SYLLABUS

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE – 21

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

15BEME601 OPERATIONS RESEARCH 3 2 0 4 100

OBJECTIVE

1. To provide knowledge and training in using optimization techniques under limited resources for the engineering and business problems.

UNIT I INTRODUCTION TO OPERATIONS RESEARCH 12

Operations research and decision–making – types of mathematical models and constructing the model – Role of computers in operations research –Linear Programming Techniques: Formulation of linear programming problem, applications and limitations, graphical method, simplex method – The Big –M method – the two–phase method.

UNIT II TRANSPORTATION PROBLEMS 12

Least cost method, North west corner rule, Vogel's approximation method, modified distribution method, optimization models, unbalance and degeneracy in transportation model.

UNIT III ASSIGNMENT MODELS AND SCHEDULING 12

Assignment models - Hungarian algorithm, unbalanced assignment problems - maximization case in assignment problems, traveling salesman problem. Scheduling – processing n jobs through two machines, processing n jobs through three machines, processing two jobs through 'm' machines, processing n jobs through m machines.

UNIT IV INVENTORY CONTROL AND QUEUING THEORY 12

Variables in inventory problems, inventory models with penalty, shortage and quantity discount, safety stock, multi item deterministic model.

Queuing Models: Queues – Notation of queues, performance measures, The M/M/1 queue, The M/M/m queue, batch arrival queuing system, queues with breakdowns.

UNIT V PROJECT MANAGEMENT, GAME THEORY, REPLACEMENT MODELS 12

Basic terminologies, constructing a project network, network computations in CPM and PERT, cost crashing –Replacement Models: Replacement of Items due to deterioration with and without time value of Money, Group replacement policy, Staff replacement

TOTAL 60

TEXT BOOKS

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Kanti Swarup, Gupta P.K and Manmohan	Operations Research	Sultan Chand and Sons, New Delhi	2010

REFERENCES

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Viswanathan N and Narahari Y	Performance Modeling of Automated Manufacturing Systems	Prentice Hall Inc, Newyork	2000
2	Prem kumar Gupta and Hira D.S	Operation Research	S Chand and Company Limited, New Delhi	2015

WEB REFERENCES

1. http://www.scienceofbetter.org/what/index.htm
2. http://www.informs.org/Pubs/OR
3. http://www.me.utexas.edu/~jensen/ORMM/models/unit/network/subunits/special_cases/transportation.html
4. http://www.projectmanagement.com/

COURSE PLAN

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE – 21

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE PLAN

Subject Name : **OPERATIONS RESEARCH**
Subject Code : **15BEME601 (Credits - 4)**
Name of the Faculty :
Designation :
Year/Semester/Section : **III YEAR / VI Semester**
Branch : **MECHANICAL ENGINEERING**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – I : INTRODUCTION TO OPERATIONS RESEARCH			
1.	1	Introduction and Fundaments about Operations Research and Decision-Making Process	**T[1], R[2], W[1]**
2.	1	Types of Mathematical Models and Constructing the Model	**T[1], R[2]**
3.	1	Role of Computers in Operations Research	**T[1], R[2]**
4.	1	Linear Programming Techniques: Formulation of Linear Programming Problem	**T[1], R[2]**
5.	1	Applications and Limitations of Linear Programming	**T[1], R[2]**
6.	1	Graphical Method	**T[1], R[2]**
7.	1	**Tutorial 1** (Problems from Graphical Method)	**R[2]**
8.	2	Simplex Method	**T[1], R[2]**
9.	1	**Tutorial 2** (Problems from Simplex Method)	**R[2]**
10.	1	The Big –M Method	**T[1], R[2]**
11.	1	The Two–Phase Method	**T[1], R[2]**
12.	1	**Tutorial 3** (Problems from Big –M Method)	**R[2]**
Total No. of Periods Planned for Unit - I			**13**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – II : TRANSPORTATION PROBLEMS			
13.	1	Introduction to Transportation Problems	**T[1], R[2], W[1]**
14.	1	Least Cost Method	**T[1], R[2]**
15.	1	North West Corner Rule	**T[1], R[2]**
16.	2	Vogel's Approximation Method - problems	**T[1], R[2]**
17.	1	**Tutorial 4** (Problems from Least Cost Method , North West Corner Rule and Vogel's Approximation Method)	**R[2]**
18.	2	Modified Distribution Method	**T[1], R[2]**
19.	1	Optimization Models	**T[1], R[2]**
20.	2	Unbalance and Degeneracy in Transportation Model	**T[1], R[2]**
21.	1	**Tutorial 5** (Problems from Optimization Models)	**R[2]**
Total No. of Periods Planned for Unit - II			**12**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – III : ASSIGNMENT MODELS AND SCHEDULING			
22.	1	Introduction to Assignment Models Difference Between Transportation & Assignment Problems	**T[1], R[2], W[1]**
23.	2	Hungarian Algorithm	**T[1], R[2]**
24.	1	Unbalanced Assignment Problems Maximization Case in Assignment Problems	**T[1], R[2]**
25.	1	Traveling Salesman Problem	**T[1], R[2]**
26.	1	**Tutorial 6** (Problems from Assignment Problems and Traveling Salesman Problem)	**R[2]**
27.	1	Introduction to Scheduling	**T[1], R[2], W[1]**
28.	1	Processing ‘n’ Jobs Through Two Machines,	**T[1], R[2]**
29.	1	Processing ‘n’ Jobs Through Three Machines	**T[1], R[2]**
30.	1	Processing Two Jobs Through ‘m’ Machines	**T[1], R[2]**
31.	1	Processing ‘n’ Jobs Through ‘m’ Machines	**T[1], R[2]**
32.	1	**Tutorial 7** (Problems from Scheduling)	**R[2]**
Total No. of Periods Planned for Unit - III			**12**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – IV : INVENTORY CONTROL AND QUEUING THEORY			
33.	1	Variables in Inventory Problems, Inventory Models with Penalty	**T[1], R[2], W[1]**
34.	1	Shortage and Quantity Discount, Safety Stock	**T[1], R[2]**
35.	2	Multi item deterministic model	**T[1], R[2]**
36.	1	**Tutorial 8** (Problems from Inventory Problems)	**R[2]**
37.	1	**Tutorial 9** (Problems from Multi item deterministic model)	**R[2]**
38.	1	Queuing Models	**T[1], R[2], W[1]**
39.	1	Queues – Notation of queues & Performance Measures	**T[1], R[2], W[1]**
40.	1	The M/M/1 Queue,	**T[1], R[2]**
41.	1	The M/M/m queue	**T[1], R[2]**
42.	1	Batch Arrival Queuing System	**T[1], R[2]**
43.	1	Queues With Breakdowns	**T[1], R[2]**
44.	1	**Tutorial 10** (Problems from Queuing Models)	**R[2]**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – IV : INVENTORY CONTROL AND QUEUING THEORY			
		Total No. of Periods Planned for Unit - IV	**13**

Sl. No.	No. of Periods	Topics to be Covered	Support Materials
UNIT – V : PROJECT MANAGEMENT, GAME THEORY, REPLACEMENT MODELS			
45.	1	Basic terminologies, Constructing a project network	**T[1], R[2], W[1]**
46.	2	Network computations in CPM	**T[1], R[2]**
47.	1	Network computations in PERT	**T[1], R[2]**
48.	1	Cost crashing	**T[1], R[2]**
49.	1	**Tutorial 11** (Problems from CPM and PERT)	**R[2]**
50.	2	Replacement Models	**T[1], R[2], W[1]**
51.	1	Replacement of Items due to deterioration with and without time value of Money	**T[1], R[2]**
52.	1	Group replacement policy	**T[1], R[2]**
53.	1	Staff replacement	**T[1], R[2]**
54.	1	**Tutorial 12** (Problems from Replacement Models)	**R[2]**
55.	2	**Discussion on Competitive Examination related Questions / University previous year questions**	**GATE, ESE QP's**
Total No. of Periods Planned for Unit - V			**14**

TOTAL PERIODS : 64

TEXT BOOKS

T [1] – Kanti Swarup, Gupta P.K & Manmohan, 2010. "Operations Research", Sultan Chand & Sons, New Delhi.

REFERENCES

R [1] - Viswanathan N andNarahari Y, 2000. "Performance Modeling of Automated Manufacturing Systems", Prentice Hall Inc, Newyork
R [2] - Prem kumar Gupta and Hira D.S, 2015. "Operation Research", S Chand and Company Limited, New Delhi

WEBSITES

W [1] - https://nptel.ac.in/courses/112106134/
W [2] - http://www.me.utexas.edu/~jensen/ORMM/models/unit/network/subunits/special_cases/transportation.html
W [3] - http://www.projectmanagement.com/

JOURNALS

J [1] – http://www.informs.org/Pubs/OR
J [2] – International Journal of Operations Research [Inderscience]

COURSE PLAN

UNIT	Total No. of Periods Planned	Lecture Periods	Tutorial Periods
I	13	10	03
II	12	10	02
III	12	10	02
IV	13	10	03
V	14	12	02
TOTAL	64	52	12

I. CONTINUOUS INTERNAL ASSESSMENT : 40 Marks

(Internal Assessment Tests: 30, Attendance: 5, Assignment/Seminar: 5)

II. END SEMESTER EXAMINATION : 60 Marks

TOTAL : 100 Marks

STAFF INCHARGE HOD / MECH DEAN / FOE

UNIT I

1. 1 Introduction - Operations Research

The term Operations Research (OR) was first coined by MC Closky and Trefthen in 1940 in a small town, Bowdsey of UK. The main origin of OR was during the second world war – The military commands of UK and USA engaged several inter-disciplinary teams of scientists to undertake scientific research into strategic and tactical military operations.

Their mission was to formulate specific proposals and to arrive at the decision on optimal utilization of scarce military resources and also to implement the decisions effectively. In simple words, it was to uncover the methods that can yield greatest results with little efforts. Thus it had gained popularity and was called “An art of winning the war without actually fighting it”

The name Operations Research (OR) was invented because the team was dealing with research on military operations. The encouraging results obtained by British OR teams motivated US military management to start with similar activities. The work of OR team was given various names in US: Operational Analysis, Operations Evaluation, Operations Research, System Analysis, System Research, Systems Evaluation and so on.

The first method in this direction was simplex method of linear programming developed in 1947 by G.B Dantzig, USA. Since then, new techniques and applications have been developed to yield high profit from least costs.

Now OR activities has become universally applicable to any area such as transportation, hospital management, agriculture, libraries, city planning, financial institutions, construction management and so forth. In India many of the industries like Delhi cloth mills, Indian Airlines, Indian Railway, etc are making use of OR activity.

1.2 Concept and Definition of OR

Operations research signifies research on operations. It is the organized application of modern science, mathematics and computer techniques to complex military, government, business or industrial problems arising in the direction and management of large systems of men, material, money and machines. The purpose is to provide the management with explicit quantitative understanding and assessment of complex situations to have sound basics for arriving at best decisions.
Operations research seeks the optimum state in all conditions and thus provides optimum solution to organizational problems.

Definition: OR is a scientific methodology – analytical, experimental and quantitative – which by assessing the overall implications of various alternative courses of action in a management system provides an improved basis for management decisions.

1.3 Characteristics of OR (Features)

The essential characteristics of OR are

1. **Inter-disciplinary team approach** – The optimum solution is found by a team of scientists selected from various disciplines.
2. **Wholistic approach to the system** – OR takes into account all significant factors and finds the best optimum solution to the total organization.

3. **Imperfectness of solutions** – Improves the quality of solution.
4. **Use of scientific research** – Uses scientific research to reach optimum solution.
5. **To optimize the total output** – It tries to optimize by maximizing the profit and minimizing the loss.

1.4 Applications of OR

Some areas of applications are

Finance, Budgeting and Investment
- Cash flow analysis , investment portfolios
- Credit polices, account procedures

Purchasing, Procurement and Exploration
- Rules for buying, supplies
- Quantities and timing of purchase
- Replacement policies

Production management
- Physical distribution
- Facilities planning
- Manufacturing
- Maintenance and project scheduling

Marketing
- Product selection, timing
- Number of salesman, advertising

Personnel management
- Selection of suitable personnel on minimum salary
- Mixes of age and skills

Research and development
- Project selection
- Determination of area of research and development
- Reliability and alternative design

1.5 Phases of OR

OR study generally involves the following major phases

1. Defining the problem and gathering data
2. Formulating a mathematical model
3. Deriving solutions from the model
4. Testing the model and its solutions
5. Preparing to apply the model
6. Implementation

Defining the problem and gathering data

The first task is to study the relevant system and develop a well-defined statement of the problem. This includes determining appropriate objectives, constraints, interrelationships and alternative course of action.

The OR team normally works in an advisory capacity. The team performs a detailed technical analysis of the problem and then presents recommendations to the management.

Ascertaining the appropriate **objectives** is very important aspect of problem definition. Some of the objectives include maintaining stable price, profits, increasing the share in market, improving work morale etc. OR team typically spends huge amount of time in gathering relevant data.

- To gain accurate understanding of problem
- To provide input for next phase.

OR teams uses Data mining methods to search large databases for interesting patterns that may lead to useful decisions.

Formulating a mathematical model

This phase is to reformulate the problem in terms of mathematical symbols and expressions. The mathematical model of a business problem is described as the system of equations and related mathematical expressions. Thus

1. **Decision variables** (x_1, x_2 … x_n) – 'n' related quantifiable decisions to be made.
2. **Objective function** – measure of performance (profit) expressed as mathematical function of decision variables. For example $P=3x_1 +5x_2 + \ldots + 4x_n$
3. **Constraints** – any restriction on values that can be assigned to decision variables in terms of inequalities or equations. For example $x_1 +2x_2 \geq 20$
4. **Parameters** – the constant in the constraints (right hand side values)

The advantages of using mathematical models are

- Describe the problem more concisely
- Makes overall structure of problem comprehensible
- Helps to reveal important cause-and-effect relationships
- Indicates clearly what additional data are relevant for analysis
- Forms a bridge to use mathematical technique in computers to analyze

Deriving solutions from the model

This phase is to develop a procedure for deriving solutions to the problem. A common theme is to search for an optimal or best solution. The main goal of OR team is to obtain an optimal solution which minimizes the cost and time and maximizes the profit.

Herbert Simon says that "Satisficing is more prevalent than optimizing in actual practice". Where satisficing = satisfactory + optimizing

Samuel Eilon says that "Optimizing is the science of the ultimate; Satisficing is the art of the feasible".

To obtain the solution, the OR team uses

Heuristic procedure (designed procedure that does not guarantee an optimal solution) is used to find a good suboptimal solution.

Metaheuristics provides both general structure and strategy guidelines for designing a specific heuristic procedure to fit a particular kind of problem.

Post-Optimality analysis is the analysis done after finding an optimal solution. It is also referred as **what-if analysis**. It involves conducting **sensitivity analysis** to determine which parameters of the model are most critical in determining the solution.

Testing the model

After deriving the solution, it is tested as a whole for errors if any. The process of testing and improving a model to increase its validity is commonly referred as **Model validation**. The OR group doing this review should preferably include at least one individual who did not participate in the formulation of model to reveal mistakes.

A systematic approach to test the model is to use Retrospective test. This test uses historical data to reconstruct the past and then determine the model and the resulting solution. Comparing the effectiveness of this hypothetical performance with what actually happened indicates whether the model tends to yield a significant improvement over current practice.

Preparing to apply the model

After the completion of testing phase, the next step is to install a well-documented system for applying the model. This system will include the model, solution procedure and operating procedures for implementation.

The system usually is computer-based. **Databases** and **Management Information System** may provide up-to-date input for the model. An interactive computer based system called **Decision Support System** is installed to help the manager to use data and models to support their decision making as needed. A **managerial report** interprets output of the model and its implications for applications.

Implementation

The last phase of an OR study is to implement the system as prescribed by the management. The success of this phase depends on the support of both top management and operating management.

The implementation phase involves several steps

1. OR team provides a detailed explanation to the operating management
2. If the solution is satisfied, then operating management will provide the explanation to the personnel, the new course of action.
3. The OR team monitors the functioning of the new system
4. Feedback is obtained
5. Documentation

<u>Introduction to Linear Programming</u>

A linear form is meant a mathematical expression of the type $a_1x_1 + a_2x_2 + \ldots + a_nx_n$, where $a_1, a_2, \ldots, a_n$ are constants and $x_1, x_2 \ldots x_n$ are variables. The term Programming refers to the process of determining a particular program or plan of action. So Linear Programming (LP) is one of the most important optimization (maximization / minimization) techniques developed in the field of Operations Research (OR).

The methods applied for solving a linear programming problem are basically simple problems; a solution can be obtained by a set of simultaneous equations. However a unique solution for a set of simultaneous equations in n-variables (x1, x2 … xn), at least one of them is non-zero, can be obtained if there are exactly n relations. When the number of relations is greater than or less than n, a unique solution does not exist but a number of trial solutions can be found.

In various practical situations, the problems are seen in which the number of relations is not equal to the number of the number of variables and many of the relations are in the form of inequalities ($\leq$ or $\geq$) to maximize or minimize a linear function of the variables subject to such conditions. Such problems are known as Linear Programming Problem (LPP).

Definition – The general LPP calls for optimizing (maximizing / minimizing) a linear function of variables called the '**Objective function**' subject to a set of linear equations and / or inequalities called the '**Constraints**' or '**Restrictions'**.

General form of LPP

We formulate a mathematical model for general problem of allocating resources to activities. In particular, this model is to select the values for $x_1, x_2 \ldots x_n$ so as to maximize or minimize

$$Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots + c_nx_n$$

subject to restrictions

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots\ldots + a_1nx_n \ (\leq \text{ or } \geq)\ b_1 \ a_{21}x_1 +$$
$$a_{22}x_2 + \ldots\ldots\ldots\ldots + a_2nx_n \ (\leq \text{ or } \geq)\ b_2$$
$$.$$
$$.$$
$$.$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}x_n \ (\leq \text{ or } \geq)\ b_m$$

and

$$x_1 \geq 0,\ x_2 \geq 0, \ldots,\ x_n \geq 0$$

Where

Z = value of overall measure of performance
x_j = level of activity (for j = 1, 2, ..., n)
c_j = increase in Z that would result from each unit increase in level of activity j
b_i = amount of resource i that is available for allocation to activities (for i = 1,2, …, m)
a_{ij} = amount of resource i consumed by each unit of activity j

Resource	Resource usage per unit of activity Activity 1 2 ………………………… n	Amount of resource available
1	$a_{11}\ a_{12} \ldots\ldots\ldots a_{1n}$	b_1
2	$a_{21}\ a_{22} \ldots\ldots\ldots a_{2n}$	b_2
.	.	.
.	.	.
.	.	.
m	$a_{m1}\ a_{m2} \ldots\ldots\ldots a_{mn}$	b_m
Contribution to Z per unit of activity	$c_1\ c_2 \ldots\ldots\ldots c_n$	

Data needed for LP model

- The level of activities $x_1, x_2 \ldots\ldots\ldots x_n$ are called **decision variables**.
- The values of the c_j, b_i, a_{ij} (for i=1, 2 … m and j=1, 2 … n) are the **input constants** for the model. They are called as **parameters** of the model.

- The function being maximized or minimized $Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$ is called **objective function**.
- The restrictions are normally called as **constraints**. The constraint $a_{i1}x_1 + a_{i2}x_2 \ldots a_{in}x_n$ are sometimes called as **functional constraint** (L.H.S constraint). $x_j \geq 0$ restrictions are called **non-negativity constraint**.

<u>Assumptions in LPP</u>

a) Proportionality
b) Additivity
c) Multiplivity
d) Divisibility
e) Deterministic

<u>Applications of Linear Programming</u>

1. Personnel Assignment Problem
2. Transportation Problem
3. Efficiency on Operation of system of Dams
4. Optimum Estimation of Executive Compensation
5. Agriculture Applications
6. Military Applications
7. Production Management
8. Marketing Management
9. Manpower Management
10. Physical distribution

<u>Advantages of Linear Programming Techniques</u>

1. It helps us in making the optimum utilization of productive resources.
2. The quality of decisions may also be improved by linear programming techniques.
3. Provides practically solutions.
4. In production processes, high lighting of bottlenecks is the most significant advantage of this technique.

<u>Formulation of LP Problems</u>

Example 1

A firm manufactures two types of products A and B and sells them at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processed on two machines G and H. Type A requires 1 minute of processing time on G and 2 minutes on H; type B requires 1 minute on G and 1 minute on H. The machine G is available for not more than 6 hours 40 minutes while machine H is available for 10 hours during any working day. Formulate the problem as a linear programming problem.

Solution

Let

x_1 be the number of products of type A
x_2 be the number of products of type B

After understanding the problem, the given information can be systematically arranged in the form of the following table.

	Type of products (minutes)		
Machine	Type A (x_1 units)	Type B (x_2 units)	Available time (mins)
G	1	1	400
H	2	1	600
Profit per unit	Rs. 2	Rs. 3	

Since the profit on type A is Rs. 2 per product, 2 x_1 will be the profit on selling x_1 units of type A. similarly, $3x_2$ will be the profit on selling x_2 units of type B. Therefore, total profit on selling x_1 units of A and x_2 units of type B is given by

Maximize $Z = 2x_1+3x_2$ (**objective function**)

Since machine G takes 1 minute time on type A and 1 minute time on type B, the total number of minutes required on machine G is given by $x_1+ x_2$.

Similarly, the total number of minutes required on machine H is given by $2x_1 + 3x_2$.

But, machine G is not available for more than 6 hours 40 minutes (400 minutes).
Therefore, $x_1+ x_2 \leq 400$ (**first constraint**)

Also, the machine H is available for 10 hours (600 minutes) only, therefore, $2x_1 + 3x_2 \leq 600$ (**second constraint**)

Since it is not possible to produce negative quantities
$x_1 \geq 0$ and $x_2 \geq 0$ (**non-negative restrictions**)

Hence

Maximize $Z = 2x_1 + 3x_2$

Subject to restrictions

$x_1 + x_2 \leq 400$
$2x_1 + 3x_2 \leq 600$

and non-negativity constraints

$x_1 \geq 0$, $x_2 \geq 0$

Example 2

A company produces two products A and B which possess raw materials 400 quintals and 450 labour hours. It is known that 1 unit of product A requires 5 quintals of raw materials and 10 man hours and yields a profit of Rs 45. Product B requires 20 quintals of raw materials, 15 man hours and yields a profit of Rs 80. Formulate the LPP.

Solution

Let

x_1 be the number of units of product A
x_2 be the number of units of product B

	Product A	Product B	Availability
Raw materials	5	20	400
Man hours	10	15	450
Profit	Rs 45	Rs 80	

Hence

Maximize $Z = 45x_1 + 80x_2$

Subject to

$5x_1 + 20x_2 \leq 400$

$10x_1 + 15x_2 \leq 450$

$x_1 \geq 0$, $x_2 \geq 0$

Example 3

A firm manufactures 3 products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 respectively. The firm has 2 machines and below is given the required processing time in minutes for each machine on each product.

	Products		
Machine	A	B	C
X	4	3	5
Y	2	2	4

Machine X and Y have 2000 and 2500 machine minutes. The firm must manufacture 100 A's, 200 B's and 50 C's type, but not more than 150 A's.

Solution

Let

x_1 be the number of units of product A
x_2 be the number of units of product B
x_3 be the number of units of product C

	Products			
Machine	A	B	C	Availability
X	4	3	5	2000
Y	2	2	4	2500
Profit	3	2	4	

Max $Z = 3x_1 + 2x_2 + 4x_3$

Subject to

$4x_1 + 3x_2 + 5x_3 \leq 2000$
$2x_1 + 2x_2 + 4x_3 \leq 2500$
$100 \leq x_1 \leq 150$ $x_2 \geq 200$
$x_3 \geq 50$

Example 4

A company owns 2 oil mills A and B which have different production capacities for low, high and medium grade oil. The company enters into a contract to supply oil to a firm every week with 12, 8, 24 barrels of each grade respectively. It costs the company Rs 1000 and Rs 800 per day to run the mills A and B. On a day A produces 6, 2, 4 barrels of each grade and B produces 2, 2, 12 barrels of each grade. Formulate an LPP to determine number of days per week each mill will be operated in order to meet the contract economically.

Solution

Let

x_1 be the no. of days a week the mill A has to work x_2 be the no. of days per week the mill B has to work

Grade	A	B	Minimum requirement
Low	6	2	12
High	2	2	8
Medium	4	12	24
Cost per day	Rs 1000	Rs 800	

Minimize $Z = 1000x_1 + 800\,x_2$

Subject to

$6x_1 + 2x_2 \geq 12$

$2x_1 + 2x_2 \geq 8$

$4x_1 + 12x_2 \geq 24$

$x_1 \geq 0$, $x_2 \geq 0$

Example 5

A company has 3 operational departments weaving, processing and packing with the capacity to produce 3 different types of clothes that are suiting, shirting and woolen yielding with the profit of Rs. 2, Rs. 4 and Rs. 3 per meters respectively. 1m suiting requires 3mins in weaving 2 mins in processing and 1 min in packing. Similarly 1m of shirting requires 4 mins in weaving 1 min in processing and 3 mins in packing while 1m of woolen requires 3 mins in each department. In a week total run time of each department is 60, 40 and 80 hours for weaving, processing and packing department respectively. Formulate a LPP to find the product to maximize the profit.

Solution

Let,

x_1 be the number of units of suiting

x_2 be the number of units of shirting

x_3 be the number of units of woolen

	Suiting	Shirting	Woolen	Available time
Weaving	3	4	3	60
Processing	2	1	3	40
Packing	1	3	3	80
Profit	2	4	3	

Maximize $Z = 2x_1 + 4x_2 + 3x_3$

Subject to

$3x_1 + 4x_2 + 3x_3 \leq 60$

$2x_1 + 1x_2 + 3x_3 \leq 40$

$x_1 + 3x_2 + 3x_3 \leq 80$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

Example 6

ABC Company produces both interior and exterior paints from 2 raw materials m1 and m2. The following table produces basic data of problem.

	Exterior paint	Interior paint	Availability
M1	6	4	24
M2	1	2	6
Profit per ton	5	4	

A market survey indicates that daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Also maximum daily demand for interior paint is 2 tons. Formulate LPP to determine the best product mix of interior and exterior paints that maximizes the daily total profit.

Solution

Let

x_1 be the number of units of exterior paint

x_2 be the number of units of interior paint

Maximize $Z = 5x_1 + 4x_2$

Subject to

$6x_1 + 4x_2 \leq 24$ $x_1 + 2x_2 \leq 6$

$x_2 - x_1 \leq 1$

$x_2 \leq 2$

$x_1 \geq 0,\ x_2 \geq 0$

b) The maximum daily demand for exterior paint is atmost 2.5 tons

$x_1 \leq 2.5$

c) Daily demand for interior paint is atleast 2 tons

$x_2 \geq 2$

d) Daily demand for interior paint is exactly 1 ton higher than that for exterior paint.

$x_2 > x_1 + 1$

Example 7

A company produces 2 types of hats. Each hat of the I type requires twice as much as labour time as the II type. The company can produce a total of 500 hats a day. The market limits daily sales of I and II types to 150 and 250 hats. Assuming that the profit per hat are Rs.8 for type A and Rs. 5 for type B. Formulate a LPP models in order to determine the number of hats to be produced of each type so as to maximize the profit.

Solution

Let x_1 be the number of hats produced by type A

Let x_2 be the number of hats produced by type B

Maximize $Z = 8x_1 + 5x_2$

Subject to

$2x_1 + x_2 \leq 500$ (labour time) $x_1 \leq 150$
$x_2 \leq 250$ $x_1 \geq 0$,
$x_2 \geq 0$

Graphical Solution Procedure

The graphical solution procedure

1. Consider each inequality constraint as equation.
2. Plot each equation on the graph as each one will geometrically represent a straight line.
3. Shade the feasible region. Every point on the line will satisfy the equation of the line. If the inequality constraint corresponding to that line is '≤' then the region below the line lying in the first quadrant is shaded. Similarly for '≥' the region above the line is shaded. The points lying in the common region will satisfy the constraints. This common region is called **feasible region.**
4. Choose the convenient value of Z and plot the objective function line.
5. Pull the objective function line until the extreme points of feasible region.
    a. In the maximization case this line will stop far from the origin and passing through at least one corner of the feasible region.
    b. In the minimization case, this line will stop near to the origin and passing through at least one corner of the feasible region.
6. Read the co-ordinates of the extreme points selected in step 5 and find the maximum or minimum value of Z.

Definitions

1. **Solution** – Any specification of the values for decision variable among (x_1, x_2… x_n) is called a solution.

2. **Feasible solution** is a solution for which all constraints are satisfied.

3. **Infeasible solution** is a solution for which atleast one constraint is not satisfied.

4. **Feasible region** is a collection of all feasible solutions.

5. **Optimal solution** is a feasible solution that has the most favorable value of the objective function.

6. **Most favorable value** is the largest value if the objective function is to be maximized, whereas it is the smallest value if the objective function is to be minimized.

7. **Multiple optimal solution** – More than one solution with the same optimal value of the objective function.

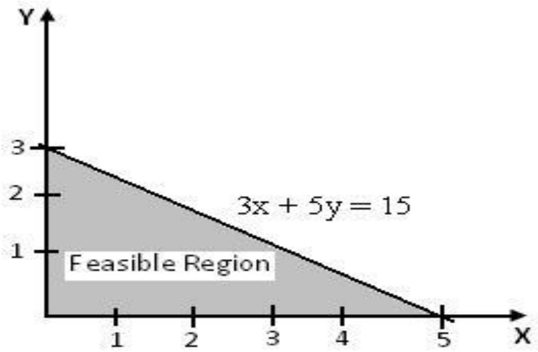
8. **Unbounded solution** – If the value of the objective function can be increased or decreased indefinitely such solutions are called unbounded solution.

9. **Feasible region** – The region containing all the solutions of an inequality

10. **Corner point feasible solution** is a solution that lies at the corner of the feasible region.

<u>Example problems</u>

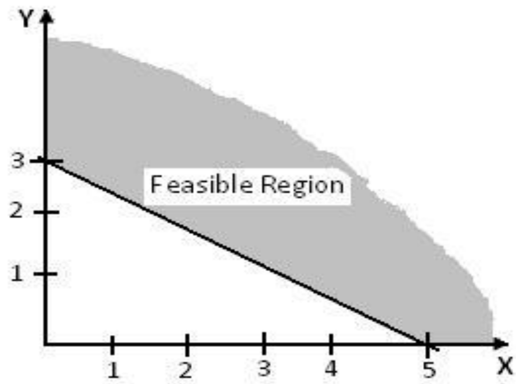
Example 1
Solve 3x + 5y < 15 graphically

Solution
- Write the given constraint in the form of equation i.e. 3x + 5y = 15
- Put x=0 then the value of y=3
- Put y=0 then the value of x=5
- Therefore the coordinates are (0, 3) and (5, 0). Thus these points are joined to form a straight line as shown in the graph.
- 

- Put x=0, y=0 in the given constraint then 0<15, the condition is true. (0, 0) is solution nearer to origin. So shade the region below the line, which is the feasible region.

Example 2
Solve 3x + 5y >15

Solution
- Write the given constraint in the form of equation i.e. 3x + 5y = 15
- Put x=0, then y=3
- Put y=0, then x=5
- So the coordinates are (0, 3) and (5, 0)
- Put x =0, y =0 in the given constraint, the condition turns out to be false i.e. 0 > 15 is false.
- So the region does not contain (0, 0) as solution. The feasible region lies on the outer part of the line as shown in the graph.



Example 3

Max Z = $80x_1 + 55x_2$

Subject to

$4x_1 + 2x_2 \leq 40$

$2x_1 + 4x_2 \leq 32$

$x_1 \geq 0$, $x_2 \geq 0$

Solution

The first constraint $4x_1 + 2x_2 \leq 40$, written in a form of equation $4x_1 + 2x_2 = 40$

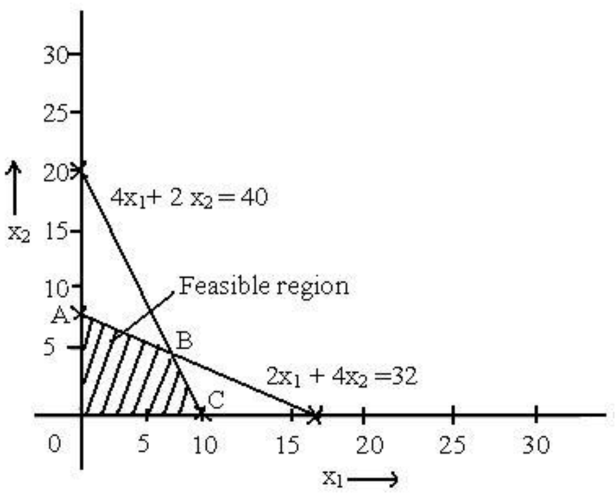
Put $x_1 = 0$, then $x_2 = 20$
Put $x_2 = 0$, then $x_1 = 10$
The coordinates are (0, 20) and (10, 0)

The second constraint $2x_1 + 4x_2 \leq 32$,
written in a form of equation $2x_1 + 4x_2 = 32$

Put $x_1 = 0$, then $x_2 = 8$
Put $x_2 = 0$, then $x_1 = 16$

The coordinates are (0, 8) and (16, 0)

The graphical representation is shown below.



The corner points of feasible region are A, B and C. So the coordinates for the corner points are A (0, 8) ,B (8, 4) (Solve the two equations $4x_1 + 2x_2 = 40$ and $2x_1 + 4x_2 = 32$ to get the coordinates) C (10, 0)

We know that Max Z = $80x_1 + 55x_2$

At A (0, 8) sub values in equation
Z = 80(0) + 55(8) = 440

At B (8, 4)
Z = 80(8) + 55(4) = 860

At C (10, 0)
Z = 80(10) + 55(0) = 800

The maximum value is obtained at the point B. Therefore Max Z = 860 and $x_1 = 8$, $x_2 = 4$

Example 4
Minimize Z = $10x_1 + 4x_2$

Subject to

$3x_1 + 2x_2 \geq 60$

$7x_1 + 2x_2 \geq 84$
$3x_1 + 6x_2 \geq 72$ $x_1 \geq 0$, x_2
≥ 0

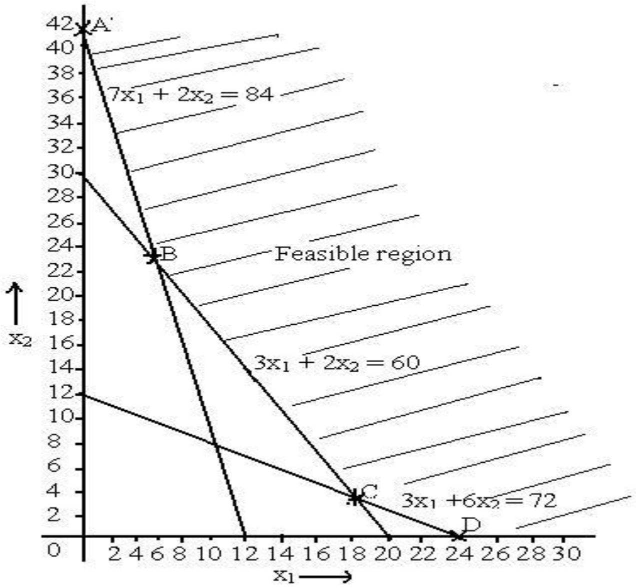
Solution

The first constraint $3x_1 + 2x_2 \geq 60$, written in a form of equation $3x_1 + 2x_2 = 60$
Put $x_1 = 0$, then $x_2 = 30$ Put $x_2 = 0$,
then $x_1 = 20$
The coordinates are (0, 30) and (20, 0)

The second constraint $7x_1 + 2x_2 \geq 84$, written in a form of equation $7x_1 + 2x_2 = 84$
Put $x_1 = 0$, then $x_2 = 42$ Put $x_2 = 0$,
then $x_1 = 12$
The coordinates are (0, 42) and (12, 0)

The third constraint $3x_1 + 6x_2 \geq 72$, written in a form of equation $3x_1 + 6x_2 = 72$
Put $x_1 = 0$, then $x_2 = 12$ Put $x_2 = 0$,
then $x_1 = 24$
The coordinates are (0, 12) and (24, 0)

The graphical representation is



The corner points of feasible region are A, B, C and D. So the coordinates for the corner points are A (0, 42) ,B (6, 21) (Solve the two equations $7x_1 + 2x_2 = 84$ and $3x_1 + 2x_2 = 60$ to get the coordinates) C (18, 3) Solve the two equations $3x_1 + 6x_2 = 72$ and $3x_1 + 2x_2 = 60$ to get the coordinates) D (24, 0)

We know that Min $Z = 10x_1 + 4x_2$

At A (0, 42)
$Z = 10(0) + 4(42) = 168$

At B (6, 21)
$Z = 10(6) + 4(21) = 144$

At C (18, 3)
$Z = 10(18) + 4(3) = 192$

At D (24, 0)
$Z = 10(24) + 4(0) = 240$

The minimum value is obtained at the point B. Therefore Min Z = 144 and $x_1 = 6$, $x_2 = 21$

Example 5

A manufacturer of furniture makes two products – chairs and tables. Processing of this product is done on two machines A and B. A chair requires 2 hours on machine A and 6 hours on machine B. A table requires 5 hours on machine A and no time on machine B. There are 16 hours of time per day available on machine A and 30 hours on machine B. Profit gained by the manufacturer from a chair and a table is Rs 2 and Rs 10 respectively. What should be the daily production of each of two products?

Solution

Let x_1 denotes the number of chairs
Let x_2 denotes the number of tables

	Chairs	Tables	Availability
Machine A	2	5	16
Machine B	6	0	30
Profit	Rs 2	Rs 10	

LPP

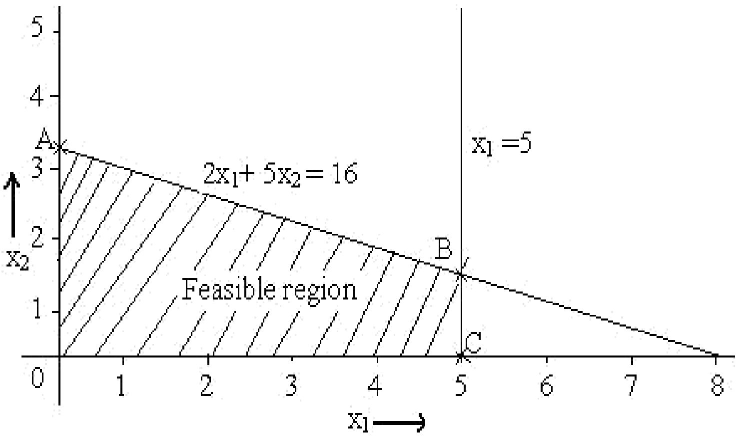
Max $Z = 2x_1 + 10x_2$

Subject to

$2x_1 + 5x_2 \leq 16$
$6x_1 + 0x_2 \leq 30$
$x_1 \geq 0$, $x_2 \geq 0$

Solving graphically

- The first constraint $2x_1 + 5x_2 \leq 16$, written in a form of equation $2x_1 + 5x_2 = 16$
- Put $x_1 = 0$, then $x_2 = 16/5 = 3.2$ and Put $x_2 = 0$, then $x_1 = 8$
- The coordinates are (0, 3.2) and (8, 0)



The second constraint $6x_1 + 0x_2 \leq 30$, written in a form of equation

$6x_1 = 30 \rightarrow x_1 = 5$

The corner points of feasible region are A, B and C. So the coordinates for the corner points are A (0, 3.2), B (5, 1.2) (Solve the two equations $2x_1 + 5x_2 = 16$ and $x_1 = 5$ to get the coordinates) C (5, 0).

We know that Max $Z = 2x_1 + 10x_2$
At A (0, 3.2)
$Z = 2(0) + 10(3.2) = 32$

At B (5, 1.2)
$Z = 2(5) + 10(1.2) = 22$

At C (5, 0)
$Z = 2(5) + 10(0) = 10$

Max $Z = 32$ and $x_1 = 0$, $x_2 = 3.2$

The manufacturer should produce approximately 3 tables and no chairs to get the max profit.

<u>Special Cases in Graphical Method</u>

<u>Multiple Optimal Solution</u>

Example 1
Solve by using graphical method

Max $Z = 4x_1 + 3x_2$

Subject to
$4x_1 + 3x_2 \leq 24$ $x_1 \leq 4.5$
$x_2 \leq 6$
$x_1 \geq 0$, $x_2 \geq 0$

Solution

The first constraint $4x_1 + 3x_2 \leq 24$, written in a form of equation $4x_1 + 3x_2 = 24$

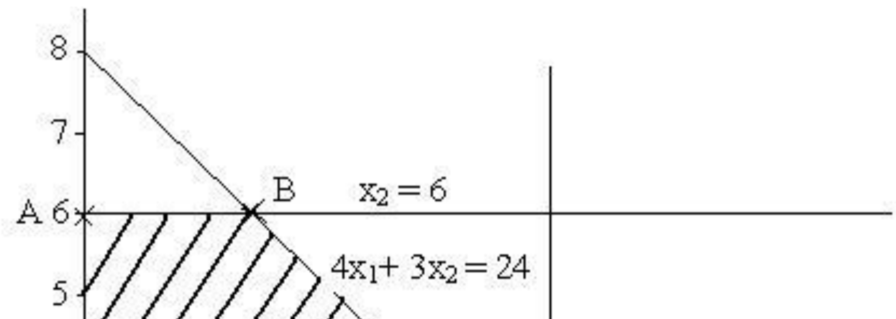
Put $x_1 = 0$, then $x_2 = 8$

Put $x_2 = 0$, then $x_1 = 6$

The coordinates are (0, 8) and (6, 0)

The second constraint $x_1 \leq 4.5$, written in a form of equation $x_1 = 4.5$

The third constraint $x_2 \leq 6$, written in a form of equation $x_2 = 6$



The corner points of feasible region are A, B, C and D. So the coordinates for the corner points are A (0, 6) , B (1.5, 6) (Solve the two equations $4x_1 + 3x_2 = 24$ and $x_2 = 6$ to get the coordinates) C (4.5, 2) (Solve the two equations $4x_1 + 3x_2 = 24$ and $x_1 = 4.5$ to get the coordinates) D (4.5, 0)

We know that Max $Z = 4x_1 + 3x_2$
At A (0, 6)
Z = 4(0) + 3(6) = 18

At B (1.5, 6)
Z = 4(1.5) + 3(6) = 24

At C (4.5, 2)
Z = 4(4.5) + 3(2) = 24

At D (4.5, 0)
Z = 4(4.5) + 3(0) = 18

Max Z = 24, which is achieved at both B and C corner points. It can be achieved not only at B and C but every point between B and C. Hence the given problem has multiple optimal solutions.

<u>No Optimal Solution</u>

Example 1
Solve graphically

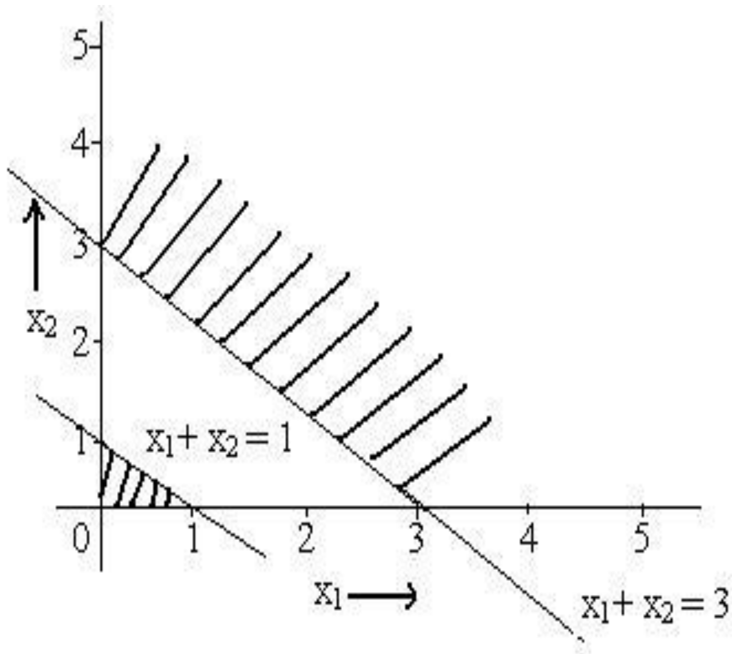
Max $Z = 3x_1 + 2x_2$

Subject to
$x_1 + x_2 \leq 1$ $x_1 + x_2 \geq$
3
$x_1 \geq 0$, $x_2 \geq 0$

Solution

The first constraint $x_1 + x_2 \leq 1$, written in a form of equation $x_1 + x_2 = 1$
Put $x_1 = 0$, then $x_2 = 1$ Put x_2
$= 0$, then $x_1 = 1$
The coordinates are (0, 1) and (1, 0)

The first constraint $x_1 + x_2 \geq 3$, written in a form of equation $x_1 + x_2 = 3$
Put $x_1 = 0$, then $x_2 = 3$ Put x_2
$= 0$, then $x_1 = 3$
The coordinates are (0, 3) and (3, 0)



There is no common feasible region generated by two constraints together i.e. we cannot identify even a single point satisfying the constraints. Hence there is no optimal solution.

Unbounded Solution

Example
Solve by graphical method

Max Z = $3x_1 + 5x_2$

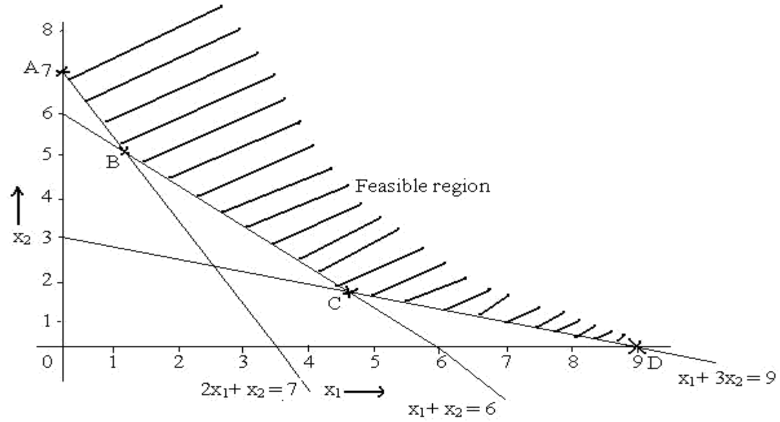
Subject to
$2x_1 + x_2 \geq 7$ $x_1 + x_2 \geq 6$
$x_1 + 3x_2 \geq 9$ $x_1 \geq 0$,
$x_2 \geq 0$

Solution
The first constraint $2x_1 + x_2 \geq 7$, written in a form of equation $2x_1 + x_2 = 7$
Put $x_1 = 0$, then $x_2 = 7$ Put $x_2 = 0$,
then $x_1 = 3.5$
The coordinates are (0, 7) and (3.5, 0)

The second constraint $x_1 + x_2 \geq 6$, written in a form of equation $x_1 + x_2 = 6$
Put $x_1 = 0$, then $x_2 = 6$ Put x_2

=0, then $x_1 = 6$
The coordinates are (0, 6) and (6, 0)

The third constraint $x_1 + 3x_2 \geq 9$, written in a form of equation $x_1 + 3x_2 = 9$
Put $x_1 = 0$, then $x_2 = 3$ Put x_2
=0, then $x_1 = 9$
The coordinates are (0, 3) and (9, 0)



The corner points of feasible region are A, B, C and D. So the coordinates for the corner points are A (0, 7), B (1, 5) (Solve the two equations $2x_1 + x_2 = 7$ and $x_1 + x_2 = 6$ to get the coordinates) C (4.5, 1.5) (Solve the two equations $x_1 + x_2 = 6$ and $x_1 + 3x_2 = 9$ to get the coordinates) D (9, 0)

We know that Max $Z = 3x_1 + 5x_2$
At A (0, 7)
$Z = 3(0) + 5(7) = 35$

At B (1, 5)
$Z = 3(1) + 5(5) = 28$

At C (4.5, 1.5)
$Z = 3(4.5) + 5(1.5) = 21$

At D (9, 0)
$Z = 3(9) + 5(0) = 27$

The values of objective function at corner points are 35, 28, 21 and 27. But there exists infinite number of points in the feasible region which is unbounded. The value of objective function will be more than the value of these four corner points i.e. the maximum value of the objective function occurs at a point at ∞. Hence the given problem has unbounded solution.

Steps to convert GLPP to SLPP (Standard LPP)

General Linear Programming Problem (GLPP)

Maximize / Minimize $Z = c_1x_1 + c_2x_2 + c_3x_3 + \ldots\ldots\ldots\ldots + c_nx_n$

Subject to constraints

$a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_n$ ($\leq$ or $\geq$) b_1 $a_{21}x_1 +$
$a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n$ ($\leq$ or $\geq$) b_2
.
.
$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots\ldots + a_{mn}x_n$ ($\leq$ or $\geq$) b_m

and

$x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$

Where constraints may be in the form of any inequality ($\leq$ or $\geq$) or even in the form of an equation (=) and finally satisfy the non-negativity restrictions.

Step 1 – Write the objective function in the maximization form. If the given objective function is of minimization form then multiply throughout by -1 and write Max $z' = \text{Min}(-z)$

Step 2 – Convert all inequalities as equations.

- If an equality of '$\leq$' appears then by adding a variable called **Slack variable.** We can convert it to an equation. For example $x_1 + 2x_2 \leq 12$, we can write as

$\mathbf{x_1 + 2x_2 + s_1 = 12.}$

- If the constraint is of '$\geq$' type, we subtract a variable called **Surplus variable** and convert it to an equation. For example $2x_1 + x_2 \geq 15$ can be written as

$\mathbf{2x_1 + x_2 - s_2 = 15}$

Step 3 – The right side element of each constraint should be made non-negative $2x_1 + x_2 - s_2 = -15$

$-2x_1 - x_2 + s_2 = 15$ (That is multiplying throughout by -1)

Step 4 – All variables must have non-negative values.

For example: $x_1 + x_2 \leq 3$

$x_1 > 0$, x_2 is unrestricted in sign

Then x_2 is written as $x_2 = x_2' - x_2''$

Therefore the inequality takes the form of equation as $x_1 + (x_2' - x_2'') + s_1 = 3$ Using the above steps, we can write the GLPP in the form of SLPP.

<u>Write the Standard LPP (SLPP) of the following</u>

Example 1

Maximize $Z = 3x_1 + x_2$

Subject to

$2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$ and

$x_1 \geq 0, x_2 \geq 0$

SLPP

Maximize $Z = 3x_1 + x_2$

Subject to

$2x_1 + x_2 + s_1 = 2$

$3x_1 + 4x_2 - s_2 = 12$

$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0$

Example 2

Minimize $Z = 4x_1 + 2x_2$

Subject to

$3x_1 + x_2 \geq 2$

$x_1 + x_2 \geq 21$

$x_1 + 2x_2 \geq 30$

and $x_1 \geq 0, x_2 \geq 0$

SLPP

Maximize $Z' = -4x_1 - 2x_2$

Subject to

$3x_1 + x_2 - s_1 = 2$
$x_1 + x_2 - s_2 = 21$
$x_1 + 2x_2 - s_3 = 30$
$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0$

Example 3

Minimize $Z = x_1 + 2x_2 + 3x_3$

Subject to

$2x_1 + 3x_2 + 3x_3 \geq -4$ $3x_1 + 5x_2 +$
$2x_3 \leq 7$

and $x_1 \geq 0, x_2 \geq 0, x_3$ is unrestricted in sign

SLPP

Maximize $Z' = -x_1 - 2x_2 - 3(x_3' - x_3'')$

Subject to

$-2x_1 - 3x_2 - 3(x_3' - x_3'') + s_1 = 4$
$3x_1 + 5x_2 + 2(x_3' - x_3'') + s_2 = 7$
$x_1 \geq 0, x_2 \geq 0, x_3' \geq 0, x_3'' \geq 0, s_1 \geq 0, s_2 \geq 0$

Some Basic Definitions

Solution of LPP

Any set of variable ($x_1, x_2 \ldots x_n$) which satisfies the given constraint is called solution of LPP.

Basic solution

Is a solution obtained by setting any 'n' variable equal to zero and solving remaining 'm' variables. Such 'm' variables are called **Basic variables** and 'n' variables are called **Non-basic variables**.

Basic feasible solution

A basic solution that is feasible (all basic variables are non negative) is called basic feasible solution. There are two types of basic feasible solution.

1. **Degenerate basic feasible solution**
   If any of the basic variable of a basic feasible solution is zero than it is said to be degenerate basic feasible solution.
2. **Non-degenerate basic feasible solution**
   It is a basic feasible solution which has exactly 'm' positive x_i, where i=1, 2, … m. In other words all 'm' basic variables are positive and remaining 'n' variables are zero.
3. **Optimum basic feasible solution**
   A basic feasible solution is said to be optimum if it optimizes (max / min) the objective function.

Introduction to Simplex Method

It was developed by G. Danztig in 1947. The simplex method provides an algorithm (a rule of procedure usually involving repetitive application of a prescribed operation) which is based on the fundamental theorem of linear programming.

The Simplex algorithm is an iterative procedure for solving LP problems in a finite number of steps. It consists in having a trial basic feasible solution to constraint-equations and testing whether it is an optimal solution and improving the first trial solution by a set of rules and repeating the process till an optimal solution is obtained.

Advantages

Simple to solve the problems
The solution of LPP of more than two variables can be obtained.

<u>Computational Procedure of Simplex Method</u>

Consider an example
Maximize $Z = 3x_1 + 2x_2$

Subject to
$x_1 + x_2 \leq 4$ $x_1 - x_2 \leq 2$
and $x_1 \geq 0, x_2 \geq 0$

Solution
Step 1 – Write the given GLPP in the form of SLPP Maximize Z
$= 3x_1 + 2x_2 + 0s_1 + 0s_2$
Subject to
$x_1 + x_2 + s_1 = 4$
$x_1 - x_2 + s_2 = 2$
$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0$

Step 2 – Present the constraints in the matrix form $x_1 + x_2 + s_1 = 4$
$x_1 - x_2 + s_2 = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Step 3 – Construct the starting simplex table using the notations

		$C_j \rightarrow$	3	2	0	0	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	Min ratio X_B / X_k
s_1	0	4	1	1	1	0	
s_2	0	2	1	-1	0	1	
	$Z = C_B X_B$		Δ_j				

Step 4 – Calculation of Z and Δ_j and test the basic feasible solution for optimality by the rules given.

$Z = C_B X_B$
$= 0 * 4 + 0 * 2 = 0$
$\Delta_j = Z_j - C_j$
$= C_B X_j - C_j$
$\Delta_1 = C_B X_1 - C_j = 0 * 1 + 0 * 1 - 3 = -3$
$\Delta_2 = C_B X_2 - C_j = 0 * 1 + 0 * -1 - 2 = -2$
$\Delta_3 = C_B X_3 - C_j = 0 * 1 + 0 * 0 - 0 = 0$
$\Delta_4 = C_B X_4 - C_j = 0 * 0 + 0 * 1 - 0 = 0$

Procedure to test the basic feasible solution for optimality by the rules given

Rule 1 – If all $\Delta_j \geq 0$, the solution under the test will be **optimal**. Alternate optimal solution will exist if any non-basic Δ_j is also zero.

Rule 2 – If atleast one Δ_j is negative, the solution is not optimal and then proceeds to improve the solution in the next step.

Rule 3 – If corresponding to any negative $_j$, all elements of the column X_j are negative or zero, then the solution under test will be **unbounded**.

In this problem it is observed that $_1$ and $_2$ are negative. Hence proceed to improve this solution

Step 5 – To improve the basic feasible solution, the vector entering the basis matrix and the vector to be removed from the basis matrix are determined.

Incoming vector

The incoming vector X_k is always selected corresponding to the most negative value of j. It is indicated by (↑).

Outgoing vector

The outgoing vector is selected corresponding to the least positive value of minimum ratio. It is indicated by (→).

Step 6 – Mark the key element or pivot element by ‘ $\boxed{}$ ‘.The element at the intersection of outgoing vector and incoming vector is the pivot element.

		Cj →	3	2	0	0	
Basic Variables	C_B	X_B	X_1 (X_k)	X_2	S_1	S_2	Min ratio X_B / X_k
s_1	0	4	1	1	1	0	4 / 1 = 4
s_2	0	2	$\boxed{1}$	-1	0	1	2 / 1 = 2 → outgoing
	$Z = C_B X_B = 0$		↑incoming $_1$= -3	$_2$= -2	$_3$=0	$_4$=0	

If the number in the marked position is other than unity, divide all the elements of that row by the key element. Then subtract appropriate multiples of this new row from the remaining rows, so as to obtain zeroes in the remaining position of the column X_k.

Basic Variables	C_B	X_B	X_1	X_2 (X_k)	S_1	S_2	Min ratio X_B / X_k
s_1	0	2	($R_1=R_1 – R_2$) 0	$\boxed{2}$	1	-1	2 / 2 = 1 → outgoing
x_1	3	2	1	-1	0	1	2 / -1 = -2 (neglect in case of negative)
	Z=0*2+3*2= 6		Δ_1=0	↑incoming Δ_2= -5	Δ_3=0	Δ_4=3	

Step 7 – Now repeat step 4 through step 6 until an optimal solution is obtained.

Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	Min ratio X_B / X_k
x_2	2	1	($R_1=R_1 / 2$) 0	1	1/2	-1/2	
x_1	3	3	($R_2=R_2 + R_1$) 1	0	1/2	1/2	
	Z = 11		Δ_1=0	Δ_2=0	Δ_3=5/2	Δ_4=1/2	

Therefore the solution is Max Z = 11, $x_1 = 3$ and $x_2 = 1$

Worked Examples

Solve by simplex method
Example 1
Maximize $Z = 80x_1 + 55x_2$

Subject to
$4x_1 + 2x_2 \leq 40$
$2x_1 + 4x_2 \leq 32$ and $x_1 \geq 0$, $x_2 \geq 0$

Solution

SLPP
Maximize $Z = 80x_1 + 55x_2 + 0s_1 + 0s_2$

Subject to
$4x_1 + 2x_2 + s_1 = 40$
$2x_1 + 4x_2 + s_2 = 32$
$x_1 \geq 0$, $x_2 \geq 0$, $s_1 \geq 0$, $s_2 \geq 0$

		$C_j \rightarrow$	80	55	0	0	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	Min ratio X_B / X_k
s_1	0	40	$\boxed{4}$	2	1	0	$40 / 4 = 10 \rightarrow$ outgoing
s_2	0	32	2	4	0	1	$32 / 2 = 16$
	$Z = C_B X_B = 0$		↑incoming $\Delta_1 = -80$	$\Delta_2 = -55$	$\Delta_3 = 0$	$\Delta_4 = 0$	
x_1	80	10	$(R_1 = R_1 / 4)$ 1	1/2	1/4	0	$10/1/2 = 20$
s_2	0	12	$(R_2 = R_2 - 2R_1)$ 0	$\boxed{3}$	-1/2	1	$12/3 = 4 \rightarrow$ outgoing
	$Z = 800$		$\Delta_1 = 0$	↑incoming $\Delta_2 = -15$	$\Delta_3 = 40$	$\Delta_4 = 0$	
x_1	80	8	$(R_1 = R_1 - 1/2R_2)$ 1	0	1/3	-1/6	
x_2	55	4	$(R_2 = R_2 / 3)$ 0	1	-1/6	1/3	
	$Z = 860$		$\Delta_1 = 0$	$\Delta_2 = 0$	$\Delta_3 = 35/2$	$\Delta_4 = 5$	

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained .Therefore the solution is Max Z = 860, $x_1 = 8$ and $x_2 = 4$

Example 2

Maximize $Z = 5x_1 + 3x_2$

Subject to

$3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

and $x_1 \geq 0$, $x_2 \geq 0$

Solution

SLPP: Maximize $Z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

Subject to

$3x_1 + 5x_2 + s_1 = 15$ $5x_1 + 2x_2 + s_2 = 10$

$x_1 \geq 0$, $x_2 \geq 0$, $s_1 \geq 0$, $s_2 \geq 0$

		$C_j \rightarrow$	5	3	0	0	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	Min ratio X_B / X_k
s_1	0	15	3	5	1	0	15 / 3 = 5
s_2	0	10	$\boxed{5}$	2	0	1	10 / 5 = 2 $\rightarrow$ outgoing
	$Z = C_B X_B = 0$		$\uparrow$incoming $\Delta_1 = -5$	$\Delta_2 = -3$	$\Delta_3 = 0$	$\Delta_4 = 0$	
s_1	0	9	$(R_1 = R_1 - 3R_2)$ 0	$\boxed{19/5}$	1	-3/5	9/19/5 = 45/19 $\rightarrow$
x_1	5	2	$(R_2 = R_2 / 5)$ 1	2/5	0	1/5	2/2/5 = 5
	$Z = 10$		$\Delta_1 = 0$	$\uparrow$ $\Delta_2 = -1$	$\Delta_3 = 0$	$\Delta_4 = 1$	
x_2	3	45/19	$(R_1 = R_1 / 19/5)$ 0	1	5/19	-3/19	
x_1	5	20/19	$(R_2 = R_2 - 2/5\ R_1)$ 1	0	-2/19	5/19	
	$Z = 235/19$		$\Delta_1 = 0$	$\Delta_2 = 0$	$\Delta_3 = 5/19$	$\Delta_4 = 16/19$	

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained. Therefore the solution is Max $Z = 235/19$, $x_1 = 20/19$ and $x_2 = 45/19$

Example 3

Maximize $Z = 5x_1 + 7x_2$

Subject to

$x_1 + x_2 \leq 4$

$3x_1 - 8x_2 \leq 24$
$10x_1 + 7x_2 \leq 35$
and $x_1 \geq 0$, $x_2 \geq 0$

Solution

SLPP

Maximize $Z = 5x_1 + 7x_2 + 0s_1 + 0s_2 + 0s_3$

Subject to

$x_1 + x_2 + s_1 = 4$
$3x_1 - 8x_2 + s_2 = 24$
$10x_1 + 7x_2 + s_3 = 35$
$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0$

		$C_j \rightarrow$	5	7	0	0	0	
Basic Variables	CB	XB	X1	X2	S1	S2	S3	Min ratio X_B / X_k
s_1	0	4	1	$\boxed{1}$	1	0	0	4 /1 = 4→outgoing
s_2	0	24	3	- 8	0	1	0	–
s_3	0	35	10	7	0	0	1	35 / 7 = 5
				↑incoming				
	$Z = C_B X_B = 0$		-5	-7	0	0	0	$\leftarrow \Delta_j$
x_2	7	4	1	1	1	0	0	
			$(R_2 = R_2 + 8R_1)$					
s_2	0	56	11	0	8	1	0	
			$(R_3 = R_3 - 7R_1)$					
s_3	0	7	3	0	-7	0	1	
	Z = 28		2	0	7	0	0	$\leftarrow \Delta_j$

Since all $\Delta_j \geq 0$, optimal basic feasible solution is obtained

Therefore the solution is Max Z = 28, $x_1 = 0$ and $x_2 = 4$

Example 4

Maximize $Z = 2x - 3y + z$

Subject to

$3x + 6y + z \leq 6$
$4x + 2y + z \leq 4$
$x - y + z \leq 3$
and $x \geq 0, y \geq 0, z \geq 0$

Solution

SLPP

Maximize $Z = 2x - 3y + z + 0s_1 + 0s_2 + 0s_3$

Subject to

$3x + 6y + z + s_1 = 6$
$4x + 2y + z + s_2 = 4$
$x - y + z + s_3 = 3$
$x \geq 0, y \geq 0, z \geq 0\ s_1 \geq 0, s_2 \geq 0, s_3 \geq 0$

Basic Variables	C_B	X_B	X	Y	Z	S_1	S_2	S_3	Min ratio X_B / X_k
		$C_j \rightarrow$	2	-3	1	0	0	0	
s_1	0	6	3	6	1	1	0	0	6 / 3 = 2
s_2	0	4	$\boxed{4}$	2	1	0	1	0	4 / 4 =1$\rightarrow$ outgoing
s_3	0	3	1	-1	1	0	0	1	3 / 1 = 3
	Z = 0		$\uparrow$incoming -2	3	-1	0	0	0	$\leftarrow\Delta_j$
s_1	0	3	0	9/2	1/4	1	-3/4	0	3/1/4=12
x	2	1	1	1/2	1/4	0	1/4	0	1/1/4=4
s_3	0	2	0	-3/2	$\boxed{3/4}$	0	-1/4	1	8/3 = 2.6$\rightarrow$
	Z = 2		0	4	$\uparrow$incoming 1/2	0	1/2	0	$\leftarrow\Delta_j$
s_1	0	7/3	0	5	0	1	-2/3	-1/3	
x	2	1/3	1	1	0	0	1/3	-1/3	
z	1	8/3	0	-2	1	0	-1/3	4/3	
	Z = 10/3		0	3	0	0	1/3	2/3	$\leftarrow\Delta_j$

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained. Therefore the solution is Max Z = 10/3, x = 1/3, y = 0 and z = 8/3

Example 5: Maximize $Z = 3x_1 + 5x_2$

Subject to

$3x_1 + 2x_2 \leq 18\ x_1 \leq 4$
$x_2 \leq 6$
and $x_1 \geq 0, x_2 \geq 0$

Solution

SLPP

Maximize $Z = 3x_1 + 5x_2 + 0s_1 + 0s_2 + 0s_3$

Subject to

$3x_1 + 2x_2 + s_1 = 18\ x_1 + s_2 = 4$
$x_2 + s_3 = 6$
$x_1 \geq 0, x_2 \geq 0, s_1 \geq 0, s_2 \geq 0, s_3 \geq 0$

		$C_j \rightarrow$	3	5	0	0	0	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	Min ratio X_B / X_k
s_1	0	18	3	2	1	0	0	18 / 2 = 9
s_2	0	4	1	0	0	1	0	4 / 0 = ∞ (neglect)
s_3	0	6	0	$\boxed{1}$	0	0	1	6 / 1 = 6→
	Z = 0		-3	↑ -5	0	0	0	←Δ_j
				$(R_1=R_1-2R_3)$				
s_1	0	6	$\boxed{3}$	0	1	0	-2	6 / 3 = 2 →
s_2	0	4	1	0	0	1	0	4 / 1 = 4
x_2	5	6	0	1	0	0	1	--
	Z = 30		↑ -3	0	0	0	5	←Δ_j
			$(R_1=R_1 / 3)$					
x_1	3	2	1	0	1/3	0	-2/3	
			$(R_2=R_2 - R_1)$					
s_2	0	2	0	0	-1/3	1	2/3	
x_2	5	6	0	1	0	0	1	
	Z = 36		0	0	1	0	3	←Δ_j

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained. Therefore the solution is Max Z = 36, $x_1 = 2$, $x_2 = 6$

Example 6

Minimize $Z = x_1 - 3x_2 + 2x_3$

Subject to

$3x_1 - x_2 + 3x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

and $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Solution

SLPP

$\text{Min}(-Z) = \text{Max } Z' = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3$

Subject to

$3x_1 - x_2 + 3x_3 + s_1 = 7$

$-2x_1 + 4x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0\ s_1 \geq 0, s_2 \geq 0, s_3 \geq 0$

		$C_j \rightarrow$	-1	3	-2	0	0	0	
Basic Variables	C_B	X_B	X_1	X_2	X_3	S_1	S_2	S_3	Min ratio X_B / X_k

s_1	0	7	3	-1	3	1	0	0	-
s_2	0	12	-2	4	0	0	1	0	3→
s_3	0	10	-4	3	8	0	0	1	10/3
				↑					
		Z' = 0	1	-3	2	0	0	0	←Δ_j
			($R_1 = R_1 + R_2$)						
s1	0	10	5/2	0	3	1	1/4	0	4→
			($R_2 = R_2 / 4$)						
x_2	3	3	-1/2	1	0	0	1/4	0	-
			($R_3 = R_3 – 3R_2$)						
s_3	0	1	-5/2	0	8	0	-3/4	1	-
			↑						
		Z' = 9	-5/2	0	0	0	3/4	0	←Δ_j
			($R_1 = R_1 / 5/2$)						
x1	-1	4	1	0	6/5	2/5	1/10	0	
			($R_2 = R_2 + 1/2 R_1$)						
x_2	3	5	0	1	3/5	1/5	3/10	0	
			($R_3 = R_3 + 5/2R_1$)						
s3	0	11	0	1	11	1	-1/2	1	
		Z' = 11	0	0	3/5	1/5	1/5	0	←Δ_j

Since all $\Delta_j \geq 0$, optimal basic feasible solution is obtained

Therefore the solution is Z' =11 which implies Z = -11, $x_1 = 4$, $x_2 = 5$, $x_3 = 0$

Computational Procedure of Big – M Method (Charne's Penalty Method)

Step 1 – Express the problem in the standard form.

Step 2 – Add non-negative artificial variable to the left side of each of the equations corresponding to the constraints of the type '≥' or '='.

When artificial variables are added, it causes violation of the corresponding constraints. This difficulty is removed by introducing a condition which ensures that artificial variables will be zero in the final solution (provided the solution of the problem exists).

On the other hand, if the problem does not have a solution, at least one of the artificial variables will appear in the final solution with positive value. This is achieved by assigning a very **large price (per unit penalty)** to these variables in the objective function. Such large price will be designated by –M for maximization problems (+M for minimizing problem), where M > 0.

Step 3 – In the last, use the artificial variables for the starting solution and proceed with the usual simplex routine until the optimal solution is obtained.

Worked Examples

Example 1
Max Z = $-2x_1 - x_2$

Subject to
$3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 6$ x_1
$+ 2x_2 \leq 4$
and $x_1 \geq 0, x_2 \geq 0$

Solution
SLPP
Max Z = $-2x_1 - x_2 + 0s_1 + 0s_2 - M\,a_1 - M\,a_2$

Subject to
$3x_1 + x_2 + a_1 = 3$
$4x_1 + 3x_2 - s_1 + a_2 = 6$ $x_1 +$
$2x_2 + s_2 = 4$
$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$

	$C_j \rightarrow$		-2	-1	0	0	-M	-M	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	A_1	A_2	Min ratio X_B / X_k
a_1	-M	3	$\boxed{3}$	1	0	0	1	0	3 /3 = 1$\rightarrow$
a_2	-M	6	4	3	-1	0	0	1	6 / 4 =1.5
s_2	0	4	1	2	0	1	0	0	4 / 1 = 4
	Z = -9M		$\uparrow$ 2 – 7M	1 – 4M	M	0	0	0	$\leftarrow \Delta_j$
x_1	-2	1	1	1/3	0	0	X	0	1/1/3 =3
a_2	-M	2	0	$\boxed{5/3}$	-1	0	X	1	6/5/3 =1.2$\rightarrow$
s_2	0	3	0	5/3	0	1	X	0	4/5/3=1.8
	Z = -2 – 2M		0	$\uparrow$ $\frac{(-5M+1)}{3}$	0	0	X	0	$\leftarrow \Delta_j$
x_1	-2	3/5	1	0	1/5	0	X	X	
x_2	-1	6/5	0	1	-3/5	0	X	X	
s_2	0	1	0	0	1	1	X	X	
	Z = -12/5		0	0	1/5	0	X	X	

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained. Therefore the solution is Max Z = -12/5, $x_1 = 3/5$, $x_2 = 6/5$

Example 2
Max Z = $3x_1 - x_2$

Subject to
$2x_1 + x_2 \geq 2$ $x_1 + 3x_2 \leq$
3 $x_2 \leq 4$
and $x_1 \geq 0, x_2 \geq 0$

Solution

SLPP
Max $Z = 3x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 - M a_1$

Subject to

$2x_1 + x_2 - s_1 + a_1 = 2$
$x_1 + 3x_2 + s_2 = 3$
$x_2 + s_3 = 4$
$x_1, x_2, s_1, s_2, s_3, a_1 \geq 0$

		$C_j \rightarrow$	3	-1	0	0	0	-M	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	A_1	Min ratio X_B / X_k
a_1	-M	2	$\boxed{2}$	1	-1	0	0	1	$2 / 2 = 1 \rightarrow$
s_2	0	3	1	3	0	1	0	0	$3 / 1 = 3$
s_3	0	4	0	1	0	0	1	0	-
	Z = -2M		↑ -2M-3	-M+1	M	0	0	0	$\leftarrow \Delta_j$
x_1	3	1	1	1/2	-1/2	0	0	X	-
s_2	0	2	0	5/2	$\boxed{1/2}$	1	0	X	$2/1/2 = 4 \rightarrow$
s_3	0	4	0	1	0	0	1	X	-
	Z = 3		0	5/2	↑ -3/2	0	0	X	$\leftarrow \Delta_j$
x_1	3	3	1	3	0	1/2	0	X	
s_1	0	4	0	5	1	2	0	X	
s_3	0	4	0	1	0	0	1	X	
	Z = 9		0	10	0	3/2	0	X	

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained. Therefore the solution is Max Z = 9, $x_1 = 3$, $x_2 = 0$

Example 3

Min $Z = 2x_1 + 3x_2$

Subject to

$x_1 + x_2 \geq 5$ $x_1 + 2x_2 \geq 6$
and $x_1 \geq 0$, $x_2 \geq 0$

Solution

SLPP
Min $Z = \text{Max } Z' = -2x_1 - 3x_2 + 0s_1 + 0s_2 - M a_1 - M a_2$

Subject to

$x_1 + x_2 - s_1 + a_1 = 5$ $x_1 + 2x_2 - s_2 + a_2 = 6$
$x_1, x_2, s_1, s_2, a_1, a_2 \geq 0$

	$C_j \rightarrow$		-2	-3	0	0	-M	-M	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	A_1	A_2	Min ratio X_B/X_k
a_1	-M	5	1	1	-1	0	1	0	5 /1 = 5
a_2	-M	6	1	$\boxed{2}$	0	-1	0	1	6 / 2 = 3→
	Z' = -11M		-2M + 2	↑ -3M+3	M	M	0	0	←Δ_j
a_1	-M	2	$\boxed{1/2}$	0	-1	1/2	1	X	2/1/2 = 4→
x_2	-3	3	1/2	1	0	-1/2	0	X	3/1/2 =6
	Z' = -2M-9		↑ (-M+1) / 2	0	M	(-M+3)/2	0	X	←Δ_j
x_1	-2	4	1	0	-2	1	X	X	
x_2	-3	1	0	1	1	-1	X	X	
	Z' = -11		0	0	1	1	X	X	

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained. Therefore the solution is Z' = -11 which implies Max Z = 11, x1 = 4, x2 = 1

Steps for Two-Phase Method

The process of eliminating artificial variables is performed in **phase-I** of the solution and **phase-II** is used to get an optimal solution. Since the solution of LPP is computed in two phases, it is called as **Two-Phase Simplex Method**.

Phase I – In this phase, the simplex method is applied to a specially constructed **auxiliary linear programming problem** leading to a final simplex table containing a basic feasible solution to the original problem.

Step 1 – Assign a cost -1 to each artificial variable and a cost 0 to all other variables in the objective function.

Step 2 – Construct the Auxiliary LPP in which the new objective function Z* is to be maximized subject to the given set of constraints.

Step 3 – Solve the auxiliary problem by simplex method until either of the following three possibilities do arise

i. Max Z* < 0 and atleast one artificial vector appear in the optimum basis at a positive level ($\Delta_j \geq 0$). In this case, given problem does not possess any feasible solution.

ii. Max Z* = 0 and at least one artificial vector appears in the optimum basis at a zero level. In this case proceed to phase-II.

iii. Max Z* = 0 and no one artificial vector appears in the optimum basis. In this case also proceed to phase-II.

Phase II – Now assign the actual cost to the variables in the objective function and a zero cost to every artificial variable that appears in the basis at the zero level. This new objective function is now maximized by simplex method subject to the given constraints.

Simplex method is applied to the modified simplex table obtained at the end of phase-I, until an optimum basic feasible solution has been attained. The artificial variables which are non-basic at the end of phase-I are removed.

Worked Examples

Example 1

Max $Z = 3x_1 - x_2$

Subject to

$2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2$

$x_2 \leq 4$

and $x_1 \geq 0, x_2 \geq 0$

Solution

Standard LPP Max Z = $3x_1 - x_2$

Subject to

$2x_1 + x_2 - s_1 + a_1 = 2$

$x_1 + 3x_2 + s_2 = 2$

$x_2 + s_3 = 4$

$x_1, x_2, s_1, s_2, s_3, a_1 \geq 0$

Auxiliary LPP

Max $Z^* = 0x_1 - 0x_2 + 0s_1 + 0s_2 + 0s_3 - 1a_1$

Subject to

$2x_1 + x_2 - s_1 + a_1 = 2$ $x_1 + 3x_2 + s_2 = 2$

$x_2 + s_3 = 4$

$x_1, x_2, s_1, s_2, s_3, a_1 \geq 0$

Phase I

		$C_j \rightarrow$	0	0	0	0	0	-1	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	A_1	Min ratio X_B / X_k
a_1	-1	2	$\boxed{2}$	1	-1	0	0	1	1→
s_2	0	2	1	3	0	1	0	0	2
s_3	0	4	0	1	0	0	1	0	-
	$Z^* = -2$		↑ -2	-1	1	0	0	0	$\leftarrow \Delta_j$
x_1	0	1	1	1/2	-1/2	0	0	X	
s_2	0	1	0	5/2	1/2	1	0	X	
s_3	0	4	0	1	0	0	1	X	
	$Z^* = 0$		0	0	0	0	0	X	$\leftarrow \Delta_j$

Since all $\Delta j \geq 0$, Max Z* = 0 and no artificial vector appears in the basis, we proceed to phase II.

Phase II

	$C_j \rightarrow$		3	-1	0	0	0	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	Min ratio X_B/X_k
x_1	3	1	1	1/2	-1/2	0	0	-
s_2	0	1	0	5/2	$\boxed{1/2}$	1	0	$2\rightarrow$
s_3	0	4	0	1	0	0	1	-
	Z = 3		0	5/2	$\uparrow$ -3/2	0	0	$\leftarrow\Delta_j$
x_1	3	2	1	3	0	1	0	
s_1	0	2	0	5	1	2	0	
s_3	0	4	0	1	0	0	1	
	Z = 6		0	10	0	3	0	$\leftarrow\Delta_j$

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained. Therefore the solution is Max Z = 6, $x_1 = 2$, $x_2 = 0$

Example 2

Max $Z = 5x_1 + 8x_2$

Subject to

$3x_1 + 2x_2 \geq 3$ $x_1 + 4x_2 \geq 4$

$x_1 + x_2 \leq 5$

and $x_1 \geq 0$, $x_2 \geq 0$

Solution

Standard LPP Max $Z = 5x_1 + 8x_2$

Subject to

$3x_1 + 2x_2 - s_1 + a_1 = 3$ $x_1 + 4x_2 - s_2 + a_2 = 4$

$x_1 + x_2 + s_3 = 5$

$x_1, x_2, s_1, s_2, s_3, a_1, a_2 \geq 0$

Auxiliary LPP

Max $Z^* = 0x_1 + 0x_2 + 0s_1 + 0s_2 + 0s_3 - 1a_1 - 1a_2$

Subject to

$3x_1 + 2x_2 - s_1 + a_1 = 3$ $x_1 + 4x_2$
$- s_2 + a_2 = 4$
$x_1 + x_2 + s_3 = 5$
$x_1, x_2, s_1, s_2, s_3, a_1, a_2 \geq 0$

Phase I

	$C_j \rightarrow$		0	0	0	0	0	-1	-1	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	A_1	A_2	Min ratio X_B / X_k
a_1	-1	3	3	2	-1	0	0	1	0	3/2
a_2	-1	4	1	$\boxed{4}$	0	-1	0	0	1	$1 \rightarrow$
s_3	0	5	1	1	0	0	1	0	0	5
	$Z^* = -7$		-4	$\uparrow$ -6	1	1	0	0	0	$\leftarrow \Delta_j$
a_1	-1	1	$\boxed{5/2}$	0	-1	1/2	0	1	x	$2/5 \rightarrow$
x_2	0	1	1/4	1	0	-1/4	0	0	x	4
s_3	0	4	3/4	0	0	1/4	1	0	x	16/3
	$Z^* = -1$		$\uparrow$ -5/2	0	1	-1/2	0	0	x	$\leftarrow \Delta_j$
x_1	0	2/5	1	0	-2/5	1/5	0	x	x	
x_2	0	9/10	0	1	1/10	-3/10	0	x	x	
s_3	0	37/10	0	0	3/10	1/10	1	x	x	
	$Z^* = 0$		0	0	0	0	0	x	x	$\leftarrow \Delta_j$

Since all $\Delta j \geq 0$, Max $Z^* = 0$ and no artificial vector appears in the basis, we proceed to phase II.

PHASE II

	$C_j \rightarrow$		5	8	0	0	0	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	S_3	Min ratio X_B / X_k
x_1	5	2/5	1	0	-2/5	$\boxed{1/5}$	0	$2\rightarrow$
x_2	8	9/10	0	1	1/10	-3/10	0	-
s_3	0	37/10	0	0	3/10	1/10	1	37
	Z = 46/5		0	0	-6/5	$\uparrow$ -7/5	0	$\leftarrow\Delta_j$
s_2	0	2	5	0	-2	1	0	-
x_2	8	3/2	3/2	1	-1/2	0	0	-
s_3	0	7/2	-1/2	0	$\boxed{1/2}$	0	1	$7\rightarrow$
	Z = 12		7	0	$\uparrow$ -4	0	0	$\leftarrow\Delta_j$
s_2	0	16	3	0	0	1	2	
x_2	8	5	1	1	0	0	1/2	
s_1	0	7	-1	0	1	0	2	
	Z = 40		3	0	0	0	4	

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained .Therefore the solution is Max Z = 40, $x_1 = 0$, $x_2 = 5$

Example 3

Max $Z = -4x_1 - 3x_2 - 9x_3$

Subject to

$2x_1 + 4x_2 + 6x_3 \geq 15$
$6x_1 + x_2 + 6x_3 \geq 12$ and $x_1 \geq 0$,
$x_2 \geq 0$, $x_3 \geq 0$

Solution

Standard LPP: Max $Z = -4x_1 - 3x_2 - 9x_3$
Subject to

$2x_1 + 4x_2 + 6x_3 - s_1 + a_1 = 15$ $6x_1 +$
$x_2 + 6x_3 - s_2 + a_2 = 12$ $x_1, x_2, s_1,$
$s_2, a_1, a_2 \geq 0$

Auxiliary LPP: Max $Z^* = 0x_1 - 0x_2 - 0x_3 + 0s_1 + 0s_2 - 1a_1 - 1a_2$
Subject to

$2x_1 + 4x_2 + 6x_3 - s_1 + a_1 = 15$ $6x_1 +$
$x_2 + 6x_3 - s_2 + a_2 = 12$ $x_1, x_2, s_1,$
$s_2, a_1, a_2 \geq 0$

Phase I

	$C_j \rightarrow$		0	0	0	0	0	-1	-1	
Basic Variables	C_B	X_B	X_1	X_2	X_3	S_1	S_2	A_1	A_2	Min ratio X_B / X_k
a_1	-1	15	2	4	6	-1	0	1	0	15/6
a_2	-1	12	6	1	$\boxed{6}$	0	-1	0	1	2→
	Z* = -27		-8	-5	↑ -12	1	1	0	0	←Δ_j
a_1	-1	3	-4	$\boxed{3}$	0	-1	1	1	X	1→
x_3	0	2	1	1/6	1	0	-1/6	0	X	12
	Z* = -3		4	↑ -3	0	1	-1	0	X	←Δ_j
x_2	0	1	-4/3	1	0	-1/3	1/3	X	X	
x_3	0	11/6	22/18	0	1	1/18	-4/18	X	X	
	Z* = 0		0	0	0	0	0	X	X	

Since all $\Delta j \geq 0$, Max Z* = 0 and no artificial vector appears in the basis, we proceed to phase II.

PHASE II

	$C_j \rightarrow$		-4	-3	-9	0	0	
Basic Variables	C_B	X_B	X_1	X_2	X_3	S_1	S_2	Min ratio X_B / X_k
x_2	-3	1	-4/3	1	0	-1/3	1/3	-
x_3	-9	11/6	$\boxed{22/18}$	0	1	1/18	-4/18	3/2→
	Z = -39/2		↑ -3	0	0	1/2	1	←Δ_j
x_2	-3	3	0	1	12/11	-3/11	1/11	
x_1	-4	3/2	1	0	18/22	1/22	-4/22	
	Z = -15		0	0	27/11	7/11	5/11	←Δ_j

Since all $\Delta j \geq 0$, optimal basic feasible solution is obtained. Therefore the solution is Max Z = -15, x1 = 3/2, x2 = 3, x3 = 0

Example 4
Min Z = $4x_1 + x_2$

Subject to

$3x_1 + x_2 = 3$
$4x_1 + 3x_2 \geq 6$ $x_1 + 2x_2$
≤ 4

and $x_1 \geq 0$, $x_2 \geq 0$

Solution

Standard LPP
Min Z = Max Z' = – $4x_1 – x_2$
Subject to

$3x_1 + x_2 + a_1 = 3$
$4x_1 + 3x_2 – s_1 + a_2 = 6$ $x_1 + 2x_2 +$
$s_2 = 4$
x_1 , x_2 , s_1, s_2, a_1, $a_2 \geq 0$

Auxiliary LPP
Max Z* = $0x_1 – 0x_2 + 0s_1 + 0s_2 –1a_1 –1a_2$
Subject to

$3x_1 + x_2 + a_1 = 3$
$4x_1 + 3x_2 – s_1 + a_2 = 6$ $x_1 + 2x_2 +$
$s_2 = 4$
x_1 , x_2 , s_1, s_2, a_1, $a_2 \geq 0$

Phase I

	$C_j \rightarrow$		0	0	0	0	-1	-1	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	A_1	A_2	Min ratio X_B / X_k
a_1	-1	3	$\boxed{3}$	1	0	0	1	0	1 →
a_2	-1	6	4	3	-1	0	0	1	6/4
s_2	0	4	1	2	0	1	0	0	4
	Z* = -9		↑ -7	-4	1	0	0	0	
x_1	0	1	1	1/3	0	0	×	0	3
a_2	-1	2	0	$\boxed{5/3}$	-1	0	×	1	6/5 →
s_2	0	3	0	5/3	0	1	×	0	9/5
	Z* = -2		0	↑ -5/3	1	0	×	0	
x_1	0	3/5	1	0	1/5	0	×	×	
x_2	0	6/5	0	1	-3/5	0	×	×	
s_2	0	1	0	0	1	1	×	×	
	Z* = 0		0	0	0	0	×	×	

Since all $\Delta_j \geq 0$, Max Z* = 0 and no artificial vector appears in the basis, we proceed to phase II.

Phase II

	$C_j \rightarrow$		-4	-1	0	0	
Basic Variables	C_B	X_B	X_1	X_2	S_1	S_2	Min ratio X_B / X_k
x_1	-4	3/5	1	0	1/5	0	3
x_2	-1	6/5	0	1	-3/5	0	-
s_2	0	1	0	0	$\boxed{1}$	1	1 →
	Z' = -18/5		0	0	↑ -1/5	0	← Δ_j
x_1	-4	2/5	1	0	0	-1/5	
x_2	-1	9/5	0	1	0	3/5	
s_1	0	1	0	0	1	1	
	Z' = -17/5		0	0	0	1/5	← Δ_j

Since all $\Delta_j \geq 0$, optimal basic feasible solution is obtained

Therefore the solution is Max Z' = -17/5

Min Z = 17/5, x_1 = 2/5, x_2 = 9/5

Example 5

Min Z = $x_1 - 2x_2 - 3x_3$

Subject to

$-2x_1 + x_2 + 3x_3 = 2$ $2x_1 + 3x_2 + 4x_3 = 1$

and $x_1 \geq 0, x_2 \geq 0$

Solution

Standard LPP

Min Z = Max Z' = $-x_1 + 2x_2 + 3x_3$

Subject to

$-2x_1 + x_2 + 3x_3 + a_1 = 2$ $2x_1 + 3x_2 + 4x_3 + a_2 = 1$

$x_1, x_2, a_1, a_2 \geq 0$

Auxiliary LPP

Max Z* = $0x_1 + 0x_2 + 0x_3 - 1a_1 - 1a_2$

Subject to

$-2x_1 + x_2 + 3x_3 + a_1 = 2$ $2x_1 + 3x_2 + 4x_3 + a_2 = 1$ $x_1, x_2, a_1, a_2 \geq 0$

Phase I

		$C_j \rightarrow$	0	0	0	-1	-1	
Basic Variables	C_B	X_B	X_1	X_2	X_3	A_1	A_2	Min Ratio X_B / X_K
a_1	-1	2	-2	1	3	1	0	2/3
a_2	-1	1	2	3	$\boxed{4}$	0	1	1/4→
	Z* = -3		0	-4	↑ -7	0	0	←Δ_j
a_1	-1	5/4	-7/4	-5/4	0	1	X	
x_3	0	1/4	1/2	3/4	1	0	X	
	Z* = -5/4		7/4	5/4	0	1	X	←Δ_j

Since for all $\Delta j \geq 0$, optimum level is achieved. At the end of phase-I Max Z* < 0 and one of the artificial variable a1 appears at the positive optimum level. Hence the given problem does not posses any feasible solution.

MCQ

Question	**opt 1**	**opt 2**	**opt 3**	**opt 4**	**answer**
Linear programming models can be applied only in those situations where the constraints and the objective function are	Linear	non-linear	either linear or non linear	both linear & non linear	Linear
LPP deals with problems involving _________ objective	single	double	multiple	all of these	single
An LPP may have _________ optimal solution	more than one	only two	only one	no	more than one
An LPP solution when permitted to be infinitely large is called ___________	unbounded	bounded	feasible	infeasible	unbounded
A basic solution is said to be _______ if one or more of the basic variable is zero	degenerate	non-degenerate	optimal	none of these	degenerate
A feasible solution which is also basic is called a ___________ solution	basic feasible	basic infeasible	degenerate	non-degenerate	basic feasible
The entering variables column is called the __________	pivot column	pivot key	pivot row	none of these	pivot column
the intersection of the pivot column and pivot row is called the ___________	pivot element	pivot key	pivot row	pivot column	pivot element
constraints involving "equal to sign" do not require use of ______ variable	slack	non-zero	zero	non-surplus	slack
Constraints involving ">=" sign are reduced to equations by introducting _________variable	surplus	non-zero	zero	non-surplus	surplus
A feasible solution which optimizes the objective function is called _____	optimal solution	infeasible solution	degenerate	non-degenerate	optimal solution
The co-efficients of artifical variables are -M in the objective function for	maximization	minimization	degenerate	all of these	maximization

__________ problem					
Decision variales in a standard LPP should have either zero or __________	positive values	negative values	either positive or negatice	all of these	positive values
The distinguishing feature of an LP model is	relationship among all variables is linear	it has single objective function and constraints	value of decision variables is non-negative	all of these	relationship among all variables is linear
Maximization of objective function in LP model maens	value occurs at allowable set of decision	highest value is chosen among allowable decision	neither of these	both highest value is chosen among allowable decision and highest value is chosen among allowable decision	value occurs at allowable set of decision
Mathematical mode of LP problem is important because	it helps in converting the verbal description and numerical data into mathematical model	decision-makers prefer to work with formal models	it captures the relevant relatonship among decision factors	it enables the use of algebraic technique	it helps in converting the verbal description and numerical data into mathematical model
A feasible solution to an LP problem	must satisfy all of the problem's constraints simultaneously	need not satisfy all of the constraints, only some of them	must be a corner point of the feasible region	must optimize the value of the objective function	must satisfy all of the problem's constraints simultaneously
An iso-profit line represents	an infinite number of	an infinite number of	an infinite number of	a boundary of the feasible	an infinite number of

	solution all of which yield the same profit	solution all of which yield the same cost	optimal solutions	region	solution all of which yield the same profit
if two constraints do not intersect in the positive quadrant of the graph ,then	the problem is infeasible	the solution is unbounded	one of the constraints is redundant	none of these	the problem is infeasible
constraints in LP problem are called active if they	represent optimal solution	at optimality do not consume all the available resources	both of represent optimal solution and at optimality do not consume all the available resources	none of these	represent optimal solution
the while solving a LP model graphically ,the area bounded by the constraints is called	feasible region	infeasible region	unbounded solution	none of these	feasible region
if a non-redundant constraint is removed from a LP problem ,then	feasible region will become larger	feasible region will become smaller	solution will become infeasible	none of these	feasible region will become larger
the dummy source or destination in a transportation problem is added to	satisfy rim conditions	prevent solution from becoming degenerate	ensure that total cost does not exceed a limit	none of these	satisfy rim conditions
An LPP that has infinity of alternatives is usually _______	solvable	unsolvable	feasible	unfeasible	unsolvable
Graphical method of linear programming is useful when the problem involves _______	three variables	two variables	one variables	none of these	two variables

A basic solution is said to be ______ if none of the basic variable is zero	degenerate	non-degenerate	optimal	none of these	non-degenerate
In solving an LPP by the simplex method ______ variable is associated with wquality type constraint	real	artifical	negative	non-zero	artifical
The method designed to over come the difficulty arising in computer implementation of Big M- Method is known as ______ method	three phase	two phase	multi phase	none of these	two phase
The co-efficients of artifical variables are +M in the objective function for __________ problem	maximization	minimization	degenerate	all of these	minimization
Non-negativity condition is an important component of LP model because	variables value should remain under the control of decision-marker	value of variables make sense and correspond to real-world problems	variables are interrelated in terms of limited resources	none of these	value of variables make sense and correspond to real-world problems
Which of the following is not a characteristic of LP model	alternative corses of action	an objective function of maximization type	limited amount of resources	non-negativity condition on the value of decision variables	an objective function of maximization type
alternative solutions exist of an LP model when	one of the constraints is redundant	objective function equation is parallel to one of the constraints	two constraints are parallel	all of these	objective function equation is parallel to one of the constraints
if one of the constraint is an equation in a LP problem with unbounded solution,then	solution to such LP problem must be degenerate	feasible region should have a line segment	alternative solutions exist	none of these	feasible region should have a line segment

the initial solution of a transportation problem can be obtained by applying any known method.however ,the only condition is that	the solution be optimal	the rim conditions are satisfied	the solution not be degenerate	all of these	the rim conditions are satisfied
LPP does not take into consideration the effect of _____ and uncertainty	resources	usability	time	both time and usability	time
__________ specifies the dependent relationship between the decision objective and the decision variables	main function	non-zero function	objective function	none of these	objective function
While plotting constraints on a graph , terminal points on both the axes are connected by a straight line because	the resources are limited in supply	the objective function is a linear function	the constraints are linear equations or in-equalities	all of the these	the constraints are linear equations or in-equalities
the solution space (region) of an LP problem is unbounded due to	an incorrect formulation of the LP model	objective function is unbounded	neither an incorrect formulation of the LP model nor objective function is unbounded	both an incorrect formulation of the LP model & objective function is unbounded	neither an incorrect formulation of the LP model nor objective function is unbounded
while solving a LP problem,infeasibility may be removed by	adding another constraint	adding another variable	removing a constraint	removing a variable	removing a constraint
The leaving variable row is called the ____________	key equation	non pivot row	pivot equation	pivot row	pivot row
A constraint in an LP model restricts_____	value of objective function	value of a decision variable	use of the available resource	all of these	all of these
Constraints in an LP model represents	limitations	requirements	balancing limitations and requirements	all of these	all of these

Before formulating a formal LP model, it is better to _____	express each constraint in words	express the objective function in words	decision variables are identified verbally	all of these	all of these
Each constraint in an LP model is expressed as an ___________	inequality with ³ sign	inequality with £ sign	equation with = sign	none of these	none of these
The best use of linear programming technique is to find an optimal use of	money	manpower	machine	all of these	all of these
Which of the following is not the characteristic of LP	resurces must be limited	only one objective function	parameters value remains constant during planning period	the problem must be of minimization type	the problem must be of minimization type
Linear programming is a	constrained optimization technique	technique for economic allocation of limited resources	mathematical technique	all of these	all of these
Non-negativity condition is an LP model implies	a (+ ve) coefficient of variables in objective function	a (+ve) coefficient of variables in any constraint	non-negative value of resources	none of these	none of these
Which of the following is the assumption of an LP model	divisibility	proportionality	additivity	all of these	all of these
Which of the following is the limitation associated with an LP model	the relationship among decision variables in linear	no guarantee to get integer valued solutions	no consideration of effect of time and uncertainity on LP model	all of these	all of these

If an iso - profit line yielding the optimal solution coincides with a constraint line, then	the solution is unbounded	the solution is infeasible	the constraint which coincides is redundant	none of these	none of these
The graphical method of LP problem uses	objective function equation	constraint equations	linear equations	all of these	all of these
Which of the following statements is true with respect to the optimal solution of an LP problem	every LP problem has an optimal solution	optimal solution of an LP problem always occurs at an extreme point	at optimal solution all resources are used completely	there will always be atleast one at a corner	there will always be atleast one at a corner
A constraint in an LP model becomes redundant because	two iso-profit line may be parallel to each other	the solution is unbounded	this constraint is not satisfied by the solution values	none of these	none of these

2 marks

1. What is a mathematical model?
2. State four applications of operations research in industry.
3. List any three practical limitations of the OR technique
4. Define slack variables and surplus variables
5. What is key column and key row? How it is selected?
6. What are the general methods for solving O.R models?
7. What is the use of artificial variable?
8. What is canonical form of a LPP?
9. Define Feasible region
10. What is O.R?
11. What are the applications of linear programming?
12. Distinguish between a resource and a constraint.
13. What are the different phases of O.R?
14. Define Feasible region
15. What is the use of artificial variable?

14 marks

1. A firm uses lathes, milling machines and grinding machines to produce two machine parts. The following table represents the machining time required for each part, the machining time available on different machines and the profit on each machine part.

Type of machines	Machining time required for the machine parts (in minutes)		Maximum time available per week (in minutes)
	I	II	
Lathes	12	6	3000
Milling machines	4	10	2000
Grinding machines	2	3	900
Profit per unit	Rs 40	Rs 100	

2. Find the number of parts I and II to be manufactured per week to maximize the profit.

   Use Two-phase simplex method to solve

   Maximize $Z = 10x_1 + 9x_2$

   Subjected to $3x_1 + 3x_2 \le 21$

   $4x_1 + 3x_2 \le 24$

   and $x_1, x_2 \ge 0$

3. Solve the following L.P.P graphically

   Maximize $Z = 100x_1 + 40x_2$

   Subject to $5x_1 + 2x_2 \le 1000$

   $3x_1 + 2x_2 \le 900$

   $x_1 + 2x_2 \le 500$

   and $x_1, x_2 \ge 0$

4. A shop can make two types of sweets (A and B). They use two resources flour and sugar. To make one packet of A they need 3kg of flour and 3kg of sugar. To make one packet of B they need 3kg of flour and 4kg of sugar. They have 21kg of flour and 28kg of sugar. These sweets are sold at Rs. 1000 and Rs. 900 per packet respectively. Find the best product mix to maximize the revenue.
5. The Standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B_1 and B_2. B_1 cost Rs 5/Kg and B_2 costs Rs 8/Kg. Strength considerations dictate that the brick contains not more than 4 kg of B_1 and minimum of 2 Kg of B_2. Since the demand for the product is likely to be related to the price of the brick, find graphically the minimum cost of the brick satisfying the above conditions.

6. Use Big-M Method to solve : Minimize $Z=7x_1+5x_2$

$$\text{Subject to } \quad x_1+x_2 \geq 4$$
$$5x_1+2x_2 \geq 10$$
$$\text{and } \quad x_1, x_2 \geq 0.$$

7. Solve the following L.P.P by the graphical method

$$\text{Maximize } Z = 3x_1+2x_2$$
$$\text{Subject to } -2x_1+x_2 \leq 1$$
$$x_1 \leq 2$$
$$x_1+x_2 \leq 3$$
$$\text{and } \quad x_1, x_2, \geq 0$$

8. A firm produces three products. These products are processed on three different machines. The time required manufacturing one unit of each of the three products and the daily capacity of the three machines are given in the table below:

Machine	Time per unit(minutes)			Machine Capacity (minutes/day)
	Product 1	Product 2	Product 3	
M1	2	3	2	440
M2	4	--	3	470
M3	2	5	--	430

9. Explain the main phases and limitations of Operation Research. & State any four applications of Operation Research.
10. A firm produces three products. These products are processed on three different machines. The time required manufacturing one unit of each of the three products and the daily capacity of the three machines are given in the table below:

Machine	Time per unit(minutes)			Machine Capacity (minutes/day)
	Product 1	Product 2	Product 3	
M1	2	3	2	440
M2	4	--	3	470
M3	2	5	--	430

Determine the number of units to be manufactured for each product daily in order to achieve maximum profit. The profit per unit for product 1,2 and 3 is Rs.4, Rs.3 and Rs.6 respectively. It is assumed that all the amounts produced are consumed in the market.

UNIT II

Introduction to Transportation Problem

The Transportation problem is to transport various amounts of a single homogeneous commodity that are initially stored at various origins, to different destinations in such a way that the total transportation cost is a minimum.

It can also be defined as to ship goods from various origins to various destinations in such a manner that the transportation cost is a minimum.

The availability as well as the requirements is finite. It is assumed that the cost of shipping is linear.

Mathematical Formulation

Let there be m origins, i^{th} origin possessing a_i units of a certain product

Let there be n destinations, with destination j requiring b_j units of a certain product

Let c_{ij} be the cost of shipping one unit from i^{th} source to j^{th} destination

Let x_{ij} be the amount to be shipped from i^{th} source to j^{th} destination

It is assumed that the total availabilities Σa_i satisfy the total requirements Σb_j i.e.

$\Sigma a_i = \Sigma b_j$ (i = 1, 2, 3 … m and j = 1, 2, 3 …n)

The problem now, is to determine non-negative of x_{ij} satisfying both the availability constraints

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad \text{for } i = 1, 2, .., m$$

as well as requirement constraints

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \text{for } j = 1, 2, .., n$$

and the minimizing cost of transportation (shipping)

$$z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}\, c_{ij} \qquad \text{(objective function)}$$

This special type of LPP is called as **Transportation Problem**.

Tabular Representation

Let 'm' denote number of factories (F_1, F_2 … F_m)

Let 'n' denote number of warehouse (W_1, W_2 … W_n)

W→					
F ↓	W_1	W_2	…	W_n	Capacities (Availability)
F_1	c11	c12	…	c1n	a_1
F_2	c21	c22	…	c2n	a_2
.	.	.	.	.	.
.	.	.	.	.	.

Fm	cm1	cm2	…	cmn	am
Required	b1	b2	…	bn	$\Sigma a_i = \Sigma b_j$

F → ↓	W1	W2	…	Wn	Capacities (Availability)
F_1	x11	x12	…	x1n	a_1
F2	x21	x22	…	x2n	a2
.	.	.	.	.	.
.	.	.	.	.	.
F_m	xm1	xm2	…	xmn	a_m
Required	b_1	b_2	…	b_n	$\Sigma a_i = \Sigma b_j$

In general these two tables are combined by inserting each unit cost c_{ij} with the corresponding amount x_{ij} in the cell (i, j). The product c_{ij} x_{ij} gives the net cost of shipping units from the factory F_i to warehouse W_j.

Some Basic Definitions

Feasible Solution

A set of non-negative individual allocations ($x_{ij} \geq 0$) which simultaneously removes deficiencies is called as feasible solution.

Basic Feasible Solution

A feasible solution to 'm' origin, 'n' destination problem is said to be basic if the number of positive allocations are m+n-1. If the number of allocations is less than m+n-1 then it is called as **Degenerate Basic Feasible Solution**. Otherwise it is called as Non-Degenerate Basic Feasible Solution.

Optimum Solution

A feasible solution is said to be optimal if it minimizes the total transportation cost.

Methods for Initial Basic Feasible Solution

Some simple methods to obtain the initial basic feasible solution are

1. North-West Corner Rule
2. Row Minima Method
3. Column Minima Method
4. Lowest Cost Entry Method (Matrix Minima Method)
5. Vogel's Approximation Method (Unit Cost Penalty Method)

North-West Corner Rule

Step 1

- The first assignment is made in the cell occupying the upper left-hand (north-west) corner of the table.
- The maximum possible amount is allocated here i.e. $x_{11} = \min(a_1, b_1)$. This value of x_{11} is

then entered in the cell (1,1) of the transportation table.

Step 2

i. If $b_1 > a_1$, move vertically downwards to the second row and make the second allocation of amount $x_{21} = \min (a_2, b_1 - x_{11})$ in the cell (2, 1).

ii. If $b_1 < a_1$, move horizontally right side to the second column and make the second allocation of amount $x_{12} = \min (a_1 - x_{11}, b_2)$ in the cell (1, 2).

iii. If $b_1 = a_1$, there is tie for the second allocation. One can make a second allocation of magnitude $x_{12} = \min (a_1 - a_1, b_2)$ in the cell (1, 2) or $x_{21} = \min (a_2, b_1 - b_1)$ in the cell (2, 1)

Step 3

Start from the new north-west corner of the transportation table and repeat steps 1 and 2 until all the requirements are satisfied.

<u>Find the initial basic feasible solution by using North-West Corner Rule</u>

1.

W→ F ↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

Solution

	W_1	W_2	W_3	W_5	Availability
F_1	5 (19)	2 (30)			7 2 0
F2		6 (30)	3 (40)		9 3 0
F_3			4 (70)	14 (20)	18 14 0
Requirement	5 0	8 6 0	7 4 0	14 0	

Initial Basic Feasible Solution
$x_{11} = 5$, $x_{12} = 2$, $x_{22} = 6$, $x_{23} = 3$, $x_{33} = 4$, $x_{34} = 14$

The transportation cost is 5 (19) + 2 (30) + 6 (30) + 3 (40) + 4 (70) + 14 (20) = Rs. 1015

Example 2.

	D1	D2	D3	D4	Supply
O_1	1	5	3	3	34
O_2	3	3	1	2	15
O3	0	2	2	3	12
O_4	2	7	2	4	19
Demand	21	25	17	17	80

Solution

	D_1	D_2	D_3	D_4	Supply	
O_1	21 (1)	13 (5)			34	13 0
O_2		12 (3)	3 (1)		15	3 0
O_3			12 (2)		12	0
O4			2 (2)	17 (4)	19	17
Demand	21	25	17	17		
	0	12	14	0		
		0	2			
			0			

Initial Basic Feasible Solution

$x_{11} = 21$, $x_{12} = 13$, $x_{22} = 12$, $x_{23} = 3$, $x_{33} = 12$, $x_{43} = 2$, $x_{44} = 17$

The transportation cost is 21 (1) + 13 (5) + 12 (3) + 3 (1) + 12 (2) + 2 (2) + 17 (4) = Rs. 221

Example 3.

From	To					Supply
	2	11	10	3	7	4
	1	4	7	2	1	8
	3	1	4	8	12	9
Demand	3	3	4	5	6	

Solution

From	To					Supply
	3 (2)	1 (11)				4 1 0
		2 (4)	4 (7)	2 (2)		8 6 2 0
				3 (8)	6 (12)	9 6 0
	3	3	4	5	6	
Demand	0	2	0	3	0	

0 0

Initial Basic Feasible Solution

$x_{11} = 3$, $x_{12} = 1$, $x_{22} = 2$, $x_{23} = 4$, $x_{24} = 2$, $x_{34} = 3$, $x_{35} = 6$

The transportation cost is 3 (2) + 1 (11) + 2 (4) + 4 (7) + 2 (2) + 3 (8) + 6 (12) = Rs. 153

Lowest Cost Entry Method (Matrix Minima Method)

Step 1

Determine the smallest cost in the cost matrix of the transportation table. Allocate $x_{ij} = \min(a_i, b_j)$ in the cell (i, j)

Step 2

If $x_{ij} = a_i$, cross out the i^{th} row of the table and decrease b_j by a_i. Go to step 3.

If $x_{ij} = b_j$, cross out the j^{th} column of the table and decrease a_i by b_j. Go to step 3.

If $x_{ij} = a_i = b_j$, cross out the i^{th} row or j^{th} column but not both.

Step 3

Repeat steps 1 and 2 for the resulting reduced transportation table until all the requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

Find the initial basic feasible solution using Matrix Minima method

Example 1.

	W_1	W_2	W_3	W_4	Availability
F1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

Solution

	W1	W2	W3	W4	
F1	(19)	(30)	(50)	(10)	7
F_2	(70)	(30)	(40)	(60)	9
F3	(40)	8 (8)	(70)	(20)	10
	5	X	7	14	

	W1	W2	W3	W4	
F_1	(19)	(30)	(50)	7 (10)	X
F_2	(70)	(30)	(40)	(60)	9

F$_3$	(40)	8 (8)	(70)	(20)	10
	5	X	7	7	

	W$_1$	W$_2$	W$_3$	W$_4$	
F$_1$	(19)	(30)	(50)	7 (10)	X
F$_2$	(70)	(30)	(40)	(60)	9
F$_3$	(40)	8 (8)	(70)	7 (20)	3
	5	X	7	X	

	W$_1$	W$_2$	W$_3$	W$_4$	
F$_1$	(19)	(30)	(50)	7 (10)	X
F$_2$	(70)	(30)	(40)	(60)	9
F$_3$	3 (40)	8 (8)	(70)	7 (20)	X
	2	X	7	X	

	W$_1$	W$_2$	W$_3$	W$_4$	
F$_1$	(19)	(30)	(50)	7 (10)	X
F2	2 (70)	(30)	7 (40)	(60)	X
F$_3$	3 (40)	8 (8)	(70)	7 (20)	X
	X	X	X	X	

Initial Basic Feasible Solution
$x_{14} = 7$, $x_{21} = 2$, $x_{23} = 7$, $x_{31} = 3$, $x_{32} = 8$, $x_{34} = 7$
The transportation cost is 7 (10) + 2 (70) + 7 (40) + 3 (40) + 8 (8) + 7 (20) = Rs. 814

Example 2.

	To					Availability
	2	11	10	3	7	4
From	1	4	7	2	1	8
	3	9	4	8	12	9
Requirement	3	3	4	5	6	

Solution

	To					
				4 (3)		4 0
	3 (1)				5 (1)	8 5 0
From		3 (9)	4 (4)	1 (8)	1 (12)	9 5 4 1 0
	3 0	3 0	4 0	5 1 0	6 1 0	

Initial Basic Feasible Solution

$x_{14} = 4$, $x_{21} = 3$, $x_{25} = 5$, $x_{32} = 3$, $x_{33} = 4$, $x_{34} = 1$, $x_{35} = 1$

The transportation cost is 4 (3) + 3 (1) + 5(1) + 3 (9) + 4 (4) + 1 (8) + 1 (12) = Rs. 78

<u>Vogel's Approximation Method (Unit Cost Penalty Method)</u>

Step1

For each row of the table, identify the **smallest** and the **next to smallest cost**. Determine the difference between them for each row. These are called **penalties**. Put them aside by enclosing them in the parenthesis against the respective rows. Similarly compute penalties for each column.

Step 2

Identify the row or column with the largest penalty. If a tie occurs then use an arbitrary choice. Let the largest penalty corresponding to the i^{th} row have the cost c_{ij}. Allocate the largest possible amount x_{ij} = min (a_i, b_j) in the cell (i, j) and cross out either i^{th} row or j^{th} column in the usual manner.

Step 3

Again compute the row and column penalties for the reduced table and then go to step 2. Repeat the procedure until all the requirements are satisfied.

<u>Find the initial basic feasible solution using vogel's approximation method</u>

Example 1.

	W_1	W_2	W_3	W_4	Availability
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

Solution

	W_1	W_2	W_3	W_4	Availability	Penalty
F1	19	30	50	10	7	19-10=9
F_2	70	30	40	60	9	40-30=10
F_3	40	8	70	20	18	20-8=12
Requirement	5	8	7	14		
Penalty	40-19=21	30-8=22	50-40=10	20-10=10		

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	(19)	(30)	(50)	(10)	7	9
F_2	(70)	(30)	(40)	(60)	9	10
F_3	(40)	**8**(8)	(70)	(20)	18/10	12
Requirement	5	8/0	7	14		
Penalty	21	22	10	10		

	W1	W2	W3	W4	Availability	Penalty
F1	**5**(19)	(30)	(50)	(10)	7/2	9
F_2	(70)	(30)	(40)	(60)	9	20
F_3	(40)	**8**(8)	(70)	(20)	18/10	20
Requirement	5/0	X	7	14		
Penalty	21	X	10	10		

	W_1	W_2	W_3	W_4	Availability	Penalty
F1	**5**(19)	(30)	(50)	(10)	7/2	40
F2	(70)	(30)	(40)	(60)	9	20
F_3	(40)	**8**(8)	(70)	**10**(20)	18/10/0	50
Requirement	X	X	7	14/4		
Penalty	X	X	10	10		

	W_1	W_2	W_3	W_4	Availability	Penalty
F_1	**5**(19)	(30)	(50)	**2**(10)	7/2/0	40
F_2	(70)	(30)	(40)	(60)	9	20
F_3	(40)	**8**(8)	(70)	**10**(20)	X	X
Requirement	X	X	7	14/4/2		
Penalty	X	X	10	50		

	W1	W2	W3	W4	Availability	Penalty
F_1	**5**(19)	(30)	(50)	**2**(10)	X	X
F2	(70)	(30)	**7**(40)	**2**(60)	X	X
F_3	(40)	**8**(8)	(70)	**10**(20)	X	X
Requirement	X	X	X	X		
Penalty	X	X	X	X		

Initial Basic Feasible Solution
$x_{11} = 5$, $x_{14} = 2$, $x_{23} = 7$, $x_{24} = 2$, $x_{32} = 8$, $x_{34} = 10$
The transportation cost is 5 (19) + 2 (10) + 7 (40) + 2 (60) + 8 (8) + 10 (20) = Rs. 779

Example 2.

		Stores				Availability
		I	II	III	IV	
	A	21	16	15	13	11
Warehouse	B	17	18	14	23	13
	C	32	27	18	41	19
Requirement		6	10	12	15	

Solution

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	(13)	11	2
Warehouse	B	(17)	(18)	(14)	(23)	13	3
	C	(32)	(27)	(18)	(41)	19	9
Requirement		6	10	12	15		
Penalty		4	2	1	10		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	11/0	2
Warehouse	B	(17)	(18)	(14)	(23)	13	3
	C	(32)	(27)	(18)	(41)	19	9
Requirement		6	10	12	15/4		
Penalty		4	2	1	10		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	X	X
Warehouse	B	(17)	(18)	(14)	**4**(23)	13/9	3
	C	(32)	(27)	(18)	(41)	19	9
Requirement		6	10	12	15/4/0		
Penalty		15	9	4	18		

Stores Availability Penalty

		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	X	X
Warehouse	B	**6**(17)	(18)	(14)	**4**(23)	13/9/3	3
	C	(32)	(27)	(18)	(41)	19	9
Requirement		6/0	10	12	X		
Penalty		15	9	4	X		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	X	X
Warehouse	B	**6**(17)	**3**(18)	(14)	**4**(23)	13/9/3/0	4
	C	(32)	(27)	(18)	(41)	19	9
Requirement		X	10/7	12	X		
Penalty		X	9	4	X		

		Stores				Availability	Penalty
		I	II	III	IV		
	A	(21)	(16)	(15)	**11**(13)	X	X
Warehouse	B	**6**(17)	**3**(18)	(14)	**4**(23)	X	X
	C	(32)	**7**(27)	**12**(18)	(41)	X	X
Requirement		X	X	X	X		
Penalty		X	X	X	X		

Initial Basic Feasible Solution

$x_{14} = 11$, $x_{21} = 6$, $x_{22} = 3$, $x_{24} = 4$, $x_{32} = 7$, $x_{33} = 12$

The transportation cost is 11 (13) + 6 (17) + 3 (18) + 4 (23) + 7 (27) + 12 (18) = Rs. 796

<u>Examining the Initial Basic Feasible Solution for Non-Degeneracy</u>

Examine the initial basic feasible solution for non-degeneracy. If it is said to be non-degenerate then it has the following two properties

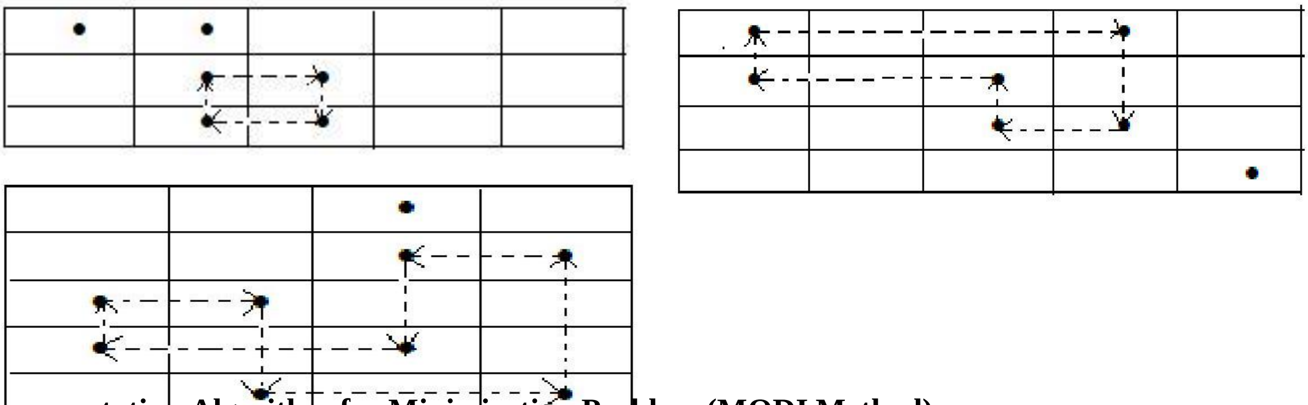
Initial basic feasible solution must contain exactly m + n – 1 number of individual allocations.
These allocations must be in independent positions

Independent Positions

•	•	•		
		•	•	•
	•			•

•				•
			•	•
		•		•

Non-Independent Positions



Transportation Algorithm for Minimization Problem (MODI Method)

Step 1

Construct the transportation table entering the origin capacities a_i, the destination requirement b_j and the cost c_{ij}

Step 2

Find an initial basic feasible solution by vogel's method or by any of the given method.

Step 3

For all the basic variables x_{ij}, solve the system of equations $u_i + v_j = c_{ij}$, for all i, j for which cell (i, j) is in the basis, starting initially with some $u_i = 0$, calculate the values of u_i and v_j on the transportation table

Step 4

Compute the cost differences $d_{ij} = c_{ij} – (u_i + v_j)$ for all the non-basic cells

Step 5

Apply optimality test by examining the sign of each d_{ij}

If all $d_{ij} \geq 0$, the current basic feasible solution is optimal

If at least one $d_{ij} < 0$, select the variable x_{rs} (most negative) to enter the basis.

Solution under test is not optimal if any d_{ij} is negative and further improvement is required by repeating the above process.

Step 6

Let the variable x_{rs} enter the basis. Allocate an unknown quantity Θ to the cell (r, s). Then construct a loop that starts and ends at the cell (r, s) and connects some of the basic cells. The amount Θ is added to and subtracted from the transition cells of the loop in such a manner that the availabilities and requirements remain satisfied.

Step 7

Assign the largest possible value to the Θ in such a way that the value of at least one basic variable becomes zero and the other basic variables remain non-negative. The basic cell whose allocation has been made zero will leave the basis.

Step 8

Now, return to step 3 and repeat the process until an optimal solution is obtained.

Worked Examples

Example 1
Find an optimal solution

	W_1	W_2	W_3	W_4	Availability
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Requirement	5	8	7	14	

Solution

1. **Applying vogel's approximation method for finding the initial basic feasible solution**

	W1	W2	W3	W4	Availability	Penalty
F1	**5**(19)	(30)	(50)	**2**(10)	X	X
F2	(70)	(30)	**7**(40)	**2**(60)	X	X
F3	(40)	**8**(8)	(70)	**10**(20)	X	X
Requirement	X	X	X	X		
Penalty	X	X	X	X		

Minimum transportation cost is 5 (19) + 2 (10) + 7 (40) + 2 (60) + 8 (8) + 10 (20) = Rs. 779

2. **Check for Non-degeneracy**
The initial basic feasible solution has m + n – 1 i.e. 3 + 4 – 1 = 6 allocations in independent positions. Hence optimality test is satisfied.

3. **Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$**

					u_i
	• (19)			• (10)	$u_1 = -10$
			• (40)	• (60)	$u_2 = 40$
		• (8)		• (20)	$u_3 = 0$
v_j	$v_1 = 29$	$v_2 = 8$	$v_3 = 0$	$v_4 = 20$	

Assign a 'u' value to zero. (Convenient rule is to select the u_i, which has the largest number of allocations in its row)

Let $u_3 = 0$, then

$u_3 + v_4 = 20$ which implies $0 + v_4 = 20$, so $v_4 = 20$

$u_2 + v_4 = 60$ which implies $u_2 + 20 = 60$, so $u_2 = 40$

$u_1 + v_4 = 10$ which implies $u_1 + 20 = 10$, so $u_1 = -10$

$u_2 + v_3 = 40$ which implies $40 + v_3 = 40$, so $v_3 = 0$

$u_3 + v_2 = 8$ which implies $0 + v_2 = 8$, so $v_2 = 8$

$u_1 + v_1 = 19$ which implies $-10 + v_1 = 19$, so $v_1 = 29$

4. **Calculation of cost differences for non basic cells $d_{ij} = c_{ij} - (u_i + v_j)$**

c_{ij}

•	(30)	(50)	•
(70)	(30)	•	•
(40)	•	(70)	•

$u_i + v_j$

•	-2	-10	•
69	48	•	•
29	•	0	•

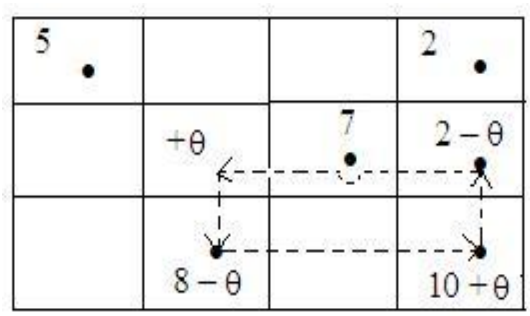
$d_{ij} = c_{ij} - (u_i + v_j)$

•	32	60	•
1	**-18**	•	•
11	•	70	•

5. **Optimality test**

$d_{ij} < 0$ i.e. $d_{22} = -18$
so x_{22} is entering the basis

6. **Construction of loop and allocation of unknown quantity Θ**



We allocate Θ to the cell (2, 2). Reallocation is done by transferring the maximum possible amount Θ in the marked cell. The value of Θ is obtained by equating to zero to the corners of the closed loop. i.e. min(8-Θ, 2-Θ) = 0 which gives Θ = 2. Therefore x_{24} is outgoing as it becomes zero.

5 (19)			2 (10)
	2 (30)	7 (40)	
	6 (8)		12 (20)

Minimum transportation cost is 5 (19) + 2 (10) + 2 (30) + 7 (40) + 6 (8) + 12 (20) = Rs. 743

7. Improved Solution

				u_i
• (19)			• (10)	$u_1 = -10$
	• (30)	• (40)		$u_2 = 22$
	• (8)		• (20)	$u_3 = 0$
$v_1 = 29$	$v_2 = 8$	$v_3 = 18$	$v_4 = 20$	v_j

c_{ij}

•	(30)	(50)	•
(70)	•	•	(60)
(40)	•	(70)	•

$u_i + v_j$

•	-2	8	•
51	•	•	42
29	•	18	•

$d_{ij} = c_{ij} - (u_i + v_j)$

•	32	42	•
19	•	•	18
11	•	52	•

Since $d_{ij} > 0$, an optimal solution is obtained with minimal cost Rs.743

Example 2

Solve by lowest cost entry method and obtain an optimal solution for the following problem

				Available
From	50	30	220	1
	90	45	170	3
	250	200	50	4
Required	4	2	2	

Solution

By lowest cost entry method

				Available
From		1(30)		1/0
	2(90)	1(45)		3/2/0
	2(250)		2(50)	4/2/0
Required	4/2/2	2/1/0	2/0	

Minimum transportation cost is 1 (30) + 2 (90) + 1 (45) + 2 (250) + 2 (50) = Rs. 855

Check for Non-degeneracy

The initial basic feasible solution has m + n – 1 i.e. 3 + 3 – 1 = 5 allocations in independent positions. Hence optimality test is satisfied.

Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$

				u_i
		• (30)		$u_1 = -15$
	• (90)	• (45)		$u_2 = 0$
	• (250)		• (50)	$u_3 = 160$
v_j	$v_1 = 90$	$v_2 = 45$	$v_3 = -110$	

Calculation of cost differences for non-basic cells $d_{ij} = c_{ij} - (u_i + v_j)$

c_{ij}

50	•	220
•	•	170
•	200	•

$u_i + v_j$

75	•	-125
•	•	-110
•	205	•

$d_{ij} = c_{ij} - (u_i + v_j)$

-25	•	345
•	•	280
•	-5	•

Optimality test

$d_{ij} < 0$ i.e. $d_{11} = -25$ is most negative So x_{11} is entering the basis

Construction of loop and allocation of unknown quantity Θ

$+\theta$	$1-\theta$	
$2-\theta$	$1+\theta$	
•		•

min(2-Θ, 1-Θ) = 0 which gives Θ = 1. Therefore x_{12} is outgoing as it becomes zero.

1(50)		
1(90)	2(45)	
2(250)		2(50)

Minimum transportation cost is 1 (50) + 1 (90) + 2 (45) + 2 (250) + 2 (50) = Rs. 830

II Iteration

Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$

				u_i
	• (50)			$u_1 = -40$
	• (90)	• (45)		$u_2 = 0$
	• (250)		• (50)	$u_3 = 160$
v_j	$v_1 = 90$	$v_2 = 45$	$v_3 = -110$	

Calculation of $d_{ij} = c_{ij} - (u_i + v_j)$

c_{ij}

•	30	220
•	•	170
•	200	•

$u_i + v_j$

•	5	-150
•	•	-110
•	205	•

$d_{ij} = c_{ij} - (u_i + v_j)$

•	25	370
•	•	280
•	-5	•

Optimality test
$d_{ij} < 0$ i.e. $d_{32} = -5$
So x_{32} is entering the basis

Construction of loop and allocation of unknown quantity Θ

•		
$1+\theta$ •	$2-\theta$ •	
• $2-\theta$	• $+\theta$	•

$2 - \Theta = 0$ which gives $\Theta = 2$. Therefore x_{22} and x_{31} is outgoing as it becomes zero.

1(50)		
3(90)	0(45)	
	2(200)	2(50)

Minimum transportation cost is 1 (50) + 3 (90) + 2 (200) + 2 (50) = Rs. 820

Iteration III

Calculation of u_i and v_j : - $u_i + v_j = c_{ij}$

				u_i
	• (50)			$u_1 = -40$
	• (90)	• (45)		$u_2 = 0$
		• (200)	• (50)	$u_3 = 155$
v_j	$v_1 = 90$	$v_2 = 45$	$v_3 = -105$	

Calculation of $d_{ij} = c_{ij} - (u_i + v_j)$

c_{ij}

•	30	220
•	•	170
250	•	•

$u_i + v_j$

•	5	-145
•	•	-105
245	•	•

$d_{ij} = c_{ij} - (u_i + v_j)$

•	25	365
•	•	275
5	•	•

Since $d_{ij} > 0$, an optimal solution is obtained with minimal cost Rs.820

MCQ

Question	**opt 1**	**opt 2**	**opt 3**	**opt 4**	**answer**
the calculation of opportunity cost in the MODI method is analogous to a	cj -zj value for non-basic variable columns in the simplex method	value of a variable in xb-column of the simplex method	variable in the B-column in the simplex method	none of these	cj -zj value for non-basic variable columns in the simplex method
The large negative opportunity cost value in an un-used cell in a transportation table is chosen to improve the current solution because	it represents per unit cost reduction	it represents per unit cost improvement	it ensure no rim improvement violation	none of these	it represents per unit cost reduction
When total supply is equal to total demand in a transportation problem, the problem is said to be	balanced	unbalanced	degenerate	none of these	balanced
An optimal assignment requires that the maximum number of lines which can be drawn through squares with zero opportunity cost be equal to the number of	rows or columns	rows and columns	rows + columns - 1	none of these	rows or columns
While solving an assignment problem, an activity is assigned to a resource through a square with zero opportunity cost because the objective is to	minimize total cost of assignment	reduce the cost of assignment to zero	reduce the cost of that particular assignment to zero	all of these	minimize total cost of assignment
The purpose of a dummy row or column in an assignment problem is called	obtain balance between total activities and total resources	prevent a solution from becoming degenerate	provide a means of representing a dummy problem	none of these	obtain balance between total activities and total resources
If there were *n* workers and *n* jobs ther would be	*n*! solutions	(*n* - 1)! Solutions	(*n* !)n solutions	*n* solutions	*n*! solutions

A transportation problem is said to be balanced if	$\Sigma a_i = \Sigma b_j$	$\Sigma a_i > \Sigma b_j$	$\Sigma a_i = o$	none of these	$\Sigma a_i = \Sigma b_j$
For any transportation problem, the coefficients of all xij in the constraints are ___	unity	integer	non-integer	none of these	unity
A solution that satisfies all conditions of supply and demand but it may or may not be optimal is called __________	initial feasible solution	feasible solution	optimal solution	all of these	initial feasible solution
In a northwest corner rule, ifthe demand in the column is satisfied, one must move to the _____ cell in the next column	right	left	center	all of these	right
Row wise and column wise difference between two minimum costs is calculated under _________- method	VAM	MODI	LP	none of these	VAM
A transportation problem can always be represented by ____________	balanced model	unbalanced	either balanced or unbalanced	none of these	balanced model
A balanced transportation model will always have a _________ solution	feasible	infeasible	optimal	all of these	feasible
In the solution of the transportation model, the amounts shipped from a dummy source to the destinations actually represents _________ at the destinations	shortage	cost	materials	resources	shortage
The transportation technique essentially uses the same steps of the ______ method	simplex	graphical	assignment	all of these	simplex

2 marks

1. What the uses of Vogel's Approximation method?
2. Explain North West Corner rule.
3. Define optimal solution in transportation problem.
4. What is the number of basic variables in an m x n balanced transportation problem?
5. Give mathematical formulation for transportation problem.
6. Define feasible solution, basic feasible solution in transportation problem
7. What is unbalanced transportation problem? How to solve it?
8. How the problems of degeneracy arise in transportation problem?
9. What is the use of MODI method?
10. Explain North West Corner rule.
11. Define optimal solution in transportation problem.
12. How do you convert an unbalanced transportation problem into balanced problem?
13. Define optimal solution in transportation problem.
14. What the difference between LPP and transportation problem?
15. What is travelling salesman problem?

14 marks

1. Find the optimal transportation cost of the following matrix using least cost method for finding the critical solution.

Market

		A	B	C	D	E	Available
Factory	P	4	1	2	6	9	100
	Q	6	4	3	5	7	120
	R	5	2	6	4	8	120
	Demand	40	50	70	90	90	

2. Solve the transportation problem with the transportation costs, demands and supplies as given below:

Destination

		D_1	D_2	D_3	D_4	Supply
Sources	S_1	6	1	9	3	70
	S_2	11	5	2	8	55
	S_3	10	12	4	7	70
Demand		85	35	50	45	

3. Obtain initial solution for the following transportation problem by using.

Destination

		A	B	C	Supply
Sources	1	2	7	4	5
	2	3	3	1	8

3	5	4	7	7
4	1	6	2	14
Demand	7	9	18	34

i) Northwest corner rule

ii) Least Cost Method

iii) Vogel's Approximation Method

4. Solve transportation problem

	1	2	3	4	Supply
I	21	16	25	13	11
II	17	18	14	23	13
III	32	27	18	41	19
Demand	6	10	12	15	

5. Solve the transportation problem

	To				Supply
	1	2	3	4	6
From	4	3	2	0	8
	0	2	2	1	10
Demand	4	6	8	6	

6. Determine the number of units to be manufactured for each product daily in order to achieve maximum profit. The profit per unit for product 1,2 and 3 is Rs.4, Rs.3 and Rs.6 respectively. It is assumed that all the amounts produced are consumed in the market.
7. Solve the transportation problem

Destination

		A	B	C	D	Supply
Source	1	11	20	7	8	50
	2	21	16	20	12	40
	3	8	12	18	9	70
	Demand	30	25	35	40	

8. Obtain initial feasible solution to the following transportation problem by using Least Cost Method and Vogel's Approximation method.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

9. Solve the transportation problem

	To				Supply
From	1	2	1	4	30

	3	3	2	1	50
	4	2	5	9	20
Demand	20	40	30	10	

10. Obtain the initial solution for the following transportation problem by using
(i) North West Corner Rule
(ii) Vogel's Approximation method

Destination

		A	B	C	D	Supply
Source	1	11	20	7	8	50
	2	21	16	20	12	40
	3	8	12	18	9	70
	Demand	30	25	35	40	

UNIT III

Introduction to Assignment Problem

In assignment problems, the objective is to assign a number of jobs to the equal number of persons at a minimum cost of maximum profit.

Suppose there are 'n' jobs to be performed and 'n' persons are available for doing these jobs. Assume each person can do each job at a time with a varying degree of efficiency. Let c_{ij} be the cost of i^{th} person assigned to j^{th} job. Then the problem is to find an assignment so that the total cost for performing all jobs is minimum. Such problems are known as **assignment problems.**

These problems may consist of assigning men to offices, classes to the rooms or problems to the research team etc.

Mathematical formulation

Cost matrix: $c_{ij}=$

$$\begin{matrix} c_{11} & c_{12} & c_{13} & \dots & c_{1n} \\ c_{21} & c_{22} & c_{23} & \dots & c_{2n} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ c_{n1} & c_{n2} & c_{n3} & \dots & c_{nn} \end{matrix}$$

$$\text{Minimize cost : } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij} \qquad i = 1, 2, .., n \quad j = 1, 2, .., n$$

Subject to restrictions of the form

$$x_{ij} = \begin{cases} 1 \text{ if ith person is assigned jth job} \\ 0 \text{ if not} \end{cases}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \text{(one job is done by the ith person, } i = 1,2,\dots n\text{)}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{(only one person should be assigned the jth job, } j = 1,2,\dots n\text{)}$$

Where x_{ij} denotes that j^{th} job is to be assigned to the i^{th} person.

This special structure of assignment problem allows a more convenient method of solution in comparison to simplex method.

Algorithm for Assignment Problem (Hungarian Method)

Step 1

Subtract the minimum of each row of the effectiveness matrix, from all the elements of the respective rows (Row reduced matrix).

Step 2

Further modify the resulting matrix by subtracting the minimum element of each column from all the elements of the respective columns. Thus first modified matrix is obtained.

Step 3

Draw the minimum number of horizontal and vertical lines to cover all the zeroes in the resulting matrix. Let the minimum number of lines be N. Now there may be two possibilities

If N = n, the number of rows (columns) of the given matrix then an optimal assignment can be made. So make the zero assignment to get the required solution.

If N < n then proceed to step 4

Step 4

Determine the smallest element in the matrix, not covered by N lines. Subtract this minimum element from all uncovered elements and add the same element at the intersection of horizontal and vertical lines. Thus the second modified matrix is obtained.

Step 5

Repeat step 3 and step 4 until minimum number of lines become equal to number of rows (columns) of the given matrix i.e. N = n.

Step 6

To make zero assignment - examine the rows successively until a row-wise exactly single zero is found; mark this zero by '‘to make the assignment. Then, mark a 'X' over all zeroes if lying in the column of the marked zero, showing that they cannot be considered for further assignment. Continue in this manner until all the rows have been examined. Repeat the same procedure for the columns also.

Step 7

Repeat the step 6 successively until one of the following situations arise

If no unmarked zero is left, then process ends

If there lies more than one of the unmarked zeroes in any column or row, then mark "" one of the unmarked zeroes arbitrarily and mark a cross in the cells of remaining zeroes in its row and column. Repeat the process until no unmarked zero is left in the matrix.

Step 8

Exactly one marked zero in each row and each column of the matrix is obtained. The assignment corresponding to these marked zeroes will give the optimal assignment.

Worked Examples

Example 1

A department head has four subordinates and four tasks have to be performed. Subordinates differ in efficiency and tasks differ in their intrinsic difficulty. Time each man would take to perform each task is given in the effectiveness matrix. How the tasks should be allocated to each person so as to minimize the total man-hours?

Subordinates

Tasks	I	II	III	IV
A	8	26	17	11
B	13	28	4	26

C	38	19	18	15
D	19	26	24	10

Solution

Row Reduced Matrix

0	18	9	3
9	24	0	22
23	4	3	0
9	16	14	0

I Modified Matrix

0	14	9	3
9	20	0	22
23	0	3	0
9	12	14	0

N = 4, n = 4

Since N = n, we move on to zero assignment

Zero assignment

$\boxed{0}$	14	9	3
9	20	$\boxed{0}$	22
23	$\boxed{0}$	3	$\cancel{0}$
9	12	14	$\boxed{0}$

Optimal assignment	A – I	B – III	C – II	D – IV
Man-hours	8	4	19	10

Total man-hours = 8 + 4 + 19 + 10 = 41 hours

Example 2

A car hire company has one car at each of five depots a, b, c, d and e. a customer requires a car in each town namely A, B, C, D and E. Distance (kms) between depots (origins) and towns (destinations) are given in the following distance matrix

	a	b	c	d	e
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

Solution

Row Reduced Matrix

30	0	45	60	70
15	0	10	40	55
30	0	45	60	75
0	0	30	30	60

20	0	35	45	70

I Modified Matrix

30	0	35	30	15
15	0	0	10	0
30	0	35	30	20
0	0	20	0	5
20	0	25	15	15

N < n i.e. 3 < 5, so move to next modified matrix

II Modified Matrix

15	0	20	15	0
15	15	0	10	0
15	0	20	15	5
0	15	20	0	5
5	0	10	0	0

N = 5, n = 5
Since N = n, we move on to zero assignment

Zero assignment

15	0	20	15	$\boxed{0}$
15	15	$\boxed{0}$	10	0
15	$\boxed{0}$	20	15	5
$\boxed{0}$	15	20	0	5
5	0	10	$\boxed{0}$	0

Route	A – e	B – c	C – b	D – a	E – d
Distance	200	130	110	50	80

Minimum distance travelled = 200 + 130 + 110 + 50 + 80 = 570 kms

Example 3
Solve the assignment problem whose effectiveness matrix is given in the table

	1	2	3	4
A	49	60	45	61
B	55	63	45	69
C	52	62	49	68
D	55	64	48	66

Solution

Row-Reduced Matrix

4	15	0	16

10	18	0	24
3	13	0	19
7	16	0	18

I Modified Matrix

1	2	0	0
7	5	0	8
0	0	0	3
4	3	0	2

N < n i.e 3 < 4, so II modified matrix

II Modified Matrix

1	2	2	0
5	3	0	6
0	0	2	3
2	1	0	0

N < n i.e 3 < 4

III Modified matrix

0	1	2	0
4	2	0	6
0	0	3	4
1	0	0	0

Since N = n, we move on to zero assignment Zero assignment

Multiple optimal assignments exists

Solution – I

$\boxed{0}$	1	2	$\cancel{0}$
4	2	$\boxed{0}$	6
$\cancel{0}$	$\boxed{0}$	3	4
1	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

Optimal assignment	A – 1	B – 3	C – 2	D - 4
Value	49	45	62	66

Total cost = 49 + 45 + 62 + 66 = 222 units

Solution – II

$\cancel{0}$	1	2	$\boxed{0}$
4	2	$\boxed{0}$	6
$\boxed{0}$	$\cancel{0}$	3	4
1	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

Optimal assignment	A – 4	B – 3	C – 1	D – 2
Value	61	45	52	64

Minimum cost = 61 + 45 + 52 + 64 = 222 units

Example 4

Certain equipment needs 5 repair jobs which have to be assigned to 5 machines. The estimated time (in hours) that a mechanic requires to complete the repair job is given in the table. Assuming that each mechanic can be assigned only one job, determine the minimum time assignment.

	J1	J2	J3	J4	J5
M1	7	5	9	8	11
M2	9	12	7	11	10
M3	8	5	4	6	9
M4	7	3	6	9	5
M5	4	6	7	5	11

Solution

Row Reduced Matrix

2	0	4	3	6
2	5	0	4	3
4	1	0	2	5
4	0	3	6	2
0	2	3	1	7

I Modified Matrix

2	0	4	2	4
2	5	0	3	1
4	1	0	1	3
4	0	3	5	0
0	2	3	0	5

N < n

II Modified Matrix

1	0	4	1	3
1	5	0	2	0
3	1	0	0	2
4	1	4	5	0
0	3	4	0	5

N = n

Zero assignment

1	$\boxed{0}$	4	1	3
1	5	$\boxed{0}$	2	$\cancel{0}$
3	1	$\cancel{0}$	$\boxed{0}$	2
4	1	4	5	$\boxed{0}$
$\boxed{0}$	3	4	$\cancel{0}$	5

Optimal assignment	M1 – J2	M2 – J3	M3 – J4	M4 – J5	M5 – J1
Hours	5	7	6	5	4

Minimum time = 5 + 7 + 6 + 5 + 4 = 27 hours

<u>Unbalanced Assignment Problems</u>

If the number of rows and columns are not equal then such type of problems are called as unbalanced assignment problems.

Example 1

A company has 4 machines on which to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table

		Machines			
		W	X	Y	Z
Jobs	A	18	24	28	32
	B	8	13	17	19
	C	10	15	19	22

Solution

18	24	28	32
8	13	17	19
10	15	19	22
0	0	0	0

Row Reduced matrix

0	6	10	14
0	5	9	11
0	5	9	12
0	0	0	0

I Modified Matrix

0	6	10	14
0	5	9	11
0	5	9	12
0	0	0	0

N < n i.e. 2 < 4

II Modified Matrix

0	1	5	9
0	0	4	6
0	0	4	7
5	0	0	0

N < n i.e. 3 < 4

III Modified Matrix

0	1	1	5
0	0	0	2
0	0	0	3
9	4	0	0

N = n

Zero assignment

Optimal assignment	W – A	X – B	Y – C
Cost	18	13	19

Multiple assignments exists Solution –I

$\boxed{0}$	1	1	5
~~0~~	~~0~~	$\boxed{0}$	2
~~0~~	$\boxed{0}$	~~0~~	3
9	4	~~0~~	$\boxed{0}$

Minimum cost = 18 + 13 + 19 = Rs 50

Solution –II

$\boxed{0}$	1	1	5
~~0~~	$\boxed{0}$	~~0~~	2
~~0~~	~~0~~	$\boxed{0}$	3
9	4	~~0~~	$\boxed{0}$

Optimal assignment	W – A	X – C	Y – B
Cost	18	17	15

Minimum cost = 18 + 17 + 15 = Rs 50

Example 2

Solve the assignment problem whose effectiveness matrix is given in the table

C1	R1	R2	R3	R4

	9	14	19	15
C2	7	17	20	19
C3	9	18	21	18
C4	10	12	18	19
C5	10	15	21	16

Solution

9	14	19	15	0
7	17	20	19	0
9	18	21	18	0
10	12	18	19	0
10	15	21	16	0

Row Reduced Matrix

9	14	19	15	0
7	17	20	19	0
9	18	21	18	0
10	12	18	19	0
10	15	21	16	0

I Modified Matrix

2	2	1	0	0
0	5	2	4	0
2	6	3	3	0
3	0	0	4	0
3	3	3	1	0

N < n i.e. 4 < 5

II Modified Matrix

1	1	0	0	0
0	5	2	5	1
1	5	2	3	0
3	0	0	5	1
2	2	2	1	0

N < n i.e. 4 < 5

III Modified Matrix

2	1	0	0	1
0	4	1	4	1
1	4	1	2	0
4	0	0	5	2
2	1	1	0	0

N = n

Zero assignment

2	1	$\boxed{0}$	$\cancel{0}$	1
$\boxed{0}$	4	1	4	1
1	4	1	2	$\boxed{0}$
4	$\boxed{0}$	$\cancel{0}$	5	2
2	1	1	$\boxed{0}$	$\cancel{0}$

Optimal assignment	C1 – R3	C2 – R1	C4 – R2	C5 – R4
Units	19	7	12	16

Minimum cost = 19 + 7 + 12 + 16 = 54 units

1.5 <u>Maximal Assignment Problem</u>

Example 1

A company has 5 jobs to be done. The following matrix shows the return in terms of rupees on assigning i^{th} (i = 1, 2, 3, 4, 5) machine to the j^{th} job (j = A, B, C, D, E). Assign the five jobs to the five machines so as to maximize the total expected profit.

Jobs

		A	B	C	D	E
	1	5	11	10	12	4
	2	2	4	6	3	5
Machines						
	3	3	12	5	14	6
	4	6	14	4	11	7
	5	7	9	8	12	5

Solution

Subtract all the elements from the highest element

Highest element = 14

9	3	4	2	10
12	10	8	11	9
11	2	9	0	8
8	0	10	3	7
7	5	6	2	9

Row Reduced matrix

7	1	2	0	8
4	2	0	3	1
11	2	9	0	8
8	0	10	3	7
5	3	4	0	7

I Modified Matrix

3	1	2	0	7
0	2	0	3	0
7	2	9	0	7
4	0	10	3	6
1	3	4	0	6

N < n i.e. 3 < 5

II Modified Matrix

2	0	1	0	6
0	2	0	4	0
6	1	8	0	6
4	0	10	4	6
0	2	3	0	5

N < n i.e. 4 < 5

III Modified Matrix

1	0	0	0	5
0	3	0	5	0
5	1	7	0	5
3	0	9	4	5
0	3	3	1	5

N = n

Zero assignment

1	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	5
$\cancel{0}$	3	$\cancel{0}$	5	$\boxed{0}$
5	1	7	$\boxed{0}$	5
3	$\boxed{0}$	9	4	5
$\boxed{0}$	3	3	1	5

Optimal assignment 1 – C 2 – E 3 – D 4 – B 5 – A
Maximum profit = 10 + 5 + 14 + 14 + 7 = Rs. 50

TRAVELLING SALESMAN PROBLEM

Just consider how a postman delivers the post to the addressee. He arranges all the letters in an order and starts from the post office and goes from addressee to addressee and finally back to his post office. If he does not arrange the posts in an order he may have to travel a long distance to clear all the posts. Similarly, a traveling sales man has to plan his visits. Let us say, he starts from his head office and go round the branch offices and come back to his head office. While traveling he will not visit the branch already visited and he will not come back until he visits all the branches.

There are different types of traveling salesman‘s problems. One is **cyclic problem**. In this problem, he starts from his head quarters and after visiting all the branches, he will be back to his head quarters. The second one is **Acyclic problem**. In this case, the traveling salesman leaves his head quarters and after visiting the intermediate branches, finally reaches the last branch and stays there. The first type of the problem is solved by Hungarian method or Assignment technique. The second one is solved by Dynamic programming method.

Point to Note: The traveling salesman's problem, where we sequence the cities or branches he has to visit is a SEQUENCING PROBLEM. But the solution is got by Assignment technique. Hence basically, the traveling salesman problem is a SEQUENCING PROBLEM; the objective is to minimize the total distance traveled.

The mathematical statement of the problem is: Decide variable xij = 1 or 0 for all values of I and j so as to:

$$\text{Minimise } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} \text{ for all } i \text{ and } j = 1,2\ldots..n \text{ Subject to}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ for } i = 1,2,\ldots.n \quad \text{(Depart from a city once only)}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ for } j = 1,2,\ldots.n \quad \text{(Arrive at a city once only)}$$

And all xij ≥ 0 for all i and j

This is indeed a statement of assignment problem, which may give two or more disconnected cycles in optimum solution. This is not permitted. That is salesman is not permitted to return to the origin of his tour before visiting all other cities in his itinerary. The mathematical formulation above does not take care of this point.

Problem 1

A salesman stationed at city A has to decide his tour plan to visit cities B, C, D, E and back to city A in the order of his choice so that total distance traveled is minimum. No sub touring is permitted. He cannot travel from city A to city A itself. The distance between cities in Kilometers is given below:

Column opportunity cost matrix

Cities	A	B	C	D	E
A	M	16	18	13	20
B	21	M	16	27	14
C	12	14	M	15	21
D	11	18	19	M	21
E	16	14	17	12	M

Total opportunity cost matrix

Cities	A	B	C	D	E
A	M	1	3	0	7
B	7	M	0	13	0
C	0	0	M	3	9
D	0	5	6	M	10
E	4	0	3	0	M

We can make only 4 assignments. Hence modify the matrix. Smallest element in the uncovered cells is 3, deduct this from all other uncovered cells and add this to the elements at the crossed cells. Do not alter the elements in cells covered by the line.

Cities	*A*	*B*	*C*	*D*	*E*
A	M	1	3	0	7
B	7	M	0	13	0
C	0	0	M	3	9
D	0	5	6	M	10
E	4	0	3	0	M

We can make only 4 assignments. Hence once again modify the matrix. Sequencing: *A* to *C*, *C* to *B*, *B* to *E*, *E* to *D*, and *D* to *A*. As there is a tie

Cities	*A*	*B*	*C*	*D*	*E*
A	M	1	0	0	4
B	10	M	0x	16	0
C	0x	0	M	3	6
D	0	5	3	M	7
E	4	0x	0	0	M

Sequencing: *A* to *C*, *C* to *B*, *B* to *E*, *E* to *D* and *D* to *A*. as there is a tie between the zero cells, the problem has alternate solution. The total distance traveled by the salesman is: 18 + 14 + 14 + 11 + 12 = 69 Km.

A to *C* to *B* to *E* to *D* to *A*, the distance traveled is 69 Km.

Problem 2: Given the set up costs below, show how to sequence the production so as to minimize the total setup cost per cycle.

Solution

We can draw five lines and make assignment. The assignment is:

Jobs	*A*	*B*	*C*	*D*	*E*
A	M	1	3	6	0
B	4	M	0	6	0x
C	4	3	M	0	3
D	8	0	1	M	1
E	0	2	0x	7	M

From *A* to *E* and From *E* to *A* cycling starts, which is not allowed in salesman problem. Hence what we have to do is select the next higher element than zero and make assignment with those elements. After assignment of next higher element is over, then come to zero for assignment. If we cannot finish the assignment with that higher element, then select next highest element and finish assigning those elements and come to next lower element and then to zero. Like this we have to finish all assignments. In this problem, the next highest element to zero is 1. Hence first assign all ones and then consider zero for assignment. Now we shall first assign all ones and then come to zero.

Total Opportunity Cost Matrix TOCM:

Jobs	*A*	*B*	*C*	*D*	*E*
A	M	1	3	6	0x
B	4	M	0	6	0x
C	4	3	M	0	3
D	8	0x	1x	M	1
E	0	2	0x	7	M

The assignment is *A* to *B*, *B* to *C*, *C* to *D* and *D* to *E* and *E* to *A*. (If we start with the element DC then cycling starts.

Now the total distance is 5 + 3 + 4 + 5 + 1 = 18 + 1 + 1 = 20 Km. The ones we have assigned are to be added as penalty for violating the assignment rule of assignment algorithm.

Problem 3

Solve the traveling salesman problem by using the data given below:

$C12 = 20$, $C13 = 4$, $C14 = 10$, $C23 = 5$, $C34 = 6$, $C25 = 10$, $C35 = 6$, $C45 = 20$ and $Cij = Cji$. And there is no route between cities '*i*' and '*j*' if a value for *Cij* is not given in the statement of the problem. (*i* and *j* are = 1,2,..5)

Solution

Cities	1	2	3	4	5
1	M	20	4	10	M
2	20	M	5	M	10
3	4	5	M	6	6
4	10	M	6	M	20
5	M	10	6	20	M

Now let us work out Column Opportunity cost Matrix(COCM)/Row Oppourtunity Cost Matrix (ROCM) and Total Opportunity cost Matrix TOCM, and then make the assignment.

Cities.	1	2	3	4	5
1	M	12	0	0x	M
2	11	M	0x	M	0
3	0x	1	M	0	1
4	0	M	0x	M	9
5	M	0	0x	8	M

The sequencing is: 1 to 3, 3 to 4, 4 to 1 and 1 to 3 etc., Cycling starts. Hence we shall start assigning with 1 the next highest element and then assign zeros. Here also we will not get the sequencing. Next we have to take the highest element 8 then assign 1 and then come to zeros.

Cities.	1	2	3	4	5
1	M	12	**0**	0	M
2	11	M	0	M	**0**
3	0	**1**	M	0	1
4	**0**	M	0	M	9
5	M	0	0	**8**	M

Sequencing is: 1 to 3, 3 to 2, 2 to 5, 5 to 4 and 4 to 1.
The optimal distance is: 4 + 10 + 5 + 10 + 20 = 49 + 1 + 8 = 58 Km.

MCQ

Question	opt 1	opt 2	opt 3	opt 4	answer
The assignment problem cab be stated in the form of a n * n matrix (c_{ij}) is called the _________	cost matrix	ineffective matrix	resources matrix	none of these	cost matrix
The assignment problem can be solved by the _____ technique	transportation	simplex	travelling salesman	all of these	transportation
The transportation problem can not be solved by the _____ technique	assignment	simplex	either assignment or simplex	all of these	assignment
In a travelling salesman problem, the salesman should not visit a city twice except the _____	starting city	middle city	all the city	all of these	starting city
the occurrence of degeneracy while solving a transportation problem means that	total supply equals total demand	the solution so obtained is noe feasible	the few allocations become negative	none of these	the solution so obtained is noe feasible
an alternative optimal solution to a minimization transportation problem exists whenever opportunity cost corresponding to unused route of transportation is	positive and greater than zero	positive with at least one equal to zero	negative with atleast one equal to zero	none of these	positive with at least one equal to zero
one disadvantage of using north-west corner rule to find initial solution to the transportation problem is that	it is complicated to use	it does not take into account cost of transportation	it leads to a degenerate initial solution	all of these	it does not take into account cost of transportation
If an opportunity cost value is used for an unused cell to test optimality, it should be	equal to zero	most negative number	most positive number any value	any value	most negative number

Every basic feasible solution of a general assignment problem having a square pay-off matrix of order *n* should have assignments equal to	2n + 1	2*n* - 1	*m* + *n* - 1	*m* + *n*	2*n* - 1
The hungarian method for solving an assignment problem can also be used to solve	a transportation problem	a travelling salesman problem	both a transportation problem & a travelling salesman problem	only a travelling salesman problem	a travelling salesman problem
To solve degeneracy, an unoccupied cell with _____ cost is converted into occupied cell by assigning Î infinitely small amount	highest	lowest	zero value	none of these	lowest
An optimum results when net cost change value of all unoccupied cells are __________	negative	non-negative	integer	zero	non-negative
The number of basic variables in the balanced transportation problem with 3 rows and 5 columns is __	5	7	8	9	7
An alternative optimal solution to a transportation problem is said to exist when one or more of the unoccupied cells have ________ value for the net cost change in the optimal solution	non-zero	zero	positive	none of these	zero
An assignment problem is said to be balanced if number of rows is equal to	number of variables	number of columns	number of non-negative variables	zero	number of columns
An assignment problem is said to be	number of	number of	number of non-	zero	number of columns

unbalanced if number of rows is not equal to	variables	columns	negative variables		
Assignment technique is essentially a ______ technique	maximization	minimization	neither maximization nor mimimization	none of these	minimization
the solution to a transportation problem with m-rows (supplies) and n-columns (destination) is feasible if number of positive allocations are	m + n	m x n	m + n - 1	m + n + 1	m + n - 1
An unoccupied cell in the transportation method is analogous to a	cj -zj value in the simplex table	variable in the B-column in the simplex table	variable not in the B-column in the simplex table	value in the XB-column in the simplex table	variable not in the B-column in the simplex table
During an iteration while moving from one solution to the next, degeneracy may occur when	the closed path indicates a diagonal move	two or more occupied cells are on the closed path but neither of them represents a corner of the path	two or more occupied cells on the closed path with minus sign are lowest circled value	either of these	two or more occupied cells on the closed path with minus sign are lowest circled value
the smallest quantity is chosen at the corners of the closed path with negative sign to be assigned at unused cell because	it improve the total cost	it does not disturb rim condition	it ensure feasible solution	all of these	it ensure feasible solution
Which of the following method is used to verify the optimality of the current solution of the transportation problem,	least cost method	vogel's approximation method	modified distribution method	all of these	modified distribution method
the method used for solving an	reduced matrix	MODI method	hungarian	none of these	hungarian method

assignment problem is called	method		method		
Maximization assignment problem is transformed into a minimization problem by	adding each entry ina column from the maximum value in that column	subtracting each entry in a column from the maximum value in that column	subtracting each entry in the table from the maximum value in that table	any one of these	subtracting each entry in the table from the maximum value in that table
An assignment problem can be solved by	simplex method	simplex method	both simplex method and transportation method	none of these	both simplex method and transportation method
For a salesman who has to visit *n* cities, following are the ways of his tour plan	*n* !	(*n* + 1) !	(*n* - 1) !	*n*	(*n* - 1) !
Only those problems where total demand equals the total supply can be solved by the technique of __________ model	assignment	linear programming	transportation	all of these	transportation
The transportation model is restricted to dealing with a ________ commodity only	double	multiple	single	all of these	single
An assignment problem is a completely _____ form of a transportation problem	optimal	simplex	degenerate	dual	degenerate
Assignment problem represents a transportation problem with all demands and supplies equal to	two	zero	one	either zero or one	one
In the optimal simplex table, $c_j - z_j$ = 0 value indicates	unbounded solution	cycling	alternative solutions exist	infeasible solution	alternative solutions exist
The degeneracy in the transportation	the problem has	the multiple	dummy	both the	the multiple optimal

problem indicates that	no feasible solution	optimal solution exist	allocations needs to be added	multiple optimal solution exist &dummy allocations needs to be added	solution exist & dummy allocations needs to be added but not the problem has no feasible solution
An assignment problem is considered as a particular case of a transportation problem because	the number of rows equals columns	all x ij = 0 or 1	all rim condition are 1	all of these	all of these
The assignment problem	requires that only one activity be assigned to each resource	is a special case of transportation problem	can be used to maximize resouces	all of these	all of these
An assignment problem is a special case of transportation problem, where	number of rows equals number of columns	all rim conditions are 1	values of each decision variable is either 0 or 1	all of these	all of these
An optimal solution of an assignment problem can be obtained only if	each row and column has only one zero element	each row and column has at least one zero element	the data are arranged in a square matrix	none of these	none of these
The number of non-basic variables in the balanced transportation problem with 4 rows and 5 columns is _______	10	13	14	12	12
An assignment problem represents a transportation problem with all demands and supplies equal to ___	2	3	4	1	1

2 marks

1. Define sequencing analysis
2. State the objective of an assignment problem
3. Explain the sequencing problem of n jobs on m machines
4. What are the assumptions made to convert a 3 machine problem to a 2 machine problem?
5. What is the objective of sequencing problem?
6. State the objective of an assignment problem
7. What are assignment problem?
8. What is objective of the travelling salesman problem?
9. Describe the mathematical formulation of assignment problem.
10. What are assignment problem?
11. Describe the mathematical formulation of assignment problem.
12. Distinguish transportation model and assignment model.
13. Distinguish transportation model and assignment model.
14. What is sequencing problem?

14 marks

1. Solve the following travelling salesman problem.

To

From		A	B	C	D
	A	-	46	16	40
	B	41	-	50	40
	C	82	32	-	60
	D	40	40	36	-

2. Solve the following sequencing problem giving an optimal solution if passing is not allowed

Machines

Jobs		M_1	M_2	M_3	M_4
	A	13	8	7	14
	B	12	6	8	19
	C	9	7	8	15
	D	8	5	6	15

3. Four different jobs can be done on four different machines. The set up and down time costs are assumed to be prohibitively high for change over. The matrix below gives the cost in rupees of processing jobs i on machine j

Machines

		M1	M2	M3	M4
Jobs	J1	5	7	11	6

J2	8	5	9	6
J3	4	7	10	7
J4	10	4	8	3

How should the jobs assigned to various machines so that total cost minimized?

4. Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order ABC. Processing times (in hours) are given in the following table.

Jobs	1	2	3	4	5	6
Machine A	8	3	7	2	5	1
Machine B	3	4	5	2	1	6
Machine C	8	7	6	9	10	9

What is the idle time for each machine?

5. The Processing Time in hours for the jobs when allocated to different machines is indicated below. Assign the machines for the jobs so that total processing time in minimum.

		Machines				
		M1	M2	M3	M4	M5
Jobs	J1	9	22	58	11	19
	J2	43	78	72	50	63
	J3	41	28	91	37	45
	J4	74	42	27	49	39
	J5	36	11	57	22	25

6. Find the sequence of the minimizes the total elapsed time required to complete the following tasks on the machines M1 and M2 in the order M1, M2. Also find the minimum total elapsed time.

Task	A	B	C	D	E	F	G	H	I
M1	2	5	4	9	6	8	7	5	4
M2	6	8	7	4	3	9	3	8	11

7. The head of the department has five jobs A, B, C, D, E and five subordinates V, W, X, Y and Z. The number of hours each man would take to perform each job is as follows:

	V	W	X	Y	Z
A	3	5	10	15	8
B	4	7	15	18	8
C	8	12	20	20	12
D	5	5	8	10	6
E	10	10	15	25	10

How should the jobs be assigned to minimize the total time?

8. Find the sequence of the minimizes the total elapsed time required to complete the following tasks on the machines in the order 1-2-3. Also find the minimum total elapsed time (hours) and the idle time on the machines.

Task	A	B	C	D	E	F	G
Machine	3	8	7	4	9	8	7
Machine	4	3	2	5	1	4	3
Machine	6	7	5	11	5	6	12

9. A company has four machines to do three jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table.

		Machines			
		1	2	3	4
	A	18	24	28	32
Jobs	B	8	13	17	19
	C	10	15	19	12

What are job assignments which will minimize the cost?

10. Solve the following sequencing problem giving an optimal solution if passing is not allowed

		Machines			
		M1	M2	M3	M4
Jobs	A	13	8	7	14
	B	12	6	8	19
	C	9	7	8	15
	D	8	5	6	15

UNIT IV

INVENTORY CONTROL

Inventory is the stock of any item or resource used in an organization. Inventory includes: raw materials, finished products, component parts, supplies, and work-in-process. An inventory system is the set of policies and controls that monitors levels of inventory and determines what levels should be maintained, when stock should be replenished, and how large orders should be.

The purposes of inventories are:
1. To maintain independence of operations
2. To meet variation in product demand
3. To allow flexibility in production scheduling
4. To provide a safeguard for variation in raw material delivery time
5. To take advantage of economic purchase order size

INVENTORY COSTS

Five types of costs need to be considered when analyzing inventory decisions:

1. Holding (or carrying) costs: storage facilities, handling, insurance, pilferage, breakage, obsolescence, depreciation, taxes, and the opportunity cost of capital.
2. Setup (or production change) costs: line conversion, equipment change-over, report preparation, etc.
3. Ordering costs: typing, calling, transportation, receiving, etc. This cost does not depend or vary on the number ordered.
4. Shortage costs (stockout costs): the loss due to losing a specific sale, customers' goodwill, or future business.
5. Cost of the item

INDEPENDENT VERSUS DEPENDENT DEMAND

Independent demand (i.e., the demand by consumers) is influenced by market conditions outside the control of operations. Independent demand calls for a replenishment philosophy. Orders are made to replenish inventory.

Dependent (or derived) demand is related to the demand for another item. For example, parts, intermediate goods, and raw materials face a demand dependent on the demand for the final goods. Dependent demand calls for a requirements philosophy. Orders are made if there is a demand or requirement for the final product.

ECONOMIC ORDER QUANTITY (EOQ)

The economic order quantity or EOQ is the certain amount to be ordered at specific intervals. It gives the perfect sawtooth pattern in a graph of inventory versus time.

EOQ is simple to understand and use but it has several restrictive assumptions which are also disadvantages in practice. Even with these weaknesses, EOQ is a good place to start to understand inventory systems. EOQ assumes:

1. Demand rate is constant, uniform, recurring, and known.
2. Lead time is constant and known.
3. Price per unit of product is constant; no discounts are given for large orders.
4. Inventory holding cost is based on average inventory.

5. Ordering or setup costs are constant.
6. All demands will be satisfied; no stockouts are allowed.
D = demand rate, units per year
S = cost per order placed, or setup cost, dollars per order
C = unit cost, dollars per unit
i = holding or carrying rate, percent of dollar value per year
Q = lot size, units
TC = total or ordering cost plus carrying cost, dollars per year

Annual purchase cost = DC

Annual ordering cost = (D/Q)S

Annual holding cost per year = HQ/2 = iCQ/2

TC = total annual cost = DC + (D/Q)S + iCQ/2

Solving for minimum TC yields the economic order quantity:

EOQ = 2SD / iC

Number of orders per year = D/Q

Number of months between orders = 12 / (D/Q)

EXAMPLE 1

A grocery store sells 10 cases of coffee each week. Each case costs $80. The cost of ordering is $10 per order. Holding or carrying cost is estimated to be 30% of the inventory value per year. So the variables are defined as:

D = 520 cases/year = 10 cases/week * 52 weeks/year
S = $10 per order
i = 30% (or 0.30)
C = $80 per case

$$EOQ = \sqrt{\frac{2SD}{iC}} = \sqrt{\frac{2*10*520}{0.3*80}} = 20.8 \approx 21 \text{cases per order}$$

How often is the coffee ordered?

520/21 = 25 orders per year (approx). Or every 15 days (365/25 = 15(approx))

The total cost of ordering and carrying inventory is:

TC = S(D/Q) + iCQ/2 = 10*(520/21) + 0.3*80*21/2 = $500 per year

If demand increased by 50% to 15 cases per week:

EOQ increases to 25 cases per order (from 21) which, please note, is NOT a 50% increase. The ordering interval drops to every 12 days. TC rises to $612.

If carrying cost was 45% (a 50% rise): (Demand is at the original 10 cases per week.) EOQ drops to 17. The ordering interval drops to 12. TC rises to $612.

Using EOQ for Determining Lot Size

An ice cream company produces 100,000 cases of black walnut & cherry ice cream each year. Switching the production line from another flavour to black walnut & cherry costs $1,000 each time for cleaning, assembling, putting the ingredients in the correct places, etc. The cost of each case is $100. The company estimates their inventory cost is 30% of inventory value per year.

D = 100,000 cases/year
S = $1000 per setup
i = 30% (or 0.30)
C = $100 per case

$$EOQ = \sqrt{\frac{2SD}{iC}} = \sqrt{\frac{2*1000*100000}{0.3*80}} = 2{,}582 \text{cases per production run}$$

How many lots or runs per year?

100000 / 2582 = 38.7 orders per year

The total cost of setup and carrying inventory is:

TC = S(D/Q) + iCQ/2 = 1000*(100000/2582) + 0.3*100*2582/2 = $77,460 per year

I doubt that the company would be so specific as to order a run of 2,582 cases. If they decided to use orders of 2,500 cases per run, their total cost would be:

TC = S(D/Q) + iCQ/2 = 1000*(100000/2500) + 0.3*100*2500/2 = $77,500 per year

For an increase of $40 per year, I suspect the company may decide that the round number of 2,500 may be easier to use on the production floor and elsewhere in the office.

FIXED-ORDER QUANTITY MODELS (or Q-models)

Fixed-order quantity models attempt to determine the specific point, R, at which an order will be placed and the size of that order, Q. The fixed-order quantity is also called the Q-system since the order quantity Q is fixed.

With a fixed-order quantity system, an order of size Q is placed when the inventory available (currently in stock and on order) reaches the point R.

R is determined as the average demand over the lead time (that is, the time between ordering and receiving) plus a safety stock to reflect variation in demand over time.

R = reorder point
$\bar{d}$ =average daily demand (constant)
L = lead time in days (constant)
z = safety factor (number of standard deviations for a specified service probability)
σ_L = standard deviation of demand over the lead time
R = $\bar{d}$ L + **z** σ_L

Service level is the percentage of customer demands satisfied from inventory. Shortage or stockout percentage = 100 - service level.

EXAMPLE USING CONTINUOUS REVIEW WITH FIXED-ORDER QUANTITIES (also called perpetual review or the Q-system, since the order quantity remains the same)

A repair shop sells tires and has assembled the following information for one specific type:
D = 500 tires/year
S = $40 per order
i = 25% (or 0.25)
C = $40 per tire
L = 4 days (250 working days per year)
σ = s.d. of daily demand = 1 tire
desired service level = 95%. So, z = 1.65

$$\text{EOQ} = \sqrt{\frac{2SD}{iC}} = \sqrt{\frac{2*40*500}{0.25*40}} = \text{63.25 tires per order}$$

Reorder Point = ROP = m + s = 8 + 3.3 = 11.3 = 12 tires (approx)

m = average demand over lead time = average demand per working day * lead time
= $\bar{d}\mathbf{L}$ = (500/250) * 4 = 8 tires

s = safety stock = $\mathbf{z}\,\sigma_L$= 1.65 * 1 * sqrt (4) = 3.3 tires

The specific Q-rule is: Continuously review stock level and order 63 tires when stock reaches the reorder point of 12 tires.

EOQ with Price Discounts

A supplier of cleaning products offers a supermarket the following deal. If the supermarket buys 29 cases or less, the cost is $25 per case. If they buy 30 or more cases, the cost will be $20 per case. The supermarket's ordering cost is $20 per order and the carrying cost is 15 percent per year.The supermarket sells 50 cases per year.

Procedure when analyzing the EOQ with discounts:
1. Calculate the EOQ for the lowest cost per unit. If the EOQ is above the price break, this is the most economical quantity.
2. If the EOQ in 1 above is below the price break, use the next lowest price to calculate the EOQ. Continue calculating EOQs until a feasible EOQ is found.
3. Calculate the total cost of the EOQ, if found, and the total cost of the higher price breaks.
4. The minimum of these total costs indicates the most economic order quantity.

FIXED-TIME PERIOD MODELS (or P-Models)

In a fixed-time period system, inventory is counted only at particular times and the size of the order varies. Compared to the Q-system, the P-system does not have a reorder point but rather a target inventory.

The P-system does not have an economic order quantity since it varies according to demand. The P-system requires a larger safety stock for the same service level.

The P system is determined by two parameters.

P = the time between orders. We can approximate P as Q/D = sqrt(2*S/iCD)

T = target inventory level = m' + s'

m' = average demand over P + L

s' = z *std. dev.(P+L)

L = lead time in days (between placing and receiving an order)

z = safety factor (number of standard deviations for a specified service probability)

EXAMPLE USING PERIODIC REVIEW (also called the fixed order interval (FOI) or the P-system, since the time period between orders remains the same)

If the same repair shop (as above) orders tires at specific intervals (say once a month or every 21 working days on average), the inventory orders are done differently. The EOQ is still the same (63.25 tires per order), but we now talk about the target inventory.

Each month's order = target inventory - stock on hand on day of order

Target inventory = m' + s' = 50 + 8.25 = 58.25 =(approx) 59 tires
m' = (500/250) * (21+4) = 50
s' = = **z*** *s* **P**+**L** 1.65 * 1 * sqrt (21+4) = 8.25

The specific P-rule is: Review the stock on, say, the first working day of every month and order the difference between the inventory on hand on that day and the target inventory of 59 tires.

USING P AND Q SYSTEMS IN PRACTICE

The choice between P and Q is not simple, but there are some conditions under which the P system may be preferred over the Q system.

1. When orders must be placed or delivered at specified intervals (e.g., canned goods in a grocery store)
2. When multiple items are ordered from the same supplier and delivered in the same shipment (e.g., different flavors of ice cream)
3. For inexpensive items which are not maintained on perpetual inventory records (e.g., nuts and bolts in a bin)

The P-system requires less record keeping, but a larger safety stock. So the Q-system may be preferred for expensive items to keep carrying costs lower.

OTHER SYSTEMS

Optional replenishment system: Reviewing on a fixed frequency and ordering a specified quantity if inventory is below a certain level. This is a mix of the P and Q systems.

Two-bin system: An amount equal to R is kept in reserve in a "second" bin. When the first bin is emptied, the second bin in emptied into the first and an order of size Q is placed.

One-bin system: This is the P-system where the one bin is reviewed at a fixed interval and inventory brought up to a certain inventory level.

ABC Inventory planning: Inventory items are classified into three groups on the basis of annual dollar volume. More attention is given to those inventory items with a high dollar volume compared to a low dollar volume.

A: high dollar volume items, say 15% of the total number of items
B: moderate dollar volume items, say the next 35%
C: low dollar volume items, the last 50%

MCQ

Question	**opt 1**	**opt 2**	**opt 3**	**opt 4**	**answer**
Sum of buffer stock, reserve stock and safety stock is equal to	recorder point	order quantity	EOQ	maximum inventory level	recorder point
Mean rate of consumption during lead time ® multiplied by mean lead time (L) is equal to	Buffer stock	reserve stock	safety stock	none of these	Buffer stock
The two-bin system is concerned with	ordering procedure	forecasting sales	production planning	none of these	ordering procedure
When order quantity increases the ordering costs will	increase	decrease	remains same	none of these	increase
Classifying items in A, B, C categories for selective control in inventory management is done by arranging items in the decresing order of	total inventory cost	item value	annual usage value	item demand	total inventory cost
Which of the following cost elements are considered while determining the economic lot size for purchase?1.Inventory carying cost,2.procurement cost,3.set up cost. Select the correct code___________	1,2,3	1,2	2,3	1,3	1,2,3
EOQ decreases when the cost of item _________	increases	decreases	both increases as well as decreases	none of these	increases
If the procurement cost per order increases 21%, the economic order quantity of the item shall increse by ________	10%	20%	30%	40%	10%
The objective of a scientific inventory is to ______ investment in various forms of inventory	increses	decreses	remain constant	all of these	increses
Review period of the time is always kept ____________ than the lead time	lower	higher	either lower or higher	none of these	lower

WIP stands for	work in progress inventories	work in process inventories	work in process industry	none of these	work in progress inventories
__________do not depend on the number of orders	variable cost	fixed cost	capital cost	none of these	variable cost
the length of time between placing an order and receipt of items is called as	order time	lead time	production time	none of these	order time
the annual inventory carrying cost is	product of average inventoryX ordering cost	product of average inventory Xcarrying cost	product of average inventory Xshortage cost	none of these	product of average inventoryX ordering cost
the inventory carriying cost _________if the quantity ordered per order is small	increases	decreases	both increases and decreases	none of these	increases
ABC analysis is a basic	arithmetic tool	analytical tool	advanced tool	all of these	arithmetic tool
fixed order quantity system is suitable for	high value items	low value items	both high value and low value items	none of these	high value items
A variable which does not appear in the basic variable (B) column of simplex table is	never equal to zero	always equal to zero	called a basic variable	none of these	always equal to zero
Release date is equal to	due date + lead time	due date - lead time	lead time - due date	none of these	due date - lead time
Small Discrete periods oftime for which production plans are made are called	time- phasing	time- buckets	lead time - due date	none of these	time- buckets
In inventory control theory, the economic order quantity is	average level of inventory	optimum lot size	lot size corresponding to break-even analysis	capacity of warehouse	optimum lot size
Economic order quantity is the quantity at	minimum	equal to the	less than the	cost of over-	equal to the

which the cost of carrying is		cost of ordering	cost of ordering	stocking	cost of ordering
Reduction in procurement cost ______ EOQ	increases	reduces	maintains	all of these	reduces
Inventory keeping takes care of ___________	manufacturing resource planning	economic fluctuation	demand	manufacturing process	economic fluctuation
inventory generally refers to the _______	materials in shop floor	materials in quality control	materials in stock	none of these	materials in quality control
tools inventory includes both standard tools and _________	used tool	unused tools	special tools	standard tools	unused tools
ROL stands for	Re-offer level	Raw material order level	Re-order level	all of these	Raw material order level
EOQ stands for	economic order quality	economic offer quantity	economic order quantity	all of these	economic offer quantity
fixed quantity system is referred as	P system	F system	Q system	S system	F system
To formulate a problem for solution by the simplex method, we must add artifical variable to ___	only equality constraints	only 'greater than' constraints	both only equality constraints and only 'greater than' constraints	none of these	both only equality constraints and only 'greater than' constraints
In the production model for determining the economic batch size, the production rate is considered as	equal to demand rate	less than demand rate	greater than demand rate	independent of demand rate	greater than demand rate
If EOQ is within the range of the lowest discounted rate offered,then	accept the discount offer and order for	reject the discount order	accept the discount offer and order at	none of these	accept the discount offer and order at

	the minimum in the range		EOQ level		EOQ level
MRP indicates	material reordering point	material reordering planning	material requirements planning	material requirement point	material requirements planning
Scarce iteams are	mostly available in indigenous market	cannot be procured easily	of short supply or imported items	all of these	of short supply or imported items
For a given annual consumption,the minimum total inventry cost is proportional to square root of the products of	ordereing cost per order	caring cost per unit per year	both ordereing cost per order and caring cost per unit per year	none of these	both ordereing cost per order and caring cost per unit per year
In inventory planning,extra inventory is unnecessarily carried to the end of the planning period when using one of the following lot size decision policies	lot-for-lot production	economic order quantity lot size	period order quantity lot size	part period total cost balancing	period order quantity lot size
An approximate percentage of 'C' items ina firm is around _________	100%	50-60%	70-75%	80-85%	70-75%
Economic order quantity results in equilization of annual procurement cost and _________	annual fixed cost	annual variable cost	annual inventory cost	all of these	annual inventory cost
If the annual consumption of an item increases by four times its EOQ will increase by ________	three times	four times	two times	none of these	two times
__________is a result of extended transportation time	tools inventory	raw materials	anticipation inventory	transportation inventory	anticipation inventory
the amount invested in an item is called as	ordering cost	purchase cost	production cost	capital cost	production cost
_________is the number of items required per	products	lead time	order cycle	demand	order cycle

unit of time					
If an optimal solution is degenerate, then ____ solution	there are alternative optimal	the solution is infeasible	the solution is of no use to the decision maker	none of these	none of these
Inventory can be in the form of	Raw materials	Supplies	in process goods	all of these	all of these
For a given level of safety stock and EOQ ordering	the reorder point depends only on the rate of consumption	the reorder point is independent of the rate of consumption	the reorder point depends only on the lead time	the reorder point depends upon the rate of consumption and lead time	the reorder point depends upon the rate of consumption and lead time
Inventory management consists of	effective running of stores	stock control system	state of merchandise method of storing and maintenance etc	all of these	all of these
to account for the fluctuations in demand and lead time there is a need of	reserve stock	sales syock	order stock	none of these	reserve stock
to avoid the ________ the organization should maintain the inventory	loss of orders	loss of stock	loss of time	a	loss of orders
inventory carrying cost is also called as	holding cost	fixed cost	ordering cost	purchase cost	holding cost
safety stock is also called as ____	buffer stock	inventory stock	maximum stock	all of these	buffer stock
procurement cost is also called as _______	ordering cost	shortage cost	capital cost	production cost	ordering cost

2 marks

1. Write any two limitations of EOQ formula.
2. Define transient state, steady state
3. Give an example of first come, last served
4. What is meant by inventory?
5. Write any two limitations of EOQ formula.
6. What are the various forms of inventory?
7. What is set up cost and holding cost?
8. Give an example of first come, first served.
9. Define lead time, Reorder level.
10. Write the Wilson's formula.
11. Define quantity discount.
12. What is meant by inventory?
13. What are the three types of inventories?
14. Write any two limitations of EOQ formula.
15. Define a Queue.

14 marks

1. A certain item costs Rs.250 per ton. The monthly requirements are 10 tons and each time the stock is replenished there is set up cost of Rs.1000. The cost carrying inventory has been estimated as 12% of the value of the stock per year. What is the optimal order quantity and how frequently should orders be placed?

2. Workers come to tool store room to enquire about special tools for accomplishing a particular project assigned to them. The average time between two arrivals is one minute and the arrivals are assumed to be in Poisson distribution. The average service time of the tool room attendant is 40 seconds. Determine
   i) Average queue length
   ii) Average number of workers in system
   iii) Mean waiting time of an arrival in the queue
   iv) Average time that a worker spends in the store room.

3. A Company producing three items has a limited storage space of averagely 750 items of all types. Determine the optimal production quantities for each item separately, when the following information is given.

Product	1	2	3
Holding cost(Rs):	0.05	0.02	0.04
Set up cost	50	40	60
Demand rate	100	120	75

4. Workers come to tool store room to enquire about special tools for accomplishing a particular project assigned to them. The average time between two arrivals is one minute and the arrivals are assumed to be in Poisson distribution. The average service time of the tool room attendant is 40 seconds. Determine
   i) Average queue length
   ii) Average number of workers in system
   iii) Mean waiting time of an arrival in the queue
   iv) Average time that a worker spends in the store room.

5. A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usage at a time. Each part costs Rs. 20. The ordering cost per order is Rs.15 and the carrying charges are 15% of the average inventor per year. You have been asked suggest a more economical purchasing policy for the company. What advice would you offer, and how much would it save company per year?

6. In a railway Marshalling yard, good train arrives at a rate of 30 Trains per day. Assuming that inter Arrival time follows an exponential distribution and the service time distribution is also exponential, with an average of 36 minutes. Calculates the following:

i) The mean Queue size (line length)
ii) The probability that Queue size exceeds 10
iii) If the input of the Train increases to an average 33 per day, what will be the changes in (i), (ii)?

7. A particular item has a demand of 9000 units/ year. The cost of one procurement is Rs.100 and the holding cost per unit is Rs 2.40 per year. The replacement is instantaneous and no shortage is allowed. Determine

i) The economic lot size
ii) The number of orders per year
iii) The time between orders
iv) The total cost per year if the cost of one unit is Rs.5

8. In a super market, the average arrival rate of customer is 10 in every 30 minutes following Poisson. The average time taken by the cashier to list and calculate the customer's purchases is 2.5 minutes, following Exponential distribution.

i) What is the probability that the Queue length exceeds 6?
ii) What is the expected time spent by a customer in the system?

9. The demand for an item in a company is 18000 units per year, and the company can produce at a rate of 3000 per month. The cost of one set up is Rs.500 and the holding cost of one unit per month is 15 paisa. The shortage cost of one unit is Rs.20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and time between set-ups.

10. A television repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is Poisson with an average rate of with an average rate of 10 per 8 hour day, what is his expected idle time day? How many jobs are ahead of the average set just brought in?

UNIT V

Introduction to Game Theory

Game theory is a type of decision theory in which one's choice of action is determined after taking into account all possible alternatives available to an opponent playing the same game, rather than just by the possibilities of several outcome results. Game theory does not insist on how a game should be played but tells the procedure and principles by which action should be selected. Thus it is a decision theory useful in competitive situations.

Game is defined as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss. The set of rules defines the **game**. Going through the set of rules once by the participants defines a **play**.

Properties of a Game

1. There are finite numbers of competitors called 'players'
2. Each player has a finite number of possible courses of action called 'strategies'
3. All the strategies and their effects are known to the players but player does not know which strategy is to be chosen.
4. A game is played when each player chooses one of his strategies. The strategies are assumed to be made simultaneously with an outcome such that no player knows his opponents strategy until he decides his own strategy.
5. The game is a combination of the strategies and in certain units which determines the gain or loss.
6. The figures shown as the outcomes of strategies in a matrix form are called 'pay-off matrix'.
7. The player playing the game always tries to choose the best course of action which results in optimal pay off called 'optimal strategy'.
8. The expected pay off when all the players of the game follow their optimal strategies is known as 'value of the game'. The main objective of a problem of a game is to find the value of the game.
9. The game is said to be 'fair' game if the value of the game is zero otherwise it s known as 'unfair'.

Characteristics of Game Theory

1. Competitive game

A competitive situation is called a **competitive game** if it has the following four properties

1. There are finite number of competitors such that $n \geq 2$. In case $n = 2$, it is called a **two-person game** and in case $n > 2$, it is referred as **n-person game**.
2. Each player has a list of finite number of possible activities.
3. A play is said to occur when each player chooses one of his activities. The choices are assumed to be made simultaneously i.e. no player knows the choice of the other until he has decided on his own.
4. Every combination of activities determines an outcome which results in a gain of payments to each player, provided each player is playing uncompromisingly to get as much as possible. Negative gain

implies the loss of same amount.

2. Strategy

The strategy of a player is the predetermined rule by which player decides his course of action from his own list during the game. The two types of strategy are

1. Pure strategy
2. Mixed strategy

Pure Strategy

If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action.

Mixed Strategy

If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus the mixed strategy is a selection among pure strategies with fixed probabilities.

3. Number of persons

A game is called 'n' person game if the number of persons playing is 'n'. The person means an individual or a group aiming at a particular objective.

Two-person, zero-sum game

A game with only two players (player A and player B) is called a 'two-person, zero-sum game', if the losses of one player are equivalent to the gains of the other so that the sum of their net gains is zero.

Two-person, zero-sum games are also called rectangular games as these are usually represented by a payoff matrix in a rectangular form.

4. Number of activities

The activities may be finite or infinite.

5. Payoff

The quantitative measure of satisfaction a person gets at the end of each play is called a payoff

6. Payoff matrix

Suppose the player A has 'm' activities and the player B has 'n' activities. Then a payoff matrix can be formed by adopting the following rules

- Row designations for each matrix are the activities available to player A

- Column designations for each matrix are the activities available to player B
- Cell entry V_{ij} is the payment to player A in A's payoff matrix when A chooses the activity i and B chooses the activity j.
- With a zero-sum, two-person game, the cell entry in the player B's payoff matrix will be negative of the corresponding cell entry V_{ij} in the player A's payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

7. Value of the game

Value of the game is the maximum guaranteed game to player A (maximizing player) if both the players uses their best strategies. It is generally denoted by 'V' and it is unique.

Classification of Games

All games are classified into

Pure strategy games
Mixed strategy games

The method for solving these two types varies. By solving a game, we need to find best strategies for both the players and also to find the value of the game.

Pure strategy games can be solved by **saddle point method.**

The different methods for solving a mixed strategy game are

Analytical method
Graphical method
Dominance rule
Simplex method

2.1 Introduction to CPM / PERT Techniques

CPM (Critical Path Method) was developed by Walker to solve project scheduling problems.

PERT (Project Evaluation and Review Technique) was developed by team of engineers working on the polar's missile programme of US navy.

The methods are essentially **network-oriented techniques** using the same principle. PERT and CPM are basically time-oriented methods in the sense that they both lead to determination of a time schedule for the project. The significant difference between two approaches is that the time estimates for the different activities in CPM were assumed to be **deterministic** while in PERT these are described **probabilistically**. These techniques are referred as **project scheduling** techniques.

2.2 Applications of CPM / PERT

These methods have been applied to a wide variety of problems in industries and have found acceptance even in government organizations. These include

- Construction of a dam or a canal system in a region
- Construction of a building or highway
- Maintenance or overhaul of airplanes or oil refinery
- Space flight
- Cost control of a project using PERT / COST
- Designing a prototype of a machine

- Development of supersonic planes

2.3 Basic Steps in PERT / CPM

Project scheduling by PERT / CPM consists of four main steps

1. **Planning**

- The planning phase is started by splitting the total project in to small projects. These smaller projects in turn are divided into activities and are analyzed by the department or section.

- The relationship of each activity with respect to other activities are defined and established and the corresponding responsibilities and the authority are also stated.
- Thus the possibility of overlooking any task necessary for the completion of the project is reduced substantially.

2. **Scheduling**

    - The ultimate objective of the scheduling phase is to prepare a time chart showing the start and finish times for each activity as well as its relationship to other activities of the project.

    - Moreover the schedule must pinpoint the critical path activities which require special attention if the project is to be completed in time.
    - For non-critical activities, the schedule must show the amount of slack or float times which can be used advantageously when such activities are delayed or when limited resources are to be utilized effectively.

3. **Allocation of resources**

    - Allocation of resources is performed to achieve the desired objective. A resource is a physical variable such as labour, finance, equipment and space which will impose a limitation on time for the project.
    - When resources are limited and conflicting, demands are made for the same type of resources a systematic method for allocation of resources become essential.
    - Resource allocation usually incurs a compromise and the choice of this compromise depends on the judgment of managers.

4. **Controlling**

- The final phase in project management is controlling. Critical path methods facilitate the application of the principle of management by expectation to identify areas that are critical to the completion of the project.
- By having progress reports from time to time and updating the network continuously, a better financial as well as technical control over the project is exercised.
- Arrow diagrams and time charts are used for making periodic progress reports. If required, a new course of action is determined for the remaining portion of the project.

2.4 Network Diagram Representation

In a network representation of a project certain definitions are used

1. Activity

Any individual operation which utilizes resources and has an end and a beginning is called activity. An arrow is commonly used to represent an activity with its head indicating the direction of progress in the project. These are classified into four categories

1. **Predecessor activity** – Activities that must be completed immediately prior to the start of another activity are called predecessor activities.
2. **Successor activity** – Activities that cannot be started until one or more of other activities are completed but immediately succeed them are called successor activities.

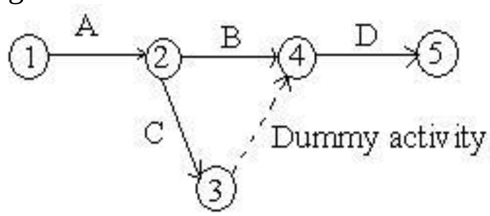
3. **Concurrent activity** – Activities which can be accomplished concurrently are known as concurrent activities. It may be noted that an activity can be a predecessor or a successor to an event or it may be concurrent with one or more of other activities.
4. **Dummy activity** – An activity which does not consume any kind of resource but merely depicts the technological dependence is called a dummy activity.

The dummy activity is inserted in the network to clarify the activity pattern in the following two situations

- To make activities with common starting and finishing points distinguishable

- To identify and maintain the proper precedence relationship between activities that is not connected by events.

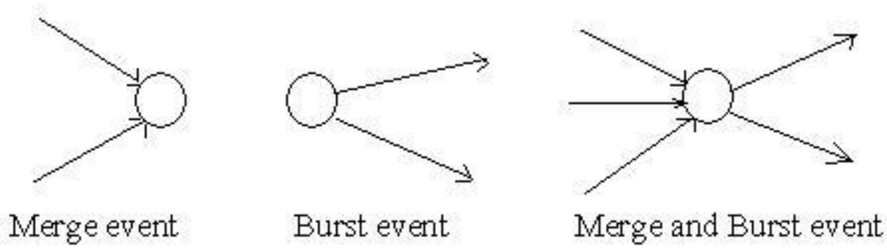
For example, consider a situation where A and B are concurrent activities. C is dependent on A and D is dependent on A and B both. Such a situation can be handled by using a dummy activity as shown in the figure.



2. Event

An event represents a point in time signifying the completion of some activities and the beginning of new ones. This is usually represented by a circle in a network which is also called a node or connector.
The events are classified in to three categories

1. **Merge event** – When more than one activity comes and joins an event such an event is known as merge event.
2. **Burst event** – When more than one activity leaves an event such an event is known as burst event.
3. **Merge and Burst event** – An activity may be merge and burst event at the same time as with respect to some activities it can be a merge event and with respect to some other activities it may be a burst event.



3. Sequencing

The first prerequisite in the development of network is to maintain the precedence relationships. In order to make a network, the following points should be taken into considerations

- What job or jobs precede it?

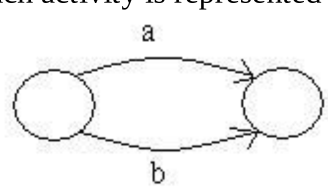
- What job or jobs could run concurrently?
- What job or jobs follow it?
- What controls the start and finish of a job?

Since all further calculations are based on the network, it is necessary that a network be drawn with full care.

2.5 <u>Rules for Drawing Network Diagram</u>

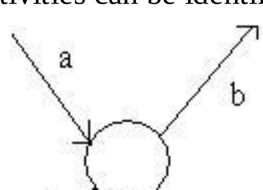
Rule 1

Each activity is represented by one and only one arrow in the network



Rule 2

No two activities can be identified by the same end events



Rule 3

In order to ensure the correct precedence relationship in the arrow diagram, following questions must be checked whenever any activity is added to the network

- What activity must be completed immediately before this activity can start?
- What activities must follow this activity?
- What activities must occur simultaneously with this activity?

In case of large network, it is essential that certain good habits be practiced to draw an easy to follow network

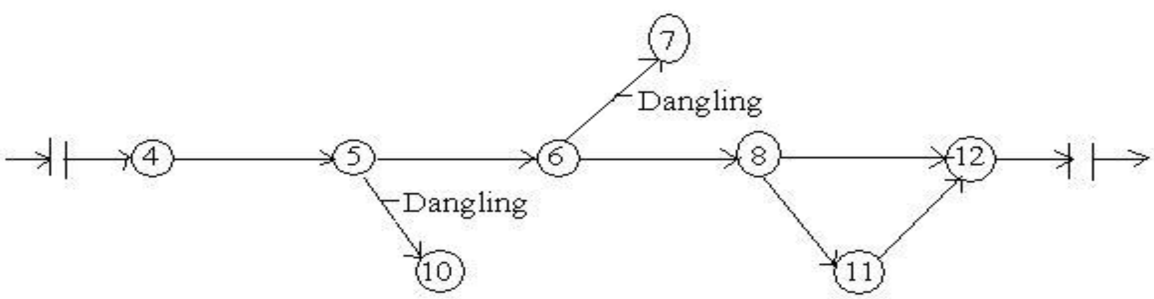
- Try to avoid arrows which cross each other Use straight arrows
- Do not attempt to represent duration of activity by its arrow length
- Use arrows from left to right. Avoid mixing two directions, vertical and standing arrows may be used if necessary.
- Use dummies freely in rough draft but final network should not have any redundant dummies.
- The network has only one entry point called start event and one point of emergence called the end event.

2.6 Common Errors in Drawing Networks

The three types of errors are most commonly observed in drawing network diagrams

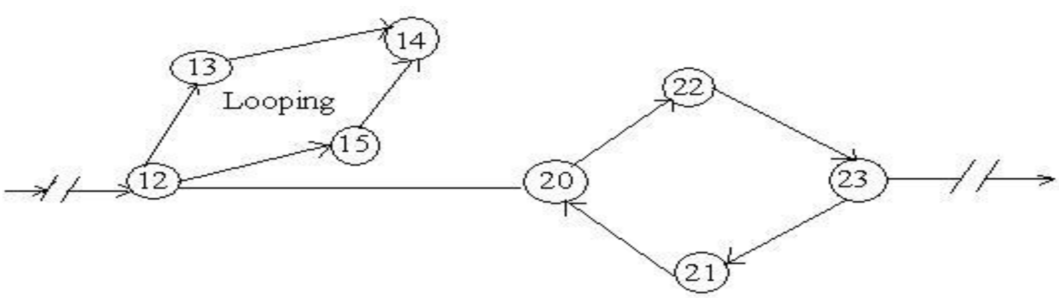
1. Dangling

To disconnect an activity before the completion of all activities in a network diagram is known as dangling. As shown in the figure activities (5 – 10) and (6 – 7) are not the last activities in the network. So the diagram is wrong and indicates the error of dangling



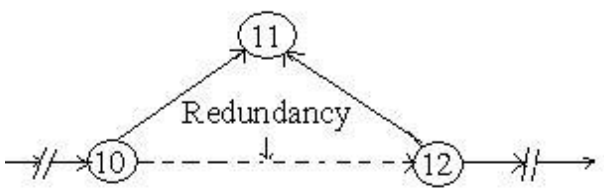
2. Looping or Cycling

Looping error is also known as cycling error in a network diagram. Drawing an endless loop in a network is known as error of looping as shown in the following figure.



3. Redundancy

Unnecessarily inserting the dummy activity in network logic is known as the error of redundancy as shown in the following diagram



<u>Critical Path in Network Analysis</u>

3.1.1 Basic Scheduling Computations

The notations used are

(i, j) = Activity with tail event i and head event j

E_i = Earliest occurrence time of event i

L_j = Latest allowable occurrence time of event j

D_{ij} = Estimated completion time of activity (i, j)

$(Es)_{ij}$ = Earliest starting time of activity (i, j)

$(Ef)_{ij}$ = Earliest finishing time of activity (i, j)

$(Ls)_{ij}$ = Latest starting time of activity (i, j)

$(Lf)_{ij}$ = Latest finishing time of activity (i, j)

The procedure is as follows

1. Determination of Earliest time (E_j): Forward Pass computation

Step 1

The computation begins from the start node and move towards the end node. For easiness, the forward pass computation starts by assuming the earliest occurrence time of zero for the initial project event.

Step 2

i. Earliest starting time of activity (i, j) is the earliest event time of the tail end event i.e. $(Es)_{ij} = E_i$
ii. Earliest finish time of activity (i, j) is the earliest starting time + the activity time i.e. $(Ef)_{ij} = (Es)_{ij} + D_{ij}$ or $(Ef)_{ij} = E_i + D_{ij}$
iii. Earliest event time for event j is the maximum of the earliest finish times of all activities ending in to that

event i.e. E_j = max [$(Ef)_{ij}$ for all immediate predecessor of (i, j)] or E_j =max [$E_i + D_{ij}$]

2. Backward Pass computation (for latest allowable time)

Step 1

For ending event assume E = L. Remember that all E's have been computed by forward pass computations.

Step 2

Latest finish time for activity (i, j) is equal to the latest event time of event j i.e. (Lf)ij = Lj

Step 3

Latest starting time of activity (i, j) = the latest completion time of (i, j) – the activity time or $(Ls)_{ij} =(Lf)_{ij} - D_{ij}$ or $(Ls)_{ij} = L_j - D_{ij}$

Step 4

Latest event time for event 'i' is the minimum of the latest start time of all activities originating from that event i.e. L_i = min [$(Ls)_{ij}$ for all immediate successor of (i, j)] = min [$(Lf)_{ij} - D_{ij}$] = min [$L_j - D_{ij}$]

3. Determination of floats and slack times

There are three kinds of floats

Total float – The amount of time by which the completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time.

Mathematically

$(Tf)_{ij}$ = (Latest start – Earliest start) for activity (i – j)
$(Tf)_{ij} = (Ls)_{ij} - (Es)_{ij}$ or $(Tf)_{ij} = (L_j - D_{ij}) - E_i$

Free float – The time by which the completion of an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent activity. Mathematically
$(Ff)_{ij}$ = (Earliest time for event j – Earliest time for event i) – Activity time for (i, j) $(Ff)_{ij} = (E_j - E_i) - D_{ij}$

Independent float – The amount of time by which the start of an activity can be delayed without effecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time.

Mathematically

$(If)_{ij} = (E_j - L_i) - D_{ij}$

The negative independent float is always taken as zero.

Event slack - It is defined as the difference between the latest event and earliest event times. Mathematically

Head event slack = $L_j - E_j$, Tail event slack = $L_i - E_i$

4. Determination of critical path

Critical event – The events with zero slack times are called critical events. In other words the event i is said to be critical if $E_i = L_i$

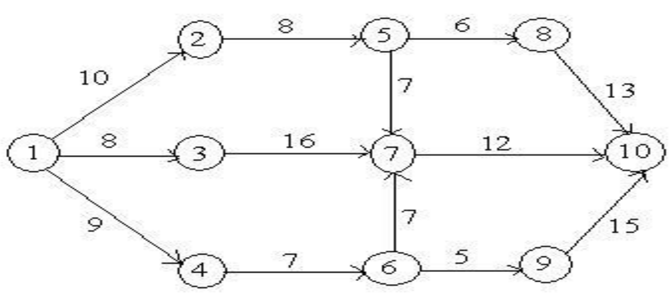
Critical activity – The activities with zero total float are known as critical activities. In other words an activity is said to be critical if a delay in its start will cause a further delay in the completion date of the entire project.

Critical path – The sequence of critical activities in a network is called critical path. The critical path is the longest path in the network from the starting event to ending event and defines the minimum time required to complete the project.

3.2 <u>Worked Examples</u>

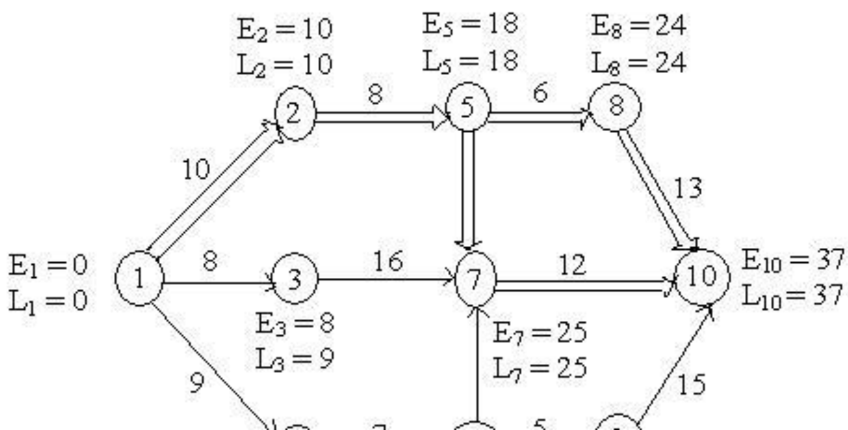
Example 1

Determine the early start and late start in respect of all node points and identify critical path for the following network.



Solution

Calculation of E and L for each node is shown in the network



Network Analysis Table

Activity(i, j)	Normal Time (D_{ij})	Earliest Time		Latest Time		Float Time $(L_i - D_{ij}) - E_i$
		Start (E_i)	Finish ($E_i + D_{ij}$)	Start ($L_i - D_{ij}$)	Finish (L_i)	
(1, 2)	10	0	10	0	10	0
(1, 3)	8	0	8	1	9	1
(1, 4)	9	0	9	1	10	1
(2, 5)	8	10	18	10	18	0
(4, 6)	7	9	16	10	17	1
(3, 7)	16	8	24	9	25	1
(5, 7)	7	18	25	18	25	0
(6, 7)	7	16	23	18	25	2
(5, 8)	6	18	24	18	24	0
(6, 9)	5	16	21	17	22	1
(7, 10)	12	25	37	25	37	0
(8, 10)	13	24	37	24	37	0
(9, 10)	15	21	36	22	37	1

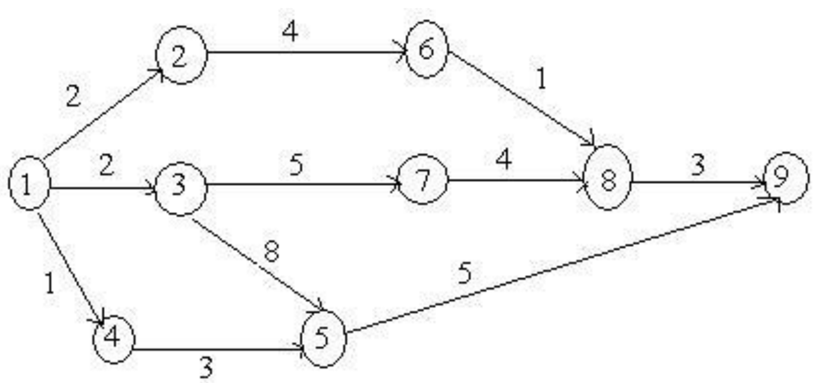
From the table, the critical nodes are (1, 2), (2, 5), (5, 7), (5, 8), (7, 10) and (8, 10)

From the table, there are two possible critical paths

i. $1 \rightarrow 2 \rightarrow 5 \rightarrow 8 \rightarrow 10$
ii. $1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 10$

Example 2

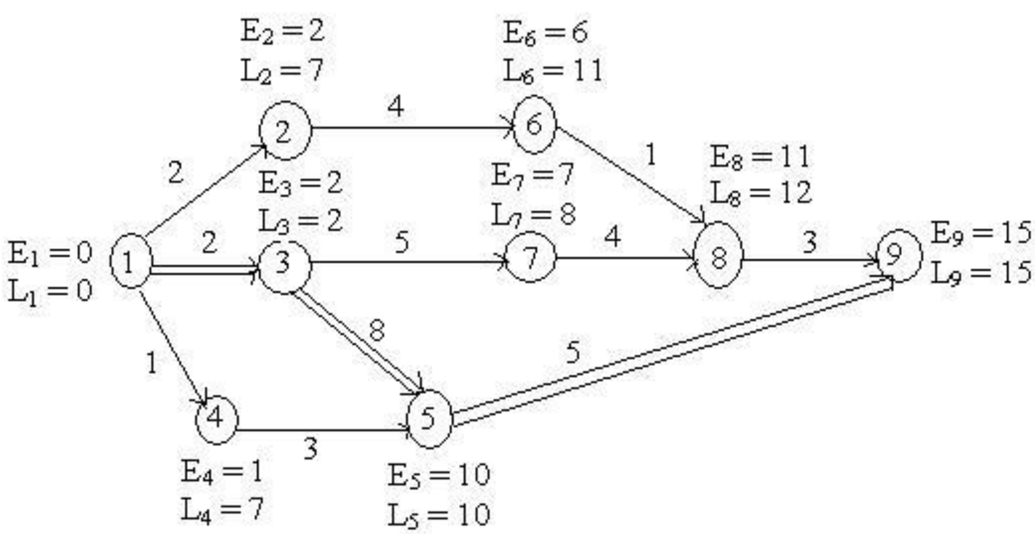
Find the critical path and calculate the slack time for the following network



Solution
The earliest time and the latest time are obtained below

Activity(i, j)	Normal Time (D_{ij})	Earliest Time Start (E_i)	Earliest Time Finish ($E_i + D_{ij}$)	Latest Time Start ($L_i - D_{ij}$)	Latest Time Finish (L_i)	Float Time ($L_i - D_{ij}$) - E_i
(1, 2)	2	0	2	5	7	5
(1, 3)	2	0	2	0	2	0
(1, 4)	1	0	1	6	7	6
(2, 6)	4	2	6	7	11	5
(3, 7)	5	2	7	3	8	1
(3, 5)	8	2	10	2	10	0
(4, 5)	3	1	4	7	10	6
(5, 9)	5	10	15	10	15	0
(6, 8)	1	6	7	11	12	5
(7, 8)	4	7	11	8	12	1
(8, 9)	3	11	14	12	15	1

From the above table, the critical nodes are the activities (1, 3), (3, 5) and (5, 9)



The critical path is 1 → 3 → 5 → 9

Example 3

A project has the following times schedule

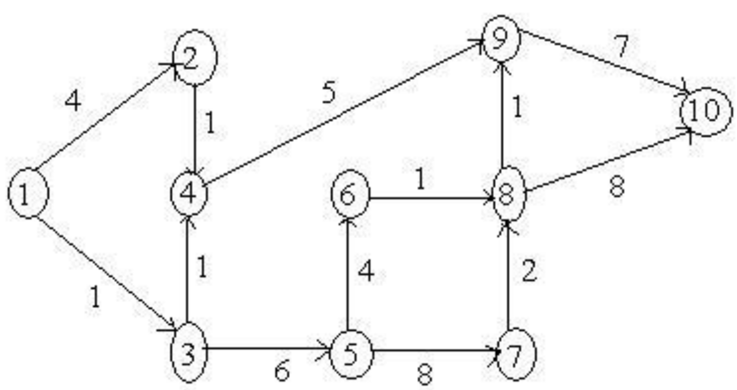
Activity	Times in weeks	Activity	Times in weeks
(1 – 2)	4	(5 – 7)	8
(1 – 3)	1	(6 – 8)	1
(2 – 4)	1	(7 – 8)	2
(3 – 4)	1	(8 – 9)	1
(3 – 5)	6	(8 – 10)	8
(4 – 9)	5	(9 – 10)	7
(5 – 6)	4		

Construct the network and compute

1. T_E and T_L for each event
2. Float for each activity
3. Critical path and its duration

Solution

The network is

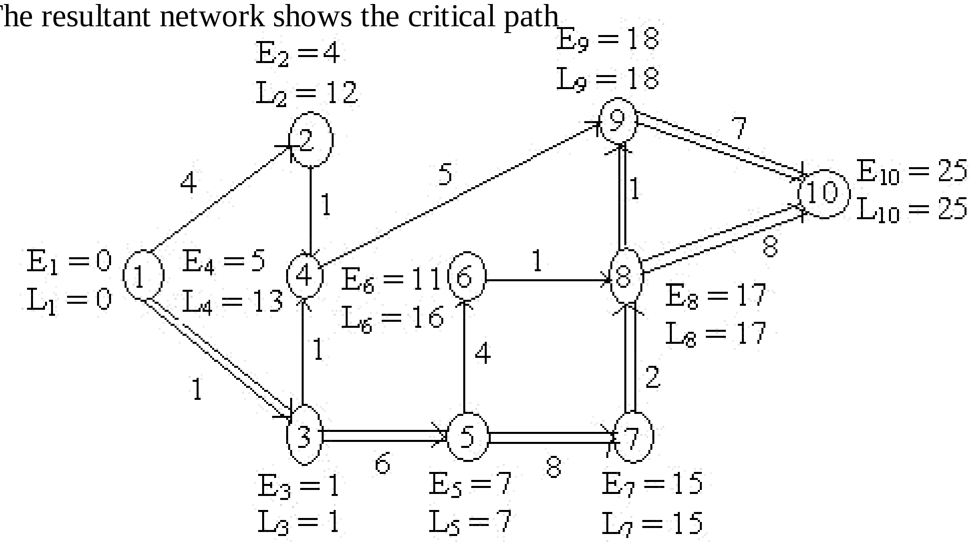


Event No.:	1	2	3	4	5	6	7	8	9	10
T_E:	0	4	1	5	7	11	15	17	18	25
T_L:	0	12	1	13	7	16	15	17	18	25

Float = T_L (Head event) – T_E (Tail event) – Duration

Activity	Duration	T_E (Tail event)	T_L (Head event)	Float
(1 – 2)	4	0	12	8
(1 – 3)	1	0	1	0
(2 – 4)	1	4	13	8
(3 – 4)	1	1	13	11
(3 – 5)	6	1	7	0
(4 – 9)	5	5	18	8
(5 – 6)	4	7	16	5
(5 – 7)	8	7	15	0
(6 – 8)	1	11	17	5
(7 – 8)	2	15	17	0
(8 – 9)	1	17	18	0
(8 – 10)	8	17	25	0
(9 – 10)	7	18	25	0

The resultant network shows the critical path



The two critical paths are

i. $1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 10$

ii. 1 → 3 → 5 → 7 → 8 →10

3.3 Project Evaluation and Review Technique (PERT)

The main objective in the analysis through PERT is to find out the completion for a particular event within specified date. The PERT approach takes into account the uncertainties. The three time values are associated with each activity

1. **Optimistic time** – It is the shortest possible time in which the activity can be finished. It assumes that every thing goes very well. This is denoted by t_0.

2. **Most likely time** – It is the estimate of the normal time the activity would take. This assumes normal delays. If a graph is plotted in the time of completion and the frequency of completion in that time period, then most likely time will represent the highest frequency of occurrence. This is denoted by t_m.

3. **Pessimistic time** – It represents the longest time the activity could take if everything goes wrong. As in optimistic estimate, this value may be such that only one in hundred or one in twenty will take time longer than this value. This is denoted by t_p.

In PERT calculation, all values are used to obtain the percent expected value.

1. **Expected time** – It is the average time an activity will take if it were to be repeated on large number of times and is based on the assumption that the activity time follows Beta distribution, this is given by

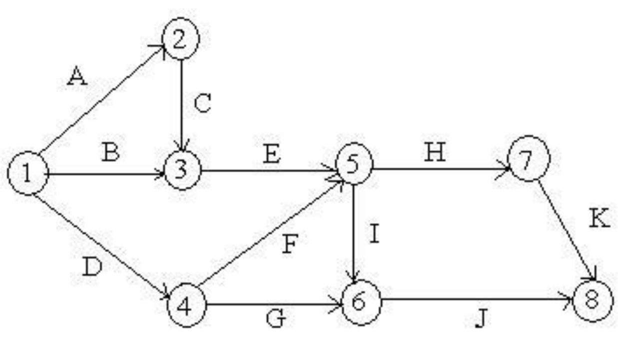
$$t_e = (t_0 + 4\, t_m + t_p) / 6$$

2. The **variance** for the activity is given by $\sigma^2 = [(t_p – t_o) / 6]^2$

3.4 Worked Examples

Example 1

For the project



Task:	A	B	C	D	E	F	G	H	I	J	K
Least time:	4	5	8	2	4	6	8	5	3	5	6
Greatest time:	8	10	12	7	10	15	16	9	7	11	13
Most likely time:	5	7	11	3	7	9	12	6	5	8	9

Find the earliest and latest expected time to each event and also critical path in the network.

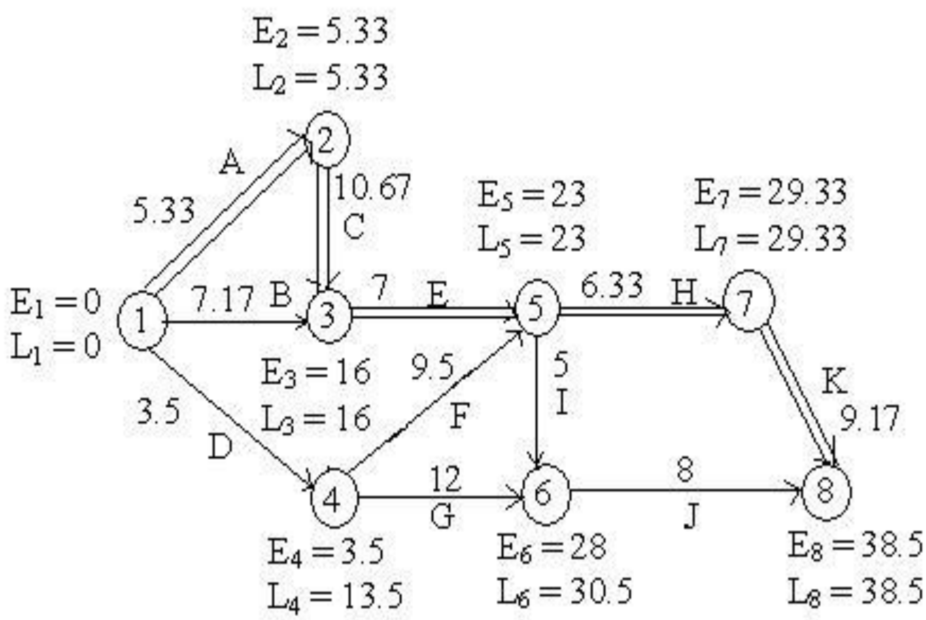
Solution

Task	Least time(t_0)	Greatest time (t_p)	Most likely time (t_m)	Expected time ($to + t_p + 4t_m$)/6
A	4	8	5	5.33
B	5	10	7	7.17
C	8	12	11	10.67
D	2	7	3	3.5
E	4	10	7	7
F	6	15	9	9.5
G	8	16	12	12
H	5	9	6	6.33
I	3	7	5	5
J	5	11	8	8
K	6	13	9	9.17

Task	Expected time (t_e)	Start		Finish		Total float
		Earliest	Latest	Earliest	Latest	
A	5.33	0	0	5.33	5.33	0
B	7.17	0	8.83	7.17	16	8.83
C	10.67	5.33	5.33	16	16	0
D	3.5	0	10	3.5	13.5	10
E	7	16	16	23	23	0
F	9.5	3.5	13.5	13	23	10
G	12	3.5	18.5	15.5	30.5	15

H	6.33	23	23	29.33	29.33	0
I	5	23	25.5	28	30.5	2.5
J	8	28	30.5	36	38.5	2.5
K	9.17	29.33	29.33	31.5	38.5	0

The network is



The critical path is A →C →E → H → K

Example 2

A project has the following characteristics

Activity	Most optimistic time (a)	Most pessimistic time (b)	Most likely time (m)
(1 – 2)	1	5	1.5
(2 – 3)	1	3	2
(2 – 4)	1	5	3
(3 – 5)	3	5	4
(4 – 5)	2	4	3
(4 – 6)	3	7	5
(5 – 7)	4	6	5
(6 – 7)	6	8	7
(7 – 8)	2	6	4
(7 – 9)	5	8	6

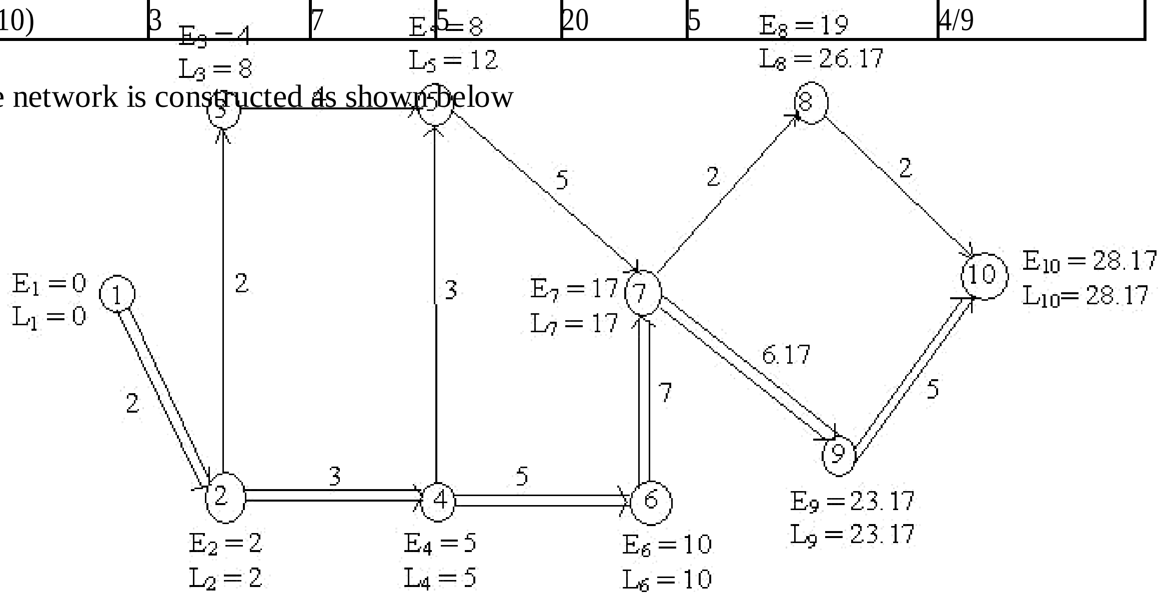
(8 – 10)	1	3	2
(9 – 10)	3	7	5

Construct a PERT network. Find the critical path and variance for each event.

Solution

Activity	(a)	(b)	(m)	(4m)	t_e (a + b + 4m)/6	v $[(b – a) / 6]^2$
(1 – 2)	1	5	1.5	6	2	4/9
(2 – 3)	1	3	2	8	2	1/9
(2 – 4)	1	5	3	12	3	4/9
(3 – 5)	3	5	4	16	4	1/9
(4 – 5)	2	4	3	12	3	1/9
(4 – 6)	3	7	5	20	5	4/9
(5 – 7)	4	6	5	20	5	1/9
(6 – 7)	6	8	7	28	7	1/9
(7 – 8)	2	6	4	16	4	4/9
(7 – 9)	5	8	6	24	6.17	1/4
(8 – 10)	1	3	2	8	2	1/9
(9 – 10)	3	7	5	20	5	4/9

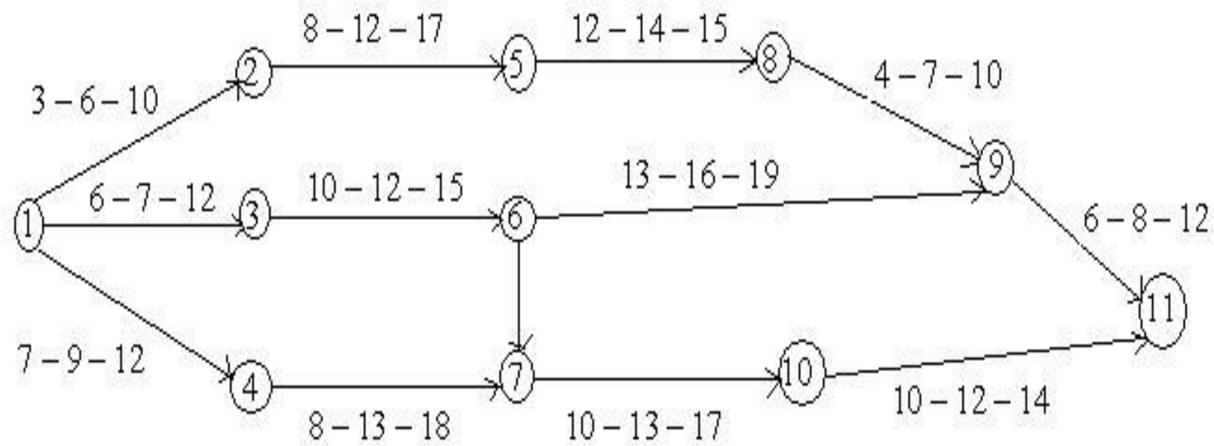
The network is constructed as shown below



The critical path = 1 → 2 → 4 → 6 → 7 →9 →10

Example 3

Calculate the variance and the expected time for each activity



Solution

Activity	(t_o)	(t_m)	(t_p)	te $(t_o + t_p + 4t_m)/6$	v $[(t_p – t_o) / 6]^2$
(1 – 2)	3	6	10	6.2	1.36
(1 – 3)	6	7	12	7.7	1.00
(1 – 4)	7	9	12	9.2	0.69
(2 – 3)	0	0	0	0.0	0.00
(2 – 5)	8	12	17	12.2	2.25
(3 – 6)	10	12	15	12.2	0.69
(4 – 7)	8	13	19	13.2	3.36
(5 – 8)	12	14	15	13.9	0.25
(6 – 7)	8	9	10	9.0	0.11
(6 – 9)	13	16	19	16.0	1.00
(8 – 9)	4	7	10	7.0	1.00
(7 – 10)	10	13	17	13.2	1.36
(9 – 11)	6	8	12	8.4	1.00
(10 – 11)	10	12	14	12.0	0.66

MCQ

Question	**opt 1**	**opt 2**	**opt 3**	**opt 4**	**answer**
Network analysis is applied projects which are	Routine or repetitive	not routine or repetitive	Used only once	used very rarely	not routine or repetitive
Network analysis is a technique to minimize some performance measures of the system such as	Total completion time of the project	overall cost of the project	The resources used for the project	all of the above	all of the above
The two major aspects of the critical path method are	Planning and scheduling	planning and implement	Planning and organization	organization and scheduling	Planning and scheduling
In the critical path method preparing time estimates for each activity comes under the aspects of	Planning	organizing	scheduling	estimating	scheduling
In CPM/PERT the critical path is the path having the	Minimum time duration	normal time duration	Most difficult to achieve time duration	none of the above	none of the above
In PERT it is assumed that the time required performing each activity	Known precisely	known approximately	known in a probabilistic sense	uncertain	uncertain
Which of the following is a time consuming effect which is necessary to complete a particular part of the project?	Event	activity	node	path	activity
Which of the following represents the point of start is a point of the completion of an activity	Node	burst	**event**	path	**event**
Which of the following in network diagram does not consume any time?	Activity	event	bottle neck activity	non critical path	event
In PERT terminology each event in a network diagram is represented by	Circle	an arrow line	a double line between two nodes	square	Circle

In CPM terminology which of the following is represented by a circle in a network diagram	An event	a path	an activity	a node	an activity
In a network diagram, an event which is beginning events for 2 or more activities is called	A node	a burst	a cluster	a group	A node
In a network diagram an event where 2 or more activities end is called	A node	a burst	a cluster	a group	a burst
In the network diagram, R.Fulkereson's rule is used to	To identify the critical activities	number the activities	to number the events	to calculate the cost slope of activities	to number the events
Which of the following indicates the allocation of ideal time and ideal resources for an activity	positive slack	negative slack	zero slack	negative float	positive slack

2 marks

1. What is meant by critical path?
2. Explain the term value of the game?
3. Define total float of an activity.
4. What are the rules for constructing a network diagram?
5. State the rule of dominance.
6. Explain the method of constructing a network diagram?
7. What are the three phases of a project?
8. What is meant by critical path?
9. What is group replacement?
10. Explain the method of constructing a network diagram?
11. What are the three time estimates involved in PERT?
12. What is salvage cost?
13. State the rule of dominance.
14. Distinguish between CPM and PERT
15. What is group replacement?
16. What is saddle point?

14 marks

1. Three time estimates (in months) of all activities of a project are as given below:

Jobs i,j	Duration		
	Optimistic	Most likely	Pessimistic
1-2	3	6	15
1-6	2	5	14
2-3	6	12	30
2-4	2	5	8
3-5	5	11	17
4-5	3	6	15
6-7	3	9	27
5-8	1	4	7
7-8	4	10	28

i) Find the expected duration and standard deviation of each activity.
ii) Construct the project network.
iii) Determine the critical path, expected project length and expected variance of the project length.

What is probability that the project will be completed two months later than expected, not more than 3 months earlier than expected, what due date has about 90% chance of being met?

2. The cost of the machine is Rs.6100/- and its scrap value is Rs.100/-. The maintenance costs found from experience are as follows:

Year	1	2	3	4	5	6	7	8
Maintenance Cost(Rs.)	100	250	400	600	900	1200	1600	2000

When should the machine be replaced?

3. A Project consists of the following activities and time estimates:

Activity	Least Time (Days)	Greatest Time (Days)	Most Likely Time (Days)
1 -- 2	3	15	6
2 -- 3	2	14	5
1 -- 4	6	30	12
2 -- 5	2	8	5
2 -- 6	5	17	11
3 -- 6	3	15	6
4 -- 7	3	27	9
5 --7	1	7	4
6 --7	2	8	5

i) Draw the network.

ii) What is the probability that the project will be completed in 27 days?

4. Using Dominance property to solve

	B			
	1	7	3	4
A	5	6	4	5
	7	2	0	3

5. A Project consists of the following activities and time estimates:

Activity	Least Time (Days)	Greatest Time (Days)	Most Likely Time (Days)
1 -- 2	3	15	6
2 -- 3	2	14	5
1 -- 4	6	30	12
2 -- 5	2	8	5
2 -- 6	5	17	11
3 -- 6	3	15	6
4 -- 7	3	27	9
5 --7	1	7	4
6 --7	2	8	5

Draw the network.
What is the probability that the project will be completed in 27 days?

6. Calculate the total float, free float and independent float for the projects whose activities are given below and determine the critical path of the project

Activity	1 -- 2	1 -- 3	1 -- 5	2 -- 3	2 -- 4
Duration (in weeks)	8	7	12	4	10
Activity	3 -- 4	3 -- 5	3 -- 6	4 -- 6	5 -- 6
Duration (in weeks)	3	5	10	7	4

7. The following table indicates the details of the project. The durations are in days. ‘a’ refers to optimistic time, ‘m’ refers to most likely time and ‘b’ refers to pessimistic time duration.

Activity	1—2	1 — 3	1—4	2—4	2—5	3—5	4—5
a	2	3	4	8	6	2	2
m	4	4	5	9	8	3	5
b	5	6	6	11	12	4	7

i) Draw the network
ii) Find the critical path
iii) Determine the expected standard deviation of the completion

8. The following failure rates have been observed for certain items.

End of the month	1	2	3	4	5
Probability of failure to date	0.10	0.30	0.55	0.85	1.00

The cost of replacing an individual item is Rs.1.25. The decision is made to replace all items simultaneously at fixed intervals and also replace individual items as they fail. If the cost of group replacement is 50 paise, what is the best interval for group replacement? At what group replacement become preferable to the adopted policy.

9. Using Dominance property to solve

		B			
		I	II	III	IV
	1	-	3	1	20
A	2	5	5	4	6
	3	-	-	0	-5

10. Three time estimates (in months) of all activities of a project are as given below:

	Time in Months		
Activity	a	m	b
1 -- 2	0.8	1.0	1.2
2 -- 3	3.7	5.6	9.9
2 -- 4	6.2	6.6	15.4
3 -- 4	2.1	2.7	6.1
4 -- 5	0.8	3.4	3.6

5 -- 6	0.9	1.0	1.1

i) Find the expected duration and standard deviation of each activity.
ii) Construct the project network.
iii) Determine the critical path, expected project length and expected variance of the project length.

What is probability that the project will be completed two months later than expected, not more than 3 months earlier than expected, what due date has about 90% chance of being met?