KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University)
(Established Under Section 3 of UGC Act 1956)
Pollachi Main Road, Eachanari (Po),
COIMBATORE – 21

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

SUBJECT NAME: HEAT AND MASS TRANSFER	SUBJECT CODE : 16BEME603	L	T	P	C
		3	2	0	4

OBJECTIVES

1. To gain knowledge on the principles and procedure for the design of Mechanical power transmission components.
2. To understand the standard procedure available for Design of Transmission of Mechanical elements
3. To learn to use standard data and catalogues

UNIT I CONDUCTION 12

Basic Concepts – Mechanism of Heat Transfer – Conduction, Convection and Radiation – General Differential equation of Heat Conduction – Fourier Law of Conduction – Cartesian and Cylindrical Coordinates – One Dimensional Steady State Heat Conduction – Conduction through Plane Wall, Cylinders and Spherical systems – Composite Systems – Conduction with Internal Heat Generation – Extended Surfaces – Unsteady Heat Conduction – Lumped Analysis – Use of Heislers Chart.

UNIT II CONVECTION 12

Basic Concepts – Convective Heat Transfer Coefficients – Boundary Layer Concept – Types of Convection – Forced Convection – Dimensional Analysis – External Flow – Flow over Plates, Cylinders and Spheres – Internal Flow – Laminar and Turbulent Flow – Combined Laminar and Turbulent – Flow over Bank of tubes – Free Convection – Dimensional Analysis – Flow over Vertical Plate, Horizontal Plate, Inclined Plate, Cylinders and Spheres.

UNIT III PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS 12

Nusselts theory of condensation–pool boiling, flow boiling, correlations in boiling and condensation. Types of Heat Exchangers – LMTD Method of heat Exchanger Analysis – Effectiveness – NTU method of Heat Exchanger Analysis – Overall Heat Transfer Coefficient – Fouling Factors.

UNIT IV RADIATION 12

Basic Concepts, Laws of Radiation – Stefan Boltzman Law, Kirchoff Law –Black Body Radiation –Grey body radiation - Shape Factor Algebra – Electrical Analogy – Radiation Shields –Introduction to Gas Radiation.

UNIT V MASS TRANSFER 12

Basic Concepts – Diffusion Mass Transfer – Fick's Law of Diffusion – Steady state Molecular Diffusion – Convective Mass Transfer – Momentum, Heat and Mass Transfer Analogy – Convective Mass Transfer Correlations

TOTAL 60

(Permitted to use standard Heat and Mass Transfer Table in the examination)

TEXT BOOKS

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Sachdeva R.C	Fundamentals of Engineering Heat and Mass Transfer	New Age International, New Delhi	2010

REFERENCES

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication

1	Frank P. Incropera and David P. DeWitt	Fundamentals of Heat and Mass Transfer	John Wiley and Sons, New Delhi	2011
2	Ozisik M.N	Heat Transfer	McGraw–Hill Book Co, New Delhi	1994
3	Kothandaraman C.P	Fundamentals of Heat and Mass Transfer	New Age International, New Delhi	2012

WEB REFERENCES

1. http://nptel.iitm.ac.in/courses/Webcourse–contents/IISc–BANG/Heat%20and%20Mass%20Transfer/New_index1.html
2. http://www.learnerstv.com/Free–Engineering–Video–lectures–ltv084–Page1.htm
3. http://en.wikipedia.org/wiki/Heat_transfer

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FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

LESSON PLAN

Subject Name : Heat and Mass Transfer **Subject Code : 16BEME603**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – I : CONDUCTION			
1.	1	Introduction to heat and mass transfer	**T[1], W[1]**
2.	1	Mechanism of heat transfer, Applications of heat transfer and Basic Modes of Heat Transfer	**T[1]**
3.	1	Fourier's law of heat conduction and general differential equation in Cartesian and cylindrical coordinates	**T[1]**
4.	1	Deriving One dimensional steady state heat conduction in plane wall, cylinders	**T[1]**
5.	1	Deriving One dimensional steady state heat conduction in hollow cylinder and sphere	**T[1]**
6.	1	Problems in One dimensional steady state heat conductions	**T[1], R[3]**
7.	1	Heat conduction through the Composite Medium & Problems in composite medium	**T[1]**
8.	1	**Tutorial-1: Problems in One dimensional steady state heat conduction in composite medium**	**T[1]**
9.	1	Critical thickness and solving their problems & Effect of variation of thermal Conductivity	**T[1]**
10.	1	Introduction of Extended Surfaces Problems in extended surfaces	**T[1], W[1]**
11.	1	Heat Conduction: Unsteady state, Lumped System Analysis Heat Transfer in Semi infinite and infinite solids	**T[1]**
12.	1	Problems in lumped system analysis Use of Transient and Temperature charts Application of numerical techniques	**T[1], R[3]**
13.	1	**Tutorial-2: Problems in lumped system analysis**	**T[1]**
Total No. of Hours Planned for Unit - I			**13**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – II : CONVECTION			
14.	1	Basic concepts and introduction to convective heat transfer coefficient and types of convection	**T[1], W[1]**
15.	1	Boundary Layer concept, Introduction to forced convection, Dimensional analysis and application of forced convection	**T[1]**

16.	1	Laminar and turbulent convective heat transfer analysis of parallel flow over flat plate	**T[1]**
17.	1	Laminar and turbulent convective heat transfer analysis in flows in cylindrical pipes and spheres	**T[1]**
18.	1	Laminar and turbulent convective heat transfer analysis in flows over bank of tubes	**T[1]**
19.	1	Laminar and turbulent combined flow with Empirical relations, application of numerical techniques in problem solving	**T[1], R[3]**
20.	1	**Tutorial-3: Problems in Forced convection flow**	**T[1]**
21.	1	Introduction to free convection, dimensional analysis Free convection in atmosphere, free convection on a vertical plate and horizontal plate	**T[1], W[1]**
22.	1	Empirical relation in free convection Problems in free convection flow over vertical plate and horizontal plate	**T[1]**
23.	1	Free convection in atmosphere, free convection on inclined plate, Cylinders and Spheres	**T[1]**
24.	1	Problems in free convection flow over inclined plate, Cylinders and Spheres	**T[1], R[3]**
25.	1	**Tutorial-4: Problems in Free convection flow**	**T[1]**
Total No. of Hours Planned for Unit - II			**12**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – III : PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS			
26.	1	Nusselts theory of condensation and pool boiling	**T[1]**
27.	1	flow boiling and correlations in boiling and condensation	**T[1]**
28.	1	Introduction to heat exchangers, classifications, Temperature Distribution	**T[1], W[1]**
29.	1	LMTD Method of Heat Exchanger Analysis	**T[1]**
30.	1	Problems solving related to LMTD Method	**T[1]**
31.	1	Problems solving related to LMTD Method	**T[1], R[3]**
32.	1	**Tutorial-5: Problems in LMTD Method**	**T[1]**
33.	1	NTU Method of Heat Exchanger Analysis and effectiveness	**T[1]**
34.	1	Problems solving related to ε-NTU Method	**T[1]**
35.	1	Problems solving related to ε-NTU Method	**T[1], R[3]**
36.	1	Overall heat transfer coefficient and fouling factors	**T[1]**
37.	1	**Tutorial-6: Problems in ε-NTU Method**	**T[1]**
Total No. of Hours Planned for Unit - III			**12**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – IV : RADIATION			
38.	1	Basic Concepts, Laws of Radiation	**T[1], W[1]**
39.	1	Stefan Boltzman Law, Kirchoff Law	**T[1]**
40.	1	Radiation properties – Emissivity and absorption of gases and gas mixtures	**T[1]**
41.	1	Heat exchange between black bodies and non-black bodies	**T[1]**
42.	1	Radiation shape factor algebra Problems related to radiation shape factor	**T[1]**
43.	1	Problems related to radiation shape factor	**T[1], R[3]**

44.	1	**Tutorial-7: Problems related to radiation shape factor**	**T[1]**
45.	1	Introduction to Radiation shields	**T[1]**
46.	1	general introduction to gas radiation	**T[1]**
47.	1	Problems solving related to radiation shield	**T[1]**
48.	1	Problems solving related to radiation shield	**T[1], R[3]**
49.	1	**Tutorial-8: Problems in radiation shield**	**T[1]**
Total No. of Hours Planned for Unit - IV			**12**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – V : MASS TRANSFER			
50.	1	Basic Concepts – Diffusion Mass Transfer, Fick's Law of Diffusion –	**T[1], W[1]**
51.	1	Steady state Molecular Diffusion	**T[1]**
52.	1	Species conservation equation – transient diffusion	**T[1]**
53.	1	Problems solving related to Mass transfer	**T[1]**
54.	1	Problems solving related to Mass transfer	**T[1], R[3]**
55.	1	**Tutorial-9: Problems in Mass Transfer**	**T[1]**
56.	1	Introduction to Convective Mass Transfer	**T[1], W[1]**
57.	1	Momentum, Heat and Mass Transfer Analogy	**T[1]**
58.	1	Different Convective Mass Transfer Correlations	**T[1]**
59.	1	Convective Mass Transfer Correlations continued	**T[1]**
60.	1	Problems solving related to Mass transfer	**T[1], R[3]**
61.	1	**Tutorial-10: Problems in Mass Transfer**	**T[1]**
62.	1	Discussion on Previous Year ESE Question Paper	**ESE Question Paper**
Total No. of Hours Planned for Unit - V			**13**

TEXT BOOKS

T [1] – Sachdeva R.C, 2010, Fundamentals of Engineering Heat and Mass Transfer, New Age International, New Delhi

REFERENCES

R [1] **-** Frank P. Incropera and David P. DeWitt, 2011, Fundamentals of Heat and Mass Transfer, John Wiley and Sons, New Delhi

R [2] **-** Ozisik M.N, 1994, Heat Transfer, McGraw–Hill Book Co, New Delhi

R [3] **-** Kothandaraman C.P, 2012, Fundamentals of Heat and Mass Transfer, New Age International, New Delhi

WEBSITES

W [1] **-** http://nptel.iitm.ac.in/courses/Webcourse–contents/IISc–BANG/Heat%20and%20Mass%20Transfer/New_index1.html

W [2] **-** http://www.learnerstv.com/Free–Engineering–Video–lectures–ltv084–Page1.htm

W [3] **-** http://en.wikipedia.org/wiki/Heat_transfer

JOURNALS

J [1] – Journal of Thermal Sciences

J [2] – Journal of Heat and Mass Transfer

TOTAL NUMBER OF COURSE HOURS : 62 Hrs

CHAPTER – 1

CONDUCTION

The science of thermodynamics deals with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, and makes no reference to how long the process will take. But in engineering, we are often interested in the rate of heat transfer, which is the topic of the science of heat transfer.

1.1. Heat Transfer

Heat transfer is defined as the transfer of heat energy from one region to the part of the same region or different region when there is a temperature difference between those regions.

We could clearly understand that temperature difference acts as the driving force for the heat transfer to occur.

The study of heat transfer is directed to

1. the estimation of rate of flow of energy as heat through the boundary of a system both under steady and transient conditions, and
2. the determination of temperature field under steady and transient conditions, which also will provide the information about the gradient and time rate of change of temperature at various locations and time. i.e. T (x, y, z, t) and dT/dx, dT/dy, dT/dz, dT/dt etc.

1.2. Modes of Heat Transfer

1.2.1. Conduction:

This is the mode of energy transfer as heat due to temperature difference within a body or between bodies in thermal contact without the involvement of mass flow and mixing. This is the mode of heat transfer through solid barriers and is encountered extensively in heat transfer equipment design as well as in heating and cooling of various materials as in the case of heat treatment.

1.2.2. Convection:

Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. The faster the fluid motion, the greater the convection heats transfer. In the absence of any bulk fluid motion, heat transfer between a solid surface and the adjacent fluid is by pure conduction.

1.2.3. Radiation:

In contrast to the mechanisms of conduction and convection, where energy transfer through a material medium is involved, heat may also be transferred through regions where a perfect vacuum exists. The mechanism in this case is electromagnetic radiation. We shall limit our discussion to electromagnetic radiation that is propagated as a result of a temperature difference; this is called thermal radiation. For example, the energy that our planet receives from the sun is a result of radiation exchange. The process of radiation heat transfer is not intuitive to most engineers.

1.3. Fourier's law of Heat conduction

In 1822, Fourier expressed the conductive heat transfer mechanism as follows,

The rate of heat conduction (Q) through a plane layer is directly proportional to the temperature difference (dT) across the layer and the area of heat transfer (A), but inversely proportional to the thickness (dx) of the layer.

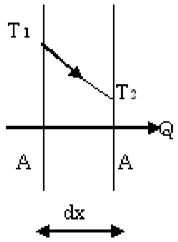


Fig.1.1

$$Q \propto A\,\frac{dT}{dx}$$

$$Q \propto A\,\frac{T_1 - T_2}{dx} \quad \Rightarrow \quad Q = -\,k\,A\,\frac{dT}{dx}$$

here the constant of proportionality k is the thermal conductivity of the material, which is a measure of the ability of a material to conduct heat. Heat is conducted in the direction of decreasing temperature, and the temperature gradient becomes negative when temperature decreases with increasing x. The negative sign ensures that heat transfer in the positive x direction is a positive quantity.

1.4. General Heat conduction equation in Cartesian coordinates

Consider an infinitesimal rectangular element of volume dx.dy.dz as shown below

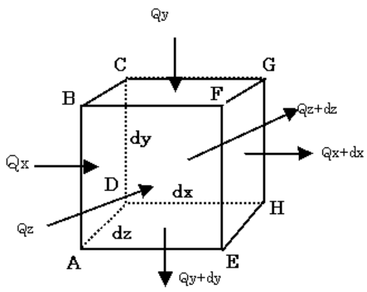


Fig.1.2

The energy balance of the above system as per first law of thermodynamics can be stated as

$$\left\{\begin{matrix}\text{Net heat conducted into}\\ \text{the element from}\\ \text{all coordinate directions}\end{matrix}\right\} + \left\{\begin{matrix}\text{Heat generated}\\ \text{within the}\\ \text{element}\end{matrix}\right\} = \left\{\begin{matrix}\text{Heat stored}\\ \text{in the}\\ \text{element}\end{matrix}\right\} ----(a)$$

Net heat conducted into the element from all the coordinate axes

1. x coordinate

Let Q_x be the heat entering the element in the direction of face ABCD and Q_{x+dx} be the heat leaving the element in the direction of face EFGH

The rate of heat flow Q_x is given by,

$$Q_x = -k_x \frac{\partial T}{\partial x} dy\, dz$$

The rate of heat flow Q_{x+dx} is given by

$$Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x}\, dx$$

$$= -k_x \frac{\partial T}{\partial x} dy\, dz + \frac{\partial \left(-k_x \frac{\partial T}{\partial x} dy\, dz\right)}{\partial x}\, dx$$

$$= -k_x \frac{\partial T}{\partial x} dy\, dz + -k_x \frac{\partial \left(\frac{\partial T}{\partial x}\right)}{\partial x}\, dx\, dy\, dz$$

$$= -k_x \frac{\partial T}{\partial x} dy\, dz + -k_x \frac{\partial^2 T}{\partial x^2}\, dx\, dy\, dz$$

The net heat conducted in x coordinate is given by

$$Q_x - Q_{x+dx} = -k_x \frac{\partial T}{\partial x} dy\, dz - \left(-k_x \frac{\partial T}{\partial x} dy\, dz + -k_x \frac{\partial^2 T}{\partial x^2}\, dx\, dy\, dz\right)$$

$$Q_x - Q_{x+dx} = -k_x \frac{\partial T}{\partial x} dy\, dz + k_x \frac{\partial T}{\partial x} dy\, dz + k_x \frac{\partial^2 T}{\partial x^2}\, dx\, dy\, dz$$

$$Q_x - Q_{x+dx} = k_x \frac{\partial^2 T}{\partial x^2}\, dx\, dy\, dz - - - - - (1)$$

Similarly for the other coordinates as

2. y coordinate

$$Q_y - Q_{y+dy} = k_y \frac{\partial^2 T}{\partial y^2}\, dx\, dy\, dz - - - - - (2)$$

3. z coordinate

$$Q_z - Q_{z+dz} = k_z \frac{\partial^2 T}{\partial z^2}\, dx\, dy\, dz - - - - - (3)$$

Hence the

$$\left\{\begin{matrix}\text{Net heat conducted into} \\ \text{the element from} \\ \text{all coordinate directions}\end{matrix}\right\} = (Q_x - Q_{x+dx}) + (Q_y - Q_{y+dy}) + (Q_z - Q_{z+dz})$$

$$= k_x \frac{\partial^2 T}{\partial x^2}\, dx\, dy\, dz + k_y \frac{\partial^2 T}{\partial y^2}\, dx\, dy\, dz + k_z \frac{\partial^2 T}{\partial z^2}\, dx\, dy\, dz$$

$$= \left(k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2}\right) dx\, dy\, dz ------(4)$$

If the thermal conductivity does not vary with coordinate direction and if it remains constant, then $k_x = k_y = k_z = k$ and the equation (4) becomes

$$= \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right).\ k\,.dx\, dy\, dz ------(5)$$

Heat generated within the element

It is given by

$$Q = \dot{q}\, dx\, dy\, dz --------(6)$$

Heat stored in the element

We know that

$$\begin{Bmatrix}\text{Heat stored}\\ \text{in the}\\ \text{element}\end{Bmatrix} = \begin{Bmatrix}\text{Mass of}\\ \text{the}\\ \text{element}\end{Bmatrix} \times \begin{Bmatrix}\text{Specific heat}\\ \text{of the}\\ \text{element}\end{Bmatrix} \times \begin{Bmatrix}\text{Rise in}\\ \text{temperature}\\ \text{of element}\end{Bmatrix}$$

$$= m \times c_p \times \frac{\partial T}{\partial t}$$

$$= \rho \times dx\, dy\, dz \times c_p \times \frac{\partial T}{\partial t}$$

$$\begin{Bmatrix}\text{Heat stored}\\ \text{in the}\\ \text{element}\end{Bmatrix} = \rho\ c_p\ \frac{\partial T}{\partial t} dx\, dy\, dz\ ---------\ (7)$$

Now substituting (5), (6) & (7) in (a), we get

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right).\ k\,.dx\, dy\, dz + \dot{q}\, dx\, dy\, dz = \rho\ c_p\ \frac{\partial T}{\partial t} dx\, dy\, dz$$

$$\left[\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right).\ k + \dot{q}\right] dx\, dy\, dz = \left[\rho\ c_p\ \frac{\partial T}{\partial t}\right] dx\, dy\, dz$$

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right).\ k + \dot{q} = \rho\ c_p\ \frac{\partial T}{\partial t}$$

Divide the above equation by k throughout

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}}{k} = \frac{\rho\ c_p}{k}\ \frac{\partial T}{\partial t}$$

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\ \frac{\partial T}{\partial t} \quad ----(8)$$

The above equation is known as the general three dimensional heat conduction equation in Cartesian coordinates.

Case: i (no heat source)

Without internal heat generation ($\dot{q} = 0$), the equation (8) becomes

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Case: ii (Steady state condition)

Under steady state condition temperature does not vary with time, hence the equation (8) becomes

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}}{k} = 0$$

The above equation is known as Poisson's equation and in the absence of internal heat generation the above equation becomes a Laplace equation

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = 0$$

1.5. General Heat conduction equation in Cylindrical coordinates

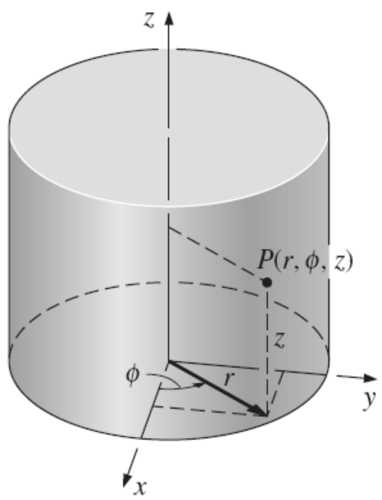


Fig.1.3

The general three dimensional heat conduction equation in cylindrical coordinate is given as follows

$$\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

If the flow is steady, one dimensional and no internal heat generation, then above equation becomes

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0$$

1.6. Steady state one dimensional heat conduction with no internal heat generation

Heat flow through a wall or a cylinder or a sphere is one dimensional when the temperature of the wall or a cylinder or a sphere varies in one direction only. The general heat conduction three dimensional equation (8) becomes

$$\frac{d^2T}{dx^2} = 0$$

1.7. Newton's law of cooling

Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the ambient temperature (i.e. the temperature of its surroundings).

$$Q \propto A_s\ (T_s - T_\infty)$$
$$Q = h\ A_s\ (T_s - T_\infty)$$

Where

h – Heat transfer coefficient, W/m^2K

A_s – surface area, m^2

T_s - Surface temperature, ºC

T_∞ - ambient temperature, ºC

1.8. Thermal conductivity & Thermal diffusivity

The thermal conductivity (k) of a material can be defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.

The thermal conductivity of a material is a measure of the ability of the material to conduct heat.

A high value for thermal conductivity indicates that the material is a good heat conductor, and a low value indicates that the material is a poor heat conductor or insulator.

The unit for thermal conductivity, k – W/m K

The thermal diffusivity (α) of a material can be defined as the ratio of the heat conducted through the material to the heat stored per unit volume.

A material that has a high thermal conductivity or a low heat capacity will obviously have a large thermal diffusivity. The larger the thermal diffusivity, the faster the propagation of heat into the medium. A small value of thermal diffusivity means that heat is mostly absorbed by the material and a small amount of heat will be conducted further.

$$Thermal\ diffusivity, \alpha = \frac{Heat\ conducted}{Heat\ stored} = \frac{k}{\rho\ c_p}$$

1.9. Steady heat conduction through plane wall

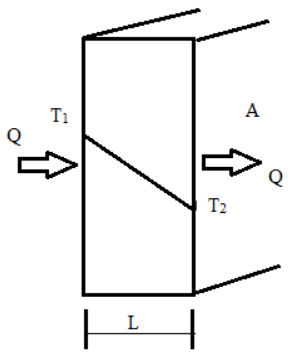


Fig.1.5 Steady heat conduction through plane wall

Considering the plane wall as shown in the fig.1.5, the rate of heat transfer Q is given by

$$Q = \frac{T_1 - T_2}{R_{wall}}, \quad watt$$

R_{wall} – Thermal resistance, K/W

$$R_{wall} = \frac{L}{k\,A}$$

L – Thickness of the wall, m

k – Thermal conductivity, W/mK

A – Heat transfer area, m^2

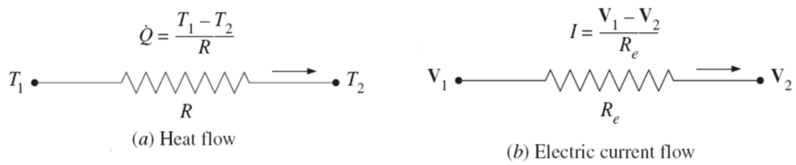


Fig. 1.6 Analogy between thermal and electrical resistance concepts

Consider convection heat transfer from a solid surface of area A_s and temperature T_s to a fluid whose temperature sufficiently far from the surface is T_∞, with a convection heat transfer coefficient h. Newton's law of cooling for convection heat transfer rate.

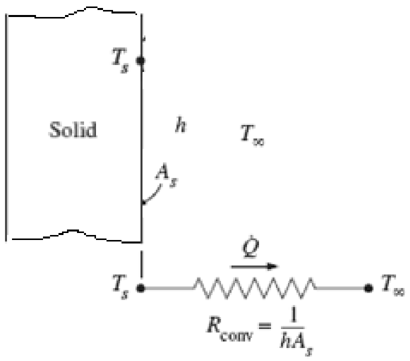


Fig.1.7. convection resistance at a surface

$$Q = h\,A_s(T_s - T_\infty)$$

It can be rearranged as

$$Q = \frac{T_s - T_\infty}{R_{conv}}$$

Where,

$$R_{conv} = \frac{1}{h\,A_s}$$

R_{conv} - Convection resistance

The rate of heat transfer for a plane wall with outer and inner convection is given by

$$Q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}, \quad watt$$

$$R_{total} = R_{conv,1} + R_{wall} + R_{conv,2}$$

$$R_{conv,1} = \frac{1}{h_1 A}$$

$$R_{wall} = \frac{L}{k\,A}$$

$$R_{conv,2} = \frac{1}{h_2 A}$$

Where,

R_{total} – Total thermal resistance, K / W

$T_{\infty 1}$ & $T_{\infty 2}$ – Fluid temperatures at inner & outer region of the plane wall, ºC

1.9.1. Composite plane wall

In practice we often encounter plane walls that consist of several layers of different materials. The thermal resistance concept can still be used to determine the rate of steady heat transfer through such composite walls.

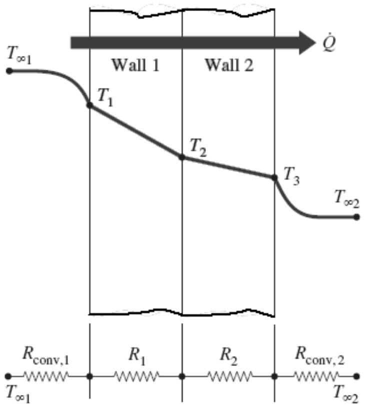


Fig. 1.9. Composite wall

Consider a plane wall that consists of two layers (such as a brick wall with a layer of insulation). The rate of steady heat transfer through this two-layer composite wall can be expressed as

$$Q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}, \quad watt ----Eq.(i)$$

$$R_{total} = R_{conv,1} + R_{wall,1} + R_{wall,2} + R_{conv,2}$$

$$R_{total} = \frac{1}{h_1 A} + \frac{L_1}{k_1\, A} + \frac{L_2}{k_2\, A} + \frac{1}{h_2 A}$$

1.10. Steady heat conduction through hollow cylinder

Fourier's law of heat conduction for heat transfer through the cylindrical layer can be expressed as

$$Q = -k\, A\, \frac{dT}{dr}$$

Consider a long cylindrical layer (such as a circular pipe) of inner radius r_1, outer radius r_2, length L, and average thermal conductivity k. The two surfaces of the cylindrical layer are maintained at constant temperatures T_1 and T_2. There is no heat generation in the layer and the thermal conductivity is constant.

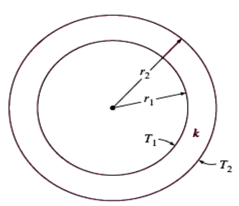


Fig.1.10

The rate of heat transfer is given by,

$$Q = \frac{T_1 - T_2}{R_{cyl}}, \quad watt$$

R_{cyl} – Thermal resistance, K/W

$$R_{cyl} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\,\pi\,k\,L}$$

L –Length of the cylinder, m

k – Thermal conductivity, W/mK

r_1 & r_2 – inner & outer radius of the cylinder, m

Similarly for a sphere,

The rate of heat transfer is given by,

$$Q = \frac{T_1 - T_2}{R_{sph}}, \quad watt$$

R_{sph} – Thermal resistance, K/W

$$R_{sph} = \frac{r_2 - r_1}{4\,\pi\,k\,r_1 r_2}$$

k – Thermal conductivity, W/mK

r_1 & r_2 – inner & outer radius of the sphere, m

Now consider steady one-dimensional heat flow through a cylindrical or spherical layer that is exposed to convection on both sides to fluids at temperatures $T_{\infty1}$ and $T_{\infty2}$ with heat transfer coefficients h_1 and h_2, respectively, as shown in Fig. 1.11.

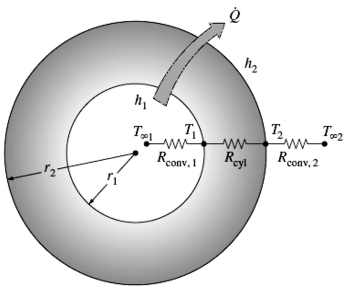


Fig.1.11

$$Q = \frac{T_{\infty1} - T_{\infty2}}{R_{total}}, \quad watt$$

$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2}$$

$$R_{total} = \frac{1}{(2\,\pi\,r_1 L)\,h_1} + \frac{ln\left(\frac{r_2}{r_1}\right)}{2\,\pi\,k\,L} + \frac{1}{(2\,\pi\,r_2 L)\,h_2}$$

Similarly for a sphere,

$$R_{total} = \frac{1}{\left(4\,\pi\,{r_1}^2\right)h_1} + \frac{r_2 - r_1}{4\,\pi\,k\,r_1 r_2} + \frac{1}{\left(4\,\pi\,{r_1}^2\right)h_2}$$

R_{total} – Total thermal resistance, K / W

$T_{\infty 1}$ & $T_{\infty 2}$ – Fluid temperatures at inner & outer region of the plane wall, ºC

1.10.1. Composite cylinder & sphere

Steady heat transfer through multilayered cylindrical or spherical shells which can be handled just like multilayered plane walls discussed earlier by simply adding an additional resistance in series for each additional layer. For example, the steady heat transfer rate through the three-layered composite cylinder of length L shown in Fig. 1.12 with convection on both sides can be expressed as

$$Q = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}, \quad watt$$

$$R_{total} = R_{conv,1} + R_{cyl,1} + R_{cyl,2} + R_{cyl,3} + R_{conv,2}$$

$$R_{total} = \frac{1}{(2\,\pi\,r_1 L)\,h_1} + \frac{ln\left(\frac{r_2}{r_1}\right)}{2\,\pi\,k_1\,L} + \frac{ln\left(\frac{r_3}{r_2}\right)}{2\,\pi\,k_2\,L} + \frac{ln\left(\frac{r_4}{r_3}\right)}{2\,\pi\,k_3\,L} + \frac{1}{(2\,\pi\,r_4 L)\,h_2}$$

Similarly, for a sphere,

$$R_{total} = R_{conv,1} + R_{sph,1} + R_{sph,2} + R_{sph,3} + R_{conv,2}$$

$$R_{total} = \frac{1}{\left(4\,\pi\,{r_1}^2\right)h_1} + \frac{r_2 - r_1}{4\,\pi\,k_1\,r_1 r_2} + \frac{r_3 - r_2}{4\,\pi\,k_2\,r_2 r_3} + \frac{r_4 - r_2}{4\,\pi\,k_3\,r_3 r_4} + \frac{1}{\left(4\,\pi\,{r_4}^2\right)h_2}$$

1.11. Critical radius of insulation

We know that adding more insulation to a wall or to the attic always decreases heat transfer. The thicker the insulation, the lower the heat transfer rate. This is expected, since the heat transfer area A is constant, and adding insulation always increases the thermal resistance of the wall without increasing the convection resistance.

Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter. The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection. The heat transfer from the pipe may increase or decrease, depending on which effect dominates.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant (Fig.1.13). The pipe is now insulated with a material whose thermal conductivity is k and outer radius is r_2. Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient h. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as (Fig. 1.14)

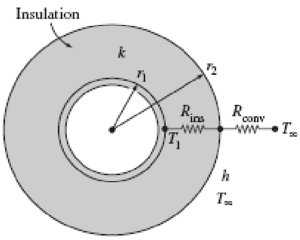


Fig. 1.13

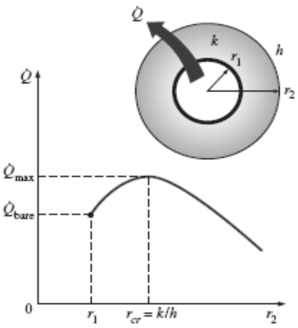


Fig. 1.14

$$Q = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\frac{ln\left(\frac{r_2}{r_1}\right)}{2\,\pi\,k\,L} + \frac{1}{(2\,\pi\,r_2 L)h}}$$

The variation of Q with the outer radius of the insulation r_2 is plotted in Fig. 1.14. The value of r_2 at which reaches a maximum is determined from the requirement that $dQ/dr_2 = 0$ (zero slope). Performing the differentiation and solving for r_2 yields the critical radius of insulation for a cylindrical body to be

$$r_c = \frac{k}{h}$$

Note that the critical radius of insulation depends on the thermal conductivity of the insulation k and the external convection heat transfer coefficient h. The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_c$, reaches a maximum when $r_2 = r_c$, and starts to decrease for $r_2 > r_c$. Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when $r_2 < r_c$.

1.12. INTERNAL HEAT GENERATION

Many practical heat transfer applications involve the conversion of some form of energy into thermal energy in the medium. Such mediums are said to involve internal heat generation, which manifests itself as a rise in temperature throughout the medium. Some examples of heat generation are resistance heating in wires, exothermic chemical reactions in a solid, and nuclear reactions in nuclear fuel rods where electrical, chemical, and nuclear energies are converted to heat.

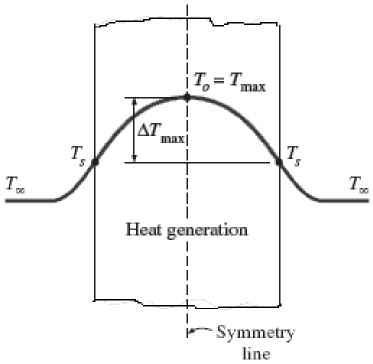


Fig.1.15.

Consider a large plane wall of thickness 2L, as shown in the Figure 1.15, which is generating heat of $\dot{q}$ W/m^3 and its surface temperature on both sides, is maintained at T_s ºC and exposed to surrounding, whose fluid temperature is T_∞ with a constant heat transfer coefficient of h. Under steady conditions, the energy balance for this solid can be expressed as,

$$\left\{\begin{matrix} Rate\ of \\ heat\ transfer \\ from\ solid \end{matrix}\right\} = \left\{\begin{matrix} Rate\ of \\ energy\ generation \\ within\ the\ solid \end{matrix}\right\}$$

This can be expressed as,

The heat transfer from the surface of the solid is by means of convection, hence

$$\begin{Bmatrix} Rate\ of \\ heat\ transfer \\ from\ solid \end{Bmatrix}, Q = h\ A_s\ (T_s - T_\infty) ----(1)$$

Whereas the rate of energy generation is given by,

$$\begin{Bmatrix} Rate\ of \\ energy\ generation \\ within\ the\ solid \end{Bmatrix}, Q = \dot{q}\ V ----(2)$$

On equating (1) & (2)

$$h\ A_s\ (T_s - T_\infty) = \dot{q}\ V$$

On simplification, the surface temperature T_s is given by,

$$T_s = T_\infty + \frac{\dot{q}\ V}{h\ A_s}$$

For a plane wall, the surface area is $2A_{wall}$ and the volume is $2L\ A_{wall}$, now on substitution in the above equation we get,

$$T_s = T_\infty + \frac{\dot{q}\ L}{h}$$

Similarly the surface temperature of a cylinder and sphere with internal heat generation is given by,

$$T_s = T_\infty + \frac{\dot{q}\ r}{2\ h} --- (cylinder)$$

$$T_s = T_\infty + \frac{\dot{q}\ r}{3\ h} --- (sphere)$$

The heat generated within the solid is conducted by the surface of the solid, hence it can be shown that,

$$\begin{Bmatrix} Rate\ of \\ energy\ generation \\ within\ the \\ solid \end{Bmatrix} = \begin{Bmatrix} Rate\ of \\ heat\ conducted \\ from\ the\ solid \\ surface \end{Bmatrix}$$

$$\dot{q}\ V = -k\ A_s \frac{dT}{dx}$$

On integrating the above equation from x = 0, where T = T_o (center line temperature), and x= L/2 where T = T_s.

$$\dot{q}\ 2\ L\ A_{wall} = -k\ A_{wall} \frac{dT}{dx}$$

$$dT = -\frac{\dot{q}\ L}{k}\ dx$$

$$\int_{T_0}^{T_s} dT = -\frac{\dot{q}\ L}{k} \int_0^{L/2} dx$$

$$T_s - T_o = -\frac{\dot{q}\,L}{k}\left(\frac{L}{2} - 0\right)$$

$$T_s - T_o = -\frac{\dot{q}\,L^2}{2\,k}$$

The centre line temperature is the maximum temperature, hence

$$T_o = T_{max} = T_s + \frac{\dot{q}\,L^2}{2\,k}$$

Similarly the maximum or center line temperature for cylinder and sphere can be expressed as follows

$$T_o = T_{max} = T_s + \frac{\dot{q}\,r^2}{4\,k} - - - - (cylinder)$$

$$T_o = T_{max} = T_s + \frac{\dot{q}\,r^2}{6\,k} - - - - (sphere)$$

1.13. HEAT TRANSFER FROM EXTENDED SURFACES

The rate of heat transfer from a surface at a temperature T_s to the surrounding medium at T_∞ is given by Newton's law of cooling as

$$Q_{conv} = h\,A_s\,(T_s - T_\infty)$$

Where A_s is the heat transfer surface area and h is the convection heat transfer coefficient.

When the temperatures Ts and T_∞ are fixed by design considerations, as is often the case, there are *two ways* to increase the rate of heat transfer: to increase the *convection heat transfer coefficient h* or to increase the *surface area As.*

1. Increasing h may require the installation of a pump or fan, or replacing the existing one with a larger one, but this approach may or may not be practical. Besides, it may not be adequate.
2. The alternative is to increase the surface area by attaching to the surface *extended surfaces* called *fins* made of highly conductive materials such as aluminum.

Finned surfaces are manufactured by extruding, welding, or wrapping a thin metal sheet on a surface. Fins enhance heat transfer from a surface by exposing a larger surface area to convection and radiation.

Finned surfaces are commonly used in practice to enhance heat transfer, and they often increase the rate of heat transfer from a surface several fold.

In the analysis of fins, we consider steady operation with no heat generation in the fin, and we assume the thermal conductivity k of the material to remain constant. We also assume the convection heat transfer coefficient h to be constant and uniform over the entire surface of the fin for convenience in the analysis.

1.13.1. FIN EUQATION

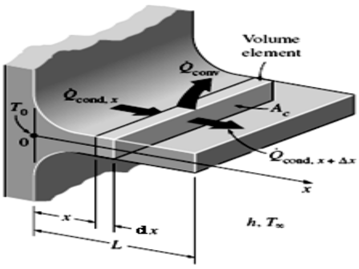


Fig.1.16

$$\left\{\begin{matrix} Rate\ of \\ heat\ conducted\ into \\ the\ element \\ at\ x \end{matrix}\right\} = \left\{\begin{matrix} Rate\ of \\ heat\ condution \\ from\ the\ element \\ at\ x+\ dx \end{matrix}\right\} + \left\{\begin{matrix} Rate\ of \\ heat\ convected \\ from \\ the\ element \end{matrix}\right\}$$

$$Q_{x,cond} = Q_{x+dx,cond} + \ Q_{conv}$$

Using Taylor series expansion, and expanding Q_{x+dx}

$$Q_{x,cond} = Q_{x,cond} + \frac{\partial Q_{x,cond}}{\partial x} dx + \ Q_{conv}$$

We know that,

$$-k\,A\frac{dT}{dx} = \ -k\,A\frac{dT}{dx} + \frac{\partial\left(-k\,A\dfrac{dT}{dx}\right)}{\partial x} dx + \ h\,A_s\,(T - \ T_\infty)$$

$$\frac{\partial\left(-k\,A\,\frac{dT}{dx}\right)}{\partial x}\,dx + \,h\,A_s\,(T - \,T_\infty) = 0$$

$$k\,A\,\frac{d^2T}{dx^2}\,dx - \,h\,A_s\,(T - \,T_\infty) = 0$$

$$k\,A\,\frac{d^2T}{dx^2}\,dx - \,h\,p\,dx\,(T - \,T_\infty) = 0$$

$$k\,A\,\frac{d^2T}{dx^2} - \,h\,p\,(T - \,T_\infty) = 0$$

If $T - T_\infty = \theta$, this implies $T = \theta$,

$$\frac{d^2T}{dx^2} - \frac{h\,p}{k\,A}\,(T - \,T_\infty) = 0$$

$$\frac{d^2\theta}{dx^2} - \,m^2\,\theta = 0$$

Where ,

$$m = \sqrt{\frac{h\,p}{k\,A}}$$

The above equation is a second order ordinary differential equation, whose general solution is of the format,

$$\theta\,(x) = \,C_1\,e^{\,m\,x} + \,C_2\,e^{-\,m\,x}$$

Where,

The constants C_1 & C_2 can be found by applying the boundary conditions.

Case (i): Long fin

For a sufficiently long fin of uniform cross section (A = constant), the temperature of the fin at the fin tip will approach the environment temperature T_∞ and thus θ will approach zero.

The boundary conditions are as follows,

1. At x= 0, $\theta(0) = \theta_b = T_b - T_\infty$
2. At x = L, $\theta(L) = \theta_L = T(L) - T_\infty = 0$ as $L \rightarrow \infty$, $T = T_\infty$

On applying the above boundary conditions,

The temperature distribution is given by

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-m\,x}$$

Where, T – temperature at 'x' distance

And the rate of heat transfer is given by,

$$Q = \sqrt{h\,p\,k\,A}\,(T_b - T_\infty)$$

Case (ii): short fin (end insulated)

Fins are not likely to be so long that their temperature approaches the surrounding temperature at the tip. A more realistic situation is for heat transfer from the fin tip to be negligible since the heat transfer from the fin is proportional to its surface area, and the surface area of the fin tip is usually a negligible fraction of the total fin area. Then the fin tip can be assumed to be insulated, and the condition at the fin tip can be expressed as

1. At x= 0, $\theta(0) = \theta_b = T_b - T_\infty$
2. At x = L, $\theta(L) = \theta_L = T(L) - T_\infty$

On applying the boundary conditions in the general solution,

The temperature distribution is given by

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh\,[m\,(L-x)]}{\cosh\,(m\,L)}$$

Where, T – temperature at ‘x’ distance

And the rate of heat transfer is given by,

$$Q = \sqrt{h\, p\, k\, A}\, (T_b - T_\infty)\, tanh(m\, L)$$

1.13.2. FIN EFFICIENCY:

Fin efficiency is defined as the ratio of actual heat transfer by the fin and the ideal heat transfer from the fin if the entire fin were at the base temperature.

$$\eta_{fin} = \frac{Q_{actual}}{Q_{max}}$$

Case (i): long fin

$$\eta_{fin} = \frac{Q_{actual}}{Q_{max}} = \frac{1}{m\, L}$$

Case (ii): short fin (end insulated)

$$\eta_{fin} = \frac{Q_{actual}}{Q_{max}} = \frac{tanh(m\, L)}{m\, L}$$

An important consideration in the design of finned surfaces is the selection of the proper fin length L. Normally the longer the fin, the larger the heat transfer area and thus the higher the rate of heat transfer from the fin.

But also the larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction. Therefore, increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost.

Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length. Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically and should be avoided. The efficiency of most fins used in practice is above 90 percent.

1.13.3. FIN EFFECTIVENESS:

Fins are used to enhance heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins.

In fact, there is no assurance that adding fins on a surface will enhance heat transfer. The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case.

The performance of fins expressed in terms of the fin effectiveness and it is defined as the ratio of heat transfer rate of a system with fin to the rate of heat transfer without fin. When this value is more than unity, the fixing up of fins for a system is beneficial or it is good to avoid fins.

$$\varepsilon_{fin} = \frac{Q_{with\,fin}}{Q_{without\,fin}} = \frac{tanh(m\,L)}{\sqrt{\frac{h\,A}{k\,p}}} \quad ---\ (shortfin\ (end\ insulated))$$

1.14. TRANSIENT HEAT CONDUCTION:

Before a barrier begins to conduct heat at steady state the barrier has to be heated or cooled to the temperature levels that will exist at steady conditions. Thus the study of transient conduction situation is an important component of conduction studies. This study is a little more complicated due to the introduction of another variable namely time to the parameters affecting conduction. This means that temperature is not only a function of location but also a function of time, τ, i.e. T = T (x, y, z, τ).

1.14.1. LUMPED PARAMETER SYSTEM:

In heat transfer analysis, some bodies are observed to behave like a "lump" whose interior temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only, T(t). Heat transfer analysis that utilizes this idealization is known as lumped system analysis, which provides great simplification in certain classes of heat transfer problems without much sacrifice from accuracy.

An energy balance of the solid for the time interval dt can be expressed as

$$\begin{Bmatrix} Heat\ trasnfer\ into\ the\ body \\ during\ dt \end{Bmatrix} = \begin{Bmatrix} increase\ in\ the\ energy \\ of\ the\ body\ during \\ the\ time\ dt \end{Bmatrix}$$

$$h\,A_s\,(T_\infty - T)\,dt = m\,C_p\,dT$$

and we know that m = ρ V and dT is nothing but d(T- T_∞)

$$h\,A_s\,(T - T_\infty)\,dt = -\rho\,V\,C_p\,d(T - T_\infty)$$

$$\frac{d(T - T_\infty)}{(T - T_\infty)} = -\frac{h\,A_s}{\rho\,V\,C_p}dt$$

On integrating the above equation at t = 0, T = T_o to t = some time, T = some temperature T, we get

$$\ln\left(\frac{T - T_\infty}{T_o - T_\infty}\right) = -\frac{h\,A_s}{\rho\,V\,C_p}\,t$$

On taking exponential on both sides we get,

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h\,A_s}{\rho\,V\,C_p}t}$$

The lumped system analysis certainly provides great convenience in heat transfer analysis, and naturally we would like to know when it is appropriate to use it. The first step in establishing a criterion for the applicability of the lumped system analysis is to define a characteristic length (L_c) as

$$L_c = \frac{V}{A_s}$$

and a Biot number B_i as

$$B_i = \frac{h\,L_c}{k}$$

When a solid body is being heated by the hotter fluid surrounding it (such as a potato being baked in an oven), heat is first convected to the body and subsequently conducted within the body.

The Biot number is the ratio of the internal resistance of a body to heat conduction to its external resistance to heat convection. Therefore, a small Biot number represents small resistance to heat conduction, and thus small temperature gradients within the body.

Lumped system analysis is applicable if

$$B_i \le 0.1$$

1.14.2. TRANSIENT HEAT CONDUCTION IN SEMI INFINITE SOLID:

Theoretically a solid which extends in both the positive and negative y and z directions to infinity and in the positive x direction to infinity is defined as a semi infinite body. There can be no such body in reality.

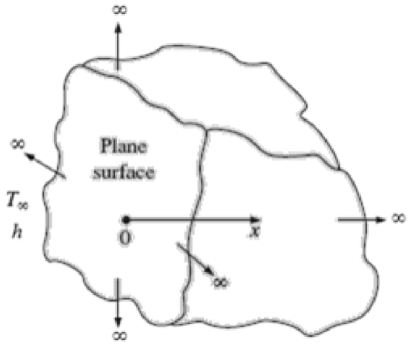


Fig.1.17 – schematic representation of a semi-infinite solid

The exact solution of the transient one-dimensional heat conduction problem in a semi-infinite medium that is initially at a uniform temperature of T_i and is suddenly subjected to convection at time t = 0 can be expressed as

$$\frac{T_x - T_o}{T_i - T_o} = erf(Z)$$

Where

$$Z = \frac{x}{2\sqrt{\alpha t}}$$

x – Distance at which the temperature T prevails

T_x – the temperature at x^{th} distance from the surface

T_o – the surface temperature

T_i – initial temperature of the semi-infinite solid

The error function or complimentary error function (erf) can be obtained from the data book for the respective Z value.

The heat flow at the surface at any time is obtained using Fourier's equation –kA (dT/dx). The surface heat flux at time t is

$$q_s = \frac{k\,(T_s - T_i)}{\sqrt{\pi \alpha t}}\ W/m^2$$

The total heat flow during a given period can be obtained by

$$q_t = 2\,k\,(T_s - T_i)\sqrt{\frac{t}{\pi\,\alpha}}\;, W/m^2$$

The heat flow at any section at a specified time is given by

$$q_x = \frac{k\,(T_s - T_i)}{\sqrt{\pi\,\alpha\,t}}\;e^{-\left(\frac{x^2}{4\,\alpha\,t}\right)}\;W/m^2$$

1.14.3. TRANSIENT HEAT CONDUCTION THROUGH INIFINITE SOLID

The variation of temperature with time and position in one-dimensional problems such as those associated with a large plane wall, a long cylinder, and a sphere are to be discussed in this part which can be assumed as bodies of infinite nature.

The solution for such condition involves the parameters x, L, t, k, α, h, T_i, and T_∞, which are too many to make any graphical presentation of the results practically. In order to reduce the number of parameters, we nondimensionalize the problem by defining the following dimensionless quantities:

$(i) Dimensionless\ temperature$: $\theta\,(x,t) = \frac{T\,(x,t) - T_\infty}{T_i - T_\infty}$

$(ii) Dimensionless\ distance\ from\ the\ centre$: $X = \frac{x}{L}$

$(iii) Dimensionless\ heat\ transfer\ coefficient$: $B_i = \frac{h\,L}{k}$

$(iii) Dimensionless\ time$: $\tau = \frac{\alpha\,t}{L^2}$

The nondimensionalization enables us to present the temperature in terms of three parameters only: X, Bi, and τ. This makes it practical to present the solution in graphical form. The dimensionless quantities defined above for a plane wall can also be used for a cylinder or sphere by replacing the space variable x by r and the half-thickness L by the outer radius r_o.

Note that the characteristic length in the definition of the Biot number is taken to be the half-thickness L for the plane wall and the radius r_o for the long cylinder and sphere instead of V/A used in lumped system analysis.

The transient temperature charts for a large plane wall, long cylinder, and sphere were presented by M. P. Heisler in 1947 and are called Heisler charts. They were supplemented in 1961 with transient heat transfer charts by H. Gröber. There are three charts associated with each geometry:

1. The first chart is to determine the temperature T_o at the center of the geometry at a given time t.
2. The second chart is to determine the temperature at other locations at the same time in terms of T_o.

3. The third chart is to determine the total amount of heat transfer up to the time t.

These plots are valid for $\tau > 0.2$.

Questions & Answers:

1. Define Heat Transfer.

Heat transfer can be defined as the transmission of energy from one region to another region due to temperature difference.

2. What are the modes of Heat Transfer?

1. Conduction
2. Convection
3. Radiation

3. Define Conduction.

Heat conduction is a mechanism of heat transfer from a region of high temperature to a region of low temperature within a medium (solid, liquid or gases) or between different medium in direct physical contact.

In condition energy exchange takes place by the kinematic motion or direct impact of molecules. Pure conduction is found only in solids.

4. Define Convection.

Convection is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperatures.

Convection is possible only in the presence of fluid medium.

5. Define Radiation.

The heat transfer from one body to another without any transmitting medium is known as radiation. It is an electromagnetic wave phenomenon.

6. State Fourier's Law of conduction.

The rate of heat conduction is proportional to the area measured – normal to the direction of heat flow and to the temperature gradient in that direction.

$$Q \alpha - A \frac{dT}{dx}$$

$$Q = -KA \frac{dT}{dx}$$

Where A – are in m^2

$\frac{dT}{dx}$ - Temperature gradient in K/m

K – Thermal conductivity W/mK.

7. Define Thermal Conductivity.

Thermal conductivity is defined as the ability of a substance to conduct heat.

8. Write down the equation for conduction of heat through a slab or plane wall.

Heat transfer $Q = \frac{\Delta T_{overall}}{R}$

Where

$\Delta T = T_1 - T_2$

$R = \frac{L}{KA}$ - Thermal resistance of slab

L = Thickness of slab

K = Thermal conductivity of slab

A = Area

9. Write down the equation for conduction of heat through a hollow cylinder.

Heat transfer $Q = \frac{\Delta T_{overall}}{R}$

Where

$\Delta T = T_1 - T_2$

$R = \frac{1}{2\pi LK} \text{ in} \left[\frac{r_2}{r_1}\right]$ thermal resistance of slab

L – Length of cylinder

K – Thermal conductivity

r_2 – Outer radius

r_1 – inner radius

10. Write down equation for conduction of heat through hollow sphere.

Heat transfer $Q = \frac{\Delta T_{overall}}{R}$

Where

$\Delta T = T_1 - T_2$

$R = \frac{r_2 - r_1}{4\pi K\ (r_1 r_2)}$ - Thermal resistance of hollow sphere.

11. State Newton's law of cooling or convection law.

Heat transfer by convection is given by Newton's law of cooling

$Q = hA\,(T_s - T_\infty)$

Where

A – Area exposed to heat transfer in m^2

h - heat transfer coefficient in W/m^2K

T_s – Temperature of the surface in K

T_∞ - Temperature of the fluid in K.

12. Write down the equation for heat transfer through a composite plane wall.

Heat transfer $Q = \frac{\Delta T_{overall}}{R}$

Where

$\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}$$

L – Thickness of slab

h_a – heat transfer coefficient at inner diameter

h_b – heat transfer coefficient at outer side.

13. Write down the equation for heat transfer through composite pipes or cylinder.

Heat transfer $Q = \frac{\Delta T_{overall}}{R}$

Where

$\Delta T = T_a - T_b$

$$R = \frac{1}{2\pi L} \quad \frac{1}{h_a r_1} + \frac{In\left[\frac{r_2}{r_1}\right]}{K_1} + \frac{In\left[\frac{r_1}{r_2}\right]L_2}{K_2} + \frac{1}{h_b r_3}.$$

14. Write down one dimensional, steady state conduction equation without internal heat generation.

$$\frac{\partial^2 T}{\partial x^2} = 0$$

15. Write down steady state, two dimensional conduction equation without heat generation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

16. Write down the general equation for one dimensional steady state heat transfer in slab or plane wall without heat generation.

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\infty}\ \frac{\partial T}{\partial t}$$

17. Define overall heat transfer co-efficient.

The overall heat transfer by combined modes is usually expressed in terms of an overall conductance or overall heat transfer co-efficient 'U'.

Heat transfer Q = UA ΔT.

18. Write down the general equation for one dimensional steady state heat transfer in slab with heat generation.

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q}{K}=\frac{1}{\alpha}\ \frac{\partial T}{\partial t}$$

19. What is critical radius of insulation (or) critical thickness.

Critical radius = r_c

Critical thickness = $r_c - r_1$

Addition of insulating material on a surface does not reduce the amount of heat transfer rate always. In fact, under certain circumstances it actually increases the heat loss up to certain thickness of insulation. The radius of insulation for which the heat transfer is maximum is called critical radius of insulation, and the corresponding thickness is called critical thickness.

20. Define fins (or) Extended surfaces.

It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surfaces used for increasing heat transfer are called extended surfaces or sometimes known as fins.

21. State the applications of fins.

The main application of fins are

1. Cooling of electronic components
2. Cooling of motor cycle engines.
3. Cooling of transformers
4. Cooling of small capacity compressors

22. Define Fin efficiency.

The efficiency of a fin is defined as the ratio of actual heat transfer by the fin to the maximum possible heat transferred by the fin.

$$\eta_{fin}=\frac{Q_{fin}}{Q_{\max}}$$

23. Define Fin effectiveness.

Fin effectiveness is the ratio of heat transfer with fin to that without fin

$$\text{Fin effectiveness} = \frac{Q_{with\ fin}}{Q_{without\ fin}}$$

24. What is meant by steady state heat conduction?

If the temperature of a body does not vary with time, it is said to be in a steady state and that type of conduction is known as steady state heat conduction.

25. What is meant by Transient heat conduction or unsteady state conduction?

If the temperature of a body varies with time, it is said to be in a transient state and that type of conduction is known as transient heat conduction or unsteady state conduction.

26. What is Periodic heat flow?

In periodic heat flow, the temperature varies on a regular basis.

Example:

1. Cylinder of an IC engine.
2. Surface of earth during a period of 24 hours.

27. What is non-periodic heat flow?

In non-periodic heat flow, the temperature at any point within the system varies non-linearly with time.

Examples:

1. Heating of an ingot in a furnace.
2. Cooling of bars.

28. What is meant by Newtonian heating or cooling process?

The process in which the internal resistance is assumed as negligible in comparison with its surface resistance is known as Newtonian heating or cooling process.

29. What is meant by Lumped heat analysis?

In a Newtonian heating or cooling process, the temperature throughout the solid is considered to be uniform at a given time. Such an analysis is called Lumped heat capacity analysis.

30. What is meant by Semi-infinite solids?

In a semi-infinite solid, at any instant of time, there is always a point where the effect of heating or cooling at one of its boundaries is not felt at all. At this point the temperature remains unchanged. In semi-infinite solids, the biot number value is ∞.

31. What is meant by infinite solid?

A solid which extends itself infinitely in all directions of space is known as infinite solid.

In semi-infinite solids, the biot number value is in between 0.1 and 100. $0< B_i < 100$.

32. Define Biot number.

It is defined as the ratio of internal conductive resistance to the surface convective resistance.

$$B_i = \frac{\text{Internal conductive resistance}}{\text{Surface convective resistance}}$$

$$B_i = \frac{hL_L}{K}.$$

33. What is the significance of Biot number?

Biot number is used to find Lumped heat analysis, semi infinite solids and infinite solids

If $B_i < 0.1$ L $\rightarrow$ Lumped heat analysis

$B_i = \infty \rightarrow$ Semi infinite solids

1. $< B_i < 100 \rightarrow$ Infinite solids.

34. Explain the significance of Fourier number.

It is defined as the ratio of characteristic body dimension to temperature wave penetration depth in time.

$$\text{Fourier Number} = \frac{\text{Characteristic body dimension}}{\text{Temperature wave penetration depth in time}}$$

It signifies the degree of penetration of heating or cooling effect of a solid.

35. What are the factors affecting the thermal conductivity?

1. Moisture
2. Density of material
3. Pressure
4. Temperature
5. Structure of material

36. Explain the significance of thermal diffusivity.

The physical significance of thermal diffusivity is that it tells us how fast heat is propagated or it diffuses through a material during changes of temperature with time.

37. What are Heisler's charts?

In Heisler chart, the solutions for temperature distributions and heat flows in plane walls, long cylinders and spheres with finite internal and surface resistance are presented. Heisler's charts are nothing but a analytical solutions in the form of graphs.

Problems:

1. A furnace wall consists of three layers. The inner layer of 10 cm thickness is made of fiber brick (k = 1.04 W/mK). The intermediate layer is made of 25 cm thickness is made of masonry brick (k=0.69 W/mK) followed by a 5 cm thick concrete wall (k=1.37 W/mK). When the furnace is in continuous operation the inner surface of the furnace is at 800˚C while the outer concrete surface is at 50˚C. Calculate the rate of heat loss per unit area of the wall, the temperature at the interface of the fiber brick and masonry brick and the temperature at the interface of the masonry brick and concrete. **(KU – Nov 2011)**

Given data:

L_1 = 10 cm = 0.1 m,

k_1= 1.04 W/mK

L_2 = 25 cm = 0.25 m,

k_2 = 0.69 W/mK

L_3 = 5 cm = 0.05 m,

k_3 = 1.37 W/mK

T_1 = 800°C & T_4 = 50°C

To find:

Q/A = ? , T_2 = ?, T_3 = ?

Solution:

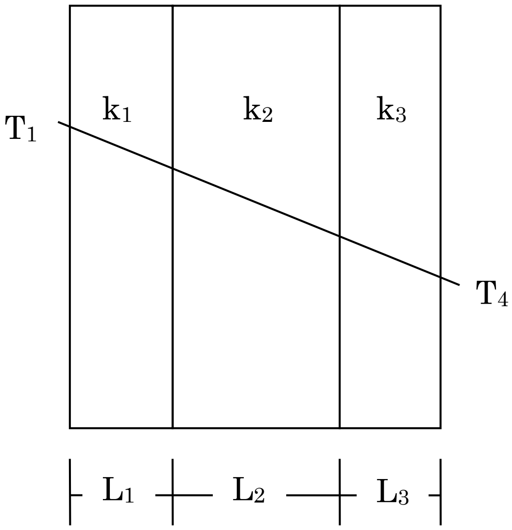
w.k.t for a composite plane wall

$$Q = \frac{T_1 - T_4}{R_{total}}, \quad \text{watt}$$

Where

$$R_{total} = \frac{L_1}{k_1\, A} + \frac{L_2}{k_2\, A} + \frac{L_3}{k_3\, A}$$

$$R_{total} = \frac{0.1}{1.04} + \frac{0.25}{0.69} + \frac{0.05}{1.37}\ , ---- (Since\ A = 1\ m^2)$$



$$R_{total} = \mathbf{0.495\ K/W}$$

$$Q = \frac{800 - 50}{0.495} \quad W$$

$$\mathbf{Q = 1515.25 \quad W}$$

To find the intermediate temperatures,

$$Q = \frac{T_1 - T_2}{R_1}, \quad \text{watt}$$

Where

$$R_1 = \frac{L_1}{k_1\ A} = \frac{0.1}{1.04} = 0.096\ K/W$$

$$1515.25 = \frac{800 - T_2}{0.096}$$

$$T_2 = 800 - (1515.25 \times 0.096)$$

$$\mathbf{T_2 = 654.5\ °C}$$

Similarly

$$Q = \frac{T_3 - T_4}{R_3}, \quad \text{watt}$$

$$R_3 = \frac{L_3}{k_3\ A} = \frac{0.05}{1.37} = 0.0365\ K/W$$

$$1515.25 = \frac{T_3 - 50}{0.0365}$$

$$T_2 = (1515.25 \times 0.0365) + 50$$

$$\mathbf{T_2 = 105.3\ °C}$$

2. An External wall of a house is made up of common brick of 10 cm thick (k = 0.7 W/mK), followed by a 4-cm thick gypsum plaster (k = 0.48 W/mK). What thickness of loosely packed insulation (k = 0.065 W/mK) should be added to reduce the heat transfer by 80%? **(KU – Nov 2012)**

Given Data:

L_1 = 10 cm = 0.1 m, k_1 = 0.7 W/mK

L_2 = 4 cm = 0.04 m, k_1 = 0.48 W/mK

k_3 = 0.065 W/mK

To find:

L_3 =? (To reduce Q by 80%)

Solution:

Let us assume that x amount of heat is transferred when there is no loosely packed insulation, there by

$$\mathrm{Q} = \frac{\Delta \mathrm{T}}{\mathrm{R_{total}}}, \quad \text{watt}$$

$$R_{total} = \frac{L_1}{k_1\, A} + \frac{L_2}{k_2\, A} = \frac{0.1}{0.7\ \times 1} + \frac{0.04}{0.48\ \times 1} = 0.226\ K/W$$

$$\therefore x = \frac{\Delta \mathrm{T}}{0.226}, \quad \text{watt} - - - - - \text{eq. 1}$$

Next let us consider that L_3 thickness of loosely packed insulation is provided there by the heat transfer is reduced by 0.8 x

$$\therefore 0.8\, x = \frac{\Delta \mathrm{T}}{0.226 + \frac{L_3}{0.065}}, \quad \text{watt} - - - - - \text{eq. 2}$$

On resolving eq.1 and eq.2 we get

$$0.226\, x = 0.8\, x \left(0.226 + \frac{L_3}{0.065}\right)$$

$$\frac{0.226}{0.8} = 0.226 + \frac{L_3}{0.065}$$

$$L_3 = \left(\frac{0.226}{0.8} - 0.226\right) 0.065$$

$$\boldsymbol{L_3 = 0.004\ m}$$

3. A composite wall consists of 10 cm thick layer of building brick, k = 0.7 W/mK and 31 cm of thick plaster, k = 0.5 W/mK. An insulating material of k = 0.08 W/mK is to be added to reduce the heat transfer through the wall by 40 %. Find its thickness. **(KU – Nov 2013, Apr 2014)**

Given Data:

L_1 = 10 cm = 0.1 m, k_1 = 0.7 W/mK

L_2 = 31 cm = 0.31 m, k_1 = 0.5 W/mK

k_3 = 0.08 W/mK

To find:

L_3 =? (To reduce Q by 40%)

Solution:

Let us assume that x amount of heat is transferred when there is no insulation, there by

$$Q = \frac{\Delta T}{R_{total}}, \quad \text{watt}$$

$$R_{total} = \frac{L_1}{k_1\, A} + \frac{L_2}{k_2\, A} = \frac{0.1}{0.7 \times 1} + \frac{0.31}{0.5 \times 1} = 0.763\ K/W$$

$$\therefore x = \frac{\Delta T}{0.763}, \quad \text{watt} - - - - - \text{eq. 1}$$

Next let us consider that L_3 thickness of loosely packed insulation is provided there by the heat transfer is reduced by 0.4 x

$$\therefore 0.4\, x = \frac{\Delta T}{0.763 + \frac{L_3}{0.08}}, \quad \text{watt} - - - - - \text{eq. 2}$$

On resolving eq.1 and eq.2 we get

$$0.763\, x = 0.4\, x \left(0.763 + \frac{L_3}{0.08}\right)$$

$$\frac{0.763}{0.4} = 0.763 + \frac{L_3}{0.08}$$

$$L_3 = \left(\frac{0.763}{0.4} - 0.763\right) 0.08$$

$$\boldsymbol{L_3 = 0.092\ m}$$

4. A temperature difference of 500 °C is applied across the fire clay brick of 10 cm thick having a thermal conductivity of 1.2 W/mK. Find the Heat flux. **(KU – Apr 2014)**

Given Data:

ΔT = 500 °C, L = 10 cm = 0.1 m, k = 1.2 W/mK

To find:

q = Q/A =?

Solution:

W.k.t

$$Q = \frac{\Delta T}{R_{total}}, \quad \text{watt}$$

where

$$R_{total} = \frac{L}{k\,A}$$

$$\therefore\ Q = \frac{\Delta T}{\frac{L}{k\,A}} \quad \Rightarrow \quad \frac{Q}{A} = q = \quad \frac{\Delta T\ k}{L}$$

$$q = \quad \frac{500\ \times 1.2}{0.1}$$

$$\mathbf{q = 6000\ W/m^2}$$

5. Find the heat flow rate through the composite wall as shown in the figure. Assume one dimensional heat flow. k_A = 150 W/m°C, k_B = 30 W/m°C, k_C = 65 W/m°C and k_D = 50 W/m°C. **(KU – April 2014)**

Given Data:

L_A = 3 cm = 0.03 m, k_A = 150 W/m°C,

A_A = 10 cm x 10 cm = 0.1 m x 0.1 m = 0.01 m^2

L_B = 8 cm = 0.08 m, k_B = 30 W/m°C,

A_B = 3 cm x 10 cm = 0.03 m x 0.1 m = 0.003 m^2

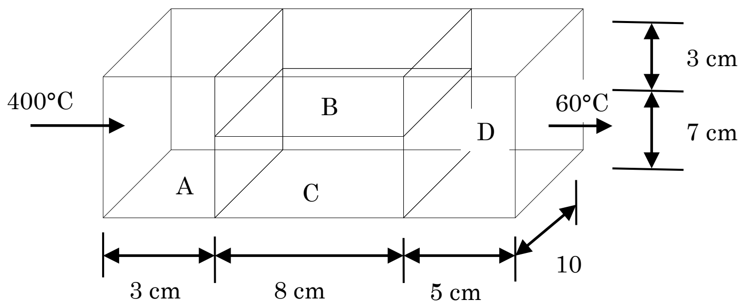
L_C = 8 cm = 0.08 m, k_C = 65 W/m°C,

A_C = 7 cm x 10 cm = 0.07 m x 0.1 m = 0.007 m^2

L_D = 5 cm = 0.05 m, k_D = 50 W/m°C,

A_D = 10 cm x 10 cm = 0.1 m x 0.1 m = 0.01 m^2

T_1 = 400 °C, T_4 = 60 °C

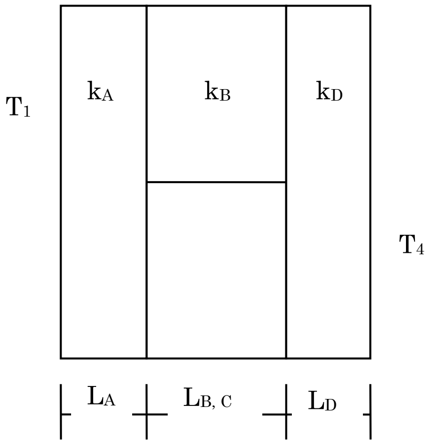


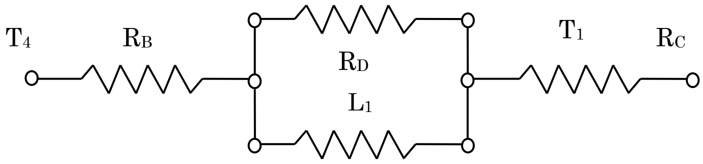
To Find:

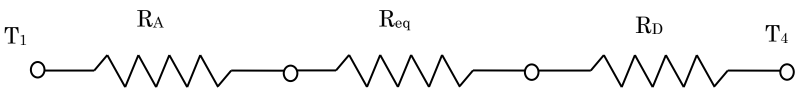
Q = ?

Solution:

w.k.t for a composite plane wall







$$Q = \frac{T_1 - T_4}{R_{total}}, \quad \text{watt}$$

Where

$$R_{total} = R_A + R_{eq} + R_D$$

$$\frac{1}{R_{eq}} = \frac{1}{R_B} + \frac{1}{R_C} = \frac{R_B + R_C}{R_B R_C}$$

$$R_{eq} = \frac{R_B R_C}{R_B + R_C}$$

$$R_A = \frac{L_A}{k_A\, A_A} = \frac{0.03}{150 \times 0.01} = 0.02\ K/W$$

$$R_B = \frac{L_B}{k_B\, A_B} = \frac{0.08}{30 \times 0.003} = 0.888\ K/W$$

$$R_C = \frac{L_C}{k_C\, A_C} = \frac{0.08}{65\ \times 0.007} = 0.176\ K/W$$

$$R_D = \frac{L_D}{k_D\, A_D} = \frac{0.05}{50\ \times 0.01} = 0.1\ K/W$$

$$\therefore\ R_{eq} = \frac{R_B R_C}{R_B +\ R_C} = \frac{0.888\ \times 0.176}{0.888 + 0.176} = 0.147\ \ K/W$$

$$R_{total} =\ 0.02 + 0.147 + 0.1$$

$$\boldsymbol{R_{total} = 0.267\ K/W}$$

$$Q = \frac{400 - 60}{0.267}\quad W$$

$$\mathbf{Q = 1273.41\quad W}$$

6. A furnace wall is composed of 220 mm of fire brick, 150 mm of common brick, 50 mm of 85% magnesia and 3 mm of steel plate on the outside. If the inside surface temperature is 1500 °C and outside surface temperature is 90°C, estimate the temperatures between the layers and calculate the heat loss in kJ/m^2h. Assume, k (for fire brick) = 4 kJ/mh°C, k (for common brick) = 2.8 kJ/mh°C, k (for 85% magnesia) = 0.24 kJ/mh°C, and k (for steel) = 240 kJ/mh°C. **(KU – August 2014)**

Given data:

L_1 = 220 mm = 0.22 m, k_1 = 4 kJ/mh°C, T_1 = 1500°C

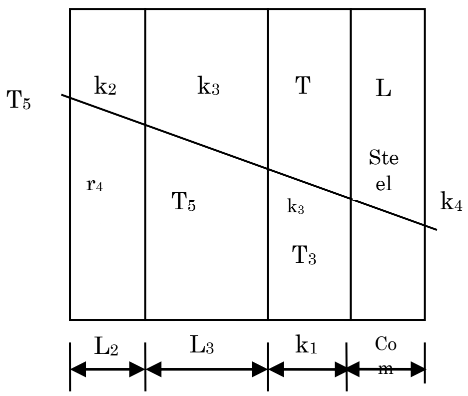
L_2 = 150 mm = 0.15 m, k_2 = 2.8 kJ/mh°C,

L_3 = 50 mm = 0.05 m, k_3 = 0.24 kJ/mh°C

L_4 = 3 mm = 0.003 m, k_4 = 240 kJ/mh°C, T_5 = 90°C

To find:

1. Q =? in kJ/m^2h
2. T_2 , T_3, T_4 =?



Solution:

Solution:

w.k.t for a composite plane wall

$$Q = \frac{T_1 - T_5}{R_{total}}$$

Where

$$R_{total} = \frac{L_1}{k_1\,A} + \frac{L_2}{k_2\,A} + \frac{L_3}{k_3\,A} + \frac{L_4}{k_4\,A}$$

$$R_{total} = \frac{0.22}{4} + \frac{0.15}{2.8} + \frac{0.05}{0.24} + \frac{0.003}{240}\ , - - - - (Since\ A = 1\ m^2)$$

$$\boldsymbol{R_{total} = 0.317\ h°C\ /\ kJ}$$

$$Q = \frac{1500 - 90}{0.317} \quad \text{kJ / h m}^2$$

$$\mathbf{Q = 4447.95\ kJ\ /\ h\ m^2}$$

To find the intermediate temperatures,

$$Q = \frac{T_1 - T_2}{R_1}, \quad \text{kJ / h m}^2$$

Where

$$R_1 = \frac{L_1}{k_1\,A} = \frac{0.22}{4} = 0.055\ h°C\ /\ kJ$$

$$4447.95 = \frac{1500 - T_2}{0.055}$$

$$T_2 = 1500 - (4447.95 \times 0.055)$$

$$\mathbf{T_2 = 1255.4\ °C}$$

Similarly for T_3

$$Q = \frac{T_2 - T_3}{R_2}, \quad \text{kJ / h m}^2$$

$$R_2 = \frac{L_2}{k_2\,A} = \frac{0.15}{2.8} = 0.054\ h°C\ /\ kJ$$

$$4447.95 = \frac{1255.4 - T_3}{0.054}$$

$$T_3 = 1255.4 - (4447.95 \times 0.054)$$

$$\mathbf{T_3 = 1015.2\ °C}$$

For T_4

$$Q = \frac{T_3 - T_4}{R_3}, \quad \text{kJ / h m}^2$$

$$R_3 = \frac{L_3}{k_3\,A} = \frac{0.05}{0.24} = 0.208\,h°C\,/\,kJ$$

$$4447.95 = \frac{1015.2 - \mathrm{T}_4}{0.208}$$

$$\mathrm{T}_4 = 1015.2 - (4447.95 \times 0.208)$$

$$\mathbf{T_4 = 90.02\ °C}$$

3. A small electric wire heating application uses wire of 2 mm diameter with 0.8 mm thick insulation (k = 0.12 W/mK). The heat transfer coefficient on the insulated surface is 35 W/m^2K. Determine the critical thickness of insulation in this case and the percentage change in the heat transfer rate if the critical thickness is used, assuming the temperature difference between the surfaces of the wire and surrounding air remains the same.**(KU – Aug 2014)**

Given Data:

d_1 = 2 mm = 0.002 m, r_1 = 0.001 m

$d_2 = d_1 + 2t$ = 0.002 m + (2 x 0.0008) = 0.0036 m, r_2 = 0.0018 m,

k = 0.12 W/mK & h_o = 35 W/m^2K

To Find:

1. Critical thickness, t_{cr} =?
2. % change in heat transfer

Solution:

We know that the critical thickness of insulation is found using

$$r_c = \frac{k}{h} = \frac{0.12}{35} = 0.0034\,m$$

$$\therefore\ t_{cr} = r_c - r_1 = 0.0034 - 0.001$$

$$\boldsymbol{t_{cr} = 0.0024\,m}$$

Case (i) Heat transfer with actual thickness of insulation (Q_{act})

We know that

$$\mathrm{Q} = \frac{\Delta \mathrm{T}}{\mathrm{R_{total}}}$$

$$\mathrm{R_{total}} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\,\pi\,k\,L} + \frac{1}{(2\,\pi\,r_2 L)\,h_o}$$

$$\mathrm{R_{total}} = \frac{ln\left(\frac{0.0018}{0.001}\right)}{2\,\pi \times\ 0.12\ \times 1} + \frac{1}{2\,\pi\ \times 0.0018\ \times 1\ \times 35}$$

$$\mathrm{R_{total}} = 3.31\ \mathrm{K/W}$$

$$\therefore\ Q = \frac{\Delta T}{3.31}$$

$$Q_{act} = 0.302\ \Delta T\ \ W$$

Case (ii) Heat transfer with critical thickness of insulation (Q_{cr})

We know that

$$Q = \frac{\Delta T}{R_{total}}$$

$$R_{total} = \frac{ln\left(\frac{r_c}{r_1}\right)}{2\ \pi\ k\ L} + \frac{1}{(2\ \pi\ r_c L)\ h_o}$$

$$R_{total} = \frac{ln\left(\frac{0.0034}{0.001}\right)}{2\ \pi \times\ 0.12\ \times 1} + \frac{1}{2\ \pi\ \times 0.0034\ \times 1\ \times 35}$$

$$R_{total} = 2.96\ K/W$$

$$\therefore\ Q = \frac{\Delta T}{2.96}$$

$$Q_{cr} = 0.337\ \Delta T\ \ W$$

% change in rate of heat transfer

$$\%\ change = 1 - \frac{Q_{act}}{Q_{cr}}$$

$$\%\ change = 1 - \frac{0.302\Delta T}{0.337\Delta T} = 0.1038$$

$$\%\ \boldsymbol{change\ in\ heat\ transfer\ rate = 10.38}\ \%$$

3. A spherical thin walled container is used to store liquid N_2 at -196 °C. The container has a diameter of 0.5 m and is covered with an evacuated reflective insulation composed of silica powder. The insulation is 25 mm thick and its outer surface is exposed to air at 27°C. The convective heat transfer coefficient on outer surface is 20 W/m^2°C. Latent heat of evaporation of N_2 is 2 x 10^5 J/kg, density of 804 kg/m^3. The thermal conductivity of silica powder is 0.007 W/m°C. Find the rate of heat transfer and rate of N_2 boil off. **(KU – Apr 2014)**

Given Data:

T_i = -196°C, d_i = 0.5 m , r_i = 0.25 m

t = 25 mm = 0.025 m, d_o = d_i + 2t = 0.55 m, r_o = 0.275 m

T_o = 27 °C, ho = 20 W/m^2°C, k = 0.007 W/m°C,

h_{fg} = 2 x 10^5 J/kg, ρ = 804 kg/m^3

To find:

1. Rate of heat transfer, Q

2. Rate of N_2 boil off, m

Solution:

We know that

$$Q = \frac{\Delta T}{R_{total}}$$

$$R_{total} = \frac{r_o - r_i}{4\,\pi\,k\,r_i r_o} + \frac{1}{\left(4\,\pi\,{r_o}^2\right)h_o}$$

$$R_{total} = \frac{0.275 - 0.25}{4\,\pi\ \times 0.007 \times 0.25 \times 0.275} + \frac{1}{(4\,\pi\ \times 0.275^2) \times 20} = 4.187\ K/W$$

$$\therefore\ Q = \frac{-196 - 27}{4.187}$$

$$\mathbf{Q = -53.26\ W}$$

We know that,

$$Q = m\ \times\ h_{fg}$$

$$\therefore m = \frac{Q}{h_{fg}} = \frac{53.26}{2\ \times\ 10^5}$$

$$\boldsymbol{m = 2.66\ \times\ 10^{-4}\ kg/sec}$$

3. A plane wall 12 mm thick generates heat at the rate of 0.05 MW/m^3, when an electric current is passed through it. The convective heat transfer coefficient between each face of the wall and the ambient air is 50 W/m^2K. Determine the surface temperature, the maximum temperature on the wall, if the ambient air temperature is 27°C and k = 15 W/mK.**(KU – Apr 2014)**

Given data:

L = 12 mm = 0.012 m, q = 0.05 MW/m^3

h = 50 W/m^2K, k = 15 W/mK, T_∞ = 27°C

To find:

1. Surface temperature, T_s
2. Maximum temperature, T_{max}

Solution:

We know that, the surface temperature T_s is given by

$$T_s = T_\infty + \frac{\dot{q}\,L}{h}$$

$$T_s = 27 + \frac{0.05\ \times 10^6\ \times 0.012}{50}$$

$$\boldsymbol{T_s = 39\ °C}$$

We know that, the maximum temperature T_{max} is given by

$$T_{max} = T_s + \frac{\dot{q}\, L^2}{2\, k}$$

$$T_{max} = 39 + \frac{0.05 \times 10^6 \times 0.012^2}{2 \times 15}$$

$$\boldsymbol{T_{max} = 39.24\ °C}$$

3. A steam pipe with inner diameter as 100 mm and 170 mm is covered with two layers of insulation. 35 mm and 50 mm thick. The thermal conductivities of the insulating materials are 0.16 W/mK and 0.085 W/mK respectively. While that of the steel pipe is 50 W/mK. The inner surface of the pipe is at 300°C and the outer surface of the insulation is 50 °C. Determine the heat loss from the pipe and layer contact temperatures. (KU – Nov 2010)

Given Data:

D_1 = 100 mm = 0.1 m, r_1 = 0.05 m, D_2 = 170 mm = 0.17 m, r_2 = 0.085 m

Insulation thickness t_1 = 35 mm = 0.035, t_2 = 50 mm = 0.05 m

Radius of first insulation, r_3 = 0.085 + 0.035 = 0.12 m

Radius of second insulation, r_4 = 0.12 + 0.05 = 0.17 m

Thermal conductivity k_1 = 50 W/mK, k_2 = 0.16 W/mK,

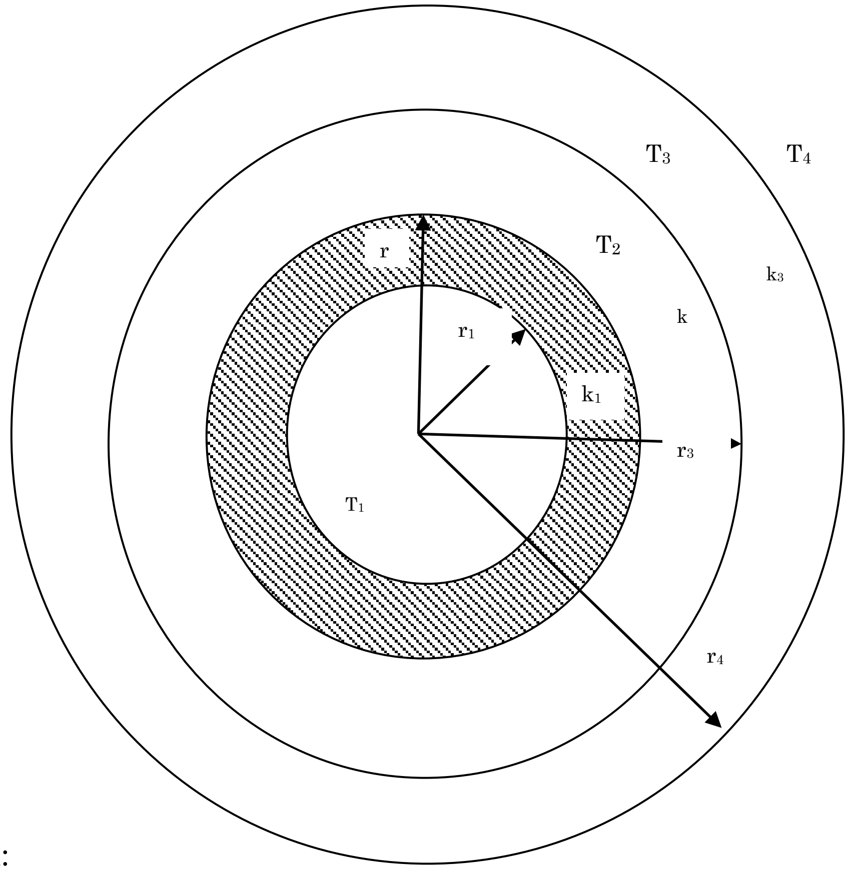
k_3 = 0.085 W/mK

Inner surface temperature, T_1 = 300°C, Outer temperature,

T_5 = 50°C

To find:

1. Q =?
2. T_2 =? & T_3 =? & T_4 =?



Solution:

We know that

$$Q = \frac{\Delta T}{R_{total}}$$

$$R_{total} = R_1 + R_2 + R_3$$

$$R_{total} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\,\pi\,k_1\,L} + \frac{ln\left(\frac{r_3}{r_2}\right)}{2\,\pi\;k_2\,L} + \frac{ln\left(\frac{r_4}{r_3}\right)}{2\,\pi\;k_3\,L}$$

$$R_{total} = \frac{ln\left(\frac{0.085}{0.05}\right)}{2\,\pi \times 50} + \frac{ln\left(\frac{0.12}{0.085}\right)}{2\,\pi \times 0.16} + \frac{ln\left(\frac{0.17}{0.12}\right)}{2\,\pi\; \times 0.085}$$

$$R_{total} = 0.0017 + 0.343 + 0.652$$

$$\mathbf{R_{total} = 0.996\ K/W}$$

$$\therefore\ Q = \frac{300 - 50}{0.996}$$

$$Q = 251\ W$$

To find the intermediate temperatures

$$Q = \frac{T_1 - T_2}{R_1}$$

$$T_2 = T_1 - (Q \times R_1)$$

$$T_2 = 300 - (251 \times 0.0017)$$

$$\mathbf{T_2 = 299.57\ °C}$$

$$Q = \frac{T_2 - T_3}{R_2}$$

$$T_3 = T_2 - (Q \times R_2)$$

$$T_3 = 299.57 - (251 \times 0.343)$$

$$\mathbf{T_3 = 213.5\ °C}$$

3. One thousand spheres made of copper of diameter 5 mm are annealed in an annealing furnace. The initial temperature of the sphere is 20 °C. Temperature of the annealing furnace is 400 °C. Take h = 30 W/m^{2}K, c_p = 0.32 kJ/kgK, ρ = 3200 kg/m^3. Find the time required for the spheres to reach a temperature of 300°C. **(KU – Nov 2010)**

Given Data:

No of spheres = 1000, d = 5 mm = 0.005 m, T_o = 20°C, T_∞ = 400°C

h = 30 W/m^{2}K, c_p = 0.32 kJ/kgK, ρ = 3200 kg/m^3

To find:

t =? For spheres to reach T = 300°C

Solution:

Volume of the sphere, V

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi \times 0.0025^3$$

$$V = 2.618 \times 10^{-5}, m^3$$

Total volume of the spheres, V = 2.168 x 10^{-5} x 1000 = 0.0262 m^3

Characteristic length,

$$L_c = \frac{r}{3} = \frac{0.0025}{3} = 0.00083\ m$$

To calculate Biot number, B_i

$$B_i = \frac{h\,L_c}{k}$$

$$B_i = \frac{30 \times 0.00083}{386} = 0.000065$$

$$B_i = 0.000065 < 0.1$$

Hence the system belongs to lumped heat analysis

We know that for a lumped parameter system

$$\frac{T - T_\infty}{T_o - T_\infty} = e^{-\frac{h\,A_s}{\rho\,V\,C_p}t}$$

$$\frac{300 - 400}{20 - 400} = e^{-\frac{30}{2300 \times 0.32 \times 10^3 \times 0.00083}t}$$

On solving

$$\boldsymbol{t = 2718.9\ seconds = 45.32\ mins}$$

4. A cylinder 1 m long and 5 cm in diameter is placed in an atmosphere at 45 °C. It is provided with 10 longitudinal straight fins of material having k = 120 W/mK. The height of 0.76 mm thick fins is 1.27 cm from the cylinder surface. The heat transfer coefficient between cylinder and atmosphere is 17 W/m^{2}K. Calculate the rate of heat transfer and the temperature at the end of fins if surface temperature of the cylinder is 150 °C. **(KU – Nov 2011, Nov 2012, Apr 2014)**

Given Data:

L = 1 m, d = 5 cm = 0.05 m, T_∞ = 45°C

No of fins = 10, k = 120 W/mK, length of the fin, l = 1.27 cm = 0.0127 m,

Thickness of fin, t = 0.76 mm = 0.76x10^{-3} m

h = 17 W/m^{2}K, T_b = 150°C

To find:

1. Rate of heat transfer, Q
2. Temperature at end of the fin, T

Solution:

To find rate of heat transfer

$$Q = \sqrt{h\,p\,k\,A}\,(T_b - T_\infty)\,tanh(m\,L)$$

$$where, \qquad m = \sqrt{\frac{hp}{k\,A}}$$

$$perimeter, \qquad p = 2 \times length\ of\ the\ cylinder$$

$$p = 2 \times 1 = 2\,m$$

$$Area\ of\ the\ fin = length\ of\ the\ cylinder \times thickness\ of\ the\ fin = L \times t$$

$$A = 1 \times 0.76 \times 10^{-3} = 0.76 \times 10^{-3}\,m^2$$

$$\therefore\ m = \sqrt{\frac{17 \times 2}{120 \times 0.76 \times 10^{-3}}} = 19.3\, m^{-1}$$

We know that for a fin the rate of heat transfer is given by,

$$Q = \sqrt{17 \times 2 \times 120 \times 0.76 \times 10^{-3}}\ (150 - 45)\, tanh(19.3 \times 0.0127)$$

$$Q = 42.643\, W\ per\ fin$$

$$therefore\ for\ 10\ longitudinal\ fins, Q = 42.643\, W\ per\ fin \times 10$$

$$\therefore \boldsymbol{Q = 426.43\ W}$$

(ii) to find the temperature at end of the fins, is given by

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{cosh\ [m\,(L - x)]}{cosh\ (m\,L)}$$

$$\frac{T - 45}{150 - 45} = \frac{cosh\ [19.3 \times (0.0127 - 0.0127)]}{cosh\ (19.3 \times 0.0127)}$$

$$\boldsymbol{T = 146.9\ °C}$$

3. An electrical wire of 10 m length and 1 mm diameter dissipates 200 W in air at 25 °C. The convection heat transfer coefficient between the wire surface and air is 15 W/m^{2}K. Calculate the critical radius of insulation and also determine the temperature of the wire if it is insulated to the critical radius of insulation. Take k = 12 W/mK

Given Data:

L = 10 m, d_1 = 1 mm = 0.001 m, r_1 = 0.0005 m, Q = 200 W, T_o = 25 °C

k = 12 W/mK & h_o = 15 W/m^{2}K

To Find:

4. Critical thickness, t_{cr} =?
5. Temperature of the wire if it is insulated to t_{cr}

Solution:

We know that the critical thickness of insulation is found using

$$r_c = \frac{k}{h} = \frac{12}{35} = 0.343\, m$$

$$\therefore\ t_{cr} = r_c - r_1 = 0.343 - 0.0005$$

$$\boldsymbol{t_{cr} = 0.3425\ m}$$

We know that

$$Q = \frac{T_i - T_o}{R_{total}}$$

$$R_{\text{total}} = \frac{ln\left(\frac{r_c}{r_1}\right)}{2\,\pi\,k\,L} + \frac{1}{(2\,\pi\,r_c L)\,h_o}$$

$$R_{\text{total}} = \frac{ln\left(\frac{0.343}{0.0005}\right)}{2\,\pi \times\ 12\ \times 10} + \frac{1}{2\,\pi\ \times 0.343\ \times 10\ \times 15}$$

$$R_{\text{total}} = 0.012\ \text{K/W}$$

$$\therefore\ Q = \frac{T_i - T_o}{0.012}$$

$$200 = \frac{T_i - 25}{0.012}$$

$$\mathbf{T_i = 27.35\ °C}$$

MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	answer
Heat is defined as ________________________	Temperature	Enthalpy	Entropy	Energy in transit	Energy in transit
Heat _____________________	Can not be seen	Can be seen	Can not be predicted	Can be predicted	Can be predicted
____________________occurs whenever two bodies at different temperatures are brought in contact with each other.	Work transfer	Heat transfer	Molecular transfer	Mass transfer	Heat transfer
______________________occurs whenever there is a temperature gradient within a body.	Mass transfer	Molecular transfer	Work transfer	Heat transfer	Heat transfer
First law of thermodynamics deals with________________________	Availability	Equilibrium states	Rate of heat transfer	temperature variation with time	Equilibrium states
Second law of thermodynamics deals with________________________	Availability	Equilibrium states	Rate of heat transfer	temperature variation with time	Availability
Heat transfer deals with________________________	Availability	Equilibrium states	Rate of heat transfer & temperature variation with time	Efficiency	Rate of heat transfer & temperature variation with time
Equation of continuity deals	Availability	Equilibrium states	Rate of heat transfer & temperature variation with time	Conservation of mass	Conservation of mass
The driving potential for heat transfer is _________________	Pressure gradient	Temperature gradient	Viscosity gradient	Molecular gradient	Temperature gradient
Heat transfer can be measure as a property called ___________________	Hotness	Enthalpy	Entropy	Temperature	Temperature
Conduction is a ________________phenomenon.	Microscopic	Macroscopic	Diffuse	Specular	Microscopic

Conduction

In a solid, transfer of energy occurs by ___________________	Lattice vibration or free electrons vibration	Density difference	Packets of photons	Electro magnetic waves	Lattice vibration or free electrons vibration
In a liquid or gas, transfer of energy occurs by_______________	Lattice vibration or free electrons vibration	Density difference	Packets of photons	Electro magnetic waves	Density difference
The value of thermal conductivity of a material indicates ____________	The temperature withstanding ability	Insulation property	Heat storage property	The ability of the material to conduct heat	The ability of the material to conduct heat
Convection is a ____________________phenomenon.	Microscopic	Macroscopic	Diffuse	Specular	Macroscopic
If two fluids at different temperatures are mixed together, heat transfer occurs by ______	Conduction	Convection	Radiation	Mass transfer	Convection
Fluid motion occurs by density difference is known as ______________	Conduction	Convection	Radiation	Mass transfer	Convection
Density difference in fluid may occur due to ___________________	Temperature difference	Pressure difference	Viscosity difference	Flow variation	Temperature difference
Fluid motion may occur by density differences caused by temperature differences is known as	Natural convection	Forced convection	Boiling	Condensation	Natural convection
Heat transfer coefficient, 'h' is	A property of the surface material	A property of the fluid	Not a property of the surface material and of the fluid	depends on flow condition	Not a property of the surface material and of the fluid
The property of the system is constant with respect to time is known as	Steady state	Unsteady state	Solid state	Liquid state	Steady state
The property of the system is varying with respect to time is known as	Steady state	Unsteady state	Solid state	Liquid state	Unsteady state
Fourier's law is defined for	Steady state, one dimensional heat flow	Steady state	One dimensional	Unsteady state	Steady state, one dimensional heat flow
Fourier's law is applicable to ____________states of matter.	Solid	Liquid	Gaseous	All	Solid

Conduction

Thermal conductivity of materials depends upon	Material structure & density of material	Moisture content and pressure	Temperature	All of these	All of these
Heat treatment of pure metals ____the value of thermal conductivity	Increases	Decreases	Does not have any effect	All	Decreases
Thermal conductivity of alloy generally___as temperature increases	Increases	Decreases	Does not have any effect	All	Increases
k / ρ cp is known as	Thermal conductivity	Thermal diffusivity	Heat capacity	Velocity temperature	Thermal diffusivity
Thermal diffusivity indicates	The ability of the material to conduct heat	The ability of the material to store heat	The ability of the material to withstand heat	The ability of the material to reject heat	The ability of the material to store heat
The unit of thermal diffusivity is	m2 /s	s / m2	kg / m3	m3 / kg	m2 /s
A wall made up of different thermal conductivity material is known as	Composite wall	Brick wall	Insulation wall	Guard wall	Composite wall
Temperature distribution is used to find out	Temperature of the atmosphere	Temperature of material	Temperature at any location in the material	Temperature of medium	Temperature at any location in the material
If the insulation material radius on a pipe is less than critical radius then the heat	loss is more	loss is less	loss is constant	will be generated	loss is more
Extended surface is used to	Increase the heat transfer surface area	Decrease the heat transfer surface area	Generate heat	Absorb heat	Increase the heat transfer surface area

CHAPTER – 2

CONVECTION

In the previous chapter, we have considered *conduction,* which is the mechanism of heat transfer through a solid or a quiescent fluid. We now consider *convection,* which is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion.

Convection is classified as *natural* (or *free*) and *forced convection,* depending on how the fluid motion is initiated. In forced convection, the fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, which manifests itself as the rise of warmer fluid and the fall of the cooler fluid. Convection is also classified as *external* and *internal,* depending on whether the fluid is forced to flow over a surface or in a channel.

2.1. PHYSICAL MECHANISM OF CONVECTION

Heat transfer through a solid is always by conduction, since the molecules of a solid remain at relatively fixed positions. Heat transfer through a liquid or gas, however, can be by conduction or convection, depending on the presence of any bulk fluid motion. Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it. Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid.

Convection heat transfer strongly depends on the fluid properties dynamic viscosity μ, thermal conductivity k, density ρ, and specific heat C_p, as well as the fluid velocity V. It also depends on the geometry and the roughness of the solid surface, in addition to the type of fluid flow (such as streamlined or turbulent). The rate of convection heat transfer is observed to be proportional to the temperature difference and is conveniently expressed by Newton's law of cooling as

$$Q = h\, A_s\, (T_s - T_\infty)$$

Where

h – Heat transfer coefficient, W/m^{2}K, A_s – surface area, m^2

T_s - Surface temperature, ºC

T_∞ - ambient temperature, ºC

The convection heat transfer coefficient h can be defined as the rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.

2.1.1. NUSSELT NUMBER:

In convection studies, it is common practice to nondimensionalize the governing equations and combines the variables, which group together into dimensionless numbers in order to reduce the number of total variables. It is also common practice to nondimensionalize the heat transfer coefficient h with the Nusselt number, defined as

$$N_u = \frac{h\,L_c}{k}$$

Where k is the thermal conductivity of the fluid and L_c is the characteristic length. The Nusselt number is named after Wilhelm Nusselt, who made significant contributions to convective heat transfer in the first half of the twentieth century, and it is viewed as the dimensionless convection heat transfer coefficient.

2.1.3. VELOCITY BOUNDARY LAYER

Consider the parallel flow of a fluid over a flat plate, as shown in Fig. 2.1. Surfaces that are slightly contoured such as turbine blades can also be approximated as flat plates with reasonable accuracy. The x-coordinate is measured along the plate surface from the leading edge of the plate in the direction of the flow, and y is measured from the surface in the normal direction. The fluid approaches the plate in the x-direction with a uniform upstream velocity of V, which is practically identical to the free-stream velocity u_∞ over the plate away from the surface (this would not be the case for cross flow over blunt bodies such as a cylinder).

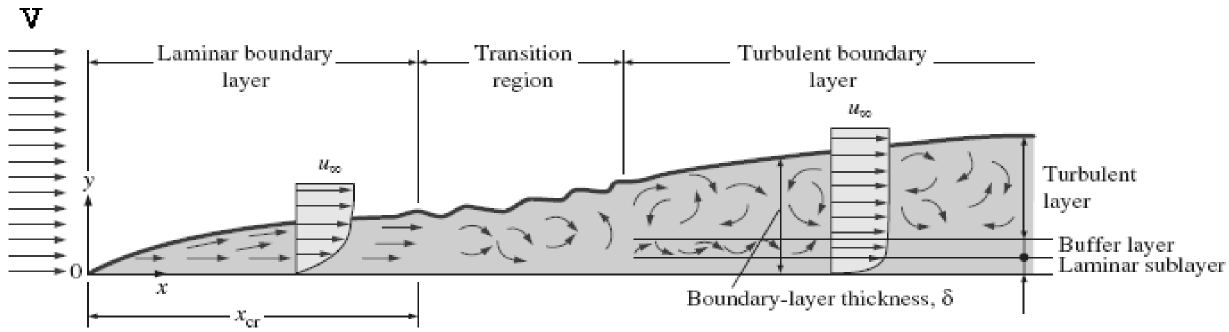


Fig .2.1. The development of the boundary layer for flow over a flat plate, and the different flow regimes.

When a fluid is forced to flow over a solid surface that is nonporous (i.e., impermeable to the fluid), it is observed that the fluid in motion comes to a complete stop at the surface and assumes a zero-velocity relative to the surface. That is, the fluid layer in direct contact with a solid surface "sticks" to the surface and there is no slip. In fluid flow, this phenomenon is known as the no-slip condition, and it is due to the viscosity of the fluid.

The region of the flow above the plate bounded by □ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the velocity boundary layer. The boundary layer thickness, □ is typically defined as the distance y from the surface at which $u = 0.99u_\infty$.

The hypothetical line of $u = 0.99u_\infty$ divides the flow over a plate into two regions: the boundary layer region, in which the viscous effects and the velocity changes are significant, and the inviscid flow region, in which the frictional effects are negligible and the velocity remains essentially constant.

2.1.4. THERMAL BOUNDARY LAYER:

Consider the flow of a fluid at a uniform temperature of T_∞ over an isothermal flat plate at temperature T_s. The fluid particles in the layer adjacent to the surface will reach thermal equilibrium with the plate and assume the surface temperature T_s. These fluid particles will then exchange energy with the particles in the adjoining-fluid layer, and so on. As a result, a temperature profile will develop in the flow field that ranges from T_s at the surface to T_∞ sufficiently far from the surface. The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the thermal boundary layer. The thickness of the thermal boundary layer □$_t$ at any location along the surface is defined as the distance from the surface at which the temperature difference T - Ts equals $0.99(T_\infty - Ts)$.

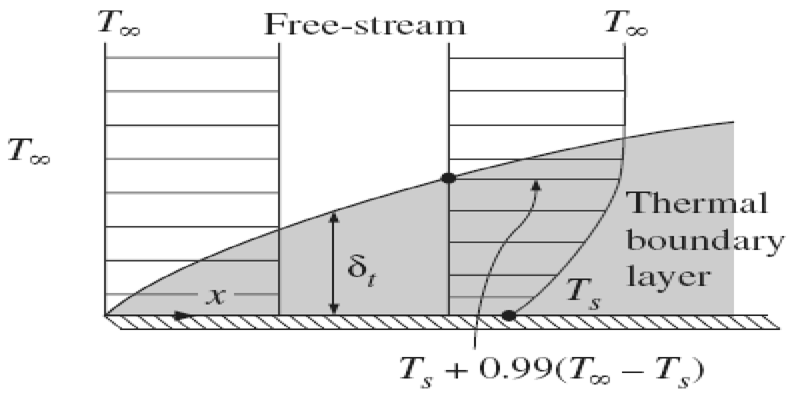


Fig.2.2. Thermal boundary layer on flat plate

2.1.5. PRANDTL NUMBER:

The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless parameter Prandtl number, defined as

$$P_r = \frac{Molecular\ diffusivity\ of\ momentum}{Molecular\ diffusivity\ of\ heat} = \frac{\nu}{\alpha} = \frac{\mu\, C_p}{k}$$

2.1.6. LAMINAR & TURBULENT FLOW

The highly ordered fluid motion characterized by smooth streamlines is called laminar. The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities characterized by velocity fluctuations is called turbulent. The flow of low-viscosity fluids such as air at high velocities is typically turbulent. The flow regime greatly influences the heat transfer rates and the required power for pumping.

2.1.7. REYNOLDS NUMBER

The transition from laminar to turbulent flow depends on the surface geometry, surface roughness, free-stream velocity, surface temperature, and type of fluid, among other things. After exhaustive experiments in the 1880s, Osborn Reynolds discovered that the flow regime depends mainly on the ratio of the inertia forces to viscous forces in the fluid. This ratio is called the Reynolds number, which is a dimensionless quantity, and is expressed for external flow as

$$R_e = \frac{Inertia\ Forces}{viscous\ forces} = \frac{V\,L_c}{\nu} = \frac{\rho\,V\,L_c}{\mu}$$

Where V is the upstream velocity (equivalent to the free-stream velocity u_∞ for a flat plate), L_c is the characteristic length of the geometry, and ν is the kinematic viscosity of the fluid. For a flat plate, the characteristic length is the distance x from the leading edge.

At large Reynolds numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid. At small Reynolds numbers, however, the viscous forces are large enough to overcome the inertia forces and to keep the fluid "in line." Thus the flow is turbulent in the first case and laminar in the second. The Reynolds number at which the flow becomes turbulent is called the critical Reynolds number. The value of the critical Reynolds number is different for different geometries.

2.2. EXTERNAL FORCED CONVECTION:

Forced convection to or from flat or curved surfaces subjected to external flow, characterized by the freely growing boundary layers surrounded by a free flow region that involves no velocity and temperature gradients.

2.2.1. FRICTION & PRESSURE DRAG

The force a flowing fluid exerts on a body in the flow direction is called drag. A stationary fluid exerts only normal pressure forces on the surface of a body immersed in it. A moving fluid, however, also exerts tangential shear forces on the surface because of the no-slip condition caused by viscous effects. Both of these forces, in general, have components in the direction of flow, and thus the drag force is due to the combined effects of pressure and wall shear forces in the flow direction. The components of the pressure and wall shear forces in the normal direction to flow tend to move the body in that direction, and their sum is called lift. The drag force F_D depends on the density ρ of the fluid, the upstream velocity V, and the size, shape, and orientation of the body, among other things.

The drag characteristics of a body is represented by the dimensionless drag coefficient C_D defined as

$$C_D = \frac{F_D}{\frac{1}{2}\,\rho\,V^2 A}$$

Where A is the frontal area (the area projected on a plane normal to the direction of flow) for blunt bodies – bodies that tends to block the flow. The frontal area of a cylinder of diameter D and length L, for example, is A = LD.

The friction drag is the component of the wall shear force in the direction of flow, and thus it depends on the orientation of the body as well as the magnitude of the wall shear stress τ_w. The friction drag is zero for a surface normal to flow, and maximum for a surface parallel to flow since the friction drag in this case equals the total shear force on the surface. Therefore, for parallel flow over a flat plate, the drag coefficient is equal to the friction drag coefficient, or simply the friction coefficient.

The fluid temperature in the thermal boundary layer varies from T_s at the surface to about T_∞ at the outer edge of the boundary. The fluid properties also vary with temperature, and thus with position across the boundary layer. In order to account for the variation of the properties with temperature, the fluid properties are usually evaluated at the so-called film temperature, defined as

$$T_f = \frac{T_s + T_\infty}{2}$$

which is the arithmetic average of the surface and the free-stream temperatures. The fluid properties are then assumed to remain constant at those values during the entire flow.

2.2.2. PARALLEL FLOW OVER FLAT PLATES

Consider the parallel flow of a fluid over a flat plate of length L in the flow direction, as shown in Figure 2.3. The x-coordinate is measured along the plate surface from the leading edge in the direction of the flow. The fluid approaches the plate in the x-direction with uniform upstream velocity V and temperature T_∞. The flow in the velocity boundary layer starts out as laminar, but if the plate is sufficiently long, the flow will become turbulent at a distance x_{cr} from the leading edge where the Reynolds number reaches its critical value for transition.

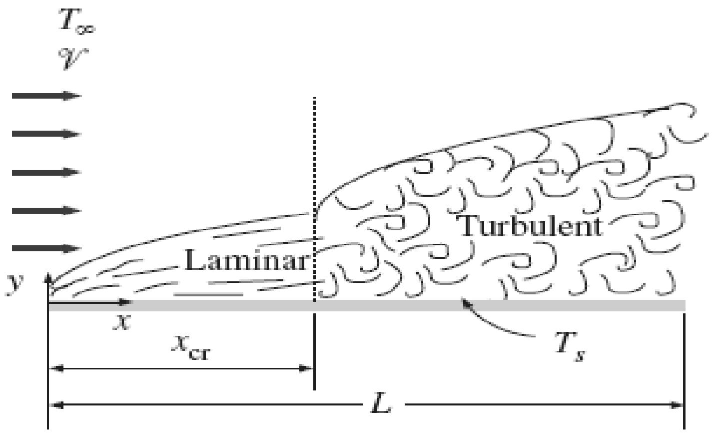


Fig. 2.3 Flow over Flat plate

The Reynolds number at a distance x from the leading edge of a flat plate is expressed as

$$R_e = \frac{V\,x}{\nu} = \frac{\rho\,V\,x}{\mu}$$

Note that the value of the Reynolds number varies for a flat plate along the flow, reaching R_{eL} = VL /ν at the end of the plate.

For flow over a flat plate, transition from laminar to turbulent is usually taken to occur at the critical Reynolds number of $R_{e,cr}$ 5 x 10^5

If $R_{e,x} < 5 \times 10^5$, the flow is laminar

If $R_{e,x} > 5 \times 10^5$, the flow is turbulent

(i) The hydrodynamic boundary layer thickness or the velocity boundary layer thickness, □$_{h,x}$

a) Laminar flow,

$$\delta_{h,x} = 5.\,x.\left(R_{e,x}\right)^{-0.5}$$

b) Turbulent flow,

$$\delta_{h,x} = 0.382\,.\,x.\left(R_{e,x}\right)^{-0.2}$$

(ii) The thermal boundary layer thickness, □$_{T,x}$

a) Laminar flow,

$$\delta_{T,x} = \delta_{h,x}(P_r)^{0.333}$$

b) Turbulent flow,

$$\delta_{T,x} = \delta_{h,x}$$

(iii) Local friction coefficient, $C_{f,x}$

a) Laminar flow,

$$C_{f,x} = 0.664\ \left(R_{e,x}\right)^{-0.5}$$

b) Turbulent flow,

$$C_{f,x} = 0.0592\ \left(R_{e,x}\right)^{-0.2}$$

(iv) Average Friction coefficient, $C_{f,L}$

a) Laminar flow,

$$C_{f,L} = 1.328\ \left(R_{e,x}\right)^{-0.5}$$

b) Turbulent flow,

$$C_{f,L} = 0.074\ \left(R_{e,x}\right)^{-0.2}$$

(v) Local Nusselt Number, $N_{u,x}$ & Local Heat transfer Coefficient, h_x

a) Laminar flow,

$$N_{u,x} = 0.332\ \left(R_{e,x}\right)^{0.5} (P_r)^{0.333}$$

$$N_{u,x} = \frac{h_x\ x}{k}$$

b) Turbulent flow,

$$N_{u,x} = 0.0296\ \left(R_{e,x}\right)^{0.8} (P_r)^{0.333}$$

$$N_{u,x} = \frac{h_x\ x}{k}$$

(vi) Average Nusselt Number, $N_{u,L}$ & Average Heat transfer coefficient, h

a) Laminar flow,

$$N_{u,L} = 2\ \left[0.332\ \left(R_{e,x}\right)^{0.5} (P_r)^{0.333}\right] = 0.664 \left(R_{e,x}\right)^{0.5} (P_r)^{0.333}$$

$$N_{u,L} = \frac{h\ L}{k}\ (or)\ \ h = 2\ h_x$$

b) Turbulent flow,

$$N_{u,L} = 1.25\ \left[0.0296\ \left(R_{e,x}\right)^{0.8} (P_r)^{0.333}\right] = 0.037\ \left(R_{e,x}\right)^{0.8} (P_r)^{0.333}$$

$$N_{u,L} = \frac{h\ L}{k}\ (or) h = 1.25\ h_x$$

For combined Laminar & Turbulent flow

(i) Average Friction coefficient, $C_{f,L}$

$$C_{f,L} = 0.074\ \left(R_{e,L}\right)^{-0.2} -\ 1742 \left(R_{e,L}\right)^{-1.0}$$

(ii) Average Nusselt Number, $N_{u,L}$ & Average Heat transfer coefficient, h

$$N_{u,L} = (P_r)^{0.333}\left[0.037\ \left(R_{e,x}\right)^{0.8} - 871\right]$$

$$N_{u,L} = \frac{h\,L}{k}$$

The rate of heat transfer can be found using the Newton's law of cooling,

$$Q = h\,A_s\,(T_s - T_\infty)$$

Where

h – Average Heat transfer coefficient, W/m^{2}K

A_s – surface area, m^2

T_s - Surface temperature, ºC

T_∞ - ambient temperature, ºC

2.2.3. FLOW OVER CYLINDERS & SPHERES

Flow across cylinders and spheres are frequently encountered in practical usage. For example, the tubes in a shell-and-tube heat exchanger involve both internal flow through the tubes and external flow over the tubes, and both flows must be considered in the analysis of the heat exchanger. Also, many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the external diameter D. Thus, the Reynolds number is defined as R_e = V D/ν where V is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $R_{e,cr}$ = 2 x10^5. That is, the boundary layer remains laminar for about $R_e \lesssim 2 \times 10^5$ and becomes turbulent for $R_e \gtrsim 2 \times 10^5$.

(i) Flow across cylinder

The Average Nusselt Number, $N_{u,L}$ & Average Heat transfer coefficient, h is given as,

$$N_{u,L} = C\,(R_e)^m (P_r)^{0.333}$$

Where the C & m are constants which varies based on the cross sectional features.

$$N_{u,L} = \frac{h\,L}{k}$$

The rate of heat transfer can be found using the Newton's law of cooling,

$$Q = h\,A_s\,(T_s - T_\infty)$$

Where

h – Average Heat transfer coefficient, W/m^{2}K

A_s – surface area, m^2 (π D L)

T_s - Surface temperature, ºC

T_∞ - ambient temperature, ºC

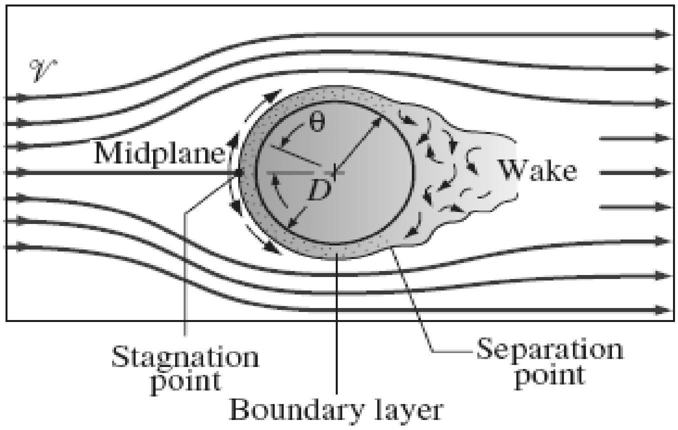


Fig.2.4. Typical flow pattern in cylinder under cross flow

(ii) Flow across sphere

The Average Nusselt Number, $N_{u,L}$ & Average Heat transfer coefficient, h is given as,

$$N_{u,L} = C\,(R_e)^m (P_r)^{0.333}$$

Where the C & m are constants which varies based on the cross sectional features.

$$N_{u,L} = \frac{h\,L}{k}$$

The rate of heat transfer can be found using the Newton's law of cooling,

$$Q = h\,A_s\,(T_s - T_\infty)$$

Where

h – Average Heat transfer coefficient, W/m^2K

A_s – surface area, m^2 ($4\,\pi\,r^2$)

T_s - Surface temperature, ºC

T_∞ - ambient temperature, ºC

2.2.4. FLOW OVER BANK OF TUBES

In heat transfer equipment such as the condensers and evaporators of power plants, refrigerators, and air conditioners. In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.

The tubes in a tube bank are usually arranged either in-line or staggered in the direction of flow, as shown in Figure 7–26. The outer tube diameter D is taken as the characteristic length. The arrangement of the tubes in the tube bank is characterized by the transverse pitch S_T, longitudinal pitch S_L, and the diagonal pitch S_D between tube centers. The diagonal pitch is determined from

$$S_D = \sqrt{{S_L}^2 + \left(\frac{S_T}{2}\right)^2}$$

As the fluid enters the tube bank, the flow area decreases from $A_1 = S_T L$ to $A_T = (S_T - D)\,L$ between the tubes, and thus flow velocity increases. In staggered arrangement, the velocity may increase further in the diagonal region if the tube rows are very close to each other. In tube banks, the flow characteristics are dominated by the maximum velocity V_{max} that occurs within the tube bank rather than the approach velocity V. Therefore, the Reynolds number is defined on the basis of maximum velocity as

$$R_{e,D} = \frac{V_{max}\, D}{\nu} = \frac{\rho\, V_{max}\, D}{\mu}$$

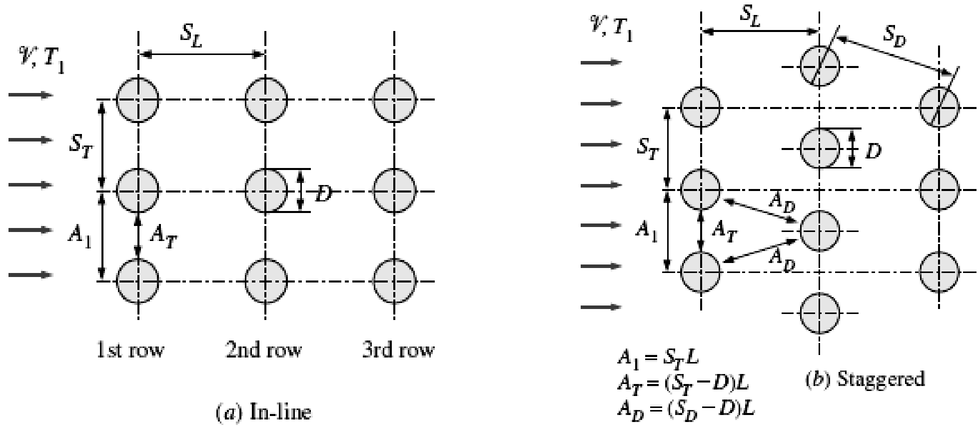


Fig. 2.5.

For inline arrangement,

$$V_{max} = \frac{S_T}{S_T - D}\, V$$

For staggered arrangement,

$$V_{max} = \frac{S_T}{2\,(S_D - D)}\, V$$

The Average Nusselt Number, $N_{u,L}$ & Average Heat transfer coefficient, h is given as,

$$N_u = 1.13\ C\,(R_e)^n (P_r)^{0.333}$$

Where the C & n are constants which can be selected from the table according to S_T/D and S_L/D

$$N_u = \frac{h\,D}{k}$$

The rate of heat transfer can be found using the Newton's law of cooling,

$$Q = h\,A_s\,(T_s - T_\infty)$$

Where

h – Average Heat transfer coefficient, W/m^{2}K

A_s – surface area, m^2 ($4\,\pi\,r^2$)

T_s - Surface temperature, ºC

T_∞ - ambient temperature, ºC

2.3. INTERNAL FORCED CONVECTION

The fluid velocity in a tube changes from zero at the surface because of the no-slip condition, to a maximum at the tube center. Therefore, it is convenient to work with an average or mean velocity V_m, which remains constant for incompressible flow when the cross-sectional area of the tube is constant.

The fluid properties in internal flow are usually evaluated at the bulk mean fluid temperature, which is the arithmetic average of the mean temperatures at the inlet and the exit.

$$T_{mf} = \frac{T_{mi} + T_{mo}}{2}$$

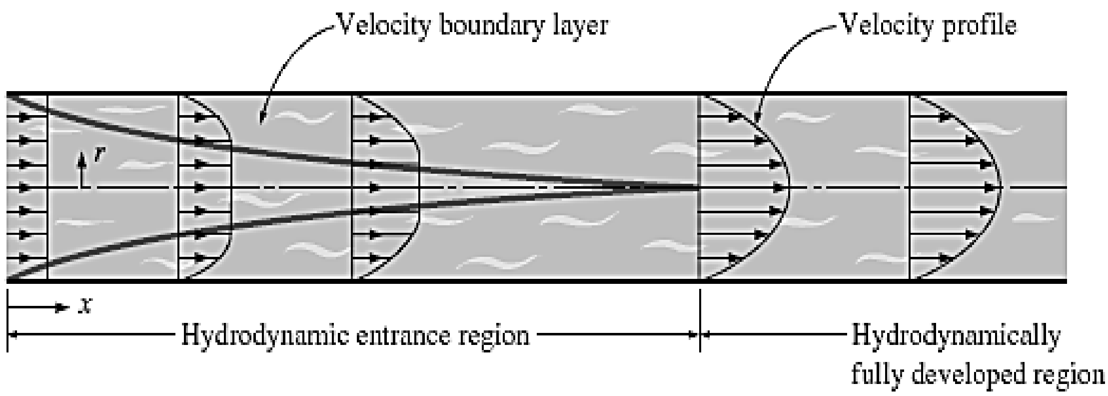


Fig.2.6. Velocity boundary for flow inside a tube

The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the hydrodynamic entrance region, and the length of this region is called the hydrodynamic entry length L_h.

Flow in the entrance region is called hydrodynamically developing flow since this is the region where the velocity profile develops. The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged is called the hydrodynamically fully developed region. The velocity profile in the fully developed region is parabolic in laminar flow and somewhat flatter in turbulent flow due to eddy motion in radial direction.

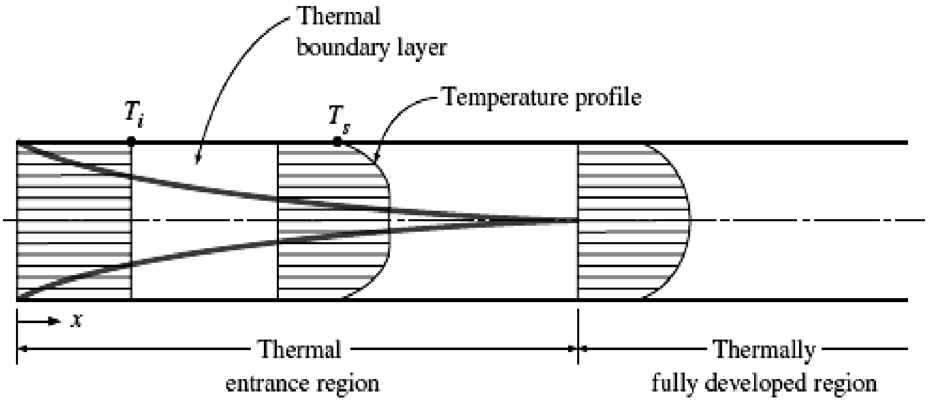


Fig.2.7. Thermal boundary layer

The region of flow over which the thermal boundary layer develops and reaches the tube center is called the thermal entrance region, and the length of this region is called the thermal entry length L_t. Flow in the thermal entrance region is called thermally developing flow since this is the region where the temperature profile develops. The region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_s - T) / (T_s - T_m)$ remains unchanged is called the thermally fully developed region. The region in which the flow is both hydrodynamically and thermally developed and thus both the velocity and dimensionless temperature profiles remain unchanged is called fully developed flow.

For flow in a circular tube, the Reynolds number is defined as

$$R_e = \frac{V\,D}{\nu} = \frac{\rho\,V\,D}{\mu}$$

where V_m is the mean fluid velocity, D is the diameter of the tube, and ν or μ/ρ is the kinematic viscosity of the fluid.

For flow through noncircular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the hydraulic diameter D_h defined as

$$D_h = \frac{4\,A_c}{P}$$

where A_c is the cross sectional area of the tube and P is its perimeter. Under most practical conditions, the flow in a tube is laminar for Re < 2300, turbulent for Re > 10,000, and transitional for 2300 > Re < 10,000.

Nusselt Number, N_u & Heat transfer Coefficient, h

a) Laminar flow,

$N_u = 4.36$, *for circular tube with constant surface heat flux*, $\dot{q}_x$

$N_u = 3.66$, *for circular tube with constant surface temperature*, T_s

b) Turbulent flow,

$$N_u = 0.023 \ (R_e)^{0.8} (P_r)^n$$

Where, n = 0.4 for heating process & n = 0.3 for cooling process, and this relation is valid when

$$0.6 < P_r < 160, \qquad R_e > 10000, \quad \& \quad \frac{L}{D} > 60$$

If the above conditions are not valid, then the other Nusselt's relation is used

$$N_u = 0.036 \ (R_e)^{0.8} (P_r)^{0.333} \left(\frac{D}{L}\right)^{0.055}$$

$$when\ R_e < 10000, \quad \& \quad 10 < \frac{L}{D} < 400$$

And the heat transfer coefficient is given as

$$N_u = \frac{h\, D_h}{k}$$

The rate of heat transfer can be found using the Newton's law of cooling,

$$Q = h\, A_s \left(T_s - T_{mf}\right)$$

$$A_s = \pi\, D_h\, L$$

or

$$Q = m\, C_p\, (T_{mo} - T_{mi})$$

The mass flow rate,

$$m = \rho\, A\, V, \qquad kg/sec$$

$$A = \frac{\pi}{4}\, {D_h}^2$$

The hydraulic diameter, D_h for various cross sections are as follows,

(i) For circular tube or pipe of diameter, D

$$D_h = D$$

(ii) For a rectangular duct of length, L and width w

$$D_h = \frac{4\,A_c}{P} = \frac{4\,(L \times w)}{(L + w)}$$

(iii) For a hollow cylindrical pipe of inner & outer diameter, D_i & D_o

$$D_h = \frac{4\,A_c}{P} = \frac{4\,\left(\frac{\pi}{4}\left[D_o{}^2 - D_i{}^2\right]\right)}{\pi\,(D_o + D_i)} = D_o + D_i$$

2.4. FREE CONVECTION

Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Some examples are cooling of electronic equipment such as power transistors, TVs, and VCRs; heat transfer from electric baseboard heaters or steam radiators; heat transfer from the refrigeration coils and power transmission lines; and heat transfer from the bodies of animals and human beings.

In a gravitational field, there is a net force that pushes upward a light fluid placed in a heavier fluid. The upward force exerted by a fluid on a body completely or partially immersed in it is called the buoyancy force. The magnitude of the buoyancy force is equal to the weight of the fluid displaced by the body.

$$F_{buoyancy} = \rho_{fluid}\, g\, V_{body}$$

where ρ_{fluid} is the average density of the fluid (not the body), g is the gravitational acceleration, and V_{body} is the volume of the portion of the body immersed in the fluid (for bodies completely immersed in the fluid, it is the total volume of the body). In the absence of other forces, the net vertical force acting on a body is the difference between the weight of the body and the buoyancy force. That is,

$$\begin{aligned} F_{Net} &= W - F_{buoyancy} \\ &= \rho_{body}\, g\, V_{body} - \rho_{fluid}\, g\, V_{body} \\ &= \left(\rho_{body} - \rho_{fluid}\right) g\, V_{body} \end{aligned}$$

Note that this force is proportional to the difference in the densities of the fluid and the body immersed in it. Thus, a body immersed in a fluid will experience a “weight loss” in an amount equal to the weight of the fluid it displaces. This is known as Archimedes’ principle.

In heat transfer studies, the primary variable is temperature, and it is desirable to express the net buoyancy force in terms of temperature differences. But this requires expressing the density difference in terms of a temperature difference, which requires knowledge of a property that represents the variation of the density of a fluid with temperature at constant pressure. The property that provides that information is the volume expansion coefficient β, defined as

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p, \quad (1/K)$$

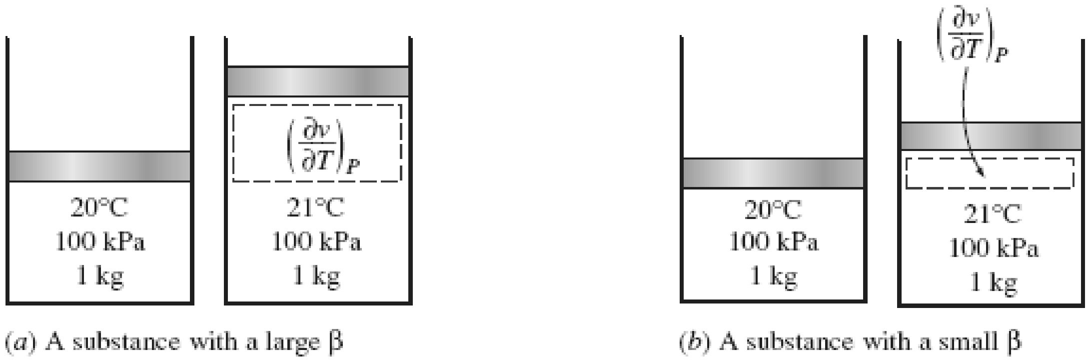


Fig.2.8. explaining the concept of volume expansion coefficient, β

The volume expansion coefficient β of an ideal gas (P = ρ RT) at a temperature T is equivalent to the inverse of the temperature

$$\beta = \frac{1}{T}, \quad (1/K)$$

where T is the absolute temperature. Note that a large value of β for a fluid means a large change in density with temperature, and that the product β ΔT represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT at constant pressure.

Also note that the buoyancy force is proportional to the density difference, which is proportional to the temperature difference at constant pressure. Therefore, the larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the larger the buoyancy force and the stronger the natural convection currents, and thus the higher the heat transfer rate.

The flow regime in natural convection is governed by the dimensionless Grashof number, which represents the ratio of the buoyancy force to the viscous force acting on the fluid and it is given as

$$G_r = \frac{F_{buoyancy}}{F_{Viscous}} = \frac{g\,\beta\,{L_c}^3\,(T_s - T_\infty)}{\nu^2}$$

Where,

g - Acceleration due to gravity

β - Volume expansion coefficient

L_c - Characteristic length

T_s - Surface temperature

T_∞ - fluid temperature

ν - Kinematic viscosity

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection. For vertical plates, for example, the critical Grashof number is observed to be about 10^9. Therefore, the flow regime on a vertical plate becomes turbulent at Grashof numbers greater than 10^9.

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermo physical properties of the fluid involved.

The simple empirical correlations for the average Nusselt number N_u in natural convection are of the form

$$N_u = \frac{h\,L_c}{k} = C\ (G_r\,P_r)^n = C\ (R_{aL})^n$$

where R_{aL} is the Rayleigh number, which is the product of the Grashof and Prandtl numbers. The values of the constants C and n depend on the geometry of the surface and the flow regime which is characterized by the range of the Rayleigh number. The value of n is usually 0.25 for laminar flow and 0.333 for turbulent flow. The value of the constant C is normally less than 1.

Case (i): Flow over vertical plate

(1) Nusselt number & heat transfer coefficient

(a) Laminar flow,

$$N_u = \frac{h\,L_c}{k} = 0.59\ (G_r\,P_r)^{0.25},\ \ 10^4 < G_r\,P_r < 10^9$$

(b) Turbulent flow

$$N_u = \frac{h\,L_c}{k} = 0.10\ (G_r\,P_r)^{0.333},\ \ 10^4 < G_r\,P_r < 10^9$$

(2) Grashof number

$$G_r = \frac{g\,\beta\,{L_c}^3\,(T_s - T_\infty)}{\nu^2}$$

(3) Rate of heat transfer

$$Q = h\,\mathrm{A}\,(T_s - T_\infty)$$

Case (ii): Flow over horizontal plate

(1) Nusselt number & heat transfer coefficient

(a) Upper surface of the plate heated,

$$N_u = \frac{h_u\,L_c}{k} = 0.54\,(G_r\,P_r)^{0.25},\;\; 2\times 10^4 < G_r\,P_r < 8\times 10^6$$

$$or$$

$$N_u = \frac{h_u\,L_c}{k} = 0.15\,(G_r\,P_r)^{0.333},\;\; 8\times 10^6 < G_r\,P_r < 10^{11}$$

(b) Lower surface of the plate heated,

$$N_u = \frac{h_l\,L_c}{k} = 0.27\,(G_r\,P_r)^{0.25},\;\; 10^5 < G_r\,P_r < 10^{11}$$

(2) Grashof number

$$G_r = \frac{g\,\beta\,{L_c}^3\,(T_s - T_\infty)}{\nu^2}$$

$$L_c = \frac{w}{2}$$

Where w is the width of the plate

(3) Rate of heat transfer

$$Q = (h_u + h_l)\,\mathrm{A}\,(T_s - T_\infty)$$

Case (iii): Flow over horizontal cylinder

(1) Nusselt number & heat transfer coefficient

$$N_u = \frac{h\,L_c}{k} = C\,(G_r\,P_r)^m$$

Where C and m are constants, which are to be taken from the table provided

(2) Rate of heat transfer

$$Q = h\,\mathrm{A}\,(T_s - T_\infty)$$

$$\mathrm{A} = \pi\,\mathrm{D}\,\mathrm{L}$$

Case (iv): Flow over sphere

(1) Nusselt number & heat transfer coefficient

$$N_u = \frac{h\,L_c}{k} = 2 + 0.43\,(G_r\,P_r)^{0.25}$$

(2) Rate of heat transfer

$$Q = h\,\mathrm{A}\,(T_s - T_\infty)$$

$$\mathrm{A} = 4\,\pi\,\mathrm{r}^2$$

The boundary layer thickness at a distance of ‘x’ from the leading edge is given as

$$\delta_x = [3.93 \times (P_r)^{-0.5} \times (0.952 + P_r)^{0.25} \times (G_r)^{-0.25}] \times x$$

The maximum velocity of fluid flow by natural convection is given as

$$V_{max} = 0.766 \times \nu \times (0.952 + P_r)^{-0.5} \times \left(\frac{g\,\beta\,(T_s - T_\infty)}{\nu^2}\right)^{0.5} \times x^{0.5}$$

The mass flow rate m, is given as

$$m = 1.7 \times \rho \times \nu \times \left[\frac{G_r}{{P_r}^2\,(0.952 + P_r)}\right]^{0.25}$$

Questions & Answers

1. What is dimensional analysis?

Dimensional analysis is a mathematical method which makes use of the study of the dimensions for solving several engineering problems. This method can be applied to all types of fluid resistances, heat flow problems in fluid mechanics and thermodynamics.

2. State Buckingham π theorem.

Buckingham π theorem states as Follows: "If there are n variables in a dimensionally homogeneous equation and if these contain m fundamental dimensions, then the variables are arranged into (n – m) dimensionless terms. These dimensionless terms are called π terms.

3. What are all the advantages of dimensional analysis?

1. It expresses the functional relationship between the variables in dimensional terms.
2. It enables getting up a theoretical solution in a simplified dimensionless form.
3. The results of one series of tests can be applied to a large number of other similar problems with the help of dimensional analysis.

4. What are all the limitations of dimensional analysis?

1. The complete information is not provided by dimensional analysis. It only indicates that there is some relationship between the parameters.
2. No information is given about the internal mechanism of physical phenomenon.
3. Dimensional analysis does not give any clue regarding the selection of variables.

5. Define Reynolds number (Re).

It is defined as the ratio of inertia force to viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

6. Define prandtl number (Pr).

It is the ratio of the momentum diffusivity of the thermal diffusivity.

$$Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

7. Define Nusselt number (Nu).

It is defined as the ratio of the heat flow by convection process under an unit temperature gradient to the heat flow rate by conduction under an unit temperature gradient through a stationary thickness (L) of metre.

$$\text{Nusselt number (Nu)} = \frac{Q_{conv}}{Q_{cond}}.$$

8. Define Grash of number (Gr).

It is defined as the ratio of product of inertia force and buoyancy force to the square of viscous force.

$$Gr = \frac{\text{Inertia force} \times \text{Buyoyancy force}}{(\text{Viscous force})^2}$$

9. Define Stanton number (St).

It is the ratio of nusselt number to the product of Reynolds number and prandtl number.

$$St = \frac{Nu}{Re \times Pr}$$

10. What is meant by Newtonion and non – Newtonion fluids?

The fluids which obey the Newton's Law of viscosity are called Newtonion fluids and those which do not obey are called non – newtonion fluids.

11. What is meant by laminar flow and turbulent flow?

Laminar flow: Laminar flow is sometimes called stream line flow. In this type of flow, the fluid moves in layers and each fluid particle follows a smooth continuous path. The fluid particles in each layer remain in an orderly sequence without mixing with each other.

Turbulent flow: In addition to the laminar type of flow, a distinct irregular flow is frequency observed in nature. This type of flow is called turbulent flow. The path of any individual particle is zig – zag and irregular. Fig. shows the instantaneous velocity in laminar and turbulent flow.

12. What is hydrodynamic boundary layer?

In hydrodynamic boundary layer, velocity of the fluid is less than 99% of free stream velocity.

13. What is thermal boundary layer?

In thermal boundary layer, temperature of the fluid is less than 99% of free stream velocity.

14. Define convection.

Convection is a process of heat transfer that will occur between a solid surface and a fluid medium when they are at different temperatures.

15. State Newton's law of convection.

Heat transfer from the moving fluid to solid surface is given by the equation

$$Q = h\,A\,(T_w - T_\infty)$$

This equation is referred to as Newton's law of cooling.

Where

h – Local heat transfer coefficient in W/m^{2}K.

A – Surface area in m^2

T_w – Surface (or) Wall temperature in K

T_∞ - Temperature of fluid in K.

16. What is meant by free or natural convection?

If the fluid motion is produced due to change in density resulting from temperature gradients, the mode of heat transfer is said to be free or natural convection.

17. What is forced convection?

If the fluid motion is artificially created by means of an external force like a blower or fan, that type of heat transfer is known as forced convection.

18. According to Newton's law of cooling the amount of heat transfer from a solid surface of area A at temperature T_w to a fluid at a temperature T_∞ is given by ________.

Ans : $Q = h\,A\,(T_w - T_\infty)$

19. What is the form of equation used to calculate heat transfer for flow through cylindrical pipes?

$$Nu = 0.023\,(Re)^{0.8}\,(Pr)^n$$

$n = 0.4$ for heating of fluids

$n = 0.3$ for cooling of fluids

20. What are the dimensionless parameters used in forced convection?

1. Reynolds number (Re)
2. Nusdselt number (Nu)
3. Prandtl number (Pr)

21. Define boundary layer thickness.

The thickness of the boundary layer has been defined as the distance from the surface at which the local velocity or temperature reaches 99% of the external velocity or temperature.

Problems:

1. A flat plate at 90°C is located in a water stream having a free stream velocity of 6 m/s and at 30 °C. The flat plate is 45 cm long and 60 cm wide. The flow in the boundary layer changes from laminar to turbulent at Re = 4 x 10^5. Find the Nusselt number for the plate and thickness of thermal boundary layer at a distance of 20 cm from leading edge. The flow is parallel to 45 cm side. Take the properties from the table. **(KU – Nov 2010)**

Given Data:

T_s = 90 °C, T_∞ = 30 °C, V = 6 m/s, L = 45 cm = 0.45 m, W = 60 cm = 0.6 m

Critical Re = 4 x 10^5, x = 20 cm = 0.2 m

To find:

(i) Nusselt number, Nu

(ii) Thickness of thermal boundary layer, δ_{Tx}

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{90 + 30}{2} = 60\ °C$$

From HMT DB pg.no. 22, the properties of water at T_f = 60°C are

ρ = 985 kg/m^3, c_p = 4183 J/kg K, ν = 0.478 x 10^{-6} m^2/sec, k = 0.6513 W/mK

Pr = 3.020

To find Reynolds number, Re at distance of 'x' m

$$R_e = \frac{V\,x}{\nu}$$

$$R_e = \frac{6 \times 0.2}{0.478 \text{ x } 10^{-6}} = 2.51 \times 10^6 > 4 \times 10^5$$

Hence the flow is Turbulent

From HMT DB pg.no. 114, the thermal boundary layer thickness is given as

$$\delta_{T,x} = \delta_{h,x}$$

Where

$$\delta_{h,x} = 0.382\,.\,x.\left(R_{e,x}\right)^{-0.2}$$

$$\delta_{h,x} = 0.382 \times 0.2 \times (2.51 \times 10^6)^{-0.2}$$

$$\delta_{h,x} = 4.01 \times 10^{-3}, \quad m$$

$$\therefore \ \boldsymbol{\delta_{T,x} = \delta_{h,x} = 4.01 \times 10^{-3}, \quad m}$$

Nusselt number, Nu is given by, from HMT DB pg.no.114

$$N_{u,x} = 0.0296 \ (R_{e,x})^{0.8} (P_r)^{0.333}$$

$$N_{u,x} = 0.0296 \ (2.51 \times 10^6)^{0.8} (3.020)^{0.333}$$

$$\boldsymbol{N_{u,x} = 5634.73}$$

2. A tube 5 m long is maintained at 100 °C by steam jacketing. A fluid flows through the tube at the rate of 175 kg/hr at 30 °C. The diameter of the tube is 2 cm. Find out average heat transfer coefficient. Take the following properties of the fluid, $\rho = 850$ kg/m^3, $c_p = 2000$ J/kg K, $\nu = 5.1 \times 10^{-6}$ m^2/sec, k = 0.12 W/mK. **(KU – Nov 2010)**

   Given Data:

   L = 5 m, T_∞ = 100 °C, m = 175 kg/hr = 0.05 kg/sec, T_s = 30 °C,

   D = 2 cm = 0.02 m

   $\rho = 850$ kg/m^3, $c_p = 2000$ J/kg K, $\nu = 5.1 \times 10^{-6}$ m^2/sec, k = 0.12 W/mK

   To find:

   h_{avg}

   Solution:

   From HMT DB pg.no. 124, for L>>D the Nusselt Number, Nu is given as

$$Nu = 3.66$$

   We know that,

$$N_u = \frac{h\,D}{k}$$

$$h = \frac{N_u k}{D} = \frac{3.66 \times 0.12}{0.02}$$

$$\boldsymbol{h = 21.96 \ W/m^2K}$$

3. Air at atmospheric pressure and 200 °C flows over a plate with a velocity of 5 m/s. The plate is 15 mm wide and is maintained at a temperature of 120 °C. Calculate the thickness of hydrodynamic and thermal boundary layers and local heat transfer coefficient at a distance of 0.5 m from the leading edge. Assume that the flow is on one side of the plate. **(KU – Nov 2011)**

   Given Data:

   T_∞ = 200 °C, T_s = 120 °C, V = 5 m/s, x = 0.5 m

To find:

δ_{hx} =?, δ_{Tx} =?, h_x =?

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{120 + 200}{2} = 160\ °C$$

From HMT DB pg.no. 34, the properties of air at T_f = 160°C are

ρ = 0.815 kg/m^3, c_p = 1017 J/kg K, ν = 30.09 x 10^{-6} m^2/sec, k = 0.03640 W/mK

Pr = 0.682

To find Reynolds number, Re at distance of 'x' m

$$R_e = \frac{V\ x}{\nu}$$

$$R_e = \frac{5\ \times 0.5}{30.06 \text{ x } 10^{-6}} = 0.832\ \times\ 10^5 < 5\ \times\ 10^5$$

Hence the flow is laminar

For laminar flow, from HMT DB pg.no.113

Hydrodynamic boundary layer thickness is given as

$$\delta_{h,x} = 5.x.\left(R_{e,x}\right)^{-0.5}$$

$$\delta_{h,x} = 5\ \times 0.5\ \times\ (0.832\ \times\ 10^5)^{-0.5}$$

$$\boldsymbol{\delta_{h,x} = 8.67\ \times\ 10^{-3}, \qquad m}$$

Thermal boundary layer thickness is given as

$$\delta_{T,x} = \delta_{h,x}(P_r)^{0.333}$$

$$\delta_{T,x} = 8.67\ \times\ 10^{-3}\ \times\ (0.682)^{0.333}$$

$$\boldsymbol{\delta_{T,x} = 7.63\ \times\ 10^{-3}, \qquad m}$$

The local Nusselt number is given as

$$N_{u,x} = 0.332\ \left(R_{e,x}\right)^{0.5}(P_r)^{0.333}$$

$$N_{u,x} = 0.332 \times\ (0.832\ \times\ 10^5)^{0.5} \times (0.682)^{0.333}$$

$$\boldsymbol{N_{u,x} = 84.3}$$

The Local heat transfer coefficient is given as

$$N_{u,x} = \frac{h_x\, x}{k}$$

$$h_x = \frac{N_{u,x}\, k}{x} = \frac{84.3\ \times 0.03640}{0.5}$$

$$\boldsymbol{h_x = 6.14\ W/m^2K}$$

4. A large vertical plate 5 m height is maintained at 100 °C and exposed to air at 30 °C. Calculate the convective heat transfer coefficient. **(KU – Nov 2011) (KU – Apr 2014)**

Given Data:

L = 5 m, T_s = 100 °C, T_∞ = 30 °C

To find:

h =?

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{100 +\ 30}{2} = 65\ °C$$

From HMT DB pg.no. 34, the properties of air at T_f = 65°C are

ρ = 1.045 kg/m^3, ν = 19.50 x 10^{-6} m^2/sec, k = 0.02931 W/mK

Pr = 0.695

From HMT DB pg.no.135

$$G_r = \frac{g\ \beta\ {L_c}^3\ (T_s - T_\infty)}{\nu^2}$$

$$\beta = \frac{1}{T_f +\ 273} = \frac{1}{65 + 273} = 2.96\ \times\ 10^{-3}\ K^{-1}$$

$$G_r = \frac{9.81\ \times\ \ 2.96\ \times\ 10^{-3}\ \times\ \ 5^3 \times\ (100 - 30)}{(19.50 \times\ 10^{-6})^2}$$

$$\boldsymbol{G_r = 6.68\ \times\ 10^{11}}$$

To find

$$G_r\, P_r = \ 6.68\ \times\ 10^{11}\ \times 0.695 = 4.64\ \times\ 10^{11}$$

To find the Nusselt number, Nu

From HMT DB pg.no. 136

$$N_u = 0.59\,(G_r\,P_r)^{0.25},\ \ 10^4 < G_r\,P_r < 10^9$$

$$N_u = 0.59\,(4.64 \times 10^{11})^{0.25}$$

$$\boldsymbol{N_u = 486.95}$$

To find the heat transfer coefficient, h

w.k.t.

$$N_u = \frac{h\,L_c}{k}$$

$$h = \frac{N_u\,k}{L_c} = \frac{486.95 \times 0.02931}{5}$$

$$\boldsymbol{h = 2.85\ W/m^2K}$$

5. Air at 20 °C is flowing along a heated plate at 134 °C at a velocity of 3 m/s. The plate is 2 m long and 1.5 m wide. Calculate the thickness of the hydrodynamic boundary layer and the skin friction coefficient at 40 cm from the leading edge of the plate. The kinematic viscosity of air at 20 °C is 15.06 x 10^{-6} m^2/sec. **(KU – Nov 2011)**

Given Data:

T_∞ = 20°C, T_s = 134°C, V = 3 m/s, L = 2 m, W = 1.5 m, x = 40 cm = 0.4 m

Kinematic viscosity of air at 20 °C, ν = 15.06 x 10^{-6} m^2/sec

To find:

Hydrodynamic boundary layer thickness, δ_{hx}

Skin friction coefficient, $C_{f,x}$

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{20 + 134}{2} = 77\ °C$$

From HMT DB pg.no. 34, the properties of air at T_f = 77°C ≈ 80°C are

ρ = 1 kg/m^3, ν = 21.09 x 10^{-6} m^2/sec, k = 0.03047 W/mK

Pr = 0.692

To find Reynolds number, Re at distance of 'x' m

$$R_e = \frac{V\,x}{\nu}$$

$$R_e = \frac{3\ \times 0.4}{21.09 \text{ x } 10^{-6}} = 0.569\ \times\ 10^5 < 5\ \times\ 10^5$$

Hence the flow is laminar

For laminar flow, from HMT DB pg.no.113

(i) Hydrodynamic boundary layer thickness is given as

$$\delta_{h,x} = 5.x.\left(R_{e,x}\right)^{-0.5}$$

$$\delta_{h,x} = 5 \times 0.4\ \times\ (0.569\ \times\ 10^5)^{-0.5}$$

$$\boldsymbol{\delta_{h,x} = 8.38\ \times\ 10^{-3}, \qquad m}$$

(ii) Local skin friction coefficient, $C_{f,x}$

$$C_{f,x} = 0.664\ \left(R_{e,x}\right)^{-0.5}$$

$$C_{f,x} = 0.664\ (0.569\ \times\ 10^5)^{-0.5}$$

$$\boldsymbol{C_{f,x} = 2.78\ \times\ 10^{-3}, \qquad m}$$

6. A steam pipe 10 cm outer diameter runs horizontally in a room at 23 °C. Take the outside surface temperature of the pipe as 165 °C. Determine the heat loss per meter length of the pipe. **(KU – Nov 2011) (KU – Apr 2014)**

Given Data:

D = 10 cm = 0.1 m, T_∞= 23°C, T_s = 165°C

To find:

Heat loss per meter length of the pipe, Q/L

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{165 +\ 23}{2} = 94\ °C$$

From HMT DB pg.no. 34, the properties of air at T_f = 94°C ≈ 95°C are

ρ = 0.959 kg/m^3, ν = 22.62 x 10^{-6} m^2/sec, k = 0.03169 W/mK

Pr = 0.689

From HMT DB pg.no.135

$$G_r = \frac{g\ \beta\ D^3\ (T_s - T_\infty)}{\nu^2}$$

$$\beta = \frac{1}{T_f + 273} = \frac{1}{95 + 273} = 2.72 \times 10^{-3}\ K^{-1}$$

$$G_r = \frac{9.81 \times 2.72 \times 10^{-3} \times 0.1^3 \times (165 - 23)}{(22.62 \times 10^{-6})^2}$$

$$\boldsymbol{G_r = 7.40 \times 10^6}$$

To find

$$G_r\ P_r = 7.40 \times 10^6 \times 0.689 = 5.09 \times 10^6$$

To find the Nusselt number, Nu

From HMT DB pg.no. 138

$$N_u = C\ (G_r\ P_r)^m$$

Where C = 0.48 & m = 0.25

$$N_u = 0.48\ (5.09 \times 10^6)^{0.25}$$

$$\boldsymbol{N_u = 22.79}$$

To find the heat transfer coefficient, h

w.k.t.

$$N_u = \frac{h\ D}{k}$$

$$h = \frac{N_u\ k}{D} = \frac{22.79 \times 0.03169}{0.1}$$

$$\boldsymbol{h = 7.22\ W/m^2K}$$

w.k.t the rate of heat transfer is given as

$$Q = h\ \mathrm{A}\ (T_s - T_\infty)$$

&

$$\mathrm{A} = \pi\ \mathrm{D}\ \mathrm{L}$$

$$Q/L = h\ \pi\ \mathrm{D}\ (T_s - T_\infty)$$

$$\frac{Q}{L} = 7.22 \times \pi \times 0.1 \times (165 - 23)$$

$$\boldsymbol{\frac{Q}{L} = 322.32\ W/m}$$

7. A square plate 60 cm sides is at 120°C which is exposed to air at 20°C. Find out the heat loss from both sides of the plates by free convection, when the plate is kept vertical and also when the plate is kept horizontal. **(KU – Nov 2012)**

   Given Data:

   L = 60 cm = 0.6 m, T_s= 120 °C, T_∞ = 20 °C

   To Find:

   Q =?

   Solution:

   To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{120 + 20}{2} = 70\ °C$$

   From HMT DB pg.no. 34, the properties of air at T_f = 70°C are

   ρ = 1.029 kg/m^3, ν = 20.02 x 10^{-6} m^2/sec, k = 0.02966 W/mK

   Pr = 0.694

   From HMT DB pg.no.135

$$G_r = \frac{g\ \beta\ L^3\ (T_s - T_\infty)}{\nu^2}$$

$$\beta = \frac{1}{T_f + 273} = \frac{1}{70 + 273} = 2.92 \times 10^{-3}\ K^{-1}$$

$$G_r = \frac{9.81 \times 2.92 \times 10^{-3} \times 0.6^3 \times (120 - 20)}{(20.02 \times 10^{-6})^2}$$

$$\boldsymbol{G_r = 1.544 \times 10^9}$$

   To find

$$G_r\ P_r = 1.544 \times 10^9 \times 0.694 = 1.07 \times 10^9 > 10^9$$

   Hence the flow is turbulent

   To find the Nusselt number, Nu

$$N_u = 0.10\,(G_r\,P_r)^{0.333},\;\; 10^4 < G_r\,P_r < 10^9$$

$$N_u = 0.10\,(1.07\times 10^9)^{0.333}$$

$$\boldsymbol{N_u = 101.57}$$

To find the heat transfer coefficient, h

w.k.t.

$$N_u = \frac{h\,L_c}{k}$$

$$h = \frac{N_u\,k}{L_c} = \frac{101.57\times 0.02966}{0.6}$$

$$\boldsymbol{h = 5.02\;W/m^2K}$$

Heat loss, Q

$$Q = h\,\mathrm{A}\,(T_s - T_\infty)$$

$$Q = h\,\mathrm{W\,L}\,(T_s - T_\infty) = 5.02\times 0.6\times 0.6\times (120-20)$$

$$Q = 180.72\,W$$

For single side of the vertical plate, hence for two sides

$$Q = 2\times 180.72\,W$$

$$\boldsymbol{Q = 361.44\,W}$$

Case (ii) if the plate is kept horizontal

For upper surface of the plate heated,

$$N_u = 0.15\,(G_r\,P_r)^{0.333},\qquad for\;\; 8\times 10^6 < G_r\,P_r < 10^{11}$$

$$N_u = 0.15\,(1.07\times 10^9)^{0.333}$$

$$N_u = 152.36$$

To find the heat transfer coefficient for upper surface heated

$$N_u = \frac{h_u\,L_c}{k}$$

$$152.36 = \frac{h_u\times 0.6}{0.02966}$$

$$\boldsymbol{h_u = 7.53\;W/m^2K}$$

For lower surface of the plate heated,

$$N_u = 0.27\,(G_r\,P_r)^{0.25},\ \ 10^5 < G_r\,P_r < 10^{11}$$

$$N_u = 0.27\,(1.07 \times 10^9)^{0.25}$$

$$N_u = 48.83$$

To find the heat transfer coefficient for lower surface heated

$$N_u = \frac{h_l\,L_c}{k}$$

$$48.83 = \frac{h_l \times 0.6}{0.02966}$$

$$\boldsymbol{h_l = 2.414\ W/m^2K}$$

To find the rate of heat transferred or heat loss

$$Q = (h_u + h_l)\ \mathrm{A}\ (T_s - T_\infty)$$

$$Q = (7.53 + 2.414) \times 0.6 \times 0.6 \times (120 - 20)$$

$$\boldsymbol{Q = 357.98\ W}$$

8. Air at 25°C flows over 1 m x 3 m (3 m long) horizontal plate maintained at 200 °C at 10 m/s. Calculate the average heat transfer coefficients for both laminar and turbulent regions. Take Re (critical) = 3.5 x 10^5. **(KU – Nov 2012) (KU – Nov 2013)**

Given Data:

T_∞ = 25 °C, L = 3 m, T_s = 200 °C, V = 10 m/s, Re_{cri} = 3.5 x 10^5

To find:

1. Average heat transfer coefficient (h) for laminar flow.
2. Average heat transfer coefficient (h) for turbulent flow.

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{200 + 25}{2} = 112.5\ °C$$

From HMT DB pg.no. 34, the properties of air at T_f = 112.5 °C are

ρ = 0.922 kg/m^3, ν = 24.29 x 10^{-6} m^2/sec, k = 0.03274 W/mK

Pr = 0.687

To find Reynolds number, Re at distance of ‘L’ m

$$R_e = \frac{V\,L}{\nu}$$

$$R_e = \frac{10 \times 3}{24.29 \text{ x } 10^{-6}} = 1.23 \times 10^6 > 5 \times 10^5$$

Hence the flow is turbulent, but the the flow is laminar upto Re = 3 x10^5

Case(i) For laminar flow

From HMT DB pg.no 113

The local Nusselt number is given as

$$N_{u,x} = 0.332 \, \left(R_{e,x}\right)^{0.5} (P_r)^{0.333}$$

$$N_{u,x} = 0.332 \times \, (3 \times 10^5)^{0.5} \times (0.687)^{0.333}$$

$$\boldsymbol{N_{u,x} = 160.47}$$

The Local heat transfer coefficient is given as

$$N_{u,x} = \frac{h_x\,x}{k}$$

$$h_x = \frac{N_{u,x}\,k}{L} = \frac{160.47 \times 0.03274}{3}$$

$$\boldsymbol{h_x = 1.75 \;\; W/m^2K}$$

w.k.t the average heat transfer coefficient, h is given as

$$h = 2 \times h_x$$

$$\therefore h = 2 \times 1.75$$

$$\boldsymbol{h = 3.5\, W/m^2K}$$

Case(ii) for turbulent flow

From HMT DB pg.no 114

The local Nusselt number is given as

$$N_{u,x} = 0.0296 \left(R_{e,x}\right)^{0.8} (P_r)^{0.333}$$

$$N_{u,x} = 0.0296 \times \, (1.23 \times 10^6)^{0.8} \times (0.687)^{0.333}$$

$$\boldsymbol{N_{u,x} = 1945}$$

The Local heat transfer coefficient is given as

$$N_{u,x} = \frac{h_x\,x}{k}$$

$$h_x = \frac{N_{u,x}\,k}{L} = \frac{1945 \times 0.03274}{3}$$

$$\boldsymbol{h_x = 21.22 \;\; W/m^2K}$$

w.k.t the average heat transfer coefficient, h is given as

$$h = 1.25 \times h_x$$

$$\therefore h = 1.25 \times 21.22$$

$$\boldsymbol{h = 26.525\ W/m^2K}$$

9. A flat plate 1 m wide and 1.5 m long is to maintained at 90°C in air with a free stream temperature of 10°C. Determine the velocity with which air must flow over flat plate along 1.5 m side, so that the rate of energy dissipation from the plate is 3.75 kW. Take the following properties of air at 50 °C. ρ = 1.09 kg/m^3, μ = 2.03 x 10^{-5} kg/m sec, k = 0.028 W/mK, Pr = 0.7, c_p= 1.007 kJ/kgK. **(KU – Nov 2013) (KU – Apr 2014)**

Given Data:

W = 1m, L = 1.5 m, T_s = 90°C, T_∞ = 10°C, Q = 3.75 kW = 3.75 x 10^3 W,

ρ = 1.09 kg/m^3, μ = 2.03 x 10^{-5} kg/m sec, k = 0.028 W/mK, Pr = 0.7,

c_p= 1.007 kJ/kgK

To find:

V = ?

Solution:

W.K.T the rate of heat transfer is given as,

$$Q = h\,A_s\,(T_s - T_\infty)$$

$$3.75 \times 10^3 = h \times (1.5 \times 1) \times (90 - 10)$$

$$\boldsymbol{h = 31.25\ W/m^2K}$$

w.k.t the local heat transfer coefficient is given as h_x

$$h_x = \frac{h}{2}$$

$$h_x = \frac{31.25}{2}$$

$$\boldsymbol{h_x = 15.625\ W/m^2K}$$

From HMT DB Pg. No 113

The local Nusselt number si given as

$$N_{u,x} = 0.332\ (R_{e,x})^{0.5}(P_r)^{0.333}$$

$$N_{u,x} = \frac{h_x\,L}{k}$$

$$\frac{h_x\,L}{k} = 0.332\ \left(R_{e,L}\right)^{0.5}(P_r)^{0.333}$$

W.k.t the Reynolds number is given as

$$R_e = \frac{V\,L}{\nu} = \frac{\rho\,V\,L}{\mu}$$

Therefore

$$\frac{h_x\,L}{k} = 0.332\ \left(\frac{\rho\,V\,L}{\mu}\right)^{0.5}\ (P_r)^{0.333}$$

$$\frac{15.625\ \times 1.5}{0.028} = 0.332\ \times \left(\frac{1.09\ \times V\ \times\ 1.5}{2.03\ \times\ 10^{-5}}\right)^{0.5}\ \times (0.7)^{0.333}$$

$$\boldsymbol{V = 100.10\ m/sec}$$

10. Atmospheric air at 275 K and a free stream velocity of 20 m/s flows over a flat plate 1.5 m long that is maintained at a uniform temperature of 325 K. Calculate the average heat transfer coefficient over the region where the boundary layer is laminar, the average heat transfer coefficient for the entire length of the plate and the total heat transfer rate from the plate to the air over the length 1.5 m and width 1 m. Assume transition occurs at Re = 2 x 10^5. **(KU – Apr 2014)**

Given Data:

T_∞ = 275 K = 2°C, V = 20 m/s, L = 1.5 m, T_s = 325 K = 52 °C, W = 1 m

Re_{cr} = 2 x 10^5

To find:

(i) h =? If the boundary layer is laminar

(iI) h =? If the boundary layer is turbulent

(iii) Q =? Rate of heat transfer

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{52 + 2}{2} = 27\ °C$$

From HMT DB pg.no. 34, the properties of air at $T_f \approx$ 25°C are

ρ = 1.185kg/m^3, ν = 15.53 x 10^{-6} m^2/sec, k = 0.02634 W/mK

Pr = 0.702

Case (i)

Considering the flow is laminar up to the given critical Reynolds number and finding the distance to which the flow is laminar

$$R_e = \frac{V\,L}{\nu}$$

$$2 \text{ x } 10^5 = \frac{20 \times L}{15.53 \text{ x } 10^{-6}}$$

$$L = 0.155\ m$$

The flow is laminar upto L = 0.155 m, after which the flow is turbulent

From HMT data book Pg. No: 113,

$$N_{u,x} = 0.332\ \left(R_{e,x}\right)^{0.5} (P_r)^{0.333}$$

$$N_{u,x} = 0.332\ \times (2 \text{ x } 10^5)^{0.5} \times (0.702)^{0.333}$$

$$N_{u,x} = 131.97$$

We know that, the local heat transfer coefficient is given as

$$N_{u,x} = \frac{h_x\,L}{k}$$

$$h_x = \frac{N_{u,x} \times k}{L} = \frac{131.97 \times 0.155}{0.02634}$$

$$\boldsymbol{h_x = 22.42}$$

Hence the average heat transfer coefficient is given as

$$\boldsymbol{h = 2 \times h_x}$$

$$\boldsymbol{h = 44.84\ W/m^2K}$$

Case (ii)

$$R_e = \frac{V\,L}{\nu}$$

$$R_e = \frac{20 \times 1.5}{15.53 \text{ x } 10^{-6}}$$

$$R_e = 1.93 \times 10^6 > 2 \times 10^5$$

$$Hence\ the\ flow\ is\ turbulent$$

Average Nusselt Number, $N_{u,L}$ & Average Heat transfer coefficient, h

$$N_{u,L} = (P_r)^{0.333} \left[0.037 \ \left(R_{e,x}\right)^{0.8} - 871\right]$$

$$N_{u,L} = (0.702)^{0.333}[0.037 \ (1.93 \times 10^6)^{0.8} - 871]$$

$$N_{u,L} = 2737.18$$

therefore

$$h = \frac{N_{u,L} \ k}{L} = \frac{2737.18 \ \times 0.02634}{1.5}$$

$$\boldsymbol{h = 48.06 \ W/m^2 K}$$

We know that

To find the rate of heat loss, Q

$$Q = h \ A_s \ (T_s - T_\infty)$$

$$Q = 48.06 \times 1 \ \times 1.5 \ \times \ (52 - 2)$$

$$\boldsymbol{Q = 3604.5 \ W}$$

11. For a particular engine, the undesirable of the crank case can be idealized as a flat plate measuring 80 cm x 20 cm. The engine runs at 80 km/hr and the crank case is cooled by air flowing past it at the same speed. Calculate the loss of heat from the crank case surface of temperature 75 °C to the ambient air temperature 25 °C. Assume the boundary layer becomes turbulent from the leading end itself. **(KU – Apr 2014)**

Given Data:

L = 20 cm = 0.8 m, W = 20 cm = 0.2 m, V = 80 km/hr = 22.2 m/sec,

T_s = 75°C, T_∞ = 25 °C

To find:

Q =?

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{75 + 25}{2} = 50 \ °C$$

From HMT DB pg.no. 34, the properties of air at T_f = 50°C are

ρ = 1.093 kg/m^3, ν = 17.95 x 10^{-6} m^2/sec, k = 0.02826 W/mK

Pr = 0.698

To find Reynolds number, Re

$$R_e = \frac{V\,L}{\nu}$$

$$R_e = \frac{22.2 \times 0.8}{17.95 \text{ x } 10^{-6}} = 0.989 \times 10^6 > 5 \times 10^5$$

Hence the flow is turbulent

From HMT DB pg.no 114, for turbulent flow, the Nusselt number is given by

$$N_{u,L} = 0.037 \; (R_e)^{0.8} (P_r)^{0.333}$$

$$N_{u,L} = 0.037 \; (0.989 \times 10^6)^{0.8} (0.698)^{0.333}$$

$$N_{u,L} = 2.05 \times 10^3$$

To find the heat transfer coefficient,

$$N_{u,L} = \frac{h\,L}{k}$$

$$h = \frac{N_{u,L}\,k}{L} = \frac{2.05 \times 10^3 \times 0.02826}{0.8}$$

$$\boldsymbol{h = 72.52\; W/m^2K}$$

To find the rate of heat loss, Q

$$Q = h\,A_s\,(T_s - T_\infty)$$

$$Q = 75.52 \times 0.8 \times 0.2 \times (75 - 25)$$

$$\boldsymbol{Q = 580.14\; W}$$

12. Air at 30 °C, 0.2 m/s flows across a 120 W electric bulb at 130 °C. Find heat transfer and power lost due to convection if bulb diameter is 70 mm. **(KU – Apr 2014)**

Given Data:

T_∞ = 30 °C, V = 0.2 m/s, Q = 120 W, T_s = 130 °C, D = 70 mm = 0.07 m

To find:

Q =?

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{130 + 30}{2} = 80\,°C$$

From HMT DB pg.no. 34, the properties of air at T_f = 80°C are

ρ = 1 kg/m^3, ν = 21.09 x 10^{-6} m^2/sec, k = 0.03047 W/mK

Pr = 0.692

To find Reynolds number, Re

$$R_e = \frac{V\ D}{\nu}$$

$$R_e = \frac{0.2 \times 0.07}{21.09 \text{ x } 10^{-6}} = 0.66 \times 10^3$$

To find the Nusselt number, Nu

From HMT DB pg.no 120

$$N_{u,L} = 0.37\ (R_e)^{0.6}$$

$$N_{u,L} = 0.37\ (0.66 \times 10^3)^{0.6}$$

$$N_{u,L} = 18.194$$

To find the heat transfer coefficient,

$$N_{u,L} = \frac{h\ D}{k}$$

$$h = \frac{N_{u,L}\ k}{D} = \frac{18.194 \times 0.03047}{0.07}$$

$$\boldsymbol{h = 7.92\ W/m^2K}$$

To find the rate of heat loss, Q

$$Q = h\ A_s\ (T_s - T_\infty)$$

$$Q = 7.92 \times 4 \times \pi \times 0.035^2 \times (130 - 30)$$

$$\boldsymbol{Q = 12.19\ W}$$

13. Air at 40 °C flows over a tube with a velocity of 30 m/s. The tube surface temperature is 120 °C, calculate the heat transfer coefficient for the following cases **(KU – Aug 2014)**

(i) Tube could be square with a side of 6 cm.

(ii) Tube is circular cylinder of diameter 6 cm.

Given Data:

T_∞ = 40°C, T_s =120 °C, V = 30 m/s

Case(i) Square tube, L = 6 cm = 0.06 m

Case(ii) Circular section tube, D = 6 cm = 0.06 m

To find

h =? For case(i) & (ii)

Solution:

To find the film temperature, T_f

$$T_f = \frac{T_s + T_\infty}{2}$$

$$T_f = \frac{120 + 40}{2} = 80\ °C$$

From HMT DB pg.no. 34, the properties of air at T_f = 80°C are

ρ = 1 kg/m^3, ν = 21.09 x 10^{-6} m^2/sec, k = 0.03047 W/mK

Pr = 0.692

To find Reynolds number, Re

$$R_e = \frac{V\ D}{\nu}$$

$$R_e = \frac{30 \times 0.06}{21.09 \text{ x } 10^{-6}} = 0.85 \times 10^5$$

Case (i)

From HMT DB pg.no 116, for non circular sections

$$N_u = 1.1\, C_1\, (R_e)^n (P_r)^{0.33}$$

From HMT DB pg.no 119 for square cross section,

C_1 = 0.092, n = 0.675

$$N_u = 1.1 \times 0.092 \times (0.85 \times 10^5)^{0.675} \times (0.692)^{0.33}$$

$$N_u = 190.45$$

To find the heat transfer coefficient,

$$N_{u,L} = \frac{h\ L}{k}$$

$$h = \frac{N_{u,L}\ k}{L} = \frac{190.45 \times 0.03047}{0.06}$$

$$\boldsymbol{h = 96.71\, \frac{W}{m^2 K}}$$

Case (ii)

From HMT DB pg.no 116, for non circular sections

$$N_u = C\, (R_e)^m (P_r)^{0.333}$$

From HMT DB pg.no 116 for square cross section,

$$C = 0.0266,\ m = 0.805$$

$$N_u = 0.0266 \times (0.85 \times 10^5)^{0.805} \times (0.692)^{0.333}$$

$$N_u = 218.69$$

To find the heat transfer coefficient,

$$N_{u,L} = \frac{h\,D}{k}$$

$$h = \frac{N_{u,L}\,k}{D} = \frac{218.69 \times 0.03047}{0.06}$$

$$\boldsymbol{h = 111.1\ W/m^2K}$$

MULTIPLE CHOICE QUESTIONS

Questions	Opt1	Opt2	Opt3	Opt4	Answer
Wet clothes are hung on a clothesline outdoors in subzero weather. The process of drying is	Vaporization	Sublimation	Melting	Condensation	Sublimation
The ratio of energy transferred by convection to that by conduction is called	Stanton number	Nusselt number	Biot number	Preclet number	Biot number
Free convection floe depends on all of the following except	Density	Coefficient of viscosity	Gravitational force	Velocity	Density
The ratio of surface convection resistance to internal conduction resistance is known as	Grashoff number	Biot number	Stanton number	Preclet number	Biot number
drop wise condensation usually occurs on	glazed surface	smooth surface	oil surface	coated surface	oil surface
provisions of fins on a given heat transfer surface will be more if there are	fewer number of thin fins	fewer number of thick fins	larger number of thin fins	larger number of thick fins	larger number of thin fins
in a heat exchanger with one fluid evaporating or condensing the surface area required least in	parallel flow	counter flow	cross flow	same in all above	same in all above
In free convection heat transfer transition from laminar to turbulent flow is governed by	Reynolds number	Grashoff number	Reynolds number , Grashoff number	Prandtl number, Grashoff number	Prandtl number, Grashoff number
Up to the critical radius of insulation	Added insulation will increase heat loss	Added insulation will decrease heat loss	Convection heat loss will be less than the conduction heat loss	Heat flux will decrease	Heat flux will decrease
Thermal boundary layer is a region where	Inertia terms are of the same order of magnitude as convection terms	Convection terms are of the same order of magnitude as dissipation terms	Convection terms are of the same order of magnitude as conduction terms	Dissipation is negligible	Inertia terms are of the same order of magnitude as convection terms
Heat pipe is widely used now a days because it acts as	As an insulator	As conductor	As a superconductor	As a fin	As conductor

Convection

In regarding nucleate boiling	The temperature of the surface is greater than the saturation temperature	Bubbles are created by expansion of entrapped gas .	The temperature is greater than that of film boiling	All of the above	The temperature is greater than that of film boiling
Heat transfer takes place according to	Zeroth law	First law	Second law	Third law	Second law
Heat is mainly transferred by conduction, convection and radiation in	insulted pipes carrying hot water	refrigerator freezer coil	boiler surfaces	condensation of seam in a condenser	boiler surfaces
For a given heat flow for the same thickness, the temperature drop across the material will be maximum for	Copper	Steel	Glass wool	Refractory brick	Glass wool
In a COAL fired boiler, the heat transfer from the fuel to the wall of the furnace is	By conduction only	By convection only	Both conduction and convection	Predominantly by radiation	Predominantly by radiation
When the thickness of insulation on a pipe exceeds the critical value THEN-----	Heat transfer rate increases	Heat transfer rate decreases	Heat transfer rate remains constant	None of the above	Heat transfer rate decreases
Temperatures near absolute zero are obtained using	Peltier effect	Thermionic emission	Azeotropes	Magnetic cooling	Magnetic cooling
Which of the following temperature measuring device will have least accuracy	Clinical thermometer	Alcohol filled thermometer	Optical pyrometer	Nitrogen filled thermometer	Optical pyrometer
In optical pyrometer the absorption filled is used	To get monochromatic light	To eliminate stray rays if light	To minimize reflection of rays from the lens surface	To enable filament operation at reduced intensity	To enable filament operation at reduced intensity
Your finger sticks to an ice tray just taken from the refrigerator. which factor has more effect .	The inside temperature of the freezer	The humidity of the air	The heat capacity of both your finger and the tray	The thermal conductivity of the tray	The thermal conductivity of the tray
With increase in temperature thermal conductivity of solid metals	Increase	Decrease	Constant	Depend on other factors	Increase

Convection

With increase in temperature thermal conductivity of water	Increase	Decrease	Constant	Depend on other factors	Depend on other factors
With increase in temperature thermal conductivity of air	Increase	Decrease	Constant	Depend on other factors	Decrease
Highest thermal conductivity is	Air	Water	Oxygen	Hydrogen	Water
Liquid metal having Highest thermal conductivity is	Sodium	Potassium	Lead	Mercury	Lead
Highest thermal conductivity is of	Solid ice	Melting ice	Water steam	Water	Water steam
Unit of thermal conductivity is	w/m.k	m2/hr	m/hr	m2/hr c	w/m.k
thermal diffusivity is	function of temperature	physical property of substance	a dimensionless parameter	All of these	function of temperature
minimum thermal diffusivity is of	aluminium	rubber	iron	lead	rubber
highest thermal diffusivity is of	iron	lead	concrete	wood	iron
critical radius of a hollow cylinder is defined as	outer radius which gives maximum heat flow	outer radius which gives minimum heat flow	inner radius which gives minimum heat flow	inner radius which gives maximum heat flow	outer radius which gives minimum heat flow

CHAPTER - III

PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS

3.1. BOILING AND CONDENSATION

Boiling is a convection process involving a change in phase from liquid to vapor. Boiling may occur when a liquid is in contact with a surface maintained at a temperature higher than the saturation temperature of the liquid.

If heat is added to a liquid from a submerged solid surface, the boiling process is referred to as pool boiling. In this process the vapor produced may form bubbles, which grow and subsequently detach themselves from the surface, rising to the free surface due to buoyancy effects. A common example of pool boiling is the boiling of water in a vessel on a stove.

In contrast, flow boiling or forced convection boiling occurs in a flowing stream and the boiling surface may itself be apportion of the flow passage. This phenomenon is generally associated with two phase flows through confined passages.

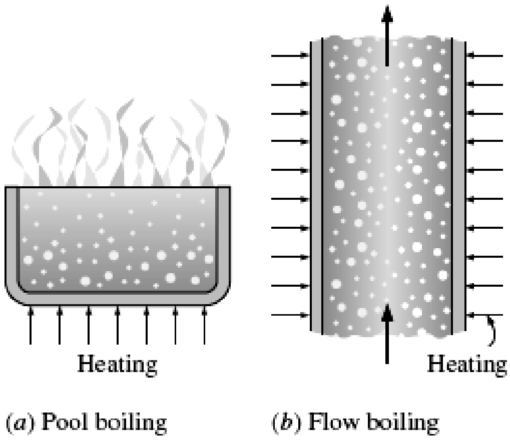


Fig.3.1.

3. 2. POOL BOILING

A necessary condition for the occurrence of pool boiling is that the temperature of the heating surface exceeds the saturation temperature of the liquid. The type of boiling is determined by the temperature of the liquid. If the temperature of the liquid is below the saturation temperature, the process is called sub cooled or local boiling. In local boiling, the bubbles formed at the surface eventually condense in the liquid. If the liquid is maintained at saturation temperature, the process is called saturated or bulk boiling.

There are various distinct regimes of pool boiling in which the heat transfer mechanism differs radically. The temperature distribution in saturated pool boiling with a liquid vapor interface is shown in the Figure 3.2.

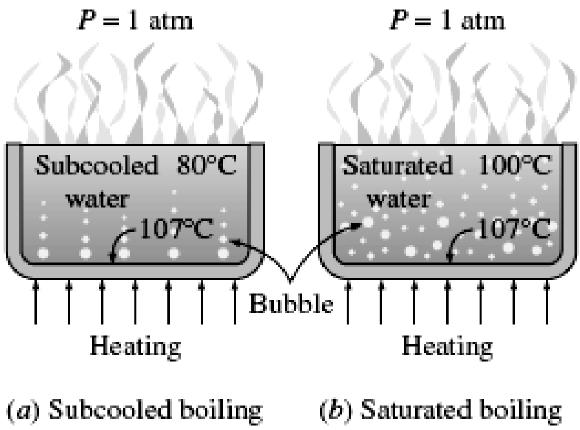


Fig. 3.2.

3.2.1. VARIOUS REGIMES OF BOILING

The different regimes of boiling are indicated in Figure 3.3. This specific curve has been obtained from an electrically heated platinum wire submerged in water by varying its surface temperature and measuring the surface heat flux q_s.

Natural Convection Boiling (to Point A on the Boiling Curve)

In thermodynamics we learned that a pure substance at a specified pressure starts boiling when it reaches the saturation temperature at that pressure. But in practice we do not see any bubbles forming on the heating surface until the liquid is heated a few degrees above the saturation temperature (about 2 to 6°C for water). Therefore, the liquid is slightly superheated in this case (a metastable condition) and evaporates when it rises to the free surface. The fluid motion in this mode of boiling is governed by natural convection currents, and heat transfer from the heating surface to the fluid is by natural convection.

Nucleate Boiling (between Points A and C)

The first bubbles start forming at point A of the boiling curve at various preferential sites on the heating surface. The bubbles form at an increasing rate at an increasing number of nucleation sites as we move along the boiling curve toward point C.

The nucleate boiling regime can be separated into two distinct regions. In region A–B, isolated bubbles are formed at various preferential nucleation sites on the heated surface. But these bubbles are dissipated in the liquid shortly after they separate from the surface.

In region B–C, the heater temperature is further increased, and bubbles form at such great rates at such a large number of nucleation sites that they form numerous continuous columns of vapor in the liquid. These bubbles move all the way up to the free surface, where they break up and release their vapor content.

The heat flux increases at a lower rate with increasing ΔT_{excess}, and reaches a maximum at point C. The heat flux at this point is called the critical (or maximum) heat flux, q_{max}. For water, the critical heat flux exceeds 1 MW/m^2.

Transition Boiling (between Points C and D on the Boiling Curve)

As the heater temperature and thus the ΔT_{excess} is increased past point C, the heat flux decreases, as shown in Figure 10–6. This is because a large fraction of the heater surface is covered by a vapor film, which acts as an insulation due to the low thermal conductivity of the vapor relative to that of the liquid. In the transition boiling regime, both nucleate and film boiling partially occur.

Nucleate boiling at point C is completely replaced by film boiling at point D. Operation in the transition boiling regime, which is also called the unstable film boiling regime, is avoided in practice. For water, transition boiling occurs over the excess temperature range from about 30°C to about 120°C.

Film Boiling (beyond Point D)

In this region the heater surface is completely covered by a continuous stable vapor film. Point D, where the heat flux reaches a minimum, is called the Leidenfrost point, in honor of J. C. Leidenfrost, who observed in 1756 that liquid droplets on a very hot surface jump around and slowly boil away.

The presence of a vapor film between the heater surface and the liquid is responsible for the low heat transfer rates in the film boiling region. The heat transfer rate increases with increasing excess temperature as a result of heat transfer from the heated surface to the liquid through the vapor film by radiation, which becomes significant at high temperatures.

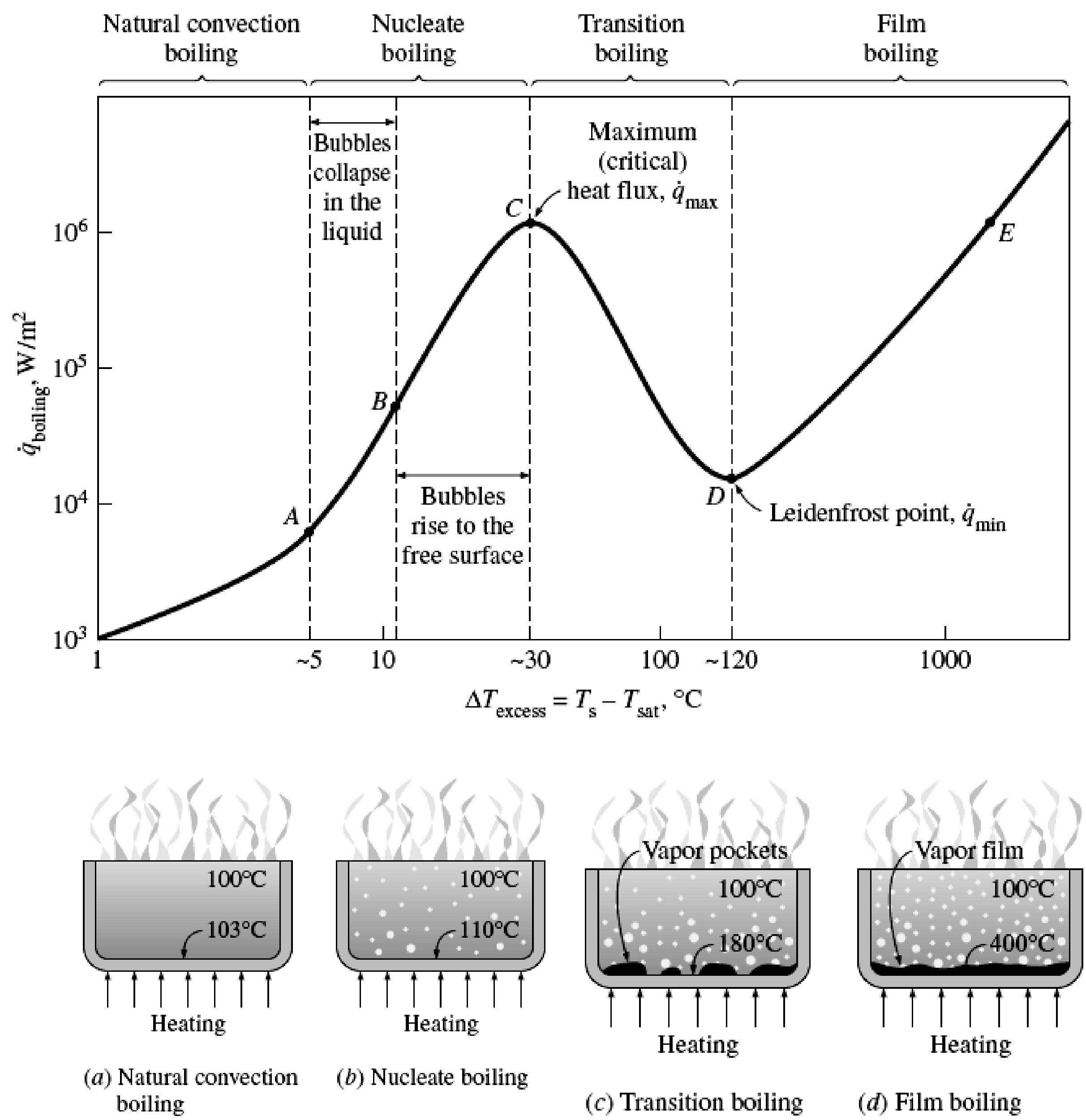


Fig.3.3. Regimes of pool boiling

3.2.2. HEAT TRANSFER CORRELATIONS IN POOL BOILING

NUCLEATE BOILING

In the nucleate boiling regime, the rate of heat transfer strongly depends on the nature of nucleation (the number of active nucleation sites on the surface, the rate of bubble formation at each site, etc.), which is difficult to predict.

The most widely used correlation for the rate of heat transfer in the nucleate boiling regime was proposed in 1952 by Rohsenow, and expressed as

$$\frac{Q}{A} = \mu_l \, h_{fg} \left[\frac{g \times (\rho_l - \rho_v)}{\sigma}\right]^{0.5} \times \left[\frac{C_{pl} \, (T_w - T_{sat})}{C_{sf} \times h_{fg} \times {P_r}^n}\right]^3$$

Where,

Q/A – heat flux, W/m^2

μ_l - Dynamic viscosity of liquid, Ns/m^2

h_{fg} - Enthalpy of evaporation, J/kg

g - Acceleration due to gravity, 9.81 m/s^2

ρ_l - Density of liquid, kg/m^3

ρ_v - Density of vapour, kg/m^3

σ - Surface tension for liquid vapour interface, N/m

C_{pl} - specific heat of liquid, J/kgK

C_{sf} - Super fluid constant

P_r - Prandtl Number

T_w - surface temperature, °C

T_{sat} - Saturation temperature, °C

n - 1 for water and 1.7 for other fluids

The maximum (or critical) heat flux in nucleate pool boiling was determined theoretically by S. S. Kutateladze in Russia in 1948 and N. Zuber in the United States in 1958 using quite different approaches, and is expressed as

$$\frac{Q}{A} = 0.18 \, h_{fg} \, \rho_v \left[\frac{\sigma \times g \, (\rho_l - \rho_v)}{{\rho_v}^2}\right]^{0.25}$$

The excess temperature ΔT_{excess} is less than 50°C for nucleate pool boiling

The rate of heat transfer Q is expressed as

$$Q = m \times h_{fg}$$

FLIM BOILING

Bromley developed a theory for the prediction of heat flux for stable film boiling on the outside of a horizontal cylinder. The heat flux for film boiling on a horizontal cylinder or sphere of diameter D is given by

$$\frac{Q}{A} = h\,(T_w - T_{sat})$$

$$h = h_{conv} + 0.75\,h_{rad}$$

$$h_{conv} = 0.62\left[\frac{{k_v}^3 \times \rho_v \times (\rho_l - \rho_v) \times g \times \left(h_{fg} + 0.4\,C_{pv}\,(T_w - T_{sat})\right)}{\mu_v\,D\,(T_w - T_{sat})}\right]^{0.25}$$

Q/A - Heat flux, W/m^2

k_v - Thermal conductivity of vapour, W/mK

μ_v - Dynamic viscosity of vapour, Ns/m^2

h_{fg} - Enthalpy of evaporation, J/kg

g - Acceleration due to gravity, 9.81 m/s^2

ρ_l - Density of liquid, kg/m^3

ρ_v - Density of vapour, kg/m^3

C_{pv} - Specific heat of liquid, J/kgK

T_w - Surface temperature, °C

T_{sat} - Saturation temperature, °C

n - 1 for water and 1.7 for other fluids

$$h_{rad} = \sigma\,\epsilon\left[\frac{{T_w}^4 - {T_{sat}}^4}{T_w - T_{sat}}\right]$$

σ - Stefan Boltzmann constant = 5.67 x 10^{-8} W/m^2 K^4

ε - Emissivity

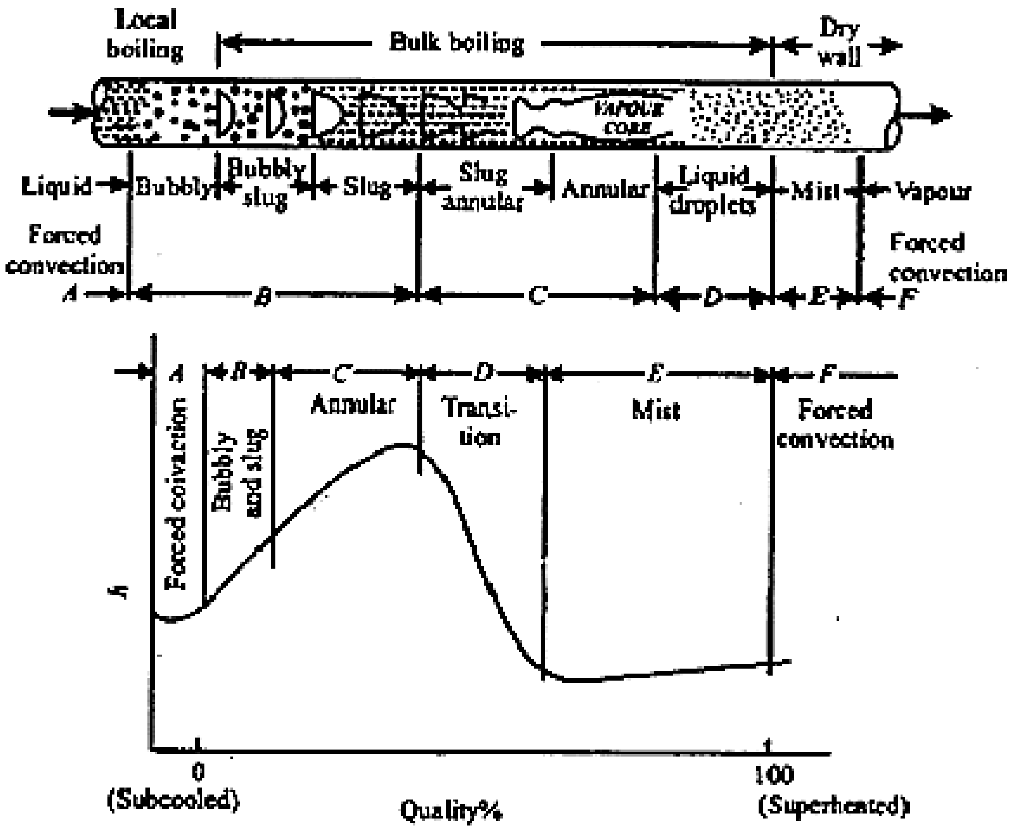
T_w - Surface temperature, °C

T_{sat} - Saturation temperature, °C

3.2. FLOW BOILING

Flow or forced convection boiling may occur when a liquid is forced through a channel or over a surface which is maintained at a temperature higher than the saturation temperature of the liquid. There are numerous applications of flow boiling in the design of steam generators for nuclear power plants and space power plants. The mechanism and hydrodynamics of flow boiling are much more complex than in pool boiling because the bubble growth and separation are strongly affected by the flow velocity. The flow is a two-phase mixture of the liquid and its vapor.

Fig.3.4. shows the various flow regimes inside a uniformly heated tube. Heat transfer to the sub cooled liquid at entry is by forced convection. This regime continues until boiling starts. The heat transfer coefficient in the boiling regime is suddenly increased. In this boiling regime, the bubbles appear on the heated surface, grow and are carried into the mainstream of the liquid, so that a bubbly flow regime prevails for some length of the tube.



Regimes for Flow Boiling Inside a Tube

Fig.3.4.

As the volume fraction of the vapor increases, the individual bubbles coalesce and plugs or slugs of vapor are formed. This regime is called the slug flow regime. As the vapor quality is increased, the flow becomes annular with a thin liquid layer on the wall and a vapor core. The vapor velocity is much higher than that of the liquid.

The heat transfer coefficient remains high as long as the liquid film wets the wall. Eventually, dry spots appear on the wall and the heat transfer coefficient drops sharply. This is called the transition region, from the annular flow to the mist or fog flow Burnout sometimes occurs at this transition because a liquid film of high thermal conductivity is replaced by a low thermal conductivity vapor at the wall.

The dry spots continue to expand until all the remaining liquid is in the form of fine droplets in the water. This is called the mist flow regime. There is little change in the heat transfer coefficient through the mist flow regime which persists until the vapor quality reaches 100%. Beyond this point the vapor is superheated by forced convection from the surface.

3.3. CONDENSATION

The process of condensation is the reverse of boiling. Whenever a saturated vapor comes in contact with a surface at a lower temperature, condensation occurs. There are two modes of condensation; film wise, in which the condensate wets the surface forming a continuous, film which covers the entire surface and drop wise in which the vapor condenses into small liquid droplets of various sizes which fall down the surface in a random fashion.

Film wise condensation generally occurs on clean uncontaminated surfaces. In this type of condensation the film covering the entire surface grows in thickness as it moves down the surface by gravity. There exists a thermal gradient in the film and so it acts as a resistance to heat transfer. In drop wise condensation a large portion of the area of the plate is directly exposed to the vapor, making heat transfer rates much larger (5 to 10 times) than those in film wise condensation.

Although drop wise condensation would be preferred to film wise condensation yet it is extremely difficult to achieve or maintain. This is because most surfaces become 'wetted' after being exposed to condensing vapors over a period of time. Drop wise condensation can be obtained under controlled conditions with the help of certain additives to the condensate and various surface coatings but its commercial viability has not yet been proved. For this reason the condensing equipment in use in designed on the basis of film wise condensation.

3.3.1. LAMINAR FILM WISE CONDENSATION ON A VERTICAL PLATE:

Film wise condensation on a vertical plate can be analyzed on lines proposed by Nusselt (1916). Unless the velocity of the vapor is very high or the liquid film very thick, the motion of the condensate would be laminar. The thickness of the condensate film will be a function of the rate of condensation of vapor and the rate at which the condensate is removed from the surface. On a vertical surface the film thickness will increase gradually from top to bottom as shown in Fig. 3.5.

Nusselt's analysis of film condensation makes the following simplifying assumptions:

1. The plate is maintained at a uniform temperature, T_s which is less than the saturation temperature, T_{sat} of the vapor.
2. The condensate flow is laminar
3. The fluid properties are constant

4. The shear stress at the liquid vapor interface is negligible
5. The acceleration of fluid within the condensate layer is neglected.
6. The heat transfer across the condensate layer is by pure conduction and the temperature distribution is linear.

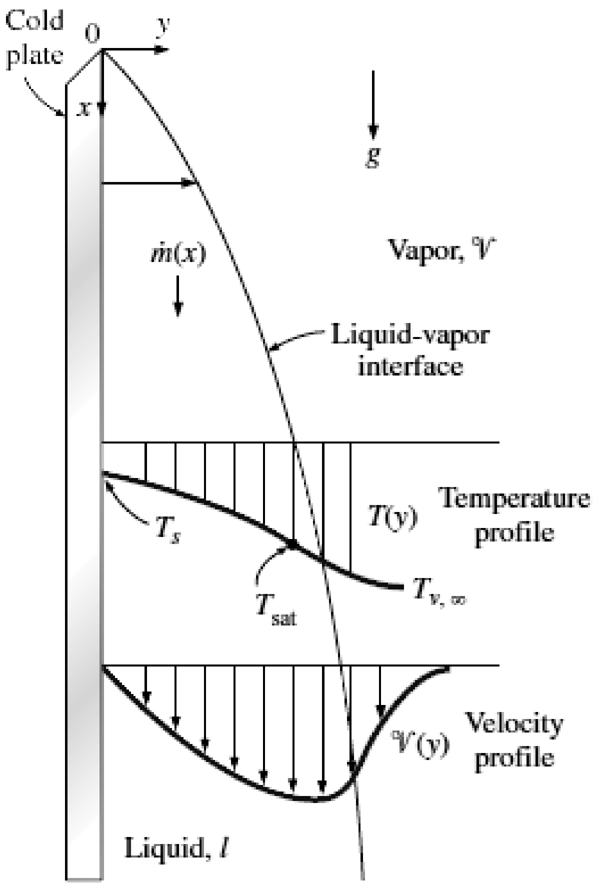


Fig.3.5.

The criterion for the flow regime (laminar or turbulent) is provided by the Reynolds number, which is defined as

$$R_e = \frac{D_h\, \rho\, u}{\mu} = \frac{4\, \delta\, \rho\, u}{\mu}$$

Where

D_h	-	Hydraulic diameter, m
δ	-	Boundary layer thickness
ρ	-	Density of the fluid, kg/m^3
u	-	Average velocity, m/sec
μ	-	Dynamic viscosity of the fluid, Ns/m^2

The hydraulic diameter D_h is given by

$$D_h = \frac{4\,A_c}{P}$$

A_c	-	Cross sectional area, m^2
P	-	Perimeter, m

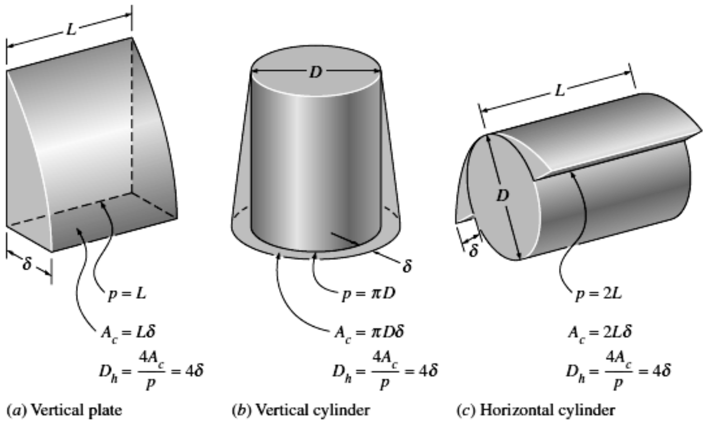


Fig. 3.6.

The above Fig.3.6. shows the calculation of hydraulic diameter for some common condensate geometry's.

3.3.2. CORRELATION FOR FILM WISE CONDENSING PROCESS

(i) Film thickness for laminar flow vertical surface

$$\delta_x = \left[\frac{4\,\mu\,k\,x\,(T_{sat} - T_w)}{g\,h_{fg}\,\rho^2}\right]^{0.25}$$

Where,

δ_x	-	Boundary layer thickness at 'x' distance, m
x	-	Distance along the surface, m
k	-	Thermal conductivity of the liquid, W/mK
h_{fg}	-	Enthalpy of evaporation, J/kg
T_w	-	Surface temperature, °C
T_{sat}	-	Saturation temperature, °C
ρ	-	Density of the fluid, kg/m^3
u	-	Average velocity, m/sec
μ	-	Dynamic viscosity of the fluid, Ns/m^2

(ii) Local Heat transfer coefficient (h_x) for vertical surface , laminar flow

$$h_x = \frac{k}{\delta_x}$$

(iii)Average Heat transfer coefficient (h) for vertical surface , laminar flow

$$h = 0.943 \left[\frac{k^3 \rho^2 g h_{fg}}{\mu L (T_{sat} - T_w)}\right]^{0.25}$$

The factor 0.943 may be replaced by 1.13 for more accurate result as suggested by Mc Adams

$$h = 1.13 \left[\frac{k^3 \rho^2 g h_{fg}}{\mu L (T_{sat} - T_w)}\right]^{0.25}$$

(iv) Average Heat transfer coefficient (h) for Horizontal surface , laminar flow

$$h = 0.728 \left[\frac{k^3 \rho^2 g h_{fg}}{\mu L (T_{sat} - T_w)}\right]^{0.25}$$

(v) Average Heat transfer coefficient (h) for Bank of tubes , laminar flow

$$h = 0.728 \left[\frac{k^3 \rho^2 g h_{fg}}{\mu N D (T_{sat} - T_w)}\right]^{0.25}$$

(vi)Average Heat transfer coefficient (h) for vertical surface , Turbulent flow

$$h = 0.0077 (R_e)^{0.4} \left[\frac{k^3 \rho^2 g}{\mu^2}\right]^{0.333}$$

3.4. HEAT EXCHANGERS:

The device used for exchange of heat between the two fluids that are at different temperatures, is called the heat exchanger. The heat exchangers are commonly used in wide range of applications, for example, in a car as radiator, where hot water from the engine is cooled by atmospheric air. In a refrigerator, the hot refrigerant from the compressor is cooled by natural convection into atmosphere by passing it through finned tubes.

In a steam condenser, the latent heat of condensation is removed by circulating water through the tubes. The heat exchangers are also used in space heating and air-conditioning, waste heat recovery and chemical processing. Therefore, the different types of heat exchangers are needed for different applications.

The heat transfer in a heat exchanger usually involves convection on each side of fluids and conduction through the wall separating the two fluids. Thus for analysis of a heat exchanger, it is very convenient to work with an overall heat transfer coefficient U, that accounts for the contribution of all these effects on heat transfer.

The rate of heat transfer between two fluids at any location in a heat exchanger depends on the magnitude of temperature difference at that location and this temperature difference varies along the length of heat exchanger. Therefore, it is also convenient to work with logarithmic mean temperature difference LMTD, which is an equivalent temperature difference between two fluids for entire length of heat exchanger.

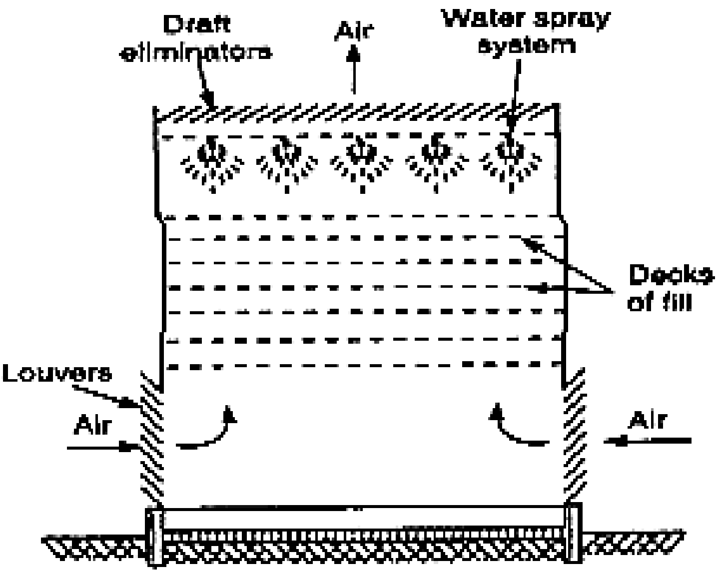
3.4.1. CLASSIFICATION OF HEAT EXCHANGER:

Heat exchangers are designed in so many sizes, types, configurations and flow arrangements and used for so many purposes. These are classified according to heat transfer process, flow arrangement and type of construction.

According to Heat Transfer Process:

(i) Direct contact type.

In this type of heat exchanger, the two immiscible fluids at different temperatures are come in direct contact. For the heat exchange between two fluids, one fluid is sprayed through the other. Cooling towers, jet condensers, desuperheaters, open feed water heaters and -scrubbers are the best examples of such heat exchangers. It cannot be used for transferring heat between two gases or between two miscible liquids. A direct contact type heat exchanger (cooling tower) is shown in Fig. 3.7.



Direct contact type heat exchanger (cooling tower)

Fig. 3.7

Transfer type heat exchangers or recuperators: In this type of heat exchanger, the cold and hot fluids flow simultaneously through the device and the heat is transferred through the wall separating them. These types of heat exchangers are most commonly used in almost all fields of engineering.

(ii) Regenerators or storage type heat exchangers.

In these types of heat exchangers, the hot and cold fluids flow alternatively on the same surface. When hot fluid flows in an interval of time, it gives its heat to the surface, which stores it in the form of an increase in its internal energy. This stored energy is transferred to cold fluid as it flows over the surface in next interval of time. Thus the same surface is subjected to periodic heating and cooling. In many applications, a rotating disc type matrix is used, the continuous flow of both the hot and cold fluids are maintained. These are preheaters for steam power plants, blast furnaces, oxygen producers etc. A stationary and rotating matrix shown in Fig. 3.8. are examples of storage type of heat exchangers.

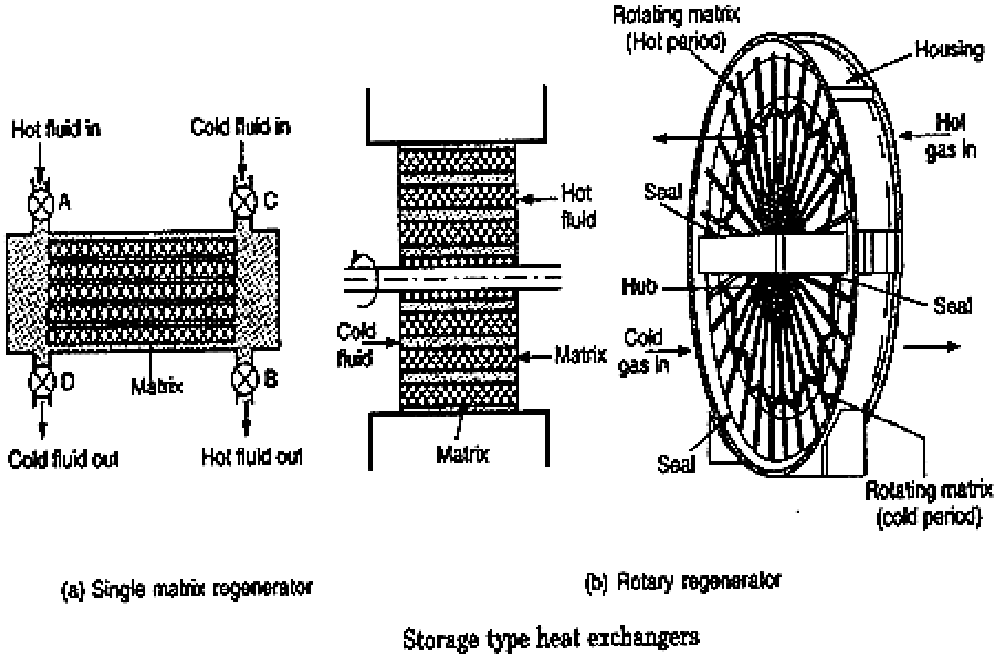
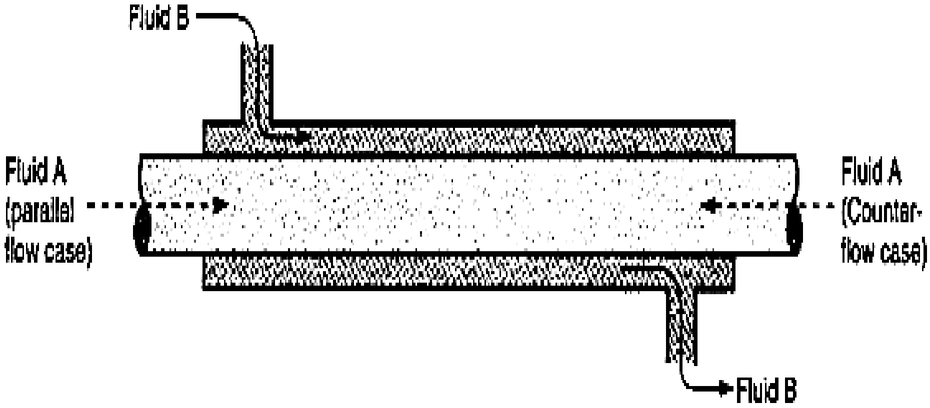


Fig.3.8

The storage type of heat exchangers is more compact than the transfer type of heat exchangers with more surface area per unit volume. However, some mixing of hot and cold fluids is always there.

<u>According to Constructional Features:</u>

(i) Tubular heat exchanger. These are also called tube in tube or concentric tube or double pipe heat exchanger as shown in Fig.3.9. These are widely used in many sizes and different flow arrangements and type.

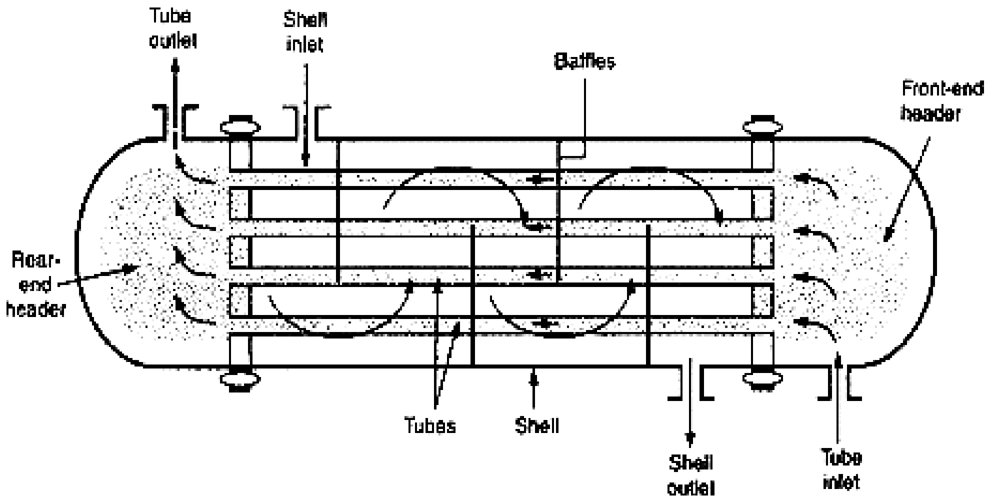


Tubular heat exchanger

Fig.3.9

(ii) Shell and tube type heat exchanger.

These are also called surface condensers and are most commonly used for heating, cooling, condensation or evaporation applications. It consists of a shell and a large number of parallel tubes housing in it. The heat transfer takes place as one fluid flows through the tubes and other fluid flows outside the tubes through the shell. The baffles are commonly used on the shell to create turbulence and to keep the uniform spacing between the tubes and thus to enhance the heat transfer rate. They are having large surface area in small volume. A typical shell and tube type heat exchanger is shown in Fig.3.10.



Shell and tube type heat exchanger : One shell and one tube pass

Fig.3.10

The shell and tube type heat exchangers are further classified according to number of shell and tube passes involved. A heat exchanger with all tubes make one U turn in a shell is called one shell pass and two tube pass heat exchanger. Similarly, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a two shell pass and four tube pass heat exchanger as shown in Fig.3.11.

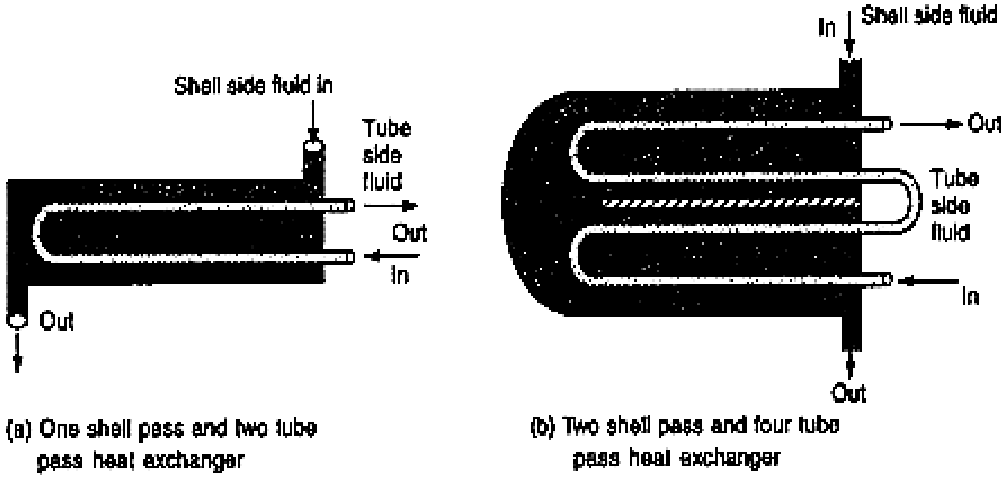


Fig.3.11.

(iii) Finned tube type.

When a high operating pressure or an enhanced heat transfer rate is required, the extended surfaces are used on one side of the heat exchanger. These heat exchangers are used for liquid to gas heat exchange. Fins are always added on gas side. The finned tubes are used in gas turbines, automobiles, aero planes, heat pumps, refrigeration, electronics, cryogenics, air-conditioning systems etc. The radiator of an automobile is an example of such heat exchanger.

(iv) Compact heat exchanger.

These are special class of heat exchangers in which the heat transfer surface area per unit volume is very large. The ratio of heat transfer surface area to the volume is called area density. A heat exchanger with an area density greater than 700 m^2/m^3 is called compact heat exchanger. The compact heat exchangers are usually cross flow, in which the two fluids usually flow perpendicular to each other. These heat exchangers have dense arrays of finned tubes or plates, where at least one of the fluid used is gas. For example, automobile radiators have an area density in order of 1100 m^2/m^3.

According to Flow Arrangement:

(i) **<u>Parallel flow:</u>** The hot and cold fluids enter at same end of the heat exchanger, flow through in same direction and leave at other end. It is also called the concurrent heat exchanger Fig 3.12.

(ii) **<u>Counter flow:</u>** The hot and cold fluids enter at the opposite ends of heat exchangers, flow through in opposite direction and leave at opposite ends Fig 3.12.

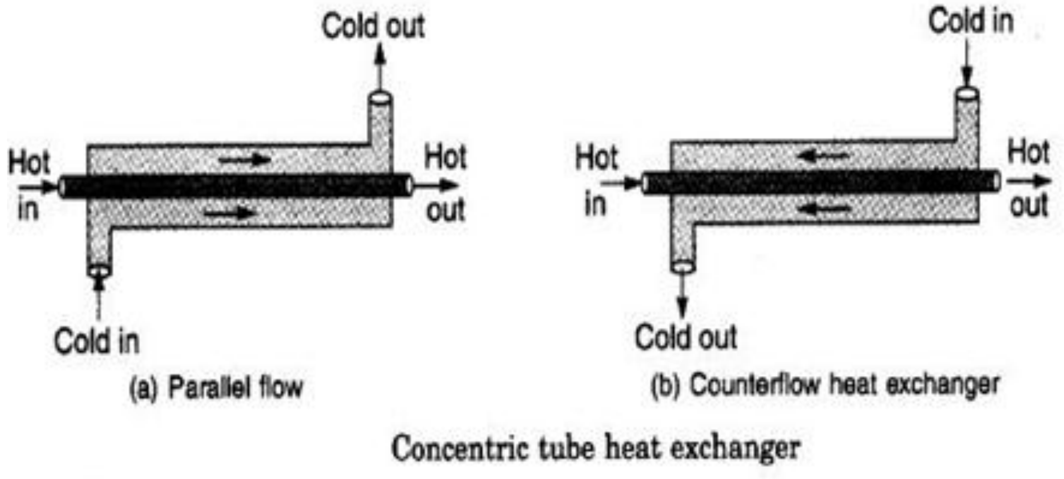


Fig.3.12.

(iii) **<u>Cross flow:</u>** The two fluids flow at right angle to each other. The cross flow heat exchanger is further classified as unmixed flow and mixed flow depending on the flow configuration. If both the fluids flow through individual channels and are not free to move in transverse direction, the arrangement is called unmixed as shown in Fig 3.13. If any fluid flows on the surface and free to move in transverse direction, then this fluid stream is said to be mixed as shown in Fig 3.13.

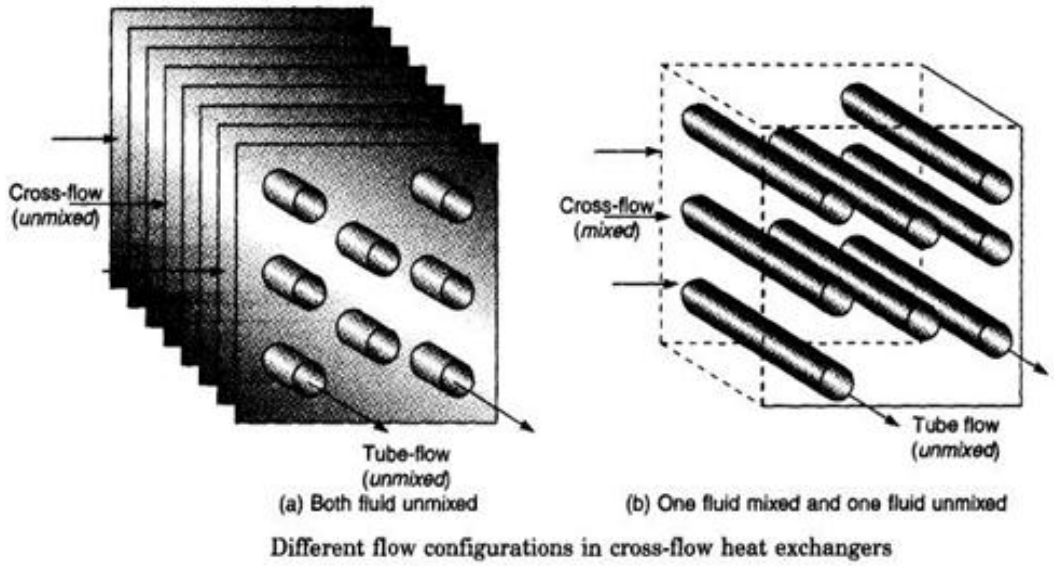


Fig.3.13

3.5. CONDENSERS AND EVAPORATORS:

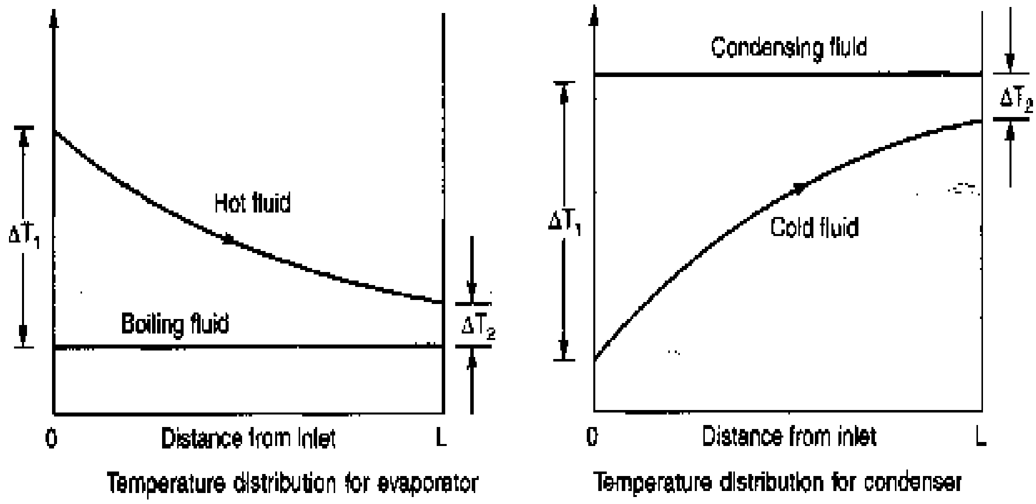


Fig.3.14.

Two special forms of heat exchangers, namely condensers and evaporators, are employed in many industrial applications. One of the fluids flowing through these exchangers changes phase. The temperature distributions in these exchangers are shown in Fig.3.14. In the case of a condenser, the hot fluid will remain at a constant temperature, provided its pressure does not change, while the temperature of the cold fluid increases.

3.6. OVERALL HEAT TRANSFER COEFFICIENT

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R, and to express the rate of heat transfer between the two fluids as

$$Q = \frac{\Delta T}{R} = U\,A_s\,\Delta T = U_i\,A_i\,\Delta T_i = U_o\,A_o\,\Delta T_o$$

where U is the overall heat transfer coefficient, whose unit is $W/m^2 \cdot °C$, which is identical to the unit of the ordinary convection coefficient h. Canceling ΔT, from the above equation, it reduces to

$$\frac{1}{U\,A_s} = \frac{1}{U_i\,A_i} = \frac{1}{U_o\,A_o} = R$$

we have two overall heat transfer coefficients U_i and U_o for a heat exchanger. The reason is that every heat exchanger has two heat transfer surface areas Ai and Ao, which, in general, are not equal to each other. Note that $U_iA_i = U_oA_o$, but $U_i \neq U_o$ unless $A_i = A_o$.

3.7. FOULING FACTOR

The performance of heat exchangers usually deteriorates with time as a result of accumulation of deposits on heat transfer surfaces. The layer of deposits represents additional resistance to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a fouling factor R_f, which is a measure of the thermal resistance introduced by fouling.

3.8. ANALYSIS OF HEAT EXCHANGERS

There are two methods in use for the analysis of heat exchangers. The first is called the Log Mean Temperature Difference method (LMTD) and the second is called the Effectiveness – NTU method (Number of transfer Units) or simply the NTU method.

3.8.1. LMTD METHOD

The heat transfer from one fluid stream to another can be written as:

To determine ΔT_m we will make the following assumptions:

- No external losses from the heat exchanger
- Negligible conduction along the tube length;
- Changes in kinetic and potential energy are negligible
- h is constant along the length of the heat exchanger
- Specific heats are constant (not a function of temperature).

Consider a parallel flow heat exchanger with the temperature distribution as shown in Figure.3.15.

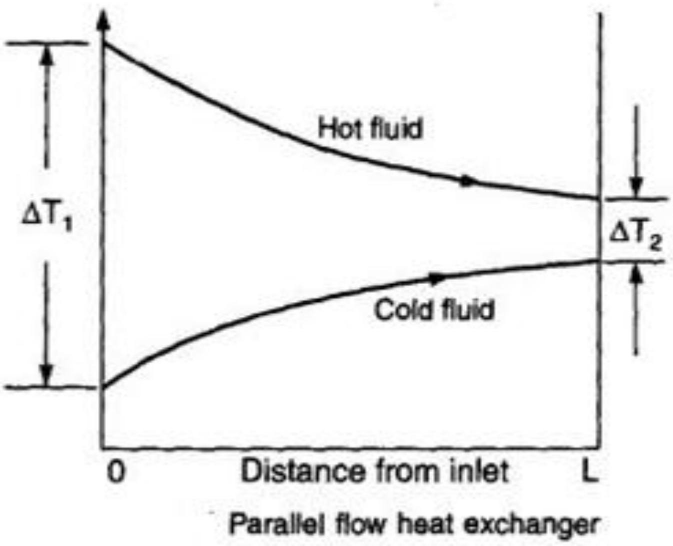


Fig.3.15

The general relationship for the heat transfer from one fluid stream to the other is given by:

$$Q = m\, C_p\, \left(T_{Fluid,in} - T_{Fluid,out}\right)$$

For hot fluid,

$$Q = m_h\, C_{ph}\, \left(T_{h,in} - T_{h,out}\right)$$

For cold fluid,

$$Q = m_c\, C_{pc}\, \left(T_{c,out} - T_{c,in}\right)$$

The Logarithmic Mean Temperature Difference, ΔT_m

For parallel flow, ΔT_m is given as

$$\Delta T_m = \frac{\left(T_{h,in} - T_{c,in}\right) - \left(T_{h,out} - T_{c,out}\right)}{ln\left(\frac{T_{h,in} - T_{c,in}}{T_{h,out} - T_{c,out}}\right)}$$

or

$$\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

Where, $\Delta T_1 = T_{h,in} - T_{c,in}$; $\Delta T_2 = T_{h,out} - T_{c,out}$

For counter flow

ΔT_m is given as

$$\Delta T_m = \frac{\left(T_{h,in} - T_{c,out}\right) - \left(T_{h,out} - T_{c,in}\right)}{ln\left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$$

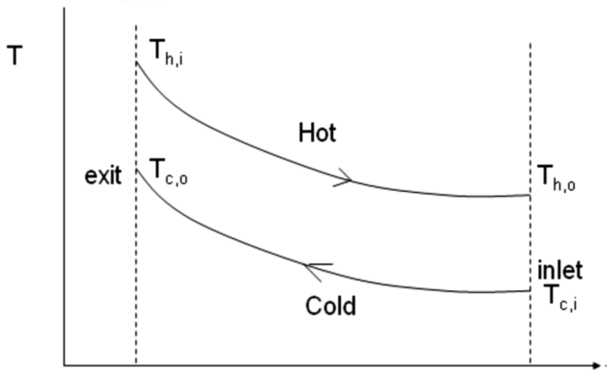


Fig.3.16

Or

$$\Delta T_m = \frac{\Delta T_2 - \Delta T_1}{ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

Where, $\Delta T_1 = T_{h,in} - T_{c,out}$; $\Delta T_2 = T_{h,out} - T_{c,in}$

The same can be extended to other types of heat exchangers such as cross flow or shell and tube using a correction factor which is a function of two other dimensionless factors which are in turn defined empirically as follows

$$\Delta T_m = (\Delta T_m)_{cf} \cdot F$$

Where,

$(\Delta T_m)_{cf}$	- LMTD for counter flow
F	- Correction factor [F = f (P,R)]

Where P and R are the empirical parameters

$$P = \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} = \frac{T_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_{h,in} - T_{h,out}}{T_{c,out} - T_{c,in}} = \frac{T_1 - T_2}{T_2 - t_1}$$

This can be taken for various types of cross flow heat exchangers using the graphs from HMT data book.

3.8.2. EFFECTIVENESS - NTU METHOD

A second kind of problem encountered in heat exchanger analysis is the determination of the heat transfer rate and the outlet temperatures of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the type and size of the heat exchanger are specified. The heat transfer surface area A of the heat exchanger in this case is known, but the outlet temperatures are not. Here the task is to determine the heat transfer performance of a specified heat exchanger or to determine if a heat exchanger available in storage will do the job.

The LMTD method could still be used for this alternative problem, but the procedure would require tedious iterations, and thus it is not practical. In an attempt to eliminate the iterations from the solution of such problems, Kays and London came up with a method in 1955 called the effectiveness–NTU method, which greatly simplified heat exchanger analysis. This method is based on a dimensionless parameter called the heat transfer effectiveness ε, defined as

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{Actual\ heat\ transfer\ rate}{maximum\ possible\ heat\ transfer\ rate}$$

The actual heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as

$$Q = C_h \left(T_{h,in} - T_{h,out}\right) = C_c \left(T_{c,out} - T_{c,in}\right)$$

Where $C_c = m_cC_{pc}$ and $C_h = m_cC_{ph}$ are the heat capacity rates of the cold and the hot fluids, respectively.

To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the maximum temperature difference in a heat exchanger is the difference between the inlet temperatures of the hot and cold fluids. That is,

$$\Delta T_{max} = T_{h,in} - T_{c,in}$$

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e., $C_c = C_h$). When $C_c \neq C_h$, which is usually the case, the fluid with the smaller heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt.

Therefore, the maximum possible heat transfer rate in a heat exchanger is

$$Q_{max} = C_{min}\left(T_{h,in} - T_{c,in}\right)$$

Where C_{min} is the smaller of $C_c = m_cC_{pc}$ and $C_h = m_cC_{ph}$.

The determination of Q_{max} requires the availability of the inlet temperature of the hot and cold fluids and their mass flow rates, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate Q can be determined from

$$Q = \varepsilon\, Q_{max} = \varepsilon\, C_{min}\left(T_{h,in} - T_{c,in}\right)$$

Effectiveness relations of the heat exchangers typically involve the dimensionless group UAs /C_{min}. This quantity is called the **number of transfer units NTU** and is expressed as

$$NTU = \frac{U\, A_s}{C_{min}}$$

Where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. Note that NTU is proportional to A_s. Therefore, for specified values of U and C_{min}, the value of NTU is a measure of the heat transfer surface area As. Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the **capacity ratio c** as

$$c = \frac{C_{min}}{C_{max}}$$

It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio c. That is,

$$\varepsilon = f\left(\frac{U\,A_s}{C_{min}},\ \frac{C_{min}}{C_{max}}\right) = f\ (NTU, c)$$

Questions & Answers

1. **What is meant by Boiling and condensation?**

   The change of phase from liquid to vapour state is known as boiling.

   The change of phase from vapour to liquid state is known as condensation.

2. **Give the applications of boiling and condensation.**

   a. Boiling and condensation process finds wide applications as mentioned below.

   b. Thermal and nuclear power plant.

   c. Refrigerating systems

   d. Process of heating and cooling

   e. Air conditioning systems

3. **What is meant by pool boiling?**

   If heat is added to a liquid from a submerged solid surface, the boiling process referred to as pool boiling. In this case the liquid above the hot surface is essentially stagnant and its motion near the surface is due to free convection and mixing induced by bubble growth and detachment.

4. **What is meant by Film wise and Drop wise condensation?**

   The liquid condensate wets the solid surface, spreads out and forms a continuous film over the entire surface is known as film wise condensation.

   In drop wise condensation the vapour condenses into small liquid droplets of various sizes which fall down the surface in a random fashion.

5. **Give the merits of drop wise condensation?**

   In drop wise condensation, a large portion of the area of the plate is directly exposed to vapour. The heat transfer rate in drop wise condensation is 10 times higher than in film condensation.

6. **What is heat exchanger?**

   A heat exchanger is defined as an equipment which transfers the heat from a hot fluid to a cold fluid.

7. **What are the types of heat exchangers?**

   The types of heat exchangers are as follows

   a. Direct contact heat exchangers

   b. Indirect contact heat exchangers

   c. Surface heat exchangers

   d. Parallel flow heat exchangers

e. Counter flow heat exchangers

f. Cross flow heat exchangers

g. Shell and tube heat exchangers

h. Compact heat exchangers.

8. What is meant by Direct heat exchanger (or) open heat exchanger?

In direct contact heat exchanger, the heat exchange takes place by direct mixing of hot and cold fluids.

9. What is meant by Indirect contact heat exchanger?

In this type of heat exchangers, the transfer of heat between two fluids could be carried out by transmission through a wall which separates the two fluids.

10. What is meant by Regenerators?

In this type of heat exchangers, hot and cold fluids flow alternately through the same space.

Examples: IC engines, gas turbines.

11. What is meant by Recupcradors (or) surface heat exchangers?

This is the most common type of heat exchangers in which the hot and cold fluid do not come into direct contact with each other but are separated by a tube wall or a surface.

12. What is meant by parallel flow heat exchanger?

In this type of heat exchanger, hot and cold fluids move in the same direction.

13. What is meant by counter flow heat exchanger?

In this type of heat exchanger hot and cold fluids move in parallel but opposite directions.

14. What is meant by cross flow heat exchanger?

In this type of heat exchanger, hot and cold fluids move at right angles to each other.

15. What is meant by shell and tube heat exchanger?

In this type of heat exchanger, one of the fluids move through a bundle of tubes enclosed by a shell. The other fluid is forced through the shell and it moves over the outside surface of the tubes.

16. **What is meant by compact heat exchangers?**

There are many special purpose heat exchangers called compact heat exchangers. They are generally employed when convective heat transfer coefficient associated with one of the fluids is much smaller than that associated with the other fluid.

17. What is meant by LMTD?

We know that the temperature difference between the hot and cold fluids in the heat exchanger varies from point in addition various modes of heat transfer are involved. Therefore based on concept of appropriate mean temperature difference, also called logarithmic mean temperature difference, also called logarithmic mean temperature difference, the total heat transfer rate in the heat exchanger is expressed as

$$Q = U\,A\,(\Delta T)m$$

Where

U – Overall heat transfer coefficient W/m^2K

A – Area m^2

$(\Delta T)_m$ – Logarithmic mean temperature difference.

18. What is meant by Fouling factor?

We know the surfaces of heat exchangers do not remain clean after it has been in use for some time. The surfaces become fouled with scaling or deposits. The effect of these deposits the value of overall heat transfer coefficient. This effect is taken care of by introducing an additional thermal resistance called the fouling resistance.

19. What is meant by effectiveness?

The heat exchanger effectiveness is defined as the ratio of actual heat transfer to the maximum possible heat transfer.

Effectiveness

$$\varepsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}}$$

$$\frac{Q}{Q_{max}}$$

1.

Problems:

1. Saturated steam under atmospheric pressure condenses on a vertical surface 100 cm high. If the temperature of the surface is maintained at 80 °C calculate at 50 cm from the top of the plate. (i) The film thickness, (ii) the mean velocity, (iii) the local heat transfer coefficient, (iv) Average heat transfer coefficient, (v) total flow per hour per meter width of the plate, (vi) Rate of condensation per hour. **KU – Nov 2010**

Given Data:

L = 100 cm = 1 m, T_w = 80°C, x = 50 cm = 0.5 m

To Find:

(i) δ_x, (ii) u, (iii) h_x, (iv) h, (v) m, (vi) Q

Solution:

We know that the saturation temperature of water is 100°C, hence from steam tables at 100°C

Enthalpy of evaporation, h_{fg} = 2256.9 kJ/kg = 2256.9 x 10^3 J/kg

Specific volume of vapour, v_g = 1.673 m^3/kg

Density of vapour, $\rho_v = 1/v_g$ = 0.597 kg/m^3

To find the film temperature, T_f

$$T_f = \frac{T_{sat} + T_w}{2}$$

$$T_f = \frac{80 + 100}{2} = 90\ °C$$

From HMT DB pg.no. 34, the properties of air at T_f = 90°C are

ρ = 0.972 kg/m^3, μ = 21.48 x 10^{-6} m^2/sec, k = 0.03128 W/mK

Pr = 0.690

(i) The film thickness, δ_x

$$\delta_x = \left[\frac{4\,\mu\,k\,x\,(T_{sat} - T_w)}{g\,h_{fg}\,\rho^2}\right]^{0.25}$$

$$\delta_x = \left[\frac{4 \times 21.48 \times 10^{-6} \times 0.03128 \times 0.5 \times (100 - 80)}{9.81 \times 2256.9 \times 10^3 \times 0.972^2}\right]^{0.25}$$

$$\boldsymbol{\delta_x = 1.89 \times 10^{-3}\,, \qquad m}$$

(ii) Average velocity, u

$$u = \frac{\rho\,g\,\delta_x{}^2}{2\,\mu}$$

$$u = \frac{0.972 \times 9.81 \times (1.89 \times 10^{-3})^2}{2 \times 21.48 \times 10^{-6}}$$

$$\boldsymbol{u = 0.8\ m/sec}$$

(iii) Local heat transfer coefficient, h_x

$$h_x = \frac{k}{\delta_x}$$

$$h_x = \frac{0.03128}{1.89 \times 10^{-3}}$$

$$\boldsymbol{h_x = 16.6\ W/m^2K}$$

(iv) Average heat transfer coefficient, h

$$h = 1.13\left[\frac{k^3\ \rho^2\ g\ h_{fg}}{\mu\ L\ (T_{sat} - T_w)}\right]^{0.25}$$

$$h = 1.13\left[\frac{0.03128^3 \times 0.972^2 \times 9.81 \times 2256.9 \times 10^3}{21.48 \times 10^{-6} \times 1 \times (100 - 80)}\right]^{0.25}$$

$$\boldsymbol{h = 39.5\ W/m^2K}$$

(v) Heat transfer, Q

$$Q = h\ A_s\ (T_{sat} - T_w)$$

$$Q = 39.5 \times 1 \times 1 \times (100 - 80)$$

$$\boldsymbol{Q = 790\ W}$$

(vi) Rate of condensation per hour, m

$$Q = m \times h_{fg}$$

$$m = \frac{Q}{h_{fg}} = \frac{790}{2256.9 \times 10^3}$$

$$\boldsymbol{m = 3.5 \times 10^{-4} kg/s}$$

$$\boldsymbol{m = 1.26\ kg/hr}$$

2. Dry saturated steam at a pressure of 2.45 bar condenses on the surface of a vertical tube of height 1 m. The tube surface temperature is kept at 117°C. Estimate the thickness of the condensate film

and the local heat transfer coefficient at a distance of 0.2 m from the upper end of the tube. **KU – Nov 2011, KU – Aug 2014**

Given Data:

p = 2.45 bar, L = 1 m, T_w = 117°C, h_x @ x = 0.2 m

To find:

(i) δ_x (ii) h_x @ x = 0.2 m

Solution:

The properties of steam at 2.45 bar, from steam tables

T_{sat} = 127 °C, h_{fg} = 2183 kJ/kg = 2183 x 10^3 J/kg

To find the film temperature, T_f

$$T_f = \frac{T_{sat} + T_w}{2}$$

$$T_f = \frac{127 + 117}{2} = 122\ °C$$

From HMT DB pg.no. 22, the properties of water at T_f = 122°C ≈ 120 °C are

ρ = 945 kg/m^3, μ = 2.33 x 10^{-4} m^2/sec, k = 0.6850 W/mK

(i) The film thickness, δ_x

$$\delta_x = \left[\frac{4\,\mu\,k\,x\,(T_{sat} - T_w)}{g\,h_{fg}\,\rho^2}\right]^{0.25}$$

$$\delta_x = \left[\frac{4 \times 2.33 \times 10^{-4} \times 0.6850 \times 0.2 \times (127 - 117)}{9.81 \times 2183 \times 10^3 \times 945^2}\right]^{0.25}$$

$$\boldsymbol{\delta_x = 1.35 \times 10^{-4}\,, \qquad m}$$

(ii) Local heat transfer coefficient, h_x

$$h_x = \frac{k}{\delta_x}$$

$$h_x = \frac{0.6850}{1.35 \times 10^{-4}}$$

$$\boldsymbol{h_x = 5074.1\ W/m^2K}$$

3. A tube of 2m length and 25mm outer diameter is to be condensing saturated steam at 100°C while the tube surface is maintained at 96°C. Estimate the average heat transfer co-efficient and the rate

of condensation of steam if the tube is kept horizontal. The steam condenses on the outside of the tube. **KU – Nov 2011**

Given Data:

L = 2 m, D = 25 mm = 0.025 m, T_{sat} = 100°C, T_w = 96°C

To Find:

(i) h =?, (ii) m =?

Solution:

We know that the saturation temperature of water is 100°C, hence from steam tables at 100°C

Enthalpy of evaporation, h_{fg} = 2256.9 kJ/kg = 2256.9 x 10^3 J/kg

To find the film temperature, T_f

$$T_f = \frac{T_{sat} + T_w}{2}$$

$$T_f = \frac{100 + 96}{2} = 98\ °C$$

From HMT DB pg.no. 22, the properties of water at T_f = 98°C ≈ 100 °C are

ρ = 961 kg/m^3, μ = 2.82 x 10^{-4} m^2/sec, k = 0.6804 W/mK

(i) Average Heat transfer coefficient (h) for Horizontal surface , laminar flow

$$h = 0.728\ \left[\frac{k^3\ \rho^2\ g\ h_{fg}}{\mu\ L\ (T_{sat} - T_w)}\right]^{0.25}$$

$$h = 0.728\ \left[\frac{0.6804^3\ \times 961^2 \times 9.81\ \times\ 2256.9\ \times\ 10^3}{2.82\ \times 10^{-4} \times 1\ \times\ (100 - 96)}\right]^{0.25}$$

$$\boldsymbol{h = 6328.3\ W/m^2K}$$

Heat transfer, Q

$$Q = h\ A_s\ (T_{sat} - T_w)$$

$$Q = 6328.3 \times \pi \times 0.025\ \times 1\ \times\ (100 - 96)$$

$$\boldsymbol{Q = 1988.0\ W}$$

(ii) Rate of condensation, m

$$Q = m \times h_{fg}$$

$$m = \frac{Q}{h_{fg}} = \frac{1988.0}{2256.9 \times 10^3}$$

$$\boldsymbol{m = 8.8 \times 10^{-4} kg/s}$$

4. An aluminium pan of 15cm diameter is used to boil water and the water depth at the time of boiling is 2.5 cm. The pan is placed on an electric stove and the heating element raises the temperature of the pan to 110°C. Calculate the power input for boiling and the rate of evaporation. Take C_{sf} = 0.0132. **KU – Nov 2012**

Given Data:

d = 15 cm = 0.15 m, x = 2.5 cm = 0.025 m, T_w= 110°C, C_{sf} = 0.0132

To find:

(i) Power input, Q

(ii) Rate of evaporation, m

Solution:

We know that the saturation temperature of water is 100°C, hence the properties of water at saturation temperature is taken from HMT DB pg.no 22

ρ_l = 961 kg/m^3, = 0.293 x 10^{-6} m^2/s, Pr = 1.740, C_{pl} = 4216 J/kgK, $\mu_l = \rho_l$ x ν = 961 x 0.293 x 10^{-6} = 281.57 x 10^{-6} Ns/m^2

From steam tables at 100°C

Enthalpy of evaporation, h_{fg} = 2256.9 kJ/kg = 2256.9 x 10^3 J/kg

Specific volume of vapour, v_g = 1.673 m^3/kg

Density of vapour, $\rho_v = 1/v_g$ = 0.597 kg/m^3

ΔT = $T_w - T_{sat}$ = 110 – 100 = 10 °C

Since ΔT is less than 50°C. So this is nucleate pool boiling.

Power input is taken from HMT DB pg.no 143

$$\frac{Q}{A} = \mu_l \, h_{fg} \left[\frac{g \times (\rho_l - \rho_v)}{\sigma}\right]^{0.5} \times \left[\frac{C_{pl}\,(T_w - T_{sat})}{C_{sf} \times h_{fg} \times {P_r}^n}\right]^3$$

Where, n = 1 for water, σ = 0.0588 N/m from HMT DB pg.no 145

$$\frac{Q}{A} = 281.57 \times 10^{-6} \times 2256.9 \times 10^3 \times \left[\frac{9.81 \times (961 - 0.597)}{0.0588}\right]^{0.5}$$

$$\times \left[\frac{4216 \times 10}{0.013 \times 2256.9 \times 10^3 \times 1.740}\right]^3$$

$$\frac{Q}{A} = 1.43 \times 10^5 \, W/m^2$$

Heat transfer,

$$Q = 1.43 \times 10^5 \times A$$

$$Q = 1.43 \times 10^5 \times \frac{\pi}{4} \times d^2$$

$$Q = 1.43 \times 10^5 \times \frac{\pi}{4} \times 0.15^2$$

$$Q = 2527 \, W = P$$

Power input for boiling, P = 2527 W

Rate of evaporation,

$$Q = m \times h_{fg}$$

$$m = \frac{Q}{h_{fg}} = \frac{2527}{2256.9 \times 10^3}$$

$$\boldsymbol{m = 1.11 \times 10^{-3} kg/s}$$

5. Dry steam at 100°C condenses on the outside surface of a horizontal pipe of outer diameter 2.5 cm. The pipe surface is maintained at 84°C by circulating water trough it. Determnine the rate of formation of condensate per meter length of the pipe. **KU – Apr 2014**

Given Data:

T_{sat} = 100 °C, D = 0.025 cm, T_w = 84°C

To Find:

m =?

Solution:

We know that the saturation temperature of water is 100°C, hence from steam tables at 100°C Enthalpy of evaporation, h_{fg} = 2256.9 kJ/kg = 2256.9 x 10^3 J/kg

To find the film temperature, T_f

$$T_f = \frac{T_{sat} + T_w}{2}$$

$$T_f = \frac{100 + 84}{2} = 92\ °C$$

From HMT DB pg.no. 22, the properties of water at T_f = 92°C ≈ 90 °C are

ρ = 967.5 kg/m^3, μ = 3.18 x 10^{-4} m^2/sec, k = 0.6746 W/mK

(i) Average Heat transfer coefficient (h) for Horizontal surface, laminar flow

$$h = 0.728\ \left[\frac{k^3\ \rho^2\ g\ h_{fg}}{\mu\ L\ (T_{sat} - T_w)}\right]^{0.25}$$

$$h = 0.728\ \left[\frac{0.6746^3\ \times 967.5^2 \times 9.81\ \times\ 2256.9\ \times\ 10^3}{3.18\ \times 10^{-4} \times 1\ \times\ (100 - 84)}\right]^{0.25}$$

$$\boldsymbol{h = 4329.1\ W/m^2K}$$

Heat transfer, Q

$$Q = h\ A_s\ (T_{sat} - T_w)$$

$$Q = 4329.1 \times \pi \times 0.025\ \times 1\ \times\ (100 - 84)$$

$$\boldsymbol{Q = 5440.1\ W}$$

(ii) Rate of condensation, m

$$Q = m\ \times\ h_{fg}$$

$$m = \frac{Q}{h_{fg}} = \frac{5440.1}{2256.9\ \times\ 10^3}$$

$$\boldsymbol{m = 2.41 \times\ 10^{-3} kg/s}$$

6. Steam enters a counter flow heat exchanger, dry saturated at 10 bar and leaves at 350 °C. The mass flow of steam is 800 kg/min. The gases enter the heat exchanger at 650 °C and mass flow rate is 1350 kg/min. If the tubes are 300 mm diameter and 3 m long. Determine the number of tubes required. Neglect the resistance offered by metallic tubes. Take C_{pg} = 1 kJ/kg °C, h_g = 250 W/m^2°C, h_s = 600 W/m^2°C. **KU – Nov 2010**

Given Data:

p = 10 bar, $T_{c,out}$ = 350°C, m_c = 800 kg/min = 13.3 kg/s, $T_{h,in}$ = 650°C, m_h = 1350 kg/min = 22.5 kg/s,

d = 300 mm = 0.3 m, L = 3 m

C_{pg} = 1 kJ/ = 1000 J/kg °C, h_g = 250 W/m^2°C, h_s = 600 W/m^2°C

To find:

No of tubes, n =?

Solution:

From steam tables pg.no 11 for a pressure of p = 10 bar, the saturation temperature of steam is given as

$$T_{c,in} = 180°C$$

We know that the rate of heat transfer is given by

$$Q = m_c\, C_{pc}\, (T_{c,out} - T_{c,in})$$

$$Q = 13.3 \times 4186 \times (350 - 180)$$

$$\boldsymbol{Q = 9.46 \times 10^6\ W}$$

We also know that

Heat lost by oil (hot fluid) = Heat gained by the water (cold fluid)

$$m_c\, C_{pc}\, (T_{c,out} - T_{c,in}) = m_h\, C_{ph}\, (T_{h,in} - T_{h,out})$$

Therefore

$$13.3 \times 4186 \times (350 - 180) = 22.5 \times 1000 \times (650 - T_{h,out})$$

$$\boldsymbol{T_{c,out} = 229.5}℃ \cong \boldsymbol{230}\ ℃$$

The rate of heat transferred is also given by,

$$Q = F\ U\ A_s\ (\Delta T)_m$$

Where $(\Delta T)_m$ for multipass shell and tube heat exchanger is taken from HMT DB pg.no. 152

$$\Delta T_m = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{ln\left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$$

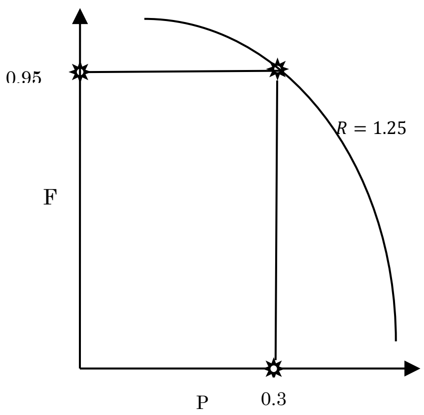
$$\Delta T_m = \frac{(650 - 350) - (230 - 180)}{ln\left(\frac{650 - 350}{230 - 180}\right)}$$

$$\boldsymbol{\Delta T_m = 139.5}℃ \cong \boldsymbol{140}℃$$

To find correction factor ‘F’

from HMT DB pg.no. 159

$$P = \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} = \frac{350 - 180}{650 - 180} = 0.4$$



$$R = \frac{T_{h,in} - T_{h,out}}{T_{c,out} - T_{c,in}} = \frac{650 - 230}{350 - 180} = 2.5$$

From the graph,

On X – axis, P = 0.4

And on curve R = 2.5

The corresponding correction factor,

F = 0.8

To find the overall heat transfer coefficient,

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$\frac{1}{U} = \frac{1}{250} + \frac{1}{600}$$

$$\boldsymbol{U = 176.5\, W/m^2K}$$

Therefore,

$$Q = F\, U\, A_s\, (\Delta T)_m$$

$$9.46 \times 10^6 = 0.8 \times 176.5 \times A_s \times 140$$

$$\boldsymbol{A_s = 478.55\, m^2}$$

Surface area of a single tube,

$$\boldsymbol{A_1 = \pi\, d\, L = \pi \times 0.3 \times 3 = 2.83\, m^2}$$

Therefore the number of tubes required is taken as

$$n = \frac{\boldsymbol{A_s}}{\boldsymbol{A_1}} = \frac{478.55}{2.83}$$

$$\boldsymbol{n = 169\ tubes}$$

7. Saturated steam at 120°C condenses on the outer tube surface of a single pass heat exchanger. Determine the surface area to heat 1000 kg/hour of water from 20°C to 90°C. Find the mass of the condensate. Take heat transfer coefficient U_o = 1800 W/m^2 and h_{fg} = 2200 kJ/kg. **KU – Nov 2013**

Given Data:

$T_{h,in}$ = 120°C, m_c = 1000 kg/hr = 0.28 kg/sec, $T_{c,in}$ = 20°C, $T_{c,out}$ = 90°C, h_{fg} = 2200 kJ/kg, U_o = 1800 W/m^2

To find:

(i) Surface area, A, (ii) Mass of the condensate, m_h =?

Solution:

The specific heat of water is taken as C_{pc} = 4186 J/kgK

For saturated steam, $T_{h,in} = T_{h,out}$= 120°C

We know that the rate of heat transfer is given by

$$Q = m_c\, C_{pc}\, \left(T_{c,out} - T_{c,in}\right)$$

$$Q = 0.28 \times 4186 \times (90 - 20)$$

$$\boldsymbol{Q = 82045.6\ W}$$

Also W.K.T,

$$Q = m_h \times h_{fg}$$

$$m_h = \frac{Q}{h_{fg}} = \frac{82045.6}{2200 \times 10^3}$$

$$\boldsymbol{m_h = 0.0373\ kg/s}$$

The rate of heat transferred is also given by,

$$Q = U\, A_s\, (\Delta T)_m$$

Where $(\Delta T)_m$ for parallel flow heat exchanger is taken from HMT DB pg.no. 152

$$\Delta T_m = \frac{\left(T_{h,in} - T_{c,in}\right) - \left(T_{h,out} - T_{c,out}\right)}{ln\left(\frac{T_{h,in} - T_{c,in}}{T_{h,out} - T_{c,out}}\right)}$$

$$\Delta T_m = \frac{(120 - 20) - (120 - 90)}{ln\left(\frac{120 - 20}{120 - 90}\right)}$$

$$\Delta T_m = 58\text{°C}$$

Therefore,

$$Q = U\, A_s\, (\Delta T)_m$$

$$82045.6 = 1800 \times A_s \times 58$$

$$\boldsymbol{A_s = 0.79\ m^2}$$

8. A parallel flow heat exchanger has hot and cold water stream running through it, the flow rates are 10 and 25 kg/min respectively. Inlet temperatures are 75˚C and 25˚C on hot and cold sides. The exit temperature on the hot side should not exceed 50˚C. Assume $h_i = h_o$ = 600W/m^{2}K. Calculate the area of heat exchanger using ε – NTU approach. **KU – Nov 2011**

Given Data:

m_h = 10 kg/min = 0.17 kg/sec, m_c = 25 kg/min = 0.42 kg/sec, $T_{h,in}$ = 75°C, $T_{c,in}$ = 25°C, $T_{c,out}$ = 50°C, $h_i = h_o$ = 600W/m^{2}K

To Find:

Heat Exchanger area, A=?

Solution:

To find the capacity rate of hot and cold fluid,

$C_c = m_c C_{pc}$ = 0.42 x 4186 = 1741.4 W/K and

$C_h = m_c C_{ph}$ = 0.17 x 4186 = 694.87 W/K

Out of the above two capacity rates, the minimum and maximum are taken as

C_{min} = 694.87 W/K and C_{max} = 1741.4 W/K

The capacity ratio, c

$$c = \frac{C_{min}}{C_{max}}$$

$$c = \frac{694.87}{1741.4} = 0.399$$

From HMT DB pg.no 152, the effectiveness

$$\varepsilon = \frac{T_1 - T_2}{T_1 - t_1} = \frac{75 - 50}{75 - 25}$$

$$\varepsilon = 0.5$$

To find NTU, from HMT DB pg.no 163 for parallele flow heat exchanger

From the graph

Y-axis = ε = 0.5 and curve, c = 0.399

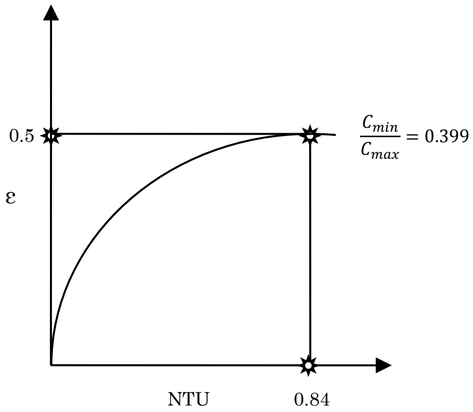
Corresponding to these, the NTU is taken as 0.84

NTU = 0.84

We also know that,

$$NTU = \frac{U\,A_s}{C_{min}}$$

To find the overall heat transfer coefficient,



$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

$$\frac{1}{U} = \frac{1}{600} + \frac{1}{600}$$

$$\boldsymbol{U = 300\ W/m^2K}$$

Therefore,

$$NTU = \frac{U\ A_s}{C_{min}}$$

$$A_s = \frac{NTU\ \times\ C_{min}}{U}$$

$$A_s = \frac{0.84\ \times\ 694.87}{300}$$

$$\boldsymbol{A_s = 1.945\ m^2}$$

9. In a double pipe counter flow heat exchanger 10000 kg/h of oil having a specific heat of 2095 J/kgK is cooled from 800ºC to 500ºC by 8000 kg/h of water entering at 25°C. Determine the heat exchanger area for an overall heat transfer coefficient of 300 W/m^{2}K. Take C_p for water as 4180 J/kgK. **KU – Nov 2011**

Given Data:

m_h = 10000 kg/h = 2.8 kg/s, C_{ph} = 2095 J/kgK, $T_{h,in}$ = 800°C, $T_{h,out}$ = 500°C, m_c = 8000 kg/h = 2.22 kg/s, $T_{c,in}$ = 25°C, U = 300 W/m^{2}K, C_{pc} = 4180 J/kgK

To Find:

Heat exchanger area, A

Solution:

Heat lost by oil (hot fluid) = Heat gained by the water (cold fluid)

$$m_c\ C_{pc}\ (T_{c,out} - T_{c,in}) = m_h\ C_{ph}\ (T_{h,in} - T_{h,out})$$

$$2.22\ \times 4180\ \times\ (T_{c,out} -\ 25) = 2.8\ \times 2095\ \times\ (800 - 500)$$

$$T_{c,out} = 215℃$$

The rate of heat transferred is given as

$$Q = m_c\ C_{pc}\ (T_{c,out} - T_{c,in})\ (or)\ m_h\ C_{ph}\ (T_{h,in} - T_{h,out})$$

$$Q = m_h\ C_{ph}\ (T_{h,in} - T_{h,out})$$

$$Q = 2.8\ \times 2095\ \times\ (800 - 500)$$

$$Q = 1.8 \times 10^6\ W$$

The rate of heat transferred is also given by,

$$Q = U\ A_s\ (\Delta T)_m$$

Where $(\Delta T)_m$ for counter flow heat exchanger is taken from HMT DB pg.no. 152

$$\Delta T_m = \frac{\left(T_{h,in} - T_{c,out}\right) - \left(T_{h,out} - T_{c,in}\right)}{ln\left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$$

$$\Delta T_m = \frac{(800 - 215) - (500 - 25)}{ln\left(\frac{800 - 215}{500 - 25}\right)}$$

$$\Delta T_m = 528°C$$

Therefore,

$$Q = U\ A_s\ (\Delta T)_m$$

$$1.8 \times 10^6 = 300 \times A_s \times 528$$

$$\boldsymbol{A_s = 11.4\ m^2}$$

10. Hot exhaust gases which enters a finned tube cross flow exchanger at 300°C and leave at 100°C, are used to heat pressurized water at a flow rate of 1 kg/s from 35 to 125°C. The exhaust gas specific heat is approximately 1000 J/kgK, and the overall heat transfer coefficient based on the gas side surface area is U_h = 100 W/m^{2}K. Determine the required gas side surface area A_h using the NTU method. Take $C_{p,\,c}$ at T_c = 80°C is 4197 J/kgK and $C_{p,\,h}$ = 1000 J/kgK. **KU – Nov 2013**

Given Data:

$T_{h,in}$ = 300°C, $T_{h,out}$ = 100°C, $T_{c,in}$ = 35°C, $T_{c,out}$ = 125°C, m_c = 1 kg/s, C_{ph} = 1000 J/kgK, U = 100 W/m^{2}K, C_{pc} = 4197 J/kgK

To Find:

A_s=?

Solution:

To find the mass flow rate of hot gas, m_h

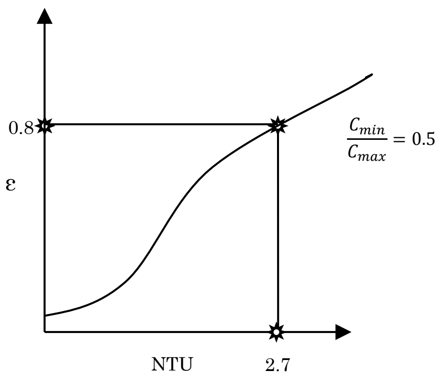
Heat lost by oil (hot fluid) = Heat gained by the water (cold fluid)

$$m_c\ C_{pc}\left(T_{c,out} - T_{c,in}\right) = m_h\ C_{ph}\left(T_{h,in} - T_{h,out}\right)$$

$$1 \times 4197 \times (125 - 35) = m_h \times 1000 \times (300 - 100)$$

$$m_h = 1.9\ kg/s$$

To find the capacity rate of hot and cold fluid,



$C_c = m_c C_{pc}$ = 1 x 4197 = 4197 W/K and

$C_h = m_h C_{ph}$ = 1.9 x 1000 = 1900 W/K

Out of the above two capacity rates, the minimum and maximum are taken as

C_{min} = 1900 W/K and C_{max} = 4197 W/K

The capacity ratio, c

$$c = \frac{C_{min}}{C_{max}}$$

$$c = \frac{1900}{4197} = 0.5$$

From HMT DB pg.no 152, the effectiveness

$$\varepsilon = \frac{T_1 - T_2}{T_1 - t_1} = \frac{300 - 100}{300 - 35}$$

$$\varepsilon = 0.8$$

To find NTU, from HMT DB pg.no 163 for parallele flow heat exchanger

From the graph

Y-axis = ε = 0.8 and

Curve, c = 0.5

Corresponding to these, the NTU is taken as 2.7

NTU = 2.7

We also know that,

$$NTU = \frac{U\,A_s}{C_{min}}$$

$$A_s = \frac{NTU \times C_{min}}{U}$$

$$A_s = \frac{2.7 \times 1900}{100}$$

$$\boldsymbol{A_s = 51.3\ m^2}$$

11. In a counter flow double pipe heat exchanger, water is heated from 25˚C to 65˚C by oil with heat of 1.45 kJ/kg K and mass flow rate is 0.9 kg/s. The oil is cooled from 230˚C to 160˚C. If the overall heat transfer coefficient is 420 W/m^2°C, calculate (i) The rate of heat transfer, (ii) The mass flow

rate, (iii) The surface area of the heat exchanger. **KU – Apr 2014**

Given Data:

m_h = 0.9 kg/s, C_{ph} = 1450 J/kgK, $T_{c,in}$ = 25°C, $T_{c,out}$ = 65°C, $T_{h,in}$ = 230°C, $T_{h,out}$ = 160°C, U = 420 W/m^{2}K, C_{pc} = 4186 J/kgK

To find:

(i) Q=? , (ii) m_c =? , (iii) A_s =?

Solution:

We know that the rate of heat transfer is given by

$$Q = m_h\, C_{ph}\, \left(T_{h,in} - T_{h,out}\right)$$

$$Q = 0.9 \times 1450 \times (230 - 160)$$

$$\boldsymbol{Q = 91350\, W}$$

We also know that

Heat lost by oil (hot fluid) = Heat gained by the water (cold fluid)

$$m_c\, C_{pc}\, \left(T_{c,out} - T_{c,in}\right) = m_h\, C_{ph}\, \left(T_{h,in} - T_{h,out}\right)$$

Therefore

$$m_c = \frac{m_h\, C_{ph}\, \left(T_{h,in} - T_{h,out}\right)}{C_{pc}\, \left(T_{c,out} - T_{c,in}\right)}$$

$$m_c = \frac{91350}{4186 \times (65 - 25)}$$

$$\boldsymbol{m_c = 0.55\, kg/s}$$

The rate of heat transferred is also given by,

$$Q = U\, A_s\, (\Delta T)_m$$

Where $(\Delta T)_m$ for counter flow heat exchanger is taken from HMT DB pg.no. 152

$$\Delta T_m = \frac{\left(T_{h,in} - T_{c,out}\right) - \left(T_{h,out} - T_{c,in}\right)}{ln\left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$$

$$\Delta T_m = \frac{(230 - 65) - (160 - 25)}{ln\left(\frac{230 - 65}{160 - 25}\right)}$$

$$\Delta T_m = 150℃$$

Therefore,

$$Q = U\, A_s\, (\Delta T)_m$$

$$91350 = 420 \times A_s \times 150$$

$$\boldsymbol{A_s = 1.45\ m^2}$$

12. A single shell pass, four tube pass heat exchanger is used to cool lubricating oil from 70°C to 45°C at a rate of 15 kg/s. Water at 25°C is used at a flow rate of 15 kg/sec. Determine the area required if the overall heat transfer coefficient has a value of 150 W/m^2°C. The oil has a specific heat of 2.3 kJ/kg °C. **KU – Apr 2014**

Given Data:

$T_{h,in}$ = 70°C, $T_{h,out}$ = 45°C, m_h = m_c = 15 kg/s, U = 150 W/m^2°C, C_{ph} = 2.3 kJ/kg °C = 2300 J/kg °C

To Find:

A_s =?

Solution:

We know that the rate of heat transfer is given by

$$Q = m_h\ C_{ph}\ (T_{h,in} - T_{h,out})$$

$$Q = 15 \times 2300 \times (70 - 45)$$

$$\boldsymbol{Q = 8.63 \times 10^5\ W}$$

We also know that

Heat lost by oil (hot fluid) = Heat gained by the water (cold fluid)

$$m_c\ C_{pc}\ (T_{c,out} - T_{c,in}) = m_h\ C_{ph}\ (T_{h,in} - T_{h,out})$$

Therefore

$$4186 \times (T_{c,out} - 25) = 2300 \times (70 - 45)$$

$$\boldsymbol{T_{c,out} = 38.7}\text{℃} \cong \boldsymbol{39}\ \text{℃}$$

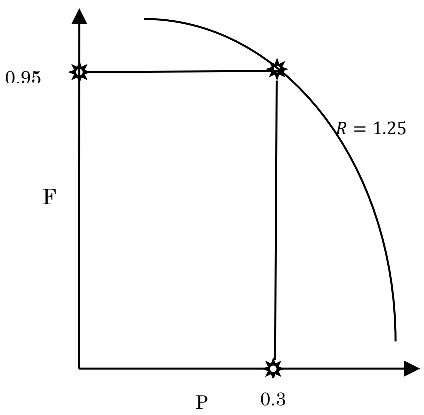
The rate of heat transferred is also given by,

$$Q = F\ U\ A_s\ (\Delta T)_m$$

Where $(\Delta T)_m$ for multipass shell and tube heat exchanger is taken from HMT DB pg.no. 152

$$\Delta T_m = \frac{(T_{h,in} - T_{c,out}) - (T_{h,out} - T_{c,in})}{ln\left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$$

$$\Delta T_m = \frac{(70 - 39) - (45 - 25)}{ln\left(\frac{70 - 39}{45 - 25}\right)}$$



$$\Delta \boldsymbol{T_m} = \mathbf{25}°\text{C}$$

To find correction factor 'F'

from HMT DB pg.no. 159

$$P = \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}} = \frac{39 - 25}{70 - 25} = 0.3$$

$$R = \frac{T_{h,in} - T_{h,out}}{T_{c,out} - T_{c,in}} = \frac{70 - 45}{45 - 25} = 1.25$$

From the graph, On X – axis, P = 0.3 and on curve R = 1.25

The corresponding correction factor,

F = 0.95

Therefore,

$$Q = F\,U\,A_s\,(\Delta T)_m$$

$$8.63 \times 10^5 = 0.95 \times 150 \times A_s \times 25$$

$$\boldsymbol{A_s = 242.25\ m^2}$$

13. Water enters a cross flow heat exchanger (both the fluids unmixed) at 5°C and flows at the rate of 4600 kg/h to cool 4000 kg/h of air initially at 40°C. Assume U = 150 W/m^{2}K, C_{pw} = 4180 J/kgK and $C_{p,air}$ = 1010 J/kgK. For an exchanger surface area of 25 m^2, calculate the exit temperature of the air and water. **KU – Apr 2014**

Given Data:

$T_{c,in}$ = 5°C, m_h = 4000 kg/h = 1.11 kg/s, m_c = 4600 kg/h = 1.28 kg/s, U = 150 W/m^{2}K, C_{pw} = 4180 J/kgK and $C_{p,air}$ = 1010 J/kgK, $T_{h,in}$ = 40°C, A_s = 25 m^2

To Find:

(i) $T_{c,out}$ =? , (ii) $T_{h,out}$ =?

Solution:

To find the capacity rate of hot and cold fluid,

$C_c = m_c C_{pc}$ = 1.28 x 4180 = 5350.4 W/K and

$C_h = m_h C_{ph}$ = 1.11x 1010 = 1121.1 W/K

Out of the above two capacity rates, the minimum and maximum are taken as

C_{min} = 1121.1 W/K and C_{max} = 5350.4 W/K

The capacity ratio, c

$$c = \frac{C_{min}}{C_{max}}$$

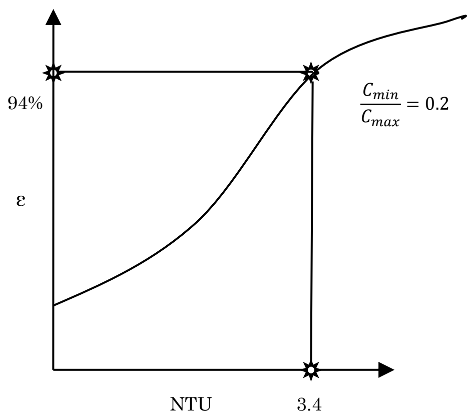
$$c = \frac{1121.1}{5350.4} = 0.2$$

We know that,

$$NTU = \frac{U\ A_s}{C_{min}}$$

$$NTU = \frac{150\ \times 25}{1121.1}$$

$$\boldsymbol{NTU = 3.4}$$



To find effectiveness, from HMT DB pg.no 166 for cross flow both the fluids unmixed heat exchanger

From the graph

Y-axis = NTU = 3.4 and curve, c = 0.2

Corresponding Y – axis for the above values, the effectiveness is taken as 94% = 0.94

$$\varepsilon = 0.94$$

The maximum heat transfer is given as

$$Q_{max} = C_{min}(T_{h,in} - T_{c,in})$$

$$Q_{max} = 1121.1 \times (40 - 5)$$

$$Q_{max} = 39238.5\ W$$

We know that,

$$\varepsilon = \frac{Q}{Q_{max}}$$

$$Q = \varepsilon \times Q_{max} = 0.94 \times 39238.5$$

$$Q = 36884.2\ W$$

We also know that the rate of heat transfer is also given by

$$Q = m_h\ C_{ph}\ (T_{h,in} - T_{h,out})$$

$$36884.2 = 1121.1 \times (40 - T_{h,out})$$

$$\boldsymbol{T_{h,out} = 7}℃$$

$$Q = m_c \, C_{pc} \left(T_{c,out} - T_{c,in}\right)$$

$$36884.2 = 5350.4 \times \left(T_{c,out} - 5\right)$$

$$\boldsymbol{T_{c,out} = 11.9 \cong 20}℃$$

14. A counter flow concentric tube heat exchanger is used to cool engine oil [C = 2130 J/kg K] from 160^0C to 60^0C with water available at 25^0C as the cooling medium. The flow rate of cooling water through the inner tube of 0.5 m is 2 kg/s while the flow rate of oil through the outer annulus, Outer diameter = 0.7 m is also 2 kg/s. If U is 250 W/m^{2}K, how long must the heat exchanger be to meet its cooling requirement? **KU – Aug 2014**

Given Data:

$m_h = m_c$ = 2 kg/s, C_{ph} = 2130 J/kgK, $T_{c,in}$ = 25°C, $T_{h,in}$ = 160°C, $T_{h,out}$ = 60°C, U = 250 W/m^{2}K, C_{pc} = 4186 J/kgK, d_{in} = 0.5 m, d_o = 0.7 m

To find:

Length of the heat exchanger, L =?

Solution:

We know that the rate of heat transfer is given by

$$Q = m_h \, C_{ph} \left(T_{h,in} - T_{h,out}\right)$$

$$Q = 2 \times 2130 \times (160 - 60)$$

$$\boldsymbol{Q = 4.26 \times 10^5 \; W}$$

We also know that

Heat lost by oil (hot fluid) = Heat gained by the water (cold fluid)

$$m_c \, C_{pc} \left(T_{c,out} - T_{c,in}\right) = m_h \, C_{ph} \left(T_{h,in} - T_{h,out}\right)$$

Therefore

$$4186 \times \left(T_{c,out} - 25\right) = 2130 \times (160 - 60)$$

$$\boldsymbol{T_{c,out} = 76}℃$$

The rate of heat transferred is also given by,

$$Q = U \, A_s \, (\Delta T)_m$$

Where $(\Delta T)_m$ for counter flow heat exchanger is taken from HMT DB pg.no. 152

$$\Delta T_m = \frac{\left(T_{h,in} - T_{c,out}\right) - \left(T_{h,out} - T_{c,in}\right)}{ln\left(\frac{T_{h,in} - T_{c,out}}{T_{h,out} - T_{c,in}}\right)}$$

$$\Delta T_m = \frac{(160 - 76) - (60 - 25)}{ln\left(\frac{160 - 76}{60 - 25}\right)}$$

$$\Delta T_m = 56°C$$

Therefore,

$$Q = U\, A_s\, (\Delta T)_m$$

$$4.26 \times 10^5 = 250 \times A_s \times 56$$

$$\boldsymbol{A_s = 30.4\, m^2}$$

To find the length of the exchanger, L

$$A_s = \pi\, d_i\, L$$

$$L = \frac{A_s}{\pi\, d_i} = \frac{30.4}{0.5} = 60.8$$

$$\boldsymbol{L \cong 61\, m}$$

MULTIPLE CHOICE QUESTIONS

Questions	Opt1	Opt2	Opt3	Opt4	Answer
In parallel flow heat exchanger, both hot and cold fluid will	enter to one side and leave at the other side	enter from opposite side parallel to each other	enter at perpendicular direction to each other	all of these	enter to one side and leave at the other side
In counter flow heat exchanger, both hot and cold fluid will	enter to one side and leave at the other side	enter from opposite side parallel to each other	enter at perpendicular direction to each other	all of these	enter from opposite side parallel to each other
Heat exchanger effectiveness is defined as the ratio of actual heat transfer to	the maximum possible heat transfer	the minimum possible heat transfer	the average heat transfer	the area heat transfer	the maximum possible heat transfer
The term NTU is related to	heat exchanger	UA/C min	the effectiveness	all of these	the effectiveness
LMTD of a cross flow heat exchanger is	higher than that of a parallel flow HE	Lighter than that of counter flow HE	zero	both higher than that of a parallel flow HE and Lighter than that of counter flow HE	both higher than that of a parallel flow HE and Lighter than that of counter flow HE
Fouling factor in a heat exchanger	increases the resistance for heat transfer	decreases the resistance for heat transfer	keep the resistance for heat transfer constant	all of these	increases the resistance for heat transfer
condensation occurs on a surface when the surface temperature is	above the saturation temperature of vapour	below the saturation temperature of vapour	both above the saturation temperature of vapour and below the saturation temperature of vapour	none of these	below the saturation temperature of vapour
The condensation process is known as film wise condensation if	the liquid wets the surface	the liquid from a film on the surface	the surface is not wetted by the liquid	both the liquid wets the surface and the liquid from a film on the surface	the liquid from a film on the surface

The condensation process is known as drop wise condensation if	the liquid wets the surface	the liquid from a film on the surface	the surface is not wetted by the liquid	both the liquid wets the surface and the liquid from a film on the surface	the surface is not wetted by the liquid
The heat transfer rate in drop wise versus film wise condensation is	greater	lower	very much lower	none of these	greater
Boiling occurs at a solid - liquid interface, when the temperature of the surface	exceeds the saturation temperature of liquid pressure	lower than the saturation of the liquid pressure	half of thesaturation temperatue of the liquid pressure	all of these	exceeds the saturation temperature of liquid pressure
A correction of LMTD is necessary in case of	parallel flow heat exchanger	counter flow heat exchanger	cross flow heat exchanger	none of these	cross flow heat exchanger
The fouling factor is	dimensionless	measures the heat transfer efficiency	virutally a factor of safety in heat exchanger design	is used only in case of Newtonian fluids	virutally a factor of safety in heat exchanger design
If the bubbles formed on a submerged hot surface get absorbed in the mass of liquid, the process of boiling is termed as	Nucleate boiling	Film boiling	pool boiling	none of these	pool boiling
In case of natural convection, the Nusselt number is function of	Reynolds number	Reynolds number and prandtl number	Grashof's number and prandtl number	Weber number and Mach number	Grashof's number and prandtl number
Heat sensitive liquids are connected	By using insulated vessel	by using deoxidisers	by using vacuum	by using low intensity heating	by using vacuum
The sinsible heat of hot industrial gases can be recovered by a	Regeneator	Recuperatpr	Either Regeneator and Recuperatpr	none of these	Either Regeneator and Recuperatpr
In a sehll and tube type heat exchanger the corrosive liquid is normally passed through	shell side	Tube side	Either shell side and Tube side	none of these	Tube side
The purpose of using multiple pass heat exchanger is to	Reduce fluid flow friction losses	Reduce pressure drop	Reduce pressure drop	increase the rate of heat transfer	increase the rate of heat transfer
The first stage of crystal formation is	Nucleation	separation	Foaming	Vortexing	Nucleation
Entrainment separator is used in evaporations to	increase the boiling effect	Reduce the boiling effect	Separate liquid droplets from vapour	prevent foaming	Separate liquid droplets from vapour
Maximum heat transfer rate can be expected in case of	Turbulent flow	Laminar flow	counter flow	Co-current flow	Turbulent flow

Steam traps are used to	Regulate the flow	Remove the condensate	Add moisture	heat steam	Remove the condensate
Dropwise condensation occurs on a	Glazed surface	smooth surface	Oily surface	coated surface	Oily surface
Agitated film evaporator is suitable for concentrating	Low temperature liquids	liquid metal	viscous liquids	insulating liquids	viscous liquids
Ditus Boelter equation is applicable in case of liquids flowing in	laminar region	Turbulent region	Transtition region	any of these	Turbulent region
In sugar mills cane juice is evaporated in	Horizontal evaporators	Long vertical evaporators	short vertical tube evaporators	zig-zag tube evaporators	Long vertical evaporators
Which evaporators is preferred for concentrating the fruit juice	Long vertical evaporators	Horizontal evaporators	short vertical tube evaporators	Falling film evaporators	Falling film evaporators
which of the following has high thermal conductivity as compared to the remaining	water	Oxygen	Air	Hydrogen	Water
In which of the following heat exchange process has higher overall heat transfer coefficient	steam to oil	Air to carbondioxide	steam condensers	Air to heavy tars	steam condensers
Baffles are provided in heat exchangers to	remove dirt	increase heat transfer rate	provide better mechanical strength	reduce heat transfer rate	increase heat transfer rate

CHAPTER - IV

RADIATION

4.1. INTRODUCTION

The process by which heat is transferred from a body by virtue of its temperature, without the aid of any intervening medium, is called thermal radiation.

Heat transfer by radiation is defined as the transfer of energy between surfaces by means of electromagnetic waves which is caused solely by a temperature difference.

Note: No medium is required for radiation. Even in vacuum, radiation heat transfer takes place.

Propagation of internal energy of an emitting body through electromagnetic waves is known as thermal radiation. Electromagnetic waves are produced due to the electromagnetic disturbances originating in the emitting radiating body. The emitted electromagnetic waves propagate in vacuum at the speed of light. These electromagnetic waves are again converted into thermal energy and absorbed by other solids.

The speed of these electromagnetic waves is equal to the speed of light (3 x 108 m/s).

4.2. QUANTUM THEORY: (POSTULATED BY PLANCK)

According to this theory, when the temperature of body is raised, the atoms become excited states. As a result, it emits energy in the form of electromagnetic radiation.

4.2.1. SURFACE EMISSION PROPERTIES

The rate of emission of radiation by a body depends on

1. The temperature of the surface
2. The nature of the surface
3. The wavelength or frequency of radiation

4.2.2. EMISSIVE POWER (E_b)

The total amount of radiation emitted by a body per unit area and time is termed as total emissive power. The unit of emissive power is W/m^2.

$$\boldsymbol{E = \int_{\lambda=0}^{\lambda=\infty} E_\lambda \, d\lambda \quad W/m^2}$$

4.2.3. MONOCHROMATIC EMISSIVE POWER [$E_{b\lambda}$]

The energy emitted by the surface at a given length per unit time per unit area in all directions is known as monochromatic emissive power.

4.2.4. STEFAN-BOLTZMANN LAW

The Stefan- Boltzmann Law states that the emissive power of black body (Eb) is proportional to the fourth power of absolute temperature.

$$E_b \alpha T^4$$

So, $E_b = \sigma T^4$

Where, $\sigma = 5.67 \times 10^{-8}$ W/m^2K^4

4.3. EMISSIVITY (ε)

Emissivity (ε) is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of the emissive power of any body to the emissive power of a black body at same temperature.

$$\text{Emissivity } (\varepsilon) = (E / E_b)$$

4.3.1. ABSORPTION, REFLECTION & TRANSMISSION

When the radiant energy falling on a body, three things happen. A part is reflected back, a part is transmitted through the surface, and the remainder is absorbed.

The radiation energy incident on a surface per unit area per unit time is called irradiation, G.

Absorptivity α: is the fraction of irradiation absorbed by the surface.

Reflectivity ρ : is the fraction of irradiation reflected by the surface.

Transmissivity τ: is the fraction of irradiation transmitted through the surface.

Radiosity J: total radiation energy streaming from a surface, per unit area per unit time. It is the summation of the reflected and the emitted radiation.

If the incident energy Q is falling on a body as shown in the above Fig.1, Q, is absorbed, Q, is reflected and Q, is transmitted, then energy balance yields,

$$Q = Q_a + Q_r + Q_r$$

Dividing the above equation by Q

$$(Q/Q) = (Qa/Q) + (Qr/Q) + (Qr/Q)$$

$$1 = \alpha + \rho + \tau$$

Absorptivity (α) = (Radiation absorbed / Incident radiation)

Reflectivity (ρ) = (Radiation reflected / Incident radiation)

Transmissivity (τ) = (Radiation transmitted / Incident radiation)

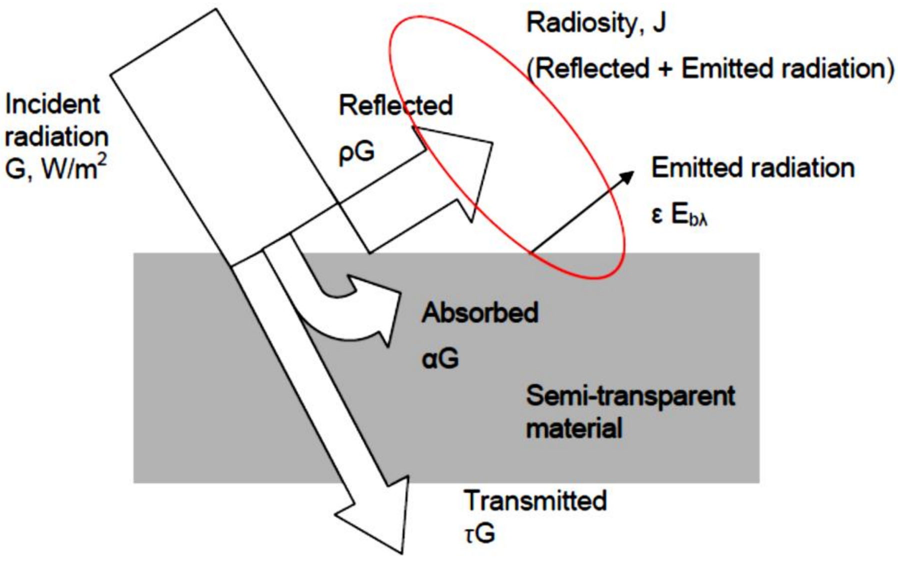


Fig.1

4.3.2. CONCEPT OF A BLACKBODY

A blackbody is a body that absorbs all the radiant energy falling on it. Here $\alpha = 0$, $\rho = 0$ & $\tau = 0$ blackbody does not exist in nature. It is used to compare the radiation characteristics of real bodies. A black body has the following properties:

1. It absorbs all the incident radiation falling on it and does not transmit or reflect regardless of wavelength and direction.
2. It emits the maximum amount of thermal radiation at all wavelengths at a given temperature.
3. The radiation emitted by a blackbody is independent of direction (i.e. it is a diffuse emitter).

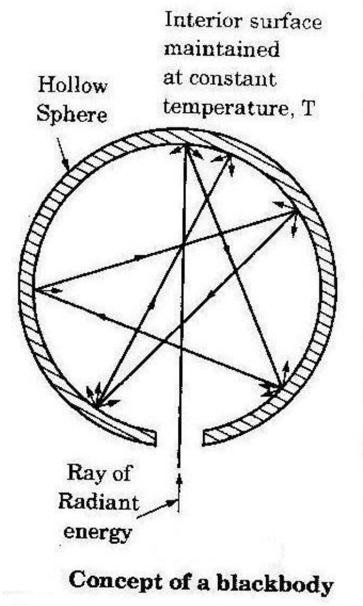


Fig. 2

Fig.2 shows a hollow sphere whose interior surface is maintained at constant temperature. A small hole is provided in the sphere. Incident radiant energy passes through the small hole, some of this energy is absorbed by the inside surface and some is reflected diffusely at the interior surface. The reflected radiation does not escape immediately from the sphere.

It strikes on and is reflected many times from the inside surface before finally escaping from the opening. At the end of the process, almost all of the incident radiation is absorbed and the energy leaving the sphere is negligible. Thus, a small hole in the hollow sphere acts like a blackbody and the entire radiation incident upon it is absorbed.

4.4. THE LAWS OF RADIATION

4.4.1. STEFAN-BOLTZMANN LAW

The Stefan- Boltzmann Law states that the emissive power of black body (E_b) is proportional to the fourth power of absolute temperature.

$$\boldsymbol{E_b} \propto T^4$$

$$\boldsymbol{E_b} = \sigma T^4$$

Where, $\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4 (σ) is the Stefan- Boltzmann constant

4.4.2. KIRCHHOFF'S LAW

The law states that at any temperature, the ratio of total emissive power [Ebl to the absorptivity [α] is constant for all substances which are in thermal equilibrium with their environment.

Let us consider a large black body of surface area A and emissive power [E_bl. A small body of area A_1, absorptivity [α_1] and emissive power per unit area [E_1] is enclosed inside the large body, as shown in Fig. 3. When the energy fall on the surface of the body at the rate [E_bl, a fraction [α] will be absorbed by small body (I), as shown in Fig. 3. When thermal equilibrium is attained, the energy absorbed by body (I) must be equal to the energy emitted [E_1].

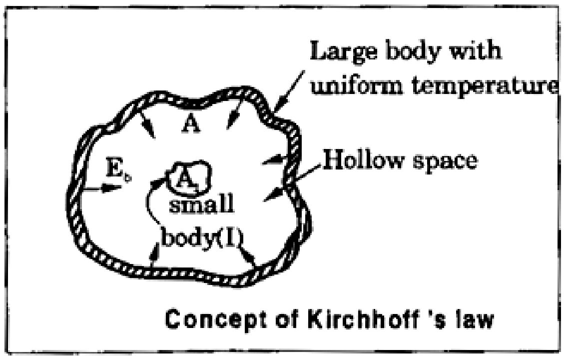


Fig. 3 Concept of Kirchhoff's law

After the thermal equilibrium,

Energy absorbed by the body A_1 = Energy emitted, E_1

$$\alpha_1 A_1 E_b = A_1 E_1$$

Where, $(E_b = E_1 / \alpha_1)$

Similarly, if we, replace body (1) by body (2) of α_2, then

$$\alpha_2 A_2 E_b = A_2 E_2$$

Where, $(E_b = E_2 / \alpha_2)$

Therefore, we can write as,

$$(E_1 / \alpha_1) = (E_2 / \alpha_2) = E_b = \text{Constant} = (E / \alpha)$$

The above equation is called Kirchoff's law

From emissivity equation,

$$E_b = (E/\varepsilon) \text{ \& also}$$

$$E_b = (E/\alpha)$$

Hence, $\varepsilon = \alpha$

Therefore Kirchhoff's law also states that the emissivity (ε) of a body is equal to its absorptivity (α) when the body remains in thermal equilibrium with its surroundings.

4.4.3. WIEN'S DISPLACEMENT LAW

This law establishes relationship between the temperature of a blackbody and the wavelength at which the maximum value of monochromatic emissive power occurs. Peak monochromatic emissive power occurs at a particular wavelength. Wien's displacement law states that the product of λ_{max} and T is constant, i.e.

$$\lambda_{max} T = \text{constant} = 2.8976 \times 10^{-3} = 2.9 \times 10^{-3} \text{ mK}$$

λ_{max} α (1/T) ("Wavelength corresponding to maximum spectral intensity is inversely proportional to absolute temperature".)

$$\lambda_{max} T = 2.8976 \times 10^{-3} = 2.9 \times 10^{-3} \text{ mK}$$

4.4.4. PLANCK'S DISTRIBUTION LAW

The relationship between monochromatic emissive power [$E_{b\lambda}$] of a black body and the wave length (λ) of a radiation at a particular temperature is given by the following expression, by Planck.

$$E_{b\lambda} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

Where, $E_{b\lambda}$ = Monochromatic emissive power, W/m^2

λ = Wave length, m

C_1 = 0.374 x 10^{-15} Wm^2

C_2 = 14.4 x 10^{-3} mK

4.5. MAXIMUM EMISSIVE POWER $[E_{b\lambda}]_{MAX}$

A combination of Planck's law and Wien's displacement law yields the condition for the maximum monochromatic emissive power for a black body.

$$[E_{b\lambda}]_{max} = C^4 T^5$$

Where, c^4 = 1.307 x 10^{-5}

4.5.1. INTENSITY OF RADIATION (I_n)

It is defined as the rate of energy leaving a surface in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

$$\boldsymbol{I_n = \frac{E_b}{\pi}}$$

4.5.2. GRAY BODY

If a body absorbs a definite percentage of incident radiation irrespective of their wave length, the body is known as gray body. The emissive power of a gray body is always less than that of the black body.

FORMULAE USED

Emissive Power (or) Total Emissive Power

$$\boldsymbol{E_b} = \sigma T^4$$

Where,

Stefan- Boltzmann constant, σ = 5.67 x 10^{-8} W/m^{2}K^4

Wien's Law

$\lambda_{max}T$ = 2898 μmK = 2.9 x 10^{-3} mK (1 μ = 10^{-6} m)

4.6. THE VIEW FACTOR (OR) SHAPE FACTOR

Radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other as well as their radiation properties and temperatures. View factor (or shape factor) is a purely geometrical parameter that accounts for the effects of orientation on radiation between surfaces.

In view factor calculations, we assume uniform radiation in all directions throughout the surface, i.e., surfaces are isothermal and diffuse. Also the medium between two surfaces does not absorb, emit, or scatter radiation.

Fi→j or Fij = the fraction of the radiation leaving surface (i) that strikes surface (j) directly.

Note the following:

1. The view factor ranges between zero and one.
2. Fij = 0 indicates that two surfaces do not see each other directly. Fij = 1 indicates that the surface (j) completely surrounds surface (i).
3. The radiation that strikes a surface does not need to be absorbed by that surface.
4. Fii is the fraction of radiation leaving surface (i) that strikes itself directly. Fii = 0 for plane or convex surfaces, and Fii ≠ 0 for concave surfaces.

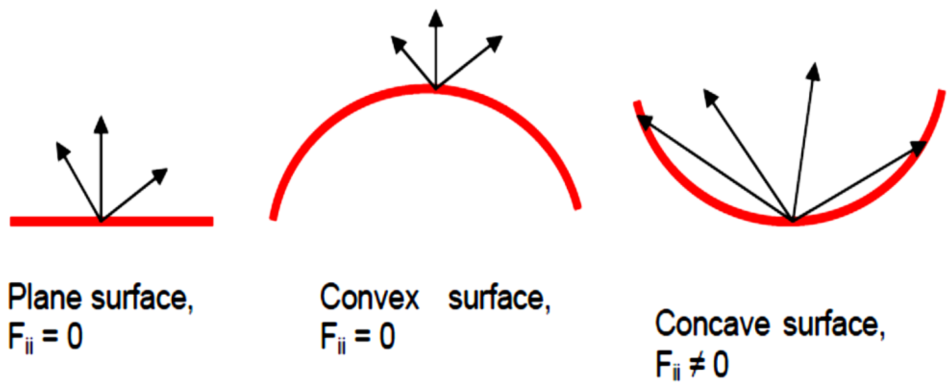


Fig. 4 View Factor between Surface and Itself

<u>4.6.1. VIEW FACTOR RELATIONS</u>

Radiation analysis of an enclosure consisting of N surfaces requires the calculations of N_2 view factors. However, all of these calculations are not necessary. Once a sufficient number of view factors are available, the rest of them can be found using the following relations for view factors.

<u>4.6.2. THE RECIPROCITY RULE</u>

The view factor F_{ij} is not equal to F_{ji} unless the areas of the two surfaces are equal. It can be shown that:

$$A_i F_{ij} = A_j F_{ji}$$

<u>4.6.3. THE SUMMATION RULE</u>

In radiation analysis, we usually form an enclosure. The conservation of energy principle requires that the entire radiation leaving any surface (i) of an enclosure be intercepted by the surfaces of enclosure. Therefore,

$$\sum_{j=1}^{N} F_{ij} = 1$$

The summation rule can be applied to each surface of an enclosure by varying i from 1 to N (number of surfaces). Thus the summation rule gives N equations. Also reciprocity rule gives 0.5 N (N-1) additional equations. Therefore, the total number of view factors that need to be evaluated directly for an N-surface enclosure becomes

$$N^2 - \left[N + \frac{1}{2}N(N-1)\right] = \frac{1}{2}N(N-1)$$

4.7. RADIATION EXCHANGE BETWEEN SURFACES

Radiant energy exchange between surfaces depends not only on the emission, absorption and reflection characteristics of the surfaces but also on their geometrical arrangement. This heat exchange will be affected further due to the presence of partially emitting and absorbing medium in between the surfaces. To account this radiation exchange, following assumptions are made.

1. All surfaces are considered to be either black or gray.
2. Radiation and reflection process are assumed to be diffuse.
3. The absorptivity of a surface is taken equal to its emissivity and independent of temperature of the source of the incident radiation.

4.7.1. RADIATION SHIELD

Radiation shields constructed from low emissivity (high reflective) materials. It is used to reduce the net radiation transfer between two surfaces.

Let us consider two parallel planes 1 and 2 each of area (A) at temperatures T_1 and T_2 respectively. A radiation shield is placed in between them as shown in Fig.5.

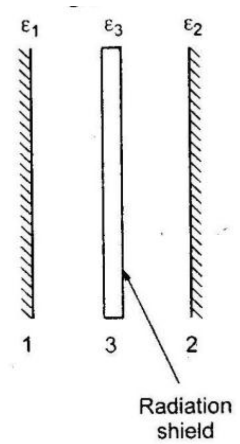


Fig. 5 Radiation Shield

FORMULAE USED

Heat exchange between two large parallel plates is given by,

$$Q_{12} = \bar{\varepsilon}\,\sigma\,A\left(T_1^4 - T_2^4\right)$$

Where

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

σ = Stefan Boltzmann constant, 5.67 x 10^{-8} W/m^{2}K^4

ε_1 & ε_2 = Emissivity of surface 1 and 2

T_1 & T_2 = Temperature of surface 1 and 2, K

Heat exchange between two large concentric cylinder (or) sphere is given by,

$$Q_{12} = \bar{\varepsilon}\,\sigma\,A_1\left(T_1^4 - T_2^4\right)$$

Where

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

For cylinder, Area, A = 2π r, m^2

For Sphere, Area, A = 4 π r^2, m^2

Heat transfer with n shield is given by,

$$Q_{12} = \frac{\sigma\,A\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2n}{\varepsilon_s} - (n+1)}$$

Where, n = Number of shields

ε_s = Emissivity of shield

4.8. RADIATION FROM GASES AND VAPOURS EMISSION AND ABSORPTION

Many gases such as N_2, 0_2, H_2, dry air etc., do not emit or absorb any appreciable amount of thermal radiation. These gases may be considered as transparent to thermal radiation. On the other hand, some gases and vapours such as CO_2, CO, H_20, SO_2, NH_3, etc., emit and absorb significant amount of radiant energy. As illustration we shall take up radiation from CO_2 and H_20, which are the most common absorbing gases present in atmosphere, industrial furnace, etc.

4.8.1. RADIATION FROM GASES DIFFERS FROM SOLIDS

The radiation from gases differs from solids in the following ways:

1. The radiation from solids is at all wavelengths, whereas gases radiate over specific wavelength ranges or bands within the thermal spectrum.
2. The intensity of radiation as it passes through an absorbing gas decreases with the length of passage through the gas volume. This is unlike solids wherein the absorption of radiation takes place within a small distance from the surface.

Questions & Answers:

1. Define Radiation.

The heat transfer from one body to another without any transmitting medium is known as radiation. It is an electromagnetic wave phenomenon.

2. Define emissive power [E]

The emissive power is defined as the total amount of radiation emitted by a body per unit time and unit area. It is expressed in W/m^2.

3. Define monochromatic emissive power. [$E_{b\lambda}$]

The energy emitted by the surface at a given length per unit time per unit area in all directions is known as monochromatic emissive power.

4. What is meant by absorptivity?

Absorptivity is defined as the ratio between radiation absorbed and incident radiation.

$$\text{Absorptivity} \quad \alpha = \frac{\text{Radiation absorbed}}{\text{Incident radiation}}$$

5. What is meant by reflectivity?

Reflectivity is defined as the ratio of radiation reflected to the incident radiation.

$$\text{Reflectivity} \quad \rho = \frac{\text{Radiation reflected}}{\text{Incident radiation}}$$

$$\text{Absorptivity} \quad \alpha = \frac{\text{Radiation absorbed}}{\text{Incident radiation}}$$

6. What is meant by transmissivity?

Transmissivity is defined as the ratio of radiation transmitted to the incident radiation.

$$\text{Transmissivity} \quad \tau = \frac{\text{Radiation transmitted}}{\text{Incident radiation}}$$

7. What is black body?

Black body is an ideal surface having the following properties.

1. A black body absorbs all incident radiation, regardless of wave length and direction.
2. For a prescribed temperature and wave length, no surface can emit more energy than black body.

8. State Planck's distribution law.

The relationship between the monochromatic emissive power of a black body and wave length of a radiation at a particular temperature is given by the following expression, by Planck.

$$E_{b\lambda} = \frac{C_1\lambda^{-5}}{e^{\left(\frac{C_2}{\lambda T}\right)} - 1}$$

Where $E_{b\lambda}$ = Monochromatic emissive power W/m^2

λ = Wave length – m

$c_1 = 0.374 \times 10^{-15}$ W m^2

$c_2 = 14.4 \times 10^{-3}$ mK

9. State Wien's displacement law.

The Wien's law gives the relationship between temperature and wave length corresponding to the maximum spectral emissive power of the black body at that temperature.

$$\lambda_{mas} T = c_3$$

Where $c_3 = 2.9 \times 10^{-3}$ [Radiation constant]

$$\Rightarrow \quad \lambda_{mas} T = 2.9 \times 10^{-3} \text{ mK}$$

10. State Stefan – Boltzmann law.

The emissive power of a black body is proportional to the fourth power of absolute temperature.

$$E_b \propto T^4$$

$$E_b = \sigma T^4$$

Where E_b = Emissive power, w/m^2

σ = Stefan. Boltzmann constant

= 5.67×10^{-8} W/m^2 K^4

T = Temperature, K

11. Define Emissivity.

It is defined as the ability of the surface of a body to radiate heat. It is also defined as the ratio of emissive power of any body to the emissive power of a black body of equal temperature.

$$\text{Emissivity } \varepsilon = \frac{E}{E_b}$$

12. What is meant by gray body?

If a body absorbs a definite percentage of incident radiation irrespective of their wave length, the body is known as gray body. The emissive power of a gray body is always less than that of the black body.

13. State Kirchoff's law of radiation.

This law states that the ratio of total emissive power to the absorbtivity is constant for all surfaces which are in thermal equilibrium with the surroundings. This can be written as

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3}$$

It also states that the emissivity of the body is always equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.

$\alpha_1 = E_1$; $\alpha_2 = E_2$ and so on.

14. Define intensity of radiation (I_b).

It is defined as the rate of energy leaving a space in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space.

$$I_n = \frac{E_b}{\pi}$$

15. State Lambert's cosine law.

It states that the total emissive power E_b from a radiating plane surface in any direction proportional to the cosine of the angle of emission

$$E_b \;\infty\; \cos\theta$$

16. What is the purpose of radiation shield?

Radiation shields constructed from low emissivity (high reflective) materials. It is used to reduce the net radiation transfer between two surfaces.

17. Define irradiation (G)

It is defined as the total radiation incident upon a surface per unit time per unit area. It is expressed in W/m^2.

18. What is radiosity (J)

It is used to indicate the total radiation leaving a surface per unit time per unit area. It is expressed in W/m^2.

19. What are the assumptions made to calculate radiation exchange between the surfaces?

1. All surfaces are considered to be either black or gray
2. Radiation and reflection process are assumed to be diffuse.
3. The absorptivity of a surface is taken equal to its emissivity and independent of temperature of the source of the incident radiation.

20. What is meant by shape factor?

The shape factor is defined as the fraction of the radiative energy that is diffused from on surface element and strikes the other surface directly with no intervening reflections. It is represented by F_{ij}. Other names for radiation shape factor are view factor, angle factor and configuration factor.

Problems:

1. Two parallel plates 3m x 2m are spaced at 1 m apart. One plate is maintained at 500°C and other at 200°C. The emissivities of the plates are 0.3 and 0.5. The plates are located in a large room and room walls are maintained at 40°C. If the plates exchange heat with each other and with the room. Find the heat lost by the hotter plate. **KU – Nov 2010.**

   Given Data:

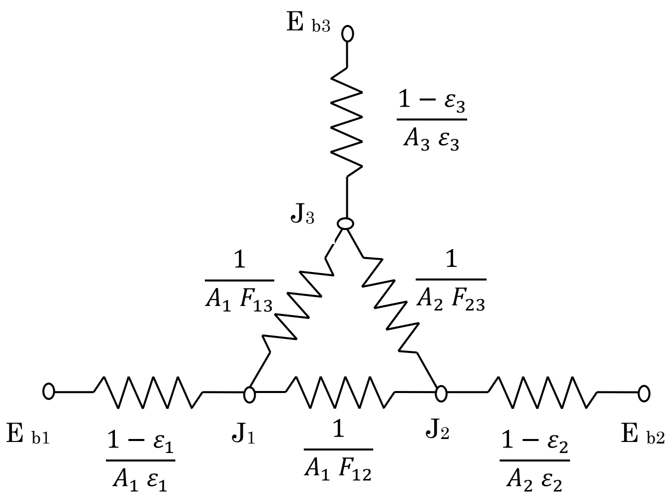
   Size of plate, 3m x 2m, distance between plates 1m, T_1 = 500°C = 773 K, T_2 = 200°C = 473 K, ε_1 = 0.3, ε_2 = 0.5, T_3 = 313 K.

   To Find:

   Heat lost by the hotter plate.

   Solution:

   This problem belongs to heat exchange between three surfaces, and hence it can be solved using electrical network analogy.



Area of the plates, $A_1 = A_2 = 3 \times 2 = 6\ m^2$ and

Room area $A_3 = \infty$

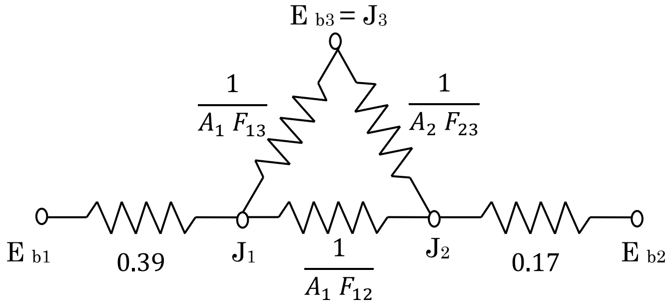
From the electrical network diagram,

$$\frac{1-\varepsilon_1}{A_1\,\varepsilon_1} = \frac{1-0.3}{6\ \times 0.3} = 0.39$$

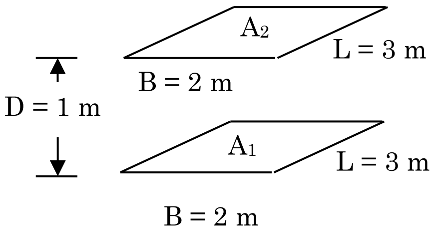
$$\frac{1-\varepsilon_2}{A_2\ \varepsilon_2} = \frac{1-0.5}{6\ \times 0.5} = 0.17$$

$$\frac{1-\varepsilon_3}{A_3\ \varepsilon_3} = 0, \qquad since\ A_3 = \infty$$

On applying the above values the network diagram becomes,



To find the shape factor F_{12}, from HMT DB pg.no 92



X = L/D = 3, Y = B/D = 2, from the graph it can be found that the corresponding shape factor F_{12} = 0.48

We know that, $F_{11} + F_{12} + F_{13} = 1$, but $F_{11} = 0$

Therefore, $F_{13} = 1 – F_{12} = 1 – 0.48 = 0.52$

$$F_{13} = 0.52$$

Similarly

$F_{21} + F_{22} + F_{23} = 1$, but $F_{11} = 0$

Therefore, $F_{23} = 1 – F_{21} = 1 – 0.48 = 0.52$

$$F_{23} = 0.52$$

From the electrical network diagram,

$$\frac{1}{A_1\ F_{13}} = \frac{1}{6\ \times 0.52} = 0.32$$

$$\frac{1}{A_2\ F_{23}} = \frac{1}{6\ \times 0.52} = 0.32$$

$$\frac{1}{A_1\ F_{12}} = \frac{1}{6\ \times 0.48} = 0.35$$

From Stefan Boltzmann law,

$$E_b = \sigma T^4$$

Therefore,

$$E_{b1} = \sigma {T_1}^4 = 5.67 \times 10^{-8} \times (773)^4$$

$$\boldsymbol{E_{b1} = 20.24 \times 10^3 \, W/m^2}$$

Similarly

$$E_{b2} = \sigma {T_2}^4 = 5.67 \times 10^{-8} \times (473)^4$$

$$\boldsymbol{E_{b2} = 2.8 \times 10^3 \, W/m^2}$$

$$E_{b3} = \sigma {T_3}^4 = 5.67 \times 10^{-8} \times (313)^4$$

$$\boldsymbol{E_{b3} = 0.544 \times 10^3 \, W/m^2}$$

The radiosities J_1 and J_2 can be calculated by using Kirchhoff's law,

The sum of the current entering the node is zero

For Node 1

$$\frac{E_{b1} - J_1}{0.39} + \frac{J_2 - J_1}{\frac{1}{A_1 F_{12}}} + \frac{E_{b3} - J_1}{\frac{1}{A_1 F_{13}}} = 0$$

$$\frac{20.24 \times 10^3 - J_1}{0.39} + \frac{J_2 - J_1}{0.35} + \frac{0.544 \times 10^3 - J_1}{0.32} = 0$$

On solving the above equation,

$$8.63 J_1 - 2.9 J_2 = 52.85 \times 10^3, ---- eq.1$$

For Node 2

$$\frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}} + \frac{E_{b3} - J_2}{\frac{1}{A_2 F_{23}}} + \frac{E_{b2} - J_2}{0.17} = 0$$

$$\frac{J_1 - J_2}{0.35} + \frac{0.544 \times 10^3 - J_2}{0.32} + \frac{2.8 \times 10^3 - J_2}{0.17} = 0$$

On solving the above equation,

$$-2.9 J_1 + 11.93 J_2 = 18.2 \times 10^3, ---- eq.2$$

On solving eq.1 and eq.2, we get

J_1 = 7353.2 W/m^2 and J_2 = 3313.0 W/m^2

Heat lost by the hottest plate, 1 is given as

$$Q = \frac{E_{b1} - J_1}{\left(\frac{1 - \varepsilon_1}{A_1 \, \varepsilon_1}\right)} = \frac{20.24 \times 10^3 - 7353.2}{0.39}$$

$$\boldsymbol{Q = 33.02 \times 10^3 \, W}$$

2. Calculate the net radiant heat exchange per m^2 area for two large parallel plates at temperature of 427^{0}C and 27ºC respectively. $\varepsilon_{[hot\ plate]}$ = 0.9 and $\varepsilon_{[cold\ plate]}$ = 0.6. If a polished aluminium shield is placed between them, find the percentage of reduction in the heat transfer. $\varepsilon_{[shield]}$ = 0.4. **KU – Nov 2011, Nov 2012, Apr 2014**

Given Data:

T_1 = 427°C = 700 K, T_2 = 27°C = 300 K, $\varepsilon_{[hot\ plate]}$ = 0.9 ,

$\varepsilon_{[cold\ plate]}$ = 0.6, . $\varepsilon_{[shield]}$ = 0.4

To Find:

percentage of reduction in the heat transfer

Solution:

Case: 1 – Heat transfer without radiation shield

$$Q_{12} = \bar{\varepsilon}\,\sigma\,A\left(T_1^{\,4} - T_2^{\,4}\right)$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.9} + \frac{1}{0.2} - 1} = 0.5625$$

$$\frac{Q_{12}}{A} = 0.5625 \times 5.67 \times 10^{-8} \times (700^4 - 300^4)$$

$$\frac{Q_{12}}{A} = 7.39 \times 10^3\ W/m^2$$

Case: 2 – Heat transfer with radiation shield

Since we don't know the radiation shield temperature, let us find the temperature T_3

Heat exchange between plate 1 and radiation shield is given as

$$Q_{13} = \bar{\varepsilon}\,\sigma\,A\left(T_1^{\,4} - T_3^{\,4}\right)$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{1}{\frac{1}{0.9} + \frac{1}{0.4} - 1} = 0.3829$$

$$\frac{Q_{13}}{A} = 0.3829 \times 5.67 \times 10^{-8} \times \left(700^4 - T_3^{\,4}\right)$$

$$\frac{Q_{13}}{A} = 2.171 \times 10^{-8} \times \left(700^4 - T_3^{\,4}\right)$$

Similarly, the heat exchange between plate 2 and radiation shield is given as

$$Q_{32} = \bar{\varepsilon}\,\sigma\,A\left(T_3^{\,4} - T_2^{\,4}\right)$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} - 1} = \frac{1}{\frac{1}{0.2} + \frac{1}{0.4} - 1} = 0.1538$$

$$\frac{Q_{32}}{A} = 0.1538 \times 5.67 \times 10^{-8} \times \left(T_3{}^4 - 300^4\right)$$

$$\frac{Q_{32}}{A} = 8.723 \times 10^{-9} \times \left(700^4 - T_3{}^4\right)$$

Also we know that,

$$Q_{13} = Q_{32}$$

$$2.171 \times 10^{-8} \times \left(700^4 - T_3{}^4\right) = 8.723 \times 10^{-9} \times \left(700^4 - T_3{}^4\right)$$

On solving,

$$T_3 = 606.55\ K$$

Therefore,

$$\frac{Q_{13}}{A} = 2.171 \times 10^{-8} \times (700^4 - 606.55^4)$$

$$\frac{Q_{13}}{A} = 2.274 \times 10^3\ W/\,m^2$$

Percentage of reduction in heat transfer is given by

$$\%\ reduction\ in\ heat\ transfer = \frac{Q_{without\ sheild} - Q_{with\ shield}}{Q_{without\ sheild}}$$

$$= \frac{\frac{Q_{12}}{A} - \frac{Q_{13}}{A}}{\frac{Q_{12}}{A}} = 1 - \frac{2.274 \times 10^3}{7.39 \times 10^3}$$

$$\%\ reduction\ in\ heat\ transfer = 69.26\ \%$$

3. Two very large parallel plates with emissivities 0.5 exchange heat. Determine the percentage reduction in the heat transfer rate if a polished aluminium radiation shield of ε = 0.04 is placed in between the plates. **KU – Nov 2011**

Given Data:

$\varepsilon_1 = \varepsilon_2 = 0.5$ and $\varepsilon_3 = 0.04$

To Find:

Percentage of reduction in heat transfer

Solution:

Case: 1 – Heat transfer without radiation shield

$$Q_{without\ sheild} = \bar{\varepsilon}\,\sigma\,A\left(T_1^{\,4} - T_2^{\,4}\right)$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.5} + \frac{1}{0.5} - 1} = 0.333$$

$$Q_{without\ sheild} = 0.333\,\sigma\,A\left(T_1^{\,4} - T_2^{\,4}\right)$$

Case: 2 – Heat transfer with radiation shield

$$Q_{with\ sheild} = \frac{\sigma\,A\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2n}{\varepsilon_s} - (n+1)}$$

$$Q_{with\ sheild} = \frac{\sigma\,A\left(T_1^{\,4} - T_2^{\,4}\right)}{\frac{1}{0.5} + \frac{1}{0.5} + \frac{2\times 1}{0.04} - (1+1)}$$

$$Q_{with\ sheild} = 0.0192\,\sigma\,A\left(T_1^{\,4} - T_2^{\,4}\right)$$

Percentage of reduction in heat transfer is given by

$$\%\ reduction\ in\ heat\ transfer = \frac{Q_{without\ sheild} - Q_{with\ shield}}{Q_{without\ sheild}}$$

$$\%\ reduction\ in\ heat\ transfer = 1 - \frac{0.0192\,\sigma\,A\left(T_1^{\,4} - T_2^{\,4}\right)}{0.333\,\sigma\,A\left(T_1^{\,4} - T_2^{\,4}\right)}$$

$$\%\ reduction\ in\ heat\ transfer = 94.2\ \%$$

4. The inner sphere of liquid oxygen container is 40 cm in diameter and outer sphere is 50 cm in diameter. Both have emissivities of 0.05. Determine the rate at which the liquid oxygen would evaporate at -183°C when the outer sphere is at 20°C. Latent heat of oxygen is 210 kJ/kg. **KU – Nov 2011**

   Given Data:

   D_1 = 40 cm = 0.4 m, r_1 = 0.2 m, D_2 = 0.5 m, r_2 = 0.25 m

   T_1 = 183°C = -183+273 = 90 K, T_2 = 20°C = 20 +273 = 293 K,

   ε_1 = 0.05, ε_2 = 0.05, h_{fg} = 210000 J/kg

   To Find:

   Rate of Evaporation

   Solution:

   Heat exchange between two large concentric cylinder (or) sphere is given by,

$$Q_{12} = \bar{\varepsilon}\, \sigma\, A_1 \left(T_1{}^4 - T_2{}^4\right)$$

Where

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

Area of the spheres,

$$A_1 = 4\,\pi\, r_1{}^2, \qquad A_2 = 4\,\pi\, r_2{}^2$$

$$\bar{\varepsilon} = \frac{1}{\frac{1}{0.05} + \frac{4\,\pi\, r_1{}^2}{4\,\pi\, r_2{}^2}\left(\frac{1}{0.05} - 1\right)} = 0.031$$

$$\boldsymbol{Q_{12} = 0.031 \times 5.67 \times 10^{-8} \times 4 \times \pi \times 0.2^2\ (90^4 - 293^4)}$$

$$Q_{12} = -6.45\, W$$

We know that the rate of evaporation is given as

$$Q = m \times h_{fg}$$

$$m = \frac{Q_{12}}{h_{fg}} = \frac{6.45}{210 \times 10^3}$$

$$\boldsymbol{m = 3.07 \times 10^{-5} kg/s}$$

5. The intensity of radiation emitted by the sun is maximum at a wavelength of 0.5 µm. As a black body, determine its surface temperature and the emissive power. **KU – Apr 2014**

   Given Data:

   λ_{max} = 0.5 x 10^{-6} m

   To Find:

   T =? And E_b =?

   Solution:

   According to Wien's Displacement law

$$\lambda_{max}T = 2.8976 \text{ x } 10^{-3} = 2.9 \text{ x } 10^{-3}\ mK$$

$$\boldsymbol{T = 5800\ K}$$

   According to Stefan Boltzmann Law

$$\boldsymbol{E_b} = \sigma\, T^4$$

$$\boldsymbol{E_b} = 5.67 \times 10^{-8} \times 5800^4$$

$$E_b = 64.1 \times 10^6 \ W/m^2$$

6. Emissivities of two large parallel plates at 800°C and 300°C are 0.5 and 0.3 respectively. Find the net energy transfer rate per square meter. **KU – Apr 2014**

   Given Data:

   T_1 = 800°C = 1073K, T_2 = 300°C = 573 K, $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.3$

   To Find:

   Q/A =?

   Solution:

$$Q_{12} = \bar{\varepsilon}\, \sigma\, A \left(T_1{}^4 - T_2{}^4\right)$$

   Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.5} + \frac{1}{0.3} - 1} = 0.231$$

$$\frac{Q_{12}}{A} = 0.231 \times 5.67 \times 10^{-8} \times (1073^4 - 573^4)$$

$$\frac{Q_{12}}{A} = 15.95 \times 10^3 \ W/m^2$$

7. A 70 mm thick metal plate with circular hole of 35 mm diameter along the thickness is maintained at a uniform temperature 250°C. Find the loss of energy to the surroundings at 27°C, assuming the two ends of the hole to be as parallel discs and the metallic surfaces and surrounding have black body characteristics. **KU – Apr 2014.**
8. An electric heating system is installed in the ceiling of a room 5m (length) x 5m (width) x 2.5m (height). The temperature of the ceiling is 315 K where as under equilibrium conditions the walls are at 295 K. if the floor is non sensitive to radiations and the emissivities of the ceiling and wall are 0.75 and 0.665 respectively calculate the radiation heat loss from the ceiling to the walls. **KU – Aug 2014.**

   Given Data:

   L = 5 m, B = 5 m and D = 2.5 m, T_1 = 315 K and T_2 = 295 K,

   $\varepsilon_1 = 0.75$, $\varepsilon_2 = 0.665$

   To Find:

   Radiation heat loss from the ceiling to the walls

   Solution:

   Heat transfer by radiation is given by

$$Q_{12} = \frac{\sigma\left(T_1{}^4 - T_2{}^4\right)}{\left(\frac{1-\varepsilon_1}{A_1\,\varepsilon_1}\right) + \frac{1}{A_1\,F_{12}} + \left(\frac{1-\varepsilon_2}{A_2\,\varepsilon_2}\right)}$$

To find the shape factor F_{12} from HMT DB pg. no 95, from the table

$$F_{12} = 0.2$$

Therefore,

$$Q_{12} = \frac{5.67 \times 10^{-8} \times (315^4 - 295^4)}{\left(\frac{1-0.75}{25 \times 0.75}\right) + \frac{1}{25 \times 0.2} + \left(\frac{1-0.665}{25\times 0.665}\right)}$$

$$Q_{12} = 551.82\ W$$

9. A square room 3m x 3m, has a floor heated to 27°C and has a ceiling at 10°C. The walls are assumed to be perfectly insulated. The height of the room is 2.5 m. The emissivity of all the surfaces is 0.8. Determine the following:
   (i) The net heat exchange between floor and ceiling
   (ii) The wall temperature

   Assuming the ceiling to floor shape factor as 0.25 **KU – Apr 2014**

10. Two large parallel planes at 800 K and 600 K have emissivities of 0.5 and 0.8 respectively. A radiation shield having an emissivity of 0.1 on one side and an emissivity of 0.05 on the other side is placed between the plates. Calculate the intermediate temperature and the heat transfer rate by radiation per square meter with and without radiation shield. **KU – Apr 2015**.

    Given Data:

    T_1 = 800 K, T_2 = 600 K, $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.8$, $\varepsilon_{3a} = 0.1$, $\varepsilon_{3b} = 0.05$

    Solution:

    Case: 1 – Heat transfer without radiation shield

$$Q_{12} = \bar{\varepsilon}\,\sigma\,A\left(T_1{}^4 - T_2{}^4\right)$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{1}{\frac{1}{0.5} + \frac{1}{0.8} - 1} = 0.444$$

$$\frac{Q_{12}}{A} = 0.444 \times 5.67 \times 10^{-8} \times (800^4 - 600^4)$$

$$\frac{Q_{12}}{A} = 7.048 \times 10^3 \, W/\, m^2$$

Case: 2 – Heat transfer with radiation shield

Since we don't know the radiation shield temperature, let us find the temperature T_3

Heat exchange between plate 1 and radiation shield is given as

$$Q_{13a} = \bar{\varepsilon}\, \sigma\, A \left(T_1{}^4 - T_3{}^4\right)$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_{3a}} - 1} = \frac{1}{\frac{1}{0.5} + \frac{1}{0.1} - 1} = 0.091$$

$$\frac{Q_{13a}}{A} = 0.091 \times 5.67 \times 10^{-8} \times \left(800^4 - T_3{}^4\right)$$

$$\frac{Q_{13a}}{A} = 5.16 \times 10^{-9} \times \left(800^4 - T_3{}^4\right)$$

Similarly, the heat exchange between plate 2 and radiation shield is given as

$$Q_{3b2} = \bar{\varepsilon}\, \sigma\, A \left(T_3{}^4 - T_2{}^4\right)$$

Where,

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_{3b}} - 1} = \frac{1}{\frac{1}{0.8} + \frac{1}{0.05} - 1} = 0.0494$$

$$\frac{Q_{3b2}}{A} = 0.0494 \times 5.67 \times 10^{-8} \times \left(T_3{}^4 - 600^4\right)$$

$$\frac{Q_{3b2}}{A} = 2.8 \times 10^{-9} \times \left(T_3{}^4 - 600^4\right)$$

Also we know that,

$$Q_{13a} = Q_{3b2}$$

$$5.16 \times 10^{-9} \times \left(800^4 - T_3{}^4\right) = 2.8 \times 10^{-9} \times \left(T_3{}^4 - 600^4\right)$$

On solving,

$$T_3 = 746.6\, K$$

Therefore,

$$\frac{Q_{13a}}{A} = 5.16 \times 10^{-9} \times (800^4 - 746.6^4)$$

$$\frac{Q_{13a}}{A} = 510.285\, W/\, m^2$$

Percentage of reduction in heat transfer is given by

$$\%\, reduction\ in\ heat\ transfer = \frac{Q_{without\ sheild} - Q_{with\ shield}}{Q_{without\ sheild}}$$

$$= \frac{\frac{Q_{12}}{A} - \frac{Q_{13}}{A}}{\frac{Q_{12}}{A}} = 1 - \frac{\mathbf{510.285}}{7.048\ \times\ 10^3}$$

$$\%\, \boldsymbol{reduction\ in\ heat\ transfer} = \ \mathbf{92.8}\,\%$$

11. A pipe of outside diameter 30 cm having emissivity 0.6 and temperature 600K is enclosed concentrically by a 40 cm square duct having emissivity 0.8 and temperature 300K. Calculate the following: (i) Net radiant heat transfer per meter length, (ii) convective heat transfer of the fluid (280K) which is surrounding the duct. **KU – Apr 2015**.

Given Data:

D = 30 cm = 0.3 m, a = 40 cm = 0.4 m,

T_1 = 600 K, T_2 = 300 K, ε_1 = 0.6 , ε_2 = 0.8

TO Find:

(i) Net radiant heat transfer per meter length,

(ii) convective heat transfer of the fluid (280K) which is surrounding the duct.

Solution:

Heat exchange between two large concentric cylinder (or) sphere is given by,

$$\boldsymbol{Q_{12} = \bar{\varepsilon}\, \sigma\, A_1\, \left(T_1{}^4 - T_2{}^4\right)}$$

Where

$$\bar{\varepsilon} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

The surface area of the pipe,

$$A_1 = \pi\, D\, L = \pi \times 0.3\ \times 1 = 0.942\ m^2$$

The surface area of the duct,

$$A_2 = 4\ aL = \ 4 \times 0.4\ \times 1 = 1.6\ m^2$$

Therefore

$$\bar{\varepsilon} = \frac{1}{\frac{1}{0.6} + \frac{0.942}{1.6}\left(\frac{1}{0.8} - 1\right)}$$

$$\bar{\varepsilon} = 0.55$$

Radiation

$$Q_{12} = 0.55 \times 5.67 \times 10^{-8} \times 0.942 \times (600^4 - 300^4)$$

$$Q_{12} = 3569.2\ W/m$$

w.k.t the rate of heat transfer is given as

$$Q = h\,\mathrm{A}\,(T_s - T_\infty)$$

$$Q_{12} = h\,\mathrm{A}\,(T_2 - T_\infty)$$

$$3569.2 = h \times 1 \times (300 - 280)$$

$$h = 178.46\ \ W/\,m^2K$$

MULTIPLE CHOICE QUESTIONS

Questions	Opt1	Opt2	Opt3	Opt4	Answer
Energy transfer through electromagnetic wave is known as	Conduction	. Convection	Radiation	None of these	Radiation
A perfect black body	Is black in color	Reflects incident energy	Absorbs all the incident energy	Transmits incident energy	Absorbs all the incident energy
If a body reflects all radiations incident on it then it's known as a	Black body	White body	Grey body	Transparent body	White body
A body transmits all radiations incident on it then its known as a	Black body	White body	Grey body	Transparent body	Transparent body
As a result of absorption of all radiations incident on a black body, the black body	Becomes good conductor of heat	Shines	Temperature increases	All of these	Becomes good conductor of heat
Wave length of radiations depends on	Temperature of the body	Material of the body	A and b	None of these	Temperature of the body
Temperature of the sun can be measured by	mercury thermometer	standard thermometer	radiation pyrometer	None of these	radiation pyrometer
Heat transfer by radiation between two surfaces can be decreased by	bringing the surface	polishing the surface	providing a radiation shield between the surfaces	All of these	providing a radiation shield between the surfaces
A radiation shield should have	zero reflectivity	low reflectivity	high reflectivity	none of these	high reflectivity
Radiation emitted by back body is known as	total radiation	full radiation	black radiation	none of these	total radiation
All bodies above absolute zero temperature emit radiation. This statement is	Weins statement	Stefans law	Planks law	Prevost theory	Planks law
Ratio of energy absorbed by the body to the total energy incident on it, is known as	Emissive power	Absorptive power	Emissivity	Transmissibility	Absorptive power
Radiation transfers through	Electromagnetic waves	signal waves	density difference	molecular vibration	Electromagnetic waves

Radiation transfers through	Discrete quanta of energy called photons	signal waves	density difference	molecular vibration	Discrete quanta of energy called photons
Which mode of heat transfer is predominant when the temperature of the body is high	Conduction	Radiation	Convection	Mass transfer	Radiation
____________ the wavelength, more powerful is the radiation	Smaller	Bigger	constant	natural frequency	Smaller
Smaller the wavelength, _________________ is the radiation	Week	Strong	Constant	Unpredictable	Strong
Solids and Liquids emit radiation in a ______________	Certain wavelength bands	Continuous spectrum	Discontinuous spectrum	a & c	Continuous spectrum
Gases and vapours radiate only in _____________________	Certain wavelength bands	Continuous spectrum	Discontinuous spectrum	a & c	Certain wavelength bands
Selective emitters are	Solids	Liquids	Solids and Liquids	Gases and vapours	Gases and vapours
Spectral means ____________	dependence on temperature	dependence on frequency	dependence on wavelength	dependence on velocity of light	dependence on wavelength
Monochromatic value is known as	value of a quantity at a given temperature	value of a quantity at a given frequency	value of a quantity at a given wavelength	value of a quantity at a given light	value of a quantity at a given wavelength
Fraction of incident radiation absorbed is known as	Absorptivity	Reflectivity	Transmissivity	Emissivity	Absorptivity
Fraction of incident radiation absorbed is known as	Absorptivity	Reflectivity	Transmissivity	Emissivity	Transmissivity
Absorption and reflection of heat rays depend on	State of the surface	Colour of the surface	Location of the surface	Climate	State of the surface
Absorptivity of surface can be increased by	Heating	Applying appropriate coating	Cooling	climate	Applying appropriate coating
For a given temperature and wavelength, energy emitted by a black body is ___ as compared to any other body	Maximum	Minimum	Equal	incomparable	Maximum

A black body absorbs all the incident radiation, of	all wavelengths and from all directions	all wavelengths and from normal directions	certain wavelengths and from all directions	certain wavelengths and from normal directions	all wavelengths and from all directions
Roughness of the surface determines	if the reflection is specular	If the reflection is diffuse	If the reflection is specular or diffuse	If the body is block or white	If the reflection is specular or diffuse
Radiation energy emitted by a black surface depends on	Wavelength	Temperature	Surface characteristics	All of these	All of these
Emissivity value varies from	0.1 to 1	0 to 1	1 to 2	2 to 3	0 to 1

CHAPTER - V

MASS TRANSFER

5.1. INTRODUCTION

Mass transfer may occur in a gas mixture, a liquid solution, or a solid solution. There are several physical mechanisms that can transport a chemical species through a phase and transfer it across phase boundaries. The two most important mechanisms are ordinary diffusion and convection.

"Mass diffusion is analogous to heat conduction and occurs whenever there is a gradient in the concentration of a species".

"Mass convection is essentially identical to heat convection: a flu id flow that transports heat may also transport a chemical species".

The similarity of mechanisms of heat transfer and mass transfer results in the mathematics often being identical, a fact that can be exploited to advantage. But there are some significant differences between the subjects of heat and mass transfer. One difference is the much greater variety of physical and chemical processes that require mass transfer analysis. Another difference is the extent to which the essential details of a given process may depend on the particular chemical system involved, and on temperature and pressure.

In a system consisting of two or more components whose concentrations vary from point to point, there is a natural tendency for species (particles) to be transferred from a region of higher concentration side (higher density side) to a region of tower concentration side (lower density side).

This process of transfer of mass as a result of the species concentration difference in a mixture is known as mass transfer.

Some examples of mass transfer are

- Humidification of air in cooling tower
- Evaporation of petrol in the carburetter of an IC engine
- The transfer of water vapour into dry air.
- Dissolution of sugar added to a cup of coffee.

5.2. MODES OF MASS TRANSFER

There are basically two modes of mass transfer given below that are similar to the conduction and convection modes of heat transfer.

1. Diffusion mass transfer
2. Convective mass transfer

5.2.1. DIFFUSION MASS TRANSFER

It may be classified into two types.

1. Molecular diffusion
2. Eddy diffusion

5.2.1.1. MOLECULAR DIFFUSION

The transport of water on a microscopic level as a result of diffusion from a region of higher concentration to a region of lower concentration in a mixture of liquids or gases is known as molecular diffusion.

5.2.1.2. EDDY DIFFUSION

When one of the diffusion fluids is in turbulent motion, eddy diffusion takes place. Mass transfer is more rapid by eddy diffusion than by molecular diffusion.

5.2.2. CONVECTIVE MASS TRANSFER

Convective mass transfer is a process of mass transfer that will occur between a surface and a fluid medium when they are at different concentrations.

5.3. CONCENTRATIONS

5.3.1. MASS CONCENTRATION OR MASS DENSITY

The mass concentration is defined as the mass of a component per unit volume of the mixture. It is expressed in kg/m^3.

Mass concentration = (Mass of a component / Unit volume of mixture)

5.3.2. MOLAR CONCENTRATION OR MOLAR DENSITY

The molar concentration is defined as the number of molecules of a component per unit volume of the mixture. It is expressed in kg-mole/m^3.

Molar concentration = (Number of molecules of component / Unit volume of mixture)

The mass concentration and molar concentration are related by the expression

$$C_A = \frac{\rho_A}{M_A}$$

Where,

ρ_A = Density of component, A

M_A = Molecular weight of component, A

5.3.3. MASS FRACTION

The mass fraction is defined as the mass concentration of species to the total mass density of the mixture.

Mass fraction = (Mass concentration of a species / Total mass density)

$$\boldsymbol{m_A = \frac{\rho_A}{\rho}}$$

5.3.4. MOLE FRACTION

The mole concentration is defined as the ratio of mole concentration of a species to the total molar concentration.

Mole fraction = (Mole concentration of a species / Total molar concentration)

$$\boldsymbol{x_A = \frac{C_A}{C}}$$

5.4. FICK'S LAW OF DIFFUSION

Consider a system shown in figure. A partition separates the gases, a and b. when the partition is removed, the gases diffuses through one other until the equilibrium is established throughout the system.

The diffusion rate is given by the fick's law, which states that molar flux of an element per unit area is directly proportional to concentration gradient.

$$\frac{m_a}{A} \propto \frac{dC_a}{dx}$$

$$\frac{m_a}{A} = -D_{ab}\frac{dC_a}{dx}$$

$$N_a = \frac{m_a}{A} = -D_{ab}\frac{dC_a}{dx}$$

Where,

$$N_a = \frac{m_a}{A} = molar\ flux, kg\ mole\ /\ sec\ m^2$$

$$D_{ab} = diffusion\ coefficient\ of\ species\ a\ and\ b, m^2/sec$$

$$\frac{dC_a}{dx} = Concentration\ gradient$$

5.5. STEADY STATE DIFFUSION THROUGH A PLANE MEMBRANE:

Mass Transfer

Consider a plane membrane of thickness L, containing fluid 'a'. The concentrations of the fluid at the opposite wall faces are C_{a1} and C_{a2} respectively.

Considering the diffusion is along X axis, then the controlling equation is

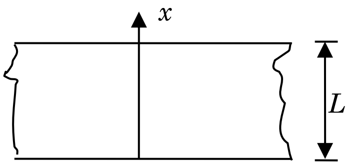


Fig.5.2.

$$\frac{d^2 C_a}{dx^2} = 0$$

Integrating above equation

$$\frac{dC_a}{dx} = C_1$$

Again integrating,

$$C_a = C_1 x + C_2 \dots\dots\dots\dots \quad eq.1$$

Apply boundary condition

At, $x = 0$

$$C_{a1} = C_2$$

At, x=L

$$C_{a2} = C_1 L + C_2$$

$$C_{a2} = C_1 L + C_{a1} \qquad [\therefore C_2 = C_{a1}]$$

$$C_1 = \frac{C_{a2} - C_{a1}}{L}$$

Substituting C_1, C_2 values in equation 1

$$C_1 = \left[\frac{C_{a2} - C_{a1}}{L}\right] x + C_{a1}$$

From Fick's law, we know that,

Molar flux,

$$\frac{m_a}{A} = -D_{ab} \frac{dC_a}{dx}$$

$$\frac{m_a}{A} = -D_{ab} \frac{d\left\{\left[\frac{C_{a2} - C_{a1}}{L}\right] x + C_{a1}\right\}}{dx}$$

$$\frac{m_a}{A} = -D_{ab}\left[\frac{C_{a2} - C_{a1}}{L}\right]$$

C_{a1} = concentration at inner side, kg mole/m^3

C_{a2} = concentration at outer side, kg mole/m^3

L = thickness, m

For cylinders,

$$L = r_2 - r_1$$

$$A = \frac{2\,\pi\, l\,(r_2 - r_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

For sphere,

$$L = r_2 - r_1$$

$$A = 4\pi r_1 r_2$$

where,

r_1 = inner radius – m

r_2 = inner radius – m

L= length - m

5.6. STEADY STATE EQUIMOLAR COUNTER DIFFUSION

Consider two large chambers a and b connected by a passage as shown in Fig.2. N_a and N_b are the steady state molar diffusion rates of components a and b respectively.

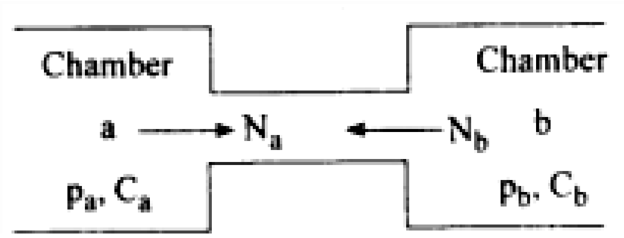


Fig . 2

Equimolar diffusion is defined as each molecule of 'a' is replaced by each molecule of 'b' and vice versa. The total pressure P = P_a + P_b is uniform throughout the system.

$$P = P_a + P_b$$

The molar flux of components a and b are given as

$$N_a = \frac{m_a}{A} = \frac{D}{GT}\left[\frac{p_{a1} - p_{a1}}{x_2 - x_1}\right]$$

$$N_b = \frac{m_b}{A} = \frac{D}{GT}\left[\frac{p_{b1} - p_{b1}}{x_2 - x_1}\right]$$

Where

G = Universal gas constant, 8.314 kJ/kg mole K

p_{a1}, p_{a2} = Partial pressures of constituents at 1 &2 in N/m^2

T = Temperature, K

5.7. ISOTHERMAL EVAPORATION OF WATER INTO AIR

Consider the isothermal evaporation of water from a water surface and its diffusion through the stagnant air layer over it as shown in Fig.3. The free surface of the water is exposed to air in the tank.

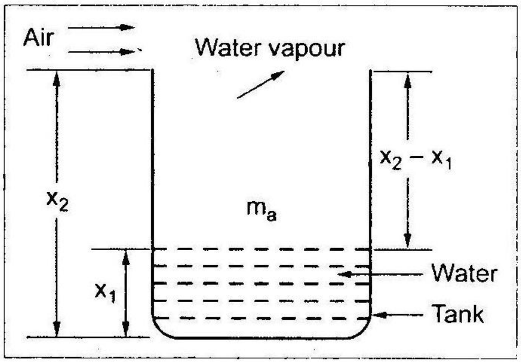


Fig . 3

For the analysis of this type of mass diffusion, following assumptions are made,

1. The system is isothermal and total pressure remains constant.
2. System is in steady state condition.
3. There is slight air movement over the top of the tank to remove the water vapour which diffuses to that point.
4. Both the air and water vapour behave as ideal gases.

From Fick's law of diffusion, the molar flux of the component can be found as

$$molar\ flux = \frac{m_a}{A} = \frac{D_{ab}}{GT}\frac{p}{(x_2 - x_1)}\ln\left[\frac{p_{a2}}{p_{a1}}\right]$$

Or

$$molar\ flux = \frac{m_a}{A} = \frac{D_{ab}}{GT}\frac{p}{(x_2 - x_1)}\ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right]$$

Where

p = Total pressure in bar

p_{w1} = partial pressure of water vapour corresponding to saturation temperature at 1 in N/m^2

p_{w2} = Partial pressure of dry air at 2 in N/m^2

5.7. CONVECTIVE MASS TRANSFER

Convective mass transfer is a process of mass transfer that will occur between a surface and a fluid medium when they are at different concentrations.

5.7.1. TYPES OF CONVECTIVE MASS TRANSFER

1. Free convective mass transfer
2. Forced convective mass transfer

5.7.1.1 FREE CONVECTIVE MASS TRANSFER

If the fluid motion is produced due to change in density resulting from concentration gradients, the mode of mass transfer is said to be free or natural convective mass transfer.

Example Evaporation of alcohol

5.7.1.2 FORCED CONVECTIVE MASS TRANSFER

If the fluid motion is artificially created by means of an external force like a blower or fan, that type of mass transfer is known as forced convective mass transfer.

Example: The evaporation of water from an ocean when air blows over it.

5.7.2. SIGNIFICANCE OF DIMENSIONLESS GROUPS

Reynolds Number (Re)

It is defined as the ratio of the inertia force to the viscous force

$$Reynolds\ number\ (Re) = \frac{Inertia\ force}{Viscous\ force}$$

$$Re = \frac{V\,x}{\nu}$$

Where,

V – velocity in m/sec

x – distance in m

ν – kinematic viscosity in m^2/sec

For flat plate,

If $Re < 5x10^5$, flow is laminar

If $Re > 5x10^5$, flow is turbulent

Schmidt Number (Sc)

It is defined as the ratio of the molecular diffusivity of momentum to the molecular diffusivity of mass.

$$Sc = \frac{Molecular\ diffusivity\ of\ momentum}{Molecular\ diffusivity\ of\ mass}$$

$$Sc = \frac{\nu}{D_{ab}}\ \ or\ \ Sc = \frac{\mu}{\rho\, D_{ab}}$$

Where,

ν = Kinematic viscosity, m^2/sec

D_{ab} = Diffusion coefficient, m^2/sec

Sherwood Number (Sh)

It is defined as the ratio of concentration gradients at the boundary.

$$Sh = \frac{h_m\, x}{D_{ab}}$$

Where,

hm = Mass transfer coefficient, m/s

x = Length, m

Dab = Diffusion coefficient, m^2/sec

FORMULAE USED FOR FLAT PLATE PROBLEMS

$$Re = \frac{V\, x}{\nu}$$

Where,

V – velocity in m/sec

x – distance in m

ν – kinematic viscosity in m^2/sec

For flat plate,

If Re < $5x10^5$, flow is laminar

If Re > $5x10^5$, flow is turbulent

For Laminar flow

[From HMT data book, page no. 175 (Sixth edition)]

Local Sherwood Number,

$$Sh_x = 0.332\ (Re_x)^{0.5}\ (Sc)^{0.333}$$

Average Sherwood Number, Sh = 0.664 (Re) 0.5 (SC) 0.333

$$Sh = 0.664\ (Re)^{0.5}\ (Sc)^{0.333}$$

For Turbulent flow

[From HMT data book, page no. 176 (Sixth edition)]

Fully turbulent from leading edge

Sherwood Number,

$$Sh = 0.0296\ (Re)^{0.8}\ (Sc)^{0.333}$$

Combined Laminar – Turbulent flow

Sherwood Number,

$$Sh = [0.037\ (Re)^{0.8} -\ 871]\ (Sc)^{0.333}$$

$$Sh = \frac{h_m\ x}{D_{ab}}$$

Where,

hm = Mass transfer coefficient, m/s

x = Length, m

Dab = Diffusion coefficient, m^2/sec

$$\boldsymbol{Sc = \frac{\nu}{D_{ab}}\ \ or\ \ Sc = \frac{\mu}{\rho\ D_{ab}}}$$

Where,

ν = Kinematic viscosity, m^2/sec

D_{ab} = Diffusion coefficient, m^2/sec

Questions & Answers:

1. What is mass transfer?

The process of transfer of mass as a result of the species concentration difference in a mixture is known as mass transfer.

2. Give the examples of mass transfer.

Some examples of mass transfer.

1. Humidification of air in cooling tower
2. Evaporation of petrol in the carburetor of an IC engine.
3. The transfer of water vapour into dry air.

3. What are the modes of mass transfer?

There are basically two modes of mass transfer,

1. Diffusion mass transfer
2. Convective mass transfer

4. What is molecular diffusion?

The transport of water on a microscopic level as a result of diffusion from a region of higher concentration to a region of lower concentration in a mixture of liquids or gases is known as molecular diffusion.

5. What is Eddy diffusion?

When one of the diffusion fluids is in turbulent motion, eddy diffusion takes place.

6. What is convective mass transfer?

Convective mass transfer is a process of mass transfer that will occur between surface and a fluid medium when they are at different concentration.

7. State Fick's law of diffusion.

The diffusion rate is given by the Fick's law, which states that molar flux of an element per unit area is directly proportional to concentration gradient.

$$\frac{m_a}{A} = -D_{ab}\frac{dC_a}{dx}$$

where,

$\frac{ma}{A}$ – Molar flux, $\frac{\text{kg -mole}}{\text{s-m}^2}$

D_{ab} Diffusion coefficient of species a and b, m^2/s

$\frac{dC_a}{dx}$ – concentration gradient, kg/m^3

8. What is free convective mass transfer?

If the fluid motion is produced due to change in density resulting from concentration gradients, the mode of mass transfer is said to be free or natural convective mass transfer.

Example : Evaporation of alcohol.

9. Define forced convective mass transfer.

If the fluid motion is artificially created by means of an external force like a blower or fan, that type of mass transfer is known as convective mass transfer.

Example: The evaluation if water from an ocean when air blows over it.

10. Define Schmidt Number.

It is defined as the ratio of the molecular diffusivity of momentum to the molecular diffusivity of mass.

$$Sc = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of mass}}$$

11. Define Sherwood Number.

It is defined as the ratio of concentration gradients at the boundary.

$$Sc = \frac{h_m x}{D_{ab}}$$

hm – Mass transfer coefficient, m/s

D_{ab} – Diffusion coefficient, m^2/s

x – Length, m

12. Define mass concentration

The mass concentration is defined as the mass of a component per unit volume of the mixture. It is expressed in kg/m^3.

Mass concentration = (Mass of a component / Unit volume of mixture)

13. Define molar concentration

The molar concentration is defined as the number of molecules of a component per unit volume of the mixture. It is expressed in kg-mole/m^3.

Molar concentration = (Number of molecules of component / Unit volume of mixture)

14. Define mass fraction.

The mass fraction is defined as the mass concentration of species to the total mass density of the mixture.

Mass fraction = (Mass concentration of a species / Total mass density)

$$\boldsymbol{m_A = \frac{\rho_A}{\rho}}$$

15. Define mole fraction.

The mole concentration is defined as the ratio of mole concentration of a species to the total molar concentration.

Mole fraction = (Mole concentration of a species / Total molar concentration)

$$\boldsymbol{x_A = \frac{C_A}{C}}$$

Problems:

1. A mixture of O_2 and N_2 with their partial pressures in the ratio 0.21 to 0.79 is in a container at 25°C. Calculate the molar concentration, the mass density, the mole fraction and the mass fraction of each species for a total pressure of 1 bar. What would be the average molecular weight of the mixture? **KU-Nov 2010, KU – Apr 2014.**

   Given Data:

   Partial pressure ratios of O_2 and N_2 – 0.21 and 0.79

   T = 25°C = 298 K, Total pressure = 1 bar

   To find:

   (i) C_{O_2}=? (ii) C_{N_2}=? (iii) ρ_{O_2}=? & ρ_{N_2}=? (iv) $\dot{m}_{O_2}$=? & $\dot{m}_{N_2}$=? (v) M

   Solution:

   Partial pressure of O_2

$$p_{O_2} = 0.21 \times total\ pressure$$

$$p_{O_2} = 0.21 \times 1 \times 10^5$$

$$p_{O_2} = 0.21 \times 10^5\ Pa$$

   Partial pressure of N_2

$$p_{N_2} = 0.79 \times total\ pressure$$

$$p_{N_2} = 0.79 \times 1 \times 10^5$$

$$p_{N_2} = 0.79 \times 10^5 \ Pa$$

We know that

$$molar\ concentration, C = \frac{p}{G\ T}$$

$$C_{O_2} = \frac{p_{O_2}}{G\ T} = \frac{0.21 \times 10^5}{8314 \times 298}$$

$$\boldsymbol{C_{O_2} = 8.46 \times 10^{-3}\ kg - mole/m^3}$$

Similarly

$$C_{N_2} = \frac{p_{N_2}}{G\ T} = \frac{0.79 \times 10^5}{8314 \times 298}$$

$$\boldsymbol{C_{N_2} = 31.88 \times 10^{-3}\ kg - mole/m^3}$$

We also know that, Molar concentration

$$C = \frac{\rho}{M}$$

Therefore

$$\rho = C \times M$$

Molecular weight of O_2 and N_2 are taken as 32 and 28 respectively

$$\rho_{O_2} = C_{O_2} \times M_{O_2}$$

$$\rho_{O_2} = 8.46 \times 10^{-3} \times 32$$

$$\boldsymbol{\rho_{O_2} = 0.271\ kg/m^3}$$

Similarly

$$\rho_{N_2} = C_{N_2} \times M_{N_2}$$

$$\rho_{N_2} = 31.88 \times 10^{-3} \times 28$$

$$\boldsymbol{\rho_{N_2} = 0.893\ kg/m^3}$$

Therefore the overall density is given as

$$\rho = \rho_{O_2} + \rho_{N_2} = 0.271 + 0.893$$

$$\boldsymbol{\rho = 1.164\ kg/m^3}$$

We know that the mass fraction is given as

$$\dot{m}_{O_2} = \frac{\rho_{O_2}}{\rho} = \frac{0.271}{1.164}$$

$$\boldsymbol{\dot{m}_{O_2} = 0.233}$$

$$\dot{m}_{N_2} = \frac{\rho_{N_2}}{\rho} = \frac{0.893}{1.164}$$

$$\boldsymbol{\dot{m}_{N_2} = 0.767}$$

Average molecular weight, M

$$M = p_{O_2}M_{O_2} + p_{N_2}M_{N_2}$$

$$M = 0.21 \times 32 + 0.79 \times 28$$

$$\boldsymbol{M = 28.84}$$

2. Estimate the Diffusion rate of water at 27°C from the surface of water in a test tube of 0.02m diameter and surface of water in the test tube is 0.05 m deep. Take diffusion coefficient of water into air as 0.26 x 10^{-4} m^2/s at 27°C and P_{sat} as 0.035 bar. **KU-Nov 2010.**

Given Data:

d = 0.02 m, $(x_2 - x_1)$ = 0.05 m, T = 27°C = 300 K,

D_{ab} = 0.26 x 10^{-4} m^2/s, p_{sat} = 0.035 bar

To Find:

Mass rate of water vapour, m_a =?

Solution:

We know that, the molar rate of water vapour

$$\frac{m_a}{A} = \frac{D_{ab}}{GT}\,\frac{p}{(x_2 - x_1)}\,\ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right]$$

We also know that

Mass rate of water vapour = molar rate of water vapour x molecular weight of steam

$$m_a = \frac{D_{ab} \times A}{GT}\,\frac{p}{(x_2 - x_1)} \times \ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right] \times 18.016$$

Where

$$A = \frac{\pi}{4}\,d^2 = \frac{\pi}{4}\,0.02^2 = 3.14 \times 10^{-4}\,m^2$$

G = universal gas constant = 8314 J/kg-mole-K

p = Total pressure = 1 atm = 1.013 bar = 1.013 x 10^5 Pa

p_{w1} = partial pressure at bottom of pan at sat temp ≈25°C

At 25°C from steam tables pg.no. 2

$$p_{w1} = 0.035 \text{ bar} = 0.035 \times 10^5 \text{ Pa}$$

p_{w2} = partial pressure at top of pan = 0

$$m_a = \frac{0.26 \times 10^{-4} \times 3.14 \times 10^{-4}}{8314 \times 300} \times \frac{1.013 \times 10^5}{0.08} \times \ln\left[\frac{1.013 \times 10^5 - 0}{1.013 \times 10^5 - 0.035 \times 10^5}\right] \times 18.016$$

$$\boldsymbol{m_a = 2.627 \times 10^{-9}\ kg/s}$$

3. Air at 25°C and at atmospheric pressure flows with a velocity of 3 m/sec inside a 10 mm diameter tube of 1 meter length. The inside surface of tube contains deposits of naphthalene. Determine the average mass transfer coefficient. (Assume the diffusion coefficient for naphthalene-air as 0.62×10^{-5} m^2/sec.) **KU-Nov 2011.**

Given Data:

T_∞ = 25°C, V = 3 m/s, D = 10 mm = 0.01 m, x = 1 m,

$D_{ab} = 0.62 \times 10^{-5}$ m^2/sec

To Find:

h_m =?

Solution:

The properties of air at T_∞ = 25°C ≈ 30°C

Kinematic viscosity of air, $\nu = 16 \times 10^{-6}$ m^2/s

We know that Reynolds number is given as

$$Re = \frac{V\,D}{\nu}$$

$$Re = \frac{3 \times 0.01}{16 \times 10^{-6}}$$

$$Re = 1875 < 2000$$

$$hence\ the\ flow\ is\ laminar$$

For laminar internal flow cylinder from HMT DB pg.no 177, the Sherwood number is given as

$$sh = 3.66$$

We also know that

$$Sh = \frac{h_m\,D}{D_{ab}}$$

$$h_m = \frac{Sh \times D_{ab}}{D} = \frac{3.66 \times 0.62 \times 10^{-5}}{0.01}$$

$$\boldsymbol{h_m = 0.0023\ m/s}$$

4. Consider air inside a tube of surface area 0.5 m^2 and wall thickness 10 mm. The pressure of air drops from 2.2 bar to 2.18 bar in 6 days. The solubility of air in the rubber is 0.072 m^2 of air per m^3 rubber at 1 bar. Determine the diffusivity of air in rubber at the operating temperature of 300 K if the volume of air in the tube is 0.028 m^3. **KU – Nov 2011.**

Given Data:

A = 0.5 m^2, L = 10 mm = 0.01 m, p_i = 2.2 bar = 2.2 x 10^5 Pa, p_d = 2.18 bar = 2.18 x 10^5 Pa, S = 0.072 m^3, T = 300 K, V = 0.028 m^3.

To find:

Diffusivity of air in rubber (D) =?

Solution:

Initial mass of air in the tube,

$$m_i = \frac{p_i\, V}{R\, T} = \frac{2.2 \times 10^5 \times 0.028}{287 \times 300}$$

$$m_i = 0.0715\ kg$$

Final mass of air in the tube

$$m_d = \frac{p_d\, V}{R\, T} = \frac{2.18 \times 10^5 \times 0.028}{287 \times 300}$$

$$m_d = 0.07089\ kg$$

Mass of the air escaped

$$m_a = m_i - m_d = 0.0715 - 0.07089$$

$$m_a = 0.00061\ kg$$

The mass flux of air escaped is given by

$$N_a = \frac{m_a}{A} = \frac{mass\ of\ air\ escaped}{time\ elapsed\ \times Area} = \frac{0.00061}{(6 \times 24 \times 3600) \times 0.5}$$

$$N_a = 2.35 \times 10^{-9}\ kg/s - m^2$$

The solubility of air should be calculated at the mean operating pressure,

$$\frac{2.2 + 2.18}{2} = 2.19\ bar$$

The solubility of air (i.e) volume at the mean inside pressure

$$S = 0.072 \times 2.19 = 0.1577\ m^3/m^3\ of\ rubber$$

The air which escapes to atmosphere will be at 1 bar pressure and its solubility will remain at 0.072 m^3 of air per m^3 of rubber.

The corresponding mass concentration at the inner and outer surfaces of the tube, from characteristic gas equation are calculated as

$$C_{a1} = \frac{p_1 V_1}{RT_1} = \frac{2.19 \times 10^5 \ \times 0.1577}{287 \ \times 300}$$

$$C_{a1} = 0.4011 \ kg/m^3$$

$$C_{a2} = \frac{p_2 V_2}{RT_2} = \frac{1 \times 10^5 \ \times 0.072}{287 \ \times 300}$$

$$C_{a1} = 0.0836 \ kg/m^3$$

The diffusion flux rate of air through the rubber is given by

$$N_a = \frac{m_a}{A} = D \left[\frac{C_{a1} - C_{a2}}{x_2 - x_1}\right] = D \left[\frac{C_{a1} - C_{a2}}{L}\right]$$

$$2.35 \times 10^{-9} = D \left[\frac{0.4011 - 0.0836}{0.01}\right]$$

$$\boldsymbol{D = 0.74 \times 10^{-10} \ m^2/s}$$

5. Dry air at 20°C (ρ = 1.2 kg/m^3, v = 15 X 10^{-6} m^2/s, D = 4.2 X 10^{-5} m^2/s) flows over flat plate of length 50 cm which is covered with a thin layer of water at a velocity of 1 m/s. Estimate the local mass transfer coefficient at a distance of 10 cm from the leading edge and the average mass transfer coefficient. **KU – Nov 2012, KU – Apr 2014**

Given Data:

T_∞ = 20°C, ρ = 1.2 kg/m^3, ν = 15 x 10^{-6} m^2/s, D_{ab} = 4.2 x 10^{-5} m^2/s, L = 50 cm = 0.5 m, U = 1 m/s and x = 10 cm = 0.1 m

To Find:

h_x at x = 0.1 m and h_m =?

Solution:

Local mass transfer coefficient at x = 0.1 m

We know that

$$Re = \frac{V\,x}{\nu}$$

$$Re = \frac{1 \times 0.1}{15 \times 10^{-6}}$$

$$Re = 6666.67 < 5 \times 10^5$$

$$hence\ the\ flow\ is\ laminar$$

for laminar flow flat plate from HMT DB pg.no 176, the Sherwood number is given as

$$sh = 0.332\ (Re)^{0.5}\ (Sc)^{0.333}$$

Where,

$$\boldsymbol{Sc = \frac{\nu}{D_{ab}}} = \frac{15 \times 10^{-6}}{4.2 \times 10^{-5}}$$

$$\boldsymbol{Sc = 0.357}$$

Therefore

$$sh = 0.332\ (6666.67)^{0.5}\ (0.357)^{0.333}$$

$$sh = 19.24$$

We also know that

$$Sh = \frac{h_x\, x}{D_{ab}}$$

$$h_x = \frac{Sh \times D_{ab}}{x} = \frac{19.24 \times 4.2 \times 10^{-5}}{0.1}$$

$$\boldsymbol{h_x = 8.08 \times 10^{-3}\ m/s}$$

To find the average mass transfer coefficient h_m for the entire length of the plate

$$Re = \frac{V\, L}{\nu}$$

$$Re = \frac{1 \times 0.5}{15 \times 10^{-6}}$$

$$Re = 3.33 \times 10^4 < 5 \times 10^5$$

$$hence\ the\ flow\ is\ laminar$$

for laminar flow flat plate from HMT DB pg.no 176, the Sherwood number is given as

$$sh = 0.664\ (Re)^{0.5}\ (Sc)^{0.333}$$

Where,

$$\boldsymbol{Sc = \frac{\nu}{D_{ab}}} = \frac{15 \times 10^{-6}}{4.2 \times 10^{-5}}$$

$$\boldsymbol{Sc = 0.357}$$

Therefore

$$sh = 0.664\ (3.33\ \times\ 10^4)^{0.5}\ (0.357)^{0.333}$$

$$sh = 85.99$$

We also know that

$$Sh = \frac{h_m\ L}{D_{ab}}$$

$$h_m = \frac{Sh \times D_{ab}}{L} = \frac{85.99\ \times 4.2 \times\ 10^{-5}}{0.5}$$

$$\boldsymbol{h_m = 0.007\ m/s}$$

6. The molecular weights of the two components A and B of a gas mixture are 24 and 28 respectively. The molecular weight of a gas mixture is found to be 30. If the mass concentration of the mixture is 1.2 kg/m^3, determine the following, 1) Density of components A and B; 2) Molar fractions; 3) Mass fractions; 4) Total pressure if the temperature of the mixture is 290 K. **KU – Nov 2013.**

Given Data:

M_A = 24 & M_B = 48, M = 30, ρ = 1.2 kg/m^3 and T = 290 K

To Find:

(i) ρ_A =? & ρ_B =? (ii) x_A & x_B =? (iii) $\dot{m}_A$ & $\dot{m}_B$ =? And (iv) P =?

Solution:

Molar concentration

$$C = \frac{\rho}{M}$$

$$C = \frac{1.2}{30}$$

$$C = 0.04$$

We know that

$$C = C_A + C_B$$

$$C_A + C_B = 0.04, \dots\dots eq.1$$

We also know that

$$\rho_A = C_A \times M_A = 24C_A$$

$$\rho_B = C_B \times M_B = 48C_B$$

Also

$$\rho = \rho_A + \rho_B$$

$$24C_A + 48C_B = 1.2, \ldots\ldots eq. 2$$

On solving eq.1 and eq.2

$$C_A = 0.03\ kg\ mole/m^3$$

$$C_B = 0.01\ kg\ mole/m^3$$

To find density

$$\rho_A = 24C_A = 24 \times 0.03$$

$$\boldsymbol{\rho_A = 0.72\ kg/m^3}$$

$$\rho_B = 48C_B = 48 \times 0.01$$

$$\boldsymbol{\rho_B = 0.48\ kg/m^3}$$

To find mole fractions

$$x_A = \frac{C_A}{C} = \frac{0.03}{0.04}$$

$$x_A = 0.75$$

$$x_B = \frac{C_B}{C} = \frac{0.01}{0.04}$$

$$x_A = 0.25$$

To find mole fractions

$$\dot{m}_A = \frac{\rho_A}{\rho} = \frac{0.72}{1.2}$$

$$\boldsymbol{\dot{m}_A = 0.6}$$

$$\dot{m}_B = \frac{\rho_B}{\rho} = \frac{0.48}{1.2}$$

$$\boldsymbol{\dot{m}_B = 0.4}$$

To find the total pressure at 290 K

$$p = \frac{m\,R\,T}{V} = \rho\,R\,T = \rho\frac{G}{M}\,T$$

Universal gas constant (G = 8314 J/kg-mole-K)

$$p = 1.2 \times \frac{8314}{30} \times 290$$

$$p = 96442\ Pa$$

7. Dry air at 27°C and 1 bar flows over a wet plate 0.5 m long at a velocity of 50 m/s. Calculate the mass transfer coefficient of water vapour in air at the end of the plate. **KU – Apr 2014**

Given Data:

T_∞ = 27°C, p = 1 bar, L = 0.5 m, V = 50 m/s.

To Find:

Mass transfer coefficient of water vapour D_{ab} =?

Solution:

The properties of air at T_∞ = 27°C ≈ 30°C

Kinematic viscosity of air, ν = 16 x 10^{-6} m^2/s

We know that

$$Re = \frac{V\,x}{\nu}$$

$$Re = \frac{50 \times 0.5}{16 \times 10^{-6}}$$

$$Re = 1.56 \times 10^6 > 5 \times 10^5$$

$$hence\ the\ flow\ is\ turbulent$$

For turbulent flow flat plate from HMT DB pg.no 177, the Sherwood number is given as

$$Sh = 0.0296\,(Re)^{0.8}\,(Sc)^{0.333}$$

Where,

$$\boldsymbol{D_{ab}} = 25.83 \times 10^{-6}\ \boldsymbol{m^2/s}, \qquad \boldsymbol{taken\ from\ HMT\ DB\ pg.no\ 181}$$

$$\boldsymbol{Sc} = \frac{\boldsymbol{\nu}}{\boldsymbol{D_{ab}}} = \frac{16 \times 10^{-6}}{25.83 \times 10^{-6}}$$

$$\boldsymbol{Sc = 0.619}$$

Therefore

$$Sh = 0.0296\,(1.56 \times 10^6)^{0.8}\,(0.619)^{0.333}$$

$$Sh = 2272.084$$

We also know that

$$Sh = \frac{h_m\,L}{D_{ab}}$$

$$h_m = \frac{Sh \times D_{ab}}{L} = \frac{2272.084 \times 25.83 \times 10^{-6}}{0.5}$$

$$\boldsymbol{h_m = 0.1174\ m/s}$$

8. An open pan 20 cm in diameter and 8 cm deep contains water at 25° C and is exposed to dry atmospheric air. If the rate of diffusion of water vapour is 8.54 x 10^{-4} kg/h. Estimate the diffusion coefficient of water in air. **KU – Apr 2014**

Given Data:

d = 0.2 m, $(x_2 – x_1)$ = 8 cm = 0.08 m, T = 25°C = 298 K, mass rate of water vapour = 8.54 x 10^{-4} kg/h = 2.37 x 10^{-7} kg/s

To Find:

Diffusion coefficient, D_{ab} =?

Solution:

We know that, the molar rate of water vapour

$$\frac{m_a}{A} = \frac{D_{ab}}{GT}\frac{p}{(x_2 - x_1)}\ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right]$$

We also know that

Mass rate of water vapour = molar rate of water vapour x molecular weight of steam

$$2.37 \times 10^{-7} = \frac{D_{ab} \times A}{GT}\frac{p}{(x_2 - x_1)} \times \ln\left[\frac{p - p_{w2}}{p - p_{w1}}\right] \times 18.016$$

Where

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}0.2^2 = 0.0314\, m^2$$

G = universal gas constant = 8314 J/kg-mole-K

p = Total pressure = 1 atm = 1.013 bar = 1.013 x 10^5 Pa

p_{w1} = partial pressure at bottom of pan at sat temp 25°C

At 25°C from steam tables pg.no. 2

p_{w1} = 0.03166 bar = 0.03166 x 10^5 Pa

p_{w2} = partial pressure at top of pan = 0

$$2.37 \times 10^{-7} = \frac{D_{ab} \times 0.0314}{8314 \times 298} \times \frac{1.013 \times 10^5}{0.08} \times \ln\left[\frac{1.013 \times 10^5 - 0}{-0.03166 \times 10^5}\right] \times 18.016$$

$$\boldsymbol{D_{ab} = 2.58 \times 10^{-5}\, m^2/s}$$

9. Air at 1 atm and 25°C, containing small quantities of iodine, flows with a velocity of 6.2 m/s inside a 35 mm diameter tube. Calculate mass transfer coefficient for iodine. The thermo physical properties of air are:

ν = 15.5 x 10^{-6} m²/s; D_{ab} = 0.82 x 10^{-5} m²/s. **KU – Apr 2014**

Given Data:

p = 1 atm = 1.013 bar, V = 6.2 m/s, D = 35 mm = 0.035 m

ν = 15.5 x 10^{-6} m^2/s, D_{ab} = 0.82 x 10^{-5} m^2/s

To Find:

h_m =?

Solution:

We know that Reynolds number is given as

$$Re = \frac{V\,D}{\nu}$$

$$Re = \frac{6.2 \times 0.035}{15.5 \times 10^{-6}}$$

$$Re = 14000 > 2000$$

$$hence\ the\ flow\ is\ Turbulent$$

For turbulent internal flow cylinder from HMT DB pg.no 177, the Sherwood number is given as

$$Sh = 0.023\,(Re)^{0.83}\,(Sc)^{0.44}$$

Where

$$\boldsymbol{Sc = \frac{\nu}{D_{ab}}} = \frac{15.5 \times 10^{-6}}{0.82 \times 10^{-5}}$$

$$\boldsymbol{Sc = 1.890}$$

Therefore

$$Sh = 0.023\,(14000)^{0.83}\,(1.890)^{0.44}$$

$$Sh = 84.07$$

We also know that

$$Sh = \frac{h_m\,D}{D_{ab}}$$

$$h_m = \frac{Sh \times D_{ab}}{D} = \frac{84.07 \times 0.82 \times 10^{-5}}{0.035}$$

$$\boldsymbol{h_m = 0.0196\ m/s}$$

10. Hydrogen gases at 3 bar and 1 bar are separated by a plastic membrane having thickness 0.25 mm. The binary diffusion coefficient of hydrogen in the plastic is 9.1 x 10^{-8} m^2/s. The solubility of hydrogen in the membrane is 2.1 x 10^{-3} kg-mole/m^3bar. An uniform

temperature condition of 20°C is assumed. Calculate (i) Molar concentration of hydrogen on both sides (ii) Molar flux of hydrogen (iii) mass flux of hydrogen. **KU – Aug 2014**

Given Data:

Inside pressure, p_1 = 3 bar , outer pressure, p_2 = 1 bar

L = 0.25 mm = 0.25 x 10^{-3} m, D_{ab} = 9.1 x 10^{-8} m^2/s

Solubility of Hydrogen = 2.1 x 10^{-3} kg-mole/m^3bar

T = 20°C = 293 K

To Find:

(i) C_{a1} and C_{a2} =? (ii) Molar flux (iii) Mass flux

Solution:

(i) Molar concentration is given as

$$C = Solubility \times Pressure$$

$$C_{a1} = 2.1 \times 10^{-3} \times 3$$

$$\boldsymbol{C_{a1} = 6.3 \times 10^{-3}\ kg - mole/m^3}$$

$$C_{a2} = 2.1 \times 10^{-3} \times 1$$

$$\boldsymbol{C_{a2} = 2.1 \times 10^{-3}\ kg - mole/m^3}$$

(ii) Molar flux

$$\frac{m_a}{A} = D_{ab} \left[\frac{C_{a1} - C_{a2}}{L}\right]$$

$$\frac{m_a}{A} = 9.1 \times 10^{-8} \left[\frac{6.3 \times 10^{-3} - 2.1 \times 10^{-3}}{0.25 \times 10^{-3}}\right]$$

$$\boldsymbol{\frac{m_a}{A} = 1.52 \times 10^{-6}}$$

(iii) Mass Flux

We know that

Mass flux = Molar flux x Molecular weight

(Molecular weight of H_2 is 2)

$$\therefore mass\ flux = 1.52 \times 10^{-6} \times 2$$

$$\boldsymbol{mass\ flux = 3.04 \times 10^{-6} \frac{kg}{s\ m^2}}$$

11. Two large tanks maintained at the same temperature and pressures are connected by a circular 0.15 m diameter duct, which is 3 m in length. One tank contains a uniform mixture

of 60 mole% ammonia and 40 mole % air and the other tank contains a uniform mixture of 20 mole % ammonia and 80 mole % air. The system is at 273 K and 1.013 x 10^{-5} Pa. Determine the rate of ammonia transfer between two tanks. Assuming steady state mass transfer. **KU – Apr 2014, KU – Aug 2014**

Given Data:

d = 0.15 m, $(x_2 – x_1)$ = 3 m,

p_{a1} = 60/100 = 0.6 bar = 0.6 x 10^5 Pa

p_{b1} = 40/100 = 0.4 bar = 0.4 x 10^5 Pa

p_{a2} = 20/100 = 0.2 bar = 0.2 x 10^5 Pa

p_{b2} = 80/100 = 0.8 bar = 0.8 x 10^5 Pa

T = 273 K, p = 1.013 x 10^5 Pa

To Find:

Mass transfer rate of ammonia =?

Solution:

We know that for Equimolar counter diffusion

$$\frac{m_a}{A} = \frac{D_{ab}}{GT}\left[\frac{p_{a1} - p_{a2}}{x_2 - x_1}\right]$$

Where

$$A = \frac{\pi}{4}\, d^2 = \frac{\pi}{4}\, 0.15^2 = 0.017\ m^2$$

G = universal gas constant = 8314 J/kg-mole-K

The diffusion coefficient for ammonia is taken from HMT DB pg.no 181

D_{ab} = 21.6 x 10^{-6} m^2/s

$$\frac{m_a}{0.017} = \frac{21.6\ \times 10^{-6}}{8314\ \times 273} \times \left[\frac{0.6\ \times 10^5 - 0.2\ \times 10^5}{3}\right]$$

Molar transfer rate of ammonia

$$m_a = 2.15\ \times 10^{-9}\ kg - mole/s$$

$$\left\{\begin{matrix} mass\ transfer \\ rate\ of\ ammonia \end{matrix}\right\} = \left\{\begin{matrix} molar\ transfer \\ rate\ of\ ammonia \end{matrix}\right\} \times \left\{\begin{matrix} Molecular\ weight \\ of\ Ammonia \end{matrix}\right\}$$

$molecular\ weight\ of\ ammonia\ is\ taken\ from\ HMT\ DB\ pg.no$ 183 as 17.03

$$\begin{Bmatrix} mass\ transfer \\ rate\ of\ ammonia \end{Bmatrix} = 2.15 \times 10^{-9} \times 17.03$$

$$\begin{Bmatrix} \boldsymbol{mass\ transfer} \\ \boldsymbol{rate\ of\ ammonia} \end{Bmatrix} = \mathbf{3.66 \times 10^{-8}}\ \boldsymbol{kg/s}$$

12. CO_2 and air experiences Equimolar counter diffusion in a circular tube whose length and diameter are 1.2 m and 60 mm respectively. The system is at a total pressure of 1 atm and temperature 273 K. The ends at the tube are connected to large chambers. Partial pressure of CO_2 at one end is 200 mm of Hg and the other end is 90 mm of Hg. Calculate (i) Mass transfer rate of CO_2 and (ii) Mass transfer rate of air. **KU – Apr 2015**

Given Data:

d = 60 mm = 0.06 m, $(x_2 - x_1)$ = 1.2 m,

T = 273 K, p = 1 atm = 1.013 x 10^5 Pa

p_{a1} = (200/760) x 1 atm = (200/760) x 1.013 x 10^5 Pa

p_{a1}= 0.27 x 10^5 Pa

p_{a2} = (90/760) x 1 atm = (90/760) x 1.013 x 10^5 Pa

p_{a2}= 0.12 x 10^5 Pa

To find:

(i) mass transfer rate of CO_2 (ii) mass transfer rate of air

Solution:

(i) Mass transfer rate of CO_2

We know that for Equimolar counter diffusion

$$\frac{m_a}{A} = \frac{D_{ab}}{GT}\left[\frac{p_{a1} - p_{a2}}{x_2 - x_1}\right]$$

Where

$$A = \frac{\pi}{4}\, d^2 = \frac{\pi}{4}\, 0.06^2 = 0.00283\ m^2$$

G = universal gas constant = 8314 J/kg-mole-K

The diffusion coefficient for CO_2 in air is taken from HMT DB pg.no 181

D_{ab} = 11.89 x 10^{-6} m^2/s

$$\frac{m_a}{0.00283} = \frac{11.89 \times 10^{-6}}{8314 \times 273} \times \left[\frac{0.27 \times 10^5 - 0.12 \times 10^5}{1.2}\right]$$

Molar transfer rate of ammonia

$$m_a = 1.51 \times 10^{-10}\ kg - mole/s$$

$$\begin{Bmatrix} mass\ transfer \\ rate\ of\ CO_2 \end{Bmatrix} = \begin{Bmatrix} molar\ transfer \\ rate\ of\ CO_2 \end{Bmatrix} \times \begin{Bmatrix} Molecular\ weight \\ of\ CO_2 \end{Bmatrix}$$

$molecular\ weight\ of\ CO_2\ is\ taken\ from\ HMT\ DB\ pg.no\ 184\ as\ 44$

$$\begin{Bmatrix} mass\ transfer \\ rate\ of\ CO_2 \end{Bmatrix} = 1.51 \times 10^{-10} \times 44$$

$$\begin{Bmatrix} \boldsymbol{mass\ transfer} \\ \boldsymbol{rate\ of\ CO_2} \end{Bmatrix} = \mathbf{6.63 \times 10^{-9}}\ \boldsymbol{kg/s}$$

(ii) Mass transfer rate of air

We know that

Molar transfer rate of air

$$m_a = -1.51 \times 10^{-10}\ kg - mole/s$$

$$\begin{Bmatrix} mass\ transfer \\ rate\ of\ air \end{Bmatrix} = \begin{Bmatrix} molar\ transfer \\ rate\ of\ air \end{Bmatrix} \times \begin{Bmatrix} Molecular\ weight \\ of\ air \end{Bmatrix}$$

$$\begin{Bmatrix} mass\ transfer \\ rate\ of\ air \end{Bmatrix} = -1.51 \times 10^{-10} \times 29$$

$$\begin{Bmatrix} \boldsymbol{mass\ transfer} \\ \boldsymbol{rate\ of\ air} \end{Bmatrix} = \mathbf{-4.38 \times 10^{-9}}\boldsymbol{kg/s}$$

13. Air at 25°C flows over a tray full of water with a velocity of 2.8 m/s. The tray measures 30 cm along the flow direction and 40 cm wide. The partial pressure of water present in the air is 0.007 bar. Calculate the evaporation rate of water if the temperature on the water surface is 15°C. Take D = 4.2 x 10^{-5} m^2/s. **KU – Apr 2015**

Given Data:

T_∞ = 25°C, V = 2.8 m/s, x = 30 cm = 0.3 m,

A = 30 cm x 40 cm = 0.3x0.4 m^2,

p_{w2} = 0.007 bar = 0.007 x 10^5 Pa,

T_w = 15°C, D_{ab} = 4.2 x 10^{-5} m^2/s

To Find:

m_w =?

Solution:

To find film temperature

$$T_f = \frac{T_w + T_\infty}{2}$$

$$T_f = \frac{15 + 25}{2}$$

$$T_f = 20°C$$

Properties of air at 20°C, from HMT DB pg.no 34

ν = 15.06 x 10^{-6} m^2/s

$$Re = \frac{V\,x}{\nu}$$

$$Re = \frac{1 \times 0.1}{15.06 \times 10^{-6}}$$

$$Re = 0557 \times 10^5 < 5 \times 10^5$$

$$hence\ the\ flow\ is\ laminar$$

for laminar flow flat plate from HMT DB pg.no 176, the Sherwood number is given as

$$sh = 0.664\ (Re)^{0.5}\ (Sc)^{0.333}$$

Where,

$$\boldsymbol{Sc = \frac{\nu}{D_{ab}}} = \frac{15.06 \times 10^{-6}}{4.2 \times 10^{-5}}$$

$$\boldsymbol{Sc = 0.358}$$

Therefore

$$sh = 0.664\ (6666.67)^{0.5}\ (0.357)^{0.333}$$

$$sh = 111.37$$

We also know that

$$Sh = \frac{h_m\,L}{D_{ab}}$$

$$= \frac{Sh \times D_{ab}}{L} = \frac{111.37 \times 4.2 \times 10^{-5}}{0.3}$$

$$\boldsymbol{h_m = 0.0155\ m/s}$$

Mass transfer coefficient based on pressure difference is given by,

$$h_{mp} = \frac{h_m}{R\,T_w} = \frac{0.0155}{287 \times 288}$$

$$\boldsymbol{h_{mp} = 1.88 \times 10^{-7}\ m/s}$$

Saturation pressure of water at 15°C, from steam tables

$$p_{w1} = 0.017 \times 10^5 \, bar$$

The evaporation rate of water is given by,

$$m_w = h_{mp} \times A \times [p_{w1} - p_{w2}]$$

$$m_w = \mathbf{1.88} \times \mathbf{10^{-7}} \times (0.3 \times 0.4) \times [0.017 \times 10^5 - 0.007 \times 10^5]$$

$$m_w = 2.25 \times 10^{-5} \, kg/s$$

Mass Transfer

MULTIPLE CHOICE QUESTIONS

Questions	Opt1	Opt2	Opt3	Opt4	Answer
Modes of mass transfer is	conduction	convection	radiation	diffusion	diffusion
Eddy diffusion is the process of	diffusion of fluids in turbulent motion	diffusion of fluids in laminar motion	diffusion of fluids without motion	none of these	diffusion of fluids in turbulent motion
Mass transfer by convection is the process of	transfer between a moving fluid and a surface	transfer between non-moving fluid and a surface	transfer between non-moving fluid and a surface	transfer between two moving fluids	transfer between a moving fluid and a surface
The mass transfer occurs due to simultaneous action of convection and diffusion is known as	heat and mass transfer	mass transfer by change of phase	heat transfer	all of these	mass transfer by change of phase
The number of moles per unit volume of the mixture is known as	molar consentration	mass consentration	mass transfer	mass fraction	molar consentration
The ratio of mass concentration to the total mass density is known as	mass fraction	mole fraction	mass concentration	molar concentration	mass fraction
The velocity of a component relative to the mass-average velocity of the mixture is know as	mass fraction	mass diffusion velocity	mole diffusion velocity	none of these	mass diffusion velocity
The velocity of a component relative to the molar-average velocity of the mixture is know as	molar diffusion velocity	mass diffusion velocity	mass diffusion	molar diffusion	molar diffusion velocity
Mass transfer caused by the existence of different velocities and concentrations is known as	mass transfer	mass fraction	mass diffusion	flux	flux
The evaporation of alcohol is an example of	convetion mass transfer	radiation mass transfer	mass transfer	none of these	convetion mass transfer
Peclet number is	Reynold number/prandtl number	prandtl number/schemidt number	prandtl number/weber number	Reynold number/schemidt number	Reynold number/prandtl number
which of the following provides meximum contact surface for a liquid vapour system	bubble cap tower	packed tower	sieve plant column	wetted wall column	wetted wall column

Mass Transfer

The process separation of solutes of different molecules in a solution using a membrane	Distillation	Leaching	adsorption	Dialysis	Dialysis
Which of the dimensionless number indicates the relative strength of the buoyant to viscous forces	prandtl number	schemidt number	weber number	Reynold number	prandtl number
A high value of Prandtl number indicates	rapid diffusion of momentum by viscous action of energy	relative heat transfer by conduction to convection	rapid heat transfer by forced convectionto natural convection	none of these	rapid diffusion of momentum by viscous action of energy
Which of the following have the same units	Boltzmann constant and plank's constant	plank's constant and stefan's constant	plank's constant and angular momentum	none of these	plank's constant and angular momentum
A diathermous body is one that	diffuses all the thermal rays in the different direction	absorbs all the thermal rays	negligible absorptivity but high refectivity	allows all the incident to pass through it	allows all the incident to pass through it
The dimensional number correlates the thickness of the hydrodynamic and thermal boundary layer	prandtl number	Nusselt number	crash off number	Mach number	prandtl number
In case of solids the heat transfer takes place according to	radiation	conduction	convection	diffusion	conduction
The overall heat co-efficients of heat transfer is used in the problems of	convection	conduction	convection and conduction	diffusion	convection and conduction
The heat of sun reaches to us according to	radiation	conduction	convection	none of these	radiation
In an air conditioning plant a cooling tower is used to cool	steam	condensed water	circulating	none of these	condensed water
In a thermal power plant cooling towers are used to cool	air	refrigerant	steam	circulating water	circulating water
Which of the following can be used to measure a temperature around -50'c	standard thermometer	radiation thermometer	Alcohol thermometer	thermocouple	Alcohol thermometer
All temperatures above the freezing point of gold are usually determined by	thermocouple	optical pyrometer	resistance bridge	gas thermometer	optical pyrometer
In constant volume hydrogen gas thermometer a correction has to applied because	hydrogen is impure	expansion of hydrogen is non-linear	density of mercury changes	none of these	expansion of hydrogen is non-linear

A thermistor is a	thermo-couple	thermometer	thermal resistance	miniature resistance	miniature resistance