17BEME401 STRENGTH OF MATERIALS 3 1 0 4 100

COURSE OBJECTIVES

1. To understand the concepts of stress and strain on deformation of solids.
2. To introduce the Concepts of safe working stresses and load carrying capacity of beams.
3. To enrich the understanding of deflection in beams and columns in engineering applications.
4. To understand the importance of the effect of torsion on shafts and springs.
5. To provide knowledge on principal stresses and analyze thin cylinders and shells subjected to pressure forces.
6. To provide knowledge on components subjected to various loadings with the help of various theories of failures.

COURSE OUTCOMES

1. Determine stress and strain on deformation of solids.
2. Compute safe working stresses and load carrying capacity of beams.
3. Estimate the deflection in beams and columns in engineering applications.
4. Analyze the effect of torsion on shafts and springs.
5. Determine principal stresses and analyze thin cylinders and shells subjected to pressure forces.
6. Design the components subjected to various loadings with the help of various theories of failures.

UNIT I STRESS, STRAIN AND DEFORMATION OF SOLIDS 12

Rigid and Deformable bodies – Strength, Stiffness and Stability – Stresses; Tensile, Compressive and Shear – Deformation of simple and compound bars under axial load – Thermal stress – Elastic constants – Strain energy and unit strain energy – Strain energy in uniaxial loads.

UNIT II BEAMS – LOADS AND STRESSES 12

Types of beams: Supports and Loads – Shear force and Bending Moment in beams – Cantilever, Simply supported and Overhanging beams – Relationship between load, shear force and bending moment – Stresses in beams – Theory of simple bending – Stress variation along the length and in the beam section – Effect of shape of beam section on stress induced – Shear stresses in beams – Shear flow.

UNIT III BEAM DEFLECTION 12

Elastic curve of Neutral axis of the beam under normal loads – Evaluation of beam deflection and slope: Macaulay Method – Columns – End conditions – Equivalent length of a column – Euler equation – Slenderness ratio – Rankine’s formula for columns

UNIT IV TORSION 12

Analysis of torsion of circular bars – Torsional Shear stress – Bars of solid and hollow circular section – Stepped shaft – Torsional rigidity – Compound shafts – Fixed and simply supported shafts – Application to close–coiled helical springs – Maximum shear stress in spring section including Wahl Factor – Deflection of helical coil springs under axial loads – Design of helical coil springs – stresses in helical coil springs under torsion loads

UNIT V ANALYSIS OF STRESSES IN TWO DIMENSIONS 12

Biaxial state of stresses – Thin cylindrical and spherical shells – Deformation in thin cylindrical and spherical shells – Biaxial stresses at a point – Stresses on inclined plane – Principal planes and stresses – Mohr’s circle for biaxial stresses – Maximum shear stress – Strain energy in bending and torsion.

TEXT BOOKS

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Punmia B.C and Jain A.K	Strength of Materials and Theory of Structures – Vol.1	Laxmi Publications New Delhi	2015
2	Ramamrutham S and Narayan R	Strength of Materials	Dhanpat Rai and Sons., New Delhi	2008

REFERENCES

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Jindal U C	Textbook on Strength of Materials	Asian Books Pvt, Ltd, Chennai	2012
2	Don H Morris, and Leroy D Sturges	Mechanics of Materials	John Wiley and Sons Inc	2006
3.	Bedi D S	Strength of Materials	S Chand and Co. Ltd., New Delhi	1984

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Established Under Section 3 of UGC Act, 1956)
Eachanari Post, Coimbatore-641021, INDIA

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING (AUTOMOBILE)

LESSON PLAN

Subject Name : **Strength of Materials**
Subject Code : **17BEAE403 (Credits - 4)**
Name of the Faculty : **Mr. G. Vignesh**
Designation : **Assistant Professor**
Year / Semester : **II / IV**
Branch : **Automobile Engineering**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
		UNIT-I: STRESS, STRAIN AND DEFORMATION OF SOLIDS	
1.	1	Rigid and deformable bodies - Strength, stiffness and stability	T2, R4
2.	1	Stresses: Tensile, compressive and shear	T2, R4
3.	2	Deformation of simple and compound bars under axial load	T2, R4
4.	1	TUTORIAL: Problems involving deformation of simple and compound bars under axial load	
5.	1	Thermal stress	T2, R4
6.	1	Elastic constants	T2, R4
7.	1	Strain energy and unit strain energy	T2, R4
8.	1	Strain energy in uniaxial loads	T2, R4
9.	1	Volumetric strains and stresses on inclined planes	T2, R4
10.	1	TUTORIAL: Problems involving thermal stress and strain energy	
11.	1	Discussion on competitive examination related questions / KAHE previous year questions	
		Total no. of hours planned for Unit-I	12

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT-II: BEAMS - LOADS AND STRESSES			
12.	1	Types of beams, supports and loads - Shear force and bending moment in beams	T2, R4
13.	1	Cantilever beams	T2, R4
14.	1	Simply supported beams	T2, R4
15.	1	Overhanging beams	T2, R4
16.	1	TUTORIAL: Problems involving shear force and bending moment in beams	
17.	1	Stresses in beams	T2, R4
18.	1	Theory of simple bending	T2, R4
19.	1	Stress variation along the length and in the beam section	T2, R4
20.	1	Effect of shape of beam section on stress induced	T2, R4
21.	1	Shear stresses in beams - Shear flow	T2, R4
22.	1	TUTORIAL: Problems involving stresses in beams	
23.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-II			12

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT-III: BEAM DEFLECTION			
24.	1	Elastic curve of neutral axis of the beam under normal loads	T2, R4
25.	1	Evaluation of beam deflection and slope	T2, R4
26.	1	Double integration method	T2, R4
27.	1	Macaulay method	T2, R4
28.	1	Moment-area method	T2, R4
29.	1	TUTORIAL: Problems involving evaluation of beam deflection and slope	
30.	1	Columns	T2, R4
31.	1	End conditions - Equivalent length of a column	T2, R4
32.	1	Euler equation - Slenderness ratio	T2, R4
33.	1	Rankine formula for columns	T2, R4
34.	1	TUTORIAL: Problems in columns	

35.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-IV			12

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT-IV: TORSION			
36.	1	Analysis of torsion of circular bars	T2, R4
37.	1	Shear stress distribution	T2, R4
38.	1	Bars of solid and hollow circular section	T2, R4
39.	1	Stepped shaft - Twist and torsion stiffness	T2, R4
40.	1	Compound shafts - Fixed and simply supported shafts	T2, R4
41.	1	TUTORIAL: Problems involving torsion of shafts	
42.	1	Application of close-coiled helical springs	T2, R4
43.	1	Maximum shear stress in spring section including Wahl factor	T2, R4
44.	1	Deflection of helical coil springs under axial loads – Design of helical coil springs	T2, R4
45.	1	Stresses in helical coil springs under torsion loads	T2, R4
46.	1	TUTORIAL: Problems involving design of helical coil springs	
47.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-III			12

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT-V: ANALYSIS OF STRESSES IN TWO DIMENSIONS			
48.	1	Biaxial state of stresses	T2, R4
49.	1	Thin cylindrical and spherical shells	T2, R4
50.	1	Deformation in thin cylindrical and spherical shells	T2, R4
51.	1	TUTORIAL: Problems in thin cylindrical and spherical shells	
52.	1	Biaxial stresses at a point	T2, R4
53.	1	Stresses on inclined plane	T2, R4

54.	1	Principal planes and stresses	T2, R4
55.	1	Mohr's circle for biaxial stresses	T2, R4
56.	1	Maximum shear stress	T2, R4
57.	1	Strain energy in bending and torsion	T2, R4
58.	1	TUTORIAL: Problems involving principal planes and stresses	
59.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-V			12

TOTAL PERIODS : 60

TEXT BOOKS:

SL. NO.	AUTHOR(S)	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1.	Popov E P	Engineering Mechanics of Solids	Prentice-Hall of India, New Delhi	1998
2.	Punmia B C, Ashok Kumar Jain and Arun Kumar Jain	Strength of Materials and Theory of Structures - Volume 2	Laxmi Publications, New Delhi	2005
3.	Ferdinand Beer E, Russell Johnston Jr., John DeWolf and David Mazurek	Mechanics of Materials	McGraw-Hill Book Co., New Delhi	2016

REFERENCE BOOKS:

SL. NO.	AUTHOR(S)	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1.	William Nash and Merle Potter	Theory and Problems in Strength of Materials Schaum Outline Series	McGraw-Hill Book Co., New York	2010
2.	Kazimi S M A	Solid Mechanics	Tata McGraw-Hill Publishing Co., New Delhi	2001
3.	Ryder G H	Strength of Materials	Macmillan India Ltd., New Delhi	2002
4.	Timoshenko S P	Elements of Strength of Materials	East West, India	2011

UNIT	**Total No. of Periods Planned**	**Lecture Periods**	**Tutorial Periods**
I	12	10	2
II	12	10	2
III	12	10	2
IV	12	10	2
V	12	10	2
TOTAL	**60**	**50**	**10**

I. CONTINUOUS INTERNAL ASSESSMENT : 40 Marks
(Internal Assessment Tests: 30, Attendance: 5, Seminar: 5)

II. END SEMESTER EXAMINATION : 60 Marks

TOTAL : 100 Marks

FACULTY **HOD / MECHANICAL** **DEAN / FOE**

UNIT-I STRESS, STRAIN AND DEFORMATION OF SOLIDS

SOLVED PROBLEMS

PROBLEM 1

A bar of 30 mm diameter is subjected to a pull of 60 kN. The measured extension on gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0039 mm. Calculate the Poisson's ratio and the value of the three moduli.

Given:

Diameter, d = 30 mm

Pull, P = 60 kN = 60×10^3 N

Length, L = 200 mm

Change in length, δL = 0.09 mm

Change in diameter, δd = 0.0039 mm

To find:

Poisson's ratio, μ

Young's modulus, E

Modulus of rigidity, G

Bulk modulus, K

Solution:

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\text{Lateral strain, } e_t = \frac{\delta d}{d} \text{ or } \frac{\delta b}{b} \text{ or } \frac{\delta t}{d}$$

$$= \frac{\delta d}{d}$$

$$= \frac{0.0039}{30}$$

$$= 1.3 \times 10^{-4}$$

$$\text{Longitudinal strain, } e_l = \frac{\delta l}{l}$$

$$= \frac{0.09}{200}$$

$$= 4.5 \times 10^{-4}$$

$$\text{Poisson's ratio, } \mu = \frac{e_t}{e_l}$$

$$= \frac{1.3 \times 10^{-4}}{4.5 \times 10^{-4}}$$

$$= 0.2889$$

Young's modulus, $E = \frac{\text{Tensile stress}}{\text{Tensile strain}}$

Young's modulus, $E = \frac{\sigma}{e_l}$

$$= \frac{P}{A\, e_l}$$

�stress, $\sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$

$$= \frac{P}{\frac{\pi}{4} d^2\, e_l}$$

$$= \frac{60 \times 10^3}{\frac{\pi}{4} \times 30^2 \times 4.5 \times 10^{-4}}$$

$$= 188628.0807 \text{ N/mm}^2$$

Young's modulus, $E = 2G\,(1 + \mu)$

Modulus of rigidity, $G = \frac{E}{2\,(1 + \mu)}$

$$= \frac{188628.0807}{2\,(1 + 0.2889)}$$

$$= 73174.6865 \text{ N/mm}^2$$

Young's modulus, $E = 3K\,(1 - 2\mu)$

Bulk modulus, $K = \frac{E}{3\,(1 - 2\mu)}$

$$= \frac{188628.0807}{3\,(1 - 2 \times 0.2889)}$$

$$= 148916.9058 \text{ N/mm}^2$$

Result:

Poisson's ratio, μ = 0.2889

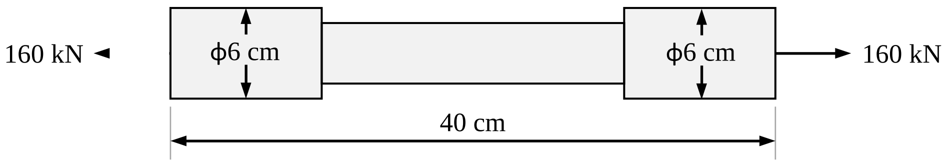
Young's modulus, E = 188628.0807 N/mm^2

Modulus of rigidity, G = 73174.6865 N/mm^2

Bulk modulus, K = 148916.9058 N/mm^2

PROBLEM 2

The bar shown in figure is subjected to a tensile load of 160 kN. If the stress in the middle portion is limited to 150 N/mm^2, determine the diameter of the middle portion. Find also the length of the middle portion if the total elongation of the bar is to be 0.2 mm. Young's modulus is equal to 2.1×10^5 N/mm^2.



Given:

Tensile load, P	= 160 kN = 160×10^3 N
Stress in middle portion, σ_2	= 150 N/mm^2
Total elongation, δL	= 0.2 mm
Total length of the bar, L	= 40 cm = 400 mm
Diameter of the left end portion, D_1	= 6 cm = 60 mm
Diameter of the right end portion, D_3	= 6 cm = 60 mm
Young's modulus, E	= 2.1×10^5 N/mm^2

To find:

Diameter of the middle portion, D_2

Length of the middle portion, L_2

Solution:

Total length of the bar = Length of left end portion + Length of middle portion + Length of right end portion

$L = L_1 + L_2 + L_3$

$L_1 + L_3 = L - L_2$

$L_1 + L_3 = 400 - L_2$ … (1)

Stress in middle portion

$$\sigma_2 = \frac{P}{A_2} \qquad \because \text{stress}, \sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

$$150 = \frac{160 \times 10^3}{\frac{\pi}{4} D_2^{\ 2}}$$

$$D_2^{\ 2} = \frac{160 \times 10^3}{\frac{\pi}{4} \times 150}$$

$$D_2 = \sqrt{\frac{160 \times 10^3}{\frac{\pi}{4} \times 150}}$$

$= 36.8527\ \text{mm}$

Total elongation = Elongation of left end portion + Elongation of middle portion + Elongation of right end portion

$\delta L = \delta L_1 + \delta L_2 + \delta L_3$

$$\delta L = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$= \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

$$= \frac{P}{E}\left(\frac{L_1}{\frac{\pi}{4}D_1^2} + \frac{L_2}{\frac{\pi}{4}D_2^2} + \frac{L_3}{\frac{\pi}{4}D_3^2}\right)$$

$$\delta L = \frac{P}{\frac{\pi}{4}E}\left(\frac{L_1}{D_1^2} + \frac{L_2}{D_2^2} + \frac{L_3}{D_3^2}\right)$$

$$0.2 = \frac{160 \times 10^3}{\frac{\pi}{4} \times 2.1 \times 10^5}\left(\frac{L_1}{60^2} + \frac{L_2}{36.8527^2} + \frac{L_3}{60^2}\right)$$

$$0.2 = \frac{160 \times 10^3}{\frac{\pi}{4} \times 2.1 \times 10^5}\left(\frac{L_1 + L_3}{60^2} + \frac{L_2}{36.8527^2}\right)$$

Substituting the value of $L_1 + L_3$ from equation (1)

$$0.2 = \frac{160 \times 10^3}{\frac{\pi}{4} \times 2.1 \times 10^5}\left(\frac{400 - L_2}{60^2} + \frac{L_2}{36.8527^2}\right)$$

$$\frac{0.2 \times \frac{\pi}{4} \times 2.1 \times 10^5}{160 \times 10^3} = \frac{400}{60^2} - \frac{L_2}{60^2} + \frac{L_2}{36.8527^2}$$

$$\frac{0.2 \times \frac{\pi}{4} \times 2.1 \times 10^5}{160 \times 10^3} - \frac{400}{60^2} = -\frac{L_2}{60^2} + \frac{L_2}{36.8527^2}$$

$0.0951 = -2.7778\times10^{-4}\ L_2 + 7.3631\times10^{-4}\ L_2$

$$L_2 = \frac{0.0951}{-2.778 \times 10^{-4} + 7.3631 \times 10^{-4}}$$

$= 207.3044\ \text{mm}$

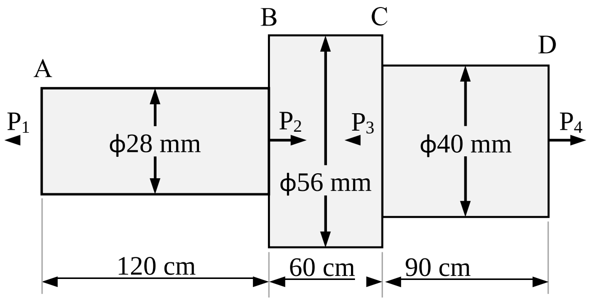
Result:

Diameter of the middle portion, D_2 = 36.8527 mm

Length of the middle portion, L_2 = 207.3044 mm

PROBLEM 3

A member ABCD is subjected to point loads P_1, P_2, P_3 and P_4 as shown in figure. Calculate the force P_2 necessary for equilibrium if P_1 = 45 kN, P_3 = 450 kN and P_4 = 130 kN. Determine the total elongation of the member, assuming the modulus of elasticity to be 2.1×10^5 N/mm^2.



Given:

Force, P_1	= 45 kN
Force, P_3	= 450 kN
Force, P_4	= 130 kN
Diameter of left end portion, D_1	= 28 mm
Diameter of middle portion, D_2	= 56 mm
Diameter of right end portion, D_3	= 40 mm
Length of left end portion, L_1	= 120 cm = 1200 mm
Length of middle portion, L_2	= 60 cm = 600 mm
Length of right end portion, L_3	= 90 cm = 900 mm

To find:

Force, P_2

Total elongation, δL

Solution:

Resolving the forces on the rod along its axis,

Forces acting towards left = Forces acting towards right

$P_1 + P_3 = P_2 + P_4$

$45 + 450 = P_2 + 130$

$P_2 = 45 + 450 - 130$

$P_2 = 365$ kN

(Taking tensile force as positive and compressive force as negative)

Consider part AB

Left hand side force = P_1 = 45 kN = 45×10^3 N

Right hand side force = $P_2 - P_3 + P_4$

= 365 – 450 + 130

= 45 kN = 45 × 10^3 N

Consider part BC

Left hand side force = $P_1 - P_2$

= 45 – 365

= – 320 kN = – 320 × 10^3 N

Right hand side force = $- P_3 + P_4$

= – 450 + 130

= – 320 kN = – 320 × 10^3 N

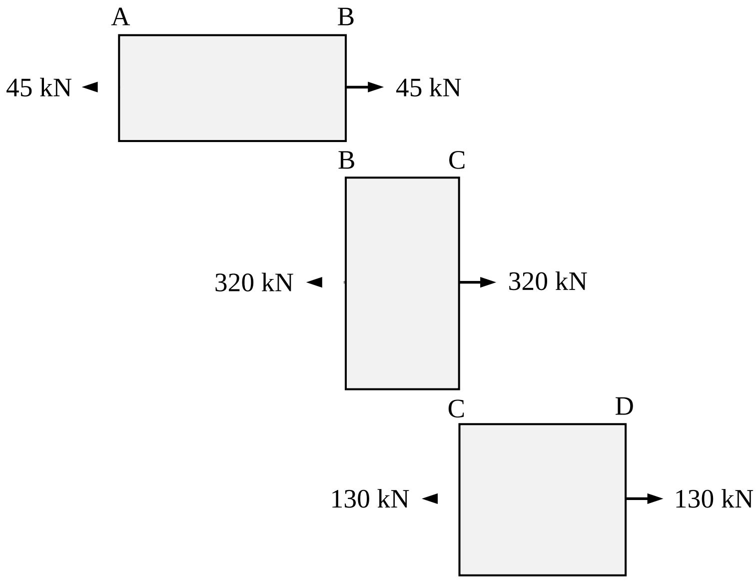
Consider part CD

Left hand side force = $P_1 - P_2 + P_3$

= 45 – 365 + 450

= 130 kN = 130 × 10^3 N

Right hand side force = P_4 = 130 kN = 130 × 10^3 N



Part AB is subjected to a tensile load of 45 kN, part BC is subjected to a compressive load of 320 kN and part CD is subjected to a tensile load of 130 kN.

Hence for part AB, there will be increase in length, for part BC there will be decrease in length and for part CD there will be increase in length.

Increase in length of AB

$$\delta L_1 = \frac{P_{AB} L_1}{A_1 E}$$

$$= \frac{P_{AB} L_1}{\frac{\pi}{4} D_1^2 E}$$

$$= \frac{45 \times 10^3 \times 1200}{\frac{\pi}{4} \times 28^2 \times 2.1 \times 10^5}$$

$= 0.4176$ mm

Decrease in length of BC

$$\delta L_2 = \frac{P_{BC} L_2}{A_2 E}$$

$$= \frac{P_{BC} L_2}{\frac{\pi}{4} D_2^{\ 2} E}$$

$$= \frac{320 \times 10^3 \times 600}{\frac{\pi}{4} \times 56^2 \times 2.1 \times 10^5}$$

$= 0.3712$ mm

Increase in length of CD

$$\delta L_3 = \frac{P_{CD} L_3}{A_3 E}$$

$$= \frac{P_{CD} L_3}{\frac{\pi}{4} D_3^{\ 2} E}$$

$$= \frac{130 \times 10^3 \times 900}{\frac{\pi}{4} \times 40^2 \times 2.1 \times 10^5}$$

$= 0.4434$ mm

Total change in length of member (Taking increase in length as positive and decrease in length as negative)

$\delta L = \delta L_1 + \delta L_2 + \delta L_3$

$= 0.4176 – 0.3712 + 0.4434$

$= 0.4898$ mm

Result:

Force, P_2 = 365 kN

Total elongation, δL = 0.4898 mm

PROBLEM 4

A tensile load of 40 kN is acting on a rod of diameter 40 mm and of length 4 m. A bore of diameter 20 mm is made centrally on the rod. To what length the rod should be bored so that the total extension will increase 30% under the same tensile load. Take $E = 2 \times 10^5$ N/mm^2.

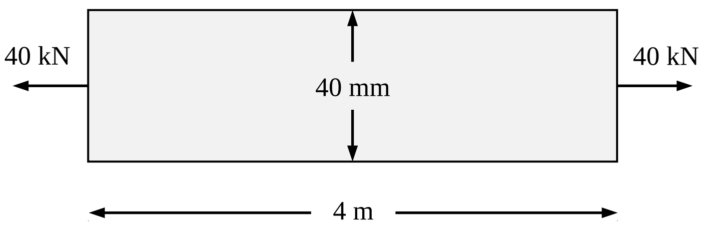
Given:

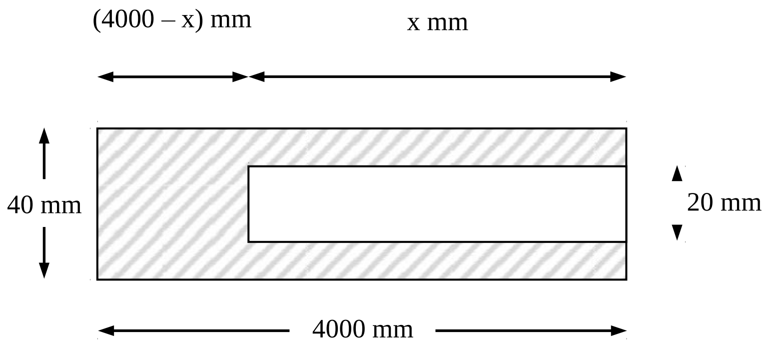
Tensile load, P	$= 40$ kN $= 40 \times 10^3$ N
Diameter of rod, D	$= 40$ mm
Length of rod, L	$= 4$ m $= 4 \times 10^3$ mm
Diameter of bore, d	$= 20$ mm
Total extension after bore	= Extension before bore + 30% Extension before bore
	$= 1.30 \times$ Extension before bore
Young's modulus, E	$= 2 \times 10^5$ N/mm^2

To find:

Length of the bore

Solution:





Area of rod

$$A = \frac{\pi}{4} D^2$$

$$= \frac{\pi}{4} 40^2$$

$$= 400\,\pi \text{ mm}^2$$

Area of bore

$$a = \frac{\pi}{4} d^2$$

$= \frac{\pi}{4} 20^2$

$= 100\ \pi\ mm^2$

Let the rod be bored to a length of 'x' mm

Then length of unbored portion = 4000 – x mm

Extension before the bore is made

$\delta L = \frac{PL}{AE}$

$= \frac{40 \times 10^3 \times 4000}{400\,\pi \times 2 \times 10^5}$

= 0.6366 mm

Extension after the bore is made = 1.30 × Extension before bore

= 1.30 × 0.6366

= 0.8276 mm … (1)

Total extension after the bore is made = Extension of unbored portion + Extension of bored portion

Extension of unbored portion

$\delta L_1 = \frac{PL_1}{A_1E}$

$= \frac{40 \times 10^3 \times (4000 - x)}{400\,\pi \times 2 \times 10^5}$

$= 0.6366 - 1.5915 \times 10^{-4}$ x mm

Extension of bored portion

$\delta L_2 = \frac{PL_2}{A_2E}$

$= \frac{PL_2}{(A - a)E}$

$= \frac{40 \times 10^3 \times x}{(400\,\pi - 100\,\pi) \times 2 \times 10^5}$

$= 2.1221 \times 10^{-4}$ x mm

Total extension after the bore is made = $\delta L_1 + \delta L_2$

$= 0.6366 - 1.5915 \times 10^{-4}\,x + 2.1221 \times 10^{-4}\,x$

$= 0.6366 + 5.3052 \times 10^{-5}\,x$ … (2)

Equating equations (1) and (2)

$0.8276 = 0.6366 + 5.3052 \times 10^{-5}\,x$

$0.8276 - 0.6366 = 5.3052 \times 10^{-5}\,x$

$x = \frac{0.8276 - 0.6366}{5.3052 \times 10^{-5}}$

x = 3600 mm

Result:

Length of bore = 3600 mm

PROBLEM 5

A steel rod of 3 cm diameter is enclosed centrally in a hollow copper tube of external diameter 5 cm and internal diameter of 4 cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of bar and tube is equal to 15 cm, determine:

i. The stresses in the rod and tube, and
ii. Load carried by the rod and tube.

Take E for steel = 2.1×10^5 N/mm^2 and for copper = 1.1×10^5 N/mm^2.

Given:

Diameter of steel rod, D_s = 3 cm = 30 mm

External diameter of copper tube, D_c = 5 cm = 50 mm

Internal diameter of copper tube, d_c = 4 cm = 40 mm

Axial pull, P = 45000 N

Length of bar and tube, L = 15 cm = 150 mm

Young's modulus of steel, E_s = 2.1×10^5 N/mm^2

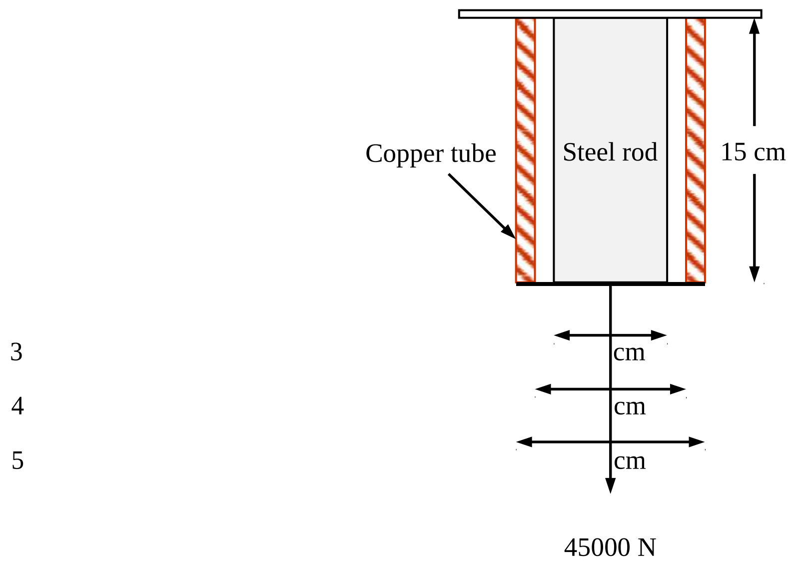
Young's modulus of copper, E_c = 1.1×10^5 N/mm^2

To find:

The stresses in the rod and tube

Load carried by each bar

Solution:



Area of steel rod

$$A_s = \frac{\pi}{4} D_s^2$$

$$= \frac{\pi}{4} \times 30^2$$

$$= 225\,\pi \text{ mm}^2$$

Area of copper tube, A_c

$$= \frac{\pi}{4}(D_c^{\ 2} - d_c^{\ 2})$$

$$= \frac{\pi}{4} \times (50^2 - 40^2)$$

$$= 225\ \pi\ \text{mm}^2$$

Change in length of steel rod = Change in length of copper tube

$\delta L_s = \delta L_c$

$$\frac{P_s L_s}{A_s E_s} = \frac{P_c L_c}{A_c E_c}$$

Lengths of the steel rod and copper tube are equal.

$L_s = L_c$

$$\frac{P_s}{A_s E_s} = \frac{P_c}{A_c E_c}$$

$$P_s = \frac{A_s E_s P_c}{A_c E_c}$$

$$= \frac{225\ \pi \times 2.1 \times 10^5 \times P_c}{225\ \pi \times 1.1 \times 10^5}$$

$P_s = 1.9091\ P_c$ … (1)

Total load = Load on steel + Load on copper

$P = P_s + P_c$

$45000 = 1.9091\ P_c + P_c$

$= 2.9091\ P_c$

$$P_c = \frac{45000}{2.9091}$$

$$= 15468.75\ \text{N}$$

Substituting the above value in equation (1)

$$P_s = 1.9091 \times 15468.75$$

$$= 29531.25\ \text{N}$$

Stress in steel rod

$$\sigma_s = \frac{P_s}{A_s}$$

$$= \frac{29531.25}{225\ \pi}$$

$$= 41.7782\ \text{N/mm}^2$$

Stress in copper tube

$$\sigma_c = \frac{P_c}{A_c}$$

$$= \frac{15468.75}{225\,\pi}$$

$$= 21.8838 \text{ N/mm}^2$$

Result:

Load carried by steel rod, P_s = 29531.25 N

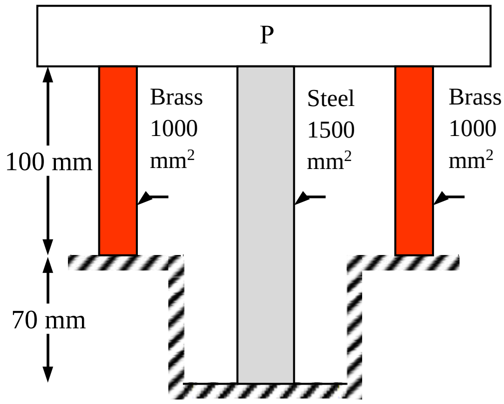
Load carried by copper tube, P_c = 15468.75 N

Stress in steel rod, σ_s = 41.7782 N/mm^2

Stress in copper tube, σ_c = 21.8838 N/mm^2

PROBLEM 6

Two brass rods and one steel rod together support a load as shown in figure. If the stresses in brass and steel are not to exceed 60 N/mm^2 and 120 N/mm^2, find the safe load that can be supported. Take E for steel = 2×10^5 N/mm^2 and for brass = 1×10^5 N/mm^2. The cross sectional size of steel rod is 1500 mm^2 and of each brass rod is 1000 mm^2.



Given:

Stress in brass, σ_b	≤ 60 N/mm^2
Stress in steel, σ_s	≤ 120 N/mm^2
Young's modulus for steel, E_s	$= 2 \times 10^5$ N/mm^2
Young's modulus for brass, E_b	$= 1 \times 10^5$ N/mm^2
Area of steel rod, A_s	= 1500 mm^2
Area of brass rod, A_b	= 1000 mm^2
Length of steel rod, L_s	= 100 + 70 = 170 mm
Length of brass rods, L_b	= 100 mm

To find:

Safe load, P

Solution:

Decrease in length of steel rod = Decrease in length of brass rods

Strain in steel × Length of steel rod = Strain in brass rods × Length of brass rods

$e_s L_s = e_b L_b$

$$\frac{e_s}{e_b} = \frac{L_b}{L_s}$$

$$= \frac{100}{170}$$

Stress in steel, $\sigma_s = e_s E_s$ … (1)

Stress in brass, $\sigma_b = e_b E_b$ … (2)

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e}$$

Dividing equation (1) by equation (2)

$$\frac{\sigma_s}{\sigma_b} = \frac{e_s E_s}{e_b E_b}$$

$$= \frac{100 \times 2 \times 10^5}{170 \times 1 \times 10^5}$$

$$= 1.1765$$

Suppose steel is permitted to reach its safe stress of 60 N/mm^2, the corresponding stress in brass will be

$$\sigma_b = \frac{\sigma_s}{1.1765}$$

$$= \frac{120}{1.1765}$$

$$= 102 \text{ N/mm}^2$$

which exceeds the safe stress of 60 N/mm^2 for brass. Therefore let brass be allowed to reach its safe stress of 60 N/mm^2. Then the corresponding stress in steel will be

$\sigma_s = 1.1765\ \sigma_b$

$= 1.1765 \times 60$

$= 70.5882$ N/mm^2

which is less than 120 N/mm^2.

Total load = Load on steel + Load on brass

$P = \sigma_s A_s + \sigma_b A_b$

$$\text{Stress, } \sigma = \frac{\text{Load}}{\text{Area}} = \frac{P}{A}$$

$P = 70.5882 \times 1500 + 60 \times (2 \times 1000)$

$= 225882.3529$ N

Result:

Safe load, P = 225882.3529 N

PROBLEM 7

A load of 270 kN is applied on a short concrete column 250 mm × 250 mm. The column is reinforced with 8 steel bars of 16 mm diameter. If the modulus of elasticity for steel is 18 times that of concrete, find the stresses in concrete and steel.
If the stress in concrete should not exceed 4 N/mm^2, find the area of steel required so that the column may support a load of 400 kN.

Given:

Load, P = 270 kN = 270×10^3 N

Area of concrete column = 250 mm × 250 mm

Number of steel bars, n = 8

Diameter of steel bars, d = 30 mm

Modulus of elasticity of steel, E_s= 18 × Modulus of elasticity of concrete = 18 E_c

Stress in concrete, σ_s ≤ 4 N/mm^2 when P = 400 kN

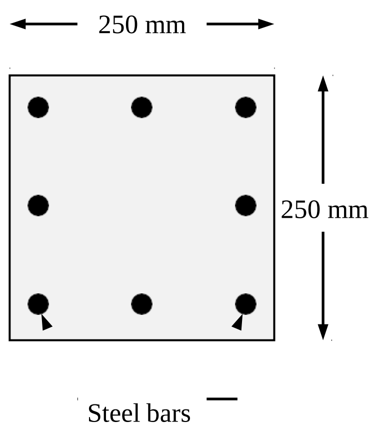
To find:

Stress in concrete, σ_c

Stress in steel, σ_s

Area of steel, A_s when P = 400 kN

Solution:



(i) When the column carries a load of 300 kN

Area of steel, A_s = Area of one steel bar × Number of steel bars

$$= \frac{\pi}{4} d^2 \times n$$

$$= \frac{\pi}{4} \times 16^2 \times 8$$

$= 512\ \pi\ mm^2$

Area of concerete, A_c = Area of concrete column – Area of steel

= (250 × 250) – 512 π

= 60891.5046 mm^2

$$E = \frac{\sigma}{e}$$

Strain in steel = Strain in concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c}\sigma_c$$

$$= \frac{18\ E_c}{E_c}\sigma_c = 18\ \sigma_c$$

$$\sigma = \frac{P}{A}$$

Total load = Load on steel + Load on concrete

$P = \sigma_s A_s + \sigma_c A_c$

$270 \times 10^3 = (18\ \sigma_c \times 512\ \pi) + (\sigma_c \times 60891.5046)$

$= (18 \times 512\ \pi + 60891.5046)\ \sigma_c$

$= 89844.4225\ \sigma_c$

$$\sigma_c = \frac{270 \times 10^3}{89844.4225}$$

= 3.0052 N/mm^2

$\sigma_s = 18\ \sigma_c$

= 15 × 3.0052

= 54.0935 N/mm^2

(ii) When the column carries a load of 400 kN

Let the area of steel bars be A_s mm^2.

Area of concerete, A_c = Area of concrete column – Area of steel

= (250 × 250) – A_s

= 62500 – A_s

Strain in steel = Strain in concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c}\sigma_c$$

$$= \frac{18\ E_c}{E_c}\sigma_c = 18\ \sigma_c$$

$\sigma_s = 18 \times 4 = 72$ N/mm^2

Total load = Load on steel + Load on concrete

$P = \sigma_s A_s + \sigma_c A_c$

$400 \times 10^3 = (72 \times A_s) + 4 \times (\ 62500 - A_s)$

$= 72\ A_s + (4 \times 62500) - 4\ A_s$

$= 68\ A_s + 250000$

$400 \times 10^3 - 250000 = 68\ A_s$

$$A_s = \frac{400 \times 10^3 - 250000}{68}$$

$= 2205.8824$ mm^2

Result:

Stress in concrete, σ_c	= 3.0052 N/mm^2
Stress in steel, σ_s	= 54.0935 N/mm^2
Area of steel, A_s (when P = 400 kN)	= 2205.8824 mm^2

PROBLEM 8

A load of 300 kN is applied on a short concrete column 250 mm × 250 mm. The column is reinforced with 8 steel bars of 30 mm diameter. If the modulus of elasticity for steel is 15 times that of concrete, find the stresses in concrete and steel.
If the stress in concrete should not exceed 4 N/mm^2, find the area of steel required so that the column may support a load of 600 kN.

Given:

Load, P = 300 kN = 300 × 10^3 N

Area of concrete column = 250 mm × 250 mm

Number of steel bars, n = 8

Diameter of steel bar, d = 30 mm

Modulus of elasticity of steel, E_s = 15 × Modulus of elasticity of concrete = 15 E_c

Stress in concrete, σ_s ≤ 4 N/mm^2 when P = 600 kN

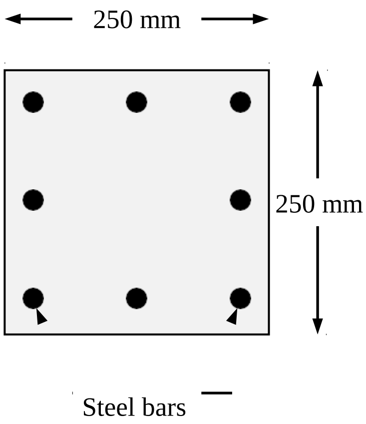
To find:

Stress in concrete, σ_c

Stress in steel, σ_s

Area of steel, A_s when P = 600 kN

Solution:



(i) When the column carries a load of 300 kN

Area of steel, A_s = Area of one steel bar × Number of steel bars

$$= \frac{\pi}{4} d^2 \times n$$

$$= \frac{\pi}{4} \times 30^2 \times 8$$

= 1800 π mm^2

Area of concerete, A_c = Area of concrete column – Area of steel

= (250 × 250) – 1800 π

= 56845.1332 mm^2

$$E = \frac{\sigma}{e}$$

Strain in steel = Strain in concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c}\sigma_c$$

$$= \frac{15\ E_c}{E_c}\sigma_c = 15\ \sigma_c$$

$$\sigma = \frac{P}{A}$$

Total load = Load on steel + Load on concrete

$P = \sigma_s A_s + \sigma_c A_c$

$300 \times 10^3 = (15\ \sigma_c \times 1800\ \pi) + (\sigma_c \times 56845.1332)$

$= (15 \times 1800\ \pi + 56845.1332)\ \sigma_c$

$= 141668.1349\ \sigma_c$

$$\sigma_c = \frac{300 \times 10^3}{141668.1349}$$

= 2.1176 N/mm^2

$\sigma_s = 15\ \sigma_c$

= 15 × 2.1176

= 31.7644 N/mm^2

(ii) When the column carries a load of 600 kN

Let the area of steel bars be A_s mm^2.

Area of concerete, A_c = Area of concrete column – Area of steel

= (250 × 250) – A_s

= 62500 – A_s

Strain in steel = Strain in concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c}\sigma_c$$

$$= \frac{15\ E_c}{E_c}\sigma_c = 15\ \sigma_c$$

$\sigma_s = 15 \times 4 = 60$ N/mm^2

Total load = Load on steel + Load on concrete

$P = \sigma_s A_s + \sigma_c A_c$

$600 \times 10^3 = (60 \times A_s) + 4 \times (62500 - A_s)$

$= 60 A_s + (4 \times 62500) - 4 A_s$

$= 56 A_s + 250000$

$600 \times 10^3 - 250000 = 56 A_s$

$$A_s = \frac{600 \times 10^3 - 250000}{56}$$

$= 6250$ mm^2

Result:

Stress in concrete, σ_c	= 2.1176 N/mm^2
Stress in steel, σ_s	= 31.7644 N/mm^2
Area of steel, A_s (when P = 600 kN)	= 6250 mm^2

PROBLEM 9

A compound bar consist of a central steel strip 25 mm wide and 6.4 mm thick placed between two strips of brass each 25 mm wide and 't' mm thick. The strips are firmly fixed together to form a compound bar of rectangular section 25 mm wide and (2t + 6.4) mm thick. Determine (i) the thickness of the brass strips which will make the apparent modulus of elasticity of compound bar 1.57×10^5 N/mm^2 and (ii) the maximum axial pull the bar can then carry if the stress is not to exceed 157 N/mm^2, in either the brass or steel. Take the values of E for steel and brass as 2.07×10^5 N/mm^2 and 1.14×10^5 N/mm^2.

Given:

Width of steel strip, w_s = 25 mm

Thickness of steel strip, t_s = 6.4 mm

Width of brass strips, w_b = 25 mm

Thickness of brass strips, t_b = t mm

Apparent Young's modulus, E = 1.57×10^5 N/mm^2

Stress in brass or steel, σ_s or $\sigma_b \leq 157$ N/mm^2

Young's modulus for steel, E_s = 2.07×10^5 N/mm^2

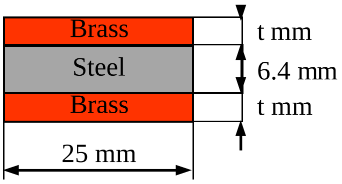
Young's modulus for brass, E_b = 1.14×10^5 N/mm^2

To find:

Thickness of brass strips, t_b

Axial pull, P

Solution:



Area of steel, $A_s = 25 \times 6.4 = 160$ mm^2

Area of brass, $A_b = 2 \times (25 \times t) = 50\,t$ mm^2

Strain in steel = Strain in brass

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\sigma_s = \frac{E_s}{E_b}\sigma_b$$

$$= \frac{2.07 \times 10^5}{1.14 \times 10^5}\sigma_b$$

$$= 1.8158\,\sigma_b$$

Total load = Load on steel + Load on brass

$P = \sigma_s A_s + \sigma_b A_b$

$= (1.8158\, \sigma_b \times 160) + (\sigma_b \times 50\, t)$

$= 290.5263\, \sigma_b + 50\, t\, \sigma_b$

$= (290.5263 + 50\, t)\, \sigma_b$

Area of the composite section

$A = A_s + A_b$

$= 160 + 50\, t$ mm^2

Strain

$$e = \frac{\sigma}{E}$$

$$= \frac{P}{AE}$$

$$= \frac{(290.5263 + 50\,t)\,\sigma_b}{(160 + 50\,t) \times 1.57 \times 10^5}$$

The above strain must be equal to the strain of brass or steel

Strain of brass

$$e_b = \frac{\sigma_b}{E_b}$$

$$= \frac{\sigma_b}{1.14 \times 10^5}$$

Equating e and e_b

$e = e_b$

$$\frac{(290.5263 + 50\,t)\,\sigma_b}{(160 + 50\,t) \times 1.57 \times 10^5} = \frac{\sigma_b}{1.14 \times 10^5}$$

$$\frac{(290.5263 + 50\,t)}{(160 + 50\,t) \times 1.57 \times 10^5} = \frac{1}{1.14 \times 10^5}$$

$$290.5263 + 50\,t = \frac{(160 + 50\,t) \times 1.57 \times 10^5}{1.14 \times 10^5}$$

$$290.5263 + 50\,t = 220.3509 + 68.8596\,t$$

$$290.5263 - 220.3509 = 68.8596\,t - 50\,t$$

$$290.5263 - 220.3509 = (68.8596 - 50)\,t$$

$$t = \frac{290.5263 - 220.3509}{68.8596 - 50}$$

$= 3.7209$ mm

Since $\sigma_s = 1.8158\, \sigma_b$ and since the stress in either steel or brass should not exceed 157 N/mm^2 (i.e. σ_s or $\sigma_b \leq 157$ N/mm^2)

Let $\sigma_s = 157$ N/mm^2

$$\sigma_b = \frac{\sigma_s}{1.8158}$$

$$= \frac{157}{1.8158}$$

$$= 86.4638 \text{ N/mm}^2$$

Load

$P = \sigma_s A_s + \sigma_b A_b$

$P = (157 \times 160) + (86.4638 \times 50 \times 3.7209)$

$P = 41206.2824$ N

Result:

Thickness of brass strips, t_s = 3.7209 mm

Axial pull, P = 41206.2824 N

PROBLEM 10

A steel tube of 30 mm external diameter and 20 mm internal diameter encloses a copper rod of 15 mm diameter to which it is rigidly joined at each end. If, at a temperature of 10°C there is no longitudinal stress, calculate the stresses in the rod and tube when the temperature is raised to 200°C. Take E for steel and copper as 2.1×10^5 N/mm^2 and 1×10^5 N/mm^2 respectively. The value of co-efficient of linear expansion for steel and copper is 11×10^{-6} per °C and 18×10^{-6} per °C respectively.

Given:

External diameter of steel tube, D_s	= 30 mm
Internal diameter of steel tube, d_s	= 20 mm
Diameter of copper rod, D_c	= 15 mm
Initial temperature, T_i	= 10°C
Final temperature, T_f	= 200°C
Young's modulus of steel, E_s	= 2.1×10^5 N/mm^2
Young's modulus of copper, E_c	= 1×10^5 N/mm^2
Coefficient of linear expansion of steel, α_s	= 11×10^{-6} per °C
Coefficient of linear expansion of copper, α_c	= 18×10^{-6} per °C

To find:

Stress in steel tube, σ_s

Stress in copper rod, σ_c

Solution:

Area of steel tube, A_s

$$= \frac{\pi}{4}(D_s^2 - d_s^2)$$

$$= \frac{\pi}{4} \times (30^2 - 20^2)$$

$= 125\,\pi$ mm^2

Area of copper rod, A_c

$$= \frac{\pi}{4} D_c^2$$

$$= \frac{\pi}{4} \times 15^2$$

$= 56.25\,\pi$ mm^2

Rise in temperature, $T = T_f - T_i$

$= 200 - 10$

$= 190$°C

Since the ends of the bars are rigidly fixed, the composite section as a whole will expand or contract. The coefficient of linear expansion of copper is more than that of steel. So, the copper will expand more than steel. But the actual expansion of the composite bar will be less than that of copper. Therefore, copper will be subjected to compressive stress, where as the steel will be subjected to tensile stress.

For the equilibrium of the system,

Tensile load on steel = Compressive load on copper

$\sigma_s A_s = \sigma_c A_c$

$$\sigma_s = \frac{A_c}{A_s} \times \sigma_c$$

$$= \frac{56.25\,\pi}{125\,\pi} \times \sigma_c$$

$= 0.45\, \sigma_c$ … (1)

We know that the steel rod and copper tube will actually expand by the same amount.

Actual expansion of steel = Actual expansion of copper … (2)

Actual expansion of steel = Free expansion of steel + Expansion due to tensile stress in steel

$$= \alpha_s\, T\, L + \frac{\sigma_s}{E_s} L$$

Actual expansion of copper = Free expansion of copper – Contraction due to compressive stress in copper

$$= \alpha_c\, T\, L - \frac{\sigma_c}{E_c} L$$

Substituting the above values in equation (2)

$$\alpha_s\, T\, L + \frac{\sigma_s}{E_s} L = \alpha_c\, T\, L - \frac{\sigma_c}{E_c} L$$

$$\alpha_s\, T + \frac{\sigma_s}{E_s} = \alpha_c\, T - \frac{\sigma_c}{E_c}$$

$$11 \times 10^{-6} \times 190 + \frac{0.45\, \sigma_c}{2.1 \times 10^5} = 18 \times 10^{-6} \times 190 - \frac{\sigma_c}{1 \times 10^5}$$

$$\frac{0.45\, \sigma_c}{2.1 \times 10^5} + \frac{\sigma_c}{1 \times 10^5} = 18 \times 10^{-6} \times 190 - 11 \times 10^{-6} \times 190$$

$$\left(\frac{0.45}{2.1 \times 10^5} + \frac{1}{1 \times 10^5}\right)\sigma_c = (18 \times 10^{-6} - 11 \times 10^{-6}) \times 190$$

$$\sigma = \frac{(18 \times 10^{-6} - 11 \times 10^{-6}) \times 190}{\left(\frac{0.45}{2.1 \times 10^5} + \frac{1}{1 \times 10^5}\right)}$$

$$\sigma_c = 109.5294 \text{ N/mm}^2$$

Substituting the above value in equation (1)

$\sigma_s = 0.45 \times 109.5294$

$\sigma_s = 49.2882$ N/mm^2

Result:

Stress in steel tube, σ_s = 49.2882 N/mm^2

Stress in copper rod, σ_c = 109.5294 N/mm^2

PROBLEM 11

A steel bar is placed between two copper bars each having the same area and length as the steel bar at 15°C. At this stage they are rigidly connected together at both the ends. When the temperature is raised to 315°C, the length of the bars increases by 1.50 mm. Determine the original length and the final stresses in the bars. Take $E_s = 2.1 \times 10^5$ N/mm^2; $E_c = 1 \times 10^5$ N/mm^2; $\alpha_s = 0.000012$ per °C; $\alpha_c = 0.0000175$ per °C.

Given:

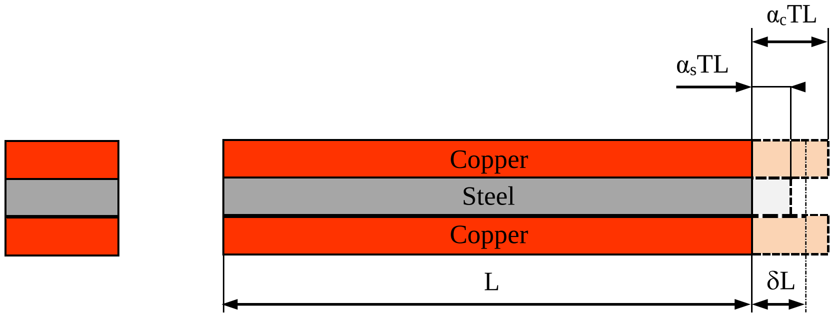
Area of copper bar	= Area of steel bar
Length of copper bar	= Length of steel bar
Initial temperature, T_i	= 15°C
Final temperature, T_f	= 315°C
Change in length, δL	= 1.50 mm
Young's modulus of steel, E_s	= 2.1×10^5 N/mm^2
Young's modulus of copper, E_c	= 1×10^5 N/mm^2
Coefficient of linear expansion of steel, α_s	= 0.000012 per °C
Coefficient of linear expansion of copper, α_c	= 0.0000175 per °C

To find:

Original length of bars, L

Stresses in bars, σ_s and σ_c

Solution:



Cross-sectional area of steel bar, A_s = A mm^2

Cross-sectional area of copper bars, A_c = 2A mm^2

Rise in temperature, $T = T_f - T_i$

$= 315 - 15$

$= 300°C$

Coefficient of linear expansion of copper is more than that of steel, hence the free expansion of copper will be more than that of steel when there is a rise in temperature. But the ends of the bars are rigidly fixed and the two bars are not free to expand. Hence the steel and brass bars will expand by the same amount. The free expansion of the copper bar will be more than the common expansion, whereas the free expansion of the steel bar will be less than the common expansion. Hence the copper bar will be subjected to compressive stress while the steel bar will be subjected to tensile stress.

For the equilibrium of the system,

Tensile load on steel = Compressive load on copper

$\sigma_s A_s = \sigma_c A_c$

$$\sigma_s = \frac{A_c}{A_s} \times \sigma_c$$

$$= \frac{2A}{A} \times \sigma_c$$

$= 2\,\sigma_c$ … (1)

We know that the steel rod and copper tube will actually expand by the same amount.

Actual expansion of steel = Actual expansion of copper … (2)

Actual expansion of steel = Free expansion of steel + Expansion due to tensile stress in steel

$$= \alpha_s\, T\, L + \frac{\sigma_s}{E_s} L$$

Actual expansion of copper = Free expansion of copper – Contraction due to compressive stress in copper

$$= \alpha_c\, T\, L - \frac{\sigma_c}{E_c} L$$

Substituting the above values in equation (2)

$$\alpha_s\, T\, L + \frac{\sigma_s}{E_s} L = \alpha_c\, T\, L - \frac{\sigma_c}{E_c} L$$

$$\alpha_s\, T + \frac{\sigma_s}{E_s} = \alpha_c\, T - \frac{\sigma_c}{E_c}$$

$$0.000012 \times 300 + \frac{2\,\sigma_c}{2.1 \times 10^5} = 0.0000175 \times 300 - \frac{\sigma_c}{1 \times 10^5}$$

$$\frac{2\,\sigma_c}{2.1 \times 10^5} + \frac{\sigma_c}{1 \times 10^5} = 0.0000175 \times 300 - 0.000012 \times 300$$

$$\left(\frac{2}{2.1 \times 10^5} + \frac{1}{1 \times 10^5}\right) \sigma_c = (0.0000175 - 0.000012) \times 300$$

$$\sigma_c = \frac{(0.0000175 - 0.000012) \times 300}{\left(\frac{2}{2.1 \times 10^5} + \frac{1}{1 \times 10^5}\right)}$$

$\sigma_c = 84.5122\ \mathrm{N/mm^2}$

Substituting the above value in equation (A)

$\sigma_s = 2 \times 84.5122$

$\sigma_s = 169.0244\ \mathrm{N/mm^2}$

Actual expansion of steel

$$\delta L = \alpha_s T L + \frac{\sigma_s}{E_s} L$$

$$\delta L = L \left(\alpha_s T + \frac{\sigma_s}{E_s} \right)$$

$$L = \frac{\delta L}{\alpha_s T + \frac{\sigma_s}{E_s}}$$

$$= \frac{1.50}{0.000012 \times 300 + \frac{169.0244}{2.1 \times 10^5}}$$

$$= 340.5316 \text{ mm}$$

Result:

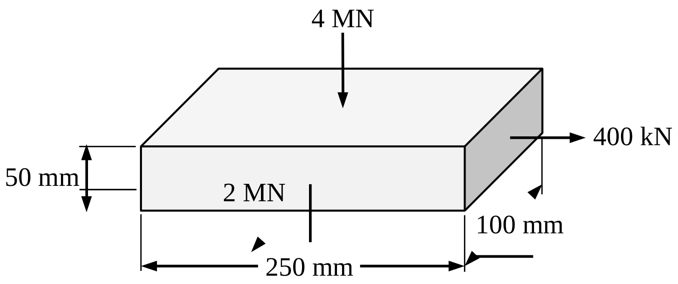
Original length of bars, L = 340.5316 mm

Stress in steel bar, σ_s = 169.0244 N/mm^2

Stress in copper bars, σ_s = 84.5122 N/mm^2

PROBLEM 12

A metallic bar 250 mm × 100 mm × 50 mm is loaded as shown in figure. Find the change in volume. Take E = 2 × 10^5 N/mm^2 and Poisson's ratio = 0.25. Also find the change that should be made in the 4 MN load, in order that there should be no change in the volume of the bar.



Given:

Dimensions of metallic bar = 250 mm × 100 mm × 50 mm

Length, x = 250 mm

Height, y = 50 mm

Width, z = 100 mm

Load in x-direction = 400 kN = 400 × 10^3 (tensile)

Load in y-direction = – 4 MN = – 4 × 10^6 (compressive)

Load in z-direction = 2 MN = 2 × 10^6 (tensile)

Young's modulus, E = 2 × 10^5 N/mm^2

Poisson's ratio, μ = 0.25

To find:

Change in volume, δV

Change in the 4 MN load when there is no change in volume of bar

Solution:

(i) Change in volume

Stress in x-direction

$$\sigma_x = \frac{\text{Load in x} - \text{direction}}{\text{Area of cross} - \text{section}}$$

$$= \frac{400 \times 10^3}{y \times z}$$

$$= \frac{400 \times 10^3}{50 \times 100}$$

$= 80\ N/mm^2$ (tension)

Stress in y-direction

$$\sigma_y = \frac{\text{Load in y} - \text{direction}}{\text{Area of cross} - \text{section}}$$

$$= \frac{-4 \times 10^6}{x \times z}$$

$$= \frac{-4 \times 10^6}{250 \times 100}$$

$= -160\ N/mm^2$ (compression)

Stress in z-direction

$$\sigma_z = \frac{\text{Load in z} - \text{direction}}{\text{Area of cross} - \text{section}}$$

$$= \frac{2 \times 10^6}{x \times y}$$

$$= \frac{2 \times 10^6}{250 \times 50}$$

$= 160\ N/mm^2$ (tension)

Change in volume

$$\frac{\delta V}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$$

$$= \frac{1}{2 \times 10^5}(80 - 160 + 160)(1 - 2 \times 0.25)$$

$$= 2 \times 10^{-4}$$

$\delta V = 2 \times 10^{-4} \times V$

$= 2 \times 10^{-4} \times x \times y \times z$

$= 2 \times 10^{-4} \times 250 \times 50 \times 100$

$= 250\ mm^3$

(ii) Change in the 4 MN load when there is no change in volume of bar

$$\frac{\delta V}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$$

If there is no change in volume, then

$$\frac{\delta V}{V} = 0$$

$$\frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu) = 0$$

But for most of materials, the value of μ lies between 0.25 and 0.33 and hence the term (1 – 2μ) is never zero.

$$\frac{1}{E}(\sigma_x + \sigma_y + \sigma_z) = 0$$

$$(\sigma_x + \sigma_y + \sigma_z) = 0$$

The stresses σ_x and σ_z are not to be changed. Only the stress corresponding to the load 4 MN (i.e. stress in y-direction) is to be changed.

$\sigma_y = -\sigma_x - \sigma_z$

$\sigma_y = -80 - 160$

$\sigma_y = -240$ N/mm^2 (compressive)

Stress in y-direction

$$\sigma_y = \frac{\text{Load in y} - \text{direction}}{\text{Area of cross} - \text{section}}$$

$$-240 = \frac{\text{Load in y} - \text{direction}}{x \times z}$$

$$= \frac{\text{Load in y} - \text{direction}}{250 \times 100}$$

Load in y-direction = – 240 × 250 × 100

= – 6000000 = – 6 MN (compressive)

But already a compressive load of 4 MN is acting.

Additional load that must be added = 6 MN – 4 MN

= 2 MN (compressive)

Result:

Change in volume, δV = 250 mm^3

Additional load in y-direction = 2 MN (compressive)

PROBLEM 13

A tension bar 5 m long is made up of two parts, 3 m of its length has a diameter of 3.5 cm while the remaining 2 m has a diameter of 5 cm. An axial load of 80 kN is gradually applied. Find the total strain energy produced in the bar and compare this value with that obtained in a uniform bar of the same length and having the same volume when under the same load. Take $E = 2 \times 10^5$ N/mm^2.

Given:

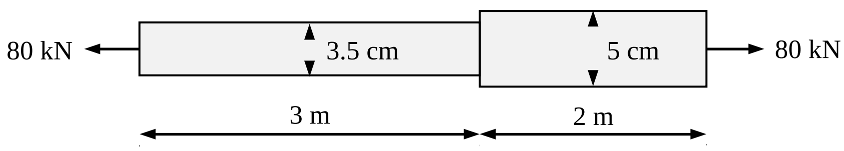
Total length of bar, L	= 5 m = 5000 mm
Length of first part, L_1	= 3 m = 3000 mm
Diameter of first part, D_1	= 3.5 cm = 35 mm
Length of second part, L_2	= 2 m = 2000 mm
Diameter of second part, D_2	= 5 cm = 50 mm
Axial load, P	= 80 kN = 80×10^3 N
Young's modulus, E	= 2×10^5 N/mm^2

To find:

Total stain energy produced in the bar, U

Strain energy in the given bar / Strain energy in the uniform bar

Solution:



	First part	Second part
Area	$A_1 = \frac{\pi}{4} D_1^2$ $A_1 = \frac{\pi}{4} 35^2$ $A_1 = 306.25\,\pi$ mm^2	$A_2 = \frac{\pi}{4} D_2^2$ $A_2 = \frac{\pi}{4} 50^2$ $A_2 = 625\,\pi$ mm^2
Volume	$V_1 = A_1 \times L_1$ $V_1 = 306.25\,\pi \times 3000$ $V_1 = 918750\,\pi$ mm^3	$V_2 = A_2 \times L_2$ $V_2 = 625\,\pi \times 2000$ $V_2 = 1250000\,\pi$ mm^3
Stress	$\sigma_1 = \frac{P}{A_1}$ $\sigma_1 = \frac{80 \times 10^3}{306.25\,\pi}$ $\sigma_1 = 83.1503$ N/mm^2	$\sigma_2 = \frac{P}{A_2}$ $\sigma_2 = \frac{80 \times 10^3}{625\,\pi}$ $\sigma_2 = 40.7437$ N/mm^2

Strain energy	$U_1 = \frac{\sigma_1^2 V_1}{2E}$ $U_1 = \frac{83.1503^2 \times 918750\,\pi}{2 \times 2 \times 10^5}$ $U_1 = 49890.2026$ Nm	$U_2 = \frac{\sigma_2^2 V_2}{2E}$ $U_2 = \frac{40.7437^2 \times 1250000\,\pi}{2 \times 2 \times 10^5}$ $U_2 = 16297.4662$ Nm

Total strain energy produced in the bar

$U = U_1 + U_2$

$= 49890.2026 + 16297.4662$

$= 66187.6687$ Nm

Strain energy stored in a uniform bar

Volume of uniform bar

$V = V_1 + V_2$

$= 918750\,\pi + 1250000\,\pi$

$= 2168750\,\pi\ \text{mm}^3$

Length of uniform bar, L = 5000 mm

Area of uniform bar

$V = A \times L$

$A = \frac{V}{L}$

$= \frac{2168750\,\pi}{5000}$

$= 1362.6658\ \text{mm}^2$

Stress in uniform bar

$\sigma = \frac{P}{A}$

$= \frac{80 \times 10^3}{1362.6658}$

$= 58.7085\ \text{N/mm}^2$

Strain energy stored in the uniform bar

$U = \frac{\sigma^2 V}{2E}$

$= \frac{58.7085^2 \times 2168750\,\pi}{2 \times 2 \times 10^5}$

$= 58708.4516$ Nm

$$\frac{\text{Strain energy in the given bar}}{\text{Strain energy in the uniform bar}} = \frac{66187.6687}{58708.4516} = 1.1274$$

Result:

Total strain energy produced in the given bar = 66187.6687 Nm

$$\frac{\text{Strain energy in the given bar}}{\text{Strain energy in the uniform bar}} = 1.1274$$

TWO MARKS QUESTIONS AND ANSWERS

1. Define stress.

When an external force acts on a body, it undergoes deformation and at the same time the body resists deformation. The magnitude of the resisting force is numerically equal to the applied force. This internal resisting force per unit area is called stress.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\sigma = \frac{P}{A}$$

2. Define strain.

When a body is subjected to an external force, there is some change of dimension in the body. Numerically the strain is equal to the ratio of change in length to the original length of the body.

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$e = \frac{\delta L}{L}$$

3. State Hooke's law.

Hooke's law states that when a material is loaded within its elastic limit, the stress is directly proportional to the strain.

$$\text{Stress} \propto \text{Strain}$$

$$\sigma \propto e$$

4. Define factor of safety.

Factor of safety is defined as the ratio of ultimate tensile stress to the permissible stress (working stress).

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Permissible stress}}$$

5. Define tensile stress and tensile strain.

When a member is subjected to equal and opposite axial pulls, the length of the member is increased. The stress is induced at any cross section of the member is called tensile stress and corresponding strain is known as tensile strain.

$$\text{Tensile stress} = \frac{\text{Tensile load}}{\text{Area}}$$

$$\text{Tensile strain} = \frac{\text{Increase in length}}{\text{Original length}}$$

6. Define compressive stress and compressive strain.

When a member is subjected to equal and opposite pushes, the length of the member is shortened. The stress produced at any cross section of the member is called compressive stress and the corresponding strain is known as compressive strain.

$$\text{Compressive stress} = \frac{\text{Compressive load}}{\text{Area}}$$

$$\text{Compressive strain} = \frac{\text{Decrease in length}}{\text{Original length}}$$

7. Define shear stress and shear strain.

   When a member is subjected to two equal and opposite forces which are acting tangentially on any cross sectional plane tend to slide one part of the body over the other part. The stress induced in that section is called shear stress and the corresponding strain is known as shear strain.

8. Define volumetric strain.

   Volumetric strain is defined as the ratio of change in volume to the original volume of the body.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$e_v = \frac{\delta V}{V}$$

9. Define modulus of elasticity or Young's modulus.

   When a body is stressed, within its elastic limit, the ratio of tensile stress to the corresponding tensile strain is constant. This ratio is known as modulus of elasticity or Young's modulus.

$$\text{Modulus of elasticity or Young's modulus} = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

$$E = \frac{\sigma}{e}$$

10. Define modulus of rigidity or shear modulus.

    When a body is stressed, within its elastic limit, the ratio of shearing stress to the corresponding shearing strain is constant. This ratio is known as modulus of rigidity.

$$\text{Modulus of rigidity or shear modulus} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\text{C or G or N} = \frac{\tau}{\phi}$$

11. Define bulk modulus.

    When a body is stressed, within its elastic limit, the ratio of direct stress to the corresponding volumetric strain is constant. This ratio is known as bulk modulus.

$$\text{Bulk modulus} = \frac{\text{Direct stress}}{\text{Volumetric strain}}$$

$$K = \frac{\sigma}{}$$

e_v

12. Define Poisson's ratio.

When a body is stressed within its elastic limit, the ratio of lateral strain to the longitudinal strain is constant for a given material.

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\mu \text{ or } \frac{1}{m} = \frac{e_t}{e_l}$$

13. State the relationship between Young's modulus and modulus of rigidity.

$$E = 2G\,(1 + \mu)$$

where, E – Young's modulus or modulus of elasticity,

G – Modulus of rigidity, and

μ – Poisson's ratio.

14. Give the relationship between Young's modulus and bulk modulus.

$$E = 3K\,(1 - 2\mu)$$

where, E – Young's modulus or modulus of elasticity,

K – Bulk modulus, and

μ – Poisson's ratio.

15. What do you understand by a composite or compound bar?

A composite or compound bar is composed of two or more different materials which are joined together so that the system is elongated or compressed as a single unit.

16. What are the types of elastic constants?

- Modulus of elasticity or Young's Modulus, E
- Shear modulus or modulus of rigidity, G
- Bulk Modulus, K

17. Write two relations used to find the forces in composite bars made of two materials subjected to tension or compression.

- Change in length of bar 1 = Change in length of bar 2
- Total load = Loads carried by bar 1 + Load carried by bar 2

18. What is strain energy?

Whenever a body is strained, the energy is absorbed in the body. The energy, which is absorbed in the body due to straining effect, is known as strain energy.

19. Define resilience.

The total strain energy stored in a body is known as resilience.

20. Define proof resilience.

The maximum strain energy stored in a body is known as proof resilience. The strain energy stored in the body will be maximum when the body is stressed up to the elastic limit.

21. Defile modulus of resilience.

Modulus of resilience is defined as the proof resilience of a material per unit volume.

$$\text{Modulus of resilience} = \frac{\text{Proof resilience}}{\text{Volume}}$$

14 MARKS QUESTIONS

1. A bar of 30 mm diameter is subjected to a pull of 60 kN. The measured extension on gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0039 mm. Calculate the Poisson's ratio and three elastic constants.

2. A hollow steel tube 3.5 m long has external diameter 120 mm. In order to determine the internal diameter, the tube was subjected to a tensile load of 400 kN and extension was measured to be 2 mm. If the modulus of elasticity for the tube material is 2×10^5 N/mm^2, determine the internal diameter of the tube.

3. Two wires, one steel and the other of copper, are of the same length and are subjected to the same tension. If the diameter of the copper wire is 2 mm, find the diameter of the steel wire, if they are elongated by the same amount. Take E for steel as 2×10^5 N/mm^2 and that for copper as 1×10^5 N/mm^2.

4. A steel bar of 3 m length and 20 mm diameter is subjected to an axial pull of 50 kN. Calculate the change in length, diameter and volume of the bar if the Poisson's ratio is 0.26. Take E = 200 kN/mm^2.

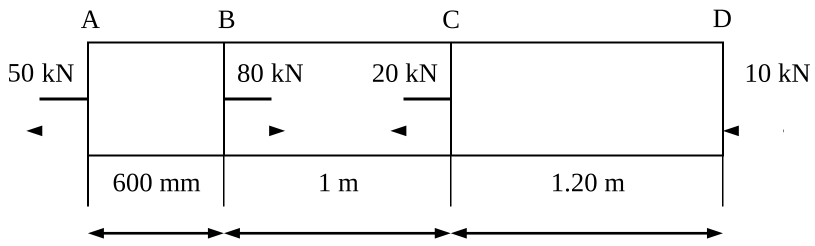
5. A 6 m hollow bar of circular section has 140 mm diameter for a length of 4 m, while it has 120 mm diameter for a length of 2 m. The bore diameter is 80 mm throughout. Find the elongation of the bar, when it is subjected to as axial tensile force of 300 kN. Take E as 2×10^5 N/mm^2.

6. A steel bar 2 m long and 40 mm in diameter is subjected to an axial pull of 80 kN. Find the length of the 20 mm diameter bore, which should be centrally carried out, so that the total elongation should increase by 20% under the same pull. Take E for the bar element as 2×10^5 N/mm^2.

7. An aluminium cylinder of diameter 60 mm located inside a steel cylinder of internal diameter 60 mm and wall thickness l5 mm. The assembly is subjected to compressive force of 200 kN. What are the forces carried and stresses developed in steel and aluminium? Take modulus of elasticity for steel as 2×10^5 N/mm^2 and aluminium as 0.75×10^5 N/mm^2.

8. A stepped bar ABCD consists of three parts AB, BC, and CD such that AB is 300 mm long and 20 mm in diameter, BC is 400 mm long and 30 mm in diameter and CD is 200 mm long and 40 mm in diameter. It was observed that the stepped bar undergoes a deformation of 0.42 mm, when it was subjected to a compressive load P. Find the value of P, if E = 2×10^5 N/mm^2.

9. A brass bar, having cross sectional area of 1000 mm^2, is subjected to axial forces as shown in figure. Find the elongation of the bar. Take E = 1.05×10^5 N/mm^2.



10. A reinforced concrete circular section of 50000 mm^2 cross sectional area carries 6 reinforcing bars whose total area is 500 mm^2. Find the safe load, the column can carry, if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio for steel and concrete as 18.

11. A reinforced concrete column 300 mm × 300 mm has six reinforcing bars of 25 mm diameter. When the column is loaded with 750 kN, find the stresses developed in the concrete and steel. Take $E_s/E_c = 15$.

12. A compound bar consists of a circular rod of steel of diameter 20 mm rigidly fitted into copper tube of internal diameter of 20 mm and external diameter of 30 mm. If the composite bar is 750 mm long and is subjected to a compressive load of 30 kN, find the stresses developed in the steel rod and copper tube. Take $E_s = 2 \times 10^5$ N/mm^2 and $E_c = 1 \times 10^5$ N/mm^2. Also find the change in the length of the bar.

13. A composite bar of 500 mm length is made up of a brass rod of 25 mm diameter enclosed in a steel tube of 50 mm external diameter and 35 mm internal diameter. The rod and tube, being coaxial and equal in length, are securely fixed at each end. If the stresses in brass and steel are not to exceed 80 MPa and 180 MPa respectively, find the load which the composite bar can safely carry? Take E for brass as 1×10^5 N/mm^2 and for steel as 2×10^5 N/mm^2.

14. A composite bar made up of aluminium and steel is held between two supports with area and length of steel bar is 1200 mm^2 and 500 mm, whereas for aluminium is 600 mm^2 and 250 mm. The bars are stress free at a temperature of 40°C. What will be the stresses in the two bars when the temperature is 18°C, if (a) the supports are unyielding (b) the supports come nearer to each other by 0.3 mm? It can be assumed that the change of temperature is uniform all along the length of the bar. Take E for steel as 2×10^5 N/mm^2, E for aluminium as 0.75×10^5 N/mm^2, coefficient of expansion for steel as 11.7×10^{-6} per °C and coefficient of expansion for aluminium as 23.4×10^{-6} per °C.

15. A flat steel bar 200 mm × 20 mm × 10 mm is placed between two aluminium bars 200 mm × 20 mm × 8 mm so as to form a composite bar. All the three bars are fastened together at room temperature. Find the stresses in each bar when the room temperature of the whole assembly is raised through 50°C. Assume Young's modulus for steel = 2×10^5 N/mm^2, Young's modulus for aluminium = 0.8×10^5 N/mm^2, coefficient of expansion for steel = 12×10^{-6} per °C and coefficient of expansion for aluminium as 24×10^{-6} per °C.

16. A steel bar 50 mm × 80 mm in cross section is 1.5 m long. It is subjected to an axial pull of 200 kN. What are the changes in length, width and volume of the bar, if the value of Poisson's ratio is 0.6? Take $E = 2.1 \times 10^5$ N/mm^2.

17. For a given material, E is 1.8×10^5 N/mm^2 and G is 0.5×10^5 N/mm^2. Find K and the lateral contraction of a round bar of 60 mm diameter and 3 m long, when stretched 3.5 mm. Take $\mu = 0.25$.

18. A round bar of 40 mm diameter is subjected to an axial pull of 80 kN and reduction in diameter was found to be 0.007775 mm. Find Poisson's ratio and Young's modulus for the material of the bar. Take value of shear modulus as 0.4×10^5 N/mm^2.

19. A bar of 20 mm diameter is tested in tension. It is observed that when a load of 40 kN is applied, the extension measured over a gauge length of 200 mm is 0.12 mm and contraction in diameter is 0.0036 mm. Find Poisson's ratio and elastic constants E, G and K.

20. Three bars made of copper, zinc and aluminium are of equal length and have cross-section 500, 750 and 1000 mm^2 respectively. They are rigidly connected at their ends. If this compound member is subjected to a longitudinal pull of 250 kN, estimate the proportion of load carried on

each rod, and the induced stresses. Take, $E_c = 1.3 \times 10^5$ N/mm^2, $E_z = 1 \times 10^5$ N/mm^2 and $E_a = 0.8 \times 10^5$ N/mm^2.

21. A rigid bar AC is supported by three rods in the same vertical plane and equidistant. The outer rods are of brass and of length 40 cm and diameter 2 cm. The central rod is of steel of 60 cm length and of 2.5 cm diameter. Calculate the forces in the bars due to an applied force P, if the bar AC remains horizontal after the load has been applied. Take $E_s/E_b = 2$.

22. A railway is laid so that there is no stress in the rails at 10°C. Calculate (a) the stress in rails at 60°C if there is no allowance for expansion, (b) the stress in the rails at 60°C if there is an expansion allowance of 10 mm per rail, (c) the expansion allowance if the stress in the rail is to be zero when the temperature is 60°C, (d) the maximum temperature to have no stress in the rails, if the expansion allowance is 13 mm per rails. Take $\alpha = 12 \times 10^{-6}$ per 1°C and $E = 2 \times 10^5$ N/mm^2. The rails are 30 m long.

23. A composite bar is made by fastening one flat bar of steel between two similar bar of aluminium alloy. The dimensions of each bar are 30 mm wide × 6 mm thick so that the cross-section of the composite bar measures 30 mm × 18 mm. If E for steel = 2×10^5 N/mm^2, and E for alloy = 0.6×10^5 N/mm^2, find the apparent value of E for the composite bar when loaded in tension. If the respective elastic limits are 230 N/mm^2 and 50 N/mm^2, find the elastic limit of the compound bar.

24. A rectangular block 250 mm × 100 mm × 80 mm is subjected to axial load as follows:
    480 kN tensile in the direction of length, 1000 kN compressive on the 250 mm × 100 mm face, and 900 kN tensile on 250 mm × 80 mm face.
    Assuming Poisson's ratio as 0.25, find in terms of modulus of elasticity of the material E, the strain in the direction of each force.
    If $E = 2 \times 10^5$ N/mm^2, find the values of the modulus of rigidity and bulk modulus for the material of the block. Also, calculate the change in volume of the block due to loading specified above.

OBJECTIVE QUESTIONS AND ANSWERS

1. Whenever some external system of forces acts on a body, it undergoes some deformation. As the body undergoes some deformation, it sets up some resistance to the deformation. This resistance per unit area to deformation, is called
   a) strain
   b) **stress**
   c) pressure
   d) modulus of elasticity

2. The unit of stress in S.I. units is
   a) N/mm^2
   b) kN/mm^2
   c) N/m^2
   d) **any one of these**

3. The deformation per unit length is called
   a) tensile stress
   b) compressive stress
   c) shear stress
   d) **strain**

4. The unit of strain is
   a) N-mm
   b) N/mm
   c) mm
   d) **unitless**

5. When a body is subjected to two equal and opposite pulls, as a result of which the body tends to extend its length, the stress and strain induced is
   a) compressive stress, tensile strain
   b) tensile stress, compressive strain
   c) **tensile stress, tensile strain**
   d) compressive stress, compressive strain

6. When a body is subjected to two equal and opposite forces, acting tangentially across the resisting section, as a result of which the body tends to shear off across the section, the stress and strain induced is
   a) tensile stress, tensile strain
   b) compressive stress, compressive strain
   c) shear stress, tensile strain
   d) **shear stress, shear strain**

7. Hook's law holds good up to
   a) yield point
   b) **elastic limit**
   c) plastic limit
   d) breaking point

8. Whenever a material is loaded within elastic limit, stress is________strain.
   a) equal to
   b) **directly proportional to**
   c) inversely proportional to
   d) less than

9. The ratio of linear stress to the linear strain is called
   a) modulus of rigidity
   b) **modulus of elasticity**
   c) bulk modulus
   d) Poisson's ratio

10. The unit of modulus of elasticity is same as those of
    a) stress, strain and pressure
    b) stress, force and modulus of rigidity
    c) strain, force and pressure
    d) **stress, pressure and modulus of rigidity**

11. When a change in length takes place, the strain is known as
    a) **linear strain**
    b) lateral strain
    c) volumetric strain
    d) shear strain

12. The change in length due to a tensile or compressive force acting on a body is given by
    a) PLA/E
    b) **PL/AE**
    c) E/PLA
    d) AE/PL

13. The modulus of elasticity for mild steel is approximately equal to
    a) 10 kN/mm^2
    b) 80 kN/mm^2
    c) 100 kN/mm^2
    d) **210 kN/mm^2**

14. Young’s modulus may be defined as the ratio of
    a) linear stress to lateral strain
    b) lateral strain to linear strain
    c) **linear stress to linear strain**
    d) shear stress to shear strain

15. The ratio of linear stress to linear strain is known as
    a) Poisson’s ratio
    b) bulk modulus
    c) modulus of rigidity
    d) **modulus of elasticity**

16. Modulus of rigidity may be defined as the ratio of
    a) linear stress to lateral strain
    b) lateral strain to linear strain
    c) linear stress to linear strain
    d) shear stress to shear strain

17. The ratio of shear stress to shear strain is called
    a) Poisson's ratio
    b) bulk modulus
    c) modulus of rigidity
    d) modulus of elasticity

18. A bar of length L metres extends by l mm under a tensile force of P. The strain produced in the bar is
    a) l/L
    b) 0.1 l/L
    c) 0.01 l/L
    d) 0.001 l/L

19. A rod of length (l) is of uniform diameter d and carries an axial tensile load P. The extension of the rod would be
    a) $\pi Pl / 4Ed^2$
    b) $4Pl / \pi Ed^2$
    c) $\pi EPl / 4d^2$
    d) $4PEl / \pi d^2$

20. The extension of a circular bar tapering uniformly from diameter d_1 at one end to diameter d_2 at the other end, and subjected to an axial pull of P is given by
    a) $4PE / \pi ld^2$
    b) $4\pi ld^2 / PE$
    c) $4Pl / \pi E\, d_1d_2$
    d) $4PlE / \pi\, d_1d_2$

21. The maximum stress produced in a bar of tapering section is at
    a) smaller end
    b) larger end
    c) middle
    d) anywhere

22. Modular ratio of the two materials is the ratio of
    a) linear stress to linear strain
    b) shear stress to shear strain
    c) their modulus of elasticities
    d) their modulus of rigidities

23. The shear modulus of most materials with respect to the modulus of elasticity is
    - **a)** equal to half
    - **b) less than half**
    - **c)** more than half
    - **d)** none of these

24. For the bars of composite section,
    - **a)** the load carried by different materials is the same as the total external load
    - **b)** the total external load is equal to the total sum of the loads carried by different materials
    - **c)** strain in all materials is equal
    - **d) both the total external load is equal to the total sum of the loads carried by different materials and strain in all materials is equal**

25. A bolt is made to pass through a tube and both of them are tightly fitted with the help of washers and nuts. If the nut is tightened, then
    - **a)** bolt and tube are under tension
    - **b)** bolt and tube are under compression
    - **c)** bolt is under compression and tube is under tension
    - **d) bolt is under tension and tube is under compression**

26. When a bar is subjected to a change of temperature and its deformation is prevented, the stress induced in the bar is
    - **a)** tensile stress
    - **b)** compressive stress
    - **c)** shear stress
    - **d) thermal stress**

27. A steel bar of 5 mm is heated from 25° C to 50° C and it is free to expand. The bar will induce
    - **a) no stress**
    - **b)** shear stress
    - **c)** tensile stress
    - **d)** compressive stress

28. When a bar is cooled to –5° C, it will develop
    - **a)** no stress
    - **b)** shear stress
    - **c) tensile stress**
    - **d)** compressive stress

29. A bar of copper and steel form a composite system which is heated to a temperature of 50° C. The stress induced in the copper bar will be
    - **a)** tensile
    - **b) compressive**
    - **c)** shear
    - **d)** zero

30. The thermal or temperature stress is a function of
    a) increase in temperature
    b) modulus of elasticity
    c) coefficient of linear expansion
    d) all of these

31. The thermal stress is given by
    a) EαT
    b) ET / α
    c) Eα / T
    d) 1 / EαT

32. Choose the wrong statement.
    a) Elongation produced in a rod (by its own weight) which is rigidly fixed at the upper end and hanging is equal to that produced by a load half its weight applied at the end.
    b) The stress at any section of a rod on account of its own weight is directly proportional to the distance of the section from the lower end.
    c) Modulus of elasticity is having the same unit as stress.
    d) If a material expands freely due to heating, it will develop thermal stresses.

33. The deformation of the bar per unit length in the direction of the force is known as
    a) linear strain
    b) lateral strain
    c) volumetric strain
    d) shear strain

34. Every direct stress is always accompanied by a strain in its own direction and an opposite kind of strain in every direction, at right angles to it. Such a strain is known as
    a) linear strain
    b) lateral strain
    c) volumetric strain
    d) shear strain

35. The ratio of the lateral strain to the linear strain is called
    a) modulus of elasticity
    b) modulus of rigidity
    c) bulk modulus
    d) Poisson's ratio

36. The Poisson’s ratio for steel varies from
    a) 0.23 to 0.27
    b) 0.25 to 0.33
    c) 0.31 to 0.34
    d) 0.32 to 0.42

37. The Poisson's ratio for cast iron varies from
    a) 0.23 to 0.27
    b) 0.25 to 0.33
    c) 0.31 to 0.34
    d) 0.32 to 0.42

38. When a bar of length l, width b and thickness t is subjected to a pull of P, its
    a) length, width and thickness increases
    b) length, width and thickness decreases
    c) length increases, width and thickness decreases
    d) length decreases, width and thickness increases

39. The ratio of change in volume to the original volume is called
    a) linear strain
    b) lateral strain
    c) volumetric strain
    d) Poisson's ratio

40. When a bar of length l, width b and thickness t is subjected to a push of P, its
    a) length, width and thickness increases
    b) length, width and thickness decreases
    c) length increases, width and thickness decreases
    d) length decreases, width and thickness increases

41. The volumetric strain is the ratio of the
    a) original thickness to the change in thickness
    b) change in thickness to the original thickness
    c) original volume to the change in volume
    d) change in volume to the original volume

42. When a rectangular bar of length l, breadth b and thickness t is subjected to an axial pull of P, then linear strain is given by
    a) P/btE
    b) btE/P
    c) bt/PE
    d) PE/bt

43. When a body is subjected to three mutually perpendicular stresses, of equal intensity, the ratio of direct stress to the corresponding volumetric strain is known as
    a) Young's modulus
    b) modulus of rigidity
    c) bulk modulus
    d) Poisson's ratio

44. The relation between Young's modulus (E) and bulk modulus (K) is given by
    a) $K = 3E(2\mu - 1)$
    b) $K = 3E(1 - 2\mu)$
    c) $E = 3K(2\mu - 1)$
    d) $E = 3K(1 - 2\mu)$

45. When a cube is subjected to three mutually perpendicular tensile stresses of equal intensity (σ), the volumetric strain is
   a) 3σ (1 – 2μ) / E
   b) E (1 – 2μ) / 3σ
   c) 3σ (2μ – 1) / E
   d) E (2μ – 1) / 3σ

46. The relation between modulus of elasticity (E) and modulus of rigidity (G) is given by
   a) E = 2G (1 + μ)
   b) E = 2G (1 – μ)
   c) G = 2E (1 + μ)
   d) G = 2E (1 – μ)

47. The relation between Young's modulus (E), shear modulus (G) and bulk modulus (K) is given by
   a) E = 3KG / (3K + G)
   b) E = 6KG / (3K + G)
   c) E = 9KG / (3K + G)
   d) E = 12KG / (3K + G)

48. The Young's modulus of a material is 125 GPa and Poisson's ratio is 0.25. The modulus of rigidity of the material is
   a) 30 GPa
   b) 50 GPa
   c) 80 GPa
   d) 100 GPa

49. Within elastic limit, shear stress is________shear strain.
   a) equal to
   b) less than
   c) directly proportional to
   d) inversely proportional to

50. Shear modulus is the ratio of
   a) linear stress to linear strain
   b) linear stress to lateral strain
   c) volumetric strain to linear strain
   d) shear stress to shear strain

51. The ratio of shear stress to shear strain is called
   a) Poisson's ratio
   b) bulk modulus
   c) modulus of rigidity
   d) modulus of elasticity

52. If D is the diameter of a sphere, then volumetric strain is equal to
   a) two times the strain of diameter
   b) 1.5 times the strain of diameter
   c) three times the strain of diameter
   d) the strain of diameter

53. If L be the length and D be the diameter of a cylindrical rod, then volumetric strain of the rod is equal to
    a) strain of length + strain of diameter
    b) strain of diameter
    c) strain of length + 2 (strain of diameter)
    d) strain of length

54. The energy stored in a body when strained within elastic limit is known as
    a) resilience
    b) proof resilience
    c) strain energy
    d) impact energy

55. The total strain energy stored in a body is termed as
    a) resilience
    b) proof resilience
    c) impact energy
    d) modulus of resilience

56. The strain energy stored in a spring, when subjected to maximum load, without suffering permanent distortion, is known as
    a) impact energy
    b) proof resilience
    c) proof stress
    d) modulus of resilience

57. The Poisson's ratio of a material which has Young's modulus of 120 GPa and shear modulus of 50 GPa, is
    a) 0.1
    b) 0.2
    c) 0.3
    d) 0.4

58. For a material, the modulus of rigidity is 100 GPa and Poisson's ratio is 0.25. The value of modulus of elasticity in GPa is
    a) 125
    b) 150
    c) 200
    d) 250

59. A steel rod of 1 sq. cm cross-sectional area is 100 cm long and has Young's modulus of 20×10^6 N/cm^2. It is subjected to an axial pull of 20 kN. The elongation of the rod will be
    a) 0.05 cm
    b) 0.10 cm
    c) 0.15 cm
    d) 0.20 cm

60. Factor of safety is defined as the ratio of
    a) ultimate stress to working stress
    b) working stress to ultimate stress
    c) breaking stress to ultimate stress
    d) ultimate stress to breaking stress

*****

UNIT-II BEAMS – LOADS AND STRESSES

SOLVED PROBLEMS

PROBLEM 1

A cantilever beam of length 2 m carries the point loads as shown in figure. Draw the shear force and bending moment diagrams for the cantilever beam.

Given:

Point load at B, W_B = 300 NPoint

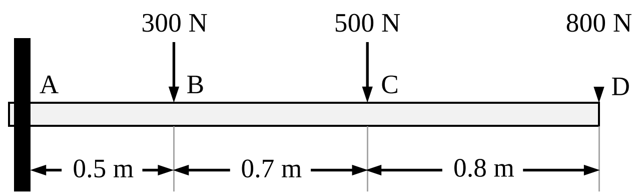
load at C, W_C = 500 NPoint load

at D, W_D = 800 N**To draw:**

Shear force diagram

Bending moment diagram

Solution:



Shear force diagram (SFD)

Shear force (SF) is the sum of forces on either left or right side of the section. Here we are considering from right side. The force is acting downwards and hence it is positive.

SF at D, $F_D = + W_D$

$= + 800$ N

SF between D and C remains constant and equal to + 800 N.

SF at C, $F_C = + W_D + W_D$

$= + 800 + 500$

$= + 1300$ N

SF between C and B remains constant and equal to + 1300 N.

SF at B, $F_B = + W_D + W_D + W_B$

$= + 800 + 500 + 300$

$= + 1600$ N

SF between B and A remains constant and equal to + 1600 N.

SF at A, $F_A = + 1600$ N

Bending moment diagram (BMD)

BM at any section = Force × Distance from the corresponding section

BM at D, $M_D = 0$

BM at C, $M_C = - W_D \times CD$

$= - 800 \times 0.8$

$= - 640$ Nm

BM at B, $M_B = - W_D \times BD - W_C \times BC$

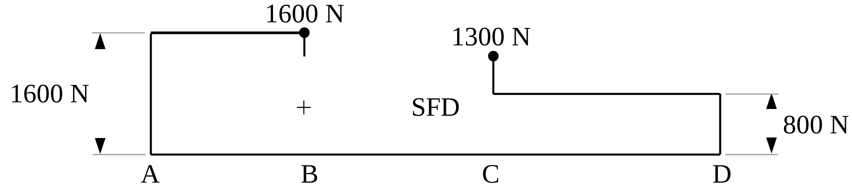
$= - 800 \times (0.8 + 0.7) - 500 \times 0.7$

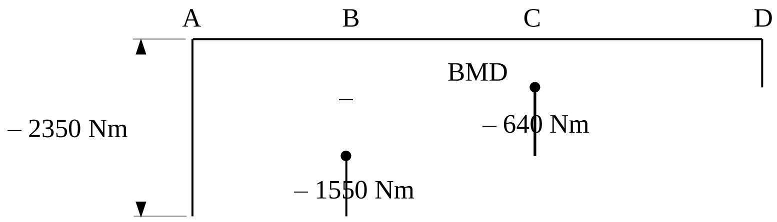
$= - 1550$ Nm

BM at A, $M_A = - W_D \times AD - W_C \times AC - W_B \times AB$

BM at A, $M_A = - 800 \times (0.8 + 0.7 + 0.5) - 500 \times (0.7 + 0.50) - 300 \times 0.5$

BM at A, $M_A = - 2350$ Nm



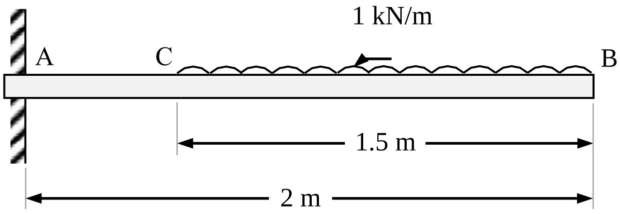


PROBLEM 2

A cantilever beam of length 2 m carries a uniformly distributed load of 1 kN/m run over a length of 1.5 m from the free end. Draw the shear force and bending moment diagrams for the cantilever.

Given:

UDL, w = 1 kN/m = 1 × 10^3 N



To draw:

Shear force diagram

Bending moment diagram

Solution:

Shear force diagram (SFD)

SF at B, $F_B = 0$

SF at C, $F_C = + w \times CB$

$= 1 \times 10^3 \times 1.5$

$= + 1500$ N

There is no load between C and A, so SF between C and A remains constant and equal to + 1500 N.

SF at A, $F_A = + 1500$ N

The points B and C are joined by an inclined line because of UDL and points C and A by a horizontal line because of no load.

Bending moment diagram (BMD)

For UDL, the BM at any section is given by,

BM = Force × Distance from the corresponding section

BM = Load/unit length × Length of load spread × Distance from the corresponding section

BM at any section between B and C at a distance x from the free end B is given by

$$M_x = -(w \times x) \times \frac{x}{2}$$

BM at B, $M_B = 0$

$$\text{BM at C, } M_c = -(w \times CB) \times \frac{CB}{2}$$

$$= -(1 \times 10^3 \times 1.5) \times \frac{1.5}{2}$$

$$= -1125 \text{ Nm}$$

BM at any section between C and A at a distance x from the free end B is given by

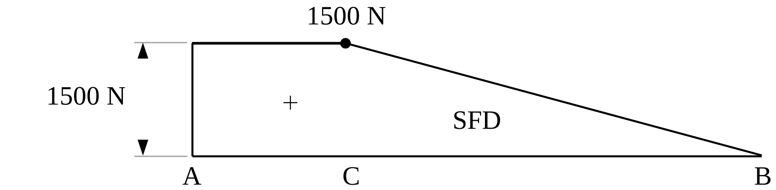
$$M_x = -\,(w \times CB) \times \left[\frac{CB}{2} + (x - CB)\right]$$

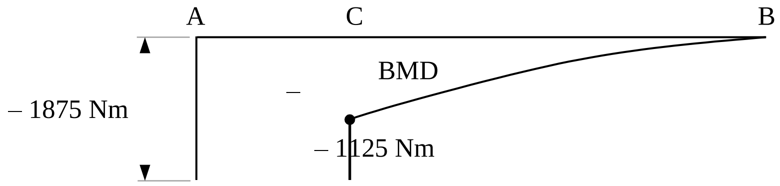
BM at A, $M_A = -\,(w \times CB) \times \left[\frac{CB}{2} + (2 - CB)\right]$

$$= -\,(1 \times 10^3 \times 1.5) \times \left[\frac{1.5}{2} + (2 - 1.5)\right]$$

$$= -\,1875 \text{ kNm}$$

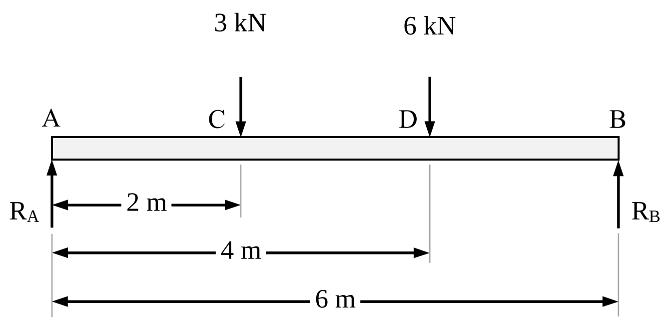
The points B and C are joined by a parabolic curve because of UDL and points C and A are joined by a straight line because of no load.





PROBLEM 3

A simply supported beam of length 6 m, carries a point load of 3 kN and 6 kN at distances of 2 m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam.



Given:

Point load at C, W_C = 3 kN Point

load at D, W_D = 6 kN **To draw:**

Shear force diagram

Bending moment diagram

Solution:

Applying ΣV = 0

$R_A - W_C - W_D + R_B = 0$

$R_A - 3 - 6 + R_B = 0$

$R_A + R_B = 9$ … (1)

Applying $\Sigma M_A = 0$

$(W_C \times AC) + (W_D \times AD) - (R_B \times AB) = 0$

$(3 \times 2) + (6 \times 4) - (R_B \times 6) = 0$

$6 R_B = (3 \times 2) + (6 \times 4)$

$$R_B = \frac{(3 \times 2) + (6 \times 4)}{6}$$

= 5 kN

Substituting the above value in equation (1)

$R_A + 5 = 9$

$R_A = 9 - 5$

= 4 kN

Shear force diagram (SFD)

SF at B, $F_B = -R_B$

$= -5$ kN

SF at D, $F_D = -R_B + W_D$

$= -5 + 6$

$= +1$ kN

SF at C, $F_C = -R_B + W_D + W_C$

$= -5 + 6 + 3$

$= +4$ kN

SF at A, $F_A = +R_A$

$= +4$ kN

Bending moment diagram (BMD)

BM at B, $M_B = 0$

BM at D, $M_D = +R_B \times DB$

$= +5 \times 2$

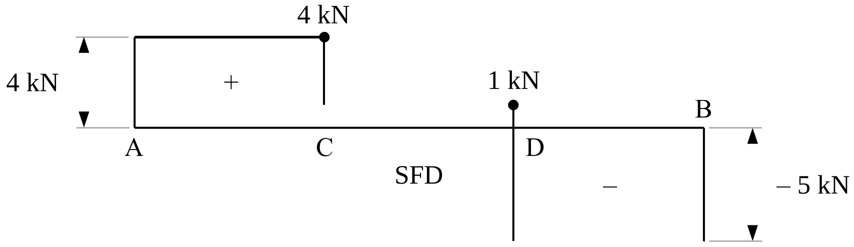
$= +10$ kNm

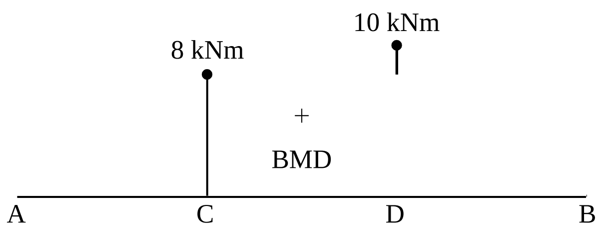
BM at C, $M_C = +(R_B \times CB) - (W_D \times CD)$

$= +(5 \times 4) - (6 \times 2)$

$= +8$ kNm

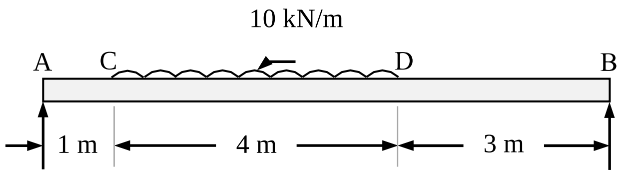
BM at A, $M_A = 0$





PROBLEM 4

Draw the shear force and bending moment diagrams for a simply supported beam of length 8 m and carrying a uniformly distributed load of 10 kN/m for a distance of 4 m as shown in figure. Also calculate the maximum bending moment.



Given:

UDL, w = 10 kN/m

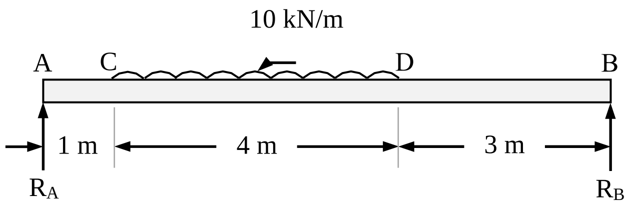
To draw / find:

Shear force diagram

Bending moment diagram

Maximum bending moment

Solution:



Applying $\Sigma V = 0$

$R_A - (w \times CD) + R_B = 0$

$R_A - (10 \times 4) + R_B = 0$

$R_A + R_B = 40$ … (1)

Applying $\Sigma M_A = 0$

$$\left(w \times CD \times \left(AC + \frac{CD}{2}\right)\right) - (R_B \times AB) = 0$$

$$\left(10 \times 4 \times \left(1 + \frac{4}{2}\right)\right) - (R_B \times 8) = 0$$

$120 - 8\,R_B = 0$

$8\,R_B = 120$

$R_B = \frac{120}{8}$

$R_B = 15$ kN

Substituting the above value in equation (1)

$R_A + 15 = 40$

R_A = 25 kN

Shear force diagram (SFD)

SF at B, $F_B = -R_B$

$= -15$ kN

SF at D, $F_D = -R_B$

$= -15$ kN

SF at C, $F_C = -R_B + (w \times CD)$

$= -15 + (10 \times 4)$

$= +25$ kN

SF at A, $F_A = +R_A = +25$ kN

The shear force at C is + 25 kN and D is – 15 kN. This means that somewhere between C and D, the shear force is equal to zero.

The shear force at any section between C and D at a distance x from C is

$F_x = -R_B + w \times (CD - x) = -15 + 10\,(5 - x)$

$F_x = -15 + 10\,(4 - x)$

Substituting the value of SF (F_x) is equal to zero in above equation

$0 = -15 + 10\,(4 - x)$

$0 = -15 + (10 \times 4) - 10x$

$10x = -15 + (10 \times 4)$

$$x = \frac{-15 + (10 \times 4)}{10}$$

$x = 2.5$ m

Hence the shear force is zero at a distance of 3.5 m from C

Bending moment diagram (BMD)

BM at B, $M_B = 0$

BM at D, $M_D = +R_B \times DB$

$= +15 \times 3$

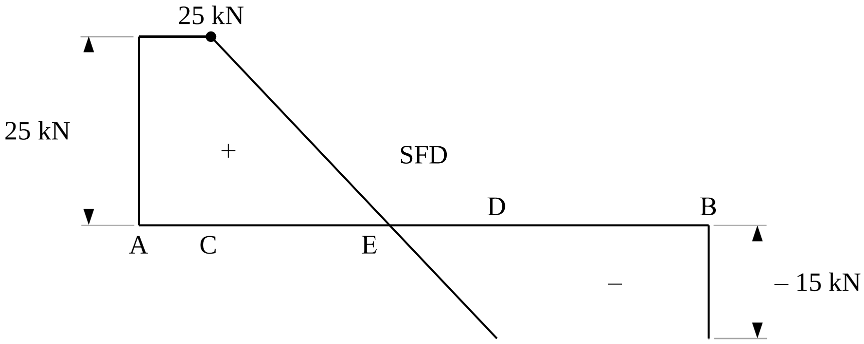
$= +45$ kNm

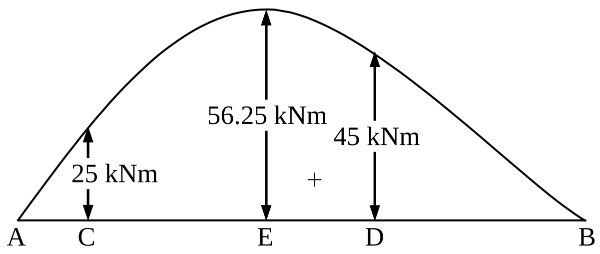
$$\text{BM at C, } M_c = +(R_B \times CB) - \left(w \times CD \times \frac{CD}{2}\right)$$

$$= +(15 \times 7) - \left(10 \times 4 \times \frac{4}{2}\right)$$

$= +25$ kNm

BM at A, $M_A = 0$





BMD

Maximum bending moment

The BM is maximum at a point where shear force changes sign. This means that the point where shear force becomes zero from positive value to the negative value or vice-versa, the BM at that point will be maximum. From shear force diagram, at point E, the shear force is zero after changing its sign. Hence BM is maximum at point E.

$$\text{BM at E, } M_E = +\,(R_B \times EB) - \left[w \times ED \times \frac{ED}{2}\right]$$

$$= +\,(R_B \times [CB - CE]) - \left[w \times [CD - CE] \times \frac{[CD - CE]}{2}\right]$$

$$= +\,(R_B \times [CB - x]) - \left[w \times [CD - x] \times \frac{[CD - x]}{2}\right]$$

$$= +(15 \times [7 - 2.5]) - \left[10 \times [4 - 2.5] \times \frac{[4 - 2.5]}{2}\right]$$

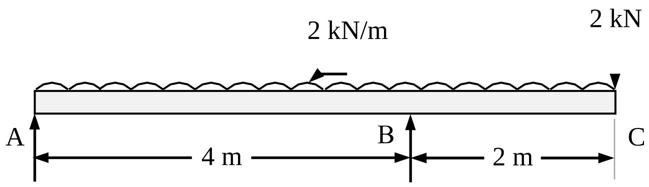
$$= +\,56.25 \text{ kNm}$$

Result:

Maximum bending moment = + 56.25 kNm

PROBLEM 5

Draw the S.F. and B.M. diagrams for the overhanging beam carrying uniformly distributed load of 2 kN/m over the entire length and a point load of 2 kN as shown in figure. Locate the point of contraflexure.



Given:

UDL, w = 2 kN/m

Point load, W = 2 kN

To draw / find:

Shear force diagram

Bending moment diagram

Point of contraflexure

Solution:

Applying $\Sigma V = 0$

$R_A - (w \times AC) - W + R_B = 0$

$R_A - (2 \times 6) - 2 + R_B = 0$

$R_A + R_B = 14$ … (1)

Applying $\Sigma M_A = 0$

$$\left(w \times AC \times \frac{AC}{2}\right) - (R_B \times AB) + (W \times AC) = 0$$

$$\left(2 \times 6 \times \frac{6}{2}\right) - (R_B \times 4) + (2 \times 6) = 0$$

$48 - 4\,R_B = 0$

$4\,R_B = 48$

$$R_B = \frac{48}{4}$$

$R_B = 12$ kN

Substituting the above value in equation (1)

$R_A + 12 = 14$

$R_A = 2$ kN

Shear force diagram (SFD)

SF at C

F_C = + W

= + 2 kN

SF at B (without reaction R_B)

F_{B1} = + W + (w × BC)

= + 2 + (2 × 2)

= + 6 kN

SF at B (with reaction R_B)

F_{B2} = + W + (w × BC) – R_B

= + 2 + (2 × 2) – 12

= – 6 kN

SF at A,

F_A = + R_A

= + 2 kN

The shear force at A is + 2 kN and B is – 6 kN. This means that somewhere between A and B, the shear force is equal to zero.

The shear force at any section between A and B at a distance x from A is

F_x = + R_A – (w × x)

F_x = + 2 – (2 × x)

F_x = 2 – 2 x

Substituting the value of SF (F_x) is equal to zero in above equation

0 = 2 – 2 x

$$x = \frac{2}{2}$$

$$x = 1 \text{ m}$$

Hence the shear force is zero at a distance of 1 m from A.

Bending moment diagram (BMD)

BM at C, M_C = 0

$$\text{BM at B, } M_B = -(W \times BC) - \left(w \times BC \times \frac{BC}{2}\right)$$

$$= -(2 \times 2) - \left(2 \times 2 \times \frac{2}{2}\right)$$

$$= -8 \text{ kNm}$$

$$\text{BM at D, } M_D = -(w \times DC) - \left[W \times DC \times \frac{DC}{2}\right] + (R_B \times DB)$$

$$= -(w \times [AC - AD]) - \left[W \times [AC - AD] \times \frac{[AC - AD]}{2}\right] + (R_B \times [AB - AD])$$

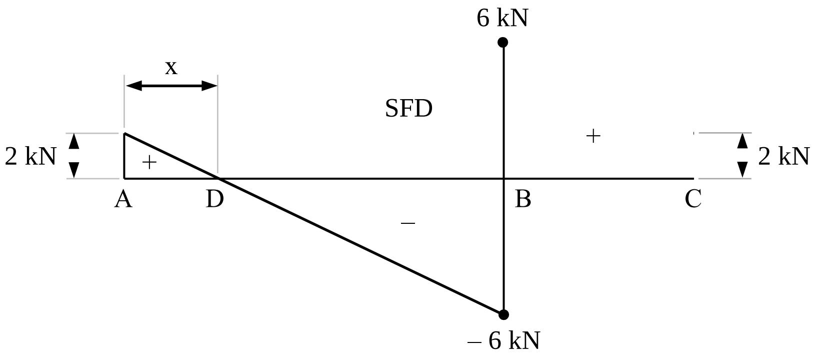
$$= -(w \times [AC - x]) - \left[W \times [AC - x] \times \frac{[AC - x]}{2}\right] + (R_B \times [AB - x])$$

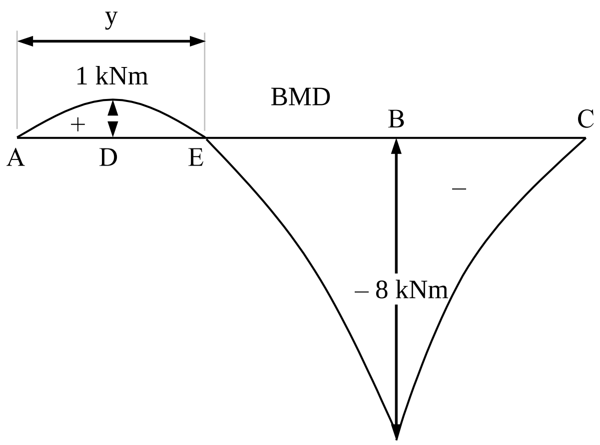
$$= -(2 \times [6 - 1]) - \left[2 \times [6 - 1] \times \frac{[6 - 1]}{2}\right] + (12 \times [4 - 1])$$

$$= -(2 \times 5) - \left[2 \times 5 \times \frac{5}{2}\right] + (12 \times 3)$$

$$= +1 \text{ kNm}$$

BM at A, $M_A = 0$





Point of contraflexure

The point is at E between A and B, where BM is zero after changing its sign.

The bending moment at any section between A and B at a distance y from A is

$$M_y = +(R_A \times y) - \left(w \times y \times \frac{y}{2}\right)$$

$$= +(2 \times y) - \left(2 \times y \times \frac{y}{2}\right)$$

$$= + 2y - y^2$$

Substituting the value of BM (M_y) is equal to zero in above equation

$0 = +2y - y^2$

$0 = y(2 - y)$

$y = 0$ m or 2 m

The point E is between A and B, so y = 2 m. Hence the bending moment is zero at a distance of 2 m from A.

Result:

Point of contraflexure is at a distance of 2 m from A.

PROBLEM 6

A simply supported timber beam of span 6 m carries a UDL of 12 kN/m over the entire span and a point load of 9 kN at 2.5 m from the left support. If the bending stress in timber is not to exceed 8 N/mm^2, design a suitable section for the beam. The depth of beam equals twice the breadth.

Given:

Length of beam, l = 6 m

UDL, w = 12 kN/m

Point load, W = 9 kN

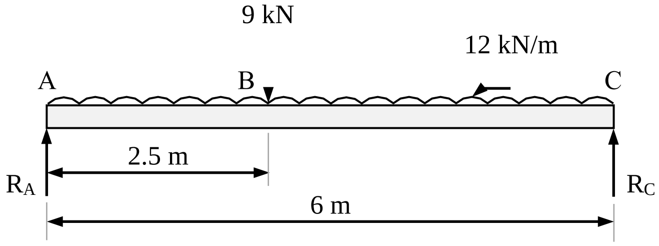
Bending stress, $(\sigma_b)_{max}$ = 8 N/mm^2

Depth of beam, d = 2 × Breadth of beam = 2b

To find:

Dimensions of the beam

Solution:



Applying $\Sigma V = 0$

$R_A - (w \times AC) - W + R_C = 0$

$R_A - (12 \times 6) - 9 + R_C = 0$

$R_A + R_C = 81$ … (1)

Applying $\Sigma M_A = 0$

$$\left(w \times AC \times \frac{AC}{2}\right) + (W \times AB) - (R_C \times AC) = 0$$

$$\left(12 \times 6 \times \frac{6}{2}\right) + (9 \times 2.5) - (R_C \times 6) = 0$$

$216 + 22.5 - 6\,R_C = 0$

$6\,R_C = 216 + 22.5$

$$R_C = \frac{216 + 22.5}{6}$$

$R_C = 39.75$ kN

Substituting the above value in equation (1)

$R_A + 39.75 = 81$

$R_A = 41.25$ kN

Shear force diagram (SFD)

SF at C

$F_C = - R_C$

$= - 39.75$ kN

SF at B (without point load)

$F_{B1} = - R_C + (w \times BC)$

$= - 39.75 + (12 \times 3.5)$

$= + 2.25$ kN

SF at B (with point load)

$F_{B2} = - R_C + (w \times BC) + W$

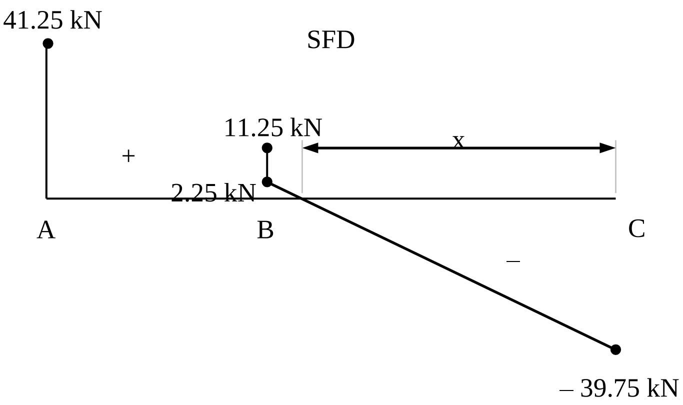
$= - 39.75 + (12 \times 3.5) + 9$

$= + 11.25$ kN

SF at A

$F_A = + R_A$

$F_A = + 41.25$ kN



The shear force at B is + 2.25 kN and C is – 39.75 kN. This means that somewhere between B and C, the shear force is equal to zero.

The shear force at any section between B and C at a distance x from C is

$F_x = - R_C + (w \times x)$

$F_x = - 39.75 + (12 \times x)$

$F_x = - 39.75 + 12x$

Substituting the value of SF (F_x) is equal to zero in above equation

$0 = - 39.75 + 12x$

$$x = \frac{39.75}{12}$$

$x = 3.3125$ m

Hence the shear force is zero at a distance of 3.3125 m from C.

Maximum bending moment

Maximum bending moment occurs where shear force is zero.

$$M_{max} = M_x = +(R_C \times x) - (w \times x \times \frac{x}{2})$$

$$= +(39.75 \times 3.3125) - (12 \times 3.3125 \times \frac{3.3125}{2})$$

$$= +65.8359 \text{ kNm}$$

$$= +65.8359 \times 10^6 \text{ Nmm}$$

For rectangular section,

Distance of extreme fibre from the neutral axis

$$y_{max} = \frac{d}{2}$$

Moment of inertia

$$I = \frac{bd^3}{12}$$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

The bending stress will be maximum, when y is maximum.

$$\frac{M_{max}}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$\frac{65.8359 \times 10^6}{\frac{bd^3}{12}} = \frac{8}{\frac{d}{2}}$$

$$\frac{65.8359 \times 10^6 \times 12}{bd^3} = \frac{8 \times 2}{d}$$

Substituting d = 2b in above equation

$$\frac{65.8359 \times 10^6 \times 12}{b(2b)^3} = \frac{8 \times 2}{2b}$$

$$\frac{b(2b)^3}{2b} = \frac{65.8359 \times 10^6 \times 12}{8 \times 2}$$

$$\frac{b \times 2^3 \times b^3}{2b} = \frac{65.8359 \times 10^6 \times 12}{8 \times 2}$$

$$b^3 = \frac{65.8359 \times 10^6 \times 12}{8 \times 2 \times 2^2}$$

$$b = \sqrt[3]{\frac{65.8359 \times 10^6 \times 12}{8 \times 2 \times 2^2}}$$

b = 231.1114 mm

d = 2 b = 462.2229 mm

Result:

Dimensions of the beam

Breadth, b = 231.1114 mm

Depth, d = 462.2229 mm

PROBLEM 7

A cast iron pipe 300 mm internal diameter, metal thickness 15 mm, is supported at two points 6 m apart. Find the maximum bending stress in the metal of the pipe when it is running full of water. Assume the specific weight of cast iron and water as 72 kN/m^3 and 10 kN/m^3 respectively.

Given:

Internal diameter of pipe, d = 300 mm

Thickness of pipe, t = 15 mm

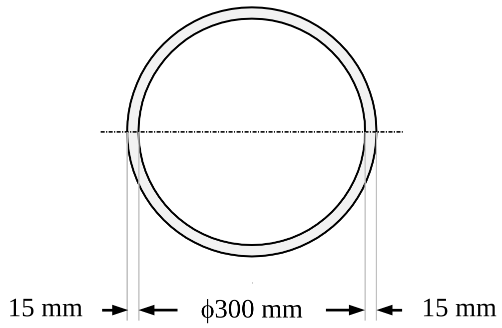
Span, l = 6 m

Specific weight of cast iron, γ_c = 72 kN/m^3

Specific weight of water, γ_w = 10 kN/m^3 **To find:**

Maximum bending stress, $(\sigma_b)_{max}$

Solution:



Outer diameter of pipe, D = d + 2 t

Outer diameter of pipe, D = 300 + (2 × 15)

Outer diameter of pipe, D = 330 mm

Weight of pipe / unit length = Area of pipe × Specific weight of cast iron

$$\text{Weight of pipe/unit length} = \frac{\pi}{4}(D^2 - d^2) \times \gamma_c$$

$$= \frac{\pi}{4}(330^2 - 300^2) \times 72$$

$$= 1068769.8210 \text{ N/m}$$

Weight of water / unit length = Area of water × Specific weight of water

$$= \frac{\pi}{4} d^2 \times \gamma_w$$

$$= \frac{\pi}{4} 300^2 \times 10$$

$$\text{Weight of water/unit length} = 706858.3471 \text{ N/m}$$

Total weight / unit length, w = Weight of pipe / unit length + Weight of water / unit length

$= 1068769.8210 + 706858.3471$

$= 1775628.1680$ N/m

Maximum bending moment for a simply supported beam carrying uniformly distributed load,

$$M_{max} = \frac{wl^2}{8}$$

$$= \frac{1775628.1680 \times 6^2}{8}$$

$$= 7990326.7550 \text{ Nm}$$

$$= 7990326.7550 \times 10^3 \text{ Nmm}$$

For hollow circular section,

Distance of extreme fibre from the neutral axis

$$y_{max} = \frac{D}{2}$$

$$= \frac{330}{2}$$

$$= 165 \text{ mm}$$

Moment of inertia

$$= \frac{\pi(D^4 - d^4)}{64}$$

$$= \frac{\pi(330^4 - 300^4)}{64}$$

$$= 184529789.4000 \text{ mm}^4$$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

The bending stress will be maximum, when y is maximum.

$$\frac{M_{max}}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$(\sigma_b)_{max} = \frac{M_{max}\, y_{max}}{I}$$

$$= \frac{7990326.7550 \times 10^3 \times 165}{184529789.4000}$$

$$= 7.1447 \text{ N/mm}^2$$

Result:

Maximum bending stress, $(\sigma_b)_{max} = 7.1447$ N/mm^2

PROBLEM 8

Find the dimensions of a timber joist span 5 m to carry a brick wall 200 mm thick and 3.2 m high, if the weight of brickwork is 19 kN/m^3 and the maximum stress is limited to 8 N/mm^2. The depth is to be twice the width.

Given:

Span, l = 5 m

Thickness of brick wall, t = 200 mm = 0.2 m

Height of brick wall, h = 3.2 m

Specific weight of brick wall, γ = 19 kN/m^3

Maximum stress, $(\sigma_b)_{max}$ = 8 N/mm^2

Depth of beam, d = 2 × Width of beam = 2b

To find:

Dimensions of timber joist

Solution:

Weight of the brick wall / unit length, w = γ × t × h

= 19 × 0.2 × 3.2

= 12.16 kN/m

The brick wall is spread over entire length of the timber joist, therefore it is considered as uniformly distributed load (UDL).

Maximum bending moment for a simply supported beam carrying uniformly distributed load,

$$M_{max} = \frac{wl^2}{8}$$

$$= \frac{12.16 \times 5^2}{8}$$

$$= 38 \text{ kNm}$$

$$= 38 \times 10^6 \text{ Nmm}$$

For rectangular section,

Distance of extreme fibre from the neutral axis

$$y_{max} = \frac{d}{2}$$

Moment of inertia

$$I = \frac{bd^3}{12}$$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

The bending stress will be maximum, when y is maximum.

$$\frac{M_{max}}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$\frac{38 \times 10^6}{\frac{bd^3}{12}} = \frac{8}{\frac{d}{2}}$$

$$\frac{38 \times 10^6 \times 12}{bd^3} = \frac{8 \times 2}{d}$$

Substituting d = 2b in above equation

$$\frac{38 \times 10^6 \times 12}{b(2b)^3} = \frac{8 \times 2}{2b}$$

$$\frac{b(2b)^3}{2b} = \frac{38 \times 10^6 \times 12}{8 \times 2}$$

$$\frac{b \times 2^3 \times b^3}{2b} = \frac{38 \times 10^6 \times 12}{8 \times 2}$$

$$b^3 = \frac{38 \times 10^6 \times 12}{8 \times 2 \times 2^2}$$

$$b = \sqrt[3]{\frac{38 \times 10^6 \times 12}{8 \times 2 \times 2^2}}$$

= 192.1251 mm

d = 2 b = 384.8501 mm

Result:

Dimensions of timber joist,

Width, b = 192.1251 mm

Depth, d = 384.8501 mm

PROBLEM 9

A T-section of a beam has the following dimensions. Width of the flange 100 mm, overall depth 80 mm, thickness of the web 10 mm and thickness of flange 10 mm. Determine the maximum bending stress in the beam, when a bending moment of 200 N-m is acting on the section.

Given:

Width of the flange = 100 mm

Overall depth = 80 mm

Thickness of the web = 10 mm

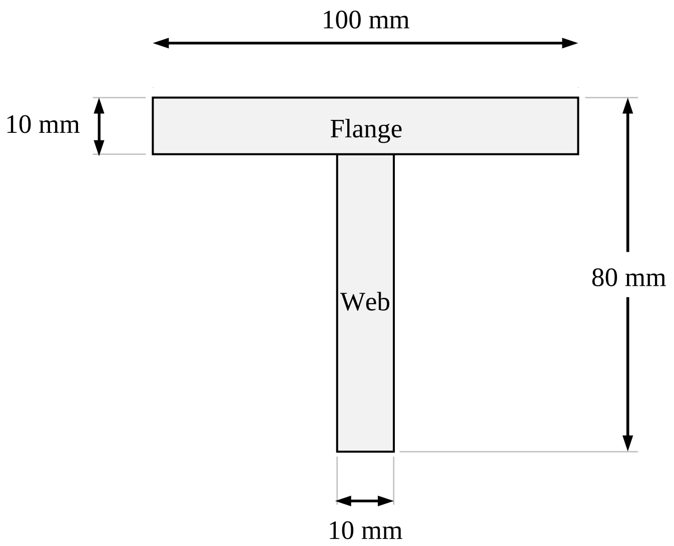
Thickness of the flange = 10 mm

Bending moment = 200 N-m = 200×10^3 N-mm

To find:

Maximum bending stress, $(\sigma_b)_{max}$

Solution:



Area of flange, $A_1 = 100 \times 10$

$= 1000 \text{ mm}^2$

Area of web, $A_2 = 10 \times 70$

$= 700 \text{ mm}^2$

Centre of gravity of flange from bottom,

$$y_1 = 80 - \frac{10}{2} = 75 \text{ mm}$$

Centre of gravity of web from bottom,

$$y_2 = \frac{70}{2} = 35 \text{ mm}$$

Location of centre of gravity of T-section from bottom,

$$y = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{1000 \times 75 + 700 \times 35}{1000 + 700}$$

$$= 58.5294 \text{ mm}$$

Moment of inertia of flange about neutral axis,

$$I_{xx1} = I_{G1} + A_1 h_1^2$$

$$= \frac{b_1 d_1^3}{12} + A_1 (y_1 - y)^2$$

$$= \frac{100 \times 10^3}{12} + 1000 \times (75 - 58.5294)^2$$

$$= 279613.6101 \text{ mm}^4$$

Moment of inertia of web about neutral axis,

$$I_{xx2} = I_{G2} + A_2 h_2^2$$

$$= \frac{b_2 d_2^3}{12} + A_2 (y - y_2)^2$$

$$= \frac{10 \times 70^3}{12} + 700 \times (58.5294 - 35)^2$$

$$= 673376.5859 \text{ mm}^4$$

Moment of inertia of T-section about neutral axis,

$$I_{xx} = I_{xx1} + I_{xx2}$$

$$= 279613.6101 + 673376.5859$$

$$= 952990.1961 \text{ mm}^4$$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

The bending stress will be maximum, when y is maximum.

Distance of centre of gravity from the top fibre, $y_t = 80 - y$

$= 80 - 58.5294$

$= 21.4706$ mm

Distance of centre of gravity from the bottom fibre, $y_b = y$

$= 58.5294$ mm

y_{max} = Maximum of y_t and y_b

$y_{max} = 58.5294$ mm

$$\frac{M_{max}}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$(\sigma_b)_{max} = \frac{M_{max}\, y_{max}}{I}$$

$$= \frac{200 \times 10^3 \times 58.5294}{952990.1961}$$

$$= 12.2833\ \mathrm{N/mm^2}$$

Result:

Maximum bending stress, $(\sigma_b)_{max}$ = 12.2833 N/mm^2

PROBLEM 10

A uniform T-section beam is 100 mm wide and 150 mm deep with a flange thickness of 25 mm and a web thickness of 12 mm. If the limiting bending stress is 160 N/mm^2 in tension, find the maximum uniformly distributed load that the beam can carry over a simply supported span of 5 m. Also determine the corresponding maximum bending stress in compression.

Given:

Width of T-section = 100 mm

Depth of T-section = 150 mm

Thickness of the flange = 25 mm

Thickness of the web = 12 mm

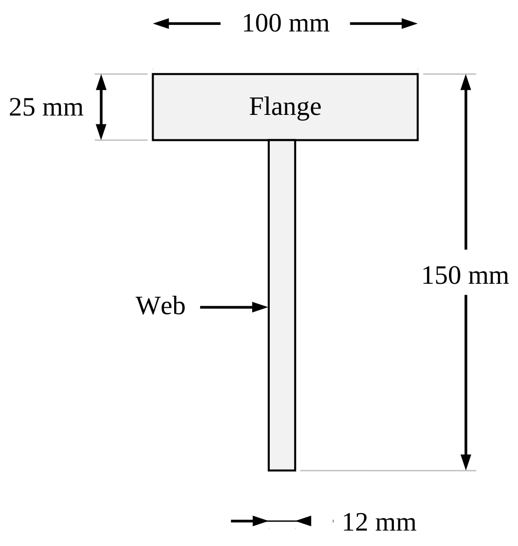
Limiting tensile bending stress, $(\sigma_{bt})_{max}$ = 160 N/mm^2

To find:

UDL, w

Maximum compressive bending stress, $(\sigma_{bc})_{max}$

Solution:



Area of flange, $A_1 = 100 \times 25$

$= 2500$ mm^2

Area of web, $A_2 = 12 \times 125$

$= 1500$ mm^2

Centre of gravity of flange from bottom,

$$y_1 = 150 - \frac{25}{2} = 137.5 \text{ mm}$$

Centre of gravity of web from bottom,

$$y_2 = \frac{125}{2} = 62.5 \text{ mm}$$

Location of centre of gravity of T-section from bottom,

$$y = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{2500 \times 137.5 + 1500 \times 62.5}{2500 + 1500}$$

$$= 109.375 \text{ mm}$$

Moment of inertia of flange about neutral axis,

$$I_{xx1} = I_{G1} + A_1 h_1^2$$

$$= \frac{b_1 d_1^3}{12} + A_1 (y_1 - y)^2$$

$$= \frac{100 \times 25^3}{12} + 2500 \times (137.5 - 109.375)^2$$

$$= 2107747.396 \text{ mm}^4$$

Moment of inertia of web about neutral axis,

$$I_{xx2} = I_{G2} + A_2 h_2^2$$

$$= \frac{b_2 d_2^3}{12} + A_2 (y - y_2)^2$$

$$= \frac{12 \times 125^3}{12} + 1500 \times (109.375 - 62.5)^2$$

$$= 5249023.438 \text{ mm}^4$$

Moment of inertia of T-section about neutral axis,

$$I_{xx} = I_{xx1} + I_{xx2}$$

$$= 2107747.396 + 5249023.438$$

$$= 7356770.833 \text{ mm}^4$$

Maximum bending moment for a simply supported beam carrying uniformly distributed load,

$$M_{max} = \frac{wl^2}{8}$$

$$= \frac{w \times 5^2}{8}$$

$$= 3.125 \text{ w N-m}$$

$$= 3.125 \times 10^3 \text{ w N-mm}$$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

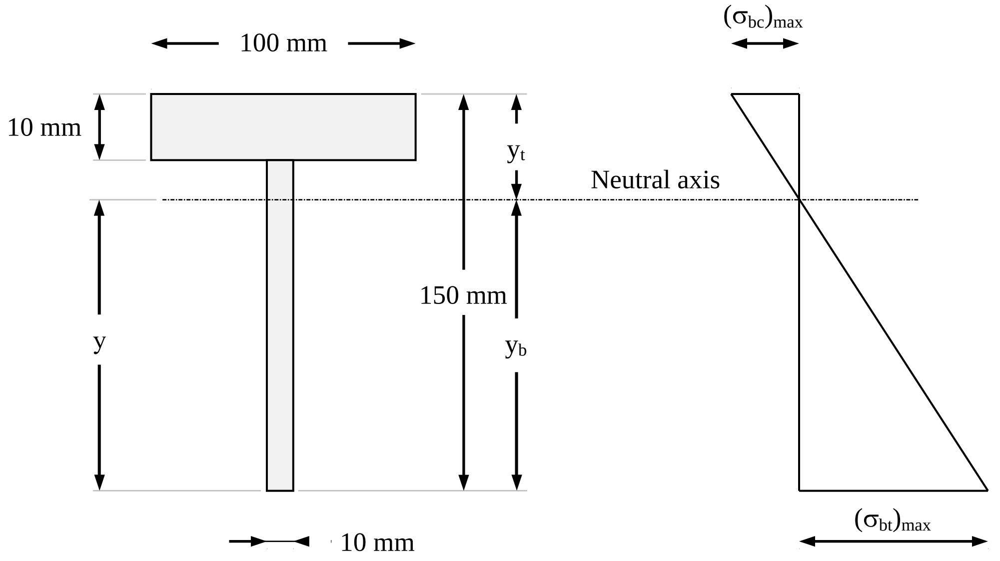
Distance of centre of gravity from the top fibre, $y_t = 150 - y$

$= 150 - 109.375$

$= 40.625$ mm

Distance of centre of gravity from the bottom fibre, $y_b = y$

$= 109.375$ mm



For a simply supported beam, the tensile stress is maximum at the extreme bottom fibre and compressive stress is maximum at the extreme top fibre.

For maximum tensile bending stress,

$$\frac{M_{max}}{I} = \frac{(\sigma_{bt})_{max}}{y_b}$$

$$\frac{3.125 \times 10^3 \text{ w}}{7356770.833} = \frac{160}{109.375}$$

$$w = \frac{160 \times 7356770.833}{109.375 \times 3.125 \times 10^3}$$

$$= 3443.8095 \text{ N/mm}$$

$$= 3443.8095 \text{ kN/m}$$

For maximum compressive bending stress,

$$\frac{M_{max}}{I} = \frac{(\sigma_{bc})_{max}}{y_t}$$

$$(\sigma_{bc})_{max} = \frac{M_{max} \times y_t}{I}$$

$$= \frac{3.125 \times 10^3 \times 3443.8095 \times 40.625}{7356770.833}$$

$$= 59.4286 \text{ N/mm}^2$$

Result:

Uniformly distributed load, w = 3443.8095 kN/m

Maximum compressive bending stress, $(\sigma_{bc})_{max}$ = 59.4286 N/mm^2

PROBLEM 11

A model beam of 50 mm diameter is broken by a transverse load of 900 N applied at the center of the spam 0.8 m. Using factor of safety of 3, calculate the safe load for a beam of 110 mm diameter, freely supported over a span of 2 m.

Given:

For model beam,

Diameter, d_m = 50 mm

Point load, W_m = 900 N

Span, l_m = 0.8 m

For actual beam,

Diameter, d = 110 mm

Span, l = 2 m

Factor of safety, F = 2

To find:

Safe load for actual beam, W

Solution:

Model beam

Maximum bending moment for a simply supported beam carrying a point load at center,

$$M_{max} = \frac{W_m l_m}{4}$$

$$= \frac{900 \times 0.8}{4}$$

$$= 180 \text{ N-m}$$

$$= 180 \times 10^3 \text{ N-mm}$$

For circular cross section,

$$y_{max} = \frac{d_m}{2}$$

$$= \frac{50}{2}$$

$$= 25 \text{ mm}$$

Moment of inertia

$$I = \frac{\pi d_m^4}{64}$$

$$= \frac{\pi \times 50^4}{64}$$

$$= 306796.1576 \text{ mm}^4$$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

The bending stress will be maximum, when y is maximum.

$$\frac{M_{max}}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$(\sigma_b)_{max} = \frac{M_{max}\, y_{max}}{I}$$

$$(\sigma_b)_{max} = \frac{180 \times 10^3 \times 25}{306796.1576}$$

$$(\sigma_b)_{max} = 14.6677 \text{ N/mm}^2$$

Actual beam

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Permissible stress or working stress}}$$

$$\text{Permissible stress} = \frac{\text{Ultimate stress}}{\text{Factor of safety}}$$

$$= \frac{14.6677}{3}$$

$$= 4.8892 \text{ N/mm}^2$$

Maximum permissible bending stress for actual beam, $(\sigma_b)_{max}$ = 4.8892 N/mm^2

Maximum bending moment for a simply supported beam carrying a point load at centre,

$$M_{max} = \frac{Wl}{4}$$

$$= \frac{W \times 2}{4}$$

$$= 0.5 \text{ W N-m}$$

$$= 0.5 \times 10^3 \text{ W N-mm}$$

For circular cross section,

$$y_{max} = \frac{d}{2}$$

$$= \frac{110}{2}$$

$$= 55 \text{ mm}$$

Moment of inertia

$$I = \frac{\pi d^4}{64}$$

$$\pi \times 110^4$$

$$= \frac{\qquad}{64}$$

$= 7186884.0690 \text{ mm}^4$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

The bending stress will be maximum, when y is maximum.

$$\frac{M_{max}}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$\frac{0.5 \times 10^3 \text{ W}}{7186884.0690} = \frac{4.8892}{55}$$

$$W = \frac{4.8892 \times 7186884.0690}{55 \times 0.5 \times 10^3}$$

$$= 1277.76 \text{ N}$$

To find:

Safe load for actual beam, W = 1277.76 N

PROBLEM 12

A mild steel tube 40 mm outside diameter and 30 mm inside diameter is used as a simply supported beam on a span of 1.8 m and it is found that the maximum safe load it can carry at mid span is 1200 N. Four of these tubes are placed in parallel to one another and firmly fixed together to form in effect a single beam, the centers of the tubes forming a square of 40 mm side with one pair of centers vertically over the other pair. Find the maximum central load which this beam can carry if the maximum stress is not to exceed that of the single tube above.

Given:

Outside diameter of tube, D = 40 mm

Inside diameter of tube, d = 30 mm

Span, l = 1.8 m

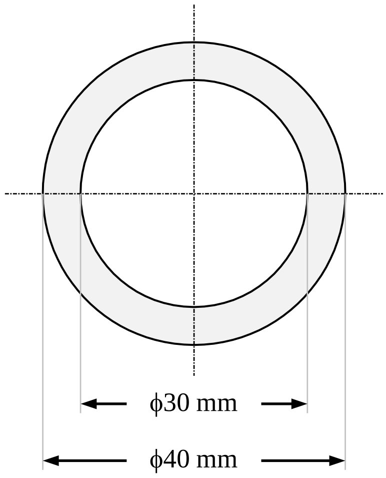
Point load at mid span, W = 1200 N

To find:

Maximum central point load of composite tube, W_c

Solution:

Single tube



Maximum bending moment for a simply supported beam carrying a point load at center,

$$M_{max} = \frac{Wl}{4}$$

$$= \frac{1200 \times 1.8}{4}$$

$$= 540 \text{ N-m}$$

$$= 540 \times 10^3 \text{ N-mm}$$

Distance of extreme fibre from the neutral axis

$$y_{max} = \frac{D}{2}$$

$$= \frac{40}{2}$$

$$= 20 \text{ mm}$$

Moment of inertia

$$I = \frac{\pi(D^4 - d^4)}{64}$$

$$= \frac{\pi(40^4 - 30^4)}{64}$$

$$= 85902.9241 \text{ mm}^4$$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

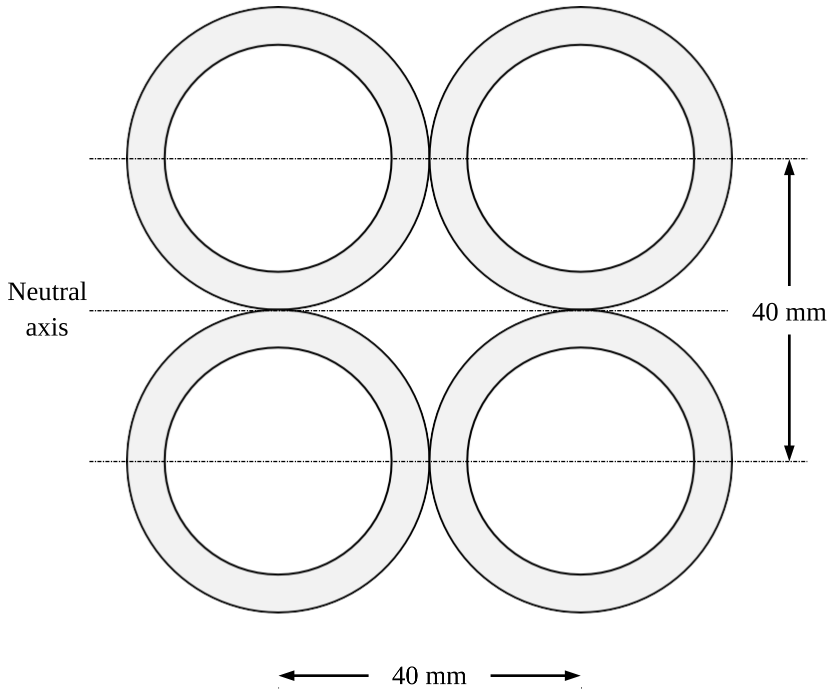
The bending stress will be maximum, when y is maximum.

$$\frac{M_{max}}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$(\sigma_b)_{max} = \frac{540 \times 10^3 \times 20}{85902.9241}$$

$$= 125.7233 \text{ N/mm}^2$$

Composite tube



Maximum bending moment for a simply supported beam carrying a point load (W_c) at center,

$$M_{max} = \frac{W_c l}{4}$$

$$= \frac{W_c \times 1.8}{4}$$

$= 0.45\ W_c$ N-m

$= 0.45 \times 10^3\ W_c$ N-mm

Distance of extreme fibre from the neutral axis

$y_{max} = D$

$= 40$ mm

Moment of inertia of single tube about neutral axis,

$I_1 = I_{G1} + A_1 h^2\ _1$

h_1^2 = Distance of centre of the tube from neutral axis

$$I_1 = \frac{\pi(D^4 - d^4)}{64} + \frac{\pi(D^2 - d^2)}{4} h_1^2$$

$$= \frac{\pi(40^4 - 30^4)}{64} + \frac{\pi(40^2 - 30^2)}{4} \times 20^2$$

$= 305814.4099\ N/mm^2$

Moment of inertia of composite tube about neutral axis,

$I = 4 \times I_1$

$= 4 \times 305814.4099$

$= 1223257.6390\ N/mm^2$

Using the relation

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

The bending stress will be maximum, when y is maximum.

$$\frac{M_{max}}{I} = \frac{(\sigma_b)_{max}}{y_{max}}$$

$$\frac{0.45 \times 10^3\ W_c}{1223257.6390} = \frac{125.7233}{40}$$

$$W_c = \frac{125.7233 \times 1223257.6390}{40 \times 0.45 \times 10^3}$$

$= 8544$ N

Result:

Maximum central point load of composite tube, W_c = 8544 N

PROBLEM 13

A timber of rectangular section is to support a load 2 tons uniformly distributed over the span of 30 cm. If the depth of the section is to be twice the breadth and maximum shear stress in timber is not to exceed 8 N/mm^2, find the dimensions of the cross section. How would you modify the cross section if it were a concentrated load placed at the center assuming the above ratio of breadth to depth?

Given:

Total load, W = 2 tons = 2000 kg = 2000 × 9.81 N = 19620 N

Span, L = 30 cm = 300 mm

Depth, d = 2 × Breadth = 2b

Maximum shear stress, τ_{max} = 8 N/mm^2

To find:

Cross-section of the beam for

- UDL
- Point load at center

Solution:

Uniformly distributed load

Shear force applied on the beam,

$$F = \frac{W}{2}$$

$$= \frac{19320}{2}$$

$$= 9810 \text{ N}$$

Average shear stress of the beam

$$\tau_{max} = 1.5 \times \tau_{ave}$$

$$\tau_{ave} = \frac{\tau_{max}}{1.5}$$

$$= \frac{8}{1.5}$$

$$= 5.3333 \text{ N/mm}^2$$

Cross-section area of the beam

$$\tau_{ave} = \frac{F}{A}$$

$$A = \frac{F}{\tau_{ave}}$$

$$b \times d = \frac{F}{\tau_{ave}}$$

$$b \times 2b = \frac{F}{\tau_{ave}}$$

$$b^2 = \frac{F}{2 \times \tau_{ave}}$$

$$b = \sqrt{\frac{F}{2 \times \tau_{ave}}}$$

$$= \sqrt{\frac{9810}{2 \times 5.3333}}$$

$$= 30.3263 \text{ mm}$$

$$d = 2b$$

$$= 60.6527 \text{ mm}$$

Point load at centre of beam

Shear force acting on the beam,

$$F = W$$

$$= 19620 \text{ N}$$

Cross section area of the beam

$$\tau_{ave} = \frac{F}{A}$$

$$A = \frac{F}{\tau_{ave}}$$

$$b \times d = \frac{F}{\tau_{ave}}$$

$$b \times 2b = \frac{F}{\tau_{ave}}$$

$$b^2 = \frac{F}{2 \times \tau_{ave}}$$

$$b = \sqrt{\frac{F}{2 \times \tau_{ave}}}$$

$$= \sqrt{\frac{19620}{2 \times 5.3333}}$$

$$= 42.8879 \text{ mm}$$

$$d = 2b$$

$$= 85.7759 \text{ mm}$$

Result:

Cross-section of the beam

UDL = 30.3263 mm × 85.7759 mm

Point load at center = 42.8879 mm × 60.6527 mm

TWO MARKS QUESTIONS AND ANSWERS

1. Define beam.

   Beam is a structural member which is supported along the length and subjected to external loads acting transversely (i.e., perpendicular to the centre line of the beam). Beam is sufficiently long as compared to the lateral dimensions.

2. What is meant by transverse loading on beams?

   If a load is acting on the beam which is perpendicular to the centre line of it, then it is called transverse loading.

3. How to classify the beams according to its supports?

   - Cantilever beam
   - Simply supported beam
   - Overhanging beam
   - Fixed beam and
   - Continuous beam

4. What is cantilever beam?

   A beam which is fixed at one end and free at the other end is called as cantilever beam.

5. What is simply supported beam?

   A beam supported or resting freely on the supports at its both ends is known as simply supported beam.

6. What is overhanging beam?

   If one or both of the end portions of a beam are extended beyond the support, then it is called as overhanging beam.

7. What is fixed beam?

   A beam whose both ends are fixed or build-in walls, is known as fixed beam.

8. What is continuous beam?

   A beam which is provided more than two supports is known as continuous beam.

9. What are the types of transverse load?

   - Point or concentrated load
   - Uniformly distributed load (UDL)
   - Uniformly varying loads (UVL)

10. What is meant by point or concentrated load?

    A load which is acting at a point is called point or concentrated load.

11. What is uniformly distributed load?

    A uniformly distributed load is one which is spread over a beam in such a manner that rate of loading is uniform along the length.

12. What is uniformly varying load?

    A uniformly varying load is one which is spread over a beam in such a manner that rate of loading varies from point to point along the length.

13. Define shear force and bending moment at a section.

    Shear force at any cross section is defined as the algebraic sum of all the forces acting either side of a beam.

    Bending moment at any cross section is the algebraic sum of the moments of all the forces which are placed either side from that point.

14. What is meant by positive or sagging bending moment?

    Bending moment is said to be positive or sagging if the moment of the force in the left side of the beam is clockwise or right side of the beam is counter clockwise. Otherwise, moment tends to bend the beam to a curvature having concavity at the top.

15. What is meant by negative or hogging bending moment?

    Bending moment is said to be negative or hogging if the moment of the force in the left side of the beam is counter clockwise or right side of the beam is clockwise. Otherwise, moment tends to bend the beam to a curvature having concavity at the bottom.

16. When will be the bending moment is maximum?

    Bending moment will be maximum when the shear force changes its sign.

17. What is the nature of curve when the beam carrying uniformly distributed load?

    The curve follows parabolic relation or parabolic curve.

18. Define the term point of contraflexure.

    The point where the bending moment is zero after changing its sign is called the point of contraflexure.

19. What are shear force and bending moment diagrams?

    Shear force diagram is one which shows the variation of forces along the length of the beam.

    Bending moment diagram is one which shows the variation of bending moment along the length of the beam.

20. Write the relationship between shear force and bending moment.

    The rate of change of bending moment is equal to the shear force at the section.

$$\frac{dM}{dx} = -F$$

21. State the theory of pure bending or simple bending.

If a beam bends only due to the application of constant bending moment and no shear force, then it is called pure bending or simple bending.

22. Write the bending equation.

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

where, M – Bending moment

I – Moment of inertia of the section

σ_b – Bending stress at that section

y – Distance from the neutral axis

E – Young's modulus of the beam

R – Radius of curvature of the beam

23. Define neutral axis.

The neutral axis of any transverse section of a beam is defined as the line of intersection of the neutral layer with the transverse section.

24. What are the assumptions made in the theory of simple bending?

- The material of beam is homogeneous and isotropic.
- The value of Young's modulus is same in tension and compression.
- The transverse sections which were plane before bending, remains plane after bending also.
- The radius of curvature is large compared to the dimensions of the cross-section of beam.
- Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

25. Define section modulus.

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of layer from the neutral axis.

$$\text{Section modulus} = \frac{\text{Moment of inertia about neutral axis}}{\text{Distance of layer from neutral axis}}$$

$$Z = \frac{I}{y}$$

26. What is moment of resistance of the section?

Moment of resistance is the product of section modulus and the bending stress at that section.

$$\text{Moment of resistance} = \text{Section modulus} \times \text{Bending stress}$$

$$M = Z \times \sigma_b$$

27. What is a composite beam or flitched beam?

A beam made up of two or more different materials assumed to be rigidly connected together and behaving like a single piece is known as a composite beam or flitched beam.

Composite beam or flithced beam is used to reinforce the lower strength material and reduce the cost.

28. What types of stresses are caused in a beam subjected to a constant shear force?

Vertical and horizontal shear stress.

29. Define shear stress distribution.

The variation of shear stress along the depth of the beam is called shear stress distribution.

30. What is the ratio of maximum shear stress to the average shear stress for the rectangular section?

$$\tau_{max} = 1.5\ \tau_{avg}$$

31. What is the ratio of maximum shear stress to the average shear stress in case of solid circular section?

$$\tau_{max} = \frac{4}{3}\tau_{avg}$$

14 MARKS QUESTIONS

25. A beam of length 10 m is simply supported at its ends carries two concentrated loads of 5 kN each at a distance of 3 m and 7 m from the left support and also a uniformly distributed load of l kN/m between the point loads. Draw the shear force and bending moment diagrams. Calculate the maximum bending moment.

26. Draw the S.F. and B.M. diagram for a simply supported beam of length 9 m and carrying a UDL of 10 kN/m for a distance of 6 m from the left end. Also calculate the maximum B.M. on the section.

27. A cantilever of length 2 m carries a uniformly distributed load of 1.5 kN/m run over the whole length and a point load of 2 kN at a distance of 0.5 m from the free end. Draw the shear force and bending moment diagrams for the cantilever.

28. A cantilever beam 2 m long carries point loads of 5 kN, 8 kN and 12 kN at 0.5 m, 1.5 m, and 2 m respectively from the free end. Draw the shear force diagram and the bending moment diagram.

29. A cantilever beam 3.5 m long is subjected to gradually varying load from 2 kN/m at the free end to 5 kN/m at the fixed end. Draw the shear force and bending moment diagram and find the maximum bending moment.

30. A horizontal cantilever 4 m long carries a UDL of 2 kN/m which extends over the middle 1.5 m of the beam and act downwards. In addition to this, a concentrated load of 5 kN acts downward at the point 1 m from the support and another concentrated load of 3 kN acts upward at the free end. Draw the shear force and bending moment diagrams.

31. A simply supported beam ABCD is of 5 m span, such that AB = 2 m, BC = 1 m, and CD = 2m. It is loaded with 5 kN/m over AB and 2 kN/m over CD. Draw the shear force and bending moment diagram.

32. The intensity of loading in a simply supported beam of 6 m span varies gradually from 30 kN/m at one end to 90 kN/m at the other end. Draw the SF and BM diagram.

33. A simply supported beam 7.5 m long is subjected to a couple of 30 kN-m in an anticlockwise direction at a distance of 5.5 m from the left support. Draw the SF and BM diagram for the beam.

34. A beam AB 5 m long is simply supported at A and B. It is loaded with point loads of 20 kN, 30 kN and 20 kN at distances of 1 m, 3 m and 4 m respectively from the A, and a uniformly distributed load at the rate of 20 kN/m over length of 2 m, the beginning of the UDL being at a distance of 2 m from A. Draw the BM and SF diagrams.

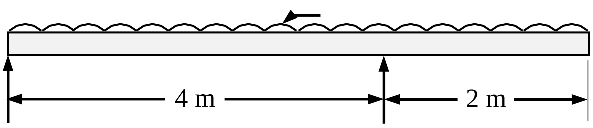
35. A beam ABC 11 m long is simply supported at A and B has an overhang BC of 3 m. It is loaded with UDL of 4 kN/m over the whole length. A concentrated load of 30 kN is acting at 6 m from A. Find the position and value of maximum bending moment and plot the BM and SF diagrams.

36. A horizontal beam, 30 m long, carries a uniformly distributed load of 10 kN/m over the whole length and a concentrated load of 30 kN at the right end. If the beam is freely supported at the left end, find the position of the second support so that the bending moment on the beam should be as small as possible. Draw the diagrams of shearing force and bending moment.

37. A beam of 6 m long rests on two supports 5 m apart. The right end is overhanging by 1 m. The beam carries a UDL of 1.5 kN/m over entire length of the beam. Draw the shear force and bending moment diagram and find the position of the maximum bending moment.

38. A lintel of 4 m span supports a concrete wall 15 cm thick. The height of the wall is 1 m at one end and increases uniformly to 4 m at the other end. Calculate the maximum bending moment on the beam if the concrete weighs 23 kN/m^3. Sketch the SF and BM diagrams.

39. Draw the SFD and BMD. Also locate the point of contraflexure.



40. A timber beam of rectangular section is to support a load of 20 kN uniformly distributed over a span of 3.6 m, when the beam is simply supported. If the depth of the section is to be twice the breadth and the stress in the timber is not to exceed 7 N/mm^2, find the breadth and depth of the cross section. How will you modify the cross-section of the beam, if it carries a concentrated load of 30 kN placed at the mid-span with the same ratio of breadth to depth.

41. A 4 m long beam with rectangular section of 10 cm width and 35 cm depth is simply supported at the ends. If it is loaded with a UDL of 4 kN/m throughout the span and a concentrated load of 2 kN placed at a distance of 1.5 m from the end. Determine the maximum bending stress in the beam.

42. A simply supported beam of cross section 50 mm × 50 mm having length of 700 mm is capable of carrying a point load of 4 KN at its center. This beam is required to be replaced by a cantilever beam of the same material having cross section 50 mm × 75 mm and length of 1200 mm. determine the maximum point load that can be placed at the free end of the cantilever.

43. A wooden beam of rectangular section 200 × 300 mm is used as simply supported beam carrying a UDL of w N/m. What is the maximum value of w, if the maximum shear stress developed in the beam section is limited to 5 N/mm^2 and span length of 6 m?

44. A floor has to carry a load of 6 kN/m^2 with a span of each joist 5 m. The section of the joist is rectangular 80 mm wide and 250 mm deep. Calculate the spacing centre to centre if the maximum permissible stress is 8 N/mm^2.

45. Calculate the dimension of the strongest rectangular beam that can be cut out of a cylindrical log of wood whose diameter is 1 m.

46. A test beam of square section 25 mm × 25 mm is broken by a transverse load of 750 N applied at the centre of span 1 m. Using a factor of safety 4, calculate the UDL for a beam of 120 mm width and 300 mm depth freely supported over a span of 5 m.

47. A rectangular beam 200 mm deep and 100 mm wide is simply supported over a span of 8 m and carries a central point load of 25 kN. Determine the maximum stress in the beam. Also calculate the value of longitudinal fibre stress at a distance of 25 mm from the surface of the beam.

48. A circular beam of 100 mm diameter is subjected to a shear force of 10 kN. Calculate the value of the maximum shear stress and sketch the shear stress distribution of the beam.

49. A beam of triangular section having base width 35 cm and height of 50 cm is subjected to a shear force of 7 kN. Find the value of maximum shear stress and sketch the shear stress distribution along the depth of the beam.

50. A 400 mm × 150 mm I girder has 20 mm thick flanges and 30 mm thick web. Calculate maximum intensity of shear stress when the shear force at the cross section is 1.6 MN. Also sketch the shear stress distribution across the depth of beam.

51. The compression flange of a cast iron girder is 10 cm wide and 3 cm deep, the tension flange 30 cm wide and 5 cm deep and the web 25 cm × 3 cm. Find (a) moment of inertia bout NA, (b) the load per metre run which may be carried over a 3 m span by a beam simply supported at its ends if the maximum permissible stresses are 95 N/mm^2 in compression and 25 N/mm^2 in tension.

52. Compare the flexural strength of the following three beams of equal weight:
    i. I-section 30 cm × 15 cm, having 2 cm thick flanges and 1.25 cm thick web.
    ii. Rectangular section having depth equal to twice the width.
    iii. Solid circular section.

OBJECTIVE QUESTIONS AND ANSWERS

1. A beam which is fixed at one end and free at the other is called
   a) simply supported beam
   b) fixed beam
   c) overhanging beam
   d) cantilever beam

2. A beam extending beyond the supports is called
   a) simply supported beam
   b) fixed beam
   c) overhanging beam
   d) cantilever beam

3. A beam encastered at both the ends is called
   a) simply supported beam
   b) fixed beam
   c) cantilever beam
   d) continuous beam

4. A beam supported on more than two supports is called
   a) simply supported beam
   b) fixed beam
   c) overhanging beam
   d) continuous beam

5. If a beam is fixed at both its ends, it is called a
   a) fixed beam
   b) built-in beam
   c) encastered beam
   d) any one of these

6. A concentrated load is one which
   a) acts at a point on a beam
   b) spreads non-uniformly over the whole length of a beam
   c) spreads uniformly over the whole length of a beam
   d) varies uniformly over the whole length of a beam

7. When a cantilever beam is loaded with concentrated loads, the bending moment diagram will be
   a) horizontal straight line
   b) vertical straight line
   c) inclined straight line
   d) parabolic curve

8. The shear force of a cantilever beam of length l carrying a uniformly distributed load of w per unit length is________at the free end.
   a) zero
   b) wl/4
   c) wl/2
   d) wl

9. The shear force of a cantilever beam of length l carrying a uniformly distributed load of w per unit length is _______ at the fixed end.
   a) zero
   b) wl/4
   c) wl/2
   d) wl

10. The shear force diagram of a cantilever beam of length l and carrying a uniformly distributed load of w per unit length will be
   a) a right angled triangle
   b) an issoscles triangle
   c) an equilateral triangle
   d) a rectangle

11. The bending moment of a cantilever beam of length l and carrying a uniformly distributed load of w per unit length is _______ at the free end.
   a) zero
   b) wl/4
   c) wl/2
   d) wl

12. The shear force and bending moment are zero at the free end of a cantilever beam, if it carries a
   a) point load at the free end
   b) point load at the middle of its length
   c) uniformly distributed load over the whole length
   d) none of the above

13. The bending moment of a cantilever beam of length l and carrying a uniform distributed load of w per unit length is________at the fixed end.
   a) wl/4
   b) wl/2
   c) wl
   d) $wl^2/2$

14. The maximum bending moment of a simply supported beam of span l and carrying a point load W at the centre of beam, is
   a) Wl/4
   b) Wl/2
   c) Wl
   d) $Wl^2/4$

15. The bending moment diagram for a simply supported beam loaded in its centre is
    a) a right angled triangle
    b) an isosceles triangle
    c) an equilateral triangle
    d) a rectangle

16. The shear force in the centre of a simply supported beam carrying a uniformly distributed load of w per unit length, is
    a) zero
    b) $wl^2/2$
    c) $wl^2/4$
    d) $wl^2/8$

17. The bending moment in the centre of a simply supported beam carrying a uniformly distributed load of w per unit length is
    a) zero
    b) $wl^2/2$
    c) $wl^2/4$
    d) $wl^2/8$

18. The shear force at the ends of a simply supported beam carrying a uniformly distributed load of w per unit length is
    a) zero at its both ends
    b) wl at one end and -wl at the other end
    c) wl/2 at one end and -wl/2 at the other end
    d) $wl^2/2$ at one end and $-wl^2/2$ at the other end

19. The shear force diagram for a simply supported beam carrying a uniformly distributed load of w per unit length, consists of
    a) one right angled triangle
    b) two right angled triangles
    c) one equilateral triangle
    d) two equilateral triangles

20. The bending moment diagram for a simply supported beam carrying a uniformly distributed load of w per unit length, will be
    a) a horizontal line
    b) a vertical line
    c) an inclined line
    d) a parabolic curve

21. The point of contraflexure is a point where
    a) shear force changes sign
    b) bending moment changes sign
    c) shear force is maximum
    d) bending moment is maximum

22. In a simply supported beam carrying a uniformly distributed load w per unit length, the point of contraflexure
    a) lies in the centre of the beam
    b) lies at the ends of the beam
    c) depends upon the length of beam
    d) does not exist

23. When there is a sudden increase or decrease in shear force diagram between any two points, it indicates that there is a
    a) point load at the two points
    b) no loading between the two points
    c) uniformly distributed load between the two points
    d) uniformly varying load between the two points

24. When the shear force diagram is a parabolic curve between two points, it indicates that there is a
    a) point load at the two points
    b) no loading between the two points
    c) uniformly distributed load between the two points
    d) uniformly varying load between the two points

25. In a beam where shear force changes sign, the bending moment will be
    a) zero
    b) minimum
    c) maximum
    d) infinity

26. The bending moment at a section, where shear force is zero, will be
    a) zero
    b) maximum
    c) minimum
    d) either minimum or maximum

27. The point of contraflexure occurs in
    a) cantilever beams
    b) simply supported beams
    c) overhanging beams
    d) fixed beams

28. The bending moment at a section tends to bend or deflect the beam and the internal stresses resist its bending. The resistance offered by the internal stresses, to the bending, is called
    a) compressive stress
    b) shear stress
    c) bending stress
    d) elastic modulus

29. The assumption, generally, made in the theory of simple bending is that
    a) the beam material is perfectly homogenous and isotropic
    b) the beam material is stressed within its elastic limit
    c) the plane sections before bending remain plane after bending
    d) all of these

30. In a simple bending theory, one of the assumptions is that the material of the beam is isotropic. This assumption means that the
    a) normal stress remains constant in all directions
    b) normal stress varies linearly in the material
    c) elastic constants are same in all the directions
    d) elastic constants varies linearly in the material

31. In a simple bending of beams, the stress in the beam varies
    a) linearly
    b) parabolically
    c) hyperbolically
    d) elliptically

32. In a simple bending theory, one of the assumptions is that the plane sections before bending remain plane after bending. This assumption means that
    a) stress is uniform throughout the beam
    b) strain is uniform throughout the beam
    c) stress is proportional to the distance from the neutral axis
    d) strain is proportional to the distance from the neutral axis

33. When a beam is subjected to a bending moment, the strain in a layer is________the distance from the neutral axis.
    a) equal to
    b) directly proportional to
    c) inversely proportional to
    d) independent of

34. The bending equation is
    a) $M/I = \sigma/y = E/R$
    b) $T/J = \tau/r = G/l$
    c) $M/y = \sigma/I = E/R$
    d) $T/r = \tau/J = G\theta/l$

35. A section of beam is said to be in pure bending, if it is subjected to
    a) constant bending moment and constant shear force
    b) constant shear force and zero bending moment
    c) constant bending moment and zero shear force
    d) none of these

36. When a beam is subjected to bending moment, the stress at any point is________the distance of the point from the neutral axis.
    a) equal to
    b) directly proportional to
    c) inversely proportional to
    d) independent of

37. The neutral axis of the cross-section a beam is that axis at which the bending stress is
    a) zero
    b) minimum
    c) maximum
    d) infinity

38. The section modulus (Z) of a beam is given by
    a) I /y
    b) Iy
    c) y/I
    d) M/I

39. The section modulus of a rectangular section about an axis through its C.G., is
    a) b/2
    b) d/2
    c) $bd^2/2$
    d) $bd^2/6$

40. The section modulus of a circular section about an axis through its C.G., is
    a) $d^2/4$
    b) $d^2/16$
    c) $d^3/16$
    d) $d^3/32$

41. A square beam and a circular beam have the same length, same allowable stress and the same bending moment. The ratio of weights of the square beam to the circular beam is
    a) 1 / 2
    b) 1
    c) 1 / √2
    d) √2

42. When a cantilever beam is loaded at its free end, the maximum compressive stress shall develop at
    a) bottom fibre
    b) top fibre
    c) neutral axis
    d) centre of gravity

43. A beam of uniform strength may be obtained by
    a) keeping the width uniform and varying the depth
    b) keeping the depth uniform and varying the width
    c) varying the width and depth
    d) any one of these

44. If the depth is kept constant for a beam of uniform strength, then its width will vary in proportional to
    a) **M**
    b) √M
    c) M^2
    d) M^3

45. A beam of uniform strength has
    a) same cross-section throughout the beam
    b) same bending stress at every section
    c) same bending moment at every section
    d) same shear stress at every section

46. The bending stress in a beam is________bending moment.
    a) equal to
    b) less than
    c) more than
    d) directly proportional to

47. At the neutral axis of a beam
    a) the layers are subjected to maximum bending stress
    b) the layers are subjected to minimum bending stress
    c) the layers are subjected to compression
    d) the layers do not undergo any strain

48. In a beam subjected to pure bending, the intensity of stress in any fibre is________the distance of the fibre from the neutral axis.
    a) equal to
    b) less than
    c) more than
    d) directly proportional to

49. The rectangular beam ‘A’ has length l, width b and depth d. Another beam ‘B’ has the same length and width but depth is double that of ‘A’. The elastic strength of beam B will be _____________ as compared to beam A.
    a) same
    b) double
    c) four times
    d) six times

50. The rectangular beam ‘A’ has length l, width b and depth d. Another beam ‘B’ has the same length and depth but width is double that of ‘A’. The elastic strength of beam B will be _____________ as compared to beam A.
    a) same
    b) double
    c) four times
    d) six times

51. The rectangular beam ‘A’ has length l, width b and depth d. Another beam ‘B’ has the same width and depth but length is double that of ‘A’. The elastic strength of beam B will be _____________ as compared to beam A.
    a) same
    b) one-half
    c) one-fourth
    d) one-eighth

52. When a rectangular beam is loaded transversely, the maximum tensile stress is developed on the
    a) top layer
    b) bottom layer
    c) neutral axis
    d) every cross-section

53. When a rectangular beam is loaded transversely, the maximum compressive stress is developed on the
    a) top layer
    b) bottom layer
    c) neutral axis
    d) every cross-section

54. At the neutral axis of a beam, the shear stress is
    a) zero
    b) minimum
    c) maximum
    d) infinity

55. The maximum shear stress developed in a beam of rectangular section is________the average shear stress.
    a) equal to
    b) 4/3 times
    c) 1.5 times
    d) twice

56. The maximum shear stress developed in a beam of circular section is________the average shear stress.
    a) equal to
    b) 4/3 times
    c) 1.5 times
    d) twice

57. A beam of T-section is subjected to a shear force of F. The maximum shear force will occur at the
    a) top of the section
    b) bottom of the section
    c) neutral axis of the section
    d) junction of web and flange

58. A flitched beam is used to
    a) change the shape of the beam
    b) effect the saving in material
    c) equalise the strength in tension and compression
    d) increase the cross-section of the beam

59. The ratio of moment of inertia about the neutral axis to the distance of the most distance point of the section from the neutral axis is called
    a) moment of inertia
    b) section modulus
    c) polar moment of inertia
    d) modulus of rigidity

60. The relation between maximum stress (σ) offered by a section, moment of resistance (M) and section modulus (Z) is given by
    a) $M = \sigma/Z$
    b) $M = Z/\sigma$
    c) $M = \sigma Z$
    d) $M = 1/\sigma Z$

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UNIT-III TORSION

SOLVED PROBLEMS

PROBLEM 1

A hollow shaft having inner diameter 0.6 times the outer diameter is to replace solid shaft of the same material to transmit 550 kW at 220 rpm. The permissible shear stress is 80 N/mm^2. Calculate the diameter of solid and hollow shafts. Also calculate the percentage saving in material.

Given:

Inner diameter of hollow shaft, d = 0.6 × Outer diameter of hollow shaft = 0.6 D_h

Power transmitted, P = 550 kW = 550 × 10^3 W

Speed, N = 220 rpm

Maximum hear stress, τ = 80 N/mm^2

To find:

Diameter of solid shaft, D

Inner diameter of hollow shaft, d

Outer diameter of hollow shaft, D_h

Percentage saving in material

Solution:

Power

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

$$= \frac{550 \times 10^3 \times 60}{2\pi \times 220}$$

$$= 23873.2415 \text{ N-m}$$

$$= 23873.2415 \times 10^3 \text{ N-mm}$$

Solid shaft

Torque transmitted by solid shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$D^3 = \frac{16 \times T}{\pi \times \tau}$$

$$D = \sqrt[3]{\frac{16 \times T}{\pi \times \tau}}$$

$$= \sqrt[3]{\frac{16 \times 23873.2415 \times 10^3}{\pi \times 80}}$$

$= 114.9733$ mm

Hollow shaft

Torque transmitted by hollow shaft,

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_h^4 - d^4)}{D_h}$$

$$= \frac{\pi}{16} \times \tau \times \frac{(D_h^4 - [0.6D_h]^4)}{D_h}$$

$$= \frac{\pi}{16} \times \tau \times \frac{D_h^4(1 - 0.6^4)}{D_h}$$

$$= \frac{\pi}{16} \times \tau \times D_h^3 \times 0.8704$$

$$D_h^3 = \frac{16 \times T}{\pi \times \tau \times 0.8704}$$

$$D_h = \sqrt[3]{\frac{16 \times T}{\pi \times \tau \times 0.8704}}$$

$$= \sqrt[3]{\frac{16 \times 23873.2415 \times 10^3}{\pi \times 80 \times 0.8704}}$$

$= 120.4179$ mm

$d = 0.6\ D_h$

$= 0.6 \times 120.4179$

$= 72.2507$ mm

Weight of solid shaft,

W_s = Weight density × Volume of solid shaft

= Weight density × Area of solid shaft × Length

$= w \times A_s \times L$

Weight of hollow shaft,

W_h = Weight density × Volume of hollow shaft

= Weight density × Area of hollow shaft × Length

$= w \times A_h \times L$

Percentage saving in material

$$= \frac{W_s - W_h}{W_s} \times 100$$

$$= \frac{w \times A_s \times L - w \times A_h \times L}{w \times A_s \times L} \times 100$$

$$= \frac{A_s - A_h}{A_s} \times 100$$

$$= \frac{\frac{\pi}{4} D^2 - \frac{\pi}{4} (D_h^2 - d^2)}{\frac{\pi}{4} D^2} \times 100$$

$$= \frac{D^2 - (D_h^2 - d^2)}{D^2} \times 100$$

$$= \frac{114.9733^2 - (120.4179^2 - 72.2507^2)}{114.9733^2} \times 100$$

$= 29.8636\%$

Result:

Diameter of solid shaft, D	= 114.9733 mm
Inner diameter of hollow shaft, d	= 72.2507 mm
Outer diameter of hollow shaft, D_h	= 120.4179 mm
Percentage saving in material	= 29.8636%

PROBLEM 2

A solid shaft transmits 240 kW at 110 rpm. If the shear stress is not to exceed 70 N/mm², find the diameter of the shaft. If this shaft is to be replaced by a hollow shaft whose internal diameter is 0.6 times outer diameter, determine the size and percentage of saving in material, the maximum shearing stress being the same.

Given:

Power, P	= 240 kW = 240 × 10^3 W
Speed, N	= 110 rpm
Maximum shear stress, τ	= 70 N/mm²
Inner diameter of hollow shaft, d	= 0.6 × Outer diameter of hollow shaft = 0.6 D_h

To find:

Diameter of solid shaft, D

Inner diameter of hollow shaft, d

Outer diameter of hollow shaft, D_h

Percentage saving in material

Solution:

Power

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

$$= \frac{240 \times 10^3 \times 60}{2\pi \times 110}$$

$$= 20834.8289 \text{ N-m}$$

$$= 20834.8289 \times 10^3 \text{ N-mm}$$

Solid shaft

Torque transmitted by solid shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$D^3 = \frac{16 \times T}{\pi \times \tau}$$

$$D = \sqrt[3]{\frac{16 \times T}{\pi \times \tau}}$$

$$= \sqrt[3]{\frac{16 \times 20834.8289 \times 10^3}{\pi \times 70}}$$

$$= 114.8737 \text{ mm}$$

Hollow shaft

Torque transmitted by hollow shaft,

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_h^4 - d^4)}{D_h}$$

$$= \frac{\pi}{16} \times \tau \times \frac{(D_h^4 - [0.6D_h]^4)}{D_h}$$

$$= \frac{\pi}{16} \times \tau \times \frac{D_h^4(1 - 0.6^4)}{D_h}$$

$$= \frac{\pi}{16} \times \tau \times D_h^3 \times 0.8704$$

$$D_h^3 = \frac{16 \times T}{\pi \times \tau \times 0.8704}$$

$$D_h = \sqrt[3]{\frac{16 \times T}{\pi \times \tau \times 0.8704}}$$

$$= \sqrt[3]{\frac{16 \times 20834.8289 \times 10^3}{\pi \times 70 \times 0.8704}}$$

$$= 120.3135 \text{ mm}$$

$d = 0.6\, D_h$

$= 0.6 \times 120.3135$

$= 72.1881 \text{ mm}$

Weight of solid shaft,

W_s = Weight density × Volume of solid shaft

= Weight density × Area of solid shaft × Length

= $w \times A_s \times L$

Weight of hollow shaft,

W_h = Weight density × Volume of hollow shaft

= Weight density × Area of hollow shaft × Length

= $w \times A_h \times L$

Percentage saving in material

$$= \frac{W_s - W_h}{W_s} \times 100$$

$$= \frac{w \times A_s \times L - w \times A_h \times L}{w \times A_s \times L} \times 100$$

$$= \frac{A_s - A_h}{A_s} \times 100$$

$$= \frac{\frac{\pi}{4} D^2 - \frac{\pi}{4}(D_h^2 - d^2)}{\frac{\pi}{4} D^2} \times 100$$

$$= \frac{D^2 - (D_h^2 - d^2)}{D^2} \times 100$$

$$= \frac{114.8737^2 - (120.3135^2 - 72.1881^2)}{114.9733^2} \times 100$$

$= 29.7951\%$

Result:

Diameter of solid shaft, D	= 114.8737 mm
Inner diameter of hollow shaft, d	= 72.1881 mm
Outer diameter of hollow shaft, D_h	= 120.3135 mm
Percentage saving in material	= 29.7951%

PROBLEM 3

Determine the diameter of a solid shaft which will transmit 300 kW at 250 rpm. The maximum shear stress should not exceed 30 N/mm^2 and twist should not be more than 1° in a shaft length of 2 m. Take modulus of rigidity = 1 × 10^5 N/mm^2.

Given:

Power transmitted, P	= 300 kW = 300 × 10^3 W
Speed, N	= 250 rpm
Maximum shear stress, τ	= 30 N/mm^2
Twist, θ	= 1° = 1° × π/180° = 0.0175 rad
Length of shaft, L	= 2 m = 2000 mm
Modulus of rigidity, G	= 1 × 10^5 N/mm^2

To find:

Diameter of the solid shaft, D

Solution:

Power

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

$$= \frac{300 \times 10^3 \times 60}{2\pi \times 250}$$

$$= 11459.1559 \text{ N-m}$$

$$= 11459.1559 \times 10^3 \text{ N-mm}$$

Strength basis

Torque transmitted by solid shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$D^3 = \frac{16 \times T}{\pi \times \tau}$$

$$D = \sqrt[3]{\frac{16 \times T}{\pi \times \tau}}$$

$$= \sqrt[3]{\frac{16 \times 11459.1559 \times 10^3}{\pi \times 30}}$$

$$= 124.8343 \text{ mm}$$

Stiffness basis

Using the relation

$$\frac{T}{J} = \frac{G\theta}{L}$$

Polar moment of inertia of solid shaft

$$J = \frac{\pi}{32} D^4$$

$$\frac{T}{\frac{\pi}{32} D^4} = \frac{G\theta}{L}$$

$$D^4 = \frac{32TL}{\pi G\theta}$$

$$D = \sqrt[4]{\frac{32TL}{\pi G\theta}}$$

$$= \sqrt[4]{\frac{32 \times 11459.1559 \times 10^3 \times 2000}{\pi \times 1 \times 10^5 \times 0.0175}}$$

$$= 107.5416 \text{ mm}$$

Higher diameter is selected to suit the both conditions and hence the suitable diameter of the shaft, D = 124.8343 mm

Result:

Diameter of the solid shaft, D = 124.8343 mm

PROBLEM 4

Design a suitable diameter for a circular shaft required to transmit 120 kW at 180 rpm. The shear stress in the shaft not to exceed 70 N/mm^2 and the maximum torque exceeds the mean by 40%. Calculate the angle of twist in a length of 2 m. Take $G = 0.8 \times 10^5$ N/mm^2.

Given:

Power transmitted, P	$= 120$ kW $= 120 \times 10^3$ W
Speed, N	$= 180$ rpm
Maximum shear stress, τ	$= 70$ N/mm^2
Maximum torque, T_{max}	$= 1.40 \times$ Mean torque $= 1.40\ T_{mean}$
Length of shaft, L	$= 2$ m $= 2000$ mm
Modulus of rigidity, G	$= 0.8 \times 10^5$ N/mm^2

To find:

Diameter of the solid shaft, D

Angle of twist, θ

Solution:

Power,

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

$$= \frac{120 \times 10^3 \times 60}{2\pi \times 180}$$

$$= 6366.1977 \text{ N-m}$$

$$= 6366.1977 \times 10^3 \text{ N-mm}$$

$$T_{mean} = T$$

$$T_{max} = 1.40\ T_{mean}$$

$$= 1.40 \times 6366.1977 \times 10^3$$

$$= 8912.6768 \times 10^3 \text{ N-mm}$$

Torque transmitted by solid shaft,

$$T_{max} = \frac{\pi}{16} \times \tau \times D^3$$

$$D^3 = \frac{16 \times T_{max}}{\pi \times \tau}$$

$$D = \sqrt[3]{\frac{16 \times T_{max}}{\pi \times \tau}}$$

$$= \sqrt[3]{\frac{16 \times 8912.6768 \times 10^3}{\pi \times 70}}$$

$$= 86.5552 \text{ mm}$$

Using the relation

$$\frac{T_{max}}{J} = \frac{G\theta}{L}$$

Polar moment of inertia of solid shaft

$$J = \frac{\pi}{32} D^4$$

$$\frac{T_{max}}{\frac{\pi}{32} D^4} = \frac{G\theta}{L}$$

$$\theta = \frac{T_{max} \times L}{\frac{\pi}{32} D^4 \times G}$$

$$= \frac{8912.6768 \times 10^3 \times 2000}{\frac{\pi}{32} \times 86.5552^4 \times 0.8 \times 10^5}$$

$$= 0.0404 \text{ rad}$$

$$= 0.0404 \times \frac{180°}{\pi}$$

$$= 2.3168°$$

Result:

Diameter of the solid shaft, D = 86.5552 mm

Angle of twist, θ = 2.3168°

PROBLEM 5

Find the diameter of the shaft required to transmit 80 kW at 150 rpm, if the maximum torque is likely to exceed the mean torque by 25% for the maximum shear stress of 60 MN/m^2. Also find the angle of twist in a length of 2.5 m. Given G = 8 × 10^4 MN/m^2.

Given:

Power, P	= 80 kW = 80 × 10^3 W
Speed, N	= 150 rpm
Maximum torque, T_{max}	= 25% more than mean torque = 1.25 T_{mean}
Maximum shear stress, τ	= 60 MN/m^2 = 60 N/mm^2
Length, L	= 2.5 m = 2500 mm
Modulus of rigidity, G	= 8 × 10^4 MN/m^2 = 8 × 10^4 N/mm^2

To find:

Diameter of the solid shaft, D

Angle of twist, θ

Solution:

Power,

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

$$= \frac{80 \times 10^3 \times 60}{2\pi \times 150}$$

$$= 5092.9582 \text{ N-m}$$

$$= 5092.9582 \times 10^3 \text{ N-mm}$$

$$T_{mean} = T$$

$$T_{max} = 1.25\ T_{mean}$$

$$= 1.25 \times 5092.9582 \times 10^3$$

$$= 6366.1977 \times 10^3 \text{ N-mm}$$

Torque transmitted by solid shaft,

$$T_{max} = \frac{\pi}{16} \times \tau \times D^3$$

$$D^3 = \frac{16 \times T_{max}}{\pi \times \tau}$$

$$D = \sqrt[3]{\frac{16 \times T_{max}}{\pi \times \tau}}$$

$$= \sqrt[3]{\frac{16 \times 6366.1977 \times 10^3}{\pi \times 60}}$$

$$= 81.4516 \text{ mm}$$

Using the relation

$$\frac{T_{max}}{J} = \frac{G\theta}{L}$$

Polar moment of inertia of solid shaft

$$J = \frac{\pi}{32} D^4$$

$$\frac{T_{max}}{\frac{\pi}{32} D^4} = \frac{G\theta}{L}$$

$$\theta = \frac{T_{max} \times L}{\frac{\pi}{32} D^4 \times G}$$

$$= \frac{6366.1977 \times 10^3 \times 2500}{\frac{\pi}{32} \times 81.4516^4 \times 8 \times 10^4}$$

$$= 0.0460 \text{ rad}$$

$$= 0.0460 \times \frac{180°}{\pi}$$

$$= 2.6379°$$

Result:

Diameter of the solid shaft, D = 81.4516 mm

Angle of twist, θ = 2.6379°

PROBLEM 6

A solid shaft P of 50 mm diameter rotates at 250 rpm. Find the power that can be transmitted for a limiting shear stress of 60 N/mm^2 in the steel. It is proposed to replace P by a hollow shaft Q of the same external diameter but with the limiting shear stress of 75 N/mm^2. Determine the internal diameter of Q to transmit the same power at the same speed.

Given:

Speed, N = 250 rpm

Solid shaft P

Diameter, D = 50 mm

Shear stress, τ = 60 N/mm^2

Hollow shaft Q

External diameter of hollow shaft, D_h = 50 mm

Shear stress, τ = 75 N/mm^2

To find:

Power, P

Internal diameter of hollow shaft, d

Solution:

Solid shaft

Torque transmitted by solid shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 60 \times 50^3$$

$$= 1472621.556 \text{ N-mm}$$

$$= 1472.6215 \text{ N-m}$$

Power,

$$P = \frac{2\pi NT}{60} \quad [\text{T in Nm}]$$

$$= \frac{2\pi \times 250 \times 1472.6215}{60}$$

$$= 38553.1422 \text{ W}$$

$$= 38.5531 \text{ kW}$$

Hollow shaft

Power transmitted by hollow shaft = Power transmitted by solid shaft

Speed of hollow shaft = Speed of solid shaft

$\therefore$ Torque transmitted by hollow shaft = Torque transmitted by solid shaft

Torque transmitted by hollow shaft,

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_h^4 - d^4)}{D_h}$$

$$1472621.556 = \frac{\pi}{16} \times 75 \times \frac{(50^4 - d^4)}{50}$$

$$\frac{1472621.556 \times 16 \times 50}{\pi \times 75} = 50^4 - d^4$$

$$d^4 = 50^4 - \frac{1472621.556 \times 16 \times 50}{\pi \times 75}$$

$$d = \sqrt[4]{50^4 - \frac{1472621.556 \times 16 \times 50}{\pi \times 75}}$$

$$= 33.4370 \text{ mm}$$

Result:

Power, P = 38.5531 kW

Internal diameter of hollow shaft, d = 33.4370 mm

PROBLEM 7

Calculate the power that can be transmitted at a 300 rpm by a hollow steel shaft of 75 mm external diameter and 50 mm internal diameter when the permissible shear stress for the steel is 70 N/mm^2 and the maximum torque is 1.3 times the mean. Compare the strength of this hollow shaft with that of solid shaft. The material, weight and length of both the shafts are the same.

Given:

Speed, N	= 300 rpm
External diameter of hollow shaft, D_h	= 75 mm
Internal diameter of hollow shaft, d	= 50 mm
Shear stress, τ	= 70 N/mm^2
Maximum torque, T_{max}	= 1.3 × Mean torque = 1.3 T_{mean}
Material of hollow shaft, $Material_h$	= Material of solid shaft, $Material_s$
Weight of hollow shaft, W_h	= Weight of solid shaft, W_s
Length of hollow shaft, L_h	= Length of solid shaft, L_s

To find:

Power, P

Strength of hollow shaft and solid shaft

Solution:

Hollow shaft

Torque transmitted by hollow shaft,

$$T_{max} = \frac{\pi}{16} \times \tau \times \frac{(D^4 - d^4)}{D_h}$$

$$= \frac{\pi}{16} \times 70 \times \frac{(75^4 - 50^4)}{75}$$

$$= 4653075.057 \text{ N-mm}$$

$$= 1.3\ T_{mean}$$

$$T_{mean} = \frac{T_{max}}{1.3}$$

$$= \frac{4653075.057}{1.30}$$

$$= 3579288.505 \text{ N-mm}$$

$$= 3579.2885 \text{ N-m}$$

$T = T_{mean}$

Power,

$$P = \frac{2\pi NT}{60} \qquad [\text{T in Nm}]$$

$$= \frac{2\pi \times 300 \times 3579.2885}{60}$$

$= 112446.6647$ W

$= 112.4467$ kW

Weight of hollow shaft, W_h = Weight of solid shaft, W_s

$w_h \times A_h \times L_h = w_s \times A_s \times L_s$ [Weight = Weight density × Area × Length]

$A_h = A_s$ [$L_h = L_s$ and $w_h = w_s$ (Material$_h$ = Material$_s$)]

$$\frac{\pi}{4}\left(D_h^2 - d^2\right) = \frac{\pi}{4} D^2$$

$$D_h^2 - d^2 = D^2$$

$$D = \sqrt{D_h^2 - d^2}$$

$$= \sqrt{75^2 - 50^2}$$

$= 55.9017$ mm

Solid shaft

Torque transmitted by solid shaft,

$$T_{max} = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 70 \times 55.9017^3$$

$= 2401059.723$ N-mm

Strength of hollow shaft, T_h = 4653075.057 N-mm

Strength of solid shaft, T_s = 2401059.723 N-mm

$$\frac{T_h}{T_s} = \frac{4653075.057}{2401059.723}$$

$$\frac{T_h}{T_s} = 1.9379$$

Result:

Power, P = 112.4467 kW

Strength of hollow shaft = 1.9379 × Strength of solid shaft

PROBLEM 8

A steel shaft ABCD having a total length of 2.4 m consists of three lengths having different sections as follows:
AB is hollow having outside and inside diameters of 80 mm and 50 mm respectively and BC and CD are solid, BC having a diameter of 80 mm and CD having a diameter of 70 mm. If the angle of twist is the same for each section, determine the length of each section and the total angle of twist if the maximum shear stress in the hollow portion is 50 N/mm^2. Take G = 8.2 × 10^4 N/mm^2.

Given:

Total length, L = 2.4 m = 2400 mm

Shaft AB

Outside diameter, D_{AB} = 80 mm

Inside diameter, d_{AB} = 50 mm

Shear stress, τ_{AB} = 50 N/mm^2

Diameter of shaft BC, D_{BC} = 80 mm

Diameter of shaft CD, D_{CD} = 70 mm

Angle of twist $\theta_{AB} = \theta_{BC} = \theta_{CD}$

Modulus of rigidity, G = 8.2 × 10^4 N/mm^2

To find:

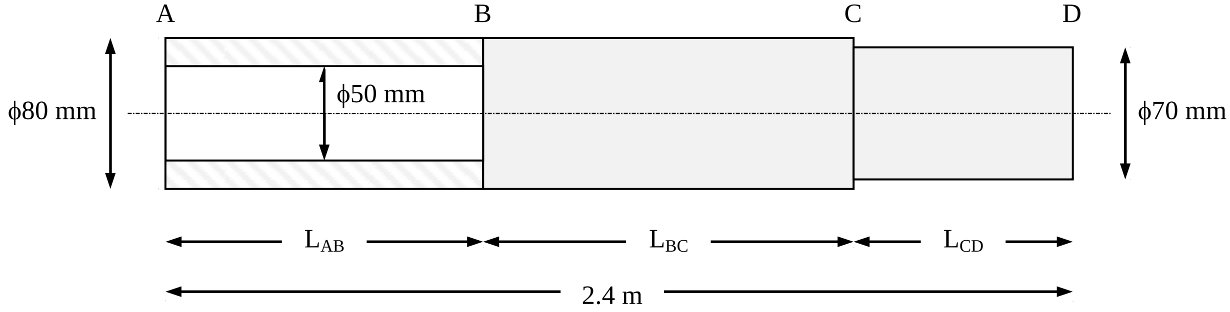
Length of shaft AB, L_{AB}

Length of shaft BC, L_{BC}

Length of shaft CD, L_{CD}

Total angle of twist, θ

Solution:



Using the relation,

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\theta = \frac{TL}{JG}$$

Torque, T and modulus of rigidity, G for are same for all sections.

$$\theta_{AB} = \frac{TL_{AB}}{J_{AB}G}$$

$$\theta_{BC} = \frac{TL_{BC}}{J_{BC}G}$$

$$\theta_{CD} = \frac{TL_{CD}}{J_{CD}G}$$

Polar moment of inertia,

$$J_{AB} = \frac{\pi}{32}(D_{AB}{}^4 - d_{AB}{}^4)$$

$$= \frac{\pi}{32}(80^4 - 50^4)$$

$$J_{BC} = \frac{\pi}{32} D_{BC}{}^4$$

$$= \frac{\pi}{32} \times 80^4$$

$$J_{CD} = \frac{\pi}{32} D_{CD}{}^4$$

$$= \frac{\pi}{32} \times 70^4$$

Angle of twist is same for all sections.

$\theta_{AB} = \theta_{BC} = \theta_{CD}$

$$\frac{TL_{AB}}{J_{AB}G} = \frac{TL_{BC}}{J_{BC}G} = \frac{TL_{CD}}{J_{CD}G}$$

$$\frac{L_{AB}}{J_{AB}} = \frac{L_{BC}}{J_{BC}} = \frac{L_{CD}}{J_{CD}}$$

$$\frac{L_{AB}}{\frac{\pi}{32}(80^4 - 50^4)} = \frac{L_{BC}}{\frac{\pi}{32} \times 80^4} = \frac{L_{CD}}{\frac{\pi}{32} \times 70^4}$$

$$\frac{L_{AB}}{80^4 - 50^4} = \frac{L_{BC}}{80^4} = \frac{L_{CD}}{70^4}$$

$$\frac{L_{AB}}{80^4 - 50^4} = \frac{L_{BC}}{80^4}$$

$$L_{BC} = \left(\frac{80^4}{80^4 - 50^4}\right) L_{AB}$$

$$\frac{L_{AB}}{80^4 - 50^4} = \frac{L_{CD}}{70^4}$$

$$L_{CD} = \left(\frac{70^4}{80^4 - 50^4}\right) L_{AB}$$

80^4

$$L_{AB} + L_{BC} + L_{CD} = L$$

$$L_{AB} + \left(\frac{}{80^4 - 50^4}\right) L_{AB} + \left(\frac{70^4}{80^4 - 50^4}\right) L_{AB} = 2400$$

$$L_{AB}\left(1+\left(\frac{80^4}{80^4-50^4}\right)+\left(\frac{70^4}{80^4-50^4}\right)\right)=2400$$

$$L_{AB}=\frac{2400}{1+\left(\frac{80^4}{80^4-50^4}\right)+\left(\frac{70^4}{80^4-50^4}\right)}$$

$$=835.7143\ \text{mm}$$

$$L_{BC}=\left(\frac{80^4}{80^4-50^4}\right)L_{AB}$$

$$=\left(\frac{80^4}{80^4-50^4}\right)\times 835.7143$$

$$=986.1958\ \text{mm}$$

$$L_{CD}=\left(\frac{70^4}{80^4-50^4}\right)L_{AB}$$

$$=\left(\frac{70^4}{80^4-50^4}\right)\times 835.7143$$

$$=578.0899\ \text{mm}$$

Using the relation,

$$\frac{\tau}{R}=\frac{G\theta}{L}$$

For shaft AB,

$$\frac{\tau_{AB}}{R_{AB}}=\frac{G\theta_{AB}}{L_{AB}}$$

$$\frac{\tau_{AB}}{\frac{D_{AB}}{2}}=\frac{G\theta_{AB}}{L_{AB}}$$

$$\theta_{AB}=\frac{\tau_{AB}\times L_{AB}}{\frac{D_{AB}}{2}\times G}$$

$$=\frac{50\times 835.7143}{\frac{80}{2}\times 8.2\times 10^4}$$

$$=0.1274\ \text{rad}$$

$$=0.1274\times\frac{180}{\pi}$$

$$=0.7299^\circ$$

Total angle of twist of the whole shaft

$\theta = \theta_{AB} + \theta_{BC} + \theta_{CD}$

$= 3 \times \theta_{AB}$ $[\theta_{AB} = \theta_{BC} = \theta_{CD}]$

$= 3 \times 0.7299$

$= 2.1898°$

Result:

Length of shaft AB, L_{AB} = 835.7143mm

Length of shaft BC, L_{BC} = 986.1958 mm

Length of shaft CD, L_{CD}= 578.0899 mm

Total angle of twist, θ = 2.1898°

PROBLEM 9

A composite shaft is made up of a solid steel shaft of 25 mm radius encased in a hollow brass tube of 40 mm inner radius and 60 mm outer radius. If both the shafts are of length 1 m, find

i. Shear stress induced in both shafts
ii. Angle of twist

Take modulus of rigidity of steel = 1×10^5 N/mm^2, modulus of rigidity of brass = 0.5×10^5 N/mm^2 and torque T = 1 kN-m.

Given:

Radius of solid steel shaft, R_s = 25 mm

Inner radius of hollow brass tube, r_b = 40 mm

Outer radius of hollow brass tube, R_b = 60 mm

Length of composite shaft, L = 1 m = 1000 mm

Modulus of rigidity of steel, G_s = 1×10^5 N/mm^2

Modulus of rigidity of brass, G_b = 0.5×10^5 N/mm^2

Torque, T = 1 kN-m = 1×10^3 N-m = 1×10^6 N-mm

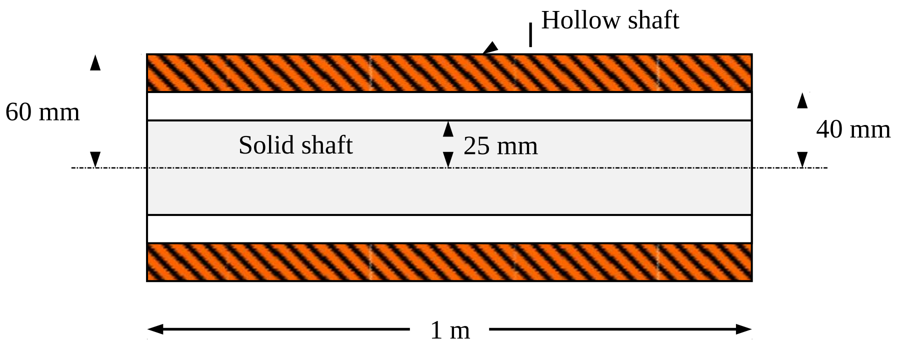
To find:

Shear stress induced in hollow brass shaft, τ_b

Shear stress induced in solid steel shaft, τ_s

Angle of twist, θ

Solution:



Diameter of solid steel shaft, $D_s = 2\ R_s = 50$ mm

Inner diameter of hollow brass tube, $d_b = 2\ r_b = 80$ mm

Outer diameter of hollow brass tube, $D_b = 2\ R_b = 120$ mm

Using the relation,

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\theta = \frac{TL}{JG}$$

For compound shaft, the angle of twist in each shaft is equal.

Angle of twist in hollow brass shaft = Angle of twist in solid steel shaft

$\theta_b = \theta_s$

$$\frac{T_b L_b}{J_b G_b} = \frac{T_s L_s}{J_s G_s}$$

$$\frac{T_b}{J_b G_b} = \frac{T_s}{J_s G_s} \qquad [L_b = L_s = L]$$

$$\frac{T_b}{J_b} = \frac{T_s}{J_s} \times \frac{G_b}{G_s} \qquad \dots (1)$$

Using the relation,

$$\frac{T}{J} = \frac{\tau}{R}$$

Substituting above value in equation (1)

$$\frac{\tau_b}{R_b} = \frac{\tau_s}{R_s} \times \frac{G_b}{G_s}$$

$$\tau_b = \tau_s \times \frac{R_b}{R_s} \times \frac{G_b}{G_s}$$

$$= \tau_s \times \frac{60}{25} \times \frac{0.5 \times 10^5}{1 \times 10^5}$$

$$= 1.2\ \tau_s$$

For compound shaft,

Total torque = Torque transmitted by hollow brass tube + Torque transmitted by solid steel tube

$$T = T_b + T_s$$

$$= \frac{J_b \times \tau_b}{R_b} + \frac{J_s \times \tau_s}{R_s} \qquad \left[T = \frac{J\tau}{R}\right]$$

$$= \frac{\frac{\pi}{32}[D_b^4 - d_b^4] \times \tau_b}{R_b} + \frac{\frac{\pi}{32} D_s^4 \times \tau_s}{R_s}$$

$$1 \times 10^6 = \frac{\frac{\pi}{32}[120^4 - 80^4] \times 1.2\ \tau_s}{60} + \frac{\frac{\pi}{32} 50^4 \times \tau_s}{25} \qquad [\tau_b = 1.2\ \tau_s]$$

$$\tau_s = \frac{1 \times 10^6}{\frac{\frac{\pi}{32}[120^4 - 80^4] \times 1.2}{60} + \frac{\frac{\pi}{32} 50^4}{25}}$$

$$= 2.8468\ \text{N/mm}^2$$

$\tau_b = 1.2\ \tau_s$

$= 1.2 \times 2.8468$

$= 3.4162\ \text{N/mm}^2$

Angle of twist

$\theta = \theta_b = \theta_s$

$= \theta_s$

$= \frac{T_s L_s}{J_s G_s}$

$= \frac{1 \times 10^6 \times 1000}{\frac{\pi}{32} \times 50^4 \times 1 \times 10^5}$

$= 0.0163 \text{ rad}$

$= 0.0163 \times \frac{180°}{\pi}$

$= 0.9338°$

Result:

Shear stress induced in hollow brass shaft, τ_b = 3.4162 N/mm^2

Shear stress induced in solid steel shaft, τ_s = 2.8468 N/mm^2

Angle of twist, θ = 0.9338°

PROBLEM 10

A closed coiled helical spring is to have a stiffness of 1.5 N/mm of compression under a maximum load of 60 N. The maximum shearing stress produced in the wire of the spring is 125 N/mm^2. The solid length of the spring is 50 mm. Find the diameter of coil, diameter of wire and number of coils. Take G = 4.5 × 10^4 N/mm^2.

Given:

Stiffness, S	= 1.5 N/mm
Load, W	= 60 N
Maximum shear stress, τ	= 125 N/mm^2
Solid length, nd	= 50 mm
Modulus of rigidity, G	= 4.5 × 10^4 N/mm^2

To find:

Diameter of the coil, D

Diameter of the wire, d

Number of coils, n

Solution:

Stiffness of spring,

$$S = \frac{Gd^4}{64R^3n}$$

$$1.5 = \frac{4.5 \times 10^4 \times d^4}{64R^3n}$$

$$\frac{d^4}{R^3n} = \frac{1.5 \times 64}{4.5 \times 10^4}$$

$$= 2.1333 \times 10^{-3} \quad \ldots (1)$$

Maximum shear stress,

$$\tau = \frac{16WR}{\pi d^3}$$

$$125 = \frac{16 \times 60 \times R}{\pi d^3}$$

$$\frac{R}{d^3} = \frac{125 \times \pi}{16 \times 60}$$

$$= 0.4091 \quad \ldots (2)$$

$$nd = 50 \text{ (given)}$$

$$d = \frac{50}{n} \quad \ldots (3)$$

Substituting value of d in equation (1)

$$\frac{\left(\frac{50}{n}\right)^4}{R^3 n} = 2.1333 \times 10^{-3}$$

$$\frac{50^4}{R^3 n^5} = 2.1333 \times 10^{-3}$$

$$R^3 n^5 = \frac{50^4}{2.1333 \times 10^{-3}}$$

$$= 2929687500 \qquad \ldots (4)$$

Substituting value of d in equation (2)

$$\frac{R}{\left(\frac{50}{n}\right)^3} = 0.4091$$

$$R = 0.4091 \times \left(\frac{50}{n}\right)^3$$

$$= \frac{51132.6929}{n^3} \qquad \ldots (5)$$

Substituting value of R in equation (4)

$$\left(\frac{51132.6929}{n^3}\right)^3 n^5 = 2929687500$$

$$\frac{51132.6929^3}{n^9} \times n^5 = 2929687500$$

$$\frac{51132.6929^3}{n^4} = 2929687500$$

$$n^4 = \frac{51132.6929^3}{2929687500}$$

$$n = \sqrt[4]{\frac{51132.6929^3}{2929687500}}$$

$$= 14.6127$$

$$\approx 15$$

Substituting value of n in equation (5)

$$R = \frac{51132.6929}{14.6127^3}$$

$$= 16.3773 \text{ mm}$$

$$D = 2R = 32.7546 \text{ mm}$$

Substituting value of n in equation (3)

$$d = \frac{50}{14.6127}$$

$$= 3.4210 \text{ mm}$$

Result:

Diameter of the coil, D = 32.7546 mm

Diameter of the wire, d = 3.4210 mm

Number of coils, n = 15

PROBLEM 11

A closely coiled helical spring of round steel wire 10 mm in diameter having 10 complete turns with a mean diameter of 12 cm is subjected to an axial load of 250 N. Determine

i. the deflection of the spring
ii. maximum shear stress in the wire
iii. stiffness of the spring and
iv. frequency of vibration.

Take $G = 0.8 \times 10^5$ N/mm^2.

Given:

Diameter of wire, d = 10 mm

Number of turns, n = 10

Mean coil diameter, D = 12 cm = 120 mm

Axial load, W = 250 N

Modulus of rigidity, $G = 0.8 \times 10^5$ N/mm^2

To find:

Deflection of the spring, δ

Maximum shear stress in the wire, τ

Stiffness of the spring, S

Frequency of vibration, f

Solution:

Mean coil radius,

$$R = \frac{D}{2}$$

$$= \frac{120}{2}$$

$$= 60 \text{ mm}$$

Deflection of the spring,

$$\delta = \frac{64WR^3n}{Gd^4}$$

$$= \frac{64 \times 250 \times 60^3 \times 10}{0.8 \times 10^5 \times 10^4}$$

$$= 43.2 \text{ mm}$$

Maximum shear stress in the wire,

$$\tau = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 250 \times 60}{\pi \times 10^3}$$

$= 76.3944\ \text{N/mm}^2$

Stiffness of the spring,

$$S = \frac{W}{\delta}$$

$$= \frac{250}{43.2}$$

$$= 5.7870\ \text{N/mm}$$

Frequency of vibration,

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{9810}{43.2}} \qquad [g = 9.81\ \text{m/s}^2 = 9810\ \text{mm/s}^2]$$

$$= 2.3984\ \text{Hz}$$

Result:

Deflection of the spring, δ	= 43.2 mm
Maximum shear stress in the wire, τ	= 76.3944 N/mm^2
Stiffness of the spring, S	= 5.7870 N/mm
Frequency of vibration, f	= 2.3984 Hz

PROBLEM 12

A safety valve which is placed on a pressure vessel has 120 mm diameter is used to blow off at a pressure of 1.23 N/mm^2 gauge. It is held by a close coiled compression spring of circular steel bar. The mean diameter is 180 mm and initial compression is 20 mm. Find the diameter of steel bar and the number of turns necessary if the shear stress allowed is 90 N/mm^2. Take $G = 0.82 \times 10^5$ N/mm^2.

Given:

Diameter of pressure vessel, D_v = 120 mm

Pressure, p = 1.23 N/mm^2

Mean coil diameter, D = 180 mm

Initial compression, δ = 20 mm

Maximum shear stress, τ = 90 N/mm^2

Modulus of rigidity, G = 0.8×10^5 N/mm^2

To find:

Diameter of steel bar, d

Number of turns, n

Solution:

Mean coil radius,

$$R = \frac{D}{2}$$

$$= \frac{180}{2}$$

$$= 90 \text{ mm}$$

Force on the safety valve,

$$W = \text{Pressure} \times \text{Area}$$

$$= p \times \frac{\pi}{4} \times D_v^2$$

$$= 1.23 \times \frac{\pi}{4} \times 120^2$$

$$= 4428\,\pi \text{ N}$$

Maximum shear stress,

$$\tau = \frac{16WR}{\pi d^3}$$

$$d^3 = \frac{16WR}{\pi\tau}$$

$$d = \sqrt[3]{\frac{16WR}{\pi\tau}}$$

$$= \sqrt[3]{\frac{16 \times 4428\,\pi \times 90}{\pi \times 90}}$$

$$= 41.3786 \text{ mm}$$

Deflection,

$$\delta = \frac{64WR^3n}{Gd^4}$$

$$n = \frac{Gd^4\delta}{64WR^3}$$

$$= \frac{0.8 \times 10^5 \times 41.3786^4 \times 20}{64 \times 4428\,\pi \times 90^3}$$

$$= 7.2270$$

$$\approx 8$$

Result:

Diameter of steel bar, d = 41.3786 mm

Number of turns, n = 8

PROBLEM 13

A composite spring has two closed coil helical springs connected in series. In both the springs, mean coil radius is 3.5 times the wire diameter. One spring is made out of 4 mm wire diameter and has 16 turns where as other spring has 12 turns. Determine the wire diameter of the second spring if the stiffness of the composite spring is 1.3 N/mm. Take $G = 0.82 \times 10^5$ N/mm^2.

Given:

For both springs,

Mean coil radius, R = 3.5 × Diameter of wire

For first spring,

Diameter of wire, d_1 = 4 mm

Number of turns, n_1 = 16

For second spring,

Number of turns, n_2 = 12

Stiffness of composite spring, S = 1.3 N/mm

Modulus of rigidity, G = 0.82×10^5 N/mm^2

To find:

Wire diameter of second spring, d_2

Solution:

When springs are connected in series, each spring is subjected to the same load and total deflection is equal to the algebraic sum of the deflection of the springs.

Stiffness,

$$S = \frac{W}{\delta}$$

$$= \frac{W}{\delta_1 + \delta_2}$$

Deflection of first spring,

$$\delta_1 = \frac{64WR_1{}^3n_1}{Gd_1{}^4}$$

$$= \frac{64W \times (3.5\ d_1)^3 n_1}{Gd_1{}^4} \qquad [R = 3.5\ d]$$

$$= \frac{64W \times (3.5 \times 4)^3 \times 16}{0.82 \times 10^5 \times 4^4}$$

$$= 0.1339\ W$$

Deflection of second spring,

$$\delta_2 = \frac{64WR_2{}^3n_2}{Gd_2{}^4}$$

$$= \frac{64W \times (3.5\,d_2)^3 n_2}{G d_2^{\,4}} \qquad [R = 3.5\ d]$$

$$= \frac{64W \times (3.5\,d_2)^3 \times 12}{0.82 \times 10^5 \times d_2^{\,4}}$$

$$= 0.4016\ \frac{W}{d_2}$$

$$S = \frac{W}{\delta_1 + \delta_2}$$

$$1.3 = \frac{W}{0.1339\,W + 0.4016\,\frac{W}{d_2}}$$

$$1.3 = \frac{1}{0.1339 + \frac{0.4016}{d_2}}$$

$$0.1339 + \frac{0.4016}{d_2} = \frac{1}{1.3}$$

$$\frac{0.4016}{d_2} = \frac{1}{1.3} - 0.1339$$

$$d_2 = \frac{0.4016}{\left(\frac{1}{1.3} - 0.1339\right)}$$

$d_2 = 0.6320$ mm

Result:

Wire diameter of second spring, $d_2 = 0.6320$ mm

PROBLEM 14

A helical spring in which the mean diameter of the coil is 8 times the wire diameter is to be designed to absorb 0.2 kN-m of energy with an extension of 100 mm. The maximum shear stress is not to exceed 125 N/mm². Determine the mean diameter of the spring, diameter of the wire and the number of turns. Also find the load with which an extension of 40 mm could be produced in the spring. Assume G = 84 kN/mm².

Given:

Mean diameter of coil, D	= 8 × Diameter of wire = 8 d
Strain energy, U	= 0.2 kN-m = 0.2 × 10^6 N-mm
Extension, δ	= 100 mm
Maximum shear stress, τ	= 125 N/mm²
Modulus of rigidity, G	= 84 kN/mm² = 84 × 10^3 N/mm²

To find:

Mean diameter of the coil, D

Diameter of the wire, d

Number of turns, n

Load for δ = 40 mm, W

Solution:

Strain energy stored by the spring,

$$U = \frac{\tau^2}{4G} \times \text{Volume of the spring}$$

$$= \frac{\tau^2}{4G} \times \text{Area of spring wire} \times \text{Total length of spring wire}$$

$$= \frac{\tau^2}{4G} \times \frac{\pi d^2}{4} \times \pi D n$$

$$0.2 \times 10^6 = \frac{125^2}{4 \times 84 \times 10^3} \times \frac{\pi d^2}{4} \times \pi \times (8\,d) \times n \qquad [D = 8\,d]$$

$$= \frac{125^2}{4 \times 84 \times 10^3} \times \frac{\pi}{4} \times \pi \times 8 \times d^3 n$$

$$d^3 n = \frac{0.2 \times 10^6}{\left(\frac{125^2}{4 \times 84 \times 10^3} \times \frac{\pi}{4} \times \pi \times 8\right)}$$

$$= 217881.0733$$

$$n = \frac{217881.0733}{d^3} \qquad \ldots (1)$$

Maximum shear stress,

$$\tau = \frac{16WR}{\pi d^3}$$

$$125 = \frac{16W \times \frac{8d}{2}}{\pi d^3} \qquad \left(R = \frac{D}{2}\right)$$

$$125 = \frac{16W \times 4}{\pi d^2}$$

$$d^2 = \frac{16W \times 4}{125\,\pi}$$

$$= 0.1630\ W \qquad \ldots (2)$$

Deflection,

$$\delta = \frac{64WR^3n}{Gd^4}$$

$$100 = \frac{64W\left(\frac{8d}{2}\right)^3\left(\frac{217881.0733}{d^3}\right)}{84 \times 10^3 \times (0.1630\ W)^2}$$

$$= \frac{64W \times 4^3 \times d^3 \times \left(\frac{217881.0733}{d^3}\right)}{84 \times 10^3 \times 0.1630^2 \times W^2}$$

$$W = \frac{64 \times 4^3 \times 217881.0733}{84 \times 10^3 \times 0.1630^2 \times 100}$$

$$= 4000\ N$$

Substituting value of W in equation (2)

$$d^2 = 0.1630 \times 4000$$

$$d = \sqrt{0.1630 \times 4000}$$

$$= 25.5323\ mm$$

D = 8 d (given)

$$= 8 \times 25.5323$$

$$= 204.2584\ mm$$

Substituting value of d in equation (1)

$$n = \frac{217881.0733}{25.5323^3}$$

$$= 13.0903$$

$$\approx 14$$

Load for δ = 40 mm

$$\delta = \frac{64WR^3n}{Gd^4}$$

$$400 = \frac{64W\left(\frac{204.2584}{2}\right)^3 \times 14}{84 \times 10^3 \times 25.5323^4}$$

$$W = 1496.0336\ N$$

Result:

Mean diameter of the coil, D	= 204.2584 mm
Diameter of the wire, d	= 25.5323 mm
Number of turns, n	= 14
Load for δ = 40 mm, W	= 1496.0336 N

TWO MARKS QUESTIONS AND ANSWERS

32. Define torsion.

A shaft is said to be in torsion, when equal and opposite torques are applied at the two ends of the shaft. The torque is equal to the product of tangential force applied and the distance between the point of application of the force and the axis of the shaft.

33. What are the assumptions made in derivation of torsion equation?

- The material of the shaft is uniform throughout.
- The twist along the shaft is uniform.
- The shaft is of uniform circular section throughout.
- Cross-sections of the shaft which are plane before twist remain plane after twist.
- All radii which are straight before twist remain straight after twist.

34. Write torsional equation.

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$$

where, T – Torque (N-mm)

J – Polar moment of inertia (mm^4)

G – Modulus of rigidity of the material of shaft (N/mm^2)

θ – Angle of twist (rad)

L – Length of shaft (mm)

τ – Shear stress (N/mm^2)

R – Radius of shaft (mm)

35. Write the expression for power transmitted by a shaft.

$$P = \frac{2\pi NT}{60}$$

where, P – Power (W)

N – Speed (rpm)

T – Torque (N-m)

36. Write the expression for torque transmitted by solid shaft and hollow shaft.

For solid shaft,

$$T = \frac{\pi}{16} \tau D^3$$

For hollow shaft,

$$T = \frac{\pi}{16} \tau \frac{\left(D_h^4 - d^4 \right)}{D_h}$$

where, T – Torque (N-m)

τ – Shear stress (N/mm^2)

D – Diameter of solid shaft (mm)

D_h – Outer diameter of hollow shaft (mm)

d – Inner diameter of hollow shaft (mm)

37. Define polar modulus.

Polar modulus is defined as the ratio of the polar moment of inertia to the radius of the shaft.

$$\text{Polar modulus} = \frac{\text{Polar moment of inertia}}{\text{Radius}}$$

$$Z_p = \frac{J}{R}$$

38. Write polar modulus for solid shaft and hollow shaft.

For solid shaft,

$$Z_p = \frac{\pi}{16} D^3$$

For hollow shaft,

$$Z_p = \frac{\pi}{16}\left(\frac{D_h^4 - d^4}{D_h}\right)$$

where, Z_p – Polar modulus (mm^3)

D – Diameter of solid shaft (mm)

D_h – Outer diameter of hollow shaft (mm)

d – Inner diameter of hollow shaft (mm)

39. Define strength of a shaft.

Strength of a shaft is the maximum torque or maximum power the shaft can transmit.

40. Define stiffness of a shaft or torsional rigidity.

Stiffness of a shaft or torsional rigidity is defined as the torque required to produce a twist of one radian per unit length of shaft.

$$\text{Torsional rigidity} = \text{Modulus of rigidity} \times \text{Polar moment of inertia}$$

41. Why hollow circular shafts are preferred when compared to solid circular shafts?

- The torque transmitted by the hollow shaft is greater than the solid shaft.
- For the same material, length and given torque, the weight of the hollow shaft will be less compared to solid shaft.

42. What is a composite or compound shaft?

A shaft made up of two or more different materials and behaving as a single shaft is known as composite or compound shaft.

43. What is meant by spring?

Springs are the elastic bodies which absorb energy due to resilience. The absorbed energy may be released as and when required.

44. Classify the types of springs.

- Bending spring
- Torsion spring

45. What is meant by stiffness of spring?

The stiffness of the spring is defined as the load that produces unit deflection.

$$\text{Stiffness} = \frac{\text{Load}}{\text{De�lection}}$$

$$S = \frac{W}{\delta}$$

46. Differentiate between close-coiled and open-coiled helical springs.

Close-coiled helical springs	**Open-coiled helical springs**
Helix angle is very small.	Helix angle is considerable.
Adjacent turns are very close to each other.	Large gap between adjacent turns.
It can carry only tensile load.	It can carry both tensile and compressive loads.

47. What kind of stress induced when an axial load acts on a close-coiled helical spring?

Shear stress.

48. What kind of stress induced when an axial load acts on an open-coiled helical spring?

Bending stress and shear stress.

49. Define spring index or spring constant.

Spring index or spring constant is the ratio of the mean diameter of the spring to the diameter of the spring wire.

$$\text{Spring index} = \frac{\text{Mean diameter of spring}}{\text{Diameter of spring wire}} = \frac{D}{d}$$

14 MARKS QUESTIONS

53. A solid shaft is subjected to a torque of 15 kN-m. Find the necessary diameter of the shaft, if the allowable shear stress is 60 N/mm^2. The allowable twist is 1° for every 20 diameters length of the shaft. Take G = 80 kN/mm^2.

54. A hollow shaft of diameter ratio 3/8 is required to transmit 3/8 is required to transmit 588 kW at 110 rpm. The maximum torque exceeds the mean by 20%. The shear stress is limited to 63 N/mm^2 and the twist should not be more than 0.0081 rad. Calculate the external diameter required satisfying both the conditions. Take G = 84 GPa. Length = 3 m.

55. Calculate the power that can be transmitted at a 300 rpm by a hollow steel shaft of 75 mm external diameter and 50 mm internal diameter when the permissible shear stress for the steel is 70 N/mm^2 and the maximum torque is 1.3 times the mean. Compare the strength of this hollow shaft with that of a solid shaft. The same material, weight and length of both the shafts are the same.

56. A hollow shaft of diameter ratio 3/8 (internal diameter to outer diameter) is to transmit 375 kW power at 100 rpm. The maximum torque being 20% greater than the mean. The shear stress is not to exceed 60 N/mm^2 and twist in a length of 4 m not to exceed 2°. Calculate its external and internal diameters which would satisfy both the above conditions. Assume modulus of rigidity, G = 0.85 × 10^5 N/mm^2.

57. A hollow shaft is to transmit 300 KW power at 80 rpm. If the shear stress is not to exceed 60 N/mm^2 and the internal diameter is 0.6 times of the external diameter, find the external and internal diameters, assuming that the maximum torque is 1.4 times the mean torque.

58. Determine the diameter of a solid steel shaft which will transmit 90 kW at 160 rpm. Also determine the length of the shaft if the twist must not exceed 1° over the entire length. The maximum shear stress is limited to 60 N/mm^2. Take the value of modulus of rigidity = 8 × 10^4 N/mm^2.

59. A hollow shaft is to transmit 400 kW at 80 rpm. If the shear stress is not to exceed 60 MN/m^2 and the internal diameter is 0.6 times the external diameter find the external and internal diameter assuming the maximum torque is 1.4 times the mean torque.

60. A hollow shaft, having an internal diameter 40% of its external diameter, transmits 562.5 kW power at 100 rpm. Determine the external diameter of the shaft if the shear stress is not to exceed 60 N/mm^2 and twist in a length of 2.5 m should not exceed 1.3°. Assume maximum torque = 1.25 mean torque and modulus of rigidity = 8 × 10^4 N/mm^2.

61. A hollow shaft is 1 m long and has external diameter 50 mm. It has 20 mm internal diameter for a part of the length and 30 mm internal diameter for the rest of the length. If the maximum shear stress in it is not to exceed 80 N/mm^2, determine the maximum power transmitted by it at a speed of 300 rpm. If the twists produced in the two portions of the shaft are equal, find the lengths of the two portions.

62. Two solid shafts AB and BC of aluminium and steel respectively are rigidly fastened together at B and attached to two rigid supports at A and C. Shaft AB is 7.5 cm in diameter and 2 m in length. Shaft BC is 5.5 cm in diameter and 1 m in length. A torque of 20000 N-cm is applied at the junction B. Compute the maximum shearing stresses in each material. What is the angle of

twist at the junction? Take modulus of rigidity of the materials as $G_a = 0.3 \times 10^5$ N/mm^2 and $G_s = 0.9 \times 10^5$ N/mm^2.

63. A composite shaft consists of copper rod of 30 mm diameter enclosed in a steel tube of external diameter 50 mm and 10 mm thick. The shaft is required to transmit a torque of 1000 N-m. Determine the shear stresses developed in copper and steel, if both the shafts have equal lengths and welded to a plate at each end, so that their twists are equal. Take modulus of rigidity for steel as twice that of copper.

64. A close coiled helical spring is required to absorb 2250 J of energy. Determine the diameter of the wire, the mean coil diameter of the spring and the number of coils necessary if (i) the maximum stress is not to exceed 400 MPa, (ii) the maximum compression of the spring is limited to 250 mm and (iii) the mean diameter of the spring is eight times the wire diameter. For the spring material, rigidity modulus is 70 GPa.

65. The stiffness of a close coiled helical spring is l.5 N/mm of compression under a maximum load of 60 N. The maximum shearing stress produced in the wire of the spring is 125 N/mm^2. The solid length of the spring (when the coils are touching) is given as 5 cm. Find
    i. diameter of wire,
    ii. mean diameter of the coils and
    iii. number of coils required.

    Take $G = 4.5 \times 10^5$ N/mm^2.

66. Design a close coiled helical spring of stiffness 20 N/mm. The maximum shear stress in the spring metal is not to exceed 80 N/mm^2 under a load of 600 N. The diameter of the coil to be 10 times the diameter of the wire. Take G = 85 kN/mm^2.

67. A truck weighing 20 kN and moving at 6 km/h has to be brought to rest by a buffer. Find how many springs each of 15 coils will be required to store the energy of motion during a compression 200 mm. The spring is made out of 25 mm diameter rod coiled to a mean diameter of 200 mm. $G = 0.945 \times 10^5$ N/mm^2.

68. Two concentric close coiled springs are employed to control a valve, which requires a force of 155 N to open the same by 12 mm. The outer spring has 14 coils of 16 mm mean radius and 4 mm wire diameter. It is compressed by 6 mm initially when the valve is closed. The free length of inner spring is 8 mm more than that of the outer one. Determine the stiffness of the inner spring and also the number of turns it has if the mean diameter of 20 mm and wire diameter of 2.5 mm. Take G = 80 GPa.

69. A weight of 160 N is dropped on to a close coiled helical spring made of 13 mm steel wire and mean radius of 60 mm with 22 coils. If the instantaneous compression is 90 mm, calculate the height of drop. Take $G = 0.88 \times 10^5$ N/mm^2.

70. A close coiled helical spring of 5 mm diameter wire which has 16 coils of 100 mm inner diameter. If the maximum shear stress is limited to 150 MPa, find the stiffness of the spring. Take G = 85 GPa.

71. Two close coiled concentric helical springs of the same length, are wound out of the same wire, circular in cross-section and supports a compressive load ‘P’. The inner spring consists of 20 turns of mean diameter 16 cm and the outer spring has 18 turns of mean diameter 20 cm.

Calculate the maximum stress produced in each spring if the diameter of wire = 1 cm and P = 1000 N.

72. A closely coiled helical spring is to carry a load of 1kN. Its mean coil diameter is to be 10 times that of wire diameter. Calculate these diameters if the maximum shear stress in the material of the spring is to be 90 N/mm^2. If the stiffness of the spring is 20 N/mm deflection and modulus of rigidity = 8.4×10^4 N/mm^2, find the number of coils.

OBJECTIVE QUESTIONS AND ANSWERS

1. The product of the tangential force acting on the shaft and its distance from the axis of the shaft (i.e. radius of shaft) is known as
   a) bending moment
   b) twisting moment
   c) torsional rigidity
   d) flexural rigidity

2. The assumption made, while determining the shear stress in a circular shaft subjected to torsion, is that
   a) the material of the shaft is uniform
   b) the twist along the shaft is uniform
   c) cross-sections of the shaft is plane and circular before and after the twist
   d) all of these

3. Choose the correct statement.
   a) Shafts of the same material and length having the same polar modulus have the same strength.
   b) For a shaft of a given material, the magnitude of polar modulus is a measure of its strength in resisting torsion.
   c) From a number of shafts of the same length and material, the shaft with greatest polar modulus will resist the maximum twisting moment.
   d) all of these

4. When a shaft is subjected to a twisting moment, every cross-section of the shaft will be under
   a) tensile stress
   b) compressive stress
   c) shear stress
   d) bending stress

5. The shear stress at the centre of a circular shaft under torsion is
   a) zero
   b) minimum
   c) maximum
   d) infinity

6. The shear stress at the outermost fibres of a circular shaft under torsion is
   a) zero
   b) minimum
   c) maximum
   d) infinity

7. The shear stress at any point of a shaft, subjected to twisting moment, is
   a) proportional to its distance from the central axis of the shaft
   b) inversely proportional to its distance from the central axis of the shaft
   c) proportional to the square of its distance from the central axis of the shaft
   d) none of these

8. When a shaft is subjected to torsion, the shear stress induced in the shaft varies from
   a) minimum at the centre to maximum at the circumference
   b) maximum at the centre to minimum at the circumference
   c) zero at the centre to maximum at the circumference
   d) maximum at the centre to zero at the circumference

9. Torsional rigidity of a shaft is equal to
   a) product of modulus of rigidity and polar moment of inertia
   b) sum of modulus of rigidity and polar moment of inertia
   c) difference of modulus of rigidity and polar moment of inertia
   d) ratio of modulus of rigidity and polar moment of inertia

10. The torsional rigidity of a shaft is defined as the torque required to produce
   a) maximum twist in the shaft
   b) maximum shear stress in the shaft
   c) minimum twist in the shaft
   d) a twist of one radian per unit length of the shaft

11. Polar modulus of a shaft section is equal to
   a) product of polar moment of inertia and maximum radius of the shaft
   b) ratio of polar moment of inertia to maximum radius of the shaft
   c) sum of polar moment of inertia and maximum radius of the shaft
   d) difference of polar moment of inertia and maximum radius of the shaft

12. The torsional rigidity of a shaft is given by
   a) T/J
   b) **T/**θ
   c) T/r
   d) T/G

13. The polar moment of inertia of a solid circular shaft of diameter is
   a) $\pi D^3 / 16$
   b) $\pi D^3 / 32$
   c) $\pi D^4 / 32$
   d) $\pi D^4 / 64$

14. The polar moment of inertia of a hollow shaft of outer diameter and inner diameter is
   a) $(D^3 – d^3) / 16$
   b) $(D^4 – d^4) / 16$
   c) $(D^4 – d^4) / 32$
   d) $(D^4 – d^4) / 64$

15. Which of the following is the correct torsion equation?
   a) $M/I = \sigma/y = E/R$
   b) $T/J = \tau/R = G\theta/l$
   c) $M/R = T/J = G\theta/l$
   d) $T/l = \tau/J = R/G\theta$

16. The torque transmitted by a solid shaft of diameter is
    a) $\pi\tau D^3 / 4$
    b) $\pi\tau D^3 / 16$
    c) $\pi\tau D^3 / 32$
    d) $\pi\tau D^3 / 64$

17. The torque transmitted, by a solid shaft of diameter 40 mm if the shear stress is not to exceed 400 N/cm^2, would be
    a) 1.6 π N-m
    b) 16 π N-m
    c) 0.8 π N-m
    d) 0.4 π N-m

18. Two solid shafts 'A' and 'B' are made of the same material. The shaft 'A' is of 50 mm diameter and shaft 'B' is of 100 mm diameter. The strength of shaft 'B' is_______as that of shaft A.
    a) one-half
    b) double
    c) four times
    d) eight times

19. In the torsion equation $T/J = \tau/R = G\theta/l$, the term J/R is called
    a) shear modulus
    b) section modulus
    c) polar modulus
    d) none of these

20. The polar modulus for a solid shaft of diameter is
    a) $\pi D^2 / 4$
    b) $\pi D^3 / 16$
    c) $\pi D^3 / 32$
    d) $\pi D^4 / 64$

21. The polar modulus for a hollow shaft of outer diameter and inner diameter is
    a) $(D^2 - d^2) / 4D$
    b) $(D^3 - d^3) / 16D$
    c) $(D^4 - d^4) / 16D$
    d) $(D^4 - d^4) / 32D$

22. The torque transmitted by a hollow shaft of outer diameter and inner diameter is
    a) $\pi\tau (D^2 - d^2) / 4D$
    b) $\pi\tau (D^3 - d^3) / 16D$
    c) $\pi\tau (D^4 - d^4) / 16D$
    d) $\pi\tau (D^4 - d^4) / 32D$

23. A shaft revolving at ω rad/s transmits torque (T) in N-m. The power developed is________watts.
    a) **Tω**
    b) 2πTω
    c) 2πTω / 75
    d) 2πTω / 4500

24. Two shafts ‘A’ and ‘B’ have the same material. The shaft ‘A’ is solid of diameter 100 mm. The shaft ‘B’ is hollow with outer diameter 100 mm and inner diameter 50 mm. The torque transmitted by shaft ‘B’ is_________________as that of shaft ‘A’.
    a) 1/6
    b) 1/8
    c) 1/4
    d) 15/16

25. Two shafts ‘A’ and ‘B’ are made of same material. The shaft ‘A’ is solid and has diameter D. The shaft ‘B’ is hollow with outer diameter D and inner diameter D/2. The strength of hollow shaft in torsion is______________as that of solid shaft.
    a) 1/16
    b) 1/8
    c) 1/4
    d) 15/16

26. Two shafts ‘A’ and ‘B’ are made of same material. The shaft ‘A’ is of diameter D and shaft ‘B’ is of diameter D/2. The strength of shaft ‘B’ is________as that of shaft ‘A’.
    a) one-eighth
    b) one-fourth
    c) one-half
    d) four times

27. When two shafts of same length, one of which is hollow, transmit equal torques and have equal maximum stress, then they should have equal
    a) polar moment of inertia
    b) polar modulus
    c) diameter
    d) angle of twist

28. A shaft can safely transmit 90 kW while rotating at a given speed. If this shaft is replaced by a shaft of diameter double the previous one and rotated at half the speed of the previous, the power that can be transmitted by the new shaft is
    a) 90 kW
    b) 180 kW
    c) 360 kW
    d) 720 kW

29. In spring balances, the spring is used
    a) to apply forces
    b) to measure forces
    c) to absorb shocks
    d) to store strain energy

30. The springs in brakes and clutches are used to
    a) to apply forces
    b) to measure forces
    c) to store strain energy
    d) to absorb shocks

31. In a watch, the spring is used to store strain energy. This energy is released
    a) to stop the watch
    b) to run the watch
    c) to change the time
    d) all of these

32. A spring used to absorb shocks and vibrations is
    a) conical spring
    b) torsion spring
    c) leaf spring
    d) disc spring

33. The load required to produce a unit deflection in a spring is called
    a) flexural rigidity
    b) torsional rigidity
    c) spring stiffness
    d) Young's modulus

34. A leaf spring is supported at the
    a) ends and loaded at the centre
    b) centre and loaded at the ends
    c) ends and loaded anywhere
    d) centre and loaded anywhere

35. When a closely-coiled helical spring is subjected to an axial load, it is said to be under
    a) bending
    b) shear
    c) torsion
    d) crushing

36. The stress induced when an axial load acts on a close coiled spring is
    a) direct stress
    b) bending stress
    c) shear stress
    d) both bending and shear stress

37. The stress induced when an axial load acts on an open coiled spring is
    a) direct stress
    b) bending stress
    c) shear stress
    d) both bending and shear stress

38. When a closely-coiled helical spring of mean diameter is subjected to an axial load (W), the deflection of the spring (δ) is given by
    a) WD^3n / Gd^4
    b) $2WD^3n / Gd^4$
    c) $4WD^3n / Gd^4$
    d) $8WD^3n / Gd^4$

39. When a closely-coiled helical spring of mean diameter is subjected to an axial load (W), the stiffness of the spring is given by
    a) Gd^4 / D^3n
    b) $Gd^4 / 2D^3n$
    c) $Gd^4 / 4D^3n$
    d) $Gd^4 / 8D^3n$

40. Two closely-coiled helical springs ‘A’ and ‘B’ of the same material, same number of turns and made from same wire are subjected to an axial load W. The mean diameter of spring ‘A’ is double the mean diameter of spring ‘B’. The ratio of deflections in spring ‘B’ to spring ‘A’ will be
    a) 1/8
    b) 1/4
    c) 2
    d) 4

41. Two closely-coiled helical springs ‘A’ and ‘B’ of the same material, same number of turns and made from same wire are subjected to an axial load W. The mean diameter of spring ‘A’ is double the mean diameter of spring ‘B’. The ratio of stiffness of spring ‘B’ to spring ‘A’ will be
    a) 2
    b) 4
    c) 6
    d) **8**

42. A closely coiled helical spring is of mean diameter and spring wire diameter (d). The spring index is the ratio of
    a) 1/d
    b) 1/D
    c) D/d
    d) d/D

43. Two closely coiled helical springs ‘A ’ and ‘B’ are equal in all aspects but the number of turns of spring ‘A’ is half that of spring ‘B’. The ratio of deflections in spring ‘A’ to spring ‘B’ is
    a) 1/8
    b) 1/4
    c) 1/2
    d) 2

44. Two closely coiled helical springs ‘A ’ and ‘B’ are equal in all aspects but the diameter of wire of spring ‘A’ is double that of spring ‘B’. The stiffness of spring ‘B’ will be________that of spring ‘A’.
    a) one-sixteenth
    b) one-eighth
    c) one-fourth
    d) one-half

45. Two closely-coiled helical springs ‘A’ and ‘B’ are equal in all aspects but the number of turns of spring ‘A’ is double that of spring ‘B’. The stiffness of spring ‘A’ will be________that of spring ‘B’.
    a) one-sixteenth
    b) one-eighth
    c) one-fourth
    d) one-half

46. A closely-coiled helical spring is cut into two halves. The stiffness of the resulting spring will be
    a) same
    b) double
    c) half
    d) one-fourth

47. A closely-coiled helical spring of stiffness k is cut into (n) equal parts. The stiffness in each part of the spring will be
    a) $k\sqrt{n}$
    b) $n\sqrt{k}$
    c) nk
    d) nk^2

48. Wire diameter, mean coil diameter and number of turns of a closely-coiled steel spring are d, D and N respectively and stiffness of the spring is k. A second spring is made of the same steel but with wire diameter, mean coil diameter and number of turns as 2d, 2D and 2N respectively. The stiffness of the new spring is
    a) **k**
    b) 2k
    c) 4k
    d) 8k

49. When a helical compression spring is cut into two equal halves, the stiffness of each of the resulting springs will be
    a) unaltered
    b) double
    c) one-half
    d) one-fourth

50. While calculating the stress induced in a close-coiled helical spring, Wahl's factor must be considered to account for
    a) the curvature and stress concentration effect
    b) shock loading
    c) poor service conditions
    d) fatigue loading

51. A length of 10 mm diameter steel wire is coiled to a close-coiled helical spring having 8 coils of 75 mm mean diameter, and the spring has a stiffness k. If the same length of the wire is coiled to 10 coils of 60 mm mean diameter, then the spring stiffness will be
    a) k
    b) 1.25 k
    c) 1.56 k
    d) 1.95 k

52. When two springs of equal lengths are arranged to form a cluster spring then which of the following statements are true:
    1. Angle of twist in both the springs will be equal
    2. Deflection of both the springs will be equal
    3. Load taken by each spring will be half the total load
    4. Shear stress in each spring will be equal
    a) 1 and 2 only
    b) 2 and 3 only
    c) 3 and 4 only
    d) 1, 2 and 4 only

53. The stiffness of a spring is 10 N/mm and the axial deflection is 5 mm. What is the axial load on the spring?
    a) 2 N
    b) 5 N
    c) 20 N
    d) 50 N

*****

UNIT-IV BEAM DEFLECTION

SOLVED PROBLEMS

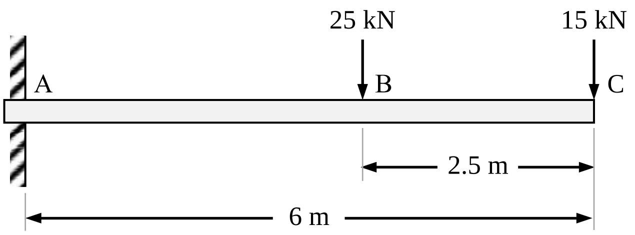
PROBLEM 1

A steel cantilever 6 m long carries two point loads, 15 kN at the free end and 25 kN at a distance of 2.5 m from the free end. Find:

i. Slope at the free end.
ii. Deflection at the free end.

Take $I = 1.3 \times 10^8$ mm^4 and $E = 2 \times 10^5$ N/mm^2.

Given:



Length, L = 6 m = 6000 mm

Point load at free end, W_1 = 15 kN = 15×10^3 N

Point load at a distance 2.5 m from free end, W_2 = 25 kN = 25×10^3 N

Moment of inertia, I = 1.3×10^8 mm^4

Young's modulus, E = 2×10^5 N/mm^2

To find:

Slope at the free end, θ_C

Deflection at the free end, y_C

Solution:

Distance AB, a = AC – BC

= 6000 – 2500

= 3500 mm

Double Integration Method

Slope at the free end due to load 15 kN alone,

$$\theta_1 = \frac{W_1 L^2}{2EI}$$

$$= \frac{15 \times 10^3 \times 6000^2}{2 \times 2 \times 10^5 \times 1.3 \times 10^8}$$

$$= 0.0104 \text{ rad}$$

Slope at the free end due to load 25 kN alone,

$$\theta_2 = \frac{W_2 a^2}{2EI}$$

$$= \frac{25 \times 10^3 \times 3500^2}{2 \times 2 \times 10^5 \times 1.3 \times 10^8}$$

$$= 5.8894 \times 10^{-3} \text{ rad}$$

Total slope at free end,

$$\theta_C = \theta_1 + \theta_2$$

$$= 0.0104 + 5.8894 \times 10^{-3}$$

$$= 0.0163 \text{ rad}$$

Downward deflection at the free end due to point load 15 kN alone,

$$y_1 = \frac{W_1 L^3}{3EI}$$

$$= \frac{15 \times 10^3 \times 6000^3}{3 \times 2 \times 10^5 \times 1.3 \times 10^8}$$

$$= 41.5385 \text{ mm}$$

Downward deflection at the free end due to point load 25 kN alone,

$$y_2 = \frac{W_2 a^3}{3EI} + \frac{W_2 a^2}{2EI}(L - a)$$

$$= \frac{25 \times 10^3 \times 3500^3}{3 \times 2 \times 10^5 \times 1.3 \times 10^8} + \frac{25 \times 10^3 \times 3500^2}{2 \times 2 \times 10^5 \times 1.3 \times 10^8}(6000 - 3500)$$

$$= 13.7420 + 14.7236$$

$$= 28.4655 \text{ mm}$$

Total deflection at free end,

$$y_C = y_1 + y_2$$

$$= 41.5385 + 28.4655$$

$$= 70.9856 \text{ mm}$$

Result:

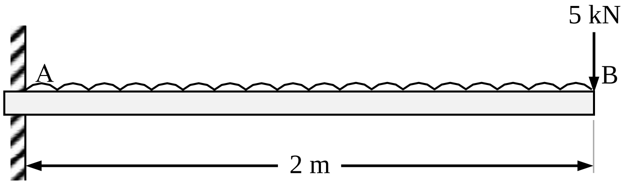
Slope at the free end, θ_C = 0.0163 rad

Deflection at the free end, y_C = 70.9856 mm

PROBLEM 2

A cantilever beam 50 mm wide and 80 mm deep is 2 m long. It carries a UDL over the entire length along with a point of 5 kN at its free end. Find the slope at the free end when the deflection is 7.5 mm at the free end. Take $E = 2 \times 10^5$ N/mm^2.

Given:



Width of beam, b = 50 mm

Depth of beam, d = 80 mm

Length of beam, L = 2 m = 2000 mm

Point load at free end, W = 5 kN = 5×10^3 N

Deflection at free end, y_B = 7.5 mm

Young's modulus, E = 2×10^5 N/mm^2

To find:

Slope at the free end, θ_B

Solution:

Moment of inertia of rectangular section,

$$I = \frac{bd^3}{12}$$

$$I = \frac{50 \times 80^3}{12}$$

$$I = 2133333.333 \text{ mm}^4$$

Double Integration Method

Downward deflection at the free end due to point load of 5 kN alone,

$$y_1 = \frac{WL^3}{3EI}$$

$$= \frac{5 \times 10^3 \times 2000^3}{3 \times 2 \times 10^5 \times 2133333.333}$$

$$= 31.25 \text{ mm}$$

Downward deflection at the free end due to uniformly distributed load of w N/mm alone,

$$y_2 = \frac{wL^4}{8EI}$$

$$= \frac{w \times 2000^4}{8 \times 2 \times 10^5 \times 2133333.333}$$

$= 4.6875\text{ w mm}$

Total deflection at the free end due to point load and UDL,

$y_B = y_1 + y_2$

$7.5 = 31.25 + 4.6875\text{ w}$

$7.5 - 31.25 = 4.6875\text{ w}$

$$w = \frac{7.5 - 31.25}{4.6875}$$

$= -5.0667\text{ N/mm}$ [Negative sign shows that UDL is upwards.]

Slope at the free end due to point load of 5 kN alone,

$$\theta_1 = \frac{WL^2}{2EI}$$

$$= \frac{5 \times 10^3 \times 2000^2}{2 \times 2 \times 10^5 \times 2133333.333}$$

$= 0.0234\text{ rad}$

Slope at the free end due to uniformly distributed load of –5.0667 N/mm alone,

$$\theta_2 = \frac{wL^3}{6EI}$$

$$= \frac{-5.0667 \times 2000^3}{6 \times 2 \times 10^5 \times 2133333.333}$$

$= -0.0158\text{ rad}$

Total slope at free end due to point load and UDL,

$\theta_B = \theta_1 + \theta_2$

$= 0.0234 - 0.0158$

$= 7.6041 \times 10^{-3}\text{ rad}$

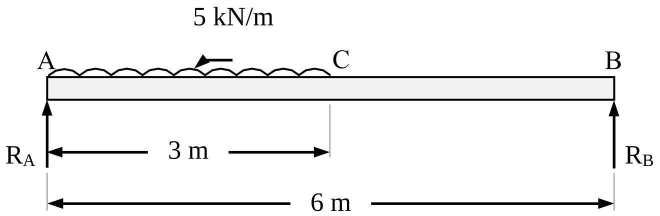
Result:

Slope at the free end, $\theta_B = 7.6041 \times 10^{-3}$ rad

PROBLEM 3

A simply supported beam of span 6 m carries UDL 5 kN/m over a length of 3 m extending from left end. Calculate deflection at mid-span. Take E = 2×10^5 N/mm^2 and I = 6.2×10^6 mm^4.

Given:



Span, L	= 6 m = 6000 mm
UDL, w	= 5 kN/m = 5 N/mm
Young's modulus, E	= 2×10^5 N/mm^2
Moment of Inertia, I	= 6.2×10^6 mm^4

To find:

Deflection at mid span, y_C

Solution:

Applying $\Sigma V = 0$

$R_A - (w \times AC) + R_B = 0$

$R_A - (5 \times 3000) + R_B = 0$

$R_A + R_B = 15000$ … (1)

Applying $\Sigma M_A = 0$

$$\left(w \times AC \times \frac{AC}{2}\right) - (R_B \times AB) = 0$$

$$\left(5 \times 3000 \times \frac{3000}{2}\right) - (R_B \times 6000) = 0$$

$22500000 - 6000\, R_B = 0$

$6000\, R_B = 22500000$

$$R_B = \frac{22500000}{6000}$$

$= 3750$ N

Substituting the above value in equation (1)

$R_A + 3750 = 15000$

$= 11250$ N

Macaulay's Method

Bending moment at any section of the beam at a distance x from the right support B is given by

$$EI\frac{d^2y}{dx^2} = R_B\,x$$ � $$- w(x - BC)\frac{(x - BC)}{2}$$

$$= R_B\,x$$ � $$- \frac{w(x - BC)^2}{2}$$

$$= 3750\,x$$ � $$- \frac{5(x - 3000)^2}{2}$$

Integrating the above equation,

$$EI\frac{dy}{dx} = \frac{3750\,x^2}{2} + C_1$$ � $$- \frac{5(x - 3000)^3}{2 \times 3}$$

Integrating the above equation,

$$EIy = \frac{3750\,x^3}{2 \times 3} + C_1x + C_2$$ � $$- \frac{5(x - 3000)^4}{2 \times 3 \times 4}$$

$$= \frac{3750\,x^3}{6} + C_1x + C_2$$ � $$- \frac{5(x - 3000)^4}{24} \quad \text{... (2)}$$

To find the values of C_1 and C_2 use boundary conditions. The boundary conditions are:

i. At x = 0, y = 0
ii. At x = 6000 mm, y = 0

Applying the first boundary condition to equation (2) and considering the equation up to first line (as x = 0 lies in the first part of the beam),

$$EI \times 0 = \frac{3750 \times 0^3}{6} + C_1 \times 0 + C_2$$

$$0 = 0 + 0 + C_2$$

$$C_2 = 0$$

Applying the second boundary condition to equation (2) and considering the complete equation (as x = 6000 mm lies in the last part of the beam),

$$EI \times 0 = \frac{3750 \times 6000^3}{6} + C_1 \times 6000 + 0 - \frac{5(6000 - 3000)^4}{24}$$

$$0 = \frac{3750 \times 6000^3}{6} + C_1 \times 6000 + 0 - \frac{5(6000 - 3000)^4}{24}$$

$$C_1 = \frac{\frac{5(6000 - 3000)^4}{24} -}{} \qquad 6000$$

3 5 $\times 6000^3$
7 0 6

$= -1.9688 \times 10^{10}$

Substituting the values of C_1 and C_2 in equation (2)

$$EIy = \frac{3750\,x^3}{6} - 1.9688 \times 10^{10}\,x \;\Big|\; - \frac{5(x - 3000)^4}{24} \qquad \text{... (3)}$$

Deflection at mid-span

Substituting x = 3000 mm in equation (3) up to first line,

$$2 \times 10^5 \times 6.2 \times 10^6 \times y_C = \frac{3750 \times 3000^3}{6} - 1.9688 \times 10^{10} \times 3000$$

$$y_C = \frac{\frac{3750 \times 3000^3}{6} - 1.9688 \times 10^{10} \times 3000}{2 \times 10^5 \times 6.2 \times 10^6}$$

$$= -34.0222 \text{ mm} \qquad \text{[Negative sign shows that deflection is downwards.]}$$

Result:

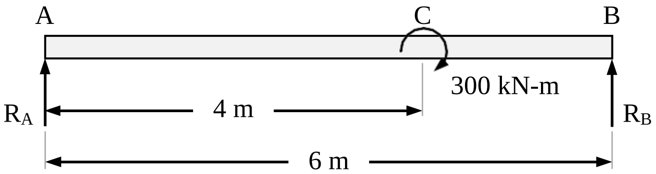
Deflection at mid span, y_C = – 34.0222 mm

PROBLEM 4

A horizontal beam AB is simply supported at A and B, 6 m apart. The beam is subjected to a clockwise couple of 300 kN-m at a distance of 4 m from the left end A. If $E = 2 \times 10^5$ N/mm^2 and $I = 2 \times 10^8$ mm^4, determine using Macaulay's method,

i. The deflection at a point where couple is acting.
ii. The maximum deflection.

Given:



Length, L = 6 m = 6000 mm

Couple, C = 300 kN-m = 300×10^6 N-mm

Young's modulus, E = 2×10^5 N/mm^2

Moment of Inertia, I = 2×10^8 mm^4

To find:

Deflection at a point where couple is acting, y_C

Maximum deflection, y_{max}

Solution:

Applying $\Sigma V = 0$

$R_A + R_B = 0$ … (1)

Applying $\Sigma M_A = 0$

$C - (R_B \times AB) = 0$

$300 \times 10^6 - (R_B \times 6000) = 0$

$$R_B = \frac{300 \times 10^6}{6000}$$

$= 50 \times 10^3$ N

Substituting the above value in equation (1)

$R_A + 50 \times 10^3 = 0$

$= -50 \times 10^3$ N

Bending moment at any section of the beam at a distance x from the left support A is given by

$$EI\frac{d^2y}{dx^2} = -R_B x \text{ � } + C$$

$$EI\frac{d^2y}{dx^2} = -50 \times 10^3\, x \text{ � } + 300 \times 10^6$$

$$EI\frac{d^2y}{dx^2} = -50 \times 10^3\, x \;|\; + 300 \times 10^6 \times (x - AC)^0$$

$$EI\frac{d^2y}{dx^2} = -50 \times 10^3\, x \;|\; + 300 \times 10^6 \times (x - 4000)^0$$

Integrating the above equation,

$$EI\frac{dy}{dx} = -\frac{50 \times 10^3\, x^2}{2} + C_1 \;|\; + 300 \times 10^6 \times (x - 4000) \qquad \ldots (2)$$

Integrating the above equation,

$$EIy = -\frac{50 \times 10^3\, x^3}{2 \times 3} + C_1x + C_2 \;|\; + \frac{300 \times 10^6\, (x - 4000)^2}{2}$$

$$= -\frac{25 \times 10^3\, x^3}{3} + C_1x + C_2 \;|\; + 150 \times 10^6\, (x - 4000)^2 \qquad \ldots (3)$$

To find the values of C_1 and C_2 use boundary conditions. The boundary conditions are:

i. At x = 0, y = 0
ii. At x = 6 m, y = 0

Applying the first boundary condition to equation (3) and considering the equation up to first line (as x = 0 lies in the first part of the beam),

$$EI \times 0 = -\frac{25 \times 10^3 \times 0^3}{3} + C_1 \times 0 + C_2$$

$$0 = -0 + 0 + C_2$$

$$C_2 = 0$$

Applying the second boundary condition to equation (3) and considering the complete equation (as x = 6000 mm lies in the last part of the beam),

$$EI \times 0 = -\frac{25 \times 10^3 \times 6000^3}{3} + C_1 \times 6000 + 0 + 150 \times 10^6 \times (6000 - 4000)^2$$

$$0 = -1.8 \times 10^{15} + 6000\, C_1 + 6 \times 10^{14}$$

$$C_1 = \frac{1.8 \times 10^{15} - 6 \times 10^{14}}{6000}$$

$$= 2 \times 10^{11}$$

Substituting the values of C_1 and C_2 in equation (3)

$$EIy = -\frac{25 \times 10^3 x^3}{3} + 2 \times 10^{11} x \text{ � } + 150 \times 10^6 \times (x - 4000)^2 \quad \ldots (4)$$

Deflection at a point where couple is acting

Substituting x = 4000 mm in equation (4),

$$2 \times 10^5 \times 2 \times 10^8 \times y_C = -\frac{25 \times 10^3 \times 4000^3}{3} + 2 \times 10^{11} \times 4000 - 150 \times 10^6 \times (4000 - 4000)^2$$

$$y_C = \frac{-\frac{25 \times 10^3 \times 4000^3}{3} + 2 \times 10^{11} \times 4000}{2 \times 10^5 \times 2 \times 10^8}$$

$= 6.6667$ mm

Maximum deflection

The slope (dy/dx) becomes zero where the maximum deflection occurs and this will occur in the larger segment.

Applying above condition in equation (2) up to first line,

$$EI \times 0 = -\frac{50 \times 10^3 x^2}{2} + 2 \times 10^{11}$$

$$-25 \times 10^3 x^2 + 2 \times 10^{11} = 0$$

$$-25 \times 10^3 x^2 = -2 \times 10^{11}$$

$$x^2 = \frac{-2 \times 10^{11}}{-25 \times 10^3}$$

$$x = \sqrt{\frac{-2 \times 10^{11}}{-25 \times 10^3}}$$

$= 2828.4271$ mm

Substituting x = 2828.4271 mm in equation (4) up to first line,

$$2 \times 10^5 \times 2 \times 10^8 \times y_{max} = -\frac{25 \times 10^3 \times 2828.4271^3}{3} + 2 \times 10^{11} \times 2828.4271$$

$$y_{max} = \frac{-\frac{25 \times 10^3 \times 2828.4271^3}{3} + 2 \times 10^{11} \times 2828.4271}{2 \times 10^5 \times 2 \times 10^8}$$

$= 9.4281$ mm

Result:

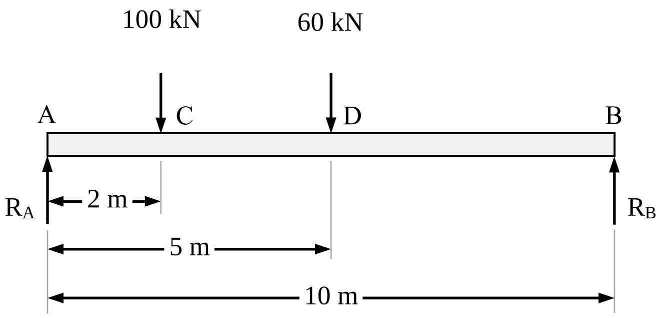
Deflection at a point where couple is acting, y_C = 6.6667 mm

Maximum deflection, y_{max} = 9.4281 mm

PROBLEM 5

A beam is simply supported at its ends over a span of 10 m and carries two concentrated loads of 100 kN and 60 kN at a distance of 2 m and 5 m respectively from the left support. Calculate (i) slope at the left support; (ii) slope and deflection under the 100 kN load. Assume EI = 36×10^4 kN-m^2.

Given:



Point load at C, W_C = 100 kN

Point load at D, W_D = 60 kN

EI = 36×10^4 kN-m^2

To find:

Slope at the left support, θ_A

Slope under 100 kN Load, θ_C

Deflection under 100 kN load, y_C

Solution:

Applying $\Sigma V = 0$

$R_A - W_C - W_D + R_B = 0$

$R_A - 100 - 60 + R_B = 0$

$R_A + R_B = 160$ … (1)

Applying $\Sigma M_A = 0$

$(W_C \times AC) + (W_D \times AD) - (R_B \times AB) = 0$

$(100 \times 2) + (60 \times 5) - (R_B \times 10) = 0$

$10\ R_B = (100 \times 2) + (60 \times 5)$

$$R_B = \frac{(100 \times 2) + (60 \times 5)}{10}$$

= 50 kN

Substituting the above value in equation (1)

$R_A + 50 = 160$

$R_A = 160 - 50$

= 110 kN

Macaulay's Method

Bending moment at any section of the beam at a distance x from the right support B is given by

$$EI\frac{d^2y}{dx^2} = R_B\,x \;|\; - W_D(x - BD) \;|\; - W_C(x - BC)$$

$$= 50\,x \;|\; - 60(x - 5) \;|\; - 100(x - 8)$$

Integrating the above equation,

$$EI\frac{dy}{dx} = \frac{50\,x^2}{2} + C_1 \;|\; -\frac{60(x-5)^2}{2} \;|\; -\frac{100(x-8)^2}{2} \qquad \text{... (2)}$$

Integrating the above equation,

$$EIy = \frac{50\,x^3}{2 \times 3} + C_1x + C_2 \;|\; -\frac{60(x-5)^3}{2 \times 3} \;|\; -\frac{100(x-8)^3}{2 \times 3}$$

$$= \frac{50\,x^3}{6} + C_1x + C_2 \;|\; -\frac{60(x-5)^3}{6} \;|\; -\frac{100(x-8)^3}{6} \qquad \text{... (3)}$$

To find the values of C_1 and C_2 use boundary conditions. The boundary conditions are:

i. At x = 0, y = 0
ii. At x = 10 m, y = 0

Applying the first boundary condition to equation (3) and considering the equation up to first line (as x = 0 lies in the first part of the beam),

$$EI \times 0 = \frac{50 \times 0^3}{6} + C_1 \times 0 + C_2$$

$$0 = 0 + 0 + C_2$$

$$C_2 = 0$$

Applying the second boundary condition to equation (3) and considering the complete equation (as x = 10 m lies in the last part of the beam),

$$EI \times 0 = \frac{50 \times 10^3}{6} + C_1 \times 10 + 0 - \frac{60(10-5)^3}{6} - \frac{100(10-8)^3}{6}$$

$$0 = \frac{25000}{3} + 10\,C_1 - 1250 - \frac{400}{3}$$

$$C_1 = \frac{-\frac{25000}{3} + 1250 + \frac{400}{3}}{10}$$

$$= -695$$

Substituting the values of C_1 and C_2 in equation (2)

$$EI\frac{dy}{dx} = \frac{50\,x^2}{2} - 695 - \frac{60(x-5)^2}{2} - \frac{100(x-8)^2}{2} \qquad \text{... (4)}$$

Slope at the left support

Substituting x = 10 m in equation (4),

$$36 \times 10^4 \times \theta_A = \frac{50 \times 10^2}{2} - 695 - \frac{60(10-5)^2}{2} - \frac{100(10-8)^2}{2}$$

$$\theta_A = \frac{\frac{50 \times 10^2}{2} - 695 - \frac{60(10-5)^2}{2} - \frac{100(10-8)^2}{2}}{36 \times 10^4}$$

$$= 2.375 \times 10^{-3} \text{ rad}$$

Slope at 100 kN load

Substituting x = 8 m in equation (4),

$$36 \times 10^4 \times \theta_A = \frac{50 \times 8^2}{2} - 695 - \frac{60(8-5)^2}{2} - \frac{100(8-8)^2}{2}$$

$$\theta_C = \frac{\frac{50 \times 8^2}{2} - 695 - \frac{60(8-5)^2}{2} - \frac{100(8-8)^2}{2}}{36 \times 10^4}$$

$$= 1.1806 \times 10^{-3} \text{ rad}$$

Deflection at 100 kN load

Substituting the values of C_1 and C_2 in equation (3)

$$EIy = \frac{50\,x^3}{6} - 695\,x - \frac{60(x-5)^3}{6} - \frac{100(x-8)^3}{6} \qquad \text{... (5)}$$

Substituting x = 8 m in equation (5),

$$36 \times 10^4 \times y_C = \frac{50 \times 8^3}{6} - 695 \times 8 - \frac{60(8-5)^3}{6} - \frac{100(8-8)^3}{6}$$

$$y_C = \frac{\frac{50 \times 8^3}{6} - 695 \times 8 - \frac{60(8-5)^3}{6} - \frac{100(8-8)^3}{6}}{36 \times 10^4}$$

$$= -4.3426 \times 10^{-3} \text{ m}$$

$$= -4.3426 \text{ mm}$$ [Negative sign shows that deflection is downwards.]

Result:

Slope at the left support, θ_A = 2.375×10^{-3} rad

Slope under 100 kN Load, θ_C = 1.1806×10^{-3} rad

Deflection under 100 kN load, y_C = – 4.3426 mm

PROBLEM 6

An I-section joist 500 mm × 250 mm × 25 mm and 5 m long is used as a column with both ends are fixed. Calculate the critical load for the column. Take $E = 2 \times 10^5$ N/mm^2.

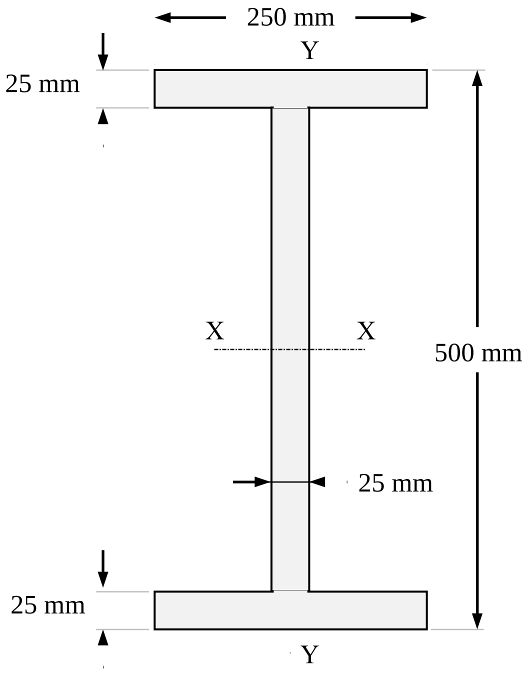
Given:

Dimensions of I-section	= 500 mm × 250 mm × 25 mm
Length, l	= 5 m = 5000 mm
End conditions of column	= Both ends fixed
Young's modulus, E	= 2×10^5 N/mm^2

To find:

Critical load, P

Solution:



Due to symmetry,

$$x = \frac{\text{Width}}{2}$$

$$= \frac{250}{2}$$

$$= 125 \text{ mm}$$

$$y = \frac{\text{Depth}}{2}$$

$$= \frac{500}{2}$$

$$= 250 \text{ mm}$$

Distance of CG of top rectangle from bottom end

$$y_1 = 500 - \frac{25}{2} = 487.5 \text{ mm}$$

Distance of CG of middle rectangle from bottom end

$$y_2 = \frac{(500 - 2 \times 25)}{2} + 25 = 250.5 \text{ mm}$$

Distance of CG of bottom rectangle from bottom end

$$y_3 = \frac{25}{12} = 12.5 \text{ mm}$$

Moment of inertia of top rectangle about X-X axis,

$$(I_{XX})_1 = \frac{b_1 {d_1}^3}{12} + A_1 {h_1}^2 \qquad [h_1 = y_1 - \bar{y}]$$

$$= \frac{250 \times 25^3}{12} + (250 \times 25) \times (487.5 - 250)^2$$

$$= 352864583.3 \text{ mm}^4$$

Moment of inertia of middle rectangle about X-X axis,

$$(I_{XX})_2 = \frac{b_2 {d_2}^3}{12} + A_2 {h_2}^2 \qquad [h_2 = y_2 - \bar{y}]$$

$$= \frac{25 \times 450^3}{12} + (25 \times 450) \times (250 - 250)^2$$

$$= 189843750 \text{ mm}^4$$

Moment of inertia of bottom rectangle about X-X axis,

$$(I_{XX})_3 = \frac{b_3 {d_3}^3}{12} + A_3 {h_3}^2 \qquad [h_3 = y_3 - \bar{y}]$$

$$= \frac{250 \times 25^3}{12} + (250 \times 25) \times (12.5 - 250)^2$$

$$= 352864583.3 \text{ mm}^4$$

Moment of inertia of the section about X-X axis,

$$I_{XX} = (I_{XX})_1 + (I_{XX})_2 + (I_{XX})_3$$

$$= 352864583.3 + 189843750 + 352864583.3$$

$$= 895572916.7 \text{ mm}^4$$

Moment of inertia of top rectangle about Y-Y axis,

$$(I_{YY})_1 = \frac{d_1 {b_1}^3}{12}$$

$$= \frac{25 \times 250^3}{}$$

1 2

$= 32552083.33 \text{ mm}^4$

Moment of inertia of middle rectangle about Y-Y axis,

$$(I_{YY})_2 = \frac{d_2 {b_2}^3}{12}$$

$$= \frac{450 \times 25^3}{12}$$

$$= 585937.5 \text{ mm}^4$$

Moment of inertia of bottom rectangle about Y-Y axis,

$$(I_{YY})_3 = \frac{d_3 {b_3}^3}{12}$$

$$= \frac{25 \times 250^3}{12}$$

$$= 32552083.33 \text{ mm}^4$$

Moment of inertia of the section about Y-Y axis,

$$I_{YY} = (I_{YY})_1 + (I_{YY})_2 + (I_{YY})_3$$

$$= 32552083.33 + 585937.5 + 32552083.33$$

$$= 65690104.17 \text{ mm}^4$$

Least value of moment of inertia is about Y-Y axis,

$$I = I_{YY} = 65690104.17 \text{ mm}^4$$

Effective length for both ends fixed,

$$L_e = \frac{l}{2}$$

$$= \frac{5000}{2}$$

$$= 2500 \text{ mm}$$

Critical load,

$$P = \frac{\pi^2 EI}{{L_e}^2}$$

$$= \frac{\pi^2 \times 2 \times 10^5 \times 65690104.17}{2500^2}$$

$$= 20746730.92 \text{ N}$$

$$= 20.7467 \text{ MN}$$

Result:

Critical load, P = 20.7467 MN

PROBLEM 7

A hollow tube 4 m long, 50 mm external diameter and 40 mm internal diameter is subjected under tensile load of 40 kN and deflects 10 mm. This tube is used as a column with both ends hinged. Determine crippling load and also safe load taking factor of safety as 3.

Given:

Length, l	= 4 m = 4000 mm
External diameter, D	= 50 mm
Internal diameter, d	= 40 mm
Tensile load	= 40 kN = 40×10^3 N
Deflection, δl	= 10 mm
End conditions of column	= Both ends hinged
Factor of safety, F	= 3

To find:

Crippling load, P

Safe load

Solution:

Area of hollow tube,

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(50^2 - 40^2)$$

$$= 225\,\pi\ \text{mm}^2$$

Tensile stress,

$$\sigma = \frac{\text{Tensile load}}{\text{Area}}$$

$$= \frac{40 \times 10^3}{225\,\pi}$$

$$= 56.5884\ \text{N/mm}^2$$

Tensile strain,

$$e = \frac{\text{Change in length}}{\text{Original length}}$$

$$= \frac{\delta l}{l}$$

$$= \frac{10}{4000}$$

$$= 2.5 \times 10^{-3}$$

Young's modulus,

$$E = \frac{\sigma}{e}$$

$$= \frac{56.5884}{2.5 \times 10^{-3}}$$

$$= 22635.3697 \text{ N/mm}^2$$

Moment of inertia of hollow tube,

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(50^4 - 40^4)$$

$$= 181132.4514 \text{ mm}^4$$

Effective length for both ends hinged,

$$L_e = l$$

$$= 4000 \text{ mm}$$

Crippling load,

$$P = \frac{\pi^2 EI}{{L_e}^2}$$

$$= \frac{\pi^2 \times 22635.3697 \times 181132.4514}{4000^2}$$

$$= 2529.0861 \text{ N}$$

$$\text{Safe load} = \frac{\text{Crippling load}}{\text{Factor of safety}}$$

$$= \frac{2529.0861}{3}$$

$$= 843.0287 \text{ N}$$

Result:

Crippling load, P = 2529.0861 N

Safe load = 843.0287 N

PROBLEM 8

A cast iron column has circular cross section of 60 mm diameter and 2 m long. If one of the ends of the column is fixed in position and other end is free, calculate the safe load by using

i. Rankine's formula, take $\sigma_c = 500$ N/mm^2 and $\alpha = 1/1600$
ii. Euler's formula, take $E = 1.2 \times 10^5$ N/mm^2

The factor of safety of the column is 3.

Given:

Diameter of column, D = 60 mm

Length, l = 2 m = 2000 mm

End conditions of column = One end fixed and other end free

Crushing stress, σ_c = 500 N/mm^2

Rankine's constant, α = 1/1600

Young's modulus, E = 1.2×10^5 N/mm^2

Factor of safety, F = 3

To find:

Safe load using Rankine's formula

Safe load using Euler's formula

Solution:

Area of cross-section,

$$A = \frac{\pi}{4} D^2$$

$$= \frac{\pi}{4} \times 60^2$$

$$= 900\,\pi \text{ mm}^2$$

Moment of inertia,

$$I = \frac{\pi}{64} D^4$$

$$= \frac{\pi}{64} \times 60^4$$

$$= 202500\,\pi \text{ mm}^4$$

Least radio of gyration,

$$= \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{202500\,\pi}{900\,\pi}}$$

$$= 15 \text{ mm}$$

Effective length for one end fixed and other end free,

$L_e = 2l$

$= 2 \times 2000$

$= 4000 \text{ mm}$

Crippling load using Rankine's formula,

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{L_e}{k}\right)^2}$$

$$= \frac{500 \times 900\,\pi}{1 + \frac{1}{1600}\left(\frac{4000}{15}\right)^2}$$

$= 31108.6803 \text{ N}$

$$\text{Safe load} = \frac{\text{Crippling load}}{\text{Factor of safety}}$$

$$= \frac{31108.6803}{3}$$

$= 10369.5601 \text{ N}$

Crippling load using Euler's formula,

$$P = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times 202500\,\pi}{4000^2}$$

$= 47090.7827 \text{ N}$

$$\text{Safe load} = \frac{\text{Crippling load}}{\text{Factor of safety}}$$

$$= \frac{47090.7827}{3}$$

$= 15696.9276 \text{ N}$

Result:

Safe load using Rankine's formula = 10369.5601 N

Safe load using Euler's formula = 15696.9276 N

PROBLEM 9

Find Euler's crippling load for a hollow cylindrical cast iron column of 200 mm external diameter, 25 mm thick and 6 m long hinged at both ends. Compare the load with crushing load calculated from Rankine's formula. Take σ_c = 550 N/mm^2, Rankine's constant = 1/1600 and E = 1.2 × 10^5 N/mm^2.

Given:

External diameter of column, D = 200 mm

Thickness, t = 25 mm

Length, l = 6 m = 6000 mm

End conditions of column = Both ends hinged

Crushing stress, σ_c = 550 N/mm^2

Rankine's constant, α = 1/1600

Young's modulus, E = 1.2 × 10^5 N/mm^2

To find:

Euler's crippling load

Ratio of Euler's and Rankine's crippling loads

Solution:

Internal diameter of column,

d = D – 2t

= 200 – 2 × 25

= 150 mm

Area of cross-section,

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(200^2 - 150^2)$$

$$= 4375\,\pi \text{ mm}^2$$

Moment of inertia,

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(200^4 - 150^4)$$

$$= 53689327.58 \text{ mm}^4$$

Least radio of gyration,

$$= \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{53689327.58}{4375\,\pi}}$$

$= 62.5$ mm

Effective length for both ends hinged,

$L_e = l$

$= 6000$ mm

Crippling load using Euler's formula,

$$P = \frac{\pi^2 EI}{{L_e}^2}$$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times 53689327.58}{6000^2}$$

$= 1766308.079$ N

$= 1.7663$ MN

Crippling load using Rankine's formula,

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{L_e}{k}\right)^2}$$

$$= \frac{550 \times 4375\,\pi}{1 + \frac{1}{1600}\left(\frac{6000}{62.5}\right)^2}$$

$= 1118262.918$ N

$= 1.1183$ MN

Ratio of Euler's and Rankine's crippling loads

$$= \frac{\text{Euler's crippling load}}{\text{Rankine's crippling load}}$$

$$= \frac{1.7663}{1.1183}$$

$= 1.5795$

Result:

Euler's crippling load = 1.7663 MN

Ratio of Euler's and Rankine's crippling loads = 1.5795

PROBLEM 10

A hollow cast iron whose outside diameter is 200 mm has a thickness of 20 mm. It is 4.5 m long and fixed at both ends. Calculate the safe load by Rankine's formula using a factor of safety of 4. Calculate the slenderness ratio and the ratio of Euler's and Rankine's critical loads. Take σ_c = 550 N/mm^2, α = 1/1600 and E = 9.4 × 10^4 N/mm^2.

Given:

Outside diameter, D	= 200 mm
Thickness, t	= 20 mm
Length, l	= 4.5 m = 4500 mm
End conditions of column	= Both ends fixed
Crushing stress, σ_c	= 550 N/mm^2
Rankine's constant, α	= 1/1600
Young's modulus, E	= 9.4 × 10^4 N/mm^2
Factor of safety, F	= 4

To find:

Safe load by Rankine's formula

Slenderness ratio

Ratio of Euler's and Rankine's critical loads

Solution:

Inside diameter,

$d = D - 2t$

$= 200 - 2 \times 20$

$= 160$ mm

Area of cross-section,

$$A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(200^2 - 160^2)$$

$$= 3600\,\pi\ \text{mm}^2$$

Moment of inertia,

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(200^4 - 160^4)$$

$$= 46369907.57\ \text{mm}^4$$

Least radio of gyration,

$$= \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{46369907.57}{3600\,\pi}}$$

$$= 64.0312 \text{ mm}$$

Effective length for both ends fixed,

$$L_e = \frac{l}{2}$$

$$= \frac{4500}{2}$$

$$= 2250 \text{ mm}$$

Crippling load using Rankine's formula,

$$P = \frac{\sigma_c A}{1 + \alpha\left(\frac{L_e}{k}\right)^2}$$

$$= \frac{550 \times 3600\,\pi}{1 + \frac{1}{1600}\left(\frac{2250}{64.0312}\right)^2}$$

$$= 3510907.177 \text{ N}$$

$$\text{Safe load} = \frac{\text{Crippling load}}{\text{Factor of safety}}$$

$$= \frac{3.5109}{4}$$

$$= 877726.7941 \text{ N}$$

$$\text{Slenderness ratio} = \frac{\text{Actual length}}{\text{Least radius of gyration}}$$

$$= \frac{l}{k}$$

$$= \frac{4500}{64.0312}$$

$$= 70.2782$$

Crippling load using Euler's formula,

$$P = \frac{\pi^2 EI}{L_e^{\,2}}$$

$$= \frac{\pi^2 \times 9.4 \times 10^4 \times 46369907.57}{2250^2}$$

$$= 8497649.09 \text{ N}$$

Ratio of Euler's and Rankine's critical loads

$$= \frac{\text{Euler's critical load}}{\text{Rankine's critical load}}$$

$$= \frac{8497649.09}{3510907.177}$$

$$= 2.4203$$

Result:

Safe load by Rankine's formula	= 877726.7941 N
Slenderness ratio	= 70.2782
Ratio of Euler's and Rankine's critical loads	= 2.4203

TWO MARKS QUESTIONS AND ANSWERS

50. Define deflection.

The deflection of a point in a loaded beam is the vertical movement of that point measured from the beam axis before loading to elastic line after loading.

51. Define slope.

The slope of a point in a loaded beam is the angle of the tangent drawn from that deflected point that makes with the beam axis.

52. What are the units of slope and deflection?

The unit of slope is radians and deflection is millimetre.

53. Name the methods for finding slope and deflection at a section in a loaded beam?

- Double integration method
- Macaulay's method
- Moment area method
- Conjugate beam method

54. Where slope and deflection will be maximum for the cantilever with point load at its free end?

Both slope and deflection will be maximum at the free end.

55. Name the method which employs BMD for the calculation of slope and deflection.

Moment area method.

56. State Mohr's theorems.

Theorem I: The change of slope between any two points is equal to the net area of the bending moment diagram between these points divided by EI.
Theorem II: The total deflection between any two points is equal to the moment of area of bending moment diagram between the two points about the last point divided by EI.

57. Where the maximum deflection will occur in a simply supported beam loaded with uniformly distributed load? And note about the slope at that point.

The maximum deflection will occur at the centre of the beam at which the slope is zero.

58. Define column.

A vertical member of a structure, which is subjected to axial compressive load and fixed at both of its ends, is known as a column, for example a vertical pillar between the roof and floor.

59. Define buckling load or critical load or crippling load.

The load at which the column just buckles (bends) is known as buckling load or critical load or crippling load.

60. State the assumption made in the Euler's column theory.

- The column is initially perfectly straight and load is applied axially.
- The cross-section of the column is uniform throughout its length.
- The column material is perfectly elastic, homogenous and isotropic and obeys Hook's law.
- The length of the column is very large compared to its lateral dimensions.
- The direct stress is very small as compared to the bending stress.
- The column will fail by bucking alone.
- The self-weight of the column is negligible.

61. What are the end conditions for long columns?

- Both the ends of the column are hinged (or pinned).
- One end is fixed and the other end is free.
- Both the ends of the column are fixed.
- One end is fixed and the other is fixed.

62. Define effective length or equivalent length of a column.

The effective length or equivalent length of a given column with given end conditions is the length of an equivalent column of the same material and cross-section with hinged ends, and having the value of the crippling load equal to that of the given column.

63. Write the effective length for various end conditions of column.

Sl. No.	**End conditions of column**	**Relation between effective length and actual length**
1	Both ends hinged	$L_e = l$
2	One end fixed and other is free	$L_e = 2l$
3	Both ends fixed	$L_e = l/2$
4	One end fixed and other is hinged	$L_e = l/\sqrt{2}$

64. Define slenderness ratio.

The ratio of the actual length of a column to the least radius of gyration of the column is known as slenderness ratio.

$$\text{Slenderness ratio} = \frac{\text{Actual length}}{\text{Least radius of gyration}} = \frac{l}{k}$$

65. State Euler's formula for crippling load.

$$P = \frac{\pi^2 EI}{L_e^2}$$

where, P – Crippling load

E – Young's modulus

I – Moment of inertia

L_e – Effective length or equivalent length

66. What is the limitation of Euler's formula?

$$\text{Crippling stress} = \frac{\pi^2 E}{\left(\frac{L_e}{k}\right)^2}$$

For a column with both ends hinged, $L_e = 1$

$$\text{Crippling stress} = \frac{\pi^2 E}{\left(\frac{l}{k}\right)^2}$$

If the slenderness ratio (l/k) is small, the crippling stress will be high. But for the column material, the crippling stress cannot be greater than the crushing stress. Hence when the slenderness ratio is less than a certain limit, Euler's formula gives a value of crippling stress greater than the crushing stress.

67. State Rankine's formula for crippling load.

$$P = \frac{\sigma_c A}{1 + \alpha \left(\frac{L_e}{k}\right)^2}$$

where, P – Crippling load

σ_c – Ultimate crushing stress

A – Area of cross-section

α – Rankine's constant

L_e – Effective length or equivalent length

k – Least radius of gyration

14 MARKS QUESTIONS

73. A simply supported beam of length 4 m is subjected to a uniformly distributed load of 30 kN/m over the whole span and deflects 15 mm at the centre. Determine the crippling loads when this beam is used as a column with the following conditions:
    i. one end fixed and other end hinged, and
    ii. both the ends pin joined.

74. Calculate the Euler's critical load for a strut of T-section, the flange width being 10 cm, overall depth 8 cm and both flange and stem 1 cm thick. The strut is 3 m long and is build in at both ends. Take $E = 2 \times 10^5$ N/mm^2.

75. Compare the crippling loads given by Euler's and Rankine's formula for a tubular steel strut 2.3 m long having outer and inner diameters 38 mm and 33 mm respectively, loaded through pin joints at each end. Take the yield stress as 335 N/mm^2, the Rankine's constant = 1/7500 and $E = 0.205 \times 10^6$ N/mm^2. For what length of strut of this cross-section does the Euler formula cease to apply?

76. A hollow cast iron column with fixed ends supports an axial load of 1000 kN. If the column is 5 m long and has an external diameter of 250 mm, find the thickness of metal required. Use Rankine's formula, taking a constant 1/6400 and assume a working stress of 80 N/mm^2.

77. A mild steel T-section having width of web as 150 mm and depth of section as 150 mm, has uniform 10 mm thickness of web and flange. This section is used as strut, 4 m long with both ends fixed. Determine, by Rankine's formula, the safe load it can carry with a factor of safety of 3. Take $\sigma_c = 330$ N/mm^2 and Rankine's constant = 1/7500.

78. A steel column made of a 4 m long channel section, 300 mm × 100 mm, is fixed at both the ends. The thickness of flange is 11.6 mm while the thickness of web is 6.8 mm. Using Rankine's formula, calculate the load it can carry with a factor of safety of 3. Take $\sigma_c = 330$ N/mm^2 and Rankine's constant = 1/7500.

79. A short length of tube, 4 cm internal diameter and 5 cm external diameter, failed in compression at a load of 240 kN. When a 2 m length of the same tube was tested as a strut with fixed ends, the load at failure was 158 kN. Assuming that σ_c in Rankine's formula is given by the first test, find the value of the constant α in the same formula. What will be the crippling load of this tube if it is used as a strut 3 m long with one end fixed and the other hinged?

80. Find the Euler crushing load for a hollow cylindrical cast iron column 20 cm external diameter and 25 mm thick if it is 6 m long and is hinged at both ends. Take $E = 1.2 \times 10^5$ N/mm^2. Compare the load with the crushing load as given by the Rankine's formula, taking $\sigma_c = 550$ N/mm^2 and $\alpha = 1/1600$. For what length of the column would these two formulae give the same crushing load?

81. Find the Euler critical load for a hollow cylindrical cast iron column 150 mm external diameter, 20 mm wall thickness if it is 6 m long with hinged at both ends. Assume Young's modulus of cast iron as 80 kN/mm^2. Compare this load with that given by Rankine formula. Using Rankine's constant, $\alpha = 1/1600$ and $\sigma_c = 567$ N/mm^2.

82. The external and internal diameter of a hollow cast iron column is 5 cm and 4 cm respectively. If the length of this column is 3 m and both of its ends are fixed, determine the crippling load using Rankine formula. Take the values of σ_c = 550 N/mm^2 and α = 1/1600 in Rankine's formula.

83. A hollow cast iron whose outside diameter is 200 mm has a thickness of 20 mm. It is 4.5 m long and is fixed at both ends. Calculate the safe load by Rankine's formula using a factor of safety of 4. Calculate the slenderness ratio and the ratio of Euler's and Rankine's critical loads. Take crushing stress = 550 N/mm^2, α = 1/1600 and E = 9.4×10^4 N/mm^2.

84. A cantilever steel beam has a free length of 3 m. The moment of inertia of the section is 30×10^6 mm^4. A concentrated load of 50 kN is acting at the free end. Find the deflection at the free end using
    i. Double integration method and
    ii. Macaulay's method.

85. A cantilever 120 mm wide and 200 mm deep is 2.5 m long. What is the uniformly distributed load which the beam can carry in order to produce a deflection of 5 mm at the free end? Take E = 200 GN/m^2.

86. A cantilever of length 2 m carries a uniformly distributed load 2 kN/m over a length of 1 m from the free end, and a point load of 1 kN at the free end. Find the slope and deflection at the free end if E = 2.1×10^5 N/mm^2 and I = 6.667×10^7 mm^4.

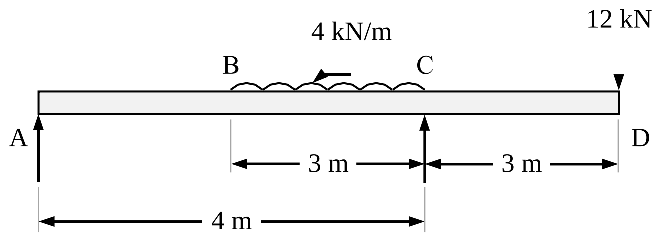
87. A cantilever of length 2 m carries a point load of 20 kN at the free end and another load of 20 kN at its centre. If E = 10^5 N/mm^2 and I = 10^8 mm^4 for the cantilever then determine by moment area method, the slope and deflection of the cantilever at the free end.

88. A beam of length 6 m is simply supported at its ends and carries two point loads of 48 kN and 40 kN at a distance of 1 m and 3 m respectively from the left support. Find:
    i. deflection under each load,
    ii. maximum deflection, and
    iii. the point at which maximum deflection occurs.

    Given E = 2×10^5 N/mm^2 and I = 85×10^6 mm^4.

89. A simply supported beam 6 m long is subjected to a 450 kN-m clockwise couple at a distance 4 meter from the left support. If the flexural rigidity (EI) of the beam is 8×10^4 kN-m^2, using any one method determine (i) the deflection at the point of application of the couple and (ii) the maximum deflection.

90. A beam ABC of length 9 m has one support of the left end and the other support at a distance of 6 m from the left end. The beam carries a point load of 1 kN at right end and also carries a uniformly distributed load of 4 kN/m over a length of 3 m as shown in figure. Determine the slope and deflection at point C. Take E = 2×10^5 N/mm^2 and I = 5×10^8 mm^4.



91. A simply supported beam of span 8 m is subjected to concentrated loads of 24 kN, 48 kN and 72 kN at 2 m, 4 m and 6 m from the left support respectively. Calculate the slope and deflection at the centre and also find maximum deflection.

92. A simply supported beam AB of span 4 m carries a point load of 100 kN at its centre. The value of I for the left half is 1×10^8 mm^4 and for the right half portion I is 2×10^8 mm^4. Find the slopes at the two supports and deflection under the load.

OBJECTIVE QUESTIONS AND ANSWERS

1. A rectangular beam of length (l) supported at its two ends carries a central point load (W). The maximum deflection occurs
   a) at the ends
   b) at l/3 from both ends
   c) at the centre
   d) none of these

2. The simply supported beam ‘A’ of length (l) carries a central point load (W). Another beam ‘B’ is loaded with a uniformly distributed load such that the total load on the beam is W. The ratio of maximum deflections between beams A and B is
   a) 5/8
   b) 8/5
   c) 5/4
   d) 4/5

3. The maximum deflection of a cantilever beam of length (l) with a point load (W) at the free end is
   a) $Wl^3 / 3EI$
   b) $Wl^3 / 8EI$
   c) $Wl^3 / 16EI$
   d) $Wl^3 / 48EI$

4. A cantilever of length (l) carries a point load (W) at the free end. The slope at the free end will be
   a) $Wl^2 / 6EI$
   b) $Wl^2 / 2EI$
   c) $Wl^2 / 24EI$
   d) $Wl^2 / 16EI$

5. The maximum deflection of a cantilever beam of length (l) with a uniformly distributed load of w per unit length is
   a) $Wl^3 / 3EI$
   b) $Wl^3 / 8EI$
   c) $Wl^3 / 16EI$
   d) $Wl^3 / 48EI$

6. A cantilever of length (l) carries a uniformly distributed load w per unit length over the whole length. The slope at the free end will be
   a) $Wl^2 / 6EI$
   b) $Wl^2 / 2EI$
   c) $Wl^2 / 24EI$
   d) $Wl^2 / 16EI$

7. A cantilever beam of length (l) carries a gradually varying load from zero at free end and w per unit length at the fixed end. The maximum deflection lies at
   a) free end
   b) fixed end
   c) mid-span
   d) none of these

8. A cantilever beam of length (l) carries a gradually varying load from zero at free end and w per unit length at the fixed end. The value of the maximum deflection is
   a) $Wl^3 / 8EI$
   b) $Wl^3 / 16EI$
   c) $Wl^4 / 30EI$
   d) $Wl^4 / 48EI$

9. A uniform simply support beam of span (l) carries a point load (W) at the centre. The downward deflection at the centre will be
   a) $Wl^3 / 8EI$
   b) $Wl^3 / 3EI$
   c) $5Wl^3 / 384EI$
   d) $Wl^3 / 48EI$

10. A uniform simply support beam of span (l) carries a point load (W) at the centre. The slope at the support will be
   a) $Wl^2 / 6EI$
   b) $Wl^2 / 2EI$
   c) $Wl^2 / 24EI$
   d) $Wl^2 / 16EI$

11. A uniformly simply supported beam of span (l) carries a uniformly distributed load w per unit length over the whole span. The downward deflection at the centre will be
   a) $Wl^3 / 8EI$
   b) $Wl^3 / 3EI$
   c) $5Wl^3 / 384EI$
   d) $Wl^3 / 48EI$

12. A simply supported beam is of rectangular section. It carries a uniformly distributed load over the whole span. The deflection at the centre is y. If the depth of the beam is doubled, the deflection at the centre would be
   a) 2y
   b) 4y
   c) y/2
   d) y/8

13. A simply supported beam carries a uniformly distributed load over the whole span. The deflection at the centre is y. If the distributed load per unit length is doubled and also depth of the beam is doubled, then the deflection at the centre would be
   a) 2y
   b) 4y
   c) y/2
   d) y/4

14. A simply supported beam ‘A’ of length l, breadth b and depth d carries a central load W. Another beam ‘B’ of the same dimensions carries a central load equal to 2W. The deflection of beam ‘B’ will be________as that of beam ‘A’.
    a) one-fourth
    b) one-half
    c) double
    d) four times

15. A simply supported beam ‘A’ of length l, breadth b, and depth d carries a central point load W. Another beam ‘B’ has the same length and depth but its breadth is doubled. The deflection of beam ‘B’ will be_____________as compared to beam ‘A’.
    a) one-fourth
    b) one-half
    c) double
    d) four times

16. The maximum deflection of a fixed beam carrying a central point load lies at
    a) fixed ends
    b) centre of beam
    c) l/3 from fixed ends
    d) none of these

17. The maximum deflection of a fixed beam of length (l) carrying a central point load (W) is
    a) $Wl^3 / 48EI$
    b) $Wl^3 / 96EI$
    c) $Wl^3 / 192EI$
    d) $Wl^3 / 384EI$

18. The maximum deflection of a fixed beam of length (l) carrying a total load W uniformly distributed over the whole length is
    a) $Wl^3 / 48EI$
    b) $Wl^3 / 96EI$
    c) $Wl^3 / 192EI$
    d) $Wl^3 / 384EI$

19. A fixed beam is a beam whose end supports are such that the end slopes
    a) are maximum
    b) are minimum
    c) are zero
    d) none of the above

20. A fixed beam of length (l) carries a point load (W) at the centre. The deflection at the centre is
    a) same as for a simply supported beam
    b) half of the deflection for a simply supported beam
    c) one-fourth of the deflection for a simply supported beam
    d) double the deflection for a simply supported beam

21. The product of Young's modulus (E) and moment of inertia (I) is known as
    a) modulus of rigidity
    b) bulk modulus
    c) flexural rigidity
    d) torsional rigidity

22. For the same loading, the maximum deflection for a fixed beam as compared to simply supported beam is
    a) more
    b) same
    c) less
    d) none of these

23. Column is defined as a
    a) member of a structure which carries a tensile load
    b) member of a structure which carries an axial compressive load
    c) vertical member of a structure which carries a tensile load
    d) vertical member of a structure which carries an axial compressive load

24. The load at which the column just buckles, is known as
    a) buckling load
    b) critical load
    c) crippling load
    d) any one of these

25. Buckling factor is defined as the ratio of
    a) equivalent length of a column to the minimum radius of gyration
    b) length of the column to the minimum radius of gyration
    c) length of the column to the area of cross-section of the column
    d) none of the above.

26. A loaded column is having the tendency to deflect. On account of this tendency, the critical load
    a) decreases with the decrease in length
    b) decreases with the increase in length
    c) first decreases then increases with the decrease in length
    d) first increases then decreases with the decrease in length

27. A loaded column fails due to
    a) stress due to direct load
    b) stress due to bending
    c) stress due to direct load and bending
    d) none of these

28. For long columns, the value of buckling load is________crushing load.
    a) equal to
    b) less than
    c) more than
    d) either equal to or more than

29. Compression members always tend to buckle in the direction of the
    a) axis of load
    b) perpendicular to the axis of load
    c) minimum cross section
    d) least radius of gyration

30. The direct stress induced in a long column is________as compared to bending stress.
    a) same
    b) more
    c) less
    d) negligible

31. A column that fails due to direct stress, is called
    a) short column
    b) long column
    c) weak column
    d) medium column

32. The assumption made in Euler's column theory is that
    a) the failure of column occurs due to buckling alone
    b) the length of column is very large as compared to its cross-sectional dimensions
    c) the column material obeys Hooke's law
    d) all of these

33. According to Euler's column theory, the crippling load for a column length (l) hinged at both ends, is
    a) $\pi^2 EI / l^2$
    b) $\pi^2 EI / 4l^2$
    c) $4\pi^2 EI / l^2$
    d) $2\pi^2 EI / l^2$

34. According to Euler's column theory, the crippling load for a column of length (l) fixed at both ends is________the crippling load for a similar column hinged at both ends.
    a) equal to
    b) two times
    c) four times
    d) eight times

35. According to Euler's column theory, the crippling load for a column of length (l) with one end fixed and the other end free is_______the crippling load for a similar column hinged at both the ends.
    a) equal to
    b) less than
    c) more than
    d) two times

36. According to Euler's column theory, the crippling load for a column of length (l) with one end fixed and the other end hinged, is
    a) $\pi^2 EI / l^2$
    b) $\pi^2 EI / 4l^2$
    c) $2\pi^2 EI / l^2$
    d) $4\pi^2 EI / l^2$

37. According to Euler's column theory, the crippling load of a column is given by $p = \pi^2 EI / Cl^2$. In this equation, the value of C for a column with both ends hinged, is
    a) 1/4
    b) 1/2
    c) 1
    d) 2

38. In the Euler's formula, the value of C for a column with one end fixed and the other end free, is
    a) 1/2
    b) 1
    c) 2
    d) **4**

39. A column of length (l) with both ends fixed may be considered as equivalent to a column of length________with both ends hinged.
    a) l/8
    b) l/4
    c) l/2
    d) l

40. A column of length (l) with both ends fixed may be considered as equivalent to a column of length________with one end fixed and the other end free.
    a) l/8
    b) l/4
    c) 1/2
    d) l

41. The buckling load for a given column depends upon
    a) area of cross-section of the column
    b) length and least radius of gyration of the column
    c) modulus of elasticity for the material of the column
    d) all of these

42. The relation between equivalent length (L) and actual length (l) of a column for both ends fixed is
    a) L = l/2
    b) $L = l/\sqrt{2}$
    c) L = l
    d) L = 2l

43. The relation between equivalent length (L) and actual length (l) of a column for one end fixed and the other end hinged is
   a) L = l/2
   b) L = l/√2
   c) L = l
   d) L = 4l

44. A vertical column has two moments of 1nert1a (i.e. I_{xx} and I_{yy}). The column will tend to buckle in the direction of the
   a) axis of load
   b) perpendicular to the axis of load
   c) maximum moment of inertia
   d) minimum moment of inertia

45. The slenderness ratio is the ratio of
   a) area of column to least radius of gyration
   b) length of column to least radius of gyration
   c) least radius of gyration to area of column
   d) least radius of gyration to length of column

46. The columns whose slenderness ratio is less than 80, are known as
   a) short columns
   b) long columns
   c) weak columns
   d) medium columns

47. A column with maximum equivalent length has
   a) both ends hinged
   b) both ends fixed
   c) one end fixed and the other end hinged
   d) one end fixed and the other end free

48. Euler's formula holds good only for
   a) short columns
   b) long columns
   c) both short and long columns
   d) medium columns

49. A column is said to be a short column, when
   a) its length is very small
   b) its cross-sectional area is small
   c) the ratio of its length to the least radius of gyration is less than 80
   d) the ratio of its length to the least radius of gyration is more than 80

50. If the slenderness ratio for a column is 100, then it is said to be a________column.
   a) long
   b) medium
   c) short
   d) weak

51. Rankine's formula is an empirical formula which is used for
    a) long columns
    b) short columns
    c) both long and short columns
    d) medium columns

52. The Rankine's formula for columns is
    a) $\sigma_c\alpha$ / [1 + A(L/k)^2]
    b) σ_cA / [1 + α (L/k)^2]
    c) $\sigma_c\alpha$ / [1 – A(L/k)^2]
    d) σ_cA / [1 – α (L/k)^2]

53. The Rankine's constant (α) in Rankine's formula is equal to
    a) π^2E / σ_c
    b) π^2 / Eσ_c
    c) Eσ_c / π^2
    d) σ_c / π^2E

54. The Rankine s constant (α) for a given material of a column depends upon the
    a) length of column
    b) diameter of the column
    c) length and diameter
    d) none of these

55. The Rankine's constant for a mild steel column with both ends hinged is
    a) 1/750
    b) 1/1600
    c) 1/7500
    d) 1/9000

56. The Rankine's formula holds good for
    a) short columns
    b) long columns
    c) both short and long columns
    d) weak columns

57. If the diameter of a long column is reduced by 20%, the percentage of reduction in Euler's buckling load is
    a) 4
    b) 36
    c) 49
    d) 59

58. With one fixed end and other free end, a column of length L buckles at load P_1. Another column of same length and same cross-section fixed at both ends buckles at load P_2. Then P_2/P_1 is
    a) 1
    b) 2
    c) 4
    d) 16

59. The buckling load will be maximum for a column, if
    a) one end of the column is clamped and other end is free
    b) both ends of the column are clamped
    c) both ends of the column are hinged
    d) one end of the column is hinged and other end is free

60. Choose the correct statement.
    a) Euler’s formula holds good only for short columns.
    b) A short column is one which has the ratio of its length to least radius of gyration more than 100.
    c) A column with both ends fixed has minimum equivalent (or effective) length.
    d) The equivalent length of a column with one end fixed and other end hinged is half of its actual length.

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UNIT-V ANALYSIS OF STRESSES IN TWO DIMENSIONS

SOLVED PROBLEMS

PROBLEM 1

A closed cylindrical drum 600 mm in diameter and 2 m long has a shell thickness of 12 mm. If it carries a fluid under a pressure of 3 N/mm^2, calculate the longitudinal and hoop stress in the drum wall and also determine the change in diameter, change in length and change in volume of the drum. Take E = 200 GPa and Poisson's ratio = 0.3.

Given:

Diameter of cylindrical drum, d = 600 mm

Length of cylindrical drum, L = 2 m = 2000 mm

Thickness of cylindrical drum, t = 12 mm

Fluid pressure, p = 3 N/mm^2

Young's modulus, E = 200 GPa = 2×10^5 N/mm^2

Poisson's ratio, μ = 0.3

To find:

Longitudinal stress, σ_l

Hoop stress, σ_c

Change in diameter, δd

Change in length, δL

Change in volume, δV

Solution:

Longitudinal stress,

$$\sigma_l = \frac{pd}{4t}$$

$$= \frac{3 \times 600}{4 \times 12}$$

$$= 37.5\ \mathrm{N/mm^2}$$

Hoop stress,

$$\sigma_c = \frac{pd}{2t}$$

$$= \frac{3 \times 600}{2 \times 12}$$

$$= 75\ \mathrm{N/mm^2}$$

Change in diameter,

$$\delta d = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right) \times d$$

$$= \frac{3 \times 600}{2 \times 12 \times 2 \times 10^5}\left(1 - \frac{0.3}{2}\right) \times 600$$

$$= 0.1913 \text{ mm}$$

Change in length,

$$\delta L = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right) \times L$$

$$= \frac{3 \times 600}{2 \times 12 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right) \times 2000$$

$$= 0.15 \text{ mm}$$

Change in volume,

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times V$$

$$= \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times \left(\frac{\pi}{4} \times d^2 \times L\right)$$

$$= \frac{3 \times 600}{2 \times 12 \times 2 \times 10^5}\left(\frac{5}{2} - 2 \times 0.3\right) \times \left(\frac{\pi}{4} \times 600^2 \times 2000\right)$$

$$= 402909.2578 \text{ mm}^3$$

Result:

Longitudinal stress, σ_l = 37.5 N/mm^2

Hoop stress, σ_c = 75 N/mm^2

Change in diameter, δd = 0.1913 mm

Change in length, δL = 0.15 mm

Change in volume, δV = 402909.2578 mm^3

PROBLEM 2

A thin cylindrical shell 3 m long, 1.2 m in diameter is subjected to an internal pressure of 1.67 N/mm². If the thickness of the shell is 13 mm, find the circumferential and longitudinal stresses. Also find the maximum shear stress and change in dimensions of the shell. Take E = 2×10^5 N/mm² and Poisson's ratio = 0.28.

Given:

Length of cylindrical shell, L = 3 m = 3000 mm

Diameter of cylindrical shell, d = 1.2 m = 1200 mm

Internal pressure, p = 1.67 N/mm²

Thickness of cylindrical shell, t = 13 mm

Young's modulus, E = 2×10^5 N/mm²

Poisson's ratio, μ = 0.28

To find:

Circumferential stress, σ_c

Longitudinal stress, σ_l

Maximum shear stress, τ_{max}

Change in diameter, δd

Change in length, δL

Change in volume, δV

Solution:

Circumferential stress,

$$\sigma_c = \frac{pd}{2t}$$

$$= \frac{1.67 \times 1200}{2 \times 13}$$

$$= 77.0769 \text{ N/mm}^2$$

Longitudinal stress,

$$\sigma_l = \frac{pd}{4t}$$

$$= \frac{1.67 \times 1200}{4 \times 13}$$

$$= 38.5385 \text{ N/mm}^2$$

Maximum shear stress,

$$\tau_{max} = \frac{\sigma_c - \sigma_l}{2}$$

$$= \frac{77.0769 - 38.5385}{2}$$

$$= 19.2692 \text{ N/mm}^2$$

Change in diameter,

$$\delta d = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right) \times d$$

$$= \frac{1.67 \times 1200}{2 \times 13 \times 2 \times 10^5}\left(1 - \frac{0.28}{2}\right) \times 1200$$

$$= 0.3977 \text{ mm}$$

Change in length,

$$\delta L = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right) \times L$$

$$= \frac{1.67 \times 1200}{2 \times 13 \times 2 \times 10^5}\left(\frac{1}{2} - 0.28\right) \times 3000$$

$$= 0.2544 \text{ mm}$$

Change in volume,

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times V$$

$$= \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times \left(\frac{\pi}{4} \times d^2 \times L\right)$$

$$= \frac{1.67 \times 1200}{2 \times 13 \times 2 \times 10^5}\left(\frac{5}{2} - 2 \times 0.28\right) \times \left(\frac{\pi}{4} \times 1200^2 \times 3000\right)$$

$$= 2536703.638 \text{ mm}^3$$

Result:

Circumferential stress, σ_c = 77.0769 N/mm^2

Longitudinal stress, σ_l = 38.5385 N/mm^2

Maximum shear stress, τ_{max} = 19.2692 N/mm^2

Change in diameter, δd = 0.3977 mm

Change in length, δL = 0.2544 mm

Change in volume, δV = 2536703.638 mm^3

PROBLEM 3

A cylindrical shell 1 m diameter and 3 m length is subjected to an internal pressure of 2 MPa. Calculate the minimum thickness if the stress should not exceed 50 MPa. Find the change in diameter and volume of the shell. Poisson's ratio = 0.3 and E = 200 kN/mm^2.

Given:

Diameter of cylindrical shell, d = 1 m = 1000 mm

Length of cylindrical shell, L = 3 m = 3000 mm

Internal pressure, p = 2 MPa = 2 N/mm^2

Circumferential stress, σ_c = 50 MPa = 50 N/mm^2 [Maximum stress]

Poisson's ratio, μ = 0.3

Young's modulus, E = 200 kN/mm^2 = 200 × 10^3 N/mm^2

To find:

Thickness of cylindrical shell, t

Change in diameter, δd

Change in volume, δV

Solution:

Circumferential stress,

$$\sigma_c = \frac{pd}{2t}$$

$$t = \frac{pd}{2\sigma_c}$$

$$= \frac{2 \times 1000}{2 \times 50}$$

$$= 20 \text{ mm}$$

Change in diameter,

$$\delta d = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right) \times d$$

$$= \frac{2 \times 1000}{2 \times 20 \times 200 \times 10^3}\left(1 - \frac{0.3}{2}\right) \times 1000$$

$$= 0.2125 \text{ mm}$$

Change in volume,

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times V$$

$$= \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times \left(\frac{\pi}{4} \times d^2 \times L\right)$$

$$= \frac{2 \times 1000}{2 \times 20 \times 200 \times 10^3}\left(\frac{5}{2} - 2 \times 0.3\right) \times \frac{\pi}{4} \quad {}^2$$

$\,_4 \times 1000$ $\times 3000$

$= 1119192.383 \text{ mm}^3$

Result:

Thickness of cylindrical shell, t = 20 mm

Change in diameter, δd = 0.2125 mm

Change in volume, δV = 1119192.383 mm^3

PROBLEM 4

A cylinder has an internal diameter of 230 mm, wall thickness 5 mm and is 1 m long. It is found to change in internal volume by 12×10^{-6} m^3 when filled with a liquid at a pressure 'p'. Taking E = 200 GPa and Poisson's ratio = 0.25, determine the stresses in the cylinder, the changes in its length and internal diameter.

Given:

Diameter of cylindrical, d	= 230 mm
Thickness of cylinder, t	= 5 mm
Length of cylindrical, L	= 1 m = 1000 mm
Change in volume, δV	$= 12 \times 10^{-6}$ m^3 $= 12 \times 10^3$ mm^3
Young's modulus, E	$= 200$ GPa $= 200 \times 10^3$ N/mm^2
Poisson's ratio, μ	= 0.25

To find:

Circumferential stress, σ_c

Longitudinal stress, σ_l

Change in length, δL

Change in diameter, δd

Solution:

Change in volume,

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times V$$

$$= \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) \times \left(\frac{\pi}{4} \times d^2 \times L\right)$$

$$p = \frac{\delta V}{\frac{d}{2tE}\left(\frac{5}{2} - 2\mu\right) \times \left(\frac{\pi}{4} \times d^2 \times L\right)}$$

$$= \frac{12 \times 10^3}{\frac{230}{2 \times 5 \times 200 \times 10^3}\left(\frac{5}{2} - 2 \times 0.25\right) \times \left(\frac{\pi}{4} \times 230^2 \times 1000\right)}$$

$$= 1.2558 \text{ N/mm}^2$$

Circumferential stress,

$$\sigma_c = \frac{pd}{2t}$$

$$= \frac{1.2558 \times 230}{2 \times 5}$$

$$= 28.8826 \text{ N/mm}^2$$

Longitudinal stress,

$$\sigma_l = \frac{pd}{4t}$$

$$= \frac{1.2558 \times 230}{4 \times 5}$$

$$= 14.4413\ \text{N/mm}^2$$

Change in length,

$$\delta L = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right) \times L$$

$$= \frac{1.2558 \times 230}{2 \times 5 \times 200 \times 10^3}\left(\frac{1}{2} - 0.25\right) \times 1000$$

$$= 0.0361\ \text{mm}$$

Change in diameter,

$$\delta d = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right) \times d$$

$$= \frac{1.2558 \times 230}{2 \times 5 \times 200 \times 10^3}\left(1 - \frac{0.25}{2}\right) \times 230$$

$$= 0.0291\ \text{mm}$$

Result:

Circumferential stress, σ_c	= 28.8826 N/mm^2
Longitudinal stress, σ_l	= 14.4413 N/mm^2
Change in length, δL	= 0.0361 mm
Change in diameter, δd	= 0.0291 mm

PROBLEM 5

A vessel in the shape of a spherical shell of 1.2 m internal diameter and 10 mm thickness is filled with a fluid at atmospheric pressure. If additional 500 cc of fluid is pumped into the shell at atmospheric pressure, find the internal pressure exerted on the wall of the shell. Find also the resulting change in volume of sphere. Take $E = 2.1 \times 10^5$ N/mm^2, Poisson's ratio = 0.28 and K for fluid as 2.4 GPa.

Given:

Diameter of spherical shell, d	= 1.2 m = 1200 mm
Thickness of spherical shell, t	= 10 mm
Additional fluid pumped	= 500 cc = 500×10^3 mm^3
Young's modulus, E	= 2.1×10^5 N/mm^2
Poisson's ratio, μ	= 0.28
Bulk modulus for fluid, K	= 2.4 GPa = 2.4×10^3 N/mm^2

To find:

Pressure exerted, p

Change in volume, δV

Solution:

Volume of sphere,

$$V = \frac{4\pi}{3} r^3$$

$$= \frac{4\pi}{3}\left(\frac{1200}{2}\right)^3 \qquad \left(r = \frac{d}{2}\right)$$

$$= 904778684.2 \text{ mm}^3$$

Change in volume of the shell,

$$\delta V = \frac{3pd}{4tE}(1-\mu) \times V$$

$$= \frac{3p \times 1200}{4 \times 10 \times 2.1 \times 10^5}(1-0.28) \times 904778684.2$$

$$= 279188.8511 \text{ p mm}^3 \qquad \ldots (1)$$

Change in volume of fluid

$$= \frac{p}{K} \times V$$

$$= \frac{p}{2.4 \times 10^3} \times 904778684.2$$

$$= 376991.1184 \text{ p mm}^3$$

The total space created in the shell = Change in volume of the shell + Change in volume of fluid

$$500 \times 10^3 = 279188.8511 \text{ p} + 376991.1184 \text{ p}$$

$$p = \frac{500 \times 10^3}{279188.8511 + 376991.1184}$$

$$= 0.7620 \text{ N/mm}^2$$

Substituting value of p in equation (1),

$$\delta V = 279188.8511 \times 0.7620$$

$$= 212738.0171 \text{ mm}^3$$

To find:

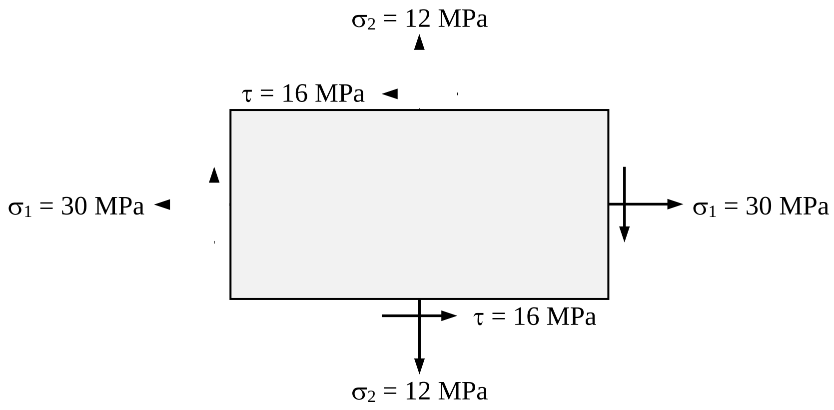
Pressure exerted, p = 0.7620 N/mm^2

Change in volume, δV = 212738.0171 mm^3

PROBLEM 6

At a point in a strained body subjected to two mutually perpendicular normal tensile stresses of magnitude 30 MPa and 12 MPa accompanied by a shear stress of 16 MPa. Locate the principal planes and evaluate the principal stresses. Also calculate the maximum intensity of shear stress and specify its planes.

Given:



Major tensile stress, σ_1 = 30 MPa = 30 N/mm^2

Minor tensile stress, σ_2 = 12 MPa = 12 N/mm^2

Shear stress, τ = 16 MPa = 16 N/mm^2

To find:

Location of principal planes

Principal stresses

Maximum shear stress

Location of maximum shear stress planes

Solution:

The planes on which principal stresses act is,

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$= \frac{2 \times 16}{30 - 12}$$

$$2\theta = \tan^{-1}\left(\frac{2 \times 16}{30 - 12}\right)$$

$$= 60.6422°$$

$$\theta = \frac{60.6422°}{2}$$

$$= 30.3211° \text{ or } 30.3211° + 90°$$

$$= 30.3211° \text{ or } 120.3211°$$

Major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{30 + 12}{2} + \sqrt{\left(\frac{30 - 12}{2}\right)^2 + 16^2}$$

$= 39.3576\ \mathrm{N/mm^2}$

Minor principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{30 + 12}{2} - \sqrt{\left(\frac{30 - 12}{2}\right)^2 + 16^2}$$

$= 2.6424\ \mathrm{N/mm^2}$

Maximum shear stress

$$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{30 - 12}{2}\right)^2 + 16^2}$$

$= 18.3576\ \mathrm{N/mm^2}$

The planes on which maximum shear stress act is,

$$\tan 2\theta = \frac{\sigma_1 - \sigma_2}{2\tau}$$

$$= \frac{30 - 12}{2 \times 16}$$

$$2\theta = \tan^{-1}\left(\frac{30 - 12}{2 \times 16}\right)$$

$$= 29.3578°$$

$$\theta = \frac{29.3578°}{2}$$

$$= 14.6789° \text{ or } 14.6789° + 90°$$

$$= 14.6789° \text{ or } 104.6789°$$

Result:

Location of principal planes	= 30.3211° or 120.3211°
Major principal stress	= 39.3576 N/mm^2

Minor principal stress = 2.6424 N/mm^2

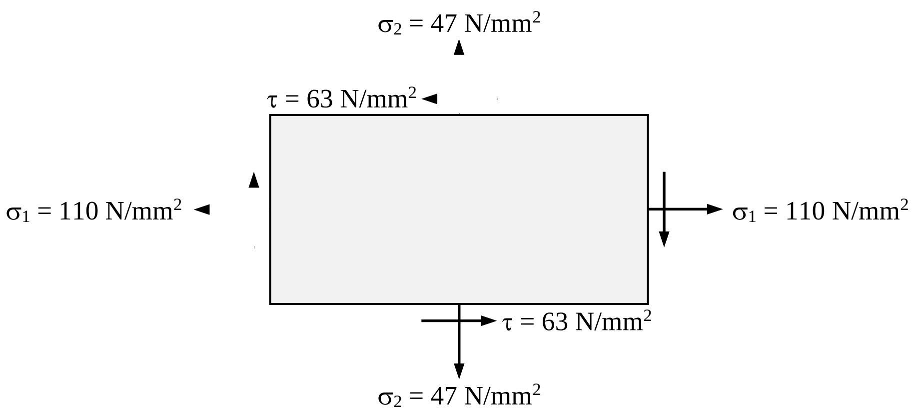
Maximum shear stress = 18.3576 N/mm^2

Location of maximum shear stress planes = 14.6789° or 104.6789°

PROBLEM 7

A rectangular block of material is subjected to a tensile stress of 110 N/mm^2 on one plane and a tensile of 47 N/mm^2 on a plane at right angles to the former. Each of the above stresses is accompanies by a shear stress of 63 N/mm^2. Determine the principal stresses, principal planes and the maximum shear stress.

Given:



Major tensile stress, $\sigma_1 = 110\ N/mm^2$

Minor tensile stress, $\sigma_2 = 47\ N/mm^2$

Shear stress, τ = 63 N/mm^2

To find:

Principal stresses

Location of principal planes

Maximum shear stress

Solution:

Major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{110 + 47}{2} + \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$

$$= 148.9361\ N/mm^2$$

Minor principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{110 + 47}{2} - \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$

$= 8.0639\ \text{N/mm}^2$

The planes on which principal stresses act is,

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$= \frac{2 \times 63}{110 - 47}$$

$$2\theta = \tan^{-1}\left(\frac{2 \times 63}{110 - 47}\right)$$

$$= 63.4349°$$

$$\theta = \frac{63.4349°}{2}$$

$$= 31.7174° \text{ or } 31.7174° + 90°$$

$$= 31.7174° \text{ or } 121.7174°$$

Maximum shear stress

$$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$

$= 70.4361\ \text{N/mm}^2$

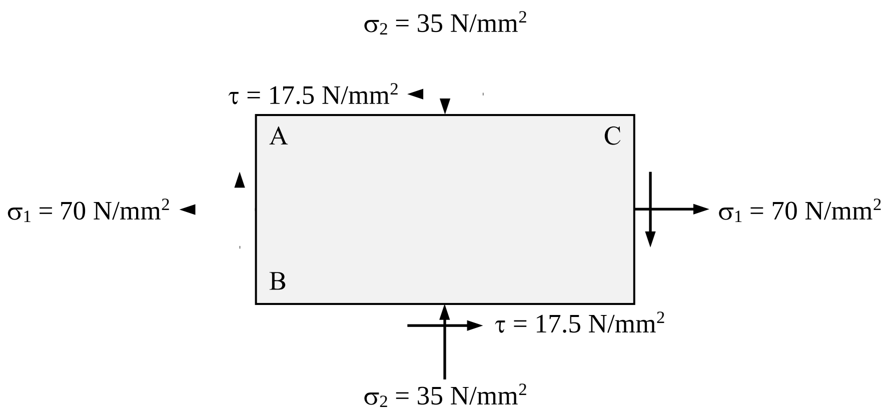
Result:

Major principal stress	= 148.9361 N/mm^2
Minor principal stress	= 8.0639 N/mm^2
Location of principal planes	= 31.7174° or 121.7174°
Maximum shear stress	= 70.4361 N/mm^2

PROBLEM 8

Two planes AB and AC which are right angles carry shear stress of intensity 17.5 N/mm^2 while these planes also carry a tensile stress of 70 N/mm^2 and a compressive stress of 35 N/mm^2 respectively. Determine the principal planes and the principal stresses. Also determine the maximum shear stress and planes on which it acts.

Given:



Shear stress, τ = 17.5 N/mm^2

Major tensile stress, σ_1 = 70 N/mm^2

Minor compressive stress, σ_2 = – 35 N/mm^2 [Negative sign due to compressive stress.]

To find:

Location of principal planes

Principal stresses

Maximum shear stress

Location of maximum shear stress planes

Solution:

The planes on which principal stresses act is,

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$$

$$= \frac{2 \times 17.5}{70 - (-35)}$$

$$2\theta = \tan^{-1}\left(\frac{2 \times 17.5}{70 - (-35)}\right)$$

$$= 18.4349°$$

$$\theta = \frac{18.4349°}{2}$$

$$= 9.2175° \text{ or } 9.2175° + 90°$$

$$= 9.2175° \text{ or } 99.2175°$$

Major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{70 + (-35)}{2} + \sqrt{\left(\frac{70 - (-35)}{2}\right)^2 + 17.5^2}$$

$= 72.8399\ \text{N/mm}^2$

Minor principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{70 + (-35)}{2} - \sqrt{\left(\frac{70 - (-35)}{2}\right)^2 + 17.5^2}$$

$= -\ 37.8399\ \text{N/mm}^2$

Maximum shear stress

$$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{70 - (-35)}{2}\right)^2 + 17.5^2}$$

$= 55.3399\ \text{N/mm}^2$

The planes on which maximum shear stress act is,

$$\tan 2\theta = \frac{\sigma_1 - \sigma_2}{2\tau}$$

$$= \frac{70 - (-35)}{2 \times 17.5}$$

$$2\theta = \tan^{-1}\left(\frac{70 - (-35)}{2 \times 17.5}\right)$$

$= 71.5651°$

$$\theta = \frac{71.5651°}{2}$$

$= 35.7825°$ or $35.7825° + 90°$

$= 35.7825°$ or $125.7825°$

Result:

Location of principal planes = 9.2175° or 99.2175°

Major principal stress = $72.8399\ N/mm^2$

Minor principal stress = $-37.8399\ N/mm^2$

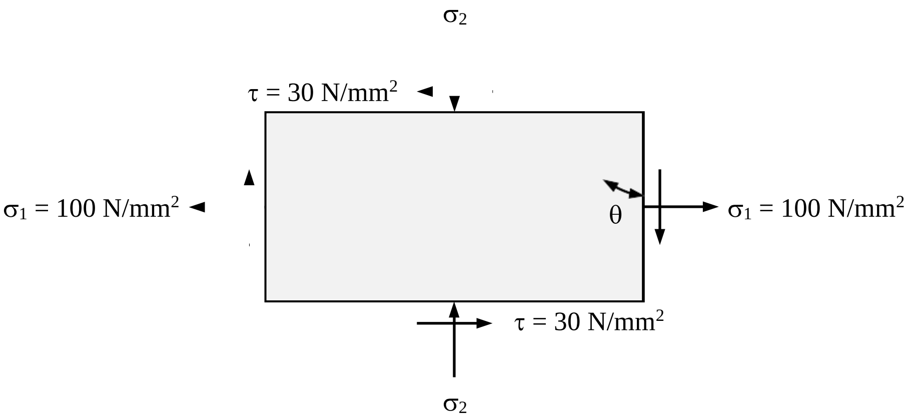
Maximum shear stress = 55.3399 N/mm^2

Location of maximum shear stress planes = 35.7825° or 125.7825°

PROBLEM 9

At a point in a strained material, there is a horizontal tensile stress of 100 N/mm^2 and an unknown vertical stress. There is also a shear stress of 30 N/mm^2 on these planes. On a plane inclined at 30° to the vertical, the normal stress is found to be 90 N/mm^2 tensile. Find the unknown vertical stress and also the principle stresses and maximum shear stress.

Given:



Horizontal tensile stress, σ_1 = 100 N/mm^2

Shear stress, τ = 30 N/mm^2

Angle of plane inclined to the vertical, θ = 30°

Normal stress, σ_n = 90 N/mm^2

To find:

Vertical stress, σ_2

Principal stresses

Maximum shear stress

Solution:

Normal stress,

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_n = \frac{\sigma_1}{2} + \frac{\sigma_2}{2} + \frac{\sigma_1}{2}\cos 2\theta - \frac{\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\sigma_n = \frac{\sigma_1}{2}(1 + \cos 2\theta) + \frac{\sigma_2}{2}(1 - \cos 2\theta) + \tau \sin 2\theta$$

$$\frac{\sigma_2}{2}(1 - \cos 2\theta) = \sigma_n - \frac{\sigma_1}{2}(1 + \cos 2\theta) - \tau \sin 2\theta$$

$$\sigma_2 = \frac{\sigma_n - \frac{\sigma_1}{2}(1 + \cos 2\theta) - \tau \sin 2\theta}{\frac{(1 - \cos 2\theta)}{2}}$$

$$\sigma_2 = \frac{90 - \frac{100}{2}\left(1 + \cos\left(2 \times 30^\circ\right)\right) - 30\sin\left(2 \times 30\right)}{\frac{\left(1 - \cos\left(2 \times 30^\circ\right)\right)}{2}}$$

$\sigma_2 = -\,43.9230\ \mathrm{N/mm^2}$

Major principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{100 + (-43.9230)}{2} + \sqrt{\left(\frac{100 - (-43.9230)}{2}\right)^2 + 30^2}$$

$= 106.0030\ \mathrm{N/mm^2}$

Minor principal stress

$$= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{100 + (-43.9230)}{2} - \sqrt{\left(\frac{100 - (-43.9230)}{2}\right)^2 + 30^2}$$

$= -\,49.9260\ \mathrm{N/mm^2}$

Maximum shear stress

$$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{70 - (-35)}{2}\right)^2 + 17.5^2}$$

$= 77.9645\ \mathrm{N/mm^2}$

To find:

Vertical stress, σ_2 = – 43.9230 $\mathrm{N/mm^2}$

Major principal stress = 106.0030 $\mathrm{N/mm^2}$

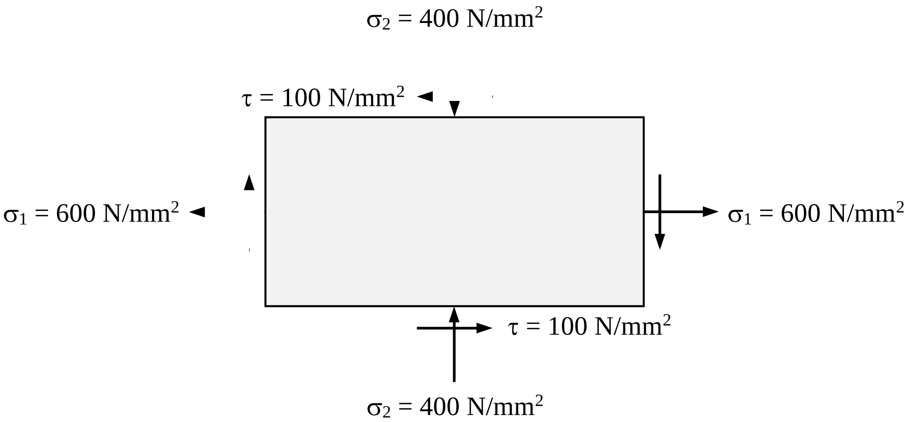
Minor principal stress = – 49.9260 $\mathrm{N/mm^2}$

Maximum shear stress = 77.9645 $\mathrm{N/mm^2}$

PROBLEM 10

A point in a strained material is subjected to mutually perpendicular stresses of 600 N/mm^2 (tensile) and 400 N/mm^2 (compressive). It is also subjected to a shear stress of 100 N/mm^2. Draw Mohr's circle and find the principal stress and maximum shear stress.

Given:



Major tensile stress, σ_1 = 600 N/mm^2

Minor compressive stress, σ_2 = – 400 N/mm^2 [Negative sign due to compressive stress.]

Shear stress, τ = 100 N/mm^2

To find:

From Mohr's circle,

Principal stresses

Maximum shear stress

Solution:

Scale

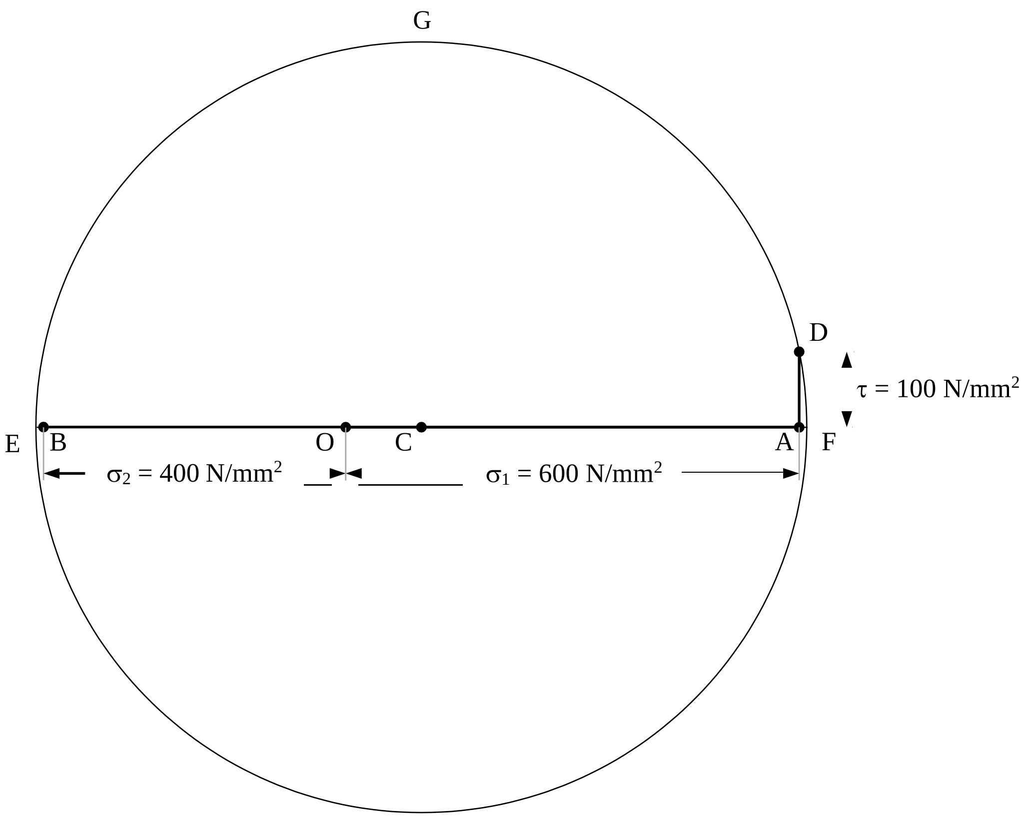
Take 1 mm = 10 N/mm^2

$$\sigma_1 = \frac{600}{10} = 60 \text{ mm}$$

$$\sigma_1 = \frac{-400}{10} = -40 \text{ mm}$$

$$\tau = \frac{100}{10} = 10 \text{ mm}$$

Take any point O as origin. Take OA = 60 mm towards right side of O and OB = – 40 mm towards left side of O. Bisect AB at C. Draw perpendicular line AD = 10 mm. Now with C as centre and radius equal to CD draw a circle. Then OF and OE represents the major and minor principal stresses respectively. And CG represents the maximum shear stress.



By measurement,

OF = 61 mm

OE = – 41 mm

CG = 51 mm

Major principal stress = OF × Scale

= 61 × 10

= 610 N/mm^2

Minor principal stress = OE × Scale

= – 41 × 10

= – 410 N/mm^2

Maximum shear stress = CG × Scale

= 51 × 10

= 510 N/mm^2

Result:

Major principal stress = 610 N/mm^2

Minor principal stress = – 410 N/mm^2

Maximum shear stress = 510 N/mm^2

TWO MARKS QUESTIONS AND ANSWERS

68. When will you call a cylinder as thin cylinder?

If the thickness of the wall of the cylindrical vessel is less than 1/20 of its internal diameter, the cylindrical vessel is known as thin cylinder.

$$\frac{t}{d} < \frac{1}{20}$$

69. In thin cylinder will the radial stress vary over the thickness of wall?

No, in case of thin cylinders, the stress distribution is assumed uniform over the thickness of wall.

70. Name the stresses set up in the material of a thin cylinder.

- Circumferential or hoop stress
- Longitudinal stress

71. What is the ratio of circumferential stress and longitudinal stress?

The circumferential stress is twice the longitudinal stress.

72. Write the expression for hoop stress in thin cylinder due to internal fluid pressure.

$$\sigma_c = \frac{pd}{2t}$$

where, σ_c – circumferential or hoop stress
p – internal fluid pressure
d – internal diameter of the thin cylinder
t – thickness of the wall of the thin cylinder

73. Write the expression for longitudinal stress in thin cylinder due to internal fluid pressure.

$$\sigma_l = \frac{pd}{4t}$$

where, σ_l – longitudinal stress
p – internal fluid pressure

d – internal diameter of the thin cylinder
t – thickness of the wall of the thin cylinder

74. Write the expression for circumferential strain in thin cylinder due to internal fluid pressure.

$$e_c = \frac{\delta d}{d} = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

where, e_c – circumferential strain

δd – change in internal diameter of the thin cylinder
d – internal diameter of the thin cylinder
p – internal fluid pressure
t – thickness of the wall of the thin cylinder
E – Young's modulus
μ – Poisson's ratio

75. Write the expression for longitudinal strain in thin cylinder due to internal fluid pressure.

$$e_l = \frac{\delta l}{l} = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

where, e_l – longitudinal strain

δ1 – change in length of the thin cylinder
l – length of the thin cylinder
p – internal fluid pressure
d – internal diameter of the thin cylinder
t – thickness of the wall of the thin cylinder
E – Young's modulus
μ – Poisson's ratio

76. Write the expression for volumetric strain in thin cylinder due to internal fluid pressure.

$$e_v = \frac{\delta V}{V} = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

where, e_v – longitudinal strain

δV – change in volume of the thin cylinder
V – volume of the thin cylinder
p – internal fluid pressure
d – internal diameter of the thin cylinder
t – thickness of the wall of the thin cylinder
E – Young's modulus
μ – Poisson's ratio

77. Write the expression for maximum shear stress in thin cylinder due to internal fluid pressure.

$$\tau_{max} = \frac{\sigma_c - \sigma_l}{2}$$

where, σ_c – circumferential or hoop stress
σ_l – longitudinal stress

78. Write the expression for hoop stress in thin spherical shell due to internal fluid pressure.

$$\sigma_c = \frac{pd}{4t}$$

where, σ_c – circumferential or hoop stress
p – internal fluid pressure
d – internal diameter of the thin cylinder
t – thickness of the wall of the thin cylinder

79. Write the expression for circumferential strain in thin spherical shell due to internal fluid pressure.

$$e_c = \frac{pd}{4tE}(1 - \mu)$$

where, e_c – circumferential strain
p – internal fluid pressure

d – internal diameter of the thin cylinder
t – thickness of the wall of the thin cylinder

E – Young’s modulus

μ – Poisson's ratio

80. Write the expression for volumetric strain in thin spherical shell due to internal fluid pressure.

$$e_v = \frac{3pd}{4tE}(1 - \mu)$$

where, e_v – circumferential strain
p – internal fluid pressure
d – internal diameter of the thin cylinder
t – thickness of the wall of the thin cylinder
E – Young's modulus
μ – Poisson's ratio

81. What do you understand by the term wire winding of thin cylinder?

In order to increase the tensile strength of a thin cylinder to withstand high internal pressure without increase in wall thickness, they are prestressed by winding with a steel wire under tension.

82. Define principal planes.

The planes, which have no shear stress, are known as principal planes.

83. Define principal stresses.

The stresses, acting on principal planes are known as principal stresses.

84. Write the expression for normal stress when a member is subjected to two mutually perpendicular direct stresses accompanied by a shear stress.

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta$$

85. Write the expression for tangential stress when a member is subjected to two mutually perpendicular direct stresses accompanied by a shear stress.

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta - \tau \cos 2\theta$$

86. What is the use of Mohr's circle?

Mohr's circle is used for finding normal, tangential and resultant stresses on an oblique plane.

87. What is the radius of Mohr's circle?

The radius of Mohr's circle is equal to the maximum shear stress.

14 MARKS QUESTIONS

93. A closed cylindrical vessel made of steel plates 4 mm thick with plane ends, carries fluid under a pressure of 3 N/mm^2. The diameter of cylinder is 25 cm and length is 75 cm, calculate the longitudinal and hoop stresses in the cylinder wall and determine the change in diameter, length and volume of the cylinder. Take $E = 2.1 \times 10^5$ N/mm^2 and $\mu = 0.286$.

94. A water main 90 cm diameter contains water at a pressure head of 3 N/mm^2. If the weight density of water is 9810 N/mm^3, find the thickness of metal required for the water main. Given the permissible stress as 22 N/mm^2.

95. A thin cylinder of internal diameter 2 m contains a fluid at an internal pressure of 3 N/mm^2. Determine the maximum thickness of the cylinder if (i) longitudinal stress is not to exceed 30 N/mm^2 and (ii) circumferential stress is not to exceed 40 N/mm^2.

96. A cylindrical vessel whose ends are closed by means of rigid flange plates is made of steel plate 3 mm thick. The length and the internal diameter of the vessel are 50 cm and 25 cm respectively. Determine the longitudinal and hoop stresses in the cylindrical shell due to an internal fluid pressure of 3 N/mm^2. Also calculate the increase in length, diameter and volume of the vessel. Take $E = 2 \times 10^5$ N/mm^2 and $\mu = 0.3$.

97. A cylindrical shell is 3 m long, 1 m internal diameter and 15 mm metal thickness. Calculate the maximum intensity of shear stress induced and also the changes in the dimensions of the shell if it is subjected to an internal pressure of 1.5 N/mm^2. Take $E = 0.204 \times 10^6$ N/mm^2 and $\mu = 0.3$.

98. A copper tube of 50 mm internal diameter, 1 m long and 1.25 mm thick has closed ends and is filled with water under pressure. Neglecting any distortion of the end plates, determine the alteration of pressure when an additional volume of 3 cm^3 of water is pumped into the tube.

99. A thin spherical shell of internal diameter 1.5 m and of thickness 8 mm is subjected to an internal pressure of 1.5 N/mm^2. Determine the increase in diameter and increase in volume. Take $E = 2 \times 10^5$ N/mm^2 and $\mu = 0.3$.

100. A seamless spherical shell of 1 m internal diameter and 5 mm thick is filled with a fluid under pressure until its volume increases by 200 cm^3. Calculate the pressure exerted by the fluid on the shell. Take $E = 2.05 \times 10^5$ N/mm^2 and $\mu = 0.3$ for the material.

101. A member is subjected to two principal stress of 80 N/mm^2 tensile and 50 N/mm^2 compressive. Find the resultant stresses on the plane making 22° with the major principal plane. Find the normal and tangential stresses on the plane.

102. Direct stresses of 120 N/mm^2 (tensile) and 90 N/mm^2 (compressive) exist on two perpendicular planes at a certain point in a body. They are also accompanied by shear stresses on the planes. The greater principal stress at the point due to these is 150 N/mm^2.
    i. Find the shear stresses on these planes.
    ii. Find also the maximum shear stress at the point.

103. At a point in a bracket the stresses on two mutually perpendicular planes are 120 N/mm^2 tensile and 60 N/mm^2 tensile. The shear stress across these planes is 30 N/mm^2. Find using the Mohr's stress circle, the principal stresses and maximum shear stress at the point.

104. At a point in an elastic material under strain, there are normal stresses of 50 N/mm^2 and 30 N/mm^2 respectively at right angles to each other with a shearing stress of 25 N/mm^2. Find the principal stresses and the position of principal planes if
    i. 50 N/mm^2 is tensile and 30 N/mm^2 is also tensile.
    ii. 50 N/mm^2 is tensile and 30 N/mm^2 is compressive.

    Find also the maximum shear stress and its plane in both the cases.

105. At a point in an elastic material, a direct tensile stress of 60 N/mm^2 and a direct compressive stress of 40 N/mm^2 are applied on planes at right angles to each other. If the maximum principal stress in the material is to be limited to 65 N/mm^2, find out the shear stress that may be allowed on the planes. Also, determine the magnitude and direction of the minimum principal stress and the maximum shear stress.

106. At a point in a material under stress, the intensity of the resultant stress on a certain plane is 60 N/mm^2 (tensile) inclined 30° to normal of that plane. The stress on a plane at right angles to this has a normal tensile component of intensity 40 N/mm^2. Find (i) the resultant stress on the second plane, (ii) the principal planes and stress, and (iii) the plane of maximum shear and its intensity.

107. At a point within a body subjected to two mutually perpendicular directions, the stresses are 80 N/mm^2 tensile and 40 N/mm^2 tensile. Each of the above stresses is accompanied by a shear stress of 60 N/mm^2. Determine the normal stress, shear stress and resultant stress on an oblique plane inclined at an angle of 45° with the axis of minor tensile stress.

108. The normal stress in two mutually perpendicular directions are 600 N/mm^2 and 300 N/mm^2 both tensile. The complimentary shear stresses in these directions are of intensity 450 N/mm^2. Find the normal and tangential stresses on the two planes which are equally inclined to the planes carrying the normal stresses mentioned above.

109. At a certain point in a strained material the principal stresses are 100 N/mm^2 and 40 N/mm^2 both tensile. Find the normal, tangential and resultant stresses across a plane through the point at 48° to the major principal plane, using Mohr's circle of stress.

110. At a certain point in a strained material, the intensities of stresses on two planes at right angles to each other are 20 N/mm^2 and 10 N/mm^2 both tensile. They are accompanied by a shear stress of magnitude 10 N/mm^2. Find graphically the location of principal planes and evaluate the principal stresses.

OBJECTIVE TYPE QUESTIONS AND ANSWERS

1. A cylindrical vessel is said to be thin if the ratio of its internal diameter to the wall thickness is
   a) less than 20
   b) equal to 20
   c) more than 20
   d) none of the above

2. A thin cylindrical shell of diameter (d), length (l) and thickness (t) is subjected to an internal pressure (p). The hoop stress or circumferential stress in the shell is
   a) pd/t
   b) pd/2t
   c) pd/4t
   d) pd/6t

3. A thin cylindrical shell of diameter length (l) and thickness (t) is subjected to an internal pressure (p). The longitudinal stress or axial stress in the shell is
   a) pd/t
   b) pd/2t
   c) pd/4t
   d) pd/6t

4. In a thin cylindrical shell subjected to an internal pressure (p), the ratio of longitudinal stress to the hoop stress is
   a) 1/2
   b) 3/4
   c) 1
   d) 1.5

5. The design of thin cylindrical shells is based on
   a) hoop stress
   b) longitudinal stress
   c) arithmetic mean of the hoop and the longitudinal stress
   d) geometric mean of the hoop and longitudinal stress

6. The hoop stress in a thin cylindrical shell is
   a) longitudinal stress
   b) compressive stress
   c) radial stress
   d) circumferential tensile stress

7. The maximum shear stress in a thin cylindrical shell subjected to internal pressure (p) is
   a) pd/t
   b) pd/2t
   c) pd/4t
   d) pd/8t

8. The hoop or circumferential stress in a thin spherical shell, when subjected to an internal pressure (p) is equal to
   a) pd / 4t
   b) pd / 2t
   c) pd / 8t
   d) 2pd / t

9. The maximum shear stress in a thin spherical shell, when subjected to an internal pressure (p) is equal to
   a) pd / 4t
   b) pd / 8t
   c) pd / 2t
   d) zero

10. The circumferential strain in case of thin cylindrical shell, when subjected to internal pressure (p), is
   a) more than diametral strain
   b) less than diametral strain
   c) equal to diametral strain
   d) twice the diametral strain

11. A thin cylindrical shell of diameter (d), length (l) and thickness (t) is subjected to an internal pressure (p). The circumferential or hoop strain is
   a) pd (1 – μ/2) / 2tE
   b) pd (1 – μ/2) / 4tE
   c) pd (1/2 – μ) /2tE
   d) pd (1/2 – μ) / 4tE

12. A thin cylindrical shell of diameter (d), length (l) and thickness (t) is subjected to an internal pressure (p). The longitudinal strain is
   a) pd (1 – μ/2) /2tE
   b) pd (1 – μ/2) / 4tE
   c) pd (1/2 – μ) / 2tE
   d) pd (1/2 – μ) / 4tE

13. A thin cylindrical shell of diameter (d), length (l) and thickness (t) is subjected to an internal pressure (p). The ratio of longitudinal strain to hoop strain is
   a) (m – 2) / (2m – 1)
   b) (2m – 1) / (m – 2)
   c) (m – 2) / (2m + 1)
   d) (2m + 1) / (m – 2)

14. When a thin cylindrical shell is subjected to an internal pressure, there will be
   a) a decrease in diameter and length of the shell
   b) an increase in diameter and decrease in length of the shell
   c) a decrease in diameter and increase in length of the shell
   d) an increase in diameter and length of the shell

15. The strain in any direction in case of a thin spherical shell, when subjected to internal pressure (p), is equal to
    a) pd (1/2 – μ) / 2tE
    b) pd (1 – μ /2) / 2tE
    c) pd (1 – μ) / 4tE
    d) 3pd (1 – μ) / 4tE

16. A thin spherical shell of diameter and thickness (t) is subjected to an internal pressure (p). The volumetric strain is
    a) pd (1 – μ) / 4tE
    b) pd (1 – μ) / 2tE
    c) 3pd (1 – μ) / 4tE
    d) pd (1 – μ) / tE

17. A thin cylindrical shell of diameter and thickness (t) is subjected to an internal pressure (p). The volumetric strain is
    a) pd (2 – μ) / tE
    b) pd (3 – 2μ) / 2tE
    c) pd (4 – 3μ) / 3tE
    d) pd (5 – 4μ) / 4tE

18. A thin cylindrical shell of diameter and thickness (t) is subjected to an internal pressure (p). The ratio of longitudinal strain to volumetric strain is
    a) (m – 1) / (2m –1)
    b) (2m – 1) / (m – 1)
    c) (m – 2) / (3m –4)
    d) (m – 2) / (5m – 4)

19. For the same internal diameter, wall thickness, material and internal pressure, the ratio of maximum stress, induced in a thin cylindrical and in a thin spherical vessel will be
    a) **2**
    b) 1/2
    c) 4
    d) 1/4

20. A metal pipe of 1 m diameter contains a fluid having a pressure of 100 N/cm^2. If the permissible tensile stress in the metal is 2 kN/cm^2, then the thickness of the metal required for making the pipe would be
    a) 5 mm
    b) 10 mm
    c) 20 mm
    d) 25mm

21. Circumferential and longitudinal strains in cylindrical boiler under internal steam pressure are e_1 and e_2 respectively. Change in volume of the boiler cylinder per unit volume will be
    a) $2e_1 – e_2$
    b) $2e_1 + e_2$
    c) $2e_2 – e_1$
    d) $2e_2 + e_1$

22. When a body is subjected to a direct tensile stress (σ) in one plane, then normal stress on an oblique section of the body inclined at an angle (θ) to the normal of the section is
    a) cos θ
    b) **$\cos^2$** θ
    c) sin θ
    d) $\sin^2$ θ

23. When a body is subjected to a direct tensile stress (σ) in one plane, the normal stress on an oblique section will be maximum, when (θ) is equal to
    a) 0°
    b) 30°
    c) 45°
    d) 90°

24. When a body is subjected to a direct tensile stress (σ) in one plane, then tangential or shear stress on an oblique section of the body inclined at an angle (θ) to the normal of the section is
    a) sin 2θ
    b) cos 2θ
    c) (σ sin 2θ) / 2
    d) (σ cos 2θ) / 2

25. The resultant stress on an inclined plane which is inclined at an angle (θ) to the normal cross-section of a body which is subjected to a direct tensile stress (σ) in one plane, is
    a) sin θ
    b) **cos** θ
    c) sin 2θ
    d) cos 2θ

26. When a body is subjected to a direct tensile stress (σ) in one plane, then maximum normal stress occurs at a section inclined at________to the normal of the section.
    a) 0°
    b) 30°
    c) 45°
    d) 90°

27. A body is subjected to a direct tensile stress (σ) in one plane. The shear stress is maximum at a section inclined at________to the normal of the section.
    a) 45° and 90°
    b) 45°and 135°
    c) 60° and 150°
    d) 30° and 135°

28. When a body is subjected to a direct tensile stress (σ) in one plane, the maximum shear stress is _______ the maximum normal stress.
    a) equal to
    b) one-half
    c) two-third
    d) twice

29. Principle plane is a plane on which the shear stress is
    a) zero
    b) minimum
    c) maximum
    d) infinity

30. If a member is subjected to an axial tensile load, the plane normal to the axis of loading carries
    a) minimum normal stress
    b) maximum normal stress
    c) maximum shear stress
    d) none of the above

31. If a member is subjected to an axial tensile load, the plane inclined at 45° to the axis of loading carries
    a) minimum shear stress
    b) maximum normal stress
    c) maximum shear stress
    d) none of the above

32. The maximum shear stress induced in a member which is subjected to an axial load is equal to
    a) maximum normal stress
    b) half of maximum normal stress
    c) twice the maximum normal stress
    d) thrice the maximum normal stress

33. When a body is subjected to a direct tensile stress (σ_x) in one plane accompanied by a simple shear stress (τ_{xy}), the maximum normal stress is
    a) $\sigma_x/2 + \sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$
    b) $\sigma_x/2 - \sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$
    c) $\sigma_x/2 + \sqrt{(\sigma^2_x - 4\tau^2_{xy})}/2$
    d) $\sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$

34. When a body is subjected to a direct tensile stress (σ_x) in one plane accompanied by a simple shear stress (τ_{xy}), the minimum normal stress is
    a) $\sigma_x/2 + \sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$
    b) $\sigma_x/2 - \sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$
    c) $\sigma_x/2 + \sqrt{(\sigma^2_x - 4\tau^2_{xy})}/2$
    d) $\sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$

35. When a body is subjected to a direct tensile stress (σ_x) in one plane accompanied by a simple shear stress (τ_{xy}), the maximum shear stress is
   a) $\sigma_x/2 + \sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$
   b) $\sigma_x/2 - \sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$
   c) $\sigma_x/2 + \sqrt{(\sigma^2_x - 4\tau^2_{xy})}/2$
   d) $\sqrt{(\sigma^2_x + 4\tau^2_{xy})}/2$

36. A body is subjected to a direct tensile stress of 300 MPa in one plane accompanied by a simple shear stress of 200 MPa. The maximum normal stress will be
   a) –100 MPa
   b) 250 MPa
   c) 300 MPa
   d) 400 MPa

37. A body is subjected to a direct tensile stress of 300 MPa in one plane accompanied by a simple shear stress of 200 MPa. The minimum normal stress will be
   a) –100 MPa
   b) 250 MPa
   c) 300 MPa
   d) 400 MPa

38. A body is subjected to a direct tensile stress of 300 MPa in one plane accompanied by a simple shear stress of 200 MPa. The maximum shear stress will be
   a) –100 MPa
   b) 250 MPa
   c) 300 MPa
   d) 400 MPa

39. When a body is subjected to bi-axial stress i.e. direct stresses (σ_x) and (σ_y) in two mutually perpendicular planes accompanied by a simple shear stress (τ_{xy}), then maximum normal stress is
   a) $(\sigma_x + \sigma_y)/2 + \sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
   b) $(\sigma_x + \sigma_y)/2 - \sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
   c) $(\sigma_x - \sigma_y)/2 + \sqrt{[(\sigma_x + \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
   d) $(\sigma_x - \sigma_y)/2 - \sqrt{[(\sigma_x + \sigma_y)^2 + 4\tau^2_{xy}]} / 2$

40. When a body is subjected to bi-axial stress i.e. direct stresses (σ_x) and (σ_y) in two mutually perpendicular planes accompanied by a simple shear stress (τ_{xy}), then minimum normal stress is
   a) $(\sigma_x + \sigma_y)/2 + \sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
   b) $(\sigma_x + \sigma_y)/2 - \sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
   c) $(\sigma_x - \sigma_y)/2 + \sqrt{[(\sigma_x + \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
   d) $(\sigma_x - \sigma_y)/2 - \sqrt{[(\sigma_x + \sigma_y)^2 + 4\tau^2_{xy}]} / 2$

41. When a body is subjected to bi-axial stress i.e. direct stresses (σ_x) and (σ_y) in two mutually perpendicular planes accompanied by a simple shear stress (τ_{xy}), then maximum shear stress is
    a) $\sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
    b) $\sqrt{[(\sigma_x + \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
    c) $\sqrt{[(\sigma_x - \sigma_y)^2 + \tau^2_{xy}]}$
    d) $\sqrt{[(\sigma_x + \sigma_y)^2 + \tau^2_{xy}]}$

42. A body is subjected to a tensile stress of 1200 MPa on one plane and another tensile stress of 600 MPa on a plane at right angles to the former. It is also subjected to a shear stress of 400 MPa on the same planes. The maximum normal stress will be
    a) 400 MPa
    b) 500 MPa
    c) 900 MPa
    d) 1400 MPa

43. A body is subjected to a tensile stress of 1200 MPa on one plane and another tensile stress of 600 MPa on a plane at right angles to the former. It is also subjected to a shear stress of 400 MP3 on the same planes. The minimum normal stress will be
    a) 400 MPa
    b) 500 MPa
    c) 900 MPa
    d) 1400 MPa

44. A body is subjected to a tensile stress of 1200 MPa on one plane and another tensile stress of 600 MPa on a plane at right angles to the former. It is also subjected to a shear stress of 400 MP3 on the same planes. The maximum shear stress will be
    a) 400 MPa
    b) 500 MPa
    c) 900 MPa
    d) 1400 MPa

45. A body is subjected to two normal stresses 20 kN/m^2 (tensile) and 10 kN/m^2 (compressive) acting perpendicular to each other. The maximum shear stress is
    a) 5 kN/m^2
    b) 10 kN/m^2
    c) 15 kN/m^2
    d) 20 kN/m^2

46. The maximum shear stress is _______ the algebraic difference of maximum and minimum normal stresses.
    a) equal-to
    b) one-fourth
    c) one-half
    d) twice

47. Mohr's circle is used to determine the stresses on an oblique section of a body subjected to
    a) direct tensile stress in one plane accompanied by a shear stress
    b) direct tensile stress in two mutually perpendicular directions
    c) direct tensile stress in two mutually perpendicular directions accompanied by a simple shear stress
    d) all of these

48. When a body is subjected to direct tensile stresses (σ_x and σ_y) in two mutually perpendicular directions, accompanied by a simple shear stress (τ_{xy}), then in Mohr's circle method, the circle radius is taken as
    a) $(\sigma_x - \sigma_y)/2 + \tau_{xy}$
    b) $(\sigma_x + \sigma_y)/2 + \tau_{xy}$
    c) $\sqrt{[(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}]} / 2$
    d) $\sqrt{[(\sigma_x + \sigma_y)^2 + 4\tau^2_{xy}]} / 2$

49. In Mohr's circle, the centre of circle from Y-axis is taken as
    a) $(\sigma_x - \sigma_y)/2$
    b) $(\sigma_x + \sigma_y)/2$
    c) $(\sigma_x - \sigma_y)/2 + \tau$
    d) $(\sigma_x + \sigma_y)/2 + \tau$

50. The extremities of any diameter on Mohr's circle represent
    a) principal stresses
    b) normal stresses on planes at 45°
    c) shear stresses on planes at 45°
    d) normal and shear stresses on a plane

51. The principal stresses σ_1, σ_2 and σ_3 at a point respectively are 80 MPa, 30 MPa and –40 MPa. The maximum shear stress is
    a) 25 MPa
    b) 35 MPa
    c) 55 MPa
    d) 60 MPa

52. Principal stresses at a point in plane stressed element are $\sigma_x = \sigma_y = 5000$ N/cm^2. Normal stress on the plane inclined at 45° to the x-axis will be
    a) 0
    b) 5000 N/cm^2
    c) 7070 N/cm^2
    d) 10000 N/cm^2

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