FLUID MECHANICS

17BTAR304

17BTAR304 **FLUID MECHANICS** **3 2 0 4 100**

OBJECTIVES:

- To understand the structure and the properties of the fluid.
- To understand and appreciate the complexities involved in solving the fluid flow problems.
- To understand the mathematical techniques already in vogue and apply them to the solutions of practical flow problems.

UNIT –I FLUID PROPERTIES AND FUNDAMENTALS OF FLOW

Brief history of fluid mechanics - Fluids and their properties - Continuum, density, viscosity, surface tension, compressibility and bulk modulus, concept of pressure ,vapor pressure and capillarity. Fluid statics - Pascal's law, Hydrostatic law -.**Buoyancy** - Archimedes' Principle- Flow characteristics -Concept of system and control volume

UNIT – II FLUID KINEMATICS AND FLUID DYNAMICS

Fluid Kinematics: Eulerian and Lagrangian description of fluid flow - Stream, streak and path lines – Classification of flows – Continuity equation (one, two and threedimensional forms) – Stream and potential functions – flow nets – Velocity measurement.

Fluid Dynamics: Euler and Bernoulli's equations – Application of Bernoulli's equation – Discharge measurement, Venturimeter,Orifices,Orificemeter, Impulse momentum relationship and its applications. Problems.

UNIT –III DIMENSIONAL ANALYSIS

Types-Rayleigh's method - Buckingham's π - theorem, Similarity parameters: Reynolds number, Froude number, Concepts of geometric, kinematic and dynamic similarity, Applications of dimensionless parameters. Model Analysis. Scale effect and distorted models.

UNIT –IV PIPE FLOW

Reynolds experiment, Darcy's equation, major and minor losses in pipes and numerical problems. Exact solutions of Navier stokes equations,Hagen-Poiseuilli law, hydraulic gradient and total energy lines, Flow though pipes in series and in parallel, branched pipes; equivalent pipe, power transmission through pipes., variation of friction factor with Reynolds number, Moody's chart.

UNIT – V BOUNDARY LAYER FLOW

Boundary layer concept, displacement, momentum and energy thickness, Von-Karman momentum integral equation, laminar and turbulent boundary layer flows, drag on a flat plate, boundary layer separation and control. Streamlined and bluff bodies, lift and drag on a cylinder and an airfoil, Problems.

TEXT BOOKS:

S.No.	AUTHOR(S)	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1.	Bruce R. Munson, Wade W. Huebsch, Alric P. Rothmayer	Fundamentals of Fluid Mechanics	John Wiley & Sons, Incorporated, New Jersey	2012
2.	White F.M.	Fluid Mechanics	Tata McGraw-Hill, New Delhi.	2016
3.	Bansal R.K.	A Textbook of Fluid Mechanics	Laxmi Publications (P) Ltd., New Delhi.	2016

REFERENCE BOOKS:

S.No.	AUTHOR(S)	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1.	Streeter V.L. and Wylie	Fluid Mechanics	McGraw-Hill Book Co, Ltd., New Delhi.	2010
2.	Kumar K.L.	Engineering Fluid Mechanics	S. Chand Publisher, New Delhi.	2008
3.	C. P. Kothandaraman	Fluid Mechanics And Machinery	New Age International, New Delhi.	2007

WEB REFERENCE:

- www.springer.com › Home ›Materials › Mechanics
- www.efunda.com/formulae/fluids/index.cfm
- nptel.iitm.ac.in/video.php?subjectId=105101082
- www.rgtudiploma.com/M02_4_sy.pdf
- www.freebookcentre.net › Physics Books

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FACULTY OF ENGINERRING

DEPARTMENT OF MECHANICAL ENGINEERING (Aerospace)

COURSE PLAN

Subject Name : **FLUID MECHANICS**
Subject Code : **17BTAR304** **(Credits - 4)**
Name of the Faculty : **ARUN PRAKSASH J**
Designation : **ASSISTANT PROFESSOR**
Year/Semester/Section : **II/III SEM**
Branch : **B.Tech Aerospace Engineering**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – I : FLUID PROPERTIES AND FUNDAMENTALS OF FLOW			
1.	2	Introduction to the Course and Brief history of fluid mechanics	T [2] ,R [1], R [3]
2.	1	Fluids and their properties - Continuum, density, viscosity	T [2] ,R [1], R [3]
3.	1	Surface tension, compressibility and bulk modulus	T [2] ,R [1], R [3]
4.	1	Fluid statics	T [2] ,R [1], R [3]
5.	2	Pascal's law and Hydrostatic law	T [2] ,R [1], R [3]
6.	1	Buoyancy	T [2] ,R [1], R [3]
7.	1	Tutorial- Fluids properties and Fluid statics	T [2] ,R [1], R [3]
8.	1	Archimedes' Principle	T [2] ,R [1], R [3]
9.	1	Flow characteristics	T [2] ,R [1], R [3]
10.	1	Concept of system and control volume	T [2] ,R [1], R [3]
11.	1	Tutorial - Flow characteristics	T [2] ,R [1], R [3]
Total No. of Hours Planned for Unit - I			**13**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – II FLUID KINEMATICS AND FLUID DYNAMICS			

12.	1	Fluid Kinematics: Eulerian and Lagrangian description of fluid flow, Stream, streak and path lines	T [1] ,R [1] ,R [3]
13.	1	Classification of flows	T [1] ,R [1] ,R [3]
14.	1	Continuity equation (one, two and three dimensional forms)	T [1] ,R [1] ,R [3]
15.	1	Stream and potential functions , flow nets	T [1] ,R [1] ,R [3]
16.	1	Velocity measurement	T [1] ,R [1] ,R [3]
17.	2	Fluid Dynamics: Euler and Bernoulli's equations , Application of Bernoulli's equation	T [1] ,R [1] ,R [3]
18.	1	Tutorial - Velocity measurement	T [1] ,R [1] ,R [3]
19.	1	Venturimeter	T [1] ,R [1] ,R [3]
20.	1	Orifice meter	T [1] ,R [1] ,R [3]
21.	1	Impulse momentum relationship and its applications	T [1] ,R [1] ,R [3]
22.	1	Tutorial – venturimeter and orificemeter	T [1] ,R [1] ,R [3]
Total No. of Hours Planned for Unit - II			**12**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – III DIMENSIONAL ANALYSIS			
23.	1	Dimensional Analysis – Introduction	T [1] ,T [2], R [1]
24.	1	Rayleigh's method	T [1] ,T [2], R [1]
25.	1	Buckingham's π - theorem	T [1] ,T [2], R [1]
26.	1	Numerical Problems	T [1] ,T [2], R [1]
27.	1	Tutorial - Buckingham's π - theorem	T [1] ,T [2], R [1]
28.	1	Similarity parameters, Reynolds number, Froude number	T [1] ,T [2], R [1]
29.	1	Concepts of geometric, kinematic and dynamic similarity	T [1] ,T [2], R [1]
30.	1	Applications of dimensionless parameters	T [1] ,T [2], R [1]
31.	2	Model Analysis , Scale effect and distorted models	T [1] ,T [2], R [1]
32.	1	Numerical Problems	T [1] ,T [2], R [1]
33.	1	Tutorial - Model Analysis	T [1] ,T [2], R [1]
Total No. of Hours Planned for Unit - III			**12**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – IV PIPE FLOW			
34.	1	Reynolds experiment and Darcy's equation	T [1] ,T [2], R [2]
35.	1	Major and minor losses in pipes	T [1] ,T [2], R [2]
36.	1	Numerical problems	T [1] ,T [2], R [2]
37.	1	Tutorial - major and minor losses in pipes	T [1] ,T [2], R [2]
38.	1	Exact solutions of Navier stokes equations	T [1] ,T [2], R [2]
39.	1	Hagen-Poiseuilli law, hydraulic gradient and total energy lines	T [1] ,T [2], R [2]
40.	1	Flow though pipes in series and in parallel	T [1] ,T [2], R [2]

41.	1	Tutorial - Flow though pipes	T [1] ,T [2], R [2]
42.	1	Branched pipes and equivalent pipe	T [1] ,T [2], R [2]
43.	2	Power transmission through pipes., variation of friction factor with Reynolds number	T [1] ,T [2], R [2]
44.	1	Moody's chart	T [1] ,T [2], R [2]
Total No. of Hours Planned for Unit - IV			**12**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – V : BOUNDARY LAYER FLOW			
45.	1	Boundary layer concept	T [2], R [1] , R [2]
46.	1	Displacement, momentum and energy thickness	T [2], R [1] , R [2]
47.	1	Von-Karman momentum integral equation	T [2], R [1] , R [2]
48.	1	Tutorial - displacement, momentum and energy thickness	T [2], R [1] , R [2]
49.	1	Laminar and turbulent boundary layer flows	T [2], R [1] , R [2]
50.	1	Drag on a flat plate	T [2], R [1] , R [2]
51.	1	Boundary layer separation and control	T [2], R [1] , R [2]
52.	1	Streamlined and bluff bodies	T [2], R [1] , R [2]
53.	2	lift and drag on a cylinder and an airfoil	T [2], R [1] , R [2]
54.	1	Numerical problems	T [2], R [1] , R [2]
55.	1	Tutorial - lift and drag	T [2], R [1] , R [2]
56.	1	**Discussion on Competitive Examination related Questions / University previous year questions**	
Total No. of Hours Planned for Unit - V			**12+1**

TOTAL PERIODS : 62

TEXT BOOKS

T [1] – Fundamentals of Fluid Mechanics by Bruce R. Munson, Wade W. Huebsch, Alric P. Rothmayer

T [2] – Fluid Mechanics by White F.M.

REFERENCES

R [1] - Fluid Mechanics by Bansal R.K.

R [2] - Fluid Mechanics and Machinery by C. P. Kothandaraman

R [3] - Fluid Mechanics by Streeter V.L. and Wylie

WEBSITES

W [1] - nptel.iitm.ac.in/

W [2] - www.freebookcentre.net › Physics Books

W [3] - www.springer.com › Home ›Materials › Mechanics

JOURNALS

J [1] - Experiments in Fluids - Springer

J [2] – Fluid Dynamics Research - IOP Publishing
J [3] – Journal of Experiments in Fluid Mechanics - China Aerodynamics Research Society
J [4] – Physics of Fluids - AIP Publishing
J [5] – Journal of Fluid Mechanics - Cambridge University Press

UNIT	Total No. of Periods Planned	Lecture Periods	Tutorial Periods
I	**13**	**9**	**3**
II	**12**	**9**	**3**
III	**12**	**9**	**3**
IV	**12**	**9**	**3**
V	**12+1**	**9**	**3**
TOTAL	**62**	**46**	**15**

I. CONTINUOUS INTERNAL ASSESSMENT : 40 Marks

(Internal Assessment Tests: 30, Attendance: 5, Assignment/Seminar: 5)

II. END SEMESTER EXAMINATION : 60 Marks

TOTAL : 100 Marks

**FACULTY COORDINATOR/ HOD /
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FLUID MECHANICS

LECTURE NOTES

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Established Under Section 3 of UGC Act 1956)

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

Compiled by
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Department of Mechanical Engineering

UNIT – I

FLUID PROPERTIES AND FUNDAMENTALS OF FLOW

INTRODUCTION

Fluid Mechanics:

Branch of science which deals with the behaviour of the fluids (liquids or gases) at rest as well as in motion.

Fluid Statics:

Study of fluid at rest

Fluid Kinematics:

Study of fluid in motion (pressure forces – not considered.)

Fluid Dynamics

Fluid in motion, pressure forces considered.

1.1 Units and Dimensions

As any quantity can be expressed in whatever way you like. It is sometimes easy to become confused as to what exactly or how much is being referred to. This is particularly true in the field of fluid mechanics. Over the years many different ways have been used to express the various quantities involved. Even today different countries use different terminology as well as different units for the same thing - they even use the same name for different things e.g. an American pint is 4/5 of a British pint

To avoid any confusion on this course we will always used the SI (metric) system - which you will already be familiar with. It is essential that all quantities be expressed in the same system or the wrong solution will results. Despite this warning you will still find that that this is the most common mistake when you attempt example questions.

1.1.1 The SI System of units

The SI system consists of six **primary** units, from which all quantities may be described. For convenience **secondary** units are used in general practice which are made from combinations of these primary units.

1.1.2 Primary Units

The six **primary** units of the SI system are shown in the table below:

Quantity	**SI Unit**	**Dimension**
Length	Metre,	L
Mass	Kilogram,	M
Time	Second,	T
Temperature	Kelvin,	t
Current	*Ampere,*	*I*
Luminosity	*Candela*	*Cd*

1.1.3 Derived Units

There are many **derived** units all obtained from combination of the above **primary** units. Those most used are shown in the table below:

Quantity	**SI Unit**		**Dimension**

velocity	m/s	ms^{-1}	LT^{-1}
Acceleration	m/s^2	ms^{-2}	LT^{-2}
Force	N $kg\ m/s^2$	$kg\ ms^{-2}$	$M\ LT^{-2}$
Energy (or work)	Joule J N m, $kg\ m^2/s^2$	$kg\ m^2s^{-2}$	ML^2T^{-2}
Power	Watt W N m/s $kg\ m^2/s^3$	Nms^{-1} $kg\ m^2s^{-3}$	ML^2T^{-3}
Pressure (or stress)	Pascal P, N/m^2, $kg/m/s^2$	Nm^{-2} $kg\ m^{-1}s^{-2}$	$ML^{-1}T^{-2}$
Density	kg/m^3	$kg\ m^{-3}$	ML^{-3}
Specific weight	N/m^3 $kg/m^2/s^2$	$kg\ m^{-2}s^{-2}$	$ML^{-2}T^{-2}$
Relative density	a ratio no units		1 no dimension
Viscosity	$N\ s/m^2$	$N\ sm^{-2}$	

	kg/m s	kg $m^{-1}s^{-1}$	M $L^{-1}T^{-1}$
Surface tension	N/m kg /s^2	Nm^{-1} kg s^{-2}	MT^{-2}

1.2 Properties of Fluids

1.2.1 Density

The density of a substance is the quantity of matter contained in a unit volume of the substance. It can be expressed in three different ways.

1.2.2 Mass Density:

Mass Density or Density is defined as ratio of mass of the fluid to its volume (P)

Density of water 1 gm/cm^3 1000 kg / m.

Density - liquids - Constant

- gases - changes with pr + temperature

$$\boxed{P = \frac{Mass\ \ of\ \ fluid}{Volume\ of\ \ fluid}}$$

$$\left(P = \frac{m}{V} \right)$$

1.2.3. Specific Weight (or) Weight Density:

It is the ratio, between weight of a fluid to its volume.

$$w = \frac{Weight\ \ of\ \ fluid}{Volume\ of\ \ fluid} = \left(\frac{Mass\ of\ \ fluid}{Volume\ of\ \ fluid} \right) \times g = p \times g$$

$$\boxed{\omega = p \times g}$$

Unit: N / m^3 in S.I Units.

W for water $= 9.81 \times 1000\ N / m^3$ in S.I units.

1.2.4 Relative Density

Relative Density, σ, is defined as the ratio of mass density of a substance to some standard mass density. For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4° c) at atmospheric pressure.

$$\sigma = \frac{\rho_{substance}}{\rho_{H_2O(at\,4^{\circ}c)}}$$

Units: None, since a ratio is a pure number.

Typical values: Water = 1, Mercury = 13.5, Paraffin Oil =0.8.

P1. Calculate density , Specific weight, and weight of 1 lt of petrol if sp gr = 0.7

Given,

Volume (v) = 1 lt = $\frac{1}{1000} = 1 \times 1000 cm^3 = \frac{1000}{10^6} m^3 = 0.001 m^3$

Sp. Gr = 0.7

1. Density $p = 3 \times 1000 kg/m^3 = 0.7 \times 1000 = 700 kg/m^3$
2. Sp. Wt $w = p \times g = 700 \times 9.81 N/m^3 = 6867 N/m^3$
3. Weight (W) : $w = \frac{w}{V} \Rightarrow \frac{W}{0.001} = 6867 \Rightarrow W = 6867 \times 0.001 = 6.867 N$

1.2.5 Viscosity:

Viscosity is defined as the property of fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid.

When two layers move one over the other at different velocities , say U and V+ du, the viscosity together with relative velocity causes a shear stress acting between the fluid layer. The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

$$\boxed{\tau \alpha \frac{du}{dy}}$$

Shear stress

This shear stress is proportional to the rate of change of velocity constant of proportionality.

(or) $$\boxed{\tau = \mu \frac{du}{dy}}$$

$\mu \Rightarrow$ Coefficient of dynamic viscosity (or) only viscosity

1.2.6 Specific Volume:

Volume per unit mass of a fluid is called specific volume

Unit : m^3 /kg. $\Rightarrow$ Commonly applied for gases.

$$Sp.\,volume = \left(\frac{Volume\ of\ a\ fluid}{Mass\ of\ fluid}\right) = \frac{1}{p} = \frac{1}{\left(\frac{mass\ of\ fluid}{volume}\right)}$$

1.2.7 Specific Gravity: (or) Relative density:

Specific gravity is the ratio between the weight of a body to the weight of equal volume of water.

Unit : Dimension less. Denoted as : ‘S’

$$S\left(_{pr.liq}\right) = \frac{Weight\ density\ of\ liquid}{Weight\ density\ of\ water}$$

$$S\left(for\ gases\right) = \frac{Weight\ density\ of\ gas}{Weight\ density\ of\ air}$$

Weight density of a liquid = $S \times wt$ density of water

$$= S \times 1000 \times \frac{9.81}{(g)} N/m^3$$

Density of a liquid = S x Density of water.

S x 1000 Kg/m^3

P2. Calculate the sp wt, density and sp gr of 1 litre of liquid which weighs 7 N.

Solution: $\begin{pmatrix} 1lt = \frac{1}{1000} m^3 \\ 1lt = 1000\ cm^3 \end{pmatrix}$

Given $V = 1ltre = \frac{1}{1000} m^3$

W = 7 N

i. Sp. Weight (w) $= \frac{weight}{volume} = \frac{7N}{\left(\frac{1}{1000}\right)m^3} = 7000\ N/m^3$

ii Density (p) $= \frac{w}{g} = \frac{7000\ N}{9.81 m^3} kg/m^3 = 713..5\ Kg/m^3$

iii. Sp. Gravity $= \frac{Density\ of\ liquid}{Density\ of\ water} = \frac{713.5}{1000}$ (Density of water = 1000 kg / m$^{3)}$

= 0.7135

g = 9.81 m / s^2

.**1.2.8 Newton's Law of Viscosity:**

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity

$$\tau = \mu \frac{du}{dy}$$

Newtonian fluid $\rightarrow$ when obeys above relation.

Non – Newtonian fluid $\Rightarrow$ doesn't obey the above relation

1.2.9 Kinematic Viscosity:

Defined as the ratio between the dynamic viscosity and density of fluid.

Represented as γ (nu). $\upsilon = \frac{Vis\cos ity}{Density} = \frac{\mu}{p}$

SI and MKS unit : m^2 / sec.

CGS : cm^2 / S. (Kinematic also known STOKE)

$$1\, Stoke = \frac{Cm^2}{S} = \left(\frac{1}{100}\right)^2 \frac{m^2}{S} = 10^{-4} m^2 / s.$$

Centistoke means $= \frac{1}{100}\, stoke$

P3. **Find the kinematic viscosity of an oil having density 981 kg/m. The shear stress at a point in oil is 0.2452 N/m² and velocity gradient at that point is 0.2 /sec.**

Mass density p = 981 kg/m³, Shear stress $\tau = 0.2452 N / m^2$

Velocity gradient $\frac{du}{dy} = 0.2$

$$\tau = \mu \frac{du}{dy} \Rightarrow 0.2452 = \mu \times 0.2 \quad \Rightarrow \mu = \frac{0.2452}{0.2} = 1.226 Ns.$$

$$\upsilon = \frac{\mu}{p} = \frac{1.226}{981} = 0.125 \times 10^{-2} m^2 / s. = 0.125 \times 10^{-2} \times 10^4 cm^2 / S \left(cm^2 / s = stoke\right)$$

$$\upsilon = 0.125 \times 10^2 cm^2 / s = 12.5 cm^2 / s = 12.5 stoke.$$

P4. Determine the sp gr of a fluid having viscosity 0.05 poise and Kinematic viscosity 0.035 stokes.

$$\text{Specific gravity of liquid} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{1428.5}{1000} = 1.428 = 1.43$$

$$\mu = 0.05\,poise = \frac{0.05}{10}\,Ns/m^2$$

$$\boxed{\upsilon = \frac{\mu}{p}}$$

$$0.035\times10^{-4} = \frac{0.05}{10}\times\frac{1}{p} \Rightarrow p = 1428.5 kg/m^3$$

1.2.10 Compressibility: (K)

Defined as the ratio of compressive stress to volumetric strain. (reciprocal of bulk modulus)

Consider a cylinder filled with a piston as shown

V $\rightarrow$ Volume of gas enclosed in the cylinder

p $\rightarrow$ Pressure of gas when volume is $\forall$

Increase in pressure = dp kgf / m^2

Decrease of volume = $d\forall$

$\therefore$ Volumetric strain $= \dfrac{-d\forall}{\forall}$

- Ve sign $\rightarrow$ Volume with $\uparrow p$

$$\therefore \quad \text{Bulk modulus } K = \frac{Increase\ of\ \Pr essure}{Volumetric\ strain} = \frac{d_p}{\dfrac{-d\forall}{\forall}} = \boxed{-\frac{d_p}{d\forall}\forall}$$

$$\boxed{Compressibility = \frac{1}{K}}$$

1.2.11 Surface Tension

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquid such that the contact surface behaves like a membrane under tension.

In MKS unit, kgf/m, SI $\Rightarrow$ N / m.

1.2.12 Capillarity:

Capillary is defined as a phenomenon of rise of a liquid surface is a small tube relative to adjacent general level of liquid when the tube is held vertically in the liquid. The resistance of liquid surface is know as capillary rise while the fall of the liquid surface is known as capillary depression.

P5. Capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension of water in contact with air = 0.0725 N/m

Capillary rise $\Rightarrow$ h = 0.2 mm = $0.2 \times 10^{3}\, m$

Surface tension $\sigma = 0.0725\, N / m$

Dia of tube = d Angel θ for water = 0 Density p for water = 1000 kg / m^2

$$h = \frac{4\sigma}{p \times g \times d} \Rightarrow 0.2 \times 10^{-3} = \frac{4 \times 0.0725}{1000 \times 9.81 \times d}$$

$$d = \frac{4 \times 0.0725}{1000 \times 9.81 \times 0.2 \times 10^{-3}} = 0.148m = 14.8cm$$

Minimum ϕ of the tube = 14.8 cm.

P6. Find out the minimum size of glass tube that can be used to measure water level o if the capillary rise in the tube is to be restricted to 2mm. Consider surface tension of water in contact with air as 0.073575 N/m.

h = 2.0 mm = $2.0 \times 10^{-3} m$ dia = d   density of water = 1000 kg / m^3

$\sigma = 0.073575 \, N/m$ angle $\theta = 0$

$$h = \frac{4\sigma}{p \times g \times d} \quad \Rightarrow 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

d = 0.015 m = 1.5 cm.

Real Fluid:

A fluid, which possesses viscosity, is known as real fluid. All fluids, in actual practice, are real fluids.

Ideal Fluid:

A fluid which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist have some viscosity.

Capillary fall.

$$h = \frac{4\sigma Cos\theta}{p \times g \times d}$$

where,

h = height of depression in tube.

d = diameter of the

σ = surface tension

ρ = density of the liquid.

θ = Angle of contact between liquid and gas.

P7. Two horizontal plates are placed 1.25 cm apart. The space between them being filled with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.

solution:

Given:

Distance between the plates, dy = 1.25 cm = 0.0125m.

Viscosity μ = 14 poise = 14

----- Ns / m^2.

10

Velocity of upper plate, u = 2.5 m/Sec.

Shear stress is given by equation as $\tau = \mu\ (du / dy)$.

Where du = change of velocity between the plates = u – 0 = u = 2.5 m/sec.

dy = 0.0125m.

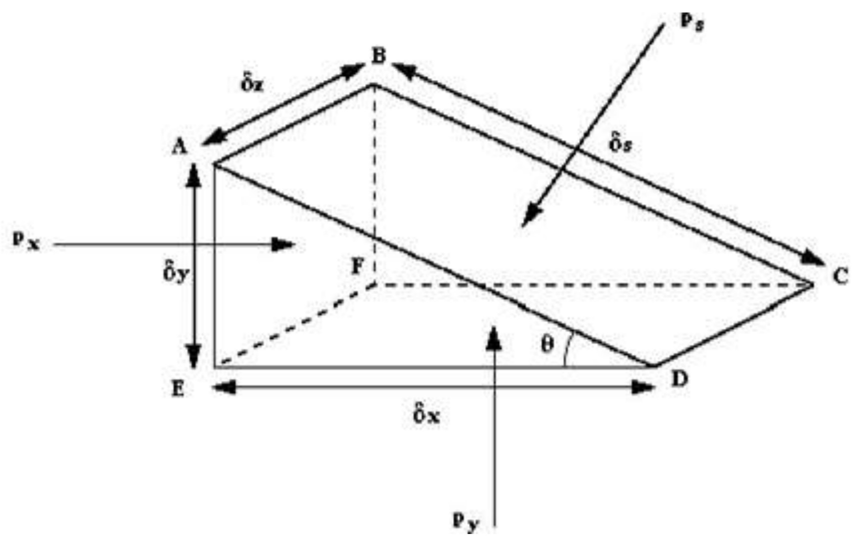
τ = (14 /10) X (2.5 / 0.0125) = 280 N/m^2.

1.3 Pascal's Law:

It stats that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

Derivation:

By considering a small element of fluid in the form of a triangular prism which contains a point P, we can establish a relationship between the three pressures p_x in the x direction, p_y in the y direction and p_s in the direction normal to the sloping face.



Triangular prismatic element of fluid

The fluid is a rest, so we know there are no shearing forces, and we know that all force are acting at right angles to

the surfaces .i.e.

p_s acts perpendicular to surface ABCD,

p_x acts perpendicular to surface ABFE and

p_y acts perpendicular to surface FECD.

And, as the fluid is at rest, in equilibrium, the sum of the forces in any direction is

zero. Summing forces in the x-direction:

Force due to p_x ,

$$F_{x_x} = p_x \times Area_{ABFE} = p_x \delta x \, \delta y$$

Component of force in the x-direction due to p_s,

$$\begin{aligned} F_{x_s} &= -p_s \times Area_{ABCD} \times \sin \theta \\ &= -p_s \delta s \, \delta z \frac{\delta y}{\delta s} \\ &= -p_s \delta y \, \delta z \end{aligned}$$

($\sin\theta = \delta y / \delta s$)

Component of force in x-direction due to p_y ,

$$F_{x_y} = 0$$

To be at rest (in equilibrium)

$$F_{x_x} + F_{x_s} + F_{x_y} = 0$$
$$p_x \delta x \delta y + \left(-p_s \delta y \delta z\right) = 0$$
$$p_x = p_s$$

Similarly, summing forces in the y-direction. Force due to p_y,

$$F_{y_y} = p_y \times Area_{EFCD} = p_y \delta x \delta z$$

Component of force due to p_s,

$$F_{y_s} = -p_s \times Area_{ABCD} \times \cos\theta$$
$$= -p_s \delta s \delta z \frac{\delta x}{\delta s}$$
$$= -p_s \delta x \delta z$$

$$\left(\cos\theta = \delta x / \delta s\right)$$

Component of force due to p_x,

$$F_{y_x} = 0$$

Force due to gravity,

$$\begin{aligned}\text{weight} &= \text{- specific weight} \times \text{volume of element}\\ &= -\rho g \times \frac{1}{2}\delta x\, \delta y\, \delta z\end{aligned}$$

To be at rest (in equilibrium)

$$F_{y_y} + F_{y_s} + F_{y_x} + \text{weight} = 0$$

$$p_y \delta x \delta y + \left(-p_s \delta x \delta z\right) + \left(-\rho g \frac{1}{2}\delta x \delta y \delta z\right) = 0$$

The element is small i.e., δx and δy are small, and so δz is very small and considered $\delta x \delta y \delta z$ negligible, hence

$$p_y = p_s$$

thus

$$p_x = p_y = p_s$$

Considering the prismatic element again, p_s is the pressure on a plane at any angle θ, the x, y and z

directions could be any orientation. The element is so small that it can be considered a point so the derived expression $p_x = p_y = p_s$. indicates that pressure at any point is the same in all directions. (The

proof may be extended to include the z axis).

Pressure at any point is the same in all directions. This is known as ***Pascal's Law*** *and applies to fluids at rest.*

1.3.1 Absolute pressure and Gauge pressure

Absolute Pressure:

It is defined as the pressure which is measured with the reference to absolute vacuum pressure.

Gauge Pressure:

It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

Manometers.

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing measuring the column of fluid by the same or another column of fluid.

1. Simple M
2. Differential M

P8 . A differential manometer is connected at the two points A and B . At B pr is 9.81 N/cm^2 (abs), find the absolute pr at A.

Pr above X – X in right limb = $1000 \times 9.81 \times 0.6 + p_B$

Pr above X – X in left limb =
$13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + P_A$

Equating the two pr head

Absolute pr at P_A = 8.887 N/cm^2

Buoyancy.

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

1.3.2 Differential Manometers

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes/ a differential manometer consists of a U – tube containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are:

1. U – tube differential manometer.
2. Inverted U – tube differential manometers.

Centre of pressure.

Is defined as the point of application of the total pressure on the surface.

The submerged surfaces may be:

1. Vertical plane surface
2. Horizontal plane surface
3. Inclined plane surface
4. Curved surface

P9. Water is flowing through two different pipes, to which an inverted differential manometer having an oil of sp. Gr 0.8 is connected the pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings.

Pr heat at $A = \frac{p_A}{pg} = 2m$ of water.

$$p_A = p \times g \times 2 = 1000 \times 9.81 \times 2$$

= 19620 N/ m^2

Pr below X – X in left limb = $P_A - p_1 gh_1$ = 19620 - 1000 x 781 x 0.3 = 16677 N/m^2

P_r below X – X in right limb

$$p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12 = P_B - 1922.76$$

Equating two pressure, we get,

$$P_B = 16677 + 1922.76 = 18599.76 \quad N/m^2 = 1.8599 \quad N/cm^2$$

P 10 . A differential manometer is connected at the two points A and B . At B pr is 9.81 N/cm^2 (abs), find the absolute pr at A.

Pr above X – X in right limb = $1000 \times 9.81 \times 0.6 + p_B$

Pr above X – X in left limb = $13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + P_A$

Equating the two pressure head

Absolute pr at P_A = 8.887 N/cm^2.

P 11. A hydraulic pressure has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic pressure when the force applied at the plunger is 500 N.

Given: Dia of ram, D = 30 cm = 0.3 m

Dia of plunger, d = 4.5 cm = 0.045 m

Force on plunger, F = 500 N

To find :

Weight lifted = W

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 m^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.045)^2 = 0.000159 m^2$

Pressure intensity due to plunger $= \frac{Force\ on\ plunger}{Area\ of\ plunger} = \frac{N}{m^2}$

$$= \frac{F}{a} = \frac{500}{0.00159} N/m^2$$

Due to Pascal law, the intensity of pressure will be equally transmitted in all distance . Hence the pressure intensity at

$$ram = \frac{500}{0.00159} = 314465.4 \quad N/m^2$$

Pressure intensity at ram $= \dfrac{weight}{Area\ of\ ram} = \dfrac{W}{A} = \dfrac{W}{0.07068}$

$$\frac{W}{0.07068} = 314465.4$$

Weight $= 314465.4 \times 0.07068 = 22222N = 22.222\ KN.$

P 12 Calculate the capillary effect in millimeters a glass tube of 4mm diameter, when immersed in (a) water (b) mercury. The temperature of the liquid is 20^0 C and the values of the surface tension of water and mercury at 20^0+ C in contact with air are 0.073575 and 0.51 N/m respectively. The angle of contact for water is zero , that for mercury 130^0 . Take density of water @ 20^0 C as equal to 998 kg

Given:

Dia of tube $\Rightarrow$ d = 4 mm = $4 \times 10^{-3}\, m$

Capillary effect (rise or depression) $\Rightarrow h = \dfrac{4\sigma \cos\theta}{p \times g \times d}$

$\sigma =$ Surface tension in kg f/m

$\theta =$ angle of contact p = density

i. Capillary effect for water

$$\sigma = 0.073575\ N/m, \quad \theta = 0^0$$

$$p = 998\ kg/m^3\ @\ 20^0\ c$$

$$h = \frac{4 \times 0.73575 \times Cos 0^0}{998 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3}\ m$$

= 7.51 mm.

Capillary effect for mercury :

$$\sigma = 0.51\ N/m, \qquad \theta = 130^0$$

$$p = sp\ gr \times 1000 = 13.6 \times 1000 = 13600\ kg/m^3$$

$$h = \frac{4 \times 0.51 \times Cos 130^0}{13600 \times 9.81 \times 4 \times 10^{-3}}$$

$= -2.46 \times 10^{-3}$ m

= - 2.46 mm.

-Ve capillary depression.

P 13. A cylinder of 0.6 m^3 in volume contains air at 50^{0}C and 0.3 N/ mm^2 absolute pressure. The air is compressed to 0.3 m^3. Find (i) pressure inside the cylinder assuming isothermal process (ii) pressure and temperature assuming adiabatic process. Take K = 1.4

Given:

Initial volume $\forall_1 = 0.36\, m^3$ Pressure P_1 = 0.3 N/mm^2

t_1 = 50^0 C $= 0.3 \times 10^6\ N/m^2$

T_1 = 273 + 50 = 323 ^{0} K $= 30 \times 10^4\ N/m^2$

$\forall_2 = 0.3 m^3$ K = 1.4

i. Isothermal Process:

$$\frac{P}{p} = Cons\tan t \quad (or)\ p\forall = Cons\tan t$$

$$p_1\forall_1 = p_2\forall_2$$

$$p_2 = \frac{p_1\forall_1}{\forall_2} = \frac{30 \times 10^4 \times 0.6}{0.3} = 0.6 \times 10^6\ N/m^2$$

= 0.6 N / mm^2

ii. Adiabatic Process:

$$\frac{p}{p^K} = Cons\tan t \quad or$$

$$\boxed{p\forall^K = cons\tan t}$$

$$p_1.\forall_1{}^K = p_2\forall_2{}^K$$

$$p_2 = p_1 \frac{\forall_1 R}{\forall_2 K} = 30\times10^4 \times\left(\frac{0.6}{0.3}\right)^{1.4} = 30\times10^4 \times 2^{1.4}$$

$$= 0.791\times10^6\, N/m^2 = 0.791 N/mm^2$$

For temperature , $p\forall = RT$ **,** $p\forall^k = cons\tan t$

$$p = \frac{RT}{\forall} \text{ and } \frac{RT}{\forall}\times\forall^k = cons\tan t$$

$$RT\forall^{k-1} = Cons\tan t$$

$$T\forall^{k-1} = Cons\tan t \qquad (\because R\ is\ also\ cons\tan t)$$

$$T_1 V_1^{k-1} = T_2 V_2^{k-1}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = 323 \quad \left(\frac{0.6}{0.3}\right)^{1.4-1.0}$$

$$= 323 \times 2^{0.4} = 426.2^0 K$$

$$t_2 = 426.2 - 273 = 153.2^0 C$$

P 14. If the velocity profile of a fluid over a plate is a parabolic with the vertex 202 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stress at a distance of 0,10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.

Given,

Distance of vertex from plate = 20 cm.

Velocity at vertex, u = 120 cm / sec.

Viscosity, $\mu = 8.5 poise = \frac{8.5}{10}\frac{Ns}{m^2} = 0.85$

Parabolic velocity profile equation

$$u = ay^2 + by + C \qquad (1)$$

a, b and c constants values determined from boundary condition.

a. at y = 0, u = 0
b. at y = 20cm, u = 120 cm/se.
c. at y = 20 cm, $\frac{du}{dy} = 0$

substituting (a) in equation (1), C = 0

substituting (b) in equation (1), $120 = a(20)^2 + b(2) = 400a + 20's$

(ii) substituting (C) in equation (1), $\frac{du}{dy} = 2ay + b$

$$0 = 2 \times a \times 20 + b = 40a + b \quad \text{(iii)}$$

$400\,a + 20\,b = 0$

$(-)$

$800\,a + 20\,b = 0$

40 a + b = 0

b = - 40 a

$$120 = 400\,a + 20\,b(-40\,a) = 400\,a - 800\,a = -400\,a$$

$$a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$b = -40 \times (-0.3) = 1.2$$

Substituting a, b and c in equation (i) $u = -0.3y^2 + 12y$

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

Velocity gradient

at y = 0, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 = 12/s.$

at y =10 cm, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 = 6/s.$

at y = 20 cm, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 = 0$

Shear Stresses:

Shear stresses is given by, $\tau = \mu \frac{du}{dy}$

Shear stress at y = 0 , $\tau = \mu \left(\frac{du}{dy} \right)_{y=0} = 0.85 \times 12.0 = 10.2 N/m^2$

ii. Shear stress at y = 10 , $\tau = \mu \left(\frac{du}{dy} \right)_{y=10} = 0.85 \times 6.0 = 5.1 N/m^2$

iii. Shear stress at y = 20 , $\tau = \mu \left(\frac{du}{dy} \right)_{y=20} = 0.85 \times 0 = 0$

P 15. A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. Both cylinder are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 rpm determine the viscosity of the fluid.

Dia of cylinder = 15 cm = 0.15 m

Dia of outer cylinder = 15.10 cm = 0.151 m

Length of cylinder $\Rightarrow$ L = 25 cm = 0.25 m

Torque T= 12 Nm   N = 100 rpm.

Viscosity = μ

Tangential velocity of cylinder $u = \frac{\pi DN}{60} = \frac{\pi \times 0.15 \times 100}{60} = 0.7854 \quad m/s$

Surface area of cylinder $A = \pi D \times L = \pi \times 0.15 \times 0.25$

= 0.1178 m^2

$\tau = \mu \frac{du}{dy}$

$$du = u - 0 = u = 0.7854\ m/s$$

$$dy = \frac{0.151 - 0.150}{2} = 0.0005\ m$$

$$\tau = \frac{\mu \times 0.7854}{0.0005}$$

Shear force, $F = Shear\ Stress \times Area = \frac{\mu \times 0.7854}{0.0005} \times 0.1178$

Torque $T = F \times \frac{D}{2}$

$$12.0 = \frac{\mu \times 0.7854}{0.0005} \times 0.1178 \times \frac{0.15}{2}$$

$$\mu = \frac{12.0 \times 0.0005 \times 2}{0.7854 \times 0.1178 \times 0.15} = 0.864 Ns / m^2$$

$$\mu = 0.864 \times 10 = 8.64 \ poise.$$

P 16. The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power last in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Given ,

$$\mu = 6 poise = \frac{6}{10} \frac{Ns}{m^2} = 0.6 \frac{Ns}{m^2}$$

D = 0.4 m $\quad L = 90mm = 90 \times 10^{-3} m$

N = 190 rpm. $\quad t = 1.5mm = 1.5 \times 10^{-3} m$

$$Power = \frac{2\pi NT}{60} W \qquad F = Shear\ stress \times Area\ N$$

$T = force \times \frac{D}{2}\ Nm. \qquad = \tau \times \pi DL$

$\tau = \mu \frac{du}{dy}\ N/m^2 \quad u = \frac{\pi\ DN}{60}\ M/s.$

Tangential Velocity of shaft, $u = \frac{\pi\ DN}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98\ m/s.$

du = change of velocity = u – 0 = u = 3.98 m/s. $dy = t = 1.5 \times 10^{-3}\ m.$

$$\tau = \mu \frac{du}{dy} \Rightarrow \quad \tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592\ N/m^2$$

Shear force on the shaft F = Shear stress x Area

$$F = 1592 \times \pi D \times L = 1592 \times \pi \times 0.4 \times 90 \times 10^{-3} = 180.05 N$$

Torque on the shaft, $T = Force \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01 \quad Ns.$

$$\text{Power lost} = \frac{2\pi \text{NT}}{60} = \frac{2\pi \times 190 \times 36.01}{60} = 716.48\,W$$

P 17. If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which U is the velocity in m/s at a distance y metere above the plate, determine the shear stress at y = 0 and y = 0.15 m. Take dynamic viscosity of fluid as 8.63 poise.

Given:

$$u = \frac{2}{3}y - y^2$$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3}$$

$$\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2\times(0.17) = 0.667 - 0.30$$

$\mu = 8.63 poise = \frac{8.63}{10}$ SI units = 0.863 Ns / m^2

$$\boxed{\tau = \mu \frac{du}{dy}}$$

i. Shear stress at y = 0 is given by

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = 0.5756\ N/m^2$$

ii. Shear stress at y = 0.15 m is given by

$$(\tau)_{y=0.15} = \mu\left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = 0.3167\ N/m^2$$

P 18. The diameters of a small piston and a large piston of a hydraulic jack at3cm and 10 cm respectively. A force of 80 N is applied on the small piston Find the load lifted by the large piston when :

a. **The pistons are at the same level**
b. **Small piston in 40 cm above the large piston.**

The density of the liquid in the jack in given as 1000 kg/m^3

Given: Dia of small piston d = 3 cm.

∴ Area of small piston , $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.068 cm^2$

Dia of large piston, D = 10 cm

$\therefore$ Area of larger piston, $A = \frac{P}{4} \times (10)^2 = 78.54 cm^2$

Force on small piston, F = 80 N

Let the load lifted = W

a. When the pistons are at the same level

Pressure intensity on small piston

$$P = \frac{F}{a} = \frac{80}{7.068} N / cm^2$$

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston $= \frac{80}{7.068}$

$\therefore$ Force on the large piston = Pressure x area

= $= \frac{80}{7.068}$ x 78.54 N = 888.96 N.

b. when the small piston is 40 cm above the large piston

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} N/cm^2$$

$\therefore$ Pressure intensity of section A – A

$= \frac{F}{a} +$ pressure intensity due of height of 40 cm of liquid. P = pgh.

But pressure intensity due to 40cm. of liquid

$$= p \times g \times h = 1000 \times 9.81 \times 0.4 N/m^2$$

$$= \frac{1000 \times 9.81 \times 0.4}{10^4} N/cm^2 = 0.3924 N/cm^2$$

$\therefore$ Pressure intensity at section

$$A - A = \frac{80}{7.068} + 0.3924$$

= 11.32 + 0.3924 = 11.71 N/cm^2

Pressure intensity transmitted to the large piston = 11.71 N/cm^2

Force on the large piston = Pressure x Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54$$

= 919. 7 N.

1.4 Continuity and Conservation of Matter

1.4.1 Mass flow rate

If we want to measure the rate at which water is flowing along a pipe. A very simple way of doing this is to catch all the water coming out of the pipe in a bucket over a fixed time period. Measuring the weight of the water in the bucket and dividing this by the time taken to collect this water gives a rate of accumulation of mass. This is know as the *mass flow rate.*

For example an empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg, then:

$$\begin{aligned} \text{mass flow rate} = \dot{m} &= \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}} \\ &= \frac{8.0 - 2.0}{7} \\ &= 0.857 kg/s \quad (kgs^{-1}) \end{aligned}$$

Performing a similar calculation, if we know the mass flow is 1.7kg/s, how long will it take to fill a container with 8kg of fluid?

$$
\begin{aligned}
\text{time} &= \frac{\text{mass}}{\text{mass flow rate}} \\
&= \frac{8}{1.7} \\
&= 4.7s
\end{aligned}
$$

1.4.2. Volume flow rate - Discharge

More commonly we need to know the volume flow rate - this is more commonly know as *discharge.* (It is also commonly, but inaccurately, simply called flow rate). The symbol normally used for discharge is

Q. The discharge is the volume of fluid flowing per unit time. Multiplying this by the density of the fluid

gives us the mass flow rate. Consequently, if the density of the fluid in the above example is 850 $kg m^3$ then:

$$
\begin{aligned}
\text{discharge,}\quad Q &= \frac{\text{volume of fluid}}{\text{time}} \\
&= \frac{\text{mass of fl uid}}{\text{density} \times \text{time}} \\
&= \frac{\text{mass flow rate}}{\text{density}} \\
&= \frac{0.857}{850} \\
&= 0.001008\, m^3/s \quad (m^3 s^{-1}) \\
&= 1.008 \times 10^{-3} m^3/s \\
&= 1.008\, l/s
\end{aligned}
$$

1.4.3. Discharge and mean velocity

An important aside about units should be made here:

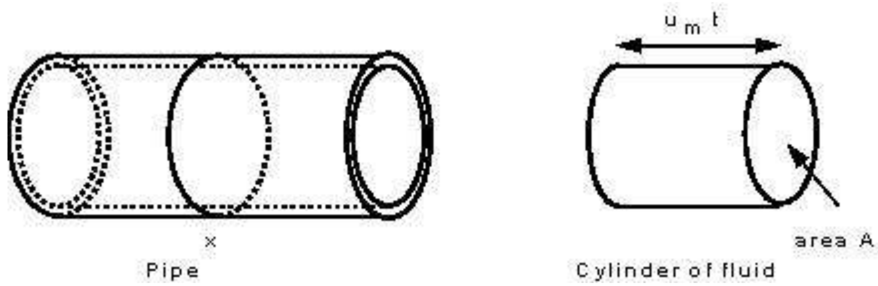
As has already been stressed, we must always use a consistent set of units when applying values to equations. It would make sense therefore to always quote the values in this consistent set. This set of units will be the SI units. Unfortunately, and this is the case above, these actual practical values are very small or very large (0.001008m^3/s is very small). These numbers are difficult to imagine physically. In these cases it is useful to use *derived units*, and in the case above the useful derived unit is the litre.

(*1 litre = 1.0* $10^{-3}m^3$). So the solution becomes $1.008 l/s$. It is far easier to imagine 1 litre than *1.0* $10^{-3}m^3$. Units must always be checked, and converted if necessary to a consistent set before using in an

equation.

1.4.3. Discharge and mean velocity

If we know the size of a pipe, and we know the discharge, we can deduce the mean velocity



Discharge in a pipe

If the area of cross section of the pipe at point X is A, and the mean velocity here is u_m. During a time t,

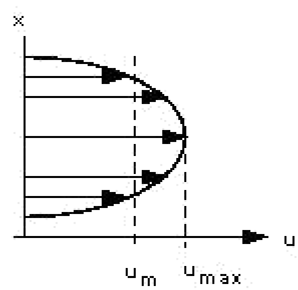
a cylinder of fluid will pass point X with a volume A u_m t. The volume per unit time (the discharge) will thus be

$$Q = \frac{\text{volume}}{\text{time}} = \frac{A \times u_m \times t}{t}$$
$$Q = Au_m$$

So if the cross-section area, A, is $1.2 \times 10^{-3} m^2$ and the discharge, Q is $24 l/s$, then the mean velocity, u_m, of the fluid is

$$\begin{aligned} u_m &= \frac{Q}{A} \\ &= \frac{2.4 \times 10^{-3}}{1.2 \times 10^{-3}} \\ &= 2.0 m/s \end{aligned}$$

Note how carefully we have called this the *mean* velocity. This is because the velocity in the pipe is not constant across the cross section. Crossing the centreline of the pipe, the velocity is zero at the walls increasing to a maximum at the centre then decreasing symmetrically to the other wall. This variation across the section is known as the velocity profile or distribution. A typical one is shown in the figure below.



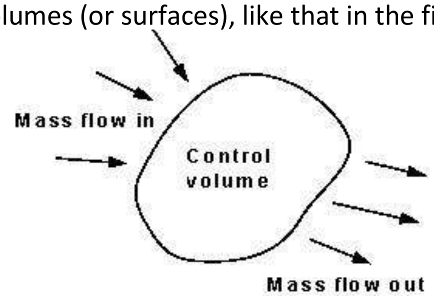
A typical velocity profile across a pipe

This idea, that mean velocity multiplied by the area gives the discharge, applies to all situations - not just pipe flow.

1.4.4. Continuity

Matter cannot be created or destroyed - (it is simply changed in to a different form of matter). This principle is know as the *conservation of mass* and we use it in the analysis of flowing fluids.

The principle is applied to fixed volumes, known as control volumes (or surfaces), like that in the figure below:



An arbitrarily shaped control volume.

For any control volume the principle of *conservation of mass* says

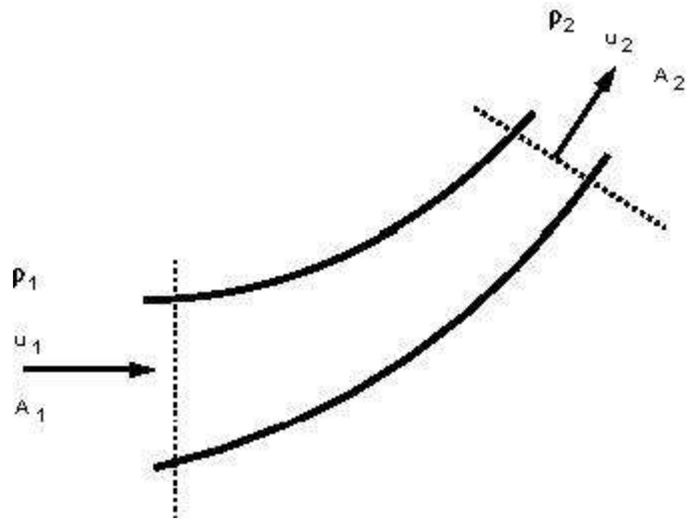
Mass entering per unit time = Mass leaving per unit time + Increase of mass in the control volume per unit time

For **steady** flow there is no increase in the mass within the control volume, so

For steady flow

Mass entering per unit time = Mass leaving per unit time

This can be applied to a streamtube such as that shown below. No fluid flows across the boundary made by the streamlines so mass only enters and leaves through the two ends of this streamtube section.



A streamtube

We can then write

$$\text{mass entering per unit time at end 1} = \text{mass leaving per unit time at end 2}$$
$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2$$

Or for steady flow,

$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2 = \text{ Constant } = \dot{m}$$

This is the equation of continuity.

The flow of fluid through a real pipe (or any other vessel) will vary due to the presence of a wall - in this case we can use the *mean* velocity and write

$$\rho_1 A_1 u_{m1} = \rho_2 A_2 u_{m2} = \text{ Constant } = \dot{m}$$

When the fluid can be considered incompressible, i.e. the density does not change [redacted]

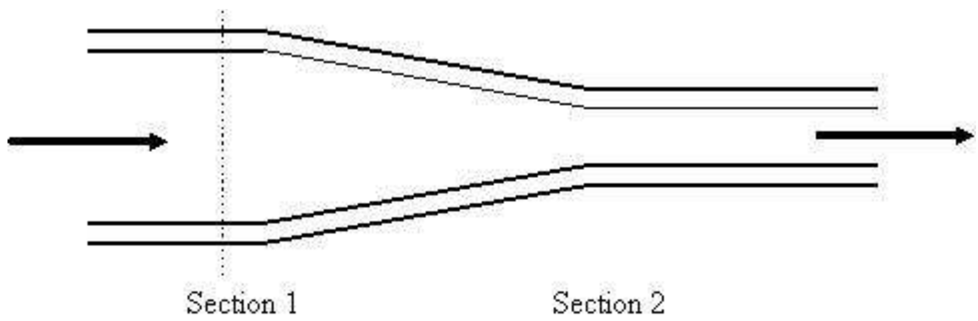
so (dropping the *m* subscript)

$$A_1 u_1 = A_2 u_2 = Q$$

This is the form of the continuity equation most often used. This equation is a very powerful tool in fluid mechanics and will be used **repeatedly** throughout the rest of this course.

Some example applications

We can apply the principle of continuity to pipes with cross sections which change along their length. Consider the diagram below of a pipe with a contraction:



A liquid is flowing from left to right and the pipe is narrowing in the same direction. By the continuity

principle, the *mass flow rate* must be the same at each section - the mass going into the pipe is equal to the mass going out of the pipe. So we can write:

$$A_1 u_1 \rho_1 = A_2 u_2 \rho_2$$

(with the sub-scripts 1 and 2 indicating the values at the two sections)

As we are considering a liquid, usually water, which is *not* very compressible, the density changes very little so we can say $\rho_1 = \rho_2 = \rho$. This also says that the *volume flow rate* is constant or that

$$\text{Discharge at section 1} = \text{Discharge at section 2}$$
$$Q_1 = Q_2$$
$$A_1 u_1 = A_2 u_2$$

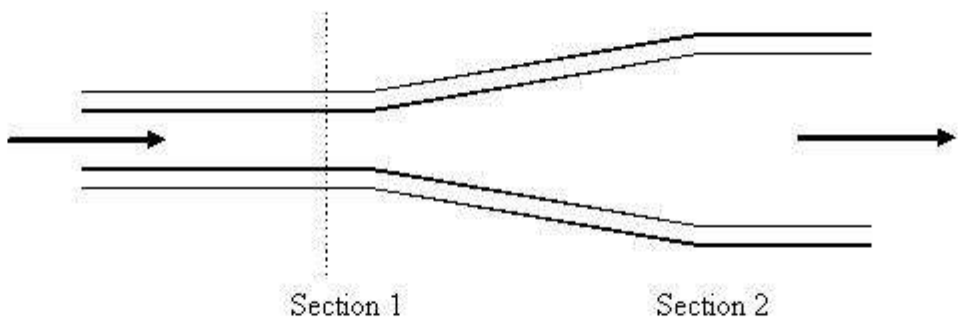
For example if the area $A_1 = 10 \times 10^{-3} m^2$ and $A_2 = 3 \times 10^{-3} m^2$ and the upstream mean velocity, $u_1 = 2.1 m/s$, then the downstream mean velocity can be calculated by

$$u_2 = \frac{A_1 u_1}{A_2}$$
$$= 7.0 m/s$$

Notice how the downstream velocity only changes from the upstream by the ratio of the two areas of the pipe. As the area of the circular pipe is a function of the diameter we can reduce the calculation further,

$$u_2 = \frac{A_1}{A_2} u_1 = \frac{\pi d_1^2 / 4}{\pi d_2^2 / 4} u_1 = \frac{d_1^2}{d_2^2} u_1$$
$$= \left(\frac{d_1}{d_2} \right)^2 u_1$$

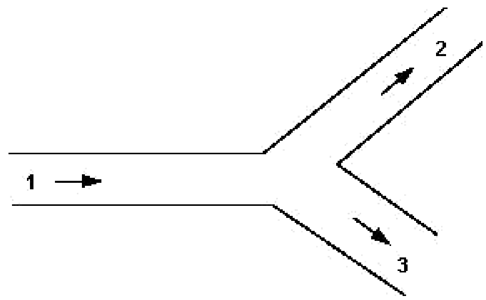
Now try this on a *diffuser,* a pipe which expands or diverges as in the figure below,



If the diameter at section 1 is $d_1 = 30mm$ and at section 2 $d_2 = 40mm$ and the mean velocity at section 2 is $u_2 = 3.0m/s$. The velocity entering the diffuser is given by,

$$\begin{aligned} u_1 &= \left(\frac{40}{30}\right)^2 3.0 \\ &= 5.3m/s \end{aligned}$$

Another example of the use of the continuity principle is to determine the velocities in pipes coming from a junction.



Total mass flow into the junction = Total mass flow out of the junction

$$_1Q_1 = {}_2Q_2 + {}_3Q_3$$

When the flow is incompressible (e.g. if it is water)

$$Q_1 = Q_2 + Q_3$$
$$A_1u_1 = A_2u_2 + A_3u_3$$

1.5 The Momentum Equation and Its Applications

We have all seen moving fluids exerting forces. The lift force on an aircraft is exerted by the air moving over the wing. A jet of water from a hose exerts a force on whatever it hits. In fluid mechanics the analysis of motion is performed in the same way as in solid mechanics - by use of Newton's laws of motion. Account is also taken for the special properties of fluids when in motion.

The momentum equation is a statement of Newton's Second Law and relates the sum of the forces acting on an element of fluid to its acceleration or rate of change of momentum. You will probably recognise the equation $F = ma$ which is used in the analysis of solid mechanics to relate applied force to acceleration. In fluid mechanics it is not clear what mass of moving fluid we should use so we use a different form of the equation.

Newton's 2nd Law can be written:

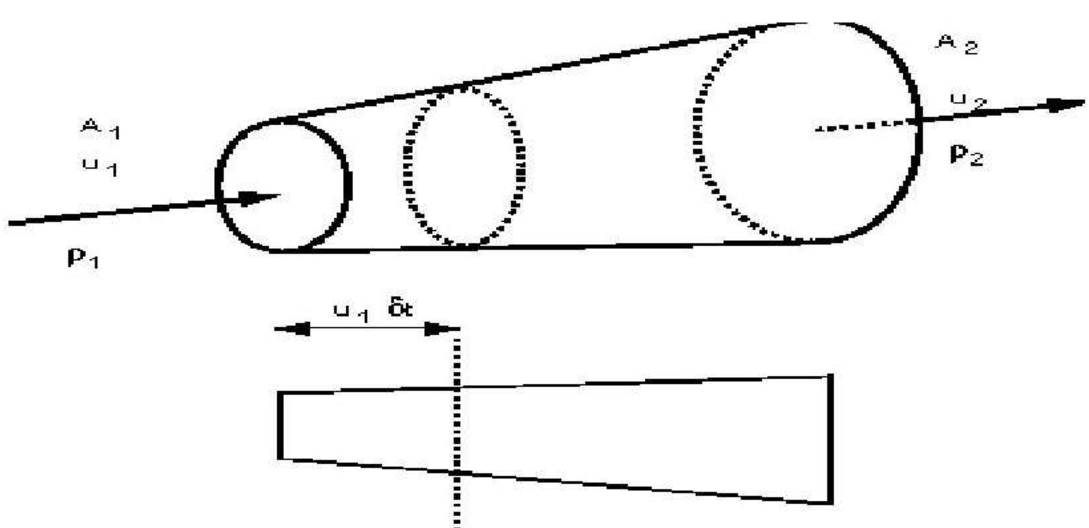
The Rate of change of momentum of a body is equal to the resultant force acting on the body, and

takes place in the direction of the force.

To determine the rate of change of momentum for a fluid we will consider a streamtube as we did for the

Bernoulli equation,

We start by assuming that we have *steady* flow which is *non-uniform* flowing in a stream tube.



A streamtube in three and two-dimensions

In time δt a volume of the fluid moves from the inlet a distance $u\delta t$, so the volume entering the streamtube in the time δt is

$$\text{volume entering the stream tube} = \text{area} \times \text{distance} = A_1 u_1 \delta t$$

this has mass,

$$\text{mass entering stream tube} = \text{volume} \times \text{density} = \rho_1 A_1 u_1 \delta t$$

and momentum

$$\text{momentum of fluid entering stream tube} = \text{mass} \times \text{velocity} = \rho_1 A_1 u_1 \delta t \, u_1$$

Similarly, at the exit, we can obtain an expression for the momentum leaving the steamtube:

$$\text{momentum of fluid leaving stream tube} = \rho_2 A_2 u_2 \delta t \, u_2$$

We can now calculate the force exerted by the fluid using Newton's 2nd Law. The force is equal to the rate of change of momentum. So

$$\text{Force} = \text{rate of change of momentum}$$
$$F = \frac{(\rho_2 A_2 u_2 \delta t\, u_2 - \rho_1 A_1 u_1 \delta t\, u_1)}{\delta t}$$

We know from continuity that $Q = A_1 u_1 = A_2 u_2$, and if we have a fluid of constant density, i.e. $\rho_1 = \rho_2 = \rho$, then we can write

$$F = Q\rho(u_2 - u_1)$$

For an alternative derivation of the same expression, as we know from conservation of mass in a stream tube that

$$\text{mass into face 1} = \text{mass out of face 2}$$

we can write

$$\text{rate of change of mass} = \dot{m} = \frac{dm}{dt} = \rho_1 A_1 u_1 = \rho_2 A_2 u_2$$

The rate at which momentum leaves face 1 is

$$\rho_2 A_2 u_2 u_2 = \dot{m} u_2$$

The rate at which momentum enters face 2 is

$$\rho_1 A_1 u_1 u_1 = \dot{m} u_1$$

Thus the rate at which momentum changes across the stream tube is

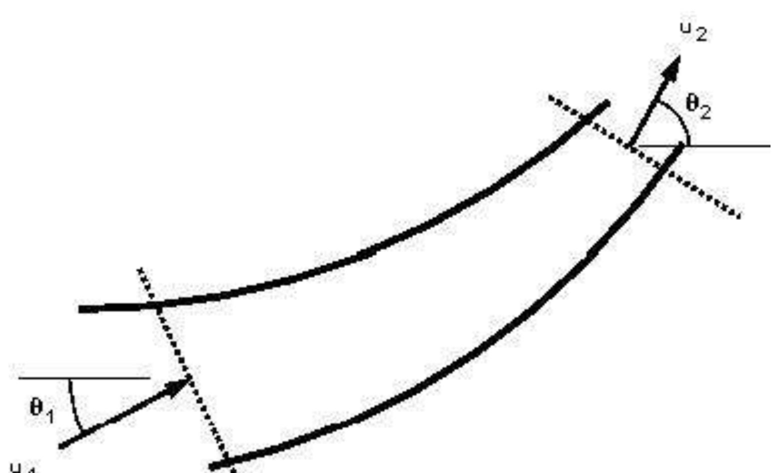
$$\rho_2 A_2 u_2 u_2 - \rho_1 A_1 u_1 u_1 = \dot{m} u_2 - \dot{m} u_1$$

$$\begin{aligned} \text{Force} &= \text{rate of change of momentum} \\ F &= \dot{m}(u_2 - u_1) \\ F &= Q\rho(u_2 - u_1) \end{aligned}$$

This force is acting in the direction of the flow of the fluid.

This analysis assumed that the inlet and outlet velocities were in the same direction - i.e. a one dimensional system. What happens when this is not the case?

Consider the two dimensional system in the figure below:



Two dimensional flow in a streamtube

At the inlet the velocity vector, u_1, makes an angle, θ_1, with the x-axis, while at the outlet u_2 make an

angle θ_2. In this case we consider the forces by resolving in the directions of the co-ordinate axes.

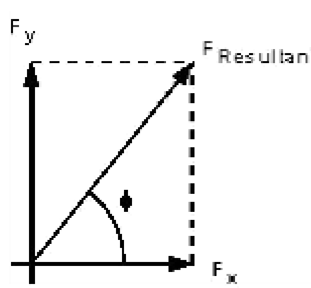
The force in the x-direction

$$
\begin{aligned}
F_x &= \text{Rate of change of momentum in x - direction} \\
&= \text{Rate of change of mass} \times \text{change in velocity in x - direction} \\
&= \dot{m}\left(u_2 \cos\theta_2 - u_1 \cos\theta_1\right) \\
&= \dot{m}\left(u_{2_x} - u_{1_x}\right) \\
&= \rho Q\left(u_2 \cos\theta_2 - u_1 \cos\theta_1\right) \\
&= \rho Q\left(u_{2_x} - u_{1_x}\right)
\end{aligned}
$$

And the force in the y-direction

$$
\begin{aligned}
F_y &= \dot{m}\left(u_2 \sin\theta_2 - u_1 \sin\theta_1\right) \\
&= \dot{m}\left(u_{2_y} - u_{1_y}\right) \\
&= \rho Q\left(u_2 \sin\theta_2 - u_1 \sin\theta_1\right) \\
&= \rho Q\left(u_{2_y} - u_{1_y}\right)
\end{aligned}
$$

We then find the **resultant force** by combining these vectorially:



$$F_{\text{resultant}} = \sqrt{F_x^2 + F_y^2}$$

And the angle which this force acts at is given by

$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

For a three-dimensional (x, y, z) system we then have an extra force to calculate and resolve in the z-direction. This is considered in exactly the same way.

In summary we can say:

$$\text{The total force } \textbf{exerted on} \text{ the fluid} = \text{rate of change of momentum through the control volume}$$
$$F = \dot{m}(u_{\text{out}} - u_{\text{in}})$$
$$= \rho Q(u_{\text{out}} - u_{\text{in}})$$

Remember that we are working with vectors so F is in the direction of the velocity. This force is made up of three components:

$F_R =$ Force exerted on the fluid by any solid body touching the control volume

$F_B =$ Force exerted on the fluid body (e.g. gravity)

$F_P =$ Force exerted on the fluid by fluid pressure outside the control volume So we say that the total force, F_T, is given by the sum of these forces:

$$F_T = F_R + F_B + F_P$$

The force exerted by the fluid on the solid body touching the control volume is opposite to F_R . So the reaction force, R, is given by

$$R = -F_R$$

1.5.1 Application of the Momentum Equation

In this section we will consider the following examples:

1. Force due to the flow of fluid round a pipe bend.
2. Force on a nozzle at the outlet of a pipe.
3. Impact of a jet on a plane surface.

1.6 Bernoulli's Equation

Bernoulli's equation is one of the most important/useful equations in fluid mechanics. It may be written,

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

We see that from applying equal pressure or zero velocities we get the two equations from the section above. They are both just special cases of Bernoulli's equation.

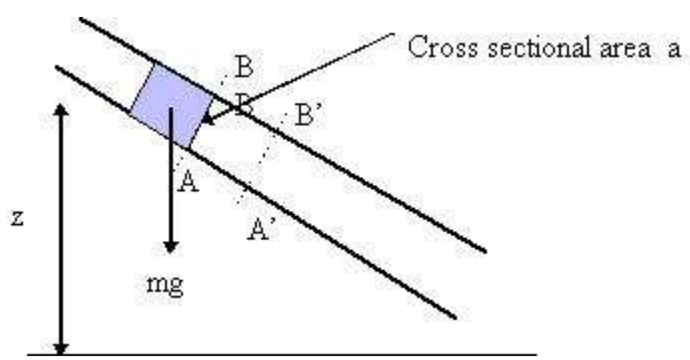
Bernoulli's equation has some restrictions in its applicability, they are:

Flow is steady;

-
- *Density is constant (which also means the fluid is incompressible); Friction losses are negligible.*
-
- *The equation relates the states at two points along a single streamline, (not conditions on two different streamlines).*

All these conditions are impossible to satisfy at any instant in time! Fortunately for many real situations where the conditions are *approximately* satisfied, the equation gives very good results.

The derivation of Bernoulli's Equation:



An element of fluid, as that in the figure above, has potential energy due to its height z above a datum and kinetic energy due to its velocity *u*. If the element has weight mg then

potential energy = mgz potential energy per unit weight = z

kinetic energy = $\frac{1}{2}mu^2$ kinetic energy per unit weight = $\frac{u^2}{2g}$

At any cross-section the pressure generates a force, the fluid will flow, moving the cross-section, so work will be done. If the pressure at cross section AB is *p* and the area of the cross-section is *a* then

force on AB = pa

when the mass *mg* of fluid has passed AB, cross-section AB will have moved to A'B' volume passing AB = $\frac{mg}{\rho g} = \frac{m}{\rho}$ therefore

distance AA' = $\frac{m}{\rho a}$

work done = force distance AA' $= pa \times \frac{m}{\rho a} = \frac{pm}{\rho}$

work done per unit weight = $\frac{p}{\rho g}$

This term is know as the pressure energy of the flowing stream.

Summing all of these energy terms gives

$$\begin{array}{c} \text{Pressure} \\ \text{energy per} \\ \text{unit weight} \end{array} + \begin{array}{c} \text{Kinetic} \\ \text{energy per} \\ \text{unit weight} \end{array} + \begin{array}{c} \text{Potential} \\ \text{energy per} \\ \text{unit weight} \end{array} = \begin{array}{c} \text{Total} \\ \text{energy per} \\ \text{unit weight} \end{array}$$

or

$$\frac{p}{\rho g} + \frac{u^2}{2g} + z = H$$

As all of these elements of the equation have units of length, they are often referred to as the following:

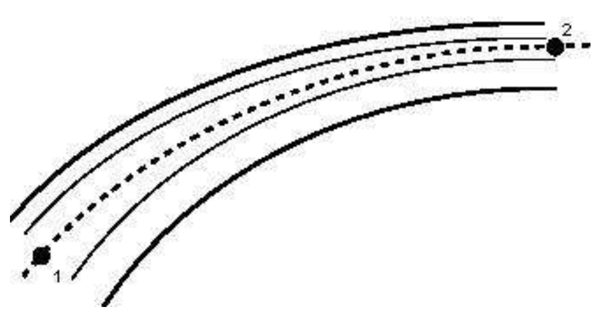
Pressure head = $\frac{p}{\rho g}$ velocity head = potential head = Z

Total head = H

By the principle of conservation of energy the total *energy* in the system does not change, Thus the total *head* does not change. So the Bernoulli equation can be written

$$\frac{p}{\rho g} + \frac{u^2}{2g} + z = H = \text{Constant}$$

As stated above, the Bernoulli equation applies to conditions along a streamline. We can apply it between two points, 1 and 2, on the streamline in the figure below



Two points joined by a streamline

Total energy per unit weight at 1 = total energy per unit weight at 2

Total head at 1 = total head at 2

$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2$$

This equation assumes no energy losses (e.g. from friction) or energy gains (e.g. from a pump) along the streamline. It can be expanded to include these simply, by adding the appropriate energy terms:

$$\begin{array}{ccccccccc} \text{Total} & & \text{Total} & & \text{Loss} & & \text{Work done} & & \text{Energy} \\ \text{energy per} & = & \text{energy per unit} & + & \text{per unit} & + & \text{per unit} & - & \text{supplied} \\ \text{unit weight at 1} & & \text{weight at 2} & & \text{weight} & & \text{weight} & & \text{per unit weight} \end{array}$$

$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2+h+w-q$$

If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter 40mm takes 30% of total discharge and pipe 3 diameter 60mm. What are the values of discharge and mean velocity in each pipe?

The Bernoulli's theorem for steady flow of an incompressible fluid.

It states that in a steady , ideal flow of an incompressible fluid, the total enrgy at any point is constant.

The total energy consists of pressure energy or datum energy. These energies per unit weight of the fluid are:

Pressure Energy = $p / \rho g$

Kinetic energy = $v^2 / 2g$

Datum Energy = z

The mathematically, Bernoulli's theorem is written as

$$(p/w) + (v^2 / 2g) + z = \text{Constant.}$$

P 19. Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0 m/s. find the total head or total energy per unit weight of the water at cross – section , which is 5 cm above the datum line.

Given:

Diameter of the pipe 5 cm = 0.5 m.

Pressure ρ = 29.43 N/cm^2 = 29.23 N/m^2

`

velocity, v = 2.0 m/s.

datum head z = 5 m

total head = Pressure head + Velocity head + Datum head

pressure head = $(p/ \rho g)$ = $(29.43X10^4/ (2X9.81))$ = 30 m

kinetic head = $(v^2/ 2g)$ = $(2X2/(2X9.81))$ = 0.204 m

Total head = $(p/ (\rho g)) + (v^2 / 2g) + z$

= 30 + 0.204 + 5 = 35.204m

P 20. Water is flowing through two different pipes, to which an inverted differential manometer having an oil of sp. Gr 0.8 is connected the pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings.

Pr heat at $A = \frac{p_A}{pg} = 2m$ of water.

$$p_A = p \times g \times 2 = 1000 \times 9.81 \times 2$$

= 19620 N/ m^2

Pr below X – X in left limb = $P_A - p_1 gh_1$ = 19620 - 1000 x 781 x 0.3 = 16677 N/m^2

P_r below X – X in right limb $p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12 = P_B - 1922.76$

Equating two pressure, we get,

$$P_B = 16677 + 1922.76 = 18599.76 \quad N/m^2 = 1.8599 \quad N/cm^2$$

P. 21 The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe section 1 is 5 m/s. determine also the velocity at section 2.

Solution. Given:

At section 1. D_1 = 10 cm = 0.1 m.

$A_1 = (\pi / 4) X D_1^2 = (\pi / 4) X (0.1)^2 = 0.007854$ m^2.

V_1 = 5 m/s.

At section 2. D_2 = 15 cm = 0.15 m.

$A_2 = (\pi / 4) X(0.15)^2 = 0.01767$ m^2.

1. Discharge through pipe is given by equation

$Q = A_1 X V_1$

$= 0.007544 X 5 = 0.03927$ m^3/ s.

using equation, We have $A_1V_1 = A_2V_2$..

$$V_2 = (A_1V_1 / A_1) = (0.007854 /0.01767) X 5 = 2.22 \text{ m/s.}$$

P 22 A pitot – static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6mm and static pressure head is 5m. calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.

Given:

Stagnation Pressure head, h_s = 6mm.

Static pressure h_t = 5 mm.

$h = 6 – 5 = 1$ m

Velocity of flow $V = Cv \sqrt{2gh} = 0.98 \sqrt{2X9.81X1} = 4.34$ m/s.

P 23. A sub-marine moves horizontally in a sea and has its axis 15 m below the surface of water. A pitot tube properly placed just in front of the sub-marine and along its axis connected to the two limbs of a U – tube containing mercury. The difference of mercury level is found to be 170 mm. find the speed of the sub-marine knowing that the sp.gr. of mercury is 13.6 and that of sea-water is 1.026 with respect fresh water.

Given :

Diff. of mercury level x = 170 mm = 0.17 m

Sp. gr. Of mercury S_g = 13.6

Sp. gr. Of sea-water S_o = 1.026

∴ h = x $[Sg/So - 1]$ = 0.17 $[(3.6/1.026) - 1]$ = 2.0834 m

V = $\sqrt{2gh}$ = $\sqrt{2X9.81X20.834}$ = 6.393 m/s.

= (6.393X60X60 / 1000) km/hr = 23.01 km/hr.

P 24. A vertical sluice gate is used to cover an opening in a dam. The opening is 2m wide and 1.2m high. On the upstream of the gate, the liquid of sp. Gr 1.45, lies upto a height of 1.5m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate and posiotn of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume the gate is hinged at the bottom.

Given,

b = 2m $$A = b \times d = 2 \times 1.2 = 2.4m^2$$

d = 1.2 m $$p_1 = 1.45 \times 1000 = 1450 kg/m^3$$

F_1 = force exerted by water on the gate

$$F_1 = p_1 g \quad A\overline{h_1}$$

$$p_1 = 1.45 \times 1000 = 1450 kg/m^2$$

$\overline{h} =$ depth of C.G of gate from free surface of liquid.

$$= 1.5 + \frac{1.2}{2} = 2.1m$$

$$F_1 = 1450 \times 9.81 \times 2.4 \times 2.1$$

= 71691 N $F_2 = p_2 g A \overline{h_2}$

P 25. Water flows through a pipe AB 1.2m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter at C, the pipe branches. Branch CD is 0.8m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.

Solution. Given:

Diameter of Pipe AB, D_{AB} = 1.2 m.

Velocity of flow through AB V_{AB} = 3.0 m/s.

Dia. of Pipe BC, D_{BC} = 1.5m.

Dia. of Branched pipe CD, D_{CD} = 0.8m.

Velocity of flow in pipe CE, V_{CE} = 2.5 m/s.

Let the rate of flow in pipe AB = Q m^3/s.

Velocity of flow in pipe BC = V_{BC} m^3/s.

Velocity of flow in pipe CD = V_{CD} m^3/s.

Diameter of pipe CE = D_{CE}

Then flow rate through CD = Q / 3

And flow rate through CE = Q – Q/3 = 2Q/3

(i). Now the flow rate through AB = Q = V_{AB} X Area of AB

= 3 X (π / 4) X $(D_{AB})^2$ = 3 X (π / 4) X $(1.2)^2$

= 3.393 m^3/s.

(ii). Applying the continuity equation to pipe AB and pipe BC,

V_{AB} X Area of pipe AB = V_{BC} X Area of Pipe BC

$3 \text{ X } (\pi / 4) \text{ X } (D_{AB})^2 = V_{BC} \text{ X } (\pi / 4) \text{ X } (D_{BC})^2$

$3 \text{ X } (1.2)^2 = V_{BC} \text{ X } (1.5)^2$

$V_{BC} = (3\text{X}1.2^2)/ 1.5^2 = 1.92$ m/s.

(iii). The flow rate through pipe

CD = $Q_1 = Q/3 = 3.393 /3 = 1.131$ m^3/s.

$Q_1 = V_{CD}$ X Area of pipe C_D X $(\pi / 4) (C_{CD})^2$

$1.131 = V_{CD} \text{ X } (\pi / 4) \text{ X } (0.8)^2$

$V_{CD} = 1.131 / 0.5026 = 2.25$ m/s.

(iv). Flow through CE, $Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262$ m^3/s.

$Q_2 = V_{CE}$ X Area of pipe CE = $V_{CE} \text{ X } (\pi / 4) (D_{CE})^2$

$2.263 = 2.5 \text{ X } (\pi / 4) (D_{CE})^2$

$D_{CE} = \sqrt{(2.263 \text{ X}4)/ (2.5 \text{ X } \pi)} = 1.0735$ m

Diameter of pipe CE = 1.0735m.

P 26. In a two – two dimensional incompressible flow, the fluid velocity components are given by u = x – 4y and v= - y – 4x. show that velocity potential exists and determine its form. Find also the stream function.

Solution.

Given:

$u = x - 4y$ and $v = - y - 4x$

$(\partial u / \partial x) = 1$ & $(\partial v / \partial y) = -1$.

$(\partial u / \partial x) + (\partial v / \partial y) = 0$

hence flow is continuous and velocity potential exists.

Let Φ = Velocity potential.

Let the velocity components in terms of velocity potential is given by

$\partial \Phi / \partial x = - u = - (x - 4y) = - x + 4y$ ---------------- (1)

$\partial \Phi / \partial y = - v = - (-y - 4x) = y + 4x$.---------------- (2)

Integrating equation(i), we get $\Phi = - (x^2 / 2) + 4xy + C$---- (3)

Where C is a constant of Integration, which is independent of ‘x’.

This constant can be a function of ‘y’.

Differentiating the above equation, i.e., equation (3) with respect to ‘y’, we get

$\partial \Phi / \partial y = 0 + 4x + \partial C / \partial y$

But from equation (3), we have $\partial \Phi / \partial y = y + 4x$

Equating the two values of $\partial \Phi / \partial y$, we get

$4x + \partial C / \partial y = y + 4x$ or $\partial C / \partial y = y$

Integrating the above equation, we get

$C = (y^2 / 2) + C_1$.

Where C_1 is a constant of integration, which is independent of ‘x’ and ‘y’.

Taking it equal to zero, we get $C = y^2/ 2$.

Substituting the value of C in equation (3), we get.

$\Phi = -(x^2/2) + 4xy + y^2/2$.

Value of stream functions

Let $\partial\psi/\partial x = v = -y - 4x$. ----------------------- (4).

Let $\partial\psi/\partial y = -u = -(x - 4y) = x + 4y$ ------(5)

Integrating equation (4) w.r.t. 'x' we get

$\Psi = -yx - (4x^2/2) + k$ -----------------(6)

Where k is a constant of integration which is independent of 'x' but can be a function 'y'.

Differentiating equation (6) w.r.to. 'y' we get,

$\partial\psi/\partial x = -x - 0 + \partial k/\partial y$

But from equation (5), we have $\partial\psi/\partial y = -x + 4y$

Equating the values of $\partial\psi/\partial y$, we get $-x + \partial k/\partial y$ or $\partial k/\partial y = 4y$.

Integrating the above equation, we get $k = 4y^2/2 = 2y^2$.

Substituting the value of k in equation (6), we get.

$\Psi = -yx - 2x^2 + 2y^2$

The Euler's Equation of motion.

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream line in which flow is taking place in s-direction as shown in fig. below. Consider a cylindrical element of cross-section dA and length dS. The forces acting on the cylindrical element are:

a. Pressure force pdA In the direction of flow.

b. Pressure force $\left(p + \frac{\partial p}{\partial s} ds \right) dA$ opposite to the direction of flow.

c. Weight of element

$$\rho g dA ds$$

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element of 's' must be equal to the mass on the fluid element X acceleration in the direction 's'.

$$pdA - \left(p + \frac{\partial p}{\partial s} \right) dA - \rho g dA ds \cos \theta$$
$$= \rho dA ds X as --------(1)$$

where a_s is the acceleration in the direction of 's'

Now, $a_s = \frac{dv}{dt}$, where v is a function of 's' and 't'.

$$= \frac{\partial v}{\partial s} \frac{dv}{dt} + \frac{\partial v}{\partial t} = \frac{v \partial v}{\partial s} + \frac{\partial v}{\partial t} \left\{ \frac{ds}{dt} = v \right\}$$

if the flow is steady, $\frac{dv}{dt} = 0$

$$a_s = \frac{v \partial v}{\partial s}$$

substituting the value of $'a_s'$ in equation (1) and sikmplifying the equation, we get

$$-\frac{\partial p}{\partial s} dsdA - \rho gdAds\cos\theta = \rho dAdsX\,\frac{v\partial v}{\partial s}$$

$$Dividingby\rho dsdA, -\frac{\partial p}{\rho\partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$$

$$or\,\frac{\partial p}{\rho\partial s} + g\cos\theta + v\frac{v\partial v}{\partial s} = 0$$

but from fig, we have cos θ = $\frac{dz}{ds}$

$$\frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = o$$

(or) $\frac{\partial p}{\rho} + gdz + vdv = 0$ -------------------------------(2).

Equation 2 is known as Euler's equation of motion

P 27. The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 lit/sec. the section 1 is 6m above datum. If the pressure at section 2 is 4m above the datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.

Given:

At section 1, D_1 = 20 cm = 0.2m

$A_1 = \frac{\Pi}{4}(0.2)^2$ = 0.314m².

P_1 = 39.24 N/cm² = 39.24 X10⁴ N/m².

Z_1 = 6.0m

At section 2, D_2 = 0.10m

$A_2 = \frac{\Pi}{4}(0.1)^2$ = 0.0785m².

P_2 = ?

Z_1 = 4.0m

Rate of flow Q = 35 lit/sec = 35/1000 = 0.035m^3/s

$Q = A_1V_1 = A_2V_2$

$V_1 = Q / A_1$ = 0.035 / 0.0314 = 1.114 m/s

$V_2 = Q / A_2$ = 0.035 / 0.0785 = 4.456 m/s.

Applying Bernoulli's Equations at sections at 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or (39.24 X10^4 / 1000X9.81) + ((1.114)^2/2X9.81) + 6.0

= (p_2 /1000X9.81)+((4.456)^2/2X9.81)+4.0

40+0.063+6.0 = (p_2/9810)+1.012+4.0

46.063 = (p_2/9810) + 5.012

(p_2/9810) = 46.063 – 5.012 = 41.051

p_2 = (41.051X9810/10^4) = 40.27 N/cm^2

P 28. In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16 cm and 8 cm respectively. A is 2 m above B. the pressure gauge readings have shown that the pressure at B is greater than at A by 0.981 N/cm². Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled

with the same liquid and connected to a U – tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.

Given:

Sp.gr.. of oil, $S_o = 0.8$

Density, $\rho = 0.8 \text{ X}1000 = 800 \text{ kg/m}^3$.

Dia at A, $D_A = 16 \text{ cm} = 0.16\text{m}$

Area at A, $A_1 = \frac{\Pi}{4} X(0.16)^2 = 0.0201 \text{ m}^2$.

Dia. At B $D_B = 8 \text{ cm} = 0.08\text{m}$

Area at B, $A_B = = \frac{\Pi}{4} X(0.08)^2 = 0.005026 \text{ m}^2$

(i). Difference of pressures, $p_B – p_A = 0.981 \text{ N/cm}^2$.

$= 0.981 \text{ X } 10^4 \text{ N/m}^2. = 9810 \text{ N/m}^2$.

Difference of pressure head $(p_B – p_A)/ \rho g = (9810 / (800\text{X}9.81)) = 1.25$

Applying Bernoulli's theorem at A and B and taking reference line passing through section B, we get

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$\frac{p_A}{\rho g} - \frac{p_B}{\rho g} + Z_A - Z_B = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$\frac{p_A - p_B}{\rho g} + 2.0 - 0.0 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$- 1.25 + 2.0 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g} \qquad \frac{p_B - p_A}{\rho g} = 1.25$$

$0.75 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$ --------------------- (i)

Now applying continuity equation at A and B, we get

$$V_A X A_1 = V_B X A_2$$

$$V_B = \frac{V_A X A_1}{A_2} = \frac{V_A X \frac{\pi}{4}(.16)^2}{\frac{\pi}{4}(.08)^2} = 4V_A$$

Substituting the Value of V_B in equation (i), we get

$$0.75 = \frac{16V_A^2}{2g} - \frac{V_A^2}{2g} = \frac{15V_A^2}{2g}$$

$$V_A = \sqrt{\frac{0.75 X 2 X 9.81}{15}} = 0.99 m/s.$$

Rate of flow, $Q = V_A X A_1$

$= 0.99 X 0.0201 = 0.01989 m^3/s.$

(ii). Difference of mercury in the U –tube.

Let h = difference of mercury level.

Then $h = x\left(\frac{S_g}{S_o} - 1\right)$

Where $h = \left(\frac{p_A}{\rho g} + Z_A\right) - \left(\frac{p_B}{\rho g} + Z_B\right) = \frac{p_A - p_B}{\rho g} + Z_A - Z_B$

= -1.25+2.0-0 = 0.75.

$$\therefore 0.75 = x\left[\frac{13.6}{0.8} - 1\right] = x X 16$$

x = (0.75 / 16) = 0.04687 cm.

Question Bank

1. Define Surface Tension and derive the expression for Surface tension on (i) Liquid droplet (ii) hollow bubble (iii) Liquid Jet

2. Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 N/cm^2 to 130 N/cm^2. The volume of the liquid decreases by 0.15 percent.

3. Calculate the density, specific weight, weight of one litre of petrol which has specific gravity of 0.7.

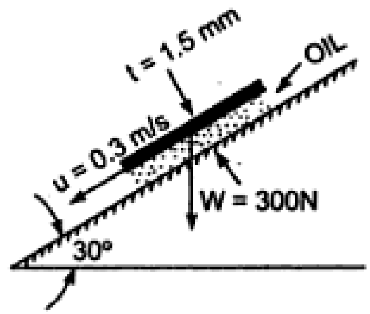
4. Define Surface Tension and derive the expression for Surface tension on (i) Liquid droplet (ii) hollow bubble (iii) Liquid Jet

5. Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take the surface tensions σ = 0.0725 N/m for water and σ = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130^0.

6. The space between the two parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm .The upper plate which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine : i) The dynamic viscosity of the oil in poise ii) the kinematic viscosity of the oil in strokes if the specific gravity of the oil is 0.95

7. One litre of crude oil weighs 9.6 N. Calculate its Specific weight, density and specific volume.

8. Calculate the dynamic viscosity of oil which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 300 as shown. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.

9. 

10. Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine. What force is required to drag a very thin plate of surface area 0.5 square metre between the two large plane surfaces at a speed of 0.6 m/s,if; i) the thin plate is in the middle of the two plane surfaces and ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces Take the dynamic viscosity of the glycerine = 0.8 x 10 -1 N s/m^2.

11. State and Prove Pascal's Law

12. State and Prove Hydrostatic law

13. Calculate the pressure due to a column of 0.3 of (a) water, (b) an oil of sp.gr. 0.8 and (c) mercury of sp.gr. 13.6. Take density of water, ρ = 1000 kg/m^3.

14. Find the volume of water displaced and the position of center of buoyancy for a wooden block of width 2.5 m and depth 1.5 m, when it floats horizontally in water. The density of the wooden block is 650 kg/m^3 and its length is 6.0m.

15. The diameters of a small piston and large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when :

    (a) the pistons are at same level.
    (b) small piston is 40 cm above the large piston.

16. The density of the liquid in the jack is given as 1000 kg/m^3

17. Find the density of a metallic body which floats at the interface of mercury of sp.gr. 13.6 and water such that 40 % of its volume is submerged in mercury and 60 % in water.

18. Calculate the specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of 6 m^3 and weight of 44 kN.

19. A plate having an area of 0.6m^2 is sliding down the inclined plane at 30^0 to the horizontal with a velocity of 0.36 m/s .There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.

20. The space between two square flat parallel plates is filled with oil. Each side of the plate is 720 mm. the thickness of the oil film is 15 mm . The upper plate, which moves at 3 m/s requires a force of 120 N to maintain the speed. Determine :
    The dynamic viscosity of the oil
    The kinematic viscosity of oil if the specific gravity of oil is 0.95.

21. The diameters of ram and plunger of an hydraulic press are 200 mm and 30 mm respectively. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400 N.

22. A wooden block of specific gravity 0.7 and having a size of 2m x 0.5 m x 0.25 m is floating in water. Determine the volume of concrete of specific weight 25 kN/m^3 , that may be placed which will immerse the (i) block completely in water and (ii) block and concrete completely in water.

UNIT – II

FLUID KINEMATICS AND FLUID DYNAMICS

Energy consideration in fluid flow:

Consider a small element of fluid in flow field. The energy in the element as it moves in the flow field is conserved. This principle of conservation of energy is used in the determination of flow parameters like pressure, velocity and potential energy at various locations in a flow. The concept is used in the analysis of flow of ideal as well as real fluids.

Energy can neither be created nor destroyed. It is possible that one form of energy is converted to another form. The total energy of a fluid element is thus conserved under usual flow conditions.

If a stream line is considered, it can be stated that the total energy of a fluid element at any location on the stream line has the same magnitude.

2.3.Forms of Energy Encountered in Fluid Flow

Energy associated with a fluid element may exist in several forms. These are listed here and the method of calculation of their numerical values is also indicated.

Kinetic Energy

This is the energy due to the motion of the element as a whole. If the velocity is *V*, then the kinetic energy for m kg is given by

$$KE = \frac{mV^2}{2g_o} \text{ Nm}$$

(…2.1.1)

The unit in the SI system will be Nm also called Joule (*J*) {(kg m^2/s^2)/(kg m/N s^2)}

The same referred to one kg (specific kinetic energy) can be obtained by dividing 2.1.1 by the mass m and then the unit will be Nm/kg.

$$KE = \frac{V^2}{2g_o}, \text{ Nm/kg}$$

….(2.1.1b)

In fluid flow studies, it is found desirable to express the energy as the head of fluid in m. This unit can be obtained by multiplying equation (6.1.1) by g_o/g.

$$\text{Kinetic head} = \frac{V^2}{2g_o}\frac{g_o}{g} = \frac{V^2}{2g}$$

..(2.1.2)

$$\text{The unit for this expression will be } \frac{m^2 s^2}{s^2 m} = \text{m}$$

Apparently the unit appears as metre, but in reality it is Nm/N, where the denominator is weight of the fluid in N.

The equation in this form is used at several places particularly in flow of liquids. But the energy associated physically is given directly only be equation 2.1.1.

The learner should be familiar with both forms of the equation and should be able to choose and use the proper equation as the situation demands.

When different forms of the energy of a fluid element is summed up to obtain the total energy, all forms should be in the same unit.

Potential Energy

This energy is due to the position of the element in the gravitational field. While a zero value for *KE* is possible, the value of potential energy is relative to a chosen datum. The value of potential energy is given by

$PE = mZ\, g/g_o$ Nm (2.1.3)

Where *m* is the mass of the element in kg, Z is the distance from the datum along the gravitational direction, in *m*. The unit will be (kg m m/s^2) × (Ns^2/kgm) *i.e.,* Nm. The specific potential energy (per kg) is obtained by dividing equation 2.1.3 by the mass of the element.
$PE = Z\, g/g_0$ Nm/kg (2.1.3. *b*)

This gives the physical quantity of energy associated with 1 kg due to the position of the fluid element in the gravitational field above the datum. As in the case of the kinetic energy, the value of *PE* also is expressed as head of fluid, *Z*. $PE = Z\,(g/g_o)\,(g_o/g) = Z\,m$.

This form will be used in equations, but as in the case of KE, one should be familiar with both the forms and choose the suitable form as the situation demands.

Pressure Energy (Also Equals Flow Energy)

The element when entering the control volume has to flow against the pressure at that location. The work done can be calculated referring Fig. 2.1.1.

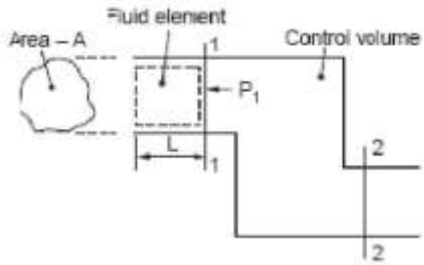


Fig.2.1: Flow work calculation

The boundary of the element of fluid considered is shown by the dotted line, Force = $P_1 A$, distance to be moved = L, work done = $P_1 AL = P_1\, mv$ as AL = volume = mass × specific volume, v.

∴ flow work = $P\,mv$. The pressure energy per kg can be calculated using $m = 1$. The flow energy is given by

$FE = P.v = P/\rho$, Nm/kg

Note: $\frac{N}{m^2}\frac{m^3}{kg} \rightarrow \frac{Nm}{kg}$

As in the other cases, the flow energy can also expressed as head of fluid.

$$FE = \frac{P}{\rho}\frac{g_o}{g}, \text{m}$$

(2.1.5)

As specific weight $\gamma = \rho\, g/g_o$, the equation is written as,

$$FE = P/\gamma, \text{ m} \tag{2.1.5b}$$

It is important that in any equation, when energy quantities are summed up consistent forms of these set of equations should be used, that is, all the terms should be expressed either as head of fluid or as energy (J) per kg. These are the three forms of energy encountered more often in flow of incompressible fluids.

Internal Energy

This is due to the thermal condition of the fluid. This form is encountered in compressible fluid flow. For gases (above a datum temperature) $IE = c_V T$ where T is the temperature above the datum temperature and c_V is the specific heat of the gas at constant volume. The unit for internal energy is J/kg (Nm/kg). When friction is significant other forms of energy is converted to internal energy both in the case of compressible and incompressible flow.

Electrical and Magnetic Energy

These are not generally met with in the study of flow of fluids. However in magnetic pumps and in magneto hydrodynamic generators where plasma flow in encountered, electrical and magnetic energy should also be taken into account.

2.4.Variation in the Relative Values of Various Forms of Energy During Flow

Under ideal conditions of flow, if one observes the movement of a fluid element along a stream line, the sum of these forms of energy will be found to remain constant. However, there may be an increase or decrease of one form of energy while the energy in the other forms will decrease or increase by the same amount. For example when the level of the fluid decreases, it is possible that the kinetic energy increases. When a liquid from a tank flows through a tap this is what happens. In a diffuser, the velocity of fluid will decrease but the pressure will increase. In a venturimeter, the pressure at the minimum area of cross section (throat) will be the lowest while the velocity at this section will be the highest.

The total energy of the element will however remain constant. In case friction is present, a part of the energy will be converted to internal energy which should cause an increase in temperature. But the fraction is usually small and the resulting temperature change will be so small that it will be difficult for measurement. From the measurement of the other forms, it will be possible to estimate the frictional loss by difference.

2.5.EULER'S Equation of Motion for Flow along a Stream Line

Consider a small element along the stream line, the direction being designated as *s*.

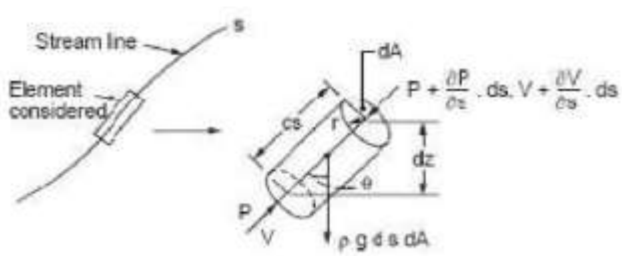


Fig.2.2: Euler's equation of Motion - Derivation

The net force on the element are the body forces and surface forces (pressure). These are indicated in the figure. Summing this up, and equating to the change in momentum.

PdA – {P + (∂P/∂*s*} *dA* – ρ*g dA ds* cos θ = ρ *dA ds as* (2.3.1) where
a_s is the acceleration along the *s* direction. This reduces to,

$$\frac{1}{\rho}\frac{\partial P}{\partial s}+g\cos\theta+a_s=0$$

(2.3.2)

(**Note**: It will be desirable to add *go* to the first term for dimensional homogeneity. As it is, the first term will have a unit of N/kg while the other two terms will have a unit of m/s^2. Multiplying by go, it will also have a unit of m/s^2).

$a_s = dV/dt$, as velocity, $V = f(s, t)$, (t = time).

$$dV=\frac{\partial V}{\partial s}ds+\frac{\partial V}{\partial t}dt \quad \text{dividing by } dt,$$

$$\frac{dV}{dt}=\frac{\partial V}{\partial s}\frac{ds}{dt}+\frac{\partial V}{\partial t} \qquad \text{As } \frac{ds}{dt}=V,$$

and as cos θ = *dz*/*ds*, equation 6.3.2 reduces to,

$$\frac{1}{\rho}\frac{\partial P}{\partial s}+g\frac{\partial z}{\partial s}+V\frac{\partial V}{\partial s}+\frac{\partial V}{\partial t}=0$$

(2.3.2 a)

For steady flow ∂*V*/ ∂*t* = 0. Cancelling ∂*s* and using total derivatives in place of partials as these are independent quantities.

$$\frac{dp}{\rho}+gdz+VdV-0$$

(2.3.3)

(**Note**: in equation 6.3.3 also it is better to write the first term as *go.dp*/ρ for dimensional homogeneity). This equation after dividing by *g*, is also written as,

$$\frac{dp}{\gamma} + d\left(\frac{V^2}{2g}\right) + dz = 0 \quad \text{or} \quad d\left[\frac{P}{\gamma} + \frac{V^2}{2g} + z\right] = 0$$

(2.3.4)

which means that the quantity within the bracket remains constant along the flow. This equation is known as Euler's equation of motion. The assumptions involved are:

1. Steady flow
2. Motion along a stream line and
3. Ideal fluid (frictionless)

In the case on incompressible flow, this equation can be integrated to obtain Bernoulli equation.

2.6.Bernoulli Equation for Fluid Flow

Euler's equation as given in 2.3.3 can be integrated directly if the flow is assumed to be incompressible.

$$\frac{dP}{\rho} + gdz + VdV = 0, \quad \text{as } \rho = \text{constant}$$

$$\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{const. or } \frac{P}{\rho} + z\left(\frac{g}{g_0}\right) + \frac{V^2}{2g_0} = \text{Constant}$$

2.4.1

The constant is to be evaluated by using specified boundary conditions. The unit of the terms will be energy unit (Nm/kg).

In SI units the numerical value of g_o = 1, kg m/N s2. Equation 2.4.1 can also be written as to express energy as head of fluid column.

$$\frac{P}{\gamma} + z + \frac{V^2}{2g} = \text{constant}$$

(2.4.2)

(γ is the specific weight N/m3). In this equation all the terms are in the unit of head of the fluid. The constant has the same value along a stream line or a stream tube. The first term represents (flow work) pressure energy, the second term the potential energy and the third term the kinetic energy. This equation is extensively used in practical design to estimate pressure/velocity in flow through ducts, venturimeter, nozzle meter, orifice meter etc. In case energy is added or taken out at any point in the flow, or loss of head due to friction occurs, the equations will read as,

$$\frac{P_1}{\rho} + \frac{V_1^{\,2}}{2g_o} - \frac{z_1 g}{g_o} + W - \frac{h_f g}{g_o} = \frac{P_2}{\rho} + \frac{V_2^{\,2}}{2g_o} + \frac{z_2 g}{g_o}$$

(2.4.3)

where *W* is the energy added and h_f is the loss of head due to friction. In calculations using SI system of units g_o may be omitting as its value is unity.

Example.1 *A liquid of specific gravity 1.3 flows in a pipe at a rate of 800 l/s, from point 1 to point 2 which is 1 m above point 1. The diameters at section 1 and 2 are 0.6 m and 0.3 m respectively. If the pressure at section 1 is 10 bar,* ***determine the pressure at section 2.***

$$\frac{10\times10^5}{9810\times1.3}+0+\frac{2.83^2}{2\times9.81}=\frac{P_2}{9810\times1.3}+1+\frac{11.32^2}{2\times9.81}$$

Using Bernoulli equation in the following form (2.4.2)

$$\frac{P}{\gamma}+z+\frac{V^2}{2g}=\text{constant},$$

Taking the datum as section 1, the pressure P_2 can be calculated. $V_1 = 0.8 \times 4/\pi \times 0.62 = 2.83$ m/s, $V_2 = 0.8 \times 4/\pi \times 0.33 = 11.32$ m/s $P_1 = 10 \times 105$ N/m^2, γ = sp. gravity × 9810.
Substituting.

Solving, **P2 = 9.092 bar** (9.092 × 105 N/m^2).

As P/γ is involved directly on both sides, gauge pressure or absolute pressure can be used without error. However, it is desirable to use absolute pressure to avoid negative pressure values (or use of the term vacuum pressure).

Example 2 *Water flows through a horizontal venturimeter with diameters of 0.6 m and 0.2 m. The guage pressure at the entry is 1 bar.* ***Determine the flow rate*** *when the throat pressure is 0.5 bar (vacuum). Barometric pressure is 1 bar.*

Using Bernoulli's equation in the form,

$$\frac{P_1}{\gamma}+Z_1+\frac{V_1^2}{2g}=\frac{P_2}{\gamma}+Z_2+\frac{V_2^2}{2g}$$

and noting $Z_1 = Z_2$, $P_1 = 2 \times 10^5$ N/m^2 (absolute)

$V1 = Q \times 4/(\pi \times 0.602) = 3.54\,Q$, $V2 = Q \times 4/(\pi \times 0.202) = 31.83Q$

$$\frac{2 \times 10^5}{9810} + 0 + \frac{3.54^2}{2 \times 9.81} Q^2 = \frac{0.5 \times 10^5}{9810} + 0 + \frac{3183^2\, Q^2}{2 \times 9.81}$$

Solving, **Q** = **0.548 m^3/*s***, V_1 = 1.94 m/s, V_2 = 17.43 m/s.

2.7. Energy Line and Hydraulic Gradient Line

The total energy plotted along the flow to some specified scale gives the energy line. When losses (frictional) are negligible, the energy line will be horizontal or parallel to the flow direction. For calculating the total energy kinetic, potential and flow (pressure) energy are considered.

Energy line is the plot of $P/\gamma + Z + V^2 g^2$ along the flow. It is constant along the flow when losses are negligible.

The plot of $P/\gamma + Z$ along the flow is called the hydraulic gradient line. When velocity increases this will dip and when velocity decreases this will rise. An example of plot of these lines for flow from a tank through a venturimeter is shown in Fig. 2.3

The hydraulic gradient line provides useful information about pressure variations (static head) in a flow. The difference between the energy line and hydraulic gradient line gives the value of dynamic head (velocity head).

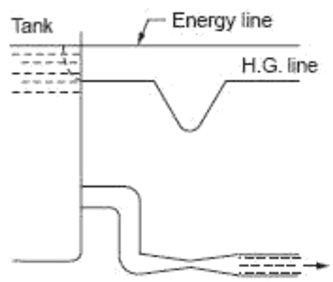


Fig.: 2.3 Energy and hydraulic gradient lines

2.8.Volume Flow through a Venturimeter

Example *Under ideal conditions show that the volume flow through a venturimeter is given by*

$$Q = \frac{A_2}{\{1-(A_2/A_1)^2\}^{0.5}}\left[2g\left(\frac{P_1-P_2}{\gamma}+(Z_1-Z_2)\right)\right]^{0.5}$$

where suffix 1 and 2 refer to the inlet and the throat.

Refer to Fig. Ex. 6.5

Volume flow $= A_1\, V_1 = A_2\, V_2$

$$\therefore \quad V_1 = \frac{A_2}{A_1} V_2,\ V_1^2 = \left(\frac{A_2}{A_1}\right)^2 . V_2^2.$$

$$\therefore \quad (V_2^2 - V_1^2) = V_2^2\left[1 \left(\frac{A_2}{A_1}\right)^2\right]$$

Applying Bernoulli equation to the flow and considering section 1 and 2,

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

Rearranging.

This is a general expression and can be used irrespective of the flow direction, inclination from horizontal or vertical position. This equation is applicable for orifice meters and nozzle flow meters also. In numerical work consistent units should be used.

Pressure should be in N/m^2, Z in m, A in m^2 and then volume flow will be m^3/s. A coefficient is involved in actual meters due to friction.

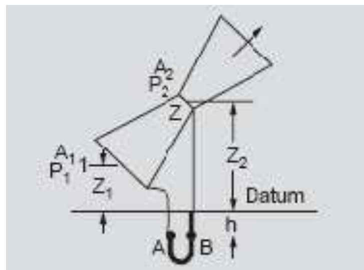


Fig 2.4

2.9.Euler and Bernoulli Equation for Flow with Friction

Compared to ideal flow the additional force that will be involved will be the shear force acting on the surface of the element. Let the shear stress be τ, the force will equal $\tau\, 2\pi r\, ds$ (where r is the radius of the element, and $A = \pi r^2$)

Refer **Para 2.3** and **Fig. 2.3.1.** The Euler equation 2.3.3 will now read as

$$\frac{dP}{\rho} + VdV + gdZ - \frac{2\tau ds}{\rho r} = 0$$

$$\frac{dP}{\gamma} + d\left(\frac{V^2}{2g}\right) + dZ - \frac{2\tau ds}{\gamma r} = 0$$

ds can also be substituted in terms of *Z* and θ

Bernoulli equation will now read as (taking *s* as the length)

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2+\frac{2\tau s}{\gamma r}$$

The last term is the loss of head due to friction and is denoted often as h_L h_f in head of fluid in metre height (check for the unit of the last term).

Example *A vertical pipe of diameter of 30 cm carrying water is reduced to a diameter of 15cm. The transition piece length is 6 m. The pressure at the bottom is 200 kPa and at the top it is 80 kPa. If frictional drop is 2 m of water head,* ***determine the rate*** *of flow.*

Considering the bottom as the datum,

$$\frac{200\times10^3}{9810}+0+\frac{V_1^2}{2g}=\frac{80\times10^3}{9810}+6+\frac{V_2^2}{2g}+2$$

$$V_2^2 = V_1^2\,(0.3/0.15)^4 = 16V_1^2$$

$\therefore$ $\dfrac{120\times10^3}{9810}-8 = 15\dfrac{V_1^2}{2g}$, Solving, $V_1 = 2.353$ and $V_2 = 9.411$ m/s

$\therefore$ **Flow rate** $= A_1\,V_1 = A_2V_2 =$ **0.166 m³/s**

2.10. Concept and Measurement of Dynamic, Static and Total Head

In the Bernoulli equation, the pressure term is known as static head. It is to be measured by a probe which will be perpendicular to the velocity direction. Such a probe is called static probe. The head measured is also called Piezometric head. (Figure 2.5 (*a*))

The velocity term in the Bernoulli equation is known as dynamic head. It is measured by a probe, one end of which should face the velocity direction and connected to one limb of a manometer with other end perpendicular to the velocity and connected to the other limb of the manometer. (Figure 2.5 (*b*))

The total head is the sum of the static and dynamic head and is measured by a single probe facing the flow direction. (Figure 2.5 (*c*))

The location of probes and values of pressures for the above measurements are shown in Fig.

2.5.

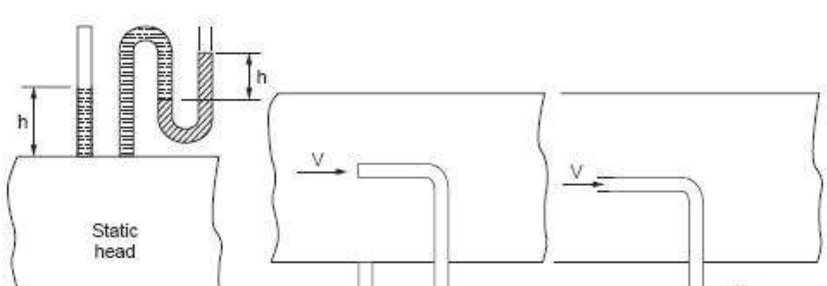


Fig.2.5: Pressure Measurement

2.11. Pitot Tube

The flow velocity can be determined by measuring the dynamic head using a device known as pitot static tube as shown in Fig. 2.6. The holes on the outer wall of the probe provides the static pressure (perpendicular to flow) and hole in the tube tip facing the stream direction of flow measures the total pressure. The difference gives the dynamic pressure as indicated by the manometer. The head will be h ($s - 1$) of water when a differential manometer is used ($s > 1$).

The velocity variation along the radius in a duct can be conveniently measured by this arrangement by traversing the probe across the section. This instrument is also called pitot– static tube.

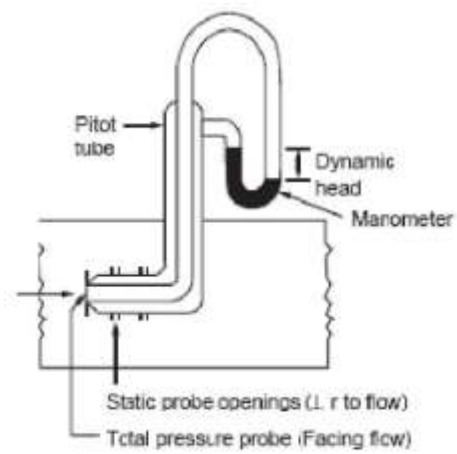


Fig. 2.6: Pitot-Static tube

Example : *The dynamic head of a water jet stream is measured as 0.9 m of mercury column.* ***Determine the height to which the jet will rise*** *when it is directed vertically upwards.* Considering the location at which the dynamic head is measured as the datum and converting the column of mercury into head of water, and noting that at the maximum point the velocity is zero,

$0.9 \times 13.6 + 0 + 0 = 0 + 0 + Z \therefore \mathbf{Z = 12.24\ m}$

Note. If the head measured is given as the reading of a differential manometer, then the head should be calculated as 0.9 (13.6 – 1) m.

Example *A diverging tube connected to the outlet of a reaction turbine (fully flowing) is called ''Draft tube''. The diverging section is immersed in the tail race water and this provides additional head for the turbine by providing a pressure lower than the atmospheric pressure at the turbine exit. If the turbine outlet is open the exit pressure will be atmospheric as in Pelton wheel. In a draft tube as shown in Fig. Ex. 2.7,* ***calculate the additional head provided by the draft tube.*** *The inlet diameter is 0.5 m and the flow velocity is 8 m/s. The outlet diameter is 1.2 m. The height of the inlet above the water level is 3 m. Also calculate the pressure at the inlet section.*

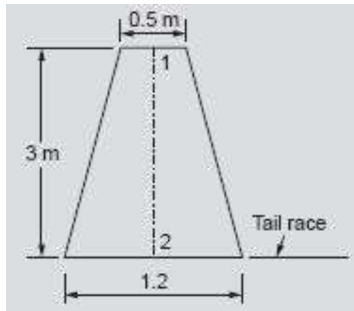


Fig. Ex.2.7 Draft tube

Considering sections 1 and 2

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}Z_1=\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_2$$

Considering tail race level, 2 as the datum, and calculating the velocities

$$V_1 = 8 \text{ m/s}, V_2 = 8\times\frac{0.5^2}{1.2^2} = 1.39 \text{ m/s.}$$

$$P_2 = \textbf{atmospheric pressure}, Z_2 = 0, Z_1 = 3$$

$$\frac{P_1}{\gamma}+\frac{8^2}{2\times 9.81}+3=\frac{1.39^2}{2\times 9.81}$$

$$\frac{P_1}{\gamma} = \mathbf{-6.16 \text{ m of water.}} \text{ (Below atmospheric pressure)}$$

Additional head provided due to the use of draft tube will equal 6.16 m of water

Note: This may cause cavitation if the pressure is below the vapor pressure at the temperature condition. Though theoretically the pressure at turbine exit can be reduced to a low level, cavitation problem limits the design pressure.

2.12. Solved Problems

Problem 6.1 *A venturimeter is used to measure the volume flow. The pressure head is recorded by a manometer. When connected to a horizontal pipe the manometer reading was h cm. If the reading of the manometer is the same when it is connected to a vertical pipe with flow upwards and (ii) vertical pipe with flow downwards, discuss in which case the flow is highest.*

Consider equation 6.6.2

$$Q = \frac{A_2}{\left[1-(A_2/A_1)^2\right]^{0.5}}\left[2gh\left(\frac{S_2}{S_1}-1\right)\right]^{0.5}$$

As long as '*h*' remains the same, the volume flow is the same for a given venturimeter as this expression is a general one derived without taking any particular inclination. This is because of the fact that the manometer automatically takes the inclination into account in indicating the value of ($Z1 - Z2$).

Problem 6.2 *Water flows at the rate of 600 l/s through a horizontal venturi with diameter 0.5 m and 0.245 m. The pressure gauge fitted at the entry to the venturi reads 2 bar.* ***Determine the throat pressure****. Barometric pressure is 1 bar*. Using Bernoulli equation and neglecting losses

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2,\ P_1 = 2 \text{ bar (gauge)} = 3 \text{ bar (absolute) } 3 \times 10^5 \text{ N/m}^2$$

$$V_1 = \frac{Q}{(\pi \times d^2/4)} = \frac{0.6}{(\pi \times 0.5^2/4)} = 3.056 \text{ m/s} \quad \text{can also use}$$

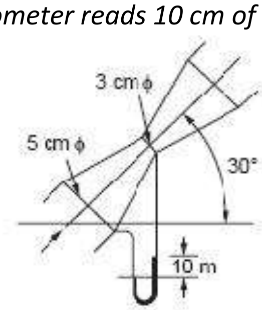
$$V_2 = V_1 \left(\frac{D_2}{D_1}\right)^2$$

$$V_2 = \frac{0.6}{(\pi \times 0.245^2/4)} = 12.732 \text{ m/s, Substituting}$$

$$\frac{3 \times 10^5}{9810} + \frac{3.056^2}{2 \times 9.81} + 0 = \frac{P_2}{9810} + \frac{12.732^2}{2 \times 9.81} + 0$$

$$\therefore \quad \mathbf{P_2 = 223617 \text{ N/m}^2 = 2.236 \text{ bar (absolute)} = 1.136 \text{ bar (gauge)}}$$

Problem 6.3 *A venturimeter as shown in Fig P. 6.3 is used measure flow of petrol with a specific gravity of 0.8. The manometer reads 10 cm of mercury of specific gravity 13.6.*



Determine the flow rate.

$$Q = \frac{A_2}{\left[1-(A_2/A_1)^2\right]^{0.5}}\left[2gh\left(\frac{S_2}{S_1}-1\right)\right]^{0.5}$$

$A_2 = (\pi/4)\ 0.03^2$ as $D_2 = 3$ cm

$\therefore \quad (A_2/A_1)^2 = (D_2/D_1)^4 = (0.03/0.05)^4$,

$h = 0.10$ m $\quad S_2 = 13.6$, $S_1 = 0.8$, Substituting,

$$Q = \frac{(\pi \times 0.03^2/4)}{\left[1-(0.03/0.05)^4\right]^{0.5}}\left[2 \times 9.81 \times 0.1\left(\frac{13.6}{0.8}-1\right)\right]^{-0.5}$$

= **4.245 × 10^{-3} m^3/s or 15.282 m^3/hr** or 4.245 l/s or 15282 l/hr or 3.396 kg/s

Problem 6.4 *A liquid with specific gravity 0.8 flows at the rate of 3 l/s through a venturimeter of diameters 6 cm and 4 cm. If the manometer fluid is mercury (sp. gr = 13.b)*

determine the value of manometer reading, h.

Using equation (6.6.2)

$$Q = \frac{A_2}{\left[1-(A_2/A_1)^2\right]^{0.5}}\left[2gh\left(\frac{S_2}{S_1}-1\right)\right]^{0.5}$$

$$A_1 = \frac{\pi \times 0.06^2}{4} = 2.83 \times 10^{-3}\ \text{m}^2\ ;$$

$$A_2 = \frac{\pi \times 0.04^2}{4} = 1.26 \times 10^{-3}\ \text{m}^2$$

$$3 \times 10^{-3} = \frac{1.26 \times 10^{-3}}{\left[1-\left(\dfrac{1.26\times 10^{-3}}{2.83\times 10^{-3}}\right)^2\right]^{0.5}}\left[2\times 9.81\times h\left(\frac{13.6}{0.8}-1\right)\right]^{0.5}$$

Solving, **h = 0.0146 m = 14.6 mm.** of mercury column.

Question Bank

1. Derive the Continuity equation in One Dimensional and Three Dimensional form using the Cartesian coordinates
2. What are the methods of describing the fluid flow?
3. With a help of a neat Sketch describe the different types of fluid flow along with their mathematical expressions

4. Water flows through a pipe AB 1.2 m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter. At C, the pipe branches. Branch CD is 0.8m in diameter and carries one-third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.

5. A 30 cm diameter pipe conveying water branches into two pipes of diameter 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s find discharge in this pipe. Also determine the velocity in 15 cm pipe if the average velocity in 20 cm pipe is 2 m/s.

6. If for a two – dimensional potential flow, the velocity potential is given by $\phi = x(2x-1)$. Determine the velocity at the point p(4,5). Determine also the value of stream function ψ at the point $\phi = x(2x-1)$.

7. Derive the Euler's equation of motion

8. Define Bernoulli's equation and derive the equation for the same

9. The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through the pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm^2 , find the intensity of pressure at section 2.

10. The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s.The pipe has a slope of 1 in 30.Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm^2.

11. Find the discharge of water flowing through a pipe 30 cm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter 0f 15 cm . The difference of pressure between the main and throat is measured by a liquid of sp.gr. 0.6 in an inverted U-tube which gives a reading of 30 cm.The loss of head between the main and throat is 0.2 times the kinetic head of the pipe.

12. A pipe 450 mm in diameter branches into two pipes of diameter 300 mm and 200 mm respectively .If the average velocity in 450 mm diameter pipe is 3 m/s find :
Discharge through 450 mm diameter pipe
Velocity in 200 mm diameter pipe if the average velocity in 300 mm pipe is 2.5 m/s

13. For a two dimensional flow the velocity function is given by the expression $\Phi = x^2 - y^2$ Determine velocity components in x and y directions.

14. Show that the velocity components satisfy the conditions of flow continuity and irrotationality.

15. Determine the stream function and the flow rate between the streamlines (2,0) and (2,2).

16. An open circuit wind tunnel draws in air from the atmosphere through a well contoured nozzle. In the test section, where the flow is straight and nearly uniform, a static pressure tap is drilled into the tunnel wall. A manometer connected to the tap shows that the static pressure within the tunnel is 45 mm of water below atmospheric pressure. Assume that air is incompressible and at 25°C, Pressure is 100 kPa (absolute).Calculate the velocity in the wind tunnel section. Density of water is 1000 kg/m^3 and characteristic gas constant for air is 287 J/kg K.
17. The water is flowing through a tapering pipe having diameters 300 mm and 150 mm at sections 1 and 2 respectively. The discharge through the pipe is 40 litres/sec. The section 1 is 10 m above datum and section 2 is 6 m above datum. Find the intensity of pressure at section 2 if that at section 1 is 400 kN/m^2.
18. A horizontal venturimeter with inlet and throat diameters 300 mm and 100 mm respectively is used to measure the flow of water. The pressure intensity at inlet is 130 kN/m^2 .The pressure intensity at inlet is 130 kN/m^2 while the vaccum pressure head at the throat is 350 mm of mercury. Assuming that 3 percent of head is lost between the inlet and throat, find :
    The value of C_d for the venturimeter
    Rate of flow

UNIT – III

DIMENSIONAL ANALYSIS

3.6 DIMENSIONAL ANALYSIS

- Need for dimensional analysis
- Methods of dimensional analysis
- Similitude and types of similitude
- Dimensionless parameters and its application
- Model analysis

3.6.1 Need for Dimensional Analysis

Dimensional analysis is a process of formulating fluid mechanics problems in terms of nondimensional variables and parameters. This helps us in the following ways,

1. Reduction of variables
2. Helps in understanding physics
3. Useful in data analysis and modeling
4. Helps us in doing similarity and model testing

3.7 Dimensional Homogeneity

Any equation describing a physical situation will only be true if both sides have the same dimensions. That is it must be **dimensionally homogenous**.

For example the equation which gives for over a rectangular weir (derived earlier in this module) is,

$$Q = \frac{2}{3} B \sqrt{2g} H^{3/2}$$

The SI units of the left hand side are $m^3 s^{-1}$. The units of the right hand side must be the same. Writing the equation with only the SI units gives

$$m^3 s^{-1} = m\left(m s^{-2}\right)^{1/2} m^{3/2}$$
$$= m^3 s^{-1}$$

i.e. the units are consistent.

To be more strict, it is the dimensions which must be consistent (any set of units can be used and simply converted using a constant). Writing the equation again in terms of dimensions,

$$L^3 T^{-1} = L\left(L T^{-2}\right)^{1/2} L^{3/2}$$
$$= L^3 T^{-1}$$

Notice how the powers of the individual dimensions are equal, (for L they are both 3, for T both -1). This property of dimensional homogeneity can be useful for:

1. Checking units of equations;
2. Converting between two sets of units;
3. Defining dimensionless relationships (see below).

3.8. Methods of Dimensional Analysis

There are two methods of Dimensional Analysis, they are

1. Rayleigh's Method (or) Indicial Method

2. Buckingham's Π Theorem Method

3.7.1 Rayleigh's Method

In this method all the variables are taken together for setting up a set of 3 indicial equations and solving them for the purpose of obtaining the dimensionless groups in the desired functional form.

Methodology

Consider X_1 as a function of X_2, X_3,X_4, ……..X_n.

$$X_1 = f(X_2, X_3, X_4 \dots \dots \dots X_n)$$

Write the equation in the above form

$$X_1 = K\left(X_2^a, X_3^b, X_4^c, \dots \dots \dots X_n^m\right)$$

Where K is Dimensionless Constant and a,b,c….m are arbiatry exponents.

Express each of the quantity in the above equation in terms of the Fundamental Dimensions

Taking the dimensions of all the quantities we can write as,

$$[X_1] = [X_2]^a[X_3]^b[X_4]^c \dots \dots \dots [X_n]^m$$

By using dimensional homogeneity, obtain a set of simultaneous equations involving the exponents *a*, *b*, *c*, ..., *m*.

Solve these equations to obtain the value of exponents *a*, *b*, *c*, ..., *m*.

Substitute the values of exponents in the main equation, and form the non-dimensional parameters by grouping the variables with like exponents.

Drawback:

The only drawback in this method is it does not provide information about the number non-dimensional groups that can be obtained as a result of dimensional analysis

Example:

Consider the force F on the propeller of an aircraft. It is known to depend upon the forward speed of aircraft U, the density of the air ρ, the viscosity of air μ, diameter of the propeller D and speed of the rotation of propeller N.

Let us apply the Rayleigh's method of Dimensional Analysis to solve this problem. The general form of the above relationship can be written as,

$$F = f(U, \rho, \mu, D, N)$$

because the force acing on the propeller of an aircraft depends upon these variables. According to the Rayleigh's method we can write the above equation as,

$$F = K\ (U^a, \rho^b, \mu^c, D^d, N^e)$$

Expressing each term in the form of fundamental dimension we can write the above equation as,

$$[MLT^{-2}] = [LT^{-1}]^a[ML^{-3}]^b[ML^{-1}T^{-1}]^c[L]^d[T^{-1}]^e$$

From the principle of dimensional homogeneity we require that the dimensions of the each term on the right hand side of the equation must be equal to the dimensions of the term F on the left hand side.

$$[MLT^{-2}] = [M]^{b+c}[L]^{a-3b-c+d}[T]^{-a-c-e}$$

Equating the indices of M, L, T the following indicial equations are obtained,

$$b + c = 1$$

$$a - 3b - c + d = 1$$

$$-a - c - e = -2$$

These are three equations for five unknowns. Therefore, three variables in the terms of the other two,

$$b = 1 - c$$

$$a = 2 - c - e$$

$$d = 2 - c + d$$

Substituting back these equations in the dimensional analysis equation we get

$$F = K.U^{2-c-e}\rho^{1-c}\mu^c D^{2-c+e}N^e$$

$$F = K\rho U^2 D^2 \left(\frac{\mu}{\rho U D}\right)^c \left(\frac{ND}{U}\right)^e$$

Where K,c,d are the unknown quantities the above equation can be written in the functional form as

$$\frac{F}{\rho U^2 D^2} = f\left(\frac{\mu}{\rho U D}, \frac{ND}{U}\right)$$

3.9 Buckingham's Π Theorem Method

The Buckingham's Π Theorem is a formalized procedure for reducing the dimensionless groups appropriate for a given fluid mechanics or other engineering problems. **The Buckingham's Π theorem is a statement of relation between a function expressed in terms of dimensional parameters and a related function expressed in terms of nondimensional parameters.** The Buckingham's Π Theorem allows us to develop the important non dimensional parameters quickly and easily.

Buckingham's Π Theorem

If there are n – variables in a physical phenomenon and those n-variables contain 'm' dimensions, then the variables can be arranged into (n-m) dimensionless groups called Π terms.

If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into (n-m) dimensionless numbers. Each term is called π term.

Let $X_1, X_2, X_3, \ldots X_n$ are the variables involved in a physical problem. Let X_1 be the dependent variable and $X_2, X_3, \ldots X_n$ are the independent variables on which X_1 depends. Then X_1 is a function of $X_2, X_3, \ldots X_n$ and mathematically it is expressed as

$$X_1 = f(X_2, X_3, \ldots X_n) \quad \text{--------------------(1)}$$

The above equation can also be written as

$$f_1(X_1, X_2, X_3, \ldots X_n) = 0 \quad \text{-------------------(2)}$$

The above (2) is a dimensionally homogeneous equation. It contains n variables. If there are m fundamental dimensions then according to Buckingham's π theorem, equation (2) can be written on terms of dimensionless groups or π- terms is equal to (n-m). Hence equation (2) becomes as

$$f_1(\pi_1, \pi_2, \pi_3, \ldots \pi_{n-m}) = 0. \quad \text{-------------(3)}$$

Each π- term is dimensionless and is independent of the system. Division or multiplication by a constant does not change the character of the π- term. Each π- term contains m+1 variables, where m is the number of fundamental dimensions and is also called repeating variables. Let in the above case X_2, X_3, and X_n are repeating variables if the fundamental dimension m (M, L,T)=3. Then each π- term is written as

$$\pi_1 = X_2^{a1}.X_3^{b1}.X_4^{c1}.X_1$$

$$\pi_2 = X_2^{a2}.X_3^{b2}.X_4^{c2}.X_5$$

$$\pi_{n-m} = X_2^{an-m}.X_3^{b\,n-m}.X_4^{cm}.X_n$$

----------------(4)

Each equation is solved by the principle of dimensional homogeneity and values of a_1, b_1, c_1 etc. are obtained. These values are substituted in equation (4) and values of π_1, π_2, π_3,……… π_{n-m} are obtained. These values of π's are substituted in equation (3). The final equation for the phenomenon is obtained by expressing any one of the π- terms as a function of others as

$$\pi_1 = \Phi[\pi_2, \pi_3, \ldots\ldots\ldots \pi_{n-m}]$$

$$\pi_2 = \Phi[\pi_1, \pi_3, \ldots\ldots\ldots \pi_{n-m}] \text{ ---------------------(5)}$$

Method of selecting Repeating variables:

The number of repeating variables are equal to the number of fundamental dimensions of the problem. The choice of repeating variables if governed by the following considerations.

1. As far as possible, the dependent variable should not be selected as repeating variable.

2. The repeating variables should be choosen in such a way that one variable contains geometric property, other variable contains flow property and third variable contains fluid property.

Variables with geometric property are (i) Length (ii) diameter (iii) Height H

Variables with flow property are (i) Velocity (ii) Acceleration

Variables with fluid property are (i) μ (ii) ρ (iii) w

3. The repeating variables selected should not form a dimensionless group.

4. The repeating variables together must have the same number of fundamental dimensions.

5. No two repeating variables should have the same dimensions.

3.10 Similitude and Types of Similitude

Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype are completely similar. Three types of similarities must exist between the model and prototype. They are

1. Geometric Similarity
2. Kinematic Similarity
3. Dynamic Similarity

3.10.1 Geometric Similarity:

The geometric similarity is said to exist between the model and the prototype if the ratio of all corresponding linear dimension in the model and prototype are equal.

L_m= Length of model , b_m= Breadth of model

D_m= Dismeter of model A_m= area of model

$\forall_m$= Volume of model

and L_p, B_p, D_p, A_p, $\forall_p$ =Corresponding values of the prototype.

For geometric similarity between model and prototype, we must have the relation,

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r$$

L_r is called the scale ratio.

For area's ratio and volume's ratio the relation should be as given below.

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^{\ 2}$$

$$\frac{\forall_p}{\forall_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3$$

3.10.2 Kinematic Similarity:

Kinematic similarity means the similarity of motion between model and prototype. Thus kinematic similarity is said to exist between the model and the prototype if the ratios of the velocity and acceleration at the corresponding points in the model and at the corresponding points in the prototype are the same. Since the velocity and acceleration are vector quantities, hence not only the ratio of magnitude of velocity and acceleration at the corresponding points in the model and prototype should be same, but the directions of velocity and accelerations at the corresponding points in the model and prototype also should be parallel.

V_{p1} = velocity of fluid at point 1 in prototype,

V_{p2}= velocity of fluid at point 2 in prototype,

a_{p1}= Acceleration of fluid at point 1 in prototype,

a_{p2}= Acceleration of fluid at point 2 in prototype,

V_{m1}, V_{m2}, a_{m1}, a_{m2} = Corresponding values at the corresponding points of fluid velocity and acceleration in the model.

For kinematic similarity, we have

$$\frac{V_{p1}}{V_{m1}} = \frac{V_{p2}}{V_{m2}} = V_r$$

where V_r is the velocity ratio.

For acceleration, we have $\frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} = a_r$

where a_r is the acceleration ratio.

Also the directions of the velocities in the model and prototype should be same.

3.10.3 Dynamic Similarity:

Dynamic similarity means the similarity of forces between the model and prototype. Thus dynamic similarity is said to exist between the model and prototype if the ratios of the corresponding forces acting at the corresponding points are equal. Also the directions of the corresponding forces at the corresponding points should be same.

$(F_i)_p$= Inertia force at a point in prototype,

$(F_v)_p$= Viscous force at the point in prototype,

$(F_g)_p$= Gravity force at the point in prototype,

$F_i)_p$, $(F_v)_p$, $(F_g)_p$ = Corresponding values of forces at the corresponding point in model.

Then for dynamic similarity, we have

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

where F_r is the force ratio.

Also the directions of the corresponding forces at the corresponding points in the model and prototype should be same.

3.11 Dimensionless Parameters and Its Applications

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or pressure force or surface tension force or elastic force. As this is a ratio of one force to the other force, it will be a dimensionless number. These dimensionless numbers are also called non-dimensional parameters.

The following are the important dimensionless numbers:

1. Reynold’s number
2. Froud’s number
3. Euler’s number
4. Weber’s number
5. Mach’s number

1. Reynold’s Number:

It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynolds number is obtained as

$$R_e = \frac{V \times d}{\nu} \quad or \quad \frac{\rho V d}{\mu}$$

2. Froude's Number (F_{es}):

The Froud's Number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravitational force. Mathematically, it is expressed as

$$\mathrm{F_e} = \sqrt{\frac{\mathrm{F_i}}{\mathrm{F_g}}} = \sqrt{\frac{\rho A V^2}{\rho A L g}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}}$$

3. Euler's number (E_u):

It is defined as the square root of the ratio of inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

$$E_u = \sqrt{\frac{F_i}{F_p}}$$

4. Weber's Number (W_e):

It is defined as the square root of the ratio of inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

$$\mathrm{W_e} = \sqrt{\frac{\mathrm{F_i}}{\mathrm{F_g}}}$$

5. Mach Number (M):

Mach Number is defined as the square root of the ratio of inertia force of a flowing fluid to the elastic force. Mathematically, it is expressed as

$$\mathrm{M} = \sqrt{\frac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\frac{\mathrm{F_i}}{\mathrm{F_e}}}$$

3.12 Model Analysis

Model is the small scale replica of the actual structure or machine. It is not necessary that models should be smaller than the prototypes (although in most of the cases it is), they may be larger than the prototypes.

Classification of model

1. Undistorted models are those models which are geometrically similar to their prototype. In other words the scale ratio for the linear dimensions of the model and its prototype are the same.

2. Distorted models are those models which are geometrically not similar to its prototype. In other words the scale ratio for the linear dimensions of the model and its prototype are not same.

Advantages:

- The performance of the machine can be easily predicted, in advance.
- With the help of dimensional analysis, a relationship between the variables influencing a flow problem in terms of dimensional parameters is obtained. This relationship helps in conducting tests on the model.
- The merits of alternative designs can be predicted with the help of model testing. The most economical and safe design may be, finally, adopted.

For the dynamic similarity between the model and the prototype the ratio of the corresponding forces acting at the corresponding points in the model and prototype should be equal. It means for dynamic similarity between the model and proto type, the dimensionless numbers should be same for model and prototype. It is quite difficult to satisfy the condition that all the dimensionless numbers are the same for the model and prototype. Therefore models are designed on the basis of equating the dimensionless number which dominates the phenomenon.

Following are the dynamic similarity laws:

1. Reynolds model law

2. Froude model law

3. Euler model law

4. Weber model law

5. Mach model law

Reynolds model law - (Pipe flow, sub-marines, aeroplane etc.)

$$[\mathrm{Re}]_m = [\mathrm{Re}]_p \Rightarrow \frac{V_m \rho_m d_m}{\mu_m} = \frac{V_p \rho_p d_p}{\mu_p}$$

Froude's model law - (Free-surface flow, jet from orifice or nozzle etc)

$$[Fr]_m = [Fr]_p \Rightarrow \frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

Euler's model law - (pressure force is a dominant force)

$$[E_u]_m = [E_u]_p \Rightarrow \frac{V_m}{\sqrt{p_m / \rho_m}} = \frac{V_p}{\sqrt{p_p / \rho_p}}$$

Weber model law - (surface tension is a dominant force)

$$[W_e]_m = [W_e]_p \Rightarrow \frac{V_m}{\sqrt{\sigma_m / \rho_m L_m}} = \frac{V_p}{\sqrt{\sigma_p / \rho_p L_p}}$$

Mach model law - (velocity of flow is comparable to the velocity of sound; compressible flow)

$$[M]_m = [M]_p \Rightarrow \frac{V_m}{\sqrt{K_m / \rho_m}} = \frac{V_p}{\sqrt{K_p / \rho_p}}$$

Question Bank

1. State the limitations of Dimensional Analysis and Explain about Model analysis and briefly discuss about its applications and advantage.
2. Explain Similitude and types of similarities and Discuss briefly about the various dimensionless numbers and their significance

3. Derive on the basis of dimensional analysis suitable parameters to develop the thrust produced by a propeller. Assume that the thrust P depends on the Angular Velocity ω ,speed of advance V, diameter D, dynamic viscosity μ , mass density ρ ,elasticity of the fluid medium which can be denoted by the speed of sound in the medium C.

4. Using the Buckingham Theorem prove

$$Q = Nd^3\phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{\rho N^2 d^3}, \frac{w}{\rho N^2 d}\right]$$

   Where d is the internal diameter of the ring , N is the rotational speed , ρ is density , μ is viscosity , σ is the surface tension and w is the specific weight of the oil.

5. The frictional torque T of a disc diameter D rotating at a speed N in a fluid of Viscosity μ and density in a turbulent flow is given by $T = D^5N^2\, \rho\Phi(\mu/D^2N\rho)$

6. Using Buckingham Theorem show that the velocity through the circular orifice is given by

$$V = \sqrt{2gH\,\phi\left[\frac{D}{H}, \frac{\mu}{\rho v H}\right]}$$

   Where H = Head Causing flow , D = Diameter of the orifice , μ = Co-Efficient of Viscosity , ρ = Mass Density and g = Acceleration due to gravity

7. The discharge Q of a Centrifugal pump depends upon the mass density of fluid ρ , the speed of the pump (N) , the diameter of the impeller (D) , the manometric head (H_m) and the viscosity of the fluid (μ). Show that $Q = ND^3\phi\left[\frac{gH}{e^2D^2}, \frac{\mu}{\rho ND^2}\right]$

8. State and explain the Buckingham 's π theorem

9. The ratio of the lengths of a submarine and its model is 30:1.The speed of submarine (prototype) is 10 m/s. The model is to be tested in a wind tunnel. Find the speed of air in the wind tunnel . Also determine the ratio of drag (resistance) between the model and its prototype .Take the value of kinematic viscosities for sea water and air as 0.012 strokes and 0.016 strokes respectively. The density for sea water and air is given as 1030 kg/m^3 and 1.24 kg/m^3 respectively.
10. A ship 300 m long moves in sea water , whose density is 1030 kg/m^3,A I : 100 model of this ship is to be tested in a wind tunnel . The Velocity of air in the wind tunnel around the model is 30 m/s and the resistance of the model is 60 N. Determine the velocity of ship in sea water and also the

resistance of the ship in sea water.The density of air is given as 1.24 kg/m^3.Take the kinematic viscosity of sea water and air as 0.012 strokes and 0.018 strokes respectively.

11. Show that the lift F_L on airfoil can be expressed as

$$F_L = \rho V^2 d^2 \Phi \left(\frac{\rho V d}{\mu}, \alpha\right)$$

Where , ρ = Mass density

V = Velocity of flow

D= A characteristic depth

α = Angle of incidence

μ = Coefficient of viscosity

12. Derive on the basis of dimensional analysis suitable parameters to present the thrust developed by a propeller. Assume that the thrust P depends upon the angular velocity ω, Speed of advance V, diameter D, dynamic viscosity μ , mass density ρ , elasticity of the fluid medium which can be denoted by the speed of sound in the medium C.

13. A torpedo shaped object 900 mm diameter is to move in air at 60 m/s and its drag is to be estimated from tests in water on a half scale model .Determine the necessary speed of the model and the drag of the full scale object if that of the model is 1140 N. Given properties : air viscosity = 1.86×10^{-5} Ns/m^2 , water viscosity = 1.01×10^{-3} Ns/m^2 ,air density = 1.2 kg/m^3,water density = 1000 kg/m^3

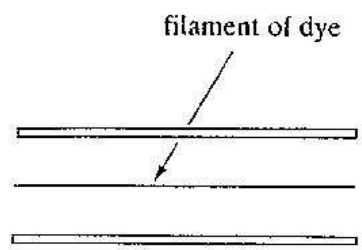
UNIT IV

PIPE FLOW

Laminar and turbulent flow

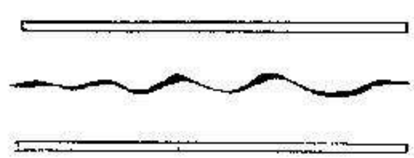
If we were to take a pipe of free flowing water and inject a dye into the middle of the stream, what would we expect to happen?

This



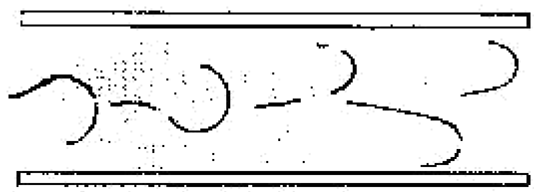
Laminar (viscous)

this



Transitional

or this

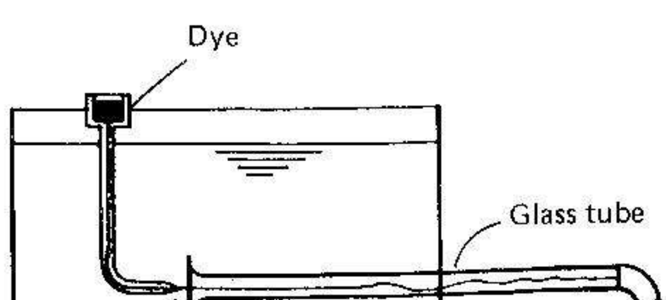


Turbulent

Actually both would happen - but for different flow rates. The top occurs when the fluid is flowing fast and the lower when it is flowing slowly.The top situation is known as **turbulent** flow and the lower as **laminar** flow. In laminar flow the motion of the particles of fluid is very orderly with all particles moving in straight lines parallel to the pipe walls.

But what is fast or slow? And at what speed does the flow pattern change? And why might we want to know this?

The phenomenon was first investigated in the 1880s by Osbourne Reynolds in an experiment which has become a classic in fluid mechanics.



He used a tank arranged as above with a pipe taking water from the centre into which he injected a dye through a needle. After many experiments he saw that this expression

$$\frac{\rho u d}{\mu}$$

would help predict the change in flow type. If the value is less than about 2000 then flow is laminar, if greater than 4000 then turbulent and in between these then in the transition zone.

This value is known as the Reynolds number, Re:

$$\mathrm{Re} = \frac{\rho u d}{\mu}$$

Laminar flow: Re < 2000

Transitional flow: 2000 < Re < 4000

Turbulent flow: Re > 4000

What are the units of this Reynolds number? We can fill in the equation with SI units:

$$\rho = kg/m^3, \quad u = m/s, \qquad d = m$$
$$\mu = Ns/m^2 = kg/ms$$

$$\mathrm{Re} = \frac{\rho u d}{\mu} = \frac{kg}{m^3}\frac{m}{s}\frac{m}{1}\frac{m}{kg} = 1$$

i.e. it has **no units**. A quantity that has no units is known as a **non-dimensional** (or dimensionless) quantity. Thus the Reynolds number, Re, is a non-dimensional number.

It can be interpreted that when the inertial forces dominate over the viscous forces (when the fluid is flowing faster and Re is larger) then the flow is turbulent. When the viscous forces are dominant (slow flow, low Re) they are sufficient enough to keep all the fluid particles in line, then the flow is laminar.

In summary:

Laminar flow

- Re < 2000
- 'low' velocity
- Dye does not mix with water
- Fluid particles move in straight lines
- Simple mathematical analysis possible
- Rare in practice in water systems.

Transitional flow

- 2000 > Re < 4000
- 'medium' velocity
- Dye stream wavers in water - mixes slightly.

Turbulent flow

- Re > 4000 'high' velocity
- Dye mixes rapidly and completely
- Particle paths completely irregular
- Average motion is in the direction of the flow
- Cannot be seen by the naked eye
- Changes/fluctuations are very difficult to detect. Must use laser.
- Mathematical analysis very difficult - so experimental measures are used
- Most common type of flow.

The types of fluid flow.

The fluid flow is classified as :

1. Steady and Unsteady flows.
2. Uniform and Non – uniform flows.
3. Laminar and turbulent flows.
4. Compressible and incompressible flows.

5. Rotational and irrotational flows
6. One, two and three dimensional flows.

3.1 Uniform Flow, Steady Flow

It is possible - and useful - to classify the type of flow which is being examined into small number of groups. If we look at a fluid flowing under normal circumstances - a river for example - the conditions at one point will vary from those at another point (e.g. different velocity) we have non-uniform flow. If the conditions at one point vary as time passes then we have unsteady flow.

Under some circumstances the flow will not be as changeable as this. He following terms describes the states which are used to classify fluid flow:

- *Uniform flow:* If the flow velocity is the same magnitude and direction at every point in the fluid it is said to be *uniform*.
- *Non-uniform:* If at a given instant, the velocity is **not** the same at every point the flow is *non-uniform*. (In practice, by this definition, every fluid that flows near a solid boundary will be non-uniform - as the fluid at the boundary must take the speed of the boundary, usually zero. However if the size and shape of the of the cross-section of the stream of fluid is constant the flow is considered *uniform*.)
- *Steady:* A steady flow is one in which the conditions (velocity, pressure and cross-section) may differ from point to point but DO NOT change with time.
- *Unsteady:* If at any point in the fluid, the conditions change with time, the flow is described as *unsteady*. (In practise there is always slight variations in velocity and pressure, but if the average values are constant, the flow is considered *steady*.

Combining the above we can classify any flow in to one of four types:

1. *Steady uniform flow*. Conditions do not change with position in the stream or with time. An example is the flow of water in a pipe of constant diameter at constant velocity.

2. *Steady non-uniform flow.* Conditions change from point to point in the stream but do not change with time. An example is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.

3. *Unsteady uniform flow*. At a given instant in time the conditions at every point are the same, but will change with time. An example is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.

4. *Unsteady non-uniform flow.* Every condition of the flow may change from point to point and with time at every point. For example waves in a channel.

3.2 Compressible or Incompressible

All fluids are compressible - even water - their density will change as pressure changes. Under steady conditions, and provided that the changes in pressure are small, it is usually possible to simplify analysis of the flow by assuming it is incompressible and has constant density. As you will appreciate, liquids are quite difficult to compress - so under most steady conditions they are treated as incompressible. In some unsteady conditions very high pressure differences can occur and it is necessary to take these into account - even for liquids. Gasses, on the contrary, are very easily compressed, it is essential in most cases to treat these as compressible, taking changes in pressure into account.

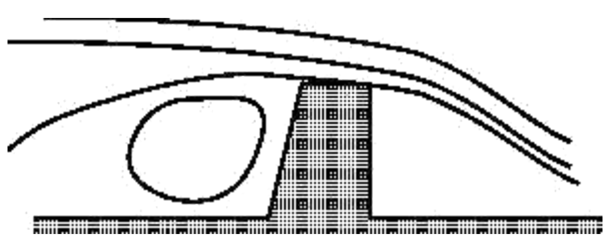
3.3 Three-dimensional flow

Although in general all fluids flow three-dimensionally, with pressures and velocities and other flow properties varying in all directions, in many cases the greatest changes only occur in two directions or even only in one. In these cases changes in the other direction can be effectively ignored making analysis much more simple.

Flow is *one dimensional* if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in time only vary in the direction of flow and not across the cross-section. The flow may be unsteady, in this case the parameter vary in time but still not across the cross-section. An example of one-dimensional flow is the flow in a pipe. Note that since flow must be zero at the pipe wall - yet non-zero in the centre - there is a difference of parameters across the cross-section. Should this be treated as two-dimensional flow? Possibly - but it is only necessary if very high accuracy is required. A correction factor is then usually applied.

One dimensional flow in a pipe.

Flow is *two-dimensional* if it can be assumed that the flow parameters vary in the direction of flow and in one direction at right angles to this direction. Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes. An example is flow over a weir foe which typical streamlines can be seen in the figure below. Over the majority of the length of the weir the flow is the same - only at the two ends does it change slightly. Here correction factors may be applied.

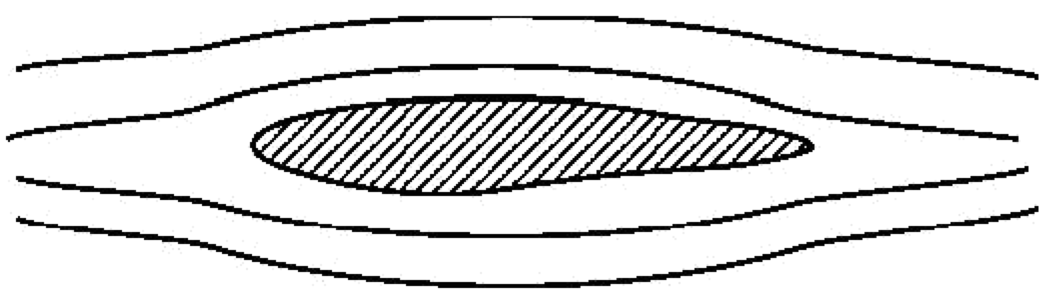


Two-dimensional flow over a weir.

In this course we will **only** be considering steady, incompressible one and two-dimensional flow.

3.4 Streamlines and stream tubes

In analysing fluid flow it is useful to visualise the flow pattern. This can be done by drawing lines joining points of equal velocity - velocity contours. These lines are know as *streamlines*. Here is a simple example of the streamlines around a cross-section of an aircraft wing shaped body:



Streamlines around a wing shaped body

When fluid is flowing past a solid boundary, e.g. the surface of an aerofoil or the wall of a pipe, fluid obviously does not flow into or out of the surface. So very close to a boundary wall the flow direction must be parallel to the boundary.

Close to a solid boundary streamlines are parallel to that boundary

At all points the direction of the streamline is the direction of the fluid velocity: this is how they are defined. Close to the wall the velocity is parallel to the wall so the streamline is also parallel to the wall. It is also important to recognize that the position of streamlines can change with time - this is the case in unsteady flow. In steady flow, the position of streamlines does not change.

3.5 Important Definitions

Laminar flow

Laminar flow is defined as that type of flow in which the fluid particles move along well defined paths or stream line and all the stream lines are straight and parallel. Thus the particles move in laminas or layers gliding over the adjacent layer. This type of flow is also called stream line flow or viscous flow

Turbulent flow

Turbulent flow is that type of flow in which the fluid particles move in a zig –zag way. Due to the movement of fluid particles in a zig – zag way.

Rate of flow or Discharge

It is defined as the quantity of a fluid flowing per second through a section of a pipe or channel. For an incompressible fluid(or liquid) the rate of flow or discharge is expressed as volume of fluid flowing across the section per section. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

The discharge (Q) = A X V

Where, A = Cross – sectional area of pipe.

V = Average velocity of fluid across the section.

Continuity Equation

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

$$A_1V_1 = A_2V_2..$$

Local acceleration

Local acceleration is defined as the rate of increase of velocity with respect to time at a given point in a flow field. In equation is given by the expression

$(\partial u / \partial t)$, $(\partial v / \partial t)$ or $(\partial w / \partial t)$ is known as local acceleration.

Convective acceleration

It is defined as the rate of change of velocity due to the change of position of fluid particles in a fluid flow. The expressions other than $(\partial u / \partial t)$, $(\partial v / \partial t)$ and $(\partial w / \partial t)$ in the equation are known as convective acceleration.

Velocity potential function.

It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is defined by Φ (Phi). Mathematically, the velocity, potential is defined as $\Phi = f(x,y,z)$ for steady flow such that.

$u = -(\partial \Phi/\partial x)$

$v = -(\partial \Phi/\partial y)$

$w = -(\partial \Phi/\partial z)$ where, u,v and w are the components of velocity in x.y and z directions respectively.

Stream function.

It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ (Psi) and only for two dimensional flow. Mathematically. For steady flow is defined as $\psi = f(x,y)$ such that ,

$$(\partial \psi / \partial x) = v$$

$$(\partial \psi / \partial y) = - u.$$

Flow net

A grid obtained by drawing a series of equipotential lines and stream lines is called a flow net. The flow net is an important tool in analyzing the two – dimensional irrotational flow problems.

The properties of stream function.

The properties of stream function (ψ) are:

1. If stream function (ψ) exists, it is possible case of fluid flow which may be rotational or irrotational.
2. If stream function (ψ) satisfies the Laplace equation, it is a possible case of irrotational flow.

The types of Motion

1. Linear Translation or Pure Translation.

2. Linear Deformation.

3. Angular deformation.

4. Rotation.

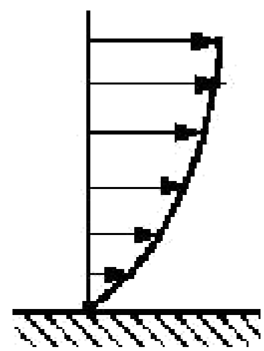
Vortex flow

Vortex flow is defined as the flow of a fluid along a curved path or the flow of a rotating mass of a fluid is known ‘ Vortex Flow’. The vortex flow is of two types namely:

1. Forced vortex flow, and
2. Free vortex flow.

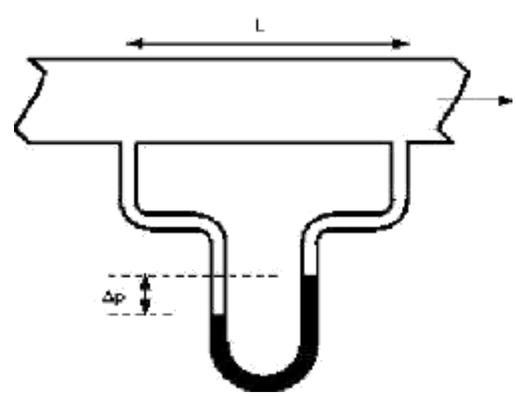
Up to this point on the course we have considered ideal fluids where there have been no losses due to friction or any other factors. In reality, because fluids are viscous, energy is lost by flowing fluids due to friction which must be taken into account. The effect of the friction shows itself as a pressure (or head) loss.

In a pipe with a real fluid flowing, at the wall there is a shearing stress retarding the flow, as shown below.

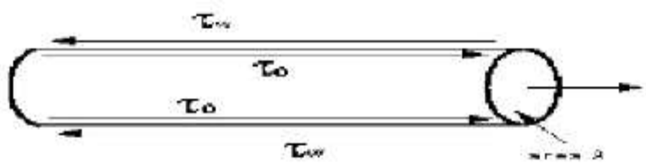


If a manometer is attached as the pressure (head) difference due to the energy lost by the fluid overcoming the shear stress can be easily seen.

The pressure at 1 (upstream) is higher than the pressure at 2.



We can do some analysis to express this loss in pressure in terms of the forces acting on the fluid. Consider a cylindrical element of incompressible fluid flowing in the pipe, as shown



The pressure at the upstream end is *p*, and at the downstream end the pressure has fallen by p to (p- p). The driving force due to pressure (F = Pressure x Area) can then be written

driving force = Pressure force at 1 - pressure force at 2

$$pA - \left(p - \Delta p\right)A = \Delta p\, A = \Delta p \frac{\pi d^2}{4}$$

The retarding force is that due to the shear stress by the walls

$$\begin{aligned} &= \text{shear stress} \times \text{area over which it acts} \\ &= \tau_w \times \text{area of pipe wall} \\ &= \tau_w \pi d L \end{aligned}$$

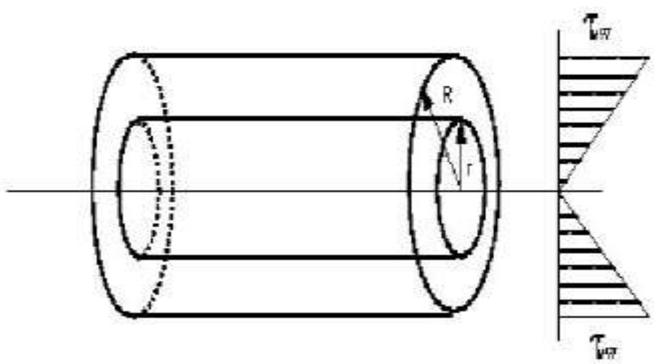
As the flow is in equilibrium,

driving force = retarding force

$$\Delta p \frac{\pi d^2}{4} = \tau_w \pi d L$$

$$\Delta p = \frac{\tau_w 4L}{d}$$

Giving an expression for pressure loss in a pipe in terms of the pipe diameter and the shear stress at the wall on the pipe.



The shear stress will vary with velocity of flow and hence with Re. Many experiments have been done with

various fluids measuring the pressure loss at various Reynolds numbers. These results plotted to show a graph of the relationship between pressure loss and Re look similar to the figure below:

This graph shows that the relationship between pressure loss and Re can be expressed as

laminar	$\Delta p \propto u$
turbulent	$\Delta p \propto u^{1.7 \text{ (or 2.0)}}$

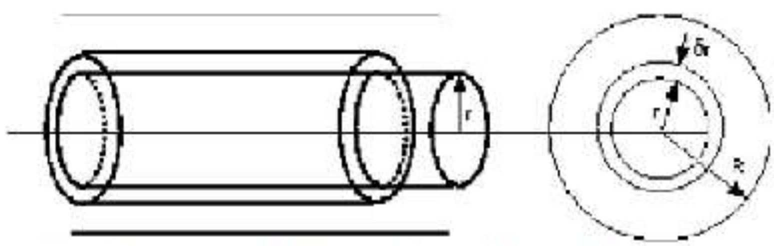
2.3 Hagen-Poiseuille equation:

As these are empirical relationships, they help in determining the pressure loss but not in finding the magnitude of the shear stress at the wall w on a particular fluid. If we knew w we could then use it to give a general equation to predict the pressure loss.

Pressure loss during laminar flow in a pipe In general the shear stress w. is almost impossible to measure. But for laminar flow it is possible to calculate a theoretical value for a given velocity, fluid and pipe dimension.

In laminar flow the paths of individual particles of fluid do not cross, so the flow may be considered as a series of concentric cylinders sliding over each other - rather like the cylinders of a collapsible pocket telescope.

As before, consider a cylinder of fluid, length L, radius r, flowing steadily in the centre of a pipe.



We are in equilibrium, so the shearing forces on the cylinder equal the pressure forces.

We are in equilibrium, so the shearing forces on the cylinder equal the pressure forces.

$$\tau \; 2\pi r \; L = \Delta p \; A = \Delta p \pi r^2$$
$$\tau = \frac{\Delta p}{L} \frac{r}{2}$$

By Newtons law of viscosity we have $\tau = \mu \frac{du}{dy}$, where y is the distance from the wall. As we are measuring from the pipe centre then we change the sign and replace y with r distance from the centre, giving

$$\tau = -\mu \frac{du}{dr}$$

Which can be combined with the equation above to give

$$\frac{\Delta p}{L}\frac{r}{2} = -\mu \frac{du}{dr}$$
$$\frac{du}{dr} = -\frac{\Delta p}{L}\frac{r}{2\mu}$$

In an integral form this gives an expression for velocity,

$$u = -\frac{\Delta p}{L}\frac{1}{2\mu}\int r\,dr$$

Integrating gives the value of velocity at a point distance r from the centre

$$u_r = -\frac{\Delta p}{L}\frac{r^2}{4\mu} + C$$

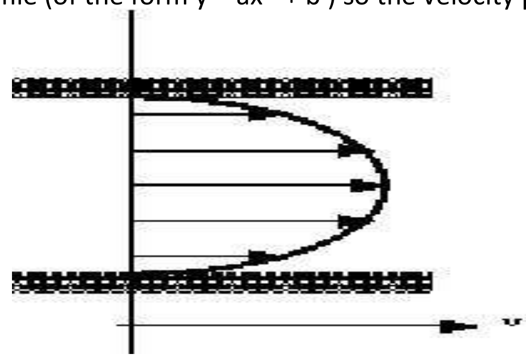
At $r = 0$, (the centre of the pipe), $u = u_{max}$, at $r = R$ (the pipe wall) $u = 0$, giving

$$C = \frac{\Delta p}{L}\frac{R^2}{4\mu}$$

so, an expression for velocity at a point r from the pipe centre when the flow is laminar is

$$u_r = \frac{\Delta p}{L}\frac{1}{4\mu}\left(R^2 - r^2\right)$$

Note how this is a parabolic profile (of the form $y = ax^2 + b$) so the velocity profile in the pipe looks similar to the figure below



Discharge in the pipe

$$\begin{aligned} Q &= u_m A \\ u_m &= \int_0^R u_r \, dr \\ &= \frac{\Delta p}{L}\frac{1}{4\mu}\int_0^R \left(R^2 - r^2\right) dr \\ &= \frac{\Delta p}{L}\frac{R^2}{8\mu} = \frac{\Delta p\, d^2}{32\mu L} \end{aligned}$$

So the discharge can be written

$$Q = \frac{\Delta p\, d^2}{32\mu L}\frac{\pi d^2}{4}$$
$$= \frac{\Delta p}{L}\frac{\pi d^2}{128\mu}$$

This is the Hagen-Poiseuille equation for laminar flow in a pipe. It expresses the discharge *Q* in terms of

the pressure gradient ($\frac{\partial p}{\partial x} = \frac{\Delta p}{L}$), diameter of the pipe and the viscosity of the fluid.

We are interested in the pressure loss (head loss) and want to relate this to the velocity of the flow. Writing pressure loss in terms of head loss h_f, i.e. $p = gh_f$

$$u = \frac{\rho g h_f d^2}{32\mu L}$$
$$h_f = \frac{32\mu L u}{\rho g d^2}$$

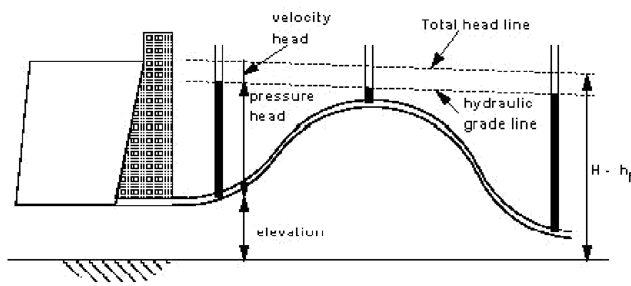
This shows that pressure loss is directly proportional to the velocity when flow is laminar. It has been validated many time by experiment.

It justifies two assumptions:

1. fluid does not slip past a solid boundary
2. Newton's hypothesis.

2.4 Losses due to friction

In a real pipe line there are energy losses due to friction - these must be taken into account as they can be very significant. How would the pressure and hydraulic grade lines change with friction? Going back to the constant diameter pipe, we would have a pressure situation like this shown below



Hydraulic Grade line and Total headlines for a constant diameter pipe with friction

How can the total head be changing? We have said that the total head - or total energy per unit weight - is constant. We are considering energy conservation, so if we allow for an amount of energy to be lost due to friction the total head will change. We have seen the equation for this before. But here it is again

with the energy loss due to friction written as a *head* and given the symbol h_f . This is often know as the *head loss due to friction*.

$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2+h_f$$

2.4.1 Hydraulic gradient line.

It is defined as the line which gives the sum of pressure head (P/ρg) and datum head (z) of a flowing fluid in a pipe with respect to some reference line or is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head

(P/ρg) of a pipe from the center of the pipe. It is briefly written as H.G.L

2.4.2 Major energy loss and minor energy loss in pipe

The loss of head or energy due to friction in pipe is known as major loss while the loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy.

2.4.3 Total Energy line

It is defined as the line which gives sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

2.4.4 Equivalent pipe line

An Equivalent pipe is defined as the pipe of uniform diameter having loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.

2.4.5 Water Hammer in pipes

In a long pipe, when the flowing water is suddenly brought to rest by closing the valve or by any similar cause, there will be a sudden rise in pressure due to the momentum of water being destroyed. A pressure wave is transmitted along the pipe. A sudden rise in pressure has the effect of hammering action on the walls of the pipe. This phenomenon of rise in pressure is known as water hammer or hammer blow.

2.5 Pipes in series

Pipes in series or compound pipes is defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line.

2.6 Pipes in parallel

The pipes are said to be parallel, when a main pipe divides into two or more parallel pipes which again join together downstream and continues as a mainline. The pipes are connected in parallel in order to increase the discharge passing through the main.

2.7 Boundary layer

When a solid body is immersed in a flowing fluid, there is a narrow region of the fluid in neighbourhood of the solid body, where the velocity of fluid varies from zero to free stream velocity. This narrow region of fluid is called boundary layer.

2.7.1 Laminar sub layer

In turbulent boundary layer region, adjacent to the solid boundary velocity for a small thickness variation in influenced by various effect. This layer is called as laminar sub layer.

2.7.2 Boundary layer thickness.

It is defined as the distance from the boundary of the solid body measured in the y – direction to the point where the velocity of the fluid is approximately equal to 0.99 times the free stream (v) velocity of the fluid.

2.7.3 Momentum thickness.

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid of boundary

$$\theta = \int_0^{\delta} u/v(1-u/v)dy$$

Incompressible flow.

It is defined as the type of flow in which the density is constant for the fluid flow. Mathematically ρ = constant. Examples : Subsonic, aerodynamics.

2.7.4 Methods of preventing the separation of boundary layers

1. Suction of slow moving fluid by suction slot
2. Supplying additional energy from a blower
3. Providing a bypass in the slotted wring
4. Rotating boundary in the direction of flow.
5. Providing small divergence in diffuser
6. Providing guide – blades in a bend.

2.7.5 Characteristics of laminar flow

(i) No slip at the boundary

(ii) Due to viscosity, there is a shear between fluid layers, which is given by
$\tau = \mu(du / dy)$ for flow in x- direction

(iii) The flow is rotational.

(iv) Due to viscous shear, there is continuous dissipation of energy and for maintaining the flow must be supplied externally.

(v) Loss of energy is proportional to first power of velocity and first power of viscosity.

(vi) No mixing between different fluid layers (except by molecular motion, which is very small)

2.7.6 Difference between laminar boundary layer and turbulent boundary layer

The boundary layer is called laminar, if the Renold number of the flow is defined as $R_e = U \mathbf{x} X / v$ is less than $3X10^5$

If the Renold number is more than $5X10^5$, the boundary layer is called turbulent boundary.

Where, U = Free stream velocity of flow

X = Distance from leading edge

v = Kinematic viscosity of fluid

2.7.7 The chezy's formula.

Chezy's formula is generally used for the flow through open channel.

$$V = C\sqrt{mi}$$

Where , C = chezy's constant, m = hydraulic mean depth and $i = h_f / L$.

P 1. A crude of oil of kinematic viscosity of 0.4 stoke is flowing through a pipe of diameter 300mm at the rate of 300 litres/sec. find the head lost due to friction for a length of 50m of the pipe.

Given :

Kinematic viscosity $v = 0.4$ stoke $= 0.4\ cm^2/s = 0.4 X 10^{-4}\ m^2/s$

Dia. Of pipe d = 300mm = 0.3m

Discharge $Q = 300$ Lit/S $= 0.3\ m^3/s$

Length of pipe L = 50m

Velocity $V = Q/\text{Area} = 0.3 / (\frac{\pi}{4}(0.3)^2) = 4.24$ m/s

Renold number $R_e = (V X d)/v = (4.24 X 0.30) / 0.4X10^{-4} = 3.18 X 10^4$

As R_e lies between 4000 and 100,000, the value of "f" is given by f = $\frac{0.079}{(R_e)^{1/4}} = \frac{0.079}{(3.18X10^4)^{1/4}} = 0.00591$

Head lost due to friction h_f = 4 f L V^2/ 2 g d

= (4 X 0.00591 X 50X 4.24^2) /(0.3 X 2 X 9.81)

= 3.61 m.

P2. Hydrodynamically smooth pipe carries water at the rate of 300 lit/s at 20°C (ρ = 1000 kg/m^3, ν = 10^{-6} m^2/s) with a head loss of 3m in 100m length of pipe. Determine the pipe diameter. Use f = 0.0032 + (0.221)/ $(R_e)^{0.237}$ equation for f where h_f = (fXLXV^2)/ 2gd and R_e = (ρVD/μ)

Given:

Discharge, Q = 300 lit/sec = 0.3m^3/s

Density ρ = 1000 kg/m^3

Kinematic viscosity ν = $10^{-6}$$m^2$/s

Head loss h_f = 3m

Length of pipe, L = 100m

Value of friction factor, f = 0.0032 + 0.221 / $(R_e)^{0.237}$

Renolds number R_e = (ρVD/μ) = (VXD) / ν ($\mu/\rho = \nu$)

VXD/ 10^{-6} = VXDX10^6

Find diameter of pipe.

Let D = diameter of pipe

Head loss in terms of friction factor is given as

h_f = (fXLXV^2)/ 2gXD

$3 = (fX100XV^2)/ 2X9.81XD$

$f = (3XDX2X9.81)/ 100V^2$

$f = 0.5886D / V^2$ -----------------------------(i)

now Q = AXV

$0.3 = \frac{\pi}{4} (D)^2 X V$ or $D^2 X V = (4 X 0.3 / \pi) = 0.382$

$V^2 = 0.382 / D^2$ --------------------------- (ii)

$f = 0.0032 + (0.221)/ (R_e)^{0.237}$

$0.5886/ D^2 = 0.0032 + (0.221)/ (VXDX10^6)^{0.237}$

{ from equation (i), $f = 0.5886D/V^2$ and $R_e = VXDX10^6$}

$$0.5886D / (0.382/D^2)^{2} = 0.0032 + \frac{0.221}{\left(\frac{0.382}{D^2} XDX10^6\right)^{0.237}}$$

{ from Equation (ii), $V = 0.382 / D^2$}

$$0.5886 \times D^5/ 0.382^2 = 0.0032 + \frac{0.221}{\left(\frac{(0.382X10^6)^{0.237}}{D^{0.237}}\right)}$$

$4.0333 D^5 = 0.0032 + 0.0015 X D^{0.237}$

$4.0333 D^5 - 0.0105 D^{0.237} - 0.0032= 0$ ---------------(iii)

the above equation (iii) will be solved hit trial method

(i). Assume D = 1m, then L.H.S of the equation (iii), becomes as

$L.H.S = 4.033 X 1^{5} - 0.0105 X 1^{0.237} - 0.0032$

$= 4.033 - 0.0105 - 0.0032 = 4.0193$

by increasing the value of D more than 1m , the L.H.S. will go on increasing. Hence decrease the value of D.

(ii) Assume D = 0.3 than L.H.S of equation (iii)

becomes as L.H.S = $4.033 \times 0.3^{0.237} - 0.0032$

$$= 0.0098 - 0.00789 - 0.0032 = -0.00129$$

as this value of negative, the values of D will be slightly more than 0.3

(iii) Assume D = 0.306 then L.H.S of equation (iii) becomes as

$$L.H.S = 4.033 \times 0.306^{0.237} - 0.0105 \times 0.306^{0.237} - 0.0032$$

$$= 0.0108 - 0.00793 - 0.0032 = -0.00033$$

This value of L.H.S is approximately equal to equal to zero. Actually the value of D will be slightly more than 0.306m say **0.308m**.

P 3. Expression for loss of head due to friction in pipes.

Or

Darcy – Weisbach Equation.

Consider a uniform horizontal pipe, having steady flow as shown figure. Let 1 -1 and 2-2 are two sections of pipe.

Let P_1 = pressure intensity at section 1-1.

Let P_2 = Velocity of flow at section 1-1.

L = length of the pipe between the section 1-1 and 2-2

d = diameter off pipe.

f^1 = Frictional resistance per unit wetted area per unit velocity.

h_f = loss of head due to friction.

And P_2, V_2 = are the values of pressure intensity and velocity at section 2-2.

Applying Bernoulli's equation between sections 1-1 & 2-2

Total head 1-1 = total head at 2-2 + loss of head due to friction between 1-1&2-2

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2+h_f$ ------------(1)

but $Z_1 = Z_1$ [pipe is horizontal]

$V_1= V_2$ [diameter of pipe is same at 1-1 & 2-2]

(1) becomes,

$(P_1/ \rho g) = (P_2/\rho g)+h_f$

$h_f = (P_1/ \rho g) - (P_2/\rho g)$

frictional resistance = frictional resistance per unit wetted area per unit velocity X

wetted area X velocity ^{2}.

$F = f^1 X \pi d l X V^2$ [Wetted area = π d X L, and Velocity $V = V_1 = V_2$]

$F_1 = f^1XPXLXV^2$ ----------- (2). [π d = wetted perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are,

1) Pressure force at section 1-1 = P_1X A

2) Pressure force at section 2-2 = P_2 X A

3). Frictional force F_1

Resolving all forces in the horizontal direction.,

$P_1 A - P_2A - F_1 = 0$

$(P_1-P_2)A = F_1 = f^1XPXLXV^2$

$(P_1 - P_2) = (f^1 X P X L X V^2 / A)$.

But from (1) we get

$P_1 – P_2 = \rho g h_f$

Equating the values of $(P_1 – P_2)$ we get

$\rho g h_f = (f^1 X P X L X V^2 / A)$.

$h_f = (f^1 / \rho g) X (P/A) X L X V^2$

$(P/A) = (\pi d / (\pi d^2/4)) = (4/d)$

hence, $h_f = (f^1 / \rho g) X (4/d) X L X V^2$.

Putting $(f^1 / \rho) = (f / 2)$, where f is the co – efficient of friction

$$h_f = \frac{4fLV^2}{2gd}$$

This equation is known as Darcy – Weisbach equation. This equation is commonly used to find loss of head due to friction in pipes

P 4. The rate of flow through a horizontal pipe is 0.25 m^3/s. the diameter of the pipe which is 200mm is suddenly enlarged to 400mm. the pressue intensity in the smaller pipe is 11.772 N/cm^2. determine (i). Loss of head due to sudden enlargement (ii). Pressure intensity in large pipe. (iii). Power lost due to enlargement.

Given :

Discharge	$Q = 0.25\ m^3/s$.
Dia. Of smaller pipe	$D_1 = 200mm = 0.2m$
Area	$A_1 = \frac{\pi}{4} (0.2)^2 = 0.03141\ m^2$.
Dia of large pipe	$D_2 = 400mm = 0.4m$

Area $A_2 = \frac{\pi}{4}\ (0.4)^2 = 0.12566\ m^2$.

Pressure in smaller pipe $p_1 = 11.772\ N/cm^2 = 11.772\ X10^4\ N/m^2$.

Now velocity $V_1 = Q / A_1 = 0.25 / 0.03414 = 7.96\ m/s$.

Velocity $V_2 = Q / A_2 = 0.25 / 0.12566 = 1.99\ m/s$.

(i). Loss of head due to sudden enlargement,

$h_e = (V_1 – V_2)^2 / 2g = (7.96 – 1.99)^2 / 2X\ 9.81 =$ **1.816 m**.

(ii). Let the pressure intensity in large pipe = p_2 .

Then applying Bernoulli's equation before and after the sudden enlargement,

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2+h_e$

but $Z_1 = Z_2$

$(P_1/\rho g) + (V_1^2 / 2g) = (P_2/\rho g) + (V_2^2 / 2g) +h_e$

or $(P_1/\rho g) + (V_1^2 / 2g) = (P_2/\rho g) + (V_2^2 / 2g) + Z_2+h_f$

$(P_2/\rho g) = (P_1/\rho g) + (V_1^2 / 2g) - (V_2^2 / 2g) - h_e$

$$= \frac{11.772X10^4}{1000X9.81} + \frac{7.96^2}{2X9.81} - \frac{1.99^2}{2X9.81} - 1.816$$

$= 12.0+3.229-0.2018-1.8160$

$= 15.229 – 20.178 = 13.21$m of water

$p_2 = 13.21\ X\ \rho g = 13.21\ X\ 100X\ 9.81\ N/m^2$

$= 13.21X1000X9.81X10^{-4}\ N/cm^2$. = **12.96N/cm².**

(iii). Power lost due to sudden enlargement,

$P = (\rho g\ \mathbf{Q\ h_e}) / 1000 = (1000X9.81X0.25X1.816)/1000 =$ **4.453kW.**

P 5. A horizontal pipe line 40m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25m of its length from the tank, the pipe is 150mm diameter is suddenly enlarged to 300mm. the height of water level in the tank is 8m

above the centre of the pipe. Considering all losses of head which occur. Determine the rate of flow. Take f = 0.01 for both sections of the pipe.

Given :

Total length of pipe, $L = 40m$

Length of 1 st pipe, $L_1 = 25m$

Dia of 1st pipe $d_1 = 150mm = 0.15m$

Length of 2nd pipe $L_2 = 40 - 25 = 15m$

Dia of 2nd pipe $d_2 = 300mm = 0.3m$

Height of water $H = 8m$

Co-effi. Of friction $f = 0.01$

Applying the Bernoulli's theorem to the surface of water in the tank and outlet of pipe as shown in fig. and taking reference line passing through the center of the pipe.

$0+0+8 = (P_2/\rho g) + (V_2^2 / 2g) + 0 + \text{all losses}$

$8.0 = 0+(V_2^2 / 2g)+h_i+ h_{f1}+ h_e+ h_{f2}$

where, h_i = loss of head at entrance = $0.5 V_1^2/ 2g$

h_{f1} = head lost due to friction in pipe 1 = $\dfrac{4XfXL_1XV_1^2}{d_1X2g}$

h_e = loss of head due to sudden enlargement = $(V_1- V_2)^2/2g$

h_{f2} = head lost due to friction in pipe 2 = $\dfrac{4XfXL_2XV_2^2}{d_2X2g}$

but from continuity equation, we have

$A_1V_1 = A_2V_2$

$$V_1 = (A_2V_2/A_1) = \frac{\frac{\pi}{4}d_2^2XV_2}{\frac{\pi}{4}d_1^2} = \left(\frac{d_2}{d_1}\right)^2 XV_2 = \left(\frac{0.3}{0.15}\right)^2 XV_2 = 4V_2$$

Substituting the value of V_1 in different head losses, we have

$h_i = 0.5\ V_i^2/\ 2g = (0.5\ X\ (4V_2)^2\ )/\ 2g = 8V_2^2/2g$

$$h_{f1} = \frac{4X\,0.01X\,25X\,(4V_2^2)}{0.15X\,2g} = \frac{4X\,0.01X\,25X16}{0.15}\,X\,\frac{V_2^2}{2g} = 106.67\frac{V_2^2}{2g}$$

$he = (V_1 - V_2)^2/\ 2g \qquad = (4V_2 - V_2)^2/2g = 9V_2^2/2g$

$$h_{f2} = \frac{4X\,0.01X15X\,(V_2^2)}{0.3X\,2g} = \frac{4X\,0.01X15}{0.3}\,X\,\frac{V_2^2}{2g} = 2.0\frac{V_2^2}{2g}$$

substituting the values of these losses in equation (i), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2X\,\frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}\,[\ 1+8+106.67+9+2] = 126.67\,\frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\frac{8.0x2xg}{126.67}} = \sqrt{\frac{8.0X\,2X\,9.81}{126.67}} = 1.113\ \text{m/s}$$

Rate of flow $Q = A_2XV_2 = \frac{\pi}{4}\ (\ 0.3)^2\ X\ 1.113 = 0.07867\ m^3/s$ = **78.67 litres/sec.**

P 6. A pipe line, 300mm in diameter and 3200m long is used to pump up 50 kg per second of an oil whose density is 950n kg/m³.and whose Kinematic viscosity is 2.1 stokes. The center of the pipe at upper end is 40m above than at the lower end. The discharge at the upper end is atmospheric. Find the pressure at the lower end and draw the hydraulic gradient and the total energy line.

Given:

Dia of pipe d = 300mm = 0.3m

Length of pipe L = 3200m

Mass M = 50kg/s = ρ. Q

Discharge $Q = 50/\ \rho = 50/950 = 0.0526\ m^3/s$

Density $\rho\ = 950\ kg/m^3$

Kinematic viscosity $\nu = 2.1$ stokes $= 2.1\ cm^2/s\ = 2.1\ X10^{-4}\ m^2/s$

Height of upper end = 40m

Pressure at upper end = atmospheric = 0

Renold number, $R_e = VXD/\nu$, where V = Discharge / Area

$$= 0.0526/\left(\frac{\pi}{4}(0.3)^2\right) = 0.744 \text{ m/s}$$

$$R_e = (0.744X0.30) / (2.1X10^{-4}) = 1062.8$$

Co – efficient of friction, $f = 16/R_e = 16 / 1062.8 = 0.015$

Head lost due to friction, h_f

$$= \frac{4XfXL\ XV^2}{d\ X2g} = \frac{4X0.015X3200X(0.744)^2}{0.3X2X9.81} = 18.05m \text{ of oil.}$$

Applying the Bernoulli's equation at the lower and upper end of the pipe and taking datum line passing through the lower end, we have

$$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2 + h_f$$

but $Z_1 = 0$, $Z_2 = 40$m., $V_1 = V_2$ as diameter is same.

$P_2 = 0$, $h_f = 18.05$m

Substituting these values, we have

$= 5400997 \text{ N/m}^2 = 54.099 \text{ N/cm}^2$.

H.G.L. AND T.E.L.

$V^2/2g = (0.744)^2/2X9.81 = 0.0282$ m

$p_1/\rho g = 58.05$ m of oil

$p_2/\rho g = 0$

Draw a horizontal line AX as shown in fig. from A draw the center line of the pipe in such way that point C is a distance of 40m above the horizontal line. Draw a vertical line AB through A such that AB = 58.05m. join B with C. then BC is the hydraulic gradient line.

Draw a line DE parallel to BC at a height of 0.0282m above the hydraulic gradient line. Then DE is the total energy line.

P 7. A main pipe divides into two parallel pipes which again forms one pipe as shown. The length and diameter for the first parallel pipe are 2000m and 1.0m respectively, while the length and diameter of 2nd parallel pipe are 2000m and 0.8m. find the rate of flow in each parallel pipe, if total flow in main is 3.0 m³/s. the co-efficient of friction for each parallel pipe is same and equal to 0.005.

Given:

Length of Pipe 1 L_1= 2000m

Dia of pipe1 d_1 = 1.0m

Length of pipe 2 L_2 = 2000m

Dia of pipe 2 d_2 = 0.8m

Total flow Q = 3.0m^3/s

$f_1 = f_2 = f = 0.005$

let Q_1 = discharge in pipe 1

let Q_2 = discharge in pipe 2

from equation, $Q = Q_1 + Q_2 = 3.0$ ---------------(i)

using the equation we have

$$\frac{4Xf_1XL_1XV_1^2}{d_1X2g} = \frac{4Xf_2XL_2XV_2^2}{d_2X2g}$$

$$\frac{4X0.005X2000XV_1^2}{1.0X2X9.81} = \frac{4X0.005X2000XV_2^2}{0.8X2X9.81}$$

$$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8} or V_1^2 = \frac{V_2^2}{0.8}$$

$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{0.894}$$

now, $Q_1 = \frac{\pi}{4} d_1^2 X V_1 = \frac{\pi}{4} (1)^2 X (V_2 / 0.894)$

and $Q_2 = \frac{\pi}{4} d_2^2 X V_2 = \frac{\pi}{4} (0.8)^2 X (V_2) = \frac{\pi}{4} (0.64) X (V_2)$

substituting the value of Q_1 and Q_2 in equation (i) we get

$\frac{\pi}{4}$ (1)^{2}X(V_2 / 0.894)+ $\frac{\pi}{4}$ (0.64)X(V_2) = 3.0 or 0.8785 V_2 + 0.5026 V_2 = 3.0

V_2[0.8785+0.5026] = 3.0 or V = 3.0 / 1.3811 = 2.17 m/s.

Substituting this value in equation (ii),

V_1 = V_2 / 0.894 = 2.17 / 0.894 m/s , Hence Q_1 = $\frac{\pi}{4}$ d_1^2XV_1 = $\frac{\pi}{4}$ 1^2X2.427 = **1.096 m^3/s**

Q_2 = Q – Q_1 = 3.0 – 1.906 = **1.904 m^3/s**.

P 8. Three reservoirs A, B,C are connected by a pipe system shown in fig. find the discharge into or from the reservoirs B and C if the rate of flow from reservoirs A is 60 litres / s. find the height of water level in the reservoir C. take f = 0.006 for all pipes.

Given:

Length of pipe AD, L_1 = 1200m

Dia of pipe AD, d_1 = 30cm = 0.3m

Discharge through AD, Q_1 = 60lit/s = 0.06 m^3/s

Height of water level in A from reference line , Z_A= 40m

For pipe DB, length L_2 = 600mm, Dia., d_2 = 20cm = 0.20m, Z_B= 38.0

For pipe DC, length L_3 = 800mm, Dia., d_3= 30cm = 0.30m,

Applying the Bernoulli's equations to point E and , $Z_A = Z_D + \frac{p_D}{\rho g} + h_f$

Where $h_f = \frac{4Xf_1XL_1XV_1^2}{d_1X2g}$, where V_1 = Q_1 / Area = 0.006 / ($\frac{\pi}{4}$ (0.3)^2) = 0.848 m/s.

$$h_f = \frac{4X0.006X1200X0.848^2}{0.3X2X9.81} = 3.518 \text{ m.}$$

$$\{Z_D + \frac{p_1}{\rho g}\} = 40.0 - 3.518 = 36.482 \text{ m}$$

Hence piezometric head at D = 36.482m. hence water flows from B to D.

Applying Bernoulli's equation to point B and D

$Z_B = \{Z_D + \frac{p_D}{\rho g}\} + h_{f2}$ or $38 = 36.482 + h_{f2}$

$h_{f2} = 38 - 36.482 = 1.518m$

but $h_{f2} = \frac{4XfXL_2XV_2^2}{d_2X2g} = \frac{4X0.006X600XV_2^2}{0.2X2X9.81}$

$1.518 = \frac{4X0.006X600XV_2^2}{0.2X2X9.81}$

$V_2 = \sqrt{\frac{1.518X0.2X2X9.81}{4X0.006X600}} = 0.643m/s$

Discharge $Q_2 = V_2X\frac{\pi}{4}(d_2)^2 = 0.643X\frac{\pi}{2}X(0.2)^2 =$ **$0.0202m^3/s$ = 20.2lit/s.**

Applying Bernoulli's equation to D and C

$\{Z_D + \frac{p_D}{\rho g}\} = Z_C + h_{f3}$

$36.482 = Z_C + \frac{4XfXL_3XV_3^2}{d_3X2g}\ where, V_3 = \frac{Q_3}{\frac{\pi}{4}d_3^2}$

but from continuity $Q_1 + Q_2 = Q_3$

$Q_3 = Q_1 + Q_2 = 0.006 + 0.0202 = 0.0802\ m^3/s$

$$V_3 = \frac{Q_3}{\frac{\pi}{4}d_3^2} = \frac{Q_3}{\frac{\pi}{4}(0.9)^2} = 1.134m/s$$

$36.482 = Z_C + \frac{4X0.006X800X1.134^2}{0.X2X9.81} = Z_C + 4.194$

$Z_C = 36.482 - 4.194 =$ **32.288m**

Question Bank

1. An oil of specific gravity 0.7mm is flowing through a pipe of diameter 500 litres/s .Find the head lost due to friction and power required to maintain the flow for a length of 1000m.Take v = .29 strokes.
2. Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500m apart is 4 m of water . Take the value of 'f' = 0.009 in the formula $h_f = \frac{4.f.L.V^2}{d\ x\ 2g}$.
3. At a sudden enlargement of water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow.
4. The rate of flow of water through a horizontal pipe is 0.25 m^3/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm.The pressure intensity in the smaller pipe is 11.772 N/cm^2.Determine : (i) Loss of head due to sudden enlargement (ii) Pressure intensity in the large pipe (iii) Power lost due to enlargement.
5. A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m . Calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank in 30 seconds.
6. An oil of viscosity 0.1 Ns/m^2 and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and 0f length 300 mm . The rate of flow of fluid through the pipe is 3.5 litres/s. Find the pressure drop in a length of 300 m and also shear stress at the pipe wall.
7. A main pipe divides into two parallel pipes which again forms one pipe.The length and diameter for the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m.Find the rate of flow in each parallel pipe , if total flow in the main is 3.0 m/s .The coefficient of friction for each parallel pipe same and equal to .005.
8. Three pipes of same length L, diameter D and Friction factor f are connected in parallel. Determine the diameter of the pipe of length L and friction factor f which will carry the same discharge for the same head loss. Use the formula $h_f = f*L*V^2/2gD$.
9. A pipe line, 300mm in diameter and 3200m long is used to pump up 50kg per second of an oil whose density is 950n kg/m^3.and whose Kinematic viscosity is 2.1 stokes. The center of the pipe at upper end is 40m above than at the lower end. The discharge at the upper end is atmospheric. Find the pressure at the lower end and draw the hydraulic gradient and the total energy line.
10. A pipeline of length 2000m is used for power transmission. If 110.3635kW power is to be transmitted through the pipe in which water having a pressure 490.5 N/cm^2 at inlet is flowing. Find the diameter of the pipe and efficiency of transmission if the pressure drop over the length of the pipe is 98.1 $N.cm^2$.Take f=.0065
11. The rate of flow of water through a horizontal pipe is 0.25 m^3/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm.The pressure intensity in the smaller pipe is 11.772

N/cm^2.Determine : (i) Loss of head due to sudden enlargement (ii) Pressure intensity in the large pipe (iii) Power lost due to enlargement.

12. Water is to be supplied to the inhabitants of a college campus through a supply main. The following data is given

    Distance of the reservoir from the campus =3000 m

    Number of inhabitants = 4000

    Consumption of water per day of each inhabitant = 180 litres

    Loss of head due to friction = 18 m

    Coefficient of friction for the pipe,f = 0.007

    If the half of the daily supply is pumped in 8 hrs , determine the size of the supply main.

13. At a sudden enlargement of water main from 240 mm to 480 mm diameter, the hydraulic gradient raises by 10 mm. calculate the rate of flow.

14. An oil of viscosity 9 poise and specific gravity 0.9 is flowing through a horizontal pipe of 60 mm diameter. If the pressure drop in 100 m length of the pipe is 1800 kN/m^2 , determine:

i) The rate of flow of oil
ii) The centre-line velocity
iii) The total frictional drag over 100 m length
iv) The power required to maintain the flow
v) The velocity gradient at the pipe wall

15. Three pipes of diameters 300 mm,200 mm and 400 mm and lengths 450 m,255 m and 315 m respectively are connected in series. The difference in water surface levels in two tanks is 18 m. Determine the rate of flow of water if coefficients of friction are 0.0075,0.0078 and 0.0072 respectively considering:

    i) Minor losses
    ii) Neglecting Minor losses

16. The main pipe divides into two parallel pipes which again forms one pipe . The data is as follows :
    First parallel pipe : Length = 1000 m , diameter = 0.8 m
    Second parallel pipe : Length = 1000 m , diameter = 0.6 m

    Coefficient of friction for each parallel pipe = 0.005

If the total rate of flow in the main is $2m^3/s$ find the rate of flow in each parallel pipe.

UNIT – V BOUNDARY LAYER FLOW

THE CONCEPT OF VISCOSITY

The behavior of a fluid in flow is very much related to two intrinsic properties of the fluid: density and viscosity. For example, a solid body moving through a gas has to overcome a certain resistance which depends on the relative velocity between fluid and solid, the shape of the solid, the density of the gas and its viscosity. The power required to move a fluid through a conduit is a function of the fluid velocity, the diameter of the conduit and the fluid density and viscosity. The existence and nature of viscosity can be demonstrated by suspending two horizontal, parallel plates in a liquid so that they are separated by a very small distance. Now, if the upper plate is kept stationary while the lower plate is set to motion with a velocity, the layer of liquid right next to this plate will also start to move. With time, the motion of the bottom layer of fluid will cause the fluid layers higher up to also move.

Newton's law of viscosity

States that *the shear stress between adjacent fluid layers is proportional to the negative value of the velocity gradient between the two layers*.

An alternative interpretation can be given by noting, from elementary physics, that

Force = mass $\times$ *acceleration = mass* $\times$ *velocity/time = momentum/time*

Therefore, *the rate of momentum transfer per unit area, between two adjacent layers of fluid, is proportional to the negative value of the velocity gradient between them.*

BOUNDARY LAYER FLOWS

Viscous internal flows have the following major boundary layer characteristics:

* An entrance region where the boundary layer grows and dP/dx $\neq$ constant,
* A fully developed region where:
    - The boundary layer fills the entire flow area.

- The velocity profiles, pressure gradient, and τ_W are constant; i.e. they are not equal to f(x),
- The flow is either laminar or turbulent over the entire length of the flow, i.e. transition from laminar to turbulent is not considered.

However, viscous flow boundary layer characteristics for external flows are significantly different as shown below for flow over a flat plate:

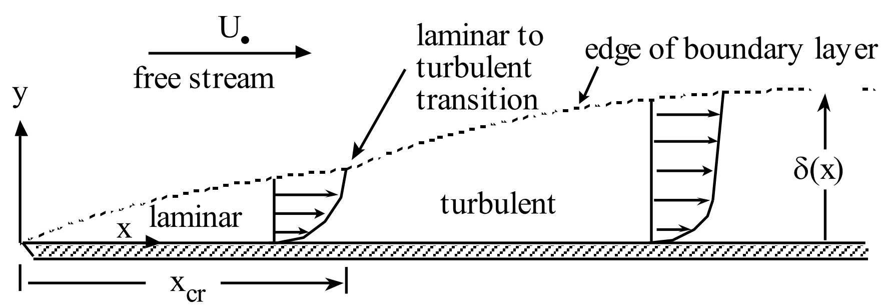


Fig. 7.1 Schematic of boundary layer flow over a flat plate

For these conditions, we note the following characteristics:

- The boundary layer thickness δ grows continuously from the start of the fluid-surface contact, e.g. the leading edge. It is a function of x, not a constant.
- Velocity profiles and shear stress τ are f(x,y).
- The flow will generally be laminar starting from x = 0.
- The flow will undergo laminar-to-turbulent transition if the streamwise dimension is greater than a distance x_{cr} corresponding to the location of the transition Reynolds number Re_{cr}.
- Outside of the boundary layer region, free stream conditions exist where velocity gradients and therefore viscous effects are typically negligible. As it was for internal flows, the most important fluid flow parameter is the local Reynolds number defined as

$$\mathrm{Re}_x = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{\upsilon}$$

where

ρ = fluid density μ = fluid dynamic viscosity

ν = fluid kinematic viscosity U_∞ = characteristic flow velocity

x = characteristic flow dimension

It should be noted at this point that all external flow applications will not use a distance from the leading edge x as the characteristic flow dimension. For example, for flow over a cylinder, the diameter will be used as the characteristic dimension for the Reynolds number.

Transition from laminar to turbulent flow typically occurs at the local transition Reynolds number, which for flat plate flows can be in the range of

$$500,000 \le \mathrm{Re}_{cr} \le 3,000,00$$

With x_{cr} = the value of x where transition from laminar to turbulent flow occurs, the typical value used for steady, incompressible flow over a flat plate is

$$\mathrm{Re}_{cr} = \frac{\rho U_\infty x_{cr}}{\mu} = 500,000$$

Thus for flat plate flows for which:

$x < x_{cr}$ the flow is laminar

$x \ge x_{cr}$ the flow is turbulent

The solution to boundary layer flows is obtained from the reduced "Navier – Stokes" equations, i.e., Navier-Stokes equations for which boundary layer assumptions and approximations have been applied.

Flat Plate Boundary Layer Theory

Laminar Flow Analysis

For steady, incompressible flow over a flat plate, the laminar boundary layer equations are:

Conservation of mass: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

'X' momentum: $$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

'Y' momentum: $$-\frac{\partial p}{\partial y} = 0$$

The solution to these equations was obtained in 1908 by Blasius, a student of Prandtl's. He showed that the solution to the velocity profile, shown in the table below, could be obtained as a function of a single, non-dimensional variable η

defined as

$$\eta = y\left(\frac{U_\infty}{\upsilon x}\right)^{1/2}$$

with the resulting ordinary differential equation:

$$f''' + \frac{1}{2} f\, f'' = 0$$

and $$f'(\eta) = \frac{u}{U_\infty}$$

Table 7.1 the Blasius Velocity Profile

$y[U/(\nu x)]^{1/2}$	u/U	$y[U/(\nu x)]^{1/2}$	u/U
0.0	0.0	2.8	0.81152
0.2	0.06641	3.0	0.84605
0.4	0.13277	3.2	0.87609
0.6	0.19894	3.4	0.90177
0.8	0.26471	3.6	0.92333
1.0	0.32979	3.8	0.94112
1.2	0.39378	4.0	0.95552
1.4	0.45627	4.2	0.96696
1.6	0.51676	4.4	0.97587
1.8	0.57477	4.6	0.98269
2.0	0.62977	4.8	0.98779
2.2	0.68132	5.0	0.99155
2.4	0.72899	∞	1.00000
2.6	0.77246		

Boundary conditions for the differential equation are expressed as follows:

at $y = 0$, $v = 0 \rightarrow f(0) = 0$; y component of velocity is zero at $y = 0$

at $y = 0$, $u = 0 \rightarrow f'(0) = 0$; x component of velocity is zero at $y = 0$

The key result of this solution is written as follows:

$$\left.\frac{\partial^2 f}{\partial \eta^2}\right)_{y=0} = 0.332 = \frac{\tau_w}{\mu U_\infty \sqrt{U_\infty / \upsilon x}}$$

With this result and the definition of the boundary layer thickness, the following key results are obtained for the laminar flat plate boundary layer:

Local boundary layer thickness $\delta(x) = \frac{5x}{\sqrt{\mathrm{Re}_x}}$

Local skin friction coefficient:

(defined below) $C_{f_x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$

Total drag coefficient for length L (integration of τ_w dA over the length of the plate, per unit area, divided by 0.5 ρ U_∞^2) $C_D = \frac{1.328}{\sqrt{\mathrm{Re}_x}}$

where by definition $C_{f_x} = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2}$ and $C_D = \frac{F_D / A}{\frac{1}{2}\rho U_\infty^2}$

With these results, we can determine local boundary layer thickness, local wall shear stress, and total drag force for laminar flow over a flat plate.

Example:

Air flows over a sharp edged flat plate with L = 1 m, a width of 3 m and U_∞ = 2 m/s . For one side of the plate, find: $\delta(L)$, $C_f(L)$, $\tau_w(L)$, C_D, and F_D.

Air: $\rho = 1.23\ kg/m^3$ $\nu = 1.46\ E\text{-}5\ m^2/s$

First check Re: $\mathrm{Re}_L = \frac{U_\infty L}{\upsilon} = \frac{2m/s * 2.15m}{1.46E-5m^2/s} = 294,520 < 500,000$

Key Point: Therefore, the flow is laminar over the entire length of the plate and calculations made for any x position from 0 - 1 m must be made using laminar flow equations.

Boundary layer thickness at x = L:

$$\delta(L) = \frac{5\,L}{\sqrt{\mathrm{Re}_L}} = \frac{5*2.15\,m}{\sqrt{294,520}} = 0.0198\,m = 1.98\,cm$$

Local skin friction coefficient at x = L:

$$C_f(L) = \frac{0.664}{\sqrt{\mathrm{Re}_L}} = \frac{0.664}{\sqrt{294,520}} = 0.00122$$

Surface shear stress at x = L:

$$\tau_w = 1/2\,\rho U_\infty^2\,C_f = 0.5*1.23\,kg/m^3*2^2\,m^2/s^2*0.00122$$

$$\tau_w = 0.0030\,N/m^2\,(Pa)$$

Drag coefficient over total plate, 0 – L:

$$C_D(L) = \frac{1.328}{\sqrt{\mathrm{Re}_L}} = \frac{1.328}{\sqrt{294,520}} = 0.00245$$

Drag force over plate, 0 – L:

$$F_D = 1/2\,\rho U_\infty^2\,C_D\,A = 0.5*1.23kg/m^3*2^2\,m^2/s^2*0.00245*2*2.15\,m^2$$

$$F_D = 0.0259\,N$$

Two key points regarding this analysis:

1. Each of these calculations can be made for any other location on the plate by simply using the appropriate x location for any .

2. Be careful not to confuse the calculation for C_f and C_D.

   C_f is a local calculation at a particular x location (including x = L) and can only be used to calculate local shear stress at a specific x, not drag force.

   C_D is an integrated average over a specified length (including any $x \leq L$) and can only be used to calculate the average shear stress over the entire plate and the integrated force over the total length.

Turbulent Flow Equations

While the previous analysis provides an excellent representation of laminar, flat plate boundary layer flow, a similar analytical solution is not available for turbulent flow due to the complex nature of the turbulent flow structure.

However, experimental results are available to provide equations for key flow field parameters.

A summary of the results for boundary layer thickness and local and average skin friction coefficient for a laminar flat plate and a comparison with experimental results for a smooth, turbulent flat plate are shown below.

Laminar	Turbulent
$\delta(x) = \dfrac{5x}{\sqrt{\mathrm{Re}_x}}$	$\delta(x) = \dfrac{0.16\,x}{\mathrm{Re}_x^{1/7}}$
$C_{f_x} = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{f_x} = \dfrac{0.027}{\mathrm{Re}_x^{1/7}}$
$C_D = \dfrac{1.328}{\sqrt{\mathrm{Re}_L}}$	$C_D = \dfrac{0.031}{\mathrm{Re}_L^{1/7}}$ for turbulent flow over entire plate, 0 – L, i.e. assumes turbulent flow in the laminar region

where $$C_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$$ local drag coefficient based on local wall shear stress (laminar or turbulent flow region) and

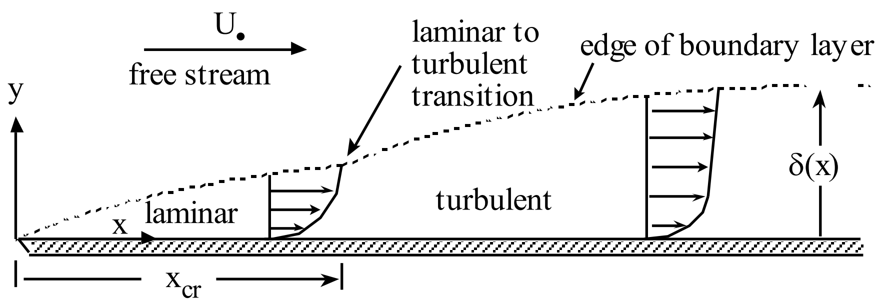
C_D = total drag coefficient based on the integrated force over the length 0 to L

$$C_D = \frac{F/A}{\frac{1}{2}\rho U_\infty^2} = \left(\tfrac{1}{2}\rho U_\infty^2 A\right)^{-1} \int_0^L \tau_w(x)\, w\, dx$$

A careful study of these results will show that, in general, boundary layer thickness grows faster for turbulent flow, and wall shear and total friction drag are greater for turbulent flow than for laminar flow given the same Reynolds number.

It is noted that the expressions for turbulent flow are valid only for a flat plate with a smooth surface. Expressions including the effects of surface roughness are available in the text.

Combined Laminar and Turbulent Flow



Flat plate with both laminar and turbulent flow sections

For conditions (as shown above) where the length of the plate is sufficiently long that we have both laminar and turbulent sections:

* Local values for boundary layer thickness and wall shear stress for either the laminar or turbulent sections are obtained from the expressions for $\delta(x)$ and Cf_x for laminar or turbulent flow as appropriate for the given region.
* The result for average drag coefficient C_D and thus total frictional force over the combined laminar and turbulent portions of the plate is given by (assuming a transition Re of 500,000)

$$C_D = \frac{0.031}{Re_L^{1/7}} = \frac{0.031}{\left(5x10^6\right)^{1/7}}$$

* Calculations assuming only turbulent flow can typically be made for two cases:

1. when some physical situation (a trip wire) has caused the flow to be turbulent from the leading edge or

2. if the total length L of the plate is much greater than the length x_{cr} of the laminar section such that the total length of plate can be considered turbulent from x = 0 to L. Note that this will over predict the friction drag force since turbulent drag is greater than laminar.

With these results, a detailed analysis can be obtained for laminar and/or turbulent flow over flat plates and surfaces that can be approximated as a flat plate.

Figure 7.6 in the text shows results for laminar, turbulent and transition regimes. Equations 7.48a & b can be used to calculate skin friction and drag results for the fully rough regime.

$$c_f \approx \left(2.87 + 1.58 \log \frac{x}{\varepsilon}\right)^{-2.5} \qquad (7.48a)$$

$$C_D \approx \left(1.89 + 1.62 \log \frac{L}{\varepsilon}\right)^{-2.5} \qquad (7.48b)$$

Equations 7.49a & b can be used to calculate total C_D for combined laminar and turbulent flow for transition Reynolds numbers of $5x10^5$ and $3x10^6$ respectively.

$$C_D \approx \frac{0.031}{Re_L^{1/7}} - \frac{1440}{Re_L} \qquad Re_{trans} = 5x10^5$$

$$C_D \approx \frac{0.031}{Re_L^{1/7}} - \frac{8700}{Re_L} \qquad Re_{trans} = 3x10^6$$

Example:

Water flows over a sharp flat plate 2.55 m long, 1 m wide, with $U_\infty = 2$ m/s. Estimate the error in F_D if it is assumed that the entire plate is turbulent.

Water: ρ = 1000 kg/m^3 ν = 1.02 E- m^2/s

Reynolds number: $\mathrm{Re}_L = \dfrac{U_\infty L}{\upsilon} = \dfrac{2\,m/s * 2.55\,m}{1.02E-6\,m^2/s} = 5E6 > 500,000$

with $\mathrm{Re}_{cr} = 500,000 \Rightarrow x_{cr} = 0.255m$ (or 10% laminar)

a. Assume that the entire plate is turbulent

$$C_D \approx \frac{0.031}{\mathrm{Re}_L^{1/7}} = \frac{0.031}{\left(5x10^6\right)^{1/7}} = 0.003423$$

$$F_D = 0.5\rho U_\infty^2\, C_D\, A = 0.5 \cdot 1000\frac{kg}{m^3} \cdot 2^2\frac{m^2}{s^2} \cdot 0.003423 \cdot 2.55m^2$$

$F_D = 17.46\,N$ **This should be high since we have assumed that the entire plate is turbulent and the first 10% is actually laminar.**

b. Now consider the actual combined laminar and turbulent flow:

$$C_D \approx \frac{0.031}{\mathrm{Re}_L^{1/7}} - \frac{1440}{\mathrm{Re}_L} = \frac{0.031}{\left(5x10^6\right)^{1/7}} - \frac{1440}{5x10^6} = 0.003135$$

Note that the C_D has decreased when both the laminar and turbulent sections are considered.

$$F_D = 0.5\rho U_\infty^2 C_D A = 0.5 \cdot 1000 \frac{kg}{m^3} \cdot 2^2 \frac{m^2}{s^2} \cdot 0.003135 \cdot 2.55 m^2$$

$F_D = 15.99\,N$ { Lower than the fully turbulent value}

$$Error = \frac{17.46 - 15.99}{15.99} \cdot 100 = 9.2\%\ high$$

Thus, the effect of neglecting the laminar region and assuming the entire plate is turbulent is as expected.

Question: Since $x_{cr} = 0.255$ m, what would your answers represent if you had calculated the Re, C_D, and F_D using $x = x_{cr} = 0.255$ m?

Answer: You would have the value of the transition Reynolds number and the drag coefficient and drag force over the laminar portion of the plate (assuming you used laminar equations).

If you had used turbulent equations you would have red marks on your paper.

Von Karman Integral Momentum Analysis

While the previous results provide an excellent basis for the analysis of flat plate flows, complex geometries and boundary conditions make analytical solutions to most problems difficult.

An alternative procedure provides the basis for an approximate solution which in many cases can provide excellent results.

The key to practical results is to use a reasonable approximation to the boundary layer profile, u(x,y). This is used to obtain the following:

a. Boundary layer mass flow: $$\dot{m} = \int_0^\delta \rho u b \ d y$$

   where b is the width of the area for which the flow rate is being obtained.

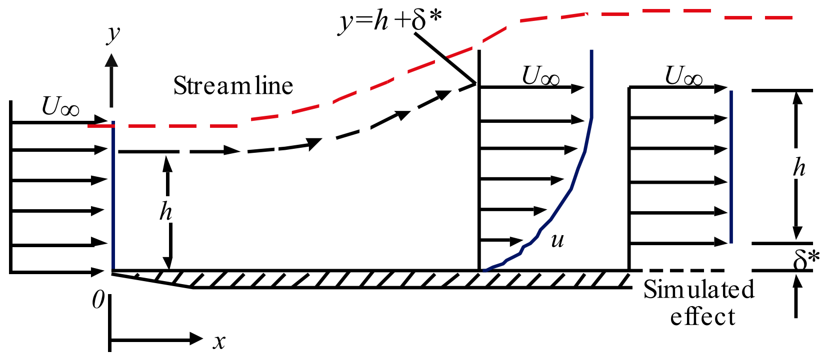
b. Wall shear stress: $$\tau_w = \mu \left.\frac{d u}{d y}\right)_{y=0}$$

You will also need the streamwise pressure gradient $\frac{d P}{d x}$ for many problems.

The Von Karman integral momentum theory provides the basis for such an approximate analysis. The following summarizes this theory.

<u>Displacement thickness</u>:

Consider the problem indicated in the adjacent figure:



A uniform flow field with velocity U_∞ approaches a solid surface. As a result of viscous shear, a boundary layer velocity profile develops.

A viscous boundary layer is created when the flow comes in contact with the solid surface.

Key Point: Compared to the uniform velocity profile approaching the solid surface, the effect of the viscous boundary layer is to displace streamlines of the flow outside the boundary layer away from the wall.

With this concept, we define δ^* = displacement thickness

δ^* = distance the solid surface would have to be displaced to maintain the same mass flow rate as for non-viscous flow.

From the development in the text, we obtain

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

Thus, the displacement thickness varies only with the local non-dimensional velocity profile. Therefore, with an expression for u / U_∞ , we can obtain $\delta^* = f(\delta)$.

Example:

Given: $\dfrac{u}{U_\infty} = 2\left(\dfrac{y}{\delta}\right) - \left(\dfrac{y}{\delta}\right)^2$ determine an expression for $\delta^* = f(\delta)$

Note that for this assumed form for the velocity profile:

1. At $y = 0$, $u = 0$ correct for no slip condition

2. At $y = \delta$, $u = U_\infty$ correct for edge of boundary layer

3. The form is quadratic

To simplify the mathematics,

let $\eta = y/\delta$, at $y = 0$, $\eta = 0$; at $y = \delta$, $\eta = 1$; $dy = \delta\, d\eta$

Therefore: $$\frac{u}{U_\infty} = 2\eta - \eta^2$$

Substituting: $$\delta^* = \int_0^1 \left(1 - 2\eta + \eta^2\right)\delta \, d\eta = \delta \left\{ \eta - \frac{2\eta^2}{2} + \frac{\eta^3}{3} \right\}_0^1$$

which yields $\delta^* = \frac{1}{3}\delta$

Therefore, for flows for which the assumed quadratic equation approximates the velocity profile, streamlines outside of the boundary layer are displaced approximately according to the equation

$$\delta^* = \frac{1}{3}\delta$$

This closely approximates flow for a flat plate.

Key Point: When assuming a form for a velocity profile to use in the Von Karman analysis, make sure that the resulting equation satisfies both surface and free stream boundary conditions as well as has a form that approximates u(y).

Momentum Thickness:

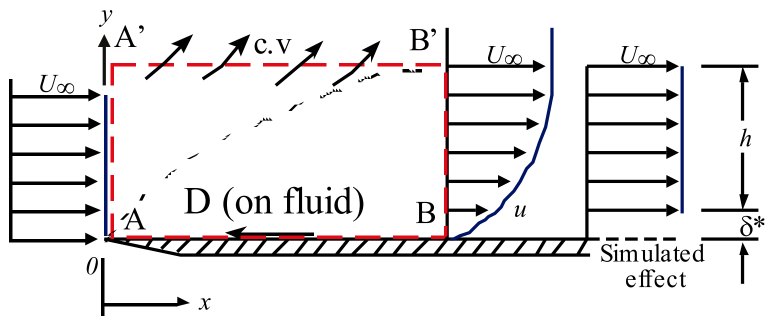
The second concept used in the Von Karman momentum analysis is that of

momentum thickness - θ

The concept is similar to that of displacement thickness in that θ is related to the loss of momentum due to viscous effects in the boundary layer.

Consider the viscous flow regions shown in the adjacent figure.

Define a control volume as shown and integrate around the control volume to obtain the net change in momentum for the control volume.



If D = drag force on the plate due to viscous flow, taking the fluid as the control volume, we can write

$$- D = \sum \text{(momentum leaving c.v.)} - \sum \text{(momentum entering c.v.)}$$

Completing an analysis shown in the text, we obtain

$$D = \rho U_\infty^2 \theta \qquad \theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Using a drag coefficient defined as $C_D = \dfrac{D/A}{\frac{1}{2}\rho U_\infty^2}$

We can also show that $C_D = \dfrac{2\,\theta(L)}{L}$

where: θ(L) is the momentum thickness evaluated over the length L.

Thus, knowledge of the boundary layer velocity distribution u = f(y) also allows the drag coefficient to be determined.

Momentum integral:

The final step in the Von Karman theory applies the previous control volume analysis to a differential length of surface. Performing an analysis similar to the previous analysis for drag D we obtain

$$\frac{\tau_w}{\rho} = \delta^* U_\infty \frac{dU_\infty}{dx} + \frac{d}{dx}\left(U_\infty^2 \theta\right)$$

This is the momentum integral for 2-D, incompressible flow and is valid for laminar or turbulent flow.

where $\delta^* U_\infty \dfrac{dU_\infty}{dx} = -\dfrac{\delta^*}{\rho}\dfrac{dP}{dx}$ Therefore, this analysis also accounts for the effect of freestream pressure gradient.

For a flat plate with non-accelerating flow, we can show that

$$P = const., \quad U_\infty = const., \quad \frac{dU_\infty}{dx} = 0$$

Therefore, for a flat plate, non-accelerating flow, the Von Karman momentum integral becomes

$$\frac{\tau_w}{\rho} = \frac{d}{dx}\left(U_\infty^2\,\theta\right) = U_\infty^2 \frac{d\theta}{dx}$$

From the previous analysis and the assumed velocity distribution of

$$\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$$

The wall shear stress can be expressed as

$$\tau_w = \mu \left.\frac{du}{dy}\right)_w = 2U_\infty \left\{\frac{2}{\delta} - \frac{2y}{\delta^2}\right\}_{y=0} = \frac{2\,\mu U_\infty}{\delta} \qquad \text{(A)}$$

Also, with the assumed velocity profile, the momentum thickness θ can be evaluated as

$$\theta = \int_0^{\delta} \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

or

$$\theta = \int_0^\delta \left(2\eta - \eta^2\right)\left(1 - 2\eta + \eta^2\right)\delta\, d\eta = \frac{2\delta}{15}$$

We can now write from the previous equation for τ_w

$$\tau_w = \rho U_\infty^2 \frac{d\theta}{dx} = \frac{2}{15}\rho U_\infty^2 \frac{d\delta}{dx}$$

Equating this result to Eqn. A we obtain

$$\tau_w = \frac{2}{15}\rho U_\infty^2 \frac{d\delta}{dx} = \frac{2\mu U_\infty}{\delta}$$

or

$$\delta\, d\delta = \frac{15\mu}{\rho U_\infty} dx$$ which after integration yields

$$\delta = \left\{\frac{30\mu x}{\rho U_\infty}\right\}^{1/2} \quad \text{or} \quad \delta = \frac{5.48}{\sqrt{\mathrm{Re}_x}}$$

Note that this result is within 10% of the exact result from Blasius flat plate theory.

Since for a flat plate, we only need to consider friction drag (not pressure drag), we can write

$$C_{f_x} = \frac{\tau_w(x)}{\frac{1}{2}\rho U_\infty^2} = \frac{2\mu U_\infty}{\delta}\frac{1}{\frac{1}{2}\rho U_\infty^2}$$

Substitute for δ to obtain

$$C_{f_x} = \frac{2\mu U_\infty}{5.48}\frac{\sqrt{\mathrm{Re}}}{\frac{1}{2}\rho U_\infty^2} = \frac{0.73}{\sqrt{\mathrm{Re}_x}}$$

Exact theory has a numerical constant of 0.664 compared with 0.73 for the Von Karman integral analysis.

It is seen that the Von Karman integral theory provides the means to determine approximate expressions for

$$\delta,\ \tau_w,\ \text{and}\ C_f$$

using only an assumed velocity profile.

Question Bank

1. With a neat sketch explain the boundary layer separation
2. Derive the expressions for Displacement thickness, Energy thickness and Momentum thickness.
3. The velocity distribution in the boundary layer is given by : $\frac{u}{U} = \frac{y}{\delta}$, where u is the velocity at the distance y from the plate and u=U at y = δ, δ being boundary layer thickness. Find
   a. The displacement thickness
   b. The Momentum thickness
   c. The Energy thickness
   d. The value of $\frac{\delta^*}{\theta}$
4. Find the displacement thickness, momentum thickness and energy thickness for the velocity distribution in boundary layer given by $\frac{u}{U}=2\left(\frac{y}{\delta}\right)-\left(\frac{y}{\delta}\right)^2$
5. Derive the Von Karmen Momentum integral equation for boundary layer flows.
6. Explain briefly about streamlined and bluff bodies.
7. For the velocity profile for laminar boundary layer flows given as $\frac{u}{U}= 2(y/\delta) - (y/\delta)^2$ find the expression for boundary layer thickness.
8. For the velocity profile for laminar boundary layer $\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$ determine the boundary layer thickness, shear stress and drag force.
9. For the velocity profile for laminar boundary layer $\frac{u}{U} = 2(y/\delta)-2(y/\delta)^3+(y/\delta)^4$ obtain the expression for boundary layer thickness, shear stress and drag force
10. The velocity distribution in the boundary layer is given by
    $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$,δ being boundary layer thickness

    Calculate the following

    i) Displacement thickness
    ii) Momentum thickness
    iii) Energy Thickness

11. Explain what is meant by separation of boundary layer. Describe with sketches the methods to control separation.

KARPAGAM UNIVERSITY

Coimbatore 21.

Faculty of Engineering

B.E – Aeronautical Engineering – III Semester

End Semester Answer Key

Subject Code : 13BEAR304 **Maximum Marks: 100**

Subject Name: Fluid Mechanics and Machinery **Time: 3 Hrs**

PART – A (15 x 2 = 30 Marks)

Answer any 15

1. What is fluid mechanics?
   Fluid mechanics is that branch of science which deals with the behaviour of the fluids at rest as well as in motion.

2. Define density.
   It is defined as the ratio of the mass of the and its volume.
   Density =mass/volume. Unit=kg/m^3

3. What are the different types of fluids?
   1) Ideal fluid.
   2) Real fluid.
   3) Newtonian fluids.
   4) Non Newtonian fluids.
   5) Ideal plastic fluid.

4. List out the pressure measuring devices.
   1) Ideal fluid.

2) Real fluid.
3) Newtonian fluids.
4) Non Newtonian fluids.
5) Ideal plastic fluid.

5. Define the term drag.
   The component of the total force in the direction of flow of fluid is known as drag.

6. Define energy thickness.
   It is defined as the distance measured perpendicular to the boundaryby which the boundary should be displaced to compensate for the reduction of kinetic energy of flowing fluid on account of boundary layer formation.

7. Write down four examples of laminar flow.
   1) Flow through pipes.
   2) Blood flow through capillaries.
   3) Laminar flow hood.
   4) Laminar flow airfoil.

8. What is meant by flow through parallel pipes?
   When a main pipeline divides into two or more parallel pipes, which again join together to form a single pipe and continue as a main line. These pipes are said to be pipes in parallel.

9. What are the uses of dimensional homogeneity?
   The law of fourier principle of dimensional homogeneity states and equation which expresses a physical phenomenon of fluid flow should be algebraically correct and dimensionally homogeneous.

10. Define Mach number.
    It is defined as the square root of the inertia force of a flowing fluid to the elastic force.

$$\mu = (\text{Inertia force}/\text{Elastic force})^{1/2}$$

11. Mention the applications of model testing.
    1) Civil engineering structures such as dams, weirs, canals etc.
    2) Design of harbour, ships and submarines.
    3) Aeroplanes, rockets and machines, missiles.

12. Define the term scale effect.
    It is impossible to product the exact behaviour of the prototype by model testing alone. The two models of same prototype behaviour will be same. So discrepancy between models and prototype will always occur. It is known as scale effect.

13. What is reciprocating pumps?

    If the fluid is displaced by reciprocating action of piston, it is known as reciprocating pumps.

14. What is hydroelectric power?

    The mechanical energy developed by a turbine is used to run an electric generator which is directly coupled to the shaft of the turbine. Thus, the mechanical energy is converted into electrical energy. This electrical power is known as hydroelectric power.

15. Write the classifications of turbine according to the quantity of water required.

    1) High head turbine.
    2) Medium head turbine.
    3) High head turbine.

16. What is Mixed flow turbine?

    In a Mixed flow turbine the water enters the blades radially and comes out axially and parallel to the turbine shaft.

17. What is the principle of reciprocating pumps?

    It operates on a principle of actual displacement of liquid by a piston or plunger, which reciprocates in a closely fitting cylinder.

18. Define suction head.

    It is the vertical height of the centre line of the pump shaft above the liquid surface in the sump from which the liquid is being raised.

19. List out the types of rotary pumps.

    1) External pumps.
    2) Internal gear pumps.
    3) Lobe pumps.
    4) Vane pumps.

20. State any two precautions against cavitations.

    1) The pressure should not be allowed to fall below its vapour pressure.
    2) Special material coatings can be given to the surface where the cavitation occurs.

PART – B (5 X 14 = 70 Marks)

Answer ALL the Questions

Question No 21 is compulsory

25. Two large plane surfaces are 2.4 cm apart. The space between the surfaces is filled With glycerine. What force is required to drag a very thin plate of surface area 0.5 Square metre between the two large plane surfaces at a speed of 0.6 m/s,if; i) the thin plate is in the middle of the two planesurfaces and ii) the thin plate is at a distance of 0.8 cm from one of the plane surfaces .Take thedynamic viscosity of the glycerine = 0.8 x 10 $^{-1}$ N s/m^2

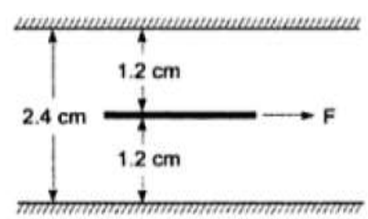


Fig. 1.7 (*a*)

Solution. Given :

Distance between two large surfaces = 2.4 cm

Area of thin plate, $A = 0.5\ m^2$

Velocity of thin plate, $u = 0.6\ m/s$

Viscosity of glycerine, $\mu = 8.10 \times 10^{-1}\ N\ s/m^2$

Case I. When the thin plate is in the middle of the two plane surfaces [Refer to Fig. 1.7 (*a*)]

Let F_1 = Shear force on the upper side of the thin plate

F_2 = Shear force on the lower side of the thin plate

F = Total force required to drag the plate

Then $F = F_1 + F_2$

The shear stress (τ_2) on the upper side of the thin plate is given by equation,

$$\tau_1 = \mu\left(\frac{du}{dv}\right)_1$$

where du = Relative velocity between thin plate and upper large plane surface
= 0.6 m/sec

dy = Distance between thin plate and upper large plane surface
= 1.2 cm = 0.012 m (plate is a thin one and hence thickness of plate is neglected)

$$\therefore \quad \tau_1 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{.012}\right) = 40.5\ N/m^2$$

Now shear force, F_1 = Shear stress × Area
$= \tau_1 \times A = 40.5 \times 0.5 = 20.25$ N

Similarly shear stress (τ_2) on the lower side of the thin plate is given by

$$\tau_2 = \mu\left(\frac{du}{dy}\right)_2 = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.012}\right) = 40.5 \text{ N/m}^2$$

∴ Shear force, $F_2 = \tau_2 \times A = 40.5 \times 0.5 = 20.25$ N

∴ Total force, $F = F_1 + F_2 = 20.25 + 20.25 =$ **40.5 N. Ans.**

Case II. When the thin plate is at a distance of 0.8 cm from one of the plane surfaces [Refer to Fig. 1.7 (*b*)].

Let the thin plate is a distance 0.8 cm from the lower plane surface.

Then distance of the plate from the upper plane surface

$$= 2.4 - 0.8 = 1.6 \text{ cm} = .016 \text{ m}$$

(Neglecting thickness of the plate)

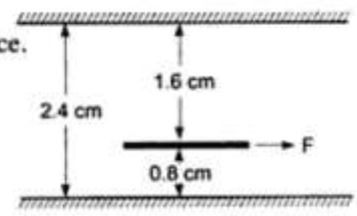


Fig.

The shear force on the upper side of the thin plate,

$$F_1 = \text{Shear stress} \times \text{Area} = \tau_1 \times A$$

$$= \mu\left(\frac{du}{dy}\right)_1 \times A = 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.016}\right) \times 0.5 = 15.18 \text{ N}$$

The shear force on the lower side of the thin plate,

$$F_2 = \tau_2 \times A = \mu\left(\frac{du}{dy}\right)_2 \times A$$

$$= 8.10 \times 10^{-1} \times \left(\frac{0.6}{0.8/100}\right) \times 0.5 = 30.36 \text{ N}$$

∴ Total force required $= F_1 + F_2 = 15.18 + 30.36 =$ **45.54 N. Ans.**

21 a)Derive the Darcy – Weisbach equation to find the loss of head due to friction in pipes.

Darcy – Weisbach Equation.

Consider a uniform horizontal pipe, having steady flow as shown figure. Let 1 -1 and 2-2 are two sections of pipe.

Let P_1 = pressure intensity at section 1-1.

Let P_2 = Velocity of flow at section 1-1.

L = length of the pipe between the section 1-1 and 2-2

d = diameter off pipe.

f^1 = Frictional resistance per unit wetted area per unit velocity.

h_f = loss of head due to friction.

And P_2, V_2 = are the values of pressure intensity and velocity at section 2-2.

Applying Bernoulli's equation between sections 1-1 & 2-2

Total head 1-1 = total head at 2-2 + loss of head due to friction between 1-1&2-2

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2 + h_f$------------(1)

but $Z_1 = Z_1$ [pipe is horizontal]

$V_1 = V_2$[diameter of pipe is same at 1-1 & 2-2]

(2) becomes,

$(P_1/ \rho g) = (P_2/\rho g) + h_f$

$h_f = (P_1/ \rho g) - (P_2/\rho g)$

frictional resistance = frictional resistance per unit wetted area per unit velocity x

wetted area x velocity ^{2}.

$F = f^1 x\ \pi\ d\ l\ x V^2$ [Wetted area = π d x L, and Velocity $V = V_1 = V_2$]

$F_1 = f^1 x P x L x V^2$ ----------- (2). [π d = wetted perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are,

1) Pressure force at section 1-1 = $P_1 x\ A$

2) Pressure force at section 2-2 = $P_2\ x\ A$

3). Frictional force F_1

Resolving all forces in the horizontal direction.,

$P_1 A - P_2 A - F_1 = 0$

$(P_1-P_2)A = F_1 = f^1 x P x L x V^2$

$(P_1\text{-}P_2) = (f^1 x P x L x V^2 / A)$.

But from (1) we get

$P_1 – P_2 = \rho g h_f$

Equating the values of $(P_1 – P_2)$ we get

$\rho g h_f = (f^1 x P x L x V^2 / A)$.

$h_f = (f^1 / \rho g) \times (P/A) \times L \times V^2$

$(P/A) = (\pi d / (\pi d^2/4)) = (4/d)$

hence, $h_f = (f^1 / \rho g) \times (4/d) \times L V^2$.

Putting $(f^1 / \rho) = (f / 2)$, where f is the co – efficient of friction

$$h_f = \frac{4fLV^2}{2gd}$$

This equation is known as Darcy – Weisbach equation. This equation is commonly used to find loss of head due to friction in pipes

or

b)A lubricating oil of viscosity 1 poise and specific gravity 0.9 is pumped through a 30 mm diameter pipe. If the pressure drop per metre length of pipe is 20 kN/m^2 , determine :

i. The mass flow rate in kg/min,
ii. The shear stress at the pipe wall,
iii. The Reynolds number of the flow , and
iv. The power required per 50 m length of the pipe to maintain the flow

Solution. Viscosity of oil, $\mu = 1$ poise $= 0{\cdot}1$ Ns/m^2

Sp. gr. of oil, $= 0{\cdot}9$

$\therefore$ Weight density, $w = 0{\cdot}9 \times 9810 = 8829$ N/m^3

Diameter of pipe, $D = 30$ mm $= 0{\cdot}03$ m

Area, $A = \frac{\pi}{4} \times 0\cdot03^2 = 7\cdot068 \times 10^{-4}$ m^2

Pressure drop per metre length of pipe, $(p_1 - p_2) = 20$ kN/m^2

(*i*) **Mass flow rate:**

Pressure drop for laminar flow through a pipeline is given by,

$$(p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2}$$

$$\therefore \qquad 20 \times 10^3 = \frac{32 \times 0\cdot01 \times \bar{u} \times 1}{(0\cdot03)^2}$$

(where, $\bar{u}$ = average velocity of flow)

or, $$\bar{u} = \frac{20 \times 10^3 \times (0\cdot03)^2}{32 \times 0\cdot1 \times 1} = 5\cdot625 \text{ m/s}$$

Flow rate, $Q = A \times \bar{u} = 7{\cdot}068 \times 10^{-4} \times 5{\cdot}625 = 0{\cdot}003975$ m^3/s

$\therefore$ Mass flow rate $= (0{\cdot}9 \times 1000) \times 0{\cdot}003975 \times 60$

$=$ **214·65 kg/min. (Ans.)**

(*ii*) **Shear stress at the wall, τ_0:**

$$\tau_0 = -\frac{\partial p}{\partial x} \cdot \frac{R}{2} = 20 \times 10^3 \times \frac{(0\cdot03/2)}{2} = \mathbf{150 \text{ N/m}^2} \text{ (Ans.)}$$

(*iii*) **Reynolds number of flow, Re:**

$$Re = \frac{\rho VD}{\mu} = \frac{(0\cdot9 \times 1000) \times 5\cdot625 \times 0\cdot03}{0\cdot1} = \mathbf{1518\cdot7} \text{ (Ans.)}$$

(where $V = \bar{u} = 5{\cdot}625$ m/s)

This is less than 2000 and hence the flow is *laminar*.

(*iv*) **Power required, *P*:**

Loss of head, $h_f = \frac{p_1 - p_2}{w} = \frac{20 \times 10^3}{8829} = 2\cdot265$ m of oil

Power reqd. per metre $= wQh_f = 8829 \times 0{\cdot}003975 \times 2{\cdot}265\ W = 79{\cdot}49\ W$

For 50 *m* length, power required,

$$P = 79{\cdot}49 \times 50 = 3974{\cdot}5\ W \text{ or } \mathbf{3{\cdot}974 \text{ kW}} \text{ (Ans.)}$$

22 a) Derive on the basis of dimensional analysis suitable parameters to develop the thrust produced by a propeller. Assume that the thrust P depends on the Angular Velocity ω ,speed of advance V, diameter D, dynamic viscosity μ , mass density ρ ,elasticity of the fluid medium which can be denoted by the speed of sound in the medium C.

Solution. Thrust P is a function of ω, V, D, μ, ρ, C

or $$P = f(\omega, V, D, \mu, \rho, C)$$

or $$f_1 = (P, \omega, V, D, \mu, \rho, C) = 0 \quad \ldots(i)$$

$\therefore$ Total number of variables, $n = 7$

Writing dimensions of each variable, we have

$$P = MLT^{-2},\ \omega = T^{-1},\ V = LT^{-1},\ D = L,$$
$$\mu = ML^{-1}T^{-1},\ \rho = ML^{-3},\ C = LT^{-1}$$

$\therefore$ Number of fundamental dimensions, $m = 3$

$\therefore$ Number of π-terms $= n - m = 7 - 3 = 4$

Hence equation (i) can be written as $f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0 \quad \ldots(ii)$

Each π-term contains $m + 1$, *i.e.*, $3 + 1 = 4$ variables. Out of four variables, three are repeating variables.

Choosing D, V, ρ as repeating variables, we get π-terms as

$$\pi_1 = D^{a_1} . V^{b_1} . \rho^{c_1} . P$$
$$\pi_2 = D^{a_2} . V^{b_2} . \rho^{c_2} . \omega$$
$$\pi_3 = D^{a_3} . V^{b_3} . \rho^{c_3} . \mu$$
$$\pi_4 = D^{a_4} . V^{b_4} . \rho^{c_4} . C$$

First π-term $\quad \pi_1 = D^{a_1} . V^{b_1} . \rho^{c_1} . P$

Writing dimensions on both sides,

$$M^0L^0T^0 = L^{a_1} . (LT^{-1})^{b_1} . (ML^{-3})^{c_1} . MLT^{-2}.$$

Equating powers of M, L, T on both sides,

Power of M, $\quad 0 = c_1 + 1, \quad \because \quad C_1 = -1$

Power of L, $\quad 0 = a_1 + b_1 - 3c_1 + 1,$

$$a_1 = -b_1 + 3c_1 - 1 = 2 - 3 - 1 = -2$$

Power of T. $\quad 0 = -b_1 - 2, \quad \therefore \quad b_1 = -2$

Substituting the values of a_1, b_1 and c_1 in π_1,

$$\pi_1 = D^{-2} . V^{-2} . \rho^{-1} P = \frac{P}{D^2V^2\rho}.$$

Second π-term $\quad \pi_2 = D^{a_2} . V^{b_2} . \Delta^{c_2} . \omega$

Writing dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} . (LT^{-1})^{b_2} . (ML^{-3})^{c_2} . T^{-1}$$

Equating the powers of M, L, T on both sides,

Power of M, $\quad 0 = c_2, \quad \therefore \quad c_2 = 0$

Power of L, $\quad 0 = a_2 + b_2 - 3c_2, \quad \therefore \quad a_2 = -b_2 + 3c_2 = 1 + 0 = 1$

Power of T, $\quad 0 = -b_2 - 1, \quad \therefore \quad b_2 = -1$

Third π-term $\pi_3 = D^{a_3} \,.\, V^{b_3} \,.\, \rho^{c_3} \,.\, \mu.$

Writing dimensions on both sides,

$$M^0L^0T^0 = D^{a_3} \,.\, (LT^{-1})^{b_3} \,.\, (ML^{-3})^{c_3} \,.\, ML^{-1}T^{-1}.$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c_3 + 1$, $\therefore$ $c_3 = -1$

Power of L, $0 = a_3 + b_3 - 3c_3 - 1$, $\therefore$ $a_3 = -b_3 + 3c_3 + 1 = 1 - 3 + 1 = -1$

Power of T, $0 = -b_3 - 1$ $\therefore$ $b_3 = -1$

Substituting the values of a_3, b_3 and c_3 in π_3,

$$\pi_3 = D^{-1} \,.\, V^{-1} \,.\, \rho^{-1} \,.\, \mu = \frac{\mu}{DV\rho}.$$

Fourth π-term $\pi_4 = D^{a_4} \,.\, V^{b_4} \,.\, \rho^{c_4}\, C.$

Substituting dimensions on both sides,

$$M^0L^0T^0 = L^{a_4} \,.\, (LT^{-1})^{b_4} \,.\, (ML^{-3})^{c_4} \,.\, LT^{-1}$$

Equating the powers of M, L, T on both sides.

Power of M, $0 = c_4$, $\therefore$ $c_4 = 0$

Power of L, $0 = a_4 + b_4 - 3c_4 + 1$, $\therefore$ $a_4 = -b_4 + 3c_4 - 1 = 1 + 0 - 1 = 0$

Power of T, $0 = -a_4 - 1$, $\therefore$ $b_4 = -1$

Substituting the values of a_4, b_4 and c_4 in π_4,

$$\pi_4 = D^0 \,.\, V^{-1} \,.\, \rho^0 \,.\, C = \frac{C}{V}.$$

Substituting the values of π_1, π_2, π_3 and π_4 in equation (*ii*),

Power of M, $0 = c_4$, $\therefore$ $c_4 = 0$

Power of L, $0 = a_4 + b_4 - 3c_4 + 1$, $\therefore$ $a_4 = -b_4 + 3c_4 - 1 = 1 + 0 - 1 = 0$

Power of T, $0 = -a_4 - 1$, $\therefore$ $b_4 = -1$

Substituting the values of a_4, b_4 and c_4 in π_4,

$$\pi_4 = D^0 \,.\, V^{-1} \,.\, \rho^0 \,.\, C = \frac{C}{V}.$$

Substituting the values of π_1, π_2, π_3 and π_4 in equation (*ii*),

$$f_1\left(\frac{P}{D^2V^2\rho}, \frac{D\omega}{V}, \frac{\mu}{DV\rho}, \frac{C}{V}\right) = 0 \quad \text{or} \quad \frac{P}{D^2V^2\rho} = f_2\left(\frac{D\omega}{V}, \frac{\mu}{DV\rho}, \frac{C}{V}\right)$$

or

$$\mathbf{P = D^2V^2\rho\; f_2\left(\frac{D\omega}{V}, \frac{\mu}{DV\rho}, \frac{C}{V}\right).}\ \textbf{Ans.}$$

or

b) A pipe of diameter 1.5 m is required to transport an oil of specific gravity 0.90 and viscosity 3 x 10^{-2} poise at the rate of 3000 litre/s. Tests were conducted on a 15 cm diameter pipe using water at 20^o C. Find the velocity and rate of flow in the model . Viscosity of water at 20^o = 0.01 poise

Solution. Given :

Dia. of prototype, $D_P = 1.5$ m

Viscosity of fluid, $\mu_P = 3 \times 10^{-2}$ poise

Q for prototype, $Q_P = 3000$ lit/s $= 3.0$ m^3/s

Sp. gr. of oil. $S_P = 0.9$

$\therefore$ Density of oil, $\rho_P = S_P \times 1000 = 0.9 \times 1000 = 900$ kg/m^3

Dia. of the model, $D_m = 15$ cm $= 0.15$ m

Viscosity of water at 20°C $= .01$ poise $= 1 \times 10^{-2}$ poise or $\mu_m = 1 \times 10^{-2}$ poise

Density of water or $\rho_m = 1000$ kg/m^3.

For pipe flow, the dynamic similarity will be obtained if the Reynold's number in the model and prototype are equal

Hence using equation (6.17), $\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_P V_P D_P}{\mu_P}$ {For pipe, linear dimension is D}

$$\therefore \quad \frac{V_m}{V_P} = \frac{\rho_P}{\rho_m} \cdot \frac{D_P}{D_m} \cdot \frac{\mu_P}{\mu_m}$$

$$= \frac{900}{1000} \times \frac{1.5}{0.15} \times \frac{1 \times 10^{-2}}{3 \times 10^{-2}} = \frac{900}{1000} \times 10 \times \frac{1}{3} = 3.0$$

But
$$V_P = \frac{\text{Rate of flow in prototype}}{\text{Area of prototype}} = \frac{3.0}{\frac{\pi}{4}(D_P)^2} = \frac{3.0}{\frac{\pi}{4}(1.5)^2}$$

$$= \frac{3.0 \times 4}{\pi \times 2.25} = 1.697 \text{ m/s}$$

$$\therefore \quad V_m = 3.0 \times V_P = 3.0 \times 1.697 = \mathbf{5.091 \text{ m/s. Ans.}}$$

Rate of flow through model, $Q_m = A_m \times V_m = \frac{\pi}{4}(D_m)^2 \times V_m = \frac{\pi}{4}(0.15)^2 \times 5.091$ m^3/s

$$= 0.0899 \text{ m}^3\text{/s} = 0.0899 \times 1000 \text{ lit/s} = \mathbf{89.9 \text{ lit/s. Ans.}}$$

23 a)A Pelton wheel is to be designed with the following specifications

Shaft Power = 11,722 kW ;

Head = 380 m ;

Speed = 750 r.p.m ;

Overall efficiency =86 % ;

Jet diameter is not to exceed one-sixth of the wheel diameter.

Determine (1) Wheel diameter (2) The number of jets required (3) Diameter of the jet

Take K_{v_1}=0.985 and K_{u_1} = 0.45

Solution. Given :

Shaft power, S.P. = 11,772 kW

Head, $H = 380$ m

Speed, $N = 750$

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \frac{d}{D} = \frac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 380} = 85.05$ m/s

The velocity of wheel, $u = u_1 = u_2$

$$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85$$

But $u = \frac{\pi DN}{60} \quad \therefore \quad 38.85 = \frac{\pi DN}{60}$

or $$D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} = \mathbf{0.989\ m.\ Ans.}$$

But $$\frac{d}{D} = \frac{1}{6}$$

$\therefore$ Dia. of jet, $$d = \frac{1}{6} \times D = \frac{0.989}{6} = \mathbf{0.165\ m.\ Ans.}$$

Discharge of one jet, $$q = \text{Area of jet} \times \text{Velocity of jet}$$

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05\ m^3/s = 1.818\ m^3/s$$

Now $$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{11772}{\frac{\rho g \times Q \times H}{1000}}$$

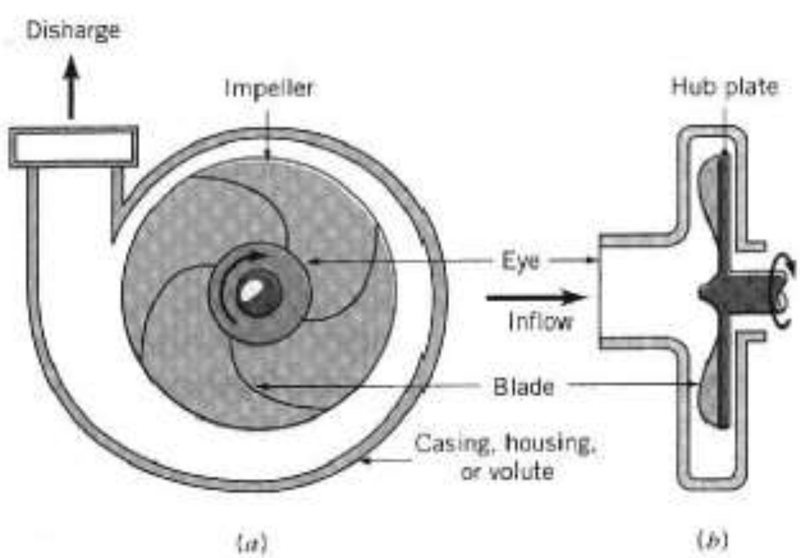
$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}, \text{ where } Q = \text{Total discharge}$$

$\therefore$ Total discharge, $$Q = \frac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672\ m^3/s$$

$\therefore$ Number of jets $$= \frac{\text{Total discharge}}{\text{Discharg of one jet}} = \frac{Q}{q} = \frac{3.672}{1.818} = \mathbf{2\ jets.\ Ans.}$$

or

b) Explain with neat sketch the Constructional details and working principle of Centrifugal pump.



Centrifugal pumps are a sub-class of dynamic axisymmetric work-absorbing turbomachinery. Centrifugal pumps are used to transport fluids by the conversion of rotational kinetic energy to the hydrodynamic energy of the fluid flow. The rotational energy typically comes from an engine or electric motor. In the typical case, the fluid enters the pump impeller along or near to the rotating axis and is accelerated by the impeller, flowing radially outward into a diffuser or volute chamber (casing), from where it exits. a centrifugal pump converts mechanical energy from a motor to energy of a moving fluid. A portion of the energy goes into kinetic energy of the fluid motion, and some into potential energy, represented by fluid pressure (Hydraulic head) or by lifting the fluid, against gravity, to a higher altitude. The transfer of energy from the mechanical rotation of the impeller to the motion and pressure of the fluid is usually described in terms of centrifugal force, especially in older sources written before the modern concept of centrifugal force as a fictitious force in a rotating reference frame was well articulated. The concept of centrifugal force is not actually required to describe the action of the centrifugal pump.

The outlet pressure is a reflection of the pressure that applies the centripetal force that curves the path of the water to move circularly inside the pump. On the other hand, the statement that the "outward force generated within the wheel is to be understood as being produced entirely by the medium of centrifugal force" is best understood in terms of centrifugal force as a fictional force in the frame of reference of the rotating impeller; the actual forces on the water are inward, or centripetal, since that is the direction of force need to make the water move in circles. This force is supplied by a pressure gradient that is set up by the rotation, where the pressure at the outside, at the wall of the volute, can be taken as a reactive centrifugal force. This was typical of nineteenth and early twentieth century writings, mixing the concepts of centrifugal force in informal descriptions of effects, such as those in the centrifugal pump.

24 a) A single acting reciprocating pump running at 50 r.p.m delivers 0.01 m^3 /s of water . The diameter of the piston is 200 mm and stroke length 400 mm.

Determine :

i) The theoretical discharge of the pump,

ii) coefficient of discharge ,

iii) Slip and Percentage of slip.

Solution. Given :

Speed of the pump, $N = 50$ r.p.m.

Actual discharge, $Q_{act} = .01\ m^3/s$

Dia. of piston, $D = 200\ mm = .20\ m$

$\therefore$ Area, $A = \frac{\pi}{4}(.2)^2 = .031416\ m^2$

Stroke, $L = 400\ mm = 0.40\ m.$

(*i*) Theoretical discharge for single-acting reciprocating pump is given by equation (20.1) as

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{.031416 \times .40 \times 50}{60} = \mathbf{0.01047\ m^3/s.}\ \textbf{Ans.}$$

(*ii*) Co-efficient of discharge is given by

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{.01047} = \mathbf{0.955.}\ \textbf{Ans.}$$

(*iii*) Using equation (20.8), we get

$$\text{Slip} = Q_{th} - Q_{act} = .01047 - .01 = \mathbf{0.00047\ m^3/s.}\ \textbf{Ans.}$$

And percentage slip

$$= \frac{(Q_{th} - Q_{act})}{Q_{th}} \times 100 = \frac{(.01047 - .01)}{.01047} \times 100$$

$$= \frac{.00047}{.01047} \times 100 = \mathbf{4.489\%}.\ \textbf{Ans.}$$

or

b)Explain with neat sketch the working principle of Lobe pump and Vane pump.

Working principle of Lobe pump:

Lobe pumps are similar to external gear pumps in operation in that fluid flows around the interior of the casing. Unlike external gear pumps, however, the lobes do not make contact. Lobe contact is prevented by external timing gears located in the gearbox. Pump shaft support bearings are located in the gearbox, and since the bearings are out of the pumped liquid, pressure is limited by bearing location and shaft deflection.

1. As the lobes come out of mesh, they create expanding volume on the inlet side of the pump. Liquid flows into the cavity and is trapped by the lobes as they rotate.

2. Liquid travels around the interior of the casing in the pockets between the lobes and the casing -- it does not pass between the lobes.

3. Finally, the meshing of the lobes forces liquid through the outlet port under pressure.

Lobe pumps are frequently used in food applications because they handle solids without damaging the product. Particle size pumped can be much larger in lobe pumps than in other PD types. Since the lobes do not make contact, and clearances are not as close as in other PD pumps, this design handles low viscosity liquids with diminished performance. Loading characteristics are not as good as other designs, and suction ability is low. High-viscosity liquids require reduced speeds to achieve satisfactory performance. Reductions of 25% of rated speed and lower are common with high-viscosity liquids.

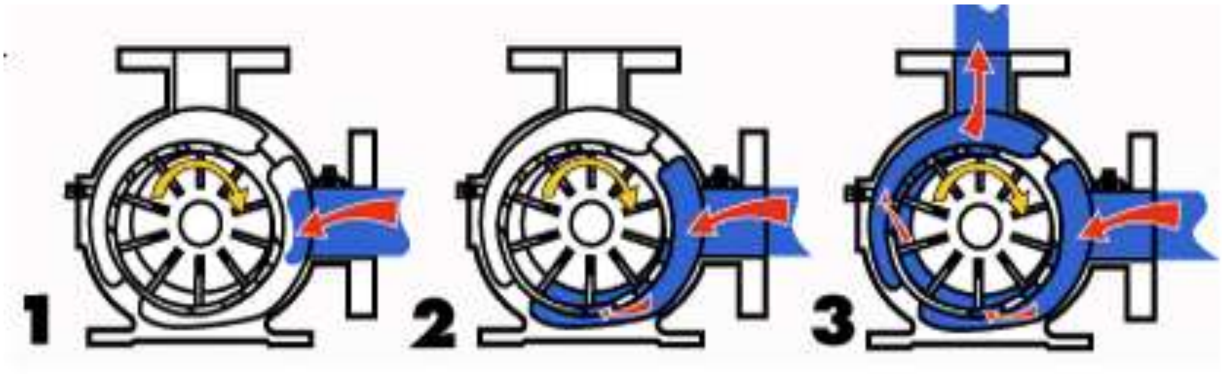
Advantages

- Pass medium solids
- No metal-to-metal contact
- Superior CIP/SIP capabilities
- Long term dry run (with lubrication to seals)
- Non-pulsating discharge

Disadvantages

- Requires timing gears
- Requires two seals
- Reduced lift with thin liquids

Working principle Vane pump:



Despite the different configurations, most vane pumps operate under the same general principle described below.

1. A slotted rotor is eccentrically supported in a cycloidal cam. The rotor is located close to the wall of the cam so a crescent-shaped cavity is formed. The rotor is sealed into the cam by two sideplates. Vanes or blades fit within the slots of the impeller. As the rotor rotates (*yellow arrow*) and fluid enters the pump, centrifugal force, hydraulic pressure, and/or pushrods push the vanes to the walls of the housing. The tight seal among the vanes, rotor, cam, and sideplate is the key to the good suction characteristics common to the vane pumping principle.

2. The housing and cam force fluid into the pumping chamber through holes in the cam (*small red arrow on the bottom of the pump*). Fluid enters the pockets created by the vanes, rotor, cam, and sideplate.

3. As the rotor continues around, the vanes sweep the fluid to the opposite side of the crescent where it is squeezed through discharge holes of the cam as the vane approaches the point of the crescent (*small red arrow on the side of the pump*). Fluid then exits the discharge port.

Advantages

- Handles thin liquids at relatively higher pressures
- Compensates for wear through vane extension
- Sometimes preferred for solvents, LPG
- Can run dry for short periods
- Can have one seal or stuffing box
- Develops good vacuum

Disadvantages

- Can have two stuffing boxes
- Complex housing and many parts

- Not suitable for high pressures
- Not suitable for high viscosity
- Not good with abrasives

ONLINE QUESTIONS

UNIT I

	Questions	opt1	opt2	opt3	opt4	opt5	opt6	Answer
1	A substance in liquid or gaseous phase is referred as ___________	Fluid	Liquid	Solid	Either liquid or Solid			Fluid
2	Fluid is a substance that _________	cannot be subjected to shear forces	always expands until it fills any container	has the same shear stress at a point regardless of its motion	cannot remain at rest under action of any shear force			cannot remain at rest under action of any shear force
3	Fluid is a substance which offers no resistance to change of ________	pressure	flow	shape	volume			shape
4	Practical fluids ______	are inviscous	possess surface tension	are Incompressible	is ideal			possess surface tension
5	In a static fluid ________	resistance to shear stress is small	fluid pressure is zero	linear deformation is small	only normal stresses can exist			only normal stresses can exist
6	A fluid is said to be ideal, if it is _______	compressible	Practically applicable	viscous and compressible	inviscous and incompressible			inviscous and incompressible
7	A real practical fluid which obeys Newton's law of viscosity is known as ______	Newtonian fluid	non Newtonian fluid	Ideal fluid	Real fluid			Newtonian fluid
8	A fluid which does not obey Newton's law of viscosity is known as _____	Newtonian fluid	non Newtonian fluid	Ideal fluid	Real fluid			non Newtonian fluid
9	An ideal flow of any fluid must fulfill the following	Newton's law of motion	Newton's law of viscosity	Pascal's law	Continuity equation			Continuity equation
10	If no resistance is encountered by displacement, such a substance is known as _________	fluid	gas	liquid	ideal fluid			ideal fluid
11	The volumetric change of the fluid caused by resistance is known as _____	volumetric strain	volumetric index	compressibility	cohesion			compressibility
12	Density of water is maximum at ______	0°C	0°K	4°C	100°C			4°C
13	Property of a fluid by which its own molecules are attracted is called ______	adhesion	cohesion	viscosity	compressibility			cohesion
14	Mercury does not wet glass. This is due to property of liquid known as ______	cohesion	surface tension	compressibility	viscosity			surface tension
15	Property of a fluid by which molecules of different kinds of fluids are attracted to each other is called _______	adhesion	cohesion	viscosity	compressibility			adhesion
16	Surface tension of water when in contact with air at 20°C is ________	0.073 N/m	0.1 N/m	0.058 N/m	0.068 N/m			0.073 N/m
17	Surface tension of mercury when in contact with air at 20°C is ________	0.073 N/m	0.1 N/m	0.058 N/m	0.068 N/m			0.1 N/m
18	A fluid having dynamic viscosity of 11 poise is equivalent to _______	9.81 Ns/m^2	1000 Ns/m^2	1 Ns/m^2	1.1 Ns/m^2			1.1 Ns/m^2
19	If 'σ' is the surface tension of fluid, 'd' be the diameter of the water droplet, then Pressure inside a water droplet, p = ____	4σ/d	8σ/d	2σ/d	σ/3d			4σ/d
20	If 'σ' is the surface tension of fluid, 'd' be the diameter of the soap bubble, then Pressure inside a soap bubble, p = ____	4σ/d	8σ/d	2σ/d	σ/3d			8σ/d

21	If 'σ' is the surface tension of fluid, 'd' be the diameter of the liquid jet, then Pressure in a liquid jet, p = ______	4σ/d	8σ/d	2σ/d	σ/3d			2σ/d
22	Which of the following is dimensionless?	specific weight	specific volume	specific speed	specific gravity			specific gravity
23	The tendency of a liquid surface to contract is due to the following property _________	cohesion	adhesion	surface tension	compressibility			surface tension
24	The bulk modulus of elasticity_________ with increase in pressure	increases	decreases	remains constant	unpredictable			increases
25	The S.I unit of surface tension is _____	N/m^2	N/m	N	Pa			N/m
26	The stress-strain relation of the Newtonian fluid is _______	linear	parabolic	hyperbolic	inverse type			linear
27	Kinematic viscosity depends upon _______	Pressure	compressibility	density	displacement			density
28	Choose the correct relationship	specific gravity = gravity x density	Dynamic viscosity = kinematic viscosity x density	Gravity = specific gravity x density	Kinematic viscosity = dynamic viscosity x density			Dynamic viscosity = kinematic viscosity x density
29	For manometer, a better liquid combination is one having	higher surface tension	lower surface tension	surface tension is no criterion	high density and viscosity			higher surface tension
30	The property of fluid by virtue of which it offers resistance to shear is called	Surface tension	Adhesion	cohesion	viscosity			viscosity
31	Choose the wrong statement	fluids are capable of flowing	fluids conform to the shape of the containing vessels	when in equilibrium, fluids cannot sustain tangential forces	when in equilibrium, fluids can sustain shear forces			when in equilibrium, fluids can sustain shear forces
32	If ‘w’ is the specific weight of liquid and ‘h’ is the depth of any point from the surface, then pressure intensity at that point will be ________	h	*w* h	*w* gh	*w* /h			*w* h
33	Choose the wrong statement	Viscosity of a fluid is that property which determines the amount of its resistance to a shearing force	Viscosity is expressed as poise, stoke, or saybolt seconds	Viscosity of liquids decreases with increase in temperature	Viscosity of liquids is appreciably affected by change in pressure			Viscosity of liquids is appreciably affected by change in pressure
34	The property by virtue of which fluids undergo a change in volume under the action of external pressure is known as ______	Specific viscosity	Viscosity index	bulk modulus	compressibility			compressibility
35	The ratio of absolute viscosity to mass density is known as	Specific viscosity	Viscosity index	Kinematic viscosity	Coefficient of viscosity			Kinematic viscosity
36	Which of the following is the unit of kinematic viscosity?	Pascal	poise	stoke	faraday			stoke
37	Which of the following is the unit of dynamic viscosity?	Pascal	poise	stoke	faraday			poise
38	Specific weight of sea water is more than that of pure water because it contains	dissolved air	dissolved salt	suspended matter	all of these			all of these
39	If 850 kg liquid occupies volume of one cubic meter, then 0.85 represents its ______	specific weight	specific mass	specific gravity	specific volume			specific gravity
40	Free surface of a liquid tends to contract to the smallest possible area due to force of _________	surface tension	specific mass	Shear force	specific gravity			surface tension

41	Falling drops of water become spheres due to the property of __________	adhesion	cohesion	surface tension	viscosity			surface tension
42	The resultant upward pressure of the fluid on an immersed body is called ______	upthrust	Buoyancy	Equilibrium of a floating body	Archimedes' principle			Buoyancy
43	Metacentric height is given as the distance between _______	the center of gravity of the body and the meta center	the center of gravity of the body and the center of buoyancy	the center of gravity of the body and the center of pressure	center of buoyancy and metacentre			the center of gravity of the body and the meta center
44	The center of gravity of the volume of the liquid displaced by an immersed body is called _______	meta-center	center of pressure	center of buoyancy	center of gravity			center of buoyancy
45	Liquids transmit pressure equally in all the directions. This is according to ______	Boyle's law	Archimedes principle	Pascal's law	Chezy's equation			Pascal's law
46	Capillary action is due to the ______	Gravitational force	Frictional resistance	Pressure	Combined action of adhesion and cohesion			Combined action of adhesion and cohesion
47	Barometer is used to measure ______	pressure in pipes, channels etc	atmospheric pressure	very low pressure	difference of pressure between two points			atmospheric pressure
48	Manometer is used to measure ________	pressure in pipes, channels etc.	atmospheric pressure	Very low pressure	Difference of pressure between two points			pressure in pipes, channels etc.
49	Rota meter is a device used to measure _______	Fluid flow rate	Pressure	Temperature	Atmospheric pressure			Fluid flow rate
50	The pressure measured above atmospheric pressure is known as ____	Absolute pressure	Static pressure	Vacuum pressure	Gauge pressure			Gauge pressure
	UNIT II							
1	When the flow parameters at any given instant remain same at every point, then flow is said to be _______	quasi static	steady state	laminar	uniform			uniform
2	The pressure at a point in a fluid will not be same in all the directions when the fluid is ________	moving	viscous	inviscous and moving	viscous and moving			viscous and moving
3	When the pipes are connected in parallel, the total loss of head due to friction _________	is equal to the sum of the loss of head in each pipe	is same as in each pipe	is equal to the reciprocal of the sum of loss of head in each pipe	is different in each pipe			is same as in each pipe
4	The boundary layer separation takes place if _______	pressure gradient is zero	Pressure gradient is positive	Pressure gradient is negative	All the other three options are wrong			Pressure gradient is positive
5	Drag is defined as the force exerted by a flowing fluid on a solid body _____	in the direction of flow	Perpendicular to the direction of flow	in the direction which is at an angle of 45 degree to the direction of flow	Opposite to the direction of flow			in the direction of flow
6	A pressure of 25 m of head of water is equal to _______	25.25 kN/m^2	245.25 kN/m^2	2500 kN/m^2	2.5 kN/m^2			245.25 kN/m^2
7	The point in the immersed body through which the resultant pressure of the liquid may be taken to act is known as ____	meta center	center of pressure	center of buoyancy	center of gravity			center of pressure

8	The resultant upward pressure of a fluid on a floating body is equal to the weight of the fluid displaced by the body. This definition is according to	Buoyancy	Equilibrium of a floating body	Archimedes' principle	Bernoulli's theorem			Archimedes' principle
9	The continuity equation is connected with ________	viscous/inviscous fluids	compressibility of fluids	law of conservation of mass	steady/unsteady flow			law of conservation of mass
10	Which of the following instrument can be used for measuring speed of a submarine moving in deep sea?	Venturimeter	Orifice plate	Hot wire anemometer	pitot tube			pitot tube
11	Piezometer is used to measure ________	pressure in pipes, channels etc	atmospheric pressure	Very low pressure at a point	difference of pressure between two points			Very low pressure at a point
12	Which of the following instrument is used to measure fluid flow on the application of Bernoulli's theorem?	Tachometer	Speedometer	Venturimeter	Barometer			Venturimeter
13	The resultant of all normal pressures acts _______	At center of pressure	At metacentre	Vertically upwards	Vertically downwards			Vertically upwards
14	The two important forces for a floating body are ______	Buoyancy, gravity	buoyancy, pressure	buoyancy, inertial	inertial, gravity			Buoyancy, gravity
15	According to Newton's second law,	Force=mass/acceleration	mass=force x acceleration	Force=mass x acceleration	Force=weight x acceleration			Force=mass x acceleration
16	A grid obtained by drawing a series of stream lines and equipotential line is known as _______	stream line	path line	flow net	streakline			flow net
17	The volume of fluid flowing across the section per second is ________	density	velocity	accelaration	Discharge			Discharge
18	Viscosity is denoted by the symbol _____	ρ	μ	ψ	Φ			μ
19	Density is denoted by the symbol ____	ρ	μ	ψ	Φ			ρ
20	Shear stress is denoted by the symbol ___	ρ	μ	Φ	τ			τ
21	Branch of fluid mechanics, which describes the fluid motion and its consequences without the consideration of nature of forces causing the motion is known as _______	Fluid Kinematics	Fluid Statics	Fluid pressure	Fluid kinetics			Fluid Kinematics
22	A stream line is a line ____________	which is along the path of a particle	which is always parallel to the main direction of flow	across which there is no flow	on which tangent drawn at any point gives the direction of velocity.			which is along the path of a particle
23	Bernoulli's equation is derived making assumptions that _________	the flow is uniform and incompressible	the flow is inviscous, uniform and steady	the flow is steady, inviscous, incompressible and irrotational	the flow is unsteady, viscous, compressible and rotational			the flow is steady, inviscous, incompressible and irrotational
24	The ratio of actual discharge of a jet of water to its theoretical discharge is known as ________	co-efficient of discharge	co-efficient of velocity	co-efficient of contraction	co-efficient of viscosity			co-efficient of discharge
25	If the Reynolds number is less than 2000, the flow in a pipe is ________	laminar flow	turbulent flow	transition flow	steady flow			laminar flow
26	If the Reynolds number is greater than 4000, the flow in a pipe is _________	laminar flow	turbulent flow	transition flow	steady flow			turbulent flow

27	If the Reynolds number is between 2000 to 4000, the flow in a pipe is _______	laminar flow	turbulent flow	flow transition	steady flow			flow transition
28	When the fluid is at rest the shear stress is _________	Maximum	Zero	Unpredictable	Minimum			Zero
29	If the density of a fluid is constant from point to point in a flow region, then it is called _______	Steady flow	Incompressible flow	Uniform flow	Rotational flow			Incompressible flow
30	If the density of a fluid changes from point to point in a flow region it is called _______	steady flow	unsteady flow	Non- uniform flow	Compressible flow			Compressible flow
31	If the velocity in a fluid flow does not changes with respect to length of direction of flow, it is called _______	Steady flow	Uniform flow	Incompressible flow	Rotational flow			Uniform flow
32	Fluid flow in a pipe takes place from ___	Lower energy to higher energy	Lower pressure to higher pressure	Higher energy level to lower energy level	None of these			Higher energy level to lower energy level
33	The term (p/ρg) from Bernoulli's equation is known as _____	Kinetic head	Potential head	Datum head	Energy			Potential head
34	The term (v^2/ 2g) from Bernoulli's equation is known as _____	Kinetic head	Potential head	Datum head	Energy			Kinetic head
35	The term (z) from Bernoulli's equation is known as ________	Kinetic head	Potential head	Datum head	Energy			Datum head
36	An ideal flow of any fluid must satisfy ______	Pascal's law	Newton's law of viscosity	boundary layer theory	continuity equation			continuity equation
37	During the opening of a valve in a pipe line, the flow is _______	steady	unsteady	uniform	laminar			unsteady
38	Uniform flow occurs when _______	the flow is steady	the flow is streamline	size and shape of the cross section in a particular length remain constant	size and cross section change uniformly along length			size and shape of the cross section in a particular length remain constant
39	The flow in which conditions do not change with time at any point, is known as _______	one dimensional flow	uniform flow	steady flow	turbulent flow			steady flow
40	General energy equation holds for	steady flow system	uniform flow system	unsteady flow system	non-uniform flow system			steady flow system
41	The fluid forces considered in the Navier Stokes equation are	gravity, pressure and viscous	gravity, pressure and turbulent	pressure, viscous and turbulent	gravity, viscous and turbulent			gravity, pressure and viscous
42	For pipes, laminar flow occurs when Reynolds number is	less than 2000	between 2000 and 4000	more than 4000	more than 2000			less than 2000
43	For pipes, turbulent flow occurs when Reynolds number is	less than 2001	between 2000 and 4001	more than 4000	more than 2000			more than 4000
44	Bernoulli equation deals with the law of conservation of ________	mass	momentum	energy	work			energy
45	Fluid flow in a well defined path is known as ______	Steady flow	Uniform flow	Turbulent flow	Laminar flow			Laminar flow
46	Fluid flow in a undefined path (zig-zag) is known as __________	Steady flow	Uniform flow	Turbulent flow	Laminar flow			Turbulent flow
47	Study of fluid at rest is termed as ___	Fluid Kinematics	Fluid Dynamics	Fluid Statics	Fluid Pressure			Fluid Statics
48	Study of fluid in motion is termed as ____	Discharge	Fluid Dynamics	Fluid Statics	Fluid Pressure			Fluid Dynamics

49	When a liquid is flowing through a pipe, the velocity of the liquid is ________	maximum at the centre and minimum near the walls	minimum at the centre and maximum near the walls	zero at the centre and maximum near the walls	maximum at the centre and zero near the walls			maximum at the centre an minimum near the walls
50	Streamlines, path lines and streak lines are virtually identical for _______	Uniform flow	Flow of ideal fluids	Steady flow	Non uniform flow			Steady flow
	UNIT III							
1	Dimension of Length is ______	M	L	T	ML			L
2	Dimension of Mass is _____	M	L	T	ML			M
3	Dimension of Time is ______	M	L	T	ML			T
4	Hydraulic gradient line (H.GL.) represents the sum of ____	pressure head and kinetic head	kinetic head and datum head	ressure head. kinetic head and datum head	Pressure head and datum head			Pressure head and datum head
5	Total energy line (T.E.L.) represents the sum of _______	pressure head and kinetic head	kinetic head and datum head	pressure head and datum head	Pressure head, kinetic head and datum head			Pressure head, kinetic head and datum head
6	Power transmission through pipes will be maximum when __________	Head lost due to friction =1/2 of the total head at the inlet of the pipe	Head lost due to friction = 1/4 of the total head at the inlet of the pipe	Head lost due to friction = total head at the inlet of the pipe	Head lost due to friction = 1/3 of the total head at the inlet of the pipe			Head lost due to friction = 1/3 of the total head at the inlet of the pipe
7	Geometric similarity between model and prototype means ______	the similarity of discharge	the similarity of linear dimensions	the similarity of motion	the similarity of forces			the similarity of motion
8	Which of the following is an example of laminar flow?	rain fall	Water showers	streamline flow of water from a kitchen faucet	Water falls from mountain			streamline flow of water from a kitchen faucet
9	__________ studied the laminar flow through a circular tube expirementally	Prandtl	Pascal	Hagen and Poiseuille	Bernoulli			Hagen and Poiseuille
10	Darcy-Weishbach equation is used to find loss of head due to ______	sudden enlargement	sudden contraction	friction	pressure difference			friction
11	The flow in a pipe is either laminar or turbulent when Reynolds number is ___	less than 2000	more than 4000	between 2000 to 4000	at 3000			between 2000 to 4000
12	The energy loss in a pipe line is due to	surface roughness only	viscous action only	friction offered by pipe wall as well as by viscous friction	pressure drop			friction offered by pipe wall as well as by viscous friction
13	Pascal-second is the unit of	pressure	kinematic viscosity	dynamic viscosity	surface tension			dynamic viscosity
14	The power transmitted through a pipe is (where w = Specific weight in N/m^3, and Q = Discharge in m^3/s, H-Total Head, hf-head loss due to friction)	w x Q x H	w x Q x hf	w x Q (H - hf)	w x Q (H + hf)			w x Q (H - hf)
15	Assertion (A): In a fluid, the rate of deformation is far more important than the total deformation itself. Reason (R): A fluid continues to deform so long as the external forces are applied.	Both A and R are individually true and R is the correct explanation of A	Both A and R are individually true but R is not the correct explanation of A	A is true but R is false	A is false but R is true			Both A and R are individually true and R is the correct explanation of A
16	The shear stress developed in lubricating oil, of viscosity 9.81 poise, filled between two parallel plates 1 cm apart and moving with relative velocity of 2 m/s is:	20 N/m^2	196.2 N/m^2	29.62 N/m^2	40 N/m^2			196.2 N/m^2

17	What is the dimension of kinematic viscosity of a fluid?	LT^{-2}	$L^{2}T^{-1}$	$ML^{-1}T^{-1}$	$L^{-2}T^{-1}$			$L^{2}T^{-1}$
18	An oil of specific gravity 0.9 has viscosity of 0.28 Stokes at 38^{0}C. What will be its viscosity in Ns/m^{2}?	0.252	0.0311	0.0252	0.0206			0.0252
19	Assertion (A): Blood is a Newtonian fluid. Reason (R): The rate of strain varies non-linearly with shear stress for blood.	Both A and R are individually true and R is the correct explanation of A	Both A and R are individually true but R is not the correct explanation of A	A is true but R is false	A is false but R is true			A is false but R is true
20	Which Property of mercury is the main reason for use in barometers?	High Density	Negligible Capillary effect	Very Low vapour Pressure	Low compressibility			Very Low vapour Pressure
21	Which one of the following sets of conditions clearly apply to an ideal fluid?	Viscous and compressible	Non-viscous and incompressible	Non-viscous and compressible	Viscous and incompressible			Non-viscous and incompressible
22	Assertion (A): If a cube is placed in a liquid with two of its surfaces parallel to the free surface of the liquid, then the pressures on the two surfaces which are parallel to the free surface, are the same. Reason (R): Pascal's law states that when a fluid is at rest, the pressure at any plane is the same in all directions.	Both A and R are individually true and R is the correct explanation of A	Both A and R are individually true but R is not the correct explanation of A	A is true but R is false	A is false but R is true			A is false but R is true
23	The reading of the pressure gauge fitted on a vessel is 25 bar. The atmospheric pressure is 1.03 bar and the value of g is $9.81m/s^{2}$. The absolute pressure in the vessel is:	23.97 bar	25.00 bar	26.03 bar	34.84 bar			26.03 bar
24	The vertical component of the hydrostatic force on a submerged curved surface is the	Mass of liquid vertically above it	Weight of the liquid vertically above it	Force on a vertical projection of the surface	Product of pressure at the centroid and the surface area			Weight of the liquid vertically above it
25	Resultant pressure of the liquid in case of an immersed body acts through which one of the following?	Centre of gravity	Centre of pressure	Metacentre	Centre of buoyancy			Centre of pressure
26	What is the vertical component of pressure force on submerged curved surface equal to?	Its horizontal component	The force on a vertical projection of the curved surface	The product of the pressure at centroid and surface area	The gravity force of liquid vertically above the curved surface up to the free surface			The gravity force of liquid vertically above the curved surface up to the free surface
27	A circular disc of radius 'r' is submerged vertically in a static fluid up to a depth 'h' from the free surface. If h > r, then the position of centre of pressure will:	Be directly proportional to h	Be inversely proportional of h	Be directly proportional to r	Not be a function of h or r			Be directly proportional to h
28	What is buoyant force?	Lateral force acting on a submerged body	Resultant force acting on a submerged body	Resultant force acting on a submerged body	Resultant hydrostatic force on a body due to fluid surrounding it			Resultant hydrostatic force on a body due to fluid surrounding it
29	The metacentric height of a passenger ship is kept lower than that of a naval or a cargo ship because ______	Apparent weight will increase	Otherwise it will be in neutral equilibrium	It will decrease the frequency of rolling	Otherwise it will sink and be totally immersed			It will decrease the frequency of rolling

30	What are the forces that influence the problem of fluid static?	Gravity and viscous forces	Gravity and pressure forces	Viscous and surface tension forces	Gravity and surface tension forces			Gravity and pressure forces
31	Existence of velocity potential implies that ______	Fluid is in continuum	Fluid is irrotational	Fluid is ideal	Fluid is compressible			Fluid is irrotational
32	A hydraulic coupling belongs to the category of _________	Power absorbing machines	Power developing machines	Energy generating machines	Energy transfer machines			Energy transfer machines
33	Which one of the following combination represents the power transmission systems?	Pump, hydraulic accumulator, hydraulic intensifier and hydraulic coupling	Pump, turbine, hydraulic accumulator and hydraulic coupling	Turbine, accumulator, intensifier and hydraulic coupling	Accumulator, intensifier, hydraulic coupling and torque converter			Accumulator, intensifier, hydraulic coupling and torque converter
34	1 Joule = _______	1 Newton-meter	2 cubic meter per kilogram	2 kilogram-meter/second squared	2 Newton second per meter squared			1 Newton-meter
35	The SI unit of Velocity is ___	meter per second	meter per second squared	radian per second	radian per second squared			meter per second
36	The SI unit of Acceleration is ___	meter per second	meter per second squared	radian per second	radian per second squared			meter per second squared
37	The SI unit of Acceleration due to gravity is ___	meter per second	meter per second squared	radian per second	radian per second squared			meter per second squared
38	The SI unit of Angular velocity is _____	meter per second	meter per second squared	radians per second	radian per second squared			radians per second
39	The SI unit of Angular acceleration is ___	meter per second	meter per second squared	radians per second	radian per second squared			radian per second squarec
40	The SI unit of Torque is ___	Newton per meter squared	Newton-meter	Pascal	Watt			Newton-meter
41	The SI unit of Pressure is ___	Newton per meter squared	Newton-meter	Joule	Watt			Newton per meter squarec
42	The SI unit of Energy is ___	Newton per meter squared	Newton	Joule	Watt			Joule
43	The SI unit of Power is ___	Newton per meter squared	Newton-meter	Joule	Watt			Watt
44	The specific gravity of an oil whose specific weight is 7.85 kN/m^3, is ______	0.8	1	1.2	1.6			0.8
45	The SI unit of surface tension is ______	J/m^2	N/m^2	J/m	J			J/m^2
46	If the specific gravity of mercury is 13.6 then its density is _________	13.6 kg/m^3	1000 N/m^3	136 kg/m^3	13600 kg/m^3			13600 kg/m^3
47	Specific gravity or Relative Density of Water is _______	1	9810	9.81	1000			1
48	Specific gravity or Relative Density of mercury is ________	13.6	13600	1	9.8			13.6
49	Density or Specific mass of water is ___	1000 N/m^3	9.81 N/m^3	9810 N/m^3	1000 kg/m^3			1000 kg/m^3
50	Specific weight of water in S.I. units is equal to _________	1000 N/m^3	9.81 N/m^3	9810 N/m^3	1000 kg/m^3			9810 N/m^3
51	If 'v' is the velocity and 'A' is the area, then according to continuity equation __	$w_1a_1 = w_2a_2$	$w_1q_1 = w_2q_2$	$A_1v_1 = A_2v_2$	$A_1/v_1 = A_2/v_2$			$A_1v_1 = A_2v_2$
52	The coefficient of discharge of venturimeter, generally, lies between _______	0.3 to 0.45	0.50 to 0.75	0.75 to 0.95	0.95 to 0.99			0.95 to 0.99

53	A nozzle placed at the end of a water pipe line discharges water at a	low pressure	high pressure	low velocity	high velocity			high velocity
54	A vessel of 4 m^3 contains an oil which weighs 30 kN. The specific weight of the oil is ________	4.5 kN/m^3	6 kN/m^3	7.5 kN/m^3	10 kN/m^3			7.5 kN/m^3
	UNIT IV							
1	Cavitation is caused by ________	high velocity	high pressure	weak material	low pressure			low pressure
2	One litre of water occupies a volume of ____	100 cm^3	10000 m^3	1000 cm^3	1 m^3			1000 cm^3
3	One litre of water occupies a volume of ____	100 cm^3	10^{-3} cm^3	10^{-3} m^3	1 cm^3			10^{-3} m^3
4	One cubic metre of water is eqivalent to ______	100 litres	250 litres	500 litres	1000 litres			1000 litres
5	A pump is defined as a device which converts _________	Hydraulic energy into mechanical energy	Heat energy into hydraulic energy	Kinetic energy into mechanical energy	mechanical energy into hydraulic energy			mechanical energy into hydraulic energy
6	If the water is in contact with both sides of the piston the reciprocating pump is called	Double acting	Double stage	Double acting and Double stage	Single acting			Double acting
7	The discharge through a double acting reciprocating pump is	Q= ALN / 60	Q= 2ALN/ 60	Q= ALN	Q= 2AL			Q= 2ALN/ 60
8	Air vessel in a reciprocating pump is used	To run the pump at a high speed without separation	To increase suction head	To increase the delivery head	To stop the pump			To run the pump at a high speed without separation
9	Among these, which one is the best example for rotodynamic pump?	Gear pump	Vane pump	Reciprocating pump	Centrifugal pump			Centrifugal pump
10	To discharge a large quantity of liquid by multistage centrifugal pumps the impellers are connected_______	in parallel	in series	in parallel & series	All of these			in parallel
11	Throttling the gate valve leads to ______	decrease in velocity	reduce power consumption	increased head & power consumption	increased flow rate			increased head & power consumption
12	One horsepower is equal to ________	500 Watts	100.7 Watts	745.7 Watts	700.7 Watts			745.7 Watts
13	The SI unit of kinematic viscosity is _______	square meter per second	cubic meter per kilogram	Newton per meter	Pascal			square meter per second
14	The SI unit of Pressure is ______	square meter per second	cubic meter per kilogram	Newton per meter	Pascal			Pascal
15	The SI unit of specific volume is _____	square meter per second	cubic meter per kilogram	Newton per meter	Pascal			cubic meter per kilogram
16	The SI unit of Bulk modulus is ______	square meter per second	cubic meter per kilogram	Newton per meter	Newton per sqauare meter			Newton per sqauare meter
17	The SI unit of vapour pressure is _____	square meter per second	cubic meter per kilogram	Newton per meter	Bar			Bar
18	The SI unit of density is _____	meter square per second	cubic meter per kilogram	Newton per meter	kilogram per cubic meter			kilogram per cubic meter
19	The SI unit of weight density is _____	meter square per second	cubic meter per kilogram	Newton per meter	Newton per cubic meter			Newton per cubic meter
20	The SI unit of Shear stress is ______	meter square per second	cubic meter per kilogram	Newton per meter squared	Newton per cubic meter			Newton per meter squared
21	The SI unit of Dynamic viscosity is ____	meter square per second	cubic meter per kilogram	Newton per meter	Newton second per meter squared			Newton second per meter squared

22	The SI unit of mass density is _____	meter square per second	cubic meter per kilogram	Newton per meter	kilogram per cubic meter			kilogram per cubic meter
23	The pressure measured below atmospheric pressure is known as ____	Absolute pressure	static pressure	vacuum pressure	gauge pressure			vacuum pressure
24	The ratio of mass to volume of the fluid at standard temperature and pressure is termed as ________	compressibility	Specific volume	Specific weight	Mass density			Mass density
25	Differential manometer is used to measure _________	pressure in pipes, channels etc	atmospheric pressure	very low pressure	difference of pressure between two points			difference of pressure between two points
26	An ideal fluid is _________	one which obeys Newton's law of viscosity	frictionless and incompressible	very viscous	frictionless and compressible			frictionless and incompressible
27	If the dynamic viscosity of a fluid is 0.5 poise and specific gravity is 0.5, then the kinematic viscosity of that fluid in stokes is ___________	0.25	0.5	1	500			1
28	The viscosity of a gas ________	decreases with increase in temperature	increases with increase in temperature	is independent of temperature	is independent of pressure for very high pressure intensities			increases with increase in temperature
29	Newton's law of viscosity relates _____	intensity of pressure and rate of angular deformation	shear stress and rate of angular deformation	shear stress, viscosity and temperature	viscosity and rate of angular deformation			shear stress and rate of angular deformation
30	If the weight of a body immersed in a fluid exceeds the buoyancy force, then the body will _______	rise until its weight equals the buoyant force	tend to move downward and it may finally sink	Float	None of these			tend to move downward and it may finally sink
31	A floating body is said to be in a state of stable equilibrium _______	when its metacentric height is zero	when the metacentre is above the centre of gravity	when the metacentre is below the centre of gravity	only when its centre of gravity is below its centre of buoyancy			when the metacentre is above the centre of gravity
32	The point in the immersed body through which the resultant pressure of the liquid may be taken to act is known as ____	center of gravity	center of buoyancy	center of pressure	metacentre			center of pressure
33	In which of the following measuring device Bernoulli's equation is used?	Tachometer	Speedometer	Venturimeter	Barometer			Venturimeter
34	In which of the following measuring device Bernoulli's equation is used?	Tachometer	Orificemeter	Manometer	Barometer			Orificemeter
35	In which of the following measuring device Bernoulli's equation is used?	Tachometer	Pitot tube	Manometer	Barometer			Pitot tube
36	The laminar flow is characterised by	existence of eddies	irregular motion of fluid particles	fluid particles moving in layers parallel to the boundary surface	All of these			fluid particles moving in layers parallel to the boundary surface
37	Laminar flow takes place at ________	very low velocities	very high velocities	Moderate velocity	High velocity			very low velocities
38	The position of center of pressure on a plane surface immersed vertically in a static mass of fluid is ______	at the centroid of the submerged area	always above the centroid of the area	always below the centroid of the area	at the centroid			always below the centroid of the area
39	In one dimensional flow, the flow ___	is steady and uniform	takes place in straight line	takes place in curve	takes place in one direction			takes place in straight line
40	The kinematic viscosity is the ______	ratio of absolute viscosity to the density of the liquid	ratio of density of the liquid to the absolute viscosity	product of absolute viscosity and density of the liquid	product of absolute viscosity and mass of the liquid			ratio of absolute viscosity to the density of the liquid

41	Coefficient of contraction is the ratio of __________	actual velocity of jet at vena contracta to the theoretical velocity	loss of head in the orifice to the head of water available at the exit of the orifice	actual discharge through an orifice to the theoretical discharge	area of jet at vena contracta to the area of orifice			area of jet at vena contracta to the area of orifice
42	A flow in which each liquid particle has a definite path, and the paths of individual particles do not cross each other, is called __________	steady flow	uniform flow	streamline flow or laminar flow	turbulent flow			streamline flow or lamina flow
43	When a liquid is flowing through a pipe, the velocity of the liquid is ________	maximum at the centre and minimum near the walls	minimum at the centre and maximum near the walls	zero at the centre and maximum near the walls	maximum at the centre and zero near the walls			maximum at the centre an minimum near the walls
44	Streamlines, path lines and streak lines are virtually identical for _______	Uniform flow	Flow of ideal fluids	Steady flow	Non uniform flow			Steady flow

UNIT V

1	Boundary layer on a flat plate is called laminar boundary layer if	Reynolds Number is less than 2000	Reynolds number is less than 4000	Reynolds number is less than 5 x 10000	Reynolds number is less than 5000			Reynolds number is less tha 5 x 10000
2	Boundary layer thickness is the distance from the surface of the solid body in the direction perpendicular to flow, where the velocity of fluid is equal to	free stream velocity	0.9 times the free stream velocity	0.99 times the free stream velocity	zero			0.99 times the free stream velocity
3	The boundary layer separation takes place if	pressure gradient is zero	Pressure gradient is positive	Pressure gradient is negative	camber is high			Pressure gradient is positiv
4	Drag is defined as the force exerted by a flowing fluid on a solid body	in the direction of flow	Perpendicular to the direction of flow	in the direction which is at an angle of 45 degree to the direction of flow	in the direction which is at an angle of 60 degree to the direction of flow			in the direction of flow
5	Lift force is defined as the force exerted by a flowing fluid on a solid body	in the direction of flow	perpendicular to the direction of flow	at an angle of 45 degree to the direction of flow	in the direction which is at an angle of 180 degree to the direction of flow			perpendicular to the direction of flow
6	Euler's number is the ratio of	inertia force to pressure force	Inertia force to elastic force	inertia force to gravity force	inertia force to viscous force			inertia force to pressure force
7	Geometric similarity between model and prototype means	the similarity of discharge	the similarity of linear dimensions	the similarity of motion	the similarity of forces.			the similarity of motion
8	Reynold's number is defined as the	ratio of inertia force to gravity force	ratio of viscous force to gravity force	ratio of viscous force to viscous force	ratio of inertia force to elastic force.			ratio of viscous force to viscous force
9	Froude's number is defined as the ratio of	Inertia force to viscous force.	inertia force to gravity force	inertia force to elastic force .	inertia force to pressure force.			inertia force to gravity force
10	Models are known undistorted model if	the prototype and model are having different scale ratios	the prototype and model are having same scale ratio	model and prototype are kinematically similar	model and prototype are similar			the prototype and model are having same scale ratio
11	Model analysis of aero planes and projectile moving at supersonic speed based on	Reynolds number	Mach number	Froude number	Euler number			Mach number
12	The boundary-layer takes place	for ideal fluids	for real fluids	for pipe flow only	for over flat plates only			for real fluids
13	Laminar sub-layer exists in.	Laminar boundary layer region	Turbulent boundary layer region	Transition zone	trailing edge			Turbulent boundary layer region

14	The laminar flow is characterised by	existence of eddies	irregular motion of fluid particles	fluid particles moving in layers parallel to the boundary surface	All the other three options are wrong			fluid particles moving in layers parallel to the boundary surface
15	Which of the following is an example of laminar flow?	underground flow	flow past tiny bodies	Flow of oil in measuring instruments	All the other three options are wrong			All the other three options are wrong
16	The pressure gradient in the direction of flow is equal to the shear gradient in the direction	parallel to the direction of flow	normal to the direction of flow	both a & b	All the other three options are wrong			normal to the direction of flow
17	___________ studied the laminar flow through a circular tube expirementally	Prandtl	Pascal	Hagen and Poiseuille	Anderson			Hagen and Poiseuille
18	A flow in which the viscosity of fluid is dominating over the inertia force is called	steady flow	unsteady flow	laminar flow	turbulent flow			laminar flow
19	Laminar flow takes place at	very low velocities	very high velocities	both (a) & (b)	All the other three options are wrong			very low velocities
20	The velocity at which the flow changes from laminar flow to turbulent flow ia called	critical velocity	velocity of approach	sub-sonic velocity	super sonic velocity			critical velocity
21	The velocity at which the laminar flow stops is known as	velocity of approach	lower critical velocity	sub-sonic velocity	super sonic velocity			lower critical velocity
22	The velocity at which the laminar flow starts is known as	velocity of approach	higher critical velocity	lower critical velocity	super sonic velocity			higher critical velocity
23	The velocity corresponding to Reynolds number of 2800, is called	velocity of approach	super sonic velocity	lower critical velocity	higher critical velocity			higher critical velocity
24	A flow is called super-sonic if the	velocity of flow is very high	discharge is difficult to measure	Mach number is between 1 and 6	All the other three options are wrong			Mach number is between 1 and 6
25	Whenever a plate is held immersed at some angle with the direction of flow of the liquid, it is subjected to some pressure. The component of this pressure, in the direction of flow of the liquid, is known as	lift	drag	stagnation pressure	thrust			drag
26	Whenever a plate is held immersed at some angle with the direction of flow of the liquid, it is subjected to some pressure. The component of this pressure, at the right angles to the direction of flow of the liquid, is known as	lift	drag	stagnation pressure	thrust			lift
27	Streamlining will reduce	form drag	induced drag	skin friction drag	parasite drag increases			form drag
28	If an aircraft has a gross weight of 3000 kg and is then subjected to a total weight of 6000 kg the load factor will be	2G	3G	9G	15G			2G
29	A constant rate of climb is determined by	weight	wind speed	excess engine power	density			excess engine power
30	Ice formed on the leading edge will cause the aircraft to	stall at the same stall speed and AoA	stall at a lower speed	stall at the same stall speed	stall at a higher speed			stall at a higher speed
31	If both wings lose lift the aircraft	pitches nose up	pitches nose down	glides on a horizontal plane	glides on a vertical plane			pitches nose up
32	Under what conditions will an aircraft create best lift?	Cold dry day at 200 ft	Hot damp day at 1200 ft	Cold wet day at 1200 ft	Cold wet day at 1800 ft			Cold dry day at 200 ft
33	If there were an increase of density, what effect would there be in aerodynamic dampening?	None	Decreased	Increased	becomes zero			Increased
34	As Mach number increases, what is the effect on boundary layer?	Becomes more turbulent	Becomes less turbulent	Decreases in thickness	increases in thickness			Becomes more turbulent

35	The stagnation point consists of	dynamic and static air pressure	static air pressure	dynamic air pressure	absolute pressure			dynamic and static air pressure
36	Yawing is a rotation around	the normal axis obtained by the elevator	the lateral axis obtained by the rudder	the normal axis obtained by the alieron	the normal axis obtained by the rudder			the normal axis obtained by the rudder
37	Sweepback of the wings will	not affect lateral stability	increase lateral stability at high speeds only	increase lateral stability at all speeds	increase directional stability			increase lateral stability at al speeds
38	With the flaps lowered, the stalling speed will	increase	become zero	remain the same	decrease			decrease
39	When flying close to the stall speed a pilot applies left rudder the aircraft will	pitch nose up	roll to the left	stall the left wing	pitch nose down			stall the left wing
40	When flaps are down it will	increase AoA and increase slow speed stability	decrease AoA and decrease slow speed stability	the AoA remains the same on both wings	increase AoA and decreases low speed stability			decrease AoA and decrease slow speed stability
41	If you have an aircraft that is more laterally stable then directionally stable it will tend to:	skid	slip	bank	yaw			skid
42	A wing section suitable for high speed would be	thick with high camber	thin with high camber	thin with little or no camber	thick with low camber			thin with little or no camber
43	As the speed of an aircraft increases the profile drag	increases	decreases	decreases at first then increase	remains constant			increases
44	The stagnation point on an aerofoil is the point where	the suction pressure reaches a maximum	the boundary layer changes from laminar to turbulent	the airflow is brought completely to rest	the suction pressure reaches zero			the airflow is brought completely to rest
45	The stalling of an aerofoil is affected by the	airspeed	angle of attack	transition speed	density of air			angle of attack
46	What gives the aircraft directional stability?	alieron	Horizontal stabiliser	Elevators	Vertical stabiliser			Vertical stabiliser
47	The most fuel efficient of the following types of engine is the	rocket	turbo-jet engine	turbo-fan engine	turboprop			turbo-fan engine
48	Forward motion of a glider is provided by	control surfaces	the weight	the drag	the engine			the weight