B.E Computer Science Engineering **2018-2019**

18BECS101 **Semester-I**

Mathematics-I **4H-4C**

(Calculus and Linear Algebra for Computer Science Engineers)

Instruction Hours/week: L:3 T:1 P:0 **Marks:** Internal:**40** External:**60** Total:**100**

End Semester Exam:3 Hours

Course Objectives

- The objective of this course is to familiarize the prospective engineers with techniques in basic calculus and linear algebra.
- It aims to equip the students with standard concepts and tools at an intermediate to advanced level that will serve them well towards tackling more advanced level of mathematics and applications that they would find useful in their disciplines.
- To develop the use of matrix algebra techniques that is needed by engineers for practical applications.
- To acquaint the student with mathematical tools needed in evaluating integrals and their usage.
- To make the student acquire sound knowledge of techniques in solving ordinary differential equations that model engineering problems.

Course Outcomes

The students will learn:

1. To apply differential and integral calculus to notions of curvature and to improper integrals. Apart from various applications, they will have a basic understanding of Beta and Gamma functions.
2. Fluency in integration using standard methods, including the ability to find an appropriate Method for a given integral.
3. The essential tools of matrices and linear algebra including linear transformations, Eigenvalues and diagonalization.
4. To apply differential and integral calculus to notions of curvature and to improper integral and proper integrals.
5. To solve the system of linear algebraic equations.
6. To analyze and evaluate the basic concepts of mathematics like matrix operation, vector spaces and calculus.

UNIT I - Matrices

Matrices, vectors: addition and scalar multiplication, matrix multiplication; Linear systems of equations, linear Independence, rank of a matrix, determinants, Cramer's Rule, inverse of a matrix, Gauss elimination and Gauss-Jordan elimination. Simple problems using Scilab.

UNIT II - Vector spaces

Vector Space, linear dependence of vectors, basis, dimension; Linear transformations (maps),range and kernel of a linear map, rank and nullity, Inverse of a linear transformation, rank nullity theorem, composition of linear maps, Matrix associated with a linear map.

UNIT III - Vector spaces

Eigen values, eigenvectors, symmetric, skew-symmetric, and orthogonal Matrices, Eigen bases. Diagonalization; Inner product spaces.

UNIT IV - Calculus

Evolutes and involutes; Evaluation of definite and improper integrals; Beta and Gamma functions and their properties; Applications of definite integrals to evaluate surface areas and volumes of revolutions.

UNIT V - Calculus

Taylor's and Maclaurin theorems with remainders; indeterminate forms and L'Hospital's rule; Maxima and minima.

SUGGESTED READINGS

1. G.B. Thomas and R.L. Finney, (2002), Calculus and Analytic geometry, 9th Edition,Pearson,.
2. Erwin kreyszig, (2006), Advanced Engineering Mathematics, 9th Edition,John Wiley & Sons.
3. Veerarajan T,(2008), Engineering Mathematics for first year,Tata McGraw-Hill, New Delhi.
4. Hemamalini. P.T.(2014), Engineering Mathematics, McGraw Hill Education (India) Private Limited, New Delhi.
5. Ramana B.V,(2010), Higher Engineering Mathematics, 11th Reprint, Tata McGraw Hill New Delhi.
6. D. Poole, (2005),Linear Algebra: A Modern Introduction, 2nd Edition, Brooks/Cole.
7. N.P. Bali and Manish Goyal, (2008), A text book of Engineering Mathematics, Laxmi Publications.
8. B.S. Grewal, (2000) Higher Engineering Mathematics, 35th Edition, Khanna Publishers,
9. V. Krishnamurthy, V.P. Mainra and J.L. Arora, An introduction to Linear Algebra, Affiliated East–West press.

Faculty In-charge **HoD**

KARPAGAM ACADEMY OF HIGHER EDUCATION
COIMBATORE-21.
FACULTY OF ENGINEERING
DEPARTMENT OF SCIENCE AND HUMANITIES

I B.E Computer Science Engineering

LECTURE PLAN

Subject : Mathematics – I (Calculus and Linear Algebra for Computer Science Engineers)
Code : 18BECS101

Unit No.	List of Topics	No. of Hours
	Matrices	
	Introduction of Matrix Algebra, Vector, Scalar and Applications.	1
	Problems based on addition and scalar multiplication.	1
	Matrix multiplication – Problems	1
	Problems based on determinants, linear Independence and rank of a matrix	1
	Problems based on determinants, linear Independence and rank of a matrix	1
UNIT – I	Problems based on inverse of a matrix	1
	Idea of solving the Linear systems of equations	1
	Linear systems of equations-Matrix inversion	1
	Linear systems of equations- Cramer's Rule	1
	Linear systems of equations- Cramer's Rule	1
	Tutorial 1: Problems based on inverse of a matrix and Cramer's Rule	1
	Concept of Gauss elimination and Gauss-Jordan Methods	1
	Problems based on Gauss elimination Methods and Gauss-Jordan Methods	1
	Problems based on Gauss elimination Methods and Gauss-Jordan Methods	1
	Tutorial 2: Problems based on Gauss elimination Methods and Gauss-Jordan Methods	1
	Simple problems using Scilab	1
	TOTAL	**16**
	Vector spaces	
	Introduction to Vector space and its applications	1
	Concepts of linear dependence of vectors- Basic problems	1
	Introduction of basic and dimension - Problems	1
	Idea of Linear transformations, range and kernal of a linear map-problems	1
	Problems based on Linear Transformations	1
	Concept of rank and Nullity-Problems	1
UNIT – II	Problems based on rank and Nullity theorem.	1
	Concept of rank and Nullity-Problems	1
	Tutorial 3: Problems based on Linear Transformations	1
	Introduction to Inverse of a linear transformation	1

	Problems based on Inverse of a linear transformation	1
	Problems based on Inverse of a linear transformation and rank Nullity theorem	1
	Concept of linear Map of composition and Matrix form	1
	Problems based on composition of linear maps	1
	Problems based on Matrix associated with a linear map	1
	Tutorial 4: Problems based on composition of linear maps and Matrix associated with a linear map	1
	TOTAL	**16**
UNIT – III	**Vector spaces**	
	Introduction of Eigen values and Eigenvectors	1
	Problems based on characteristic Equation	1
	Problems based on Eigen values and Eigenvectors of a real matrix	1
	Problems based on Eigen values and Eigenvectors of a real matrix	1
	Type of Matrix: symmetric, skew-symmetric and orthogonal Matrices	1
	Problems based on symmetric, skew-symmetric and orthogonal Matrices	1
	Problems based on Eigen bases	1
	Problems based on Eigen bases	1
	Tutorial 5: Problems based on types of Matrix and Eigen bases	1
	Concept of Diagonalization in Matrix	1
	Problems based on Diagonalization in Matrix	1
	Problems based on Diagonalization	1
	Concept of Inner product spaces	1
	Problems based on Inner product spaces	1
	Problems based on Inner product spaces	1
	Tutorial 6: Problems based on Inner product spaces	1
	TOTAL	**16**
UNIT – IV	**Calculus**	
	Introduction to Calculus, Differentiation and Integration	1
	Concepts of involutes and Evolutes.	1
	Problems based on Evolutes	1
	Problems based on Evolutes	1
	Basic problems in integration	1
	Evaluation of definite and improper integrals	1
	Evaluation of definite and improper integrals	1
	Concepts of Beta and Gamma functions and their properties	1
	Problems based on Beta and Gamma functions	1
	Problems based on Beta and Gamma functions	1
	Tutorial 7: Problems in improper integrals and Beta and Gamma functions.	1
	Applications of definite integrals to evaluate surface areas	1
	Applications of definite integrals to evaluate surface areas-Problems	1
	Applications of definite integrals to evaluate volume of revolution	1
	Applications of definite integrals to evaluate volume of revolution-Problems	1
	Tutorial 8: Applications of definite integrals to evaluate surface areas and volume of revolution -Problems	1
	TOTAL	**16**
	Calculus	
	Introduction of ordinary and partial differential equations	1

UNIT – V	Differentiation rule-Basic Problems	1
	Concepts of Taylor's theorem and problems	1
	Idea of Maclaurin theorem with remainders	1
	Problems based on Taylor's and Maclaurin theorems	1
	Problems based on Taylor's and Maclaurin theorems	1
	Concepts of indetermine forms in calculus	1
	Problems based on L'Hospital's rule	1
	Problems based on L'Hospital's rule	1
	Tutorial 9: Problems based on Taylor's, Maclaurin theorems and L'Hospital's rule	1
	Concepts of Maxima and minima for the functions of two variables.	1
	Problems based on Maxima and minima for the functions of two variables	1
	Problems based on Maxima and minima for the functions of two variables	1
	Problems based on Maxima and minima for the functions of two variables	1
	Tutorial 10: Problems based on Maxima and minima for the functions of two variables	1
	Discussion of previous years ESE Questions	1
	TOTAL	**16**
	TOTAL NO. OF HOURS	**80**

FACULTY IN-CHARGE **HOD**

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE - 641 021

DEPARTMENT OF SCIENCE AND HUMANITIES

I B.E COMPUTER SCIENCE AND ENGINEERING

MATHEMATICS - I (18BECS 101)

(Calculus and Linear Algebra)

QUESTION BANK

UNIT - I (MATRICES)

PART - C

1) i) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ shows that $A^2 - 4A - 5I = 0$

Soln

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$4A = 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$$

$5I \Rightarrow 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

Proof:

$\Rightarrow A^2 - 4A - 5I = 0$

$\Rightarrow \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$

Hence Proved.

ii) Verify $A(\text{adj} A) = (\text{Adj} A)A = |A| I_3$, where

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

Soln:

~~adj. A~~

$A_{ij} = \begin{bmatrix} +(6-3) & -(3+6) & +(-1-4) \\ -(3+1) & +(3-2) & -(-1-2) \\ +(-3-2) & -(-3-1) & +(2-1) \end{bmatrix}$

$= \begin{bmatrix} 3 & -9 & -5 \\ -4 & 1 & 3 \\ -5 & 4 & 1 \end{bmatrix}$

$\text{adj}.A = \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$

$|A| = 1(6-3) - 1(3+6) + 1(-1-4)$

$= 1(3) - 1(9) + 1(-5)$

$= -3-9-5 = -11$

$$|A|I_3 = -11\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}-11&0&0\\0&-11&0\\0&0&-11\end{bmatrix}$$

$$A(adj.A) = \begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}\begin{bmatrix}3&-4&-5\\-9&1&4\\-5&3&1\end{bmatrix}$$

$$= \begin{bmatrix}-11&0&0\\0&-11&0\\0&0&-11\end{bmatrix} \quad - (2)$$

$$(adj.A) = \begin{bmatrix}3&-4&-5\\-9&1&4\\-5&3&1\end{bmatrix}\begin{bmatrix}1&1&1\\1&2&-3\\2&-1&3\end{bmatrix}$$

$$= \begin{bmatrix}-11&0&0\\0&-11&0\\0&0&-11\end{bmatrix} \quad - (3)$$

From (1), (2), (3)

$$A(adj.A) = (adj.A)A = |A|I_3$$

Hence the Result.

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(2) Find inverse of matrix

i) $A = \begin{bmatrix}1&1&1\\2&2&3\\2&4&9\end{bmatrix}$

Soln.

$$|A| = \begin{vmatrix}1&1&1\\2&2&3\\2&4&9\end{vmatrix}$$

$= 1(18-12) - 1(18-6) + 1(8-4)$

$= 1(6) - 1(12) + 1(4)$

$= 6 - 12 + 4$

$= -2 \neq 0$

$\therefore A^{-1}$ exists

$$[A_{ij}] = \begin{bmatrix} +(18-12) & -(18-6) & +(8-4) \\ -(9-4) & +(9-2) & -(4-2) \\ +(3-2) & -(3-2) & +(2-2) \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -12 & 4 \\ -5 & 7 & -2 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\text{adj}\cdot A = [A_{ij}]^T$$

$$= \begin{bmatrix} 6 & -5 & 1 \\ -12 & 7 & -1 \\ 4 & -2 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\,\text{adj}\cdot A$$

$$= \frac{1}{-2}\begin{bmatrix} 6 & -5 & 1 \\ -12 & 7 & -1 \\ 4 & -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 5/2 & -1/2 \\ 6 & -7/2 & 1/2 \\ -2 & 1 & 0 \end{bmatrix}$$

Hence the result.

(ii) Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{vmatrix}$$

$$= 2(-1-2) - 4(0-2) + 3(0-2)$$

$$= 2(-3) - 4(-2) + 3(-2)$$

$$= -6+8-6$$

$$= -12+8$$

$$= -4 \neq 0$$

$\therefore A^{-1}$ exists.

$$[A_{ij}] = \begin{bmatrix} +(-1-2) & -(0-2) & +(0-2) \\ -(4-6) & +(-2-6) & -[4-8] \\ +(4-3) & -(2-0) & +(2-0) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 & -2 \\ 2 & -8 & 4 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\text{adj} \cdot A = [A_{ij}]^T$$

$$= \begin{bmatrix} -3 & 2 & 1 \\ 2 & -8 & -2 \\ -2 & 4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{adj} \cdot A$$

$$= \frac{1}{-4} \begin{bmatrix} -3 & 2 & 1 \\ 2 & -8 & -2 \\ -2 & 4 & 2 \end{bmatrix}$$

$$= 1 \begin{bmatrix} -3/-4 & 2/-4 & 1/-4 \\ 2/-4 & -8/-4 & -2/-4 \\ -2/-4 & 4/-4 & 2/-4 \end{bmatrix}$$

$$= \begin{bmatrix} 3/4 & 1/-2 & -1/4 \\ -1/2 & 2 & 1/2 \\ 1/2 & -1 & -2 \end{bmatrix}$$

Hence the result.

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3) Find rank of following matrices

i). $A = \begin{bmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{bmatrix}$

soln

$$A = \begin{bmatrix} 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \\ 1 & 5 & -7 & 2 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 0 & 7 & -8 & 14 \\ 0 & 7 & -8 & 7 \end{bmatrix} \quad \begin{array}{l} R_2 = R_2 - 3R_1 \\ R_3 = R_3 - R_1 \end{array}$$

$R_2 = 3 \quad 1 \quad -5 \quad -1$
$(-)\ 3R_1 = 3 \quad -6 \quad 3 \quad -15$
$\quad\quad - \quad + \quad - \quad +$
$0 \quad 7 \quad -8 \quad 14$

$R_3 = 1 \quad 5 \quad -7 \quad 2$
$(-)\ R_1 = 1 \quad -2 \quad 1 \quad -5$
$\quad\quad - \quad + \quad - \quad +$
$0 \quad 7 \quad -8 \quad 7$

$$\sim \begin{bmatrix} 1 & -2 & 1 & -5 \\ 0 & 7 & -8 & 16 \\ 0 & 0 & 0 & -9 \end{bmatrix} \quad R_3 = R_3 - R_2$$

$$\begin{array}{rrrrrr} & R_3 = & 0 & 7 & -8 & 7 \\ (-) & R_2 = & 0 & 7 & -8 & 16 \\ & & - & & + & - \\ \hline & & 0 & 0 & 0 & -9 \end{array}$$

$\boxed{\rho(A) = 3}$ = no of non zero rows

Hence the result

ii) $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 4 & 6 & -2 \\ 3 & 6 & 9 & -3 \end{bmatrix}$

Soln

$$A \sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 3R_1 \end{array}$$

$$\begin{array}{rrrrrr} & R_2 = & 2 & 4 & 6 & -2 \\ (-) & 2R_1 = & 2 & 4 & 6 & -2 \\ & & - & - & - & + \\ \hline & & 0 & 0 & 0 & 0 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{rrrrrr} & R_3 = & 3 & 6 & 9 & -3 \\ (-) & 3R_1 = & 3 & 6 & 9 & -3 \\ & & - & - & - & - \\ \hline & & 0 & 0 & 0 & 0 \end{array}$$

$\therefore \boxed{\rho(A) = 1}$ = no of non zero rows

Hence the result.

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4. Solve by Matrix inversion method,

i) $2x - y + 3z = 9, \quad x + y + z = 6, \quad x - y + z = 2.$

Soln.

The given system of equation can be written as $A \cdot X = B$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$A \cdot x = B$

$x = A^{-1}B$

$|A| = 2(1+1) + 1(1-1) + 3(-1-1)$

$= 2(2) + 1(0) + 3(-2)$

$= 4 + 0 - 6$

$= -2 \neq 0$

A^{-1} exists

$$[A_{ij}] = \begin{bmatrix} +(1+1) & -(1-1) & +(1-1) \\ -(-1+3) & +(2-3) & -(-2+1) \\ +(-1-3) & -(2-3) & +(2+1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -2 \\ -2 & -1 & 1 \\ -4 & 1 & 3 \end{bmatrix}$$

$$\text{adj}.A = [A_{ij}]^T = \begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{ adj}.A$$

$$= \frac{1}{-2} \begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 2 & -2 & -4 \\ 0 & -1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 18-12-8 \\ 0-6+2 \\ -18+6+6 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -2 \\ -4 \\ -6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1, 2, 3$$

$x = 1, \quad y = 2, \quad z = 3$

Hence the result.

Solve by Matrix inversion method

(ii) $2x + y + 3z = 3, \quad 2y + z = 2, \quad x + y + 2z = 1$

Soln:

The given system of equation can be written as matrix form.

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$A \cdot X = B$$

$$|A| = 2(4-1) - 1(0-1) + 3(0-2)$$

$$= 2(3) - 1(-1) + 3(-2)$$

$$= 6 + 1 - 6$$

$$= 1$$

$$[A_{ij}] = \begin{bmatrix} +(4-1) & -(0-1) & +(0-2) \\ -(2-3) & +(4-3) & -(2-1) \\ +(1-6) & -1(2-0) & +(4-0) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & -2 \\ 1 & 1 & -1 \\ -5 & -2 & 4 \end{bmatrix}$$

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$\text{adj } A = [A_{ij}]^T$

$$= \begin{bmatrix} 3 & 1 & -5 \\ 1 & 1 & -2 \\ -2 & -1 & 4 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 3 & 1 & -5 \\ 1 & 1 & -2 \\ -2 & -1 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2-5 \\ 3+2-2 \\ -6-2+4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ -4 \end{bmatrix}$$

$x = 6, \quad y = 3, \quad z = -4$

Hence the Result.

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5 Solve by

i) Matrix Inversion method

$x + z = 2, \quad -x + 2z = 1, \quad y + 3z = 3$

Soln, The given system of equation can be written as matrix form.

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$A \cdot X = B$

$X = A^{-1}B$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ -1 & 0 & 2 \\ 0 & 1 & 3 \end{vmatrix}$$

$= 1(0-2) - 0(\ ) + 1(-1-0)$

$= -3 \qquad |A| \neq 0$

A^{-1} exists

$$A_{ij} = \begin{bmatrix} +(0-2) & -(-3-0) & +(-1-0) \\ -(0-3) & +(3-0) & -(1-0) \\ +(0-0) & -(2+1) & +(0-0) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 3 & -1 \\ 3 & 3 & -1 \\ 0 & -3 & 0 \end{bmatrix}$$

$$\text{adj}.A = [A_{ij}]^T = \begin{bmatrix} -2 & 3 & 0 \\ 3 & 3 & -3 \\ -1 & -1 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}.A$$

$$= \frac{1}{-3} \begin{bmatrix} 2 & 3 & 0 \\ 3 & 3 & -3 \\ -1 & -1 & 0 \end{bmatrix}$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} -2 & 3 & 0 \\ 3 & 3 & -3 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$= -\frac{1}{3} \begin{bmatrix} -4+3+0 \\ 6+3-9 \\ -2-1+0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/-3 \\ 0 \\ -3/-3 \end{bmatrix}$$

$x = 1/3, \ y = 0, \ z = 1$

Hence the result.

ii) Solve by Gauss elimination method

$3x + y - z = 3$, $2x - 8y + z = -5$, $x - 2y + 9z = 8$.

The eqn is form of $AX = B$

$$\begin{bmatrix} 3 & 1 & -1 \\ 2 & -8 & 1 \\ 1 & -2 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$$

The arguement matrix is

$$[A|B] = \left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ -8 & 2 & 1 & -5 \\ -2 & 1 & 9 & 8 \end{array}\right] \quad C_1 \leftrightarrow C_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & 26 & -7 & 19 \\ 0 & 7 & 7 & 14 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + 8R_1 \\ R_3 \to R_3 + 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & 26 & -7 & 19 \\ 0 & 1 & 1 & 2 \end{array}\right] \quad R_3 \to R_3/7$$

$$\sim \left[\begin{array}{ccc|c} 1 & 3 & -1 & 3 \\ 0 & +26 & -7 & 19 \\ 0 & 0 & 33 & 33 \end{array}\right] \quad R_3 \to 26R_3 - R_2$$

We can write equ

$33z = 33$

$z = 33/33$

$\boxed{z = 1}$

$26y - 7z = 19$

$26y = 19 + 7$

$26y = 26$

$y = 26/26$

$\boxed{y = 1}$.

$x + 3y - z = 3$

$x + 3(1) - 1 = 3$

$x + 3 - 1 = 3$

$x + 2 = 3$

$x = 3 - 2$

$\boxed{x = 1}$

$\boxed{x = 1}$ $\boxed{y = 1}$ $\boxed{z = 1}$

6) Solve by cramer's rules method for the following system of equations.

if $x+y+z=4$, $x-y+z=2$, $2x+y-z=1$.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1(1-1) - 1(-1-2) + 1(1+2)$$
$$= 0+3+3 = 6.$$

$$\Delta x = \begin{vmatrix} 4 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 4(1-1) - 1(-2-1) + 1(2+1)$$
$$= 0+3+3 = 6.$$

$$\Delta y = \begin{vmatrix} 1 & 4 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 1(-2-1) - 4(-1-2) + 1(1-4)$$
$$= -3+12-3 = 6.$$

$$\Delta z = \begin{vmatrix} 1 & 1 & 4 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = 1(-1-2) - 1(1-4) + 4(1+2)$$
$$= -3+3+12$$

$$x = \frac{\Delta x}{\Delta}$$
$$= 6/6$$
$$\boxed{x=1}.$$

$$y = \frac{\Delta y}{\Delta}$$
$$= 6/6$$
$$\boxed{y=1}.$$

$$z = \frac{\Delta z}{\Delta}$$
$$= 12/6$$
$$\boxed{z=2}$$

ii] $2x+y+z=5$, $x+y+z=4$, $x-y+2z=1$.

$$A=\begin{bmatrix}2&1&1\\1&1&1\\1&-1&2\end{bmatrix} \qquad B=\begin{bmatrix}5\\4\\1\end{bmatrix}$$

$$\Delta=\begin{vmatrix}2&1&1\\1&1&1\\1&-1&2\end{vmatrix}=2(2+1)-1(2-1)+1(-1-1)$$
$$=6-1-2=3\neq 0.$$

$$\Delta x=\begin{vmatrix}5&1&1\\4&1&1\\1&-1&2\end{vmatrix}=5(2+1)-1(8-1)+1(-4-1)$$
$$=15-7-5=3$$

$$\Delta y=\begin{vmatrix}2&5&1\\1&4&1\\1&1&2\end{vmatrix}=2(8-1)-5(2-1)+1(1-4)$$
$$=14-5-3=6.$$

$$\Delta z=\begin{vmatrix}2&1&5\\1&1&4\\1&-1&1\end{vmatrix}=2(1+4)-1(1-4)+5(-1-1)$$
$$=10+3-10=3.$$

$$x=\Delta x/\Delta$$
$$=3/3$$
$$\boxed{x=1}$$

$$y=\Delta y/\Delta$$
$$=6/3$$
$$\boxed{y=2}$$

$$z=\Delta z/\Delta$$
$$z=3/3$$
$$\boxed{z=1}$$

Solve the Equation by using Gauss elimination and Gauss Jordon method

$x+y+z=1$, $4x+3y-z=6$, $3x+5y+3z=4$

Solution:-

Gauss Elimination method:-

The Equation of form is $AX=B$,

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

The arugment matrix

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & 2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_3 \to R_2 - 4R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & 0 & -10 & 5 \end{array}\right] \quad R_3 \to R_3 + 2R_2$$

$x+y+z=1$ — ①

$-y-5=2$ — ②

$-10z=5$ — ③

$-10z = 5$

$z = \cancel{5}/\cancel{10}2 = -1/2$ //

Sub $z = -1/2$ in ②

$-y + 5/2 = 2$

$-y = 2 - 5/2$

$-y = \frac{4-5}{2} = -1/2$ //

$y = 1/2$ //

Sub $y = \frac{1}{2}$ $z = -\frac{1}{2}$ in ①

$$x + \frac{1}{2} - \frac{1}{2} = 1$$

$$x + 0 = 1$$

$$x = 1$$

The Solution is $(x, y, z) = (1, \frac{1}{2}, -\frac{1}{2})$

<u>By Gauss Jordon method.</u>

$$\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 4 & 3 & -1 & 6 \\ 3 & 5 & 3 & 4 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & -5 & 2 \\ 0 & 2 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & -1 & -5 & 2 \\ 0 & 0 & -10 & 5 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + R_2 \\ R_3 \to R_3 + 2R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4 & 3 \\ 0 & 1 & +5 & -2 \\ 0 & 0 & 1 & -\frac{1}{2} \end{array}\right] \quad \begin{array}{l} R_2 \to -R_2 \\ R_3 \to \frac{-R_3}{10} \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & +\frac{1}{2} \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 5R_3 \\ R_1 \to R_1 + 4R_3 \end{array}$$

$$\sim [I | x]$$

The Solution (x, y, z) is $(1, +\frac{1}{2}, -\frac{1}{2})$

8) Solve the system of equations by using Gauss elimination and Gauss Jordan's method.

$x+2y-4z=-4$, $2x+5y-9z=-10$, $3x-2y+3z=11$.

The equation is form $AX=B$

$$\begin{bmatrix} 1 & 2 & -4 \\ 2 & 5 & -9 \\ 3 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -10 \\ 11 \end{bmatrix}$$

The argument method.

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 2 & 5 & -9 & -10 \\ 3 & -2 & 3 & 11 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & -8 & 15 & 23 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -1 & -1 \end{array}\right] R_3 \to R_3 + 8R_2$$

$-z = -1$

$\boxed{z=1}$.

$y-(1) = -2$

$y = -2+1$

$\boxed{y=-1}$

$x+2(-1)-4(1) = -4$

$x = -4+6$

$\boxed{x=2}$

Gauss Jordan's method

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -4 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] R_2 \to R_2 + R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -4 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] R_1 \to R_1 - 2R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] R_1 \to R_1 + 4R_3$$

$$\sim \left[I / X \right]$$

The soln is $\boxed{x = 2}$ $\boxed{y = -1}$ $\boxed{z = 1}$.

9] Solve the system of equation by using Gauss. elimination and Gauss Jordan's method

$x - y + z = 1$, $-3x + 2y - 3z = -6$, $2x - 5y + 4z = 5$

The equ is form of $AX = B$

$$\begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -3 \\ 2 & -5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \\ 5 \end{bmatrix}$$

The argument matrix is

$$\left[A \mid B\right] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ -3 & 2 & -3 & -6 \\ 2 & -5 & 4 & 5 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & -3 & 2 & 3 \end{array}\right] \begin{array}{l} R_2 \to R_2 + 3R_1 \\ R_3 \to R_3 - R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & -3 & 2 & 3 \end{array}\right]$$

$$\boxed{y = 3}$$

$$-3(3) + 2(z) = 3$$

$$2z = 3 + 9$$

$$\boxed{z = 6}$$

$$x - (3) + 6 = 1$$

$$x = 1 - 3$$

$$\boxed{x = -2}$$

Gauss Jordan's method

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 12 \end{array}\right] \begin{array}{l} R_1 \to R_1 + R_2 \\ R_3 \to R_3 + 3R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 6 \end{array}\right] R_3 \to R_3/2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 6 \end{array}\right] R_1 \to R_1 - R_3$$

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$\sim \left[I \mid X \right]$

The soln is $\boxed{x = -2}$ $\boxed{y = 3}$ $\boxed{z = 6}$

Questions	opt1	opt2
A square matrix A is said to be ----if the determinant value of A is zero.	singular	non singular
A square matrix A is said to be ----if the determinant value of A is not equal to zero.	singular	non singular
A square matrix A is said to be singular if the determinant value of A is ----.	1	2
A square matrix A is said to be non singular if the determinant value of A is ----.	1	2
A square matrix in which all the elements below the leading diagonal are zeros,it is called an -----matrix.	upper triangular	lower triangular
A square matrix in which all the elements above the leading diagonal are zeros,it is called an -----matrix.	upper triangular	lower triangular
A unit matrix is a ----matrix.	scalar	lower triangular
A system of equation is said to be consistent if they have	one solution	one or more solution
If rank of A is equal to the rank of [A,B] then the system of equations is ------	Consistent	inconsistent
If rank of A is not equal to the rank of [A,B] then the system of equations is ------	Consistent	inconsistent
A square matrix A which satisfies the relation A^2 = A is called	nilpotient	idempotient
A matrix is idempotient if ___	A^3 = A	A^2 = 0
If the rank of A is 2, then the rank of A^ (-1) is	3	2
If A is an mxn matrix, then A^T is an___ matrix.	mxn	nxm
Let A and B be two matrices, then (A+B)^T =	(A^T)+(B^T)	A^T
Let A be mxn matrix and B be nxp matrix. Then (AB)^T=	(A^T)+(B^T)	(AB)^T
Let A and B be two matrices with entries from C. Then A=conjugate of A iff all entries of A are _____	complex	real
A diagonal matrix in which all the entries of the principal diagonal are equal is called a ___ matrix.	scalar	lower triangular
A square matrix is a ___ matrix iff it is both lower triangular and upper triangular.	scalar	lower triangular
The product of any two non-singular matrices is __	scalar	lower triangular
If A and B are two nxn matrices then det(AB)=	det(A)*det (B)	det(A)+det (B)
The transpose of the co-factor matrix is called the ____ of the matrix A.	adjoint	inverse
If A is a square matrix of order n then adj A is a square matrix of order ___	0	n
If A is square matrix then adj(A)^T =	A^T	adj A
A square matrix A of order n is non-singular iff A is ___	adjoint	invertible
Let A be any square matrix of order n, then (adj A)A=A(adj A) = ___	adj A	A
If A is a non-singular matrix, then (A^T)^(-1)=	(A^-1)^(T)	(A^-1)
Let A and B be non-singular matries of order n, then (AB) is ___ matrix	adjoint	invertible
Let A and B be singular matries of order n, then (AB) is ___ matrix	adjoint	invertible

opt3	opt4	opt5	opt6	Answer	
symmetric	non symmetric			singular	
symmetric	non symmetric			non singular	
non zero	zero			zero	
non zero	zero			non zero	
symmetric	non symmetric			upper triangular	
symmetric	non symmetric			lower triangular	
symmetric	non symmetric			scalar	
no solution	infinite solution			one or more solution	
symmetric	non symmetric			Consistent	
symmetric	non symmetric			inconsistent	
Hermitian	Skew - Hermitian			idempotient	
A^1 =A	A^2 = A			A^2 = A	
4	1			2	
nxn	mxm			nxm	
(A^T)*(B^T	(A^T)-(B^T)			(A^T)+(B^T)	
(B^T)*(A^T	(A^T)-(B^T)			(B^T)*(A^T)	
rational	irrational			real	
symmetric	non symmetric			scalar	
symmetric	diagonal			diagonal	
singular	non-singular			non-singular	
det(A)/det (B)	det(A)-det (B)			det(A)*det (B)	
scalar	minor			adjoint	
1	n^2			n	
(adj A)^T	(- adj A)			(adj A)^T	
singular	non-singular			invertible	
(det A)*I	det A			(det A)*I	
(A^T)	(- A^T)			(A^-1)^(T)	
singular	non-singular			non-singular	
singular	non-singular			singular	

Let A be a singular matrix and B be a non-singular matrix of order n, then (AB) is ___ matrix	adjoint	invertible
A matrix obtained form the idenity matrix by applying a single elmentary row or column operation is called ___ matrix	scalar	an elementary
Any elementary matrix is ___ matrix.	scalar	invertible
Any non-singular square matrix A of order n is equivalent to the __ matrix of order n.	identity	scalar
The row rank and the column rank of any matrix are ____	different	equal
The ___ of a matrix A is the common value of its row and column rank.	adjoint	inverse
Any non singular square matrix of order n is equivalent to ___	the identity matrix of order n	a diagonal matrix of order n
If A is m x n matrix and B is n x k matrix, what is the order of AB?	mxn	nxk
___ method is a modified form of Gauss elimination method.	Cramer's	Matrix inversion
If A and B are of the same order matrices then tr (AB) =____.	tr A	tr B
The rank of a null matrix is defined to be_____.	1	(-1)
A determinant has ____ value.	numerical	zero
The determinant is possible only for a_____ matrix.	null	square
If each diagonal element of a scalar matrix is unity, the matrix is called a___ matrix.	scalar	unit
The determinant of every square sub matrix of a given matrix A is called a ___ of the matrix A.	minor	major
A system of linear equations in n unknowns with augmented matrix M, then the system has a solution iff rank (A)=__	rank (M)	n
A system of linear equations in n unknowns with augmented matrix M, then the solution is unique iff rank (A)=__	n	1
A system of linear equations in n unknowns with augmented matrix M, then the solution is _____ iff rank (A) = n.	consistent	inconsistent

singular	non-singular			singular	
singular	non-singular			an elementary	
singular	non-singular			non-singular	
singular	square			identity	
diagonal matrix	square matrix			equal	
rank	equal			rank	
scalar matrix of order n	the zero matrix of order n			the identity matrix of order n	
mxk	kxm			mxk	
Gauss Jordon	Echelon form			Gauss Jordon	
tr A+tr B	tr BA			tr BA	
0	2			0	
row	column			numerical	
row	column			square	
null	row			unit	
rank	inverse			minor	
0	n^2			rank (M)	
0	n^2			n	
different	unique			unique	

Questions	opt1	opt2	opt3
The sum of the main diagonal elements of a matrix is called----	trace of a matrix	quadratic form	eigen value
Every square matrix satisfies its own ----------	characteristic polynomial	characteristic equation	orthogonal transformation
The orthogonal transformation used to diagonalise the symmetric matrix A is----	NT AN	NT A	NAN-1
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then kλ1 , kλ2, kλ3,……… ,kλ n are the eigen values of --------------	kA	kA2	kA-1
Diagonalisation of a matrix by orthogonal reduction is true only for a ----- matrix.	diagonal	triangula	real symmetric
In a modal matrix, the columns are the -----------	eigen vectors of A	eigen vectors of adj A	eigen vectors of inverse ofA
If atleast one of the eigen values of A is zero, then det A = -----	0	1	10
If the canonical form of a quadratic form is 5y12 - 6 y22 , then the index is --------	4	0	2
det (A- λI) represents------	characteristic polynomial	characteristic equation	quadratic form
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then 1/λ1 , 1/λ2, 1/λ3,……… ,1/λ n are the eigen values of --------------	A^(-1)	A	A^n
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then λ1p , λ 2 p , ………. λ n p are the eigen values of	A^(-1)	A^2	A^(-p)
Cayley -Hamilton theorem is used to find ------------	inverse and higher powers of A	eigen va	eigen vectors
The eigen values of a --------------------- matrix are its diagonal	diagonal	symmet	skew-ma
In an orthogonal transformation NT AN = D , D refers to a ---------- matrix.	diagonal	orthogo	symmetr
In a modal matrix, the columns are the eigen vectors of----------	A-1	A2	A
If the eigen values of 8x12 + 7 x22 +3 x32 –12 x1 x2 – 8 x2 x3 +4 x3x1 are 0,3 & 15, then its nature is-----------	positive definite	positive semidefinite	indefinit

opt4	opt5	opt6	Answer
canonical form			trace of a matrix
canonical form			characteristic equation
NA			NT AN
A-1			kA
scalar			real symmetric
eigen values of A			eigen vectors of A
5			0
1			1
canonical form			characteristic polynomial
A^p			A^(-1)
A^p			A^p
quadratic form			inverse and higher powers of A
triangular			triangula
skew-symmetric			diagonal
adj A			A
negative definite			positive semidefinite

The elements of the matrix of the quadratic form x12 + 3 x22 + 4 x1 x2 are ------------	a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3	a11 = -1, a12 = -2 , a 21 = 2 , a 22 = 3	a11 = 1, a12 = 4 , a 21 = 4 , a 22 = 3
If the sum of two eigen values and trace of a 3x3 matrix A are equal, then det A = ---------	λ1 λ2 λ3	0	1
If 1,5 are the eigen values of a matrix A, then det A = -------	5	0	25
If the canonical form of a quadratic form is 5y12 + 6 y22 , then the rank is --------	4	0	2
The non –singular linear transformation used to transform the quadratic form to canonical form is ----------	X= NTY	X= NY	Y= NX
The eigen vector is also known as-------	latent value	latent vector	column value
If 1,3,7 are the eigen values of A, then the eigen values of 2A are ------------	1,3,7	1,9,21	2,6,14
If the eigen values of 2A are 2, 6, 8 then eigen values of A are _	1,3,4	2,6,8	1,9,16
The number of positive terms in the canonical form is called the	rank	index	Signatur
If all the eigenvalues of A are positive then it is called as______	Positive definite	Negative definite	Positive semidefinite
If all the eigenvalues of A are negative then it is called as_____	Positive definite	Negative definite	Positive semidefinite
A homogeneous polynomial of the second degree in any number of variables is called the _______	characteristic polynomial	characteristic equation	quadratic form
The Set of all eigen values of the matrix A is called the ____________ of A	rank	index	Signatur
A Square matrix A and its transpose have ________ eigen values	different	Same	Inverse
The sum of the ___________ of a matrix A is equal to the sum of the principal diagonal elements of A.	characteristic polynomial	characteristic equation	eigen values
The product of the eigenvalues of a matrix A is equal to______	Sum of main diagonal	Determinant of A	Sum of minors of Main diagonal
The eigenvectors of a real symmetric are ________	equal	unequal	real
When the quadratic form is reduced to the canonical form, it will contain only r terms, if the _____ of A is r.	rank	index	Signatu

a11 = 1, a12 = 4 , a 21 = 3 , a 22 = 1			a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3
2			0
6			5
1			2
NXA			X= NY
orthogonal value			latent vector
1,9,49			2,6,14
12,4,3			1,3,4
indefinite			index
Negative semidefinite			Positive definite
Negative semidefinite			Negative definite
canonical form			quadratic form
spectrum			spectrun
Transpose			Same
eigen vectors			eigen values
Sum of the cofactors of A			Determinant of A
symmetric			real
spectrum			rank

The excess of the number of positive terms over the number of negative terms in the canonical form is called the ____________ of the quadratic form.	rank	index	Signatu
If all the eigen values of A are less than zero and atleast one eigen value is zero then the quadratic form is said to be ____________	Positive definite	Negative	Positive
If all the eigen values of A are greater than zero and atleast one eigen value is zero then the quadratic form is said to be ____________	Positive definite	Negati ve definite	Positiv e semide finite
If the quadratic form has both positive and negative terms then it is said to be ____________	Positive definite	Negati ve definite	Positiv e semide finite

spectrum			Signatu
Negative semidefinite			Negative semidefinite
Negative semidefinite			Positive semidefinite
indefinite			indefini

KARPAGAM ACADEMY OF HIGHER EDUCATION

DEPARTMENT OF SCIENCE & HUMANITIES

MATHEMATICS - I [18 BECS 101]

UNIT-II [VECTOR SPACES]

PART-C [14 MARKS]

① Let $T: R^3 \to R^3$ be the linear transformation defined by $T(x,y,z) = (x+2y-z,\ y+z,\ x+y-2z)$ Find the basis and dimension of (a) Image of T (b) Kernal of T (c) Prove Rank Nullity Theorem.

Sol:- (i) The Image of T :-

Let $v_1 = x+2y-z$

$v_2 = 0x+y+z$

$v_3 = x+y-2z$

Let
$$M = \begin{array}{cc} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix} & \begin{array}{c} x \\ y \\ z \end{array} \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - R_2$$

Thus $(1,0,1)$ & $(0,1,-1)$ form a basis

$\therefore$ Rank$(T) = 2$

(ii) Kernal of T:-

Let $v_1 = x+2y-z$

$v_2 = y+z$

$v_3 = x+y-2z$

$$\text{Let } N = \begin{array}{c} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \\ \begin{matrix} x & \; y & \; z \end{matrix} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 + R_2 \end{matrix}$$

(ie) $x+2y-z = 0.$

$y+z = 0.$

Then we have to assume a variable free,

put $z=1$,

$y+1 = 0$

$$\boxed{y = -1}$$

$x + 2(-1) - 1 = 0$

$x - 3 = 0$

$\boxed{x = 3}$

$\therefore (3, -1, 1)$ form the basis.

Nullity $(T) = 1$;

$\dim(\text{Ker } T) = 1$;

(iii) <u>By Rank Nullity Theorem</u> :-

$\dim V = \dim(\text{Im } T) + \dim(\text{Ker } T)$

$= 2 + 1$

$= 3$

$\therefore$ The domain is R^3.

---

(2) Let $T: R^4 \rightarrow R^3$ be the linear mapping defined by $T(x, y, z, t) = (x + 2y + z + t),$
$(2x - 2y + 3z + 4t),$
$(3x - 3y + 4z + 5t)$

Find the basis and dimension of

(a) image of T.

(b) Kernal of T.

(c) Rank nullity Theorem.

Sol. (i) The image of T:

Let $v_1 = x - y + z + t.$

$v_2 = 2x - 2y + 3z + 4t.$

$v_3 = 3x - 3y + 4z + 5t.$

$$\text{Let } M = \begin{array}{c} x \\ y \\ z \\ t \end{array} \begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{array}{l} \\ R_2 \to R_2 + R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \\ \\ R_4 \to R_4 - 2R_3 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

Then $(1, 2, 3)$ & $(0, 1, 1)$ form a basis.

$\therefore$ rank $(T) = 2.$

$\therefore \dim(\operatorname{Im} T) = 2$

(ii) The Kernal of T :-

$$\text{Let } N = \begin{array}{c} \begin{matrix} x & y & z & t \end{matrix} \\ \begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & -2 & 3 & 4 \\ 3 & -3 & 4 & 5 \end{bmatrix} \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - R_2 . \end{matrix}$$

(ie) $x - y + z + t = 0$.

$z + 2t = 0$.

There are four variable and two free variable for that two equations (z, f)

(i) $z = 0, y = 1 \Rightarrow 2t = 0$

$\boxed{t = 0}$

$x - 1 + 0 + 0 = 0$

$\boxed{x = 1}$

$\therefore$ The set $(x, y, z, t) = (1, 1, 0, 0)$

(ii) $z=1,\ y=0 \Rightarrow 1+2t=0$

$2t=-1$

$\boxed{t=-1/2}$

$\Rightarrow x-0+1-\frac{1}{2}=0$

$x+\frac{1}{2}=0.$

$\boxed{x=-1/2}$

$\therefore$ The set $(x,y,z,t) = \left[-\frac{1}{2}, 0, 1, -\frac{1}{2}\right]$

Then $(1,1,0,0)$ & $\left(-\frac{1}{2}, 0, 1, -\frac{1}{2}\right)$ form a basis $\therefore$ Nullity $(T) = 2.$

$\therefore \dim(\operatorname{Ker} T) = 2.$

(iii) By RNT:-

$\dim V = \dim(\operatorname{Im} T) + \dim(\operatorname{Ker} T)$

$= 2+2$

$\dim V = 4.$

$\therefore R^4$ is the domain of T.

3\) Let $T: R^4 \to R^3$ be the linear transformation defined by $T(x, y, z, t) = (x - y + z + t,\ x + 2z - t,\ x + y + 3z - 3t)$

Find the basis and dimension of (a) Image of T (b) Kernal of T (c) Prove RNT :-

sol :-

$$M = \begin{array}{c} x \\ y \\ z \\ t \end{array} \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & -2 & -4 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_3 \to R_3 - R_2 \\ R_4 \to \frac{R_4}{-2} - R_2 \end{array}$$

$\therefore$ $(1, 1, 1)$ & $(0, 1, 2)$ form a basis

$\therefore$ $\dim(\operatorname{Im} T) = 2$.

(ii) Ker T:

$$N = \begin{array}{c} \begin{array}{cccc} x & y & z & t \end{array} \\ \begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 0 & 2 & -1 \\ 1 & 1 & 3 & -3 \end{bmatrix} \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 2 & 2 & -4 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & 1 & -2 \end{bmatrix} R_3 \to R_3/2$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore x - y + z + t = 0.$

$y + z - 2t = 0.$

put $y = 0, z = 1;$

$0 + 1 - 2t = 0$

$-2t = -1$

$\boxed{t = 1/2}$

$x - 0 + 1 + 1/2 = 0$

$x = -1 - 1/2$

$\boxed{x = -3/2}$

$\therefore (x, y, z, t) = (-3/2, 0, 1, 1/2)$

put $y=1, z=0$;

$\Rightarrow 1+0-2t=0$

$-2t=-1$

$\boxed{t=1/2}$

$\Rightarrow x-1+0+1/2=0$

$x-1=-1/2$

$x=-1/2+1$

$\boxed{x=+1/2}$ $\quad \therefore (x,y,z,t)=(1/2, 1, 0, 1/2)$

$\therefore (-3/2, 0, 1, 1/2)$ & $(1/2, 1, 0, 1/2)$ form a basis.

Nullity $(T) = 2$.

$\therefore \dim(\text{Ker } T) = 2$.

(iii) Prove RNT:-

$\dim V = \dim(\text{Im } T) + \dim(\text{Ker } T)$

$= 2+2$

$= 4$ $\quad [R^4$ is domain of $T]$

(4) Find the basis and dim of

(a) The image of T,

(b) Kernal of T,

(c) Prove RNT.

for the matrix Mapping $A: R^4 \to R^3$

where $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$

Sol:-

At first we have to write the values in a equation type and then find the above conditions

$V_1 = x + 2y + 3z + t.$

$V_2 = x + 3y + 5z - 2t.$

$V_3 = 3x + 8y + 13z - 3t.$

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(i) To find the image of T :-

Let $V_1 = x + 2y + 3z + t$

$V_2 = x + 3y + 5z - 2t$.

$V_3 = 3x + 8y - 13z + 3t$.

The matrix $M = \begin{array}{c} x \\ y \\ z \\ t \end{array} \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & 8 \\ 3 & 5 & -13 \\ 1 & -2 & 3 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & +4 \\ 0 & -3 & -6 \end{bmatrix} \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1 \end{array}$

$\sim \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \\ \\ R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 + 3R_2 \end{array}$

$\therefore$ $(1,1,3)$ & $(0,1,2)$ form the basis

$\therefore$ Rank $(T) = 2$.

$\therefore$ dim $(\text{Im } T) = 2$.

(ii) <u>To find the Kernal of T:-</u>

$V_1 = x + 2y + 3z + t.$

$V_2 = x + 3y + 5z - 2t.$

$V_3 = 3x + 8y + 13z - 3t.$

$$N = \begin{array}{c} \begin{matrix} x & y & z & t \end{matrix} \\ \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix} \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & -3 \\ 0 & 2 & 4 & -6 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 2R_2 \end{matrix}$$

(ie) $x + 2y + 3z + t = 0.$

$y + 2z - 3t = 0.$

There are 4 variables but 2 eqn. Therefore put two free variable.

put $y=0,\ z=1$;

$$0+2-3t=0.$$

$$-3t=-2$$

$$\boxed{t=2/3}$$

$$x+2(0)+3(1)+2/3=0.$$

$$x+3=-2/3$$

$$x=-2/3-3$$

$$x=\frac{-2-9}{3} \Rightarrow \boxed{x=-11/3}$$

$\therefore (x,y,z,t)=\left(-11/3,0,1,2/3\right)$

(ii) put $z=0,\ y=1$;

$$1+0-3t=0.$$

$$-3t=-1$$

$$\boxed{t=1/3}$$

$$x+2(1)+3(0)+1/3=0.$$

$$x+2=-1/3$$

$$x=-1/3-2$$

$$x=\cancel{-1-6/3}=-7/3 \quad \therefore (x,y,z,t)=\left(-7/3,1,0,\frac{1}{3}\right)$$

Thus $\left(-11/3,0,1,2/3\right)$ & $\left(-7/3,1,0,1/3\right)$ form a basis $\therefore$ Nullity $(T)=2$ $\therefore$ $\dim(\ker T)=2$

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(iii) RNT :-

$$\dim V = \dim(\operatorname{Im} T) + \dim(\operatorname{Ker} T)$$

$$= 2 + 2$$

$$= 4$$

$\therefore R^4$ is the domain of T.

---

⑤ Check wheather the following vectors are linearly independent (or) not.

[EACH QUESTION CARRIES 7 MARKS]

(i) $(1,1,0), (1,1,1), (0,+1,-1)$

sol:-

The L.C is $av_1 + bv_2 + cv_3 = 0$.

$$a\begin{bmatrix}1\\1\\0\end{bmatrix} + b\begin{bmatrix}1\\1\\1\end{bmatrix} + c\begin{bmatrix}0\\1\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right] \quad R_2 \to R_2 - R_1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_3 \leftrightarrow R_2 .$$

$$\boxed{c = 0}$$

$$b - c = 0$$

$$b - 0 = 0$$

$$\boxed{b = 0}$$

$$a + b = 0 .$$

$$\boxed{a = 0}$$

$\therefore$ V_1, V_2, V_3 are linearly Independent.

(ii) $(1, -2, 1)$ $(2, 1, -1)$ and $(7, -4, 1)$

Sol:-

The L.C is $av_1 + bv_2 + cv_3 = 0$.

$$a\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + b\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + c\begin{bmatrix} 7 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 7 & 0 \\ -2 & 1 & -4 & 0 \\ 1 & -1 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 7 & 0 \\ 0 & 5 & 10 & 0 \\ 0 & 3 & 6 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 7 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2/5 \\ R_3 \to R_3/3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 7 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0. \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$a + 2b + 7c = 0.$$
$$b + 2c = 0.$$

The Echelon form system has only 2 non zero equation in 3 unknown. It has no solution. So it is linearly dependent.

Check whether the foll. vectors are L.I or Not.

③ (1,1,2), (2,3,1) and (4,5,5)

Sol:- The L.C is $av_1 + bv_2 + cv_3 = 0$.

$$a\begin{bmatrix}1\\1\\2\end{bmatrix} + b\begin{bmatrix}2\\3\\1\end{bmatrix} + c\begin{bmatrix}4\\5\\5\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 1 & 3 & 5 & 0 \\ 2 & 1 & 5 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -3 & -3 & 0 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \to R_3 + 3R_2$$

$a + 2b + 4c = 0$.

$b + c = 0$.

∴ The Echlen form system has only 2 non zero equation in 3 unknown.

∴ It is linearly dependent.

(4) (1, 2, 1), (2, 1, 0) (1, -1, 2)

Sol:

The L.C is $av_1 + bv_2 + cv_3 = 0$.

$$\Rightarrow a\begin{bmatrix}1\\2\\1\end{bmatrix} + b\begin{bmatrix}2\\1\\0\end{bmatrix} + c\begin{bmatrix}1\\-1\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c}1 & 2 & 1 & 0\\2 & 1 & -1 & 0\\1 & 0 & 2 & 0\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c}1 & 2 & 1 & 0\\0 & -3 & -3 & 0\\0 & -2 & 1 & 0\end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1\\ R_3 \to R_3 - R_1\end{array}$$

$$\sim \left[\begin{array}{ccc|c}1 & 2 & 1 & 0\\0 & -3 & -3 & 0\\0 & 0 & 9 & 0\end{array}\right] R_3 \to 3R_3 - 2R_2$$

$9c = 0$

$\boxed{c = 0}$

$-3b - 3c = 0.$

$-3b - 3(0) = 0$

$-3b = 0$

$\boxed{b = 0}$

$a + 2b + c = 0$

$a + 2(0) + 0 = 0$

$\boxed{a = 0}$

$\therefore$ The given vectors v_1, v_2, v_3 are Linearly Independent.

6. (i) Express the vector (1, −2, 5) as the linear combination of (1, 1, 1), (1, 2, 3), (2, −1, 1) in R^3, where R is a field of real numbers.

Soln.

Let linear combination is $V = aV_1 + bV_2 + cV_3$

$$\Rightarrow \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = a\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$a + b + 2c = 1$

$a + 2b - c = -2$

$a + 3b + c = 5$

The Matrix $[A/B] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 5 & 10 \end{array}\right] \begin{array}{l} \\ \\ R_3 \to R_3 - 2R_2 \end{array}$$

$5c = 10$

$c = \frac{10}{5}$

$\boxed{c = 2}$

$b - 3c = -3$

$b - 3(2) = -3$

$b - 6 = -3$

$b = -3 + 6 = 3$ $\boxed{b = 3}$

$$a + b + 2c = 1$$
$$a + 3 + 2(2) = 1.$$
$$a + 7 = 1.$$
$$a = 1 - 7$$
$$\boxed{a = -6}$$

Hence $(1, -2, 5) = -6V_1 + 3V_2 + 2V_3$.

(ii) Express the vector $(3, 7, -4)$ in R^3 as a linear combination of vector $V_1 = (1, 2, 3)$, $V_2 = (2, 3, 7)$ & $V_3 = (3, 5, 6)$, where R is field of real numbers.

The linear combination is

$$V = aV_1 + bV_2 + cV_3.$$

$$\begin{bmatrix} 3 \\ 7 \\ -4 \end{bmatrix} = a\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + b\begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix} + c\begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$$

$$a + 2b + 3c = 3$$
$$2a + 3b + 5c = 7.$$
$$3a + 7b + 6c = -4.$$

The matrix $[A|B] = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 3 & 5 & 7 \\ 3 & 7 & 6 & -4 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & -3 & -13 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & -1 & +1 \\ 0 & 0 & -4 & -12 \end{array}\right] R_3 \to R_3 + R_2$$

$$-4c = -12$$
$$c = \frac{\cancel{-12}}{\cancel{-4}} \; 3 \qquad \boxed{c = 3}$$

$-b-c=1.$ $\qquad$ $a+2b+3c=3$

$-b-3=1.$ $\qquad$ $a+2(-4)+3(3)=3$

$-b=1+3$ $\qquad$ $a-8+9=3.$

$-b=4.$ $\qquad$ $a+1=3$

$\boxed{b=-4}$ $\qquad$ $\boxed{a=2}$

Hence $(3,7,-4)=2V_1-4V_2+3V_3.$

7) Find the inverse of a linear transformation for the mapping $T:R^2\to R^2$ defined by

(i) $T(x,y)=(2x+y,3x+2y).$

We set $T(x,y)=(s,t)$

$(x,y)=T^{-1}(s,t).$

We have

$T(x,y)=(2x+y,3x+2y).$

$(s,t)=(2x+y,3x+2y).$

(i,e) $s=2x+y\to ①$ $\qquad$ $t=3x+2y\to ②.$

Solve that

$① \times 2 \Rightarrow 4x+2y=2s$

$② \times 1 \Rightarrow \underset{(-)}{3x}\underset{(-)}{+2y}=\underset{(-)}{t}$

$x=2s-t$

Sub x in ①.

$2x+y=s.$

$2(2s-t)+y=s.$

$4s-2t+y=s$

$y=s-4s+2t.$

$y=-3s+2t.$

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$T^{-1}(s,t)=(x,y)$

$T^{-1}(s,t)=(2st, -3s+2t)$

$T^{-1}(x,y)=(2x-y, -3x+2y)$.

(ii) $T(x,y)=(x+2y, 2x+3y)$.

Sol.

We set $T(x,y)=(s,t)$

$(x,y)=T^{-1}(s,t) \longrightarrow (1)$.

we have.

$T(x,y)=(x+2y, 2x+3y)$.

$(s,t)=(x+2y, 2x+3y)$.

i.e., $s=x+2y, \quad 2x+3y$.

$$\begin{array}{r} 2x+4y=2s. \\ 2x+3y=t. \\ \hline y=2s-t \\ \hline \end{array}$$

$s=x+2s-t$.

$\boxed{x=t-s}$

$T^{-1}(s,t)=(x,y)$.

$T^{-1}(s,t)=(t-s, 2s-t)$.

$T^{-1}(x,y)=(y-x, 2x-y)$.

8) (i) Let $T: R^2 \to R^2$ and $S: R^2 \to R^2$ be the linear ~~tran~~transformation, defined by $T(x,y) = (2x, x+y)$, $S(x,y) = (2y, x)$ Find

(i) $T \circ S$ (ii) $S \circ T$

(i) $TOS = T(2(x,y))$

$= T(2y, x)$

$= (4y, 2y+x)$

$\because T(x,y) = (2x, x+y).$

$T(2y, x) = (2(2y), 2y+x)$

$= (4y, 2y+x)$

(ii) $SOT = S(T(x,y))$

$= 2x+2y, 2x$

$S(x,y) = (x+y, 2x)$

$S(2x, x+y) = (2(x+y), 2x)$

$= (2x+2y, 2x)$

(ii) Let $F: R^3 \to R^2$ and $G: R^2 \to R^2$ and $H: R^2 \to R^2$ be the linear transformation defined by $F(x,y,z) = (y, x+z)$, $G(x,y,z) = (2z, x-y)$ and $H(x,y) = (y, 2x)$ Find (a) HOF and HOG

(b) $HO(F+G)$ and $(HOF + HOG)$

Sol:

(i) $(HOF)(x,y,z) = H(F(x,y,z)).$

$= H(y, x+z).$

$= (x+z, 2y).$

$(HOG)(x,y,z) = H(G(x,y,z)).$

$= H(2z, x-y).$

$= (x-y, 2(2z)).$

$= (x-y, 4z).$

(ii). $H \circ (F+G) = H \circ F + H \circ G.$

$= H \circ F(x, y, z) + (H \circ G)(x, y, z).$

$= H(F(x, y, z)) + H(G(x, y, z)).$

$= (x+z, 2y) + (x-y, 4z).$

F, G & H are linear.

9) (i) Obtain the matrix represent the linear transformation, $T: R^3 \to R^3$ given by $T(x,y,z) = (3x+z, -2x+y, x+2y+4z)$. with respect of the basis $\{e_1, e_2, e_3\}$.

Soln: Let get the standard basis $\{e_1, e_2, e_3\}$.

$= \{(1,0,0), (0,1,0), (0,0,1)\}$

$T(e_1) = (1,0,0) = \{3(1)+0, -2(1)+0, 1+2(0)+4(0)\}$

$= (3+0, -2, 1)$

$= (3, -2, 1)$

$T(e_2) = (0,1,0) = \{3(0)+0, -2(0)+1, 0+2(1)+4(0)\}$

$= \{0, 1, 2\}$

$T(e_3) = (0,0,1) = \{0+1, 0, 4(1)\}$

$= (1, 0, 4)$.

The matrix form is $\begin{bmatrix} 3 & -2 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 4 \end{bmatrix}$

(ii) Find the linear transformation $T: R^3 \to R^3$ determined by the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ with respect of the matrix standard basis (e_1, e_2, e_3).

Sol: $T(e_1) = (1,2,1) = e_1 + 2e_2 + e_3$

$T(e_2) = (0,1,1) = e_2 + e_3$

$T(e_3) = (-1,3,4) = -e_1 + 3e_2 + 4e_3$.

$T(x,y,z) = x(T(e_1)) + y(T(e_2)) + z(T(e_3))$

$= x(1,2,1) + y(0,1,1), + z(-1,3,4)$

$= (x, 2x, x) + (0, y, y) + (-z, 3z, 4z)$

$T(x,y,z) = (x+0-z), (2x+y+3z), (x+y+4z)$

10) (i) Show that mapping $T: R^2 \to R^2$ is defined by $T(x,y) = (x+y, x)$ is a linear transformation.

Sol: Let $u = (a,b)$, $V = (c,d)$ $\alpha \in F$.

(i) T.P $T(u+V) = T(u) + T(V)$.

$T(u+V) = T((a,b) + (c,d))$.

$= T(a+c, b+d)$

$= ((a+c)+(b+d), (a+c))$ $\left[\begin{array}{l} \text{Put } x = a+c \\ y = b+d \end{array}\right]$

$= (a+b+c+d, a+c)$

$= (a+b, a) + (c+d, c)$

$= T(a.b) + T(c.d)$

$= T(u) + T(V)$

$\Rightarrow T(u+V) = T(u) + T(V)$.

(ii) T.P $T(\alpha u) = \alpha T(u)$

$T(\alpha u) = T(\alpha(a,b))$.

$= T(\alpha a, \alpha b)$.

$= (a\alpha + \alpha b, \alpha a)$

$= \alpha(a+b, a)$.

$= \alpha T(a,b)$.

$\Rightarrow T(\alpha, u) = \alpha T(u)$.

$\therefore$ T is a linear transformation

(ii) Show that mapping $T: R^2 \to R^2$ is defined by $T(x, y) = (x+y, x-y)$ is a linear transformation.

(i) $T(u+v) = T(u) + T(v)$.

$$T(u+v) = T(a, b) + (c, d)$$

$$= (a+c, b+d)$$

$$\left[\text{Put } x = a+c \quad y = b+d\right]$$

$$= (a+c+b+d, a+c-b+d).$$

$$= ((a+b)+(a-b)+((c+d)+(c-d)).$$

$$= T((a, b)(c, d)).$$

$$T(u+v) = T(u) + T(v)$$

(ii) $T(\alpha u) = \alpha T(u)$

$$= T(\alpha (a, b)).$$

$$= T(\alpha a, \alpha b).$$

$$= T(\alpha a, +\alpha b, \alpha a - \alpha b)$$

$$= \alpha (a+b; a-b)$$

$$= \alpha (T(a, b))$$

$$T(\alpha u) = \alpha T(u).$$

$\therefore$ T is linear transformation.

Questions	opt1	opt2
The set of all linear combinations of finite sets of elements of S is called	linear depende	spanning set
The vector space {0} then the dimension is ___.	0	1
The rank nullity theorem is dim V = ___.	rank(T)+nullit	rank(T)-nullit
The kernel of T is named as_____.	dim (Im T)	dim (ker T)
___ denotes the null space of A	Ker A	Rank A
The ____ of two subspaces of a vector space is a subspace.	union	intersection
The intersection of any number of subspaces of a vectors space V is a _	subspace	basis
Row equivalence matrices have the same ___ space.	column	null
The nonzero rows of a matrix in echelon form are ____.	linearly depen	linearly indep
Any subset of a linearly independent set is ___.	linearly depen	linearly indep
A set S of vectors is a ___ of V if it satisfies span and linearly independe	subspace	basis
___ denotes the column space of A	Ker A	Im A
Let V be a vector space then any n+1 or more vectors in V are___.	linearly depen	linearly indep
The ___ of T is defined to be the dimension of images.	rank	kernel
Let V be a vector space of finite dimenstion n. Then any n+1 or more ve	linearly depen	linearly indep
Let V be a vector space of finite dimenstion n. Then any ___ or more ve	n+1	n
Let V be a vector space of finite dimenstion n. Then any ___ set S with	linearly depen	linearly indep
Let V be a vector space of finite dimenstion n. Then any linearly indepe	linearly depen	basis
Let V be a vector space of finite dimenstion n. Then any spanning set T	linearly depen	basis
Let V be a vector space of finite dimenstion n. Then any ____T of V wi	linearly depen	spanning set
A vector space with an inner product defined on V is called ___	column space	elementary sp
An inner product space is called ____ space	column	an elementary
An inner product of <u, v+w> =	<u,v>+<u,w>	<u,v>-<u,w>
An inner product of <u,0> is ___	1	2
Let V be an inner product space and let x in V. The norm of x is defined	<x,x>	<x,0>

opt3	opt4	opt5	opt6	Answer	
linear span	linear combination			linear span	
2	3			0	
rank(T).nulli	basis			rank(T)+nullity(T)	
dim V	linear transformation			dim (ker T)	
Im A	dim A			Ker A	
complement	rank			intersection	
dimension	rank			subspace	
row	kernel			row	
linearly span	linearly combination			linearly independent	
linearly span	linearly combination			linearly independent	
dimension	rank			basis	
dim A	Rank A			Im A	
linearly span	linearly combination			linearly dependent	
basis	linear map			rank	
linearly span	linearly combination			linearly dependent	
n-1	n+2			n+1	
linearly span	linearly combination			linearly independent	
linearly span	linearly combination			basis	
linearly span	linearly combination			basis	
linearly span	linearly combination			spanning set	
an inner proc	row space			an inner product space	
an unitary	row			an unitary	
<u,v>*<u,w	<u,v>/<u,w>			<u,v>+<u,w>	
(-1)	0			0	
sqrt<x,x>	0			sqrt<x,x>	

Let V be an inner product space and let x in V. The x is called a unit vec	0	1
Let V be an inner product space and let x in V. The x is called a ___vect	row	column
The sum of two vectors is a ___	scalar	vector
The product of a scalar and a vector is a ___	scalar	vector
{0} and V are subspaces of any vector space V. They are called the ___	scalar	vector
Let V and W be vector space over a field F, then T from V to W defined	scalar	vector
Let V and W be vector space over a field F, then T from V to W defined	scalar	vector
Let V be a vector space and A and B are subspaces of V then __ is a sub	A+B	A-B
Let V be a vector space and A and B are subspaces of V then A is a subs	A+B	A-B
Let V be a vector space and A and B are subspaces of V then B is a subs	A+B	A-B
Let S be a non-empty subset of a vector space V. Then the set of all ___	linearly depen	linearly indep
The Linear span is denoted by___	dim V	dim S
Let V be a vector space over a field F and S be a non-empty subset of V	linear span	linear indeper
L[L(S)] = __	dim V	dim S
Any vector space is an abelian group with respect to vector___	addition	subtraction
Any finite dimensional vector spce over R can be provided with ___	scalar	vector
In an inner product space, every vector has a ___	scalar	vector
The norm of the vector (1,2,3) in V with standard inner product is __	6	14
In R, let S = {1}. Then L(S) =	S	C
In C, let S = {1, i}. Then L(S) =	S	C

2	3			1	
scalar	unit			unit	
unit	inner product			vector	
unit	inner product			vector	
unit	trivial			trivial	
identity	trivial			trivial	
identity	trivial			identity	
A*B	A/B			A+B	
A*B	A/B			A+B	
A*B	A/B			A+B	
linear span	linear combinations			linear combinations	
L(S)	S			L(S)	
linear depen	subspace			subspace	
L(S)	S			L(S)	
multiplicatio	division			addition	
unit	an inner product			an inner product	
unit	norm			norm	
sqrt(14)	1			sqrt(14)	
R	Q			R	
R	{a+bi}			C	

Questions	opt1	opt2	opt3
The sum of the main diagonal elements of a matrix is called----	trace of a matrix	quadratic form	eigen value
Every square matrix satisfies its own ----------	characteristic polynomial	characteristic equation	orthogonal transformation
The orthogonal transformation used to diagonalise the symmetric matrix A is----	NT AN	NT A	NAN-1
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then kλ1 , kλ2, kλ3,……… ,kλ n are the eigen values of --------------	kA	kA2	kA-1
Diagonalisation of a matrix by orthogonal reduction is true only for a ----- matrix.	diagonal	triangula	real symmetric
In a modal matrix, the columns are the -----------	eigen vectors of A	eigen vectors of adj A	eigen vectors of inverse ofA
If atleast one of the eigen values of A is zero, then det A = -----	0	1	10
If the canonical form of a quadratic form is 5y12 - 6 y22 , then the index is --------	4	0	2
det (A- λI) represents------	characteristic polynomial	characteristic equation	quadratic form
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then 1/λ1 , 1/λ2, 1/λ3,……… ,1/λ n are the eigen values of --------------	A^(-1)	A	A^n
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then λ1p , λ 2 p , ………. λ n p are the eigen values of	A^(-1)	A^2	A^(-p)
Cayley -Hamilton theorem is used to find ------------	inverse and higher powers of A	eigen va	eigen vectors
The eigen values of a --------------------- matrix are its diagonal	diagonal	symmet	skew-ma
In an orthogonal transformation NT AN = D , D refers to a ---------- matrix.	diagonal	orthogo	symmetr
In a modal matrix, the columns are the eigen vectors of----------	A-1	A2	A
If the eigen values of 8x12 + 7 x22 +3 x32 –12 x1 x2 – 8 x2 x3 +4 x3x1 are 0,3 & 15, then its nature is-----------	positive definite	positive semidefinite	indefinit

opt4	opt5	opt6	Answer
canonical form			trace of a matrix
canonical form			characteristic equation
NA			NT AN
A-1			kA
scalar			real symmetric
eigen values of A			eigen vectors of A
5			0
1			1
canonical form			characteristic polynomial
A^p			A^(-1)
A^p			A^p
quadratic form			inverse and higher powers of A
triangular			triangula
skew-symmetric			diagonal
adj A			A
negative definite			positive semidefinite

The elements of the matrix of the quadratic form x12 + 3 x22 + 4 x1 x2 are ------------	a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3	a11 = -1, a12 = -2 , a 21 = 2 , a 22 = 3	a11 = 1, a12 = 4 , a 21 = 4 , a 22 = 3
If the sum of two eigen values and trace of a 3x3 matrix A are equal, then det A = ---------	λ1 λ2 λ3	0	1
If 1,5 are the eigen values of a matrix A, then det A = -------	5	0	25
If the canonical form of a quadratic form is 5y12 + 6 y22 , then the rank is --------	4	0	2
The non –singular linear transformation used to transform the quadratic form to canonical form is ----------	X= NTY	X= NY	Y= NX
The eigen vector is also known as-------	latent value	latent vector	column value
If 1,3,7 are the eigen values of A, then the eigen values of 2A are ------------	1,3,7	1,9,21	2,6,14
If the eigen values of 2A are 2, 6, 8 then eigen values of A are _	1,3,4	2,6,8	1,9,16
The number of positive terms in the canonical form is called the	rank	index	Signatur
If all the eigenvalues of A are positive then it is called as______	Positive definite	Negative definite	Positive semidefinite
If all the eigenvalues of A are negative then it is called as_____	Positive definite	Negative definite	Positive semidefinite
A homogeneous polynomial of the second degree in any number of variables is called the _______	characteristic polynomial	characteristic equation	quadratic form
The Set of all eigen values of the matrix A is called the ____________ of A	rank	index	Signatur
A Square matrix A and its transpose have ________ eigen values	different	Same	Inverse
The sum of the ___________ of a matrix A is equal to the sum of the principal diagonal elements of A.	characteristic polynomial	characteristic equation	eigen values
The product of the eigenvalues of a matrix A is equal to______	Sum of main diagonal	Determinant of A	Sum of minors of Main diagonal
The eigenvectors of a real symmetric are ________	equal	unequal	real
When the quadratic form is reduced to the canonical form, it will contain only r terms, if the _____ of A is r.	rank	index	Signatu

a11 = 1, a12 = 4 , a 21 = 3 , a 22 = 1			a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3
2			0
6			5
1			2
NXA			X= NY
orthogonal value			latent vector
1,9,49			2,6,14
12,4,3			1,3,4
indefinite			index
Negative semidefinite			Positive definite
Negative semidefinite			Negative definite
canonical form			quadratic form
spectrum			spectrun
Transpose			Same
eigen vectors			eigen values
Sum of the cofactors of A			Determinant of A
symmetric			real
spectrum			rank

The excess of the number of positive terms over the number of negative terms in the canonical form is called the ___________ of the quadratic form.	rank	index	Signatu
If all the eigen values of A are less than zero and atleast one eigen value is zero then the quadratic form is said to be ___________	Positive definite	Negative	Positive
If all the eigen values of A are greater than zero and atleast one eigen value is zero then the quadratic form is said to be ___________	Positive definite	Negative definite	Positive semidefinite
If the quadratic form has both positive and negative terms then it is said to be ___________	Positive definite	Negative definite	Positive semidefinite

spectrum			Signatu
Negative semidefinite			Negative semidefinite
Negative semidefinite			Positive semidefinite
indefinite			indefini

KARPAGAM ACADEMY OF HIGHER EDUCATION

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COIMBATORE - 641021

DEPARTMENT OF SCIENCE AND HUMANITIES

I B.E COMPUTER SCIENCE AND ENGINEERING

MATHEMATICS - I (18BECS101)

Calculus and Linear Algebra

QUESTION BANK

UNIT - III

(VECTOR SPACES)

1. Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

Solution:

Step 1:

To find the characteristic Equation

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

S_1 = sum of the main diagonal element

$= 1+2+3$

$= 6$

$$S_2 = \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$$

$= (2-0) + (6-2) + (3+2)$

$= 2+4+5$

$= 11$

$S_3 = |A|$

$$= \begin{vmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{vmatrix}$$

$= 1(6-2) + 0(3-2) - 1(2-4)$

$= 1(4) + 0 - 1(-2)$

$= 4 + 2$

$= 6$

The characteristic Equation is,

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

Step : 2

To find the Eigenvalues.

$$\begin{array}{c|cccc}
1 & 1 & -6 & 11 & -6 \\
 & \downarrow & 1 & -5 & 6 \\
\hline
2 & 1 & -5 & 6 & \boxed{0} \\
 & \downarrow & 2 & -6 & \\
\hline
3 & 1 & -3 & \boxed{0} & \\
 & \downarrow & 3 & & \\
\hline
 & 1 & \boxed{0} & &
\end{array}$$

The Eigen Value is $(1, 2, 3)$

$$\lambda_1 = 1, \quad \lambda_2 = 2, \quad \lambda_3 = 3$$

Step: 3 → To find the Eigen Vectors $(A - \lambda I) X = 0$

case: (i) $\lambda = 1$

$$\begin{bmatrix} 1-1 & 0 & -1 \\ 1 & 2-1 & 1 \\ 2 & 2 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_3 = 0 \quad \text{——— (1)}$$

$$x_1 + x_2 + x_3 = 0 \quad \text{——— (2)}$$

$$2x_1 + 2x_2 + 2x_3 = 0 \quad \text{——— (3)}$$

(2) & (3) are same

So therefore

$$x_3 = 0$$

$$x_1 + x_2 + x_3 = 0 \Rightarrow x_1 + x_2 = 0$$

$$x_1 = -x_2$$

$$\frac{x_1}{-1} = \frac{x_2}{1}$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

Case (ii) $\lambda = 2$

$$\begin{bmatrix} 1-2 & 0 & -1 \\ 1 & 2-2 & 1 \\ 2 & 2 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$= \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 - x_3 = 0$ ———— (4)

$x_1 + x_3 = 0$ ———— (5)

$2x_1 + 2x_2 + x_3 = 0$ ———— (6)

(4) & (5) are same

		x_1	x_2	x_3
1	0	1	1	0
2	2	1	2	2

$$\frac{x_1}{(0-2)} = \frac{x_2}{(2-1)} = \frac{x_3}{(2-0)}$$

$$\frac{x_1}{-2} = \frac{x_2}{1} = \frac{x_3}{2}$$

$$X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Case (iii) $\lambda = 3$

$$\begin{bmatrix} 1-3 & 0 & -1 \\ 1 & 2-3 & 1 \\ 2 & 2 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & -1 \\ 1 & -1 & 1 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 - x_3 = 0 \quad \text{---} \quad (7)$$

$$x_1 - x_2 + x_3 = 0 \quad \text{---} \quad (8)$$

$$2x_1 + 2x_2 = 0 \quad \text{---} \quad (9)$$

$$\begin{array}{ccccc} & & x_1 & x_2 & x_3 \\ \hline -2 & 0 & -1 & -2 & 0 \\ 1 & -1 & 1 & 1 & -1 \end{array}$$

$$\cancel{\frac{x_1}{(0-1)} = \frac{x_2}{(+0-2)}} =$$

$$\frac{x_1}{0-1} = \frac{x_2}{-1+2} = \frac{x_3}{2-0}$$

$$\frac{x_1}{-1} = \frac{x_2}{1} = \frac{x_3}{2}$$

$$X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ The Eigen Value is 1, 2, 3 and the Eigen Vectors are

$$X_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \quad X_3 = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

2. Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Sollution:

Step: 1

To find the characteristic Equation.

$$\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$$

$S_1 = 1+5+1 = 7$

$S_2 = (5-1) + (5-1) + (1-9)$

$= 4 + 4 - 8$

$= 0$

$S_3 = |A| = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{vmatrix}$

$= -36$

The characteristic equation is

$$\lambda^3 - 7\lambda^2 + 36 = 0$$

Step: 2

To find the Eigen Values.

$$\begin{array}{r|rrrr}
-2 & 1 & -7 & 0 & 36 \\
 & & -2 & 18 & -36 \\
\hline
3 & 1 & -9 & 18 & \boxed{0} \\
 & & 3 & -18 & \\
\hline
6 & 1 & -6 & \boxed{0} & \\
 & & 6 & & \\
\hline
 & 1 & \boxed{0} & &
\end{array}$$

The Eigen values are $-2, 3, 6$

$\lambda_1 = -2, \quad \lambda_2 = 3, \quad \lambda_3 = 6$

Step : 3

To find the Eigen vectors

$$(A - \lambda I) X = 0$$

Case (i) $\lambda = -2$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_1 + x_2 + 3x_3 = 0$ ——— (1)

$x_1 + 7x_2 + x_3 = 0$ ——— (2)

$3x_1 + x_2 + 3x_2 = 0$ ——— (3)

(1) & (3) are same

	x_1		x_2	x_3
3	1	3	3	1
1	7	1	1	7

$$\frac{x_1}{1-21} = \frac{x_2}{3-3} = \frac{x_3}{21-1}$$

$$\frac{x_1}{-20} = \frac{x_2}{0} = \frac{x_3}{20}$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -20 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Case (ii) $\lambda = 3$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 + 3x_3 = 0$ ——— (4)

$x_1 + 2x_2 + x_3 = 0$ ——— (5)

$3x_1 + x_2 - 2x_3 = 0$ ——— (6)

$$\begin{array}{ccccc} & x_1 & x_2 & x_3 & \\ \hline -2 & 1 & 3 & -2 & 1 \\ 1 & 2 & 1 & 1 & 2 \end{array}$$

$$\frac{x_1}{1-6} = \frac{x_2}{3+2} = \frac{x_3}{-4-1}$$

$$\frac{x_1}{-5} = \frac{x_2}{5} = \frac{x_3}{-5}$$

$$X_2 = \begin{bmatrix} -5 \\ 5 \\ -5 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

Case (iii) $\lambda = 6$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5x_1 + x_2 + 3x_2 = 0$ ——— (7)

$x_1 - x_2 + x_3 = 0$ ——— (8)

$3x_1 + x_2 - 5x_3 = 0$ ——— (9)

	x_1	x_2	x_3	
-5	1	3	-5	1
1	-1	1	1	-1

$$\frac{x_1}{1+3} = \frac{x_2}{3+5} = \frac{x_3}{5-1}$$

$$\frac{x_1}{4} = \frac{x_2}{8} = \frac{x_3}{4}$$

$$X_3 = \begin{bmatrix} 4 \\ 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Step : 4

To find the normalized matrix

$$M = \begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & +1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 / \sqrt{x_1^2+x_2^2+x_3^2} \\ x_2 / \sqrt{x_1^2+x_2^2+x_3^3} \\ x_3 / \sqrt{x_1^2+x_2^2+x_3^2} \end{bmatrix}$$

$$N = X_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$X_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/\sqrt{6} \\ +2/\sqrt{6} \\ +1/\sqrt{6} \end{bmatrix}$$

$$N = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & 2/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{3} & +1/\sqrt{6} \end{bmatrix}$$

Step 5 :

$$N^T \Rightarrow \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \\ 1/\sqrt{6} & 2/\sqrt{6} & +1/\sqrt{6} \end{bmatrix}$$

Step : 6

$N^T D N$

$$= \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \\ 1/\sqrt{6} & 2/\sqrt{6} & +1/\sqrt{6} \end{bmatrix} \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & 2/\sqrt{6} \\ 1/\sqrt{2} & -1/\sqrt{3} & +1/\sqrt{6} \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

3. Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}$

Solution:

Step: 1

To find the characteristic equation,

$$\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$$

$s_1 = 2+6+2 = 10$

$s_2 = (12-0) + (12-0) + (4-16)$

$= \cancel{12} + 12 - \cancel{12}$

$= 12$

$s_3 = |A| = 2(12-0) - 0 + 4(0-24)$

$= 24 - 96$

$= -72$

The characteristic eqn is

$$\lambda^3 - 10\lambda^2 + 12\lambda + 72 = 0$$

Step: 2

To find the Eigen Value

$$\begin{array}{r|rrrr}
-2 & 1 & -10 & 12 & 72 \\
 & \downarrow & -2 & 24 & -72 \\
\hline
+6 & 1 & -12 & 36 & \boxed{0} \\
 & \downarrow & +6 & -36 & \\
\hline
6 & 1 & -6 & \boxed{0} & \\
 & \downarrow & 6 & & \\
\hline
 & 1 & \boxed{0} & &
\end{array}$$

The Eigen Value is $-2, 6, 6$

$\lambda_1 = -2, \lambda_2 = 6, \lambda_3 = 6$

Step: 3

To find the Eigenvectors $(A-\lambda I)x=0$

Case (i) $\lambda=-2$

$$\begin{bmatrix} 4 & 0 & 4 \\ 0 & 8 & 0 \\ 4 & 0 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

		x_1		x_2	x_3
4	0	4		4	0
0	8	0		0	8

$$\frac{x_1}{0-32} = \frac{x_2}{0-0} = \frac{x_3}{32-0}$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Case (ii) $\lambda=6$

$$\begin{bmatrix} -4 & 0 & 4 \\ 0 & 0 & 0 \\ 4 & 0 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-4x_1+4x_3=0$

$\Rightarrow \quad x_1 - x_3 = 0 \quad \text{—— } (1)$

$4x_1-4x_3=0$

$\Rightarrow \quad x_1 - x_3 = 0 \quad \text{—— } (2)$

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② $\Rightarrow$ $x_1 - x_3 = 0$

$$\frac{x_1}{1} = \frac{x_3}{1}$$

$$X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Case (iii) $\lambda = 6$

Let $X_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

(i) $X_1^T X_3 = 0 \Rightarrow \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$ ——— ③

(ii) $X_2^T X_3 = 0 \Rightarrow \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$ ——— ④

③ $\Rightarrow$ $a + 0 + c = 0$ ——— ⑤

(ii) $\Rightarrow$ $a + 0 + c = 0$ ——— ⑥

		a		b	c	
−1	0	1		−1	0	
1	0	1		1	0	

$$\frac{a}{0} = \frac{b}{2} = \frac{c}{0}$$

$$X_3 = \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Step 4 :

To find the normalized matrix

$$V_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad N_1 = \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad N_2 = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad N_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \sqrt{1} \Rightarrow 1$$

$$N = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Step 5

To find N^T

$$\begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}$$

Step 6

To find the N^TAN

$$\begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

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The Eigen value is 0, 3, 15

$\lambda_1 = 0, \lambda_2 = 3, \lambda_3 = 15$

Step: 3

To find the Eigenvectors $(A - \lambda I)X = 0$

Case i)

$\lambda_1 = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$8x_1 - 6x_2 + 2x_3 = 0$ ——— (1)

$-6x_1 + 7x_2 - 4x_3 = 0$ ——— (2)

$2x_1 - 4x_2 + 3x_3 = 0$ ——— (3)

$$\begin{array}{ccccc} & x_1 & x_2 & x_3 & \\ \hline 8 & -6 & 2 & 8 & -6 \\ -6 & 7 & -4 & -6 & 7 \end{array}$$

$$\frac{x_1}{24 - 14} = \frac{x_2}{-12 + 32} = \frac{x_3}{56 - 36}$$

$$\frac{x_1}{10} = \frac{x_2}{20} = \frac{x_3}{20}$$

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Case (ii) $\lambda = 3$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 8 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 - 6x_2 + 2x_3 = 0$ ———— (4)

$-6x_1 + 4x_2 - 4x_3 = 0$ ———— (5)

$8x_1 - 4x_2 = 0$ ———— (6)

$$\begin{array}{ccccc} & x_1 & x_2 & x_3 & \\ \hline 5 & -6 & 2 & 5 & -6 \\ -6 & 4 & -4 & -6 & 4 \end{array}$$

$$\frac{x_1}{24-8} = \frac{x_2}{-12+20} = \frac{x_3}{20-36}$$

$$\frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16}$$

$$X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

Case (iii) $\lambda = 15$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-7x_1 - 6x_2 + 2x_3 = 0$$

$$-6x_1 - 8x_2 - 4x_3 = 0$$

$$2x_1 - 4x_2 - 12x_3 = 0$$

	x_1		x_2	x_3
-7	-6	2	-7	-6
-6	-8	-4	-6	-8

$$\frac{x_1}{24+16} = \frac{x_2}{-12-28} = \frac{x_3}{56-36}$$

$$\frac{x_1}{40} = \frac{x_2}{-40} = \frac{x_3}{20}$$

$$X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

step. 4

To find the normalized matrix N

$$N = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

step : 5

To find the N^T

$$N^T = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

step : 6

To find N^TAN

$$\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 6 & -1 & 2 \\ -1 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

(5) Diagonalize the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Step: 1

To find the characteristic equation

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$S_1 = 6+3+3 = 12$$

$$S_2 = \begin{bmatrix} 6 & -2 \\ -2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$$

$$= (18-4) + (9-1) + (18-4)$$

$$= 14+8+14$$

$$= 36$$

$$S_3 = |A| = \begin{vmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

$$= 6(9-1) + 2(-6+2) + 2(2-6)$$

$$= 6(8) + 2(-4) + 2(-4)$$

$$= 48 - 16$$

$$= 32$$

$\therefore$ The Characteristic eqn $\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$.

Step: 2

To find the Eigen value

$$\begin{array}{r|rrrr}
2 & 1 & -12 & 36 & -32 \\
 & \downarrow & 2 & -20 & 32 \\
\hline
 & 1 & -10 & 16 & \boxed{0} \\
2 & \downarrow & 2 & -16 & \\
\hline
 & 1 & -8 & \boxed{0} & \\
8 & \downarrow & 8 & & \\
\hline
 & 1 & \boxed{0} & &
\end{array}$$

$\lambda_1 = 2 \quad \lambda_2 = 2 \quad \lambda_3 = 8$

step: 3

To find the Eigen vectors

$(A - \lambda I)X = 0.$

case(i) $\lambda = 8$

$$\begin{bmatrix} 6-8 & -2 & 2 \\ -2 & 3-8 & -1 \\ 2 & -1 & 3-8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{cccccc} & x_1 & & x_2 & & x_3 \\ \hline -2 & -2 & 2 & -2 & -2 & \\ -2 & -5 & -1 & -2 & -5 & \end{array}$$

$$\frac{x_1}{12} = \frac{x_2}{-6} = \frac{x_3}{6}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

case(ii) $\lambda = 2$

$$\begin{bmatrix} 6-2 & -2 & 2 \\ -2 & 3-2 & -1 \\ 2 & -1 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

three eqn are same.

$$2x_1 - x_2 + x_3 = 0$$

Put $x_1 = 0$

$$2(0) - x_2 + x_3 = 0$$

$$-x_2 + x_3 = 0$$

$$-x_2 = -x_3$$

$$\frac{x_2}{-1} = \frac{x_3}{-1}$$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$$

case(iii) $\lambda = 2$

Put $x_2 = 0$

$$2x_1 - 0 + x_3 = 0$$

$$2x_1 = -x_3$$

$$\frac{x_1}{-1} = \frac{x_3}{2}$$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

Step: 4

To find Normalized Matrix 'N'

$$M = \begin{bmatrix} 2 & 0 & -1 \\ -1 & -1 & 0 \\ 1 & -1 & 2 \end{bmatrix}$$

Eigen vectors

N

$$x_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \qquad n_1 = \begin{bmatrix} 2/\sqrt{(2)^2+(-1)^2+(1)^2} \\ -1/\sqrt{(2)^2+(-1)^2+(1)^2} \\ 1/\sqrt{(2)^2+(-1)^2+(1)^2} \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \qquad n_2 = \begin{bmatrix} 0 \\ -1/\sqrt{(0)^2+(-1)^2+(-1)^2} \\ -1/\sqrt{(0)^2+(-1)^2+(-1)^2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

$$x_3 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \qquad n_3 = \begin{bmatrix} -1/\sqrt{(-1)^2+(0)^2+(2)^2} \\ 0 \\ 2/\sqrt{(-1)^2+(0)^2+(2)^2} \end{bmatrix} = \begin{bmatrix} -1/\sqrt{5} \\ 0 \\ \frac{2}{\sqrt{5}} \end{bmatrix}$$

$$\therefore N = \begin{bmatrix} \frac{2}{\sqrt{6}} & 0 & \frac{-1}{\sqrt{5}} \\ \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{2}{\sqrt{5}} \end{bmatrix}$$

Step: 5

To find N^T

$$N^T = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} \end{bmatrix}$$

Step: 6

To find Diagonalization of Matrix

$D = N^T A N$

$$D = \begin{bmatrix} \frac{2}{\sqrt6} & \frac{-1}{\sqrt6} & \frac{1}{\sqrt6} \\ 0 & \frac{-1}{\sqrt2} & \frac{-1}{\sqrt2} \\ \frac{-1}{\sqrt5} & 0 & \frac{2}{\sqrt5} \end{bmatrix} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} \frac{2}{\sqrt3} & 0 & -\frac{1}{\sqrt5} \\ \frac{-1}{\sqrt6} & \frac{-1}{\sqrt2} & 0 \\ \frac{1}{\sqrt6} & \frac{-1}{\sqrt2} & \frac{2}{\sqrt5} \end{bmatrix}$$

$$D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(6)

(i) Expand using inner product space (a) $\langle 5u_1+8u_2, 6v_1-7v_2\rangle$, (b) $\langle 3u+5v, 4u-6v\rangle$, (c) $\|2u-3v\|^2$.

(a) $\langle 5u_1+8u_2, 6v_1-7v_2\rangle$

$= \langle 5u_1, 6v_1\rangle + \langle 5u_1 - 7v_2\rangle + \langle 8u_2, 6v_1\rangle + \langle 8u_2 - 7v_2\rangle$

$= 30\langle u_1, v_1\rangle - 35\langle u_1, v_2\rangle + 48\langle u_2, v_1\rangle$
$- 56\langle u_2, v_2\rangle$

(b) $\langle 3u+5v, 4u-6v\rangle$

$= \langle 3u, 4u\rangle + \langle 3u, -6v\rangle + \langle 5v, 4u\rangle + \langle 5v - 6v\rangle$

$= 12\langle u,u\rangle - 18\langle u,v\rangle + 20\langle v,u\rangle - 30\langle v,v\rangle$

$(\because \langle u,v\rangle = \langle v,u\rangle)$

$= 12\langle u,u\rangle + 2\langle u,v\rangle - 30\langle v,v\rangle$

$\therefore \langle u,u\rangle = \|u\|^2$

$\langle v,v\rangle = \|v\|^2$

$= 12\|u\|^2 + 2\langle u,v\rangle - 30\|v\|^2$

(c) $\|2u - 3v\|^2$

$= \langle 2u-3v, 2u-3v\rangle$

$= \langle 2u, 2u\rangle + \langle 2u, -3v\rangle + \langle -3v, 2u\rangle + \langle -3v, -3v\rangle$

$= 4\langle u,u\rangle - 6\langle u,v\rangle - 6\langle v,u\rangle + 9\langle v,v\rangle$

$= 4\|u\|^2 - 12\langle u,v\rangle + 9\|v\|^2$

(ii) If $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Check whether the Matrix A is Orthogonal ?

$AA^T = A^TA = I$

$A^T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$AA^T = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta + 0 & -\sin\theta\cos\theta + \sin\theta\cos\theta + 0 & 0+0+0 \\ -\sin\theta\cos\theta + \cos\theta\sin\theta + 0 & \sin^2\theta + \cos^2\theta & 0 \\ 0+0+0 & 0+0+0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad \text{———} \quad (1)$$

$$A^T A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta + 0 & \cos\theta\sin\theta - \sin\theta\cos\theta & 0 \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \cos^2\theta + \sin^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I \quad \text{———} \quad (2) \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$AA^T = A^T A = I$$

$\therefore$ The Matrix A is a orthogonal.

(4)

(i) consider vectors $u = (1, 2, 4)$ $v = (2, -3, 5)$, $w = (4, 2, -3)$ in R^3. Find (a) $u \cdot v$ (b) $u \cdot w$. (c) $v \cdot w$

(d) $(u+v) \cdot w$ (e) $\|u\|$ (f) $\|v\|$

(a) $u \cdot v = (1, 2, 4) \cdot (2, -3, 5)$

$= (1 \times 2) + (2 \times -3) + (4 \times 5)$

$= 2 - 6 + 20$

$= 16$

(b) $u \cdot w = (1, 2, 4) \cdot (4, 2, -3)$

$= 4 + 4 - 12$

$= -4.$

(c) $v \cdot w = (2, -3, 5) \cdot (4, 2, -3)$

$= 8 - 6 - 15$

$= -13$

(d) $(u+v) \cdot w = u \cdot w + v \cdot w$

$= -4 + (-13)$

$= -17$

(e) $\|u\| = \sqrt{1^2 + (2)^2 + (4)^2}$

$= \sqrt{21}$

$\|U\|^2 = 21$

(f) $\|v\| = \sqrt{(2)^2 + (-3)^2 + (5)^2}$

$= \sqrt{38}$

$\|v\|^2 = 38$

(ii) Show that the matrix $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is Orthogonal.

$$BB^T = B^TB = I$$

$$B^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$BB^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (\because \cos^2\theta + \sin^2\theta = 1)$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & -\sin\theta\cos\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BB^T = I \quad \text{——— } (1)$$

$$B^TB = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad \text{——— } (2)$$

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Diagonaling the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Sollution.

Step. 1

To find the characteristic Equation

$$\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$$

$s_1 = 8 + 7 + 3$

$= 18$

$s_2 = (56 - 36) + (21 - 16) + (24 - 4)$

$= 20 + 5 + 20$

$= 45$

$s_3 = 0$

The characteristic Equation is

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0$$

Step : 2

To find the Eigen Values

0	1	-18	45	0
	↓	0	+0	0
3	1	-18	45	$\boxed{0}$
	↓	3	-45	
15	1	-15	$\boxed{0}$	
	↓	15		
	1	$\boxed{0}$		

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Questions	opt1	opt2
The sum of the main diagonal elements of a matrix is called------	trace of a matrix	quadratic form
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then kλ1 ,kλ2, kλ3, ……… ,kλ n are the eigen values of --------------	kA	kA^2
In a modal matrix, the columns are the -----------	eigen vectors of A	eigen vectors of adj A
If atleast one of the eigen values of A is zero, then det A = -----	0	1
det (A- λI) represents------	characteristic polynomial	characteristic equation
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then 1/λ1 ,1/λ2, 1/λ3,……… ,1/λ n are the eigen values of --------------	A^(-1)	A
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then λ1p , λ 2 p , ………. λ n p are the eigen values of	A^(-1)	A^2
The eigen values of a --------------------- matrix are its diagonal elements	diagonal	symmetric
If the sum of two eigen values and trace of a 3x3 matrix A are equal, then det A = ---------	λ1 λ2 λ3	0
If 1,5 are the eigen values of a matrix A, then det A = -------	5	0
The eigen vector is also known as-------	latent value	latent vector
If 1,3,7 are the eigen values of A, then the eigen values of 2A are ------------	1,3,7	1,9,21
If the eigen values of 2A are 2, 6, 8 then eigen values of A are ________	1,3,4	2,6,8
The Set of all eigen values of the matrix A is called the ___________ of A	rank	index
A Square matrix A and its transpose have _______ eigen values.	different	Same
The sum of the __________ of a matrix A is equal to the sum of the principal diagonal elements of A.	characteristic polynomial	characteristic equation
The product of the eigenvalues of a matrix A is equal to_________	Sum of main diagonal	Determinant of A
The eigenvectors of a real symmetric are _______	equal	unequal
If all the eigen values of a matrix are distinct, then the corresponding eigen vectors______	linearly dependent	unique
A matrix is called symmetric if and only if ----------	A=A^T	A=A^-1
If a matrix A is equal to A^T then A is a -------- matrix.	symmetric	non symmetric
A matrix is called skew-symmetric if and only if ---------	A=A^T	A=A^-1
If a matrix A is equal to -A^T then A is a -------- matrix.	symmetric	non symmetric
A matrix is called orthogonal if and only if -----	A^T=A^-1	A^T=-A^-1

opt3	opt4	opt5	opt6	Answer
eigen value	canonical form			trace of a matrix
kA^(-1)	A^(-1)			kA
eigen vectors of inverse ofA	eigen values of A			eigen vectors of A
10	5			0
quadratic form	canonical form			characteristic polynomial
A^n	A^p			A^(-1)
A^(-p)	A^p			A^p
skew-matrix	triangular			triangular
1	2			0
25	6			5
column value	orthogonal value			latent vector
2,6,14	1,9,49			2,6,14
1,9,16	12,4,3			1,3,4
Signature	spectrum			spectrum
Inverse	Transpose			Same
eigen values	eigen vectors			eigen values
Sum of minors of Main diagonal	Sum of the cofactors of A			Determinant of A
real	symmetric			real
not unique	linearly independent			linearly independent
A=-A^T	A=A			A=A^T
skew-symmetric	singular			symmetric
A=-A^T	A=A			A=-A^T
skew-symmetric	singular			skew-symmetric
A^T=A^-2	A^T=-A^-2			A^T=A^-1

A matrix is called ------------if and only if $A^T=A^{-1}$.	orthogonal	square
The equation det $(A-\lambda I) = 0$ is used to find ----------	characteristic polynomial	characteristic equation
If the characteristic equation of a matrix A is $\lambda^2 - 2 = 0$, then the eigen values are -------	2,2	(-2,-2)
Eigen value of the characteristic equation $\lambda^2-4 = 0$ is	2, 4	2, -4
Eigen value of the characteristic equation $\lambda^3-6\lambda^2+11\lambda-6 = 0$ is	1,2,3	1, -2,3
Largest Eigen value of the characteristic equation $\lambda^3-3\lambda^2+2\lambda = 0$ is	1	0
Smallest Eigen value of the characteristic equation $\lambda^3-7\lambda^2+36 = 0$ is	-3	3
Sum of the principal diagonal elements =	product of eigen values	product of eigen vectors
Product of the eigen values =	(-|A|)	1/|A|
A Square matrix A and its transpose have ________ eigen values.	different	Same
If 1 and 2 are the eigen values of a 2X2 matrix A, then the eigen values of A^2 is	2, 4	3,4
If 1 and 2 are the eigen values of a 2X2 matrix A, then the eigen values of $A^{-1}$is	2,1/2	1,1/2
If a real symmetric matrix of order 2 has ---------then the matrix is a scalar matrix.	equal eigen vectors	different eigen vectors
If A and B are nxn matrices and B is a non singular matrix then A and $B^{(-1)}$ AB have	same eigen vectors	different eigen vectors
The eigenvalues of the matrix I_2 are ___	(1,-1)	(-1,-1)
For any square matrix A, then $A*(A^T)$ is	symmetric	non symmetric
For any square matrix A, then $A+(A^T)$ is	symmetric	non symmetric
For any square matrix A, then $A-(A^T)$ is	symmetric	non symmetric
Any orthogonal matrix is ___	symmetric	skew-symmetric
Let A and B be symmetrix matrices of order n. Then AB+BA is __	symmetric	non symmetric
Let A and B be symmetrix matrices of order n. Then AB is symmetric iff ___	AB=BA	BA
Let A be orthogonal matrix of order n. Then A^T is __	symmetric	orthogonal
Let A and B be orthogonal matrices of the same order. Then AB is __	symmetric	orthogonal

non symmetric	triangular			orthogonal
eigen values	eigen vectors			characteristic equation
(2^(1/2),-2^(1/2))	(2i,-2i)			(2^(1/2),-2^(1/2))
2, -2	2, 2			2,-2
1,2,-3	1,-2,-3			1,2,3
2	4			2
-2	6			-2
sum of eigen values	sum of eigen vectors			sum of eigen values
(-1/|A|)	|A|			|A|
Inverse	Transpose			Same
5,6	1, 4			1, 4
1,2	4,1/2			1,1/2
equal eigen values	different eigen values			equal eigen values
same eigen values	different eigen values			same eigen values
(-1,1)	(1,1)			(1,1)
skew-symmetric	singular			symmetric
skew-symmetric	singular			symmetric
skew-symmetric	singular			skew-symmetric
non-singular	singular			non-singular
skew-symmetric	singular			symmetric
AB=0	AB=n			AB=BA
skew-symmetric	singular			orthogonal
skew-symmetric	singular			orthogonal

Questions	opt1	opt2	opt3
The sum of the main diagonal elements of a matrix is called----	trace of a matrix	quadratic form	eigen value
Every square matrix satisfies its own ----------	characteristic polynomial	characteristic equation	orthogonal transformation
The orthogonal transformation used to diagonalise the symmetric matrix A is----	NT AN	NT A	NAN-1
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then kλ1 , kλ2, kλ3,……… ,kλ n are the eigen values of --------------	kA	kA2	kA-1
Diagonalisation of a matrix by orthogonal reduction is true only for a ----- matrix.	diagonal	triangula	real symmetric
In a modal matrix, the columns are the -----------	eigen vectors of A	eigen vectors of adj A	eigen vectors of inverse ofA
If atleast one of the eigen values of A is zero, then det A = -----	0	1	10
If the canonical form of a quadratic form is 5y12 - 6 y22 , then the index is --------	4	0	2
det (A- λI) represents------	characteristic polynomial	characteristic equation	quadratic form
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then 1/λ1 , 1/λ2, 1/λ3,……… ,1/λ n are the eigen values of --------------	A^(-1)	A	A^n
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then λ1p , λ 2 p , ………. λ n p are the eigen values of	A^(-1)	A^2	A^(-p)
Cayley -Hamilton theorem is used to find ------------	inverse and higher powers of A	eigen va	eigen vectors
The eigen values of a --------------------- matrix are its diagonal	diagonal	symmet	skew-ma
In an orthogonal transformation NT AN = D , D refers to a ---------- matrix.	diagonal	orthogor	symmetr
In a modal matrix, the columns are the eigen vectors of----------	A-1	A2	A
If the eigen values of 8x12 + 7 x22 +3 x32 –12 x1 x2 – 8 x2 x3 +4 x3x1 are 0,3 & 15, then its nature is-----------	positive definite	positive semidefinite	indefinit

opt4	opt5	opt6	Answer
canonical form			trace of a matrix
canonical form			characteristic equation
NA			NT AN
A-1			kA
scalar			real symmetric
eigen values of A			eigen vectors of A
5			0
1			1
canonical form			characteristic polynomial
A^p			A^(-1)
A^p			A^p
quadratic form			inverse and higher powers of A
triangular			triangula
skew-symmetric			diagonal
adj A			A
negative definite			positive semidefinite

The elements of the matrix of the quadratic form x12 + 3 x22 + 4 x1 x2 are ------------	a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3	a11 = -1, a12 = -2 , a 21 = 2 , a 22 = 3	a11 = 1, a12 = 4 , a 21 = 4 , a 22 = 3
If the sum of two eigen values and trace of a 3x3 matrix A are equal, then det A = ---------	λ1 λ2 λ3	0	1
If 1,5 are the eigen values of a matrix A, then det A = -------	5	0	25
If the canonical form of a quadratic form is 5y12 + 6 y22 , then the rank is --------	4	0	2
The non –singular linear transformation used to transform the quadratic form to canonical form is ----------	X= NTY	X= NY	Y= NX
The eigen vector is also known as-------	latent value	latent vector	column value
If 1,3,7 are the eigen values of A, then the eigen values of 2A are ------------	1,3,7	1,9,21	2,6,14
If the eigen values of 2A are 2, 6, 8 then eigen values of A are _	1,3,4	2,6,8	1,9,16
The number of positive terms in the canonical form is called the	rank	index	Signatur
If all the eigenvalues of A are positive then it is called as______	Positive definite	Negative definite	Positive semidefinite
If all the eigenvalues of A are negative then it is called as_____	Positive definite	Negative definite	Positive semidefinite
A homogeneous polynomial of the second degree in any number of variables is called the ______	characteristic polynomial	characteristic equation	quadratic form
The Set of all eigen values of the matrix A is called the ____________ of A	rank	index	Signatur
A Square matrix A and its transpose have _______ eigen values	different	Same	Inverse
The sum of the ___________ of a matrix A is equal to the sum of the principal diagonal elements of A.	characteristic polynomial	characteristic equation	eigen values
The product of the eigenvalues of a matrix A is equal to______	Sum of main diagonal	Determinant of A	Sum of minors of Main diagonal
The eigenvectors of a real symmetric are ________	equal	unequal	real
When the quadratic form is reduced to the canonical form, it will contain only r terms, if the _____ of A is r.	rank	index	Signatu

a11 = 1, a12 = 4 , a 21 = 3 , a 22 = 1			a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3
2			0
6			5
1			2
NXA			X= NY
orthogonal value			latent vector
1,9,49			2,6,14
12,4,3			1,3,4
indefinite			index
Negative semidefinite			Positive definite
Negative semidefinite			Negative definite
canonical form			quadratic form
spectrum			spectrun
Transpose			Same
eigen vectors			eigen values
Sum of the cofactors of A			Determinant of A
symmetric			real
spectrum			rank

The excess of the number of positive terms over the number of negative terms in the canonical form is called the ___________ of the quadratic form.	rank	index	Signatu
If all the eigen values of A are less than zero and atleast one eigen value is zero then the quadratic form is said to be ___________	Positive definite	Negative	Positive
If all the eigen values of A are greater than zero and atleast one eigen value is zero then the quadratic form is said to be ___________	Positive definite	Negative definite	Positive semidefinite
If the quadratic form has both positive and negative terms then it is said to be ___________	Positive definite	Negative definite	Positive semidefinite

<table>
<tr><td colspan="2">spectrum</td><td></td><td>Signatu</td></tr>
<tr><td>Negative semidefinite</td><td></td><td></td><td>Negative semidefinite</td></tr>
<tr><td>Negative semidefinite</td><td></td><td></td><td>Positive semidefinite</td></tr>
<tr><td colspan="2">indefinite</td><td></td><td>indefini</td></tr>
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KARPAGAM ACADEMY OF HIGHER EDUCATION

DEPARTMENT OF SCIENCE AND HUMANITIES.

I. B.E. COMPUTER SCIENCE AND ENGINEERING.

MATHEMATICS - I (18BECS101)

UNIT - IV CALCULUS

PART - C.

1. Find the equation of evolute of the parabola $y^2 = 4ax$.

solution:

step 1 The Parametric form.

$$x = at^2 \quad ; \quad y = 2at.$$

$$\frac{dx}{dt} = 2at. \quad ; \quad \frac{dy}{dt} = 2a.$$

$$y_1 = dy/dx = 2a/2at = 1/t. \Rightarrow \boxed{y_1 = 1/t}.$$

$$y_2 = \frac{d^2y}{dx^2} = d/dt\,(dy/dx)\,dt/dx$$

$$= d/dt\left[1/t\right](1/2at)$$

$$= -1/t^2 \cdot 1/2at.$$

$$\boxed{y_2 = -1/2at^3.}$$

step 2 : To find $(\bar{x}, \bar{y})$

Let (X, Y) be centre of curvature, then

$$X = x - \left[\frac{y_1(1+y_1^2)}{y_2}\right]$$

$= 2at$

$$= at^2 - \left[\frac{(1/t)(1+1/t^2)}{-1/2at^3}\right]$$

$= at^2 + 2at^3(1/t)(1+1/t^2)$

$= at^2 + 2at^2\left[1+1/t^2\right]$

$= at^2 + 2at^2 + 2a\cancel{t^2}/\cancel{t^2}.$

$\boxed{X = 3at^2+2a}$ $\longrightarrow$ (1).

$$Y = y + \left[\frac{1+y_1^2}{y_2}\right]$$

$$= 2at + \left[\frac{1+1/t^2}{-1/2at^3}\right]$$

$$= 2at - 2at^3 - \frac{2at^{\cancel{3}}}{\cancel{t^2}}$$

$= 2at - 2at - 2at^3$

$\boxed{Y = -2at^3}$ $\longrightarrow$ (2)

step 3: To eliminate 't' from (1) & (2)

(1) → $x = 3at^2 + 2a$

$x - 2a = 3at^2$

$\left(\frac{x-2a}{3a}\right) = t^2$.

$(t^2)^3 = \left[\frac{x-2a}{3a}\right]^3$

$t^6 = \frac{(x-2a)^3}{27a^3} \rightarrow (3)$

(2) → $y = -2at^3$

$\frac{y}{-2a} = t^3$.

$(t^3)^2 = \left(\frac{-y}{2a}\right)^2$

$t^6 = \frac{y^2}{4a^2}$. $\rightarrow (4)$

From (3) & (4)

$\frac{(x-2a)^3}{27a^3} = \frac{y^2}{4a^2}$,

$4(x-2a)^3 = 27ay^2$.

step 4: Locus of (x,y)

$4(x-2a)^3 = 27ay^2$, which is the required evolute of the parabola $y^2 = 4ax$.

———— x ————

2) Find the evolute of parabola $x^2 = 4ay$.

solution:

step 1 the Parametric form

$x = 2at \qquad y = at^2$.

$dx/dt = 2a \qquad dy/dt = 2at$.

$$\therefore y_1 = \frac{dy}{dx} = \frac{2at}{2a} = t.$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right)\left(\frac{dt}{dx}\right)$$

$$= d/dt\,(t)\,(1/2a)$$

$$\boxed{y_2 = 1/2a}$$

<u>step 2</u>: To find (X, Y)

$$X = x - \left[\frac{y_1(1+y_1^2)}{y_2}\right]$$

$$= 2at - \left[\frac{t(1+t^2)}{1/2a}\right]$$

$$= 2at - 2at(1+t^2)$$

$$= 2at - 2at - 2at^3.$$

$$\boxed{X = -2at^3.} \longrightarrow (1)$$

$$Y = y + \left[\frac{1+y_1^2}{y_2}\right]$$

$$= at^2 + \left[\frac{1+t^2}{1/2a}\right]$$

$$= at^2 + 2a(1+t^2) = at^2 + 2a + 2at^2.$$

$$\boxed{Y = 3at^2 + 2a} \longrightarrow (2)$$

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step 3: To eliminate 't' from (1) & (2).

(1) $\rightarrow x = -2at^3$

$\frac{x}{-2a} = t^3$

$(t^3)^2 = \left(\frac{-x}{2a}\right)^2$

$t^6 = \frac{x^2}{4a^2} \rightarrow (3)$

(2) $\rightarrow y = 3at^2 + 2a$.

$y - 2a = 3at^2$

$\left[\frac{y-2a}{3a}\right] = t^2$.

$(t^2)^3 = \left[\frac{y-2a}{3a}\right]^3$.

$t^6 = \frac{(y-2a)^3}{27a^3} \rightarrow (4)$.

$\frac{x^2}{4a^2} = \frac{(y-2a)^3}{27a^3}$.

$27ax^2 = 4(y-2a)^3$.

step 4: Locus of (x, y)

$27ax^2 = 4(y-2a)^3$, which is the required evolute of parabola $x^2 = 4ay$.

—— x ——

3) Find the evolute of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

solution:

step 1 The Parametric Form.

$x = a\cos\theta$; $y = b\sin\theta$.

$$\frac{dx}{d\theta} = -a\sin\theta \qquad dy/d\theta = b\cos\theta$$

$$y_1 = \frac{dy}{dx} = -b/a \cot\theta.$$

$$y_2 = d/d\theta \left(d/dx\right) d\theta/dx.$$

$$= d/d\theta \left(-b/a \cot\theta\right)\left(-1/\sin\theta\right)$$

$$= \frac{-b}{a^2}\left(-\operatorname{cosec}^2\theta\right)\left(-\operatorname{cosec}\theta\right)$$

$$\boxed{y_2 = -b/a^2 \operatorname{cosec}^3\theta.}$$

Step 2: To find (X, Y)

$$X = x - \left[\frac{y_1(1+y_1^2)}{y^2}\right]$$

$$= a\cos\theta - \left[\frac{-b/a \cot\theta \left(1 + b^2/a^2 \cot^2\theta\right)}{-b/a^2 \operatorname{cosec}^3\theta}\right]$$

$$= a\cos\theta - \frac{a^2}{b}\sin^3\theta\left[-\frac{b}{a}\cot\theta\right]\left[1+\frac{b^2}{a^2}\cot^2\theta\right]$$

$$= a\cos\theta - \cancel{a^2}\sin^{\cancel{3}}\theta\left[-b/\cancel{a}\frac{\cos\theta}{\cancel{\sin\theta}}\right]\left[1 + \frac{b^2}{a^2}\frac{\cos^2\theta}{\sin^2\theta}\right]$$

$$= a\cos\theta - a\sin^2\theta\cos\theta\left(1 + b^2/a^2 \cos^2\theta/\sin^2\theta\right]$$

$$= a\cos\theta - a\sin^2\theta\cos\theta - \cancel{a}\cancel{\sin^2}\theta\cos\theta\left[b^2/a^2 \cos^2\theta/\cancel{\sin^2}\theta\right]$$

$$= a\cos\theta - a\sin^2\theta\cos\theta - b^2/a\,\cos^3\theta$$

$$= a\cos\theta\,[1-\sin^2\theta] - b^2/a\,\cos^3\theta$$

$$= a\cos^3\theta - \frac{b^2}{a}\cos^3\theta$$

$$= \left[a - \frac{b^2}{a}\right]\cos^3\theta$$

$$\boxed{X = \left[\frac{a^2-b^2}{a}\right]\cos^3\theta.} \longrightarrow (1)$$

$$Y = y + \left[\frac{1+y_1^2}{y_2}\right]$$

$$= b\sin\theta + \left[\frac{1+b^2/a^2\,\cot^2\theta}{-b/a^2\,\operatorname{cosec}^3\theta}\right]$$

$$= b\sin\theta - \frac{a^2}{b}\sin^3\theta\left(1+b^2/a^2\,\frac{\cos^2\theta}{\sin^2\theta}\right)$$

$$= b\sin\theta - \frac{a^2}{b}\sin^3\theta - \frac{\cancel{a^2}}{\cancel{b}}\sin^3\theta\cdot\frac{b^{\cancel{2}}}{\cancel{a^2}}\,\frac{\cos^2\theta}{\cancel{\sin^2\theta}}.$$

$$= b\sin\theta - a^2/b\,\sin^3\theta - b\sin\theta\cos^2\theta.$$

$$= b\sin\theta\,[1-\cos^2\theta] - a^2/b\,\sin^3\theta.$$

$$= b\sin^3\theta - a^2/b\,\sin^3\theta$$

$$\boxed{Y = \left[\frac{b^2-a^2}{b}\right]\sin^3\theta} \longrightarrow (2)$$

step 3: To eliminate 'θ'

$(1) \longrightarrow x^{2/3} = \left(\frac{a^2-b^2}{a}\right)^{2/3} (\cos^3\theta)^{2/3}$.

$$x^{2/3} = \frac{(a^2-b^2)^{2/3}}{a^{2/3}} \cos^2\theta.$$

$$(xa)^{2/3} = (a^2-b^2)^{2/3} \cos^2\theta. \quad \to (3)$$

$(2) \longrightarrow y^{2/3} = \left(\frac{b^2-a^2}{b}\right)^{2/3} (\sin^3\theta)^{2/3}$

$$y^{2/3} = \frac{(b^2-a^2)^{2/3}}{b^{2/3}} \sin^2\theta.$$

$$(yb)^{2/3} = (b^2-a^2)^{2/3} \sin^2\theta \quad \to (4)$$

$(3)+(4) \Rightarrow (xa)^{2/3}+(yb)^{2/3} = (a^2-b^2)^{2/3}\cos^2\theta + (b^2-a^2)^{2/3}\sin^2\theta.$

$$= (a^2-b^2)^{2/3}(\cos^2\theta+\sin^2\theta)$$

$$(xa)^{2/3}+(yb)^{2/3} = (a^2-b^2)^{2/3}.$$

step 4: Locus of (x, y)

$(xa)^{2/3}+(yb)^{2/3} = (a^2-b^2)^{2/3}$ which is the required evolute of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

—— x ——

4) Find the evolute of hyperpola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

sol

step 1: Parametric form.

$$x = a\sec\theta, \quad y = b\tan\theta$$

$$\frac{dx}{d\theta} = a\sec\theta \cdot \tan\theta. \qquad dy/d\theta = b\sec^2\theta$$

$$y_1 = \frac{b}{a}\frac{\sec^2\theta}{\sec\theta \cdot \tan\theta} = \frac{b}{a\sin\theta}.$$

$$\boxed{y_1 = b/a \ \operatorname{cosec}\theta}$$

$$y_2 = \frac{b/a\ [-\operatorname{cosec}\theta \cdot \cot\theta]}{a\sec\theta \cdot \tan\theta}.$$

$$\boxed{y_2 = -b/a\ \cot^3\theta}$$

step 2: (x, y).

$$X = x - \frac{y_1(1+y_1^2)}{y_2}$$

$$= a\sec\theta - \frac{b/a\operatorname{cosec}\theta\ (1+b^2/a^2\operatorname{cosec}^2\theta)}{-b/a^2\ \cot^3\theta}.$$

$$= a\sec\theta + 1/a\ \frac{\operatorname{cosec}\theta\,(a^2+b^2\operatorname{cosec}^2\theta)}{\cot^3\theta}.$$

$$= a\sec\theta + 1/a\ 1/\sin\theta\ \frac{\sin^3\theta}{\cos^3\theta}\left(a^2+b^2\ 1/\sin^2\theta\right)$$

$$= a\sec\theta + \frac{a\sin^2\theta}{\cos^3\theta} + \frac{b^2}{a\cos^3\theta}$$

$$= a\sec\theta + \frac{a(1-\cos^2\theta)}{\cos^3\theta} + b^2/a\ \sec^3\theta$$

$$= a\sec\theta + a\sec^3\theta - a\sec\theta + b^2/a\ \sec^3\theta$$

$$= \frac{a^2\sec\theta + a^2\sec^3\theta - a^2\sec\theta + b^2\sec^3\theta}{a}$$

$$ax = a^2\sec\theta + a^2\sec^3\theta - a^2\sec\theta + b^2\sec^3\theta$$

$$\Rightarrow ax = (a^2+b^2)\sec^3\theta$$

$$\boxed{(ax)^{2/3} = (a^2+b^2)^{2/3}\sec^2\theta} \longrightarrow (1)$$

$$Y = y + \frac{1+y_1^2}{y^2}$$

$$= b\tan\theta + \left(\frac{1 + b^2/a^2 \operatorname{cosec}^2\theta}{-b/a^2 \cot^3\theta}\right)$$

$$= b\tan\theta - \frac{\frac{a^2+b^2}{a^2}\frac{1}{\sin^2\theta}}{b/a^2 \frac{\cos^3\theta}{\sin^3\theta}}$$

$$= b\tan\theta - \left(a^2 + \frac{b^2}{\sin^2\theta}\right)\cdot \frac{1}{b}\frac{\sin^3\theta}{\cos^3\theta}$$

$$= b\tan\theta - a^2/b\tan^3\theta - b\tan\theta\cdot\sec^2\theta$$

$$bY = -(a^2+b^2)\tan^3\theta$$

$$\boxed{(bY)^{2/3} = (a^2+b^2)^{2/3}\tan^2\theta} \longrightarrow (2).$$

step 3: Eliminating θ.

$$(1)-(2) = (ax)^{2/3} - (bY)^{2/3} = (a^2+b^2)^{2/3}(\sec^2\theta - \tan^2\theta)$$

$$(ax)^{2/3} - (by)^{2/3} = (a^2+b^2)^{2/3}.$$

step 4: Locus.

locus of (x, y) is $(ax)^{2/3} - (by)^{2/3} = (a^2+b^2)^{2/3}$ which gives the equation of evolute of given hyperbola.

x

5. Find the surface area of the solid generated by revolving the arc of parabola $y^2 = 4ax$, bounded by x-axis and (0,0) to (a,0)

$$\text{Surface area} = 2\pi \int_a^b y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx.$$

$$y^2 = 4ax.$$

$$y = 2\sqrt{a}\sqrt{x}.$$

Differentiating $y^2 = 4ax$,

$$2y\, \frac{dy}{dx} = 4a.$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{4a^2}{y^2}$$

$$= \frac{4a^2}{4ax} = \frac{a}{x}.$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+a/x}.$$

$$= \frac{\sqrt{x+a}}{\sqrt{x}}.$$

$$S.A = 2\pi \int_0^a 2\sqrt{a}\sqrt{x}\, \frac{\sqrt{x+a}}{\sqrt{x}}\, dx.$$

$$= 4\pi\sqrt{a} \int_0^a (x+a)^{1/2}\, dx.$$

$\not\leq$ 8

$$= 4\pi\sqrt{a}\left[\frac{(x+a)^{3/2}}{3/2}\right]_0^a$$

$$= 4\pi\sqrt{a} \times 2/3 \left[(2a)^{3/2} - (a)^{3/2}\right]$$

$$= 8/3\,\pi\sqrt{a}\left[2\sqrt{2}\,.\,a\sqrt{a} - a\sqrt{a}\right]$$

$$= 8/3\,\pi\sqrt{a}\,.\sqrt{a}\,a\left[2\sqrt{2} - 1\right]$$

$$= 8/3\,\pi a^2\left[2\sqrt{2} - 1\right] \text{ sq. units.}$$

6. Find the Surface area of the Solid obtained by revaluation by arc of the Curve $y = \sin x$ from $x = 0$ to $x = \pi$ about x-axis.

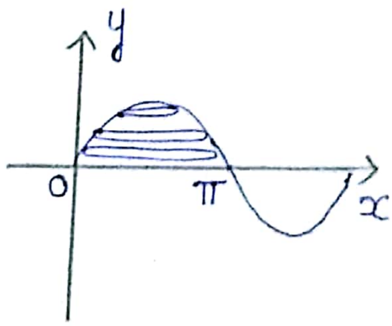
Solution.

$$\boxed{\text{Surface Area} = 2\pi \int_a^b y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx}$$

Given,

$$y = \sin x.$$

Differentiate with respect to x,



$$\frac{dy}{dx} = \cos x.$$

$$\left(\frac{dy}{dx}\right)^2 = \cos^2 x.$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \cos^2 x}$$

$$= 2\pi \int_0^{\pi} \sin x \sqrt{1+\cos^2 x} \cdot dx$$

$$= 2\pi \int_1^{-1} \sqrt{1+t^2}\,(-dt)$$

$$= 2\pi \int_{-1}^{1} \sqrt{1+t^2}\; dt$$

Put $t = \cos x$

$\frac{dt}{dx} = -\sin x$

$dt = -\sin x\, dx$

x	0	π
t	1	-1

$$= 2\pi \times 2 \int_0^1 \sqrt{1+t^2}\; dt$$

$$= 4\pi \int_0^1 \sqrt{1+t^2}\; dt$$

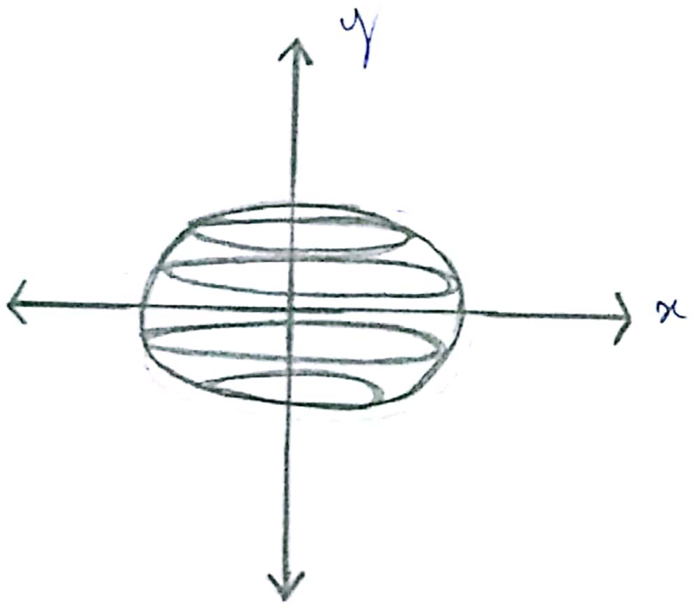
$$= 4\pi \left[\frac{t}{2}\sqrt{1+t^2} + \frac{1}{2} \log\left[t + \sqrt{1+t^2}\right] \right]_0^1$$

$$= 4\pi \left[\left\{ \frac{1}{2}\sqrt{2} + \frac{1}{2}\log\left[1+\sqrt{2}\right] \right\} - \{0\} \right]$$

$$= \frac{4\pi}{2}\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$$

$$= 2\pi\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right] \text{ Square. Units.}$$

7. Find the Volume of the Solid when the region enclosed by the Curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b > 0)$ is revolved about the y-axis.



The Volume,

$$\boxed{V = \pi \int_c^d x^2 \, dy}$$

Given : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{a^2} = 1 - \frac{y^2}{b^2}$$

$$x^2 = a^2\left(1 - \frac{y^2}{b^2}\right)$$

y - axis $\rightarrow$ The limits b to $-b$

$$V = \pi \int_{-b}^{b} a^2 \left(1 - \frac{y^2}{b^2}\right) dy.$$

$$= 2\pi a^2 \int_{0}^{b} \left(\frac{b^2 - y^2}{b^2}\right) dy.$$

$$= \frac{2\pi a^2}{b^2} \int_{0}^{b} (b^2 - y^2)\, dy.$$

$$= \frac{2\pi a^2}{b^2} \left[b^2 y - \frac{y^3}{3} \right]_{y=0}^{y=b}$$

$$= \frac{2\pi a^2}{b^2} \left[\left(b^2 b - \frac{b^3}{3} \right) - (0 - 0) \right]$$

$$= \frac{2\pi a^2}{b^2} \left(b^3 - \frac{b^3}{3} \right)$$

$$= \frac{2\pi a^2}{b^2}\left(\frac{3b^3 - b^3}{3}\right)$$

$$= \frac{2\pi a^2}{3b^2}\left(2b^3\right)$$

$$V = \frac{4\pi a^2 b}{3} \text{ Cubic. Units.}$$

Volume of the Solid $= \dfrac{4\pi a^2 b}{3}$ Cubic. Units.

8. Evaluate $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

Let, $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\frac{\sin x}{\cos x}}}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{\left(\frac{\sqrt{\cos x + \sin x}}{\sqrt{\cos x}}\right)}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x + \sin x}} \cdot dx \longrightarrow (1)$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/3 + \pi/6 - x)} \cdot dx}{\sqrt{\cos(\pi/3 + \pi/6 - x) + \sin(\pi/3 + \pi/6 - x)}}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos(\pi/2 - x)}\, dx}{\sqrt{\cos(\pi/2 - x)} + \sqrt{\sin(\pi/2 - x)}}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \longrightarrow (2)$$

$$\left[\begin{array}{l} \because \cos(90-\theta) = \sin\theta \\ \sin(90-\theta) = \cos\theta \end{array}\right]$$

(1) + (2),

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \cdot dx$$

$$2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} \cdot dx.$$

$$2I = \int_{\pi/6}^{\pi/3} dx$$

$$I = \frac{1}{2} \left[x\right]_{\pi/6}^{\pi/3}.$$

$$= \frac{1}{2}\left[\frac{\pi}{3} - \frac{\pi}{6}\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{6}\right]$$

$$= \frac{\pi}{12} \text{ //.}$$

$I = \pi/12$ //

9. Prove that,

$$\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

We know that,

$$\boxed{\Gamma(m) = \int_0^\infty e^{-t}\, t^{m-1}\,.\,dt}$$

Put $t = x^2$

$$\frac{dt}{dx} = 2x$$

$$dt = 2x\,.\,dx$$

$$\Gamma(m) = \int_0^\infty e^{-x^2}\,.\,x^{2(m-1)}\,.\,2x\,dx.$$

$$= 2\int_0^\infty e^{-x^2}\, x^{2m-2}\,.\,x^1\,dx.$$

$$= 2\int_0^\infty e^{-x^2} \cdot x^{2m}\, x^{-2}\, x^{1}\, dx.$$

$$= 2\int_0^\infty e^{-x^2} \cdot x^{2m}\, x^{-1} \cdot dx$$

$$\Gamma(m) = 2\int_0^\infty e^{-x^2} \cdot x^{2m-1} \cdot dx.$$

IIIly,

$$\Gamma(n) = 2\int_0^\infty e^{-y^2}\, y^{2n-1} \cdot dy.$$

$$\Gamma(m) \cdot \Gamma(n) = 2\int_0^\infty e^{-x^2}\, x^{2m-1} \cdot dx \times$$

$$2\int_0^\infty e^{-y^2}\, y^{2-1}\, dy.$$

$$= 4\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\, x^{2m-1} \cdot y^{2n-1}.$$

$$dx\, dy \longrightarrow (1)$$

Transform to Polar co-ordinates.

Put $x = r\cos\theta$

$y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$,

$x^2 + y^2 = r^2$ and,

θ Varies from 0 to $\pi/2$

r Varies from 0 to ∞

$$① \rightarrow \Gamma(m)\cdot\Gamma(n) = 4\int_0^{\pi/2}\int_0^{\infty} e^{-r^2}(r\cos\theta)^{2m-1} \cdot (r\sin\theta)^{2n-1}\, r\,dr\,d\theta$$

$$= 4\int_0^{\pi/2}\int_0^{\infty} e^{-r^2}\, r^{2m-1}\cos^{2m-1}\theta \cdot r^{2n-1} \cdot \sin^{2n-1}\theta \cdot r^1\,dr\,d\theta.$$

$$= 4\int_0^{\pi/2}\int_0^{\infty} e^{-r^2}\, r^{2m-1+2n\cancel{-1}\cancel{+1}} \cdot \sin^{2n-1}\theta \cdot \cos^{2m-1}\theta\, dr\,d\theta.$$

$$= 4\int_0^{\pi/2}\int_0^{\infty} e^{-r^2}\, r^{2m+2n-1} \cdot \sin^{2n-1}\theta \cdot \cos^{2m-1}\theta \cdot dr\,d\theta.$$

$$= 2\int_0^{\infty} e^{-r^2} \cdot r^{2(m+n)-1}\, dr \times 2\int_0^{\pi/2} \sin^{2n-1}\theta \cdot \cos^{2m-1}\theta \cdot d\theta.$$

$$\Gamma(m)\cdot\Gamma(n) = \Gamma(m+n) \times \beta(m,n) \longrightarrow ②$$

[i) $\Gamma(n) = \int_0^{\infty} e^{-x} \cdot x^{n-1}\, dx.$

$$\Gamma(m+n) = 2\int_0^{\infty} e^{-r^2} \cdot r^{2(m+n)-2}\, r \cdot dr.$$

$$\Gamma(m+n) = 2\int_0^{\infty} e^{-r^2} \cdot r^{2(m+n)-1} \cdot dr.$$

ii) $\beta(m,n) = \int_0^{\pi/2} 2\sin^{2n-1}\theta \cdot \cos^{2m-1}\theta \cdot d\theta$]

$$② \rightarrow \Gamma(m)\,\Gamma(n) = \Gamma(m+n)\cdot\beta(m,n)$$

$$\frac{\Gamma(m)\cdot\Gamma(n)}{\Gamma(m+n)} = \beta(m,n)$$

Hence Proved.

10. <u>Evaluate</u>.

i) $\int_0^{\pi/2} e^{2x} \cdot \cos x \cdot dx$.

$$\int_0^{\pi/2} e^{2x} \cdot \cos x \, dx = \left[\left(\frac{e^{2x}}{2^2 + 2} \right) \left(2\cos x + \sin x \right) \right]_{x=0}^{x=\pi/2}$$

$$= \left[\frac{e^{2(\pi/2)}}{5} \left(2\cos(\pi/2) + \sin(\pi/2) \right) \right] - \left[\frac{e^0}{5} \, 2\cos 0 + \sin 0 \right]$$

$$= \frac{e^{\pi}}{5}(0+1) - \frac{1}{5}(2 \times 1 + 0)$$

$$= \frac{e^{\pi}}{5}(1) - \frac{1}{5}(2)$$

$$= \frac{1}{5}(e^{\pi} - 2)$$

ii) $\displaystyle\int_0^a \sqrt{a^2 - x^2}\,dx$

$$\int_0^a \sqrt{a^2-x^2}\,dx = \left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_{x=0}^{x=a}$$

$$= \left[\frac{a}{2}\sqrt{a^2-a^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right)\right] - \left[\frac{0}{2}\sqrt{a^2-0} + \frac{0^2}{2}\sin^{-1}\frac{0}{a}\right]$$

$$= \frac{a^2}{2}\sin^{-1}(1) - \frac{a^2}{2}\sin^{-1}(0)$$

$$= \frac{a^2}{2}\cdot\pi/2 - \frac{a^2}{2}(0)$$

$$\cancel{\pi/4} \quad = \frac{\pi a^2}{4}$$ //.

iii) $\int_0^{\pi/2} \cos^8 x \, dx.$

$$= \frac{7}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}.$$

$$= \frac{105\pi}{768}$$

($n = 8$ is even)

W.K.T,

$$\int_0^{\infty} x^n \cdot e^{-ax} dx = \frac{n!}{a^{n+1}}.$$

---

iv) $\int_0^{\pi/2} \sin^7 x \, dx.$

$$= \frac{6}{7} \cdot \frac{4}{5} \cdot \frac{2}{3} \cdot 1 \quad (n = 7 \text{ is odd}).$$

$$= \frac{48}{105}.$$

W.K.T,

$$\int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots, & \text{if } n \text{ is odd} \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2} & \text{If } n \text{ is even} \end{cases}$$

Questions	opt1	opt2
What is the value of Gamma of one ?	0	1
gamma (n+1)=___________	(n+1)!	n gamma (n+1
what is the value of gamma(1 /2)?	pi	0
Which one of the following statement is true?	gamma(2)=gan	gamma(1/2)= (
Which one of the following statement is false?	gamma(2)=gan	gamma(1)= 1
gamma(1/4). gamma(3/4)=_____	2pi	pi√2
The values of gamma(4)=______	1!	2!
If C ′ is the evolute of the curve C then C is called the______ of the curve C ′	involute	curvature
_______ of a curve is the envelope of the normals of that curve.	involute	curvature
The parametric coordinates of the parabola x^2=4ay are_____.	(x=at^2 y=2at)	(x=at y=at)
The parametric coordinates of the ellipse is given by _________.	(x=acos theta y=bsin theta)	(x=asin theta y=bcos theta)
The parametric coordinates of the hyperbola is given by _________.	(x=acos theta y=bsin theta)	(x=asin theta y=bcos theta)
The parametric coordinates of the parabola y^2=4ax are_____.	(x=at^2 y=2at)	(x=at y=at)
The locus of the centre of curvature for a curve is called its evolute and the curve is called an __________ of its evolute.	involute	evolute
The locus of the centre of curvature for a curve is called its______.	involute	evolute
If y=1/x, then y1=________	-1/x^2	1/x
If y=x^2, then y1=________	x^2	1/x
If y=x^2, then y2=________	x^2	1/x
If x=2at then dx/dt=	2at	2a
If x=at^2 then dx/dt=	2at	2a
If y=ax^2+2ax then dy/dx at (3,2) is ______	8a	4ax
If y=ax^2+2ax then dy/dx at (2,2) is ______	8a	4ax
If y=ax^2+2ax then dy/dx is ______	8ax+2a	4ax+2
If y=ax^2+2ax then second derivative is ______	2a	4ax
The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=a^2 about its diameter is..........	(4/3) pi a^3	(2/3) pi a^3
The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=2^2 about its diameter is..........	(32/3)pi	(1/3)pi
The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=3^2 about its diameter is..........	16 pi	9 pi
The Volume of a sphere of radius 'a' is.................	2/3 8 pi a^3	4/3 pi a^3
The surface are of the sphere of radius 'a' is.................	4 pi a^2	pi a^2
The Volume of a sphere of radius '2' is.................	16/3 pi	32/3 pi
The surface area of the sphere of radius '3' is.................	36 pi	9 pi
int dx=.............	x+C	1

opt3	opt4	opt5	opt6	Answer	
2	3			1	
gamma (n-1)	n gamma (n)			n gamma (n)	
1	root(pi)			root(pi)	
gamma (1/2)=	gamma(1/2)= 0			gamma(2)=gamma(1)	
gamma(1/2)=	gamma (n+1)=n+1			gamma (n+1)=n+1	
√(2pi)	1			pi√2	
3!	4!			3!	
radius of curvature	centre of curvature			involute	
radius of curvature	evolute			evolute	
(x=2at y=at^2)	(x=a y=t)			(x=2at y=at^2)	
(x=atan theta y=bsec theta)	(x=asec theta y=btan theta)			(x=acos theta y=bsin theta)	
(x=atan theta y=bsec theta)	(x=asec theta y=btan theta)			(x=asec theta y=btan theta)	
(x=2at y=at^2)	(x=a y=t)			(x=at^2 y=2at)	
envelope	curvature			involute	
envelope	curvature			evolute	
ax	bx			-1/x^2	
2x	x			2x	
2x	2			2	
2t	0			2a	
2t	0			2at	
2ax	6a			8a	
2ax	6a			6a	
2ax+2a	6a			2ax+2a	
6ax	6a			2a	
(1/3) pi a^3	pi a^3			(4/3) pi a^3	
(2/3)pi	pi			(32/3)pi	
36 pi	pi			36 pi	
1/3 pi a^3	pi a^3			4/3 pi a^3	
3 pi a^2	2 pi a^2			4 pi a^2	
8/3 pi	8 pi			32/3 pi	
27 pi	18 pi			36 pi	
0	x^2			x+C	

int cdx=…………….	cx+C	0
int 5dx=.............	x+C	5x+C
int x^n dx=.............	x^(n+1)/ (n+1)-	x^(n-1)/ (n-1)+
int xdx=…..	x^2+C	x^2/2+C
int x^ (2) dx=.............	(x^(2)/2)+C	(x^(3)/3)+C
int 3x^(2) dx=.............	3x^(2)+C	x+C
int (1/x) dx=.............	1+ C	log x+C
int e^(x) dx=.............	(-e^x)+ C	e^(-x) + C
int e^(-x) dx=.............	(-e^x)+ C	e^(-x) + C
int e^(2x) dx=.............	(-e^2x)/2+ C	e^(-2x)/2 + C
int e^(-2x) dx=.............	(-e^(-2x))/2+ C	e^(-2x)/2 + C
int cosx dx=.............	sinx + C	cosx + C
int sinx dx=.............	sinx + C	cosx + C
int cosmx dx=.............	(sinmx)/m + C	(cosmx)/m + C

1	x+C			cx+C	
x^2+C	5+C			5x+C	
nx^ (n-1)+ C	(n+1) x^ (n+1)+ C			x^(n+1)/ (n+1)+ C	
x^3/2+C	x^2/2+C			x^2/2+C	
x+C	2x+C			(x^(3)/3)+C	
x^2+C	x^(3) +C			x^(3) +C	
(-1)+C	(-log x)+ C			log x+C	
(-e^(-x))+C	e^x + C			e^x + C	
(-e^(-x))+C	e^x + C			(-e^(-x))+C	
(-e^(-2x))/2+(	e^2x/2+ C			e^2x/2 + C	
(-e^(-2x))/2+(	e^(-2x)/2+ C			e^2x/2 + C	
(-cosx)+C	(-sinx)+C			sinx + C	
(-cosx)+C	(-sinx)+C			(-cosx)+C	
(-cosmx)/m+(	(-sinmx)/m+C			(sinmx)/m+ C	

Questions	opt1	opt2	opt3
The sum of the main diagonal elements of a matrix is called----	trace of a matrix	quadratic form	eigen value
Every square matrix satisfies its own ----------	characteristic polynomial	characteristic equation	orthogonal transformation
The orthogonal transformation used to diagonalise the symmetric matrix A is----	NT AN	NT A	NAN-1
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then kλ1 , kλ2, kλ3,……… ,kλ n are the eigen values of --------------	kA	kA2	kA-1
Diagonalisation of a matrix by orthogonal reduction is true only for a ----- matrix.	diagonal	triangula	real symmetric
In a modal matrix, the columns are the -----------	eigen vectors of A	eigen vectors of adj A	eigen vectors of inverse ofA
If atleast one of the eigen values of A is zero, then det A = -----	0	1	10
If the canonical form of a quadratic form is 5y12 - 6 y22 , then the index is --------	4	0	2
det (A- λI) represents------	characteristic polynomial	characteristic equation	quadratic form
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then 1/λ1 , 1/λ2, 1/λ3,……… ,1/λ n are the eigen values of --------------	A^(-1)	A	A^n
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then λ1p , λ 2 p , ………. λ n p are the eigen values of	A^(-1)	A^2	A^(-p)
Cayley -Hamilton theorem is used to find ------------	inverse and higher powers of A	eigen va	eigen vectors
The eigen values of a --------------------- matrix are its diagonal	diagonal	symmet	skew-ma
In an orthogonal transformation NT AN = D , D refers to a ---------- matrix.	diagonal	orthogoı	symmetı
In a modal matrix, the columns are the eigen vectors of----------	A-1	A2	A
If the eigen values of 8x12 + 7 x22 +3 x32 –12 x1 x2 – 8 x2 x3 +4 x3x1 are 0,3 & 15, then its nature is----------	positive definite	positive semidefinite	indefinit

opt4	opt5	opt6	Answer
canonical form			trace of a matrix
canonical form			characteristic equation
NA			NT AN
A-1			kA
scalar			real symmetric
eigen values of A			eigen vectors of A
5			0
1			1
canonical form			characteristic polynomial
A^p			A^(-1)
A^p			A^p
quadratic form			inverse and higher powers of A
triangular			triangula
skew-symmetric			diagonal
adj A			A
negative definite			positive semidefinite

The elements of the matrix of the quadratic form x12 + 3 x22 + 4 x1 x2 are ------------	a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3	a11 = -1, a12 = -2 , a 21 = 2 , a 22 = 3	a11 = 1, a12 = 4 , a 21 = 4 , a 22 = 3
If the sum of two eigen values and trace of a 3x3 matrix A are equal, then det A = ---------	λ1 λ2 λ3	0	1
If 1,5 are the eigen values of a matrix A, then det A = -------	5	0	25
If the canonical form of a quadratic form is 5y12 + 6 y22 , then the rank is --------	4	0	2
The non –singular linear transformation used to transform the quadratic form to canonical form is ----------	X= NTY	X= NY	Y= NX
The eigen vector is also known as-------	latent value	latent vector	column value
If 1,3,7 are the eigen values of A, then the eigen values of 2A are ------------	1,3,7	1,9,21	2,6,14
If the eigen values of 2A are 2, 6, 8 then eigen values of A are _	1,3,4	2,6,8	1,9,16
The number of positive terms in the canonical form is called th	rank	index	Signatur
If all the eigenvalues of A are positive then it is called as______	Positive definite	Negative definite	Positive semidefinite
If all the eigenvalues of A are negative then it is called as_____	Positive definite	Negative definite	Positive semidefinite
A homogeneous polynomial of the second degree in any number of variables is called the ______	characteristic polynomial	characteristic equation	quadratic form
The Set of all eigen values of the matrix A is called the ____________ of A	rank	index	Signatur
A Square matrix A and its transpose have _______ eigen value	different	Same	Inverse
The sum of the __________ of a matrix A is equal to the sum of the principal diagonal elements of A.	characteristic polynomial	characteristic equation	eigen values
The product of the eigenvalues of a matrix A is equal to______	Sum of main diagonal	Determinant of A	Sum of minors of Main diagonal
The eigenvectors of a real symmetric are _______	equal	unequal	real
When the quadratic form is reduced to the canonical form, it will contain only r terms, if the _____ of A is r.	rank	index	Signatu

a11 = 1, a12 = 4 , a 21 = 3 , a 22 = 1			a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3
2			0
6			5
1			2
NXA			X= NY
orthogonal value			latent vector
1,9,49			2,6,14
12,4,3			1,3,4
indefinite			index
Negative semidefinite			Positive definite
Negative semidefinite			Negative definite
canonical form			quadratic form
spectrum			spectrun
Transpose			Same
eigen vectors			eigen values
Sum of the cofactors of A			Determinant of A
symmetric			real
spectrum			rank

The excess of the number of positive terms over the number of negative terms in the canonical form is called the ___________ of the quadratic form.	rank	index	Signatu
If all the eigen values of A are less than zero and atleast one eigen value is zero then the quadratic form is said to be ___________	Positive definite	Negative	Positive
If all the eigen values of A are greater than zero and atleast one eigen value is zero then the quadratic form is said to be ___________	Positive definite	Negati ve definite	Positiv e semide finite
If the quadratic form has both positive and negative terms then it is said to be ___________	Positive definite	Negati ve definite	Positiv e semide finite

spectrum			Signatu
Negative semidefinite			Negative semidefinite
Negative semidefinite			Positive semidefinite
indefinite			indefini

KARPAGAM ACADEMY OF HIGHER EDUCATION

DEPARTMENT OF SCIENCE AND HUMANITIES

I - B.E. COMPUTER SCIENCE AND ENGINEERING

MATHEMATICS - I (Calculus and Linear Algebra) - 18BECS101

QUESTION BANK

UNIT-V (CALCULUS)

Part - C

1) (i) Obtain the Taylor's series expansion for $f(x) = \cos x$ at $x = \frac{\pi}{2}$

Sol.:

Given that

$f(x) = \cos x$	$f(\pi/2) = \cos \pi/2 = 0$
$f'(x) = -\sin x$	$f'(\pi/2) - \sin \pi/2 = -1$
$f''(x) = -\cos x$	$f''(\pi/2) - \cos \pi/2 = 0$
$f'''(x) = \sin x$	$f'''(\pi/2) = \sin \pi/2 = 1$
$f^{iv}(x) = \cos x$	$f^{iv}(\pi/2) = \cos \pi/2 = 0$

The Taylor Series is,

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

$$f(x) = \cos x = f(\pi/2) + \frac{f'(\pi/2)}{1!}(x-\pi/2) + \frac{f''(\pi/2)}{2!}(x-\pi/2)^2 + \dots$$

$$= 0 + \frac{(-1)}{1!}(x-\pi/2) + 0 + \frac{1}{3!}(x-\pi/3)^3 + 0 + \dots$$

$$f(x) = \cos x = -(x-\pi/2) + \frac{1}{3!}(x-\pi/2)^3 + \dots$$

(ii) Obtain the Taylor series Expansion for $f(x) = \sin x$ about $x = \pi/2$

<u>Sol.</u>:

Given that $f(x) = \sin x$ | $f(\pi/2) = \sin \pi/2 = 1$

$f'(x) = \cos x$ | $f'(\pi/2) = \cos \pi/2 = 0$

$f''(x) = -\sin x$ | $f''(\pi/2) = -\sin \pi/2 = -1$

$f'''(x) = -\cos x$ | $f'''(\pi/2) = -\cos \pi/2 = 0$

$f^{IV}(x) = \sin x$ | $f^{IV}(\pi/2) = \sin \pi/2 = 1$

The Taylor series is,

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

$$f(x) = \sin x = f(\pi/2) + \frac{f'(\pi/2)}{1!}(x-\pi/2) + \frac{f''(\pi/2)}{2!}(x-\pi/2)^2 + \dots$$

$$= 1 + 0 + \frac{(-1)}{2!}(x-\pi/2)^2 + 0 + \frac{1}{4!}(x-\pi/2)^4 + \dots$$

$$= 1 - \frac{1}{2!}(x-\pi/2)^2 + \frac{1}{4!}(x-\pi/2)^4 + \dots$$

2.) (i) Obtain the Maclaurin's series expansion for $f(x) = \tan^{-1} x$

Sol. :

Given that : $f(x) = \tan^{-1} x$

$f(0) = \tan^{-1} 0 = 0$

$f'(x) = \sec^2 x$

$= 1 + \tan^2 x$

$= 1 + [f(x)]^2$

$f'(0) = 1 + 0^2 = 1$

$f''(x) = 2 f(x) . f'(x)$

$f''(0) = 2[f(0) . f'(0)] = 2(0.1) = 0$

$f'''(x) = 2[f(x) . f''(x) + f'(x) . f'(x)]$

$f'''(0) = 2[0 . 0 + 1 . 1] = 2$

$f^{IV}(x) = 2[f(x) . f'''(x) + f'(x) f''(x) + 2 f'(x) . f''(x)]$

$f^{IV}(0) = 2[0 . 2 + 0 . 1 + 2 . 1 . 0]$

$= 0$

The Maclaurin's series is

$$f(x) = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \dots\dots$$

$f(x) = \tan^{-1} x \quad = 0 + \frac{1}{1!} x + 0 + \frac{2}{3!} x^3 + 0 + \dots\dots$

$f(x) = \tan x \quad = x + \frac{2}{3!} x^3 + \dots\dots$ //

(2)

(ii) Obtain the Maclaurin's series expansion for $f(x) = \tan^{-1}x$

$f(x) = \tan^{-1}x$	$f(0) = \tan^{-1}(0) = 0$
$f'(x) = \frac{1}{1+x^2} + 1 - x^2 + x^4 - x^6 + \dots$	$f'(0) = 1$
$f''(x) = -2x + 4x^3 - 6x^5 + \dots$	$f''(0) = 0$
$f'''(x) = -2 + 12x^2 - 30x^4 + \dots$	$f'''(0) = -2$
$f^{IV}(x) = 24x - 120x^3 + \dots$	$f^{IV}(0) = 0$
$f^{V}(x) = 24 - 360x^2 + \dots$	$f^{V}(0) = 24$

The Maclaurin's series is,

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots$$

$$f(x) = \tan^{-1}x = 0 + \frac{1}{1!}x + 0 - \frac{2}{3!}x^3 + 0 + \frac{24}{5!}x^5 + \dots$$

$$f(x) = \tan^{-1}x = x - \frac{2}{3!}x^3 + \frac{24}{120}x^5 + \dots \quad //$$

3(i) Find the absolute maximum and absolute minimum values of

$f(x) = x^3 - 3x^2 + 1 \quad , \quad -1/2 \le x \le 4.$

Sol.:

Step ①:

$f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

To find critical numbers,

$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0$

$3x = 0 \quad (or) \quad x - 2 = 0$

$\boxed{x = 0} \quad (or) \quad \boxed{x = 2}$

Step ②:

Put $x = 0$,

$f(0) = 0 - 0 + 1$

$f(0) = 1$

Put $x = 2$,

$f(2) = 2^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$

Put $x = -1/2$,

$f(-1/2) = (-1/2)^3 - 3(-1/2)^2 + 1 = -1/8 - \frac{3}{4} + 1$

$f(-1/2) = \frac{-1 - 6 + 8}{8} = 1/8$

Put $x = 4$

$f(4) = 4^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 17$

Step ③:

The absolute max. is $f(4) = 17$

The absolute min. is $f(2) = -3$

(ii) Find the absolute maximum and minimum values of

$f(x) = \cancel{x^4 - 3x^3 + 8}\ x^3 - 12x + 1 \quad , [-3, 5]$

Sol.:

Step ①: $f(x) = x^3 - 12x + 1$

$f'(x) = 3x^2 - 12$

To find critical numbers

$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0 \Rightarrow 3(x^2 - 4) = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow x^2 = 4$

$x = \sqrt{4} \Rightarrow \boxed{x = +2 \quad -2}$

Step ②:

Put $x = 2$

$f(2) = 2^3 - 12(2) + 1 = 8 - 24 + 1 = -15$

Put $x = -2$

$f(-2) = (-2)^3 - 12(-2) + 1 = -8 + 24 + 1 = 17$

Put $x = -3$

$f(-3) = (-3)^3 - 12(-3) + 1 = -27 + 36 + 1 = 10$

Put $x = 5$

$f(5) = 5^3 - 12(5) + 1 = 125 - 60 + 1 = 66$

Step ③:

The absolute max. is $f(5) = 66$

The absolute min. is $f(2) = -15$ //

(4)
Find the local maximum and local minimum values of

(i) $f(x) = x^4 - 3x^3 + 3x^2 - x$

SOLUTION:-

$$f(x) = x^4 - 3x^3 + 3x^2 - x$$

$$f'(x) = 4x^3 - 9x^2 + 6x - 1$$

To find critical number :-

$$f'(x) = 0$$

$$4x^2 - 9x^2 + 6x - 1 = 0$$

$$x = 1, 1, 1/4$$

1	4	-9	6	-1
	↓	4	-5	1
	4	-5	1	0
1	↓	4	-1	
	4	-1	0	
1/4	↓	1		
	4	0		

Put $x = 1$;

$$f(1) = (1)^4 - 3(1)^3 + 3(1)^2 - 1$$

$$= 1 - 3 + 3 - 1$$

$$= 0$$

Put $x = 1/4$;

$$f(1/4) = \left(\frac{1}{4}\right)^4 - 3(1/4)^3 + 3(1/4)^2 - 1/4$$

$$= \frac{1}{256} - 3/64 + 3/16 - 1/4$$

$$= \frac{1 - 12 + 48 - 64}{256} = \frac{-27}{256}$$

The Stationary points $(1, 0)$ $\left(1/4, \frac{-27}{256}\right)$

$$f''(x) = 12x^2 - 18x + 6$$

Put $f''(1) = 12(1) - 18(1) + 6$

$$= 12 - 18 + 6 = 0$$

$(1, 0)$ is can't be Extermum point

Put $x = 1/4$

$$f''(1/4) = 12\left(\frac{1}{4}\right)^3 - 18\left(1/4\right) + 6$$

$$= \frac{12}{16} - \frac{18}{4} + 6$$

$$= \frac{3}{4} - \frac{9}{2} + 6$$

$$= \frac{3-18}{4} + 6$$

$$= \frac{-15}{4} + 6$$

$$= \frac{-15+24}{4} = \frac{9}{4} \ (+ive)$$

$\left((1/4), (-27/256)\right)$ is the local minimum.

11)

Find the Local Maximum and local minimum values of

$f(x) = 2x^3 + 5x^2 - 4x$

SOLUTION:-

$f'(x) = 6x^2 + 10x - 4$

$= 3x^2 + 5x - 2$

$f'(x) = 0$

$3x^2 + 5x - 2 = 0$

$(3x-1)(x+2) = 0$

$x = -2 \mid x = 1/3$

Put $x = -2$

$f(-2) = 2(-2)^3 + 5(-2)^2 - 4(-2)$

$= -16 + 20 + 8$

$f(-2) = 12$ //

Put $x = 1/3$

$f(1/3) = 2(1/3)^3 + 5(1/3)^2 - 4(1/3)$

$= \frac{2}{27} + 5/9 - 4/3 = \frac{2+15-36}{27} = \frac{-19}{27}$ //

The Stationary point $(-2, 12)$ $(1/3, -19/27)$

$f''(x) = 12x + 10$

Put $f''(-2) = 12(-2) + 10 = -24 + 10 = -14$ // (-ive)

Put $f(1/3) = \cancel{12}^{4}(1/\cancel{3}_{1}) + 10 = 4 + 10 = 14$ (+ive)

Positive $(1/3, -19/27)$ is local minimum

Negative $(-2, 12)$ is local Maximum //

5) Evaluate $\lim_{x \to \frac{\pi}{2}} (\tan x)^{\cos x}$ by using l'hospital's rule

SOLUTION:-

Let $y = (\tan x)^{\cos x}$ $(\infty^0 \text{ form})$

TAKE log on both sides

$$\log y = \log \tan x^{\cos x}$$

$$\log y = \cos x \log \tan x \qquad (\because \log a^x = x \log a)$$

$$\lim_{x \to \pi/2} \log y = \lim_{x \to \pi/2} \cos x \log \tan x.$$

By using l'hopital's rule:

$$\lim_{x \to \pi/2} \log y = \lim_{x \to \pi/2} \frac{\log \tan x}{\sec x} \qquad \left(\because \cos x = \frac{1}{\sec x}\right)$$

$$\lim_{x \to \pi/2} \log y = \lim_{x \to \pi/2} \frac{1}{\tan x} \sec^2 x . \frac{1}{\sec x \tan x}.$$

$$\left[\begin{array}{l} \because \log \tan x = \frac{1}{\tan x} \sec^2 x \\ \sec x = \sec x \tan x \\ \tan x = \sec^2 x \\ \tan^2 x = \frac{\sin^2 x}{\cos^2 x} \\ \sec x = \frac{1}{\cos x} \end{array}\right]$$

$$\lim_{x \to \pi/2} \log y = \lim_{x \to \pi/2} \frac{\sec x}{\tan^2 x}$$

$$= \lim_{x \to \pi/2} \frac{1}{\cancel{\cos x}} \frac{\cos^{\cancel{2}} x}{\sin^2 x}$$

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6

$$\lim_{x \to \pi/2} \log y = \lim_{x \to \pi/2} \frac{\cos x}{\sin^2 x}$$

$$= \lim_{x \to \pi/2} \frac{\cos x}{\sin^2 x}$$

$$= \frac{\cos \pi/2}{(\sin \pi/2)^2} \qquad \left(\because \begin{array}{l} \cos 90^\circ = 0 \\ \sin 90 = 1 \end{array} \right)$$

$$= \frac{0}{1^2} = \frac{0}{1} = 0$$

$$\lim_{x \to \pi/2} \log y = 0$$

By composite function:

$$\lim_{x \to \pi/2} \log y = 0$$

$$\log \lim_{x \to \pi/2} y = 0$$

$$\lim_{x \to \pi/2} y = e^0 \qquad \left(\because e^0 = 1 \right)$$

$$\lim_{x \to \pi/2} (\tan x)^{\cos x} = 1$$

(6) Evaluate $\lim_{x \to 0} (\cos x)^{1/x}$ by using l'hospital's rule

Solution;-

Let $y = (\cos x)^{1/x}$

Taking log on both sides

$$\log y = \log \left[(\cos x)^{1/x}\right]$$

$$\log y = \frac{1}{x} \log(\cos x)$$ $(\because \log a^x = x.\log a)$

$$\lim_{x \to 0} \log y = \lim_{x \to 0} \cdot \frac{1}{x} \log(\cos x)$$

By using l'hopital rule

$$\lim_{x \to 0} \log y = \lim_{x \to 0} \frac{1}{1} \frac{-\sin x}{\cos x}$$

$$\lim_{x \to 0} \log y = \lim_{x \to 0} \frac{-\sin x}{\cos x}$$

$$\lim_{x \to 0} \log y = \lim_{x \to 0} -\tan x$$

$$\lim_{x \to 0} \log y = -\tan 0 = 0 //.$$

$\because \log x = \frac{1}{x} \cdot 1$
$\cos x = -\sin x$
$\log \cos x = \frac{1}{\cos x} -\sin x$

By composite function

$$\lim_{x \to 0} \log y = 0$$

$$\log \lim_{x \to 0} y = 0$$

$$\lim_{x \to 0} y = e^0$$

$$x \to 0 (\cos x)^{1/x} = 1 //.$$

$\boxed{\because e^0 = 1}$

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7) Evaluate $\lim_{x \to 0} (\cos x)^{\sin x}$ by using l'hopital rule

SOLUTION:- (1^0 form)

Let $y = (\cos x)^{\sin x}$

Take log on both sides

$\log y = \log (\cos x)^{\sin x}$

$\log y = \sin x \log (\cos x)$

$\left[\because \log a^x = x \log a \right.$
$\left. \log(\cos x)^{\sin x} = \sin x \log \cos x \right]$

$\lim_{x \to 0} \log y = \lim_{x \to 0} \sin x \log (\cos x)$

By using l' hopital rule.

$\lim_{x \to 0} \log y = \lim_{x \to 0} \cos x \frac{1}{\cos x} - \sin x$

$\left[\because \sin x = \cos x\right.$
$\left.\log(\cos x) = \frac{1}{\cos x} - \sin x \right]$

$\lim_{x \to 0} \log y = \lim_{x \to 0} -\sin x$

$\lim_{x \to 0} \log y = -\sin 0 = 0$ //

By COMPOSITE FUNCTION:-

$\lim_{x \to 0} \log y = 0$

$\log \lim_{x \to 0} y = 0$

$(\because e^0 = 1)$

$\lim_{x \to 0} y = e^0$

$\lim_{x \to 0} (\cos x)^{\sin x} = 1$ //

8) Evaluate $\lim_{x \to 0+} x^{\sin x}$ by using l'hopital's rule

SOLUTION:-

(0^0 form)

Let $y = x^{\sin x}$

Taking log on both sides

$$\log y = \log x^{\sin x} \qquad \left(\because \log a^x = x \log a\right)$$

$$\log y = \sin x \log x$$

$$\lim_{x \to 0+} \log y = \lim_{x \to 0+} \sin x \log x$$

By l'hopital's rule

$$\lim_{x \to 0} \log y = \lim_{x \to 0_+} \frac{\log x}{\operatorname{cosec} x}$$

$$\lim_{x \to 0} \log y = \lim_{x \to 0_+} \frac{1}{x} \frac{1}{-\operatorname{cosec} x \cot x}$$

$\left(\because \sin x = \frac{1}{\operatorname{cosec} x}\right.$

$\log x = \frac{1}{x}$

$\left.\operatorname{cosec} x = -\operatorname{cosec} x \cot x\right)$

$$\lim_{x \to 0} \log y = \lim_{x \to 0+} \frac{1}{x} \frac{-\sin x}{1} \frac{\sin x}{\cos x}$$

$$= \lim_{x \to 0_+} \frac{1}{x} \frac{-\sin^2 x}{\cos x}$$

$$= \lim_{x \to 0_+} \frac{-2\sin x \cdot \cos x}{-x \sin x + \cos x \cdot 1}$$

$$= \frac{-2(0) \cdot 0}{0+0} = 0 //$$

$\left(\because \cot x = \frac{\cos x}{\sin x}\right.$

$\sin^2 x = 2\sin x \cdot \cos x$

$\left.uv = uv' + vu'\right)$

BY COMPOSITE FUNCTION:-

$$\lim_{x \to 0_+} \log y = 0$$

$$\lim_{x \to 0_+} \log y = 0$$

$$\log \lim_{x \to 0+} y = 0$$

$$\lim_{x \to 0} y = e^0 \qquad \therefore e^0 = 1$$

$$\lim_{x \to 0} x^{\sin x} = 1$$

Questions	opt1	opt2
The Taylor series of f(x) about the point 0 is ___________ series.	Maclaurins	Taylor
The expansion of f(x) by Taylor series is __________	zero	unique
The point at which function f(x) is either maximum or minimum is known as ______ point	Stationary	Saddle point
A function f has ___ at 'c' if f(c) ≥ f(x) for all 'x' in D, where D is domain of 'f'.	an absolute maximum	an absolute minimum
If f(x) = x2, then f(0) = 0 is the ___ value of f.	an absolute maximum	an absolute minimum
A function f has a ____ at 'c' if there is an open interval I containing 'c' such that f(c) ≥ f(x) for all 'x' in I.	an absolute maximum	an absolute minimum
A function f has a ____ at 'c' if there is an open interval I containing 'c' such that f(c) ≤ f(x) for all 'x' in I.	an absolute maximum	an absolute minimum
If 'f' has a ___ at 'c' and if f'(c) exists then f'(c)=0.	critical number	stationary point
A function 'f' has ___ at 'c' if f(c) ≤ f(x) for all 'x' in D, where D is domain of 'f'.	an absolute maximum	an absolute minimum
If 'f' has a local extremum at 'c' and if f'(c) exists then f'(c)=___.	0	1
Evaluate: limit x tends to 0 (x / tan x) =	1	2
Evaluate: limit x tends to infinity (x^2 / e^x) =	1	2
L'Hopital's rule can be applied only to differentiable functions for which the limis are in the _____ form	real	indeterminat e
L'Hopital's rule can be applied only to ___functions for which the limis are in the indeterminate form	differentiable	real
If f(x) = x^3, then the function has ___	eiher an absolute maximum or an absolute minimum	neiher an absolute maximum nor an absolute minimum
A ___ of a function f is a number c in the domain of f such that either f'(c) =0 or f'(c) does not exist	critical number	stationary point
___ are critical numbers c in he domain of f, for which f'(c)=0	Critical number	Stationary points
If f has a local extremum at c, then c is a ___ of f	critical number	stationary point
If f has a ____ at c, then c is a critical number of f	critical number	stationary point
If f(x)=x^2 - 4x+5 on [0,3] then the absolute maximum value is __	2	3
Find the critical numbers, for the function f(x)=x^3 - 3x^2 +1.	(1 2)	(0 2)
Find the critical numbers, for the function f(x)=x^3 - 3x +1.	(1 1)	(-1 1)
Find the critical number, for the function f(x)=2x - 3x^2 .	(1/2)	(1/3)
Find the critical number, for the function f(x)=x^2 - 2x +2.	0	1
Find the critical number, for the function f(x)=1-2x-x^2.	0	1
Find the critical numbers, for the function f(x)=x^3 - 12x +1.	(0 1)	(0 2)

opt3	opt4	opt5	opt6	Answer
power	binomial			Maclaurins
minimum	maximum			unique
extremum	implicit			Stationary
local maximam	locam minimum			an absolute maximum
local maximam	an absolute and local minimum			an absolute and local minimum
local maximam	locam minimum			local maximam
local maximam	local minimum			local minimum
local extremum	an absolute maximum			local extremum
local maximam	locam minimum			an absolute minimum
c	(-1)			1
3	0			1
3	0			0
complex	extremum			indeterminate
complex	extremum			differentiable
local maximam	locam minimum			neiher an absolute maximum nor an absolute minimum
local extremum	an absolute maximum			critical number
Local extremum	An absolute maximum			Stationary points
local extremum	an absolute maximum			critical number
local extremum	an absolute maximum			local extremum
4	5			5
(2 2)	(1 3)			(0 2)
(0 1)	(-1 -1)			(-1 1)
(1/4)	1			(1/3)
2	3			1
2	3			1
(0 3)	(0 4)			(0 4)

Find the stationary point of the function f(x)=2x - 3x^2	(1 1)	(1 2)
Find the stationary point of the function f(x)=x^3 - 3x +1	(1 -1) and (-1 3)	(1 -1)
Find the absolute maximum of the function f(x) = x^2-2x+2, [0,3]	1	3
Find the absolute minimum of the function f(x) = x^2-2x+2, [0,3]	1	3
Find the absolute maximum of the function f(x) = 1-2x-x^2 [-4,1]	1	2
Find the absolute minimum of the function f(x) = 1-2x-x^2 [-4,1]	1	2

(1/3 1/3)	(1/2 1)			(1/3 1/3)
(-1 3)	(1 1) and (1 3)			(1 -1) and (-1 3)
5	8			5
5	8			1
7	8			2
(-7)	(-8)			(-7)

Questions	opt1	opt2	opt3
The sum of the main diagonal elements of a matrix is called----	trace of a matrix	quadratic form	eigen value
Every square matrix satisfies its own ----------	characteristic polynomial	characteristic equation	orthogonal transformation
The orthogonal transformation used to diagonalise the symmetric matrix A is----	NT AN	NT A	NAN-1
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then kλ1 , kλ2, kλ3,……… ,kλ n are the eigen values of --------------	kA	kA2	kA-1
Diagonalisation of a matrix by orthogonal reduction is true only for a ----- matrix.	diagonal	triangula	real symmetric
In a modal matrix, the columns are the -----------	eigen vectors of A	eigen vectors of adj A	eigen vectors of inverse ofA
If atleast one of the eigen values of A is zero, then det A = -----	0	1	10
If the canonical form of a quadratic form is 5y12 - 6 y22 , then the index is --------	4	0	2
det (A- λI) represents------	characteristic polynomial	characteristic equation	quadratic form
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then 1/λ1 , 1/λ2, 1/λ3,……… ,1/λ n are the eigen values of --------------	A^(-1)	A	A^n
If λ1, λ2, λ3,……… λ n are the eigen values of A ,then λ1p , λ 2 p , ………. λ n p are the eigen values of	A^(-1)	A^2	A^(-p)
Cayley -Hamilton theorem is used to find ------------	inverse and higher powers of A	eigen va	eigen vectors
The eigen values of a --------------------- matrix are its diagonal	diagonal	symmet	skew-ma
In an orthogonal transformation NT AN = D , D refers to a ---------- matrix.	diagonal	orthogo	symmetı
In a modal matrix, the columns are the eigen vectors of----------	A-1	A2	A
If the eigen values of 8x12 + 7 x22 +3 x32 –12 x1 x2 – 8 x2 x3 +4 x3x1 are 0,3 & 15, then its nature is----------	positive definite	positive semidefinite	indefinit

opt4	opt5	opt6	Answer
canonical form			trace of a matrix
canonical form			characteristic equation
NA			NT AN
A-1			kA
scalar			real symmetric
eigen values of A			eigen vectors of A
5			0
1			1
canonical form			characteristic polynomial
A^p			A^(-1)
A^p			A^p
quadratic form			inverse and higher powers of A
triangular			triangula
skew-symmetric			diagonal
adj A			A
negative definite			positive semidefinite

The elements of the matrix of the quadratic form x12 + 3 x22 + 4 x1 x2 are ------------	a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3	a11 = -1, a12 = -2 , a 21 = 2 , a 22 = 3	a11 = 1, a12 = 4 , a 21 = 4 , a 22 = 3
If the sum of two eigen values and trace of a 3x3 matrix A are equal, then det A = ---------	λ1 λ2 λ3	0	1
If 1,5 are the eigen values of a matrix A, then det A = -------	5	0	25
If the canonical form of a quadratic form is 5y12 + 6 y22 , then the rank is --------	4	0	2
The non –singular linear transformation used to transform the quadratic form to canonical form is ----------	X= NTY	X= NY	Y= NX
The eigen vector is also known as-------	latent value	latent vector	column value
If 1,3,7 are the eigen values of A, then the eigen values of 2A are ------------	1,3,7	1,9,21	2,6,14
If the eigen values of 2A are 2, 6, 8 then eigen values of A are _	1,3,4	2,6,8	1,9,16
The number of positive terms in the canonical form is called th	rank	index	Signatur
If all the eigenvalues of A are positive then it is called as______	Positive definite	Negative definite	Positive semidefinite
If all the eigenvalues of A are negative then it is called as_____	Positive definite	Negative definite	Positive semidefinite
A homogeneous polynomial of the second degree in any number of variables is called the ______	characteristic polynomial	characteristic equation	quadratic form
The Set of all eigen values of the matrix A is called the ____________ of A	rank	index	Signatur
A Square matrix A and its transpose have ________ eigen value	different	Same	Inverse
The sum of the ___________ of a matrix A is equal to the sum of the principal diagonal elements of A.	characteristic polynomial	characteristic equation	eigen values
The product of the eigenvalues of a matrix A is equal to______	Sum of main diagonal	Determinant of A	Sum of minors of Main diagonal
The eigenvectors of a real symmetric are ________	equal	unequal	real
When the quadratic form is reduced to the canonical form, it will contain only r terms, if the _____ of A is r.	rank	index	Signatu

a11 = 1, a12 = 4 , a 21 = 3 , a 22 = 1			a11 = 1,a12 =2 , a 21 = 2 , a 22 = 3
2			0
6			5
1			2
NXA			X= NY
orthogonal value			latent vector
1,9,49			2,6,14
12,4,3			1,3,4
indefinite			index
Negative semidefinite			Positive definite
Negative semidefinite			Negative definite
canonical form			quadratic form
spectrum			spectrun
Transpose			Same
eigen vectors			eigen values
Sum of the cofactors of A			Determinant of A
symmetric			real
spectrum			rank

The excess of the number of positive terms over the number of negative terms in the canonical form is called the ___________ of the quadratic form.	rank	index	Signatu
If all the eigen values of A are less than zero and atleast one eigen value is zero then the quadratic form is said to be ___________	Positive definite	Negative	Positive
If all the eigen values of A are greater than zero and atleast one eigen value is zero then the quadratic form is said to be ___________	Positive definite	Negati ve definite	Positiv e semide finite
If the quadratic form has both positive and negative terms then it is said to be ___________	Positive definite	Negati ve definite	Positiv e semide finite

spectrum			Signatu
Negative semidefinite			Negative semidefinite
Negative semidefinite			Positive semidefinite
indefinite			indefini