17BEME302 ENGINEERING MECHANICS 4 H – 4 C

Instruction hours / week L: 3 T: 1 P: 0 Marks: Internal: 40 External: 60 Total: 100
End Semester Exam: 3 Hours

COURSE OBJECTIVE:

1. To develop capacity to predict the effect of force and motion in the course of carrying out the design functions of engineering.

COURSE OUTCOMES:

At the end of the course the students will be able to

1. Draw free body diagrams and determine the resultant of forces and/or moments.
2. Determine the centroid and second moment of area of sections.
3. Apply laws of mechanics to determine efficiency of simple machines with consideration of friction.
4. Analyze statically determinate planar frames.
5. Analyze the motion and calculate trajectory characteristics.
6. Apply Newton's laws and conservation laws to elastic collisions and motion of rigid bodies.

UNIT I STATICS OF PARTICLES

Forces – system of forces – concurrent forces in plane and space– resultant – problems involving the equilibrium of a particle–free body diagram–equilibrium of particle in space.

UNIT II STATICS OF RIGID BODIES IN TWO DIMENSIONS

Rigid bodies–moment of force about an axis–moments and couples–equivalent system of coplanar forces– Rigid body in equilibrium–problems involving equilibrium of rigid body–types of supports–reactions of beams.

UNIT III CENTROID, CENTRE OF GRAVITY AND MOMENT OF INERTIA

Centroids of areas, composite areas, determination of moment of inertia of plane figures, polar moment of inertia – radius of gyration – mass moment of inertia of simple solids.

UNIT IV KINEMATICS OF PARTICLES

Introduction – plane, rectilinear motion – time dependent motion – rectangular coordinates – projectile motion. IMPULSE AND MOMENTUM: Concept of conservation of momentum – Impulse–Momentum principle– Impact – Direct central impact – Oblique central impact – Impact of elastic bodies.

UNIT V KINETICS OF PARTICLES AND FRICTION

Equations of motion–rectilinear motion–Newton's II law – D'Alembert's principle – Energy – potential energy–kinetic energy–conservation of energy–work done by a force – work energy method. Laws of friction – coefficient of friction–problems involving dry friction – wedge and ladder friction.

SUGGESTED READINGS

1. Dr. N. Kottiswaran, 2010, "Engineering Mechanics", Ninth Edition - reprint, Sri Balaji Publications.
2. S Timoshenko, D H Young and J V Rao, 2008. "Engineering Mechanics", Revised fourth Edition, Tata McGraw Hill education pvt. Ltd
3. S Rajasekaran, G Sankarasubramanian, 2009, "Engineering Mechanics", Third Edition, Vikas Publishing House Pvt Ltd.
4. Beer F P and Johnston E.R. 2009, "Vector Mechanics for Engineers–Statics and Dynamics", Tata Mc–Graw Hill Publishing Co. Ltd., New Delhi.
5. Bansal R K, 2006, "Engineering Mechanics", Laxmi Publications Pvt. Ltd., New Delhi.

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE – 641021

FACULTY OF ENGINEERING

DEPARMENT OF MECHANICAL ENGINEERING

LECTURE PLAN

Subject : **ENGINEERING MECHANICS**
Code : **17BEME302**
Class/Semester/Branch : **II Year / III Semester – B.E – Mechanical Engineering**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT I - Statics of Particles			
1.	1	• Introduction to Engineering Mechanics, Scalar, Vector, Distance, Displacement, Speed, Velocity, Force, Work done, Energy and Power.	T[1]- pp. 2-4 T[2] -pp. 1-3 T[3] -pp. 1-3
2.	1	• Rigid body & Deformable body • Classification of Force System	T[1]- pp. 5-16 T[2] -pp. 4-10
3.	1	• Particle • Newton's Law, Principle of Transmissibility.	T[3] -pp. 3-8
4.	1	• Triangular Law, Polygon Law & Parallelogram Law • Resultant, Equilibrium condition & Equilibrant	T[1]- pp. 43-48 T[1]- pp. 5-16
5.	1	• Lami's Theorem • Problems related to Lami's Theorem	T[2] -pp. 10-12
6.	1	• **Tutorial 1** (Problems related to Parallelogram Law & Lami's Theorem)	T[1]- pp. 5-16
7.	1	• Resolution of Forces • Freebody Diagram • Problems related to Concurrent force system acting on particles which is in Equilibrium condition in plane & Freebody diagram.	T[1]- pp. 18-24 T[3] -pp. 35-50
8.	1	• Vector, Force vector, Unit vector & Position vector	T[1]- pp. 31-42
9.	1	• Angle of inclination with X, Y & Z axes. • Components of the force along the axes.	T[3] -pp. 51-68
10.	1	• Magnitude of Resultant vector in space • Direction of Resultant vector in space	T[1]- pp. 30-53 T[2] -pp. 43-54 T[3] -pp. 23-34
11.	1	• Solving problems on Concurrent force system acting on particles which is in Equilibrium condition in space.	T[1]- pp. 60-65
12.	1	• **Tutorial 2** (Problems from Concurrent force system acting on particles which is in Equilibrium condition in space)	T[1]- pp. 60-65 T[2] -pp. 43-54
Total no. of Hours planned for unit - I			**12**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT II		**Statics of Rigid Bodies in Two Dimensions**	
1.	1	• Introduction of Moment • Moment of a force • Varignon's Theorem	**T[1]- pp. 138-140** **T[2] -pp. 160-172** **T[3] -pp. 206-207**
2.	1	• Magnitude of Resultant force • Direction of Resultant force	**T[1]- p. 140** **T[2] -pp. 172-184**
3.	1	• Location of Resultant force using Varignon's Theorem. • Problems related to Non concurrent parallel forces.	**T[3] -pp. 207-208**
4.	1	• Non concurrent non parallel forces - Magnitude of Resultant force, Direction of Resultant force, Location of Resultant force. • Problems related to Non concurrent Non parallel forces.	**T[1]- p. 141** **T[2] -pp. 184-190** **T[3] -pp. 209-214**
5.	1	• Couple, Difference between Moment and Couple. • Conversion of Force to Force – Couple System. • Problems related to Force – Couple System.	**T[1]- pp. 142-148** **T[2] -pp. 190-220** **T[3] -pp. 218-235**
6.	1	• **Tutorial 3** (Problems from Non concurrent parallel forces, Non - parallel forces and Force – Couple System)	**T[1]- pp. 176-179** **T[2] -pp. 190-220**
7.	1	• Introduction of Beam, Difference between Frame and Beam • Types of Supports and Reactions.	**T[1]- pp. 179-185** **T[2] -pp. 283-301**
8.	1	• Types of Loads • Conversion of UDL & UVL to point load.	**T[1]- pp. 179-185** **T[3] -pp. 261-266**
9.	1	• Concept of Beam with Roller & Hinged support • Problems related to force acting on beam with Roller & Hinged support.	**T[1]- pp. 185-200** **T[2] -pp. 301-312** **T[3] -pp. 263-270**
10.	1	• Concept of Beam with two Hinged support • Problems related to force acting on beam with two Hinged support	**T[1]- pp. 254-270** **T[3] -pp. 271-289**
11.	1	• Concept of Beam with inclined support. • Problems related to force acting on beam with inclined support.	**T[1]- pp. 270-275** **T[3] -pp. 366-382**
12.	1	• **Tutorial 4** (Problems from Beams) & revision for CIA I test.	**T[1]- pp. 270-275**
		Total no. of Hours planned for unit - II	**12**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT III		**Centroid, Centre of Gravity and Moment of Inertia**	
1.	1	• Introduction of Centroid and Centre of Gravity. • Centroid of simple plane figures • Problems related to Centroid of simple plane figures	**T[1]- pp. 456-465** **T[2] -pp. 759-768** **T[3] -pp. 724-727**
2.	1	• Centroid of Composite plane figures • Symmetrical about X axis • Symmetrical about Y axis	**T[1]- pp. 480-484** **T[2] -pp. 768-785**
3.	1	• Not Symmetrical about any axis • Problems related to Centroid of Composite plane figures	**T[1]- pp. 489-491** **T[2] -pp. 765-785**
4.	1	• **Tutorial 5** (Problems from Centroid of simple & Composite plane figures)	**T[1]- pp. 489-491**
5.	1	• Centre of Gravity of simple solids • Problems related to Centre of Gravity of simple solids	**T[1]- pp. 409-411**
6.	1	• Centre of Gravity of simple solids (same materials) • Problems related to Centre of Gravity of simple solids (same materials)	**T[1]- pp. 409-411**
7.	1	• Centre of Gravity of simple solids (Different materials) • Problems related to Centre of Gravity of Composite solids (Different materials)	**T[1]- pp. 491-500** **T[2] -pp. 822-837**
8.	1	• **Tutorial 6** (Problems from Centre of Gravity of simple solids)	**T[1]- pp. 491-500**
9.	1	• Introduction to Moment of Inertia • Moment of Inertia of simple plane by Integration (Rectangle, Triangle & Circle).	**T[1]- pp. 502-510** **T[2] -pp. 838-842** **T[3] -pp. 826-830**
10.	1	• Parallel axis theorem • Perpendicular axis theorem	**T[1]- pp. 510-515** **T[2] -pp. 822-842**
11.	1	• Radius of Gyration • Mass Moment of Inertia of simple solids.	**T[1]- pp. 523-525** **T[1] -pp. 535-542**
12.	1	• Problems related to Mass Moment of Inertia of simple solids	**T[1]- pp. 523-525**
Total no. of Hours planned for unit - III			**12**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT IV		**Kinematics of Particles**	
1.	1	• Introduction to Kinematics • Characteristics of kinematics	**T[1]- pp. 290-291** **T[2] -pp. 441-444** **T[3] -pp. 391-401**
2.	1	• Types of Plane motion • Types of Rectilinear motion	**T[1]- pp. 292-336** **T[2] -pp. 402-444**
3.	1	• Equation of motion in a straight line • Problems related to straight line motion	**T[1]- pp. 292-336** **T[2] -pp. 402-444**

4.	1	• Distance travelled in n^{th} second • Motion of particle under Gravity • Problems related to Distance travelled in n^{th} second & Motion of particle under Gravity	**T[1]- pp. 362-365**
5.	1	• Rectangular coordinates • Problems related to Rectangular coordinates	**T[1]- pp. 362-365**
6.	1	• Projectile Motion • Path of Projectile motion	**T[1]- pp. 362-365**
7.	1	• Standard result in Projectile Motion • Problems related to Projectile Motion	**T[1]- pp. 525-528** **T[2] -pp. 982-990**
8.	1	• **Tutorial 7** (Problems from Rectilinear motion & Projectile Motion)	**T[1]- pp. 292-365**
9.	1	• Introduction to Impulse – Momentum method • Impulse of a force • Momentum	**T[1]- pp. 525-536** **T[2] -pp. 992-1005** **T[3] -pp. 931-942**
10.	1	• Impulse – Momentum equation • Problems related to Impulse – Momentum equation	**T[1]- pp. 525-536** **T[2] -pp. 992-1005** **T[3] -pp. 931-942**
11.	1	• Impulse – Momentum equation (Motion of Connected bodies) • Bodies have same velocity & different velocity • Problems related to Motion of Connected bodies	**T[1]- pp. 562-564** **T[2] -pp. 995-1000**
12.	1	• **Tutorial 8** (Problems from Impulse – Momentum equation)	
Total no. of Hours planned for unit - IV			**12**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT V Kinetics of Particles and Friction			
1.	1	• Introduction to Kinetics • Mass, Weight & Momentum • Newton's second law of motion • Equations of Motion • Problems related to linear motion (using Motion equation)	**T[1]- pp. 112-115** **T[2] -pp. 850-872**
2.	1	• Work • Energy (Potential & Kinetic energy) • Conservation of energy	**T[1]- pp. 112-115**
3.	1	• Work done by force • Work – Energy Equation	**T[1]- pp. 115-118**
4.	1	• **Tutorial 9** (Problems from Work – Energy Method)	**T[1]- pp. 112-118** **T[3] -pp. 91-101**
5.	1	• Introduction of Friction • Role of frictional force • Types of friction	**T[1]- pp. 564-566**
6.	1	• Limiting friction • Co-efficient of friction and angle of friction	**T[2] -pp. 843-860**
7.	1	• Law's of static friction • Law's of Dynamic friction	**T[2] -pp. 945-955**
8.	1	• Impending motion • Basic concepts	**T[1]- pp. 607-610**

9.	1	• Angle of Repose • Angle of static friction • Problems related to static friction	**T[2] -pp. 843-860**
10.	1	• Ladder friction • Problems related to Ladder friction	**T[1]- pp. 72-75** **T[2] -pp. 843-860** **T[3] -pp. 91-94**
11.	1	• Wedge friction • Problems related to Wedge friction	**T[1]- pp. 72-75**
12.	1	• **Tutorial 10** (Problems from Ladder & Wedge friction)	**T[1]- pp. 76-96** **T[2] -pp. 860-872**
Total no. of Hours planned for unit - V			**12**

Text Books :

[1] Dr. N. Kottiswaran, 2010, “Engineering Mechanics”, Ninth Edition - reprint, Sri Balaji Publications.
[2] S Timoshenko, D H Young and J V Rao, 2008. “Engineering Mechanics”, Revised fourth Edition, Tata McGraw Hill education pvt. ltd
[3] S Rajasekaran, G Sankarasubramanian, 2009, “Engineering Mechanics”, Third Edition, Vikas Publishing House Pvt Ltd.

Reference Books:

[4] Beer F P and Johnston E.R. 2009, “Vector Mechanics for Engineers–Statics and Dynamics”, Tata Mc–Graw Hill Publishing Co. Ltd., New Delhi.
[5] Bansal R K, 2006, “Engineering Mechanics”, Laxmi Publications Pvt. Ltd., New Delhi.

Website : http://nptel.ac.in/courses/Webcourse-contents/IIT-%20Guwahati/engg_mechanics/index.htm
http://nptel.ac.in/courses/112103108/
http://www.indiabix.com/mechanical-engineering/engineering-mechanics/

UNIT	Total No. of Periods Planned	Lecture Periods	Tutorial Periods
I	**12**	**10**	**02**
II	**12**	**10**	**02**
III	**12**	**10**	**02**
IV	**12**	**10**	**02**
V	**12**	**10**	**02**
TOTAL	**60**	**50**	**10**

I. CONTINUOUS INTERNAL ASSESSMENT : 40 Marks

(Internal Assessment Tests: 30, Attendance: 5, Seminar: 5)

II. END SEMESTER EXAMINATION : 60 Marks

TOTAL : 100 Marks

UNIT I - STATICS OF PARTICLES

MECHANICS:

Mechanics can be defined as the branch of physics concerned with the state of rest or motion of bodies that subjected to the action of forces. **OR**

It may be defined as the study of forces acting on body when it is at rest or in motion is called mechanics.

Classification of Mechanics

The engineering mechanics are classified as shown

Engineering Mechanics

Mechanics of Rigid bodies | Mechanics of Deformed bodies | Mechanics of fluid

Mechanics of Rigid bodies: Statics, Dynamics

Mechanics of fluid: Statics, Dynamics

BRANCHES OF MECHANICS:

Mechanics can be divided into two branches.

1. Static. 2. Dynamics.

a) Statics

It is the branch of mechanics that deals with the study of forces acting on a body in equilibrium. Either the body at rest or in uniform motion is called statics

b) Dynamics:

It is the branch of mechanics that deals with the study of forces on body in motion is called dynamics. It is further divided into two branches.

i) Kinetics ii) kinematics.

i) Kinetics

It is the branch of the dynamics which deals the study of body in motion under the influence of force i.e. is the relationship between force and motion are considered or the effect of the force are studied

ii) Kinematics:

It is the branch of the dynamics that deals with the study of body in motion with out considering the force.

Fundamental concept

The following are the fundamental concept used in the engineering mechanics

1. Force

In general force is a Push or Pull, which creates motion or tends to create motion, destroy or tends to destroys motion. In engineering mechanics force is the action of one body on another. A force tends to move a body in the direction of its action,

A force is characterized by its point of application, magnitude, and direction, i.e.

a force is a vector quantity.

Units of force

The following force units are frequently used.

A. Newton

The S.I unit of force is Newton and denoted by N. which may be defined as $1N = 1$ kg. 1 m/s^2

B. Dynes

Dyne is the C.G.S unit of force. 1 Dyne = 1 g. 1 cm/s^2

One Newton force = 10□ dyne

C. Pounds

The FPS unit of force is pound. $1\ lb_f = 1\ lb_m$. 1ft/s^2

One pound force = 4.448 N

One dyne force = 2.248×10^{-6} lbs

2. Space

Space is the geometrical region occupied by bodies whose positions are described by linear and angular measurement relative to coordinate systems. For three dimensional problems there are three independent coordinates are needed. For two dimensional problems only two coordinates are required.

3. Particle

A particle may be defined as a body (object) has mass but no size (neglected), such body cannot exists theoretically, but when dealing with problems involving distance considerably larger when compared to the size of the body. For example a bomber aeroplane is a particle for a gunner operating from ground.

In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that it analyze as a mass concentrated at a point. A body may tread as a particle when its dimensions are irrelevant to describe its position or the action of forces applied to it. For example the size of earth is insignificant compared to the size of its orbits and therefore the earth can be modeled as a particle when studying its orbital motion. When a body is idealized as a particle, the principles of mechanics reduce to rather simplified form since the geometry of the body will not be involved in the analysis of the problem.

4. Rigid Body

A rigid body may be defined a body in which the relative positions of any two particles do not change under the action of forces means the distance between two points/particles remain same before and after applying external forces.

As a result the material properties of any body that is assumed to be rigid will not have to be considered while analyzing the forces acting on the body. In most cases the actual deformations occurring in the structures, machines, mechanisms etc are relatively small and therefore the rigid body assumption is suitable for analysis

Basic quantities

In engineering mechanics length, mass, time and force are basic quantities

1. Length

In engineering mechanics length is needed to locate the position of a particle and to describe the size of physical system. Some important length conversions factors

1cm = 10 mm 1 m = 100 cm 1 m = 1000 mm

1 m = 3.2808' (feet)   1 m = 39.37 Inch   1 Mile = 1.609 km

2. Mass

Mass is the property of matter by which we can compare the action of one body with that of another. This property manifests itself as gravitational attraction between two bodies and provides a quantitative measure of the resistance of matter to a change in velocity. Some important mass conversion factors are given below

$$1\ Kg = 2.204\ lb_m$$

3. Time

Time is the measure of the succession of events and is a basis quantity in dynamic. Time is not directly involved in the analysis of statics problems but it has importance in dynamics.

Systems of units

In engineering mechanics length, mass, time and force are the basic units used therefore; the following are the units systems are adopted in the engineering mechanics

1. International System of Units (SI):

In SI system of units the basic units are length, time, and mass which are arbitrarily defined as the meter (m), second (s), and kilogram (kg). Force is the derived unit.

$1N = 1\ kg.\ 1\ m/s^2$

2. CGS systems of units

In CGS system of units, the basic units are length, time, and mass which are arbitrarily defined as the centimeter (cm), second (s), and gram (g). Force is the derived units $1\ Dyne = 1\ g.\ 1\ cm/s^2$

3. British systems of units

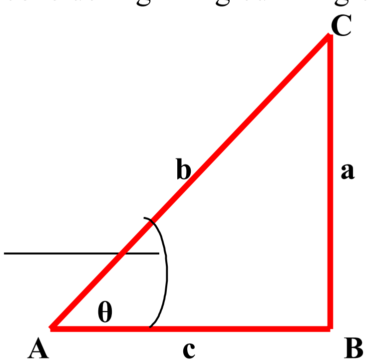
In CGS system of units, the basic units are length, time, and mass which are arbitrarily defined as the centimeter (cm), second (s), and gram (g). Force is the derived units $1\ lb = 1lbg.\ 1ft/s^2$

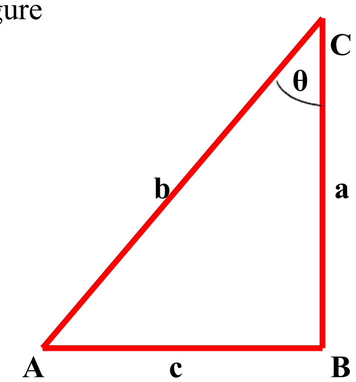
4. U.S. Customary Units

The basic units are length, time, and force which are arbitrarily defined as the foot (ft), second (s), and pound (lb). Mass is the derived unit,

Trigonometry

The measurement of the triangle sides and angles is called trigonometry. Let us consider right-angled triangle ABC as shown in figure





Than the following ratio can be considered for both the triangles

Sin θ = per/hyp = a/b   Sin θ = per/hyp = c/b

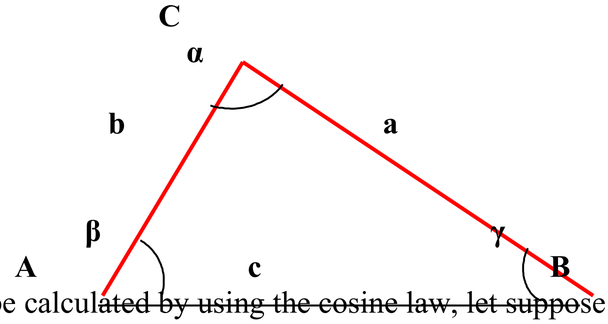
Cos θ = base/hyp = c/b   Cos θ = base/hyp =a/b

Tan θ = per/base = a/c   Tan θ = per/base = c/a

The any side of the right angled triangle may be calculated by

$b^2 = a^2 + b^2$

Similarly consider the following Triangle



The any side of the triangle can be calculated by using the cosine law, let suppose we have to calculate the side "AC" that is "b" then

$$b = \sqrt{a^2 + c^2 - (2bc)\cos \gamma}$$

Similarly, to calculate sides "AB" that is "c" and "AC" that is "a" then by using the cosine lay as below

$$c = \sqrt{a^2 + b^2 - 2ab\cos \alpha}$$

And

$$a = \sqrt{c^2 + b^2 - 2cb\cos \beta}$$

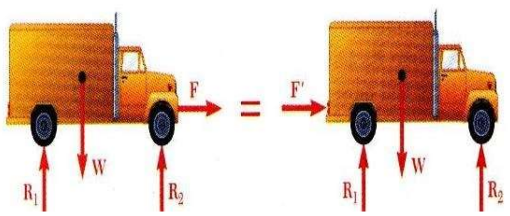
The sides of the triangle ABC can be calculated by using the sin law

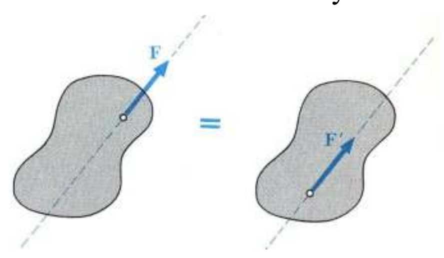
$$\frac{a}{\text{Sin } \beta} \qquad \frac{b}{\text{Sin } \gamma} \qquad \frac{c}{\text{Sin } \alpha}$$

<u>Principle of transmissibility of forces</u>

The state of rest of motion of a rigid body is unaltered if a force acting in the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.

For example the force F acting on a rigid body at point A. According to the principle of transmissibility of forces, this force has the same effect on the body as a force F applied at point B.





The following two points should be considered while using this principle.

1. In engineering mechanics we deal with only rigid bodies. If deformation of the body is to be considered in a problem. The law of transmissibility of forces will not hold good.

2. By transmission of the force only the state of the body is unaltered, but not the internal stresses which may develop in the body

Therefore this law can be applied only to problems in which rigid bodies are involved

SCALAR AND VECTOR QUANTITY

Scalar quantity

Scalar quantity is that quantity which has only magnitude (numerical value with suitable unit) **or**

Scalars quantities are those quantities, which are completely specified by their magnitude using suitable units are called scalars quantities. For example mass, time, volume density, temperature, length, age and area etc

The scalars quantities can be added or subtracted by algebraic rule e.g. 7kg + 8kg = 15 kg sugar Or 4 sec + 5 sec = 9 sec

Vector quantity

Vector quantity is that quantity, which has magnitude unit of magnitude as well as direction, is called vector quantity. **Or**

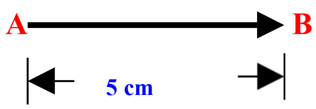
Vector quantities are those quantities, which are completely specified by their magnitude using suitable units as well directions are called vector quantities. For example velocity, acceleration, force, weight, displacement, momentum and torque etc are all vector quantities. Vector quantity can be added, subtracted, multiplied and divided by particular geometrical or graphical methods.

VECTOR REPRESENTATION

A vector quantity is represented graphically by a straight line the length of line gives the magnitude of the vector and arrowhead indicates the direction.

For example we consider a displacement (d) of magnitude 10 km in the direction of east. Hence we cannot represent 10 km on the paper therefore we select a suitable scale shown in fig. Scale 1 cm = 2 km

So we draw a line of length 5 cm which show the magnitude of vector quantity that is 10 km while the arrow indicates the direction form origin to east ward as shown in fig.



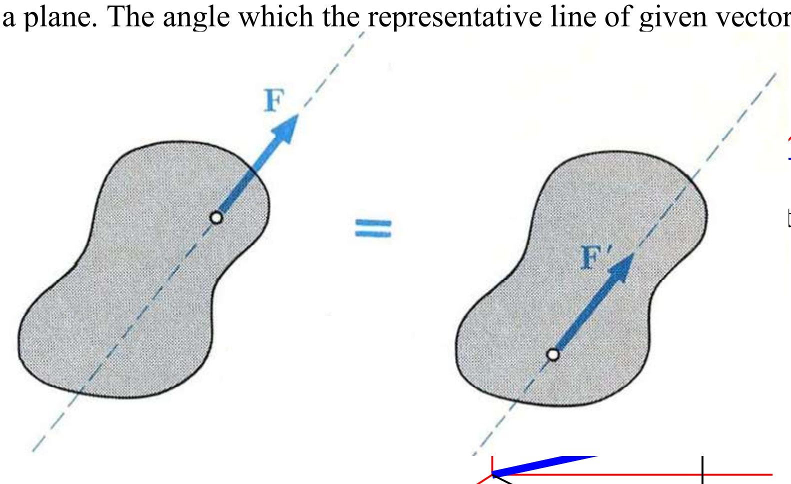
Point A is called tail that shows the origin.

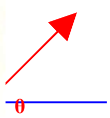
Point B is called head, which shows the direction of vector quantity.

The length of line is the magnitude of the vector quantity.

RECTANGULAR CO-ORDINATE SYSTEM

Two lines at right angle to each other are known as co-ordinate axes and their point of intersection is called origin. The horizontal line is called x-axis while vertical line is called y-axis. Two co ordinate systems are used to show the direction of a vector is a plane. The angle which the representative line of given vector makes with + ve x axis in





that is Z-axis. The direction

of the vector in space is specified by three angles named α, β, and γ with X, Y Z axes respectively as show

Z

Y

X

EXERCISE 1

Show the following vectors graphically from 1 to 6

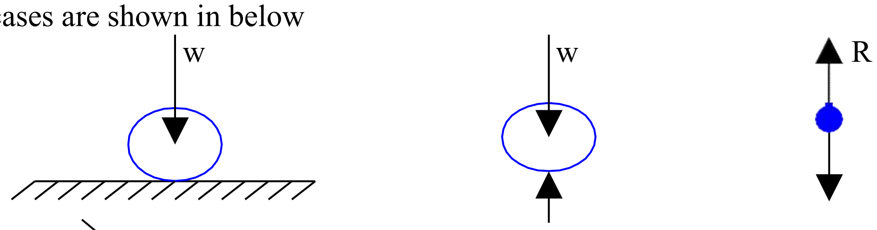
1.	Force	15 kN	45^0	with x-axes.
2.	Displacement	75 km	30^o	north of east
3.	Velocity	60 km\h	90^o	with x-axes.
4.	Velocity	5 km\h	45^o	with horizontal axes
5.	Force	20 kN	135^o	with x-axes.
6.	Displacement	40 k m		north-east.

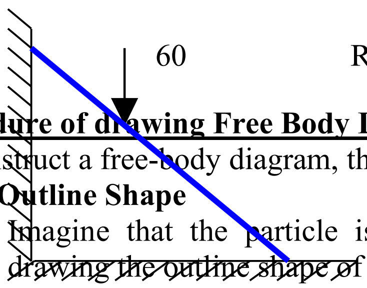
7. A crow flies northward from pole A to pole B and covers distance of 8 km. It then flies eastward to pole C and covers 6 km. find the net displacement and direction of its flight. **Ans: 10 km 53º north of east**

8. A traveler travels 10 km east 20 km north 15 km west and 8 km south. Find the displacement of the traveler from the starting point. **Ans: 13 km 23º north west**

Free body diagram

A diagram or sketch of the body in which the body under consideration is freed from the contact surface (surrounding) and all the forces acting on it (including reactions at contact surface) are drawn is called free body diagram. Free body diagram for few cases are shown in below





Procedure of drawing Free Body Diagram

To construct a free-body diagram, the following steps are necessary:

Draw Outline Shape

Imagine that the particle is cut free from its surroundings or isolated by drawing the outline shape of the particle only

Show All Forces

Show on this sketch all the forces acting on the particle. There are two classes of forces that act on the particle. They can be active forces, which tend to set the particle in motion, or they can be reactive forces which are the results of the constraints or supports that tend to prevent motion.

Identify Each Force

The forces that are known should be labeled complete with their magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are not known.

Method of Problem Solution

Problem Statement

Includes given data, specification of what is to be determined, and a figure showing all quantities involved.

Free-Body Diagrams

Create separate diagrams for each of the bodies involved with a clear indication of all forces acting on each body.

Fundamental Principles

The six fundamental principles are applied to express the conditions of rest or motion of each body. The rules of algebra are applied to solve the equations for the unknown quantities.

Solution Check:

1. Test for errors in reasoning by verifying that the units of the computed results are correct
2. Test for errors in computation by substituting given data and computed results into previously unused equations based on the six principles.
3. Always apply experience and physical intuition to assess whether results seem "reasonable"

Numerical Accuracy

The accuracy of a solution depends on

1. Accuracy of the given data.
2. Accuracy of the computations performed. The solution cannot be more accurate than the less accurate of these two.
3. The use of hand calculators and computers generally makes the accuracy of the computations much greater than the accuracy of the data. Hence, the solution accuracy is usually limited by the data accuracy.

SYSTEM OF FORCES:

Force

In general force is a Push or Pull, which creates motion or tends to create motion, destroy or tends to destroys motion. In engineering mechanics force is the action of one body on another. A force tends to move a body in the direction of its action,

A force is characterized by its point of application, magnitude, and direction, i.e. a force is a vector quantity.

Force exerted on body has following two effects

1. The **external effect**, which is tendency to change the motion of the body or to develop resisting forces in the body
2. The **internal effect**, which is the tendency to deform the body.

If the force system acting on a body produces no external effect, the forces are said to be in **balance** and the body experience no change in motion is said to be in **equilibrium**.

Units of force

The following force units are frequently used.

A. Newton

The S.I unit of force is Newton and denoted by N. which may be defined as

$1N = 1$ kg. $1\ m/s^2$

B. Dynes

Dyne is the C.G.S unit of force.

1 Dyne = 1 g. 1 cm/s^2

One Newton force = 10□ dyne

C. Pounds

The FPS unit of force is pound.

$1\ lb_f = 1\ lb_m.\ 1 ft/s^2$

One pound force = 4.448 N

One dyne force = 2.248×10^{-6} lbs

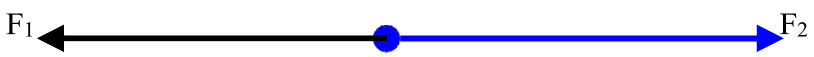
Systems of forces

When numbers of forces acting on the body then it is said to be system of forces

Types of system of forces

1. Collinear forces:

In this system, line of action of forces act along the same line is called collinear forces. For example consider a rope is being pulled by two players as shown in figure

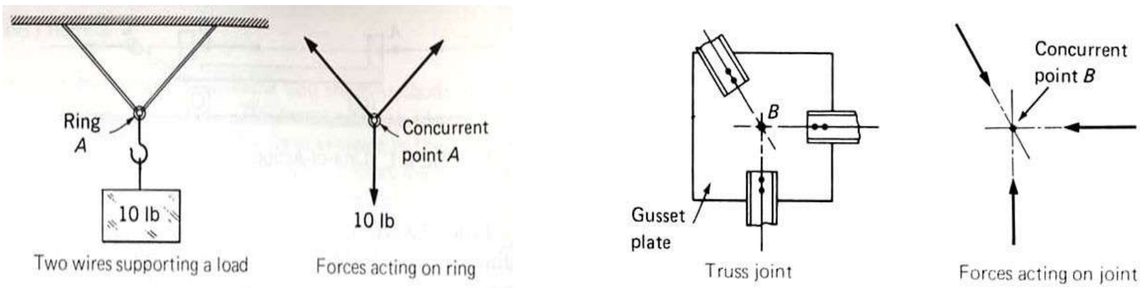


2. Coplanar forces

When all forces acting on the body are in the same plane the forces are coplanar

3. Coplanar Concurrent force system

A concurrent force system contains forces whose lines-of action meet at same one point. Forces may be **tensile (pulling)** or Forces may be ***compressive (pushing)***



4. <u>Non Concurrent Co-Planar Forces</u>

A system of forces acting on the same plane but whose line of action does not pass through the same point is known as non concurrent coplanar forces or system for example a ladder resting against a wall and a man is standing on the rung but not on the center of gravity.

5. <u>Coplanar parallel forces</u>

When the forces acting on the body are in the same plane but their line of actions are parallel to each other known as coplanar parallel forces for example forces acting on the beams and two boys are sitting on the sea saw.

6. <u>Non coplanar parallel forces</u>

In this case all the forces are parallel to each other but not in the same plane, for example the force acting on the table when a book is kept on it.

<u>ADDITION OF FORCES</u>

<u>ADDITION OF (FORCES) BY HEAD TO TAIL RULE</u>

To add two or more than two vectors (forces), join the head of the first vector with the tail of second vector, and join the head of the second vector with the tail of the third vector and so on. Then the resultant vector is obtained by joining the tail of the first vector with the head of the last vector. The magnitude and the direction of the resultant vector (Force) are found graphically and analytically.

RESULTANT FORCE

A resultant force is a single force, which produce same affect so that of number of forces can produce is called resultant force

COMPOSITION OF FORCES

The process of finding out the resultant Force of given forces (components vector) is called composition of forces. A resultant force may be determined by following methods

1. Parallelogram laws of forces or method
2. Triangle law of forces or triangular method
3. polygon law of forces or polygon method

A) PARALLELOGRAM METHOD

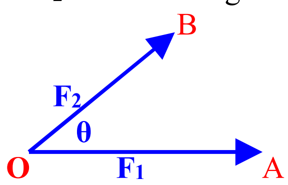
According to parallelogram method 'If two forces (vectors) are acting simultaneously on a particle be represented (in magnitude and direction) by two adjacent sides of a parallelogram, their resultant may represent (in magnitude and direction) by the diagonal of the parallelogram passing through the point. OR

When two forces are acting at a point such that they can by represented by the adjacent sides of a parallelogram then their resultant will be equal to that diagonal of the parallelogram which passed through the same point.

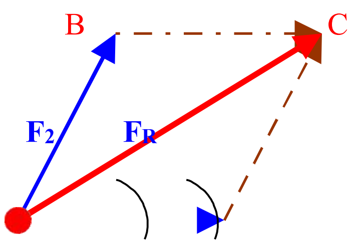
The magnitude and the direction of the resultant can be determined either graphically or analytically as explained below.

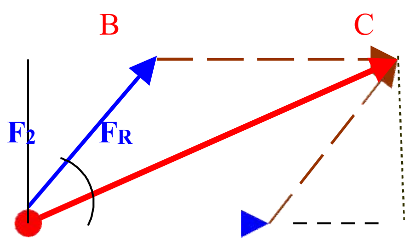
Graphical method

Let us suppose that two forces F_1 and F_2 acting simultaneously on a particle as shown in the figure (a) the force F_2 makes an angle θ with force F_1



First of all we will draw a side OA of the parallelogram in magnitude and direction equal to force F_1 with some suitable scale. Similarly draw the side OB of parallelogram of same scale equal to force F_2, which makes an angle θ with force F_1. Now draw sides BC and AC parallel to the sides OA and BC. Connect the point O to Point C which is the diagonal of the parallelogram passes through the same point O and hence it is the resultant of the given two forces. By measurement the length of diagonal gives the magnitude of resultant and angle α gives the direction of the resultant as shown in fig (A).





θ α θ α θ

O F_1 A O F_1 A D

fig (A) Fig (B)

Analytical method

In the paralleogram OABC, from point C drop a perpendicular CD to meet OA at D as shown in fig (B)

In parallelogram OABC,

$OA = F_1$ $OB = F_2$ Angle $AOB = \theta$

Now consider the ΔCAD in which

Angle $CAD = \theta$ $AC = F_2$

By resolving the vector F_2 we have,

$CD = F_2 \text{ Sin } \theta$ and $AD = F_2 \text{ Cosine } \theta$

Now consider ΔOCD

Angle $DOC = \alpha$. Angle $ODC = 90^o$

According to Pythagoras theorem

$(Hyp)^2 = (per)^2 + (base)^2$

$OC^2 = DC^2 + OD^2$.

$OC^2 = DC^2 + (OA + AD)^2$

$F_R{}^2 = F^2 \text{ Sin}^2\theta + (F_1 + F_2 \text{ Cosine } \theta)^2$

$F_R{}^2 = F^2{}_2 \text{ Sin}^2\theta + F^2{}_1 + F^2{}_2 \text{ Cos}^2\theta + 2 F_1 F_2 \text{ Cosine}\,\theta$.

$F_R{}^2 = F^2{}_2 \text{ Sin}^2\theta + F^2{}_2 \text{ Cos}^2\theta + F^2{}_1 + 2 F_1 F_2 \text{ Cosine } \theta$.

$F_R{}^2 = F^2{}_2 (\text{Sin}^2\theta + \text{Cos}^2\theta) + F^2{}_1 + 2 F_1 F_2 \text{ Cosine}\,\theta$.

$F_R{}^2 = F^2{}_2 (1) + F^2{}_1 + 2 F_1 F_2$ Cosine

θ. $F_R{}^2 = F^2{}_2 + F^2{}_1 + 2 F_1 F_2 \text{ Cosine } \theta$.

$F_R{}^2 = F^2{}_1 + F^2{}_2 + 2 F_1 F_2 \text{ Cosine } \theta$.

$$\mathbf{F_R = \sqrt{F^2{}_1 + F^2{}_2 + 2 F_1 F_2 \text{ Cosine } \theta}.}$$

The above equation gives the magnitude of the resultant vector.

Now the direction of the resultant can be calculated by

$$\text{Sin } \alpha = \frac{CD}{OC} = \frac{F_2 \text{ Sin } \theta}{F_R} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \quad 1 \qquad \text{OR}$$

$$\text{Tan } \alpha = \frac{CD}{OD} = \frac{F_2 \text{ Sin}}{F_1 + F2 \text{ Cosine } \theta} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} \quad 2$$

The above two equation gives the direction of the resultant vector that is α.

B) TRIANGLE METHOD OR TRIANGLE LAW OF FORCES

According to triangle law or method" If two forces acting simultaneously on a particle by represented (in magnitude and direction) by the two sides of a triangle taken in order their resultant is represented (in magnitude and direction) by the third side of triangle taken in opposite order. OR

If two forces are acting on a body such that they can be represented by the two adjacent sides of a triangle taken in the same order, then their resultant will be equal to the third side (enclosing side) of that triangle taken in the opposite order.

The resultant force (vector) can be obtained graphically and analytically or trigonometry.

Graphically

Let us consider two forces F_1 and F_2 acting on the particle the force F_1 is horizontal while the force F_2 makes an angle θ with force F_1 as shown in fig (A). Now draw lines OA and AB to some convenient scale in magnitude equal to F_1 and $F_{2.}$ Join point O to point B the line OB will be the third side of triangle, passes through the same point O and hence it is the resultant of the given two forces. By measurement the length

of OB gives the magnitude of resultant and angle α gives the direction of the resultant as shown in fig (B).

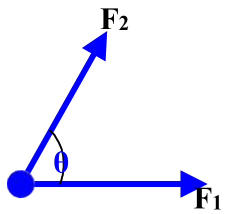


Fig (A)

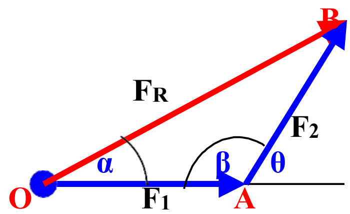


Fig (B)

ANALYTICAL OR TRIGONOMETRIC METHOD

Now consider Δ AOB in which

Angle AOB = α which is the direction of resultant vector OB makes with horizon anal axis.

Angle OAB = 180º - θ. As we know

Angle AOB + Angle OAB + Angle ABO = 180º.

By putting the values we get

α + 180º -θ + angle ABO = 180º

Angle ABO = α-θ

By applying the sine law to the triangle ABO

$$\frac{OA}{\text{Sin } B} = \frac{AB.}{\text{Sin } O} = \frac{OB}{\text{Sin } A}$$

$$\frac{\mathbf{F_1}}{\mathbf{Sin\ (\theta - \alpha)}} = \frac{\mathbf{F_2}}{\mathbf{Sin\ \alpha}} = \frac{\mathbf{F_R}}{\mathbf{Sin\ (180 - \theta)}}$$

Note

It is better to calculate the resultant of F_1 and F_2 by using cosine law we get

$$F_R = \sqrt{F^2_1+F^2_2 + 2\,F_1\,F_2\,\text{Cosine}\,\beta.}$$

Where $\beta = 180 - \theta$

And the direction of resultant may be determined by using sine law

$$\frac{F_1}{\text{Sin}\,\gamma} = \frac{F_2}{\text{Sin}\,\alpha} = \frac{F_R}{\text{Sin}\,\beta}.$$

C) POLYGON METHOD

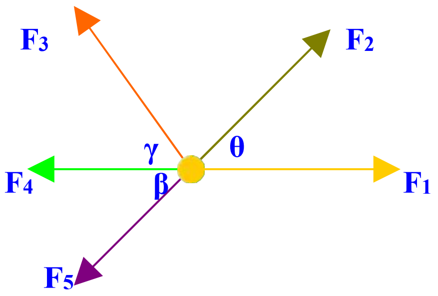
According to this method" if more then two forces acting on a particle by reprehend by the sided of polygon taken in order their resultant will be represented by the closing side of the polygon in opposite direction" OR

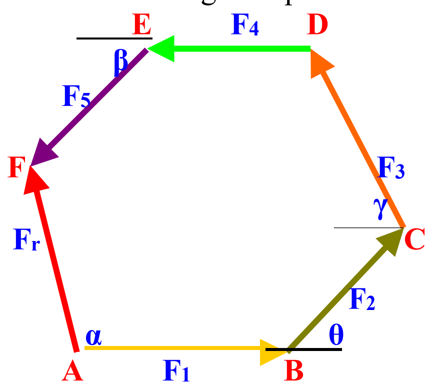
If more than two forces are acting on a body such that they can by represented by the sides of a polygon Taken in same order, then their resultant will be equal to that side of the polygon, which completes the polygon (closing side taken in opposite order.

The resultant of such forces can be determined by graphically and analytically.

Graphically:

Consider the following diagram in which number of forces acting on a particle.





Starting from A the five vectors are plotted in turns as shown in fig by placing the tail end of each vector at the tip end of the preceding one. The arrow from A to the tip of the last vector represents the resultant of the vectors with suitable scale.In this polygon the side AF represents the resultant of the given components and α shows the direction. By measurement of AF will give the resultant and α give direction of given scale

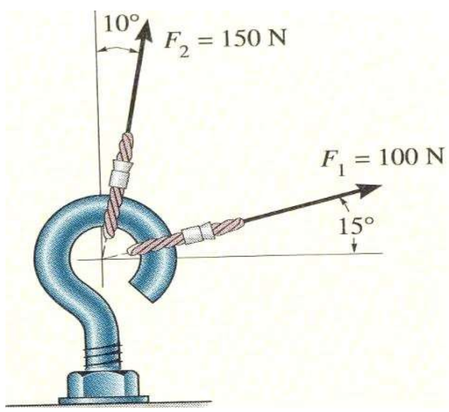
Analytically

The resultant and direction can be determined by solving it step-by-step analytically using formulas of parallelogram, triangle law or trigonometry

EXAMPLE

The screw eye is subjected to two forces F_1 and F_2 as shown in fig. Determine the magnitude and direction of the resultant force by parallelogram by using the graphical or analytical method.

Draw the free body diagram of the given fig.



Given $F_1 = 100$ N $F_2 = 150$ N $\theta_1 = 15^o$ $\theta_2 = 10^o$

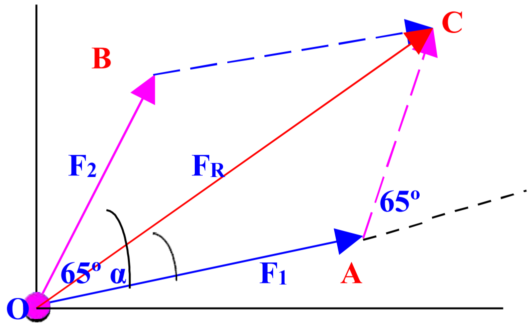
Required Resultant = F_R =?

Solution Angle AOB = 90 -15 – 10 = 65º

A) Graphically

Scale 20 N = 1 cm.

Now draw parallelogram OABC with rule and protractor according to scale as shown in diagram.



By measuring

$OC = F_R = 10.6\ cm = 10.6 \times 20 = 212\ N$

$\alpha = 54^o$ with x axis

Result **Resultant = 212 N** **Direction = 54 with x axis**

B Analytical method

We know that

$$F_r = \sqrt{F^2_1 + F^2_2 + 2\,F_1\,F_2 \qquad \theta.}$$

Putt the value and $\theta = 65^o$

$$F_r = \sqrt{(100)^2 + (150)^2 + 2\,(100)\,(150)\,\text{Cosine}65^o}$$

$F_r = 212.55$ N.

We also know that $\quad \sin\alpha = \dfrac{F_2 \sin\theta.}{R}$

$$\sin\alpha = \frac{150 \sin 65^o}{212.55}$$

$$\alpha = \sin^{-1}\frac{150 \sin 65^o}{212.5}$$

$\alpha = 39.665^o$ with force F_1

$39.665^o + 15^o$

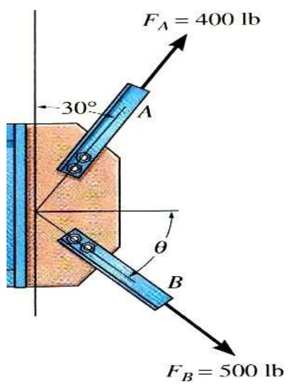
$= 54.665^o$ with x axis.

Result **Resultant = 212.55 N** **Direction = 54.665° with x axis**

.

EXAMPLE 3

The plate is subjected to the forces acting on member A and B as shown. If θ = 60° determine the magnitude of the resultant of these forces and its direction measured from clockwise from positive x-axis. Adopt triangle method graphically and analytically.



Given

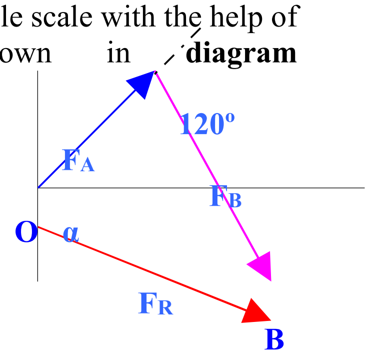
$F_A = 400N$ $F_B = 500N$ $\theta_1 = 30^o$ with Y axis $\theta_2 = 60^o$ with positive x axis

Required Resultant F_R =? Direction = α =?

Solution the angle between two forces 60 + (90 – 30) = 120º

<u>A: Graphically</u> Scale 100 lb = 1 cm

Now draw triangle OAB with suitable scale with the help of scale and protractor as shown in **diagram A**



By measurement we get,

OB = F_R = 4.6 cm x 100 = 460 lb Angle BOA = 70º α = 10º

Result Resultant = 460 lb Direction = 10º

B Analytically:

According to cosine law for given triangle AOB

$$F_R = \sqrt{F^2_A + F^2_B - 2(F_A)(F_B)(\text{cosine } \theta)}$$

$$F_R = \sqrt{(400)^2 + (500)^2 - 2(400)(500)(\text{cosine } (180-120)}$$

F_R = 458.257 lb

According to sine law for given triangle AOB

$$\frac{\mathbf{F_B}}{\mathbf{Sin}\ \alpha} = \frac{\mathbf{F_R}}{\mathbf{Sin}\ (180-\theta)}$$

$$\frac{500}{\text{Sin}\ \alpha} = \frac{458.257}{\text{Sin}\ (180-\theta)}$$

$$\text{Sin}\ \alpha = \frac{500\ \text{Sin}\ (180-\theta)}{458.257}$$

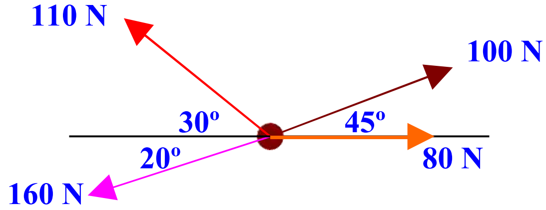
α = 70.89º with force F_A

And α = 70.89º -60º = 10 with x axis

Result Resultant = 458.257 lb & Direction = 10.89º

Example 4

Four forces act on a body at point O as shown in fig. Find their resultant.



Given

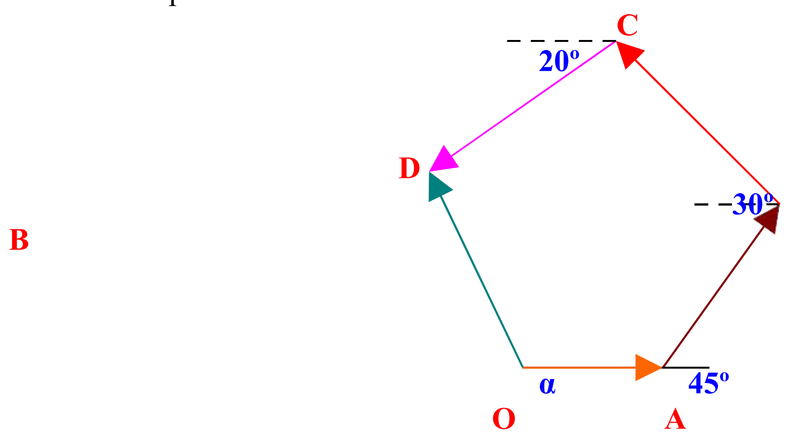
$F_1 = 80$ N	$\theta_0 = 0$	at x axis
$F_2 = 100$ N	$\theta_1 = 45^\circ$	with x axis
$F_3 = 110$ N	$\theta_2 = 30^\circ$	with –ve x axis
$F_4 = 160$ N	$\theta_3 = 20^\circ$	with –x axis

Required

Resultant = F_R =? Direction =α =?

Sol: Graphically Scale 20 N = 1 cm.

Starting from O the four vectors are plotted in turn as shown in fig by placing the tail end of each vector at the tip end of the preceding one. The arrow from O to the tip of the last vector represents the resultant of the vectors.



By measurement

The resultant OB = F_R = x 20 = 124 N

The direction of the resultant = = 143º with + ve x axis.

Result: Resultant = 119 N Direction = 143º

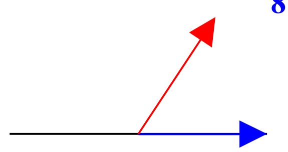
EXERCISE 2.1

1. Find the resultant and the direction of the following diagram.

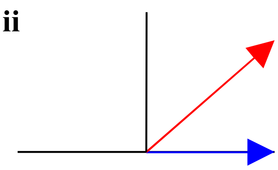
i

6lb

8 N



ii



42º

60º

5 N

4 lb

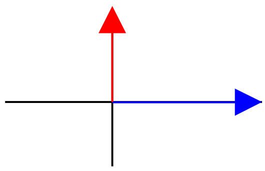
Ans: 12.18 N & 26.07º

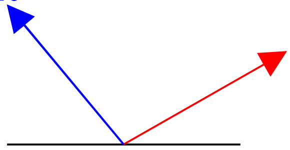
Ans: 8.718 lb & 36.585º

iii **iv**

24 N **20**

N





30 N

10 N

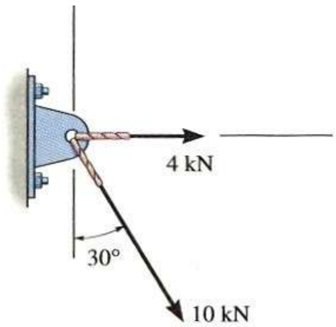
140º

30º

Ans 26 N & 67.38º **Ans: 29.826 N & 69.059º with x-axis.**

2 Determine the magnitude and direction of the resultant force as shown in fig

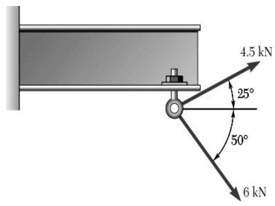
Ans: 12.489 N & 43.902º



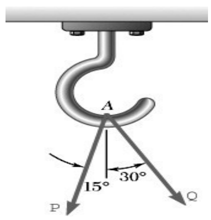
3 Determine the magnitude and the direction of the resultant of two forces 7 N and 8 N acting at a point with an included angle of 60º with between them. The force of 7 N being horizontal

4. Determine the magnitude and direction of the resultant of two forces 20 N and 30 N acting at a point with an included angle of 40º between them. The force 30 N being horizontal

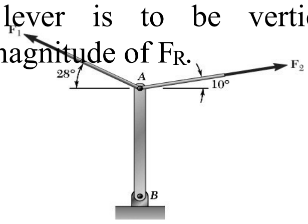
5. Two forces are applied to an eye bolt fastened to a beam. Determine the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.



6. Two forces **P** and **Q** are applied as shown at point *A* of a hook support. Knowing that *P* =15 lb and *Q* = 25 lb, determine the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

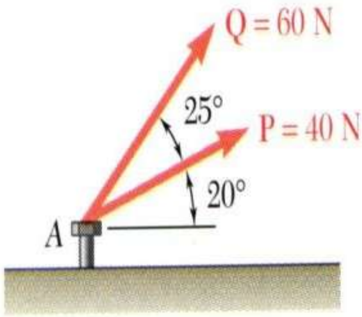


7. Two control rods are attached at A to lever AB. knowing that the force in the left-hand rod is F_1 = 120 N, determine (a) the required force F_2 in the right-hand rod if the resultant of the forces exerted by the rods on the lever is to be vertical, (b) the corresponding magnitude of F_R.

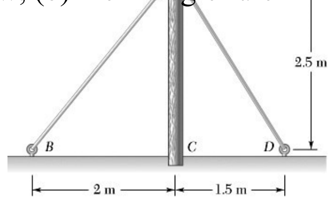


8. The two forces P and Q act on bolt A as shown in diagram. Find their resultant and direction

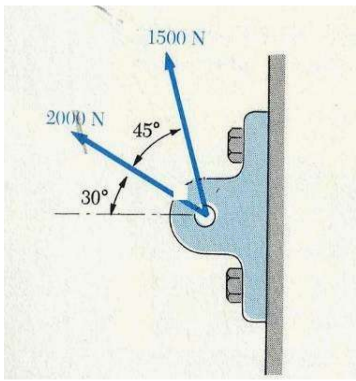
9. The cable stays AB and AD help support pole AC.



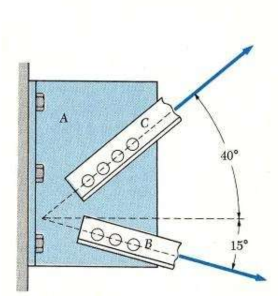
Knowing that the tension is 500 N in AB and 160 N in AD, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at A using (a) the parallelogram law, (b) the triangle rule



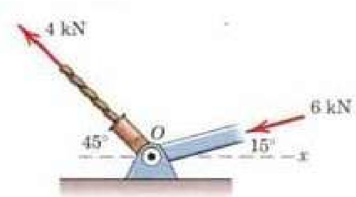
10. Determine the magnitude and direction of the resultant of the two forces.



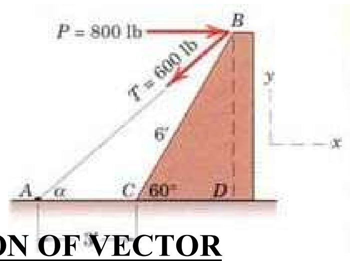
11. Two structural members B and C are riveted to the bracket A. Knowing that the tension in member B is 6 kN and the tension in C is 10 kN, determine the magnitude and direction of the resultant force acting on the bracket.



12. The two structural member one in tension and other in compression, exerts on point O, determine the resultant and angle θ



13. The force P and T act on body at point B replace them with a single force

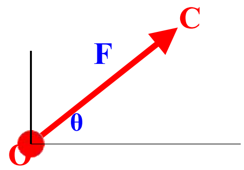


RESOLUTION OF VECTOR

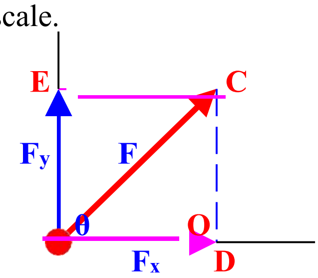
The processes of finding the components of given vector (resultant) is called resolution of vector. Or The processes of splitting up of single vector into two or more vector is called resolution of the vector A vector can be resolved into two or more vectors which have the same combined affect as that the effect of original vector

RESOLUTION OF VECTOR INTO RECTANGULAR COMPONENTS

If vector is resolved into such components which are at right angles (perpendicular) to each other then they are called the rectangular components of that vector, now let us consider a resultant vector F to be resolved into two components which makes an angle θ with horizontal axes as shown in fig.



Now draw a line OC to represent the vector in magnitude, which makes an angle θ with x-axis with some convenient scale. Drop a perpendicular CD at point C which meet x axis at point D, now join point O to point D, the line OD is called horizontal component of resultant vector and represents by F_x in magnitude in same scale. Similarly draw perpendicular CE at point C, which will meet y-axis at point E now join O to E. The line OE is called vertical component of resultant vector and represents by F_y in magnitude of same scale.



Analytically or trigonometry

In ΔCOD Angle COD = θ Angle ODC = 90° OC = F

OD = F_x OE = CD = F_y

We know that

Cosine θ = $\frac{OD.}{OC}$ Cosine θ = $\frac{F_x}{F}$

And **F_x = F Cosine θ**

Similarly we have

$$\text{Sin } \theta = \frac{DC}{OC} \qquad \text{Sin } \theta = \frac{F_y}{F}$$

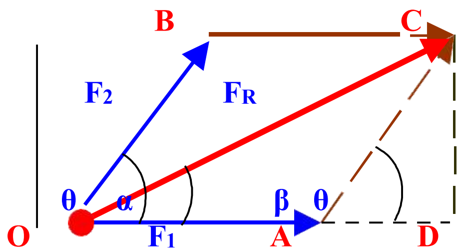
And $$\mathbf{F_y = F \text{ Sine } \theta}$$

RESOLVING OF A FORCE INTO TWO COMPONENTS WHICH ARE NOT MUTUALLY AT RIGHT ANGLE TO EACH OTHER

If a force or vector is to be required to resolved into such components which are not at right angle to each other then it can be determined in reverse manner as we find the resultant vector of given components by Parallelogram method, Triangle method or Trigonometry

A) Parallelogram method

Now consider a force F_R, which is resolved into components F_1 and F_2. The force F makes an angle α with force F_1 and force F_2 makes an angle θ with component F_1, so we can make a parallelogram with suitable scale as shown in fig.



We can also determine the components of force F by analytically as we know that direction of the resultant vector can be determined by

$$\text{Sin } \alpha = \frac{F_2 \text{ Sin } \theta.}{F_R} \qquad \text{OR} \qquad (1)$$

$$\text{Tan } \alpha = \frac{F_2 \text{ Sin } \theta}{F_1 + F2 \text{ Cosine } \theta} \qquad (2)$$

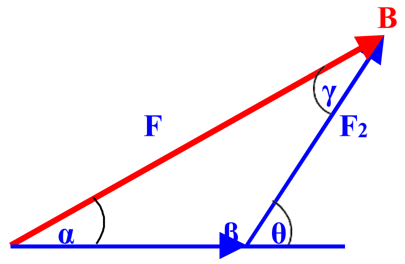
So we can find F_2 from equation 1

$$F_2 = \frac{F_R \text{ Sin } \alpha}{\text{Sin } \theta}$$

Similarly from equation 2

$$F_1 = \frac{F_2 \text{ Sin } \theta - F2 \text{ Cosine } \theta}{\text{Tan } \alpha}$$

B) Triangle method: Now consider a force F, which is resolved into components F_1 and F_2. The force F makes an angle α with force F_1 and force F_2 makes an angle θ with component F_1, so we can make a triangle with some suitable scale as shown in fig.



O F1 A

By measurements we get the components F_1 and F_2.
Similarly we can find the components F_1 and F_2 by using the following formula

$$\frac{F_1}{\sin\gamma} = \frac{F_2}{\sin\alpha} = \frac{F_R}{\sin\beta}$$

For component F_1

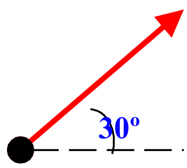
$$F_1 = \frac{F_R \sin\gamma}{\sin\beta}$$

For component F_2

$$F_2 = \frac{F_R \sin\alpha}{\sin\beta}$$

EXAMPLE 5

Resolve the force 200 N into components along x and y direction and determine the magnitude of components.



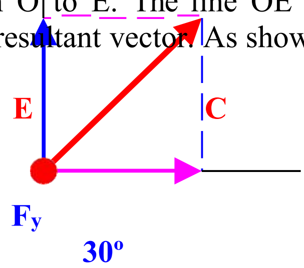
Given: Force = F = 200 N Direction = θ = 30°

Required Horizontal components = F_x =?
Vertical components = F_y =?

Solution

A) **Graphically** Scale 1 cm = 20 N

Now draw a line OC to represent t vector in magnitude with given scale, which makes an angle 30° with x-axis. Drop a perpendicular CD at point C which meet x axis at point D, now join point O to point D, the line OD is called horizontal component (F_x) of resultant vector. Similarly draw perpendicular CE at point C, which will meet y-axis at point E now join O to E. The line OE is called vertical component (F_y) of resultant vector. As shown in fig



O **F_x** **D**

By measuring we get

$OD = F_x = 8.6 \text{ cm} \times 20 = 172 \text{ N}$

$OE = Fy = 5 \text{ cm} \times 20 = 100 \text{ N}$

Result: **$F_x = 173.20$ N** **$F_y = 100$ N**

B) <u>Analytically</u>

We know that $F_x = F \text{ cosine } \theta = 200 \text{ cosine} 30$

$F_x = 173.20$ N

We also know that

$F_y = F \text{ Sin } \theta = 200 \text{ Sin } 30$ **$F_y = 100$**
N

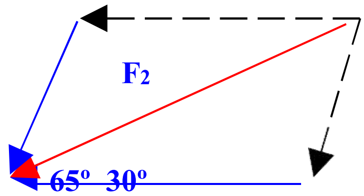
Result: **$F_x = 173.20$ N** **$F_y = 100$ N**

<u>EXAMPLE 6</u>

A push of 40 N acting on a point and its line of action are inclined at an angle of 30º with the horizontal. Resolve it along horizontal axis and another axis which is inclined at an angle of 65º with the horizontal.

C

B



F

A

D

Given Force = F = 40 N Direction = θ = 30º

Direction = α = 65º

Required Force component = F_1 =? Force component = F_2 =?

Solution **Graphical Method**

Let Scale 10 N = 1 cm

Now draw the parallelogram ABCD with given scale as shown in fig

By measurement $AD = F_1 = 2.5 \times 10 = 25 \text{ N}$

$AC = F_2 = 2.3 \times 10 = 23 \text{ N}$

Result **$F_1 = 25$ N** **$F_2 = 23$ N**

Analytically

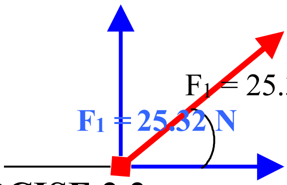
We have $F_2 = \dfrac{F \text{ Sin } \alpha}{\text{Sin } \theta} = \dfrac{40 \text{ Sin } 30}{\text{Sin } 65^\circ}$

$F_2 = 22.06 \text{ N}$

Similarly from equation

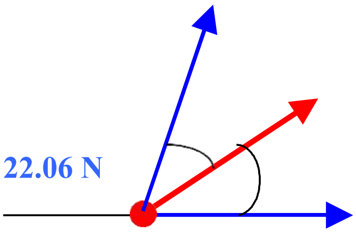
$F_1 = F_2 \sin \theta$ - F2

$F_1 = \underline{22.06 \sin 65}$ - 22.06 Cosine 65
Tan 30



$F_1 = 25.32$ N

Result **$F_1 = 25.32$ N** **$F_2 = 22.06$ N**



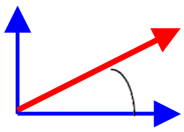
EXERCISE 2.2

1. Resolve the given forces as shown in following diagrams into components F_1 and F_2

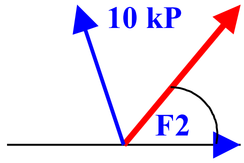
i **ii**

–

200 N



10 kP



F2

30º

25º

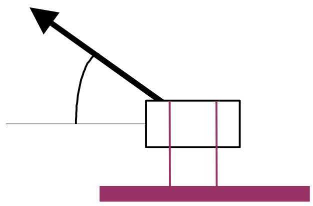
30º **F_1**

F1

iii **iv**

156 lb

F_2 **100 N** **F_2**

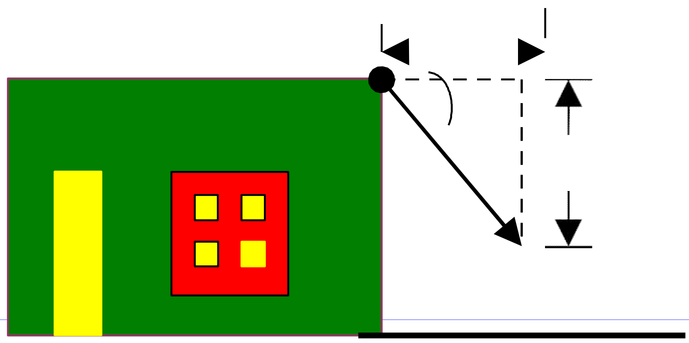


30º

26º

F_1

34º **F_1**



2. A force of 800 N is exerted on a bolt A as shown in fig. Determine the horizontal and vertical components of force.

800 N

Ans: 655.32 N & 458.816N

35º

4. A man pull with force of 300 N on a rope attached to a building as shown in fig, what are the horizontal and vertical components of the force exerted by the rope at point A

Ans: 180 N & 36.87º

5 While emptying a wheel barrow, a gardener exerts on each handle AB a force **P** directed along line CD. Knowing that **P** must have a 135-N horizontal component, determine (a) the magnitude of the force **P**, (b) its vertical component

6 Member *CB* of the vise shown exerts on block *B* a force P directed along line *CB*. Knowing that P must have a The guy wire BD exerts on the telephone pole AC a force P directed along BD. Knowing that P has a 450-N component along line AC, determine (a) the magnitude of the force P, (b) its component in a direction perpendicular to AC.

ENGINEERING MECHANICS

UNIT 1 - MCQ

Questions	opt1	opt2	opt3	opt4	answer
The study of a body at rest is known as ____________	statics	dynamics	force	displacement	statics
The study of a body at motion is known as ____________	dynamics	angle	statics	force	dynamics
A quantity which is completely specified by magnitude & direction is known as ___________	scalar	vector	vector & scalar	quantity	vector
A ___________ is a body of infinitely small volume & is considered to be concentrated at a point	dynamics	scalar	coplanar force	particle	particle
If two forces P & Q act at a point & the angle between the two forces be 'α' , then the resultant is given by R = _________	$P^2 + Q^2$	P+Q+2PQcosα	$\sqrt{(P^2+Q^2+2PQ\cos\alpha)}$	$\sqrt{(P^2+Q^2+2PQ\cos\theta)}$	$\sqrt{(P^2+Q^2+2PQ\cos\alpha)}$
Direction, θ = ________________	$\tan^{-1}(P/Q)$	$\tan^{-1}(\alpha/\beta)$	$\tan^{-1}$ (Q.sinα/ P+Qcosθ)	$\tan^{-1}$ (Q.sinα/ P+Qcosα)	$\tan^{-1}$ (Q.sinα/ P+Qcosα)
If two forces P & Q are equal & act at a point & the angle between the two forces be 'α' , then the resultant is given by R = ____________	R= P+Q	R=P-Q	R=2P.cos(α/2)	R=cosα	R=2P.cos(α/2)
If the two forces P & Q are equal & are acting at an angle α between them, then the angle made by resultant is given by ____________	θ=(α/2)	α=2	θ=2	θ=α	θ=(α/2)
According to lame's theorem, " if three forces acting at a point are equilibrium, each force will be proportional to the _________ of the angle between the other two forces.	cos	tan	sin	tan -1	sin

ENGINEERING MECHANICS					
Moment = Force X ________	volume	distance	force	area	distance
The force causes ________ displacement	linear	angular	distance	moment	linear
A body will be in equilibrium, if the ________ in any direction is zero.	angle	displacement	resultant force	force	resultant force
The moment causes ____________ displacement	angular	linear	moment	displacement	angular
If the forces are acting in one plane, then the forces are called ________	coplanar forces	collinear forces	concurrent	forces	coplanar forces
If the forces are intersecting at a common point, then the forces are called ________	concurrent forces	coplanar	collinear	current forces	concurrent forces
If the forces are having same line of action, then the forces are called	current forces	collinear forces	concurrent	coplanar	collinear forces
The resultant R of three (or) more forces acting at a point is given by, R = ________	$\sqrt{(\Sigma H)^2 + (\Sigma V)^2}$	$\Sigma H + \Sigma V$	$\Sigma H + V$	H / V	$\sqrt{(\Sigma H)^2 + (\Sigma V)^2}$
The ________ forces are parallel to each other & are acting in the same direction.	unlike forces	unlike parallel forces	like parallel forces	forces	like parallel forces
The ________ forces are acting in the opposite direction.	unlike parallel forces	parallel forces	like forces	moment	unlike parallel forces
If the resultant of a no. of parallel forces is zero, then the system may have a ________	resultant couple or may be in equilibrium	moment	couple	force	resultant couple or may be in equilibrium
If the algebraic sum of moments of all forces about any point is not zero, then system will have a ____________	resultant couple	moment	force	equilibrium	resultant couple
If the algebraic sum of moments of all forces about any point is zero, then system will	equilibrium	constant	angular displacement	linear	equilibrium

have a ________					
The study of a body in motion, when the forces which cause the motion are not considered, is called ________	kinematics	dynamics	scalar	kinetics	kinematics
The study of a body in motion, when the forces which cause the motion are considered, is called ________	kinematics	dynamics	scalar	kinetics	kinetics
Lame's theorem= __________	(P/sinα) = (Q/sin β) = (R/sinγ)	P=Q=R	(P/cos α) = (Q / cos β) = (R / cos γ)	P = Q =R = sin α	(P/sinα) = (Q/sin β) = (R/sinγ)
Unit for moment of force is ______________	N	m	N-m	m2	N-m
Unit for work (or) energy ____________	joule	N-m	N	second	joule
Unit vector = _____________	F/ √(Fx^2 + Fy^2 + Fz^2)	√(Fx^2 + Fy^2 + Fz^2)	F/ (Fx^2 + Fy^2 + Fz^2)	F	F/ √(Fx^2 + Fy^2 + Fz^2)
If two forces are acting on a particle, the particle will be in equilibrium, when the two forces are equal, ________ & ________	Opposite & collinear	Concurrent	collinear	Opposite & concurrent	Opposite & collinear
If three forces are acting on a particle, the particle will be in equilibrium, when the three forces are ________	collinear	Concurrent	parallel	forces	Concurrent
unit for acceleration = __________	m/s^2	m/s	$m.s^2$	m.s	m/s^2
Which of the following is a scalar quantity?	Force	Speed	Velocity	Acceleration	Speed
The forces, which meet at one point and their lines of action also lie on the same plane, are known as	coplaner concurrent forces	coplaner non-concurrent forces	non-coplaner concurrent forces	non-coplaner non-concurrent forces	coplaner concurrent forces
The unit of force in S.I. system of units is	dyne	kilogram	newton	watt	newton
If the resultant of two equal forces has the	30°	60°	90°	120°	120°

same magnitude as either of the forces, then the angle between the two forces is					
The angle between two forces when the resultant is maximum and minimum respectively are	0° and 180°	180° and 0°	90° and 180°	90° and 0°	0° and 180°
The unit of power in S.I. units is	Newton meter	Watt	Joule	Pascal	Watt
The unit of force in S.I. units is	Dyne	Watt	Newton	kilogram-force	Newton
Forces are called coplanar when all of them acting on body lie in	One point	One plane	Two points	Different planes	One plane
The unit of work or energy in S.I. units is	Watt	Newton	kilogram-force	Joule	Joule
Effect of a force on a body depends upon	magnitude	direction	position or line of action	all of the given	all of the given
If a number of forces act simultaneously on a particle, it is possible	Not replace them by a single force	To replace them by a single force	To replace them by a single force through C.G.	Not replace them by a couple	To replace them by a single force
A force is completely defined when we specify	magnitude	direction	point of application	all of the given	all of the given
The algebraic sum of the resolved parts of a number of forces in a given direction is equal to the resolved part of their resultant in the same direction. This is as per the principle of	independence of forces	dependence of forces	balance of force	resolution of forces	resolution of forces
The resolved part of the resultant of two forces inclined at an angle 9 in a given direction is equal to	The algebraic sum of the resolved parts of the forces in the given direction	The sum of the resolved parts of the forces in the given direction	The difference of the forces multiplied by the cosine of 9	The sum of the forces multiplied by the sine of 9	The algebraic sum of the resolved parts of the forces in the given direction
Which of the following is not the unit of distance ?	Angstrom	Micron	millimetre	Milestone	Milestone
The weight of a body is due to	centripetal force of earth	gravitational force of attraction towards the	forces experienced by body in atmosphere	force of attraction experienced by particles	gravitational force of attraction towards the center of the earth

		center of the earth			
The forces, which meet at one point, but their lines of action do not lie in a plane, are called	coplanar non-concurrent forces	non-coplanar concurrent forces	non-coplanar non-concurrent forces	none of the given	non-coplanar concurrent forces
Which of the following is not a scalar quantity	mass	Volume	acceleration	density	acceleration
According to principle of transmissibility of forces, the effect of a force upon a body is	maximum when it acts at the center of gravity of a body	different at different points in its line of action	the same at every point in its line of action	minimum when it acts at the C.G. of the body	the same at every point in its line of action
Which of the following is a vector quantity	Mass	momentum	energy	speed	momentum
A number of forces acting at a point will be in equilibrium if	Their total sum is zero	Two resolved parts in two directions at right angles are equal	sum of resolved parts are zero	all of them are inclined equally	sum of resolved parts are zero
Which of the following is not a vector quantity	weight	velocity	acceleration	force	weight
According to law of triangle of forces	three forces acting at a point will be in equilibrium	three forces acting at a point can be represented by a triangle, each side being proportional to force	if three forces acting upon a patticle are represented in magnitude and direction by the sides of a triangle, taken in order, they will be in equilibrium	if three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two	if three forces acting upon a patticle are represented in magnitude and direction by the sides of a triangle, taken in order, they will be in equilibrium
According to lami's theorem	three forces acting at a point will be in equilibrium	three forces acting at a point can be represented by a triangle, each side being proportional to force	if three forces acting upon a patticle are represented in magnitude and direction by the sides of a triangle, taken in order, they will be in equilibrium	if three coplanar forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two	if three coplanar forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two
If a rigid body is in equilibrium under the action of three forces, then	these forces are equal	the lines of action of these forces meet in a point	the lines of action of these forces are parallel	the lines of action of these forces meet in a point & parallel	the lines of action of these forces meet in a point & parallel

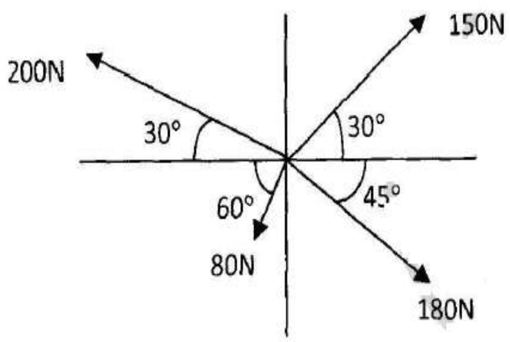
The necessary condition for forces to be in equilibrium is that these should be	Coplanar	meet at one point	both Coplanar and meet at one point	None of the given	both Coplanar and meet at one point
If three forces acting in different planes can be represented by a triangle, these will be in	non-equilibrium	partial equilibrium	equilibrium	Unpredictable	non-equilibrium
tan θ= --------------	ΣV / ΣH	V	ΣH / ΣV	H / V	ΣV / ΣH

2 MARKS

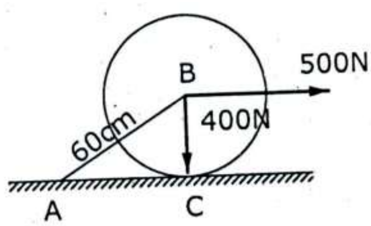
1. Define dynamics.
2. Define like collinear coplanar forces.
3. State varignon's theorem.
4. Define statics.
5. State Newton's laws of motion.
6. Define coplanar concurrent forces.
7. Define kinetics.
8. State classification of force system.
9. Define parallelogram law.
10. What is like parallel force?
11. What are fundamental and derived units? Give examples.
12. Two forces of magnitude 50kN and 80kN are acting on a particle, such that angle between the two is 135°. If both the forces are acting away from the particle, calculate the resultant and find its direction.
13. Define non coplanar forces.
14. The sum of two concurrent forces F1 and F2 is 300N and their resultant is 200 N. the angle between the forces F1 and resultant is 90°. Find the magnitude of each force.
15. Define laws of mechanics.

14 MARKS

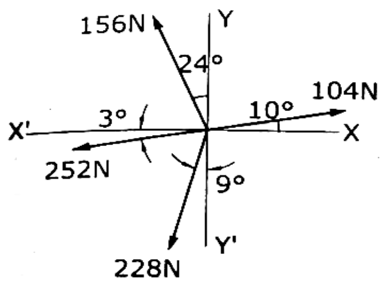
1. Determine the resultant of the concurrent force system shown in figure.



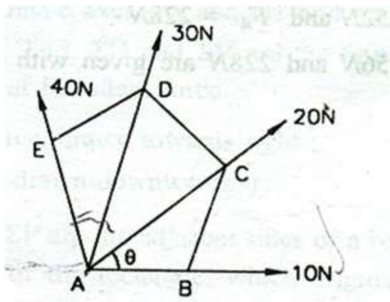
2. A circular roller of radius 20 cm and of weight 400N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 60 cm as shown in fig. A horizontal force of 500 N is acting at B. Find the tension in the bar AB and reaction at C.



3. Four coplanar forces are acting at a point as shown in fig. determine the resultant and magnitude and direction.

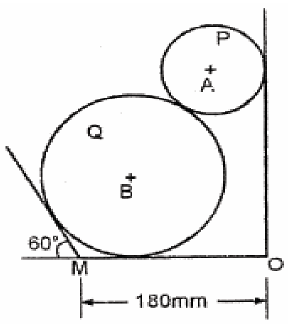


4. The forces 10N, 20N, 30N and 40N are acting on one of the vertices of a regular Pentagon, towards the other four vertices taken in order. Find the Magnitude and direction of the resultant force R.

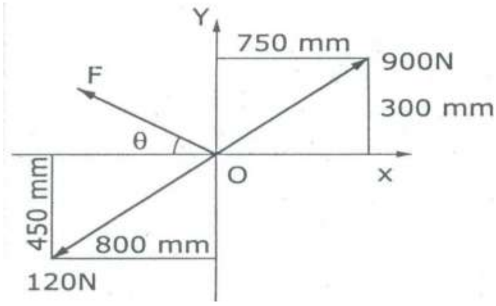


5. A force of magnitude 30 KN, 30 KN, 60kN and 50 KN are acting on a particle O. the angles made by the forces with axis are 25°, 80°, 140° and 230° respectively. All the angles measured in anticlockwise direction. Find the Magnitude and direction of Equilibrant.

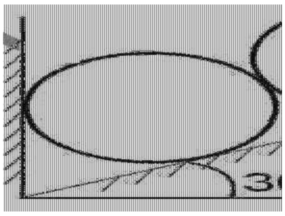
6. Two cylinders rest in channel shown in fig.the cylinder P has diameter of 100mm and weight 200N and the cylinder Q has diameter of 180mm and weight 500N. if the bottom width of the box is 180mm, with one side vertical and other side inclined at 60°.find all the reactions at contact points.



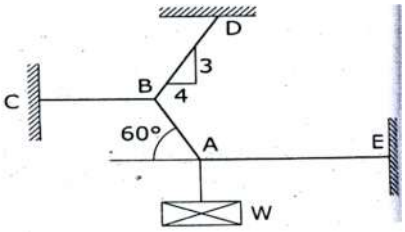
7. The resultant force of coplanar concurrent force system shown in fig. is zero. Determine the force F and its angle θ.



8. Two identical rollers, each of weight 500 N, are supported by an inclined plane making an angle of 30º to the horizontal and a vertical wall as shown in the figure.



9. A load 300N is supported at A by a system of four chords as shown in fig. Determine the tension in each chord for equilibrium.

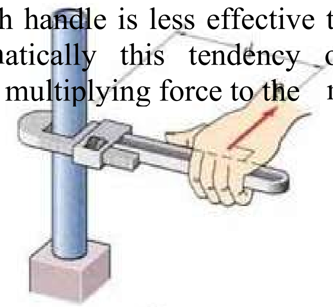


UNIT – 2: STATICS OF RIGID BODIES IN TWO DIMENSIONS

Moment of a force

The tendency of a force to move the body in the direction of its application a force can tend to rotate a body about an axis. This axis may be any line which is neither intersects nor parallel to the line of the action of the force. This rational tendency of force is know as the moment of force.

As a familiar example of the concept of moment, consider the pipe wrench as shown in figure (a). One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude of the force and the effective length d of the wrench handle. Common experience shown that a pull which is not perpendicular to the wrench handle is less effective than the right angle pull. Mathematically this tendency of force (moment) is calculated by multiplying force to the moment arm (d)



(a)

Moment about a point

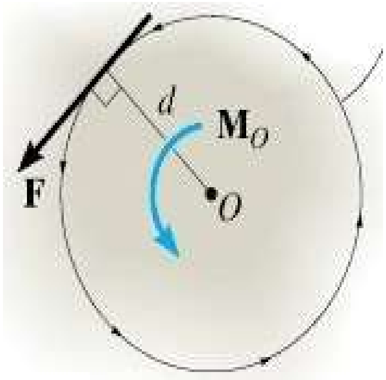
Consider following body (two dimensional) acted by a force F in its plane. The magnitude of moment or tendency of the force to rotate the body about the axis O_O perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, therefore magnitude of the moment is defined as the product of force and moment arm.

Moment = Force x moment arm

M = Fd

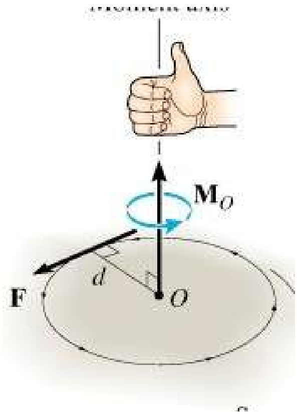
Where d = moment arm and F = magnitude of force

Moment arm is defined as the perpendicular distance between axis of rotation and the line of action of force.



Direction of moment of a force

The direction Mo is specified using the “right-hand rule”. To do this the fingers of the right hand are curled such that they follow the sense of rotation, which would occur if the force could rotate about point O. The thumb then point along the moment axis so that it gives the direction and sense of the moment vector, which is upward and perpendicular to the shaded plane containing **F** and **d**.



CLOCK WISE AND ANTI CLOCK WISE MOMENTS

The moment are classified as clockwise and anticlockwise moment according to the direction in which the force tends to rotate the body about a fixed point

Clockwise Moment

When the force tends to rotate the body in the same direction in which the hands of clock move is called clockwise moment the clockwise moment is taken as positive or other wise mentioned.

Anticlockwise Moment

When the force tends to rotate the body in the opposite direction in which the hands of clock move is called anti clockwise moment which is taken as negative or other wise mentioned

Unit of moment

S.I unit	is	N.m.	(Newton. meter)
F.P.S unit	is	lb. ft	(Pound. foot)
G.G.S unit	is	dyne.cm	(dyne. Centimeter) etc

Example 1

Determine the moment of the force about point "O" for following diagram.

1 **Given** Force=100 N
Moment arm=2m

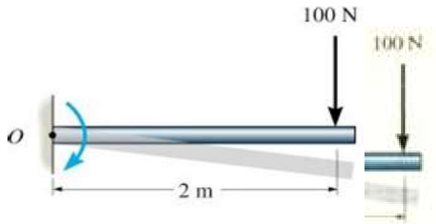
Required Mo=?

Working formula: - M_O=Force x Moment arm.

Sol putt the values in first w, f

Mo= F x r = 100 x 2

Mo= 200N.m.



(a)

Result: - Moment = 200N.m Direction = clock wise

2

Given

Force = 40lb

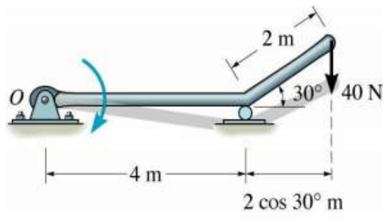
Required; M_0 =?

W.F, Mo = F x d.

Sol

By geometry of fig

Moment arm = 4ft + 2cos 30º = 5.73ft



(c)

Put the value in W.F.

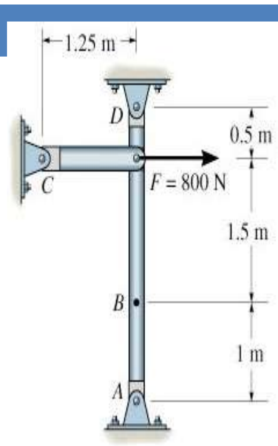
Mo = F x r

Mo = 40 x 5.73

Mo = 229.282lb.ft

Resultant Moment = 229.282 lb.ft Direction = clock wise

Example2



Determine the moment of the force 800 N acting on the frame about points A, B, C and D.

Given

Force = F = 800 N

Required M_A=? M_B? M_C=? M_D=?

Working formula

Moment =force x moment arm.

Sol Solve this question step by step

Now first consider the Point A.

M_A = F x r

M_A = 800 x (1.5+1)

M_A = 2000 N.m clock wise________I

Now Moment about B

M_B = F x r = 800x 5

M_B = 1200 N m clock wise_________(2)

From (1) and (2) it is evidence that when force remain constant then moment varies with moment arm that is moment depends upon moment arm. Similarly it can be proved that moment about any point varies with force when moment arm remain same.

Now consider point C

Moment = Force x distance

Mc = 800 x 0

Mc = 0._________(3)

As the line of action of force passes through point C that is point of application it shows that the line of action should be perpendicular to the point i.e. "C"

Now consider the point D.

M_D = F x r.

M_D = 800 x 0.5

M_D= 400 N.m

Result

M_A =2000 N.m clock wise Or M_A = + 200 N.m

M_B = 1200 N.m clock wise Or M_B = + 1200 N.m

M_C = O. Mc = O

M_D =. 400 N.m anti clock wise M_D = - 400N.m

Note: - The positive sign shows that the moment is clock wise direction and it is also proved that moment defends upon following two factors.

1. The magnitude of the force
2. The perpendicular distance from the line of action of the force to the fixed point or line of the body about which it rotates.

PRINCIPLE OF MOMENT/ VARIGNON'S THEOREM

It is stated that the moment of a force about a point is equal to the sum of the moments of the force components about the point. Or the moment produce by the resultant force is equal to the moment produce by the force components.

Mathematically $\quad M_{F_o} = \sum M_o$

Moment produce by the force F about any point O = Moment produce due to force components. Let us consider a force F acting at a point A and this force create the moment about point O which is r distance away from point A as shown in fig (a)

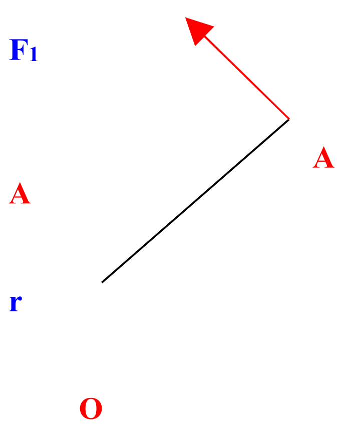


fig (a)

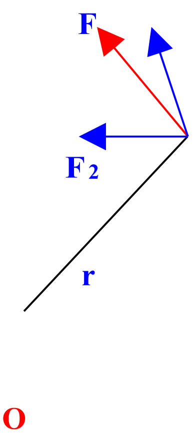


fig (b)

The moment produce due to Force F is given by

$$M_{F_o} = F \times r \_\_\_\_\_\_\_\_\_\_ \quad 1$$

Now resolve the force into its components F_1 and F_2 in such a way that

$$F = F_1 + F_2 \text{ as shown in fig (b)}$$

The moment produce by these components about O is given by

$$\sum M_o = 0$$

$\sum M_o$ = moment produce due to force F_1 + moment produce due to force component F_2

$$\sum M_o = F_1 \times r + F_2 \times r = (F_1 + F_2)\, r$$

Put $F = F_1 + F_2$ in the above formula

$\sum M_o = F \times r$ 2

By comparing the equation 1 and by equation 2

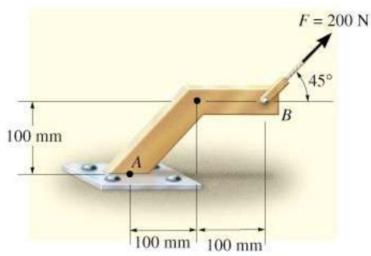
$$M_{F_o} = \sum M_o$$

The above equation shows that moment produce by the Force (resultant) is equal to the moment produce by components F_1 and F_2.

Note the above equation is important application to solution of problems and proofs of theorems. Such it is often easier to determine the moments of a force's components rather than the moment of the force.

EXAMPLE 3

A 200 N force acts on the bracket as shown determine the moment of force about "A"



Given $F = 200N$ $\theta = 45^o$

Required $M_A = ?$

Solution Resolve the force into components F_1 am F_2

$F_1 = F \cos\theta$ $F_1 = 200$ cosine 45^o

$\mathbf{F_1 = 141.42N.}$

$F_2 = F \sin\theta$ $F_2 = 200 \sin 45^o$

$\mathbf{F_2 = 2.468N.}$

We know that $M_A = 0$

M_A = moment produce due to component F_1+ moment produce due to component F_2.

$M_A = F_1 \times r_1 + F_2 \times r_2$.

Let us consider that clock wise moment is + ve.

$M_A = F_1 \times r_1 + F_2 \times r_2$

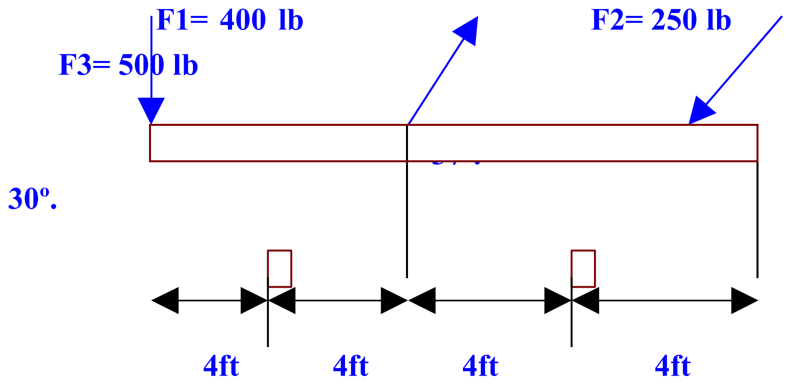
$M_A = -141.42 \times 0.1 + 2.468 \times (0.1 + 0.1)$

$M_A = -13.648$ N

$M_A =$ 13 .648 N anti clock wise.

EXAMPLE 2.4

Determine the moment of each of three forces about B on the beam.



Given

$F_1 = 400lb$ $F_2 = 250$ lb $F_3 = 500lb$

$r_1 = 4$ Ft $r_2 = 4$ Ft $r_3 = 4$ Ft $r_4 = 4$ Ft

Required Moment about B = M_B =?

Solution

Moment due to force F_1 about B:

Consider clockwise moment is positive

$M_B = 400 \times (4+4+4)$

M_B = 48, 00 lb .ft

Moment due to vertical component of F2

$M_B = F2 \sin\theta \times r$

$M_B = 250 \text{ Sin } 37 \times 4$

M_B = 601.815lb ft clock wise

Moment due to vertical component of F3

$M_B = F3 \sin\theta \times R$

$M_B = 500 \times \text{Sin } 30 \times 4$

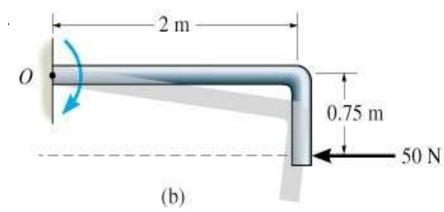
M_B = 601.815lb clock wise

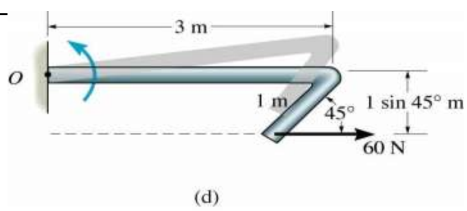
Result **M_B = 48, 00 lb .ft 601.815lb, 601.815lb**

EXERCISE

1. Find the moment of the force about "O" as shown in diagram

i **ii**





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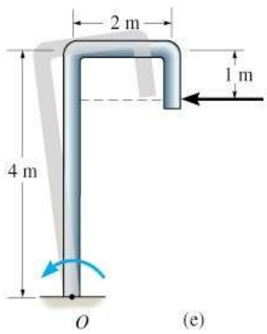


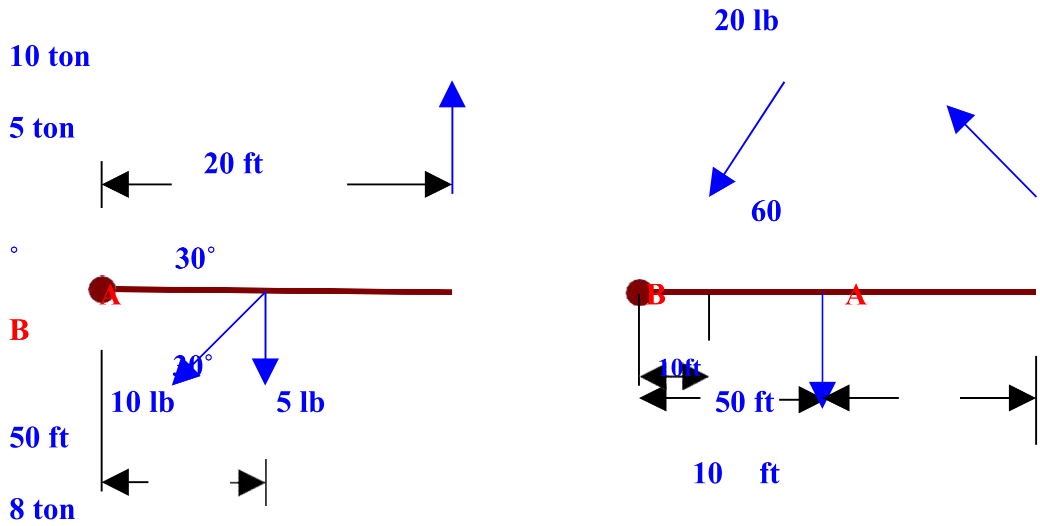
ise

Ans : 42.426

Ans: 21 kN m

2. Find the moment of each force about A as shown in the following force system.

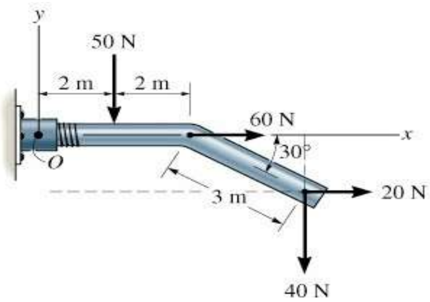
i **ii**



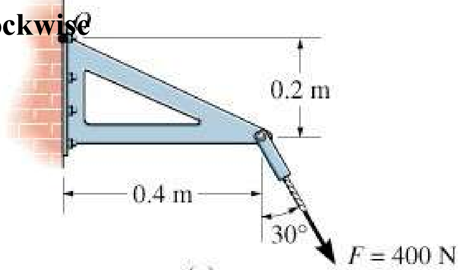
Ans: 300 lb ft anti clockwise **Ans: 236.603 ton ft anti clock wise**

3. Determine the resultant moment of four forces acting on the rod about "O" as shown is diagram.

Ans: 333.92 N m clock wise

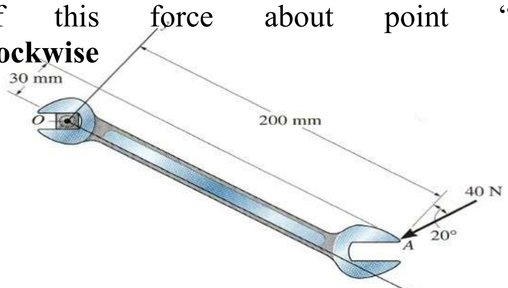


4. The Force F acts at the end of angle bracket shown determine the moment of forces about "O" **Ans : 98.56 clockwise**

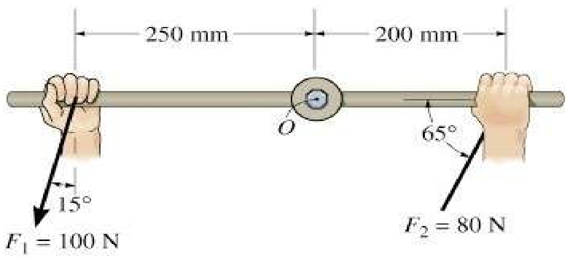


(a)

5. A force of 40N is applied to the wrench. Determent the moment of this force about point "O" **Ans: 7.107 clockwise**



6. The wrench is used to loosen the bolt. Determine the moment of each force about the bolt's axis passing through point O. **(Ans: 24.1 N-m, 14.5 N-m)**



pf
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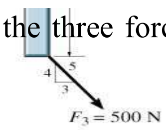
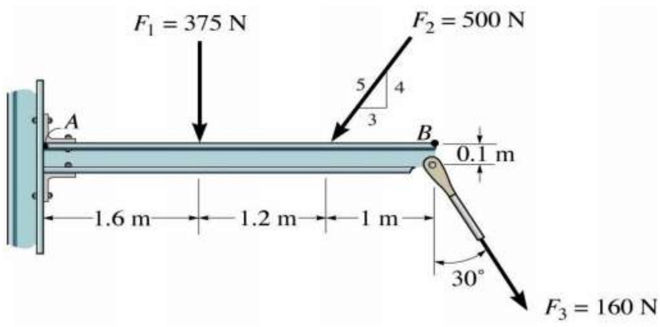
An
-m,

7. Determine the moment of each of the three forces about point A. orce as a whole, an ts. s: 433 N- 800 N-

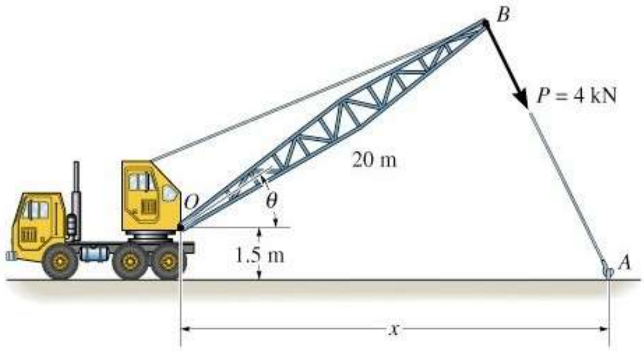


8. Determine the moment about point A of each of the three forces

Ans: 600 N-m, 1.12 KN-m, 518 N-m



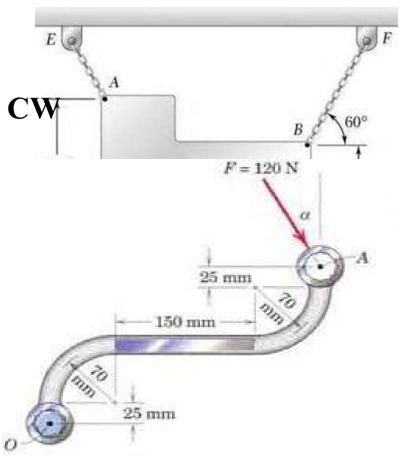
9. The towline exerts a force of P = 4 kN at the end of the 20 m long crane boom. If θ = 30º, determine the displacement x of the hook at A so that the force creates a maximum moment about point O. What is this moment? **(Ans: 24.0 m, 80 kN-m)**



10. $\alpha = 30°$, calculate the moment of F about the center O of the bolt. Determine the value of α which would maximize the moment about O state the value of this maximum moment

Ans: 41.5 N=

m CW 32.2° 41.6 N-m CW



PARALLEL FORCES

When the lines of action of Forces are parallel to each other are called parallel forces the parallel forces never meet to each other. There are two types of parallel forces as discussed as under

1. **Like parallel forces**

   When two parallel forces acing in such away that their directions remain same are called like parallel forces

2. **Un like parallel forces**

   When two parallel forces acing in such away that their directions are opposite to each other called like parallel forces

.

COUPLE

When two parallel forces that have the same magnitude but opposite direction is known as couple. The couple is separated by perpendicular distance. As matter of fact a couple is unable to produce any straight-line motion but it produces rotation in the body on which it acts. So couple can be defined as unlike parallel forces of same magnitude but opposite direction which produce rotation about a specific direction and whose resultant is zero

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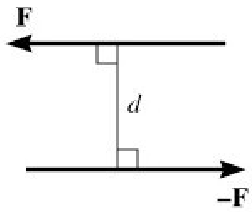


APPLICATION OF COUPLE

1. To open or close the valves or bottle head, tap etc
2. To wind up a clock.
3. To Move the paddles of a bicycle
4. Turning a key in lock for open and closing.

Couple Arm

The perpendicular distance between the lines of action of the two and opposite parallel forces is known as arm of the couple.

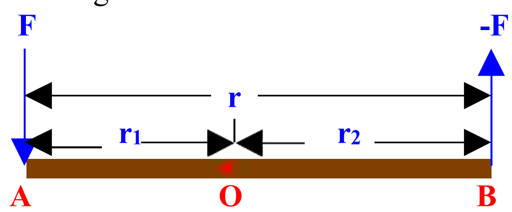
Moment of couple or couple moment

The moment of the couple is the product of the force (one of the force of the two equal and opposite parallel forces) and the arm of the couple. Mathematically

Moment of couple = force x arm of couple

Moment of couple = F x r

Let us find the resultant moment of couple about a point O on the couple arm AB as shown in fig



Moment about O

$\sum M$ = Moment about O due to F + moment about O due to –F

$\sum M = -F \times r_1 + (-F \times r_2)$

$\sum M = -F \times r_1 - F \times r_2$

$\sum M = -F (r_1 + r_2)$

$\sum M = F (r_1 + r_2)$ ____________ 1

From diagram $r = r_1 + r_2$ put in equation 1

$\sum M = F \times r$

So the moment produce by the two unlike parallel forces is equal to moment produce by one of the force of the two equal and opposite parallel forces.

Therefore

The moment of couple = force x couple arm.

Direction of couple

The direction and sense of a couple moment is determined using the right hand rule, where the thumb

CLASSIFICATION OF COUPLE

The couplet are classified as clockwise couple and anticlockwise couple

1. Clockwise couple

A couple whose tendency is to rotate the body in a clockwise direction is known as clockwise couple

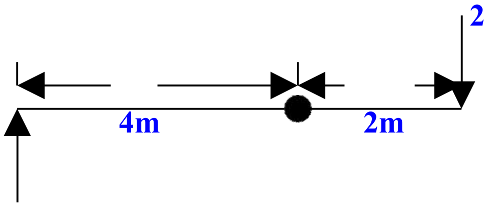
2. Anticlockwise couple

A couple whose tendency is to rotate the body in anticlockwise direction is known as anticlockwise couple

EXAMPLE 8

Determine the moment of couple acting on the moment shown

00 N



200 N

Given

F_1=200 N L_1=4m F_2=200 N L_2 = 2m.

Required Moment of couple = M =?
Working Formula M = F x r.
Solution
Put the values in working formula

$M= 200(4+2)$

M=1200 N. m

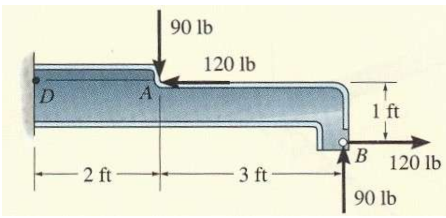
Result **M= 1200 N. m**

EXAMPLE 9

Determine the moment of

couple acting on the moment

shown.



Given $F_1=F_2=90lb$ $F_3 = F_4 = 120lb.$
Required Moment of couple = M=?
Solution The moment of couple can be determined at any point for example at A, B or D.
Let us take the moment about point B

$M_B = \sum F R.$

$M_B = -F_1 \times r_1 - F_2 \times r_2 .$

$M_B = - 90(3) - 120 (1)$

$M_B = - 390$ lb ft

Result **$M_B = M_A=M_D =390$ lb .ft counter clock wise.**

Moment of couple = 390 lb.ft count cloche wise

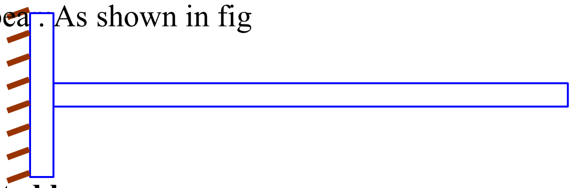
BEAM A beam is a long straight bar having a constant cross-sectional area. Beams are classified as

1 Cantilever beam 2 Simply supported beam
3 Over hanging beam 4 Rightly fixed or built in beam

5 Continuous beam.

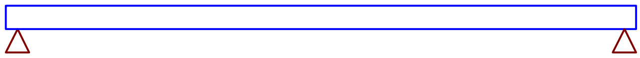
1. Cantilever beam

A beam, which is fixed at one and free at the other end, is called cantilever bea . As shown in fig



2. Simply supported beam

A beam which is pinned (pivoted) at one end and roller support at other end is called simply supported beam. As shown in fig

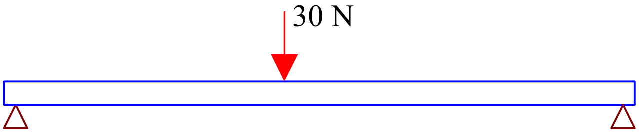


LOAD

The external applied force is called load. Load is in the form of the force or the weight of articles on the body is called load.

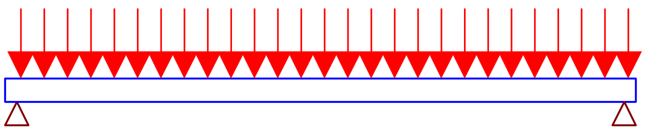
1. Concentrated or Point load

A load, which is applied through a knife-edge, is called point or concentrated load.



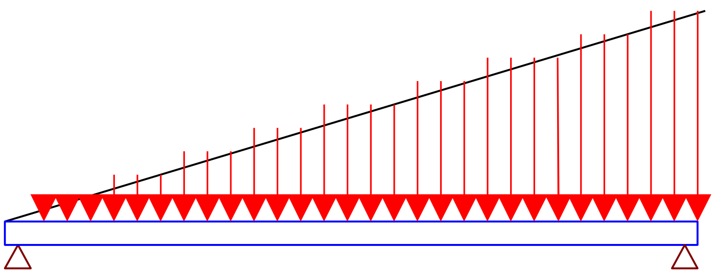
2. Uniformly distributed load

A load which is evenly distributed over a part or the entire length of beam is called uniformly distributed load or U D.L



3. Uniformly varying load

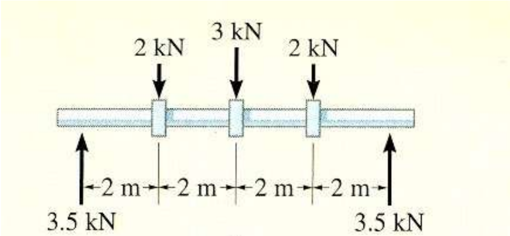
The load whose intensity varies lineally along the length of beam over which it is applied is called uniformly varying load.



Note

Any beam may be point, uniformly distributed and uniformly varying load

EXAMPLE 10 Find the reaction of the shaft at point shown**.**



(b)

Given Span = L = 8m x = 2m, y = 2m, Z = 2m

F_1 = 2 KN F_2 = 3 KN F_3= 2 KN.

Required Shear force and moment diagram

Solution Take moment about "A" also consider the upward force and clock wise moment is positive

$\sum M_A = 0$

$R_E (L) – F3 (x + y + z) – F_2 (x + y) – F_1 (x) + R_A (0) = 0.$

$R_E (8) - 2 (6) – 3 (4) – 2 (2) + 0 = 0$

R_E = 3.5 KN

Now for R_A we can calculate by

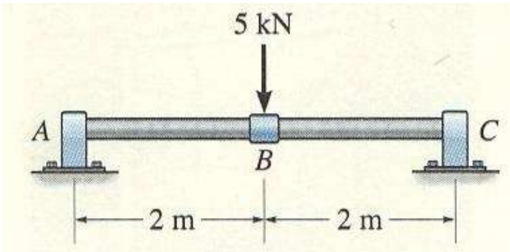
$\sum F = 0$

$R_A - F_1 - F_2 - F_3 + R_E = 0$

$R_A - 2 - 3 - 2 + 3.5 = 0$

EXAMPLE 2.11

Find reaction at A and C for shaft shown. The



support at A is a thrust bearing and support C is a Journal bearing. Also draw shear force bending moment diagram.

Given Span = L = 4m. Load = P = 5 kN.
Required R_A =? R_C =?

Solution Take moment about "A" also considers upward force and clockwise moment is positive.

$\sum M_A = 0$

$R_c (L) - P (x) + R_A (0) = 0.$

$R_c (4) - 5 (2) = 0$

$\mathbf{R_c = 2.5\ k\ N}$

To calculate the reaction at point A

$\sum F = 0$

$R_A - P + R_c = 0$

$R_A - 5 + 2.5 = 0$ $\mathbf{R_A = 2.5\ k\ N}$

<u>**EXAMPLE 2**</u>

Find the reaction of a simply supported beam 6m long is carrying a uniformly distributed load of 5kN/m over a length of 3m from the right hand.

Given_

P = 5 k N /m L = 6 m Y = 3m, Z = 3m.

Required Reaction at A & B = R_A & R_B =?

Solution first of all we will change the uniformly distributed load into the point load

= 5 x 3= 15 kN

Take moment about A also consider that the upward force or load and clockwise moment is positive.

$\sum M_A = 0$

$R_c (L) - P (y + z/2) + R_A (0) = 0$

$R_B (6) - (15) (3 + 1.5) + R_A (0) = 0$

$\mathbf{R_B = 11.\ 25\ kN}$

To calculate the reaction at point A

$\sum F = 0$

$R_A - P + R_B = 0$

$R_A - 15 - 11.25$

$\mathbf{R_A = 3.75.kN}$

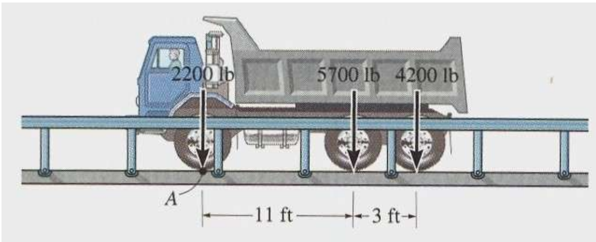
<u>Exercise 2</u>

6. Find the moment of couple shown what must the force of a couple balancing this couple having arm of length of 6ft. **Ans: 36 lb ft, 6 lb**

7. The tires of a truck exert the forces shown on the deck of the bridge replace this system of forces by an equivalent

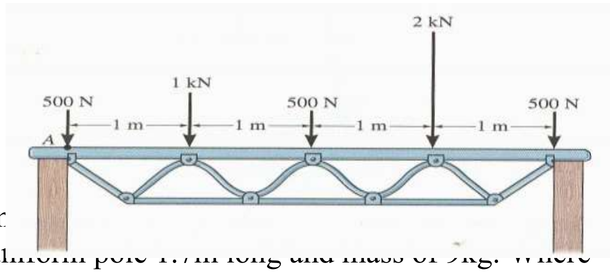
resultant force and specify its measured form point A.

Ans: 12.1 kip, 10.04 ft



8. The system of parallel forces acts on the top of the Warne truss. Determine the equivalent resultant force of the system and location measured from point A

Ans: 4.5



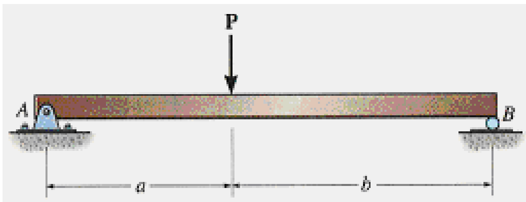
9. A man ar

means of a u

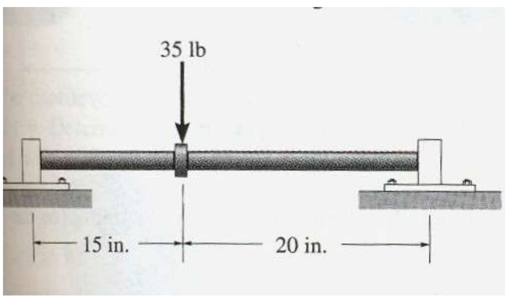
the weight must placed so that the man may carry twice as mush of weight as that boy.
Ans: 111.18 N, .04646 m

10. Two unlike parallel forces of magnitude 400 N and 100 N acting in such a way that their lines of action are 150 mm apart. Determine the magnitude of the resultant force and the point at which it acts.

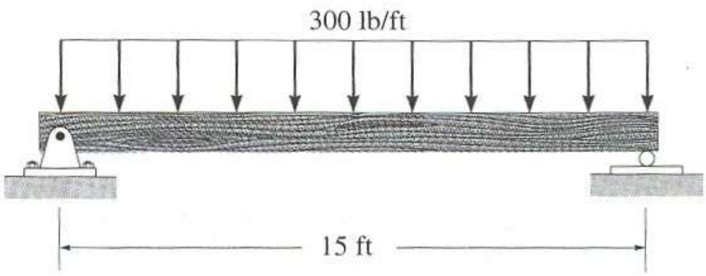
Ans: 300 N & 50 mm
11. Find reaction at point A and B for the beam shown set P= 600lb a = 5ft b = 7ft.



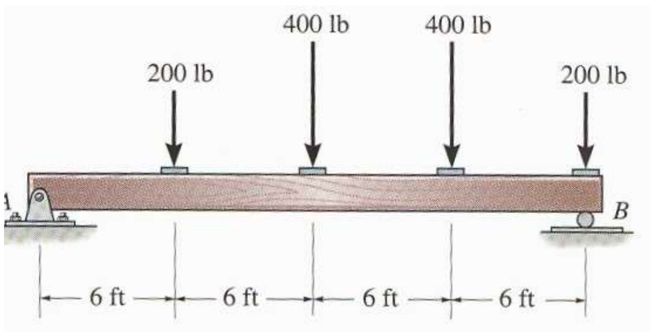
12. Find the reaction at the points for the beam as shown



13 Find the reaction at the points as shown in diagram



14 Find the reaction at the points as shown in diagram



EQUILIBRIUM OF PARTICLE AND BODY

Equilibrium of a Particle

When the resultant of all forces acting on a particle is zero, the particle is said to be in equilibrium.

A particle which is acted aupon two forces

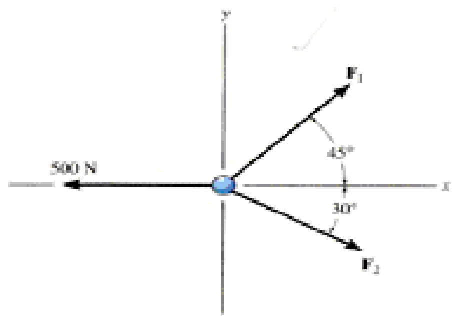
Newton's First Law:

If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.

Exercise

1. Determine the magnitude of F_1 and F_2 so that the partial is

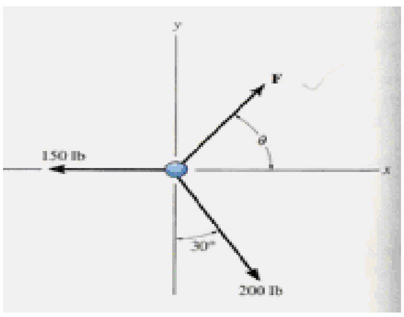
in equilibrium



12. Determine the magnitude and direction of F_1 and F_2 so that the partial is in equilibrium

Ans:

42.567 lb & 54.723 lb



EQUILIBRIUM

A particle is in equilibrium if it is at rest if originally at rest or has a constant velocity if originally in motion. The term equilibrium or static equilibrium is used to describe an object at rest. To maintain equilibrium it is necessary to satisfy Newton's first law of motion, which requires the resultant force acting on particle to be equal to zero. That is

$$\sum F = 0 \longrightarrow A$$

Where $\sum F$ = Sum of all the forces acting on the particle which is necessary condition for equilibrium. This follows from Newton's second law of motion, which can be written as

$$\sum F = ma.$$

Put in equation A $\quad ma = 0$

Therefore the particle acceleration a = 0. Consequently the particle indeed moves with constant velocity or at rest.

METHODS FOR THE EQUILIBRIUM OF FORCES

There are many methods of finding the equilibrium but the following are important

1. Analytical Method 2. Graphical Method

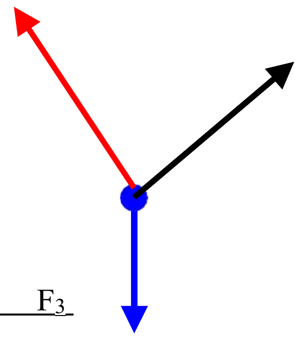
1. Analytical method for the equilibrium of forces

The equilibrium of forces may be studied analytically by Lami's theorem as discussed under

LAMI'S THEOREM

It states, "If there are three forces acting at a point be in equilibrium then each force is proportional to the sine of the angle between the other two forces".

Let three force F_1, F_2 and F_3 acting at a point and the opposite angles to three forces are γ , β, and α as shown in figure



F_2

F_1

Mathematically

α

$$\frac{F_1}{\sin \beta} = \frac{F_2}{\sin \gamma} = \frac{F_3}{\sin \alpha}$$

β γ

F_3

EXAMPLE 7

UNIT 2 - MCQ

Questions	opt1	opt2	opt3	opt4	answer
A stationary body will be in equilibrium if the algebraic sum of all forces is -----------------	0	equal	not equal	not equal to zero	0
A stationary body will be in -------------- if the algebraic sum of all forces is zero.	0	equal	equilibrium	1	equilibrium
When a body is subjected to two forces, the body will be in equilibrium if the two forces are ---------------, equal and opposite	collinear	concurrent	coplanar	parallel	collinear
Moment = ---------------	Force X distance	Force/distance	distance	Fx+Fy	Force X distance
If three concurrent forces are acting on a body & the body is in equilibrium, then the resultant of two forces should be --------------- & opposite to the third force	normal	tangent	equal	parallel	equal
The reaction at the knife edge support will be --------------- to the surface of the beam.	collinear	equal	parallel	normal	normal
The reaction in case of roller support will be normal to the surface of --------------------	Roller base	base	parallel	equal	Roller base
For a smooth surface, the reaction is always -------- to the support	normal	parallel	angular	equal	normal
A load, acting at a point on a beam is known as -----------------	Roller base	UDL	parallel	couple	Roller base
If each unit length of the beam carries same intensity of load, then that type of load is known as ---------------	normal	couple	parallel	UDL	UDL
The reaction on a roller support is at ------------ to the roller base.	right angles	parallel	~~equal~~	force	right angles
If three forces act at a joint & two of them are acting the same straight line then third force would be ---------	zero	equal	opposite	normal	zero
Varignon's theorem states that the moment of a force about any point is equal to the ------------ of the moments of its components about that point	algebraic sum	moment	couple	square	algebraic sum
The effect of couple is to produce --------------- about an axis normal to the plane of force which constitute couple.	pure rotation	normal	vertical	force	pure rotation
M = --------------	r.F	|rXF|	rXF	r.F/r	rXF

ENGINEERING MECHANICS					
For two dimensional bodies, the forces are generally resolved into ---------------- & vertical components	vertical	normal	sum	horizontal	horizontal
For stable equilibrium of a body, -----------------	ΣF=0, ΣM=0	ΣF=0	ΣM=0	ΣM=ΣF	ΣF=0, ΣM=0
The beam which carries load in such a way that the rate of loading on each unit length of the beam varies uniformly, this type of load is known as ----------------	UVL	UDL	point load	moment	UVL
The moment is the product of the force & the ------------------ between the line of action of the force & the point about which moment is to be taken	displacement	force	perpendicular distance	moment	perpendicular distance
When two equal & opposite parallel forces act on a body at some distance apart, the two forces form a -----------------	normal	force	moment	couple	couple
The couple has a tendency to --------- the body.	rotate	normal	vertical	move	rotate
Moment of the couple = -------------------------	FXS	Fxa	FXH	MXF	Fxa
The reaction at the hinged end may be either vertical or inclined depending upon the -----------	moment	reaction	type of loading	support	type of loading
If the load is vertical, then the reaction will also be -----------------	horizontal	support	couple	vertical	vertical
If the load is inclined, then the reaction will also be -----------------	inclined	horizontal	vertical	couple	inclined
The normal at any point on the surface of the sphere will always pass through the ------------- of the sphere.	parallel	normal	inclined	centre	centre
For two dimentional bodies, the forces are written as ----------------	ΣF=0, ΣM=0	ΣM=0	ΣM=ΣF	ΣM/ΣF	ΣF=0, ΣM=0
For three-dimensional bodies, the forces are written as ---------------	ΣM=0	ΣF=0, ΣM=0	ΣF=0, ΣM=0, ΣFz=0	ΣF=0	ΣF=0, ΣM=0, ΣFz=0
The given beam is drawn to a suitable scale along with the loads & the reactions R_A & R_B. This step is known as ------------	Bow's notations	vector diagram	space diagram	couple	space diagram
If the end portion of a beam is extended beyond the support, then the beam is known as -----------------	overhanging beam	simply supported beam	cantilever beam	continuous beam	overhanging beam
In case of roller supported beams, the reaction on the roller end is always -------------------- to the suppport.	normal	parallel	sine	vertical	normal
Overhanging portion may be at one end of the beam or at ------------- of the beam.	end	support	both ends	normal	both ends

ENGINEERING MECHANICS					
The main advantage of such a roller support is that beam, due to change in temperature can move easily towards left or right, on account of expansion or -------------------	rise	move	decrease	contration	contration
In case of hihged supported beam, the reaction on the hinged end may be either vertical or inclined depending upon the -------------------	reactions	moments	ends	types of loading	types of loading
The main advantage of a highed end is that the beam remains ------------------------	normal	move	angle	stable	stable
If three forces act at a joint & two of them are along the same straight line then then for the equilibrium of the joint, the third force should be equal to ---------	zero	normal	1	-1	zero
The forces in the members of cantilever truss can be obtained by starting the calculations from the --------- of cantilever	middle	support	top	free end	free end
The horizontal reaction will be obtained by adding algebraically all the ---------------	vertical loads	normal	angular	horizontal loads	horizontal loads
When trying to turn a key into a lock, following is applied	coplanar force	non-coplanar forces	couple	Moment	couple
Two non-collinear parallel equal forces acting in opposite direction	constitute a moment	constitute a couple	constitute a moment of couple	constitute a resultant couple.	constitute a moment
According to principle of moments	if a system of coplanar forces is in equilibrium, then their algebraic sum is zero	if a system of coplanar forces is in equilibrium, then the algebraic sum of their moments about any point in their plane is zero	the algebraic sum of the moments of any two forces about any point is equal to moment of theiwesultant about the same point	positive and negative couples can be balanced	if a system of coplanar forces is in equilibrium, then their algebraic sum is zero
Two coplanar couples having equal and opposite moments	balance each other	produce a couple and an unbalanced force	are equivalent	can not balance each other	can not balance each other
The product of either force of couple with the arm of the couple is called	resultant couple	moment of the forces	resulting couple	moment of the couple	moment of the couple
The center of gravity of a uniform lamina lies at	the center of heavy portion	the bottom surface	the mid point of its axis	all of the given	the mid point of its axis
The algebraic sum of moments of the forces forming couple about any point in their plane is	equal to the moment of the couple	constant	both of above are correct	both of above are wrong	equal to the moment of the couple
A single force and a couple acting in the same plane upon a rigid body	balance each other	cannot balance each other	produce moment of a couple	are equivalent	cannot balance each other
If three forces acting in one plane upon a rigid body, keep it in	meet in a point	be all parallel	at least two of them must meet	all the givens are correct	all the givens are correct

equilibrium, then they must either					
The maximum frictional force which comes into play when a body just begins to slide over another surface is called	limiting friction	sliding friction	rolling friction	kinematic friction	limiting friction
Sum of moment of all the forces will be equal to moment about a point. This is according to	Resolution of froces	Newton	Varignon	Superposition of forces	Varignon
For the roller support , which of the following is not possible	Rotation about a pin	Rotation about a hinge	Vertical movement	Horizontal movement	Vertical movement
Couple is also a	Coplanar force	Non coplanar, like parallel force	Non coplanar, unlike parallel force	Non coplanar force	Non coplanar, unlike parallel force
A hinged support will always have	Horizontal reaction	Vertcial reaction	Rotational reaction	Both horizontal and vertical reactions	Both horizontal and vertical reactions
The effect of a force remains unaltered along its line of action. This is according to	Resolution	Newton	Superposition of forces	Varignon	Superposition of forces
According to the principles of transmissibility of forces, when a force acts upon a body, its effect is	Minimum when it acts at C.G of the body	Maximum when it acts at C.G of the body	Different at different points of the body	Same at every point in its line of action	Same at every point in its line of action
The third unknown force of coplanar concurrent system in equilibrium is defined by	Triangle law of forces	Polygon law of forces	Moment	Couple	Triangle law of forces
If the sum of all the forces acting on a body is zero, it may be concluded that the body	must be in equilibrium	may be in equilibrium provided the forces are concurrent	can not be in equilibrium	may be in equilibrium provided the forces are parallel	may be in equilibrium provided the forces are concurrent
If the cross product of two vectors is zero, then	the vectors must be collinear	either of the vectors or both must be zero	the vectors must be perpendicular to each other	the vectors must be parallel to each other	the vectors must be parallel to each other
The rectangular components of force in space in x axis is	Fcosθ	Fsinθ	Ftanθ	Fcotθ	Fcosθ
A force of magnitude 200N makes an angle of 35 degrees with x axis. Its rectangular component in x axis is	165.83N	163.83N	163.93N	169.83N	163.83N
A force F = 10i+5j-4K acts through the origin. Its magnitude is	10.87N	12.87N	11.87N	13.87N	11.87N

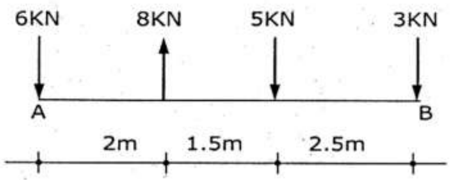
2 MARKS

1. Define resolution of forces.
2. Write short notes about fixed support with sketch.
3. Sketch the different types of couples.
4. State the principle of transmissibility.

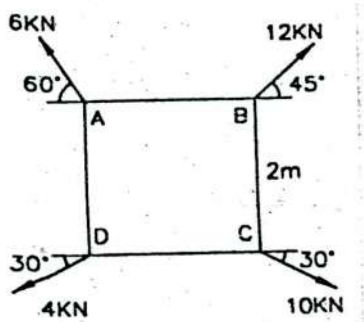
5. State Lami's theorem.
6. What is a couple?
7. Two forces 50 kN and 10 kN act at a point 'O'. The included angle between them is 60o. Find the magnitude and the direction of the resultant.
8. Write short notes on hinged support.
9. Define principal axes and principal moment of inertia.
10. Explain free body diagram with one example.
11. Define point load.
12. A vector A is equal to 2i- 3j +2k. find the projections of this vector on the line joining the point P(-3,2,1) and Q(2,-2,-1).
13. Find the unit vector of a force F=4i-5j+8k

14 MARKS

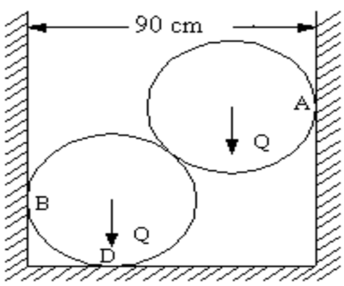
1. A system of parallel forces is acting on a rigid bar as shown in figure. Reduce the system into a single force and a force-couple system at A and B.



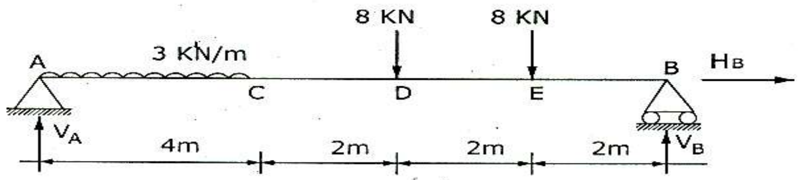
2. Four forces are acting on a square ABCD as shown in fig. Calculate the magnitude and direction of the resultant force.



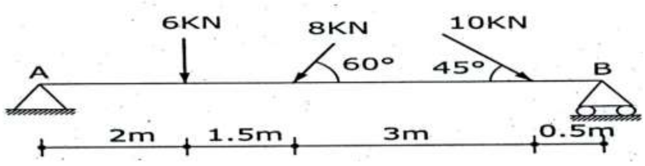
3. Two spheres, each of weight 1000 N and radius of 25 cm rest in horizontal channel of width 90 cm as shown in figure. Find the reactions on the points of contact A, B and D



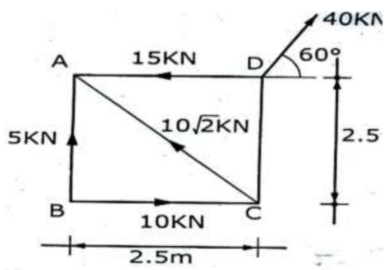
4. A Beam AB of span 10m span is loaded as shown in fig. Determine the reactions' at A and B.



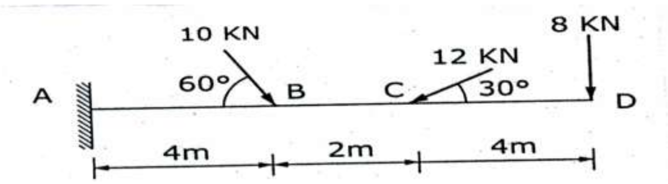
5. Find the support reactions of a simply supported beam shown in figure.



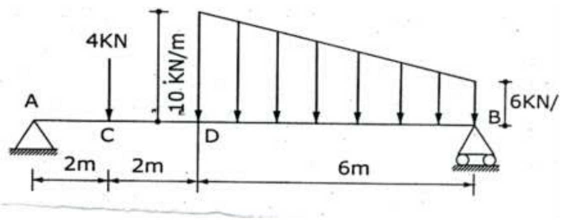
6. Calculate the resultant moment about the corner B shown in figure.



7. Find the support reactions of a beam shown in figure.



8. Find the support reactions of a beam shown in figure.



UNIT – 3: CENTRIOD, CENTRE OF GRAVITY AND MOMENT OF INERTIA

CENTRE OF GRAVITY

The center of gravity is a point where whole the weight of the body act is called center of gravity. As we know that every particle of a body is attracted by the earth towards its center with a magnitude of the weight of the body. As the distance between the different particles of a body and the center of the earth is the same, therefore these forces may be taken to act along parallel lines. A point may be found out in a body, through which the resultant of all such parallel forces acts. This point, through which the whole resultant (weight of the body acts, irrespective of its position, is known as center of gravity (briefly written as C.G). It may be noted that every body has one and only one center of gravity.

CENTROID

The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The center of area of such figures is known as Centroid. The method of finding out the Centroid of a figure is the same as that of finding out the center of gravity of a body.

AXIS OF REFERENCE

The center of gravity of a body is always calculated with referrer to some assumed axis known as axis of reference. The axis of reference, of plane figures, is generally taken as the lowest line of the figure for calculating y and the left line of the figure for calculating x.

METHODS FOR CENTRE OF GRAVITY OF SIMPLE FIGURES

The center of gravity (or Centroid) may be found out by any one of the following methods

I. By geometrical considerations

2. By moments method

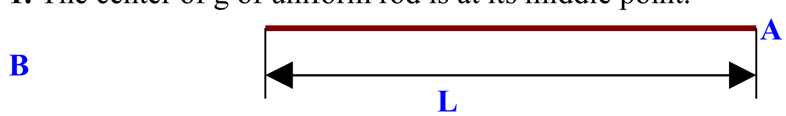
3. By graphical method

1 Center of Gravity by Geometrical Considerations

The center of gravity of simple figures may be found out from the geometry of the figure

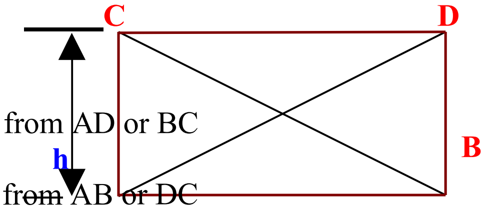
A) The center of gravity of plane figure

1. The center of g of uniform rod is at its middle point.



Center of gravity = L / 2 from point A or B

2. The center of gravity of a rectangle is at a point, where its diagonals meet each other. It is also a mid point of the length as well as the breadth of the rectangle as shown in fig



L x

from AD or BC

from AB or DC

h

A

G

=

G

=

h

/

2

A

r

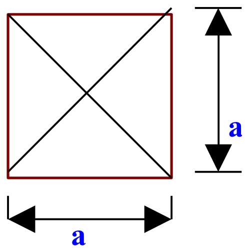
e

a

=

L

3. The center of gravity of a square is a point, where its diagonals meet each other. It is a mid point of its side as shown in fig

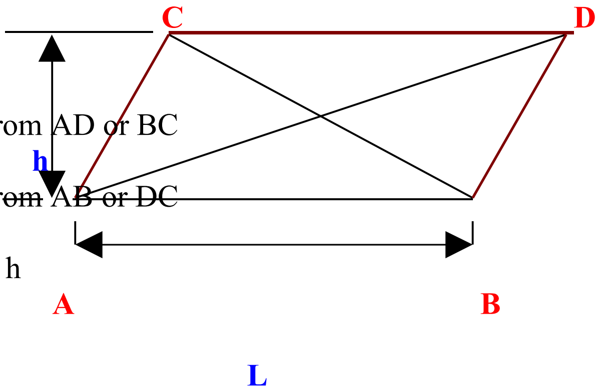


G = a/2 from any

side

Area = 2x a

4. The center of gravity of a parallelogram is at a point, where its diagonals meet each other. It is also a mid point of the length as well as the height of the parallelogram as shown in fig



G = L/2

from AD or BC

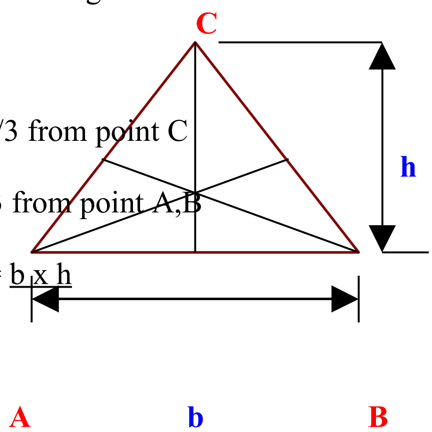
G = h/2

from AB or DC

Area = L

x h

5. The center of gravity of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet as shown in Fig.



G

= 2h/3 from point C

G

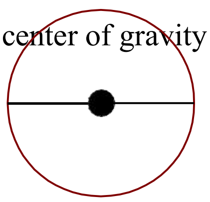
= h/3 from point A,B

A

rea = b x h

2

6. The center of gravity of the circle is the center of the circle



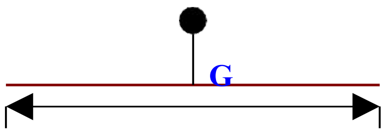
G = r or d/2 from any point from the circumference

Area = π x r^2

7. The center of gravity of the semi circle is at a distance 4 r/3 π from diameter AB

G = 4 r/3 π from diameter AB

Area = $\frac{\pi \times r^2}{2}$

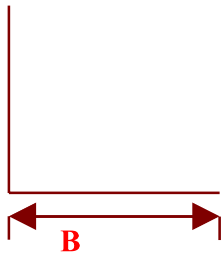


A B

8. The center of gravity of quarter circular at a distance 4 r/3 π from diameter AC

C

from radius AC



$G = 4r/3\pi$

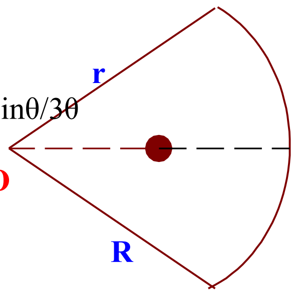
Area = $\frac{\pi \times r^2}{4}$

A B

r

9. The center of gravity of sector is at a distance 2rsinθ/3θ from center c.

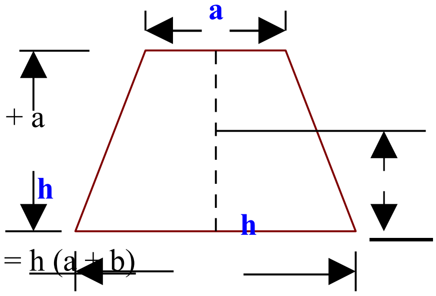
2rsinθ/3θ



G =

Area = $\theta \times r^2$

10. The center of gravity of a trapezium is at a distance of h/3x [b+2a/b+a] form the side AB as shown in Fig.



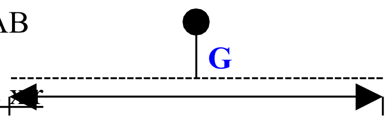
$G = \frac{h}{3} \frac{b+2a}{b+a}$

G

Area = $\frac{h(a+b)}{2}$

11. The center of gravity semi circular arc is at distance 2 r/ π from AB

$G = 2r/\pi$ from AB



Length of Arc =

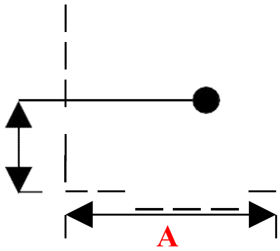
2

π

A B

8. The center of gravity of quarter arc is at a distance 2 r/ π

B



G = 2 r/ π
Length of arc AB =

rπ /2

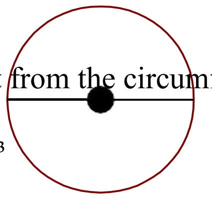
G

A

r

B) THE CENTRE OF GRAVITY OF SOLID BODY

1. The center of gravity of a sphere is at a distance r from any point



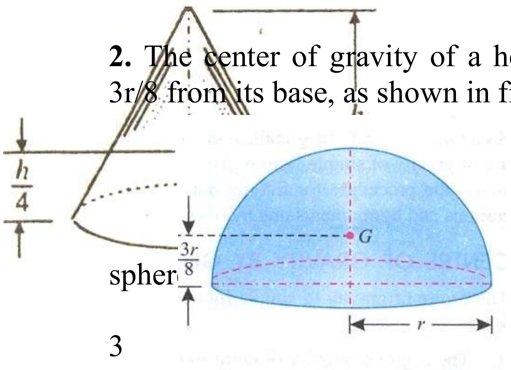
G = r or d/2 from any

point from the circumference

Volume of sphere = $\frac{4}{3}$ x

π x r^3

2. The center of gravity of a hemisphere is at a distance of 3r/8 from its base, as shown in fig.



G = $\frac{3}{8}$ x r

Volume of

spher

3

3. The gravity of right circular solid cone is at a distance h/4 from its base, measured along the vertical axis

G = h/4
Volume of cone = $\frac{1}{3}$

x π x r^2 x h

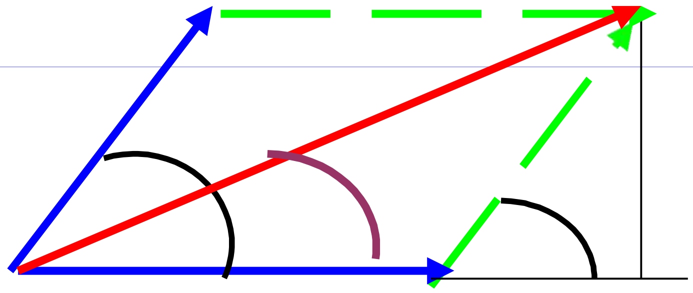
4. The center of gravity of a cube is at a distance of h/4 from every face (where h is the length of each side).

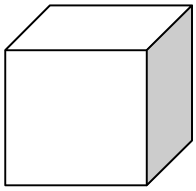
G = h/4
Volume of cube = length x width x height

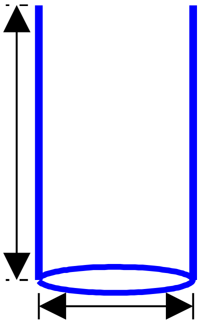
5. The center of gravity of a cylinder is h/2 from diameter AB

G = h/2
Volume of cylinder = π x r^2 x h

h

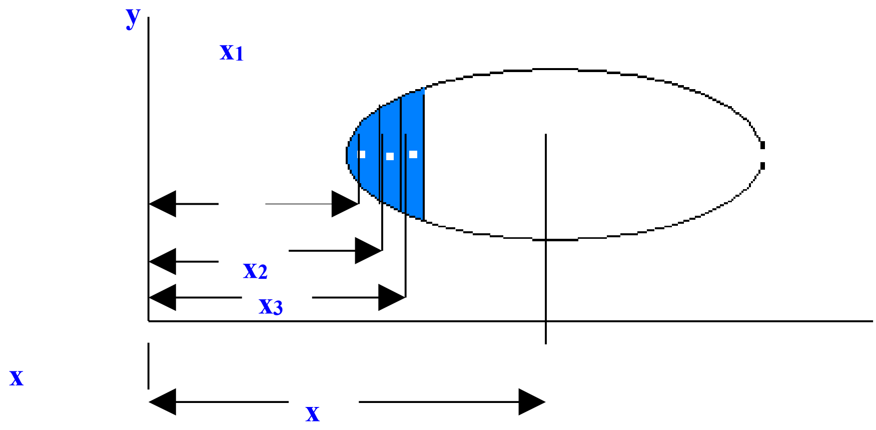






CENTRE OF GRAVITY BY MOMENTS

The center of gravity of a body may also be found out by moments as discussed below. Consider a body of mass M whose center of gravity is required to be found out. Now divide the body into small strips of masses whose centers of gravity are known as shown in fig



Let

m_1, m_2, m_3 ……. = mass of strips 1, 2, 3,

x_1, x_2, and x_3… = the corresponding perpendicular distance or the center of gravity of strips from Y axis

According to principal of moment

$$M\,x = m_1 x_1 + m_2 x_2 + m_3 x_3$$

$$M\,x = \sum m\,x$$

$$\mathbf{x = \frac{\sum m\,x}{M}} \qquad (1)$$

Where $\sum m = m_1 + m_2 + m_3 + \ldots$

And $\sum x = x_1 + x_2 + x_3 + \ldots$

Similarly

$$\mathbf{y = \frac{\sum m\,y}{M}} \qquad (2)$$

The plane geometrical figures (such as T-section, 1-section, L-section etc.) have only areas but no mass the center of gravity of such figures is found out in the same way as that of solid bodies. Therefore the above two equations will become

$$\mathbf{x = \frac{\sum a\,x}{A}}$$

Or $$\mathbf{x = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}}$$

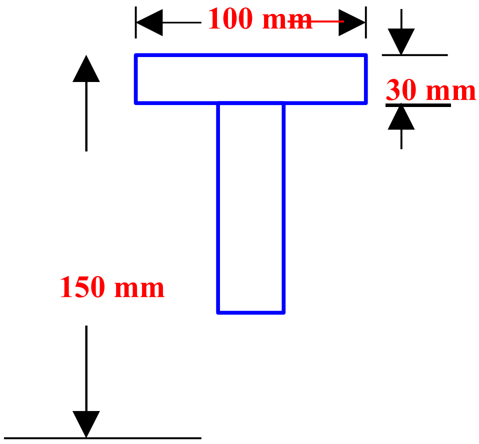
$$\mathbf{y = \frac{\sum a\,y}{A}}$$

Or $$\mathbf{y = a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots}$$

$\mathbf{a_1 + a_2 + a_3 + \ldots\ldots\ldots}$

EXAMPLE 4

Find the center of gravity of a 100 mm x 150 mm x 30 mm T-section. As shown in the fig



Given Height = 150 mm width = 100 mm
thick ness = 30 mm

Required center of gravity = y =?

Working formulae $y = \dfrac{\sum a\,y}{A}$ or $y = \dfrac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots\ldots\ldots}{a_1 + a_2 + a_3 + \ldots\ldots\ldots}$

Solution

#	Body	Area mm²	Distance (y) mm	Area x y
1	Rectangular ABCD	a_1 = 100 x 30 = 3000	30/2 = 15	3000 x 15 = 45
2	Rectangular EFGH	a_2 = (150 – 30) x 30 = 3600	150-30/2 = 135	3600 x135= 48
		∑ = 9600		∑ = 531000

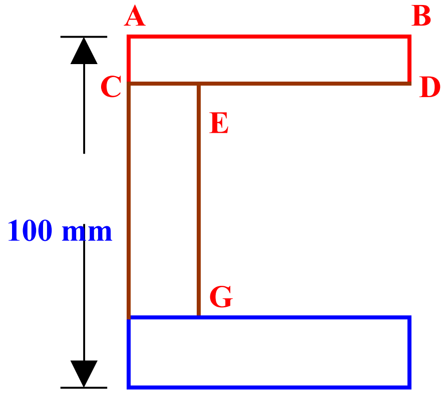
Put in the working formula

$$y = \frac{\sum a\,y}{A} = \frac{531000}{9600} \qquad Y = 94.09 \text{ mm}$$

Result center of gravity = 94.09 mm

EXAMPLE 2

Find the center of gravity of a channel section 100 mm x 50mm x 15 mm.



Solution Consider the rectangle ABC

Area = a_1 = 50 x 15 = 750 mm² x_1 = 50 / 2 = 25 mm

Consider the rectangle CEFG

Area = a_2 = (100 -15-15) x 15 = 1050 mm²

x_{21} = 15 / 2 = 7.5 mm

Consider the rectangle FHIJ

Area = a_3 = 50 x 15 = 750 mm² x_3 = 50 / 2 = 25 mm

Put the values in the working formula

$$x = \frac{a_1\,x_1 + a_2x_2 + a_3\,x_3}{a_1 + a_2 + a_3} = \frac{750 \text{ x } 25 + 1050 \text{ x } 7.5 \text{ x } 750 \text{ x } 25}{25 + 7.5 + 25}$$

x = 17 .8 mm

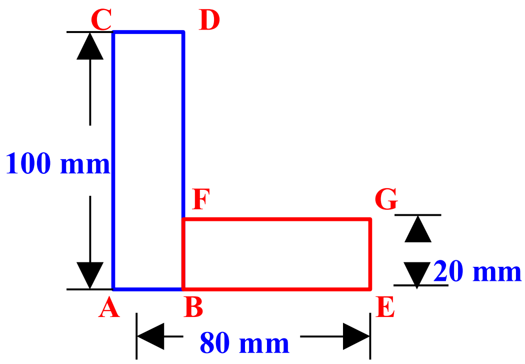
Result **Center of gravity = 17.8 mm**

CENTRE OF GRAVITY OF UNSYMMETRICAL SECTIONS

Sometimes, the given section, whose center of gravity is required to be found out, is not symmetrical either about x-axis or y-axis. In such cases, we have to find out both the values of center of gravity of x and y which means with reference to x axis and y axis

EXAMPLE 3

Find the centroid of an unequal angle section 100 mm x 80 mm x 20mm.



Required center of gravity =?

Working formula $x = \dfrac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$

$$y = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

#	Body	Area mm²	Distance (x) mm	Distance (y)
1	Rectangular ABCD	$a_1 = 100 \times 20 = 2000$	$x_1 = 20/10 = 10$	$y_1 = 100/2 = 50$

2	Rectangular BEFG	$a_2 = (80 - 20) \times 20 = 1200$	$x_2 = 20 - 60/2 = 50$	$y_2 = 20/2 = 10$

Put the value in the first working formula

$$x = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{10 + 60} \qquad x = 25 \text{ mm}$$

$$y = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{10 + 60} \qquad y = 35 \text{ mm}$$

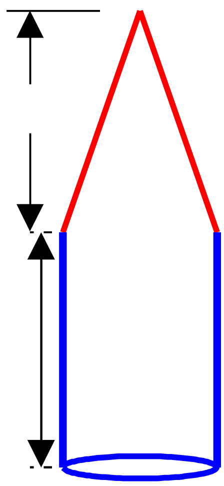
Result **x = 25 mm** **y = 35 m**

CENTRE OF GRAVITY OF SOLID BODIES

The center of gravity of solid bodies (such as hemisphere, cylinder, right circular solid cone etc) is found out in the same way as that of the plane figures. The only difference between the plane and solid bodies is that in the case of solid bodies we calculate volumes instead of areas

EXAMPLE 4

A solid body formed by joining the base of a right circular cone of height H to the equal base of right circular cylinder of height h. calculate the distance of the center of gravity of the solid from its plane face when H = 120 mm and h = 30 mm



Given cylinder height = h = 30 mm
Right circular cone = H = 120 mm

Required center of gravity =?

120 mm

Working formula

$$y = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2}$$

Solution

Consider the cylinder
30 mm

Volume of cylinder = π x r^2 x 30 = 94.286 r^2

C.G of cylinder = y_1 = 30/2 = 15mm

Now consider the right circular cone

Volume of cone = π/3 x r^2 x 120 = 377.143 r^2

C.G of cone = y_2 = 30 + 120/4 = 60 mm

Put the values in the formula

$$y = \frac{v_1 y_1 + v_2 y_2}{v_1 + v_2} = \frac{94.286 r^2 \times 15 + 377.143 r^2 \times 60}{94.286 r^2 + 377.143 r^2}$$

y = 40.7 mm

Result center of gravity = 40.7 mm

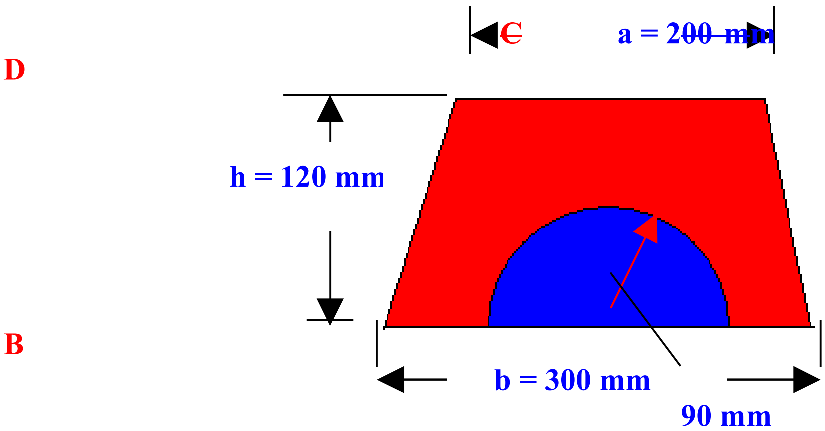
CENTRE OF GRAVITY OF SECTIONS WITH CUT OUT HOLES

The center of gravity of such a section is found out by considering the main section; first as a complete one and then deducting the area of the cut out hole that is taking the area of the cut out hole as negative. Now substituting the area of the cut out hole as negative, in the general equation for the center of gravity, so the equation will become

Or $$y = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

EXAMPLE 5

A semicircles of 90 mm radius is cut out from a trapezium as shown in fig find the position of the center of gravity



Given

Trapezium ABCD

b = 300 mm a = 200 mm h = 120 mm

Semicircle radius = r = 90 mm

Working Formula $y = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$

Solution

Area of trapezium = $\frac{a + b}{2} \times h = \frac{200 + 300}{2} \times 120 = 30000$ mm²

centre of gravity of trazezium = $y_1 = \frac{h}{3} = \left[\frac{b + 2a}{b + a}\right]$
4

$y_1 = \frac{120}{3}\left[\frac{300 + 2 \times 200}{300 + 200}\right] = 56$ mm
4

Area of semicircle = $\frac{\text{area of the circle}}{2} = \frac{\pi}{2} r^2 = \frac{\pi}{2} 90^2 =$ 89100 mm²

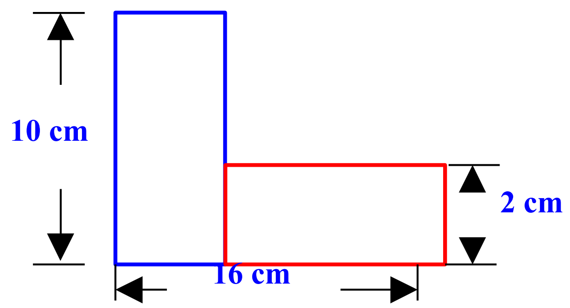
Center of gravity of the semicircle = $\frac{4r}{3\pi} = \frac{4\ 90}{3\pi} = 38.183$

Put the values in working formula

$$y = \frac{30000 \times 56 - 89100 \times 38.183}{30000 - 89100}$$

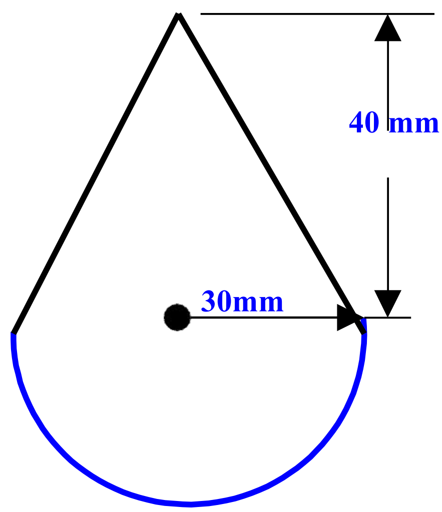
Result **Center of the gravity = 69.1 mm**

1. Find the center of gravity of an unequal angle section 10 cm x 16 cm x 2 cm



Ans: 5.67 mm and 2.67 mm

2. A body consists of a right circular solid cone of height 40 mm and radius 30 mm placed on a solid hemisphere of radius 30 mm of the same material find the position of the center of gravity of the body

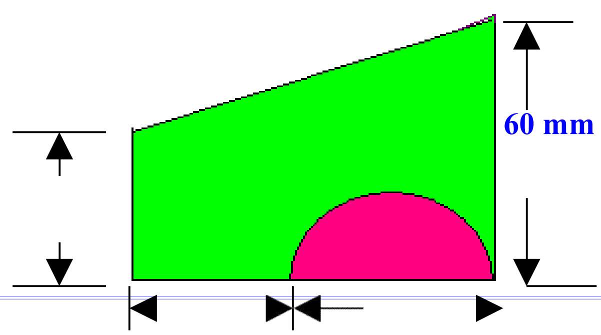


Ans: 28.4 mm

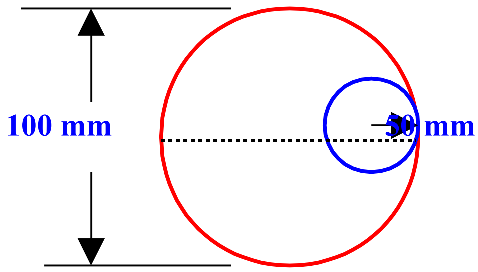
3. A hemisphere of 60 mm diameter is placed on the top of the cylinder having 60 mm diameter. Find the center of gravity of the body from the base of the cylinder if its height is 100 mm.

Ans: 60.2 mm

4. A semicircular area is removed from a trapezium as shown in fig determine the position of the center of gravity

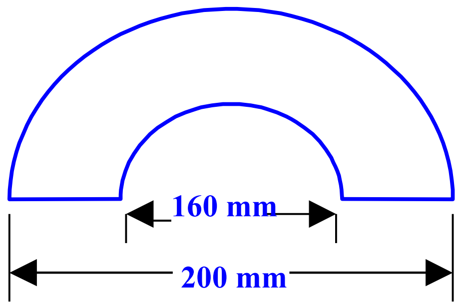


5. A circular hole of 50 mm diameter is cut out from a circular disc of 100 mm diameter as shown in fig find the center of gravity of the section



Ans: 41.7 mm

6. Find the center of gravity of a semicircular section having outer and inner diameters of 200 mm and 160 mm respectively as shown in fig.



Ans: 57.5 mm

UNIT 3 - MCQ

Questions	opt1	opt2	opt3	opt4	answer
The point, through which the whole weight of the body acts, is known as ---------------	centre of gravity	moment of inertia	normal	force	centre of gravity

ENGINEERING MECHANICS					
The point, which the total area of the plane figure is assumed to be concentrated is known as -------------- of that area	moment of inertia	centroid	force	moment	centroid
The centriod & centre of gravity are at the -----------------	different point	normal	same point	equal	same point
The centre of gravity of a uniform rod lies at a --------------	different point	same point	end point	middle point	middle point
The C.G. of a triangle lies at a point where the three medians of a triangle -------------	meet	same point	cross	normal	meet
The C.G. of a parallelogram or a rectangle is at a point where its diagonal meet -----------------	same point	different point	each other	point	each other
The C.G. of a circle lies at its --------------	end point	diameter	centre	area	centre
The C.G. of a body consisting of different areas is given by ---------------	a_1x_1/a_1	$a_1x_1+a_2x_2+$----- / $(a_1+a_2+$----$)$	$a_1x_{1+}a_1$	$a_1x_{1-}a_1$	$a_1x_1+a_2x_2+$----- / $(a_1+a_2+$-----$)$
If a given section is symmetrical about x-x axis or y-y axis, the C.G. of the section will lie on the ---------------	Axis symmetry	centre	end	area	Axis symmetry
The ------------------ of an area about an axis is the product of area & square of the distance of the C.G. of the area from that axis.	moment of inertia	moment	area	force	moment of inertia
The ---------------- of a body is the distance from an axis of reference where the mass of the given body is assumed to be concentrated.	moment	moment of inertia	radius	radius of gyration	radius of gyration
Radius of gyration ---------------------	$\sqrt{(I/E)}$	I/A	$\sqrt{(I/A)}$	A/I	$\sqrt{(I/A)}$
According to theorem of perpendicular axis ---------------	Izz = Ixx + Iyy	$I_{AB}= I_G +Ah^2$	Izz = Ixx	Ixx / Iyy	Izz = Ixx + Iyy
According to theorem of parallel axis ---------------	Izz = Ixx + Iyy	$I_{AB}= I_G +Ah^2$	Ixx / Iyy	Izz = Ixx	$I_{AB}= I_G +Ah^2$
Moment of inertia of a rectangular section about an horizontal axis passing through base =-------------------	bd/12	$bd^3/12$	bd/6	$bd^3/6$	$bd^3/12$
Moment of inertia of a circular section = --------------------------	$\pi D/64$	$\pi D^3/64$	$\pi D^4/64$	$\pi D^3/6$	$\pi D^4/64$
Moment of inertia of a triangular section about the base ---------------	$bd^3/12$	bh/12	$bh^3/6$	$bh^3/12$	$bh^3/12$
Moment of inertia of a triangular section about an axis passing through C.G. & parallel to the base ---------------	$bh^3/12$	$bh^3/36$	$bh^2/36$	bh/12	$bh^3/36$
The C.G. of an area by integration method is given by -----------------	$\int x^*.\, dA/ \int dA$	$x^*/\int dA$	x/dA	$dA/\int dA$	$\int x^*.\, dA/ \int dA$

ENGINEERING MECHANICS					
The C.G. of a straightt or curved line is given by =----------------	∫x*. dA/ ∫dA	∫y*. dA/ ∫dA	y*/∫dA	∫x*. dL/ ∫dL	∫x*. dL/ ∫dL
Surfaces are a --------------------- bodies.	Three dimensional	normal	two dimensional	surface	two dimensional
The solids are ------------------ bodies.	Three dimensional	normal	two dimensional	surface	Three dimensional
For two dimensional bodies -------------- is to be determined.	volume	surface	area	force	area
For three dimensional bodies -------------- is to be determined.	volume	area	equal	force	volume
For rectangular section y = ---------------------	d/2	b/2	x/2	area	d/2
For rectangular section x = ---------------------	d/2	b/2	x/2	area	b/2
Polar moment of inertia Izz = ------------------	Ixx	Iyy	Ixx+Iyy	Ixx-Iyy	Ixx-Iyy
The product of inertia of the plane area is obtained if an elemental area is multiplied by the ------------ of its co-ordinated & is integrated for entire area	product	equal	normal	sum	product
The product of inertia may be positive, negative or ------------------ depending upon distance 'x' & 'y' which could be positive, negative or zero.	equal	zero	2	1	zero
The product of inertia with respect to ------------------- axis will be zero.	vertical	perpendicular	centroidal	horizontal	centroidal
If area is symmetrical with respect to one or both of the axes, the product of inertia will be -------------------------	zero	equal	2		zero
The principal axes are the axes about which the ----------- of inertia is zero.	sum	product	zero	force	product
The moment of inertia is always ---------------	positive	negative	sum	zero	positive
The product of inertia may be ---------------, negative or zero.	negative	positive	sum	zero	positive
The product of inertia	vertical axis	principal axis	perpendicular	normal	principal axis
The product of inertia depends upon the -------------- of the axes.	normal	orientation	equal	perpendicular	orientation
As the product of inertia is zero, about symmetrical axis, hence symmetrical axis is the --------------- of inertia for the area.	vertical	product	principal axis	horizontal axis	principal axis
The mass moment of inertia of the rectangular plate about x-x axis passing through the C.G. of the plate is given by -------------------	$bd^3/12$	$Md^2/22$	$Md^2/12$	$Mb^2/12$	$Md^2/12$

ENGINEERING MECHANICS					
The mass moment of inertia of the rectangular plate about y-y axis passing through the C.G. of the plate is given by ------------------	$bd^3/12$	$Md^2/22$	$Md^2/12$	$Mb^2/12$	$Mb^2/12$
Mass moment of inertia of the rectangular plate about a line passing through the base ------------------	$Md^3/3$	$Md^2/22$	$Md^4/4$	Md/2	$Md^3/3$
Mass moment of inertia of a hollow rectangular plate ---------------------	1/12(Md2) - 1/12 (md1 2)	Md^2	Md/12	$Md^2/12$	1/12(Md2) - 1/12 (md1 2)
Mass moment of inertia of a circular plate ---------------	$Md^2/12$	$MR^2/4$	MR^2	$MD^2/4$	$MR^2/4$
Mass moment of inertia of a hollow circular cylinder ----------------	$M/4\ (Ro^2 + Ri^2)$	$MR^2/4$	MR/4	$Md^2/12$	$M/4\ (Ro^2 + Ri^2)$
Mass moment of inertia of a right circular cone of base Radius R, height H & mass M about its axis -----------------	$3/10\ (MR^2)$	$MR^2/10$	MR/2	$MR^2/2$	$3/10\ (MR^2)$
First moment of area is also called as --------------------	second moment of area	moment of inertia	moment of area	radius of gyration	moment of area
Second moment of area is also called as --------------	Area moment of inertia	moment of area	radius	force	Area moment of inertia
If instead of area, the mass (m) of the body is taken into consideration then the second moment is known as ----------------	moment of inertia	radius	second moment of mass	force	second moment of mass
The second moment of mass is also known as --------------	moment of inertia	force	radius	mass moment of inertia	mass moment of inertia
Centroid of volume is the point at which the total volume of a body is assumed to be -----------------	normal	equal	concentrated	centre	concentrated
The point, through which the whole weight of the body acts, irrespective of its position, is known as	moment of inertia	centre of gravity	centre of percussion	centre of mass	centre of gravity
Pick up the incorrect statement from the following	The C.G. of a circle is at its center	The C.G. of a triangle is at the intersection of its medians	The C.G. of a rectangle is at the inter-section of its diagonals	The C.G. of a semicircle is at a distance of r/2 from the center	The C.G. of a semicircle is at a distance of r/2 from the center
The center of percussion of a solid cylinder of radius r resting on a horizontal plane will be	r/2	2r/3	r/A	3r/2	3r/2
The units of moment of inertia of mass are	kg m2	m4	kg/m2	kg m	kg m2
The center of gravity of a triangle lies at the point of	concurrence of the medians	intersection of its altitudes	intersection of bisector of angles	intersection of diagonals	concurrence of the medians
The center of gravity of a uniform lamina lies at	the center of heavy portion	the bottom surface	the mid point of its axis	all of the given	the mid point of its axis
The units of moment of inertia of an area are	kg m2	m4	kg/m2	kg/m4	m4
The C.G. of a plane lamina will not be at its geometrical centre in the	right angled triangle	equilateral triangle	Square	circle	right angled triangle

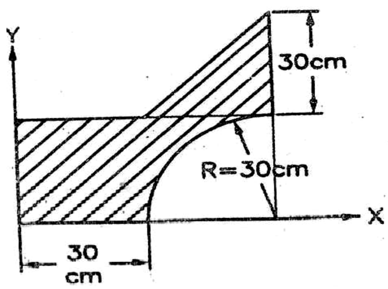
case of a					
The C.G. of a right circular solid cone of height h lies at the following distance from the base	h/2	J/3	h/6	h/4	h/4
The M.I. of hollow circular section about a central axis perpendicular to section as compared to its M.I. about horizontal axis is	same	double	Half	four times	double
The angle which an inclined plane makes with the horizontal when a body placed on it is about to move down is known as angle of	Friction	limiting friction	Repose	static friction	Repose

2 MARKS

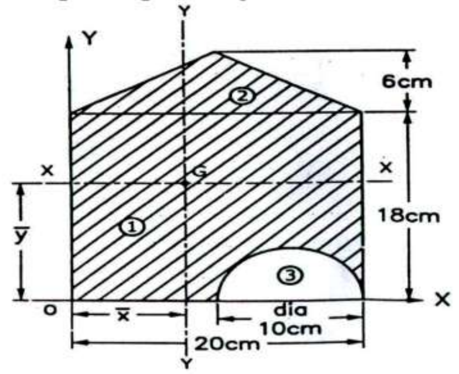
1. State parallel axis theorem.
2. State perpendicular axis theorem.
3. Write down the equations of motion of a particle under gravitation.
4. Define centroid.
5. Define moment of inertia.
6. State the principle of work and energy.
7. Locate the centroid and calculate the moment of inertia about centroidal axes of a semicircular lamina of radius 2m.
8. A semicircular area having a radius of 100 mm is located in the XY-plane such that its Diameter coincides with Y-axis. Determine the X-coordinate of the center.
9. Distinguish between centroid and center of gravity.
10. A stone is projected in space at an angle of 45° to horizontal at an initial velocity of 10 m/sec. Find the range of the projectile.
11. Define triangle law of forces.
12. Explain will you reduce a force into an equivalent force-couple system with an example

14 MARKS

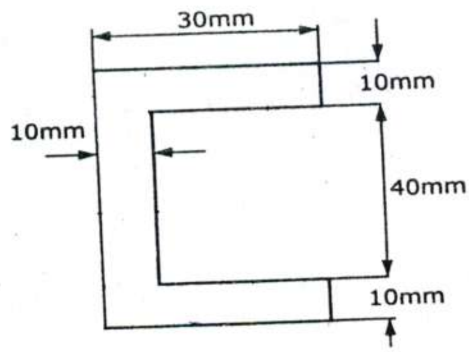
1. Find the MI of an I section about XX and YY axes through its centroid. Dimensions are, Top flange: 150mm x 12mm, Web: 200mm x 10mm, Bottom flange: 150mm x 12mm.
2. Locate the centroid of the shaded area shown in figure.



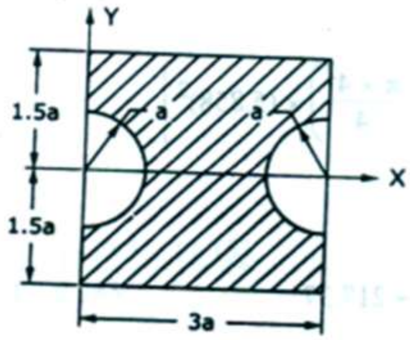
3. Determine Moment of Inertia of the composite plane figure shown about its bottom edge.



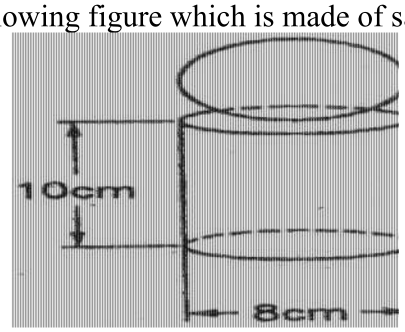
4. Locate the centroid of the section shown in fig.



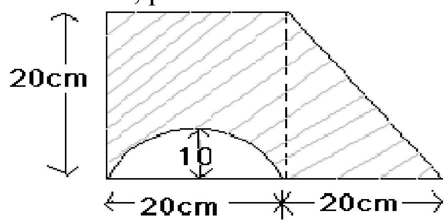
5. Locate the centroid of the area shown in figure. If the value of ‘a’ is taken as 20mm.



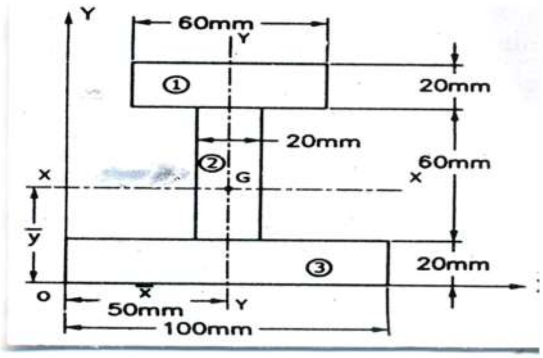
6. Locate the center of gravity of the following figure which is made of same material.



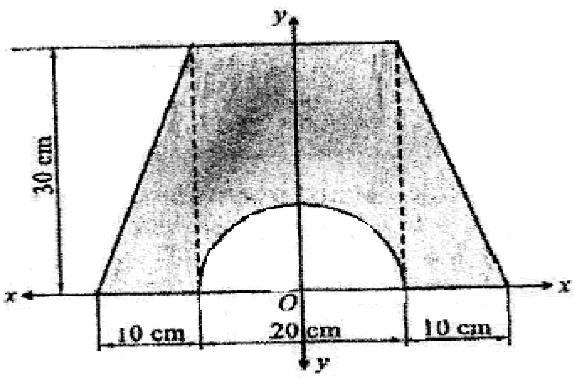
7. Find the moment of inertia for the shaded area, parallel to x – axis.



8. Determine the Moment of Inertia of an I section about its centroidal axes.



9. Locate the centroid of the shaded area shown in figure below. The dimensions are in mm.



UNIT-5: KINEMATICS OF PARTICLES AND FRICTION

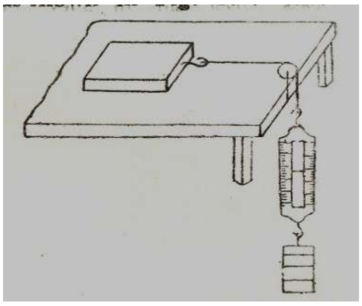
FRICTION

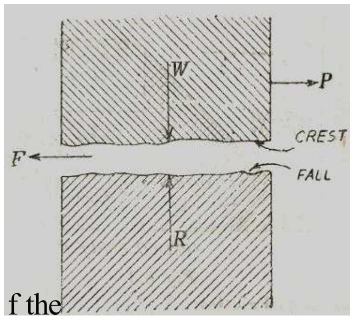
A force which prevents the motion or movement of the body is called friction or force of friction and its direction is opposite to the applied external force or motion of the body. Friction is a force of resistance acting on a body which prevents or retards motion of the body. Or

When a body slides upon another body, the property due to which the motion of one relative to the other is retarded is called friction. This force always acts tangent to the surface at points of contact with other body and is directed opposite to the motion of the body.

Explanation

Consider a block resting on, a horizontal plane surface. Attach a string to one side of the block as shown in Fig.





The other end o f the string is connected to the spring balance. Apply an external force on the balance. Gradually increase the magnitude of the external force. Initially the body will not move and the effect of the applied force is nullified. This is because there acts a force on the block which opposes the motion or movement of the block. The nature of this opposing force is called friction. It depends upon many factors. The major cause of friction is the microscopic roughness of the contact surfaces. No surface is perfectly smooth. Every surface is composed of crests and falls as shown in fig b. It is the interlocking of the crests of one surface into the falls of the other surface which produces the resistance against the movement of one body over the other body. When the force exerted is sufficient to overcome the friction, the movement ensures and the crests are being sheared off. This gives rise to heat and raises the local temperature. This is also the reason of the wear of the contact surfaces. This phenomenon of friction necessitates the presence o fluid film between the two surfaces to avoid wear of surfaces. The process of creating the fluid film is called lubrication.

TYPES OF FRICTION

Friction is of the following two types.

1. Static Friction

It is the friction acting on the body when the body is at the state of rest or the friction called into play before the body tends to move on the surface is called static friction. The magnitude of the static friction is equal to the applied force. It varies from zero to maximum until the movement ensures.

2. Dynamic Friction

It is the friction acting on the body when body is in motion is called dynamic friction. Dynamic friction is also known as kinetic friction. The magnitude of the dynamic friction is constant.

The dynamic friction has two types

i. Sliding Friction ii. Rolling Friction

i. Sliding friction

The sliding friction acts on those bodies, which slide over each other for example the friction between piston, and cylinder will slide friction because the motion of the motion of the piston in cylinder is sliding and there is surface contact between piston and cylinder.

ii. Rolling Friction

The rolling friction acts on those bodies which have point contact with each other for example the motion of the wheel on the railway track is the example of rolling motion and the friction between the wheel and railway track is rolling friction. It is experimentally found that the magnitude of the sliding friction is more than the rolling friction because in the rolling friction there is a point contact rather than surface contact.

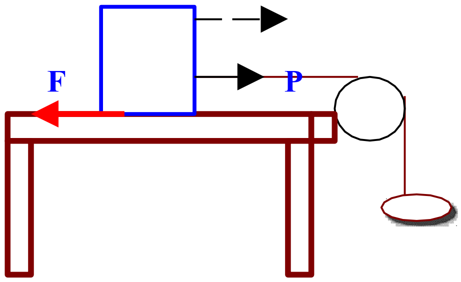
LIMITING FRICTION

The maximum friction (before the movement of body) which can be produced by the surfaces in contact is known as limiting friction

It is experimentally found that friction directly varies as the applied force until the movement produces in the body. Let us try to slide a body of weight w over another body by a force P as shown in fig

Motion of the body

Pan



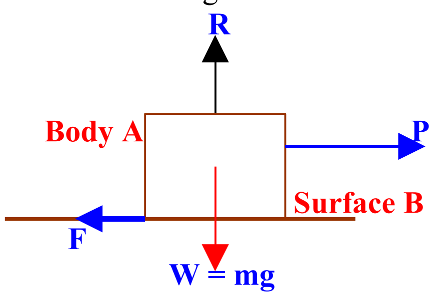
A little consideration will show that the body will not move because the friction F which prevents the motion. It shows that the applied force P is exactly balanced by the force of friction acting in the opposite direction of applied force P. if we increase the force P by increasing the weight in the pan, the friction F will adjust itself according to applied

force P and the body will not move. Thus the force of friction has a property of adjusting its magnitude to become exactly equal and opposite to the applied force which tends to produce the motion.

There is however a limit beyond which the friction cannot increase. If the applied force increases this limit the force of friction cannot balance applied force and body begins to move in the direction of applied force. This maximum value of friction, which acts on body just begin to move, is known as limiting friction. It may be noted that when the applied force is less than the limiting friction the body remains at rest, and the friction is called static friction, which may have any values zero to limiting friction.

<u>NORMAL REACTION</u>

Let us consider a body A of weight “W” rest over another surface B and a force P acting on the body to slide the body on the surface B as shown in fig



A little concentration will show that the body A presses the surface B downward equal to weight of the body and in reaction surface B lift the body in upward direction of the same magnitude but in opposite direction therefore the body in equilibrium this upward reaction is termed as normal reaction and it is denoted by R or N.

Note

It is noted the weight W is not always perpendicular to the surface of contact and hence normal reaction R is not equal to the weight W of body in such a case the normal reaction is equal to the component of weight perpendicular to surface.

CO EFFICIENT OF FRICTION

The ratio of limiting friction and normal reaction is called coefficient of friction and is denoted by μ.

Let R = normal reaction

And F = force of friction (limiting friction)

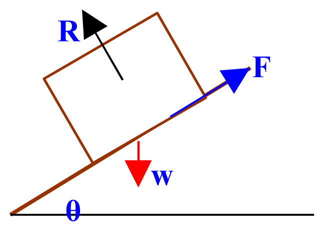
μ = Co efficient of friction

$$\frac{F}{R} = \mu$$

$$\mathbf{F = \mu R}$$

ANGLE OF FRICTION

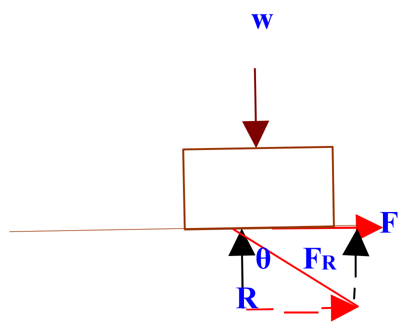
The angle of a plane at which body just begins to slide down the plane is called angle of frication. Consider a body resting on an inclined plane as shown in diagram.



The body is in equilibrium under the Acton of the following forces

1. Weight of the body acting vertically downwards = w
2. Friction force acting along upwards = F
3. Normal reaction acting at right angle to the plane =R

Let the angle of inclination be gradually increased till the body just starts sliding down the plane. This angle of inclined plane at which a body just begins to slide down the plane is called the angle of friction. And it is equal to the angle between normal reaction R and the resultant between frictional force F and normal reaction R



From diagram

$$\text{Tan } \theta = F / R$$

But $F / R = \mu$

Where μ is the co-efficient of friction,

$$\text{Tan } \alpha = \mu$$

LAWS OF FRICTION

These laws are listed below:

1. Laws of Static Friction

1 The force of friction always acts in a direction opposite to that in which the body tends to move.

2 The magnitude of force of static friction is just sufficient to prevent a body from moving and it is equal to the applied force.

3. The force of static friction does not depend upon, shape, area, volume, size etc. as long as normal reaction remains the same.

4. The limiting force of friction bears a constant ratio to normal reaction and this constant ratio is called coefficient of static friction.

2. Laws of Dynamic Friction

1 When a body is moving with certain velocity, it is opposed by a force called force of dynamic friction.

2 The force of dynamic friction comes into play during the motion of the body and as soon as the body stops, the force of friction disappears.

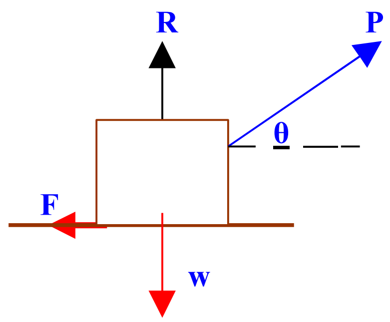
3 The force of dynamic friction is independent of area, volume, shape, size etc. of the body so long the normal reaction remains the same.

However, to some extent it varies with the magnitude of velocity of the body. Force of dynamic friction is high for low speeds and low for very high speeds.

4 The ratio of force of dynamic friction and normal reaction on the body is called coefficient of dynamic friction.

EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE

We know that a body lying on a rough horizontal plane will remain in equilibrium but when ever a force is applied on the body it will tend to move in the direction of force. Consider a body moving on a horizontal Plane under the influence of force P which is inclined at an angle θ to the surface. As shown in fig



Where

w = weight of the body
P = applied force
α = Angle of Repose
F = friction
θ = angle of inclination of the plane the horizontal

Resolve the applied force P into its component that is

Horizontal component = $P \cos\theta$ Vertical component = $P \sin\theta$

Now consider the horizontal & vertical equilibrium condition of the body then

$$F = P \cos\theta \qquad (1)$$

And $$w = R + P \sin\theta \qquad (2)$$

The value of P can be determined by following formula

$$P = \frac{w \sin\alpha}{\cos(\theta - \alpha)}$$

For minimum force P

$$P = W \sin \alpha$$

MOTION OF BODY ON INCLINED PLANE IN UPWARD DIRECTION

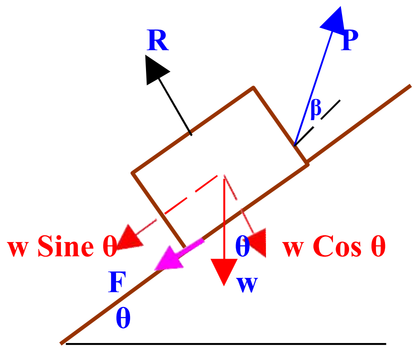
Let

W = weight of the body   P = applied force

α = Angle of Repose   θ = angle of inclination of the plane the horizontal

Now consider the following two cases

Case 1) When angle of inclination of the force to plane is β



Consider the forces acting on body which are parallel to the plane also consider the equilibrium of body

$$P \text{ cosine } \beta = w \sin \theta + F$$

$$P \text{ cosine } \beta = w \sin \theta + \mu R \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_1}$$

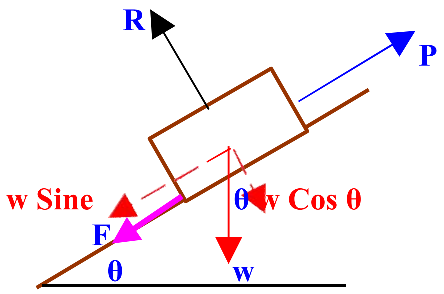
Similarly the forces acting on body normal to the plane and consider the equilibrium condition

$$R + P \sin \beta = w \text{ cosine } \theta \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_2}$$

The magnitude of the force P can be calculated by the following formula

$$P = \frac{W \operatorname{Sin} (\theta + \alpha)}{\operatorname{Cosine} (\beta - \alpha)}$$

Case 2) When the force is parallel to the plane



By considering the equilibrium of the forces parallel and normal to the plane we have

P = w Sine θ + F

P = w Sine θ + μR____________________1

And R = w Cosine θ______________________2

The force P can be calculated by the following formula

$$P = \frac{W \operatorname{Sin} (\theta + \alpha)}{\operatorname{Cos} \alpha}$$

Motion of body on Inclined plane in downward direction

Let

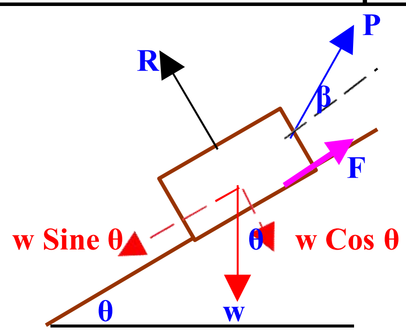
W = weight of the body P = applied force

θ = angle of inclination of the plane the horizontal

α = Angle of Repose β = angle of force P

Now consider the following two cases

Case 1 When angle of inclination of the force to plane is β



Now consider the forces acting parallel to the plane also the equilibrium of forces

P cosine β + F = w sin θ

P cosine β +μR = w sin θ__________________1

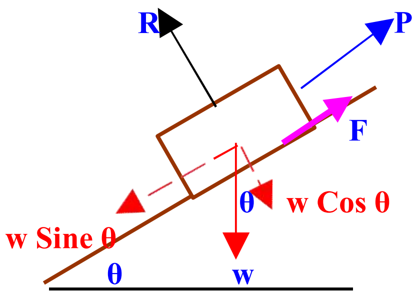
Similarly consider the force normal to the plane

R + P sin β = w cos θ__________________2

The magnitude of the force P can be calculated by the following formula

$$P = \frac{W \sin(\theta - \alpha)}{\cos(\beta - \alpha)}$$

<u>Case 2 when the force is parallel to the plane</u>



From diagram we have

$P + F = w \sin\theta$

$P + \mu R = w \sin\theta$ ____________________ 1

Similarly $R = w \cos\theta$ ____________________ 2

The force P can be calculated by following formula

$$P = \frac{W \sin(\theta - \alpha)}{\cos \alpha}$$

<u>EQUILIBRIUM OF LADDER</u>

A ladder is a device which is used to climb up or down to the roof or walls. It consists of two long uprights and number of rungs which makes the steps of the ladder.

Consider a ladder which is resting on ground and leaning against walls as shown in the fig. Let

L = Length of ladder

w_1 = Weight of ladder acts at middle of the ladder

w_2= Weight of man climbing up acts at the distance x from the lower end

μ_f = co efficient of friction between floor and ladder

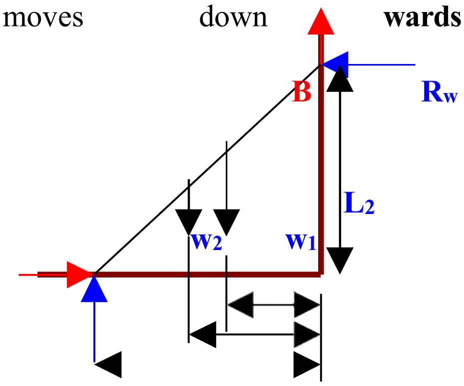
μ_w = co efficient of friction between ladder and wall

Let us suppose ladder slips down wards

F_f = friction produce between floor and ladder towards wall as ladder moves away from the wall.

F_w = friction produce between wall and ladder upwards as ladder moves down **wards**

F_w



A θ ~~C~~

F_f L_4

L_3

L_1

For the sake of convince we consider that the friction at B is zero i.e. the wall is perfectly smooth. Now take the moment about B.

$$R_f \times L_1 = F_f \times L_2 + w_2 \times L_3 + w_1 \times L_4$$

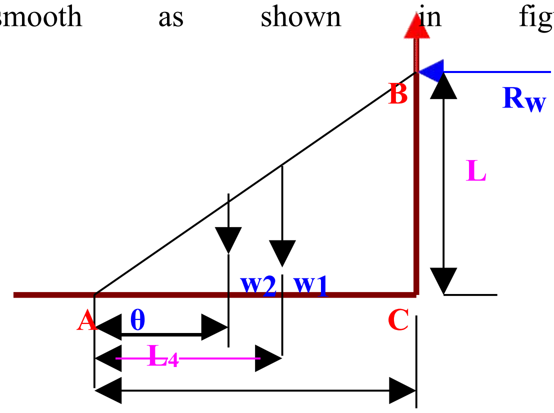
Where $F_f = \mu_f \times R_f$

$$\mathbf{R_f \times L_1 = (\mu_f \times R_f \times L_2) + w_2 \times L_3 + w_1 \times L_4}$$

________________A

Similarly consider the friction at A is zero i.e. the floor is perfectly smooth as shown in figure.

F_W



L_3

L_1

Therefore $R_w \times L_2 = F_w \times L_1 + w_1 \times L_3 + w_2 \times L_4$

Where $F_w = \mu_w \times R_w$

$$\mathbf{R_w \times L_2 = (\mu_w \times R_w \times L_1) + w_1 \times L_3 + w_2 \times L_4}$$

________________A

<u>EXAMPLE 1</u>

A horse exerts a pull of 3 KN just to move a carriage having a mass of 800 kg. Determine the co efficient of friction between the wheel and the ground

Take $g = 10\ m/sec^2$

Given $P = 3\ KN$ Mass $= m = 800\ Kg$ $g = 10\ m/sec^2$

Required co efficient of friction $= \mu = ?$

Working formula $\mathbf{F = \mu R}$

Solution we know that $W = mg$

$$W = 800 \times 10 = 8000\ N$$

A little consideration will show that the weight of the carriage is equal to the normal reaction because that the body is horizontal to the plane as shown in fig

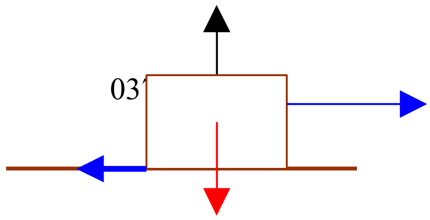
Therefore $W = R$ and $P = F$

R

Put the values in working formula we get

$$300 = \mu\ 8000$$

$$\mu = 03$$



P

Result co efficient of friction = 0.375

F

w = mg

EXAMPLE 3

A body of mass 100 Kg rests on horizontal plane the co efficient of friction between body and the plane 0.40. Find the work done in moving the body through a distance of 20 m along the plane.

Given $m = 100$ Kg $\mu = 0.40$ $d = 20$ m

Required work done =?

Working formula 1 $W = F \times d$

2 $Fs = \mu R$

Solution we know that $R = W = mg$

$R = W = 10 \times 9.81 = 98.1$ N

Put the values in 2nd working formula we get

$Fs = 0.40 \times 98.1$

$Fs = 39.24$ N

Now put the values in 1st working formula

$W = 39.24 \times 20$

$W = 748.8$ N

Resultant **weight = 748.8 N**

EXAMPLE 4

A weight of 50 N is resting on the horizontal table and can be moved by a horizontal force of 20 N. Find the co efficient of friction, the direction and magnitude of the resultant between normal reaction and frictional force

Given $W = 50$ N $P = 20$ N

Required co efficient of friction = μ =?

Direction = θ =?

Resultant = S =?

Working formula 1 $F = \mu R$

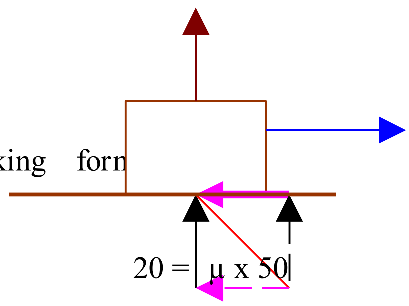
w = 50 N

2 $S = \sqrt{R^2 + Fs^2}$

3 $\tan\theta = \mu$

Solution put the value in 1st working form

P = 20 N



$Fs = \mu R$

F

R

$20 = \mu \times 50$

$\mu = 0.4$

put the value in the 2nd working formula
S

$$S = \sqrt{50^2 + 20^2}$$
$$\mathbf{S = 53.85\ N}$$

Put the value in the 3rd working formula

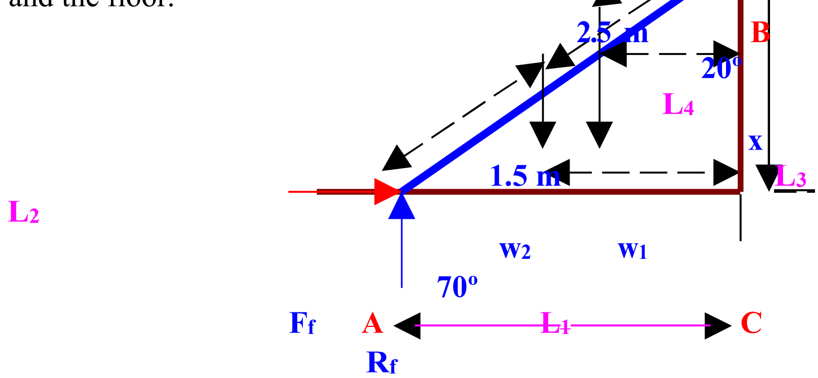
$$\tan\theta = \mu$$

$$\tan\theta = 0.4$$
$$\theta = 21.801^\circ$$

Result Co efficient of friction = μ = 0.4
Direction = θ = 21.801º
Resultant = S = 53.85 N

EXAMPLE 5

A ladder 5 m long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70º with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750 N stands on a rung 1.5 m from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor.



Given Length of leader = L = 5 m weight of leader = w_1 = 900 N

Weight of man = w_2 = 750 N inclination of leader = θ = 70º

Distance covered by man from bottom = 1.5 m

Required coefficient of frication between ladder and floor = μ_f =?

Working formula $R_f \times L_1 = (\mu_f \times R_f \times L_2) + w_2 \times L_3 + w_1 \times L_4$

Solution we know that

$$R_f = w_1 + w_2$$
$$R_f = 900 + 750$$
$$R_f = 1650\ N$$

We can calculate L_1, L_2, by considering the geometry of the figure. Now consider the triangle ABC

$\cos 70 = L_1/L = L_1/5$ $\quad L_1 = 1.7101$ m

And $\quad \sin 70 = L_2/L = L_2/5$ $\quad L_2 = 4.698$ m

Similarly we can calculate the L_3 & L_4 by considering the geometry of the figure

$\sin 20 = L_4/2.5$ $\quad L_4 = 0.85$ m

And $\quad \sin 20 = L_3/5-1.5$ $\quad L_3 = 1.197$ m

Put the values in the working formula to calculate the coefficient of friction between the floor and ladder

$$R_f \text{ x } L_1 = (\mu_f \text{ x } R_f \text{ x } L_2) + w_2 \text{ x } L_3 + w_1 \text{x } L_4$$

$$1650 \text{ x } 1.7101 = \mu_f \text{ x } 1650 \text{ x } 4.698 + 750 \text{ x } 1.197 + 900 \text{ x } 0.85$$

$\mu_f = 0.149$

Resultant **Coefficient of friction = μ_f = 0.15**

ENGINEERING MECHANICS					
Questions	opt1	opt2	opt3	opt4	answer
Force of friction always acts in the direction -------------- to the direction of motion.	equal	same	opposite	normal	opposite
The max value of frictional force acting on a body, when the body is on the point of motion is called ---------------	limiting force of friction	moment	mass	resistance	limiting force of friction
The force of friction acting on a body when the body is moving is called ----------------	static friction	dynamic friction	force	moment	dynamic friction
The ratio of limiting force of friction to the normal reaction between two bodies is known as ----------------	friction	dynamic friction	coefficient of friction	force	coefficient of friction
The angle made by the resultant of the normal reaction & the limiting force of friction with the normal reaction is known as --------------------	friction	coefficient of friction	force	angle of friction	angle of friction
The relation between angle of friction & coefficient of friction is expressed as -----------	$\tan \varphi=\mu$	$\tan \mu = \varphi$	$\tan^{-1}\varphi=\mu$	$\varphi=\mu$	$\tan \varphi=\mu$
If a ladder is leaniing against a smooth vertical wall, the force of friction between ladder & vertical wall will be --------------	$\tan \varphi$	$\cos\varphi$	$\cos\theta$	0	0
if a body is placed on a rough inclined plane & the angle of inclination of the plane is gradually increased, till the body just starts sliding down the -------------	plane	normal	end	vertical	plane
The angle of the inclined plane at which the body just begins to slide down the plane, is called ----------------	plane	angle	angle of repose	normal	angle of repose
Angle of repose is equal to --------------------	angle of friction	angle	normal angle	column angle	angle of friction
If the inclination of the plane, with the horizontal is less than angle of friction, the body placed on the inclined plane will be always in ------------- without any external force	normal	move	friction	equilibrium	equilibrium
The min. force required to drag a body of weight 'W' placed on a rough horizontal plane, when the force is applied at an angle 'θ' with the horizontal is equal to -------------	$W\cos\theta$	$W\sin\theta$	W	$W\tan\theta$	$W\sin\theta$
The angle 'θ' will be equal to ------------- for the least force.	friction	angle of repose	angle	angle of friction	angle of friction
When the body is on the point of moving up the plane, P = ---------------	$W\sin\varphi$	$W\sin(\varphi+\alpha)$	$W. \sin (\alpha+\varphi)/ \cos (\theta-\varphi)$	W	$W. \sin (\alpha+\varphi)/ \cos (\theta-\varphi)$
For the body is on the point of moving down the plane, P=-------------	$W. \sin (\alpha+\varphi)/ \cos (\theta+\varphi)$	$W. \sin (\alpha-\varphi)/ \cos (\theta+\varphi)$	$\sin(\theta+\varphi)$	$\cos(\theta-\varphi)$	$W. \sin (\alpha-\varphi)/ \cos (\theta+\varphi)$
Wedge is a piece of metal or wood which is usually of triangular ----------	trapezoidal	rectangular	square	triangular	trapezoidal

-- in cross section					
Wedge is used either lifting loads for slight adjustments in the position of a body for tighting fits or keys for ---------------	wedge	screw	shafts	keys	shafts
A screw jack is a device used for lifting heavy weights/loads with the help of a small effort applied at its ----------------	handle	screw	wedge	keys	handle
The angle of screw in terms of pitch of the screw & mean diameter of the screw is given by tanα= ---------------	P.π	P/π.d	P.d	π.d	P/π.d
The effort applied horizontally at the mean radius of the screw jack to lift a load 'W' is given by P=----------------	W.tanφ-α)	W.tanφ	W.tan(φ+α)	.tanφ-α)	W.tan(φ+α)
The effort applied at the end of the handle of a screw jack is given by =----------------	wd tan(α+φ)/2L	tan μ = φ	Wd/2L	2L/Wd	wd tan(α+φ)/2L
Torque required to work the jack for lifting a load 'W' is given by -----------------	Wd tan α/2	Wd/2	Wd tanφ/2	Wd tan (α+φ)/2	Wd tan (α+φ)/2
Efficiency of a screw jack for raising a load 'W' is given by ---------------	tanα	.tan(φ+α)	tanα/tanφ	tanα / tan (α+φ)	tanα / tan (α+φ)
The efficiency of the screw jack is ------------- of the weight lifted applied	independent	normal	dependent	force	independent
The efficiency of the screw jack will be max if α= --------------------	45°	φ/2	45°-φ/2	45°- θ/2	45°-φ/2
The max efficiency of a screw jack is given by ----------------	1-sinφ	1+sinφ	1-sinφ / sinφ	1-sinφ / (1+sinφ)	1-sinφ / (1+sinφ)
The efficiency of a machine in terms of ideal & actual effort ----------	Ideal effort X actual effort	Ideal effort/actual effort	Ideal effort	Actual effort	Ideal effort/actual effort
Torque = -------------------	P'Xd/2	P'Xd	P'Xd/3	P'Xd/4	P'Xd/2
The simple screw jack, which consists of a nut, a screw with square threads & a handle fitted to the load of the -----------------	Keys	screw	Jack	threads	screw
When the handle is rotated through one complete turn, ----------------- is also rotated through one turn.	Keys	Jack	threads	screw	screw
The forces normal to the inclined plane R* = ----------------------------------	W.cosα	Wsinα	W.tanα	W(1-sinθ)	W(1-sinθ)
Friction acts -------------- to the surface of contact & depends upon the nature of surface of contact.	Perpendicular	parallel	normal	vertical	parallel
Study of geometry of motion of bodies without considering forces is	Kinematics	Kinetics	Statics	Projectile motion	Kinematics
Motion of a particle along a straight line is called	Curvilinear motion	Rectilinear motion	Projectile motion	Kinematics	Rectilinear motion
Study of geometry of motion of bodies considering forces is	Kinematics	Kinetics	Statics	Projectile motion	Kinetics

ENGINEERING MECHANICS					
The difference in the position of a particle in a given time interval is known as	Acceleration	Velocity	Displacement	Intantaneous velocity	Displacement
SI unit of average velocity is	s/m	m/s^2	m.s	m/s	m/s
SI unit for average acceleration is	s/m^2	m/s^2	$m.s^2$	m/s	m/s^2
Rate of change of velocity is known as	Acceleration	Average velocity	Average acceleration	Intantaneous velocity	Acceleration
Acceleration is a derivative of	Average velocity	Velocity	Average acceleration	Intantaneous velocity	Velocity
In rectilinear motion, when the velocity of the particle is constant, the motion is called as an	Curvilinear motion	uniform velocity	uniform acceleration	Uniform rectilinear motion	Uniform rectilinear motion
The motion of a particle is $s = 2t^3-6t^2+10$. Acceleration of the particle when v =0 is	12 m/s^2	13m/s^2	14m/s^2	15m/s^2	12 m/s^2
In rectilinear motion, when the acceleration of the particle is constant, the motion is called as an	Curvilinear motion	uniformly accelerated rectilinear motion	uniform acceleration	Uniform rectilinear motion	uniformly accelerated rectilinear motion
When the body starts from rest, its initial velocity is	3m/s	2 m/s	0	3 m/s	0
In retardation, the acceleration is	negative	positive	half of acceleration	none of the above	negative
When the body comes to rest, its initial velocity is	3 m/s	2 m/s	0	1 m/s	0
Initial velocity 40 kmph is equal to	22.22 m/s	11.11 m/s	33.33 m/s	5.5 m/s	11.11 m/s
An athlete covers a distance of 100m in 10.59s. The acceleration is	1.883 m/s^2	1.983 m/s^2	1.783 m/s^2	0	1.783 m/s^2
A stone is dropped from the top of a tower 200m high. Time required to strike the ground is	6.38sec	6.48 sec	6.58 sec	6.68 sec	6.38sec
The path of the projectile is a	curve	parabola	hyperbola	Ellipse	parabola
Time taken by the projectile to complete the projectile motion is called as	Trajectory	Projectile	Time of flight	Velocity of projection	Time of flight
Time of flight is given by	usinα / g	3usinα / g	2usinα / g	4usinα / g	2usinα / g
Horizontal range of the projectile is given by	u^2sin2α/g	3usinα / g	2usinα / g	4usinα / g	u^2sin2α/g
Projection angle for maximum range is	30°	45°	60°	10°	45°
Maximum height reached by the projectile is	usinα / g	3usinα / g	2usinα / g	u^2sin2α / 2g	u^2sin2α / 2g
Linear momentum is given by	m.a	$m.a^2$	m.v	$m.v^2$	m.v
SI unit for workdone is	Newtons	Joules	N/m	Kg.m	Joules
SI unit for spring constant is	Newtons	Joules	N/m	Kg.m	N/m
Impulse is given by	F.d	F.t	F/d	F/t	F.t
SI unit for momentum is	Kg.m/s	Kg/s	m/s	m/Kg	Kg.m/s

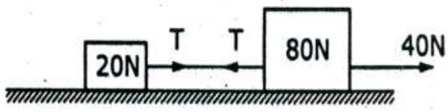
2 MARKS

1. State coulomb's laws of dry friction.
2. A car accelerates uniformly from a speed of 30 km/h to a speed of 75 km/h in 5 s. Determine the acceleration of the car and also the distance travelled during 5 s.

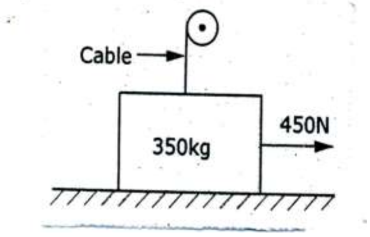
3. How do you find out the resultant force for coplanar non-concurrent parallel force system?
4. Find the unit vector of AB, coordinates A (1, 2, 3) and B (5, 8, 12).
5. The equation of motion of a particle moving in a straight line is given by S = 18t +3t2 –2t3, where S is in meters and t in seconds. Find the velocity and acceleration at starts. Also find time when particle reaches its maximum velocity.
6. Define coefficient of static friction.
7. Define work.
8. Write work energy equation of rigid body. Mention the meaning for all parameters used in the equation.
9. Define cone of friction.
10. A car runs with an initial velocity of 30 m/s and uniform acceleration of 3 m/s2. Find its velocity after 5 seconds.
11. A car starts from rest with a constant acceleration of 4m/sec2. Determine the distance traveled in the 7th second.
12. A body is moving with velocity of 4 m/s. After five seconds the velocity of the body becomes 14m/s. find the acceleration of the body.
13. Define Co-efficient of friction and angle of friction

14 MARKS

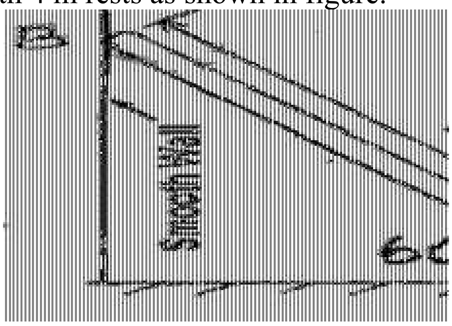
1. Two weights 80N and 20N are connected by a thread and move along a plane under the action of force 40N applied to the first weight of 80N as shown in figure. The co-efficient of friction between sliding surfaces of the weights and the plane is 0.3. Determine the acceleration of the weights and the tension in the thread.



2. A man can pull horizontally with a force of 450N. A mass of 350 kg is resting on a horizontal surface for which the co-efficient of friction is 0.20. The vertical cable of a crane is attached to the top of the block as shown in fig. What will be the tension in the cable if the man is just able to start the block to the right?

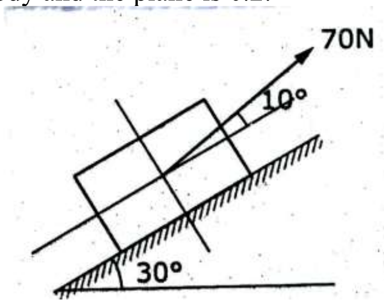


3. A ladder of weight 1000 N and length 4 m rests as shown in figure.

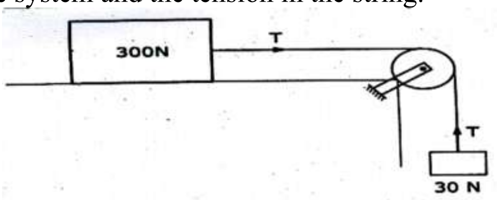


If a 750 N weight is applied at a distance of 3 m from the top of ladder, it is at the point of sliding. Determine the coeffcient of friction between ladder and the floor.

4. Determine the total work done on a 5 kg body, which is pulled 6m up on a rough inclined plane as shown in fig. Take co-efficient of kinetic friction between the body and the plane is 0.2.

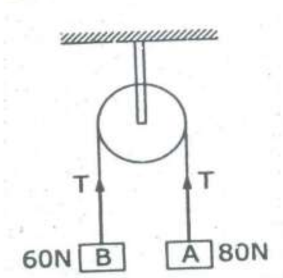


5. The figure shows a body of weight 300N on a smooth horizontal plane is attached by a string to a 30N weight, which hangs vertically. Find the acceleration of the system and the tension in the string.

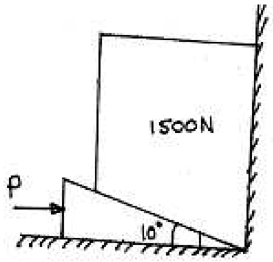


6. A uniform ladder of weight 1000KN and of length 4m rest on a smooth vertical wall. The ladder makes an angle of 60° with horizontal. When a man of weight 750 N stands on a ladder at a distance 3m from the top of the ladder is at the point of sliding. Determine the co-efficient of friction between the ladder and the floor.

7. Two blocks A and B of weight 80N and 60N are connected by a string passing through a smooth pulley as shown in fig. Calculate the acceleration of the body and the tension in the string. Use Newton's laws of motion.



8. A block overlying a 10° wedge on a horizontal floor and leaning against a vertical wall and weighing 1500N is to be raised by applying a horizontal force to the wedge. Assuming the coefficient of friction to be 0.3, determine the minimum horizontal force to be applied to raise the block. As shown in the Figure.



9. A 7m long ladder rests against a vertical wall, which it makes an angle of 45° and on a floor. If a man whose weight is one half that of the ladder climbs it, at what distance along the ladder will he be, when the ladder is about to slip? Take coefficient of friction between the ladder and the wall is 0.33 and that between the ladder and the floor is 0.5.