG- Necking Zone

G- Failure of Mild Steel.

Note!- * *

Any Material which do not have a distinct yield point (D-E) (—) than the yield stress is taken as "Residual strain" equal to 0.2% after removal of the load, also known as 0.2% proof stress. Generally it happens in case of "Aluminium, Cast Iron and Higher strength steel.

* Lin

7. Elasticity :-

It is defined as the property of a material due to which the material regains its original shape after removal of External load.

** 8. Poisson's Ratio :-

It is defined as the ratio of lateral strain to longitudinal (or) linear strain. its symbol is μ (or) $\frac{1}{m}$ (or) ν

$$\boxed{\mu = \frac{1}{m} = \nu = \frac{\epsilon_{lateral}}{\epsilon_{linear}} = \frac{\frac{\delta b}{b}}{\frac{\delta l}{l}}}$$

Unit - No dimension.

$\mu = 0.3 \rightarrow$ Mild steel in Elastic range

$\mu = 0.5 \rightarrow$ Mild steel in plastic range.

$\mu = 0.33 \rightarrow$ for Aluminium

$\mu = 0.2 \rightarrow$ Concrete

** 9. Hooke's Law :-

According to Hooke's law the linear strain developed in a material which is loaded upto "Limit of proportionality" is directly proportional to stress.

$\epsilon \propto f$ (or) $\epsilon \propto \sigma$

$\boxed{\epsilon = \frac{1}{K} \times f}$ $\quad \epsilon = \frac{1}{K} \times \sigma$

$$\epsilon = \frac{1}{E} \times f$$

$$\sigma = \boxed{f = \epsilon \times E}$$

$$\frac{P}{A_0} = \frac{\delta l}{l} \times E$$

Change in length

$$\boxed{\delta l = \frac{Pl}{A_0 E}}$$

where,

E → Modulus of Elasticity.

→ Young's Modulus of Elasticity

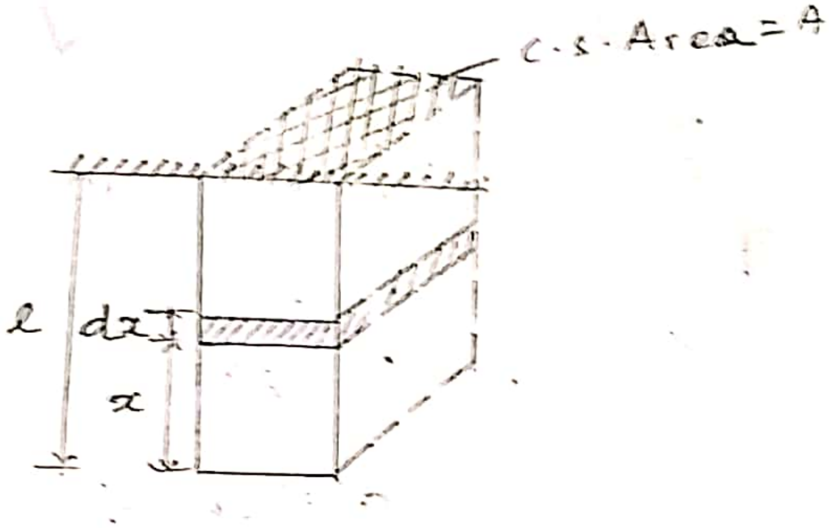
→ 2×10^5 N/mm² for Mild steel (MPa)

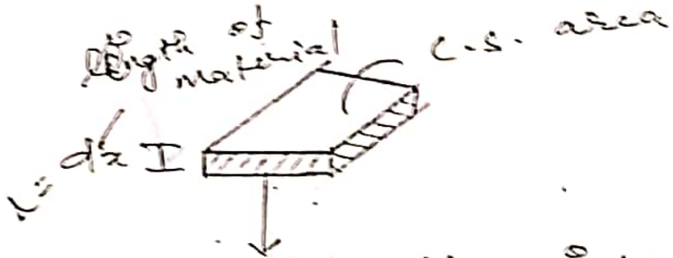
AE → Axial Rigidity.

→ Newton (SI)

→ MLT^{-2} (Dimension)

① Derive an expression for Elongation due to Self weight :-





$\gamma = \frac{W}{V}$

$W = \gamma \times V$

$W = \gamma \times A \times l$

P = Self weight of hanging portion

$= \gamma \times \text{volume} = \gamma \times A \times x.$

For Elemental strip :-

$$\boxed{\delta l_{(x)} = \frac{P_x \, l_x}{A_x \times E_x}}$$

$l_x \rightarrow$ length of the elemental strip.

$$\delta l = \frac{(\gamma \times A \times x) \times dx}{A \times E}$$

$$\delta l = \int_0^l \frac{\gamma A}{AE} \times (x \, dx)$$

$$= \frac{\gamma A}{AE} \left[\frac{x^2}{2} \right]_0^l$$

$$= \frac{\gamma \times A}{AE} \times \left[\frac{l^2}{2} \right]$$

$$= \frac{[\gamma \times A \times l] \times l}{2AE}$$

$$\boxed{\delta l = \frac{W_{total} \times l}{2AE}}$$ ✓

(or)

W – Load varies from 0 to W

$$\delta l = \frac{Pl}{AE}$$

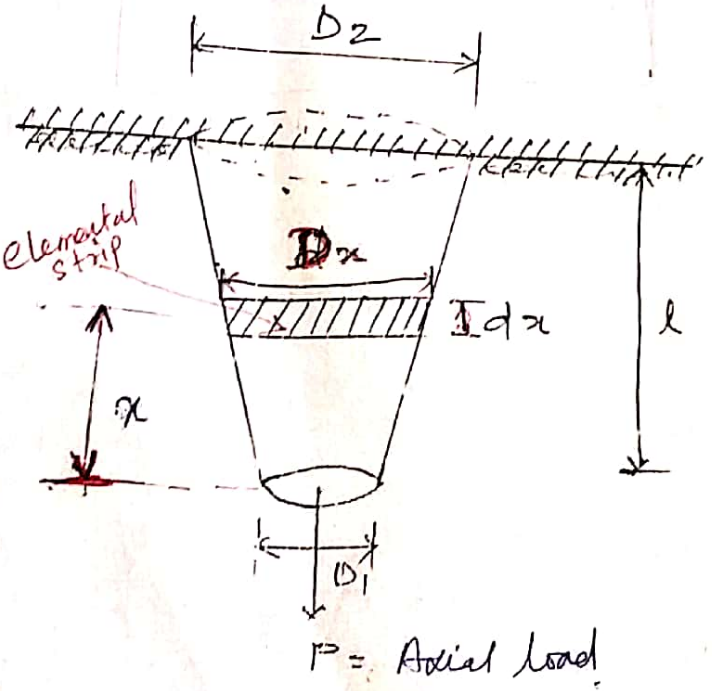
take Avg load

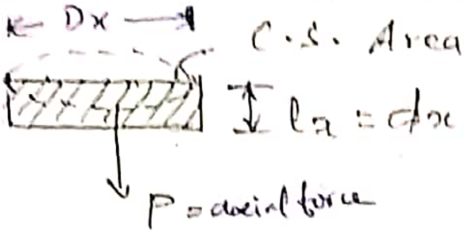
$$= \frac{\frac{(0+W) \times l}{2}}{AE}$$

$$\boxed{\delta l = \frac{Wl}{2AE}}$$ ✓

★★ 2. Expression for Elongation of a Circular tapping Section Subjected to an axial load 'P'.

Derivation





C.S. Area $= \dfrac{\pi D_x^2}{4}$

Dia of the elemental Strip $\Big\}$ $D_x = D_1 + \left[\dfrac{D_2 - D_1}{l}\right] \times x$

Dia of the elemental strip $\Big\}$ $\boxed{D_x = D_1 + kx}$ Take $\left\{ k = \dfrac{D_2 - D_1}{l} \right\}$

For Elemental strip:-

$\boxed{\delta l = \dfrac{Pl}{AE}}$

$$\boxed{\delta l_x = \frac{P_x l_x}{A_x E_x}}$$

$$\delta l_x = \frac{P_x l_x}{\frac{\pi D_x^2}{4} E_x}$$

$$\int \quad \delta l = \frac{P\,dx}{\frac{\pi}{4}(D_1 + kx)^2 \cdot E}$$

$$\delta l = \int_0^l \frac{P\,dx}{\frac{\pi}{4}(D_1 + kx)^2 E}$$

$$= \frac{4P}{\pi E} \int_0^l \frac{dx}{(D_1 + kx)^2}$$

$$= \frac{4P}{\pi E} \int_0^l (D_1 + kx)^{-2} \cdot dx$$

$$= \frac{4P}{\pi E} \left[\frac{(D_1 + kx)^{-1}}{(-1) \times (k)} \right]_0^l$$

$$= -\frac{4P}{\pi E} \left[\frac{1}{k(D_1 + kx)} \right]_0^l$$

$$= -\frac{4P}{k\pi E}\left[\frac{1}{D_1+kl} - \frac{1}{D_1}\right]$$

$$= -\frac{4P}{\cancel{k}\pi E}\left[\frac{\cancel{D_1} - \cancel{D_1} - kl}{D_1 \times (D_1 + kl)}\right]$$

$$\cancel{= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1^2 + D_1 kl}}$$

$$\cancel{= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1^2 + D_1 kl}}$$

$$\cancel{= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1^2 + D_1\left[\left(\frac{D_2 - D_1}{l}\right) l\right]}}$$

$$\cancel{= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1^2 + D_1 D_2}}$$

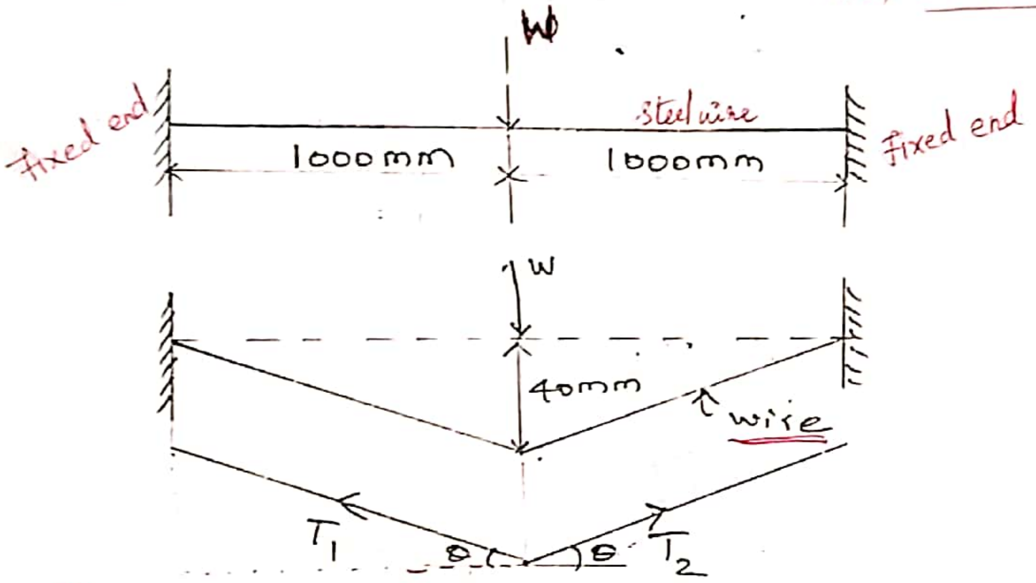
$$= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1 (D_1 + kl)}$$

$$= \frac{4Pl}{\pi E \times D_1} \cdot \frac{1}{\left[\cancel{D_1} + \frac{D_2 - \cancel{D_1}}{\cancel{l}} \times \cancel{l}\right]}$$

$$\boxed{\delta l = \frac{4Pl}{\pi E D_1 \times D_2}}$$

(3). A steel wire 1mm in diameter is 2m long and fixed at the ends. If it is subjected to a central point load (W) such that the central deflection is 40mm. Determine the magnitude of central point load (W) and the stress developed in the wire.

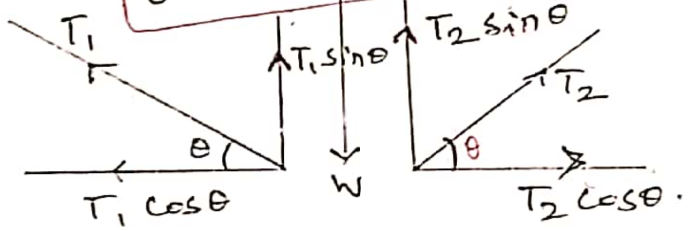
Take, $E = 2\times10^5\ N/mm^2$



$\tan\theta = \frac{Opp}{Adj}$

$$\tan\theta = \frac{40}{1000}$$

$$\boxed{\theta = 2.291^\circ}$$



Law of equilibrium equation:-

$\boxed{\Sigma H = 0}$ $T_1 \cos\theta = T_2 \cos\theta$.

$$\boxed{T_1 = T_2} \longrightarrow (1)$$

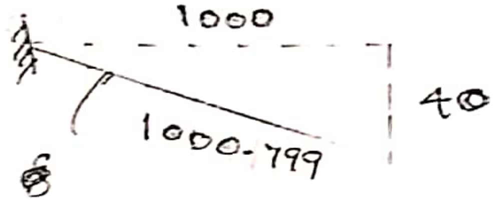
$\boxed{\Sigma V = 0}$

$$T_1 \sin\theta + T_2 \sin\theta = W$$

$$2T_1 = W/\sin\theta$$

$$T_1 = \frac{W}{2\sin 2.291} \rightarrow (2)$$

According to Hooke's law



$$\delta l_1 = 1000.799 - 1000$$
$$= 0.799 \text{ mm.}$$

$\frac{\sigma}{\varepsilon} = E$

$\sigma = \varepsilon E$

$\sigma = \frac{\delta l}{l} \times E$

$$f = \epsilon E$$
$$= \frac{\delta l}{l} \times E$$
$$= \frac{0.799}{1000} \times E$$
$$= 7.99 \times 10^{-4} \times 2 \times 10^5$$

$$\sigma = \boxed{f = 159.8 \text{ N/mm}^2}$$

$$\frac{P}{A} = 159.8$$

$$\frac{P}{\frac{\pi \times (1)^2}{4}} = 159.8$$

$$T_1 = T_2 = \boxed{P = 125.51 \text{ N}}$$ — Tensile force

$$125.51 = \frac{W}{2 \sin 2.91}$$

$$\boxed{W = 10.03 \text{ N}}$$ //

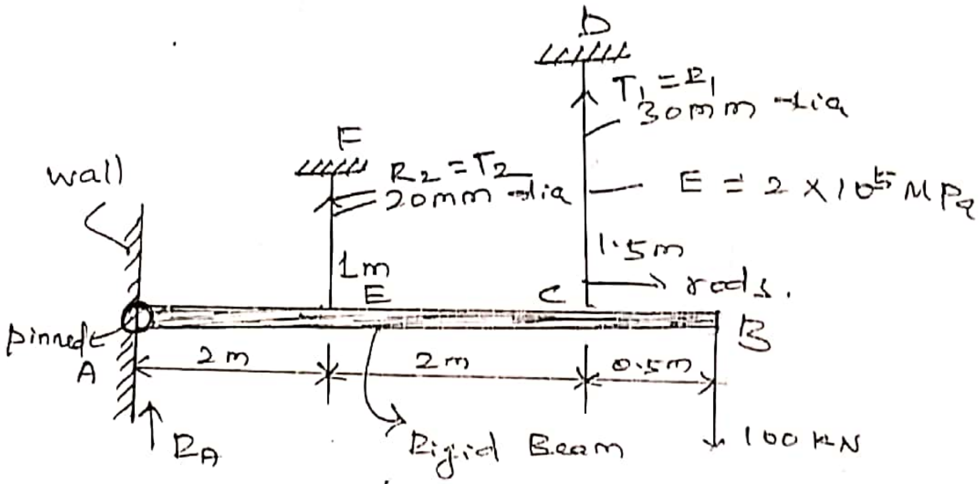
$$\cancel{T_1 = \frac{125.51}{2\sin 2.291}}$$

$$\cancel{T_1 = 1569.86\,N = T_2}$$

$$125.51 = \frac{w}{2\sin 2.91}$$

$$\boxed{w = 10.03\,N}$$

4. A rigid beam is shown in figure below. Determine stresses developed in the rods supporting the rigid beam.



Laws of moment,

$\Sigma M_A = 0$ (B'cos of pinned Support)

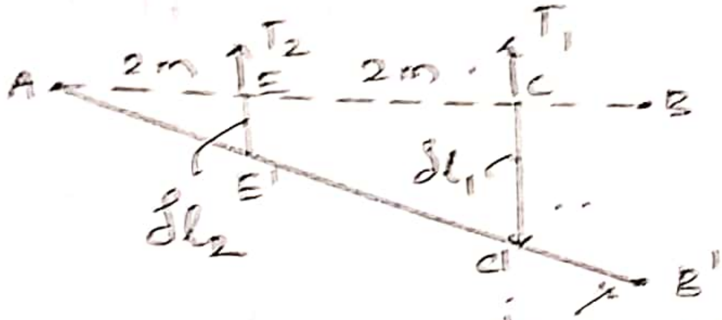
$$R_1 \times 4 + R_2 \times 2 = 100 \times 4.5$$

$$4R_1 + 2R_2 = 450 \quad \text{——— (1)}$$

$\Sigma V = 0$

$$R_A + R_1 + R_2 = 100. \quad \text{——— (2)}$$

Scanned by CamScanner



due to rigid nature it follows straight line.

$$\frac{\delta l_1}{\delta l_2} = \frac{4}{2} = 2$$

$$\frac{\delta l_1}{\delta l_2} = 2$$

$$f_1 = E \times \epsilon_1 = E \times \frac{\delta l_1}{L_1}$$

$$f_2 = E \times \epsilon_2 = E \times \frac{\delta l_2}{l_2}$$

$$\delta l_1 = 2\,\delta l_2$$

$$\frac{R_1 l_1}{A_1 E_1} = \frac{2R_2 l_2}{A_2 E_2}$$

$$\frac{R_1(1\cdot 5)}{\frac{\pi}{4}\times 30^2 \cancel{E}} = \frac{2R_2(1)}{\frac{\pi}{4}\times 20^2 \cancel{E}}$$

$$1\cdot 5 R_1 \times 20^2 = 2R_2 \times 30^2$$

$$600 R_1 = 1800 R_2$$

$$R_1 = 3R_2$$

$4R_1 + 2R_2 = 450$

$4(3R_2) + 2R_2 = 450$

$14R_2 = 450$

$\boxed{R_2 = 32.14 \text{ KN}}$

$\boxed{R_1 = 96.43 \text{ KN}}$

$R_A = -28.57 \text{ KN}$

$= 28.57 (\downarrow) \text{ KN.}$

Stress developed in bars

$$f_1 = \frac{R_1}{A}$$

$$= \frac{96.43}{\frac{\pi}{4} \times 30^2} =$$

$$= 0.1364 \text{ KN/mm}^2$$

$$= 136.4 \text{ N/mm}^2$$

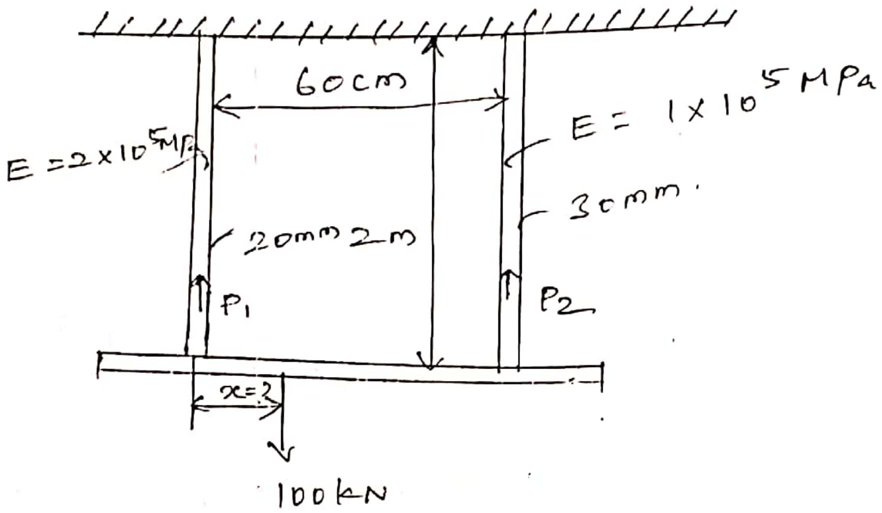
$$f_2 = \frac{R_2}{A}$$

$$= \frac{32.14}{\frac{\pi}{4} \times 20^2}$$

$$= 102.30 \text{ N/mm}^2$$

5. Two metallic bars are used to support a load as shown in figure below. Determine the position of load such that the bottom supporting member remains horizontal. Also find the stress in ~~bottom~~ bars.

$\delta = \frac{Pl}{AE}$



$\Sigma V = 0,$

$P_1 + P_2 = 100$ —— ①

Due to horizontal condition of the bottom support.

$\delta l_1 = \delta l_2$

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2}$$

$$\frac{P_1 \times 2000}{\frac{\pi}{4}(20)^2 \times 2\times 10^5} = \frac{P_2 \times 2000}{\frac{\pi}{4}(30)^2 \times 1 \times 10^5}$$

$900 P_1 = 800 P_2$

$P_1 = 8/9 \, P_2$

$8/9\,P_2 + P_2 = 100$

$$\boxed{P_2 = 52.94 \text{ kN}}$$

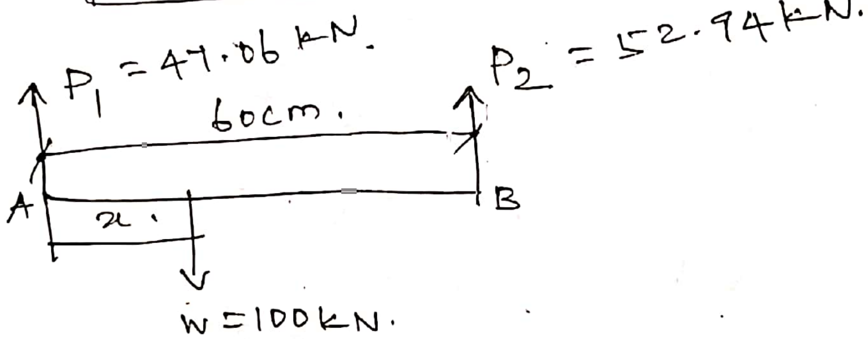
$$\boxed{P_1 = 47.06 \text{ kN}}$$

$$f_1 = \frac{P_1}{A_1} = \frac{47.06}{\frac{\pi}{4} \times 20^2}$$

$$\boxed{f_1 = 149.79 \text{ N/mm}^2}$$

$$f_2 = \frac{P_2}{A_2} = \frac{52.94}{\frac{\pi}{4} \times 30^2}$$

$$\boxed{f_2 = 74.89 \text{ N/mm}^2}$$



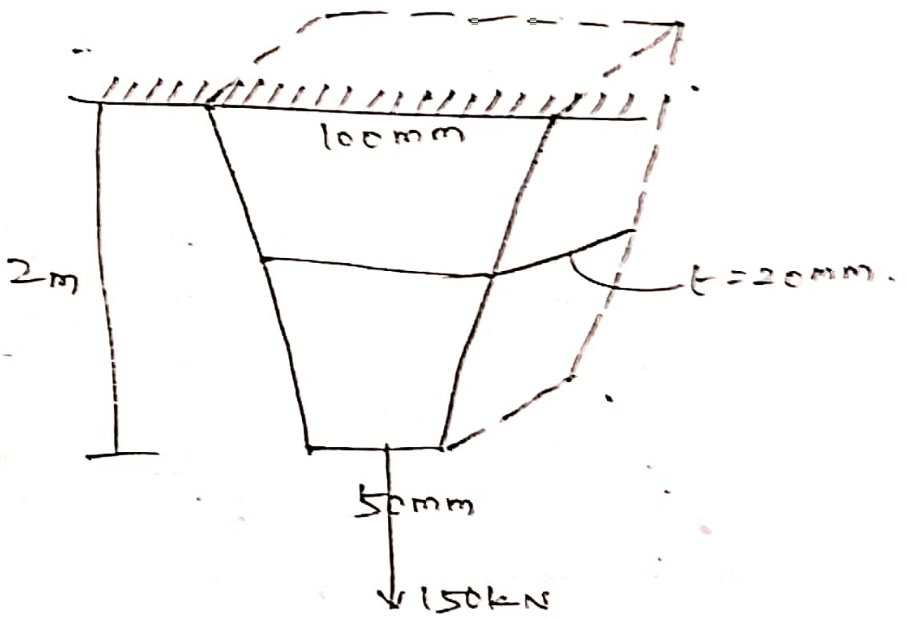
Law of moment,

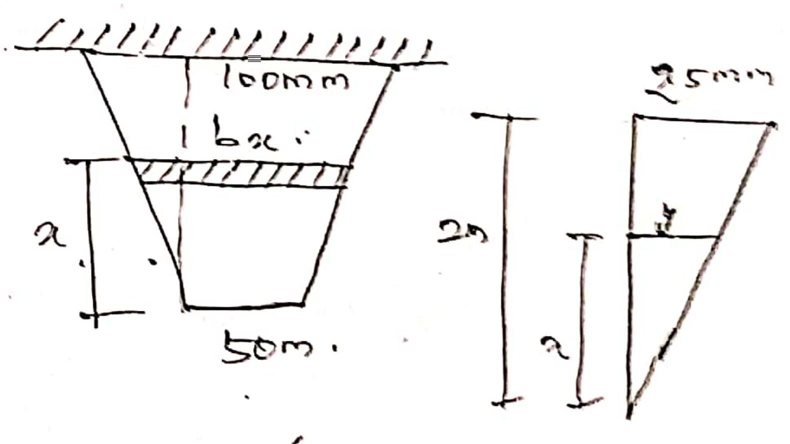
$\Sigma M_A = 0.$

$52.94 \times 60 = 100x.$

$$\boxed{x = 31.76 \text{ cm}}$$

6. A metallic bar having length 2m is subjected to an axial tensile load of 150kN. The width of the bar varies (tapers) from 100mm at the top to 50mm at the bottom. The thickness of the bar is 20mm (constant). Determine the elongation of the bar $E = 2\times10^5$ MPa.





$b_x = (50 + 75x)$

$\frac{x}{y} = \frac{2}{25}$

$y = \frac{25x}{2000}$

$b_x = 50 + \frac{50}{2000}x$

$= 50 + \frac{1}{40}x$

$$\delta l_x = \int_0^{2} \frac{Pl}{AE}$$

$$\delta l = \int_0^{2000} \frac{150 \times dx}{(50 + \frac{1}{40} x) \times 20 \times 2 \times 10^5}$$

$$= \frac{150 \times 1000}{40 \times 10^5} \int_0^{2} \left[50 + \frac{1}{40} x\right]^{-1} dx$$

$$= \frac{150000}{40 \times 10^5} \left[\frac{(50 + \frac{1}{40} x)^0}{(\frac{1}{40})(-1+1)}\right]_0^{2000}$$

$$= \frac{150 \times 1000}{40 \times 10^5 \times \frac{1}{40}} \left[\log_e \left(50 + \frac{1}{40} x\right)\right]_0^{2000}$$

$$= 1.5 \left[4.605 - 3.912\right]$$

$$\boxed{\delta l = 1.0395 \text{ mm}}$$

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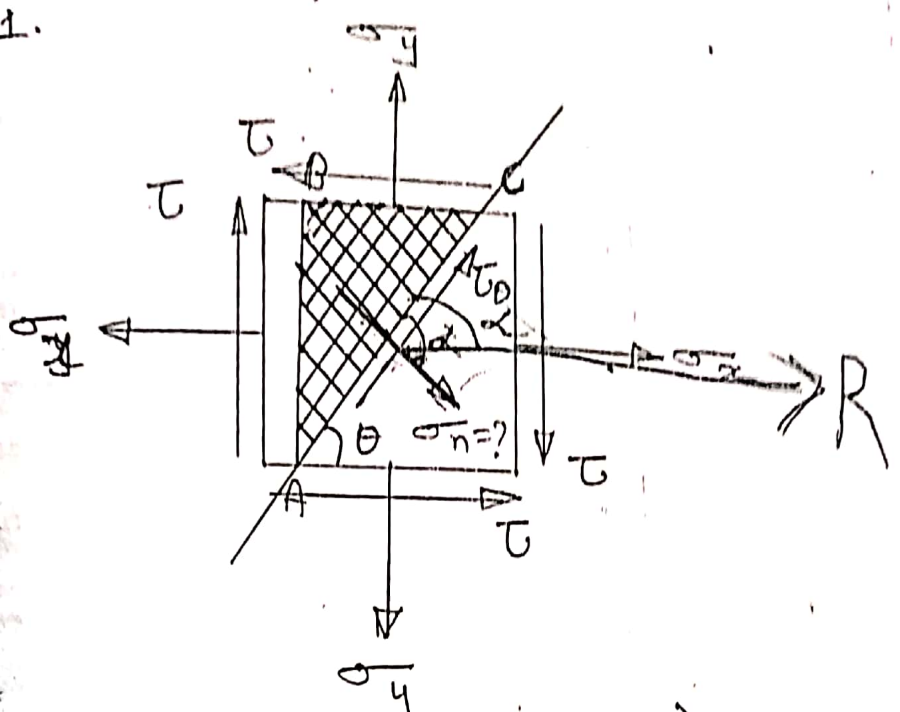
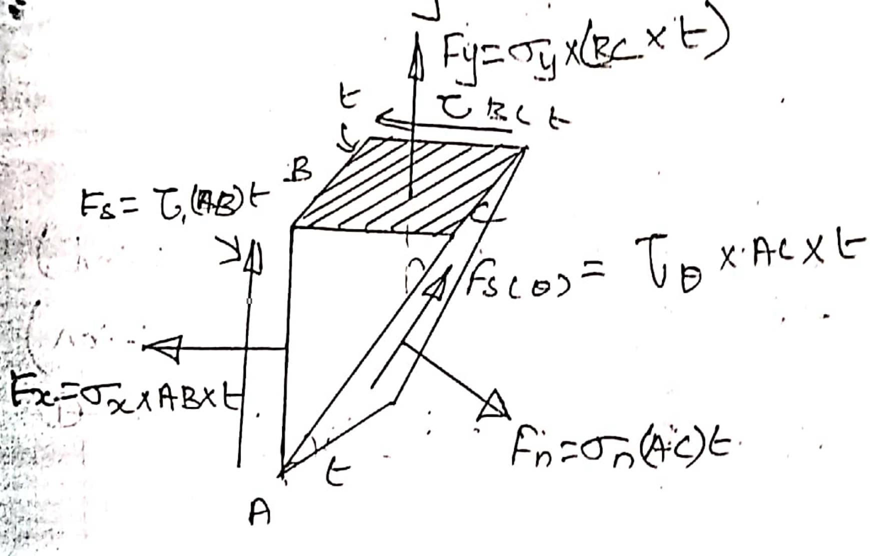
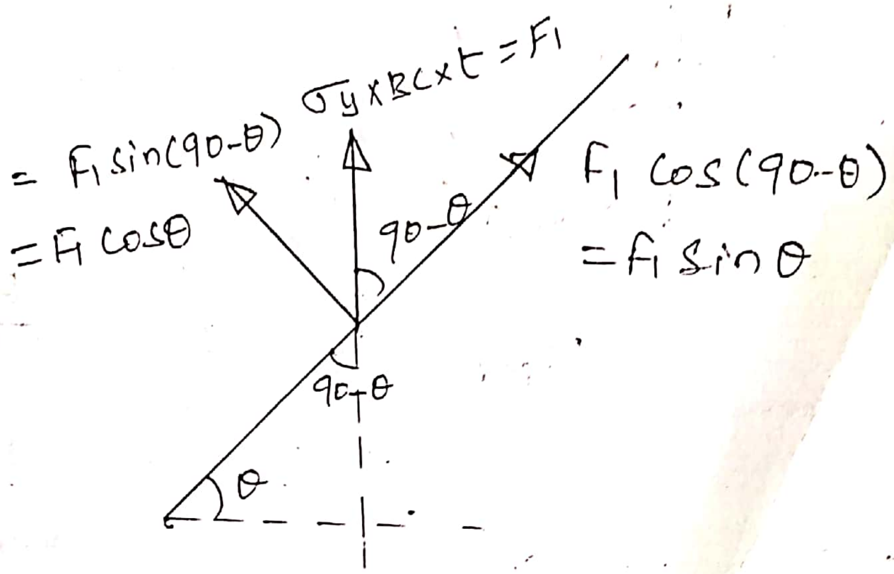
19/08/10

II. COMPLEX STRESS

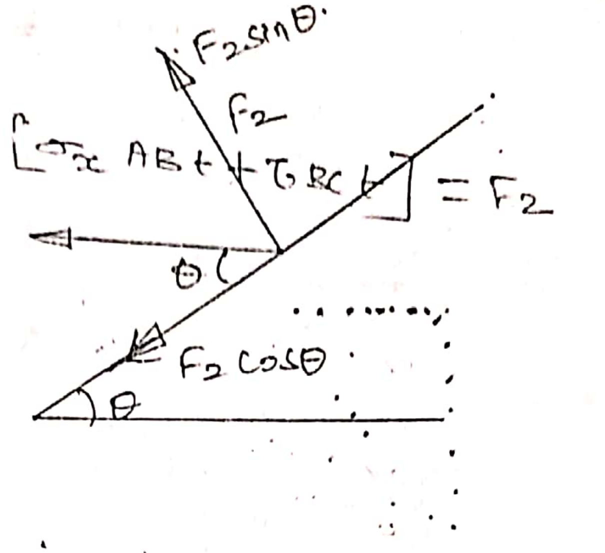
1.

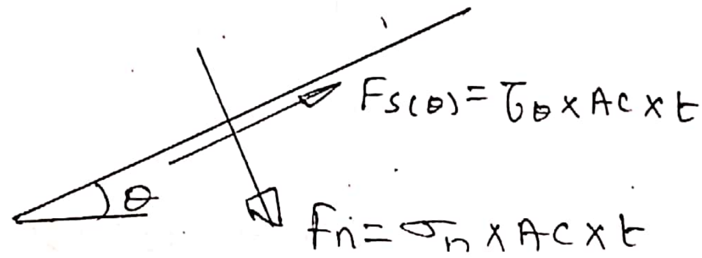






$$\sigma_y \times BC \times t + \tau \times AB \times t = F_1$$





$\sum F_{\text{along the plane}} = 0$ (For No vibration) (or) (stable condition)

$F_1 \sin\theta - F_2 \cos\theta + F_{s(\theta)} = 0$ —— ①

$\sum F_{\cancel{\perp r}\ \text{to plane}} = 0.$

$F_1 \cos\theta + F_2 \sin\theta - F_n = 0$ —— ②

$\sin\theta = \frac{AB}{AC}$, $\cos\theta = \frac{BC}{AC}$

$\sigma_y BC t \sin\theta + \tau AB t \sin\theta + \sigma_x AB t \cos\theta$

$- \tau BC t \cos\theta + \tau_\theta AC t = 0.$

$\tau_{(\theta)} = \sigma_x \left(\frac{AB}{AC}\right) \cos\theta - \sigma_y \left(\frac{BC}{AC}\right) \sin\theta$

$+ \tau \left(\frac{BC}{AC}\right) \cos\theta - \tau \left(\frac{AB}{AC}\right) \sin\theta$

$= \sigma_x \sin\theta \cos\theta - \sigma_y \sin\theta \cos\theta$

$+ \tau \cos^2\theta - \tau \sin^2\theta.$

** $= (\sigma_x - \sigma_y) \sin\theta \cos\theta + \tau(\cos^2\theta - \sin^2\theta)$

$$\boxed{\tau_{(\theta)} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta}$$

$F_n = F_1 \cos\theta + F_2 \sin\theta.$ tangential stress

$\sigma_n \times AC \times t = \sigma_y BC t \cos\theta + \tau AB t \cos\theta$

$+ \sigma_x AB t \sin\theta + \tau BC t \sin\theta$

$\sigma_n = \sigma_y \cos^2\theta + \tau \cos\theta \sin\theta$

$+ \sigma_x \sin^2\theta + \tau \cos\theta \sin\theta.$

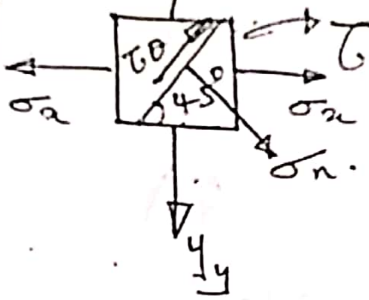
**

$$\boxed{\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \sin 2\theta}$$

$$\sigma_R = \sqrt{\sigma_n^2 + \tau_\theta^2}$$

$$\alpha = \tan^{-1}\left(\frac{\sigma_n}{\tau_\theta}\right)$$

For Maximum Value of τ_θ (Tangential stress)

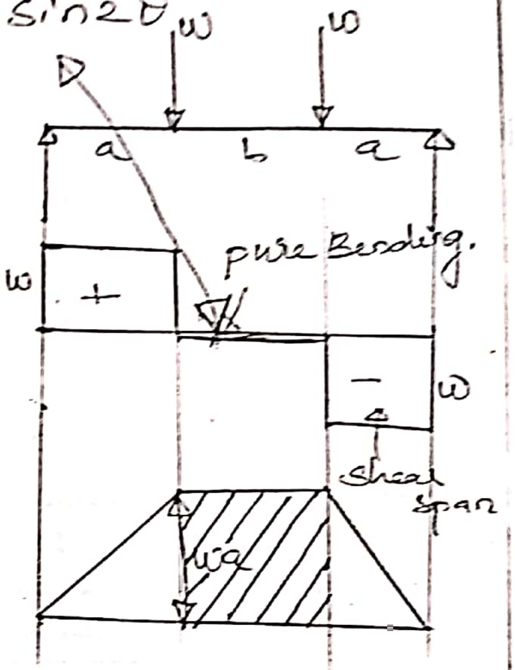


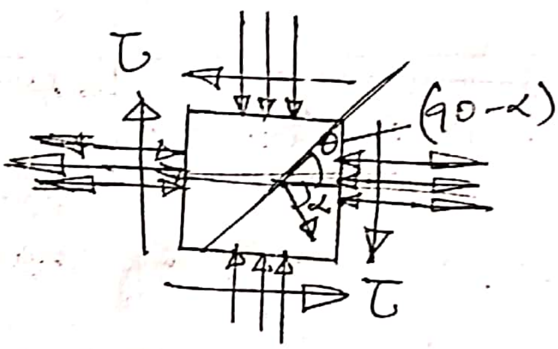
$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

$$\sin 2\theta = 1$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$



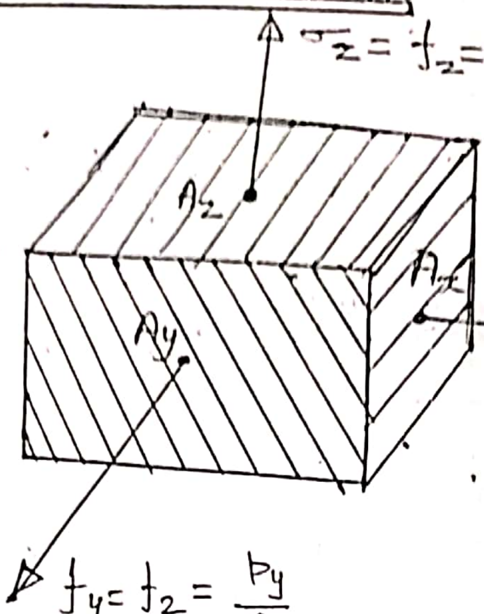


$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \, \overset{\sin 2\theta}{\cancel{\cos}}$$

$$\sigma_n = \sigma_x \sin^2\theta - \sigma_y \cos^2\theta + \tau \sin 2\theta$$

$$\tau_\theta = \frac{\sigma_x + \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta.$$

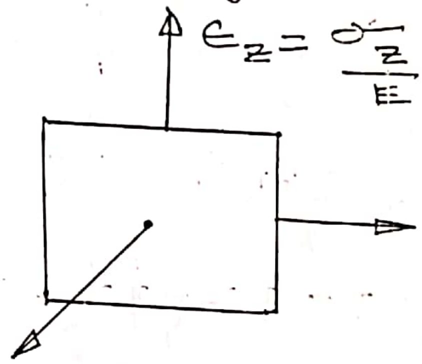
COMPLEX STRAIN



$\sigma_z = f_z = f_3 = \frac{P_z}{A_z}$

$\sigma_x = f_x = f_1 = \frac{P_x}{A_x}$

$f_y = f_2 = \frac{P_y}{A_y}$



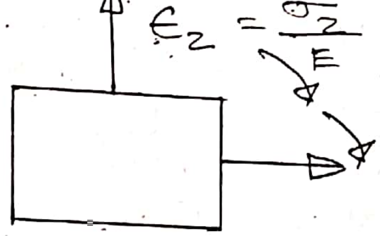
$\epsilon_z = \frac{\sigma_z}{E}$ (Hooke's Law)

$\epsilon_x = \frac{\sigma_x}{E}$

$\epsilon_y = \frac{\sigma_y}{E}$

Poisson's Ratio,

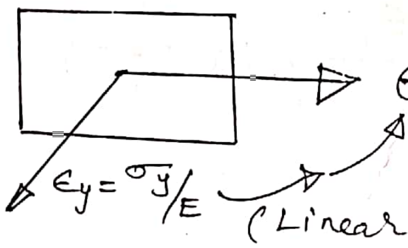
$$\mu = \frac{\epsilon_{lateral}}{\epsilon_{linear}}$$



$\epsilon_z = \frac{\sigma_z}{E}$ (Linear)

$\epsilon_x' = \mu \epsilon_z$

For z, x is called Lateral.



$\epsilon_x'' = \mu \epsilon_y$

$\epsilon_y = \sigma_y / E$ (Linear for y)

Final strain in 'X'-direction,

$$\epsilon_x = \epsilon_1 + \epsilon_1' + \epsilon_1''$$

$$= \frac{\sigma_x}{E} + \left(-\mu\frac{\sigma_z}{E}\right) + \left(-\mu\frac{\sigma_y}{E}\right)$$

$$\boxed{\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E}} \quad \text{—} ①$$

lllly

$$\boxed{\epsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E}} \quad \text{—} ②$$

lllly

✓ $$\boxed{\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}} \quad \text{—} ③$$

Total volumetric strain,

✓ $$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\sigma_x}{E} - \frac{\sigma_y}{E}\mu - \frac{\sigma_z}{E}\mu + \frac{\sigma_y}{E} - \frac{\sigma_x}{E}\mu - \frac{\sigma_z}{E}\mu + \frac{\sigma_z}{E} - \frac{\sigma_x}{E}\mu - \frac{\sigma_y}{E}\mu$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}[1 - 2\mu]$$

✓ $$\frac{\delta V}{V} = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\mu)$$

$$\boxed{\delta V = \frac{V \times (\sigma_x + \sigma_y + \sigma_z)}{E}(1-2\mu)}$$

derivation

$$\epsilon_V = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

if $\sigma_x = \sigma_y = \sigma_z = \sigma = p$

$$\epsilon_V = \frac{3p}{E}(1-2\mu)$$

Bulk modulus

$$E = 3\frac{p}{\epsilon_V}(1-2\mu)$$

$$\boxed{E = 3K(1-2\mu)}$$

$E = \frac{f}{\epsilon_{linear}}$;	$K = \frac{p}{\epsilon_{vol}}$;	$G = C = N = \frac{\tau}{\phi}$

✓ $E = 3K(1-2\mu)$

✓ $E = 2G(1+\mu)$

$$\frac{E}{2G} = 1+\mu \; ; \; \boxed{\mu = \frac{E}{2G} - 1}$$

$$E = 3K\left(1 - \frac{2E}{2G} + 2\right)$$

$$E = 3K\left(3 - \frac{E}{G}\right)$$

$$E = 9K - \frac{3KE}{G}$$

$$E + \frac{3kE}{G} = 9k.$$

$$E(G + 3k) = 9kG$$

✓ $$\boxed{E = \frac{9kG}{(3k + G)}}$$

$E = k,$

$E = 2G(1 + \mu)$

$E = 3k(1 - 2\mu).$

$$\cancel{E} = 3\cancel{E}(1 - 2\mu)$$

$$3 - \cancel{6}\mu = 1$$

$$3\cancel{6}\mu = \cancel{2}$$

$$\boxed{\mu = 1/3}$$

$k = G.$

$$3k(1 - 2\mu) = 2G(1 + \mu).$$

$$3\cancel{G}(1 - 2\mu) = 2\cancel{G}(1 + \mu).$$

$$3 - 6\mu = 2 + 2\mu.$$

$$8\mu = 1$$

$$\boxed{\mu = 1/8}$$

if $E = G$

$$E = 2G(1+\mu)$$

$$\cancel{E} = 2\cancel{E}(1+\mu)$$

$$\boxed{\mu = -1/2}$$

Important Formula :→ 17/09/10

1. $\sigma = f = \frac{P}{A}$
2. $\sigma = f = \epsilon \times E$
3. $\epsilon = \frac{\delta l}{l}$
4. $E = \frac{\text{Longitudinal stress}}{\text{Linear strain (Main strain) (Longitudinal strain)}}$

5. Poisson ratio $(\frac{1}{m})$ (μ),

$$\checkmark \mu = \frac{\text{Lateral strain or, Transverse strain}}{\text{Linear strain or, Longitudinal strain}}$$

$$= \frac{\delta b/b}{\delta l/l}$$

** 6. The value of poisson's Ratio (μ or $\frac{1}{m}$)

a) Concrete – 0.2 ✓

b) Cast iron – 0.27

c) Wrought iron – 0.278

d) Steel – 0.288

e) Stainless steel – 0.3 ✓

* f) Aluminium – 0.33 ✓

* g) Brass – 0.34

* h) Bronze – 0.35

i) Copper – 0.355

7. $f_{temp} = \epsilon \times E$

$$= \frac{\delta l}{l} \times E$$

$$= \frac{l \alpha \Delta t}{l} \times E$$

$$= \alpha (\Delta t) \times E$$

8. Young's Modulus (Modulus of Elasticity)

$$E = \frac{f_{linear}}{\epsilon_{linear}} \quad (N/mm^2)$$

$$E = \frac{P/A}{\delta l / l} \Rightarrow \delta l = \frac{Pl}{AE}$$

9. Bulk Modulus = $\frac{\text{Change in pressure}}{\text{Volumetric strain}}$

$$K = \frac{p}{\epsilon_v} = \frac{p}{\Delta V / V} \quad (N/mm^2)$$

10. Modulus of Rigidity (shear modulus)

$$= \frac{\text{Shear stress}}{\text{Shear strain}}$$ — Angular Deformation

$$C = G = N = \frac{\tau}{\phi} \quad (MPa)$$

11. Three Moduli :-

E, K, G

12. $E = 3K(1 - 2\mu)$

$$= 3K\left(1 - \frac{2}{m}\right)$$

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13. $E = 2G(1+\mu)$

$= 2G\left(1+\frac{1}{m}\right)$

14. $E = \frac{9KG}{3K+G}$

15. strain in 'x' - direction,

$$\epsilon_x = \frac{f_x}{E} - \frac{f_y}{mE} - \frac{f_z}{mE}$$

$$= \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

** 16. strain in 'y' - direction,

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

17. strain in 'z' - direction,

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

18. If $\epsilon_x, \epsilon_y, \mu$ are known, then

$\sigma_x = ?$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$$

$$\frac{\sigma_y}{E} = \epsilon_y + \frac{\mu\sigma_x}{E}$$

$$\sigma_y = \left(\epsilon_y + \frac{\mu\sigma_x}{E}\right)E$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu}{\cancel{E}}\left(\epsilon_y + \frac{\mu\sigma_x}{E}\right)\cancel{E}$$

$$\epsilon_x = \frac{\sigma_x}{E} - \mu\epsilon_y - \frac{\mu^2\sigma_x}{E}$$

$$(\epsilon_x + \mu\epsilon_y) = \sigma_x\left(\frac{1}{E} - \frac{\mu^2}{E}\right)$$

$$\sigma_x = \frac{\epsilon_x + \mu\epsilon_y}{\left(\frac{1}{E} - \frac{\mu^2}{E}\right)}$$

$$= \frac{(\epsilon_x + \mu\epsilon_y)E}{1 - \mu^2}$$

19. $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z = \epsilon_1 + \epsilon_2 + \epsilon_3$

$$= \left(\frac{f_1}{E} - \frac{f_2}{mE} - \frac{f_3}{mE}\right) + \left(\frac{f_2}{E} - \frac{f_1}{mE} - \frac{f_3}{mE}\right)$$

$$+ \left(\frac{f_3}{E} - \frac{f_1}{mE} - \frac{f_3}{mE}\right)$$

$$= \frac{f_1 + f_2 + f_3}{E} - \frac{2}{mE}(f_1 + f_2 + f_3)$$

$$= \frac{f_1 + f_2 + f_3}{E}\left(1 - \frac{2}{m}\right)$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\mu)$$

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where,

Tensile Compressive.

$$\sigma_x = +\frac{P_x}{A_x} \quad or, \quad -\frac{P_x}{A_x}$$

$$\sigma_y = +\frac{P_y}{A_y} \quad or, \quad -\frac{P_y}{A_y}$$

$$\sigma_z = +\frac{P_z}{A_z} \quad or, \quad -\frac{P_z}{A_z}$$

20. Change in volume,

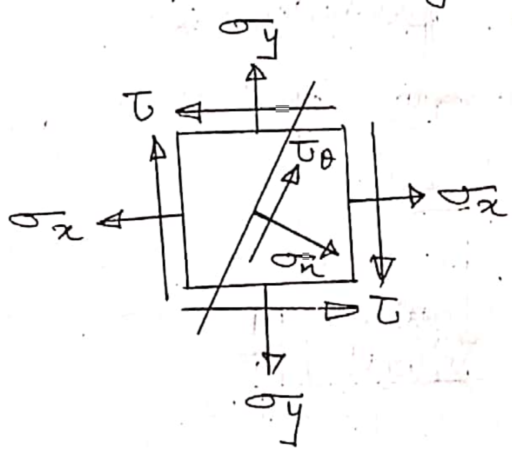
$$\epsilon_v = \frac{\delta v}{v}$$

$$\delta v = \epsilon_v \times v$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu) \times l \times b \times d$$

Normal stress,

21. $\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \sin 2\theta$



$$\sigma_n = \sigma_x\left(\frac{1-\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1+\cos 2\theta}{2}\right)$$
$$+ \tau \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau \sin 2\theta$$

22. Tangential stress (shear stress)

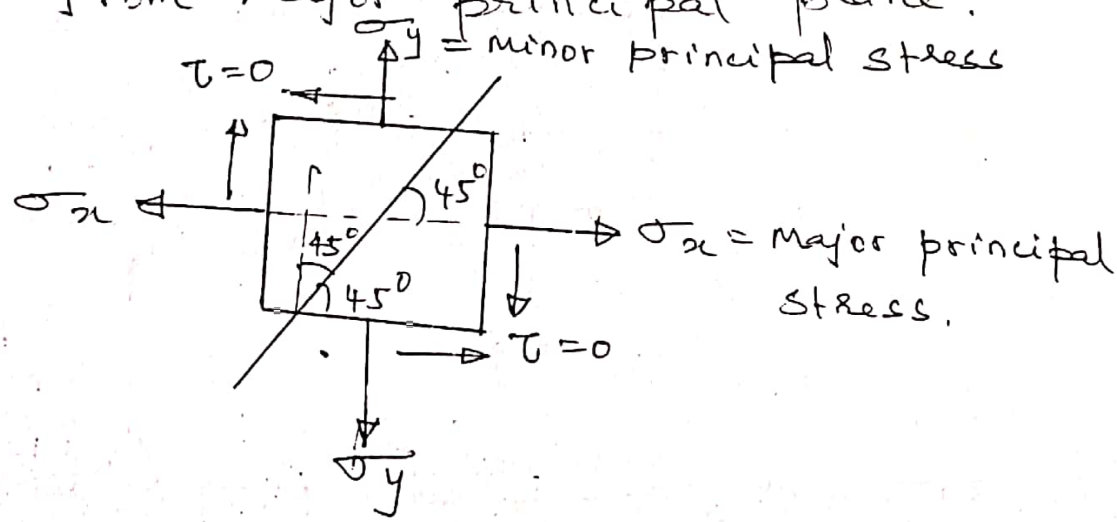
$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau\cos 2\theta$$

***23. Maximum shear stress,

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$$

$$= \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$$

24. The plane on which maximum tangential stress ($\tau_{max} = \tau_\theta$) takes place is at 45° from the direction of major principal stress or 45° from major principal plane.



25. Strain energy stored upto elastic limit is called resilience.

26. The maximum strain energy upto elastic Limit is called proof Resilien[ce]

27. The strain energy per unit volume is called Modulus of Resilience $\left(\frac{f^2}{2E}\right)$

28. Strain energy due to axial load

$$U = \frac{P^2 l}{2AE} = \frac{f^2}{2E} \times \text{Volume.}$$

29. Strain energy due to shear stress,

$$U = \frac{\tau^2}{2G} \times \text{volume}$$

30. Strain energy due to Volumetric Strain,

$$U = \frac{\sigma^2}{2K} \times \text{volume.}$$

31. Strain energy due to Torsion,

$$U = \frac{T^2 L}{2GJ} = \frac{\tau^2}{4G} \times \text{volume}$$

32. Strain energy due to bending,

$$U = \int \frac{M^2 dx}{2EI}$$

33. AE = Axial Rigidity

= kN, N

$= MLT^{-2}$

34. Strain energy due to shear force,

$$U = \int \frac{V^2 dx}{2GA}$$

35. GA = shear rigidity

$= N$

$= MLT^{-2}$

36. GJ = Torsional rigidity

$= N-m^2$

$= ML^3T^{-2}$

37. EI = Flextural rigidity

$= N-m^2$

$= ML^3T^{-2}$

38. $\frac{AE}{L}$ = Axial stiffness

$= N/m$

$= ML^0T^{-2}$

39. $\frac{GA}{L}$ = shear stiffness

$= N/m$

$= ML^0T^{-2}$

40. $\frac{EI}{L}$ = Flextural stiffness

$= N-m$

$= ML^2T^{-2}$

41. $\frac{GJ}{L}$ = Torsional stiffness

$= N-m$

$= ML^2T^{-2}$

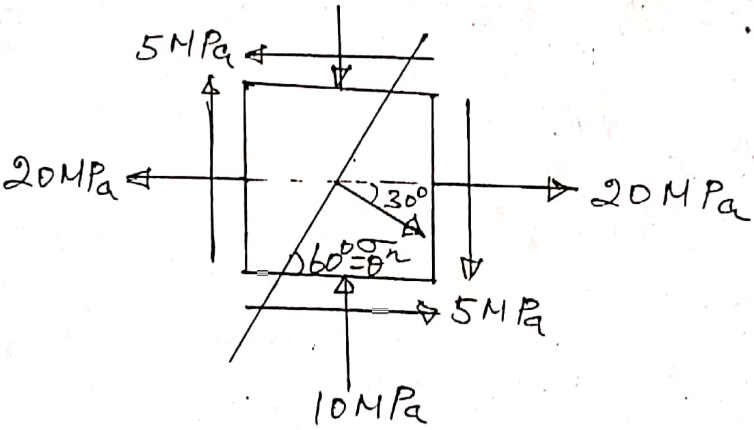
*

42. Effect due to impact load stress

= 2 × Effect due to st

Numericals :→

1. Determine Normal & tangential stress along an inclined plane which is subjected to two tr stresses 20 MPa tensile along x-direction and 10 MPa compressive along y-direction. The shear stress is 5 MPa. The Normal stress on the inclined plane makes an angle 30° with the direction of tensile stress & also determine resultant stress & its direction.



$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \sin 2\theta$$

$$= +20 \sin^2 60^\circ + (-10) \cos^2 60^\circ + 5 \sin 2\times 60^\circ$$

·5 2·5

$$= +16.83 \text{ MPa}$$

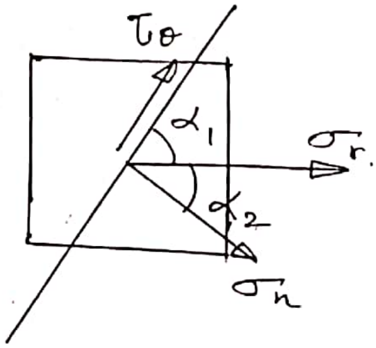
$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$$

$$= \frac{20-(-10)}{2} \sin 2\times 60 + 5 \cos 2\times 60$$

$$= +10.49 \text{ MPa.}$$

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_\theta^2}$$

$$= 19.83 \text{ MPa.}$$



$$\alpha_1 = \tan^{-1}\left(\frac{\sigma_n}{\tau_\theta}\right)$$

$$= 58.07^\circ$$

$$\alpha_2 = \tan^{-1}\left(\frac{\tau_\theta}{\sigma_n}\right)$$

$$= 31.93^\circ.$$

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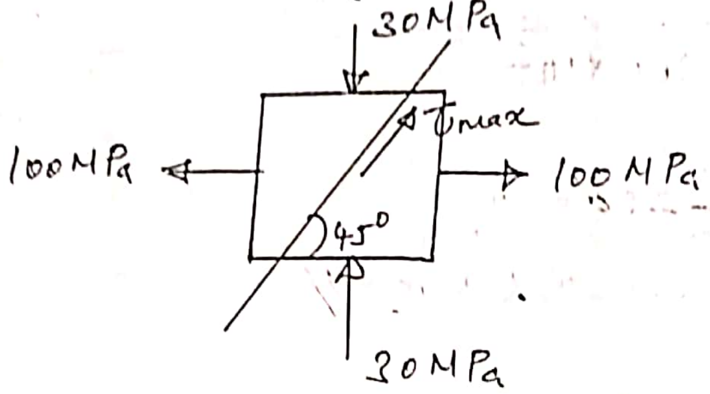
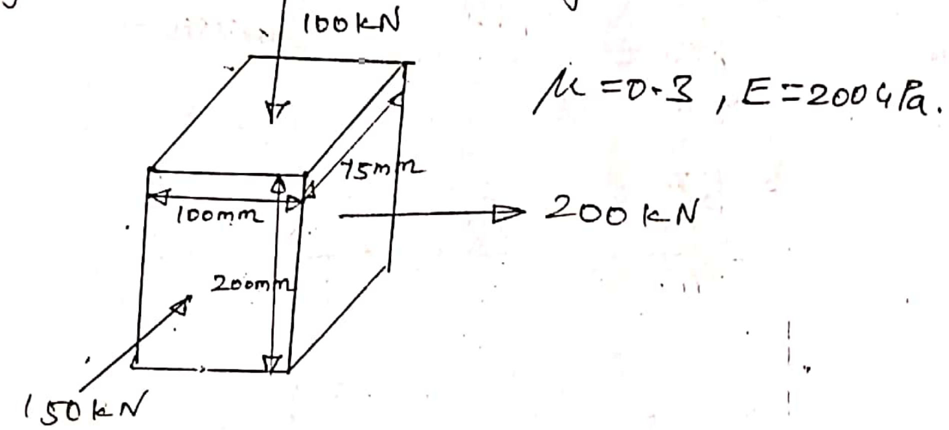
2a. Determine maximum shear stress & its plane of a loaded member as shown in figure below.



$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2} = \frac{100-(-30)}{2}$$

$$= 65 \text{ MPa}.$$

3. Determine change in volume of an object as shown in figure below.



$$\epsilon_v = \frac{\cancel{200}\ \cancel{100}}{\cancel{75 \times 100}}$$

$$\Delta V = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1-2\mu) \times l \times b \times d$$

$$\sigma_x = \frac{200 \times 10^3}{200 \times 75} = 13.33 \text{ MPa}$$

$$\sigma_y = -\frac{100 \times 10^3}{100 \times 75} = -13.33 \text{ MPa}$$

$$\sigma_z = \frac{150\times10^3}{100\times200} = -7.5\ MPa.$$

$$\Delta V = \frac{(13.33-13.33-7.5)}{200\times10^3}\times(1-2\times0.3)\times100\times200\times75$$

$$= -2225\ mm^3$$

$$\Delta V = -22.5\ mm^3$$

4. Determine change in length in x-direction if $\sigma_x = 100\ MPa$ tensile & $\sigma_y = 50\ MPa$ compressive. take $\frac{1}{m} = 0.3$ & total length 700mm.

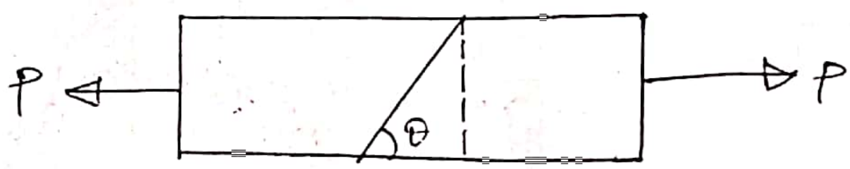
$$\epsilon_x = \frac{\overset{\text{tension}}{\sigma_x}}{E} - \frac{\overset{\text{compressive}}{\sigma_y}}{mE}$$

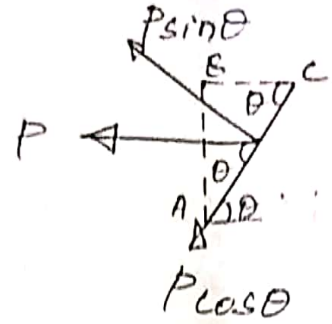
$$\frac{\delta l}{700} = \frac{100}{2\times10^5} + \frac{50}{2\times10^5}\times0.3$$

$$\delta l = 0.4025\ mm$$

$$\epsilon_x = 5.75\times10^{-4}$$

5. Eg # Determine the Normal stress developed on a plane as shown in figure below.





$F_n = P \sin\theta$

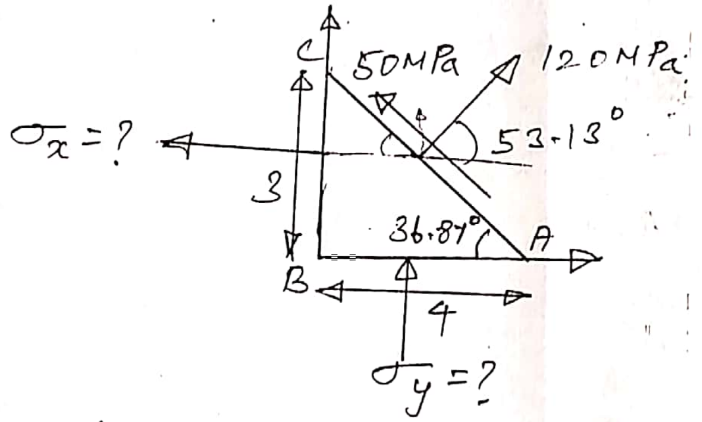
$$\sigma_n = \frac{F_n}{A} = \frac{P \sin\theta}{AC \times 1} \text{ — unit thickness}$$

$$= \frac{\sigma_x \times AB \times 1 \times \sin\theta}{AC}$$

$$= \sigma_x \sin\theta \times \sin\theta$$

$$\boxed{\sigma_n = \sigma_x . \sin^2\theta}$$

6. Determine the Normal stresses on two ⊥r direction as shown in figure below.



$\Sigma H = 0.$

$$\sigma_x \times 3 \times 1 + 50 \times 5 \times 1 \cos 36.87^0 - 120 \times 5 \times 1 \cos 53.13^0 = 0$$

$$3\sigma_x = -199.99 + 360.00$$

$$\sigma_x = +53.34 \text{ MPa (Tensile)}$$

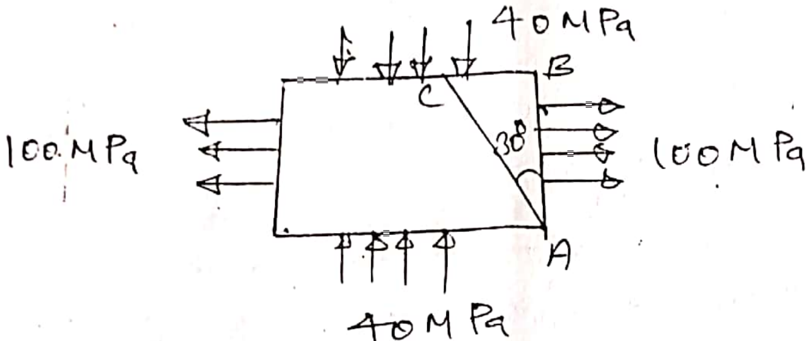
$\Sigma V = 0_y$

$\sigma_y \times 4 \times 1 + 50 \times 5 \times 1 \times \sin 36.87^0 + 120 \times 5 \times 1 \times \sin 53.13^0 = 0$

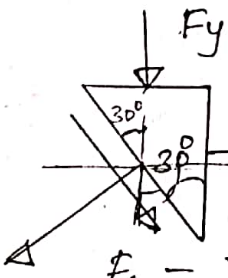
$4\sigma_y = -150.00 - 479.99$

$\sigma_y = -157.499$ MPa (Tensile)

7. Determine resultant stress on a given plane as shown in figure below.



$F_y = 40 \times BC \times 1$



$F_x = 100 \times AB \times 1$

$F_s = \tau_\theta \times AC \times 1$

$F_n = \sigma_n \times AC \times 1$

ΣF along plane, $= 0$;

$\tau_\theta \times AC \times 1 + 40 \times BC \times 1 \cos 30^0 + 100 \times AB \times 1 \times \cos 60 = 0.$

$AC\,\tau_\theta = -40\,BC \cos 30^0 - 100\,AB \cos 60^0$

$\tau_\theta = -40 \cos 30 \sin 30 - 100 \cos 60 \cos 30$

$= -17.32 - 43.30$

$\tau_\theta = -60.62$ MPa (direction reversed)

$\Sigma F_{\perp r\ to\ plane} = 0,$

$$\sigma_n \times AC \times 1 = -40 \times BC \times 1 \times \sin 30^\circ + 100 \times AB \times 1 \times \sin 60$$

$$\sigma_n = -40 \frac{BC}{AC} \sin 30^\circ + 100 \frac{AB}{AC} \sin 60^\circ$$

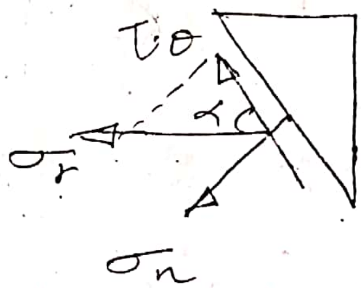
$$= -40 \sin 30^\circ \times \sin 30^\circ + 100 \cos 30^\circ \sin 60^\circ$$

$$= -10 + 75$$

$$\sigma_n = 65\ MPa$$

$$\sigma_r = \sqrt{65^2 + 60.62^2}$$

$$\sigma_r = 88.88\ MPa$$



$$\alpha = \tan^{-1}\left(\frac{65}{60.62}\right)$$

$$= \cancel{66}.\ 46.99^\circ$$

* 8. A metallic bar having size ~~200~~ 300 mm in length & 40 mm square in cross section is subjected an axial load of 160 kN. The change in length is 0.12 mm & change in breadth is 0.005 mm. Determine,

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i) Young's modulus

ii) poisson ratio

iii) Bulk modulus

iv) Modulus of rigidity.

v) Volumetric strain

vi) change in volume.

$$\mu = \frac{\delta b/b}{\delta l/l} = \frac{0.005/40}{0.12/\cancel{6000}\ 300} = \cancel{0.2}$$

$$= \frac{1.25\times10^{-4}}{6\times10^{-5}}$$

$$= \cancel{2.08}\ \cancel{0.625} = 0.3125$$

$$f = \frac{P}{A} = \frac{160\times10^{3}}{40\times40} = 100\,MPa$$

$$E = \frac{f}{e} = \frac{100}{0.12/\cancel{6000}\ 300} = \cancel{16.67}\ 2.5 \times10^{5}\,MPa$$

$$E = 3K(1-2\mu)$$

$$2.5\times10^{5} = 3k(1-2\times0.3125)$$

$$k = 2.22\times10^{5}\,MPa$$

$$E = 2G(1+\mu)$$

$$2.5\times10^{5} = 2G(1+0.3125)$$

$$G = 0.95\times10^{5}\,MPa$$

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

$$= \frac{100+0+0}{25\times10^5}(1-2\times0.3125)$$

$$\epsilon_v = 1.5\times10^{-4}$$

$$\epsilon_v = \frac{\Delta v}{V}$$

$$\Delta v = \epsilon_v \times V$$

$$= 1.5\times10^{-4}\times 40\times40\times300$$

$$= 72\ mm^3$$ (increase in volume)

*****

III · PRINCIPAL STRESSES & THEORIES OF FA

1. The principal stress is the Normal Stress acting on any plane where shear stress is zero

2. The plane where only Normal stress acts [& Tangential stress = 0] is called The principal plane.

3. There are two principal planes:-
   a) Major principal plane
   b) Minor principal plane

4. The principal planes always meet at Right angle to each other [90°]

** 5. The maximum shear stress is equal to

$$= \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$$

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}}$$

* 6. The plane along which maximum shear stress [Tangential stress] acts makes an angle 45° from the principal plane

7. To determine the Location of principal plane, the tangential stress [shear stress] [τ_0] is made equal to zero. from where 'θ'- can be determined.

$$\boxed{\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}}$$

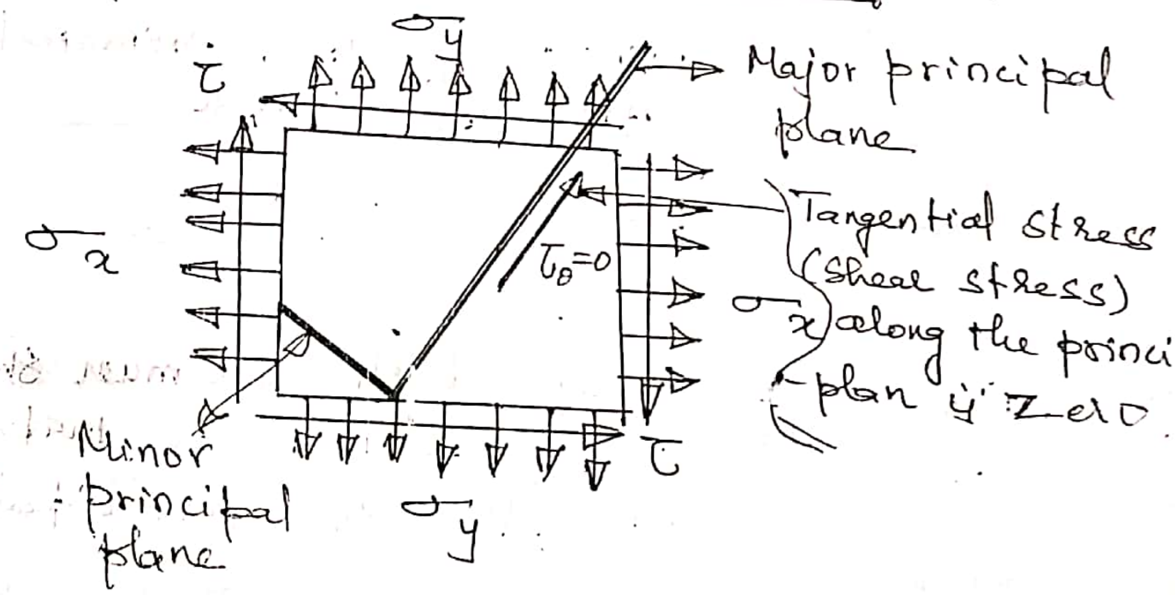
**
8. The major & minor principal stresses are given by formula:—

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$\frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$ — Radius of Mohr Circle.

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

9. Location of principal planes:→



$$\tau_\theta = 0$$

$$\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta = 0.$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{-\tau}{(\sigma_x - \sigma_y)/2}$$

$$\boxed{\tan 2\theta = \frac{-2\tau}{(\sigma_x - \sigma_y)}}$$

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}.$$

$$180 - 2\theta = \tan^{-1}\left(\frac{2\tau}{\sigma_x - \sigma_y}\right)$$

$$\boxed{\theta = 90 - \frac{1}{2}\tan^{-1}\left(\frac{2\tau}{\sigma_x - \sigma_y}\right)}$$

10. a) If $\sigma_x > \sigma_y$, $\theta_{p.p} = 45^0$ to 90^0 from σ_x-direction

b) If $\sigma_x = \sigma_y$, $\theta_{pp} = 45^\circ$ from σ_x direction

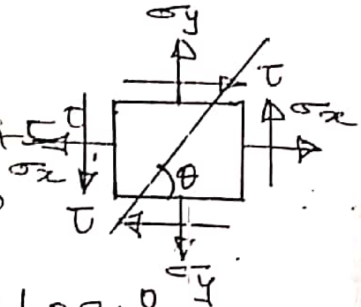
c) If $\sigma_x < \sigma_y$, $\theta_{pp} = 0$ to 45^0 from σ_x-direction

If τ - Reverse direction,

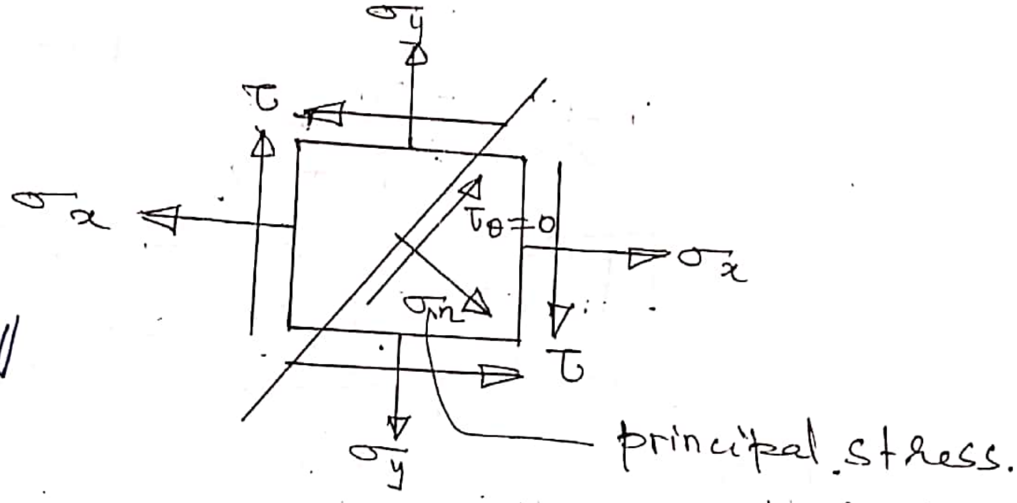
a) $\sigma_x > \sigma_y$, $\theta_{p.p} = 0$ to 45^0

b) $\sigma_x = \sigma_y$, $\theta_{pp} = 45^0$

c) $\sigma_x < \sigma_y$, $\theta_{pp} = 45^0$ to 90^0 from σ_x-direction



10. The Magnitude of principal stress [Analytically]



$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \sin 2\theta$$

$$= \sigma_x \left[\frac{1 - \cos 2\theta}{2}\right] + \sigma_y \left[\frac{1 + \cos 2\theta}{2}\right] + \tau \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau \sin 2\theta$$

$$\tan 2\theta = \frac{-2\tau}{(\sigma_x - \sigma_y)} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin 2\theta = \frac{-2\tau}{\pm\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

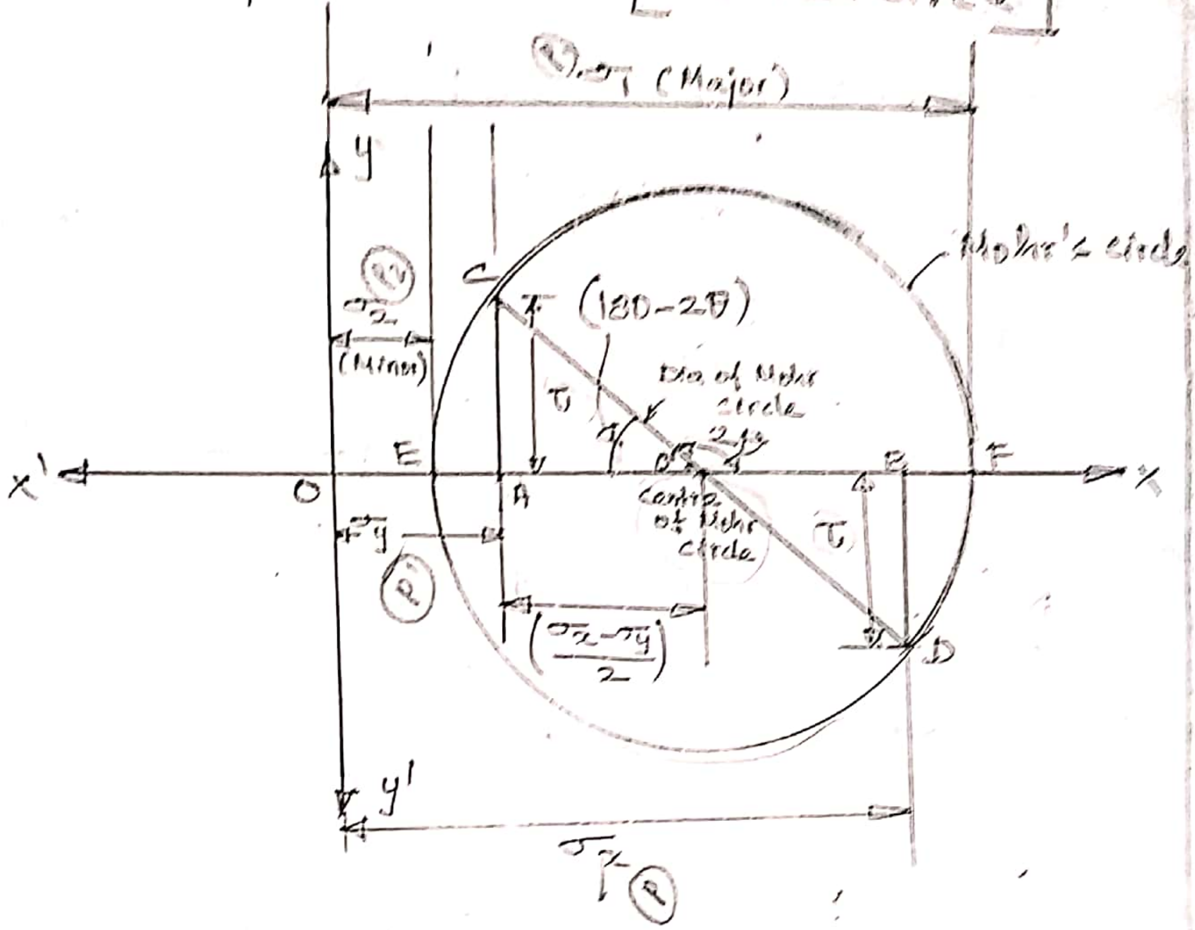
$$\left[\text{hyp} = \pm\sqrt{(-2\tau)^2 + (\sigma_x - \sigma_y)^2} = \pm\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}\right]$$

$$\cos 2\theta = \frac{(\sigma_x - \sigma_y)}{\pm\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} - \left(\frac{\sigma_x-\sigma_y}{2}\right) \times \frac{\sigma_x-\sigma_y}{\pm\sqrt{4\tau^2+(\sigma_x-\sigma_y)^2}}$$

$$+ \tau \times \frac{-2\tau}{\pm\sqrt{4\tau^2+(\sigma_x-\sigma_y)^2}} \times 2 +$$

Graphical method [Mohr's Circle]



Minor principal stress,

$$OE = \sigma_2 = OO' - O'E$$

$$O'E = \sqrt{\tau^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$O'C = \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

= Radius of Mohr circle.

$$OE = \sigma_2 = \sigma_y + \frac{\sigma_x - \sigma_y}{2} - \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\boxed{\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

Major principle stress,

$$OF = \sigma_1 = OO' + O'F$$

$$= \sigma_y + \frac{\sigma_x - \sigma_y}{2} + \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\boxed{\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

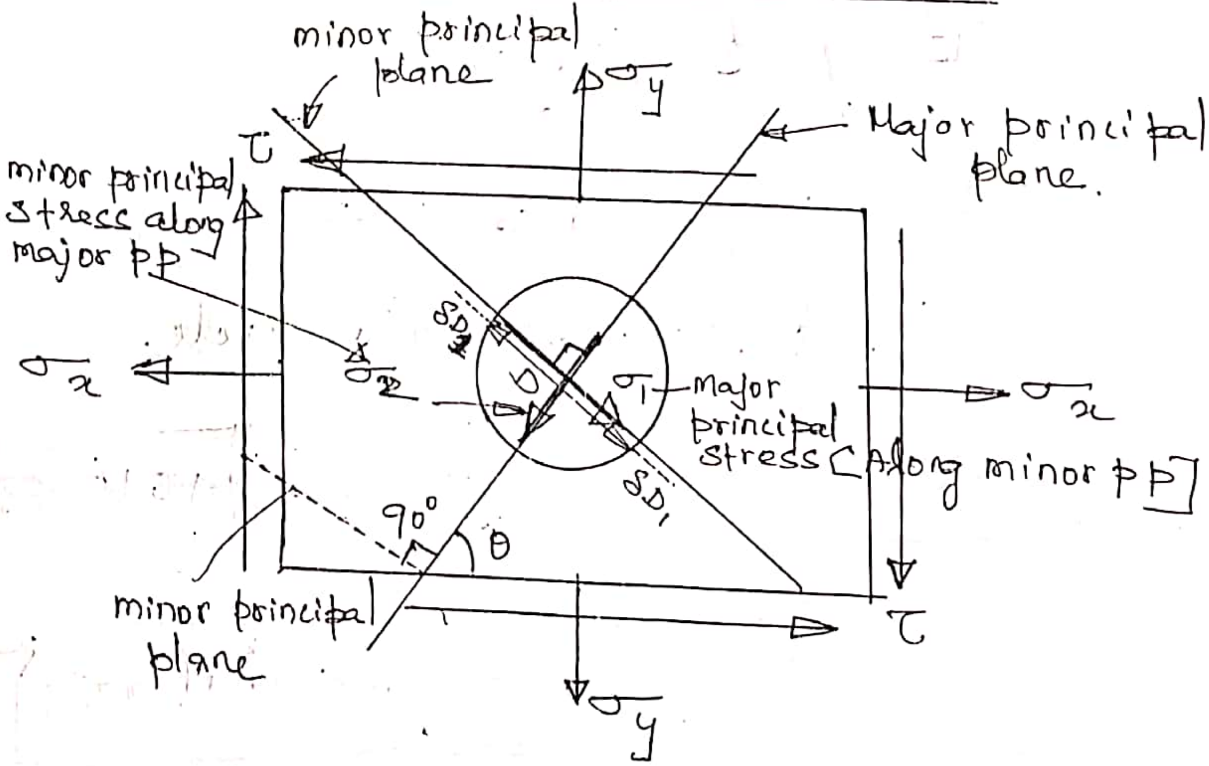
$\triangle^{le}$ O'CA,

$$\tan(180 - 2\theta) = \frac{\tau}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

$$\boxed{\tan(180 - 2\theta) = \frac{2\tau}{(\sigma_x - \sigma_y)}}$$

11-10-10

Circular Hole Become Ellipse:-



Major principal strain,

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$\frac{\delta D_1}{D} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

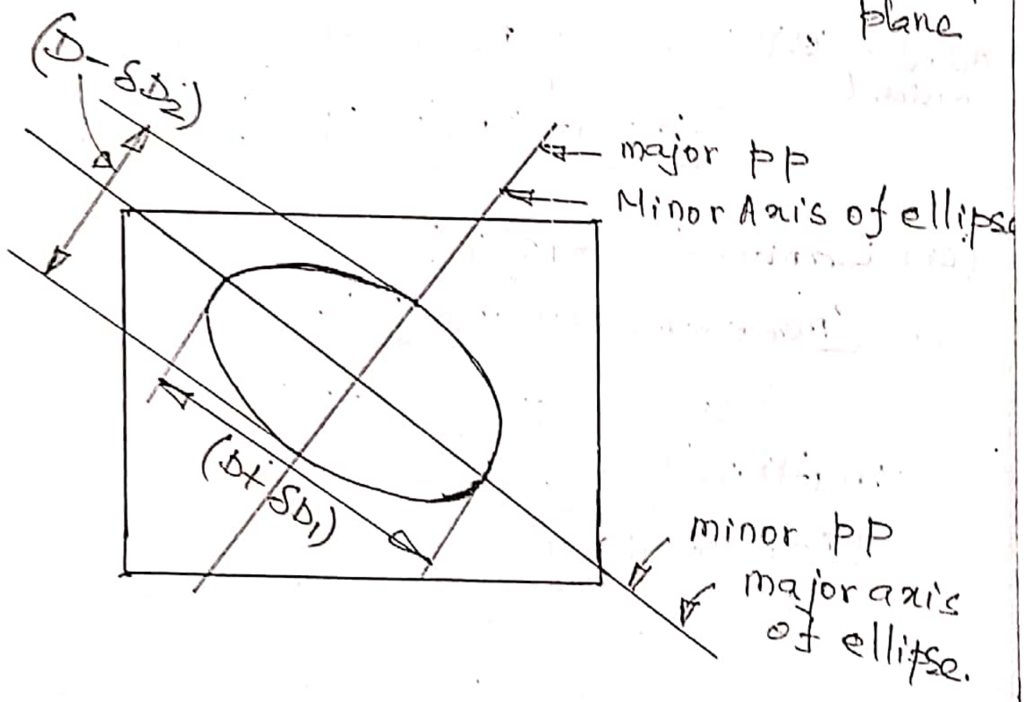
$$\boxed{\delta D_1 = D\left[\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}\right]} \Rightarrow \text{Along minor principal plane}$$

Minor principal strain,

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$\frac{\delta D_2}{D} = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$\boxed{\delta D_2 = D\left[\frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}\right]} \Rightarrow \text{Along major principal plane}$$



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THEORIES OF FAILURE :→

There are five theories of failure:-
[Based on Elastic condition of loading]
[Hooke's Law is obeyed]

Brittle material — (a) Rankine's theory
[Maximum principal stress theory]

Brittle — (b) Saint vinent's theory
[Maximum principal strain theory]

Ductile material — (c) Tresca or, Guest theory
[Maximum shear stress theory]

Ductile — (d) Haig's theory
[Total strain energy theory]

General material — (e) Von misses theory
[Shear-strain energy theory]

(a) Rankine's theory :→
[Maximum principal stress theory]

It is mainly applicable for Brittle material [$f_y = 0.002\epsilon$] [Do not have distinct yield point]

$$\sigma_1 > \sigma_0$$

$$\sigma_1 > \frac{f_y}{F.O.S} \rightarrow \text{for failure}$$

For no failure,

$$\sigma_1 \le \sigma_0$$

$$\boxed{\frac{\sigma_x+\sigma_y}{2} + \frac{1}{2}\sqrt{4\tau^2+(\sigma_x-\sigma_y)^2} \le \frac{f_y}{F.O.S}}$$

(b) Saint Vinent theory

[Maximum principal strain theory]

It is mainly applicable for General material [Neither Brittle nor Ductile]

$\epsilon_1 > \epsilon_0$, for failure

For No failure,

$$\epsilon_1 \le \epsilon_{0\,(\text{permissible})}$$

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \le \frac{\sigma_0}{E} \rightarrow \text{As per Hooke's law.}$$

$$\sigma_1 - \mu\sigma_2 \le \sigma_0$$

$$\boxed{\sigma_1 - \mu\sigma_2 \le \frac{f_y}{F.O.S}}$$

(c) Tresca or, Guest theory :-

[Maximum shear stress theory]

It is mainly applicable for Ductile material.

$\tau_{max} > \tau_{0\,(perm)}$ — for failure for Safety, No failure,

$$\tau_{max} \le \tau_{0\,(perm)}$$

$$\frac{\sigma_1-\sigma_2}{\cancel{2}} \le \frac{\sigma_0}{\cancel{2}}$$

$$\sigma_1-\sigma_2 \le \sigma_0.$$

$$\boxed{\sigma_1-\sigma_2 \le \frac{f_y}{F.O.S}}$$

(d) Haig theory:-

[Total Strain energy theory]

It is applicable for Ductile material & thick Cylinder.

Equivalent stress;

$$\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-\frac{2}{m}(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)} \le \left[\sigma_0 = \frac{f_y}{FOS}\right]$$

For No failure

(e) Von mises theory:-

[Shear strain energy theory]

For General material

$$\sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}} \le \left[\sigma_0 = \frac{f_y}{FOS}\right]$$

For No failure

Numericals:—

1. A circular hole is drilled in a mild steel plate which is subjected to stresses $\sigma_x = 100$ MPa (tensile), $\sigma_y = 30$ MPa (tensile), $\tau = 40$ MPa. The dia of hole is 300mm. Determine,

(i) Major principal stress & Minor principal stress.

(ii) Maximum principal strain

(iii) principal plane location

(iv) Maximum shear stress

(v) Major axis of ellipse

take $\mu = 0.3$ & $E = 210$ GPa.

(i)

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$= \frac{100+30}{2} \pm \frac{1}{2}\sqrt{4 \times 40^2 + (100-30)^2}$$

$$= 65 \pm 53.15$$

$\sigma_1 = (+)118.15$ MPa (tensile)

$\sigma_2 = (+)11.85$ MPa (tensile)

ii) ~~$\theta_{N/2}$~~

$$\tan(180-2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$= \frac{2\times 40}{100-30}$$

$$\theta_1 = 65.59^\circ$$

$$\theta_2 = 90^\circ + \theta_1$$

$$\theta_2 = 155.59^\circ$$

iii) $$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{118.15 - 11.85}{2}$$

$$\tau_{max} = 53.15\,MPa$$

[Radius of Mohr Circle]

(iv) $$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$= \frac{1}{2.1\times10^5}(118.15 - 0.3\times 11.85)$$

$$\epsilon_1 = 5.457\times10^{-4}$$

$$\frac{\delta D_1}{D} = 5.457\times10^{-4}$$

$$\delta D_1 = 5.457\times10^{-4}\times 300$$

$$\delta D_1 = 0.164\,mm$$

Major axis of Ellipse,

$$D_1 = D + \delta D_1$$

$$= 300 + 0.164$$

$$= 300.164 \text{ mm}.$$

Minor axis of ellipse,

$$\epsilon_2 = (11.85 - 0.3 \times 118.15) \frac{1}{2.1 \times 10^5}$$

$$\frac{\delta D_2}{D} = -1.124 \times 10^{-4}$$

$$\delta D_2 = -1.124 \times 10^{-4} \times 300$$

$$\delta D_2 = -0.0337 \text{ mm}.$$

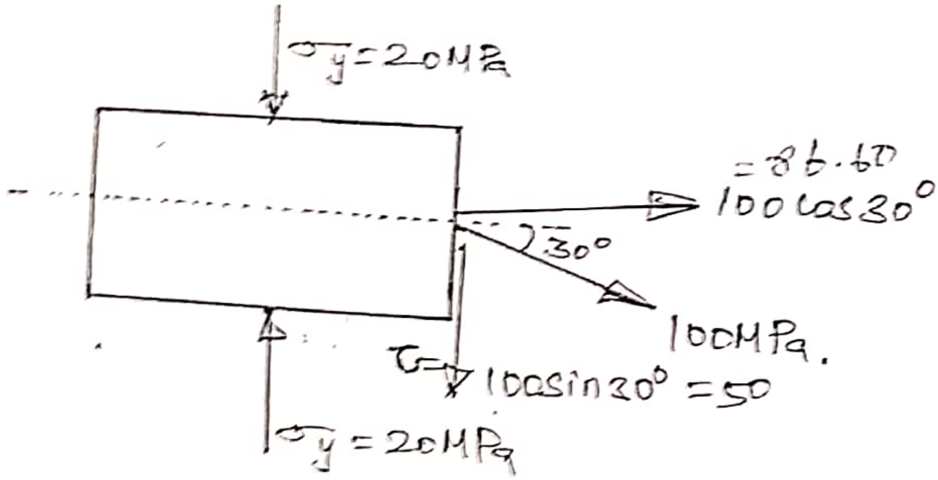
Minor axis of Ellipse,

$$D_2 = D + \delta D_2$$

$$= 300 + (-0.0337)$$

$$D_2 = 299.966 \text{ mm}$$

2. A steel material is subjected to direct stress at an angle 30° with horizontal having tensile nature of value 100MPa the material is subjected to compressive stress 20MPa in ⊥r direction. Determine principal stresses, maximum shear stress & principal planes.



$$\sigma_{1,2} = \frac{100\cos 30^\circ + (-20)}{2} \pm \frac{1}{2}\sqrt{4\times(100\sin 30)^2 + [100\cos 30^\circ - (-20)]^2}$$

$= 33.30 \pm 73.08$

$\sigma_1 = 106.38$ MPa (tensile)

$\sigma_2 = -39.78$ (compressive)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 73.08 \ MPa$$

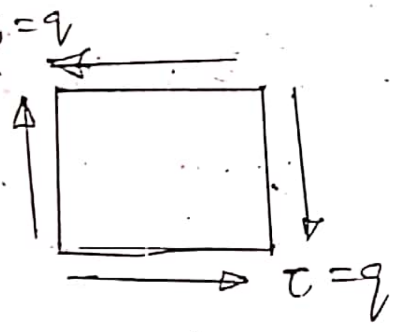
at 45° from P.P

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$= \frac{2\times 50}{86.60 - (-20)}$$

$\theta = 68.42^\circ$

3. A steel material is subjected to pure shear [No Bending] 50MPa. Determine principal stresses, principal plane & Dia of Mohr's circle,



$$\sigma_{1,2} = \frac{0+0}{2} \pm \frac{1}{2}\sqrt{4\times 50^2 + (0-0)^2}$$

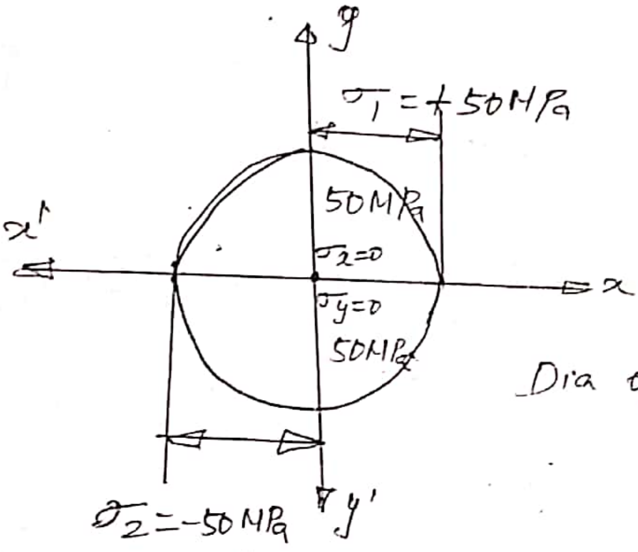
$$= 0 \pm 50$$

$$\sigma_1 = +50\,MPa$$

$$\sigma_2 = -50\,MPa$$

$$\tan(180 \cdot 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2\tau}{0} = \infty$$

$$\theta = 45^0$$



Dia of Mohr's circle $= 100MPa$

$= 2\tau_{max}$

$= 2 \times R_{mohr}$

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4. Determine principal strains & Dia of Mohr circle of strain if $\epsilon_x = 2\times10^{-3}$, $\epsilon_y = 3\times10^{-4}$, $\phi_{xy} = 10^{-4}$.

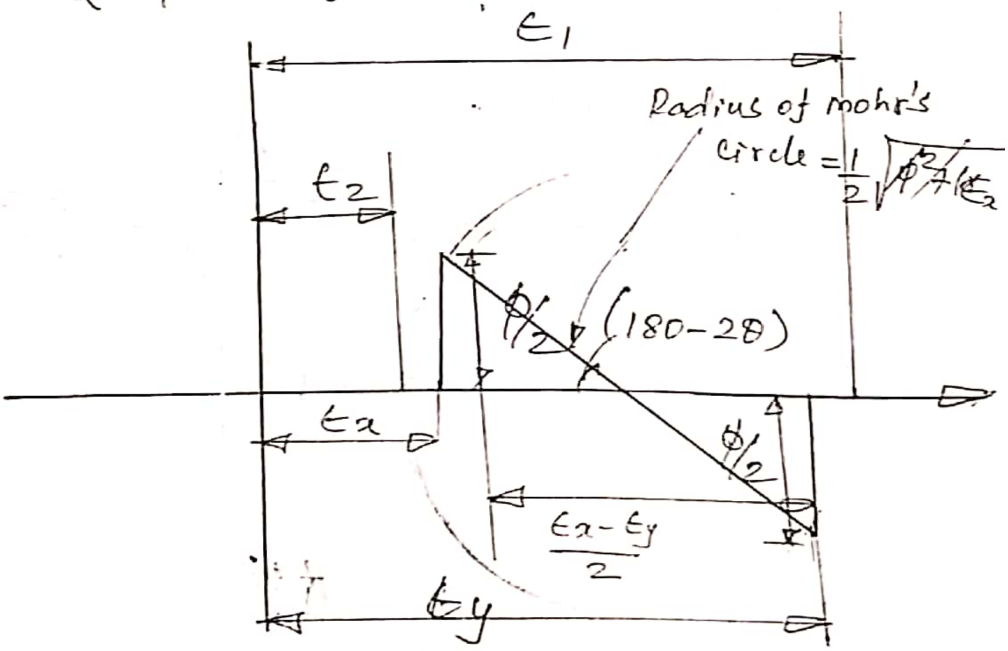
$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2}\sqrt{\phi_{xy}^2 + (\epsilon_x - \epsilon_y)^2}$$

$$= \frac{2\times10^{-3} + 3\times10^{-4}}{2} \pm \frac{1}{2}\sqrt{(10^{-4})^2 + (2\times10^{-3} - 3\times10^{-4})^2}$$

$$= 1.15\times10^{-3} \pm 8.515\times10^{-4}$$

$$\epsilon_1 = 2.00\times10^{-3}$$

$$\epsilon_2 = 2.985\times10^{-4}$$



Radius of Mohr's circle $= \frac{1}{2}\sqrt{\phi^2 + (\epsilon_x - \epsilon_y)^2}$

$$= 8.515\times10^{-4}$$

$$\text{Dia} = \sqrt{\phi^2 + (\epsilon_x - \epsilon_y)^2}$$

$$= 1.703\times10^{-3}$$

$$\tan(180-2\theta) = \frac{\phi/2}{(\epsilon_x-\epsilon_y)/2}$$

$$\tan(180-2\theta) = \frac{\phi}{(\epsilon_x-\epsilon_y)}$$

$$= \frac{10^{-4}}{(2\times10^{-3}-3\times10^{-4})}$$

$$\theta = 88.32^\circ$$

5. A steel material is subjected to stresses, $\sigma_x = 90$ MPa, (tensile), $\sigma_y = 40$ MPa (compre) & $\tau = 30$ MPa. Determine the F.O.S if yield stress 270 MPa using Haig theory. take $\mu = 0.3$

$$\sigma_{1,2} = \frac{90+(-40)}{2} \pm \frac{1}{2}\sqrt{30^2\times4+(90-(-40))^2}$$

$$= 25 \pm 71.59$$

$$\sigma_1 = 96.59 \text{ MPa}.$$

$$\sigma_2 = -46.59 \text{ MPa}.$$

Haig theory,

$$\sqrt{96.59^2+(-46.59)^2-\frac{2\times0.3}{\cancel{0.2}}(96.59\times(-46.59)) + 0+0} \le \frac{\sigma_y}{F.os.}$$

$$\frac{119.17}{\cancel{208.72}} \le \frac{270}{F.O.S}$$

$$F.O.S \le 2.27$$

6. A steel material is subjected to principal stresses 80 MPa tensile & 30 MPa compressive,

(i) equivalent stress based on saint vinent theory & Tresca theory, also Determine F.O.S based on Rankine theory & Von mises theory, take $\mu = 0.3$ & yield stress $= 270$ MPa.

Saint Vinent,

$$\sigma_1 - \mu\sigma_2 \le \cancel{\frac{f_y}{F.O.S}}\ \sigma_0$$

$$80 - 0.3\times(-30) \le \cancel{270}\ \sigma_0$$

$$\sigma_0 \ge 89 \text{ MPa.}$$

Tresca,

$$\sigma_1 - \sigma_2 \le \sigma_0$$

$$80 - (-30) \le \sigma_0.$$

$$\sigma_0 \ge 110 \text{ MPa.}$$

Rankine,

$$\sigma_1 \le \sigma_o$$

$$+80 \le \frac{270}{F}$$

$$F \le 270/80$$

$$F.O.S \le 3.375$$

Von mises,

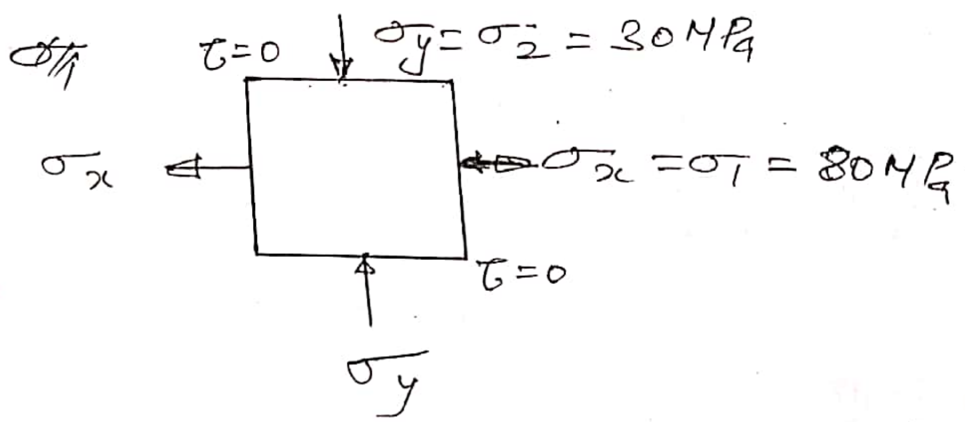
$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]} \le \frac{f_y}{F.O.S}$$

$$\sqrt{\frac{1}{2}\left[(80-(-30))^2+(-30-0)^2+(0-80)^2\right]} \le \frac{270}{F.O.S}$$

$$F.O.S \le \frac{270}{98.49}$$

$$F.O.S \le 2.741$$

7. If σ_2 is 30 MPa (compressive) & σ_1 is 80 MPa (tensile). Determine σ_x & σ_y, if $\tau = 0$.

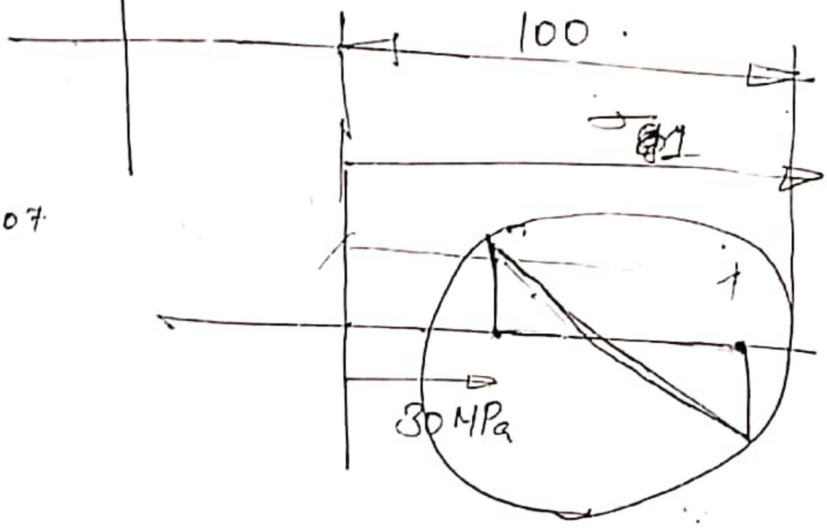


8. If $\sigma_1 = 100$ MPa (tensile), $\sigma_y = 30$ MPa (Comp), $\tau = 20$ MPa. Determine minor principal stress.

$$100 = \frac{\sigma_x + (-30)}{2} + \frac{1}{2}\sqrt{4\times 20^2 + (\sigma_x - 30)^2}$$

$$= \frac{\sigma_x}{2} - 15 + \frac{1}{2}\sqrt{1600 + (\sigma_x - 30)^2}$$

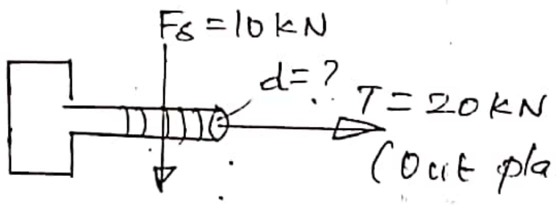
$\sigma_2^{(-)} = 33.07$



9. A mild bolt is subjected to transverse shear 10kN & axial tension 20kN. yield stress is 270MPa & F.O.S = 3. Determine Dia of bolt by using,

1) Rankine theory
2) Vinent theory
3) Tresca theory
4) Haig theory
5) Von misses theory,   take $\mu = 0.3$

$E = 210\,GPa$.



$$\tau = \frac{F_s}{A} = \frac{10\times10^3}{\pi/4 \times d^2} =$$

$$\sigma_a = \frac{T}{A} = \frac{20\times10^3}{\pi/4 \times d^2}$$

$\sigma_y = 0$

$$\left(\frac{\tau_{vf}}{\tau_d}\right)^2 + \left(\frac{\sigma_{at}}{\sigma_d}\right)^2 \le 1$$

$$\sigma_{1,2} = \frac{20000}{\frac{\pi}{4}\times d^2} \times \frac{1}{2} \pm \frac{1}{2}\sqrt{4\times\left(\frac{10000}{A}\right)^2 + \left(\frac{20000}{A}\right)^2}$$

$$= \frac{10000}{A} \pm \frac{28284.27}{2}/A$$

$\sigma_1 = 24142.14/A.$

$\sigma_2 = -4142/A$

1) Rankine,

$\sigma_1 \le \sigma_0.$

$$\frac{24142}{\cancel{A}\ \pi/4 d^2} \le \frac{270}{F.O.S = 3}.$$

$d \ge 18.48$ mm.

2) Vienant theory,

$(\sigma_1 - \mu\sigma_2) \le (\sigma_0 = f_y/FOS)$

$$\left[\frac{24142}{\cancel{A}\ \pi/4 d^2} - 0.3 \times \left(-\frac{4142}{A}\right)\right] \le \frac{270}{3}.$$

$d \ge 18.95$ mm.

3) Tresca,

$\sigma_1 - \sigma_2 \le \sigma_0.$

$$\frac{24142}{A} - \left(-\frac{4142}{A}\right) \le \frac{270}{3}$$

$d \ge 20.00$ mm

4) Haig theory,

$$\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)} \le \sigma_0$$

$$\sqrt{\left(\frac{24142}{A}\right)^2+\left(-\frac{4142}{A}\right)^2+0-2\times0.3\left(\frac{24142}{A}\right)\left(-\frac{4142}{A}\right)+0+0} \le \sigma_0$$

$$\frac{25690.27}{\pi/4\, d^2} \le \frac{270}{3}$$

$$d \ge 19.06\,mm$$

5) Vonmises theory,

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]} \le \frac{f_y}{F}$$

$$\sqrt{\frac{1}{2}\left[\left[\frac{24142}{A}-\left(-\frac{4142}{A}\right)\right]^2+\left(-\frac{4142}{A}\right)^2+\left(-\frac{24142}{A}\right)^2\right]} \le \frac{f_y}{3}$$

$$\frac{26457.3}{\pi/4\times d^2} \le \frac{270}{3}$$

$$d \ge 19.35\,mm.$$

IV. TORSION & PRINCIPAL STRESSES & THEORIES OF FAILURE

1. Torque is the moment applied about the axis of the member [about z-axis] & this phenomenon is called Torsion.
2. The Torque develops shear stress.
3. The torque is represented by symbol 'T'

T = Force × radius

$$\boxed{T = F \times r}$$

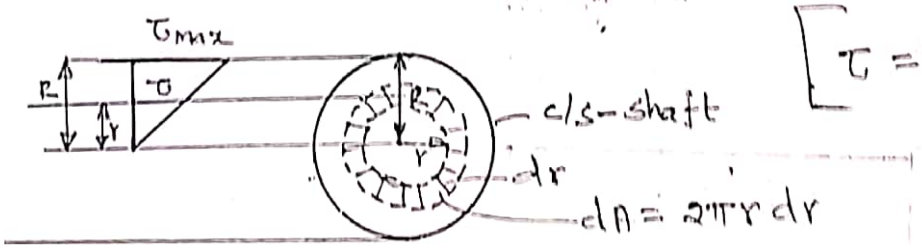
Unit – N-m ; Dimension: ML^2T^{-2}

4. Torsion is mainly used for screw-driver & to transmit power on the shaft.

5. Expression for Torque:–

From 11r Δle

$$\left[\tau = \frac{\tau_{max}}{R} \times r\right]$$



$$dT = dF \times r$$
$$= (\tau \times dA) \times r$$
$$= \tau \times 2\pi r \times dr \times r$$

$$dT = \tau_{max} \times \frac{r}{R} \times 2\pi r dr \times r.$$

$$T = \int_0^R \frac{\tau_{max}}{R} \times 2\pi r^3 dr$$

$$= \frac{2\pi \tau_{max}}{R}\left[\frac{r^4}{4}\right]_0^R = \frac{2\pi \tau_{max} \times R^4}{4R}$$

$$\boxed{T = \frac{\pi \tau_{max} R^3}{2}}$$

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$$R = \frac{D}{2}$$

$$T = \frac{\pi \tau_{max}}{2}\left(\frac{D}{2}\right)^3$$

$$= \frac{\pi \tau_{max}}{2} \times \frac{D^3}{8} \times \frac{D}{D}$$

$$= \frac{\pi}{2} \times \frac{\tau_{max}}{8} \times \frac{D^4}{2R}$$

$$= \frac{\pi}{32} \times \tau_{max} \times D^4 \times \frac{1}{R}$$

$$= \frac{\pi}{32} \times D^4 \times \tau_{max} \times \frac{1}{R} \qquad \left[\frac{\pi}{32} D^4 = J\right]$$

$$T = \tau_{max} \times J \times \frac{1}{R}$$

$$\boxed{\frac{T}{J} = \frac{\tau_{max}}{R}} \quad ***$$

Polar M.I $= I_p = J =$ M.I about zz axis

$$= I_{xx} + I_{yy}$$

$$= \frac{\pi}{64} D^4 + \frac{\pi}{64} D^4$$

$$\boxed{J = I_p = \frac{\pi}{32} D^4}$$ — solid shaft

$$\boxed{J = I_p = \frac{\pi}{32} [D^4 - d^4]}$$ — Hollow shaft

$$T = \frac{\tau_{max}}{R} \times J$$

$$= \frac{\tau_{max}}{\cancel{D}/\cancel{2}} \times \frac{\pi \cancel{D^4}^{\,3}}{\cancel{32}\,16}$$

$$\boxed{T = \frac{\pi}{16} D^3 \tau_{max}}$$

$$\boxed{\tau_{max} = \frac{16T}{\pi D^3}}$$

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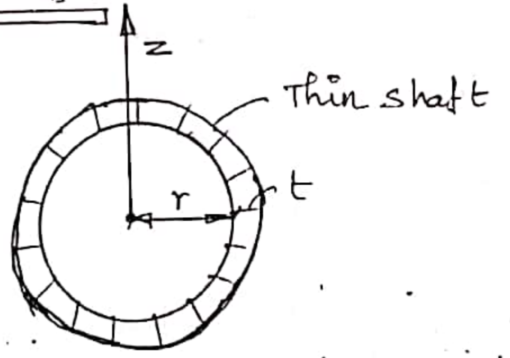
For thin shaft:-



polar M.I = Moment of moment of Area

$= \text{Second moment of area}$

$= (2\pi r t) \times r \times r$

$= 2\pi t r^3$

$= 2\pi t \times \left(\frac{D}{2}\right)^3$

$= 2\pi \frac{t D^3}{8}$

$$\boxed{I_{polar} = \frac{\pi D^3 t}{4}}$$

For thin shaft:-

$$\frac{T}{J} = \frac{\tau_{max}}{R}$$

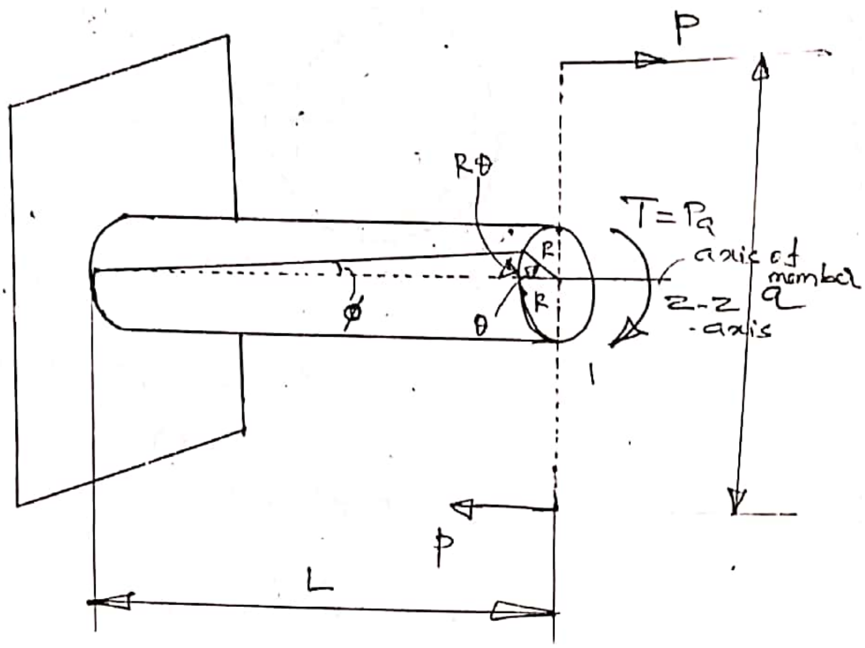
$$\frac{T}{\frac{\pi}{4} D^3 t} = \frac{\tau_{max}}{\frac{D}{2}}$$

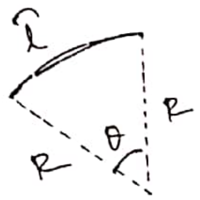
$$\boxed{\frac{T}{\frac{\pi}{4} D^3 t} = \frac{\tau_{max}}{\frac{D}{2}}}$$

Note:-

Incase of Hollow shaft, the shear stress distribution is non-uniform across the thickness. ~~But~~ therefore Law of integration is required. But incase of thin shaft the shear stress distribution is almost uniform. Therefore there is no need of applying integration.

Expression for Angle of twist :-





$$\theta = \frac{l}{R}$$

$$l = R\theta$$

$$\tan\phi = \frac{R\theta}{L}$$

$$\boxed{\phi = \frac{R\theta}{L}} \quad \because \left[\begin{array}{l} \phi = \text{is very small} \\ \tan\phi \simeq \phi \end{array}\right]$$

ϕ = shear strain

shear stress = Shear strain x Modulus of Elasticity

$$\tau = \phi \times G$$

$$\phi = \frac{\tau}{G} \qquad [C = G = N]$$

$$\frac{R\theta}{L} = \frac{\tau}{G}$$

$$** \quad \boxed{\frac{G\theta}{L} = \frac{\tau}{R}}$$

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}}$$

$$\frac{T}{J} = \frac{G\theta}{L}$$

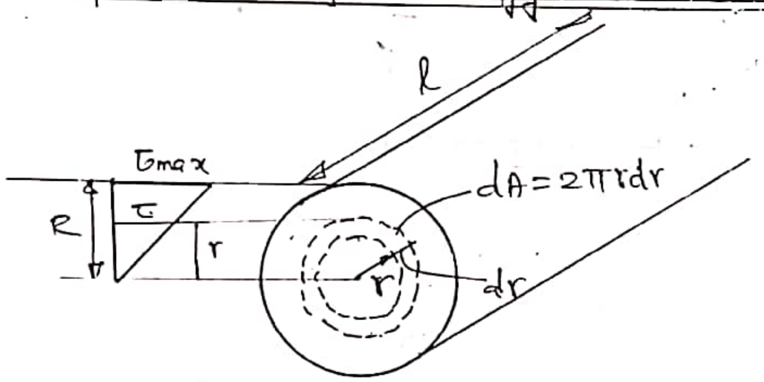
$$\boxed{\frac{TL}{\theta} = GJ} = \text{Tortional Rigidity.}$$

$$\boxed{\frac{T}{\frac{\pi}{32}D^4} = \frac{\tau_{max}}{D/2}} \qquad \boxed{\frac{\tau_{max}}{D/2} = \frac{G\theta}{L}}$$

$D = ?$

Adopt greater value of 'D'.

Expression for Energy due to Torsion:-



$$U = \frac{f^2}{2E} \times \text{volume}$$

$$\partial U = \frac{\tau^2}{2G} \times dV$$

$$= \frac{\tau^2}{2G} \times (dA \times l)$$

$$= \frac{\tau^2}{2G} \times 2\pi r\,dr \times l$$

$$\partial U = \left(\tau_{max} \times \frac{r}{R}\right)^2 \times \frac{2\pi r\,dr \times l}{2G}$$

$$U = \int_0^R \frac{\tau_{max}^2}{\cancel{2}GR^2} \times \cancel{2}\pi r^3 l\,dr$$

$$= \frac{\tau_{max}^2 \times \pi l}{GR^2} \int_0^R r^3\,dr$$

$$= \frac{\pi l \tau_{max}^2}{GR^2} \left[\frac{r^4}{4} \right]_0^R$$

$$U = \frac{\pi l \tau_{max}^2}{GR^2} \times \frac{R^4}{4}$$

$$= \frac{\tau_{max}^2}{4G} \times \pi R^2 \times l$$

$$= \frac{\tau_{max}^2}{4G} \times A \times l$$

$$\boxed{U = \frac{\tau_{max}^2}{4G} \times Volume}$$

$$U = \frac{\tau_{max}^2}{4G} \times Volume$$

Note:- Modulus of resilience

$U = \frac{f^2}{2E} \times volume$ – Axially applied load

$= \frac{\tau^2}{4G} \times Volume$ – Torsion effect ~~Axially loaded member~~

$= \frac{\tau^2}{2G} \times volume$ – ~~Torsion effect~~ Sheal stress (SF)

$= \frac{p^2}{2k} \times volume$ – Volumetric change due to Hydrostatic force (Bulk mod)

$= \int \frac{M^2 ds}{2EI}$ – Bending (flexural)

$= \int \frac{V^2 ds}{2GA}$ – Shear force

EI – flexural rigidity.

GA – shear rigidity.

GJ – Torsional rigidity.

AE – Axial rigidity.

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$$U = \frac{\tau^2_{max}}{4G} \times volume$$

$$\left[\frac{T}{J} = \frac{\tau_{max}}{R}\right]$$

$$= \frac{\left[\frac{T}{J} \times R\right]^2}{4G} \times volume$$

$$= \frac{T^2 R^2}{J^2 \times 4G} \times \pi R^2 \times L$$

$$= \frac{T^2 L}{J^2 \times 4G} \times \pi R^4$$

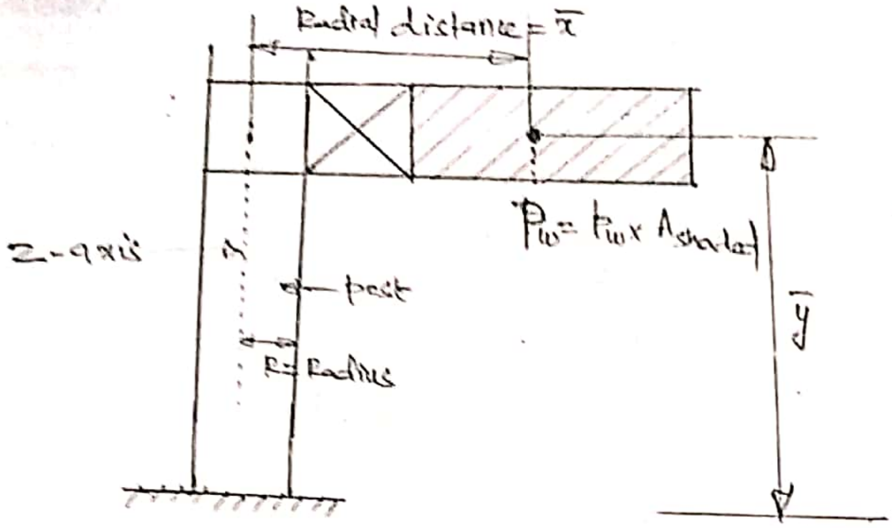
$$= \frac{T^2 L}{\left(\frac{\pi}{32} \times D^4\right)^2 \times 4G} \times \pi \times \left(\frac{D}{2}\right)^4$$

$$= \frac{T^2 L}{J^2 \times 2G} \times \frac{\pi D^4}{32}$$

$$= \frac{T^2 L}{J^2 \times 2G} \times J$$

$$\boxed{U = \frac{T^2 L}{2GJ}}$$

Expression for Equivalent Torque & equivalent Bending moment due to combined effect of Torsion & Bending moment :-



$$P_w = p_w \times A_{shaded}$$

$$\text{Torque}, T = \left[P_w \times \bar{x}\right]$$

$$BM, M = P_w \times \bar{y}$$

Due to torque, shear stress develops,

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \tau = \frac{T}{J} \times R$$

$$= \frac{T \times \frac{D}{2}}{\frac{\pi}{32} D^4}$$

✓ $$\boxed{\tau = \frac{16T}{\pi D^3}}$$

Due to BM, Bending stress develops,

$$\frac{M}{I} = \frac{f}{y}$$

$$f = \frac{M}{I} \times y$$

$$\sigma_x = \frac{M}{\frac{\pi D^4}{64}} \times \frac{D}{2}$$

✓ $$\boxed{\sigma_x = \frac{32M}{\pi D^3}}$$; $\sigma_y = 0$

[other direction]
[fall down]

Principal stress,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$= \frac{\frac{32M}{\pi D^3} + 0}{2} \pm \frac{1}{2}\sqrt{\left(\frac{32M}{\pi D^3} - 0\right)^2 + 4 \times \left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{16M}{\pi D^3} \pm \frac{1}{2}\sqrt{\left(\frac{16}{\pi D^3}\right)^2 \left[(2M)^2 + 4 \times T^2\right.}$$

✓ $$\sigma_{1,2} = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

Major principal stress,

$$\sigma_1 = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

= Resultant Bending stress
[Equating bending stress].

$$\frac{M}{I} = \frac{f}{y}.$$

$$\frac{M_{equ}}{\frac{\pi}{64} D^4} = \frac{\sigma_1}{\frac{D}{2}}$$

$$\frac{32 M_{equi}}{\pi D^3} = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

✓ $$\boxed{M_{equi} = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \text{Radius of Mohr's circle}$$

$$= \left[\frac{16}{\pi D^3}\left[M + \sqrt{M^2+T^2}\right] - \frac{16}{\pi D^3}\left[M - \sqrt{M^2+T^2}\right]\right] \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{16}{\pi D^3}\left[2\sqrt{M^2+T^2}\right]$$

$$\tau_{max} = \frac{16}{\pi D^3}\left[\sqrt{M^2+T^2}\right] \quad \text{—} ①$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{T_{equi}}{\frac{\pi D^4}{32}} = \frac{\tau_{max}}{\frac{D}{2}}$$

$$\tau_{max} = \frac{16 T_{equ}}{\pi D^3} \quad \text{—} ②$$

$$① = ② \Rightarrow$$

$$\frac{16 T_{equi}}{\pi D^3} = \frac{16}{\pi D^3}\left[\sqrt{M^2+T^2}\right]$$

✓ $$\boxed{T_{equi} = \sqrt{M^2+T^2}}$$

power transmitted by shaft :-

$$\text{Power} = \frac{W.D}{\text{time}} = \frac{F \times \text{distance}}{t}$$

$$= F \times v_{(Linear)}$$

$$= F \times r\omega$$

$$\text{power} = T\omega$$

$$\boxed{\text{power} = \frac{2\pi NT}{60}}$$

Theories of Failure :—

1. Rankine's theory :– [Maximum principal stress theory]
[Brittle material]

$$\sigma_1 \leq \sigma_0$$

$$\frac{16}{\pi D^3}\left[M + \sqrt{M^2+T^2}\right] \leq \left[\sigma_0 = \frac{f_y}{FOS}\right]$$

2. Saint vinent theory :– [Maximum principal strain theory]

$$\epsilon_1 \leq \epsilon_0$$

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \leq \frac{\sigma_0}{E}$$

$$\sigma_1 - \mu\sigma_2 \leq \frac{f_y}{FOS}$$

3. Tresca, or, Guest Theory :– [Ductile material]
[Maximum shear stress theory]

$$\frac{\sigma_1 - \sigma_2}{2} \leq \frac{\sigma_0}{2}$$

$$\sigma_1 - \sigma_2 \leq \frac{f_y}{FOS}$$

4. Haig theory :– [Total strain energy theory]
[Ductile material] [thin cylinder]

$$\sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2)} \leq \sigma_0 = \frac{f_y}{FOS}$$

5. Von misses theory :– [~~Max~~ shear strain energy]
[Distortion]

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]} \leq \left[\sigma_0 = f_y\right.$$

Numericals :→

1. A solid shaft has to transmit mean power of 1000kW at 300 rpm. The maximum shear stress is 60MPa. Determine the dia of shaft if the maximum torque is 25% more than the mean torque. Also determine % saving in material if Hollow shaft having inner dia 0.6 times of outer is used.

$$\frac{T}{J} = \frac{\tau}{R} \quad ; \quad P = \frac{2\pi N T_{mean}}{60}$$

$$1000 \times 10^3 = \frac{2\pi \overset{300}{N} T_{mean}}{60}$$

$$T_{mean} = 31830.99 \text{ N-M}$$

$$T_{max} = 25\% \times T_{mean} + T_{mean}$$

$$= 0.25 \times 31830.99 + 31830.99$$

$$= 7957.75 + 31830.99$$

$$= 39788.74 \text{ N-M}$$

$$\frac{39788.74}{\frac{\pi}{16} \times D^3} = \frac{60 \times 10^6}{\frac{D}{2}}$$

$$D = 0.15 \text{ M} = 150 \text{ MM}.$$

Hollow shaft.

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{39788.74}{\frac{\pi}{32}(D^4 - d^4)} = \frac{60 \times 10^6}{\frac{D}{2}}$$

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$$\frac{39788.74}{\frac{\pi}{16}\left(D^4-(0.6D)^4\right)} = \frac{60\times10^6}{D}$$

$$D = 0.157\,m$$

$$D_H = 157\,mm$$

$$\%\text{ saving in material} = \frac{\text{wt. of Solid shaft} - \text{wt. of Hollow shaft}}{\text{wt. of Solid shaft}}$$

$$= \frac{W_S - W_H}{W_S}\times100$$

$$= \frac{\gamma_S A_S L_S - \gamma_H A_H L_H}{\gamma_S A_S L_S}$$

$$= \frac{A_S - A_H}{A_S}$$

$$= \frac{\frac{\pi}{4}D_S^2 - \frac{\pi}{4}(D^2-d^2)}{\frac{\pi}{4}D_S^2}$$

$$= \frac{\overset{150}{D_S^2} - \overset{157}{D^2} + (0.6D)^2_{157}}{D_S^2\ 150}$$

$$= 29.76\%$$

2. A thin cylinder having dia 300mm & thickness 10mm is subjected to external torque 100N-m & inner fluid pressure 1MPa. Determine,

i) ~~Maximum~~ shear stress due to torque

ii) Maximum principal stress

iii) Maximum shear stress

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iv) Absolute maximum shear stress.

Soln:-

$t = 10\,mm$

$T = 100 \times 1000\ N\text{-}mm$

$R = D/2 = 300/2 = 150\,mm$

$\tau = ?$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{T}{\frac{\pi D^3 t}{4}} = \frac{\tau}{\frac{D}{2}}$$

$$\frac{100 \times 10^3}{\frac{\pi \times 300^2\, 10}{4}} = \frac{\tau}{\frac{300}{2}}$$

$$\tau = 0.071\ N/mm^2$$

$\cancel{\sigma_1 = \frac{16}{\pi D^3} \sqrt{M}}$

σ_x = Hoop stress $= \frac{pD}{2t} = \frac{1 \times 300}{2 \times 10}$

$= 15\,MPa$

σ_y = Longitudinal stress $= \frac{pD}{4t} = 7.5\,MPa$

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$= \frac{15+7.5}{2} \pm \frac{1}{2}\sqrt{4 \times 0.071^2 + (15-7.5)^2}$$

$$= 11.25 \pm 3.75$$

$\sigma_1 = 15.00\,MPa$

$\sigma_2 = 7.50\,MPa$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 3.75 \text{ MPa}$$

Absolute maximum shear $= \frac{\sigma_1}{2}$

$= 15/2$

$= 7.5 \text{ MPa}$

V. SFD & BMD

1. Beam is a structural member which is subject to load ⊥r to the axis of the member [Transverse load] & it transfers load to the support through Bending only. Beam is a bending member which is design on the basis of the maximum Bending moment & maximum Shear force.

2. There are following types of load :-
   a) point load [concentrated load] [W] [KN]
   b) Uniformly distributed load (udl) (w) (KN/m) or Rectangular load.
   c) Gradually varying load [G.V.L] or Triangular load [0 to w] [KN/m]

3. The algebraic Sum of all moments consider from extreme end to any section of the beam is called Bending moment at that section

BM

4. The graphical representation of Bending moment along with its nature (+ or –) called Bending moment diagram [BMD] which is very-very essential to find position of steel bars in RCC structures [Beams].

If the Bending moment is consider from left to right then clockwise moment is plus & Anticlockwise called minus.

But if the moment is consider from right to left then clockwise called minus & anti-clockwise called plus.

5. The algebraic sum of all forces consider from extreme end to any section of the beam is called shear force at that section. The graphical representation of shear force along with its nature is called shear force diagram (SFD) which is very very essential to determine the position of stirrups in RCC beams.

SF ✓ ⊛

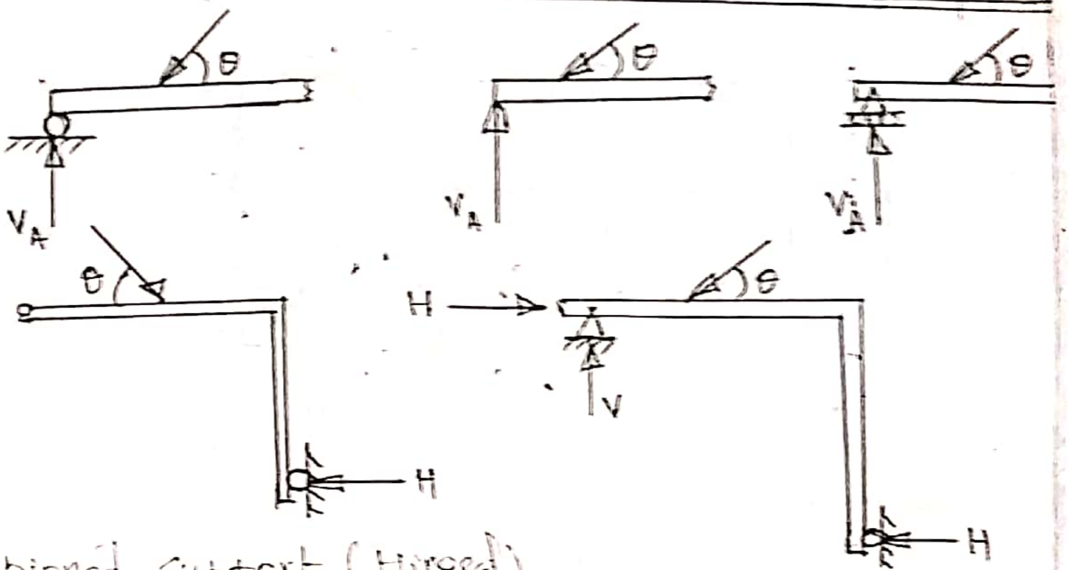
If the force is upward, consider from left to right, is considered as plus & downward force is taken as minus but if it is consider from right to left then upward force is taken as minus & downward force is taken as plus.

Any force developing clockwise moment from left to right then that force is consider as plus & if it develops anticlockwise moment [left to right] then that force is consider as negative.

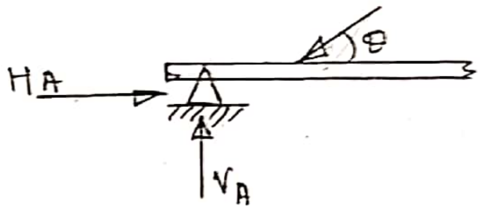
6.

6. Support reaction [Type of support]

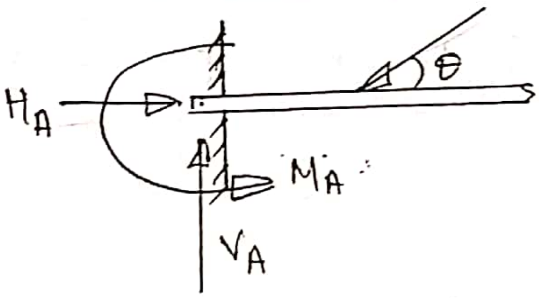
a) Roller support, simple support, Free support



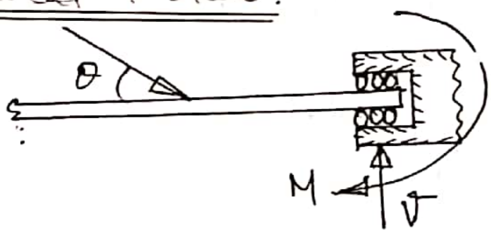
b) pinned support (Hinged)



c) Fixed support [Built-in] [Encastered]

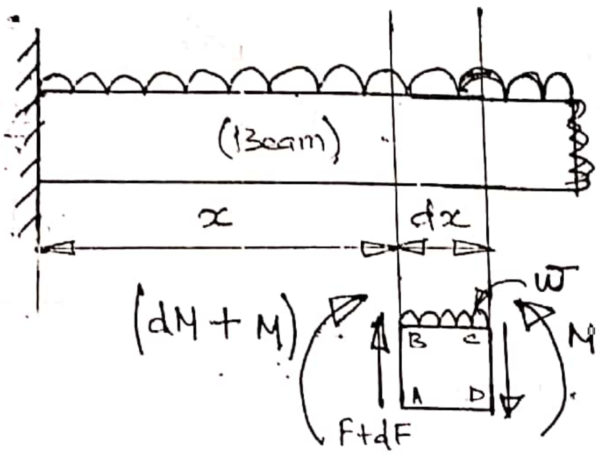


d) Guided Roller:-



7. Relation among shear force, Bending moment & Load:-

7. Relation among shearforce, Bending moment & Load :-



- Free Body Diagram

Law of Equilibrium;

$$\Sigma V = 0$$

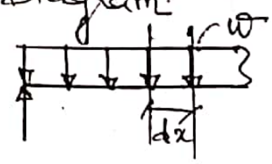
$$+(F+dF) - F - w\,dx = 0.$$

$$dF = w\,dx$$

$$\boxed{\frac{dF}{dx} = w}$$ – intensity of load

$$dF = w\,dx$$

$$\boxed{F = \int w\,dx}$$ → Area of loading Diagram



$$\Sigma M_A = 0;$$

$$-F\,dx + M - w\,dx \times \frac{dx}{2} - (dM + M) = 0$$

$$-F\,dx - \cancel{w\frac{(dx)^2}{2}} - dM = 0$$

neglected – dx – very small, $(dx)^2$ – very very small.

$$dM = -F\,dx$$

$$\boxed{\frac{dM}{dx} = -F}$$ → Rate of change of BM with respect to distance is called shear force

$$\boxed{\frac{dM}{dx} = S.F}$$

For maximum Bending moment,

$$\frac{dM}{dx} = 0.$$

shear force $= 0$.

$\boxed{V_{xx} = 0.}$ — for maximum BM

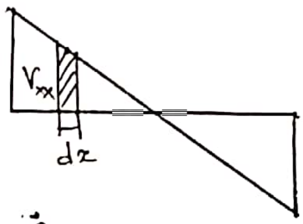
$$\frac{dM}{dx} = V_{xx}$$

$$dM = V_{xx}\,dx$$

$$\int dM = \int V_{xx}\,dx$$

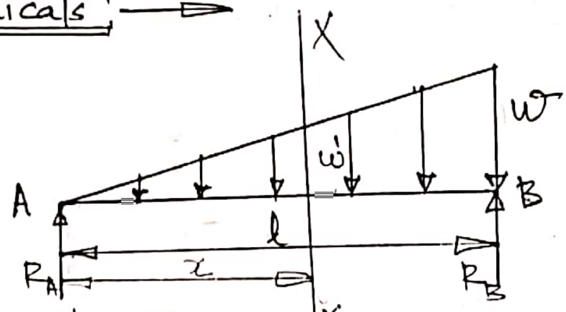
$$M = \int V_{xx}\,dx$$

$\boxed{BM = \text{Area of SFD}}$



Numericals :—

1.



(Triangular Load)

Total load, $= \frac{1}{2} w \times l$

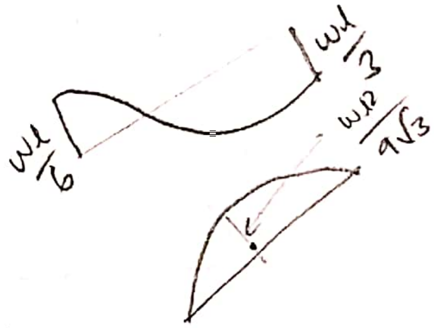
$\Sigma M_B = 0.$

$$R_A \times l = \frac{1}{2} \times w \times l \times \frac{l}{3}$$

$$R_A = \frac{wl}{6} \;;\; R_B = \frac{1}{2} \times w \times l - \frac{wl}{6} = \frac{wl}{3}$$

For Maximum BM,

$$V_{xx} = 0$$



$$\frac{wl}{6} - \frac{1}{2} \times w' \times x = 0.$$

$$\frac{wl}{6} - \frac{1}{2} \times \frac{w \times x}{l} \times x = 0.$$

parabolic SFD

$$\frac{\cancel{w} x^2}{\cancel{2} l} = \frac{\cancel{w} l}{\cancel{6}\, 3}$$

$$x^2 = \frac{l^2}{3}$$

$$\boxed{x = l/\sqrt{3}} = 0.577\,l$$

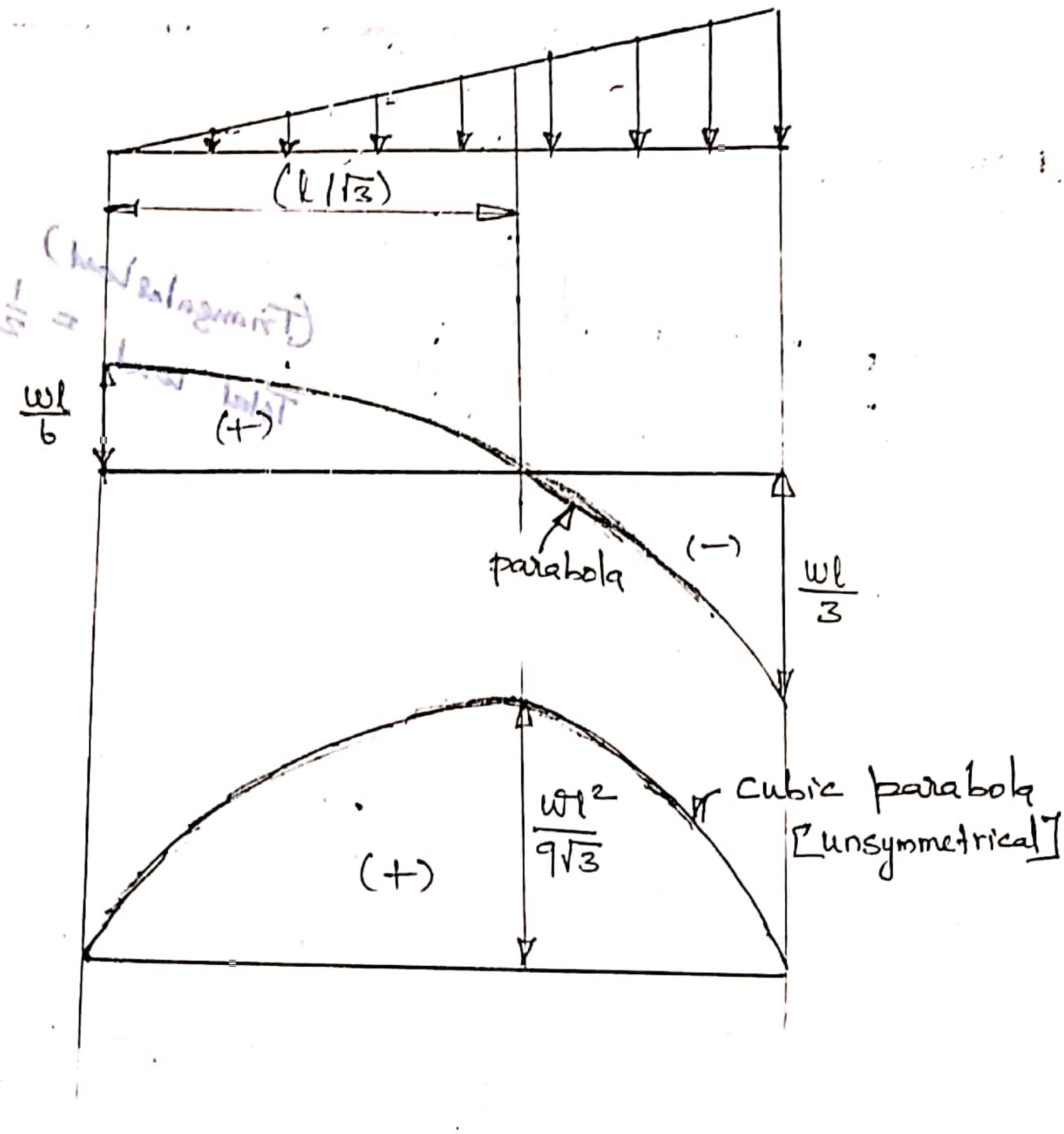
$$M_{xx} = M_{max} = \frac{wl}{6}\times x - \frac{1}{2}\times\frac{wx}{l}\times x\times\frac{x}{3}$$

$$= \frac{wlx}{6} - \frac{wx^{3}}{6l}$$ — cubic parabolic BMD

$$= \frac{wl\times l}{\sqrt{3}\times 6} - \frac{w}{6l}\times\left(\frac{l}{\sqrt{3}}\right)^{3}$$

$$= \frac{wl^{2}}{6\sqrt{3}} - \frac{wl^{2}}{6\times 3\times\sqrt{3}}$$

$$\boxed{M_{max} = \frac{wl^{2}}{9\sqrt{3}}}$$



2. A beam is supported by a strut as shown in figure below. Determine,

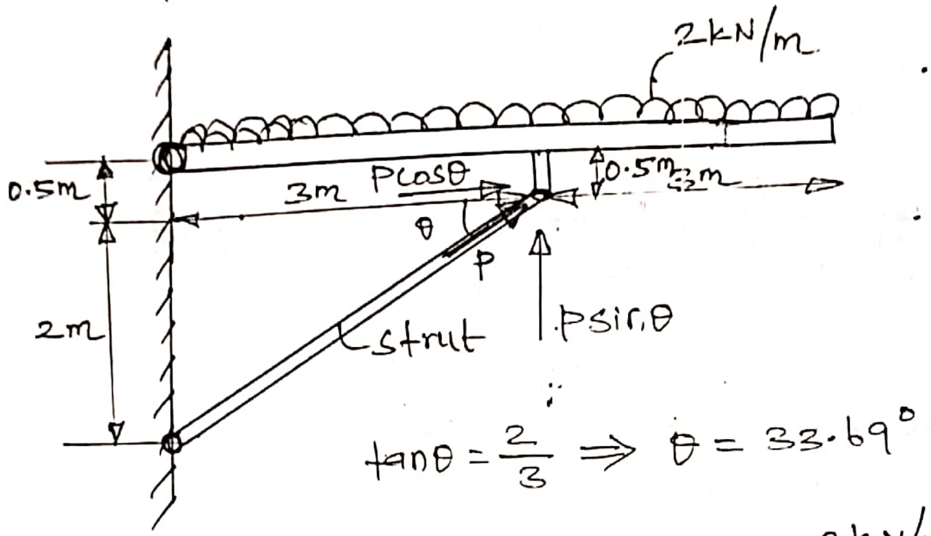
(i) Force in the strut

(ii) Thrust in the beam

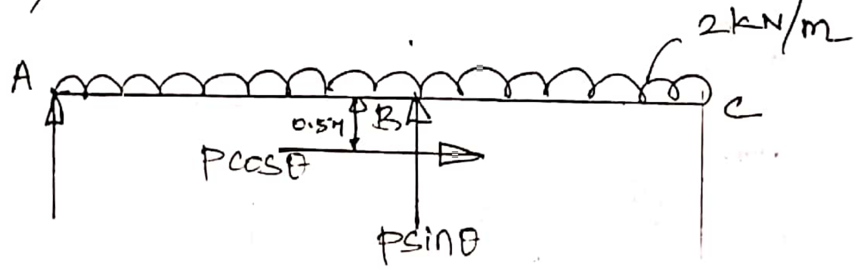
(iii) Maximum bending moment in the beam

(iv) Maximum shear force in the beam.

(v) SFD & BMD



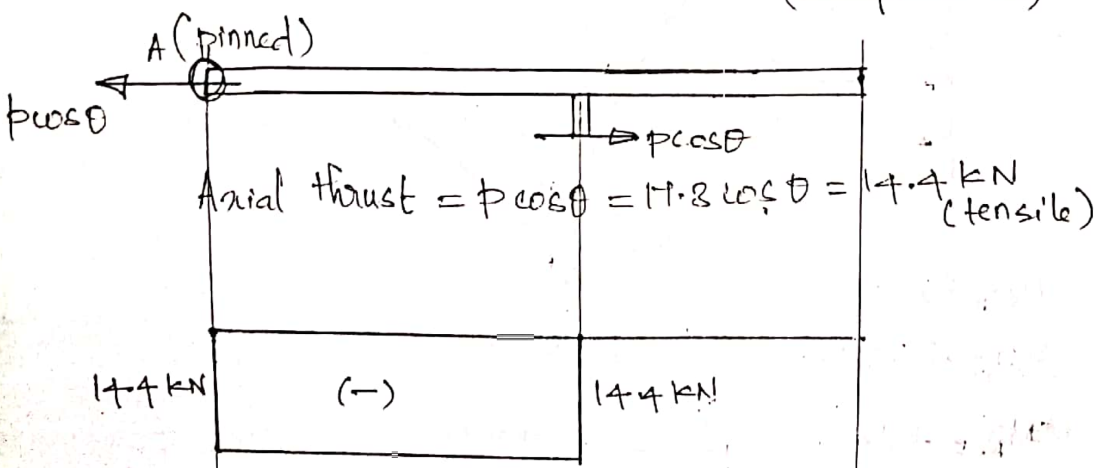
$$\tan\theta = \frac{2}{3} \Rightarrow \theta = 33.69^\circ$$



$\Sigma M_A = 0$;

$$-\frac{2\times 6^2}{2} + P\cos\theta \times 0.5 + P\sin\theta \times 3 = 0$$

Force in the strut, $\boxed{P = +17.30\text{ kN}}$ (compressive)



Axial thrust $= P\cos\theta = 17.3\cos\theta = 14.4$ kN (tensile)

Thrust diagram.

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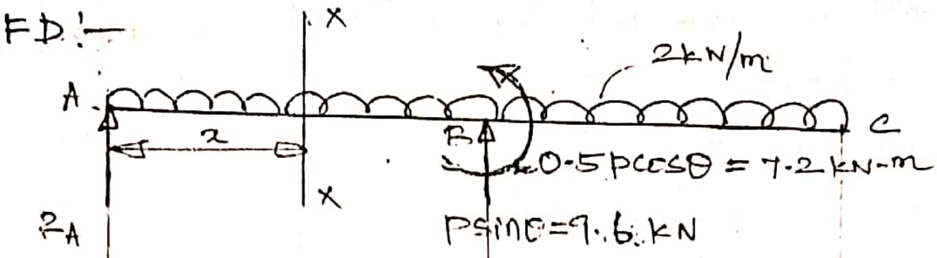
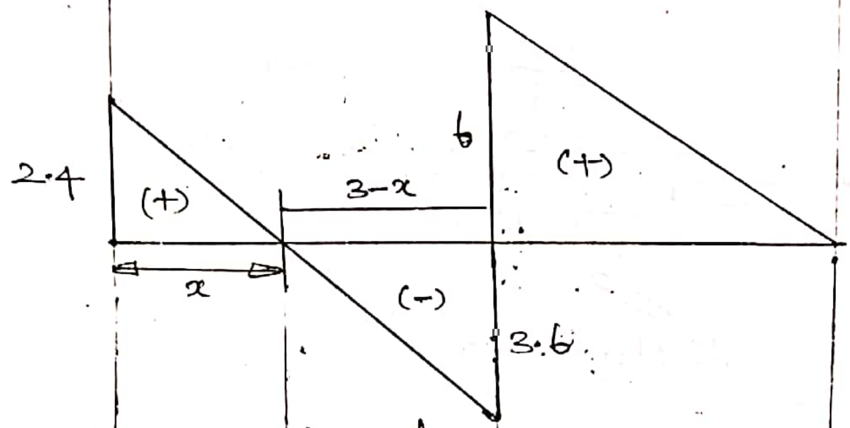
SFD:-



$\Sigma V = 0;\ R_A = 2 \times 6 - P\sin\theta = 2.4\,kN$



Maximum shear force = 6 kN

$$\frac{x}{2.4} = \frac{3-x}{3.6} \Rightarrow 3.6x = 7.2 - 2.4x$$

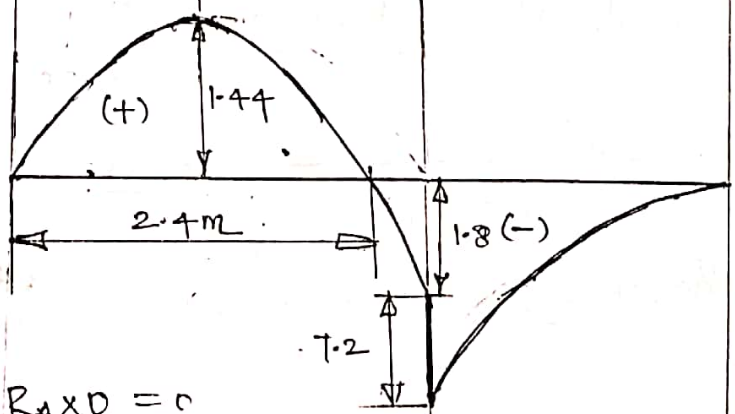
$$\boxed{x = 1.2\,m}$$

(or)

For max^m BM, $V_{xx} = 0;\ R_A - 2x = 0$

$$\boxed{x = 1.2\,m}$$

BMD:-



$M_A = R_A \times 0 = 0$

$M_{xx} = R_A \times x - \frac{2x^2}{2} = 1.44\,kN\text{-}m$

$M_{A-B} = R_A \times 3 - \frac{2 \times 3^2}{2} = -1.8\,kN\text{-}m$

$M_B = -1.8 - 7.2 = -9\ kN\text{-}m.$

$M_C = 0$

For zero BM, $M_{xx} = 0$

$$R_A x - \frac{2x^2}{2} = 0$$

$$2.4x - x^2 = 0.$$

$$x = 0, 2.4\ m.$$

$\boxed{x = 2.4\ m}$ – point of inflection.

3. A structural member is shown in figure below. Determi

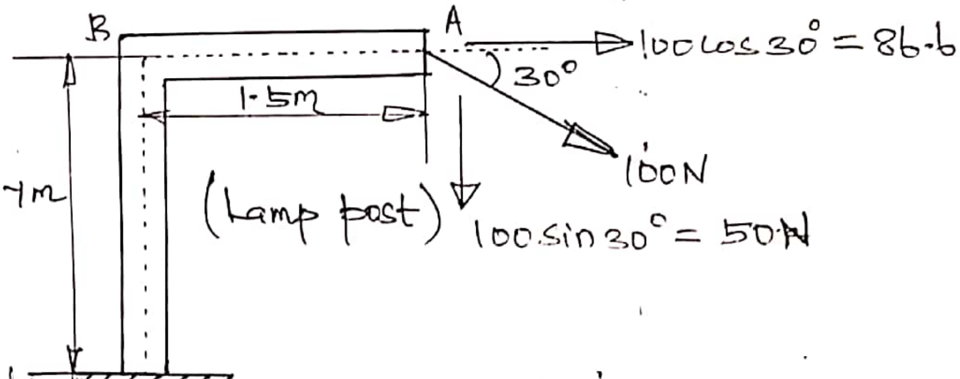
(i) Maximum shear force for the beam,

(ii) Maximum BM for the beam,

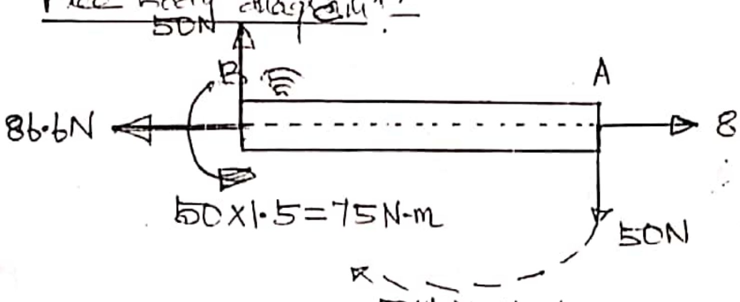
(iii) Maximum thrust in the beam,

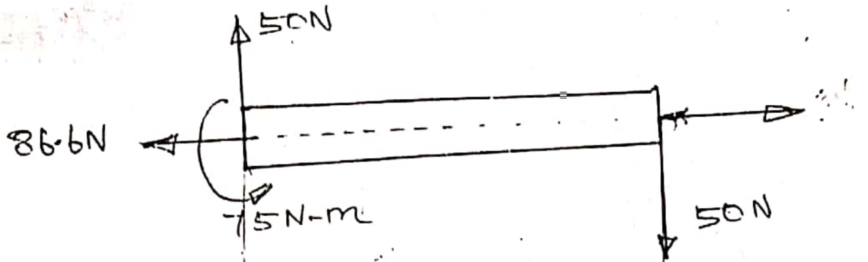
(iv) Maximum BM & thrust in the column,

(V) SFD & BMD & thrust diagram.

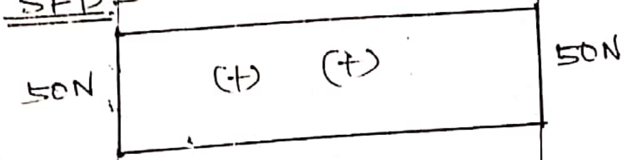


Free body diagram:-

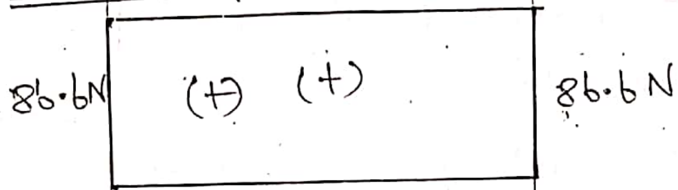




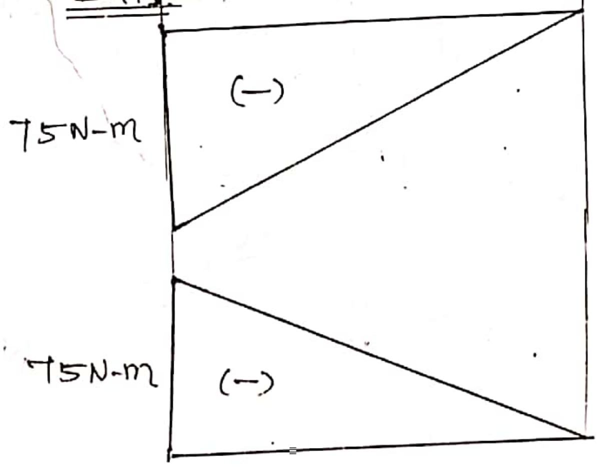
SFD:-

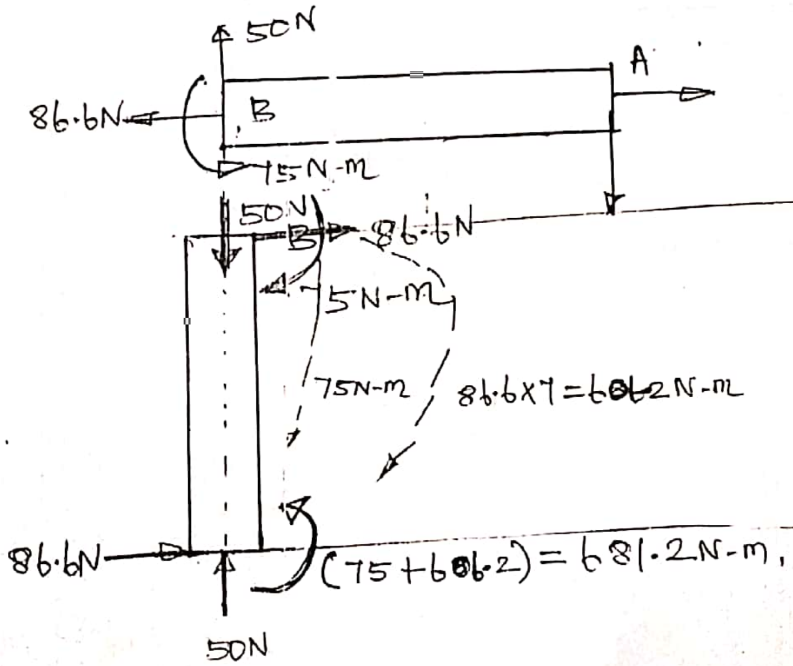


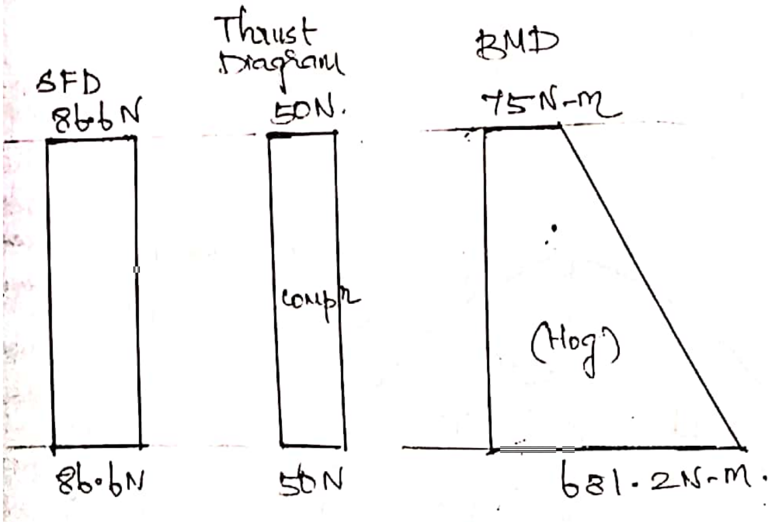
Thrust diagram:-



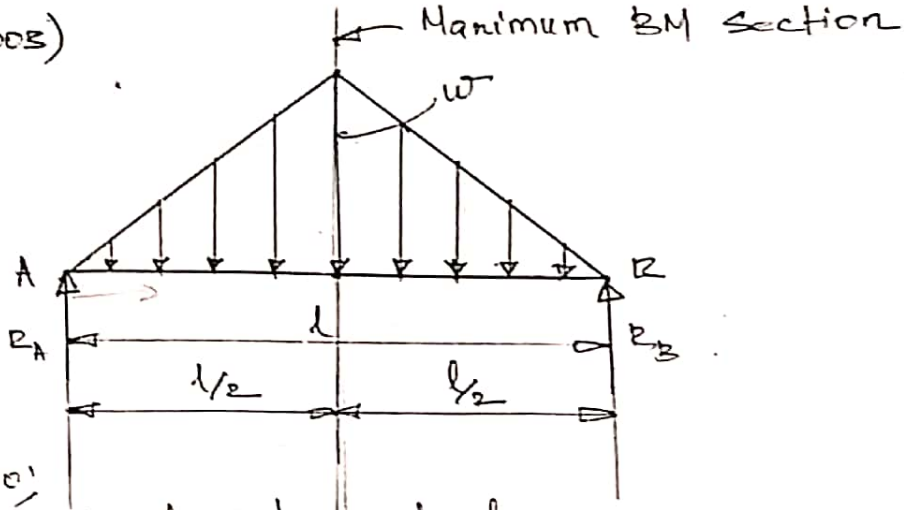
BMD:-






SFD
866N
86.6N
Thrust Diagram
50N.
compr
50N
BMD
75N-m
(Hog)
681.2N-m.

4. (IIT-M - 2003)

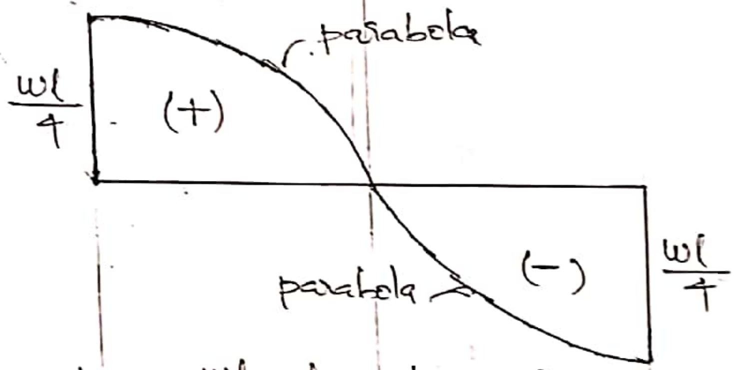


$\Sigma M_B = 0$;

$$R_A \times l = \frac{1}{2} \times w \times l \times \frac{l}{2}$$

$$R_A = \frac{wl}{4}\ ; \quad R_B = \frac{wl}{4}$$

<u>SFD</u>:-

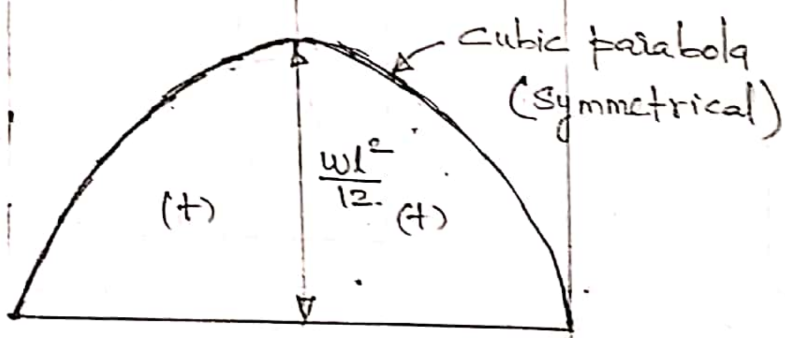


$$M_{max} = M_{mid} = \frac{wl}{4} \times \frac{l}{2} - \frac{1}{2} \times w \times \frac{l}{2} \times \frac{1}{3} \times \frac{l}{2}$$

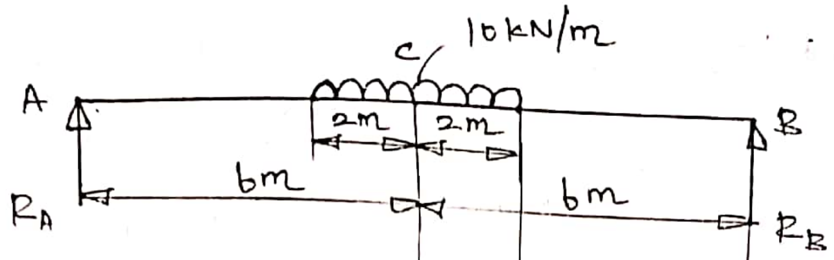
$$= \frac{wl^2}{8} - \frac{wl^2}{24}$$

$$M_{max} = \frac{wl^2}{12}$$

<u>BMD</u>:-



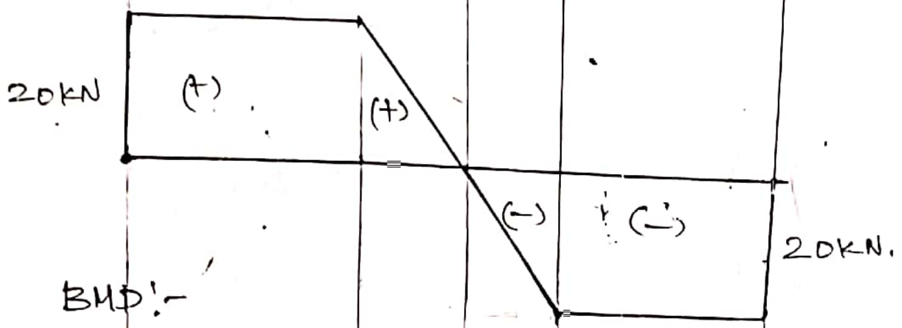
5.



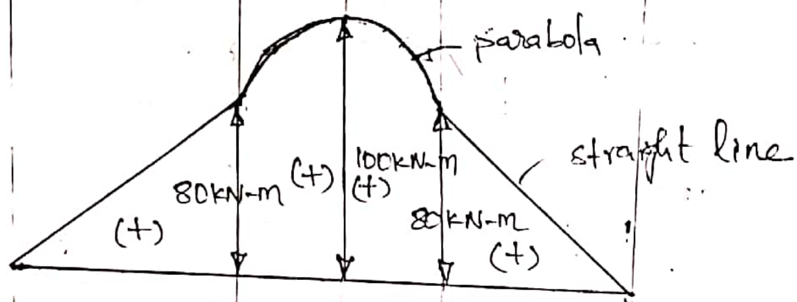
$$R_A = R_B = \frac{10\times4}{2} = 20\,kN$$

$$M_{max} = M_{mid} = 20\times6 - \frac{10\times2^2}{2} = 100\,kN\text{-}m$$

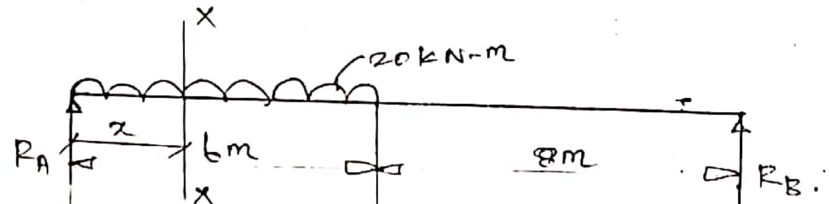
SFD:-



BMD:-



6.

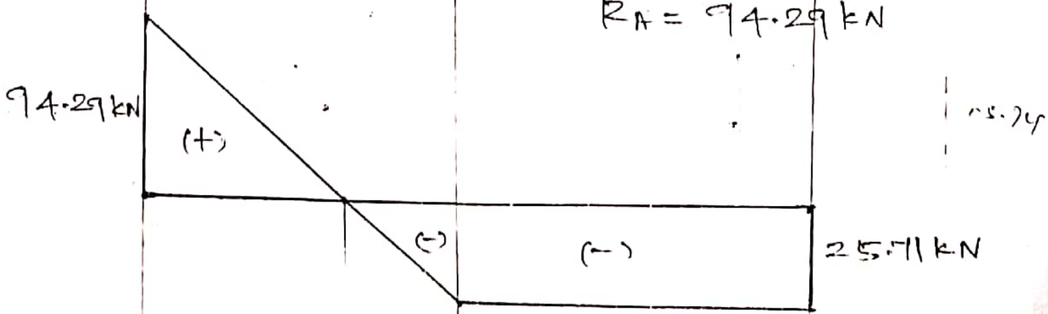


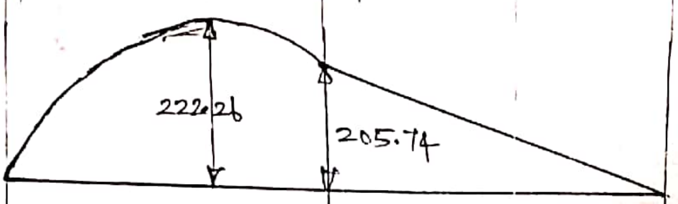
$\Sigma M_A = 0$

$$R_B\times14 - \frac{20\times6^2}{2} = 0$$

$R_B = 25.71\,kN$

$R_A = 94.29\,kN$





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$V_{xx} = 0$; $\quad R_A - 20x = 0$

$$x = \frac{94.29}{20}$$

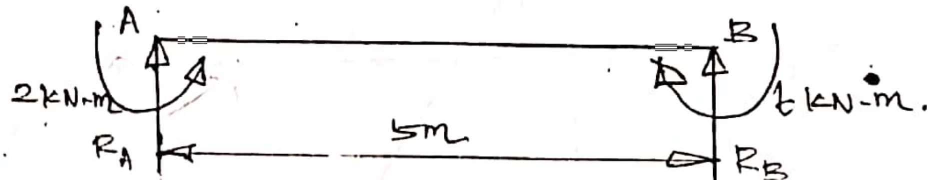
$$x = 4.71m.$$

$$M_{max} = M_{x-x} = 94.29 \times x - 20 \times \frac{x^2}{2}$$

$$= 94.29 \times 4.71 - 20 \times \frac{4.71^2}{2}$$

$$= 222.26 \text{ kN-m}$$

7. (HW)

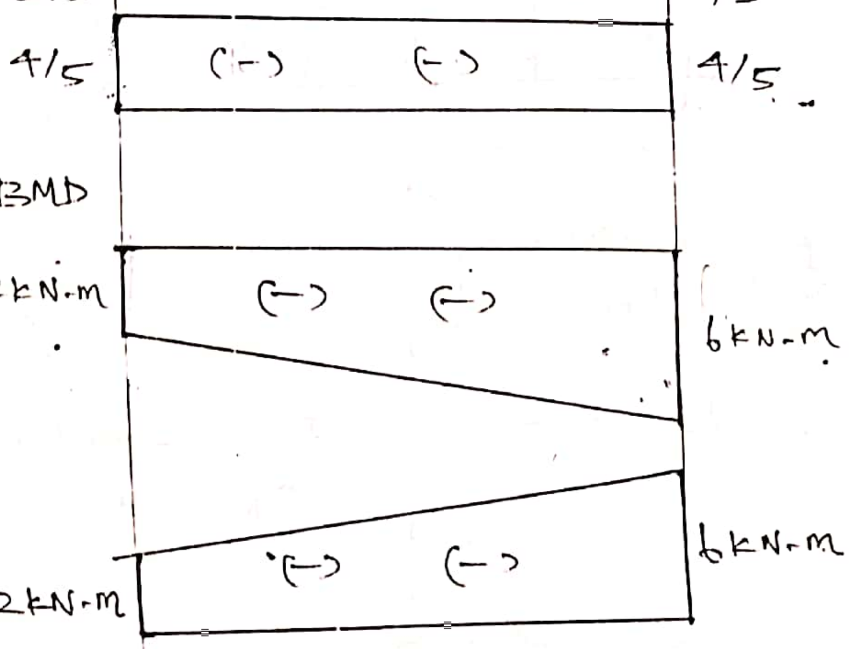


$R_A + R_B = 0$;

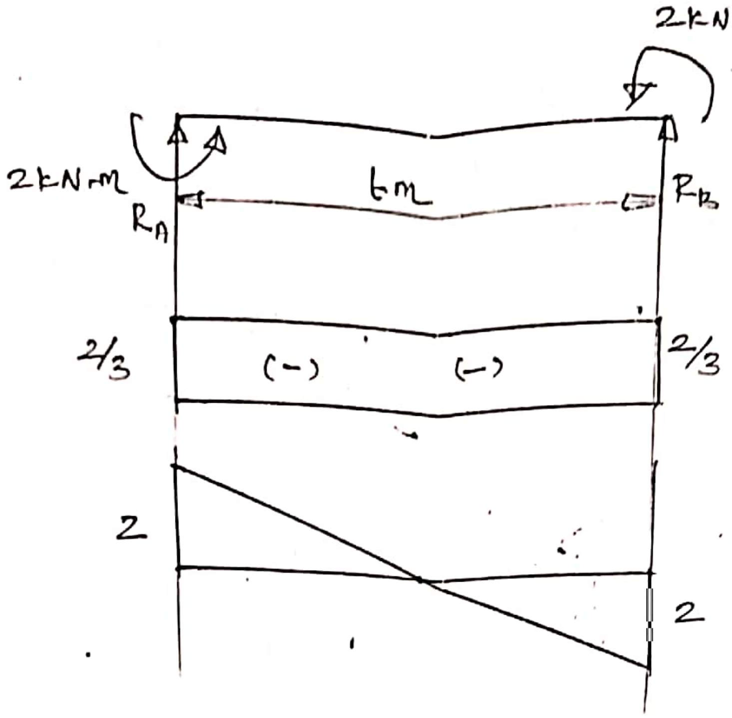
$\Sigma M_B = 0$; $\quad R_A \times 5 - 2 + 6 = 0.$

$R_A = -4/5$ kN

$R_B = +4/5$ kN



8.



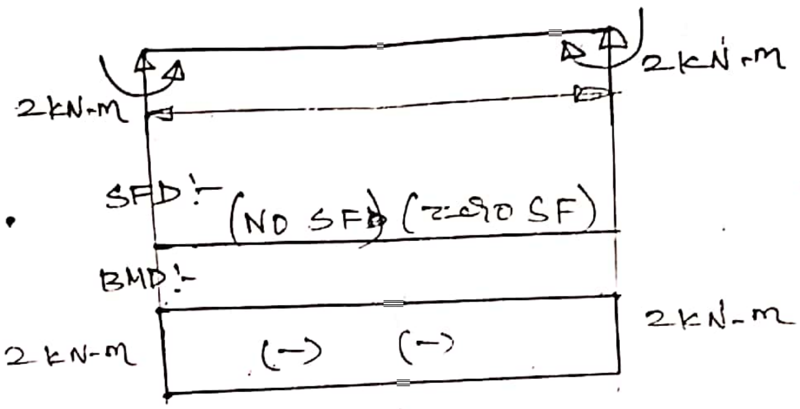
$R_A + R_B = 0$.

$\Sigma M_B = 0$;

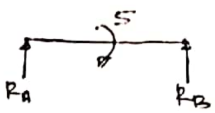
$R_A \times 6 = 2 + 2$

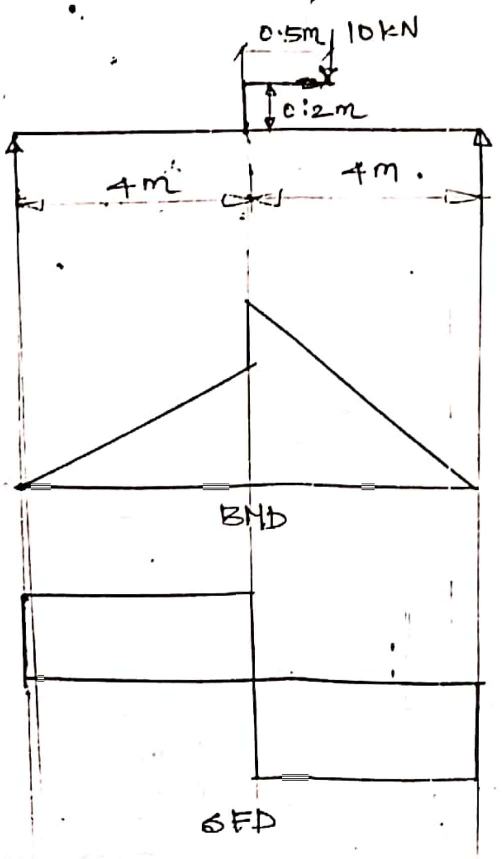
$R_A = 2/3$

9.



10.





$R_A + R_B = 10$

$R_A \times 8 = 10 \times 3.5 - 10 \times 0.5$

$R_A = 3.75$