18BEEE102 **Waves, Optics And Introduction To Quantum Mechanics (Theory & Lab.)** **7H-5C**

Instruction Hours/week: L:3 T:1 P:3 **Marks:** Internal:**40** External:**60** Total:**100**

End Semester Exam:3 Hours

(i) Theory

Course Objective:

The goal is to develop an awareness and understanding of wave motion; skills in the use of optical devices and also to understand the idea of wave function.

Course Outcomes

1. The students will study about the waves on strings and other transverse waves.
2. The students will have the knowledge on the optical applications
3. The students will understand the uncertainty relations as well as solve Schrodinger equation for simple potentials

Unit 1 - Waves

Mechanical and electrical simple harmonic oscillators, damped harmonic oscillator, impedance, steady state motion of damped Harmonic oscillator

Non-dispersive transverse and longitudinal waves:

Transverse wave on a string, the wave equation on a string, Harmonic waves, reflection and transmission of waves at a boundary, impedance matching, standing waves and their Eigen frequencies, longitudinal waves and the wave equation for them, acoustics waves.

Unit 2 - Wave Optics

Huygens' principle, superposition of waves and interference of light by wave front splitting and amplitude splitting; Young's double slit experiment, Newton's rings, Michelson interferometer, Farunhofer diffraction from a single slit and a circular aperture, the Rayleigh criterion for limit of resolution and its application to vision; Diffraction gratings and their resolving power

Unit 3 - Lasers

Einstein's theory of matter radiation interaction and A and B coefficients; amplification of light by population inversion, different types of lasers: gas lasers (He-Ne, CO_2), solid-state lasers (Neodymium), Properties of laser beams: mono-chromaticity, coherence, directionality and brightness-application of lasers in science, engineering and medicine.

Unit 4 - Introduction to Quantum Mechanics

Introduction to quantum theory – Black body radiation **-** dual nature of matter and radiation – de Broglie wavelength, uncertainty principle –Schrödinger's wave equation – time dependent and time independent equations – particle in one dimensional box- physical significance of wave function, scanning electron microscope.

Unit 5 - Introduction to Solids and Semiconductors

Free electron theory of metals, Fermi level, density of energy states, Bloch's theorem for particles in a periodic potential, Kronig-Penney model and origin of energy bands. Types of electronic materials: metals, semiconductors, and insulators**-** Intrinsic and extrinsic semiconductors(no need derivation).

SUGGESTED READINGS

1. H. J. Pain(2006), The physics of vibrations and waves**,** Wiley.
2. A.Ghatak(2012) , Optics, McGraw Hill Education.

3. M.N. Avadhanulu and PG Kshirsagar,(2011), A Text book of Engineering Physics, S.Chand and company, Ltd., New Delhi.
4. Ganesan.S and Baskar.T,(2015), Engineering Physics I, GEMS Publisher, Coimbatore-641001.
5. I.G. Main,(1993)**,** Vibrations and waves in physics**,** Cambridge University Press
6. Gaur, R.K. and Gupta,(2011), S.C, Engineering Physics, Dhanpat Rai Publications,New Delhi.
7. E. Hecht,(2008), Optics, Pearson Education.
8. D. J. Griffiths,(2014)**,** Quantum mechanics**,** Pearson Education.
9. D. A. Neamen,(1997), Semiconductor Physics and Devices, Times Mirror High Education Group.
10. B.G. Streetman,(1995), Solid State Electronic Devices, Prentice Hall of India.

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DEPARTMENT OF SCIENCE AND HUMANITIES

LECTURE PLAN

Subject : WAVES, OPTICS AND INTRODUCTION TO QUANTUM MECHANICS
Code : 18BEEE102

Unit No.	List of Topics	No. of Hours
	Waves	
	Introduction to waves	1
	Mechanical and electrical simple harmonic oscillators	1
	Damped harmonic oscillator, Impedance	1
	Steady state motion of damped Harmonic oscillator	1
	Transverse wave on a string, the wave equation on a string	1
	Tutorial-1	1
UNIT- I	Harmonic waves, Reflection and transmission of waves at a boundary	1
	Impedance matching	1
	Standing waves and their Eigen frequencies	1
	longitudinal waves and the wave equation for them	1
	Acoustics waves	1
	Tutorial-2	1
	Total	**12**
	Wave Optics	
	Introduction to wave optics, Huygens' principle	1
	Superposition of waves and interference of light by wave front splitting and amplitude splitting	1
	Young's double slit experiment	1
	Newton's rings	2
UNIT – II	Tutorial-1	
	Michelson interferometer	2
	Farunhofer diffraction from a single slit and a circular aperture	2
	The Rayleigh criterion for limit of resolution and its application to vision	1
	Diffraction gratings and their resolving power	1
	Tutorial-2	1
	Total	**12**
	Lasers	
	Introduction to Laser	1
	Einstein's theory of matter radiation interaction and A and B coefficients	1
	Amplification of light by population inversion	1
	Different types of lasers: gas lasers (He-Ne)	2
UNIT – III	Gas lasers –(CO_2)	2
	Tutorial-1	1
	Solid-state lasers (Neodymium),	1
	Properties of laser beams: mono-chromaticity, coherence, directionality and brightness	1
	Application of lasers in science, engineering and medicine.	1
	Tutorial-2	1

	Total	**12**
UNIT – IV	**Introduction to Quantum Mechanics**	
	Introduction to quantum theory	1
	Black body radiation	1
	Dual nature of matter and radiation	1
	de Broglie wavelength	1
	Uncertainty principle	1
	Tutorial-1	1
	Schrödinger's wave equation – time dependent equation	1
	Time independent equations	1
	Particle in one dimensional box	1
	Physical significance of wave function	1
	Scanning electron microscope	1
	Tutorial-2	1
	Total	**12**
UNIT – V	**Introduction to Solids and Semiconductors**	
	Introduction to solids, Free electron theory of metals	1
	Fermi level	1
	Density of energy states	2
	Bloch's theorem for particles in a periodic potential	1
	Tutorial-1	1
	Kronig-Penney model and origin of energy bands	2
	Types of electronic materials: metals, semiconductors, and insulators	2
	Intrinsic and extrinsic semiconductors (no need derivation).	1
	Tutorial	1
	Total	**12**
	TOTAL NO OF HOURS	**60**

TEXT BOOK& REFERENCES:

S.NO	AUTHOR(S) NAME	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1	H. J. Pain,	The physics of vibrations and waves	Wiley	2006
2	A. Ghatak	Optics	McGraw Hill Education,	2012
3	M.N. Avadhanulu and PG Kshirsagar	A Text book of Engineering Physics	S.Chand and company, Ltd., New Delhi	2011
4	Ganesan.S and Baskar.T	Engineering Physics I	GEMS Publisher, Coimbatore-641 001	2015
5	I. G. Main	Vibrations and waves in physics	Cambridge University Press	1993
6	Gaur, R.K. and Gupta, S.C	Engineering Physics	Dhanpat Rai Publications,New	2011
7	E. Hecht	Optics	Pearson Education	2008
8	D. J. Griffiths	Quantum mechanics	Pearson Education	2014
9	D. A. Neamen	Semiconductor Physics and Devices	Times Mirror High Education Group	1997
10	B.G. Streetman	Solid State Electronic Devices	Prentice Hall of India	1995

WEBSITES:

1. https://www.youtube.com/watch?v=x1-SibwIPM4
2. https://www.youtube.com/watch?v=TcmGYe39XG0
3. www. nptel.ac.in

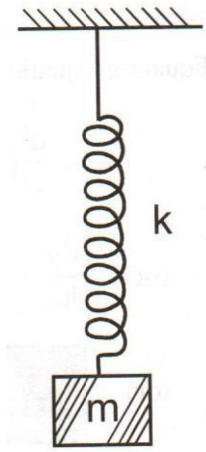
STAFF IN-CHARGE **HOD**

UNIT-1

Waves

DIFFERENTIAL EQUATION AND ITS SOLUTION FOR DAMPED OSCILLATOR

Let us consider a spring in a which a body of mass ‘m’ is suspended as shown in figure.



Fig

Here, two types of forces act on it,

1. A restoring force that act on the opposite direction to the displacement in order to restore its original position.
   i.e. $F_1 \alpha - y$

   (or) . $F_1 = -ky$ ---- (1)

Where k is the force constant and

Y is the displacement

Here, the negative sign indicates that the restoring force is acting in the opposite direction.

2. A frictional force (or) damping force due to air resistance, which also acts opposite to the direction of motion.

Damping force $F_2 = -r\frac{dy}{dt}$ -------- (2)

Where, r is the frictional force and

$\frac{dy}{dt}$ is the velocity.

Here negative sign indicates that the damping force act along the opposite direction.

The total force (F) in the opposite direction is

$$F = F_1+F_2$$

$$F = -ky-r\frac{dy}{dt} \quad \text{--------------- (3)}$$

We know Force = mass x acceleration

$$F = m\frac{d^2y}{dt^2} = -ky-r\frac{dy}{dt}$$

(or)

$$\frac{d^2y}{dt^2} = \frac{-ky}{m} - \frac{r}{m}\frac{dy}{dt}$$

(or)

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{ky}{m} = 0 \quad \text{---------------(5)}$$

$let\ \frac{r}{m} = 2b\ \&\ \frac{K}{M} = w^2$

we can write equation (5)*as*

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + w^2y = 0 \quad \text{---------- (6)}$$

equation (6)is the second order differential equation, for which the solution can be written as

$$y = Ae^{\alpha t} \quad \text{--------- (7)}$$

where

A and α are arbitrary constants

Differentiating equation (7) twice with respect to time we get

$$\frac{dy}{dt} = Ae^{\alpha t}\alpha \quad \text{---- (8)}$$

$\frac{d^2y}{dt^2} = Ae^{\alpha t}\alpha^2$ ------ (9)

substituting the values of (7),(8) and (9) in (6) we get

$Ae^{\alpha t}\alpha^2 + 2b\, Ae^{\alpha t}\alpha + w^2 Ae^{\alpha t} = 0$

$Ae^{\alpha t}\,(\alpha^2 + 2b\,\alpha + w^2) = 0$

$Here, Ae^{\alpha t} \neq 0$

$\alpha^2 + 2b\,\alpha + w^2 = 0$

solving this we get $\alpha = -b \pm \sqrt{b^2 - w^2}$ ------------ (10)

substituting the value of α *in equation* (7)*we get*

$y = A \exp[-b \pm \sqrt{b^2 - w^2}]t$

$y = A_1 \exp[-b + \sqrt{b^2 - w^2}]t + A_2 \exp[-b - \sqrt{b^2 - w^2}]t$ ---- (11)

where $A_1$and A_2 are arbitrary constants.

Equation (11) represents the general solution, which is based on the value of 'b' and 'w'.

SPECIAL CASES

Case i. When $b^2 > w^2$

whenb^2*is greater than* w^2, then $\sqrt{b^2 - w^2}$ will be real and less than b. Therefore the terms $-b + \sqrt{b^2 - w^2}$ & $-b - \sqrt{b^2 - w^2}$ becomes negative. Thus the displacement decreases drastically, without performing any oscillatory motion.

This type of motion is called over-damped oscillation (or) dead beat as shown in figure.

Ex. 1. Pendulum moving in a very thick coil media.

2. Dead beat moving coil galvanometer.

Case ii. When $\sqrt{b^2 = w^2}$

When b^2*is* equal to w^2*, then* $\sqrt{b^2 - w^2}$ is not exactly zero, but has a very small value say 'β'

We can write equation (11) as

$y = A_1 \exp[-b + \beta]t + A_2 \exp[-b - \beta]t$

(or) $y = A_1[e^{-bt}e^{\beta t}] + A_2[e^{-bt}e^{-\beta t}]$

(or) $y = e^{-bt}[A_1 e^{\beta t} + A_2 e^{-\beta t}]$

(or) y = e^{-bt}[(A_1+ A_2)+βt(A_1- A_2)+---]

Let A_1+ A_2 = p and β(A_1- A_2) = q

y = $e^{-bt}[p + qt]$ --------------- (12)

from equation (12) we can see that, when ‘t’ increases (p+qt) increases. Whereas e^{-bt} decreases. Therefore the displacement ‘y’ approaches zero more rapidly.

This type of motion is called critical damped motion (or) critical oscillation, as shown in figure.

Ex.

1. Movement of pointer in voltmeter, ammeter etc.,
2. Sensitive galvanometers.

Case iii. When $b^2 < w^2$

When $b^2 is\ less\ than\ w^2$, then $\sqrt{b^2 - w^2}$ becomes imaginery,

i.e. $\sqrt{b^2 - w^2} = i\sqrt{w^2 - b^2}$ = iγ

where, γ = $\sqrt{w^2 - b^2}$

Equation (11) becomes

y = A_1exp[-b+i$\gamma]t + A_2$exp[-b-i$\gamma]t$

y = $A_1(e^{-bt}e^{i\gamma t})$ + $A_2(e^{-bt}e^{-i\gamma t})$

y =$e^{-bt}(A_1e^{i\gamma t} + A_2e^{-i\gamma t})$

y = e^{-bt}[(A_1+A_2) cosγt + i(A_1 – A_2) sinγt]

let A_1+A_2 = a sin∅ and i(A_1 – A_2) = a cos∅

y = e^{-bt}[a sin∅ cosγt + a cos∅ sinγt]

y = e^{-bt} a sin(γt + ∅)

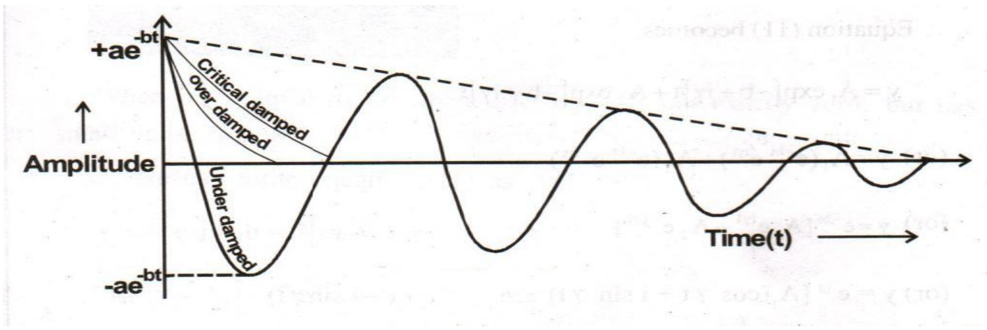
y = e^{-bt} a sin [($\sqrt{b^2 - w^2}$)t + ∅] --------------- (13)

from equation (13) we can see that the amplitude ae^{-bt} decreses continuously. Further , as the sin $[(\sqrt{b^2 - w^2})t + \emptyset]$ value can vary between +1 to -1, the amplitude also varies between + ae^{-bt}. here the decay of amplitude depends on the damping factor ‘b’.

This type of motion is called ‘ under damped motion’ as shown in figure.

Ex.

1. Motion of a simple pendulum in air
2. Motion of the coil in ballistic galvanometer.



Fig

2.5 DIFFERENTIAL EQUATION AND ITS SOLUTION FOR FORCED OSCILLATOR:

Let us consider a mass ‘m’ connected to a spring and an external force is applied as shown in figure.

Here three types of forces acxt on it , viz

1. restoring force $F_1 = -ky$
2. Damping force $F_2 = -r\frac{dy}{dt}$ and
3. External force $F_3 = F_0 \sin wt$

$\therefore$ Total force F= F_1+ F_2+ F_3

$\therefore$ F = -ky- r$\frac{dy}{dt}$ + F_0sinwt

We know force = mass x acceleration

F = m$\frac{d^2y}{dt^2}$

From equation (2) and (3) we get

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} + F_0 \sin wt$$

$$\frac{d^2y}{dt^2} = \frac{-k}{m}y - \frac{r}{m}\frac{dy}{dt} + \frac{\text{F0sinwt}}{m}$$

(or)

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{\text{F0sinwt}}{m} \text{ -------------------- } (3)$$

Let

$\frac{r}{m}$ = 2b: $\frac{k}{m} = w_0{}^2$ and $\frac{F0}{m}$ = f

$we\ can\ write\ equation\ (3) as$

$$\frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + w_0{}^2 y = \text{fsinwt --------- } (4)$$

$equation\ (4) is\ the\ second\ order\ differential\ equation, for\ which\ solution\ can\ b$

Y = A sin (wt-θ) ---------- (5)

Where A is the steady amplitude of vibration and

θ is the angle at which the displacement ‘y’ lag behind the applied force.

Differentiating equation (5) twice with respect to time , we get

$\frac{dy}{dt}$ = A cos (wt-θ) w ------- (6)

$\frac{d^2y}{dt^2}$ = - A sin (wt- θ) w^2 ------------ (7)

$substituting\ equation\ (5), (6) \&\ (7) in\ equation\ (4) we\ get$

-Asin(wt- θ) w^2+2b A cos (wt-θ) w+$w_0{}^2$ A sin (wt- θ) = fsin[(wt-θ)+θ] ------ (8)

Equation (8) holds good for all values of 't'. Therefore the coefficients of sin (wt- θ) and cos (wt- θ) must be equal on both the sides.

Therefore we can write

$A(w_0{}^2 - w^2) = f\cos\theta$ -------- (9)

Similarly we can write

$2bwA = f\sin\theta$ ------- (10)

Squaring and adding equation (9) and (10) we get

$A^2(w_0{}^2 - w^2)^2 + 4b^2w^2A^2 = f^2(\cos^2\theta + \sin^2\theta)$

(or)

$$A^2[(w_0{}^2 - w^2)^2 + 4b^2w^2] = f^2$$

Therefore $A = \frac{f}{\sqrt{(w_0{}^2 - w2)2 + 4b^2w^2}}$ -------- (11)

Dividing equation (10) by equation (9) we get

$\frac{2bwA}{A(w_0{}^2 - w2)} = \frac{f\sin\theta}{f\cos}$

(or)

$\mathrm{Tan}\theta = \frac{2bw}{(w_0{}^2 - w2)}$

(or)

$\theta = \tan^{-1}[\frac{2bw}{(w_0{}^2 - w2)}]$ -----------------(12)

from equations (11)*and* (12), *it is* clear that the amplitude and phase of the forced oscillation depend on $w_0{}^2 - w^2$, i.e., it depends on the driving frequency w and the natural frequency w_0 of the oscillator.

SPECIAL CASES

Case i. When $w_0^2 \gg w^2$

Amplitude

When the driving force w is very much less than the natural frequency w_0, then we can write equation (11) as

Amplitude $A = \frac{f}{\sqrt{(w_0{}^2 - w2)2 + 4b2w2}} \approx \frac{f}{w_0{}^2}$

Since f = $\frac{F0}{m}$, we can write

Amplitude A = $\frac{F0}{k}$ ---------------- (13)

Thus the amplitude depends on the force constant k of the spring and the magnitude of the applied force F_0.

Phase

When $w_0^2 \gg$ w then the equation (12) becomes

θ = tan^{-1}[$\frac{2bw}{(w_0{}^2-w2)}$]

since $w_0^2 \gg$ w; $\frac{2bw}{w_0{}^2}$ becomes zero [$\frac{1}{\infty}$ = 0]

∴ $\theta = 0$ ---------------- (14)

Therefore under this situation the displacement and the driving force will be in phase.

Case ii. When $w_0^2 = w^2$

Amplitude

When the driving force w is equal to the natural frequency w_0, then this condition is called resonant frequency.

Now equation (11) becomes

$$A = \frac{f}{\sqrt{(w_0{}^2-w2)2+4b2w2}} \approx \frac{f}{\sqrt{4b2w2}}$$

(or)

$$A = \frac{f}{2bw}$$

Since f = $\frac{F0}{m}, 2b = \frac{r}{m}, we\ can\ write$

$$A = \frac{\frac{F_0}{m}}{\frac{r}{m}w}$$

Amplitude A = $\frac{\mathbf{F0}}{\boldsymbol{rw}}$ ------------------------ (15)

Thus the amplitude depends on the damping force (r) and applied force F_0

Phase

When $w_0^2 = w^2$, then equation (12) becomes

$$\boldsymbol{\theta} = \tan^{-1}[\frac{2bw}{0}]$$

$$\boldsymbol{\theta} = \tan^{-1} \infty$$

$\therefore\ \boldsymbol{\theta = \frac{\pi}{2}}$ **--------------------- (16)**

Therefore under this situation, the displacement lags behind the driving force by a phase of $\frac{\pi}{2}\ (or) 90°$

Case iii. When $w_0^2 \ll w^2$

Amplitude :

When the driving force w is very much larger than the natural frequency w_0, then we can write equation (11) as

$$\text{Amplitude } A = \frac{f}{\sqrt{(w_0^2 - w2)2 + 4b2w2}} \approx \frac{f}{\sqrt{(w^2)^2}}$$

$$\text{(or) } A = \frac{F}{W^2}$$

Since $f = \frac{F0}{m}$, we can write (or)

$$\text{Amplitude } A = \frac{F0}{mw^2} \text{ ---------------- (17)}$$

Thus the amplitude depends on the applied force $F_0$and the mass of the body (m).

Phase

When $w_0^2 \ll$ w, then equation (12) becomes

$$\boldsymbol{\theta} = \tan^{-1}\left[\frac{2bw}{-w^2}\right]$$

$$\textbf{(or) } \boldsymbol{\theta} = \tan^{-1}\left[\frac{2b}{-w}\right] \quad \text{(since w is very large : } \boldsymbol{\frac{1}{w} = 0})$$

$$\boldsymbol{\theta} = \tan^{-1}(-0)$$

$\therefore\ \boldsymbol{\theta = \pi}$ **------------------------- (18)**

Therefore under this situation, the displacement lags behind the driving force by a phase of $\pi\ (or) 180°$.

questions	opt1	opt2	opt3	opt4	answer
Time taken to complete a wave is termed as	span	period	life	duration	**period**
Any two shortest points in a wave that are in phase are termed as	wave distance	wavelength	phase length	amplitude	**wavelength**
Direction of waves is parallel to distance of vibration in	transverse waves	longitudinal waves	both transverse and longitudinal waves	none of the waves	**longitudinal waves**
Sound is a good example of	transverse waves	longitudinal waves	both transverse and longitudinal waves	none of the waves	**longitudinal waves**
Motion that is repeated at regular intervals is termed as	Vibration	Oscillation	Ventilation	Periodic motion	**Periodic motion**
Ups and downs in longitudinal waves are termed as	compression and rarefaction	crests and rarefactions	compressions and troughs	crests and troughs	**compression and rarefaction**
A pendulum bob is a good example of	Vibration	Oscillation	Ventilation	Periodic motion	**Periodic motion**
Sound is a bad example of	transverse waves	longitudinal waves	both transverse and longitudinal waves	none of the waves	**transverse waves**
If we increase wavelength frequency would	increase	decrease	remain same	may increase or decrease	**decrease**
Waves transfer energy from one point to other.	It's true	Its false	its neutral	None of others	**It's true**
Light wave is a good example of	transverse waves	longitudinal waves	both transverse and longitudinal waves	sine waves	**transverse waves**
Direction of waves is perpendicular to direction of vibration in	transverse waves	longitudinal waves	both transverse and longitudinal waves	sine waves	**transverse waves**
Ups and downs in transverse waves are termed as	compression and rarefaction	crests and rarefactions	compressions and troughs	crests and troughs	**crests and troughs**
A source of any wave is	Ventilation	Oscillation	Energy	Force	**Oscillation**
Energy in waves is transfer and medium is	also transferred	not transferred	medium does not exist	may transfer or may not transfer	**not transferred**
Height of crest or depth of trough from center is called	wave distance	wavelength	phase length	amplitude	**amplitude**
Types of waves is/are	latitudinal and longitudinal	transverse and latitudinal	transverse only	transverse and longitudinal	**transverse and longitudinal**
When we decrease wavelength frequency	increases	decreases	remains same	may increase or decrease	**increases**
If we wave a rope, medium would be	hand by which the rope is waved	the rope itself	the other side to which the rope is tied	air	**the rope itself**
Two points on same line at same distance and speed are said to be in	parallel	phase	displacement	pair	**phase**
One oscillation is also known as	One vibration	One ventilation	One semi-circulation	semi vibration	**One vibration**
A wave is made up of	air molecules	vibrations	periodic motions	oscillations	**periodic motions**
Which of the following statements is wrong	Sound travels in a straight line	Sound travels as waves	Sound is a from of energy	Sound travels faster in vacuum that then in air	**Sound travels faster in vacuum that then in air**
When a compression is incident on rigid wall it is reflected as	Compression with a phase change of p	Compression with no phase change	Rarefaction with a phase change of p	Rarefaction with no phase change	**Compression with a phase change of p**
In the longitudinal waves the direction of vibration in medium of particle is	Perpendicular to propagation of wave	Parallel to propagation	Different from each other	Variable for time to time.	**Parallel to propagation**
With the propagation of a longitudinal wave through a material medium the quantities transmitted in the propagation direction are	Energy, momentum and mass	Energy	Energy and mass	Energy and linear momentum	**Energy and linear momentum**
Loudness of a note of sound is	Directly proportional to amplitude of the wave	Directly proportional to square of amplitude of wa	Directly proportional to velocity of the wave	Directly proportional to square of velocity of the v	**Directly proportional to square of amplitude of wave**
The velocity of sound is maximum in	Water	Air	Vacuum	Metal	**Metal**
The velocity of sound in any gas depends upon	Wavelength of sound only	Density and elasticity of gas	Intensity of sound waves only	Amplitude and frequency of sound	**Density and elasticity of gas**
Ultrasonic waves are those waves	To which man can hear	Man can not hear	Are of high velocity	Of high amplitude	**Man can not hear**
If the pressure amplitude in a sound wave is tripled. Then by what factor the intensity of the sound wave increased	3	6	9	12	9
Which of the following phenomenon cannot take place with sound wave?	Reflection	Interference	Diffraction	Polarization	**Polarization**
Transverse waves can propagate	In a gas but not in a metal	In a metal but not in gas	Neither in a gas nor in a metal	Either in a gas or in a metal	**In a metal but not in gas**
Transverse waves can travel through	Air, water and a copper wire	Air and copper wire but not through water	A copper wire but not through air and water	Water but not through air and a copper wire	**A copper wire but not through air and water**
A astronaut can't hear the explosion on the surface of the moon because	Frequency of explosion is out of audible range	Temperature at that point is very low	There is no medium on moon	None of above	**There is no medium on moon**
An astronaut can't hear his companion at the surface of the moon because	Produced frequencies are above the radio frequencies	There is no medium for sound propagation	Temperature is too low during night and too high during day moon	None of above	**There is no medium for sound propagation**
Elastic waves in a solid are	Only transverse	Only longitudinal	Either transverse or longitudinal	Neither transverse nor longitudinal	**Either transverse or longitudinal**
The velocity of sound in air is not affected by change in	Moisture content of air	Temperature of air	Atmospheric pressure	Composition of air	**Atmospheric pressure**
Doppler effect is applicable for	Sound wave	Light waves	Eletro magnetic waves	Both 'a' and 'b'	**Both 'a' and 'b'**
X-ray waves, television waves and radio waves are examples of	mechanical waves	transverse waves	longitudinal waves	electromagnetic waves	**electromagnetic waves**
Waves which require medium for propagation are	electromagnetic	mechanical	transverse	longitudinal	**mechanical**
Energy can be transferred from one place to another through	linear motion	circular motion	force	waves	**waves**
Water waves and sound waves are examples of	transverse waves	longitudinal waves	mechanical waves	electromagnetic waves	**mechanical waves**
Wave motion in a medium transfers	energy only	both mass and energy	mass only	neither mass nor energy	**energy only**
Maximum displacement from equilibrium position is	frequency	amplitude	wavelength	period	**amplitude**
In s.h.m, velocity at equilibrium position is	minimum	constant	maximum	zero	**maximum**
Natural frequency of a guitar string can be changed by changing it's	area	diameter	length	stiffness	**length**
In cars, springs are damped by	shock absorbers	engine	tyres	brake pedals	**shock absorbers**
A force that acts to return mass to it's equilibrium position is called	frictional force	restoring force	normal force	contact force	**restoring force**
Oscillations become damped due to	normal force	friction	tangential force	parallel force	**friction**
Angular frequency of s.h.m is equal to	2pie	2pief	2f	1/T	**2πf**
For a resonating system it should oscillate	bound	only for some time	freely	for infinite time	**freely**
Velocity at equilibrium position is	constant	minimum	maximum	zero	**maximum**
Number of oscillations per unit time is	amplitude	wavelength	frequency	period	**frequency**
When displacement x equal to 0, then kinetic energy of system is	minimum	maximum	constant	zero	**maximum**

Unit 2

Wave Optics

Fraunhoffer Diffraction :

In the case of Fraunhoffer diffraction, both the source and slit are very far from the diffracting aperture. In other words, this is equivalent to saying that the light originating from the source (where it has a spherical wavefront) when reaches at the aperture, the wavefront is nearly plane. The intensity of light at each point of aperture is thus nearly same. (If the distance between source and slit is not large, the wavefront at the aperture will be spherical and intensity of light at each point of aperture will be diffrent).

After diffraction, each point of wavefront at aperture, will act as a new source of light and will spread as spherical wavefront .When the screen is very far from the aperture, the wavefront reaching the screen wiil again be planar and the wavefronts converging on any point on the screen will have same amplitude, and thus we need not worry about difference in field strength from different wavefronts.

Diffraction by a narrow single slit

Consider a single slit Fig. 2a(a rectangular aperture, whose length is larger compared to its breadth, and breadth is quite narrow, comparable to the wavelength of light $=0.1$ mm for visible light) placed in front of a monochromatic light source as shown in figure 2(b).

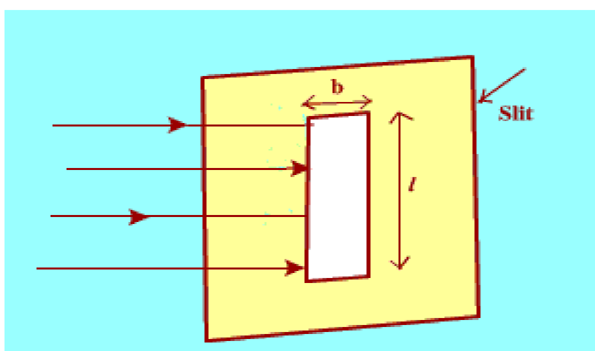


Fig. 2a: Single Slit

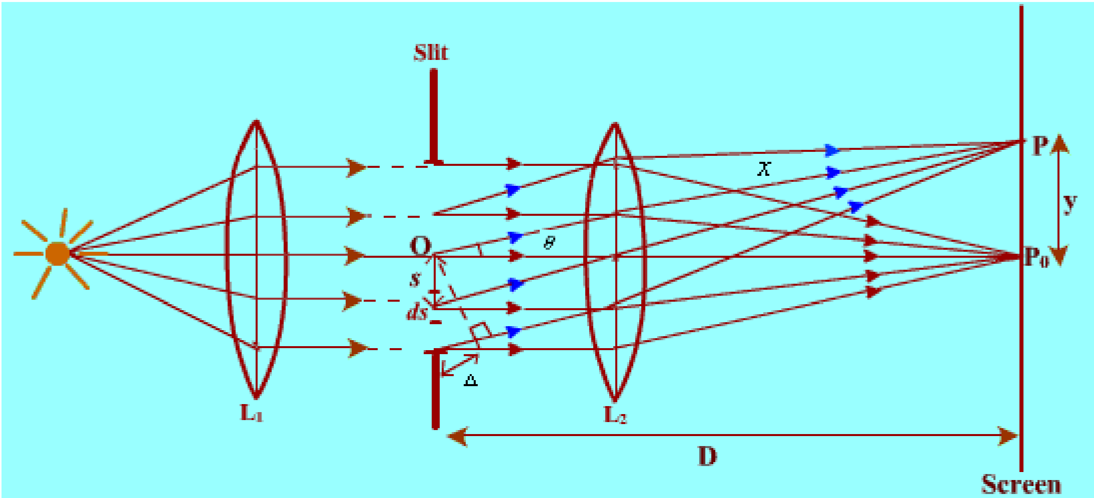


Fig: 2(b): Experimental arrangement and geometrical construction to understand the distribution of intensity on a screen in the single slit pattern

When a plane wave front is incident on the slit, each point on the wave front acts as the source of spherical secondary wave fronts. Lets divide the wave front incident on the slit in a large number of elements, each of width *ds*, (infinitesimaly small).

The part of each secondary wave, originating from these small elements and travelling normal to the plane of slit will be focussed at P_0 while those travelling at an angle θ will reach *P* (Fig.2b). Considering first the wavelet emitted by the element *ds* situated at the center of slit (let us call it origin). Its amplitude will be directly proportional to length *ds*and inversely proportional to the distance *x* (it will produce spherical wave front). At *P*, it will produce an infinitesimal amplitude (electric field) which is expressed as

$$dA_0 = \frac{ads}{x} Sin(\omega t - kx)$$

where '*a*' is the amplitude of incident wave, ω and k are its frequency and wave vector respectively.

The wave fronts reaching *P* from other elements *ds* will vary in phase due to extra path travelled by them. The displacements produced by another element ds at a distance *s* from the center(origin) will be given by

$$dA_s = \frac{ads}{x} Sin[\omega t - k(x + \Delta)]$$

$$= \frac{ads}{x} Sin(\omega t - kx - ks\ Sin\theta)$$

Where $\Delta(= sSin\theta)$ is the path difference.

The net amplitude at *P* will be sum of effect due to all the elements *ds* and can be obtained by integrating dE_s from *s*=*-d/2* to *d/2* where *d* is the width of the slit. In doing so, we can first sum the amplitude produced by the symmetrically placed elements *ds* and then integrate it from *s*=0 to *d/2.* The contribution due to a pair of symmetrically placed element *ds* is

$$dA = dA_{-s} + dA_{+s}$$

$$= \frac{ads}{x} [Sin(\omega t - kx - ks\ Sin\theta) + Sin(\omega t - kx + ks\ Sin\theta)]$$

Using the trigonomerical identity $Sin\alpha + Sin\beta = 2Cos\left(\frac{\alpha - \beta}{2}\right) Sin\left(\frac{\alpha + \beta}{2}\right)$ We have

$$dA = \frac{ads}{x} [2Cos(ks\ Sin\theta)\ Sin(\omega t - kx)]$$

Net effect at *P* is now

$$A = \int_0^{d/2} dA$$

We can treat x constant,

$$A = \frac{2a}{x} Sin(\omega t - kx) \int_0^{d/2} Cos(ksSin\theta) ds$$

$$= \frac{2a}{x} \left[\frac{Sin(ksSin\theta)}{kSin\theta}\right]_0^{d/2} Sin(\omega t - kx)$$

$$= \frac{ad}{x} \frac{Sin\beta}{\beta} Sin(\omega t - kx)$$

where $\beta = \frac{1}{2} kd\ Sin\theta$

The resultant wave reaching at P , is therefore, a simple harmonic one. The amplitude of this varies as position P is varied (θ varies). The resultant amplitude is given by

$$A = A_0 \frac{Sin\beta}{\beta}$$

Where $$A_0 = \frac{ad}{x}$$

The intensity of light at any point on screen is, thus given by

$$I = A^2 = A_0{}^2 \frac{Sin^2\beta}{\beta^2}$$

An intensity pattern as shown in fig. 3. is observed on screen

Analysis of Diffraction pattern due to single slit

The intensity on the screen for diffraction due to single slit is

$$I = A_0^2 \frac{Sin^2\beta}{\beta^2}$$ where $$\beta = \frac{1}{2}kd\ Sin\theta$$.

The intensity will be **minimum** (or zero) when $\sin\beta = 0$

or $\beta = \pm m\pi$ m=1,2,......................................

or $$\frac{1}{2}kd\ Sin\theta = \pm m\pi$$

$$\Rightarrow Sin\theta = \pm m\frac{\lambda}{d} \qquad \left[k = \frac{2\pi}{\lambda}\right]$$

If slit is far from the screen, θ is small such that $\sin\theta = \tan\theta = y/D$ where y is distance of point P from center point P_0

and D is distance between slit and screen.

$$\therefore \frac{y}{D} = \pm m\frac{\lambda}{d}$$
$$or \;\; d = \pm\frac{m\lambda D}{y}$$

Thus knowing *m*(order of minima), λ, *D* and *y* (separation of *m*'th minima from principal maxima) the slit width *'d'* can be determined.

Now, let us see, when we will observe **maxima** in diffraction pattern.

At the center of screen, the path traveled by the secondary waves originating from one point of slit is same as that traveled by another wave on the opposite side of center of slit at the same distance and hence path difference is zero, ie all the waves meet in phase thus we observe a maxima at this point (Fig 3). This can also be seen mathematically for the intensity distribution equation for point $P_0, here\ \theta = 0 \Rightarrow \beta = 0$, $Sin\beta = 0$ and $Sin\beta / \beta = 1$ (for small β, the series expansion of $Sin\beta$ will contain only first term β). Thus the intensity at the center will be

$$I_0 = A_0^2$$

Now we will determine the position of the maxima.

$$I = A_0^2 Sin^2\beta / \beta^2$$
$$\frac{dI}{d\beta} = A_0^2\left[\frac{2Sin\beta Cos\beta}{\beta^2} - \frac{2Sin^2\beta}{\beta^3}\right] = 0$$

$\Rightarrow$ either $Sin\beta = 0$ **or** $Cos\beta - \frac{Sin\beta}{\beta} = 0 \Rightarrow \tan\beta = \beta$

for $Sin\beta = 0, \beta = \pm m\pi$ corresponds to minima as discussed earlier. The second condition : $\tan\beta = \beta$ gives the position of maxima.

Solution of transcendental equation $\tan\beta = \beta$ can be determined graphically, where two functions $f_1(\beta) = \tan\beta$ and $f_2(\beta) = \beta$ are plotted against β (**Fig.4**). The point where these two curves intersects, satisfies $\tan\beta = \beta$ and these values of β corresponds to secondary maxima in the diffraction pattern. The corresponding values of $\beta = \pm 1.4303\pi, \pm 2.4590\pi, \pm 3.4707\pi, \cdots$ .This clearly shows that the secondary maximas are not exactly midway between the two minimas but are slightly displaced towards the principle maxima. The intensity at the mid way between two minima is $4I_0^2/(2n+1)^2\pi^2$ where *n=1,2,3.....* and is slightly less than the intensity at the secondary maxima. The intensity of the first secondary maxima is $\frac{4}{9\pi^2}$ times the intensity of principal maxima.

Resolving Power of Grating and other Image forming system

Resolution of image forming system

An optical system is used to form the image of objects. These objects may be situated at very large distances (like stars in galaxy) or very fine in size (microscopic size) The quality of image depends upon the quality of lenses used (Abberations). Even if we have perfect lens systems, the quality of image (sharpness) is limited by the diffraction at the aperture (entrance slit of the optical instruments). It means that if there are more than two light sources (illuminated objects) nearby, whether the images of these two sources (intensity distribution on screen) is separate or overlaps on each other will depend upon intensity distribution due to either sources on screen.

Till now we discuss the cases where monochromatic light was coming from a distant source (so that when it reaches the slit the wave front is plane). We discussed the intensity distribution on screen after the light passes through a slit or a system of slits. Clearly, the intensity distribution will change, if light is coming from some other source also(at some angle of incidence not equal to 0^0) or having more than one wavelength. Resolution or resolving power an optical instrument is the ability to separate the image pattern arising due to two nearby sources or due to two closely spaced wavelengths.

Let us consider the diffraction pattern for monochromatic light coming from two distant sources, whose angular separation from slit is α (**Fig.12.4.1.**). The parallel wave front reaching the slit will be inclined at an angle α with respect to each other and the central maxima for these two set of rays will also be separated by α.

The resultant diffraction pattern on the screen will be superposition of the patterns from these two sources as shown in **(Fig.12.4.2)**.

From figure, we can see that if the angular separation between the two central maxima is large, the diffraction pattern is well separated. However, if the angular separation is very small, the diffraction patterns overlap and we can not separate the two images. The two patterns are just resolved if the central maxima of one falls on first minima of other. This is known as Rayleigh criterion of resolution. In this case $dSin\theta = \lambda$ (first order minima of one pattern), where *d* is slit width.

For small θ, the angular separation between the two principal maxima, when these are just resolved will be $\alpha = \theta = \lambda / d$. If a lens of focal length *f* is used to form the image at the screen and the lens is placed close to the slit then the linear separation between the two maxima will be y , such that $\theta = \frac{y}{f} = \lambda / d$ or $y = f\lambda / d$

Resolving Power of a grating

We have seen that in the case of grating the position of principal maxima other than the central maxima depends upon the wavelength of light used (**Fig.12.4.3.**). Now if we are using a white light source, the grating should be able to resolve the diffraction pattern due to two nearby λ and $\lambda + \Delta\lambda$ if the angular separation between the corresponding order maxima due to one wavelength is equal to the half angular width of the principal maxima of other wavelength (i.e. the first minima after m^{th} order principal maxima of wavelength λ falls on the m^{th} order principal maxima of $\lambda + \Delta\lambda$; Rayleigh criterion)

i;e $b Sin\theta = m\lambda + \frac{\lambda}{N}$ (minima of wavelength λ)

$b Sin\theta = m(\lambda + \Delta\lambda)$ (maxima of wavelength $\lambda + \Delta\lambda$)

or $$m\lambda + \frac{\lambda}{N} = m(\lambda + \Delta\lambda)$$

or $$\frac{\lambda}{N} = m\Delta\lambda$$

or $$\frac{\lambda}{\Delta\lambda} = mN$$

The quantity $(\lambda / \Delta\lambda)$ is known as resolving power of grating. Clearly, Grating can resolve two nearby wavelengths if both m & N are large.

An earliest method to obtain two coherent waves (source) by division of wave front of parent wave is young's double slit method and is described below.

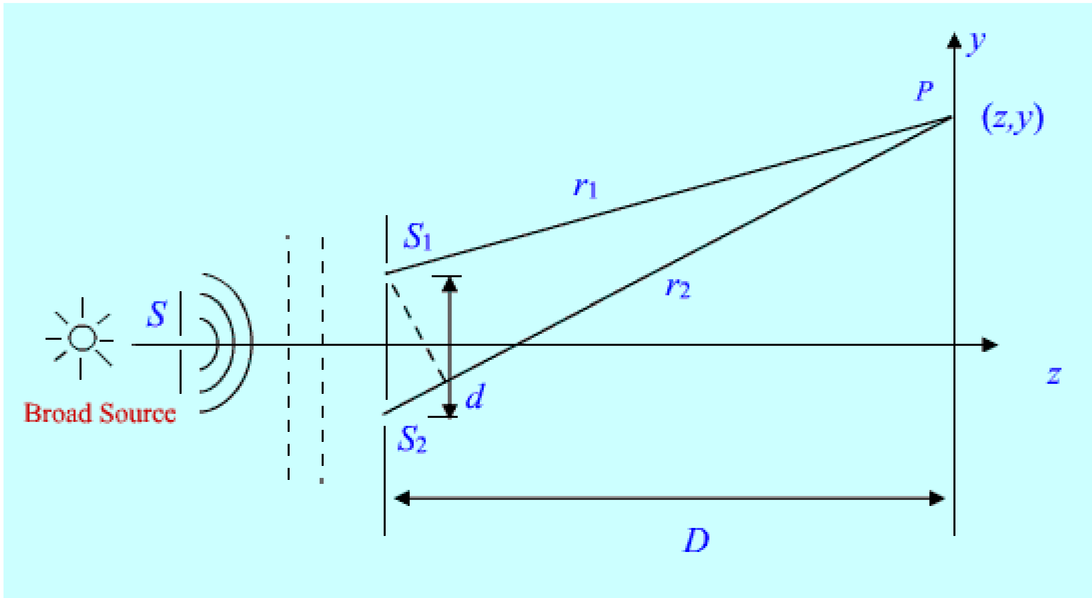


Fig.10.2.1: Derivation of two coherent waves by method division of wavefront (Young's dubble slit experiment)

In this method, light from broad source is made to fall on a small aperture slot S. Since the aperture is small it acts as a source of cylindrical wave. Now at all points of the wave front, the phase of the wave remains the same. When the light (wave front) falls on Symmetrically placed slits S_1 & S_2 , these select two different regions of the wave front and light emerging out of these slits have no phase difference. The slits S_1 and S_2 thus behave as coherent sources. When waves from these sources S_1& S_2 travel forward and fall on a screen, at any point on screen additional phase difference may arise between two waves depending on the path these waves travel. However resultant phase difference at any point doesn't change with time. The intensity at the point of superposition is decided by the relative phase difference between the two waves.

For a point $P(x, y, z)$ on the screen, the two waves (from S_1 and S_2) travel distances r_1 and r_2 respectively. The path difference Δ is given by

$$\Delta = (r_2 - r_1) \qquad \text{--------------} (10.2.16a)$$

Where $r_2^2 = x^2 + \left(y + \frac{d}{2}\right)^2 + z^2$ and $r_1^2 = x^2 + \left(y - \frac{d}{2}\right)^2 + z^2$

d is the separation between slits S_1 and S_2 .

If the screen is placed at a distance D from the center of double slits ($z=0$) and $D>>d$, we have

$$\Delta = \frac{yd}{D} \qquad \text{---------------------------------------}(10.2.16b)$$

Thus the phase difference between the two waves at the screen will be

$$\phi = \frac{2\pi}{\lambda} \cdot \frac{yd}{D}$$ --(10.2.17)

The waves will interfere constructively (bright fringe) or destructively (dark fringe) depending on whether ϕ is an even or odd multiple of π. The m^{th} order bright fringe will be at points on the screen where

$$y_m = \frac{mD\lambda}{d}$$ --(10.2.18)

m=0,1,2,3----------

The central fringe (y=0) is always bright.

The fringes are equidistant and parallel to x axis. The fringe width is given by

$$y = y_{m+1} - y_m = \frac{D\lambda}{d}$$ --(10.2.19)

The intensity distribution on screen for these superimposing waves is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$$

Assuming that the two light waves are of equal intensity I_0

$$I = I_0 + I_0 + 2I_0\cos\varphi = 4I_0\cos^2\frac{\varphi}{2}$$ ------------------------------ (10.2.20)

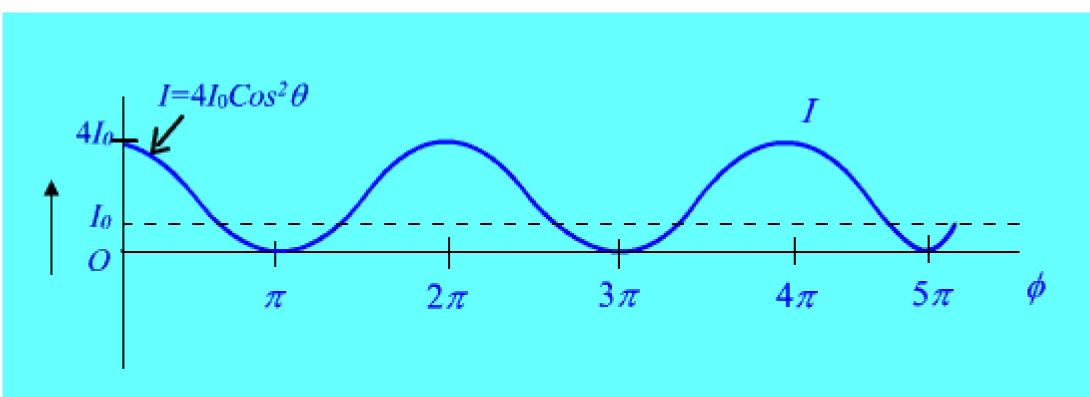


Fig.10.2.2: Resultant intensity distribution as a function of phase difference for two intensity waves of same amplitude.

. Newton's Ring : In a Newton's ring set up, the two super imposing light waves are produced by division of amplitude. The set up consists of a plano convex lens placed on a flat (plane) glass surface as shown in figure. The light from a broad monochromatic source is made to fall on a glass plate inclined at 45° . The reflected rays are incident on plano convex lens and are reflected from the top and bottom surface of air film formed between the convex surface and flat surface of lens and glass plate respectively and interference pattern (circular rings) is observed on the screen (eyepiece) with their center at 'O'.

Let t be the thickness of air film at point P and r be the distance of point P from O. The thickness of this film remains constant along a circle of radius x

and center at O. If λ is the wavelength of light used and 'μ' is the refractive index of the medium between plano convex lens and flat glass plate, then the point P, and therefore the circle with radius OP is dark or bright according to

$$2\mu t Cos(r+\theta) = 2n\lambda/2 \quad \text{(dark ring)}$$

$$2\mu t Cos(r+\theta) = (2n+1)\lambda/2$$

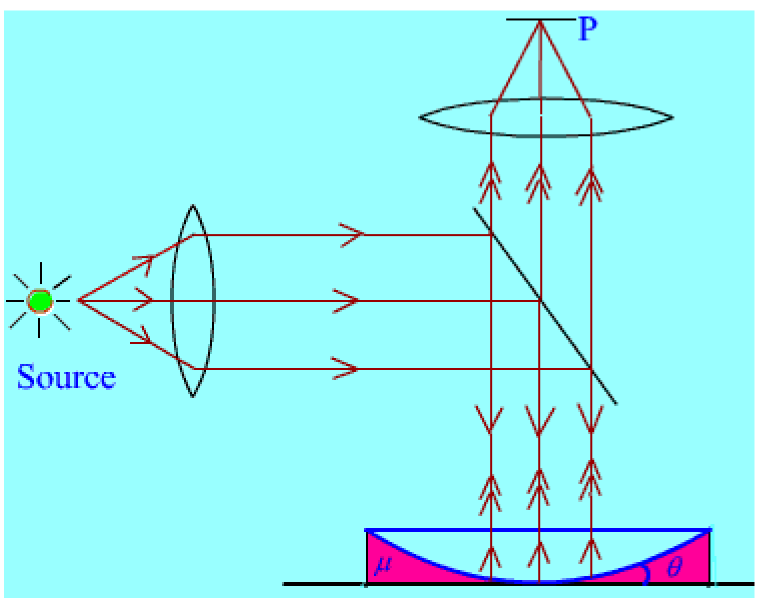


Fig.11.1.4: Newton's Ring Set-Up .

where r is angle of refraction for the ray in to the film and θ is angle of film. n is the order of ring.

For normal incidence ($r = 0$) and small θ (if radius of plano convex is large), then

$$2\mu t = 2n\lambda / 2 \quad \text{(dark)}$$

$$2\mu t = (2n+1)\lambda / 2 \quad \text{(bright)}$$

from the geometry of (Fig.11.1.5)

$$R^2 = (R-t)^2 + x^2$$

$$\text{or} \quad x^2 = R^2 - (R-t)^2 = t(2R-t)$$

$$\text{or} \quad t = \frac{x^2}{2R} \quad \text{for } t << 2R$$

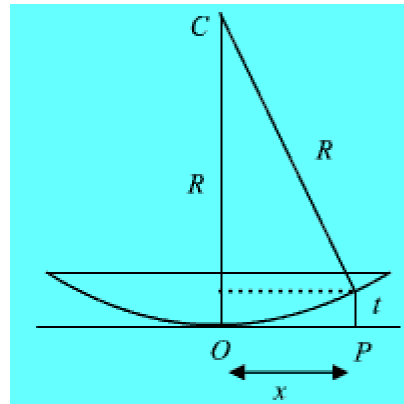


Fig.11.1.5

The radius on n'th dark or bright fringe is, therefore given by

$$x_n^2 = \frac{n\lambda R}{\mu} \quad \text{(dark ring)}$$

$$x_n^2 = \frac{(2n+1)\lambda R}{2\mu} \quad \text{(bright ring)}$$

Central spot '*O*', is dark although the film thickness is zero there, This is because of the phase difference of π (equivalent path difference = $\lambda / 2$) introduced in the ray reflected by flat glass plate.

The radius of the n^{th} dark or bright ring can be measured by measuring the diameter of the ring. In an experimental set up the cross wire of eye piece is focused on the n^{th} order ring on both side the center spot respectively by moving the micrometer screw and noting down the corresponding reading on micrometer scale. For an accurate determination of λ the diameter *'D'* (=2*x*) of every 5^{th} ring is measured, while the space between the plano convex lens and flat surface has only air film $(\mu = 1)$. In this case, for bright rings .

$$D_n^2 = 2(2n+1)\lambda R$$

and $$D_{n+p}^2 = 2(2n+2p+1)\lambda R$$

thus $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

One can also plot D_n^2 Vs n and fit the experimental data to a straight line ($Y = MX+C$) using least square fitting method. From the intercept 'C' the wavelength of light can be determined.

To determine the refractive index of liquid using newton's ring method, first the diameter of different order bright rings are measured using air film. Then liquid is inserted between the glass surface and diameter is again measured. The ratio

$$\left(D_{n+p}^2 - D_n^2\right)_{\text{air}} / \left(D_{n+p}^2 - D_n^2\right)_{\text{liquid}}$$

gives the refractive index of liquid.

Michelson interferometer

In Michelson interferometer the two coherent sources are derived from the principle of division of amplitude. The parallel light rays from a monochromatic source are incident on beams splitter (glass plate) G_1 which is semi silvered on its back surface and mounted at 45° to the axis. Light ray incident 'O' is refracted into the glass plate and reaches point A , where where it is partially reflected (ray 1) and partially transmitted ray 2. These rays then fall normally on mirrors M_1 (movable) and M_2 (fixed) and are reflected back. These reflected rays reunite at point A again and follow path AT. Since these two rays are derived from same source(at A) and are therefore coherent, can interfere and form interference pattern.

In this geometry, the reflected ray 1, travels an extra optical path, a compensating plate G_2 of same thickness as plate G_1) is inserted in the path of ray 2 such that G_2 is parallel to G_1 . This introduces the same optical path in glass medium for ray 2 as ray 1 travels in plate G_1 (therefore is called a compensating plate). Any optical path difference between the ray 1 and ray 2 is now equal to actual path difference between them.

To understand, how the fringes are formed, refer to fig. An observer at 'T' will see the images of mirror M_2 and source S (M'_2 and S' respectively) through beam splitter along with the mirror M_1. S_1 and S_2 are the images of source in mirrors M_1 and M_2 respectively. The position of these elements in figure depend upon their relative distances from point A .

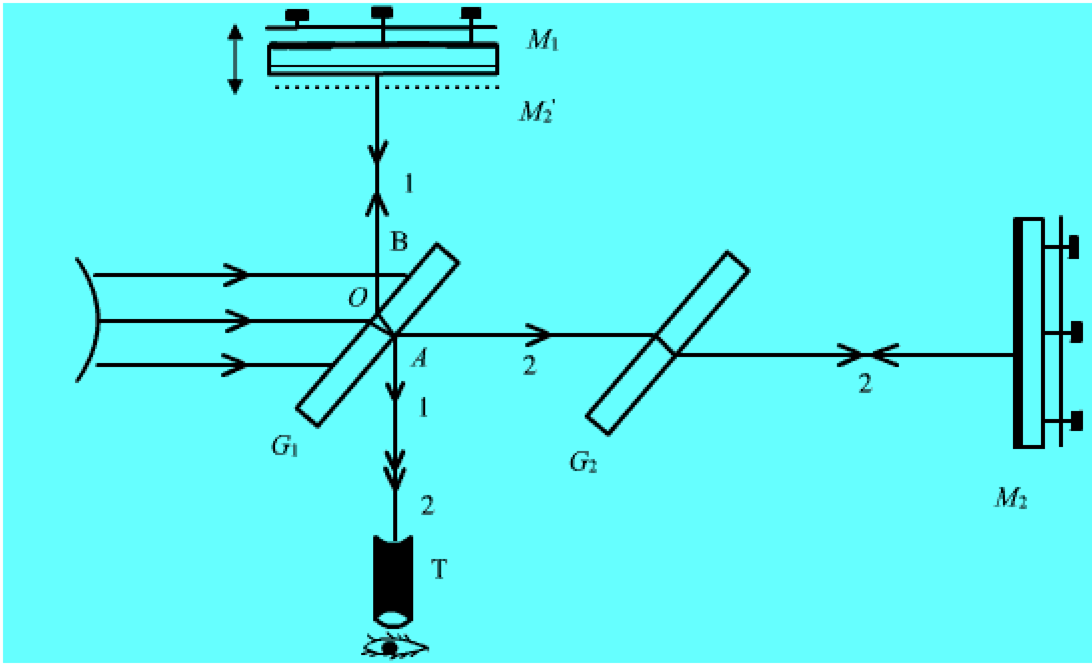


Fig.11.2.1: Michelson Inferometer (Experimental Set-up)

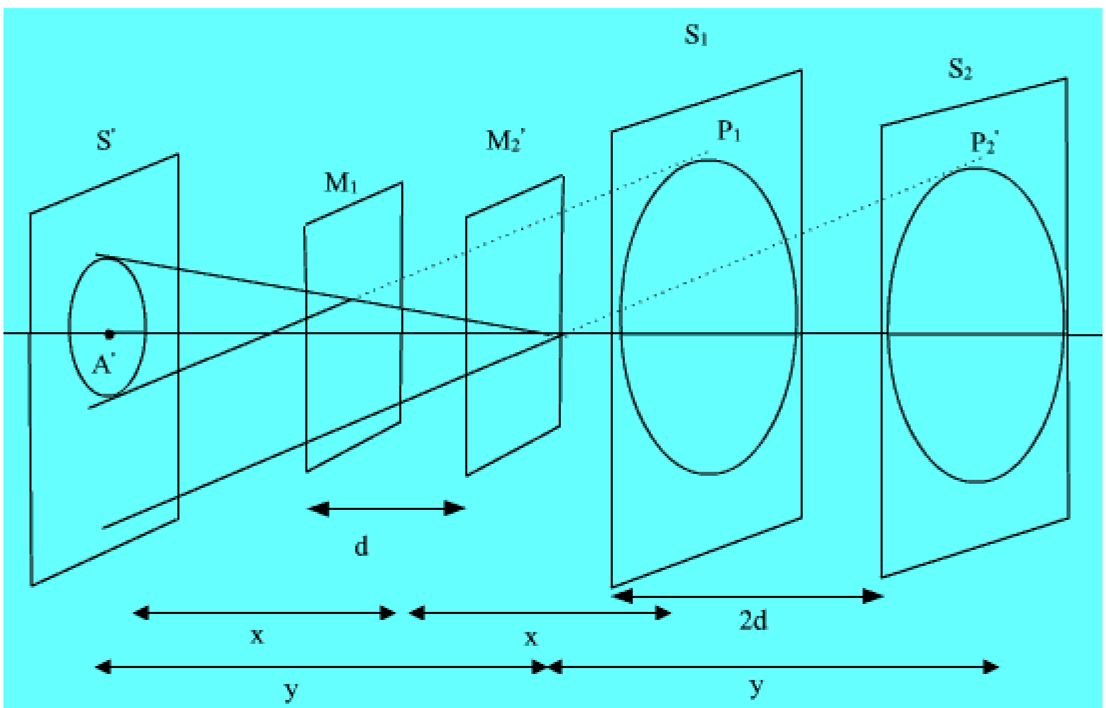


Fig. 11.2.2 : Formation of Fringes Light from a point (say P) from extended source appears to come from corresponding coherent points P_1 and P_2 on S_1 and S_2 .

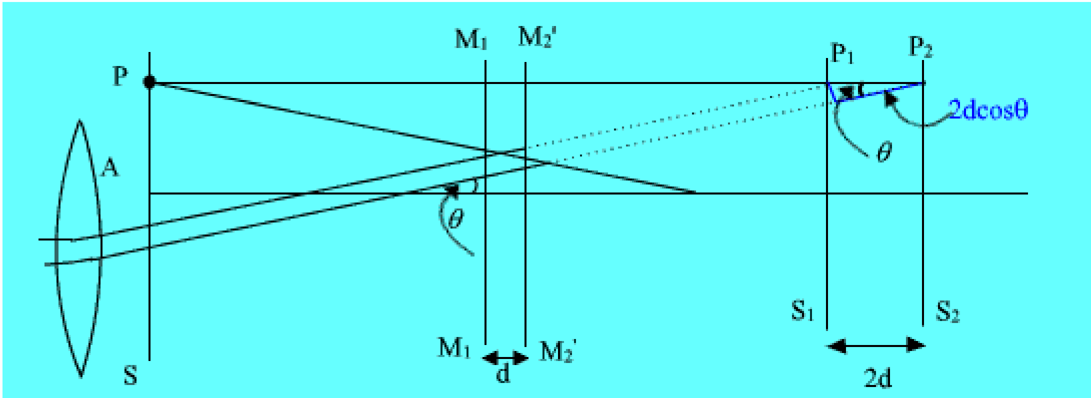


Fig.11.2.3

If 'd ' is the separation between mirrors M_1 and M_2' then '$2d$' is the separation between virtual sources S_1 and S_2 The path difference between the two parallel rays coming from point P_1 and P_2 respectively and reaching the eyepiece is equal to $2dCos\theta$.

$$2dCos\theta = n\lambda \text{ (bright)}$$

$$2dCos\theta = (2n+1)\lambda/2 \text{ (dark)}$$

These fringes are concentric rings or straight line depending upon the mutual inclination of mirrors M_1 and M_2(M_2'). If mirrors M_1 and M_2 are parallel to each other the case similar to the air film between two parallel plate and fringes formed are concentric rings.

Michelson interferometer is used to determine the wavelength of monochromatic source, the difference between two wavelengths, determination of thickness/refractive index of thin transparent sheet. The experimental procedure is described below.

Determination of wavelength(λ)

(A) If the mirror M_2 is not exactly perpendicular to mirror M_1 (in this case mirror M_1 and image M'_2 of mirror M_2 will not be exactly parallel), a wedge shaped air film is formed between M_1 and M'_2. The two reflected waves from M_1 and $M_2$are no longer parallel but appear to diverge from some point near M_1 (see fig) and are localised fringes.

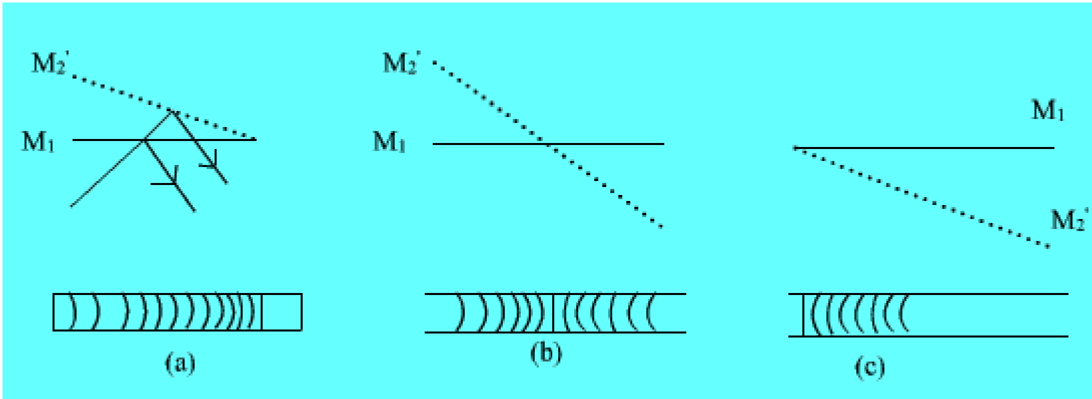


Fig.11.2.4

These localized fringes are equidistant straight lines, parallel to the edge of wedge provided '*d*' is small so that variation in path difference is practically due to variation in film thickness only. If *d* is increased, the fringes will not be exactly straight due to some variation of path difference with the angle between M_1 and M'_2. In this case fringes

become convex towards the edge of wedge.(fig 9). If the separation between M_1 and M'_2 is decreased the fringes move across the field towards the thick part of wedge. As *d* is changed by $\lambda / 2$, a new fringe crosses the center. At this time, fringes gradually become straighten.

Now if we change the position of movable mirror M_2, then the path difference is changed. When the distance between the mirror is changed by $\lambda / 2$, the next order bright ring appears at the center. Thus by recording the position of movable mirror and the number *N* of central bright rings moved, can determine λ using following relation.

$$x = N\lambda / 2$$

where *x* is the difference between the position of movable mirror during which '*n*' new bright ring appear at the center.

Determination of difference in wavelength :

When the light coming from the source consists of two closely spaced wavelength (such as D_1 and D_2 lines of sodium vapour lamp $\lambda_1 = 5896^0\pi , D_2 = 5890^0\pi$) each wavelength produces its own fringe pattern. When the separation between M_1 and M'_2 is small. The rings due to λ and λ_2 almost coincide.

When separation '*d*' between the mirrors M_1 and M'_2 is increased the two rings patterns have different spacing (fringe width due to two are different) and rings of λ_1 is gradually separated by those due to λ_2. At certain spacing ' d ' between mirrors, the dark ring of λ_1 coincides with bright rings of λ and the rings have maximum indistinctness.

As the mirror M_1 is moved further away (d increases) the rings due to λ_1 and λ_2 become most distinct and indistinct periodically. Let x is the distinct by which the mirror M_1 is moved for two consecutive situations when ring due to $\lambda_1$and λ_2 are maximum indistinct. During the movement, n fringes of λ_1 and λ_2 fringes of λ_2 have appeared at the center (only then dark ring of λ_1 will again coincide with bright ring of λ_2). Since each time mirror M_1 is moved by $\lambda/2$, a new ring appear at the center hence,

$$x = n\frac{\lambda_1}{2} = (n+1)\frac{\lambda_2}{2}$$

or, $$n = \frac{2x}{\lambda_1}$$ and $$(n+1) = \frac{2x}{\lambda_2}$$

$$\Rightarrow \quad \lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2x}$$

Since $\lambda_1$and $\lambda_2$are close together, $\lambda_1\lambda_2$product can be replaced by $\lambda^2\left(\lambda = \frac{\lambda_1+\lambda_2}{2x}\right)$. Thus we have

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}$$

knowing λ and x, we can determine the difference in wavelength of two very close wavelengths using michelson inferometer.

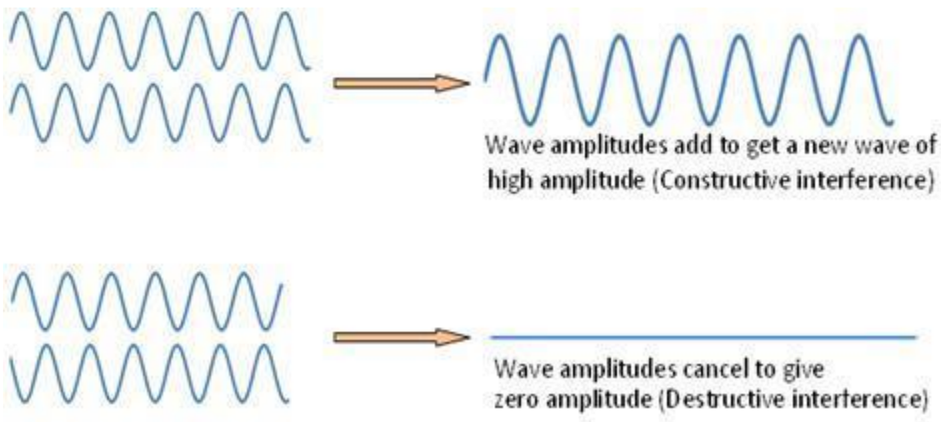
Theory:

Interferometers are used to precisely measure the wavelength of optical beams through the creation of interference patterns . The Michelson interferometer is a historically important device which provides simple interferometric configuration, useful for introducing basic principles.

Interference theory:

Light is a transverse wave. When two waves of same wavelength and amplitude travel through same medium, their amplitudes combine. A wave of greater or lesser amplitude than the original will be the result. The addition of amplitudes due to superposition of two waves is called interference. If the crest of one wave meets with the trough of the other, the resultant intensity will be zero and the waves are said to interfere destructively. Alternatively, if the crest of one wave meets with the crest of the other, the resultant will be maximum intensity and the waves are said to interfere constructively.

Suppose two coherent (*i.e.* their initial phase relationship remains constant) waves start from the same point and travel different paths before coming back together and interfering with each other. Suppose also that the re-combined waves illuminate a screen where the position on the screen depends on the difference in the lengths of the paths traveled by the two waves. Then the resulting alternating bright and dark bands on the screen are called interference fringes.



In constructive interference, a bright fringe (band) is obtained on the screen. For constructive interference to occur, the path difference between two beams must be an integral multiple $\boldsymbol{m\lambda}$ of the wavelength $\boldsymbol{\lambda}$, where $\boldsymbol{m}$ is the order, with $\boldsymbol{m}$ =0,1,2...

If the path difference between two waves is $(m+\frac{1}{2})\lambda$, the interference between them is destructive, and a dark fringe appears on the screen.

Michelson Interferometer:

The Michelson interferometer is the best example of what is called an amplitude-splitting interferometer. It was invented in1893 by Albert Michelson, to measure a standard meter in units of the wavelength of the red line of the cadmium spectrum. With an optical interferometer, one can measure distances directly in terms of wavelength of light used, by counting the interference fringes that move when one or the other of two mirrors are moved. In the Michelson interferometer, coherent beams are obtained by splitting a beam of light that originates from a single source with a partially reflecting mirror called a beam splitter. The resulting reflected and transmitted waves are then re-directed by ordinary mirrors to a screen where they superimpose to create fringes. This is known as interference by division of amplitude. This interferometer, used in 1817 in the famous Michelson- Morley experiment, demonstrated the non-existence of an electromagnetic-wave-carrying ether, thus paving the way for the Special theory of Relativity.

A simplified diagram of a Michelson interferometer is shown in the fig: 1.

Light from a monochromatic source S is divided by a beam splitter (BS), which is oriented at an angle 45° to the beam, producing two beams of equal intensity. The transmitted beam (T) travels to mirror M_1 and it is reflected back to BS. 50% of the returning beam is then reflected by the beam splitter and strikes the screen, E. The reflected beam (R) travels to mirror M_2, where it is reflected. 50% of this beam passes straight through beam splitter and reaches the screen.

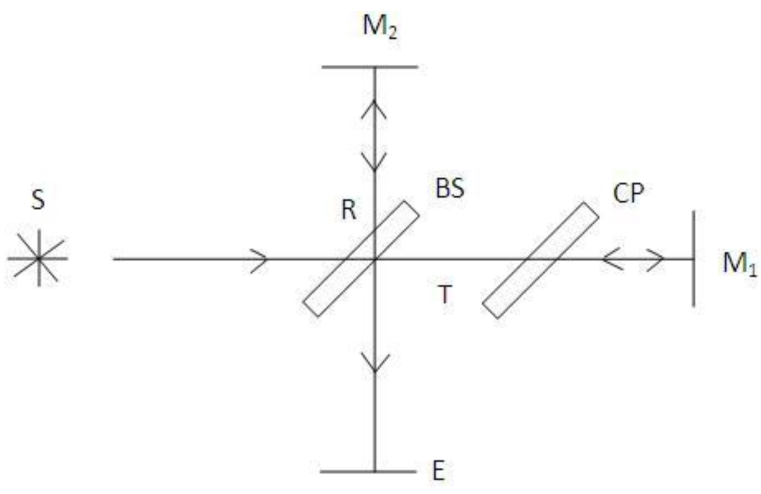


Fig. 1

Fig. 2

Since the reflecting surface of the beam splitter BS is the surface on the lower right, the light ray starting from the source S and undergoing reflection at the mirror M_2 passes through the beam splitter three times, while the ray reflected at M_1 travels through BS only once. The optical path length through the glass plate depends on its index of refraction, which causes an optical path difference between the two beams. To compensate for this, a glass plate CP of the same thickness and index of refraction as that of BS is introduced between M_1 and BS. The recombined beams interfere and produce fringes at the screen E. The relative phase of the two beams determines whether the interference will be constructive or destructive. By adjusting the inclination of M_1 and M_2, one can produce circular fringes, straight-line fringes, or curved fringes. This lab uses circular fringes, shown in Fig. 2.

From the screen, an observer sees M_2 directly and the virtual image M_1' of the mirror M_1, formed by reflection in the beam splitter, as shown in Fig. 3. This means that one of the interfering beams comes from M_2 and the other beam appears to come from the virtual image M_1'. If the two arms of the interferometer are equal in length, M_1' coincides with M_2. If they do not coincide, let the distance between them be *d*, and consider a light ray from a point S. It will be reflected by both M_1' and M_2, and the observer will see two virtual images, S_1 due to reflection at M_1', and S_2 due to reflection at M_2. These virtual images will be separated by a distance 2*d*. If θ is the angle with which the observer looks into the system, the path difference between the two beams is $2d\cos\theta$. When the light that comes from M_1 undergoes reflection at BS, a phase change of π occurs, which corresponds to a path difference of $\lambda/2$.

Therefore, the total path difference between the two beams is,

$$\Delta = 2d\, Cos\,\theta + \frac{\lambda}{2}$$

The condition for constructive interference is then,

$$\Delta = 2d\, Cos\,\theta + \frac{\lambda}{2} = m\lambda, \quad m = 0, 1, 2... \qquad (1)$$

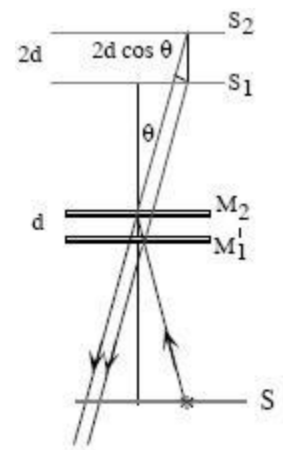


Fig. 3

For a given mirror separation *d*, a given wavelength λ, and order *m*, the angle of inclination θ is a constant, and the fringes are circular. They are called *fringes of equal inclination*, or *Haidinger fringes*. If M_1' coincides with M_2, d = 0, and the path difference between the interfering beams will be $\lambda/2$. This corresponds to destructive interference, so the center of the field will be dark.

If one of the mirrors is moved through a distance $\lambda/4$, the path difference changes by $\lambda/2$ and a maximum is obtained. If the mirror is moved through another $\lambda/4$, a minimum is obtained; moving it by another $\lambda/4$, again a maximum is obtained and so on. Because *d* is multiplied by $\cos\theta$, as *d* increases, new rings appear in the center faster than the rings already present at the periphery disappear, and the field becomes more crowded with thinner rings toward the outside. If *d* decreases, the rings contract, become wider and more sparsely distributed, and disappear at the center.

For destructive interference, the total path difference must be an integer number of wavelengths plus a half wavelength,

$$\Delta_{destr} = 2d\ Cos\,\theta + \frac{\lambda}{2} = (m + \frac{1}{2})\lambda, \quad \text{m= 0, 1, 2...}$$

If the images S_1 and S_2 from the two mirrors are exactly the same distance away, d=0 and there is no dependance on θ. This means that only one fringe is visible, the zero order destructive interfrence fringe, where

$$\Delta_{destr} = \frac{\lambda}{2} = (m + \frac{1}{2})\lambda, \text{ m= 0}$$

and the observer sees a single, large, central dark spot with no surrounding rings.

Measurement of wavelength:

Using the Michelson interferometer, the wavelength of light from a monochromatic source can be determined. If M_1 is moved forward or backward, circular fringes appear or disappear at the centre. The mirror is moved through a known distance *d* and the number *N* of fringes appearing or disappearing at the centre is counted. For one fringe to appear or disappear, the mirror must be moved through a distance of $\lambda/2$. Knowing this, we can write,

$$d = \frac{N\lambda}{2}$$

so that the wavelength is,

$$\lambda = \frac{2\,d}{N} \qquad (2)$$

Applications

1. The Michelson - Morley experiment is the best known application of Michelson Interferometer.

2. They are used for the detection of gravitational waves.

3. Michelson Interferometers are widely used in astronomical Interferometry.

questions	opt1	opt2	opt3	opt4
For destructive interference, path difference is	odd number of half wavelengths	Even number of half wavelengths	whole number of half wavelengths	even whole number of half wavelengths
Constructive interference happens when two waves are	out of phase	zero amplitude	in phase	in front
Two waves with phase difference 180° have resultant of amplitude	one	zero	same as the single wave	doubles as the single wave
If two waves are in phase and have same amplitude then resultant wave has	half of amplitude of single wave	same amplitude of single wave	twice of amplitude of single wave	thrice of amplitude of single wave
Extra distance travelled by one of waves compared with other is called	Path	displacement	phase difference	path difference
Interference of light is evidence that	the speed of light is very large	light is a transverse wave	light is electromagnetic in character	light is a wave phenomenon
Scattering of white light in to constitute colours is called	Diffraction	Refraction	Reflection	Dispersion
With diffraction grating, angles are	small	Greater	Zero	close to zero
Effect of diffraction is greatest if waves pass through a gap with width equal to	Frequency	Wavelength	amplitude	none of the above
Spreading of wave as it passes through a gap or around an edge is called as	Diffraction	Refraction	Reflection	Superposition
When a thin film of oil or soap bubbles is illuminated with white light, multiple colours appears this is	Diffraction	Refraction	Reflection	Dispersion
In Fresnel bi prism experiment the central fringe is	Dark	Bright	black	White
Interference pattern is caused by superposition of --------------waves.	One	Two	Three	none of the above
Interference pattern all maxima have	same wavelength	same intensity	same frequency	coherence
In fraunhofer diffraction, the diffraction wavefront is	Circle	spherical	elliptical	plane
For interference pattern	width of dark and bright bands are equal	width of dark and bright bands are not equal	width of dark bands only	width of bright bands only
In Fresnel diffraction, the diffraction wave front is	planar	spherical	cylindrical	none of the above
Fringes are referred to as	minima	maxima	nodes	normal points
Color depends on what characteristic of light?	its frequency	its wavelength	both of these	neither of these
Diffraction is a result of	Refraction	Reflection	Dispersion	Interference
Waves diffract the most when their wavelength is	short	long	both diffract the same	none of the above
The effect in which white light separates into different colors is called	Magnification	Reflection	Dispersion	Refraction
Which principle explains the phenomenon of diffraction?	Principle of Simultaneity	Pascal’s Principle	Archimedes’ Principle	Huygen’s principle
Which of the following explains the concept of diffraction loss?	Fresnel zone	Principle of Simultaneity	Pascal’s Principle	Archimedes’ Principle
Diffraction allows radio signals to propagate around ________	Continuous surface	Smooth surface	Curved surface of Earth	Does not allow propagation
The penetration of waves into the regions of the geometrical shadow is	interference	Dispersion	polarisation	diffraction
Interference occurs due to _______ of light	Wave nature	particle nature	wave and particle	light
Superposition of crest and trough results in	Constructive interference	Destructive interference	polarisation	diffraction
Interference due to reflected light is also called _________ law	sine law	cosine law	Tangent law	cotangent law
The diffraction phenomenon is	Rectilinear propagation of light	Bending of light around an obstacle	Oscillation of wave in one direction	none of them
The interference occurs in	longitudiunal wave	transverse wave	both a and b	none of these
Which of the following changes in interference of light?	Velocity	intensity	frequency	wavelength
To observe the diffraction pattern no lenses are used in	Fresnel	Fraunhoffer	diffraction	Michelson’s interferometer
A line on diffraction grating is	an opaque space	air	both a and b	vaccum
Which of the following depends on the total number of lines on the grating	intensity of principle maxima	intensity of principle minima	both a and b	zero intensity
Diffraction appears if the size of obstacle in path of rays is the order of	1mm	2mm	4mm	0
Interference of light is evidence that	The speed of light is very large	light is a transverse wave	light is electromagnetic in character	Light is a wave phenomenon
In Fresnel diffraction	source of light is kept at infinite distance from	source of light is kept at finite distance from tl	Convex lens used	aperture width is selected so that it can acts as
The dark lines constituting the absorption spectrum exhibited by sunlight are frequently called	. Fresnel lines	Fraunhofer lines	Fermi lines	Franklin lines
Light is	an electromagnetic wave	a form of energy visible to the human eye	the same type of energy as an X ray	all of the above
Colors in thin films are because of	Dispersion	. Interference	Compton effect	diffraction
Light travels fastest	In a vacuum	through water	Through glass	through diamond
Fraunhofer diffraction is observed when __________.	Only screen is placed at finite distance	source is placed at finite distance	neither source nor screen is at finite distance	None of these
The grating used to observe, diffraction of visible light can have approximately __________.	300 lines per cm	3000 lines per cm	15000 lines per cm	30 lines per cm
X-ray diffraction can be observed by using __________.	Diffraction Grating	Rock salt crystal	Convex lens	Michelson’s interferometer
There are two types of diffraction Fresnel and __________.	Michelson	Fraunhofer	Huygens	De Broglie
Why are diffraction gratings used instead of single slit or double slits to measure the wavelength of light?	Gratings provide a much wider diffraction ma	Gratings provide equal spacing between the	Gratings provide very narrow diffraction	None of these
For diffraction gratings, 'd' represents	Spaces between antinodes in a diffraction patt	Width of each one of the slits in the grating	The distance between the grating and the	Distance between each one of the slits in the
when white light is incident on a diffraction grating, the light that light will be deviated from central image	yellow	red	Violet	green
dispersive power of a grating can be defined as	the increase in the angle of refraction	the increase in the angle of diffraction	the increase in the angle of incidence	None of these
a diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by	no change	diffraction bands become narrower and crowc	bands become broader and apart	bands is disappear

answer
odd number of half wavelengths
in phase
zero
twice of amplitude of single wave
path difference
light is a wave phenomenon
Dispersion
Greater
Wavelength
Diffraction
Diffraction
Bright
Two
same intensity
plane
width of dark and bright bands are equal
cylindrical
maxima
both of these
Interference
long
Dispersion
Huygen's principle
Fresnel zone
Curved surface of Earth
diffraction
wave nature
Destructive interference
cosine law
Bending of light around an obstacle
longitudiunal wave
velocity
Fresnel
an opaque space
intensity of principle maxima
1mm
Light is a wave phenomenon
source of light is kept at finite distance from the aperture
Fraunhofer lines
all of the above
Interference
In a vacuum
None of these
15000 lines per cm
Diffraction Grating
Fraunhofer
Gratings provide very narrow diffraction maxima which can be accurately measured
Width of each one of the slits in the grating
Red
the increase in the angle of diffraction corresponding to a chnge in wavelength
diffraction bands become narrower and crowded together

UNIT-III

LASER

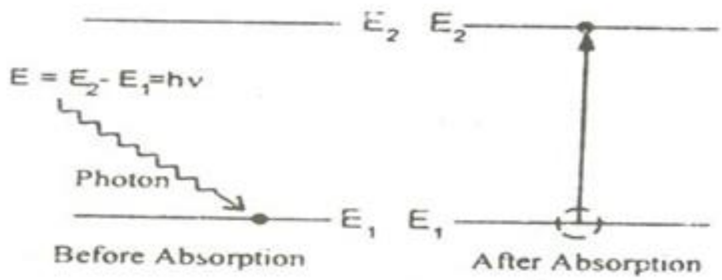
PRINCIPLE OF SPONTANEOUS AND STIMULATED EMISSION EINSTEIN'S QUANTUM THEORY OF RADIATION

When light is absorbed by the atoms or molecules, then it goes from the lower energy level (E_1) to the higher energy level (E_2) and during the transition from higher energy level (E_2) to lower energy level (E_1), the light is emitted from the atoms or molecules.Let us consider an atom exposed to (light) photons of energy E_2-E_1= h^γ, three distinct processes takes place.

i. Absorption
ii. Spontaneous emission
iii. Stimulated emission

i. Absorption

An atom in the lower energy level or ground state energy level E_1absorbs the incident photon radiation of energy (h^γ) and goes to the higher energy level or excited energy state E_2 as shown in fig 5.1. This process is called as absorption.



If there are many number of atoms in the ground state then each atom will absorb the energy from the incident photon and goes to the excited state then,

The rate of absorption (R_{12}) is proportional to the following factors.

i.e. R_{12} α Energy density of incident radiation (ρ_v)

α No of atoms in the ground state (N_1)

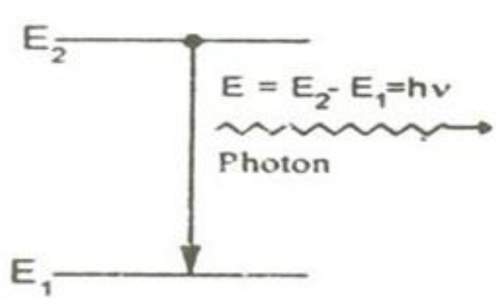
i.e. $R_{12} \; \alpha \; \rho_v N_1$

or $R_{12} = B_{12} \rho_v N_1$ ----(1)

Where, B_{12} is a constant which gives the probability of absorption transition per unit time.

ii. Spontaneous emission

The atom in the excited state returns to the ground state by emitting a photon of energy $E = (E_2 - E_1) = h^{\gamma}$, spontaneously without any external triggering as shown in figure.



This process is known as spontaneous emission. Such an emission is random and is independent of incident radiation.

If N_1 and N_2 are the numbers of atoms in the ground state (E_1) and excited state (E_2) respectively, then

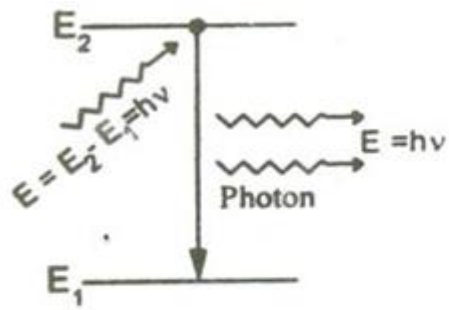
The rate of spontaneous emission is $R_{21}(Sp) \propto N_2$

$$\text{(or) } R_{21}(Sp) = A_{21}N_2 \qquad \text{-----(2)}$$

Where, A_{21} is a constant which gives the probability of spontaneous emission transition per unit time.

iii. Stimulated emission

The atom in the excited state can also return to the ground state by external triggering (or) inducement of photon there by emitting a photon of energy equal to the energy of the incident photon, known as stimulated emission. Thus results in two photons of same energy, phase difference and of same directionality as shown in figure.



The rate of stimulated emission is $R_{21}(St) \propto \rho_v N_2$

$$\text{(or) } \quad R_{21}(St) = B_{21}\rho_v N_2 \qquad \text{----(3)}$$

Where, B_{21} is a constant which gives the probability of stimulated emission transition per unit time.

Einstein's theory

Einstein's theory of absorption and emission of light by an atom is based on Planck's theory of radiation. Also under thermal equilibrium, the population of energy levels obeys the Boltzmann's distribution law.

i.e. under thermal equilibrium

The rate of absorption = The rate of emission

$$B_{12}\rho_v N_1 = A_{21}N_2 + B_{21}\rho_v N_2$$

$$\rho_v [B_{12} N_1 - B_{21} N_2] = A_{21}N_2$$

$$\therefore \rho_v = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$\therefore \rho_v = \frac{A_{21}}{B_{12}(N_1/N_2) - B_{21}} \qquad \text{----(4)}$$

We know from Boltzmann distribution law

$$N_1 = N_0 e^{-E_1/K_BT}$$

Similarly

$$N_2 = N_0 e^{-E_2/K_BT}$$

Where

K_B - Boltzmann Constant

N_0 - Number of atoms at absolute zero

T- Absolute temperature

At equilibrium, we can write the ratio of population levels as follows,

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/K_BT}$$

Since $E_2 - E_1 = h\vartheta$, we have

$$\frac{N_1}{N_2} = e^{h\vartheta/K_BT} \qquad \text{----(5)}$$

Sub eqn (5) in (4), we get

$$\rho_v = \frac{A_{21}}{B_{12}(e^{h\vartheta/K_BT}) - B_{21}}$$

$$\rho_v = \frac{A_{21}}{B_{21}} \frac{1}{(B_{12}/B_{21}) e^{h\vartheta/K_BT} - 1} \qquad \text{----(6)}$$

This equation has a very good agreement with plank's energy distribution radiation law

$$\rho_v = \frac{8\pi h\vartheta^3}{C^3} \frac{1}{e^{h\vartheta/K_BT} - 1} \qquad \text{---(7)}$$

Therefore comparing (6) and (7), we have

$$B_{12} = B_{21} = B$$

and

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\vartheta^3}{C^3} \qquad \text{----(8)}$$

Taking $A_{21} = A$

The constants A and B are called as Einstein Coefficients, which accounts for spontaneous and stimulated emission probabilities. Ratio of magnitudes of stimulated and spontaneous emission rates are as follows,

From eqn (2) and (3) we have

$$\frac{R_{21}(st)}{R_{21}(sp)} = \frac{B_{21}\rho_v N_2}{A_{21}N_2}$$

$$\frac{R_{21}(st)}{R_{21}(sp)} = \frac{B_{21}\rho_v}{A_{21}} \quad \text{---(9)}$$

Rearranging eqn (6) we can write

$$\frac{B_{21}\rho_v}{A_{21}} = \frac{1}{(B_{12}/B_{21})e^{h\vartheta/K_BT}-1}$$

Since $B_{12} = B_{21}$, we have

$$\frac{1}{e^{h\vartheta/K_BT}-1} = \frac{B_{21}\rho_v}{A_{21}} \quad \text{--- (10)}$$

Comparing (9) and (10) we get

$$\frac{R_{21}(st)}{R_{21}(sp)} = \frac{1}{e^{h\vartheta/K_BT}-1} = \frac{B_{21}\rho_v}{A_{21}}$$

In simpler way the ratio can be written as

$$\mathbf{R} = \frac{\mathbf{B_{21}\rho_v}}{\mathbf{A_{21}}}$$

Generally spontaneous emission is more predominant in the optical region (ordinary light). To increase the number of coherent photons stimulated emission should dominate over spontaneous emission.

DIFFERENCE BETWEEN SPONTANEOUS AND STIMULATED EMISSION OF RADIATION

S. No	**Stimulated emission**	**spontaneous emission**
1.	An atom in the excited state is induced to return to ground state, thereby resulting in two photons of same frequency and energy is called stimulated emission.	The atom in the excited state returns to ground state thereby emitting a photon, without any external inducement is called spontaneous emission.
2.	The emitted photons move in same direction and is highly directional	The emitted photons move in all directions and are random.
3.	The radiation is high intense, monochromatic and coherent.	The radiation is less intense and is incoherent.
4.	The photons are in phase (i.e.) there is a constant phase difference.	The photons are not in phase (ie.) there is no phase relationship between them.
5.	The rate of transition is given by $R_{21}(St) = B_{21}\rho_v N_2$	The rate of transition is given by $R_{21}(Sp) = A_{21}N_2$

POPULATION INVERSION

Consider two energy level systems E_1 and E_2. Suppose a photon of energy equal to the energy difference between the two energy levels, incident on the system, then there is equal chances for stimulated emission and absorption to occur. At this situation the chance for emission or absorption depends only on the number of atoms in the ground state and in the excited state.

Let N_1 be the number of atoms in ground state and N_2 be the number of atoms in excited state. Then,

If $N_1 > N_2$ there is more chance for absorption takes place.

If $N_2 > N_1$ there is more chance for stimulated emission takes place.

Therefore, the number of atoms in the excited state should be increased by some means. Thus the state of achieving more number of atoms in the excited state compared to the ground state atoms is called population inversion.

We know from Boltzmann distribution law $N_1/ N_2 = e^{(E_2-E_1)/K_BT}$

Case (i): If T is +ve

$$N_1 = N_2\, e^{+ve}$$

For example if $N_2 = 5$ and if $(E_2 - E_1) / k_BT \approx 2$,

Then, $N_1 = 5.\ e^{+2} = 36.9$

$\mathbf{N_1 > N_2}$ **since 36.9 >5**

Case (ii) If T is -ve

$$N_1 = N_2\, e^{-ve}$$

For example If $N_2 = 5$ and if $(E_2 - E_1) / k_BT \approx 2$,

$N_1 = 5.\ e^{-2} = 0.6766$

$\mathbf{N_2 > N_1}$ **since 5>0.6766**

This shows that number of atoms in excited state can be made more than number of atoms in the ground state only under negative temperature. But, the negative temperature is practically not possible. Therefore population inversion can be achieved by some other artificial process known as pumping process.

Active medium

The medium in which the population inversion takes place is called as active medium.

Active centre

The material in which the atoms are raised to excited state to achieve population inversion is called as active centre.

PUMPING METHODS

Pumping

The process of raising more number of atoms to excited state by artificial means is called as pumping process. There are several methods by which the population inversion (pumping) can be achieved. Some of the most commonly used methods are as follows,

(i) Optical pumping

(ii) Electric discharge

(iii) Inelastic atom – atom collision.

(iv) Direct conversion

(v) Chemical process

(i) Optical pumping

The atoms are excited with help of photons emitted by an external optical source. The atoms absorb energy from the photons and raises to excited state.

Ex. Ruby laser, Nd-YAG laser.

(ii) Electric discharge

The electrons are accelerated to very high velocity by strong electric field and they collide with gas atoms and these atoms are raised to excited state (e.g) argon laser, Helium-Neon laser, CO_2 Laser etc…

(iii) Inelastic atom-atom collision

In this method a combination of two types of gases are used. Say A and B, either having same (or) nearly coinciding excited states A*and B*.

During electric discharge ‘A’ atoms get excited due to collision with electrons. The excited A* atoms now collide with ‘B’ atoms so that B goes to excited state B*(e.g), Helium-Neon laser, CO_2 Laser.

$$e^- + A^* \rightarrow A^*$$

$$A^* + B \rightarrow B^* + A$$

(iv) Direct conversion

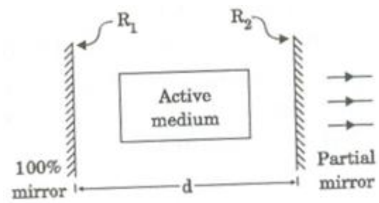
Due to electrical energy applied in direct band gap semiconductor like GaAs etc., the combination of electrons and holes take place and electrical energy is converted into light energy directly. (e.g) Semiconductor laser.

(v) Chemical process

Due to some chemical reactions, the atoms may be raised to excited state**.** (e.g) Dye laser

OPTICAL RESONATOR

The Optical resonator constitutes an active medium kept in between a 100% reflecting mirror and a partially reflecting mirror as shown in figure 5.4. This optical resonator acts as a feedback system in amplifying the light emitted from the active medium, by making it to undergo multiple reflections between the 100% mirror and partial mirror. Here the light bounces back and forth between the two mirrors and hence the intensity of the light is increased enormously. Finally the intense, amplified beam called LASER is allowed to come out through the partial mirror as shown in figure 5.4.



TYPES OF LASERS

Based on the type of active medium, Laser systems are broadly classified into the following categories.

S.No	TYPES OF LASER	EXAMPLES
1.	Solid State Laser	Ruby Laser Nd:YAG laser
2.	Gas laser	He-Ne Laser, CO_2 Laser, Argon – ion Laser
3.	Liquid Laser	SeOCL2 Laser, Europium Chelate Laser
4.	Dye Laser	Rhodamine 6G laser, Coumarin dye laser
5.	Semiconductor Laser	GaAs laser, GaAsP laser

CARBON – DI - OXIDE LASER

Characteristics of co_2 laser

Type – Molecular Gas laser

Active medium – Mixture of CO_2 , N_2 and helium or Water vapour

Active centre – Co_2

Pumping method – Eletric discharge method

Optical resonator – Metallic mirror of gold or silicon mirrors coated with aluminium

Power output – 10Kw

Nature of output – continuous or pulsed

Wavelength emitted – 9.6µm & 10.6 µm

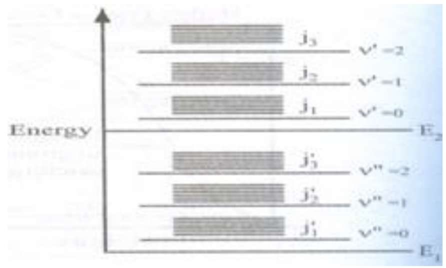
Introduction

An Indian engineer C.K.N designed the co_2 laser we know I n the case if atoms, electrons can be excited to higher energy levels of the molecule eg. He - Ne laser. Besides these electronic energy levels, the molecule can have other energy levels also due to rotation and vibration of the molecule (co_2) they give rise to various vibrational and rotational energy levels as shown in figure 5.6.

Where, E_1E_2 – electronic energy levels

v’, v’’ – vibrational energy levels

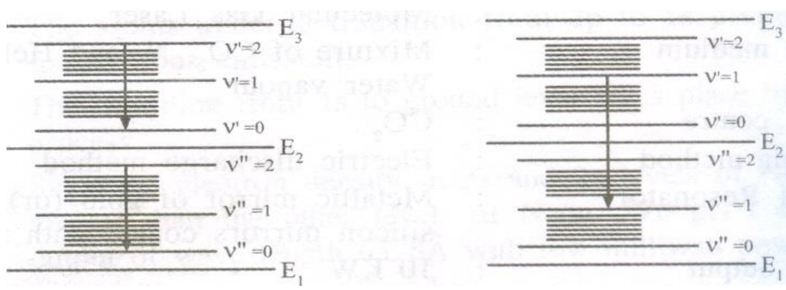
j,j’ – rotational energy levels.



Principle

The transition between these vibrational and rotational energy levels leads to the construction of molecular gas laser. Here the nitrogen atoms are initially raised to excited stare. The nitrogen atoms deliver the energy to co_2 atoms which has closest energy level to it. Then, the transition takes place between the vibrational energy 7 levels of the CO_2 atoms and hence laser beam is emitted. The molecular gas laser can have two types of transitions such as,

i. Transition between vibrational levels of same electronic state as shown in figure.
ii. Transition between vibrational levels of different electronic state as shown in figure.



CO_2 laser satisfies them first condition. i.e. here the laser transition occurs between vibrational levels of same electronic state.

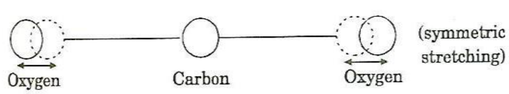
Fundamental modes of vibration of the co_2 molecule.

There are three fundamental modes of vibration.

1. Symmetric stretching mode ($10^{\circ}0$)
2. Bending mode ($01^{\circ}0$, $02^{\circ}0$)
3. Asymmetric stretching mode ($00^{\circ}1$, $00^{\circ}2$)

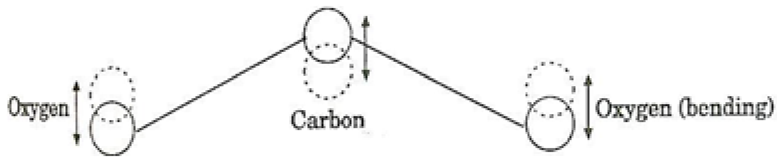
Symmetric stretching mode ($10^{\circ}0$)

The carbon atom is stationary and the oxygen atoms oscillate or vibrate along the axis of the molecule as shown in figure. The state of vibration is given by 3 integers ($mn^{l}q$) here ($10^{\circ}0$), which corresponds, to the degree of excitation.



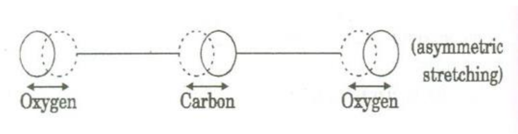
Bending mode ($01^{\circ}0$, $02^{\circ}0$)

Here the atoms will not be linear, rather the atoms will vibrate perpendicular to the molecular axis as shown in figure. This gives rise to two quanta of frequency represented by ($01^{\circ}0$, $02^{\circ}0$).



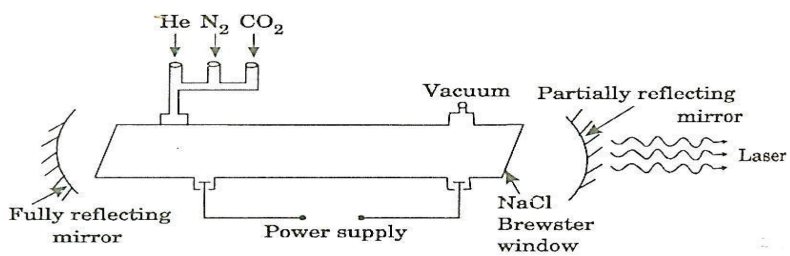
Asymmetric stretching mode ($00^{\circ}1$, $00^{\circ}2$)

Here all the atoms will vibrate. Here the oxygen atoms vibrate in the opposite direction to the vibration direction of carbon atom as shown in figure 5.10. This gives the quanta of frequency ($00^{\circ}1$, $00^{\circ}2$).



Construction

It consists of a discharge tube in which co_2 is taken along with nitrogen and helium gases with their pressure level of 0.33:1.2:7 mm of Hg for co_2, nitrogen and the He respectively. Nitrogen helps to increase the population of atoms in the upper level of co_2, while helium helps to depopulate the atoms in the lower level of co_2 and also to cool the discharge tube.



The discharge is produced by DC excitation. At the ends of the tube sodium chloride/Brewster windows are placed as shown in figure. Confocal silicon mirrors coated with aluminium or metallic mirror of gold is employed for proper reflection, which form the resonant cavity. The output power can be increased by increasing the diameter of the tube.

Working

(i) The discharge is passed through the tube first, the nitrogen atoms are raised to excited state $e^- + N_2 \rightarrow N_2^*$

(ii) The excited N_2 atoms undergo resonant energy transfer with co_2 atom and raises co_2 ($00^{\circ}1$) to excited state due to closer energy level of co_2 ($00^{\circ}1$) and nitrogen.

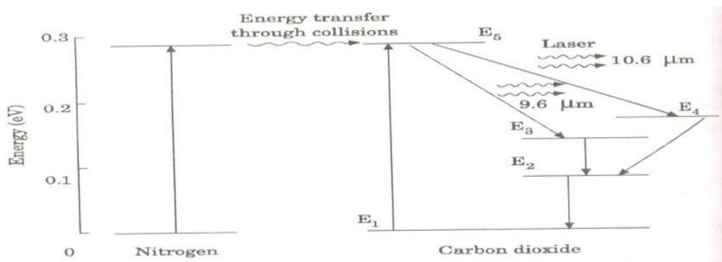
$$N_2^* + CO_2 + CO_2^* + N_2$$

(iii) When transition takes place between 00°1 to 10°0, the laser of wavelength 10.6 µm is emitted as shown in figure.

(iv) Similarly when transition takes place between 00°1 and 02°0 laser beam of wavelength 9.6 µm is emitted as shown in figure.

(v) Since 00°1 to 10°0 has higher gain than 00°1 to 02°0 transition, usually the laser beam of wavelength 10.6µm is produced more.

(vi) When the gas flow is longitudinal power output is 50 to 60 watts but if the gas flow is perpendicular to the discharge tube the output power may be raised to 10 kilo watt/m.



(vii) This type of co_2 laser is known as TEA laser.

(i.e.), (Transversely Excited Atmospheric Pressure laser).

(viii)The contamination of carbon monoxide and oxygen will also have some effect on the laser action. To avoid these unused gases can be pumped out and fresh CO_2 must be inside the discharge tube.

Application of CO_2 laser

(i) This laser has applications in medical field such as neurosurgery. Microsurgery, treatment of liver, lungs and also in bloodless operations.

(ii) It is widely used in open air communication.

(iii) This laser also has wide applications over military field.

HOMOJUNCTION SEMICONDUCTOR LASER

Characteristics of Homojunction laser

Type – Homojunction semiconductor laser

Active medium – PN junction diode

Active centre – Recombination of electrons and holes

Pumping method – Direct pumping

Optical resonator – junctions of diodes- polished

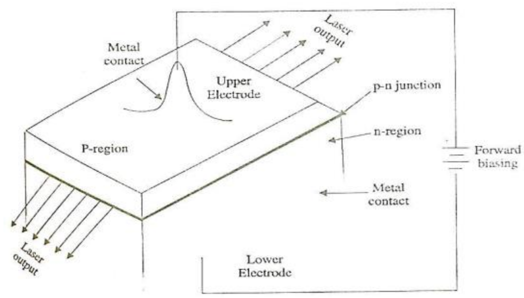
Power output – The power output from this laser is 1mW.

Nature of output – The nature of output is continuous wave or pulsed output

Wavelength emitted – 8400 – 8600 A°

Principle

The electron in conduction band combines with a hole in the valence band and hence the recombination of electron and hole produces energy in the form of light. This photon, in turn may induce another electron in the conduction band to valence band and there by stimulate the emission of another photon.

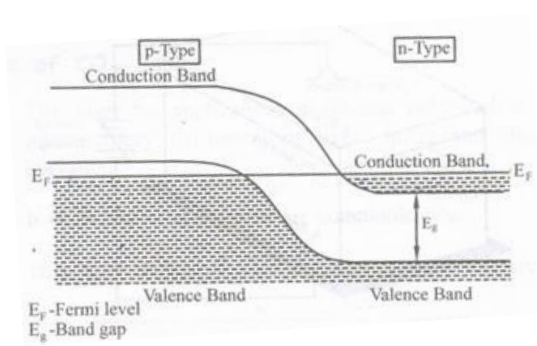


Construction

The active medium is a p-n-junction diode made from a single crystalline material ie. Gallium Arsenide, in which p-region is doped with germanium and n-region with tellurium. The thickness of the p-n-junction layer is very narrow so that the emitted laser radiation has large divergence. The junctions of the p and n are well polished and are parallel to each other as shown in figure. Since the refractive index of GaAs is high, it acts as optical resonator so that the external mirrors are not needed. The upper and lower electrodes fixed in the p and n region helps for the flow of current to the diode while biasing.

Working

(i) The population inversion in a p-n-junction is achieved by heavily doping p and n materials, so that the Fermi level lies within the conduction band of n type and within the valence band of p type as shown in figure.

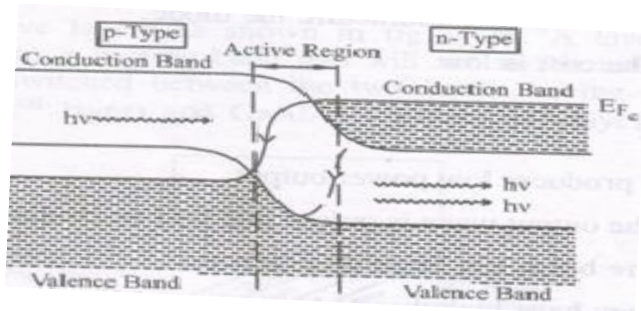


(ii) If, the junction is forward biased with an applied voltage nearly equal to the band gap voltage, direct conduction takes place. Due to high current density, active region is generated near the depletion region.

(iii) At this junction, if a radiation having frequency (γ) is made to incident on the p-n-junction then the photon emission is produced as shown in figure.

Thus the frequency of the incident radiation should be in the range

$$E_g < \gamma < \frac{(E_{F_C} - E_{F_V})}{h}$$



(iv) Further the emitted photon increases the rate of recombination of injected electrons from the n region and holes in p region by inducing more recombination. Hence the emitted photons have the same phase and frequency as that of original inducing photons and will be amplified to get intense beam of laser.

(v) The wavelength of emitted radiation depends on i. the band gap and ii. The concentration of donor and acceptor atoms in GaAs,

1. Calculation of wavelength

Band gap of GaAs = 1.44ev

$$E_g = h\,\gamma = h\frac{c}{\lambda}$$

$$\lambda = \frac{hc}{E_g}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{1.44 \times 1.6 \times 10^{-19}}$$

$$= 8626\ A^{\circ}$$

The wavelength is near IR region.

Advantages

(i) It is easy to manufacture the diode.

(ii) The cost is low.

Disadvantages

(i) It produces low power output.

(ii) The output wave is pulsed and will be continuous only for some time.

(iii) The beam has large divergence.

(iv) They have high threshold current density.

HETEROJUNCTION SEMICONDUCTOR LASER

Characteristics of Hetero junction semiconductor laser

Type - Hetero junction semiconductor laser

Active medium - p-n-junctions with various layers

Active centre – Recombination of electrons and holes

Pumping method – Direct pumping

Optical resonator – Junctions of diodes- polished

Power output – The power output from this laser is 10mW.

Nature of output – continuous wave form

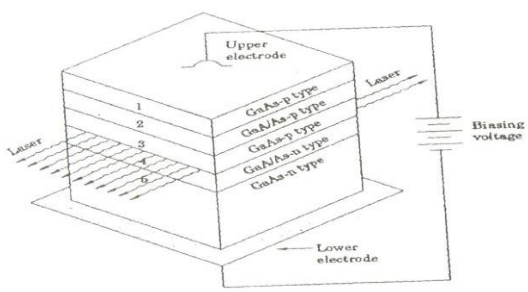
Band gap – 1.55ev

Principle

The electron in conduction band combines with a hole in the valence band and hence the recombination of electron and hole produces energy in the form of light. This photon, in turn may induce another electron in the conduction band to valence band and there by stimulate the emission of another photon.

Construction

It consists of five layers as shown in figure. A layer of GaAs- p-type (3rd layer) which has narrow band gap will act as the active region. This layer (3rd layer) is sandwitched between the two layers having wider band gap viz. GaAlAs – p-type (2nd layer) and GaAlAs – n-type (4th layer).



A contact layer made of GaAs – p-type (1st layer) is made to form at the top of the 2nd layer for necessary biasing. All these four laters are grown over the substrate (5th layer) made of GaAs-n-type. The junctions of GaAs – p-type (3rd layer) and GaAlAs – n-type (4th layer) are well polished and hence it acts as an optical resonator. The upper and lower electrodes help in forward biasing the diode.

Working

Working of a heterojunction laser is similar to that of the working of a homojunction laser.

(i) The diode is forward biased with the help of upper and lower electrodes.

(ii) Due to forward biasing the charge carriers are produced in the wide band gap layers (2 and 4).

(iii) These charge carriers are injected into the active region (layer 3).

(iv) The charge carriers are continuously injected from 2nd and 4th layer to the 3rd layer, until the population inversion is achieved.

(v) At this state some of the injected charge carriers recombines and produces spontaneously emitted photons.

(vi) These spontaneously emitted photons stimulate the injected charge carriers to emit photons.

(vii) As a result more number of stimulated emissions arises and thus large number of photons is produced.

(viii) These photons are reflected back and forth at the junction and hence an intense, coherent beam of LASER emerges out from the p-N junctions of active region ie. Between layer-3 and layer-4 as shown in figure.

(ix) The wavelength of the emitted radiation is given by $\lambda = \frac{hc}{E_g}$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{1.55 \times 1.6 \times 10^{-19}}$$

$$= 8014\ A^{\circ}$$

The wavelength lies IR region.

Advantages

i. Power output is high.
ii. It produces continuous wave output.
iii. It has high directionality and high coherence.
iv. It has low threshold current density compared to homojunction laser.
v. These diodes are highly stable and have longer life time.

Disadvantages

i. Cost is higher than homojunction laser.
ii. Practical difficulties arise while growing the different layers of p-n junction.

INDUSTRIAL APPLICATIONS [LASERS IN WELDING, HEAT TREATMENT AND CUTTING]

Laser heat treatment

Laser is a light beam of high intensity, directionality and coherence. So, when laser light is focused on a particular area, even of micrometer size, for a very longer time, then that particular area alone will be heated and the other area will remain as such. This is called thermal effect or laser heat treatment. In this process the light energy is converted into heat energy.

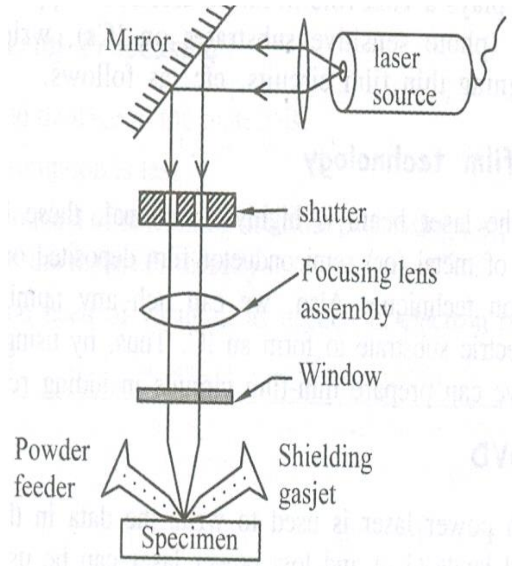
Instrumentation technique

Principle

The technique of laser heat treatment is used in engineering applications like surface hardening, coating, glazing, alloying, cutting, welding, drilling and perforating holes in the materials and hence this process is called material processing. In general ruby laser , Nd-YAG laser and co_2 laser are used for this purpose.

Instrumentation

The Instrumentation for materials processing consists of a laser source to produce laser beam, shutter to control the intensity of the laser beam and an assembly of lenses to effectively focus the laser onto the specimen as shown figure.



Apart from these Instrumentation, separate control arrangements are made for removing the molten materials, smokes, fumes etc.. with the help of a shielding gas jet, which consist of the assisting gases such as sir, N_2, o_2,Ar etc. the powder feeder is used feed the metal powder, wherever necessary.

Processing

Laser source is switched on. The light by the plane mirror is made to pass through the shutter. The intensity of the laser beam is controlled by the shutter and the controlled laser beam is allowed to fall on the focusing lens assembly. This lens assembly focuses the light effectively onto the window and is made to incident on the specimen.

Now the specimen gets heated, giving rise to smokes, fumes and molten materials. These smokes, fumes and molten materials are removed immediately by blowing the assisting gas from the shielding gas jet and this in turn makes the laser beam to continuously fall on the specimen, thereby increasing the cutting rate. Thus the materials can be drilled, cut, put holes etc. using this technique effectively and easily. In case of alloying, cladding, molding, welding etc. the power feeder will be used to spray the metal power over the specimen, during the focusing of laser beam on to the specimen.

Applications

Laser in Microelectronics

Laser plays a vital role in micro-electronics applications, such as making photos masks, writing/reading CDs and DVDs, designing thin film circuits, etc as follows.

(i) Thin film technology

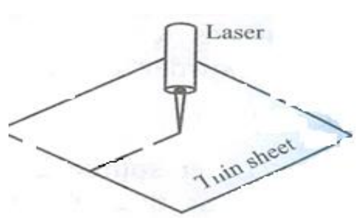
As the laser beam is highly directional, these beams are used to trim off a portion of metal or semiconductor film deposited on the dielectric substrate, by evaporation technique. Also, we can itch any number of micro components over the dielectric substrate to form an IC. Thus, by using as accurately controlled laser beam we can prepare thin film circuits including resistors, capacitors etc…

(ii) CD/DVD

High power laser is used to write the data in the CD/DVD by creating pits (0's) and lands (1's) and low power laser can be used to read the data.

Laser cutting

Laser is used as a tool to cut thin metal sheets by properly focusing the laser onto any particular area to be cut, for a longer time. Thus due to thermal effect the sheet is cut as shown in figure.



Laser drilling and perforating holes

The same technique as used for cutting will be adopted for drilling and perforating holes, even upto 0.2 to 0.5 µm of thickness.

Laser welding

In ordinary welding process heat will be made to fall on the area to be welded, so that the material in that area will go to molten state. This on cooling will join the material. In this process the heat will spread all over the surroundings and will affect the other areas of the material and hence the material gets damaged. To avoid this difficulty, laser is used for welding. Due to its high directionality, it is focused on to that particular area alone, even of very small size and the other area remains unaffected. Thus due to thermal effect the parts can be welded. This process is also called Micro-Welding.

questions	opt1	opt2	opt3	opt4	answer
Which of the following is a unique property of laser?	Directional	Speed	Coherence	Wavelength	Coherence
Which of the following is an example of optical pumping?	Ruby laser	Helium-Neon laser	Semiconductor laser	Dye laser	Ruby laser
Which of the following can be used for generation of laser pulse?	Ruby laser	Carbon dioxide laser	Helium neon laser	Nd- YAG laser	Nd- YAG laser
What is the need to achieve population inversion?	To excite most of the atoms	To bring most of the atoms to ground state	To achieve stable condition	To reduce the time of production of laser	To excite most of the atoms
Which of the following is used in atomic clocks?	Laser	Quartz	Maser	Helium	Maser
Which of the following can be used in vibrational analysis of structure?	Maser	Quartz	Laser	Helium	Laser
A device which converts electrical energy in the form of a current into optical energy is called as	Optical source	Optical coupler	Optical isolator	Circulator	Optical source
How many types of sources of optical light are available?	One	Two	Three	Four	Three
The radiation emission process (emission of a proton at frequency) can occur in __________ ways.	Two	Three	Four	One	Two
Which process gives the laser its special properties as an optical source?	Dispersion	Stimulated absorption	Spontaneous emission	Stimulated emission	Stimulated emission
The lower energy level contains more atoms than upper level under the conditions of ________________	Isothermal packaging	Population inversion	Thermal equilibrium	Pumping	Thermal equilibrium
__________________ in the laser occurs when photon colliding with an excited atom causes the stimulated emission of a second photon.	Light amplification	Attenuation	Dispersion	Population inversion	Light amplification
An electron can never be found inside nucleus, this statement is according to	Heisenberg uncertainty principle	Bernoulli's equation	bohrs model	both a and b	Heisenberg uncertainty principle
Potential energy source for inducing fusion reaction is	x-ray	laser	ultraviolet	microwave	laser
Principle of laser is	spontaneous absorption	simulated emission	induced emission	both b and c	both b and c
For an electron to be confined to a nucleus, its speed relative to speed of light would have to be	equal	less	greater	equal to infinity	greater
Electron in atom are held in atom due to	coulombs force	nuclear force	atomic force	both a and b	coulombs force
What does the acronym LASER stand for?	Light Absorption by Stimulated Emission of Radiation	Light Amplification by Stimulated Emission of Radiation	Light Alteration by Stimulated Emission of Radiation	Light Amplification by spontaneous Emission of Radiation	Light Amplification by Stimulated Emission of Radiation
What does the acronym MASER stand for?	Microwave Amplification by Stimulated Emission of Radiation	Molecular Absorption by Stimulated Emission of Radiation	The name of Albert Einstein's dog	Molecular Absorption by Spontaneous Emission of Radiation	Microwave Amplification by Stimulated Emission of Radiation
What is one way to describe a Photon?	Solid as a rock	A wave packet	A torpedo	star	A wave packet
What determines the color of light?	its intensity	its wavelength	its source	its luminance	its wavelength
Which scientist first came up with the idea of stimulated emission ?	Alexander Graham Bell	Isaac Newton	Arthur Schalow	Albert Einstein	Albert Einstein
Which laser is considered "eye safe"?	Laser bar-code scanners	The eximer laser	Communications lasers	Industrial purpose lasers	Laser bar-code scanners
Why are lasers used in fiber optic communications systems	The government has mandated it	They can be pulsed with high speed data	They are very inexpensive	They are very expensive	They can be pulsed with high speed data
What type of laser is used in CD and DVD players?	Semiconductor	YAG	Alexandrite	ruby laser	Semiconductor
Why are lasers used in Laser Printers	They can be focused down to very small spot sizes for high resolution	They are cheap	They are impossible to damage	They are very storng	They can be focused down to very small spot sizes for high resolution
As wavelength gets longer, the laser light can be focused to…	Larger spot sizes	Smaller spot sizes	High intensity	Low intensity	Smaller spot sizes
Which color of light has the shortest wavelength ?	Yellow	Blue	Red	Green	Blue
What property of laser light is used to measure strain in roadways?	Intensity	Power	Coherence	Monochromacity	Coherence
What is the type of laser used most widely in industrial materials processing applications?	Dye Laser	YAG laser	Ruby Laser	Carbon Dioxide Laser	Carbon Dioxide Laser
The Eximer laser produces light with what wavelength?	Visible	Ultraviolet	Infrared	X-Ray	Infrared
Laser energy is used to break up kidney or gallstones in process called?	Trbecularplasty	Lithotripsy	Viscocanalostomy	gastrography	Viscocanalostomy
The National Ignition Facility will use what type of laser for fusion power experimentation?	Neodymium-glass	Argon gas	Rhodamine Dye	He gas	Neodymium-glass
Chemical lasers use____ to produce their beams.	Excessive amounts of electrical power	Small amounts of electrical power	No electrical power	High electrical power	No electrical power
What type of laser could cause skin cancer if not used properly?	Red semiconductor laser	Blue semiconductor	Eximer laser	YAG laser	Eximer laser
Which of the following is unique property of Laser?	High frequency	Speed	coherence	Wavelength	coherence
Which of the following is an example of optical pumping?	Nd-YAG laser	He-Ne laser	Semiconductor laser	Dye laser	Nd-YAG laser
What is the need to achieve population inversion?	To excite most of the atoms	To bring most of the atoms ground state	To achieve stable condition	To reduce the time of production of laser	To excite most of the atoms
What kind of pumping system used in He-Ne laser	optical	direct conversion	electrical	chemical	electrical
CO_2 is what type of laser?	gas laser	liquid laser	solid laser	molecular gas laser	molecular gas laser
Which of the following is an example of optical pumping?	Nd-YAG laser	He-Ne laser	Semiconductor laser	Dye laser	Nd-YAG laser
Laser is called as __________________	non-material knife.	source	wave	particle	non-material knife.
The number of TV channels that can be accommodated using laser is	8 million	40 million	80 million	4 million	80 million
The light from a laser source is monochromatic because all the photons	are in phase	have same energy	have same amplitude	are in the same direction	have same energy
In normal population, the number of atoms in the	excited state is more	ground state is more	both states are equal	ground state is zero	ground state is more
In ruby laser the atoms are excited by	ruby rod	flash tube	silvered mirror	semi-transparent mirror	flash tube
In ruby laser, a large number of atoms occupy	ground state	excited state	metastable state	normal state	metastable state
The process of population inversion is to increase the number of atoms in the	excited state	ground state	both state	intermediate state	excited state
Laser can be used in the	fission reaction	polarization	thermo-nuclear fusion	production of white light	thermo-nuclear fusion

UNIT IV

QUANTUM MECHANICS

INTRODUCTION TO QUANTUM THEORY

Laws of thermodynamics and classical laws of electricity and magnetism provide the basis for explanation of all phenomena in classical physics. It was general belief of the scientists that these laws would suffice to account for any subsequent discovered phenomena. Classical mechanics successfully explained the motion of the objects, which are directly observable. When the objects are not observable, then the concept of classical mechanics cannot be applied.

The phenomena in the realm of the atoms, nuclei and elementary particles are commonly referred to as quantum phenomena and subject matter containing all these phenomena constitutes what is known as Quantum Physics.

Inadequacy of classical mechanics:

According to the classical mechanics, if we consider the case of an electron moving round the nucleus, its energy should decrease (because the accelerated charged particle loses energy in the form of electromagnetic waves) and therefore its velocity should decrease continuously. The ultimate result is that the electron comes closer and closer to the nucleus until it collapses. This shows the instability of the atom; it is in contradiction to the observed fact of the stability of an atom. Thus the classical mechanics fails to explain the stability of an atom.

The classical mechanics also failed to explain the spectrum of the hydrogen atom. According to the classical theory, the excited atoms of hydrogen emit electromagnetic radiations of all wavelengths continuously, while it is observed that they emit the radiation of certain wavelengths only.

Difficulties with classical theories of Black Body Radiation and Origin of Quantum Theory of Radiation:

We know that when bodies radiate energy, their temperature falls until the loss of energy is compensated by an external source. In case of heat radiation, we can obtain the thermal equilibrium by maintaining the body at a fixed temperature with the help of some heat-giving source. In this case the body gives as much radiation as it receives. If the body absorbs all the incident radiation, then it is called Black Body radiation. In actual practice, it is not possible to realize a perfectly black body, but an enclosure provided with a small opening serves the purpose because the radiation entering the enclosure will be reflected many times inside the enclosure and ultimately absorbed.

BLACK BODY RADIATION

Perfect black body:

A perfect black body is one which absorbs and emits in all the radiations (corresponding to all wavelengths) that fall on it. The radiation given out by a perfect black body is called Black body radiation.

Kirchoff's law:

Ratio of emissive power to the coefficient of absorption of any given wavelength is the same for all bodies at a given temperature and is equal to the emissive power of the black body at that temperature.

$$E_\lambda = \frac{e_\lambda}{a_\lambda}$$

Experiment:

In practice a perfect black body is not available. Therefore let us consider a hollow sphere coated with lamp black on its inner surface.

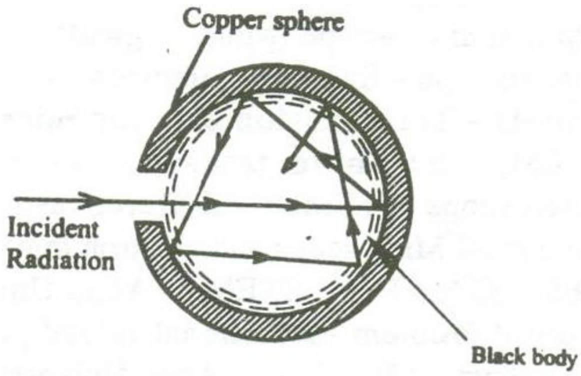


Fig 3.1

A fine hole is made for radiations to enter into the sphere as shown in the fig 3.1.

Now when the radiations are made to pass through the hole it undergoes multiple reflections and are completely absorbed. Thus the black body acts as a perfect absorber. Now when the black body is placed in a temperature bath of fixed temperature, the heat radiations will come out only through the hole in the sphere and not through the walls of the sphere.

Therefore, we can conclude that the radiations are emitted from the inner surface of the sphere and not from the outer surface of the sphere. Thus a perfect black body is a perfect absorber and also a perfect radiator of all wavelengths.

Energy spectrum:

When a perfect black body is allowed to emit radiations at different temperatures, then the distribution of the energy for different wavelengths at various temperatures is obtained as shown in the fig 3.2.

From figure the following results are formulated.

i. The energy distribution is not uniform for a given temperature.

ii. The intensity of radiation (E) increases with respect to the increase in wavelength at particular wavelength in becomes maximum (λ_m) and after this it starts decreasing with respect to the increase in wavelength.

iii. When the temperature is increased, the maximum wavelength (λ_m) decreases.

iv. For all the wavelengths an increase in its temperature causes increase in energy.

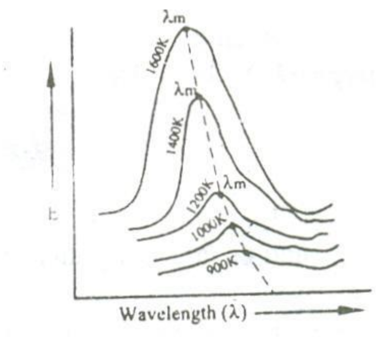


Fig 3.2

v. The total energy emitted at any particular temperature can be calculated from the area under that particular curve.

PHOTON AND ITS PROPERTIES:

According to the Quantum theory of radiation, we know that the exchange of energy values between the light radiation and particles have discrete energy values.

Photon

The discrete energy values in the form of small packets (or) bundles (or) quantas of definite frequency or wavelength are called photon. These photons are propagates like a particle like a particle but with the speed of light ($3x10^8$m/s).

Properties of photon:

- Photons are similar to that of electrons.
- We know for electrons the definite quantities are 'e' and 'm'. Similarly for photons the definite quantities are 'h' and 'c'.
- Photons will not have any charge. They are neutral and hence they are not affected by magnetic (or) electric fields.
- They do not ionize gases.
- The energy of the photon is given by E = hν, which varies with respect to the type of radiation frequencies.
- The momentum of photon is given by p = mc, where 'm' is the mass of the photon and 'c' is the velocity of light.
- The relation between energy and the momentum of the photon is given by E = pc.

[i.e., $E = mc^2 = mc(c) = pc$].

DUAL NATURE OF RADIATION (LIGHT) AND MATTER (PARTICLES) – MATTER WAVES

De- Broglie concept of dual nature:

The universe is made of radiation (light) and matter (particles). The light exhibits the dual nature (ie.) it can behave both a wave (interference, diffraction, phenomenon) and as a particle (Compton Effect, photo electric effect etc.)

Since the nature loves symmetry, in 1923 Louis debroglie suggested that an electron or any other material particle must exhibit wave like properties in addition to particle nature.

The waves associated with a material particle are called as matter waves.

De-Broglie wavelength:

From the theory of light, considering a photon as a particle the total energy of the photon is given by

$$E = mc^2 \qquad \text{---(1)}$$

Where, 'm' is the mass of the particle and 'c' is the velocity of light.

Considering the photon as a wave, the total energy is given by

$$E = h\nu \qquad \text{--- (2)}$$

Where, 'h' is the Planck's constant and 'ν' is the frequency of the radiation.

From equations (1) and (2)

$$E = mc^2 = h\nu \qquad \text{--- (3)}$$

We know Momentum = Mass × velocity

$$p = mc$$

$\therefore$ Equation (3) becomes $h\nu = pc$

$$p = \frac{h\nu}{c}$$

Since $\lambda = \frac{c}{\nu}$ we can write $\quad p = \frac{h}{\lambda}$

(or) The wavelength of a photon $\lambda = \frac{h}{p}$ --- (4)

de-Broglie suggested that the equation 3 can be applied both for photons and material particles. If m is the mass of the particle and v is the velocity the particle, then

Momentum $p = mv$

$\therefore$ de-Broglie wavelength $\quad \lambda = \frac{h}{mv}$ --- (5)

De-Broglie wavelength in terms of energy:

We know kinetic energy $E = \frac{1}{2}mv^2$

Multiplying by 'm' on both sides we get

$$Em = \frac{1}{2}m^2v^2$$

$$2Em = m^2v^2$$

(or) $mv = \sqrt{2mE}$

∴ de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$ --- (6)

De-Broglie wavelength in terms of Voltage

If a charged particle of charge 'e' is accelerated through a potential difference 'v', Then the Kinetic Energy of the particle = $\frac{1}{2}mv^2$ --- (7)

Also we know energy = eV --- (8)

Equating (7) and (8), we get,

$$\frac{1}{2}mv^2 = eV$$

Multiplying by 'm' on both sides we get

$$\frac{1}{2}m^2v^2 = meV$$

$$m^2v^2 = 2meV$$

(or) $mv = \sqrt{2meV}$ --- (9)

Substituting equation (9) in equation (5), we get

∴ de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2meV}}$ --- (10)

De-Broglie wavelength in terms of Temperature:

When a particle like neutron is in thermal equilibrium at temperature T, then they possess Maxwell distribution of velocities.

∴ Their kinetic energy $E_k = \frac{1}{2}mv_{rms}^2$ --- (11)

Where, 'v_{rms}' is the Root mean square velocity of the particle.

Also, we know energy = $\frac{3}{2}K_BT$ --- (12)

Where, 'K_B' is the Boltzmann constant.

Equating (11) and (12) we get

$$\frac{1}{2}mv^2 = \frac{3}{2}K_BT$$

Multiplying by 'm' on both sides we get

$$\frac{1}{2}m^2v^2 = \frac{3}{2}mK_BT$$

$$m^2v^2 = 3mK_BT$$

(or) $mv = \sqrt{3mK_BT}$

∴ De-Broglie wavelength $\lambda = \frac{h}{\sqrt{3mK_BT}}$

PROPERTIES OF MATTER WAVES:

- Matter waves are not electromagnetic waves.
- Matter waves are new kind of waves in which due to the motion of the charged particles, electromagnetic waves are produced.
- The wave and particle aspects cannot appear together.
- Locating the exact position of the particle in the wave is uncertain.
- Lighter particles will have high wavelength.

- Particles moving with less velocity will have high wavelength.
- The velocity of matter wave is not a constant; it depends on the velocity of the particle.
- The velocity of matter wave is greater than the velocity of light.

UNCERTAINITY PRINCIPLE

According to classical ideas, it is possible for a particle to occupy a fixed position and have a definite momentum. Hence we can predict exactly its position and momentum, at any time.

But according to quantum mechanics, there is an inherent uncertainity in the determination of the position and momentum of the particle. According to Heisenberg's principle the position and momentum of a particle cannot be determined simultaneously to any degree of accuracy.

Statement

It is impossible to determine precisely and simultaneously the values of both the position and momentum of a particle.

Example

Considering the position and momentum as a pair of physical variables. These quantiTIes are related as

$$\Delta x \Delta p \approx \frac{h}{2\pi}$$

where Δx is the error in determining position

Δp is error in determining momentum of the particle.

Similarly we have

$$\Delta E \Delta t \approx \frac{h}{2\pi}$$

$$\Delta J \Delta \theta \approx \frac{h}{2\pi}$$

where ΔE and Δt is the error in determining energy and time respectively and ΔJ and $\Delta\theta$ are the error in determining the angular momentum and angle respectively.

SCHROEDINGER WAVE EQUATION:

Schroedinger wave equation describes the wave nature of a particle in the mathematical form. It is the basic equation of motion of matter waves.

If the particle has wave properties, then there should be some sort of wave equation to describe the behavior of that particle.

Schrodinger connected the expression of de-Broglie's wavelength with the classical wave equation for a moving particle and he obtained a new wave equation

FORMS OF SCHROEDINGER WAVE EQUATION

There are two forms of Schroedinger wave equation. They are

a. Time independent wave equation

b. Time dependent wave equation

SCHROEDINGER TIME INDEPENDENT WAVE EQUATION:

Consider a wave associated with a moving particle.

Let x, y, z be the coordinates of the particle and ψ wave function for de – Broglie's waves at any given instant of time't'.

The classical differential equation for wave motion is given by

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{--- (1)}$$

Here, 'v' is wave velocity.

The eqn (1) is written as

$$\nabla^2\psi=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{--- (2)}$$

Where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is a Laplacian's operator.

The solution of eqn (2) gives ψ as a periodic variations in terms of 't',

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t} \quad \text{--- (3)}$$

Here, $\psi_0(x, y, z)$ is a function of x, y, zonly, which is the amplitude at the point considered. 'ω' is angular velocity of the wave.

Differentiating eqn (3) with respect to 't', we get

$$\frac{\partial\psi}{\partial t}=-i\omega\psi_0 e^{-i\omega t}$$

Again differentiating with respect to 't', we have

$$\frac{\partial^2\psi}{\partial t^2}=(-i\omega)(-i\omega)\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2}=i^2\omega^2\psi_0 e^{-i\omega t}$$

$$\frac{\partial^2\psi}{\partial t^2}=-\omega^2\psi \quad \text{--- (4)}$$

Where, $i^2 = -1$ and $\psi= \psi_0 e^{-i\omega t}$

Substituting eqn (4) in eqn (2), we have

$$\nabla^2\psi=-\frac{\omega^2}{v^2}\psi$$

$$\nabla^2\psi+\frac{\omega^2}{v^2}\psi = 0 \quad \text{--- (5)}$$

We know that angular frequency $\omega = 2\pi\nu$

$$\omega = 2\pi\frac{v}{\lambda}$$

Where, $\nu = \frac{v}{\lambda}$, ν is the frequency and v is the wave velocity

$$\frac{\omega}{v} = \frac{2\pi}{\lambda} \qquad \text{--- (6)}$$

Squaring the eqn (6) on both sides, we get

$$\frac{\omega^2}{v^2} = \frac{2^2\pi^2}{\lambda^2}$$

$$\frac{\omega^2}{v^2} = \frac{4\pi^2}{\lambda^2} \qquad \text{--- (7)}$$

Substituting eqn (7) in eqn (5), we get

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \qquad \text{--- (8)}$$

On substituting, $\lambda = \frac{h}{mv}$ in eqn (8), we get

$$\nabla^2\psi + \frac{4\pi^2}{\left(\frac{h}{mv}\right)^2}\psi = 0$$

$$\nabla^2\psi + \frac{4\pi^2}{\frac{h^2}{m^2v^2}}\psi = 0$$

$$\nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0 \qquad \text{--- (9)}$$

If 'E' is the total energy of the particle, 'V' is potential energy and $\frac{1}{2}mv^2$ is kinetic energy, then Total energy = Potential energy + Kinetic energy

$$E = V + \frac{1}{2}mv^2$$

$$E - V = \frac{1}{2}mv^2$$

$$2(E - V) = mv^2$$

Multiplying by 'm' on both sides, we have

$$2m(E - V) = m^2v^2 \qquad \text{--- (10)}$$

Substituting eqn (10) in eqn (9), we get

$$\nabla^2\psi + \frac{4\pi^2 2m\,(E - V)}{h^2}\psi = 0$$

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \qquad \text{--- (11)}$$

The eqn (11) is known as Schroedinger time independent wave equation.

Let us know introduce $\hbar = \frac{h}{2\pi}$ in eqn (11),

$$\hbar^2 = \frac{h^2}{2^2\pi^2}$$

$$\hbar^2 = \frac{h^2}{4\pi^2} \qquad \text{--- (12)}$$

where, 'ħ' is a reduced Planck's constant.

The eqn (11) is modified by substituting ħ,

$$\nabla^2\psi + \frac{m\,(E - V)}{\frac{h^2}{8\pi^2}}\psi = 0$$

$$\nabla^2\psi + \frac{m\,(E-V)}{\frac{h^2}{2x4\pi^2}}\psi = 0$$

$$\nabla^2\psi + \frac{2m\,(E-V)}{\frac{h^2}{4\pi^2}}\psi = 0 \qquad \text{--- (13)}$$

On substituting eqn (12) in eqn (13), Schroedinger time independent wave equation is written as,

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E-V)\,\psi = 0 \qquad \text{--- (14)}$$

Special case:

If we consider one dimensional motion i.e., particle moving along only X-direction, then Schroedinger time independent wave equation (14) reduces to

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\,\psi = 0 \qquad \text{--- (15)}$$

SCHROEDINGER TIME DEPENDENT WAVE EQUATION:

Schoredinger time independent wave equation is derived from Schroedinger time independent wave equation.

The solution of classical differential equation of the wave motion is given by

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t} \qquad \text{--- (1)}$$

Differentiating eqn (1) with respect to 't', we get

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t} \qquad \text{--- (2)}$$

$$\frac{\partial\psi}{\partial t} = -i(2\pi\nu)\psi_0 e^{-i\omega t}$$

Where, $\omega = 2\pi\,\nu$

$$\frac{\partial\psi}{\partial t} = -i(2\pi\nu)\psi \qquad \text{--- (3)}$$

Where, $\psi = \psi_0 e^{-i\omega t}$

$$\frac{\partial\psi}{\partial t} = -2\pi i\,\frac{E}{h}\psi$$

Where, $E = h\nu$ (or) $\nu = \frac{E}{h}$

$$\frac{\partial\psi}{\partial t} = -i\,\frac{E}{\frac{h}{2\pi}}\psi$$

We know that $\hbar = \frac{h}{2\pi}$

$$\therefore\ \frac{\partial\psi}{\partial t} = -i\,\frac{E}{\hbar}\psi \qquad \text{--- (4)}$$

Multiplying 'i' on both sides in eqn (4), we get

$$i\,\frac{\partial\psi}{\partial t} = -i^2\,\frac{E}{\hbar}\psi$$

We know that $i^2 = -1$

$$\therefore\ i\,\frac{\partial\psi}{\partial t} = -(-1)\,\frac{E}{\hbar}\psi$$

$$i\,\frac{\partial\psi}{\partial t} = \frac{E}{\hbar}\psi$$

$$i\hbar\,\frac{\partial\psi}{\partial t} = E\psi \qquad \text{--- (5)}$$

Schroedinger time independent wave equation is given by

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\,\psi = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E\psi - V\psi) = 0 \qquad \text{--- (6)}$$

Substituting eqn (5) in eqn (6), we get

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right) = 0$$

$$\nabla^2\psi = -\frac{2m}{\hbar^2}\left(i\hbar\frac{\partial\psi}{\partial t} - V\psi\right)$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t} - V\psi$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad \text{--- (7)}$$

$$\text{(or) } H\psi = E\psi \qquad \text{--- (8)}$$

Where, H = $\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)$ is Hamiltotnian operator and E = $i\hbar\frac{\partial}{\partial t}$ is energy operator

The eqn (8) is known as Schroedinger time dependent wave equation

PHYSICAL SIGNIFICANCE OF WAVE FUNCTION:

Wave function:

It is the variable quantity that is associated with a moving particle at any position (x, y, z) and at any time‘t’ and it relates the probability of finding the particle at that point and at that time.

- It relates the particle and the wave statistically (i.e.,) $\psi = \psi_0 e^{-i\omega t}$
- Wave function gives the information about the particle behavior.
- Ψ is a complex quantity and individually it does not have any meaning.
- $|\psi|^2 = \psi^*\psi$ is real and positive, it has physical meaning. This concept is similar to light. In light, amplitude may be positive (or) negative but the intensity, which is square amplitude, is real and is measurable.
- $|\psi|^2$ represents the probability density (or) probability of finding the particle per unit volume.
- For a given volume $d\tau$, the probability of finding the particle is given by

  Probability (P) = $\iiint |\psi|^2 d\tau$

  Where, $d\tau = dx.dy.dz$
- The probability will have any value between zero and one. (i.e.,)
  1. If P = 0 then there is no chance for finding the particle (i.e.,) there is no particle, within the given limits.
  2. If P = 1 then there is 100% chance for finding the particle (i.e.,) the particle is definitely present, within the given limits.

iii. If P = 0.7 then there is 70% chance for finding the particle and 30% there is no chance for finding the particle, within the given limits.

Example:

If a particle is definitely present within a one dimensional box (x-direction) of length ‘l’, then the probability of finding the particle can be written as

$$P = \int_0^l |\psi|^2 \, dx = 1$$

PARTICLE IN A ONE DIMENSIONAL BOX:

Let us consider particle (electron) of mass ’m’ moving along the x - axis, enclosed in a one dimensional potential box as shown in the figure 3.10.

Since the walls are of infinite potential the particle does not penetrate out from the box.

Also, the particle is confined between the length ‘l’ of the box and has elastic collisions with the walls. Therefore, the potential energy of the electron inside the box is constant and can be taken as zero for simplicity.

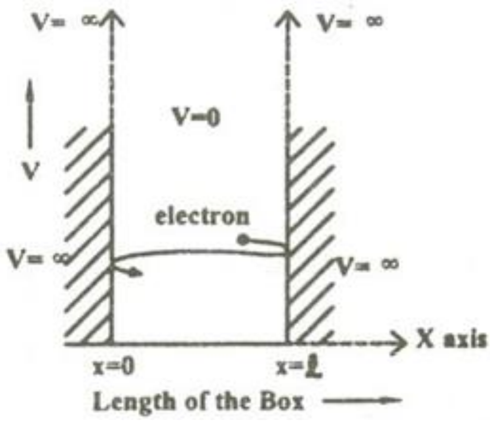


Fig 3.10

∴ We can say that the Outside the box and on the wall of the box, the potential energy V of the electron is ‘α’.

Inside the box the potential energy (V) of the electron is zero.

In other words we can write the boundary condition as

$$V(x) = 0 \text{ when } 0 < x < l$$

$$V(x) = \alpha \text{ when } 0 \geq x \geq l$$

Since the particle cannot exist outside the box the wave function $\psi = 0$ when$0 \geq x \geq l$.

To find the wave function of the particle within the box of length ‘l’, let us consider the Schroedinger one dimensional time independent wave equation(i.e.,)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

Since the potential energy inside the box is zero [i.e.V = 0], the particle has kinetic energy alone and thus it is named as a free particle (or) free electron

∴ For a free particle (electron), the Schroedinger wave equation is given by

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0$$

Or $$\frac{\partial^2 \psi}{\partial x^2} + k^2 E\psi = 0 \qquad \text{--- (1)}$$

Where, $$k^2 = \frac{2mE}{\hbar^2} \qquad \text{--- (2)}$$

Equation (1) is a second order differential equation; therefore, it should have solution with two arbitrary constants.

∴ The solution for equation (1) is given by

$$\Psi(x) = A \sin kx + B \cos kx \quad \text{--- (3)}$$

Where, A and B are called Arbitrary constants, which can be found by applying the boundary conditions.

(i.e.,) $V(x) = \alpha$ when $x = 0$ and $x = l$

Boundary condition (i) at x=0 Potential energy $V = \alpha$. ∴ There is no chance for finding the particle at the walls of the box, ∴ $\psi(x) = 0$

∴ Equation (3) becomes

$$0 = A \sin 0 + B \cos 0$$

$$0 = 0 + B\,(1)$$

$$\therefore \mathbf{B = 0}$$

Boundary condition (ii) at $x = l$ Potential energy $V = \alpha$. ∴ There is no chance for finding the particle at the walls of the box, ∴ $\psi(x) = 0$

∴ Equation (3) becomes

$$0 = A \sin kl + B \cos kl$$

Since B = 0 (from the first boundary condition), we have

$$0 = A \sin kl$$

Since $A \neq 0$; $\sin kl = 0$

We know $\sin n\pi = 0$

Comparing these two equations,

We can write $kl = n\pi$

Where, n is an integer.

$$\text{(or) } k = \frac{n\pi}{l} \quad \text{--- (4)}$$

Substituting the value of B and k in equation 3 we can write the wave function associated with the free electron confined in a one dimensional box as

$$\Psi_n(x) = A \sin \frac{n\pi}{l} \quad \text{--- (5)}$$

Energy of the particle (Electron)

We know from equation (2),

$$k^2 = \frac{2mE}{\hbar^2}$$

Where, $\hbar^2 = \frac{h^2}{4\pi^2}$

$$k^2 = \frac{2mE}{\left(\frac{h^2}{4\pi^2}\right)}$$

$$\text{(or)} \quad k^2 = \frac{8\pi^2 mE}{h^2} \quad \text{--- (6)}$$

Squaring eqn (4), we get

$$k^2 = \frac{n^2\pi^2}{l^2} \qquad \text{--- (7)}$$

Equating (6) and (7), we can write

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{l^2}$$

$$E = \frac{n^2\cancel{\pi^2}h^2}{8\cancel{\pi^2}ml^2}$$

∴ Energy of the particle (electron) $E_n = \frac{n^2h^2}{8ml^2}$ --- (8)

∴ From equations (8) and (5) we can say that, for each value of 'n', there is an energy level and the corresponding wave function.

Thus we can say that, each value of E_n is known as Eigen value and the corresponding value of ψ_n is called Eigen function.

Energy levels of an electron

For various values of 'n' we get various energy values of the electron. The lowest energy value or ground state energy value can be got by substituting n = 1 in equation (8)

∴ When n= 1 we get $E_1 = \frac{1^2h^2}{8ml^2} = \frac{h^2}{8ml^2}$

Similarly we can get the other energy values

When n= 2, we get $E_2 = \frac{2^2h^2}{8ml^2} = \frac{4h^2}{8ml^2} = 4E_1$

When n = 3, we get $E_3 = \frac{3^2h^2}{8ml^2} = \frac{9h^2}{8ml^2} = 9E_1$

When n = 4, we get $E_4 = \frac{4h^2}{8ml^2} = \frac{16h^2}{8ml^2} = 16E_1$

∴ In general we can write the energy Eigen function as

$$\mathbf{E_n = n^2E_1} \qquad \text{--- (9)}$$

It is found that from the energy levels E_1, E_2, E_3 etc the energy levels of an electron are discrete.

This is the great success which is achieved in Quantum Mechanics than classical mechanics, in which the energy levels are found to be continuous.

The various energy Eigen values and their corresponding Eigen functions of an electron enclosed in a one dimensional box is as shown in the fig 3.11.

Thus we have discrete energy values.

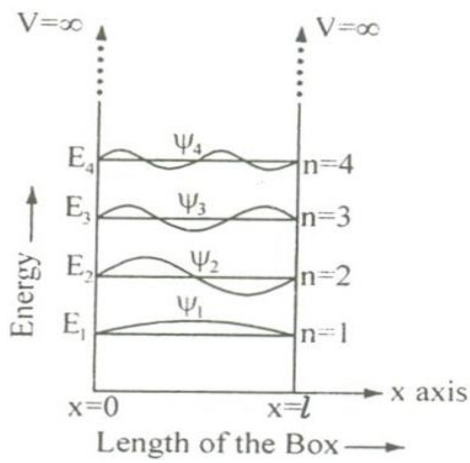


Fig 3.11

Normalization of the wave function:

Normalization:

It is the process by which the probability (P) of finding the particle (electron) inside the

$$\mathbf{A} = \sqrt{\frac{2}{l}}$$

Substituting the Value of A in equation (5),

The normalized wave function can be written as

$$\Psi_n = \sqrt{\frac{2}{l}} \sin \frac{n\pi}{l}$$

The normalized wave function and their energy values are as shown in the fig 3.12

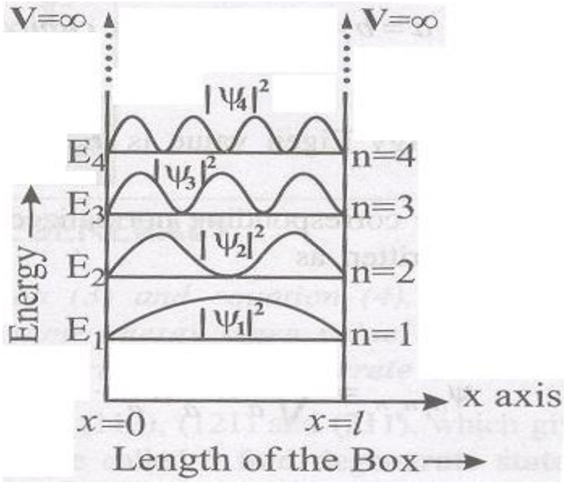


Fig 3.12

box can be done.

We know that the total probability (P) is equal to 1 means then there is a particle inside the box. Therefore, for a one dimensional potential box of length 'l' the probability

$$P = \int_0^l |\psi|^2 \, dx = 1 \qquad \text{--- (10)}$$

(Since the particle is present inside the well between the length 0 to 'l' the limits are chosen between 0 to l)

Substituting equation (5) in equation (10), we get

$$P = \int_0^l A^2 \sin^2 \frac{n\pi}{l} \, dx = 1$$

$$\text{(or) } A^2 \int_0^l \left[\frac{1-\cos 2\left(\frac{n\pi x}{l}\right)}{2}\right] dx = 1$$

$$A^2\left[\frac{x}{2} - \frac{1}{2}\frac{\sin\left(\frac{2n\pi x}{l}\right)}{\left(\frac{2n\pi}{l}\right)}\right]_0^l = 1$$

$$A^2\left[\frac{l}{2} - \frac{1}{2}\frac{\sin\left(\frac{2n\cancel{l}\pi}{\cancel{l}}\right)}{\left(\frac{2n\pi}{l}\right)}\right] = 1$$

$$A^2\left[\frac{l}{2} - \frac{1}{2}\frac{\sin(2n\pi)}{\left(\frac{2n\pi}{l}\right)}\right] = 1$$

We know $\sin n\pi = 0$ ∴ $\sin 2n\pi$ is also $= 0$

∴ Equation 11 can be written as

$$A^2\left[\frac{l}{2}\right] = 1$$

$$A^2 = \frac{2}{l}$$

DEGENERACY AND NON-DEGENERACY:

Degeneracy:

It is seen from equation (3) and equation (3), for several combination of quantum numbers we have same energy Eigen value but different Eigen functions. Such states and energy levels are called Degenerate state.

The three combinations of quantum numbers (112), (121) and (211) which gives same Eigen value but different Eigen functions are 3 fold degenerate state.

Non –degeneracy:

For various combinations of quantum number if we have same energy value and same (one) Eigen function then such states and energy levels are called Non – degenerate state.

BASICS OF A MICROSCOPE:

A microscope is a device which is used to view the magnified image of a smaller object, which cannot be clearly seen through a naked eye.

In general we can classify the microscope as simple and compound microscope. A simple microscope is made up of a single biconvex magnifying lens held in a simple frame. A compound microscope is made up of two lenses (or) system of lenses for better magnification.

Depending on the field of application, many other microscopes such as phase contrast microscope, UV microscope, metallurgical microscope, electron microscope, etc are designed. These types of microscopes give a stereoscopic vision and reduce the strain of our eyes.

Magnifying power:

The magnifying power (M) of a microscope is defined as the ratio between the angle subtended by the final image at the eye (β) to the angle subtended by the object at the eye (α), placed at the near point.

$$M = \frac{\beta}{\alpha}$$

Resolving power:

It is the ability of an optical instrument to form a distinct and separable image of the two point objects which are close to each other.

If‘d’ is the least distance between two close point objects, then we can write

$$d = \frac{\lambda_0}{2\,NA}$$

∴ Resolving power = $\frac{I}{d} = \frac{2NA}{\lambda_0}$

Where, NA be the numerical aperture of the objective of the microscope and λ_0 be the wavelength of light through vacuum.

Therefore the resolving power of a microscope can be increased by decreasing the value of λ_0. Thus, by using UV light and quartz lenses, the resolving power can be increased.

ELECTRON MICROSCOPE

It is a type of microscope in which instead of light beam, a beam of electrons are used to form a large image of very small object. These microscopes are widely used in the field of engineering and medicine.

Principle:

A stream of electrons is passed through the object and the electron which carries the information about the object are focused by electric and magnetic fields.

Since the resolving power is inversely proportional to the wavelength, the electron microscope has high resolving power because of its shorter wavelength.

Construction:

An electron microscope is similar to that of an optical microscope. Here the focusing of electrons can be done either by magnetic lens or by electrostatic lens. Normally in electron microscope magnetic lenses are used for focusing.

In general, the magnetic lenses are made of two coils C1 and C2 enclosed inside the iron cases which have one hole as shown in fig 3.13. When the holes face each other, the magnetic field in space between the two coils focuses the electrons emerging out from the electron gun. Similarly the divergence of the electrons can also be made by adjusting the position of the holes in the iron cases.

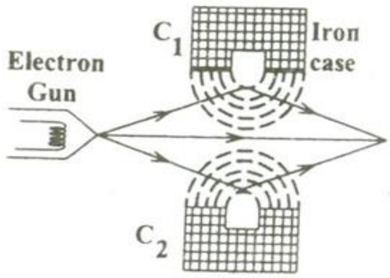
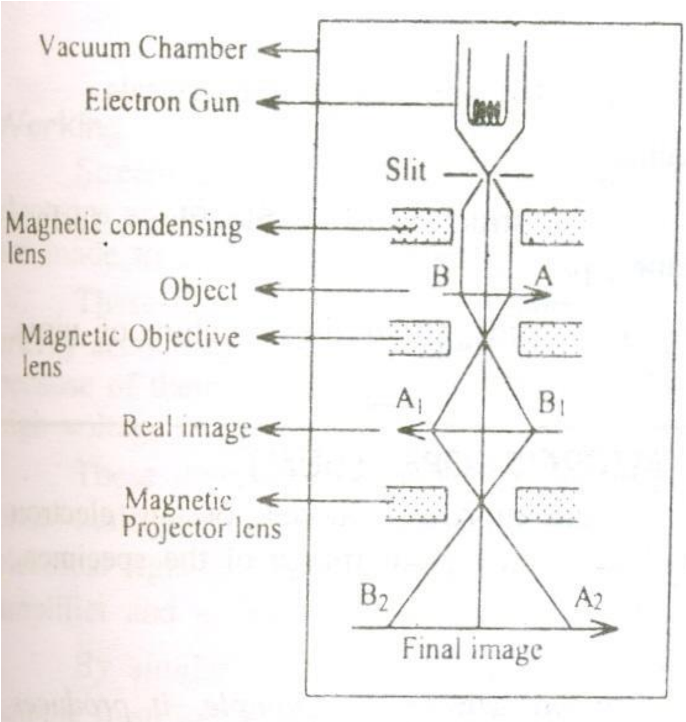


Fig 3.13

The essential parts of an electron microscope are as shown in the fig 3.14 and for comparison an optical microscope is also shown in fig 3.15.



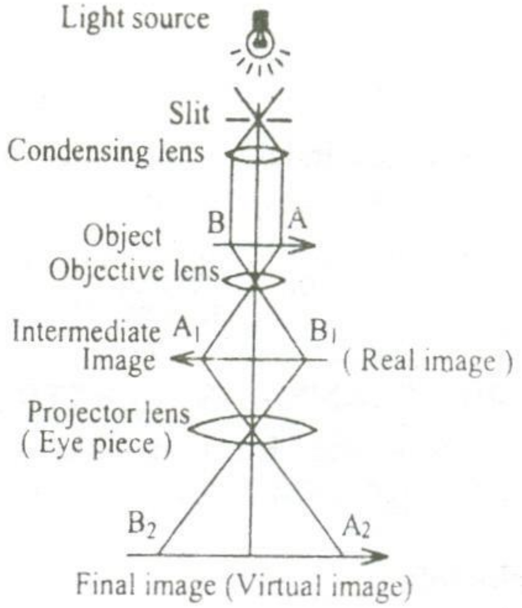


Fig 3.14 Fig 3.15

The electron microscope consists of an electron gun to produce the stream of electrons. Similar to the condensing lens, objective and eye piece in an optical microscope here three magnetic lenses are used.

- Magnetic condensing lens
- Magnetic objective lens
- Magnetic projector lens

The whole arrangement is kept inside a vacuum chamber to allow the passage of electron beam.

Working:

Stream of electrons are produced and accelerated by the electron gun. The electron beam is made to pass through the center of the doughnut shaped magnetic condensing lens. These electrons are made as parallel beam and are focused on to the object AB (fig 3.14). The electrons are transmitted more in the less dense region of the object and is transmitted less (i.e.,) absorbed by the denser region of the object. Thus the transmitted electron beam on the falling over the magnetic objective lens, resolves the structure of the object to form a magnified real image of the object. Further the image can be magnified by the magnetic projector lens and the final image is obtained on the fluorescent screen.

In order to make a permanent record of the image of the object, the final image can also be obtained on a photographic plate.

Advantages:

- It can produce magnification as high as 1, 00,000 times as that of the size of the object.

- The focal length of the microscopic system can be varied.

Applications:

It has a very wide area of applications (Eg.) in biology, metallurgy, physics, chemistry, medicine, engineering etc.

- It is used to determine the complicated structure of the crystals.
- It is used in the study of the colloids.
- In industries it is used to study the structure of textile fibers, surface of metals, composition of paper, paints etc.
- In the medical field it is used to study about the structure of virus, bacterial etc which are of smaller size.

SCANNING ELECTRON MICROSCOPE

Scanning electron microscope is an improved model of an electron microscope. SEM is used to study the three dimensional image of the specimen.

Principle:

When the accelerated primary electron strikes the sample, it produces secondary electrons. These secondary electrons are collected by a positive charged electron detector which in turn gives a 3- dimensional image of the sample.

Construction:

It consists of an electron gun to produce high energy electron beam. A magnetic condensing lens is used to condense the electron beam and a scanning coil is arranged in-between magnetic condensing lens and the sample.

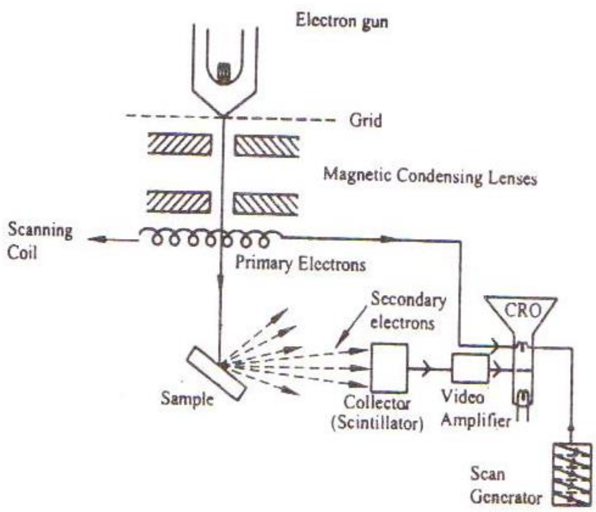


Fig 3.16

The electron detector (Scintillator) is used to collect the secondary electrons and can be converted into electrical signal. These signals can be fed into CRO through video amplifier as shown in fig 3.16.

Working:

Stream of electrons are produced by the electron gun and these primary electrons are accelerated by the grid and anode. These accelerated primary electrons are made to be incident on the sample through condensing lenses and scanning coil.

These high speed primary electrons on falling over the sample produce low energy secondary electrons. The collections of secondary electrons are very difficult and hence a high voltage is applied to the collector.

These collected electrons produce scintillations on to the photo multiplier tube are converted into electrical signals. These signals are amplified by the video amplifier and are fed to the CRO.

By similar procedure the electron beam scans from left to right and again right to left etc., similar to we read a book (fig 3.17) and the whole picture of the sample is obtained in the CRO screen.

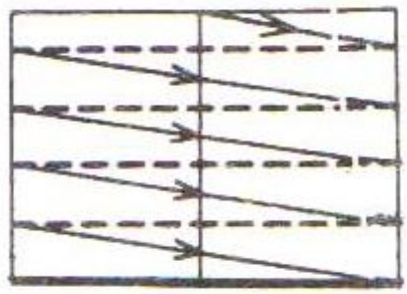


Fig 3.17

Advantages:

- It can be used to examine specimens of large thickness.
- It has large depth of focus.
- It can be used to get a three dimensional image of the object.
- Since the image can be directly viewed in the screen, structural details can be resolved in the precise manner.
- The magnification may be upto 3, 00,000 times greater than that of the size of the object.

Disadvantages:

The resolution of the image is limited to about 10-20 nm, hence it is very poor.

Applications:

- It is used to examine the structure of very large specimens in a three dimensional view.
- Similar to the application of electron microscope this SEM also has applications over various fields such as Biology, Industries, Engineering, Physics, Chemistry, etc.

PROBLEMS

1. **Calculate de-Broglie wavelength associated with a proton moving with a velocity equal to $\frac{1}{20}^{th}$ of the velocity of light.**

   Mass of proton = 1.675×10^{-27}kg

   Given data:

   Mass of proton m = 1.675×10^{-27}kg

   Velocity of proton v = $\frac{1}{20}$ x velocity of light

   $v = \frac{1}{20} \times 3\times10^{8}$

   $v = 15 \times 10^{6}$ m/s

   Solution:

   de-Broglie wavelength $\lambda = \frac{h}{mv}$

   $\lambda = \frac{6.63\times10^{-34}}{1.675\times10^{-27}\times15\times10^{6}}$

   $\boldsymbol{\lambda = 2.64 \times 10^{-14}}$ **m**

2. **Calculate the de-Broglie wavelength of an electron of energy 100 eV.**

   Given data:

   Energy of electron (E) = 100 eV =$100 \times 1.6 \times 10^{-19}$ Joules

   $E = 1.6 \times 10^{-17}$ Joules

   Solution:

   de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

   $\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-17}}}$

   $\lambda = 1.227 \times 10^{-10}$ m

   $\boldsymbol{\lambda = 1.227 A^{\circ}}$

3. **An electron is accelerated by a potential of 150 V. what is the wavelength of that electron wave?**

   Given data:

   Accelerated potential of an electron (V) = 150 V

   Solution:

   de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2meV}}$

   $\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times150}}$

   $\lambda = 1.0018 \times 10^{-10}$m

   $\boldsymbol{\lambda = 1 A^{\circ}}$

4. **Calculate the de-Broglie wavelength corresponding to the root mean square velocity of hydrogen molecules at 27˚C.**

Given data:

Temperature T = 27˚C = 300K

Mass of hydrogen = mass of proton = 1.678 $\times 10^{-27}$kg

Solution:

de-Broglie wavelength $\lambda = \frac{h}{\sqrt{3mK_BT}}$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{3\times1.678\times10^{-27}\times1.38\times10^{-23}\times300}}$$

λ = 1.451 A˚

5. **An electron is confined to a one dimensional box of side 10^{-10}m. Obtain the first two Eigen values of the electron.**

Given data:

Length of one dimensional box 'l' = 10^{-10}m

Solution:

1st Eigen value,

Eigen Energy of the particle (electron) $E_1 = \frac{1^2(6.63\times10^{-34})^2}{8\times9.1\times10^{-31}\times(10^{-10})^2}$

$E_1 = 6.022 \times 10^{-18}$J

(or) $\boldsymbol{E_1 = \frac{6.022\times10^{-18}}{1.6\times1.6^{-19}}}$ **= 37.63 eV**

2nd Eigen value,

Eigen Energy of the particle (electron) $\boldsymbol{E_2 = 2^2E_1}$ **= 2.408 x 10^{-17}J = 150 eV**

(or) $$E' = \frac{7.8002\times10^{-14}}{1.6\times10^{-19}}$$

$$\mathbf{E' = 0.4875 \times 10^6\ eV}$$

6. **Calculate the magnifying power of a microscope. Give that the angle subtended by the final image (β) is 40˚ at eye and the angle subtended by the object at the eye kept at the near point (α) is 10˚.**

Given data:

$\beta = 40^\circ$; $\alpha = 10^\circ$

Solution:

Magnifying power $M = \frac{\beta}{\alpha}$

Or $M = \frac{tan\beta}{tan\alpha}$

$= \frac{tan40^\circ}{tan10^\circ}$

∴ **Magnifying power M = 4.758**

questions	opt1	opt2	opt3	opt4	answer
Waves associated with electrons are referred to as	plasma waves	UV waves	gamma rays	matter waves	matter waves
Frequency below which no electrons are emitted from metal surface is	minimum frequency	angular frequency	maximum frequency	threshold frequency	threshold frequency
Loss of energy of an electron results in	absorption of photon	emission of photon	destruction of photon	formation of photon	emission of photon
According to Newton, light travels as	particles	waves	both A and B	dust	particles
In electron diffraction, rings behave as	particles	waves	both A and B	rays	waves
Energy absorbed by electron is used in	escaping the metal	increasing kinetic energy	both A and B	increasing frequency	both A and B
Diffraction of slow moving electrons is used to estimate	arrangement of atoms in metals	nature of atoms	number of atoms in metals	position of atoms in metalloids	arrangement of atoms in metals
Energy of photon is directly related to the	wavelength	wave number	frequency	amplitude	frequency
When a charged particle is accelerated through a potential difference V, it's kinetic energy	decreases	remains same	increases	varies depending on resistance of wire	increases
Energy of an electron in an atom is	quantized	continuous	radial	randomized	quantized
In dark, LDR has	low resistance	high current	high resistance	both A and B	high resistance
Electrons show diffraction effects because their de Broglie wavelength is similar to	spacing between atomic layers	no. of atomic layers	nature of atomic layers	positioning of atomic layers	spacing between atomic layers
Plank's constant has units	J	s	J /s	J s	J s
Gas atoms that exert negligible electrical forces on each other are	molecules	compounds	isotopes	isolated atoms	isolated atoms
Quantum of electromagnetic energy is called	particles	photons	waves	energy	surface
In photoelectric effect, electrons should be removed from the	inner shells	surface	from core	the nucleus	surface
Light interacts with matter as	wave	particle	both A and B	rays	particle
When white light is passed through cool gases, spectra observed is called	line spectra	continuous spectra	emission line spectra	absorption line spectra	absorption line spectra
Wavelength of ultraviolet region of electromagnetic spectrum is	121 nm	120 nm	119 nm	130 nm	121 nm
In an insulator, valence band is	fully occupied	fully empty	half filled	half charged	fully occupied
Most energetic photons are	alpha	beta	gamma	x-rays	gamma
Which of the following colors is associated with the lowest temperature of a black body radiator?	Violet	Blue	Green	Red	Red
Classical physics could not explain the behavior of a black body radiator at very short wavelengths. What was this problem called?	Absorption failure	Ultraviolet Explosion	Wavelength decrease	Photoelectric Effect	Ultraviolet Explosion
What did Max Planck propose to solve the black body radiator problem?	Radiation is made up of waves.	Light changes its speed in different media.	Light comes in packets of energy.	Light has a continuous energy profile.	Light comes in packets of energy.
The energy of a photon depends on its:	Amplitude	Speed		Frequency	Frequency
How does the energy of a photon change if the wavelength is doubled?	Doubles	Quadruples	Stays the same	Is cut to one-half	Is cut to one-half
How does the momentum of a photon change if the wavelength is halved?	Doubles	Quadruples	Stays the same	Is cut to one-half	Doubles
The photoelectric effect was explained by Albert Einstein by assuming that:	light is a wave.	light is a particle.	an electron behaves as a wave.	an electron behaves as a particle.	light is a particle.
The kinetic energy of photoelectrons depends on the:	speed of light.	angle of illumination.	intensity of the light.	photon frequency.	photon frequency.
When an electron falls from an orbit where n = 2 to n = 1:	A photon is emitted.	A photon is absorbed.	No change in atomic energy.	The atomic energy increases.	A photon is emitted.
When an electron jumps from an orbit where n = 1 to n = 4, its energy in terms of the energy of the ground level (E_1) is:	E1/9	2 E 0	2 E1	16 E1	16 E1
The Compton Effect supports which of the following theories?	Special Theory of Relativity.	Light is a wave.	Thomson model of the atom.	Light is a particle.	Light is a particle.
Which one of the following objects, moving at the same speed, has the greatest de Broglie wavelength?	Neutron	Electron	Tennis ball	Bowling ball	Electron
Which theory explains the interaction of photons with matter (electrons)?	Quantum Chromodynamics	The Standard Model	String Theory	Quantum Electrodynamics	Quantum Electrodynamics
Which theory explains the attraction between protons and neutrons?	Quantum Chromodynamics	The Grand Unified Theory	The Standard Model.	String Theory	Quantum Chromodynamics
How much of the universe is comprised of matter and energy that is explained by current Physics theory?	95 percentage	75percentage	50percentage	5 percentage	5 percentage
A perfect black body is one which ________________ all the radiations.	absorbs	emits	absorbs and emits	reflects	absorbs and emits
The classical theory was not able to explain the_______	diffraction	interference	emission of black body radiation	diffraction and interference	emission of black body radiation
The wave nature associated with a material particle is called as __________	standing wave	progressive wave	transverse wave	matter wave	matter wave
The relation between energy and the momentum of the photon is_______	P is equal to EC	Eis equal toP/C	C is equal toEP	E is equal to PC	E is equal toPC
According to de-broglie wave equation, when velocity of the particle increases wavelength will be___________.	doubles	increases	decreases	zero	decreases
A particle in one dimensional box at the walls of the box, the wave function will be___________.	zero	increases	decreases	Infinity	zero
A perfect black body is a perfect absorber and radiator of___________ radiation.	monochromatic	all wavelengths of the given	coherent	polychromatic	all wavelengths of the given
The source used in the SEM is ___________.	electrical source	chemical source	neutron gun	electron gun	electron gun
For a free particle, the potential energy is___________.	0	1	2	3	0
According to ___________ theory, the hydrogen spectrum is a discrete spectrum.	classical	electromagnetic	quantum	wave	quantum
The equation of motion of matter wave was derived by	Heisenberg	Bohr	de Broglie	Schrodinger	Schrodinger

UNIT -V
ELECTRONIC MATERIALS

INTRODUCTION

Materials can be broadly classified into three types based on conductivity. They are,

1. Conductors (Example: metals),
2. Semi – conductors (Example: germanium, silicon) and
3. Insulators (Example: wood, mica, glass).

Conductors:

- Conductivity is the ability or power to conduct or transmit heat, electricity, or sound.
- Conductors are materials that electricity easily passes through, that do not resist the flow of electricity.
- Examples are copper, aluminum, steel, silver, gold, electrolytes.
- Low resistive materials are generally called as conducting materials.
- The conducting property of the solid is due to valence electrons or free electrons.

ELECTRON THEORY OF METALS

The electron theory of metals explain the following concepts

- Structural, electrical and thermal properties of materials.
- Elasticity, cohesive force and binding in solids.
- Behaviour of conductors, semi conductors, insulators etc.

So far **three electron theories** have been proposed.

1. Classical Free electron theory

- It is a macroscopic theory.
- Proposed by Drude and Loretz in 1900.
- It explains the free electrons in lattice
- It obeys the laws of classical mechanics.

2. Quantum Free electron theory

- It is a microscopic theory.
- Proposed by Sommerfield in 1928.
- It explains that the electrons move in a constant potential.
- It obeys the Quantum laws.

3. **Brillouin Zone theory or Band theory:**

- Proposed by Bloch in 1928.
- It explains that the electrons move in a periodic potential.
- It also explains the mechanism of semi conductivity, based on bands and hence called band theory.

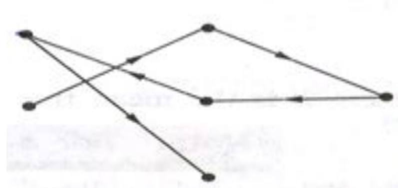
CLASSICAL FREE ELECTRON THEORY OF METALS

After the discovery of electron, Drude and Lorentz attempted for an explanation of electrical and thermal conductivities based on the assumption that metal contains a certain number of free electrons and hence called **free electron theory**. All the free electrons freely move in the crystal like molecules of a gas in a container. Mutual repulsion between electrons is ignored and hence potential energy is taken as zero. Therefore the total energy of the electron is equal to its kinetic energy.

Postulates of Classical free electron theory:

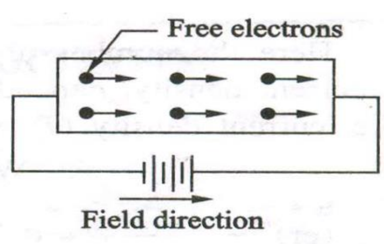
In the absence of Electric field:

All the atoms are composed of atoms. Each atom have central nucleus around which there are revolving electrons. The electrons are free to move in all possible directions about the whole volume of metals. In the absence of an electric field the electrons move in random directions making collisions from time to time with positive ions which are fixed in the lattice or other free electrons. All the collisions are elastic i.e.; no loss of energy.



In the presence of Electric field:

When an external field is applied the free electrons are slowly drifting towards the positive potential. Since the electrons are assumed to be a perfect gas they obey classical kinetic theory of gasses. Classical free electrons in the metal obey Maxwell-Boltzmann statistics.



Drift velocity (V_d):

The average velocity acquired by the free electron in a particular direction due to the application of electric field is called as Drift velocity.

Mobility (μ):

Mobility is defined as the drift velocity acquired by the free electrons per unit electric field (E) applied to it.

$$\mu = \frac{V_d}{E}$$

Where, l is the distance travelled by the electron.

Relaxation time (τ):

It is the time taken by the free electron to reach its equilibrium position from its disturbed position in the presence of applied field.

$$\tau = \frac{l}{V_d}$$

Collision time (τ_c):

It is the average time taken by the free electron between two successive collision.

$$\tau = \frac{\lambda}{V}$$

Where, λ is the mean free path.

Mean free path:

The average distance travelled between two successive collisions is called mean free path.

(i.e) $\lambda = \bar{c}\tau_c$, where $\bar{c}$ is the root mean square velocity of the electron.

QUANTUM FREE ELECTRON THEORY

Classical free electron theory could not explain many physical properties. In 1928, Sommerfeld developed a new theory applying quantum mechanical concepts and Fermi-Dirac statistics to the free electrons in the metal. This theory is called quantum free electron theory. Classical free electron theory permits all electrons to gain energy. But quantum free electron theory permits only a fraction of electrons to gain energy.

According to Classical free electron theory the particles (electrons) of a gas at zero kelvin will have zero kinetic energy and hence all the particles are found to be in rest. But according to quantum theory when all the particles are at rest, all of them should be filled only in the ground state energy level, which is impossible and is controversial to the Pauli's exclusion principle.

Thus inorder to fill the electrons in a given energy level, we should know the following.

i. Energy distribution of electrons
ii. Number of available energy states
iii. Number of filled energy states
iv. Probability of filling an electron in a given energy state

As the "free electron gas" obeys Fermi-Dirac statistics, all the above can be very easily determined using it.

Fermi energy level (E_F)

Fermi energy level is the maximum energy level upto which the electrons can be filled at 0K.

Importance:

1. Fermi energy level act as a reference level which seperates the vacant and filled states at 0K.
2. It gives the information about the filled electron states and the empty states.
3. At 0K, below Fermi energy level electrons are filled at above Fermi energy level it will be empty.
4. When the temperature is increased, few electron gains the thermal energy and it goes to higher energy levels.

FERMI DISTRIBUTION FUNCTION

Fermi distribution function F (E) represents the probability of an electron occupying a given energy state. To find out the energy states actually occupied by the free electron at any temperature (T) we can apply the Fermi – Dirac statistics. The Fermi – Dirac statistics deals with the particles (Electrons) having half integral spin, named as Fermions.

$$F(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{K_BT}\right)}} \qquad \ldots(1)$$

Where E_F is the Fermi energy

K_B is the Boltzmann constant

EFFECT OF TEMPERATURE ON FERMI FUNCTION

The effect of temperature on Fermi function F (E) can be discussed with respect to equation (1).

(i) At 0 kelvin

At 0 kelvin, the electrons can be filled only upto a maximum energy level called Fermi energy (E_{F_0}), above E_{F_0} all the energy levels will be empty. It can be proved from the following conditions.

(i) When $E < E_F$, equation (1) becomes,

$$F(E) = \frac{1}{1+e^{-\infty}} = \frac{1}{1} = 1$$

[i.e., 100% chance for the electron to be filled within the Fermi energy level]

(ii) When $E>E_F$, equation (1) becomes,

$$F(E) = \frac{1}{1+e^{\infty}} = \frac{1}{\infty} = 0$$

[i.e Zero % chance for the electron not to be filled within the Fermi energy level]

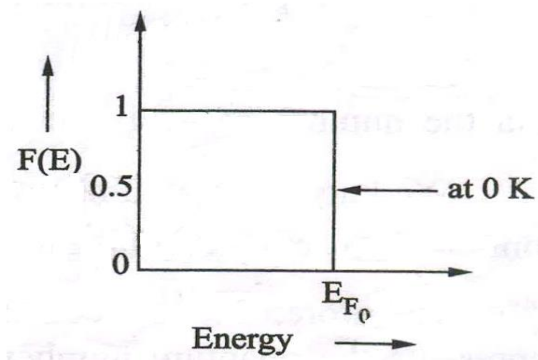
(i) When $E=E_F$, equation (1) becomes,

$$F(E) = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

[i.e 50% chance for the electron to be filled and not to be filled within the Fermi energy level]

This clearly shows that 0 kelvin all the energy states below E_{F_0} are filled and all those above it are empty.

The Fermi function at 0 kelvin (E_{F_0}) can also be represented graphically as shown in fig. It is seen from the figure that the curve has step-like character at 0 kelvin.

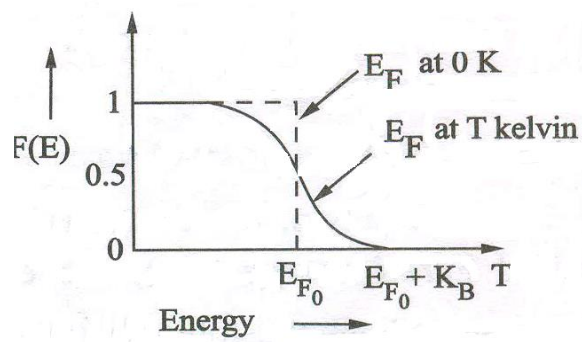


(i.e) F (E) = 1 below (E_{F_0})

and F (E) = above (E_{F_0})

(i) At any temperatue T kelvin:

When the temperature is raised slowly from absolute zero, the Fermi distribution function smoothly decreases to zero as shown in fig.



Explanation:

Due to the supply of thermal energy the electrons within the range of K_BT below the Fermi level (E_{F_0}) alone takes the energy (=K_BT) and goes to higher energy state. Hence at any temperature (T), empty states will also be available belowE_{F_0}.

DENSITY OF STATES AND CARRIER CONCENTRATION IN METALS

The Fermi Function F (E) gives only the probability of filling up of electrons in a given energy state, it does not gives the information about the number of electrons that can be filled in a given energy state. To know that we should know the number of available energy states so called density of states.

Definition:

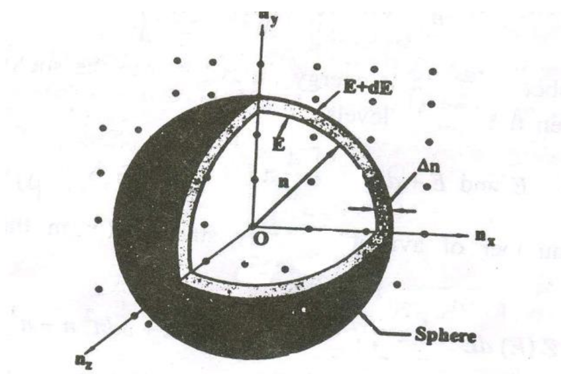
Density of States Z (E) dE is defined as the number of available energy states per unit volume in an energy interval.(dE).

Explanation:

In order to fill the electrons in an energy state we have to first find the number of available energy states within a given energy interval.

We know that a number of available energy levels can be obtained for various combinations of quantum numbers n_x, n_y and n_z ((i.e) $n^2 = n^2_x+n^2_y+n^2_z$)

Therefore, let us construct a three dimensional space of points which represents the quantum numbers as shown in fig. In this space each point represents an energy level.



Number of energy levels in a cubical metal piece:

To find the energy levels in a cubical metal piece and to find the number of electrons that can be filled in a given energy level, let us construct a sphere of radius 'n' in the space.

The sphere is further divided into many shells and each of this shell represents a particular combination of quantum numbers (n_x, n_y and n_z) and therefore represents a particular energy value.

Let us consider two energy values E and E+dE. The number of energy states between E and E+dE can be found by finding the number of energy states between the shells of radius n and n+, from the origin.

The number of available energy states within the sphere of radius

$$\text{‘ n ‘} = [\tfrac{4}{3}\pi n^3]$$

Since will have only positive values, we have to take only one octant of the sphere (i.e) 1/8th of the sphere volume.

The number of available energy states within the sphere of radius

$$\text{‘ n ‘} = \frac{1}{8}[\frac{4}{3}\pi n^3]$$

Similarly, the number of available energy states within the sphere of radius

$$n + dn = \frac{1}{8}[\frac{4}{3}\pi\ (n + dn)^3]$$

Therefore, the number of available energy states between the shells of radius n and n + dn (or) between the energy levels E and E + d E

$$= \frac{1}{8}[\frac{4}{3}\pi(n+dn)^3 - \frac{4}{3}\pi n^3]$$

(i.e) The number of available energy states between the energy interval d E is

$$Z(E)\ d\ E = \frac{1}{8}\frac{4\pi}{3}(n^3 + dn^3 + 3n^2dn + 3dn^2dn - n^3)$$

Since the higher powers of dn is very small, dn2 and dn3 terms can be neglected.

$$Z(\ E\ )\ d\ E = \frac{\pi}{6}\ 3n^2dn$$

(or)

$$Z(E)\,dE = \frac{\pi}{2} n^2 dn \qquad \dots (1)$$

We know the energy of the electron in a cubical metal piece of sides ‘l’,

$$E = \frac{n^2 h^2}{8ml^2}$$

$$\text{(or)} \quad n^2 = \frac{8ml^2 E}{h^2} \qquad \dots(2)$$

$$n = \left[\frac{8ml^2 E}{h^2}\right]^{1/2} \qquad \dots(3)$$

Differentiating equation (2) we get

$$2ndn = \frac{8ml^2}{h^2}\, dE$$

$$ndn = \frac{8ml^2 E}{2h^2}\, dE \qquad \dots(4)$$

Equation (1) can be written as

$$Z(E)\,dE = \frac{\pi}{2} n(ndn)$$

Substituting equation (3) and (4) in the above equation we have

$$Z(E)\,dE = \frac{\pi}{2}\left[\frac{8ml^2 E}{h^2}\right]^{1/2} \left[\frac{8ml^2}{2h^2}\right] dE$$

$$= \frac{\pi}{2} \cdot \frac{1}{2}\left[\frac{8ml^2 E}{h^2}\right]^{1/2} \left[\frac{8ml^2}{2h^2}\right] dE$$

$$= \frac{\pi}{4}\left[\frac{8ml^2}{h^2}\right]^{3/2} E^{1/2}\, dE$$

$$Z(E)\,dE = \frac{\pi}{4}\left[\frac{8m}{h^2}\right]^{3/2} E^{1/2}\, l^3 dE$$

Here l^3 represents the volume of the metal piece.

If $l^3 = 1$, then we can write that

The number of available energy states per unit volume (i.e) Density of states.

$$Z(E)dE = \frac{\pi}{4}\left[\frac{8m}{h^2}\right]^{3/2} E^{1/2}\, dE \qquad \dots (5)$$

Since each energy level provides 2 electron states one with spin up and another with spin down (pauli’s exclusion principle), we have

Density of states

$$Z(E)dE = 2.\frac{\pi}{4}\left[\frac{8m}{h^2}\right]^{3/2} E^{1/2}\, dE$$

$$Z(E)dE = \frac{\pi}{2} [\frac{8m}{h^2}]^{3/2} \; E^{1/2} \, dE \qquad \ldots(6)$$

CARRIER CONCENTRATION IN METALS

Let N (E) d E represents the number of filled energy states between the interval of energy d E. Normally all the energy states will not be filled. The probability of filling of electrons in a given energy state is given by Fermi Function F (E).

$$N (E)\, dE = Z(E)\, dE \,.\, F(E) \qquad \ldots (7)$$

Substituting equation (6) in equation (7), we get

Number of filled energy states per unit volume

$$N (E)\, dE = \frac{\pi}{2} [\frac{8m}{h^2}]^{3/2} \; E^{1/2} \, d E .F(E) \qquad \ldots(8)$$

N (E) is known as carrier distribution function (or) Carrier concentration in metals.

Fermi Energy at 0 Kelvin:

We know at 0K maximum energy level that can occupied by the electron is called Fermi energy level (E_{F_0})

(i.e) at 0 Kelvin for E< EF and

Therefore F (E) = 1

Integrating equation (8) within the limits 0 to E_{F_0} we can get the number of energy states electrons (N) within the Fermi energy E_{F_0}

$$\int N(E)d\,E = \frac{\pi}{2} [\frac{8m}{h^2}]^{3/2} \int_0^{E_{F_0}} E^{\frac{1}{2}} \,\mathrm{E.dE}$$

$$= \frac{\pi}{2} \left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \frac{E_{F_0}{}^{\frac{3}{2}}}{\frac{3}{2}}$$

(or) Number of filled energy states at zero $= \frac{\pi}{3} \left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E_{F_0}{}^{\frac{3}{2}}$ ...(9)

(or)

$$E_{F_0}{}^{\frac{3}{2}} = \frac{3N}{\pi} \left(\frac{h^2}{8m}\right)^{3/2}$$

Fermi energy $$E_{F_0} = \frac{h^2}{8m} (\frac{3N}{\pi})^{2/3} \qquad ...(10)$$

Average energy of an electron at 0K:

Average energy of an electron

$$(E_{ave}) = \frac{\text{Total energy of the electrons at 0K } (E_T)}{\text{Number of energy states at 0K (N)}} \qquad \ldots(11)$$

Here, the total energy of the electrons at 0K = $(Number\ of\ energy\ states\ at\ 0K)$ X $(Energy\ of\ the\ electron)$

$$\text{(i.e) } E_T = \int_0^{EF_0} N(E)dE\ .E$$

$$= \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \int_0^{E_{F_0}} E^{\frac{1}{2}}\ \text{E.dE} \qquad [\text{F (E) = 1}]$$

$$= \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} \frac{E_{F_0}{}^{\frac{5}{2}}}{\frac{5}{2}}$$

$$E_T = \frac{\pi}{5}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E_{F_0}{}^{\frac{5}{2}} \qquad \ldots(12)$$

Substituting equation 9 and 12 in 11 we get

$$E_{ave} = \frac{\frac{\pi}{5}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E_{F_0}{}^{\frac{5}{2}}}{\frac{\pi}{3}\left(\frac{8m}{h^2}\right)^{\frac{3}{2}} E_{F_0}{}^{\frac{3}{2}}}$$

$$E_{ave} = \frac{3}{5} E_{F_0}{}^{\frac{5}{2}} E_{F_0}{}^{\frac{-3}{2}}$$

The average energy of an electron at 0K is $E_{ave} = \frac{3}{5} E_{F_0}$

ENERGY BAND THEORY OF SOLIDS

Energy band theory of solids plays a very important role in determining whether a solid is a conductor, insulator or a semiconductor. This theory explains how an energy band occurs in a solid.

i. FREE AND BOUND ELECTRONS

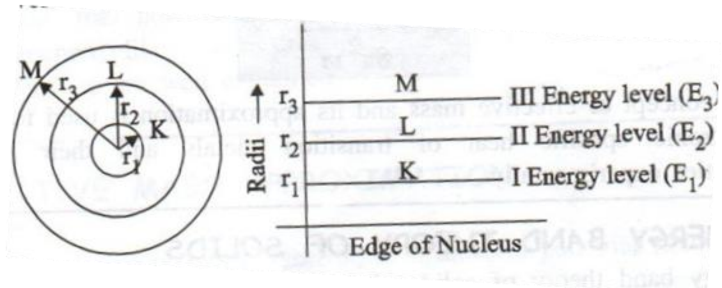
In an isolated atom all the electron are tightly bounded within the central positive nucleus and revolves around various orbits. The number of electrons at outer most orbit are called valence electrons. In the outer most orbits, the attractive force between the nucleus and electrons will be very less, so that electrons can be easily detached from the nucleus. These detached electrons from the outer shell orbits are called free electrons. But in innermost orbits electrons are tightly bound to the nucleus, and hence they are called as bound electrons. The free and bound electrons as shown in figure.

ii. **ENERGYLEVELS**

We know that each orbit of an atom has fixed amount of energy associated with it. The electrons moving in a particular orbit possess the energy of orbit. The larger the orbit, the greater is its energy. So, the outermost orbit of electrons possess more energy than inner orbit electrons.

The energy of different orbits are called energy levels. As shown in figure.

Let E_1 be the energy level of K-shell, E_2 be the energy level of M-shell, E_3 be the energy level of L-shell and so on.

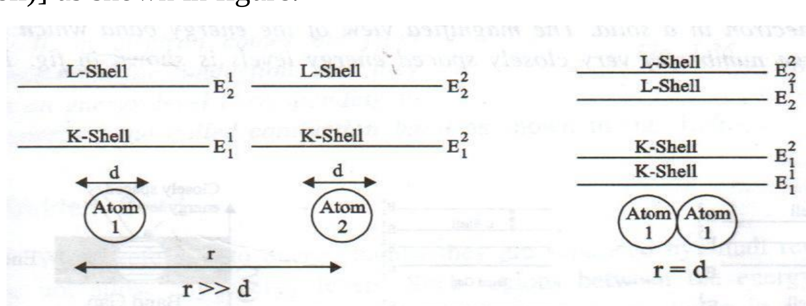


From the figure, it is clear that the electrons can revolve only in certain permitted orbits of radii r_1, r_2, r_3 .... and not in arbitrary orbit.

1. Therefore the electrons cannot filled in spacing of energy levels.
2. Electrons fill the lowest energy levels first. A specific quantity of energy must be supplied to move an electron to the next higher level.
3. Pauli Exclusion Principle states that no two electrons can occupy the same quantum state. Not more than two electrons can occupy any one energy level.

iii. ENERGY BANDS:

A set of closed spaced energy levels is called an energy band. let us consider two identical atoms of diameter (d) separated at a diatance (r), so that leletronic energy levels of one atom [E_1^1(K-shell) and E_2^1 (L-shell)] do not affect the electronic energy levels of the other atom [E_1^2(K-shell) and E_2^2 (L-shell)] as shown in figure.



Now, when we bring the atoms close together, some force of attraction occurs between them and according to quantum mechanics, their wave functions will start overlapping, therefore the two atoms are brought closer, it does not remain as two independent atoms, rather it forms a single two-atom system with two different energy levels to form energy band as shown in figure.

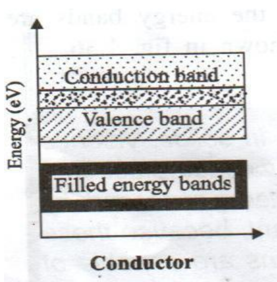
CLASSIFICATION OF MATERIALS INTO METALS, SEMICONDUCTORS AND INSULATORS ON THE BASIS OF BAND THEORY:

Based on band theory, and on the basis of presence of forbidden band gap the materials are classified into three catogories,viz.

i. Metals (or) Conductors
ii. Semiconductors
iii. Insulators

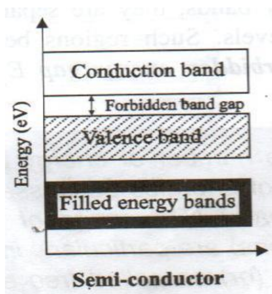
i. Metals (or) Conductors

In conductors, there is no forbidden energy gap. Here the valence band and conduction band overlap each other as shown in figure a. In metals the availability of free electrons will be very high due to the overlapping of conduction band and valence band. Hence even small field is applied to it, the electrons in valence band freely enters in to the conduction band and produces current. **EX. Copper, Aluminium, Iron.,**



ii. Semiconductors

In Semiconductors, the forbidden energy gap is very small, in the order 0.5ev to 1.5ev and hence there will be a very small gap between the valence and conduction band as shown in figure b. **Ex. Ge, Si.**

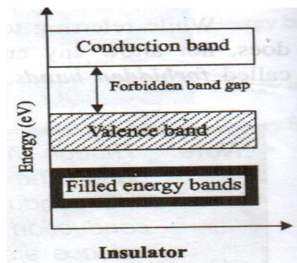


Generally, in Semiconductors the availability of free electrons in conduction band will be less compared to metals, due to the presence of forbidden band gap between the valence band and conduction band. Therefore, when external field of energy, equal to (or) greater than forbidden band gap energy (E_g) is applied to a semiconductor, immediately the conduction will take place.

iii. Insulators

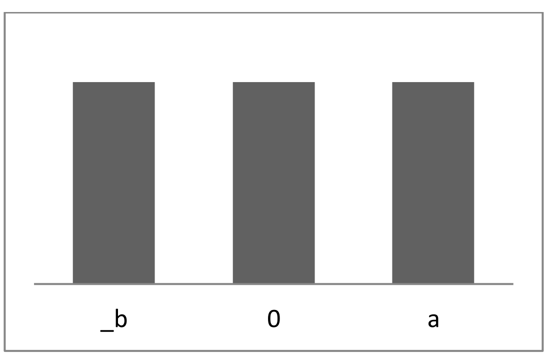
Insulators, the forbidden band gap is very wide, in the range of 3 ev to 5.47 ev and hence there will be very large gap between the valence band and conduction band as shown in figure c. Since the forbidden band gap is very very high in the case of insulators, so that very large amount of external field is required for conduction to occur.

Ex: Diamond, Dielectrics etc..



KRONIG PENNY MODEL

The Kronig Penny model is simplified model for an electron in one dimensional periodic potential the possible states that the electron can occupy are determined by Schrodinger equation.



The corresponding Schroedinger equation for the two regions I and II

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2} E\Psi = 0 \qquad , \; 0 < x < a$$

$$\frac{d^2\Psi}{dx^2} + \alpha^2\Psi = 0 \qquad \text{\_\_\_\_\_\_\_(1)}$$

$$\alpha^2 = \frac{8\pi^2 m}{h^2} \text{ E} \qquad \_\_\_\_\_\_(2)$$

$$\frac{d^2\Psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\Psi = 0 \qquad -b < x < 0$$

$$\frac{d^2\Psi}{dx^2} + \beta^2 \Psi = 0 \qquad \_\_\_\_\_\_(3)$$

$$\beta^2 = \frac{8\pi^2 m}{h^2} (\text{E} - V_0) \qquad \_\_\_\_\_\_(4)$$

The general solution of the above equation (1) , (3)

$$\Psi_1(x) = Ae^{i\alpha x} + Be^{-i\alpha x} \qquad \_\_\_\_\_\_(5)$$

$$\Psi_2(x) = Ce^{\beta x} + De^{-\beta x} \qquad \_\_\_\_\_\_(6)$$

Where A, B, C, D are arbitrary constants. The expected solution of the above Schroedinger equation must have the same form as that of Bloch function.

Now applying boundary condition we obtain the following modified equation

A + B = C + D

iα (A – B) = β (C- D)

$$Ce^{\beta b} + De^{-\beta b} = e^{ik(a+b)}[Ae^{i\alpha a} + Be^{-i\alpha a}]$$

$$\beta Ce^{\beta b} + \beta De^{-\beta b} = i\alpha e^{ik(a+b)}[Ae^{i\alpha a} - Be^{-i\alpha a}]$$

The solution of above equation can be found by following determinant.

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ i\alpha & -i\alpha & -\beta & \beta \\ -e^{ik(a+b)+i\alpha a} & -e^{ik(a+b)-i\alpha a} & e^{-\beta b} & e^{\beta b} \\ -i\alpha e^{ik(a+b)+i\alpha a} & iae^{ik(a+b)-i\alpha a} & \beta e^{-\beta b} & \beta e^{\beta b} \end{vmatrix} = 0 \qquad \_\_\_\_\_\_(7)$$

On simplifying this determinant we obtain

$$coska = \left(\frac{\beta^2 - \alpha^2}{2\alpha\beta}\right)\beta b sin\alpha a\ sinh\beta b + cos\alpha a\ cosh\beta b \qquad \_\_\_\_\_\_(8)$$

In order to simplify above equation Kronig Penny assumes that the potential energy is 0 at lattice sites and equals to V_0 inbetween them hence equation (8) becomes

$$coska = \left(\frac{\beta^2 - \alpha^2}{2\alpha\beta}\right)\beta b sin\alpha a + cos\alpha a \qquad \_\_\_\_\_\_(9)$$

$$\beta^2 - \alpha^2 = \frac{8\pi^2 m}{h^2}(\text{E} - V_0) - \frac{8\pi^2 m}{h^2} E$$

$$= \frac{8\pi^2 m V_0}{h^2} \qquad \_\_\_\_\_\_(10)$$

Sub. Eqn (10) in (9)

$$coska = \frac{8\pi^2 mV_0}{2\alpha\beta h^2}\beta bsin\alpha a + cos\alpha a$$

$$coska = Pb\frac{sin\alpha a}{\alpha a} + cos\alpha a \quad \_\_\_\_\_\_(11)$$

Where $P = \frac{4\pi^2 mV_{0\,b}}{h^2}$

$$P\frac{sin\alpha a}{\alpha a} = 0$$

$$\frac{sin\alpha a}{\alpha a} = 0$$

$$sin\alpha a = 0$$

$$\alpha a = sin^{-1}(0) = n\pi$$

$$\alpha = \frac{n\pi}{a}$$

$$\alpha^2 = \frac{n^2\pi^2}{a^2} \quad \_\_\_\_\_\_(12)$$

Comparing equations (2) and (12)

$$\frac{n^2\pi^2}{a^2} = \frac{8\pi^2 m}{h^2}\text{ E} \quad \_\_\_\_\_\_(13)$$

$$E = \frac{n^2\pi^2}{8a^2 m}$$

Effective mass of electron:

The mass acquired by an electron when it is accelerated in a periodic potential is called effective mass of an electron.

When an electron is accelerated by an electrical or magnetic field in a periodic potential, the mass of an electron is not a constant. But, it varies with respect to the field applied. This varying mass is called effective mass (m*).

Consider a crystal subjected to an electric field of intensity ‘E’. Due to this applied field , the electron gains a velocity which can be

PROBLEMS

1. Energy required to remove an electron from sodium metal is 2.3e V. Does Sodium exhibit photoelectric effect from an orange light having wavelength 2800Å?

   Solution:

   Given: 2800 Å, E_g = 2.3

   Energy required to remove an electron from sodium = 2.3

$$E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{6.625\times10^{-34}\times3\times10^{8}}{2800\times10^{-10}\times1.6\times10^{-19}}$$

   By using orange light having wavelength 2800 the energy produced is

   Since, the above calculated energy 4.4 e V i.e energy produced is greater than the required (i.e) 2.3e V. **Sodium exhibits photo-electric effect** from an orange light having wavelength 2800 Å

2. Calculate the drift velocity of electrons in copper and current density in wire of diameter 0.16cm which carries a steady current of 10A.Given n = 8.46×10^{28}/m^3

   Solution:

   Current density J = $\frac{\text{Current}}{\text{Area of cross section}}$

$$= \frac{10}{\frac{\pi\times(0.16\times10^{-2})^2}{4}}$$

$$J = 497.3 \times 10^4 \text{ A/m}^2$$

questions	opt1	opt2	opt3	opt4	answer
Commonly used conducting materials are	copper	aluminium	both a and b	copper and silver	both a and b
Which of the following materials is preferred for transmitting electrical energy over long distance ?	Copper	Aluminium	Steel reinforced copper	Steel reinforced aluminium	Steel reinforced aluminium
The kinetic energy of a bounded electron is	less than that of unbounded electron	greater than that of unbounded electron	equal to that of unbounded electron	infinite	less than that of unbounded electron
The conductivity of a conductor can be increased by	decreasing its temperature	increasing its temperature	decreasing its vibration	increasing its vibration	decreasing its temperature
Mercury as an electric contact material is	a liquid	a metal	a metal liquid	a gas	a metal liquid
Which of the following resistive materials has the lowest temperature co-efficient of resistance ?	Nichrome	Constantan	Kanthal	Molybdenum	Nichrome
The conductors have transport phenomena of electrons due to	electric field	magnetic field	electromagnetic field	none of the above	electric field
Which of the following variety of copper has the best conductivity ?	Induction hardened copper	Hard drawn copper	Pure annealed copper	Copper containing traces of silicon	Pure annealed copper
Which of the following is the poorest conductor of electricity ?	Carbon	Steel	Silver	Aluminium	Carbon
Carbon electrodes are not used in	GLS lamps	electric arc furnace	dry cells	cinema projectors	GLS lamps
Solder is an alloy of	copper and aluminium	tin and lead	nickel, copper and zinc	silver, copper and lead	tin and lead
Overhead telephone wires are made of	aluminium	steel	ACSR conductors	copper	steel
A good electric contact material should have all of the following properties except	high resistivity	high resistance to corrosion	good thermal conductivity	high melting point	high resistivity
Most of the common metals have _____ structure.	linear	hexagonal	orthorhombic	cubic	cubic
Which of the following affect greatly the resistivity of electrical conductors ?	Composition	Pressure	Size	Temperature	Composition
In graphite, bonding is	covalent	metallic	Vander Waals	Vander Waals and covalent	Vander Waals and covalent
Which of the following material is not a semiconductor ?	Silica	Germanium	Selenium	Gallium-arsenide	Silica
Substances whose specific resistance abruptly decreases at very low temperature are called	insulators	conductors	semiconductors	superconductors	superconductors
Which of the following materials is the best conductor of electricity?	Tungsten	Aluminium	Copper	Carbon	Copper
The conduction of electricity, in semiconductors, takes place due to movement of	positive ions only	negative ions only	positive and negative ions	electrons and holes	electrons and holes
Which of the following is a semiconductor material ?	Phosphorous	Rubber	Silicon	Aluminium	Silicon
Selenium is _____ semiconductor.	extrnisic	intmisic	N-type	P-type	intmisic
The forbidden gap in an insulator is	large	small	nil	any of the above	large
In conductors conduction of electricity takes place due to movement of	electrons only	positive ions only	negative ions only	positive and negative ions	electrons only
The conductivity of an extrinsic semiconductor with temperature	decreases	increases	remains constant	Zero	remains constant
The current due to electron flow in conduction band is the hole current in valence band.	equal to	less than	greater than	any of the above	greater than
is an element used in semiconductors whose atoms have three valence electrons.	An acceptor	A donor	Germanium	Silicon	An acceptor
The minority carrier concentration is largely a function of	forward biasing voltage	reverse biasing voltage	temperature	the amount of doping	temperature
For germanium the forbidden energy gap is	0.15 eV	0.25 eV	0.5eV	0.7eV	0.7eV
Intrinsic semiconductors are those	Which are made of semiconductor material in its purest form	Which have zero energy gap	Which have more electrons than holes	Which are available locally	Which are made of semiconductor material in its purest form
The energy gap is much more in silicon than in germanium because	It has less number of electrons	It has high atomic mass number	Its crystal has much stronger bonds called ionic bonds	Its valence electrons are more tightly bound to their parent nuclii	Its valence electrons are more tightly bound to their parent nuclii
A semiconductor in its purest form is called _____	Insulator	Superconductor	Intrinsic semiconductor	Extrinsic semiconductor	Intrinsic semiconductor
A P-type semiconductor results when	A pentavalent impurity is added to an intrinsic semiconductor	A trivalent impurity is added to an intrinsic semiconductor	Either a pentavalent or trivalent impurity is added to an intrinsic semiconductor	None of the above	A trivalent impurity is added to an intrinsic semiconductor
An intrinsic semiconductor at absolute zero _____	Becomes extrinsic semiconductor	Behaves like an insulator	Disintegrates into pieces	Becomes superconductor	Behaves like an insulator
A semiconductor has.... temperature co-efficient of resistance.	Zero	Positive	Negative	None of the above	Negative
A doped semiconductor is also known as	Intrinsic semiconductor	Extrinsic semiconductor	Diffused semiconductor	None of the above	Extrinsic semiconductor
In semiconductor the forbidden energy gap lies	Just below the conduction band	Just above the conduction band	Either above or below the conduction band	Between the valence band and conduction band	Between the valence band and conduction band
In a N-type semiconductor, the positive of the Fermi level	Is at the centre of the energy gap	Is lower than the centre of energy gap	Is higher than the centre of energy gap	Can be anywhere depending upon the doping concentration	Is higher than the centre of energy gap
As the temperature of a semiconductor increases its	Conductivity increases	Resistivity increases	Atomic number decreases	Temperature co-efficient becomes zero	Conductivity increases
Which of the following doping elements would not be suitable for converting intrinsic semiconductor	Phosphorous	Indium	Antimony	Arsenic	Indium
A semiconductor is formed by _____ bonds.	Covalent	Electrovalent	Co-ordinate	None of the above	Covalent
The most commonly used semiconductor is _____	Germanium	Silicon	Carbon	Sulphur	Silicon
A semiconductor has generally _____ valence electrons.	2	3	6	4	4
The strength of a semiconductor crystal comes from _____	Forces between nuclei	Forces between protons	Electron-pair bonds	None of the above	Electron-pair bonds
When a pentavalent impurity is added to a pure semiconductor, it becomes _____	An insulator	An intrinsic semiconductor	p-type semiconductor	n-type semiconductor	n-type semiconductor
Addition of pentavalent impurity to a semiconductor creates many _____	Free electrons	Holes	Valence electrons	Bound electrons	Free electrons
Addition of trivalent impurity to a semiconductor creates many _____	Holes	Free electrons	Valence electrons	Bound electrons	Holes
A hole and electron in close proximity would tend to _____	Repel each other	Attract each other	Have no effect on each other	no force is acting	Attract each other
The random motion of holes and free electrons due to thermal agitation is called _____	Diffusion	Pressure	Ionisation	polarisation	Diffusion
A reverse bias pn junction has _____	Very narrow depletion layer	Almost no current	Very low resistance	Large current flow	Almost no current