17BEME304- FLUID MECHANICS AND MACHINERY

Objective

- To enrich the understanding of fluid properties
- To make the students conversant with types of flow and calculate Major and minor loses in pipes.
- To acquaint the student with the concepts of Buckingham's π theorem.
- To explain the working of different pumps
- To explain the working of different turbines.
- To equip students with skills to produce analytical solutions to various simple problems

Outcome

- Demonstrate basic knowledge of fluid properties
- Find types of flow and calculate Major and minor loses in pipes.
- Apply Buckingham's π theorem for problem solving.
- Understand the working of different pumps
- Understand the working of different turbines.
- produce analytical solutions to various simple problems

UNITI FLUID PROPERTIES AND FLOW CHARACTERISTICS 12

Fluid properties: Mass density, weight density, specific gravity, viscosity, compressibility, surface tension and capillarity. Buoyancy and floatation– metacentre and metacentric height (definition only)

Flow characteristics: concepts of system and control volume, application of control volume to continuity equation, energy equation, momentum equation and moment ofmomentum equation.

UNITII FLOW THROUGH CIRCULAR PIPES 12

Hydraulic and energy gradient – Types of fluid flow – Laminar flow through circular conduits – Boundary layer concepts – types of boundary layer thickness – Darcy Weisbach equation –friction factor – Moody diagram – commercial pipes – minor losses – Flow through pipes in series and parallel.

UNITIII DIMENSION ALANALYSIS 12

Dimension and units, dimensional homogeneity, applications of Buckingham's π theorem, model and similitude, similaritylaws.

UNITIV HYDRAULIC TURBINES 12

Classification of turbines – heads and efficiencies – velocity triangles.Axial, radial and mixed flow turbines. Pelton wheel, Francis turbine and Kaplan turbines- working principles - work done by water on the runner – draft tube. Specific speed - unit quantities – performance curves for turbines – governing of turbines.

UNITV HYDRAULIC PUMPS 12

Classification of pumps – centrifugal pump–working principle–head, discharge, efficiencies and losses – performance curves – specific speed. Reciprocating pump–components and working–slip–indicator diagram – air vessel – Jet pump – Gear pump – Submersible pump.

TOTAL 60

TEXT BOOKS

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Streeter V.L, Wylie E.B	Fluid Mechanics	McGraw–Hill, New Delhi	1998
2	Kumar K.L	Engineering Fluid Mechanics	S. Chand	2010

REFERENCES

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Bansal. R.K	Fluid Mechanics and Hydraulics Machines	Laxmi publications (P) Ltd, New Delhi	2015
2	White. F.M	Fluid Mechanics	Tata McGraw–Hill, New Delhi	2010
3	Fox and McDonald	Fluid Mechanics	John Wiley	2015

WEB REFERENCES

1. www.imeche.org
2. openlibrary.org
3. nptel.iitg.ernet.in
4. www.tecquipment.com

KARPAGAM ACADEMY OF HIGHER EDUCATION
COIMBATORE – 21
FACULTY OF ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING

LECTURE PLAN

Subject Name : **Fluid Mechanics & Fluid Machinery**
Subject Code : **17BEME304**
Name of the Faculty : **Mr.R.Karuppasamy**
Designation : **Assistant Professor**
Year/Semester/Section : **II / IV / A**
Branch : **B.E - Mechanical Engineering**

SL. NO.	LECTURE DURATION	TOPICS TO BE COVERED	SUPPORT MATERIALS
		UNIT-I FLUID PROPERTIES AND FLOW CHARACTERISTICS	
1.	1	Introduction to fluid & its types – History behind, introduction to fluid mechanics & its classification, applications, units & measurement	T[3]
2.	1	Fluid properties – Density (mass & weight), Specific volume, Specific gravity or relative density	T[3]
3.	1	Velocity variation near a solid boundary (velocity profile), fluid properties – Viscosity (dynamic & kinematic)	T[3]
4.	1	**Tutorial 1:** Numerical problems from fluid properties (Density, specific volume, specific gravity, viscosity)	T[3]
5.	1	Fluid properties – Adhesion, cohesion, surface tension, capillarity, compressibility (bulk modulus), vapour pressure & cavitation	T[3]
6.	1	Pressure (atmospheric, vacuum & gauge) & its measurement, Pascal's Law	T[3]
7.	1	Buoyancy & Centre of Buoyancy, Meta-Centre and Meta-Centric Height (Definition Only)	T[3]
8.	1	System, Control volume and its application	T[3]
9.	1	Energy Equation, Continuity equation & Momentum Equation	T[3]
10.	1	Moment of Momentum Equation	T[3]
11.	1	**Tutorial 2:** Numerical problems from fluid properties (surface tension, capillarity, compressibility) & Pressure measurement	T[3]
		Total no. of Hours planned for unit – I	**11**

SL. NO.	LECTURE DURATION	TOPICS TO BE COVERED	SUPPORT MATERIALS
UNIT – II FLOW THROUGH CIRCULAR PIPES			
12.	1	Introduction to fluid Kinematics, Types of fluid flow (classification),	**T[3]**
13.	1	Laminar flow through circular tubes (Hagen Poiseulle's), Hydraulic and energy gradient	**T[3**
14.	1	Boundary layer concepts (boundary layer flows, boundary layer thickness, boundary layer separation, drag and lift coefficients)	**T[3]**
15.	1	Types of boundary layer thickness	**T[3]**
16.	1	(Darcy Weisback's equation), pipe roughness, friction factor, Moody's diagram	**T[3]**
17.	1	**Tutorial 3:** Numerical problems from pipe roughness and friction factor & boundary layer thickness.	**T[3]**
18.	1	Flow through circular conduits – Compound pipes.	**T[3]**
19.	1	Flow through Parallel pipe. Head loss due to minor losses in flow through pipes (Sudden expansion, sudden contraction)	**T[3]**
20.	1	Head loss due to minor losses in flow through pipes (Obstacles, inlet, exit, bend)	**T[3]**
21.	1	**Tutorial 4:** Numerical problems from Flow through pipes in series & parallel & Head loss due to minor losses.	**T[3]**
Total no. of Hours planned for unit – II			**10**

SL. NO.	LECTURE DURATION (Hr)	TOPICS TO BE COVERED	SUPPORT MATERIALS
UNIT-III DIMENSIONAL ANALYSIS			
22.	1	Introduction to Dimensional analysis, Dimension and units	**T[3]**
23.	1	Secondary Quantities, Dimensional homogeneity	**T[3]**
24.	1	Methods of Dimensional Analysis	**T[3]**
25.	1	Buckingham's π Theorem - Method of selecting repeating variables, Buckingham's π Theorem – Procedure for solving problems	**T[3]**
26.	1	**Tutorial 5:** Numerical problems from Buckingham's π Theorem	**T[3]**
27.	1	Model and its analysis, Similitude and its types	**T[3]**
28.	1	Model Laws or Similarity laws – Reynold's Model law	**T[3]**

29.	1	Similarity laws – Froude & Euler's laws	T[3]
30.	1	Similarity laws – Weber Model and Mach Model law	T[3]
31.	1	**Tutorial 6:** Numerical problems from Similarity laws	T[3]
Total no. of Hours planned for unit – III			**10**

SL. NO.	LECTURE DURATION (Hr)	TOPICS TO BE COVERED	SUPPORT MATERIALS
UNIT-IV HYDRAULIC TURBINES			
32.	1	Introduction to Fluid machines, definition and classification, exchange of energy, Euler's equation for turbo machines	T[3]
33.	1	Construction of velocity vector diagrams, head and specific work, Components of energy transfer, degree of reaction	T[3]
34.	1	Introduction to Hydro Turbines, Classification of hydro Turbines – impulse, reaction turbine (Pelton wheel turbine, Francis turbine, Propeller turbine , Kaplan turbine)	T[3]
35.	1	Velocity triangles, Work done, Specific speed, efficiencies, Performance curve for turbines	T[3]
36.	1	Pelton wheel turbine (Working principles, Velocity triangles, Work done, Specific speed, efficiencies, Performance curve for turbines)	T[3]
37.	1	**Tutorial 7:** Numerical problems from Pelton wheel turbine	T[3]
38.	1	Francis turbine (Working principles, Velocity triangles, Work done, Specific speed, efficiencies, Performance curve for turbines)	T[3]
39.	1	Propeller turbine (Working principles, Velocity triangles, Work done, Specific speed, efficiencies, Performance curve for turbines)	T[3]
40.	1	Kaplan turbine (Working principles, Velocity triangles, Work done, Specific speed, efficiencies, Performance curve for turbines)	T[3]
41.	1	**Tutorial 8:** Numerical problems from Propeller turbine, Kaplan turbine	T[3]
Total no. of Hours planned for unit – IV			**10**

SL. NO.	LECTURE DURATION (Hr)	TOPICS TO BE COVERED	SUPPORT MATERIALS
UNIT-V HYDRAULIC PUMPS			
42.	1	Introduction to Hydraulic Pumps, Classification of Pumps	T[3]

		(rotodynamic and positive displacement)	
43.	1	Introduction to rotodynamic pump – radial flow - Centrifugal pump – construction and working principle	**T[3]**
44.	1	Centrifugal pump – velocity triangle - Performance characteristic curves (head, discharge, efficiencies and losses) and Specific speed	**T[3]**
45.	1	**Tutorial 9:** Numerical problems from Centrifugal pump	**T[3]**
46.	1	Introduction to positive displacement pumps - Reciprocating pump construction and working principle	**T[3]**
47.	1	Reciprocating pump – Slip, indicator diagram, air vessel	**T[3]**
48.	1	**Tutorial 10:** Numerical problems from reciprocating Pump	**T[3]**
49.	1	Rotodynamic pump - Jet pump - construction, Working, advantages, disadvantages	**T[3]**
50.	1	Rotodynamic pump - Submersible pump - construction, working principle, advantages, disadvantages	**T[3]**
51.	1	Positive displacement pump - Gear pump - construction, working principle, advantages, disadvantages	**T[3]**
52.	1	Pervious year question discussion	**T[3]**
Total no. of Hours planned for unit – V			**11**

SUGGESTED READINGS:

1. Victor L Streeter, E. Benjamin Wylie and K.W. Bedford, Fluid Mechanics, 9edition, McGraw–Hill,NewDelhi,2010.
2.Prof. Kumar K.L, Engineering Fluid Mechanics,1stEdition, S. Chand publishers, 2016
3. Bansal. R.K, A Text book of Fluid Mechanics and Hydraulics Machines, 10th edition, Laxmi publications(P)Ltd,NewDelhi,2018.
4.White.F.M, Fluid Mechanics, 8thedition, Tata McGraw–Hill, New Delhi, 2016
5. Fox and McDonald, Fluid Mechanics, 8thediton, John Wiley, 2015

WEBSITE REFERENCES:

W[1]. http://nptel.ac.in/courses/Webcourse-contents/IIT-KANPUR/machine/ui/Course_home-lec33.htm
W[2]. http://nptel.ac.in/courses/112103174/22
W[3]. http://www.liquid-dynamics.com/animations/pumps.htm
W[4]. http://www.hydrauliconline.com/hydraulic-pumps/how-do-hydraulic-pumps-work/

UNIT	Total No. of Periods Planned	Lecture Periods	Tutorial Periods
I	11	9	2

II	**10**	**8**	**2**
III	**10**	**8**	**2**
IV	**10**	**8**	**2**
V	**11**	**8**	**2**
TOTAL	**52**	**42**	**10**

I. CONTINUOUS INTERNAL ASSESSMENT : 40 Marks

(Internal Assessment Tests: 30, Attendance: 5, Assignment: 5)

II. END SEMESTER EXAMINATION : 60 Marks

TOTAL : 100 Marks

FACULTY **HOD / MECH** **DEAN / FOE**

UNIT I: FLUID PROPERTIES AND FLOW CHARACTERISTICS

UNITS AND MEASUREMENTS

Physical Quantity	Symbol	SI Unit	SI Unit Abbreviation
Mass	m	kilogram	kg
Length	l	meter	m
Time	t	second	s
Temperature	T	Kelvin	K
Area	A	square meter	m^2
Volume	v	cubic meter	m^3
Velocity	V	meter per second	m/s
Acceleration	a	meter per second squared	m/s^2
Acceleration due to gravity	g	meter per second squared	m/s^2
Angular velocity	ω	radian per second	rad/s
Angular acceleration	α	radian per second squared	rad/ s^2
Force	F	kilogram-meter/second^2 (or) Newton	$kg{\cdot}m/s^2$ (or) N
Pressure	p	Newton per square meter (or) Pascal	N/m^2 (or) P
Shear stress	τ	Newton per meter squared	N/m^2
Torque	T	Newton-meter	N-m
Energy	E	Newton-meter (or) Joule	N-m (or) J
Power	P	Newton-meter per second (or) Joule per second (or) Watt	N-m/s (or) J/s (or) W
Mass density or density	ρ	kilogram per cubic meter	kg/m^3
Weight density or Specific weight	w	Newton per cubic meter	N/m^3
Specific Volume	v	cubic meter per kilogram	m^3/ kg
Dynamic viscosity (or) Absolute viscosity	μ	Newton second per meter squared	Ns/m^2
Kinematic viscosity	υ	meter square per second	m^2/s
Surface tension	σ	Newton per meter	N/m
Capillarity	h	meter	m
Bulk modulus	K	Newton per meter squared	N/m^2

EQUIVALENT UNITS	
1Newton = 1 kilogram-meter/second^2 (= kg·m/s^2)	1 litre = 10^{-3} m^3
1kgf = 9.81 Newton	1 ton= 1000 kg
1Pascal = 1 Newton/metre^2	1 poise = 0.1 Ns/m^2
1bar = 10^5 Newton/metre^2	1stoke = 10^{-4} m^2/s
1 atm = 1.013 x 10^5 Newton/metre^2	
1 Joule = 1 Newton-meter	
1 Watt = 1 Newton metre/second or Joule/second	

MULTIPLES AND SUBMULTIPLES OF SI UNITS

Symbol	Name	Factor		Symbol	Name	Factor
T	tera	10^{12}		p	Pico	10^{-12}
G	giga	10^9		n	nano	10^{-9}
M	mega	10^6		μ	micro	10^{-6}
k	kilo	10^3		m	milli	10^{-3}
h	hecto	10^2		c	centi	10^{-2}
da	deca	10^1		d	deci	10^{-1}

INTRODUCTION

Mechanics is the science that deals with both stationary and moving bodies under the influence of forces. The branch of mechanics that deals with bodies at rest is called statics, while the branch that deals with bodies in motion is called dynamics. Fluid mechanics is defined as the science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics), and the interaction of fluids with solids or other fluids at the boundaries.

Fluid mechanics itself is also divided into several categories. The study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as hydrodynamics. A subcategory of hydrodynamics is hydraulics, which deals with liquid flows in pipes and open channels. Gas dynamics deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds. The category aerodynamics deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds. Some other specialized categories such as meteorology, oceanography and hydrology deal with naturally occurring flows.

Fluid mechanics is widely used both in everyday activities and in the design of modern engineering systems from vacuum cleaners to supersonic aircraft. Therefore, it is important to develop a good understanding of the basic principles of fluid mechanics.

To begin with, fluid mechanics plays a vital role in the human body. The heart is constantly pumping blood to all parts of the human body through the arteries and veins, and the lungs are the sites of airflow in alternating directions. All artificial hearts, breathing machines, and dialysis systems are designed using fluid dynamics.

An ordinary house is, in some respects, an exhibition hall filled with applications of fluid mechanics. The piping systems for cold water, natural gas, and sewage for an individual house and the entire city are designed primarily on the basis of fluid mechanics. The same is also true for the piping and ducting network of heating and air-conditioning systems. A refrigerator involves tubes, through which the refrigerant flows, a compressor that pressurizes the refrigerant, and two heat exchangers where the refrigerant absorbs and rejects heat. Fluid mechanics plays a major role in the design of all these components. Even the operation of ordinary faucets is based on fluid mechanics.

There are numerous applications of fluid mechanics in an automobile. All components associated with the transportation of the fuel from the fuel tank to the cylinders—the fuel line, fuel pump, fuel injectors, or carburetors—as well as the mixing of the fuel and the air in the cylinders and the purging of combustion gases in exhaust pipes are analyzed using fluid mechanics. Fluid mechanics is also used in the design of the heating and air-conditioning system, the hydraulic brakes, and the power steering, automatic transmission, and lubrication systems, the cooling system of the engine block including the radiator and the water pump, and even the tires. The sleek streamlined shape of recent model cars is the result of efforts to minimize drag by using extensive analysis of flow over surfaces.

On a broader scale, fluid mechanics plays a major part in the design and analysis of aircraft, boats, submarines, rockets, jet engines, wind turbines, biomedical devices, the cooling of electronic components, and the transportation of water, crude oil, and natural gas. It is also considered in the design of buildings, bridges, and even billboards to make sure that the structures can withstand wind loading. Numerous natural phenomena such as the rain cycle, weather patterns, and the rise of ground water to the top of trees, winds, ocean waves, and currents in large water bodies are also governed by the principles of fluid mechanics

On the basis of the spacing between the molecules, the matter can be classified into three states i.e. Solid, Liquid and Gaseous state.

In solids, the molecules are very closely spaced whereas in liquids the spacing between the different molecules is relatively large and in gases the spacing between the molecules is still large. It means that inter-molecular cohesive forces are large in solids, smaller in liquids and extremely small in gases, and on account of this fact, solids possess compact and rigid form, liquid molecules can move freely within the liquid mass and the molecules of gases have greater freedom of movement so that the gases fill the container completely in which they are placed.

Liquids and gases exhibit different characteristics. The liquids under ordinary conditions are quite difficult to compress (and therefore they may for most purposes be regarded as incompressible) whereas gases can be compressed much readily under the action of external pressure (and when the external pressure is removed the gases tend to expand indefinitely).

A substance in liquid or gaseous phase is referred as fluid. A fluid is a substance which deforms continuously when subjected to external shearing force.

A fluid has the following characteristics:

1. It has no definite shape of its own, but conforms to the shape of the containing vessel.
2. Even a small amount of shear force exerted on a liquid/fluid will cause it to undergo a deformation which continues as long as the force continues to be applied.

1. PROPERTIES OF FLUIDS

Some important properties of fluids are:

1. Mass density or density
2. Weight density or specific weight
3. Specific volume
4. Specific gravity
5. Viscosity
6. Cohesion
7. Adhesion
8. Surface tension
9. Capillarity
10. Compressibility
11. Vapour pressure

1.1 Mass Density or Density or Specific mass

Mass density (simply known as density) of a liquid may be defined as the mass per unit volume at a standard temperature and pressure. It is usually denoted by 'ρ' (rho). Its unit is kg/m^3.

Mass Density = mass/volume

$\rho = m/v \quad (kg/m^3)$

1.2 Weight Density or Specific Weight

Weight density or specific weight of a fluid can be defined as the weight per unit volume at standard temperature and pressure. It is usually denoted by 'w'. Its unit is N/m^3.

Weight density or specific weight = weight/volume

$w = W/v = mg/v = \rho g \quad (N/m^3)$

1.3 Specific volume

It is defined as volume per unit mass of fluid. It is reciprocal of mass density. It is denoted by the symbol, $\bar{v}$.

Specific volume = volume/mass

$\bar{v} = v/m \quad (m^3/kg)$

1.4 Specific Gravity (or) Relative Density

Specific gravity or relative density is defined as the ratio of density of a fluid to the density of a standard fluid. It is a dimensionless quantity and is denoted by 'S'.

S = density of any liquid / density of standard liquid (water)
S = density of any gas / density of standard gas (air)

Note:

For liquid, water is considered as standard fluid and

Density of standard liquid (water) = 1000 kg/m^3

For gas, air is considered as standard fluid

Density of standard gas (air) = 1 kg/m^3

P1. Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7N.

Given data < kg, N, m>

Volume, V = 1 litre = $0.001m^3$
Weight, W = 7N

Solution

W=mg
7= m x 9.81
$m = \frac{7}{9.81} = 0.7135$ kg

Specific weight, w $= \frac{\text{Weight(W)}}{\text{Volume(v)}} = \frac{7}{0.001} = 7000\ N/m^3$

Density, ρ $= \frac{\text{mass(m)}}{\text{Volume(v)}} = \frac{0.713}{0.001} = 713\ kg/m^3$

Specific volume, $\bar{v}$ $= \frac{\text{volume(v)}}{\text{mass(m)}} = \frac{0.001}{0.713} = 1.4025\times10^{-3} m^3/kg$

Specific gravity, S $= \frac{\rho\ \text{liquid}}{\rho\ \text{water}} = \frac{713}{1000} = 0.713$ (no unit)

P2. Calculate density, specific weight and weight of one litre petrol of specific gravity 0.7

Given data <kg, N, m>

Specific gravity, s = 0.7

Volume, V = 1 litre = $0.001m^3$

Solution

Specific gravity, S $= \frac{\rho\ \text{liquid}}{\rho\ \text{water}}$

$0.7 = \frac{\rho\ \text{petrol}}{1000}$

Density, ρ_{petrol} $= \rho = 0.7 \times 1000 = 700\ kg/m^3$

Specific weight, w $= \rho g = 700 \times 9.81 = 6867\ N/m^3$

Weight, W $= wV = 6867 \times 0.001 = 6.867\ N$

P3. Calculate the specific weight, density, specific volume and specific gravity of a liquid having a volume of 6m³ and weight of 44 kN.

Given data <kg, N, m>

Weight, W $= 44\ kN = 44 \times 10^3\ N$

Volume, v $= 6\ m^3$

Solution:

$W = mg$

$44 \times 10^3 = m \times 9.81$

$m = \frac{44 \times 10^3}{9.81} = 4485.2\ kg$

Specific weight, w $= \frac{\text{Weight(W)}}{\text{Volume(v)}} = \frac{44 \times 10^3}{6} = 73333.33\ N/m^3$

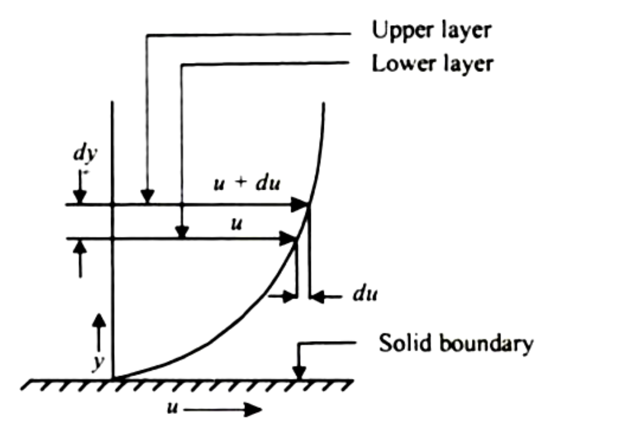
Density, ρ $= \frac{\text{mass(m)}}{\text{Volume(v)}} = \frac{4485.2}{6} = 747.53\ kg/m^3$

Specific volume, $\bar{v}$ $= \frac{\text{volume(v)}}{\text{mass(m)}} = \frac{6}{4485.2} = 1.34 \times 10^{-3} m^3/kg$

Specific gravity, S $= \frac{\rho\text{liquid}}{\rho\text{water}} = \frac{747.53}{1000} = 0.74753$

1.5 Viscosity

It is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. Viscosity of fluid is due to cohesion and interaction between the fluid particles.



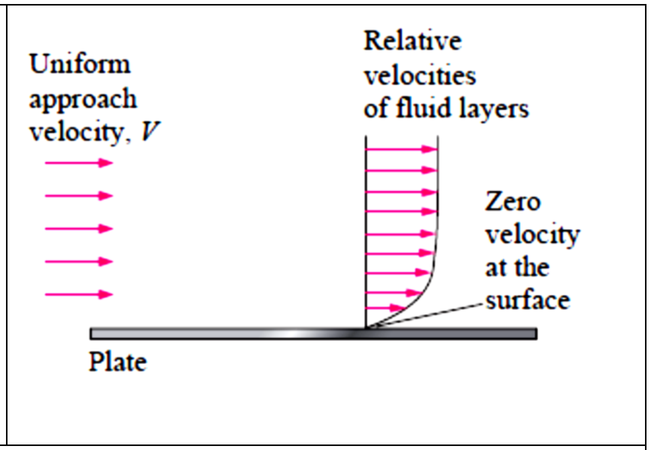


Figure 1: Velocity Variation near boundary layer (relative velocity)

When two layers of fluid, at a distance *"dy'* apart, move one over the other at different velocities, say *u* and *u + du,* the viscosity together with relative velocity causes a shear stress acting between the fluid layers. The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

Newton's Law of Viscosity

Newton's law of viscosity states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient of viscosity.

$$\tau \propto (du/dy)$$

$$\tau = \mu\, (du/dy)$$

Where, μ (called mu) is a constant and is known as coefficient of viscosity or dynamic viscosity.

Where, τ = Shear stress in N/m^2, μ - Dynamic viscosity in Ns/m^2, (du/dy) – Velocity gradient in s^{-1}.

Dynamic Viscosity

The amount of shear stress (τ) required to produce unit shear strain (du/dy) is called as dynamic viscosity.

$$\mu = \tau / (du/dy)$$

The SI unit of dynamic viscosity is Ns/m^2.

Kinematic Viscosity

Kinematic viscosity is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by υ (called nu).

$$\nu = \frac{Dynamic\ viscosity}{Density} = \frac{\mu}{\rho}$$

The SI unit of kinematic viscosity is m^2/s.

Viscosity is affected by temperature. The viscosity of liquids decreases but that of gases increases with increase in temperature. This is due to the reason that in liquids the shear stress is due to the inter-molecular cohesion which decreases with increase of temperature. In gases the inter-molecular cohesion is negligible and the shear stress is due to exchange of momentum of the molecules, normal to the direction of motion. The molecular activity increases with rise in temperature and so does the viscosity of gas.

The viscosity under ordinary conditions is not appreciably affected by the changes in pressure. However, the viscosity of some oils has been found to increase with increase in pressure.

Based on viscosity, the fluids can be classified into the following types.

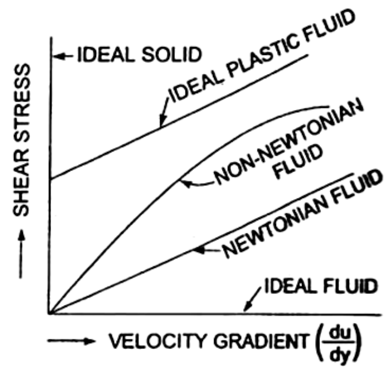


Figure 2: Types of fluids

Ideal fluid: An ideal fluid is one which has no viscosity and incompressible. In true sense no such fluid exists in nature. However fluids which have low viscosities such as water and air can be treated as ideal fluids under certain conditions. The assumption of ideal fluids helps in simplifying the mathematical analysis.

Real fluid: A real practical fluid is one which has viscosity, surface tension and compressibility in addition to the density. The real fluids are actually available in nature.

Newtonian fluid: A real practical fluid which obeys Newton’s law of viscosity is known as Newtonian fluid.

Non Newtonian fluid: A fluid which does not obey Newton’s law of viscosity is known as non Newtonian fluid.

Ideal Plastic fluid: An ideal plastic (or Bingham plastic) has a definite yield stress and a constant linear relation between shear stress and the rate of angular deformation, Examples: Sewage sludge, drilling muds etc.

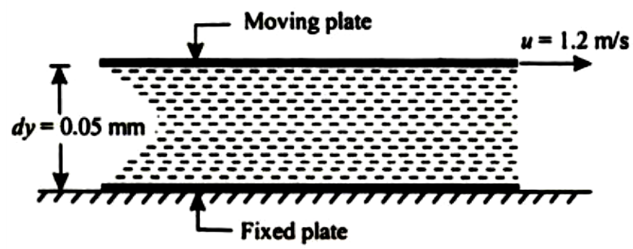
P1. **A plate 0.05 mm distant from a fixed plate moves at 1.2 m/s and requires a shear stress of 2.2 N/m^2 to maintain this speed. Find the viscosity of the fluid between the plates.**

Given data < N, m, s>

Velocity of the moving plate, du = 1.2 m/s

Distance between the plates. dy = 0.05 mm = 0.05 x 10^{-3} m

Shear stress on the moving plate, τ = 2.2 N/m^2



Solution

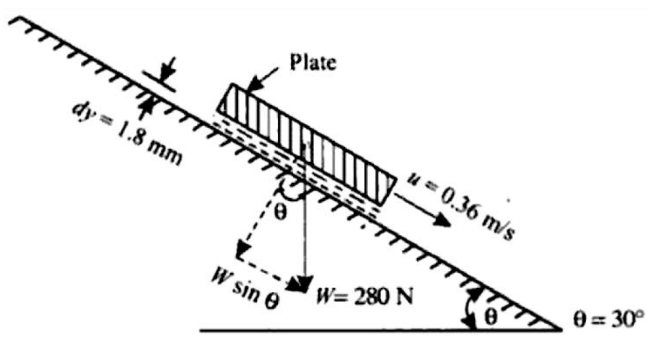
Shear stress, $\tau = \mu\frac{du}{dy}$

$$2.2 = \mu\frac{1.2}{0.05\ X\ 10^{-3}}$$

$\mu = 9.16 \times 10^{-6}$ Ns/m^2

P2. **A plate having an area of 0.6 m^2 is sliding down the inclined plane at 30^0 to the horizontal with a velocity of 0.36 m/s. There is a cushion of fluid 1.8 mm thick between the plane and the plate. Find the viscosity of the fluid if the weight of the plate is 280 N.**

Given data:



Area of plate, A = 0.6 m^2

Weight of plate, W = 280 N

Velocity of plate, u = 0.36 m/s

Thickness of film, t= dy = 1.8 mm = 1.8 x 10^{-3} m

W along the plate = Wsin θ = 280 sin 30^0=140 N

Solution:

∴ Shear force on the bottom surface of the plate, F = 140 N and shear stress,

$$\tau = \frac{F}{A} = \frac{140}{0.6} = 233.33\text{N/m}^2$$

We know, $\tau = \mu \cdot \frac{du}{dy}$

Where, du = change of velocity = $u - 0$ = 0.36 m/s

$dy = t = 1.8 \times 10^{-3}$ m

∴ $$233.33 = \mu \times \frac{0.36}{1.8 \times 10^{-3}}$$

or $$\mu = \frac{233.33 \times 1.8 \times 10^{-3}}{0.36} = 1.166\,\text{N.s/m}^2 = \textbf{11.66 poise (Ans.)}$$

P3. The velocity distribution for flow over a plate is gives by u = 2y-y² where u is the velocity in m/s at a distance 'y' metres above the plate. Determine the velocity gradient and shear stress at the boundary and 0.15m from it. Take dynamic viscosity of fluid as 0.9 Ns/m².

Soluton. $u = 2y - y^2$ ...(given) ∴ $\frac{du}{dy} = 2 - 2y$

(*i*) **Velocity gradient,** $\frac{du}{dy}$:

At the boundary : At $y = 0$, $\left(\frac{du}{dy}\right)_{y=0} = \mathbf{2s^{-1}}$ **(Ans.)**

At 0.15 m from the boundary: At $y = 0.15$ m, $\left(\frac{du}{dy}\right)_{y=0.15} = 2 - 2 \times 0.15 = \mathbf{1.7\ s^{-1}}$ **(Ans.)**

(*ii*) **Shear stress, τ:**

$$(\tau)_{y=0} = \mu \cdot \left(\frac{du}{dy}\right)_{y=0} = 0.9 \times 2 = \mathbf{1.8\,N/m^2}\ \textbf{(Ans.)}$$

and $$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.9 \times 1.7 = \mathbf{1.53\,N/m^2}\ \textbf{(Ans.)}$$

[Where μ = 0.9 N.s/m² ... (given)]

P4. Two plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerin of viscosity 8.1 poise. What force is required to drag a thin plate of surface area 0.5 m² between the plane surfaces at a speed of 0.6 m/s if

(i) thin plate is in the middle of two plane surfaces (ii) thin plate is at a distance of 0.8cm from one of the plane surfaces?

Given data

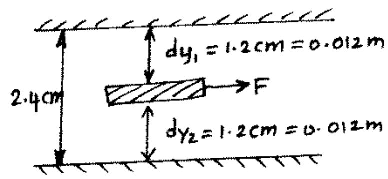
Area of plate A =0.5m²

Distance between plane surfaces =2.4cm =0.024m

Velocity of plate, du = 0.6m/s

Viscosity μ = 8.1 poise $= \frac{8.1}{10} \frac{Ns}{m^2}\, 0.81 \frac{Ns}{m^2}$

Solution



Case (i)

F_1 – Shear force on the upper side of the plate

F_2 – Shear force on the lower side of the plate

F – Total shear force

$\tau_1 = \mu \frac{du}{dy_1} = \frac{0.81*0.6}{0.012} = 40.5$ N/m^2

$\frac{F_1}{A} = 40.5$ N/m^2

F_1=40.5 x 0.5 = 20.25 N

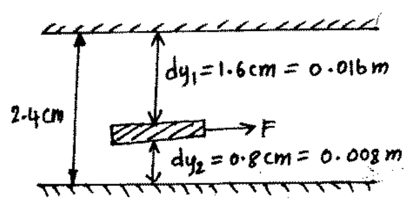
$\tau_2 = \mu \frac{du}{dy_2} = \frac{0.81*0.6}{0.012} = 40.5$ N/m^2

$\frac{F_2}{A} = 40.5$ N/m^2

F_2=40.5 x 0.5 = 20.25N

F = F_1+F_2 =20.25 +20.25 = 40.5 N

Case (ii)



$\tau_1 = \mu \frac{du}{dy1} = \frac{0.81*0.6}{0.016} = 30.375$ N/m^2

$\frac{F_2}{A} = 30.375$ N/m^2

F_1=30.375 x 0.5 = 15.18N

$\tau_2 = \mu \frac{du}{dy2} = \frac{0.81*0.6}{0.008} = 60.75$ N/m^2

$\frac{F_2}{A} = 60.75$ N/m^2

F_2=60.75 x 0.5 = 30.36N

F = F_1+F_2 =15.18 +30.36 = 45.54 N

P5. A 400mm diameter shaft is rotating at 200 rpm in a bearing of length 120mm. If the thickness of oil film is 1.5 mm and the dynamic viscosity of the oil is 0.7 Ns/m², determine:

(i) Torque required to overcome the friction oil bearing

(ii) Power utilized in overcoming the viscous resistance

Assume a linear velocity profile.

Given data:

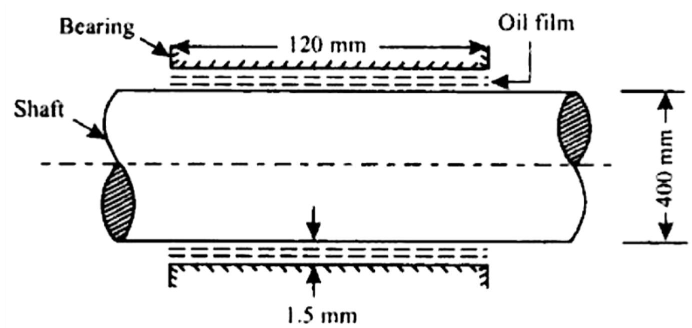
Diameter of the shaft, d =400 mm = 0.4 m

Speed of the shaft, N =200 rpm,

Thickness of the oil film, t = 1.5 mm = 1.5×10^{-3} m

Length of the bearing, l = 120 mm = 0.12 m

Dynamic viscosity, μ=0.7 Ns/m²



Tangential velocity of the shaft,

$$u = \frac{\pi d N}{60} = \frac{\pi \times 0.4 \times 200}{60} = 4.19 \text{ m/s}$$

(*i*) Torque required to overcome friction, *T*:

We know, $\tau = \mu . \frac{du}{dy}$

where du **= change of velocity =** $u - 0 = 4.19$ **m/s**

$$dy = t = 1.5 \times 10^{-3}\ \text{m}$$

$$\therefore \qquad \tau = 0.7 \times \frac{4.19}{1.5 \times 10^{-3}}$$

$$= 1955.3\ \text{N/m}^2.$$

$\therefore$ **Shear force,** F = shear stress × area

$$= \tau \cdot \pi\, dl$$

$$= 1955.3 \times \pi \times 0.4 \times 0.12$$

$$= 294.85\ \text{N}$$

Hence, viscous torque, $= F \times d/2 = 294.85 \times \dfrac{0.4}{2}$

$$= \mathbf{58.97\ Nm\ (Ans.)}$$

(*ii*) Power utilised, P:

$$P = T \times \frac{2\pi N}{60}\ \text{watts, where } T \text{ is in Nm}$$

$$\therefore \qquad P = 58.97 \times \frac{2\pi \times 200}{60} = 1235\ \text{W} \ \text{ or } \mathbf{1.235\ kW\ (Ans.)}$$

1.6 Cohesion

Cohesion means intermolecular attraction between molecules of the same liquid. It enables a liquid to resist small amount of tensile stresses. Cohesion is a tendency of the liquid to remain as one assemblage of particles. "Surface tension" is due to cohesion between particles at the free surface.

1.7 Adhesion

Adhesion means attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid. This property enables a liquid to stick to another body.

1.8 Surface tension

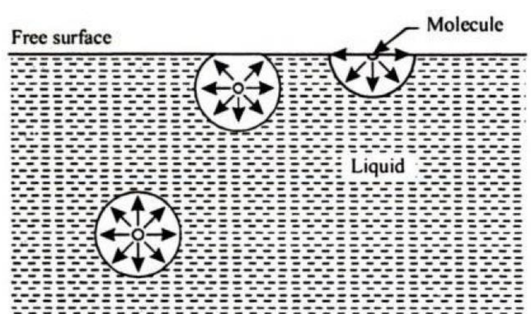


Figure 3: Surface tension

Surface tension is caused by the force of cohesion at the free surface. A liquid molecule in the interior of the liquid mass is surrounded by other molecules all around and is in equilibrium. At the free surface of the liquid, there are no liquid molecules above the surface to balance the force of the molecules below it. Consequently, as shown in the above Figure 3, there is a net inward force on the molecule. The force is normal to the liquid surface. At the free surface, a thin layer of molecules is formed. This is because of this film that a thin small needle can float on the free surface (the layer acts as a membrane). It is usually expressed in N/m.

Some important examples of phenomenon of surface tension are as follows:

i. Rain drops (A falling rain drop becomes spherical due to cohesion and surface tension).
ii. Rise of sap in a tree.
iii. Bird can drink water from ponds.
iv. Capillary rise and capillary siphoning.
v. Collection of dust particles on water surface.
vi. Break up of liquid jets.

The value of surface tension depends upon the following factors:

i. Nature of the liquid,
ii. Nature of the surrounding matter (e.g.. solid, liquid or gas), and
iii. Kinetic energy (and hence the temperature of the liquid molecules).

NOTE: As compared to pressure and gravitational forces, surface tension forces arc generally negligible; but become quite significant when there is a free surface and the boundary conditions are small as in the case of small scale models of hydraulic engineering structures.

Case I: Water droplet

Let p - Pressure inside the droplet.
d - Diameter of the droplet and

$$Surface\ tension, \sigma = \frac{pd}{4}$$

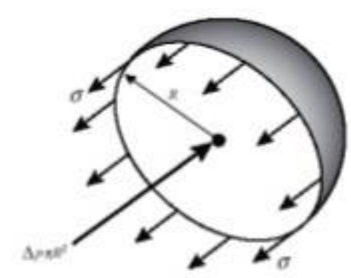


Figure 4: Water droplet

Case II: Soap (hollow) bubbles

$$Surface\ tension, \sigma = \frac{pd}{8}$$

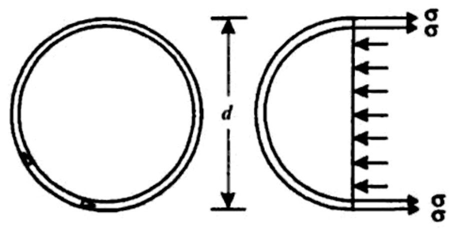


Figure.5: Pressure inside a soap bubble

Case III: A liquid jet

$$Surface\ tension, \sigma = \frac{pd}{2}$$

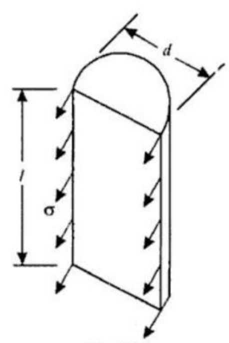


Figure 6: Forces on liquid jet

P1: If the surface tension at air-water interface is 0.069 N/m, what is the pressure difference between inside and outside of an air bubble of diameter 0.009mm?

Solution. *Given*: $\sigma = 0.069$ N/m; $d = 0.009$ mm

An air bubble has only one surface. Hence,

$$p = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.069}{0.009 \times 10^{-3}} = 30667 \text{ N/m}^2$$

$$= \mathbf{30.667 \text{ kN/m}^2 \text{ or kPa (Ans.)}}$$

1.9 Capillarity

Capillarity is a phenomenon by which a liquid (depending upon its specific gravity) rises into a thin glass tube above or below its general level. This phenomenon is due to the combined effect of cohesion and adhesion of liquid particles.

Let h-capillary rise or depression,

σ-surface tension,

θ-interface angle,

w-weight density (=ρg),

d-capillary tube diameter.

$$Capillarity\ rise\ or\ depression, h = \frac{4\sigma cos\theta}{wd}$$

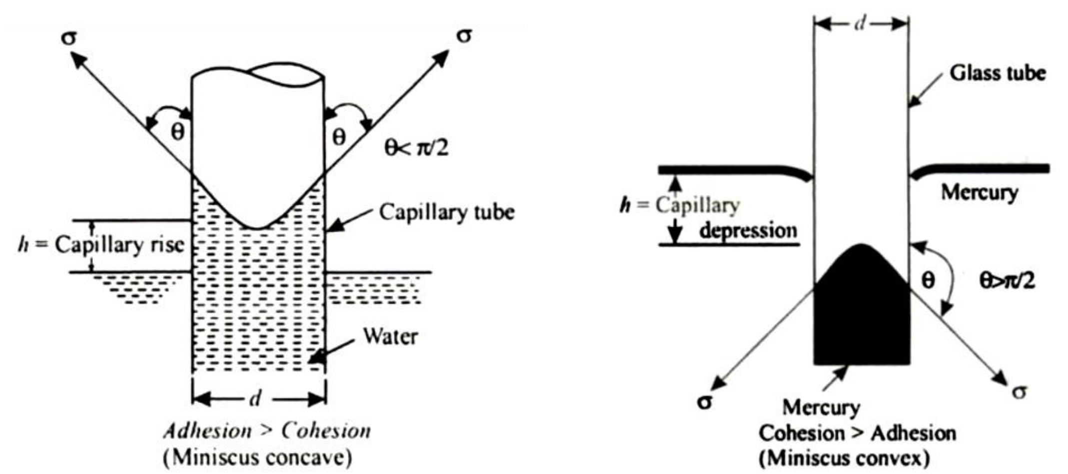


Figure 7: Capillary Effect

Note: For water, $\theta=0^0$ and $w=9810\ N/m^3$

For mercury, $\theta=128^0$ and $w=13600\ N/m^3$

P1. A clean tube of diameter 2.5mm is immersed in a liquid with a coefficient of surface tension =0.4 N/m. The angle of contact of the liquid with the glass call be assumed to be 135⁰. Tire density of the liquid = 13600 kg/m³. What would be the level of the liquid ill the tube relative to the free surface of the liquid inside the tube?

Given data:

d = 2.5 mm; σ = 4 N/m, θ = 135°; ρ = 13600 kg/m³

Solution:

Level of the liquid in the tube, h:

The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall),

$$h = \frac{4\sigma \cos\theta}{wd}$$

$$= \frac{4 \times 0.4 \times \cos 135°}{(9.81 \times 13600) \times 2.5 \times 10^{-3}}$$

$$= -3.39 \times 10^{-3} \text{m or } -3.39 \text{ mm}$$

Negative sign indicates that there is a capillary **depression (fall) of 3.39 mm. (Ans.)**

P2: Calculate the capillarity effect in millimeters in a glass tube of 2.5 mm diameter, when immersed in (i) water (ii) mercury the temperature of the liquid is 20° C and the values of surface tension of water and mercury at 20° C in contact with air are 0.0725 N/m and 0.52 N/m respectively. The contact angle for water is θ = 0⁰ and for mercury is θ = 130⁰.

Given Data < N, m >

Diameter d= 2.5mm = 2.5 x 10^{-3}m = 0.0025m

Surface tension of water, σ_{water} =0.0725 N/m

Contact angle, $\theta = 0^0$

Surface tension of Mercury, $\sigma_{mercury}$ = 0.52 N/m

θ = 130^0

Solution

(i) For water, capillary effect, h $= \frac{4\sigma\cos\theta}{wd}$

$= \frac{4*0.0725*\cos\theta}{9810*0.0025}$ = **<u>0.0118m</u>** (capillary rise)

(ii) For mercury, capillary depression h $= \frac{4\sigma\cos\theta}{wd}$

$= \frac{4*0.052*\cos 130}{133416*0.0025}$ = **<u>-0.004m</u>**

(Capillary depression)

1.10 Compressibility

The property by virtue of which fluids undergo a change in volume under the action of external pressure is known as **compressibility.** It decreases with the increases in pressure of fluid as the volume modulus increases with the increase of pressure.

The variation in volume of water, with variation of pressure, is so small that for all practical purposes it is neglected. Thus, the water is considered to be an incompressible liquid. However in case of water flowing through pipes when sudden or large changes in pressure (e.g. water hammer) take place, the compressibility cannot be neglected. The compressibility in Fluid Mechanics is considered mainly when the velocity of flow is high enough reaching 20 percent of speed of sound in the medium.

Elasticity of fluids is measured in terms of **bulk modulus of elasticity (K)** which is defined as the ratio of compressive stress to the volumetric strain. Compressibility is the reciprocal of bulk modulus of elasticity.

Consider a cylinder fitted with a piston;

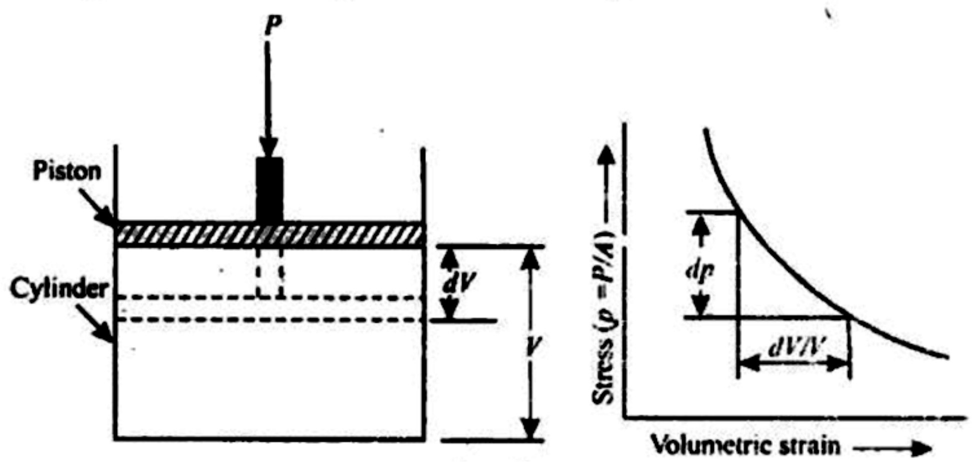


Figure 8: Cylinder fitted with a piston

Let V - Volume of gas enclosed in the cylinder

dV - Change in Volume

p - Pressure of gas when volume is V

$$\therefore \text{ Volumetric strain } = -\frac{dV}{V}$$

(Negative sign indicates *decrease in volume with increase of pressure*)

$$\therefore \text{ Bulk modulus, } K = \frac{dp\,(\text{increase of pressure})}{-dV/V\,(\text{volumetric strain})}$$

i.e.,

$$K = \frac{dp}{-dV/V}$$

$$\left(\text{Compressibility} = \frac{1}{K}\right)$$

P1. When the pressure of liquid is increased from 3.5 MN/m² to 6.5 MN/m² its volume is found to decrease by 0.08 percent. What is the bulk modulus of elasticity of the liquid?

Given data:

Initial pressure = 3.5 MN/m^2

Final pressure = 6.5 MN/m^2

:. Increase in pressure, dp = 6.5 - 3.5 = 3.0 MN/m^2

Solution:

Decrease in volume = 0.08 percent $\therefore\ -\dfrac{dV}{V} = \dfrac{0.08}{100}$

Bulk modulus (K) is given by:

$$K = \frac{dp}{-\frac{dV}{V}} = \frac{3 \times 10^6}{\frac{0.08}{100}} = 3.75 \times 10^9 \text{ N/m}^2 \text{ or } 3.75 \text{ GN/m}^2$$

Hence $\quad K = \mathbf{3.75\ GN/m^2}$ **(Ans.)**

1.11 Vapour Pressure

All liquids have a tendency to evaporate or vaporize (i.e., to change from the liquid to the gaseous state). Molecules are continuously projected from the free surface to the atmosphere. These ejected molecules are in a gaseous state and exert their own partial vapour pressure on the liquid surface. This pressure is known as the vapour pressure of the liquid (p_v). If the surface above the liquid is confined, the partial vapour pressure exerted by the molecules increases till the rate at which the molecules re-enter the liquid is equal to the rate at which they leave the surface. When the equilibrium condition is reached, the vapour pressure is called saturation vapour pressure (p_{vs}).

2. PRESSURE

When a fluid is contained in a vessel, it exerts force at all points on the sides and bottom and top of the container. The force per unit area is called pressure.

If F is the force and A is area on which the force acts, then intensity of pressure, P = F/A. The pressure of a fluid on a surface will always act normal to the surface.

2.1 Pascal's Law

"The intensity of pressure at any point in a liquid at rest is the same in all directions".

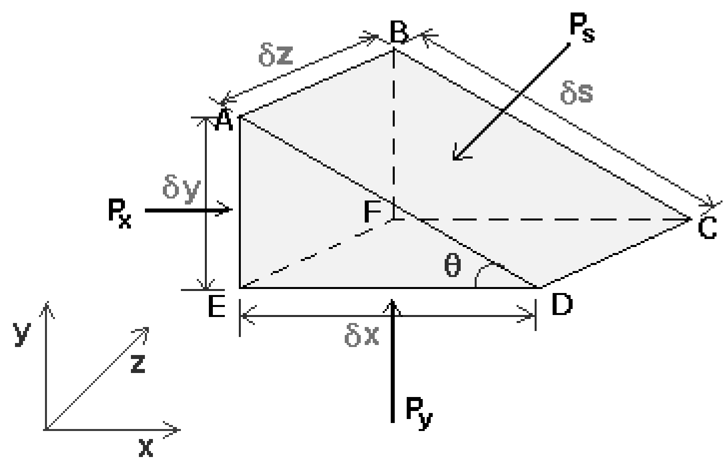


Figure 9: Pressure on a fluid element at rest

By considering the equilibrium of a small fluid element in the form of a triangular prism ABCDEF surrounding a point in the fluid, a relationship can be established between the pressures P_x in the x direction, P_y in the y direction, and Ps normal to any plane inclined at any angle θ to the horizontal at this point.,

Px is acting at right angle to ABEF, Py at right angle to CDEF and similarly Ps at right angle to ABCD.

Since there can be no shearing forces for a fluid at rest, and there will be no accelerating forces, the sum of the forces in any direction must therefore, be zero. The forces acting are due to the pressures on the surrounding and the gravity force.

Force due to $P_x = P_x$ x Area ABEF $= P_x\delta y\delta z$

Horizontal component of force due to P_s,

$P_s = -$ (P_s x Area ABCD) $\sin(\theta) = - P_s\delta s\delta z\delta y/\delta s = -P_s\delta y\delta z$

As P_y has no component in the x direction, the element will be in equilibrium, if $P_x\delta y\delta z + (-P_s\delta y\delta z) = 0$

i.e. $P_x = P_s$

Similarly in the y direction, force due to $P_y = P_y\delta x\delta z$

Component of force due to $P_s = -$ (P_s x Area ABCD) $\cos(\theta) = - P_s\delta s\delta z\ \delta x/\delta s = - P_s\delta x\delta z$

Force due to weight of element $= - mg = - \rho Vg = - \rho(\delta x\delta y\delta z/2)g$

Since δx, δy and δz are very small quantities, $\delta x \delta y \delta z$ is negligible in comparison with other two vertical force terms, and the equation reduces to,

$P_y = P_s$

Therefore, $P_x = P_y = P_s$

i.e. pressure at a point is same in all directions. This is **Pascal's law**. This applies to fluid at rest.

2.2 Atmospheric, Absolute and Gauge Pressures

Atmospheric pressure: The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact, and it is known as atmospheric pressure. The atmospheric pressure is also known as 'Barometric pressure'. The atmospheric pressure at sea level (above absolute zero) is called 'Standard atmospheric pressure',

Gauge pressure: It is the pressure, measured with the help of pressure measuring instrument in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.Gauges record pressure above or below the local atmospheric pressure, since they measure the difference in pressure of the liquid to which they are connected and that of surrounding air. If the pressure of the liquid is below the local atmospheric pressure, then the gauge is designated as 'vacuum gauge' and the recorded value indicates the amount by which the pressure of the liquid is below local atmospheric pressure, i.e. negative pressure.
(Vacuum pressure is defined as the pressure below the atmospheric pressure).

Absolute pressure: It is necessary to establish an absolute pressure scale which is independent of the changes in atmospheric pressure. A pressure of absolute zero can exist only in complete vacuum. Any pressure measured above the absolute zero of pressure is termed as all absolute pressure.

Absolute pressure = Atmospheric pressure + Gauge pressure
Absolute pressure = Vacuum pressure – Atmospheric pressure

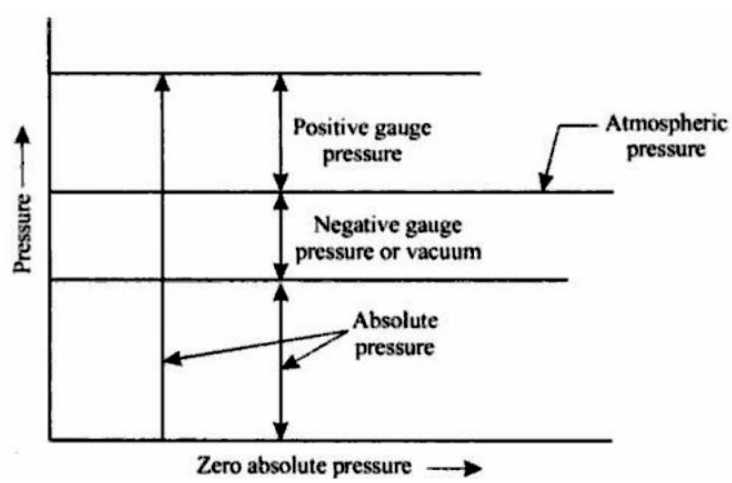


Figure 10: Relationship between pressures

2.3 Measurement of Pressure

The pressure of a fluid may be measured by the following devices:

1) Manometers:

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of liquid. These are classified as follows:

(a) Simple manometers:

(i) Piezometer (ii) U-tube manometer and (iii) Single column manometer.

(b) Differential manometers.

2) Mechanical gauges:

These are the devices in which the pressure is measured by balancing the fluid column by spring (elastic clement) or dead weight. Generally these gauges are used for measuring high pressure and where high precision is not required. Some commonly used mechanical gauges are:

(i) Bourdon tube pressure gauge (ii) Diaphragm pressure gauge
(iii) Bellow pressure gauge and (iv) Dead-weight pressure gauge.

2.3.1 Manometers:

(a) Simple manometers:

A simple manometer is one which consists of a glass tube whose one end is connected to a point where pressure is to be measured and the other end remains open to atmosphere. Common types of simple manometers are discussed below:

(i) Piezometer:

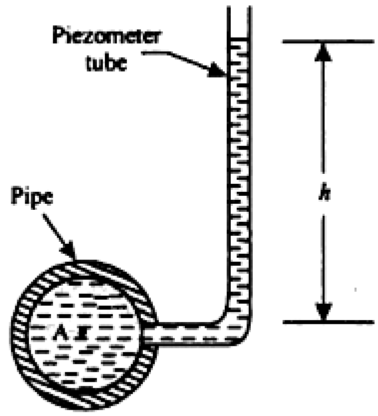


Figure 11: Piezometer

A piezometer is the simplest form of manometer which can be used for measuring moderate pressures of liquids. It consists of a glass tube (Fig. 11) inserted in the wall of a vessel or of a pipe, containing liquid whose pressure is to be measured. The tube extends vertically upward to such a height that liquid can freely rise in it without overflowing. The pressure at any point in the liquid is indicated by the height of the liquid in the tube above that point, which can be read on the scale attached to it Thus if 'w' is the specific weight of the liquid, then the pressure at point A(P) is given by,

$$\boxed{P = wh = \rho g h}$$

Piezometers measure gauge pressure only (at the surface of the liquid), since the surface of the liquid in the tube is subjected to atmospheric pressure. A piezometer tube is not suitable for measuring negative pressure; as in such a case the air will enter in pipe through the tube.

(ii) U-tube manometer:

Piezometers cannot be employed when large pressures in the lighter liquids are to be measured, since this would require very long tubes, which cannot be handled conveniently. Furthermore gas pressures cannot be measured by the piezometers because a gas forms no free atmospheric surface. These limitations can be overcome by the use of U-tube manometers.

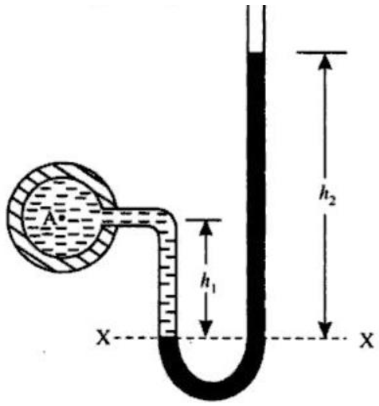


Figure 12: U-Tube manometer measuring gauge pressure

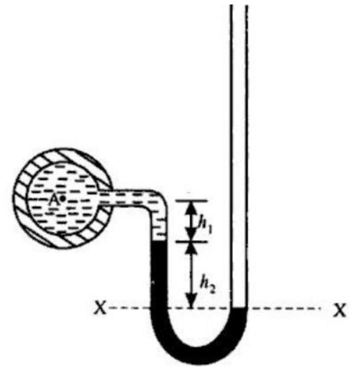


Figure 13: U-Tube manometer measuring vacuum pressure

For Gauge pressure (positive pressure) (Figure 12)

Let, A be the point at which pressure is to be measured,
X-X is the datum line as shown in Figure 12.
h_1 - Height of the light liquid in the left limb above the datum line,
h_2 - Height of the heavy liquid in the right limb above the datum line,
h - Pressure in pipe, expressed in terms of head,
S_1 - Specific gravity of the light liquid and
S_2 - Specific gravity of the heavy liquid,

The pressures in the left limb and right limb above the datum line X-X are equal (as the pressures at two points at the same level in a continuous homogeneous liquid are equal).

Pressure head above X-X in the left limb = $h + h_1S_1$
Pressure head above X-X in the right limb = $h_2 S_2$
Equating these two pressures, we get

$$\boxed{\begin{array}{l} h + h_1S_2 = h_2S_2 \\ h = h_2S_2 - h_1S_1 \end{array}}$$

For Vacuum pressure (Negative pressure) (Figure 13)
Pressure head above X-X in the left limb = $h + h_1S_1 + h_2S_2$
Pressure head above X-X in the right limb =0
Equating these two pressures, we get

$$\boxed{h + h_1S_1 + h_2S_2 = 0 \text{ or } h = -(h_1S_1 + h_2S_2)}$$

(b) Differential Manometers

A differential manometer is used to measure the difference in pressures between two points in a pipe or in two different pipes. In its simplest form a differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is required to be found out. Following are the most commonly used types of differential manometers.

i. U-tube differential manometer.

ii. inverted U-tube differential manometer.

(i) U-tube differential manometer:

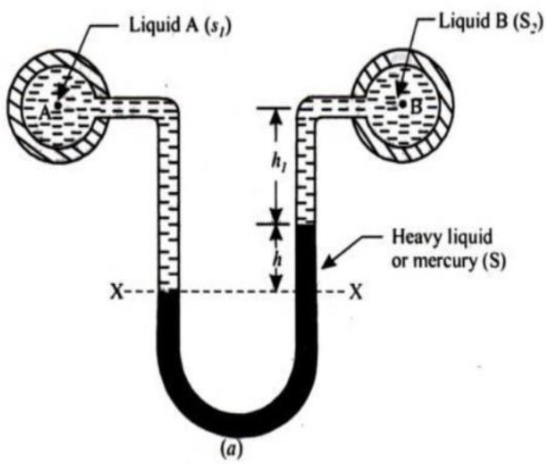


Figure 14: Two pipes at same level

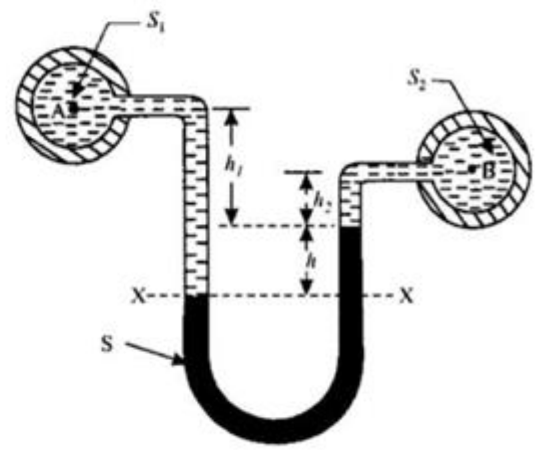


Figure 15**:** Two pipes at different level

Case I. Figure 14 **shows a differential manometer whose two ends are connected with two different points A and B at the same level and containing same liquid.**

Let, h = Difference of the centre of A, from the mercury level in the right limb,

$S_1 (= S_2)$ = Specific gravity of liquid at the two points A and B

S = Specific gravity of heavy liquid or mercury in the U-tube,

h_A = Pressure head at A, and

h_B = Pressure head at B,

We know that the pressures in the left limb and right limb, above the datum line, are equal.

Pressure head in the *left limb*

$$= h_A + (h_1 + h)\, S_1$$

Pressure head in the *right limb*

$$= h_B + h_1 \times S_1 + h \times S$$

$$h_A + (h_1 + h)S_1 = h_B + h_1 S_1 + hS$$

or,
$$h_A - h_B = h_1 S_1 + hS - (h_1 + h)\, S_1$$
$$= h_1 S_1 + hS - h_1 S_1 + hS_1 = h\,(S - S_1)$$

i.e., Difference of pressure head,

$$h_A - h_B = h\,(S - S_1)$$

Case II. Figure 15 **shows a differential manometer whose two ends are connected to two different points A and B at different levels and containing different liquids.**

Let, h = Difference of mercury level (heavy liquid) in the U-tube,

h_1 = Distance of the centre of A, from the mercury level in the left limb,

h_2 = Distance of the centre of B, from the mercury level in the right limb,

S_1 = Specific gravity of liquid in pipe A,

S_2 = Specific gravity of liquid in pipe B,

S = Specific gravity of heavy liquid or mercury,

h_A = Pressure head at A, and

h_B = Pressure head at B.

Considering the pressure heads above the datum line X–X, we get

Pressure head in the *left limb* $= h_A + (h_1 + h) S_1$

Pressure head in the *right limb* $= h_B + h_2 \times S_2 + h \times S$

Equating the above pressure heads, we get

$$h_A + (h_1 + h) S_1 = h_B + h_2 \times S_2 + h \times S$$

$$(h_A - h_B) = h_2 \times S_2 + h \times S - (h_1 + h) S_1$$

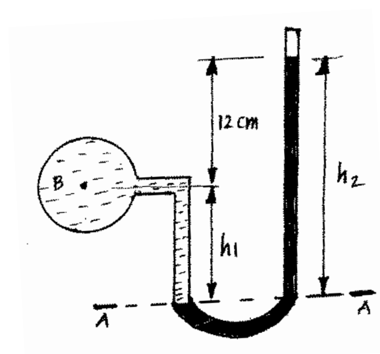
$$= h_2 \times S_2 + h \times S - h_1 S_1 - hS_1 = h(S - S_1) + h_2 S_2 - h_1 S_1$$

i.e., Difference of pressure head at A and B,

$$h_A - h_B = h(S - S_1) + h_2 S_2 - h_1 S_1$$

P1: The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of specific gravity 0.9 is flowing. The centre of pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Given data < kg, N, m >



Specific gravity of fluid, S_1 = 0.9

$\frac{\rho 1}{\rho water}$ = 0.9

ρ_1 = 0.9 x 1000 = 900 kg/m^3

Specific gravity of mercury, S_2 = 13.6

$\frac{\rho 2}{\rho water}$ = 13.6

ρ_2 = 13.6 x 1000 = 13600 kg/m^3

h_2 = 20cm = 0.2 m

h_1 = 20cm – 12 cm = 8 cm = 0.08 m

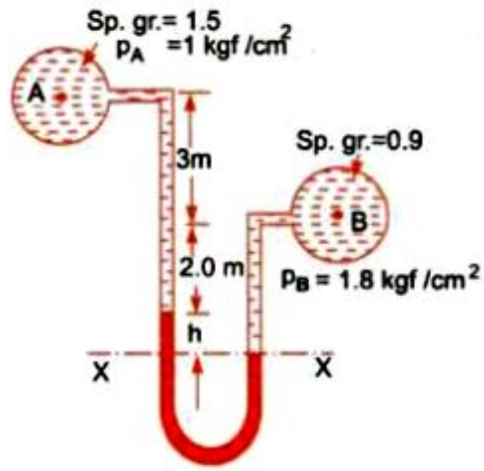
Solution

$P = \rho_2 g h_2 - \rho_1 g h_1$

$= 13600 \times 9.81 \times 0.2 - 900 \times 9.81 \times 0.08$

$= 25977\ N/m^2$

P2 **A differential manometer is connected at the two points A and B of two pipes as shown in the figure below. The pipe A contains a liquid of specific gravity of 1.5 while pipe B contains a liquid of specific gravity of 0.9. The pressure at A and B are 1 kgf/cm^2 and 1.8 kgf/cm^2 respectively. Find the difference in mercury level in the manometer.**



Solution:

Sp. gr. of liquid at A, $S_1 = 1.5$ $\therefore$ $\rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9$ $\therefore$ $\rho_2 = 900$

Pressure at A, $p_A = 1\ \text{kgf/cm}^2 = 1 \times 10^4\ \text{kgf/m}^2$

$= 10^4 \times 9.81\ \text{N/m}^2$ ($\because$ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8\ \text{kgf/cm}^2$

$= 1.8 \times 10^4\ \text{kgf/m}^2$

$= 1.8 \times 10^4 \times 9.81\ \text{N/m}^2$ ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000\ \text{kg/m}^3$

Taking X-X as datum line.

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$
$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb = $900 \times 9.81 \times (h + 2) + p_B$
$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get
$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4 = 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get
$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$
or $$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$
or $$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$
$$\therefore \quad h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1 \text{ cm. Ans.}}$$

Buoyancy is the ability or tendency of something to float in water or other fluid.

Floatation is the action of floating in a liquid or gas.

Metacentric height

Metacentric height is the distance between the centre of gravity and the metacentre of a floating body, as of a vessel.

TWO MARK QUESTIONS & ANSWERS

1. **Define fluid.**
   A substance in liquid or gas phase is referred as fluid. A fluid is a substance which deforms continuously when subjected to external shearing force.
2. **What are the types of fluids?**

   The different types of fluids are Ideal fluid, Real Fluid, Newtonian fluid, Non-Newtonian fluid, Ideal plastic fluid.
3. **Define ideal fluid.**

   An ideal fluid is one which has no viscosity, surface tension and is incompressible. It is also known as imaginary fluid because such fluid does not exist in nature.

   Water is assumed to be an ideal fluid.
4. **Define real fluid.**

   A real practical fluid is one which has viscosity, surface tension and is compressible. All fluid in actual practice is real fluids.
5. **Define density.**
   Density (ρ) of a fluid can be defined as the mass (m) per unit volume (V) at standard temperature and pressure. Its unit is kg/m^3.

   Density, ρ = (mass/Volume)

6. Define specific weight or weight density.

Specific weight or weight density of a fluid can be defined as the Weight (W) per unit volume (V) at standard temperature and pressure. Its unit is N/m^3.

Weight density or Specific weight, w = (Weight/Volume)

Weight density or Specific weight, w = ρg

7. Define specific gravity or relative density.

Specific gravity or relative density is defined as the ratio of the weight density of a fluid to the weight density of a standard fluid. It is a dimensionless quantity and is denoted by S.

S = weight density of liquid / weight density of water
S = weight density of gases / weight density of air

NOTE:

- For liquid, Water is considered as standard fluid and
- For gas; air is considered as standard fluid

8. What is viscosity?

It is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

9. State Newton's law of viscosity.

Newton's law of viscosity states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the coefficient of viscosity.

$\tau \propto (du/dy)$

$\tau = \mu\, (du/dy)$

Where, τ = Shear stress in N/m^2, μ - Dynamic viscosity in Ns/m^2, (du/dy) – Velocity gradient in s^{-1}.

10. Define Newtonian fluid.

A real practical fluid which obeys Newton's law of viscosity is known as Newtonian fluid.

11. Define non Newtonian fluid.

A fluid which does not obey Newton's law of viscosity is known as non Newtonian fluid.

12. What is kinematic viscosity?

It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by greek symbol (ν) called 'nu'. Its S.I unit is m^2/s.

Kinematic viscosity (ν) =viscosity (μ) /density (ρ)

13. Define compressibility.

It can be defined as the ratio of compressive stress to volumetric strain. It is the reciprocal of bulk modulus.

14. Define pressure and list out the devices used for its measurement.

Pressure can be defined as the amount of force acting per unit area.

$$P=F/A$$

The devices used for pressure measurements are-

a) Manometer
b) Mechanical gauges

15. List the different types of manometers?

i. Simple manometer
   a. Piezometer
   b. U-tube manometer
   c. Single column manometer
ii. Differential manometer
   a. Differential U-tube manometer
   b. Inverted differential U-tube manometer

16. Define buoyancy and floatation.

Buoyancy is the ability or tendency of something to float in water or other fluid.
Floatation is the action of floating in a liquid or gas.

17. What is meant by Metacentric height?

Metacentric height is the distance between the centre of gravity and the metacentre of a floating body, as of a vessel.

EXERCISE PROBLEMS

1. A liquid having specific gravity of 0.82 is filled in a vessel, calculate its density & specific weight.
2. Calculate the specific weight and specific mass of a liquid having a volume of 6 m^3 and weight of 44 kN.
3. A liquid having specific gravity of 0.82 is filled in a vessel, calculate its specific volume.
4. Find kinematic viscosity of a liquid having dynamic viscosity 11.4 poise and specific gravity 1.9.
5. Define specific weight & Calculate the specific weight of mercury if its specific gravity is 13.6.
6. If the volume of the air occupied in a room is 39.64 m^3. Calculate the mass of the air in the room.
7. How much litres of water can be stored in a sump of size (3.048 m * 3.048 m * 3.658 m)?
8. Define specific weight & Calculate the specific weight of an liquid which has an specific gravity 0.732.
9. Determine the viscosity of a liquid having kinematic viscosity 6 stokes and density 1900 kg/m^3.
10. The velocity distribution for flow over a flat plate is given by u $=\frac{3}{4}y - y^2$, where u is velocity in metre/second at a distance y metre above the plate. Determine the shear stress at the boundary of the flat plate and at 15 cm from the plate. Assume dynamic viscosity as 8.6 poise.

11. A soap bubble 62.5mm diameter has an internal pressure in excess of the outside pressure of 20 N/m^2. What is surface tension in the soap film?
12. Two plane surfaces are 2.4 cm apart. The space between the surfaces is filled with glycerine of viscosity 8.1 poise. What force is required to drag a thin plate of surface area 0.5 m^2 between the plane surfaces at a speed of 0.6 m/s if (a) thin plate is at a distance of 0.8cm from one of the plane surfaces (b) thin plate is in the middle of two plane surfaces
13. The dynamic viscosity of oil used for lubrication between a shaft and sleeve is 6 poise. The shaft diameter of 0.4 m and rotates at 190 rpm. Calculate the torque required to overcome the friction and the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.
14. The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of specific gravity 0.9 is flowing. The centre of pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.
15. A U-tube manometer used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to the atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m^2, calculate the new difference in the level of mercury. Sketch the arrangements in both cases.
16. Find out the differential reading 'h' of an inverted U-tube manometer containing oil of specific gravity 0.7 as the manometric fluid when connected across pipes A and B, Conveying liquids glycerol and water having specific gravities 1.2 and 1.0 at pipes A and B respectively and immiscible with manometric fluid. Pipes A and B are located at the same level and assume the pressures at A and B to be equal. The rise of Glycerol is 30cm above from the centre of pipe.

UNIT 2
FLOW THROUGH CIRCULAR PIPES

2.1 FLUID KINEMATICS

Fluid kinematics gives the geometry of fluid motion. It is a branch of fluid mechanics, which describes the fluid motion, and its consequences without consideration of the nature of forces causing the motion.

Fluid kinematics is the study of velocity as a function of space and time in the flow field. From velocity, pressure variations and hence, forces acting on the fluid can be determined.

Velocity Field: Velocity at a given point is defined as the instantaneous velocity of the fluid particle, which at a given instant is passing through the point. It is represented by V=V(x,y,z,t). Vectorially, V=ui+vj+wk where u,v,w are three scalar components of velocity in x,y and z directions and (t) is the time. Velocity is a vector quantity and velocity field is a vector field.

2.2 FLOW PATTERNS

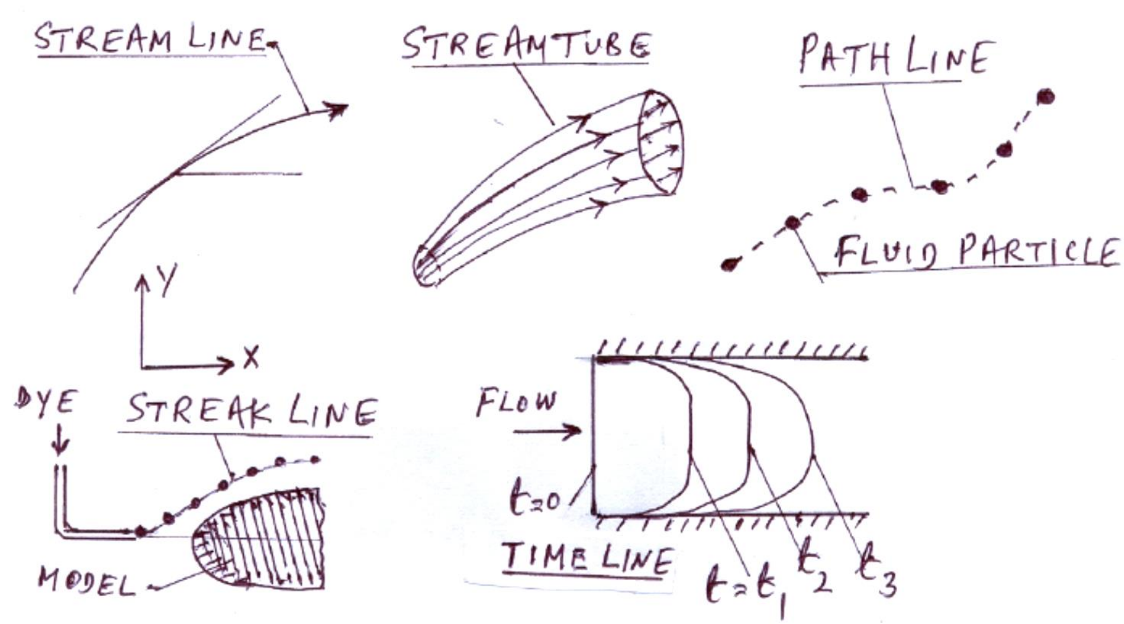


Figure 1: Flow patterns

Fluid Mechanics is a visual subject. Patterns of flow can be visualized in several ways. Basic types of line patterns used to visualize flow are streamline, path line, streak line and time line.

a) Stream line is a line, which is everywhere tangent to the velocity vector at a given instant.
b) Path line is the actual path traversed by a given particle.
c) Streak line is the locus of particles that have earlier passed through a prescribed point.
d) Time line is a set of fluid particles that form a line at a given instant.

Streamline is convenient to calculate mathematically.

Other three lines are easier to obtain experimentally. Streamlines are difficult to generate experimentally. Streamlines and Time lines are instantaneous lines. Path lines and streak lines are generated by passage of time. In a steady flow situation, streamlines, path lines and streak lines are identical. In Fluid Mechanics, the most common mathematical result for flow visualization is the streamline pattern – It is a common method of flow pattern presentation.

Streamlines are everywhere tangent to the local velocity vector. For a stream line, (dx/u) = (dy/v) = (dz/w). Stream tube is formed by a closed collection of streamlines. Fluid within the stream tube is confined there because flow cannot cross streamlines. Stream tube walls need not be solid, but may be fluid surfaces

2.3 METHOD OF DESCRIBING FLUID MOTION

Two methods of describing the fluid motion are: (a) Lagrangian method & (b) Eularian method.

2.3.1 Lagrangian Method

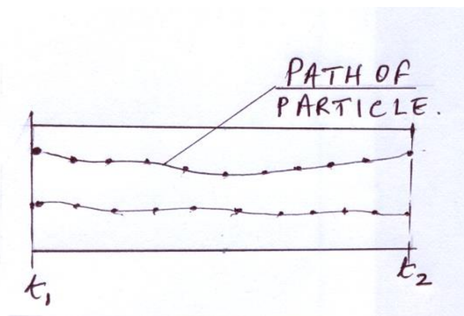


Figure 2: Lagrangian method

A single fluid particle is followed during its motion and its velocity, acceleration etc. are described with respect to time. Fluid motion is described by tracing the kinematics behavior of each and every individual particle constituting the flow. We follow individual fluid particle as it moves through the flow. The particle is identified by its position at some instant and the time elapsed since that instant. We identify and follow small, fixed masses of fluid. To describe the fluid flow where there is a relative motion, we need to follow many particles and to resolve details of the flow; we need a large number of particles. Therefore, Langrangian method is very difficult and not widely used in Fluid Mechanics.

2.3.2 Eularian Method

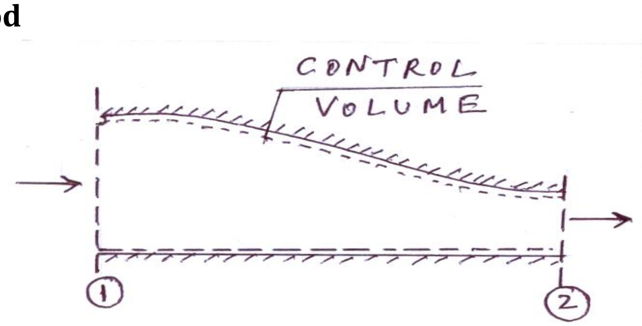


Figure 3: Eulerian Method

The velocity, acceleration, pressure etc. are described at a point or at a section as a function of time. This method commonly used in Fluid Mechanics. We look for field description, for Ex.; seek the velocity and its variation with time at each and every location in a flow field. Ex., V=V(x,y,z,t). This is also called control volume approach. We draw an imaginary box around a fluid system. The box can be large or small, and it can be stationary or in motion.

2.4 TYPES OF FLUID FLOW

1) Steady and Un-steady flows
2) Uniform and Non-uniform flows
3) Laminar and Turbulent flows
4) Compressible and Incompressible flows
5) Rotational and Irrotational flows
6) One, Two and Three dimensional flows

2.4.1 Steady and Unsteady Flows

Steady flow is the type of flow in which the various flow parameters and fluid properties at any point do not change with time. In a steady flow, any property may vary from point to point in the field, but all properties remain constant with time at every point. $[\partial V/\partial t]_{x,y,z} = 0$; $[\partial p/\partial t]_{x,y,z} = 0$. Ex.: V=V(x,y,z); p=p(x,y,z). Time is a criterion.

Unsteady flow is the type of flow in which the various flow parameters and fluid properties at any point change with time. $[\partial V/\partial t]_{x,y,z} \neq 0$; $[\partial p/\partial t]_{x,y,z} \neq 0$, Eg.:V=V(x,y,z,t), p=p(x,y,z,t) or V=V(t), p=p(t) . Time is a criterion

2.4.2 Uniform and Non-Uniform Flows

Uniform Flow is the type of flow in which velocity and other flow parameters at any instant of time do not change with respect to space. Eg., V=V(x) indicates that the flow is uniform in 'y' and 'z' axis. V=V (t) indicates that the flow is uniform in 'x', 'y' and 'z' directions. Space is a criterion.

Uniform flow field is used to describe a flow in which the magnitude and direction of the velocity vector are constant, i.e., independent of all space coordinates throughout the entire flow field (as opposed to uniform flow at a cross section). That is, $[\partial V/\partial s]_{t=constant} = 0$, that is 'V' has unique value in entire flow field.

Non-uniform flow is the type of flow in which velocity and other flow parameters at any instant change with respect to space. $[\partial V/\partial s]_{t=constant}$ is not equal to zero. Distance or space is a criterion

2.4.3 Laminar and Turbulent Flows

Laminar Flow is a type of flow in which the fluid particles move along well-defined paths or stream-lines. The fluid particles move in laminas or layers gliding smoothly over one another. The behavior of fluid particles in motion is a criterion.

Turbulent Flow is a type of flow in which the fluid particles move in zigzag way in the flow field. Fluid particles move randomly from one layer to another. Reynolds number is a criterion. We can assume that for a flow in pipe, for Reynolds No. less than 2000, the flow is laminar; between 2000-4000, the flow is transitional; and greater than 4000, the flow is turbulent.

2.4.4 Compressible and Incompressible Flows

Incompressible Flow is a type of flow in which the density (ρ) is constant in the flow field. This assumption is valid for flow Mach numbers with in 0.25. Mach number is used as a criterion. Mach Number is the ratio of flow velocity to velocity of sound waves in the fluid medium

Compressible Flow is the type of flow in which the density of the fluid changes in the flow field. Density is not constant in the flow field. Classification of flow based on Mach number is given below:

M < 0.25 – Low speed
M < unity – Subsonic
M around unity – Transonic
M > unity – Supersonic
M > > unity, (say 7) – Hypersonic

2.4.5 Rotational and Irrotational Flows

Rotational flow is the type of flow in which the fluid particles while flowing along stream-lines also rotate about their own axis.

Ir-rotational flow is the type of flow in which the fluid particles while flowing along stream-lines do not rotate about their own axis.

2.4.6 One, Two and Three Dimensional Flows

The number of space dimensions needed to define the flow field completely governs dimensionality of flow field. Flow is classified as one, two and three- dimensional depending upon the number of space co-ordinates required to specify the velocity fields.

One-dimensional flow is the type of flow in which flow parameters such as velocity is a function of time and one space coordinate only. For Ex., V=V(x,t) – 1D, unsteady ; V=V(x) – 1D, steady

Two-dimensional flow is the type of flow in which flow parameters describing the flow vary in two space coordinates and time. For Ex., V=V(x,y,t) – 2D, unsteady; V=V(x,y) – 2D, steady.

Three-dimensional flow is the type of flow in which the flow parameters describing the flow vary in three space coordinates and time. For Ex., V=V(x,y,z,t) – 3-D, unsteady; V=V(x,y,z) – 3D, steady

2.5 CONTINUITY EQUATION

Rate of flow or discharge (Q) is the volume of fluid flowing per second.

For incompressible fluids flowing across a section, Volume flow rate, Q= A×V m^3/s where A=cross sectional area and V= average velocity.

For compressible fluids, rate of flow is expressed as mass of fluid flowing across a section per second. Mass flow rate (m) = (ρAV) kg/s where, ρ = density.

Figure 4: Continuity Equation

Continuity equation is based on Law of Conservation of Mass, For a fluid flowing through a pipe, in a steady flow, the quantity of fluid flowing per second at all cross- sections is a constant.

Let V_1=average velocity at section [1], ρ_1=density of fluid at [1], A_1=area of flow at [1];

Let V_2, ρ_2, A_2 be corresponding values at section [2].

Rate of flow or Discharge at section [1] = $\rho_1 A_1 V_1$ ---------------------- (1)

Rate of flow at section [2] = $\rho_2 A_2 V_2$ ---------------------- (2)

Equating [1] = [2]

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

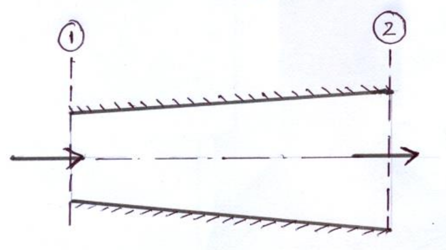
This equation is applicable to steady compressible or incompressible fluid flows and is called Continuity Equation. If the fluid is incompressible, $\rho_1 = \rho_2$ and the continuity equation reduces to $A_1 V_1 = A_2 V_2$

For steady, one dimensional flow with one inlet and one outlet,

$$\rho_1 A_1 V_1 - \rho_2 A_2 V_2 = 0 \quad \text{---------------------} (3)$$

P1 The diameters of the pipe at sections (1) and (2) are 15cm and 20cm respectively. Find the discharge through the pipe if the velocity of water at section (1) is 4m/s. Determine also the velocity at section (2)

(Answers: $0.0706 m^3/s$, 2.25m/s)



P2 Water flows through a pipe AB 1.2 m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter. At C, the pipe branches. Branch CD is 0.8m in diameter and carries one-third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.

Given data

$D_1 = 1.2m$

$A_1 = \frac{\pi D_1^2}{4} = 1.131m^2$

$V_1 = 3\ m/s$

$D_2 = 1.5\ m$

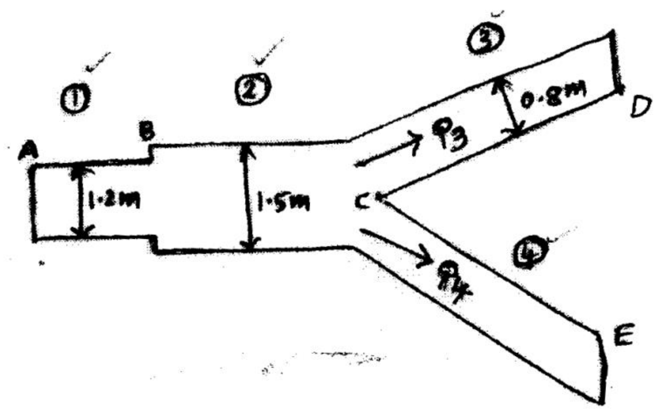
$A_2 = \frac{\pi D_2^2}{4} = 1.767m^2$

$D_3 = 0.8m$

$A_3 = \frac{\pi D_3^2}{4} = 0.5026\ m^2$

$Q_3 = \frac{1}{3}\ Q_1$

$V_4 = 2.5\ m/s$



Solution

$Q_1 = A_1V_1 = 1.131\ x3 = 3.393\ m^3/s$

$\underline{Q_2 = Q_1 = 3.393\ m^3/s}$

$A_2V_2 = A_1V_1$

$V_2 = \frac{A1V1}{A2} = \frac{1.131x3}{1.767} = 1.92\ m/s$

$Q_3 = \frac{1}{3} Q_1$

$A_3V_3 = \frac{1}{3} x\ 3.393$

$V_3 = \frac{3.393}{3x0.5026} = 2.25m/s$

$Q_2 = Q_3 + Q_4$

$3.393 = A_3V_3 + Q_4$

$\underline{Q_4 = 2.262\ m^3/s}$

$A_4V_4=2.262$

$\frac{\pi}{4} D_4^2 x2.5=2.262$

D_4=1.0735m

2.5.1 Continuity Equation In 3-Dimensions (Cartesian Coordinates):

Consider infinitesimal control volume as shown of dimensions dx, dy and dz in x,y,and z directions.

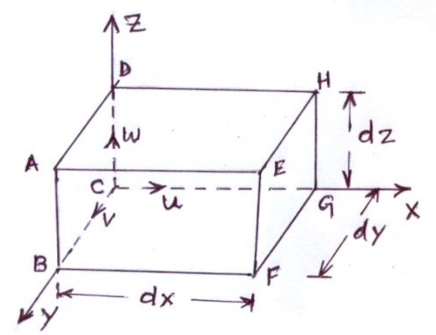


Figure 5: Continuity Equation in Three Dimensions

Let u,v,w are the velocities in x,y,z directions.

Considering mass flow rate in x-direction

Mass of fluid entering the face ABCD = Density ×velocity in x-direction ×Area ABCD

$= \rho u dy.dz$ ------ (1)

Mass of fluid leaving the face EFGH $= \rho u dy.dz + [\partial (\rho u dy.dz)/ \partial x](dx)$ ------ (2)

Therefore, net rate of mass flow rate in x-direction= (2) – (1)

$= -[\partial (\rho u dy.dz)/ \partial x](dx)$

$= -[\partial (\rho u)/ \partial x](dxdydz)$ ------ (3)

Similarly, the net rate of mass flow rate in,

y-direction= $- [\partial (\rho v)/ \partial y](dxdydz)$ ------ (4)

z-direction= $- [\partial (\rho w)/ \partial z](dxdydz)$ ------ (5)

The rate of accumulation of mass within the control volume

$=\partial (\rho dV)/ \partial t = \rho\partial/\partial t\, (dV)$ ------ (6)

Where, dV=Volume of the element=dxdydz and dV is invariant with time.

Note: *From conservation of mass, the net rate of efflux = Rate of accumulation of mass within the control volume.*

$\Rightarrow$ (3)+(4)+(5) = (6)

$- [\partial(\rho u)/ \partial x + \partial (\rho v)/ \partial y + \partial (\rho w)/ \partial z](dxdydz) = \rho\partial/\partial t(dxdydz)$ OR

$\rho\partial/\partial t + \partial (\rho u)/ \partial x +\partial (\rho v)/ \partial y + \partial (\rho w)/ \partial z =0$ ------ (7)

Equation (7) is known as Continuity Equation in Catesian Coordinates.

Continuity Equation in Catesian Coordinates is applicable for,

a) Steady and unsteady flows
b) Uniform and non-uniform flows
c) Compressible and incompressible flows.

For steady flows, $(\partial/\partial t) = 0$ and $[\partial (\rho u) /\partial x + \partial (\rho v)/ \partial y + \partial (\rho w)/ \partial z] =0$
If the fluid is incompressible, ρ= constant $[\partial u/\partial x + \partial v/\partial y + \partial w/\partial z] =0$
This is the continuity equation for 3-D flows.
For 2-D flows, w=0 and $[\partial u/\partial x + \partial v/\partial y] = 0$

2.6 VELOCITY POTENTIAL

Velocity Potential Function is a Scalar Function of space and time co-ordinates such that its negative derivatives with respect to any direction give the fluid velocity in that direction.

2.7 STREAM FUNCTION (ψ)

Stream Function is defined as the scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. Stream function is defined only for two dimensional flows and 3-D flows with axial symmetry.

2.8 FLOW NET

A grid obtained by drawing a series of equi-potential lines and streamlines is called a Flow Net. A Flow Net is an important tool in analyzing two-dimensional ir-rotational flow problems. Examples: Uniform flow, Line source and sink, Line vortex

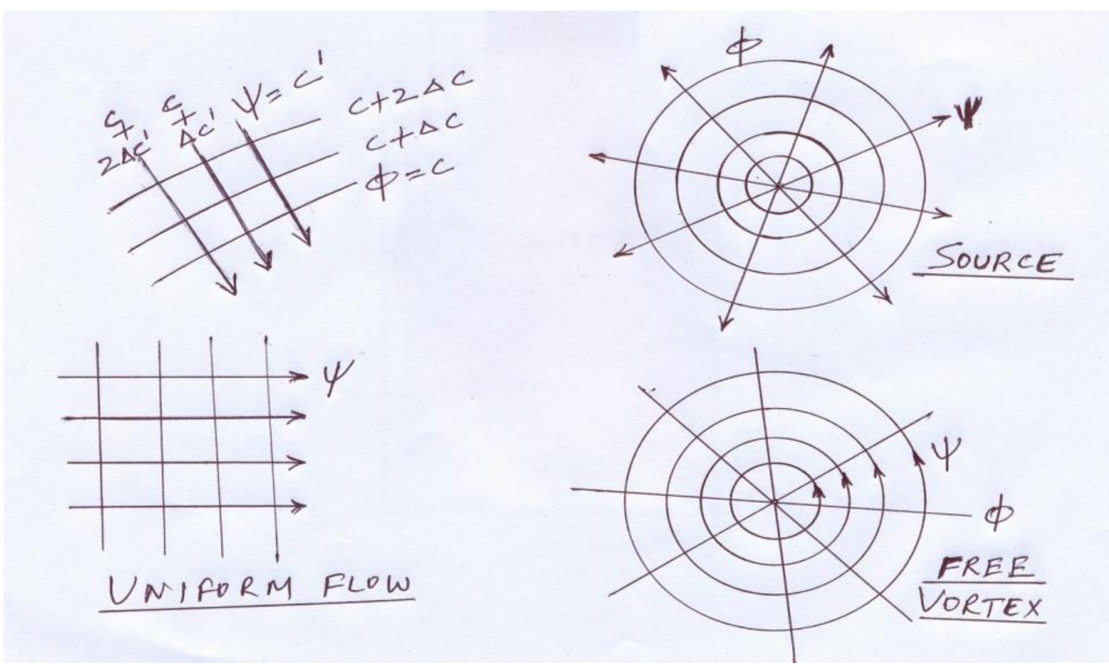
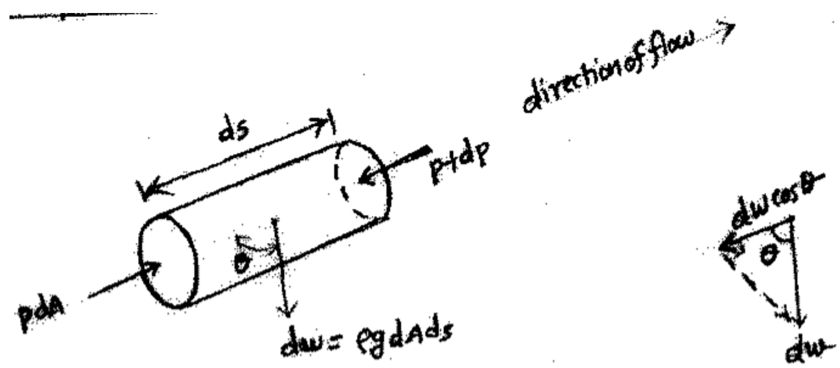


Figure 6: Flow Nets

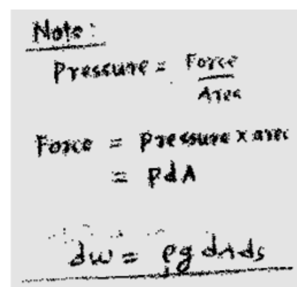
2.9 FLUID DYNAMICS

Fluid dynamics is a subdiscipline of fluid mechanics that deals with fluid flow—the natural science of fluids (liquids and gases) in motion. It has several subdisciplines itself, including aerodynamics (the study of air and other gases in motion) and hydrodynamics (the study of liquids in motion). Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines, predicting weather patterns, understanding nebulae in interstellar space and modelling fission weapon detonation. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid, and crowd dynamics.

2.9.1 Euler's equation for the flow of an incompressible frictionless fluid.



Let, P – pressure on element at one side
P+dp – pressure on element at other side
dA – cross sectional area of element
θ – Angle between direction of flow and weight of element (dw)



Number of forces acting on the fluid element

1. Pressure force (pdA) in the direction of flow
2. Presure force (P+dp)dA in the opposite direction of flow
3. Weight of element (dw)

Net force acting on the fluid element along the direction of flow

$$F = PdA - (p+dp)dA - dw\cos\theta$$

$$F = PdA - pdA - dpdA - \rho g dA ds \cos\theta$$

$$F = -dpdA - \rho g dA dz$$

Mass of fluid element, $m = \rho dA ds$

Acceleration of fluid element, a = $\frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = V\frac{dv}{ds}$

According to Newton's second law

F = ma

-dpdA – ρgdAdz = ρdAds . V $\frac{dv}{ds}$

Divide ρdA on both sides

= $\frac{dp}{\rho}$ - gdz = Vdv

$\frac{dp}{\rho}$ + Vdv + gdz = 0 --- (1)

Equation (1) is known as Euler's equation.

2.9.2 Bernoulli's Equation

By integrating Euler's Equation (1), we can derive Bernoulli's equation,

$\frac{dp}{\rho}$ + Vdv + gdz = 0

$\frac{1}{\rho}\int dp + \int Vdv + g\int dz = 0$

$\frac{p}{\rho} + \frac{V^2}{2}$ +gz = constant

Divide by 'g'

$\frac{p}{\rho g} + \frac{V^2}{2g}$ +z = Constant

$\frac{p}{w} + \frac{V^2}{2g}$ +z = Constant --- (2)

In steady, ideal flow of an incompressible fluid sum of pressure energy, kinetic energy and potential energy is constant

$\frac{p}{w} + \frac{V^2}{2g}$ + z = constant

Where, $\frac{p}{w}$ - pressure energy

$\frac{V^2}{2g}$ - Kinetic energy

Z - Potential energy (or) datum energy

Assumptions made in deriving Bernouilli's equation:

The fluid is ideal

The flow is steady.

The flow is incompressible.

The flow is irrotational.

Applications of Bernouilli's equation:

It has the application on the following measuring devices.

- **i.** Venturimeter.
- **ii.** Orifice meter.
- **iii.** Pitot tube

VENTURIMETER

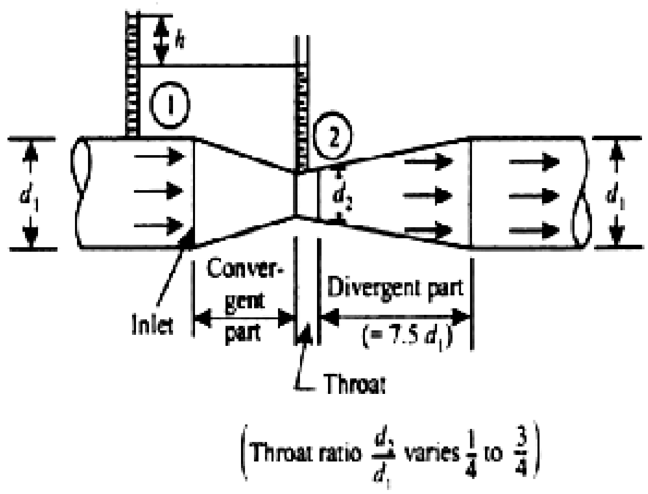


Figure 7: Venturimeter

A venturimeter is one of the most important practical applications of Bernoulli's theorem. The device venturimeter has been named after the 18th century Italian engineer Venturi. It is a device used to measure the rate of flow or discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know the discharges there. It is suitable for large pipes and high rate of flow. A venturimeter consists of three main parts: A short converging part, Throat, and Diverging part (Figure 7 Venturimeter).

Due to the reduction in area at the throat, the velocity of fluid increases and consequently pressure is reduced. This reduction in pressure is measured by means of either piezometer tubes or differential manometers. This pressure drop enables the determination of discharge through pipe. The device Venturimeter uses Bernoulli's equation to determine the flow through pipe.

DISCHARGE, Q_{th}

$$Q_{th} = \frac{a_1 a_2 \times \sqrt{2gH}}{\sqrt{a_1^2 - a_2^2}} \quad m^3/s$$

Where,

a_1 = Area of inlet pipe in m^2

a_2 = Area of the orifice in m^2

H = Manometric head in terms of flowing liquid in m

CO-EFFICIENT OF DISCHARGE (Cd)

$$Cd = Qact / Qth$$

ORIFICEMETER

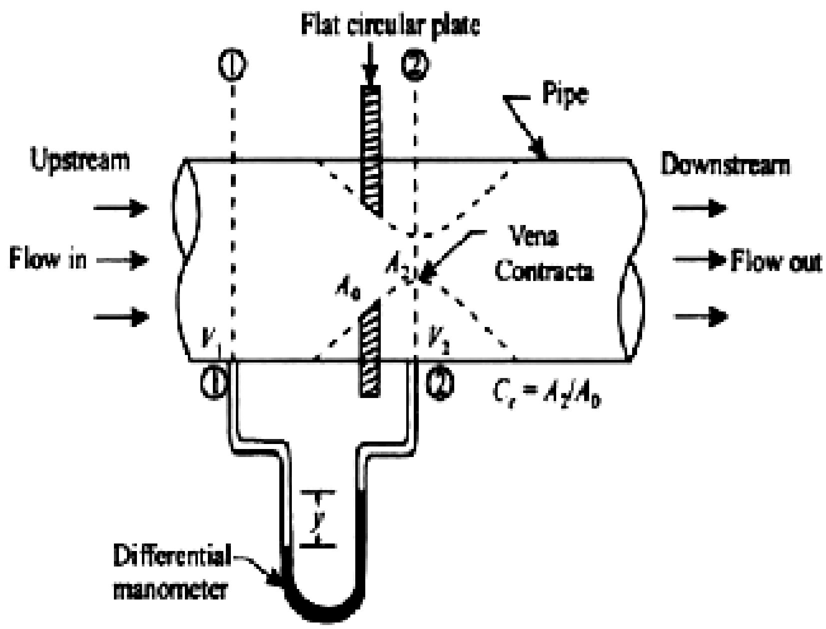


Figure 7: Orificemeter

Orifice meter a device employed for measuring the discharge or rate of flow of liquid flowing through a pipe. Orifice meter consist of a flat circular plate having a circular sharp edged hole called orifice concentric with the pipe. The diameter of the orifice may vary from 0.4 to 0.8 times the diameter of the pipe but its value is generally chosen as 0.5. A differential manometer is connected at section (1) which is at the distance of 1.5 to 2 times the pipe diameter upstream from the orifice plate, and at section (2) which is at a distance of about half the diameter of the orifice from the orifice plate on the downstream side

Orifice meter works on the principle that the reducing the cross sectional area of the flow passage, a pressure difference between the two sections is developed and the measurement of the pressure difference enable the determination of the discharge through the pipe. It uses the Bernoulli's equation to determine the flow through the pipe.

DISCHARGE, Q_{th}

$$Q_{th} = \frac{a_1 a_2 \times \sqrt{2gH}}{\sqrt{a_1^2 - a_2^2}} \quad m^3/s$$

Where,

a_1 = Area of inlet pipe in m^2

a_2 = Area of the orifice in m^2

H = Manometric head in terms of flowing liquid in m

CO-EFFICIENT OF DISCHARGE (Cd)

Cd = Qact/ Qth

PITOT TUBE

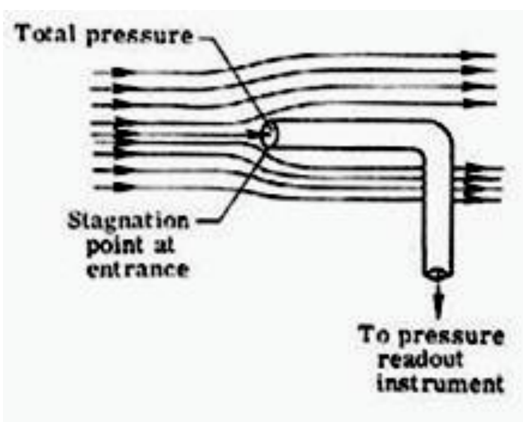


Figure 8: Pitot tube

A pitot is a pressure measurement instrument used to measure fluid flow velocity. The pitot tube was invented by the French engineer Henri Pitotin the early 18th century and was modified to its modern form in the mid-19th century by French scientist Henry Darcy. It is widely used to determine the airspeed of an aircraft, water speed of a boat, and to measure liquid, air and gas flow velocities in industrial applications. The pitot tube is used to measure the local flow velocity at a given point in the flow stream and not the average flow velocity in the pipe or conduit.

The basic pitot tube consists of a tube pointing directly into the fluid flow. As this tube contains fluid, a pressure can be measured; the moving fluid is brought to rest (stagnates) as there is no outlet to allow flow to continue. This pressure is the stagnation pressure of the fluid, also known as the total pressure or (particularly in aviation) the pitot pressure.

The measured stagnation pressure cannot itself be used to determine the fluid flow velocity (airspeed in aviation). However, Bernoulli's equation states:

Stagnation pressure = static pressure + dynamic pressure

P3 A horizontal venturimeter with inlet and throat diameters 30cm and 15cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and throat is 20 cm of mercury. Determine the rate of flow. Take C_d=0.98.

Given data <m, s>

d_1= 30cm = 0.3m $\quad a_1 = \frac{\pi d_1^2}{4}$ = 0.0707 m^2

d_2= 15cm = 0.15m $\quad a_1 = \frac{\pi d_2^2}{4}$ = 0.01777 m^2

Coefficient of Discharge, C_d = 0.98

x= 20cm of mercury= 0.2 m of mercury

Specific gravity of manometric fluid (mercury), S_m= 13.6

Specific gravity of liquid (water) in pipe, S =1

Differential pressure head, h= $x \frac{S_m}{S} - 1 = 0.2 * \frac{13.6}{1} - 1$ = 2.52m

Solution

$$Q_{act} = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh} \qquad = \frac{0.98\text{x}0.0707\text{x}0.0177}{\sqrt{0.07072 - 0.01772}} \text{ x } \sqrt{2\text{x}9.81\text{x}2.52}$$

$$\mathbf{Q_{act} = 0.1259\ m^3/s}$$

Viscous flow is a type of fluid flow in which there is a continuous steady motion of the particles, the motion at a fixed point always remaining constant.

The flow of a fluid in a pipe may be laminar flow or it may be turbulent flow. Osborne Reynolds (1842 - 1912), a British scientist and mathematician, was the first to distinguish the difference between these two classifications of flow by using a simple apparatus as shown in Fig.1.

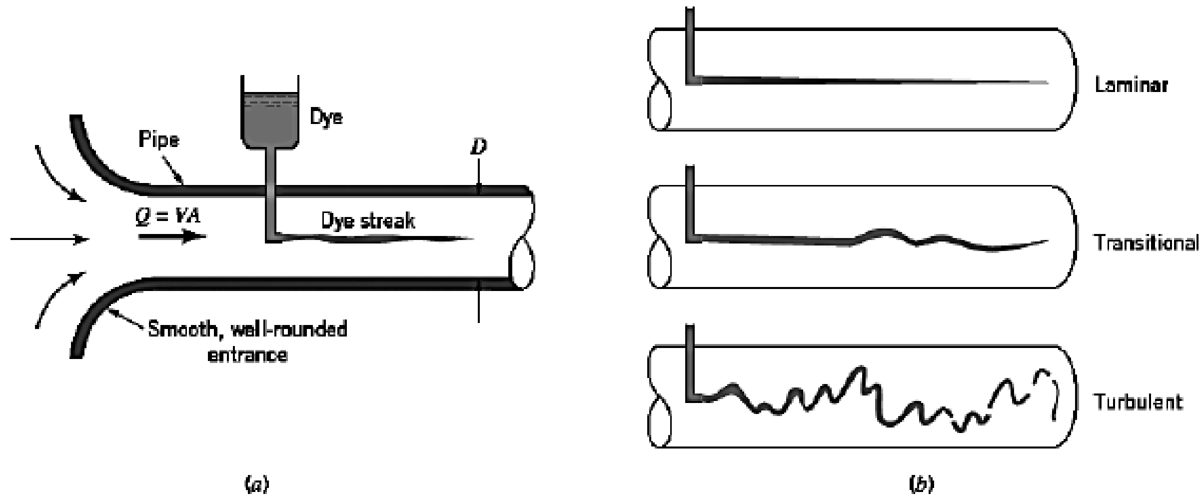


Figure 1: (a) Reynolds' experiment using water in a pipe to study transition to turbulence, (b) Typical dye streak

For the flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of pressure for a given length is to be determined. The flow through circular pipe will be viscous or laminar, if the Reynold's number is less than 2000. The expression for Reynold's number is given by

$$R_e = \frac{\rho v d}{\mu}$$

Where, ρ - Density of fluid flowing through pipe,

V - Average velocity of fluid,

D -Diameter of pipe and,

μ - Viscosity of fluid

Hagen Poiseuille Flow

- Consider **fully developed** laminar flow through a straight tube of circular cross-section as in Fig.2. Rotational symmetry is considered to make the flow two-dimensional axisymmetric.
- Let us take z-axis as the axis of the tube along which all the fluid particles travel, i.e.

$$v_z \neq 0, v_r = 0, v_\theta = 0$$

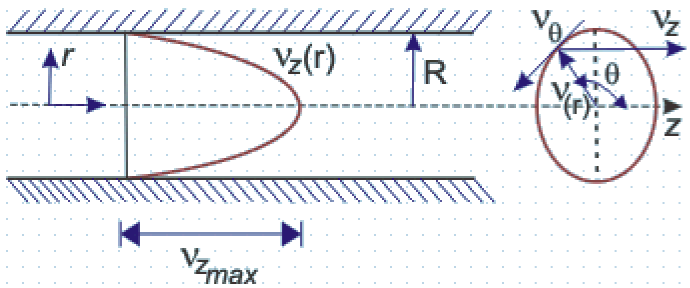


Figure 2: Hagen-Poiseuille flow through a pipe

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in figure. Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius (r+dr). Let the length of fluid element be Δx. If 'p' is the intensity of pressure on the face AB, then the intensity of pressure on the face CD will be $\left(p+\frac{\partial p}{\partial x}\Delta x\right)$.

The the forces acting on the fluid element are:

1. The pressure force, p x πr^2 on face AB

2. The pressure force $\left(p+\frac{\partial p}{\partial x}\Delta x\right)$. πr^2 on face CD

3. The shear force, $\tau \times 2\pi r \Delta x$ on the surface of fluid element. As there is no acceleration, hence the summation of all forces in the direction of flow must be zero.

$$p\ \pi r^2 - \left(p+\frac{\partial p}{\partial x}\Delta x\right).\ \pi r^2\ - \tau \times 2\pi r \Delta x\ =0$$

$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \ \tau \times 2\pi r \Delta x =0$$

$$-\frac{\partial p}{\partial x} r - 2\tau = 0$$

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{-------------------- (1)}$$

The shear stress τ across a section varies with 'r' as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress across a section is linear as shown in figure.

(i) velocity Distribution: To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \frac{\partial u}{\partial y}$ is substituted in equation (1)

But in the relation $\tau = \mu \frac{\partial u}{\partial y}$, y is measured from the pipe wall. Hence

$$y = R - r \quad \text{and} \quad dy = -dr$$

$$\tau = \mu \frac{\partial u}{-\partial r} = -\mu \frac{du}{dr}$$

substituting this value in equation (1)

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x} \frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{2\mu} \frac{\partial p}{\partial x} r$$

Integrating the equation w.r.t 'r' we get

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 + C \text{ ------------- (2)}$$

where C is the constant of integration and its value is obtained from the boundary condition that at r=R, u=0

$$0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 + C$$

$$C = -\frac{1}{4\mu} \frac{\partial p}{\partial x} r^2$$

Substituting this value of C in equation (2), we get

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} \left[R^2 - r^2\right] \text{ ----------------------- (3)}$$

In equation (3) values of μ, $\frac{\partial p}{\partial x}$ and r are constant, which means the velocity u, varies with the square of r. Thus the equation (3) is a equation of parabola. This shows that the velocity distribution across the section of a pipe is parabolic. This velocity distribution is shown in fig.

(ii) Ratio of Maximum velocity to average velocity:

The velocity is maximum, when r =0 in equation (3). Thus maximum velocity, U_{max} is obtained as

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2 \text{ ------------------- (4)}$$

The average velocity, $\bar{u}$, is obtained by dividing the discharge of the fluid across the section by the area of the pipe (πR^2). The discharge (Q) across the section is obtained by considering the through a ring element of radius r and thickness. The fluid flowing per second through the elementary ring.

dQ= velocity at a radius r x area of ring element

$$= u \times 2\pi r dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] \times 2\pi r dr$$

$$Q = \int_o^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] \times 2\pi r dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \int_0^R \left(R^2 - r^2\right) r dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \int_0^R \left(R^2 r - r^3\right) dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{4}\right] = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi R^4$$

Average velocity, $\bar{u} = \frac{Q}{Area} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^4}{\pi R^2}$

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2 \qquad \text{--------------- (5)}$$

Dividing equation (4) by equation (5)

$$\frac{U_{max}}{\bar{u}} = \frac{\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2} = 2.0$$

Ratio of maximum velocity to average velocity = 2.0

(iii) Drop of pressure for a given length (L) of a pipe:

From equation (5), we have

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2 \qquad \text{or} \qquad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

Integrating the above equation w.r.t . x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}dx$$

$$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_1]$$

$$[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2}L \qquad \{x_2 - x_1 = L \text{ from equation (3)}\}$$

$$= \frac{8\mu\overline{uL}}{\left(D/2\right)^2} \qquad \{R = \frac{D}{2}\}$$

$$[p_1 - p_2] = \frac{32\mu\overline{uL}}{D^2}, \quad \text{Where } p_1 - p_2 \text{ is the drop of pressure}$$

Loss of pressure head $= \dfrac{p_1 - p_2}{\rho g}$

$$\frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu \bar{u} L}{\rho g D^2} \quad \text{---------------------} \; (6)$$

Equation (6) is called Hagen Poiseuille Formula.

ENERGY LOSSES IN PIPE FLOW

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which happens some energy loss.

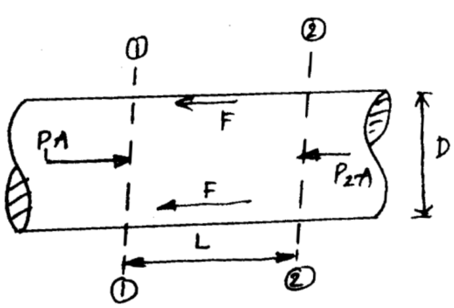
Energy losses

Major energy loss
(due to friction)

Minor energy losses
a. Sudden expansion
b. Sudden contraction
c. Bend in pipe
d. Pipe fitting
e. An obstruction in pipe

Major energy loss (Loss of energy due to friction)

Darcy - Weisbach Equation (Loss of head due to friction)



Note:

Energy, $E = FL$

$F = \frac{E}{L}$

$F \propto \frac{\frac{1}{2} m v^2}{L}$

$\propto \frac{\frac{1}{2} \cdot \rho v^2 \cdot V}{L}$

$\propto \frac{1}{2} \cdot \rho A' v^2$

$F = f \cdot \frac{1}{2} \rho A' v^2$

Let, P_1 - pressure at (1)

P_2 - pressure at (2)

V - Velocity of flow

D - Diameter of pipe

A – Cross sectional area of the pipe (=$\frac{\pi D^2}{4}$)

A_1 - Wetted area (=πDL)

f – Co-efficient of friction (or) friction factor

F - Frictional force

h_f - loss of head due to friction

The forces acting on the fluid element

1. Pressure of force(=P_1A)
2. Internal force(P_2A)
3. Dynamic frictional force ($F=f.\frac{1}{2}.\rho A_1V_2$)

Under equilibrium, ε F=0

$P_1A-P_2A-F=0$

$A (P_1-P_2) =F$

$P_1-P_2=\frac{F}{A} = \frac{f.\frac{1}{2}.\rho A1V^2}{\frac{\pi D^2}{4}} = \frac{f.\frac{1}{2}.\rho \pi DLV^2}{\frac{\pi D^2}{4}}$

$P_1-P_2=\frac{2f\rho LV^2}{D}$

$$\frac{P1-P2}{\rho g} = \frac{2f\rho LV^2}{D\rho g}$$

$$\mathbf{h_f=\frac{4fLV^2}{2Dg}} \text{ -- } (1)$$

Equation (1) is known as darcy's equation for head loss due to friction.

Where, $f=\frac{16}{Re}$ if Re<2000 (Laminar flow)

$f=\frac{0.079}{Re^{0.25}}$ if Re>4000 (Turbulent flow)

Head loss due to friction,

$$h_f = f\frac{L}{d}\frac{V^2}{2g}$$

Where,

L: length of the pipe
V: mean velocity of the flow
d: diameter of the pipe

f is the friction factor for fully developed laminar flow:

$$f = \frac{64}{\text{Re}} \ (for \ \text{Re} < 2000) \qquad \text{Re} = \frac{\rho u_{avg} d}{\mu}$$

For turbulent flow:

$$\frac{1}{\sqrt{f}} = 1.74 - 2.0 \ \log_{10}\left[\frac{\varepsilon_p}{R} + \frac{18.7}{\text{Re}\sqrt{f}}\right]$$

Moody's Diagram

R: radius of the pipe

ε_p : degree of roughness (for smooth pipe, $\varepsilon_p = 0$)

$\text{Re} \to \infty$: completely rough pipe

P1 A fluid viscosity 7 poise and specific gravity 1.3 is flowing through a pipe of 100 mm diameter at a velocity of 3.5 m/s. Calculate loss of head due to friction for the pipe length of 50m.

Given Data

Viscosity, =7poise $=\frac{7}{10}$Ns/m^2 =0.7Ns/m^2

Specific gravity, S =1.3

$\frac{\rho}{\rho water} = 1.3$

=1.3x1000=1300 kg/m^3

Diameter, D=100mm=0.1m

$A = \frac{\pi D^2}{4} = \frac{\pi(0.1)2}{4} = 7.85 \times 10^{-3} m^2$

Velocity, V= 3.5m/s

Length, L= 50m

Solution

Reynold's No., $Re = \frac{\rho VD}{\mu} = \frac{1300x3.5x0.1}{0.7} = 650$

If Re<2000, flow is laminar

$f = \frac{16}{Re} = \frac{16}{650} = .0246$

Loss of head due to friction, $h_f = \frac{4fLV^2}{2gD} = \frac{4x0.0246x50x3.5x3.5}{2x9.81x0.1} = 30.7186m$

P2 Water is flowing through a pipe of 250mm diameter and 60m long at 0.3m^3/s. Find the head loss due to friction. Assume kinematic viscosity of water as 0.012 stokes.

Given Data

$D=250mm = 0.25m$

$L=60m$

$Q=0.3\ m^3/s$

$\gamma=0.012$ stoke $= 0.01x10^{-4}m^2/s$

$A = \frac{\pi D^2}{4} = 0.049m^2$

$V = \frac{Q}{A} = 6.11\ m/s$

Solution

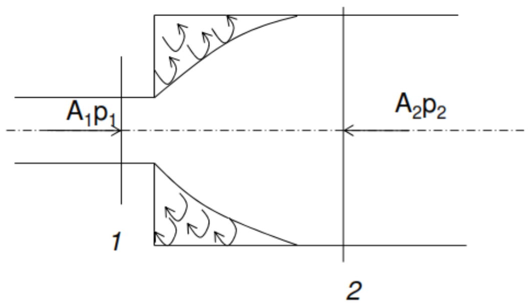
$Re=\frac{VD}{\gamma} = \frac{6.11*0.25}{0.01*10^{-4}} = 1.527x10^6$

If Re>4000, flow is turbulent

$f= \frac{0.079}{Re^{0.25}} = \frac{0.079}{(1.527x10^6)^{\wedge}0.25} = 0.0022$

Loss of head due to friction $hf=\frac{4fLV^2}{2gD} = \frac{4x0.0022x60x6.11^2}{2x9.81x0.25} = 4.1m$

MINOR ENERGY HEAD LOSS: Sudden expansion



Let A_1 = Area at section 1.
p_1 = pressure at section 1
v_1 = *velocity* at section 1

A_2, p_2, v_2 are the corresponding values at section 2.

Applying Bernoulli's equations for real fluid at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{v_1^{\,2}}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^{\,2}}{2g} + z_2 + h_e$$

But $z_1 = z_2$ as the pipe is horizontal.

$$\Rightarrow h_e = \frac{p_1 - p_2}{\rho g} + \frac{v_1^{\,2} - v_2^2}{2g} \qquad (1)$$

Consider a control volume of liquid between sections 1 and 2. Resolving the forces acting on the liquid inside the control volume, we get

$$F_x = p_1 A_1 - p_2 A_2 + p'(A_2 - A_1)$$

where p' is pressure of the liquid eddies in the area (A_2-A_1). Experimentally it is known that $p' = p_1$, hence $F_x = (p_1 - p_2)A_2$

Momentum of liquid/sec at section 1= $\rho A_1 v_1^2$

Momentum of liquid/sec at section 2= $\rho A_2 v_2^2$

Change of momentum of liquid/sec = $\rho A_2 v_2^2 - \rho A_1 v_1^2 = \rho A_2 \left(v_2^2 - v_1 v_2\right) = F_x$

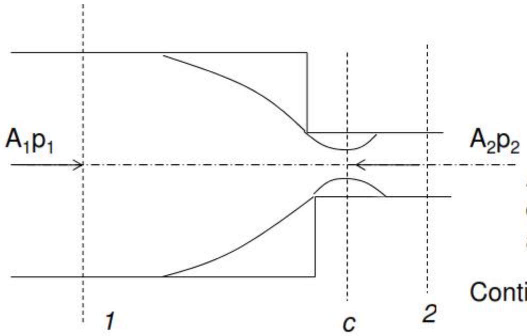
(Using the continuity equation)

$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2 - v_1 v_2}{g}$$

Thus from (1), *we obtain*

$$h_e = \frac{(v_1 - v_2)^2}{2g}$$

Minor energy head loss: Sudden contraction



Let A_1 = Area at section 1.
p_1 = pressure at section 1
v_1 = *velocity* at section 1

A_2, p_2, v_2 *and* A_c, p_c, v_c are the corresponding values at section 2 and c, respectively.

Continuity equation:

$$A_c v_c = A_2 v_2 \Rightarrow \frac{v_c}{v_2} = \frac{A_2}{A_c} = \frac{1}{C_c}$$

Head loss due to expansion from section c to 2

$$h_c = \frac{(v_c - v_2)^2}{2g} = \frac{v_2^2}{2g}\left(\frac{v_c}{v_2} - 1\right)^2 = \frac{v_2^2}{2g}\left(\frac{1}{C_c} - 1\right)^2 = k\frac{v_2^2}{2g}$$

where, $k \equiv \left(\frac{1}{C_c} - 1\right)^2$. The value of k is 0.5 to 0.7.

Head loss at the entrance of the pipe:

$$h_i = 0.5\frac{v^2}{2g}$$

Where, ***v*** is the velocity of the liquid in the pipe.

Head loss at the exit of the pipe:

$$h_0 = \frac{v^2}{2g}$$

Where, ***v*** is the velocity of the liquid at the outlet of the pipe.

Head loss due to bend in pipe:

$$h_b = \frac{kv^2}{2g}$$

Where, ***v*** is the velocity of the flow, k is the coefficient of the bend which depends on the angle of the bend, radius of curvature of the bend and diameter of pipe.

Head loss due to pipe fittings:

$$h_f = \frac{kv^2}{2g}$$

Where, $\boldsymbol{v}$ is the velocity of the flow, k is the coefficient of pipe fitting.

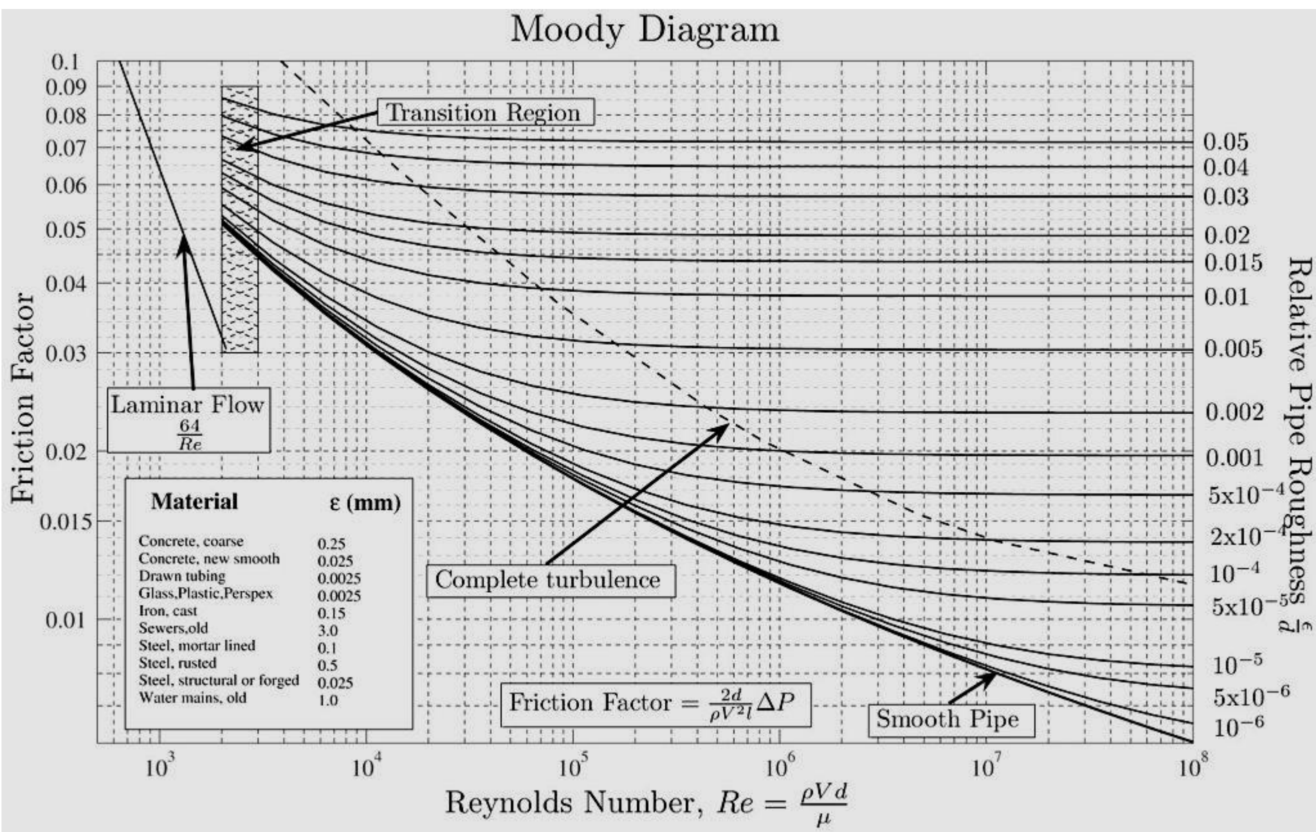
MOODY'S DIAGRAM

The Moody's chart or Moody's diagram is a graph in non-dimensional form that relates the Darcy-Weisbach friction factor, Reynolds number and relative roughness for fully developed flow in a circular pipe. It can be used for working out pressure drop or flow rate down such a pipe.

This dimensionless chart is used to work out pressure drop, δp or head loss, h_f and flow rate through pipes. Head loss can be calculated using the Darcy–Weisbach equation:

$h_f = (2fLv^2) / dg$

Where: hf = head loss (m) f = friction factor L = length of pipe work (m) d = inner diameter of pipe work (m) v = velocity of fluid (m/s) g = acceleration due to gravity (m/s^2)



FLOW THROUGH PIPES IN SERIES OR FLOW THROUGH COMPOUND PIPES:

$$\text{Head, } H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

FLOW THROUGH PARALLEL PIPES:

Loss of head for branch pipe 1= Loss of head for branch pipe 2

or $$\frac{4f_1L_1V_1^2}{d_1 \times 2g} = \frac{4f_2L_2V_2^2}{d_2 \times 2g}$$

If f_1=f_2, then $$\frac{L_1V_1^2}{d_1 \times 2g} = \frac{L_2V_2^2}{d_2 \times 2g}$$

POWER TRANSMISSION BY A PIPELINE

In certain occasions, hydraulic power is transmitted by conveying fluid through a pipeline. For example, water from a reservoir at a high altitude is often conveyed by a pipeline to an impulse hydraulic turbine in an hydroelectric power station. The hydrostatic head of water is thus transmitted by a pipeline. Let us analyse the efficiency of power transmission under this situation.

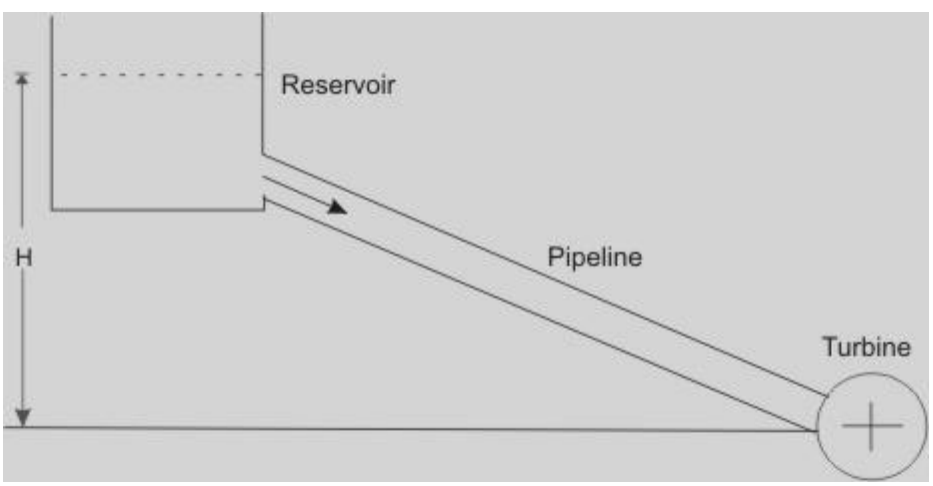


Figure 3: Transmission of hydraulic power by a pipeline to a turbine

The potential head of water in the reservoir = H (the difference in the water level in the reservoir and the turbine center)

The head available at the pipe exit (or at the turbine entry), $H_E = H - h_f$

Where, h_f is the loss of head in the pipeline due to friction.

Therefore, the power available *P* at the exit of the pipeline becomes

$$P = \rho g Q H_E$$

P1 The rate of flow of water through a horizontal pipe is 0.25 m³/s. The diameter of the pipe is suddenly enlarged from 200mm to 400mm. The pressure intensity in the smaller pipe is 11.772 N/cm². Determine: (i) Loss of head due to sudden enlargement (ii) Pressure intensity in the large pipe and (iii) Power lost due to enlargement.

Given Data

$Q = 0.25 m^3/s$

$D_1=200mm=0.2m$ $\quad A_1=\frac{\pi D_1^2}{4}=\frac{\pi 0.2^2}{4}=0.0314m^2$

$D_2=400mm=0.4m$ $\quad A_2=\frac{\pi D_2^2}{4}=\frac{\pi 0.4^2}{4}=0.1256m^2$

$P_1=11.772N/cm^2 = 11.722x10^4N/m^2$

Solution

$Q = A_1V_1 =A_2V_2$

$V_1=\frac{Q}{A1} = \frac{0.25}{0.0314} = 7.96m/s$

$V_2=\frac{Q}{A2} = \frac{0.25}{0.1256} = 7.99m/s$

(i) Loss of head due to sudden enlargement, $h_e = \frac{(V1-V2)^2}{2g}$

$= \frac{(7.96-1.99)^2}{2x9.81}$

(ii) $\frac{P1}{pg}+\frac{V_1^2}{2g}+Z_1 = \frac{P2}{pg}+\frac{V_2^2}{2g}+Z_2$ {for horizontal pipe, $Z_1=Z_2$}

$\frac{11.772x10^4}{1000x9.81}+\frac{7.96^2}{2x9.81}=\frac{P2}{1000x9.81}+\frac{1.99^2}{2x9.81}$

$P_2 = 12.96x10^4N/m^2$

(iii) Power lost = ρgQh_e = 1000x9.81x0.25x1.816 = 4453 Nm/s (or) Watts

P2 The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300m, 170m and 210m and of diameters 300mm, 200mm, 400mm respectively, is 12m. Determine the rate of flow of water if coefficients of friction are 0.005, 0.0052, 0.0048 respectively, considering (i) minor losses (ii) neglecting minor losses.

Given Data

H = 12m

L_1 = 300m, D_1 = 300mm = 0.3m, f_1 = 0.005;

L_2 = 170m, D_2 = 200mm = 0.2m, f_2 = 0.0052;

L_3 = 210m, D_3 = 400mm = 0.4m, f_3 = 0.0048;

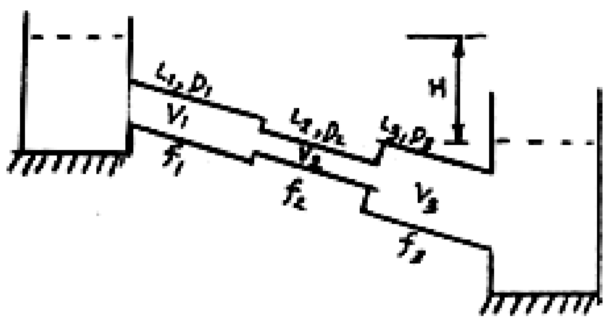
Applying Continuity equation,

$Q_1=Q_2=Q_3$

$A_1V_1=A_2V_2=A_3V_3$

$V_2=2.25\ V_1$

$V_3=0.565\ V_1$



Solution

(i) Discharge calculation - Considering minor losses also

$$H = \frac{4f_1L_1V_1^2}{2gD_1} + \frac{0.375V_2^2}{2g} + \frac{4f_2L_2V_2^2}{2gD_1} + \frac{(V2-V3)^2}{2g} + \frac{4f_3L_3V_3^2}{2gD_3}$$

$$12 = \frac{4x0.005x300xV_1^2}{2x9.81x0.3} + \frac{0.375x2.25^2xV_1^2}{2x9.81} + \frac{4x0.0052x170x2.25^2xV_1^2}{2x9.81x0.2} + \frac{(2.25-0.565)^2V_1^2}{2x9.81} + \frac{4x0.0048x210x0.565^2xV_1^2}{2x9.81x0.4}$$

$12 = 5.9867\ V_1^2$

V_1 = 1.415 m/s

$Q = A_1V_1 = \pi/4 \text{ x } D_1^2 \text{ x } V_1 = \pi/4 \text{ x } 0.32 \text{ x } 1.415 =$ **<u>0.1m³/s</u>**

(ii) Discharge calculation – Neglecting minor losses

$$H = \frac{4f_1L_1V_1^2}{2gD_1} + \frac{4f_2L_2V_2^2}{2gD_1} + \frac{4f_3L_3V_3^2}{2gD_3}$$

$$12 = \frac{4x0.005x300xV_1^2}{2x9.81x0.3} + \frac{4x0.0052x170x2.25^2xV_1^2}{2x9.81x0.2} + \frac{4x0.0048x210x0.565^2xV_1^2}{2x9.81x0.4}$$

$12=5.745V_1^2$

V_1=1.445m/s

$Q = A_1V_1 = \pi/4 \times D_1^2 \times V_1 = \pi/4 \times 0.32 \times 1.445 =$ **0.102m³/s**

TWO MARK QUESTION & ANSWERS

1. **What is the cause for major energy loss and how it can be calculated?**
   Major energy loss is due to friction and it can be calculated by Darcy weisbach formula and Chezy's formula.
2. **What are minor energy losses and what is its cause?**
   Minor energy losses in flow through pipes is due to,
   a) Sudden expansion in pipe.
   b) Sudden contraction in pipe.
   c) Bend in pipe.
   d) Due to obstruction in pipe.
3. **Write the expression for head loss due to friction using Darcy's formula.**

$$h_f = 4fLV^2 / 2gd$$

Where, h_f =head loss due to friction

f = Coefficient of friction in pipe L = Length of the pipe

d = Diameter of pipe V = velocity of the fluid

4. **What are the factors involved in the frictional loss in flow through pipes?**
   f = Coefficient of friction in pipe, L = Length of the pipe, d = Diameter of pipe, V = velocity of the fluid.
5. **Write the expression for loss of head due to sudden enlargement of the pipe.**

$$h_e = (V_1-V_2)^2 /2g$$

Where,

h_e = Loss of head due to sudden enlargement of pipe .

V_1 = Velocity of flow at the inlet of the pipe

V_2 = Velocity of flow at the outlet of the pipe

6. **Write the expression for loss of head due to sudden contraction.**

$$h_c =0.5 V^2/2g$$

Where,

h_c = Loss of head due to sudden contraction .

V = Velocity at outlet of pipe.

7. **Write the expression for loss of head at the entrance of the pipe.**

$$h_e =0.5 V^2/2g$$

Where,

h_e = Loss of head at the entrance of pipe.

V = Velocity at entrance of the pipe.

8. **Write the expression for loss of head at the exit of the pipe.**

$$h_e = V^2/2g$$

Where,

h_e = Loss of head at the exit of pipe.

V = Velocity at outlet of pipe.

9. **Write the expression for Reynold's number in flow through pipes.**

$$Re=(\rho VD)/\mu$$

Where,

Re=Reynold's number

ρ=density in kg/m^3

V=velocity in m/s

D=diameter of the pipe

μ=dynamic viscosity in Ns/m^2

10. **Give the dimensions for the terms.**

a) Density, b) Dynamic viscosity

Density in kg/m^3 – $\mathbf{ML^{-3}}$

Dynamic viscosity in Ns/m^2 – $\mathbf{ML^{-1}T^{-1}}$

11. **Give the dimensions for the terms.**

b) Specific weight, b) Pressure

Specific weight in N/m^3 – $\mathbf{ML^{-2}\,T^{-2}}$

Pressure in N/m^2 – $\mathbf{ML^{-1}T^{-2}}$

12. **What are the factors to be determined when viscous fluid flows through the circular pipe?**

The factors to be determined are;

a) Velocity distribution across the section.
b) Ratio of maximum velocity to the average velocity.
c) Shear stress distribution.
d) Drop of pressure for a given length.

13. **Write the expression to calculate the efficiency of power transmission through pipes.**

$$\textbf{Efficiency, } \eta = \{(H-h_f)/H\} \times 100\%$$

Where,

H-Total head in metre

h_f-head loss due to friction in metre

14. **What is the condition for maximum power transmission through pipes and the corresponding maximum efficiency?**

$$h_f = (H/3)$$

Where,

H-Total head in metre
h_f-head loss due to friction in metre

Corresponding Maximum efficiency is **$\eta = \{(H-h_f)/H\} \times 100\% = 66.67\%$**

15. What is flow through pipes in series or flow through compound pipes?

Pipes of different lengths and different diameters connected end to end in series to form a pipe line is known as compound pipes.

EXERCISE PROBLEMS

1. Derive Darcy – Weisbach equation to find the loss of head due to friction in pipes.
2. Water is flowing through a pipe of 250mm diameter and 60m long at a rate of 0.3m^3/s. Find the head loss due to friction. Assume the kinematic viscosity of water as 0.012 stokes.
3. An oil of viscosity 0.1 Ns/m^2 and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and of length 300m. The rate of flow of fluid through the pipe is 0.0035 m^3/s. Find the pressure drop in a length of 300m and also the shear stress at the pipe wall.
4. A pipe line of length 2000 m is used for power transmission. If 110.3625 kW power is to be transmitted through the pipe in which water having a pressure of 490.5 N/cm^2 at inlet is flowing. Find the diameter of the pipe and efficiency of transmission if the pressure drop over the length of pipe is 98.1 N/cm^2. Take f = 0.0065.
5. Water at 15^{0}C flows between two large parallel plates at a distance of 1.6mm apart. Determine (a) The maximum velocity (b) The pressure drop per unit length and (c) The shear stress at the walls of the plates if the average velocity is 0.2 m/s. The viscosity of water at 15^{0}C is given as 0.01 poise.
6. A main pipe divides into two parallel pipes which again forms one pipe. The length and diameter for the first parallel pipe are 2000m and 1 m respectively, while the length and diameter of second parallel pipe are 2000m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3 m^3/s .the coefficient of friction for each parallel pipe is same and equal to 0.005.

UNIT 3

DIMENSIONAL ANALYSIS

Dimensional analysis is a method by which one obtains certain information about a given phenomenon on the assumption that the phenomenon is governed by a dimensionally homogeneous equation amongst the dependent and the independent variables. After the dimensional analysis is carried out one gets a complete set of dimensionless parameters. ***"If the dimensions of the terms on the left hand side and right hand side of a given equation are identical the equation is said to be dimensionally homogeneous".***

All dimensionally homogeneous equations can be expressed in the form of equations involving only dimensionless groups. If the equation is dimensionally homogeneous it can be used in any consistent system of units, e.g. COS. SI or FPS and the numerical constants appearing in the equation have no dimensions; as such they remain unchanged.

Dimensional analysis is used in checking the correctness of a given equation changing the equation from one system of units to the other, simplifying theoretical analysis, reducing the number of variables with which one has to deal with in studying a problem, analysis of experimental data and in the analysis of models.

The fundamental or primary dimensions used in fluid mechanics problems are Mass - M, Length – L and Time – T.

3.1 BUCKINGHAM'S π THEOREM

If there are 'n' variables which govern a certain phenomenon and if these variables involve 'm' primary units, then the phenomenon can be described by (n - m) independent dimensionless parameters.

$$\pi = (n - m)$$

Quantity	SI Unit					Dimension
Velocity, v	m/s			ms^{-1}		LT^{-1}
Acceleration, a	m/s^2			ms^{-2}		LT^{-2}
Force, F	Newton, N	kg m/s^2		kg ms^{-2}		$M LT^{-2}$
Energy (or work)	Joule, J	N m,	kg m^2/s^2	kg m^2s^{-2}		ML^2T^{-2}
Power, P	Watt, W	N m/s	kg m^2/s^3	Nms^{-1}	kg m^2s^{-3}	ML^2T^{-3}
Pressure, p (or Shear Stress, τ)	Pascal, Pa	N/m^2	kg/ms^2	Nm^{-2}	kg $m^{-1}s^{-2}$	$ML^{-1}T^{-2}$
Density, ρ	kg/m^3			kg m^{-3}		ML^{-3}
Specific weight, w	N/m^3	$kg/m^2/s^2$		kg $m^{-2}s^{-2}$		$ML^{-2}T^{-2}$

Relative density, S	a ratio no units				no dimension
Viscosity. μ	N s/m^2	kg/m s	N sm^{-2}	kg $m^{-1}s^{-1}$	M $L^{-1}T^{-1}$
Surface tension, σ	N/m	kg /s^2	Nm^{-1}	kg s^{-2}	MT^{-2}

Quantity	Notation	Mass system
		Geometric variables
Area	a, A	L^2
Angle	α, θ	$M^0L^0T^0$
Length	l	L
Volume	$\forall$	L^3
		Variables related to fluid
Dynamic viscosity	μ	$ML^{-1}T^{-1}$
Kinematic viscosity	ν	L^2T^{-1}
Mass density	ρ	ML^{-3}
Specific weight	γ	$ML^{-2}T^{-2}$
Surface tension	σ	MT^{-2}
Volume modulus of elasticity	E	$ML^{-1}T^{-2}$
		Variables related to flow
Acceleration	a	LT^{-2}
Angular acceleration	—	T^{-2}
Angular momentum	—	ML^2T^{-1}
Angular velocity	ω	T^{-1}
Force	F	MLT^{-2}
Frequency	f	T^{-1}
Moment	—	ML^2T^{-2}
Momentum	M	MLT^{-1}
Power	P	ML^2T^{-3}
Pressure or stress	p	$ML^{-1}T^{-2}$

Rotation	—	T^{-1}
Strain	—	$M^0L^0T^0$
Torque	T	ML^2T^{-2}
Velocity	U, V	LT^{-1}
Work or energy	W	ML^2T^{-2}

DETERMINING THE π GROUPS

Regardless of the method to be used to determine the dimensionless parameters, one begins by listing all dimensional parameters that are known (or believed) to affect the given flow phenomenon. Some experience admittedly is helpful in compiling the list.

Students, who do not have this experience, often are troubled by the need to apply engineering judgment in an apparent massive dose. However, it is difficult to go wrong if a generous selection of parameters is made. If you suspect that a phenomenon depends on a given parameter, include it. If your suspicion is correct, experiments will show that the parameter must be included to get consistent results. If the parameter is extraneous, an extra Π parameter may result, but experiments will later show that it may be eliminated from consideration. Therefore, do not be afraid to include all the parameters that you feel are important.

The six steps listed below outline a recommended procedure for determining the π parameters:

Step 1: List all the dimensional parameters involved. (Let n be the number of parameters.)

If the pertinent parameters are not all included, a relation may be obtained, but it will not give the complete story. If parameters that actually have no effect on the physical phenomenon are included, either the process of dimensional analysis will show that these do not enter the relation sought, or one or more dimensionless groups will be obtained that experiments will show to be extraneous.

Step 2: Select a set of fundamental (primary) dimensions, e.g., MLT.

Step 3: List the dimensions of all parameters in terms of primary dimensions. (Let r be the number of primary dimensions.) Either force or mass may be selected as a primary dimension.

Step 4: Select a set of r dimensional parameters that includes all the primary dimensions. These parameters will all be combined with each of the remaining parameters, one of those at a time, and .so will be called repeating parameters. No repeating parameter should have dimensions that are a power of the dimensions of another repeating parameter; for example, do not include both a length (L) and a moment of inertia of an area (L^4) as repeating parameters. The repeating parameters chosen may appear in all the dimensionless groups obtained; consequently, do not include the dependent parameter among those selected in this step.

Step 5: Set up dimensional equations, combining the parameters selected in Step 4 with each of the other parameters in turn, to form dimensionless groups. (There will be n - m equations.) Solve the dimensional equations to obtain the n - m dimensionless groups.

Step 6: Check to see that each group obtained is dimensionless. If mass was initially selected as a primary dimension, it is wise to check the groups using force as a primary dimension, or vice versa.

METHOD OF REPEATING VARIABLES

After the variables governing the phenomenon are identified, choose repeating variables equal in number to the number of primary units (or rank of dimensional matrix) used in describing the variables. Do not choose the dependent variable as one of the repeating variables. The repeating variables should contain the entire primary units among themselves and they should not combine among themselves to form a dimensionless group. For most of the problems in fluid mechanics, a characteristic geometric variable such as length or area, one fluid property such as mass density, and one flow characteristic such as velocity are a good choice. Combine each non-repeating variable with repeating variables to form dimensionless groups.

SIMILITUDE

Model studies are usually conducted to find solutions to numerous complicated problems in hydraulic engineering and fluid mechanics. In order that results obtained in the model studies correctly represent the behaviour of the prototype (actual structure), the following three similarities must be ensured between the model and the prototype.

Geometric similarity

For geometric similarity to exist between the model and the prototype the ratios of corresponding lengths in the model and in the prototype must be same and the included angles between two corresponding sides must be the same. Models which are Dot geometrically similar are known as geometrically distorted models.

Kinematic similarity

Kinematic similarity is similarity of motion, If at the corresponding (or homologous) points in the model and in the prototype the velocity or acceleration ratios are same and there the velocity or acceleration vectors point in the same direction, the two flows are said to be kinematically similar. When kinematic similarity exists the flow patterns of the two flows can be superimposed by change of length scale.

Dynamic similarity

Dynamic similarity is the similarity of forces. The flows in the model and in the prototype are dynamically similar if at all the corresponding points identical types of forces are parallel and. bear the same ratio. In dynamic similarity, the force polygons of the two flows can be superimposed by change in force scale.

IMPORTANT DIMENSIONLESS PARAMETERS

Dimensional analysis will yield various dimensionless parameters. Some will be ratios of length, mass densities specific weights and velocities etc. 1be force ratios obtained in dimensional analysis acquire great significance. Their structure and significance are given below:

$$\text{Reynolds number } Re = \frac{\text{Inertial force/volume}}{\text{Viscous force/volume}} = \frac{Ul\rho}{\mu}$$

$$\text{Froude number } Fr = \sqrt{\frac{\text{Inertial force/volume}}{\text{Gravity force/volume}}} = \frac{U}{\sqrt{gl}}$$

$$= \frac{\text{Flow velocity}}{\text{Celerity of a small gravity wave}}$$

$$\text{Euler number } E = \sqrt{\frac{\text{Inertial force/volume}}{\text{Pressure force/volume}}} = \frac{U}{\sqrt{\Delta p/\rho}}$$

$$\text{Drag coefficient } C_D = \frac{\text{External force/volume}}{\text{Inertial force/volume}} = \frac{F}{l^2\rho U^2/2}$$

$$\text{Weber number } W = \frac{\text{Inertial force/volume}}{\text{Surface tension force/volume}} = \frac{\rho U^2 l}{\sigma}$$

$$= \left(\frac{\text{Flow velocity}}{\text{Velocity of a small ripple on liquid surface}}\right)^2$$

$$\text{Mach number } M = \sqrt{\frac{\text{Inertial force/volume}}{\text{Compressibility force/volume}}} = \frac{U}{\sqrt{E/\rho}}$$

$$= \text{flow velocity/velocity of pressure wave in fluid}$$

P1 Find an expression for the drag force for a smooth sphere of diameter (D) moving with uniform velocity (V) in fluid density (ρ) and dynamic viscosity (μ).

Given data

Type of variable	Physical Quantity	Dimension	Type of Property
Dependent	Drag force(F)	MLT^{-2}	-
Independent	Diameter(D)	L	Geometric
	Velocity(V)	ML^{-1}	Kinematic
	Density(ρ)	ML^{-3}	Dynamic
	Viscosity(ρ)	$ML^{-1}T^{-1}$	Dynamic

Solution

F=f (D, V, ρ, μ)

f (F, D, V, ρ, μ)=0

No. of variables, n=5

No. of fundamental dimensions, m=3

No. of repeating variable=3

Repeating variables = {D,V,ρ} other variables={F, μ}

No. of π terms=m-n=5-3=2

F (π_1, π2) =0

$\pi_1=D^{a1}V^{b1}\rho^{c1}F$

$\pi_2=D^{a2}V^{b2}\rho^{c2}\mu$

Π1 term

$\pi_1=D^{a1}V^{b1}\rho^{c1}F$

$M^0L^0T^0=L^{a1}\ [LT^{-1}]^{b1}\ [ML^{-3}]^{c1}\ MLT^{-2}$

Power of M -> 0 = c1+1

Power of L-> 0 = a1+b1-3c1+1

Power of T-> 0=-b1-2

a1=-2 b1=-2 c1=-1

$\pi_1=D^{-2}V^{-2}\rho^{-1}F = \frac{F}{D2V2\rho}$

Π1 term

$\pi2=D^{a2}V^{b2}\rho^{c2}F$

$M^0L^0T^0=L^{a2}\ [LT^{-1}]^{b2}\ [ml^{-3}]^{c2}\ ML^{-1}T^{-1}$

Power of M -> 0 = c2+1

Power of L-> 0 = a2+b2-3c2-1

Power of T-> 0=-b2-1

a1=-1   b1=-1   c1=-1

$\pi_2=D^{-1}V^{-1}\rho^{-1}\mu = \frac{\mu}{DV\rho}$

f (π1,π2)=0

f $\left(\frac{F}{D2V2\rho}, \frac{\mu}{DV\rho}\right)$=0

$\frac{F}{D2V2\rho}$ = f $\left(\frac{\mu}{DV\rho}\right)$

$$\mathbf{F=D_2V_2\rho\ f\left(\frac{\mu}{DV\rho}\right)}$$

P2 Obtain an expression in non-dimensional form for the pressure gradient in a horizontal pipe of circular cross-section. Show how this relates to the familiar expression for frictional head loss.

Step 1. Identify the relevant variables.

$$\mathrm{d}p/\mathrm{d}x,\ \rho,\ V,\ D,\ k_s,\ \mu$$

Step 2. Write down dimensions.

$$\frac{\mathrm{d}p}{\mathrm{d}x} \qquad \frac{[force/area]}{length} = \frac{\mathrm{MLT^{-2}\times L^{-2}}}{\mathrm{L}} = \mathrm{ML^{-2}T^{-2}}$$

ρ	$\mathrm{ML^{-3}}$
V	$\mathrm{LT^{-1}}$
D	L
k_s	L
μ	$\mathrm{ML^{-1}T^{-1}}$

Step 3. Establish the number of independent dimensions and non-dimensional groups.

Number of relevant variables: $n = 6$

Number of independent dimensions: $m = 3$ (M, L and T)

Number of non-dimensional groups (Πs): $n - m = 3$

Step 4. Choose m (= 3) dimensionally-independent scaling variables.

e.g. geometric (D), kinematic/time-dependent (V), dynamic/mass-dependent (ρ).

Step 5. Create the Πs by non-dimensionalising the remaining variables: $\mathrm{d}p/\mathrm{d}x$, k_s and μ.

$$\Pi_1 = \frac{\mathrm{d}p}{\mathrm{d}x} D^a V^b \rho^c$$

Considering the dimensions of both sides:

$$\mathrm{M^0L^0T^0} = (\mathrm{ML^{-2}T^{-2}})(\mathrm{L})^a(\mathrm{LT^{-1}})^b(\mathrm{ML^{-3}})^c$$

$$= \mathrm{M}^{1+c}\mathrm{L}^{-2+a+b-3c}\mathrm{T}^{-2-b}$$

Equate powers of primary dimensions. Since M only appears in [ρ] and T only appears in [V] it is sensible to deal with these first.

M: $0 = 1 + c \quad \Rightarrow \quad c = -1$

T: $0 = -2 - b \quad \Rightarrow \quad b = -2$

L: $0 = -2 + a + b - 3c \quad \Rightarrow \quad a = 2 - b + 3c \ = 1$

Hence,

$$\Pi_1 = \frac{\mathrm{d}p}{\mathrm{d}x} D V^{-2} \rho^{-1} = \frac{D\dfrac{\mathrm{d}p}{\mathrm{d}x}}{\rho V^2}$$ (Check: OK – ratio of two pressures)

$$\Pi_2 = \frac{k_s}{D}$$ (by inspection, since k_s is a length)

$$\Pi_3 = \mu D^a V^b \rho^c$$

In terms of dimensions:

$$M^0L^0T^0 = (ML^{-1}T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c$$
$$= M^{1+c}L^{-1+a+b-3c}T^{-1-b}$$

Equating exponents:

M: $0 = 1 + c \quad \Rightarrow \quad c = -1$

T: $0 = -1 - b \quad \Rightarrow \quad b = -1$

L: $0 = -1 + a + b - 3c \quad \Rightarrow \quad a = 1 - b + 3c \quad = -1$

Hence,

$$\Pi_3 = \frac{\mu}{\rho V D}$$ (Check: OK – this is the reciprocal of the Reynolds number)

Step 6. Set out the non-dimensional relationship.

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

or

$$\frac{D\frac{dp}{dx}}{\rho V^2} = f(\frac{k_s}{D}, \frac{\mu}{\rho V D})$$

Step 7. Rearrange (if required) for convenience.

We are free to replace any of the Πs by a power of that Π, or by a product with the other Πs, provided we retain the same number of independent dimensionless groups. In this case we recognise that Π_3 is the reciprocal of the Reynolds number, so it looks better to use $\Pi_3' = (\Pi_3)^{-1} = \text{Re}$ as the third non-dimensional group. We can also write the pressure gradient in terms of head loss: $\frac{dp}{dx} = \rho g \frac{h_f}{L}$. With these two modifications the non-dimensional relationship (*) then becomes

$$\frac{g h_f D}{L V^2} = f(\frac{k_s}{D}, \text{Re})$$

or

$$h_f = \frac{L}{D} \times \frac{V^2}{g} \times f(\frac{k_s}{D}, \text{Re})$$

Since numerical factors can be absorbed into the non-specified function, this can easily be identified with the Darcy-Weisbach equation

$$h_f = \lambda \frac{L}{D} \frac{V^2}{2g}$$

where λ is a function of relative roughness k_s/D and Reynolds number Re, a function given (Topic 2) by the Colebrook-White equation.

TWO MARK QUESTION & ANSWERS

1. **What are the types of fluid flow?**
   - **a)** Steady & unsteady fluid flow
   - **b)** Uniform & Non-uniform flow
   - **c)** Laminar & Turbulent flow
   - **d)** Compressible & Incompressible flow
   - **e)** One dimensional, two-dimensional & three-dimensional flows
   - **f)** Rotational & Irrotational flow
2. **Name the different types of forces present in the fluid flow.**
   - **a)** Inertia force
   - **b)** Viscous force
   - **c)** Surface tension force
   - **d)** Gravity force
3. **What are the assumptions made in deriving Bernouilli's equation?**
   - **a)** The fluid is ideal
   - **b)** The flow is steady.
   - **c)** The flow is incompressible.
   - **d)** The flow is irrotational.
4. **List the applications of Bernouillie's equation.**

   It has the application on the following measuring devices.
   - **iv.** Orifice meter.
   - **v.** Venturimeter.
   - **vi.** Pitot tube.
5. **State Buckingham's Π theorem**

   It states that if there are 'n' variables in a dimensionally homogeneous equation and if these variables contain 'm' fundamental dimensions (M,L,T), then they are grouped into (n-m), dimensionless independent Π-terms.

   $$\Pi = (n-m)$$
6. **What is the need of studying dimensional analysis?**

   Dimensional analysis is a mathematical technique which makes use of the study of dimensions as an aid to solution of several engineering problems. It plays an important role in research work.
7. **Mention the methods available for dimensional analysis.**
   - **a)** Rayleigh method,
   - **b)** Buckingham π method
8. **Define dimensional homogeneity.**

   An equation is said to be dimensionally homogeneous if the dimensions of the terms on its Left Hand Side are same as the dimensions of the terms on its Right Hand Side. **Eg:** $LT^{-1} = LT^{-1}$

9. List the fundamental quantities in dimensional analysis.

Mass (M), Length (L), Time (T) are the fundamental quantities.

10. What is Laminar flow or viscous flow or stream line flow?

The flow in a well defined path is known as laminar flow or stream line flow or viscous flow.

11. What is turbulent flow?

The flow in an undefined path is known as turbulent flow. It is otherwise known as zig-zag flow.

12. Mention the range of Reynold's number for laminar and turbulent flow in a pipe.

a) If the Reynold's number is less than 2000, the flow is laminar.

b) If the Reynold's number is greater than 4000, the flow is turbulent flow.

c) If the Reynold's number is between 2000 to 4000 the flow is either laminar or turbulent.

13. What is the difference between laminar flow and turbulent flow?

Sl. No.	Laminar flow	Turbulent flow
1.	The flow in a well defined path	The flow in an undefined path
2.	Reynold's number is less than 2000, the flow is laminar	Reynold's number is greater than 4000, the flow is turbulent

11. Define Steady flow.

Steady flow means velocity (V), pressure (p), density (ρ) of the fluid particle does not changes with respect to time.

$$\partial V/\partial t = 0, \partial p/\partial t = 0, \partial \rho/\partial t = 0$$

12. Define unsteady flow.

Unsteady flow means velocity (V), pressure (p), density (ρ) of the fluid particle changes with respect to time.

$$\partial V/\partial t \neq 0 \quad \partial p/\partial t \neq 0 \quad \partial \rho/\partial t \neq 0$$

14. Define discharge or rate of flow.

Discharge is defined as the quantity of fluid flowing per second through a section of a pipe or channel.

15. Define Compressible flow.

If the density (ρ) of the fluid changes point to point the flow is said to be compressible flow.

$$\rho \neq constant$$

16. Define Incompressible flow.

If the density (ρ) of the fluid does not change point to point the flow is said to be incompressible flow.

$$\rho = constant$$

EXERCISE PROBLEMS

1. The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water following through the pipe at section 1 is 5 m/s.
2. A 30 cm diameter pipe conveying water branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in 30 cm diameter pipe is 2.5 m/s, find the discharge in the pipe. Also, determine velocity in 15 cm pipe if average velocity in 20 cm diameter pipe is 2 m/s.
3. Derive Euler's equation for the flow of an incompressible frictionless fluid from consideration of momentum.
4. Water flows through a pipe AB 1.2 m diameter at 3 m/s and then passes through a pipe BC 1.5m diameter. At C, the pipe branches. Branch CD is 0.8 m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.
5. Derive Bernoulli's equation for the flow of an incompressible frictionless fluid from consideration of momentum.
6. A 30cm diameter pipe, conveying water, branches into two pipes of diameters 20 cm and 15 cm respectively. If the average velocity in the 30 cm diameter pipe is 2.5 m/s. Find the discharge in this pipe, also determine the velocity in 15 cm pipe if the average velocity in 20 cm diameter pipe is 2 m/s.
7. A pressure difference (ΔP) in a pipe of diameter (D) and length (L) due to turbulent flow depends on velocity (V), viscosity (μ), density (ρ) and surface roughness (k). Using Buckingham π theorem, derive an expression for ΔP.
8. Using Buckingham's π theorem, find an expression for the drag force for a smooth sphere of diameter (D) moving with uniform velocity (V) in fluid density (ρ) and dynamic viscosity (μ).
9. The resisting force (R) of a supersonic plane during flight can be considered as dependent upon length of the aircraft (L), Velocity (V), air viscosity (μ), air density (ρ) and bulk modulus (K). Express the functional relationship between these variables and resisting force.

UNIT 4

HYDRAULIC TURBINES

The device which converts hydraulic energy into mechanical energy or vice versa is known as **Hydraulic Machines.** The hydraulic machines which convert hydraulic energy into mechanical energy are known as **Turbines** and that convert mechanical energy into hydraulic energy is known as **Pumps.**

The hydraulic turbines are also known as 'water turbines' since the fluid medium used in them is water.

1. CLASSIFICATION OF HYDRAULIC TURBINES

The hydraulic turbines are classified as follows:

1. According type of energy at inlet of the turbine
   a. Impulse turbine
   b. Reaction turbine
2. According to the direction of the flow of water
   a. Tangential flow turbine
   b. Radial flow turbine
   c. Axial flow turbine
   d. Mixed flow turbine
3. According to the head at the inlet of the turbine
   a. High head turbine e.g: Pelton turbine.
   b. Medium head turbine e.g : Francis turbine.
   c. Low head turbine e.g: Kaplan turbine, propeller turbine.
4. According to the specific sped of the turbine
   a. Low specific speed turbine
   b. Medium specific speed turbine
   c. High specific turbine

If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as **impulse turbine**. As the water flows over the vanes, the pressure is atmospheric from inlet to outlet of the turbine. In the impulse turbine, all the potential (pressure) energy of water is converted into kinetic (velocity) energy in the nozzle before striking the turbine wheel buckets. Hence an impulse turbine requires high head and low discharge at the inlet. The water as it flows over the turbine blades will be at the atmospheric pressure. The impulse turbine may be radial flow or tangential flow type.

(Impulse Turbine - Example: Pelton Wheel Turbine)

If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as **reaction turbine**. As the waters flows through the runner, the water is under pressure and the pressure energy goes on changing into kinetic energy. The runner is completely enclosed in an air tight casing and the runner and casing is completely full of water.

(Reaction Turbine – Example: Francis Turbine, Kaplan Turbine)

If the water flows along the tangent of the runner, the turbine is known as tangential flow turbine. If the water flows in the radial direction through the runner, the turbine is called radial flow turbine. If the water flows from outwards to inwards, radially the turbine is called inward radial flow turbine, on the other hand, if the water flows radially from inwards to outwards, the turbine is known as outward radial flow turbine.

Table 1 Classification of hydraulic turbines based on application

Turbine Name	Turbine Type	Type of energy	Head	Discharge	Direction of flow	Specific Speed
Pelton Wheel	Impulse	Kinetic	High Head > 250m to 1000m	Low	Tangential to runner	Low <35 Single jet 35 – 60 Multiple jet
Francis Turbine	Reaction Turbine	Kinetic + Pressure	Medium 60 m to 150 m	Medium	Radial flow / Mixed Flow	Medium 60 to 300
Kaplan Turbine	Reaction Turbine	Kinetic + Pressure	Low < 30 m	High	Axial Flow	High 300 to 1000

If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called axial flow turbine. If the water flows through the runner in radial direction but leaves in the direction parallel to axis of rotation of the runner, the turbine is called mixed flow turbine.

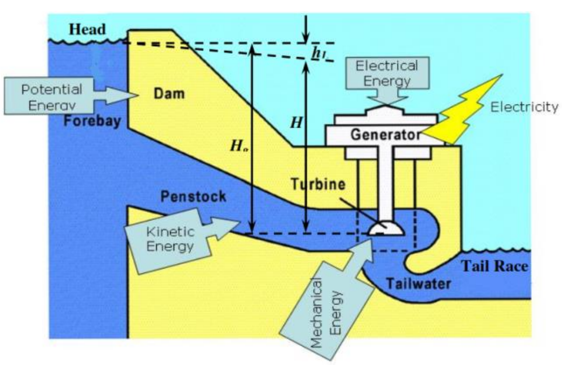


Fig.1a General Layout of Hydro electric power plant

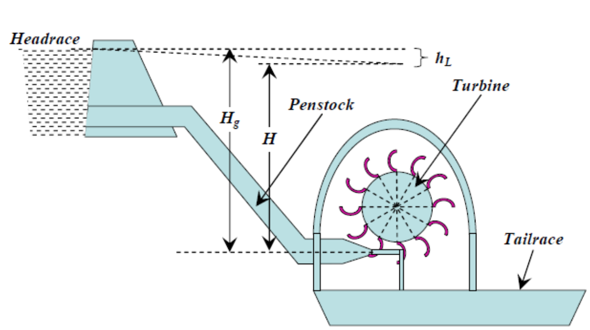


Fig 1b Jet of water striking on a turbine

Figure 1a and 1b, shows the general layout of a hydro electric power plant. A hydro electric power plant consists of the following:

- A ***Dam*** constructed across a river or a channel to store water. The reservoir is also known as ***Headrace.***
- Pipes of large diameter called ***Penstocks*** which carry water under pressure from storage reservoir to the turbines. These pipes are usually made of steel or reinforced concrete.

- ***Turbines*** having different types of vanes or buckets or blades mounted on a wheel called runner.
- ***Tailrace,*** which is a channel carrying water away from the turbine after the water has worked on the turbines. The water surface in the tailrace is also referred to as tailrace.

2. IMPORTANT TERMS:

Gross Head (H_g): It is the vertical difference between headrace and tailrace.

Net Head :(H): Net head or effective head is the actual head available at the inlet of the to work on the turbine.

$$H = H_g - h_L$$

Where h_L is the total head loss during the transit of water from the headrace to tailrace which is mainly head loss due to friction, and is given by,

$$h_f = \frac{4 f L V^2}{2 g d}$$

Where,

f is the coefficient of friction of penstock depending on the type of material of penstock

L is the total length of penstock

V is the mean flow velocity of water through the penstock

D is the diameter of penstock and

g is the acceleration due to gravity

3. TYPES OF EFFICIENCIES

Depending on the considerations of input and output, the efficiencies can be classified as

i. Hydraulic Efficiency
ii. Mechanical Efficiency
iii. Overall efficiency

(i) Hydraulic Efficiency: (η_h)

It is the ratio of the power developed by the runner of a turbine to the power supplied at the inlet of a turbine. Since the power supplied is hydraulic, and the probable loss is between the striking jet and vane it is rightly called hydraulic efficiency.

If R.P. is the Runner Power and W.P. is the Water Power

$$\eta_h = \frac{\text{R.P.}}{\text{W.P.}}$$

(ii) Mechanical Efficiency: (η_m)

It is the ratio of the power available at the shaft to the power developed by the runner of a turbine. This depends on the slips and other mechanical problems that will create a loss of energy between the runner in the annular area between the nozzle and spear, the amount of water reduces as the spear is pushed forward and vice-versa and shaft which is purely mechanical and hence mechanical efficiency.

If S.P. is the Shaft Power

$$\eta_m = \frac{\text{S.P.}}{\text{R.P.}}$$

(iii) Overall Efficiency: (η)

It is the ratio of the power available at the shaft to the power supplied at the inlet of a turbine. As this covers overall problems of losses in energy, it is known as overall efficiency. This depends on both the hydraulic losses and the slips and other mechanical problems that will create a loss of energy between the jet power supplied and the power generated at the shaft available for coupling of the generator.

$$\eta = \frac{\text{S.P.}}{\text{W.P.}}$$

(Or)

$$\boldsymbol{\eta = \eta_h \times \eta_m}$$

4. SPECIFIC SPEED (Ns)

The specific speed of a turbine is defined as, the speed of a geometrically similar turbine that would develop unit power when working under a unit head (1m head). It is prescribed by the relation,

$$N_S = \frac{N\sqrt{P}}{H^{5/4}}$$

Where,

Ns – Specific Speed, N – Speed in rpm, H – Head in metre

5. PELTON WHEEL TURBINE

The Pelton wheel or Pelton turbine is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmosphere. This turbine is used for high heads and is named after L.A. Pelton, an American Engineer.

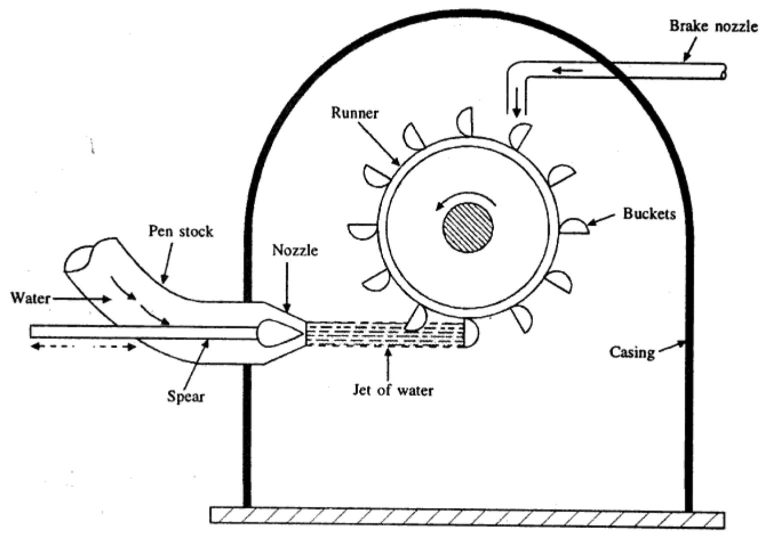


Figure 2: Pelton wheel turbine

A pelton wheel consists of a rotor, at the periphery of which is mounted equally spaced double hemispherical or double ellipsoidal buckets. Water is transferred from a high head source through penstock which is fitted with a nozzle, through which the water flows out as a high speed jet. A needle spear moving inside the nozzle controls the water flow through the nozzle and at the same time provides a smooth flow with negligible energy loss. All the available potential energy is thus converted into kinetic energy before the jet strikes the buckets of the runner. The pressure all over the wheel is constant and equal to atmosphere, so that energy transfer occurs due to purely impulse action.

The pelton turbine is provided with a **casing** the function of which is to prevent the splashing of water and to discharge water to the tail race.

When the nozzle is completely closed by moving the spear in the forward direction the amount of water striking the runner is reduced to zero but the runner due to inertia continues revolving for a long time. In order to bring the runner to rest in a short time, a nozzle (brake) is provided which directs the jet of water on the back of buckets; this jet of water is called **braking jet**.

Speed of the turbine runner is kept constant by a governing mechanism that automatically regulates the quantity of water flowing through the runner in accordance with any variation of load.

Figure 2 shows a schematic diagram of a pelton wheel. The jet emerging from the nozzle hits the splitter symmetrically and is equally distributed into the two halves of hemispherical bucket as shown. The bucket centre line cannot be made exactly like a mathematical cusp, partly because of manufacturing difficulties and partly because the jet striking the cusp invariably carries particles of sand and other abrasive material which tend to wear it down.

WORKING OF A PELTON WHEEL TURBINE

Water at high pressure from the penstock pipe enters the nozzle provided with a spear. The pressure energy of water is converted into velocity energy, as it flows through the nozzle. By rotating the hand wheel, the spear is moved to control the quantity of water flowing out of the nozzle. When the spear is pushed forward into the nozzle, the amount of water striking the buckets is reduced.

The jet of water at high velocity from the nozzle strikes the buckets at the center of the cup. The impulsive force of the jet striking on the buckets causes the rotation of the wheel in the direction of the striking jet. Thus, pressure energy of the water is converted into mechanical energy. The pressure inside the casing is atmospheric.

The pelton wheel operates under a high head of water. Therefore it requires less quantity of water. Draft tubes are not usually used with it.

6. REACTION TURBINE

If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as reaction turbine. As the waters flows through the runner, the water is under pressure and the pressure energy goes on changing into kinetic energy. The runner is completely enclosed in an air tight casing and the runner and casing is completely full of water.

If the water flows along the tangent of the runner, the turbine is known as tangential flow turbine. If the water flows in the radial direction through the runner, the turbine is called radial flow turbine. If the water flows from outwards to inwards, radially the turbine is called inward radial flow turbine, on the other hand, if the water flows radially from inwards to outwards, the turbine is known as outward radial flow turbine.

6.1. FRANCIS TURBINE

Francis turbine was developed by the American engineer Francis in 1850. It is an inward flow radial type reaction turbine. It operates under medium head.

The main parts of a radial flow reaction turbine are: Casing, guide mechanism, runner and draft tube.

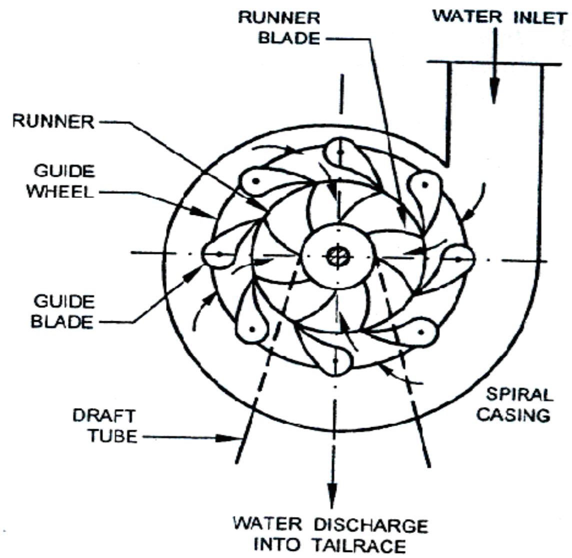


Figure 3: Francis Turbine

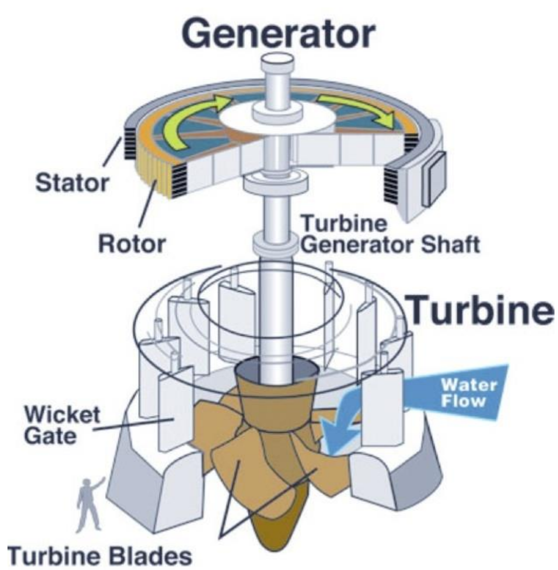


Figure 4: Cut Section of Francis Turbine

Casing: As mentioned above that in case of reaction turbine, casing and runner are always full of water. The water from the penstocks enters the casing which is of spiral shape in which area of cross section of the casing goes on decreasing gradually. The casing completely surrounds the runner of the turbine. The casing as shown in fig.3 is made of spiral shape, so that the water may enter the runner at constant velocity throughout the circumference of the runner. The casing is made of concrete, cast steel or plate steel.

Guide mechanism: It consists of a stationary circular wheel all round the runner of the turbine. The stationary guide vanes are fixed on the guide mechanism. The guide vanes allow the water to strike the vanes fixed on the runner without shock at inlet. Also by a suitable arrangement, the width between two adjacent vanes of guide mechanism can be altered so that the amount of water striking the runner can be varied.

Runner: It is a circular wheel on which a series of radial curved vanes are fixed. The surfaces of the vanes are made very smooth. The radial curved vanes are so shaped that the water enters and leaves the runner without shock. The runner is made of cast steel, cast iron or stainless steel. They are keyed to the shaft.

Draft tube: The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. The water at exit cannot be directly discharged to the tail race. A tube or pipe of gradually increasing area is used for discharging water from the exit of the turbine to the tail race. This tube of increasing area is called draft tube.

Working of Francis Turbine: First, water enters the guide blades, which guide the water to enter the moving blades. In the moving blades, part of the pressure energy is converted into kinetic energy, which causes rotation of the runner. Water leaving the moving blades is at a low pressure. Thus, there is a pressure difference between the entrance and the exit of the moving blades. This difference in pressure is called reaction. Pressure acts on moving blades and causes the rotation of the wheel in the opposite direction.

6.2 KAPLAN TURBINE

The reaction turbine developed by an Australian Professor Victor Kaplan in the year 1913 is an improved version of the older propeller turbine. It is particularly suitable for generating hydropower in locations where large quantities of water are available under a relatively low head. Consequently the specific speed of these turbines is high, viz., 300 to 1000. As in the case of a Francis turbine, the Kaplan turbine is provided with a spiral casing, guide vane assembly and a draft tube. The blades of a Kaplan turbine, three to eight in number are pivoted around the central hub or boss, thus permitting adjustment of their orientation for changes in load and head. This arrangement is generally carried out by the governor which also moves the guide vane suitably. For this reason, while a fixed blade propeller turbine gives the best performance under the design load conditions, a Kaplan turbine gives a consistently high efficiency over a larger range of heads, discharges and loads. The facility for adjustment of blade angles ensures shock-less flow even under non-design conditions of operation. Water entering radially from the spiral casing is imparted a substantial whirl component by the wicket gates. Subsequently, the curvature of the housing makes the flow become axial to some extent and finally then relative flow as it enters the runner, is tangential to the leading edge of the blade as shown in Fig 5, Energy transfer from fluid to runner depends essentially on the extent to which the blade is capable of extinguishing the whirl component of fluid. In most Kaplan runners as in Francis runners, water leaves the wheel axially with almost zero whirl or tangential component.

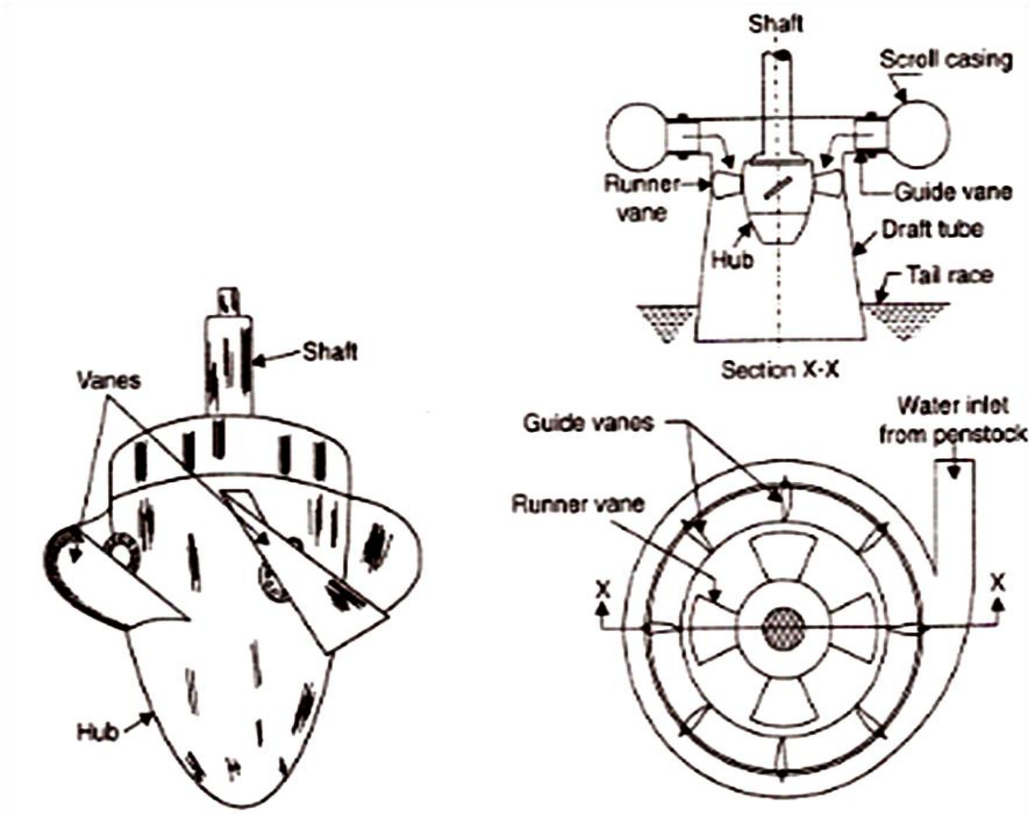


Figure 5: Kaplan Turbine

Working Principle: Water at high pressure enters the spiral casing through the inlet and flows over the guide blades. The water from the guide blades strokes the runner blades axially. Thus, the kinetic energy is imparted by water to the runner blades, causing the rotation of the runner. The runner has only 4 or 6 blades. The water discharges at the center of the runner in the axial direction into the draft tube. The draft tube is of L shape with its discharging end immersed into the tail race.

P1 A Pelton wheel is having a mean bucket diameter of 1m and is running at 1000rpm. The net head on the Pelton wheel is 700m. If the side clearance angle is 15^0 and discharge through nozzle is 0.1 m^3/s. Find

(i) Power available at the nozzle and
(ii) Hydraulic efficiency of the turbine.

Given data

Mean bucket diameter, D = 1m

Speed, N =1000m

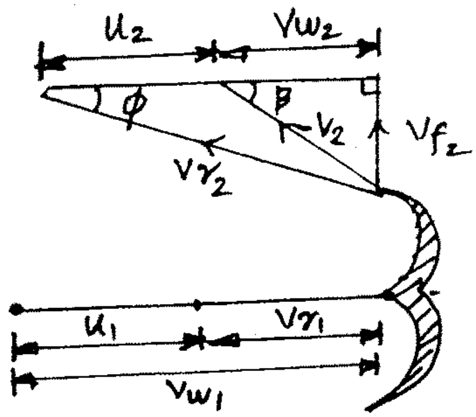
Head, H =700m

Clearance angle, Φ $=15^0$

Discharge, Q $=0.1 m^3/s$

Jet velocity, v_1 $= Cv\sqrt{2gH}$ $=1\sqrt{2x9.81x700}$

=117.19m/s



Solution

Vane velocity, $u_1 = u_2 = \frac{\pi DN}{60}$ $= \frac{\pi x1x1000}{60}$ = 52.36m/s

Relative velocity, $V_{r1} = v_1 - u_1$ =117.19-52.36 =64.83m/s

$V_{r1} = V_{r2}$ =64.83m/s

Whirl velocity, V_{w1} $= u_1 + v_{r1}$ =117.19m/s

$Cos\Phi = \frac{u_{2+}v_{w2}}{v_{r2}}$

$cos15 = \frac{52.36+v_{w2}}{64.83}$

V_{w2} =10.26m/s

(i) Power available at the nozzle $=\frac{1}{2}$ PQV_1^2 $=\frac{1}{2}$ x1000x0.1x117.292=686674.8 Watts

(ii) Hydraulic efficiency, $\eta_h = \frac{2(v_{w1}+v_{w2})u_1}{v_1^2} = \frac{2(117.91+10.26)36}{117.19^2}$ = 0.97 = 0.97*100 = 97%

P2 A Pelton wheel is to be designed for the following specifications: Shaft power = 11,772 kW; Head = 380 meters; Speed = 750 rpm; Overall efficiency = 86%; Jet diameter is not to exceed one- sixth of the wheel diameter. Determine (i) The wheel diameter (ii) The no. of jets required and Diameter of the jet. Take K_{v1} = 0.98 and K_{u1} = 0.45

Given Data

Shaft Power =11772 kW =11772 * 1000 W

H = 3 80m

N = 750rpm

η_o = 80% =0.86

d = $\frac{1}{6}$D

Cv =0.98, ku =0.45

Solution

V_1 =Cv $\sqrt{2gH}$ =0.98$\sqrt{2x9.81x380}$ =85.05m/s

u_1=u_2 =ku $\sqrt{2gH}$ = 0.45 $\sqrt{2x9.81.380}$ = 3338.85m/s

U_1 =$\frac{\pi DN}{60}$

38.85 =$\frac{\pi Dx750}{60}$

The wheel diameter, <u>D =0.989 m</u>

Diameter of the jet, d=$\frac{1}{6}$D =$\frac{1}{6}$x0.989 <u>= 0.165 m</u>

Discharge of single jet, q = av_1 =$\frac{\pi d^2}{4}$.v_1 = $\frac{\pi 0.165^2}{4}$x85.05 =1.818 m^3/s

η_o =$\frac{s.p}{w.p}$

0.86 =$\frac{11772x10^3}{w.p}$

w.p =13688372

$\frac{1}{2}$xpQv_1^2 =13688372

$\frac{1}{2}$x1000 x q x 85.05^2 =13688372

Q = 3.789 m^3/s

No. of jets required = $\frac{Total\ Discharge}{Discharge\ by\ jet}$ = $\frac{3.789}{1.818}$ = **<u>2jets</u>**

P3 A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 litres/s under a head of 30 m. The buckets deflect the jet through an angle of 160^0. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.

Given Data

u_1=u_2=10 m/s

Q=700litres/s=0.7m^3/s

H=30m

Φ=180^0-160^0=20^0

Cv = 0.98

Solution

V_1 =$C_v\sqrt{2gH}$ = 0.98√2x9.81x30 = 23.77m/s

V_{r1} =v_1-u_1 =23.77-10 =13.77m/s

V_{w1} =u1-v_{r1} =10+13.77 =23.77m/s

CosΦ =$\frac{u_2+v_{w2}}{v_{r2}}$

Cos20 =$\frac{10+v_{w2}}{13.77}$

V_{w2} =2.94m/s

(i) Power of runner =ρQ(v_{w1}+v_{w2})=1000x0.7(23.77+2.94)10=186970 Watts

(ii) Hydraulic efficiency, η_h =$\frac{2(v_{w1}+v_{w2})u1}{v_1^2}$ = $\frac{2(23.77+2.94)10}{23.772}$ =0.945*100 = 94.5%

P4 A Francis turbine with an overall efficiency of 75% is required to produce 148.25 kW power. It is working under a head of 7.62 m. The peripheral velocity = 0.26$\sqrt{2gH}$ and the radial velocity of flow at inlet is 0.96$\sqrt{2gH}$. The wheel runs at 150 rpm and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge, determine (i) The guide blade angle (ii) The wheel vane angle at inlet (iii) Diameter of the wheel at inlet and Width of the wheel at inlet

Given Data

η_o =0.75

s.p =148.25kw =148.25x10^3 W

H =7.62m

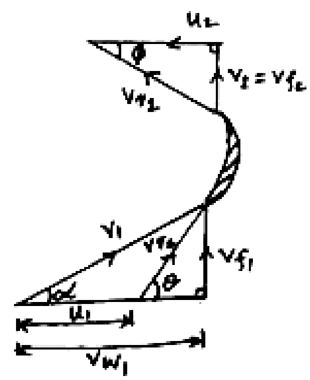
u_1 =0.26$\sqrt{2gH}$ =0.26$\sqrt{2x9.81x7.62}$ =3.179m/s

Vf_1 =0.96$\sqrt{2gH}$ =0.96$\sqrt{2x9.81x7.62}$ =11.738m/s

N =150rpm

Hydraulic losses =20%

Hydraulic efficiency, η_h =80% =0.8



Solution

η_h = 0.8

$\frac{v_{w1}u_1}{gH}$ =0.8

$\frac{v_{w1}x3.179}{9.81x7.62}$ =0.8

V_{w1} =18.34m/s

$\tan\alpha$ $=\frac{v_{f1}}{v_{w1}}$ $=\frac{11.738}{18.34}$ =0.64

A =$\tan^{-1}(0.64)$ =32.620 (Guide blade angle)

$\tan\theta$ $=\frac{v_{f1}}{v_{w1}-u_1}$ $=\frac{11.738}{18.34-3.179}$ =0.774

θ =$\tan^{-1}(0.774)$ =37.740 (vane angle at inlet)

u_1 $=\frac{\pi DN}{60}$

3.179 $=\frac{\pi x D x 150}{60}$

D =0.4047m (Diameter of wheel)

η_o $=\frac{s.p}{w.p}$

0.75 $=\frac{148.25x10^3}{\rho gQH}$ $=\frac{148.25x10^3}{1000x9.81xQx7.62}$

Q =2.644 m^3/s

$D_1B_1Vf_1$=2.664

B_1 =0.177m (width at inlet)

TWO MARK QUESTION & ANSWERS

1. **What are fluid machines or Hydraulic machines?**

   The machines which use the liquid or gas for the transfer of energy from fluid to rotor or from rotor to fluid are known as fluid machines.

2. **How are fluid machines classified?**

   Fluid machines are classified into two categories depending upon transfer of energy:

   a. **Turbines** – hydraulic energy is converted to mechanical energy and then electrical energy.

   b. **Pumps** – electrical energy is converted to mechanical energy and then hydraulic energy.

3. **What are called turbines?**

   Hydraulic turbines are the machines which use the energy of water and convert it into mechanical energy. The mechanical energy developed by a turbine is used in running the electrical generator which is directly coupled to the shaft.

4. **Define Gross Head of a turbine.**

   The difference between head race level and tail race level is known as Gross Head

5. **Define Net head of a turbine.**

   It is also called effective head and is defined as the head available at the inlet of the turbine. {$\mathbf{H = H_g - h_f}$}.

6. **Name the efficiencies to be accounted for finding turbine performance.**

   Hydraulic efficiency, Mechanical efficiency, volumetric efficiency, Overall efficiency

7. **What is an impulse turbine?**

   If at the inlet of the turbine, the energy available is only kinetic energy, the turbine is known as impulse turbine. The pressure at the inlet of the turbine is atmosphere. This turbine is used for high heads. The water strikes the bucket along the tangent of the runner.

   Eg: Pelton Wheel Turbine.

8. **What is a reaction turbine?**

   If at the inlet of the turbine, the water possesses kinetic energy as well as pressure energy, the turbine is known as reaction turbine. As the water flows through the runner, the water is under pressure and the pressure energy goes on changing into kinetic energy. The runner is completely enclosed in an air-tight casing and the runner and casing is completely full of water. This turbine is used for medium heads.
   Eg: Francis Turbine.

9. **Define Jet Ratio.**

   It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by (m) and is given as $\mathbf{m = D/d}$

10. **Classify hydraulic turbines.**

- **a)** Based on type of energy available at inlet
- **b)** Based on head available at inlet
- **c)** Based on specific speed
- **d)** Based on direction of flow through runner

11. What is meant by Draft Tube?

The draft tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race. One end of the draft tube is connected to the outlet of the runner while the other end is sub-merged below the level of water in the tail race.

12. What are the uses of draft tube?

- **a)** Discharges water to tail race safely
- **b)** Converts a large proportion of rejected kinetic energy into useful pressure energy
- **c)** Net head of the turbine is increased.

13. Define specific speed of a turbine.

It is defined as the speed of the turbine which is geometrically similar and it will develop unit power when working under unit head.

$$N_s = N\sqrt{P} / (H)^{1.25}$$

14. List the characteristic curves of Hydraulic turbine.

- **a)** Main Characteristic Curves (or) Constant Head Curves
- **b)** Operating Characteristic Curves (or) Constant Speed Curves
- **c)** Constant Efficiency Curves

15. Give an example for high head turbine, medium head turbine and low head turbine.

- **a)** High head turbine – [> 250 m] - (Pelton wheel)
- **b)** Medium head turbine – [60 to 250 m] - (Francis turbine)
- **c)** Low head turbine – [< 30 m] – (Kaplan turbine, Propeller turbine)

EXERCISE

1. A Pelton wheel has a mean bucket speed of 35 m/s with a jet of water flowing at the rate of 1 m^3/s under a head of 270m. The buckets deflect the jet through an angle of 170^0. Calculate the power delivered to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity at 0.98.

2. A Pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 litres/s under a head of 30 m. The buckets deflect the jet through an angle of 160^0. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.

3. A Kaplan turbine runner is to be designed to develop 7357.5 kW shaft power. The net available head is 5.50 m. Assume that the speed ratio is 2.09 and flow ratio is 0.68, and the overall efficiency is 60%. The diameter of the boss is $1/3^{rd}$ of the diameter of the runner. Find the diameter of the runner, its speed and its specific speed.

4. A Pelton wheel is having a mean bucket diameter of 1m and is running at 1000rpm. The net head on the Pelton wheel is 700m. If the side clearance angle is 15^0 and discharge through nozzle is 0.1 m^3/s. Find: (i) Power available at the nozzle and (ii)Hydraulic efficiency of the turbine.

UNIT V: HYDRAULIC PUMPS

INTRODUCTION

A hydraulic pump is a mechanical source of power that converts mechanical power into hydraulic energy (hydrostatic energy i.e. flow, pressure). It generates flow with enough power to overcome pressure induced by the load at the pump outlet. When a hydraulic pump operates, it creates a vacuum at the pump inlet, which forces liquid from the reservoir into the inlet line to the pump and by mechanical action delivers this liquid to the pump outlet and forces it into the hydraulic system.

5.1 CLASSIFICATION OF PUMPS

On the basis of transfer of mechanical energy, the pumps can be broadly classified as follows:

1. Rotodynamic pumps

    -Radial flow pumps

    -Axial flow pumps

    -Mixed flow pumps

2. Positive displacement pumps

In rotodynamic pumps, increase in energy level is due to a combination of centrifugal energy and kinetic energy. The energy transfer, in a radial flow pump, occurs mainly when the flow in its radial path. In an axial flow pump, the energy transfer occurs when the flow in its axial direction. The energy transfer in a mixed flow pump takes place when the flow comprises radial as well as axial components. The radial flow type pumps arc commonly called centrifugal pumps.

5.2 CENTRIFUGAL PUMP

Centrifugal pump works on the principle that a fluid of mass is given a force it is thrown outward radially. The main parts of the centrifugal pump include;

1. Impeller
2. Casing
3. Suction pipe
4. Discharge pipe

The suction pipe is connected to the sump or a ground level tank from where the fluid has to be pumped. The suction pipe at the sump is connected with strainer thus restricting any foreign particles entering into the pump. Generally as the length of the suction pipe is less the friction loss also will be less.

The other end of the suction pipe is connected to the suction eye of the pump. The suction eye is the first point of entry of water into pump. The discharge pipe is connected to the above level where the fluid has to be delivered.

Since the length of the discharge pipe is long the friction loss will also be higher at the discharge end.

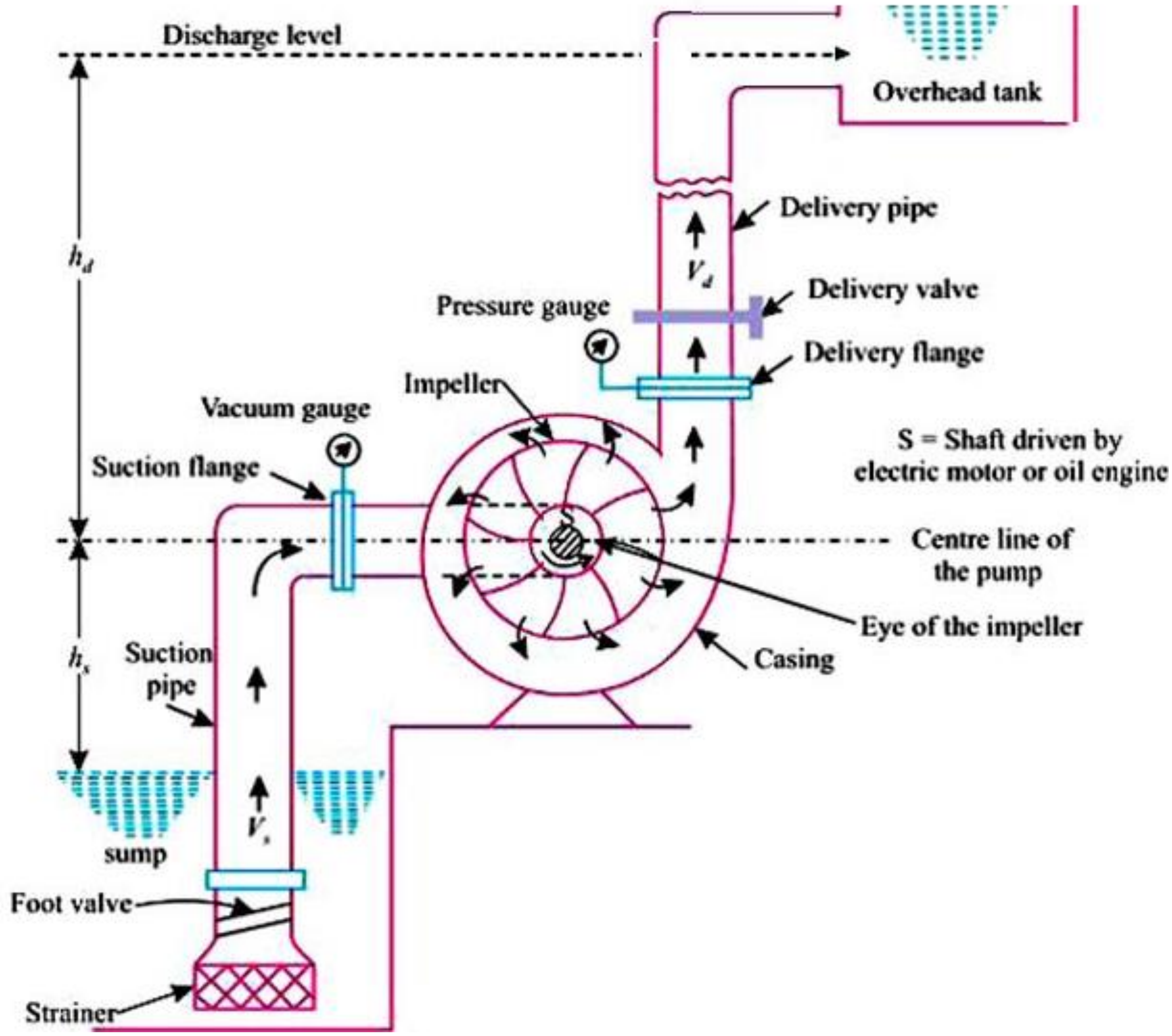


Figure 1: Components of a Centrifugal Pump

The casing of the pump is designed of gradually increasing cross sectional area. It means that velocity of the fluid is decreased in order to attain pressure energy. So the casing does the work of reducing the velocity of the fluid.

Impeller: The impeller rotates to create a pressure drop in the suction side. The pressure drop is much below the atmospheric pressure and hence the water gets into the pump. In the pump the impeller provides the centrifugal action to the fluid. So this impeller can be of various types depending on the fluid on which it is operated. The types of impeller are

- Open impeller
- Closed impeller
- Semi-closed impeller

Open impeller is used for pumping of solid waste or sewage from water treatment plants. Semi closed impeller is also used for water and other fluid applications. In case of pumping any corrosive or hazardous material closed type impeller can be used. **Casing:** The pump casing can also be differed in different ways

- Volute casing
- Vortex casing
- Volute casing with diffuser

In order to increase the performance of the pump a small chamber called vortex is provided around the impeller. This helps in increasing the static pressure energy of the fluid thus increase in pressure.

Diffuser is a set of stationary vanes surrounding the impeller. The function of the diffuser is to increase the efficiency by gradual increase in annular space hence reducing velocity and thus increasing pressure.

Suction head (H_s): The height difference between sump level and centre of pump

Delivery head (H_d): The height difference between centre of pump and delivery point.

Net head (H): The sum of suction and delivery heads

$$H=H_s + H_d$$

Manometric head (H_m): The head against which the pump has to work.

$$H_m= H_s + H_d + h_{fs} + h_{fd}$$

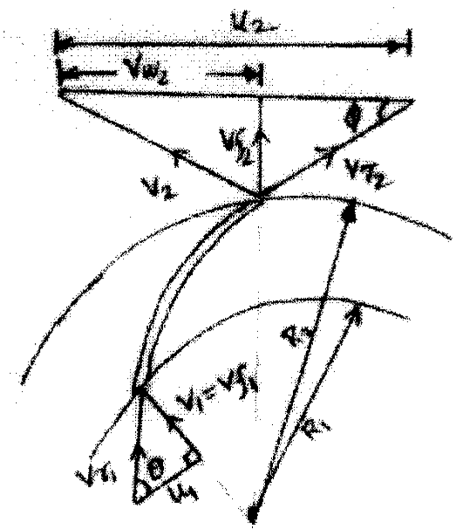
Priming

Priming is the process in which the impeller of a centrifugal pump will get fully sub merged in liquid without any air trap inside.

Cavitation

In designing any installation in which a centrifugal pump is used, careful attention must be paid to check the minimum pressure which will arise at any point. If this pressure is less than the vapor pressure at the pumping temperature, vaporization will occur and the pump may not be capable of developing the required suction head. Moreover, if the liquid contains gases, these may come out of solution giving rise to packets of gas. This phenomenon is known as cavitation and may result in mechanical damage to the pump as the bubbles collapse. The onset of cavitation is accompanied by a marked increase in noise and vibration as the bubbles collapse, and a loss of head.

Work done by a Centrifugal Pump



Let

N- Speed in rpm

D_1- Diameter of impeller at inlet (=$2R_1$)

u_1- Tangential velocity at inlet (=$\frac{\pi D1N}{60}$)

V_1- velocity of water at inlet

V_{f1}- flow velocity at inlet

V_{w1}- whirl velocity at inlet

V_{r1}- relative velocity at inlet

B_1- width of impeller at inlet

θ- Vane angle at inlet

D_2, U_2, V_2, Vf_1, V_{w2}, V_{r2}, B_2, Φ- corresponding at outlet of the pump

1. Discharge, $Q=D_1B_1V_{f1}=D_2B_2V_{f2}$
2. Weight of water, W=gQ
3. Work done per sec=$\frac{w}{g}V_{w2}u_2$
4. Work done per unit weight=$\frac{1}{g}V_{w2}u_2$
5. Manometric efficiency, $\eta_{man}=\frac{gH_m}{vw_2u_2}$

5.3 RECIPROCATING PUMP

If the mechanical energy is converted into hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forwards), which exerts the thrust on the liquid and increases its hydraulic energy (pressure energy), the pump is known as reciprocating pump.

Working principle:

The working of the reciprocating pump is very simple and just like an I.C engine. First of all the piston has the function of providing the suction force, so that the liquid can be lift up or can be sucked in with great force. After that comes the compression part which will impart the required pressure energy to the fluids. In this part of the phase the piston have to do a great work so that the liquid can be compressed properly and its pressure can increased to the desired level. The inlet and the outlet valve open at a certain pressure which is set by the manufacturer.

If the piston is of single acting type which means it can suck from one side and transmit to the same side only. But we can have the double reciprocating pump too which have the function of the giving suction and discharge simultaneously in each stroke. This pump can be used as the compressor also but for that we have to have a good valve arrangement which can operate with good frequency.

Note: It is to be noted that the reciprocating pump is a positive displacements pump which means that the fluid can only move in one direction and can never reverse back. So due to this the pump is always started with outlet valve open otherwise the pressure will keep on building and this will lead to rupturing of the pipeline or even the pump itself. But if relief valve is fitted then this pressure will come down.

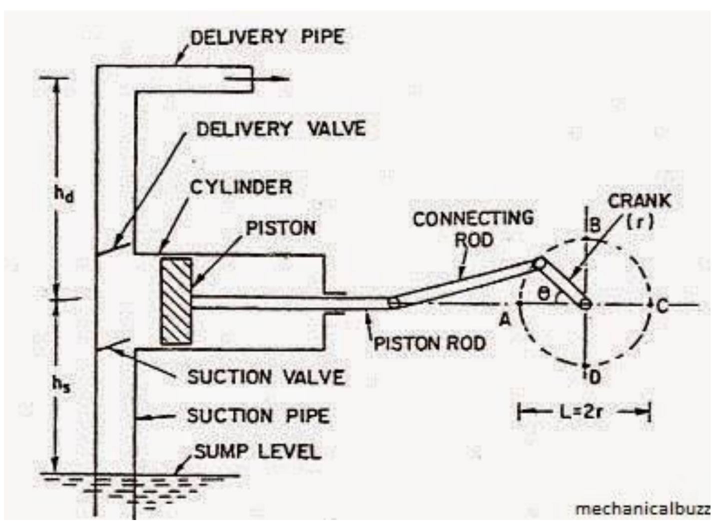


Figure 2: Single acting reciprocating pump

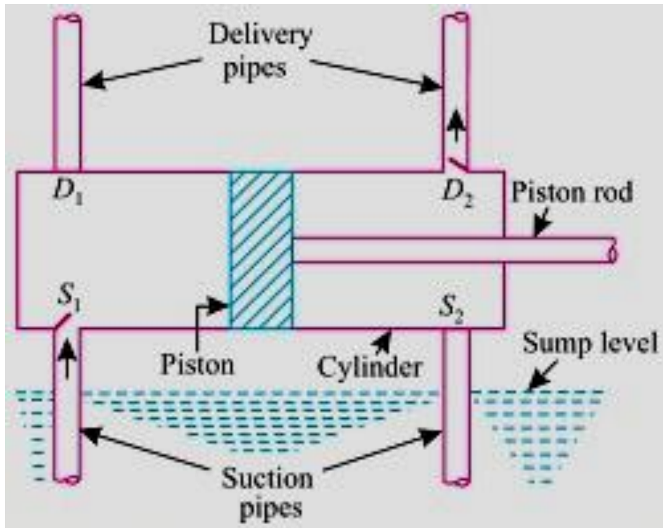


Figure 3: Double acting reciprocating pump

Reciprocating pumps are very important part of the ships machinery and any other industry which is present in the world. High pressure is the main characteristic of this pump and this high pressure output are being used in places like starting of the engine or you can say the building of pressure in the fluids. But these are used in limited application because they require lot of maintenance. These pumps are positive displacements pumps and that is the reason they do not require any type of priming for their functioning in the starting period of the pump.

Components of Reciprocating pump

Following are the parts of the centrifugal pump:

1. Piston
2. Liner
3. Inlet and outlet valve
4. Motor
5. Shaft
6. Relief valve
7. Piston rings

These are some of the parts of the reciprocating pumps and this works same like an I.C engine but there is no combustion in this but whole process is same as we have in the I.C engine.

Material Used:

Piston: Generally made of the mild steel
Liner: Made of Mild steel
Valves: There are also made of steel but little improved form which has the stiffness and less wear and tear are used and brass can also be used.

Maintenance required:

- The piston rings which are used are always in direct contact with the liner body and hence they wear a lot. So we have to change them time to time.
- The valves used in this pump have to be taken care properly as it will lead leakage if they go bad. So they have to take care off while maintaining of the pump.
- The gland packing from where the shaft comes out of the pump has to be taken care in order to check its leakage.
- In coupling or the crosshead of the pump by which the piston get the liner motion also have to checked for any type of misalignment and wear and tear.

Advantages:

1. Advantages of the reciprocating pump are as given below:
2. Gives high pressure at outlet.
3. Gives high suction lift.
4. Priming is not required in this pump.
5. They are used for air also.

Disadvantages:

1. High wear and tear, so requires lot maintenance.
2. The flow is not uniform, so we have to fit a bottle at both ends.
3. The flow is very less and cannot be used for high flow operations.
4. More heavy and bulky in shape.
5. Initial cost is much more in this pump.

$$Discharge\ of\ pump\ per\ second = A \times L \times \frac{N}{60}$$

$$Weight\ of\ water\ delivered\ per\ second = \frac{\rho g A L N}{60}$$

$$Work\ done\ per\ second = \frac{\rho g \times ALN}{60} \times (h_s + h_d)$$

$$Power\ required\ to\ drive\ the\ pump = \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000} kW$$

$$Discharge\ of\ pump\ per\ second = \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L \times \frac{N}{60}$$

$$Power\ required\ by\ double-acting\ reciprocating\ pump = \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000} kW$$

Slip: Slip of a reciprocating pump is defined as the difference between the theoretical discharge and the actual discharge of the pump.

$$Slip = Q_{th} - Q_{act}$$

$$percentage\ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

5.4 JET PUMP

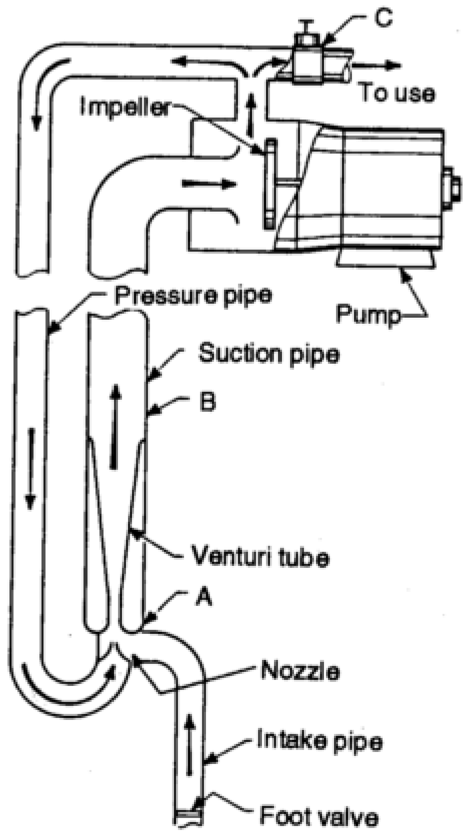


Fig 4 Jet Pump

Jet Pump: This pump is a combination of a surface centrifugal pump, down-hole nozzle, and venturi arrangement (Figure 4). It can be used in small diameter wells that require a lift of 100 feet or less. The pump supplies water, under pressure, to the nozzle. The increase in velocity at the nozzle results in a decrease in pressure at that point, which in turn draws water through the foot valve into the intake pipe. The combined flow then enters the venturi where the velocity is gradually decreased and the pressure head recovered. The excess flow is discharged at the surface through a control valve, which also maintains the required recirculating flow to the nozzle.

A jet pump's efficiency is low compared to an ordinary centrifugal pump. However, other features make the jet pump a desirable pump. They are,

- Adaptability to wells as small as 2 inches in diameter.
- Easy accessibility to all moving parts at the ground surface.
- Simple design resulting in relatively low purchase and maintenance costs.

Applications

Jet pumps are designed to assist in a variety of situations in the oil and gas industry, but are extremely useful for low flow and pressure wells. They are used in the conventional manner of oil and gas retrieval, as well as in more complicated applications like slimhole wells and wells that have faulty or weak casing. The versatility of a jet pump makes it a staple in the oil and gas industry.

5.5 SUBMERSIBLE PUMP

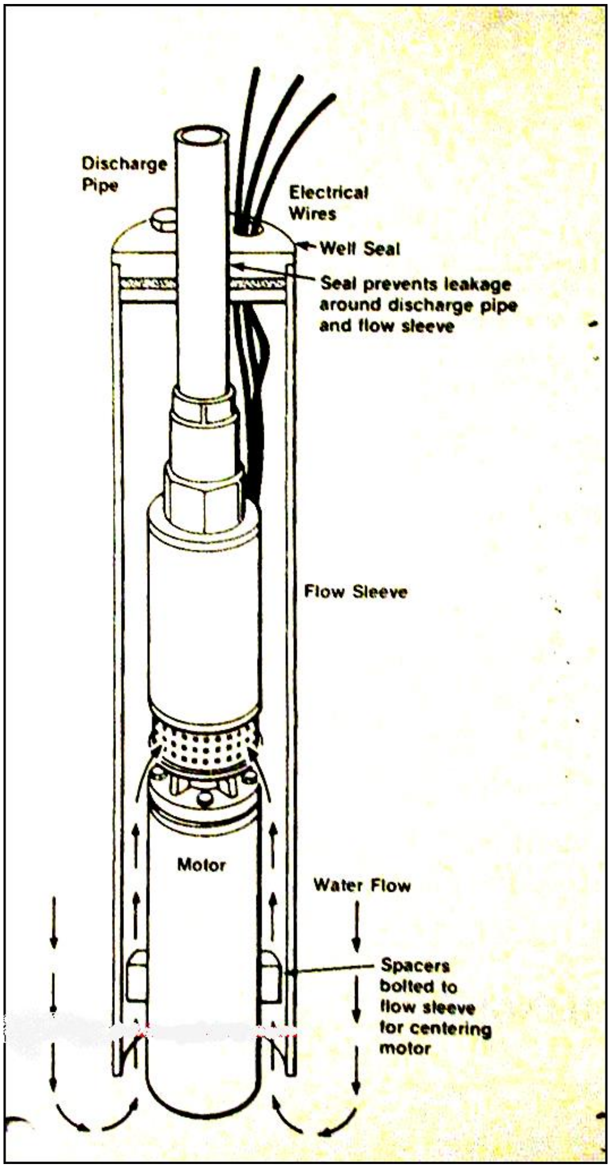


Fig 5 Sectional view of a Submersible Pump

Working Principle

The submersible pumps mostly used are multistage centrifugal pumps operating in a vertical position. Although their constructional and operational features underwent a continuous evolution over the years, their basic operational principle remained the same. Produced liquids, after being subjected to great centrifugal forces caused by the high rotational speed of the impeller, lose their kinetic energy in the diffuser where a conversion of kinetic to pressure energy takes place. This is the main operational mechanism of radial and mixed flow pumps.

The pump shaft is connected to the gas separator or the protector by a mechanical coupling at the bottom of the pump. When fluids enter the pump through an intake screen and are lifted by the pump stages. Other parts include the radial bearings (bushings) distributed along the length of the shaft providing radial support to the pump shaft turning at high rotational speeds. An optional thrust bearing takes up part of the axial forces arising in the pump but most of those forces are absorbed by the protector's thrust bearing.

Applications

Submersible pumps are found in many applications. Single stage pumps are used for drainage, sewage pumping, general industrial pumping and slurry pumping. They are also popular with pond filters. Multiple stage submersible pumps are typically lowered down a borehole and most typically used for residential, commercial, municipal and industrial water extraction (abstraction), water wells and in oil wells.

Other uses for submersible pumps include sewage treatment plants, seawater handling, fire fighting (since it is flame retardant cable), water well and deep well drilling, offshore drilling rigs, artificial lifts, mine dewatering, and irrigation systems.

Special attention to the type of submersible pump is required when using certain types of liquids. Pumps used for combustible liquids or for water that may be contaminated with combustible liquids must be designed not to ignite the liquid or vapors.

5.6 GEAR PUMP

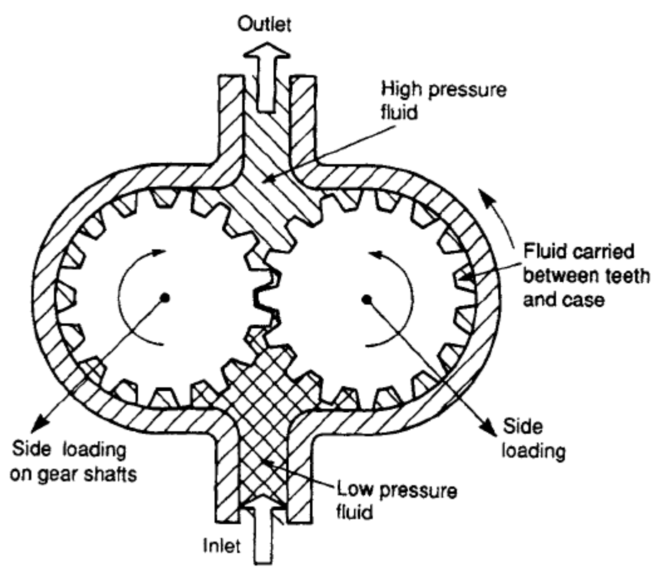


Figure 6: Gear Pump

The simplest and most robust positive displacement pump. External gear pumps in particular can be engineered to handle even the most aggressive corrosive liquids.

The main parts of the pump are

1. Externals (head, casing, bracket)
2. Internals (shafts)
3. Internals (gears)
4. Bushing
5. Shaft Seal

The external gear pump uses two identical gears rotating against each other, one gear is driven by a motor and it in turn drives the other gear. Each gear is supported by a shaft with bearings on both sides of the gear.

- As the gears come out of mesh, they create expanding volume on the inlet side of the pump. Liquid flows into the cavity and is trapped by the gear teeth as they rotate.
- Liquid travels around the interior of the casing in the pockets between the teeth and the casing, it does not pass between the gears.
- Finally, the meshing of the gears forces liquid through the outlet port under pressure.

Because the gears are supported on both sides, external gear pumps are quiet-running and are routinely used for high-pressure applications such as hydraulic applications. With no overhung bearing loads, the rotor shaft can't deflect and cause premature wear.

Advantages

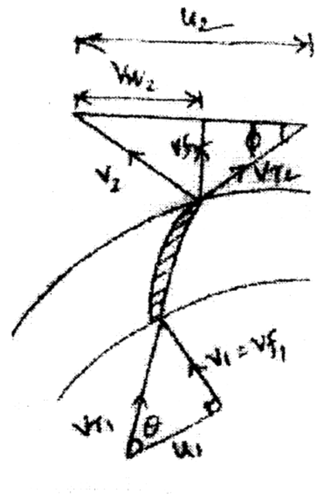
- High speed
- High pressure
- No overhung bearing loads
- Relatively quiet operation
- Design accommodates wide variety of materials

Applications

Common external gear pump applications include, but are not limited to:

- Various fuel oils and lubricants.
- Chemical additive and polymer metering
- Chemical mixing and blending
- Industrial and mobile hydraulic applications (log splitters, lifts, etc.)
- Acids and caustic (stainless steel or composite construction)
- Low volume transfer

P1 The internal and external diameters of the impeller of a centrifugal pump are 300mm and 600mm respectively. The pump is running at 1000 rpm. The vane angles at inlet and outlet are 20^0 and 30^0 respectively. The water enters the impeller radially and velocity of the flow is constant. Determine the work done by the impeller per unit weight of water.



Given Data

D_1 =300mm =0.3m

D_2 =600m =0.6m

N =1000rpm

θ $=20^0$

Φ $=30^0$

V_{f1} $=V_{f2}$

Solution

$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi x0.3x1000}{60}$ **=** 15.71m/s

$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi x0.6x1000}{60}$ =31.42m/s

$\tan\theta = \frac{Vf_1}{u1}$

$\tan 20^0 = \frac{Vf_1}{15.71}$ V_{f1}=5.72m/s

V_{f1} $=V_{f2}$ =5.72m/s

$\tan\Phi = \frac{Vf_2}{u2 - v_{w2}}$

$\tan 30^0 = \frac{5.72}{31.42 - v_{w2}}$ V_{w2}=21.51m/s

Work done per unit weight $= \frac{1}{g} V_{w2} u_2$ $= \frac{1}{9.81}$x21.51x31.42 = 68.9

TWO MARK QUESTION & ANSWERS

1. **What is a pump?**
   A hydraulic equipment which converts electrical energy into mechanical energy and then to hydraulic energy is known as a pump
2. **What is a roto dynamic pump?**

When the increase in pressure is developed by rotating impeller or by action of centrifugal force then the pump is called as roto dynamic pump.

3. **How does a Centrifugal pump works?**

   In a centrifugal pump, mechanical energy is converted into pressure energy means of centrifugal force acting on the fluid.

4. **Mention the main components of a centrifugal pump.**

   Impeller, Casing, Suction pipe, Delivery pipe, Foot valve with strainer

5. **What is meant by priming in a centrifugal pump?**

   Through the delivery pipe water should be completely filled until portion of delivery pipe, casing and suction pipe, so no air pocket is left inside the pump.

6. **Define manometric head.**

   It is the head against which a centrifugal pump has to work.

7. **Define mechanical efficiency of a centrifugal pump.**

   It is defined as the ratio of the power actually delivered by the impeller to the power supplied to the shaft.

8. **What is a reciprocating pump?**

   Reciprocating pump is a positive displacement pump. This means the liquid is first sucked into the cylinder and then displaced or pushed by the thrust of a piston.

9. **Mention the main components of a reciprocating pump.**

   Cylinder, piston, piston rod, connecting rod, suction pipe with suction valve, delivery pipe with delivery valve.

10. **What is single acting pump reciprocating pump?**

    If the water is in contact with one side of the piston the pump then it is known as single acting reciprocating pump. For one complete revolution one suction stroke and one delivery stroke occurs.

11. **Write the expression for slip and % slip.**

    The difference between the theoretical discharge (Q_T) and actual discharge (Q_{act}) is known as slip of the pump.

    Slip = $Q_T - Q_{act}$

    % Slip = $[(Q_T - Q_{act})/Q_T] \times 100$ %

    - If Q_{act} is more than the Q_T then slip will be –ive.
    - If Q_{act} lesser than Q_T then the slip will be +ive.

12. **Define indicator diagram.**

    The indicator diagram for a reciprocating pump is defined as the graph drawn between the pressure head in the cylinder and the distance traveled by the piston for one complete revolution of the crank.

13. **What is an air vessel?**

    An air vessel is a closed chamber containing compressed air in the top portion and liquid at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid may flow into the vessel or out from the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out of the vessel, the air will expand into the chamber.

14. What is the purpose of an air vessel fitted in the pump?

- **a.** To obtain a continuous supply of liquid at a uniform rate.
- **b.** To save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipes, and
- **c.** To run the pump at a high speed without separation.

15. Define Cavitation.

If the pressure in the cylinder is below the vapour pressure, the dissolved gases will be liberated from the liquid and air bubbles are formed. This process is termed as cavitation.

EXERCISE PROBLEMS

1. A centrifugal pump is to discharge 0.118 m^3/s at a speed of 1450 rpm against a head of 25m. The impeller diameter is 250mm, its width at outlet is 50mm and manometric efficiency is 75%. Determine the vane angle at the periphery of the impeller.
2. A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm work against a total head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine: (i) Vane angle at inlet (ii) Work done by impeller on water per second.
3. The diameters of an impeller of a centrifugal pump at inlet and outlet are 30cm and 60cm respectively. The velocity of flow at outlet is 2m/s and the vanes are set back at an angle of 45^0 at the outlet. Determine the minimum starting speed of the pump if the manometric efficiency is 70%.
4. A centrifugal pump delivers water against a net head of 14.5m and a design speed of 1000 rpm. The vanes are curved back to an angle of 30^0 with the periphery. The impeller diameter is 300 mm and outlet width 50mm. Determine the discharge of the pump if the manometric efficiency is 95%.
5. A double – acting reciprocating pump running at 50 rpm is discharging 900 litres of water per minute. The pump has stroke of 400mm. The diameter of piston is 250mm. The delivery and suction heads are 25m and 4m respectively. Find the slip of the pump and power required to drive the pump.
6. The internal and external diameters of the impeller of a centrifugal pump are 300mm and 600mm respectively. The pump is running at 1000 rpm. The vane angles at inlet and outlet are 20^0 and 30^0 respectively. The water enters the impeller radially and velocity of the flow is constant. Determine the work done by the impeller per unit weight of water.

UNIT 5

HYDRAULIC PUMPS

INTRODUCTION

A hydraulic pump is a mechanical source of power that converts mechanical power into hydraulic energy (hydrostatic energy i.e. flow, pressure). It generates flow with enough power to overcome pressure induced by the load at the pump outlet. When a hydraulic pump operates, it creates a vacuum at the pump inlet, which forces liquid from the reservoir into the inlet line to the pump and by mechanical action delivers this liquid to the pump outlet and forces it into the hydraulic system.

5.1 CLASSIFICATION OF PUMPS

On the basis of transfer of mechanical energy, the pumps can be broadly classified as follows:

3. Rotodynamic pumps
   - -Radial flow pumps
   - -Axial flow pumps
   - -Mixed flow pumps
4. Positive displacement pumps

In rotodynamic pumps, increase in energy level is due to a combination of centrifugal energy and kinetic energy. The energy transfer, in a radial flow pump, occurs mainly when the flow in its radial path. In an axial flow pump, the energy transfer occurs when the flow in its axial direction. The energy transfer in a mixed flow pump takes place when the flow comprises radial as well as axial components. The radial flow type pumps arc commonly called centrifugal pumps.

5.2 CENTRIFUGAL PUMP

Centrifugal pump works on the principle that a fluid of mass is given a force it is thrown outward radially. The main parts of the centrifugal pump include;

5. Impeller
6. Casing
7. Suction pipe
8. Discharge pipe

The suction pipe is connected to the sump or a ground level tank from where the fluid has to be pumped. The suction pipe at the sump is connected with strainer thus restricting any foreign particles entering into the pump. Generally as the length of the suction pipe is less the friction loss also will be less.

The other end of the suction pipe is connected to the suction eye of the pump. The suction eye is the first point of entry of water into pump. The discharge pipe is connected to the above level where the fluid has to be delivered.

Since the length of the discharge pipe is long the friction loss will also be higher at the discharge end.

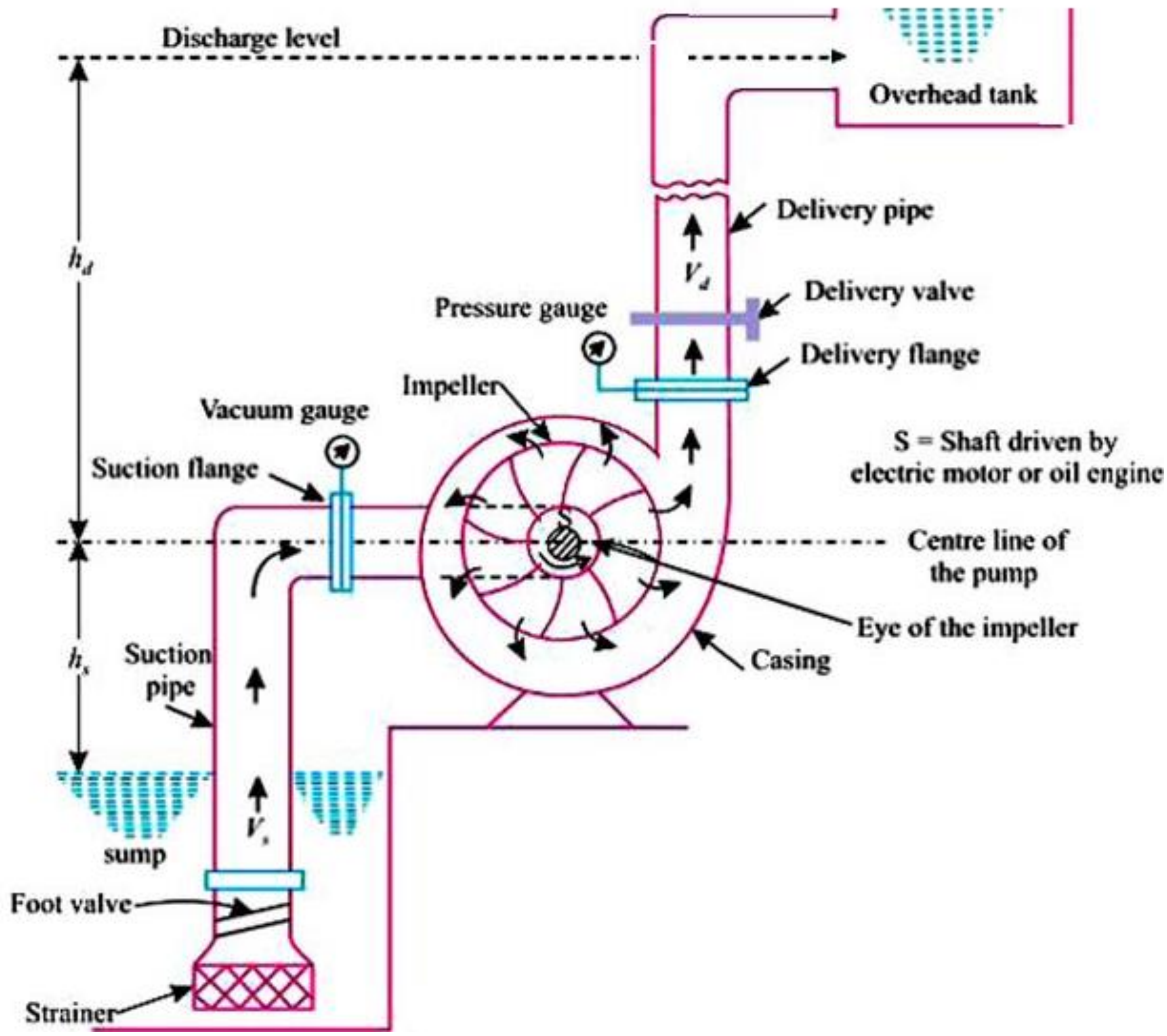


Figure 1: Components of a Centrifugal Pump

The casing of the pump is designed of gradually increasing cross sectional area. It means that velocity of the fluid is decreased in order to attain pressure energy. So the casing does the work of reducing the velocity of the fluid.

Impeller: The impeller rotates to create a pressure drop in the suction side. The pressure drop is much below the atmospheric pressure and hence the water gets into the pump. In the pump the impeller provides the centrifugal action to the fluid. So this impeller can be of various types depending on the fluid on which it is operated. The types of impeller are

- Open impeller
- Closed impeller
- Semi-closed impeller

Open impeller is used for pumping of solid waste or sewage from water treatment plants. Semi closed impeller is also used for water and other fluid applications. In case of pumping any corrosive or hazardous material closed type impeller can be used. **Casing:** The pump casing can also be differed in different ways

- Volute casing
- Vortex casing
- Volute casing with diffuser

In order to increase the performance of the pump a small chamber called vortex is provided around the impeller. This helps in increasing the static pressure energy of the fluid thus increase in pressure.

Diffuser is a set of stationary vanes surrounding the impeller. The function of the diffuser is to increase the efficiency by gradual increase in annular space hence reducing velocity and thus increasing pressure.

Suction head (H_s): The height difference between sump level and centre of pump

Delivery head (H_d): The height difference between centre of pump and delivery point.

Net head (H): The sum of suction and delivery heads

$$H = H_s + H_d$$

Manometric head (H_m): The head against which the pump has to work.

$$H_m = H_s + H_d + h_{fs} + h_{fd}$$

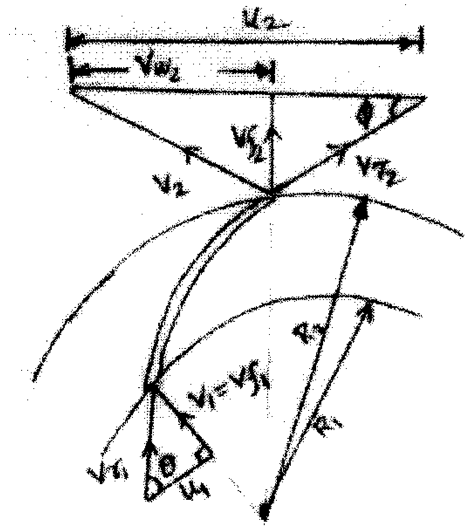
Priming

Priming is the process in which the impeller of a centrifugal pump will get fully sub merged in liquid without any air trap inside.

Cavitation

In designing any installation in which a centrifugal pump is used, careful attention must be paid to check the minimum pressure which will arise at any point. If this pressure is less than the vapor pressure at the pumping temperature, vaporization will occur and the pump may not be capable of developing the required suction head. Moreover, if the liquid contains gases, these may come out of solution giving rise to packets of gas. This phenomenon is known as cavitation and may result in mechanical damage to the pump as the bubbles collapse. The onset of cavitation is accompanied by a marked increase in noise and vibration as the bubbles collapse, and a loss of head.

Work done by a Centrifugal Pump



Let

N- Speed in rpm

D_1- Diameter of impeller at inlet (=$2R_1$)

u_1- Tangential velocity at inlet (=$\frac{\pi D1N}{60}$)

V_1- velocity of water at inlet

V_{f1}- flow velocity at inlet

V_{w1}- whirl velocity at inlet

V_{r1}- relative velocity at inlet

B_1- width of impeller at inlet

θ- Vane angle at inlet

D_2, U_2, V_2, Vf_1, V_{w2}, V_{r2}, B_2, Φ- corresponding at outlet of the pump

6. Discharge, $Q=D_1B_1V_{f1}=D_2B_2V_{f2}$
7. Weight of water, $W=gQ$
8. Work done per sec=$\frac{w}{g}V_{w2}u_2$
9. Work done per unit weight=$\frac{1}{g}V_{w2}u_2$
10. Manometric efficiency, $\eta_{man}=\frac{gH_m}{vw_2u_2}$

5.3 RECIPROCATING PUMP

If the mechanical energy is converted into hydraulic energy (or pressure energy) by sucking the liquid into a cylinder in which a piston is reciprocating (moving backwards and forwards), which exerts the thrust on the liquid and increases its hydraulic energy (pressure energy), the pump is known as reciprocating pump.

Working principle:

The working of the reciprocating pump is very simple and just like an I.C engine. First of all the piston has the function of providing the suction force, so that the liquid can be lift up or can be sucked in with great force. After that comes the compression part which will impart the required pressure energy to the fluids. In this part of the phase the piston have to do a great work so that the liquid can be compressed properly and its pressure can increased to the desired level. The inlet and the outlet valve open at a certain pressure which is set by the manufacturer.

If the piston is of single acting type which means it can suck from one side and transmit to the same side only. But we can have the double reciprocating pump too which have the function of the giving suction and discharge simultaneously in each stroke. This pump can be used as the compressor also but for that we have to have a good valve arrangement which can operate with good frequency.

Note: It is to be noted that the reciprocating pump is a positive displacements pump which means that the fluid can only move in one direction and can never reverse back. So due to this the pump is always started with outlet valve open otherwise the pressure will keep on building and this will lead to rupturing of the pipeline or even the pump itself. But if relief valve is fitted then this pressure will come down.

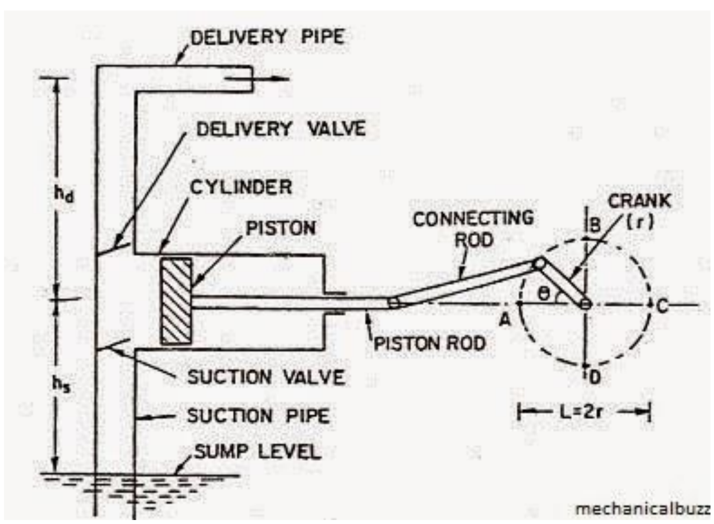


Figure 2: Single acting reciprocating pump

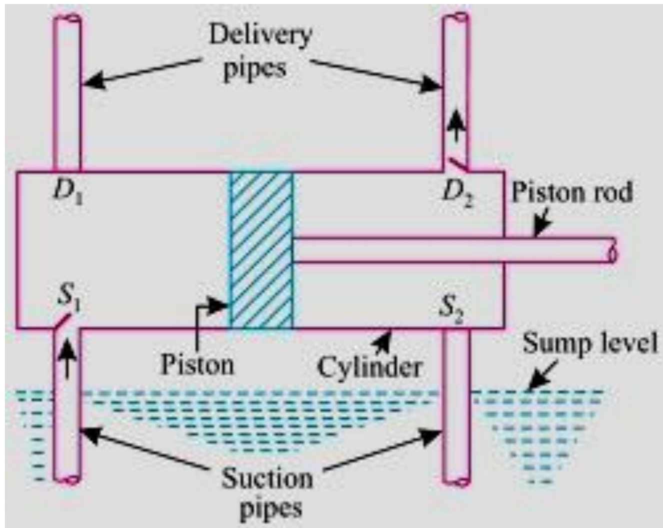


Figure 3: Double acting reciprocating pump

Reciprocating pumps are very important part of the ships machinery and any other industry which is present in the world. High pressure is the main characteristic of this pump and this high pressure output are being used in places like starting of the engine or you can say the building of pressure in the fluids. But these are used in limited application because they require lot of maintenance. These pumps are positive displacements pumps and that is the reason they do not require any type of priming for their functioning in the starting period of the pump.

Components of Reciprocating pump

Following are the parts of the centrifugal pump:

8. Piston
9. Liner
10. Inlet and outlet valve
11. Motor
12. Shaft
13. Relief valve
14. Piston rings

These are some of the parts of the reciprocating pumps and this works same like an I.C engine but there is no combustion in this but whole process is same as we have in the I.C engine.

Material Used:

Piston: Generally made of the mild steel
Liner: Made of Mild steel
Valves: There are also made of steel but little improved form which has the stiffness and less wear and tear are used and brass can also be used.

Maintenance required:

- The piston rings which are used are always in direct contact with the liner body and hence they wear a lot. So we have to change them time to time.
- The valves used in this pump have to be taken care properly as it will lead leakage if they go bad. So they have to take care off while maintaining of the pump.
- The gland packing from where the shaft comes out of the pump has to be taken care in order to check its leakage.
- In coupling or the crosshead of the pump by which the piston get the liner motion also have to checked for any type of misalignment and wear and tear.

Advantages:

6. Advantages of the reciprocating pump are as given below:
7. Gives high pressure at outlet.
8. Gives high suction lift.
9. Priming is not required in this pump.
10. They are used for air also.

Disadvantages:

6. High wear and tear, so requires lot maintenance.
7. The flow is not uniform, so we have to fit a bottle at both ends.
8. The flow is very less and cannot be used for high flow operations.
9. More heavy and bulky in shape.
10. Initial cost is much more in this pump.

$$Discharge\ of\ pump\ per\ second = A \times L \times \frac{N}{60}$$

$$Weight\ of\ water\ delivered\ per\ second = \frac{\rho g A L N}{60}$$

$$Work\ done\ per\ second = \frac{\rho g \times ALN}{60} \times (h_s + h_d)$$

$$Power\ required\ to\ drive\ the\ pump = \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000} kW$$

$$Discharge\ of\ pump\ per\ second = \left[\frac{\pi}{4} D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L \times \frac{N}{60}$$

$$Power\ required\ by\ double-acting\ reciprocating\ pump = \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000} kW$$

Slip: Slip of a reciprocating pump is defined as the difference between the theoretical discharge and the actual discharge of the pump.

$$Slip = Q_{th} - Q_{act}$$

$$percentage\ slip = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

5.4 JET PUMP

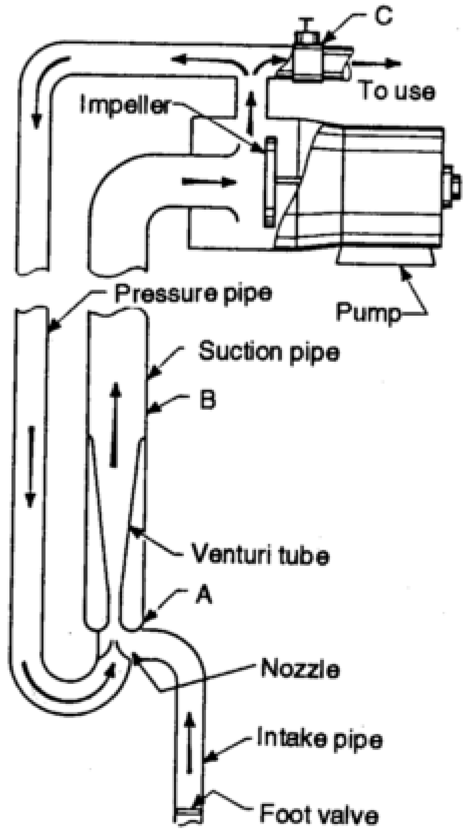


Fig 4 Jet Pump

Jet Pump: This pump is a combination of a surface centrifugal pump, down-hole nozzle, and venturi arrangement (Figure 4). It can be used in small diameter wells that require a lift of 100 feet or less. The pump supplies water, under pressure, to the nozzle. The increase in velocity at the nozzle results in a decrease in pressure at that point, which in turn draws water through the foot valve into the intake pipe. The combined flow then enters the venturi where the velocity is gradually decreased and the pressure head recovered. The excess flow is discharged at the surface through a control valve, which also maintains the required recirculating flow to the nozzle.

A jet pump's efficiency is low compared to an ordinary centrifugal pump. However, other features make the jet pump a desirable pump. They are,

- Adaptability to wells as small as 2 inches in diameter.
- Easy accessibility to all moving parts at the ground surface.
- Simple design resulting in relatively low purchase and maintenance costs.

Applications

Jet pumps are designed to assist in a variety of situations in the oil and gas industry, but are extremely useful for low flow and pressure wells. They are used in the conventional manner of oil and gas retrieval, as well as in more complicated applications like slimhole wells and wells that have faulty or weak casing. The versatility of a jet pump makes it a staple in the oil and gas industry.

5.5 SUBMERSIBLE PUMP

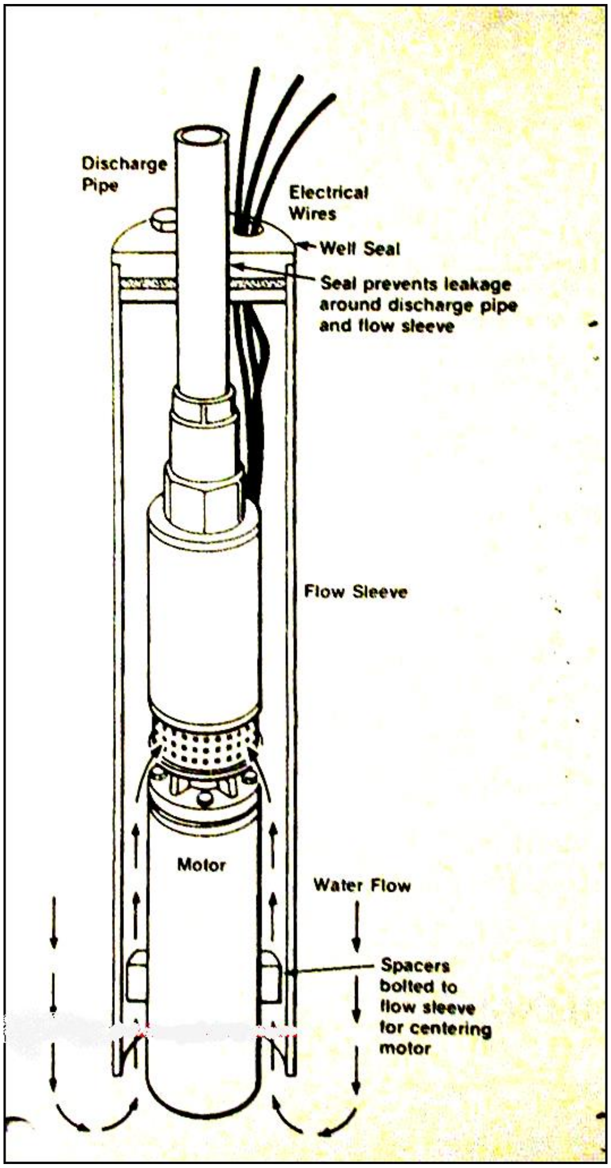


Fig 5 Sectional view of a Submersible Pump

Working Principle

The submersible pumps mostly used are multistage centrifugal pumps operating in a vertical position. Although their constructional and operational features underwent a continuous evolution over the years, their basic operational principle remained the same. Produced liquids, after being subjected to great centrifugal forces caused by the high rotational speed of the impeller, lose their kinetic energy in the diffuser where a conversion of kinetic to pressure energy takes place. This is the main operational mechanism of radial and mixed flow pumps.

The pump shaft is connected to the gas separator or the protector by a mechanical coupling at the bottom of the pump. When fluids enter the pump through an intake screen and are lifted by the pump stages. Other parts include the radial bearings (bushings) distributed along the length of the shaft providing radial support to the pump shaft turning at high rotational speeds. An optional thrust bearing takes up part of the axial forces arising in the pump but most of those forces are absorbed by the protector's thrust bearing.

Applications

Submersible pumps are found in many applications. Single stage pumps are used for drainage, sewage pumping, general industrial pumping and slurry pumping. They are also popular with pond filters. Multiple stage submersible pumps are typically lowered down a borehole and most typically used for residential, commercial, municipal and industrial water extraction (abstraction), water wells and in oil wells.

Other uses for submersible pumps include sewage treatment plants, seawater handling, fire fighting (since it is flame retardant cable), water well and deep well drilling, offshore drilling rigs, artificial lifts, mine dewatering, and irrigation systems.

Special attention to the type of submersible pump is required when using certain types of liquids. Pumps used for combustible liquids or for water that may be contaminated with combustible liquids must be designed not to ignite the liquid or vapors.

5.6 GEAR PUMP

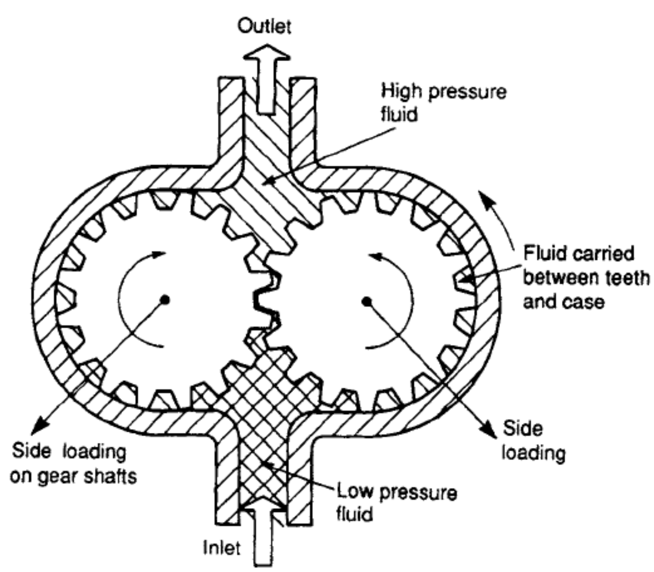


Figure 6: Gear Pump

The simplest and most robust positive displacement pump. External gear pumps in particular can be engineered to handle even the most aggressive corrosive liquids.

The main parts of the pump are

6. Externals (head, casing, bracket)
7. Internals (shafts)
8. Internals (gears)
9. Bushing
10. Shaft Seal

The external gear pump uses two identical gears rotating against each other, one gear is driven by a motor and it in turn drives the other gear. Each gear is supported by a shaft with bearings on both sides of the gear.

- As the gears come out of mesh, they create expanding volume on the inlet side of the pump. Liquid flows into the cavity and is trapped by the gear teeth as they rotate.
- Liquid travels around the interior of the casing in the pockets between the teeth and the casing, it does not pass between the gears.
- Finally, the meshing of the gears forces liquid through the outlet port under pressure.

Because the gears are supported on both sides, external gear pumps are quiet-running and are routinely used for high-pressure applications such as hydraulic applications. With no overhung bearing loads, the rotor shaft can't deflect and cause premature wear.

Advantages

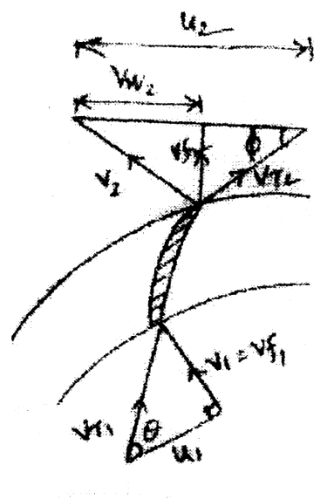
- High speed
- High pressure
- No overhung bearing loads
- Relatively quiet operation
- Design accommodates wide variety of materials

Applications

Common external gear pump applications include, but are not limited to:

- Various fuel oils and lubricants.
- Chemical additive and polymer metering
- Chemical mixing and blending
- Industrial and mobile hydraulic applications (log splitters, lifts, etc.)
- Acids and caustic (stainless steel or composite construction)
- Low volume transfer

P1 The internal and external diameters of the impeller of a centrifugal pump are 300mm and 600mm respectively. The pump is running at 1000 rpm. The vane angles at inlet and outlet are 20^0 and 30^0 respectively. The water enters the impeller radially and velocity of the flow is constant. Determine the work done by the impeller per unit weight of water.



Given Data

D_1 =300mm =0.3m

D_2 =600m =0.6m

N =1000rpm

θ $=20^0$

Φ $=30^0$

V_{f1} $=V_{f2}$

Solution

$U_1 = \frac{\pi D_1 N}{60} = \frac{\pi x0.3x1000}{60}$ **=** 15.71m/s

$U_2 = \frac{\pi D_2 N}{60} = \frac{\pi x0.6x1000}{60}$ =31.42m/s

$\tan\theta = \frac{Vf_1}{u1}$

$\tan 20^0 = \frac{Vf_1}{15.71}$ V_{f1}=5.72m/s

$V_{f1} = V_{f2} = 5.72$m/s

$\tan\Phi = \frac{Vf_2}{u2 - v_{w2}}$

$\tan 30^0 = \frac{5.72}{31.42 - v_{w2}}$ V_{w2}=21.51m/s

Work done per unit weight $= \frac{1}{g} V_{w2} u_2$ $= \frac{1}{9.81}$x21.51x31.42 = 68.9

<u>TWO MARK QUESTION & ANSWERS</u>

13. What is a pump?

A hydraulic equipment which converts electrical energy into mechanical energy and then to hydraulic energy is known as a pump

14. What is a roto dynamic pump?

When the increase in pressure is developed by rotating impeller or by action of centrifugal force then the pump is called as roto dynamic pump.

15. How does a Centrifugal pump works?

In a centrifugal pump, mechanical energy is converted into pressure energy means of centrifugal force acting on the fluid.

16. Mention the main components of a centrifugal pump.

Impeller, Casing, Suction pipe, Delivery pipe, Foot valve with strainer

17. What is meant by priming in a centrifugal pump?

Through the delivery pipe water should be completely filled until portion of delivery pipe, casing and suction pipe, so no air pocket is left inside the pump.

18. Define manometric head.

It is the head against which a centrifugal pump has to work.

19. Define mechanical efficiency of a centrifugal pump.

It is defined as the ratio of the power actually delivered by the impeller to the power supplied to the shaft.

20. What is a reciprocating pump?

Reciprocating pump is a positive displacement pump. This means the liquid is first sucked into the cylinder and then displaced or pushed by the thrust of a piston.

21. Mention the main components of a reciprocating pump.

Cylinder, piston, piston rod, connecting rod, suction pipe with suction valve, delivery pipe with delivery valve.

22. What is single acting pump reciprocating pump?

If the water is in contact with one side of the piston the pump then it is known as single acting reciprocating pump. For one complete revolution one suction stroke and one delivery stroke occurs.

23. Write the expression for slip and % slip.

The difference between the theoretical discharge (Q_T) and actual discharge (Q_{act}) is known as slip of the pump.

Slip = $Q_T - Q_{act}$

% Slip = $[(Q_T - Q_{act})/Q_T] \times 100$ %

- If Q_{act} is more than the Q_T then slip will be –ive.
- If Q_{act} lesser than Q_T then the slip will be +ive.

24. Define indicator diagram.

The indicator diagram for a reciprocating pump is defined as the graph drawn between the pressure head in the cylinder and the distance traveled by the piston for one complete revolution of the crank.

16. What is an air vessel?

An air vessel is a closed chamber containing compressed air in the top portion and liquid at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid may flow into the vessel or out from the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out of the vessel, the air will expand into the chamber.

17. What is the purpose of an air vessel fitted in the pump?

- **a.** To obtain a continuous supply of liquid at a uniform rate.
- **b.** To save a considerable amount of work in overcoming the frictional resistance in the suction and delivery pipes, and
- **c.** To run the pump at a high speed without separation.

18. Define Cavitation.

If the pressure in the cylinder is below the vapour pressure, the dissolved gases will be liberated from the liquid and air bubbles are formed. This process is termed as cavitation.

EXERCISE PROBLEMS

7. A centrifugal pump is to discharge 0.118 m^3/s at a speed of 1450 rpm against a head of 25m. The impeller diameter is 250mm, its width at outlet is 50mm and manometric efficiency is 75%. Determine the vane angle at the periphery of the impeller.

8. A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm work against a total head of 40 m. The velocity of flow through the impeller is constant and equal to 2.5 m/s. The vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at outlet is 50 mm, determine: (i) Vane angle at inlet (ii) Work done by impeller on water per second.

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10. A centrifugal pump delivers water against a net head of 14.5m and a design speed of 1000 rpm. The vanes are curved back to an angle of 30^0 with the periphery. The impeller diameter is 300 mm and outlet width 50mm. Determine the discharge of the pump if the manometric efficiency is 95%.

11. A double – acting reciprocating pump running at 50 rpm is discharging 900 litres of water per minute. The pump has stroke of 400mm. The diameter of piston is 250mm. The delivery and suction heads are 25m and 4m respectively. Find the slip of the pump and power required to drive the pump.

12. The internal and external diameters of the impeller of a centrifugal pump are 300mm and 600mm respectively. The pump is running at 1000 rpm. The vane angles at inlet and outlet are 20^0 and 30^0 respectively. The water enters the impeller radially and velocity of the flow is constant. Determine the work done by the impeller per unit weight of water.