15BTAR702 HYPERSONIC AERODYNAMICS 3 1 0 4 100

OBJECTIVES:

To study the basic concepts of hypersonic flows and their effects on flight vehicles

UNIT I FUNDAMENTALS OF HYPERSONIC AERODYNAMICS

Introduction to hypersonic aerodynamics, differences between hypersonic aerodynamics and supersonic aerodynamics, concept of thin shock layers, hypersonic flight paths, hypersonic Similarity parameters, shock wave and expansion wave relations of inviscid hypersonic flows.

UNIT II SIMPLE SOLUTION METHODS FOR HYPERSONICINVISCID FLOWS

Local surface inclination methods, Newtonian theory, modified Newtonian law, tangent wedge and tangent cone methods, shock expansion methods, approximate theory-thin shock layer theory.

UNIT III VISCOUS HYPERSONIC FLOW THEORY

Boundary layer equation for hypersonic flow-hypersonic boundary layers, self-similar and non self-similar boundary layers, solution methods for non-self-similar boundary layers aerodynamic heating.

UNIT IV VISCOUS INTERACTIONS IN HYPERSONIC FLOWS

Introduction to the concept of viscous interaction in hypersonic flows, strong and weak viscous interactions, hypersonic viscous interaction similarity parameter, introduction to shock wave boundary layer interactions.

UNIT V INTRODUCTION TO HIGH TEMPERATURE EFFECTS

Nature of high temperature flows, chemical effects in air-real and perfect gases- Gibb’s free energyand entropy-chemically reacting mixtures-recombinationand dissociation.

TEXT BOOK:

S.NO.	Author(s)	Title of the Book	Publisher	Year of Publication
1.	John. D. Anderson	Hypersonic and High Temperature Gas Dynamics	American Institute Of Aeronautics and Astronautics, Washington.D.C	2006
2.	Wallace D. Hayes, Ronald F. Probstein, Ronald R. Probstein	Hypersonic Inviscid Flow	Courier Corporation,Massachusetts, United States.	2004

REFERENCES BOOKS:

S.NO.	Author(s)	Title of the Book	Publisher	Year of Publication
1.	Anderson Jr D	Modern compressible flows	McGraw-Hill Book Co., NewYork.	2004
2.	John. T Bertin	Hypersonic Aerothermodynamics	American Institute Of Aeronautics and Astronautics, Washington.D.C	1994
3.	G. K. Mikhailov, V. Z. Parton	Super- and Hypersonic Aerodynamics and Heat Transfer	CRC Press,Florida, United States.	1992

WEB REFERENCES:

http://nptel.ac.in/courses/101103003/
https://www.grc.nasa.gov/www/BGH/
http://www.lr.tudelft.nl/en/departments/aerodynamics-wind-energy-flight-performance-and propulsion
https://www.aiaa.org/Secondary.aspx?id=908
www.myopencourses.com › Courses › Aerospace Engineering

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DEPARTMENT OF MECHANICAL ENGINEERING (Aerospace)

COURSE PLAN

Subject Name : HYPERSONIC AERODYNAMICS
Subject Code : 15BTAS702 (Credits - 3)
Name of the Faculty : B.AKILAN
Designation : ASSISTANT PROFESSOR
Year/Semester/Section : IV Year/VII SEM
Branch : B.Tech Aerospace Engineering

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – I : FUNDAMENTALS OF HYPERSONIC AERODYNAMICS			
1.	1	Introduction to hypersonic aerodynamics	
2.	1	Fundamentals of Hypersonic aerodynamics	T [1] ,R [1] ,R [2]
3.	2	Differences between hypersonic aerodynamics and supersonic aerodynamics	T [1] ,R [1] ,R [2]
4.	2	Concept of thin shock layers	T [1] ,R [1] ,R [2]
5.	1	Hypersonic flight paths	T [1] ,R [1] ,R [2]
6.	1	Hypersonic Similarity parameters	T [1] ,R [1] ,R [2]
7.	1	Shock wave relations of inviscid hypersonic flows	T [1] ,R [1] ,R [2]
8.	1	Expansion wave relations of inviscid hypersonic flows	T [1] ,T [2]] ,R [1]
9.	1	Objective type Questions discussion	
Total No. of Hours Planned for Unit - I			**11**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – II : SIMPLE SOLUTION METHODS FOR HYPERSONICINVISCID FLOWS			
10.	1	Local surface inclination methods	T [2] ,R [1] ,R [2]
11.	2	Newtonian theory	T [2] ,R [1] ,R [2]
12.	1	Modified Newtonian law	T [2] ,R [1] ,R [2]
13.	1	Tangent wedge method	
14.	1	Tangent cone method	T [2] ,R [1] ,R [2]

15.	1	Shock expansion methods	T [2] ,R [1] ,R [2]
16.	2	Approximate theory	T [2] ,R [1] ,R [2]
17.	1	Thin shock layer theory.	T [2] ,R [1] ,R [2]
18.	1	Objective type Questions discussion	
Total No. of Hours Planned for Unit - II			**11**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – III : VISCOUS HYPERSONIC FLOW THEORY			
19.	1	Concept of Boundary layer equation	T [2] ,R [1] ,R [2]
20.	2	Boundary layer equation for hypersonic flow.	T [2] ,R [1] ,R [2]
21.	1	Hypersonic boundary layers	T [2] ,R [1] ,R [2]
22.	2	Self-similar and non self-similar boundary layers	T [2] ,R [1] ,R [2]
23.	2	Solution methods for non-self-similar boundary layers	T [2] ,R [1] ,R [2]
24.	1	Aerodynamic heating	T [2] ,R [1] ,R [2]
25.	1	Objective type Questions discussion	
Total No. of Hours Planned for Unit - III			**10**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – IV : VISCOUS INTERACTIONS IN HYPERSONIC FLOWS			
26.	1	Introduction to the concept of viscous interaction in hypersonic flows	T [1] ,R [1] ,R [2]
27.	2	Strong viscous interactions	T [1] ,R [1] ,R [2]
28.	2	Weak viscous interactions	T [1] ,R [1] ,R [2]
29.	2	Hypersonic viscous interaction similarity parameter,	T [1] ,R [1] ,R [2]
30.	1	Introduction to shock wave boundary layer interactions	T [1] ,R [1] ,R [2]
31.	1	Types of Interaction	T [1] ,R [1] ,R [2]
32.	1	Objective type Questions discussion	
Total No. of Hours Planned for Unit - IV			**10**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – V : INTRODUCTION TO HIGH TEMPERATURE EFFECTS			
33.	1	Introduction to High Temperature effects	T [1] ,R [1] ,R [2]
34.	2	Nature of high temperature flows	T [1] ,R [1] ,R [2]
35.	1	Chemical effects in air	T [1] ,R [1] ,R [2]
36.	1	Real and perfect gases	T [1] ,R [1] ,R [2]
37.	1	Gibb’s free energy	T [1], T [2], R [1]
38.	1	Gibb’s free entropy	T [1], T [2], R [1]

39.	1	Chemically reacting mixtures	T [1], T [2], R [1]
40.	1	Recombination and dissociation	T [1], T [2], R [1]
41.	1	Objective type Questions discussion	
42.	1	Previous Year Question paper Discussion	
Total No. of Hours Planned for Unit - V			**10+1**

TOTAL PERIODS : 53

TEXT BOOKS

T [1] – Hypersonic and High Temperature Gas Dynamics - John. D. Anderson

T [2] - Hypersonic Inviscid Flow - Wallace D. Hayes, Ronald F.

REFERENCES

R [1] - Modern compressible flows – Anderson Jr D

R [2] - Hypersonic Aerothermodynamics – John. T Bertin,

JOURNALS

J [1] - Aerospace Science and Technology - Journal - Elsevier

J [2] –Journal of Aerospace Engineering | ASCE Library

J [3] –The Aeronautical Journal - Royal Aeronautical Society

UNIT	Total No. of Periods Planned	Lecture Periods	Tutorial Periods
I	11	10	1
II	11	10	1
III	10	09	1
IV	10	09	1
V	09+2	09	2
TOTAL	53	47	6

I. CONTINUOUS INTERNAL ASSESSMENT : 40 Marks

(Internal Assessment Tests: 30, Attendance: 5, Assignment/Seminar: 5)

II. END SEMESTER EXAMINATION : 60 Marks

TOTAL : 100 Marks

FACULTY **COORDINATOR/ AERO** **HOD / MECH** **DEAN / FOE**

UNIT IV

VISCOUS INTERACTIONS IN HYPERSONIC FLOWS

On the basis of the knowledge of the properties of stream and potential functions and Cauchy-Riemann equations (C-R equations) the following analysis of various ideal fluid flows are discussed. Equation of stream lines

$$\psi = \text{constant}$$

$$= uy - vx$$

Equation of potential lines $\phi = \text{constant}$

$$= ux + vy$$

2.1 Uniform flow

For a steady uniform irrotational flow inclined α to x-axis with a velocity U,

$u = U\cos\alpha = d\phi/dx = d\psi/dy$; $\quad -v = U\sin\alpha = -\dfrac{d\phi}{dy} = \dfrac{d\psi}{dx}$

$\therefore \quad \phi = U(x\cos\alpha + y\sin\alpha) = \text{constant}$

(Cartesian)

$$= U(r\cos\theta.\cos\alpha + r\sin\theta.\sin\alpha)$$

$$= U r \cos(\theta - \alpha)$$

(Polar)

$$= ux+vy \quad \therefore x = -y\tan\alpha + \text{constant}/U\cos\alpha$$

$\psi = U(y\cos\alpha - x\sin\alpha) = \text{constant}$

(Cartesian)

$$= U(r\sin\theta.\cos\alpha - r\cos\theta.\sin\alpha)$$

$$= Ur\sin(\theta - \alpha)$$

(Polar)

$$= uy - vx \quad \therefore y = x\tan\alpha + \text{constant}/U\cos\alpha$$

From above, flows parallel to x and y-axes can be seen as follows.

a) For flows parallel to x - axis, $\alpha = 0$

$\psi = Uy + \text{constant}$, stream lines are parallel to x-axis

and $\phi = Ux + \text{constant}$, potential lines are perpendicular to x - axis.

b) For flow parallel to y - axis, $\alpha = 0$

$\psi = Ux + \text{constant}$, stream lines are parallel to y - axis.

$\phi = - Uy +$ constant, potential lines are perpendicular to y - axis.

2.2 Source or sink

Source is a point from which fluid is emitted uniformly in all direction. A sink flow is a reversed source flow and represents a fluid flow radially inwards to a point. Note that in either case there is only radial velocity to fluid particles.

Strength of source / sink is defined as the volume rate of flow Q per unit depth per unit time. Instantaneous radial velocity at a radius $r = u_r =$ volume rate of flow / circumferential area for unit depth i.e., $u_r = Q / 2\pi r \times 1 = Q / 2\pi r$

Stream and potential functions :

Using C - R equation

$$u_r = \frac{Q}{2\pi r} = \frac{1}{r}\frac{d\psi}{d\theta} \quad \therefore \ \psi = \frac{Q}{2\pi}\theta = \frac{Q}{2\pi}\tan^{-1}(y/x)$$

$$u_r = \frac{Q}{2\pi r} = \frac{d\phi}{dr} \quad \therefore \ \phi = \frac{Q}{2\pi}\ln r = \frac{Q}{4\pi}\ln(x^2 + y^2)$$

Stream lines of source are ψ = constant i.e., $\frac{Q}{2\pi}\theta$ = constant or $\frac{Q}{2\pi}\tan^{-1}(y/x)$ = constant. Stream lines are radial lines through origin if the source is at origin.

Potential lines of source are ϕ = constant is $\frac{Q}{2\pi}\ln r$ = constant or $\frac{Q}{4\pi}\ln(x^2 + y^2)$ = constant. Potential lines are concentric circles with centre origin if the source is at origin.

The instantaneous velocities in Cartesian coordinates can also be expressed.

$$u = u_r \cos\theta = \frac{Q}{2\pi r}\cos\theta = \frac{Q}{2\pi}\frac{r\cos\theta}{r^2} = \frac{Q}{2\pi}\frac{x}{(x^2 + y^2)}$$

$$v = u_r \sin\theta = \frac{Q}{2\pi r}\sin\theta = \frac{Q}{2\pi}\frac{r\sin\theta}{r^2} = \frac{Q}{2\pi}\frac{y}{(x^2 + y^2)}$$

For sink $\quad u_r = -\frac{Q}{2\pi r} \quad ; \quad \psi = -\frac{Q}{2\pi}\theta = -\frac{Q}{2\pi}\tan^{-1}(y/x)$

and $\quad \phi = -\frac{Q}{2\pi}\ln r = \frac{-Q}{4\pi}\ln(x^2 + y^2)$

2.3 Vortex

A circulatory motion of fluid about an axis perpendicular to free surface is vortex flow. There is only tangential velocity to fluid particles in this case. Flow is irrotational and vorticily is zero. The tangential component of velocity u_θ varies in versely as the distance from center(free vortex). The equation for free vortex flow u_θ - r = constant and $u_r = 0$

Stream and potential functions :

Using C - R equation

$$u_\theta = \frac{1}{r}\frac{d\phi}{d\theta} \quad \text{or} \quad u_\theta r = \frac{d\phi}{d\theta} \quad \therefore \ \phi = u_\theta r \theta \qquad \text{(i)}$$

As the circulation Γ of for vortex is = $2\pi r\, u_\theta$ at any radius r, $u_\theta r = \Gamma/2\pi$.

Substituting in (i) $\phi = \frac{\Gamma}{2\pi} \ \theta = \frac{\Gamma}{2\pi} \tan^{-1}(y/x)$

$$u_\theta = \frac{d\psi}{dr} \quad \text{put } u_\theta = \Gamma/2\pi r$$

$$\therefore \ \psi = \frac{-\Gamma}{2\pi} \ln r = \frac{-\Gamma}{4\pi} (x^2 + y^2)$$

Potential lines of vortex $\frac{\psi}{2\pi} \ \theta$ = constant.

These are radial lines through origin if the vortex is at origin.

Stream lines of vortex $\frac{\Gamma}{2\pi} \ln r$ = constant.

These are concentric circles with center origin if the vortex is at origin.

Note 1 : The strength of vortex is defined as the circulation Γ of the vortex and is taken conventionally as positive for anticlockwise flow.

Note 2 : Compare stream and potential lines of source with that of vortex.

2.4 Source sink combination :

A source at -a and sink at + a from origin equal strengths are placed along x-axis forms the combination.

Stream and potential functions :

$$\psi = \psi \text{ of source} + \psi_2 \text{ of sink}$$

$$= \frac{Q}{2\pi}\theta_1 - \frac{Q}{2\pi}\theta_2 = \frac{Q}{2\pi}(\theta_1 - \theta_2)$$

$\theta_1 = \tan^{-1}$ [y/(x-a)] and $\theta_2 = \tan^{-1}$ [y/(x+a)]

$$\tan(\theta_1 - \theta_2) = \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 . \tan\theta_2} = \frac{2ay}{x^2 + y^2 - a^2}$$

$\therefore$ Stream lines of source - sink combination are

$$\psi = \text{constant} = -\frac{Q}{2\pi} \tan\text{-}1 \frac{2ay}{x^2 + y^2 - a^2} \quad \text{(i)}$$

Potential lines of source - sink combination are

$$\phi = \text{constant} = \frac{Q}{2\pi} \ln (r_1 / r_2) \quad \text{(ii)}$$

where $r_1^2 = (a + r\cos\theta)^2 + (r\sin\theta)^2$; $\ln r_1 = ½ \ln (r_1^2)$

$r_2^2 = (r\cos\theta - a)^2 + (r\sin\theta)^2$; $\ln r_2 = ½ \ln (r_2^2)$

Stream lines are circles with common chord of length 2a and with centres on y-axis. Potential lines are also circles with common chord and centers on x - axis as shown in figure.

2.5 Doublet :

Source and sink of numerically equal strengths are considered to approach one another under such a condition that the distance between them approaches zero so that the product of strength and distance between them has a constant value,

i.e., $\frac{Q}{2\pi}$ 2a = constant,

As 2a approaches zero i.e., $a \to o$.

Stream and potential functions of doublet :

From source sink combination stream function

i) as $a \to o$ $\quad \psi = \frac{-Q}{2\pi} \tan^{-1} \frac{2ay}{x^2 + y^2 - a^2}$

becomes $\psi = \frac{-Q}{2\pi} \tan^{-1} \frac{y}{x^2 + y^2}$

For small values of angle this is approximated to

$$\psi = \frac{-Q}{2\pi} \frac{y}{x^2 + y^2} = \frac{-C y}{x^2 + y^2} = \frac{-C \sin\theta}{r}$$

Here C is taken as the strength of doublet :

Similarly for source sink combination the potential function also approximated to

$$\phi = \frac{C\,y}{x^2 + y^2} = \frac{C\cos\theta}{r} \qquad \text{by suitable approximation}$$

$$\phi = \frac{Q}{2\pi} \ln \frac{r_1}{r_2} = \frac{Q}{2\pi} [\ln r_1 - \ln r_2]$$

As $r_1^2 = (a + r\cos\theta)^2 + (r\sin\theta)^2 \qquad \ln r_1 = ½ \ln (r_1^2)$

$r_2^2 = (r\cos\theta - a)^2 + (r\sin\theta)^2 \qquad \ln r_2 = ½ \ln (r_2^2)$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots$$

$$\ln(1 - x) = x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \ldots$$

$\ln(1 - x) - \ln(1 + x) = 2x + \ldots$ Using these, simplify $(\ln r_1 - \ln r_2)$ to get the above result

<u>Stream and potential lines of doublet :</u>

$\dfrac{-C\sin\theta}{r}$ = constant represents the stream lines of doublet ;

$\dfrac{-C\cos\theta}{r}$ = constant represents the potential lines of doublet.

Stream lines are circles with centres on y-axis and tangential to x-axis; while potential lines are circles with centers on x-axis and tangential to y-axis.

The instantaneous velocities in polar coordinates can also expressed.

$$u_r = \frac{1}{r}\frac{d\psi}{d\theta} = +\frac{Q}{2\pi}\frac{\cos\theta}{r^2} \; ; \quad u_\theta = -\frac{d\psi}{dr} = -\frac{Q}{2\pi}\frac{\sin\theta}{r^2}$$

Resultant velocity $\quad U = \sqrt{u_r^2 + u_\theta^2} = \dfrac{Q}{2\pi}\dfrac{1}{r^2}$

2.5 Source with uniform flow

A source placed in the origin and uniform flow in positive x - direction has the stream line in the approaching uniform flow, divides at the stagnation point to two braches encloses the whole flow from source.

Potential and stream functions :

$$\phi = Ux + \frac{Q}{2\pi} \ln (x^2 + y^2)^{½}$$

$$= Ux + \frac{Q}{4\pi} \ln (x^2 + y^2)$$

$$= Ur \cos\theta + \frac{Q}{2\pi} \ln r \qquad \text{(i)}$$

$$\psi = Uy + \frac{Q}{2\pi} \tan^{-1} (y/x)$$

$$= Ur \sin\theta + \frac{Q}{2\pi} \theta \qquad \text{(ii)}$$

Stagnation point

This is located at x - axis putting $u_r = 0$

$$u_r = \frac{1}{r} \frac{d\psi}{d\theta} - \frac{1}{r} \left(Ur \cos\theta + \frac{Q}{2\pi} \right) = 0$$

i.e., $\frac{Q}{2\pi} = - Ur \cos\theta$, $\quad r = \frac{-Q}{2\pi U} \cos\theta$

On x-axis $\theta = 0$ or π $\quad \therefore\ r \ \frac{-Q}{2\pi U} \cos\theta$

The coordinates of stagnation point in Cartesian coordinates $\left(\frac{-Q}{2\pi U}, 0 \right)$ and in polar coordinates $\left(\frac{-Q}{2\pi U}, \pi \right)$

The dividing line equation can be obtained by substituting the coordinates of stagnation point to equation of stream line (ii)

$$\psi = Ur \sin\theta + \frac{Q}{2\pi} \theta$$

$$= U \frac{Q}{2\pi U} \sin\pi + \frac{Q}{2\pi} \pi$$

$$\psi = \frac{Q}{2} \text{ or } Uy + \frac{Q}{2\pi} \theta = \frac{\theta}{2}$$

$$y = \frac{Q}{2U} \left(1 - \frac{\theta}{\pi} \right) \text{ and } r = \frac{Q}{2\pi} \frac{(1 - \theta/\pi)}{\sin\theta}$$

The shape of the curve traced by the dividing line is called Rankine half body.

The principal dimensions of the Rankine half body are also obtained as follows.

at $\theta = 0$, $y_{max} = \frac{Q}{2U}$ is the maximum ordinate

at $\theta = \pi/2$, $y = \frac{Q}{4U}$ is the upper ordinate at origin

at $\theta = \pi$, $y = 0$ leading edge (stagnation point) $x = \frac{Q}{2\pi U}$

at $\theta = \frac{3\pi}{2}$, $y = \frac{-Q}{4U}$ is the lower ordinate at origin

Velocity at any point on Rankine half body

$$u = u_1 + u_2 = U + \frac{Q}{2\pi} \frac{\cos\theta}{r}$$

$$v = v_1 + v_2 = 0 + \frac{Q}{2\pi} \frac{\sin\theta}{r}$$

$$V^2 = u^2 + v^2 = U^2 + \frac{Q^2}{4\pi^2 r^2} + \frac{2QU}{2\pi r}\cos\theta$$

Pressure distribution on Rankine half body

If p_∞ is the free stream pressure and U velocity and p the pressure at any point on Rankine half body,

$$Cp = \frac{p_\infty - p}{\frac{1}{2}\rho U^2} = 1 - \frac{V^2}{U^2}$$

$$= \frac{-Q}{2\pi U}\left(\frac{Q}{2\pi U r^2} + \frac{2\cos\theta}{r}\right)$$

The pressure coefficient on the Rankine half body surface is obtained by putting

$$r = \frac{Q(1 - \theta/\pi)}{2\pi \sin\theta}$$

$$Cp = \frac{\sin\theta}{\pi - \theta}\left(\frac{\sin\theta}{\pi - \theta} + 2\cos\theta\right)$$

The point on Rankine half body where pressure is free stream pressure is obtained by putting $p_\infty = p$ and solving for θ yield to 113.3 degrees.

2.7 Source sink combination in uniform flow

Source and sink of equal strengths are placed at -a and +a from origin along x-axis and a uniform stream is introduced in positive x-direction

Potential and stream functions :

$$\phi = \frac{Q}{2\pi}\ln r_1 - \frac{Q}{2\pi}\ln r_2 + Ur\cos\theta$$

$$= \frac{Q}{2\pi}\ln(r_1/r_2) + Ur\cos\theta \qquad \text{(in polar coordinates)} \qquad \text{(i)}$$

$$= \frac{Q}{2\pi}\ln\left\{\frac{[(x+a)^2 + y^2]^{1/2}}{[(x-a)^2 + y^2]^{1/2}}\right\} + Ux \quad \text{(in Cartesian coordinates)}$$

$$= \frac{Q}{4\pi} \ln \left\{ \frac{(x+a)^2 + y^2}{(x-a)^2 + y^2} \right\} + Ux = \text{constant}$$

$$\psi = \frac{Q}{2\pi} (\theta_1 - \theta_2) + Ur \sin \theta \qquad \text{(in Polar coordinates)} \qquad \text{(ii)}$$

$$= \frac{-Q}{2\pi} \tan^{-1} \frac{2ay}{x^2 + y^2 - a^2} + Uy = \text{constant} \quad \text{(in Cartesian coordinates)}$$

i) <u>is the equation for potential lines and</u>

ii) <u>is the equation for stream lines</u>

<u>Dividing stream line corresponds</u> to $\psi = 0$

$$-\frac{Q}{2\pi} \tan^{-1} \frac{2ay}{x^2 + y^2 - a^2} + Uy = 0$$

This can be shown to be an oval known as Rankine body.

<u>Stagnation points</u>

Using C.R equation

$$u = -\frac{d\psi}{dy} = -U + \frac{Q}{2\pi} \left(\frac{x+a}{(x+a)^2 + y^2} - \frac{x-a}{(x-a)^2 + y^2} \right)$$

At stagnation point $u = 0$

$$0 = -U + \frac{Q}{2\pi} \left(\frac{x+a}{(x+a)^2 + y^2} - \frac{x-a}{(x-a)^2 + y^2} \right)$$

Stagnation point in x - axis where $y = 0$

$$0 = -U + \frac{Q}{2\pi} \left(\frac{x+a}{(x+a)^2} - \frac{x-a}{(x-a)^2} \right)$$

$$= -U + \frac{Q}{2\pi} \left(\frac{1}{x+a} - \frac{1}{x-a} \right)$$

$$= -U + \frac{Q}{2\pi} \left(\frac{2a}{x^2 - y^2} \right)$$

$$\therefore \quad x = \pm \sqrt{\left(a^2 + \frac{Qa}{\pi U} \right)}$$

The stagnation point are on x-axis at $\pm \sqrt{\left(a^2 + \frac{Qa}{\pi U} \right)}$ from origin

<u>Rankine body dimensions</u>

Since at stagnation point $\theta_1 = \theta_2 = \pi$, $\psi = 0$ from (ii)

$$0 = \frac{Q}{2\pi} (\theta_1 - \theta_2) + Ur \sin\theta$$

$$\therefore \quad r = \frac{Q}{2\pi} \quad \frac{(\theta_2 - \theta_1)}{U \sin\theta}$$

This gives the <u>equation to profile of Rankine body :</u>

The spacing of source and sink being 2a, the half length r_s of the body can be determined from consideration of the stagnation point S for which $r_1 = r_s - a \;\; r_s = r_s + a$. Since the velocity at any point is the vector sum of velocities of components patterns the velocity at S is

$$V = U - \frac{Q}{2\pi r_1} + \frac{Q}{2\pi r_2} = 0$$

$$U - \frac{Q}{2\pi} \left(\frac{1}{r_s - a} + \frac{1}{r_s + a} \right) = 0$$

$$r_s = a \sqrt{\left(\frac{2Q}{2\pi a U} + 1 \right)} = a \sqrt{\left(\frac{Q}{\pi a U} + 1 \right)}$$

$\therefore$ 2 r_s is the length of oval

From figure for the point 'p' $\theta_1 = \alpha, \theta_2 = \pi - \alpha \;\; \theta = \frac{\pi}{2} \quad \therefore \;\; h = \frac{Q}{2\pi U} (\pi - 2\alpha)$

$$\therefore \; \alpha = \frac{\pi}{2} - \frac{Uh}{2Q/2\pi} = \frac{\pi}{2} - \frac{Uh}{Q/\pi}$$

$$h = a \tan\alpha$$

$$= a \tan \left(\frac{\pi}{2} - \frac{Uh}{Q/\pi} \right)$$

$$= a \cot \frac{Uh}{Q/\pi}$$

$\therefore$ 2h is the width of oval

2.8 Doublet with uniform flow :

Doublet on x axis at origin with uniform flow in the positive x - direction forms this configuration.

<u>Stream and potential functions</u>

$$\psi = \frac{-C \sin\theta}{r} + Ur \sin\theta = \frac{-Q}{2\pi} \frac{y}{x^2 + y^2} + Ux$$

$$= U (r - \frac{C}{Ur}) \sin\theta$$

$$= U\left(r - \left(\sqrt{\frac{C}{Ur}}\right)^2 \frac{1}{r}\right) \sin\theta$$

where $\sqrt{\frac{C}{U}}$ is the cylinder radius

$$\phi = \frac{C \cos\theta}{r} + Ur\cos\theta = \frac{Q}{2\pi}\ \frac{x}{x^2 + y^2} + Ux.$$

$$= U\left(r + \frac{C}{Ur}\right)\cos\theta$$

<u>Dividing stream line</u> is obtained by putting $\psi = 0$

i.e., $\frac{-C}{r}\sin\theta + Ur\sin\theta = 0$

$$r^2 = \frac{c\sin\theta}{U\sin\theta} = \frac{C}{U}$$

i.e., $x^2 + y^2 = \left(\frac{C}{U}\right)^2 = \left(\sqrt{\frac{Q}{2\pi U}}\right)^2$

<u>The circle of radius $\sqrt{C/U}$ is the dividing stream line</u>

<u>Stagnation points :</u>

Stagnation points are on x - axis (y = 0) at $\pm\ \sqrt{C/U}$ from origin.

<u>Doublet with uniform flow resembles the flow past a cylinder of radius $\sqrt{C/U}$.</u>

<u>Velocity distribution</u>

Using C.R equation

$$u_r = \frac{1}{r}\ \frac{d\psi}{d\theta} = \frac{1}{r}\frac{d}{d\theta}\left(Ur\sin\theta\left(1 - \frac{Q}{2\pi Ur^2}\right)\right)$$

$$= U\cos\theta\left(1 - \frac{Q}{2\pi Ur^2}\right)$$

$$= U\cos\theta\left(1 - \frac{a^2}{r^2}\right)$$

where a is the radius of cylinder $\sqrt{Q/2\pi U}$. For r = a, $u_r = 0$.

This means that there is no radial velocity with in or on the cylinder surface.

$$u_\theta = -\frac{d\psi}{dr} = \frac{d}{dr}\left(Ur\sin\theta\left(1 - \frac{Q}{2\pi Ur^2}\right)\right)$$

$$= - U \sin\theta \left(1 + \frac{Q^2}{r^2}\right)$$

Put r = a so that tangential velocity on the cylinder surface u_θ = - 2 U sin θ.

The maximum tangential velocity is when θ = π/2 or 3π/2 i.e., on top and bottom of cylinder.

<u>Pressure distribution :</u>

<u>a) Cylinder in a uniform flow :</u>

Let the pressure on the cylinder at any point θ inchined to positive x-axis be p, velocity there being $\vec{u_r} + \vec{u_\theta} = 0 - 2U \sin\theta$. Applying Bernoullis theorem to a point in free stream (pressure p_∞ and velocity U) and to the point on the cylinder surface

$$p_\infty + \frac{1}{2}\rho U^2 = p + \frac{1}{2} \rho (-2 U \sin\theta)^2$$

$$p - p_\infty = \frac{1}{2} \rho U^2 (1 - 4 \sin^2\theta)$$

Pressure co-efficient (Euler number) $C_p = \dfrac{p - p_\infty}{½ \rho U^2}$ at various points on the cylinder plotted gives the figure.

The points on the cylinder surface where pressure is the same as free stream flow are obtained by setting $p = p_\infty$ so that $1 - 4 \sin^2\theta = 0$ i.e., when θ equals 30°, 150°, 120°, 210° and 330°. The maximum pressure occurs when $1 - 4 \sin^2\theta$ is maximum and θ equals π/2 or 3π/2 corresponding to pressure coefficients are - 3.

The figure shows the distribution of the pressure coefficient. C_p on the surface of the cylinder. It is symmetrical about the axis and so this body does not offer any resistance to motion or there is no drag force. For a stationary cylinder kept with its axis perpendicular to the flow of a fluid no lift or drag force is felt. **(D' Alembert's Paradox).**

This aspect is, however, on the assumption that the fluid is ideal, (perfect) i.e., it has zero viscosity. Due to viscous nature of the real fluids, the fluid moving ove the surface of a cylinder experience frictional forces which retard the flow. Eventually, the flow separates over the down stream of cylinder and actual pressure distribution departs radically from what obtained by ideal fluid flow analysis. This destroys the symmetry about the axis and drag force is setup.

<u>b) Spinning cylinder in a uniform flow :</u>

For a flow past a cylinder with circulation. (or the cylinder being rotating about its own axis) the pressure distribution can be arrived from the ideal fluid flow analysis. The force exerted in the direction of flow can be shown equal to zero. But the force exerted on the cylinder perpendicular to the flow direction in finite. This is called lift force. Thus in a uniform flow with circulation around a cylinder there is force only in one direction. This is called the "Magnus Effect".

c) Aerofoils :

The upper and lower surfaces of an aerofoil are made of different cambers for unsymmetrical aerofoils. It is found that the major part of the top surface will be at a lower pressure than the bottom surface. This is due to the downward curvature of the aerofoil surface. The resultant pressure acting towards upper side contributes lift. In aerofoils the top surface contributes the major part of the lift force than bottom surface. This is because the top surface is mainly responsible for down wash of fluid.

In subsonic the major part of lift come from the leading edge of the aerofoil. An aerofoil kept in an air stream the centre of pressure has a tendency to move forward towards the leading edge of aerofoil from the geometrical centre of the chord.

The distribution of the pressure on ordinary aerofoils vary with the angle of attack at which it is set to the relative direction of flow. Consequently there will be a movement of the centre of pressure. It can be noticed that at a negative angle of incidence, and even at zero degree, the pressure on the top surface is decreased, this cause a loop in the pressure distribution diagram for these region of angle of attack (negative to zero) nose portion being pushed downwards and tail portion being pushed upwards for which the centre of pressure of the aerofoil is long way back. As the angle of attack is slowly increased up to say stalling the centre of pressure is seen to move gradually forward until it is less than one third the chord length from the leading edge, while above stalling angle the centre of pressure beings to move backwards again.

The distribution of C_p at the middle section of an aerofoil for various angles of attack from negative to near stalling is shown in the figure. The total force exterted on the wing either on vacuum or on pressure side us expressed by the area of the diagram. It can seen in figure that major part of the lift is caused by vacuum action on the tojpside of the wing. After α =14.6 degree lift starts to decrease.

From the pressure distribution measurements, the total lift on a wing can be calculated by integration process first across the aerofoil section and then across the span of the wing.

2.9 Doublet, vortex and uniform flow :

The flow pattern from left to right past a cylinder of radius a (doublet with uniform flow) with an addition of clockwise rotational vortex ($-\Gamma$) represents the flow past a spinning cylinder.

The theory of circulation on immersed bodies provides mathematical explanation for the occurrence of lifting forces on aerofoils and spinning balls in flight. The basis of the analysis of aerodynamic forces on aerofoils is by the elementary analysis of flow past spinning cylinders.

Stream and potential functions :

$\psi = \psi_1$ of uniform flow + ψ_2 of doublet + ψ_3 of vortex

$$= Ur\sin\theta - \frac{C\sin\theta}{r} + \frac{\Gamma}{2\pi}\ln r$$

$$= U\left(1 - \frac{a^2}{r^2}\right) r\sin\theta + \frac{\Gamma}{2\pi}\ln r \quad \text{as } \sqrt{C/U} = a \qquad \text{(i)}$$

$\phi = \phi_1$ of uniform flow + ϕ_2 of doublet + ϕ_3 of vortex

$$= Ur\cos\theta + \frac{C\cos\theta}{r} - \frac{\Gamma}{2\pi}\theta \qquad \text{(ii)}$$

Stagnation points

Using C - R equation

$$u_r = \frac{1}{r}\frac{d\psi}{dr} = U\cos\theta\left(1 - \frac{a^2}{r^2}\right) + 0$$

$$u_\theta = -\frac{d\psi}{dr} = -U\sin\theta\left(1 + \frac{a^2}{r^2}\right) + \frac{\Gamma}{2\pi r}$$

On the surface of cylinder where r = a; $u_r = \theta$; $u_\theta = -2U\sin\theta + \Gamma/2\pi a$. At stagnation point both $u_r + u_\theta = 0$ $\therefore \sin\theta = \Gamma/4\pi aU$. This means that according to the position i.e., θ, the value of $\Gamma/4\pi aU$ may be negative, zero or positive (i.e., from -1 to +1)

Case 1 : For $\sin\theta$ is a negative quantity $\Gamma < 4\pi aU$. This is the situation of subcritical circulation. Sine angle is negative in 3rd and 4th quadrants only. Hence there will be two stagnation points on the third and fourth quadrants of circle.

Case 2: For $\sin\theta$ is zero Γ must be equal to zero. This is the case of flow past a cylinder without circulation. This is already discussed with 2 stagnation points on x-axis.

Case 3: For $\sin\theta$ equal to 1, $\Gamma = 4\pi aU$ for which there will be only one stagnation point at $\theta = 3\pi/2$. This is the situation of critical circulation.

Case 4: For $\sin\theta$ a positive quantity other than unity $\Gamma > 4\pi aU$. The stagnation point moves out into the flow enclosing fluid inside the stream line as shown in figure. This is Super critical circulation.

Pressure distribution - Kutta - Joukowski theorem

Let the pressure on the cylinder at any point inclined θ to positive x-axis be p, the velocity there being $- 2U\sin\theta \quad \Gamma/2\pi a$. Applying Bernoullis theorem to a point in the free stream (pressure p_∞ and velocity U) and to the point on the cylinder surface (note Γ is taken negative as it is clockwise).

$$p_\infty + \frac{1}{2}\rho U^2 = P + \frac{1}{2}\rho\left(-2U\sin\theta - \frac{\Gamma}{2\pi a}\right)^2$$

$$p - p_\infty = \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta - \frac{4\Gamma}{2\pi aU}\sin\theta - \frac{\Gamma^2}{4\pi^2a^2U^2}\right)$$

Resulting force on the elemental area ($a\,d\theta.1$) (per unit length of cylinder) on the cylinder surface = $(p - p_\infty)\ a\,d\theta$

The component of this elemental force is x-direction $= (p - p_\infty)\ a\cos\theta\,d\theta$

i) $\therefore$ Total force is x-direction (force in negative x-direction is called drag)

$$= \int_{\pi/2}^{3\pi/2} (p - p_\infty)\ a\cos\theta\,d\theta$$

$$= \int_{\pi/2}^{3\pi/2} \frac{1}{2}\rho U^2\left(1 - 4\sin^2\theta \quad \frac{4\Gamma}{2\pi aU} - \frac{\Gamma^2}{4\pi^2a^2U^2}\right) a\cos\theta d\theta = 0$$

$\therefore$ Drag force is zero

$$\text{As} \int_{\pi/2}^{3\pi/2} \cos\theta\,d\theta = \int_0^{2\pi} \cos\theta\,d\theta = \left(\sin\theta\right)_0^{2\pi} = 0$$

and $\int_{\pi/2}^{3\pi/2} \sin^2\theta . \cos\theta \, d\theta = \int_0^{2\pi} (1 - \cos^2\theta) \cos\theta \, d\theta = \int_0^{2\pi} (\cos\theta - \cos^3\theta) d\theta$

$$= \int_0^{2\pi} [\cos\theta - 1/4 \ (3\cos\theta - \cos3\theta)] \, d\theta$$

$$= \int_0^{2\pi} (\cos\theta - 3/4 \cos\theta + 1/4 \ \cos3\theta) \, d\theta$$

$$= \int_0^{2\pi} 1/4 \ (\cos\theta + \cos3\theta) \, d\theta$$

$$= 1/4 \left(\sin\theta + \frac{\text{Sin}3\theta}{3} \right)_0^{2\pi} = 0$$

and $\int_{\pi/2}^{3\pi/2} \sin\theta.\cos\theta \, d\theta = \int_0^{2\pi} \frac{1}{2} \sin2\theta \, d\theta = \frac{1}{2} \left(\frac{-\cos2\theta}{2} \right)_0^{2\pi}$

$$= \frac{1}{2} \left(- \frac{1}{2} + \frac{1}{2} \right)$$

$$= 0$$

Component of this elemental force on y-direction

$$= (p - p_\infty) \ a \sin\theta \, d\theta$$

ii) $\therefore$ Total force in y-direction

(force in positive y-direction is called lift)

$$= \int_0^{2\pi} (p - p_\infty) \ a \sin\theta \, d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \rho\, U^2 \left(1 - 4 \sin^2\theta \quad \frac{4\Gamma}{2\pi a U} \sin\theta \; - \frac{\Gamma^2}{4\pi^2 a^2 U^2} \right) a \sin\theta \, d\theta$$

$$= \frac{1}{2} \rho U^2 \left(-4 \frac{\Gamma\pi}{2\pi U} \right)$$

$= - \rho\, U\Gamma$ per unit length of cylinder.

$\therefore$ Lift force $L = - \rho\, U\Gamma$

$$\text{As} \int_0^{2\pi} \sin\theta \, d\theta = \left[- \cos\theta \right]_0^{2\pi} = 0$$

$$\text{and} \int_0^{2\pi} \sin^2\theta \, d\theta = \frac{1}{2} \int_0^{2\pi} (1 - \cos^2\theta)\, d\theta = \frac{1}{2} \left[\theta \frac{1}{2} \sin 2\theta \right]_0^{2\pi} = \pi$$

$$\text{and} \int_0^{2\pi} \sin^3\theta \, d\theta = -\int_0^{2\pi} \sin^2\theta \sin\theta \, d\theta = -\int_0^{2\pi} (1 - \cos^2\theta)\, d(\cos\theta)$$

$$= - \left[\cos\theta \; - \frac{\cos 3\theta}{3} \right]_0^{2\pi} = 0$$

Note : U is positive, Γ is negative (clockwise), lift L becomes positive i.e., in the upword direction of y-axis $L = \rho\, U\Gamma$.

In a flow past a cylinder with circulation (or cylinder being rotated about its own axis) in ideal fluid flow analysis the force exerted in the direction of flow (drag) is shown to the equal to zero. But the force exerted on cylinder perpendicular to the flow (lift) is finite. Thus in a uniform flow with circulation around a cylinder there is force only in one direction. This cross force which is known as the Magnus effect, is independent of the cylinder size. In fact, Kutta and Joukowski each showed that the force is independent of the shape of the body, and in theory, is always equal to the product of the density, the circulation and the velocity, per unit length.

In a real fluid, surface resistance and separation effects produce a finite drag force D. For the development of the cross force, L a circulatory flow of fluid in the region of the cylinder wall can be produced by rotation of the cylinder, which drags the fluid in contact around with it. The resulting local circulatory motion, superimposed upon the translatory flow past the cylinder, develops regions of high and low velocity on the opposite sides of the cylinder and a lift force results. This is one explanation of the lateral deflection during flight of tennis and golf balls which have a spin. In addition, the occurrence of early separation on the low velocity side and late separation on the high velocity side of the ball, results in an unsymmetrical wake, which may produce an appreciable lateral force.

The coefficient of lift C_L is defined by the equation

$$D = C_L \tfrac{1}{2} \rho U^2 A$$

where L = the lift force and A = the area of the projection of the object on a plane normal to the flow direction. (In the case of aerofoils, the projection is conventionally taken on the plane of the chord.)

For a cylinder of unit length and diameter d,

$$Y = C_L \tfrac{1}{2} \rho U^2 d$$

$$\therefore\ C_L = \frac{L}{\tfrac{1}{2} \rho U^2 d}$$

For potential flow, $L = \rho U\Gamma$ and

$$CL = \frac{\rho U\Gamma}{\tfrac{1}{2} \rho U^2 d} = \frac{\Gamma}{\tfrac{1}{2} Ud} = \frac{\pi d V_\theta}{\tfrac{1}{2} Ud} = \frac{2\pi V_\theta}{U}$$

where V_θ is the circulation velocity at the cylinder surface.

From tests carried out on a cylinder rotating in a fluid with a surface velocity V_θ, the values of C_L based on measured values L are much lower than those computed for irrotational flow and it appears that the local circulation induced by surface drag is only half as effective as the constant circulation of irrotational flow. The maximum lift occurs, in practice, when V_θ is about 4U.

<u>Drag on a circular cylinder:</u>

For ideal, non viscous flow of fluid past a circular cylinder, because of the flow symmetry ,the pressure at the corresponding points on the front and back of the cylinder are equal. No unbalance pressure force acts on the cylinder and consequently the pressure drag is zero. But for real fluids viscosity effects creep in. Consequently the symmetrical pressure distribution is destroyed. The flow separating at top and bottom of cylinder surfaces. Beyond the point of flow separation, eddies and vortices form which persist for some distance before they are finally damped out by viscous forces.

Boundary layer becomes turbulent beyond $R_e = 5x10^5$, the separation points shift downstream towards rear of the cylinder and that diminishes the width of the wake region.

The flow pattern and pressure distribution on the front portion of the cylinder are identical both for irrotational and real flows with laminar and turbulent boundary layers. The difference exists mainly on the back of cylinder. There is acceleration of flow in the front portion of the cylinder and the boundary layer adheres to the surface. However, the adverse pressure gradient in the decelerating zone on the rear side affects the boundary layer growth and its eventual separation which leads to changes in the flow pattern.

At very low $R_e < 0.5$, intertial forces are negligible as compared with viscous forces. Pressure gradients which depend on square of velocity are insignificant and so the boundary layer do not detach from the cylinder surfaces. Obviously at low R_e, skin fraction accounts for a large part of total drag.

At R_e ranging from 2 to 40, the laminar flow separates. The separation is however, symmetrical and characterized by the formation of two eddies or vortices which rotate in opposite directions. Beyond the eddies the streamlines close together and there by limit the size of the wake.

For further increase in R_e above 90 vortices detach from body and form staggered rows of uniformly spaced vortices called Karman vortices. At greater than Re = 5000 complete turbulent mixing takes place, vortices disintegrate and disappear.

Alternate spreading of vortices gives rise to pressure fluctuations which set up vibrations with frequency equal to that of vortex sheding. Severe damage can thus result when the natural frequency of vibration of the body attains resonance with the frequency of vortex sheding. Occurrence of resonance is evident in the singing of telephone or power wires in the wind; flutter of aircraft wings; vibrations setup in chimmeys and suspension bridges exposed to high winds.

The variation with Reynolds number of the total drag coeeficient for flow past a smooth cylinder and stream lined strut are obtained form several sets. Of experiments.

At any low values of R_e ($R_e < 1$) inertical effects are negligible and the drag force is essentially due to viscous effects. With R_e tending to zero, skin friction drag becomes about two third of total drag. This flow is called <u>creeping flow</u> and drag is referred to as <u>deformation drag ;</u> body pushes itself through the fluid which is deformed by it. The drag coefficient varies inversely with R_e and is indicated by the straight line part of the graph.

With gradual growth in R_e vortex trails are fully established and pressure drag makes a proportionately large contribution. Slope of the curve decrease and drag – R_e relationship deviates form straight line. At $R_e \approx 200$ pressure drag account 75% of total drag and $C_D \approx 0.95$. There-after C_D gradually increases to a steady value of 1.2 till the boundary layer is laminar R_e 10^5. For $10^3 < R_e < 10^5$ viscous shear at surface of cylinder becomes insignificant; pressure drag contributes for almost whole of the total drag.

With subsequent increase in R_e from 10^5 to 5×10^5, there is a sharp drop in the drag form 1.2 to 0.35. This is due to the change of flow pattern in the boundary layer transition from laminar to turbulent. Up to $R_e < 10^5$, boundary layer is laminar and it separates at widest part of the cylinder. Entire rear half of the cylinder is subjected to relatively low pressure, and the large unbalanced pressure force across the cylinder gives relatively high value of drag coefficient.

For $R_e > 5 \times 10^5$, transition to turbulence occurs before separation and the boundary layer becomes fully turbulent. It has more capacity for mixing and absorbs energy from main stream than laminar boundary layer. Turbulent being more rough it is less likely to separate from surfaces.

With increase in surface roughness, instability and transition are triggered sooner. i.e., the boundary layers are forced to become turbulent at low R_e and consequently C_D diminishes. A similar trend is evident with high turbulence levels which bring up early onset of turbulent boundary layers.

With in a range of $5 \times 10^5 < R_e > 3 \times 10^6$ C_D rises to about 0.7. There-after the viscous effects are relatively small and CD becomes practically independent of Reynolds number.

Compared to cylinder a stream-lined body has a considerably lower values of drag coefficient as shown.

2.10 Alternate approach to ideal fluid flow problem using complex potential :

With a minimum knowledge of complex variable and its properties the following discussions are carried out.

$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$;

$1/z = 1/r(\cos\theta - i\sin\theta)$; r is modulus ; θ amplitude

If $z = x + iy$ and $w = f(z) = \phi + i\psi$

$$\frac{dw}{dz} = u - iv.$$

2.11 Uniform flow

a) Parallel to x- axis velocity U

$w = Uz = U(x + iy) = \phi + i\psi$

$\therefore \phi = -Vy$; $\psi = Vx$ represent potential and stream lines respectively and = U $\frac{dw}{dz}$ u - iv. This means imaginary part is zero. is u = U and v = 0

b) Parallel to y-axis velocity V

$w = iVz = iV(x + iy) = \phi + i\psi$

$\therefore \phi = -Vy$; $\psi = Vx$ represent potential and stream times respectively and $\frac{dw}{dz}$ = iV = u - iv. This means real part is zero. i.e., u = o and -v = V

c) Inclined $\propto$ to postivie x - axis Velocity U

$$\begin{aligned} w = Uze^{-i\alpha} &= U(x + iy)(\cos\alpha + i\sin\alpha)^{-1} \\ &= U(x + iy)(\cos\alpha - i\sin\alpha) \\ &= U\left[(x\cos\alpha + y\sin\alpha) + i(-x\sin\alpha + y\cos\alpha)\right] \\ &= \phi + i\psi \end{aligned}$$

$$\begin{aligned} \therefore \phi &= U(x\cos\alpha + y\sin\alpha) \\ &= U(r\cos\theta\cos\alpha + r\sin\theta\sin\alpha) \\ &= Ur\cos(\theta - \alpha) \end{aligned}$$

$$\psi = U\,(y\cos\alpha + x\sin\alpha)$$
$$= U\,(r\sin\theta\cos\alpha + r\cos\theta\sin\alpha)$$
$$= Ur\,\sin(\theta - \alpha)$$

2.12 Source or sink at origin

$$w = m\ln z = \phi + i\,\psi$$
$$= m\ln(re^{i\theta}) = m\,(\ln r + i\,\theta)$$

$\therefore\ \phi = m\ln r\ ;\ \psi = m\,\theta$

$$\frac{dw}{dz} = \frac{d}{dz}(m\ln z) = \frac{m}{z} = \frac{m}{r}e^{-i\theta}$$

$$= \frac{m}{r}(\cos\theta - i\sin\theta)$$
$$= u - iv$$

$$\therefore\ u = \frac{m}{r}\cos\theta;\quad v = \frac{m}{r}\sin\theta$$

$$u = \frac{m\ r\cos\theta}{r^2}\ ;\quad v = \frac{m}{r^2}\ r\sin\theta$$

$$= \frac{m\ x}{x^2 + y^2} \qquad = \frac{m\ y}{x^2 + y^2}$$

For sink $w = -m\ln z$ will yield to

$$\phi = -\,m\ln r\ ;\ \psi = -\,m\theta \ \text{ and}$$
$$u = -\frac{m}{r}\cos\theta\ ;\quad u = -\frac{m}{r}\sin\theta$$

2.13 Vortex at origin

$$w = \frac{-i\ \Gamma\ln z}{2\pi} = \phi + i\,\psi$$

$$= \frac{-i\,\Gamma}{2\pi}\ln(re^{i\theta}) = \frac{-i\,\Gamma}{2\pi}(\ln r + i\theta)$$

$$= \frac{\Gamma\theta}{2\pi} - \frac{i\ \Gamma}{2\pi}\ln r$$

$$\phi = \frac{\Gamma}{2\pi}\theta\ ;\ \psi = -\frac{\Gamma}{2\pi}\ln r$$

$$\frac{dw}{dz} = \frac{d}{dz}\left(\frac{-i\ \Gamma \ln z}{2\pi}\right) = \frac{-i\,\Gamma}{2\pi z} = \frac{-i\,\Gamma}{2\pi r} \ (\cos\theta + i \sin\theta)^{-1}$$

$$= \frac{i\ \Gamma}{2\pi r} \cos\theta + \frac{\Gamma}{2\pi r} \sin\theta$$

$$= u - iv$$

$$\therefore\ u = \frac{\Gamma}{2\pi r}\sin\theta\ ; \qquad v = \frac{\Gamma}{2\pi r}\cos\theta$$

$$= u_\theta \sin\theta \qquad\qquad = u_\theta \cos\theta$$

$$\therefore\ u_\theta = \frac{\Gamma}{2\pi r} \quad (u_r = o;\ \text{but } v \text{ is not zero})$$

2.14 Source and sink combination :

Source at -a from origin $\quad w = m \ln (z - [-a])$

$= m \ln (z + a)$

Sink at +a from origin $\quad w = m \ln (z - [+a])$

$= m \ln (z - a)$

Source sink combination $\quad w = m \ln(z+a) – m \ln (z - a)$

$$= m \ln \frac{z + a}{z - a}$$

$$= m \ln \frac{r_1\, e^{i\theta_1}}{r_2 e^{i\theta_2}}$$

$$= m \ln \frac{r_1}{r_2} + i\, m\, (\theta_1 - \theta_2)$$

$$= \phi + i\,\psi$$

$$\therefore\ \phi = m \ln \frac{r_1}{r_2} \ \text{ and } \psi = m\, (\theta_1 - \theta_2)$$

2.15 Source with uniform flow :

Source at origin and uniform flow in positive x - direction

$$w = m \ln z + Uz = m \ln re^{i\theta} + Ure^{i\theta}$$

$$= m \ln r + m \ln e^{i\theta} + Ure^{i\theta}$$

$$= m \ln r + m\, i\theta + Ur\ (\cos \theta + i \sin \theta)$$

$$= (m \ln r + Ur \cos\theta) + i\ (m\, \theta + Ur \sin \theta)$$

$$= \phi + i\ \psi$$

$\therefore\ \phi = m \ln r + Ur\cos\theta\ ;\ \psi = m\theta + Ur\sin\theta$

$$\frac{dw}{dz} = \frac{d}{dz}(m\ln z + Uz)$$

$$= \frac{m}{z} + U$$

$$= \frac{m}{r}(\cos\theta + i\sin\theta)^{-1} + U$$

$$= \frac{m}{r}(\cos\theta - i\sin\theta) + U$$

$$= u - iv$$

$$u = \frac{m}{r}\cos\theta + U \text{ and } v = \frac{m}{r}\sin\theta$$

2.16 Doublet :

Source and sink of equal strength are situated at arbitrary points on a line inclined α to positive x-direction at $-a_1$ and $+a_1$ as shown in fig such that $a_1 = h(\cos\alpha + i\sin\alpha) = h e^{i\alpha}$ h being the modulus and α the amplitude.

w of source $= m\ln(z + a_1) = m\ln(z + he^{i\alpha})$

w of sink $= -m\ln(z - a_1) = -m\ln(z - he^{i\alpha})$

w of combination $= m\left[\ln(z + he^{i\alpha}) - \ln(z - he^{i\alpha})\right]$

$$= m\left[\ln\left(1 + \frac{he^{i\alpha}}{z}\right)z - \ln\left(1 - \frac{he^{i\alpha}}{z}\right)z\right]$$

$$= m\left[\ln\left(1 + \frac{he^{i\alpha}}{z}\right) + \ln z - \ln\left(1 - \frac{he^{i\alpha}}{z}\right) - \ln z\right]$$

$$= m\left[\ln\left(1 + \frac{he^{i\alpha}}{z}\right) - \ln\left(1 - \frac{he^{i\alpha}}{z}\right)\right]$$

$$= m\left[2\frac{he^{i\alpha}}{z} + \frac{2}{3}\left(\frac{he^{i\alpha}}{z}\right)^3 + \frac{2}{5}\left(\frac{he^{i\alpha}}{z}\right)^5 + \ldots\right]$$

$$w = m\frac{2e^{i\alpha}}{z}\left[1 + \frac{1}{3}\left(\frac{he^{i\alpha}}{z}\right)^2 + \frac{1}{5}\left(\frac{he^{i\alpha}}{z}\right)^4 + \ldots\right]$$

For doublet when $h \to o$ while $m.2h = \mu$, being a constant.

$$w = \frac{\mu e^{i\alpha}}{z}[1 + 0 + 0 + 0 + \ldots\ldots\ldots..]$$

The line form $-h$ to $+h$ forms the axis of doublet. When this axis coincides with x-axis $\alpha = 0$ and $e^{i\alpha} = e^{\circ} = 1$

<u>$\therefore$ w of doublet with axis on x axis is</u> $\frac{\mu}{z}$

$$w = \frac{\mu}{z} = \frac{\mu}{r}(\cos\theta + i\sin\theta)^{-1}$$

$$= \frac{u}{r} (\cos\theta - i\sin\theta) = \phi + i\psi$$

$$\therefore \phi = \frac{\mu}{r}\cos\theta \text{ and } \psi = -\frac{\mu}{r}\sin\theta$$

2.17 Source and sink with uniform flow :

A source and sink of equal strengths at -a and +a from origin along X-axis with uniform flow in the positive x - direction.

$$w = m\ln(z+a) - m\ln(a-a) + Uz$$

$$= m\ln\frac{r_1 e^{i\theta}}{r_2 e^{i\theta}} + Ur(\cos\theta + i\sin\theta)$$

$$= m\ln\left(\frac{r_1}{r_2}\right) + im(\theta_1 - \theta_2) + Ur\cos\theta + Ur\, i\sin\theta$$

$$= \phi + i\psi$$

$$\phi = m\ln\left(\frac{r_1}{r_2}\right) + Ur\cos\theta; \quad \psi = m(\theta_1 - \theta_2) + Ur\sin\theta$$

$$\frac{dw}{dz} = \frac{d}{dz}\left(ml(z+z) - m\ln(z-a) + Uz\right)$$

$$= \frac{m}{z+a} - \frac{m}{z-a} + U = u - iv$$

At stagnation point $u - iv = 0$

i.e., $\frac{m}{z+a} - \frac{m}{z-a} + U = 0$

$$(z+a)(z-a) = \frac{2am}{U}$$

$$z^2 = \frac{2am}{U} + a^2 \quad \therefore z = \pm\sqrt{a^2 + \frac{2am}{U}} = x + iy$$

This means stagnation points are at y = 0 i.e., x - axis at $\sqrt{a^2 \cdot \frac{2am}{U}}$ distant from origin.

2.18 Doublet with uniform flow

A doublet on at origin with uniform flow in the positive x-direction.

$$w = \frac{\mu}{z} + Uz$$

$$= \frac{\mu}{r}(\cos\theta + i\sin\theta)^{-1} + Ur(\cos\theta + i\sin\theta)$$

$$= \frac{\mu}{r}\ (\cos - i\sin\theta) + Ur\ (\cos\theta + i\sin\theta)$$

$$= \phi + i\,\psi$$

$$\phi = \frac{\mu}{r}\cos\theta + Ur\cos\theta\ ;$$

$$= \cos\theta\ \left(\frac{\mu}{r} + Ur\right)\ ;$$

$$\psi = -\frac{\mu}{r}\sin\theta + Ur\sin\theta = Ur\sin\theta\left(1 - \frac{\mu}{r^2U}\right)$$

$$\frac{dw}{dz} = \frac{d}{dz}\left(Uz + \frac{\mu}{z}\right) = U - \frac{\mu}{z^2} = u - iv$$

At stagnation point, $U - \frac{\mu}{z^2} = 0$

$$z = \pm\sqrt{\frac{\mu}{U}} = x + iy$$

This means the stagnation points are at $y = 0$ i.e., x - axis at $x = \pm\frac{\mu}{U}$ distant from origin. This is the radius of the cylinder $a = \sqrt{\frac{\mu}{U}}$

2.19 Doublet vortex and uniform flow

A doublet at origin with a uniform flow in the positive x - direction along with a clockwise vortex ($-\Gamma$)

$$w = \frac{\mu}{z} + Uz + \frac{i\Gamma}{2\pi}\ \ln z$$

$$= \frac{\mu}{r}\ (\cos\theta + i\sin\theta)^{-1} + Ur\,(\cos\theta + i\sin\theta) + \frac{i\Gamma}{2\pi}\ \ln(re^{i\theta})$$

$$= \frac{\mu}{r}\ (\cos\theta - i\sin\theta) + Ur\cos\theta + i\sin\theta) + \frac{i\Gamma}{2\pi}\ \ln r - \frac{i\Gamma}{2\pi}\,\theta$$

$$= \left(\frac{\mu}{r}\cos\theta + Ur\cos\theta - \frac{i\Gamma}{2\pi}\,\theta\right) + i\left(\frac{-\mu}{r}\sin\theta + Ur\sin\theta + \frac{\Gamma}{2\pi}\ \ln r\right)$$

$$\therefore\ \phi = \frac{\mu}{r}\cos\theta + Ur\cos\theta - \frac{\Gamma}{2\pi}\,\theta$$

$$\psi = \frac{-\mu}{r}\sin\theta + Ur\sin\theta + \frac{\Gamma}{2\pi}\ln r$$

$$= Ur\sin\theta\left(1 - \frac{\mu}{Ur^2}\right) + \frac{\Gamma}{2\pi}\ln r$$

$$= Ur\sin\theta\left(1 - \left(\sqrt{\frac{\mu}{U}}\right)^2\frac{1}{r^2}\right) + \frac{\Gamma}{2\pi}\ln r$$

$$= Ur\sin\theta\left(1 - \frac{a^2}{r^2}\right) + \frac{\Gamma}{2\pi}\ln r$$

2.20 Importance of the analysis of ideal fluid flows in practical situations.

Sources and sinks are convenient mathematical concepts with no exact counterparts in nature, for the source involves continual creation of fluid at a point, the sink involves continual annihilation, and the velocities in the region of the points approach infinite values. However, a radial flow to an outlet, such as an artesian well, may resemble the sink pattern, except in the sink's central region of very high velocity. The chief value of the concept of sources and sinks lies in the fact that, in combination with other simple patterns, they produce more complex patterns which closely resemble flow patterns occurring in nature.

A flow pattern in which the streamlines are concetric circles is known as a circular vortex. If the fluid particles rotate as they revolve around the vortex center, as they do in a rotating cup of water, the vortex is said to be rotational or 'forced'. If the particles do not rotate, the vortex is irrotational or 'free' and it is this type which is now considered. Natural occurrences which approach the condition of irrotational vortex, are the vortex which forms as a container is drained through an orifice in its base and the air vortex known as the tropical hurricane or tornado.

Examples of physical patterns resembling that of the source-sink combination are the unsteady pattern of flow produced by the motion of an elongated body through an initially stationary fluid and the steady pattern of percolation to a pumped well from a 'recharge' well. In the latter case, water is pumped from a well continuously, for use, for example, as cooling water in an industrial plant and then returned to the ground through a neighboiuring well, to recharge the ground water supply. In a confined aquifer, the recharge well resembles the source and the pumped well the sink. However, the amount of recirculation and the pattern of flow may be affected by lack of uniformity of the aquifer or by a superimposed general movement of ground water in the region of the wells.

In flow past a half body branch lines can be regarded together as the solid boundary of a round-nosed body such as an island or bridge pier, which extends downstream to a distance large compared with its width. The upper half of the pattern might be regarded as a plan of the flow adjacent to a side-contraction in a wide channel; or, in elevation, as flow of water over a rising bed or of wind up a hillside. In each case, stagnation occurs in theory at S but, in fact, the central streamline may separate a small distance upstream, with the result that two stagnation vortices are formed near S. Any streamline can be regarded as a solid boundary and the velocity and pressure distributions along it can be determined analytically.

In a doublet the pattern is that of flow issuing from a point, moving initially in the positive direction of the axis, spreading out to flow in the reverse direction and finally returning to the point. This pattern has not exact counterpart in nature, but the unsteady pattern of flow produced by a cylinder moving through an otherwise stationary fluid corresponds to that protion of a doublet pattern which lies outside the cylinder boundary. Moreover, the combination of a doublet with uniform flow yields the pattern of steady flow past a cylinder.

It is appropriate here to consider what is known as the virtual mass of a solid moving through a fluid. The total kinetic energy of the solid, and of the fluid set in motion by it, can be regarded as the kinetic energy of a solid of the same dimensions, but of increased mass, the increase being known as the ‘virtual mass’.

For the cylinder moving with a velocity, U, through a fluid, initially at rest, the fluid velocity at any point has the magnitude,

$$V = \sqrt{u_r^2 + u_\theta^2} = \frac{\mu}{r^2} \quad \frac{a^2}{r^2} U$$

At the instant the center of the cylinder is at the origin, and the fluid at infinity is seen to be still at rest.

The total kinetic energy of the fluid per unit length of cylinder is

$$T^1 = \int_a^\infty \tfrac{1}{2} V^2 \, dM$$

$$= \int_a^\infty \tfrac{1}{2} \rho V^2 2\pi r dr$$

$$T^1 = \rho\pi a^4 U^2 \int_{r=a}^{\infty} \frac{dr}{r^3}$$

$$= \tfrac{1}{2} \rho\pi a^2 U^2$$

i.e., $T^1 = \tfrac{1}{2} M' U^2$

where $M' = \rho\pi a^2$, the mass of fluid with a volume equal to the cylinder volume.

The total kinetic energy of the fluid and cylinder is

$$R = \tfrac{1}{2} (M + M') U^2$$

In irrotational flow in which the velocity potential is everywhere single valued, no motion of the fluid is possible if the fluid is at rest at infinity, the interior boundaries are at rest and there are no singularities. Hence if the body is brought to rest, the whole body of fluid will also come to rest at the same instant. The work expended in stopping the body will be equal therefore to the total kinetic energy of the body and the fluid, T.

Similarly, in accelerating or retarding the cylinder, since the work done equals the change in total kinetic energy, the effective mass to be considered is the actual mass plus the virtual mass, and the additional resistance to accelerative forces is

$$F' = M' \frac{du}{dt}$$

In the case of the real fluid, the irrotational flow conditions do not apply absolutely and there is a time lag between the change in velocity of the body and the attaining of the new equilibrium motion by the fluid. Nevertheless, the virtual mass effect is a physical fact. It is an important factor to be allowed for in the moving and docking of ships.

UNIT V

1 INTRODUCTION TO HIGH TEMPERATURE EFFECTS

1.1 Analysis of Fluid Flow

The analysis of paths of individual fluid particles is referred as ':Langrangian method' of analysis. The method deals with the positions, velocities and accelerations of individual particles, the co-ordinates of particles being variables which are functions of the initial positions of the particles and of time. In the 'Eulerian method' the characteristics at a general point in the flow or for simplicity a section in the fluid is considered. The co-ordinates describe a general point in the flow and they do not vary with time. Instead of considering the variation of velocities and accelerations of particles as they follow their various paths, the velocities and accelerations of particles as they cross the general point is being analysed in this approach. As an example of this method, the most general statement of velocity of a particle in a three dimensional space can be mathematically expressed as

$$u = f_1 (x, y, z, t) \text{ in } x\text{ - direction}$$
$$v = f_2 (x, y, z, t) \text{ in } y\text{ - direction}$$
$$w = f_3 (x, y, z, t) \text{ in } z\text{ - direction}$$

This means that the velocity of the particle 'u' in the x - direction is a function of space co - ordinates x, y, and z and of time t while 'v' the velocity in y - direction is another function of x, y, z and t and 'w' the velocity in z - direction is a third function of x, y, z and t. For a 'steady flow' the velocity at a point is not a function of time.

$$\therefore \quad u = f_1 (x, y, z)$$
$$v = f_2 (x, y, z)$$
$$w = f_3 (x, y, z)$$

The acceleration in the three directions can be obtained as follows.

$$\frac{du}{dt} = u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta w} + \frac{\delta u}{\delta t}$$

$$\frac{dv}{dt} = u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y} + w\frac{\delta v}{\delta w} + \frac{\delta v}{\delta t}$$

$$\frac{dw}{dt} = u\frac{\delta w}{\delta x} + v\frac{\delta w}{\delta y} + w\frac{\delta w}{\delta w} + \frac{\delta w}{\delta t}$$

In the above relation the partial differentials, of u, v, and 'w' with respect to t, $\left(\frac{\delta u}{\delta t}, \frac{\delta v}{\delta t}, \frac{\delta w}{\delta t}\right)$ are called 'local accelerations' and remaining terms on the right hand side are called 'convective accelerations'. For a steady flow, local acceleration is zero.

i.e., $$\frac{\delta u}{\delta t} = \frac{\delta v}{\delta t} = \frac{\delta w}{\delta t} = o$$

1.2 Equation of Continuity

Consider an infinitesimal parallelepiped in the body of a fluid of sides dx, dy and dz. Consider the mass flow per unit time in x-direction. Mass entering through left face is $\rho u \; dy \; dz$ and mass leaving right face is $\left(\rho u + \frac{\delta}{\delta x}(\rho u)\, dx\right) dy \; dz$ where ρ is the mass density of fluid.

Net gain of mass in x-direction is $\frac{\delta}{\delta x}(\rho u)$ dx, dy, dz.

Similarly considering y and z directions of the parallelepiped, net mass efflux

$$= \frac{\delta}{\delta x}(\rho u)\, dx \; dy \; dz + \frac{\delta}{\delta y}(\rho v)\, dx \; dy \; dz + \frac{\delta}{\delta z}(\rho w)\, dx \; dy \; dz$$

i.e., $$= \left(\frac{\delta}{\delta x}(\rho u) + \frac{\delta}{\delta y}(\rho v) + \frac{\delta}{\delta z}(\rho w)\right) dx \; dy \; dz$$

In the above case only space variation is considered. In a most general approach the time gain also is to be taken into account.

$\therefore$ The above relation modifies to

$$\left(\frac{\delta}{\delta x}(\rho\, u) + \frac{\delta}{\delta y}(\rho v) + \frac{\delta}{\delta z}(\rho w)\right) dx\ dy\ dz + \frac{\delta}{\delta t}(\rho)\ dx\ dy\ dz.$$

Net mass efflux must be zero as there is no creation of mass within the element.

$$\therefore \left(\frac{\delta}{\delta x}(\rho u) + \frac{\delta}{\delta y}(\rho v) + \frac{\delta}{\delta z}(\rho w) + \frac{\delta}{\delta t}(\rho)\right) dx\ dy\ dz = 0$$

$$\frac{\delta}{\delta x}(\rho\, u) + \frac{\delta}{\delta y}(\rho v) + \frac{\delta}{\delta z}(\rho w) + \frac{\delta}{\delta t}(\rho) = 0 \qquad 1.5$$

This is the general equation of continuity for a fluid in motion.

If the fluid is incompressible 'ρ' remains constant with respect to space and time. Therefore the equation simplifies to

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0 \qquad 1.6$$

This is applicable to both steady and unsteady flow.

For a steady flow of a compressible fluid the equation 1.5 can be simplified by assuming $\frac{\delta \rho}{\delta t} = 0$. The equation becomes

$$u\frac{\delta \rho}{\delta x} + v\frac{\delta \rho}{\delta y} + w\frac{\delta \rho}{\delta z} + \rho\left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}\right) = 0 \qquad 1.7$$

1.3 Euler's Hydrodynamic Equations

Assumption made in this derivation are

i) fluid is compressible

ii) flow is frictionless i.e., flow is non-viscous.

iii) flow is irrotational and unsteady.

iv) body forces acting on the fluid particle remain constant.

In a three dimensional space consider a fluid particle. Let the components of the body forces such as gravity force or magnetic force in the three directions be X, Y and Z respectively per unit mass of fluid. The force acting in the x-direction pdydz from left side and

$$\left(p + \frac{\delta p}{\delta x} \right) \quad \text{dydz from right side.}$$

$\therefore$ Net force acting in the x-direction $= - \dfrac{\delta p}{\delta x}$ dxdydz ___ (i)

Mass of the parallelepiped under consideration $= \rho$ dxdydz

By Newton's second law of motion, force = mass . acceleration

$$= \rho \ dxdydz \ \frac{du}{dt}$$

Equating (i) and (ii) and expanding $\dfrac{du}{dt}$ by using 1.1 and simplifying,

$$-\frac{1}{\rho}\frac{\delta p}{\delta x} = u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta z} + \frac{\delta u}{\delta t}$$

Total force acting in x – direction taking into account of the body force also.

$$X - \frac{1}{\rho}\frac{\delta p}{\delta x} = u\frac{\delta u}{\delta x} + v\frac{\delta u}{\delta y} + w\frac{\delta u}{\delta z} + \frac{\delta u}{\delta t} \qquad 1.8$$

Similarly in y and z directions

$$Y - \frac{1}{\rho}\frac{\delta \rho}{\delta x} = u\frac{\delta v}{\delta x} + v\frac{\delta v}{\delta y} + w\frac{\delta v}{\delta z} + \frac{\delta v}{\delta t} \qquad 1.9$$

$$Z - \frac{1}{\rho}\frac{\delta \rho}{\delta x} = u\frac{\delta w}{\delta x} + v\frac{\delta w}{\delta y} + w\frac{\delta w}{\delta z} + \frac{\delta w}{\delta t} \qquad 1.10$$

1.8, 1.9, 1.10 are called Euler's hydrodynamic equations in three dimensional space.

1.4 Navier - Stoke's Equations

Euler's equations derived above are modified, taking into account the effect of viscosity of the fluid also in these equations

$$\frac{1}{\rho}\quad\frac{\delta p}{\delta x}\qquad\frac{\mu}{\rho}\left(\frac{\delta^2 u}{\delta x^2}\qquad\frac{\delta^2 u}{\delta v^2}\qquad\right)$$

$$X - \quad + \quad + \quad + \frac{\delta^2 u}{\delta z^2}$$

1.11

$$= u \frac{\delta u}{\delta x} + v \frac{\delta u}{\delta v} + w \frac{\delta u}{\delta z} + \frac{\delta u}{\delta t} \qquad 1.12$$

$$Y - \frac{1}{\rho} \frac{\delta p}{\delta x} + \frac{\mu}{\rho} \left(\frac{\delta^2 v}{\delta x^2} + \frac{\delta^2 v}{\delta y^2} + \frac{\delta^2 v}{\delta z^2} \right)$$

$$+ u \frac{\delta u}{\delta x} + v \frac{\delta v}{\delta v} + w \frac{\delta v}{\delta z} + \frac{\delta v}{\delta t} \qquad 1.12$$

$$Z - \frac{1}{\rho} \frac{\delta p}{\delta x} + \frac{\mu}{\rho} \left(\frac{\delta^2 w}{\delta x^2} + \frac{\delta^2 w}{\delta v^2} + \frac{\delta^2 w}{\delta z^2} \right)$$

$$= u \frac{\delta u}{\delta x} + v \frac{\delta w}{\delta v} + w \frac{\delta w}{\delta z} \quad \frac{\delta w}{\delta t} \qquad 1.13$$

1.5 Equation of State

This is a well known relation pertaining to the variation of the density with the changes of pressure and temperature. For a perfect gass we know pv = RT for unit weight of the fluid. And also $v = 1/\rho$

$$\therefore \quad P = \rho \ R \ T \text{ absolute units}$$

or $$P = g \ \rho \ R \ T \text{ gravitational units}$$

This equation can be written giving suffixes.

$$\frac{\rho_2}{\rho_1} = \frac{\rho_2}{\rho_1} \cdot \frac{T_1}{T_2}$$

1.6 Momentum Theory

This is another form of the Newtonian analysis of force. Newton's second law of motion states that the rate of change of momentum is directly proportional to the external impulsive force. Both the momentum and impulse are vector quantities. Therefore considering one dimensional analysis of a flow of fluid the momentum equation in x-direction can be written as

$$F_x dt = d \ (MV)_x$$

Mass of the fluid in the elemental length 'ds' $= \rho . \ dA. \ ds$

Let 'dF_x' be the differential force acting on an elemental length 'ds' of a streamtube of cross section 'dA'

$$\therefore \; dF_x\, dt = d\,(\rho \; dA \; ds \; V)_x$$

For a steady flow of an incompressible fluid, the above equation can be rewritten in the following form.

$$dF_x dt = \rho \, dA \; ds \; \frac{dV_x}{ds} \, ds$$

$$= \rho \, dA \, ds \; \frac{dV_x}{ds} \, V dt$$

or $$dF_x = \rho dA \; ds \; \frac{dV_x}{ds} \, V$$

$$= \rho \, ds \frac{dV_x}{ds} \, dQ$$

where 'dQ' stands for the rate of flow given by 'VdA'

For a steady flow 'dQ' is constant. Hence integrating above equation between two points '1' and '2' in the streamtube.

we get, $$\Sigma F_x = \rho \int \left[(V_x)_1 - (V_x)_2 \right] dQ$$

Here 'ΣF_x' stands for the sum of all the impulsive forces per unit time due to all the forces acting in x-direction and the expression on the right hand side stands for the resulting rate of change in the x-component of its momentum. Similar equations can be formed in the case of two dimensional analysis of the flow of fluid.

1.7 Energy Equation

This is another name given to the law of conservation of energy. The equation is derived

by the direct application of the law of conservation of energy to a fluid flowing through a duct as shown in the fig. 3

At the section '1' the total force acting is 'p_1A_1'. Assume a small interval of time 'dt'. This section moves a short distance 'ds_1' making to do an external work on the moving fluid, given by the force multiplied by the distance moved. $= p_1A_1 \,.\, ds_1$

Similarly at section '2' external work done by the force in moving the fluid through ds_2 is '$p_1A_2 . ds_2$'

As both the force and displacement in section '2' are in opposite directions to the force and displacement in section '1' vectorially the work done by the pressure force at section '2' equals ($--p_2 A_2 \quad ds_2$). For moving the fluid from section '1' to section '2' work must be done on the fluid by the some external agency (say by a prime-mover) and this work done on the fluid adds mechanical energy to it. This energy is denoted by 'M' work units per unit weight of the fluid moving. Consider also an amount of heat 'Q' heat units per units weight of the fluid moving transferred from the fluid to the surroundings. 'Q' will be negative if the transfer of energy is into the fluid. The weight of the fluid entering the pipe and leaving the pipe during the interval of time is the same, as there is steady flow through the pipe.

$$\therefore \; w_1. \; A_1. \; ds_1 \;\; = w_2. \; A_2. \; ds_2$$

The total energy entering the pipe during an interval of time 'dt' is given by the weight of fluid entering during the interval times the sum of the potential energy, kinetic energy and internal energy; so too the total energy leaving. The net change of energy of flowing fluid is given by,

$$w_1A_1ds_1 \left\{ (Z_2 - Z_1) + \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) + J \; (U_2 - U_1) \right\}$$

Here $(Z_2 - Z_1)$ is for net potential energy,

$\left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right)$ is for net kinetic energy,

and $J \; (U_2 - U_1)$ is for net internal energy, all being reckoned in work units.
The net external work done by fluid and transfer of thermal energy

$$* \; w_1A_1ds_1 \left\{ \frac{p_1}{w_1} - \frac{p_2}{w_2} + (M\text{-}JQ) \right\} \quad \text{work units}$$

Now according to the fundamental law of conservation of energy, equating the work done plus transfer of thermal energy to net change of energy of flowing fluid,

$$w_1A_1ds_1 \quad \left\{ \frac{p_1}{w_1} - \frac{p_2}{w_2} + (M\text{-}JQ) \right\} =$$

$$w_1A_1ds_1 \quad \left\{ (Z2 - Z1) + \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g} \right) + J \; (U_2 - U_1) \right\}$$

Simplifying ,

$$\frac{p_1}{w_1} - \frac{p_2}{w_2} + M - JQ = (Z_2 - Z_1) + \left(\frac{V_2^2}{2g} \quad \frac{V_1^2}{2g} \right) + J(U_2 - U_1)$$

This is the general energy equation for steady flow of any fluid.

* w : is taken as the specific weight of fluid

1.8 Bernoulli's Theorem

This states that the sum of the potential energy, pressure energy and kinetic energy of a continuously flowing fluid in a stream tube remains content at any section in its flow. In the case of gases potential energy will be practically negligible compared to the other two as the density in very low (i.e., for unit volume $\rho\ g\ \ h \cong O$ since $\rho \cong O$) Hence in the following analysis potential energy is not taken into account.

In the fig.4 a small element of length 'ds' of the fluid is considered in a streamtube of varying cross-sectional area. Since the elemental strip has a very small length, the cross section 'A' of the stream tube at the section considered can be assumed to be uniform for the length 'ds'. Assume that diameter of the tube is so small that there is no appreciable change of pressure in the section considered. Let the uniform intensity of pressure acting from left be 'p' and (p + dp) from right. Let 'V' be the velocity of the stremlines on the elemental strip. Due to the diference of pressure acting on the strip it must accelerate from 'V' to (V + dV) to right. As we have assumed the case of a gas the weight of the moving particles can be neglected.

Mass of fluid flowing per second = volume per sec. mass density

$$= A\ \ ds\ .\ \rho$$

$$\text{Acceleration} \quad = \frac{(V + dV)\ - V}{dt}$$

$$= \frac{dV}{dt}$$

Unbalanced external force is caused by the difference of pressure on left and right of the elemental strip.

$$\therefore\ \text{Unbalanced force} \quad = \left\{\ p\ -\ (p + dp)\ \right\}\ A$$

$$= -\ A\ \ dp$$

From Newton's second law, the unbalanced external force is equal to the product of mass and acceleration.

$$\therefore \qquad - A \; dp = \rho A \; ds \; . \; \frac{dV}{dt}$$

$$\text{i.e.,} \qquad \frac{-dp}{\rho} \qquad \frac{dV}{dt} \, ds$$

$$\text{But,} \qquad \frac{ds}{dt} = V$$

$$\therefore \qquad \frac{dp}{\rho} = V \; dV$$

$$\left.\begin{aligned} &\text{or} \qquad V \; dV + \frac{dp}{\rho} = O \\ &\text{On integrating this,} \\ &\frac{V^2}{2} + \int \frac{dp}{\rho} = \text{constant} \end{aligned}\right\} \qquad 1.14$$

<u>Case 1. Incompressible fluid</u>

Consider incompressible fluid for which we know 'ρ' is constant.

$$\therefore \quad \text{Form 1.14,} \qquad \rho \frac{V^2}{2} + p = \text{constant} \qquad 1.15$$

$\rho \dfrac{V^2}{2}$ stands for the kinetic energy known also as "reference pressure" or "dynamic pressure" and 'p' stands for the pressure energy of unit volume of the fluid.

<u>Case 2. Compressible fluid</u>

For a compressible fluid, the mass density is a variable depending on the variations of pressure and temperature. Assume reversible adiabatic process.

$$pv^{\gamma} = \text{Constant} = C$$

$$\text{i.e.,} \qquad \left\{\frac{C}{p}\right\}^{\frac{1}{\gamma}} = \frac{1}{\rho} \qquad 1.16$$

In this case the integration of the terms of 1.14 is carried out taking the variations of 'ρ' also into consideration Substituting 1.16 in 1.14

$$\frac{V^2}{2} + \int \left\{\frac{C}{p}\right\}^{\frac{1}{\gamma}} dp = \text{constant}$$

Integrating this,

$$\frac{V^2}{2} + C^{\frac{1}{\gamma}} \left\{ \frac{p^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} \right\} = \text{Constant}$$

Simplifying and substituting for $C^{\frac{1}{\gamma}}$ from 1.16.

$$\frac{V^2}{2} + \left\{ \frac{\gamma}{\gamma - 1} \right\} p \left(\frac{C}{p} \right) = \text{constant}$$

i.e., $$\frac{V^2}{2} + \frac{\gamma}{\gamma - 1} \frac{p}{\rho} = \text{constant} \qquad 1.17$$

1.9 Flow Analysis : One Dimensional Approximation

All fluid properties vary or depend up on one dimension, the distance along the flow. In this type of analysis variations of properties of the flow of fluid normal to the direction of flow are neglected. So long as the rate of change of properties are not much or changes are not rapid, one dimensional approach is accurate.

1.9.1 Equation of Continuity

Consider x-direction of fluid flow. For mass density ρ, velocity U and area of cross section A

$$\rho UA = \text{constant}$$

Differencing $d\rho UA + \rho\partial UA + \rho U\partial A = 0$

Dividing by ρUA $$\frac{\partial\rho}{\rho} + \frac{\partial U}{U} + \frac{\partial A}{A} = 0 \qquad 1.18$$

- For incompressible flow $\frac{\partial\rho}{\rho} = 0$

$$\therefore \quad \frac{\partial U}{U} + \frac{\partial A}{A} = 0 \qquad 1.19$$

- This is true only when the speed is not high i.e., in subsonic flow.

1.9.2 Continuity Equation in Unsteady Flow

Referring fig. 5, the rate of fluid entering minus rate of fluid leaving the stream tube is equal to the rate of mass accumulation in the stream tube.

$$\frac{\partial}{\partial s}(\rho UA)\ ds = -\frac{\partial}{\partial t}(\rho\,\partial s\,A) \qquad \text{as } \frac{\partial s}{\partial t} = U$$

$$\frac{\partial}{\partial s}(\rho UA) = -\frac{\partial}{\partial t}(\rho A)$$

$$\frac{\partial}{\partial s}(\rho UA + \frac{\partial}{\partial t}(\rho A) = 0 \qquad 1.20$$

When the flow is steady second term vanishes and so $\frac{\partial}{\partial s}(\rho UA) = 0$

i.e., ρUA = constant 1.20 a

1.9.3 Equation of Motion Along a Stream Line

Let velocity "v" of fluid along a stream line at the position is the space say defined by "s" at time "t".

$\therefore$ v = function of s and t

$$v = v(s,t)$$

$$dv = \frac{\partial v}{\partial s}ds + \frac{\partial v}{\partial t}dt$$

Rearranging $\frac{dv}{dt} = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}$

$$= v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}$$

As explained above first term is convective acceleration and second term is local acceleration.

Consider a stream line in the fluid flow as shown is fig 6. The forces on this fluid element ds are its weight and pressure forces. By Newton's second law, mass times acceleration must be equal to the impressed forces. Note that changes along stream line is being considered.

$$(\rho A ds)\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s}\right) = -\left(A\frac{\partial p}{\partial s}ds + \rho g A\, ds.\cos\theta\right)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s} = -\left(\frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds}\right)$$

Rewritting,

$$\frac{1}{\rho}$$

$$\frac{\partial v}{\partial t} ds + v\partial v + \frac{\quad}{\quad} \partial p + gdz = 0$$

or $$\frac{\partial v}{\partial t} ds + \partial \left(\frac{v^2}{2} + \frac{p}{\rho} + gz \right) = 0$$

For steady flow

$$\partial \left(\frac{v^2}{2} + \frac{p}{\rho} + gz \right) = 0$$

Along stream line, $$\frac{v^2}{2} + \frac{p}{\rho} + gz = \text{constant} \qquad 1.21$$

Writing this with terms of pressure,

$$\rho \frac{v^2}{2} + p + \rho gz = \text{constant say , B} \qquad 1.22$$

The constant 'B' is referred as Bernoullis constant.

Writing this with terms of energy of fluid per unit mass,

$$\frac{v^2}{2} + \frac{p}{\rho} + gz = \text{constant} \qquad 1.23$$

Writing this with terms of height (head)

$$\frac{v^2}{2g} + \frac{p}{w} + z = \text{Constant} \qquad 1.24$$

Assumptions of above analysis :

i. The fluid element is only acted upon by the forces due to normal pressure on the ends and self weight of element, any forces on the sides of element is ignored. This shows that there is no shear forces for the fluid and hence the effect of viscosity is neglected.

   For non-viscous, or frictionless flow the Bernoullis equation is valid i.e., ideal fluid flow.

ii. As the particle is considered moving along the same stream line, the flow must be steady.

iii. As the change of density is not accounted and hence the fluid is incompressible.

iv. Bernoullis equation is applied to two adjacent stream lines when there is no heat transfer between them.

v. Benoullis equation is applicable to irrotational flow field.

(For the proof of 4th and 5th assumptions refer worked out problems)

Hence Bernoullis equation is only applicable when the flow is steady frictionless, incompressible, irrotational and without heat transfer.

UNIT I

FUNDAMENTALS OF HYPERSONIC AERODYNAMICS

- ✓ Introduction to hypersonic aerodynamics.
- ✓ Differences between hypersonic aerodynamics and supersonic aerodynamics.
- ✓ Concept of thin shock layers.
- ✓ Hypersonic flight paths.
- ✓ Hypersonic Similarity parameters.
- ✓ Shock wave and expansion wave relations of inviscid hypersonic flows.

Introduction to hypersonic aerodynamics:

The development of aeronautics and spaceflight, from its practical beginnings with the Wright Brothers' first airplane flight on 17 December 1903 and Robert H. Goddard's first liquid-fueled rocket launch on 16 March 1926, has been driven by one primary urge—the urge to always fly faster and higher. Anyone who has traced advancements in aircraft in the 20th century has seen an exponential growth in both speed and altitude, starting with the 35-mph Wright flyer at sea level in 1903, progressing to 400-mph fighters at 30,000 ft in World War II, transitioning to 1200-mph supersonic aircraft at 60,000 ft in the 1960s and 1970s, highlighted by the experimental X-15 hypersonic airplane, which achieved Mach 7 and an altitude of 354,200 ft on 22 August 1963, and finally capped by the space shuttle—the ultimate in manned airplanes with its Mach 25 reentry into the Earth's atmosphere from a 200-mile low Earth orbit. (See [1] for graphs which demonstrate the exponential increase in both aircraft speed and altitude over the past 100 years.) Superimposed on this picture is the advent of high-speed missiles and spacecraft: for example, the development of the Mach 25 intercontinental ballistic missile in the 1950s; the Mach 25 Mercury, Gemini, and Vostok manned orbital spacecraft of the 1960; and of course the historic Mach 36 Apollo spacecraft, which returned men from the moon starting in 1969. The point here is that the extreme high-speed end of the flight spectrum has been explored, penetrated, and utilized since the 1950s. Moreover, flight at this end of the spectrum is called hypersonic flight, and the aerodynamic and gas dynamic characteristics of such flight are classified under the label of hypersonic aerodynamics.

The hypersonic flight regime includes atmospheric entry and re-entry, ground testing, and flight for both powered and unpowered vehicles. In the present Lecture Series, the main interest is on sustained and controlled hypersonic flight, whether for military or civil transport application. Even though it is not currently certified for flight, there is one operational hypersonic vehicle: the space shuttle of NASA. At least 20 years before the development of the shuttle a significant activity in hypersonic flight research was conducted by the US Air Force in their X-15 program. This vehicle has reached a flight Mach number of 6.7 on its final flight, which also used to test a hypersonic ramjet engine.

Differences between hypersonic aerodynamics and supersonic aerodynamics

Regime	(Mach number)	(mph)	(km/h)	(m/s)	General plane characteristics
Subsonic	<0.8	<614	<988	<274	Most often propeller-driven and commercial turbofan aircraft with high aspect-ratio (slender) wings, and rounded features like the nose and leading edges.
Transonic	0.8–1.2	614–921	988–1,482	274–412	Transonic aircraft nearly always have swept wings that delay drag-divergence, and often feature designs adhering to the principles of the Whitcomb area rule.
Supersonic	1.2–5.0	921–3,836	1,482–6,174	412–1,715	Aircraft designed to fly at supersonic speeds show large differences in their aerodynamic design because of the radical differences in the behavior of fluid flows above Mach 1. Sharp edges, thin airfoil-sections, and all-moving tail plane/canards are common. Modern combat aircraft must compromise in order to maintain low-speed handling; "true" supersonic designs include the F-104 Starfighter and BAC/Aerospatiale Concorde.
Hypersonic	5.0–10.0	3,836–7,673	6,174–12,348	1,715–3,430	Cooled nickel or titanium skin; highly integrated (due to domination of interference effects: non-linear behaviour means that superposition of results for separate components is invalid)[clarification needed], small wings, see X-51A Waverider, HyperSoar and WU-14 (DF-ZF).

High-hypersonic	10.0–25.0	7,673–19,182	12,348–30,870	3,430–8,575	Thermal control becomes a dominant design consideration. Structure must either be designed to operate hot, or be protected by special silicate tiles or similar. Chemically reacting flow can also cause corrosion of the vehicle's skin, with free-atomic oxygenfeaturing in very high-speed flows. Examples include the 53T6 *ABM-3 Gazelle* (Mach 17) anti-ballistic missile, the DF-41 (Mach 25) intercontinental ballistic missile and the Russian Avangard hypersonic vehicle (Mach 20). Hypersonic designs are often forced into blunt configurations because of the aerodynamic heating rising with a reduced radius of curvature.
Re-entry speeds	>25.0	>19,181.7	>30,869.95	>8,575	Ablative heat shield; small or no wings; blunt shape.

Concept of thin shock layers:

Thin Shock Layer Theory Thin shock layer theory is based on the assumption that the shock is very much closer to the body which in turn leads to small volume between shock and body. This situation is typical of very high Mach number flows over generic hypersonic configurations. In such situations, we can assume that, $M\infty \rightarrow \infty$ and$\gamma \rightarrow 1$. As it has been already observed that the shock angle and Mach angle are almost equal for hypersonic flow regime, we can express this fact as β θ → . For such high Mach condition within the shock layer, we will have same equation for shock, body and any streamline in the shock. This is the basic assumption of thin shock layer theory. Consider the body and the shock as shown in Fig. 19.1. Here the co-ordinate system is such that *x* axis is parallel to the shock while *y* axis is perpendicular to the shock. Let *u* and *v* be the components of velocity in the x and y directions respectively. Let us assume the flow to be two-dimensional flow for the present illustration.

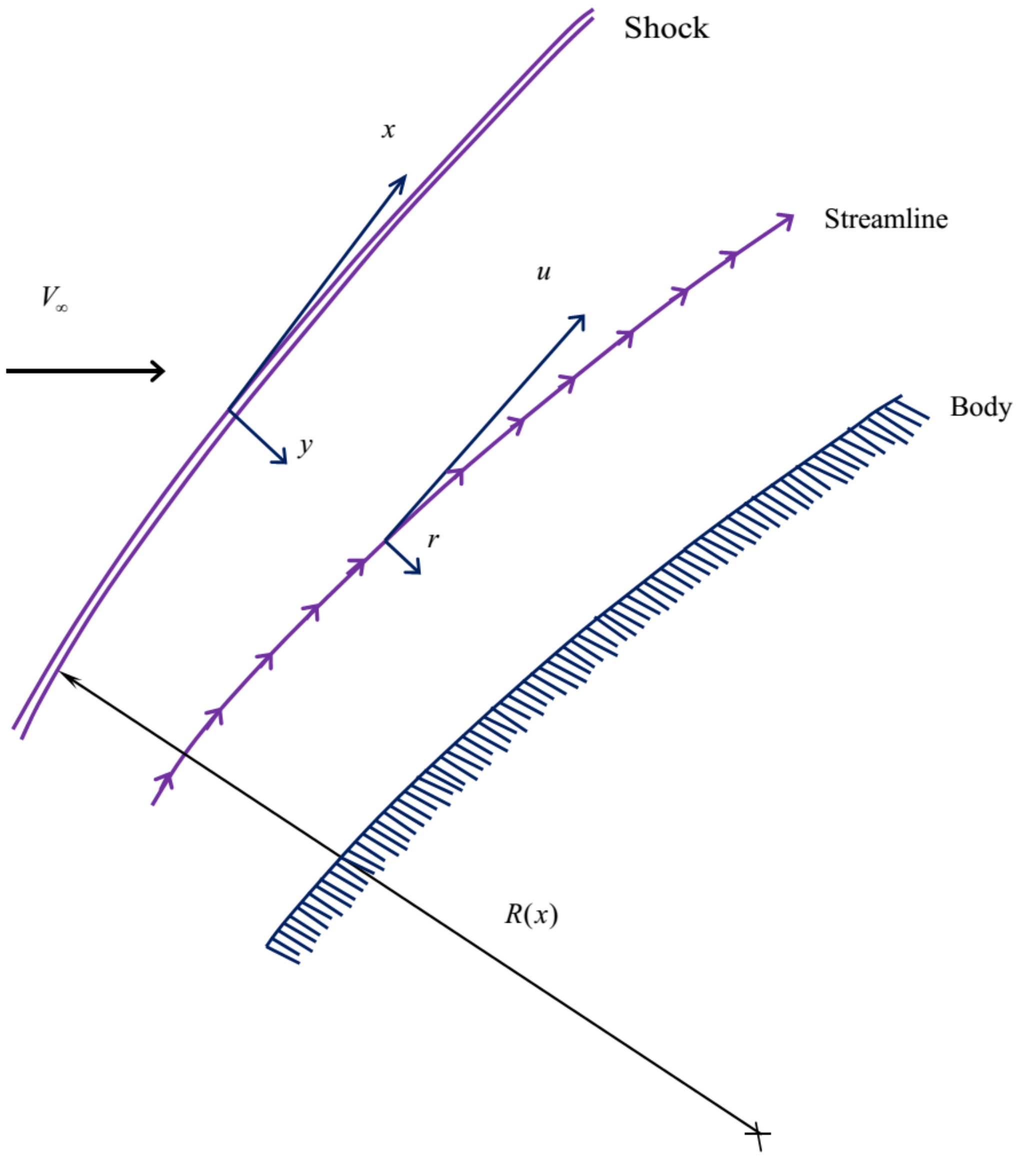

Fig. 19.1: Illustration for thin shock layer theory [1]

The momentum equation for the present coordinate system is

2

u p

R y

ρ ∂ = ∂ Since our assumptions include thin shock layer and same equation for shock, streamlines and body. Here, R is the local streamline radius of curvature. For the thin shock-layer assumptions,

UNIT II

SIMPLE SOLUTION METHODS FOR HYPERSONICINVISCID FLOWS

1. Show that potential lines are concentric circles and stream lines are radial lines for a source

Potential lines, ϕ = constant

$$= \frac{Q}{2\pi} \ln r = \frac{Q}{2\pi} \ln (x^2 + y^2)$$

$$\therefore \frac{\text{Constant}.4\pi}{Q} = \ln (x^2 + y^2)$$

$$x^2 + y^2 = e^{\text{constant } 4\pi/Q}$$

$$= \text{another constant}$$

$\therefore$ Potential lines are concentric circles centre at origin if the source is at the origin, (0,0) and radius is square root of $e^{\text{constant } 4\pi/Q}$.

Stream lines, ψ = constant

$$= \frac{Q}{2\pi} \theta = \frac{Q}{2\pi} \tan^{-1} (y/x)$$

$$\tan\left(\frac{2\pi \text{ constant}}{Q}\right) = \frac{y}{x}$$

$$\therefore \quad y = x \tan\left(\frac{2\pi \text{ constant}}{Q}\right)$$

This is of the form y = mx + o. which is the equation for straight line through origin. If the source is at origin, the slope of the line being tan (2π constant / Q).

2**. Show that potential lines are radial lines and stream lines are concentric circles for a vortex.**

It has been already stated that potential lines of source are stream lines of vortex and stream lines of source are potential lines of vortex. Refer the previous problem for proof.

3. Show that both stream lines and potential lines are circles in the case of source-sink pair of same strength.

A source at -a and sink at +a from origin form a source-sink pair of same strength.

Stream lines, $\psi = \psi_1 + (-\psi_2) = \psi_1 - \psi_2 = \text{constant}$

$$= \frac{-Q}{2\pi}(\theta_1 - \theta_2) = \frac{Q}{2\pi}(\theta_2 - \theta_1) = \frac{Q}{2\pi}\theta$$

The locus of any point satisfying this condition is a circular are (see figure). The chord of length 2a subtends θ on the circular arc. Therefore stream lines are circles with common chord of length 2a and radius of a cosec $\psi/(Q/2\pi)$. The center is located at $[0, -a \cot \psi/(Q/2\pi)]$

Potential lines, $\phi = \phi_1 + (-\phi_2) = \frac{Q}{2\pi}\ln r_1 - \frac{Q}{2\pi}\ln r_2$

$$= \frac{Q}{2\pi}\ln\left(\frac{r_1}{r_1}\right) = \text{constant}$$

Equipotential lines are also circles whose radii can be shown to be equal to a cosech $\phi/(Q/2\pi)$ with centre at $\left(a \coth \phi/(Q/2\pi), 0\right)$

4. Show that stream lines are circles tangential to x-axis and potential lines are circles tangential to y-axis for a doublet at origin.

Stream lines of doublet $-\frac{C\sin\theta}{r} = \text{constant.}$

For any point P (r, θ) on the circle, the diameter of circle is $r/\sin\theta$. i.e., the circles are tangential to x-axis.

$\therefore$ Stream lines of doublet are circle with centers $(0, r/2\sin\theta)$ which is on y-axis and tangential to x-axis.

Potential lines of doublet $\frac{C\cos\theta}{r} = \text{constant.}$

Any point P (r, θ) on the circle the diameter of circle is $r/\cos\theta$ i.e., the circle are tangential to y-axis.

$\therefore$ Potential lines of doublet are circles with centers (r/2 cosθ, 0) which are on x-axis and tangential to y-axis.

Note : Sketch the stream and potential lines of the complex potential w $\frac{\mu}{z}$

$$w = \frac{\mu}{z} = \mu\,(re^{i\theta})^{-1}$$

$$= \frac{\mu}{r}\; e^{i\theta} = \frac{\mu}{r}\;(\cos\theta - i\sin\theta)$$

$$= \phi + i\,\psi$$

$$\therefore\; \phi = \text{constant} = \frac{\mu}{r}\cos\theta$$

$$\psi = \text{constant} = -\frac{\mu}{r}\sin\theta$$

Potential lines are circles with centers $\left(\frac{r}{2\cos\theta}, 0\right)$ and radii $\frac{r}{2\cos\theta}$. The circles have the centre on x-aixs and circles are tangential to y-axis.

Stream lines are circles with centres $\left(0, \frac{r}{2\sin\theta}\right)$ and radii $\frac{r}{2\sin\theta}$. The circles have the centers as y-axis and circles are tangential to x-axis.

5. A spiral vortex has potential and stream lines of equiangular spirals — prove.

A combination of source and vortex of same strength at the origin produces the pattern of outward spiral flow

$$\text{Resultant}\;\; \phi = \frac{Q}{2\pi}\ln r + \frac{\Gamma}{2\pi}\theta$$

$$\text{and}\;\; \psi = \frac{Q}{2\pi}\theta - \frac{\Gamma}{2\pi}\ln r$$

For ψ = constant

$$\therefore\; \ln r = \frac{Q\theta/2\pi - \text{constant}}{\Gamma/2\pi}$$

$$r = e^{\frac{Q}{2\pi}(\theta - \text{Constant})\,/\left(\frac{\Gamma}{2\pi}\right)}$$

$$= C_1 e^{\frac{Q\theta}{\Gamma}}$$

This is equation for equiangular spiral. By similar method potential lines are also can be proved to be equiangular spirals.

6. Show that source - sink pair with uniform flow will generate a symmetrical oval by the dividing stream line.

The stream function of source sink pair at -a and + a from origin with a uniform flow parallel to x-axis is shown already as

$$\therefore \quad \psi = \frac{Q}{2\pi} \tan^{-1}\left(\frac{-2ay}{x^2 + y^2 - a^2}\right) + Uy \qquad \text{(i)}$$

Dividing stream line is $\psi = 0$ i.e., only when y = 0

$$\therefore \quad 0 = \frac{Q}{2\pi} \tan\text{-}1\left(\frac{-2ay}{x^2 + y^2 - a^2}\right) + Uy$$

$$\text{i.e.,} \quad \tan\left(\frac{Uy2\pi}{Q}\right) = \frac{2ay}{x^2 + y^2 - a^2}$$

$$\cot\left(\frac{Uy2\pi}{Q}\right) = \frac{x^2 + y^1 - a^2}{2ay}$$

$$x^2 + y^2 = a^2 + 2ay \cot\left(\frac{2\pi Uy}{Q}\right)$$

$$\frac{x^2 + y^2}{a^2} = 1 + \frac{2y}{a} \cot\left(\frac{2\pi Uy}{Q}\right) \qquad \text{(ii)}$$

The right hand side of the above equation is always positive even for negative value of y, as cot (-y) . (-y) = +ve and

cot (+y) . (+y) = + ve

$\therefore$ The oval is symmetric about y = 0 i.e., x-axis.

This equation (ii) represents equation of oval, the stagnation points determines the major axis .

Note : Pattern of flow past Rankine oval become flow past a cylinder when source sink-pair becomes doublet.

7. Show that for a doublet with uniform flow the dividing stream line is a circle at origin.

The stream function of a doublet at origin with uniform flow parallel to x-axis is

$$\psi = \frac{-Cy}{x^2 + y^2} + Uy$$

$$= -C \frac{r \sin\theta}{r^2} + Ur \sin\theta$$

$$= U\left(r - \frac{C}{Ur}\right) \sin\theta$$

$$= U\left(r - \frac{a^2}{r}\right) \sin\theta$$

where $a = \sqrt{C/U}$; When $\psi = 0$ $\sin\theta = 0$ only when $\theta = 0$ or π

$$\therefore r \frac{a^2}{r} = 0 ; \quad r^2 = a^2 \quad \text{i.e.,} \quad x^2 + y^2 = a^2$$

This is evidently a circle with centre at origin and radius of a. Hence the dividing stream line $\psi = 0$ will generate a circle at (0, 0) and radius of $\sqrt{C/U}$.

8. Why there should have a minimum circulation strength for (a) a cylinder (b) for aerofoils. What are the limits of circulation strength.

a) Cylinder

At stagnation point $u_r = u_\theta = 0$; $-2U \sin\theta + \dfrac{\Gamma}{2\pi a} = 0$; $\sin\theta = \dfrac{+\Gamma}{4\pi aU}$

i. For no circulation $\Gamma = 0$, $\sin\theta = 0$ $\therefore$ $\theta = 0$ or π There are two stagnation points. (a, 0) and (-a, 0) where a is the radius of cylinder.

ii. When $\sin\theta$ is negative, $\Gamma < 4\pi aU$. There can have two stagnation points on cylinder depending on the above relation.

iii. When $\Gamma = -4\pi aU$ i.e., when $\sin\theta = -1$ i.e., when $\theta = 3\pi/2$ and stagnation point is single only (-a, 0).

iv. When $\Gamma > 4\pi aU$ (which is only of physical interest) position stagnation point will never be on the cylinder. It will be in the fluid. This causes the fluid motion in the space between cylinder and stagnation point.

<u>b) Aerofoil</u>

The magnitude of the lift is $\rho U\Gamma$. This relationship, known as the Kutta-Joukowki law, can be shown to apply not only to circular cylinders and to aerofoils but to any form in two-dimensional irrotational flow. In the case of a real fluid, surface resistance and separation may produce effects markedly different from that predicted by the law but, for streamlined profiles, it is in fair agreement with experimental determinations for angles of attack up to about 10°. In fact this law forms the basis of the <u>circulation theory of lift</u> on aeroplane wings, the thrust of fan and propeller blades, and the transverse forces on unsymmetrical solid bodies, and on rotating balls and cylinders moving through a fluid. The principal problem in the application of law is the determination of the appropriate value for the circulation, Γ. In the case of the aerofoil, it is that value of Γ which makes the trailing edge a stagnation point.

9. Show that the slope of lift curve of a symmetrical aerofoil may be approximated to 2π assuming Kutta condition of flow.

Kutta condition states that the circulation developed on an aerofoil so that the stream lines at the trailing edge is tangential to the aerofoil Γ equal to π x chord x free stream velocity times $\sin\alpha$ where α is the angle of attack.

The magnitude of lift force is $\rho U\Gamma$ as per Kutta - Joukowski theorem per unit length.

Lift force $L = \rho U \Gamma$ x length

$= \rho U (\pi \times c \times U \times \sin\alpha)$ length

$$C_L \frac{\rho}{2} \text{ area} \times U^2 = \rho U \pi c U \sin\alpha \times \text{length}$$

$$C_L \frac{\rho}{2} \times \text{length} \times U^2 = \rho U^2 \pi \rho \sin\alpha \times \text{length}$$

$$C_L = 2\pi \sin\alpha$$

C_L

α

For small value of α, the lift curve slope becomes $= \frac{}{} = 2\pi$ as $\sin\alpha \approx \alpha^c$.

Note : This result will be shown in the thin aerofoil theory.

10. Briefly explain the vector velocity magnitude and direction in the following ideal fluid flow.

- **i. Uniform flow along x-aixs.**
- **ii. Source at origin**
- **iii. Vortex at origin**
- **iv. Doublet at origin.**

It z is a complex variable $w = f(z)$; $\frac{dw}{dz} = u - iv$; $\left|\frac{dw}{dz}\right| = \sqrt{u^2+v^2} = V$; and $\arg\left|\frac{dw}{dz}\right| = \tan^{-1}\left(\frac{-v}{u}\right) = \pi - \tan^{-1}\left(\frac{v}{u}\right) = \pi - \delta$ where δ is $\tan^{-1}\left(\frac{v}{u}\right)$ which the velocity vector V makes with positive x-axis.

$\therefore \frac{dw}{dz} = V e^{i(\pi-\delta)} = - Ve^{-i\delta}$ and $\frac{dw}{dz}$ is called the complex velocity of flow.

i. Uniform flow in x-axis direction

Let U the velocity is the direction $w = Uz$ so that $\frac{dw}{dz} = U = Ue^{\circ} \therefore \delta = 0$

ii. Source at origin

$$w = \phi + i\psi = \frac{Q}{2\pi} \ln r + i\frac{Q}{2\pi}\theta = \frac{Q}{2\pi} \ln(r + i\theta)$$

$$= \frac{Q}{2\pi} \ln re^{i\theta} = \frac{Q}{2\pi} \ln z$$

$$\frac{dw}{dz} = \frac{Q}{2\pi z} = \frac{Q}{2\pi} e^{-i\theta}$$

$$\left|\frac{dw}{dz}\right| = V = \sqrt{u^2 + v^2} = \frac{Q}{2\pi r} \text{ and } \arg\left(\frac{dw}{dz}\right) = \frac{Q}{2\pi r} e^{-i\theta} \therefore \delta = 0$$

iii. Vortex at origin

$$w = \phi + i\psi = \frac{i\Gamma}{2\pi} \ln z = \frac{i\Gamma}{2\pi} \ln re^{i\theta}$$

$$\frac{dw}{dz} = \frac{i\Gamma}{2\pi z} = \frac{i\Gamma}{2\pi r} e^{-i\theta} = \frac{i\Gamma}{2\pi r} e^{i(\pi/2 - \theta)}$$

$$= u - iv = \frac{\Gamma}{2\pi r} e^{i(\pi/2 - \theta)}$$

$$\frac{\Gamma}{2\pi r}\left(\qquad \frac{\pi}{2} \qquad \right)$$

$$= \overline{\quad} \cos\left(\frac{\pi}{2} - \theta\right) + i \sin\left(\overline{\quad} - \theta\right)$$

$$= \frac{\Gamma}{2\pi} (\cos\theta + i \sin\theta)$$

$$\left|\frac{dw}{dz}\right| = V = \sqrt{u^2 + v^2} = \left(\sqrt{\cos^2\theta + \sin^2\theta}\right)\frac{\Gamma}{2\pi r} = \frac{\Gamma}{2\pi r}$$

$$\arg\left(\frac{dw}{dz}\right) = \frac{\Gamma}{2\pi r} e^{\pi/2 - \theta} \quad \text{i.e.,} \quad \frac{\pi}{2} - \theta = \pi - \delta$$

$$\delta = \frac{\pi}{2} + \theta$$

iv. Doublet at origin

$$w = \frac{\mu}{2\pi z} = \frac{\mu}{2\pi r e^{i\theta}}$$

$$\frac{dw}{dz} = \frac{-\mu}{2\pi z^2} = \frac{-\mu}{2\pi r^2} e^{-2i\theta} \quad \text{from which}$$

$$V = \frac{\mu}{2\pi r^2} \quad \text{and} \quad \delta = 2\theta$$

11. Obtain the stream function for a source and a sink (in two-dimensional flow).

Show that the streamlines due to a source and sink of equal strength m, distance 2s apart, are circles. A source and sink, each of strength 150m^2 /sec, are situated at points M and N on the x-axis 8m apart with origin midway between them. Taking the x-axis (excluding the portion MN) as $\psi = 0$, find the radius of the circle given by $\psi = 25$m^2/sec.

A source is a point from which fluid is flowing out equally in all directions. In two dimensions the flow is restricted to one plane and to allow for the application of the results to three-dimensional flow the term line source is sometimes used. A three-dimensional point source gives a different flow. In the two-dimensional case, the radii from the source are streamlines. The total flux across any circle having the source as centre is equal to the entire output of the source. This is called the strength and is denoted by m. It has the units m^2/sec. Suppose A is on the radius chosen for the streamline $\psi = 0$. The stream function for any other point P on the same circle as A equals the flux across the arc AP. Its ratio to the total output equals the ratio of the arc to the total circumference. Hence

$$\psi = \frac{\text{arc AP}}{\text{circumference for circle through A}} \times \text{total flux}$$

$$= \theta \times r/2\pi r \times m$$

$$= m\theta/2\pi \qquad \text{(i)}$$

where θ is the angle POA in radians. By convention θ is restricted to the range $\pm\pi$. The velocity is radially outwards in direction and, at any radius r, is given by

$$u = \text{total flux / circumference} = m/2\pi r \qquad \text{(ii)}$$

The velocity at $r = 0$ is infinite and this is an example of a <u>singular point.</u>

A sink is a point towards which fluid is flowing equally from all directions and at which it is disappearing. It can be regarded as a negative source, and thus for a strength m its stream function is

$$\psi = - m\theta/2\pi \qquad \text{(iii)}$$

Although sources and sinks are only theoretical concepts in themselves they may be used in combination with other flows to represent cases of practical interest.

Suppose an origin 0 is taken midway between the source and sink in the given combination. Take the x-axis through M, the source, and N, the sink. Suppose P is any point in the plane and θ_1, θ_2 are the angles made by PM, PN with the positive direction of the x-axis. Using this notation we have, from the combination

$$\psi = m\theta_1/2\pi - m\theta_2/2\pi = (m/2\pi)(\theta_1 - \theta_2)$$

But θ_1, the exterior angle of the triangle PMN, equals the sum of the interior and opposite angle of the triangle θ_2 and α. Thus $\theta_1 - \theta_2 = \alpha$

$$\psi = m\alpha/2\pi$$

The streamlines are therefore curves for which α is constant and this condition is satisfied by circles passing through M and N. The centre C of any such circle lies on the y-axis, and $\angle$NCM (the angle at the centre) is twice $\angle$NPM (the angle at the circumference). Thus $\angle NCM = 2\alpha$ and $\angle OCM = \alpha$.

Using the numerical data we have, $\alpha = 2\pi\psi/m = 2\pi \times 25/150$

$$= \pi/3 \text{ rad or } 60°$$

The required radius is

$$CM = OM \operatorname{cosec} \alpha = 4 \operatorname{cosec} 60°$$
$$= 4.62 \text{ m.}$$

12. Show how to construct (by a graphical method) the streamlines representing the flow of a source in the neighbourhood of a plane wall. Also deduce an analytical expression for the streamlines.

Suppose the wall is represented by the y-axis and the source is situated on the x-axis at $x = s$. Any streamline can be replaced by a solid boundary since there is no flow across it. In the present case, the flow due to the source alone must be modified in such a way that the y-axis becomes a streamline. This is done by introducing a second source of equal strength at the point $(-s, 0)$ as shown. The streamlines for each source alone are radii and those for the combined flow pass through their points of intersection as explained (for the case of uniform flow). The broken lines in fig. show the method of constructing the resultant streamlines.

Analytically, the method is similar to that of previous problem.

In the notation of fig. with sources, each of strength m, at M and N, the stream function at P is

$$\psi = m\theta_1/2\pi + m\theta_2/2\pi = (m/2\pi)(\theta_1 + \theta_2) \qquad \text{(i)}$$

If P is the point (x, y) we have

$$\tan\theta_2 = PR / NR = y/(x + s) \text{ and } \tan\theta_1 = y/(x - s)$$

By the trigonometrical identity for the tangent of the sum of two angles.

$$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2} = \frac{[y/(x - s)] + [y/(x + s)]}{1 - [y/(x - s)][y/(x + s)]}$$

$$= \frac{y(x + s) + y(x - s)}{(x - s)(x + s) - y^2}$$

$$= \left(\frac{2yx}{(x^2 - y^2 - s^2}\right)$$

Substituting in (i) the required stream function is

$\psi = (m/2\pi) \tan^{-1} [2xy/(x^2 - y^2 - s^2)]$

13. A line source of strength σ is at the origin in an otherwise uniform stream of an inviscid incompressible fluid of velocity - U parallel to the x-axis. Write down the resulting stream function for the combined flow and determine the equation of the streamline which branches at the stagnation point. In particular, determine in terms of σ and U the maximum distance measured parallel to the y-axis between the braches. What is the value of the pressure coefficient on this streamline at the points where the y-axis cuts it?

Discuss very briefly how this solution can be used to describe the flow past a half-body.

The streamlines for the combined flow can be obtained graphically as shown in the upper half of fig. (This does not form part of the present solution.) The uniform flow (velocity - U) is from right to left and the broken lines are the streamlines due to the source. There is a stagnation point on the x-axis where the velocity (left to right) due to the source equals that of the uniform flow. The streamline $\psi = 0$ consists of the x-axis to the right of S together with two curved branches which meet at S and are symmetrical about the x-axis.

The symbol m will be retained for source strength. Adding the stream functions for the uniform flow, - Uy, and the source,

$$\psi = - Uy + m\theta/2\pi \qquad \text{(i)}$$

At any point having the coordinates (x, y), $\tan \theta = y/x$ and thus,

$$\psi = -Uy + (m/2\pi) \tan^{-1} (y/x) \qquad \text{(ii)}$$

The streamline $\psi = 0$ is given by

$$0 = - Uy + m\theta/2\pi \quad \text{or} \quad y = m\theta/2\pi U \qquad \text{(iii)}$$

The maximum possible value of y clearly corresponds to $\theta = \pi$ and thus the maximum distance between the branches is

$$t = 2y_{max} = 2 \times m\pi/2\pi U = m/U \qquad \text{(iv)}$$

The y-axis cuts the stream where $\theta = \pi/2$. At this point $y = r$ (the radial distance from the source) and,

$y \text{ (or } r) = m/2\pi U \times \pi/2 = m/4U$

The corresponding velocity due to the source alone is, from (i)

$u = m/2\pi r = (m/2\pi)(4U/m) = 2U/\pi$

This is a vertical velocity and, by compounding it with the horizontal uniform flow velocity, the resultant is given by

$$q^2 = (-U)^2 + (2U/\pi)^2 = U^2[1 + (2/\pi)^2]$$

$$= 1.405\, U^2$$

The pressure coefficient is

$$C_p = 1 - q^2/U^2 = 1 - 1.405 = -0.405$$

The streamline $\psi = 0$ can be replaced by a solid boundary without affecting the form of the flow. Thus the flow is identical with that of an otherwise uniform stream past a body whose shape is that of the streamline $\psi = 0$. The solution has also been used to describe the flow over a cliff due to a wind. The boundary $\psi = 0$ contains the entire flow from the source which is merely a mathematical device for representing the effect of the body on the uniform flow.

14. A guard for supporting the strut of a wind tunnel is designed by the combination of a source at origin with a free stream of uniform velocity U_o. Show the pressure distribution on the surface of guard is $p - p_o = \frac{1}{2}\rho U_o^2 \left(\frac{\sin 2\theta}{\theta} - \left(\frac{\sin\theta}{\theta}\right)^2\right)$ where p is the pressure on the surface and p_o at free stream.

Total stream function $\psi = \psi$ uniform flow $+ \psi$ source

$$U_0 = U_o y + \frac{m\theta}{2\pi}$$

$$= U_o r \sin\theta + \frac{m\theta}{2\pi} \qquad \text{(i)}$$

Velocity components $u_r = \frac{1}{r}\left(\frac{\partial\psi}{\partial\theta}\right)$

$$= U_o \cos\theta + \frac{m\theta}{2\pi r}$$

and $u_\theta = -\frac{\partial\psi}{\partial r}$

$$= -U_o \sin\theta$$

Resultant velocity $U^2 = u_r^2 + u_\theta^2$

$$= \left(Uo \cos\theta + \frac{m}{2\pi r}\right)^2 + \left(-Uo \sin\theta\right)^2$$

$$= U_o^2 \cos^2\theta + \frac{m^2}{4\pi^2 r^2} + \frac{m}{\pi r} U_o \cos\theta + U_o^2 \sin^2\theta \quad \text{(ii)}$$

For the solid boundary line i.e., at dividing line $\psi = 0$

i.e., $U_o \, r \sin\theta + \frac{m\theta}{2\pi} = 0 \quad \therefore \quad r = \frac{-m\theta}{2\pi U_o \sin\theta}$

Substituting this in (ii)

$$U^2 = U_o^2 \cos^2\theta + \frac{m^2}{4\pi^2}\left(\frac{-2\pi \, Uo \sin\theta}{m\theta}\right)^2 + \frac{-mU_o \cos\theta}{\pi}\left(\frac{-2\pi \, U_o \sin\theta}{m\theta}\right) + U_o^2 \sin^2\theta$$

$$= U_o^2 (\cos^2\theta + \sin^2\theta) + U_o^2 \left(\frac{\sin\theta}{\theta}\right)^2 - U_o^2 \left(\frac{-2\sin\theta \, .\cos\theta}{m\theta}\right)$$

$$= U_o^2 \left(1 + \left\{\frac{\sin\theta}{\theta}\right\}^2 - \frac{\mathrm{Sin}2\theta}{\theta}\right)$$

Applying Bernoullis theorem $p_o + ½ \rho U_o^2 = p + ½ \rho U^2$

$$p - p_o = ½ \rho U_o^2 \left(1 - \frac{U}{U_o}^2\right) = ½ \rho U_o^2 \left(\frac{\sin 2\theta}{\theta} - \left\{\frac{\sin\theta}{\theta}\right\}^2\right)$$

15. A source with strength 0.25m^2/s and vortex with strength 1m^2/s (anticlockwise) are located at origin. Determine stream and velocity potential. Calculate radial and tangential velocity components at (1, 0.5).

Stream function of combination $\psi = \psi_{\text{source}} + \psi_{\text{vortex}}$

$$= \frac{m}{2\pi}\theta - \frac{\Gamma}{2\pi} \ln r$$

$$= 1/2\pi \; (0.25\theta - 1\text{x} \ln r)$$

Potential function of combination $\phi = \phi_{\text{source}} + \phi_{\text{vortex}}$

$$= \frac{m}{2\pi} \ln r + \frac{\Gamma}{2\pi}\theta$$

$$u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} = \frac{1}{r}\,\frac{1}{2\pi}(0.25 - 0) = \frac{1}{8\pi r}$$

$$u_\theta = -\frac{\partial\psi}{\partial r} = \frac{-1}{2\pi}\left(0 - \frac{1}{r}\right) = \frac{1}{2\pi r}$$

At point (1, 0.5) $\quad r = \sqrt{1^2 + 0.5^2} = 1.117 \text{ m}$

$$\frac{1}{8\pi \;\, 1.117}$$

$$\therefore \quad u_r = \frac{}{} = 0.0356 \text{ m/s}$$

$$u_\theta = \frac{1}{2\pi \ 1.117} = 0.1425 \text{ m/s}$$

16. For a source at origin in an otherwise uniform stream of a fluid of velocity U in the negative x-direction write down(i) the stream line which branches at the stagnation point (ii) what is the maximum distance parallel to y-axis between these branches (iii) what is the value of pressure coefficient on this stream line at points where y-axis cuts it.

i. At any point (x, y) $\quad \psi = -Uy + \frac{m\theta}{2\pi} \quad$ where $\theta = \tan^{-1}(y/x)$

At dividing stream line $\psi = 0$ or $Uy = \frac{m\theta}{2\pi}$; $y = \frac{m\theta}{2\pi U}$

ii. The maximum possible value of y is when $\theta = 2\pi$ and maximum distance between branches = $2y_{max} = m/U$

y axis cuts the stream line when $\theta = \pi/2$, where the radial distance from source $r = m/2\pi U \times \pi/2 = m/4U$

corresponding velocity $u_r = \frac{\partial \phi}{\partial r} = \frac{\partial}{\partial r}\left(\frac{m}{2\pi}\ln r\right) = \frac{m}{2\pi r} = 2U/\pi$

The resultant velocity $\quad V^2 = (-U)^2 + \left(\frac{2U}{\pi}\right)^2 = 1.405U^2$

iii. $\therefore C_p = 1 - (V/U)^2 = 1 - 1.405 = -0.405$

17. A two dimensional irrotational flow is produced by a source 200 m^2/s in a stream 40 m/s together with sink of equal strength 2m down from source. Find the fineness ratio of the oval represented by the dividing stream line.

The dividing stream line $\psi = 0$ forms an oval curve of 2a and 2b major and minor axes respectively.

The distance from source and sink to the stagnation points are

Source to $S_2 = a + 1$

Sink to $S_1 = a - 1$

Velocity due to source = $\frac{-m}{2\pi(a+1)}$

$$\frac{m}{2\pi(a-1)}$$

Velocity due to sink $=$ $\overline{}$

For stream velocity U $=$ - 40 m/s

$\therefore$ At stagnation point $\dfrac{m}{2\pi(a+1)} + \dfrac{m}{2\pi(a-1)} + U = 0$

i.e., $\dfrac{-200}{2\pi(a+1)} + \dfrac{200}{2\pi(a-1)} - 40 = 0$

Solving $a = 1.61$ m

At P (0, b) $\psi = \dfrac{m}{2\pi}(\theta_1 - \theta_2) = \dfrac{m\,\alpha}{2\pi}$ due to source and sink

$\alpha = 2\tan^{-1}(1/b)$ from the configuration

Total ψ due to source, sink and uniform flow is

$\psi = -40\,b + \dfrac{200}{2\pi} \times 2\tan^{-1}(1/b)$ which is equal to zero at the dividing stream line.

$\therefore\ -\ 40\,b + \left(\dfrac{200}{2\pi}\right) 2\tan^{-1}(1/b) = 0$

$\tan^{-1}(1/b) = \pi b/5$ or $b = \cot(\pi b/5)$

This equation is solved graphically to get value of b. $\therefore$ b = 1.14m

The fineness ratio $= \dfrac{2b}{2a} = \dfrac{1.14}{1.61} = 0.708$

18. A circular cylinder of 2.0 m diameter and 12m length is rotated at 300 rpm about its axis when it is kept in an air stream of 40 m/s velocity, with its axis perpendicular to the flow. Determine (i) circulation around the cylinder, (ii) theoretical lift, (iii) position of stagnation points and (iv) actual drag, lift and resultant force on the cylinder, Take $C_D = 0.52$, $C_L = 1.0$ and $\rho = 1.208$ kg/m^3.

Periforal velocity u_c $= \dfrac{\pi DN}{60} = \dfrac{\pi \times 2 \times 300}{60}$

$= 31.42$ m/s

i. Circulation Γ $= 2\pi R\,u_c = 2\pi \times 1 \times 31.42$

$$= 197.44\ m^2/s$$

ii. Theoretical lift L $= \rho\, U\, \Gamma$ x length

$= 1.208 \times 40 \times 197.44 \times 12$

$= 114.485 \times 10^3$ N

iii. Position of stagnation points :

Velocity on the surface u $= 2U \sin\theta + \Gamma / 2\pi R$

For stagnation point u $= o \;\therefore\; \sin\theta = \pi / 4\pi UR$

$\therefore\ \theta = \sin^{-1} (\Gamma / 4\pi UR)$

$= \sin^{-1} (197.44 / 4\pi \times 40 \times 1)$

$= -23.13°$ and $203.13°$

iv. Actual lift L $= C_L \times (\text{Dia} \times \text{length}) \times \rho \dfrac{U^2}{2}$

$= 1 \times 2 \times 12 \times 1.208 \times \dfrac{40^2}{2}$

$= 23.194 \times 10^3$ N

Actual drag D $= C_D \times (\text{Dia} \times \text{Length}) \times \rho \dfrac{U^2}{2}$

$= 0.52 \times 2 \times 12 \times 1.208 \times \dfrac{40^2}{2}$

$= 12.061 \times 10^3$ N

Resultant force on the cylinder= $\sqrt{(L^2 + D^2)} = \sqrt{\left[(23.194 \times 10^3)^2 + (12.061 \times 10^3)^2\right]}$

$= 26.142 \times 10^3$ N

The resultant force is inclined $\tan^{-1}$ (L/D) with the direction of flow i.e., $\tan^{-1}(L/D) = \tan^{-1}(23.194 / 12.061) = 27.41°$

19. Wings of an aeroplane are replaced by two cylinders of 1m diameter and 4m length and it is proposed to make use of the lift caused by their rotation to lift the aeroplane. If the plane travels at 250 km/hr speed and weighs 80,000 N, determine the speed of rotation of the cylinders and the power required to overcome rotor friction. Assume C_D = 0.80 and surface velocity of cylinder 1.5 times free stream velocity.

Assuming level flight, lift balances the weight. W = L

$$\frac{U^2}{2}$$

$$80{,}000 = C_L \times A \times \rho \; \frac{\quad}{\quad}$$

where $A = 2 \times \text{Dia} \times \text{Length of cylinder} = 2 \times 1 \times 4 \text{ m}^2$

$\rho = 1.208 \text{ kg/m}^3$ at sea level conditions.

$$U = 250 \text{ km/hr} = \frac{250 \times 1000}{60 \times 60} = 69.44 \text{ m/s}$$

From which $C_L = 3.433$.

Periforal velocity is 1.5times free stream velocity $\frac{\pi DN}{60}$ =1.5x 69.44

$\therefore$ Speed of rotation $N = 1990$ rpm

Power required = Drag x distance moved

$$= 0.8 \times (2 \times 4 \times 1) \times 1.208 \times \frac{69.44^3}{2}$$

$$= 1294.56 \times 10^3 \text{ watt}$$

20. An aeroplane weighing 22,500 N has a wing area of 22.5 m^2 and span of 12m. What is the lift coefficient if it travels at 320 km/hr in the horizontal direction? Also compute the theoretical value of circulation and angle of attack measured from zero lift axis.

Assuming level flight, lift exactly balances weight. $W = L = C_L A \rho \frac{U^2}{2}$

$$22500 = C_L \times 22.5 \times 1.208 \times \left(\frac{320 \times 1000}{60 \times 60}\right)^2$$

From which $C_L = 0.2095$

Assuming lift curve slope as 2π i.e., $\frac{C_L}{\alpha} = 2\pi$

The angle of incidence $= \frac{0.2095}{2\pi} \times \frac{180}{\pi} = 2^0$

Circulation $\Gamma = \pi \text{ chord } U \alpha$

$$= \pi \times \frac{22.5}{12} \times 88.89 \times \frac{2}{180/\pi}$$

$$= 17.5 \text{ m}^2/\text{s}$$

21. Show that lift coefficient of a circular cylinder of radius R kept in a flow V_∞ with circulation Γ is $C_L = \Gamma / RV_\infty$. For such a flow, calculate peak pressure coefficient for $C_L = 5$

The velocity of flow on the surface of a spinning cylinder

$$v_\theta = V = 2V_\infty \sin\theta + \Gamma / 2\pi R$$

At any point $\quad C_P = 1 - (V / V_\infty)^2$

$$= 1 - (2V_\infty \sin\theta + \Gamma / 2\pi R)^2 \times 1 / V_\infty^2$$

$$= 1 - 4\sin^2\theta + \frac{2\Gamma \sin\theta}{\pi R V_\infty} + \left(\frac{\Gamma}{2\pi R V_\infty}\right)^2$$

The coefficient of lift of an aerofoil can be evaluated as

$$C_L = \frac{1}{c}\int_0^c c_{pl}\, dx + \int_0^c \frac{1}{c} c_{pu}\, dx$$

The suffix l and u stand for lower and upper surface.

Put $x = R\cos\theta$; $\quad dx = (-R\sin\theta)\, d\theta \quad c = 2R$

$$C_L = \frac{1}{2R}\int_\pi^{2\pi} c_{pl}(-R\sin\theta)\, d\theta + \frac{1}{2R}\int_\pi^{\pi} c_{pu}(-R\sin\theta)\, d\theta$$

$$= \frac{-1}{2}\int_\pi^{2\pi} c_{pl}\sin\theta\, d\theta - \frac{1}{2}\int_0^{\pi} c_{pu}\sin\theta\, d\theta$$

$$= 2 \times \left(\frac{-1}{2}\right)\int_0^{2\pi} c_p \sin\theta\, d\theta$$

For cylinder $c_{pl} = c_{pu} \quad \therefore \; C_L = -1\displaystyle\int_0^{2\pi} c_p \sin\theta\, d\theta$

i.e., $$= -1\int_0^{2\pi}\left(1 - 4\sin 2\theta + \frac{2\Gamma\sin\theta}{\pi R V_\infty} + \frac{\Gamma}{2\pi R V_\infty}\right)\sin\theta\, d\theta$$

$$\int_0^{2\pi} \qquad \int_0^{2\pi}$$

As $\int \sin\theta . d\theta = 0$; $\int \sin^2\theta . d\theta = \pi$ and $\int_0^{2\pi} \sin^3\theta . d\theta = 0$

$$C_L = \Gamma / RV_\propto$$

C_p is peak for $\sin\theta = 1$ i.e., at $\theta = 90°$ $\therefore$ $V = 2V_\propto + \Gamma / 2\pi R$

Given $C_L = 5 = \Gamma/RV_\propto$ $\therefore$ $\Gamma / R = 5 V_\propto$ $\therefore$ $V = 2V_\propto + \frac{5V_\propto}{2\pi} = 2.796 V_\propto$

$$\therefore C_p = 1 - (V / V_\propto)^2 = 1 - 2.796^2$$

$$= -6.82$$

22. A ship has two vertical rotors, 2.5 m in diameter and 1m high. When the rotors spin at 240 rpm, the relative motion of air to the ship results in 50 km/hr of wind. Calculate the force exerted on the ship by the spinning rotors. Take density of air as 1.24 kg/m^3.

Relative velocity of wind U = 50 km/hr

= 13.89 m/s

Circumferential velocity of rotor u_c = $\frac{\pi \text{ x dia x N}}{60}$

= $\frac{\pi \text{ x } 2.5 \text{ x } 240}{60}$

= 31.4 m/s

Circulation around cylinder = $\Gamma = 2\pi R u_c$

= $2\pi \text{ x } \frac{2.5}{2} \text{ x } 31.4$

= 246.5 m^2/s

$\therefore$ Lift on one cylinder = $\rho U\Gamma$ length of cylinder

= 1.24 x 13.89 x 246.5 x 8

= 33891.6 N

Total force exerted by two spinning rotors

= 2 x 33891.6

= 67783.2 N

23. A cylinder whose axis is perpendicular to the stream of air having a velocity of 20 m/s, rotates at 300 rpm. The cylinder is 2 m in diameter and 10 m long. Find (i) the circulation, (ii) the theoretical lift force per unit length, (iii) the position of stagnation points, and (iv) the actual lift, drag and direction of resultant force. For determining actual drag and lift, assume $\frac{u_o}{U_o}$ = 1.57 ; C_L = 3.4 C_D = 0.65 ; Where u_o represents the periforal velocity due to circulation and for air ρ = 1.24 kg/m^3.

Periforal speed of cylinder $u_c = \dfrac{\pi \text{ Dia } N}{60}$

$= \dfrac{\pi \times 2 \times 300}{60} = 31.4$ m/s

i. Circulation $\Gamma = 2\pi R u_c$

$= 2\pi \times 1 \times 31.4 = 197.3$ m^2/s

ii. Theoretical lift / unit length $= \rho U \Gamma$

$= 1.24 \times 20 \times 197.3$

$= 4888.24$ N

Net velocity of flow on the surface of cylinder is sum of circulation and free stream velocity

i.e., $u_c = 2U \sin\theta + \Gamma / 2\pi R$

At stagnation point, $u_c = 0$

$\therefore \quad 0 = 2U \sin\theta + \Gamma / 2\pi R$

$\therefore \quad \sin\theta = -\Gamma / 4\pi RU$

$= (-197.3 / 4\pi) \times 1 \times 20$

$= -0.785$

$\therefore \quad \theta = 231.75° \text{ and } 308.25°$

iii. The stagnation points located at 231.7° and 308.25°in anticlockwise direction of circulation

iv. Actual lift $L = C_L \; A \; \rho \dfrac{U^2}{2}$

$= C_L \times (\text{length} \times \text{dia}) \times \rho \dfrac{U^2}{2}$

$= 3.4 \times (2 \times 10) \times 1.24 \times ½ \times 20^2$

$= 16846.2$ N

Actual drag $D = C_D \; A \; \rho \dfrac{U^2}{2}$

$= 0.65 \times (2 \times 10) \times 1.24 \times ½ \times 20^2$

$= 2628.8 \; N$

Direction of resultant force $\tan\theta = \dfrac{L}{D} = \dfrac{16846.2}{2628.8} = 5.23$

$\therefore \; \theta = 79.1°$

v. For $\sin\theta = 1$; in $\sin\theta = \Gamma / 4\pi RU$, there will be only one stagnation point

$\therefore \; \Gamma = 4\pi RU$

$= 4\pi \times 1 \; 20$

$= 251.32 \; m^2/s$

$\therefore$ Periforal speed $u_c = \dfrac{\Gamma}{2\pi R} = \dfrac{251.32}{2\pi \times 1} = 40$ m/s

$= \dfrac{\pi DN}{60} = \dfrac{\pi \times 2 \times N}{60}$

$\therefore \; N = 382$ rpm

There will be only one stagnation point for cylinder rotating at 382 rpm.

24. A fluid motion is described by the stream function

$$\boldsymbol{\psi = w\, a^2 \left(1 - \frac{r^2}{a^2}\right) \quad \text{for} \quad 0 \le r \le a}$$

and $\psi = -\, w\, a^2 \;\ln \dfrac{r}{a}$ for $r \ge$ a where w and a are constants.

(i) Show the motion is irrotational for r ≥ a.

(ii) Determine the pressure distribution for the entire flow field.

Condition for irrotationality of flow in terms of stream function in polar coordinates is

$$\frac{1}{r}\;\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\;\frac{\partial\psi}{\partial\theta} = 0$$

for $r \ge a \quad \psi = wa^2 \;\ln\left(\dfrac{r}{a}\right)$

<u>Case 1:</u> When $r = a$, $\psi = -wa^2 \ln\left(\dfrac{a}{a}\right) = 0$

Rotation $\dfrac{1}{r}\;\dfrac{\partial}{\partial r}(0) + \dfrac{\partial^2}{\partial r^2}(0) + \dfrac{1}{r^2}\;\dfrac{\partial}{\partial\theta}(0) = 0$

Case 2 : When r > a,

$$\text{Rotation}\quad \frac{1}{r}\frac{\partial}{\partial r}\left(-wa^2(\ln r-\ln a)\right)+\frac{\partial^2}{\partial r^2}\left(-wa^2(\ln r-\ln a)\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(-wa^2(\ln r-\ln a)\right)$$

$$=\frac{1}{r}\left(-wa^2\left(\frac{1}{r}-0\right)\right)+wa^2\frac{1}{r^2}+0$$

$$=-\frac{wa^2}{r^2}+w\frac{a^2}{r^2}=0$$

(i) ∴ Condition for irrotationality is satisfied for $r \ge a$.

$$u_r=\frac{1}{r}\frac{d\psi}{d\theta},\quad u_\theta=\frac{-d\psi}{dr}$$

for $o \le r \le a$ $\quad u_r=\frac{1}{r}\frac{d}{d\theta}\left\{wa^2\left(1-\frac{r^2}{a^2}\right)\right\}=0$

$$u_\theta=\frac{-d}{d\theta}\left\{wa^2\left(1-\frac{r^2}{a^2}\right)\right\}$$
$$=2\,wr$$

$$V^2=u_r^2+u_\theta^2=(2\,wr)^2$$

for $r \ge a$ $\quad u_r=\frac{1}{r}\frac{d}{d\theta}\left\{-wa^2\ln\frac{r}{a}\right\}=0$

$$u_\theta=-\frac{d}{dr}\left\{-wa^2\ln\frac{r}{a}\right\}$$

$$V^2=u_r^2+u_\theta^2=\left(w\frac{a^2}{r}\right)^2$$

<u>(ii) Pressure distribution</u>

$$C_p=\frac{p\quad p_\infty}{\tfrac{1}{2}\rho U^2}=1-\left(\frac{V}{U}\right)^2$$

for $0 \le r \le a$ $\quad C_p=1-\left(\frac{2wr}{r}\right)^2=1-\frac{4w^2r^2}{U^2}$

$$\left(\quad\right)\qquad \frac{w^2r^4}{r^2U^2}$$

for $0 \le r \le a$ $\quad C_p = 1 - \left(\frac{wa^2}{rU}\right)^2 = 1 -$

25. Sketch ϕ and ψ lines for $w = \mu/z$ (see note under problem 4)

26. Derive expression for C_p on the surface of a cylinder kept in a uniform flow. (Refer 2.8 pressure distribution)

27. If $w = \pm\left(Uz + \frac{Ua^2}{z} + \frac{ik}{2\pi}\ln z\right)$ sketch the stream lines; mark stagnation point (s) ; mark the lift force.

$$w = Uz + \frac{Ua^2}{z} + \frac{ik}{2\pi}\ln z$$

Here w represents the complex potential for flow past a circular cylinder of radius 'a' kept at origin and rotating about its own axis.

The first term is the complex potential for uniform flow in the positive x-direction; second term a doublet at origin; third term for anticlockwise vortex of strength k at the origin due to spinning of cylinder.

$$w = -\left(Uz + \frac{Ua^2}{z} + \frac{ik}{2\pi}\ln z\right)$$

The first term is the complex potential for uniform flow in the negative x-direction; second term a doublet at origin; third term for clockwise vortex of strength k at the origin due to spinning of cylinder.

Lift force : The direction of lift acting on such a case are to be noted. This is obtained by the actual velocity of flow of fluid past the cylinder. Velocity of fluid increases when uniform flow and circulation are in the same direction and velocity decreases when uniform flow and circulation are in opposite directions. Thus at one half of cylinder due to addition of velocity, velocity is more while on the other half of cylinder due to deduction of velocity, velocity is less. Where there is more velocity there will be less pressure and vice versa The lift force will act from higher pressure half to lower pressure half.

Stagnation point (s) These will be in the portion of cylinder where there less velocity. The number of stagnation point depends on the relative strength of vortex and velocity of flow which is discussed in theory.

28. A circular cylinder of radius of placed at the origin in a 2D uniform flow of ideal fluid approaching with velocity U at an angle α to x-axis. Write the complex potential for the flow. What strength of circulation is required to make the point (a, 0) on the cylinder a stagnation point.

This is a case of doublet at origin with a uniform flow at α inclination to the x-axis.

The complex potential for this is

$$w = Uze^{-i\alpha} + \frac{\mu}{z}$$

If circulation is added to this flow the complex potential changes to

$$w = Uze^{-i\alpha} + \frac{\mu}{z} + \frac{ik}{2\pi} \ln z$$

The complex potential can be written as (Refer Problem No.)

$$w = Uz + \frac{m}{2\pi} \ln (z + he^{i\alpha}) - \frac{ik}{2\pi} \ln z$$

$$= Uz + \frac{m}{2\pi z} 2\, he^{i\alpha} - \frac{ik}{2\pi} \ln (r\, e^{i\theta})$$

$$= Ur (\cos\theta + i \sin\theta) + \frac{mh}{\pi} (\cos\alpha + i \sin\alpha) \frac{1}{r} (\cos\theta - i \sin\theta) - \frac{ik}{2\pi} (\ln r + i\theta)$$

putting $2mh = \mu$

$$w = Ur (\cos\theta + i \sin\theta) + \frac{\mu}{2\pi r} \cos\alpha.\cos\theta - i \cos\alpha.\sin\theta + i \sin\alpha.\cos\alpha + \sin\alpha.\sin\theta) - \frac{ik}{2\pi} (\ln r + i\,\theta)$$

$$= Ur (\cos\theta + i \sin\theta) + \frac{\mu}{2\pi r} \left\{\cos(\alpha - \theta) - i \sin(\alpha - \theta)\right\} - \frac{ik}{2\pi} \ln r + \frac{k}{2\pi} \theta$$

$$= \phi + i\,\psi$$

$$\therefore \psi = Ur \sin\theta - \frac{\mu}{2\pi r} \sin(\alpha - \theta) \frac{k}{2\pi} \ln r$$

$$u_\theta = -\frac{d\psi}{dr}$$

$$= -\frac{d}{dr}\left(Ur \sin\theta - \frac{\mu}{2\pi r} \sin(\alpha - \theta) - \frac{k}{2\pi} \ln r\right)$$

$$\left(\quad \frac{\mu}{2\pi r^2} \quad \frac{k}{2\pi r} \right)$$

$$= - U \sin\theta + \frac{\qquad}{\qquad} - \frac{\qquad}{\qquad}$$

On the surface of the cylinder $r = a$, $u_r = 0$. At stagnation point $u_r = 0$ and $u_\theta = 0$. Given point being $(a, 0)$ $r = a$ on the surface of cylinder and $\theta = 0$ because $y = 0$ corresponding to x-axis.

$$\therefore \quad u_\theta = \left(U \sin\theta + \frac{\mu}{2\pi a^2} - \frac{k}{2\pi a} \right) = 0$$

$$\text{i.e.,} \quad k = \frac{\mu}{a}$$

The circulation strength should be μ/a.

Short Question and Answer

Unit - II

1. Define stream function

Flow per unit time (flux) across the line joining two stream lines is called stream function of stream line.

The unit of stream function is m^2/sec.

Per unit dimension perpendicular to the phane of stream line stream function gives volume rate of flow.

2. State the equation of stream line in differential form

$udy - vdx = 0$ cartesian coordinates

$u_r\, rd\theta - u_\theta\, dr = 0$ polar coordinates

3. State the equation of potential line in differential form

$udx + vdy = 0$ in cartesian coordinates

4. State the condition for single stagnation point on a rotating cylinder

$\Gamma = 4\pi UR$

5. Define sub critical and super critical circulation

See notes

6. Define the circulation of fluid around a rotating cylinder in terms of periforal velocity of cylinder

circulation = circumstance x periforal velocity

7. What is D'Alemberts Paradox?

For a stationary cylinder kept with axis perpendicular to the flow of an ideal fluid, no lift or drag force is felt.

8. What is magnus effect?

For a spinning cylinder kept with its axis perpendicular to the flow of an ideal fluid, there is force only in one direction which is lift force. This is called magnus effect. Magnus effect is independent of the cylinder size.

9. State Kutta - Joukowski theorem

For a spinning cylinder kept with its axis perpendicular to the flow of fluid there is force in the direction perpendicular to axis. The magnitude of this force is $L = \rho V\Gamma$ per unit length of cylinder where ρ is the mass density of fluid, V the velocity of fluid and Γ is the circulation. This force is independent of the cylinder diameter.

10. How much is the circulation for (i) uniform flow (ii) source / sink.

(i) The circulation around any closed curve in uniform flow is zero.

(ii) The circulation is zero associated with source / sink flow.

11. What is ideal fluid (perfect fluid)

Ideal fluid is one which is frictionless and effect of viscosity is negligible in fluid mechanies.

A perfect fluid is one which obeys Boyles and Charlas law in thermodynamics

12. What is a rotational flow.

A fluid flow in which every fluid element rotates about its own centre.

13. What is vortex line and vortex tube?

Vortex line is the vector line of the vorticity field.

Vortex tube is a vector tube filled with fluid and formed by vortex lines.

14. What is the common condition to have both stream and potential function to exists for a flow.

Flow should be continuous.

15. What is a free vortex flow?

A flow field with circular stream lines with absolute value of velocity varying inversely with the distance from centre. The flow is irrotational at every point except at the centre.

16. What does a free vortex flow mean?

A flow which is free of vorticity except at the centre.

17. What is meant by bound vortex of a wing?

The vortex that represents circulatory flow around the wing is called the bound vortex. The vortex remains stationary with respect to the general flow.

18. Define velocity with respect to a potential line.

There is no velocity vector tangential to a potential line, the velocity is perpendicular to the potential line.

19. a) What is a forced vortex flow?

A flow in which each fluid particle moves in a circular path with speed varying directly as the distance from the axis of rotation.

19. b) Sketch velocity distribution of a forced vortex

20. Why tornado is highly destructive at or near the centre?

Tornado is a free vortex flow such that velocity multiplied by distance from centre is constant. Therefore the velocity is maximum at the cnetre. Hence it is highly destructive.

21. Specify the stream and potential lines for a doublet

Stream lines $\psi = r / \sin\theta$; Stream lines are circles tangent to x-axis.

Potential lines $\phi = r/\cos\theta$; Potential lines circles tangent to y-axis.

22. Specify the stream and potential line for a source or sink.

Stream lines $\psi = m\theta / 2\pi$; Stream lines are radial lines from centre

Potential line $\phi = \ln r$; Potential lines are circles.

23. Compare the stream lines and potential lines of source / sink with that of a vortex flow

The stream lines of source / sink and potential lines of vortex are similar.

The potential lines of source / sink and stream lines of vortex are similar.

24. State the properties of a stagnation point in a fluid flow.

The sudden change of momentum of fluid from a finite value to stagnant value impresses pressure force at the point of stagnation, thus whole of the velocity gets converted to pressure.

25. What is Rankine half body ?

The dividing stream line $\psi = m/2$ of source uniform flow combination forms the shape of Rankine half body. $\frac{m}{2} = U_o\, y + \frac{m\theta}{2\pi}$

26. What is Rankine oval?

The dividing stream line ($\psi = 0$) of doublet uniform flow combination forms the shape of Rankine oval.

27. How transverse force can be introduced to a flow around a cylinder?

Add a circulatory flow along with the uniform flow to get a tranverse force. Spin the cylinder about its own axis to get circulatory flow.

28.Compare vortex with source / sink flow pattern.

See question 23

29. How will be the stream and potential lines in a source - vortex combination?

Stream and potential lines in a source vortex combination are both equiangular spirals. The change of direction of radial movements of fluid particles will be equal in magnitude while opposite in direction to the change in tangential movement so that curves are equiangular spirals.

30. State the stream function for uniform flow of velocity u parallel to positive x - direction.

Stream function $\psi = - U_y$

31. State the stream function for uniform flow of velocity V parallel to positive y-direction.

Stream function $\psi = - V_x$

32. What is the diameter of a circular cylinder which is obtained by combination of doublet of strength "μ" at origin and uniform flow U parallel to X-axis.

Diameter a $= \sqrt{\mu/2\pi U}$

33. How a line source differs from a point source?

A two dimensional source is a point source from which the fluid is assumed to flow out radially in all directions. As this flow is restricted to one plane and to allow for the application of the results to three dimensional flow, the term line source is sometimes used.

34. How are stream lines of doublet

A family of circles tangent to x-axis with centers in y-axis.

35. How are the obential lines of doublet

A family of circles tangent to y-axis with centers in x-axis.

36. Point out the position of the generators on a cylinder at which the pressure is equal to that at an undisturbed stream when an incompressible fluid flows past an infinite circular cylinder.

When the angles are $\pm\ 30°$ or $\pm\ 150°$ to the direction of flow, locates four generators. (See notes for the proof).

37. Define potential flow of a fluid.

The irrotational motion of an incompressible fluid is called potential flow.

38. "If the fluid flow is in concentric circles, the circulation of such a flow is constant" Give expression for circulation.

Circulation $\Gamma = v \times 2\pi r$, where v is the tangential velocity and r radius of circle.

39. Relate vorticity and circulation

Vorticity is the circulation around an element divided by its area.

40. Relate the vorticity and angular velocity

Vorticity is equal to twice angular velocity.

Therefore, Circulation = 2 x rotation x area

41. What is meant by Karman vortex sheet?

A body moving in real fluid leaves double row of vortices from the sides of body. These vortice are rotating in opposite directions and gradually dissipated by viscosity as if move down stream. If the vortice are stable, for a distance between vortices 'h' and for pitch 'l' of the vortices $h/l = 0.2806$ for Karman vortex sheet.

42. What is the relation between circulation and strength of a vortex?

The circulation calculated around a stream line of an irrotational vortex is a measure of the intensity of vortex.

$$\text{Circulation } \Gamma = \oint V.ds$$

$$= V_\theta \oint ds = 2\pi r V_\theta = K$$

(as $r V_\theta$ is a constant)

43. How are the stream lines in a source sink pair?

The stream lines are circles with centre on y-axis for a source sink pair. Stream lines are circles with common chord.

44. What is a vortex pair?

Two vortices of equal strength, but of opposite sign or with opposite direction of rotation constitute a vortex pair.

45. What is meant by complex potential ?

If stream function ψ and potential function ϕ are combined in a single function w such that $w(z) = \phi + i\psi$ then w (z) is called complex potential.

46. What is meant by conformal transformation.

A transformation is a mathematical process by which a figure may be distorted or altered in size and shape. This is done by means of algebric relationship between the original coordinates and coordinates of new position, the pair of coordinates being represented by complex variables.

The transformation is said to be conformal if small elements of area are unaltered in shapes (though they are, in general, altered in size, position and orientation)

47. What is Joukowski transformation?

Joukowski assumes the relation $w(z) = z + \frac{a^2}{z}$ so that second term is small when z is large. Thus at great distances from the origin the flow is undisturbed by the transformation.

48. What is thickness ratio (fineness ratio) of a Rankine oval

It is the ratio of maximum thickness to chord of Rankine oval.

49. How are the stream line of source sink combination pair?

Stream line of source sink combination pair series of circles with centers in y-axis and passing through source and sink. (flow is from source to sink). They have a common chord of source sink distance.

50. How are the potential lines of source sink pair?

The potential lines of source sink pair are eccentric non intersecting circles with centre along x-axis.

24. A fluid motion is described by the stream function

$$\psi = w\,a^2\left(1-\frac{r^2}{a^2}\right) \quad \textbf{for} \quad 0 \le r \le a$$

and $\psi = -w\,a^2 \ln \dfrac{r}{a}$ **for** $r \ge a$ **where w and a are constants.**

(ii) Show the motion is irrotational for r ≥ a.

(ii) Determine the pressure distribution for the entire flow field.

Condition for irrotationality of flow in terms of stream function in polar coordinates is

$$\frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial\psi}{\partial\theta} = 0$$

for $r \ge a$ $\psi = wa^2 \ln\left(\frac{r}{a}\right)$

<u>Case 1:</u> When $r = a$, $\psi = -wa^2 \ln\left(\frac{a}{a}\right) = 0$

Rotation $\frac{1}{r}\frac{\partial}{\partial r}(0) + \frac{\partial^2}{\partial r^2}(0) + \frac{1}{r^2}\frac{\partial}{\partial\theta}(0) = 0$

<u>Case 2 :</u> When $r > a$,

$$\text{Rotation} \quad \frac{1}{r}\frac{\partial}{\partial r}\left(-wa^2(\ln r - \ln a)\right) + \frac{\partial^2}{\partial r^2}\left(-wa^2(\ln r - \ln a)\right)$$

$$+ \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(-wa^2(\ln r - \ln a)\right)$$

$$= \frac{1}{r}\left(-wa^2\left(\frac{1}{r}-0\right)\right) + wa^2\frac{1}{r^2} + 0$$

$$= -\frac{wa^2}{r^2} + w\frac{a^2}{r^2} = 0$$

(i) $\therefore$ Condition for irrotationality is satisfied for $r \ge a$.

$$u_r = \frac{1}{r^2}\frac{d\psi}{d\theta}, \quad u_\theta = \frac{-d\psi}{dr}$$

for $o \le r \le a$ $\quad u_r = \dfrac{1}{r}\dfrac{d}{d\theta}\left\{wa^2\left(1-\dfrac{r^2}{a^2}\right)\right\} = 0$

$$u_\theta = \frac{-d}{d\theta}\left\{wa^2\left(1-\frac{r^2}{a^2}\right)\right\}$$

$$= 2\,wr$$

$$V^2 = u_r^2 + u_\theta^2 = (2\,wr)^2$$

for $r \geq a$ $\quad u_r = \dfrac{1}{r}\dfrac{d}{d\theta}\left\{-wa^2 \ln \dfrac{r}{a}\right\} = 0$

$$u_\theta = -\frac{d}{dr}\left\{-wa^2 \ln \frac{r}{a}\right\}$$

$$V^2 = u_r^2 + u_\theta^2 = \left(w\frac{a^2}{r}\right)^2$$

(ii) Pressure distribution

$$C_p = \frac{p \quad p_\infty}{\frac{1}{2}\rho U^2} = 1-\left(\frac{V}{U}\right)^2$$

for $0 \leq r \leq a$ $\quad C_p = 1-\left(\dfrac{2wr}{r}\right)^2 = 1-\dfrac{4w^2r^2}{U^2}$

for $0 \leq r \leq a$ $\quad C_p = 1-\left(\dfrac{wa^2}{rU}\right)^2 = 1-\dfrac{w^2r^4}{r^2U^2}$

25. Sketch ϕ and ψ lines for $w = \mu/z$

26. Derive expression for C_p on the surface of a cylinder kept in a uniform flow.

(Refer 2.8 pressure distribution)

27. If $w = \pm\left(Uz + \dfrac{Ua^2}{zr} + \dfrac{ik}{2\pi}\ln z\right)$ sketch the stream lines; mark stagnation point (s) ; mark the lift force.

$$w = Uz + \frac{Ua^2}{zr} + \frac{ik}{2\pi}\ln z$$

Here w represents the complex potential for flow past a circular cylinder of radius ‘a’ kept at origin and rotating about its own axis.

The first term is the complex potential for uniform flow in the positive x-direction; second term a doublet at origin; third term for anticlockwise vortex of strength k at the origin due to spinning of cylinder.

$$w = -\left(Uz + \frac{Ua^2}{z} + \frac{ik}{2\pi}\ln z\right)$$

The first term is the complex potential for uniform flow in the negative x-direction; second term a doublet at origin; third term for clockwise vortex of strength k at the origin due to spinning of cylinder.

Lift force : The direction of lift acting on such a case are to be noted. This is obtained by the actual velocity of flow of fluid past the cylinder. Velocity of fluid increases when uniform flow and circulation are in the same direction and velocity decreases when uniform flow and circulation are in opposite directions. Thus at one half of cylinder due to addition of velocity, velocity is more while on the other half of cylinder due to deduction of velocity, velocity is less. Where there is more velocity there will be less pressure and vice versa The lift force will act from higher pressure half to lower pressure half.

Stagnation point (s) These will be in the portion of cylinder where there less velocity. The number of stagnation point depends on the relative strength of vortex and velocity of flow which is discussed in theory.

28. A circular cylinder of radius of placed at the origin in a 2D uniform flow of ideal fluid approaching with velocity U at an angle α to x-axis. Write the complex potential for the flow. What strength of circulation is required to make the point (a, 0) on the cylinder a stagnation point.

This is a case of doublet at origin with a uniform flow at α inclination to the x-axis

The complex potential for this is

$$w = Uze^{-i\alpha} + \frac{\mu}{z}$$

If circulation is added to this flow in the anticlockwise direction the complex potential changes to

$$w = Uze^{-i\alpha} + \frac{\mu}{z} + \frac{ik}{2\pi}\ln z$$

$$= Ur\left[\cos(\theta - \alpha) + i\sin(\theta - \alpha)\right] + \frac{\mu}{r}(\cos\theta - i\sin\theta) + \frac{ik}{2\pi}(\ln r + i\theta)$$

$$= Ur\cos(\theta - \alpha) + \frac{\mu}{r}\cos\theta - \frac{k\theta}{2\pi} + i\left[Ur\sin(\theta - \alpha) - \frac{\mu}{r}\sin\theta + \frac{k}{2\pi}\ln r\right]$$

$$= \phi + i\psi$$

$$\therefore\ \psi = Ur\sin(\theta - \alpha) - \frac{\mu}{r}\sin\theta + \frac{k}{2\pi}\ln r$$

$$u_\theta = -\frac{d\psi}{dr}$$

$$= -\frac{d}{dr}\left(Ur\sin(\theta - \alpha) - \frac{\mu}{r}\sin\theta + \frac{k}{2\pi}\ln r\right)$$

$$= -\left[U\sin(\theta - \alpha) + \frac{\mu}{r^2}\sin\theta + \frac{k}{2\pi r}\right]$$

To make (a, 0) to be stagnation point u_r and u_θ at this point must be zero. This point being on the cylinder surface $u_r = 0$. The polar coordinate of this point is also (a, 0) $\therefore$ $r = a$; $\theta = 0$

Substituting this value of polar coordinates in the expression for u_θ and equating to zero.

$$-\left(U\sin(0 - \alpha) + \frac{\mu}{a^2}\sin 0 + \frac{k}{2\pi a}\right) = 0$$

$\therefore$ $k = 2\pi a U \sin\alpha$ $\therefore$ The strength of circulation must be equal to $2\pi aU\sin\alpha$ in the anticlock wise direction.

29. If the axis of doublet is inclined α to the x-axis and with a uniform flow is the positive x-direction, what is the complex potential. Bring out the circulation required in the clockwise direction to make (a, 0) stagnation point.

The complex potential can be written as

$$w = Uz + \frac{m}{2\pi z} 2\,he^{i\alpha} - \frac{ik}{2\pi} \ln (r\, e^{i\theta})$$

$$= Ur(\cos\theta + i\sin\theta) + \frac{mh}{\pi}(\cos\alpha + i\sin\alpha)\frac{1}{r}(\cos\theta - i\sin\theta) - \frac{ik}{2\pi}(\ln r + i\theta)$$

putting $2mh = \mu$

$$w = Ur(\cos\theta + i\sin\theta) + \frac{\mu}{2\pi r}\cos\alpha.\cos\theta - i\cos\alpha.\sin\theta + i\sin\alpha.\cos\theta + \sin\alpha.\sin\theta) - \frac{ik}{2\pi}(\ln r + i\theta)$$

$$= Ur(\cos\theta + i\sin\theta) + \frac{\mu}{2\pi r}\left\{\cos(\alpha - \theta) - i\sin(\alpha - \theta)\right\} - \frac{ik}{2\pi}\ln r + \frac{k}{2\pi}\theta$$

$$= \phi + i\psi$$

$$\therefore\ \psi = Ur\sin\theta - \frac{\mu}{2\pi r}\sin(\alpha - \theta) - \frac{k}{2\pi}\ln r$$

$$u_\theta = -\frac{d\psi}{dr}$$

$$= -\frac{d}{dr}\left(Ur\sin\theta - \frac{\mu}{2\pi r}\sin(\alpha - \theta) - \frac{k}{2\pi}\ln r\right)$$

$$= -\left(U\sin\theta + \frac{\mu}{2\pi r^2} - \frac{k}{2\pi r}\right)$$

On the surface of the cylinder $r = a$, $u_r = 0$. At stagnation point $u_r = 0$ and $u_\theta = 0$. Given point being (a, 0) $r = a$ on the surface of cylinder and $\theta = 0$ because $y = 0$ corresponding to x-axis.

$$\therefore\quad u_\theta = -\left(U\sin\theta + \frac{\mu}{2\pi a^2}\sin\alpha - \frac{k}{2\pi a}\right) = 0$$

i.e., $$k = \frac{\mu\sin\alpha}{a}$$

$\therefore$ The circulation strength should be $\dfrac{\mu\sin\alpha}{a}$

UNIT III

VISCOUS HYPERSONIC FLOW THEORY

1.a) Transformation $w = z^2$,

We have $u + iv = (x + iy)^2 = x^2 - y^2 + 2ixy$

$$u = x^2 - y^2 \text{ and } v = 2xy$$

Case 1

If u is constant (say, a), then $x^2 - y^2 = a$ which is a rectangular hyperbola. Similarly, if v is constant (say, b), than $xy = b/2$ which also represents a rectangular hyperbola.

Hence a pair of lines $u = a$, $v = b$ parallel to the axes in the w - plane, map into pair of orthogonal rectangular hyperbolae in the z-plane as shown in Fig.

Case 2

Again if x is constant (say, c), then $y = v/2c$ and $y^2 = c^2 - u$. Elimination of y from these equations gives $v^2 = 4c^2 (c^2 - u)$, which represents a parabola. Similarly, if y is a constant (say, d), then elimination of x from the equation (i) gives $v^2 = 4d^2 (d^2 + u)$ which is also a parabola.

Hence the pair of line $x = c$ and $y = d$ parallel to the axes in the z-plane map into orthogonal parabolas in the w-plane as show in fig.

Also since $\frac{dw}{dz} = 2z = 0$ for $z = 0$, therefore, it is a critical point of the mapping.

Taking $z = re^{i\theta}$

and $w = Re^{i\phi}$

then in polar form $w = z^2$ becomes $Re^{i\phi} = r^2 e^{2i\theta}$.

This shows that upper half of the z-plane $o < \theta < \pi$ transforms into the entire w-plane $0 \le \phi < 2\pi$. The same is true of the lower half.

Note : 1. Taking the axes to represent two walls, a single quadrant could be used to represent fluid flow at a corner wall. This transformation can also represent the electrostatic field in the vicinity of a corner conductor.

Note 2 For the transformation $w = z^n$, n being a positive integer, we have $dw/dz = 0$ at $z = 0$.

Also $$Re^{i\phi} = r^n e^{in\theta}$$

$\therefore$ $$R = r^n \quad \text{and} \quad \phi = \theta^n,$$

when $0 < \theta < \pi/n$, correspondingly $0 < \phi < \pi$.

Hence $w = z^n$ gives a conformal mapping of the z-plane everywhere except at the origin and that if forms out a sector of z-plane of central angle π/n to cover the upper half of the w-plane.

1 b). Transformation $w = e^z$.

Writing $z = x + iy$ and $w = \rho e^{i\phi}$, we have $\rho e^{i\phi} = e^{x + iv} = e^x . e^{iy}$

whence $$\rho = e^x \tag{i}$$

and $$\phi = y \tag{ii}$$

From (i), it is clear that the lines parallel to y-axis (x = const.) map into circles (ρ = const.) in the w-plane, their radii being $\lesseqgtr 1$ according as $x \lesseqgtr 0$

Similarly, it follows from (ii) that the lines parallel to the x-axis (y = const.) map into the radial lines (ϕ = const.) of the w-plane. Thus any horizonal strip of height 2π in the z-plane will cover once the entire w-plane.

This transformation can be used to obtain the circulation of a liquid around a cylindrical obstacle, the electrostatic field due to a charged circular cylinder etc.

1.c) Transformation $w = \cosh z$.

We have $u + iv = \cosh (x + iy)$

$$= \cosh x \cos y + i \sinh x \sin y$$

so that $$u = \cosh x \cos y \text{ and } v = \sinh x \sin y.$$

Elimination of x from these equations gives

$$\frac{u^2}{\cos^2 y} - \frac{v^2}{\sin^2 y} = 1 \tag{i}$$

while elimination of y gives

$$\frac{u^2}{\cosh^2 y} + \frac{v^2}{\sinh^2 y} = 1 \tag{ii}$$

Equation (i) shows that the lines parallel to the x-axis (i.e. y = const.) in the z-plane map into hyperbolae in the w - plane.

Equation (ii) shows that the lines parallel to the y-axis (i.e. x = const.) in the z-plane map into ellipses in the w-plane (fig.)

This transformation can be used

(i) to obtain the circulation of liquid around an elliptic cylinder ;

(ii) to determine the electrostatic field due to a charged cylinder ;

(iii) to determine the potential between two confocal elliptic (or hyperbolic) cylinders.

2. a) Show that points on a circle $x^2 + y^2 = a^2$ are transformed to points on the ellipse by Joukowski transformation. How this ellipse transforms to a flat plate.

If $z = x + iy$ and $w = \phi + i\psi$; $w = f(z)$ in the form $w = z + \frac{c^2}{z}$ which is Joukowski transformation .

$$\phi + i\psi = z + \frac{c^2}{z}$$

$$= x + iy + \frac{c^2}{x + iy} \cdot \frac{x - iy}{x - iy}$$

$$= \frac{x(x^2+y^2+c^2) + iy(x^2+y^2-c^2)}{x^2 + y^2}$$

$$\therefore \frac{\phi}{x} = \frac{x^2+y^2+c^2}{x^2+y^2} \quad ; \quad \frac{\psi}{y} = \frac{x^2+y^2-c^2}{x^2+y^2}$$

As for the circle $x^2 + y^2 = a^2$ subsisting this in above we get the corresponding points on the transferred figure i.e.,

$$\frac{\phi}{x} = \frac{a^2 + c^2}{a^2} \quad ; \quad \frac{\psi}{y} = \frac{a^2 - c^2}{a^2}$$

substituting these values of x, y to the equation of circle

$$\frac{\phi^2}{\left(\frac{a^2 + c^2}{a^2}\right)} + \frac{\psi^2}{\left(\frac{a^2 - c^2}{a^2}\right)} = a^2$$

$$\frac{\phi^2 a^4}{(a^2 + c^2)^2} + \frac{\psi^2 a^4}{(a^2 + c^2)^2} = a^2$$

$$\frac{\phi^2}{\left(a^2 + \frac{c^2}{a}\right)^2} + \frac{\psi^2}{\left(a^2 + \frac{c^2}{a}\right)^2} = 1$$

This is equation to an ellipse of semi-axes

$$a + \frac{c^2}{a} \quad \text{and} \quad a - \frac{c^2}{a}$$

Note : It a = c this above ellipse become a line ellipse (it is called so) or a flat plate of length 4a.

2 b) Joukowski's transformation $w = z + 1/z$.

Since $\dfrac{dw}{dz} = \dfrac{(z+1)(z-1)}{z^2}$

the mapping is conformal except at the point $z = 1$ and $z = -1$ which correspond to the points $w = 2$ and $w = -2$ of the w-plane.

Changing to polar co-ordinates,

$$W = u + iv = r(\cos\theta + i\sin\theta) + \frac{1}{r(\cos\theta + i\sin\theta)}$$

$$= r(\cos\theta + i\sin\theta) + \frac{1}{r}(\cos\theta - i\sin\theta)$$

$\therefore$ $u = (r + 1/r)\cos\theta$ and $v = (r - 1/r)\sin\theta$.

Elimination of θ gives

$$\frac{u^2}{(r+1/r)^2} + \frac{v^2}{(r-1/r)^2} = 1 \qquad \text{(i)}$$

while the elimination of r gives

$$\frac{u^2}{4\cos^2\theta} - \frac{v^2}{4\sin^2\theta} = 1 \qquad \text{(ii)}$$

From (i), it follows that the circles r =constant of z-plane transform into a family of ellipses of the w-plane. These ellipses are confocal for $(r+1/r)^2 - (r-1/r)^2 = 4$, i.e, a constant.

In particular, the unit circle (r=1) in the z-plane flattens out to become the segment u=-2 to u=2 of the real axis in w-plane. Thus the exterior of the unit circle in the z-plane maps into the entire w-plane.

From (ii), it is clear that the radial lines θ = constant of the z-plane transform into a family of hyperbolae which are also confocal.

This transformation is used to map the exterior of the profile of an aeroplane wing on the exterior of a nearly circular region.

3. What is a conformal transformation and what is its magnification?

Apply Joukowski's transformation $t = z + a^2/z$ to a circle in the z-plane, radius a, and centre the origin. Hence obtain an expression for the velocity at any point on the surface of a cylinder in an otherwise uniform stream. You may assume that the velocity q at point in the z-plane and the velocity q' at the corresponding point in the w-plane are related by

$$q = q' \sqrt{[1 - 2(a/r)^2 \cos 2\theta + (a/r)^4]} \qquad \text{(i)}$$

where r and θ and the polar coordinates in the z-plane.

(N.B. - The derivation of (i) is given below)

A transformation is a mathematical process by which a figure or network may be distorted or altered in size. It is effected by means of an algebraic relationship between the original coordinates of any point (x and y and the coordinates of its new position, ϕ, ψ. The pairs of coordinates (x, y) and (ϕ,ψ) are conveniently represented by complex variables z and w respectively.

The transformation is conformal if small elements of area are unaltered in shape (though they are, in general, altered in size, position and orientation). Suppose the small triangular element ABC in the z-plane transforms to the triangle A'B'C' in the w-plane. If the transformation is conformal these triangles are geometrically similar. In the notation of fig. The magnification is the ratio.

$$A'B' / AB = \sqrt{[(\delta\phi)^2 + (\delta\psi)^2]} / \sqrt{[(\delta x)^2 + (\delta y)^2]}$$

Joukowski's transformation is conformal and, as shown in below, it can be expressed in the form -

$$\left.\begin{aligned} \phi &= (r + a^2/r) \cos\theta \\ 4 &= (r - a^2/r) \sin\theta \end{aligned}\right\} \qquad \text{(ii)}$$

where r and θ are the polar coordinates in the z-plane. For a circle in the z-plane, radius a and centre the origin, r = a (constant). Thus from (ii) the coordinates in the w-plane are

$$\phi = 2a\cos\theta \qquad \text{and } \psi = 0$$

As θ varies from 0° to 180°, φ varies from 2a to - 2a. Thus the given circle transforms to a straight line (that part of the φ-axis in the range ± 2a).

Provided certain conditions are fulfilled a conformal transformation converts the entire flow pattern in the z-plane to that in the w-plane. In Joukowski's transformation the second term (a^2/z)is small when z is large and thus at great distances from the origin w = z. This means that the flow of the undisturbed stream is unaltered by the transformation. In the present case it related the flow past a circular cylinder to that past a flat plate set parallel to the stream as shown if fig..

Let U be the velocity of the undisturbed stream in each plane. Then the velocity at any point on the plate (in the w-plane) is

$$q' = U$$

At any point on the surface of the cylinder, r = a and hence, by the relationship given in the question, the velocity on the surface of the cylinder is -

$$\begin{aligned} q &= q' \sqrt{(1 - 2\cos 2\theta + 1)} \\ &= U \sqrt{[2(1 - cs\, 2\theta)]} \\ &= U \sqrt{[2(2\sin^2\theta)]} \qquad (\text{since } \cos 2\theta = 1 - 2\sin^2\theta) \\ &= 2U \sin\theta \end{aligned}$$

<u>Derivation of (i)</u>

Joukowski's transformation relates the complex variables

z = x + iy and z = φ + iψ by the equation

$$w = z + a^2/z$$

Substituting for w, z and 1/z we have

$$\phi + i\psi = r(\cos\theta + i\sin\theta) + (a^2/r)(\cos\theta - i\sin\theta)$$

$$= (r + a^2/r)\cos\theta + i\,(r - a^2/r)\sin\theta$$

Equating real and imaginary parts

$$\phi = (r + a^2/r)\cos\theta \quad ; \quad \psi = (r - a^2/r)\sin\theta$$

The magnification of the transformation is the ratio of the length of small corresponding vectors. Using the definition of magnification ratio we have,

$$\text{Magnification} = \frac{\sqrt{[(\delta\phi)^2 + (\delta\psi)^2]}}{\sqrt{[(\delta x)2 + (\delta y)^2]}} = \frac{|\delta w|}{|\delta z|}$$

or $\dfrac{|dw|}{|dz|}$ in the limiting case

If a transformation is applied to a fluid motion, the distances between streamlines are increased by the magnification factor and the velocities are correspondingly decreased. Thus if q and q' are the velocities at corresponding points in the z - and w-planes respectively -

$$q = q' \frac{|dw|}{|dz|}$$

In the case of Jouowski's transformation -

$$\begin{aligned} dw/dz &= 1 - a^2/z^2 \\ &= 1 - (a^2/r^2)(\cos\theta - i\sin\theta)^2 \\ &= 1 - (a^2/r^2)(\cos^2\theta - 2i\cos\theta\sin\theta - \sin^2\theta) \\ &= 1 - (a^2/r^2)(\cos^2\theta - \sin^2\theta) + 2i\,(a^2/r^2)(\cos\theta\sin\theta) \\ &= 1 - (a^2/r^2)\cos 2\theta + i\,(a^2/r^2)\sin 2\theta \end{aligned}$$

The modulus of this expression is -

$$\begin{aligned} \frac{|dw|}{|dz|} &= \sqrt{\left\{ \left(1 - \frac{a^2}{r^2}\cos 2\theta\right)^2 + \left(\frac{a^2}{r^2}\sin 2\theta\right)^2 \right\}} \\ &= \sqrt{[\,1 - 2(a^2/r^2)\cos 2\theta + (a^4/r^4)\cos^2 2\theta + (a^4/r^4)\sin^2 2\theta\,]} \\ &= [\,1 - 2(a/r)^2\cos 2\theta + (a/r)^4\,] \end{aligned}$$

The velocities are therefore related by -

$$q = q'\sqrt{[1 - 2(a/r)^2\cos 2\theta + (a/r)^4]}$$

4. A long flat plate of constant width c is set normally to a stream flowing at speed u_0. Assuming the flow irrotational, obtain an expression giving the distribution of normal pressure on the plate and hence locate two lines on the plate along which the pressure is the same as that of the undisturbed stream.

Fig.(a) shows the flow past a circle in the z-plane (radius a and centre the origin) in the which the undisturbed velocity u_0 is parallel to the y-axis. As in the previous example this circle can be transformed into a line length 4a in the w-plane. In the present case, however, this line is normal to the stream, as shown in Fig. (b).

Thus Joukowski's transformation can be used to obtain the flow past a flat plate set normal to the stream from that past a circular cylinder. Let P be any point on the cylinder whose polar coordinate is θ, measured from the x-axis. Its angular displacement from the front stagnation point is $(90° - \theta)$ and the velocity at P is

$$q = 2u_o \sin(90° - \theta) = 2u_o \cos\theta$$

At any point on the surface of the cylinder $r = a$ and thus by the relationship given in the previous question,

(i) $$q = q' \sqrt{(1 - 2\cos 2\theta + 1)}$$

or $$2u_0 \cos\theta = q' \sqrt{[2(1 - \cos 2\theta)]}$$

$$= q' (2\sin\theta)$$

and the velocity at any point on the plate ($\psi = 0$) is

$$q' = 2u_0 \cos\theta / (2\sin\theta) = u_0 \cos\theta / \sqrt{(1 - \cos^2\theta)} \qquad \text{(i)}$$

Also, putting $r = a$ in (ii) of previous problem, $\phi = 2a \cos\theta = (c/2)\cos\theta$ (since $c = 4a$, the width of the plate). Substituting in (i),

$$q' = \frac{u_0(2\phi/c)}{\sqrt{[1 - (2\phi/c)^2]}} = \frac{2u_0}{\sqrt{(c^2 - 4\phi^2)}} \qquad \text{(ii)}$$

If p is the pressure at the point (ϕ, 0) on the plate and p_1 is the pressure of the undisturbed stream then, by Bernoullis equation,

$$p + \tfrac{1}{2}\rho q'^2 = p_1 + \tfrac{1}{2}\rho u_0^2$$

and, using (ii), the pressure coefficient is

$$C_p = (p - p_1)/\tfrac{1}{2}\rho u_0^2 = 1 - q'^2/u_0^2$$

$$= 4\phi^2/(c^2 - 4\phi^2)$$

The variation of C_p is shown in fig. 8(c). Putting $C_p = 0$, we have -

$$1 - 4\phi^2/(c^2 - 4\phi^2) = 0$$

or $$8\phi^2 = c^2$$

and $$\phi/c = 1/\sqrt{8} = 0.3535$$

Hence the lines along which the pressure equals that of the undisturbed stream are parallel to the centre line of the plate and 0.3535c from it. The pressure distributions on the front and back of the plate are identical and there is no resultant drag.

5. A long elliptic cylinder of thickness ratio 1/7 is set at zero incidence in an airstream with velocity 60 m/sec. Calculate the pressure difference between the stagnation point and the point of maximum thickness. Assume irrotational flow.

Suppose Joukowski's transformation is applied to a circle in the z-plane, centre at origin, whose radius b is greater than a (the radius of transformation) as shown in fig.9(a). For this circle r = b (constant) and, by (ii)of problem 3 , the coordinates in the w-plane are -

$$\phi = (b + a^2/b)\cos\theta \quad \text{and} \quad \psi = (b - a^2/b)\sin\theta$$

or $$\frac{\phi}{b + a^2/b} = \cos\theta \text{ and } \frac{\psi}{b + a^2/b} = \sin\theta$$

Squaring and adding

$$\frac{\phi^2}{(b + a^2/b)^2} + \frac{\psi^2}{(b + a^2/b)^2} = \cos^2\theta + \sin^2\theta = 1$$

This is the equation of an ellipse in which $b + a^2/b$ and $b - a^2/b$ are the semi-axes and the thickness ratio is -

$$\frac{\text{minor axis}}{\text{major axis}} = \frac{2(b - a^2/b)}{2(b + a^2/b)} = \frac{b^2 - a^2}{b^2 + a^2} \qquad \text{(i)}$$

It follows that the flow past a circular cylinder transforms to that past an elliptic cylinder at zero incidence with the same undisturbed velocity U. Using the thickness ratio given in the question we have, from (i),

$$(b^2 - a^2) / (b^2 + a^2) = 1/7 \qquad \text{or} \quad b^2/a^2 = 4/3 \qquad \text{(ii)}$$

The maximum thickness of the elliptic cylinder occurs at $\phi = 0$. By (ii) of problem 3 the corresponding point of the circular cylinder is given by $\theta = 90°$ and the velocity there is $q = 2U \sin\theta = 2U = 120$ m/sec. Substituting in and using (ii) the velocity q' at the maximum thickness point of the elliptic cylinder is given by -

$$q = q' \sqrt{[1 - 2 (a/b)^2 \cos 180° + (a/b)^4]}$$

or $$120 = q' \sqrt{[1 + (2 \times ¾) + (3/4)^2]}$$

$$= 7q'/4$$

Thus $q' = 4/7 \times 120 = 68.57$ m/sec.

At the stagnation point the velocity is zero and hence by Bernoulli's equation the pressure difference between the stagnation point and the point of maximum thickness is -

$$½ \rho q'^2 = ½ \times 0.125 \times (68.57)^2$$

$$= 293.86 \text{ kg/m}^2$$

6. Obtain an expression for the thickness ratio of the symmetrical aerofoil section transformed from a circle of radius b by means of the formula $w = z + a^2/z$

Consider a circle whose centre is on the x-axis at a small distance from the origin 0 as shown in fig. (b). Suppose its radius b is slightly greater than a and is such that it intersects the negative branch of the x-axis at the same point as the circle of transformation. The application of Joukowski's transformation results in a symmetrical aerofoil section in the w-plane.

Suppose a quantity m is defined by the relationship -

$$b = a(1 + m) \qquad \text{(i)}$$

Then, since b is only slightly greater than a, m is small and m^2, m^3, ….will be neglected in comparison with 1. Referring to Fig. (b) and using polar coordinates we have, in the z-plane -

$$PN = r\sin\theta, \qquad ON = r\cos\theta$$

$$OC = b - a = a(1 + m) - a = am$$

and $$CN = ON - OC = r\cos\theta - am$$

Applying Pythagoras' theorem to triangle PCN,

$$(PC)^2 = (PN)^2 + (CN)^2$$

or $$b^2 = (r\sin\theta)^2 + (r\cos\theta)^2 + (r\cos\theta - am)^2$$

and, using (i),

$$a^2(1 + m)^2 = r^2\sin^2\theta + r^2\cos^2\theta - 2amr\cos\theta + a^2m^2$$

or $$a^2(1 + 2m) = r^2 - 2amr\cos\theta$$

from which $(r/a)^2 - 2m\cos\theta.\ (r/a) - (1 + 2m) = 0$

This is a quadratic equation for r/a and, using the binomial theorem and neglecting m^2, m^3,… in comparison with 1, the solution is -

$$\frac{r}{a} = \frac{2m\cos\theta \pm \sqrt{[4m^2\cos^2\theta + 4(1 + 2m)]}}{2}$$

$$= m\cos\theta \pm (1 + 2m)^{1/2}$$

$$= m\cos\theta \pm (1 + \tfrac{1}{2}2m + \ldots)$$

$$= 1 + m(1 + \cos\theta) \qquad \text{(ii)}$$

(taking the plus sign because r is greater than a). From (ii), to the same degree of approximation -

$$a/r = 1/[1 + m(1 + \cos\theta)] = [1 + m(1 + \cos\theta)]^{-1}$$

$$= 1 - m(1 + \cos\theta) \qquad \text{(iii)}$$

Adding (ii) and (iii), $r/a + a/r = 2$

Subtracting (iii) from (ii),

$$r/a - a/r = 2m(1 + \cos\theta)$$

Hence, using (ii) of problem (3) the coordinates of the section in the w-plane are

$$\phi = (r + a^2/r) \cos\theta = a(r/a + a/r) \cos\theta$$
$$= 2a \cos\theta \qquad \text{(iv)}$$
$$\psi = (r - a^2/r) \sin\theta = a(r/a - a/r) \sin\theta$$
$$= 2am(1 + \cos\theta) \sin\theta \qquad \text{(v)}$$

The leading and trailing edges are given by $\theta = 0°$ and $\theta = 180°$ respectively. To the present degree of approximation these values give $\phi = \pm 2a$, and hence the chord length is 4a.

The maximum ordinate is found by differentiating (v).

Thus $d\psi/d\theta = 2am[(1 + \cos\theta) \cos\theta + \sin\theta(-\sin\theta)] = 0$

This is zero if $\theta = 60°$ or $180°$ but the latter value refers to the trailing edge. Putting $\theta = 60°$, $\cos\theta = ½$ and $\sin\theta = \sqrt{3}/2$. Substituting in (v) the maximum ordinates is -

$$\psi_{max} = 2am(1 + \cos 60°) \sin 60°$$
$$= 2am(1 + ½)\sqrt{3}/2$$
$$= 3\sqrt{3}am/2$$

Since the section is symmetrical the thickness ratio is -

$$2\psi_{max} / \text{chord} = 3\sqrt{3}am/4a = 3\sqrt{3}m/4$$
$$\text{or } 1.3m \text{ (approximately)} \qquad \text{(vi)}$$

7. The thickness ratio of a symmetrical Joukowski section is 0.156. Estimate the pressure coefficient midway along the chord for two-dimensional incompressible flow at zero incidence.

Using (vi) of the previous problem and given the thickness ratio, we have -

$$1.3m = 0.156 \text{ and } m = 0.12 \qquad \text{(i)}$$

Hence, $b = a(1 + m) = 1.12a$ (ii)

The equation of the circle in the z-plane is, see fig. (b)

$$(x - OC)^2 + y^2 = b^2$$

or $$(x - am)^2 + y^2 = b^2 \qquad \text{(iii)}$$

At the mid-chord point of the Joukowski section $\phi = 0$ and from (iv) of previous problem, $\theta = 90°$. At the corresponding point in the z-plane, x= 0 and,from(i)and (iii)

$$y^2 = b^2 - a^2m^2 = a^2(1 + m)^2 - a^2m^2$$
$$= a^2(1 + 2m)$$
$$= 1.24a^2$$

or $$y = 1.114a \qquad \text{(iv)}$$

θ now refers to $\angle PCN$, the velocity at the point P is -

$$q = 2U \sin \angle PCN = 2U.\ PN / CP = 2U.\ y/b$$

and with the values of b and y from (ii) and (iv) -

$$q = 2U.1.114a/1.12a = 1.99U \qquad \text{(v)}$$

As in the previous example $a/r = 1 - m(1 + \cos\theta)$, and , if $\theta = 90°$,

$$a/r = 1 - m = 0.88$$

Thus, from (i) of problem (3), the velocities at the corresponding points in the two planes are related by -

$$q = q' \sqrt{[1 - 2 \times (0.88)^2 \times (-1) + (0.88)^4]}$$
$$= 1.774q'$$

Hence, using (v),

$$q' = q/1.774 = 1.99U/1.774 = 1.122U$$

and, by Bernoulli's theorem, the pressure coefficient is -

$$C_p = (p - P)/ \tfrac{1}{2}\rho U^2 = 1 - (q'/U)^2$$
$$= 1 - (1.122)^2$$
$$= -0.259$$

8. Explain how the Joukowski transformation is used to obtain a circular are aerofoil and obtain an expression for its radius.

Write down expressions for the coordinates of the general Joukowski aerofoil and explain briefly why sections of this type are not used in modern aircraft.

Suppose, fig.(c) of problem (5), the centre of the circle to be transformed lies on the y-axis, the radius b being such that the circle intersects the x-axis at the same points as the circle of transformation. Let $\angle CBO = \beta$. Then, from triangle CBO -

$$OC = b \sin\beta = a \tan\beta$$

$$CN = ON - OC = r \sin\theta - b \sin\beta$$

$$NP = r \cos\theta$$

Applying Pythagoras' theorem to triangle CNP,

$$(r \cos\theta)^2 + (r \sin\theta - b \sin\beta)^2 = b^2$$

$$r^2(\cos^2\theta + \sin^2\theta) - 2rb \sin\theta \sin\beta + b^2 \sin^2\beta = b^2$$

$$r^2 - 2rb \sin\theta \sin\beta = b^2 (1 - \sin^2\beta)$$

$$= b^2 \cos^2\beta$$

$$= a^2$$

Hence $r^2 - a^2 = 2rb \sin\theta \sin\beta$

or, dividing by r, $r - a^2/r = 2a \sin\theta \tan\beta$

Thus, using (ii) of problem 3, the ordinate of the resulting section is -

$$\psi = (r - a^2/r) \sin\theta = 2a \sin^2\theta \tan\beta \qquad \text{(i)}$$

If θ is replaced by $-\theta$ in this result, the ordinate is unchanged. Hence the result of the transformation is a single curve as shown in fig. (c). The maximum value of ψ occurs when $\theta = 90°$ and $\sin\theta = 1$. Thus the centre-line camber is -

$$\psi_{max} /\text{chord length} = (2a \tan\beta)/4a = \tfrac{1}{2} \tan\beta \qquad \text{(ii)}$$

It can be shown that the transformed curve is a circular arc. Suppose its radius is R. Then applying Pythagoras' theorem to a triangle formed by the centre of the circular arc, the origin in the w-plane and one end of the arc, we have -

$$R^2 = (R - \psi_{max})^2 + (2a)^2$$

or $2R\psi_{max} = (\psi_{max})^2 + 4a^2$

and $R = \psi_{max}/2 + 2a^2/\psi_{max}$

$$= [a(\tan\beta)/2 + 2a^2/ (2a \tan\beta)$$

$$= [a(\tan^2\beta + 1)]/\tan\beta$$

$$= a \sec^2\beta/\tan\beta \qquad (\text{since } 1 + \tan^2\beta = \sec^2\beta)$$

$$= a/(\cos\beta \sin\beta)$$

$$= 2a \operatorname{cosec} 2\beta \qquad (\text{since } \cos\beta \sin\beta = \tfrac{1}{2} \sin 2\beta)$$

The result of transforming a circle whose centre is slightly displaced from both axes is a cambered aerofoil, fig. (d). It can be sown that (to the same degree of approximation as before) the ψ coordinate at any point is the sum of the ordinates due to the separate displacements of the centre C from the x- and y-axes.

Thus, the coordinates of any point on the cambered aerofoil are -

From (iii) problem (6) $\quad \phi = 2a \cos\theta$

From (iv) of problem (6) $\quad \psi = 2am(1 + \cos\theta)\sin\theta + 2a\sin^2\theta\tan\beta$

$$= 2a\sin\theta\,[m(1+\cos\theta) + \beta\sin\theta]$$

since, for small angles, $\tan\beta = \beta$ rad (approximately).

The thickness ratio is 1.3m and the centre-line camber is ½ β. Joukowski aerofoils have no particular advantages over any other type apart from the comparative simplicity with which they are derived. All sections of this family have a <u>cusped trailing edge, i.e. the upper and lower surfaces have a common tangent there.</u> This makes practical construction difficult but small modifications near the trailing edge have little effect on the theoretical properties.

With some other transformation Karman-Treftiz and von Mises the upper and lower surface have distinct tangents at the trailing edge.

In irrotational flow the drag of the aerofoil is zero but in a real fluid this is no longer true. Recent investigations have shown that some new families of aerofoils are far superior to the Joukowski and there "conventional" types in the matter of profile drag. For this reason the simple Joukowski sections are no longer used but conformal transformation (in an extended form) is still a powerful method of aerofoil design.

The problem of finding a transformation which will enable a given section to be obtained from a circle is more difficult than that of determining the section when the transformation is given. In general it can only be solved by successive approximations. It is possible by such methods to determine the pressure distribution round any given section but recent work has been directed towards deriving a section which will possess a given pressure distribution.

9. Calculate the lift coefficient of a thin Joukowski aerofoil, camber 0.05, at an incidence of 7°, explaining your method.

By (i) problem 3 a stagnation point S in the z-plane ($q = 0$) transforms to a stagnation point in the w-plane ($q' = 0$), unless q'/q is infinite. In the case of the symmetrical aerofoil (fig. problem 5), S coincides with B and S' with B', the trailing edge. For the unsymmetrical cases (fig. c & d of problem 5), S' is a head of trailing edge B'. The flow is then of the type illustrated in fig. problem 6. At the trailing edge $r = a$, $\theta = 180°$, and by (i) of problem 3, $q/q' = 0$. if q has a finite value at B then q' is infinite at B'. (A point at which q/q' is zero or infinite is called a singular point and the transformation ceases to be conformal there.)

In order that q' is finite at B', q must be zero at B. This condition can be obtained by adding a circulation k to the flow past the circle in the z-plane of such magnitude that S coincides with B. It follows that S' coincides with the trailing edge. This method of specifying the magnitude of the circulation is known as Joukowski's hypothesis.

As shown in Fig of problem 5 (c) and (d), the stagnation point S on the cylinder must be displaced through an angular distance β. From above, in which a is now replaced by b and θ by β, the required circulation is -

$$k = 4\pi bU \sin\beta \qquad \text{(i)}$$

This result applies to zero incidence and the effect of incidence on lift is most easily found by supposing the aerofoil fixed and the undisturbed stream inclined at the required angle α as shown in fig. It follows that the stagnation point must now be displaced by $\angle SCB = \alpha + \beta$ and the circulation according to Joukowski's hypothesis is, from (i),

$$k = 4\pi bU \sin(\alpha + \beta)$$

$$= 4\pi bU(\alpha + \beta) \qquad \text{(ii)}$$

since, for small angles, $\sin(\alpha + \beta) = (\alpha + \beta)$ radian approximately.

The circulation is the same for the aerofoil and its lift per unit length is ρUK. The corresponding lift coefficient is

$$C_L = \rho Uk/(\tfrac{1}{2}\rho U^2.c) = \rho U.4\pi bU(\alpha + \beta)/(\tfrac{1}{2}\rho U^2.4a)$$

$$= 2\pi(\alpha + \beta) \qquad \text{(iii)}$$

since a and b are approximately equal. For a given aerofoil β is constant and the theoretical lift slope is -

$$dC_L/d\alpha = 2\pi$$

With the data given in the question $\tfrac{1}{2}\beta = 0.05$ and $\beta = 0.1$, $\alpha = 0.1222$ radian. Thus, from (iii) -

$$C_L = 2\pi(0.1222 + 0.1) = 1.396$$

10. Explain, with the aid of diagrams, how to obtain a stream line of the irrotational flow past a Joukowski aerofoil of symmetrical section from a given streamline of the flow past a circular cylinder.

Joukowski's transformation is $w = z + a^2/z$ where w is the complex coordinate in the aerofoil plane and z is the complex coordinate in the circle plane.

Suppose P is any point on a streamline of the flow past the circle in the z-plane, as shown in fig. Then z, the complex coordinate of P is represented by the vector OP. Suppose Q is a point such that OQ represents a^2/z. OP and OQ therefore correspond to the two terms in the transformation formula. If OP and OQ are added vectorially the resulting vector OP' represents the complex coordinate w, and P' is therefore the transformation of the point P.

It is shown in previously that the length of OQ is a^2/OP and that OP and OQ are on opposite sides of the x-axis and equally inclined to it. The geometrical construction for P' is as follows -

Draw OQ, of length a^2/OP, so that P and Q are on opposite sides of the x-axis and $\angle POx = \angle QOx$. Complete a parallelogram with OP and OQ as adjacent sides. Then P', the transformation of P, is the opposite end of the diagonal through O.

By repeating the construction for a series of points the required streamline is obtained. In Fig. the w-plane and z-plane are shown together. By applying the construction to the circle, centre C and radius b, the aerofoil itself can be drawn and, in this case, it is found that the points such as Q lie on a circle. This auxiliary circle is first drawn by locating one or two positions of Q. It passes through B and, in the symmetrical case, its centre lies on the x-axis.

11. When will conformal transformation break down? When will Joukowski transformation fails?

Conformal transformation is the best choise for 2D flows of low speeds. This can not be extended to 3D flows or flows of high speeds.

Joukowski aerofoils have no direct use in practical design of aerofoils. The overall lift is proportional to the circulation generated and the magnitude of circulation should be such as to keep the velocity finite of the trailing edge (Kuta condition).

12. Use Blasius' theorem, to determine the force on a circular cylinder in a uniform stream, with circulation Γ.

The flow is simulated by a uniform stream of velocity U, a doublet of strength equal to $2\pi Ua^2$, at the origin, with its axis in the negative x direction, and a vortex of strength - Γ at the origin. The complex potential for the whole flow is therefore given by

$$w = +Uz + \frac{Ua^2}{z} - \frac{i\Gamma}{2\pi} \ln z$$

$$\frac{dw}{dz} = + U\left(1 - \frac{a^2}{z^2}\right) - \frac{i\Gamma}{2\pi z}$$

$$\left(\frac{dw}{dz}\right)^2 = \frac{i\Gamma}{\pi z} \quad \text{plus other terms which are powers of z.}$$

Now

$$\oint_c z^n dz = \left(\frac{1}{n+1} z^{n+1}\right)_c = 0, \qquad \text{if } n \neq -1$$

$$= [\ln z]c = [\ln re\ i\theta]_c = [\ln r]_c + [i\theta]^{2\pi} =_c 2\pi i, \text{ if } n = -1$$

Thus all the powers of z in the above expression for $(dw/dz)^2$ contribute nothing when integrated round c, except for the term in 1/z. Thus

$$\oint_c \left(\frac{dw}{dz}\right)^2 dz = 2\pi i \frac{iU\Gamma}{\pi} = -2U\Gamma$$

so that $X - iY = ½ i\rho\ (-2U\Gamma) = - i\rho U\Gamma$.

Hence $X = 0$, which is the paradox of D' Alembert,

and $Y = \rho U\Gamma$, which is the Kutta - Joukowski result.

Also

$$z\left(\frac{dw}{dz}\right)^2 = \frac{2U^2a^2}{z} + \frac{\Gamma^2}{4\pi^2 z}$$

plus other powers of z. Thus the term in 1/z is real, so that

$$\oint_c z\left(\frac{dw}{dz}\right)^2 dz$$

is imaginary. Thus M = 0. This implies that the lift acts through the origin.

or

$$\therefore \frac{dw}{dz} = U\left(1 - \frac{a^2}{z^2}\right) - \frac{i\Gamma}{2\pi z}$$

$$\left(\frac{dw}{dz}\right)^2 = U^2 \left(1 - \frac{a^2}{z^2}\right)^2 + \frac{i^2\Gamma^2}{4\pi^2 z^2} - 2U\left(1 - \frac{a^2}{z^2}\right)\frac{i\Gamma}{2\pi z}$$

$$= U^2 - \frac{iU\Gamma}{\pi z} - \left(2U^2a^2 - \frac{\Gamma}{4\pi^2}\right)\frac{1}{z^2} + \frac{iUa^2\Gamma}{\pi z^3} + \frac{U^2a^2}{z^4}$$

$$= A_0 + \frac{A_1}{z} + \frac{A_2}{z^2} + \ldots$$

whence $A_1 = -\dfrac{iU\Gamma}{\pi}$, $A_2 = -\left(2U^2a^2 - \dfrac{\Gamma}{4\pi^2}\right)$

$$X - iY = -\pi\rho A_1 = \pi\rho\frac{iU\Gamma}{\pi} = i\rho U\Gamma$$

$$\therefore X = 0, \ Y = -\rho U\Gamma$$

that is, the drag force is zero and the lift force is $-\rho U\Gamma$. When U is positive and Γ is negative (clockwise circulation), Y is positive, that is, upwards.

$M + iN = -i\pi\rho A_2$ (wholly imaginary)

$\therefore$ moment M is zero.

13. Use Blasius theorem to prove Kutta – Jowkowski equation for an aerofoil. (Two-dimensional flow past a profile of any cross section, with circulation)

The velocity at a great distance from the cylinder being U, the complex velocity for the pattern of flow around the cylinder can be written

$$\frac{dw}{dz} = Ue^{i\alpha} + \frac{A}{z} + \frac{B}{z^2} + \ldots$$

where α is the angle of incidence or angle of attack

$$\therefore w = Ue^{i\alpha} z + A \ln z - \frac{B}{z} + \ldots$$

since there is a clockwise circulation K, the resulting component in w must be

$$-\frac{iK}{2\pi} \ln z$$

$$\therefore \quad A = -\frac{iK}{2\pi}$$

$$\therefore \left(\frac{dw}{dz}\right)^2 = U^2 e^{2i\alpha} - \frac{iKUe^{i\alpha}}{\pi z} - \ldots = A_0 + \frac{A_1}{z^2} + \frac{A_1}{z^2} + \ldots$$

from Blasius's theorem, since $A_1 = -\dfrac{iKUe^{i\alpha}}{\pi}$

$$X - iY = -\pi\rho A_1 = \pi\rho \frac{iKUe^{i\alpha}}{\pi} = iK\rho Ue^{i\alpha}$$

If $\alpha = 0$, irrespective of the shape or orientation of the profile

$$X = 0, \; Y = -\rho UK = -\rho U\Gamma$$

14. Using Joukowki transformation how a circle in x-y plane is transformed to a flat plate in w-plane.

(1) How should the circle be located with respect to coordinate axes (ii) What will be the chord length (iii) Determine (a) velocity and (b) pressure distribution after application of Kutta trailing edge condition.

Joukowski transformation is $w = z + a^2/z$ where a is the radius of circle of transformation in z-plane. It r, θ are the polar coordinates in z-plane corresponding ϕ, ψ coordinates in w-plane in terms of polar coordinates may be evaluated as follows

$$\begin{aligned} z &= x + iy \\ &= r(\cos\theta + i\sin\theta) \\ w &= z + a^2/z \\ &= r(\cos\theta + i\sin\theta) + a^2/r(\cos\theta - i\sin\theta) \\ &= (r + a^2/r)\cos\theta + i\,(r - a^2/r)\sin\theta \\ &= \phi + i\,\psi \end{aligned}$$

Coordinates of w-plane are

$$\phi = (r + a^2/r)\cos\theta \text{ and } \psi = (r - a^2/r)\sin\theta$$

For the circle in the z-plane, radius a and centre at origin r = a and the coordinates in w-plane becomes

$$\phi = (a + a^2/a)\cos\theta = 2a\cos\theta$$

$$\psi = (a - a^2/a)\sin\theta = 0$$

As θ varies from 0 the π, ϕ varies from 2a to –2a. Thus the circle in z-plane transforms to a straight line (the part of ϕ - axis in the range $\pm$ 2a)

(i) The circle is located at origin in z-plane with respect to (r, θ) coordinates for transformation to w-plane.

(ii) The chord length of flat plate is 4 times the radius of the circle transformed.

Velocity and pressure distribution

a) $w = z + a^2/z$

$$\frac{dw}{dz} = 1 - a^2/z^2$$

$$= 1 - a^2/r^2 (\cos\theta - i \sin\theta)^2$$

$$= 1 - a^2/r^2 \cos 2\theta + i\, a^2/r^2 \sin 2\theta$$

$$= u + i v$$

$$\therefore \quad u = 1 - a^2/r^2 \cos 2\theta \text{ ; and } v = a^2/r^2 \sin 2\theta$$

$$V = \sqrt{u^2 + v^2}$$

$$= \sqrt{(1 - a^2/r^2 \cos 2\theta)^2 + (a^2/r^2 \sin 2\theta)^2}$$

$$= \sqrt{1 + (a^2/r^2 \cos 2\theta)^2 - 2\, a^2/r^2 \cos 2\theta + (a^2/r^2 \sin 2\theta)^2}$$

$$= \sqrt{1 - 2a^2/r^2 \cos 2\theta + (a^2/r^2)^2}$$

This represents the velocity distribution on the transformed flat plate.

b) If U is the free stream velocity at an undisturbed point

$$C_p = 1 - (V/U)^2$$

$$= 1 - \frac{1 - 2\, a^2/r^2 \cos 2\theta + (a^2/r^2)^2}{U^2}$$

This represents the pressure distribution on the transformed flat plate.

15. Indicate clearly how the circle is to be placed for Joukowski transformation to get (a) symmetrical Joukowski aerofoil profile. (b) circular are aerofoil (c) cambered aerofoil.

a) Symmetrical Joukowski aerofoil profile

Circle of radius b, slightly greater than a is placed with its centre C on x-axis at a small positive distance from origin such that OC = b – a = a x m where m is small. This circle will be tangential at negative x-axis.

b) Circular are aerofoil :

Circle of radius b, slightly greater than a is placed with its centre C on y-axis at a small positive distance from origin so that both these circles intersect a distance of +a and –a on x-axis.

c) Cambered aerofoil :

Circle of radius b, slightly greater than a is placed with its centre C, slightly displaced at a positive distance from both axes.

18. How circulation and lift coefficient over cambered Joukowski aerofoil profile is determined.

For a cylinder at origin of radius a with uniform velocity U and circulation strength k tangential velocity of fluid on the surface of cylinder at angle $\theta = 2U \sin\theta - k / 2\pi a$.

At stagnation point the tangential velocity is also zero.

$\therefore \quad k \quad = \quad 4\pi aU \sin\theta$

To obtain cambered Joukowski aerofoil profile, cylinder radius is changed from a to b and centre is shifted from origin. Hence the stagnation point on the cylinder is displaced through an angular distance β. Hence the required circulation strength

$k = 4\pi\, b\, U \sin\beta$.

If the undistured stream is at an angle of incidence α the displacement of stagnation point is by $\alpha + \beta$.

$$\therefore \quad k = 4\pi\, b\, U \sin(\alpha + \beta)$$

Using Kutta – Joukowski equation equating lift per unit span of aerofoil

$$L = C_L\, \rho/2\, (4a \times 1)\, U^2 = \rho^{Uk}$$

$$= \rho^{U\, 4\pi\, b\, U \sin(\alpha + \beta)}$$

Taking $a \approx b$ and $\sin(\alpha + \beta) = (\alpha + \beta)$

$$C_L = 2\pi\, (\alpha + \beta)$$

CONFORMAL TRANSFORMATION

UNIT - III

Complex variables - Application in fluid machines

Complex variables and its analysis is a powerful analytical method of determining two-dimensional patterns of irrotational flow. This approach extends greatly the range of boundary forms which can be treated and provides, for each pattern, a single expression embodying both the stream function and the potential function. The method is known as conformal transformation or conformal mapping and it requires some knowledge of complex variable theory, elements of which are introduced below.

3.1 Complex numbers

A number consisting of two distinct scalar parts, a and b, and written in the form a+ ib, where i is $\sqrt{-1}$, is called a complex number. The first part, a, is said to be the real part and the second part, b, the imaginary part of the number. The distinctive names of the parts, and the use of I, may be regarded as devices for maintaining the separate identities of the two parts in various mathematical operations.

Complex numbers occur in the solution of some algebraic equations. For example the two roots of the quadratic equation $x^2 - 4x + 13 = 0$ are the complex numbers $x = 2 \pm 3i$. We shall be concerned here with the use of complex numbers in specifying the location of points on planes. In the Argand diagram (fig.3) the position of any point is specified by one variable, z, instead of the two variables x and y of Cartesian geometry. We regard z as a complex number whose real part is x and whose imaginary part is y. This complex number, which really represents the position vector $\vec{Oz}$, is therefore

$$z = x + iy \tag{3.1}$$

The modulus, or absolute value, of z is the magnitude, Oz, of the position vector and it is designated by r, and sometimes by |z|.

Hence

$$r = |z| = \sqrt{(x^2 + y^2)} \tag{3.2}$$

The argument of z is the direction, θ, of the position vector measured from the positive x-axis in an anticlockwise direction. It therefore has the value

$$\theta = \tan^{-1} \frac{y}{x} \tag{3.3}$$

and it is normally restricted to the range $-\pi < \theta < \pi$.

Alternative modes of expressing z result from the substitution of polar co-ordinates (r, θ) in Eq. 3.1

$$z = r\cos\theta + ir\sin\theta = r(\cos\theta + i\sin\theta)$$

and, since

$$e^x = [1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{i\theta} = 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \dots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots\right)$$

$$= \cos\theta + i\sin\theta$$

$$\therefore \quad r(\cos\theta + i\sin\theta) = re^{i\theta}$$

The three forms for expressing z are thus

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta} \tag{3.4}$$

It follows that

$$z^n = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$$

For two complex numbers to be equal there must be equality of the real parts and also of the imaginary parts, that is, equality of moduli and also of arguments. It is meaningless to state that one complex number is greater than another. Such a statement can only be made regarding corresponding parts of two complex numbers.

3.1 Functions of a complex variable

If x and y are variables, the complex number $z = x + iy$ is called a complex variable. Suppose that another variable, w, is defined as, say, $w = z^2$ or $w = \ln z$, or $w = \cosh^{-1} z$. Then w is said to be a function of the complex variable z, that is, $w = f(z)$, and w itself is a complex variable with a real part and an imaginary part. The real part is usually designated φ and the imaginary part, ψ, both φ and ψ being, in general, functions of x and y. Hence

$$w = \phi + i\psi = f_1(x, y) + i f_2(x, y) \tag{3.6}$$

Just as $z = x + iy$ defines a point on a diagram which has x as abscissa and y as ordinate, so $w = \phi + i\psi$ defines a point on another diagram which has ϕ as abscissa and ψ as ordinate (fig. 3.2). The two diagrams are referred to as the z-plane and the w-plane respectively. For each point (x, y) on the z-plane, there will be a corresponding point (ϕ, ψ) on the w-plane, since the value of ϕ and ψ are each determined by the values x and y.

Similarly, for any line on the z-plane, there will be a corresponding line on the w-plane, changed in form in the ‘transformation ‘ from the z-plane to the w-plane. There is no question of geometrical projection from one plane to the other. The relationship is simply one of correspondence of points, a line being regarded as a series of points.

The family of lines $\phi = x^2 - y^2 =$ constant on the z-plane are rectangular hyperbolae, orthogonal to the lines ψ = constant; on the w-plane, the corresponding family of lines, with the same equations, ϕ = constant, are parallel to the ψ-axis and hence again orthogonal to the liens ψ = constant (figs. 3.5 and 3.3).

If ψ be regarded as a stream function and ϕ as the associated velocity potential function, the pattern of ψ - lines and ϕ-lines on the w-plane (fig.3.2) clearly represents uniform flow parallel to the ϕ-axis in the positive ϕ-direction. On the z-plane (fig. 3.3) the corresponding ψ- and ϕ-lines represent the pattern of irrotational flow at a 90° corner. For some function other than $w = z^2$, the z-plane pattern will be different from that of fig. 3.3

Whatever form the function $w = f(z)$ may take, the pattern on the w-plane is always that of parallel flow from left to right, as in fig. 3.2 and pattern on the z-plane is always the physical pattern under investigation. The function $w = f(z)$ may be regarded as transforming the z-plane pattern to the uniform flow pattern of the w-plane. Once the transforming function, $w = f(z)$ for a particular physical pattern, is known, its real part, $\phi = f_1(x,y)$, equated to a constant yields the equation ;of the equipotential lines in the physical or z-plane; and its imaginary part, $\psi = f_2(x, y)$, equated to a constant, yields the equation of the streamlines in the z-plane. Each line has its own particular constant.

A transformation $w = f(z)$ expressed in its inverse form, $z = f^{-1}(w)$ can be regarded as transforming the parallel flow of the w-plane to the pattern of flow on the z-plane. Consideration of the inverse form will frequently enable a visual concept of the z-plane pattern to be obtained, through the use of polar co-ordinates. For example, given the function $w = z^2$ the z-plane pattern is determined by examination of the inverse form $z = w^{1/2}$. If $z = re^{i\theta}$ and $w = r_1e^{i\theta}$, then $re^{i\theta} = r^{1/2}\ \rho^{\,i(\theta_1/2)}$ and, equating moduli and arguments,

$$r = r_1^{1/2}\ ,\ \theta = \frac{\theta_1}{2}$$

so that this transformation can be pictured as forming the z-plane pattern by 'folding' the upper half of the w-pattern clockwise about the origin in such a manner that all polar angles, θ_1, are halved; and at the same time 'shrinking' it differentially so that all polar distances, r_1, are reduced to the square roots of their original values (figs 3.2, 3.3). The streamline $\psi = 0$ ($A_1O_1B_1$) takes up the new position A O B and the pattern on the z plane is flow at a 90° corner, if we consider the pattern of one quadrant only. (In fact, in the complete z-plate pattern, each quadrant contains a corner flow pattern).

Of the infinite number of functions of the complex variable, many provide transformations such as the corner flow pattern, which are of practical interest to the hydraulic engineer.

3.3 Analytic functions

The condition that the transformed pattern of ϕ-lines in the z-plane does in fact represent a possible pattern of irrotational flow is that the function $\phi = f_1(x, y)$ satisfies the conditions of continuity and irrotationality or, in other words, that ϕ satisfies the Laplace equation which embodies these two conditions. Similarly, the ψ lines will represent a possible irrotational flow pattern if ψ satisfies the Laplace equation.

It will be seen in this section that these limitations on ϕ and ψ restrict w to a class of function known in the theory of the complex variable as analytic. The term 'function of a complex variable' is conventionally restricted, in fact, to analytic functions. It may be stated that a function w = f(z) may be analytic for all values of z, or possibly for all but one or some finite number of values of z; that is, for all points, or all but a finite number of points in the z-plane. The exceptions are called singularities or singular points. Hence the statement that function is analytic 'within a region' or domain implies that there are no singular points in the area under consideration.

A function, w = f(z), is said to be analytic within a region of the z-plane only, if for each point in that region (that is for each value of z):

(a) there is one and only one corresponding value of w and that value is finite, and

(b) $\frac{dw}{dz}$ is single - valued and neither zero nor infinite.

At singular points, where these conditions are not satisfied, the transformation process is not applicable, although at a very small distance from them it may be. In diagrams, singular points are frequently encircled, to indicate the fact that they are isolated from the transformation or mapping process. Examples of analytic functions with singular points are -

(i) $w = \frac{1}{z - a}$ which is analytic except at z = = a where w is infinite

(ii) w = 1n z which is analytic except at z = 0, where w is infinite

(iii) $w = z^2$, which is analytic except at z = 0 where $\frac{dw}{dz}$ is zero.

From a practical viewpoint, singular points are generally points of theoretically infinite or zero velocity on the physical plane. The points of infinite velocity include the centers of sources, sinks, vortices, and doublets, and sharp corners where boundaries are deflected away from the flow. At these points in a real fluid, not only are infinite velocities impossible but, with the necessarily high velocity gradient involved, viscous effects become appreciable and significant departures form the irrotational flow pattern, such as rotational vortex flow or separation, result.

The points of zero velocity are stagnation point such as those which occur where a streamline branches on the upstream face or edge of a submerged body; where two streamlines unite, near the downstream face or edge; or where at a sharp corner, a boundary is deflected into the flow. The pattern of stagnation flow is approached in some cases of real fluid flow such as in the flow near the tip of a pitot tube; whereas in others, such as in flow at a boundary corner, separation may result in a noticeable departure from the irrotational pattern.

It will now be demonstrated that a function which is analytic, that is, one-valued with a one-valued derivative, does yield a possible pattern of irrotational flow. Condition (a), that $w = \phi + i\psi$ should have one value for a given value of z, results in ϕ and ψ having only one value at a point P in the z-plane. This is in accord with the requirement that only one possible pattern of flow can exist for a given set of boundary conditions.

Condition (b), that the derivative $\frac{dw}{dz}$ should have only one value at any point P in the z-plane, results, as will be now shown in ϕ and ψ being solutions to the Laplace equation, which embodies the conditions that the continuity equation is satisfied and that flow is irrotational. Consider, again, the example of flow at a 90°corner (fig. 3.4 and 3.5). It can be seen that in the region of a point P on the z-plane, a small change, $\delta z = \delta x + i\delta y$, in z corresponds to a small change, $\delta w = \delta\phi + i\delta\psi$, in w on the w-plane. These changes have both magnitude and direction. The relationship between δw and δz can be expressed in terms of the rate of change of w with z, thus

$$\frac{dw}{dz} = \lim_{\delta z \to 0} \frac{\delta w}{\delta z} = \lim_{\delta z \to 0} \frac{\delta\phi + i\delta\psi}{\delta x + i\delta y}$$

Condition (b) requires that, at any point, $\frac{dw}{dz}$ shall have only one value, whatever the direction of δz. It can be shown that it will have only one value for all directions, provided that it has the one value for any pair of directions at right angles to one another, for example the x-and y-directions, at any point P (Fig. 3.4) Hence condition (b) will be satisfied if $\frac{dw}{dz}$ has the same value, in the limit, for $\delta z_1 = \delta x + i0 = \delta x$ as it has for $\delta z_2 = 0 + i\delta y = i\delta y$.

In the first case

$$\frac{dw}{dz} = \lim_{\delta z_1 \to 0} \frac{\delta w}{\delta z_1} = \lim_{\delta x \to 0} \frac{\delta w}{\delta x} = \frac{\partial w}{\partial x} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x} \qquad 3.7a$$

$$\frac{dw}{dz} = \lim_{\delta z_2 \to 0} \frac{\delta w}{\delta z_2} = \lim_{\delta y \to 0} \frac{\delta w}{i\delta y} = \frac{1}{i}\frac{\partial w}{\partial y} + \frac{1}{i}\left(\frac{\partial \phi}{\partial y} + i\frac{\partial \psi}{\partial y}\right) \qquad 3.7b$$

For these two values of $\frac{dw}{dx}$ to be equal, the real parts must be equal and the imaginary parts must be equal, that is

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \qquad 3.8a$$

$$\frac{\partial\phi}{\partial y} = \frac{\partial\psi}{\partial x} \qquad 3.8b$$

These equations, the consequence of the condition of one-valued derivatives, are called the Cauchy-Riemann equations. Provided that the partial derivatives $\frac{\partial\phi}{\partial x}$, $\frac{\partial\phi}{\partial y}$, $\frac{\partial\psi}{\partial x}$ and $\frac{\partial\psi}{\partial y}$ exist, and are continuous, the Cauchy-Riemann equations are the sufficient conditions that a continuous one-valued function $w = f(z)$ is analytic.

The complex variable, $w = \phi + i\psi$, in which ϕ and ψ satisfy the Cauchy-Riemann equations, is called the complex potential. Differentiating eq.3.8a with respect to x, and eq. 3.8b with respect to y, and adding, results in

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad 3.9$$

Similarly, differentiating eq. 3.8a with respect to y, and eq. 3.8b with respect to x and subtracting, yields

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \qquad 3.10$$

Hence the real and the imaginary parts of any function of a complex variable satisfy the Laplace equation and are therefore possible velocity potential functions or stream functions for two-dimensional irrotational flow. This is directly evident from the Cauchy- Riemann equations, for, irrotational flow,

$$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} = u \quad \text{and} \quad \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} = v$$

3.4 Significance of dw/dz

The derivative $\frac{dw}{dz}$ can be regarded either as a complex operator or as a complex velocity.

(a) The complex operator

An infinitesimal line δz on the z-plane is transformed into a corresponding line δw

on the w-plane (or vice versa) according to the relationship

$$\delta w \;=\; \frac{dw}{dz}\;\delta z \qquad 3.11$$

for δw and δz are complex numbers and $\frac{dw}{dz}$ can be regarded as a complex number or operator which transforms δz, by rotation and stretching (or shrinking) to produce δw. Since $\frac{dw}{dz}$ has only one value at any point, infinitesimal lines in all directions in the region of that point will be equally affected as regards rotation and scale change. Hence infinitesimal figures will be transformed, by rotation and change of scale, without distortion, into the w-plane and any two lines intersecting at a particular angle on the z-plane will, after transformation, intersect at the same angle on the w-plane. These transformations, owing to their retention of angular form, are called conformal. Since $\frac{dw}{dz}$ is, in general, a function of z and therefore varies from point to point, the amounts of rotation and change of scale vary for different points of the z-plane.

Large figures therefore become distorted in transformation but angles formed by the intersecting lines of those figures do not. In fig.s 3.6 and 3.7, the small triangle, LMN on the z-plane is seen to be rotated and altered in size without appreciable difference of form on the w-lplane. The large triangle vertices, L, M’ and N’, are rotated with respect to the origin by different amounts, the triangle is enlarged and distorted but the angles have not been altered in the transformation to the w-plane. Angles at the origin are altered, for the origin is a singular point at which the mapping process breaks down.

(b) The complex velocity

Since, from eq. 3.7 a, $\quad \frac{dw}{dz} \;=\; \frac{\partial w}{\partial z}$

$$\frac{dw}{dz} \;=\; \frac{\partial}{\partial x}\,(\phi \;+\; i\psi) \;=\; \frac{\partial \phi}{\partial x} + i\,\frac{\partial \psi}{\partial x} \;=\; u \;-\; iv$$

$$\therefore \quad \frac{dw}{dz} \;=\; u \;-\; iv \qquad 3.12$$

The derivative $\frac{dw}{dz}$ can be regarded therefore as a complex velocity, the real part equaling the x-component, and the imaginary part the negative of the y-component of the velocity V at any point (fig. 3.8).

The absolute value of the complex velocity is $\left|\frac{dw}{dz}\right| = \sqrt{(u^2 \;+\; v^2)}$, the speed at the point z; and its argument is $-\propto$ where $\propto = \tan^{-1} \dfrac{u}{v}$

$$\therefore \frac{dw}{dz} = |V| [\cos(-\propto) + i \sin(-\alpha)]$$

$$\therefore \frac{dw}{dz} = |V| e^{-\propto} \qquad 3.13$$

where |V| is the absolute magnitude of the velocity V. Provided the w = f(z) function is known, Eq. 3.13 provides a ready means of determining the velocity at any point of the flow pattern in the z-plane. In particular, at stagnation points $\dfrac{dw}{dz}$ equals zero.

CONFORMAL TRANSFORMATION

UNIT - III

3.1 Some simple transformations

The following simple transformations are but a few of the many useful analytic functions which have been investigated. They are here treated with sufficient detail to enable to develop proficiency in identifying the flow patterns, in some cases an alternative, rapid approach, which gives a qualitative idea of the transformation pattern, is provided.

	Transformation	Flow pattern in the z-plane
1.	$w = Az$	Uniform flow
2.	(i) $w = m \ln (z - a)$	Source at $z = a$
	(ii) $w = \frac{-iK}{2\pi} \ln (z - a)$	Vortex at $z = a$
	(iii) $w = \left(m - \frac{iK}{2\pi}\right) \ln (z - a)$	Spiral vortex at $z = a$
3.	$w = \frac{\mu}{z - a}$	Doublet at $z = a$
4.	$w = m \ln \frac{z + a}{z - a}$	Source at (-a, 0), sink at (a, 0)
5.	$w = Az^n$	Flow at a wall angle, $\theta = \frac{\pi}{n}$
6.	$z = c \cosh w$	Flow through an aperture (inverse function)
7.	$z = e^{-w} - w$	Flow into a rectangular channel(inverse function)
8.	(i) $w = U\left(z + \frac{a^2}{z}\right)$	Flow past a cylinder of radius a
	(ii) $w = U\left(z + \frac{a^2}{z}\right) - \frac{iK}{2\pi} \ln z$	Flow past a cylinder with circulation.

3.2 Flow past a cylinder

(i) Without circulation

The combination of a doublet with uniform flow yield the pattern of uniform flow past a circular cylinder. The complex potential for this pattern is simply the sum of the individual complex potentials. For uniform flow with a velocity U in the positive x-direction and a doublet with its axis in the negative x-direction.

$$
\begin{aligned}
w &= w_{\text{uniform}} + w_{\text{doublet}} \\
&= Uz + \frac{\mu}{z} \\
&= U\left(z + \frac{\mu}{Uz}\right)
\end{aligned}
$$

$$\therefore \quad w = U\left(z + \frac{a^2}{z}\right) \qquad 3.1\text{ a}$$

where $a = \sqrt{\dfrac{\mu}{U}}$, the radius of the cylinder

Where z is very large, w approaches the value Uz, that is flow is practically uniform at large distances from the origin. With z expressed in polar coordinates

$$\phi + i\psi = U\left(re^{i\theta} + \frac{a^2}{r}e^{-i\theta}\right)$$

$$= U\left(r + \frac{a^2}{r}\right)\cos\theta + iU\left(r - \frac{a^2}{r}\right)\sin\theta$$

$$\therefore \quad \phi = U\left(r + \frac{a^2}{r}\right)\cos\theta \qquad 3.1\text{b}$$

$$\psi = U\left(r - \frac{a^2}{r^3}\right)\sin\theta \qquad 3.1\text{ c}$$

The complex velocity

$$\frac{dw}{dz} = U\left(1 - \frac{a^2}{z^2}\right) = |V|\, e^{-i\propto}$$

is zero at the stagnation points $z = \pm\ a$ and has the maximum value of 2U at $z = \pm\, ia$.

(ii) With circulation

Addition of the complex potential for a clockwise vortex of strength K yields the complex potential for flow in the positive x-direction past a cylinder with circulation Γ equal to K. In this instance, K has a negative value.

$$w = U\left(z + \frac{a^2}{z}\right) - \frac{iK}{2\pi} \ln z \qquad 3.2\text{ a}$$

$$\therefore \phi = U\left(r + \frac{a^2}{r}\right)\cos\theta + \frac{K}{2\pi}\theta \qquad 3.2\text{ b}$$

$$\psi = U\left(r - \frac{a^2}{r}\right)\sin\theta - \frac{K}{2\pi} \ln r \qquad 3.2\text{ c}$$

3.2.1 Transformation of the circle

The pattern of flow past a circular cylinder, obtained by means of eq. 3.1, can itself be transformed into other patterns. It can be treated, therefore, as the pattern on an intermediate plane, say the z_1 - plane, and the transformation $w = z_1 + \frac{a^2}{z_1}$ can be regarded as the first step in the transformation the w-plane to the physical or z-plane. Consideration is now directed to the several possible transformations from this z_1-plane to the z-plane. The useful physical patterns which can be obtained include those of flow past plates, streamlined struts, arcs and aerofoils. In each case, the z_1 - plane circle of radius a is transformed into the new profile. For simplicity, the undisturbed velocity on the z-plane is taken as unity and in the positive x-direction, except where otherwise stated.

The terms inverse point, and image or optical reflection, are defined for the present purposes as follows. The inverse point of any point P (fig. 3.1a) with respect to a circle of centre 0 and radius a is Q where Q lies on OP and OP.OQ = a^2, that is $OQ = \frac{a^2}{OP}$. The image, or reflection, of any point Q in the x-axis is R where OR = OQ and the angles xOR, xOQ are of equal magnitude.

Hence R is he image in the x-axis of the inverse of P. If P is defined by the radius vector $r_1e^{i\theta_1}$, then R is defined by $\frac{a^2}{r_1}e^{-i\theta_1}$ since $OR = OQ = \frac{a^2}{r_1}$. The vector sum of OP and OR is $OS = re^{i\theta}$, that is, $\vec{OP} + \vec{OR} = \vec{OP} + \vec{PS} = \vec{OS}$

$$re^{i\theta} = r_1e^{i\theta_1} + \frac{a^2}{r_1}e^{-i\theta_1} \qquad 3.3$$

S being located geometrically by the parallelogram rule of addition. It will be seen that, if $re^{i\theta}$ be denoted by z, and $r_1e^{i\theta_1}$ by z_1, the above equation can be written

$$z = z_1 + \frac{a^2}{z_1} \qquad 3.4$$

Also, if P lies on the circle of radius a, so does R, so that the locus of S is the x-axis between (-2a, 0) and (2a, 0) (fig. 3.1b).

These geometrical relationship will be used in the transformation now to be considered.

3.2.2 Flow parallel to a flat plate

The transformation

$$w = z_1 + \frac{a^2}{z_1} \qquad 3.5\text{ a}$$

or, expressed in its inverse form,

$$z_1 = \tfrac{1}{2}\left[w \pm \sqrt{(w^2 - 4a^2)}\right] \qquad 3.5\text{ b}$$

transforms the uniform flow of the w-plane to flow past a circle of radius a, in the positive x_1 - direction in the z_1 -plane. Since $\frac{dw}{dz} = 1 - \frac{a^2}{z_1^2}$ is zero at $z_1 = \pm a$, these are singular points.

The next transformation equation

$$z = z_1 + \frac{a^2}{z_1^2} \qquad 3.6$$

being similar in form to eqn. 3.5 reproduces the w-plane pattern of uniform flow, parallel to the x-axis, in the z-plane. The streamlines of the z_1 - plane become lines parallel to the x-axis. The circle of radius a, henceforth called the a-circle, becomes a line extending from (-2a, 0) to (2a, 0) and can be regarded as a flat plate of length 2l, and of negligible thickness, set parallel to the flow. The transformations are shown in Fig. 3.2.

The geometrical construction which transforms the streamlines and the a-circle can be used also to transform circles of radius greater than a, centred at the origin of the z_1 -plane. They form, in the z-plane, a series of confocal ellipses, the foci being at (-2a, 0) and (2a, 0). One such circle is shown in broken line in Fig. 3.2.

3.2.3 Flow normal to a flat plate

If the direction of flow past the a circle in the z_1- plane is changed by means of a 'rotating' transformation, the final transformation to the z- plane changes the circle to a straight line as before but the flow is no longer uniform and parallel to it. For example suppose that the direction of flow pattern is rotated through $-\frac{\pi}{2}$ by means of the transformation $z_2 = - iz_1$.

On the z_2-plane the direction of flow is in the negative y-direction (fig. 3.1c). The final transformation

$$z = z_2 + \frac{a^2}{z_2}$$

yields the physical pattern of flow in the negative y-direction normal to a flat plate of length 4a.

Since $\frac{dz}{dz_2} = 1 - \frac{a^2}{z_2^2} = 0$ at $z_2 = \pm a$, these are, once again, singular points.

The successive transformations relating the w- and z-patterns,

$$w = z_1 + \frac{a^2}{z_1} \qquad 3.7\text{ a}$$

$$z_2 = - iz_1 \qquad 3.7\text{ b}$$

$$z = z_2 + \frac{a^2}{z_2} \qquad 3.7\text{ c}$$

can be combined by the elimination of z_1 and z_2

$$z_1 = -\frac{z_2}{i} = iz_2$$

$$\therefore \quad w = iz_2 + \frac{a^2}{iz_2} = i\left(z_2 - \frac{a^2}{z_2}\right)$$

$$\therefore \quad w^2 = -\left(z_2 - \frac{a^2}{z_1}\right)^2$$

also $$z^2 = \left(z_2 + \frac{a^2}{z_2}\right)^2$$

$$\therefore \quad w^2 + z^2 = 4a^2$$

$$\therefore \quad w = i\sqrt{(z^2 - 4a^2)}$$

If $l = 2a$, the half width of the plate,

$$w = i\sqrt{(z^2 - l^2)} \qquad 3.8$$

which is the required transformation.

The transformation for horizontal flow normal to a vertical plate (fig. 3.2) is obtained by rotating the z-plane pattern through $\frac{\pi}{2}$ by multiplication of z by i, the new physical pattern being on the z'-plane

$$z' = iz$$

$$\therefore \quad z = \frac{z'}{i}$$

$$\therefore \quad w = i\sqrt{\left\{\left(\frac{z'}{i}\right)^2 - l^2\right\}}$$

$$\therefore \quad w = \sqrt{(z'^2 + l^2)} \qquad 3.9$$

3.2.4 Flow past an ellipse

In case (i) above, the final transformation $z = z_1 + \frac{a^2}{z_1}$ transformed the a-circle into a straight line of length 4a, and larger, concentric circles into ellipses. If, instead, the final transformation equation is

$$z = z_1 + \frac{b^2}{z_1} \qquad 3.10$$

where b is less than a, then a circle of radius b with centre origin, the b-circle, is the circle in the z_1-plane which is transformed into a straight line in the z-plane. The line extends from (-2b, 0) to (2b, 0), its extremities being singular points (fig. 3.3). The a-circle which is larger and concentric with the b-circle, is transformed into an elliptical profile with the singular points as foci. The method of graphical construction is apparent if eqn. 3.10 is expressed in polar co-ordinates and applied to the a-circle.

$$re^{i\theta} = ae^{i\theta} + \frac{b^2}{a}e^{-i\theta} \qquad 3.11$$

The radius vector $\frac{b^2}{a}e^{-i\theta}$ in the z_1-plane defines points on a circle of radius $\frac{b^2}{a}$, which is the inverse of the a-circle in the b-circle, since $\frac{b^2}{a} \times a = b^2$ (Fig. 3.4). For each point P_1 on the a-circle (with the image, P'_1, of its inverse, on the $\frac{b^2}{a}$ circle), the corresponding transformed point P in the z-plane is obtained by vectorial addition. The resulting ellipse has a length of $2\left(a + \frac{b^2}{a}\right)$ and a width of $2\left(a - \frac{b^2}{a}\right)$. The successive stages in the transformation from the w-plane to the z-plane are shown in fig. 3.3. The profiles obtained by this transformation and treated in this and the following three subsections are called Joukowski profiles.

3.2.5 Flow past a streamlined strut

If the a-circle is not centred upon the origin of the z_1-plane some of the symmetry of its transformed profile disappears. Displacement in the horizontal direction destroys the symmetry about the vertical axis. In Figs. 3.5 and 3.6c the a-circle centre is displaced from the origin to C, where OC = m, a small distance in the positive x-direction, (by the transformation from z_1 plane to z_2 plane)

$$z_2 = z_1 + m\,e^{i\theta} = z_1 + m$$

Now, the transformation

$$z = z_2 + \frac{b^2}{z_2} \qquad 3.12$$

transforms the b-circle O and radius $OB_1 = b = a - m$ in the z_2 - plane into the straight lines BD of length 4b in the z-plane. The circle of radius $OA_1 = a + m$ is transformed into the ellipse with major half axis OA and foci B and D. The a-circle of radius a and centre C, is transformed into a profile which extends from B to A and has the characteristics of the straight line, that is a cusp of zero angle, at B; and of the ellipse, that is, a round nose, at A. The profile is that of a symmetrical Joukowski streamlined strut and it can be developed by the normal graphical procedure. The successive transformations are shown in Fig. 3.7, the flow being in the negative x-direction.

3.2.6 Flow past a circular arc

The centre of the a-circle (Figs. 3.8 and 3.9) is displaced along the y-axis to C where OC = m by the transformation from the z_1 - plane to the z_2 - plane

$$z_2 = z_1 + m\,e^{i(\pi/2)} = z_1 + im \qquad 3.13$$

The next transformation

$$z = z_2 + \frac{b^2}{z_2} \qquad 3.14$$

where b is selected so that it equals OB_1 (fig. 3.8), transforms the b-circle into the straight line AB of length 4b on the z-plane. The two arcs $B_1D_1A_1$ and $B_1E_1A_1$ of the a-circle in the z_2 - plane are each transformed into the one circular arc BA on the z-plane.

The geometrical construction in Fig. 3.8 shows why the two transformed arcs coincide, for

$$P_1O \times OE_1 = B_1O \times OA_1$$

$$r_1 \times OE_1 = b^2$$

$$\therefore \; OE_1 = \frac{b^2}{r_1}$$

Hence, by symmetry, $OP'_1 = OE_1 = \dfrac{b^2}{r_2}$ that is, the image, P'_1, of the inverse of the a-circle point P in the b-circle lies itself on the a-circle. When P_1 is one the arc $B_1E_1A_1$, then P'_1 is on the arc $B_1D_1A_1$ so that, in the process of vectorial addition, both arcs yield the one transformed arc.

The camber of the transformed arc, OD, is seen to equal 2m, for, when P_1 is at D_1, in the z_2-plane, $D_1D = OF_1 = \dfrac{b^2}{r_1}$

$$\therefore \; OD = OD_1 - D_1D = OD_1 - OF_1$$

$$= (a + m) - (a - m)$$

$$= 2m$$

The transformations from the w-plane to the z-plane for flow past a circular arc are sketched in Fig. 3.9. The positions of the stagnation points are evident in each of the three z-planes. The ends of the arc in the final plane are singularities where the velocity of flow is theoretically infinite.

3.2.7 Flow past an aerofoil

The centre of the a-circle (Figs. 3.10 and 3.11) is displaced into the first quadrant to C such that OC = m and the angle COX = δ, by the transformation

$$z_2 = z_1 + me^{i\delta} \qquad 3.15$$

The transformation to the z-plane,

$$z = z_2 + \frac{b^2}{z_2} \qquad 3.16$$

where $b = OB_1$, transforms the b-circle into the straight line of length 4b. The a-circle is transformed into aerofoil profile which has the rounded nose and cusped tail of the strut and the camber of the arc previously treated. The normal geometrical construction can be used, as indicated in Fig. 3.10. It can be shown that the locus of the image, P'_1, of the inverse of P_1 is a circle with radius $C'B_1$ and centre C' on the line B_1C such that the angles $C'Oy_2$ and y_2 OC are equal. Use of this fact simplifies the location of P'_1 in the graphical method.

A circle in the z_2 - plane with centre E_1 and radius E_1B_1 (Fig. 3.10b) would transform into the circular arc of span 4b which is the skeleton of the aerofoil, shown in broken line in the z-plane. The thickness of the aerofoil increases with increase in the distance E_1C' and the camber, with increase in the distance OE_1. The stages in the transformation from the w-plane to the z-plane are shown in Fig. 3.11.

The meanings of some terms commonly used in connection with aerofoils are indicated in Fig. 3.12 ; the angle of attack being defined in some cases as α and in some cases as the more easily measured α'.

The cusped trailing edge where the angle of intersection of the upper and lower surfaces is zero is unsatisfactory in a practical aerofoil for constructional reasons. A modified profile (fig. 3.13) with a finite angle, τ, at the trailing edge can be obtained by means of the transformation, due to Karman and Trefftz

$$\frac{z + nb}{z - nb} = \left(\frac{z_2 + b}{z_2 - b}\right)^n \qquad 3.17$$

The angle τ equals $(2 - n)\pi$, n being selected as a little less than 2. When n = 2, the transformation is

$$\frac{z + 2b}{z - 2b} = \left(\frac{z_2 + b}{z_2 - b}\right)^2$$

which simplifies to the ordinary Joukowski transformation,

$$z = z_2 + \frac{b^2}{z_2}$$

The transformation functions so far considered, and the corresponding geometrical constructions, are applicable of course to all of the streamlines, the profile being merely a streamline of particular importance. From the stream pattern in the z-plane, the distribution of pressure around the profile is readily determined.

3.3 Kelvin's Circulation Theorem

Theorem due to Kelvin, it follows that the circulation, and hence the vortex strength, does not vary with time, under certain conditions.

These conditions are

(i) the fluid is non-viscous ;

(ii) its density is either constant or a function of pressure only; and

(iii) the body forces are derivable form a single-valued potential (such as the gravity force potential, Ω, introduced in unit I problem 18).

But the condition (i) no real fluid is invicid (ii) is true in homogenous liquid which can be treated as incompressible and for isentropic flow of gas (iii) is true of the gravitational field.

Proof of Kelvin's Circulation Theorem rests on the demonstration that the value of $\frac{d\Gamma}{dt}$, in a circuit which moves with the fluid elements composing it, is zero.

The circulation in the circuit shown in Fig. 3.14 is the summation of the line integrals of velocity along elements such as AB, whose projections on the co-ordinate planes are dx, dy and dz. From the expression for Γ

$$\Gamma = \int (u\,dx + v\,dy + w\,dz) \qquad 3.18$$

the rate of change of Γ is

$$\frac{D\Gamma}{Dt} = \int \frac{D\Gamma}{Dt}(u\,dx + v\,dy + w\,dz) \qquad 3.19$$

The term

$$\frac{D(u\,dx)}{Dt} = dx\frac{Du}{Dt} + u\frac{Ddx}{Dt}$$

and $\frac{Ddx}{Dt}$ is the rate of increase of dx, that is, it equals the difference, du, in the x-components of the velocities of A and B. From problem 18 of unit I

$$\therefore \frac{D(u\,dx)}{Dt} = dx\frac{Du}{Dt} + u\,du$$

$$= \left(X - \frac{1}{\rho}\frac{\partial p}{\partial x}\right)dx + u\,du$$

$$= -\left(\frac{\partial \Omega}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x}\right)dx + u\,du$$

$$= -\frac{\partial}{\partial x}\left(\Omega + \int\frac{dp}{\rho}\right)dx + u\,du$$

In a similar way $\frac{D\,(v\,dy)}{\partial x}$ and $\frac{D\,(w\,dz)}{Dt}$ can be evaluated, and, by addition, we obtain

$$\frac{D}{Dt}\,(udx + v\,dy + w\,dz) \;=\; -\,d\left(\Omega + \int \frac{d\rho}{\rho}\right) + u\,du + v\,dv + w\,dw$$

$$= -\,d\left(\Omega + \int \frac{d\rho}{\rho}\right) + d\,(V^2) \qquad 3.20$$

In order to evaluate Eqn.3.19, this must be integrated around the circuit. Since Ω is a single-valued potential, and the density is either a constant or a function of pressure only, integration of the right hand term yields zero. Therefore

$$\frac{D\Gamma}{Dt} \;=\; \int \frac{D\Gamma}{Dt}\,(udx + v\,dy + w\,dz) \;=\; 0 \qquad 3.21$$

that is, Γ is constant in a circuit which moves with the fluid.

3.4 Circulation theory of lift

A stagnation point occurs at S_1 on the nose and at S_2 on the upper surface near the tail of the profile (fig 3.). At the tail, the air in theory makes an impossibly sharp turn as it flows from below around the surface to the rear stagnation points S_2.

In practice, not only is this pattern of air flow around the tail not possible, but a lift force is experienced by all aerofoils, arcs, and inclined plates — in fact by any profile which is not symmetrically formed or situated with respect to the direction of flow. Joukowski' hypothesis, proposed with the object of bringing theory into agreement with experiment, supposes that in all such flows there is a set up about the profile a circulation of such magnitude that the rear stagnation point is moved thereby to the trailing edge. This hypothesis not only disposes of the difficulty of the infinite velocity at the trailing edge but it explains the presence of the lift force and provides a means of determining analytically the magnitude of the circulation and hence of the lift force itself.

In this theoretical approach it is assumed that the aerofoil is of inifinite length so that flows around its ends do not affect the pattern, which is therefore two-dimensional; further it is assumed that separation does not occur. In the case of streamlined profiles at small angles of incidence with the flow, separation effects are small and the predictions of pressure distributions and lift forces made on the basis of Joukowski's hypothesis agree well with experiment.

It is easy to visualize a rotating cylinder setting up a circulatory flow by means of viscous drag on the air close to its surface but the method of estabilishment of circulation around a non-rotating aerofoil is less obvious. It is to be remembered that the mean motion of a low viscosity fluid such as air resembles irrotational flow, except in regions where the velocity gradient is high. The establishment of the circulation around the aerofoil is explained by this dual character of the air.

According to Kelvin's Circulation Theorem, in an ideal fluid subject to body forces derivable from a single-valued potential, the circulation in a circuit which moves with the fluid remains constant. Hence the circulation around a vortex does not change with time, regardless of the fluid's motion. In effect, this means that a vortex in a non-viscous fluid can neither be created nor destroyed. If it exists, it must always have existed and will continue to do so. Hence, if a fluid initially at rest is set in motion by movement of its boundaries, no vortices can develop and the circulation around any curve in the fluid remains at zero. An aerofoil which is set in motion in a non-viscous fluid initially at rest would therefore produce no vortex and no circulatory flow around itself.

In a real fluid, at the commencement of motion, the flows from the upper and lower edges meet at the rear stagnation point s (fig. 3.15a). As the velocity increases, flow from the lower surface around the tip cannot be maintained and separation occurs at the tip, viscous action producing a clockwise 'starting' vortex (cast off vortex) with a circulation of its own. Application of Kelvin's circulation theorem to a closed curve surrounding the aerofoil, and lying in regions of practically irrotational flow, suggests that the circulation around this curve must remain at zero. This can only be possible if a second circulatory flow of equal magnitude to that of the starting vortex, but opposite in sign, has been simultaneously established within the closed curve. This is the anticlockwise circulation around the aerofoil, according to Joukowski's hypothesis (fig. 3.15b)

As the speed increases, so also does the strength of the circulation and this increasing strength causes the forward stagnation point to move downwards and the rear stagnation pint to move back towards the tip. When it reaches this point, the cause of the development of the starting vortex no longer exists. When the speed becomes steady, the vortex ceases to grow and is detached and swept downstream to be dissipated by viscous action, leaving the circulation around the aerofoil undiminished (fig. 3.15c). This briefly, is the basis of <u>circulation theory of lift</u>.

The magnitude of the lift is $\rho U\Gamma$. This relationship, known as the <u>Kutta-Joukowski Law</u>, can be shown to apply not only to circular cylinders and to aerofoils but to any form in two-dimensional irrotational flow. In the case of a real fluid, surface resistance and separation may produce effects markedly different from that predicted by the law but, for streamlined profiles, it is in fair agreement with experimental determinations for angles of attack up to about 10°. In fact this law forms the basis of the circulation theory of lift on aeroplane wings, the thrust of fan and propeller blades, and the transverse forces on unsymmetrical solid bodies, and on rotating balls and cylinders moving through a fluid. The principal problem in the application of the law is the determination of the appropriate value for the circulation Γ. In the case of the aerofoil, it is that value of Γ which makes the trailing edge a stagnation point.

3.5 Theorem of Blasius

Let w(z) be the complex potential of a two-dimensional inviscid flow past a body of any given shape and attitude. Let X and Y be the components in the x and y directions respectively of the force per unit span on the body. Let M be the anti-clockwise moment per unit span about the point z = 0. Then

$$X - iY = \frac{i\rho}{2} \int_C \left(\frac{dw}{dz}\right)^2 dz$$

and

$$* \qquad M = - Re \left\{ \tfrac{1}{2}\rho \int_C z \left(\frac{dw}{dz}\right)^2 dz \right\}$$

where C is the curve representing the boundary of the body. This result is known as the <u>theorem of Blasius</u>, and may be proved as follows, by considering a solid body in a general flow, as depicted in Fig. 3.16. Consider the small element δs in the boundary of the solid. Then $\delta x = -\sin\beta\,\delta s$ and $\delta y = \cos\beta\,\delta s$, while

$$\delta X = -p.\ \delta s \cos\beta = -p\,\delta y$$
$$\delta Y = -p.\ \delta s \sin\beta = p\,\delta x.$$

* Re - real part

Then

$$\int \qquad\qquad \int$$

$$X - iY = -\int_C p\,(dy + idx) = -i\int_C p(dx - idy) \qquad 3.22$$

Now,

$$\int_C dx = \int_C dy = 0,$$

so that

$$\int_C \text{constant}\,(dx - idy) = 0.$$

Also, $p = \text{constant} - \tfrac{1}{2}\rho q^2$, from Bernoulli's Theorem, so that

$$\int_C P\,(dx - idy) = \int_C \text{constant}\,(dx - idy) - \tfrac{1}{2}\rho\int_C q^2\,(dx - idy).$$

Equation 3.22 then becomes

$$X - iY = \tfrac{1}{2} i\rho\int_C q^2\,(dx - idy) \qquad 3.23$$

Now

$$\begin{aligned} q^2\,(dx - idy) &= (u^2 + v^2)\,(dx - idy) \\ &= (u^2 + v^2)\,(dx + idy) - 2i\,(u^2 + v^2)\,dy \\ &= (u^2 - v^2)\,(dx + idy) + 2v^2dx - 2iu^2dy \\ &= (u - iv)^2\,(dx + idy) + 2v\,(v + iu)\,dx - 2u\,(v + iu)\,dy. \end{aligned}$$

But the contour C is a streamline of the flow, so that, on C, $vdx = udy$ and $2vdx\,(v + iu) = 2udy\,(v + iu)$. Thus equation 3.23 becomes

$$\begin{aligned} X - iY &= \tfrac{1}{2} i\rho\int_C (u - iv)^2\,(dx + idy) \\ &= \tfrac{1}{2} i\rho\int_C \left(\frac{dw}{dz}\right)^2 dz. \qquad 3.24 \end{aligned}$$

The sum of the moments of dX and dY about the origin is

$$dM = -\,ydX + x\,dY = p\,(xdx + ydy)$$

which is the real part of pz dz since

$$\begin{aligned} pzdz = p\,(x + iy)\,(dx - idy) &= p\,(x\,dx + ydy) + i\,(ydx - xdy) \\ &= d\,(M + iN), \ \text{(say)} \end{aligned}$$

$$\therefore\ d\,(M + iN) = pzdz = -\tfrac{1}{2}\rho z\left(\frac{dw}{dz}\right)^2 dz$$

$$\therefore \quad M + iN = -\tfrac{1}{2}\rho \int z\left(\frac{dw}{dz}\right)^2 dz \qquad 3.25$$

Thus

$$M = \int_c p\,(xdx + ydy) = -\tfrac{1}{2}\rho \int_c q^2\,(xdx + ydy), \qquad 3.26$$

since

$$p = \text{constant} - \tfrac{1}{2}\rho q^2,$$

and

$$\int_c \text{constant}\,(xdx + ydy) = 0.$$

Now

$$z\left(\frac{dw}{dz}\right)^2 dz = (u - iv)^2\,(x + iy)\,(dx + idy)$$
$$= (u^2 - v^2 - 2iuv)\,(xdx - ydy + iydx + ixdy)$$

so that

$$\mathcal{R}e\left\{z\left(\frac{dw}{dz}\right)^2 dz\right\} = (u^2 - v^2)\,(xdx - ydy) + 2uv\,(xdy + ydx) \qquad 3.27$$

Since C is a streamline, on C udx = udy, so that the second term on the right-hand side of equation (3.27) is equal to

$$2v^2xdx + 2u^2ydy$$

Equation (3.27) thus becomes

$$\mathcal{R}e\left\{z\left(\frac{dw}{dz}\right)^2 dz\right\} = (u^2 + v^2)\,(xdx + ydy) = q^2\,(xdx + ydy) \qquad 3.28$$

So that equation (3.26) now gives

$$M = -\tfrac{1}{2}\rho\,\mathcal{R}e\left\{\int_c z\left(\frac{dw}{dz}\right)^2 dz\right\} \qquad 3.29$$

which, together with equation (3.24, 3.25) constitutes the theorem of Blasius.

In order to evaluate these line integrals, Equation 3.24, 3.25 use is made of the Cauchy Integral Theorem, the integrands being both analytic functions of z. Each integral is taken around a circular path of very large radius, with the origin as centre, instead of around the cylinder boundary, a procedure justified by the corollary of the Cauchy theorem, since there are no singularities in the space surrounding the cylinder.

For the integration of Eqn.3.23 let

$$\left(\frac{dw}{dz}\right)^2 = A_0 + \frac{A_1}{z} + \frac{A_2}{z^2} + \frac{A_3}{z^3} + \ldots$$

then $$X - iY = \tfrac{1}{2} i\rho \int \left(A_0 + \frac{A_1}{z} + \frac{A_2}{z^2} + \frac{A_3}{z^3} + \ldots \right) dz = \tfrac{1}{2} i\rho 2\pi i A_1$$

$$\therefore\ X - iY = -\pi\rho A_1$$

For he integration of Eqn. 3.26

$$M + iN = -\tfrac{1}{2}\rho \int_c \left(A_0 z + A_1 + \frac{A_2}{z} + \frac{A_3}{z^3} + \ldots \right) dz$$

$$= \tfrac{1}{2}\rho 2\pi i A_2$$

$$\therefore\ M + iN = -i\pi\rho A_2$$

Short Question and Answer

Unit – III

1. What do you mean by singular points or singularity

A function w = f (z) is said to be analytic within a region of z plane if (i) each point in the region ie, for each value of z there is one and only one corresponding value of w and the value should be finite (ii) dw / dz must be single valued and neither zero nor infinite. The area where above two conditions are not applicable is called singularity or singular points.

2. Give examples of analytic functions with singular points.

(i) w = 1/(z-a) which is analytic except at z = a where it is infinite

(ii) w = ln z which is analytic except at z = 0 where w is infinite

(iii) w = z^2 which is analytic except at z = 0 where dw/dz = 0.

3. When conformal transformation break down.

It aerodynamic design to involve only two dimensional flows at low speeds, the design method based on conformal transformation theory would be best choice. This technique can not be extended to three dimensional flows or high speed flows. For this reason it is no longer widely used.

3. State Joukowski hypothesis.

In order that the velocity at the trailing edge of aerofoil is zero or stagnation point at the trailing edge, the circulation is increased to the flow past the aerofoil to such a magnitude that the stagnation point coincides with the trailing edge. This method of specifying the magnitude of circulation is known as Joykowski hypothesis.

4. How the circulation is limited in real fluid flows past on aerofoil

For a given aerofoil, at a given angle of attack, there is single value of lift at a particular circulation. This statement is justified by the actual stream line pattern at the trailing edge of the aerofoil.

When the flow starts it will curl around sharp trailing edge from bottom surface to top surface. The velocity becomes infinitely large at the sharp corner.

So this pattern of flow is not tolerated for long period. The stagnation point 2 on the upper surface moves towards the trailing edge. The stream line at the trailing edge forms a starting vortex down stream. Thus the flow will be smoothly leaving the top and bottom surfaces at the trailing edge.

In establishing a steady flow over an aerofoil at a given angle of attack, nature adopts that particular value of circulation, say Γ_2, which results in the flow leaving smoothly the trailing edge.

6. Why there is a limit to the amount of circulation on an aerofoil kept in a flow.

See the previous question.

7. Explain the features of finite angle trailing edge and cusped trailing edge.

For finite angle trailing edge the velocities V_1 and V_2, parallel to top and bottom surfaces respectively at a, are finite, but they are in different directions as shown in figure. This is impossible physically and so $V_1 = V_2 = 0$ at a. Hence trailing edge become stagnation point.

For a cusped trailing edge V_1 and V_2 are in the same direction at 'a' and so V_1 and V_2 are finite. From consideration Bernoulli's theorem, at 'a' for pressure Pa

$$Pa + \tfrac{1}{2} \rho V_1^2 = Pa + \tfrac{1}{2} c V_2^2 \text{ given } V_1 = V_2 \neq 0$$

Hence the velocities leaving top and bottom are finite and same in magnitude and direction. The trailing edge becomes no longer a stagnation point.

8. Show that as per Kutta-condition the strength of vortex at trailing edge is zero.

(i) For finite trailing edge angle $V_1 = V_2 = 0$ $\therefore$ Strength of vortex at a = γ (T. E) = 0

ii) For cusped trailing edge $V_1 = V_2 \neq 0$ But as $V_1 = V_2$ strength of vortex at a = γ (T.E) = 0

Kutta condition makes the strength of vortex at trailing edge must be equal to zero.

9. What is Kutta- trailing edge condition

See previous question.

10. When Joukowski transformation brake down

For finite angle trailing edge of an aerofoil, the trailing edge becomes stagnation point.

For cusped trailing edge, trailing edge of aerofoil no longer becomes stagnation point.

By method Joukowski transformation the trailing edge becomes cusped and so the Kutta- condition is not satisfied. This is the draw back of Joukowski transformation.

11. Explain the roll of the starting vortex (cast off vortex) in establishing the lift on an aerofoil.

The starting vortex builds and grows up to just the right strength such that equal and opposite clockwise circulation around the aerofoil leads to smooth flow from trailing edge (at this situation Kutta - condition is exactly satisfied). When the point is reached the vorticity shed from the leading edge becomes zero, the starting vortex no longer grows in strength, and steady circulation exists around the aerofoil.

12. "The circulation on an aerofoil is equal and opposite to the circulation around starting vortex" prove.

First figure shows the flow field some time after a steady flow has been established over the aerofoil with starting vortex down stream. The fluid is enclosed in C_2 and let its circulation be Γ_2. By Kelvins theorem the circulation Γ_2 around C_2 (which encloses aerofoil and starting vortex) is the same a that of C_1 ie, $\Gamma_1 = \Gamma_2 = 0$

Let us subdivide C_2 into two loops by making a cut bd, thus forming curves C_3 and C_4, the curve C_3 encloses starting vortex and C_4 encloses aerofoil. The anticlockwise circulation Γ_3 is around starting vortex and clockwise circulation Γ_4 is around the aerofoil. Sum of these two should be equal to Γ_2.

i.e., $\Gamma_3 + \Gamma_4 = \Gamma_2$ as $\Gamma_2 = 0$, $\Gamma_4 = -\Gamma_3$

Hence the circulation on aerofoil is equal and opposite to the circulation around starting vortex.

13. Explain the origin of lift on an aerofoil which starts from rest in a real fluid.

For no relative wind there is no circulation. When flow starts from zero velocity, at low velocity there is potential flow without circulation (fig a) and stagnation point is on the upper surface forward of the trailing edge. Hence fluid has to flow round the trailing edge from bottom to top surface. As the flow velocity increases there will be a large velocity gradient at the trailing edge and correspondingly large viscous forces are brought to play. Due to this, a surface discontinuity develops which rolls up into a vortex between the original stagnation point and the trailing edge (fig b). This grows to starting vortex and it leaves the aerofoil extending towards down stream (fig c).

The starting vortex can be regarded as the missing side of the simple horse shoe vortex on the aeroplane wing. Because it does not move with aerofoil, it has no influence in the steady case. Its strength is equal and opposite to that of bound vortex which represents the wing. Hence total circulation around any circuit containing both the starting and bound vortices remains at zero.

14. State the importance of Kelvins circulation

The circulation around a curve is unaltered is transformation.

15. What is a barotropic fluid.

A fluid having density either constant or a function of its pressure is said to be a barotropie fluid.

16. Give example of body force field which is derivable from single valued potential

Gravitational field and magnetic field.

17. State the importance of Blasius theorem in aerodynamics.

For an assumed complex potential of a two dimensional flow past a body of given shape and orientation the aerodynamic forces per span length and moments per span length of the body are obtained in terms of complex variables of the flow.

Questions		opt1		opt2		opt3		opt4	opt5	opt6	Answer
Fixed control volume in space is known as ________method	a.	Lagrangian	b.	Eulerian	c.	Rayleigh	d.	Fanno			**Eulerian**
Thermodynamic properties do not change with respect to time in ________.	a.	Steady flow	b.	Compressible Flow	c.	Incompressible flow	d.	Stream line flow			**Steady flow**
A vortex line in a fluid has constant	a.	straight line	b.	circulation	c.	Streamlines	d.	equipotential lines			**circulation**
The __________ relates the velocity induced by a vortex filament to its strength and orientation	a.	Second law of TD	b.	first law of TD	c.	Biot-Savart law	d.	Zeroth law			**Biot-Savart law**
__________ is the science of fluid motion	a.	Fluid dynamics	b.	structural dynamics	c.	Aerodynamic	d.	kinematics			**Fluid dynamics**
For an over expanding nozzle, ____________.	a.	$P_b<P_e$	b.	$P_b>P_e$	c.	$P_b=P_e$	d.	$P_b \leq P_e$			$\mathbf{P_b<P_e}$
__________ are curves that are everywhere tangent to the velocity vector	a.	pathline	b.	Streamlines	c.	equipotential lines	d.	circulation			**Streamlines**
A ______ is the trajectory followed by an individual particle.	a.	pathline	b.	Streamlines	c.	equipotential lines	d.	circulation			**pathline**
________is a measure of the degree of local rotation in the fluid.	a.	Vorticity	b.	circulation	c.	Streamlines	d.	equipotential lines			**Vorticity**
When $P_b= P_e$ _______.	a.	Flow takes place slowly	b.	Flow takes place rapidly	c.	No flow takes place	d.	Flow takes place at isentropic			**Flow takes place at isentropic**
Supersonic flow occurs in_______.	a.	Convergent nozzle	b.	Divergent nozzle	c.	Under expanding nozzle	d.	Choked nozzle			**Under expanding nozzle**
Equation for state for perfect gas is _______.	a.	Pv=RT	b.	P/v=RT	c.	P+v=R+T	d.	P-v=R-T			**Pv=RT**
The shock can always from supersonic to subsonic because of _______.	a.	First law of thermodynamics	b.	Newton's law of motion	c.	Second law of thermodynamics	d.	Newton's third law			**Second law of thermodynamics**
Shock is a_______across fluid flow properties	a.	Continuity	b.	discontinuity	c.	both(a) and(b)	d.	all of the above			**discontinuity**
a* refers to _______.	a.	M<1	b.	M>1	c.	M=1	d.	M=0			**M=1**
Normal shock occurs only from_______.	a.	$V_2=V_1$	b.	$V_2>V_1$	c.	$V_1=V_2=1$	d.	$V_2<V_1$			$\mathbf{V_2<V_1}$
The sonic velocity is the ______ property of the fluid	a.	Static	b.	Hydrodynamic	c.	Thermo dynamic	d.	Aerodynamic			**Thermodynamic**
For isentropic flows, stagnation enthalpy and stagnation temperature are_______.	a.	Zero	b.	constants	c.	variables	d.	unity			**constants**
The change in pressure with respect to the change in velocity is always_______.	a.	Zero	b.	positive	c.	unity	d.	negative			**negative**
In a converging area duct with a subsonic flow, the fluid flows out with a_______.	a.	high velocity	b.	high density	c.	high pressure	d.	high viscosity			**high velocity**
In stagnation state, the fluid is brought to a state of zero velocity in_______.	a.	Adiabatic manner	b.	isentropic manner	c.	polytrophic manner	d.	isobaric manner			**isentropic manner**
In a finite wing, the strong vortices produced at ________ .	a.	tip of the wing	b.	centre of the wing	c.	no vortices produced	d.	edge of the wing			**tip of the wing**
What is the advantage of using blow-down wind tunnel?	a.	no need of pressure regulator	b.	high mach capability	c.	noise free operation	d.	longer			**high mach capability**
The Mach Number at which the aerodynamic drag increases rapidly is known as_______.	a.	Upper Critical Mach Number	b.	Lower Critical Mach Number	c.	Drag divergence	d.	Center of pressure			**Drag divergence**
The chord-wise length about which the pitching moment is independent of the lift co-efficient and the angle of attack is known as_______.	a.	Aerodynamic center	b.	Center of pressure	c.	Stagnation point	d.	a and b			**Aerodynamic center**
The small perturbation equations for subsonic and supersonic flows are_______.	a.	Linear	b.	non-linear	c.	Rotational	d.	irrotational			**Linear**
Diffusers are used to ________ the velocity of inlet air.	a.	decrease	b.	increase	c.	Zero	d.	constant			**decrease**
Subsonic flows through a diverging nozzle will have________ velocity	a.	Steady flow	b.	Compressible Flow	c.	Incompressible flow	d.	Streamline flow			**Incompressible flow**
Supersonic flow can be converted in to subsonic using _________ nozzle	a.	Diverging	b.	Expanding	c.	Converging	d.	Converging-diverging			**Converging**
For under expanding nozzle ________.	a.	$P_b < P_e$	b.	$P_b > P_e$	c.	$P_b = P_e$	d.	All of the above			$\mathbf{P_b < P_e}$
Subsonic flow over the wedge body produces ________ shock.	a.	Oblique	b.	normal	c.	expansion	d.	No			**Oblique**
Total temperature is constant across a stationary ________ shock.	a.	Oblique	b.	normal	c.	expansion	d.	No			**b**
A gas with constant specific heats is known as________.	a.	Thermally perfect gas	b.	Calorically perfect gas	c.	Ideal gas	d.	Mach			**normal**
Prandtl relation for normal shock is________.	a.	$V_1*V_2=(a*)^2$	b.	$V_1/V_2=(a*)^2$	c.	$V_1+V_2=(a*)^2$	d.	$V_1-V_2=(a*)^2$			$\mathbf{V_1*V_2=(a*)^2}$
Normal shock is _________ to flow direction.	a.	parallel	b.	perpendicular	c.	inclined	d.	all of the above			**perpendicular**
Lift produce is directly proportional to _____ pressure.	a.	Dynamic	b.	Static	c.	total	d.	all of the above			**Static**
When the velocity increases lift produced will _____.	a.	decrease	b.	constant	c.	increase	d.	independent			**increase**
The expansion takes place across a continuous succession of Mach waves is _____.	a.	isentropic	b.	adiabatic	c.	isothermal	d.	isobaric			**isentropic**
When the oblique shock is reflected from the solid surface,it will produce ________ shock.	a.	Oblique	b.	normal	c.	expansion	d.	No			**Oblique**
In a finite wing, the strong vortices produced at ________.	a.	tip of the wing	b.	centre of the wing	c.	no vortices produced	d.	edge of the wing			**tip of the wing**
Can isentropic condition be applied across a normal shock wave?	a.	yes	b.	no because of T_{to} is constant	c.	no because of loss in P_{to}	d.	no because of P_{to} is constant			**no because of T_{to} is constant**
Back pressure is the pressure ________.	a.	outside the nozzle plane	b.	inside the nozzle plane	c.	inlet pressure	d.	exit pressure			**outside the nozzle plane**
The critical mach number is increased by using ________.	a.	Thinner and Swept wing	b.	Thicker Delta wing	c.	Thinner wing only	d.	Thicker wing only			**Thinner and Swept wing**
In a blow down wind tunnel, _________ is used to produce air flow.	a.	compressor	b.	Turbine	c.	Duct fan	d.	All the above			**Duct fan**
The feature of blow down configuration is _________.	a.	High Mach Capability	b.	Less noise operation	c.	Continuous operation	d.	All the above			**High Mach Capability**
There is no need for _________ in the indraft tunnel.	a.	Pressure regulator	b.	Vacuum Chamber	c.	Noise reducer	d.	Plenum Chamber			**Noise reducer**
Nozzles are used to ________ the velocity of inlet air.	a.	decrease	b.	increase	c.	zero	d.	constant			**increase**
Subsonic flows through a converging nozzle will ________ velocity.	a.	decrease	b.	increase	c.	zero	d.	neutralize			**increase**
When Pb<Pe ,Convergent-Divergent duct is known as ____________ nozzle.	a.	Under expanding	b.	Expanding	c.	Converging	d.	over expanding			**Under expanding**
In a converging – diverging nozzle, mach no in the throat is always _________.	a.	M = 0	b.	M = 1	c.	M > 1	d.	M < 1			**M = 1**
Shock will always occur in flow with_________.	a.	M = 0	b.	M = 1	c.	M > 1	d.	M < 1			**M > 1**
When flow cross the normal shock wave, total pressure will be _________.	a.	decreased	b.	increased	c.	zero	d.	Constant			**decreased**
Detached shock wave occurs in a flow over _______ body.	a.	blunt	b.	sharp	c.	wedge	d.	Constant			**blunt**
Flow across the expansion wave , velocity will be ________.	a.	decreased	b.	increased	c.	zero	d.	Constant			**increased**
In a compressible flow density is ________.	a.	decreased	b.	increased	c.	Variable one	d.	Constant			**Constant**
The Weak pressure disturbance is known as ________.	a.	Normal shock	b.	Oblique shock	c.	Sound Wave	d.	Mach wave			**Sound Wave**

The lift produced by the airfoil is due to ________ of velocity.	a.	decreased	b.	increased	c.	zero	d.	Constant	**increased**
Relation between the mach no(M) and mach angle (μ) _________.	a.	. $\mu = \sin^{-1} M$	b.	$\sin \mu = M$	c.	$\sin \mu = 1/M$	d.	$\mu = M$	**$\sin \mu = 1/M$**
The Value Of the Characteristic Mach Number will be ________ if Mach Number tends to infinity.	a.	0	b.	2.449	c.	2.65	d.	1	**2.449**
Critical Mach number is defined as_________.	a.	$M^*=a/c^*$	b.	$M = c/a^*$	c.	$M^*=c/a^*$	d.	$M^* =c^*/a^*$	**$M^* =c^*/a^*$**
Normal shocks are always_______.	a.	one-dimensional	b.	two-dimensional	c.	three-dimensional	d.	all the above	**one-dimensional**
An incident shock gets reflected as a shock from a solid boundary is called ________.	a.	like reflections	b.	simple regions	c.	unlike reflections	d.	non-simple regions	**like reflections**
The advantage of Blow down wind tunnel is_________.	a.	High Mach Capability	b.	Shock –free operation	c.	Muffler	d.	Continuous operation	**High Mach Capability**
The Shock wave may occur in _________.	a.	Blow down	b.	Induction tunnel	c.	Shock tube	d.	All the above	**All the above**
The supersonic wind tunnel produces_________.	a.	$M = 0$	b.	$M = 1$	c.	$M > 1$	d.	$M < 1$	**$M > 1$**
The Component used to straighten the airflow in the wind tunnel is________	a.	Settling chamber	b.	Pressure regulator	c.	diffuser	d.	test section	**Settling chamber**
In supersonic flow, converging nozzle is used to ________ velocity.	a.	decrease	b.	increase	c.	zero	d.	constant	**increase**
Velocity of sound(a) is give by________.	a.	γRT	b.	RT	c.	$\sqrt{(\gamma RT)}$	d.	$(\gamma RT)^2$	**$\sqrt{(\gamma RT)}$**
According to continuity equation, in a flow ________.	a.	Total energy remains constant	b.	Mass flow rate remains constant	c.	Velocity remains constant	d.	Density remains constant	**Mass flow rate remains constant**
In a converging – diverging nozzle, condition of M = 1 occur at ________.	a.	inlet	b.	exit	c.	throat	d.	Every where	**throat**
When flow cross the normal shock wave total temperature ________.	a.	decrease	b.	increase	c.	zero	d.	constant	**constant**
Which one of the following condition is true when flow cross normal shock wave_____	a.	$V_1=V_2$	b.	$V_1>V_2$	c.	$V_1<V_2$	d.	$V_1=V_2=1$	**$V_1>V_2$**
Oblique shock is ___________ to flow direction	a.	parallel	b.	perpendicular	c.	Inclined	d.	All the above	**Inclined**
One dimensional flow with friction is known as ___________.	a.	Fanno flow	b.	Steady flow	c.	Rayleigh flow	d.	Laminar flow	**Fanno flow**
The three-dimensional Shock is known as ________.	a.	Normal shock	b.	Oblique shock	c.	Expansion wave	d.	Mach wave	**Expansion wave**
Lift produced in a body due to flow over it is directly proportion all to________.	a.	V	b.	V^2	c.	V^3	d.	All the above	**V^2**
Aerodynamic center is the point at which the moment produced will________.	a.	Always vary	b.	Remains constant	c.	Not consider	d.	be zero	**Remains constant**
The mach no (M) and mach angle (μ) can be related as ________.	a.	. $\mu = \sin^{-1} M$	b.	$\sin \mu = M$	c.	$\sin \mu = 1/M$	d.	$\mu = M$	**$\sin \mu = 1/M$**
The Process inside the Sound wave is ________.	a.	isentropic	b.	adiabatic	c.	isobaric	d.	isothermal	**isothermal**
One-dimensional Flow with heat addition is known as________.	a.	Rayleigh Flow	b.	Fanno Flow	c.	Irreversible flow	d.	No wave	**Rayleigh Flow**
The one-dimensional motion of a shock wave in a tube of constant area are called________.	a.	wave motion	b.	plane waves	c.	wave speed	d.	All the above	**plane waves**
In a finite wing, the strong vortices produced at ________.	a.	tip of the wing	b.	centre of the wing	c.	no vortices produced	d.	edge of the wing	**tip of the wing**
The small perturbation equations are ________ upto M≤1.	a.	linear	b.	non-linear	c.	rotational	d.	irrotational	**linear**
Critical velocity in converging-diverging nozzle occur at _________.	a.	Velocity at the exit	b.	Maximum velocity	c.	Velocity at the throat	d.	Velocity at the inlet	**Velocity at the throat**
Smaller loads on model during startup because of faster starts is a feature of_________	a.	Indraft	b.	Blow down	c.	Gun tunnel	d.	Helium Tunnel	**Blow down**
Flow across the normal shock wave is ________.	a.	isentropic	b.	iso baric	c.	non isentropic	d.	reversible adiabatic	**non isentropic**
For over expanding nozzle ____________.	a.	$P_b<P_e$	b.	$P_b>P_e$	c.	$P_b=P_e$	d.	$P_b \leq P_e$	**$P_b<P_e$**
Thermodynamic properties do not change with respect to time in ________.	a.	Steady flow	b.	Compressible Flow	c.	Incompressible flow	d.	Streamline flow	**Steady flow**
Constant Pressure process is known as ________.	a.	Isobaric	b.	Isothermal	c.	Adiabatic	d.	Isentropic	**Isobaric**
When $P_b= P_e$ _______.	a.	Flow takes place slowly	b.	Flow takes place rapidly	c.	No flow takes place	d.	Flow takes place at isentropic	**Flow takes place at isentropic**
Diffuser are used to ________ the velocity of inlet air.	a.	decrease	b.	increase	c.	zero	d.	constant	**decrease**
Subsonic flows through a diverging nozzle will ________ velocity.	a.	Steady flow	b.	Compressible Flow	c.	Incompressible flow	d.	Streamline flow	**Incompressible flow**
Subsonic can be converted in to Supersonic flow using _________ nozzle	a.	Convergent nozzle	b.	Divergent nozzle	c.	convergent - divergent nozzle	d.	Choked nozzle	**convergent - divergent nozzle**
For under expanding nozzle ________.	a.	$P_b<P_e$	b.	$P_b>P_e$	c.	$P_b=P_e$	d.	$P_b \leq P_e$	**$P_b>P_e$**
When velocity of flow over an airfoil increases, then lift produced by the airfoil will	a.	decrease	b.	increase	c.	zero	d.	constant	
Upper critical Mach number is defined as the free stream mach number for which the entire flow around the body is____________.	a.	sonic	b.	Subsonic	c.	Transonic	d.		**Transonic**
Normal shocks are always created at ____________.	a.	subsonic flow	b.	supersonic flow	c.	Transonic flow	d.	hypersonic flow	**supersonic flow**
At ________the airflow over some point of the aircraft reaches the speed of sound .	a.	Critical Mach number	b.	subcritical	c.	normal	d.	lower critical	**Critical Mach number**
The small perturbation equations for subsonic and supersonic flows are________.	a.	linear	b.	non-linear	c.	Drag divergence	d.	irrotational	**linear**
Critical velocity in converging-diverging nozzle occur at _________.	a.	Velocity at the exit	b.	Maximum velocity	c.	rotational	d.	Velocity at the inlet	Velocity at the throat
For analysis of flows past ballistic missiles,____________is used.	a.	Blow down	b.	Gun tunnel	c.	Velocity at the throat	d.	Helium tunnel	Gun tunnel
When flow pass the normal shock wave, it follows _______ process.	a.	isentropic	b.	iso baric	c.	In draft tunnel	d.	reversible adiabatic	non isentropic
psi is a unit of	a.	pressure	b.	temperature	c.	non isentropic	d.	energy	pressure
A high-lift subsonic aerofoil section has a thickness/chord ratio approximatel	a.	12%	b.	6%	c.	15%	d.	25%	12%
Lift increase with	a.	air density	b.	AOA, within the stalling limit	c.	18%	d.	all of the above	all of the above
A rectangular wing with unsymmetrical aerofoil section stalls in clean configuration when the AOA is	a.	between 4° and 8°	b.	between 12° and	c.	Aspecr Ratio	d.	between 45° and 60°	
The direction of lift is	a.	vertically upward	b.	90° to the roll axis	c.	between 30° and 35°	d.	90° to the flight path	vertically upward
Sound propagation is	a.	isothermal process	b.	Isentropic process	c.	90° to the wing mean chord	d.	Isochoric process	Isentropic process
When the Mach number ahead of a normal shock is infinity the Mach number behind the normal shock is	a.	infinity	b.	high supersonic	c.	Isobaric process	d.	low subsonic	low subsonic
The lowest value of shock angle for oblique shocks is	a.	Zero	b.	12.5 deg	c.	Zero	d.	15 deg	Mach angle
The maximum possible turning angle through Prandtl_Meyer expansion is	a.	130.5 deg	b.	180 deg	c.	Mach angle	d.	165 deg	130.5 deg
Isothermal compressibility of a compressible gas at a given pressure is	a.	Directly proportional to the absolute temperature of the gas	b.	Reciprocal of the pressure	c.	145 deg	d.	All of the above	Reciprocal of the pressure
is a pressure drag resulting from static pressure components located to either side of compression or shock waves	a.	pressure drag	b.	Wave Drag	c.	Inversely proportional to the square of the pressure	d.	All of the above	Wave Drag
are airfoils developed to increase the increment between the critical Mach number and the drag divergent Mach number.	a.	critical	b.	subcritical	c.	skinfr	d.	optimum	Supercritical airfoils
The free stream Mach number at which sonic speed is first achieved somewhere on the surface of a body it is called	a.	critical Mach number.	b.	subcritical	c.	Supercritical airfoils	d.	transonic	critical Mach number.
The difference in effective downwash angle for the two tail spans for sweptback wings at high angles of attack when the tips stall is	a.	tip loss	b.	vorticity	c.	Drag divergence	d.	incident	tip effects
The free stream Mach number at which the entire flow around the surface is in subsonic flow is called as	a.	Lower critical Mach number.	b.	upper critical Mach number.	c.	tip effects	d.	super critical	Lower critical Mach number.
The free stream Mach number at which the entire flow around the surface is in supersonic flow is called as	a.	Lower critical Mach number.	b.	subcritical	c.	subcritical	d.	upper critical Mach number.	upper critical Mach number.
is a simple statement that the cross sectional area of the body should have a smooth variation along the longitudinal distance of the body	a.	supersonic	b.	Transonic are rule	c.	super critical	d.	supercritical	Transonic are rule
Swept wings are used to increase the	a.	critical Mach number	b.	critical Mach number.	c.	subsonic	d.	Drag divergence	critical Mach number
is a device to produce high speed flow with high temperatures, by the rupture of a diaphragm separating a high pressure gas from a low pressure gas.	a.	shock tube	b.	baurdon tube	c.	subcritical	d.	pressure transducers	shock tube
is a powerful tool in experimental fluid mechanics	a.	Flow Visualisation	b.	Pressure regulator	c.	pitot tube	d.	pressure transducers	Flow Visualisation

separation point and wake formation can be analysed due to increase in angle of attack.	a.	subsonic wind tunnel	b.	supersonic wind tunnel	c.	pitot tube	d.	cascade tunnel	Flow Visualisation
A ____________ is a wind tunnel that produces supersonic speeds ($1.2<M<5$)	a.	subsonic wind tunnel	b.	supersonic wind tunnel	c.	Flow Visualisation	d.	Interferometer	supersonic wind tunnel
_______system gives a picture or an image representative of the variation of density throughout the complete flow field.	a.	supersonic wind tunnel	b.	Schlieren	c.	Schlieren	d.	Schlieren	Schlieren
is a flow visualization technique meant for high speed flows with transonic and supersonic Mach number. This employed for field with strong shock waves.	a.	Schlieren	b.	Interferometer	c.	Interferometer	d.	supersonic wind tunnel	Shadowgraph
It is an optical technique to visualize high speed flows in the ranges of transonic and supersonic Mach numbers. This gives a qualitative estimate of flow density in the field.	a.	Schlieren	b.	Interferometer	c.	Shadowgraph	d.	supersonic wind tunnel	Interferometer
The model must be accommodated inside the rhombus formed by the incident and reflected shocks, for proper measurements is known as	a.	geometric test	b.	dynamic test	c.	Shadowgraph	d.	kinematic	principles of model testing
is a device used to measure forces, moments, pressure, shear stress, heat transfer and flow field	a.	optical visualization	b.	Wind tunnel	c.	principles of model testing	d.	particle velocimetry	Wind tunnel
The instruments used to measure the fluctuating pressures are	a.	optical visualization	b.	Turbulence sphere	c.	dye injection	d.	particle velocimetry	Turbulence sphere
are normally used from high subsonic to high supersonic flow conditions. The air is pumped into a closed high pressure chamber upstream of the plenum.	a.	Induction	b.	Wind tunnel	c.	dye injection	d.	Blow down tunnels	Blow down tunnels
type energy stored in the form of pressure in a reservoir	a.	wind tunnel	b.	Induction	c.	dye injection	d.	Blow down tunnels	Intermittent
type energy stored in the form of Vacuum in a reservoir.	a.	Induction	b.	Intermittent	c.	Intermittent	d.	vaccum tunnel	Induction
The pressure to be measured are usually to large to be measured instead of water or alcohol.	a.	shock tube	b.	baurdon tube	c.	blow down	d.	pressure transducers	pressure transducers
is used for testing medium bathing aircraft and reentering spacecraft	a.	shock tube	b.	Helium tunnel	c.	pitot tube	d.	Intermittent	Helium tunnel
Sensitive to the change of Displacement	a.	Shadowgraph	b.	Schileren	c.	Induction	d.	dye injection	Shadowgraph
Sensitive to the change of Angular deflection	a.	dye injection	b.	Shadowgraph	c.	Interferometer	d.	Interferometer	Schileren
Sensitive to the change of Phase change	a.	Shadowgraph	b.	Schileren	c.	Schileren	d.	Interferometer	Interferometer
The starting shock crossing the diffuser throat and remaining in its divergent part is called the	a.	Drying	b.	Liquefaction	c.	dye injection	d.	Mach reflection.	swallowing of the starting shock.
the working fluid is the best way to avoid condensation.	a.	swallowing of the starting shock.	b.	Drying	c.	swallowing of the starting shock.	d.	sublimation	Drying
How can condensation be avoided/reduced in supersonic wind tunnels?	a.	vapourisation	b.	sublimation	c.	Liquefaction	d.	Liquefaction	Drying
troubles might start around $M=4$ if high pressure air is expanded from room temperature.	a.	vapourisation	b.	sublimation	c.	Drying	d.	Liquefaction	Liquefaction
Intersection of normal shock and the right running oblique shock gives rise to a reflected left running oblique shock in order to bring the flow into the original direction are called	a.	Mach absoption	b.	Mach reflection.	c.	Liquefaction	d.	simple	Mach reflection.
Supersonic expansion or compression with mach lines which are straight is called	a.	simple region	b.	non simple region	c.	injection	d.	curved	simple region
The intersection of mach lines of different form, lies leads to______________ where all the mach lines are not straight but curved.	a.	curve	b.	simple region	c.	Mach reflection.	d.	Mach reflection.	non simple region
Assumptions made to derive Normal Shock relation	a.	Flow is 1-D, invisid	b.	Flow is 2-D, invisid	c.	non simple region	d.	Flow is 3-D, invisid	Flow is 1-D, invisid
is the weak limit of an oblique shock wave	a.	Mach wave	b.	normal shock	c.	Flow is 2-D, visicous	d.	shock	Mach wave
shock waves in the flow over the wing and tail plane are sufficient to stall the wing, make control surfaces ineffective or lead to loss of control such as	a.	Mach angle	b.	Mach tuck	c.	Mach angle	d.	normal shock	Mach tuck
The phenomena associated with problems at the critical Mach number became known as	a.	compressibility	b.	incompressible	c.	Mach wave	d.	vapourisation	compressibility
_____ is a wing platform with a wing root to wingtip direction angled beyond the span wise axis, generally used to delay the drag rise caused by fluid compressibility	a.	delta wing	b.	Straight wing	c.	expansion	d.	Swept wings	**Swept wings**
______as a means of reducing wave drag were first used on jet fighter aircraft	a.	delta wing	b.	Straight wing	c.	eliptical wing	d.	eliptical wing	**Swept wings**
The four-engine propeller-driven ______ aircraft has swept wings.	a.	A-10	b.	A380	c.	Swept wings	d.	TU-95	**TU-95**
The ______ is that free-stream Mach number at which sonic flow is first encountered on the airfoil.	a.	subcritical	b.	critical Mach number	c.	pushpak	d.	supercritical	**critical Mach number**
The_______gradient induced by the shock tends to separate the boundary layer on the top surface, causing a large pressure drag	a.	adverse pressure	b.	reverse pressure	c.	Drag divergence	d.	high pressure	**adverse pressure**
________is the Mach number at which the aerodynamic drag on an airfoil or airframe begins to increase rapidly as the Mach number continues to increase	a.	critical	b.	drag divergence	c.	favourable pressure	d.	supercritical	**drag divergence**
can cause the drag coefficient to rise to more than ten times its low speed value.	a.	subcritical	b.	supercritical	c.	subcritical	d.	drag divergence	**drag divergence**
The value of the drag divergence Mach number is typically greater than	a.	0.3	b.	1	c.	critical	d.	2	**0.6**
A change of state is called a _____________ in thermodynamics	a.	steady state	b.	Process	c.	0.6	d.	cycle	**Process**
is a branch of natural science concerned with heat and its relation to energy and work.	a.	cycle	b.	steady state	c.	unsteady state	d.	unsteady state	**Thermodynamics**
is defined as a quantity of matter or a region in space chosen for study.	a.	cycle	b.	system	c.	Thermodynamics	d.	steady state	**system**
The mass or region outside the system is called the ____________	a.	system	b.	Process	c.	process	d.	surroundings	**surroundings**
The______ is the total energy contained by a thermodynamic system	a.	electrical energy	b.	heat energy	c.	boundary	d.	potential energy	**internal energy**
_______the real or imaginary surface that separates the system from its surroundings. The boundaries of a system can be fixed or movable.	a.	system	b.	Boundary	c.	internal energy	d.	wall	**Boundary**
A closed system that does not communicate with the surroundings by any means_____________	a.	Isolated system	b.	closed	c.	surrounding	d.	cycle	**Isolated system**
A closed system that communicates with the surroundings by heat Only	a.	flexible	b.	adiabatic	c.	open	d.	Rigid system	**Rigid system**
A closed or open system that does not exchange energy with the surroundings by heat	a.	Isolated system	b.	Adiabatic system	c.	insulated system	d.	Rigid system	**Adiabatic system**
are those that are independent of the size (mass) of a system, such as temperature, pressure, and density.	a.	Intensive properties	b.	Adiabatic system	c.	insulated system	d.	Isolated system	**Intensive properties**
values that are dependant on size of the system such as mass, volume, and total energy U.	a.	Adiabatic system	b.	Extensive properties	c.	Extensive properties	d.	Isolated system	**Extensive properties**
when the temperature is the same throughout the entire system.	a.	enthalphy	b.	entropy	c.	Intensive properties	d.	Thermal equilibrium	**Thermal equilibrium**
is a process during which the temperature remains constant	a.	isentropic	b.	Isobaric	c.	equilibrium	d.	Isochoric process	**Isothermal**
is a process during which the pressure remains constant	a.	Isothermal	b.	Isobaric	c.	Isothermal	d.	Isochoric process	**Isobaric**
is process during which the specific volume remains constant	a.	isentropic	b.	Isochoric process	c.	isentropic	d.	polytropic	**Isometric**
states that if two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other.	a.	charles law	b.	Second law of thermodynamics	c.	Isometric	d.	First law of thermodynamics	**0th law of thermodynamics**
It simply states that during an interaction, energy can change from one form to another but the total amount of energy remains constant.	a.	First law of thermodynamics	b.	0th law of thermodynamics	c.	0th law of thermodynamics	d.	boyles law	**First law of thermodynamics**
energy has quality as well as quantity, and actual processes occur in the direction of decreasing quality of energy.	a.	First law of thermodynamics	b.	Second law of thermodynamics	c.	Second law of thermodynamics	d.	charles law	**Second law of thermodynamics**
The speed of sound as a function of	a.	pressure	b.	density	c.	0th law of thermodynamics	d.	Static temperature	**temperature**
it is proportional to the square root of the temperature	a.	speed of light	b.	speed of sound	c.	Total temperature	d.	change in density	**speed of sound**
the speed of sound is about ________ of the average molecular velocity.	a.	three-quarters	b.	one-quarters	c.	molecular velocity	d.	quarters	**three-quarters**
Flow is said to be subsonic flow if	a.	$M<1$	b.	$M=1$	c.	two-quarters	d.	$M>5$	**$M<1$**
Flow is said to be sonic flow if	a.	$M<1$	b.	$M=1$	c.	$M>1$	d.	$M>5$	**$M=1$**
Flow is said to be supersonic flow if	a.	$M<1$	b.	$M=1$	c.	$M>1$	d.	$M>5$	**$M>1$**
Flow is said to be Hypersonic flow if	a.	$M=1$	b.	$M>1$	c.	$M>1$	d.	$M<1$	**$M>5$**
Mach number was defined as the ratio of	a.	$V/2a$	b.	$2V/a$	c.	$M>5$	d.	V/a	**V/a**
For calorically perfect gas the square of the ______ is proportional to the ratio of kinetic to internal energy	a.	Static temperature	b.	Mach number	c.	$3V/a$	d.	density	**Mach number**
For $0<M<1$ (subsonic flow), an increase in velocity (positive du) is associated with a decrease in	a.	diverging duct	b.	velocity	c.	pressure	d.	area	**area**

For M > 1 (supersonic flow), an increase in ____ is associated with an increase in area	a.	converging duct	b.	velocity	c.	converging duct	d.	diverging duct		**velocity**
For supersonic flow, the velocity increases in a diverging duct and decreases in a	a.	velocity	b.	converging duct	c.	area	d.	diverging duct		**converging duct**
A expansion fan is a centered expansion process, which turns a supersonic flow around a	a.	convex corner	b.	concave	c.	area	d.	curved		**convex corner**
Across the expansion fan, the flow accelerates and the Mach number ______	a.	increases	b.	decrease	c.	sharp corner	d.	negative		**increases**
Across the expansion fan, tthe static pressure, temperature and density ____	a.	decrease	b.	constant	c.	constant	d.	negative		**decrease**
The process is isentropic, the stagnation properties remain constant across the	a.	expansion fan	b.	Normal shock	c.	increases	d.	reflected wave		**expansion fan**
The fan consists of an infinite number of Mach waves, diverging from a	a.	sharp corner	b.	convex corner	c.	oblique shocks	d.	flat		**sharp corner**
Straight ________ are also attached to the tip of a sharp cone in supersonic flow	a.	expansion fan	b.	Normal shock	c.	concave	d.	reflected wave		**oblique shocks**
velocities on a graph which uses Vx and Vy velocity components is called the	a.	hodograph plane	b.	shock polars	c.	oblique shocks	d.	subsonic		**hodograph plane**
In the shock polars inside the circle, all velocities are subsonic, outside it, all velocities are ________	a.	supersonic	b.	less	c.	Intersection of shock waves	d.	hypersonic		**supersonic**
In the shock polars inside the circle, all velocities are_____,outside it, all velocities are supersonic	a.	supersonic	b.	less	c.	subsonic	d.	hypersonic		**subsonic**
The pressure is continuous but the entropy is discontinuous at the slip line in	a.	shock polars	b.	Intersection of shock waves	c.	subsonic	d.	hypersonic		**Intersection of shock waves**
Reflected waves form a like pattern throughout the exhaust jet	a.	rectangle	b.	diamond	c.	subsonic	d.	concave		**diamond**
_________ co-ordinates are more convenient for elongated bodies and bodies of revolution	a.	Cylindrical	b.	spherical	c.	convex corner	d.	rectangular		**Cylindrical**
__________theory is frequently linear theory	a.	linearised flow theory	b.	Small-perturbation	c.	cartesian	d.	Compressible Flow theory		**Small-perturbation**
The ______ where the assumption of small perturbations allowed a linearized solution	a.	Small-perturbation	b.	Compressible Flow theory	c.	Small-perturbation	d.	linearised flow theory		**acoustic theory**
___________for a symmetrical airfoil in supersonic flow is predicted at the mid-chord point	a.	Aerodynamic center	b.	center of pressure	c.	acoustic theory	d.	coefficient of drag		**center of pressure**
____________ is a design technique used to reduce an aircraft's drag at transonic	a.	Whitcomb area rule	b.	area rule	c.	coefficient of pressure	d.	thick aerofoil		**Whitcomb area rule**
_______ is a design technique used to reduce an aircraft's drag at transonic and supersonic speeds, particularly between Mach 0.75 and 1.2	a.	subsonic area rule	b.	subcritical	c.	thumbrule	d.	transonic area rule		**transonic area rule**
To reduce the number of these shock waves, an aerodynamic shape should change in cross sectional area as smoothly as possible. This leads to a perfect aerodynamic shape known as the	a.	shark lets	b.	Sears-Haack body	c.	Supercritical	d.	winglet		**Sears-Haack body**
The Mach number in the test section of blow down tunnel is determined by pressure and temperature in the	a.	settling chamber	b.	diaphragm	c.	Wing tip vortices	d.	filter		**plenum**
Test times are limited in ____________ wind tunnels	a.	suction	b.	blowdown	c.	plenum	d.	indraft tunnels		**blowdown**
A ______ is often employed downstream of the test section to shock down the supersonic flow to subsonic before entering the low pressure chamber.	a.	second throat	b.	first throat	c.	subsonic	d.	settling chamber		**second throat**
A closed configuration with both high pressure and low pressure chambers is shown in the figure, but there are other configurations of blowdwon tunnels. Some blowdown tunnels, called	a.	suction	b.	blowdown	c.	third throat	d.	subsonic		**indraft tunnels**
High subsonic wind tunnels opertated at	a.	(1.2<M<5)	b.	(0.4 < M < 0.75)	c.	indraft tunnels	d.	(0.75 < M < 1.2)		**(0.4 < M < 0.75)**
transonic wind tunnels opertated at	a.	(1.2<M<5)	b.	(0.4 < M < 0.75)	c.	M=1	d.	(0.75 < M < 1.2)		**(0.75 < M < 1.2)**
A supersonic wind tunnel is a wind tunnel that produces supersonic speeds	a.	(1.2<M<5)	b.	(0.4 < M < 0.75)	c.	M=1	d.	M=1		**(1.2<M<5)**
__________ have short test times (usually less than one second), relatively high Reynolds number, and low power requirements	a.	Ludwieg tube	b.	shock tubes	c.	(0.75 < M < 1.2)	d.	bourdan tube		**Ludwieg tube**
Stagnation temperatures of at pressures of several hundred atmospheres provide test Mach numbers from 6 to 15 for run durations on the order of 1 minute	a.	3500° F	b.	3500°C	c.	density tube	d.	2500° F		**3500° F**
allow the study of fluid flow at temperatures and pressures that would be difficult to obtain in wind tunnels	a.	pitot tube	b.	Shock tubes	c.	1500° F	d.	bourdan tube		**Shock tubes**
Aerodynamics of a spinning cricket ball is related to	a.	Bernoulli's principle	b.	Magnus effect	c.	density tube	d.	Newton's second law		**Magnus effect**
Velocity potential is valid for	a.	Viscous flow	b.	Real flow	c.	Kutta condition	d.	Irrotational flow		**Irrotational flow**
Streamlined body is one for which	a.	Pressure drag is more than skin friction drag	b.	Induced drag is more than profile drag	c.	Rotational flow	d.	All of the above		**Skin friction drag is more than ressure drag**
Stalling in an incompressible flow is due to	a.	sudden expansion	b.	flow separation	c.	Skin friction drag is more than ressure drag	d.	Isentropic expansion		**flow separation**
Lifting flow over circular cylinder is obtained by the combination of	a.	Uniform flow + source + vortex	b.	Uniform flow + sink + vortex	c.	Adiabatic compression	d.	Uniform flow + doublet + vortex		**Uniform flow + doublet + vortex**
NACA 0014 implies the airfoil is	a.	Symmetric	b.	Positively cambered	c.	Source + Sink + Uniform flow	d.	Cusped		**Symmetric**
Kutta-Joukowski theorem gives the dependence of lift per unit span on	a.	Total pressure	b.	Temperature	c.	Negatively cambered	d.	All of the above		**Circulation**
Aerodynamic center of an airfoil is the point about which	a.	Pitching moment is zero	b.	Pitching moment is constant	c.	Circulation	d.	Pitching moment is negative		**Pitching moment is zero**
Sound propagation is	a.	Isothermal process	b.	Isentropic process	c.	Pitching moment is positive	d.	Isochoric process		**Isentropic process**
When the Mach number ahead of a normal shock is Infinity the Mach number behind the normal shock is	a.	Infinity	b.	High supersonic	c.	Isobaric process	d.	Low subsonic		**Low subsonic**
The lowest value of shock angle for oblique shocks is	a.	Zero	b.	12.5 deg	c.	Zero	d.	115 deg		**Mach angle**
The maximum possible turning angle through Prandtl_Meyer expansion is	a.	130.5 deg	b.	180 deg	c.	Mach angle	d.	115 deg		**130.5 deg**
When the Mach number ahead of a normal shock is infinity the ratio of static density before and after the normal shock is	a.	infinity	b.	Finite	c.	145 deg	d.	Zero		**Finite**
For supersonic flow of Mach number = 2 flowing over a compression corner of turning angle nearly equal to zero the shock angle is	a.	Zero	b.	45 deg	c.	Inversely proportional to the square of the pressure	d.	12.5 deg		**30 deg**
Transonic area rule is applied to the following component of airplane	a.	Wings	b.	Tail	c.	30 deg	d.	Power plants		**Fuselage**
Prandtl's relation for a normal shock is an equation consisting of	a.	Characteristic Mach numbers	b.	Static pressures	c.	Fuselage	d.	density		**Characteristic Mach numbers**
The maximum possible value of Characteristic Mach number is	a.	Infinity	b.	2.45	c.	Total pressures	d.	1		**2.45**
Mach angle is the lowest possible value of	a.	Flow turning angle	b.	shock angle	c.	2.45	d.	Characteristic Mach numbers		**shock angle**
Prandtl-Glauert rule gives the relation between	a.	Viscous and inviscid flow	b.	line integral and surface integral	c.	Angle of incidence	d.	incompressible and compressible flow characteristics		**incompressible and compressible flow characteristics**
Small perturbation theory is applicable for	a.	circular cylinders	b.	slender bodies	c.	Streamlines and equipotential lines	d.	incompressible and compressible flow characteristics		**slender bodies**
In free vortex flow the tangential velocity is	a.	directly proportional to radial distance	b.	inversely proportional to radial distance	c.	spherical bodies	d.	Zero		**inversely proportional to radial distance**
Incompressible inviscid flow can be represented by	a.	second order polynomial	b.	Fourier series	c.	independent of radial distance	d.	**Laplace equation**		**Laplace equation**

Curl of velocity vector is	a.	Acceleration	b.	normal component of velocity	c.	Laplace equation	d.	momentum	**Vorticity**
Viscosity of gases	a.	increases with temperature	b.	decreases with increasing temperature	c.	Vorticity	d.	constant with increasing temperature	**increases with temperature**
Stream function is related to	a.	Volume flow rate	b.	Circulation	c.	is independent of temperature	d.	momentum	**Volume flow rate**
Sweep back results in	a.	less directional stability	b.	less longitudinal stability	c.	Angular velocity	d.	higher directional stability	**higher directional stability**
Supercritical airfoils are characterized by	a.	sharp leading edge	b.	highly cambered upper surface	c.	stronger longitudinal stability	d.	conical upper surface	**flattened upper surface**
A compressible fluid when brought to rest generates greater pressure than an	a.	orthotropic fluid	b.	incompressible fluid	c.	flattened upper surface	d.	isotropic fluid	**incompressible fluid**
Air at lower density is more compressible than air at higher density and therefore compressibility error increases with	a.	increase in altitude	b.	decrease in altitude	c.	barotropic fluid	d.	increase in pressure	**increase in altitude**
is lighter than air and it is present only in the lower layers of the atmosphere.	a.	moisture	b.	liquid	c.	decrease in pressure	d.	Water vapour	**Water vapour**
The speed of sound is directly proportional to the	a.	absolute viscosity	b.	Kinematic viscosity	c.	ice	d.	absolute pressure	**square root of the absolute temperature**
is the ratio of absolute viscosity to density	a.	absolute viscosity	b.	Kinematic viscosity	c.	square root of the absolute temperature.	d.	square root of the absolute temperature.	**Kinematic viscosity**
is a point on the aerofoil chord line through which the resultant aerodynamic force acts.	a.	Centre of pressure	b.	coefficient of pressure	c.	absolute pressure	d.	coefficient of momentum	**Centre of pressure**
is that fixed point on the aerofoil around which the coefficient of pitching moment is a constant	a.	coefficient of momentum	b.	Aerodynamic centre	c.	Aerodynamic centre	d.	coefficient of pressure	**Aerodynamic centre**
The sum of the static and dynamic pressure is called total head pressure and it remains ___________.	a.	constant	b.	decrease	c.	Centre of pressure	d.	greater	**constant**
_________ would form only when the wing is producing lift and would disappear when the wing is not producing lift	a.	Drag	b.	wash in	c.	increase	d.	washout	**Wing tip vortices**
Zero lift drag___________ because of the forward movement of the transition and separation points with increase in lift.	a.	constant	b.	decrease	c.	Wing tip vortices	d.	greater	**increases**
is a layer of retarded air in contact with the surface of the wing	a.	Boundary layer	b.	displacement	c.	increase	d.	compressibility	**Boundary layer**
_____ drag is caused due to the effect of the boundary layer and it increases with increase in speed.	a.	induced	b.	Skin friction	c.	viscosity	d.	interference	**Skin friction**
________ method of preventing wing tips stalling on swept back wings	a.	wing lets	b.	sharklets	c.	pressure	d.	canard	**Boundary layer fences**
Boundary layer control also reduces____________drag	a.	induced	b.	skin friction	c.	Boundary layer fences	d.	interference	**skin friction**