KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University)

(Established Under Section 3 of UGC Act, 1956)

EachanariPost, Coimbatore-641021.Tamilnadu,India.

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING – AEROSPACE ENGINEERING

SUBJECT CODE: 16BTAR305	SEMESTER : III
SUBJECT NAME: SOLID MECHANICS	L T P C = 3 0 0 3

Course Objectives:

- To gain knowledge of simple stresses, strains and deformation in components due to external loads.
- To assess stresses and deformations through mathematical models of beams, twisting bars or combinations of both.
- Effect of component dimensions and shape on stresses and deformations are to be understood.
- The study would provide knowledge for use in the design courses

UNIT I INTRODUCTION TO MECHANICS

Rigid bodies-two dimensional structure-moment of force about an axis-moment of a couple equivalent system of coplanar forces- problems involving beams and frames. Roof trusses-Method of joints, method of sections, Introduction-plane, rectilinear motion - time dependent motion rectangular coordinates-projectile motion.

UNIT II STRESS, STRAIN AND DEFORMATION OF SOLIDS

Rigid and Deformable bodies – Strength, Stiffness and Stability – Stresses; Tensile, Compressive and Shear – Deformation of simple and compound bars under axial load – Thermal stress – Elastic constants – Strain energy ,potential energy and unit strain energy – Strain energy in uni-axial loads.

UNIT III BEAMS - LOADS AND STRESSES

Types of beams: Supports and Loads – Shear force and Bending Moment in beams – Cantilever, Simply supported and Overhanging beams – Stresses in beams – Theory of simple bending – Stress variation along the length and in the beam section – Effect of shape of beam section on stress induced – Shear stresses in beams – Shear flow.

UNIT IV TORSION AND BEAM DEFLECTION

Analysis of torsion of circular bars – Shear stress distribution – Bars of Solid and hollow circular section – Stepped shaft – Twist and torsion stiffness – Compound shafts – Fixed and simply supported shafts –

Application to close-coiled helical springs – Maximum shear stress in spring section including Wahl Factor – Elastic curve of Neutral axis of the beam under normal loads – Evaluation of beam deflection and slope: Double integration method, Macaulay Method

UNIT V ANALYSIS OF STRESSES IN TWO DIMENSIONS

Biaxial state of stresses – Thin cylindrical and spherical shells – Deformation in thin cylindrical and spherical shells – Biaxial stresses at a point – Stresses on inclined plane – Principal planes and stresses – Mohr‘s circle for biaxial stresses – Maximum shear stress - Strain energy in bending and torsion.

TEXT BOOKS:

S.No.	AUTHOR(S)	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1	R. K. Bansal	A Textbook of Strength of Materials	Laxmi Publications. New Delhi.	2010
2	R. S. Khurmi	Strength of Material	S. Chand Publications. New Delhi.	2013
3	Ramamrutham S and R. Narayan	Strength of Materials	DhanpatRai and Sons. New Delhi.	2011

REFERENCE BOOKS:

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1	James M. Gere, Barry J. Goodno	Mechanics of Materials	Prentice Hall Inc. New Jersey.	2008
2	Hearn E. J	Mechanics of Materials	Pergamon Press, Oxford.	1977
3	Bedi D.S	Strength of Materials	S Chand and Co. Ltd., New Delhi	1984
4	Singh D.K	Strength of Materials	ANE Books. New Delhi.	2007
5	Jindal U.C	Textbook on Strength of Materials	Asian Books Pvt Ltd., New Delhi.	2007

WEB REFERENCE:

- www.engineersedge.com
- http://en.wikiversity.org
- www.globalsources.com
- www.clag.org.uk/beam.html
- nptel.iitm.ac.in/courses/IIT.../Strength_of_Materials/index.php

ESE MARKS ALLOCATION

S.No.	Particulars	Marks
1.	Section – A (20 × 1 = 20) Online Test – MCQ type	20
2.	Section – B (5 × 2 = 10)	10
3.	Section – C (5 × 14 = 70) Either ‘A’ or ‘B’ type	70
Total		**100**

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FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING – AEROSPACE ENGINEERING

Subject Name : **Solid Mechanics**
Subject Code : **16BTAR305** **(Credits - 3)**
Name of the Faculty : **Mr.C.Nithiyapathi**
Designation : **Assistant Professor**
Year/Semester/Section : **II / III / -**
Branch : **Aerospace Engineering**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – I : INTRODUCTION TO MECHANICS			
1.	1	Fundamentals - Division of Mechanics	T[1], T[2], R[1]
2.	1	Fundamentals – Terminologies, Definitions & Basic Concepts	T[1], T[2], R[1]
3.	1	Rigid bodies-two dimensional structure	T[1], T[2], R[1]
4.	2	Moment of force about an axis, moment of a couple equivalent system of coplanar forces	T[1], T[2], R[1]
5.	1	Problem and solution – moment of force & couple	T[1], T[2], R[1]
6.	2	Problems involving beams and frames	T[1], T[2], R[1]
7.	1	Tutorial - Roof trusses-Method of joints	T[1], T[2], R[1]
8.	1	Problem and solution - Method of joints	
9.	1	Roof trusses- Method of sections	T[1], T[2], R[1]
10.	1	Introduction – plane, Rectilinear motion, Time dependent motion, Rectangular Coordinates, Projectile motion	T[1], T[2], R[1]
		Total No. of Hours Planned for Unit – I : 12	

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – II : STRESS, STRAIN AND DEFORMATION OF SOLIDS			
11.	1	Rigid and Deformable bodies – Strength, Stiffness and Stability	T[1], T[2], R[1]
12.	1	Stresses: Tensile, Compressive and Shear	T[1], T[2], R[1]
13.	2	Deformation of simple and compound bars under axial load	T[1], T[2], R[1]
14.	1	Problem and solution – Stress and Deformation	T[1], T[2], R[1]
15.	1	Problem and solution – Stress and Deformation - compound bars under axial load	T[1], T[2], R[1]
16.	1	Thermal stress - Problem and solution	T[1], T[2], R[1]
17.	2	Elastic constants, Strain energy ,potential energy and unit strain energy	T[1], T[2], R[1]
18.	1	Strain Energy in uni-axial loads	T[1], T[2], R[1]
19.	1	Tutorial - Problem and solution - simple and compound bars under axial load	T[1], T[2], R[1]

20.	1	Problem and solution - Strain energy	T[1], T[2], R[1]
		Total No. of Hours Planned for Unit – II : 12	

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – III : BEAMS - LOADS AND STRESSES			
21.	1	Types of beams: Supports and Loads	T[1], T[2], R[3]
22.	2	Shear force and Bending Moment in beams	T[1], T[2], R[3]
23.	1	Cantilever, Simply supported and Overhanging beams	T[1], T[2], R[3]
24.	1	Stresses in beams	T[1], T[2], R[3]
25.	1	Problem and solution - Shear force and Bending Moment	T[1], T[2], R[3]
26.	1	Theory of simple bending	T[1], T[2], R[3]
27.	1	Stress variation along the length and in the beam section	T[1], T[2], R[3]
28.	1	Effect of shape of beam section on stress induced	T[1], T[2], R[3]
29.	1	Tutorial - Problem and solution – Stress Determination	T[1], T[2], R[3]
30.	2	Shear stresses in beams – Shear flow	T[1], T[2], R[3]
		Total No. of Hours Planned for Unit – III: 12	

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – IV : TORSION AND BEAM DEFLECTION			
31.	1	Introduction, Analysis of torsion of circular bars	T[1], T[2], R[3]
32.	1	Shear stress distribution – Bars of Solid and hollow circular section – Stepped shaft	T[1], T[2], R[3]
33.	1	Twist and torsion stiffness	T[1], T[2], R[3]
34.	1	Compound shafts – Fixed and simply supported shafts	T[1], T[2], R[3]
35.	1	Problem and solution - Analysis of torsion & Shear stress distribution	T[1], T[2], R[3]
36.	1	Torsion - Application to close-coiled helical springs	T[1], T[2], R[3]
37.	1	Maximum shear stress in spring section including Wahl Factor	T[1], T[2], R[3]
38.	1	Elastic curve of Neutral axis of the beam under normal loads	T[1], T[2], R[3]
39.	1	Evaluation of beam deflection and slope: Double integration method	T[1], T[2], R[3]
40.	1	Tutorial - Problem and solution - Double integration method	T[1], T[2], R[3]
41.	2	Evaluation of beam deflection and slope: Macaulay Method	T[1], T[2], R[3]
		Total No. of Hours Planned for Unit – IV: 12	

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – V : ANALYSIS OF STRESSES IN TWO DIMENSIONS			
42.	1	Biaxial state of stresses, Biaxial stresses at a point	T[1], T[2], R[3]
43.	1	Thin cylindrical and spherical shells	T[1], T[2], R[3]
44.	2	Deformation in thin cylindrical and spherical shells	T[1], T[2], R[3]
45.	1	Stresses on inclined plane – Principal planes	T[1], T[2], R[3]
46.	1	Tutorial - Problem and solution – Deformations in thin cylindrical and spherical shells	T[1], T[2], R[3]

47.	2	Stresses – Mohr‘s circle for biaxial stresses – Maximum shear stress	T[1], T[2], R[3]
48.	1	Strain energy in bending and torsion	T[1], T[2], R[3]
49.	1	Problem and solution - Mohr‘s circle for biaxial stresses	T[1], T[2], R[3]
50.	1	Problem and solution - Strain energy in bending and torsion	T[1], T[2], R[3]
51.	1	**Discussion on Competitive Examination related Questions / University previous year questions - Tutorial**	
		Total No. of Hours Planned for Unit – V: 12	

TOTAL PERIODS : 60

TEXT BOOKS

T [1] – R. K. Bansal (2010), “A Textbook of Strength of Materials, Laxmi Publications, New Delhi.

T [2] – R. S. Khurmi (2013), “Strength of Material”, S. Chand Publications. New Delhi

T [3] - Ramamrutham S and R. Narayan (2011), “Strength of Materials”, DhanpatRai and Sons.New Delhi.

REFERENCES

R [1] - James M. Gere, Barry J. Goodno (2008), “Mechanics of Materials”, Prentice Hall Inc. New Jersey.

R [2] - Hearn E. J (1997). “Mechanics of Materials”, Pergamon Press, Oxford.

R [3] - Bedi D.S (1984), “Strength of Materials”, S Chand and Co. Ltd., New Delhi

R [4] - Singh D.K (2007), “Strength of Materials”, ANE Books. New Delhi.

R [5] - Jindal U.C (2007), “Textbook on Strength of Materials”, Asian Books Pvt Ltd., New Delhi.

WEBSITES

W [1] - http://nptel.ac.in/

W [2] - https://web.mst.edu

W [3] - https://apm.iitm.ac

JOURNALS

J [1] - Mechanics of Materials- An International Journal - www.journals.elsevier.com

J [2] – Journal of Material Sciences & Engineering - www.omicsgroup.org

J [3] – MMSE Journal (Mechanics, Materials science and Engineering)

J [4] – International Journal of Materials, Mechanics and manufacturing - http://www.ijmmm.org/

UNIT	Total No. of Periods Planned	Lecture Periods	Tutorial Periods
I	12	11	1
II	12	11	1
III	12	11	1
IV	12	11	1
V	12	10	2
TOTAL	60	54	6

I. CONTINUOUS INTERNAL ASSESSMENT : 40 Marks

(Internal Assessment Tests: 30, Attendance: 5, Assignment / Seminar: 5)

II. END SEMESTER EXAMINATION : 60 Marks

TOTAL : 100 Marks

16BTAR305 SOLID MECHANICS 3 0 0 3 100

UNIT I INTRODUCTION TO MECHANICS

Rigid bodies-two dimensional structure-moment of force about an axis-moment of a couple equivalent system of coplanar forces- problems involving beams and frames. Roof trusses-Method of joints, method of sections, Introduction-plane, rectilinear motion - time dependent motion rectangular coordinates-projectile motion.

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Info:

Prepared & Compiled by,

Mr.C.Nithiyapathi
Assistant Professor,
Department of Mechanical Engineering,
Karpagam Academy of Higher Education.

Method of Sections

- The Method of Sections involves analytically *cutting* the truss into sections and solving for static equilibrium for each section.

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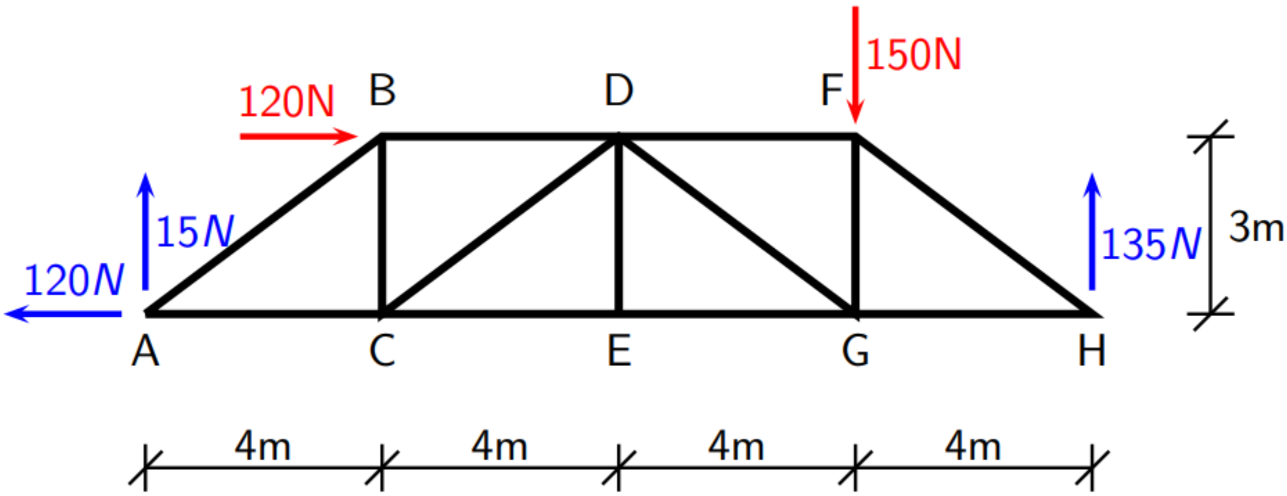
- The sections are obtained by *cutting* through some of the members of the truss to expose the force inside the members.
- In the Method of Joints, we are dealing with static equilibrium at a point. This limits the static equilibrium equations to just the two force equations. A section has finite size and this means you can also use moment equations to solve the problem. This allows solving for up to three unknown forces at a time.

- The Method of Sections involves analytically *cutting* the truss into sections and solving for static equilibrium for each section.
- The sections are obtained by *cutting* through some of the members of the truss to expose the force inside the members.
- In the Method of Joints, we are dealing with static equilibrium at a point. This limits the static equilibrium equations to just the two force equations. A section has finite size and this means you can also use moment equations to solve the problem. This allows solving for up to three unknown forces at a time.
- Since the Method of Sections allows solving for up to three unknown forces at a time, you should choose sections that involve cutting through no more than three members at a time.
- When a member force points toward the joint it is attached to, the member is in compression. If that force points away from the ioint it is attached to. the member is in tension.

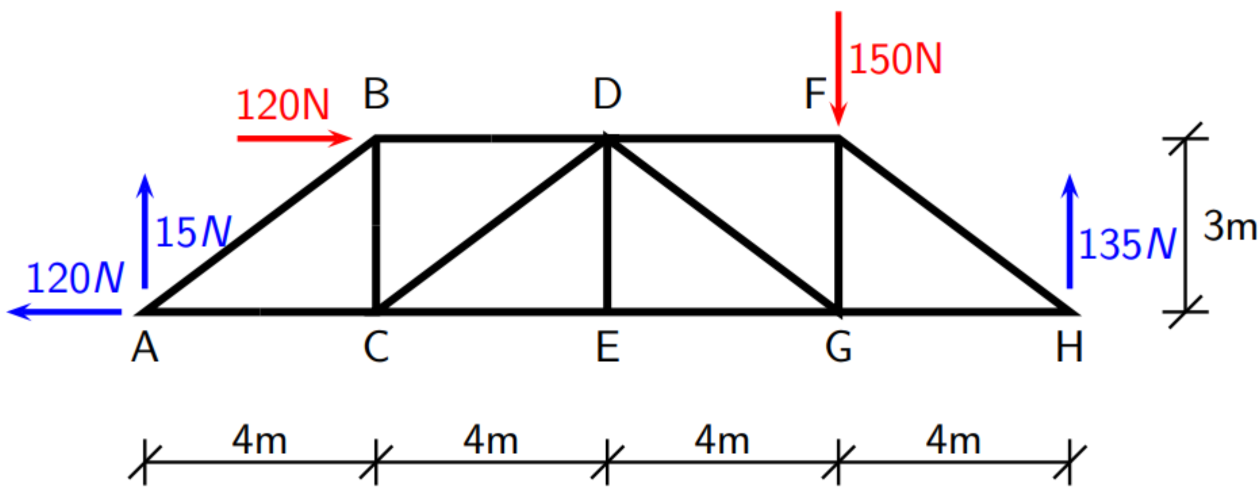
Method of Sections

Refer back to the end of the "truss-initial-analysis.pdf" file to see what has been solved so far for the truss. This is what has been solved for so far:



Method of Sections - Cutting through AC, BC and BD

Let's create a section by *cutting* through members AC, BC and BD. Recall that we want to cut through at most three members.



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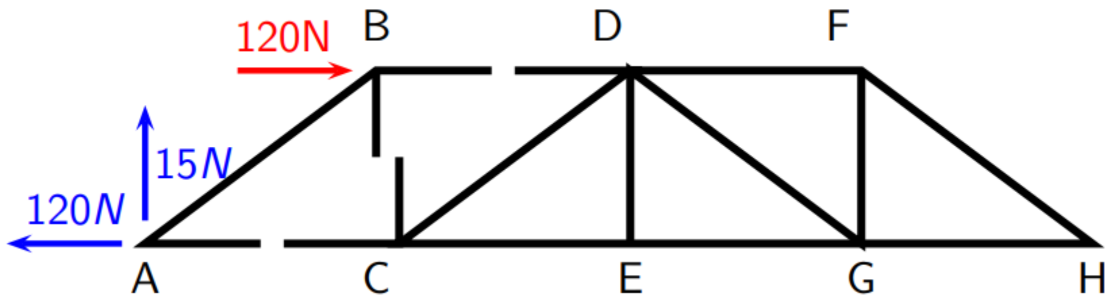
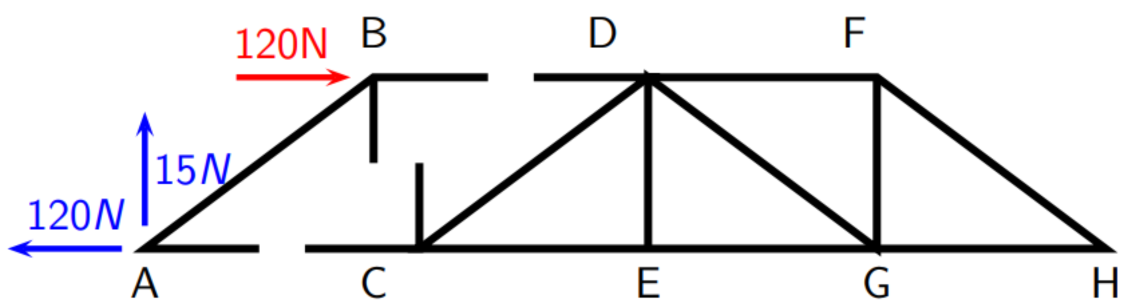


Let's slide the rest of the truss out of the way.

Method of Sections - Cutting through AC, BC and BD

Let's create a section by *cutting* through members AC, BC and BD. Recall that we want to cut through at most three members.



Let's slide the rest of the truss out of the way.

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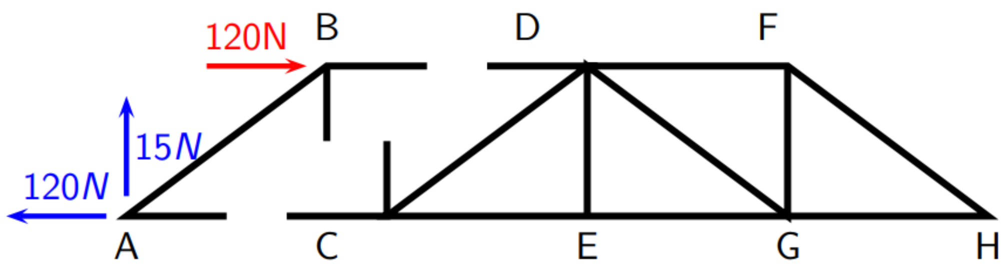

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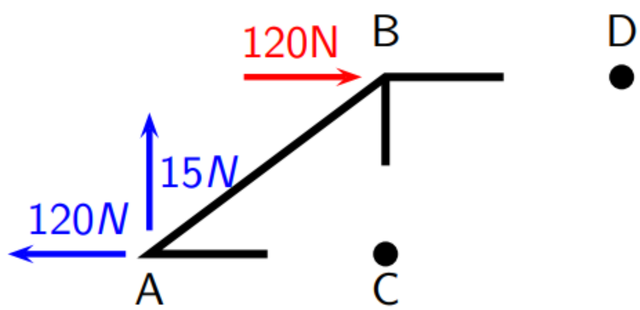
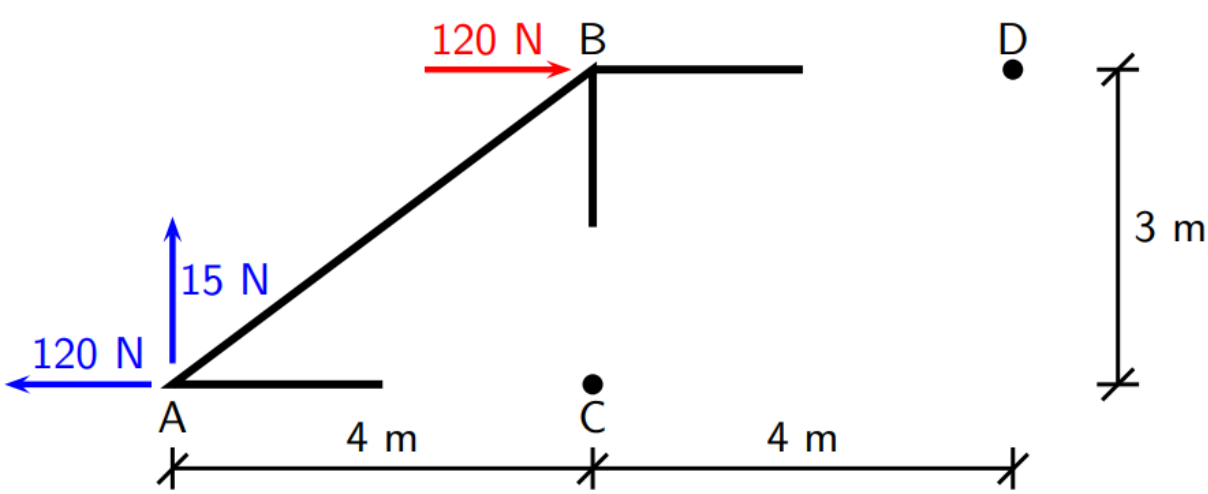
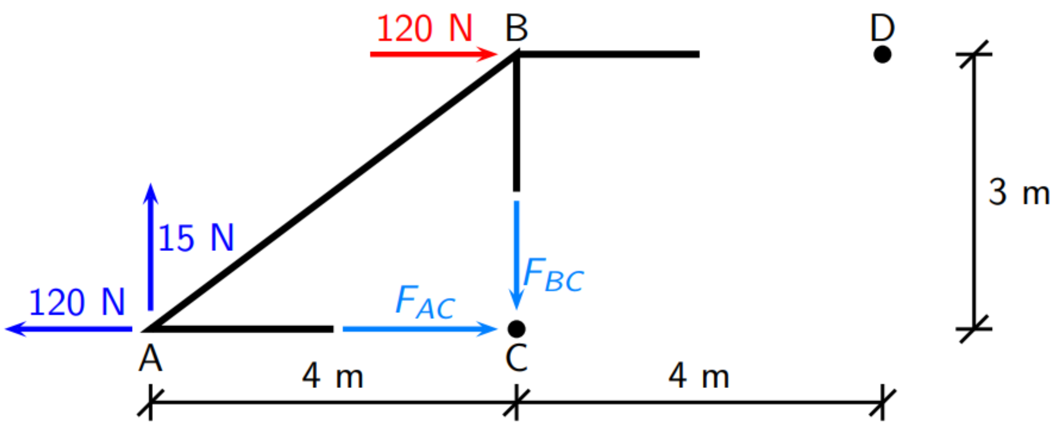
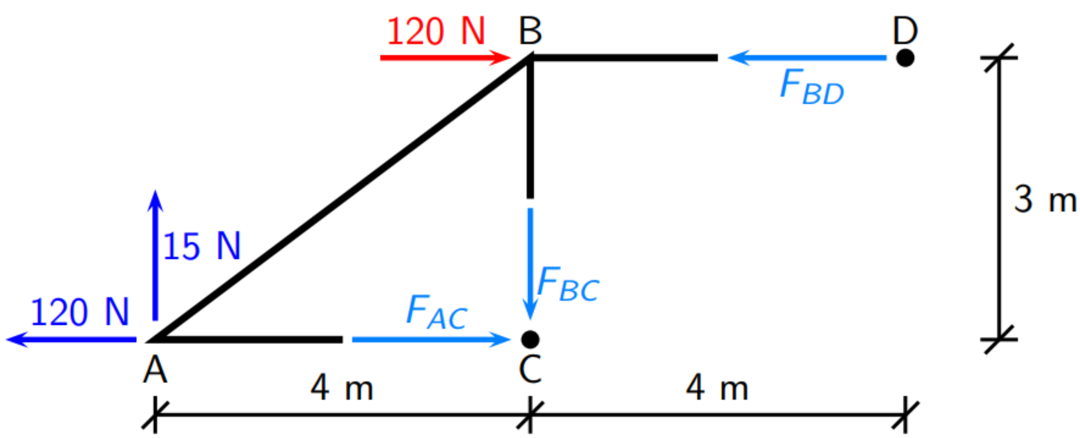






Since F_{BC} is the only force that has a vertical component, it must point down to balance the 15 N force (A_y).

Taking moments about point B has both forces at A giving clockwise moments. Therefore, F_{AC} must point to the right to provide a counter-clockwise moment.



Since F_{BC} is the only force that has a vertical component, it must point down to balance the 15 N force (A_y).

Taking moments about point B has both forces at A giving clockwise moments. Therefore, F_{AC} must point to the right to provide a counter-clockwise moment.

Taking moments about point C has the 15 N force acting at A and the 120 N acting at B giving clockwise moments. Therefore, F_{BD} must point to the left to provide a counter-clockwise moment.

Solving in the order of the previous page:

$$\uparrow^{+} \sum F_y = +15N - F_{BC} = 0$$
$$F_{BC} = 15N \text{ (tension)}$$

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$$\uparrow^{+} \sum F_y = +15N - F_{BC} = 0$$
$$F_{BC} = 15N \text{ (tension)}$$

$$\circlearrowleft^{+} \sum M_B = -(120N)(3m) - (15N)(4m) + F_{AC}(3m) = 0$$
$$F_{AC} = \frac{(360 + 60)Nm}{3m} = 140N \text{ (tension)}$$

Solving in the order of the previous page:

$$\uparrow^{+} \sum F_y = +15N - F_{BC} = 0$$
$$F_{BC} = 15N \text{ (tension)}$$

$$\circlearrowleft^{+} \sum M_B = -(120N)(3m) - (15N)(4m) + F_{AC}(3m) = 0$$
$$F_{AC} = \frac{(360 + 60)Nm}{3m} = 140N \text{ (tension)}$$

$$\circlearrowleft^{+} \sum M_C = -(15N)(4m) - (120N)(3m) + F_{BD}(3m) = 0$$
$$F_{BD} = \frac{(60 + 360)Nm}{3m} = 140N \text{ (compression)}$$

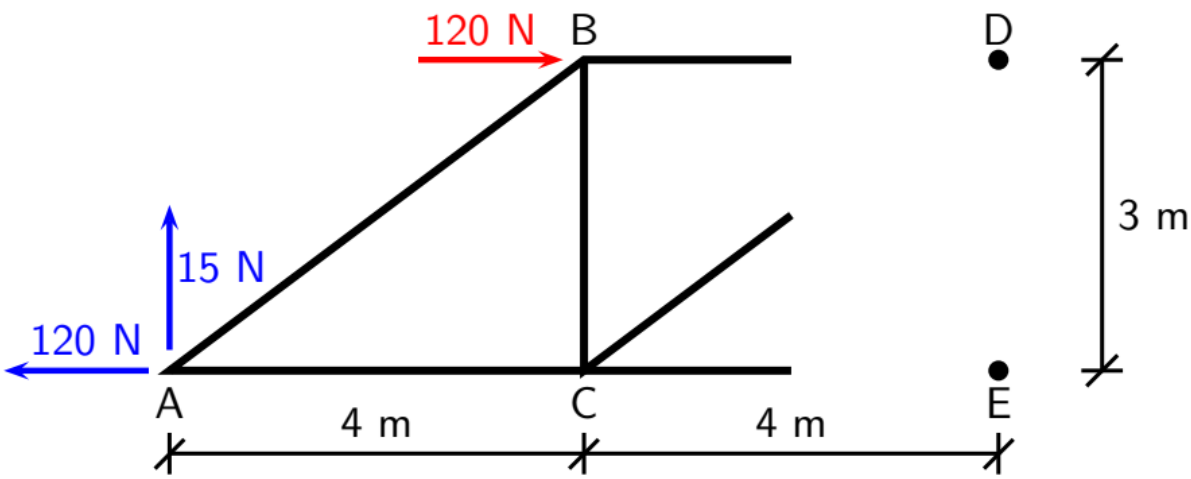
- When drawing your sections, include the points that the cut members would have connected to if not cut. In the section just looked at, this would be points C and D.

- When drawing your sections, include the points that the cut members would have connected to if not cut. In the section just looked at, this would be points C and D.
- Each member that is cut represents an unknown force. Look to see if there is a direction (horizontal or vertical) that has only one unknown. If this true, you should balance forces in that direction. In the section just looked at, this would be the forces in the vertical direction since only F_{BC} has a vertical component.

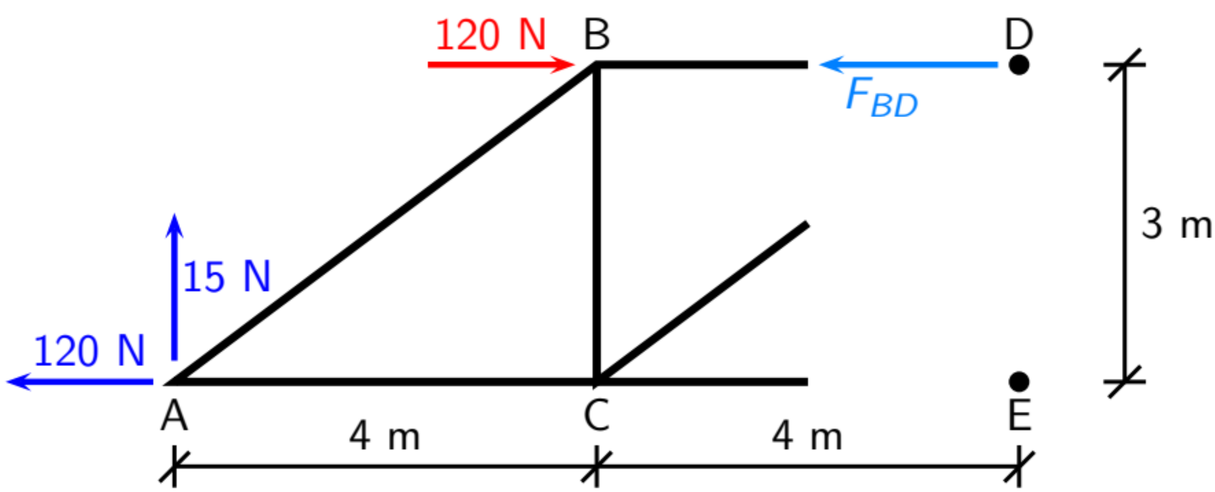
- When drawing your sections, include the points that the cut members would have connected to if not cut. In the section just looked at, this would be points C and D.
- Each member that is cut represents an unknown force. Look to see if there is a direction (horizontal or vertical) that has only one unknown. If this true, you should balance forces in that direction. In the section just looked at, this would be the forces in the vertical direction since only F_{BC} has a vertical component.

- If possible, take moments about points that two of the three unknown forces have lines of forces that pass through that point. This will result in just one unknown in that moment equation. In the section just looked at, taking moments about point B eliminates the unknowns F_{BC} and F_{BD}. Similarly, taking moments about point C eliminates the unknowns F_{BC} and F_{AC} from the equation.

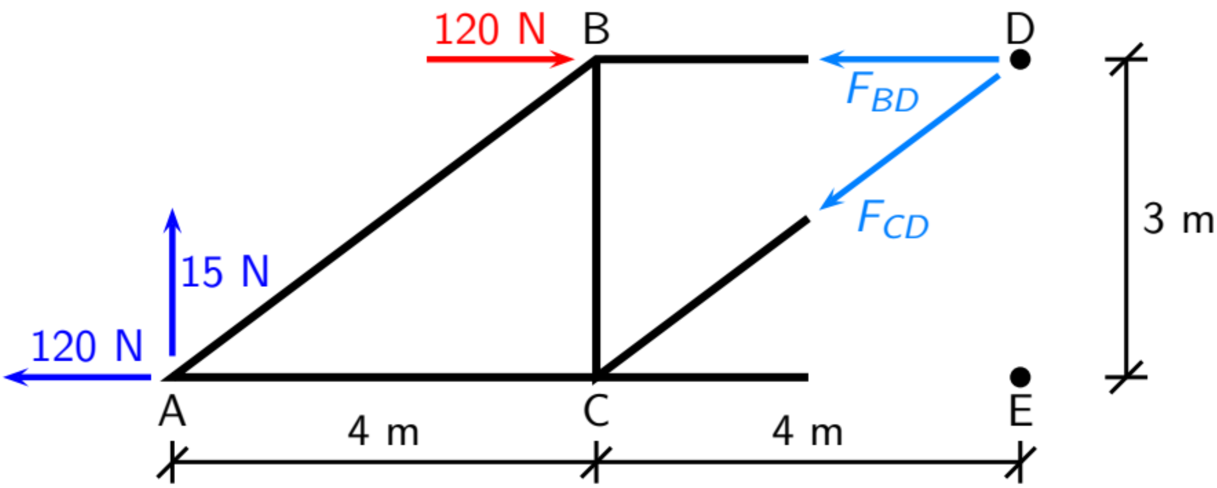
Method of Sections - Cutting through BD, CD and CE



Method of Sections - Cutting through BD, CD and CE

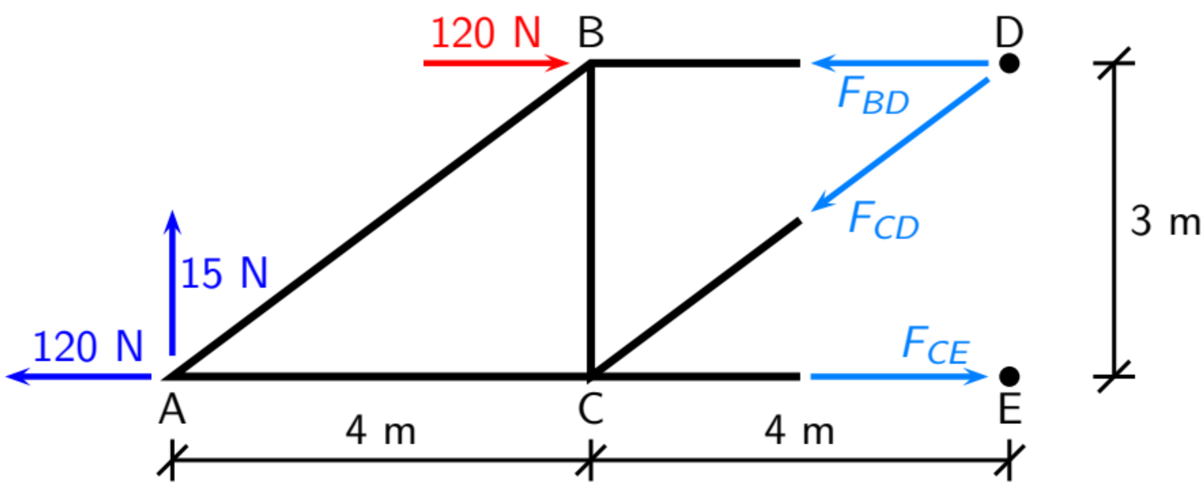


Since we know (from the previous section) the direction of F_{BD} we draw that in first. We could also reason this direction by taking moments about point C.



Since we know (from the previous section) the direction of F_{BD} we draw that in first. We could also reason this direction by taking moments about point C.

Since F_{CD} is the only force that has a vertical component, it must point down to balance the 15 N force (A_y).



Since we know (from the previous section) the direction of F_{BD} we draw that in first. We could also reason this direction by taking moments about point C.

Since F_{CD} is the only force that has a vertical component, it must point down to balance the 15 N force (A_y).

Taking moments about point D has the 120 N force and 15 N force acting at A giving clockwise moments. Therefore F_{CE} must point to the right to give a counter-clockwise moment to balance this out.

Solving in the order of the previous page:

$$\uparrow^{+} \sum F_y = +15N - \frac{3}{5}F_{CD} = 0$$

$$F_{CD} = \frac{5}{3}(15N) = 25N \text{ (compression)}$$

Solving in the order of the previous page:

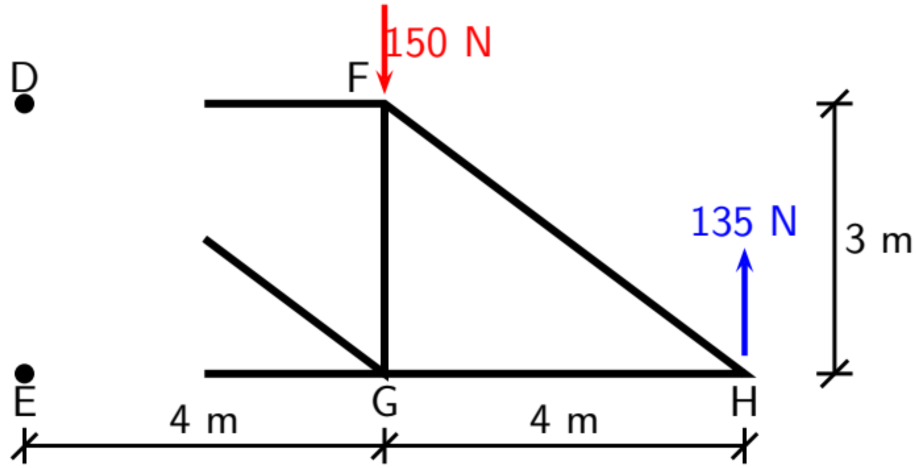
$$\uparrow^{+} \sum F_y = +15N - \frac{3}{5}F_{CD} = 0$$

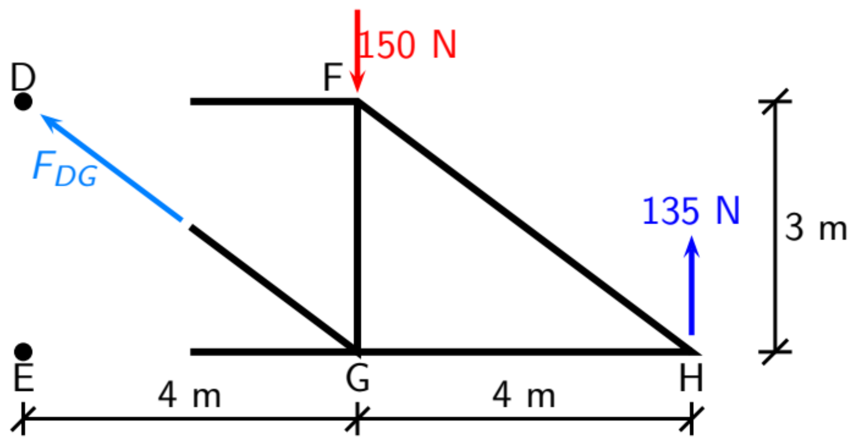
$$F_{CD} = \frac{5}{3}(15N) = 25N \text{ (compression)}$$

$$\circlearrowleft^{+} \sum M_D = -(120N)(3m) - (15N)(8m) + F_{CE}(3m) = 0$$

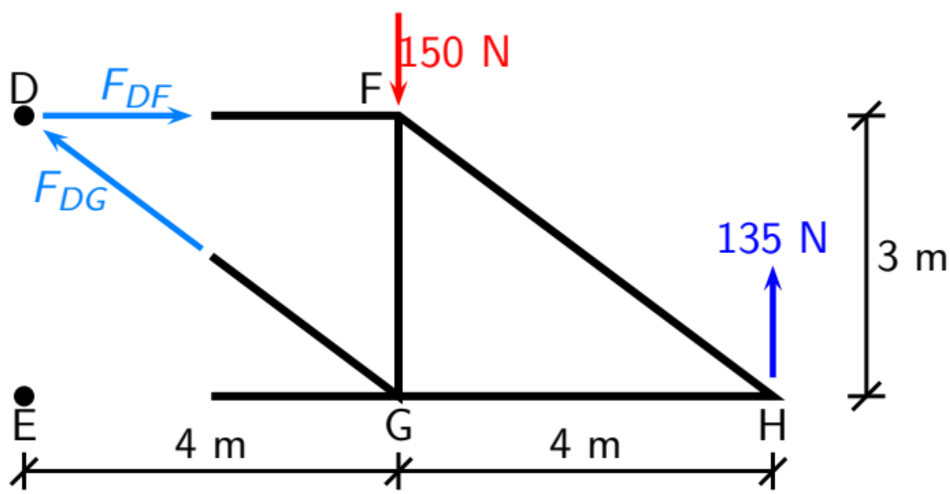
$$F_{CE} = \frac{(360 + 120)Nm}{3m} = 160N \text{ (tension)}$$

Method of Sections - Cutting through DF, DG and EG



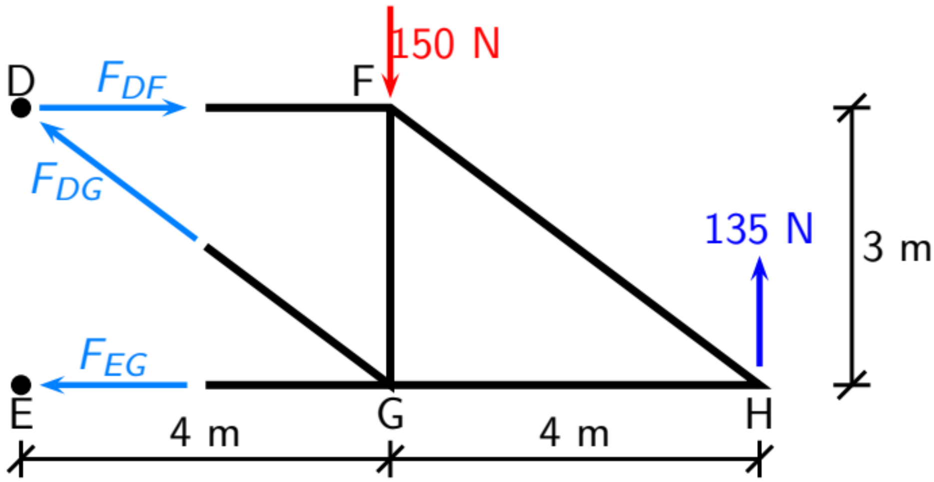


Since F_{DG} is the only unknown with a vertical component, it must point up since the 150 N force at F is bigger the 135 N force at H.



Since F_{DG} is the only unknown with a vertical component, it must point up since the 150 N force at F is bigger the 135 N force at H.

Taking moments about point G has the 135 N force at H giving a counter-clockwise moment. Therefore F_{DF} must point to the right to give a clockwise moment about point G to balance this out.



Since F_{DG} is the only unknown with a vertical component, it must point up since the 150 N force at F is bigger the 135 N force at H.

Taking moments about point G has the 135 N force at H giving a counter-clockwise moment. Therefore F_{DF} must point to the right to give a clockwise moment about point G to balance this out.

Taking moments about point D has the 150 N force acting clockwise and the 135 N force acting counter-clockwise. The 135 N force has twice the moment arm so F_{EG} must point left to give a clockwise moment to balance this out.

Solving in the order of the previous page:

$$\uparrow^{+} \sum F_y = -150N + 135N + \frac{3}{5} F_{DG} = 0$$

$$F_{DG} = \frac{5}{3}(150N - 135N) = \frac{5}{3}(15N) = 25N \text{ (tension)}$$

Solving in the order of the previous page:

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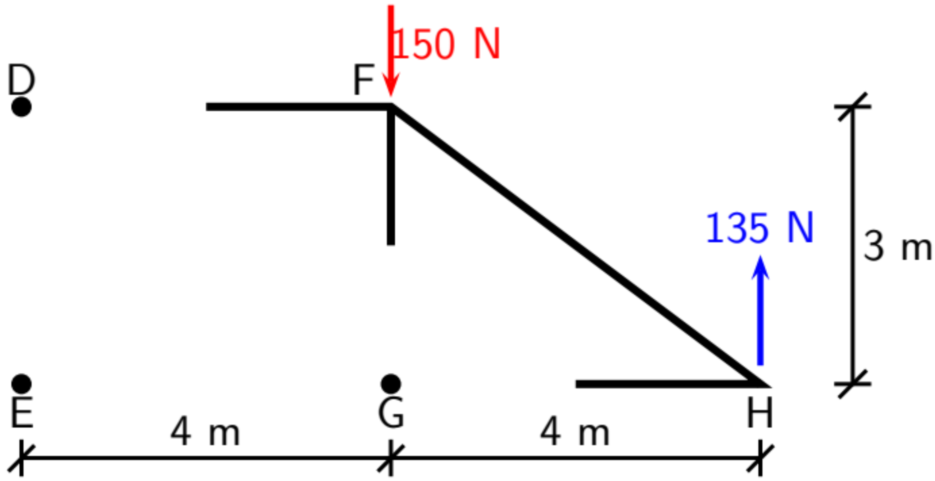
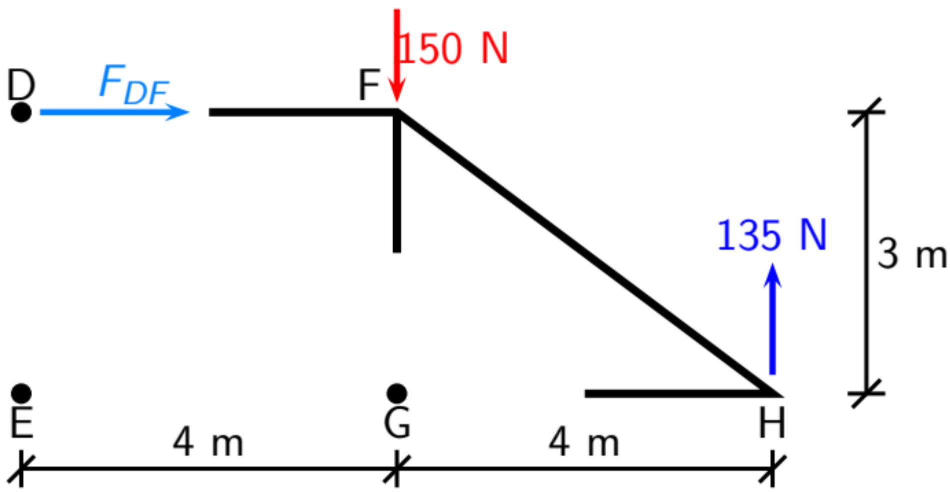
$$F_{DF} = \frac{540Nm}{3m} = 180N \text{ (compression)}$$

$$\circlearrowleft^{+} \sum M_D = -(150N)(4m) + (135N)(8m) - F_{EG}(3m) = 0$$

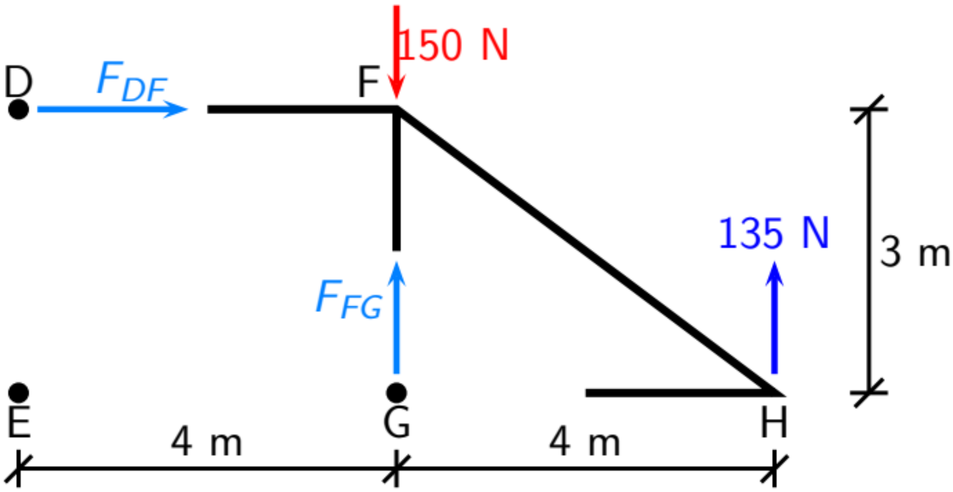
$$F_{EG} = \frac{(-600 + 1080)Nm}{3m} = \frac{480Nm}{3m} = 160N \text{ (tension)}$$

Method of Sections - Cutting through DF, FG and GH





From the previous section, we know F_{DF} points right. Taking moments about G would also give this result.



From the previous section, we know F_{DF} points right. Taking moments about G would also give this result.

Since F_{FG} is the only unknown with a vertical component, it must point up since the 150 N force at F is bigger than the 135 N force at H.

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Since F_{FG} is the only unknown with a vertical component, it must point up since the 150 N force at F is bigger than the 135 N force at H.

Taking moments about point F has the 135 N force acting counter-clockwise. The means that F_{GH} must point left to give a clockwise moment to balance this out.

Solving in the order of the previous page:

$$\uparrow^{+} \sum F_y = -150N + 135N + F_{FG} = 0$$

$$F_{FG} = 150N - 135N = 15N \text{ (compression)}$$

Solving in the order of the previous page:

$$\uparrow^{+} \sum F_y = -150N + 135N + F_{FG} = 0$$
$$F_{FG} = 150N - 135N = 15N \text{ (compression)}$$

$$\circlearrowleft^{+} \sum M_F = +(135N)(4m) - F_{GH}(3m) = 0$$
$$F_{GH} = \frac{540Nm}{3m} = 180N \text{ (tension)}$$

Method of Sections - Remaining members

- For the rest of the members, AB, DE and FH, the only sections that would cut through them amount to applying the Method of Joints.

- To solve for the force in member AB, you would cut through AB and AC. This is equivalent to applying the method of joints at joint A.
- To solve for the force in member FH, you would cut through FH and GH. This is equivalent to applying the method of joints at joint H.
- To solve for the force in member DE, you would cut through CE, DE and EG. This is equivalent to applying the method of joints at joint E.

16BTAR305 **SOLID MECHANICS** **3 0 0 3 100**

UNIT II STRESS, STRAIN AND DEFORMATION OF SOLIDS

Rigid and Deformable bodies – Strength, Stiffness and Stability – Stresses; Tensile, Compressive and Shear – Deformation of simple and compound bars under axial load – Thermal stress – Elastic constants – Strain energy ,potential energy and unit strain energy – Strain energy in uni-axial loads.

TEXT BOOKS

T [1] – R. K. Bansal (2010), "A Textbook of Strength of Materials, Laxmi Publications, New Delhi.

T [2] – R. S. Khurmi (2013), "Strength of Material", S. Chand Publications. New Delhi

REFERENCES

R [1] - James M. Gere, Barry J. Goodno (2008), "Mechanics of Materials", Prentice Hall Inc. New Jersey.

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10. Stress strain diagrams

- Bar or rod – the longitudinal direction is considerably greater than the other two, namely the dimensions of cross section.
- For the design of the m/c components we need to understand about “mechanical behavior” of the materials.
- We need to conduct experiments in laboratory to observe the mechanical behavior.
- The mathematical equations that describe the mechanical behavior is known as “constitutive equations or laws”
- Many tests to observe the mechanical behavior- tensile test is the most important and fundamental test- as we gain or get lot of information regarding mechanical behavior of metals
- Tensile test Tensile test machine, tensile test specimen, extensometer, gage length, static test-slowly varying loads, compression test.

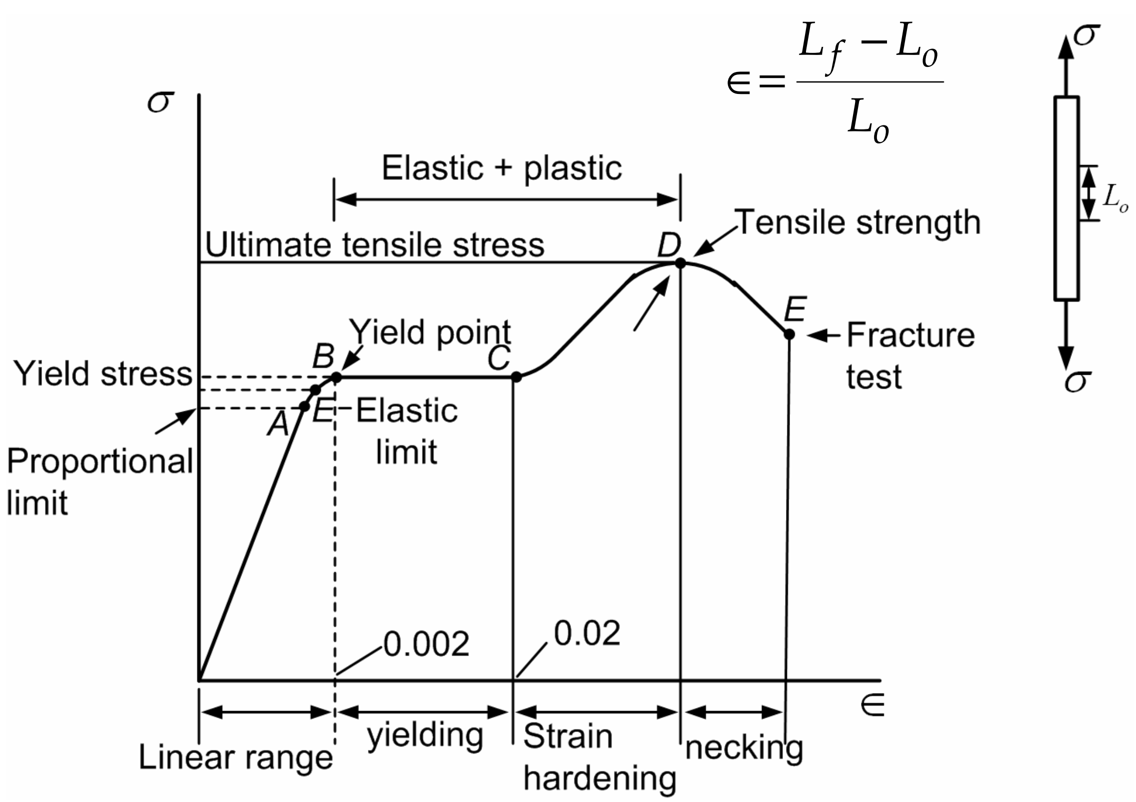
<u>Stress -strain diagrams</u>

After performing a tension or compression tests and determining the stress and strain at various magnitudes of load, we can plot a diagram of stress Vs strain.

Such is a characteristic of the particular material being tested and conveys important information regarding mechanical behavior of that metal.

We develop some ideas and basic definitions using $\sigma - \in$ curve of the mild steel.

Structural steel = mild steel = 0.2% carbon=low carbon steel



Region O-A

(1) σ and $\in$ linearly proportional.

(2) *A*- Proportional limit

σ_p - proportionality is maintained.

(3) Slope of *AO* = modulus of elasticity "*E*" – N/m²,Pa

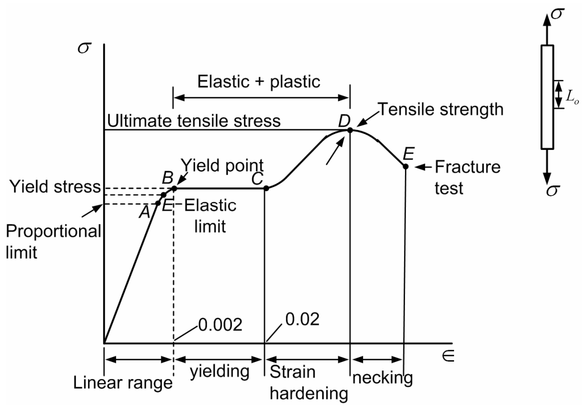
(4) Strains are infinites ional.

Region A-B

(1) Strain increases more rapidly than σ

(2) Elastic in this range

Proportionality is lost.



Region B-C

(1) The slope at point B is horizontal.

(2) At this point B, $\in$ increases without increase in further load. I.e no noticeable change in load.

(3) This phenomenon is known as yielding

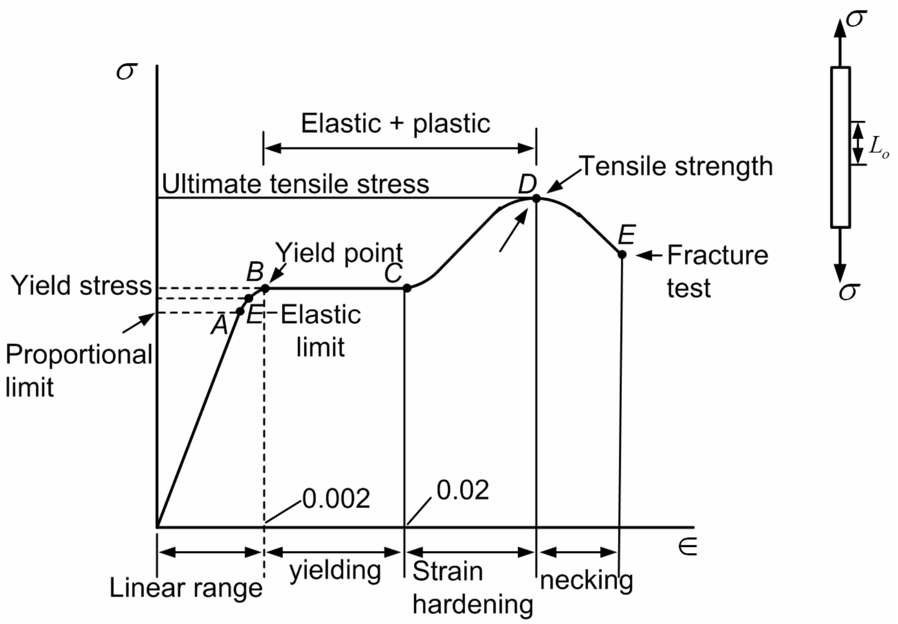
(4) The point B is said to be yield points, the corresponding stress is yield stress σ_{ys} of the steel.

(5) In region B-C material becomes "perfectly plastic i.e which means that it deforms without an increase in the applied load.

(6) Elongation of steel specimen or $\in$ in the region BC is typically 10 to 20 times the elongation that occurs in region OA.

(7) $\in_s$ below the point A are said to be small, and $\in_s$ above A are said to be large.

(8) $\in_s < \in_A$ are said to be elastic strains and $\in > \in_A$ are said to be plastic strains = large strains = deformations are permanent.



Region C-D

(1)The steel begins to "strain harden" at "C" . During strain hardening the material under goes changes in its crystalline structure, resulting in increased resistance to the deformation.

(2)Elongation of specimen in this region requires additional load,

$\therefore$ $\sigma - \in$ diagram has + ve slope C to D.

(3) The load reaches maximum value – ultimate stress.

(4)The yield stress and ultimate stress of any material is also known as yield strength and the ultimate strength .

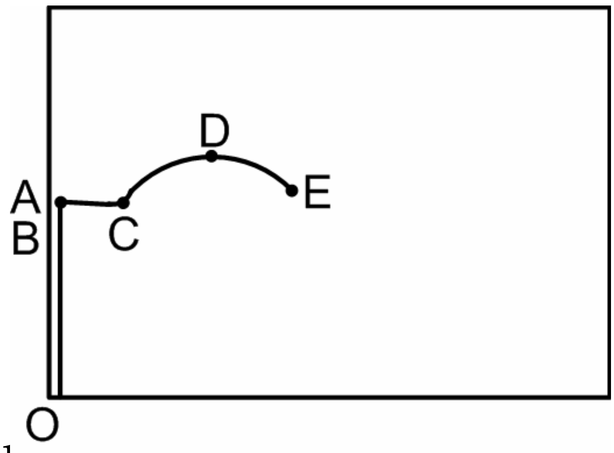
(5) σ_u is the highest stress the component can take up.

Region-DE

Further stretching of the bar is needed less force than ultimate force, and finally the component breaks into two parts at E.

Look of actual stress strain diagrams

$\in_{C\,to\,E} > \in_{B\,to\,C} > \in_{O\,to\,A}$



(1) Strains from O to A are so small in comparison to the strains from A to E that they cannot be seen.

(2) The presence of well defined yield point and subsequent large plastic strains are characteristics of mild – steel.

(3) Metals such a structural steel that undergo large permanent strains before failure are classified as ductile metals.

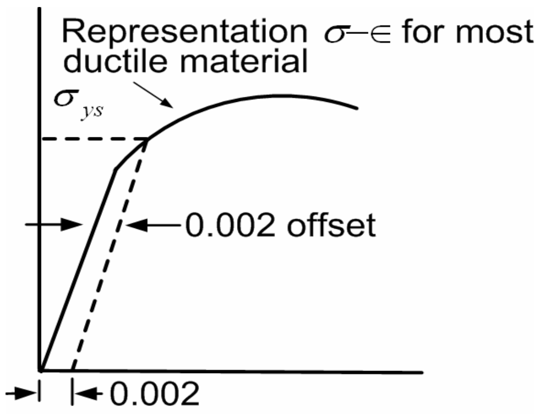
Ex. Steel, aluminum, copper, nickel, brass, bronze, magnesium, lead etc.

Aluminum alloys – Offset method

(1) They do not have clear cut yield point.

(2) They have initial straight line portion with clear proportional limit.

(3) All does not have obvious yield point, but undergoes large permanent strains after proportional limit.



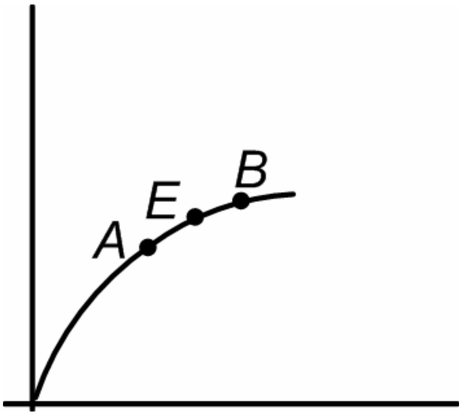
(4) Arbitrary yield stress is

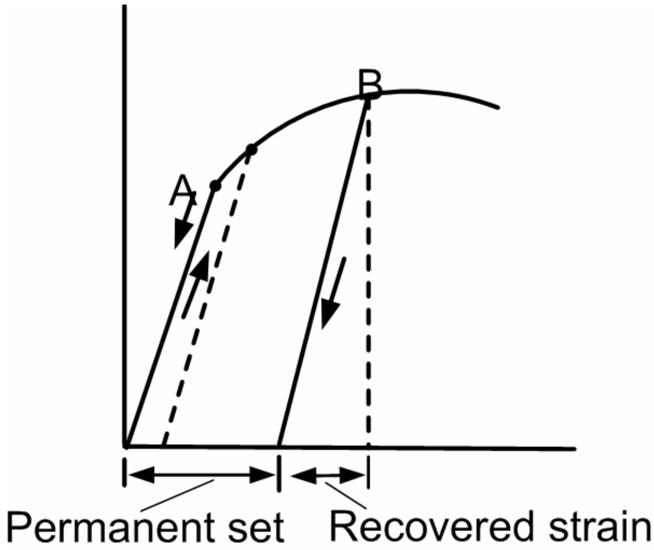
determined by off- set method.

(5) Off-set yield stress is not material property

Elasticity & Plasticity

(1) The property of a material by which it (doesn't) returns to its original dimensions during unloading is called (plasticity) elasticity and the material is said to be elastic (plastic).



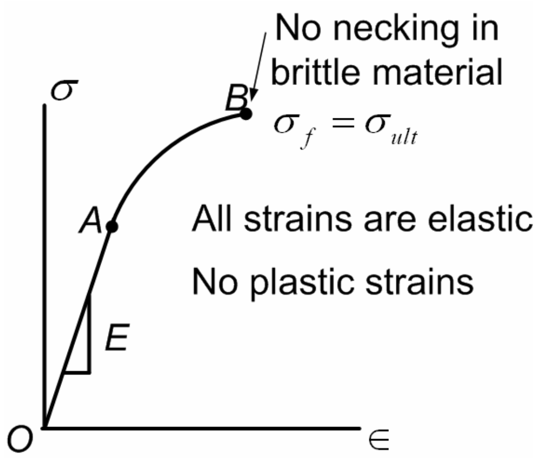


(2) For most of the metals proportional limit = elastic limit.

(3) For practical purpose proportional limit = elastic limit = yield stress

(4)All metals have some amount of straight line portion.

Brittle material in tension



(1) Materials that fail in tension at relatively low values of strain are classified or brittle materials.

(2) Brittle materials fail with only little elongation (elastic) after the proportional limit.

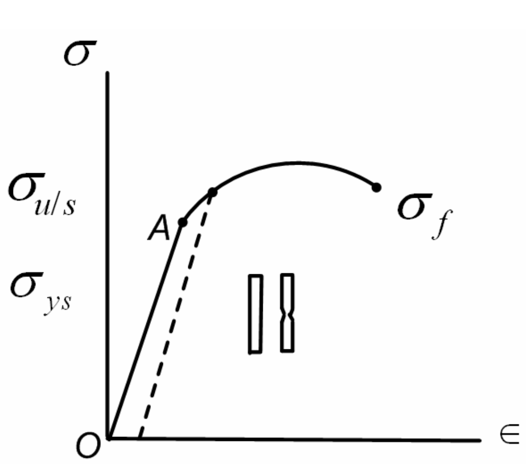
(3)Fracture stress = Ultimate stress for brittle materials

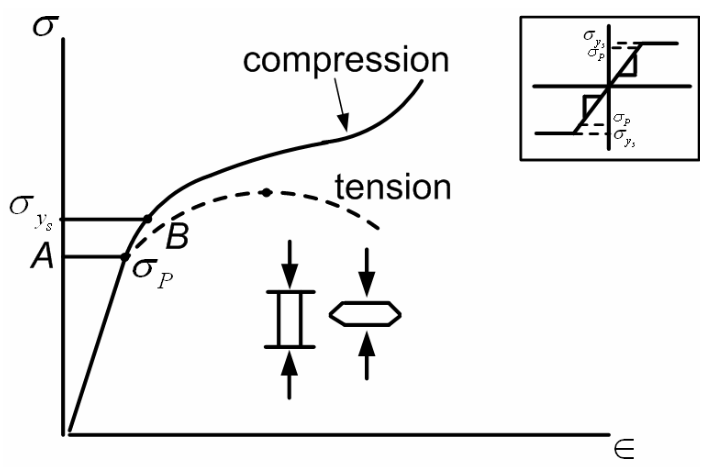
(4)Up to *B*, i.e fracture strains are elastic.

(5)No plastic deformation in case of brittle materials.

Ex. Concrete, stone, cast iron, glass, ceramics

Ductile metals under compression





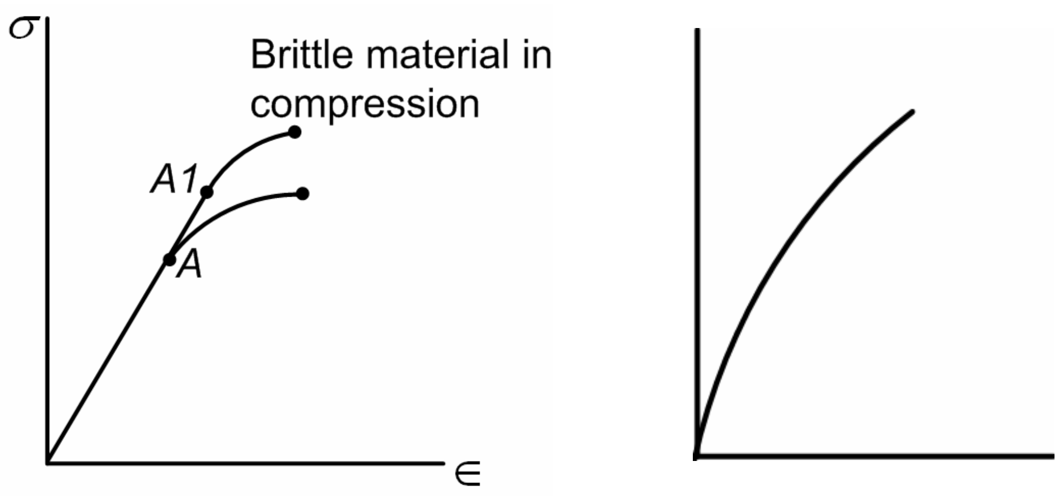
(1) $\sigma-\in$ curves in compression differ from $\sigma-\in$ in tension.

(2)For ductile materials, the proportional limit and the initial portion of the $\sigma-\in$ curve is same in tension and compression.

(3)After yielding starts the behavior is different for tension and compression.

(4)In tension after yielding – specimen elongates – necking and fractures or rupture. In compression – specimen bulges out- with increasing load the specimen is flattened out and offers greatly increased resistance.

Brittle materials in compression



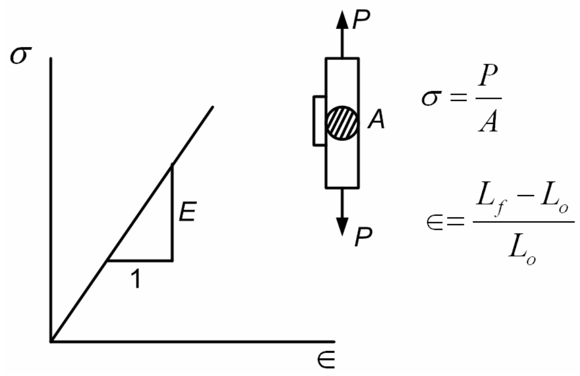
(1)Curves are similar both in tension and compression

(2)The proportional limit and ultimate stress i.e fracture stress are different.

(3)In case of compression both are greater than tension case

(4)Brittle material need not have linear portion always they can be non-linear also.

11. Generalized Hooke's Law



(1) A material behaves elastically and also exhibits a linear relationship between σ and $\in$ is said to be linearly elastic.

(2) All most all engineering materials are linearly elastic up to their corresponding proportional limit.

(3) This type of behavior is extremely important in engineering – all structures designed to operate within this region.

(4) Within this region, we know that either in tension or compression

$$\sigma = E\in$$
$$Stress\,in\,particular\,direction = strain\,in\,that\,dir.\,X\;E$$

E =Modulus of elasticity $-Pa, N/m^2$

= Young's modulus of elasticity.

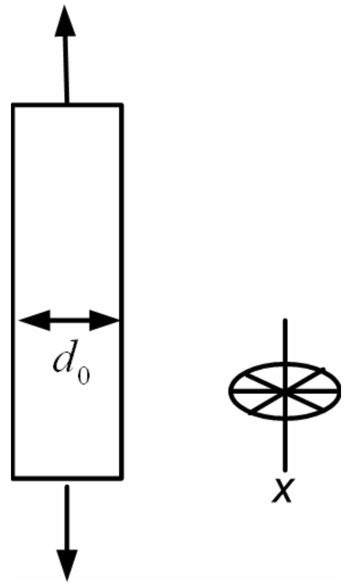
(5) $\sigma_x = E\in_x$ or $\sigma_y = E\in_y$

(6) $\sigma = E\in$ is known as Hooke's law.

(7) Hooke's law is valid up to the proportional limit or within the linear elastic zone.

Poisson's ratio

When a prismatic bar is loaded in tension the axial elongation is accompanied by lateral contraction.



Lateral contraction or lateral strain

$$\in' = \frac{d_f - d_o}{d_o}$$ this comes out to be –ve

$$Poisson's\ ratio\ = \nu\,(nu) = -\frac{lateral\,strain}{axial\,strain} = \frac{-\in'}{\in}$$

$\in'$ *is perpendicular to* $\in$

If a bar is under tension $\in$ +ve, $\in'$ -ve and $\nu = +$

If a bar is under compression $\in$ -ve, $\in'$ +ve and $\nu = +$

ν =always +ve = material constant

For most metals $\nu = 0.25\ to\ 0.35s$

Concrete $\nu = 0.1\ to\ 0.2$

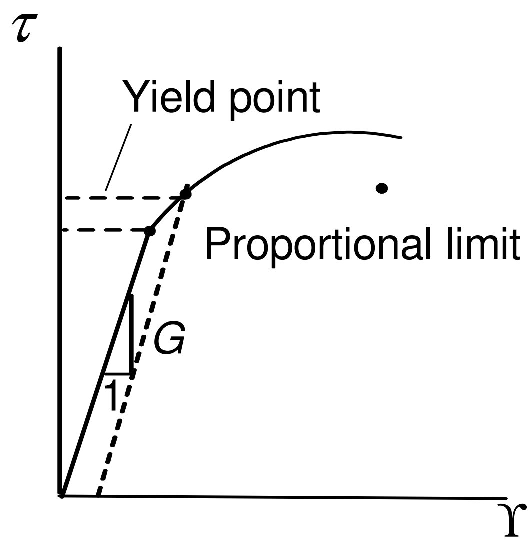
Rubber $\nu = 0.5$

ν is same for tension and compression

ν is constant within the linearly elastic range.

Hooks law in shear

(1)To plot τ, Y the test is twisting of hollow circular tubes



(2)τ, Y diagrams are (shape of them) similar in shape to tension test diagrams $(\sigma \, Vs \in)$ for the same material, although they differ in magnitude.

(3)From $\tau - Y$ diagrams also we can obtain material properties proportional limit, modulus of elasticity, yield stress and ultimate stress.

(4)Properties are usually ½ of the tension properties.

(5)For many materials, the initial part o the shear stress diagram is a st. line through the origin just in case of tension.

$\tau = GY$ - Hooke's law in shear

G =Shear modulus of elasticity or modulus of rigidity.

$= Pa \; or \; N / m^2 s$

Proportional limit
Elastic limit
Yield stress
Ultimate stress

} Material properties

$E, v, and\ G \rightarrow$ material properties – elastic constants - elastic properties.

Basic assumptions solid mechanics

Fundamental assumptions of linear theory of elasticity are:

(a) The deformable body is a continuum

(b) The body is homogeneous

(c) The body is linearly elastic

(d) The body is isotropic

(e) The body undergoes small deformations.

Continuum

Completely filling up the region of space with matter it occupies with no empty space.

Because of this assumption quantities like

$$u = u(x, y, z)$$
$$\sigma = \sigma(x, y, z)$$
$$\in = \in(x, y, z)$$

Homogeneous

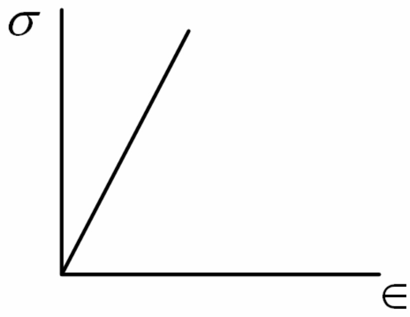
Elastic properties do not vary from point to point. For non-homogenous body

$$E = E(x, y, z)$$
$$v = v(x, y, z)$$
$$G = G(x, y, z)$$

Linearly elastic

Material follows Hooke's law

$$\sigma = E\epsilon$$
$$\tau = GY$$
$$v = Constant$$



Isotropic

Material properties are same in all directions at a point in the body

$$E = C_1 \qquad \text{for all } \theta$$
$$\nu = C_2 \qquad \text{for all } \theta$$
$$G = C_3 \qquad \text{for all } \theta$$

The meaning is that

$$\sigma_x = E\epsilon_x$$
$$\sigma_y = E\epsilon_y$$

The material that is not isotropic is anisotropic

$$E = E(\theta)$$
$$\nu = \nu(\theta)$$
$$G = G(\theta)$$

The meaning is that

$$\sigma_x = E_1\epsilon_x$$
$$\sigma_y = E_2\epsilon_y$$
$$E_1 \neq E_2$$

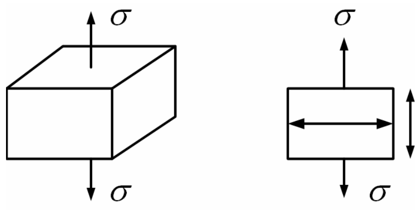
Small deformations

(a) The displacements must be small

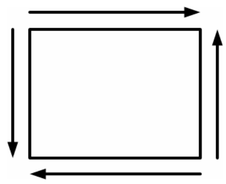
(b) The strains must also be small

Generalized Hooke's law for isotropic material

We know the following quantities from the tension and shear testing.



$$\left.\begin{aligned}\sigma &= E\in \\ v &= -\frac{\in'}{\in}\end{aligned}\right\} Tensile\,test$$



$\tau = GY$ - Shear test or torsion test.

What are the stress –strain relation for an element subjected to $3D$ state of stress. i.e what is the generalized Hooke's law.

Hooke's law – when only one stress is acting

Generalized Hooke's law – when more than one stress acting

We assume that

Material is linearly elastic, Homogeneous, Continuum, undergoing small deformations and isotropic.

For an isotropic material the following are true

(1)Normal stress can only generate normal strains.

- Normal stresses for reference xyz cannot produce Y of this reference

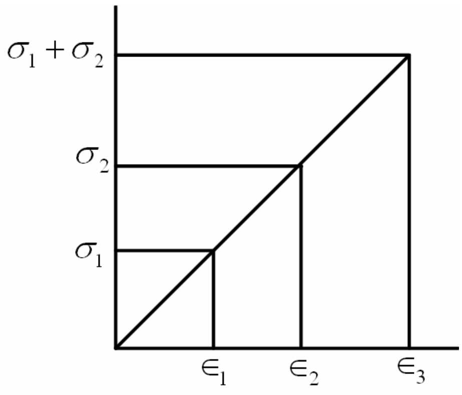
(2)A shear stress say τ_{xy} can only produce the corresponding shear strain Υ_{xy} in the same coordinate system.

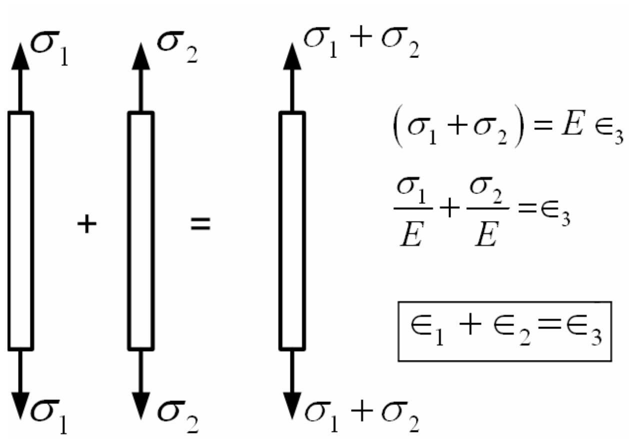
Principal of superposition:

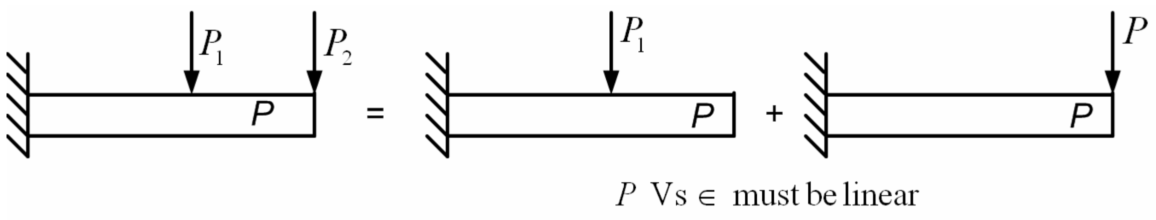
This principle states that the effect of a given combined loading on a structure can be obtained by determining separately the effects of the various loads individually and combining the results obtained, provided the following conditions are satisfied.

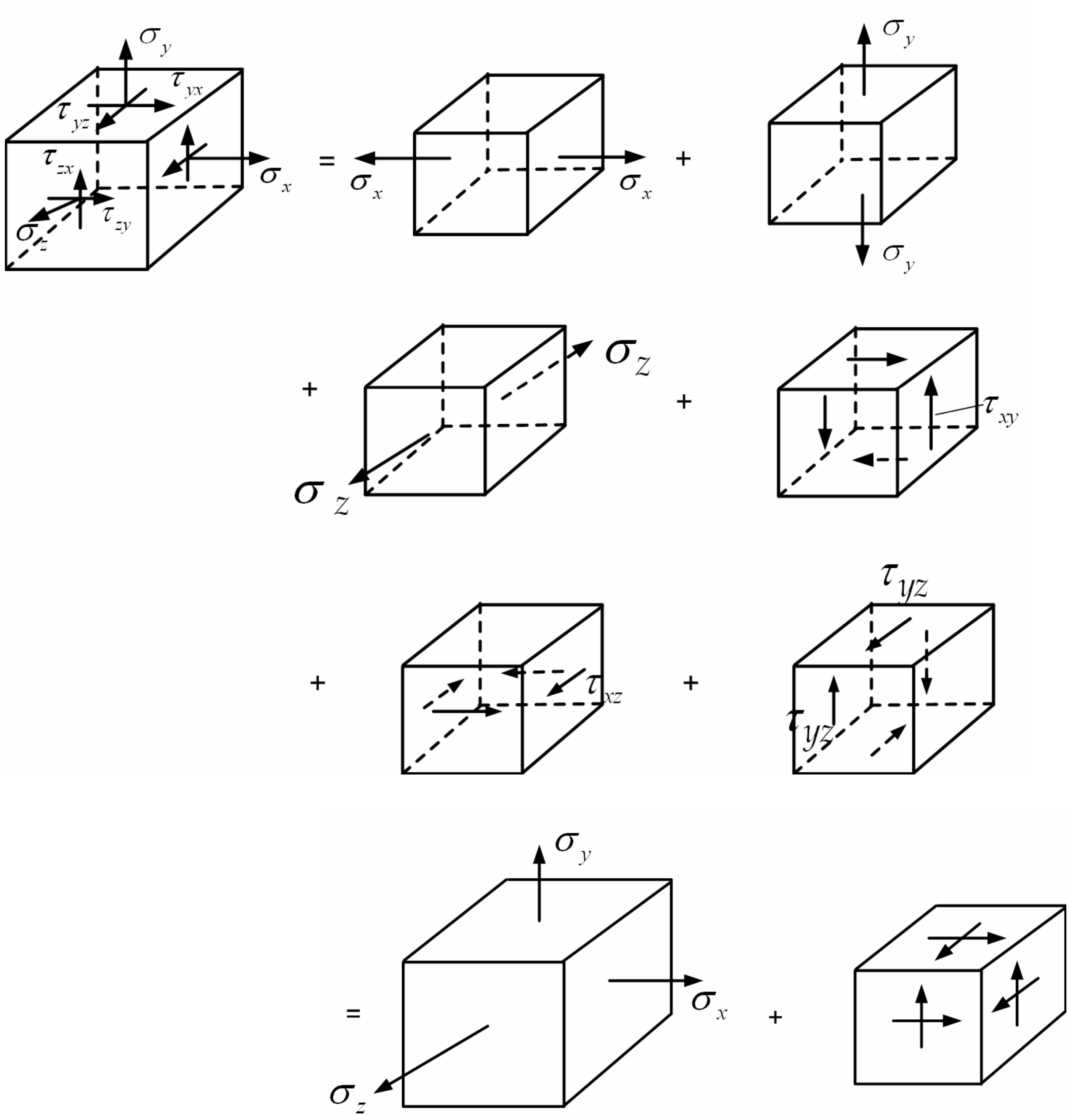
(1)Each effect is linearly related to the load that produces it.

(2)The deformations must be small.

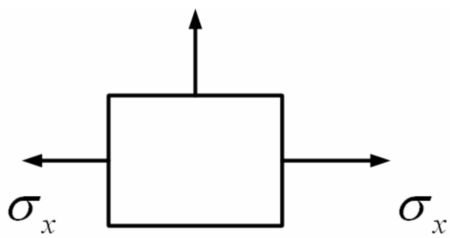








Let us know consider only σ_x is applied to the element. From Hooke's we can write



$$\in_x = \frac{\sigma_x}{E}$$

$$\in_y = -v\frac{\sigma_x}{E}$$

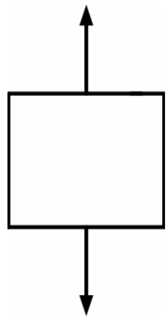
$$\in_z = -v\frac{\sigma_x}{E}$$

Only σ_y applied

$$\in_y = \frac{\sigma_y}{E}$$

$$\in_x = -v\frac{\sigma_y}{E}$$

$$\in_z = -v\frac{\sigma_y}{E}$$

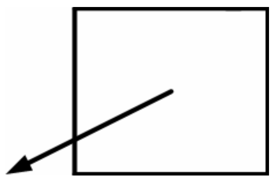


Similarly, σ_z alone is applied

$$\in_z = \frac{\sigma_z}{E}$$

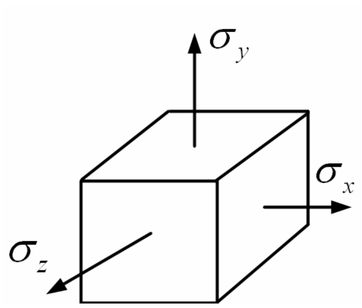
$$\in_x = -v\frac{\sigma_z}{E}$$

$$\in_y = -v\frac{\sigma_z}{E}$$



Contribution to $\in_x$ due to all three normal stresses is

$$\in_x = \frac{\sigma_x}{E} - \frac{v\sigma_y}{E} - \frac{v\sigma_3}{E}$$



Therefore

$$\in_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right]$$

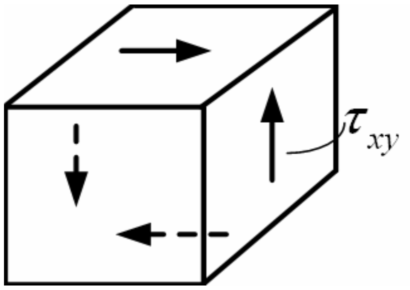
$$\in_y = \frac{1}{E}\left[\sigma_y - v\left(\sigma_x + \sigma_z\right)\right]$$

$$\in_x = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right]$$

Normal strains are not affected by shear stresses

Now let us apply only τ_{xy}

$$\Upsilon_{xy} = \frac{\tau_{xy}}{G}$$



Similarly because of τ_{yz} *and* τ_{xz}

$$\Upsilon_{yz} = \frac{\tau_{yz}}{G}$$

$$\Upsilon_{xz} = \frac{\tau_{xz}}{G}$$

Therefore, when all six components of stresses and strains are acting on an infinitesimal element or at a point then the relation between six components of stresses and strains is

$$\in_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right]$$

$$\in_y = \frac{1}{E}\left[\sigma_y - v\left(\sigma_x + \sigma_z\right)\right]$$

$$\in_x = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right]$$

$$\Upsilon_{xy} = \frac{\tau_{xy}}{G}$$

$$\Upsilon_{yz} = \frac{\tau_{yz}}{G}$$

$$\Upsilon_{xz} = \frac{\tau_{xz}}{G}$$

These six equations are known as generalized Hooke's law for isotropic materials.

Matrix representation of generalized Hooke's law for isotropic materials is therefore,

$$\begin{Bmatrix} \in_x \\ \in_y \\ \in_z \\ Y_{xy} \\ Y_{yz} \\ Y_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-v}{E} & \frac{-v}{E} & 0 & 0 & 0 \\ \frac{-v}{E} & \frac{1}{E} & \frac{-v}{E} & 0 & 0 & 0 \\ \frac{-v}{E} & \frac{-v}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}$$

<u>Stress components in terms of strains</u>

$$\in_x + \in_y + \in_z = \frac{1}{E}\left(\sigma_x + \sigma_y + \sigma_z\right) - \frac{2v}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)s$$

$$e = \left(\sigma_x + \sigma_y + \sigma_z\right)\left[\frac{1-2v}{E}\right]$$

$$\in_x + \in_y + \in_z = e$$

$$\in_x = \frac{1}{E}\left[\sigma_x - v\sigma_x - v\left(\sigma_y + \sigma_z\right)\right]$$

$$= \frac{1}{E}\left[\sigma_x - v\left(\sigma_x + \sigma_y + \sigma_z\right) + v\sigma_x\right]$$

$$= \frac{1}{E}\left[\sigma_x(1+v) - v\left(\sigma_x + \sigma_y + \sigma_z\right)\right]$$

$$= \frac{1}{E}\left[\sigma_x(1+v) - \frac{veE}{(1-2v)}\right]$$

$$= \frac{\sigma \times (1+v)}{E} - \frac{ve}{(1-2v)}$$

$$\therefore \sigma_x = \left[\in_x + \frac{ve}{1-2v}\right]\frac{E}{1+v}$$

$$\frac{E}{1+v} = \mu \text{ (mu)} \qquad \text{where} \qquad \lambda = \frac{Ev}{(1+v)(1-2v)}$$

λ, μ are Lames constants

$$\sigma_x = e\lambda + \in_x \mu$$
$$\sigma_y = e\lambda + \in_y \mu$$
$$\sigma_z = e\lambda + \in_z \mu$$
$$\tau_{xy} = Y_{xy}G = 2\mu Y_{xy}$$
$$\tau_{xy} = Y_{yz}G = 2\mu Y_{yz}$$
$$\tau_{xy} = Y_{zx}G = 2\mu Y_{zx}$$

Lame's constants have no physical meaning

Stress-strain relations for plane stress

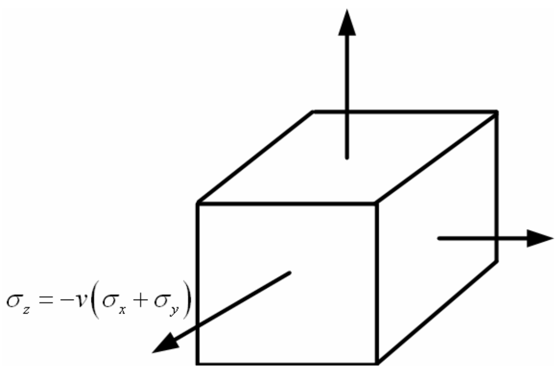
$$\sigma_x = \sigma_x(x,y)$$
$$\sigma_y = \in_y(x,y)$$
$$\tau_{xy} = \tau_{xy}(x,y)$$
$$\sigma_z = \tau_{yz} = \tau_{zx} = 0$$

$$\in_x = \frac{1}{E}\left(\sigma_x - v\sigma_y\right)$$
$$\in_y = \frac{1}{E}\left(\sigma_y - v\sigma_x\right)$$
$$\in_z = -\frac{v}{E}\left(\sigma_x + \sigma_y\right) = \frac{-v}{1-v}\left(\in_x + \in_y\right)$$
$$Y_{xy} = \frac{\tau_{xy}}{G}$$
$$Y_{yz} = Y_{xz} = 0$$

Stress- strain relations for plane strain

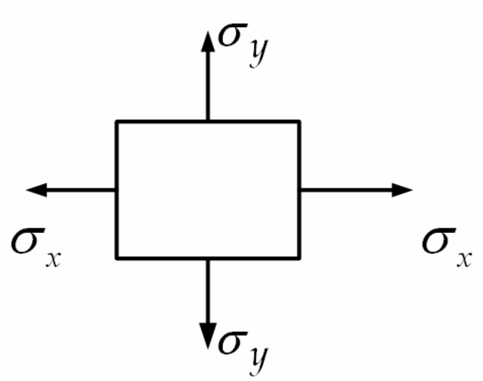
$$\in_x = \in_x(x,y)$$
$$\in_y = \in_y(x,y)$$
$$Y_{xy} = Y_{xy}(x,y)$$
$$\in_3 = Y_{xz} = Y_{yz} = 0$$
$$e = \in_x + \in_y$$

$$\sigma_x = e\lambda + \mu\in_x = \sigma_x(x,y)$$
$$\sigma_y = e\lambda + \mu\in_y = \sigma_y(x,y)$$
$$\sigma_z = -v\left(\sigma_x + \sigma_y\right) = \sigma_z(x,y)$$
$$= -v(2e\lambda + \mu e)$$
$$= -ve(2\lambda + \mu)$$
$$= -v(2\lambda + \mu)\left(\in_x + \in_y\right)$$
$$\tau_{xy} = GY_{xy}$$
$$\tau_{xz} = \tau_{yz} = 0$$



- Therefore, the stress transformation equations for plane stress can also be used for the stresses in plane strain.
- The transformation laws for plane strain can also be used for the strains in plane stress. $\in_z$ does not effect geometrical relationships used in derivation.

Example of Generalized Hooke's law



$$\in_x = \frac{1}{E}\left[\sigma_x - v\sigma_y\right]$$

$$\sigma_x = \lambda e + \mu \in_x$$

$$\in_y = \frac{1}{E}\left[\sigma_y - v\sigma_x\right]$$

$$\sigma_y = \lambda e + \mu \in_y$$

$$\sigma_x = 2\sigma_y$$

$$\sigma_x = -\sigma_y$$

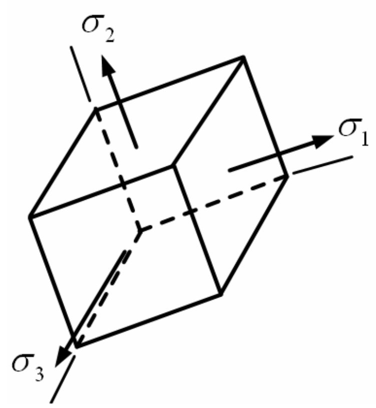
$$\in_x = \frac{1}{E}\left[\sigma_x - v\lambda e - v\mu \in_y\right]$$

$$\in_x = \frac{1}{E}\left[\sigma_x + v\sigma_y\right]$$

$$= \sigma_x\left(\frac{1+v}{E}\right)$$

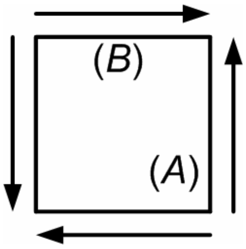
Principal stress and strain directions of isotropic materials

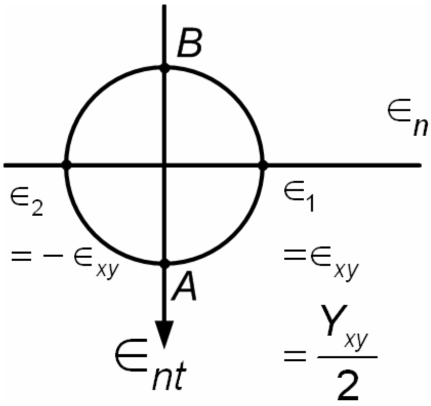
τ is zero along those planes, therefore γ is also zero along these planes i.e normal strains of the element are principal strains. For isotropic materials - the principal strains and principal stresses occurs in the same direction.

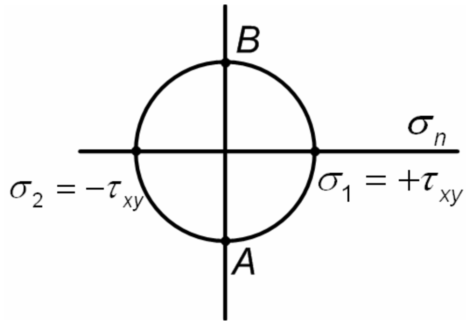


12. Volumetric strain and Bulk modulus

Relation between E, v and G







$$\sigma_1 = \tau_{xy} \qquad \in_1 = \frac{1}{E}(\sigma_1 - v\sigma_2)$$

$$\sigma_2 = -\tau_{xy} \qquad \in_2 = \frac{1}{E}(\sigma_2 - v\sigma_1)$$

$$\in_1 = \frac{1}{E}\left(\tau_{xy} + v\tau_{xy}\right) = \frac{\tau_{xy}(1+v)}{E}$$

$$\in_2 = \frac{-\tau_{xy}(1+v)}{E}$$

$$\in_1 = \in_{xy} = \frac{Y_{xy}}{2} = \frac{\tau_{xy}}{2G}$$

$$\in_2 = \frac{-\tau_{xy}}{2G}$$

$$\frac{\tau_{xy}(1+v)}{E} = \frac{\tau_{xy}}{2G} \qquad \Rightarrow$$

$$G = \frac{E}{2(1+v)}$$

Only two elastic constants are independent.

Volumetric strain-dilatation

Consider a stress element size dx, dy, dz

$$dv = dxdydz$$

After deformations

$$dx^* = (1+\in_x)dx$$
$$dy^* = (1+\in_y)dy$$
$$dz^* = (1+\in_z)dz$$

In addition to the changes of length of the sides, the element also distorts so that right angles no longer remain sight angles. For simplicity consider only Y_{xy}.

The volume dv^* of the deformed element is then given by

$$dv^* = Area(OA^*B^*C^*)\times dz^*$$

$$Area(OA^*B^*C^*) = dx^*(dy^* Cos Y_{xy})$$
$$= dx^* dy^* Cos Y_{xy}$$

$$\therefore \quad dv^* = dx^* dy^* dz^* Cos Y_{xy}$$

For small Y_{xy} $Cos Y_{xy} \approx 1$

$$\therefore \quad dv^* = dx^* dy^* dz^* - Volume\,change\,doesn't\,depend\,on\,Y$$
$$= (1+\in_x)(1+\in_y)(1+\in_z)dxdydz$$

dropping all second order infinitesimal terms

$$dv^{*} = \left(1 + \in_x + \in_y + \in_z\right) dxdydz$$

Now, analogous to normal strain, we define the measure of volumetric strain as

$$Volumetric\ strain = \frac{final\ volume\text{-}initial\ volume}{initialvolume}$$

$$e = \frac{dv^{*} - dv}{dv}$$

$$e = \in_x + \in_y + \in_z$$

- e =volumetric strain = dilatation. This expression is valid only for infinitesimal strains and rotations
- $e = \in_x + \in_y + \in_z = J_1 = first\ in\ variance\ of\ strain.$
- Volumetric strain is scalar quantity and does not depend on orientation of coordinate system.
- Dilatation is zero for state of pure shear.

Bulk modulus of elasticity

$$\in_x + \in_y + \in_z = \frac{(1-2v)}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\text{Mean stress} = \bar{\sigma} = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$e = \frac{3(1-2v)}{E}\bar{\sigma}$$

$$\bar{\sigma} = Ke$$

Where $K = \dfrac{E}{3(1-2v)}$ bulk modulus of elasticity.

- Bulk modulus is widely used in fluid mechanics.
- From physical reasoning $E > 0, G > 0, K \geq 0$

Steel : E = 200 Gpa

v = 0.3

Al : E = 70 Gpa

v = 0.33

Copper: E = 100 Gpa

v = 0.35

$$G = \frac{E}{2(1+v)} SinG \quad E and \quad G > 0$$

$$(1+v) > 0 \rightarrow \quad v > -1$$

Similarly $SinG \quad E > 0 \;\; \& \;\; K \geq 0$

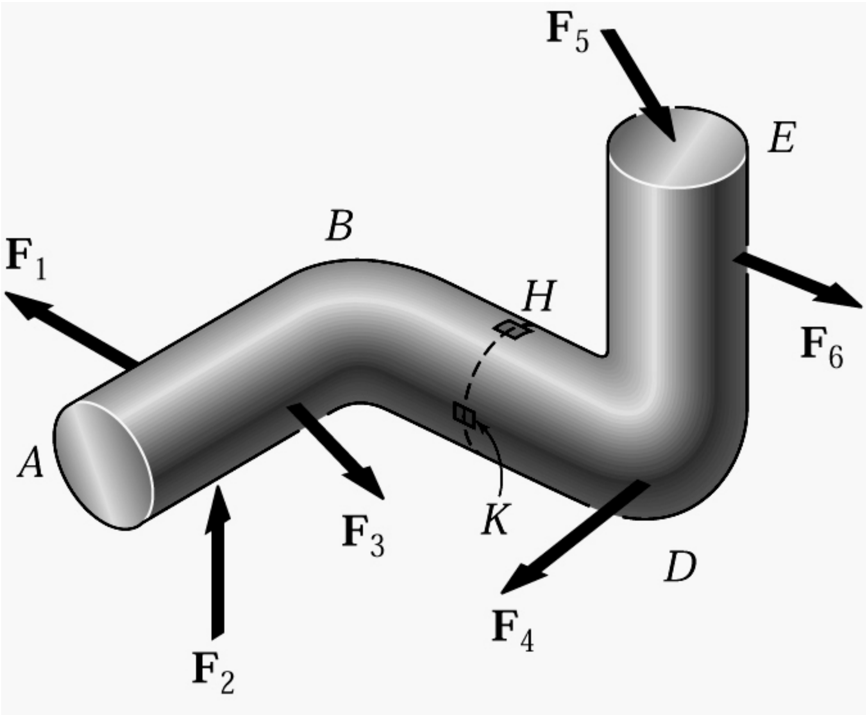
$$K = \frac{E}{3(1-2v)} \rightarrow 1-2v \geq 0 \rightarrow v \leq 0.5$$

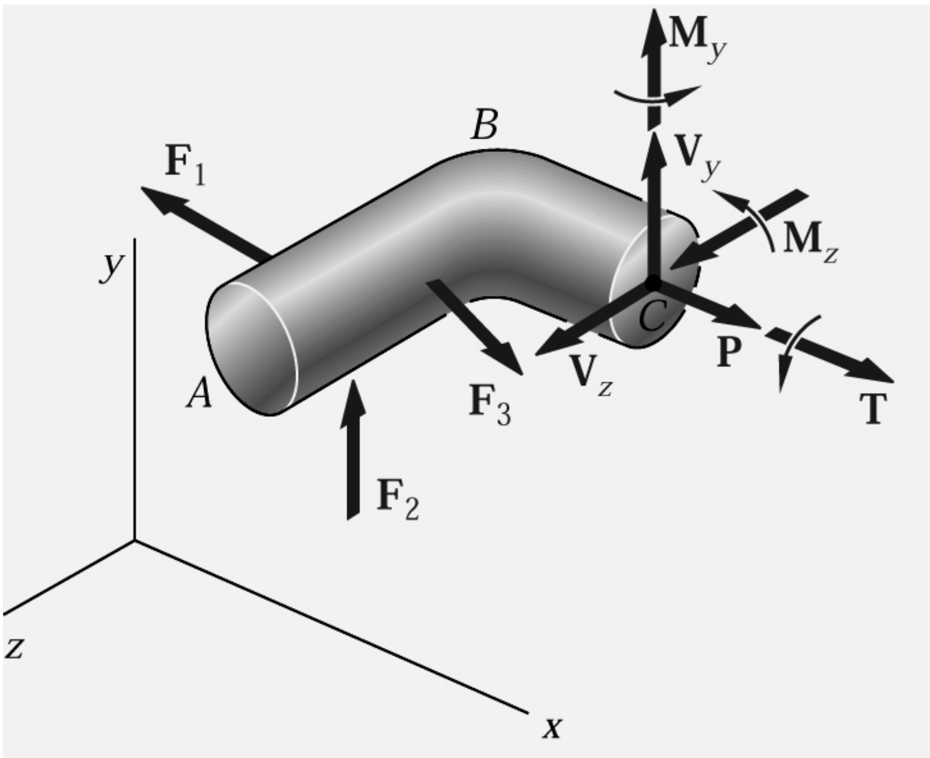
$\therefore$ Theoretical bounds on v are

$$-1 < v \leq 0.5$$

as $v \rightarrow 0.5 \quad K \rightarrow \alpha$ and $c \rightarrow 0$ material is incompressible.

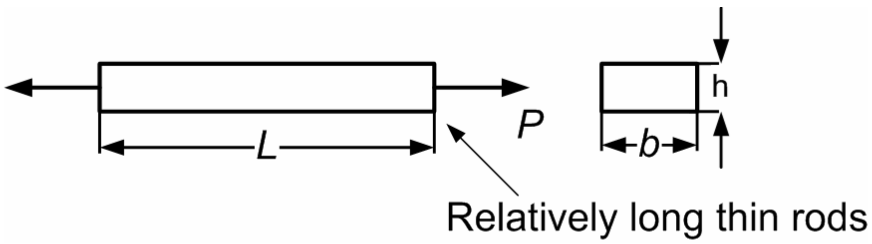
13. Axially loaded members



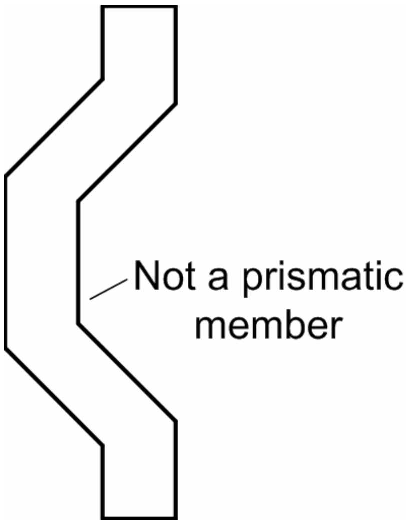


Geometry, locating and material properties

- A prismatic bar is subjected to axial loading
- A prismatic bar is a st. structural member having constant cross-section through out it length.
- Bar or rod →length of the member is ≫ cross sectional dimensions.

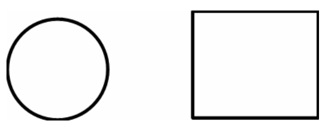


$$L \gg h$$
$$L \gg b$$

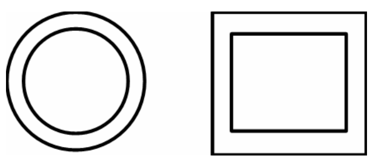


Axial force is a load directed along the axis of the member – can create tension or compression in the member.

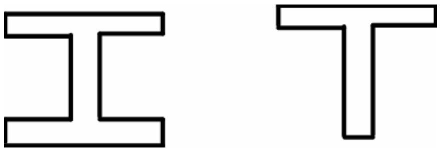
Typical cross sections of the members



- Solid Sections



- Hollow Sections

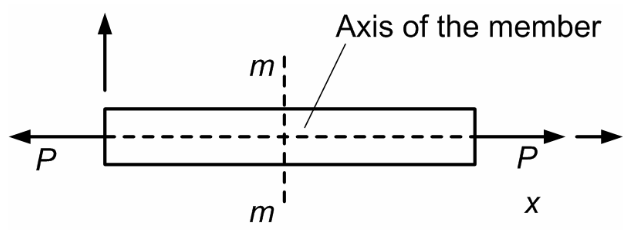


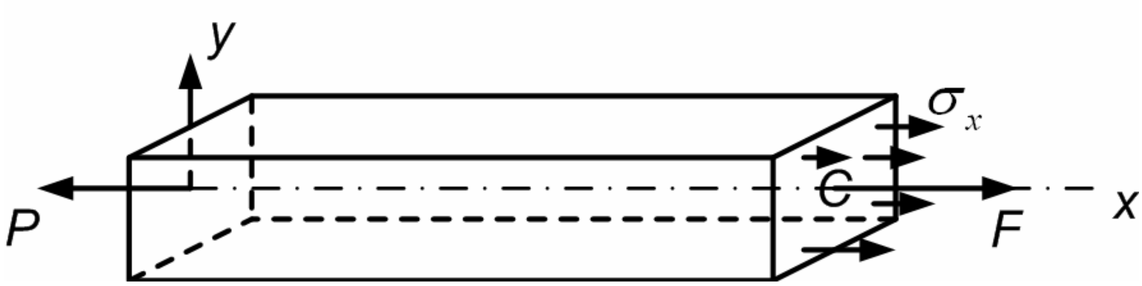
- Other sections

Material properties: The member is homogenous linearly elastic and isotropic material.

<u>Stresses, strains and deformations</u>

Consider a prismatic bar of constant cross-sectional area A and length L, with material properties A & v. Let the rod be subjected to an axial force "p", which acts along x-axis.





$$F = P$$
$$M_x = M_y = M_z = 0$$
$$V_y = V_z = 0$$

The right of the section m-m exerts elementary forces or stresses on to the left of the section to maintain the equilibrium. Sum of all these elementary forces must be equal to the resultant F.

$$\int_A \sigma_x dA = F$$

$$M_y = \int \sigma_x z dA = 0$$

$$M_z = -\int \sigma_x y dA = 0$$

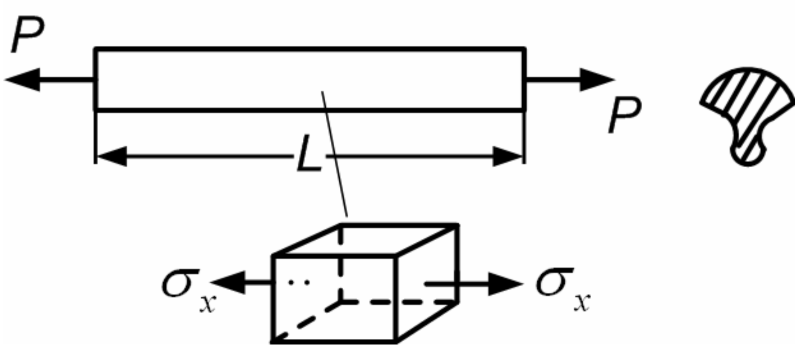
Above equation must be satisfied at every cross-section, however, it does not tell how σ_x is distributed in the cross-section.

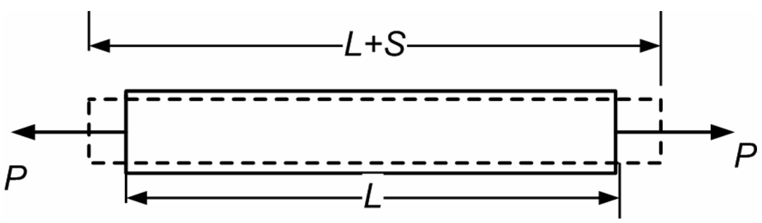
The distribution cannot determine by the methods of static or equations of equilibrium- statically indeterminate

To know about the distribution of σ_x in any given section, it is necessary to consider the deformations resulting from the application of loads.

Since the body needs to develop only σ_x component in order to maintain equilibrium, therefore the state of stress at any point of prismatic rod is

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



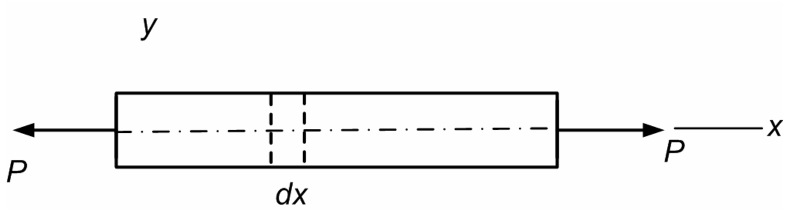


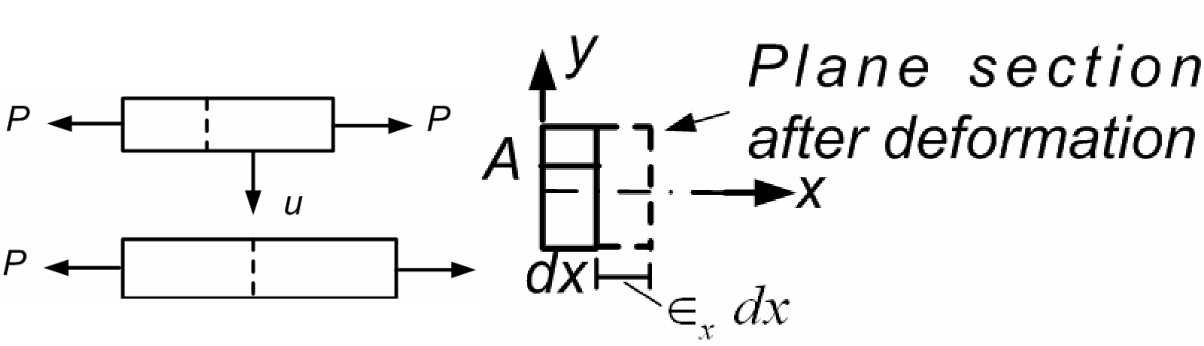
We make the following assumptions on deformation based on experimental evidence

(1)The axis of the bar remains straight after deformation

(2)All plane cross-sections remain plane and perpendicular to the axis of the bar

Key kinematical assumptions





- As a result of the above kinematic assumptions all points in a given y-z plane have the same displacements in the x-direction.
- Any line segment AB undergoes same strain $\in_x$ therefore $\in_x$ cannot be a function of y or z, but at most is a function of x- only.

In the present case situation is same at all cross-sections of the prismatic bar, therefore

$$\in_x = Constant$$

at all points of the body i.e $\in_x$ is also no a function of x.

Since we are studying a homogenous, linearly elastic and isotropic prismatic bar

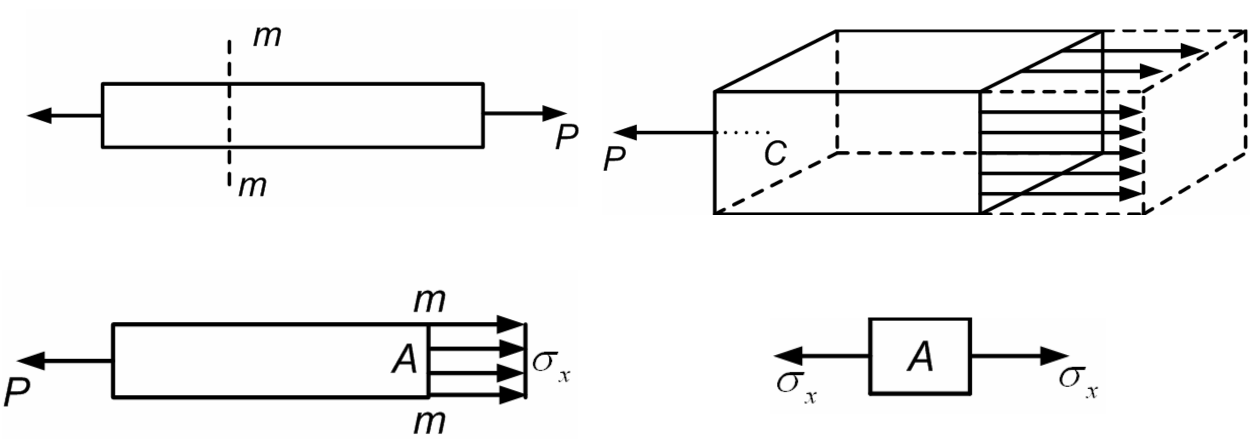
$$\in_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y - \sigma_z\right)\right] \rightarrow \qquad \in_x = \frac{\sigma_x}{E}$$

$$\in_y = \frac{1}{E}\left[\sigma_y - v\left(\sigma_x - \sigma_z\right)\right] \rightarrow \qquad \in_y = -\frac{V}{E}\sigma_x$$

$$\in_z = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x - \sigma_y\right)\right] \rightarrow \qquad \in_z = -\frac{V}{E}\sigma_x$$

In the present case, $\in_x$ is independent of y and z coordinates, therefore σ_x is also independent of y and z coordinates i.e

σ_x is uniformly distributed in a cross-section



Moreover $\sigma_x = E\in_x = Constant$ throughout the bar.

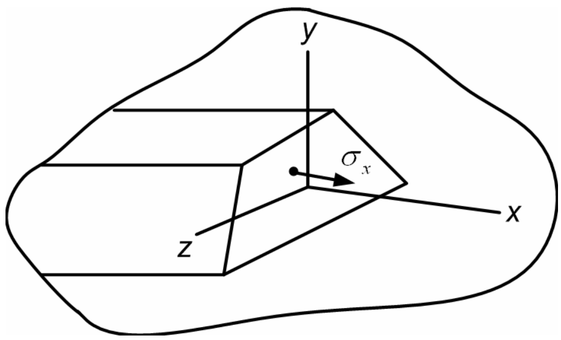
We know that internal resultant force

$$F = \int_A \sigma_x dA$$

Since σ_x is a independent of y & z

$$F = \sigma \int_A da = \sigma A$$

$$\therefore \quad \sigma = \frac{F}{A} = \frac{P}{A}$$

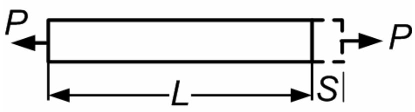


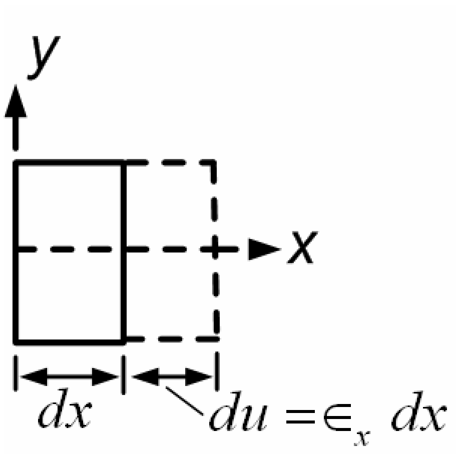
$$M_y = \int_A \sigma_x . z dA = 0 \Rightarrow \int_A z dA$$

$$M_z = -\int_A \sigma_x . y dA = 0 \Rightarrow \int_A y dA = 0 \qquad (1)$$

Eq. (1) indicates that moment are taken about the centroid of the cross-section.

Elongation or Contraction





$$\in_x = \frac{\sigma_x}{E} = \frac{P}{AE}$$

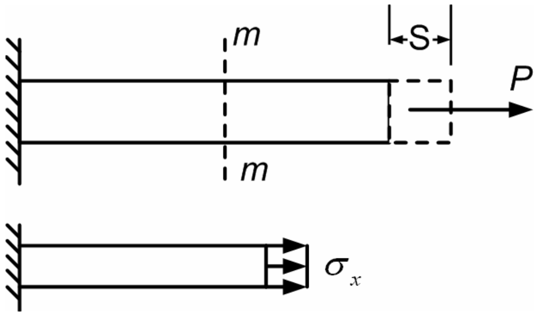
Total elongation of the rod

$$u(L) - u(0) = \delta = \int_0^L \in_x da = \int_0^L \frac{P}{AE} dx = \frac{PL}{AE}$$

$$\sigma_x = \frac{P}{A}$$

$$\delta = \frac{PL}{AE}$$

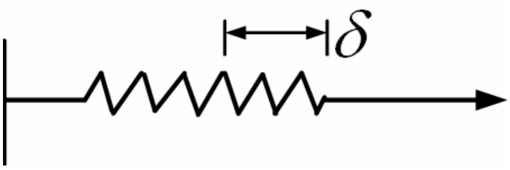
$$AE = Axial\, rigidity$$



If A,E and P are functions of x then

$$\delta = \int_0^L \frac{P(x)}{A(x)E(x)}dx$$

Stiffness and flexibility



$$P = kS$$

$$S = fP$$

$$k = \frac{1}{f}$$

$$k = \frac{AE}{L} \qquad f = \frac{L}{AE}$$

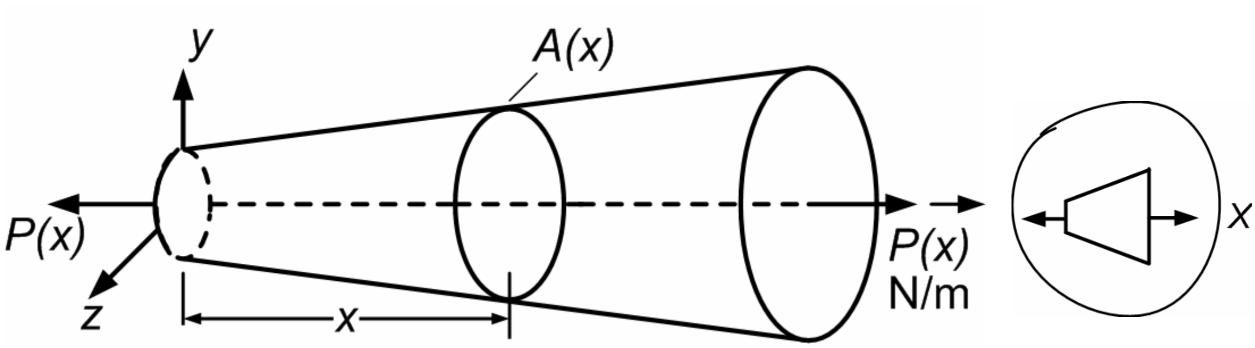
These are useful in computer analysis of structural members.

Extension of results: Non-uniform bars (non-prismatic)

For a prismatic bar

$$\sigma_x = \frac{P}{A} \;\; \& \;\; \delta = \frac{PL}{AE}$$

This is exact solution for prismatic bar.



$$\sigma_x = \frac{P(x)}{A(x)} = \frac{F(x)}{A(x)}$$

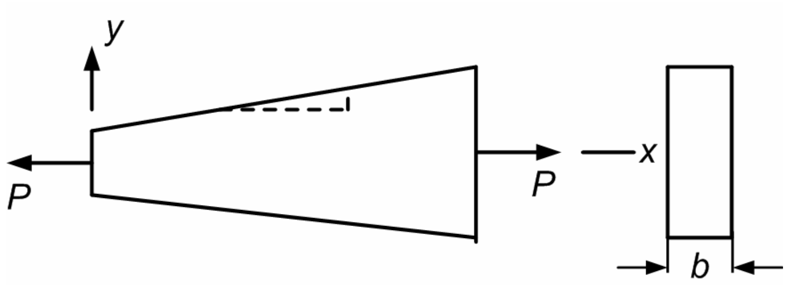
$$S = \int_0^L \frac{P(x)}{A(x)E(x)} dx$$

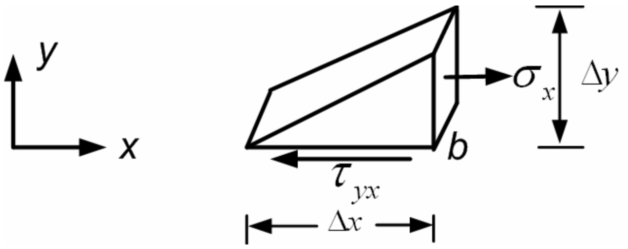
Approximate expression

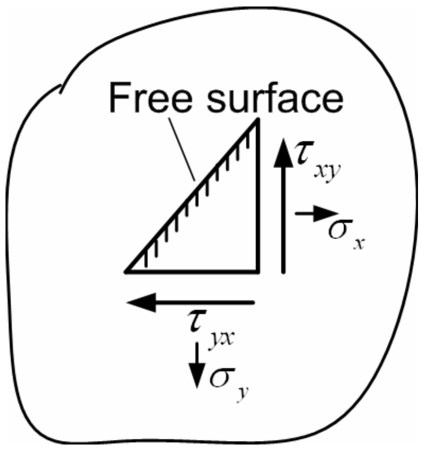
The above formula becomes a good approximation for uniformly varying cross-sectional area $A(x)$ member.

Above formula is quite satisfactory if the angle of taper is small

Plane sections remain plane and perpendicular to the x- axis is no longer valid for the case of non-prismatic rods.





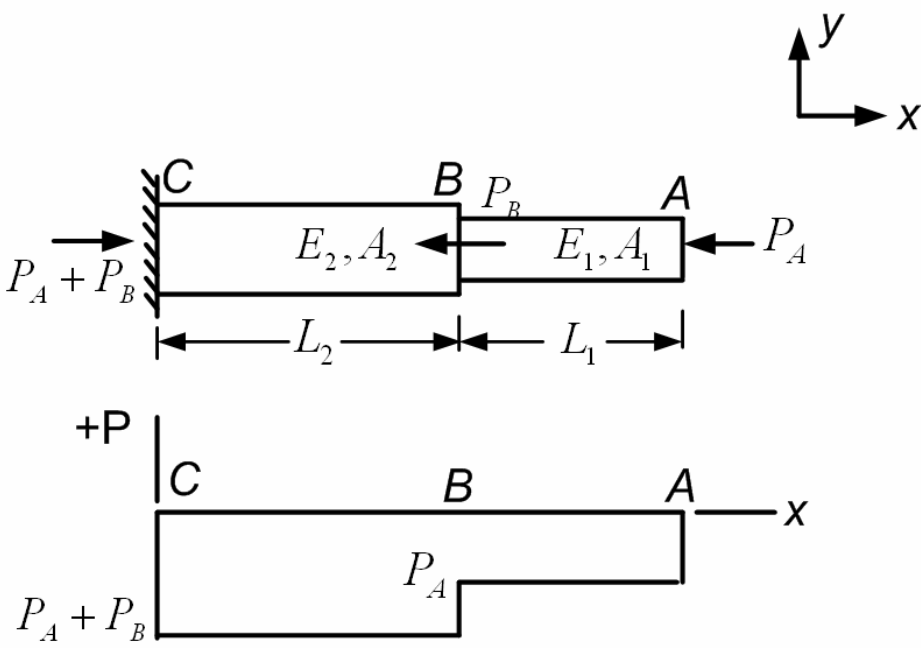


$$\Sigma F_x = 0 \Rightarrow \sigma_x (b\Delta y) - \tau_{yx} (b\Delta x) = 0$$

$$\tau_{xy} = \tau_{yx} = \sigma_x (x).\frac{\Delta y}{\Delta x} s$$

Taking $\Delta x \to 0$, we note that $\tau_{yx} \to 0$ only if $\frac{\Delta y}{\Delta x} \to 0$ i.e at the slope of the upper surface of the rod tends to zero.

Case2



$$\delta_{BC} = \frac{PL}{AE} = \frac{-(P_A + P_B)L_2}{A_2E_2}$$

$$\delta_{AB} = \frac{PL}{AE} = \frac{-P_AL_1}{A_1E_1}$$

$$\sigma_{BC} = -\frac{(P_A + P_B)}{A_2}$$

$$\sigma_{AB} = -P_A / A_1$$

$$\delta_{CA} = S_{BC} + S_{AB}$$

This method can be used when a bar consists of several prismatic segments each having different material, each having different axial forces, different dimensions and different materials. The change in length may be obtained from the equation

$$\delta = \sum_{i=1}^{n} \frac{P_iL_i}{A_iE_i} \quad and \quad \sigma_i = \frac{P_i}{A_i}$$

Statically indeterminate problems

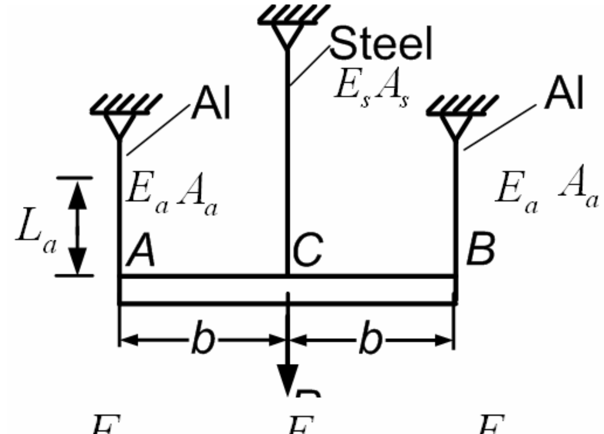
Equilibrium

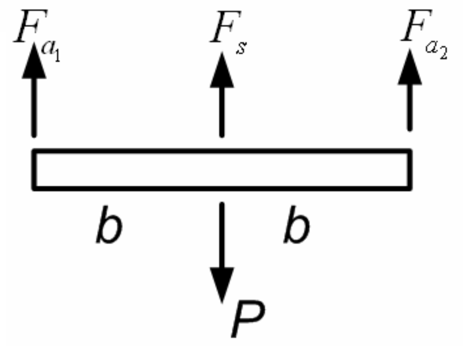
$$\left[\Sigma F_y = 0\right]$$

$$F_{a1} + F_{a2} + F_s - P = 0$$

$$[\Sigma M_C = 0]$$

$$bF_{a1} - bF_{a2} = 0$$



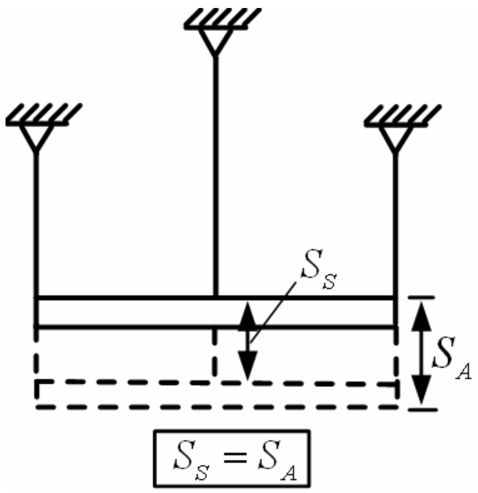


$$F_{a1} = F_{a2}$$

$$2F_a + F_s = P \qquad (1)$$

For statically indeterminate problems we must consider the deformation of the entire system to obtain "compatibility equation"

The rigid plate must be horizontal after deformation



$$\delta_s = \delta_A \Rightarrow \text{ geometric compatibility equation}$$

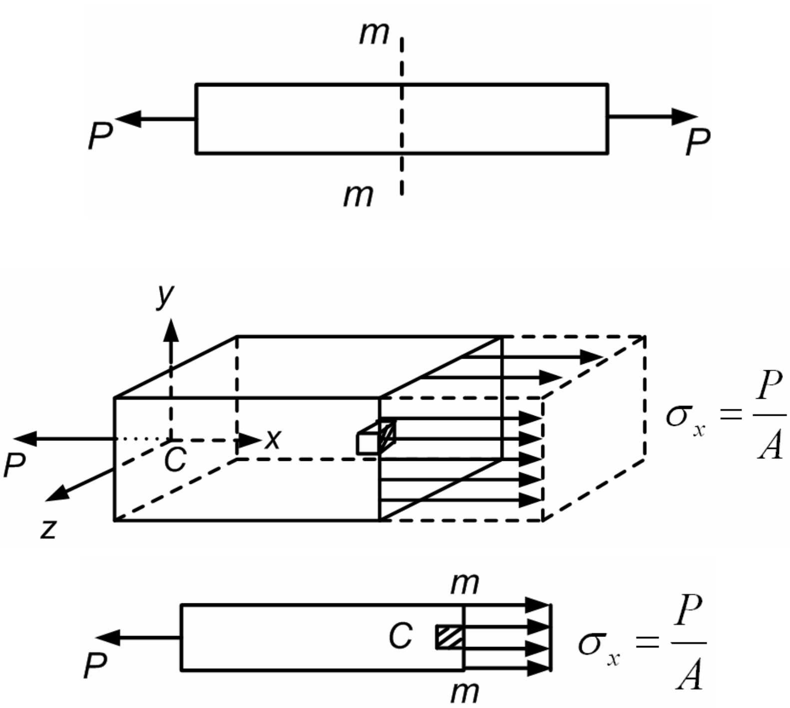
$$\delta_s = \frac{F_s L_s}{A_s E_s} \qquad and \qquad \delta_A = \frac{F_A L_A}{E_A A_A}$$

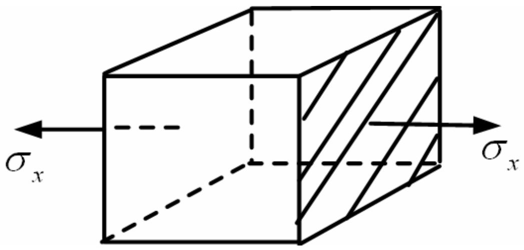
Then using the geometry compatibility

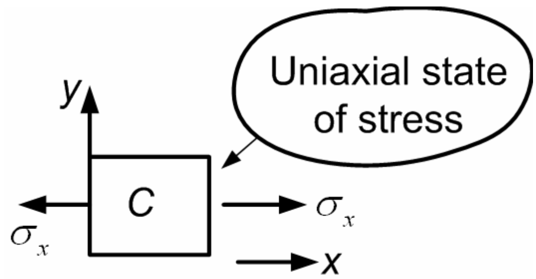
$$\delta_s = \delta_A \Rightarrow \frac{F_A L_A}{E_A A_A} = \frac{F_s L_{As}}{E_s A_s} \qquad (2)$$

By solving (1) & (2) we can obtain internal forces F_s & F_A

Stresses in axially loaded members





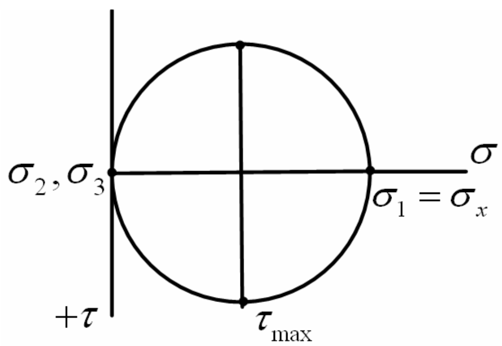


Uniaxial state stress is a special case of plane stress

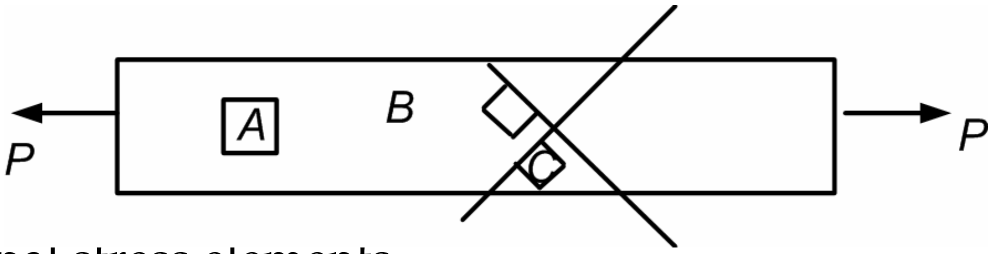
$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_x & 0 \\ 0 & 0 \end{bmatrix}$$

$$\sigma_1 = \sigma_x$$

$$\tau_{max} = \left|\frac{\sigma_1}{2}\right| = \left|\frac{\sigma_x}{2}\right|$$

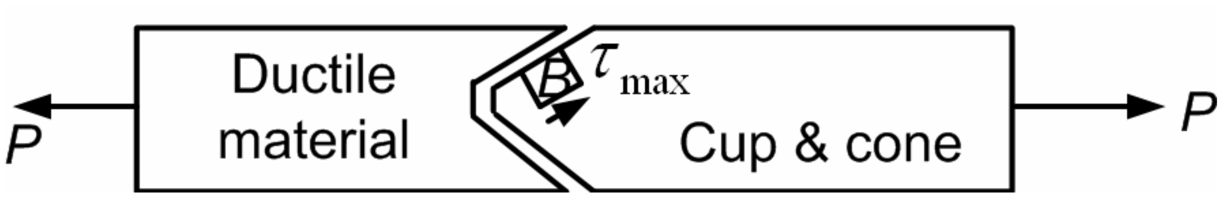


Occurs at 45° to $x-y$ or $x-z$ planes.

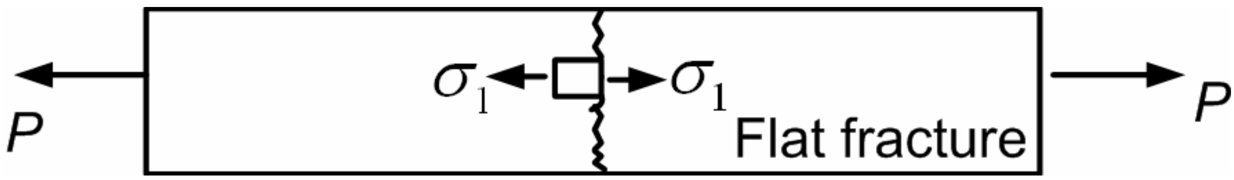


A –Principal stress elements

B,C –maximum shear stress elements.



Ductile material are weak in shear. They fail along τ_{max} planes.



Brittle materials weak in normal tensile stresses. They fail along σ_1 planes.

Limitations of analysis

$$\sigma_x = \frac{P}{A} \quad \& \quad S = \frac{PL}{AE}$$

(1)They are exact for long prismatic bars of any cross-section, when axial force is applied at the centroid of the end cross-sections.

(2)They should not be employed (especially $\sigma_x = \frac{P}{A}$) at concentrated loads and in the regions of geometric discontinuity.

(3)They provide good approximation if the taper is small.

(4)Above equations should not be applied for the case of relatively short rods.

(5)They are exact for relatively short members under compressive loading.

References:

Solid Mechanics – INTERNET SOURCES & E-BOOKS

16BTAR305 **SOLID MECHANICS** **3 0 0 3 100**

UNIT III BEAMS - LOADS AND STRESSES

Types of beams: Supports and Loads – Shear force and Bending Moment in beams – Cantilever, Simply supported and Overhanging beams – Stresses in beams – Theory of simple bending – Stress variation along the length and in the beam section – Effect of shape of beam section on stress induced – Shear stresses in beams – Shear flow.

TEXT BOOKS

T [1] – R. K. Bansal (2010), "A Textbook of Strength of Materials, Laxmi Publications, New Delhi.

T [2] – R. S. Khurmi (2013), "Strength of Material", S. Chand Publications. New Delhi

REFERENCES

R [3] - Bedi D.S (1984), "Strength of Materials", S Chand and Co. Ltd., New Delhi

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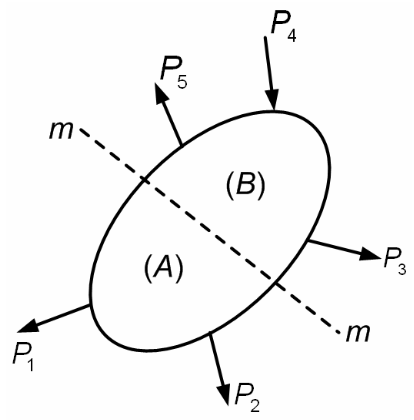
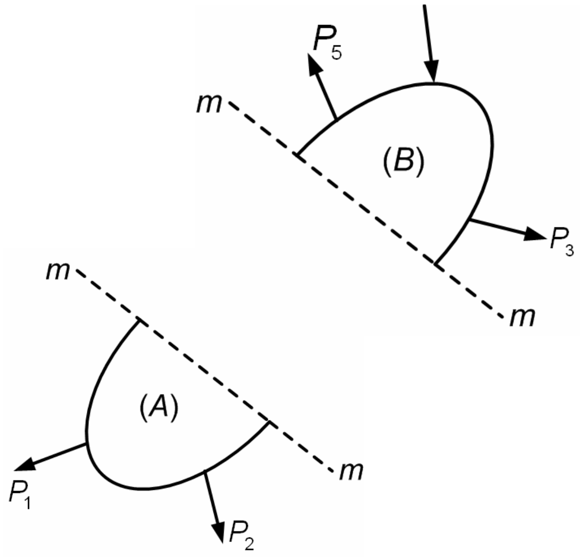
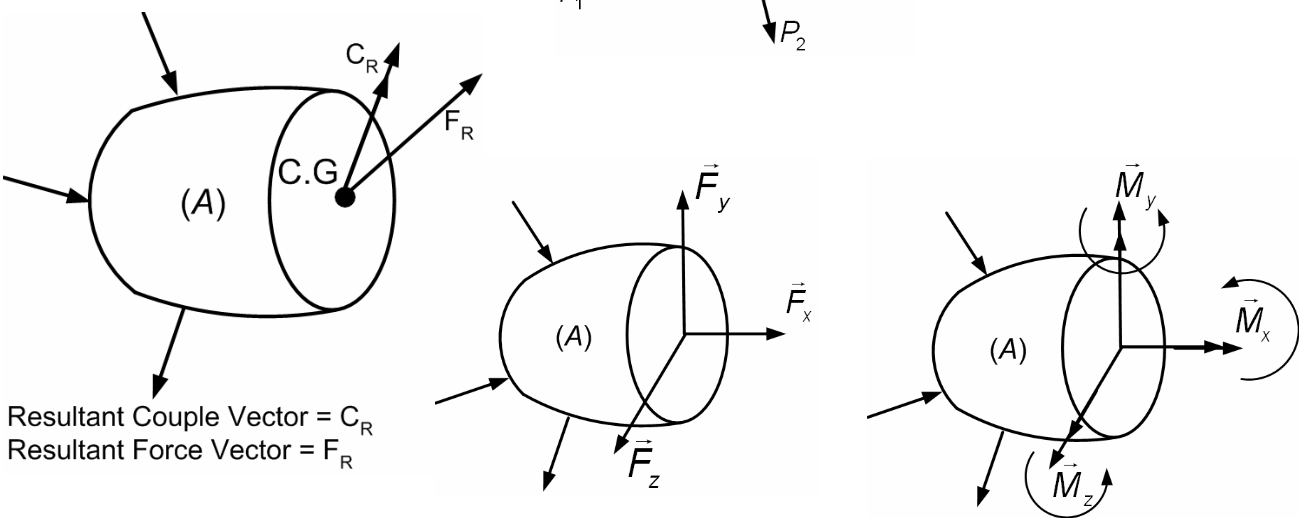
1. Shear force and bending moment diagrams

Internal Forces in solids







Sign conventions

- Shear forces are given a special symbol on $V_y \frac{1}{2}$ and V_z
- The couple moment along the axis of the member is given

$$M_x = T = \text{Torque}$$

$$M_y = M_z = \text{bending moment.}$$

We need to follow a systematic sign convention for systematic development of equations and reproducibility of the equations

The sign convention is like this.

If a face (i.e. formed by the cutting plane) is +ve if its outward normal unit vector points towards any of the positive coordinate directions otherwise it is –ve face

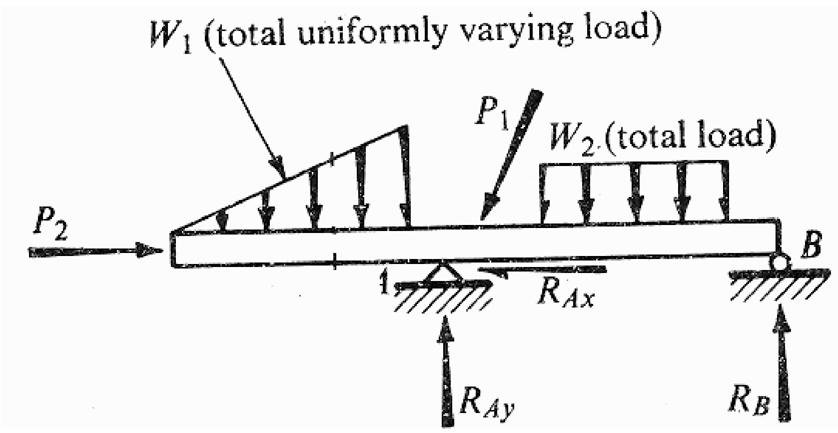
- *A force component on a +ve face is +ve if it is directed towards any of the +ve coordinate axis direction. A force component on a –ve face is +ve if it is directed towards any of the –ve coordinate axis direction. Otherwise it is –v.*

Thus sign conventions depend on the choice of coordinate axes.

Shear force and bending moment diagrams of beams

Beam is one of the most important structural components.

- *Beams are usually long, straight, prismatic members and always subjected forces perpendicular to the axis of the beam*



Two observations:

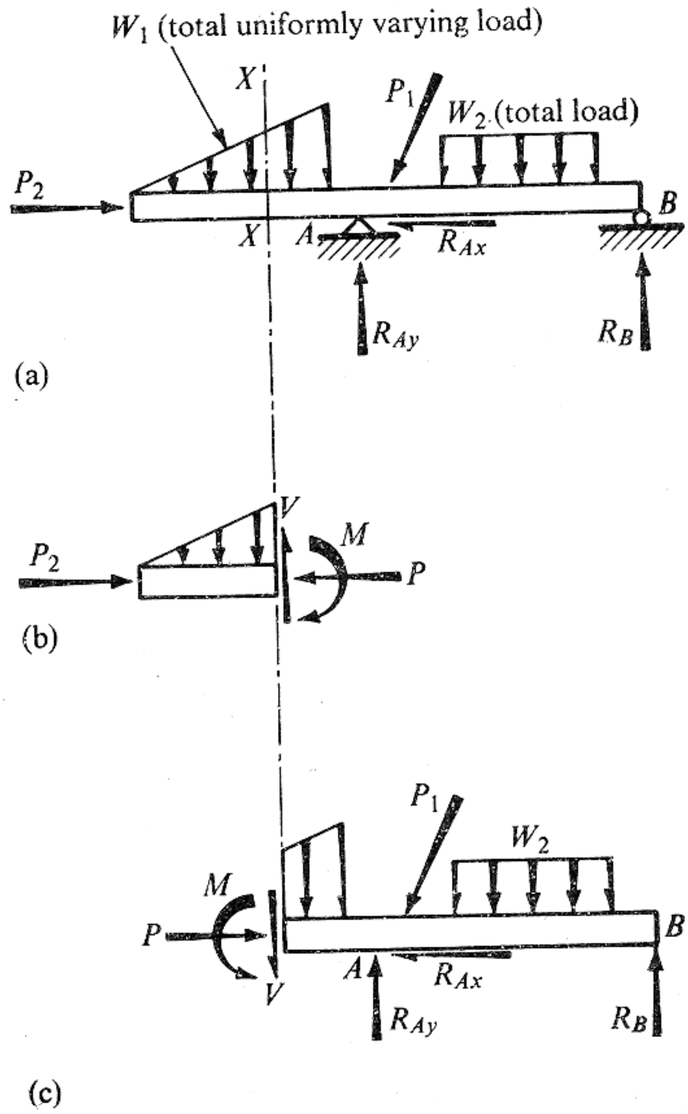
(1) Forces are coplanar

(2) All forces are applied at the axis of the beam.

Application of method of sections

What are the necessary internal forces to keep the segment of the beam in equilibrium?

$$\sum F_x = 0 \Rightarrow P$$
$$\sum F_y = 0 \Rightarrow V$$
$$\sum F_z = 0 \Rightarrow M$$

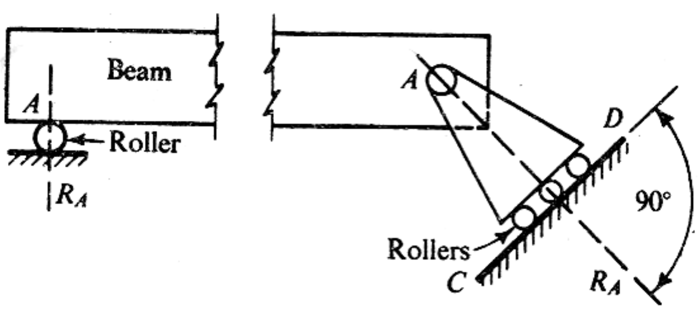


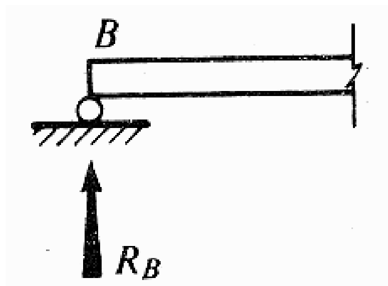
- The shear for a diagram (SFD) and bending moment diagram(BMD) of a beam shows the variation of shear force and bending moment along the length of the beam.

These diagrams are extremely useful while designing the beams for various applications.

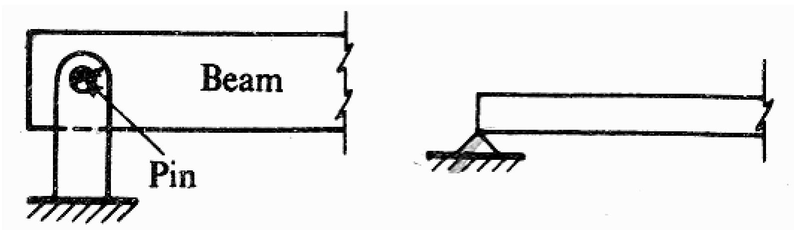
Supports and various types of beams

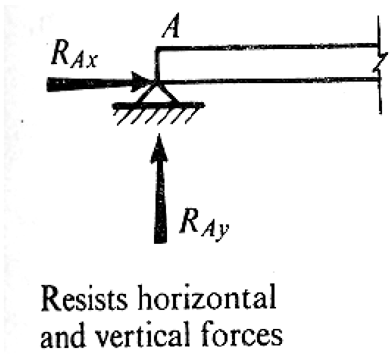
(a) *Roller Support* – resists vertical forces only





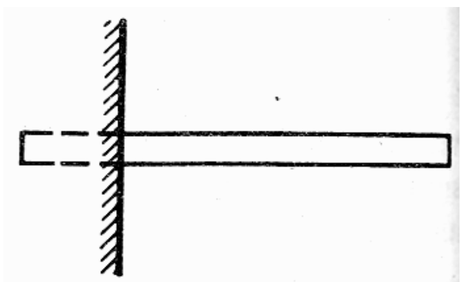
(b) *Hinge support or pin connection* – resists horizontal and vertical forces

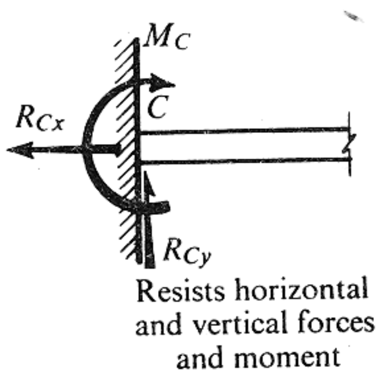




Hinge and roller supports are called as simple supports

(c) *Fixed support or built-in end*



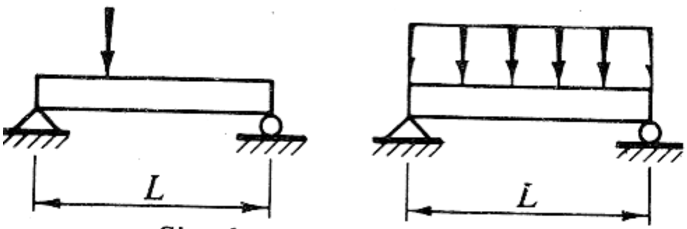


The distance between two supports is known as “span”.

Types of beams

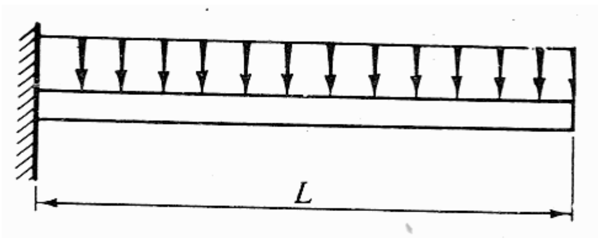
Beams are classified based on the type of supports.

(1) *Simply supported beam*: A beam with two simple supports

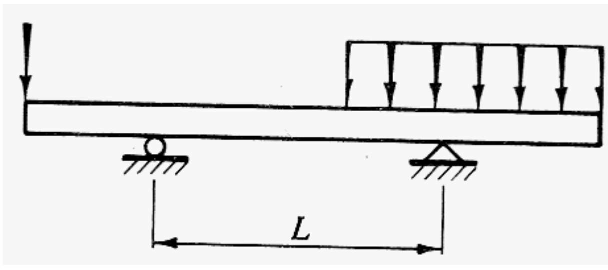


Simply supported beams

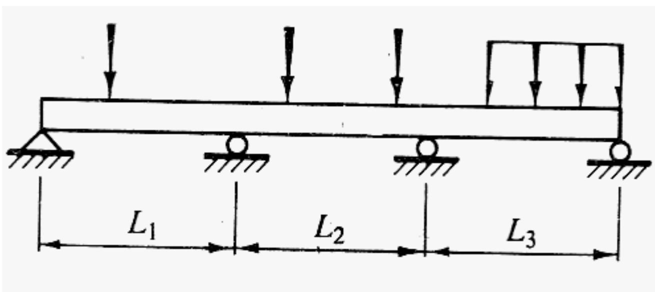
(2) *Cantilever beam*: Beam fixed at one end and free at other



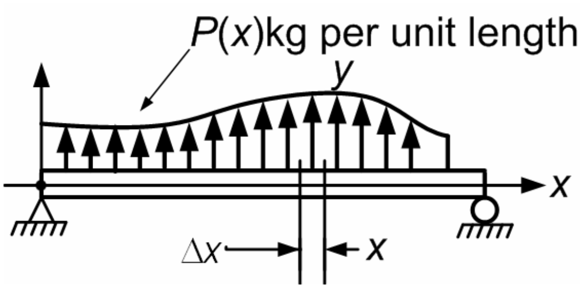
(3) *Overhanging beam*

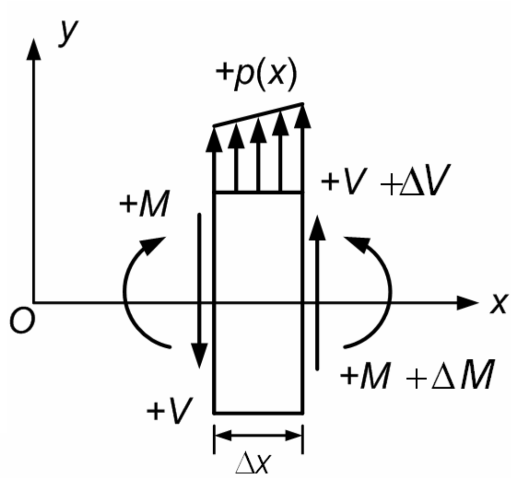


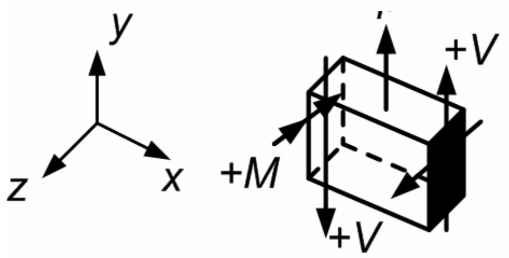
(4) *Continuous beam*: More than two supports



Differential equations of equilibrium







Beam sign convention

$$[\Sigma F_x = 0 \rightarrow +]$$

$$\left[\Sigma F_y = 0 \uparrow +\right]$$

$$V + \Delta V - V + P\Delta x = 0$$

$$\Delta V = -P\Delta x$$

$$\frac{\Delta V}{\Delta x} = -P$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = \frac{dV}{dx} = -P$$

$$[\Sigma M_A = 0] \;\; V\Delta x - M + M + \Delta M - \frac{P\Delta x^2}{2} = 0$$

$$V\Delta x + \Delta M - \frac{P\Delta x^2}{2} = 0$$

$$\frac{\Delta M}{\Delta x} + V - \frac{P\Delta x}{2} = 0$$

$$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = \frac{dM}{dx} = -V$$

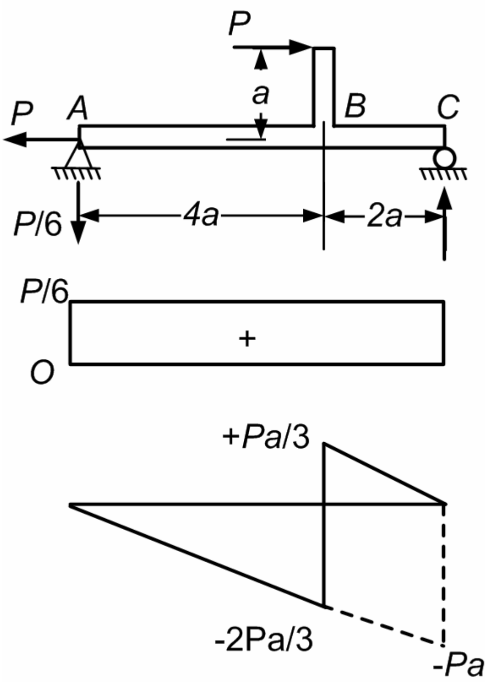
From equation $\dfrac{dV}{dx} = -P$ we can write

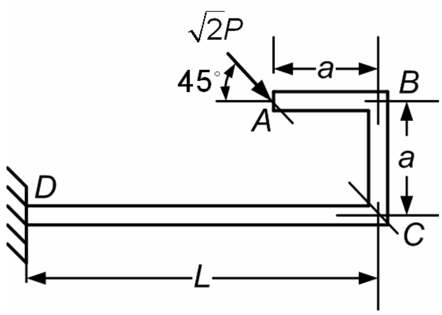
$$V_D - V_C = -\int_{X_C}^{X_D} P dx$$

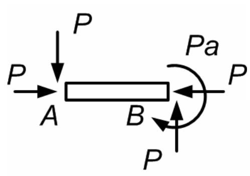
From equation $\dfrac{dM}{dx} = -V$

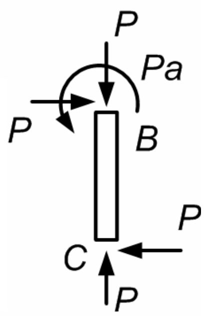
$$M_D - M_C = -\int V dx$$

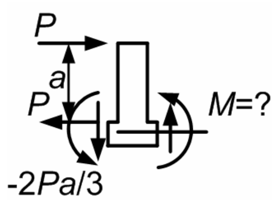
Special cases:

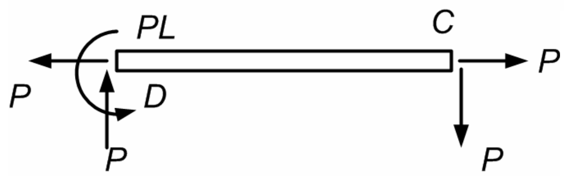


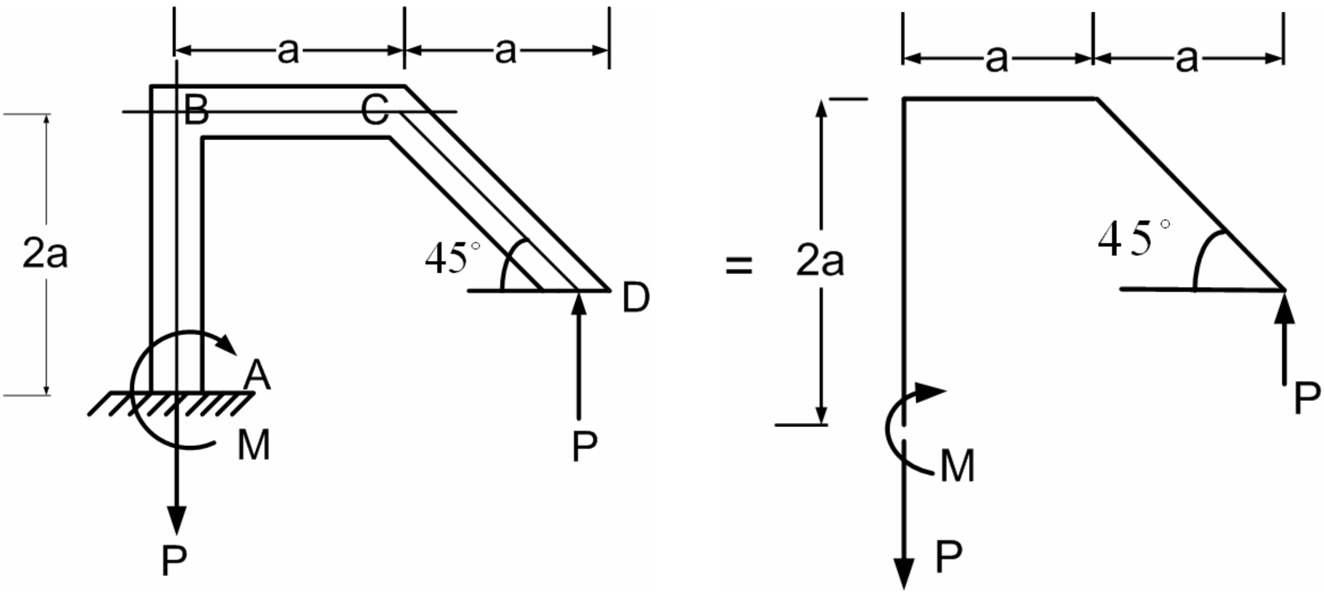








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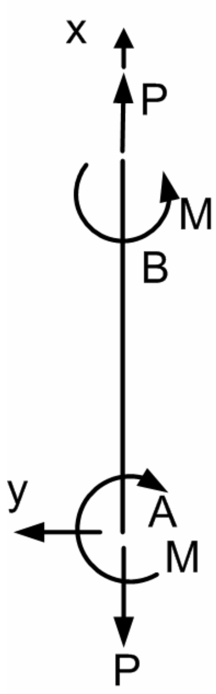
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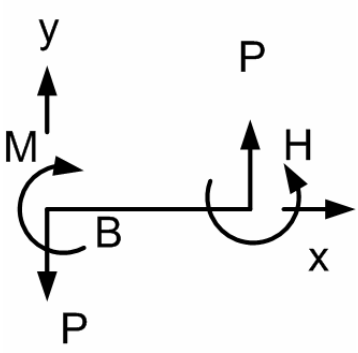
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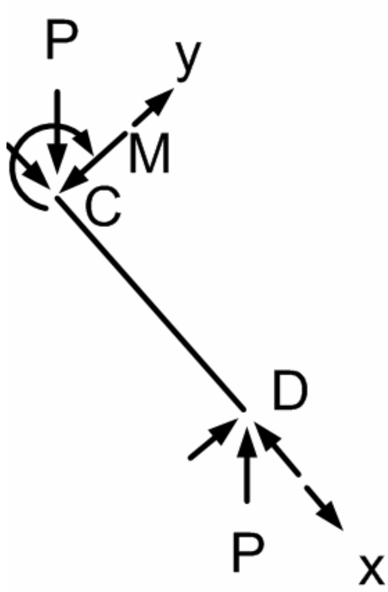
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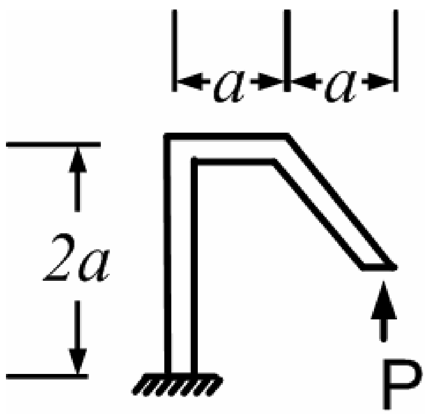
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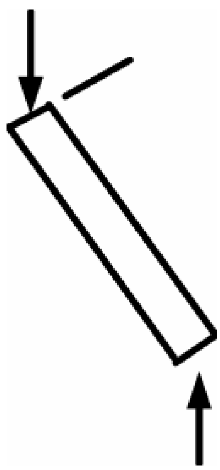

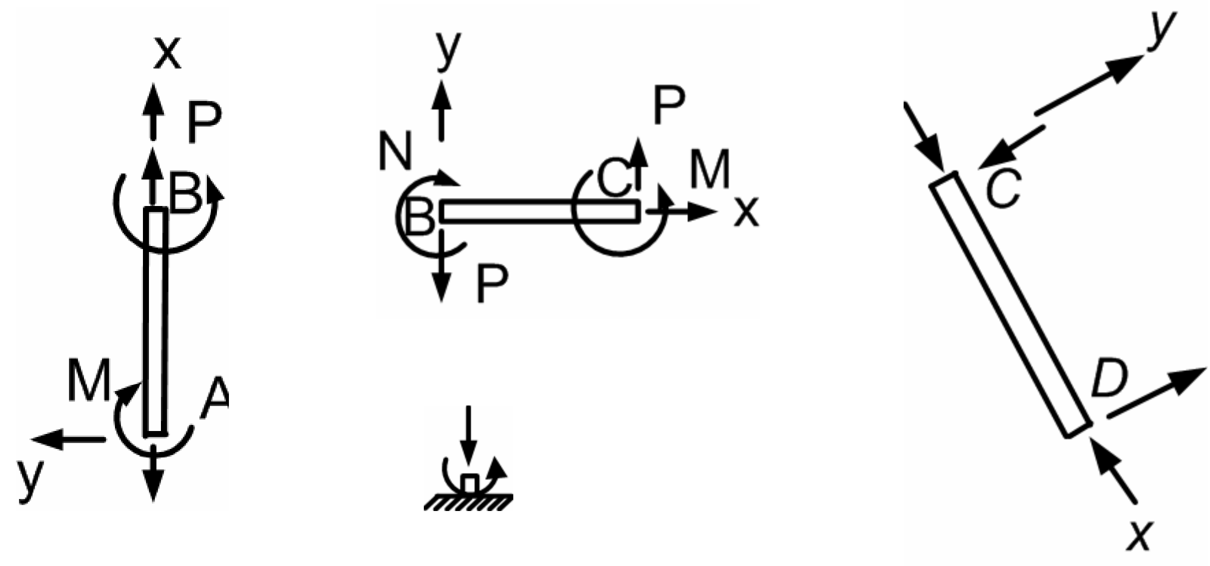
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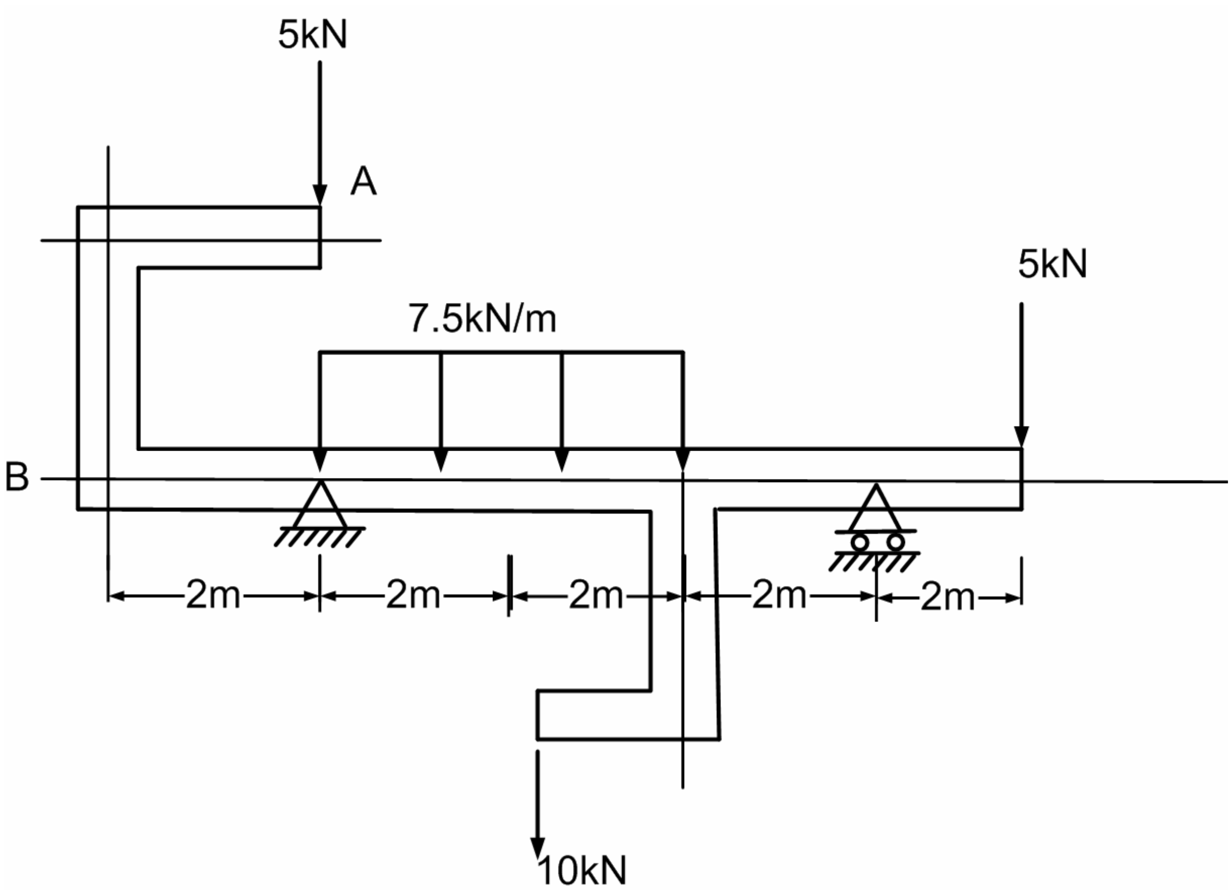
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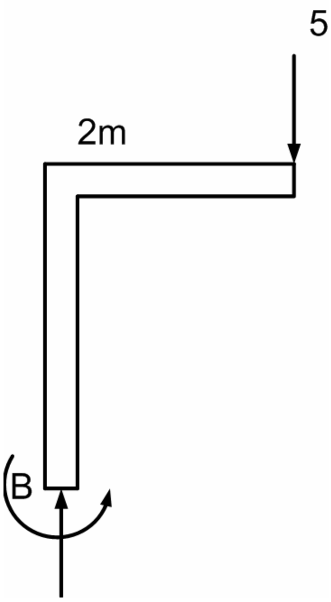
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2m
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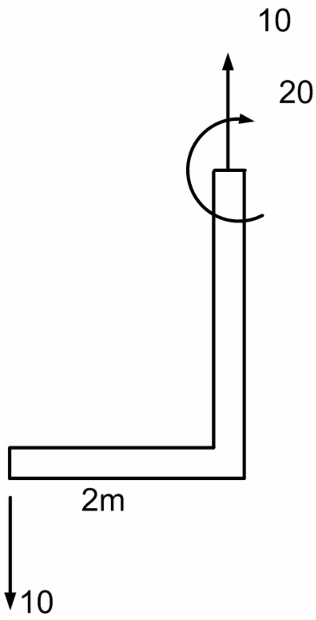
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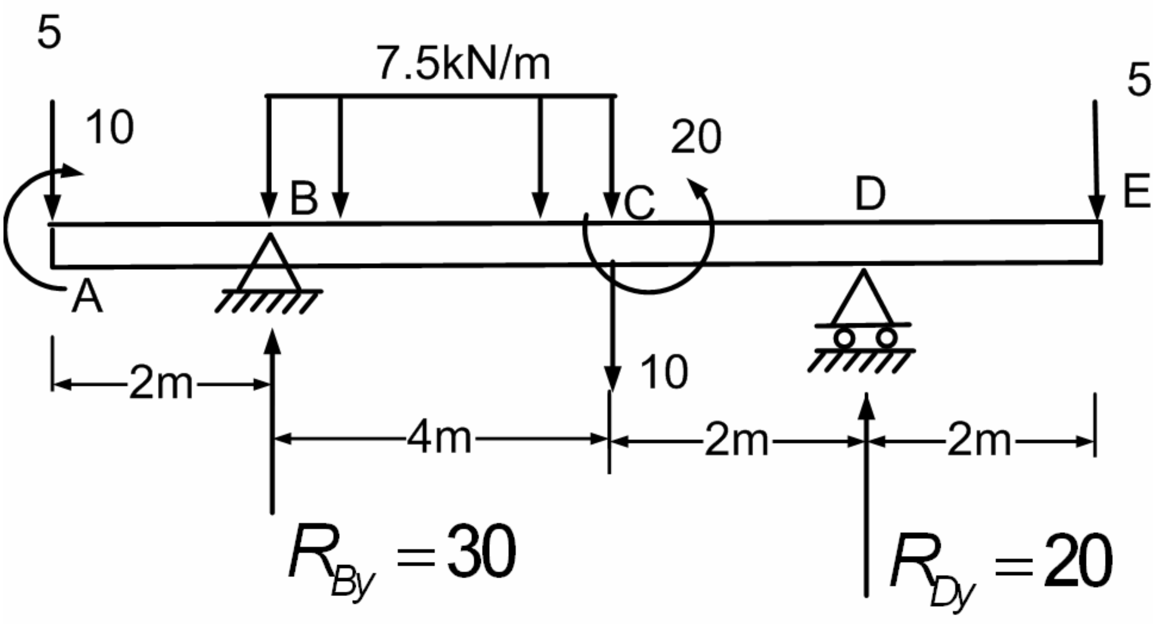
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7.5kN/m
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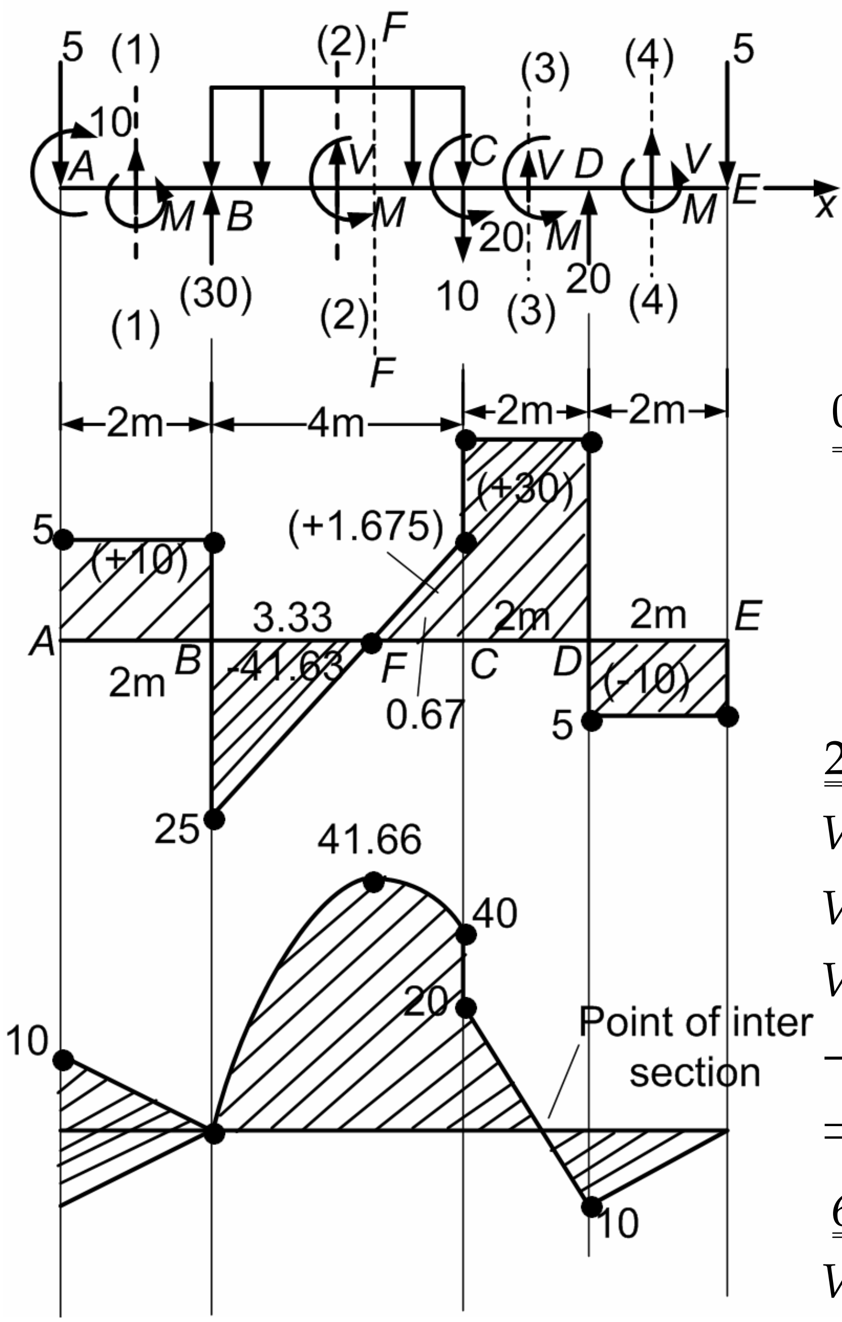
$R_{By} = 30$
$R_{Dy} = 20$



$0 \le x \le 2 - (1) - (1)$

$2 \le x \le 6 - (2 - (2)$

$6 \le x \le 8 - (3) - (3)$

$8 \le x \le 10 - (4) - (4)$

$\underline{\underline{0 \le x \le 2}}$ $(1)-(1)$

$$V - 5 = 0$$

$$V = 5$$

$$V_A = 5;\ V_B = 5$$

$\underline{\underline{2 \le x \le 6}}$ $(2)-(2)$

$$V - 5 + 30 - 7.5(x-2) = 0$$

$$V = 5 - 30 + 7.5(x-2)$$

$$V_B = -25;\ V_C = 5$$

$$-25 + 7.5(x-2) = 0$$

$$\Rightarrow x = 5.33$$

$\underline{\underline{6 \le x \le 8}}$ $(3)-(3)$

$$V - 5 + 30 - 30 - 10 = 0$$

$$V = +15$$

$$V_C = +15;\quad V_D = +15$$

$\underline{\underline{8 \le x \le 10}}$ $(4)-(4)$

$$V - 5 + 30 - 30 - 10 + 20 = 0$$

$$V + 5 = 0$$

$$V = -5$$

$$V_D = -5;\quad V_E = -5$$

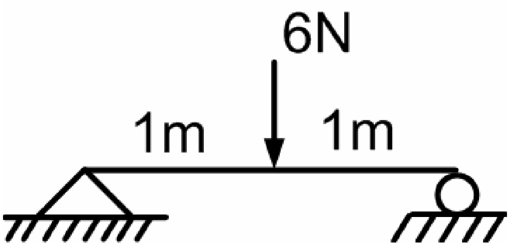
$$\underline{\underline{0 \le x \le 2}} \quad -(1)-(1)$$
$$M - 10 + 5x = 0$$
$$M = -5x + 10$$
$$M_A = +10; \quad M_B = 0$$

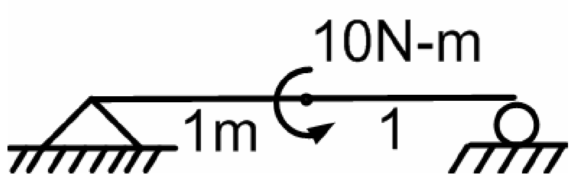
$$\underline{\underline{2 \le x \le 6}} \quad -(2)-(2)$$
$$M - 10 + 5x - 30(x-2) + \frac{7.5(x-2)^2}{2} = 0$$
$$M = 10 - 5x + 30(x-2) - \frac{7.5(x-2)^2}{2}$$
$$M_E\big|_{x=5.33} = 41.66 +$$
$$M_C\big|_{x=6} = 40$$

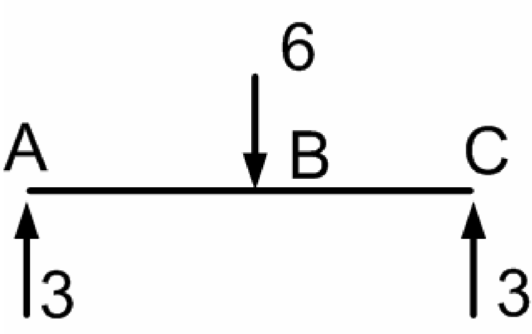
$$\underline{\underline{6 \le x \le 8}} \quad -(3)-(3)\,[C-D]$$
$$M - 10 + 5x - 30(x-2) + 30(x-4) + 10(x-6) + 20 = 0$$
$$M_C\big|_{x=6} = 20 +$$
$$M_D\big|_{x=8} = -10$$

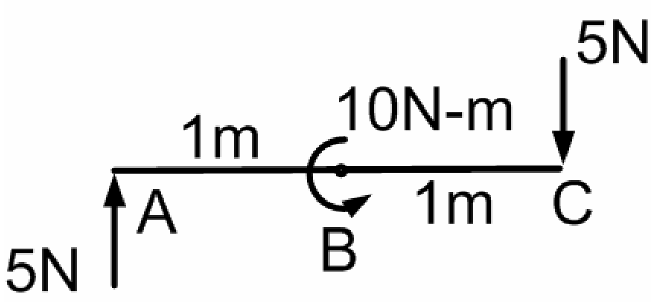
$$\underline{\underline{8 \le x \le 10}} \quad [D-E] \quad (4)-(4)$$
$$M - 10 + 5x - 30(x-2) + 30(x-4) + 10(x-6) + 20 - 20(x-8) = 0$$
$$M_E\big|_{x=8} = 0$$

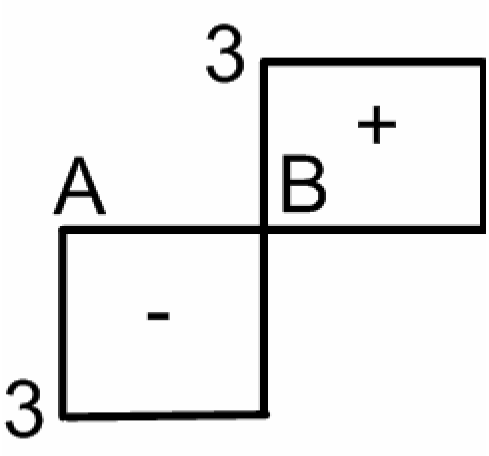
Problems to show that jumps because of concentrated force and concentrated moment

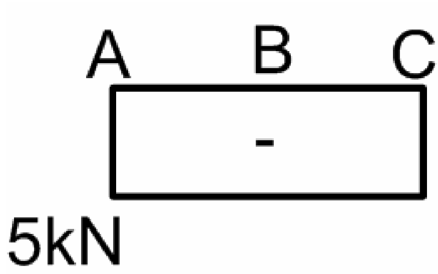


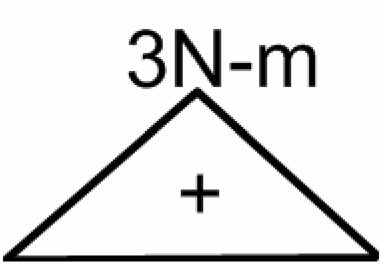


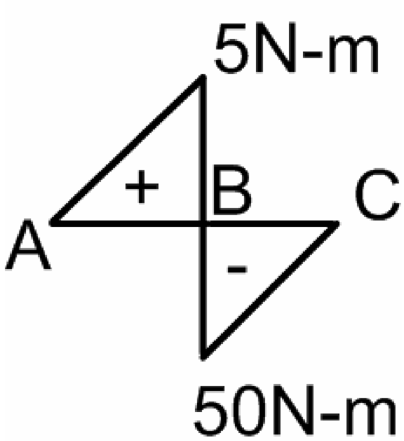






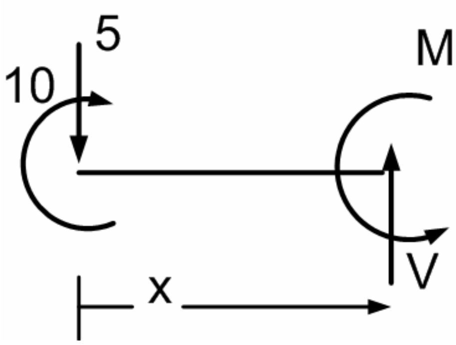






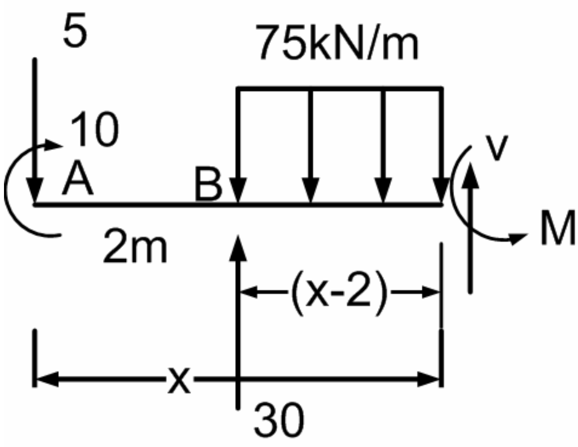
We can also demonstrate internal forces at a given section using above examples. This should be carried first before drawing SFD and BMD.

$\underline{\underline{0 \le x \le 2}}$ $[A - B]$



$$V-5=0 \qquad M-10+5x=0$$
$$V=5 \qquad M=10-5x$$
$$V_A=5 \qquad M_A=10;\ M_B=0$$
$$V_B=5$$

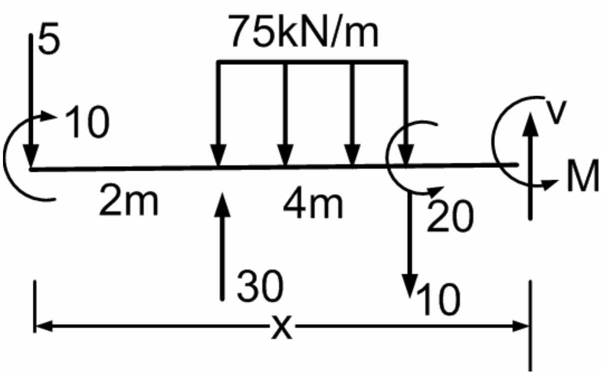
$\underline{\underline{2\le x\le 6}}\ [B-C]$



$$V-5+30-7.5(x-2)=0$$
$$V=7.5(x-2)+5-30$$
$$V_B=-25;\ \ V_C=5$$
$$-25+7.5(x-2)=0$$
$$x=5.33$$

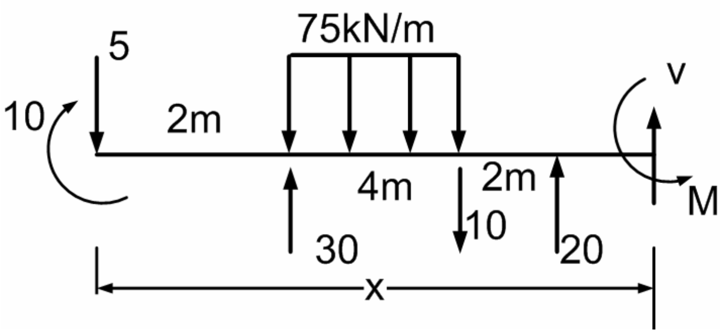
$$M-10+5x-30(x-2)+7.5\frac{(x-2)^2}{2}=0$$
$$x=6$$
$$M_C=40$$
$$M_E|x=5.33=41.66$$
$$x=2$$
$$M_B=0$$

$\underline{\underline{6\le x\le 8}}\ [C-D]$

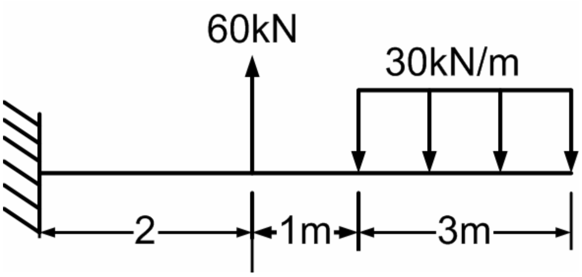


$$V-5+30-10-30=0$$
$$V=15$$
$$V_C=15,\quad V_D=15$$

$\underline{\underline{8 \le x \le 10}}\ [D-E]$



$$V-5+30-10-30+20=0$$
$$V=-5$$
$$V_D=-5, \quad V_E=-5$$



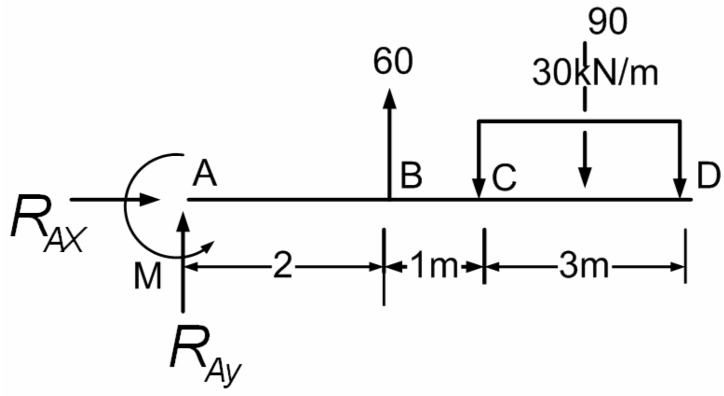
$$\left[\sum F_x \rightarrow + = 0\right] \Rightarrow R_{Ax} = 0$$

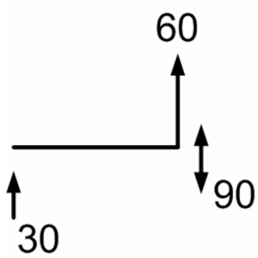
$$\left[\sum F_y \uparrow + = 0\right] \Rightarrow R_{Ay} + 60 - 90 = 0$$

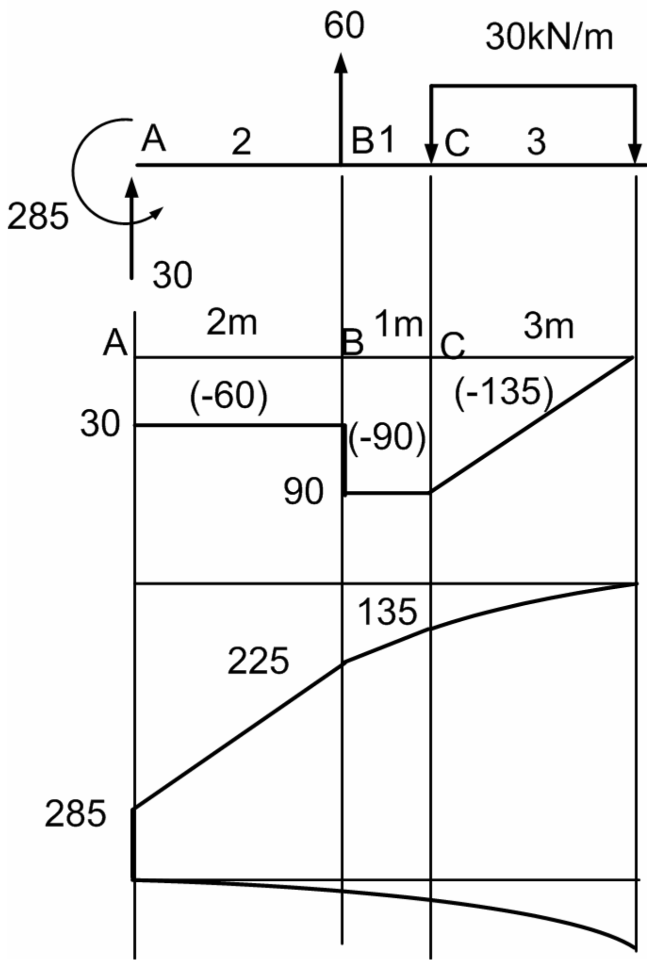
$$R_{Ay} = 30kN \uparrow$$

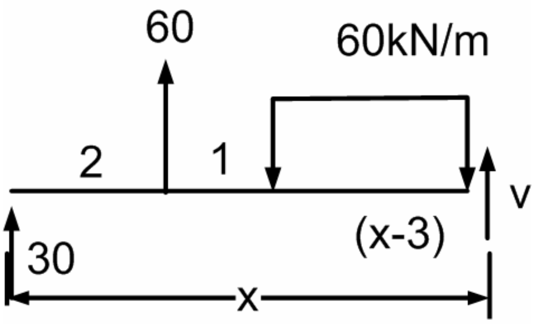
$$\left[\sum M_A = 0\right] \Rightarrow M + 60 - 90 \times 4.5 = 0$$

$$M = 285 k - m$$









$$30 + V + 60 - 30(x-3) = 0$$

$$V = 30(x-3) - 90$$

$$= 30 \times 3 - 90$$

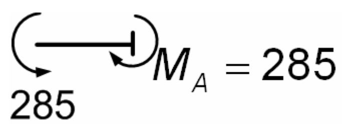
$$= 90 - 90$$

$$= 0$$

$$M_B - M_A = -(-60)$$

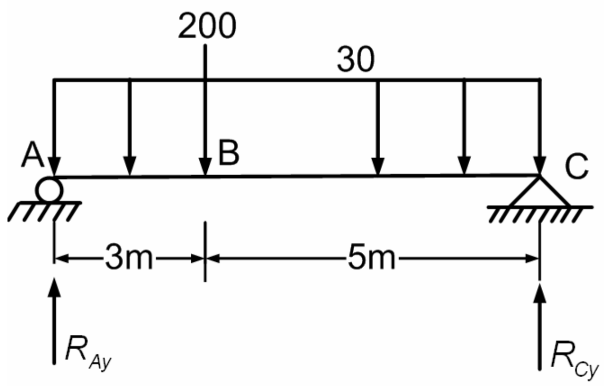
$$M_B = 60 + M_A = 60 - 285$$

$$= -225$$



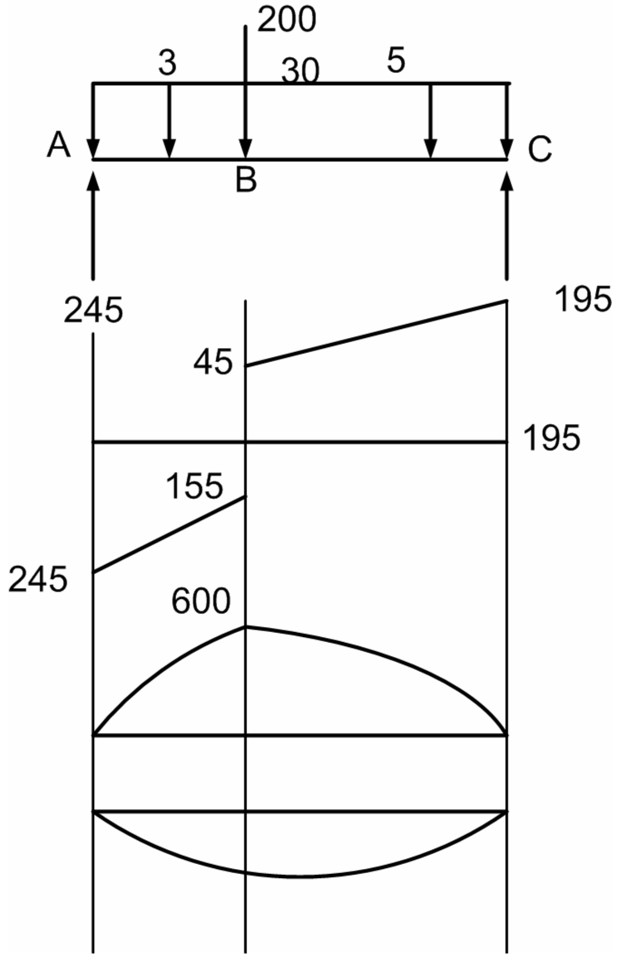
$$M_C - M_B = -(-90)$$
$$M_C = M_B + 90 = -225 + 90$$
$$= -135$$

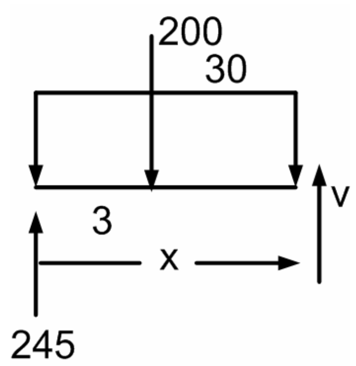
$$M_D - M_C = -(-135)$$
$$M_D = M_C + 135 = -135 + 135 = 0$$



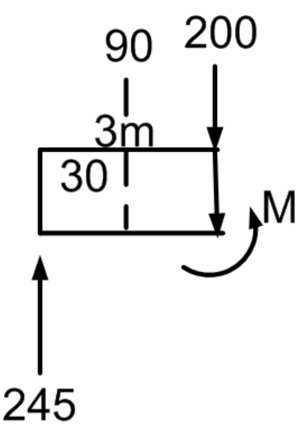
$$\left[\sum F_y \uparrow + = 0\right]$$
$$R_{Ay} + R_{Cy} - 200 - 240 = 0$$
$$R_{Ay} + R_{Cy} = 440 \qquad (1)$$

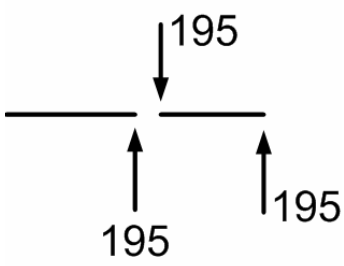
$$[\sum M_A = 0]$$
$$-200 \times 3 - 240 \times 4 + R_{Cy} \times 8 = 0$$
$$R_{Cy} = 195\, kN \uparrow$$
$$R_{Ay} = 245\, kN \uparrow$$





$$V + 245 - 200 - 30x = 0$$
$$V = 30x - 45$$
$$V = 30 \times 8 - 45 = 240 - 45$$
$$V = 195$$

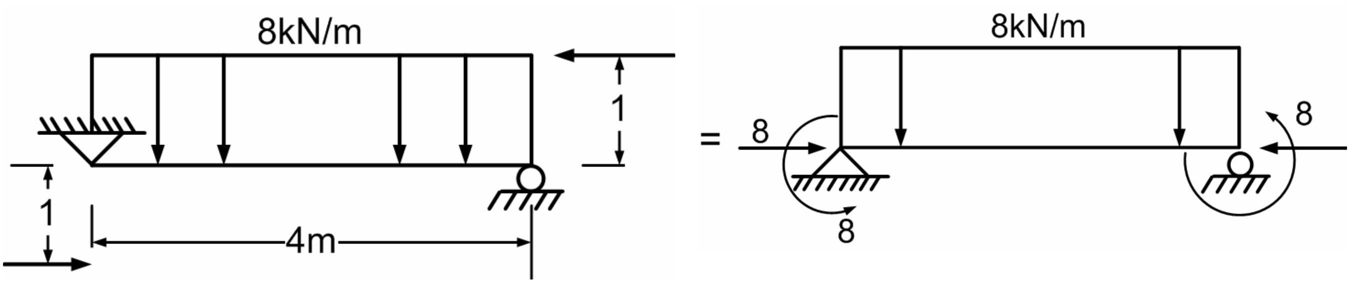


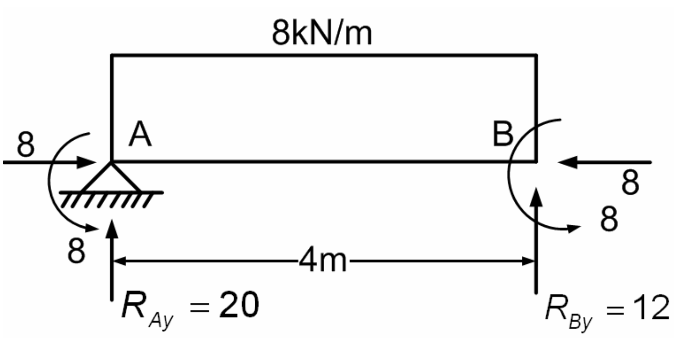


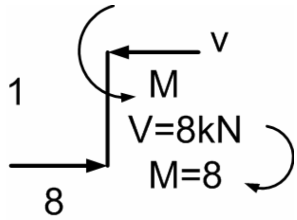
$$M - 245\times 3 + 90\times 1.5$$

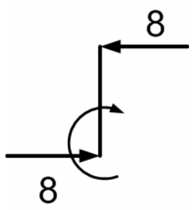
$$M = 245\times 3 - 90\times 1.5$$

$$M = 600$$









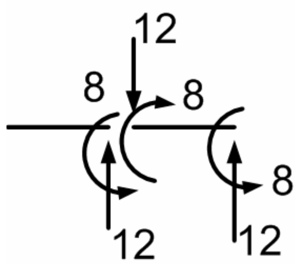
$$R_{Ay} + R_{By} = 32$$

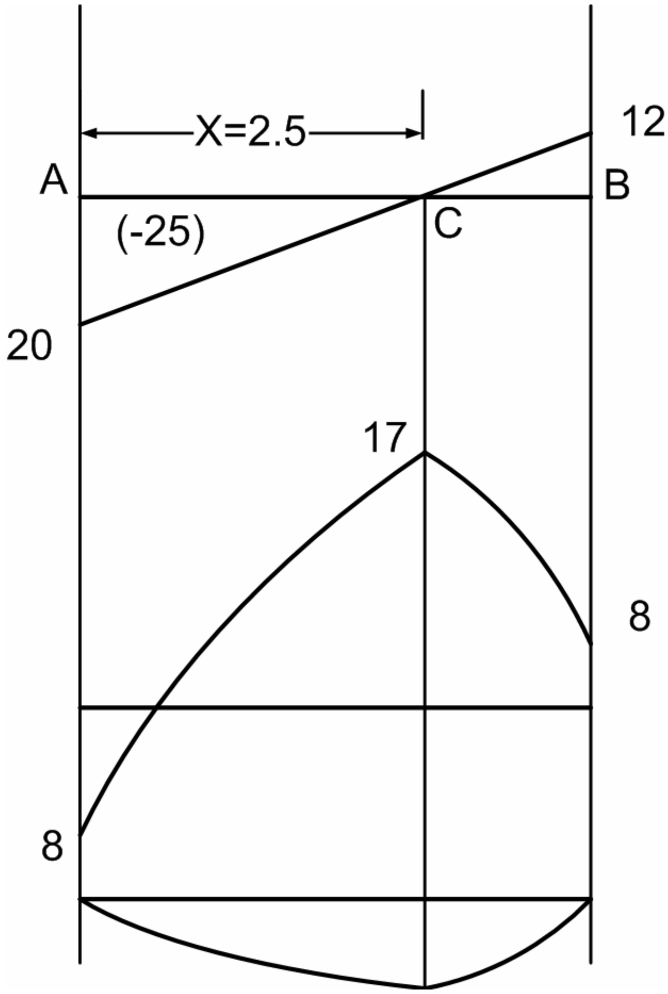
$$[\Sigma M_A = 0] - 32\times 2 + 18 + 8 + 4R_{By} = 0$$

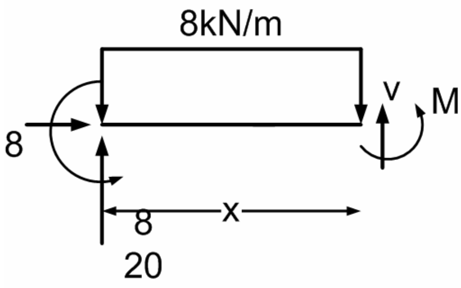
$$-64 + 16 + 4R_{By} = 0$$

$$R_{By} = 12kN$$

$$R_{Ay} = 20kN$$



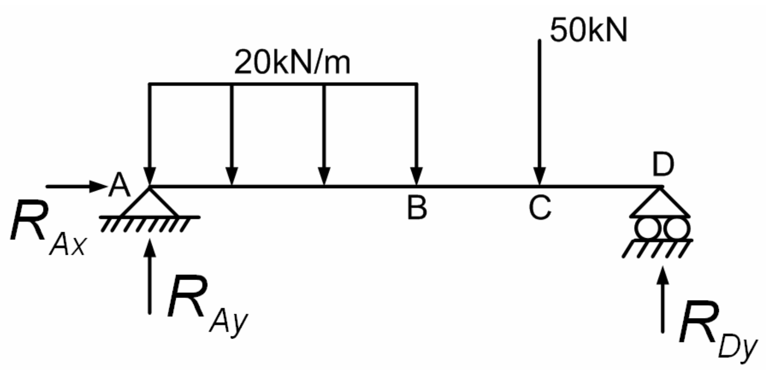




$$V + 20 - 8x = 0$$
$$V = 8x - 20$$
$$8x - 20 = 0$$
$$x = 20/8 = 2.5$$

$$M_C - M_A = -(-50)$$
$$M_C = M_A + 50 = -8 + 25 = 17$$

Problem:



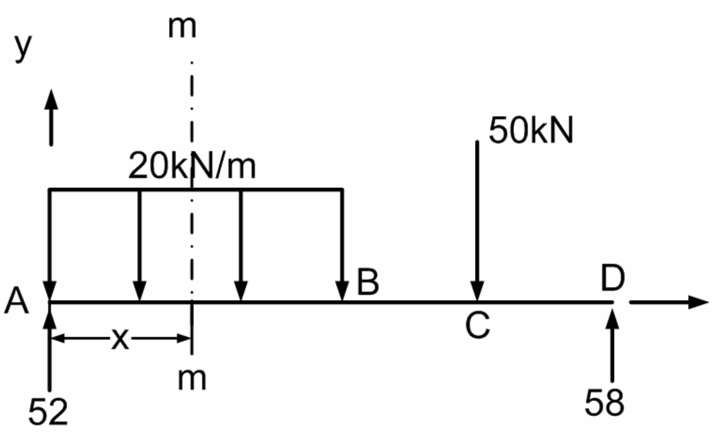
$$[\Sigma F_x \rightarrow + = 0]$$
$$R_{Ax} = 0$$

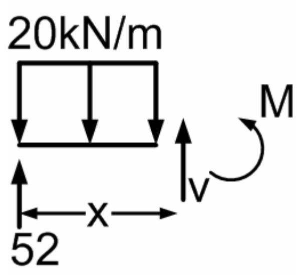
$$\left[\Sigma F_y = 0 \uparrow +\right] R_{Ay} + R_{Dy} - 60 - 50 = 0 \Rightarrow R_{Ay} + R_{Dy} = 110 \qquad (1)$$

$$[\Sigma M_A = 0] - 60 \times 1.5 - 50 \times 4 + R_{Dy} \times 5 = 0$$

$$R_{Dy} = \frac{290}{5} = 58kN \uparrow$$

$$R_{Ay} = 52\,kN \uparrow$$





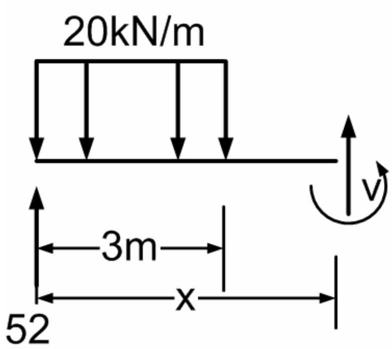
$$\left[\sum F_y = 0 \uparrow +\right] \quad V + 52 - 20x = 0$$

$$V = 20x - 52 \qquad \left(0 \leq x \leq \overset{(B)}{3m}\right)$$

$$[\sum M = 0]$$

$$M + \frac{20x^2}{2} - 52x = 0$$

$$M = 52x - \frac{20x^2}{2} (0 \leq x \leq 3m)$$



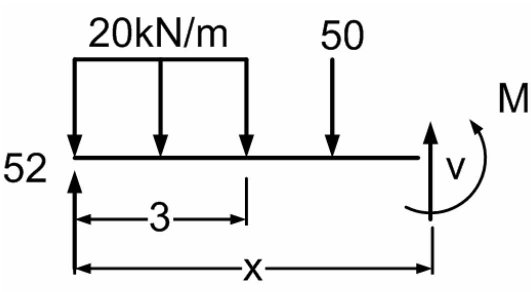
$$\left[\sum F_y = 0 \uparrow +\right]$$

$$V + 52 - 60 = 0$$

$$V = 8kN \uparrow \qquad \left(\overset{B}{3} \leq x \leq \overset{C}{4m}\right)$$

$$[\sum M = 0] \quad M - 52x + 60(x - 1.5) = 0$$

$$M = 52x - 60(x - 1.5) \quad \left(\overset{B}{3} \leq x \leq \overset{C}{4m}\right)$$



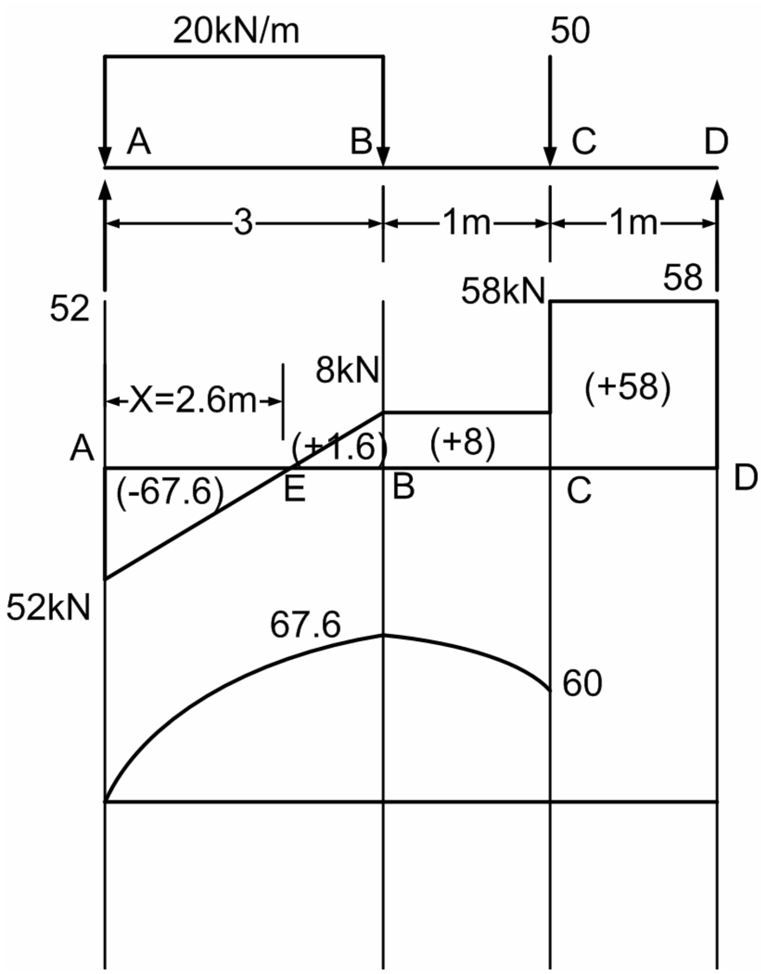
$$\left[\sum F_y = 0 \uparrow +\right]$$

$$V + 52 - 60 - 50 = 0$$

$$V = 58\,kN \qquad (4 \le x \le 5)$$

$$[\sum M = 0]\;\; M - 52x + 60(x - 1.5) + 50(x - 4) = 0$$

$$M = 52x - 60(x - 1.5) - 50(x - 4) \;\; (4 \le x \le 5)$$



$$\frac{dM}{dx} = -V$$

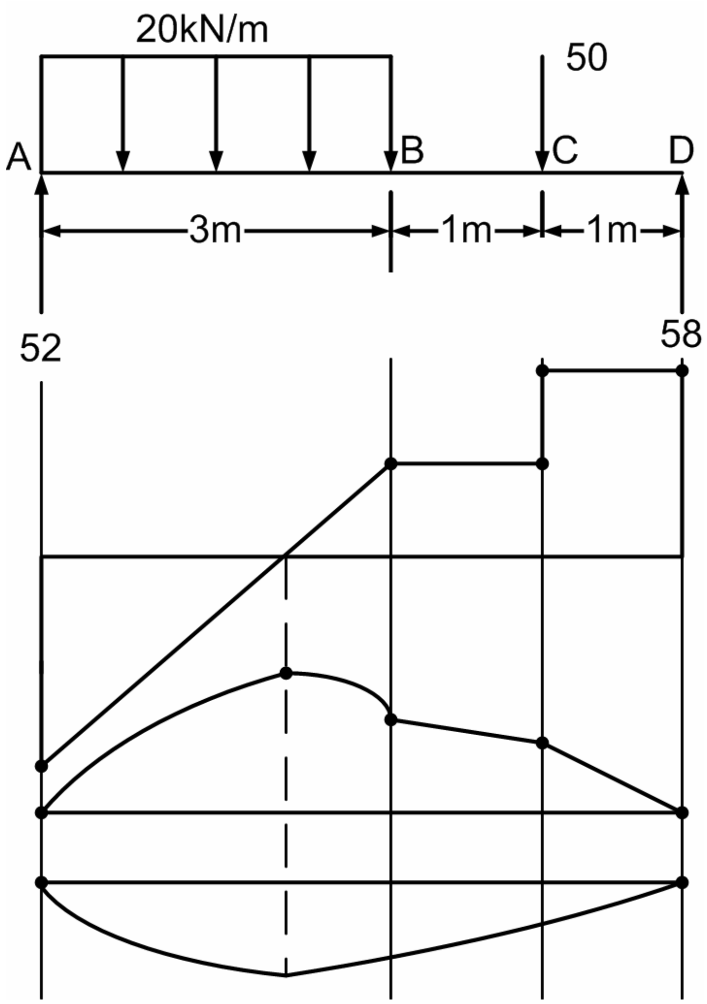
$$\frac{dV}{dx} = -P$$

$$20 \times -52 = 0$$

$$x = 52 / 20 = 2.6\,m$$

$$M_B - M_E = -1.6$$

$$M_B = -1.6 + 67.6$$



$$M_B - M_A = -\int V dx$$

$$\frac{dM}{dx} = -V$$

$$\frac{dV}{dx} = -P$$

$$20\times -52 = 0$$

$$x = 52/20 = 2.6$$

$$M_B - M_E = -1.6$$

$$M_B = -1.6 + M_E = -1.6 + 67.6$$

$$= 66$$

$$M_C - M_B = -8$$

$$M_C = -8 + M_B$$

$$= -8 + 66 = 58$$

$$M_D - M_C = -58$$

$$M_D = M_C + 58$$

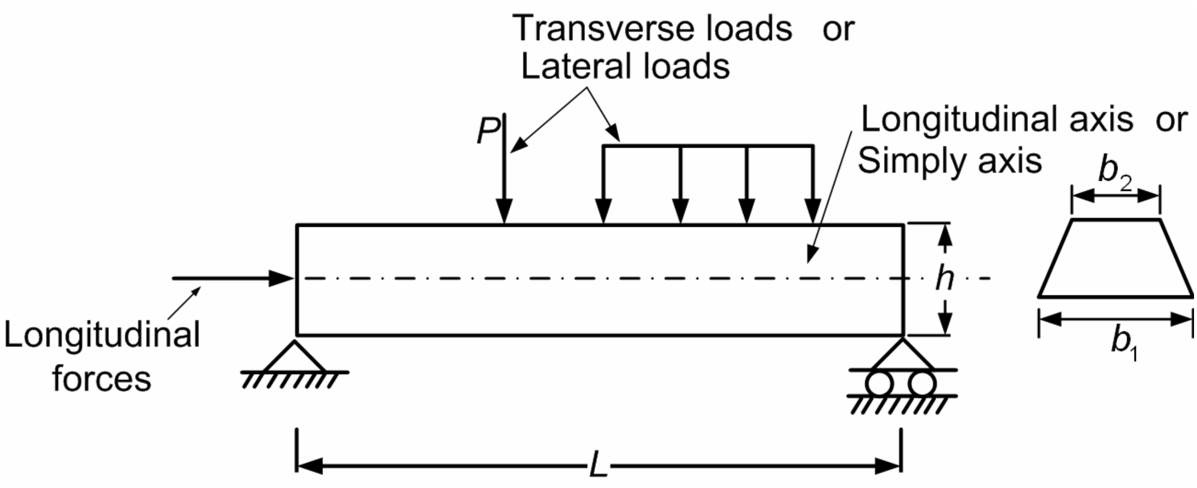
$$= 58 - 58 = 0$$

References:

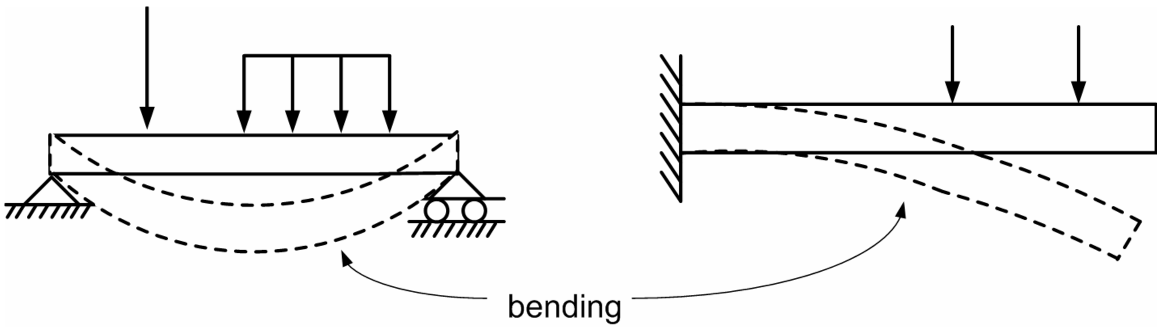
Solid Mechanics – INTERNET SOURCES & E-BOOKS

15. Symmetrical bending of beams

Some basics



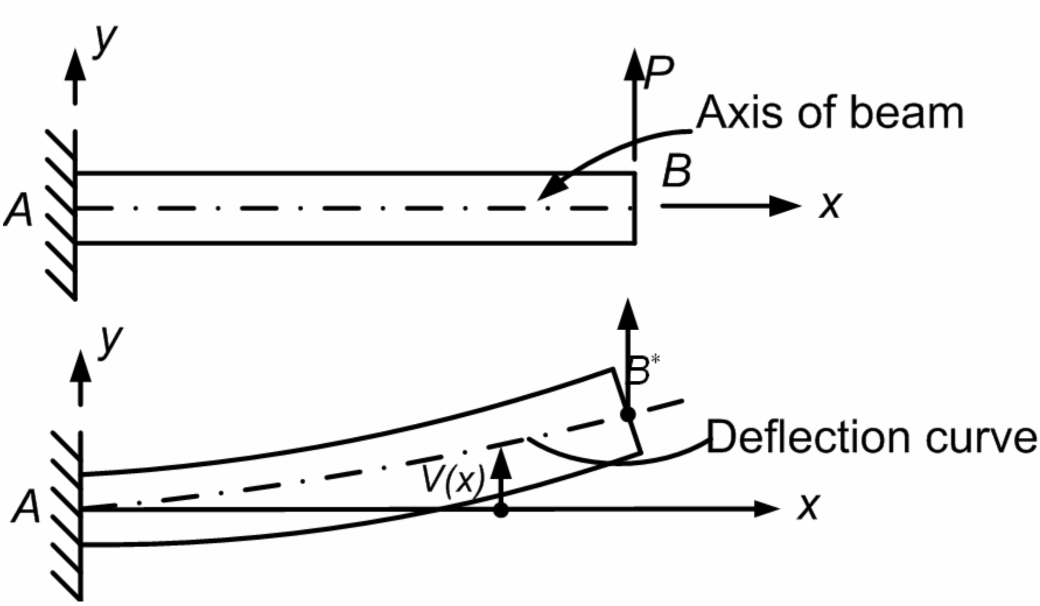
- Transverse loads or lateral loads: Forces or moments having their vectors perpendicular to the axis of the bar.
- Classification of structural members.
- Axially loaded bars :- Supports forces having their vectors directed along the axis of the bar.
- Bar in tension:- Supports torques having their moment vectors directed along the axis.
- Beams :- Subjected to lateral loads.
- Beams undergo bending (flexure) because of lateral loads.



Roughly speaking, “bending” refers to a *change in shape from a straight configuration to a non straight configuration*.

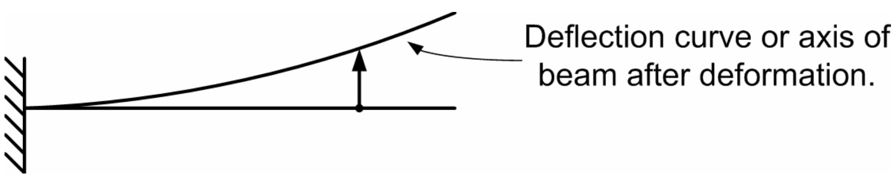
Bending moments i.e M_z and M_y are responsible for bending of beams.

The loads acting on a beam cause the beam to bend or flex, thereby ***deforming its axis into a curve-known as “ deflection curve” of the beam***.



If all points in $x-y$ plane remain in the $xy-$ plane after deformation i.e after bending then $xy-$ plane is known as “plane of bending”.

If a beam bend in a particular plane, then the deflection curve is a plane curve lying in the plane of bending.



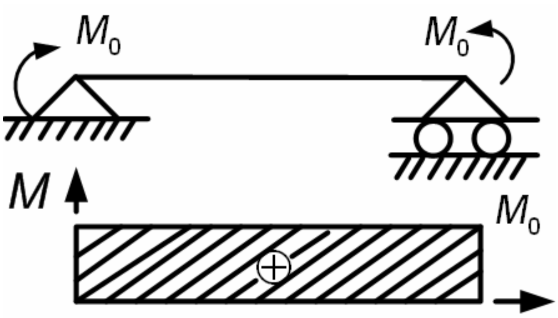
The $y-$direction displacement [i.e. $v-$component] of any point along its axis is known as the "deflection of the beam".

Pure bending and non-uniform bending

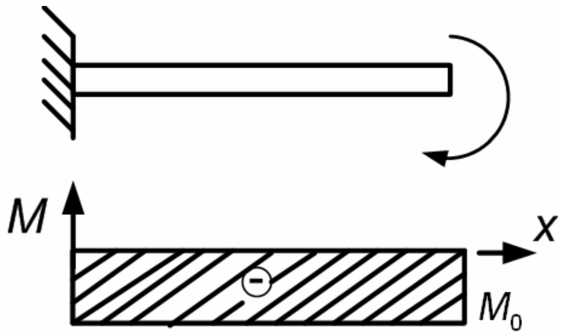
If the internal bending moment is constant at all sections then beam is said to be under "pure bending".

$$\frac{dM}{dx} = -V$$

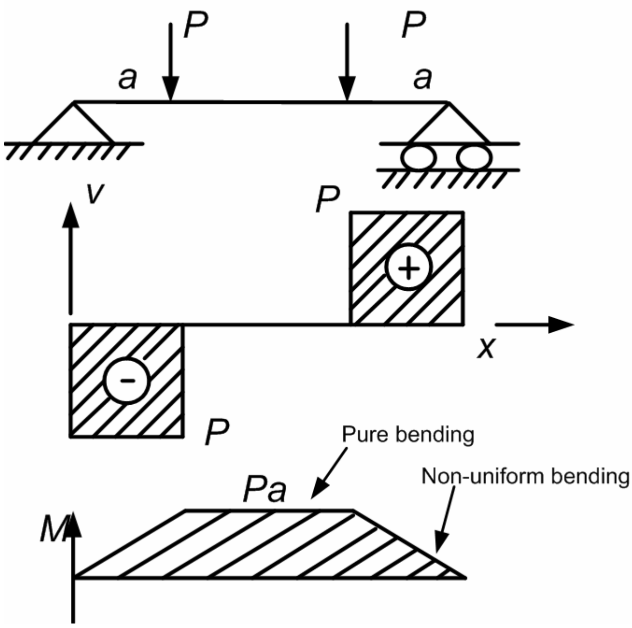
Pure bending (i.e., M=constant) occurs only in regions of a beam where the shear force is zero.



Uniform bending or pure bending



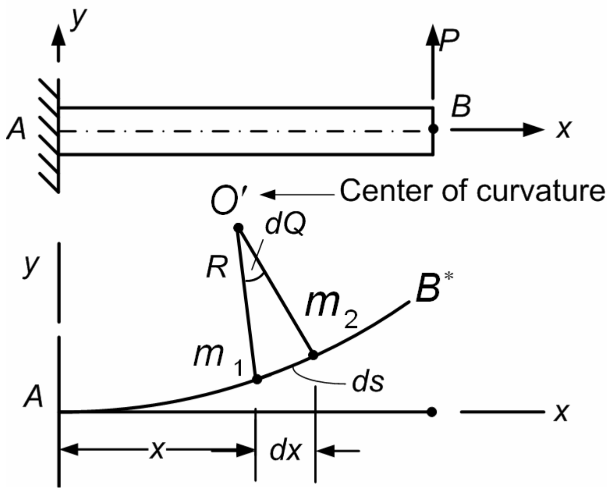
Uniform bending or pure bending



If $M = M(x)$ it is non- uniform bending

Curvature of a beam

When loads are applied to the beam, if it bends in a plane say xy −plane, then its longitudinal axis is deformed into a curve.



$O-$ Center of curvature

$R-$ Radius of curvature

$k=\frac{1}{R}=$ Curvature

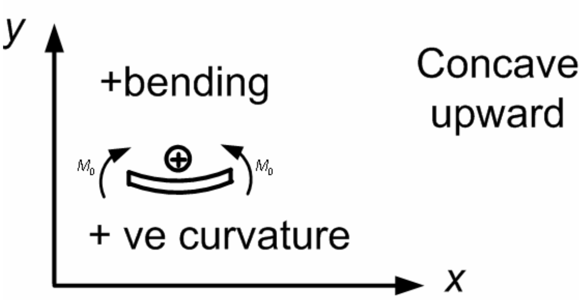
in general $R=R(x)$ and $k=k(x)$.

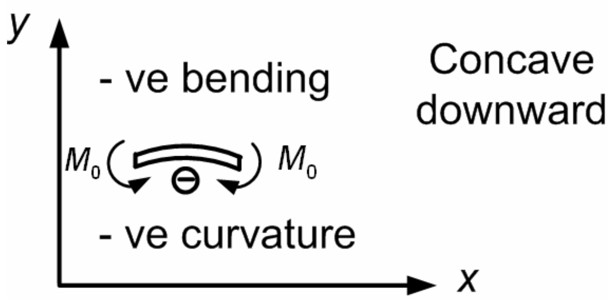
$RdQ=dS$

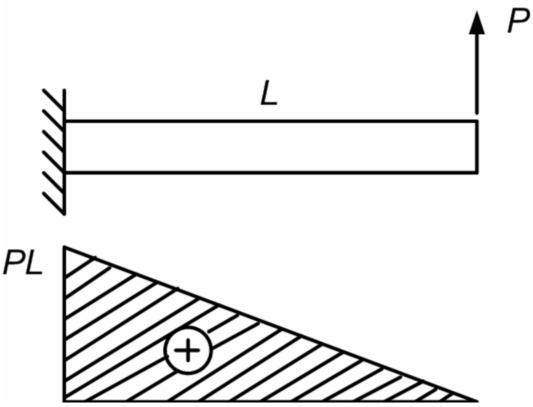
$k=\frac{1}{R}=\frac{dQ}{dS}$ for any amount of R

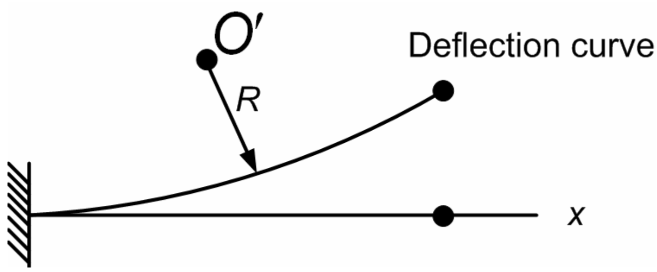
The deflections of beams are very small under small deformation condition. small deflections means that the deflection curve is nearly flat.

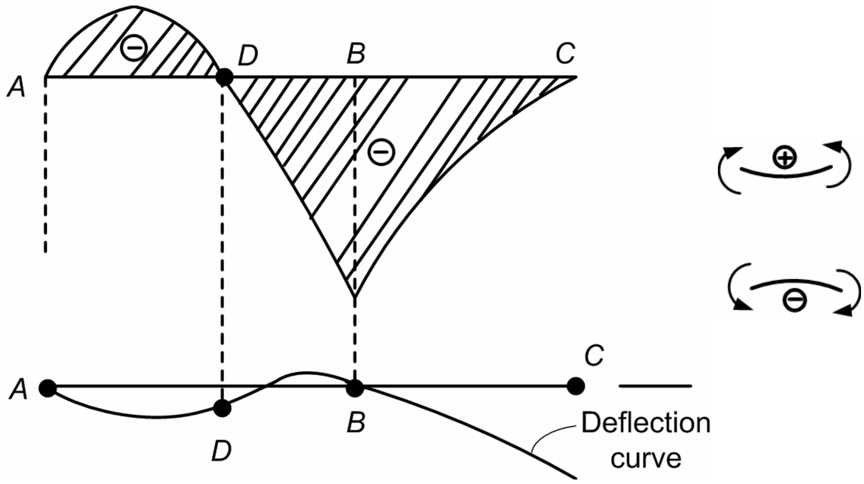
$k=\frac{1}{R}=\frac{dQ}{dX}$ nder small deformations.









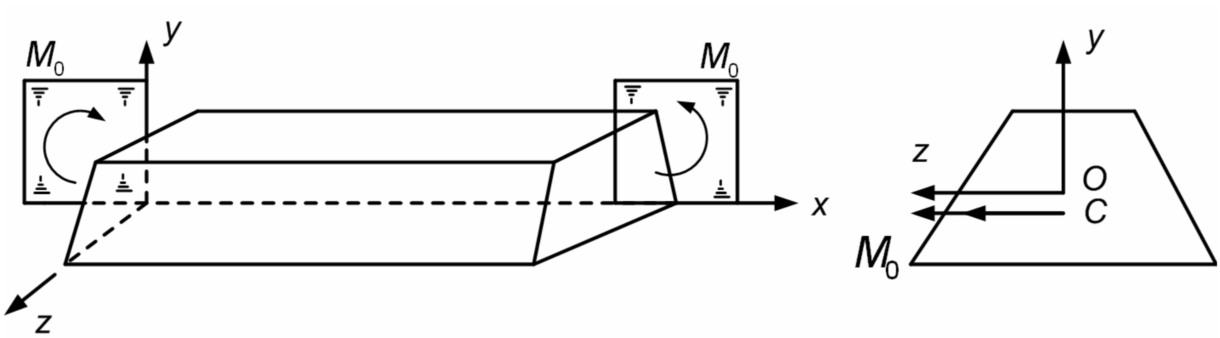


It is given that deflections at A and B should be zero.

Symmetrical bending of beams in a state of pure bending

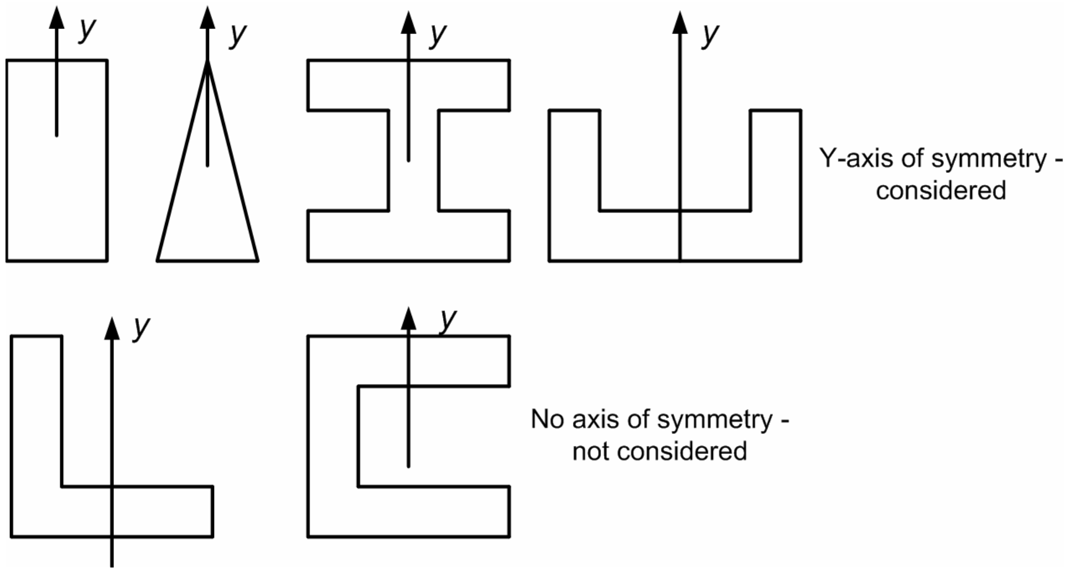
Geometry, loading and material properties

A long prismatic member possess a plane of symmetry subjected to equal and opposite couples M_0(or bending moments) acting in the same plane of symmetry.



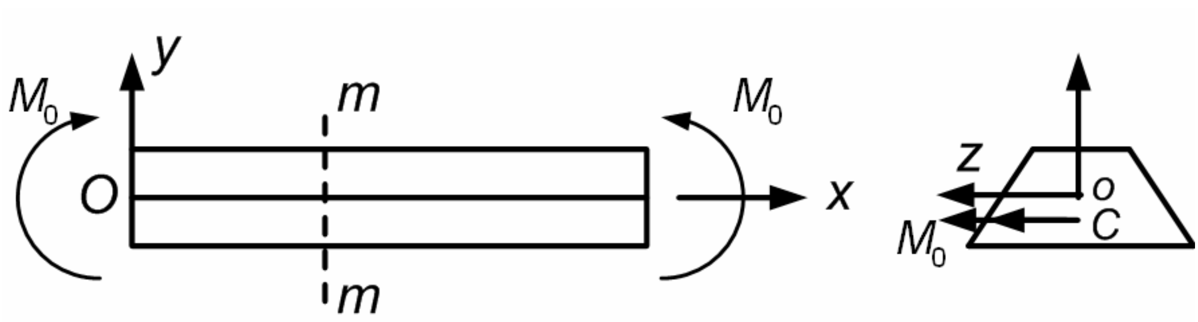
Initially we choose origin of the coordinate system "O" is not at the centroid of the cross-section.

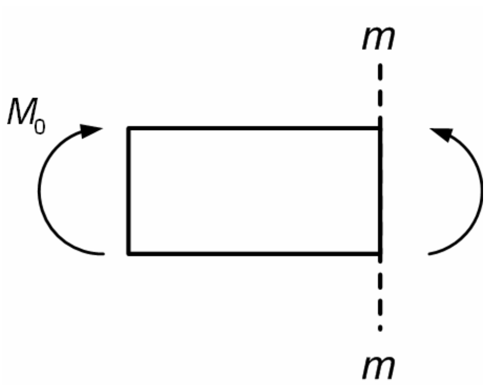
The $y-$axis passing through the cross-section is an axis of symmetry. The XY plane is the plane of symmetry.



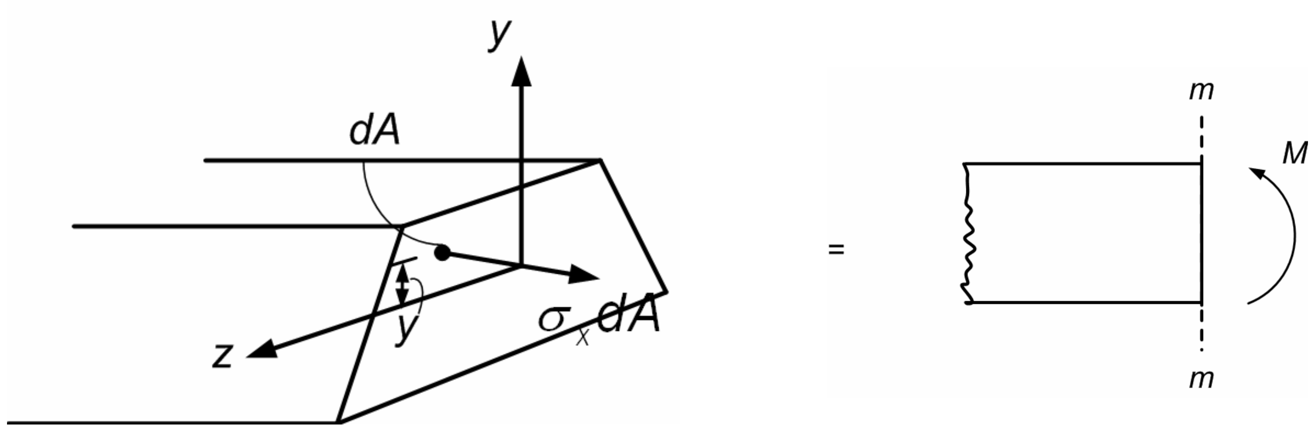
Material is homogeneous, linearly elastic and isotropic undergoing small deformations.

Stresses in symmetric member in pure bending





$$F_x = V_y = V_z = 0$$
$$M_x = M_y = 0$$
$$M_z = M = M_0$$



$$M = -\int y\sigma_x dA$$

Therefore, $\sigma_x dA$ are the only elementary forces that are required to be developed by right of the section on to the left of the section.

The distribution of σ_X any section should satisfy

$$F_x = 0 \Rightarrow \int \sigma_x dA = 0$$

$$M_y = 0 \Rightarrow \int z\sigma_x dA = 0$$

$$M_z = M \Rightarrow -\int y\sigma_x dA = M$$

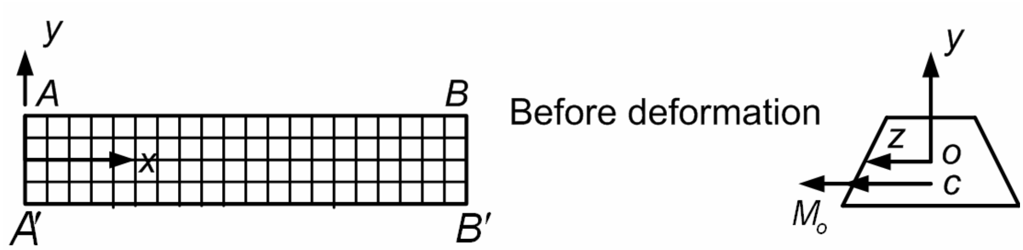
Actual distribution of stresses - cannot by statics - statically indeterminate - deformations should be considered.

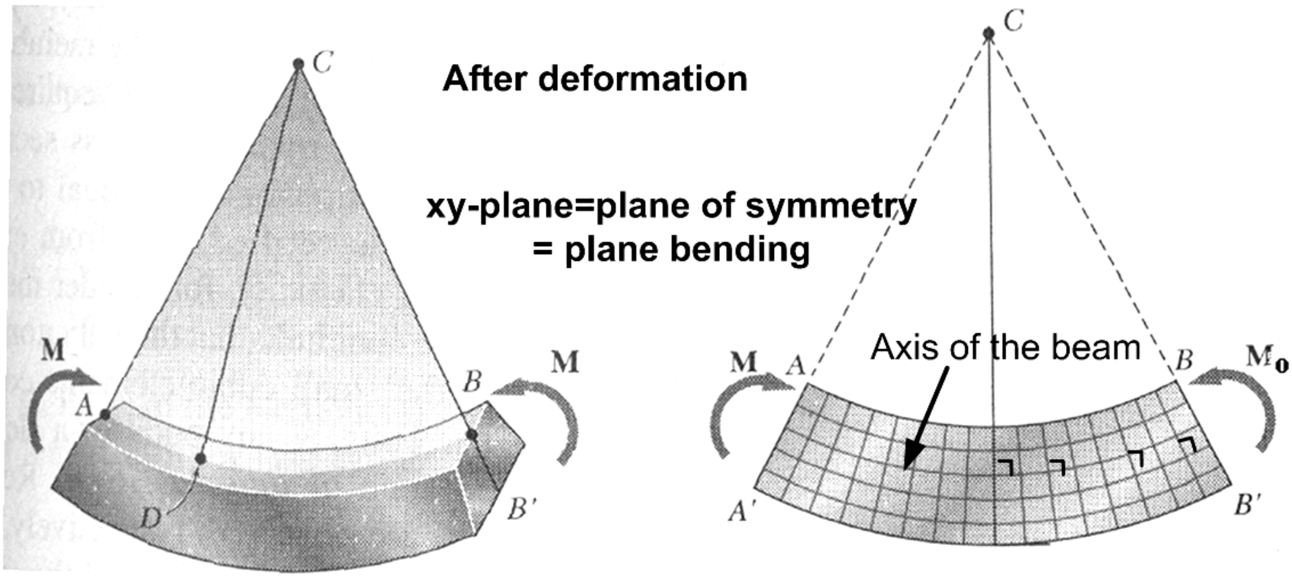
Thus, the state of stress at any point within a prismatic beam (cross-section having an axis of symmetry) subjected to pure bending is a uniaxial state of stress.

$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Deformations in a symmetric member in pure bending

Since the member is subjected to bending moments, it will bend under the action of these couples.





Since, the prismatic member possessing a plane of symmetry (i.e xy- plane) and subjected to equal and opposite couples M_0 acting in the plane of symmetry, the member will bend in the plane of symmetry (i.e xy plane).

The curvature k at a particular point on the axis of the beam depends on the bending moment at that point. Therefore a prismatic beam in pure bending will have constant curvature.

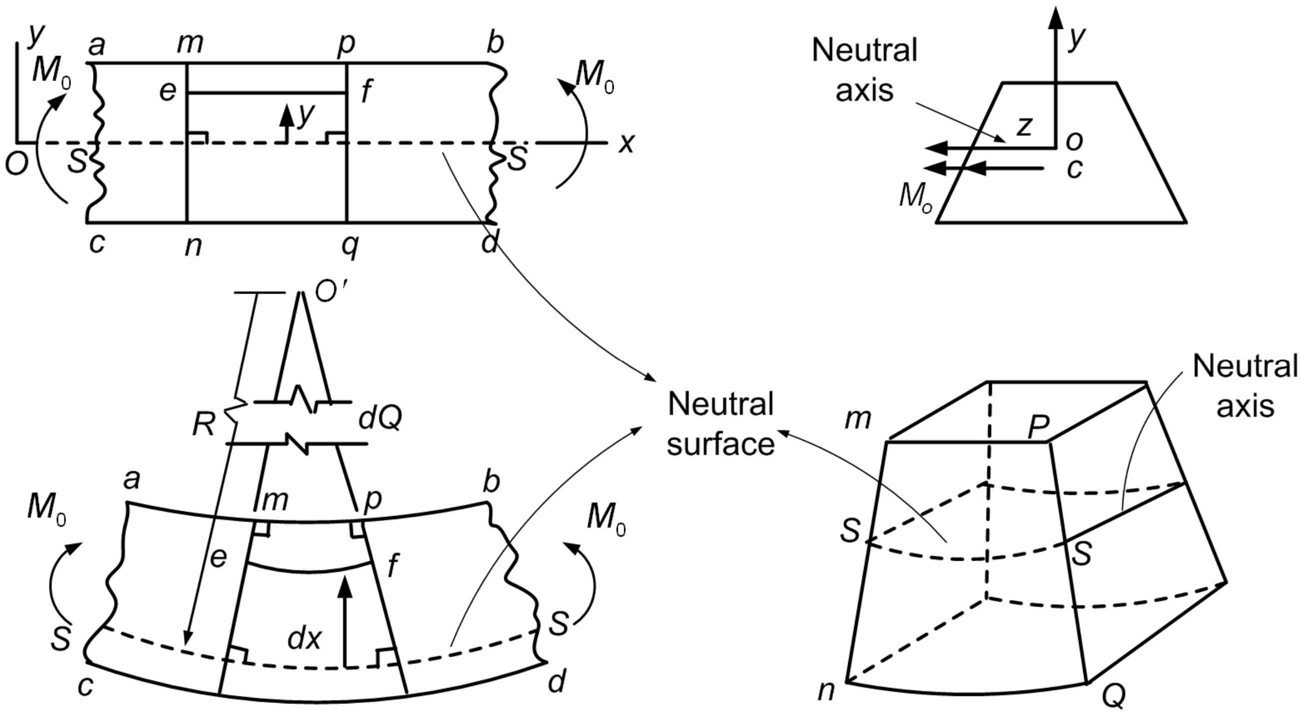
The line AB, which was originally a straight line, will be transformed in to a circle of center O and so the line $A'B'$.

Decrease in length of AB and increase in length of $A'B'$ in positive bending.

Cross-sections which are plane and ⊥ to the axis of the undeformed beam, remain plane and remain ⊥ to the axis of the deformed beam i.e to the deflection curve. Kinematic assumption

Variation of strain and $M-\kappa$ relation

Elementary theory of bending or Euler-Bernoulli theory



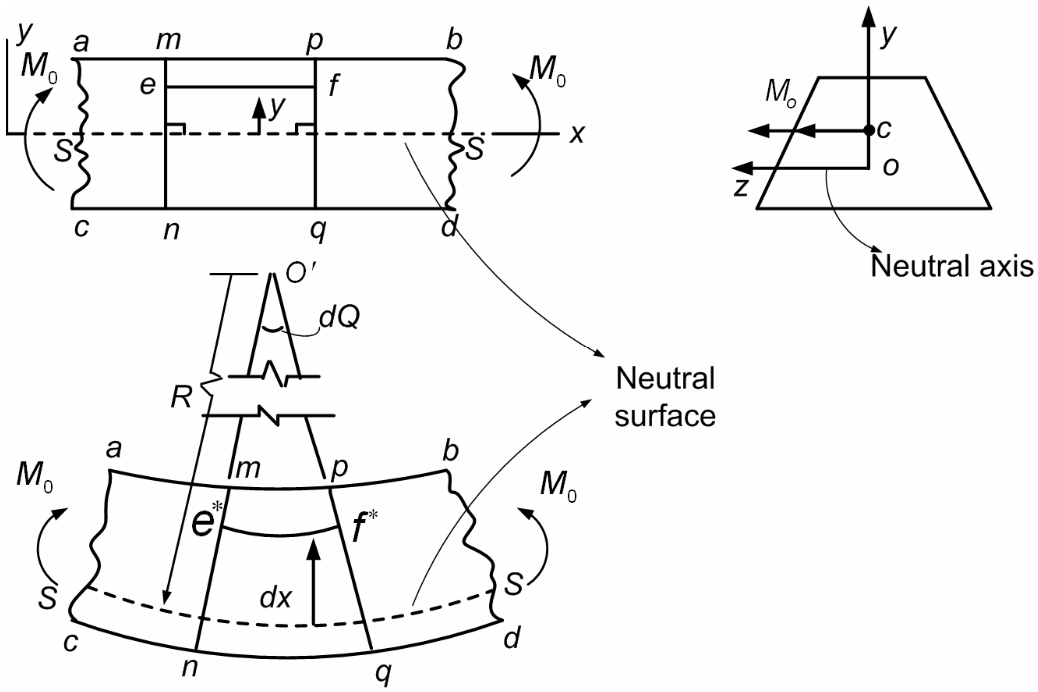
Under the action of M_0, the beam deflects in the xy – plane (plane of symmetry) and any longitudinal fibers such as SS bent into a circular curve. The beam is bent concave upward (due to +ve bending) upon which is a +ve curvature.

Cross-sections mn and pq remain plane and normal to the longitudinal axis of the beam. Cross-sections mn and pq rotate with respect to each other about z-axis.

Lower part of the beam is intension and upper part is in compression.

The x- axis lies along the neutral surface of undeformed beam

Variation of strain and M-k relations (contd.)



Initial length of fiber $ef = dx$

Final length of $ef = \overline{e^* f^*} = (R - y)dQ$

The distance dx between two planes is unchanged at the neutral surface,

$$RdQ = dx \Rightarrow k = \frac{1}{R} = \frac{dQ}{dx}$$

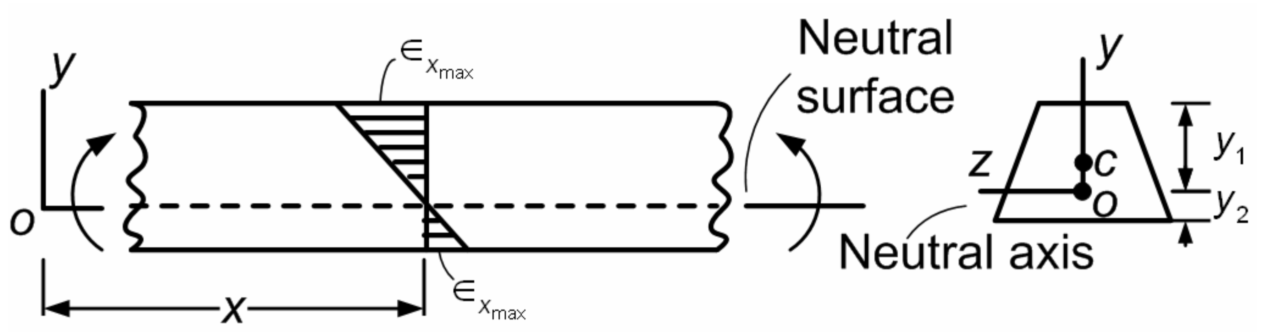
Therefore, the longitudinal strain i.e $\in_x$ at a distance "y" from the neutral axis is

$$\in_x = \frac{e^* f^* - \overline{ef}}{\overline{ef}} = \frac{(R-y)dQ - dx}{dx} = \frac{-y}{R}$$

$$\therefore \in_x = -\frac{y}{R} \Rightarrow \in_x = -ky$$

In case of pure bending $\in_x \neq \in_x (x \; and \; z), \in_x = \in_x (y)$

The preceding equation shows that the longitudinal strains $(\in_x)$ in the beam (in pure bending) are proportional to the curvature and vary linearly with the distance y from the neutral axis or neutral surface.



$\in_x = 0$ at the neutral surface

Maximum compressive $\in_x = \frac{-y_1}{R}$

Maximum tensile $\in_x = \frac{+y_2}{R}$

However, we still do not know the location of neutral axis or neutral surface.

Stresses in beams in pure bending :- For linearly elastic and isotropic beam material

$$\in_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right] \qquad \Upsilon_{xy} = \frac{\tau_{xy}}{G}$$

$$\in_y = \frac{1}{E}\left[\sigma_y - v\left(\sigma_x + \sigma_z\right)\right] \qquad \Upsilon_{yz} = \frac{\tau_{yz}}{G}$$

$$\in_z = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right] \qquad \Upsilon_{xz} = \frac{\tau_{zx}}{G}$$

The state of the stress at any point within a prismatic beam in pure bending is

$$\left[\sigma_{ij}\right] = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \quad \sigma_x = E\in_x = \frac{-Ey}{R} = -Eky$$

$$\in_y = -\frac{V}{E}\sigma_x = -V\in_x$$

$$\in_z = -\frac{V}{E}\sigma_x = -V\in_x$$

From the above equation

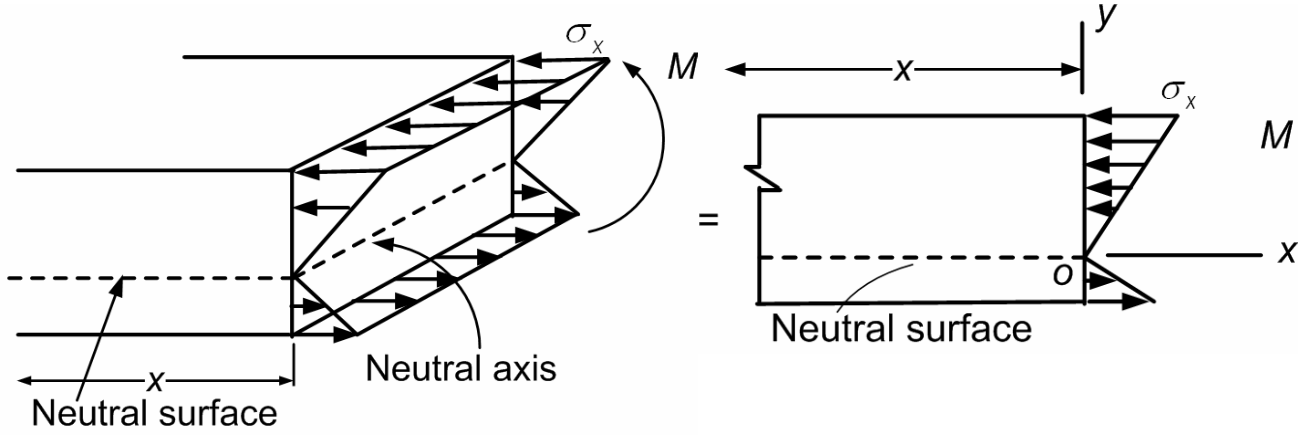
$$\sigma_x \neq \sigma(x,z)$$

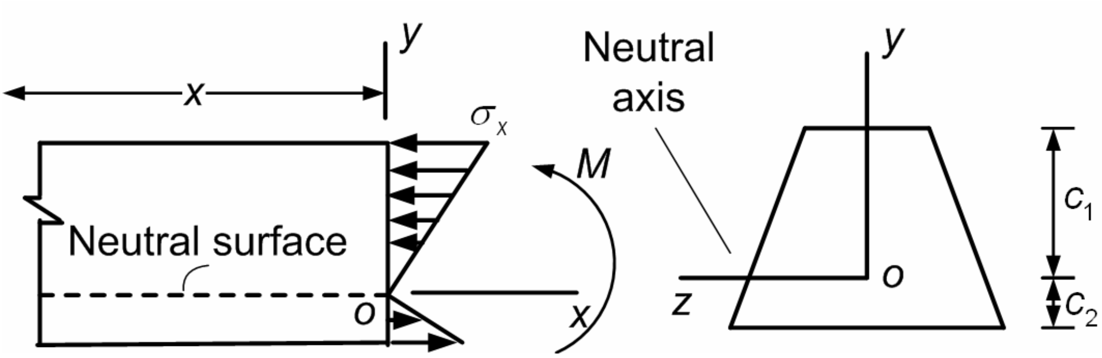
$$\sigma_x = \sigma(y) \quad \because \in_x = \in_x(y)$$

$$\because \in_x = linear\ f(y)$$

$$\therefore \sigma_x = linear\ f(y)$$

i.e., var y linearly with the distance y from the neutral surface





σ_x at $y = 0$ i.e on the neutral surface = 0

$$\text{Maximum compressive } \sigma_x = -\frac{EC_1}{R}$$

$$\text{Maximum tensile} \qquad \sigma_x = \frac{EC_2}{R}$$

Maximum normal stress σ_x occurs at the points farthest from the neutral axis.

In order to compute the stresses and strain we must locate the neutral axis of the cross-section.

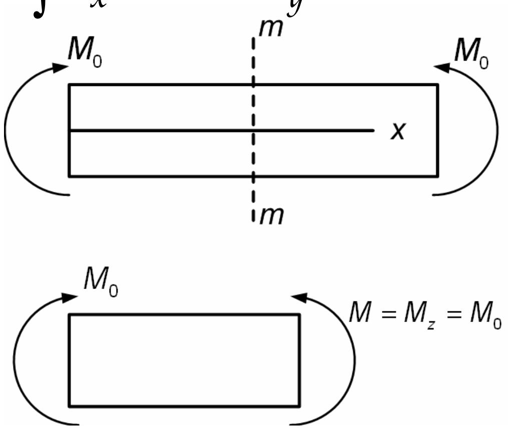
Location of neutral axis

We must satisfy the following equations at any given section m-m

$$\int \sigma_x dA = 0$$

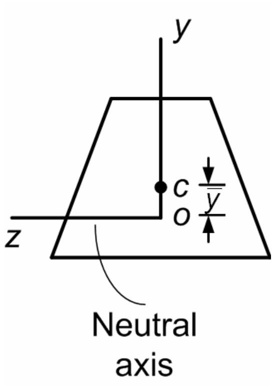
$$-\int \sigma_x y dA = M = M_0 = M_z$$

$$\int \sigma_x z dA = M_y = 0$$



Considering first equation

$$\int_A \sigma_x dA = -\int_A \frac{Ey}{R} = 0$$



$$\int_A y dA = 0$$

The above equation shows that the distance $\bar{y}$ between neutral axis and centroid "C" of a cross-section is zero.

In other words, the neutral axis i.e *z*-axis pass through the centroid of the cross-section, provided if the material follows Hooke's law.

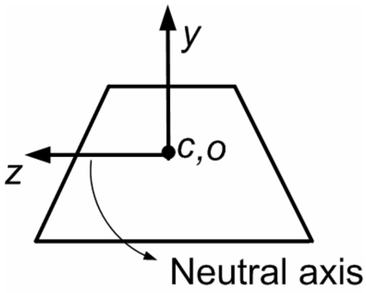
The origin 'O' of coordinates is located at the centroid of the cross-sectional area.

Thus, when a prismatic beam of linearly elastic material is subjected to pure bending, the y and z (neutral axis) axes are principal centroidal axes.

Moment – Curvature relationship

$$M = -\int_A \sigma_x y dA$$

$$M = +\int_A \frac{Ey}{R} y dA$$



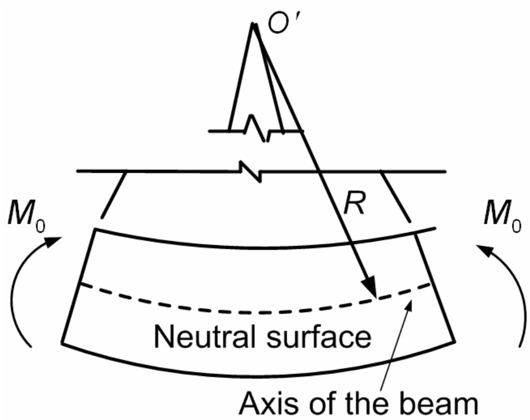
$$M = \frac{E}{R}\int_A y^2 dA$$

$\int_A y^2 dA = I_{zz} =$ Moment of inertia of cross-sectional area about neutral axis

$$\therefore \quad M = \frac{EI}{R}$$

$$k = \frac{1}{R} = \frac{M}{EI}$$

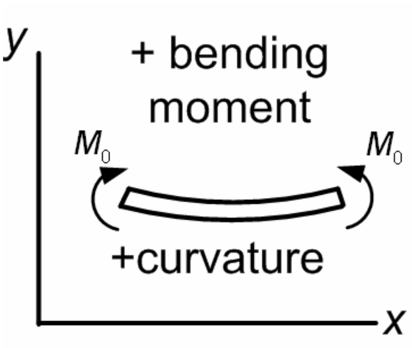
$$k = \frac{1}{R} = \frac{M_0}{EI}$$

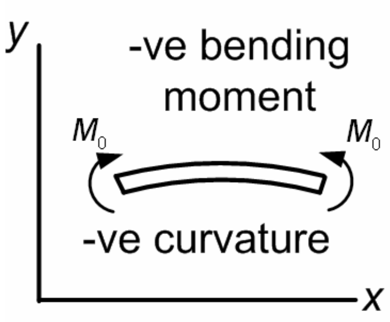


Moment-Curvature relation

Curvature k is directly proportional to M- internal bending moment and inversely proportional to EI- flexural rigidity of the beam.

Flexural rigidity is a measure of the resistance of a beam to bending.





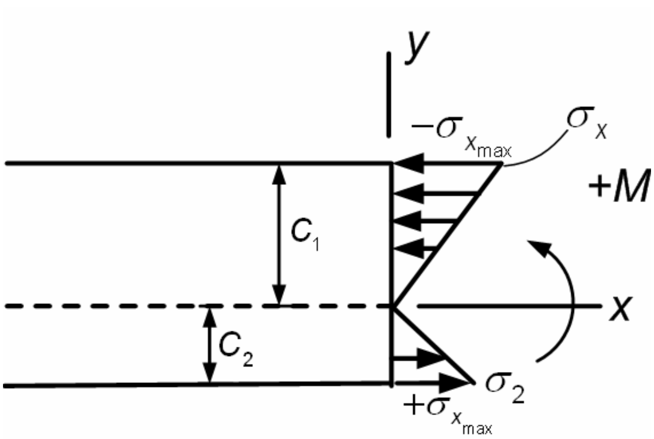
Relation between σ_x and M - Flexure formula

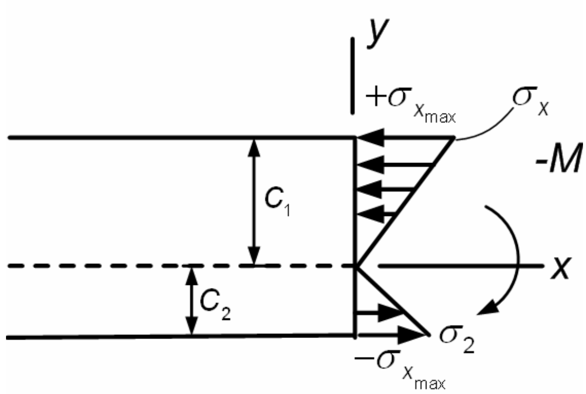
$$\sigma_x = -Eky$$

and $k = \dfrac{M}{EI}$

$$\therefore \quad \sigma_x = -\frac{My}{I}$$ - flexure formula.

Stresses evaluated from flexure formula are called bending stresses or flexural stresses.





The maximum tensile and compressive bending stresses occur at points located farthest from the neutral axis.

The maximum normal stresses are

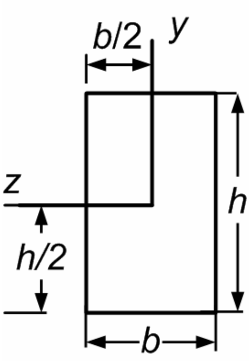
$$\sigma_1 = \frac{-MC_1}{I} = -\frac{M}{S_1} \qquad \sigma_2 = \frac{MC_2}{I} = \frac{M}{S_2}$$

$$S_1 = \frac{I}{C_1} \quad and \quad S_2 = \frac{I}{C_2}$$ -Section moduli

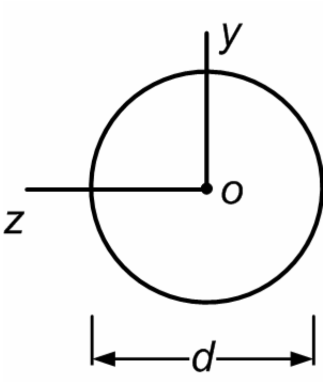
S =Section modulus

Cross- sectional properties of some common shapes

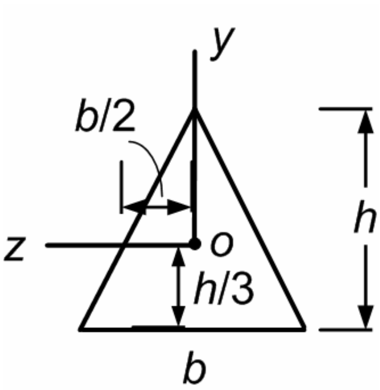
$z-$ axis – neutral axis



$$I_{zz} = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6}$$

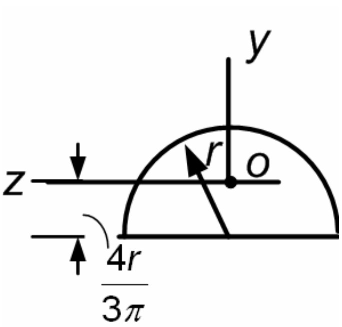


$$I_{zz} = \frac{\pi}{64} d^4 \qquad S = \frac{\pi d^3}{32}$$



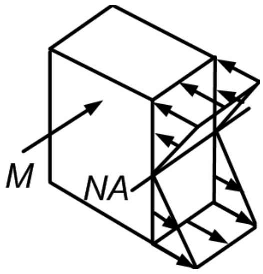
$$I_{zz} = \frac{bh^3}{36}$$

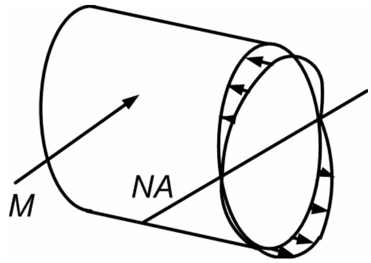
$$h = \sqrt{3}\, b / 2 \quad \textit{for eqilateral triangle}$$

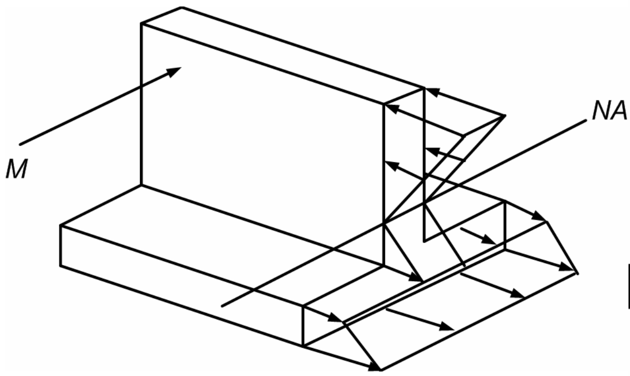


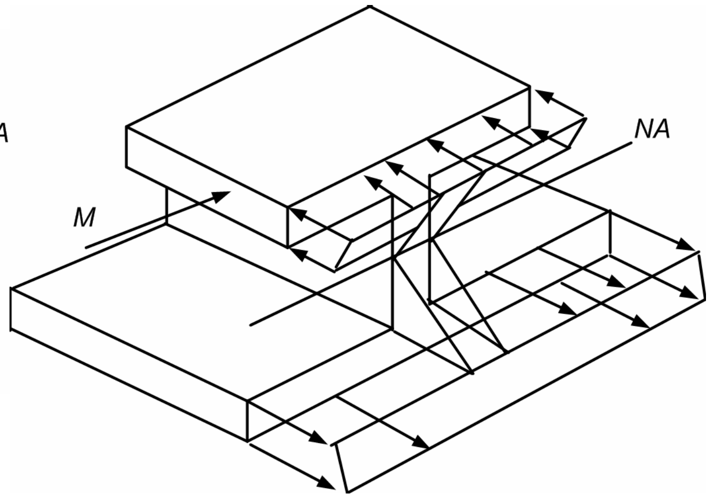
$$I_{zz} = 0.1098 r^4$$

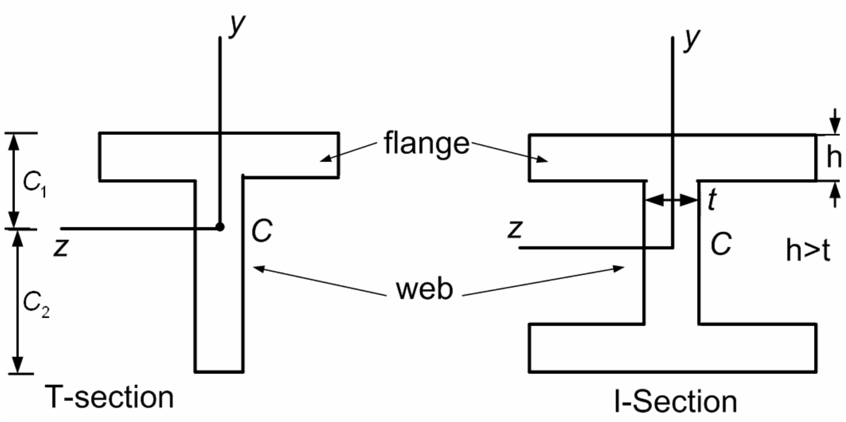
Distribution σ_x on various cross-sections







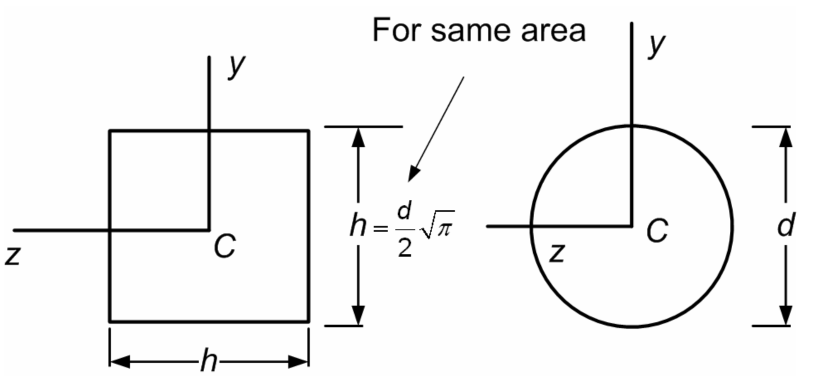




$$\sigma_{max} = \frac{M}{S}$$

$$S = \frac{I}{y_{max}}$$

$$M = \sigma_{alllow} S$$

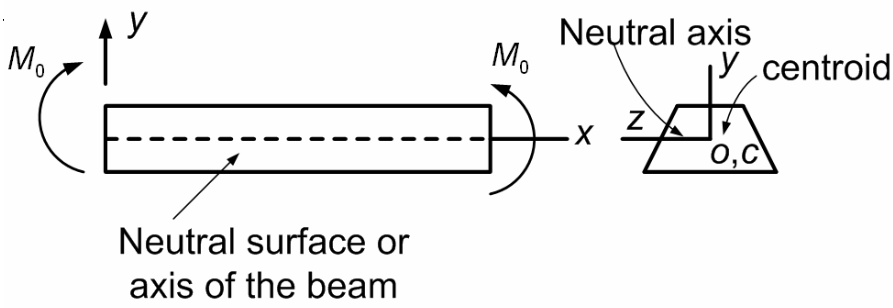


$$\frac{S_{square}}{S_{circle}} = 1.18$$

- This result shows that a beam of square cross-section is more efficient in resisting bending then circular beam of same area.
- A circle has a relatively larger amount of material located near the neutral axis. This material is less highly stresses.
- I- Section is more efficient then a rectangular cross-section of the same area and height, because I- section has most of the material in the flanges at the greatest available distance from the neutral axis.

Extension of results

Long prismatic beam under pure bending, and symmetrical



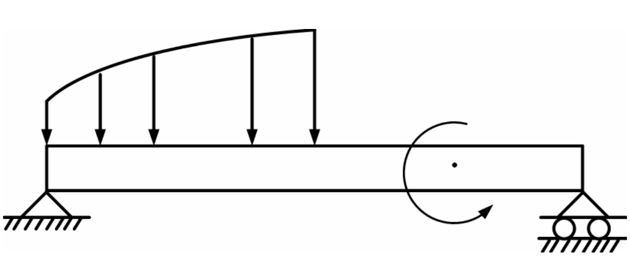
$$M \neq M(x)$$
$$M = Constant$$

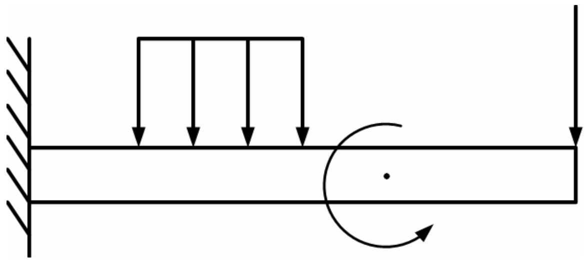
$$\sigma_x(y) = -\frac{My}{I}$$
$$I = I_{zz}$$
$$k = \frac{1}{R} = \frac{M}{EI}$$

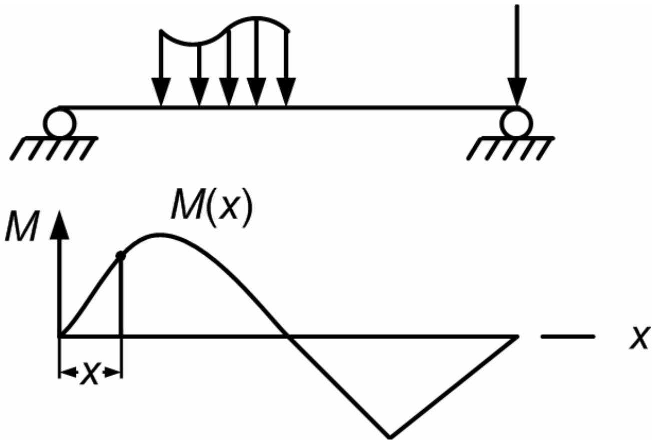
$$\in_x = \frac{\sigma_x}{E}$$
$$\in_y = -v \in_x$$
$$\in_z = -v \in_z$$

Elementary theory of bending

Bending of beams due to applied lateral loads







$$\frac{dM}{dx} = -V$$

Consider now a beam subjected to typical arbitrary transverse loads acting. In this case the interval bending moment $M = M(x)$ and $V(x) \neq 0$, and thus we have non-uniform bending.

Non-uniform bending is a result of presence of transverse shear force $V(y)$. If $V(y) = 0$ then $M = constant$.

It can be shown that the above results can also be used for non-uniform bending problems.

$$\sigma_x(x,y) = \frac{-M(x)y}{I}$$

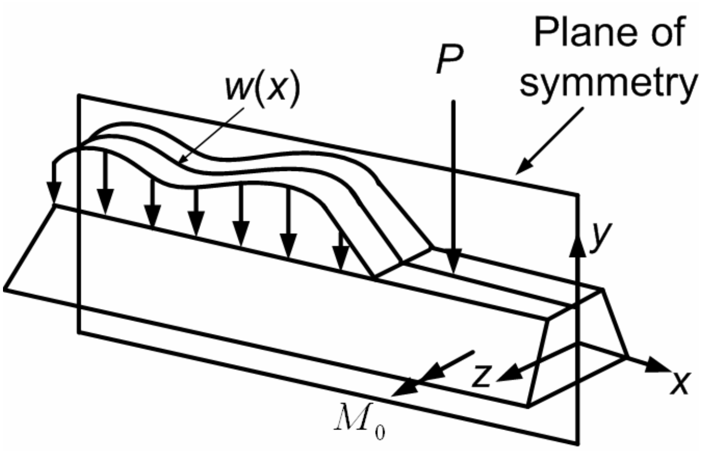
$$k = \frac{1}{R(x)} = \frac{M(x)}{EI}$$

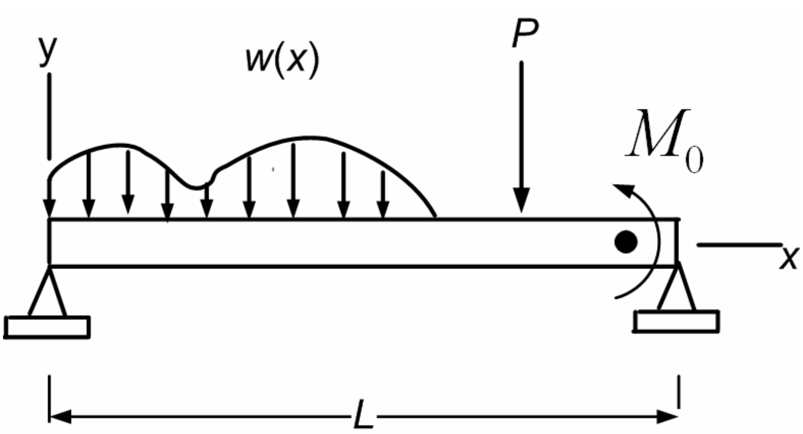
$$\in_x(x,y) = \frac{\sigma_x(x,y)}{E}$$

$$\in_y = -\nu \in_x$$

$$\in_z = -\nu \in_x$$

The above results can also be used for non-uniform bending problems provided if they satisfy the following conditions.





- The cross-sections should have y-axis of symmetry
- All applied transverse or lateral loads should lie in the x-y plane of symmetry and all applied couples act about z-axis only.
- $L \gg h \quad --long\ slender\ beams$
- *Bending that conforms to conditions (i) and (ii) is called symmetrical bending.*

If these three conditions are satisfied then one can employ the following expressions for non-uniform bending as-well

$$\sigma_x(x,y) = -\frac{M(x)y}{I}$$
$$I = I_{zz}$$
$$k(x) = \frac{1}{R(x)} = \frac{M(x)}{EI}$$

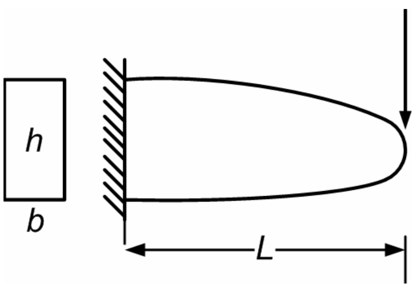
$$\in_x(x,y) = \frac{\sigma_x}{E}$$
$$\in_y(x,y) = -v\in_x$$
$$\in_z(x,y) = -v\in_z$$

Application of above equations to the non-uniform bending problems is equivalent to the following two assumptions.

(a)That even under such loading conditions, plane sections still remain plane after deformation and they remain ⊥ to the deformed longitudinal axis or neutral surface.

<u>Bending stresses in a non-prismatic beam</u>

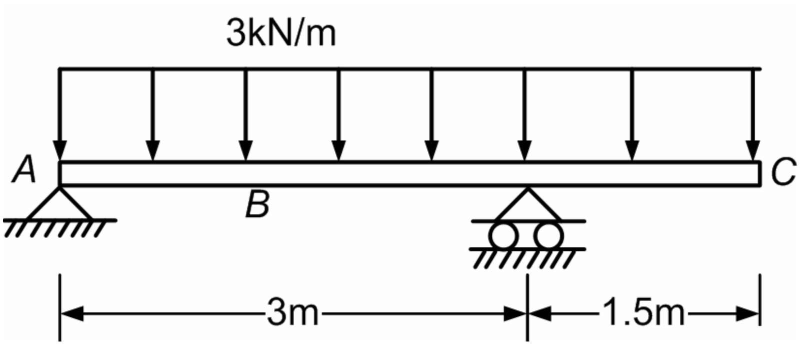
The above equation can also be applied to the case of non-prismatic beam subjected to either pure or non-uniform bending, provided cross-sectional properties do not vary sharply.

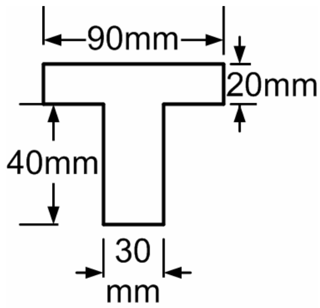


$$\sigma_x = -\frac{M(x)y}{I(x)}$$
$$k(x) = \frac{1}{R(x)} = \frac{M(x)}{EI(x)}$$

Problem

Determine the maximum tensile and compressive stresses in the beam due to the uniform load.



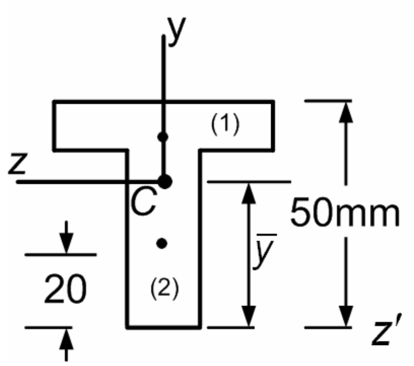


Solution

Centroid :-

	$A\ mm^2$	$\bar{y}$	$\bar{y}A\ mm^3$
1	$20\times 90 = 1800$	50	90×10^3
2	$40\times 30 = 1200$	20	24×10^3

$A = \Sigma A = 3000 \quad \Sigma\bar{y}A = 114\times 10^3$

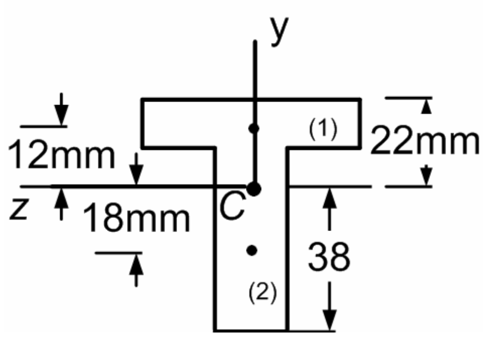


$$A\bar{y} = \Sigma\bar{y}A \Rightarrow \bar{y}3000 = 114\times 10^3 \Rightarrow \bar{y} = 38mm$$

$$I_{zz} = I = \Sigma\left(\bar{I} + Ad^2\right)s$$

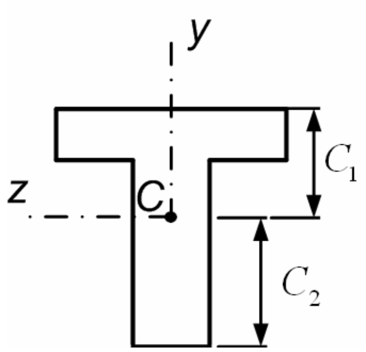
$$= \Sigma\left(\frac{bh^3}{12} + Ad^2\right)$$

$$= \frac{1}{12}90\times 20^3 + 1800\times 12^2 + \frac{1}{12}\times 30\times 40^2 + 1200\times 18^2$$



$$I_{zz} = I = 868\times 10^3\ \ mm^4 = 868\times 10^{-9} m^4$$

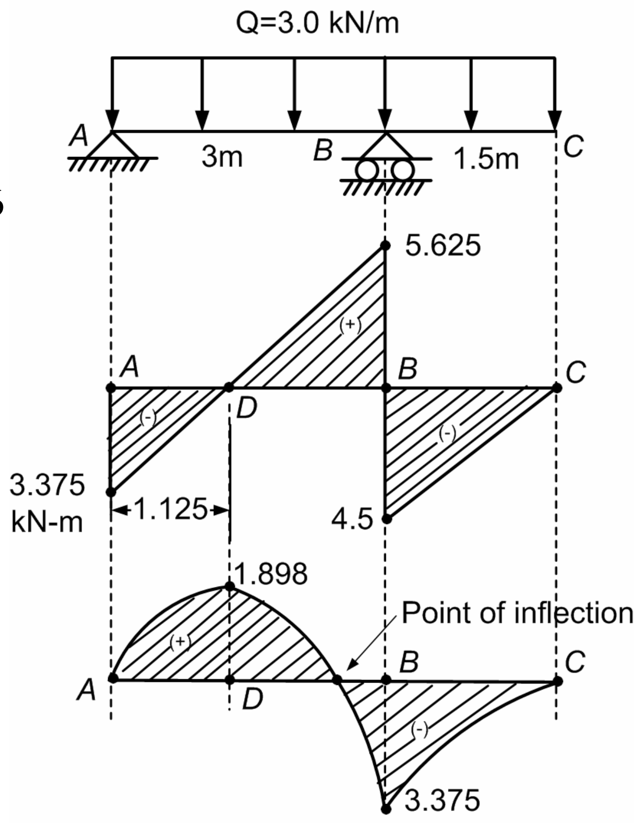
$C_1 = 22\,mm$ and $C_2 = 38\,mm$



$$\sigma_x = -\frac{My}{I}$$

$$\sigma_{max} = \frac{M}{S} : S = \frac{I}{y_{max}}$$

At maximum +ve bending moment i.e at (D)



$$S_1 = \frac{I}{C_1} = \frac{868\times10^{-9}}{22\times10^{-3}} = 39.45\times10^{-6}$$

$$S_2 = \frac{I}{C_2} = \frac{868\times10^{-9}}{38\times10^{-3}} = 22.84\times10^{-6}$$

at D:

$$\sigma_{t_{max}} = \frac{M}{s_2} = \frac{1.898}{22.84\times10^{-6}}$$

$$\sigma_{t_{max}} = 83.1\,MPa$$

$$\sigma_{C_{max}} = \frac{M}{s_1} = \frac{1.898}{39.45\times10^{-6}}$$

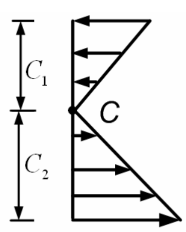
$$\sigma_{C_{max}} = 48.11\,MPa$$

At maximum -ve moment i.e at (B)

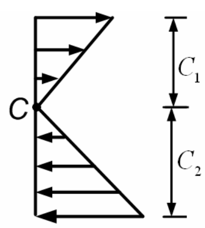
$$\sigma_{t_{max}} = \frac{M}{s_1} = \frac{3.375}{39.45\times10^{-6}} = 85.55\ MPa$$

$$\sigma_{C_{max}} = \frac{M}{s_2} = \frac{3.375}{22.84\times10^{-6}} = 147.8\ MPa$$

$$\sigma_{t_{max}} = 85.55 \ \ and \ \ \sigma_{C_{max}} = 147.8\ MPa$$



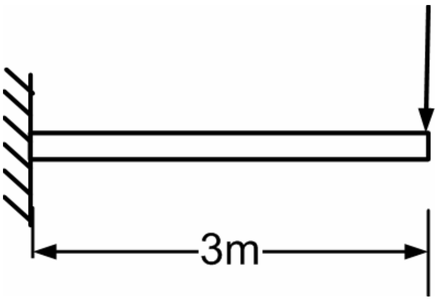
Cross-section (D)

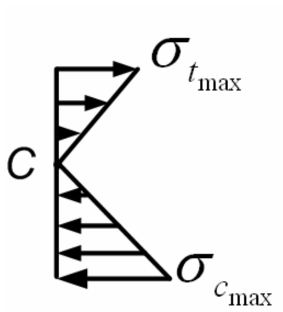


Cross-section (B)

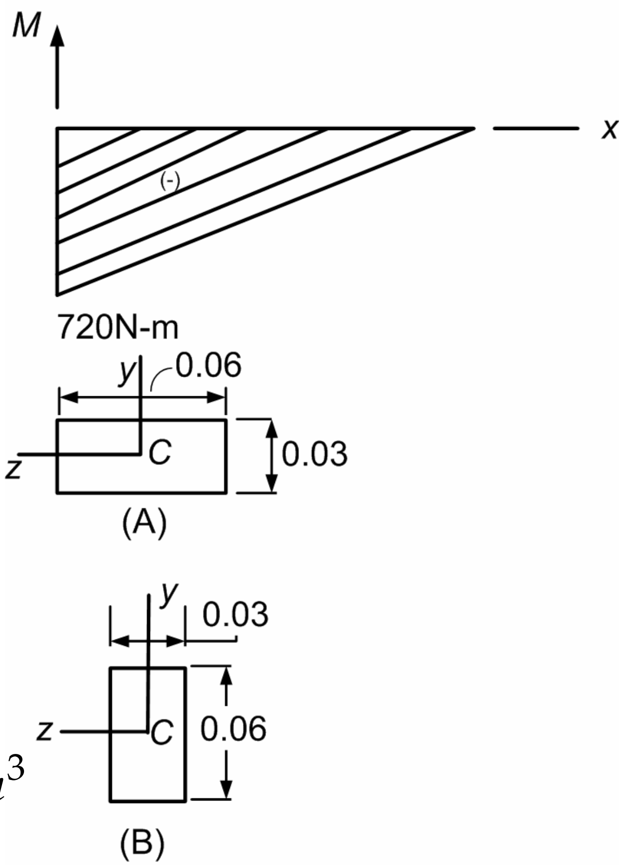
Problem

a wooden member of length $L = 3\text{m}$ having a rectangular cross-section 3 cm × 6 cm is to be used as a cantilever with a load $P = 240\text{N}$ acting at the free end. Can the member carry this load if the allowable flexural stress both in tension and in compression is $\sigma_{allow} = 50$ Mpa ?





Solution



$$M_{max} = 720\ N\text{-}m$$

$$S_A = \frac{1}{12}\frac{0.06 \times 0.03^3}{0.015} = 9 \times 10^{-6} m^3$$

$$\sigma_{t_{max}} = \sigma_{C_{max}} = \frac{M}{S_A} = \frac{PL}{S_A}$$

$$\sigma_{t_{max}} = \sigma_{C_{max}} = \sigma_{allow}$$

$$P_{alow} = \frac{\sigma_{allow} \times S_A}{L} = 150N$$

$$S_B = \frac{1}{12}\frac{0.03 \times 0.06^3}{0.03} = 1.8 \times 10^{-5} m^3$$

$$P_{alow} = \frac{\sigma_{allow} \times S_B}{L} = 300N$$

$\therefore$ The beam can carry $P = 240N$ only when oriented as in (B)

Limitations

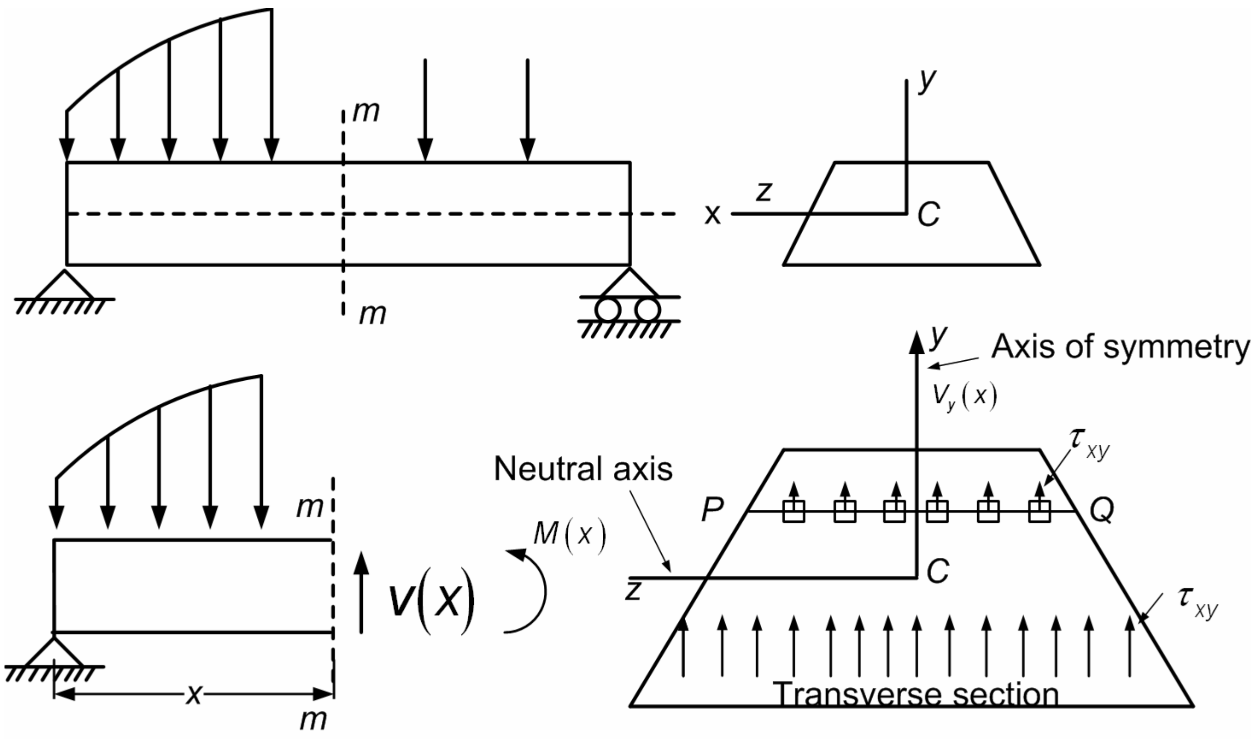
(1)The flexure formula is exact for a prismatic beam in pure bending.

(2)It provides very good approximation of σ_x for long slender beams $(L >> h)$ under symmetrical bending.

(3)The flexure formula can be employed for any shape of the cross-section, provided the cross-section has y-axis of symmetry.

(4)It should not be employed in regions close to geometric discontinuities and concentrated loads.

16. Shear Stresses in Beams



$$V_y(x) = \int_A \tau_{xy}\, dA$$

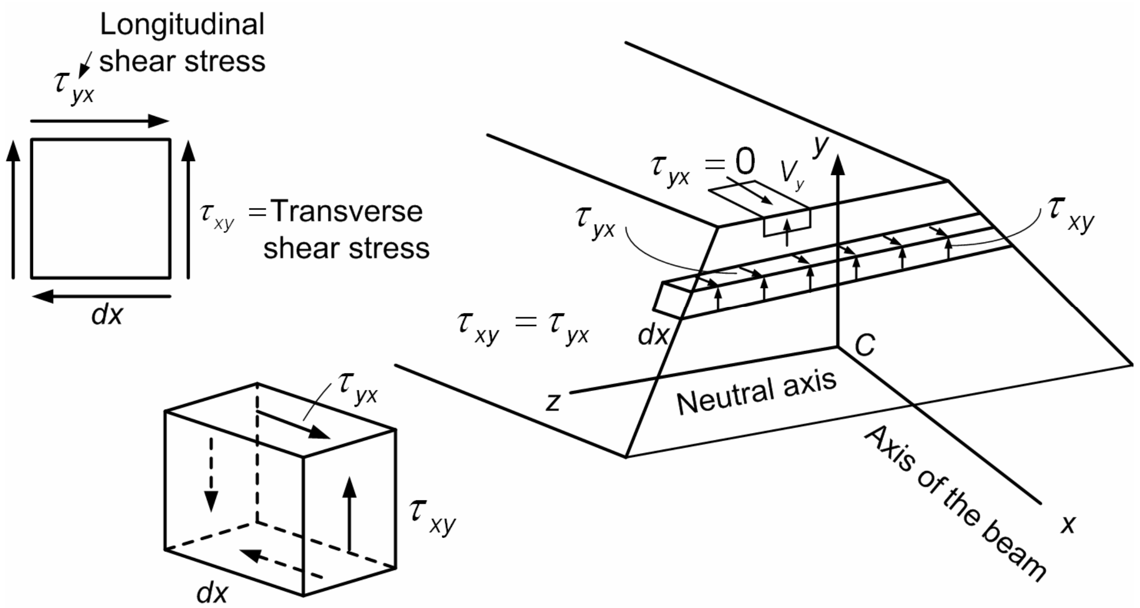
It is reasonable to assume that

(1)The shear stresses acting on the cross-section are parallel to the shear force $V_y(x)$ i.e $\perp$ to the line $\overline{PQ}$

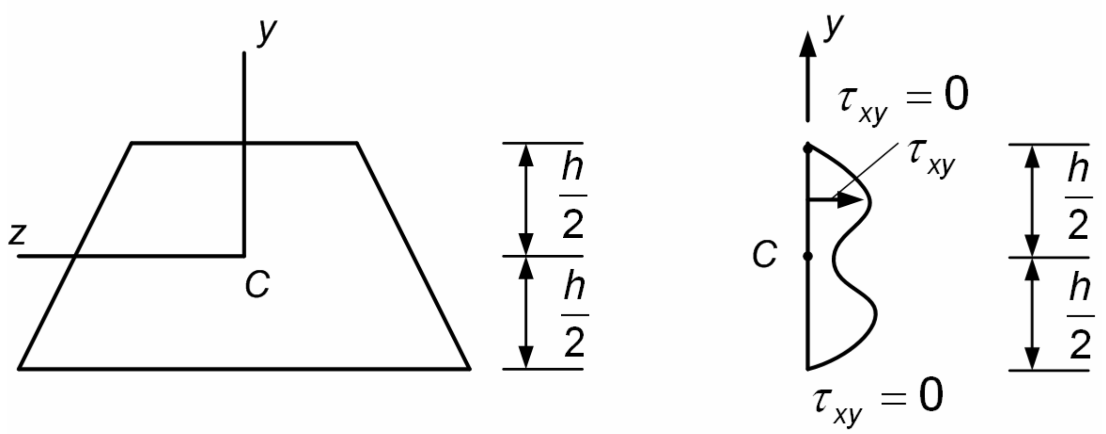
(2)It is also reasonable to assume that the shear stresses τ_{xy} are uniformly distributed across the width of the beam, so that $M_x = T = 0$ for symmetrical bending

$$\therefore\quad \tau_{xy} = \tau_{xy}(x, y) \quad \textit{such thats}$$

$$V_y(x) = \int_A \tau_{xy}(x, y)\, dA$$

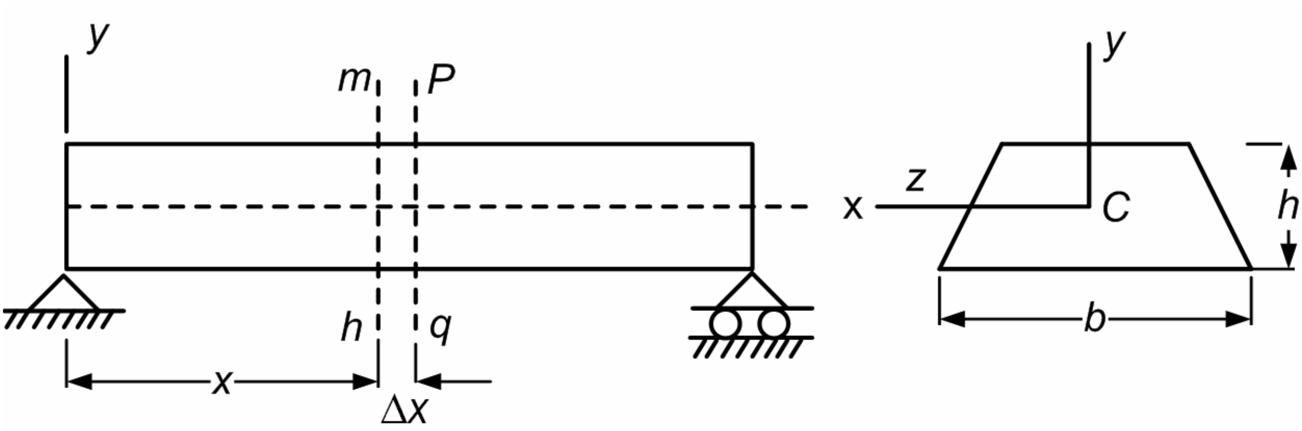


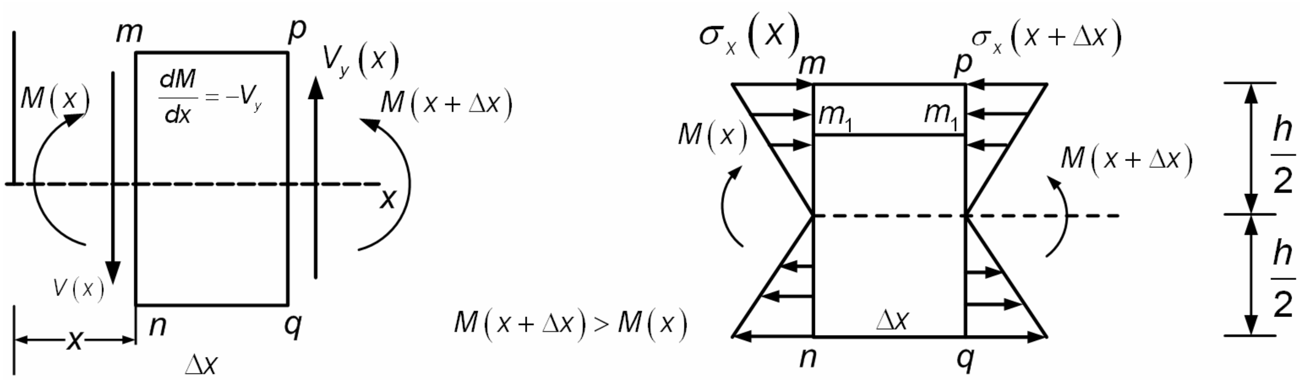
- Thus, there are horizontal shear stresses (or longitudinal shear stresses) acting between horizontal layers of the beam as well as vertical shear stresses acting on the cross-sections.
- At any point of the beam $\tau_{xy} = \tau_{yx}$
- Pattern of distribution of τ_{xy} =pattern of distribution of τ_{yx}
- Since $\tau_{xy} = \tau_{yx}$, it follows that the vertical shear stresses τ_{xy} must vanish at $y = \pm\frac{h}{2}$, if the beam is subjected only lateral loads.

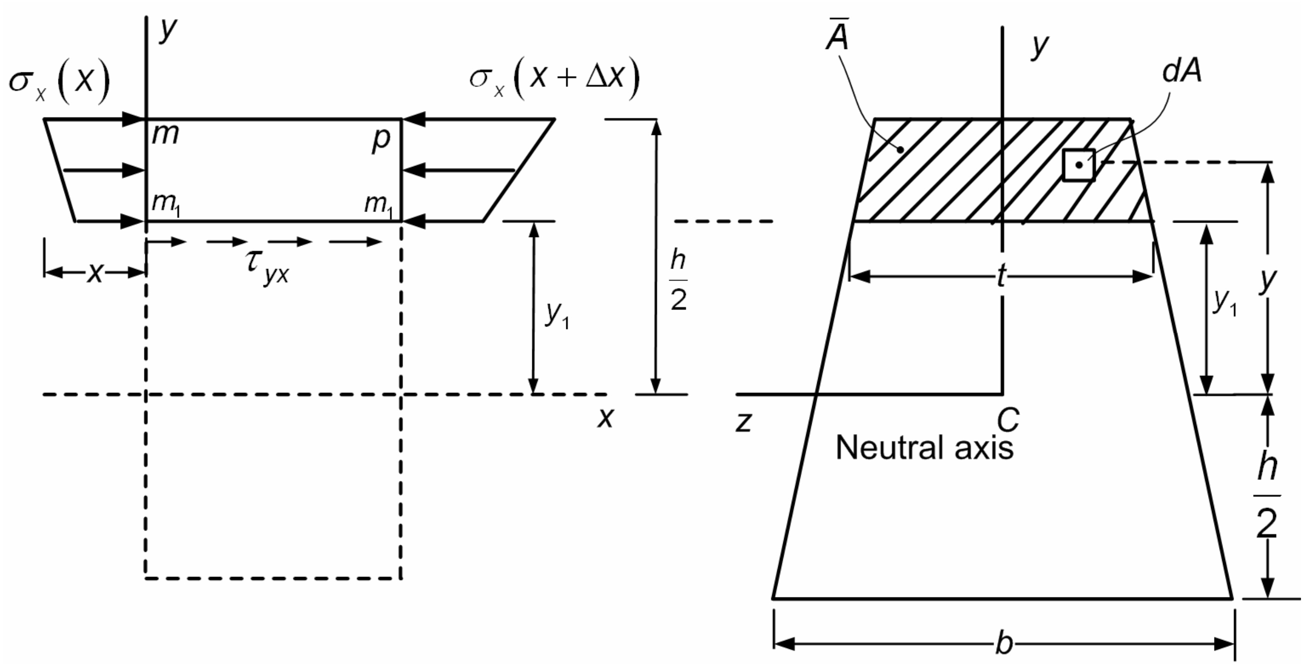


Derivation of shear stress formula

Beam under non-uniform bending i.e $M = M(x)$







t = width or thickness of the beam at $y = y_1$

We now wish to satisfy equilibrium in the x- direction.

Taking $[\Sigma F_x \rightarrow + = 0]$ we have then

$$-\int_A \sigma_x(x+\Delta x, y)\,dA + \int_A \sigma_x(x,y)\,dA + \tau_{yx} t \Delta x = 0$$

$$\tau_{yx} t = \frac{1}{\Delta x}\left[\int_A \sigma_x(x+\Delta x, y)\,dA - \int_A \sigma_x(x,y)\,dA\right]$$

$$\sigma_x(x,y) = \frac{-M(x)y}{I}$$

$$\tau_{yx} t = \frac{1}{\Delta x}\left[-\frac{1}{I}\int_A M(x+\Delta x)y\,dA + \frac{1}{I}\int_A M(x)y\,dA\right]$$

$$\tau_{yx} t = -\frac{1}{\Delta x I}\left[M(x+\Delta x) - M(x)\int_A y\,dA\right]$$

$$\tau_{yx} = \frac{-1}{It}\left[\frac{M(x+\Delta x) - M(x)}{\Delta x}\right]\int_A y\,dA$$

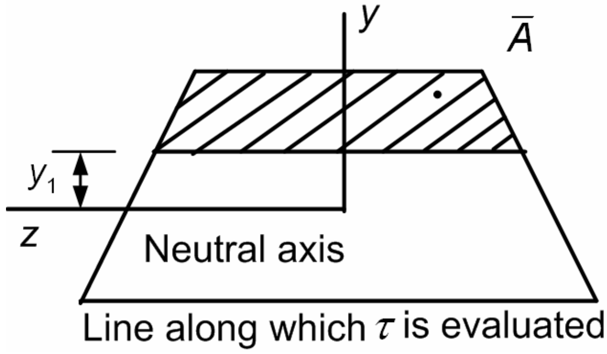
taking limit as $\Delta x \rightarrow 0$

$$\tau_{yx} = \frac{-1}{It}\lim_{\Delta x \rightarrow 0}\frac{M(x+\Delta x) - M(x)}{\Delta x}\int_A y\,dA$$

$$\tau_{yx} = \frac{-1}{It}\frac{dM}{dx}\int_A y\,dA$$

$$\frac{dM}{dx} = -V_y(x)$$

$$\therefore \quad \tau_{yx} = \frac{V_y(x)}{It}\int_A y\,dA$$



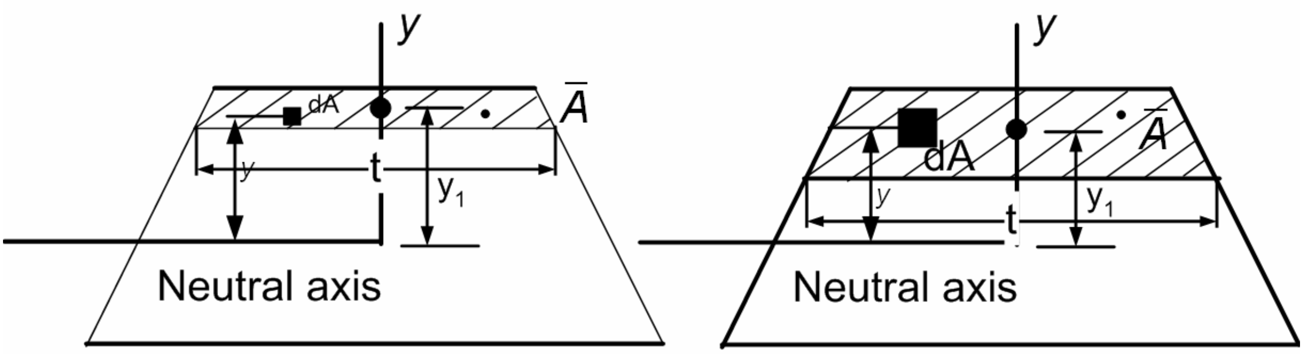
The above integral is by definition the first moment of are $\bar{A}$ about the z-axis, we denote it by symbol Q.

$$Q = \int_{\bar{A}} y dA$$

$$\therefore \quad \tau_{yx} = \tau_{xy} = \tau = \frac{V_y Q}{It} \qquad (1)$$

shear formula

in the above equation $I = I_{zz}$ stands for the moment of inertia of the entire cross sectional area around the neutral axis.



$$Q = \int_{\bar{A}} y\, dA = \bar{A}\, y_1 \qquad\qquad Q = \int_{\bar{A}} y\, dA = \bar{A}\, y_1$$

From (1)

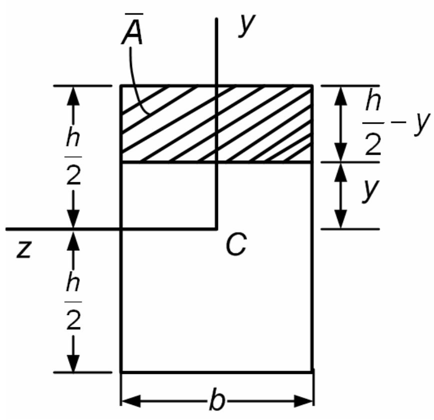
$$\tau_{yx} t = f = \frac{V_y Q}{I} = \frac{VQ}{I}$$

The quantity "f" is known as the "shear flow".

Shear flow is the horizontal shear force per unit distance along the longitudinal axis of the beam.

Distribution of shear stresses in a Rectangular beam

An example of application of equations



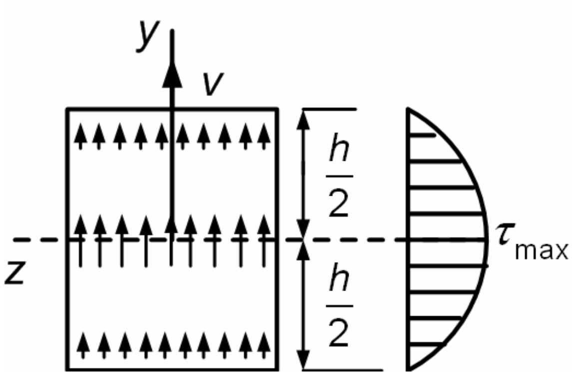
$$Q=\int_{\bar{A}} u\,dA=b\left(\frac{h}{2}-y\right)\left[y+\frac{h/2-y}{2}\right]s$$

$$Q=\frac{b}{2}\left(\frac{h^2}{4}-y^2\right)$$

$$I=\frac{1}{12}bh^3$$

$$\tau_{xy}=\tau_{yx}=\frac{VQ}{It}=\frac{V}{2I}\left(\frac{h^2}{4}-y^2\right)$$

at $y=\pm\frac{h}{2}$ $\quad \tau_{xy}=\tau_{yx}=0$

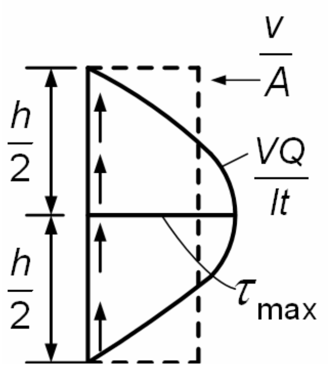


The shear stresses in a rectangular beam vary quadratically with the distance y from the neutral axis.

Maximum value of shear stress occurs at the neutral axis where Q is maximum.

$$\tau_{xy_{max}}=\tau_{yx_{max}}=\frac{Vh^2}{8I}=\frac{3}{2}\frac{V}{A}$$

Thus τ_{max} in a beam of rectangular cross-section is 50% larger than the average shear stress $\frac{V}{A}$



It is always possible to express the maximum shear stress τ_{xy} as

$$\tau_{xy_{max}} = K\frac{V}{A}$$

for most of the cross-sectional areas

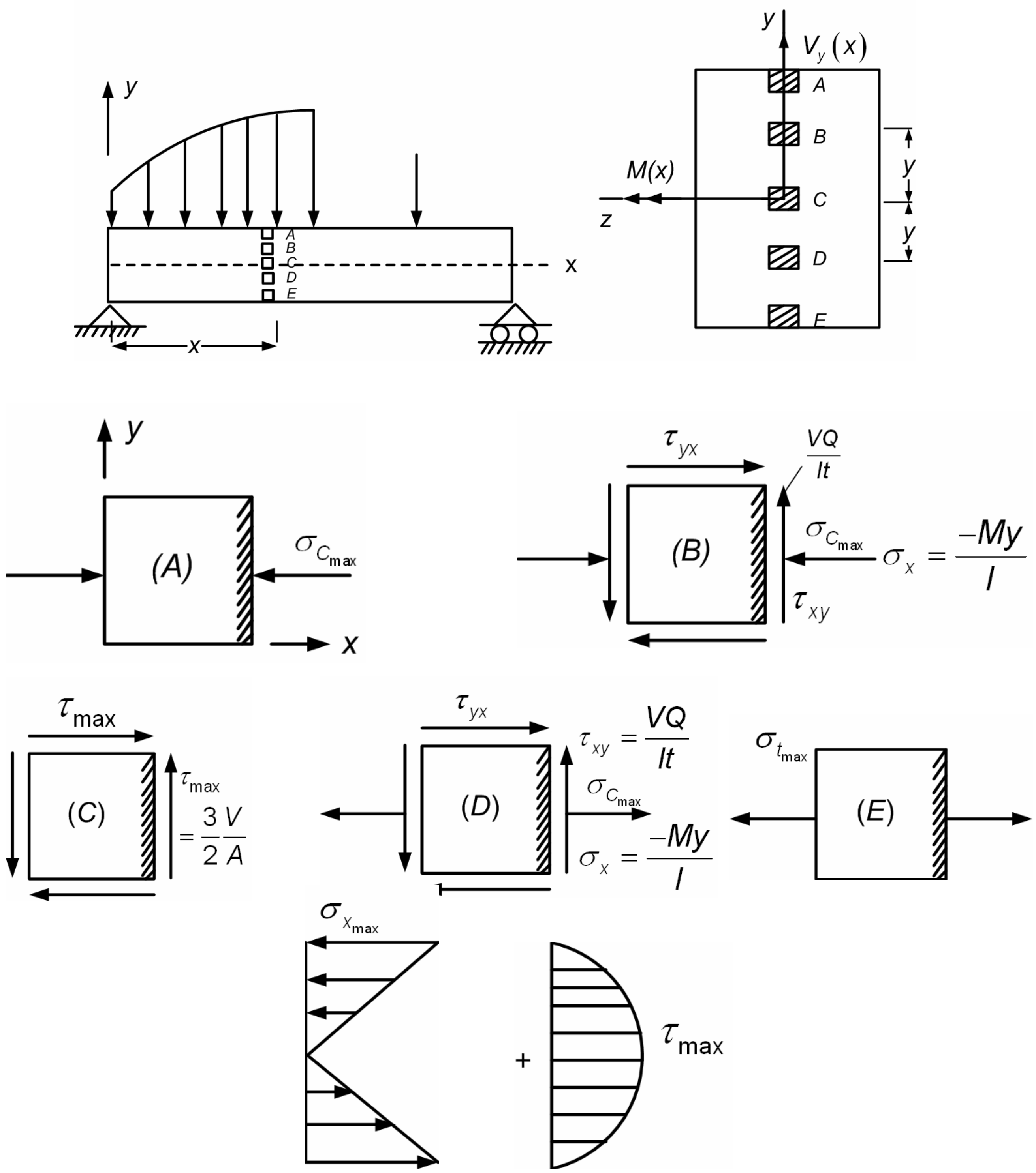
$$K = \frac{3}{2} \qquad Rec\tan gular$$

$$K = \frac{4}{3} \qquad Circular$$

$$K = \frac{3}{2} \qquad Triangular$$

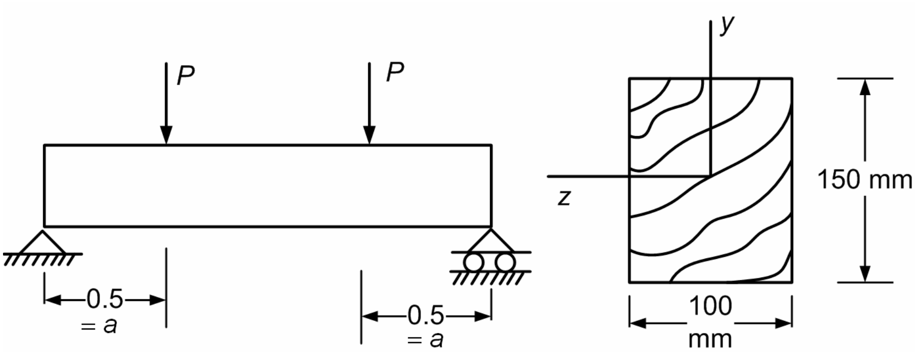
For most of the cross-section τ_{max} occurs at the neutral axis. This is not always true.

Stress elements in non-uniform bending



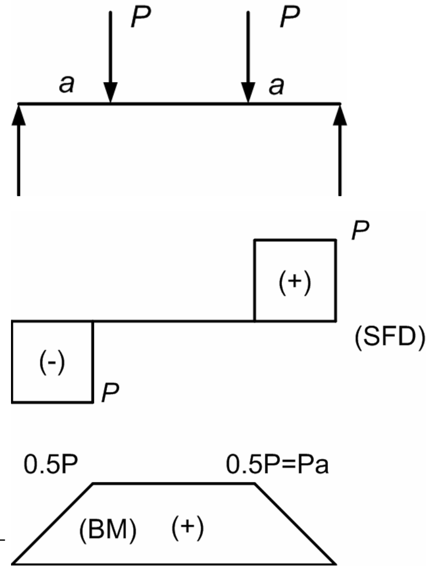
Problem

A wood beam AB is loaded as shown in the figure. It has a rectangular cross –section (see figure). Determine the maximum permissible value p_{max} of the loads if the allowable stress is bending is $\sigma_{allow} = 11MPa$ (for both tension and compression) and allowable stress in horizontal shear is $\tau_{allow} = 1.2MPa$



Solution

V_{max} occurs at supports and maximum BM occurs in between the loads.



$$V_{max} = P \qquad M_{max} = 0.5P = Pa$$

$$S = \frac{bh^2}{6} \qquad A = bh$$

$$\therefore \quad \sigma_{max} = \frac{M_{max}}{S} = \frac{6Pa}{bh^2}$$

$$\tau_{xy_{max}} = \tau_{yx_{max}} = \tau_{max} = \frac{3}{2}\frac{V_{max}}{A} = \frac{3}{2}\frac{P}{A} = \frac{3}{2}\frac{P}{bh}$$

Therefore, the maximum permissible values of the load P in dending and shear respectively are

$$P_{allow}\big|_b = \frac{\sigma_{allow}bh^2}{6a} \qquad P_{allow}\big|_s = \frac{2\tau_{allow}bh}{3}$$

Substituting numerical values into these formulas,

$$P_{allow}\big|_b = 8.25kN$$

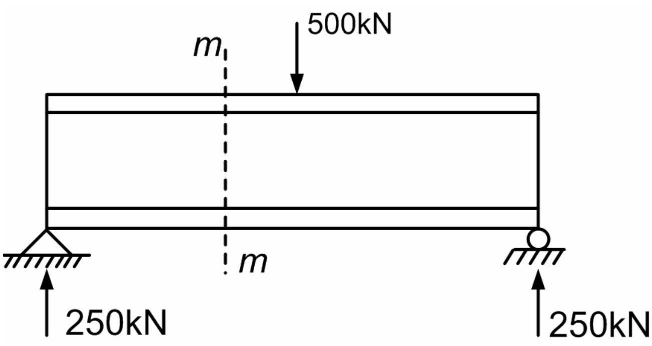
$$P_{allow}\big|_s = 8.25kN$$

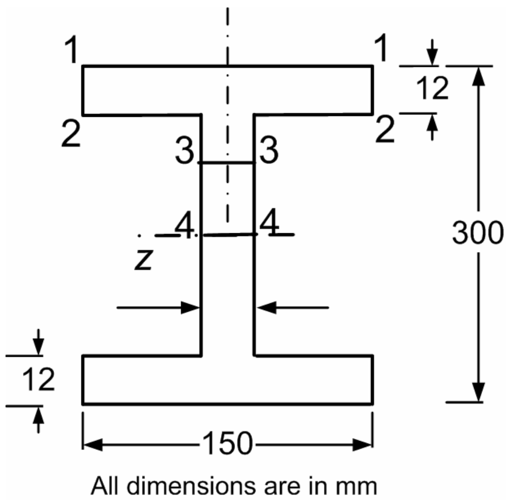
Thus bending governs the design and the maximum allowable load is

$$P_{max} = 8.25\,kN$$

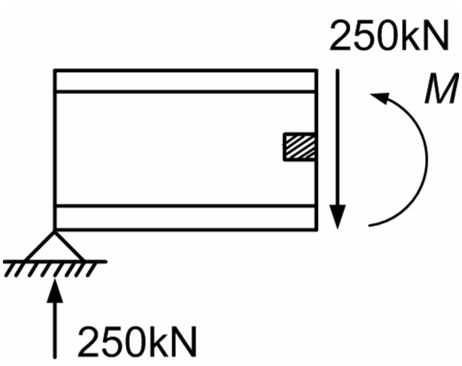
Problem

An I –beam is loaded as in figure. If it has the cross-section as shown in figure, determine the shearing stresses at the levels indicated. Neglect the weight of the beam.

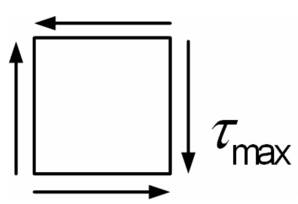


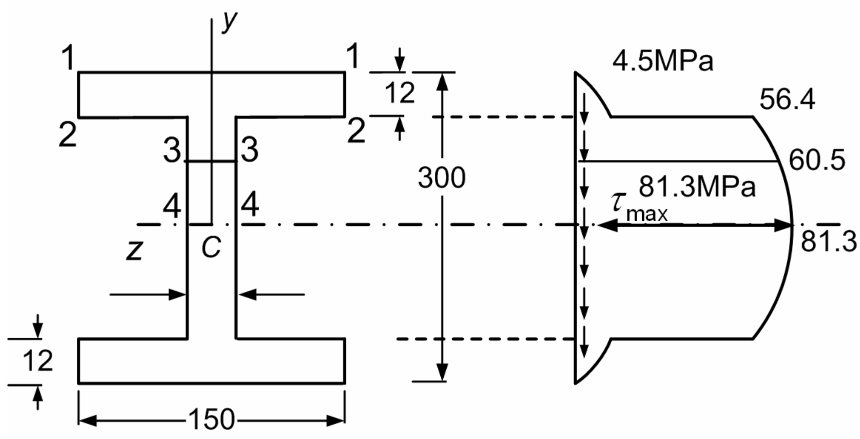


Solution



Vertical shear is same at all sections





$$I_{zz} = I = \frac{(150)(300)^3}{12} - \frac{(138)(276)^3}{12} = 95.7\times10^6 mm^4 s$$

The ratio $\frac{V}{I} = \frac{250\times10^3}{95.7\times10_6} = 2.61\times10^{-3} N/mm^4 s$

Level	$\bar{A}(mm^2)$	$\bar{y}$ mm	$Q=\bar{A}\bar{y}$ $\times10^3 mm^3$	t mm	$\tau_{xy}=\frac{VQ}{It}$ MPa
1-1	0	150	0	150	0
2-2	12×150 $=1800$	144	259.2	150 12	4.5 56.4
3-3	12×150 $=1800$ 12×12 $=144$	144 132	259.2 19.0 } 278.2	12	60.5
4-4	12×150 $=1800$ 12×138 $=1656$	144 69	259.2 114.3 } 373.5	12	81.3

$$\tau_{max} = 81.3\ MPa$$

16BTAR305 **SOLID MECHANICS** **3 0 0 3 100**

UNIT IV TORSION AND BEAM DEFLECTION

Analysis of torsion of circular bars – Shear stress distribution – Bars of Solid and hollow circular section – Stepped shaft – Twist and torsion stiffness – Compound shafts – Fixed and simply supported shafts – Application to close-coiled helical springs – Maximum shear stress in spring section including Wahl Factor – Elastic curve of Neutral axis of the beam under normal loads – Evaluation of beam deflection and slope: Double integration method, Macaulay Method

TEXT BOOKS

T [1] – R. K. Bansal (2010), "A Textbook of Strength of Materials, Laxmi Publications, New Delhi.

T [2] – R. S. Khurmi (2013), "Strength of Material", S. Chand Publications. New Delhi

REFERENCES

R [3] - Bedi D.S (1984), "Strength of Materials", S Chand and Co. Ltd., New Delhi

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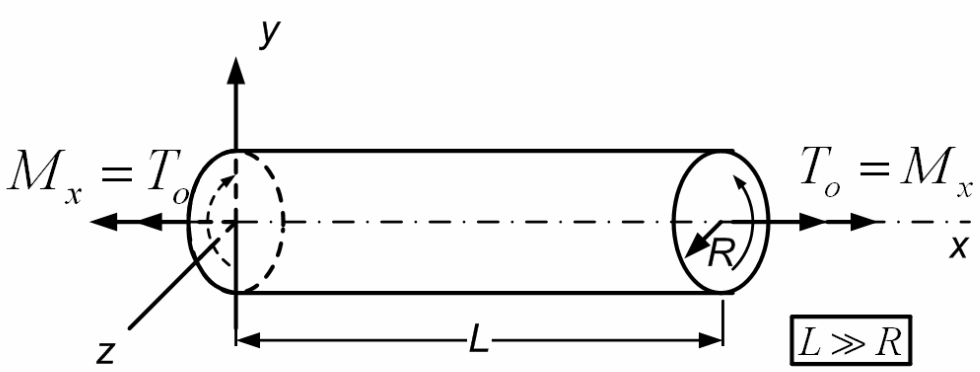
14. Torsion of circular bars

Geometry, loading and Material properties

A prismatic bar of circular cross- section subjected to equal and opposite torques acting at the ends.

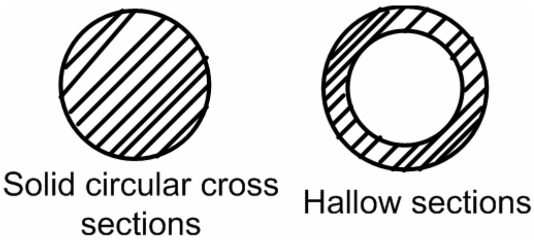


Whenever torques act on a member, then it will be twisted.

Torsion refers to the twisting of a straight bar when it is loaded by torques.

Material: Homogeneous, linearly elastic, and isotropic undergoing small deformations.

Presently theory is valid only for

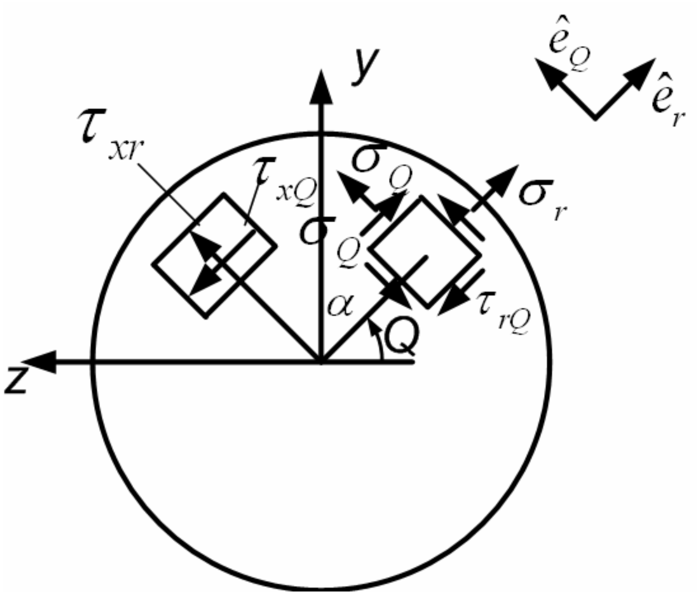


Stresses and strains in polar coordinates

Stresses, strains and displacements in polar coordinates.

Since we are dealing with a circular member it is preferable to use polar coordinates

$$\left[\sigma_{ij}\right]=\begin{bmatrix}\sigma_r & \tau_{r\theta} & \tau_{rx}\\ \tau_{\theta r} & \sigma_\theta & \tau_{\theta x}\\ \tau_{xr} & \tau_{x\theta} & \sigma_x\end{bmatrix}$$



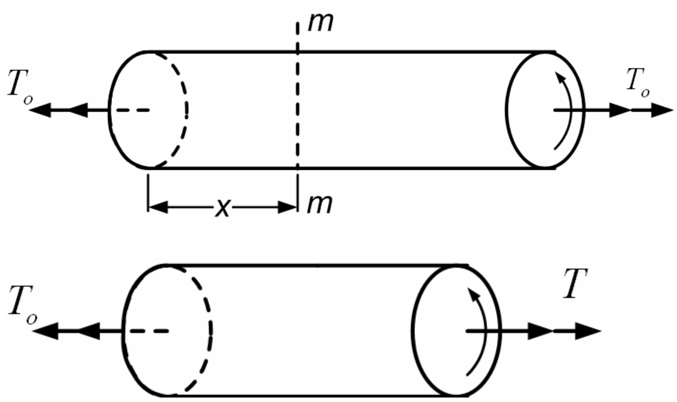
$$\in_x=\frac{1}{E}\left[\sigma_x-v\left(\sigma_r+\sigma_\theta\right)\right]$$

$$\in_r=\frac{1}{E}\left[\sigma_r-v\left(\sigma_x+\sigma_\theta\right)\right]$$

$$\in_x=\frac{1}{E}\left[\sigma\theta-v\left(\sigma_r+\sigma_x\right)\right]$$

$$Y_{r\theta}=\frac{\tau_{rQ}}{G};\ Y_{x\theta}=Y_{\theta x}=\frac{\tau_{x\theta}}{G};\ Y_{xr}=Y_{rx}=\frac{\tau_{rx}}{G}$$

Equilibrium and elementary forces

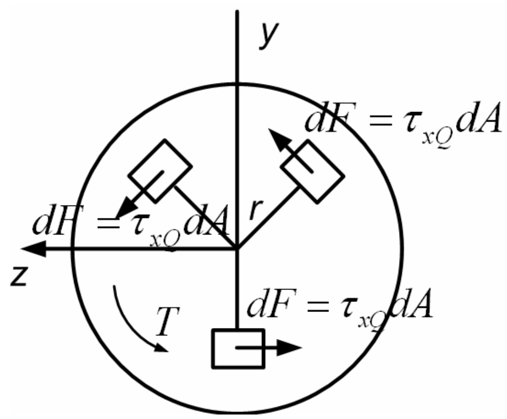


$$F_x=V_y=V_z=M_y=M_z=0$$

$$M_x=T=T_0$$

Since every cross-section of the bar is identical and since every cross-section is subjected to the same internal torque "T", then the bar is said to be under "pure torsion"

To keep the body under equilibrium, elementary forces $dF=\tau_{x\theta}dA$ are only forces that are required to be exerted by the other section, so that



$$dT = dF \times r = \sigma_{x\theta} r dA$$
$$T = \int_A \tau_{x\theta} r dA \qquad (1)$$
$$T = T_0$$

Direction of $\tau_{z\theta}$ can be obtained from the direction of internal torque T at that section.

The state of stress in pure torsion is therefore

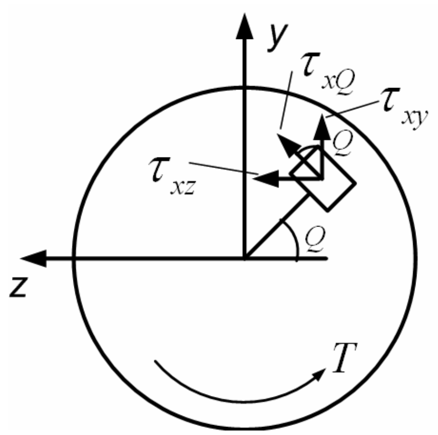
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \tau_{\theta x} \\ 0 & \tau_{x\theta} & 0 \end{bmatrix}$$

While the relation in (1) express an important condition that must be satisfied by the shearing stresses τ_{xQ} in any given cross-section of the bar it does not tell how these stresses are distributed in the cross-section.

The actual distribution of stresses under a given load is statically indeterminate. So we must know about the deformation of the bar.

Presence of $\tau_{x\theta}$ in polar coordinates means, presence of

$$\tau_{xy} = \tau_{xQ} Cos\theta$$
$$\tau_{xz} = \tau_{xQ} Sin\theta$$



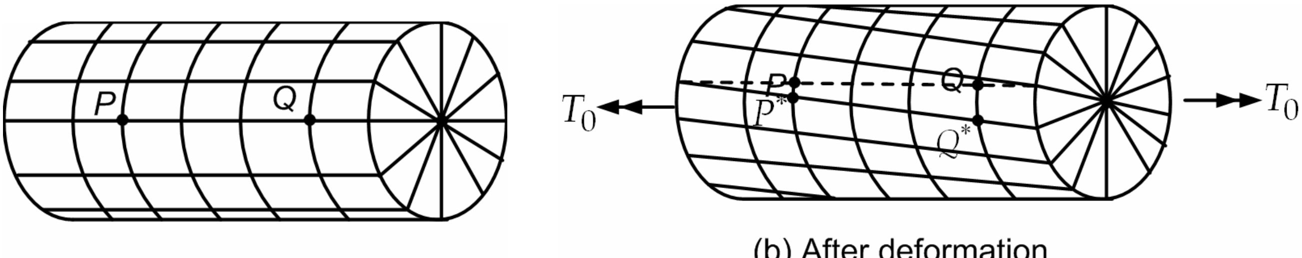
Therefore the state of stress in case pure torsion in terms of rectangular stress components is then

$$\begin{bmatrix} 0 & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & 0 & 0 \\ \tau_{zx} & 0 & 0 \end{bmatrix} \text{- state of pure shear.}$$

We must then ensure that

$$V_y = \int \tau_{xy} dA = 0$$
$$V_z = \int \tau_{xz} dA = 0$$

<u>Deformation in pure torsion</u>



(a) Before deformation (b) After deformation

Following observations can be made from the deformation of a circular bar subjected to equal and opposite end torques.

(1)The cross-sections of the bar do not change in shape i.e they remain circular.

(2)A line parallel to the x- axis or longitudinal line become a helical curve.

(3)All cross-sections remain plane.

(4)All cross-sections rotate about the axis of the bar as a solid rigid slab.

(5)However, various cross-sections along the bar rotate through different amount.

(6)The radial lines remain radial lines after deformation

(7)Neither the length of the bar nor the length of radius will change.

These are especially of circular bars only. Not true for non-circular bars.

Assumptions on deformation for pure torsion

(1)All cross –sections rotate with respect to the axis of the circular bar i.e x-axis.

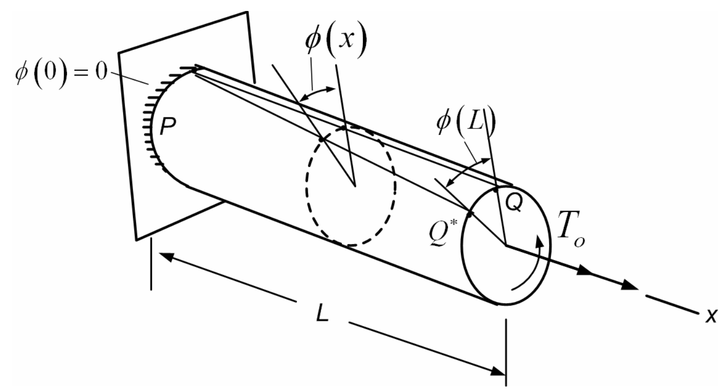
(2)All cross-sections remain plane and remain perpendicular to the axis of the bar.

(3)Radial lines remain straight after the deformation.

(4)Neither the length of the bar nor its radius will change during the deformation.

These assumptions are correct only if the circular bar undergoes "small deformations" only.

Variation of shear strain ($\Upsilon_{x\theta}$)

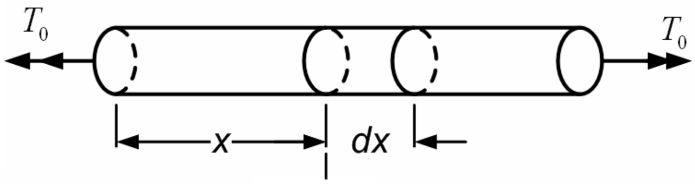


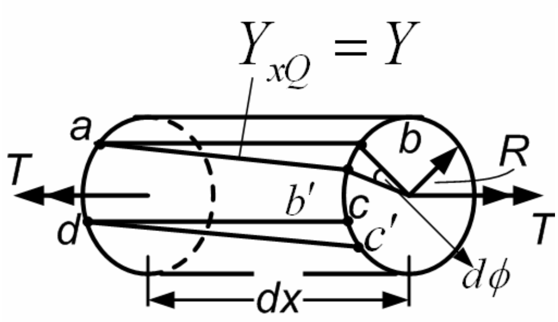
Because of T_0, the right end will rotate through an infinitesimal angle

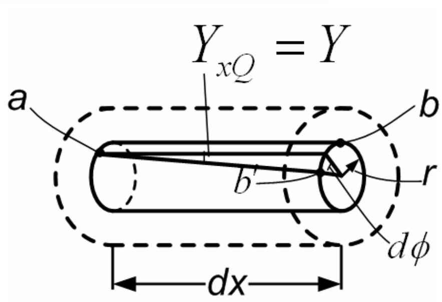
ϕ- angle of twist.

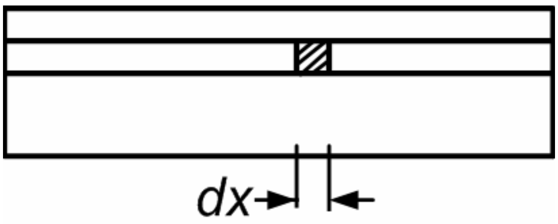
*ϕ - varies along the axis of the bar.

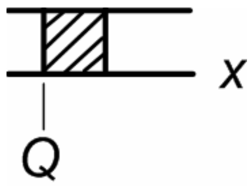
$$\frac{d\phi}{dx} = \odot - \text{rate of twist}$$ angle of twist per unit length.

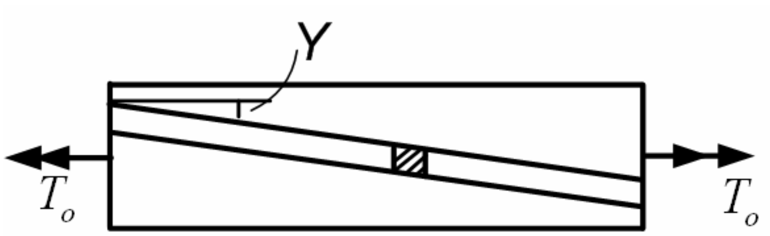


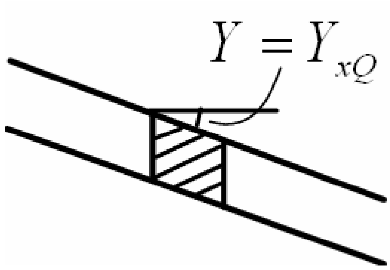




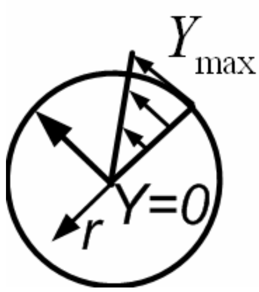








Y_{xQ} is independent of x and



$$Y_{xQ}dx = Ydx = rd\phi$$

$$Y = r\frac{d\phi}{dx}$$

In case of pure torsion the shear strain Υ varies linearly with "r"

Maximum shear strain Υ occurs at the outer surface of the circular bar i.e., $r = R$

$$\Upsilon_{max} = R\frac{d\phi}{dx}$$

Shear strain is zero at the center of the bar.

The equation $\Upsilon = r\dfrac{d\phi}{dx}$ is strictly valid to circular bars having small deformations.

If the material is linearly elastic

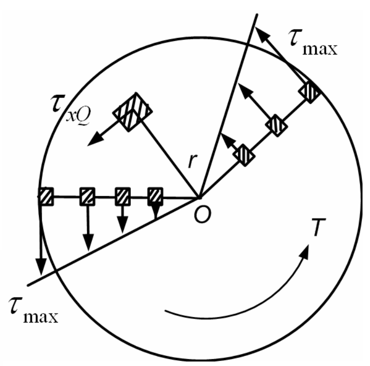
$$\tau = G\Upsilon$$

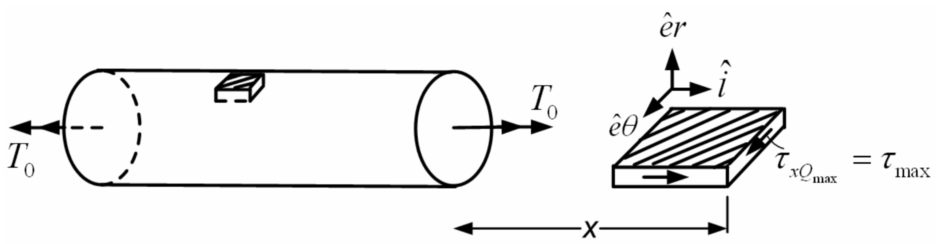
Therefore, variation of shear stress τ_{xQ} in pure torsion is given by

$$\tau = \tau_{xQ} = G\Upsilon_{xQ} = G\Upsilon\frac{d\phi}{dx}$$

Shear stress τ is only function of "r" and varies linearly with radius r of the circular bar.

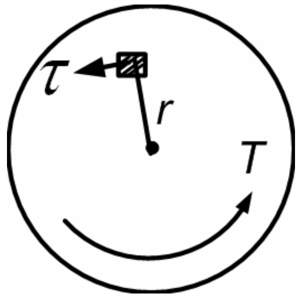
$$\tau_{max} = \tau_{xQ_{max}} = RG\frac{d\phi}{dx}$$





The torsion formula

Relation between internal torque T and shear stress τ



$$T = \int_A \tau r dA$$

$$T = \int G r \frac{d\phi}{dx} r dA$$

Since G & $\frac{d\phi}{dx}$ are independent of area A then

$$T = G \frac{d\phi}{dx} \int_A r^2 dA$$

$$I_P = \int_A r^2 dA$$

Polar moment of inertia of across – sec tion

For solid circular bar, $I_P = \frac{\pi}{32} D^4$ $I_P = \frac{\pi}{2} R^4$

$$\therefore \quad T = G I_P \frac{d\phi}{dx}$$

$$\therefore \quad \frac{d\phi}{dx} = \odot = \frac{T}{G I_P}$$

But $\tau = G r \frac{d\phi}{dx}$

$$\frac{\tau}{Gr} = \frac{T}{G I_P}$$

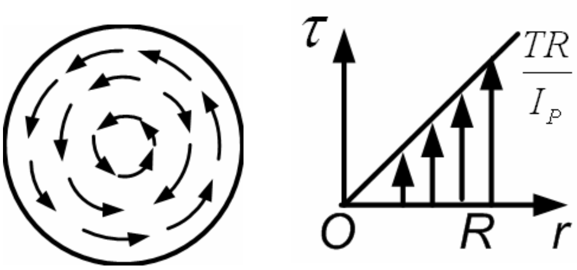
$$\tau = \frac{Tr}{I_P}$$

Torsion formula

This is the relation between shear stresses τ_{xQ} and torque T existing at the section.

Torsion formula is independent of material property.

$$\tau_{max} = \tau_{xQ_{max}} = \frac{TR}{I_P}$$

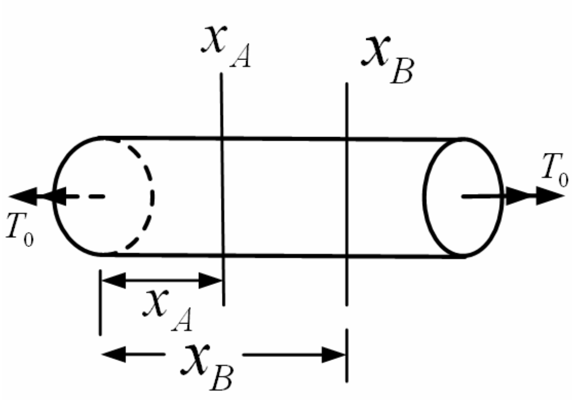


$$\tau_{max} = \frac{16T}{\pi D^3}$$

for solid circular bars

Angles of twist

We now determine the relative rotation of any two cross-sections



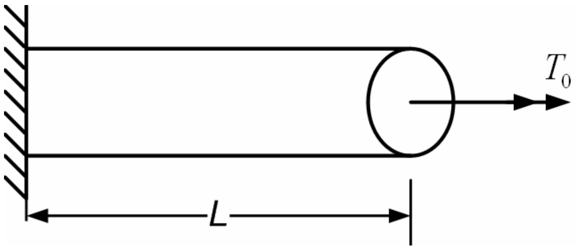
$$\odot = \frac{d\phi}{dx} = \frac{T}{GI_P}$$

$$\phi_{B/A} = \phi_B - \phi_A = \int_{x_A}^{x_B} \frac{T}{GI_P} dx$$

In case of prismatic circular bar subjected to equal opposite torques at the ends

$$\phi_{B/A} = \phi_B - \phi_A = \frac{TL}{GI_P} n$$
$$if \quad x_B - x_A = L$$
$$pure\, torsion$$

Direction of ϕ at a section is same as that of T



$$\phi = \frac{TL}{GI_P} = \frac{T_0 L}{GI_P}$$

Since $\odot = \frac{d\phi}{dx} = \frac{T}{GI_P}$ then, in case of pure torsion.

$$\odot = \frac{d\phi}{dx} = \frac{\phi}{L} = constant$$

Thus in case of pure torsion $\phi(x)$ varies linearly with x

In case of torsion

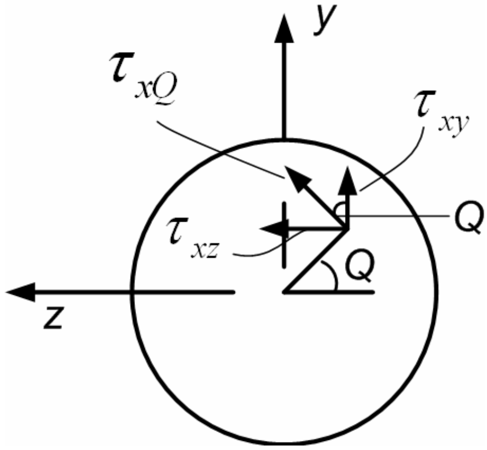
$$\phi = \frac{TL}{GI_P}$$

Load; displacement

$$k = \frac{GI_P}{L}; \; f = \frac{L}{GI_P}$$

The product GI_P – Torsional rigidity

$$\tau_{xy} = \tau_{xQ} Cos\theta$$
$$\tau_{xz} = \tau_{xQ} Sin\theta$$



We should ensure that distribution of τ_{xQ} should also gives $V_y = V_z = 0$

$$V_y = \int_A \tau_{xy} dA = \int_A \tau_{x\theta} Cos\theta dA$$

$$V_y = \int_0^{2\pi} \int_0^R \frac{Tr}{I_P} Cos\theta dr d\theta$$

$$= \frac{T}{I_P} \int_0^{2\pi} \int_0^R r Cos\theta dr d\theta = 0$$

$$\therefore \quad V_y = 0$$

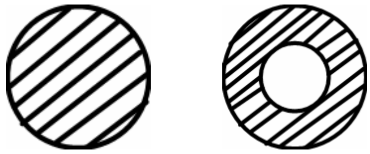
$$V_z = \frac{T}{I_P} \int_0^{2\pi} \int_0^R r Sin\theta dr d\theta = 0$$

$$\therefore \quad V_z = 0$$

Hollow circular bars: The deformation of hollow circular bars and solid circular bars are same. The key kinematic assumptions are valid for any circular bar, either solid or hollow. Therefore all equations of solid circular bars can be employed for hollow circular bars, instead of using

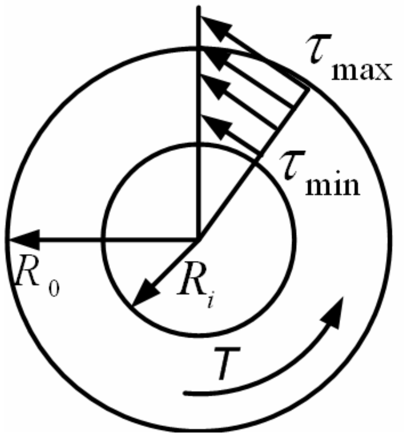
$$I_P = \frac{\pi}{32} D^4 - \ Soild$$

$$I_P = \frac{\pi}{32}\left(D_o{}^4 - D_i^4\right) - \ hollow$$



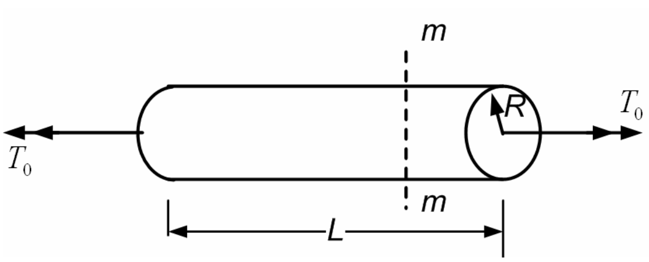
$$\tau_{max} = \frac{TR_o}{I_P}$$

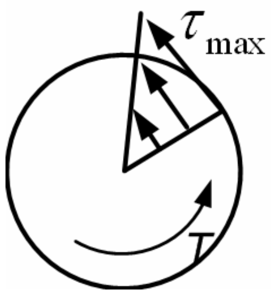
$$\tau_{min} = \frac{TR_i}{I_P}$$

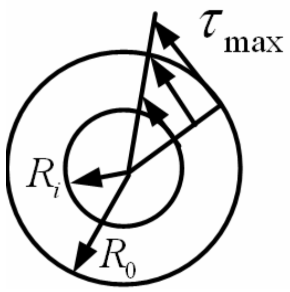


Hollow bars are move efficient than solid bars of same "A".

- Most of the material in soild shaft is stressed below the maximum stress and also have smaller moment arm "r".
- In hollow tube most of the material is near the outer boundary, where τ is maximum values and has large moment arms "r".







$$\tau = \frac{Tr}{I_P}$$

$$I_P = \frac{\pi}{32} D^4 - \ solid$$

$$= \frac{\pi}{32}\left(D_o^4 - D_i^4\right) - hollow$$

$$\tau_{max} = \frac{TR}{I_P};\ \frac{TR_o}{I_P}$$

$$\tau_{min} = \frac{TR_i}{I_P}$$

$$Y = \frac{\tau}{G}$$

$$\tau, Y - f(r)$$

$$\odot = \frac{d\phi}{dx} = \frac{T}{GI_P}$$

$$\phi|_{B/A} = \phi_B - \phi_A = \frac{TL}{GI_P}$$

$$L = x_B - x_A$$

$$\odot = constant$$

$$\phi = linearly\ with\ x$$

(4) If weight reduction and savings of materials are important, it is advisable to use a circular tube.

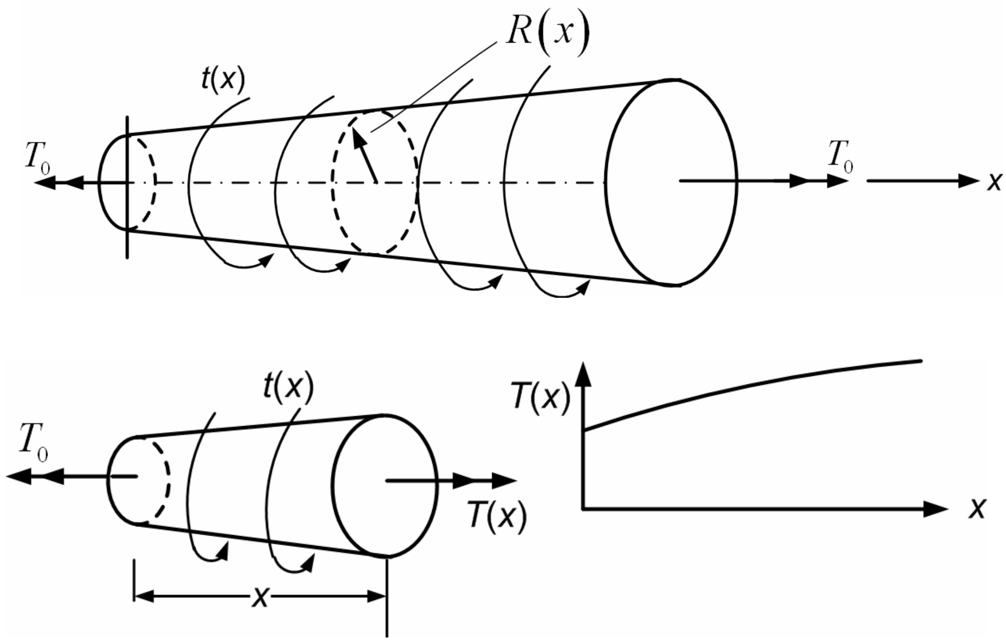
(5) Ex large drive shafts, propeller shafts, and generator shafts usually have hollow circular cross sections.

<u>Extension of results</u>

<u>Case-I</u> Bar with continuously varying cross-sections and continuously varying torque

- Pure torsion refers to torsion of prismatic bar subjected to torques acting only at the ends.

- All expressions are developed based on the key kinematic assumptions, these are therefore, strictly valid only for prismatic circular bars.



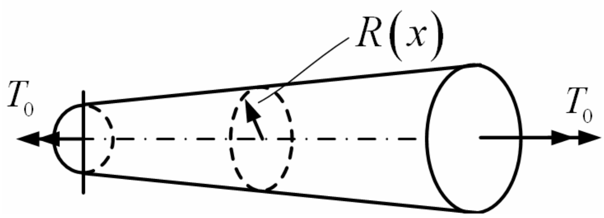
$$\tau(x) = \frac{T(x)r}{I_P(x)}$$

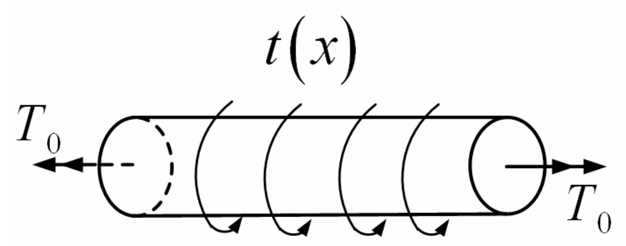
$$\odot(x) = \frac{d\phi}{dx} = \frac{T(x)}{GI_P(x)}$$

$$\phi_B - \phi_A = \phi_{B/A} = \int_{x_A}^{x_B} \frac{T(x)}{GI_P(x)} dx$$

The above equations yield good approximations to the exact solution, provide if $R(x)$ doesn't vary sharply with x.

Some special cases

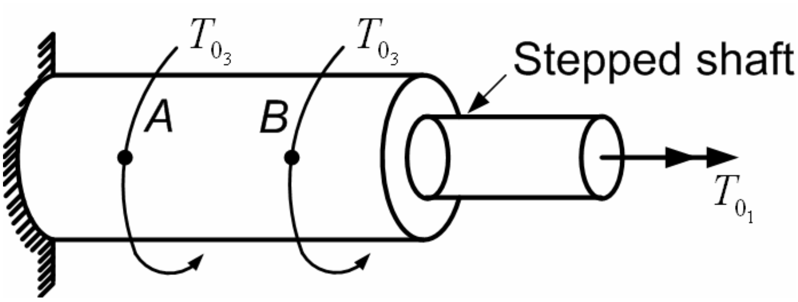




$$\tau(x) = \frac{Tr}{I_P(x)} \qquad \tau(x) = \frac{T(x)r}{I_P}$$

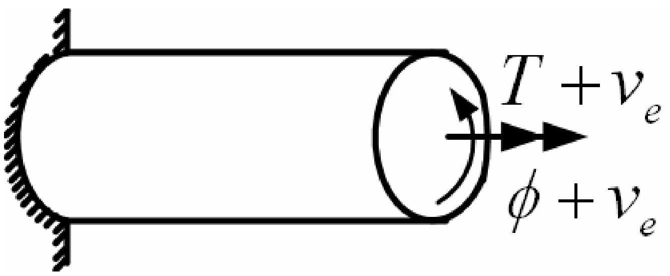
$$\odot(x) = \frac{T}{GI_P(x)} \qquad \odot(x) = \frac{T(x)}{GI_P}$$

Case II

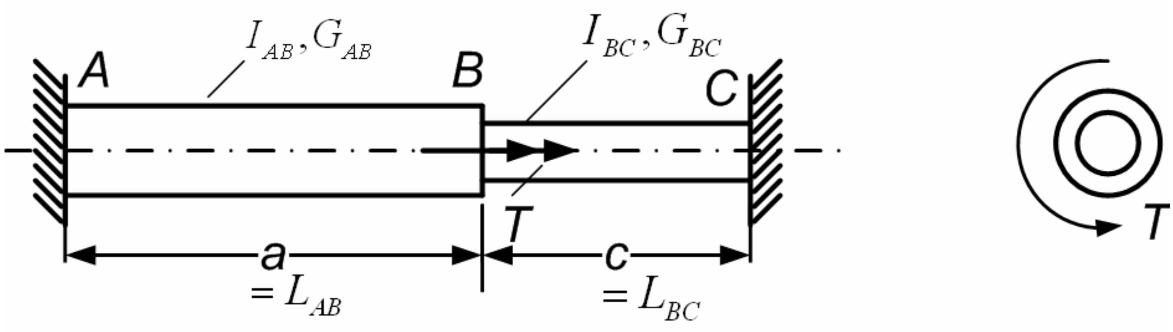


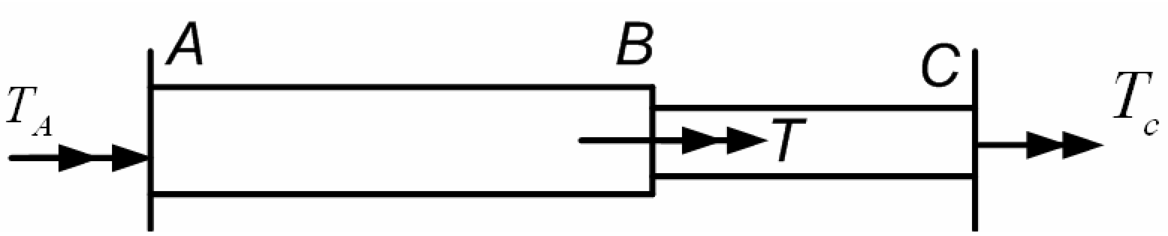
$$\tau_i = \frac{T_i r_i}{I_{P_i}}$$

$$\phi_{B/A} = \sum_{i=1}^{n} \frac{T_i L_i}{G_i I_{P_i}}$$



Statically indeterminate problems





$[\Sigma M_x = 0]$ $\quad T_A + T_C + T = 0 \quad$ (1)

We note that within $AB, T = T_A$ and

within $BC \quad T = T_C$

- To solve the problem we must consider geometry of deformation to formulate the compatibility equation.
- Clearly the rotation of section B with respect to A must be same as that with respect to C i.e

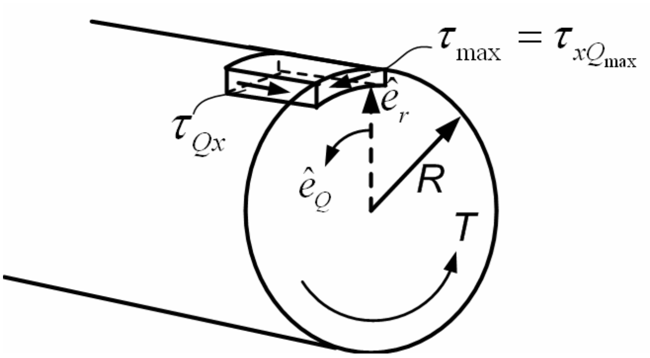
$$\phi_{B/A} = \phi_{B/C}$$

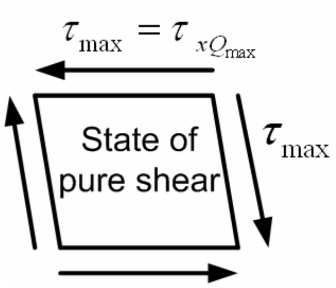
Compatibility equation

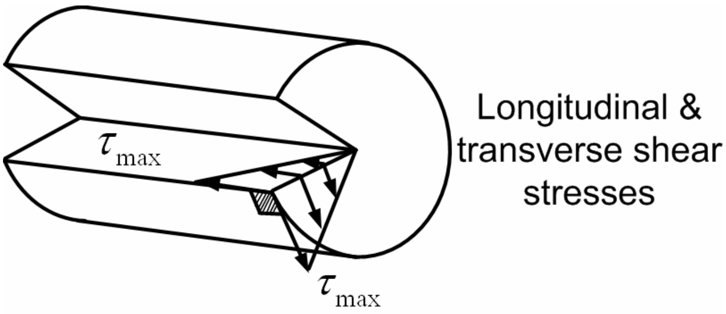
$$\phi_{B/A} = \frac{T_A L_{AB}}{G_{AB} I_{P_{AB}}} \; ; \quad \phi_{B/C} = \frac{T_C L_{BC}}{G_{BC} I_{P_{BC}}}$$

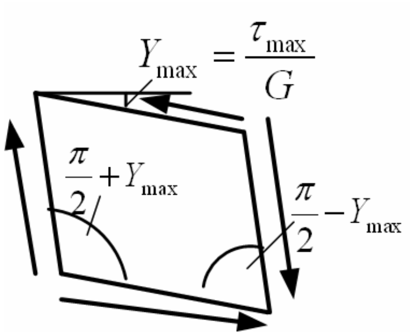
$$\frac{T_A L_{AB}}{G_{AB} I_{P_{AB}}} = \frac{T_C L_{BC}}{G_{BC} I_{P_{BC}}} \quad (2)$$

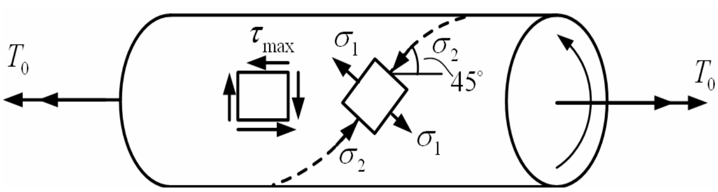
Stresses in pure torsion

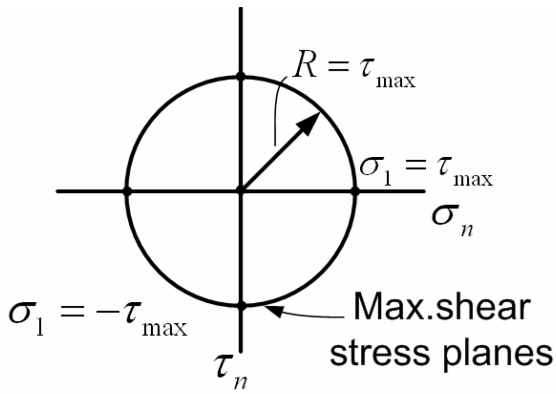






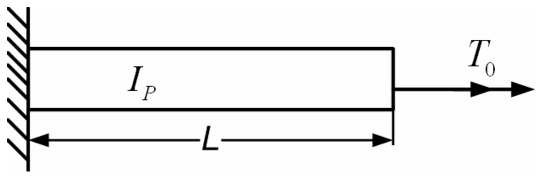




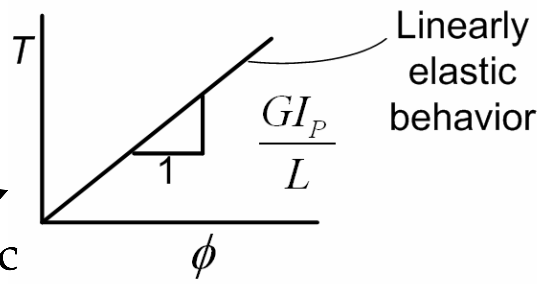


If a torsion bar is made up of brittle material, which is generally weak in tension, failure will occur in tension along a helix inclined at 45° to the axis.

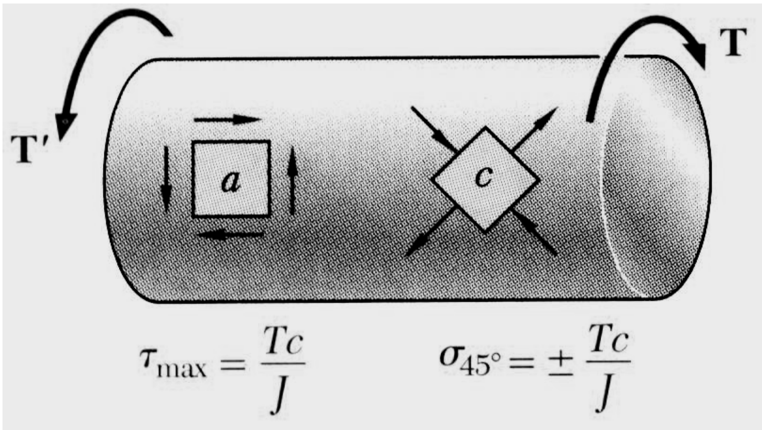
Ductile materials generally fail in shear. When subjected to torsion, a ductile circular bar breaks along a plane perpendicular to its longitudinal axis or x – axis.

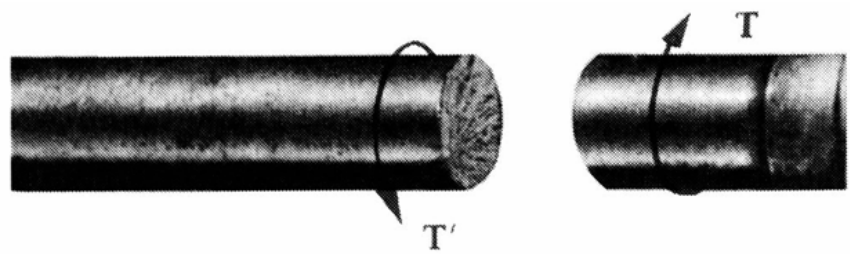


$$\sigma_x = \frac{P}{A}$$

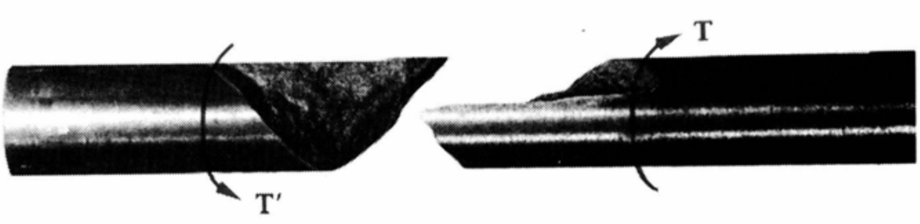


Torsion testing m/c



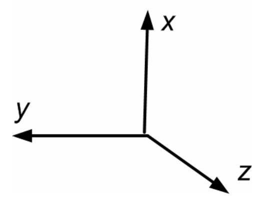


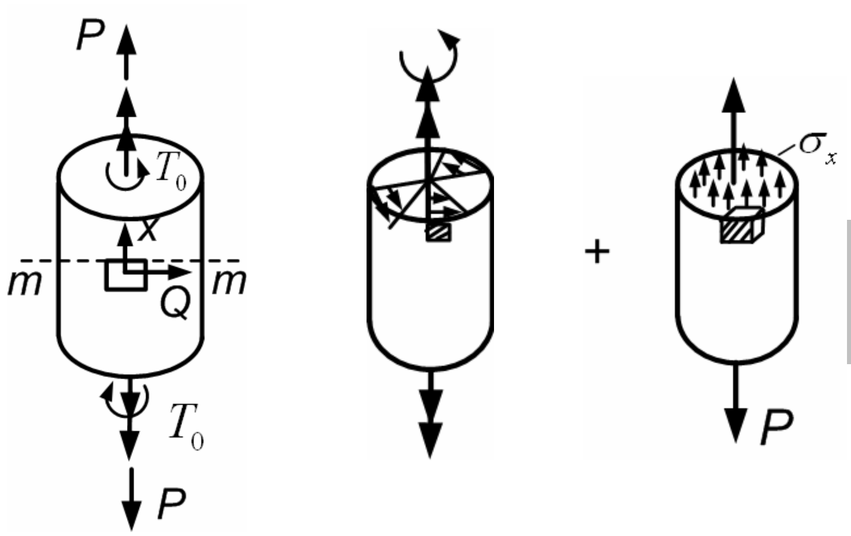
Ductile materials



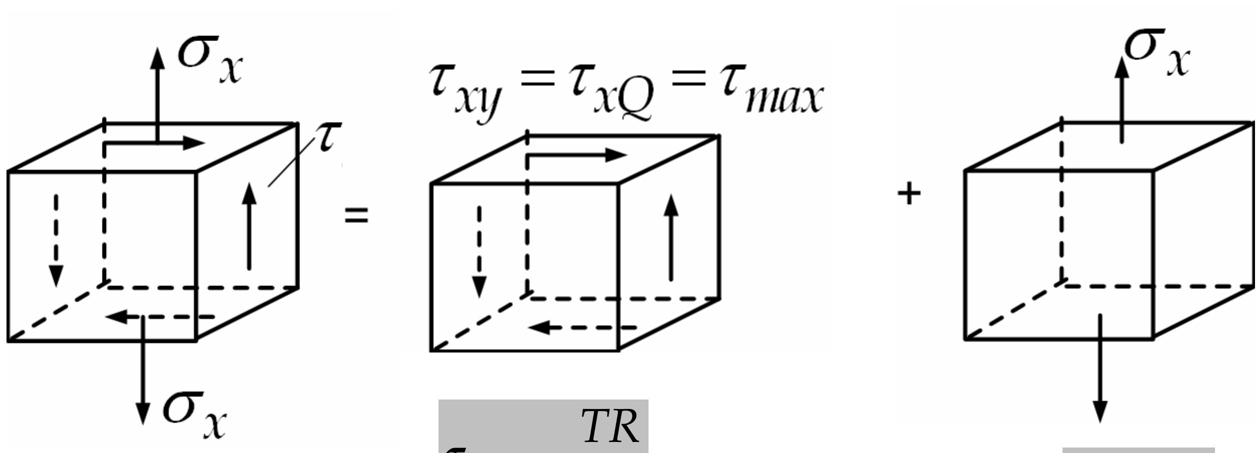
Brittle materials

<u>*Combined loading or combined stress*</u>



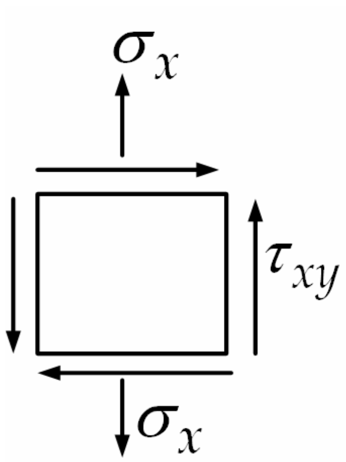


Principal of superposition

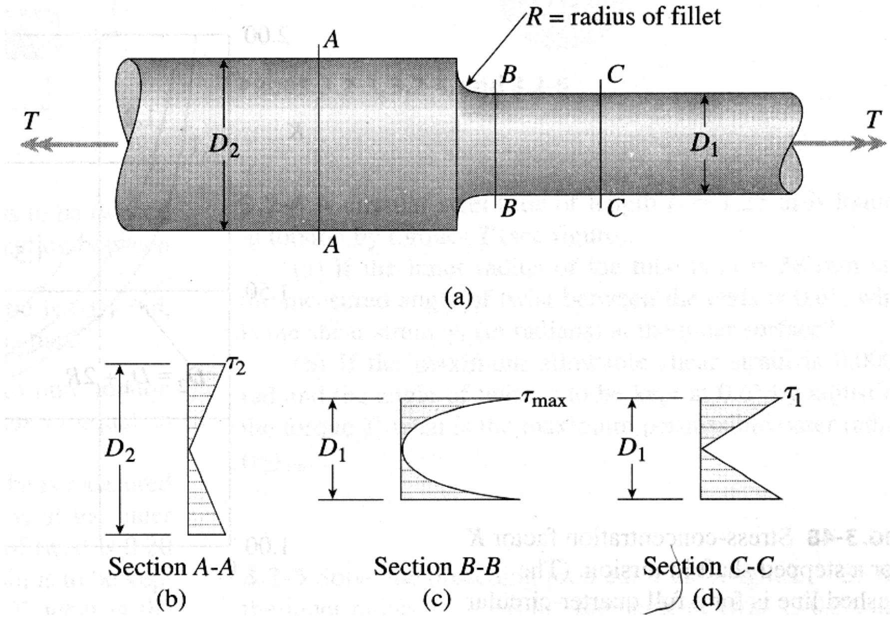


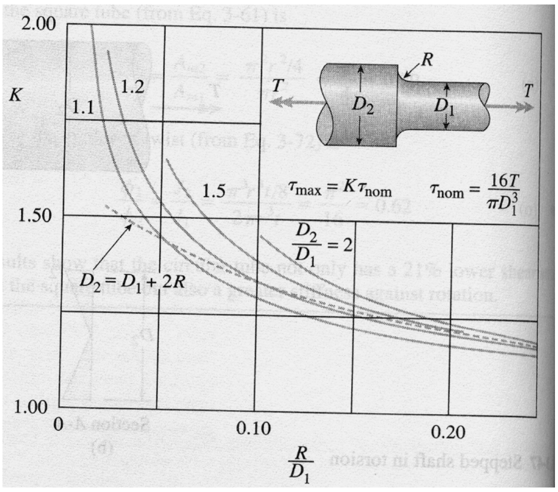
$$\tau_{max} = \frac{TR}{I_P}$$

$$\sigma_x = \frac{P}{A}$$



Stress concentrations in torsion

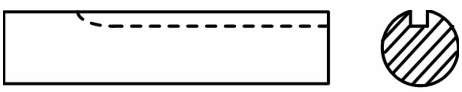




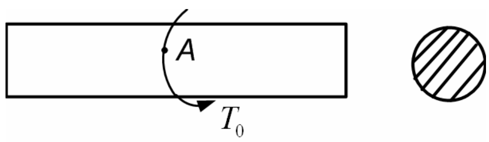
Stress concentration effect is greatest at section *B-B*

$$\tau_{max} = K\tau_{avg} = K\tau_{nom}$$

$$\tau_{avg} = \tau_{nom} = K\tau_1 = K\left(\frac{16T}{\pi D_1^3}\right)$$



Key way in shaft



$$\tau = \frac{Tr}{I_P}, \odot = \frac{T}{GI_P}; \phi = \frac{TL}{GI_P}; \Upsilon = r\frac{d\phi}{dx}$$

Limitations of torsion formulae

(1)The above solutions are exact for pure torsion of circular members (solid or hollow section)

(2)Above equations can be applied to bars (solid or hollow) with varying cross-sections only when changes in $R(x)$ are small and gradual.

(3)Stresses determined from the torsion formula are valid in regions of the bar away from stress concentrations, which are high localized stresses that occur whenever diameter changes abruptly and whenever concentrated torque are applied.

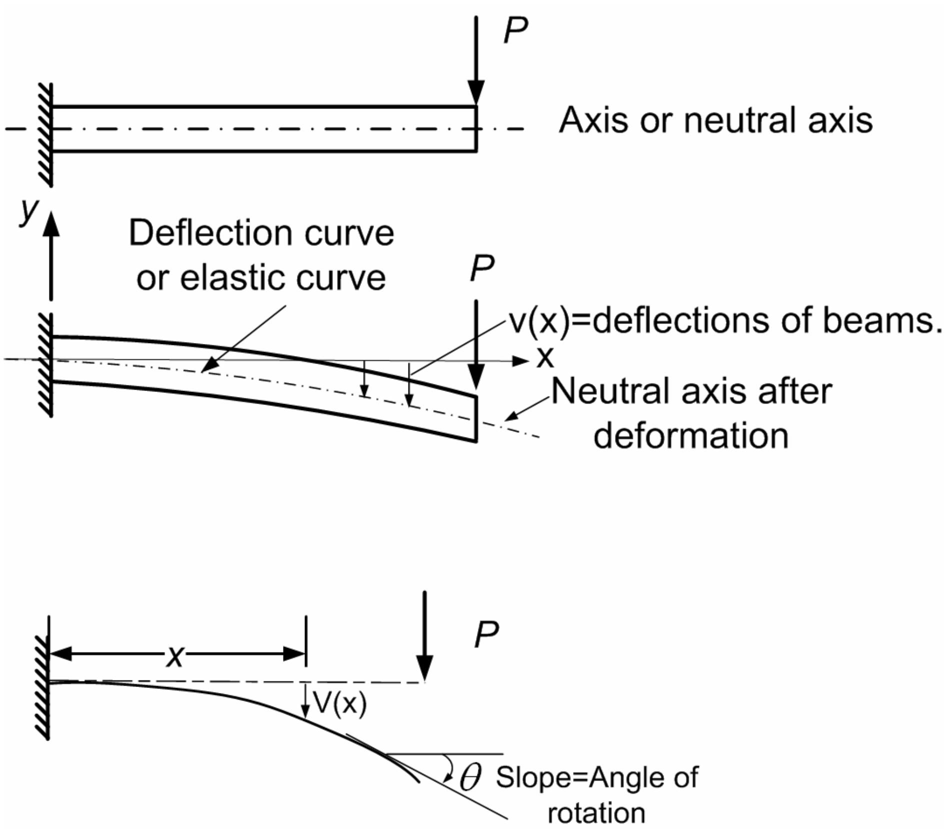
(4)It is important to recognize that, the above equation should not be used for bars of other shapes. Noncircular bars under torsion are entirely different from circular bars.

References:

Solid Mechanics – INTERNET SOURCES & E-BOOKS

20. Deflection of beams

When a beam with a straight longitudinal axis is loaded by lateral loads, the axis is deformed into a curve, called the "deflection curve" or "elastic-curve"



Deflections: means u ,v displacement of any particle. In case of beams deflection means *v* displacement of particles located on the axis of the beam.

Deflection calculation is an important part of component design

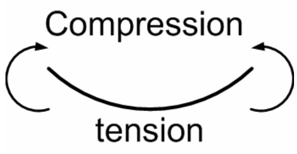
Deflections -- useful in vibration, analysis of various engineering components ex. Earthquake loading.

Undesirable vibrations are due to excessive deflections.

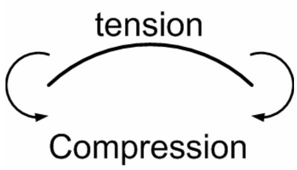
Approximate sketches of deflection curves

Approximate sketches of the deflection curve can be drawn if BM diagram is available for a given loading.

We know that +BM means

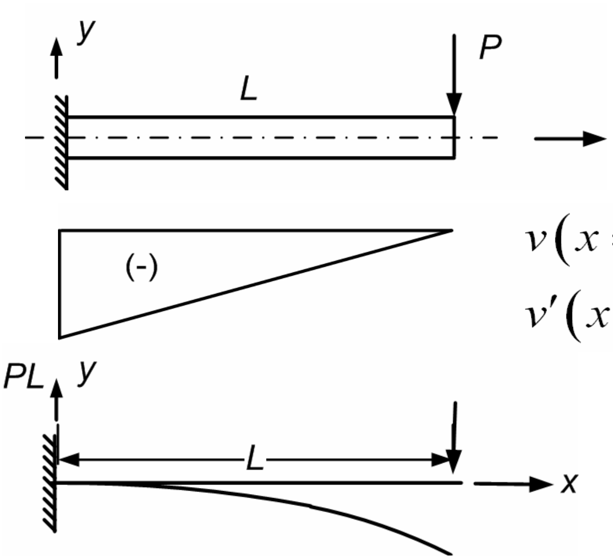


- BM means



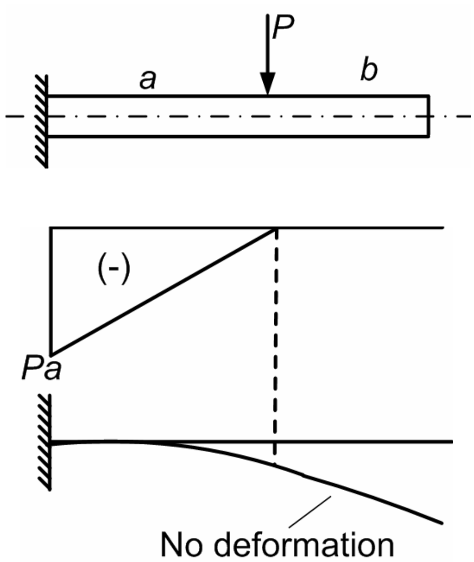
Examples

(1)



$$v(x=0)=0$$

$$v'(x=0)=0$$



The objective is to find the shape of the elastic curve or deflection curve for given loads i.e., what is the function $v(x)$.

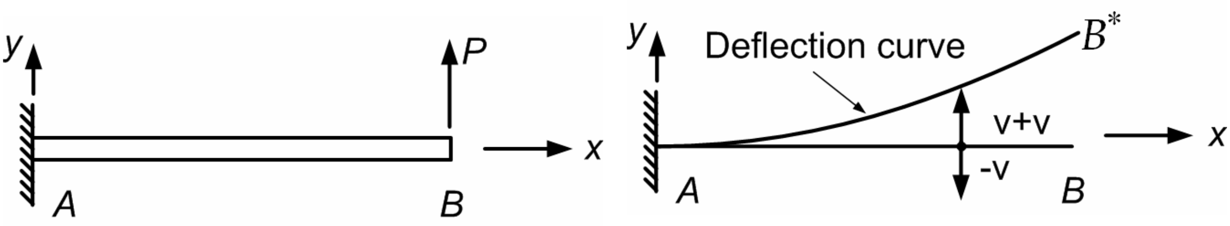
There are two approaches

(1) Differential equations of the deflection curve

(2) Moment-area method

Differential equations of the deflection curve

Consider a cantilever beam: The axis of the beam deforms into a curve as shown due to load P.

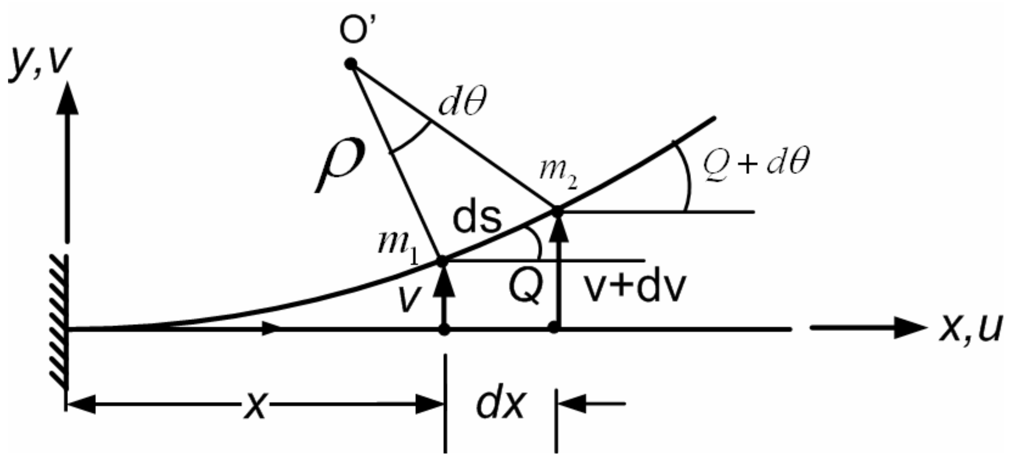


Here we assume only symmetrical bending case. The xy plane is the plane of bending.

$\downarrow -v$ deflection of the beam.

$\uparrow v + ve$ and. $\downarrow -v$

To obtain deflection curve we must express v as a function of x.



When the beam is bent, there is not only a deflection at each point along the axis but also a rotation.

The angle of rotation θ of the axis of the beam is the angle between x – axis and the tangent to the deflection curve at a point.

For given x-y coordinate system

$$\theta \rightarrow +ve \rightarrow \ anticlockwise$$

$$O' = Center\ of\ curvature$$

$$\rho = Radius\ of\ curvature$$

From geometry $\rho d\theta = ds$

$$k = \frac{1}{\rho} = \frac{d\theta}{ds}$$

$$curvature\ of\ the\ deflection\ curve$$

k - curvature - +ve when angle of rotation increases as we move along the beam in the +ve x – direction.

$$Slope\ of\ the\ deflection\ curve = \frac{dv}{dx} = tan\,\theta$$

Slope $\frac{dv}{dx}$ is positive when the tangent to the curve slopes upward to the right.

The deflection curves of most beams have very small angles of rotations, very small deflection and very small curvatures. That is they undergo small deformations.

When the angle of rotation θ is extremely small, the deflection curve is nearly horizontal

$$ds \approx dx$$

This follows from the fact that

$$ds = \sqrt{dx^2 + dv^2} = \sqrt{1 + (v')^2}\,dx$$

for small θ $(v')^2$ can be neglected compared to 1

$$\therefore ds \approx dx$$

Therefore, in small deflection theory no difference in length is said to exist between the initial length of the axis and the arc of the elastic curve.

$$k = \frac{1}{\rho} = \frac{d\theta}{dx}$$

Since θ is small $tan\,\theta \approx \theta$

$$\therefore \frac{dv}{dx} = \theta$$

$$\therefore k = \frac{1}{\rho} = \frac{d\theta}{dx} = \frac{d^2 v}{dx^2}$$

$$\left.\begin{aligned} k &= \frac{d^2 v}{dx^2} = v'' \\ \theta &= \frac{du}{dx} = u' \end{aligned}\right\} \text{only in small deformation theory}$$

If the material of the beam is linearly elastic and follows Hooke's law, the curvature is

$$k = \frac{1}{\rho} = \frac{M}{EI}$$

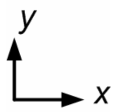
$+M \rightarrow$ leads to +K and so on

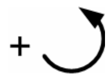
$$\therefore \frac{d^2v}{dx^2} = \frac{M}{EI} \text{ or}$$

$$EI\frac{d^2v}{dx^2} = M$$

The basic differential equations of the deflection curve.

Sign conventions used in the above equation:

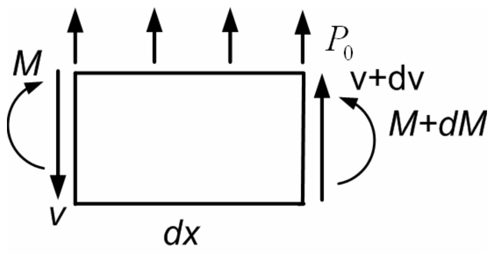
(a) The 

(b) $\frac{dv}{dx}$ and θ are 

(c) k is + 
(d) M is +ve if beam bends 

Another useful equations can be obtained by noting that

$$\frac{dM}{dx} = -V$$
$$\frac{dV}{dx} = -p$$



Non-prismatic beams

$$EI(x)\frac{d^2v}{dx^2} = M(x)$$

$$\left(EI(x)v''\right)' = -v(x)$$

$$\left(EI(x)v''\right)'' = +P(x)$$

For prismatic beams.

$$EIv'' = M(x) \qquad BM\ equation\ (2^{nd}\ order)$$

$$EIv''' = -V(x) \qquad Shear\ force\ equation\ (3^{rd}\ order)$$

$$EIv'''' = +P(x) \qquad Load\ equation\ (4^{th}\ order)$$

Integrating the equations and then evaluating constants of integration from boundary conditions of the beam.

Assumptions involved in the above equations

(a) Material obeys Hooke's law

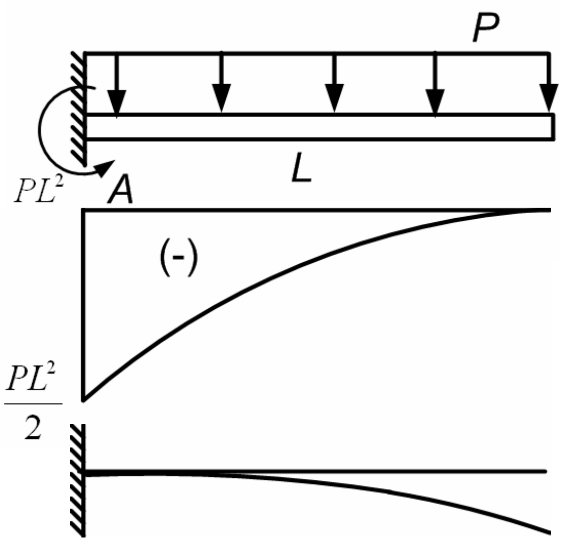
(b) Slope of deflection curve small – small deformations

(c) Deformations due to bending only – shear neglected

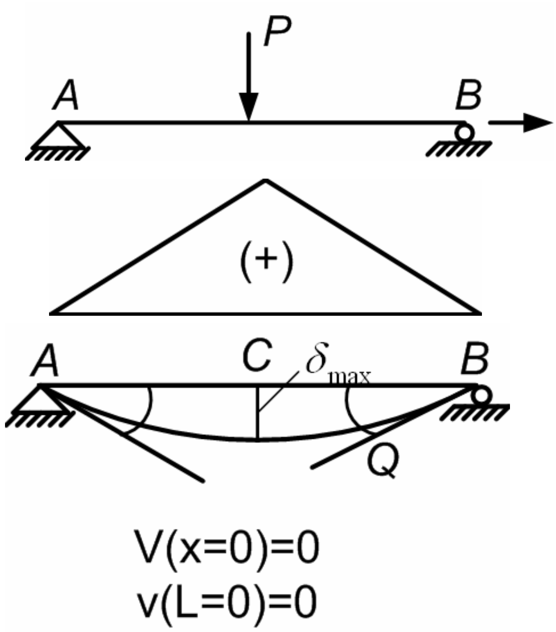
When sketching deflection curve we greatly exaggerate the deflection for clarity. Otherwise they actually are very small quantities.

Approximate sketching

(3)



(4)



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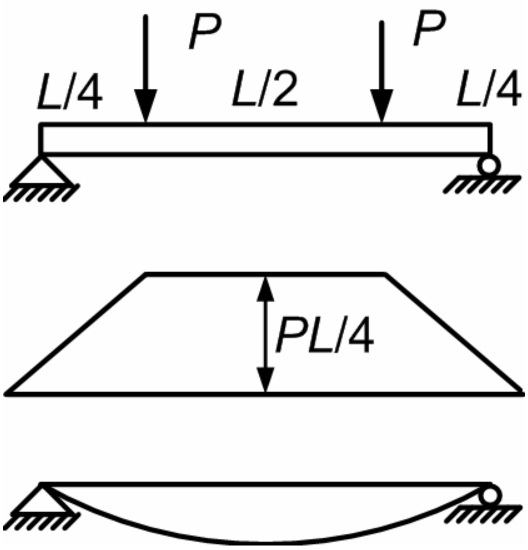

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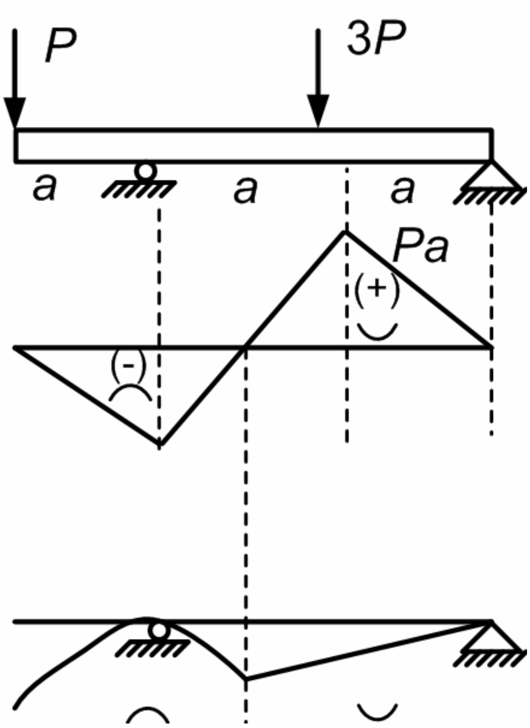


Boundary conditions

(1)Boundary conditions

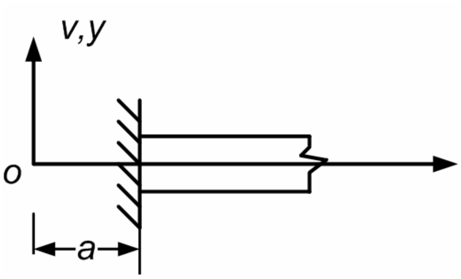
(2)Continuity conditions

(3)Symmetry conditions

Boundary conditions

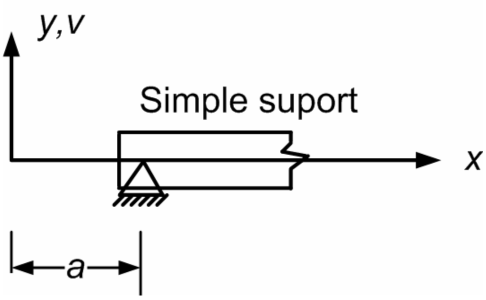
Pertain to the deflections and slopes at the supports of a beam:

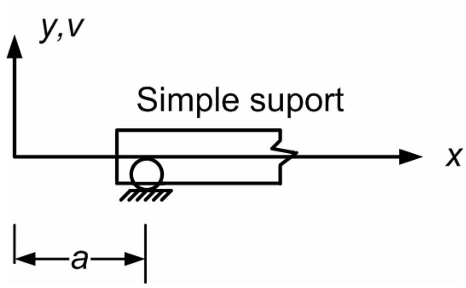
(i)Fixed support or clamped support



$$v(a)=0$$
$$\theta(a)=v'(a)=0$$

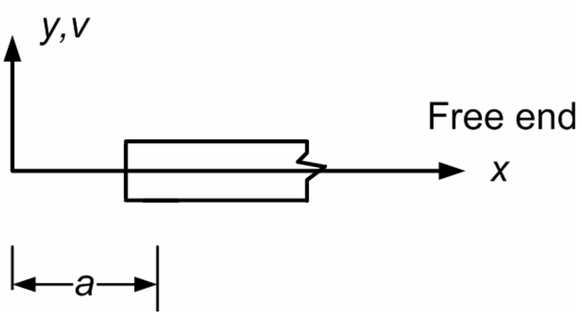
(ii)





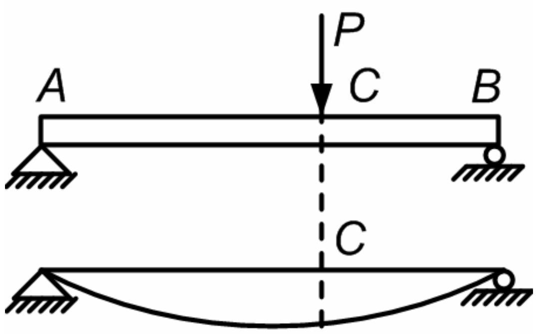
$$v(a)=0$$
$$M(a)=EIv''(a)=0 \Rightarrow v''(a)=0$$

(iii)



$$M(a)=EIv''(a)=0$$
$$V(a)=-EIv''(a)=0$$

Continuity conditions



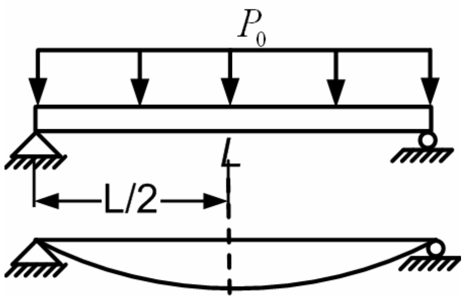
All deflection curves are physically continuous. Therefore

$$v(c)\Big|_{from\ side\ AC} = v(c)\Big|_{from\ side\ BC}$$

Similarly at "C"

$$v'(c)\Big|_{from\ side\ AC} = v'(c)\Big|_{from\ side\ BC}$$

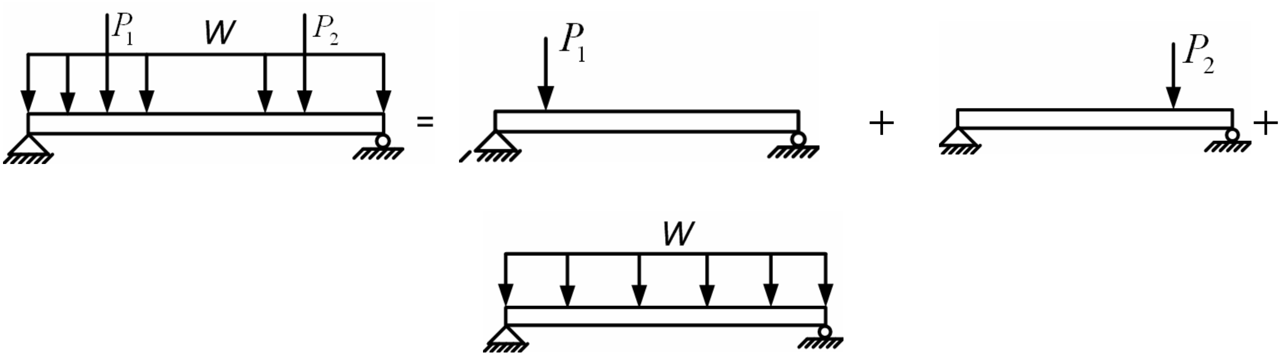
Symmetry conditions



$v'\left(\frac{L}{2}\right) = 0$ because of loading and beam. This we should load in advance.

The method for finding deflection using differential equations is known as " method of successive integration".

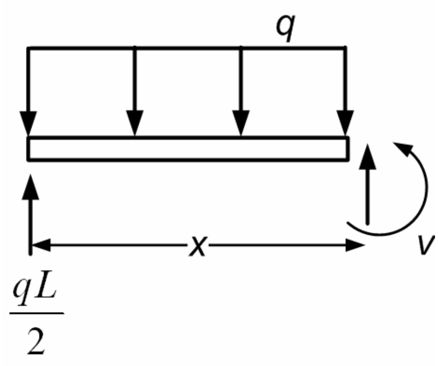
Application of principle of superposition: Numerous problems with different loadings have been solved and readily available. Therefore in practice the deflection of beam subjected to several or complicated loading conditions are solved using principle of superposition.

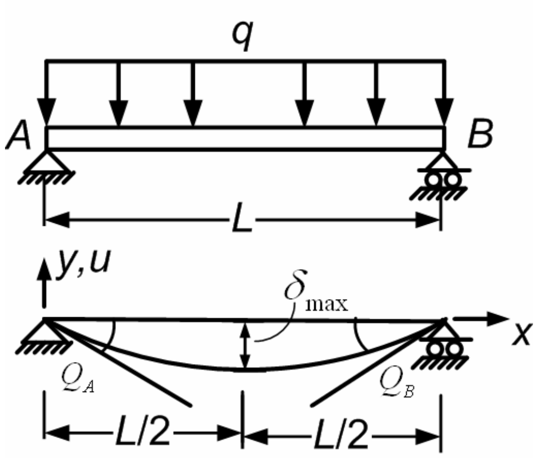


Problem 1

Determine the equation of the deflection curve for a simple beam AB supporting a uniform load of intensity of acting through out the span of the beam. Also determine maximum deflection δ_{max} at the mid point of the beam and the angles of rotation Q_A and Q_B at the supports. Beam has constant EI.

Solution





$$V+\frac{qL}{2}-qx=0$$

$$V=qx-\frac{qL}{2} \qquad (1)$$

$$M-\frac{qL}{2}x+\frac{qx^2}{2}=0$$

$$M=\frac{qLx}{2}-\frac{qx^2}{2} \qquad (2)$$

Differential equation of deflection curve.

$$EIv''=M(x)$$

$$EIv''=\frac{qLx}{2}-\frac{qx^2}{2}$$

Slope of the beam

$$EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} + C_1$$

$BC \rightarrow$ Symmetry conditions

$$v'\left(x = \frac{L}{2}\right) = 0$$

$$0 = \frac{qLL^2}{16} - \frac{qL^3}{48} + C_1$$

$$0 = \frac{qL^3}{16} - \frac{qL^3}{48} + C_1$$

$$C_1 = -\frac{qL^3}{24}$$

Slope equation is

$$EIv' = \frac{qLx^2}{4} - \frac{qx^3}{6} - \frac{qL^3}{24}s$$

$$v' = \frac{-q}{24EI}\left(L^3 - 6L^2 + x^3\right)$$

Deflection of the beam

$$EIv = \frac{qLx^3}{12} - \frac{qx^4}{24} - \frac{qL^3}{24}x + C_2$$

B.C.

$v(x=0)=0$

$0 = 0 - 0 - 0 + C_2 \Rightarrow$ $C_2 = 0$

$$EIv = \frac{qLx^3}{12} - \frac{qx^4}{24} - \frac{qL^3}{24}x$$

$$\therefore v = \frac{-q}{24EI}\left(L^3x - 2Lx^3 + x^4\right)$$

$$v = \frac{-q}{24EI}\left(x^4 + L^3x - 2Lx^3\right)$$

you can check $v = 0$ at $x = 0$ and $L = 0$

(b) From symmetry maximum deflection occurs at the midpoint $x = \frac{L}{2}$

$$v\left(x = \frac{L}{2}\right) = \frac{-5qL^4}{384EI}$$

-ve sign means that deflection is downward as expected.

$$\delta_{max} = \left|v\left(x = \frac{L}{2}\right)\right| = \frac{5qL^4}{384EI}s$$

$$Q_A = v'(0) = \frac{-qL^3}{24EI}$$

-ve sign indicates clock wise rotation as expected.

$$Q_B = v'(x = L) = \frac{qL^3}{4EI} - \frac{qL^3}{6EI} - \frac{qL^3}{24EI}$$

$v'(L) = \frac{qL^3}{24EI}$ + ve sign means anticlockwise direction.

since the problem is symmetric, $|v'(0)| = |v'(L)|$

Problem: 2

Above problem using third order equation

$$EIv''' = -V(x)$$

$$EIv''' = -\left(qx - \frac{qL}{2}\right) = \frac{qL}{2} - qx$$

Moment equation

$$EIv'' = \frac{qLx}{2} - \frac{qx^2}{2} + C_1$$

B.C.

$$M(x=0) = 0 \Rightarrow EIv''(x=0) = 0$$
$$\Rightarrow C_1 = 0$$
$$EIv'' = \frac{qLx}{2} - \frac{qx^2}{2}$$

Problem 3

Above problem using fourth order differential equation

$$P = q$$
$$EIv'''' = -q$$

Shear for a equation

$$EIv''' = -qx + C_1$$

From symmetry conditions

$$V\left(x=\frac{L}{2}\right)=0 \Rightarrow EIv'''\left(x=\frac{L}{2}\right)=0$$

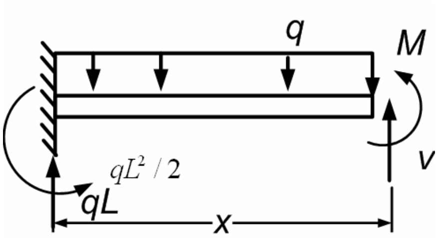
$$0=-q\frac{L}{2}+C_1 \Rightarrow C_1=+\frac{qL}{2}$$

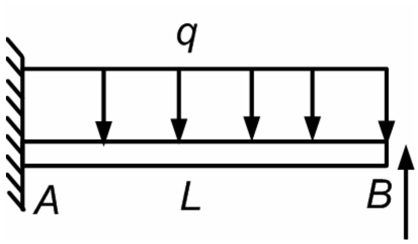
$$\therefore EIv'''=-qx+\frac{qL}{2}$$

Problem 4

Determine the equation of the deflection curve for a cantilever beam AB subjected to a uniform load of intensify q. Also determine the angle of rotation and deflection at the free end. Beam has constant EI.

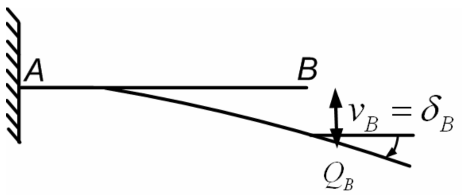
Solution:





$$V+qL-qx=0$$

$$V=qx-qL$$



$$M+\frac{qL^2}{2}-qLx+\frac{qx^2}{2} \Rightarrow M=qLx-\frac{qL^2}{2}-\frac{qx^2}{2}$$

Differential equation of deflection curve

$$EIv''=M(x)$$

$$EIv''=-\frac{qL^2}{2}+qLx-\frac{qx^2}{2}$$

Slope equation: $EIv' = -\frac{qL^2x}{2} + \frac{qLx^2}{2} - \frac{qx^3}{6} + C_1$

BC: $v'(x=0) = C_1 = 0$

$$EIv' = -\frac{qL^2x}{2} + \frac{qLx^2}{2} - \frac{qx^3}{6}$$

Deflection equation

$$EIv = -\frac{qL^2x^2}{4} + \frac{qLx^3}{6} - \frac{qx^4}{24} + C_2$$

$v(x=0)=0$

$0 = 0 + 0 - 0 + C_2 \Rightarrow$ $C_2 = 0$

$$\therefore\ EIv = -\frac{qL^2x^2}{4} + \frac{qLx^3}{6} - \frac{qx^4}{24}$$

$$v = \frac{-q}{24EI}\left[6L^2x^2 \overset{-}{+} 4Lx^3 \overset{+}{\odot} x^4\right]$$

$v'(x=L) \Rightarrow$

$$EIv' = \frac{-qL^3}{2} + \frac{qL^3}{2} - \frac{qL^3}{6} = \frac{-qL^3}{6}$$

$$\therefore v' = Q_B = -\frac{qL^3}{6EI}$$

$v(x=L) \Rightarrow$

$$v = \frac{-q}{24EI}\left[6L^4 - 4L^4 + L^4\right] = \frac{-3qL^4}{24EI}$$

$v(x=L) = \frac{-3qL^4}{24EI} \Rightarrow v = \frac{qL^4}{8EI}$ -maximum deflection also.

Problem 5

Above problem using third order equation

$$EIv'' = -V(x)$$
$$EIv''' = qL - qx$$

Moment equation

$$EIv'' = qLx - \frac{qx^2}{2} + C_1$$

B.C. $M(x=L)=0 \Rightarrow EIv''(x=L)=0$

$$\Rightarrow 0 = qL^2 - \frac{qL^2}{2} = \frac{qL^2}{2} \Rightarrow 4 = -\frac{qL^2}{2}$$

$$EI'v'' = qLx - \frac{qx^2}{2} + \frac{qL^2}{2}$$

$$EIv'' = qLx - \frac{qx^2}{2} + \frac{qL^2}{2}$$

Problem 6

Above problem with fourth order equation

$$EIv'''' = P(x)$$

$$\therefore EIv'''' = \bar{\oplus} q$$

Shear force equation

$$EIv''' = -qx + C_1$$

$B.C$ $V(x=L)=0 \Rightarrow EIv'''(x=L)=0$

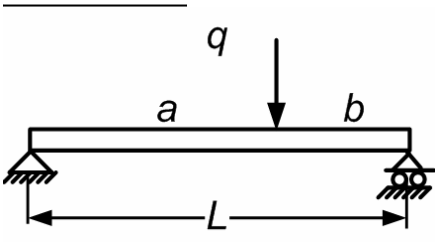
$$0 = -qL + C_1 \Rightarrow C_1 = +qL$$

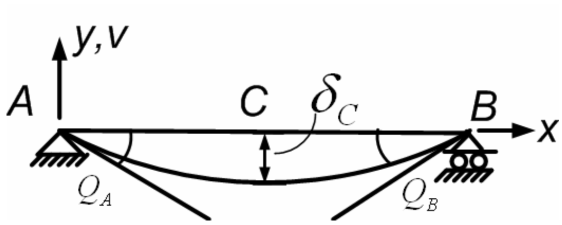
$$\therefore\ EIv''' = -qx + qL$$

Problem 7

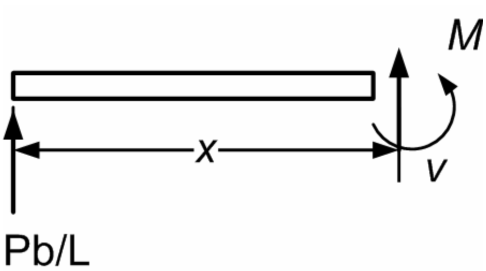
A simple beam AB supports a $EI=$ concentrated load P acting at distances a and b from the left-hand and right-hand supports respectively. Determine the equations Q_A of the Q_B deflection curve, the angles of rotation and δ_{max} at the supports, the δ_L maximum deflection and the deflection at the midpoint C of the beam. Constant EI

Solution





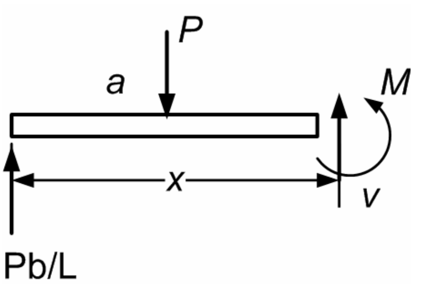
$$\frac{Pb}{L} + x = P \Rightarrow x = P - \frac{Pb}{L}$$



$$V + \frac{Pb}{L} = 0$$

$$V = -\frac{Pb}{L}$$

$$M - \frac{Pb}{L}x = 0 \Rightarrow H = \frac{Pbx}{L}$$



$$V + \frac{Pb}{L} - P = 0$$

$$V = P - \frac{Pb}{L}$$

$$M+P(x-a)-\frac{Pbx}{L}$$

$$M=\frac{Pbx}{L}-P(x-a)$$

$$M=\frac{Pbx}{L}-Px+Pa=-\frac{Pxa}{L}+Pa$$

Differential equation of deflection curve

$$EIv''=\frac{Pbx}{L} \qquad 0\le x\le a$$

$$EIv''=-\frac{Pxa}{L}+Pa \qquad a\le x\le L$$

Slope equations:

$$EIv'=\frac{Pbx^2}{2L}+C_1 \quad o\le x\le a$$

$$EIv'=\frac{-Px^2a}{2L}+Pax+C_2 \quad a\le x\le L$$

B.C. $v'(x=a)\big|_{AP}=v'(x=a)\big|_{PB}$

$$\frac{P(L-a)a^2}{2L}+C_1=\frac{-Pa^3}{2L}+Pa^2+C_2$$

$$\frac{PLa^2}{2L}-\frac{Pa^3}{2L}+C_1=-\frac{Pa^3}{2L}+Pa^2+C_2$$

$$\Rightarrow C_1=\frac{Pa^2}{2}+C_2$$

Deflection curve equations:

$$EIv' = \frac{Pbx^3}{6L} + C_1x + C_3 \qquad 0 \le x \le a$$

$$EIv = \frac{-Px^3a}{6L} + \frac{Pax^2}{2} + C_2x + C_4 \quad a \le x \le L$$

B.C: $v(x=0)=0$ and $v(x=L)=0$

$$0 = 0 + 0 + C_3 \quad \Rightarrow \qquad C_3 = 0$$

$$0 = -\frac{PL^3a}{6L} + \frac{PaL^2}{2} + C_2L + C_4$$

$$0 = -\frac{PL^2a}{6} + \frac{PaL^2}{2} + C_2L + C_4$$

$$= \frac{PaL^2}{3} + C_2L + C_4$$

$$C_4 = -\frac{PaL^2}{3} - C_2L$$

$$v(x=a)\big|_{AP} = v(x=a)\big|_{PB}$$

$$\frac{P(L-a)a^3}{6L} + C_1a = \frac{-Pa^4}{6L} + \frac{Pa^3}{2} + C_2a + C_4$$

$$\frac{PLa^3}{6L} + \cancel{\frac{Pa^4}{6L}} + C_1a = \cancel{\frac{-Pa^4}{6L}} + \frac{Pa^3}{2} + C_2a + C_4$$

$$\frac{Pa^3}{6} + C_1a = \frac{Pa^3}{2} + C_2a + C_4$$

$$C_1a = \frac{Pa^3}{3} + C_2a - \frac{PaL^2}{3} - C_2L$$

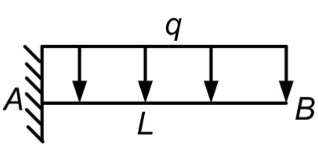
$$\frac{Pa^3}{2}+\cancel{C_2a}=\frac{Pa^3}{3}+\cancel{C_2a}-\frac{PaL^2}{3}-C_2L$$

$$\frac{Pa^3}{6}=-\frac{PaL^2}{3}-C_2L \Rightarrow C_2L=-\frac{PaL^2}{3}-\frac{Pa^3}{6}$$

$$C_2=-\frac{PaL}{3}-\frac{Pa^3}{6}$$

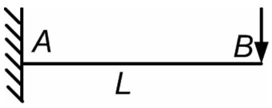
Some important formulae to remember

(1)



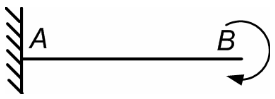
$$\delta_B=\frac{qL^4}{8EI},Q_B=\frac{qL^3}{6EI}$$

(2)



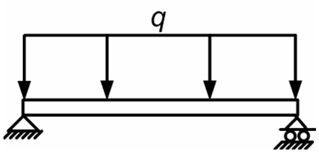
$$\delta_B=\frac{PL^3}{3EI},\quad Q_B=\frac{PL^2}{2EI}$$

(3)



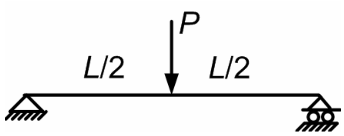
$$\delta_B=\frac{M_0L^2}{2EI},\quad Q_B=\frac{M_0L}{EI}$$

(4)



$$\delta_c=\delta_{max}=\frac{5qL^4}{384EI};\quad Q_A=Q_B=\frac{qL^3}{24EI}$$

(5)

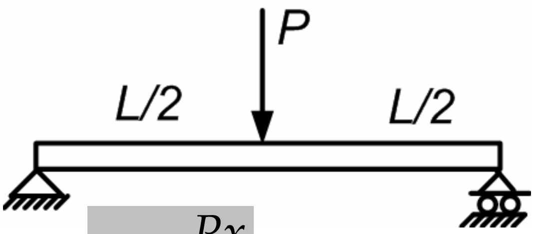


$$\delta_c=\delta_{max}=\frac{PL^3}{48EI};Q_A=Q_B=\frac{PL^2}{16EI}$$

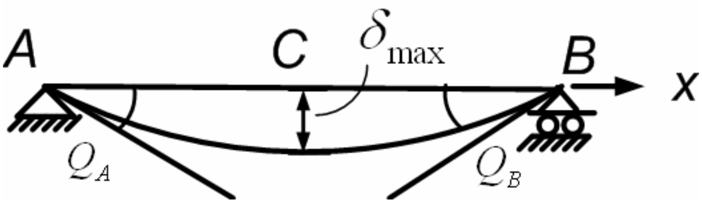
Problem 8

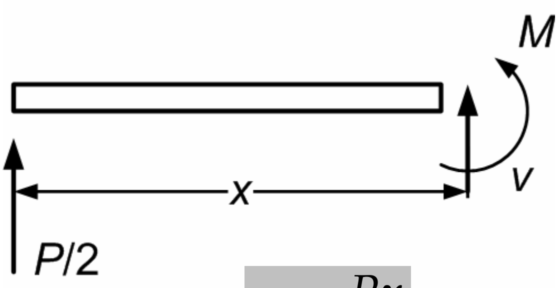
A simple beam AB supports a concentrated load P acting at the center as shown. Determine the equations of the deflection curve, the angles of rotation Q_A and Q_B at the supports, the maximum deflection δ_{max} of the beam.

Solution



$$M = \frac{Px}{2}$$



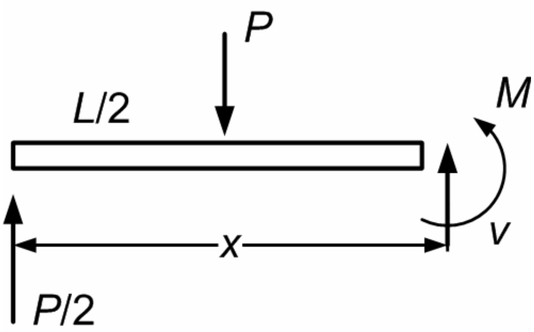


$$M = \frac{Px}{2}$$

$$V = -\frac{P}{2}$$

$$M - \frac{P}{2}x = 0$$

$$M = \frac{Px}{2}$$



$$V + \frac{P}{2} - P = 0$$

$$V = P/2$$

$$M - \frac{Px}{2} + P\left(x - \frac{L}{2}\right) = 0$$

$$M = \frac{Px}{2} - P\left(x - \frac{L}{2}\right) = \frac{Px}{2} - Px + \frac{PL}{2} = \frac{PL}{2} - \frac{Px}{2}$$

$$M = \frac{PL}{2} - \frac{Px}{2}$$

Differential equation deflection curve

$$EIv'' = \frac{Px}{2} \qquad 0 \le x \le L/2$$

$$EIv'' = \frac{PL}{2} - \frac{Px}{2} \qquad \frac{L}{2} \le x \le L$$

Slope equations

$$EIv' = \frac{Px^2}{4} + C_1 \qquad 0 \le x \le L/2$$

$$EIv' = \frac{PLx}{2} - \frac{Px^2}{4} + C_2 \qquad \frac{L}{2} \le x \le L$$

$$v'\left(x = \frac{L}{2}\right)\bigg|_{AP} = v'\left(x = \frac{L}{2}\right)\bigg|_{PB}$$

$$\frac{PL^2}{16} + C_1 = \frac{PL^2}{4} - \frac{PL^2}{16} + C_2$$

$$C_1 = C_2 + \frac{PL^2}{4} - \frac{PL^2}{8} = C_2 + \frac{PL^2}{8}$$

$$C_1 = C_2 + \frac{PL^2}{8}$$

Deflection equations:

$$EIv = \frac{Px^3}{12} + C_1 x + C_3 \qquad 0 \le x \le L/2$$

$$EIv = \frac{PLx^2}{4} - \frac{Px^3}{12} + C_2 x + C_4 \qquad L/2 \le x \le L$$

B.C: $v(x=0)=0$ and $v(x=L)=0$

$$0 = 0 + 0 + C_3 \quad \Rightarrow \qquad C_3 = 0$$

$$0 = \frac{PL^3}{4} - \frac{PL^3}{12} + C_2 L + C_4$$

$$= \frac{PL^3}{6} + C_2 L + C_4$$

$$C_4 = -\frac{PL^3}{6} - C_2 L$$

$$v'\left(x = \frac{L}{2}\right)\bigg|_{AP} = v'\left(x = \frac{L}{2}\right)\bigg|_{PB}$$

$$\frac{PL^3}{96} + \frac{C_1 L}{2} = \frac{PL^3}{16} - \frac{PL^3}{96} + C_2 \frac{L}{2} + C_4$$

$$C_1 \frac{L}{2} = \frac{PL^3}{16} - \frac{PL^3}{48} + C_2 \frac{L}{2} + C_4$$

$$C_1 \frac{L}{2} = \frac{PL^3}{24} + C_2 \frac{L}{2} + C_4$$

$$C_2 \frac{L}{2} + \frac{PL^3}{16} = \frac{PL^3}{24} + C_2 \frac{L}{2} - \frac{PL^3}{6} - C_2 L$$

$$\frac{PL^3}{24} - \frac{PL^3}{6} - \frac{PL^3}{16} = C_2 L \Rightarrow C_2 = \frac{(2-8-3)PL^2}{48}$$

$$C_2 = -\frac{9PL^2}{48} = \frac{-3PL^2}{16}$$

$$C_2 = -\frac{3PL^2}{16}$$

$$\therefore C_1 = -\frac{3PL^2}{16} + \frac{PL^2}{8} = -\frac{PL^2}{16}$$

$$C_1 = -\frac{PL^2}{16}$$

$$\therefore C_4 = -\frac{PL^3}{6} - L\left(\frac{-3PL^2}{16}\right)$$

$$= \frac{-PL^3}{6} + \frac{3PL^3}{16} = \frac{(-8+9)PL^3}{48}$$

$$C_4 = -\frac{PL^3}{48}$$

Deflection curves

$$EIv = \frac{Px^3}{12} - \frac{PL^2}{16}x + C_3 \qquad 0 \le x \le \frac{L}{2}$$

$$EIv = \frac{PLx^2}{4} - + \frac{Px^3}{12} - \frac{3PL^2}{16}x + \frac{PL^3}{48} \quad \frac{L}{2} \le x \le L$$

$$EIv\Big|_{x=\frac{L}{2}} = \frac{PL^3}{96} - \frac{PL^3}{32} = \frac{-PL^3}{48}$$

$$\therefore \; v\Big|_{x=\frac{L}{2}} = -\frac{PL^3}{48EI}$$

$$EIv\Big|_{x=\frac{L}{2}} = \frac{PL^3}{16} - \frac{PL^3}{96} - \frac{3PL^3}{32} + \frac{PL^3}{48} = \frac{(6-1-9+2)PL^3}{96}$$

$$= -\frac{PL^3}{48}$$

$$v = -PL^3 / 48EI$$

Slope equations:

$$EIv' = \frac{Px^2}{4} - \frac{PL^2}{16} \qquad 0 \le x \le \frac{L}{2}$$

$$EIv' = \frac{PLx}{2} - \frac{Px^2}{4} - \frac{3PL^2}{16} \qquad \frac{L}{2} \le x \le L$$

$$EIv'(x=0) = 0 - \frac{PL^2}{16} = -\frac{PL^2}{16}$$

$$\therefore\ v'(x=0) = Q_A = -\frac{PL^2}{16EI}(-)\ \ Clock\,wise$$

$$EIv'(x=L) = \frac{PL^2}{2} - \frac{PL^2}{4} - \frac{3PL^2}{16} = \frac{(8-4-3)PL^2}{16} = \frac{PL^2}{16}$$

$$\therefore v'(x=L) = Q_B = \frac{PL^2}{16EI}(+ve,\ CCW\ from\ \ x\text{-}axis)$$

Problem 9

A cantilever beam AB supports load of intensity of acting over part of the span and a concentrated load P acting at the free end. Determine the deflections δ_B and angle of rotation Q_B at end B of the beam. Beam has constant EI. Use principle of superposition.

Solution

$$\delta_{B_1} = \frac{qa^3}{24EI}(4L-a), \qquad Q_{B_1} = \frac{qL^3}{6EI}$$

$$\delta_{B_2} = \frac{PL^3}{3EI}, \qquad Q_{B_2} = \frac{PL^2}{2EI}$$

$$\delta_B = \delta_{B_1} + \delta_{B_2} = \frac{qa^3}{24EI}(4L - a) + \frac{PL^3}{3EI}$$

$$Q_B = Q_{B_1} + Q_{B_1} = \frac{qa^3}{6EI} + \frac{PL^2}{2EI}$$

21. Moment- Area Method

This method is based upon two theorems related to the area of the bending moment diagram it is called moment-area method.

First moment area theorem

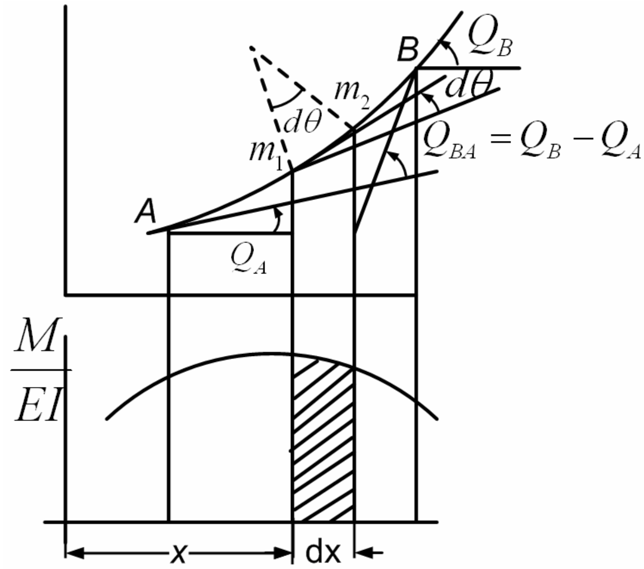
Consider segment AB of the deflection curve of a beam in region of + ve curvature.

The equation

$\frac{d^2\theta}{dx^2} = \frac{M}{EI}$ can be written as

$$\frac{d^2\theta}{dx^2} = \frac{d\theta}{dx} = \frac{M}{EI}$$

$$d\theta = \frac{M}{EI}dx$$



The quantity $\frac{M}{EI}dx$ corresponds to an infinitesimal area of the $\frac{M}{EI}$ diagram. According to the above equation the area is equal to the arrange in angle between two adjacent point m_1 and m_2. Integrating the above equation between any two points A & B gives.

$$\int_A^B d\theta = \theta_B - \theta_A = \Delta\theta_{BA} = \int_A^B \frac{M}{EI}dx$$

This states that the arrange in angle measured in radius between the two tangents at any two points A and B on the elastic curve is equal to the area of $\frac{M}{EI}$ diagram between A & B , If θ_A is known then

$$\theta_B = \theta_A + \Delta\theta_{BA}$$

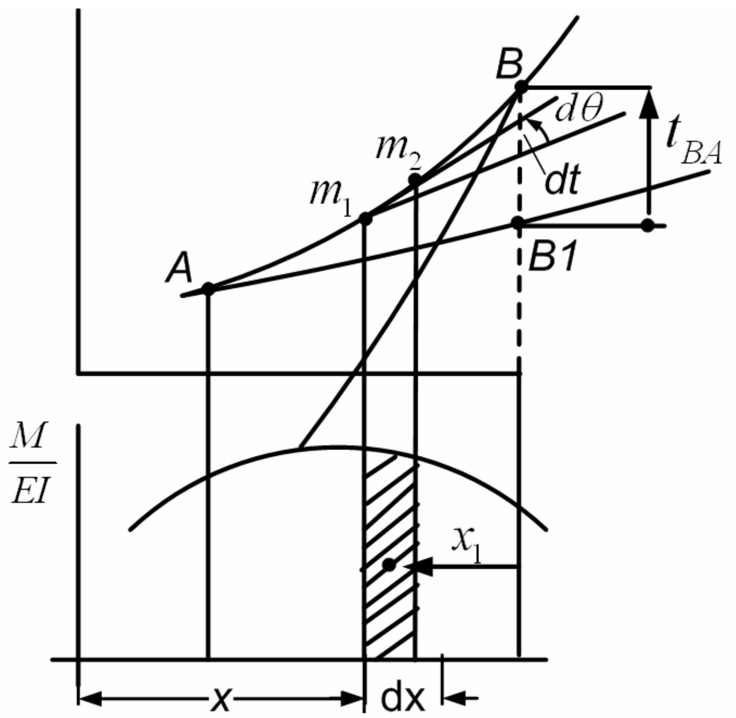
In performing above integration, areas corresponding to the $+M$ are taken + ve, area corresponding to the – ve M are taken –ve

If $\int_A^B \frac{M}{EI}dx$ is +ve- tangent B rotates c.c.w from A or θ_B is algebraically larger than A.

If – ve – tangent B rotates c.w from A.

<u>Second moment-area theorem</u>

This is related to the deflection curve between A and B.



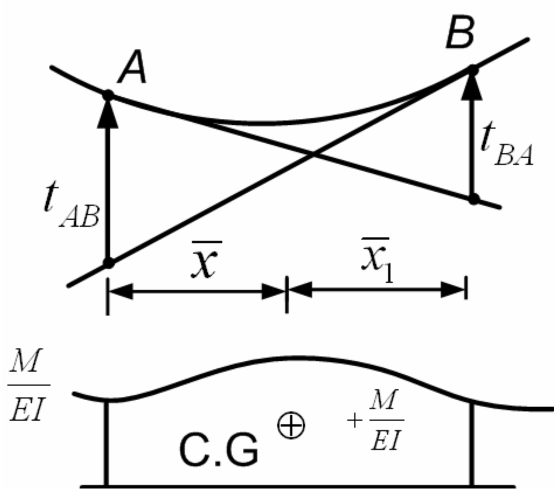
We see that dt is a small contribution to t_{BA}. Since the angles between the tangents and x-axis are very small we can take

$$dt = x_1 d\theta = x_1 \frac{M}{EI} dx$$

The expression $x_1 \frac{M}{EI} dx =$ first moment of infinitesimal area $\frac{M}{EI} dx$ w.r.t. a vertical line through B.

Integrating between the point A & B

$t_{BA} = \int_A^B dt = \int_A^{B'} x_1 \frac{M}{EI} dx =$ First moment of the area of the $\frac{M}{EI}$ diagram between points A & B, evaluated w.r.t. B.



$$t_{BA} = \phi \bar{x}_1$$

$$t_{AB} = \phi \bar{x}$$

$$where \ \phi = \int_A^B \frac{M}{EI} dx$$

if M is +ve ϕ=+ve

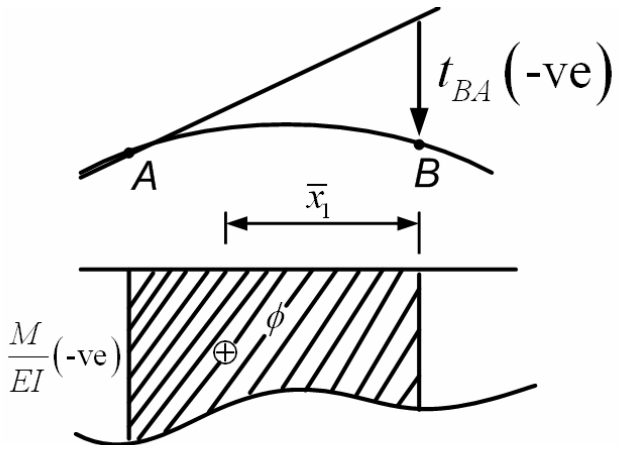
if M is -ve ϕ=-ve

$\bar{x}$ and $\bar{x}_1$ are always taken +ve quantities.

$\therefore$Sign of tangential deviation depends on sign of M.

A positive value of tangential deviation- point B is above A and vice versa – ve value means point B is below the point A.

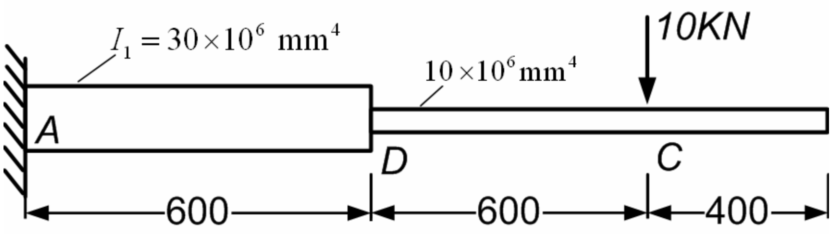
In applying the moment area method a carefully prepared sketch of the elastic curve is always necessary.



Problem:1

Consider an aluminum cantilever beam 1600 mm long with a 10 –kN for a applied 400 mm from the free end for a distance of 600 mm from the fixed end, the beam is of greater depth than it is beyond, having $I_1 = 50\times10^6\,mm^4$. For the remaining 1000 mm of the beam $I_2 = 10\times10^6\,mm^4$. Find the deflection and angular rotation of the free end. Neglect weight of the beam and $E = 70GPa$

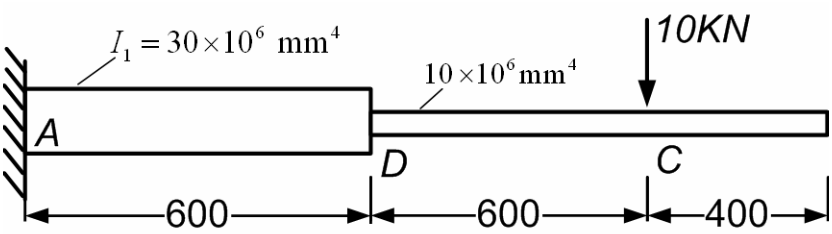
Solution:

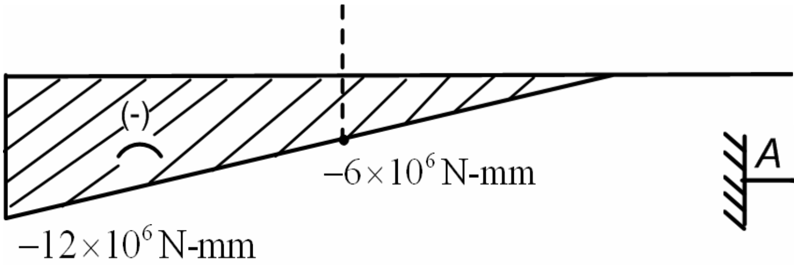


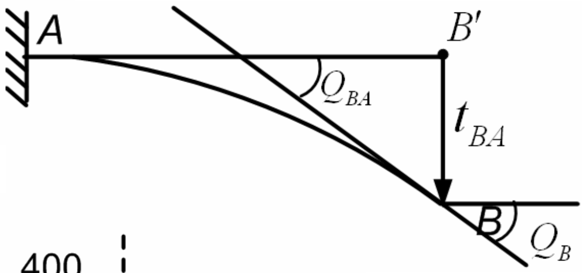
$EI = 3.5\times10^{24}$

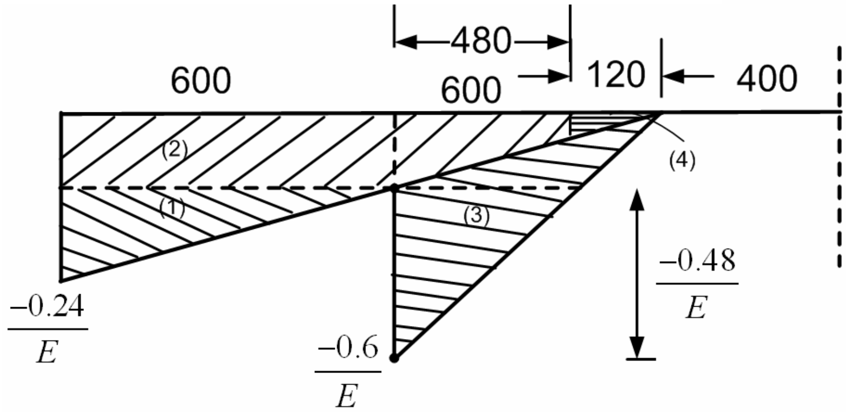
$70\times10^9\times10^{-6}\ N/mm^2$

$= 70\times10^3 N/mm^2$









$$\frac{-0.6}{E\times 600}=\frac{-0.48}{E.x}$$

$x = 480\text{mm}$

$$A_1=\frac{1}{2}bh=\frac{1}{2}\times 600\times\left(\frac{-0.12}{E}\right)=-\frac{36}{E}$$

$$A_2=bh=-\frac{129.6}{E}$$

$$A_3=\frac{1}{2}bh=\frac{1}{2}\times 480\times\left(\frac{-0.48}{E}\right)=-\frac{115.2}{E}$$

$$A_4=\frac{1}{2}bh=\frac{1}{2}\times 120\times\left(\frac{-0.12}{E}\right)=-\frac{7.2}{E}$$

$$\Delta Q_{BA}=Q_B-Q_A=\int_A^B\frac{M}{EI}dx=A_1+A_2+A_3+A_4$$

$$Q_B=-\frac{36}{E}-\frac{129.6}{E}-\frac{115.2}{E}-\frac{7.2}{E}=-\frac{288}{E}$$

$$Q_B = -\frac{288}{E} = -\frac{288}{70\times10^3} = -4.14\times10^{-3}\ rad$$

$Q_B = 4.14\times10^{-3}\ rad$ ↺ from tangent at A.

$$t_{BA} = \delta_B$$

$\bar{x}_2 = 1060mm; \bar{x}_1 = 1400; \bar{x}_3 = 840mm; \bar{x}_4 = 480mm$

$$t_{BA} = \delta_B = A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 + A_4\bar{x}_4$$

$$= \left(\frac{-36}{E}\right)1400 + \left(\frac{-129.6}{E}\right)1060 + \left(\frac{-115.2}{E}\right)840 + \left(\frac{-7.2}{E}\right)480$$

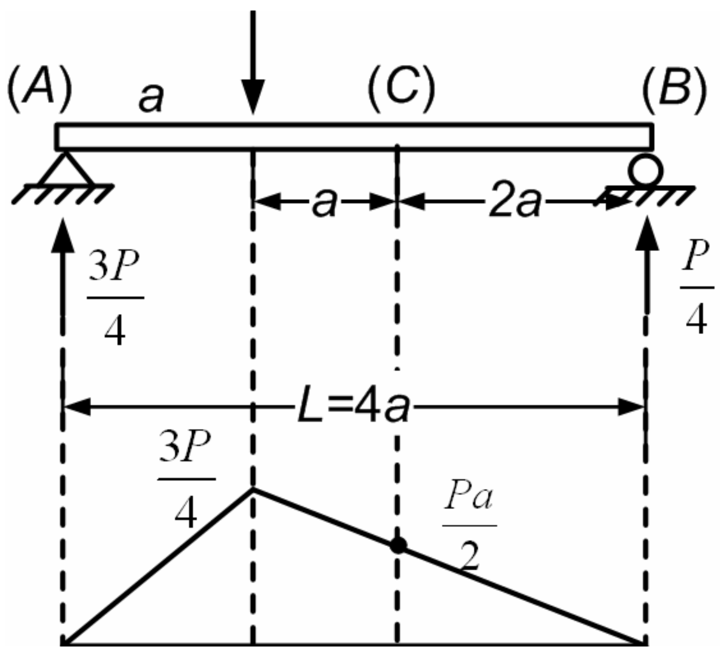
$$= \frac{-288000}{E} = -4.11mm$$

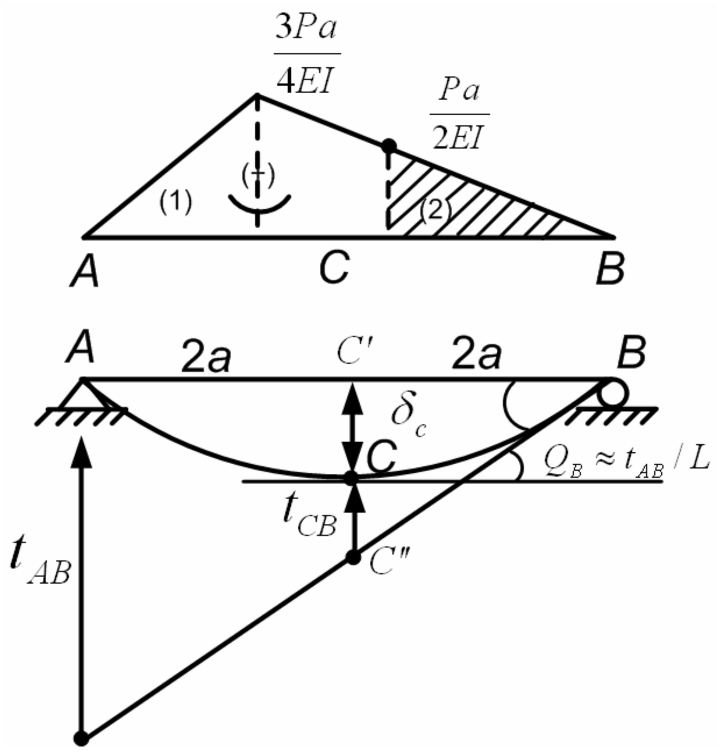
$\delta_B = -4.11mm$ below the tangent at point A.

Problem 2

Find the deflection due to the concentrated force P applied as soon as figure, at the center of a simply supported beam EI constant.

Solution:





Since EI is constant $\frac{M}{EI}$ diagram is same as M diagram.

$$v_c = c''c' - t_{CB}$$

$$c''c' = \frac{1}{2} t_{AB}$$

$$A_1 = \frac{1}{2}bh = \frac{1}{2} \times a \times \frac{3Pa}{4EI} = \frac{3Pa^2}{8EI} s$$

$$A_2 = \frac{1}{2}bh = \frac{1}{2} \times 3a \times \frac{3Pa}{4EI} = \frac{9Pa^2}{8EI}$$

$$\bar{x}_1 = \frac{2}{3}a\,;\ \bar{x}_2 = 2a$$

$$t_{AB} = A_1\bar{x}_1 + A_2\bar{x}_2 = \frac{3Pa^2}{8EI}\frac{2}{3}a + \frac{9Pa^2}{8EI}2a$$

$$= \frac{Pa^3}{4EI} + \frac{9Pa^3}{4EI} = \frac{10Pa^3}{4EI} = \frac{5Pa^3}{2EI}(+ve)$$

$$t_{CB} = \frac{1}{2} \times 2a \times \frac{Pa}{2EI} \times \left(\frac{2a}{3} \right) = \frac{Pa^3}{3EI} s$$

$$c''c' = t_{AB/2} = \frac{5Pa^3}{4EI}$$

$$\therefore v_c = \frac{5Pa^3}{4EI} - \frac{Pa^3}{3EI} = \frac{(15-4)Pa^3}{4EI} = \frac{11Pa^3}{12EI}$$

$$v_c = \frac{11Pa^3}{12EI}$$

The +ve sign of t_{AB} & t_{CA} indicate points A & C above the tangent through B.

(a) The slope of the elastic curve at C can be found from the slope of one of the ends as:

$$\Delta Q_{BC} = Q_B - Q_C \Rightarrow Q_C = Q_B - \Delta Q_{BC}$$

$$\Delta Q_{BC} = Q_B - Q_C \int_C^B \frac{M}{EI} dx = \frac{1}{2} \times 2a \times \frac{Pa}{2EI} = \frac{Pa^2}{2EI} s$$

$$Q_B \approx t_{AB} / L = \frac{5Pa^3}{2EI} \frac{1}{4a} - \frac{Pa^2}{2EI} = \frac{5Pa^2}{8EI} - \frac{Pa^2}{2EI}$$

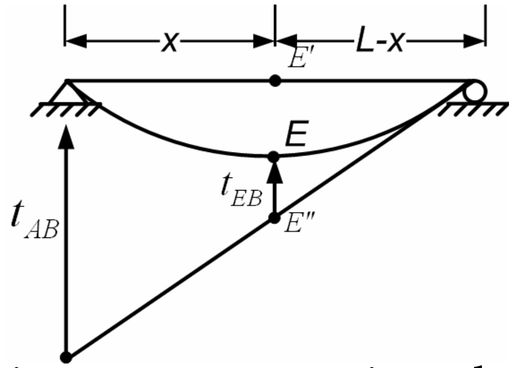
$$Q_c = \frac{Pa^2}{8EI}$$

(b) If the deflection curve equations is wanted then by selecting an ordinary point E at a distance x

$$v_E = E''E' - EE''$$

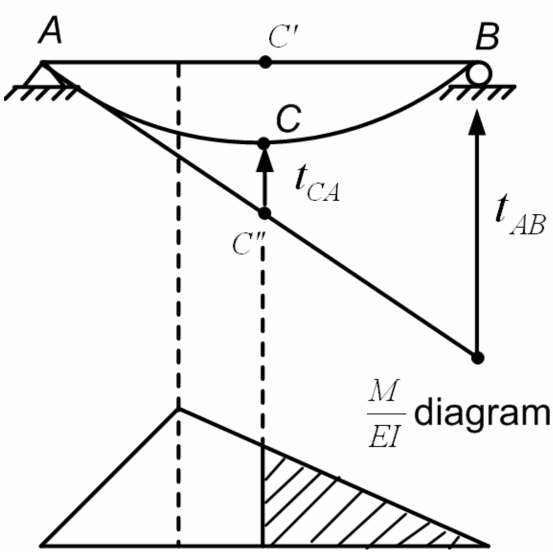
$$v_E = \left(\frac{L-x}{L}\right) t_{AB} - t_{EB}$$

In this way one can obtain equation of the deflection curve.



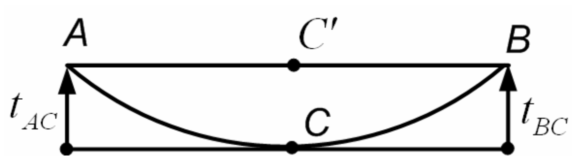
(c) To simplify the calculations some care in selecting the tangent at a support must be considered.

In this approach to find t_{CA} we need to consider unhatched region which is more difficult.



(d) The deflection at C can also be calculated as follows.

$$v_c = \frac{t_{AC} + t_{BC}}{2}$$

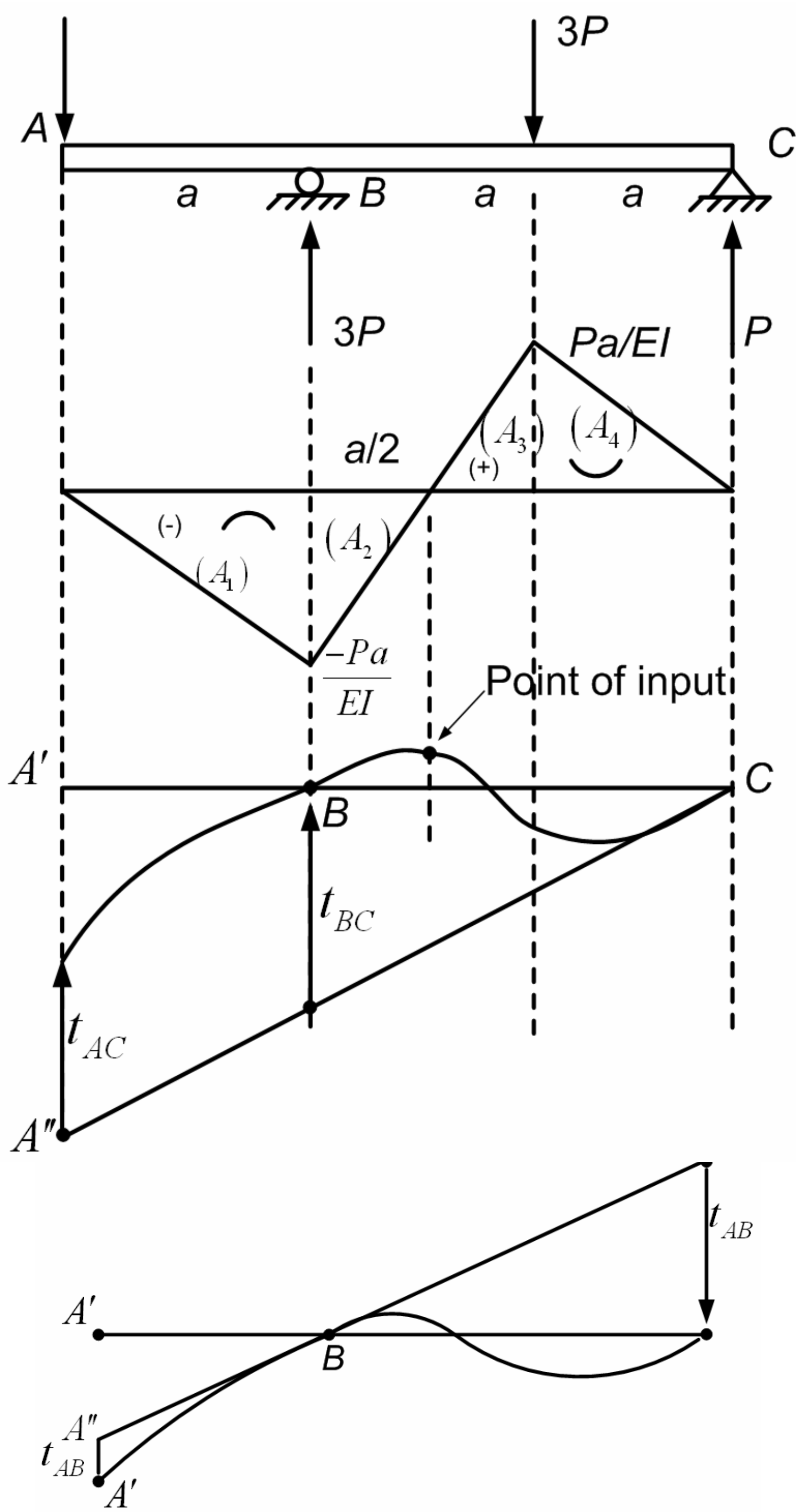


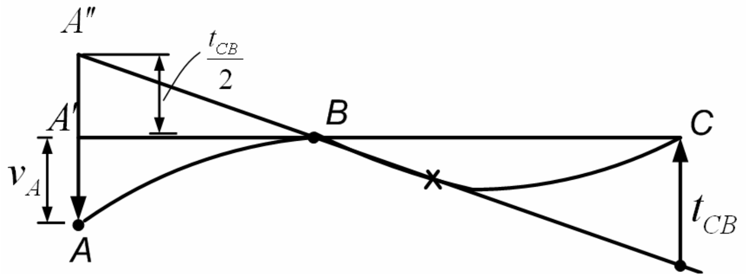
∴C is at the center of the beam. However, this is also move complicated approach compared to first, as to find t_{CA} we again need to consider unhatched region.

Problem 3

Find the deflection of the end A of the beams shown in figure caused by the applied forces. The EI is constant.

Solution





$$A_1 = \frac{1}{2}bh = \frac{1}{2} \times a \times \left(\frac{-Pa}{EI}\right) = -\frac{Pa^2}{2EI}$$

$$A_2 = \frac{1}{2} \times \frac{a}{2} \times \left(-\frac{Pa}{EI}\right) = -\frac{Pa^2}{4EI}$$

$$A_3 = \frac{Pa^2}{4EI} \quad and \quad A_4 = \frac{Pa^2}{2EI}$$

$$\bar{x}_1 = \frac{a}{3} + 2a = \frac{7a}{3};\ \bar{x}_2 = \frac{2}{3}\frac{a}{3} + \frac{a}{2} + a = \frac{11a}{6}$$

$$\bar{x}_3 = \frac{1}{3}\frac{a}{2} + a = 7a/6;\ \bar{x}_4 = \frac{2a}{3}$$

$$t_{CB} = A_2\bar{x}_2 + A_3\bar{x}_3 + A_4\bar{x}_4$$

$$= -\frac{Pa^2}{4EI} \times \frac{11a}{6} + \frac{Pa^2}{4EI} \times \frac{7a}{6} + \frac{Pa^2}{2EI} \times \frac{2a}{3}$$

$$= -\frac{11Pa^3}{24EI} + \frac{7Pa^3}{24EI} + \frac{Pa_3}{3EI} = \frac{(-11+7+8)Pa^3}{24EI}$$

$$t_{CB} = \frac{4Pa^3}{24EI} = \frac{Pa^3}{6EI}$$

The + sign of t_{CB} indicates that the point C is above the tangent through B. Hence corrected sketch of the elastic curve is made.

$$t_{AB} = -\frac{Pa^2}{2EI} \times \frac{2}{3} a = -\frac{Pa^3}{3EI}$$

$$\therefore\ v_A = \left| t_{AB} \right| - A''A'$$

$$= \frac{Pa^3}{3EI} - \frac{Pa^3}{12EI} = \frac{Pa^3}{4EI}$$

$$v_A = \frac{Pa^3}{4EI}$$

Note: Another method to find v_A is shown. This may be simpler method than the present one.

References:

Solid Mechanics – INTERNET SOURCES & E-BOOKS

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UNIT V ANALYSIS OF STRESSES IN TWO DIMENSIONS

Biaxial state of stresses – Thin cylindrical and spherical shells – Deformation in thin cylindrical and spherical shells – Biaxial stresses at a point – Stresses on inclined plane – Principal planes and stresses – Mohr‘s circle for biaxial stresses – Maximum shear stress - Strain energy in bending and torsion.

TEXT BOOKS

T [1] – R. K. Bansal (2010), "A Textbook of Strength of Materials, Laxmi Publications, New Delhi.

T [2] – R. S. Khurmi (2013), "Strength of Material", S. Chand Publications. New Delhi

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R [3] - Bedi D.S (1984), "Strength of Materials", S Chand and Co. Ltd., New Delhi

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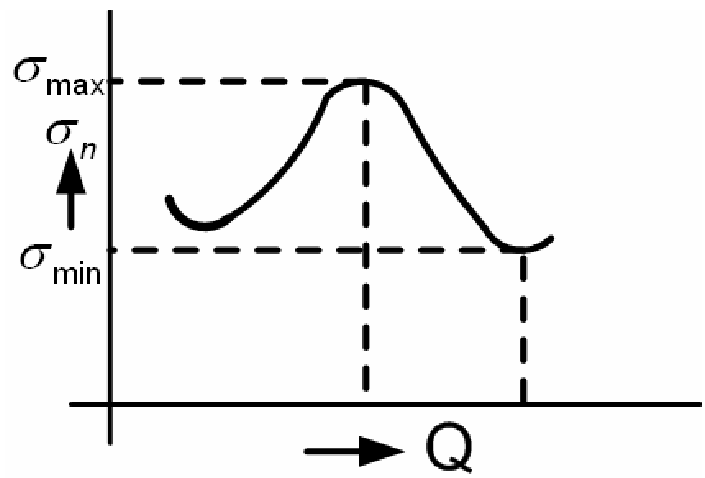
4. Principal Stresses

Principal Stresses

Now we are in position to compute the direction and magnitude of the stress components on any inclined plane at any point, provided if we know the state of stress (Plane stress) at that point. We also know that any engineering component fails when the internal forces or stresses reach a particular value of all the stress components on all of the infinite number of planes only stress components on some particular planes are important for solving our basic question i.e under the action of given loading whether the component will ail or not? Therefore our objective of this class is to determine these plane and their corresponding stresses.

$$(1)\ \sigma_n = \sigma_n(\theta) = \frac{\sigma_n + \sigma_y}{2} + \frac{\sigma_n - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$



(2) Of all the infinite number of normal stresses at a point, what is the maximum normal stress value, what is the minimum normal stress value and what are their

corresponding planes i.e how the planes are oriented ? Thus mathematically we are looking for maxima and minima of $\sigma_n(Q)$ function..

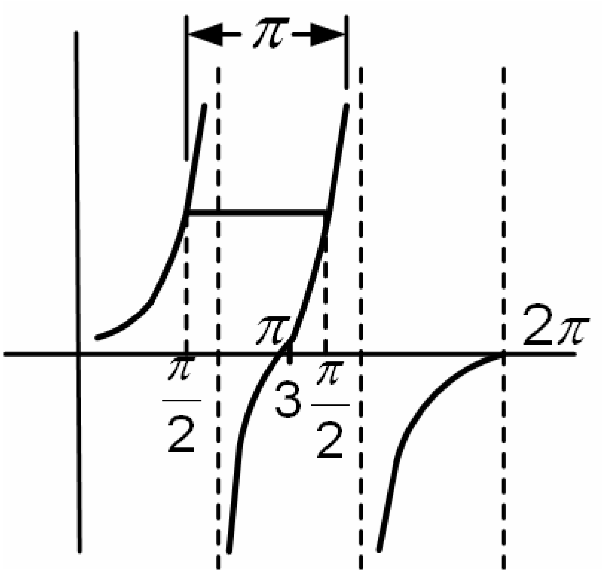
$$(3)\quad \sigma_n = \frac{\sigma_n + \sigma_y}{2} + \frac{\sigma_n - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

For maxima or minima, we know that

$$\frac{d\sigma_n}{d\theta} = 0 = -\left(\sigma_x - \sigma_y\right) Sin2\theta + 2\tau_{xy} Cos2\theta$$

$$tan\,2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

(4) The above equations has two roots, because *tan* repeats itself after π. Let us call the first root as θ_{P_1}



$$tan\,2\theta_{P_1} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$tan\,2\theta_{P_2} = tan\left(2\theta_{P_1} + \pi\right) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\theta_{P_2} = \theta_{P_1} + \frac{\pi}{2} s$$

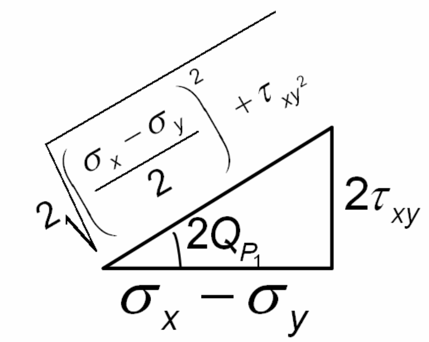
(5) Let us verify now whether we have minima or minima at θ_{P_1} and θ_{P_2}

$$\frac{d^2\sigma_n}{d\theta^2} = -2(\sigma_x - \sigma_y)Cos2\theta - 4\tau_{xy}Sin2\theta$$

$$\therefore \left.\frac{d^2\sigma_n}{d\theta^2}\right|_{\theta=\theta_{P_1}} = -2(\sigma_x - \sigma_y)Cos2\theta_{P_1} - 4\tau_{xy}Sin2\theta_{P_1}$$

We can find $Cos2\theta_{P_1}s$ and $Sin2\theta_{P_1}s$ as

$$Cos2\theta_{P_1} = \frac{\sigma_x - \sigma_y}{2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$



$$Sin2\theta_{P_1} = \frac{2\tau_{xy}}{2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}} = \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

Substituting $Cos2\theta_{P_1}$ and $Sin2\theta_{P_1}$

$$\left.\frac{d^2\sigma_n}{d\theta^2}\right|_{\theta=\theta_{P_1}} = \frac{-2\left(\sigma_x-\sigma_y\right)\left(\sigma_x-\sigma_y\right)}{2\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}} - \frac{4\tau_{xy}\tau_{xy}}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

$$= \frac{-\left(\sigma_x-\sigma_y\right)^2}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}} - \frac{4\tau_{xy}^2}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

$$= \frac{-4}{\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}\left[\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}\right]$$

$$\therefore\ \frac{d^2\sigma_n}{d\theta^2} = -4\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} \qquad \text{(-ve)}$$

$$\left.\frac{d^2\sigma_n}{d\theta^2}\right|_{\theta=\theta_{P_2}=\theta_{P_1}+\frac{\pi}{2}} = 2\left(\sigma_x-\sigma_y\right)Cos\left(2\theta_{P_1}+\pi\right) - 4\tau_{xy}Sin\left(2\theta_{P_1}+\pi\right)$$

$$= 2\left(\sigma_x-\sigma_y\right)Cos2\theta_{P_1} + 4\tau_{xy}Sin2\theta_{P_1}$$

Substituting $Cos2\theta_{P_1}$ & $Sin2\theta_{P_1}$ m we can show that

$$\therefore\ \left.\frac{d^2\sigma_n}{d\theta^2}\right|_{\theta=\theta_{P_2}} = -4\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2 s} \qquad \text{(+ve)}$$

Thus the angles $\theta_{P_1}s$ and $\theta_{P_2}s$ define planes of either maximum normal stress or minimum normal stress.

(6) Now, we need to compute magnitudes of these stresses

We know that,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

$$\sigma_n\big|_{\theta=\theta_{P_1}} = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta_{P_1} + \tau_{xy} Sin2\theta_{P_1}$$

Substituting $Cos2\theta_{P_1}s$ and $Sin2\theta_{P_1}$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Max. Normal stress because of + sign

Similarly,

$$\sigma_n\big|_{\theta=\theta_{P_2}=\theta_{P_1}=\frac{\pi}{2}} = \sigma_2 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos\left(2\theta_{P_1} + \pi\right) +$$
$$\tau_{xy} Sin\left(2\theta_{P_1} + \pi\right)$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} Cos2\theta_{P_1} - \tau_{xy} Sin2\theta_{P_1}$$

Substituting $Cos2\theta_{P_1}$ and $Sin2\theta_{P_1}$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$Min. normal\, sress\, because\, of -ve\, sign$$

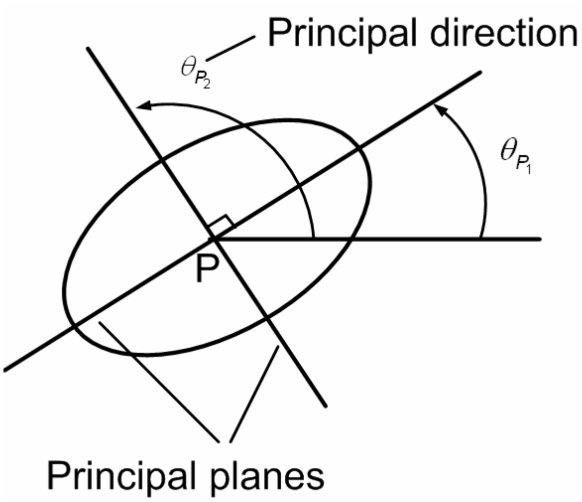
We can write

$$\sigma_1\, or\, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(7) Let us se the properties of above stress.

(1) $\theta_{P_2} = \theta_{P_1} + \frac{\pi}{2}s$ - planes on which maximum normal stress and minimum normal stress act are ⊥ to each other.

(2) Generally maximum normal stress is designated by σ_1 and minimum stress by σ_2. Also $\theta_{P_1} \to \sigma_1 ; \theta_{P_2} \to \sigma_2$



$$\sigma_1 > \sigma_2 \quad algebraically \;\; i.e.,$$

$$0 - \quad \sigma_1$$

$$-1000 - \sigma_2$$

(4) maximum and minimum normal stresses are collectively called as principal stresses.

(5) Planes on which maximum and minimum normal stress act are known as principal planes.

(6) θ_{P_1} and θ_{P_2} that define the principal planes are known as principal directions.

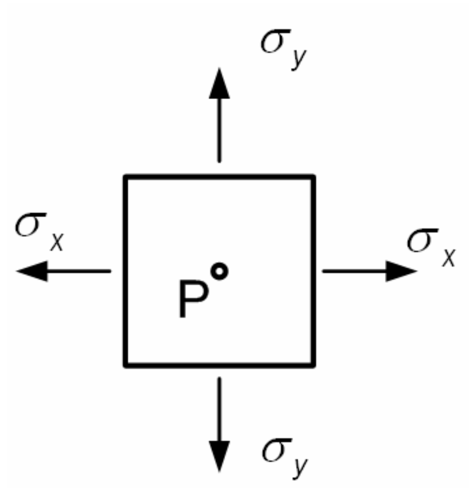
(8) Let us find the planes on which shearing stresses are zero.

$$\tau_{nt} = 0 = -\left(\sigma_x - \sigma_y\right) Sin2\theta + \tau_{xy} Cos2\theta$$

$$tan\,2\theta = \frac{2\tau_{xy}}{\sigma_x = \sigma_y}$$

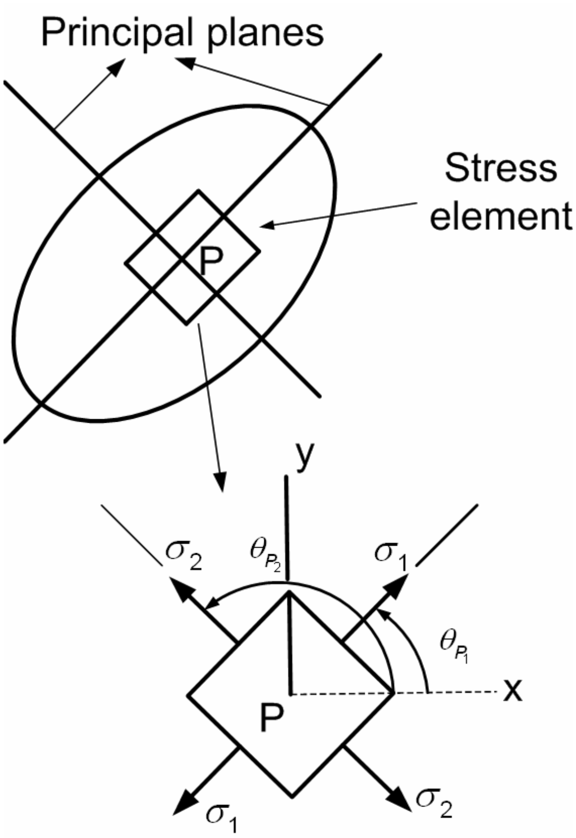
$$= directions\ of\ principal\ plans$$

Thus on the principal planes no shearing stresses act. Conversely, the planes on which no shearing stress acts are known as principal planes and the corresponding normal stresses are principal stresses. For example the state of stress at a point is as shown.



Then σ_x and σ_y are principal stresses because no shearing stresses are acting on these planes.

(9) Since, principal planes are ⊥ to each other at a point P, this also means that if an element whose sides are parallel to the principal planes is taken out at that point P, then it will be subjected to principal stresses. Observe that no shearing stresses are acting on the four faces, because shearing stresses must be zero on principal planes.



(10) Since σ_1 and σ_2 are in two ⊥ directions, we can easily say that

$$\sigma_x + \sigma_y = \sigma_1 + \sigma_2 = \sigma_{x'} + \sigma_{y'} = I_1$$

5. Maximum shear stress

Maximum and minimum shearing stresses

So far we have seen some specials planes on which the shearing stresses are always zero and the corresponding normal stresses are principal stresses. Now we wish to find what are maximum shearing stress plane and minimum shearing stress plane. We approach in the similar way of maximum and minimum normal stresses

$$(1)\quad \tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) Sin 2\theta + \tau_{xy} Cos 2\theta$$

$$\frac{d\tau_{nt}}{d\theta} = -\left(\sigma_x - \sigma_y\right) Cos 2\theta + \tau_{xy} Cos 2\theta$$

For maximum or minimum

$$\frac{d\tau_{nt}}{d\theta} = 0 = -\left(\sigma_x - \sigma_y\right) Cos 2\theta - 2\tau_{xy} Sin 2\theta$$

$$\Rightarrow tan\, 2\theta = \frac{-\left(\sigma_x - \sigma_y\right)}{2\tau_{xy}}$$

This has two roots

$$tan\, 2\theta_{S_1} = -\frac{\left(\sigma_x - \sigma_y\right)}{2\tau_{xy}}$$

$s - stands\ for\ shear\ stress$

$p - stands\ for\ principal\ stresses.$

$$\tan 2\theta_{S_2} = \tan\left(2\theta_{S_1} + \pi\right) = \frac{-\left(\sigma_x - \sigma_y\right)}{2\tau_{xy}}$$

$$\therefore \quad \theta_{S_2} = \theta_{S_1} + \frac{\pi}{2}$$

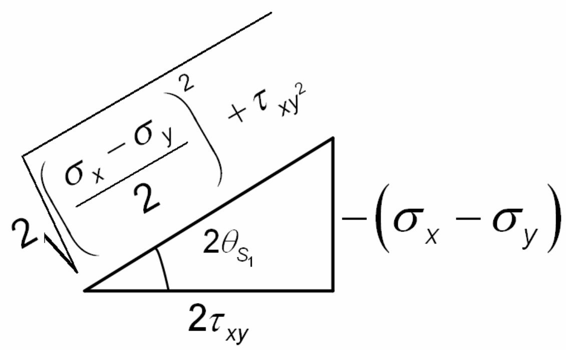
Now we have to show that at these two angles we will have maximum and minimum shear stresses at that point.

Similar to the principal stresses we must calculate

$$\frac{d^2\tau_{nt}}{d\theta^2} = 2\left(\sigma_x - \sigma_y\right)Sin2\theta - 4\tau_{xy}Cos2\theta$$

$$\left.\frac{d^2\tau_{nt}}{d\theta^2}\right|_{\theta=\theta_{S_1}} = 2\left(\sigma_x - \sigma_y\right)Sin2\theta_{S_1} - 4\tau_{xy}Cos2\theta_{S_1}$$

$$Cos2\theta_{S_1} = \frac{2\tau_{xy}}{2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$



$$Sin2\theta_{S_1} = \frac{-\left(\sigma_x - \sigma_y\right)}{2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

Substituting above values in the above equation we can show that

$$\left.\frac{d^2\tau_{nt}}{d\theta^2}\right|_{\theta=\theta_{S_1}} = \text{- ve}$$

Similarly we can show that

$$\left.\frac{d^2\tau_{nt}}{d\theta^2}\right|_{\theta=\theta_{S_2}=\theta_{S_1}+\frac{\pi}{2}} = \text{+ ve}$$

Thus the angles θ_{S_1} and θ_{S_2} define planes of either maximum shear stress or minimum shear stress. Planes that define maximum shear stress & minimum shear stress are again ⊥ to each other.. Now we wish to find out these values.

$$\tau_{nt} = -\frac{(\sigma_x - \sigma_y)}{2} Sin2\theta + \tau_{xy} Cos2\theta$$

$$\tau_{nt}\big|_{\theta=\theta_{S_1}} = -\frac{(\sigma_x - \sigma_y)}{2} Sin2\theta_{S_1} + \tau_{xy} Cos2\theta_{S_1}$$

Substituting $Cos2\theta_{S_1}$ and $Sin2\theta_{S_1}s$, we can show that

$$\tau_{max} = +\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{nt}\big|_{\theta=\theta_{S_2}=\theta_{S_1}+\frac{\pi}{2}} = -\frac{(\sigma_x - \sigma_y)}{2} Sin\left(2\theta_{S_1}+\pi\right) + \tau_{xy} Cos\left(2\theta_{S_1}+\pi\right)$$

Substituting $Cos2\theta_{S_1}$ and $Sin2\theta_{S_1}$

$$\tau_{min} = -\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

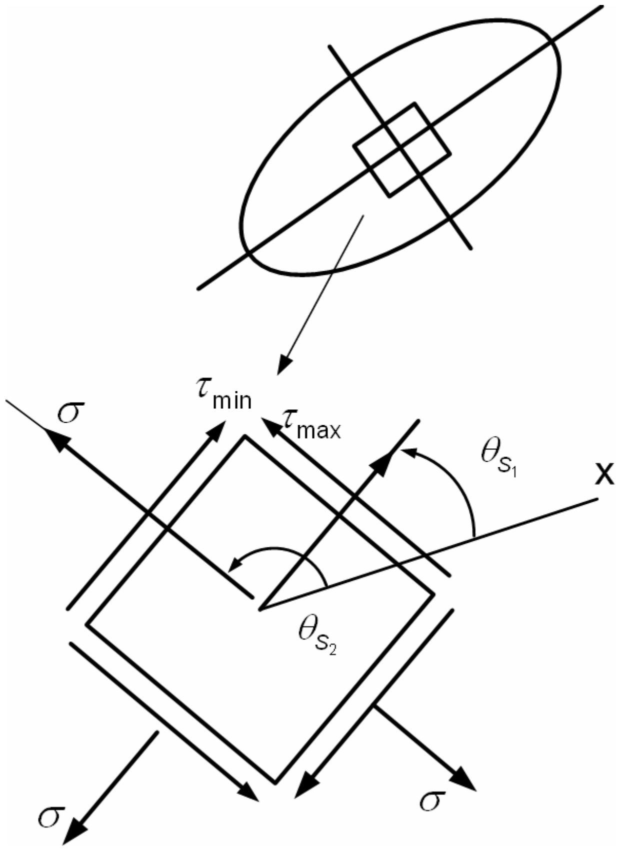
τ_{max} is algebraically $> \tau_{min}$, however their absolute magnitude is same. Thus we can write

$$\tau_{max} \; or \; \tau_{min} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Generally

$$\tau_{max} - \theta_{S_1}$$
$$\tau_{min} - \theta_{S_2}$$

Q. Why τ_{max} and τ_{min} are numerically same. Because θ_{S_1} & θ_{S_2} are $\perp$ planes.



(2) Unlike the principal stresses, the planes on which maximum and minimum shear stress act are not free from normal stresses.

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta s$$

$$\sigma_n \big|_{\theta=\theta_{S_1}} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta_{S_1} + \tau_{xy} Sin2\theta_{S_1}$$

Substituting $Cos2\theta_{S_1}$ and $Sin2\theta_{S_1}$

$$\sigma = \sigma_n \big|_{\theta=\theta_{S_1}} = \frac{\sigma_x + \sigma_y}{2}$$

$$\sigma_n \big|_{\theta=\theta_{S_2}=\theta_{S_1}+\frac{\pi}{2}} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos\left(2\theta_{S_1} + \pi\right)$$

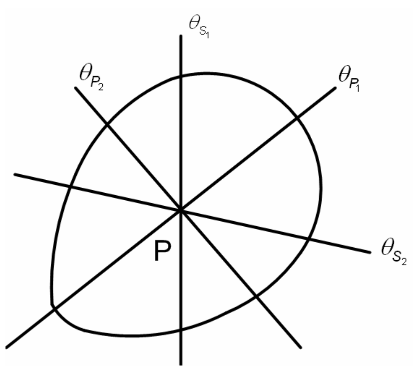
$$+\tau_{xy} Sin\left(2\theta_{S_1} + \pi\right)$$

Simplifying this equation gives

$$\sigma = \sigma_n \big|_{\theta=\theta_{S_2}} = \frac{\sigma_x + \sigma_y}{2}$$

Therefore the normal stress on maximum and minimum shear stress planes is same.

(3) Both the principal planes are ⊥ to each other and also the planes of τ_{max} and τ_{min} are also ⊥ to each other. Now let us see there exist any relation between them.

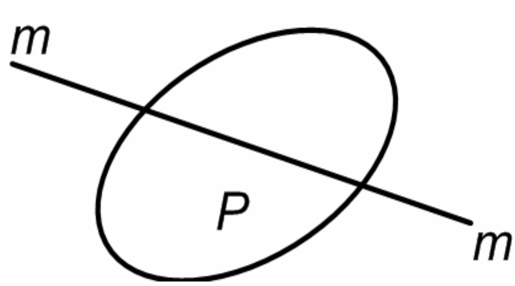


6. Mohr's circle

Mohr's circle for plane stress

So far we have seen two methods to find stresses acting on an inclined plane

(a) Wedge method
(b) Use of transformation laws.



Another method which is purely graphical approaches is known as the Mohr's circle for plane stress.

A major advantage of Mohr's circle is that, the state of the stress at a point, i.e the stress components acting on all infinite number of planes can be viewed graphically.

Equations of Mohr's circle

We know that, $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$

This equation can also be written as

$$\sigma_n - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2} Cos2\theta + \tau_{xy} Sin2\theta$$

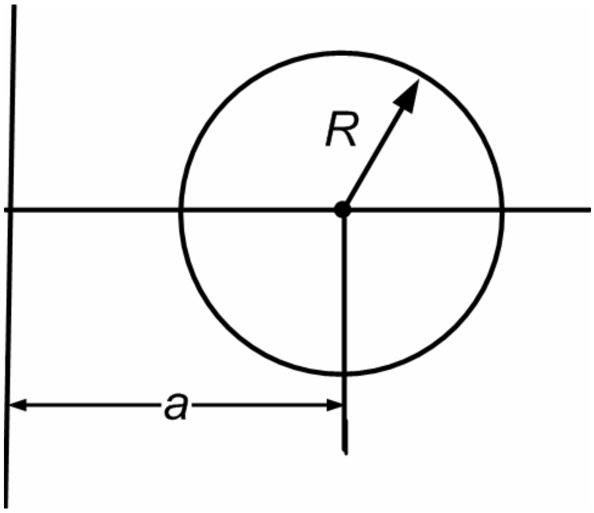
$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) Sin2\theta + \tau_{xy} Cos2\theta$$

$$\left[\sigma_n - \left(\frac{\sigma_x + \sigma_y}{2}\right)\right]^2 + \tau_{nt}^2 = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

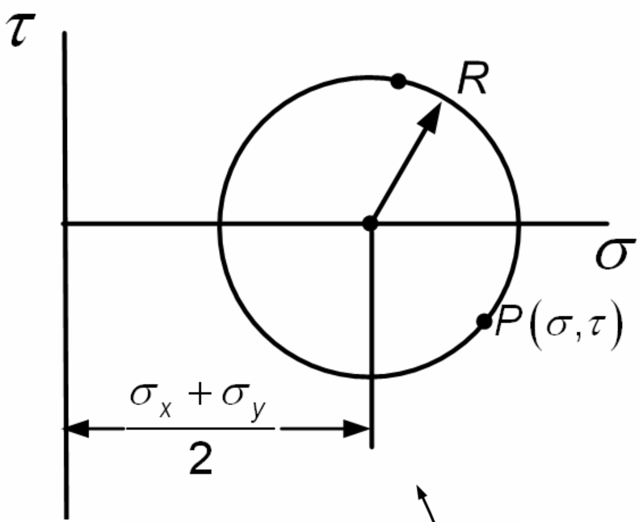
$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$(x - a)^2 + y^2 = R^2$$

The above equation is clearly an equation of circle with center at $(a,0)$ on $\tau-\sigma$ plane it represents a circle with center at $\left(\frac{\sigma_x+\sigma_y}{2},0\right)$ and having radius



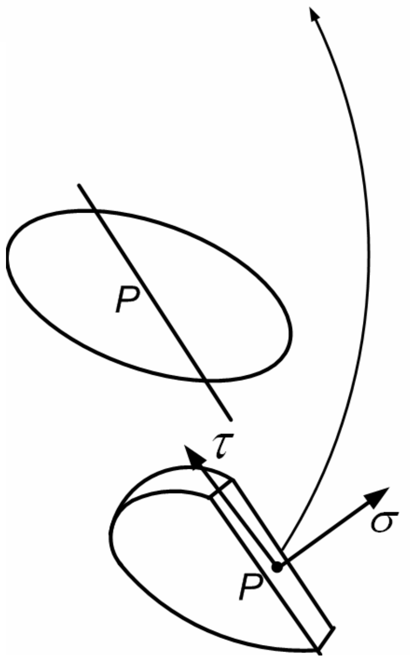
$$R=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)+\tau_{xy}^2}$$



This circle on $\sigma-\tau$ plane- Mohr's circle.

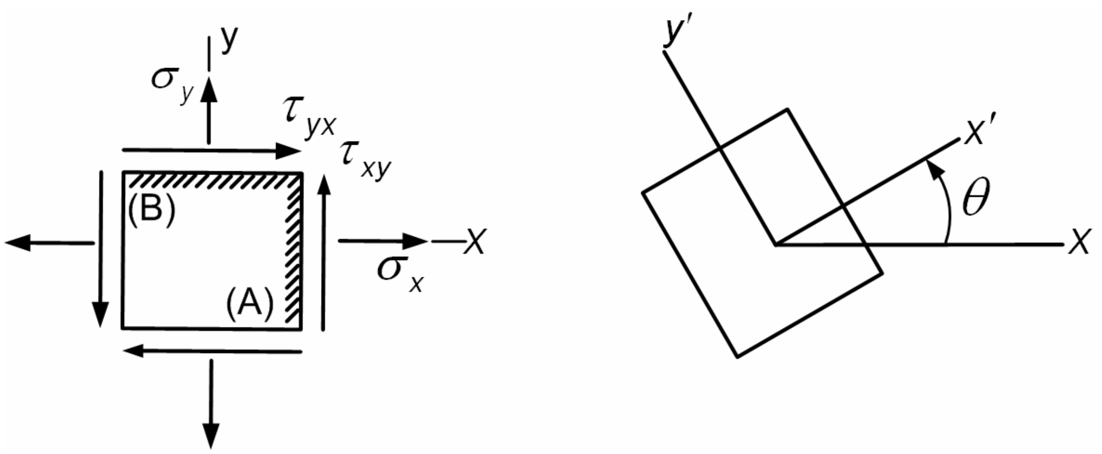
From the above deviation it can be seen that any point P on the Mohr's circle represents stress which are acting on a plane passing through the point.

In this way we can completely visualize the stresses acting on all infinite planes.

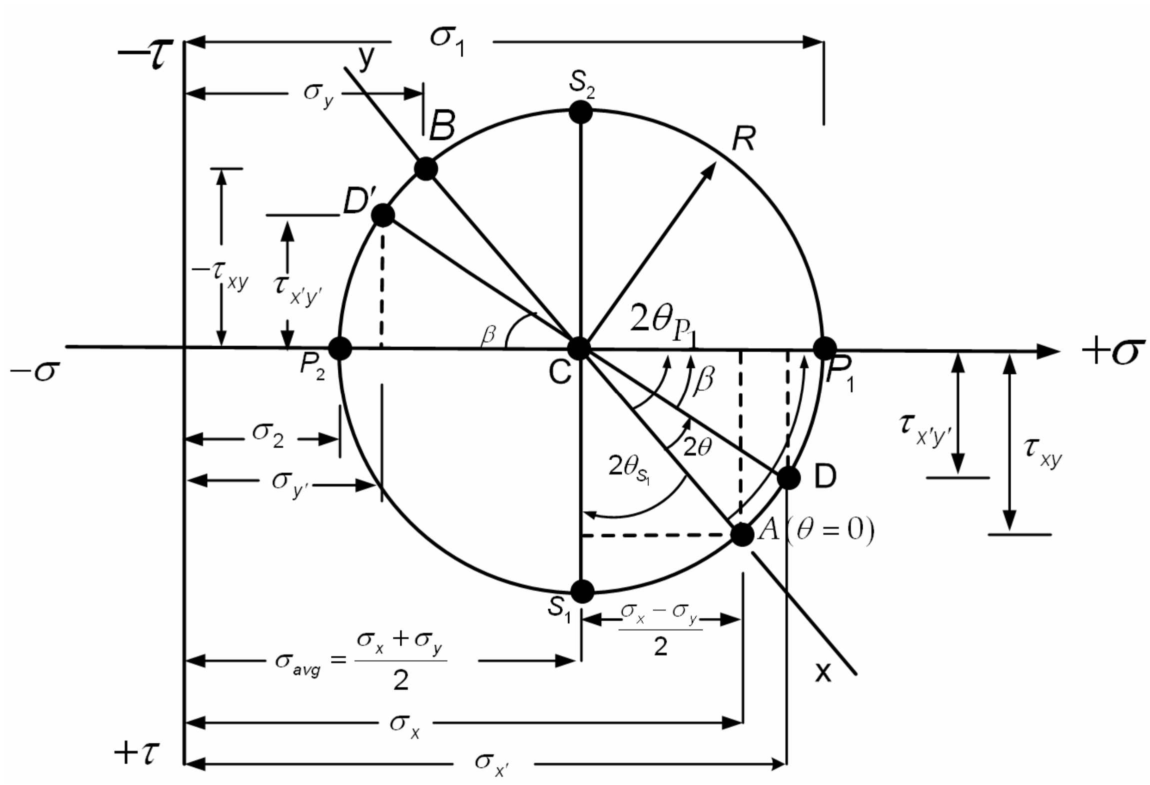


(3) *Construction of Mohr's circle*

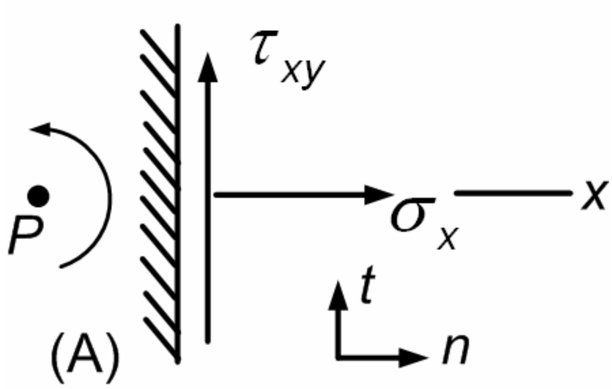
Let us assume that the state of stress at a point is given



A typical problem using Mohr's circle i.e given $\sigma_{x'}, \sigma_{y'}$ and $\tau_{x'y'}$ on an inclined element. For the sake of clarity we assume that, $\sigma_{x'}, \sigma_{y'}s$ and $\tau_{x'y'}$ all are positive and $\sigma_x > \sigma_y$

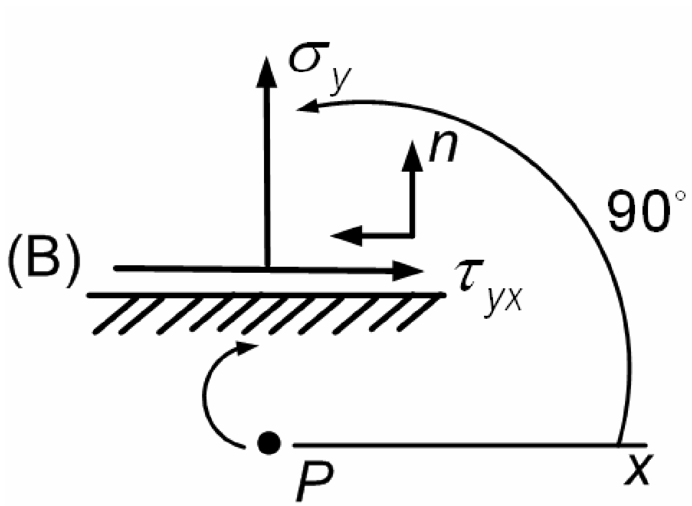


- Since any point on the circle represents the stress components on a plane passing through the point. Therefore we can locate the point A on the circle.
- The coordinates of the plane $A = \left(+\sigma_x, +\tau_{xy}\right)$



Therefore we can locate the point A on the circle with coordinates $\left(+\sigma_x, +\tau_{xy}\right)s$

- Therefore the line AC represents the x-axis. Moreover, the normal of the A-plane makes 0° w.r.t the x-axis.
- In a similar way we can locate the point B corresponding to the plane B.



The coordinates of $B = \left(+\sigma_y, -\tau_{xy}\right)s$

Since we assumed that for the sake of similarity $\sigma_y < \sigma_x s$. Therefore the point B diametrically opposite to point A.

- The line BC represents y- axis. The point A corresponds to $Q = 0^\circ$, and pt. B corresponds to $Q = 90^\circ(+\text{ve})$ of the stress element.

At this point of time we should be able to observe two important points.

- The end points of a diameter represents stress components on two ⊥ planes of the stress element.
- The angle between x- axis and the plane B is 90° (c.c.w) in the stress element. The line CA in Mohr's circle represents x- axis and line CB represents y-axis or plane B. It can be seen that, the angle between x-axis and y-axis in the Mohr's circle is 180° (c.c.w). Thus 2Q in Mohr's circle corresponds to Q in the stress element diagram.

<u>Stresses on an inclined element</u>

- Point A corresponds to $Q = 0$ on the stress element. Therefore the line CA i.e x-axis becomes reference line from which we measure angles.
- Now we locate the point "D" on the Mohr's circle such that the line CD makes an angle of 2Q c.c.w from the x-axis or line CA. we choose c.c.w because in the stress element also Q is in c.c.w direction.

- The coordinates or stresses corresponding to point D on the Mohr's circle represents the stresses on the x'- face or D on the stress element.

$$\sigma_{x'} = \sigma_{avg} + RCos\beta$$
$$\tau_{x'y'} = RSin\beta$$
$$\sigma_{y'} = \sigma_{avg} - RCos\beta$$

Since D & D′ are ⊥ planes in the stress element, then they become diametrically opposite points on the circle, just like the planes A & B did

Calculation of principal stress

The most important application of the Mohr's circle is determination of principal stresses.

The intersection of the Mohr's circle --- with normal stress axis gives two points P_1 and P_2. Thus P_1 and P_2 represents points corresponding to principal stresses. In the current diagram the coordinates the of

$$P_1 = \sigma_1, 0s$$
$$P_2 = \sigma_2, 0$$

$$\sigma_1 = \sigma_{avg} + R$$

$$\sigma_2 = \sigma_{avg} - R$$

The principal direction corresponding to σ_1 is now equal to $2\theta_{p_1}$, in c.c.w direction from the x-axis.

$$\theta_{p_2} = \theta_{p_1} \pm \frac{\pi}{2}$$

We can see that the points P_1 and P_2 are diametrically opposite, this indicate that principal planes are ⊥ to each other in the stress element. This fact can also be verified from the Mohr's circle.

<u>In- plane maximum shear stress</u>

What are points on the circle at which the shearing stress are reaching maximum values numerically? Points S_1 and S_2 at the top and bottom of the Mohr's circle.

- The points S_1 and S_2 are at angles $2\theta = 90^\circ$ from points P_1 P_2 and, i.e the planes of maximum shear stress are oriented at $\pm 45^\circ$ to the principal planes.
- Unlike the principal stresses, the planes of maximum shear stress are not free from the normal stresses. For example the coordinates of

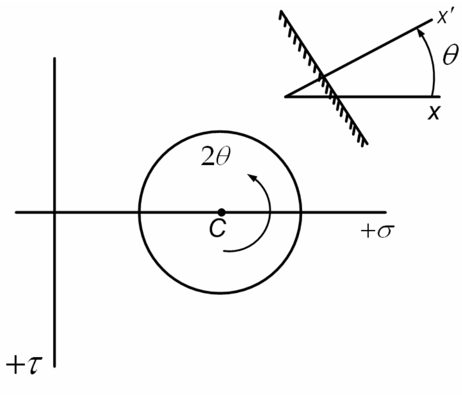
$$S_1 = +\tau_{max}, \sigma_{avg} s$$

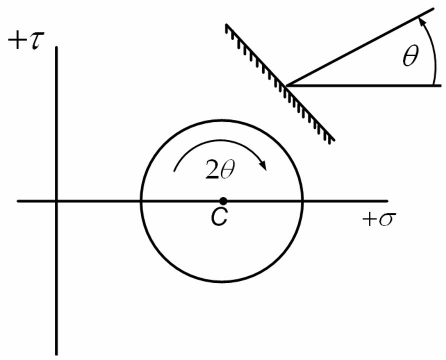
$$S_2 = -\tau_{max}, \sigma_{avg}$$

$$\tau_{max} = \pm R$$

$$\sigma = \sigma_{avg}$$

Mohr's circle can be plotted in two different ways. Both the methods are mathematically correct.

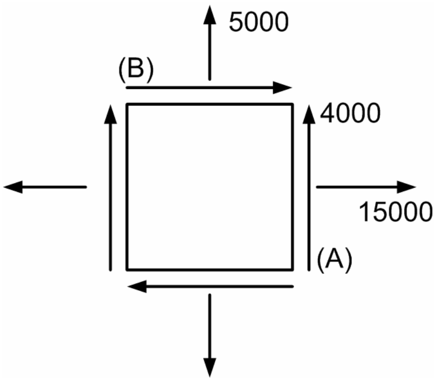


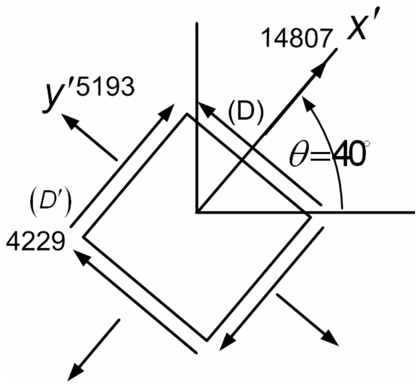


Finally

- Intersection of Mohr's circle with the σ-axis gives principal stresses.
- The top and bottom points of Mohr's circle gives maximum –ve shear stress and maximum +ve shear stress.
- Do not forget that all these inclined planes are obtained by rotation about z-axis.

Mohr' circle problem





Solution:

$$\frac{\sigma_x + \sigma_y}{2} = \frac{15000 + 5000}{2} = 10000\, MPa$$

A - (15000,4000)

B - (5000,-4000)

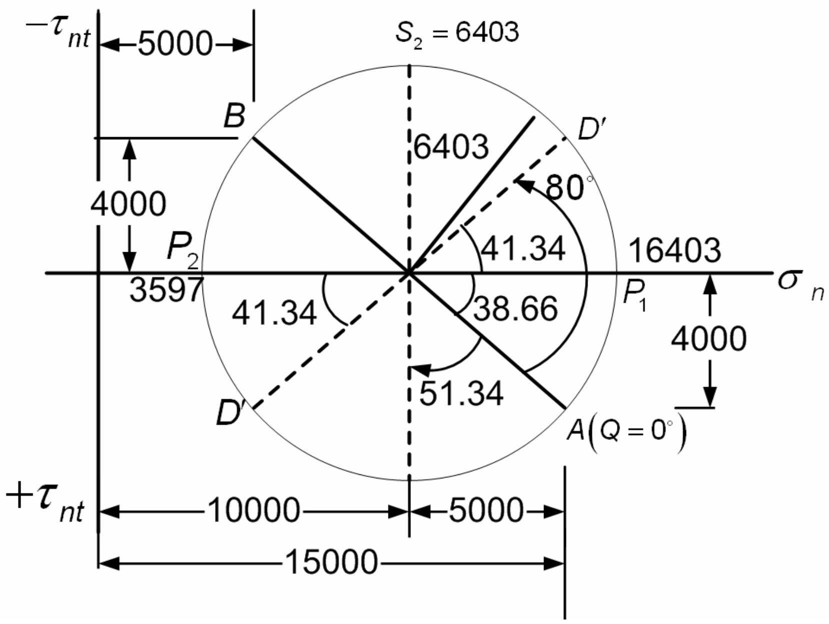
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{15000 - 5000}{2}\right)^2 + 4000^2}$$

$$= \sqrt{5000^2 + 4000^2}$$

$$R = 6403\, MPa$$

$$\frac{\sigma_x - \sigma_y}{2} = 5000$$

(a)



Point D: $\sigma_{x'} = 10000 + 6403Cos41.34 = 14807\,MPa$

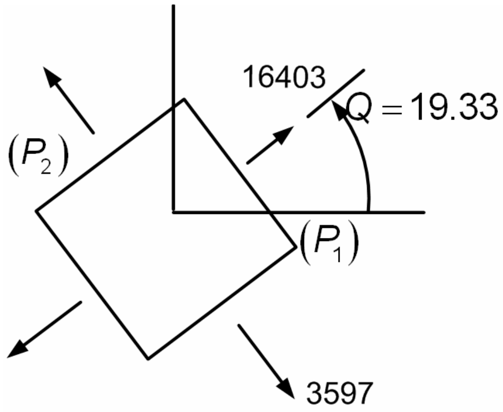
$\tau_{x'y'} = -6403Sin41.34 \qquad = -4229\,MPa$

Point D': $\sigma_n = \sigma_{y'} = 10000 - 6403Cos41.34 = 593\ MPa$

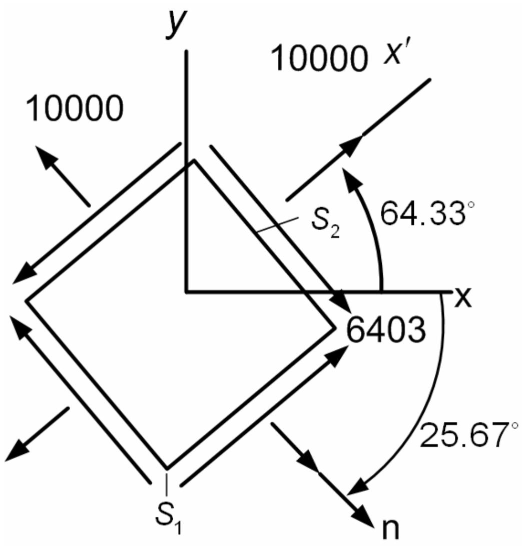
$\tau_{nt} = \tau_{x'y'} = 6403Sin41.34 \qquad = 4229$

b) $\sigma_1 = 16403 \ \ ; \ \theta_{P_1} = \dfrac{38.66}{2} = 19.33$

$\sigma_2 = 3597\,MPa$

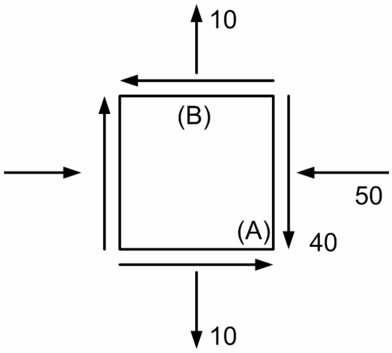


c) $\tau_{max} = 6403\ MPa - \theta_{S_1} = 25.67\ = -25.67^\circ$



(2) $\theta = 45^\circ$

Principal stresses and principal shear stresses.



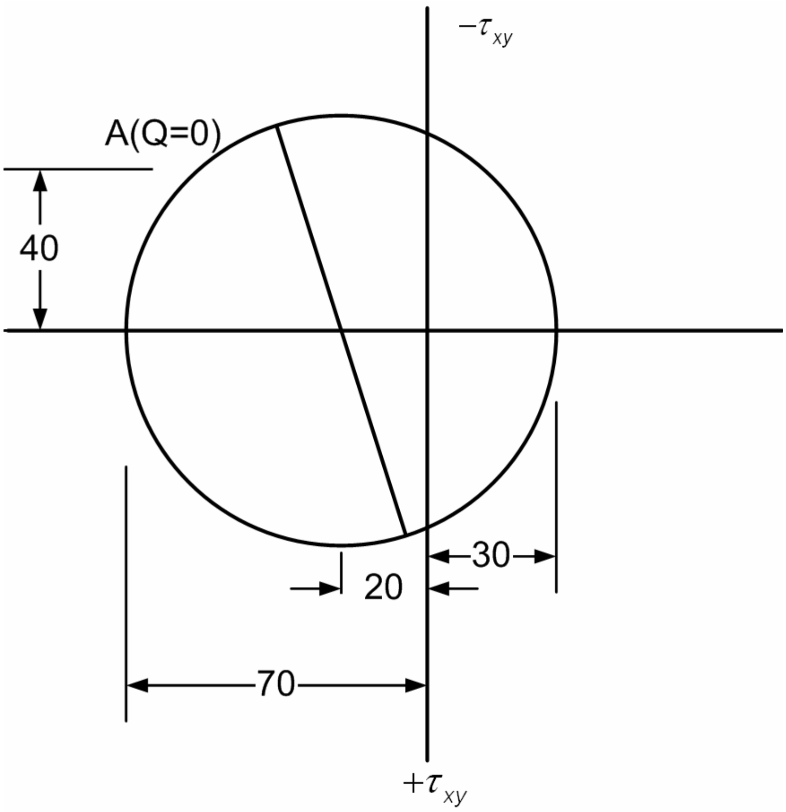
Solution:

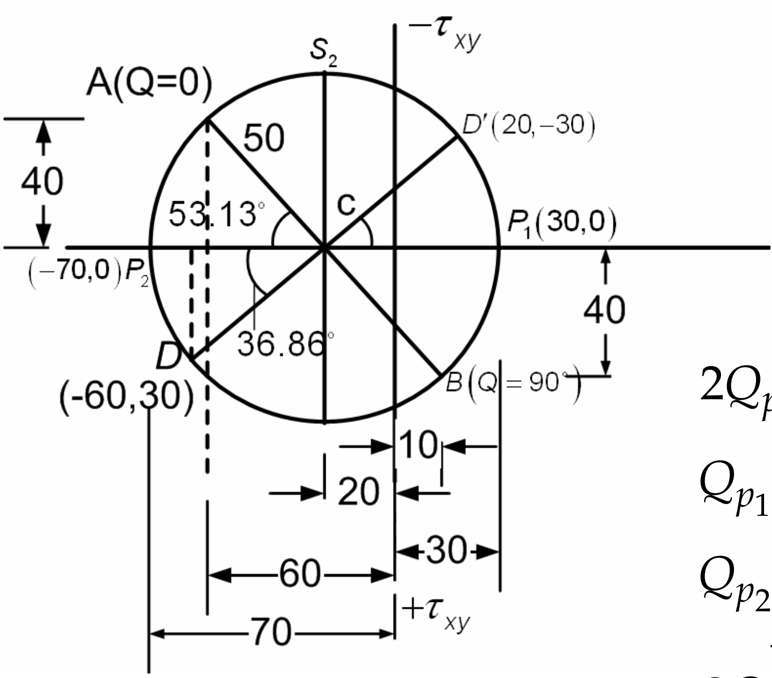
$$\frac{\sigma_x + \sigma_y}{2} = \frac{-50+10}{2} = -20$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-50-10}{2}\right)^2 + (-40)^2} = 50\,MPa$$

$$A \rightarrow (-50,-40) \qquad p_1 = \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + R = -20 + 50 = 30s$$

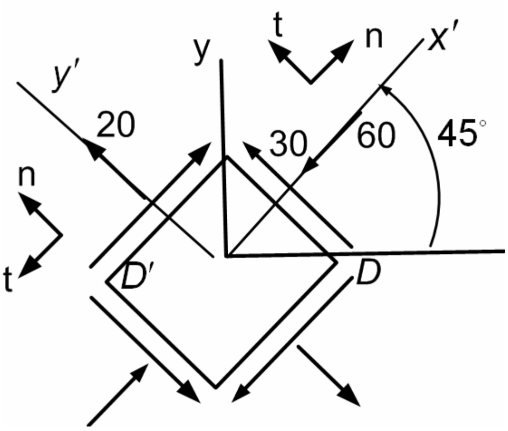
$$B \rightarrow (10,40) \qquad p_2 = \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - R = -20 - 50 = -70$$

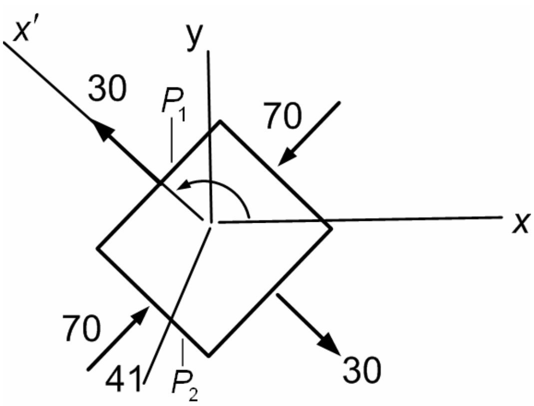


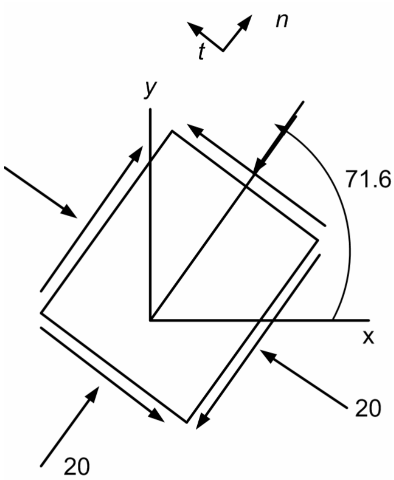


$$2Q_{p_1} = 233.13$$
$$Q_{p_1} = 116.6^\circ$$
$$Q_{p_2} = 206.6^\circ$$

$$2Q_{s_1} = 143.13$$
$$Q_{s_1} = 71.6$$
$$Q_{s_2} = 161.6$$







Q. $\sigma_x = 31\,MPa,\ \sigma_y = -5\ MPa\ \ and\ \ \tau_{xy} = 33\ MPa$

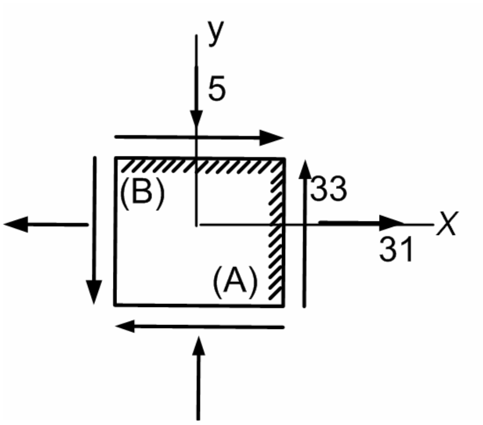
Stresses on inclined element $\theta = 45^\circ$

Principal stresses and maximum shear stress.

Solution:

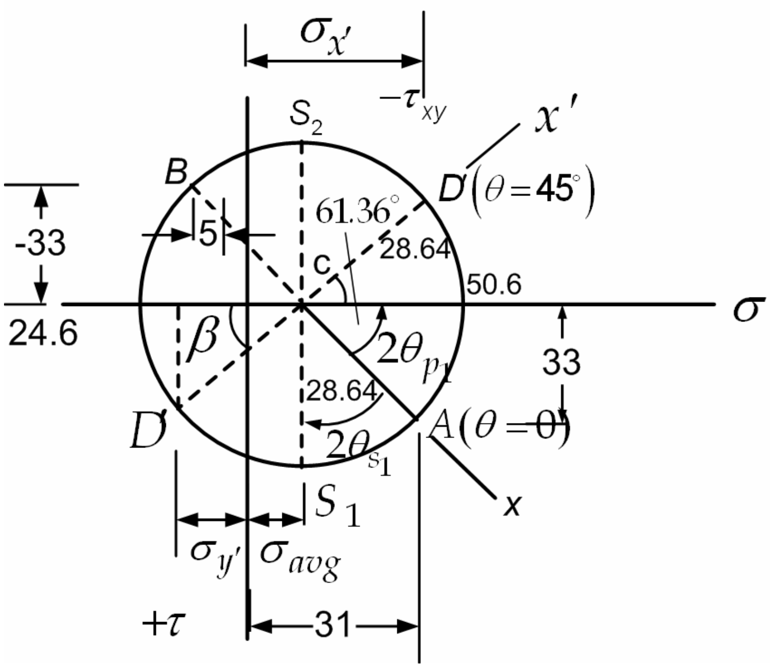
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{31-5}{2} = 13\ \ MPa$$

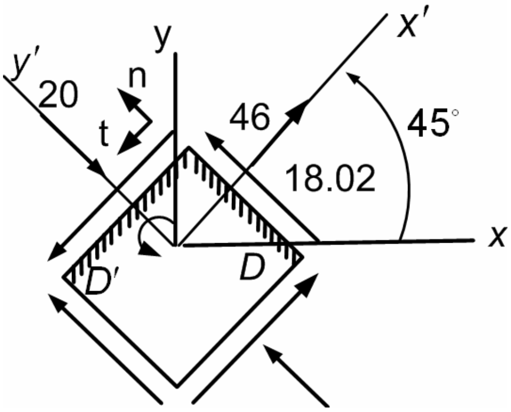
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 37.6\ \ MPa$$



$A(31,33)$

$B(-5,-33)$





$$\sigma_{x'} = RCos\beta + \sigma_{avg} s$$
$$= 37.6Cos28.64 + 13 = 46\ \ MPa$$

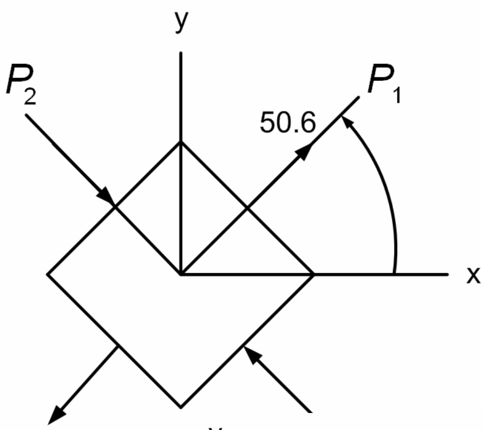
$$\tau_{x'y'} = -RSin\beta = -37.6\ \ 28.64 = -18.02$$

$$\sigma_{y'} = RCos\beta - \sigma_{avg}$$
$$= -20\ \ MPa$$

$\therefore\ \sigma_1 = 50.6\ MPa$

$\sigma_2 = -24.6\ MPa$

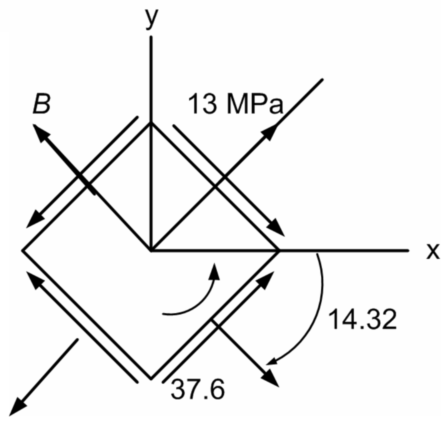
$\theta_{p_1} = 30.68$



$\tau_{max} = 37.6\ MPa - \theta_{s_1} = -14.32$

$\tau_{min} = -37.6\ MPa$

$\sigma = \sigma_{avg} = 13\ MPa$



16BTAR305 **SOLID MECHANICS** **3 0 0 3 100**

Multiple Choice Based Questions & Answers

(for all the FIVE units)

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UNIT I INTRODUCTION TO MECHANICS

S.No.	Questions	opt1	opt2	opt3	opt4	Answer
1	If a force acts on a body, it sets up some resistance to the deformation. This resistance is known as.	stress	strain	elasticity	modulus of elasticity	**stress**
2	The term deformation per unit length is applied for	stress	strain	modules of elasticity	Bulk Modulus	**strain**
3	Strain energy is the	energy stored in a body when strained within elastic limits	energy stored in a body when strained upto the breaking of a specimen	maximum strain energy which can be stored in a body	proof resilience per unit volume of a material	**energy stored in a body when strained within elastic limits**
4	Modulus of elasticity is the ratio of	stress to strain	stress to original length	deformation to original length	strain to the original length	**stress to strain**
5	The total change in length of bar of different sections is equal to the	Sum of changes in the length of different sections	Average of changes in the lengths of different sections	Difference of changes in the lengths of different sections	deformation to original length	**Sum of changes in the length of different sections**
6	A circular bar of length (l) uniformly Area (A). If the bar is subjected to an axial tensile load (P), then its elongation is equal to an axial tensile load (P), then its elongation is equal to	Pl/AE	Pl/A_1A_2E	$4Pl/л E d_1 d_2$	$Pl/4 л E d_1 d_2$	**Pl/AE**
7	The maximum stress produced in a bar of tapering section is at	Larger end	Smaller end	Middle	All of these	**Smaller end**
8	In a composite section, the number of different materials is	One only	Two only	More than two	Any where	**More than two**
9	A composite section contains 4 different materials. The stresses in all the different materials will be	Zero	Equal	Different	In the ratio of their areas	**Different**
10	Thermal stress is caused when the temperature of a body	is increased	is decreased	remains constant	either a or b	**either a or b**
11	When the temperature of a body is increases the stress induced will be	tension	compression	both a and b	neither a nor b	**compression**
12	If the ends of a body yield, the magnitude of thermal stress will	increase	decrease	decrease	constant	**decrease**
13	The maximum thermal stress in a circular tapering section is	directly proportional to the bigger diameter	directly proportional to the smaller diameter	directly proportional to the smaller diameter	both b and c	**directly proportional to the bigger diameter**
14	If a composite bar is cooled, then the nature of stress in a part with high coefficient of thermal expansion will be	tensile	zero	compressive	shear	**tensile**
15	The ratio of lateral strain to the linear strain is called	modulus of elasticity	modulus of rigidity	compressive	poison’s ratio	**modulus of elasticity**

16	When a rectangular bar is subjected to a tensile stress, then the volumetric strain is equal to	0	ϵ[1+2/m]	ϵ[2-1/m]s	ϵ[2-1/m]	**ϵ[1+2/m]**
17	The bulk modulus of a body is equal to	m E/3(m-2)	ϵ[1+2/m]	m E/2(m-2)	m E/2(m+2)	**m E/3(m-2)**
18	A body returns to its original shape after removal of the force, is called	tensile	elasticity	ductility	malleability	**elasticity**
19	The neutral axis of the cross-section a beam is that axis at which the bending stress is	zero	minimum	maximum	infinity	**zero**
20	Euler's formula holds good only for	short columns	long columns	both short and long columns	weak columns	**long columns**
21	A steel bar of 5 mm is heated from 15° C to 40° C and it is free to expand. The bar Will induce	thermal stress	shear stress	tensile stress	compressive stress	**thermal stress**
22	The stress induced in a body, when suddenly loaded, is __________ the stress induced when the same load is applied gradually.	equal to	one-half	twice	four times	**twice**
23	The law which states that within elastic limits strain produced is	Bernoulli’s law	plastic point	Hooke’s law	elasticity	**Hooke’s law**
24	When equal and opposite forces applied to a body, tend to elongate it, the stress so produced, is called	shear stress	Stress law	tensile stress	transverse stress	**tensile stress**
25	If the slenderness ratio for a column is 100, then it is said to be a __________ column.	long	short	medium	very long	**long**
26	In a bar of large length when held vertically and subjected to a load at its lower end, its own-weight produces additional stress. The maximum stress will be	at the lower cross-section	shear strain	volumetric strain	at every point of the bar	**at every point of the bar**
27	The bending moment at a point on a beam is the algebraic __________ of all the moments on either side of the point.	diffrence	sum	division	average	**sum**
28	Strain resetters are used to	measure shear strain	measure linear strain	measure volumetric strain	relieve strain	**measure linear strain**
29	The maximum stress produced in a bar of tapering section is at	smaller end	larger end	middle	anywhere	**smaller end**
30	The phenomenon of slow extension of materials having constant load ie increase with the time is called	creeping	fatigue	fracture	crack	**creeping**

31	The energy stored in a body when strained within elastic limit is known as	resilience	proof resilience	strain energy	impact energy	**strain energy**
32	In compression test, the fracture in cast iron specimen would occur along	the axis of load	an oblique plane	at right angles to the axis of specimen	would not occur	**an oblique plane**
33	When a bar is cooled to - 5°C, it will develop	no stress	shear stress	tensile stress	compressive stress	**tensile stress**
34	The total strain energy tored in a body is knowns as	resilience	proof resilience	strain energy	impact energy	**resilience**
35	unit of energy in SI unit	N	Watt	joule	pascal	**joule**
36	Match the following a) modulas of elasticity 1) Direct stress/ volumetric strain b) Rigidity modulus 2) Axial stress / Long strain c) Bulk modulus 3) Shear stress / shear strain d) Poisson ratio 4) Lateral strain /Long strain	a – 1 , b – 2 , c – 3 , d – 4	a – 2 , b – 3 , c – 1 , d – 4	c) a – 4 , b – 2 , c – 3 , d –1	d) a – 4 , b – 1 , c – 3 , d – 4	**a – 2 , b – 3 , c – 1 , d – 4**
37	Match the following a) Linear strain 1.Change in dia / Original dia b) Lateral strain 2. Stress/ strain c) Volumetric strain 3. Change in length/Original length d)Youngs modulus 4.Change in volume / Original volume	a – 1 , b – 2 , c – 3 , d – 4	a – 2 , b – 3 , c – 1 , d – 4	c) a – 3, b – 1 , c –4 , d –2	d) a – 4 , b – 1 , c – 3 , d – 4	**c) a – 3, b – 1 , c –4 , d –2**
38	Match the following a) Elongation 1) 2WL/AE b) Stress 2) change in length/original length c) Strain 3) P/A d) Elongation due to self weight 4) PL/AE	a – 1 , b – 2 , c – 3 , d – 4	a – 2 , b – 3 , c – 1 , d – 4	c) a – 4 , b – 2 , c – 3 , d –1	d) a – 4 , b – 1 , c – 3 , d – 4	**d) a – 4 , b – 1 , c – 3 , d – 4**
39	The value of Poisson's ratio for cast iron is	0.1	0.23-0.27	0.4-0.6	0.45-0.46	**0.23-0.27**
40	Match the following a.torsion 1 N-m b.Youngs modulus 2 mm c.Strain 3.N/mm2 d. Elongation 4. No unit	a –1 , b – 2 , c – 3 , d – 4	a – 2 , b – 3 , c – 1 , d – 4	c) a – 4 , b – 2 , c – 3 , d –1	d) a – 1 , b – 3 , c – 4 , d –2	**d) a – 1 , b – 3 , c – 4 , d –2**
41	Elongation of bar due to tensile load	PL/AE	AE/PL	Strain /Stress	du/dL	**PL/AE**
42	Elongation due to own weight	WL/AE	PL/AE	2WL/AE	PLAE	**WL/AE**
43	Bulk modulus is the ratio of	stress to strain	stress to original length	deformation to original length	Direct stress/ volumetric strain	**Direct stress/ volumetric strain**
44	Lateral strain is the ratio of	Change dia /Original dia	Change in width / Original width	Change in depth/Original depth	All the given options	**All the given options**
45	The value of possion’s ratio always remains	. greater than one	less than one	equal to one	zero	**less than one**

46	Unit of stress is	KN	N/m2	KN. mm2	KN/mm	**N/m2**
47	Types of primary strains are	3	2	Linear strain	4	**3**
48	The property of a material which allows it to be drawn into a smaller section is called	plasticity	elasticity	ductility	malleability	**ductility**
49	The ratio of the moment of inertia of a circular plate and that of a square plate for equal depth, is	less than one	equal to one	more than one	equal to 3π/16	**equal to 3π/16**
50	As compared to uniaxial tension or compression, the strain energy stored in bending is only	1\2	1\5	1\4	1\3	**1\3**
51	The ratio of elongations of a conical bar due to its own weight and that of a prismatic bar of the same length, is	1\2	1\5	1\4	1\3	**1\3**
52	The maximum twisting moment a shaft can resist, is the product of the permissible shear stress and	polar modulus	moment of inertia	polar moment of inertia	modulus of rigidly.	**polar modulus**
53	A three-hinged arch is said to be	a bent beam	statically indeterminate structure	statically determinate structure	column	**statically determinate structure**
54	for a given load(P), if area increases	length increases	stress increases	stress decreases	length decreases	**stress increases**
55	bulk modulus (K) of a body is given by	K =E/(3(1-2μ))	K =(3(1-2μ))/E	K =E/(3(1+2μ))	K =(3(1+2μ))/E	**K =E/(3(1-2μ))**
56	Change in volume /original volume	Linear strain	Lateral strain	Volumetric strain	strain	**Volumetric strain**
57	The elongation of a bar due to its own weight	WL/3E	WL/E	WL/2E	2E/WL	**WL/2E**
58	Elongation of bar due to tensile load	PL/AE	AE/PL	P/AE	PA/LE	**PL/AE**

UNIT II STRESS, STRAIN AND DEFORMATION OF SOLIDS

S.No.	Questions	opt1	opt2	opt3	opt4	Answer
1	If a cantilever beam is subjected to a point load at its free end then the shear force under the point load is	zero	less than the load	equal to the load	more than the load	**equal to the load**
2	The bending moment at the free end of a cantilever beam carrying a uniformly distributed load is	zero	minimum	maximum	equal to the load.	**zero**
3	The B.M. at the centre of a simply supported beam carrying a uniformly distributed load is	wl	wl/2	$wl^2/4$	$wl^2/8$	**$wl^2/8$**
4	A simply supported beam AB of span (l) Carries a point load (w) at a distance from the left end A, such that a<b. The maximum deflection will be	at C	between A and C	between C and B	any where between A and B	**between C and B**
5	The point of contra flexure is a point where shear force ______	changes sign	shear force is maximum	bending moment is maximum	zero	**changes sign**
6	A simply supported beam carriers a point load at its centre. The slope at its supports is	Wl2/16EI	WI3/16EI	WI2/48EI	WI3/48E	**Wl2/16EI**
7	A simply supported beam of span (l) is subjected to a uniformly distributed load of (w) per unit length over the whole span. The maximum deflections at the centre of the beam is	$5wl^5/48EI$	$5wl^4/96EI$	$5wl^4/192EI$	5wl3/384EI	**5wl3/384EI**
8	Two simply supported beams of the same span carry the same load. If the first beam carries the total load as a point load at its centre and the other uniformly distributed over the whole span then ration of maximum	1:01	1.1.5	1.5:1	d. 2:1	**1.5:1**
9	A simply supported beam of span L carries a uniformly distributed load W per m length. The maximum bending moment M is	WL/2	WL/4	$WL^2/8$	WL/12	**$WL^2/8$**
10	A simply supported beam of span L carries concentrated load W at its mid-span. The maximum bending moment M is	WL/2	WL/4	WL/8	WL/12	**WL/4**
11	A simply supported beam carries two equal concentrated loads W at distances L/3 from either support. The maximum bending moment M is	WL/3	WL/4	SWL/8	3WL/12	**WL/3**
12	The shape of the bending moment diagram over the length of a beam, having no external load, is always	linear	parabolic	cubical	circular	**linear**
13	The shape of the bending moment diagram over the length of a beam, carrying a uniformly distributed load is always	linear	parabolic	cubical	circular	**parabolic**

14	The shape of the bending moment diagram over the length of a beam, carrying a uniformly increasing load is always	linear	parabolic	cubical	circular	**cubical**
15	For a simply supported beam with a central load, the bending moment is	least at the centre	least at the supports	maximum at the support	maximum at the centre	**maximum at the centre**
16	For a cantilever with a uniformly distributed load W over its entire length L, the maximum bending moment is	WL	½ WL	1/3 WL	½ W2L	**½ WL**
17	A cantilever beam carrying point load W on its free end, the maximum bending moments is	WL/4	WL	WL/2	WL/3	**WL**
18	The bending moment is maximum on a section where shearing force	is maximum	is minimum	is equal	changes sign	**changes sign**
19	A beam is said to be of uniform strength, if	B.M. is same throughou	Shear stress is same throughout the	Deflection is same throughout the beam	Bending stress is same at every section along its	**Bending stress is same at every section**
20	In a continuous bending moment curve the point where it changes sign, is called	point of inflexion	point of contra flexture	point of virtual hinge	all the above	**all the above**
21	Pick up the correct assumption of the theory of simple bending	The value of the Young's	Trverse section of a beam remains plane	The material of the beam is homogeneous and	All the above	**All the above**
22	Along the neutral axis of a simply supported beam	fibres do not undergo	fibres undergo minimum strain	fibres undergo maximum strain	undergo shear	**fibres do not undergo strain**
23	In a loaded beam, the point of contraflexure occurs at a section where	bending moment is	bending moment is zero or	bending moment is maximum	shearing force is maximum	**bending moment is zero or changes sign**
24	In a beam, the neutral plane	may be at its centre	passes through the C.G. of the area of cross-	does not change during deformation	none of these	**does not change during deformation**
25	The point of contra flexure occurs in	cantilever beams only	continuous beams only	over hanging beams only	B and C only	**B and C only**
26	For a beam with rectangularsection ,the ratio of maximum shear stress to average shear stress is	a.2 : 1	b.1.5 : 1	c.3 : 2	d.4:3	**b.1.5 : 1**
27	loaded beam, the point of contraflexure occurs at a section where	bending moment is	bending moment is zero or	bending moment is maximum	shearing force is maximum	**bending moment is zero or changes sign**

28	The moment carrying capacity of any beam section is proportional to its	moment of inertia	flexural rigidity	principal moment of inerctia	section modulas	**flexural rigidity**
29	Bending moment is equal to	Force X load	Force X distance	Distance X Seconds	Moment x distance	**Force X distance**
30	The moment diagram for a cantilever whose free end is subjected to a bending moment, will be a	triangle	rectangle	parabola	cubic parabola	**rectangle**
31	The moment diagram for a cantilever which is subjected to a uniformly distributed load will be a	triangle	rectangle	parabola	cubic parabola	**parabola**
32	The moment diagram for a cantilever carrying concentrated load at its free end, will be	triangle	rectangle	parabola	cubic parabola	**triangle**
33	Shear force for a cantilever carrying a uniformly distributed load over its whole length, is	triangle	rectangle	parabola	cubic parabola	**rectangle**
34	.If the shear force along a section of a beam is zero, the bending moment at the section is	zero	maximum	minimum	average of maximum –minimum	**maximum**
35	Hooke's law states that stress and strain are	directly proportional	inversely proportional	curvilinear related	none of these	**directly proportional**
36	determine all 1the reactive forces at the supports, the structure is said to be	determinate	statically determinate	statically indeterminate	none of these	**statically indeterminate**
37	elements at the supports. These can be determined by using the following fundamental equation of statics	∑H = 0	∑V= 0	∑H = 0 ; H = 0	d). ∑H = 0 ; ∑V = 0; ∑ M = 0	**d). ∑H = 0 ; ∑V = 0; ∑ M = 0**
38	A bending moment may be defined as :	Arithmetic sum of the	Arithmetic sum of the moments on	Algebraic sum of the moments of all the forces on	Arithmetic sum of the forces on either side of the	**Algebraic sum of the moments of all**
39	section, the shear force F and bending moment M at a section are related by	F = My/I	F = M/Z	F = dM/dx	F = ∫Mdx	**F = dM/dx**
40	. A simply supported beam of span L carries a uniformly distributed load W per meter length and point load W. The maximum bending moment M is	WL/2	WL/4 + WL2/8	WL/8	WL/12	**WL/4 + WL2/8**
41	The shape of the bending moment diagram over the length of a beam, carrying a uniformly distributed load is always	linear	parabolic	cubical	circular	**linear**

42	The section modulus of a rectangular section is proportional to	area of the section	square of the area of the section	product of the area and depth	product of the are and width	**area of the section**
43	For a cantilever with a uniformly distributed load w N/m length and point load at its free end , the maximum bending moment is	a) WL	b) ½ WL +WL	c) 1/3 WL	½ W2L	**b) ½ WL +WL**
44	S1 :Moment is equal to product of force and distance S2 : Moment is zero at point of contraflexure	S1 is correct	S2 is wrong	Both S1 & S2 are correct	Both S1 & S2 are correct	**Both S1 & S2 are correct**
45	S1 : Sagging moment is negative moment S2 : Hogging moment is positive moment	S1 is correct	S2 is wrong	Both S1 & S2 are correct	Both S1 & S2 are correct	**Both S1 & S2 are correct**
46	.Match the following a. Simply supported 1. One end Fixed other end free b. Cantilever 2.both ends are simply supported	a – 1 , b – 2 , c – 3 , d – 4	a – 2 , b – 3 , c – 1 , d – 4	a – 2 , b –1 , c – 4 , d –3	a – 3 , b – 4 , c – 4 , d –1	**a – 2 , b –1 , c – 4 , d –3**
47	If the member is subjected to a uniform bending moment(M), the radius of curvature of the deflected form of the member is given by	M/R=E/I	M/I=R/E	M/I=E/R	M/E=RL	**M/I=E/R**
48	bending stress is maximum at a point, which is ______ distance from the neutral axis	zero	constant	maximum distance	minimum distance	**maximum distance**
49	if the beam is subjected to point load , then the B.M diagram is drawn with	straight line	cubic curve	parabolic curve	uniformly varying staight line	**uniformly varying staight line**
50	in pure bending beam is assumed to bend with	R is not constant over	R is constant over the thickness	R linearly varies along the thickness	R=0	**R is constant over the thickness**
51	bending moment will vary in __________ from neutral axis to any layer	linear line	parabolic curve	cubic curve	any	**linear line**
52	for a simply supported beam point of contraflexure	occure at mid-point	occure at the support	does not occure	at L/3	**does not occure**
53	at the free end of any beam shear force will be	maximum	minimum	zero	depend on point load	**depend on point load**
54	maximum B.M occure at a point where	shear force is maximum	shear force is minimum	shear force changes sign	shear force is not zero	**shear force changes sign**
55	in pure bending of beam, strength of the beam can be increased by increasing	young's modulus	modulus of rigidity	length	radius of curvature	**young's modulus**

56	A simple supported beam of span (L) carries a uniformly distributed load over the whole span. The bending moment diagram will be	a rectangula r	a traingular	parabolic	cubic	**parabolic**
57	The point of contra-flexure occurs only in	continuou s beams	cantilever beam	overhanging beams	simply supported beams	**overhanging beams**
58	A sinply supported beam carries a uniformly distributed load of w N per unit length over the whole span(L). The shear force at the center is	wL/2	wL/8	wL/4	zero	**zero**
59	The B.M. at the centre of a simply supported beam carrying a point load(W) is	wl	wl/4	$wl^2/4$	wl/8	**wl/4**

UNIT III BEAMS - LOADS AND STRESSES

S.No.	Questions	opt1	opt2	opt3	opt4	Answer
1	Torque transmitted by a solid shaft of diameter (D), when subjected to a shear stress(τ)is equal to	л/16 x τ x D^2	л/16 x τ x D^3	л/32 x τ x D^2	л/16 x τ x D^3	**л/32 x τ x D^2**
2	A shaft revolving at r.p.m. transmits torque (T) in kg. m. The power developed is	2 л NT kW	2 л NT /30(kW)	2 л NT /60(kW)	2 л NT /120(kW)	**2 л NT /60(kW)**
3	Polar moment of inertia of a solid shaft of diameter (D) is	л/16 x D^2	л/16 x D^4	л /32 x D^3	л /32 x D^4	**л /32 x D^4**
4	When a solid shaft is subjected to torsion the shear stress induced in the shaft at in centre is,	Zero	Minimum	maximum	average	**Zero**
5	Strain energy stored in a hollow shaft of external diameter D and internal diameter (d) when subjected to a shearing stress (τ) is equal to	$\tau^2/C(D^2+d^2/D)$	$\tau^2/4C(D^2+d^2/D)$	$\tau^2/C(D^2-d^2/D)$	$\tau^2/4C(D^2-d^2/D)$	**$\tau^2/4C(D^2+d^2/D)$**
6	In a leaf spring, maximum bending stress developed in the plates is	Wl/nbt^2	$2Wl/nbt^2$	$3Wl/nbt^2$	$3Wl/2nbt^2$	**$3Wl/2nbt^2$**
7	The maximum deflection at the centre of a leaf spring is	σ_b l/Et	σ_b $l/2Et^2$	σ_b $l/3Et^2$	σ_b $l/3Et^2$	**σ_b $l/3Et^2$**
8	When a closely coiled spring is subjected to an axial load, it is said to be under	bending	Shear	torsion	all of these	**torsion**
9	The deflection of a closely coiled helical spring of diameter (D) subjected to an axial load (W) is	64 WR^3 n/Cd^4	64 WR^2 n/Cd^4	64 WR n/Cd^4	64 WRn^2/Cd^4	**64 WR^3 n/Cd^4**
10	The law which states that within elastic limits strain produced is proportional to the stress producing it, is known as	Bernoulli's law	Stress law	Hooke's law	Poisson's law	**Hooke's law**
11	When equal and opposite forces applied to a body, tend to elongate it, the stress so produced, is called	shear stress	compressive stress	Direct stress	all the above	**Direct stress**
12	In a leaf spring, maximum bending stress developed in the plates is	Wl/nbt^2	$2Wl/nbt^2$	$3Wl/nbt^2$	$3Wl/2nbt^2$	**$3Wl/2nbt^2$**
13	The maximum deflection at the centre of a leaf spring is	σ_b l/Et	σ_b $l/2Et^2$	σ_b $l/3Et^2$	σ_b $l/3Et^2$	**σ_b $l/3Et^2$**
14	When a closely coiled spring is subjected to an axial load, it is said to be under	bending	Shear	torsion	all of these	**torsion**
15	The deflection of a closely coiled helical spring of diameter (D) subjected to an axial load (W) is	64 WR^3 n/Cd^4	64 WR^2 n/Cd^4	64 WR n/Cd^4	64 WRn^2/Cd^4	**64 WR^3 n/Cd^4**
16	The shear stress at any section of a shaft is maximum	At the centre of the section	At a distance r/2 from thecentre	At the top of the surface	At a distance 3/4r from the center	**At the top of the surface**

17	The following assumption is not true in the theory of pure torsion	The twist along the shaft is uniform	The shaft is of uniform circular section	c/s is plane before twist remins plane	all radii get twisted due to torsion	**all radii get twisted due to torsion**
18	In a shaft shear stress intensity at a point is not	directly proportional to the distange from the axis	inversely proportional to the distance from	inversely proportional to the polar M.I.	directly proportional to the applied torque	**inversely proportional to the distance from the axis**
19	The maximum twisting moment a shaft can resist,is the permissible shear stress and	moment of inertia	polar moment of inertia	polar modulus	modulus of rigidity	**polar modulus**
20	A shaft turning 150 r.p.m. is subjected to a torque of 150 kgm. Horse power transmitted by the shaft is	π	10 π	π^2	1/ π	**10 π**
21	A shaft 9m long is subjected to a torque 30 t-m at a point 3m distant from either end.The reactive torque at the nearer end will be	5 tonnes meter	10 tonnes meter	15 tonnes meter	20 tonnes meter	**20 tonnes meter**
22	If a shaft is simultaneously subjected to a torque T and a bending moment M, the ratio of maximum bending stress and shearing stress is	M/T	T/M	2M/T	2T/M	**2M/T**
23	A member which does not regain its original shape after removal of load producing deformation is said	plastic	elastic	rigid	none of the above	**plastic**
24	Strain energy of any member may be defined as work done on it	to deform it	to resist elongation	to resist shortening	all the above	**all the above**
25	S1: Torque is a twisting moment, S2: When a solid shaft is subjected to torsion,the shear stress induced in the shaft at is center iszero	S1 is right S2 is wrong	S2 is right S1 is wrong	Both S1 & S2 are right	Both S1 & S2 are wrong	**Both S1 & S2 are right**
26	S1:Unit of torque is KNM, S2:Moment unit is KN	S1 is right S2 is wrong	S2 is right S1 is wrong	Both S1 & S2 are right	Both S1 & S2 are wrong	**S1 is right S2 is wrong**
27	The following assumption is not true in the theory of pure torsion	The shaft is of uniform circular section throughout	Cross-section of the shaft, which is plane before	All radii get twisted due to torsion.	The twist along the shaft is uniform	**All radii get twisted due to torsion.**
28	S1:Helical spring are also called torsion spring, S2:Bending spring and Tension spring are two type of spring	S1 is right S2 is wrong	S2 is right S1 is wrong	Both S1 & S2 are right	Both S1 & S2 are wrong	**S1 is right S2 is wrong**
29	Choose the correct abbreviation	Moment of inertia - I	Radius of gyration - Y	Max bending moment - B	Shear - S	**Moment of inertia - I**
30	Joule is the unit of	Work	force	power	energy	**Work**
31	the safe twisting moment for a compound shaft is equal to the	maximum calculated avlue	minimum calculated avlue	mean value	extreme value	**minimum calculated avlue**
32	the ratio of the maximum shear stress to maximum normal stress at any point in a solid circular shaft is	1	0-Jan	2	3-Feb	**1**
33	the torsional rigidity of a shaft is expressed by the	maximum torque it can transmit	number of cycle it undergoes gefore failure	elastic limit upto which it resists torsion, shear	torque required to produce a twist of one radian per	**torque required to produce a twist of one radian per unit length of shaft**
34	strain energy stored in a solid circular shaft is proportional to	GJ(torsional rigidity of shaft)	1/GJ	(GJ)2	1/(GJ)2	**1/GJ**

35	The value of shear stress which is induced in the shaft due to the applied couple varies	from maximum at the centre to zero at the circumfrence	from zero at the centre to maimum at the	from maximum at the centre to minimum at the	from minimum at the centre to maximum at the	**from zero at the centre to maimum at the circumfrence**
36	A key is subjected to side pressure as well at shearing forces. These pressure are called	bearing stresses	fatigue stresses	crushing stress	resultant stresses	**bearing stresses**
37	in a belt drive, the pulley diameter is doubled, the belt tension and pully width remaining same. The changes required in key will be	increase key length	increase keydepth	increase key width	double all the dimensions	**increase key width**
38	Shear stress induced ina shaft subjected to tension will be	maximum at periphery and zero at centre	maximum at centre	uniform thoughout	none of the above	**none of the above**
39	in the design of pully, Key and shaft	all there are designed for same strength	key is made weaker link	pulley is made weaker	shaft is made weaker	**key is made weaker link**
40	the elongation produced in a tapered shaft with end diameters d1,d2 due to tensile or compressive axial load is proportional to	d1+d2	1/d1+d2	d1*d2	1/d1*d2	**1/d1*d2**
41	units of strain are	dimensionless	cm/cm	kg/cm2/cm	kg/cm	**dimensionless**
42	a cylindrical bar of L metres deforms by l cm. The strain in bar is	l/L	0.1l/L	0.01l/L	100l/L	**0.01l/L**
43	A composite bar made of steel and copper is heated up. The stresses developed in steel and copper will be	compressive and will be	compressive and bending	bending and tensile	tensile and compressive	**tensile and compressive**
44	Two solid shafts are made of same material and have their dimeters D and D/2. The ratio of strength of bigger shaft to smaller one in torsion is	4	2	8	16	**8**
45	The strain energy stored in a hollow shaft of outer and linner diameters D and d subjected to shear stress Ss and having modulus of rigidity C is equal to	S2s/4c(D2-d2/D)*volume	S2s/2c(D2-d2/D)*volume	Ss/4c(D2-d2/D)*volume	S2s/4c(D2-d2/D2)*volume	**S2s/4c(D2-d2/D)*volume**
46	compare the strengths of solid and hollw shaft having inside diameter of D/2 in torsion. The ratio of strength of solid to hollow shafts in torsion will be	0.5	0.75	15/16	0.25	**15/16**
47	Torsion bars are in series	if same torque acts in each	if they have equal angles of twist and an	are not possible	if their ends are welded together	**if same torque acts in each**
48	100 kW is to be transmitted by each of two separate shafts. A is turning at 250 rpm and B at 300 rpm. Which shaft must have greater diameter	A	B	Both will have same diameter	unpredictable	**A**
49	torsionla rigidity of a solid circular sfaht of diameter 'd' is proportional to	d	d2	1/d2	d4	**d4**
50	the elongation of a close colid helical spring subjected to tensile load is proportional to	mean diameter of spring	reciprocal of length of spring	diameter of wire of coil	shear modulus of the material or spring	**mean diameter of spring**
51	the minimum thickmess of a fiange forged at the end of shaft is determined by the	compresion between two flanges	tightening of bolts	fact that it must be sufficient to prevent the shaft	any one of the above	**fact that it must be sufficient to prevent the shaft from shearing out of**
52	torsion bars are in parallel	if same torque acts in each	if they have equal angles of twist and an	are not possible	if their ends connected together	**if they have equal angles of twist and an applied torque apporioned between them**

53	proof load for springs is the maximum load that it can undertake	without producing permanent deformation in spring	upto elastic limit	upto yield point	to straighten fully the leafs of a carrrage spring	**without producing permanent deformation in spring material**
54	A torsion bar with a spring constant k is cut into n equal lengths. What is the spring constant of each portion	k/n	n√k	kn	nk	**nk**
55	two idetical spring of spring constant k in series are attached in series with a parallel combination of two indentical springs of spring constant k. The overall equivalent spring constant is	2.5 k	1.25 k	0.4 k	0.75 k	**0.4 k**
56	Two identical leaf spring of spring constant k are attached at free end by a spring of spring constant of combination is	2.5 k	1.5 k	0.4 k	0.75 k	**1.5 k**
57	if D be the diameter of coil of a close colied helical spring and total angle of twist in full length be Φ, then deflection of spring is equal to	DΦ	(D/2)θ	2DΦ	DΦ2	**(D/2)θ**
58	A coil is cut into two halves, the stiffness of cut coils will be	double	half	same	something else	**double**
59	A hollow shaft of same cross-section ared as solid shaft transmits	same torque	less torque	more torque	more or less depending on external diameter	**more torque**
60	Torque in a solid shaft of diametre d and shear strength of Ss is given by	π/Ss*d3	(π/16)*Ss*d3	πd4/16	πd3/32	**(π/16)*Ss*d3**

UNIT IV TORSION AND BEAM DEFLECTION

S.No.	Questions	opt1	opt2	opt3	opt4	Answer
1	The ratio of the effective length of a column and minimum radius of gyration of its cross sectional area, is known as	Buckling factor	Slenderness ratio	Crippling factor	Buckling stress	**Slenderness ratio**
2	A vertical column has two moments of inertia (i.e. Ixx and Iyy). The column will tend to buckle in the direction of the	axis of load	perpendicular to the axis	maximum moment of inertia	minimum moment of inertia	**minimum moment of inertia**
3	A column is known as medium size of its slenderness ratio is between	20 to 32	32to 120	120to160	160 to 180	**32to 120**
4	Euler’s formula states that the buckling load P for a column of length l, both ends hinged and whose least moment of inertia and	$p = л^2EI / l^2$	$p = л l^2 / EI$	$p = лEI / l^2$	$p = л^2EI / l^3$	**$p = л^2EI / l^2$**
5	A long vertical member, subjected to an axial compressive load, is called	A column	A strut	A tie	A stanchion	**A column**
6	A vertical column has two moments of inertia (i.e. Ixx and Iyy). The column will tend to buckle in the direction of the	axis of load	perpendicular to the axis	maximum moment of inertia	minimum moment of inertia	**minimum moment of inertia**
7	The neutral axis of the cross-section a beam is that axis at which the bending stress is	zero	minimum	maximum	infinity	**zero**
8	Euler's formula holds good only for	short columns	long columns	both short and long	weak columns	**long columns**
9	The object of caulking in a riveted joint is to make the joint	free from corrosion	stronger in tension	free from stresses	leak-proof	**leak-proof**
10	A steel bar of 5 mm is heated from 15° C to 40° C and it is free to expand. The bar Will induce	no stress	shear stress	tensile stress	compressive stress	**no stress**

11	A body is subjected to a tensile stress of 1200 MPa on one plane and another tensile stress of 600 MPa on a plane at right angles	400 MPa	500 MPa	900 MPa	1400 MPa	**1400 MPa**
12	Two shafts 'A' and 'B' transmit the same power. The speed of shaft 'A' is 250 r.p.m. and that of shaft 'B' is 300 r.p.m. The shaft	greater diameter	smaller diameter	same diameter	greater than 100	**greater diameter**
13	The stress induced in a body, when suddenly loaded, is __________ the stress induced when the same load is applied	equal to	one-half	twice	four times	**twice**
14	If the slenderness ratio for a column is 100, then it is said to be a __________ column.	long	medium	short	all of the given	**long**
15	The value of Rankines constant for mild steel is	1\9000	1/7500	1/1600	1/750	**1/1600**
16	The maximum diameter of the hole that can be punched from a plate of maximum shear stress 1/4th of its maximum crushing stress	t	2t	4t	8t	**4t**
17	Two closely coiled helical springs 'A' and 'B' are equal in all respects but the number of turns of spring 'A' is half that of spring	1\8	1\4	1\2	2	**1\2**
18	The deformation per unit length is called	tensile stress	compressive stress	shear stress	strain	**strain**
19	In the torsion equation the term J/R is called	shear modulus	section modulus	polar modulus	none of these	**polar modulus**
20	Strain resetters are used to	measure shear strain	measure linear strain	measure volumetric strain	relieve strain	**measure linear strain**
21	The torque transmitted by a solid shaft of diameter (D) is (where τ = Maximum allowable shear stress)	π/4*T*D^3	π/16*T*D^3	π/32*T*D^3	π/64*T*D^3	**π/16*T*D^3**

22	When a rectangular beam is loaded transversely, the maximum compressive stress is developed on the	top layer	bottom layer	neutral axis	every cross-section	**bottom layer**
23	The point of contraflexure is a point where	shear force changes sign	bottom layer	shear force is maximum	bending moment is maximum	**bottom layer**
24	The bending stress in a beam is ___________ section modulus.	directly proportional to	bottom layer	equal to	zero	**directly proportional to**
25	In order to know whether a column is long or short, we must know its	slenderness ratio	Buckling factor	Crippling factor	strain energy	**slenderness ratio**
26	Resilience is the	energy stored in a body when strained within	energy stored in a body	maximum strain energy	none of the above	**none of the above**
27	If the depth is kept constant for a beam of uniform strength, then its width will vary in proportional to (where M = Bending	M	√M	M2	M3	**M**
28	A concentrated load is one which	acts at a point on a beam	spreads non-uniformly	spreads uniformly over the	varies uniformly over the whole length of a beam	**acts at a point on a beam**
29	In a simple bending of beams, the stress in the beam varies	linearly	parabolically	hyperbolically	elliptically	**linearly**
30	The stress at which the extension of the material takes place more quickly as compared to the increase in load, is called	elastic limit	yield point	ultimate point	breaking point	**yield point**
31	Pick up the correct assumption of the theory of simple bending	The value of the Young’s modulus is the	Transverse section of a beam	The material of the	all of the given option	**all of the given option**
32	If Z and I are the section modulus and moment of inertia of the section, the shear force F and bending moment M at a section	F = My/I	F = M/Z	F = dM/dx	wl/12	**F = My/I**

33	A load which is spread over a beam in such a manner that it varies uniformly over the whole length of abeam is called uniformly	distributed	varying	resulting	none of the above	**varying**
34	Which of the following statement is correct?	The energy stored in a body, when strained	The maximum strain	The proof resilience per unit	all of the above	**all of the above**
35	Compression members always tend to buckle in the direction of the	axis of load	perpendicular to the axis	minimum cross section	least radius of gyration	**least radius of gyration**
36	The maximum tangential stress in a thick cylindrical shell is always __________ the internal pressure acting on the shell.	equal to	less than	greater than	infinity	**greater than**
37	A thin spherical shell of diameter (d) and thickness (t) is subjected to an internal pressure (p). The stress in the shell material	pd/t	pd/2t	pd/4t	pd/8t	**pd/4t**
38	Principle plane is a plane on which the shear stress is	zero	minimum	maximum	infinity	**zero**
39	The maximum tangential stress in a thick cylindrical shell is always __________ the internal pressure acting on the shell.	equal to	less than	greater than	infinity	**greater than**
40	A thin spherical shell of diameter (d) and thickness (t) is subjected to an internal pressure (p). The stress in the shell material	pd/t	pd/2t	pd/4t	pd/8t	**pd/4t**
41	Principle plane is a plane on which the shear stress is	zero	minimum	maximum	infinity	**zero**
42	The bending moment in the centre of a simply supported beam carrying a uniformly distributed load of w per unit	zero	wl2/2	wl2/4	wl2/8	**wl2/8**
43	When a thin cylindrical shell is subjected to an internal pressure, there will be	a decrease in diameter and length of the shell	an increase in	a decrease in	an increase in diameter and length of the	**an increase in diameter and length**

44	The point of contraflexure occurs in	cantilever beams	simply supported beams	overhanging beams	fixed beams	**overhanging beams**
45	A beam of uniform strength has	same cross-section throughout the	same bending stress at	same bending moment	same shear stress at every section	**same bending stress at**
46	In a simple bending theory, one of the assumption is that the material of the beam is isotropic. This assumption means that the	normal stress remains constant in all directions	normal stress varies	elastic constants are same	elastic constants varies linearly in the material	**elastic constants are same in**
47	The polar modulus for a solid shaft of diameter (D) is	π/16*D^4	π/16*D^3	π/32*D^3	π/32*D^4	**π/16*D^3**
48	The extremeties of any diameter on Mohr's circle represent	principal stresses	normal stresses on planes	shear stresses on planes	normal and shear stresses on a plane	**normal stresses on planes at 45°**
49	The bending moment of a cantilever beam of length l and carrying a gradually varying load from zero at free end and w per unit	wl/2	wl	wl2/2	wl2/6	**wl2/6**
50	The Rankine's theory for active earth pressure is based on the assumption that	the retained material is homogeneous	the frictional resistance	the failure of the	all of the above	**all of the above**
51	The strain energy stored in a spring, when subjected to maximum load, without suffering permanent distortion, is known as	impact energy	proof resilience	proof stress	modulus of resilience	**proof resilience**
52	The resultant stress on an inclined plane which is inclined at an angle θ to the normal cross-section of a body which is subjected	σ sin θ	σ cos θ	σ sin 2θ	σ cos 2θ	**σ cos θ**
53	The ratio of change in volume to the original volume is called	linear strain	lateral strain	volumetric strain	Poisson's ratio	**volumetric strain**
54	In a beam where shear force changes sign, the bending moment will be	zero	minimum	maximum	infinity	**maximum**

55	The rectangular beam 'A' has length l, width b and depth d. Another beam 'B' has the same length and width but depth is	same	double	four times	six times	**four times**
56	When a closely-coiled helical spring is subjected to an axial load, it is said to be under	bending	shear	torsion	crushing	**torsion**
57	According to Euler's column theory, the crippling load for a column of length (l) with one end fixed and the other end free is	equal to	less than	more than	none of the above	**less than**
58	If percentage reduction in area of a certain specimen made of material 'A' under tensile test is 60% and the percentage reduction in	the material A is more ductile than material B	the material B is more	the ductility of	the material A is brittle and material B is	**the material A is more ductile**

UNIT V ANALYSIS OF STRESSES IN TWO DIMENSIONS

S.No.	Questions	opt1	opt2	opt3	opt4	Answer
1	The hoop stress is also known as __________	Longitudinal stress	Circumferential stress	Bending stress	Compressive stress	**Circumferential stress**
2	For a thin cylindrical shell of diameter d and thickness t, being subjected to a fluid pressure p, hoop stress is given by	Pd/3t	Pd/2t	Pd/5t	Pd/4t	**Pd/2t**
3	For a thin cylindrical shell of diameter d and thickness t, being subjected to a fluid pressure p, longitudinal stress is given by	Pd/3t	Pd/8t	Pd/5t	Pd/4t	**Pd/4t**
4	For a thin cylindrical shell longitudinal stress is equal to ________	Hoop stress	Two times the hoop stress	Three times hoops stress	Half of the hoop stress	**Half of the hoop stress**
5	The hoop stress is considered as _________	Compressive stress	Bending stress	Minor principal stress	Major principal stress	**Major principal stress**
6	The hoop stress and the longitudinal stress act at the following angle to each other	45 degree	60 degree	90 degree	120 degree	**90 degree**
7	The difference between hoop stress & longitudinal stress For a thin cylindrical shell of diameter d and thickness t, being	Pd/16t	Pd/8t	Pd/5t	Pd/4t	**Pd/4t**
8	The hoop strain for a thin cylindrical shell of diameter d, thickness t, Poisson's ratio v, and being subjected to pressure	Pd(1- v)/4tE	Pd(1- $2v$)/4tE	Pd(1- 0.5v)/2tE	Pd(1+ v)/4tE	**Pd(1- 0.5v)/2tE**
9	The longitudinal strain for a thin cylindrical shell of diameter d, thickness t, Poisson's ratio v, and being subjected	Pd(1- v)/4tE	Pd(1- $2v$)/4tE	Pd(2- v)/4tE	Pd(1+ v)/4tE	**Pd(1- $2v$)/4tE**
10	The volumetric strain for a thin cylindrical shell of diameter d, thickness t, Poisson's ratio v, and being subjected	Pd(5- $3v$)/4tE	Pd(1- $2v$)/4tE	Pd(5- $4v$)/4tE	Pd(1+ v)/4tE	**Pd(5- $4v$)/4tE**
11	For a thin spherical shell ____	Hoop stress is only present	Longitudinal stress is two times the hoop stress	Hoop stress is equal to one half of the	Hoop and longitudinal stresses are	**Hoop stress is only present**
12	The volumetric strain for a thin spherical shell of diameter d, thickness t, Poisson's ratio v, and being subjected to	Pd(5- $3v$)/4tE	3Pd(1- v)/4tE	Pd(5- $4v$)/4tE	Pd(1+ v)/4tE	**3Pd(1- v)/4tE**

13	The hoop stress for a thin spherical shell of diameter d, thickness t, Poisson's ratio v , and being subjected to pressure p, is	Pd/3t	Pd/8t	Pd/5t	Pd/4t	**Pd/4t**
14	A pressure vessel is said to be a thick shell, when	it is made of thick sheets	the internal pressure is very high	the ratio of wall thickness of the vessel to its	the ratio of wall thickness of the vessel to its	**the ratio of wall thickness of the vessel to its diameter is less than**
15	A thin cylindrical shell of diameter (d), length (l) and thickness (t)is subjected to an internal pressure (p). The hoop stress	pd/t	pd/2t	pd/4t	pd/6t	**pd/2t**
16	Two closely-coiled helical springs 'A' and 'B' are equal in all respects but the number of turns of spring 'A' is double	one-sixteenth	one-eighth	one-fourth	one-half	**one-half**
17	Pressure vessels are made of _____	Non ferrous materials	Sheet metal	Cast iron	Any of the given	**Any of the given**
18	The shear stress at the centre of a circular shaft under torsion is	zero	minimum	maximum	infinity	**zero**
19	Which of the following are usually called as thin cylinders -----______	Boilers	Steam pipes	Tanks	All of them	**All of them**
20	Longitudinal stress act ______ to the longitudinal axis of the shell	Parallel	Perpendicular	Either of the given	transverse	**Parallel**
21	Thin cylinders are frequently required to operate under pressures upto _____	5 MPa	15 MPa	30 MPa	250 MPa	**30 MPa**
22	The hoop stress of spherical shell for a built up edge is given by ______(n efficiency)	Pd/3tn	Pd/8t	Pdn/5t	Pd/4tn	**Pd/4tn**
23	The design of a thin cylindrical shell is based on _____	Internal pressure	Diameter of the shell	Longitudinal stress	All of these	**All of these**
24	In order to strengthen a cylindrical shell against bursting force ____ is done	Thickness is increased	Diameter is increased	Wire is wound on the cylinder	length is increased	**Wire is wound on the cylinder**
25	A spring used to absorb shocks and vibrations is	conical spring	torsion spring	leaf spring	disc spring	**leaf spring**

26	A closely coiled helical spring is of mean diameter (D) and spring wire diameter (d). The spring index is the ratio of	1/d	1/D	D/d	d/D	**D/d**
27	Principal planes are planes of _______	Maximum shear stress	Mimimum shear stress	Maximum normal stress	Zero shear stress	**Zero shear stress**
28	Principal stresses are basically _______	Shear stresses	Bending stresses	Normal stresses	None of these	**Normal stresses**
29	The planes of maximum shear stress are located at the following angle to the principal planes	45 degree	60 degree	90 degree	30 degree	**45 degree**
30	Principal planes are separated by _______	45 degree	60 degree	90 degree	180 degree	**90 degree**
31	Maximum shear stress is equal to _________	Half the algebraic difference of principal stress	The algebraic difference of principal stresses	The sum of the principal stresses	None of these	**Half the algebraic difference of principal stress**
32	When a bar of length l, width b and thickness t is subjected to a push of P, its	length, width and thickness increases	length, width and thickness decreases	length increases, width and thickness	length decreases, width and	**length decreases, width and tickness increases**
33	Mohr's circle is used to determine the stresses on an oblique section of a body subjected to	direct tensile stress in one plane accompanied by a	direct tensile stress in two mutually	direct tensile stress in two mutually	all of the above	**all of the above**
34	The radius of the Mohr's circle indicates _______	Maximum principal stress	Minimum principal stress	Maximum shear stress	Minimum shear stress	**Maximum shear stress**
35	In case one principal stress is zero, the other principal stress is equal to___	Maximum shear stress	Two times the maximum shear stress	Three times the maximum shear stress	None of these	**Two times the maximum shear stress**
36	In Mohr's circle the tensile stress will be reckoned___ and will be plotted to ______ of the origin	Negative , right	Positive, left	Negative, left	Positive, right	**Positive, right**
37	In Mohr's circle the compressive stress will be reckoned___ and will be plotted to ______ of the origin	Negative , right	Positive, left	Negative, left	Positive, right	**Positive, right**
38	The resultant stress makes an angle normal to the plane and is called ____	Slant angle	Principal angle	Obliquity	None of these	**Obliquity**

39	For a thin walled shell the diameter thickness ratio is _______	<20	>20	20	None of these	**>20**
40	For a thick walled shell the diameter thickness ratio is _______	<20	>20	20	None of these	**<20**
41	A beam encastered at both the ends is called	simply supported beam	fixed beam	cantilever beam	continuous beam	**fixed beam**
42	If a member is subjected to an axial tensile load, the plane inclined at an angle of 45 degree to the axis of loading	Maximum shear stress	Maximum normal stress	Maximum shear stress	None of these	**Maximum normal stress**
43	The maximum shear stress induced in a member which is subjected to an axial load is equal to	Maximum normal stress	Half of the maximum normal stress	Twice the maximum normal stress	Thrice the maximum normal stress	**Half of the maximum normal stress**
44	If a member, whose tensile strength is more than two times the shear strength, is subjected to an axial load upto failure	Maximum normal stress	Maximum shear stress	Normal stress or shear stress	None of these	**Maximum shear stress**
45	The normal stress on an oblique plane at an angle (t ta)to the cross section of a body which is subjected to a direct	(sigma /2)sin 2 (t ta)	sigma cos (t ta)	sigma cos sqr (t ta)	sigma sin sqr (t ta)	**sigma cos sqr (t ta)**
46	The load required to produce a unit deflection in a spring is called	flexural rigidity	torsional rigidity	spring stiffness	Young's modulus	**spring stiffness**
47	The ratio of bulk modulus to Young's modulus for a Poisson's ratio of 0.25 will be	1\3	2\3	1	3\2	**2\3**
48	Choose the correct statement	The hoop stress in a thin cylindrical shell is compressive stress	The shear stress in a thin spherical shell is more than	The design of thin cylindrical shell is based on	The ratio of hoop stress to longitudinal	**The design of thin cylindrical shell is based on hoop stress**
49	A water main 1m in diameter contains a fluid having pressure 1Mpa . If the maximum permissible tensile stress in	2 cm	2.5 cm	1 cm	0.5 cm	**2.5 cm**
50	The circumferential strain in case of thin cylindrical shell, when subjected to internal pressure p is,	More than diametric strain	Less than diametric strain	Equal to diametric strain	None of these	**Equal to diametric strain**
51	In case of cylinders which have to carry high internal fluid pressure, the method adopted is to	Wind strong steel wire under tension on the cylinder	Shrink one cylinder over the other	Both a & b	None of these	**Both a & b**

52	If a prismatic bar be subjected to an axial tensile stress (sigma), the shear stress induced at a plane inclined at (t ta) with	(sigma/2)sin 2q	(sigma/2) cos (t ta)	(sigma /2)cos sqr (t ta)	(sigma /2) sin sqr (t ta)	**(sigma/2)sin 2q**
53	In case of biaxial state of normal stresses, the normal stress on 45 degree plane is equal to	Sum of normal stresses	Difference of normal stresses	Half the sum of normal stresses	Half the difference of normal stresses	**Half the sum of normal stresses**
54	Circumferential and longitudinal strains in cylinder boiler under internal steam pressure are e1 and e2 respectively.	e1+2e2	e1(e2)sqr	2e1+e2	(e1)sqr.e2	**2e1+e2**
55	Principal stresses at a point in plane stressed element are sigma(x)=sigma (y)= 5000 N/cm2. Normal stress on the	0	5000 N/cm2	7070 N/cm2	10000 N/cm2	**5000 N/cm2**
56	A thin cylinder of radius r and thickness t when subjected to an internal hydrostatic pressure p causes a radial displacement	du/dr	(1/r). (du/dr)	u/r	2u/r	
57	The principal stresses sigma1, sigma2, and sigma3 at a point respectively are 80 MPa, 30 MPa and -40 MPa. The	25 MPa	35MPa	55 MPa	60 MPa	**60 MPa**