KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Pollachi main Road, Coimbatore-641021

DEPARTMENT OF CIVIL ENGINEERING

SYLLABUS

15BECE8E02 /SEISMIC DESIGN OF REINFORCED CONCRETE STRUCTURES

4 0 0 4 100

UNIT I **12**

Single Degree Of Freedom Systems: Formulation of equation of motion, Free and forced vibrations, Damping, Types of Damping – Damped and undamped vibrations, Response to dynamic loading. Introduction of Free and forced vibration of undamped and damped MDOF systems

UNIT II **12**

Engineering Seismology: Elements of Engineering Seismology, Characteristics of Earthquake Engineering, Earthquake History, Indian Seismicity. Performance of structures under past earthquakes, Lessons learnt from past earthquakes

UNIT III **12**

Seismic Analysis: Seismic Design Concepts- Calculation of base shear as per IS1893- Lateral Load analysis of building frames by Portal method and Cantilever method

UNIT IV **12**

Earthquake Resistant Design: Concept of Earthquake Resistant Design, Provisions of Seismic Code IS 1893 (Part I), Response Spectrum, Design Spectrum, Design of Buildings

UNIT V **12**

Ductile Detailing: Ductility- Assessment of Ductility- Member/ Element ductility, Structural Ductility- Factors affecting ductility-Ductile Detailing, Provisions of IS 13920.for beams, columns and footings- Special Confining Requirements

Total: 45 Hrs+15 Hrs Tutorial

TEXT BOOKS:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Earthquake Resistant Design of Structures,	Agarwal and Shrikhande	Prentice Hall of India	2007

REFERENCES:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Structural Dynamics – Theory and Computations, Third Edition	Mario Paz	CBS publishers	2007
2	Design of Earthquake Resistant Buildings	AgarwalPankaj and Shrikhande Manish	Mc- Graw Hill Book Company, New York	2006
3	Dynamics of Structures	Humar J	Prentice Hall	2012
4	Dynamics of structures – Theory and applications to Earthquake Engineering	Anil K Chopra	Prentice Hall Inc	2001
5	Earthquake Tips	C V R Moorthy	NICEE,IIT Kanpur	2004
6	Dynamics of Structures, Second Edition	Clough R.W, and Penzien J,	McGraw – Hill International Edition	*2003*

LIST OF WEBSITES

- http://www.icivilengineer.com
- http://www.engineeringcivil.com/
- http://www.aboutcivil.com/
- http://www.engineersdaily.com
- http://www.asce.org/
- http://www.cif.org/
- http://icevirtuallibrary.com/
- http://www.ice.org.uk/
- http://www.engineering-software.com/ce/
- http://nptel.ac.in/

Staff In charge HOD/CIVIL

LECTURE PLAN

Lecturer : Mr. V. Johnpaul
Semester : VII
Subject Code : **15BECE8E02**
Subject Name : Seismic Design of Reinforced Concrete Structures
No of Credits : 4

Sl.No	Hours	Topics	Book	Page No
		UNIT I-Single Degree Of Freedom Systems		
1.	1	Introduction to Seismic Design- Vibration, Loading, Degrees of Freedom	T1	111,191-193
2.	2	Formulation of equation of motion (Introduction, Vibration Analysis, Mathematical modeling of an SDOF System)	T1	112,115-119
3.	2	Derivation of Equation of Motion	T1	115,260
4.	1	Tutorial-1		
5.	2	Free and forced vibrations	T1	116-126
6.	2	Damping, Types of Damping	www.nptel.ac.in	
7.	1	Response to dynamic loading	springer.com	
8.	1	Introduction of Free and forced vibration of undamped and damped MDOF systems	T1	162-163
9.	1	Tutorial-2		
			Total	13
		UNIT II - Engineering Seismology		
10.	1	Elements of Engineering Seismology	T1	38-41
11.	3	Characteristics of Earthquake Engineering(Elastic Rebound theory, Plate tectonics, seismic waves, Earthquake size, magnitude, Intensity)	T1	3-12
12.	2	Earthquake History	T1	Wikipedia.com
13.	1	Tutorial-3		
14.	1	Indian Seismicity	T1	34-38
15.	1	Performance of structures under past earthquakes	T1	nptel.ac.in
16.	1	Lessons learnt from past earthquakes	T1	nptel.ac.in
17.	1	Tutorial-4		
			Total	11
		UNIT III - Seismic Analysis		
18.	1	Introduction to the IS codal provision	IS1893:2002	
19.	2	Seismic Design Concepts	IS1893:2002	
20.	2	Calculation of base shear as per IS1893:2002	T1	256-259 & IS1893
21.	1	Tutorial-5		
22.	3	Lateral Load analysis of building frames by Portal method	T1	407-418
23.	3	Lateral Load analysis of building frames by Cantilever	T1	410-412

		method		
24.	1	Tutorial-6		
			Total	12
UNIT IV- Earthquake Resistant Design				
25.	1	Concept of Earthquake Resistant Design	T1	IS1893:2002
26.	3	Provisions of Seismic Code IS 1893 (Part I)	T1	IS1893:2002
27.	1	Tutorial-7		
28.	3	Response Spectrum	T1	IS1893:2002
29.	2	Design Spectrum	T1	101-105
30.	1	Design of Buildings	T1	nptel.ac.in
31.	1	Tutorial-8		
			Total	12
UNIT V- Ductile Detailing				
32.	2	Ductility	T1	244-245,341-342
33.	1	Assessment of Ductility	T1	342-343
34.	1	Member/ Element ductility	T1	343-345
35.	1	Structural Ductility	T1	345-346
36.	1	Factors affecting ductility	T1	346-347
37.	1	Tutorial-9		
38.	3	Ductile Detailing(IS 13920:1993)- Provisions of IS 13920.for beams, columns and footings	T1	348-358
39.	2	Special Confining Requirements	T1	358-362
40.	1	Tutorial-10		
			Total	12

Total:60 Hours

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Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Earthquake Resistant Design of Structures,	Agarwal and Shrikhande	Prentice Hall of India	2007

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6	Dynamics of Structures, Second Edition	Clough R.W, and Penzien J,	McGraw – Hill International Edition	*2003*
7	Reinforced Concrete Structures-Vol-II	Dr.B.C.Punmia, Ashok K. Jain, ArunK.jain	Laxmi Publications (P) Ltd	2011

Dr.N.Balasundaram **Dr.D.Lakshmanan**

Concepts from vibrations

NEWTON'S LAWS

First law:

If there are no forces acting upon a particle, then the particle will move in a straight line with constant velocity.

Second law:

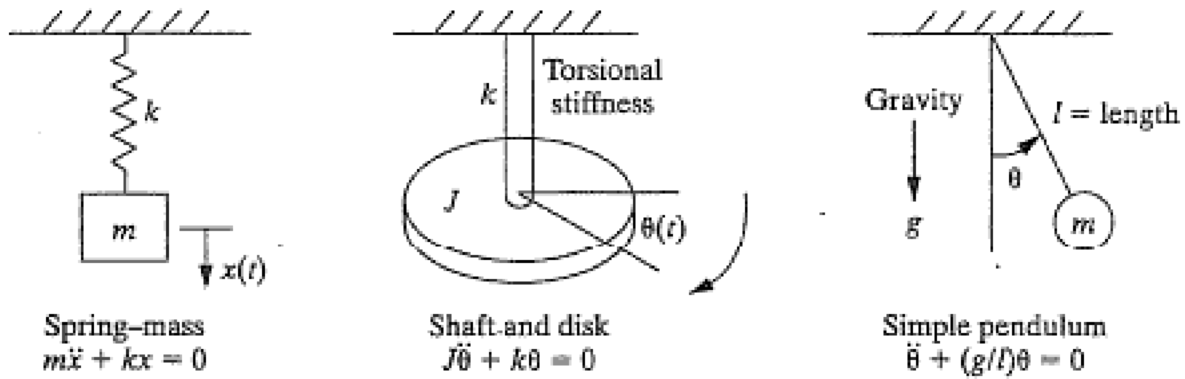
A particle acted upon by a force moves so that the force vector is equal to the time rate of change of the linear momentum vector.

Third law:

When two particles exert forces upon one another, the forces lie along the line joining the particles and the corresponding force vectors are the negative of each other.

Definition

- The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time defines the degree of freedom of the system. **A single degree of freedom system** requires only one coordinate to describe its position at any instant of time.

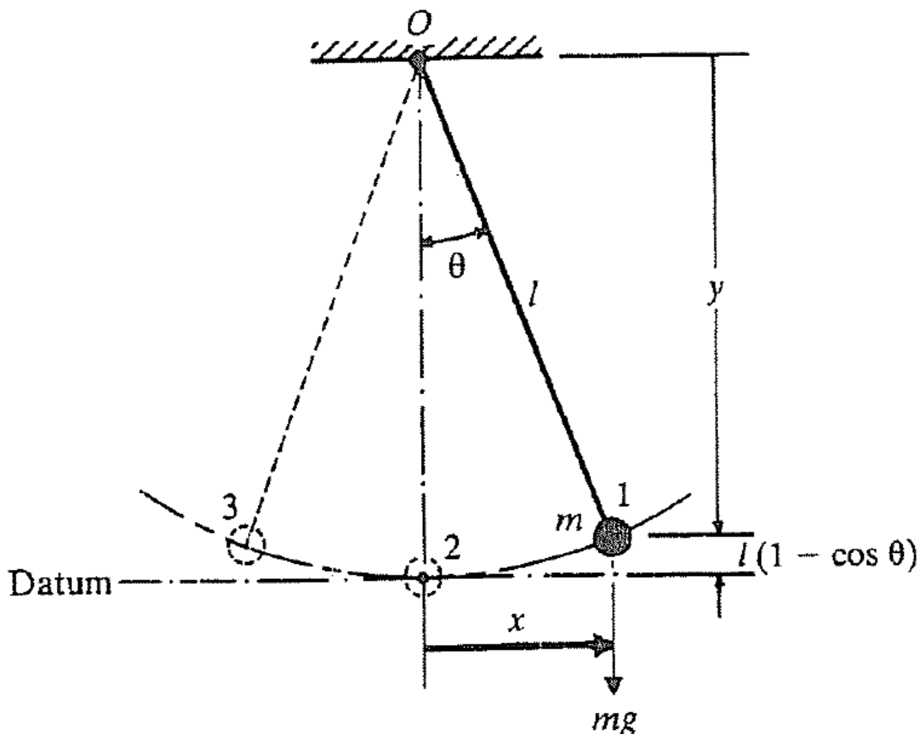


Single degree of freedom system

- For the simple pendulum in the figure, the motion can be stated either in terms of θ or x and y. If the coordinates x and y are used to describe the motion, it must be recognized that these coordinates are not independent. They are related to each other through the relation

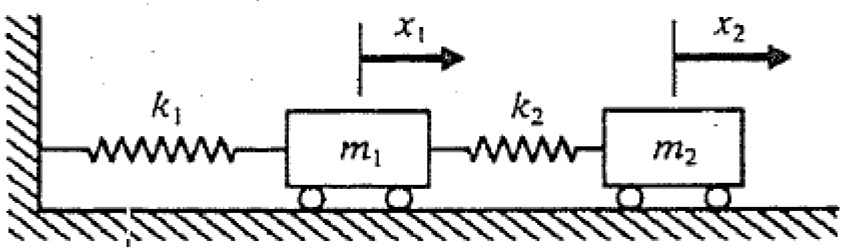
$$x^2 + y^2 = l^2$$

where l is the constant length of the pendulum. Thus any one coordinate can describe the motion of the pendulum. In this example, we find that the choice of θ as the independent coordinate will be more convenient than the choice of x and y.

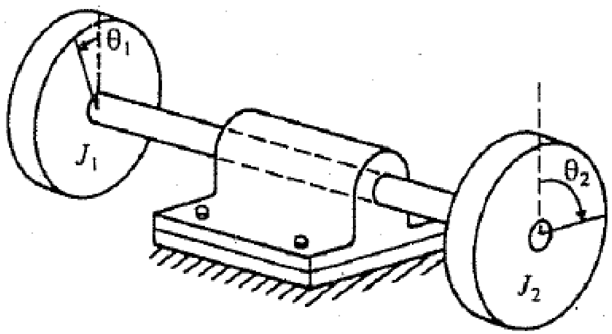


Two degree of freedom system

- Some examples of two degree of freedom systems are shown in the figure. The first figure shows a two mass – two spring system that is described by two linear coordinates x_1 and x_2. The second figure denotes a two rotor system whose motion can be specified in terms of $\theta 1$ and $\theta 2$. The motion of the system in the third figure can be described completely either by X and θ or by x,y and X.



(a)



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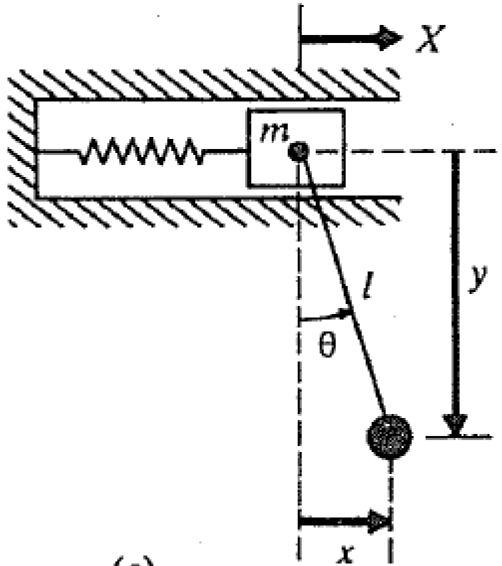
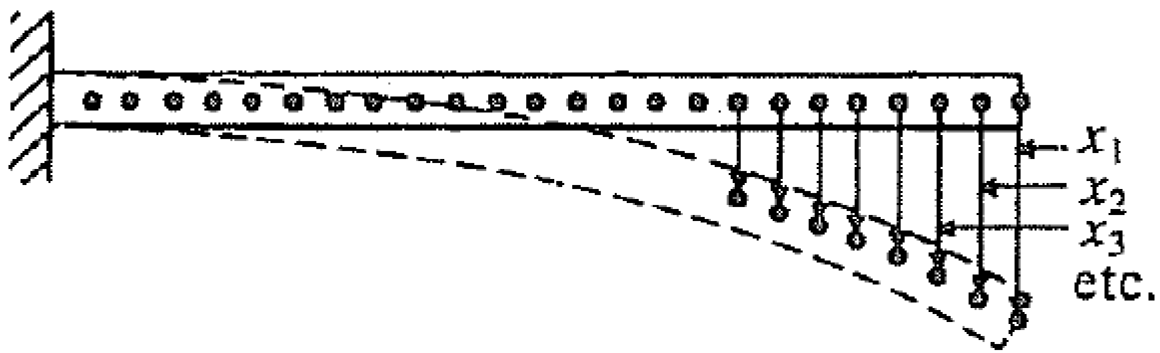


(c)

Discrete and continuous systems

- A large number of practical systems can be described using a finite number of degrees of freedom, such as the simple system shown in the previous slides.

- Some systems, especially those involving continuous elastic members, have an infinite number of degrees of freedom as shown in the figure. Since the beam in the figure has an infinite number of mass points, we need an infinite number of coordinates to specify its deflected configuration. The infinite number of coordinates defines its elastic deflection curve. Thus, the cantilever beam has infinite number of degrees of freedom.



Discrete and continuous systems

- Systems with a finite number of degrees of freedom are called **discrete** or **lumped parameter systems**, and those with an infinite number of degrees of freedom are called **continuous** or **distributed systems**.

- Most of the time, continuous systems are approximated as discrete systems, and solutions are obtained in a simple manner. Although treatment of a system as continuous gives exact results, the analytical methods available for dealing with continuous systems are limited to a narrow selection of problems, such as uniform beams, slender rods and thin plates.

- Hence, most of the practical systems are studied by treating them as finite lumped masses, springs and dampers. In general, more accurate results are obtained by increasing the number of masses, springs and dampers- that is by increasing the number of degrees of freedom.

Classification of vibration

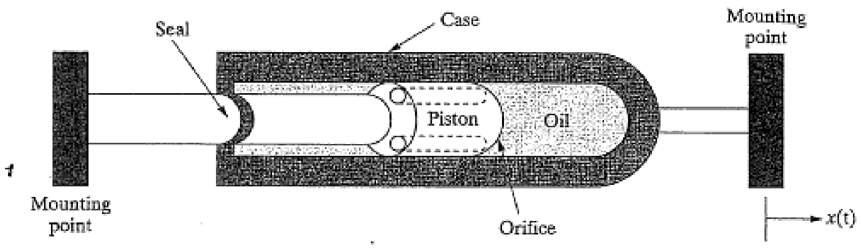
- **Free vibration:** If a system, after an initial disturbance is left to vibrate on its own, the ensuing vibration is known as free vibration. No external force acts on the system. The oscillation of a simple pendulum is an example of free vibration.

- **Forced vibration:** If a system is subjected to an external force (often a repeating type of force), the resulting vibration is known as forced vibration.

  - If the frequency of the external force coincides with one of the natural frequencies of the system, a condition known as resonance occurs, and the system undergoes dangerously large oscillations. Failures of such structures as buildings, bridges, turbines, and airplane wings have been assoicated with then occurrence of resonance.

Classification of vibration

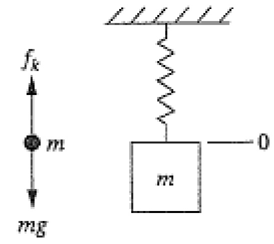
- **Undamped vibration:** If no energy is lost or dissipated in friction or other resistance during oscillation, the vibration is known as undamped vibration.
- If any energy is lost in this way however, it is called **damped vibration**.

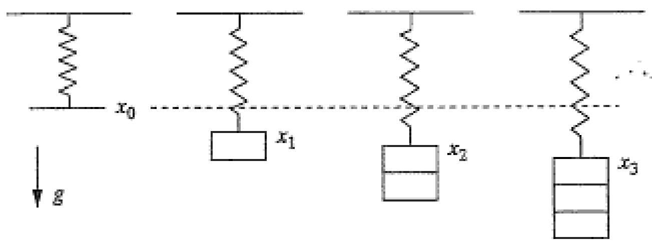


- While the spring forms a physical model for storing kinetic energy and hence causing vibration, the dashpot, or damper, forms the physical model for dissipating energy and damping the response of a mechanical system. A dashpot consists of a piston fit into a cylinder filled with oil. This piston is perforated with holes so that motion of the piston in the oil is possible. The laminar flow of the oil through the perforations as the piston moves causes a damping force on the piston.

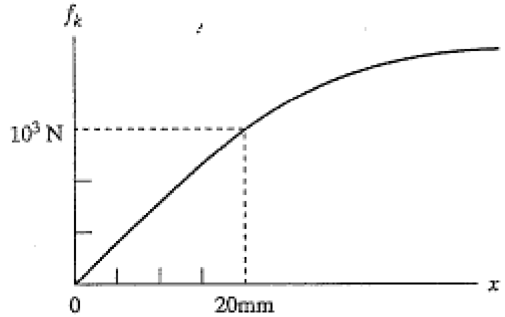
Classification of vibration

- **Linear vibration:** If all the basic components of a vibratory system-the spring, the mass, and the damper, behave linearly, the resulting vibration is known as linear vibration. The differential equations that govern the behaviour of vibratory linear systems are linear. Therefore, the principle of superposition holds.





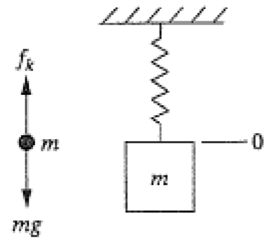
- **Nonlinear vibration:** If however, any of the basic components behave nonlinearly, the vibration is called '**nonlinear vibration**'. The differential equations that govern the behaviour of vibratory non-linear systems are non-linear. Therefore, the principle of superposition does not hold.

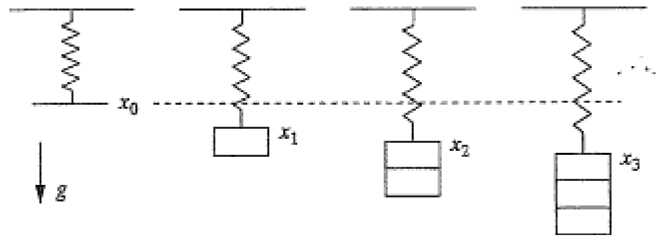


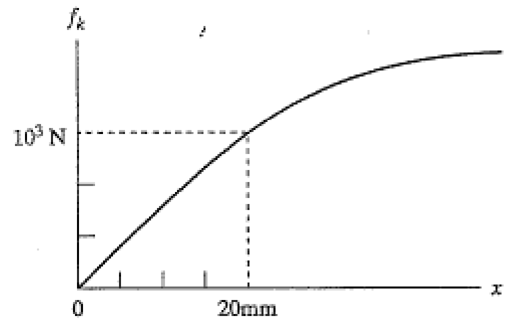
Classification of vibration

Linear and nonlinear vibrations contd:

- The nature of the spring force can be deduced by performing a simple static experiment. With no mass attached, the spring stretches to a position labeled as $x_o=0$ in the figure.
- As successively more mass is attached to the spring, the force of gravity causes the spring to stretch further. If the value of the mass is recorded, along with the value of the displacement of the end of the spring each time more mass is added, the plot of the force (mass denoted by m, times the acceleration due to gravity, denoted by g), versus this displacement denoted by x, yields a curve similar to that shown in the figure.





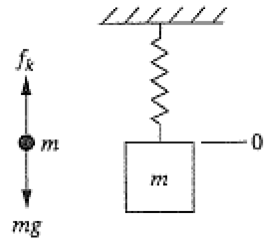


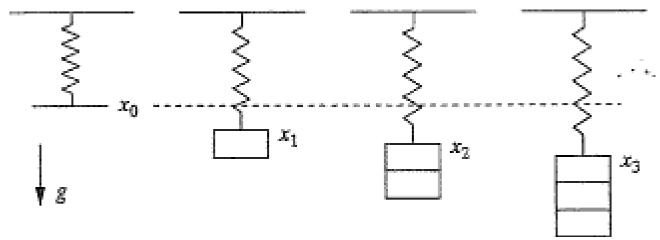
Classification of vibration

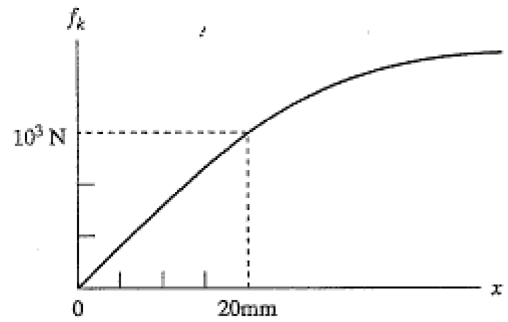
Linear and nonlinear vibrations contd:

- Note that in the region of values for x between 0 and about 20 mm, the curve is a straight line. This indicates that for deflections less than 20 mm and forces less than 1000 N, the force that is applied by the spring to the mass is proportional to the stretch of the spring.

- The constant of proportionality is the slope of the straight line.



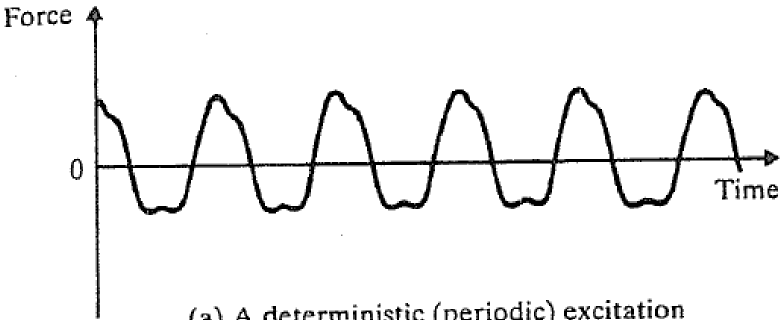




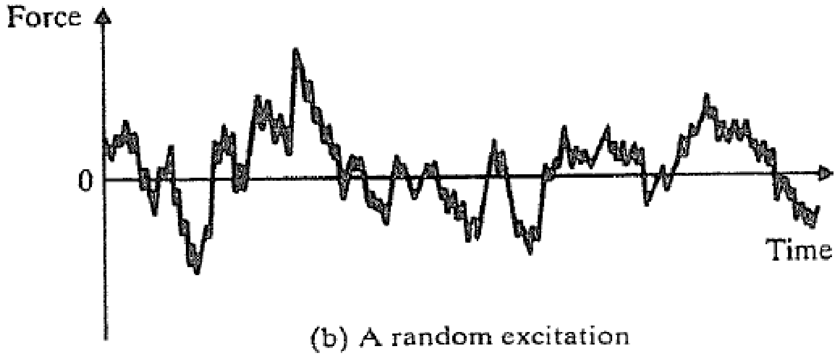
Classification of vibration

- **Deterministic vibration:** If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time, the excitation is called '**deterministic**'. The resulting vibration is known as '**deterministic vibration**'.

- **Nondeterministic vibration:** In some cases, the excitation is **non-deterministic** or **random**; the value of excitation at a given time cannot be predicted. In these cases, a large collection of records of the excitation may exhibit some statistical regularity. It is possible to estimate averages such as the mean and mean square values of the excitation.



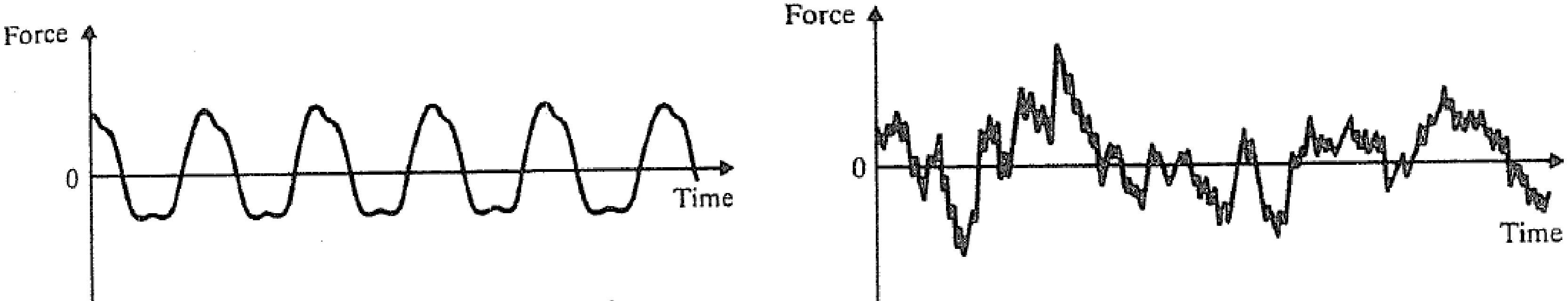
(a) A deterministic (periodic) excitation



(b) A random excitation

Classification of vibration

- Examples of random excitations are wind velocity, road roughness, and ground motion during earthquakes.
- If the excitation is random, the resulting vibration is called **random vibration**. In the case of random vibration, the **vibratory response** of the system is also **random**: it can be described only in terms of statistical quantities.



(a) A deterministic (periodic) excitation

(b) A random excitation

Free Vibration of Single Degree of Freedom Systems

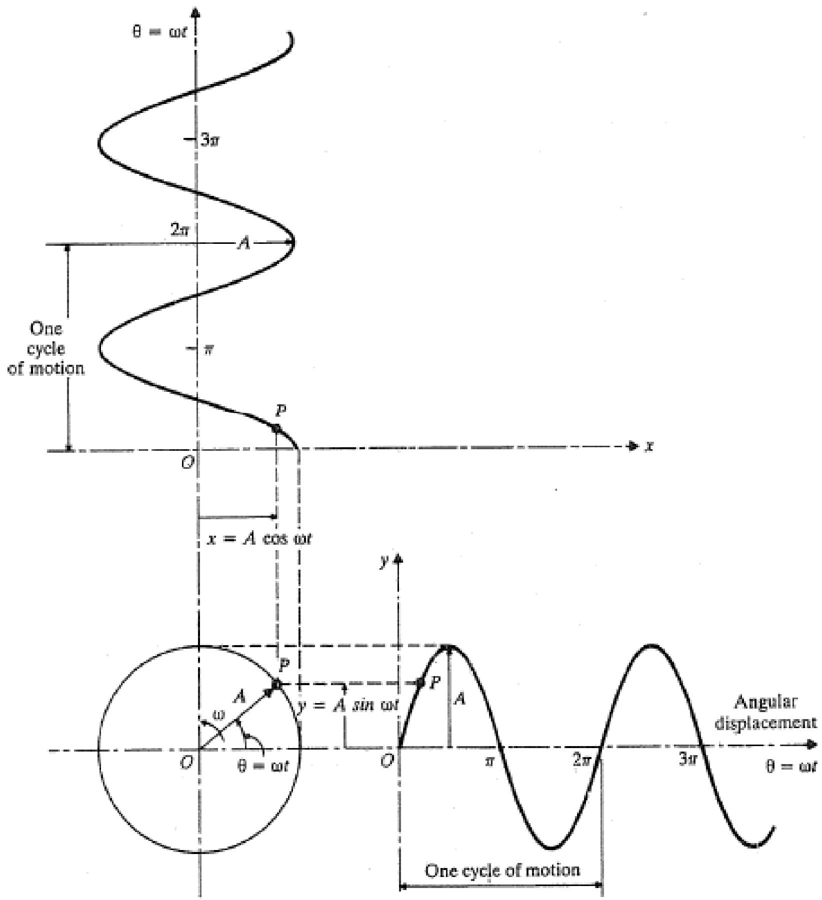
- Harmonic Motion
- Free vibration of undamped SDOF systems
- Free vibration of damped SDOF systems

Harmonic motion

- Oscillatory motion may repeat itself regularly, as in the case of a simple pendulum, or it may display considerable irregularity, as in the case of ground motion during an earthquake.
- If the motion is repeated after equal intervals of time, it is called **periodic motion**.
- The simplest type of **periodic motion** is **harmonic motion**.

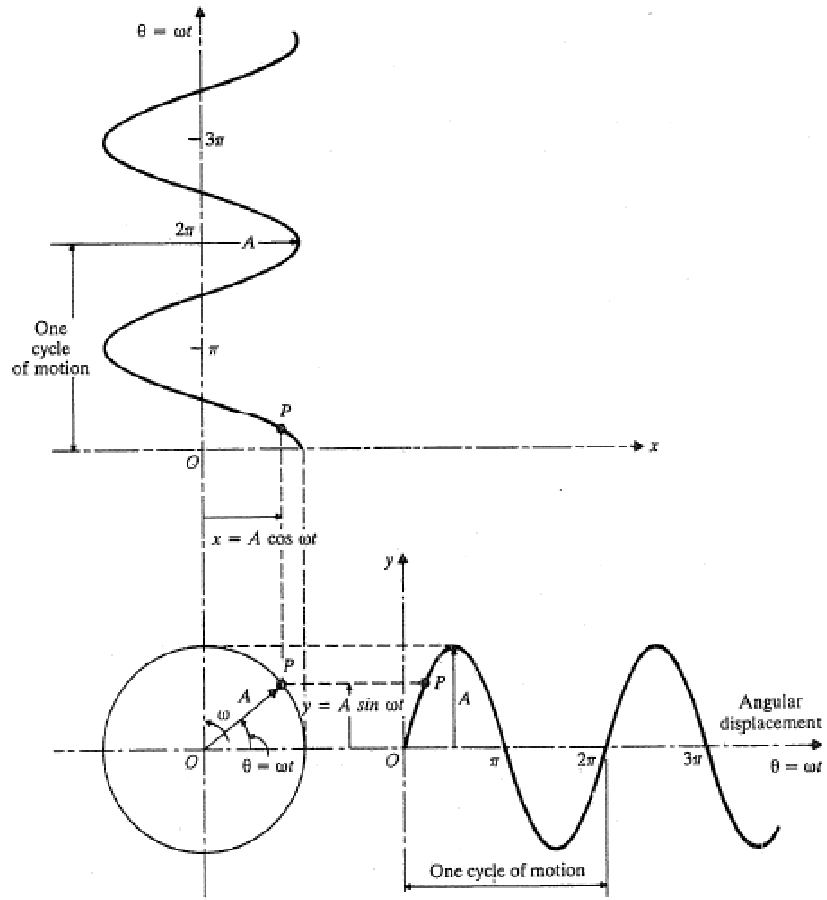
Harmonic motion

- Shown in the figure is a vector OP that rotates counterclockwise with constant angular velocity ω.
- At any time t, the angle that OP makes with the horizontal is $\theta=\omega t$.
- Let y be the projection of OP on the vertical axis. Then y=A sin ωt. Here y, a function of time is plotted versus ωt.



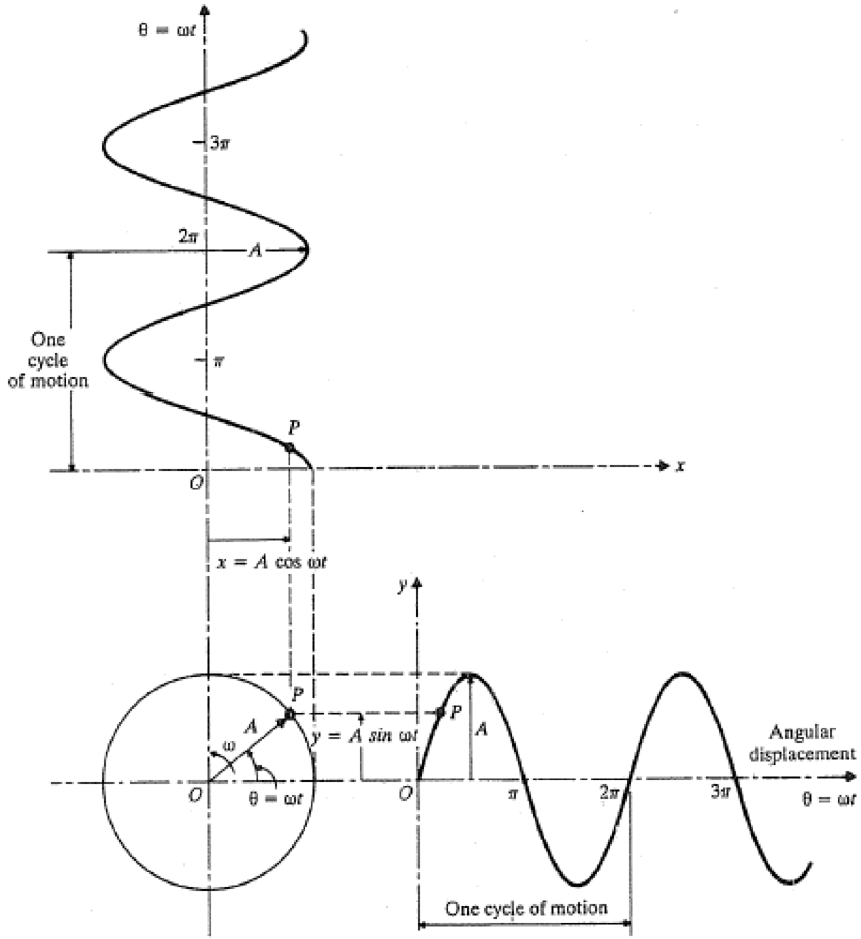
Harmonic motion

- A particle that experiences this motion is said to have **harmonic motion**.
- The maximum displacement of a vibrating body from its equilibrium position is called the **amplitude of vibration**. Amplitude A is shown in the figure.
- Range 2A is the peak to peak displacement.
- Now consider the units of θ. Let C be the circumference of the circle shown in the figure.



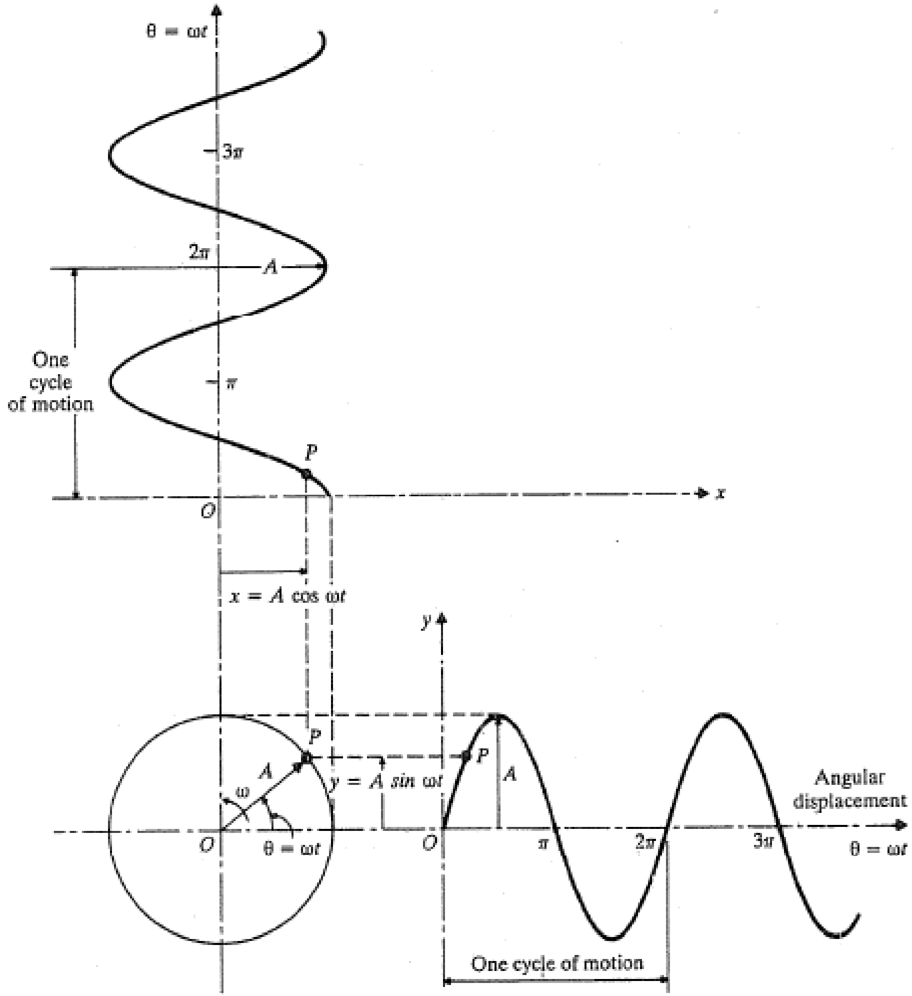
Harmonic motion

- Thus $C=2\pi A$. Or we can write $C=A\theta$, where $\theta=2\pi$ for one revolution. Thus defined, θ is said to be in radians and is equivalent to 360°. Therefore, one radian is approximately equal to 58.3°.
- In general, for any arc length,
  $s=A\,\theta$, where θ is in radians. It follows that ω in the figure would be in radians per second.
- As seen in the figure, the vectorial method of representing harmonic motion requires the description of both the horizontal and vertical components.
- The time taken to complete one cycle of motion is known as the period of oscillation or time period and is denoted by τ. The period is the time for the motion to repeat (the value of τ in the figure).



Harmonic motion

- Note that $\omega\ \tau = 2\ \pi$ where ω denotes the angular velocity of the cyclic motion. The angular velocity ω is also called the **circular frequency**.
- The movement of a vibrating body from its undisturbed or equilibrium position to its extreme position in one direction, then to the equilibrium position, then to its extreme position in the other direction, and back to equilibrium position is called **a cycle of vibration**.
- One revolution (i.e., angular displacement of 2π radians) of the pin P in the figure or one revolution of the vector OP in the figure constitutes a **cycle**. Cycle is the motion in one period, as shown in the figure.



Harmonic motion

- Frequency is the number of cycles per unit time.
- The most common unit of time used in vibration analysis is seconds. Cycles per second is called Hertz.
- The time the cycle takes to repat itself is the period T. In terms of the period, the frequency is:

$$f = \frac{1}{\tau}$$

- The frequency f is related to ω: $f = \frac{\omega}{2\pi}$

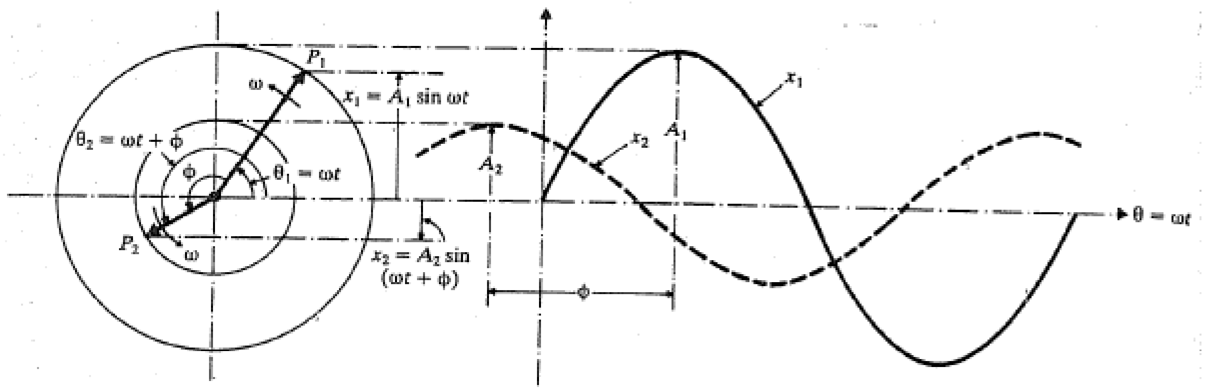
$$\omega = 2\pi f$$

Harmonic motion

- Phase angle: Consider two vibratory motions denoted by:

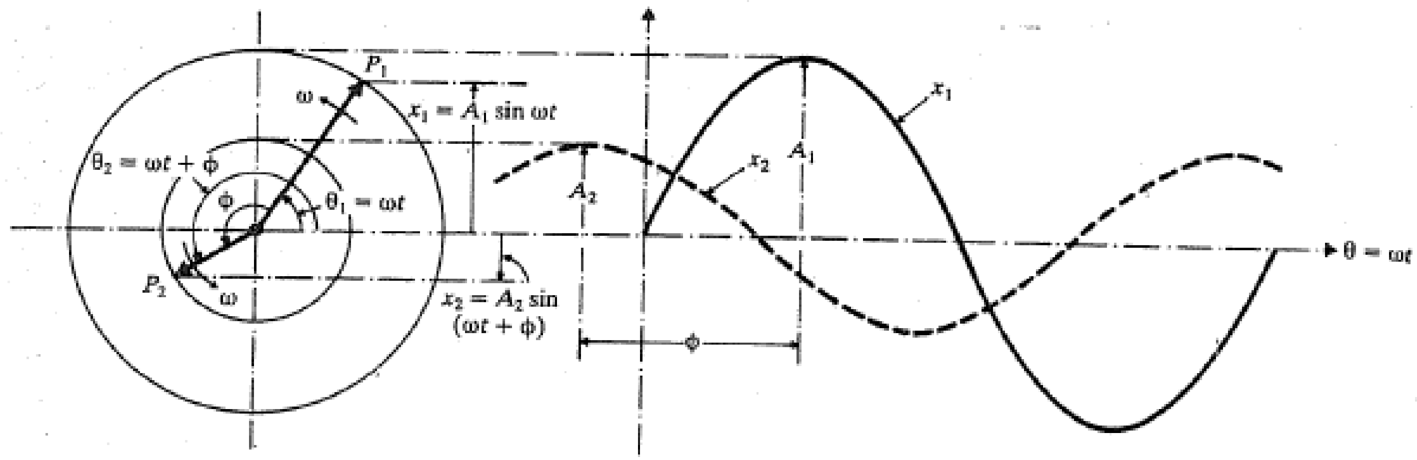
$$x_1 = A_1 \sin \omega t$$
$$x_2 = A_2 \sin(\omega t + \phi)$$

- These two harmonic motions are called **synchronous** because they have the same frequency or angular velocity ω. Two synchronous oscillations need not have the same amplitude, and they need not attain their maximum values at the same time as shown in the figure.



Harmonic motion

- In this figure, the second vector OP_2 leads the first one OP_1 by an angle ϕ known as the **phase angle**. This means that the maximum of the second vector would occur ϕ radians earlier than that of the first vector. These two vectors are said to have a phase difference of ϕ.



Harmonic motion

- From introductory physics and dynamics, the fundamental kinematical quantities used to describe the motion of a particle are displacement, velocity and acceleration vectors.
- The acceleration of a particle is given by:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$$

- Thus, displacement, velocity, and acceleration have the following relationships in harmonic motion:

$$x = A\sin\omega t$$

$$v = \dot{x} = A\omega\cos\omega t$$

$$a = \ddot{x} = -A\omega^2\sin\omega t$$

Operations on harmonic functions

- Using complex number representation, the rotating vector $\vec{X}$ can be written as:

$$\vec{X} = A\,e^{i\omega t}$$

where ω denotes the circular frequency (rad/sec) of rotation of the vector $\vec{X}$ in counterclockwise direction. The differentiation of the harmonics given by the above equation gives:

$$\frac{d\vec{X}}{dt} = \frac{d}{dt}(Ae^{i\omega t}) = i\omega A e^{i\omega t} = i\omega\vec{X}$$

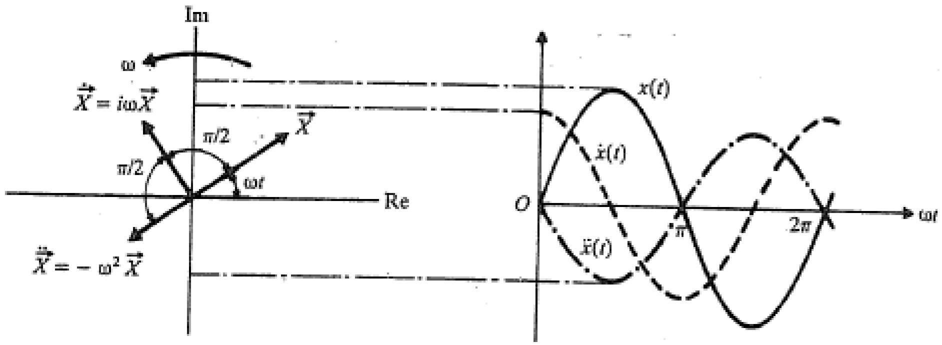
$$\frac{d^2\vec{X}}{dt^2} = \frac{d}{dt}(i\omega A e^{i\omega t}) = -\omega^2 A e^{i\omega t} = -\omega^2\vec{X}$$

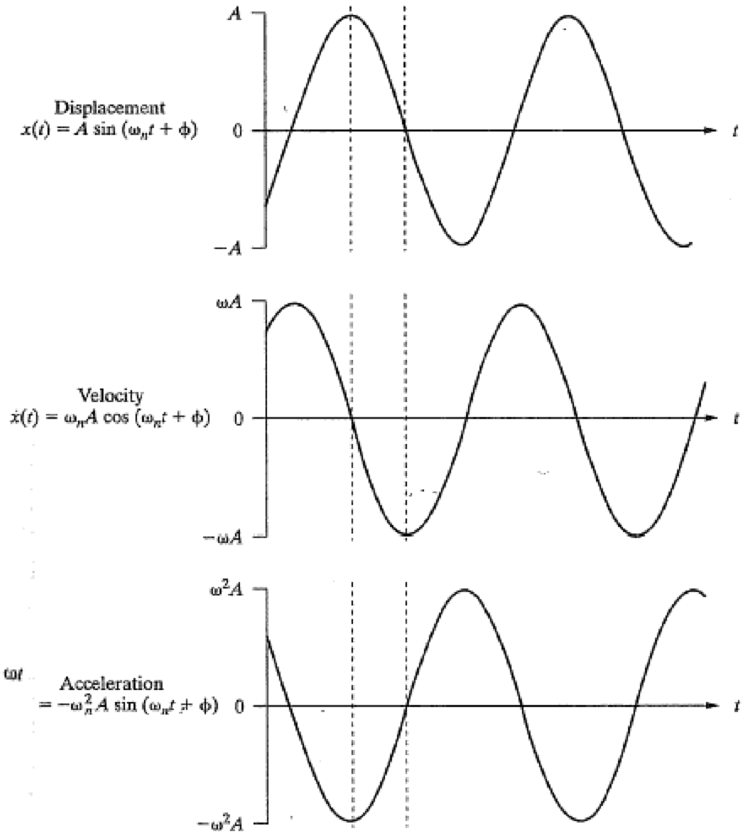
- Thus the displacement, velocity and acceleration can be expressed as:

$$\begin{aligned}
\text{displacement} &= \mathrm{Re}[Ae^{i\omega t}] &&= A\cos\omega t\\
\text{velocity} &= \mathrm{Re}[i\omega A e^{i\omega t}] &&= -\omega A\sin\omega t\\
& &&= \omega A\cos(\omega t + 90^\circ)\\
\text{acceleration} &= \mathrm{Re}[-\omega^2 A e^{i\omega t}] &&= -\omega^2 A\cos\omega t\\
& &&= \omega^2 A\cos(\omega t + 180^\circ)
\end{aligned}$$

Operations on harmonic functions

- It can be seen that the acceleration vector leads the velocity vector by 90 degrees and the velocity vector leads the displacement vector by 90 degrees.





Harmonic motion

- **Natural frequency:** If a system, after an initial disturbance, is left to vibrate on its own, the frequency with which it oscillates without external forces is known as its **natural frequency**. As will be seen, a vibratory system having n degrees of freedom will have, in general, n distinct **natural frequencies of vibration**.
- **Beats:** When two harmonic motions, with frequencies close to one another, are added, the resulting motion exhibits a phenomenon known as **beats**. For example if:

$$x_1(t) = X\cos\omega t$$
$$x_2(t) = X\cos(\omega+\delta)t$$

where δ is a small quantity.

The addition of these two motions yield:

$$x(t) = x_1(t) + x_2(t) = X[\cos\omega t + \cos(\omega+\delta)t]$$

Harmonic motion

Beats:

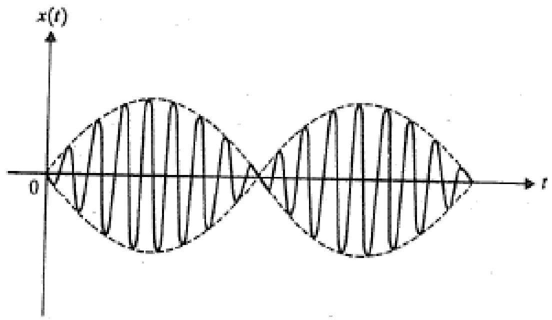
$$x(t) = x_1(t) + x_2(t) = X[\cos \omega t + \cos(\omega + \delta)t]$$

Using the relation

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

The first equation can be written as:

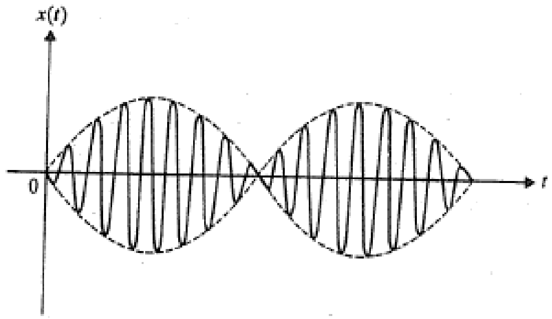
$$x(t) = 2X\cos\frac{\delta t}{2}\cos(\omega + \frac{\delta}{2})t$$



Harmonic motion

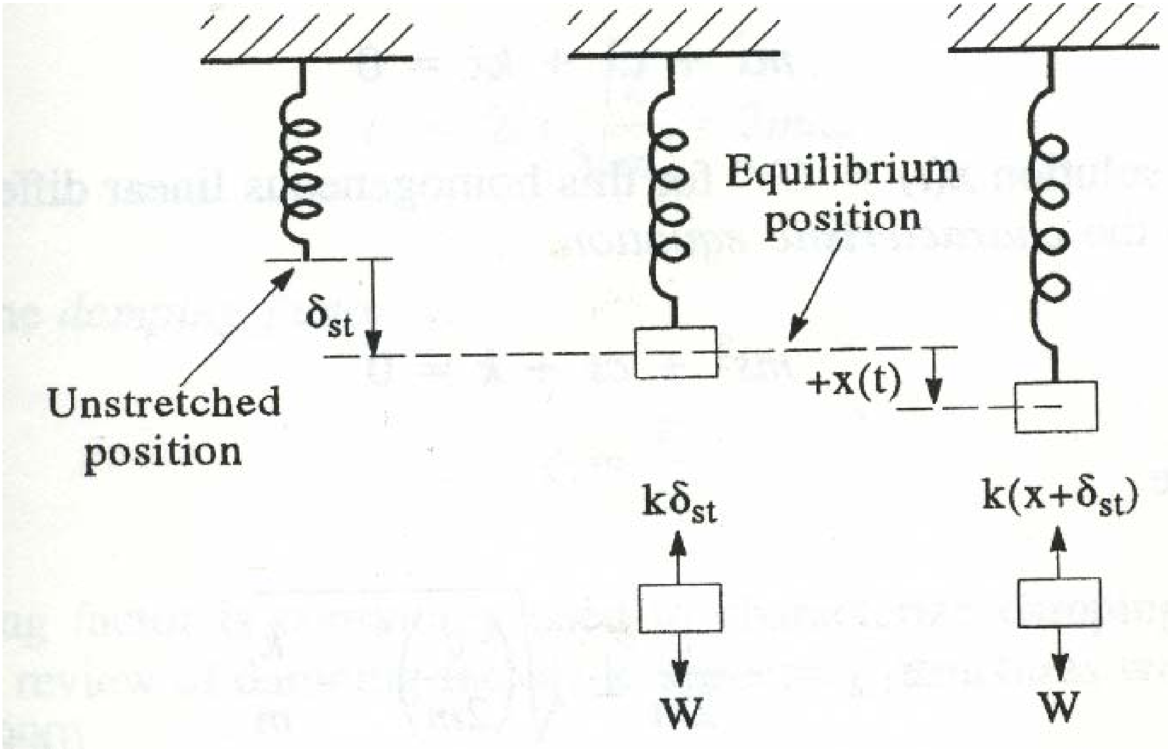
Beats:

- It can be seen that the resulting motion x(t) represents a cosine wave with frequency $\omega+\frac{\delta}{2}$ which is approximately equal to ω and with a varying amplitude $2X\cos\frac{\delta t}{2}$. Whenever, the amplitude reaches a maximum it is called a beat.
- In machines and in structures,the beating phenomenon occurs when the forcing frequency is close to the natural frequency of the system. We will later return to this topic.



Free vibration of undamped SDOF systems

- The stiffness in a spring can be related more directly to material and geometric properties of the spring. A spring like behaviour results from a variety of configurations, including longitudinal motion (vibration in the direction of the length), transverse motion (vibration perpendicular to the length), and torsional motion(vibration rotating around the length).

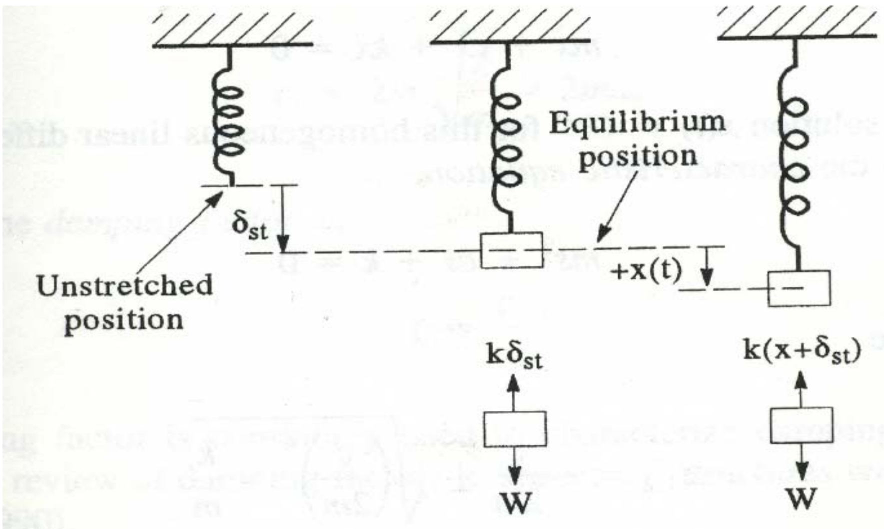


Free vibration of undamped SDOF systems

- A spring is generally made of an elastic material. For a slender elastic material of length l, cross-sectional area A and elastic modulus E (or Young's modulus), the stiffness of the bar for vibration along its length is given by:

$$k = \frac{EA}{l}$$

- The modulus E has the units of Pascal (denoted Pa) which are N/m2.



Free vibration of undamped SDOF systems

- When the mass m (weight W) is applied, the spring will deflect to a static equilibrium position $\delta_{st.}$

- At this position, we find that:

$$W = mg = k\delta_{st}$$

- If the mass is perturbed and allowed to move dynamically, the displacement *x*, measured from the equilibrium position, will be a function of time. Here, x(t) is the absolute motion of the mass and the force in the spring can be expresssed as:

$$-k(x+\delta_{st})$$

- To determine the position as a function of time, the equations of motion are employed; the free body diagrams are drawn as shown in the figure. Note that x is measured positive downward.

Free vibration of undamped SDOF systems

- Applying Newton's second law,

$$W - k(x + \delta_{st}) = m\ddot{x}$$

- But from the static condition, note that W=k$\delta_{st.}$ Thus, the equation of motion becomes:

$$m\ddot{x} + kx = 0$$

- With the standard form of: $$\ddot{x} + \frac{k}{m}x = 0$$

- This is Case III that has complex roots where the general solution was computed as:

$$x = e^{-ax/2}(A\cos\omega t + B\sin\omega t)$$

- Since in the above equation:

$$a = 0 \qquad \omega^2 = b - \frac{1}{4}a^2$$

Free vibration of undamped SDOF systems

- It is shown that

$$x(t) = A\cos\omega_n t + B\sin\omega_n t$$

where A and B are constants of integration and

$$\omega_n = \sqrt{\frac{k}{m}} \qquad (rad/\sec)$$

Here ω_n defines the natural frequency of the mass. This is the frequency at which the mass will move regardless of the amplitude of the motion as long as the spring in the system continues to obey Hooke's law. The natural frequency in Hertz is:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad (Hz)$$

Free vibration of undamped SDOF systems

- The initial conditions $x = x_o$ at t=0 and $\dot{x} = \dot{x}_o$ at $t = 0$ are used to evaluate the constants of integration A and B. When substituted into the equation

$$x(t) = A\cos\omega_n t + B\sin\omega_n t$$

we get:

$A = x_o$ and $B = \dfrac{\dot{x}_o}{\omega_n}$ and consequently $x(t) = x_o \cos\omega_n t + \dfrac{\dot{x}_o}{\omega_n}\sin\omega_n t$

- The sum in the equation

$$x(t) = A\cos\omega_n t + B\sin\omega_n t$$

can also be combined to a phase shifted cosine with amplitude $C = \sqrt{A^2 + B^2}$ and phase angle ϕ=arctan(B/A). For this purpose let:

$$A = C\cos\phi \text{ and } \mathrm{B} = C\sin\phi$$

Free vibration of undamped SDOF systems

- Introducing the new values of A and B into

$$x(t) = A\cos\omega_n t + B\sin\omega_n t$$

we get:

$$x(t) = C\cos\phi\cos\omega_n t + C\sin\phi\sin\omega_n t$$

- Since

$$\cos(x-y) = \cos x\cos y + \sin x\sin y$$

- x(t) can be expressed as:

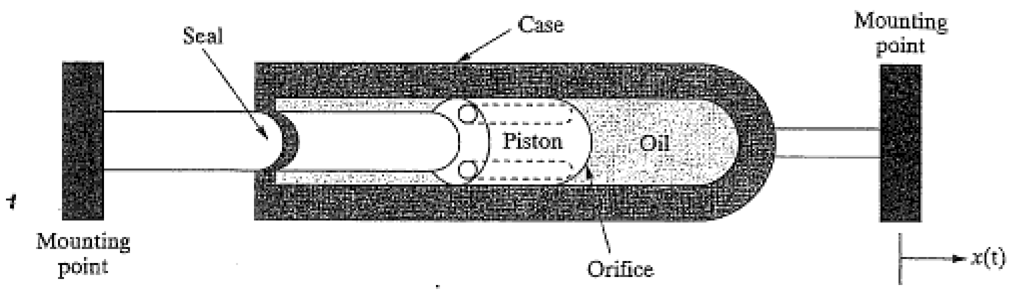
$$x(t) = C\cos(\omega_n t - \phi)$$

- Where ϕ=arctan(B/A) and consequently:

$$\phi = \arctan\left(\frac{\dot{x}_o}{\omega_n x_o}\right) \text{ and } C = \sqrt{A^2 + B^2} = \sqrt{x_o^2 + \left(\frac{\dot{x}_o}{\omega_n}\right)^2}$$

Damping

- **Undamped and damped vibration:** The response of a spring-mass model predicts that the system will oscillate indefinitely. However, everyday observation indicates that most freely oscillating systems eventually die out and reduce to zero motion.
- The choice of representative model for the observed decay in an oscillating system is based partially on physical observation and partially on mathematical convenience. The theory of differential equations suggests that adding a term to equation $m\ddot{x}(t)+kx(t)=0$ of the form $c\dot{x}$, where c is a constant, will result in a solution x(t) that dies out.
- Physical observation agrees fair well with this model and it is used very successfully to model the damping or decay in a variety of mechanical systems.
- This type of damping is called the **viscous damping**.

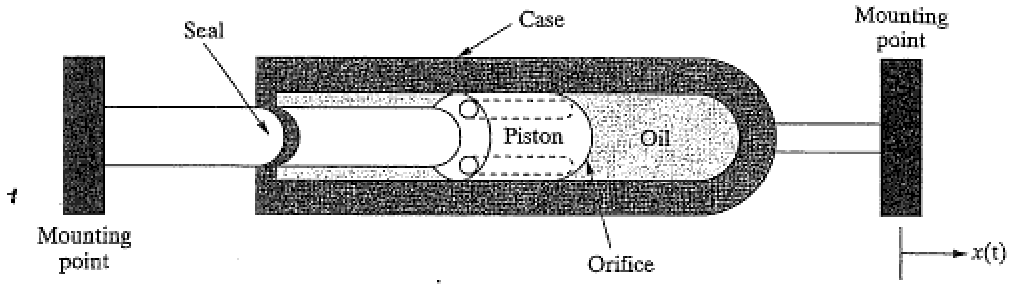


Damping

- The laminar flow of the oil through the perforations as the piston moves causes a damping force on the piston.
- The force is proportional to the velocity of the piston, in a direction opposite that of the piston motion. This damping force has the form:

$$f_c = c\dot{x}(t)$$

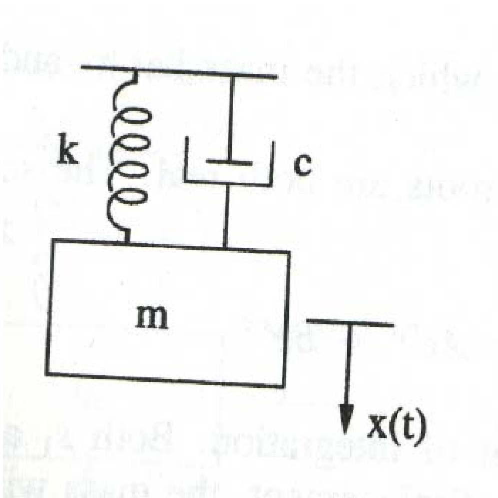
where c is a constant of proportionality related to the oil viscosity. The constant c, called the damping coefficient, has units of Ns/m, or kg/s.

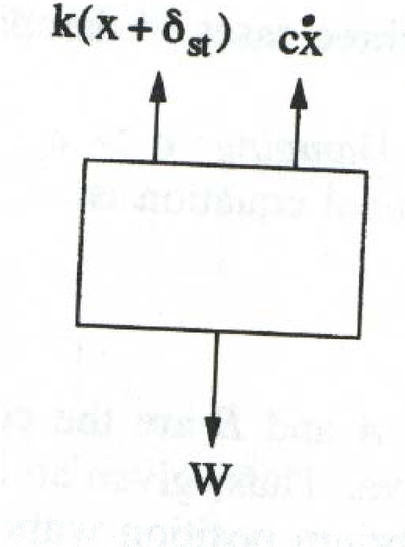


Damped free vibration of SDOF system

- Consider the spring-mass system with an energy dissipating mechanism described by the damping force as shown in the figure. It is assumed that the damping force F_D is proportional to the velocity of the mass, as shown; the damping coefficient is *c*. When Newton's second law is applied, this model for the damping force leads to a linear differential equation,

$$m\ddot{x} + c\dot{x} + kx = 0$$





Damped free vibration of SDOF system

- The ODE is homogeneous linear and has constant coefficients. The characteristic equation is found by dividing the below equation by m:

$$s^2 + \frac{c}{m}s + \frac{k}{m} = 0$$

- By the roots of a quadratic equation, we obtain:

$$s_1 = -\alpha + \beta, \quad s_2 = -\alpha - \beta,$$

where

$$\alpha = \frac{\mathrm{c}}{2\mathrm{m}} \quad \text{and} \quad \beta = \frac{1}{2m}\sqrt{c^2 - 4mk}$$

- It is now most interesting that depending on the amount of damping (much, medium or little) there will be three types of motion corresponding to the three cases I, II and III.

Three cases of damping

- Heavy damping when $c > c_c$
- Critical damping $c = c_c$
- Light damping $0 < c < c_c$

UNIT-II
ENGINEERING SEISMOLOGY

Elements of Engineering Seismology, Characteristics of Earthquake Engineering, Earthquake History, Indian Seismicity. Performance of structures under past earthquakes, Lessons learnt from past earthquakes

What is an Earthquake?

• Unpredictable natural phenomenon of vibration of the ground

• It becomes one of the most devastating natural hazard only if it's considered in relation with structures

Earthquakes ⇐⇒ Structures

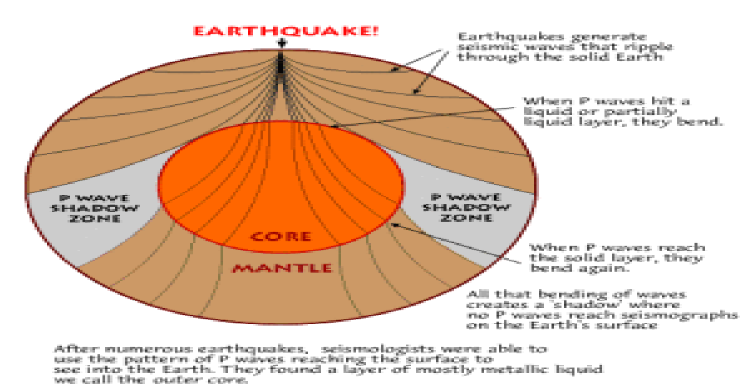
What is Earthquake Engineering? • The earthquake has begun to become a problem for humans since they started to build structures • The deaths and the damage to buildings that they cause have several economic, social, psychological and even political effects A general study of earthquakes involves many scientific disciplines that deal with the problem:

Seismology ⇐⇒ Engineering ⇐⇒ Economy ⇐⇒ Psychology

Earthquake Engineering =⇒ Branch of engineering devoted to mitigating earthquake hazards . It covers the investigation and solutions of the problems created by damaging structures.

Causes of earthquake:

- It is caused due to release of energy from faults and fractures in the Earth's crust. A fault in earth's crust is basically a sharp break in crustal rocks.
- This energy release generates waves which travel in all directions.
- The point where energy is released is called the focus / hypocenter. It is generally located at the depth of 60 km.
- The point on surface where the energy waves reach the surface is called the epicenter. The earthquake waves travel in all directions, but they have highest intensity at the epicenter. Hence they cause maximum damage there. The intensity of Earthquake decreases as one moves away from the epicenter.

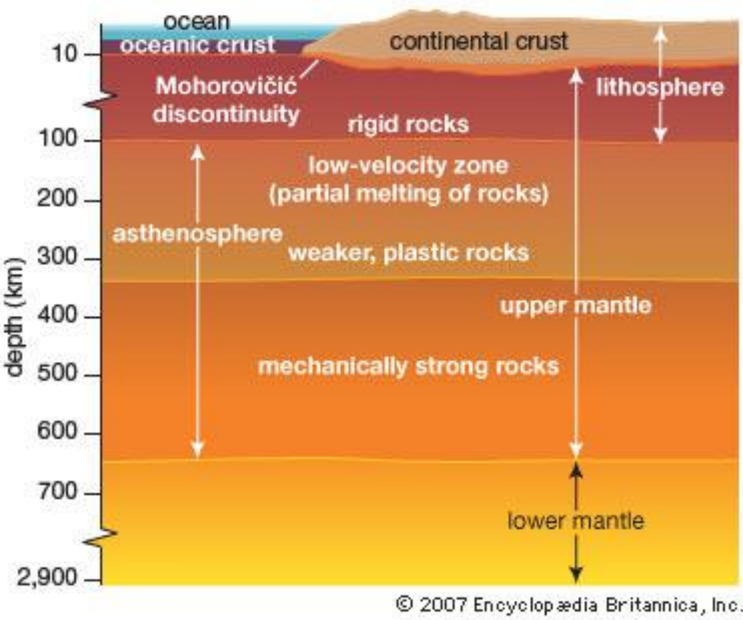


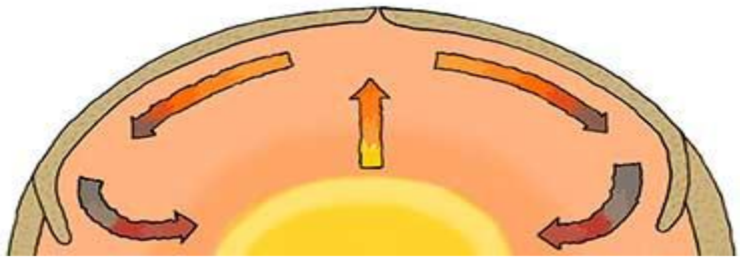
What Is Plate Tectonics?

From the deepest ocean trench to the tallest mountain, plate tectonics explains the features and movement of Earth's surface in the present and the past.
Plate tectonics is the theory that Earth's outer shell is divided into several plates that glide over the mantle, the rocky inner layer above the core. The plates act like a hard and rigid shell compared to Earth's mantle. This strong outer layer is called the lithosphere, which is 100 km (60 miles) thick, according to Encyclopedia Britannica. The lithosphere includes the crust and outer part of the mantle. Below the lithosphere is the asthenosphere, which is malleable or partially malleable, allowing the lithosphere to move around. How it moves around is an evolving idea

Principles of Plate Tectonics

In essence, plate-tectonic theory is elegantly simple. Earth's surface layer, 50 to 100 km (30 to 60 miles) thick, is rigid and is composed of a set of large and small plates. Together, these plates constitute the lithosphere, from the Greek *lithos*, meaning "rock." The lithosphere rests on and slides over an underlying partially molten (and thus weaker but generally denser) layer of plastic partially molten rock known as the asthenosphere, from the Greek *asthenos*, meaning "weak." Plate movement is possible because the lithosphere-asthenosphere boundary is a zone of detachment. As the lithospheric plates move across Earth's surface, driven by forces as yet not fully understood, they interact along their boundaries, diverging, converging, or slipping past each other. While the interiors of the plates are presumed to remain essentially undeformed, plate boundaries are the sites of many of the principal processes that shape the terrestrial surface, including earthquakes, volcanism, and orogeny (that is, formation of mountainranges).





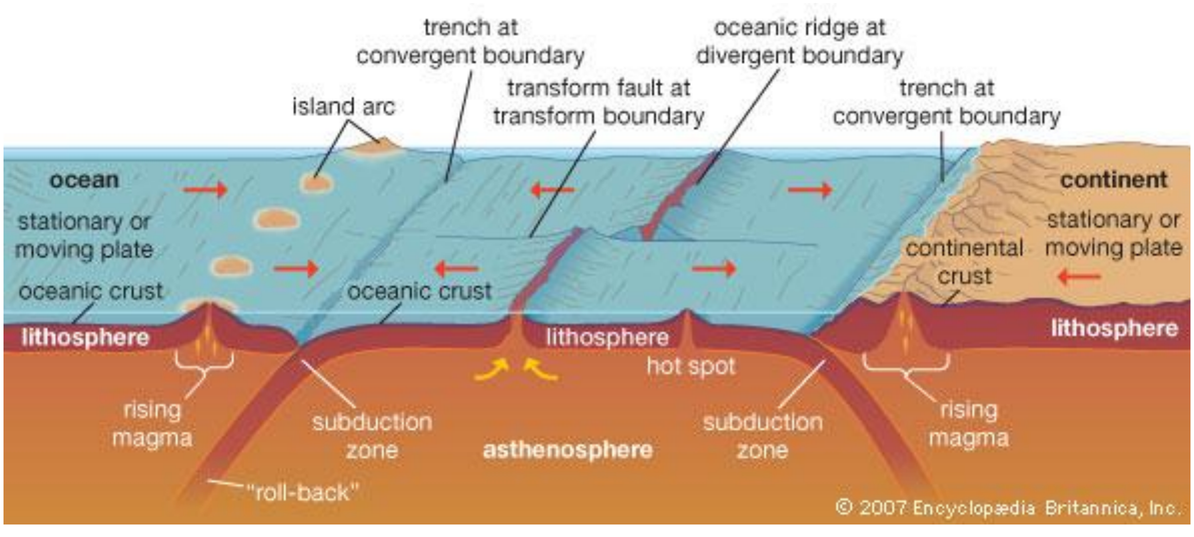
A cross section of Earth's outer layers, from the crust through the lower mantle.

platetectonicsThe roles that convection currents and other forces play in the movement of Earth's tectonic plates

The process of plate tectonics may be driven by convection in Earth's mantle, the pull of heavy old pieces of crust into the mantle, or some combination of both. For a deeper discussion of plate-driving mechanisms, *see* Plate-driving mechanisms and the role of the mantle.

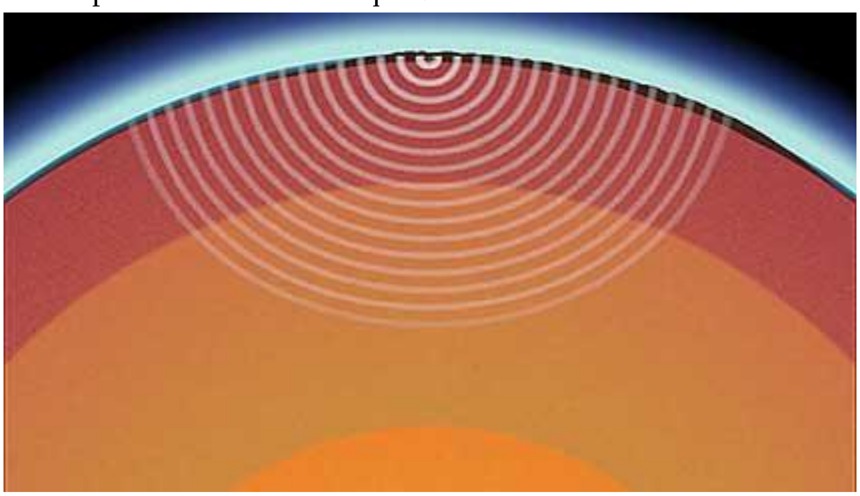
Earth's layers

Knowledge of Earth's interior is derived primarily from analysis of the seismic waves that propagate through Earth as a result of earthquakes. Depending on the material they travel through, the waves may speed up, slow down, bend, or even stop if they cannot penetrate the material they encounter.



crustal generation and destruction*Three-dimensional diagram showing crustal generation and destruction according to the theory of plate tectonics; included are the three kinds of plate boundaries—divergent, convergent (or collision), and strike-slip (or transform)*

There are two types of crust, continental and oceanic, which differ in their composition and thickness. The distribution of these crustal types broadly coincides with the division into continents and ocean basins, although continental shelves, which are submerged, are underlain by continental crust. The continents have a crust that is broadly granitic in composition and, with a density of about 2.7 grams per cubic cm (0.098 pound per cubic inch), is somewhat lighter than oceanic crust, which is basaltic (i.e., richer in iron and magnesium than granite) in composition and has a density of about 2.9 to 3 grams per cubic cm (0.1 to 0.11 pound per cubic inch). Continental crust is typically 40 km (25 miles) thick, while oceanic crust is much thinner, averaging about 6 km (4 miles) in thickness. These crustal rocks both sit on top of the mantle, which is ultramafic in composition (i.e., very rich in magnesium and iron-bearing silicate minerals). The boundary between the crust (continental or oceanic) and the underlying mantle is known as the Mohorovičićdiscontinuity(also called Moho), which is named for its discoverer, Croatian seismologist AndrijaMohorovičić. The Moho is clearly defined by seismic studies, which detect an acceleration in seismic waves as they pass from the crust into the denser mantle. The boundary between the mantle and the core is also clearly defined by seismic studies, which suggest that the outer part of the core is a liquid.



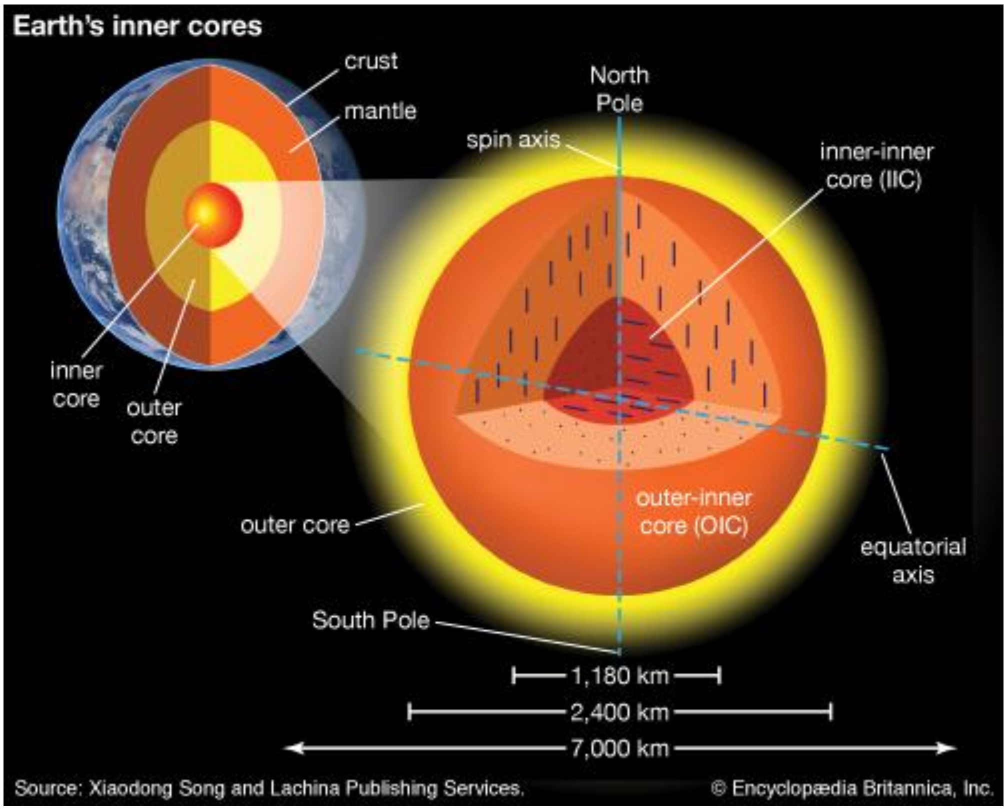
seismicwave*The shifting rock in an earthquake causes vibrations called seismic waves that travel within Earth or along its surface. The four main types of seismic waves are P waves, S waves, Love waves, and Rayleigh waves.*

The effect of the different densities of lithospheric rock can be seen in the different average elevations of continental and oceanic crust. The less-dense continental crust has greater buoyancy, causing it to float much higher in the mantle. Its average elevation above sea level is 840 metres (2,750 feet), while the average depth of oceanic crust is 3,790 metres (12,400 feet). This density difference creates two principal levels of Earth's surface.

The lithosphere itself includes all the crust as well as the upper part of the mantle (i.e., the region directly beneath the Moho), which is also rigid. However, as temperatures increase with depth, the heat causes mantle rocks to lose their rigidity. This process begins at about 100 km (60 miles) below the surface. This change occurs within the mantle and defines the base of the lithosphere and the top of the asthenosphere. This upper portion of the mantle, which is known as the lithospheric mantle, has an average density of about 3.3 grams per cubic cm (0.12 pound per cubic inch). The asthenosphere, which sits directly below the lithospheric mantle, is thought to be slightly denser at 3.4–4.4 grams per cubic cm (0.12–0.16 pound per cubic inch).

In contrast, the rocks in the asthenosphere are weaker, because they are close to their melting temperatures. As a result, seismic waves slow as they enter the asthenosphere. With increasing depth, however, the greater pressure from the weight of the rocks above causes the mantle to become gradually stronger, and seismic waves increase in velocity, a defining characteristic of the lower mantle. The lower mantle is more or less solid, but the region is also very hot, and thus the rocks can flow very slowly (a process known as creep).

At a depth of about 2,900 km (1,800 miles), the lower mantle gives way to Earth's outer core, which is made up of a liquid rich in iron and nickel. At a depth of about 5,100 km (3,200 miles), the outer core transitions to the inner core. Although it has a higher temperature than the outer core, the inner core is solid because of the tremendous pressures that exist near Earth's centre. Earth's inner core is divided into the outer-inner core (OIC) and the inner-inner core (IIC), which differ from one another with respect to the polarity of their iron crystals. The polarity of the iron crystals of the OIC is oriented in a north-south direction, whereas that of the IIC is oriented east-west.

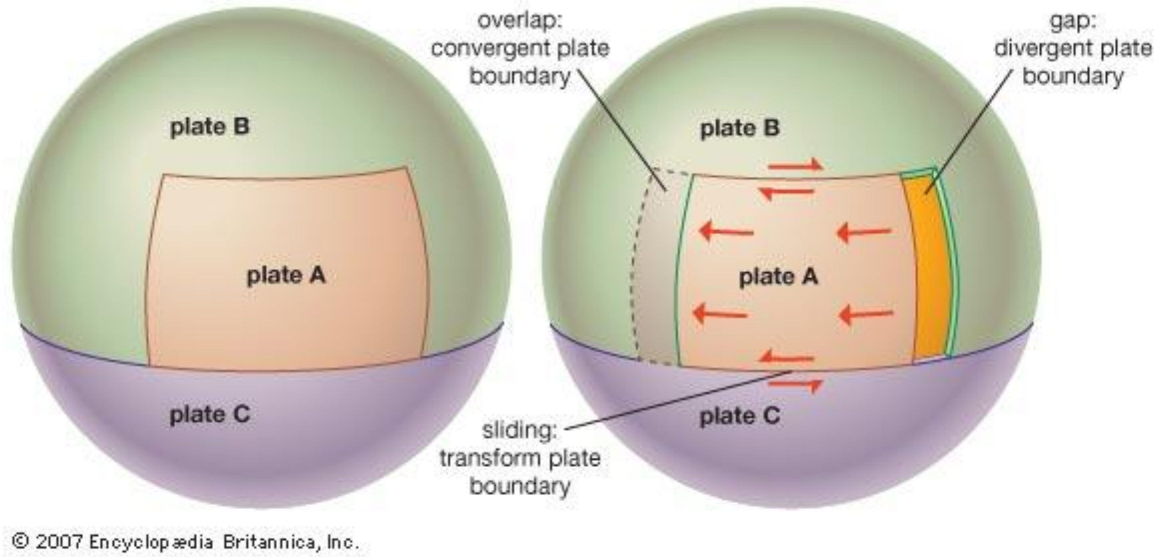


Earth's core*The internal layers of Earth's core, including its two inner cores.*

Plate boundaries

Lithospheric plates are much thicker than oceanic or continental crust. Their boundaries do not usually coincide with those between oceans and continents, and their behaviour is only partly influenced by whether they carry oceans, continents, or both.

In a simplified example of plate motion shown in the figure, movement of plate A to the left relative to plates B and C results in several types of simultaneous interactions along the plate boundaries. At the rear, plates A and B move apart, or diverge, resulting in extension and the formation of a divergent margin. At the front, plates A and B overlap, or converge, resulting in compression and the formation of a convergent margin. Along the sides, the plates slide past one another, a process called shear. As these zones of shear link other plate boundaries to one another, they are called transform faults.



Theoretical diagram showing the effects of an advancing tectonic plate on other adjacent, but stationary, tectonic plates. At the advancing edge of plate A, the overlap with plate B creates a convergent boundary. In contrast, the gap left behind the trailing edge of plate A forms a divergent boundary with plate B. As plate A slides past portions of both plate B and plate C, transform boundaries develop.

Divergent margins

As plates move apart at a divergent plate boundary, the release of pressure produces partial melting of the underlying mantle. This molten material, known as magma, is basaltic in composition and is buoyant. As a result, it wells up from below and cools close to the surface to generate new crust. Because new crust is formed, divergent margins are also called constructive margins.

Continental rifting

Upwelling of magma causes the overlying lithosphere to uplift and stretch. (Whether magmatism [the formation of igneous rock from magma] initiates the rifting or whether rifting decompresses the mantle and initiates magmatism is a matter of significant debate.) If the diverging plates are capped by continental crust, fractures develop that are invaded by the ascending magma, prying the continents farther apart. Settling of the continental blocks creates a rift valley, such as the present-day East African Rift Valley. As the rift continues to widen, the continental crust becomes progressively thinner until separation of the plates is achieved and a new ocean is created. The ascending partial melt cools and crystallizes to form new crust. Because the partial melt is basaltic in composition, the new crust is oceanic, and an ocean ridge develops along the site of the former continental rift. Consequently, diverging plate boundaries, even if they originate within continents, eventually come to lie in ocean basins of their own making.

Divergence and creation of oceanic crust are accompanied by much volcanic activity and by many shallow earthquakes as the crust repeatedly rifts, heals, and rifts again. Brittle earthquake-prone rocks occur only in the shallow crust. Deep earthquakes, in contrast, occur less frequently, due to the high heat flow in the mantle rock. These regions of oceanic crust are swollen with heat and so are elevated by 2 to 3 km (1.2 to 1.9 miles) above the surrounding seafloor. The elevated topography results in a feedback scenario in which the resulting gravitational force pushes the crust apart, allowing new magma to well up from below, which in turn sustains the elevated topography. Its summits are typically 1 to 5 km (0.6 to 3.1 miles) below the ocean surface. On a global scale, these ridges form an interconnected system of undersea “mountains” that are about 65,000 km (40,000 miles) in length and are called oceanic ridges.

Convergent margins

Given that Earth is constant in volume, the continuous formation of Earth’s new crust produces an excess that must be balanced by destruction of crust elsewhere. This is accomplished at convergent plate boundaries, also known as destructive plate boundaries, where one plate descends at an angle—that is, is subducted—beneath the other.

Because oceanic crust cools as it ages, it eventually becomes denser than the underlying asthenosphere, and so it has a tendency to subduct, or dive under, adjacent continental plates or younger sections of oceanic crust. The life span of the oceanic crust is prolonged by its rigidity, but eventually this resistance is overcome. Experiments show that the subducted oceanic lithosphere is denser than the surrounding mantle to a depth of at least 600 km (about 400 miles).

The mechanisms responsible for initiating subduction zones are controversial. During the late 20th and early 21st centuries, evidence emerged supporting the notion that subduction zones preferentially initiate along preexisting fractures (such as transform faults) in the oceanic crust. Irrespective of the exact mechanism, the geologic record indicates that the resistance to subduction is overcome eventually.

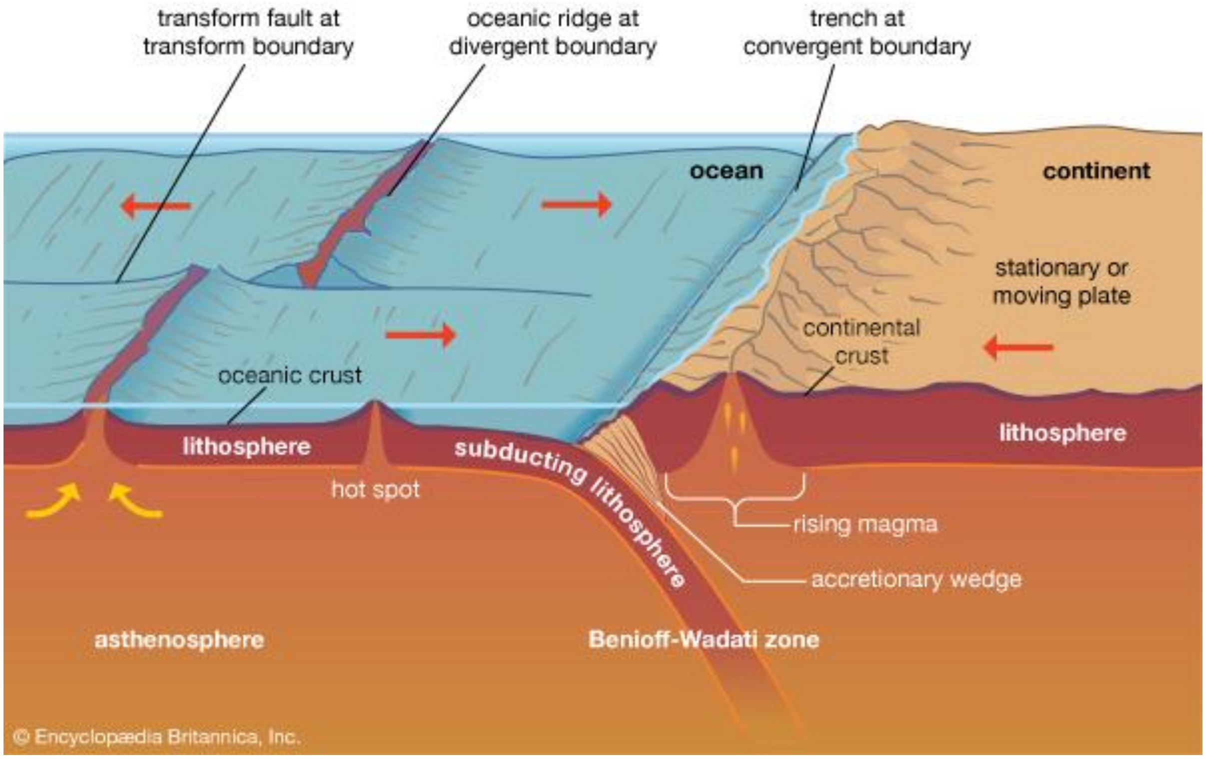
Where two oceanic plates meet, the older, denser plate is preferentially subducted beneath the younger, warmer one. Where one of the plate margins is oceanic and the other is continental, the greater buoyancy of continental crust prevents it from sinking, and the oceanic plate is preferentially subducted. Continents are preferentially preserved in this manner relative to oceanic crust, which is continuously recycled into the mantle. This explains why ocean floor rocks are generally less than 200 million years old whereas the oldest continental rocks are more than 4 billion years old. Before the middle of the 20th century, most geoscientists maintained that continental crust was too buoyant to be subducted. However, it later became clear that slivers of continental crust adjacent to the deep-sea trench, as well as sediments deposited in the trench, may be dragged down the subduction zone. The recycling of this material is detected in the chemistry of volcanoes that erupt above the subduction zone.

Two plates carrying continental crust collide when the oceanic lithosphere between them has been eliminated. Eventually, subduction ceases and towering mountain ranges, such as the Himalayas, are created. *See below* Mountains by continental collision.

Because the plates form an integrated system, it is not necessary that new crust formed at any given divergent boundary be completely compensated at the nearest subduction zone, as long as the total amount of crust generated equals that destroyed.

Subduction zones

The subduction process involves the descent into the mantle of a slab of cold hydrated oceanic lithosphere about 100 km (60 miles) thick that carries a relatively thin cap of oceanic sediments. The path of descent is defined by numerous earthquakes along a plane that is typically inclined between 30° and 60° into the mantle and is called the Wadati-Benioff zone, for Japanese seismologist KiyooWadati and American seismologist Hugo Benioff, who pioneered its study. Between 10 and 20 percent of the subduction zones that dominate the circum-Pacific ocean basin are subhorizontal (that is, they subduct at angles between 0° and 20°). The factors that govern the dip of the subduction zone are not fully understood, but they probably include the age and thickness of the subducting oceanic lithosphere and the rate of plate convergence.

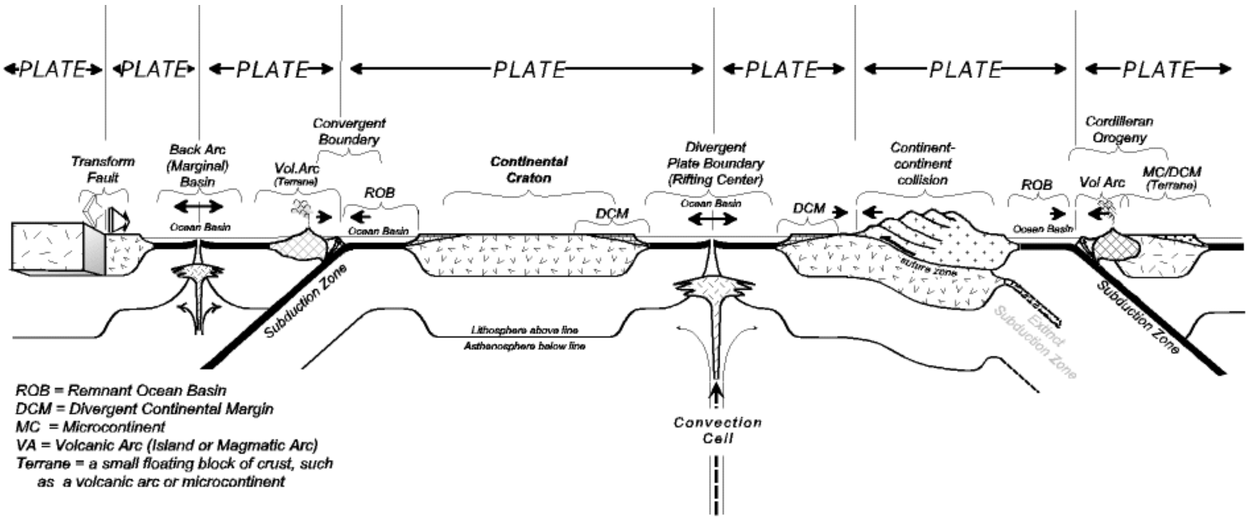


Transform faults

Along the third type of plate boundary, two plates move laterally and pass each other along giant fractures in Earth's crust. Transform faults are so named because they are linked to other types of plate boundaries. The majority of transform faults link the offset segments of oceanic ridges. However, transform faults also occur between plate margins with continental crust—for example, the San Andreas Fault in California and the North Anatolian fault system in Turkey. These boundaries are conservative because plate interaction occurs without creating or destroying crust. Because the only motion along these faults is the sliding of plates past each other, the horizontal direction along the fault surface must parallel the direction of plate motion. The fault surfaces are rarely smooth, and pressure may build up when the plates on either side temporarily lock. This buildup of stress may be suddenly released in the form of an earthquake.

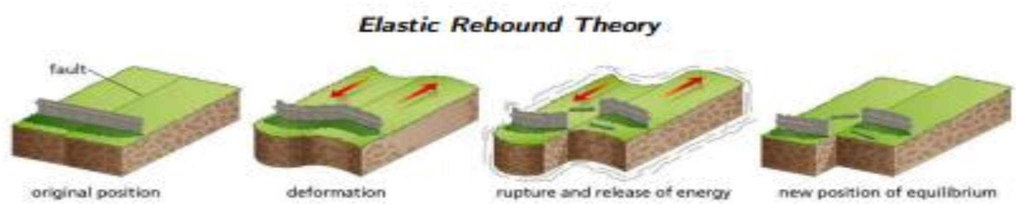
Many transform faults in the Atlantic Ocean are the continuation of major faults in adjacent continents, which suggests that the orientation of these faults might be inherited from preexisting weaknesses in continental crust during the earliest stages of the development of oceanic crust. On the other hand, transform faults may themselves be reactivated, and recent geodynamic models suggest that they are favourable environments for the initiation of subduction zones.

Plate Tectonic Theory: Plate Boundaries and Interplate Relationships



Elastic Rebound Theory

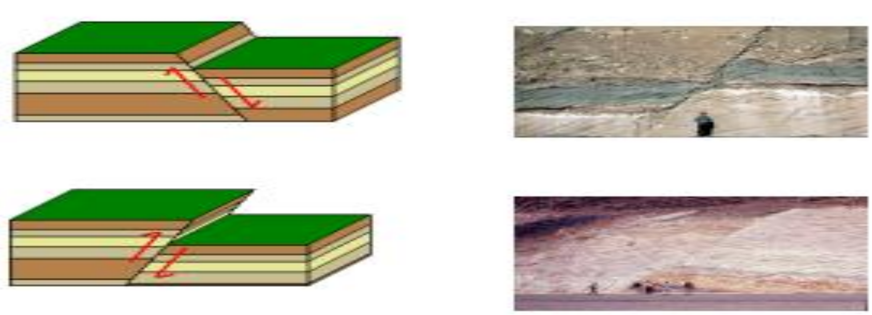
• Earthquakes are ground vibrations that are caused mainly by the fracture of the crust of the earth or by the sudden movement along an already existing fault. • The fracture or the slippage emits large amounts of energy in the form of seismic waves that travel through the interior of the earth and cross the surface. • Cracks along which rocks slip are called faults; they may break through the ground surface, or remain deed within the earth.



The most common mechanisms of earthquake sources are:

• Normal faults:The block above the fault moves down relative to the block below the fault. This fault motion is caused by tension forces and results in extension

• Reverse faults: The block above the fault moves up relative to the block below the fault. This fault motion is caused by compression forces and results in shortening.

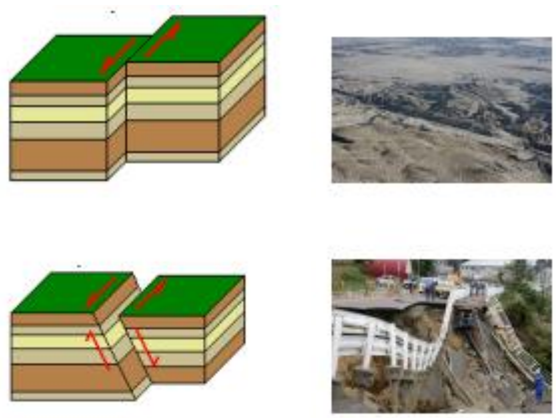


Normal faults & Reverse faults

The most common mechanisms of earthquake sources are:

• Strike-Slip faults: The movement of blocks is horizontal. This fault motion is caused by shearing forces.

• Oblique-Slip faults: Oblique-slip faulting suggests both dip-slip faulting and strike-slip faulting. It is caused by a combination of shearing and tension or compressional forces.



Oblique-Slip faults & Strike-Slip faults

Earthquake and Seismic Waves

The study of seismic activity gives valuable information about the interior structure of the Earth. Earthquake is a natural event and involves shaking of the ground. A thorough study and observations of this phenomenon has helped in understanding of innermost parts where this occurs . An Earthquake is the shaking or vibration/tremors in the Crust, caused as a result of internal forces and volcanism in the Earth.

Seismic Waves

Seismic waves are the waves of energy caused by the sudden breaking of rock within the earth or an explosion. They are the energy that travels through the earth and is recorded on seismographs.

Types of Seismic Waves

There are several different kinds of seismic waves, and they all move in different ways. The two main types of waves are **body waves** and **surface waves**. Body waves can travel through the earth's inner layers, but surface waves can only move along the surface of the planet like ripples on water. Earthquakes radiate seismic energy as both body and surface waves.

Body Waves

- Traveling through the interior of the earth, **body waves** arrive before the surface waves emitted by an earthquake. These waves are of a higher frequency than surface waves.

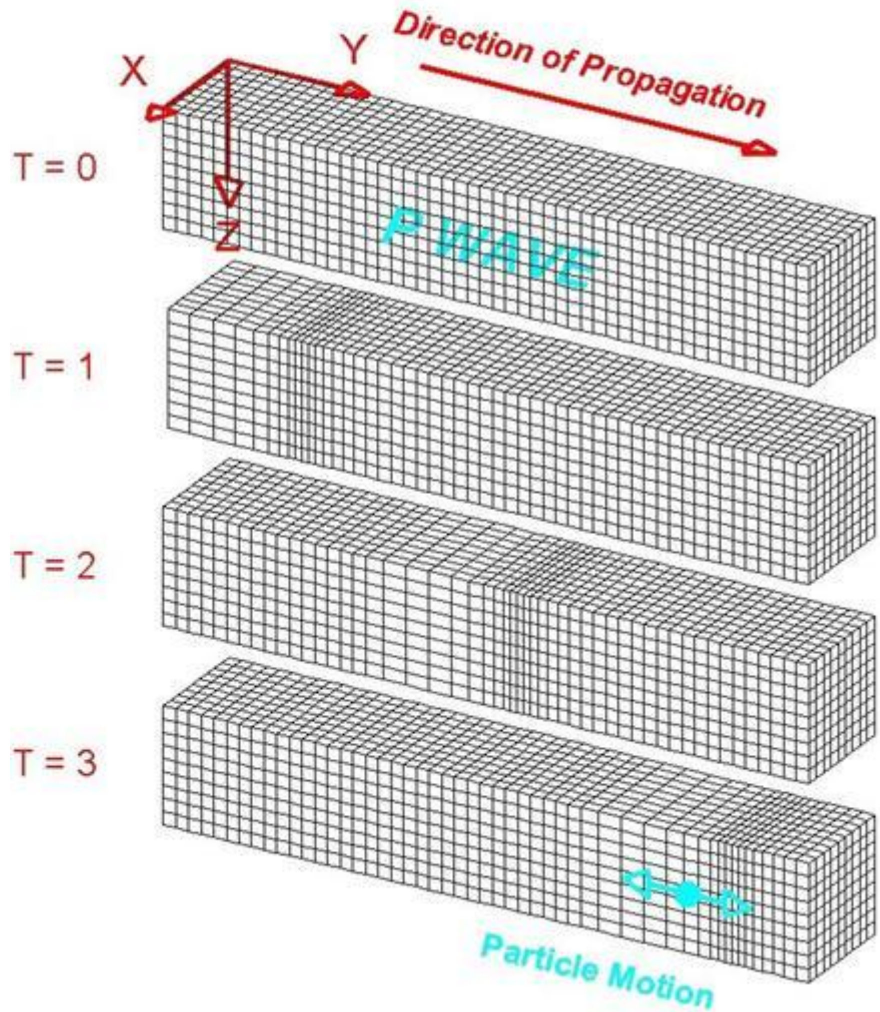
- **Body waves** are generated by the energy released at the focus /hypocenter. They move in all directions through the body of Earth. They interact with the surface rocks and generate Surface waves, which move along the surface. Body waves are further divided in two types: P – waves and S – waves

P waves

The first kind of body wave is the **P wave** or **primary wave**. This is the fastest kind of seismic wave, and, consequently, the first to 'arrive' at a seismic station. The P wave can move through solid rock and fluids, like water or the liquid layers of the earth. It pushes and pulls the rock it moves through just like sound waves push and pull the air. Have you ever heard a big clap of thunder and heard the windows rattle at the same time? The windows rattle because the sound waves were pushing and pulling on the window glass much like P waves push and pull on rock. Sometimes animals can hear the P waves of an earthquake. Dogs, for instance, commonly begin barking hysterically just before an earthquake 'hits' (or more specifically, before the surface waves arrive). Usually people can only feel the bump and rattle of these waves.

P waves are also known as **compressional waves**, because of the pushing and pulling they do. Subjected to a P wave, particles move in the same direction that the the wave is moving in, which is the direction that the energy is traveling in, and is sometimes called the 'direction of wave propagation.

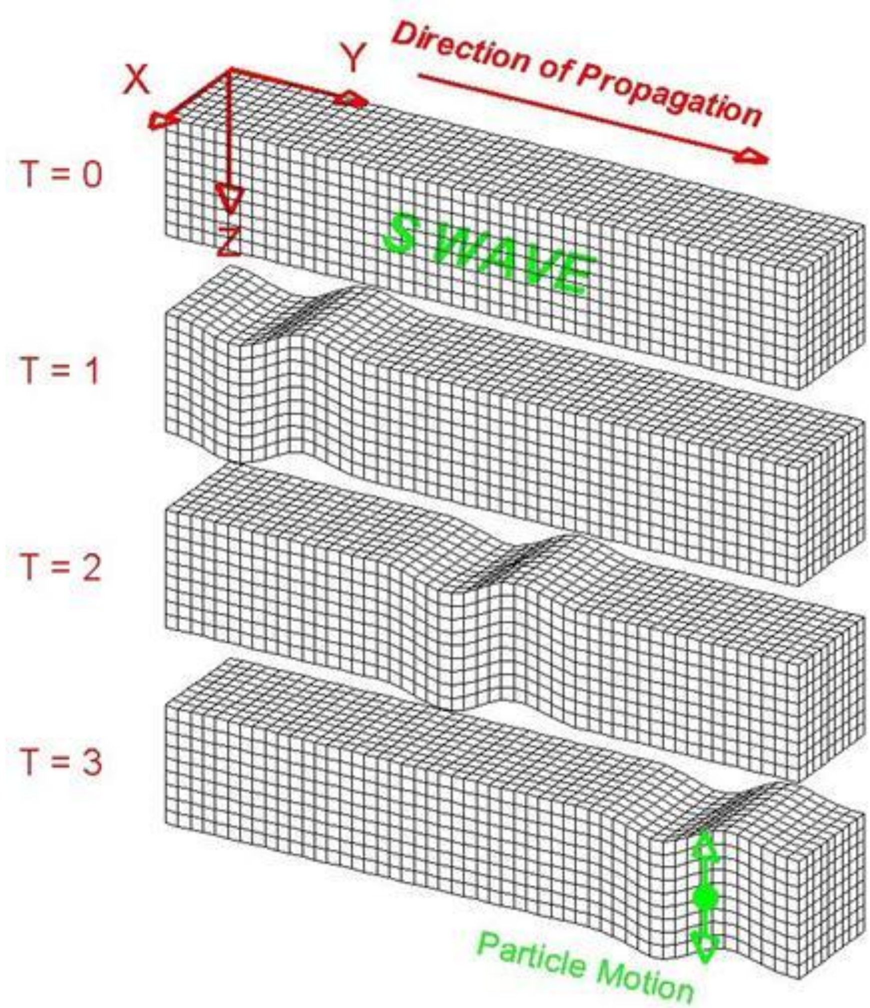
- These waves are known as Primary waves as they are first one to arrive at the surface.
- Their characteristics are similar to Sound waves, as they travel through all three mediums- solid, liquid and gases.
- P-waves have a tendency to vibrate parallel to the direction of wave propagation. this causes density differences in the material through which they travel.
- These waves are responsible for stretching and squeezing of material.
- Shadow zone: these are specific areas where waves are not reported on the seismograph. P-waves appears as around the Earth at 105-145 degrees away from the epicenter.

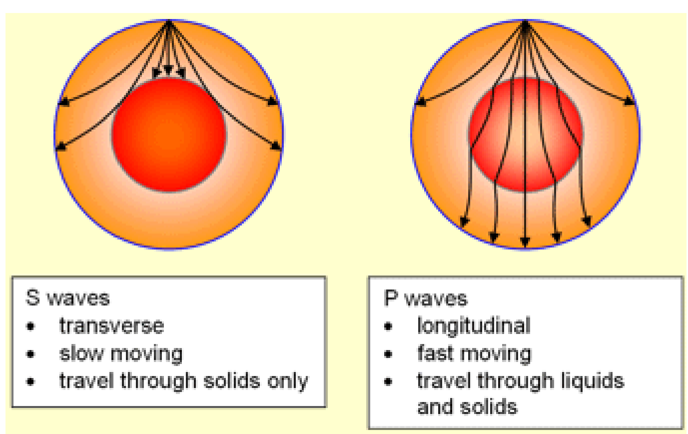


S wave or **secondary wave**

The second type of body wave is the **S wave** or **secondary wave**, which is the second wave you feel in an earthquake. An S wave is slower than a P wave and can only move through solid rock, not through any liquid medium. It is this property of S waves that led seismologists to conclude that the Earth's **outer core** is a liquid. S waves move rock particles up and down, or side-to-side--perpindicular to the direction that the wave is traveling in (the direction of wave propagation).

- These waves arrive after some time delay; hence they are called secondary waves.
- An important characteristic of these s-waves is that they travel only through solid medium. This is important because this information helped in understanding the structure of interior of Earth.
- The direction of vibration of these S – wave is perpendicular to the direction of wave propagation, thereby creating crests and troughs in material of their transmission.
- Shadow zone: Beyond 105 degrees from the epicenter no S-waves are reported.





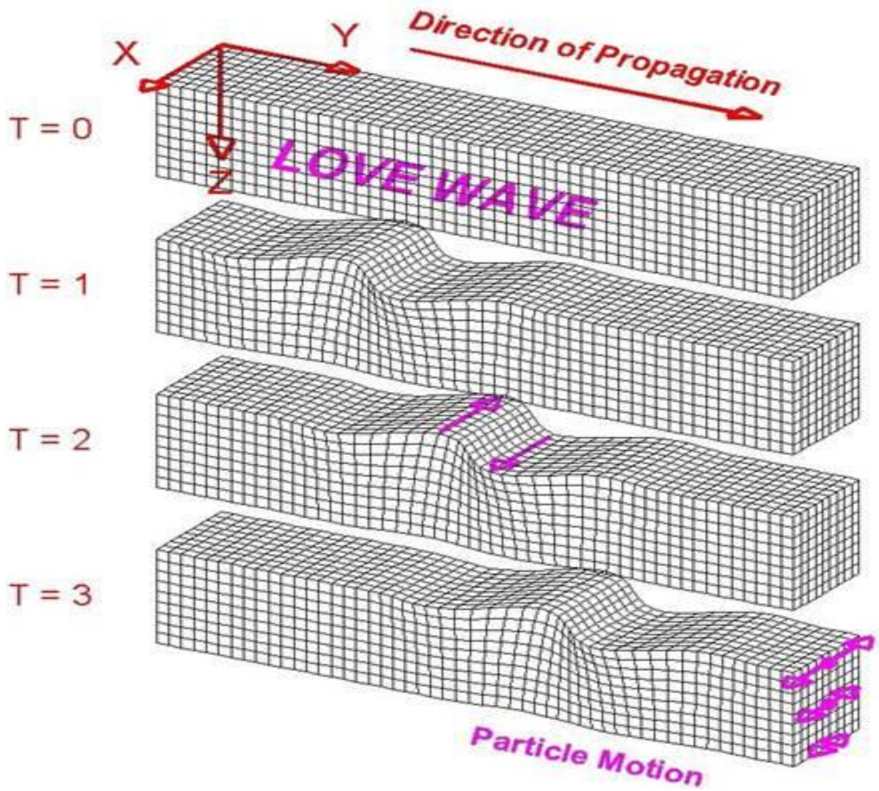
SURFACE WAVES

- Travelling only through the crust, **surface waves** are of a lower frequency than body waves, and are easily distinguished on a seismogram as a result. Though they arrive after body waves, it is surface waves that are almost enitrely responsible for the damage and destruction associated with earthquakes. This damage and the strength of the surface waves are reduced in deeper earthquakes.

Surface waves move along the surface.The velocity of these waves vary with the material through which they travel, the denser the material, the higher the velocity of these waves. They change their direction as they reflect and refract after coming across

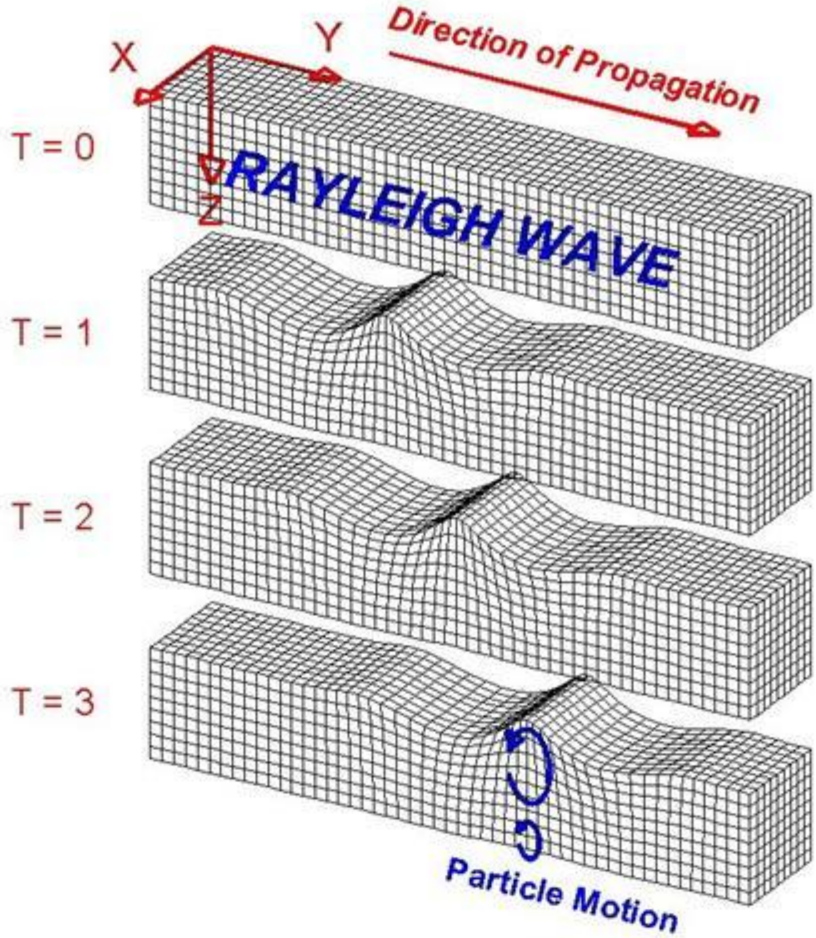
Love Waves

- The first kind of surface wave is called a **Love wave**, named after A.E.H. Love, a British mathematician who worked out the mathematical model for this kind of wave in 1911. It's the fastest surface wave and moves the ground from side-to-side. Confined to the surface of the crust, Love waves produce entirely horizontal motion.



RAYLEIGH WAVES

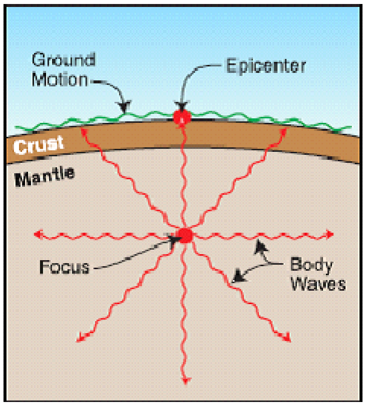
The other kind of surface wave is the **Rayleigh wave**, named for John William Strutt, Lord Rayleigh, who mathematically predicted the existence of this kind of wave in 1885. A Rayleigh wave rolls along the ground just like a wave rolls across a lake or an ocean. Because it rolls, it moves the ground up and down, and side-to-side in the same direction that the wave is moving. Most of the shaking felt from an earthquake is due to the Rayleigh wave, which can be much larger than the other waves.



Measurement:

All earthquakes are different in their intensity and magnitude. The instrument for measurement of the vibrations is known as Seismograph.

1. **Magnitude scale:** Richter scale. Energy released during a quake is expressed in absolute numbers of 0-10.
2. **Intensity scale:** Mercalli scale. It measures the visible damage caused due o the quake. It is expressed in the range of 1-12.



Magnitude / Intensity Comparison

Magnitude and Intensity measure different characteristics of earthquakes. Magnitude measures the energy released at the source of the earthquake. Magnitude is determined from measurements on seismographs. Intensity measures the strength of shaking produced by the earthquake at a certain location. Intensity is determined from effects on people, human structures, and the natural environment.

INTENSITY

The Modified Mercalli Intensity Scale

- The effect of an earthquake on the Earth's surface is called the intensity. The intensity scale consists of a series of certain key responses such as people awakening, movement of furniture, damage to chimneys, and finally - total destruction. Although numerous *intensity scales* have been developed over the last several hundred years to evaluate the effects of earthquakes, the one currently used in the United States is the Modified Mercalli (MM) Intensity Scale. It was developed in 1931 by the American seismologists Harry Wood and Frank Neumann. This scale, composed of increasing levels of intensity that range from imperceptible shaking to catastrophic destruction, is designated by Roman numerals. It does not have a mathematical basis; instead it is an arbitrary ranking based on observed effects.
- The Modified Mercalli Intensity value assigned to a specific site after an earthquake has a more meaningful measure of severity to the nonscientist than the magnitude because intensity refers to the effects actually experienced at that place.
- The **lower** numbers of the intensity scale generally deal with the manner in which the earthquake is felt by people. The **higher** numbers of the scale are based on observed structural damage. Structural engineers usually contribute information for assigning intensity values of VIII or above.
- The following is an abbreviated description of the levels of Modified Mercalli intensity.

Intensity	Shaking	Description/Damage

Intensity	**Shaking**	**Description/Damage**
I	Not felt	Not felt except by a very few under especially favorable conditions.
II	Weak	Felt only by a few persons at rest,especially on upper floors of buildings.
III	Weak	Felt quite noticeably by persons indoors, especially on upper floors of buildings. Many people do not recognize it as an earthquake. Standing motor cars may rock slightly. Vibrations similar to the passing of a truck. Duration estimated.
IV	Light	Felt indoors by many, outdoors by few during the day. At night, some awakened. Dishes, windows, doors disturbed; walls make cracking sound. Sensation like heavy truck striking building. Standing motor cars rocked noticeably.
V	Moderate	Felt by nearly everyone; many awakened. Some dishes, windows broken. Unstable objects overturned. Pendulum clocks may stop.
VI	Strong	Felt by all, many frightened. Some heavy furniture moved; a few instances of fallen plaster. Damage slight.
VII	Very strong	Damage negligible in buildings of good design and construction; slight to moderate in well-built ordinary structures; considerable damage in poorly built or badly designed structures; some chimneys broken.
VIII	Severe	Damage slight in specially designed structures; considerable damage in ordinary substantial buildings with partial collapse. Damage great in poorly built structures. Fall of chimneys, factory stacks, columns, monuments, walls. Heavy furniture overturned.
IX	Violent	Damage considerable in specially designed structures; well-designed frame structures thrown out of plumb. Damage great in substantial buildings, with partial collapse. Buildings shifted off foundations.
X	Extreme	Some well-built wooden structures destroyed; most masonry and frame structures destroyed with foundations. Rails bent.

Magnitude / Intensity Comparison

- The following table gives intensities that are typically observed at locations near the epicenter of earthquakes of different magnitudes.

Magnitude	**Typical Maximum Modified Mercalli Intensity**
1.0 - 3.0	**I**
3.0 - 3.9	**II - III**
4.0 - 4.9	**IV - V**
5.0 - 5.9	**VI - VII**
6.0 - 6.9	**VII - IX**
7.0 and higher	**VIII** or higher

Abbreviated Modified Mercalli Intensity Scale

- **I**. Not felt except by a very few under especially favorable conditions.
- **II**. Felt only by a few persons at rest, especially on upper floors of buildings.
- **III**. Felt quite noticeably by persons indoors, especially on upper floors of buildings. Many people do not recognize it as an earthquake. Standing motor cars may rock slightly. Vibrations similar to the passing of a truck. Duration estimated.
- **IV**. Felt indoors by many, outdoors by few during the day. At night, some awakened. Dishes, windows, doors disturbed; walls make cracking sound. Sensation like heavy truck striking building. Standing motor cars rocked noticeably.
- **V**. Felt by nearly everyone; many awakened. Some dishes, windows broken. Unstable objects overturned. Pendulum clocks may stop.
- **VI**. Felt by all, many frightened. Some heavy furniture moved; a few instances of fallen plaster. Damage slight.
- **VII**. Damage negligible in buildings of good design and construction; slight to moderate in well-built ordinary structures; considerable damage in poorly built or badly designed structures; some chimneys broken.

- **VIII**. Damage slight in specially designed structures; considerable damage in ordinary substantial buildings with partial collapse. Damage great in poorly built structures. Fall of chimneys, factory stacks, columns, monuments, walls. Heavy furniture overturned.
- **IX**. Damage considerable in specially designed structures; well-designed frame structures thrown out of plumb. Damage great in substantial buildings, with partial collapse. Buildings shifted off foundations.
- **X**. Some well-built wooden structures destroyed; most masonry and frame structures destroyed with foundations. Rails bent.
- **XI**. Few, if any (masonry) structures remain standing. Bridges destroyed. Rails bent greatly.
- **XII**. Damage total. Lines of sight and level are distorted. Objects thrown into the air

MAGNITUDE

Measuring the Size of an Earthquake

- Seismic waves are the vibrations from earthquakes that travel through the Earth; they are recorded on instruments called seismographs. Seismographs record a zig-zag trace that shows the varying amplitude of ground oscillations beneath the instrument. Sensitive seismographs, which greatly magnify these ground motions, can detect strong earthquakes from sources anywhere in the world. The time, location, and magnitude of an earthquake can be determined from the data recorded by seismograph stations.
- Modern seismographic systems precisely amplify and record ground motion (typically at periods of between 0.1 and 100 seconds) as a function of time.
- Earthquakes with magnitude of about 2.0 or less are usually called microearthquakes; they are not commonly felt by people and are generally recorded only on local seismographs. Events with magnitudes of about 4.5 or greater - there are several thousand such shocks annually - are strong enough to be recorded by sensitive seismographs all over the world. Great earthquakes, such as the 1964 Good Friday earthquake in Alaska, have magnitudes of 8.0 or higher. On the average, one earthquake of such size occurs somewhere in the world each year.

The Richter Scale

- Although similar seismographs had existed since the 1890's, it was only in 1935 that Charles F. Richter, a seismologist at the California Institute of Technology, introduced the concept of earthquake magnitude. His original definition held only for California earthquakes occurring within 600 km of a particular type of seismograph (the Woods-Anderson torsion instrument). His basic idea was quite simple: by knowing the distance from a seismograph to an earthquake and observing the maximum signal amplitude recorded on the seismograph, an empirical quantitative ranking of the earthquake's inherent size or strength could be made. Most California earthquakes occur within the top

16 km of the crust; to a first approximation, corrections for variations in earthquake focal depth were, therefore, unnecessary.

- The Richter magnitude of an earthquake is determined from the logarithm of the amplitude of waves recorded by seismographs. Adjustments are included for the variation in the distance between the various seismographs and the epicenter of the earthquakes. On the Richter Scale, magnitude is expressed in whole numbers and decimal fractions. For example, a magnitude 5.3 might be computed for a moderate earthquake, and a strong earthquake might be rated as magnitude 6.3. Because of the logarithmic basis of the scale, each whole number increase in magnitude represents a tenfold increase in measured amplitude; as an estimate of energy, each whole number step in the magnitude scale corresponds to the release of about 31 times more energy than the amount associated with the preceding whole number value.
- The Richter Scale is not commonly used anymore, except for small earthquakes recorded locally, for which ML and Mblg are the only magnitudes that can be measured. For all other earthquakes, the moment magnitude scale is a more accurate measure of the earthquake size.

Magnitude

- Richter's original magnitude scale (M_L) was extended to observations of earthquakes of any distance and of focal depths ranging between 0 and 700 km. Because earthquakes excite both body waves, which travel into and through the Earth, and surface waves, which are constrained to follow the natural wave guide of the Earth's uppermost layers, two magnitude scales evolved - the m_b and M_S scales.
- The standard body-wave magnitude formula is
- $m_b = \log_{10}(A/T) + Q(D,h)$,
- where A is the amplitude of ground motion (in microns); T is the corresponding period (in seconds); and $Q(D,h)$ is a correction factor that is a function of distance, D (degrees), between epicenter and station and focal depth, h (in kilometers), of the earthquake. The standard surface-wave formula is
- $M_S = \log_{10} (A/T) + 1.66 \log_{10} (D) + 3.30$.
- There are many variations of these formulas that take into account effects of specific geographic regions, so that the final computed magnitude is reasonably consistent with Richter's original definition of M_L. Negative magnitude values are permissible.
- A rough idea of frequency of occurrence of large earthquakes is given by the following table:
- The original m_b scale utilized compressional body P-wave amplitudes with periods of 4-5 s, but recent observations are generally of 1 s-period P waves. The M_S scale has consistently used Rayleigh surface waves in the period range from 18 to 22 s.
- When initially developed, these magnitude scales were considered to be equivalent; in other words, earthquakes of all sizes were thought to radiate fixed proportions of energy

at different periods. But it turns out that larger earthquakes, which have larger rupture surfaces, systematically radiate more long-period energy. Thus, for very large earthquakes, body-wave magnitudes badly underestimate true earthquake size; the maximum body-wave magnitudes are about 6.5 - 6.8. In fact, the surface-wave magnitudes underestimate the size of very large earthquakes; the maximum observed values are about 8.3 - 8.7. Some investigators have suggested that the 100 s mantle Love waves (a type of surface wave) should be used to estimate magnitude of great earthquakes. However, even this approach ignores the fact that damage to structure is often caused by energy at shorter periods. Thus, modern seismologists are increasingly turning to two separate parameters to describe the physical effects of an earthquake: seismic moment and radiated energy.

Fault Geometry and Seismic Moment, M_O

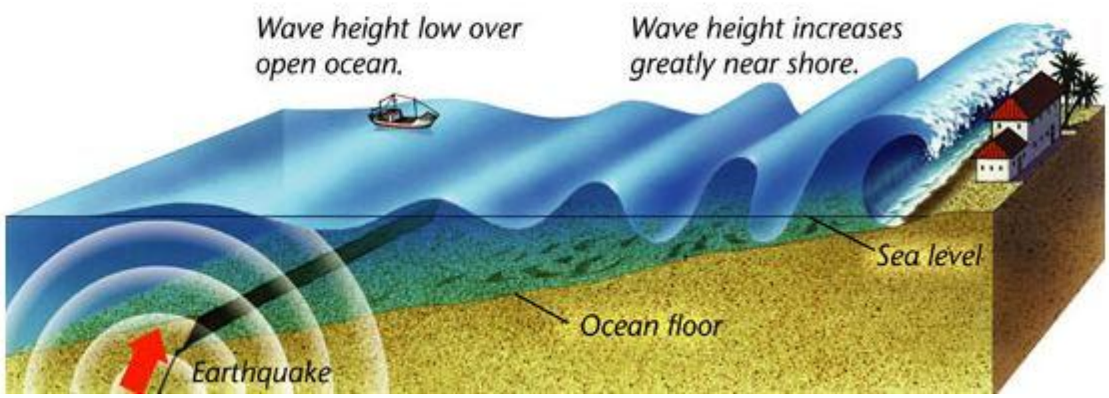
- The orientation of the fault, direction of fault movement, and size of an earthquake can be described by the fault geometry and seismic moment. These parameters are determined from waveform analysis of the seismograms produced by an earthquake. The differing shapes and directions of motion of the waveforms recorded at different distances and azimuths from the earthquake are used to determine the fault geometry, and the wave amplitudes are used to compute moment. The seismic moment is related to fundamental parameters of the faulting process.
- $M_O = \mu S \langle d \rangle$,
- where μ is the shear strength of the faulted rock, S is the area of the fault, and <d> is the average displacement on the fault. Because fault geometry and observer azimuth are a part of the computation, moment is a more consistent measure of earthquake size than is magnitude, and more importantly, moment does not have an intrinsic upper bound. These factors have led to the definition of a new magnitude scale M_W, based on seismic moment, where
- $M_W = 2/3 \log_{10}(M_O) - 10.7$.
- The two largest reported moments are 2.5×10^{30} dyn·cm (dyne·centimeters) for the 1960 Chile earthquake (M_S 8.5; M_W 9.6) and 7.5×10^{29} dyn·cm for the 1964 Alaska earthquake (M_S 8.3; M_W 9.2). M_S approaches its maximum value at a moment between 10^{28} and 10^{29} dyn·cm.

Energy, *E*

- The amount of energy radiated by an earthquake is a measure of the potential for damage to man-made structures. Theoretically, its computation requires summing the energy flux over a broad suite of frequencies generated by an earthquake as it ruptures a fault. Because of instrumental limitations, most estimates of energy have historically relied on the empirical relationship developed by Beno Gutenberg and Charles Richter:
- $\log_{10}E = 11.8 + 1.5M_S$

- where energy, *E,* is expressed in ergs. The drawback of this method is that M_S is computed from an bandwidth between approximately 18 to 22 s. It is now known that the energy radiated by an earthquake is concentrated over a different bandwidth and at higher frequencies. With the worldwide deployment of modern digitally recording seismograph with broad bandwidth response, computerized methods are now able to make accurate and explicit estimates of energy on a routine basis for all major earthquakes. A magnitude based on energy radiated by an earthquake, M_e, can now be defined,
- $M_e = 2/3 \log_{10}E - 2.9$.
- For every increase in magnitude by 1 unit, the associated seismic energy increases by about 32 times.
- Although M_w and M_e are both magnitudes, they describe different physical properites of the earthquake. M_w, computed from low-frequency seismic data, is a measure of the area ruptured by an earthquake. M_e, computed from high frequency seismic data, is a measure of seismic potential for damage. Consequently, M_w and M_e often do not have the same numerical value.

TSUNAMI



How Tsunami works.

- It is a Japanese term which means" harbour waves".
- These are waves generated by the tremors and vibration of earth. These are not earthquake themselves but an effect of earthquake.
- Occurrence: When the epicenter of an earthquake is below the oceanic waves and magnitude of the quake is above 5 on the Richter scale.

INTRODUCTION TO SEISMIC ZONES

Seismic Zonation may be termed as the geographic delineation of areas having different potentials for hazardous effects from future earthquakes. Seismic zonation can be done at any scale, national, regional, local, or site.

The term Zoning implies that the parameter or parameters that characterize the hazard have a constant value in each zone. If, for example, for practical reasons, the number of zones is

reduced (from five as is the case in large majority of national codes), we obtain a rather simplified representation of the hazard, which in reality has continuous variation.
A seismic zone is a region in which the rate of seismic activity remains fairly consistent. This may mean that seismic activity is incredibly rare, or that it is extremely common.
Some people often use the term “seismic zone” to talk about an area with an increased risk of seismic activity, while others prefer to talk about “seismic hazard zones” when discussing areas where seismic activity is more frequent. Many nations have government agencies concerned with seismic activity. These agencies use the data they collect about seismic activity to divide the nation into various seismic zones. A number of different zoning systems are used, from numerical zones to colored zones, with each number or color representing a different level of seismic activity.

A seismic zoning map for engineering use is a map that specifies the levels of force or ground motions for earthquake-resistant design, and thus it differs from a seismicity map, which provides only the occurrence of earthquake information. The task of seismic zoning is multidisciplinary and involves the best of input from geologist, seismologist, geotechnical, earthquake and structural engineers.

Need for Seismic Zonation

These maps identify the regions of a country or province in which various intensities of ground shaking may have occurred or may be anticipated. Maps of probabilistic hazard give an idea of the underlying statistical uncertainty, as is done in calculating insurance rates. These maps give, for example, the odds at which specified earthquake intensity would be exceeded at a site of interest within a given time span. Seismic zoning is used to reduce the human and economic losses caused by earthquakes, thereby enhancing Economic development and Political stability. New probabilistic maps have been developed as the basis of seismic design provisions for building practice. These usually give the expected intensity of ground shaking in terms of peak acceleration. The peak acceleration can be thought of as the maximum acceleration in earthquakes on firm ground at the frequencies that affect sizable structures. The losses due to damaging earthquakes can be mitigated through a comprehensive assessment of seismic hazard and risk. Seismic zonation of vulnerable areas for bedrock motion thus becomes important so that the planners and administrators can make use of it after applying appropriate amplification factors to take into account the local soil conditions, for better land use planning and safe development.

First Seismic Code

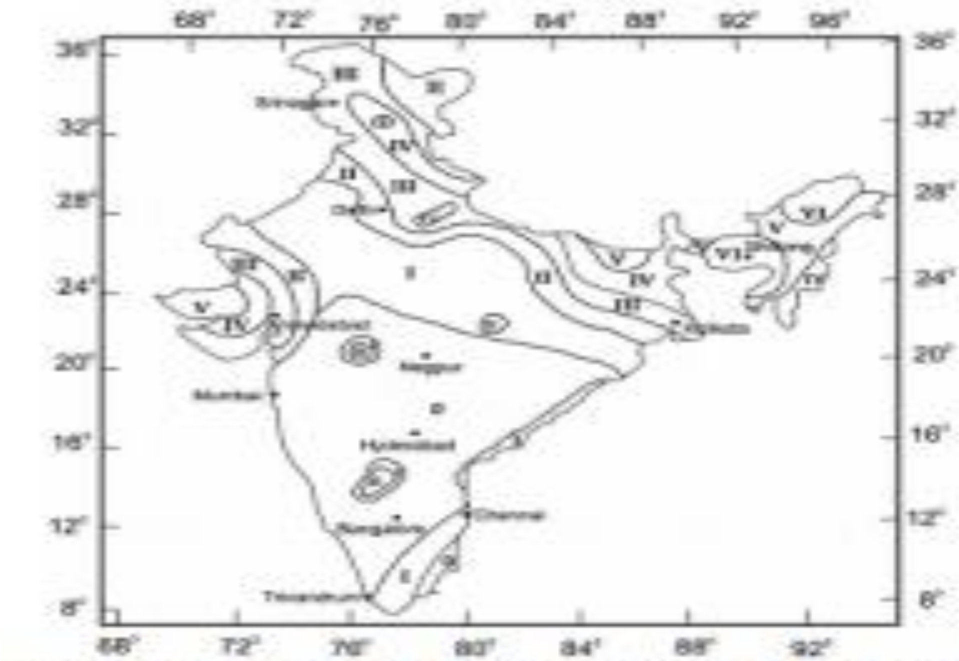
Some of the largest earthquakes of the world have occurred in India and the earthquake engineering developments in the country started rather early. After the 1897 Assam earthquake a new earthquake resistant type of housing was developed which is still prevalent in the north-east India. The Baluchistan earthquakes of 1930‟s led to innovative earthquake resistant constructions and to the development of first seismic zone map. The earthquake historical records in India are available only for about 200 years. The records of the 19th century as mentioned above are mostly newspaper descriptions of what happened at different places during a particular earthquake.

The 1935 Quetta earthquake was interesting from several view points. For the first time, serious and systematic efforts were made in the country at earthquake resistant constructions and for developing earthquake codes. The Geological Survey of India (GSI) first came up with the national seismic hazard map of India in 1935 after the 1934 Bihar-Nepal earthquake. In 1962, the BIS published the seismic zonation map of India (BIS, 1962) based on earthquake epicenters and the isoseismal map published by the GSI in 1935 (Table 10.2). The earthquakes of magnitude 5 and above with maximum Modified Mercalli Intensity (MMI) scale ranging from V to IX was considered.

Seismic zone	Probable maximum intensities (MMI scale)
0	Below V
I	V
II	VI
III	VII
IV	VIII
V	IX
VI	X and above

Seven seismic zones based on BIS 1962 and 1966 with its expected maximum

MMI Seismic zone Probable maximum intensities (MMI scale) 0 Below V I V II VI III VII IV VIII V IX VI X and above The first Indian seismic code was published in 1962 (IS:1893-1962), It provides seismic design criteria for buildings, bridges, liquid retaining tanks, stacks, gravity dams, earth dams, and retaining walls. In the first zoning exercise (1959-1962) it was thought that since a map is being presented for the first time to the general public, it should conform to historicalindications only. It was thus a useful historical picture of the severity of seismicity of a region. In this zoning map, the Himalaya and northeast India were graded into zones VII-IV, and the Deccan Plateau was marked as zone zero.



Seismic zonation map of India prepared in 1962 (BIS, 1962)

Fig A

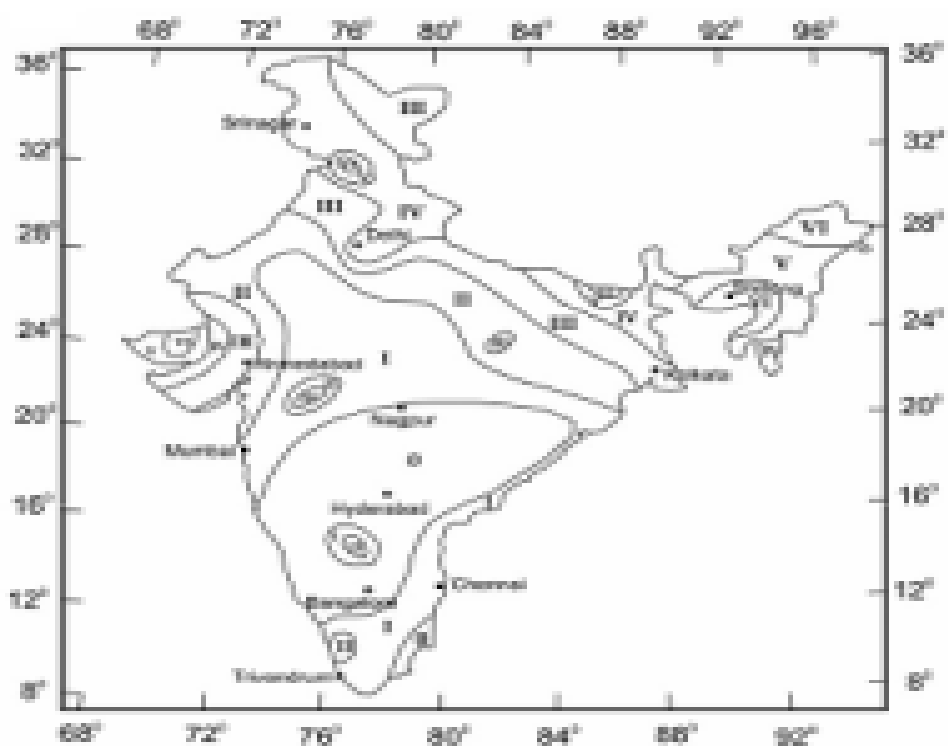


Fig B :Seismic zonation map of India prepared in 1966 (BIS, 1966)

The first formal zone map (IS: 1893-1962) shown in Fig a) divided the country into seven seismic zones (0 to VI) corresponding to areas liable to MM intensity of: less than V, V, VI, VII, VIII, IX, X and above, respectively. Fig 10.2: Seismic zonation map of India prepared in 1962 (BIS, 1962)

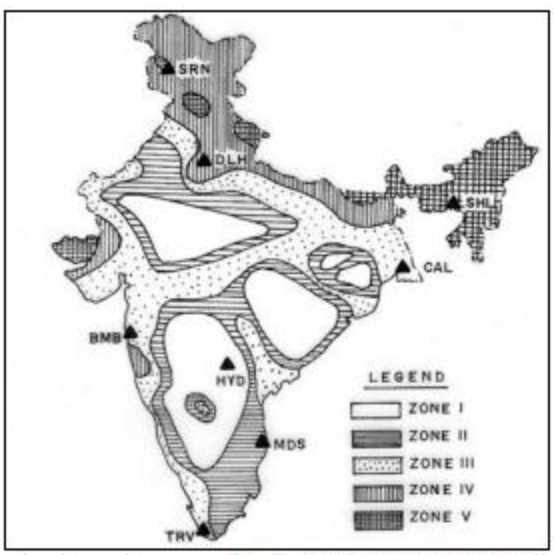
Fig B Seismic zonation map of India prepared in 1966 (BIS, 1966)divided the country into seven seismic zones (0 to VI) corresponding to areas liable to MM intensity of: less than V, V, VI, VII, VIII, IX, X and above, respectively.

Third Zonation Map

Another code (IS: 4326) was published in 1967 (revised in 1976 and 1993): it outlines the aseismic design and construction requirements for buildings. a comprehensive earthquake catalogue was published in 1983 (ISET, 1983). The 1967 Koyna earthquake, M 6.7, Necessity arose to review the zoning, particularly in the Deccan Plateau. Several updated geological, geophysical and seismological information were considered. The zone map went through a major revision (IS: 1893-1970). It reduced the number of zones from seven to five (I to V). Table 10.3 shows the zones and the corresponding probable maximum intensities.

FIVE SEISMIC ZONES

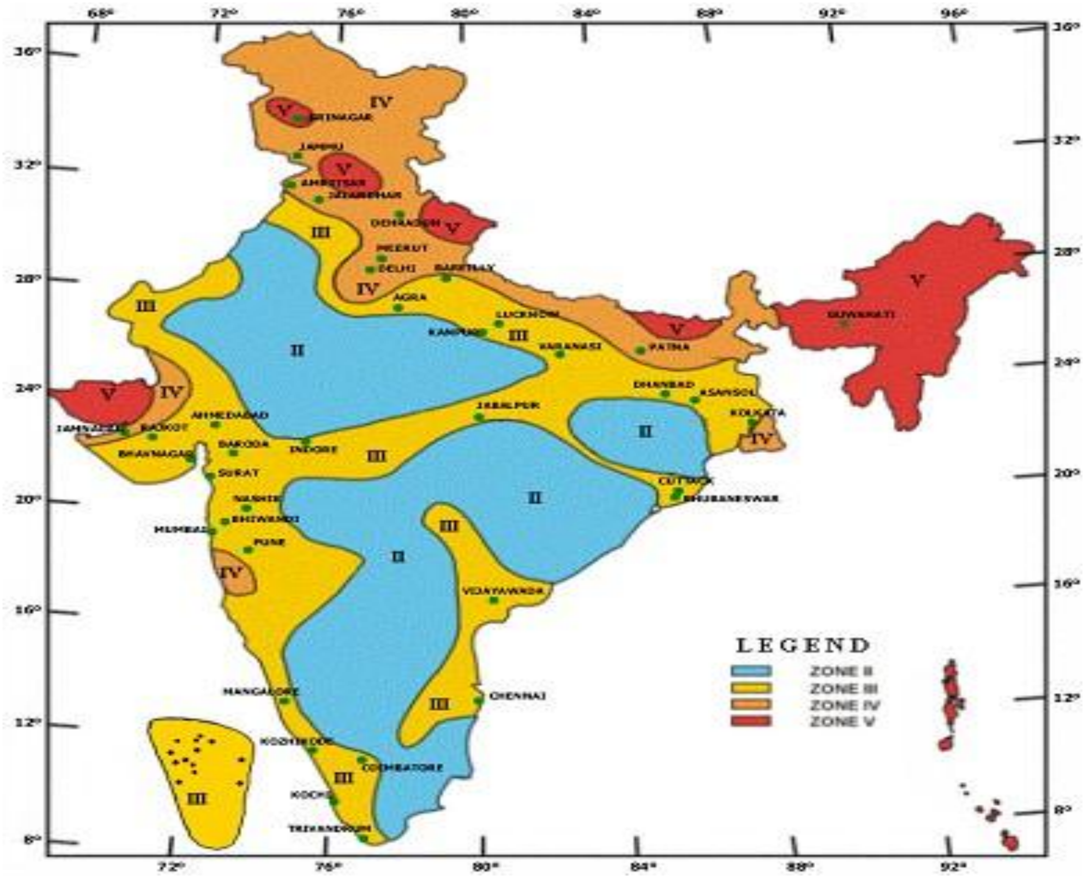
Seismic zone	Probable maximum intensities(MMI scale)
I	V
II	VI
III	VII
IV	VIII
V	IX or more



Seismic Zoning Map of India

The Geological Survey of India (G. S. I.) first published the seismic zoning map of the country in the year 1935. With numerous modifications made afterwards, this map was initially based on the amount of damage suffered by the different regions of India because of earthquakes. Color coded in different shades of the color red, this map shows the four distinct seismic zones of India. Following are the varied seismic zones of the nation, which are prominently shown in the map:

- Zone - II: This is said to be the least active seismic zone.
- Zone - III: It is included in the moderate seismic zone.
- Zone - IV: This is considered to be the high seismic zone.
- Zone - V: It is the highest seismic zone.



STRUCTURAL PERFORMANCE DURING EARTHQUAKES

EARTHQUAKE EFFECTS

There are four basic causes of earthquakeinduced damage: ground shaking, ground failure, tsunamis and fire. The principal cause of earthquake-induced damage is ground shaking. As the earth vibrates, all buildings on the ground surface will respond to that vibration in varying degrees. Earthquake induced accelerations, velocities and displacements can damage or destroy a building unless it has been designed and constructed or strengthened to be earthquake resistant. Therefore, the effect of ground shaking on buildings is a principal area of consideration in the design of earthquake resistant buildings. Seismic design loads are extremely difficult to determine due to the random nature of earthquake motions. However, experiences from past strong earthquakes have shown that reasonable and prudent practices can keep a building safe during an earthquake. 2.2.2 Ground failure Earthquake-induced ground failure has been observed in the form of ground rupture along the fault zone, landslides, settlement and soil liquefaction. Ground rupture along a fault zone may be very limited or may extend over hundreds of kilometers. Ground displacement along the fault may be horizontal, vertical or both, and can be measured in centimeters or even metres. Obviously, a building directly astride such a rupture will be severely damaged or collapsed. While landslide can destroy a building, the settlement may only damage the building. Soil liquefaction can occur in low density saturated sands of relatively uniform size. The phenomenon of liquefaction is particularly important for dams, bridges, underground pipelines, and buildings standing on such ground.

Effect of site conditions on building damage

Past earthquakes show that site condition significantly affects the building damage. Earthquake studies have almost invariably shown that the intensity of a shock is directly related to the type of soil layers supporting the building. Structures built on solid rock and firm soil frequently fares better than buildings on soft ground. This was dramatically demonstrated in the 1985 Mexico City earthquake, where the damage on soft soils in Mexico City, at an epicentral distance of 400 km, was substantially higher than at closer locations. From studies of the July 28, 1957 earthquake in Mexico City, it was already known for example that the damage on the soft soils

OTHER FACTORS AFFECTING DAMAGE

The extent of damage to a building depends much on the strength, ductility, and integrity of a building and the stiffness of ground beneath it in a given intensity of the earthquake motions. Almost any building can be designed to be earthquake resistant provided its site is suitable. Buildings suffer during an earthquake primarily because horizontal forces are exerted on a structure that often meant to contend only with vertical stresses. The principal factors that influence damage to buildings and other man-made structures are listed below:

Building configuration An important feature is regularity and symmetry in the overall shape of a building. A building shaped like a box, as rectangular both in plan and elevation, is inherently stronger than one that is L-shaped or Ushaped, such as a building with wings. An irregularly shaped building will twist as it shakes, increasing the damage.

Opening size In general, openings in walls of a building tend to weaken the walls, and fewer the openings less the damage it will suffer during an earthquake. If it is necessary to have large openings through a building, or if an open first floor is desired, then special provisions should be made to ensure structural integrity.

Rigidity distribution: The rigidity of a building along the vertical direction should be distributed uniformly.Therefore, changes in the structural system of a building from one floor to the next will increase the potential for damage, and should be avoided. Columns or shear walls should run continuously from foundation to the roof, without interruptions or changes in material.

Ductility By ductility is meant the ability of the building to bend, sway, and deform by large amounts without collapse. The opposite condition in a building is called brittleness arising both from the use of materials that are inherently brittle and from the wrong design of structures using otherwise ductile materials. Brittle materials crack under load; some examples are adobe, brick and concrete blocks. It is not surprising that most of the damage during the past earthquakes was to unreinforced masonry structures constructed of brittle materials, poorly tied together. The addition of steel reinforcements can add ductility to brittle materials.

Reinforced concrete, for example, can be made ductile by proper use of reinforcing steel and closely spaced steel ties.

Foundation Buildings, which are structurally strong to withstand earthquakes sometimes, fail due to inadequate foundation design. Tilting, cracking and failure of superstructures may result from soil liquefaction and differential settlement of footing. Certain types of foundations are more susceptible to damage than others. For example, isolated footings of columns are likely to be subjected to differential settlement particularly where the supporting ground consists of different or soft types of soil. Mixed types of foundations within thesame building may also lead to damage due to differential settlement. Very shallow foundations deteriorate because of weathering, particularly when exposed to freezing and thawing in the regions of cold climate.

Construction quality In many instances the failure of buildings in an earthquake has been attributed to poor quality of construction, substandard materials, poor workmanship, e. g., inadequate skill in bonding, absence of through stones or bonding units, and improper and inadequate construction.

Calculation of Design Seismic Force by Static Analysis Method

Problem Statement:

Consider a four-storey reinforced concrete office building shown in Fig. 1.1. The building is located in Shillong (seismic zone V). The soil conditions are medium stiff and the entire building is supported on a raft foundation. The R. C. frames are infilled with brick-masonry. The lumped weight due to dead loads is 12 kN/m^2 on floors and 10 kN/m^2 on the roof. The floors are to cater for a live load of 4 kN/m^2 on floors and 1.5 kN/m^2 on the roof. Determine design seismic load on the structure as per new code.

[Problem adopted from Jain S.K, "A Proposed Draft for IS:1893 Provisions on Seismic Design of Buildings; Part II: Commentary and Examples", Journal of Structural Engineering, Vol.22, No.2, July 1995, pp.73-90]

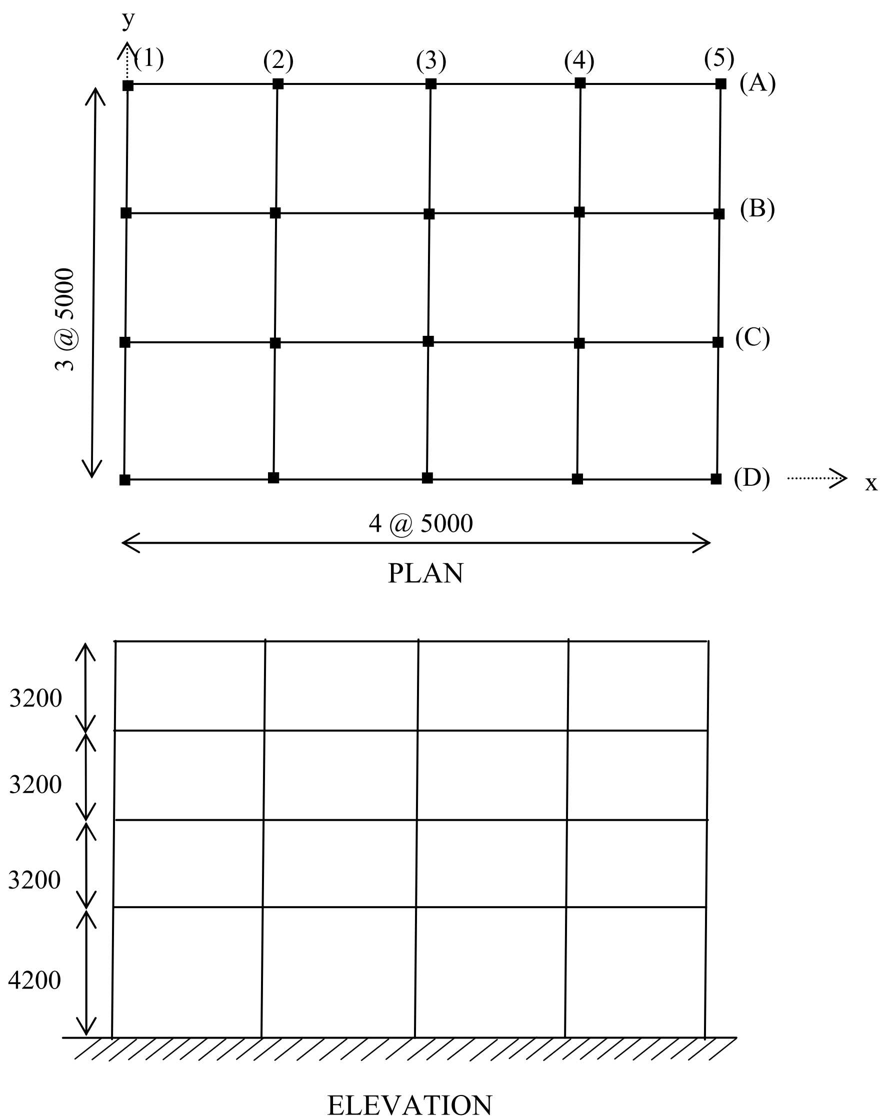


Figure 1.1 – Building configuration

Solution:

Design Parameters:

For seismic zone V, the zone factor Z is 0.36 (Table 2 of IS: 1893). Being an office building, the importance factor, *I*, is 1.0 (Table 6 of IS: 1893). Building is required to be provided with moment resisting frames detailed as per IS: 13920-1993. Hence, the response reduction factor, *R*, is 5.

(Table 7 of IS: 1893 Part 1)

Seismic Weights:

The floor area is 15×20=300 sq. m. Since the live load class is 4kN/sq.m, only 50% of the live load is lumped at the floors. At roof, no live load is to be lumped. Hence, the total seismic weight on the floors and the roof is:

Floors:

$$W_1 = W_2 = W_3 = 300 \times (12 + 0.5 \times 4) = 4{,}200 \text{ kN}$$

Roof:

$$W_4 = 300 \times 10 = 3{,}000 \text{ kN}$$

(clause7.3.1, Table 8 of IS: 1893 Part 1)

Total Seismic weight of the structure,

$$W = \sum W_i = 3 \times 4{,}200 + 3{,}000 = 15{,}600 \text{ kN}$$

Fundamental Period:

Lateral load resistance is provided by moment resisting frames infilled with brick masonry panels. Hence, approximate fundamental natural period:

(Clause 7.6.2. of IS: 1893 Part 1)

EL in X-Direction:

$$T = 0.09h/\sqrt{d}$$

$$= 0.09(13.8)/\sqrt{20}$$

$$= 0.28 \text{ sec}$$

The building is located on Type II (medium soil). From Fig. 2 of IS: 1893, for T=0.28 sec, S_a/g = 2.5

$$A_h = \frac{ZI}{2R}\frac{S_a}{g}$$

$$= \frac{0.36 \times 1.0}{2 \times 5} \times 2.5$$

$$= 0.09$$

(Clause 6.4.2 of IS: 1893 Part 1)

Design base shear

$$V_B = A_h W$$

$$= 0.09 \times 15{,}600$$

$$= 1{,}440 \text{ kN}$$

(Clause 7.5.3 of IS: 1893 Part 1)

Force Distribution with Building Height:

The design base shear is to be distributed with height as per clause 7.7.1. Table 1.1 gives the calculations. Fig. 1.2(a) shows the design seismic force in X-direction for the entire building.

EL in Y-Direction:

$$T = 0.09\,h/\sqrt{d}$$

$$= 0.09(13.8)/\sqrt{15}$$

$$= 0.32 \text{ sec}$$

$$\frac{S_a}{g} = 2.5;$$

$$A_h = 0.09$$

Therefore, for this building the design seismic force in Y-direction is same as that in the X-direction. Fig. 1.2(b) shows the design seismic force on the building in the Y-direction.

Table 1.1 – Lateral Load Distribution with Height by the Static Method

Storey Level	$W_i\ (kN)$	h_i (m)	$W_i h_i^2 \times (1000)$	$\frac{W_i h_i^2}{\sum W_i h_i^2}$	Lateral Force at i^{th} Level for EL in direction (kN)	
					X	Y
4	3,000	13.8	571.3	0.424	611	611
3	4,200	10.6	471.9	0.350	504	504
2	4,200	7.4	230.0	0.171	246	246
1	4,200	4.2	74.1	0.055	79	79
Σ			**1,347.3**	**1,000**	**1,440**	**1,440**

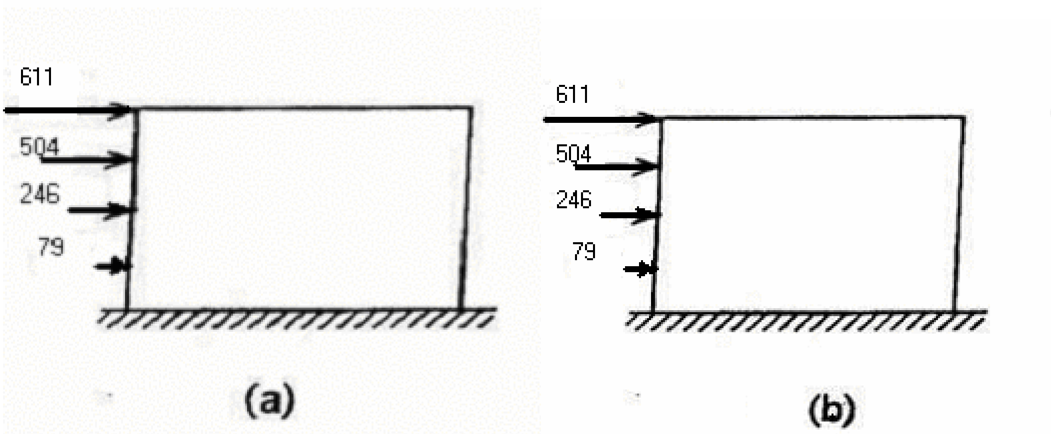


Figure 1.2 -- Design seismic force on the building for (a) X-direction, and (b) Y-direction.

Example 2 – Calculation of Design Seismic Force by Dynamic Analysis Method

Problem Statement:

For the building of Example 1, the dynamic properties (natural periods, and mode shapes) for vibration in the X-direction have been obtained by carrying out a free vibration analysis (Table 2.1). Obtain the design seismic force in the X-direction by the dynamic analysis method outlined in cl. 7.8.4.5 and distribute it with building height.

Table 2.1 – Free Vibration Properties of the building for vibration in the X-Direction

	Mode 1	Mode 2	Mode 3
Natural Period (sec)	0.860	0.265	0.145
	Mode Shape		
Roof	1.000	1.000	1.000
3rd Floor	0.904	0.216	-0.831
2nd Floor	0.716	-0.701	-0.574
1st Floor	0.441	-0.921	1.016

[Problem adopted from, Jain S.K, "A Proposed Draft for IS: 1893 Provisions on Seismic Design of Buildings; Part II: Commentary and Examples", Journal of Structural Engineering, Vol.22, No.2, July 1995, pp.73-90]

Solution:

Table 2.2 -- Calculation of modal mass and modal participation factor (clause 7.8.4.5)

Storey Level i	Weight $W_i(kN)$	Mode 1			Mode 2			Mode 3		
4	3,000	1.000	3,000	3,000	1.000	3,000	3,000	1.000	3,000	3,000
3	4,200	0.904	3,797	3,432	0.216	907	196	-0.831	-3,490	2,900
2	4,200	0.716	3,007	2,153	-0.701	-2,944	2,064	-0.574	-2,411	1,384
1	4,200	0.441	1,852	817	-0.921	-3,868	3,563	1.016	4,267	4,335
Σ	15,600		11,656	9,402		-2,905	8,822		1,366	11,620
$M_k = \dfrac{\left[\sum w_i\ \phi_{ik}\right]^2}{g\sum w_i\ \phi_{ik}^2}$		$\dfrac{11{,}656^2}{9{,}402g} = \dfrac{14{,}450kN}{g}$ = 14,45,000 kg			$\dfrac{2{,}905^2}{8{,}822g} = \dfrac{957kN}{g}$ =95,700 kg			$\dfrac{1{,}366^2}{11{,}620g} = \dfrac{161kN}{g}$ = 16,100 kg		
% of Total weight		92.6%			6.1%			1.0%		
$P_k = \dfrac{\sum w_i\ \phi_{ik}}{\sum w_i\ \phi_{ik}^2}$		$\dfrac{11{,}656}{9{,}402} = 1.240$			$\dfrac{-2{,}905}{8{,}822} = -0.329$			$\dfrac{1{,}366}{11{,}620} = 0.118$		

It is seen that the first mode excites 92.6% of the total mass. Hence, in this case, codal requirements on number of modes to be considered such that at least 90% of the total mass is excited, will be satisfied by considering the first mode of vibration only. However, for illustration, solution to this example considers the first three modes of vibration.

The lateral load Q_{ik} acting at i^{th} floor in the k^{th} mode is

$$Q_{ik} = A_{hk}\phi_{ik}\ P_k\ W_i$$

(clause 7.8.4.5 c of IS: 1893 Part 1)

The value of A_{hk} for different modes is obtained from clause 6.4.2.

Mode 1:

$T_1 = 0.860$ sec;

$(S_a/g) = \dfrac{1.0}{0.86} = 1.16$;

$$A_{h1} = \frac{ZI}{2R}(S_a/g) = \frac{0.36\times 1}{2\times 5}\times(1.16) = 0.0418$$

$$Q_{i1} = 0.0418 \times 1.240 \times \phi_{i1} \times W_i$$

Mode 2:

$T_2 = 0.265$ sec;

$(S_a/g) = 2.5$;

$$A_{h2} = \frac{ZI}{2R}(S_a/g) = \frac{0.36\times 1}{2\times 5}\times(2.5) = 0.09$$

$$Q_{i1} = 0.09 \times (-0.329) \times \phi_{i2} \times W_i$$

Mode 3:

$T_3 = 0.145$ sec;

$(S_a/g) = 2.5$;

$$A_{h3} = \frac{ZI}{2R}(S_a/g) = \frac{0.36\times 1}{2\times 5}\times(2.5) = 0.09$$

$$Q_{i3} = 0.09 \times (0.118) \times \phi_{i3} \times W_i$$

Table 2.3 summarizes the calculation of lateral load at different floors in each mode.

Table 2.3 – Lateral load calculation by modal analysis method (earthquake in X-direction)

Floor Level i	Weight W_i (kN)	Mode 1			Mode 2			Mode 3		
		ϕ_{i1}	Q_{i1}	V_{i1}	ϕ_{i2}	Q_{i2}	V_{i2}	ϕ_{i3}	Q_{i3}	V_{i3}
4	3,000	1.000	155.5	155.5	1.000	-88.8	-88.8	1.000	31.9	31.9
3	4,200	0.904	196.8	352.3	0.216	-26.8	-115.6	-0.831	-37.1	-5.2
2	4,200	0.716	155.9	508.2	-0.701	87.2	-28.4	-0.574	-25.6	-30.8
1	4,200	0.441	96.0	604.2	-0.921	114.6	86.2	1.016	45.4	14.6

Since all of the modes are well separated (clause 3.2), the contribution of different modes is combined by the SRSS (square root of the sum of the square) method

$V_4 = [(155.5)^2 + (88.8)^2 + (31.9)^2]^{1/2} = 182$ kN

$V_3 = [(352.3)^2 + (115.6)^2 + (5.2)^2]^{1/2} = 371$ kN

$V_2 = [(508.2)^2 + (28.4)^2 + (30.8)^2]^{1/2} = 510$ kN

$V_1 = [(604.2)^2 + (86.2)^2 + (14.6)^2]^{1/2} = 610$ kN

(Clause 7.8.4.4a of IS: 1893 Part 1)

The externally applied design loads are then obtained as:

$Q_4 = V_4 = 182$ kN
$Q_3 = V_3 - V_4 = 371 - 182 = 189$ kN
$Q_2 = V_2 - V_3 = 510 - 371 = 139$ kN
$Q_1 = V_1 - V_2 = 610 - 510 = 100$ kN

(Clause 7.8.4.5f of IS: 1893 Part 1)

Clause 7.8.2 requires that the base shear obtained by dynamic analysis (V_B = 610 kN) be compared with that obtained from empirical fundamental period as per Clause 7.6. If V_B is less than that from empirical value, the response quantities are to be scaled up.

We may interpret "base shear calculated using a fundamental period as per 7.6" in two ways:

1. We calculate base shear as per Cl. 7.5.3. This was done in the previous example for the same building and we found the base shear as 1,404 *kN*. Now, dynamic analysis gives us base shear of 610 *kN* which is lower. Hence, all the response quantities are to be scaled up in the ratio (1,404/610 = 2.30). Thus, the seismic forces obtained above by dynamic analysis should be scaled up as follows:

$Q_4 = 182 \times 2.30 = 419$ kN
$Q_3 = 189 \times 2.30 = 435$ kN
$Q_2 = 139 \times 2.30 = 320$ kN

$Q_1 = 100 \times 2.30 = 230$ kN

2. We may also interpret this clause to mean that we redo the dynamic analysis but replace the fundamental time period value by T_a (= 0.28 sec). In that case, for mode 1:

$T_1 = 0.28$ sec;
$(S_a / g) = 2.5$;

$$A_{h1} = \frac{ZI}{2R}(S_a / g)$$
=0.09

Modal mass times A_{h1}
= 14,450 × 0.09
= 1,300 kN

Base shear in modes 2 and 3 is as calculated earlier: Now, base shear in first mode of vibration =1300 kN, 86.2 kN and 14.6 kN, respectively.

Total base shear by SRSS

$$= \sqrt{1300^2 + 86.2^2 + 14.6^2}$$
= 1,303 kN

Notice that most of the base shear is contributed by first mode only. In this interpretation of Cl 7.8.2, we need to scale up the values of response quantities in the ratio (1,303/610 = 2.14). For instance, the external seismic forces at floor levels will now be:

$Q_4 = 182 \times 2.14 = 389$ kN
$Q_3 = 189 \times 2.14 = 404$ kN
$Q_2 = 139 \times 2.14 = 297$ kN
$Q_1 = 100 \times 2.14 = 214$ kN

Clearly, the second interpretation gives about 10% lower forces. We could make either interpretation. Herein we will proceed with the values from the second interpretation and compare the design values with those obtained in Example 1 as per static analysis:

Table 2.4 – Base shear at different storeys

Floor Level *i*	*Q* (static)	*Q* (dynamic, scaled)	*Storey Shear V* (static)	*Storey ShearV* (dynamic, scaled)	Storey Moment, *M* (Static)	Storey Moment, *M* (Dynamic)
4	611 kN	389 kN	611 kN	389 kN	1,907 kNm	1,245 kNm
3	504 kN	404 kN	1,115kN	793 kN	5,386 kNm	3,782 kNm
2	297 kN	297 kN	1,412kN	1,090 kN	9.632 kNm	7,270 kNm
1	79 kN	214 kN	1,491 kN	1,304 kN	15,530 kNm	12,750 kNm

Notice that even though the base shear by the static and the dynamic analyses are comparable, there is considerable difference in the lateral load distribution with building height, and therein lies the advantage of dynamic analysis. For instance, the storey moments are significantly affected by change in load distribution.

Example 3 – Location of Centre of Mass

Problem Statement:

Locate centre of mass of a building having non-uniform distribution of mass as shown in the figure 3.1

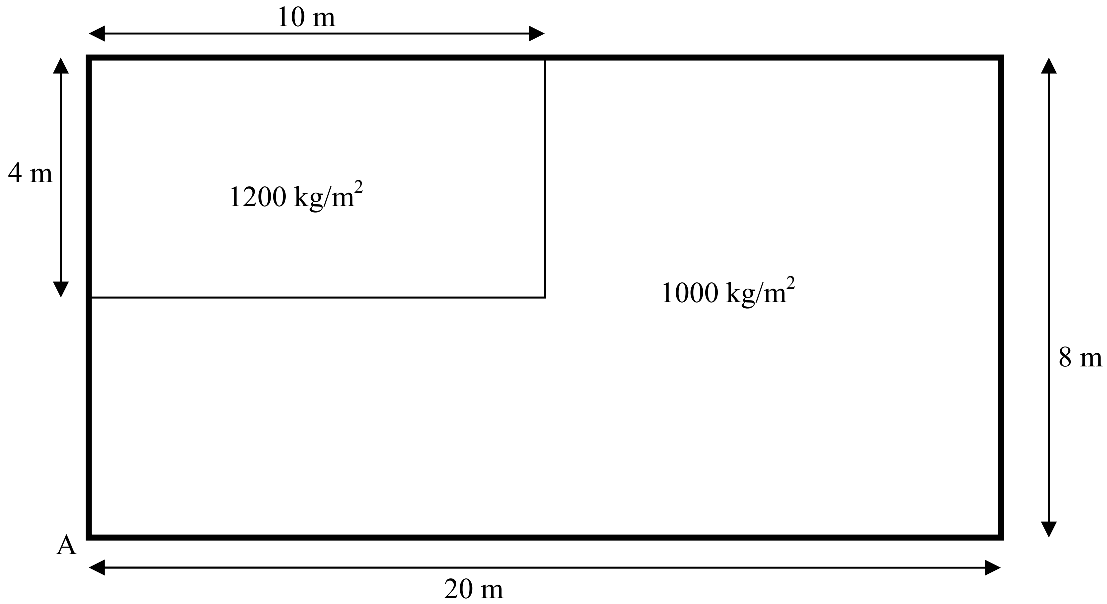


Figure 3.1 –Plan

Solution:

Let us divide the roof slab into three rectangular parts as shown in figure 2.1

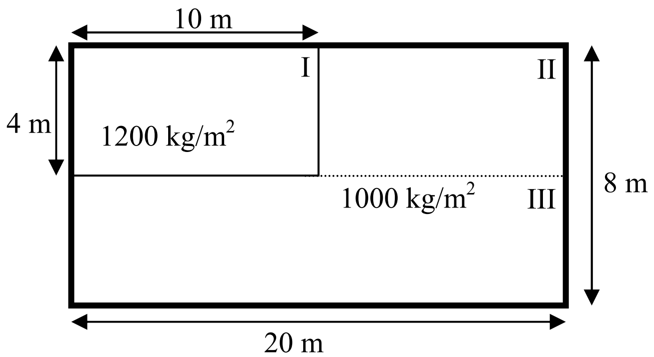


Figure 3.2

Mass of part I is 1200 kg/m^2, while that of the other two parts is 1000 kg/m^2.

Let origin be at point A, and the coordinates of the centre of mass be at (X, Y)

$$X = \frac{(10\times4\times1200)\times5+(10\times4\times1000)\times15+(20\times4\times1000)\times10}{(10\times4\times1200)+(10\times4\times1000)+(20\times4\times1000)}$$

= 9.76 m

$$Y = \frac{(10\times4\times1200)\times6+(10\times4\times1000)\times6+(20\times4\times1000)\times2}{(10\times4\times1200)+(10\times4\times1000)+(20\times4\times1000)}$$

= 4.1 m

Hence, coordinates of centre of mass are (9.76, 4.1)

Example 4 – Location of Centre of Stiffness

Problem Statement:

The plan of a simple one storey building is shown in figure 3.1. All columns and beams are same. Obtain its centre of stiffness.

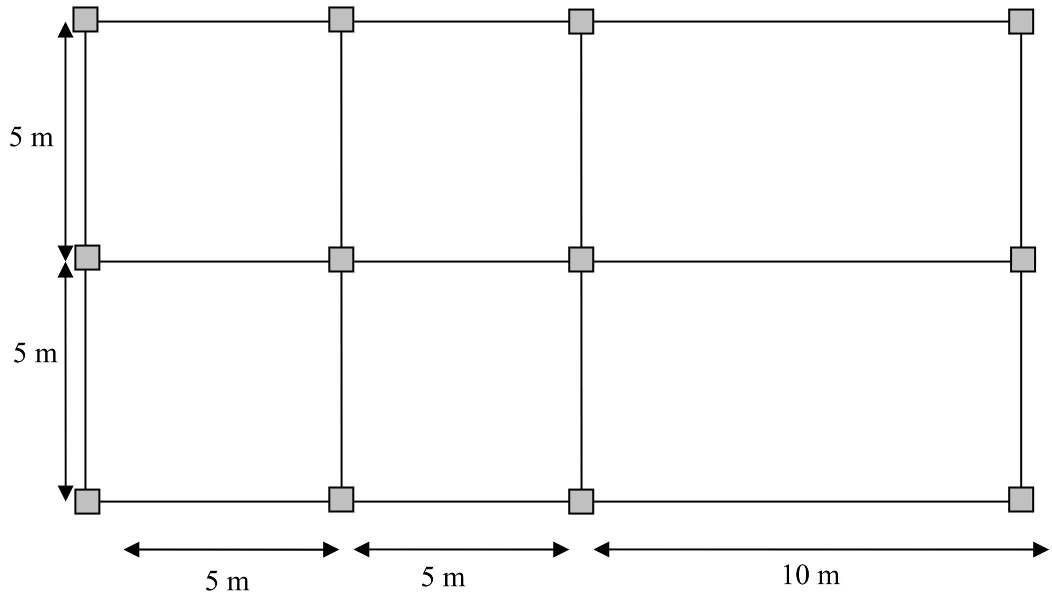


Figure 4.1 –Plan

Solution:

In the X-direction there are three identical frames located at uniform spacing. Hence, the y-coordinate of centre of stiffness is located symmetrically, i.e., at 5.0 m from the left bottom corner.

In the Y-direction, there are four identical frames having equal lateral stiffness. However, the spacing is not uniform. Let the lateral stiffness of each transverse frame be *k*, and coordinating of center of stiffness be (X, Y).

$$X = \frac{k \times 0 + k \times 5 + k \times 10 + k \times 20}{k + k + k + k} = 8.75 \text{ m}$$

Hence, coordinates of centre of stiffness are (8.75, 5.0).

Example 5 –Lateral Force Distribution as per Torsion Provisions of IS 1893-2002 (Part 1)

Problem Statement:

Consider a simple one-storey building having two shear walls in each direction. It has some gravity columns that are not shown. All four walls are in M25 grade concrete, 200 thick and 4 m long. Storey height is 4.5 m. Floor consists of cast-in-situ reinforced concrete. Design shear force on the building is 100 kN in either direction.

Compute design lateral forces on different shear walls using the torsion provisions of 2002 edition of IS 1893 (Part 1).

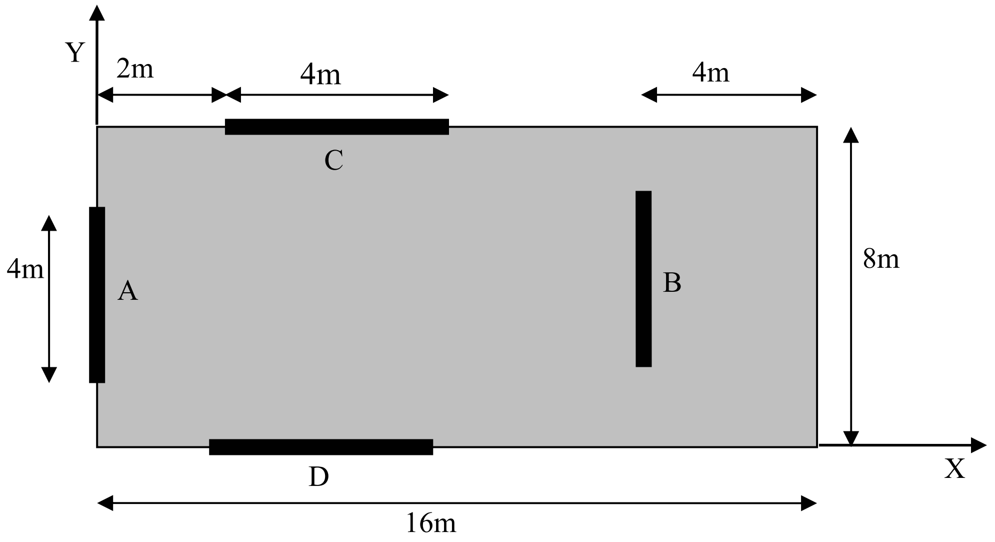


Figure 5.1 – Plan

Solution:

Grade of concrete: M25

$E = 5000\sqrt{25} = 25000 \text{ N/mm}^2$

Storey height h = 4500 m

Thickness of wall t = 200 mm

Length of walls L = 4000 mm

All walls are same, and hence, spaces have same lateral stiffness, k.

Centre of mass (CM) will be the geometric centre of the floor slab, i.e., (8.0, 4.0).

Centre of rigidity (CR) will be at (6.0, 4.0).

EQ Force in X-direction:

Because of symmetry in this direction, calculated eccentricity = 0.0 m

Design eccentricity:

$$e_d = 1.5 \times 0.0 + 0.05 \times 8 = 0.4,$$

and

$$e_d = 0.0 - 0.05 \times 8 = -0.4$$

(Clause 7.9.2 of IS 1893:2002)

Lateral forces in the walls due to translation:

$$F_{CT} = \frac{K_C}{K_C + K_D} F = 50.0 \text{ kN}$$

$$F_{DT} = \frac{K_D}{K_C + K_D} F = 50.0 \text{ kN}$$

Lateral forces in the walls due to torsional moment:

$$F_{iR} = \frac{K_i r_i}{\sum\limits_{i=A,B,C,D} K_i r_i^2} (Fe_d)$$

where r_i is the distance of the shear wall from CR.

All the walls have same stiffness, $K_A = K_B = K_C = K_D = k$, and

r_A = -6.0 m

r_B = -6.0 m

r_C = 4.0 m
r_D = -4.0 m,
and $e_d = \pm 0.4$ m

Therefore,

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k}\left(Fe_d\right)$$

$= \pm 2.31\,\text{kN}$

Similarly,
$F_{BR} = \pm 2.31\,\text{kN}$
$F_{CR} = \pm 1.54\,\text{kN}$
$F_{DR} = \pm 1.54\,\text{kN}$

Total lateral forces in the walls due to seismic load in X direction:

F_A = 2.31 kN
F_B = 2.31 kN
F_C = Max (50 ± 1.54) = 51.54 kN
F_D = Max (50 ± 1.54) = 51.54 kN

EQ Force in Y-direction:

Calculated eccentricity= 2.0 m
Design eccentricity:
$e_d = 1.5 \times 2.0 + 0.05 \times 16 = 3.8\,\text{m}$
or $= 2.0 - 0.05 \times 16 = 1.2\,\text{m}$

Lateral forces in the walls due to translation:

$$F_{AT} = \frac{K_A}{K_A + K_B}F = 50.0 \text{ kN}$$

$$F_{BT} = \frac{K_B}{K_A + K_B}F = 50.0 \text{ kN}$$

Lateral force in the walls due to torsional moment: when e_d = 3.8 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k}\left(Fe_d\right) = -$$

21.92 kN

Similarly,
F_{BR} = 21.92 kN
F_{CR} = -14.62 kN
F_{DR} = 14.62 kN

Total lateral forces in the walls:
F_A = 50 - 21.92= 28.08 kN
F_B = 50 +20.77= 71.92 kN
F_C = -14.62 kN
F_D = 14.62 kN

Similarly, when e_d = 1.2 m, then the total lateral forces in the walls will be,
F_A = 50 – 6.93 = 43.07 kN
F_B = 50 + 6.93 = 56.93 kN
F_C = - 4.62 kN
F_D = 4.62 kN

Maximum forces in walls due to seismic load in Y direction:
F_A = Max (28.08, 43.07) = 43.07 kN;
F_B = Max (71.92, 56.93) = 71.92 kN;
F_C = Max (14.62, 4.62) = 14.62 kN;
F_D = Max (14.62, 4.62) = 14.62 kN;

Combining the forces obtained from seismic loading in X and Y directions:
F_A = 43.07 kN
F_B =71.92 kN
F_C =51.54 kN
F_D =51.54 kN.

However, note that clause 7.9.1 also states that “However, negative torsional shear shall be neglected”. Hence, wall A should be designed for not less than 50 kN.

Example 6 – Lateral Force Distribution as per New Torsion Provisions

Problem Statement:

For the building of example 5, compute design lateral forces on different shear walls using the torsion provisions of revised draft code IS 1893 (part 1), i.e., IITK-GSDMA-EQ05-V2.0.

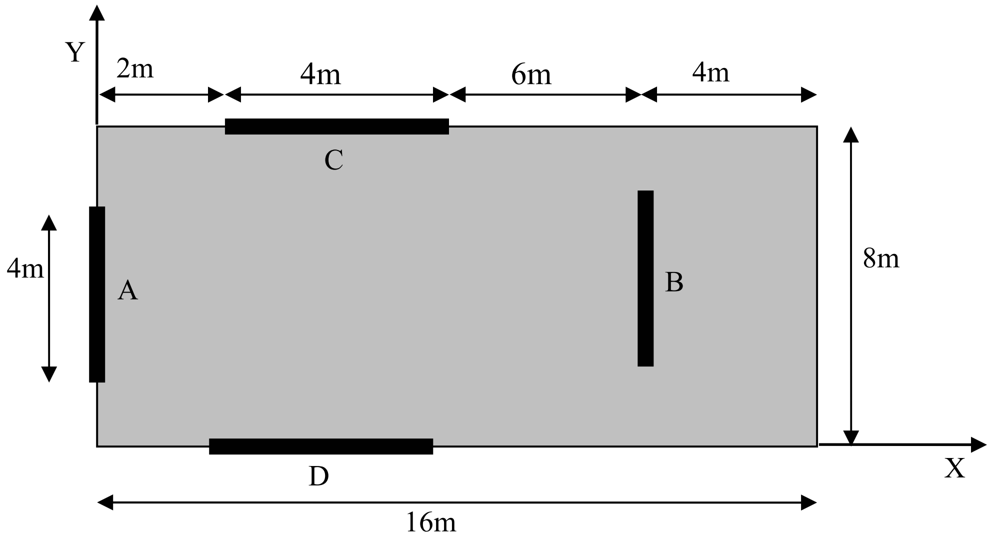


Figure 6.1 – Plan

Solution:

Grade of concrete: M25

$E = 5000\sqrt{25} = 25000$ N/mm^2

Storey height h = 4500 m

Thickness of wall t = 200 mm

Length of walls L = 4000 mm

All walls are same, and hence, same lateral stiffness, k.

Centre of mass (CM) will be the geometric centre of the floor slab, i.e., (8.0, 4.0).

Centre of rigidity (CR) will be at (6.0, 4.0).

EQ Force in X-direction:

Because of symmetry in this direction, calculated eccentricity = 0.0 m

Design eccentricity, $e_d = 0.0 \pm 0.1 \times 8 = \pm 0.8$
(clause 7.9.2 of Draft IS 1893: (Part1))

Lateral forces in the walls due to translation:

$$F_{CT} = \frac{K_C}{K_C + K_D} F = 50.0 \text{ kN}$$

$$F_{DT} = \frac{K_D}{K_C + K_D} F = 50.0 \text{ kN}$$

Lateral forces in the walls due to torsional moment:

$$F_{iR} = \frac{K_i r_i}{\sum\limits_{i=A,B,C,D} K_i r_i^2} (Fe_d)$$

where r_i is the distance of the shear wall from CR

All the walls have same stiffness, $K_A = K_B = K_C = K_D = k$

r_A= -6.0 m
r_B= -6.0 m
r_C= 4.0 m
r_D= -4.0 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k} (Fe_d)$$

= - 4.62 kN

Similarly,

F_{BR} = 4.62 kN
F_{CR} = 3.08 kN
F_{DR} = -3.08 kN

Total lateral forces in the walls:

F_A = 4.62 kN
F_B = - 4.62 kN
F_C = 50+3.08 = 53.08 kN
F_D = 50-3.08 = 46.92 kN

Similarly, when e_d= - 0.8 m, then the lateral forces in the walls will be,

F_A = - 4.62 kN
F_B = 4.62 kN
F_C = 50-3.08 = 46.92 kN
F_D = 50+3.08 = 53.08kN

Design lateral forces in walls C and D are:
F_C= F_D= 53.05 kN

EQ Force in Y-direction:

Calculated eccentricity= 2.0 m
Design eccentricity,

$$e_d = 2.0 + 0.1 \times 16 = 3.6\ m$$

or

$$e_d = 2.0 - 0.1 \times 16 = 0.4\ m$$

Lateral forces in the walls due to translation:

$$F_{AT} = \frac{K_A}{K_A + K_B} F = 50.0 \text{ kN}$$

$$F_{BT} = \frac{K_B}{K_A + K_B} F = 50.0 \text{ kN}$$

Lateral force in the walls due to torsional moment: when e_d= 3.6 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k}\left(Fe_d\right) = -20.77 \text{ kN}$$

Similarly,

F_{BR} = 20.77 kN
F_{CR} = 13.85 kN
F_{DR} = -13.8 kN

Total lateral forces in the walls:

F_A = 50-20.77= 29.23 kN
F_B = 50+20.77= 70.77 kN
F_C = 13.85 kN
F_D = -13.85 kN

Similarly, when e_d= 0.4 m, then the total lateral forces in the walls will be,

F_A = 50-2.31= 47.69 kN
F_B = 50+2.31= 53.31 kN
F_C = 1.54 kN
F_D = - 1.54 kN

Maximum forces in walls A and B
F_A =47.69 kN, F_B =70.77 kN

Design lateral forces in all the walls are as follows:

F_A =47.69 kN
F_B =70.77 kN
F_C =53.05 kN
F_D =53.05 kN.

Calculation of Design Seismic Force by Static Analysis Method

Problem Statement:

Consider a four-storey reinforced concrete office building shown in Fig. 1.1. The building is located in Shillong (seismic zone V). The soil conditions are medium stiff and the entire building is supported on a raft foundation. The R. C. frames are infilled with brick-masonry. The lumped weight due to dead loads is 12 kN/m^2 on floors and 10 kN/m^2 on the roof. The floors are to cater for a live load of 4 kN/m^2 on floors and 1.5 kN/m^2 on the roof. Determine design seismic load on the structure as per new code.

[Problem adopted from Jain S.K, “A Proposed Draft for IS:1893 Provisions on Seismic Design of Buildings; Part II: Commentary and Examples”, Journal of Structural Engineering, Vol.22, No.2, July 1995, pp.73-90]

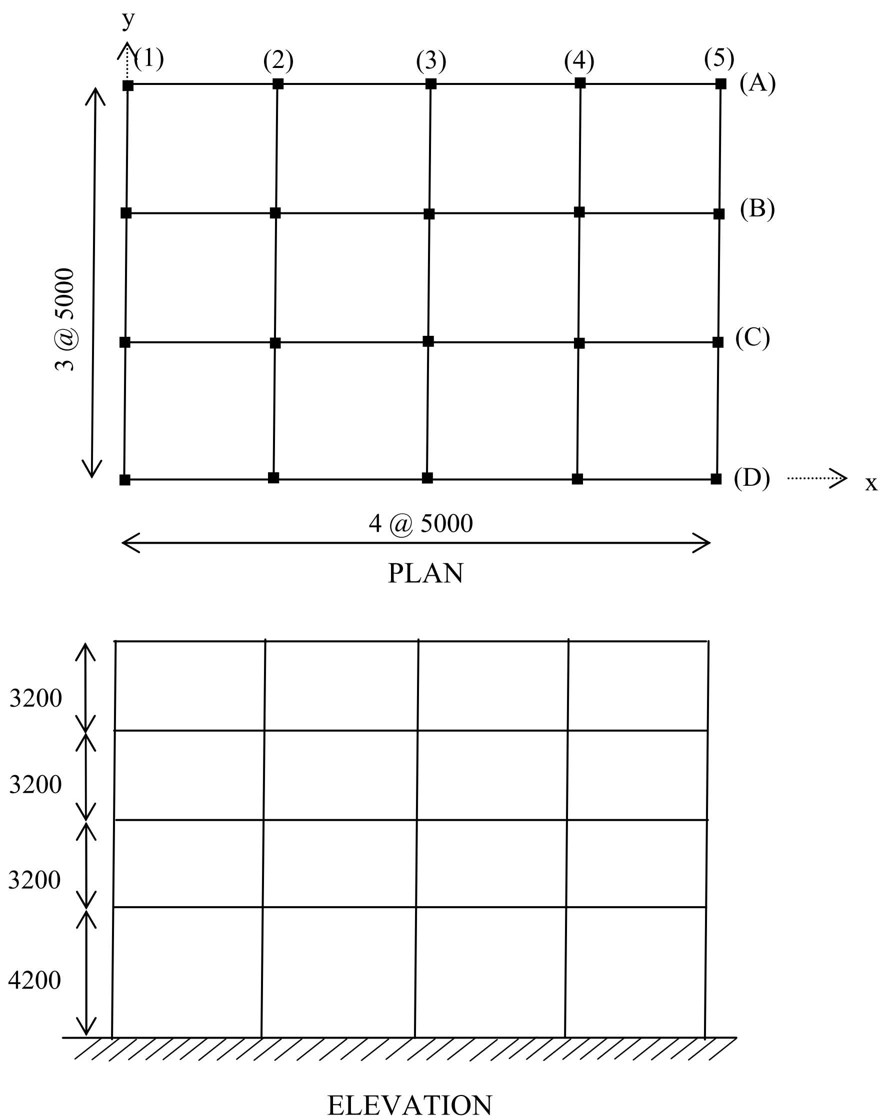


Figure 1.1 – Building configuration

Solution:

Design Parameters:

For seismic zone V, the zone factor Z is 0.36 (Table 2 of IS: 1893). Being an office building, the importance factor, *I*, is 1.0 (Table 6 of IS: 1893). Building is required to be provided with moment resisting frames detailed as per IS: 13920-1993. Hence, the response reduction factor, *R*, is 5.

(Table 7 of IS: 1893 Part 1)

Seismic Weights:

The floor area is 15×20=300 sq. m. Since the live load class is 4kN/sq.m, only 50% of the live load is lumped at the floors. At roof, no live load is to be lumped. Hence, the total seismic weight on the floors and the roof is:

Floors:

$W_1 = W_2 = W_3 = 300\times(12+0.5\times4)$
$= 4{,}200$ kN

Roof:

$W_4 = 300\times10$
$= 3{,}000$ kN

(clause7.3.1, Table 8 of IS: 1893 Part 1)

Total Seismic weight of the structure,

$W = \sum W_i = 3\times4{,}200 + 3{,}000$
$= 15{,}600$ kN

Fundamental Period:

Lateral load resistance is provided by moment resisting frames infilled with brick masonry panels. Hence, approximate fundamental natural period:

(Clause 7.6.2. of IS: 1893 Part 1)

EL in X-Direction:

$T = 0.09h/\sqrt{d}$
$= 0.09(13.8)/\sqrt{20}$
$= 0.28$ sec

The building is located on Type II (medium soil). From Fig. 2 of IS: 1893, for T=0.28 sec, S_a/g = 2.5

$$A_h = \frac{ZI}{2R}\frac{S_a}{g}$$
$$= \frac{0.36\times1.0}{2\times5}\times2.5$$
$$= 0.09$$

(Clause 6.4.2 of IS: 1893 Part 1)

Design base shear

$$V_B = A_h W$$
$$= 0.09\times15{,}600$$
$$= 1{,}440 \text{ kN}$$

(Clause 7.5.3 of IS: 1893 Part 1)

Force Distribution with Building Height:

The design base shear is to be distributed with height as per clause 7.7.1. Table 1.1 gives the calculations. Fig. 1.2(a) shows the design seismic force in X-direction for the entire building.

EL in Y-Direction:

$$T = 0.09h/\sqrt{d}$$
$$= 0.09(13.8)/\sqrt{15}$$
$$= 0.32 \text{ sec}$$

$\frac{S_a}{g} = 2.5;$

$A_h = 0.09$

Therefore, for this building the design seismic force in Y-direction is same as that in the X-direction. Fig. 1.2(b) shows the design seismic force on the building in the Y-direction.

Table 1.1 – Lateral Load Distribution with Height by the Static Method

Storey Level	$W_i\ (kN)$	h_i (m)	$W_i h_i^2 \times (1000)$	$\frac{W_i h_i^2}{\sum W_i h_i^2}$	Lateral Force at i^{th} Level for EL in direction (kN)	
					X	Y
4	3,000	13.8	571.3	0.424	611	611
3	4,200	10.6	471.9	0.350	504	504
2	4,200	7.4	230.0	0.171	246	246
1	4,200	4.2	74.1	0.055	79	79
Σ			**1,347.3**	**1,000**	**1,440**	**1,440**

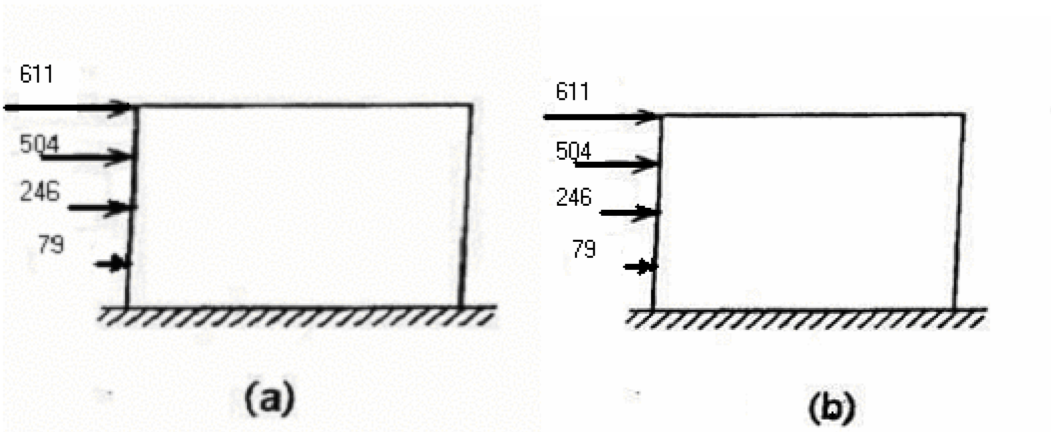


Figure 1.2 -- Design seismic force on the building for (a) X-direction, and (b) Y-direction.

Example 2 – Calculation of Design Seismic Force by Dynamic Analysis Method

Problem Statement:

For the building of Example 1, the dynamic properties (natural periods, and mode shapes) for vibration in the X-direction have been obtained by carrying out a free vibration analysis (Table 2.1). Obtain the design seismic force in the X-direction by the dynamic analysis method outlined in cl. 7.8.4.5 and distribute it with building height.

Table 2.1 – Free Vibration Properties of the building for vibration in the X-Direction

	Mode 1	Mode 2	Mode 3
Natural Period (sec)	0.860	0.265	0.145
	Mode Shape		
Roof	1.000	1.000	1.000
3rd Floor	0.904	0.216	-0.831
2nd Floor	0.716	-0.701	-0.574
1st Floor	0.441	-0.921	1.016

[Problem adopted from, Jain S.K, "A Proposed Draft for IS: 1893 Provisions on Seismic Design of Buildings; Part II: Commentary and Examples", Journal of Structural Engineering, Vol.22, No.2, July 1995, pp.73-90]

Solution:

Table 2.2 -- Calculation of modal mass and modal participation factor (clause 7.8.4.5)

Storey Level i	Weight $W_i(kN)$	Mode 1			Mode 2			Mode 3		
4	3,000	1.000	3,000	3,000	1.000	3,000	3,000	1.000	3,000	3,000
3	4,200	0.904	3,797	3,432	0.216	907	196	-0.831	-3,490	2,900
2	4,200	0.716	3,007	2,153	-0.701	-2,944	2,064	-0.574	-2,411	1,384
1	4,200	0.441	1,852	817	-0.921	-3,868	3,563	1.016	4,267	4,335
Σ	15,600		11,656	9,402		-2,905	8,822		1,366	11,620
$M_k = \dfrac{\left[\sum w_i\ \phi_{ik}\right]^2}{g\sum w_i\ \phi_{ik}^2}$		$\dfrac{11{,}656^2}{9{,}402g} = \dfrac{14{,}450kN}{g}$ = 14,45,000 kg			$\dfrac{2{,}905^2}{8{,}822g} = \dfrac{957kN}{g}$ =95,700 kg			$\dfrac{1{,}366^2}{11{,}620g} = \dfrac{161kN}{g}$ = 16,100 kg		
% of Total weight		92.6%			6.1%			1.0%		
$P_k = \dfrac{\sum w_i\ \phi_{ik}}{\sum w_i\ \phi_{ik}^2}$		$\dfrac{11{,}656}{9{,}402} = 1.240$			$\dfrac{-2{,}905}{8{,}822} = -0.329$			$\dfrac{1{,}366}{11{,}620} = 0.118$		

It is seen that the first mode excites 92.6% of the total mass. Hence, in this case, codal requirements on number of modes to be considered such that at least 90% of the total mass is excited, will be satisfied by considering the first mode of vibration only. However, for illustration, solution to this example considers the first three modes of vibration.

The lateral load Q_{ik} acting at i^{th} floor in the k^{th} mode is

$$Q_{ik} = A_{hk}\phi_{ik}\ P_k\ W_i$$

(clause 7.8.4.5 c of IS: 1893 Part 1)

The value of A_{hk} for different modes is obtained from clause 6.4.2.

Mode 1:

$T_1 = 0.860$ sec;

$(S_a / g) = \dfrac{1.0}{0.86} = 1.16$;

$$A_{h1} = \frac{ZI}{2R}(S_a / g) = \frac{0.36 \times 1}{2 \times 5} \times (1.16) = 0.0418$$

$$Q_{i1} = 0.0418 \times 1.240 \times \phi_{i1} \times W_i$$

Mode 2:

$T_2 = 0.265$ sec;

$(S_a / g) = 2.5$;

$$A_{h2} = \frac{ZI}{2R}(S_a / g) = \frac{0.36 \times 1}{2 \times 5} \times (2.5) = 0.09$$

$$Q_{i1} = 0.09 \times (-0.329) \times \phi_{i2} \times W_i$$

Mode 3:

$T_3 = 0.145$ sec;

$(S_a / g) = 2.5$;

$$A_{h3} = \frac{ZI}{2R}(S_a / g) = \frac{0.36 \times 1}{2 \times 5} \times (2.5) = 0.09$$

$$Q_{i3} = 0.09 \times (0.118) \times \phi_{i3} \times W_i$$

Table 2.3 summarizes the calculation of lateral load at different floors in each mode.

Table 2.3 – Lateral load calculation by modal analysis method (earthquake in X-direction)

Floor Level i	Weight W_i (kN)	Mode 1			Mode 2			Mode 3		
		ϕ_{i1}	Q_{i1}	V_{i1}	ϕ_{i2}	Q_{i2}	V_{i2}	ϕ_{i3}	Q_{i3}	V_{i3}
4	3,000	1.000	155.5	155.5	1.000	-88.8	-88.8	1.000	31.9	31.9
3	4,200	0.904	196.8	352.3	0.216	-26.8	-115.6	-0.831	-37.1	-5.2
2	4,200	0.716	155.9	508.2	-0.701	87.2	-28.4	-0.574	-25.6	-30.8
1	4,200	0.441	96.0	604.2	-0.921	114.6	86.2	1.016	45.4	14.6

Since all of the modes are well separated (clause 3.2), the contribution of different modes is combined by the SRSS (square root of the sum of the square) method

$V_4 = [(155.5)^2 + (88.8)^2 + (31.9)^2]^{1/2} = 182$ kN

$V_3 = [(352.3)^2 + (115.6)^2 + (5.2)^2]^{1/2} = 371$ kN

$V_2 = [(508.2)^2 + (28.4)^2 + (30.8)^2]^{1/2} = 510$ kN

$V_1 = [(604.2)^2 + (86.2)^2 + (14.6)^2]^{1/2} = 610$ kN

(Clause 7.8.4.4a of IS: 1893 Part 1)

The externally applied design loads are then obtained as:

$Q_4 = V_4 = 182$ kN
$Q_3 = V_3 - V_4 = 371 - 182 = 189$ kN
$Q_2 = V_2 - V_3 = 510 - 371 = 139$ kN
$Q_1 = V_1 - V_2 = 610 - 510 = 100$ kN

(Clause 7.8.4.5f of IS: 1893 Part 1)

Clause 7.8.2 requires that the base shear obtained by dynamic analysis (V_B = 610 kN) be compared with that obtained from empirical fundamental period as per Clause 7.6. If V_B is less than that from empirical value, the response quantities are to be scaled up.

We may interpret "base shear calculated using a fundamental period as per 7.6" in two ways:

1. We calculate base shear as per Cl. 7.5.3. This was done in the previous example for the same building and we found the base shear as 1,404 *kN*. Now, dynamic analysis gives us base shear of 610 *kN* which is lower. Hence, all the response quantities are to be scaled up in the ratio (1,404/610 = 2.30). Thus, the seismic forces obtained above by dynamic analysis should be scaled up as follows:

$Q_4 = 182 \times 2.30 = 419$ kN
$Q_3 = 189 \times 2.30 = 435$ kN
$Q_2 = 139 \times 2.30 = 320$ kN

$Q_1 = 100 \times 2.30 = 230$ kN

2. We may also interpret this clause to mean that we redo the dynamic analysis but replace the fundamental time period value by T_a (= 0.28 sec). In that case, for mode 1:

$T_1 = 0.28$ sec;
$(S_a / g) = 2.5$;

$$A_{h1} = \frac{ZI}{2R}(S_a / g)$$
$=0.09$

Modal mass times A_{h1}
= 14,450 × 0.09
= 1,300 kN

Base shear in modes 2 and 3 is as calculated earlier: Now, base shear in first mode of vibration =1300 kN, 86.2 kN and 14.6 kN, respectively.

Total base shear by SRSS

$$= \sqrt{1300^2 + 86.2^2 + 14.6^2}$$
= 1,303 kN

Notice that most of the base shear is contributed by first mode only. In this interpretation of Cl 7.8.2, we need to scale up the values of response quantities in the ratio (1,303/610 = 2.14). For instance, the external seismic forces at floor levels will now be:

$Q_4 = 182 \times 2.14 = 389$ kN
$Q_3 = 189 \times 2.14 = 404$ kN
$Q_2 = 139 \times 2.14 = 297$ kN
$Q_1 = 100 \times 2.14 = 214$ kN

Clearly, the second interpretation gives about 10% lower forces. We could make either interpretation. Herein we will proceed with the values from the second interpretation and compare the design values with those obtained in Example 1 as per static analysis:

Table 2.4 – Base shear at different storeys

Floor Level *i*	*Q* (static)	*Q* (dynamic, scaled)	*Storey Shear V* (static)	*Storey ShearV* (dynamic, scaled)	Storey Moment, *M* (Static)	Storey Moment, *M* (Dynamic)
4	611 kN	389 kN	611 kN	389 kN	1,907 kNm	1,245 kNm
3	504 kN	404 kN	1,115kN	793 kN	5,386 kNm	3,782 kNm
2	297 kN	297 kN	1,412kN	1,090 kN	9.632 kNm	7,270 kNm
1	79 kN	214 kN	1,491 kN	1,304 kN	15,530 kNm	12,750 kNm

Notice that even though the base shear by the static and the dynamic analyses are comparable, there is considerable difference in the lateral load distribution with building height, and therein lies the advantage of dynamic analysis. For instance, the storey moments are significantly affected by change in load distribution.

Example 3 – Location of Centre of Mass

Problem Statement:

Locate centre of mass of a building having non-uniform distribution of mass as shown in the figure 3.1

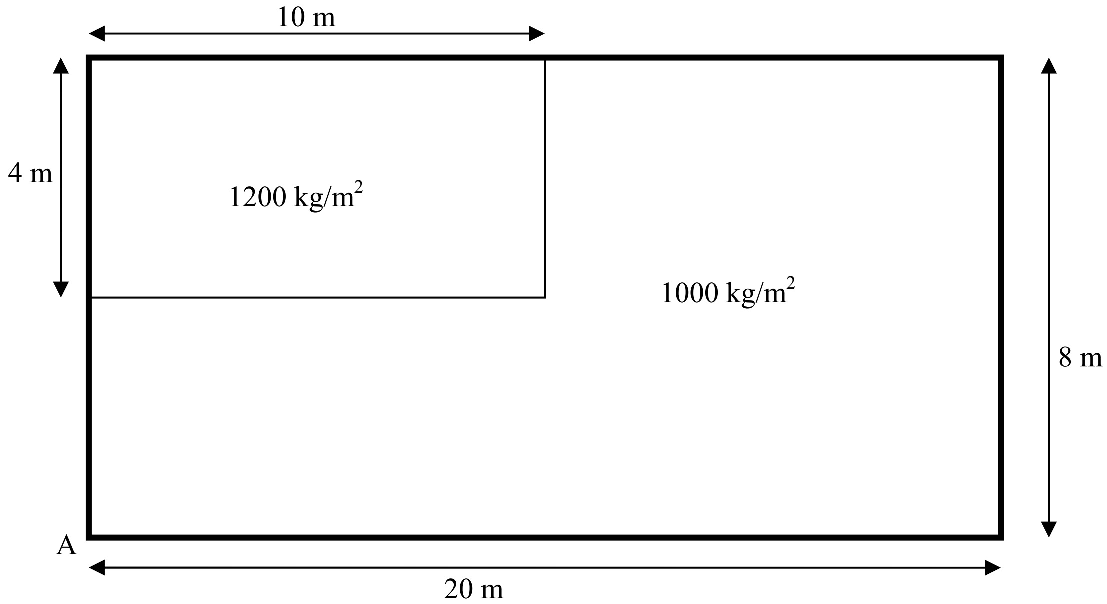


Figure 3.1 –Plan

Solution:

Let us divide the roof slab into three rectangular parts as shown in figure 2.1

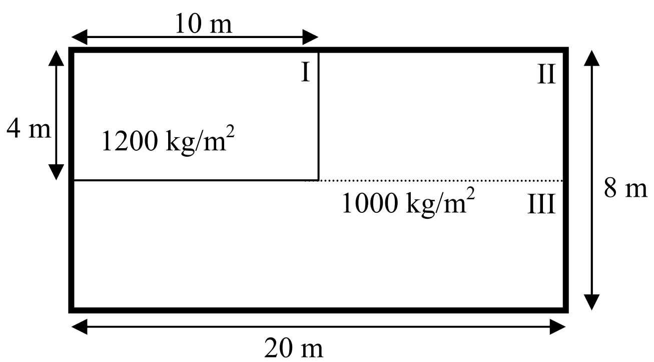


Figure 3.2

Mass of part I is 1200 kg/m^2, while that of the other two parts is 1000 kg/m^2.

Let origin be at point A, and the coordinates of the centre of mass be at (X, Y)

$$X = \frac{(10\times4\times1200)\times5+(10\times4\times1000)\times15+(20\times4\times1000)\times10}{(10\times4\times1200)+(10\times4\times1000)+(20\times4\times1000)}$$

= 9.76 m

$$Y = \frac{(10\times4\times1200)\times6+(10\times4\times1000)\times6+(20\times4\times1000)\times2}{(10\times4\times1200)+(10\times4\times1000)+(20\times4\times1000)}$$

= 4.1 m

Hence, coordinates of centre of mass are (9.76, 4.1)

Example 4 – Location of Centre of Stiffness

Problem Statement:

The plan of a simple one storey building is shown in figure 3.1. All columns and beams are same. Obtain its centre of stiffness.

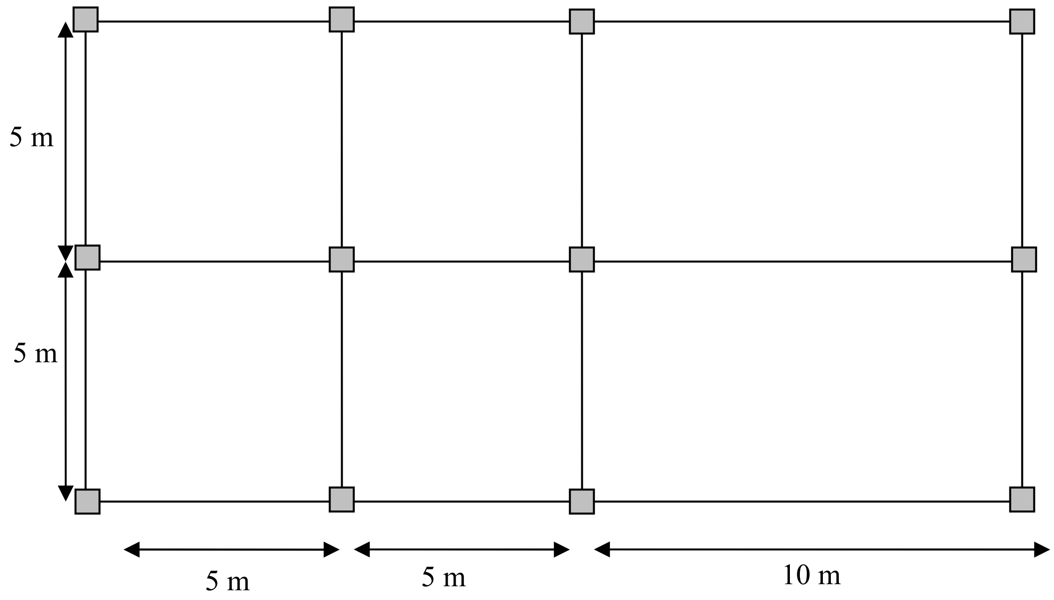


Figure 4.1 –Plan

Solution:

In the X-direction there are three identical frames located at uniform spacing. Hence, the y-coordinate of centre of stiffness is located symmetrically, i.e., at 5.0 m from the left bottom corner.

In the Y-direction, there are four identical frames having equal lateral stiffness. However, the spacing is not uniform. Let the lateral stiffness of each transverse frame be *k*, and coordinating of center of stiffness be (X, Y).

$$X = \frac{k \times 0 + k \times 5 + k \times 10 + k \times 20}{k + k + k + k} = 8.75 \text{ m}$$

Hence, coordinates of centre of stiffness are (8.75, 5.0).

Example 5 –Lateral Force Distribution as per Torsion Provisions of IS 1893-2002 (Part 1)

Problem Statement:

Consider a simple one-storey building having two shear walls in each direction. It has some gravity columns that are not shown. All four walls are in M25 grade concrete, 200 thick and 4 m long. Storey height is 4.5 m. Floor consists of cast-in-situ reinforced concrete. Design shear force on the building is 100 kN in either direction.

Compute design lateral forces on different shear walls using the torsion provisions of 2002 edition of IS 1893 (Part 1).

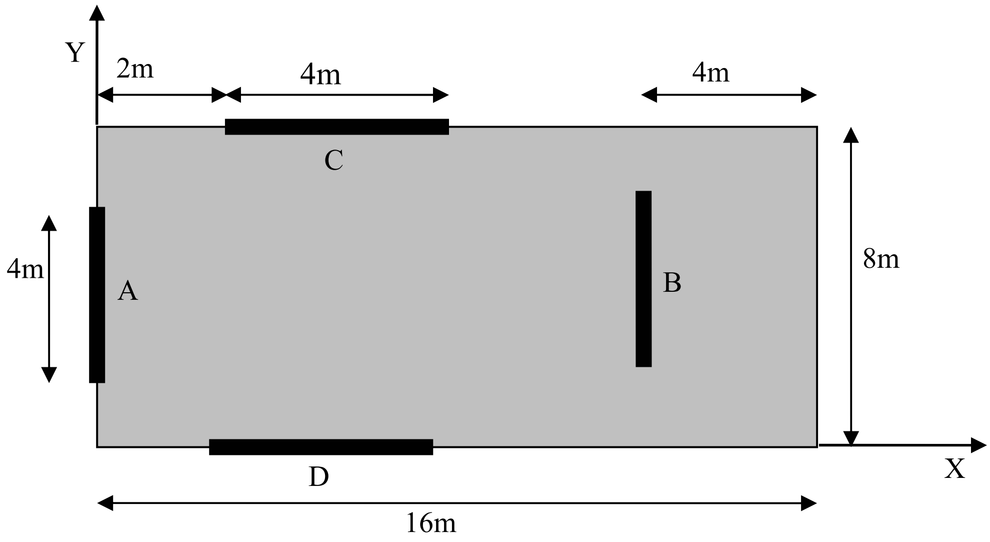


Figure 5.1 – Plan

Solution:

Grade of concrete: M25

$E = 5000\sqrt{25} = 25000\ \text{N/mm}^2$

Storey height h = 4500 m
Thickness of wall t = 200 mm
Length of walls L = 4000 mm
All walls are same, and hence, spaces have same lateral stiffness, k.

Centre of mass (CM) will be the geometric centre of the floor slab, i.e., (8.0, 4.0).

Centre of rigidity (CR) will be at (6.0, 4.0).

EQ Force in X-direction:

Because of symmetry in this direction, calculated eccentricity = 0.0 m

Design eccentricity:

$$e_d = 1.5 \times 0.0 + 0.05 \times 8 = 0.4,$$

and

$$e_d = 0.0 - 0.05 \times 8 = -0.4$$

(Clause 7.9.2 of IS 1893:2002)

Lateral forces in the walls due to translation:

$$F_{CT} = \frac{K_C}{K_C + K_D} F = 50.0\ \text{kN}$$

$$F_{DT} = \frac{K_D}{K_C + K_D} F = 50.0\ \text{kN}$$

Lateral forces in the walls due to torsional moment:

$$F_{iR} = \frac{K_i r_i}{\sum\limits_{i=A,B,C,D} K_i r_i^2} (F e_d)$$

where r_i is the distance of the shear wall from CR.

All the walls have same stiffness, $K_A = K_B = K_C = K_D = k$, and

r_A = -6.0 m
r_B = -6.0 m

r_C = 4.0 m
r_D = -4.0 m,
and $e_d = \pm 0.4$ m

Therefore,

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k}\left(Fe_d\right)$$

$= \pm 2.31\,\text{kN}$

Similarly,
$F_{BR} = \pm 2.31\,\text{kN}$
$F_{CR} = \pm 1.54\,\text{kN}$
$F_{DR} = \pm 1.54\,\text{kN}$

Total lateral forces in the walls due to seismic load in X direction:

F_A = 2.31 kN
F_B = 2.31 kN
F_C = Max (50 ± 1.54) = 51.54 kN
F_D = Max (50 ± 1.54) = 51.54 kN

EQ Force in Y-direction:

Calculated eccentricity= 2.0 m
Design eccentricity:
$e_d = 1.5 \times 2.0 + 0.05 \times 16 = 3.8\,\text{m}$
or $= 2.0 - 0.05 \times 16 = 1.2\,\text{m}$

Lateral forces in the walls due to translation:

$$F_{AT} = \frac{K_A}{K_A + K_B}F = 50.0\ \text{kN}$$

$$F_{BT} = \frac{K_B}{K_A + K_B}F = 50.0\ \text{kN}$$

Lateral force in the walls due to torsional moment: when e_d = 3.8 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k}\left(Fe_d\right) = -$$
21.92 kN

Similarly,
F_{BR} = 21.92 kN
F_{CR} = -14.62 kN
F_{DR} = 14.62 kN

Total lateral forces in the walls:
F_A = 50 - 21.92= 28.08 kN
F_B = 50 +20.77= 71.92 kN
F_C = -14.62 kN
F_D = 14.62 kN

Similarly, when e_d = 1.2 m, then the total lateral forces in the walls will be,
F_A = 50 – 6.93 = 43.07 kN
F_B = 50 + 6.93 = 56.93 kN
F_C = - 4.62 kN
F_D = 4.62 kN

Maximum forces in walls due to seismic load in Y direction:
F_A = Max (28.08, 43.07) = 43.07 kN;
F_B = Max (71.92, 56.93) = 71.92 kN;
F_C = Max (14.62, 4.62) = 14.62 kN;
F_D = Max (14.62, 4.62) = 14.62 kN;

Combining the forces obtained from seismic loading in X and Y directions:
F_A = 43.07 kN
F_B =71.92 kN
F_C =51.54 kN
F_D =51.54 kN.

However, note that clause 7.9.1 also states that “However, negative torsional shear shall be neglected”. Hence, wall A should be designed for not less than 50 kN.

Example 6 – Lateral Force Distribution as per New Torsion Provisions

Problem Statement:

For the building of example 5, compute design lateral forces on different shear walls using the torsion provisions of revised draft code IS 1893 (part 1), i.e., IITK-GSDMA-EQ05-V2.0.

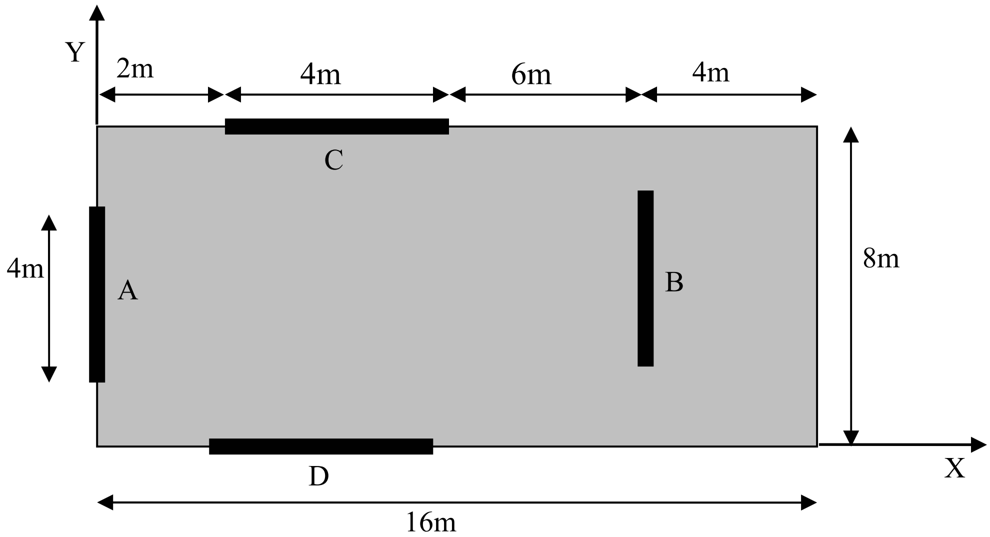


Figure 6.1 – Plan

Solution:

Grade of concrete: M25

$E = 5000\sqrt{25} = 25000 \text{ N/mm}^2$

Storey height h = 4500 m

Thickness of wall t = 200 mm

Length of walls L = 4000 mm

All walls are same, and hence, same lateral stiffness, k.

Centre of mass (CM) will be the geometric centre of the floor slab, i.e., (8.0, 4.0).

Centre of rigidity (CR) will be at (6.0, 4.0).

EQ Force in X-direction:

Because of symmetry in this direction, calculated eccentricity = 0.0 m

Design eccentricity, $e_d = 0.0 \pm 0.1 \times 8 = \pm 0.8$

(clause 7.9.2 of Draft IS 1893: (Part1))

Lateral forces in the walls due to translation:

$$F_{CT} = \frac{K_C}{K_C + K_D} F = 50.0 \text{ kN}$$

$$F_{DT} = \frac{K_D}{K_C + K_D} F = 50.0 \text{ kN}$$

Lateral forces in the walls due to torsional moment:

$$F_{iR} = \frac{K_i r_i}{\sum_{i=A,B,C,D} K_i r_i^2} (Fe_d)$$

where r_i is the distance of the shear wall from CR

All the walls have same stiffness, $K_A = K_B = K_C = K_D = k$

r_A= -6.0 m
r_B= -6.0 m
r_C= 4.0 m
r_D= -4.0 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k} (Fe_d)$$

= - 4.62 kN

Similarly,

F_{BR} = 4.62 kN
F_{CR} = 3.08 kN
F_{DR} = -3.08 kN

Total lateral forces in the walls:

F_A = 4.62 kN
F_B = - 4.62 kN
F_C = 50+3.08 = 53.08 kN
F_D = 50-3.08 = 46.92 kN

Similarly, when e_d= - 0.8 m, then the lateral forces in the walls will be,

F_A = - 4.62 kN
F_B = 4.62 kN
F_C = 50-3.08 = 46.92 kN
F_D = 50+3.08 = 53.08kN

Design lateral forces in walls C and D are:
F_C= F_D= 53.05 kN

EQ Force in Y-direction:

Calculated eccentricity= 2.0 m
Design eccentricity,

$$e_d = 2.0 + 0.1 \times 16 = 3.6 \; m$$

or

$$e_d = 2.0 - 0.1 \times 16 = 0.4 \; m$$

Lateral forces in the walls due to translation:

$$F_{AT} = \frac{K_A}{K_A + K_B} F = 50.0 \text{ kN}$$

$$F_{BT} = \frac{K_B}{K_A + K_B} F = 50.0 \text{ kN}$$

Lateral force in the walls due to torsional moment: when e_d= 3.6 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k} \left(Fe_d\right) = -$$

20.77 kN

Similarly,

F_{BR} = 20.77 kN
F_{CR} = 13.85 kN
F_{DR} = -13.8 kN

Total lateral forces in the walls:

F_A = 50-20.77= 29.23 kN
F_B = 50+20.77= 70.77 kN
F_C = 13.85 kN
F_D = -13.85 kN

Similarly, when e_d= 0.4 m, then the total lateral forces in the walls will be,

F_A = 50-2.31= 47.69 kN
F_B = 50+2.31= 53.31 kN
F_C = 1.54 kN
F_D = - 1.54 kN

Maximum forces in walls A and B
F_A =47.69 kN, F_B =70.77 kN

Design lateral forces in all the walls are as follows:

F_A =47.69 kN
F_B =70.77 kN
F_C =53.05 kN
F_D =53.05 kN.

Design of RC Structures II

UNIT- II

Building Frames

Framed Structure:-

* A building frame may contain a no. of bays, and may have several storeys.
* A multi-storyed, multi-panelled frame is a complicated statically indeterminate structure.
* It consists of no. of beams and columns built monolithically.
* In framed structure floors and the walls are supported on beams which transmit the loads to the columns.
* Building frame is subjected to both vertical and horizontal loads.

⇒ Vertical Loads:

① Dead load → self wt of beams, slabs, columns etc.

② Live load.

⇒ horizontal loads:-

① wind forces.

② Earth quake forces.

⇒ Practically all major buildings are framed structures. The building frames are highly Indeterminate structures

upto 2 storeys - Load bearing wall construction

upto 5 storeys - Approx. analysis procedures are useful.

exceeding 5 storeys - computer based analysis procedures are useful.

⇒ Building frames analysis:-

In earlier period to analyse the no. of storey of the building frames by * two cycle moment distribution method (or)

* substitute frame method.

Substitute frame :-

* what ever be the no. of storey it is customary (and permissible) to analyse only a part of a frame, termed as Substitute frame.

(or)

* A simple method of analysis, accurate enough for practical purpose, is used by analysing a small portion of the frame, called substitute frame, rather than analysis of the whole frame.

⇒ It is based on the assumption that the moments in one floor have negligible effect of the moments of the floors above and below.

⇒ This substitute frame may be moved from floor to floor.

Analysis for vertical loads (or) gravity loads In substitute frame method:-

* A building structure may be assumed to be consisting of two sets of plane frames crossing each other at right angles.

* The vertical members are common to both these sets of frames.

* Each set of frames are analysed separately.

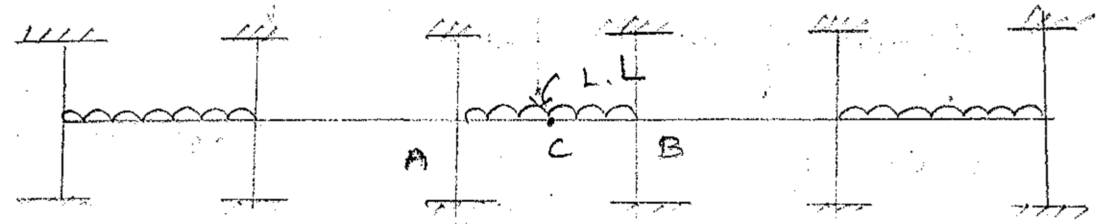
* Moments in the vertical members occur in two planes, the stresses in columns should be found for moments acting in two planes simultaneously and the corresponding vertical loads.

~~⇒ The beam should be loaded with Live Loads as follow~~

i) <u>Maximum Bending moments in beams:-</u>

The beam should be loaded with live loads as follows for maximum effects.

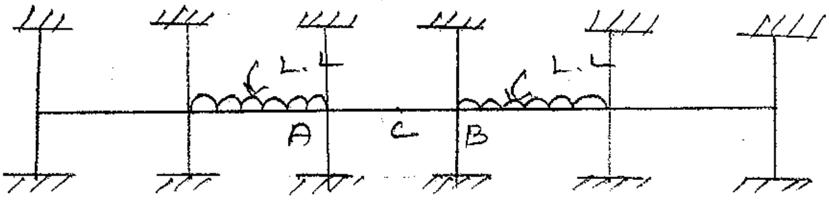
a) <u>For max. positive B.M @ mid span of C</u>



@ mid pt C of a span AB, the loads should be placed on the span and on alternative span as show in fig.

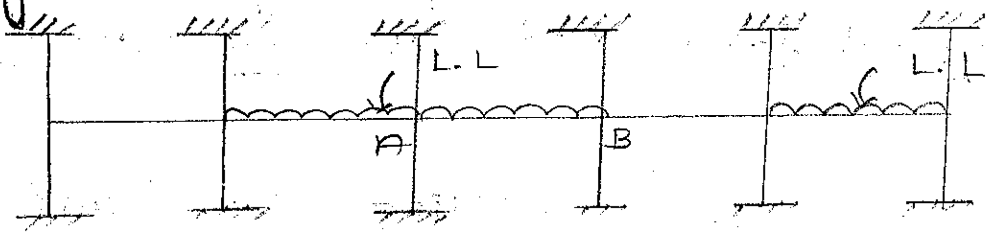
b). <u>For max. -ve B.M @ mid span of c :-</u>

@ the mid point C of a span AB, the span AB should be unloaded while load should be placed on spans adjacent to the span under consideration, as shown in fig.



c). For max. -ve B.M @ support A:-

for max -ve B.M @ support A, Loads should be placed on the two spans adjacent to the support, as shown in fig.

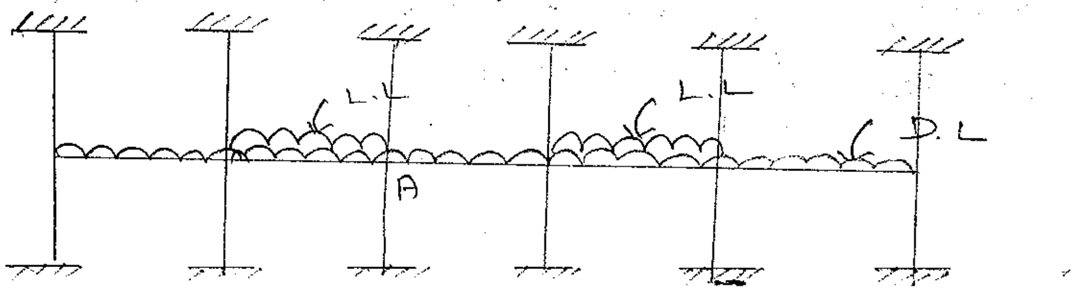


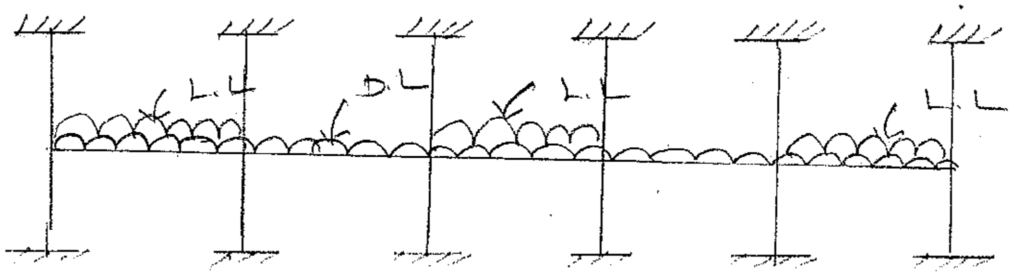
⇒ The B.M due to Dead Loads (D.L) are found separately.

⇒ The B.M for D.L and L.L are then added and the beam is designed.

ii). Max. B.M in columns:-

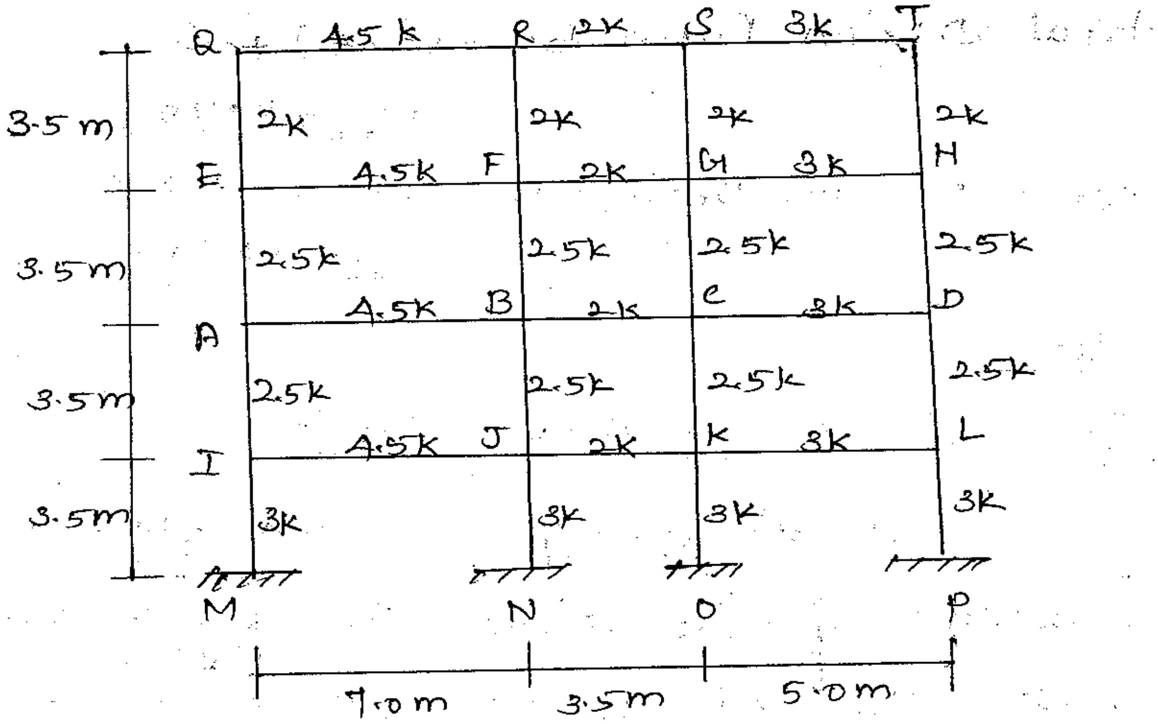
max. B.M in column @ A when the alternate spans are loaded. They are 2 sets of alternate loading as shown in fig.



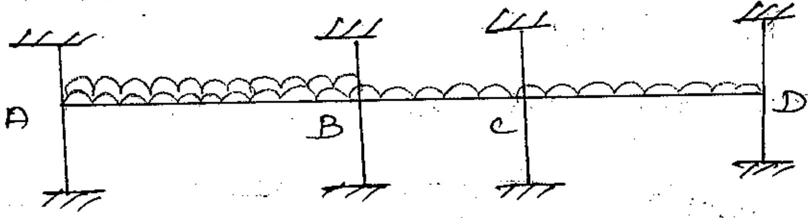


* The corresponding axial loads are found.
* The column is designed to resist the stresses provided by every combination of axial load and the corresponding mt.

① In multi-storey building, the frame shown in fig. are spaced @ 4 m intervals. Analysis the beam AB & BC, CD for mid span +ve BM taking L.L of 4 KN/m² and DL as 3 KN/m², 3.25 KN/m² and 2.75 KN/m² for the panel AB, BC & CD respectively. the self wt of the beam may be taken as,
beams of 7m span = 5 KN/m,
beams of 50 m span = 3.5 KN/m,
beams of 3.5 m span = 2.5 KN/m. The relative stiffness of the members are marked on the fig. it self.



Sol:- i) max +ve B.M in midspan of AB



Step1:- Calculation of Load

The frames are spaced @ 4m interval the L.L transfored from the floor.

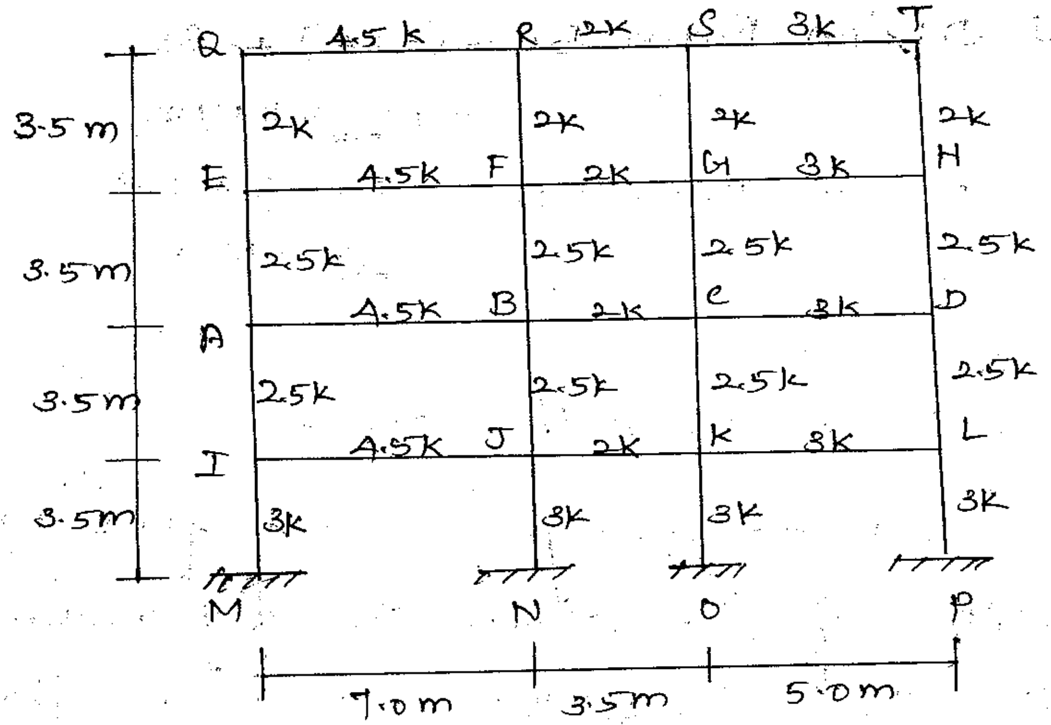
= L.L × spacing

= 4 × 4 = 16 kN/m

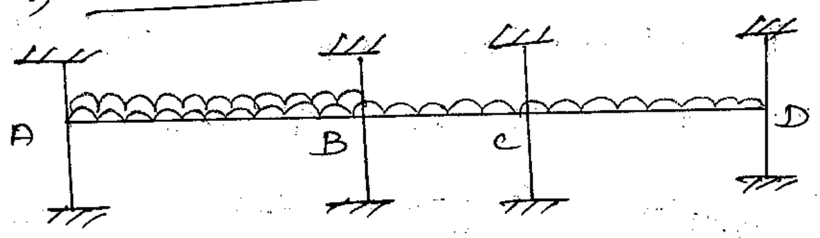
Total D.L on a beam = The D.L from the floors + D.L due to self wt of the beam.

total D.L on beam AB = (D.L × spacing) + self wt of beam

= (3 × 4) + 5 = 17 kN/m



Sol:- i) max +ve B.M in midspan of AB



Step1:- Calculation of Load

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= L.L x spacing

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Total D.L on a beam = The D.L from the floors + D.L due to self wt of the beam.

total D.L on beam AB = (D.L x spacing) + self wt of beam

= (3x4) + 5 = 17 kN/m

for span CD total load = D.L + L.L

$= 14.5 + 16 = 30.5\ kN/m$

$$MF_{AB} = -\frac{wl^2}{12} = -\frac{33 \times 7^2}{12} = -134.75\ kN.m$$

$$MF_{BA} = \frac{wl^2}{12} = 134.75\ kN.m$$

$$MF_{BC} = -\frac{wl^2}{12} = -\frac{15.5 \times 3.5^2}{12} = -15.8281\ kN.m$$

$$MF_{CB} = \frac{wl^2}{12} = 15.823\ kN.m$$

$$MF_{CD} = -\frac{wl^2}{12} = -\frac{(30.5) \times 5^2}{12} = -63.542\ kN.m$$

$$MF_{DC} = \frac{wl^2}{12} = 63.542\ kN.m$$

Step 4:-

Distribution Factor (D.F) calculation:-

joint	members	Relative stiffness (K)	Σk	$D.F = \frac{k}{\Sigma k}$
A	AE	2.5k	9.5k	0.263
	AB	4.5k		0.474
	AI	2.5k		0.263
B	BF	2.5k	11.5k	0.217
	BC	2k		0.174
	BJ	2.5k		0.217
	BA	4.5k		0.391

C	CB	2k	10k	0.2
	CD	3k		0.3
	CG	2.5k		0.25
	CK	2.5k		0.25
D	DC	3k	8k	0.375
	DH	2.5k		0.3125
	DL	2.5k		0.3125

Step 5:-

<u>Formation of mt distribution table:-</u>

joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	0.474	0.392	0.174	0.20	0.30	0.375
F.E.M	-134.75	+134.75	-15.83	+15.83	-63.542	+63.542
Balance	+63.87	-46.62	-20.692	+9.542	+14.314	-23.823
carry over	-23.31	+31.94	+4.771	-10.346	-11.9115	+7.157
Balance	+11.049	-14.39			+6.680	-2.684
Final mts	-83.141	+105.68			-54.45	+44.192

<u>Balance:-</u> (1st cycle)

$AB = (-134.75 \times 0.474) = -63.87$

$BA = (134.75 - 15.83) \times 0.392 = 46.62$

$BC = (134.75 - 15.83) \times 0.174 = 20.692$

$CB = (15.83 - 63.542) \times 0.2 = -9.542$

$CD = (15.83 - 63.542) \times 0.3 = -14.314$

Carry over:-

$AB = -\frac{46.62}{2} = -23.31$

$BA = +\frac{63.87}{2} = +31.94$

$BC = \frac{9.542}{2} = 4.771$

$CB = -\frac{20.692}{2} = -10.346$

$CD = \frac{-23.823}{2} = -11.9115$

$Dc = +\frac{14.314}{2} = +7.157$

Balance :- (2nd cycle)

$AB = -23.31 \times 0.474 = -11.049$

$BA = (+31.94 + 4.77) \times 0.392 = +14.39$

$CB = -(10.346 + 11.9115) \times 0.2 = -4.45$

$CD = -(10.346 + 11.9115) \times 0.3 = -6.677$

$DC = 7.157 \times 0.375 = +2.684$

Final mts:-

$AB = -134.75 + 63.87 - 23.31 + 11.049$

$= -83.141$ KN.m

$BA = 134.75 - 46.62 + 31.94 - 14.39$

$= +105.68$ KN.m

ii). For max +ve B.M in mid span of CD :-

Let us analyse second floor ABCD substitute frame shown in fig. for max mid span +ve B.M in CD, the L.L is placed on AB & CD. The DL is placed on all the spans.

It is same in previous calculation method.

Final mts:- in

$CD = -63.542 + 14.314 - 11.9115 + 6.680$

$= -54.45$ KN.m

$DC = 63.542 - 23.823 + 7.157 - 2.684$

$= +44.20 \ kN.m$

Step 6:- To Draw B.M Diagram

a). for span AB:-

Final Fixed end moments

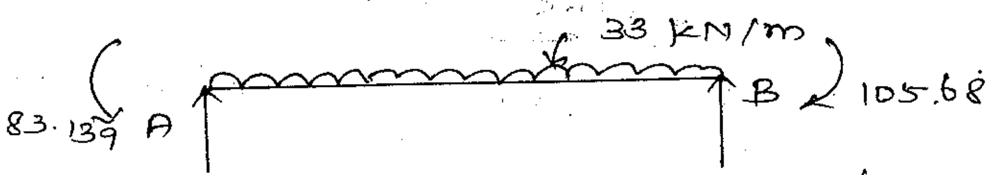
$M_{FAB}' = -83.139$

$M_{FBC}' = +105.68$

free BM @ mid span of $AB = \frac{wl^2}{8}$

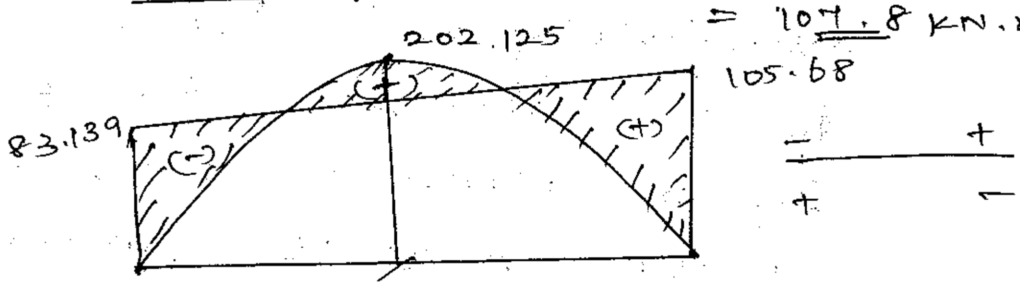
$= \frac{(D.L + L.L)l^2}{8}$

$= \frac{33 \times 7^2}{8} = 20.2125 \ kN\text{-}m$



Net B.M @ the centre of AB $= 20.2125 - \left(\frac{83.139 + 105.68}{2}\right)$

$= 107.8 \ kN.m$



B.M.D

b). for span CD:-

Final Fixed end moments

$M_{FCD}' = -54.45 \ kN.m$

$M_{FDC}' = +44.192 \ kN.m$

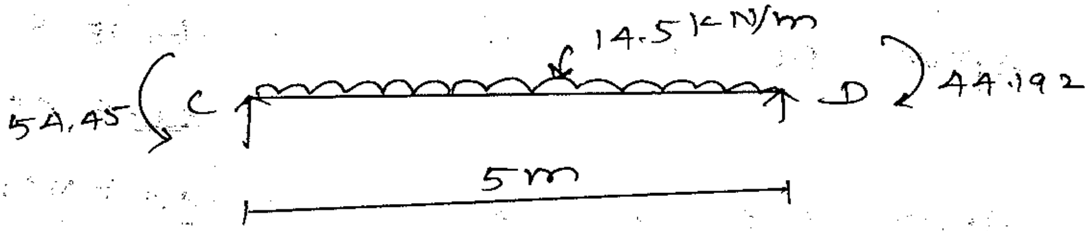
iii). Max +ve B.M in mid span of BC :-

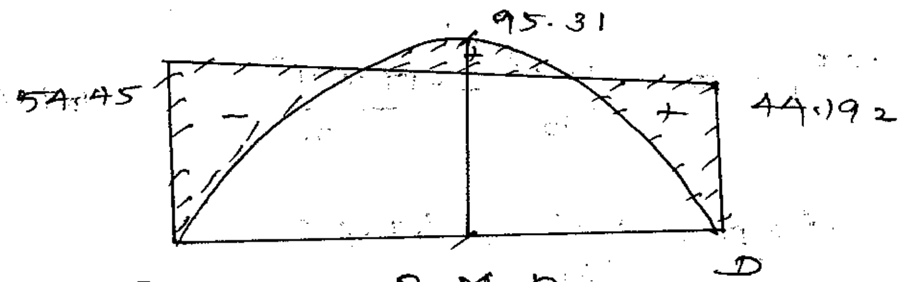
free B.M @ mid span of CD $= \frac{wl^2}{8}$

$= \frac{(14.5+16) \times 5^2}{8} = 95.31$ kN.m

Net B.M @ the centre of CD $= 95.31 - \frac{(54.45 + 44.192)}{2}$

$= 45.98$ kN.m





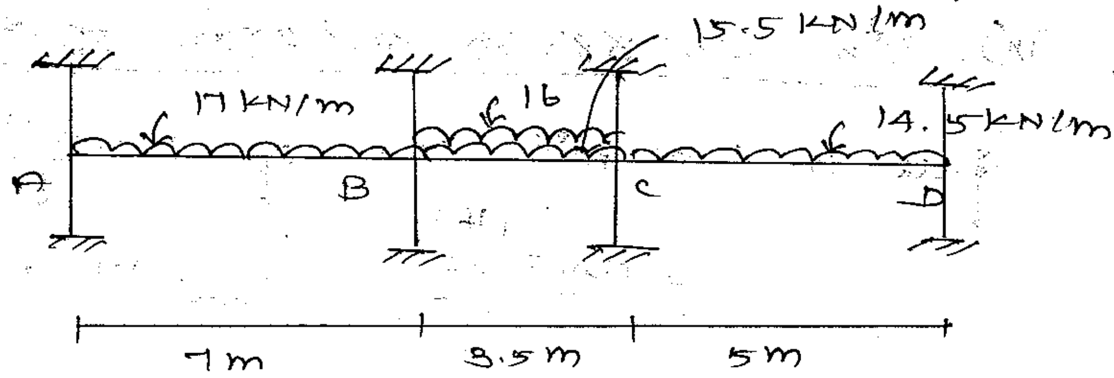
B.M.D

iii). Max (+ve) B.M in mid span of BC :-

Step 1:-

Formation of substitute frame:-

For max +ve B.M in mid span of BC. the L.L is placed on BC and D.L placed on all the spans.



Step 8:-

Fixed end mt calculation:-

Span AB total load $w = 17 KN/m$

span BC " " $= 16 + 15.5$

$= 31.5 KN/m$

span CD " " $= 14.5 KN/m$

$$M_{FAB} = -\frac{wl^2}{12} = \frac{-17 \times 7^2}{12} = -69.42$$

$$M_{FBA} = \frac{wl^2}{12} = 69.42$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-31.5 \times 3.5^2}{12} = -32.16$$

$$M_{FCB} = \frac{wl^2}{12} = 32.16$$

$$M_{FCD} = -\frac{wl^2}{12} = -\frac{14.5 \times 5^2}{12} = -30.21$$

$$M_{FDC} = \frac{wl^2}{12} = 30.21$$

Step 9:-

Distribution Factor (DF) calculation:-

joint	members	Relative stiffness K	ΣK	$D.F = \frac{K}{\Sigma K}$
A	AE AB AI	2.5k 4.5k 2.5k	9.5k	0.263 0.474 0.263
B	BF BC BJ BA	2.5k 2k 2.5k 4.5k	11.5k	0.217 0.174 0.217 0.391
C	CB CD CG CK	2k 3k 2.5k 2.5k	10k	0.2 0.3 0.25 0.25
D	DC DH DL	3k 2.5k 2.5k	8k	0.375 0.3125 0.3125

Step 10:-

Formation of mt distribution Table:-

joint	A	B		C		D
member	AB	BA	BC	CB	CD	DC
D.F	0.474	0.392	0.174	0.2	0.3	0.375
FEM	-69.42	+69.42	-32.16	+32.16	-30.21	+30.21
Balance	+32.9	-14.6	-6.48	-0.34	-0.585	-11.33
CO		+16.45	-0.2	-3.24	-5.66	
Balance			-2.83	-1.78		
Final mts			-41.67	+30.31		

Balance 1st cycle:-

AB = -69.4 × 0.474 = -32.9

BA = (+69.42 - 32.16) × 0.392 = 14.6

BC = (69.42 − 32.16) × 0.174 = 6.48

CB = (32.16 − 30.21) × 0.2 = 0.39

CD = (32.16 − 30.21) × 0.3 = 0.585

DC = 30.21 × 0.375 = 11.33

Balance (2nd cycle)

BC = (−0.2 + 16.45) × 0.174 = 2.83

CB = (−3.24 − 5.66) × 0.2 = 1.78

Step 11:-

To Draw B.M.D of BC :-

Final end moments:-

$MF_{BC'} = -41.67$ kN.m

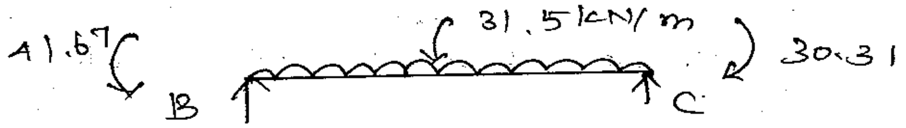
$MF_{CB'} = +30.31$ kN.m

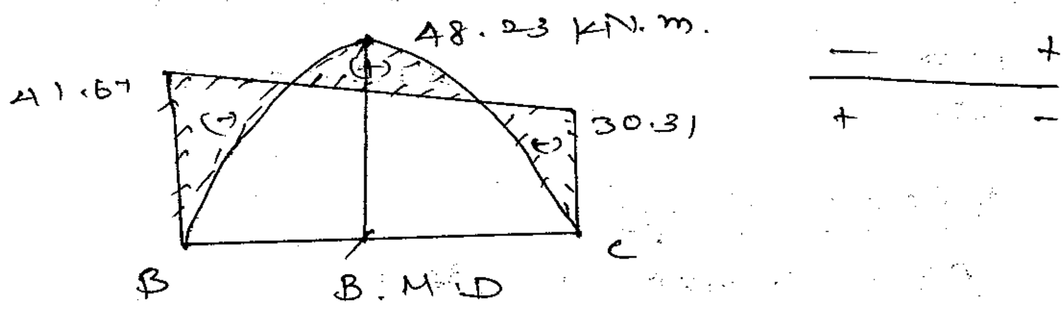
free B.M @ centre of span BC $= \frac{wl^2}{8}$

$= \frac{31.5 \times 3.5^2}{8} = 48.23$ kN.m

Net B.M @ centre BC $= 48.23 - \frac{41.67 + 30.31}{2}$

$= 12.24$ kN.m





B.M.D

Analysis of frames subjected to horizontal Forces

* A building frame is subjected to horizontal forces due to Wind pressure & seismic effects

* These horizontal forces cause axial forces in columns and bending moment in all the members of the frame.

The following approximate methods are commonly used for the analysis of building frames subjected to lateral forces.

⇒ Portal method

⇒ Cantilever method.

→ Factor method.

Portal method:-

* It is more suited for low rise building frames.

In this method, following assumptions are made,

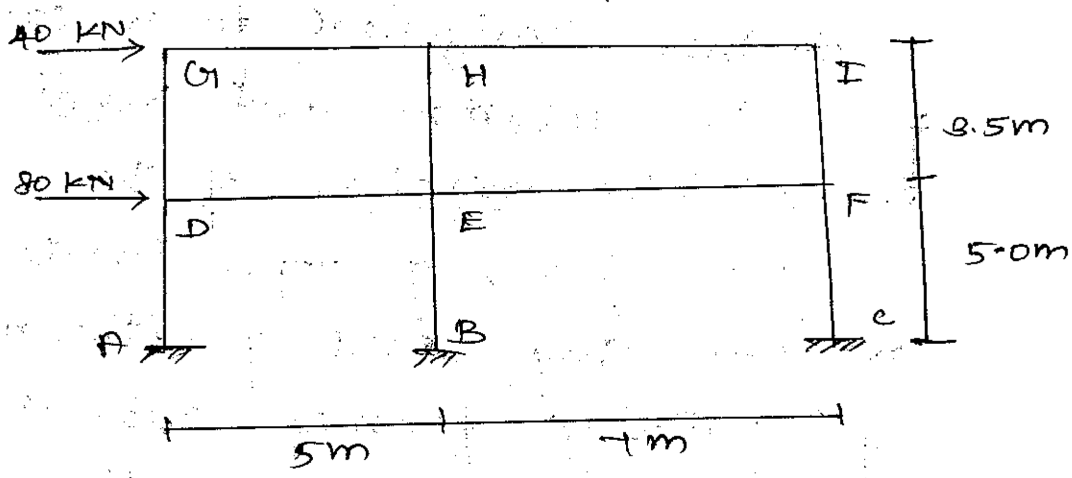
1. The point of contraflexure is located at the centre of each beam.

2. The point of contraflexure is located @ the 'c' of each column.

3. horizontal shear taken by each interior column is double the horizontal

Shear taken by each of exterior column.

(d) Analyse the building frame, subjected to horizontal forces, as shown in fig. Use portal method. sketch the B.M.D.



⇒ Point of contraflexure (P.O.C) will be assumed to occur @ mid span / mid height of all the beams / columns.

Step 1:-

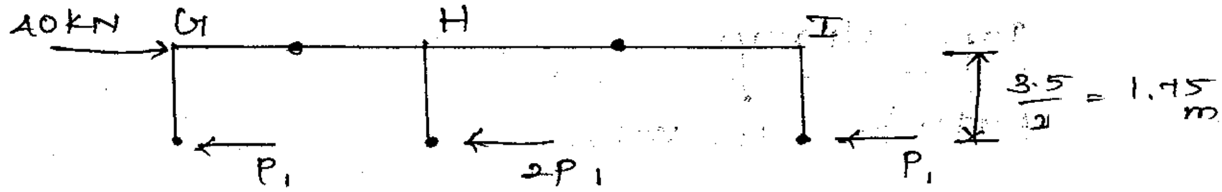
<u>column shear (or) horizontal shear calculation:-</u>

Horizontal shear in interior column is assumed to be twice that in the exterior columns.

$P_1, P_2 \ldots$ = horizontal shear in exterior columns of a storey.

$2P_1, 2P_2 \ldots$ = shear in interior columns of the respective storey.

For top storey:-



Sum of shear in columns = Total external shear

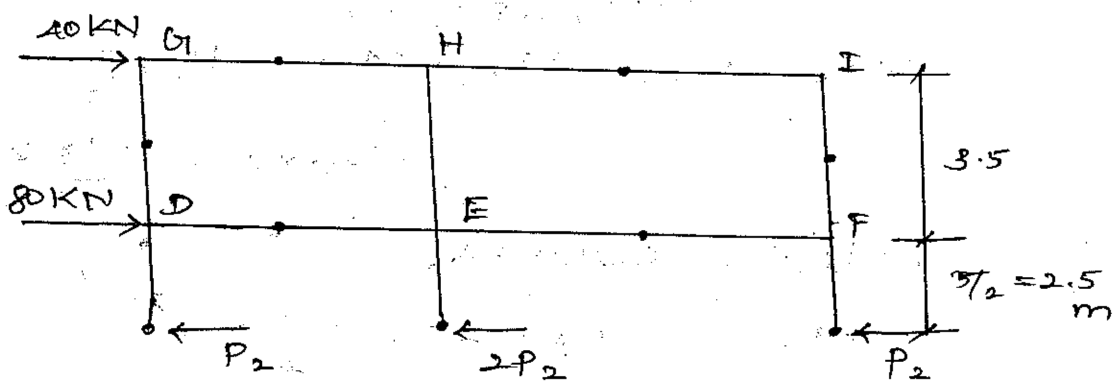
$P_1 + 2P_1 + P_1 = 40$ KN

$4P_1 = 40$

$\boxed{P_1 = 40/4 = 10 \text{ KN}}$

$\boxed{\therefore 2P_1 = 20 \text{ KN}}$

For the bottom storey:-



Sum of shear in columns = total external shear

$P_2 + 2P_2 + P_2 = 40 + 80$

$4P_2 = 120$

$\boxed{P_2 = 30 \text{ KN}}$

$\boxed{2P_2 = 60 \text{ KN}}$

Step 2 :- moments @ the ends of columns :-

Top storey :-

Exterior columns,

$$M_{GD} = M_{DG} = M_{IF} = M_{FI} = P_1 \times \frac{3.5}{2}$$

$$= 10 \times \frac{3.5}{2} = \underline{\underline{17.5}} \text{ kN.m } (\circlearrowright)$$

Interior columns,

$$M_{HE} = M_{EH} = 2P_1 \times \frac{3.5}{2} = 20 \times 1.75 = \underline{\underline{35}} \text{ kN.m } (\circlearrowright).$$

For the bottom storey :-

Exterior columns

$$M_{DA} = M_{AD} = M_{FC} = M_{CF} = P_2 \times \frac{5}{2}$$

$$= 30 \times 2.5 = \underline{\underline{75}} \text{ kN.m } (\circlearrowright)$$

Interior columns,

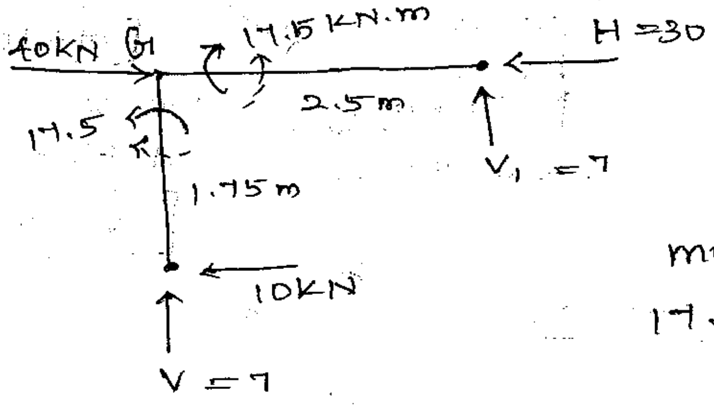
$$M_{EB} = M_{BE} = 2P_2 \times \frac{5}{2} = 60 \times 2.5 = 150 \text{ kNm } (\circlearrowright)$$

Step 3 :- Calculation of mts @ the ends of Beams and Beam shears :-

⇒ Beam shears are evaluated by considering various free bodies bounded by hinges. [rotational equilibrium].

While working out the support mts on each member, we have to remember that the support moment plus external moment is zero.

consider joint G,



$\Sigma H = 0;$

$40 - 10 + (-H) = 0$

$\boxed{H = 30\ kN}$ (←)

moment,

$17.5 = V_1 \times 2.5$

$V_1 = \frac{17.5}{2.5} = 7\,kN$ (↑)

$\boxed{V_1 = 7\ kN}$

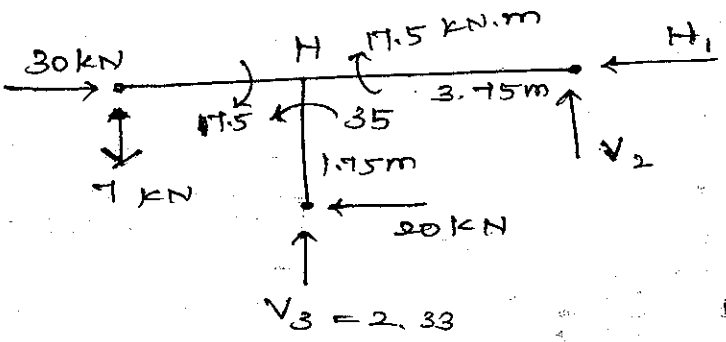
$\Sigma V = 0;$

$V_1 - V = 0$

$V = V_1$

$\boxed{V = 7\ kN}$

consider joint H, :-



$\Sigma H = 0,$

$30 - 20 - H_1 = 0$

$\boxed{H_1 = 10\,kN}$ (←)

moment,

$17.5 = V_2 \times 3.75$

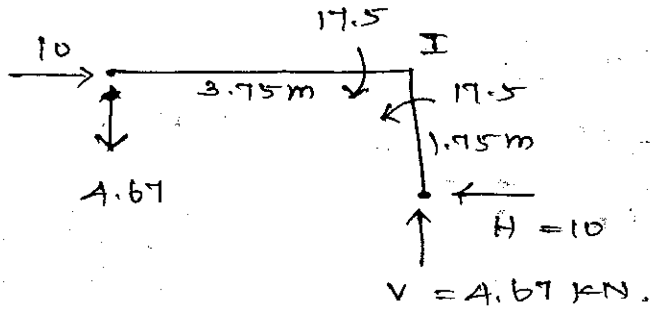
$\boxed{V_2 = 4.67\ kN}$

$\Sigma V = 0$

$-7 + 4.67 + V_3 = 0$

$\boxed{V_3 = 2.33\ kN}$

consider joint I;-



$\Sigma H = 0$

$10 - H = 0$

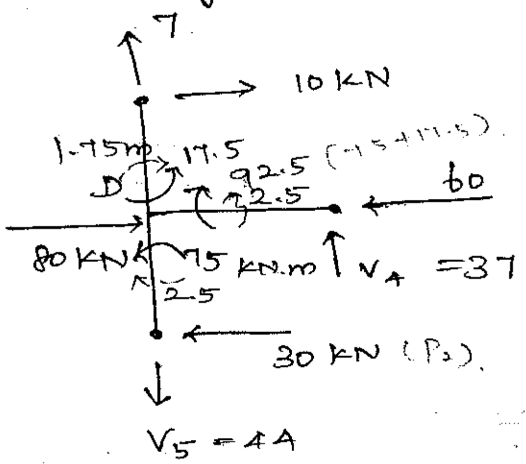
$\boxed{H = 10}$

$\Sigma V = 0$

$4.67 - V = 0$

$\boxed{V = 4.67}$

consider joint D;-



$92.5 = V_A \times 2.5$

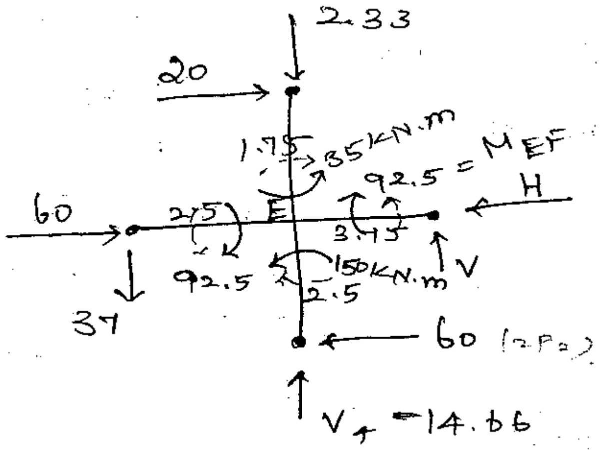
$\boxed{V_A = 37 \text{ kN}}$

$\Sigma V = 0$

$7 + 37 - V_5 = 0$

$\boxed{V_5 = 44 \text{ kN}}$

consider joint E:-



$-35 - 150 + 92.5 + M_{EF} = 0$

$\boxed{M_{EF} = 92.5 \text{ kN.m}}$

$92.5 = V \times 3.75$

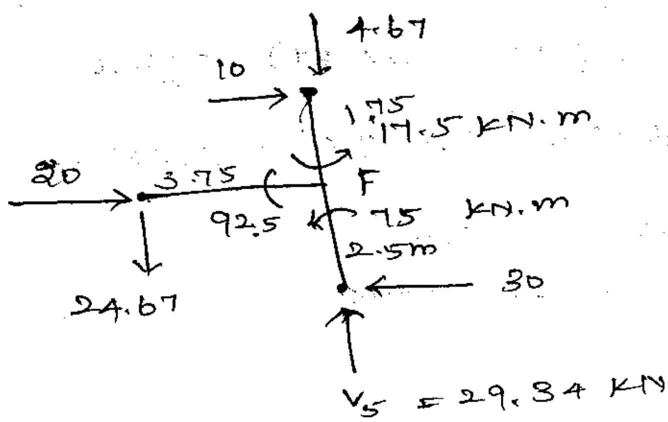
$\boxed{V = 24.67 \text{ kN}}$

$\Sigma V = 0$

$2.33 - V_4 + 37 - 24.67 = 0$

$\boxed{V_4 = 14.66 \text{ kN}}$

consider Joint F

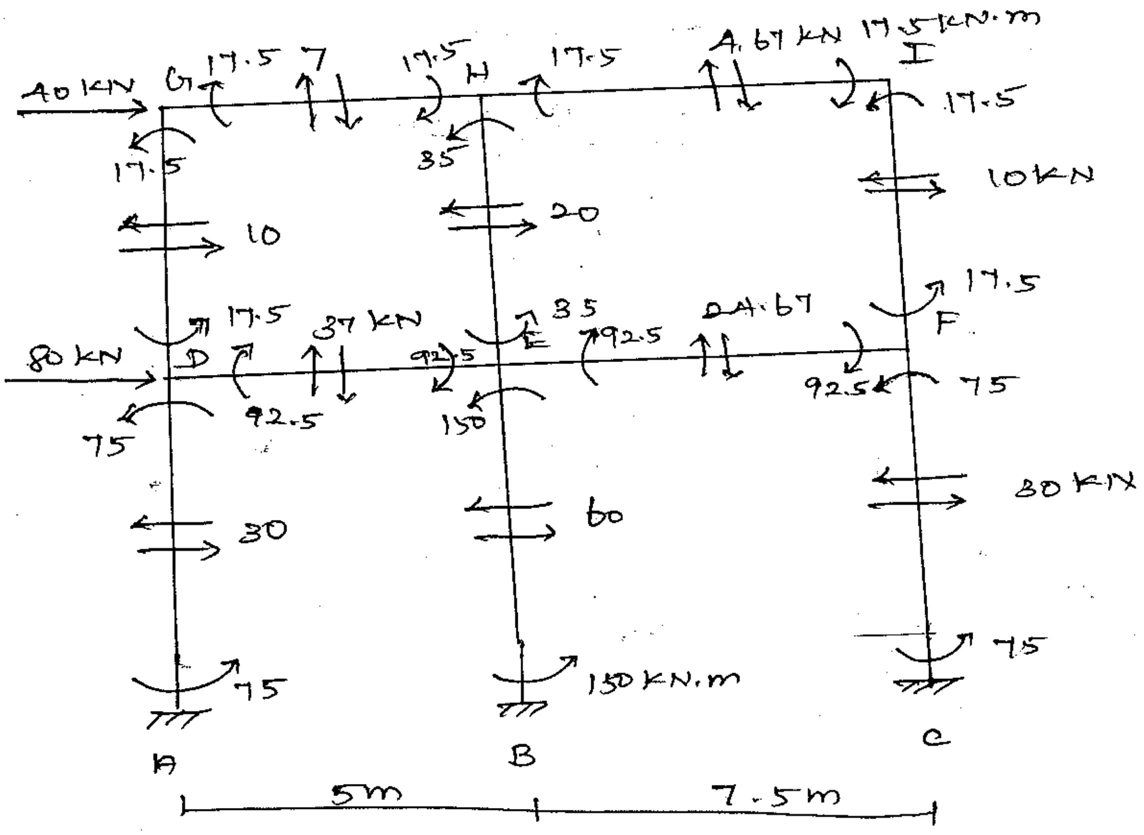


$\Sigma V = 0$

$4.67 - V_5 + 24.67 = 0$

$\boxed{V_5 = 29.34 \text{ kN}}$

Step 4:- Calculation of Axial Forces in Column:-



Axial Forces in columns:-

P_{GD} = Shear in Beam GH = 7 kN (tension)

P_{HE} = $F_{HG} - F_{HI}$ = 7 − 4.67 = 2.33 kN (comp.)

P_{IF} = shear in Beam IH = 4.67 kN (comp.)

P_{DA} = Axial Force in GD + shear in DE
= 7 + 37 = 44 kN (tens.)

$P_{EB} = P_{HE} + (F_{DE} - F_{EF})$

$= 2.33 + (37 - 24.67) = \underline{14.66\ KN}$ (comp.)

P_{FC} = axial Force in IF + shear in FE

$= 4.67 + 24.67$

$= \underline{\underline{29.34\ KN}}$ (comp)

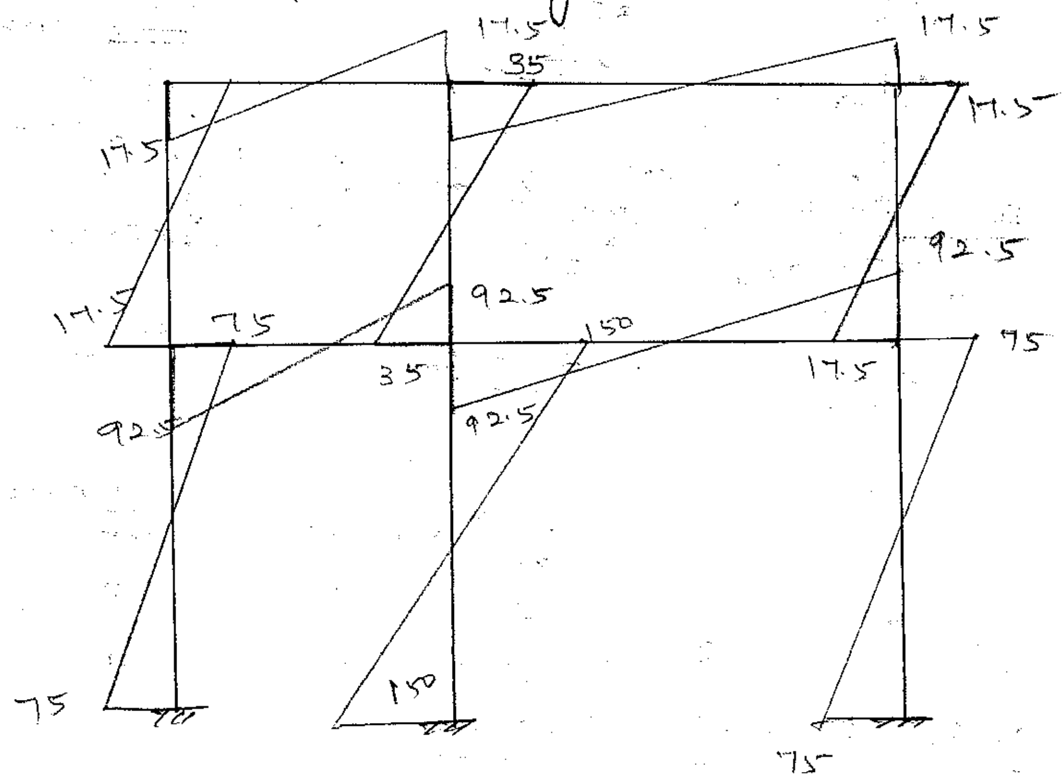
<u>check:-</u>

Total axial F @ the base,

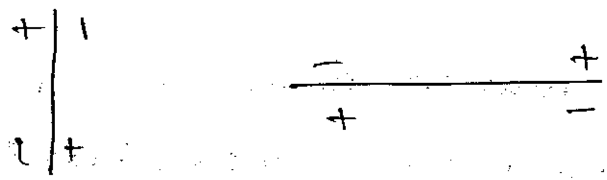
$= +44 - 14.66 - 29.34$

$= 0$ (zero).

<u>step 5:-</u> <u>B. M Diagram:-</u>



<u>sing convention:-</u>



Cantilever Methods:-

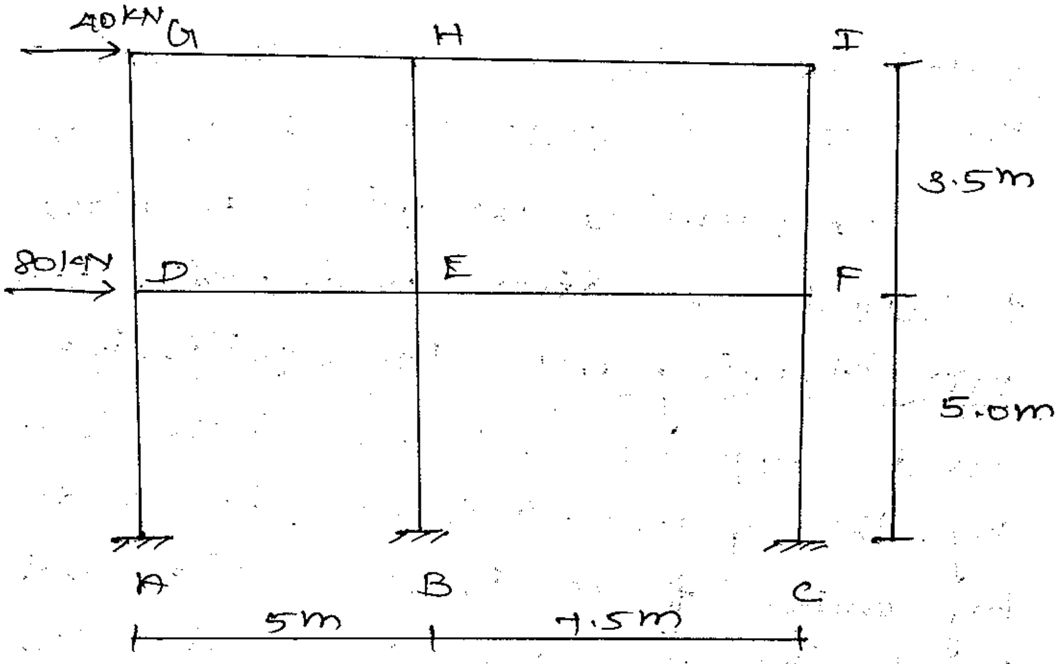
This method assumes the building frame as a vertical cantilever fixed @ the base and free @ the top and subj. to Lateral Lods. Hence the col.s on the windward side will be in tension and those on the leeward side will be in compression.

Assuming the wind to blow from left to right, the wind load will cause a C.W over turning mt. For equilibrium an equal and opposite mt will have to be developed by the frame. This will be made available by axial forces in the col.s, which will be tensile in the wind ward col.s & comp. in the leeward col.s and will be of such magnitude as to create an Anti C.W moment required for the equilibrium.

In this method the following assumptions are made in the analysis:-

1. There is a P.O.C @ the centre of each beam.
2. There is a P.O.C @ the centre of each column.
3. The direct stresses (axial stress) in the columns due to horizontal Forces, are directly proportional to their distance from the centroidal vertical axis of the frame.

Q. Analyse the frame subj. to horizontal Forces as shown in fig. below by cantilever method assuming that all the col.s have the same area of c/s.



Sol:-

Step 1:-

Location of centroidal axis of the col.s:-

Let the centroidal axis be @ a dist. of $\bar{x}$ from the windward (W.W) col. GDA. Taking mt of areas of the col.s about GDA. The c/s area of all the col.s are assumed to be same area 'A'.

$$\bar{x} = \frac{\Sigma x A}{\Sigma A}$$

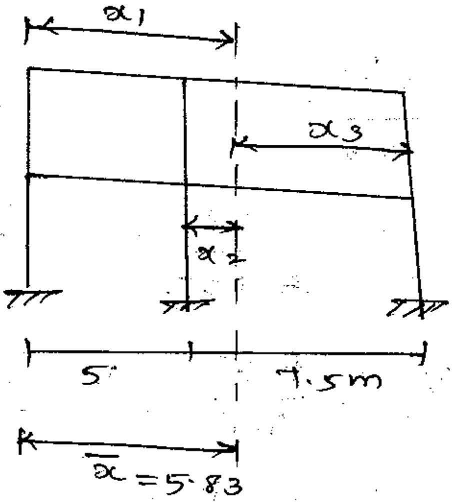
$$= \frac{(0 \times A) + (5 \times A) + (12.5 \times A)}{A + A + A} = \frac{17.5 \cancel{A}}{3 \cancel{A}}$$

$$\boxed{\bar{x} = 5.83 \text{ m}}$$

$\therefore x_1 = 5.83\,m$

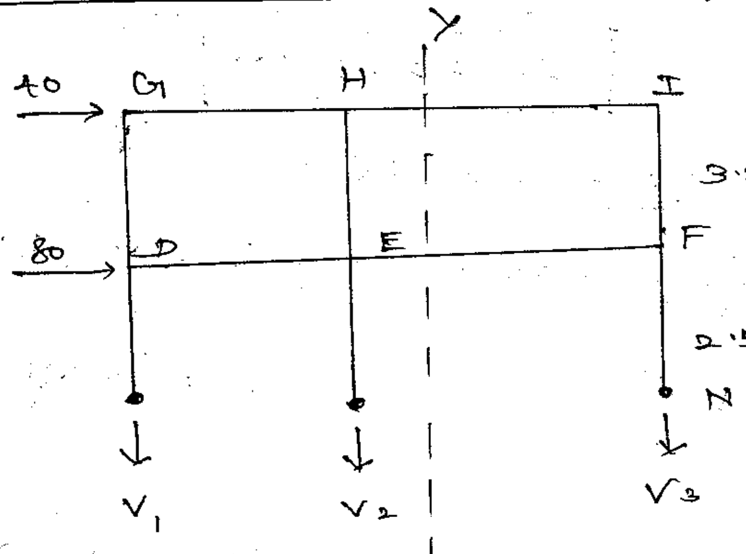
$x_2 = 5.83 - 5 = 0.83\,m$

$x_3 = 7.5 - 0.83 = 6.67\,m$



Step 2:-

Axial Forces in the col.s of First Storey:-



taking mt about 'N' (+↺)

$$(40 \times \overset{(3.5+2.5)}{6}) + (80 \times 2.5) - V_1 \times \overset{(5+7.5)}{12.5} - V_2 \times 7.5 = 0$$

$$\boxed{240 + 200 - 12.5\,V_1 - 7.5\,V_2 = 0} \quad -(1)$$

Let the axial Force in the col. DA,

$$\boxed{V_1 = V}$$

Since the areas are equal the A.F in the other col.s will be in ~~par~~ proportion to their dist. from the centroidal axis.

$$\frac{V}{x_1} = \frac{V_2}{x_2} = \frac{V_3}{x_3}$$

$$\therefore V_2 = \left(\frac{x_2}{x_1}\right) V = \frac{0.83}{5.83} V \quad (\downarrow).$$

$$V_3 = \left(\frac{x_3}{x_1}\right) V = \left(\frac{6.67}{5.83}\right) V \quad (\uparrow)$$

Sub. V_2 & V_3 in eqn ①

$$\Rightarrow 240 + 200 - 12.5V - 7.5\left(\frac{0.83}{5.83}\right) V = 0$$

$$\boxed{V = 32.43 \text{ kN}}$$

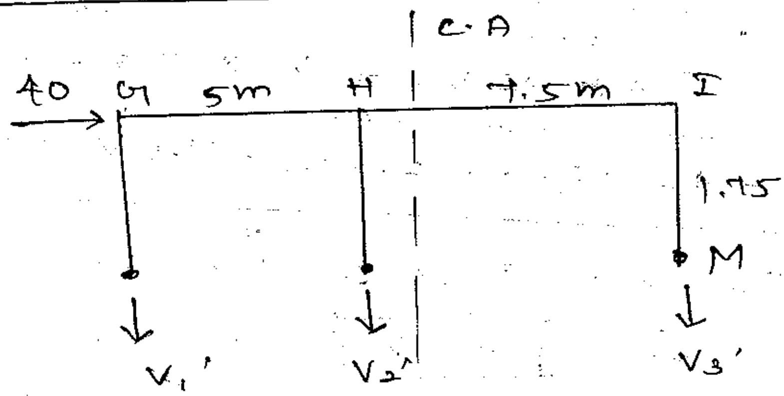
$$V = V_1 = \underline{32.43 \text{ kN}} \quad (\downarrow).$$

$$V_2 = \frac{0.83}{5.83} \times 32.43 = \underline{4.62 \text{ kN}} \quad (\downarrow)$$

$$V_3 = \frac{6.67}{5.83} \times 32.43 = \underline{37.102} \text{ kN} (\uparrow).$$

Step 3:-

Calculation of A.F in the col.s of 2nd storey:-



taking mt about M, (+↻)

$$40 \times 1.75 - V_1' \times \overset{(5+7.5)}{12.5} - V_2' \times 7.5 = 0$$

$$\boxed{70 - 12.5\, V_1' - 7.5\, V_2' = 0} \quad \text{—— (2)}$$

Let $V_1' = V_1 = $ A.F in the col. AF
axial Force.

$$\frac{V_1}{x_1} = \frac{V_2'}{x_2} = \frac{V_3'}{x_3}$$

$$V_2' = \left(\frac{x_2}{x_1}\right) V_1 \ (\downarrow). = \left(\frac{0.83}{5.83}\right) V_1$$

$$V_3' = \left(\frac{x_3}{x_1}\right) V_1 \ (\uparrow). = \left(\frac{6.67}{5.83}\right) V_1$$

sub. V_2' & V_3' value in eq (2),

$$70 - 12.5\, V_1 - 7.5 \left(\frac{0.83}{5.83}\right) V_1 = 0$$

$$\boxed{V_1 = 5.16 \text{ kN}} \ (\downarrow).$$

$$\boxed{V_2' = 0.735 \text{ kN} \ (\downarrow)}$$

$$\boxed{V_3' = 5.903 \text{ kN} \ (\uparrow)}$$

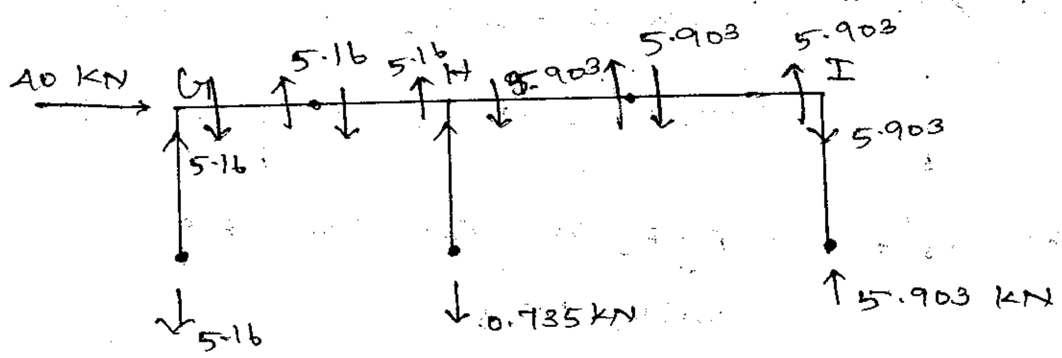
Step 4:-

Calculation of S.F in the colums:-

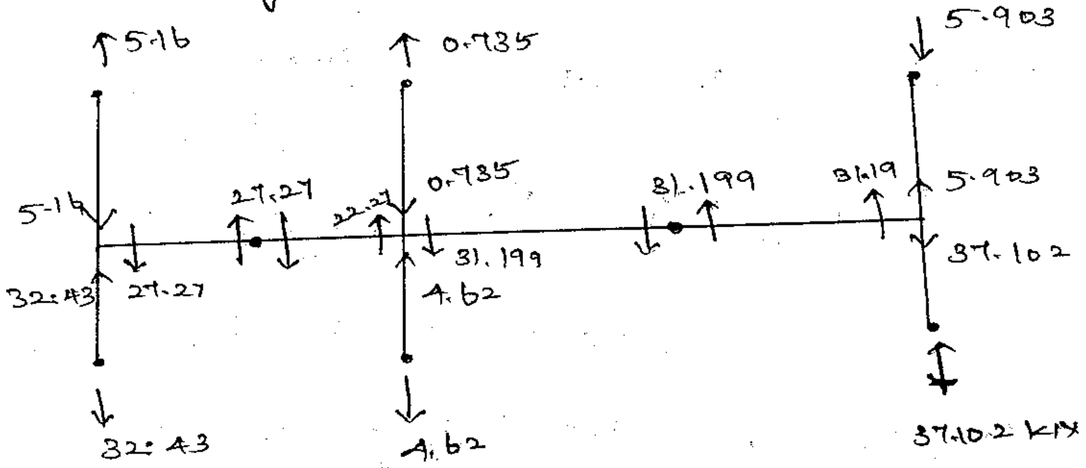
It can be determined from the free bodies of the beams / columns as shown in fig. below.

The beams shear @ the ends of each beams are also the shears @ the hinges @ mid-span.

IInd storey:-



First storey:-



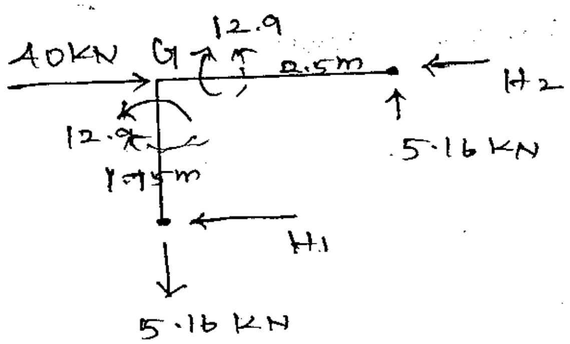
The S.F @ all the joints are calculated and marked above.

Step 5:-

calculation of B.M, S.F in Beams & columns:-

consider the free body diagram of each joint apply equilibrium equations

consider joint G,



$12.9 = H_1 \times 1.75$

$\boxed{H_1 = 7.37\ KN}$

$M_{GH} = M_{HG} = 5.16 \times 2.5$

$= \underline{12.9\ KN.m}$

$M_{GD} = M_{DG} = \underline{12.9}\ KN.m$

$\Sigma H = 0$

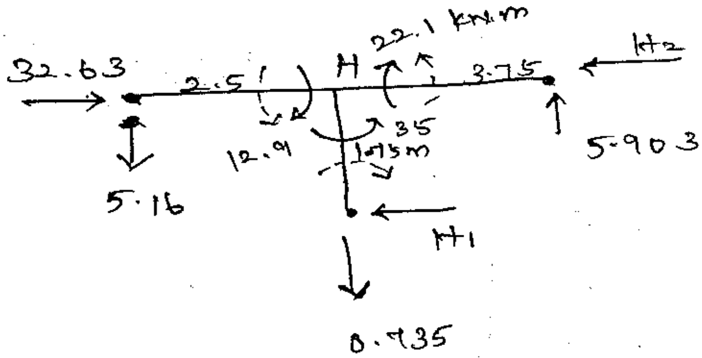
$40 - 7.37 - H_2 = 0$

$\boxed{H_2 = 32.63 \text{ kN}}$

Consider joint H:-

$M_{IH} = M_{HI} = 5.903 \times 3.75$

$= 22.1 \text{ kN.m} (\circlearrowright)$



$M_{HE} = M_{EH}$

$= 22.1 + 12.9$

$= 35 (\circlearrowleft)$

$35 = H_1 \times 1.75$

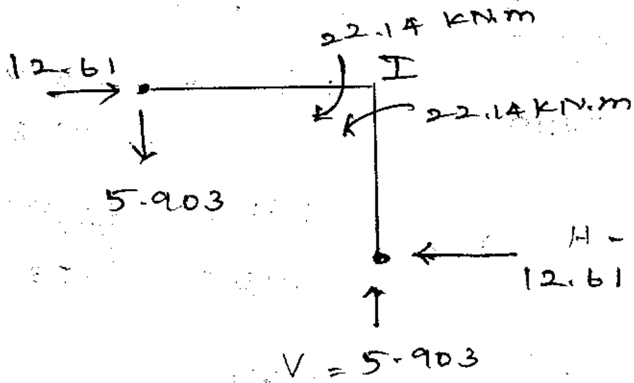
$\boxed{H_1 = 20.02 \text{ kN}}$

$\Sigma H = 0$

$32.63 - 20.02 = H_2$

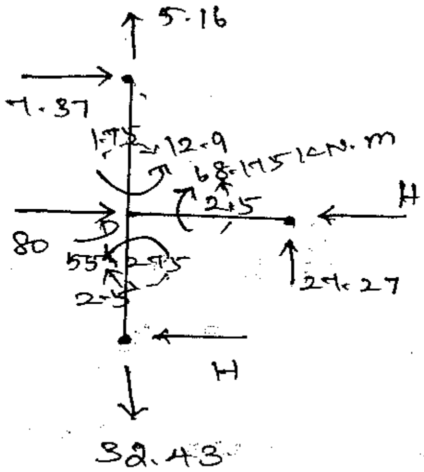
$\boxed{H_2 = 12.61 \text{ kN}}$

joint I:-



$M_{FI} = M_{IF} = 22.14 \text{ kN.m}$

consider joint D:-



$M_{ED} = M_{DE} = 27.27 \times 2.5$

$= 68.175 \text{ kN.m}$

$M_{DA} = M_{AD}$

$= 68.175 - 12.9$

$= 55.275 \text{ kN.m}$

$55.275 = H \times 2.5$

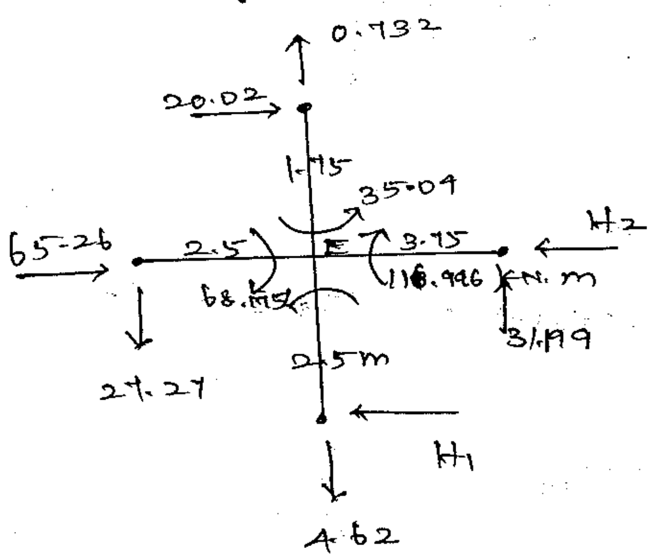
$\boxed{H = 22.11 \text{ kN}}$

$\Sigma H = 0$

$80 + 7.37 - 22.11 - H_1 = 0$

$\boxed{H_1 = 65.26 \text{ kN}}$ $(\leftarrow)$

consider joint E:-



$M_{EF} = 31.99 \times 3.75$

$= 119.996 \text{ kN m}$

$M_{EF} = M_{FE}$

$\Sigma M_E = 0$

$116.996 - 35.04 + 68.175 = M_{EB}$

$M_{EB} = 150.131 \text{ kN m}$

$M_{EB} = M_{BE}$

$150.131 = H_1 \times 2.5$

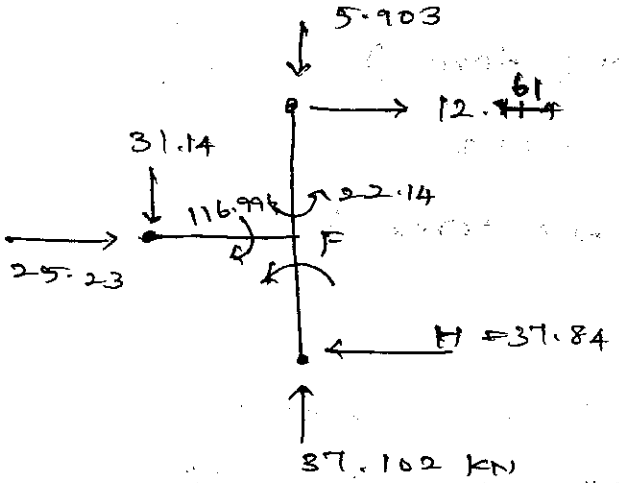
$\boxed{H_1 = 60.05 \text{ kN}}$;

$\Sigma H = 0$

$65.26 + 20.02 - 60.05 = H_2$

$\boxed{H_2 = 25.23 \text{ kN}}$

Consider joint F:-



$M_{FC} = 116.6996 - 22.14$

$= 94.86 \text{ kN m}$

$\Sigma H = 0$

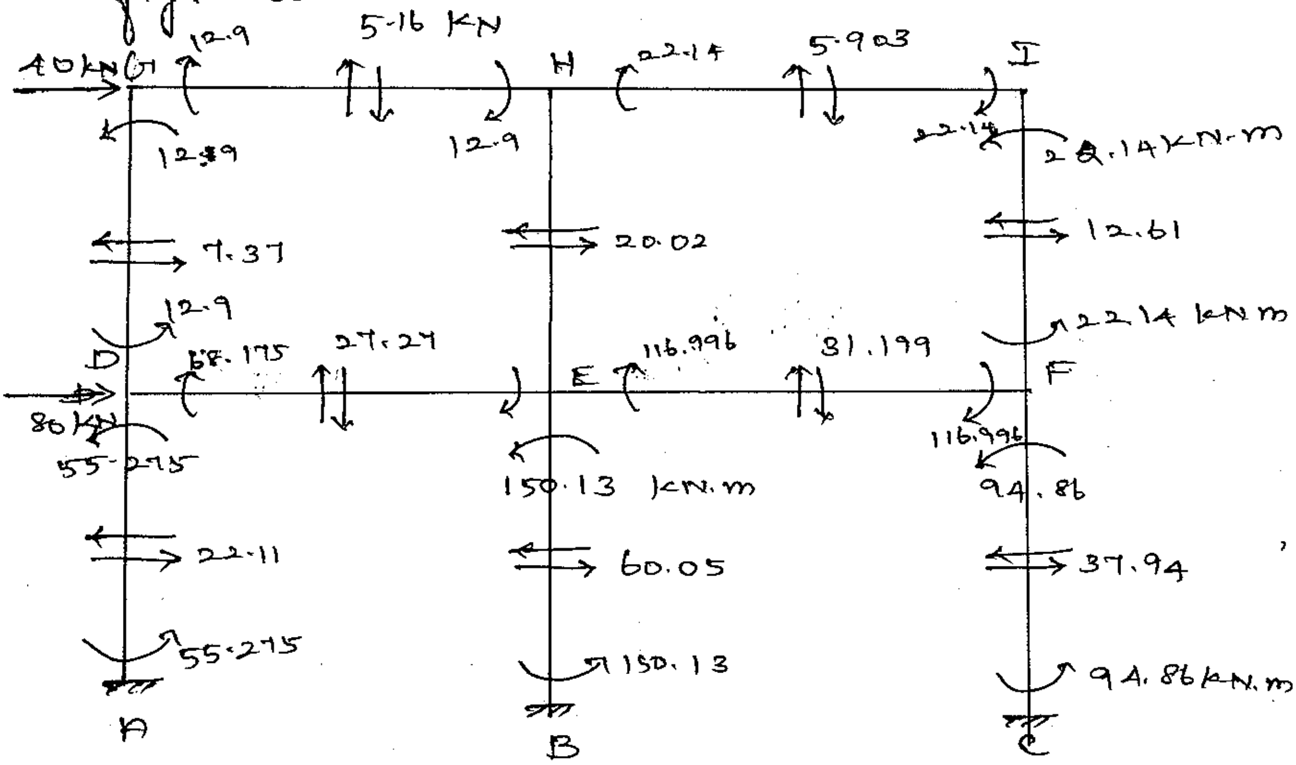
$25.23 + 12.61 - H = 0$

$\boxed{H = 37.84 \text{ kN}}$

Step 6:-

Calculation of A.F in the columns:-

mts & shears in cols & Beams are shown in fig. below.



Axial Forces in columns:-

$P_{GD} = 5.16 \text{ kN}$ (tension)

$P_{HE} = 0.735 \text{ kN}$ (tens.)

$P_{IF} = 5.903$ kN (comp.)

$P_{DA} = 32.43$ kN (tens.)

$P_{EB} = 4.62$ kN (tens.)

$P_{FC} = 37.102$ kN (comp.)

Check :-

total axial Force @ base.

$= -32.43 - 4.62 + 37.102$

$= 0.052 \simeq 0.$ (zero)

Calculation of Design Seismic Force by Static Analysis Method

Problem Statement:

Consider a four-storey reinforced concrete office building shown in Fig. 1.1. The building is located in Shillong (seismic zone V). The soil conditions are medium stiff and the entire building is supported on a raft foundation. The R. C. frames are infilled with brick-masonry. The lumped weight due to dead loads is 12 kN/m^2 on floors and 10 kN/m^2 on the roof. The floors are to cater for a live load of 4 kN/m^2 on floors and 1.5 kN/m^2 on the roof. Determine design seismic load on the structure as per new code.

[Problem adopted from Jain S.K, "A Proposed Draft for IS:1893 Provisions on Seismic Design of Buildings; Part II: Commentary and Examples", Journal of Structural Engineering, Vol.22, No.2, July 1995, pp.73-90]

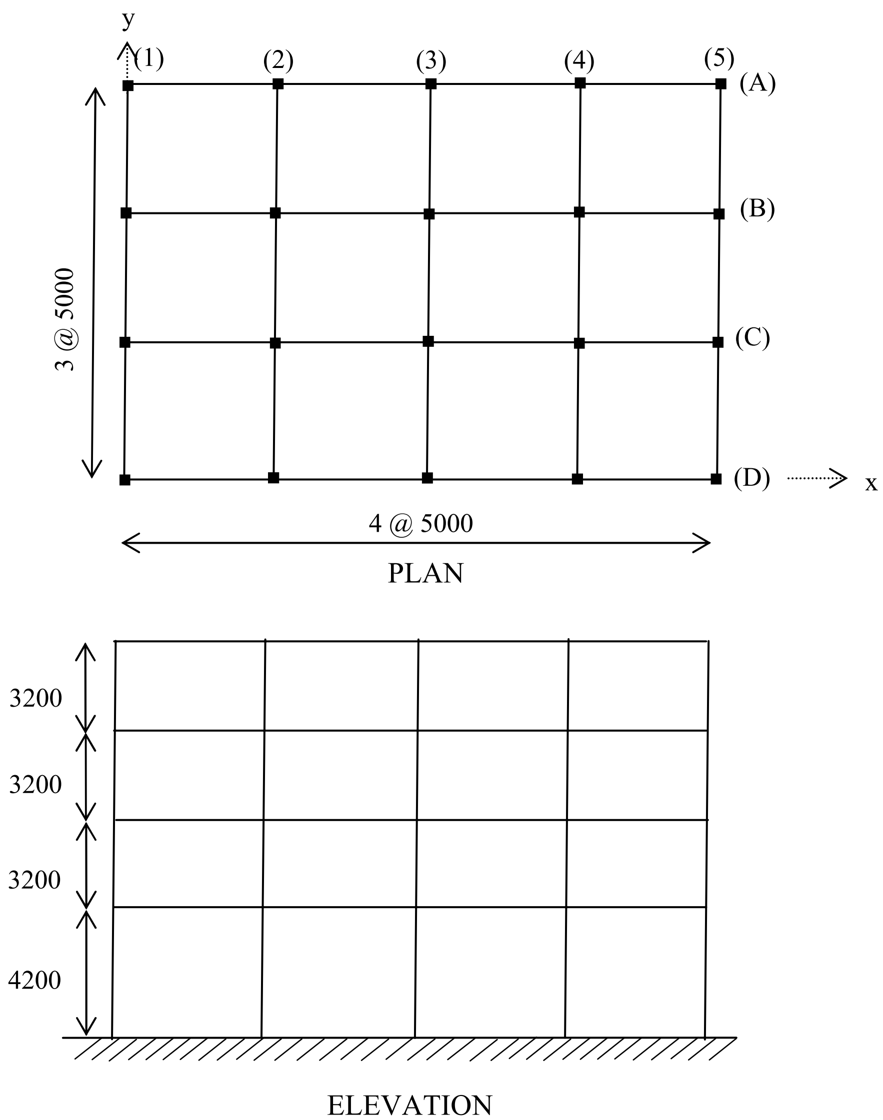


Figure 1.1 – Building configuration

Solution:

Design Parameters:

For seismic zone V, the zone factor Z is 0.36 (Table 2 of IS: 1893). Being an office building, the importance factor, *I*, is 1.0 (Table 6 of IS: 1893). Building is required to be provided with moment resisting frames detailed as per IS: 13920-1993. Hence, the response reduction factor, *R*, is 5.

(Table 7 of IS: 1893 Part 1)

Seismic Weights:

The floor area is 15×20=300 sq. m. Since the live load class is 4kN/sq.m, only 50% of the live load is lumped at the floors. At roof, no live load is to be lumped. Hence, the total seismic weight on the floors and the roof is:

Floors:

$W_1 = W_2 = W_3 = 300 \times (12 + 0.5 \times 4)$
$= 4{,}200$ kN

Roof:

$W_4 = 300 \times 10$
$= 3{,}000$ kN

(clause7.3.1, Table 8 of IS: 1893 Part 1)

Total Seismic weight of the structure,

$W = \sum W_i = 3 \times 4{,}200 + 3{,}000$
$= 15{,}600$ kN

Fundamental Period:

Lateral load resistance is provided by moment resisting frames infilled with brick masonry panels. Hence, approximate fundamental natural period:

(Clause 7.6.2. of IS: 1893 Part 1)

EL in X-Direction:

$T = 0.09h/\sqrt{d}$
$= 0.09(13.8)/\sqrt{20}$
$= 0.28$ sec

The building is located on Type II (medium soil). From Fig. 2 of IS: 1893, for T=0.28 sec, $S_a/g = 2.5$

$$A_h = \frac{ZI}{2R}\frac{S_a}{g}$$
$$= \frac{0.36 \times 1.0}{2 \times 5} \times 2.5$$
$$= 0.09$$

(Clause 6.4.2 of IS: 1893 Part 1)

Design base shear

$$V_B = A_h W$$
$$= 0.09 \times 15{,}600$$
$$= 1{,}440 \text{ kN}$$

(Clause 7.5.3 of IS: 1893 Part 1)

Force Distribution with Building Height:

The design base shear is to be distributed with height as per clause 7.7.1. Table 1.1 gives the calculations. Fig. 1.2(a) shows the design seismic force in X-direction for the entire building.

EL in Y-Direction:

$$T = 0.09\,h/\sqrt{d}$$
$$= 0.09(13.8)/\sqrt{15}$$
$$= 0.32 \text{ sec}$$

$\frac{S_a}{g} = 2.5;$

$A_h = 0.09$

Therefore, for this building the design seismic force in Y-direction is same as that in the X-direction. Fig. 1.2(b) shows the design seismic force on the building in the Y-direction.

Table 1.1 – Lateral Load Distribution with Height by the Static Method

Storey Level	$W_i\ (kN)$	h_i (m)	$W_i h_i^2 \times (1000)$	$\dfrac{W_i h_i^2}{\sum W_i h_i^2}$	Lateral Force at i^{th} Level for EL in direction (kN)	
					X	Y
4	3,000	13.8	571.3	0.424	611	611
3	4,200	10.6	471.9	0.350	504	504
2	4,200	7.4	230.0	0.171	246	246
1	4,200	4.2	74.1	0.055	79	79
Σ			**1,347.3**	**1,000**	**1,440**	**1,440**

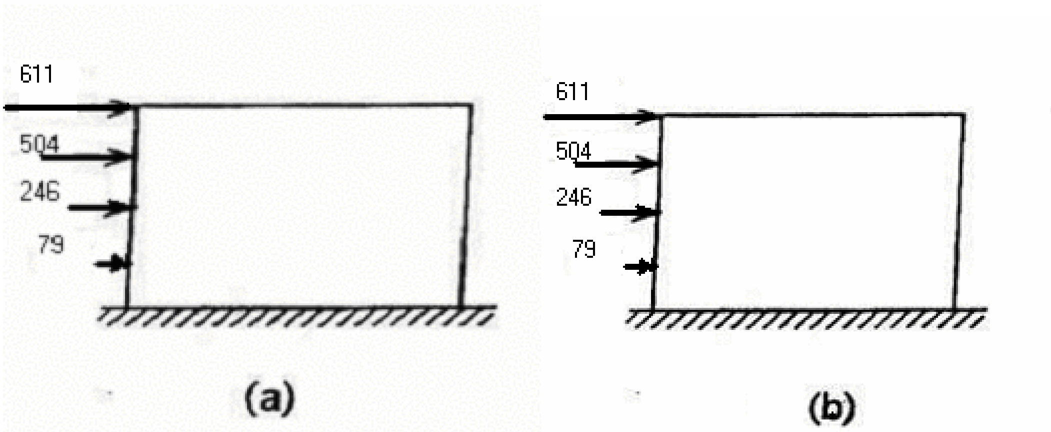


Figure 1.2 -- Design seismic force on the building for (a) X-direction, and (b) Y-direction.

Example 2 – Calculation of Design Seismic Force by Dynamic Analysis Method

Problem Statement:

For the building of Example 1, the dynamic properties (natural periods, and mode shapes) for vibration in the X-direction have been obtained by carrying out a free vibration analysis (Table 2.1). Obtain the design seismic force in the X-direction by the dynamic analysis method outlined in cl. 7.8.4.5 and distribute it with building height.

Table 2.1 – Free Vibration Properties of the building for vibration in the X-Direction

	Mode 1	Mode 2	Mode 3
Natural Period (sec)	0.860	0.265	0.145
	Mode Shape		
Roof	1.000	1.000	1.000
3^{rd} Floor	0.904	0.216	-0.831
2^{nd} Floor	0.716	-0.701	-0.574
1^{st} Floor	0.441	-0.921	1.016

[Problem adopted from, Jain S.K, "A Proposed Draft for IS: 1893 Provisions on Seismic Design of Buildings; Part II: Commentary and Examples", Journal of Structural Engineering, Vol.22, No.2, July 1995, pp.73-90]

Solution:

Table 2.2 -- Calculation of modal mass and modal participation factor (clause 7.8.4.5)

Storey Level i	Weight $W_i(kN)$	Mode 1			Mode 2			Mode 3		
4	3,000	1.000	3,000	3,000	1.000	3,000	3,000	1.000	3,000	3,000
3	4,200	0.904	3,797	3,432	0.216	907	196	-0.831	-3,490	2,900
2	4,200	0.716	3,007	2,153	-0.701	-2,944	2,064	-0.574	-2,411	1,384
1	4,200	0.441	1,852	817	-0.921	-3,868	3,563	1.016	4,267	4,335
Σ	15,600		11,656	9,402		-2,905	8,822		1,366	11,620
$M_k = \dfrac{\left[\sum w_i\, \phi_{ik}\right]^2}{g\sum w_i\, \phi_{ik}^2}$		$\dfrac{11{,}656^2}{9{,}402g} = \dfrac{14{,}450kN}{g}$ = 14,45,000 kg			$\dfrac{2{,}905^2}{8{,}822g} = \dfrac{957kN}{g}$ =95,700 kg			$\dfrac{1{,}366^2}{11{,}620g} = \dfrac{161kN}{g}$ = 16,100 kg		
% of Total weight		92.6%			6.1%			1.0%		
$P_k = \dfrac{\sum w_i\, \phi_{ik}}{\sum w_i\, \phi_{ik}^2}$		$\dfrac{11{,}656}{9{,}402} = 1.240$			$\dfrac{-2{,}905}{8{,}822} = -0.329$			$\dfrac{1{,}366}{11{,}620} = 0.118$		

It is seen that the first mode excites 92.6% of the total mass. Hence, in this case, codal requirements on number of modes to be considered such that at least 90% of the total mass is excited, will be satisfied by considering the first mode of vibration only. However, for illustration, solution to this example considers the first three modes of vibration.

The lateral load Q_{ik} acting at i^{th} floor in the k^{th} mode is

$$Q_{ik} = A_{hk}\phi_{ik}\, P_k\, W_i$$

(clause 7.8.4.5 c of IS: 1893 Part 1)

The value of A_{hk} for different modes is obtained from clause 6.4.2.

Mode 1:

$T_1 = 0.860$ sec;

$(S_a / g) = \frac{1.0}{0.86} = 1.16$;

$$A_{h1} = \frac{ZI}{2R}(S_a / g) = \frac{0.36 \times 1}{2 \times 5} \times (1.16) = 0.0418$$

$$Q_{i1} = 0.0418 \times 1.240 \times \phi_{i1} \times W_i$$

Mode 2:

$T_2 = 0.265$ sec;

$(S_a / g) = 2.5$;

$$A_{h2} = \frac{ZI}{2R}(S_a / g) = \frac{0.36 \times 1}{2 \times 5} \times (2.5) = 0.09$$

$$Q_{i1} = 0.09 \times (-0.329) \times \phi_{i2} \times W_i$$

Mode 3:

$T_3 = 0.145$ sec;

$(S_a / g) = 2.5$;

$$A_{h3} = \frac{ZI}{2R}(S_a / g) = \frac{0.36 \times 1}{2 \times 5} \times (2.5) = 0.09$$

$$Q_{i3} = 0.09 \times (0.118) \times \phi_{i3} \times W_i$$

Table 2.3 summarizes the calculation of lateral load at different floors in each mode.

Table 2.3 – Lateral load calculation by modal analysis method (earthquake in X-direction)

Floor Level i	Weight W_i (kN)	Mode 1			Mode 2			Mode 3		
		ϕ_{i1}	Q_{i1}	V_{i1}	ϕ_{i2}	Q_{i2}	V_{i2}	ϕ_{i3}	Q_{i3}	V_{i3}
4	3,000	1.000	155.5	155.5	1.000	-88.8	-88.8	1.000	31.9	31.9
3	4,200	0.904	196.8	352.3	0.216	-26.8	-115.6	-0.831	-37.1	-5.2
2	4,200	0.716	155.9	508.2	-0.701	87.2	-28.4	-0.574	-25.6	-30.8
1	4,200	0.441	96.0	604.2	-0.921	114.6	86.2	1.016	45.4	14.6

Since all of the modes are well separated (clause 3.2), the contribution of different modes is combined by the SRSS (square root of the sum of the square) method

$V_4 = [(155.5)^2 + (88.8)^2 + (31.9)^2]^{1/2} = 182$ kN

$V_3 = [(352.3)^2 + (115.6)^2 + (5.2)^2]^{1/2} = 371$ kN

$V_2 = [(508.2)^2 + (28.4)^2 + (30.8)^2]^{1/2} = 510$ kN

$V_1 = [(604.2)^2 + (86.2)^2 + (14.6)^2]^{1/2} = 610$ kN

(Clause 7.8.4.4a of IS: 1893 Part 1)

The externally applied design loads are then obtained as:

$Q_4 = V_4 = 182$ kN
$Q_3 = V_3 - V_4 = 371 - 182 = 189$ kN
$Q_2 = V_2 - V_3 = 510 - 371 = 139$ kN
$Q_1 = V_1 - V_2 = 610 - 510 = 100$ kN

(Clause 7.8.4.5f of IS: 1893 Part 1)

Clause 7.8.2 requires that the base shear obtained by dynamic analysis (V_B = 610 kN) be compared with that obtained from empirical fundamental period as per Clause 7.6. If V_B is less than that from empirical value, the response quantities are to be scaled up.

We may interpret "base shear calculated using a fundamental period as per 7.6" in two ways:

1. We calculate base shear as per Cl. 7.5.3. This was done in the previous example for the same building and we found the base shear as 1,404 *kN*. Now, dynamic analysis gives us base shear of 610 *kN* which is lower. Hence, all the response quantities are to be scaled up in the ratio (1,404/610 = 2.30). Thus, the seismic forces obtained above by dynamic analysis should be scaled up as follows:

$Q_4 = 182 \times 2.30 = 419$ kN
$Q_3 = 189 \times 2.30 = 435$ kN
$Q_2 = 139 \times 2.30 = 320$ kN

$Q_1 = 100 \times 2.30 = 230$ kN

2. We may also interpret this clause to mean that we redo the dynamic analysis but replace the fundamental time period value by T_a (= 0.28 sec). In that case, for mode 1:

$T_1 = 0.28$ sec;
$(S_a / g) = 2.5$;

$$A_{h1} = \frac{ZI}{2R}(S_a / g)$$
$=0.09$

Modal mass times A_{h1}
$= 14{,}450 \times 0.09$
$= 1{,}300$ kN

Base shear in modes 2 and 3 is as calculated earlier: Now, base shear in first mode of vibration =1300 kN, 86.2 kN and 14.6 kN, respectively.

Total base shear by SRSS

$$= \sqrt{1300^2 + 86.2^2 + 14.6^2}$$
$= 1{,}303$ kN

Notice that most of the base shear is contributed by first mode only. In this interpretation of Cl 7.8.2, we need to scale up the values of response quantities in the ratio (1,303/610 = 2.14). For instance, the external seismic forces at floor levels will now be:

$Q_4 = 182 \times 2.14 = 389$ kN
$Q_3 = 189 \times 2.14 = 404$ kN
$Q_2 = 139 \times 2.14 = 297$ kN
$Q_1 = 100 \times 2.14 = 214$ kN

Clearly, the second interpretation gives about 10% lower forces. We could make either interpretation. Herein we will proceed with the values from the second interpretation and compare the design values with those obtained in Example 1 as per static analysis:

Table 2.4 – Base shear at different storeys

Floor Level *i*	*Q* (static)	*Q* (dynamic, scaled)	*Storey Shear V* (static)	*Storey ShearV* (dynamic, scaled)	Storey Moment, *M* (Static)	Storey Moment, *M* (Dynamic)
4	611 kN	389 kN	611 kN	389 kN	1,907 kNm	1,245 kNm
3	504 kN	404 kN	1,115kN	793 kN	5,386 kNm	3,782 kNm
2	297 kN	297 kN	1,412kN	1,090 kN	9.632 kNm	7,270 kNm
1	79 kN	214 kN	1,491 kN	1,304 kN	15,530 kNm	12,750 kNm

Notice that even though the base shear by the static and the dynamic analyses are comparable, there is considerable difference in the lateral load distribution with building height, and therein lies the advantage of dynamic analysis. For instance, the storey moments are significantly affected by change in load distribution.

Example 3 – Location of Centre of Mass

Problem Statement:

Locate centre of mass of a building having non-uniform distribution of mass as shown in the figure 3.1

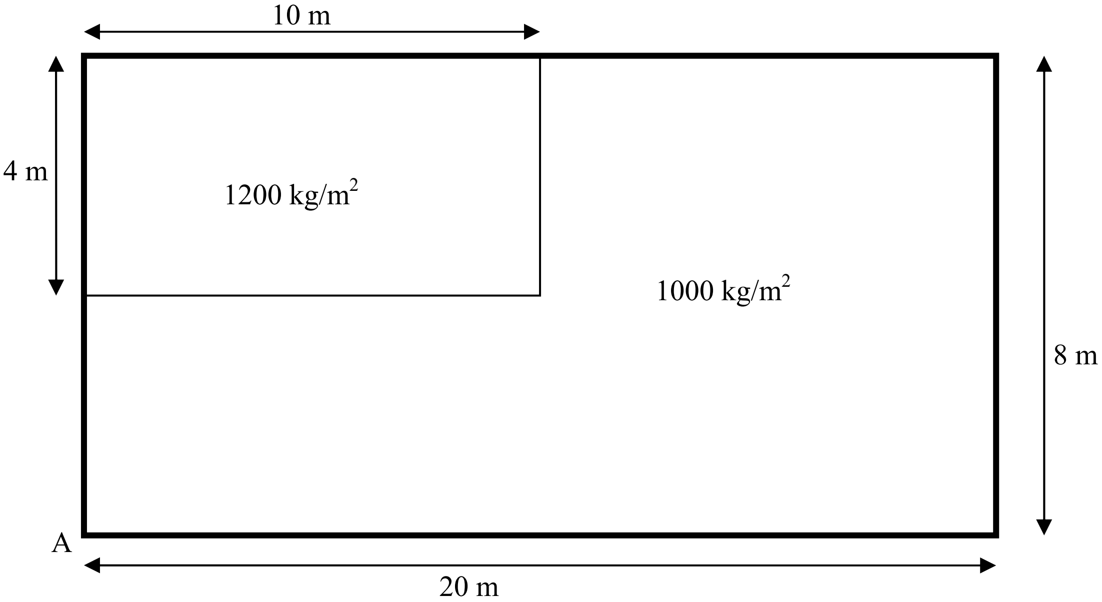


Figure 3.1 –Plan

Solution:

Let us divide the roof slab into three rectangular parts as shown in figure 2.1

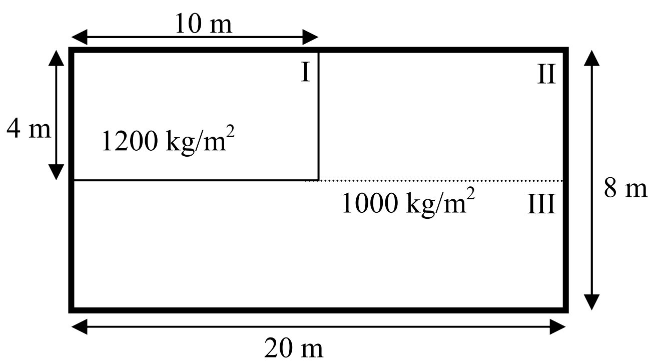


Figure 3.2

Mass of part I is 1200 kg/m^2, while that of the other two parts is 1000 kg/m^2.

Let origin be at point A, and the coordinates of the centre of mass be at (X, Y)

$$X = \frac{(10\times 4\times 1200)\times 5 + (10\times 4\times 1000)\times 15 + (20\times 4\times 1000)\times 10}{(10\times 4\times 1200) + (10\times 4\times 1000) + (20\times 4\times 1000)}$$

= 9.76 m

$$Y = \frac{(10\times 4\times 1200)\times 6 + (10\times 4\times 1000)\times 6 + (20\times 4\times 1000)\times 2}{(10\times 4\times 1200) + (10\times 4\times 1000) + (20\times 4\times 1000)}$$

= 4.1 m

Hence, coordinates of centre of mass are (9.76, 4.1)

Example 4 – Location of Centre of Stiffness

Problem Statement:

The plan of a simple one storey building is shown in figure 3.1. All columns and beams are same. Obtain its centre of stiffness.

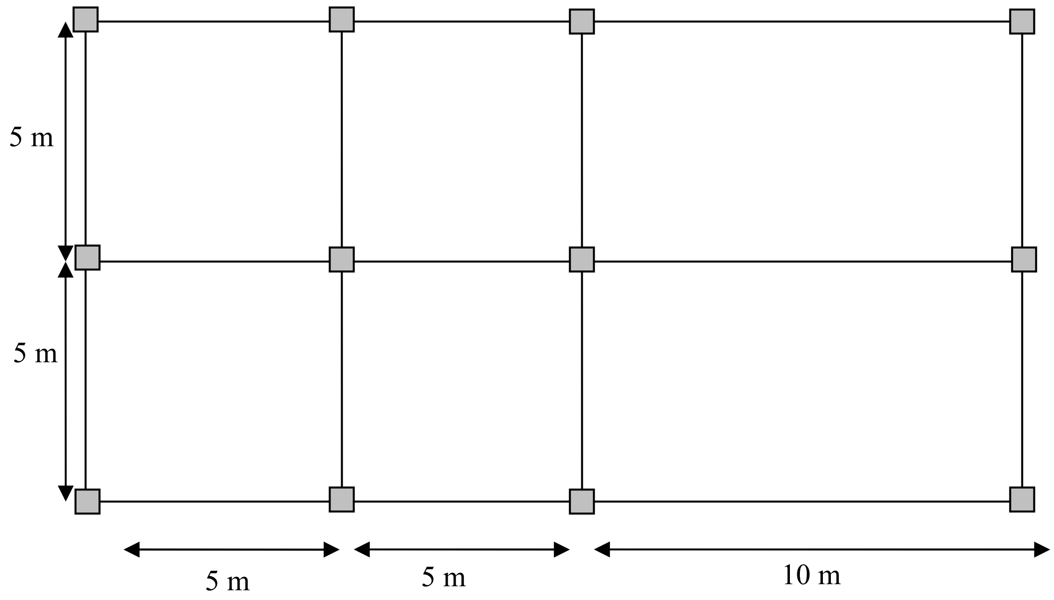


Figure 4.1 –Plan

Solution:

In the X-direction there are three identical frames located at uniform spacing. Hence, the y-coordinate of centre of stiffness is located symmetrically, i.e., at 5.0 m from the left bottom corner.

In the Y-direction, there are four identical frames having equal lateral stiffness. However, the spacing is not uniform. Let the lateral stiffness of each transverse frame be *k*, and coordinating of center of stiffness be (X, Y).

$$X = \frac{k \times 0 + k \times 5 + k \times 10 + k \times 20}{k + k + k + k} = 8.75 \text{ m}$$

Hence, coordinates of centre of stiffness are (8.75, 5.0).

Example 5 –Lateral Force Distribution as per Torsion Provisions of IS 1893-2002 (Part 1)

Problem Statement:

Consider a simple one-storey building having two shear walls in each direction. It has some gravity columns that are not shown. All four walls are in M25 grade concrete, 200 thick and 4 m long. Storey height is 4.5 m. Floor consists of cast-in-situ reinforced concrete. Design shear force on the building is 100 kN in either direction.

Compute design lateral forces on different shear walls using the torsion provisions of 2002 edition of IS 1893 (Part 1).

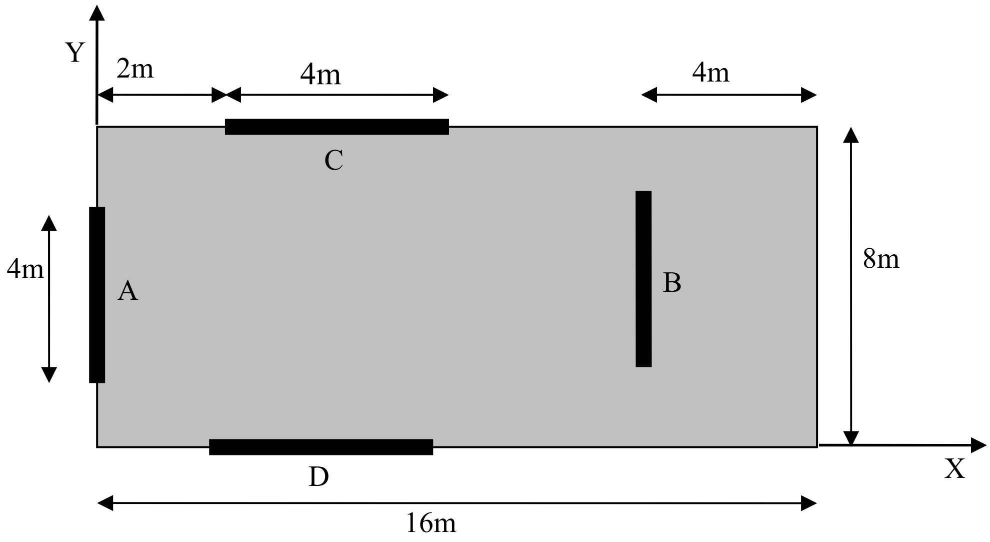


Figure 5.1 – Plan

Solution:

Grade of concrete: M25

$E = 5000\sqrt{25} = 25000$ N/mm^2

Storey height h = 4500 m

Thickness of wall t = 200 mm

Length of walls L = 4000 mm

All walls are same, and hence, spaces have same lateral stiffness, k.

Centre of mass (CM) will be the geometric centre of the floor slab, i.e., (8.0, 4.0).

Centre of rigidity (CR) will be at (6.0, 4.0).

EQ Force in X-direction:

Because of symmetry in this direction, calculated eccentricity = 0.0 m

Design eccentricity:

$$e_d = 1.5 \times 0.0 + 0.05 \times 8 = 0.4,$$

and

$$e_d = 0.0 - 0.05 \times 8 = -0.4$$

(Clause 7.9.2 of IS 1893:2002)

Lateral forces in the walls due to translation:

$$F_{CT} = \frac{K_C}{K_C + K_D} F = 50.0 \text{ kN}$$

$$F_{DT} = \frac{K_D}{K_C + K_D} F = 50.0 \text{ kN}$$

Lateral forces in the walls due to torsional moment:

$$F_{iR} = \frac{K_i r_i}{\sum_{i=A,B,C,D} K_i r_i^2} (Fe_d)$$

where r_i is the distance of the shear wall from CR.

All the walls have same stiffness, $K_A = K_B = K_C = K_D = k$, and

r_A = -6.0 m

r_B = -6.0 m

r_C = 4.0 m
r_D = -4.0 m,
and $e_d = \pm 0.4$ m

Therefore,

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k}\left(Fe_d\right)$$

$= \pm 2.31\,\text{kN}$

Similarly,
$F_{BR} = \pm 2.31\,\text{kN}$
$F_{CR} = \pm 1.54\,\text{kN}$
$F_{DR} = \pm 1.54\,\text{kN}$

Total lateral forces in the walls due to seismic load in X direction:

F_A = 2.31 kN
F_B = 2.31 kN
F_C = Max (50 ± 1.54) = 51.54 kN
F_D = Max (50 ± 1.54) = 51.54 kN

EQ Force in Y-direction:

Calculated eccentricity= 2.0 m
Design eccentricity:
$e_d = 1.5 \times 2.0 + 0.05 \times 16 = 3.8\,\text{m}$
or $= 2.0 - 0.05 \times 16 = 1.2\,\text{m}$

Lateral forces in the walls due to translation:

$$F_{AT} = \frac{K_A}{K_A + K_B} F = 50.0 \text{ kN}$$

$$F_{BT} = \frac{K_B}{K_A + K_B} F = 50.0 \text{ kN}$$

Lateral force in the walls due to torsional moment: when e_d = 3.8 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k}\left(Fe_d\right) = -$$

21.92 kN

Similarly,
F_{BR} = 21.92 kN
F_{CR} = -14.62 kN
F_{DR} = 14.62 kN

Total lateral forces in the walls:
F_A = 50 - 21.92= 28.08 kN
F_B = 50 +20.77= 71.92 kN
F_C = -14.62 kN
F_D = 14.62 kN

Similarly, when e_d = 1.2 m, then the total lateral forces in the walls will be,
F_A = 50 – 6.93 = 43.07 kN
F_B = 50 + 6.93 = 56.93 kN
F_C = - 4.62 kN
F_D = 4.62 kN

Maximum forces in walls due to seismic load in Y direction:
F_A = Max (28.08, 43.07) = 43.07 kN;
F_B = Max (71.92, 56.93) = 71.92 kN;
F_C = Max (14.62, 4.62) = 14.62 kN;
F_D = Max (14.62, 4.62) = 14.62 kN;

Combining the forces obtained from seismic loading in X and Y directions:
F_A = 43.07 kN
F_B =71.92 kN
F_C =51.54 kN
F_D =51.54 kN.

However, note that clause 7.9.1 also states that "However, negative torsional shear shall be neglected". Hence, wall A should be designed for not less than 50 kN.

Example 6 – Lateral Force Distribution as per New Torsion Provisions

Problem Statement:

For the building of example 5, compute design lateral forces on different shear walls using the torsion provisions of revised draft code IS 1893 (part 1), i.e., IITK-GSDMA-EQ05-V2.0.

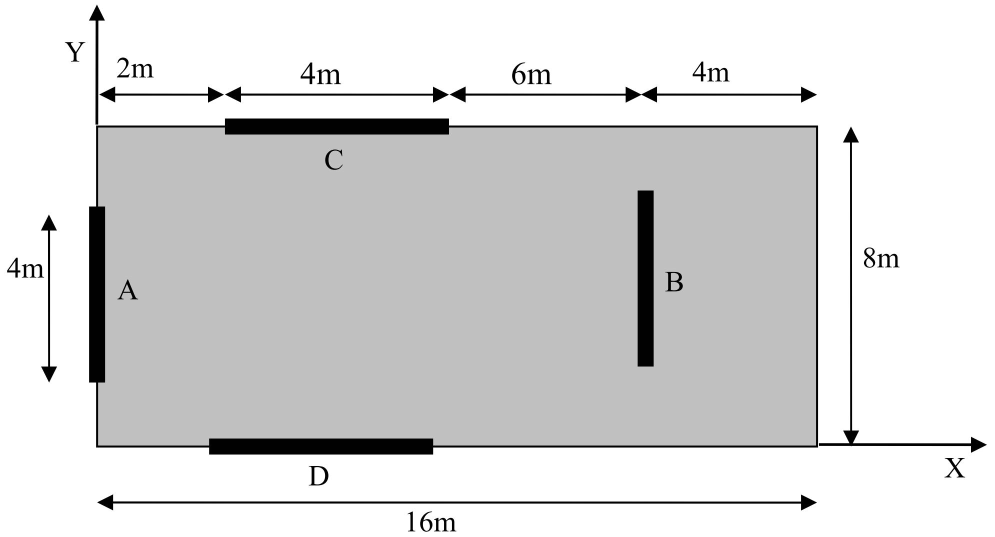


Figure 6.1 – Plan

Solution:

Grade of concrete: M25

$E = 5000\sqrt{25} = 25000\ \text{N/mm}^2$

Storey height h = 4500 m

Thickness of wall t = 200 mm

Length of walls L = 4000 mm

All walls are same, and hence, same lateral stiffness, k.

Centre of mass (CM) will be the geometric centre of the floor slab, i.e., (8.0, 4.0).

Centre of rigidity (CR) will be at (6.0, 4.0).

EQ Force in X-direction:

Because of symmetry in this direction, calculated eccentricity = 0.0 m

Design eccentricity, $e_d = 0.0 \pm 0.1 \times 8 = \pm 0.8$

(clause 7.9.2 of Draft IS 1893: (Part1))

Lateral forces in the walls due to translation:

$$F_{CT} = \frac{K_C}{K_C + K_D} F = 50.0\ \text{kN}$$

$$F_{DT} = \frac{K_D}{K_C + K_D} F = 50.0\ \text{kN}$$

Lateral forces in the walls due to torsional moment:

$$F_{iR} = \frac{K_i r_i}{\sum\limits_{i=A,B,C,D} K_i r_i^2} (Fe_d)$$

where r_i is the distance of the shear wall from CR

All the walls have same stiffness, $K_A = K_B = K_C = K_D = k$

r_A= -6.0 m
r_B= -6.0 m
r_C= 4.0 m
r_D= -4.0 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k} (Fe_d)$$

= - 4.62 kN

Similarly,

F_{BR} = 4.62 kN
F_{CR} = 3.08 kN
F_{DR} = -3.08 kN

Total lateral forces in the walls:

F_A = 4.62 kN
F_B = - 4.62 kN
F_C = 50+3.08 = 53.08 kN
F_D = 50-3.08 = 46.92 kN

Similarly, when e_d= - 0.8 m, then the lateral forces in the walls will be,

F_A = - 4.62 kN
F_B = 4.62 kN
F_C = 50-3.08 = 46.92 kN
F_D = 50+3.08 = 53.08kN

Design lateral forces in walls C and D are:
F_C= F_D= 53.05 kN

EQ Force in Y-direction:

Calculated eccentricity= 2.0 m
Design eccentricity,

$$e_d = 2.0 + 0.1 \times 16 = 3.6\ m$$

or

$$e_d = 2.0 - 0.1 \times 16 = 0.4\ m$$

Lateral forces in the walls due to translation:

$$F_{AT} = \frac{K_A}{K_A + K_B} F = 50.0 \text{ kN}$$

$$F_{BT} = \frac{K_B}{K_A + K_B} F = 50.0 \text{ kN}$$

Lateral force in the walls due to torsional moment: when e_d= 3.6 m

$$F_{AR} = \frac{r_A k}{\left(r_A^2 + r_B^2 + r_C^2 + r_D^2\right)k}\left(Fe_d\right) = -20.77 \text{ kN}$$

Similarly,

F_{BR} = 20.77 kN
F_{CR} = 13.85 kN
F_{DR} = -13.8 kN

Total lateral forces in the walls:

F_A = 50-20.77= 29.23 kN
F_B = 50+20.77= 70.77 kN
F_C = 13.85 kN
F_D = -13.85 kN

Similarly, when e_d= 0.4 m, then the total lateral forces in the walls will be,

F_A = 50-2.31= 47.69 kN
F_B = 50+2.31= 53.31 kN
F_C = 1.54 kN
F_D = - 1.54 kN

Maximum forces in walls A and B
F_A =47.69 kN, F_B =70.77 kN

Design lateral forces in all the walls are as follows:

F_A =47.69 kN
F_B =70.77 kN
F_C =53.05 kN
F_D =53.05 kN.

Codal Provisions IS 1893 (Part 1) 2002

Dynamic Analysis

Dynamic analysis shall be performed to obtain the design seismic forces and its distribution to different levels along the height of building and to the various lateral load resisting elements in following cases:

(1) Regular Building – Greater than 40 m height in zone IV and V and those greater than 90 m in height in zone II and III.
(2) Irregular building – All framed buildings higher than 12 m in zone IV and V, and those greater than 40 m height in zone II and III.
(3) For irregular building lesser than 40 m in height in zone II and III, dynamic analysis even though not mandatory, is recommended.

Method of Dynamic Analysis:

Buildings with regular, or nominally irregular plan configuration may be modeled as a system of masses lumped at floor levels with each mass having one degree of freedom, that of lateral displacement in the direction under consideration.

Undamped free vibration analysis of entire building modeled as spring – mass model shall be performed using appropriate masses and elastic stiffness of the structural system to obtain natural periods (T) and mode shapes $\{\phi\}$ of those of its modes of vibration that needs to be considered. The number of modes to be used should be such that the sum of total of modal masses of all modes considered is at least 90% of total seismic mass.

In dynamic analysis following expressions shall be used for the computation of various quantities:

(a) Modal mass (M_k) – Modal mass of the structure subjected to horizontal or vertical as the case may be, ground motion is a part of the total seismic mass of the structure that is effective in mode k of vibration. The modal mass for a given mode has a unique value, irrespective of scaling of the mode shape.

$M_k = (\Sigma W_i\phi_{ik})^2/(g\,\Sigma W_i\phi_{ik}^2)$
Where

g = acceleration due to gravity,
ϕ_{ik} = mode shape coefficient at floor i in mode k
W_i = Seismic weight of floor i.

(b) Modal Participation factor (P_k) – Modal participation factor of mode k of vibration is the amount by which mode k contributes to the overall vibration of the structure under horizontal or vertical earthquake ground motions. Since the amplitudes of 95 percent mode shape can be scaled arbitrarily, the value of this factor depends on the scaling used for the mode shape.

$$P_k = (\Sigma W_i\phi_{ik}) / (\Sigma W_i\phi_{ik}^2)$$

(c) Design lateral force at each floor in each mode – The peak lateral force (Q_{ik}) at floor i in mode k is given by

$$Q_{ik} = A_k\phi_{ik} P_k W_i$$

Where

A_k = Design horizontal spectrum value using natural period of vibration (T_k) of mode k.
$= (Z I S_a) / (2 R g)$

(d) Storey shear forces in each mode – The peak shear force (V_{ik}) acting in storey i in mode k is given by

$$V_{ik} = \Sigma Q_{ik}$$

(e) Storey shear force due to all modes considered – The peak storey shear force (V_i) in storey i due to all modes considered is obtained by combining those due to each mode as per following rules:

(i) CQC method: The peak response quantities shall be combined as per Complete Quadratic Combination (CQC) method

$$\lambda = \sqrt{\sum_{1}^{r} \sum_{1}^{r} \lambda_i \rho_{ij} \lambda_j}$$

Where,

r = Number of modes being considered,
ρ_{ij} = Cross-modal coefficient
λ_i = Response quantity in mode i including sign
λ_j = Response quantity in mode j including sign

$$\rho_{ij} = \frac{8\,\zeta^2\,(1+\beta)\,\beta^{1.5}}{(1-\beta^2)^2 + 4\,\zeta^2\,\beta\,(1+\beta)^2}$$

ζ = Modal damping ratio (in fraction) 2% and 5% for steel and reinforced concrete building respectively
β = Frequency ratio = ω_i/ω_j
ω_i = Circular frequency in i^{th} mode and
ω_j = Circular frequency in j^{th} mode

(ii) SRSS method : If the bulding does not have closely spaced modes, than the peak response quantity (λ) due to all modes considered shall be obtained as per Square Root of Sum of Square method

$$\lambda = \sqrt{\sum_{1}^{r} (\lambda_k)^2}$$

Where
λ_k = Absolute value of quantity in mode k and
r = Number of modes being considered

Closely spaced modes of a structure are those of its natural modes of vibration whose natural frequencies differ from each other by 10 percent or less of the lower frequency.

(iii) SAV: If the building has a few closely spaced modes, then the peak response quantity (λ^*) due to these modes shall be obtained as

$$\lambda^* = \sum_{c}^{r}(\lambda_k)$$

Where the summation is for the closely spaced modes only. This peak response quantity due to the closely spaced modes (λ^*) is then combined with those of the remaining well separated modes by the method of SRSS.

The analytical model for dynamic analysis with unusual configuration should be such that it adequately models the types of irregularities present in the building configuration. Building with plan irregularities like torsion irregularities, re-entrant corners, diaphragm discontinuity, out-of plane offset, non parallel systems as defined in IS 1893 can not be modeled for dynamic analysis as discussed above.

The design base shear (V_B) shall be compared with base shear ($\overline{V}_B$) calculated using a fundamental period T_a, as given by empirical formula of clause 7.6 of IS 1893. Where V_B is less than $\overline{V}_B$, all the response quantities shall be multiplied by $\overline{V}_B/V_B$.

To illustrate above procedure an example is solved as follows.

Example of Dynamic analysis as per IS 1893 (Part I) -2002

Analyse a 15 storey RC building as shown in fig. The live load on all the floors is 200 kg/m^2 and soil below the building is hard. The site lies in zone V. All the beams are of size 40 × 50 cm and slabs are 15 cm thick. The sizes of columns are 60 × 60 cm in all the storeys and wall alround is 12 cm thick. (Data from SP 22 –1982)

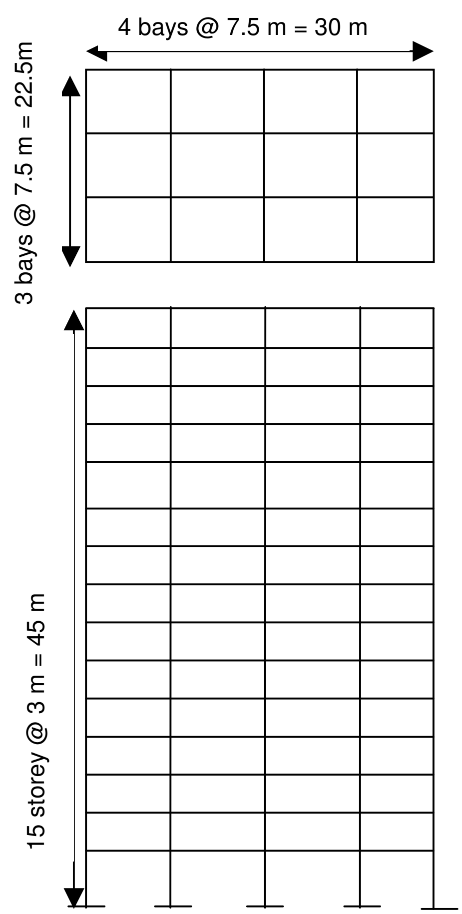
Analysis of the building

(a) Calculation of dead load, live load and storey stiffness: Dead loads and live loads at each floor are computed and lumped. Stiffness in a storey is lumped assuming all the columns to be acting in parallel with each column contributing stiffness corresponding to $K_c = 12EI/L^3$, where I is the moment of inertia about bending axis, L is the column height, and E the elastic modulus of the column material. The total stiffness of storey is thus ΣK_c. The lumped mass at all floor level is 52.43 (t-s^2/m) and at roof level is 40 (t-s^2/m). The values of I, K_c and ΣK_c for all the floors / storeys is 1.08×10^8 cm^4, 9024 t/m and 180480 t/m, respectively. The value of modulus of elasticity of column material considered is 1880000 t/m^2.

(b) The first three natural frequencies and the corresponding mode shape are determined using Stodola Vienello iteration procedure and are given in as below:

Periods and modes shape coefficients at various levels for first three modes

Mode No.	1	2	3
Period in seconds	1.042	0.348	0.210
Mode shape coefficient at various floor levels			
$\phi_1^{(r)}$	0.037	0.108	0.175
$\phi_2^{(r)}$	0.073	0.206	0.305
$\phi_3^{(r)}$	0.108	0.285	0.356
$\phi_4^{(r)}$	0.143	0.336	0.315
$\phi_5^{(r)}$	0.175	0.356	0.192
$\phi_6^{(r)}$	0.206	0.342	0.019
$\phi_7^{(r)}$	0.235	0.296	-0.158
$\phi_8^{(r)}$	0.261	0.222	-0.296
$\phi_9^{(r)}$	0.285	0.127	-0.355
$\phi_{10}^{(r)}$	0.305	0.019	-0.324
$\phi_{11}^{(r)}$	0.323	-0.089	-0.208
$\phi_{12}^{(r)}$	0.336	-0.190	-0.039
$\phi_{13}^{(r)}$	0.347	-0.273	0.140
$\phi_{14}^{(r)}$	0.353	-0.330	0.283
$\phi_{15}^{(r)}$	0.356	-0.355	0.353



(c) The next step is to obtain seismic forces at each floor level in each individual mode as per IS 1893

(i) Calculate mode participation factors $P_k = \Sigma\, W_i\phi_{ik} / \Sigma\, W_i\phi_{ik}^2$

(ii) Calculation of modal mass $M_k = (\Sigma\, W_i\phi_{ik})^2/(g\, \Sigma\, W_i\phi_{ik}^2)$

(iii) Calculate design lateral force at each floor in each mode

$Q_{ik} = A_k\phi_{ik}\, P_k W_i$

A_k is design horizontal spectrum value depending upon natural period of vibration (T_k) of mode k.

(iv) Calculate storey shear force in each mode $V_{ik} = \Sigma\, Q_{ik}$

(v) Calculate the storey shear forces due to all mode considered by combining those due to each mode as per clause 7.8.4.4 of IS 1893

As per CQC method $\lambda = (\Sigma\Sigma\lambda_i\rho_{ij}\lambda_j)^{1/2}$

As per SRSS method $\lambda = (\Sigma\lambda_k^2)^{1/2}$

As per SAV method $\lambda = (\Sigma\lambda_k)$

These calculations are tabulated as below:

Computation of mode participation factor P_1 Lateral force (Q_{i1}) and shear force (V_{i1}) for mode 1

Floor No.	Weight W_i	Mode coefficient ϕ_{i1}	$W_i\phi_{i1}$	$W_i\phi_{i1}^2$	$Q_{i1} = A_1\phi_{i1} P_1 W_i$	V_{i1}
1	514.34	0.037	19.030518	0.704129	2.348176	219.497345
2	514.34	0.073	37.546696	2.740909	4.632888	217.149170
3	514.34	0.108	55.548538	5.999242	6.854136	212.516281
4	514.34	0.143	73.550385	10.517706	9.075383	205.662140
5	514.34	0.175	90.009201	15.751610	11.106238	196.586761
6	514.34	0.206	105.953697	21.826462	13.073628	185.480530
7	514.34	0.235	120.869507	28.404333	14.914091	172.406906
8	514.34	0.261	134.242310	35.037243	16.564161	157.492813
9	514.34	0.285	146.586426	41.777130	18.087301	140.928650
10	514.34	0.305	156.873184	47.846321	19.356586	122.841354
11	514.34	0.323	166.131287	53.660408	20.498943	103.484772
12	514.34	0.336	172.817673	58.066738	21.323977	82.985825
13	514.34	0.347	178.475403	61.930965	22.022083	61.661846
14	514.34	0.353	181.561417	64.091179	22.402868	39.639763
15	392.40	0.356	139.694397	49.731205	17.236895	17.236895
Total			1778.890625	498.085571		

Mode participation factor $P_1 = \Sigma\, W_i\phi_{i1} / \Sigma\, W_i\phi_{i1}^2 = 1778.890625 / 498.085571 = 3.571456$

$A_1 = (Z\, I\, S_a) / (2\, R\, g)$

$Z = 0.36$ (zone V), $I = 1.0$, $R = 5.0$, $S_a/g = 1/1.042 = 0.9596$ (for hard soil)

$A_1 = (0.36 \times 1.0 \times 0.9596) / (2 \times 5.0) = 0.0345456$

Modal mass = $(\Sigma W_i\phi_{i1})^2 / \Sigma W_i\phi_{i1}^2 = (1778.890625)^2 / 498.085571 = 6353.23$

% of total mass = 6353.23/ 7593.16 = 83.67 %

Computation of mode participation factor P_2 Lateral force (Q_{i2}) and shear force (V_{i2}) for mode 2

Floor No.	Weight W_i	Mode coefficientϕ_{i2}	$W_i\phi_{i2}$	$W_i\phi_{i2}^2$	$Q_{i2}= A_2\phi_{i2} P_2W_i$	V_{i2}
1	514.34	0.108	55.548538	5.999242	5.903646	62.543804
2	514.34	0.206	105.953697	21.826462	11.260656	56.640160
3	514.34	0.285	146.586426	41.777130	15.579063	45.379501
4	514.34	0.336	172.817673	58.066738	18.366896	29.800436
5	514.34	0.356	183.104446	65.185181	19.460165	11.433540
6	514.34	0.342	175.903702	60.159069	18.694878	-8.026625
7	514.34	0.296	152.244141	45.064266	16.180361	-26.721502
8	514.34	0.222	114.183105	25.348650	12.135271	-42.901863
9	514.34	0.127	65.320969	8.295763	6.942250	-55.037132
10	514.34	0.019	9.772428	0.185676	1.038604	-61.979382
11	514.34	-0.089	-45.776112	4.074074	-4.865041	-63.017986
12	514.34	-0.190	-97.724281	18.567614	-10.386043	-58.152946
13	514.34	-0.273	-140.414368	38.333122	-14.923103	-47.766903
14	514.34	-0.330	-169.731659	56.011448	-18.038918	-32.843800
15	392.40	-0.355	-139.301987	49.452206	-14.804882	-14.804882
Total			588.486694	498.346680		

Mode participation factor $P_2 = \Sigma W_i\phi_{i2} / \Sigma W_i\phi_{i2}^2 = 588.486694 / 498.346680 = 1.180878$

$A_2 = (Z\, I\, S_a) / (2\, R\, g)$

$Z = 0.36$ (zone V), $I = 1.0$, $R = 5.0$, $S_a/g = 2.5$ (for hard soil)

$A_2 = (0.36 \times 1.0 \times 2.5) / (2 \times 5.0) = 0.09$

Modal mass = $(\Sigma W_i\phi_{i2})^2 / \Sigma W_i\phi_{i2}^2 = (588.486694)^2 / 498.346680 = 694.91$

% of total mass = 694.91/ 7593.16 = 9.15 %

Computation of mode participation factor P_3 Lateral force (Q_{i3}) and shear force (V_{i3}) for mode 3

Floor No.	Weight W_i	Mode coefficientϕ_{i3}	$W_i\phi_{i3}$	$W_i\phi_{i3}^2$	$Q_{i3} = A_3\phi_{i3} P_3W_i$	V_{i3}
1	514.34	0.175	90.009201	15.751610	5.631668	21.699989
2	514.34	0.305	156.873184	47.846321	9.815193	16.068321
3	514.34	0.356	183.104446	65.185181	11.456422	6.253129
4	514.34	0.315	162.016571	51.035221	10.137002	-5.203293
5	514.34	0.192	98.752960	18.960569	6.178744	-15.340295
6	514.34	0.019	9.772428	0.185676	0.611438	-21.519039
7	514.34	-0.158	-81.265457	12.839943	-5.084592	-22.130478
8	514.34	-0.296	-152.244141	45.064266	-9.525564	-17.045887
9	514.34	-0.355	-182.590103	64.819481	-11.424240	-7.520324
10	514.34	-0.324	-166.645615	53.993179	-10.426631	3.903916
11	514.34	-0.208	-106.982376	22.252335	-6.693640	14.330547
12	514.34	-0.039	-20.059195	0.782309	-1.255057	21.024187
13	514.34	0.140	72.007362	10.081031	4.505334	22.279245
14	514.34	0.283	145.557739	41.192841	9.107211	17.773911
15	392.40	0.353	138.517197	48.896568	8.666700	8.666700
Total			346.824188	498.886536		

Mode participation factor $P_2 = \Sigma\, W_i\phi_{i3} / \Sigma\, W_i\phi_{i3}^2$ =346.824188 / 498.886536= 0.695197

$A_k = (Z\ I\ S_a) / (2\ R\ g)$

Z = 0.36 (zone V), I = 1.0, R = 5.0, S_a/g = 2.5 (for hard soil)

$A_k = (0.36 \times 1.0 \times 2.5) / (2 \times 5.0) = 0.09$

Modal mass = $(\Sigma\, W_i\phi_{i3})^2 / \Sigma\, W_i\phi_{i3}^2 = (346.824188)^2 / 498.886536$= 241.37

% of total mass = 241.37/ 7593.16 = 3.18 %

Storey shear due to modes considered

Floor No.	V_{i1}	V_{i2}	V_{i3}	SAV	SRSS	CQC
1	219.497345	62.543804	21.699989	303.741119	229.263397	229.911057
2	217.149170	56.640160	16.068321	289.857666	224.989029	225.523697
3	212.516281	45.379501	6.253129	264.148926	217.397263	217.745331
4	205.662140	29.800436	-5.203293	240.665863	207.875092	208.027100
5	196.586761	11.433540	-15.340295	223.360596	197.515579	197.521500
6	185.480530	-8.026625	-21.519039	215.026199	186.897095	186.828293
7	172.406906	-26.721502	-22.130478	221.258881	175.863403	175.763092
8	157.492813	-42.901863	-17.045887	217.440567	164.119217	163.973633
9	140.928650	-55.037132	-7.520324	203.486115	151.481110	151.230743
10	122.841354	-61.979382	3.903916	188.724655	137.646942	137.233459
11	103.484772	-63.017986	14.330547	180.833313	122.007080	121.423195
12	82.985825	-58.152946	21.024187	162.162949	103.491203	102.803680
13	61.661846	-47.766903	22.279245	131.707993	81.118584	80.450851
14	39.639763	-32.843800	17.773911	90.257477	54.460423	53.949886
15	17.236895	-14.804882	8.666700	40.708481	24.318855	24.075233

As per clause 7.8.4.4 of IS 1893, if the building does not have closely spaced modes, the peak response quantity due to all modes considered shall be obtained as per SRSS method. Closely spaced modes of the structure are those of its natural modes of vibration whose natural frequencies differ from each other by 10 percent or less of the lower frequency. In this example as shown below, the frequencies in each mode differ by more than 10%, so building is not having closely spaced modes and so, SRSS method can be used.

Mode	Time period	Natural frequency 2π / T
1	1.042	6.03
2	0.348	18.06
3	0.210	29.92

The comparison of storey shear using SRSS method and CQC method is shown above.

As per clause 7.8.2 of IS 1893 the design base shear (V_B) shall be compared with base shear (V_B) calculated using a fundamental period T_a . When V_B is less than V_B, all the response quantities (e.g. member forces, displacements, storey forces, storey shear and base reactions) shall be multiplied by V_B/V_B.

For this example

$T_a = 0.075\ h^{0.75}$ for RC frame building

$T_a = 0.075\ (45)^{0.75} = 1.3031$ sec

For hard soil $S_a/g = 1.00/T_a = 1/1.3031 = 0.7674$

$\overline{V}_B = A_h\ W$

$W = 514.34 \times 14 + 392.4 = 7593.16$

$A_h = (Z\ I\ S_a) / (2\ R\ g)$

$Z = 0.36$ (for zone V)

$I = 1.0$

$R = 5.0$ (considering SMRF)

$A_h = (0.36 \times 1 \times 0.7674) / (2 \times 5.0) = 0.0276$

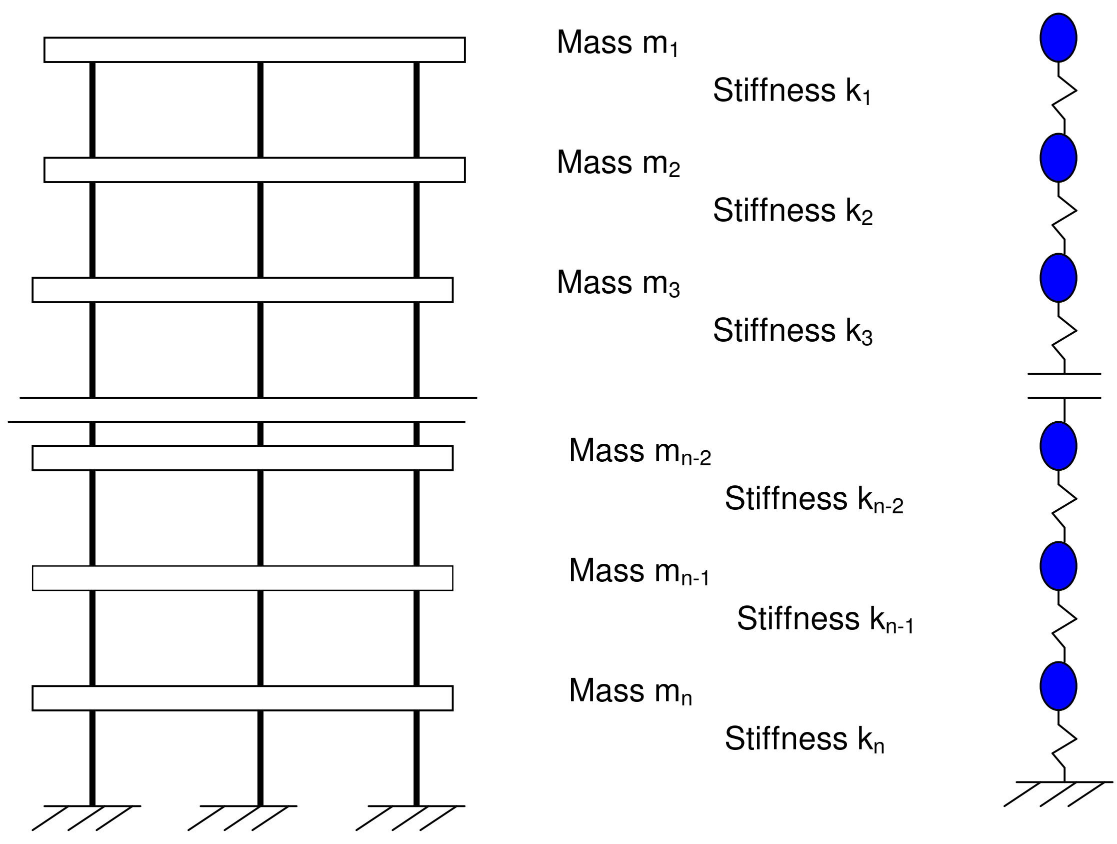
Base shear $\overline{V}_B = 0.0276 \times 7593.16 = 209.77$ t

Base shear from dynamic analysis $V_B = 229.9$ t

So, $V_B > \overline{V}_B$, response quantities need not required to be modified.

Appendix: Procedure to calculate time period and mode shape factors

In many structural system a number of independent displacement coordinates are required to describe the displacement of the mass of structure at any instant of the time. In order to simplify the solution, it is usually assumed that the mass of the structure is lumped at the centre of mass of the individual storey levels. This results in a diagonal matrix of mass properties in which translational mass is located on the main diagonal. It is also convenient to develop the structural stiffness matrix in terms of structural matrices of the individual storey level. This idealization is based on assumption: (i) Floor diaphragm is rigid in its own plane, (ii) the girders are rigid relative to column, (iii) the columns are flexible in horizontal direction but rigid in the vertical. Under these assumptions the building structure is idealized as having three dynamic degrees of freedom at each storey level: a translational degree of freedom in two orthogonal directions and a rotation about vertical axis through the centre of mass. For the simplest idealization, (building having symmetrical plan) each storey level has one translational degree of freedom. Then the stiffness matrix will have following form:



$$\text{Mass matrix } [M] = \begin{pmatrix} m_1 & 0 & 0 & \ldots\ldots. & 0 & 0 & 0 \\ 0 & m_2 & 0 & \ldots\ldots. & 0 & 0 & 0 \\ 0 & 0 & m_3 & \ldots\ldots. & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots\ldots & & m_i & \ldots. & & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots\ldots & m_{n-2} & 0 & 0 \\ 0 & 0 & 0 & \ldots\ldots & 0 & m_{n-1} & 0 \\ 0 & 0 & 0 & \ldots\ldots & 0 & 0 & m_n \end{pmatrix}$$

$$\text{Stiffness matrix } [K] = \begin{pmatrix} k_1 & -k_1 & 0 & 0 & 0 & 0 & \ldots\ldots & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & 0 & 0 & \ldots\ldots & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 & 0 & 0 & \ldots\ldots & 0 \\ \ldots\ldots & & & & & & & \\ 0 \;\ldots\ldots\ldots.. & 0 & -k_{i-1} & k_{i-1}+k_i & -ki & 0 & 0 & 0 \quad 0 \\ \ldots\ldots.. & & & & & & & \\ 0 \;\ldots\ldots\ldots & & & & 0 & -k_{n-2} & k_{n-2}+k_{n-1} & -k_{n-1} \\ 0 \;\ldots\ldots\ldots. & & & & 0 & 0 & -k_{n-1} & k_n \end{pmatrix}$$

Equation of motion for undamped free vibration of a multi degree of freedom system can be written in matrix form as

$$[M]\{y''\} + [k]\{y\} = \{0\}$$

Since motion of a system in free vibration are simple harmonic, the displacement vector can be represented as

$$\{y\} = \{A\} \sin\omega t$$

differentiating twice with respect to time results in

$$\{y''\} = -\omega^2 \{y\}$$

Substituting in equation of motion

$$([K] - \omega^2 [m])\{y\} = \{0\}$$

The classical solution to the above equation derives from the fact that in order for a set of homogeneous equation to have a non trivial solution, the determinant of the coefficient matrix must be zero:

$\text{Det}([K] - \omega^2 [m]) = \{0\}$

Expanding the determinant by minors results in a polynomial of degree N, which is called frequency equation. The N roots of the polynomial represents the frequencies of the N modes of the vibration. The mode having lowest frequency (largest time period) is called the first or fundamental mode. Once the frequencies are known they can be substituted one at a time into the equilibrium equation, which can be solved for relative amplitudes of motion for each of the displacements components in the particular mode of the vibration. To solve this eigen value problem for calculation of eigen values (natural frequencies) and eigen vector (mode shape) for large size problem various computer aided techniques are to be used.

Some of the methods for the solution of eigen value problem are as:

(1) Vector iteration method

Power method
Inverse iteration method
Forward iteration

(2) Transformation methods

Jacobi method
Givens method
Householder Q-R method

(3) Approximate solution techniques

Static condensation
Dynamic condensation
Rayleigh-Ritz method

(4) Sub-space iteration method