KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act, 1956)

Coimbatore – 641021

FACULTY OF ENGINEERING

DEPARTMENT OF AUTOMOBILE ENGINEERING

CLASS : **III YEAR B.E. AUTOMOBILE ENGINEERING**

SEMESTER : **6**

COURSE CODE : **17BEAE602**

COURSE NAME : **INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH**

17BEAE602 INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH 3 0 0 3 100

INTENDED OBJECTIVES:

- To create awareness about optimization in utilization of resources.
- To understand and apply operations research techniques to industrial operations.

UNIT I INTRODUCTION

Evolution of industrial engineering, fields and functions of industrial engineering. Methods engineering process charts, motion study, work sampling and work measurement.

UNIT II PRODUCTION PLANNING AND CONTROL

Introduction, objectives, components of PPC, manufacturing systems, plant layout, types of layouts, forecasting, product planning, loading and scheduling, dispatching, production control, material handling principles, case studies.

Human engineering- Ergonomics, design of controls and displays, heating, ventilation, glare, airflow, influence of factory environment on productivity, industrial safety.

Cost analysis - Cost structure of a product-labor, material, overhead. Overhead absorption, machine hour rate, cost computation for simple machined components, learning curve, 'Make-or-Buy' decision.

UNITIII LINEAR PROGRAMMING TECHNIQUES

Operations research and decision-making, types of mathematical models and constructing the model. Role of computers in operations research, formulation of linear programming problem, applications and limitations, simplex method, variants in simplex method (analytical and graphical).

UNIT IV DISTRIBUTION METHODS AND ASSIGNMENT MODELS

Vogel's approximation method, modified distribution method, optimization models, unbalance and degeneracy in transportation model. Hungarian algorithm, traveling salesman problem, routing problems, processing 'n' jobs through two machines and three machines, processing two jobs through 'm' machines.

UNIT V INVENTORY CONTROL

Variables in inventory problems, inventory models with penalty, storage and quantity discount, safety stock, inventory models with probability, lead time, demand, multi item deterministic model.

TEXT BOOKS

SL.NO	AUTHOR(S)	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1.	Frederick S.Hillier and Gerald J.Lieberman	Introduction to Operations Research	Tata McGraw Hill Publishing Company Ltd., New Delhi	2006
2.	Chase R.B, Jacob F.R.E.D and Aquilano N.J	Operations Management for Competitive Advantage, 10th Edition	Tata McGraw Hill, New Delhi	2004
3.	Elwood S. Buffa	Modern Production /Operations Management 8th Edition	Wiley Eastern, New Delhi.	2007
4.	KantiSwarup Gupta P.K and Manmohan	Operations Research	Sultan Chand and Sons, New Delhi.	1995

REFERENCES

SL.NO.	AUTHOR(S)	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1.	Srinath.C	PERT and CPM – Principles and Applications 3rd Edition	East West Press, New Delhi	2001
2.	Dharani Venkatakrishnan.S	Operations Research	Keerthi Publication House, Coimbatore	1991
3.	Kannappan.D, Par anthaman.D, Augu stine.A.G	Mechanical Estimating and Costing	Tata mcGraw Hill, (New Delhi :)	2003
4.	Saravanan.R	Manufacturing optimization through intelligent techniques	CRC Press, Florida.	2006
5.	Gupta.P.K and Hira.D.S	Operations Research	S. Chand & Co, New Delhi.	2012
6.	Panneerselvam.R	Production and Operations Management, 2nd Edition	Prentice Hall of India (P) Ltd.	2007

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Pollachi Main Road, Eachanari Post, Coimbatore – 641 021, Tamilnadu, India.

FACULTY OF ENGINEERING

DEPARTMENT OF AUTOMOBILE ENGINEERING

LECTURE PLAN

Subject Name : **Industrial Engineering and Operations Research**
Subject Code : **17BEAE602 (Credits - 3)**
Name of the Faculty : **Sathish Kumar K**
Designation : **Assistant Professor**
Year / Semester : **III / 6**
Branch : **Automobile Engineering**

UNIT-I: INTRODUCTION			
Sl. No.	**No. of Periods**	**Topics to be Covered**	**Support Materials**
1.	1	Evolution of industrial engineering	T[1] 3–4
2.	1	Fields and functions of industrial engineering	T[1] 5–6
3.	1	Methods, engineering process charts	R[2] 499–507
4.	1	Motion study	T[1] 47–52 W[1] M1L6
5.	2	Work sampling and work measurement	R[2] 514–515, 509–510 W[1] M1L10, M1L8
6.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-I			7

UNIT-II: PRODUCTION PLANNING AND CONTROL			
Sl. No.	**No. of Periods**	**Topics to be Covered**	**Support Materials**
7.	1	Introduction, objectives, components of PPC	R[2] 573, 374–376
8.	1	Manufacturing systems, plant layout, types of layouts	R[2] 159–162
9.	1	Forecasting, product planning, loading and scheduling	R[2] 91–109, 376–377
10.	1	Dispatching, production control	T[1] 320–321, 300–301

11.	1	Material handling principles, case studies	T[1] 129
12.	1	Human engineering - Ergonomics, design of controls and displays, heating, ventilation, glare, airflow	T[1] 157–166
13.	1	Influence of factory environment on productivity, industrial safety	T[1] 165
14.	1	Cost analysis - Cost structure of a product - labor, material, overhead	T[1] 355–360
15.	1	Overhead absorption, machine hour rate, cost computation for simple machined components	T[1] 360–362
16.	1	Learning curve, ‘Make-or-Buy’ decision	R[2] 43–48
17.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-II			11

UNIT-III: LINEAR PROGRAMMING TECHNIQUES			
Sl. No.	**No. of Periods**	**Topics to be Covered**	**Support Materials**
18.	1	Operations research and decision-making	R[1] 11–12
19.	1	Types of mathematical models and constructing the model	R[1] 20–25
20.	1	Role of computers in operations research	R[1] 34
21.	1	Formulation of linear programming problem	T[1] 396, R[1] 43
22.	1	Applications and limitations	R[1] 41–43, 108–109
23.	1	Simplex method	T[1] 401–402, R[1] 158–163
24.	2	Variants in simplex method (analytical and graphical)	R[1] 172–177, 183–187 W[2] W2
25.	2	Tutorial: Problems in linear programming techniques	
26.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-III			11

UNIT-IV: DISTRIBUTION METHODS AND ASSIGNMENT MODELS			
Sl. No.	**No. of Periods**	**Topics to be Covered**	**Support Materials**
27.	1	Vogel’s approximation method	R[1] 242–245
28.	1	Modified distribution method	R[1] 246–248 W[2] W6
29.	1	Optimization models, unbalance and degeneracy in transportation model	R[1] 252–253
30.	1	Hungarian algorithm	T[1] 312–313, R[1] 334–336
31.	1	Traveling salesman problem	R[1] 392–393

32.	1	Routing problems	R[1] 392–393
33.	1	Processing ‘n’ jobs through two machines and three machines	T[1] 304–305, 307, R[1] 419–420, 426
34.	1	Processing two jobs through ‘m’ machines	R[1] 431
35.	2	Tutorial: Problems in distribution methods and assignment models	
36.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-IV			11

UNIT-V: INVENTORY CONTROL			
Sl. No.	**No. of Periods**	**Topics to be Covered**	**Support Materials**
37.	1	Variables in inventory problems	R[1] 1038–1039
38.	3	Inventory models with penalty, shortage and quantity discount	R[1] 1041–1044, R[2] 256–259, 261–262
39.	1	Safety stock, inventory models with probability	T[1] 328–335, R[1] 1078
40.	1	Lead time demand	R[1] 1086
41.	1	Multi item deterministic model	R[1] 1095–1097
42.	2	Tutorial: Problems in inventory control	
43.	1	Discussion on competitive examination related questions / KAHE previous year questions	
Total no. of hours planned for Unit-V			10

TOTAL PERIODS : 50

TEXT BOOKS:

T[1]. Martand Telsang, “Industrial Engineering and Production Management”, S. Chand, New Delhi, 2017.

REFERENCE BOOKS:

R[1]. P. K. Gupta and D.S. Hira, “Operations Research”, S. Chand and Company Limited, New Delhi, 2012.

R[2]. R. Panneerselvam, "Production and Operations Management", Prentice-Hall of India Private Limited, 2007.

WEBSITES:

W[1]. https://nptel.ac.in/courses/112107142/

W[2]. https://nptel.ac.in/courses/110106062/

UNIT	Lecture Periods (A)	Tutorial Periods (B)	Total No. of Periods Planned (A+B)
I	7	0	7
II	11	0	11
III	9	2	11
IV	9	2	11
V	8	2	10
TOTAL	**44**	**6**	**50**

SCHEME OF EXAMINATION:

I. CONTINUOUS INTERNAL ASSESSMENT : 40 marks

Internal Assessment Tests : 25 marks
Attendance : 5 marks
Seminar : 5 marks

II. END SEMESTER EXAMINATION : 60 marks

TOTAL : 100 marks

Signature of Faculty **Signature of HOD** **Signature of Dean**

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FACULTY OF ENGINEERING

DEPARTMENT OF AUTOMOBILE ENGINEERING

INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH

UNIT-I

INTRODUCTION

CONTENTS

1 INTRODUCTION TO INDUSTRIAL ENGINEERING

1.1 DEFINITION

American Institute of Industrial Engineers (AIIE) defines Industrial Engineering as follows:

Industrial Engineering is concerned with the design, improvement and installation of an integrated system of men, materials and equipment. It draws upon specialised knowledge and skills in the mathematical, physical sciences together with the principles and methods of engineering analysis and design to specify, predict and evaluate the results to be obtained from such systems.

The prime objective of industrial engineering is to increase productivity by eliminating waste and non-value adding (unproductive) operations and improving the effective utilisation of resources.

1.2 HISTORY AND DEVELOPMENT OF INDUSTRIAL ENGINEERING

History of industrial engineering dates back to the industrial revolution and it has passed through various phases to reach the present advanced and developed stage. Though Frederick Taylor is named as the father of scientific management and Industrial Engineering, there are many others who contributed to the industrial engineering field before Taylor and then they got associated with industrial engineering.

Adam Smith's concept of Division of labour through his book *The Wealth of the Nation* in 1776 is important as it influenced the factory system.

James Watt, Arkwright, Bolton Matthew and Robinson obtained a place in the history of industrial engineering because of their progressive and scientific attitude towards the improvements in the performance of machines and industries.

The period between 1882–1912 was the critical period in the history of industrial engineering.

The important works during this period are:

1. Factory system and owner, engineer and manager concept.
2. Equal work, equal pay and incentive schemes.
3. Scheduling and Gantt charts.
4. Engineers started taking interest in cost control and accounting. The most often quoted and acknowledged investigator that has lead to being a discipline of industrial engineering in present form was F.W. Taylor, who took interest in human aspects of production and productivity.

The modem industrial engineering techniques had their origin during the period between 1940 to 1946. Predetermined time standards (PMTS), value analysis and system analysis are a few prominent ones. They were expanded, refined and applied in subsequent years. Operation Research technique has brought a revolution and changed and expanded the scope of industrial engineering activities. The computers have added dimension to the industrial engineering activities.

Present State of Industrial Engineering

Industrial engineering has not remained restricted to manufacturing activities but has extended its services to service industries also. The development of techniques like

- Value Engineering,

- Operation Research,
- CPM and PERT,
- Human Engineering (Ergonomics).
- Systems Analysis,
- Advances in Information Technology and Computer Packages, and
- Mathematical and Statistical Tools.

have expanded the scope of activities of industrial engineering. Thus industrial engineering has taken a firm position in the organisation and it is contributing maximum towards increasing productivity and efficiency in particular and Quality of Work Life (QWL) in general.

1.3 OBJECTIVES OF INDUSTRIAL ENGINEERING

The basic objectives of the industrial engineering department are:

1. To establish methods for improving the operations and controlling the production costs, and
2. To develop programmes for reducing those costs.

Industrial engineering department exists primarily to provide specialised services to production departments. The services offered depend on the type of organisation. Normally the services include such functions as method study, establishing time standards, development of wage-incentive schemes, job evaluation and merit rating. In some cases, industrial engineering department is assigned to head projects.

1.4 FUNCTIONS OF AN INDUSTRIAL ENGINEER

- Developing the simplest work methods and establishing one best way of doing the work. (Standard Method)
- Establishing performance standards as per the standard methods. (Standard Time)
- To develop a sound wage and incentive schemes.
- To aid in the development and designing of sound inventory control, determination of economic lot size and work-in-process for each stage of production.
- To assist and aid in preparing a detailed job description, and job specification for each job and to evaluate them.
- Development of cost reduction and cost control programmes, and to establish a standard costing system.
- Sound selection of site and developing a systematic layout for the smooth flow of work without any interruptions.
- Development of standard training programmes for various levels of organisation for effective implementation of various improvement programmes.

1.5 TECHNIQUES OF INDUSTRIAL ENGINEERING

The tools and techniques of industrial engineering aim at improving the productivity of the organisation by optimum utilisation of organisation's resources, i.e., men, materials and machines. The various tools and techniques of industrial engineering are:

Method study: To establish a standard method of performing a job or an operation after a thorough analysis of the jobs and to establish the layout of production facilities to have a uniform flow of material without backtracking.

Time study (Work measurement): This is a technique used to establish a standard time for a job or for an operation.

Motion economy: This is used to analyse the motions employed by the operators to do the work. The principles of motion economy and motion analysis are very useful in mass production or for short cycle repetitive jobs.

Financial and non-financial incentives: These helps to evolve at a rational compensation for the efforts of the workers.

Value analysis: It ensures that no unnecessary costs are built into the product and it tries to provide the required functions at the minimum cost. Hence, helps to enhance the worth of the product.

Production, planning and control: This includes the planning for the resources (like men, materials and machines), proper scheduling and controlling production activities to ensure the right quantity, quality of product at the predetermined time and pre-established cost.

Inventory control: To find the economic lot size and the reorder levels for the items so that the item should be made available to the production at the right time and quantity to avoid stock out situation and with minimum capital lock-up.

Job evaluation: This is a technique which is used to determine the relative worth of jobs of the organisation to aid in matching jobs and personnel and to arrive at sound wage policy.

Material handling analysis: To scientifically analyse the movement of materials through various departments to eliminate unnecessary movement to enhance the efficiency of material handling.

Ergonomics (Human engineering): It is concerned with the study of the relationship between man and his working conditions to minimise mental and physical stress. It is concerned with the man-machine system.

System analysis: It is the study of various sub-systems and elements that make a system, their interdependencies in order to design, modify and improve them to achieve greater efficiency and effectiveness.

Operation research techniques: These techniques aid to arrive at the optimal solutions to the problems based on the set objective and constraints imposed on the problems. The techniques that are more often used are:

- Linear programming problems,
- Simulation models,
- Queuing models,
- Network analysis (CPM and PERT),
- Assignment, sequencing and transportation models,
- Dynamic and integer programming, and
- Games theory.

The other techniques: It includes statistical process control techniques, group technology. organisation and methods (O & M).

2 WORK-STUDY

2.1 INTRODUCTION

Work-study deals with the techniques of method study and work measurement, which are employed to ensure the best possible use of the human, machine and material resources in carrying out a specified activity.

2.1.1 OBJECTIVE

Work-study is concerned with finding better ways of doing work and avoiding waste in all its forms. As such the objective of work-study is to assist management to obtain the optimum use of the human, machine and material resources available to the organization for the accomplishment of the work upon which it is engaged.

The objective has three aspects:

- The most effective use of plant and equipment.
- The most effective use of human effort.
- The evaluation of human work.

Work-study has two broad areas viz., method study and time study.

Method Study is concerned with finding the facts about a situation and after a critical examination of these facts, developing a new and better method of doing that work. It is defined as the existing and proposed ways of doing work and the development and application of easier and more productive methods.

Time Study is concerned with the establishment of time standards for a qualified worker to perform a specified job at a defined level of performance.

Method study must precede time study before any attempt is made to measure and set standards for the various jobs concerned.

2.2 METHOD STUDY

It is the systematic recording, analysis and critical examination of existing and proposed ways of doing work and the development and application of easier and new production methods.

2.2.1 OBJECTIVES OF METHOD STUDY

Method study is essentially concerned with finding better ways of doing things. It adds value and increases efficiency by eliminating unnecessary operations, avoidable delays and other forms of waste.

The improvement in efficiency is achieved through:

- Improved layout and design of workplace.
- Improved and efficient work procedures.
- Effective utilisation of men, machines and materials.
- Improved design or specification of the final product.

The objectives of method study techniques are:

(i) To present and analyse true facts concerning the situation.
(ii) To examine those facts critically.
(iii) To develop the best answer possible under given circumstances based on critical examination of facts.

2.2.2 AREAS OF APPLICATION OF METHOD STUDY

It can be applied to any field of work, but the most important areas where it plays a major role in improving productivity are as follows.

- Improved layout of office, working areas of factories
- Improved design of plant and equipment
- Improved use of materials, plant, equipment and manpower
- Most effective handling of materials
- Improved flow of work
- Standardization of methods and procedures
- Improved safety standards
- Better working conditions

2.2.3 STEPS IN METHOD STUDY

Methods improvement involves a systematic, orderly and scientific approach to problems. One should have an open mind, maintain a questioning attitude, collect all relevant facts, consult others including workers, list reasons/causes for various effects.

The decision must be taken after listing out all alternatives and evaluating them critically.

Based on these guidelines, the steps in the method study are explained below:

1. *Select:* Select the work to be studied.
2. *Record:* Record all the relevant facts of the present (or proposed) method by direct observation.
3. *Examine:* Examine the facts critically in sequence, using a special critical examination sheet.
4. *Develop:* Develop the best method i.e. the most practical, economical and effective method, under prevailing circumstances.
5. *Install:* Install that method as standard practice.
6. *Maintain:* Maintain that standard practice by regular routine check.

2.2.3.1 SELECT

Cost is the main criteria for selection of a job, process, or department for methods analysis. To carry out the method study, a job is selected such that the proposed method achieves one or more of the following results:

- Improvement in quality with lesser scrap.
- Increased production through better utilisation of resources.
- Elimination of unnecessary operations and movements.
- Improved layout leading to a smooth flow of material and a balanced production line.
- Improved working conditions.

The job should be selected for the method study based upon the following considerations:

2.2.3.1.1 ECONOMIC ASPECT

The method study involves cost and time. If sufficient returns are not attained, the whole exercise will go waste. Thus the money spent should be justified by the savings from it. The following guidelines can be used for selecting a job:

- Bottleneck operations which are holding up other production operations.
- Operations involving excessive labour.
- Operations producing a lot of scrap or defectives.
- Operations having poor utilisation of resources.
- Backtracking of materials and excessive movement of materials.

2.2.3.1.2 TECHNICAL ASPECTS

The method study man should be careful enough to select a job in which he has the technical knowledge and expertise. A person selecting a job in his area of expertise is going to do full justice. Other factors which favour selection in technical aspect are:

- Job having inconsistent quality.
- Operations generating a lot of scraps.
- Frequent complaints from workers regarding the job.

2.2.3.1.3 HUMAN CONSIDERATIONS

Method study means a change as it is going to affect the way in which the job is done presently and is not fully accepted by the workman and the union. Human consideration plays a vital role in method study. These are some of the situations where human aspect should be given due importance:

- Workers complaining about unnecessary and tiring work.
- More frequency of accidents.
- Inconsistent earnings.

2.2.3.2 RECORD

In order to carry out any investigation, data or relevant facts pertaining to the existing method must be collected and recorded. There are a number of recording techniques developed to simplify and standardize the work.

The recording may trace the movements of men, material or details of various processes. The principle is to use the simplest technique which will contain all relevant information needed for an investigation.

The different recording techniques are charts, diagrams, models and photographic aids. The most commonly used recording techniques to cover most of the activities are shown in the following table.

The recording techniques are designed to simplify and standardise the recording work.

2.2.3.2.1 RECORDING TECHNIQUES

Recording Technique	Information Recorded
(a) Charts	
1. Outline process chart	Principle operations and inspection of the processes.
2. Flow process chart	Activities of men, material or equipment are analyzed into five events viz., operation, transport, inspection, delay and storage.
3. Two-handed process chart	Movement of two hands or limbs of the operator.
4. Multiple activity chart	Simultaneous/interrelated activities of operators and/or machines on a common time scale.
5. Simultaneous motion cycle (SIMO) chart	Movement of body members of the operator, expressed in terms of therbligs on a common time scale.
(b) Diagrams and Models	
1. Flow diagram	Path of men, materials and equipments on a scale model.
2. String diagram	Same as above except for the variation that it uses string to trace the path.
(c) Photographic Aids	
1. Cycle graph	Movement of hand obtained by exposing a photographic plate to the light emitted from small bulbs attached to the operator's fingers.
2. Chrono-cycle graph	Modification of cycle graph in which recording is made using flashlights.

2.2.3.2.2 SYMBOLS USED IN PROCESS CHART

In order to make the presentation of the facts clearly, without any ambiguity and to enable to grasp them quickly and clearly, it is useful to use symbols instead of written description.

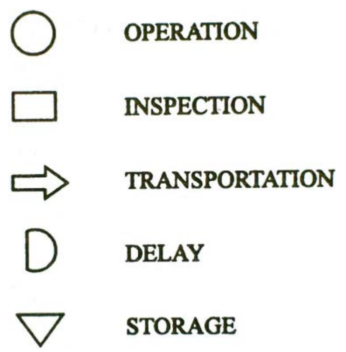


Figure 1 Symbols used in process chart

Operation

A circle indicates an operation. An operation occurs when an object is intentionally changed in one or more of its characteristics (physical or chemical). It indicates the main steps in a process, the method of procedure, usually the part, material or product concerned which is modified or changed during the operation.

An operation always takes the object one stage ahead towards completion.

Examples:

- Turning, drilling, milling, etc.
- A chemical reaction.
- Welding, brazing and riveting.
- Lifting, loading, unloading.
- Getting instructions from supervisor.
- Taking dictation.

Inspection

A square indicates an inspection. It indicates any type of inspection, check, measurement, visual scrutiny for quality and/or quantity.

Examples:

- Visual observations for the finish.
- Count of the quantity of incoming material.
- Checking the dimensions.

Transport

An arrow indicates transport. It indicates movement of workers, material or equipment from place to place.

Examples:

- Movement of materials from one workstation to another.
- Workers travelling to bring tools.

Delay or Temporary Storage

A capital 'D' indicates a delay. It indicates a delay in the sequence of events.

Examples:

- Work waiting between consecutive operations.
- Workers waiting at tool cribs.
- Operators waiting for instructions from supervisor.

Storage

An equilateral triangle standing on its vertex indicates storage. It indicates controlled storage in which material is received into or issued from stores under some form of authorization or an item is retained for reference purposes. For example, materials kept in stores to be distributed to various work centres.

Combined Activity

Two symbols can be combined when activities are performed concurrently as one activity. A circle in a square indicates a combined activity of operation and inspection.

Examples:

- Reading and recording a gauge.
- Watering the garden while walking.

2.2.3.2.3 CHARTS

This is the most popular method of recording the facts. The activities comprising the jobs are recorded using method study symbols. Great care is to be taken in preparing the charts so that the information it shows is easily understood and recognised. The following information should be given in the chart:

- Adequate description of the activities.
- Whether the charting is for the present or proposed method.
- Specific reference to when the activities will begin and end.
- Time and distance scales used wherever necessary.
- The date of charting and the name of the person who does charting.

2.2.3.2.3.1 OPERATION PROCESS CHART

It is also called outline process chart. An operation process chart gives the bird's-eye view of the whole process by recording only the major activities and inspections involved in the process. Operation process chart uses only two symbols, i.e., operation and inspection. Operation process chart is helpful to:

- Visualise the complete sequence of operations and inspections in the process.
- Know where the operation selected for detailed study fits into the entire process.

In the operation process chart, the graphic representation of the points at which materials are introduced into the process and what operations and inspections are carried on them are shown.

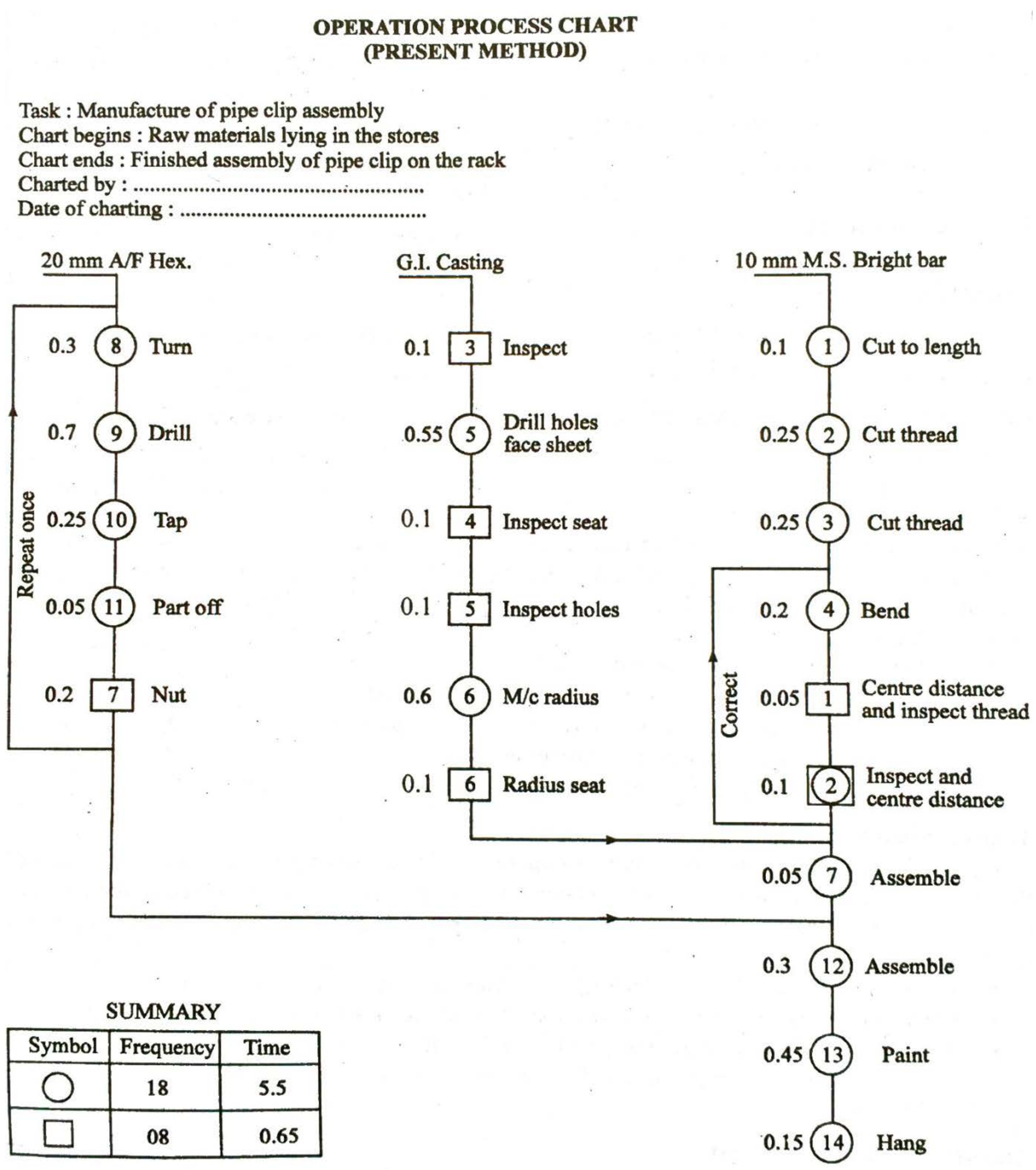


Figure 2 Operation process chart

Charting conventions:

A few conventions used in the chart are briefed below.

- The chart should carry a title at the top. It shall include the name of the chart, task charted, etc., as shown in the figure.
- The component having more number of operations is charted on the extreme right side of the paper
- Vertical flow lines are used to indicate the sequence of events. Process chart symbols are used. (operations and inspections only)

- Horizontal flow lines feeding into the vertical lines are used to show the description of purchased materials or components.
- If it is necessary to the cross vertical line and horizontal line, draw a semi-circle in the horizontal line at the point where the vertical line crosses.
- To the right of the symbol, a brief description of the event is recorded. To the left of the symbol, the time required for the operation may be recorded.
- Operations taking place are numbered in one serial order, in accordance with sequence. Similarly, all inspections are numbered in another serial order. The numbering of events is to be commenced with the major process (main component) and continued until a point of entry of a second component is reached. The sequence of numbering is then continued from the start of the second component and proceeded down until the next point of entry of another component and then repeated.
- When the chart is completed, a summary of the chart is given at the end of the chart. The summary contains the total number of each activity. The total time taken for the performance of all events may also be shown.

2.2.3.2.3.2 FLOW PROCESS CHART

A flow process chart is a process chart which shows the sequence of flow of all activities which occur while producing a product/component or executing a procedure. The flow process chart can be classified into three types as listed below:

- Flow process chart (material type)
- Flow process chart (man type)
- Flow process chart (equipment type)

The flow process chart (material type) shows the sequence of flow of materials of a product/ component while manufacturing/servicing it. The flow process chart (man type) shows the movement of employee(s) related to production/inspection of a product. The flow process chart (equipment type) shows the sequence of usage of equipment(s).

The flow process chart is useful

- To reduce the distance travelled by men (or materials).
- To avoid waiting time and unnecessary delays.
- To reduce the cycle time by combining or eliminating operations.
- To fix up the sequence of operations.
- To relocate the inspection stages.

Like the operation process chart, the flow process chart is constructed by placing symbols one below another as per the occurrence of the activities and are joined by a vertical line. A brief description of the activity is written on the right hand-side of the activity symbol and time or distance is given on the left-hand side.

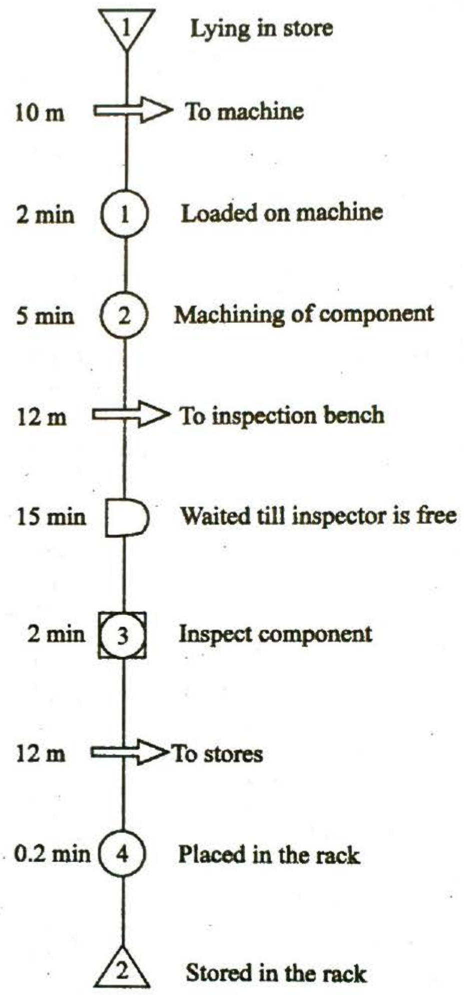
Illustration of flow process chart (material type):

1. One of the components involves the following activities.
2. The component was brought from stores 10 m away.
3. The component was loaded on the machine (2 min).
4. It was machined (5 min).
5. It was then moved to inspection bench 12 m away.
6. It has to wait for 15 min for the inspector to be free from the previous job.

7. The component was checked for accuracy (2 min).
8. It was then moved back to store 12 m away from inspection bench and stored in a rack.

FLOW PROCESS CHART (Material type)
(PRESENT METHOD)

Task : Machining of the component
Chart begins : Component lying in the stores
Chart ends : The machined component lying in the stores
Charted by : ....
Date of charting : ....



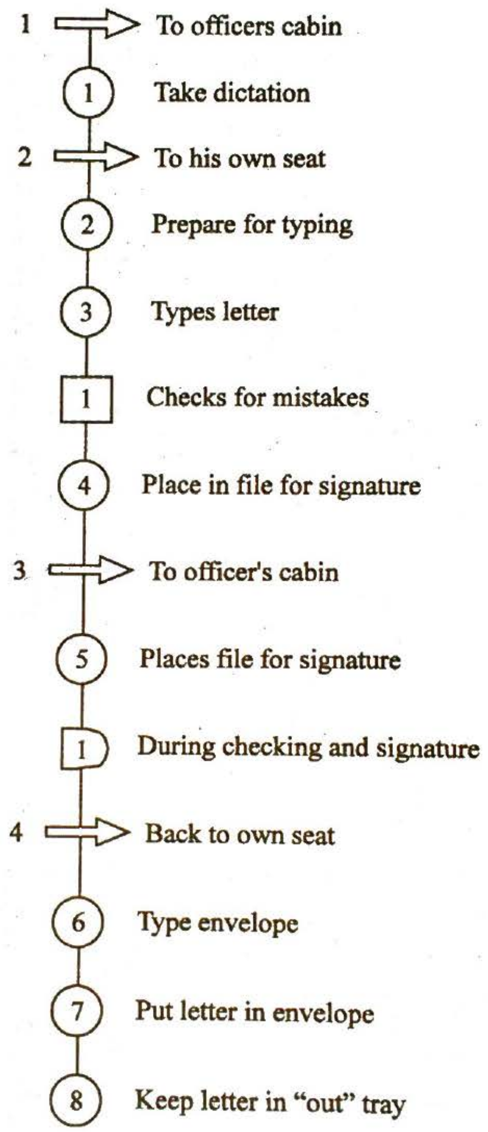
SUMMARY

Symbol	Frequency	Time	Distance
○	4	9.2 min	–
⇨	3	–	34 m
□	1	2 min	–
D	1	15 min	–
▽	2	–	–

Figure 3 Flow process chart (material type)

FLOW PROCESS CHART (Man type)
(PRESENT METHOD)

Task : Writing a letter
Chart begins : Typist in his chiar at his office
Chart ends : Typist puts letter in "out tray"
Charted by : ....
Date of charting : ....



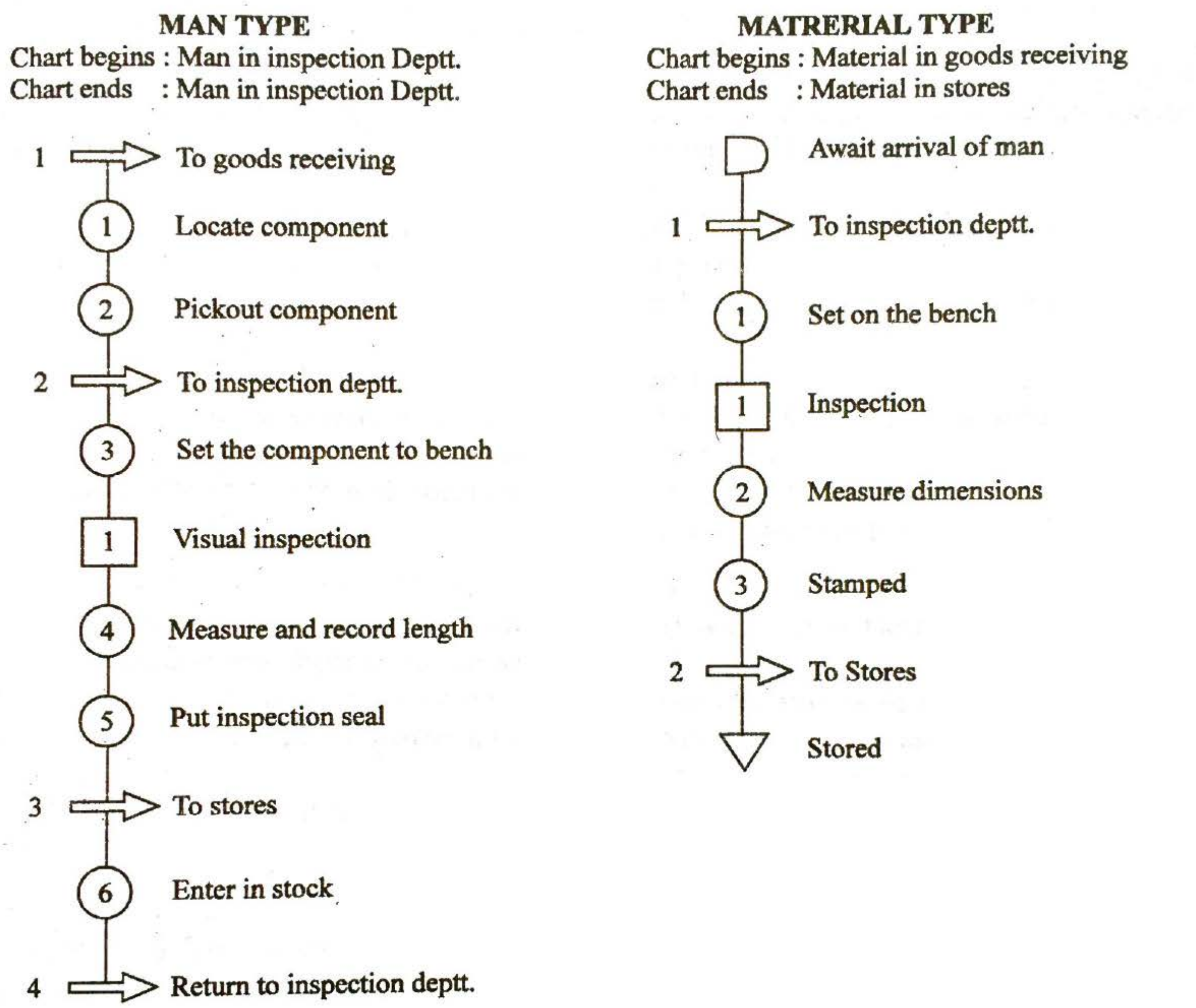
SUMMARY

Symbol	○	⇨	□	▽	D
Frequency	08	04	01	–	01

Figure 4 Flow process chart (man type)

FLOW PROCESS CHART (Man and Material type)
(PRESENT METHOD)

Task : Inspection of component



SUMMARY

Symbol	○	⇨	□
Frequency	6	4	1

Symbol	○	⇨	□	D	▽
Frequency	3	2	1	1	1

Figure 5 Flow process chart (man and material type)

2.2.3.2.3.3 TWO-HANDED PROCESS CHART

Two-handed process chart records the sequence of activities performed by both hands of an operator side by side. This chart is very much useful to analyze the activities of the operator and come out with an improved method of performing the integrated task of importance. This chart uses the same symbols which are used in two-handed process charts with the following modified meanings.

○ This symbol is used to represent the activities performed by an operator which are equivalent to operation. These activities include grasp, position, use, release, etc. of a tool or material or subassembly.

→ This symbol is used to represent the movement of the hand/limb of the operator while performing the work. The hand/limb may be moved from or to assembly/ work, tool or material.

D This symbol is used to represent the idling time of the hand/limb of the operator which will happen during a certain part of the work cycle.

▽ This symbol is used to represent the operation of holding the assembly/work, tool or material by one hand while the other hand does some other activity.

The two-handed process chart consists of two charts, one for the left hand and other for the right hand. The simultaneous activities are recorded opposite to each other on the chart. This helps to analyse what left hand will be doing when the right hand is working or vice versa at any point in time.

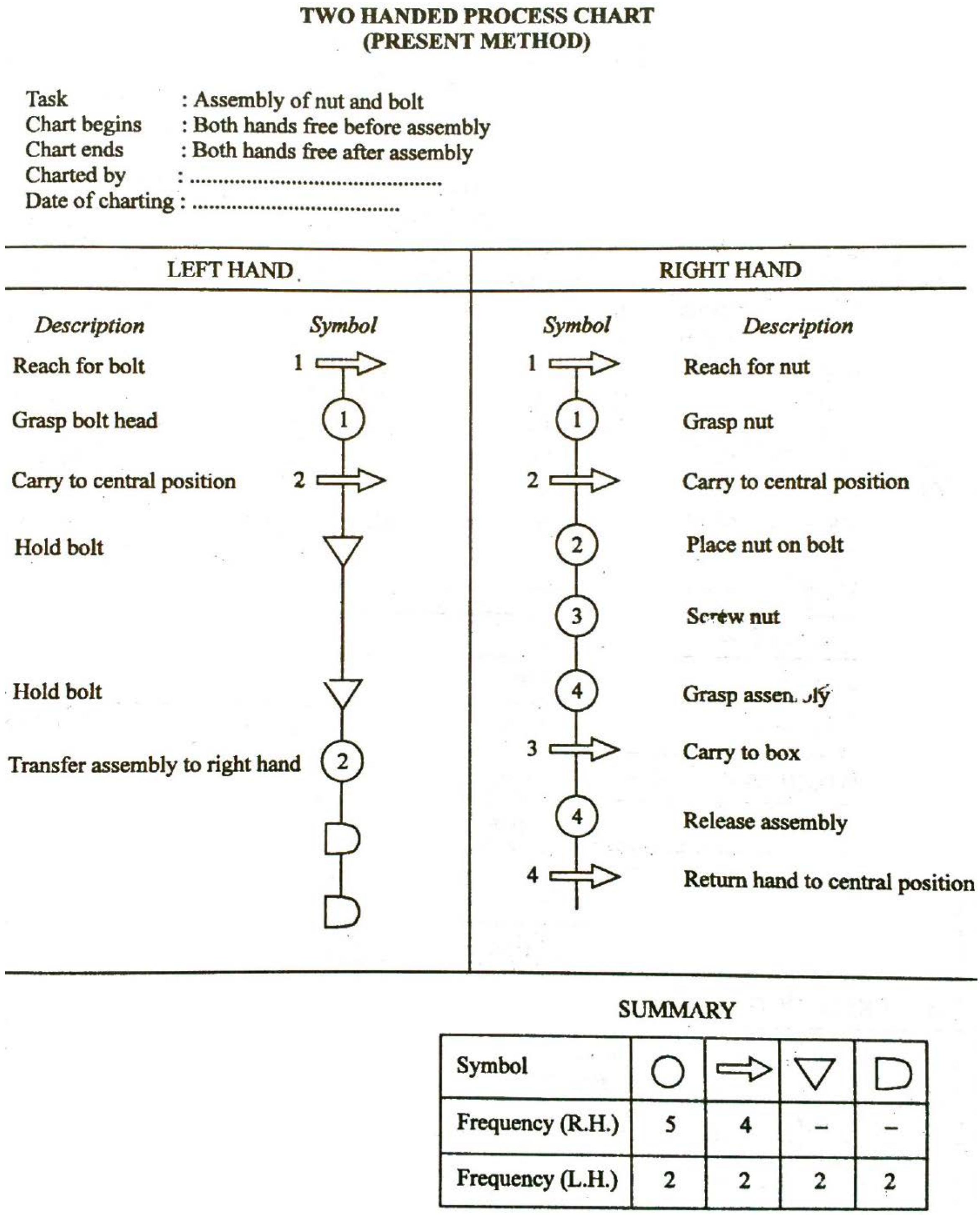


Figure 6 Two-handed process chart

2.2.3.2.3.4 MULTIPLE ACTIVITY CHART

In many situations, certain facilities/machines/equipments will be managed by a single individual or by a group of individuals. A multiple activity chart is a pictorial representation of activities of individuals and associated facilities/machines/equipments simultaneously on a common time scale.

Multiple activity chart is made to

- Study idle time of the man and machines.
- Determine the number of machines handled by one operator.
- Determine the number of operators required in teamwork to perform the given job.

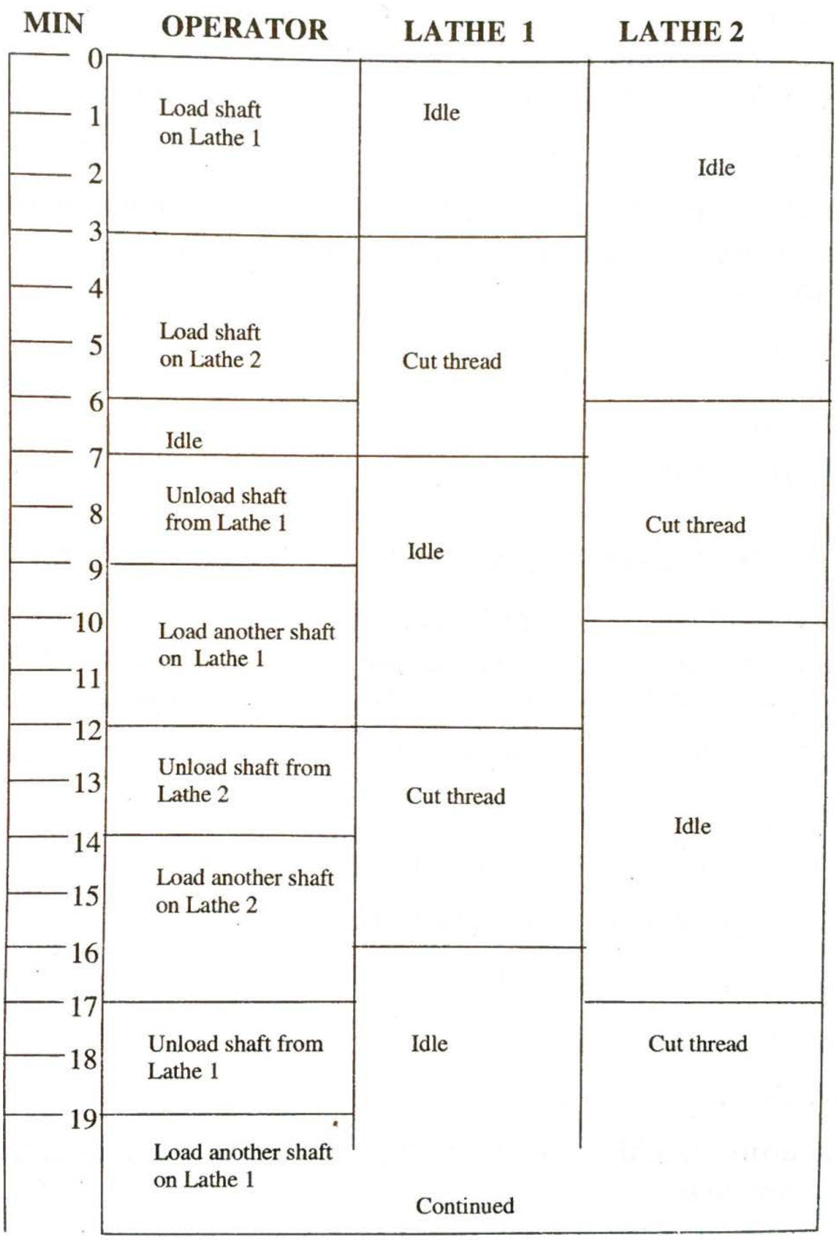


Figure 7 Multiple activity chart (Man-machines chart)

Consider an example of an operator performing thread cutting on one end of a shaft using a lathe. The sequence of activities involved in thread cutting is listed below.

1. Loading the shaft in the lathe (3 minutes)
2. Thread cutting (4 minutes)
3. Unloading the shaft from the lathe (2 minutes)

Since the operator will be idle for a certain portion of the above work cycle that is during thread cutting, he can be assigned more than one lathe. Let us consider the assignment of two lathes for one operator, then the corresponding multiple activity chart is known as the man-machines chart which is shown in the figure. In the figure, when the operator is loading/unloading a shaft on/from a lathe, the respective lathe will be idle. These are explained with few cases as presented below.

The operator is loading a shaft on lathe 1 during the first three minutes and he is loading a shaft on lathe 2 during the next three minutes (from the fourth minute to the sixth minute). So, the lathe 1 keeps idling during the first three minutes and the lathe 2 keeps idle during the first six minutes. Similarly, when both the lathes are performing thread cutting operation, the operator has to be idle for that duration. During the seventh minute, both the lathes are performing thread cutting operation. Hence, the operator is idle during this interval. This chart gives a complete sequence of activities for each entity in the chart which will help the process to be executed as per expected schedule. A careful analysis will help to assign the optimal number of machines to an operator.

2.2.3.2.4 DIAGRAMS

The flow process chart shows the sequence and nature of movement but it does not clearly show the path of movements. In the paths of movements, there are often undesirable features such as congestion, backtracking and unnecessary long movements. To record these unnecessary features, representation of the working area in the form of flow diagrams, string diagrams can be made:

- To study the different layout plans and thereby select the most optimal layout.
- To study traffic and frequency over different routes of the plant.
- Identification of backtracking and obstacles during movements.

Diagrams are of two types:

1. Flow Diagram, and
2. String Diagram

2.2.3.2.4.1 FLOW DIAGRAM

A flow diagram is a drawing, substantially to scale, of the working area, showing the location of the various activities identified by their numbered symbols and are associated with particular flow process chart either man type or material type.

The routes followed in transport are shown by joining the symbols in sequence by a line which represents as nearly as possible the paths or movement of the subject concerned.

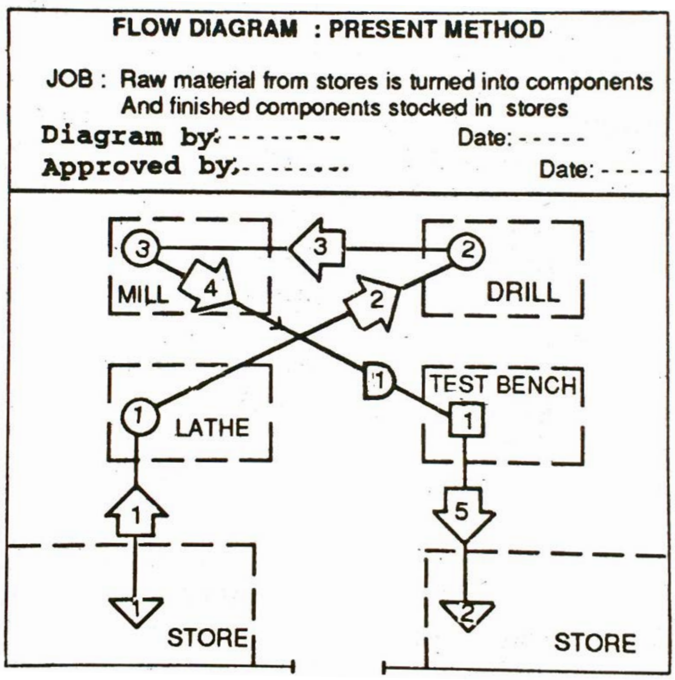


Figure 8 Flow diagram

The procedure to make the flow diagram:

1. The Layout of the workplace is drawn to scale.
2. Relative positions of the machine tools, work benches, storage, inspection benches, etc. are marked to the scale.
3. The path followed by the subject under study is traced by drawing lines.
4. Each movement is serially numbered and indicated by the arrow for direction.
5. Different colours are used to denote different types of movements.

2.2.3.2.4.2 STRING DIAGRAM

The string diagram is a scale layout drawing on which length of a string is used to record the extent as well as the pattern of movement of a worker working within a limited area during a certain period of time. It is especially valuable where the journeys are so irregular in distance and frequency to see exactly what is happening.

The primary function of a string diagram is to produce a record of an existing set of conditions so that the job of seeing what is actually taking place is made as simple as possible. One of the most valuable features of the string diagram is the way it enables the actual distance travelled during the period of study to be calculated by relating the length of the thread used to the scale of the drawing. Thus it helps to make a very effective comparison between different layouts or methods of doing a job in terms of the travelling involved.

The main advantage of the string diagram compared to flow diagram is that repetitive movements between workstations which are difficult to be traced on the flow diagram can be conveniently shown on string diagram.

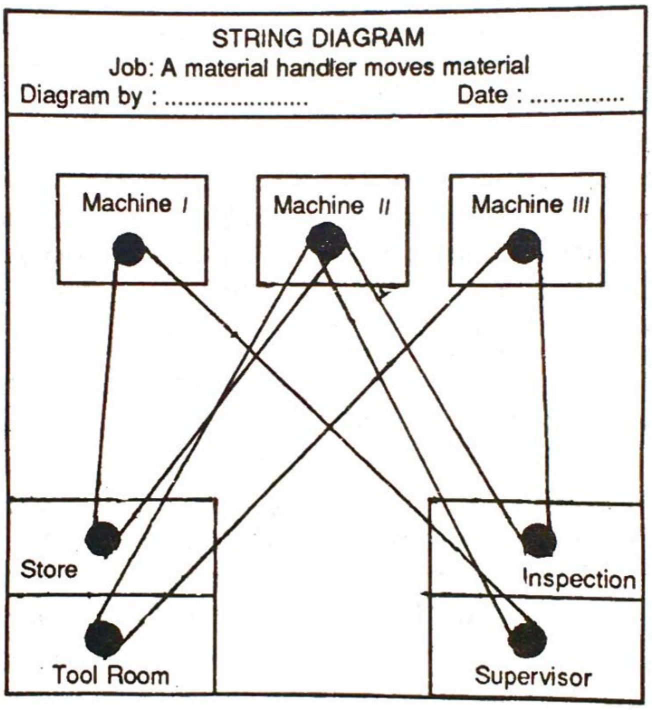


Figure 9 String diagram

Procedure to draw string diagram:

1. A layout of the workplace or factory is drawn to scale on a soft board.
2. Pins are fixed into boards to mark the locations of workstations, pins are also driven at the turning points of routes.
3. A measured length of thread is taken to trace the movement (path).
4. The distance covered by the object is obtained by measuring the remaining part of the thread and subtracting it from the original length.

By using different coloured threads, diagrams can be prepared to record movements of different workers working within the same area.

2.2.3.2.5 MICRO-MOTION STUDY

Micro-motion study provides a technique for recording and timing an activity. Micro-motion study is a set of techniques intended to divide the human activities into a group of movements or micro-motions (called as therbligs) and the study of such movements helps to find for an operator one best pattern of movement that consumes less time and requires less effort to accomplish the task. Therbligs were suggested by Frank B. Gilbreth, the founder of motion study. The applications of micro-motion study include the following:

- As an aid in studying the activities of two or more persons on group work.
- As an aid in studying the relationship between the activities of the operator and the machine as a means of timing operations
- As an aid in obtaining motion time data for time standards.
- Acts as a permanent record of the method and time of activities of the operator and the machine.

The micro-motion group of techniques is based on the idea of dividing human activity into divisions of movements or groups of movements (therbligs) according to the purpose for which they are made. Gilbreth differentiated 17 fundamental hand or hand and eye motions to which an eighteenth has subsequently been added.

Therbligs refer primarily to motions of the human body at the workplace and to the mental activities associated with it. They permit a much more precise and detailed description of the work than any other recording techniques.

Micro-motion study involves the following steps:

1. *Filming the operation to be studied:*

   Micro-motion study consists of taking motion pictures of the activity while being performed by an operator. The equipment required to make a film or videotape of the operation consists of 16 mm movie camera, 16 mm film, wink counter (micro-chronometer) and other usual photographic aids.

   Micro-chronometer (or wink counter) is a timing device placed in the field of view while filming. Time is recorded in winks (1 wink = 1/2000 of a minute).

2. *Analysis of the data from the films:*

   Once the operation has been filmed and the film is processed, then the film is viewed with help of projector for analysis of micro-motions. The film is analysed in the following way:
   (i) The film is run at a normal speed so as to get familiar with the pattern of movement involved.
   (ii) A typical work cycle is selected from amongst the filmed cycles.
   (iii) The film is run at a very low speed and is usually stopped or reversed frequently to identify the motions (therbligs)
   (iv) Therbligs after identification are entered in analysis sheet.

3. *Recording of the data using SIMO chart:*

   Simultaneous motion cycle chart (SIMO chart) is a recording technique for micro-motion study. A SIMO chart is a chart, based on the film analysis, used to record simultaneously on a common time scale the therbligs or a group of therbligs performed by different parts of the body of one or more operators.

2.2.3.2.6 MEMO-MOTION STUDY

Memo motion photography is a form of time-lapse photography which records activity by the use of cine camera adapted to take the picture at longer intervals than normal (time interval normally lies between 1/2 sec to 4 sec).

There are many jobs that have activities which do not need to be examined in fine detail and are still too fast or intricate to be recorded accurately without the help of a film. There are cases where the micro-motion study is not justified like smaller production quantities, job cycle may exceed 4-minute duration.

The filming of these various classes of works can be performed efficiently and economically by a method of time-lapse cine photography known as memo motion.

This is carried out by attaching an electric time lapse unit to the cine camera so that a picture is taken at an interval of time set at any convenient unit between 1/2 sec to 4 sec in frequency. A camera is placed with a view over the whole working area to take pictures at the rate of one or two per second instead of 24 frames a second. The result is that the activities of 10 or 20 minutes may be compressed into one minute and a very rapid survey of the large movements giving rise to wasted efforts can be detected and steps are taken to eliminate them. It is economical compared to micro-motion study.

2.2.3.2.7 CYCLE GRAPH AND CHRONOCYCLE GRAPH

These are the photographic techniques for the study of the path of movements of an operator’s hands, fingers, etc. These are used especially for those movements which are too fast to be traced by the human eye.

A *cycle graph* is a record of the path of movement usually traced by a continuous source of light on a photograph. A small electric bulb is attached to hand, finger or other parts of the body of the operator performing the operation. A photograph is taken by a still camera and the light source shows the path of the motion and the path of the photograph is called "cycle graph".

Cycle graph has a limitation. It will not give the direction or the speed of movements. This limitation is overcome by a chronocycle graph.

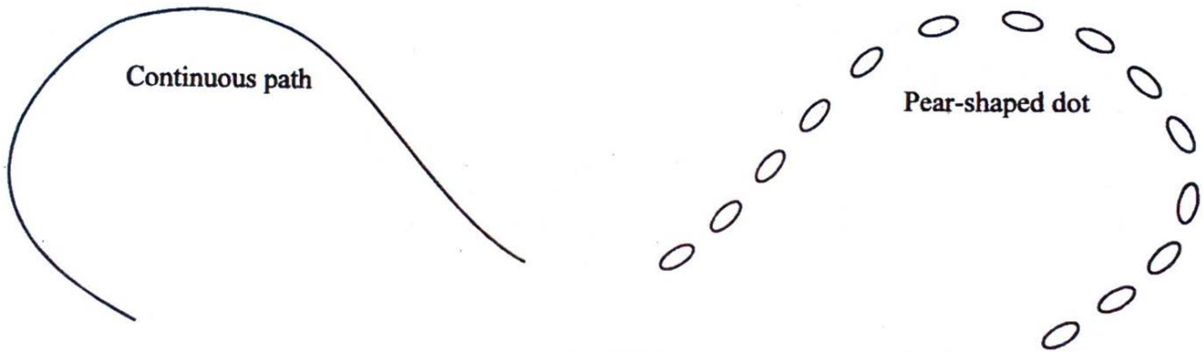


Figure 10 Cycle graph and chronocycle graph

A *chronocycle graph* is a special form of cycle graph in which the light source is suitably interrupted so that the path appears as a series of pear-shaped dots, the pointed end indicating the direction of movement and the spacing indicating the speed of movement. The time taken for the movement can be determined by knowing the rate at which the light source is being interrupted and by counting the number of dots.

2.2.3.3 EXAMINE

This is the most important step in the method study. This step aims: to eliminate the activity altogether if it to combine it with other activities; to change the sequence of activities so that work delay is reduced and to simplify the activity to reduce the work content or time consumption.

In this step, we will have to ask a series of questions. The questions may be classified into primary questions and secondary questions.

Primary questions

Purpose What is the purpose of the event? Why is it necessary?

Place Where does the event take place? Why there?

Sequence When does it occur? Why then?

Person	Who carries out the work? Why that person?
Means	How is the purpose achieved? Why that particular way?

Secondary questions

Purpose	What else could be done?
Place	Where else could it be done?
Sequence	When else could it be done?
Person	Who else could do it?
Means	How else could it be done?

Alternatives

When each of the above questions is applied to any event, a number of alternatives would emerge. When these have been established, ask the following questions:

Purpose	What should be done?
Place	Where should it be done?
Sequence	When should it be done?
Person	Who should do it?
Means	How should it be done?

2.2.3.4 DEVELOP

Critical examination gives rise to a number of creative ideas. Since all the ideas are not practicable, some of the ideas are required to be discarded and others are to be refined and developed.

Development involves the analysis of the three phases: evaluation, investigation and selection. For a development of the new method, the following approaches can be considered:

- Eliminate all unnecessary operations.
- Combine operations and elements.
- Change the sequence of operations.
- Simplify the necessary operations.

2.2.3.4.1 EVALUATION

Evaluation phase tests the true worth of each alternative and thereby decide whether an idea should be pursued or discarded. It is, therefore, an exercise to shortlist the creative ideas.

2.2.3.4.2 INVESTIGATION

Investigation explores as to how the ideas cleared at the evaluation stage suitably can be converted into practical suggestions. Investigation usually involves preparation of drawings, making prototypes, conducting trial runs. The aim is to test each idea for its economic and technical feasibility so that each suggestion is definite and supported by evidence of practicability. An investigation involves the testing of technical and economic feasibility.

2.2.3.4.3 SELECTION

Each alternative needs to be evaluated against a set of specific factors. The most commonly selected factors are investment required, production rate, manufacturing cost per piece, return on investment. Using point system, weights are then assigned to each of the factors, the performance of each factor is then predicted for each alternative. This step is followed by an evaluation process of each alternative against each specific factor.

To select a preferred alternative, the points scored by each alternative against each specific factors are added. The alternative scoring the maximum is selected.

2.2.3.5 INSTALL

Installation refers to the implementation of the proposed method and it serves the following objectives:

1. Preparation of change proposal to management.
2. Steps to prepare its implementation on acceptance of a proposal.
3. To get formal approval from management.
4. To implement the accepted proposal.

Installation is composed of two steps:

1. *Recommendation phase:* The formal written report should be prepared for the changed method, present the recommendations to the management. Also, provide information on the implementation plan and get the approval of the management.
2. *Implementation phase:* The entire study effort will go waste if the proposal is not implemented. Though the responsibility of implementation is that of top management yet the assistance of method study man is required to:
    - tackle problems at implantation phase.
    - minimise delay in the implementation process.
    - the proposal is implemented in its entirety.

2.2.3.6 MAINTAIN

A method change does not end up with implementation of the proposal. Follow-up after the implementation is equally important. The maintenance of the proposed method involves:

- Monitoring and control.
- Audit of the savings.
- Review of the approach.
- Evaluation of the effectiveness of the proposed method.

2.3 WORK MEASUREMENT

2.3.1 DEFINITION

Work measurement is also called by the name "Time study". Work measurement is absolutely essential for both the planning and control of operations. Without measurement data, we cannot determine the capacity of facilities or it is not possible to quote delivery dates or costs. We are not in a position to determine the rate of production and also labour utilisation and efficiency. It may not be possible to introduce incentive schemes and standard costs for budget control.

Time study has been defined by British Standard Institution as “The application of techniques designed to establish the time for a qualified worker to carry out a specified job at a defined level of performance.”

2.3.2 OBJECTIVES OF WORK MEASUREMENT

The use of work measurement as a basis for incentives is only a small part of its total application. The objectives of work measurement are to provide a sound basis for:

- Comparing alternative methods.
- Assessing the correct initial manning (manpower requirement planning).
- Planning and control.
- Realistic costing.
- Financial incentive schemes.
- The delivery date of goods.
- Cost reduction and cost control.
- Identifying substandard workers.
- Training new employees.

2.3.3 TECHNIQUES OF WORK MEASUREMENT

Various techniques of work measurement are:

- Time study (stopwatch technique)
- Synthesis
- Work sampling
- Analytical estimating
- Predetermined motion and time study

Time study and work sampling involve direct observation and the remaining are data-based and analytical in nature.

2.3.3.1 Time Study

A work measurement technique for recording the times and rates of working for the elements of a specified job carried out under specified conditions and for analysing the data so as to determine the time necessary for carrying out the job at the defined level of performance.

2.3.3.2 SYNTHETIC DATA

A work measurement technique for building up the time for a job or parts of the job at a defined level of performance by totalling element times obtained previously from time studies on other jobs containing the elements concerned or from synthetic data.

2.3.3.3 Work Sampling

A technique in which a large number of observations are made over a period of time of one or group of machines, processes or workers. Each observation records what is happening at that instant and the percentage of observations recorded for a particular activity, or delay, is a measure of the percentage of time during which that activities or delay occurs.

2.3.3.4 ANALYTICAL ESTIMATING

A work measurement technique, being a development of estimating, whereby the time required to carry out elements of a job at a defined level of performance is estimated partly from knowledge and practical experience of the elements concerned and partly from synthetic data.

2.3.3.5 PREDETERMINED MOTION TIME STUDY (PMTS)

A work measurement technique whereby times established for basic human motions (classified according to the nature of the motion and conditions under which it is made) are used to build up the time for a job at the defined level of performance. The most commonly used PMTS is known as Methods Time Measurement (MTM).

2.3.3.6 WORK MEASUREMENT TECHNIQUES AND THEIR APPLICATION

Work Measurement Technique	Application
1. Time study	Short cycle repetitive jobs. Widely used for direct work.
2. Synthetic data	Short cycle repetitive jobs.
3. Work sampling	Long cycle jobs/heterogeneous operations.
4. Analytical estimating	Short cycle non-repetitive jobs.
5. MTM	Manual operations confined to one work.

2.3.4 STEPS IN MAKING TIME STUDY

Stopwatch time is the basic technique for determining accurate time standards. They are economical for the repetitive type of work. Steps in taking the time study are:

1. Select the work to be studied.
2. Obtain and record all the information available about the job, the operator and the working conditions likely to affect the time study work.
3. Breakdown the operation into elements. An element is a distinct part of a specified activity composed of one or more fundamental motions selected for convenience of observation and timing.
4. Measure the time by means of a stopwatch, taken by the operator to perform each element of the operation. Either continuous method or snap back method of timing could be used.

5. At the same time, assess the operator's effective speed of work relative to the observer's concept of "normal" speed. This is called performance rating.
6. Adjust the observed time by rating factor to obtain normal time for each element.

$$\text{Normal time} = \frac{\text{Observed time} \times \text{Performance rating } (\%)}{100}$$

7. Add the suitable allowances to compensate for fatigue, personal needs, contingencies, etc., to give standard time for each element.
8. Compute allowed time for the entire job by adding elemental standard times considering the frequency of occurrence of each element.
9. Make a detailed job description describing the method for which the standard time is established.
10. Test and review standards where necessary.

The basic steps in time study are:

Select	The job to be timed.
Obtain & Record	The details regarding a method, operator, job and working conditions.
Define	The elements, break the job into elements convenient for timing.
Measure	Time duration for each element and assess the rating.
Extend	Observed time into normal time (basic time).
Determine	Relaxation and personal allowances.
Compute	Standard time for the operation for defined job or operation.

2.3.4.1 SELECTING JOB FOR TIME STUDY

The reasons for which time study may be done:

- The job in question is a new one or not previously carried out.
- Change in the method of the existing time standard.
- Complaint received from workers or unions regarding the time standard.
- A particular operation becomes bottle-neck operation which holds up a number of subsequent activities.
- Change in the management policy regarding how time standards are used, i.e., general purpose or wage incentive plans.

The general guidelines for selecting the job for time study:

- Bottle-neck operations.
- Repetitive jobs.
- Jobs using a greater deal of manual labour.
- Jobs with longer cycle time.
- Sections/department frequently working overtime.

2.3.4.2 OBTAINING AND RECORDING INFORMATION

During this step, all the relevant and necessary information regarding the method, operator and details of working conditions are recorded:

- The accuracy of time standards depends upon the correctness of the method employed by the operators. So wrong methods should not be timed. The method is to be standard and the time required to carry out the job as per the standard method is to be timed.

- The selection of an operator refers to choosing an operator amongst many operators doing the same job. He should be a representative worker with a normal pace neither too fast nor too slow. So the details of the operator are essential to be recorded before starting actual time study.
- Information to enable the identification details such as part number and name, machine no. speed and feed, materials, operator details, etc.
- Working conditions under which an operator carries out the job like temperature, dust, smoke, vibrations, noise, etc.
- Working position such as standing, sitting, bending, etc., and weights handled, protective clothing, etc.

2.3.4.3 BREAKING THE JOBS INTO ELEMENTS

Once the recording of the basic information regarding the job and the operator are done, the next step is breaking the job into elements.

An *element* is a distinct part of a specified job selected for convenience of observation, measurement and analysis.

Work cycle is a complete sequence of elements necessary to perform a specified activity or job to yield one unit of production. It may also include the elements which do not occur with every cycle.

Reasons for Breaking the Jobs into Elements

- To ensure that productive time is separated from unproductive activities (separating effective time and ineffective time).
- To permit the rate of performance to be assessed more accurately than would be possible if the assessment were made over a complete cycle.
- To enable different types of elements to be identified and distinguished so that each element is given appropriate treatment.
- To ensure elements involving a high degree of fatigue to be isolated and to make the allocation of fatigue allowances more accurately.
- To enable the detailed work specification to be produced.
- To enable machine elements to be distinguished from "human" elements.
- To enable time standards to be checked or modified at later date, omissions and errors to be rectified.
- For accuracy of rating.
- To enable time values for frequently recurring elements, such as the loading/unloading of jobs into the fixture, machine adjustment to be extracted and used in the compilation of standard data.

Types of Elements

- A *repetitive element* is an element which occurs in every work cycle of the job. *Examples:* Picking up a part for assembly, an element of locating a workpiece in a holding device, etc.

- An *occasional element* is one that does not occur in every work cycle of the job or which may occur at regular intervals. *Examples:* Tool changing after some time, adjusting tension or machine setting, instruction from the supervisor, etc. An occasional element is useful work to be included in standard time.

- A *constant element* is an element for which the basic time remains constant whenever it is performed. *Examples:* Switch on a machine, measure the diameter, insert cutting tools, etc.

- A *variable element* is an element for which the basic time varies in relation to some basic characteristics of the product, equipment or process. *Examples:* Dimensions, weight, quality, etc.

- A *manual element* is an element performed by a worker.

- A *machine element* is an element automatically performed by a power-driven machine. *Examples:* Press working parts, annealing tubes, etc.

- *Governing element* is an element occupying a longer time than that of any other element which is being performed concurrently. *Example:* Gauge dimensions while turning diameter (turning diameter will be a governing element)

- A *foreign element* is one that is observed during the study but does not form part of the given activity of the cycle. *Examples:* Dropping work on the floor, operator talking to his colleague, etc.

Guidelines for Breaking Jobs into Elements

- Elements should be easily identified.
- Each element should have a definite beginning and end.
- Manual elements should be separated from variable elements.
- Occasional elements should be timed separately.
- Elements should be as short as can be conveniently timed by a trained observer.
- Elements should be chosen so that they represent naturally unified and recognisably distinct segments of the operation.

2.3.4.4 MEASURE DURATION OF EACH ELEMENT

When elements have been selected, the next step is starting the timing of operations. There are two principal methods of timing with the stopwatch:

- *Cumulative timing:* In cumulative timing, the watch runs continuously throughout the study. It is started at the beginning of the first element of the first cycle to be timed and is not stopped until the whole study is completed. At the end of each element, the watch reading is recorded and individual element times are obtained by successive subtractions after the study is completed.

- *Fly back timing:* In fly back timing the hands of the stopwatch are returned to zero at the end of each element and allowed to start immediately, the time for each element is obtained directly.

While recording the time of the elements, the operator’s speed of working is assessed and recorded on the observation sheet. Rating is the time study engineer’s assessment of the operator’s pace of working in relation to the concept of standard or normal. Rating is used to convert the observed time into normal time.

2.3.4.5 EXTEND OBSERVED TIME INTO NORMAL TIME

The representative time established from the observation data is the time which an operator has taken while working at a certain pace. The observed time is converted into basic or normal time by multiplying it by rating factor.

$$\text{Normal time} = \frac{\text{Observed time} \times \text{Performance rating (\%)}}{100}$$

2.3.4.6 DETERMINE RELAXATION AND OTHER ALLOWANCES

Normal times of elements added together give normal time for the operation. But this will not be equal to standard time as the operators cannot work continuously. Some additional time added to normal time to arrive at the standard time. The additional time is needed to:

- Relaxation allowance
- Interference allowance
- Contingency allowance
- Policy allowance

2.3.4.7 CALCULATE STANDARD TIME FOR THE JOB

Standard time is the time allowed to an operator to carry out the specified task under specified conditions and defined level of performance.

The various allowances are added to the normal time as applicable to get the standard time. Thus basic constituents of standard time are:

- Elemental (observed time).
- Performance rating to compensate for the difference in pace of working.
- Relaxation allowance.
- Interference and contingency allowance.
- Policy allowance.

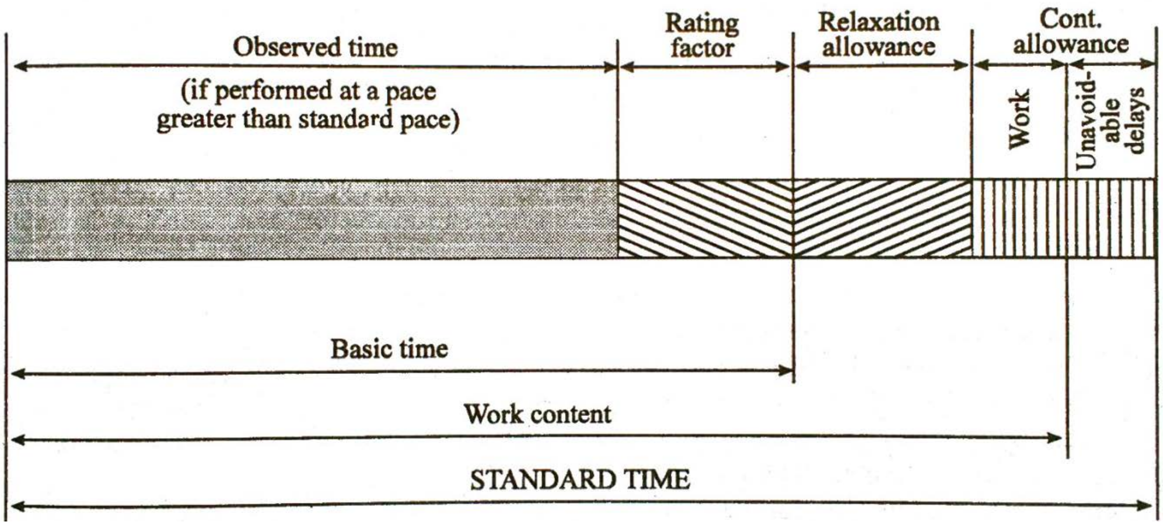


Figure 11 Computation of standard time

2.3.5 TIME STUDY EQUIPMENTS

Basic time study equipment required to make the time study are:

- Time study board,
- Stopwatch, and
- Time study forms.

Time study board: Time study board is simply a flat board, usually of plywood or of any suitable plastic sheet and it should have fittings to hold a stopwatch and time study forms. The use of board provides support and resting face while writing observations on the shop-floor and makes the hands free to write and operate stopwatch.

Stopwatch: Stop is the measuring instrument to observe the elemental timings and usually a decimal watch is used. A decimal minute stop watch has two hands. The small hand represents minutes on the dial and completes one revolution in 30 minutes. The large hand represents centi-minutes (1/100th minute) and completes one revolution in one minute and each division on large dial represents 0.01 minute. Two commonly used types of stopwatches are:

- *Cumulative stopwatch:* The watch is started by pressing the winding knob located on the head of the watch and is stopped by pressing the winding knob. Pressing winding knob the third time snaps the hands back to zero. Once started it will run until the required number of cycles have been timed.

- *Fly back stopwatch:* This is most commonly used watch. In this type of watch, the movement is started and stopped by a slide at the side of the winding knob. Pressure on the top of the winding knob causes both the hands to fly back to zero without stopping the mechanism from which point they move forward immediately. This type of watch is used for either fly back or cumulative timing method.

Electronic timers are most widely used timing devices for time study. The electronic timer which performs the same function as the stopwatch is sometimes referred to as electronic stopwatch. Electronic data collectors and computers and motion picture camera (with a constant speed motor drive) are also used for the purpose.

Time study forms: Time study forms are usually printed forms of standard size. The use of standard forms is desirable as the constant information's such as part number and part name, operation description, observers name and other description are pre-printed on the top of the form which eliminates the possibility of any details being missed. As the size of the forms is standardised they can be easily filed for future reference.

2.3.6 NUMBER OF CYCLES TO BE STUDIED

- The number of cycles through which any particular job should be observed varies directly as the amount of variations in the times of the elements of the job.
- The number of cycles to be observed will depend on the degree of accuracy desired. This, in turn, will depend on the length of the run of the job and the number of people engaged in it.
- The study should be continued through a sufficient number of cycles to ensure that occasional elements such as handling boxes of finished parts, periodical cleaning of machines, etc., can be observed several times.
- Where more than one operator is engaged on the same job it is preferable to take a short study on each of several operators rather than timing too long on a single operator.

2.3.7 PERFORMANCE RATING

The performance rating is the process of adjusting the actual pace of working of an operator by comparing it with the mental picture of the pace of an operator working at normal speed.

$$\text{Performance rating} = \frac{\text{Observed time} \times 100}{\text{Normal time}}$$

In other words, the rating is a levelling factor to convert observed timings into normal timings.

2.3.7.1 FACTORS AFFECTING PERFORMANCE RATING

There exists a variation from element to element and even among the elements in the same operation. This is due to the inconsistency in the speed of the working of the operator.

Each worker by nature has different temperament and attitudes towards the work. Some workers by their nature are fast (above the speed of the average worker) and some are by nature slow. Both these workers will not represent a normal worker.

The variation in actual times for a particular element may be due to the factors both internal and external. The external factors which are not in control of work-study man are:

- Variation in the quality or other characteristics of the material used even though it is in prescribed tolerance limit.
- Changes in the operating efficiency of tools and equipment within their useful life.
- Unavoidable changes in methods or conditions of operations.
- Change in working conditions like heat, light, dust, etc.

Factors which are within the control (internal factors) are:

- Acceptable variation, in the quality of the product.
- Variation due to operator's ability.
- Variation due to his attitude of mind.

The various methods of performance rating are:

- Speed rating.
- Westing house system of rating.
- Synthetic rating, and
- Objective rating.

Speed rating: In this technique, the speed of the movements of the operator is the only factor considered for performance rating. The speed rating is found by the observer by comparing the pace of operators working with his own concept of normal pace. An average worker is rated at 100%, better than average worker is rated at a figure higher than 100% and below average worker will be rated below 100%. If a worker is rated at 125% it means that the speed is 25% higher than the observer's concept of normal and rating of 80% means the worker is working 20% below the observer's concept of a normal worker.

In speed rating, the process of the rating is confined to the comparison of the speed of movements with a concept of normal speed. On the basis of this assumption, the rating process is made simpler and with training in developing the concept of normal pace, the observer can become quite proficient in his judgement.

Westing house method of rating: Westing house system utilises a set of criteria to measure the performance of the operators. The factors are:

- *Skill:* Measures the worker's proficiency in adhering to a given method, coordination of proper hand and eye movements, rhythm of the movements. The skill has been classified into six degrees, each degree indicating a specified class of skill within which an operator performs the task.
- *Effort:* Measures the speed with which the skill is applied. The effort is also divided into six degrees.
- *Consistency:* Measures factors which affect the consistency of the operator to perform the work cycle repeatedly within the same time. Elements which affect the consistency are variations in materials, hard spots, the presence of foreign elements, etc. Consistency is subdivided into six classes.
- *Conditions:* Measure the extent to which the conditions like temperature, vibrations, light and noise affect the operator's performance.

Synthetic rating: The performance rating under this method is established by comparing the observed time of some of the manual elements with those of known time values of the elements from predetermined motion and time studies (PMTS).

The procedure is to make the time study in a usual manner and then compare the actual time for the elements with predetermined time values for the same elements.

A ratio is computed between the predetermined time value for the element and actual time value for the element. This ratio is the performance index or rating factor for the operator for the particular element.

$$\text{Performance rating factor} = \frac{\text{Predetermined time for elements}}{\text{Average actual time value for the same element}}$$

Objective rating: In this method, the operator's speed is rated against a single standard pace which is independent of job difficulty. The observer merely rates speed of movement or activity, paying no attention to the job itself. After the pace rating is made, an allowance or a secondary adjustment is added to the pace rating to take care of job difficulty.

Job difficulty is divided into six classes, and a percentage is provided for each of these factors.

The job difficulties as per the founder of this system, M.E. Mundel have been categorised into six classes as follows:

1. Amount of body used.
2. Foot pedals.
3. Bi-manualness.
4. Eye-hand coordination.
5. Handling requirements.
6. Weight.

2.3.8 ALLOWANCES

The normal time for an operation does not contain any allowances for the worker. It is impossible to work throughout the day even though the most practical, effective method has been developed. Even under the best working method situation, the job will still demand the expenditure of human effort and some allowance must therefore be made for recovery from fatigue and for relaxation. Allowances must also be made to enable the worker to attend to his personal needs. The allowances are categorised as:

2.3.8.1 RELAXATION ALLOWANCE

Relaxation allowances are calculated so as to allow the worker to recover from fatigue.

Relaxation allowance is an addition to the basic time intended to provide the worker with the opportunity to recover from the physiological and psychological effects of carrying out specified work under specified conditions and to allow attention to personal needs. The amount of allowance will depend on the nature of the job.

Relaxation allowances are of two types:

- **Fixed allowance**
    - *Personal needs allowance:* It is intended to compensate the operator for the time necessary to leave the workplace to attend to personal needs like drinking water, washing hands, etc. Women require longer personal allowance than men. A fair personal allowance is 5% for men and 7% for women.
    - *Allowances for basic fatigue:* This allowance is given to compensate for energy expended during working. A common figure considered as the allowance is 4% of the basic time.
- **Variable Allowance**

  A variable allowance is allowed to an operator who is working under poor environmental conditions that cannot be improved, added stress and strain in performing the job.

  The variable fatigue allowance is added to the fixed allowance to an operator who is engaged on medium and heavy work and working under abnormal conditions. The amount of variable fatigue allowance varies from organisation to organisation.

2.3.8.2 INTERFERENCE ALLOWANCE

It is an allowance of time included into the work content of the job to compensate the operator for the unavoidable loss of production due to simultaneous stoppage of two or more machines being operated by him. This allowance is applicable for machine or process controlled jobs.

Interference allowance varies in proportion to the number of machines assigned to the operator. The interference of the machine increases the work content.

2.3.8.3 CONTINGENCY ALLOWANCE

A contingency allowance is a small allowance of time which may be included in a standard time to meet legitimate and expected items of work or delays, the precise measurement of which is uneconomical because of their infrequent or irregular occurrence.

This allowance provides for small unavoidable delays as well as for occasional minor, extra work.

Some of the examples calling for contingency allowance are:

- Tool breakage involving removal of the tool from the holder and all other activities to insert a new tool into the tool holder.
- Power failures of small duration.
- Obtaining the necessary tools and gauges from the central tool store.

Contingency allowance should not exceed 5%.

2.3.8.4 POLICY ALLOWANCE

Policy allowances are not the genuine part of the time study and should be used with utmost care and only in clearly defined circumstances.

The usual reason for making the policy allowance is to line up standard times with requirements of wage agreement between employers and trade unions.

The policy allowance as defined by ILO:

"A policy allowance is an increment, other than bonus increment, applied to standard time (or to some constituent part of it, e.g., work content) to provide a satisfactory level of earnings for a specified level of performance under exceptional circumstances. Policy allowance is sometimes made as imperfect functioning of a division or part of a plant."

2.4 WORK SAMPLING

Work sampling is also a technique for establishing standard times for activities. This method was introduced by L.H.C. Tippett in 1934. This method is more suitable for analyzing group activities and repetitive activities which take longer duration.

If a given individual performs more than one activity, then the time standard for each activity may be computed using this method.

For example, on a printing press, a single worker will be doing composing, proofreading, printing, cutting etc. The time standards for these activities may be determined using the work sampling method.

In this method, the standard time of a specified activity is determined by computing the proportion of time the worker spends on that activity.

The percentage of the total observation a person is engaged in a given activity approximates the percentage of the total work time spent in that activity. Assume for example, that during an 8-hour study period an analyst made a total of 100 observations and 10 of those observations showed that the worker was inspecting the size of components produced on his machine. It would be assumed that 10/100 or 10% of the time was spent on inspection. Ten percent of 8 hours is 48 minutes. The proportion of time is computed by observing the operator at random intervals. Let us assume that the operator has produced 40 components during the 8-hour duration. Then the average time spent for inspection on one component is 1.2 minute (48/40 min).

This time is taken as the observed time. Later, the analyst must adjust this time for a performance rating of the operator and allowances.

The steps for the work-sampling method are listed below:

1. Perform the following preliminary tasks.
    a. Select the job which is to be studied.
    b. Inform the workers about the method study.
    c. Prepare a detailed list of activities of the job
2. Decide the total duration of observation.
   Total duration of observation = No. of hours/shifts × No. of shifts per day × No. of days
3. Determine the number of observations to be made.

4. Prepare a tour schedule to take the specified number of observations over the specified time duration.
5. Record performance rating of the workers and their activities as per the schedule.
6. Determine the acceptable number of units produced during the period.
7. Compute the percentage of working on a given task using the following formula.

$$\text{Percentage of working} = \frac{\text{Frequency of performance of a task}}{\text{Total number of observations}} \times 100$$

8. Determine the normal time.

$$\text{NT} = \frac{(\text{Total time})\ (\%\ \text{of working})\ \text{PR}}{\text{No. of acceptable units produced}}$$

9. Determine the standard time

$$\text{ST} = \text{NT} \times \text{Allowance fraction}$$

$$\text{ST} = \text{NT} \times \frac{1}{1 - \text{Percentage allowance}}$$

3 SOLVED PROBLEMS

1 In a welding shop, a direct time study was done on a welding operation. One inexperienced industrial engineer and one experienced industrial engineer conducted the study simultaneously. They agreed precisely on cycle time (shown below) but their opinion on rating the worker differed. The experienced engineer rated the worker 100% and the other engineer rated the worker 120%. They used a 10% allowance.

Cycle Time in Minutes	Number of Times Observed
20	2
24	1
29	1
32	1

From the above statement,

(i) Determine the standard time using the experienced industrial engineer's worker rating.

(ii) Find the standard time using the worker rating of the inexperienced industrial engineer.

(iii) Comment on the reliability of time study engineers.

SOLUTION:

(i) Rating the worker at 100% by the experienced industrial engineer

Average cycle time,

$$\mathrm{ACT} = \frac{\Sigma \mathrm{T}}{\text{No. of cycles}}$$

$$= \frac{(20 \times 2) + (24 \times 1) + (29 \times 1) + (32 \times 1)}{2 + 1 + 1 + 1}$$

$= 25$ min

Normal time = Average cycle time × Performance rating

NT = ACT × PR

= 25 × 100% = 25 min

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\mathrm{ST} = \frac{\mathrm{NT}}{1 - \%(\mathrm{A})}$$

$$= \frac{25}{1 - 0.10}$$

$= 27.7778$ min

(ii) Rating the worker at 120% by the inexperienced industrial engineer

Average cycle time,

$$\text{ACT} = \frac{\Sigma \text{T}}{\text{No. of cycles}}$$

$$= \frac{(20 \times 2) + (24 \times 1) + (29 \times 1) + (32 \times 1)}{2 + 1 + 1 + 1}$$

$$= 25 \text{ min}$$

Normal time = Average cycle time × Performance rating

NT = ACT × PR

= 25 × 120% = 30 min

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\text{ST} = \frac{\text{NT}}{1 - \%(\text{A})}$$

$$= \frac{30}{1 - 0.10}$$

$$= 33.3333 \text{ min}$$

(iii) Comment on the reliability of time study engineers

The task of estimating performance rating of a worker requires certain experience. So, we can rely on the results obtained by the experienced industrial engineer.

2 The time study engineer of a company was asked to fix the standard time of making a spindle using a lathe. The data of the time study are shown in the following table. The performance rating of the worker is 105%. Find the standard time for the spindle by assuming an allowance percentage of 10%.

Cycle Time in Minutes	Frequency
36	1
37	3
38	3
39	2
40	1

SOLUTION:

Average cycle time per spindle,

$$\text{ACT} = \frac{\Sigma\text{T}}{\text{No. of cycles}}$$

$$= \frac{(36 \times 1) + (37 \times 3) + (38 \times 3) + (39 \times 2) + (40 \times 1)}{1 + 3 + 3 + 2 + 1}$$

$= 37.9$ min

Normal time = Average cycle time × Performance rating

NT = ACT × PR

= 25 × 105% = 39.795 min

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\text{ST} = \frac{\text{NT}}{1 - \%(\text{A})}$$

$$= \frac{39.795}{1 - 0.10}$$

$= 44.2167$ min

3 A component has two operations, viz. turning and external thread cutting. These operations are performed by the same operator on a lathe using a continuous rod which is fed through the headstock of the lathe. After completing the operations, the component will be cut from the rod and its time is included in the thread cutting operation. Then the operator will start processing for the next piece of that component. A direct time study was done for each of the operations and the data are furnished in following table. The performance rating of the worker is 125% for the turning operation, and it is 110% for the thread cutting operation. Assuming an allowance of 10% for each of the operations, find the standard time per piece of the component.

Operation	Cycle Time in Minutes	Number of Times Observed
Turning	5	2
	6	4
	7	2
	8	2
Thread cutting	10	3
	11	5
	12	2

SOLUTION:

Operation	Cycle Time in Minutes	Number of Times Observed	Performance Rating
Turning	5	2	125%
	6	4	
	7	2	
	8	2	
Thread cutting	10	3	110%
	11	5	
	12	2	

Determination of Standard Time for Turning Operation

Average cycle time,

$$\text{ACT} = \frac{\Sigma \text{T}}{\text{No. of cycles}}$$

$$= \frac{(5 \times 2) + (6 \times 4) + (7 \times 2) + (8 \times 2)}{2 + 4 + 2 + 2}$$

$$= 6.4 \text{ min}$$

Normal time = Average cycle time × Performance rating

NT = ACT × PR

= 6.4 × 125% = 8 min

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\mathrm{ST} = \frac{\mathrm{NT}}{1 - \%(\mathrm{A})}$$

$$= \frac{8}{1 - 0.10}$$

$$= 8.8889 \text{ min}$$

Determination of Standard Time for Thread Cutting Operation

Average cycle time,

$$\mathrm{ACT} = \frac{\Sigma \mathrm{T}}{\text{No. of cycles}}$$

$$= \frac{(10 \times 3) + (11 \times 5) + (12 \times 2)}{3 + 5 + 2}$$

$$= 10.9 \text{ min}$$

Normal time = Average cycle time × Performance rating

NT = ACT × PR

= 10.9 × 110% = 11.99 min

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\mathrm{ST} = \frac{\mathrm{NT}}{1 - \%(\mathrm{A})}$$

$$= \frac{11.99}{1 - 0.10}$$

$$= 13.3222 \text{ min}$$

Standard time per piece of the component

= Standard time for turning operation per piece + Standard time for thread cutting operation per piece

= 8.8889 + 13.3222

= 22.2111 min

4 In a battery manufacturing company, two batteries are packed in a cover. This process is done manually and it involves the following operations:

1. Picking a plastic cover in left hand.
2. Wide opening the cover using both hands.
3. Picking two batteries from the running conveyor and placing them in the cover.
4. Sealing the cover.
5. Dropping the sealed cover onto another running conveyor.

It is observed that the operator who is performing these operations works at the same pace for all the operations. The performance rating of the employee is 120%. A direct time study was performed and the data on the cycle time to complete all the operations for packing the two batteries in a cover are summarized in the following table. Find the standard time to pack one packet of batteries by assuming an allowance of 12%.

Cycle Time in Seconds	Frequency
20	1
21	4
22	3
23	2

SOLUTION:

Average cycle time,

$$\text{ACT} = \frac{\Sigma \text{T}}{\text{No. of cycles}}$$

$$= \frac{(20 \times 1) + (21 \times 4) + (22 \times 3) + (23 \times 2)}{1 + 4 + 3 + 2}$$

$$= 21.6 \text{ s}$$

Normal time = Average cycle time × Performance rating

NT = ACT × PR

= 21.6 × 120% = 25.92 s

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\text{ST} = \frac{\text{NT}}{1 - \%(\text{A})}$$

$$= \frac{25.92}{1 - 0.12}$$

$$= 29.4545 \text{ s}$$

5 A time study engineer has studied the time taken to machine crankshafts. He has taken 40 observations and these are summarized in the form of frequency distribution as shown below:

Time (Minutes)	Frequency
20	15
21	10
22	10
23	5

The performance rating of the operator machining the crankshaft is 110%. Find the standard time for machining the crankshaft by assuming an allowance of 15%.

SOLUTION:

Average cycle time,

$$\mathrm{ACT} = \frac{\Sigma \mathrm{T}}{\mathrm{No.\ of\ cycles}}$$

$$= \frac{(20 \times 15) + (21 \times 10) + (22 \times 10) + (23 \times 5)}{15 + 10 + 10 + 5}$$

$= 21.125$ min

Normal time = Average cycle time × Performance rating

NT = ACT × PR

= 21.125 × 110% = 23.2375 min

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\mathrm{ST} = \frac{\mathrm{NT}}{1 - \%(\mathrm{A})}$$

$$= \frac{23.2375}{1 - 0.15}$$

$= 27.3382$ min

6 The observed times and the performance ratings for the five elements are given in the following table. Compute the standard time by assuming allowance as 17% of the basic time.

Element	1	2	3	4	5
Observed time (seconds)	12	5	30	8	6
Performance rating	85%	80%	90%	85%	80%

SOLUTION:

Normal time = Observed time × Performance rating

Element	Observed Time (seconds)	Performance Rating	Normal time (seconds)
1	12	85%	12 × 0.85 = 10.2
2	5	80%	5 × 0.80 = 4
3	30	90%	30 × 0.90 = 27
4	8	85%	8 × 0.85 = 6.8
5	6	80%	6 × 0.80 = 4.8
			Total = 52.8

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\text{ST} = \frac{\text{NT}}{1 - \%(\text{A})}$$

$$= \frac{52.8}{1 - 0.17}$$

$$= 63.6145 \text{ s}$$

7 A job consists of three work elements and all are performed by the same operator. An analyst conducted work sampling to determine the standard time for the job. The duration of the study is one shift with 400 minutes of effective time. The details of observations are summarized in the following table. The total number of acceptable units produced during the study period is 150 units. Determine the standard time by assuming an allowance of 10%.

Work Element Number	Frequency of Performance	Performance Rating
1	70	80%
2	80	120%
3	50	110%

SOLUTION:

$$\text{Percentage of working} = \frac{\text{Frequency of performance of a task}}{\text{Total number of observations}} \times 100$$

$$\text{Normal time} = \frac{\text{Total time} \times \text{Percentage of working} \times \text{Performance rating}}{\text{No. of acceptable units produced}}$$

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

Work Element Number	Percentage of Working	Normal Time (minutes)	Standard Time (minutes)
1	$\frac{70}{70+80+50} \times 100 = 35\%$	$\frac{400 \times 0.35 \times 0.8}{150} = 0.7467$	$\frac{0.7467}{1-0.10} = 0.8296$
2	$\frac{80}{70+80+50} \times 100 = 40\%$	$\frac{400 \times 0.40 \times 1.2}{150} = 1.28$	$\frac{1.28}{1-0.10} = 1.4222$
3	$\frac{50}{70+80+50} \times 100 = 25\%$	$\frac{400 \times 0.25 \times 1.1}{150} = 0.7333$	$\frac{0.7333}{1-0.10} = 0.8148$
			Total = 3.0667

The standard time of the job per unit = 3.0667 min

8 A job consists of two operations and both are performed by the same operator. The time study engineer decided to conduct work sampling to fix the standard time for the job. The effective shift time is 450 minutes and the number of acceptable units of the job produced is 15. The details of the observations of the work sampling are shown in the following table. Determine the standard time of the job by assuming an allowance of 10%.

Work Element Number	Frequency of Performance	Performance Rating
1	20	90%
2	30	110%

SOLUTION:

$$\text{Percentage of working} = \frac{\text{Frequency of performance of a task}}{\text{Total number of observations}} \times 100$$

$$\text{Normal time} = \frac{\text{Total time} \times \text{Percentage of working} \times \text{Performance rating}}{\text{No. of acceptable units produced}}$$

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

Work Element Number	Percentage of Working	Normal Time (minutes)	Standard Time (minutes)
1	$\frac{20}{20+30} \times 100 = 40\%$	$\frac{450 \times 0.40 \times 0.90}{15} = 10.8$	$\frac{10.8}{1-0.10} = 12$
2	$\frac{30}{20+30} \times 100 = 60\%$	$\frac{450 \times 0.60 \times 1.10}{15} = 19.8$	$\frac{19.8}{1-0.10} = 22$
			Total = 34

The standard time of the job per unit = 34 min

9 In a binding press, binding of large size books consists of the following operations:

1. Stitching of the pages
2. Trimming the book
3. Cover making
4. Joining the book with the cover and finishing

The first operation is carried by Mr. Ramesh and the rest of the operations are performed by Mr. Suresh. It is decided to use conventional time study technique for the first operation and work sampling method for the rest of the operations. The effective time available per shift is 480 minutes and the number of acceptable book binding done during this period is 8. The details of the time study data of the first operation are shown in the table. The performance rating of Mr. Ramesh is 105%. The details of the work sampling data are shown in the table.

Time Study Data for Operation 1

Cycle time (minutes)	18	19	20	21	22
Frequency	2	3	5	4	1

Work Sampling Data

Operation	Frequency of Performance	Performance Rating
2	7	120%
3	21	125%
4	22	95%

Find the standard time of the entire job of book binding per book by assuming an allowance of 10% to all the operations.

SOLUTION:

Determination Standard Time of Operation 1

Time Study Data for Operation 1

Cycle Time (minutes)	Frequency	Performance Rating
18	2	
19	3	
20	5	105%
21	4	
22	1	

Average cycle time,

$$\mathrm{ACT} = \frac{\Sigma \mathrm{T}}{\text{No. of cycles}}$$

$$= \frac{(18 \times 2) + (19 \times 3) + (20 \times 5) + (21 \times 4) + (22 \times 1)}{2 + 3 + 5 + 4 + 1}$$

$$= 19.9333 \text{ min}$$

Normal time = Average cycle time × Performance rating

NT = ACT × PR

= 19.9333 × 105% = 20.93 min

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

$$\text{ST} = \frac{\text{NT}}{1 - \%(\text{A})}$$

$$= \frac{20.93}{1 - 0.10}$$

$= 23.2556$ min

Determination Standard Time for the Operations 2 to 4

$$\text{Percentage of working} = \frac{\text{Frequency of performance of a task}}{\text{Total number of observations}} \times 100$$

$$\text{Normal time} = \frac{\text{Total time} \times \text{Percentage of working} \times \text{Performance rating}}{\text{No. of acceptable units produced}}$$

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

Work Element Number	Percentage of Working	Normal Time (minutes)	Standard Time (minutes)
1	$\frac{7}{7+21+22} \times 100 = 14\%$	$\frac{480 \times 0.14 \times 1.20}{8} = 10.08$	$\frac{10.08}{1-0.10} = 11.2$
2	$\frac{21}{7+21+22} \times 100 = 42\%$	$\frac{480 \times 0.42 \times 1.25}{8} = 31.5$	$\frac{31.5}{1-0.10} = 35$
3	$\frac{22}{7+21+22} \times 100 = 44\%$	$\frac{480 \times 0.44 \times 0.95}{8} = 25.08$	$\frac{25.08}{1-0.10} = 27.8667$
			Total = 74.0667

The standard time of the job per unit

= Standard time of operation 1 + Standard time of operations from 2 to 4

= 23.2556 + 74.0667

= 97.3222 min

10 A job consists of four work elements and all are performed by the same operator. An analyst conducted work sampling to determine the standard time for the job. The duration of the study is one day with two shifts. Each shift has 420 minutes of effective time. The details of observations are summarized in the following table. The total number of acceptable units produced during the study period is 115 units. Determine the standard time by assuming an allowance of 12%.

Work Element Number	Frequency of Performance	Performance Rating
1	50	90%
2	90	150%
3	75	100%
4	85	115%

SOLUTION:

$$\text{Percentage of working} = \frac{\text{Frequency of performance of a task}}{\text{Total number of observations}} \times 100$$

$$\text{Normal time} = \frac{\text{Total time} \times \text{Percentage of working} \times \text{Performance rating}}{\text{No. of acceptable units produced}}$$

$$\text{Standard time} = \frac{\text{Normal time}}{1 - \text{Percentage allowance}}$$

Work Element Number	Percentage of Working	Normal Time (minutes)	Standard Time (minutes)
1	$\frac{50}{50+90+75+85} \times 100 = 16.67\%$	$\frac{420 \times 0.1667 \times 0.90}{115} = 0.5478$	$\frac{0.5478}{1-0.12} = 0.6225$
2	$\frac{90}{50+90+75+85} \times 100 = 30\%$	$\frac{420 \times 0.30 \times 1.50}{115} = 1.6435$	$\frac{1.6435}{1-0.12} = 1.8676$
3	$\frac{75}{50+90+75+85} \times 100 = 25\%$	$\frac{420 \times 0.25 \times 1.00}{115} = 0.9130$	$\frac{0.9130}{1-0.12} = 1.0375$
4	$\frac{85}{50+90+75+85} \times 100 = 28.33\%$	$\frac{420 \times 0.2833 \times 1.15}{115} = 1.19$	$\frac{1.19}{1-0.12} = 1.3522$
			Total = 4.8799

The standard time of the job per unit = 4.8799 min

4 TWO MARKS QUESTIONS AND ANSWERS

1. **Define industrial engineering.**

   American Institute of Industrial Engineers (AIIE) defines Industrial Engineering as follows: Industrial Engineering is concerned with the design, improvement and installation of integrated system of men, materials and equipment. It draws upon specialised knowledge and skills in the mathematical, physical sciences together with the principles and methods of engineering analysis and design to specify, predict and evaluate the results to be obtained from such systems.

2. **Define method study.**

   Method study is the systematic recording and critical examination of existing and proposed ways of doing work, as a means of developing and applying easier and more effective methods and reducing costs.

3. **Name the various steps involved in method study.**

   i. Select
   ii. Record
   iii. Examine
   iv. Develop
   v. Define
   vi. Install
   vii. Maintain

4. **Define work measurement or time study.**

   Work measurement is the application of techniques designed to establish the time for a qualified worker to carry out a specified job at a defined level of performance.

5. **What are the various techniques of work measurement?**

   - Time study
   - Synthesis
   - Work sampling
   - Analytical estimating
   - Predetermined motion and time study

6. **Name the various steps involved in time study.**

   i. Select
   ii. Obtain & record
   iii. Define
   iv. Measure
   v. Extend
   vi. Determine
   vii. Compute

7. **Name the various timing methods in stop watch study.**

   - Fly back or snap back method
   - Cumulative or continuous method

8. What are the different types of allowances?

- Relaxation allowance
- Interference allowance
- Contingency allowance
- Policy allowance

9. Define ergonomics.

Ergonomics is the study of the man in relation to his work. ILO defines it as "The application of human biological sciences along with engineering sciences to achieve optimum mutual adjustment of men and his work, the benefits being measured in terms of human efficiency and well-being."

10. What are the various approaches for make or buy decision?

- Simple cost analysis
- Economic analysis
- Break even analysis

11. State any five principles of material handling.

- Planning principle
- Systems principle
- Space utilisation principle
- Unit load principle
- Gravity principle
- Material flow principle
- Simplification principle
- Safety principle
- Mechanisation principle
- Standardisation principle
- Flexibility principle
- Equipment selection principle
- Dead weight principle
- Motion principle
- Idle time principle
- Maintenance principle
- Obsolescence principle
- Capacity principle
- Control principle
- Performance principle

KARPAGAM ACADEMY OF HIGHER EDUCATION

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FACULTY OF ENGINEERING

DEPARTMENT OF AUTOMOBILE ENGINEERING

INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH

UNIT-II

PRODUCTION PLANNING AND CONTROL

CONTENTS

1 PRODUCTION PLANNING AND CONTROL

1.1 INTRODUCTION

Production activity constitutes the transformation of materials into a desirable output (products). The production consists of a series of sequential operations to produce a desirable product acceptable to the customer and meets the customer demand, with respect to the quality and intended function. Production is an organised activity which has got specific objectives. "The efficiency of a production system is stated in terms of its ability to produce the products with the required quantity and specified quality at a predetermined cost and pre-established time." Production planning as control is a tool available to the management to achieve the stated objectives. Thus, a production system is encompassed by the four factors, i.e., quantity, quality, cost and time. Production planning starts with the analysis of the given data, i.e., demand for products, delivery schedule, etc., and on the basis of the information available, a scheme of utilisation of firms resources like machines, materials and men are worked out to obtain the target in the most economical way.

Once the plan is prepared, then operations (execution of plan) are performed in line with the details given in the plan. Production control comes into action if there is any deviation between the actual and planned. The corrective action is taken so as to achieve the targets set as per plan by using control techniques.

Thus production planning and control can be defined as the "direction and coordination of firms resources towards attaining the prefixed goals." Production planning and control helps to achieve uninterrupted flow of materials through production line by making available the materials at right time and required quantity.

1.2 NEED FOR PPC

Production planning and control serves as a useful tool to coordinate the activities of the production system by proper planning and control system.

Production system can be compared to the nervous system with PPC as a brain. Production planning and control is needed to achieve:

- Effective utilisation of the firm's resources.
- To achieve the production objectives with respect to quality, quantity, cost and timeliness of delivery.
- To obtain the uninterrupted production flow in order to meet customers varied demand with respect to quality and committed delivery schedule.
- To help the company to supply good quality products to the customer on a continuous basis at competitive rates.

1.3 PRODUCTION PLANNING AND PRODUCTION CONTROL

1.3.1 PRODUCTION PLANNING

Production planning is a pre-production activity. It is the pre-determination of manufacturing requirements such as manpower, materials, machines and manufacturing process.

Ray wild defines "Production planning is the determination, acquisition and arrangement of all facilities necessary for future production of products". It represents the design of a production system. Apart from planning the resources, it is going to organise the production.

Based on the estimated demand for the company's products, it is going to establish the production programme to meet the targets set using the various resources.

1.3.2 PRODUCTION CONTROL

In spite of planning to the minute details, yet always (most of the time) it is not possible to achieve production 100% as per the plan. There may be innumerable factors which affect the production system and because of which there is a deviation from the actual plan. Some of the factors that affect are:

- Non-availability of materials (due to the shortage, etc.).
- Plant, equipment and machine breakdown.
- Changes in demand and rush orders.
- Absenteeism of workers.
- Lack of coordination and communication between various functional areas of business.

Thus, if there is a deviation between actual production and planned production, the control function comes into action. Production control through control mechanism tries to take corrective action to match the planned and actual production. Thus production control reviews the progress of the work and takes corrective steps in order to ensure that programmed production takes place. The essential steps in control activity are:

1. Initiating the production.
2. Progressing.
3. Corrective action based upon the feedback and reporting back to the production planning.

1.4 OBJECTIVES OF PPC

- Systematic planning of production activities to achieve the highest efficiency in the production of goods/services.
- To organise the production facilities like machines, men, etc., to achieve stated production objectives with respect to quantity and quality time and cost.
- Optimum scheduling of resources.
- Coordinate with other departments relating to production to achieve regularly balanced and uninterrupted production flow.
- To conform to delivery commitments.
- Materials planning and control.
- To be able to make adjustments due to changes in demand and rush orders.

1.5 FUNCTIONS OF PPC

The functions of production planning and controlling are depicted in the figure. Pre-planning is a macro level planning and deals with the analysis of data and is an outline of the planning policy based upon the forecasted demand, market analysis and product design and development. This stage is concerned with process design (new processes and developments, equipment policy and replacement and workflow (plant layout). The preplanning function of PPC is concerned with decision-making with respect to methods, machines and workflow with respect to availability, scope and capacity.

The planning function starts once the task to be accomplished is specified, with the analysis of 4 M's, i.e., Machines, Methods, Materials and Manpower. This is followed by process planning (routing). Both short-term (near future) and long-term planning are considered. Standardisation, simplification of products and processes are given due consideration.

Control phase is effected by dispatching, inspection and expediting. Finally, evaluation makes the PPC cycle complete and corrective actions are taken through feedback from the analysis. A good communication and feedback system is essential to enhance and ensure the effectiveness of PPC.

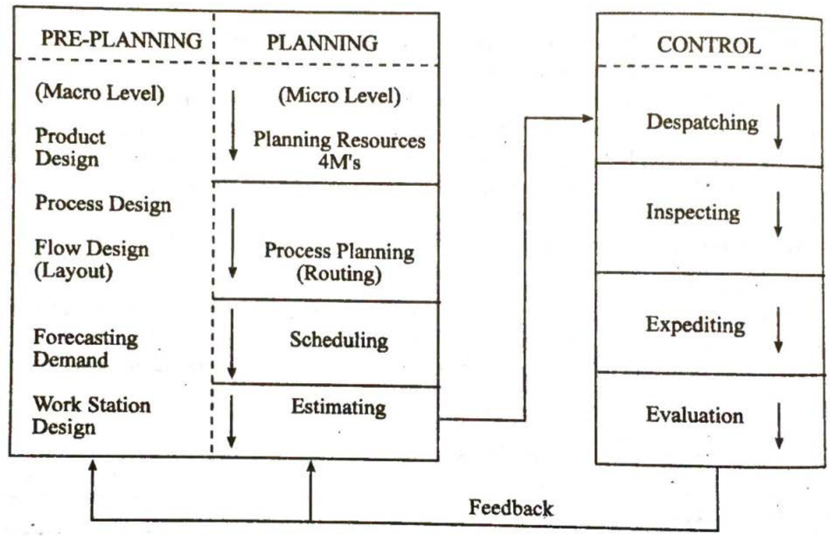


Figure 1 Functions of PPC

The Main Functions of PPC

1. *Materials function:* Raw materials, finished parts and bought out components should be made available in the required quantities and at required time to ensure the correct start and end for each operation resulting in uninterrupted production. The function includes the specification of materials (quality and quantity), delivery dates, variety reduction (standardisation), procurement and make or buy decisions.
2. *Machines and equipment:* This function is related to the detailed analysis of available production facilities, equipment downtime, maintenance policy procedure and schedules. Thus, the duties include the analysis of facilities and making their availability with minimum downtime because of breakdowns.
3. *Methods:* This function is concerned with the analysis of alternatives and selection of the best method with due consideration to constraints imposed. Developing specifications for processes is an important aspect of PPC and determination of the sequence of operations.
4. *Process planning (Routing):* It is concerned with the selection of path or route which the raw material should follow to get transformed into the finished product. The duties include:
   - Fixation of a path of travel giving due consideration to the layout.
   - Breaking down of operations to define each operation in detail.
   - Deciding the setup time and process time for each operation.
5. *Estimating:* Once the overall method and sequence of operations are fixed and process sheet for each operation is available, then the operations times are estimated. This function is carried out using extensive analysis of operations along with methods and routing and standard time for operation are established using work measurement techniques.
6. *Loading and scheduling:* Scheduling is concerned with the preparation of machine loads and fixation of starting and completion dates for each of the operations. Machines have to be loaded according to their capability of performing the given task and according to their capacity. Thus the duties include:
   - Loading the machines as per their capability and capacity.
   - Determining the start and completion times for each operation.

- To coordinate with sales department regarding delivery schedules.

7. *Dispatching:* This is the execution phase of planning. It is the process of setting production activities in motion through the release of orders and instructions. It authorises the start of production activities by releasing materials, components, tools, fixtures and instruction sheets to the operator. The activities involved are:
    - To assign definite work to definite machines, work centres and men.
    - To issue required materials from stores.
    - To issue jigs, fixtures and make them available at correct point of use.
    - Release necessary work orders, time tickets, etc., to authorise the timely start of operations.
    - To record the start and finish time of each job on each machine or by each man.
8. *Expediting:* This is the control tool that keeps a close observation of the progress of the work. It is a logical step after dispatching which is called "follow-up" or "progress". It coordinates extensively to execute the production plan. The progressing function can be divided into three parts, i.e., follow up of materials, follow up of work-in-process and follow up of assembly. The duties include:
    - Identification of bottlenecks and delays and interruptions because of which the production schedule may be disrupted.
    - To devise action plans (remedies) for correcting the errors.
    - To see that production rate is in line with schedule.
9. *Inspection:* It is a major control tool. Though the aspects of quality control are the separate function, this is of very much important to PPC both for the execution of the current plans and its scope for future planning. This forms the basis for knowing the limitations with respects to methods, processes, etc., which is very much useful for the evaluation phase.
10. *Evaluation:* This stage though neglected is crucial to the improvement of productive efficiency. A thorough analysis of all the factors influencing the production planning and control helps to identify the weak spots and the corrective action with respect to preplanning and planning will be effected by feedback. The success of this step depends on communication, data and information gathering and analysis.

2 PLANT LAYOUT

2.1 DEFINITION

Plant layout refers to the physical arrangement of production facilities. It is the configuration of departments, work centres and equipment in the conversion process.

According to Moore "Plant layout is a plan of an optimum arrangement of facilities including personnel, operating equipment, storage space, material handling equipment and all other supporting services along with the design of the best structure to contain all these facilities."

The overall objective of plant layout is to design a physical arrangement that meets the required output quality and quantity most economically.

2.2 OBJECTIVES OF PLANT LAYOUT

The primary goal of the plant layout is to maximise the profit by the arrangement of all the plant facilities to the best advantage of total manufacturing of the product.

Thus the objective of plant planning is the best relationship between output, space and manufacturing cost.

The objectives of plant layout are:

- Streamline the flow of materials through the plant.
- Facilitate the manufacturing process.
- Maintain high turnover of in process inventory.
- Minimise materials handling.
- Effective utilisation of men, Equipment and space.
- Make effective utilisation of cubic space.
- The flexibility of manufacturing operations and arrangements.
- Provide for employee convenience, safety and comfort.

2.3 PRINCIPLES OF PLANT LAYOUT

- *Principle of integration:* A good layout is one that integrates men, materials, machines and supporting services and others in order to get the optimum utilisation of resources and maximum effectiveness.
- *Principle of minimum distance:* This principle is concerned with the minimum travel (or movement) of man and materials. The facilities should be arranged such that, the total distance travelled by the men and materials should be minimum and as far as possible straight line movement should be preferred.
- *Principle of cubic space utilisation:* The good layout is one that utilise both horizontal and vertical space. It is not only enough if only the floor space is utilised optimally but the third dimension, i.e., the height is also to be utilised effectively.
- *Principle of flow:* A good layout is one that makes the materials to move in the forward direction towards the completion stage, i.e., there should not be any backtracking.
- *Principle of maximum flexibility:* The good layout is one that can be altered without much cost and time, i.e., future requirements should be taken into account while designing the present layout.
- *Principle of safety and security and satisfaction:* A good layout is one that gives due consideration to workers safety and satisfaction and safeguards the plant and machinery against fire, theft, etc.
- *Principle of minimum handling:* A good layout is one that reduces the material handling to the minimum.

2.4 ADVANTAGES OF PLANT LAYOUT

- *Advantages to the worker:* A good layout will reduce the effort of the workers and minimises the manual material handling. It reduces the number of accidents and provides better working conditions.
- *Advantages to the management:* Effective plant layout reduces the labour costs and enhances productivity thus ultimately reducing the cost per unit. This helps the management to gain competitiveness in manufacturing.
- *Advantages to manufacturing:* Minimises the movement between work centres and also results in the reduced manufacturing cycle.
- *Advantages to production control:* A good layout facilitates production through uniform and uninterrupted flow of materials and helps to carry out production activities within the predetermined time period and with effectiveness.

2.5 FACTORS INFLUENCING PLANT LAYOUT

- Type of production - engineering industry, process industry.
- Production system - job shop, batch production, mass production.
- Scale of production.

- Availability of the total area.
- Arrangement of the material handling system.
- Type of building - single storey or multi-storey.
- Future expansion plan.
- Type of production facilities - dedicated or general purpose.

2.6 TYPES OF MANUFACTURING SYSTEMS

The production system (facility, equipment and operating methods) that a company uses depends upon the type of the product that is offered to the customer and the strategy that it employees to serve its customers.

Make to stock production: In this type, the products are manufactured and placed in stock before the customer's order is received. The product is dispatched to the customer "Off the shelf" from finished goods inventory after receipt of a customer order.

Examples: Manufactures of standard items like bearings, nut and bolts, etc., produce and keep the finished goods inventory.

Make to order production: Some companies make the product to order and manufacture the product after receipt of a customer order. Here the lead time to deliver the item to the customer will be more as the production activity starts only after the receipt of the firm order.

Types of Production

According to volume and standardisation of the production of the products, the manufacturing systems are classified as:

- Job type production,
- Batch production, and
- Continuous production.

2.6.1 JOB TYPE PRODUCTION

It is characterised by manufacturing of one or few quantities of products designed and produced as per the specifications of the customers within the prefixed time and cost, i.e., this type of production is distinguished by high variety and low volume.

Characteristics of job type production:

- High variety and low volume.
- General purpose machines and equipment to perform a wider range of operations.
- The flow of materials and components between different workstation is highly discontinuous due to an imbalance in work content.
- Manufacturing cycle time is more.
- A highly skilled workforce is required.
- Highly competent and qualified supervisors are required.
- Very large work-in-process inventory.
- Flexible material handling system with a capability to move objects of various sizes and shapes along widely varying paths.

- Difficulty in planning, scheduling and coordinating the productions of numerous components of a wide variety.

2.6.2 BATCH PRODUCTION

Batch production is characterised by the manufacture of a limited number of products produced at regular intervals and stocked at warehouses awaiting sales.

Examples: Pharmaceutical industry, chemical industry, assembly shops such as machine tools, subcontractors who take component for processing from a large manufacturer.

Characteristics of batch production:

- Short production
- The plant and machineries set-up is used for a limited number of parts and then it is used to make a different product.
- More number of set-ups.
- The workers are expected to possess skill in one particular manufacturing operation.
- The amount of supervision required is less compared to job type.
- Plant and machineries are flexible.
- Manufacturing cycle time is comparatively lower than job production.
- Large work-in-process inventory.
- Flexible material handling system.

2.6.3 CONTINUOUS (MASS) PRODUCTION

This is characterised by high volume and low variety. This manufactures several standard products produced and stocked in the warehouses as finished goods waiting to be despatched.

Examples: Plastic goods, manufacture and assembly shops of automobiles, etc.

Characteristics of continuous production:

- The flow of material is continuous.
- Special purpose machines are used.
- Material handling system is mechanised most of the time by conveyers, etc.
- Relatively lower skilled persons can manage work.
- Shorter cycle time.
- Work-in-process is comparatively low because of line balancing.
- Higher inventory of raw materials.
- Less flexibility of equipment and machines.

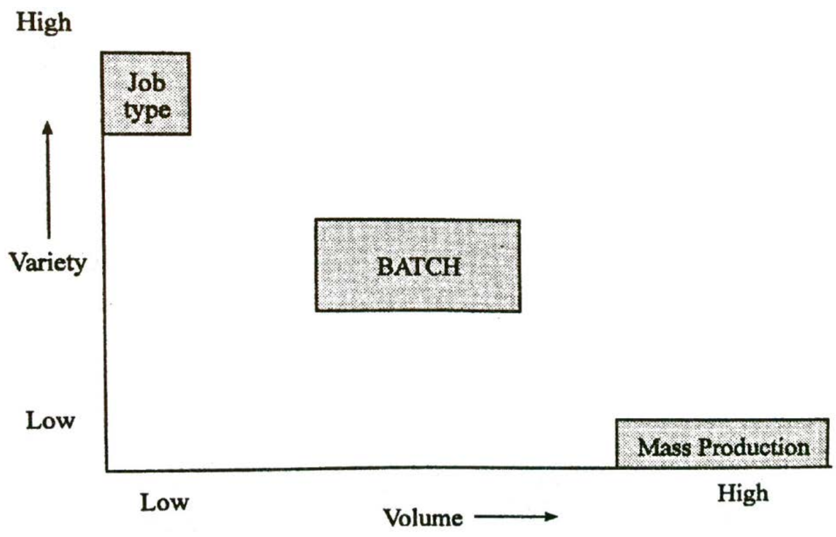


Figure 2 Types of manufacturing systems

2.7 TYPES OF LAYOUT

Layouts can be classified into the following categories.

- Process layout or functional layout
- Product layout or line layout
- Group layout or combination layout
- Fixed position layout or project type of layout

2.7.1 PROCESS LAYOUT (FUNCTIONAL LAYOUT)

In a process layout, similar machines and services are located together. e.g., all lathes, milling machines, etc., are grouped in the shop will be clustered in like groups. Process layout is normally used when the production volume is not sufficient to justify a product layout. Typically, job shops employ process layouts due to the variety of products manufactured and their low production volumes.

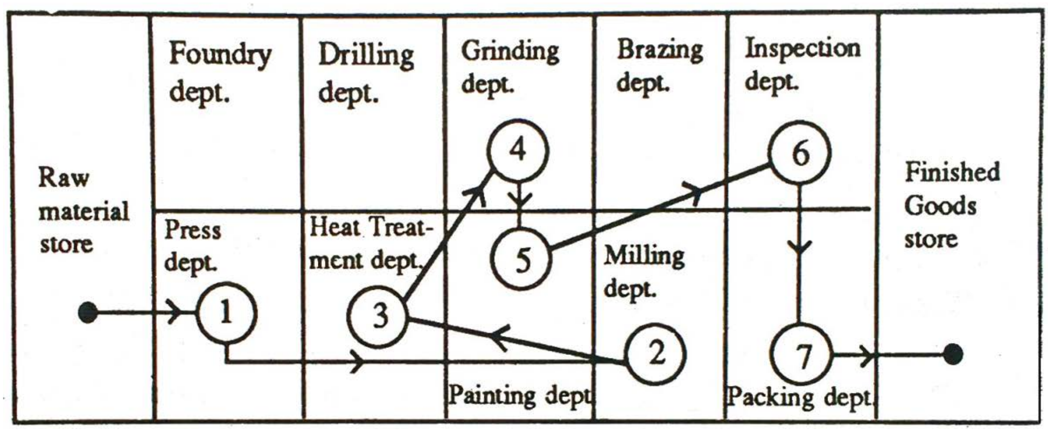


Figure 3 Process layout

Advantages

- Machines are better utilized; fewer machines are required.

- A high degree of flexibility in terms of task allocation to machines exists.
- Comparatively low investment in machines is required.
- The diversity of tasks offers a more interesting and satisfying occupation for the operator.

Limitations

- Material handling cost will be high.
- Production planning and control systems are more involved.
- Throughput time is longer.
- Large amounts of in-process inventory will result.
- Space and capital are tied up by work in process.
- Higher grades of skill are required.

2.7.2 PRODUCT LAYOUT (LINE LAYOUT)

In this type of layout, the machines are arranged in the sequence as required by the product. The product layout is selected when the volume of production of a product is high such that a separate production line to manufacture it can be justified. In a strict product layout, machines are not shared by different products. Therefore the production volume must be sufficient to achieve satisfactory utilization of the equipment.

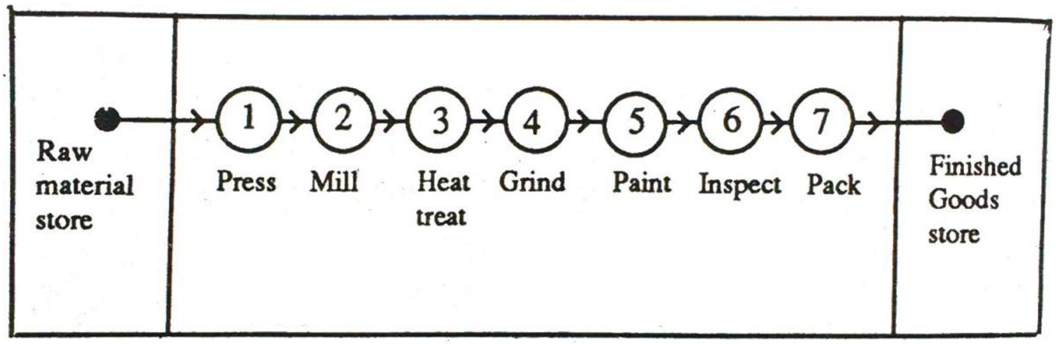


Figure 4 Product layout

Advantages

- The flow of product will be smooth and logical in flow lines.
- In-process inventory is less.
- Throughput time is less.
- Material handling cost is minimum.
- Operators need not be skilled.
- Simple production planning and control systems are possible.
- Less space is occupied by work in transit and for temporary storage.

Limitations

- A breakdown of one machine in a product line may cause stoppages of machines in the downstream of the line.
- A change in product design may require major alterations in the layout.
- The line output is decided by the bottleneck machine.
- Comparatively high investment in equipment is required.

2.7.3 GROUP LAYOUT (COMBINATION LAYOUT)

A group layout is a combination of the product layout and process layout. It combines the advantages of both layout systems. If there are *m* machines and *n* components, in a group layout, the *m*-machines and *n*-components will be divided into a distinct number of machine-component cells (groups) such that all the components assigned to a cell are almost processed within that cell itself. Here, the objective is to minimize the inter-cell movements.

The basic aim of a group technology layout is to identify families of components that require similar processing on a set of machines. In turn, these machines are grouped into cells. Each cell is capable of satisfying all the requirements of the component family assigned to it.

The layout design process considers mostly a single objective while designing layouts, In process layout, the objective is to minimize the total cost of materials handling. Because of the nature of the layout, the cost of equipments will be the minimum in this type of layout. In product layout, the cost of materials handling will be at the absolute minimum. But the cost of equipments would not be at the minimum if the equipments are not fully utilized.

In group technology layout, the objective is to minimize the sum of the cost of transportation and the cost of equipments. So, this is called as multi-objective layout.

2.7.4 FIXED POSITION LAYOUT (PROJECT TYPE OF LAYOUT)

In this type of layout, the material, or major components remain in a fixed location and tools, machinery, men and other materials are brought to this location. The shipbuilding industry commonly employs this type of layout.

This type of layout is suitable when one or few pieces of identical heavy products are to be manufactured and when the assembly consists of a large number of heavy parts, the cost of transportation of these parts is very high.

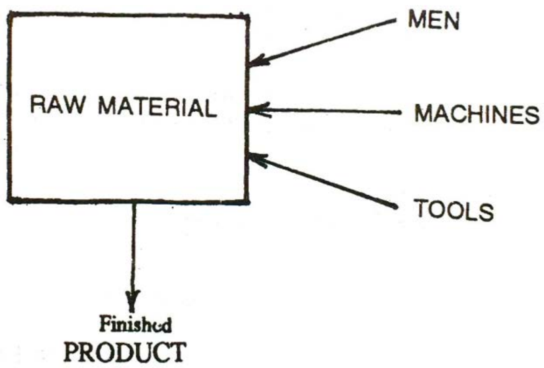


Figure 5 Fixed position layout

Advantages

- Helps in job enlargement and upgrades the skills of the operators.
- The workers identify themselves with a product in which they take interest and pride in doing the job.
- Greater flexibility with this type of layout.
- Layout capital investment is lower.

3 FORECASTING

3.1 INTRODUCTION

A forecast is an estimate of an event which will happen in future. The event may be the demand of a product, rainfall at a particular place, the population of a country, or growth of technology. The forecast value is not a deterministic quantity. Since it is only an estimate based on the past data related to a particular event, proper care must be given in estimating it.

In any industrial enterprise, forecasting is the first level decision activity. That is the demand of a particular product must be available before taking up any other decision problems like materials planning, scheduling, type of production system (mass or batch production) to be implemented, etc.

So, forecasting provides a basis for coordination of plans for activities in various parts of a company. All the functional managers in any organization will base their decisions on the forecast value. So, it is vital information for the organization. Due to these reasons, proper care should be exercised while estimating forecast values.

In business, forecasts may be classified into technology forecasts, economic forecasts and demand forecasts.

Forecasting as defined by American Marketing Association is: “An estimate of sales in physical units (or monetary value) for a specified future period under a proposed marketing plan or programme and under the assumed set of economic and other forces outside the organisation for which the forecast is made.”

3.2 DEMAND FORECASTING

- Majority of the activities of the industries depend upon future sales.
- Projected demand for the future assists in decision-making with respect to investment in plant and machinery, market planning and programmes.
- To schedule the production activity to ensure optimum utilisation of plant’s capacity.
- To prepare material planning to take up replenishment action to make the materials available at right quantity and right time.
- To provide information about the relationship between demands for different products in order to obtain a balanced production in terms of the quantity required of different products as a function of time.
- Forecasting is going to provide a future trend which is very much essential for product design and development.

Thus, in this changing and uncertain techno-economic and marketing scenario, forecasting helps to predict the future with accuracy.

3.3 LONG-TERM AND SHORT-TERM FORECASTS

Depending upon the period for which the forecast is made, it is classified as long-term forecasting and short-term forecasting. Forecasts which cover the periods less than one year is termed as short-term forecasting, and which cover the period over one year (5 years or 10 years) future are termed as long-term forecasts. Short-term forecasts are made for the purpose of materials control, loading and scheduling and budgeting. Long-term forecasts are made for the purposes of product diversification, sales and advertising budgets, capacity planning and investment planning.

3.4 FACTORS AFFECTING FORECAST (DEMAND)

The factors affecting forecast are given below:

- Business cycle
- Random variation
- Customer's plan
- Product's life cycle
- Competition's efforts and prices
- Customer's confidence and attitude
- Quality
- Credit policy
- Design of goods or services
- Reputation for service
- Sales effort
- Advertising

3.5 FORECASTING MODELS

The forecasting techniques can be classified into qualitative techniques and quantitative techniques. Qualitative techniques use subjective approaches. These are useful where no data is available and are useful for new products. Quantitative techniques are based on historical data. These are more accurate and computers can be used to speed up the process.

Quantitative forecasting techniques

- Simple moving average
- Single exponential smoothing
- Double moving average
- Double exponential smoothing
- Simple regression
- Semi-average method
- Multiple regression
- Box Jenkins

Qualitative forecasting techniques

- Delphi type method
- Market surveys

3.6 MEASURES OF FORECAST ACCURACY

Demand forecast influences most of the decisions in all the functions. Hence, it must be estimated with the highest level of precision. Some common measures are inevitable to measure the accuracy of a forecasting technique. This measure may be an aggregate error (deviation) of the forecast values from the actual demands. The different types of errors which are generally computed are as presented below.

Mean Absolute Deviation (MAD): It is the mean of absolute deviations of forecast demands from actual demand values. The MAD is sometimes called the mean absolute error (MAE).

$$\mathrm{MAD} = \frac{\sum_{t=1}^{n} |D(t) - F(t)|}{n}$$

where D(t) – Demand for period *t*

F(t) – Forecast demand for the period *t*

n – Number of time period used

Mean Square Error (MSE): Mean square error is the mean of the squares of the deviations of the forecast demands from the actual demand values. Usually, the effects on operations of small errors are not serious. These errors may be smoothed out by inventory or overtime work. It will be difficult to have smoothed values for forecast even if there are few large errors. Consequently, a method of measuring errors that penalizes large errors more than small errors is sometimes desired. The mean square error (MSE) provides this type of measure of forecast error.

$$\mathrm{MSE} = \frac{\sum_{t=1}^{n} [D(t) - F(t)]^2}{n}$$

Mean Forecast Error (MFE): Mean forecast error (MFE) is the mean of the deviations of the forecast demands from the actual demands.

$$\mathrm{MFE} = \frac{\sum_{t=1}^{n} [D(t) - F(t)]}{n}$$

Mean Absolute Percent Error (MAPE): Mean absolute percentage error (MAPE) is the mean of the percent deviations of the forecast demands from the actual demands.

$$\mathrm{MAPE} = \frac{1}{n} \sum_{t=1}^{n} \frac{|D(t) - F(t)|}{D(t)} \times 100$$

4 PRODUCTION CONTROL

4.1 INTRODUCTION

Production control provides the foundation on which most of the other controls are based. Control is described as the constraining the activities to follow the plans. "Production control is the function of management which plans, directs and controls the material supply and processing activities of an enterprise, so that specified products are produced by specified methods to meet an approved sales programme." These activities were carried out in such a manner that the labour, plant and capital available are used to the best advantage. The production control specifies three levels – programming, ordering and dispatching.

- Programming plans the production output of products.
- Ordering plans the output of components from the suppliers and departments which is necessary to meet the programme.
- Dispatching considers each department in turn and plans the output from machines and work centres necessary to carry out the orders.

4.2 OUTLINE OF PRODUCTION CONTROL

4.2.1 LOADING AND SCHEDULING

The sales department will issue works order which will authorise the manufacture of a product or group of products. This order is the starting point for all activities of the production control department concerned with the manufacturing of products. The master production schedule (MPS) is prepared which involves assessing labour, and material requirements and availability and determining the dates by which major functions must be completed. The loading of various work centres is carried out. A copy of the master schedule will be passed to the material control which will check material availability.

4.2.2 MATERIAL CONTROL

The function of a material control section of production control is that of assessing the need for material and then taking an appropriate step (actions) to meet the requirements.

4.2.3 DISPATCH AND PROGRESS

Manufacturing is actually initiated at an appropriate time which collects together all relevant documents, verifies availability of each of the factors of production and authorises the start of production activities by an issue of authorising documents. The progress section will monitor performance and verifying that the requirements of the master schedule are being fulfilled. Any deviations from this schedule are brought to the notice of the concerned persons and corrective actions are devised to keep the deviation at a minimum.

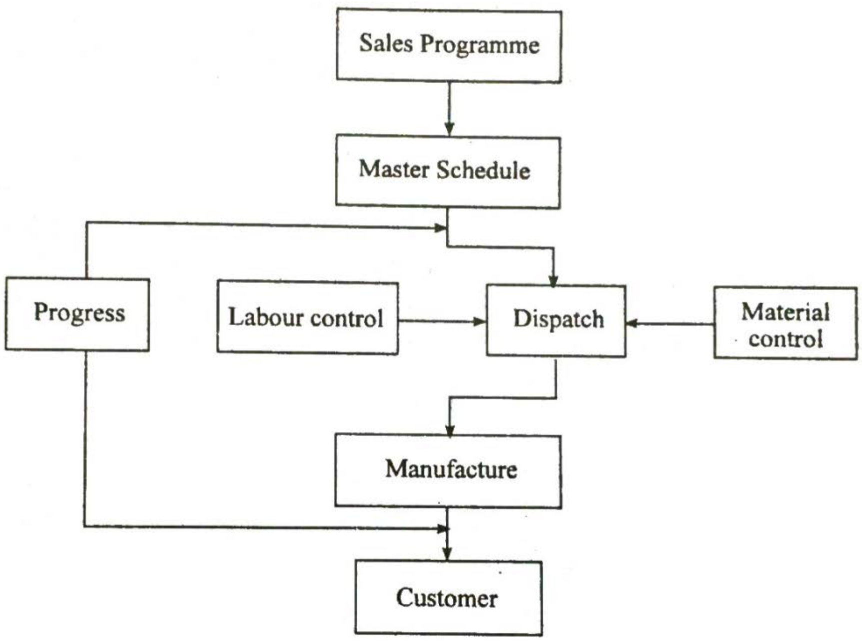


Figure 6 Outline of functions of production control

4.3 LOADING, SEQUENCING AND SCHEDULING

Output plans specify when products are needed but these specifications must be transformed into operational terms to be implemented on the shop floor. The operations scheduling system is shown in the figure.

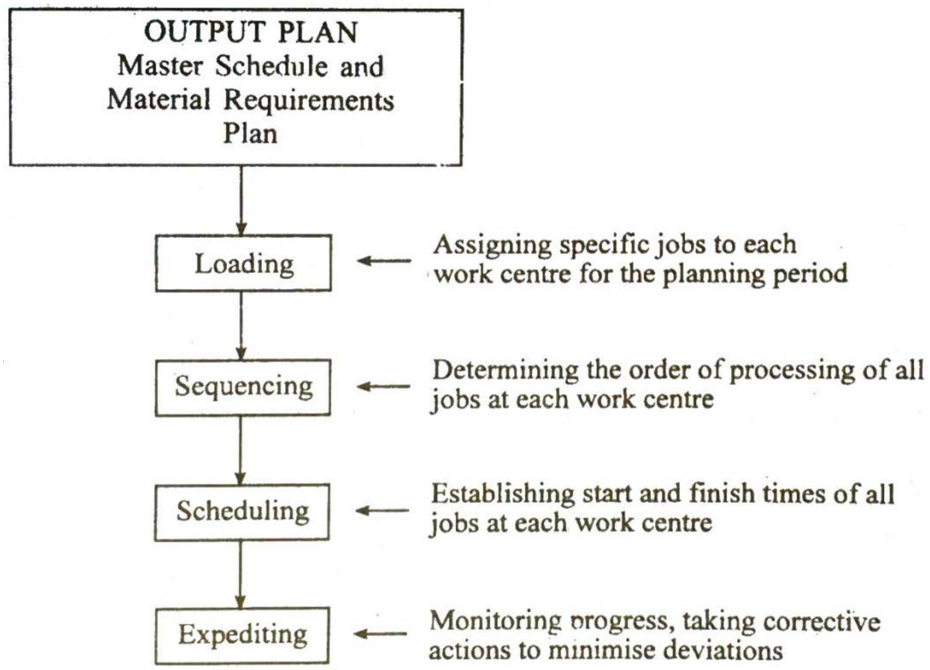


Figure 7 Operations scheduling system

4.4 LOADING

Each job may have a unique product specification and has a unique routing through various work centres, when the job orders are released, they are allocated to the work centres thus establishing the quantity of load each work centre should carry during the specific planned period. This assignment is called loading. A load is the work assigned to a machine or an operator and capacity is the volume of output capable of being produced in any convenient period of time.

Loading is the study of the relationship between load and capacity at work centres. Gantt load charts and visual load profiles are helpful for the evaluation of the current loading

4.4.1 GANTT CHART

It is a principal tool used for both loading and scheduling. The chart was originated by the American engineer, Henry L. Gantt and consists of a simple rectangular grid, divided a by series of parallel horizontal and vertical lines. Vertical lines divide the chart into units of time. The scale units can be years, months, weeks or days or hours according to duty for which chart is required. The horizontal lines divide the chart into sections, which can be used to represent either work tasks or work centres. The Gantt chart offers the advantages of ease and clarity in communicating important shop information. The Gantt chart must be updated periodically to account for new jobs.

Example: There are four jobs which are to be processed on 3 work centres. Jobs A, B, C and D require sheet metal, and paintwork. A, C and D require electrical. These are presented in the form of load chart shown in the figure.

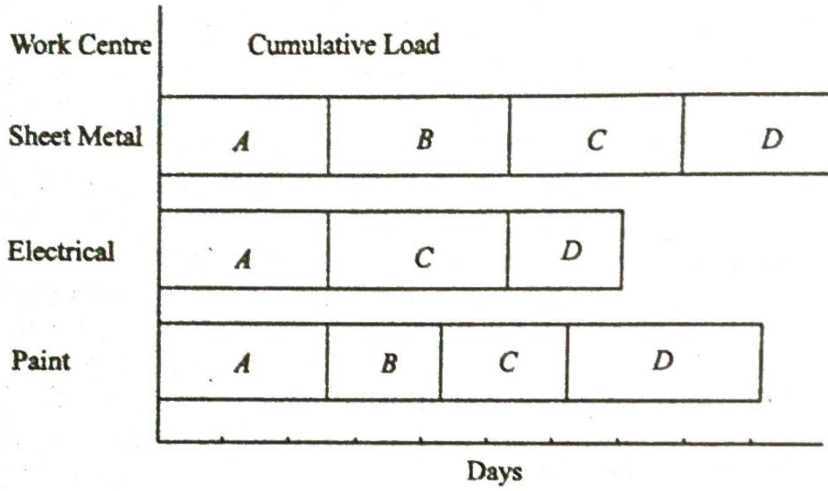


Figure 8 Load chart

4.4.2 VISUAL LOAD PROFILES

It is a graph comparing workloads and capacities on a timescale. In the manual scheduling system, the load consists of open orders (existing orders from customers) assigned to the work centre.

In computer-based scheduling systems, the load consists of open orders and planned orders (prospective orders from customers).

When there are overloads, some of the load will be shifted to alternative work centres. There are two ways this can be done:

- *Lot splitting:* In which a job order is split and only part of it is processed now and other are processed later.
- *Operations splitting:* Processing part of the job in one work centre and the rest at another.

4.5 SCHEDULING

Scheduling can be defined as "prescribing of when and where each operation necessary to manufacture the product is to be performed." It is also defined as "establishing of times at which to begin and complete each event or operation comprising a procedure." The principal aim of scheduling is to plan the sequence of work so that production can be systematically arranged towards the end of completion of all products by the due date.

4.5.1 PRINCIPLES OF SCHEDULING

- *The principle of optimum task size:* Scheduling tends to achieve maximum efficiency when the task sizes are small, and all tasks are of the same order of magnitude.
- *Principle of optimum production plan:* The planning should be such that it imposes an equal load on all plants.
- *Principle of optimum sequence:* Scheduling tends to achieve the maximum efficiency when the work is planned so that work hours are normally used in the same sequence.

4.5.2 INPUTS TO SCHEDULING

- Performance standards: The information regarding the performance standards (standard times for operations) helps to know the capacity in order to assign required machine hours to the facility.
- Units in which loading and scheduling are to be expressed.
- The effective capacity of the work centre.
- Demand pattern and extent of flexibility to be provided for rush orders.
- Overlapping of operations.
- Individual job schedules.

4.5.3 SCHEDULING STRATEGIES

Scheduling strategies vary widely among firms and range from “no scheduling” to very sophisticated approaches. The strategies are grouped into four classes:

- Detailed scheduling.
- Cumulative.
- Cumulative detailed.
- Priority decision rules.

Detailed scheduling for specific jobs that are arrived from customers is impracticable in an actual manufacturing situation. Changes in orders, equipment breakdown, and unforeseen events deviate the plans.

Cumulative scheduling of total workload is useful especially for long-range planning of capacity needs. This may load the current period excessively and under load future periods. It has some means to control the jobs.

The cumulative detailed combination is both feasible and practical approach if the master schedule has fixed and flexible portions.

Capacities are planned on a broad basis first in terms of total labour and machine requirements per week at key work centres. As changes occur during the weeks prior to manufacturing, the computer updates material and capacity requirements automatically. Capacity may then be allocated to specific jobs later a few days before the commencement of job. The shortest scheduling unit-for job shop is one day.

Priority decision rules are scheduling guides that are used independently and conjunction with one of the above strategies, e.g., first come first serve. These are useful in reducing work-in-process (WIP) inventory.

4.5.3.1 FORWARD SCHEDULING AND BACKWARD SCHEDULING

Forward scheduling (Set forward): It is commonly used in job shops where customers place their orders on “needed as soon as possible” basis. Forward scheduling determines the start and finish times of next priority job by assigning it the earliest available time slot and from that time, determines when the job will be finished in that work centre. Since the job and its components stall as early as possible, they will typically be completed before they are due at the subsequent work centres in the routing. The set forward method generates in process inventory that is needed at subsequent work centres and higher inventory cost.

Forward scheduling is simple to use and it gets jobs done in shorter lead times, compared to backward scheduling.

Backward scheduling (Set backward): It is often used in assembly type industries and commit in advance to specific delivery dates. Backward scheduling determines the start and finish times for waiting jobs by assigning them to the latest available time slot that will enable each job to be completed just when it is due, but not before. By assigning jobs as late as possible, backward scheduling minimises inventories since a job is not completed until it must go directly to the next work centre on its routing.

Bill of materials (BOM) and lead time estimates are maintained for all work centres otherwise the system breaks down and due dates are violated.

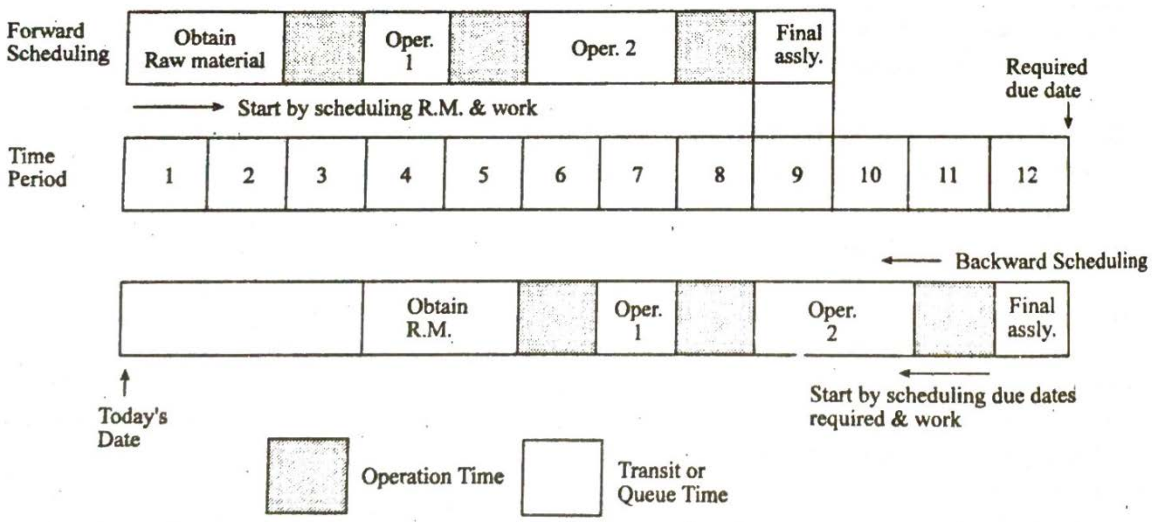


Figure 9 Forward and backward scheduling

4.5.3.2 FINITE LOADING

It is a scheduling procedure that assigns jobs into work centres and determines their starting and completion dates by considering work centre capacities.

This is an alternative scheduling technique that combines into a single system the loading, sequencing and scheduling. Finite loading systems start with a specified capacity for each work centre and list of jobs. The work centres capacity is then allocated unit by unit (e.g., labour hours, machine hours, etc.) to the jobs by simulating job starting and completion dates. Thus, the system creates a detailed schedule for each job and each work centre based on centres capacities.

4.5.3.3 CRITICAL RATIO SCHEDULING

It is a technique for establishing and maintaining priorities among the jobs in a factory. The critical ratio which is an index and sets a relationship between when a product is required and when it can be supplied.

$$\text{Critical ratio} = \frac{\text{Time the job is needed (Demand time)}}{\text{Supply lead time}}$$

For example,

$$\text{Critical ratio} = \frac{\text{Product is required in 20 days}}{\text{Product can be made available in 10 days}} = 2$$

A critical ratio of 2 means that time available to deliver the product is twice the time required to manufacture the product.

Critical ratio of

- greater than one means that there is sufficient time and job can be completed ahead of the schedule.
- equal to one implies that the job is just on schedule and requires close watch.
- less than one suggests that the job is critical and needs to be expedited to complete on schedule.

4.5.3.4 INDEX METHOD

It is a subjective method of evaluation. It is simple and easy to operate and provides considerable improvements over conventional scheduling techniques. Normally jobs are assigned to the best machine until it is loaded to its full capacity and remaining jobs to the next best machine. This does not result in optimum loading. The best way of assigning jobs to machines is on the basis of the relative effectiveness of machines.

4.5.3.5 CRITICAL PATH METHOD

It is a useful technique to determine the schedule of "one-off" jobs, Critical path analysis overcomes the deficiencies of Gantt Chart and used for scheduling large projects, where relation between various activities of the project are more complicated.

4.6 DISPATCHING

Dispatching is the routine of setting productive activities in motion through the release of orders and instruction in accordance with previously planned times and sequences in the route sheets.

4.6.1 THE FUNCTIONS OF DISPATCHING

- The primary function of dispatching is to prepare manufacturing orders which consist of shop orders, move orders, tool orders, etc. These are to be issued at the right time to the concerned persons.
- The release of necessary order and production forms so that the operations can be started.
- Withdraw required quantity of material from the stores and deliver to the work centre where the first operation is to be completed through stores issue order.
- The issue of tools required for production.
- Inter-departmental transport (move order).
- Stage inspection.
- Coordination with scheduling.
- Forwarding materials to dispatch or to finished parts stores.

4.6.2 DOCUMENTS RAISED BY THE DISPATCHER

- *Material requisitions*.
- *Job cards:* Which authorise the workman to start the work on certain material, indicate what to do and also serves as a means of production progress.
- *Labour cards:* Which are used to report labour time utilised and quantity of work performed and to supply other information which is required in the preparation of production reports and payrolls.
- *Move cards:* Which authorise the movement of materials as per the requirements of the job and used in production progress reports.

4.6.3 DUTIES OF A DISPATCHER

- The receipt and filing of all shop orders and associated documents.
- Selection of jobs for the issue, in the most favourable sequence.
- The issue of job cards or other forms of instructions to the operations.
- The issue of instructions to setters regarding what machines are to be set up, for which jobs and when.
- The issue of instructions concerning the movement of materials between work centres.
- The issue of instructions concerning the issue and return to stores of special tooling.
- The maintenance of records of production.

5 MATERIAL HANDLING

5.1 INTRODUCTION

In order to convert the raw materials into finished products, it is essential that one of the three basic elements of production, i.e., material, men or machines should move. In a majority of the industrial processes, it is the material that moves from raw material stage to finished goods stage. Because the material is more widely moved rather than the men or machines, hence the name "Material Handling".

Haynes defines "Material handling embraces the basic operations in connection with the movement of bulk, packaged and individual products in a semi-solid or solid state by means of gravity manually or power-actuated equipment and within the limits of individual producing, fabricating, processing or service establishment."

Material handling does not add any value to the product but adds to the cost of the product and hence it will cost the customer more. So the handling should be kept at a minimum.

Material handling amounts to 15–25% of total cost of a product according to American material handling society. Material handling is the art and science involving the movement, handling and storage of materials during different stages of manufacturing.

Out of the total time .spent for manufacturing a product, 20% of the time is utilised for actual processing on them while the remaining 80% of the time is spent in moving from one place to another, waiting for the processing or storage (temporary).

5.2 OBJECTIVES OF MATERIAL HANDLING

- Minimise cost of material handling.
- Minimise delays and interruptions by making available the materials at the point of use at right quantity and at right time.
- Increase the productive capacity of the production facilities by effective utilisation of capacity and enhancing productivity.
- Safety in material handling through improvement in working condition.
- Maximum utilisation of material handling equipment.
- Prevention of damages to materials.
- Lower investment in in-process inventory.

5.3 ELEMENTS OF MATERIAL HANDLING

- *Motion:* Move in most economical, safe and efficient manner.
- *Time:* Provide materials on time.
- *Quantity:* Ensure the supply of correct quantity continuously at each manufacturing organisation.
- *Space:* Ensure optimum use of cubic space.

5.4 MATERIAL HANDLING ACTIVITIES AND FUNCTIONS

In a manufacturing organisation, the handling activity encompasses:

1. Transportation and handling at suppliers end.
2. Material handling at manufacturing plant.
3. Transportation and handling from warehouse to the customer (physical distribution).

The material handling activity in the plant starts with the unloading of the material after receipt from suppliers and extends throughout the processing from raw material stage till it is manufactured and stored in the warehouse to be dispatched to the customer.

The various activities and functions in manufacturing organisation are

1. Unloading
2. Receiving and temporary storage location
3. Storing
4. Issuing
5. Workplace handling
6. In-process handling and storage
7. Inter-department
8. Intra-plant
9. Packaging
10. Warehousing of finished goods
11. Loading and shipping

Relationship between Plant Layout and Material Handling

There is a close relationship between plant layout and material handling. A good layout ensures minimum material handling and eliminates rehandling.

- The material movement does not add any value to the product so, the material handling should be kept at a minimum though not avoid it. This is possible only through the systematic plant layout. Thus a good layout minimises handling.
- The productive time of workers will go without production if they are required to travel a long distance to get the material, tools, etc. Thus a good layout ensures minimum travel for workman thus enhancing the production time and eliminating the hunting time and travelling time.
- Space is an important criterion. Plant layout integrates all the movements of men, material through a well-designed layout with the material handling system.
- Good plant layout helps in building efficient material handling system. It helps to keep material handling shorter, faster and economical. A good layout reduces the material backtracking, unnecessary workmen movement ensuring effectiveness in manufacturing.

Thus a good layout always ensures minimum material handling.

5.5 PRINCIPLES OF MATERIAL HANDLING

- *Planning principle:* All handling activities should be planned.
- *Systems principle:* Plan a system integrating as many handling activities as possible and coordinating the full scope of operations (receiving, storage, production, inspection, packing, warehousing, supply and transportation).
- *Space utilisation principle:* Make optimum use of cubic space.
- *Unit load principle:* Increase quantity, size and weight of load handled.
- *Gravity principle:* Utilise gravity to move a material wherever practicable.
- *Material flow principle:* Plan an operation sequence and equipment arrangement to optimise material flow.
- *Simplification principle:* Reduce combine or eliminate unnecessary movement and/or equipment.
- *Safety principle:* Provide for safe handling methods and equipment.
- *Mechanisation principle:* Use mechanical or automated material handling equipment.
- *Standardisation principle:* Standardise method, types, size of material handling equipment.
- *Flexibility principle:* Use methods and equipment that can perform a variety of task and applications.
- *Equipment selection principle:* Consider all aspect of material and method to be utilised.
- *Dead weight principle:* Reduce the ratio of dead weight to payload in mobile equipment.
- *Motion principle:* Equipment designed to transport material should be kept in motion.
- *Idle time principle:* Reduce idle time/unproductive time of both material handling equipment and manpower.
- *Maintenance principle:* Plan for preventive maintenance or scheduled repair of all handling equipment.
- *Obsolescence principle:* Replace obsolete handling methods/equipment when more efficient method/equipment will improve operation.
- *Capacity principle:* Use handling equipment to help achieve its full capacity.
- *Control principle:* Use material handling equipment to improve production control, inventory control and other handling.
- *Performance principle:* Determine the efficiency of handling performance in terms of cost per unit handled which is the primary criterion.

5.6 SYMPTOMS OF BAD MATERIAL HANDLING

- A frequent interruption in production due to delay in handling and supplying materials to the point of use.
- Skilled labour performing duties like storing, movement and handling of materials.
- Damages to materials in handling.
- Accumulation of work-in-process and materials in different locations.
- Reworking and rejections due to handling defects.
- Crowded floor space with scrap and materials.
- Congestion at receipt, production and inspection areas.
- Long waiting for material handling equipment to pick up and deliver materials.

6 ERGONOMICS

6.1 INTRODUCTION AND DEFINITION

The word *ergonomics* has its origin in two Greek words *ergon* meaning work and *nomos* meaning laws. So it is the study of the man in relation to his work. The word ergonomics is used commonly in Europe. In the USA and other countries, it is called by the name *human engineering* or it is also called *human factors engineering*.

ILO defines human engineering as, "The application of human biological sciences along with engineering sciences to achieve optimum mutual adjustment of men and his work, the benefits being measured in terms of human efficiency and well-being." The human factors or human engineering is concerned with the man-machine system. Thus another definition which highlights the man-machine system is: "The design of human tasks, man-machine system, and effective accomplishment of the job, including displays for presenting information to human sensors, controls for human operations and complex man-machine systems."

Human engineering focuses on human beings and their interaction with products, equipment facilities and environments used in the work. Human engineering seeks to change the things people use and the environment in which they use the things to match in a better way the capabilities, limitations and needs of people.

6.2 OBJECTIVES OF HUMAN ENGINEERING

Human engineering (ergonomics) has two broader objectives:

- To enhance the efficiency and effectiveness with which the activities (work) is carried out so as to increase the convenience of use, reduced errors and increase in productivity.
- To enhance certain desirable human values including safety, reduced stress and fatigue and improved quality of life.

Thus, in general, the scope and objective of ergonomics is "designing for human use and optimising working and living conditions".

Thus human factors (ergonomics) discovers and applies information about human behaviour, abilities and limitations and other characteristics to the design of tools, machines, systems, tasks, jobs and environment for productive, safe, comfortable and effective human use.

6.3 ERGONOMICS IS MULTIDISCIPLINARY

The various disciplines that are going to have an influence on human factors are:

Engineering:	Design of work system suitable to the worker.
Physiology:	Study of man and his working environment.
Anatomy:	Study of body dimensions and relations for work design.
Psychology:	Study of adaptive behaviour and skills of people.
Industrial hygiene:	Occupational hazards and workers health.

6.4 STUDY OF HUMAN ENGINEERING AREAS

- Anthropometry and biomechanics.
- Control of physical work environment.
- Design of man-machine system.
- Design of controls and displays.
- Accidents fatigue and safety.
- Workplace design.

6.5 DISPLAY DESIGN

Displays are necessary extensions to man's senses and provide both prime and supplemental information needed by operators in making decisions and in effecting control responses.

Information presented by displays can be considered dynamic or static. Dynamic information continuously changes or is subjected to change through time e.g., traffic signals, charts or graphs.

The more detailed classification:

- *Quantitative information:* Display presentations that reflect the approximate value of some variable such as temperature, speed, etc.
- *Qualitative information:* Display presentations that reflect the approximate value, trend, rate of change, direction of change, etc.
- *Status information:* Display presentations that indicate the condition or status of the system such as ON-OFF condition, etc.
- *Warning and signal indicators:* Display the emergency, unsafe condition.
- *Representational information:* Pictorial or graphic representation.
- *Identification information.*
- *Time-phased information.*

6.5.1 TYPES OF DISPLAYS

6.5.1.1 VISUAL DISPLAYS

Depending on the use, visual displays classified as:

- *Quantitative display:* To read a precise numeric value e.g., display for pressure measurements, display for speed measurement, angle measurement. Conventional quantitative displays are mechanical devices of the following types:
    - Fixed scale with moving points e.g., pressure gauge, automobile speedometer.
    - Moving scale with fixed pointer e.g., weighing machine to measure human weight.

  The modern technology has made it possible to present electronically generated features. e.g., digital displays.

- *Qualitative display:* The display is used to read an approximate value or to indicate the rate of change, change in direction, etc. e.g., the increase or decrease in pressure.
- *Check display:* The display gives information about the parameters whether they are normal.

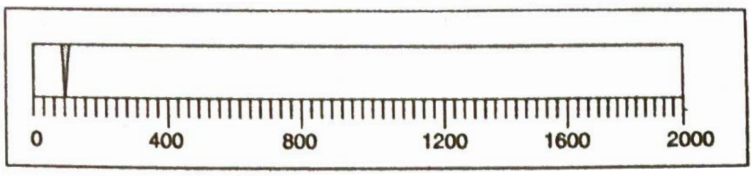
6.5.1.2 AUDITORY DISPLAYS

As compared to visual displays, auditory displays can make monitoring performance superior. So these devices are suitable as warning devices. Following are some of the situations in which auditory displays are more suitable:

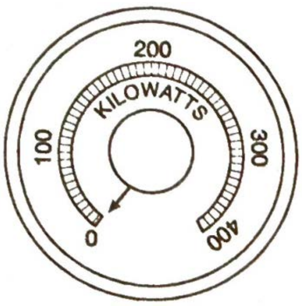
- When the message is simple and short.
- When the message calls for immediate attention.
- When the receiver moves from one place another.
- When continuously changing in the information of some type is presented.

6.5.2 CONSIDERATIONS IN DISPLAY DESIGN

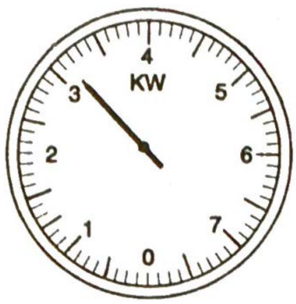
- What is the information to be transmitted? What is its purpose of function?
- What type of display is to be used?
- Nature of the visual environment in which information is to be transmitted.
- Detailed design characteristics of the type of display chosen.



Example of poorly designed scale



Original



Redesigned

The meter at the right would be easier to read because it is bolder and less cluttered than one at the left. It has fewer graduations and the double arc line has been eliminated. The scale length is increased by placing the markers closer to the perimeter, although this requires that the numerals be placed inside the scale, the clear design and the fact that the numerals are upright probably would partially offset this disadvantage.

Figure 10 Display design

6.6 DESIGN OF CONTROLS

The selection of control should be considered with regard to the functional requirements of the system. Controls are the means by which information is transmitted to the machine from the man.

6.6.1 TYPES OF CONTROL

- *Hand controls:* The anthropometric data for the human hand can be used as an aid to design dimensions of hand controls.
- *Hand levers:* Levers give a quick control action and can accommodate large forces. They are not suitable for fine adjustments. Levers can provide efficient ON/OFF or step by step control.
- *Hand wheels:* Hand wheel provide a controlling torque via both hands and they are used for heavy loads. They can provide good accuracy of adjustments.
- *Cranks:* Cranks are intended to provide torque via one hand. Smaller cranks are used for fast control.
- *Knobs:* Rotating knobs are recommended for light loading control with either finger or with the whole hand. Knobs are typically used in applications such as instrument control panels.
- *Push buttons and toggle switches:* These are essentially used as light load ON/OFF controls and are normally designed for operation by one finger.
- *Joysticks:* This is a type of hand control now extensively use in computer applications including CNC machines.
- *Foot pedals:* These are used for fast action control with medium or heavy loading capacity. They lack in accuracy and range which may be obtained with hand controls. These are used in the sitting position.

6.6.2 SPECIFIC CONTROL RECOMMENDATIONS

The various functional requirements and design features are given below:

- Push buttons should have positive, snap (click) action for operator feedback and concave or rough surface top to aid in fingering.
- Toggle switches provide a quick mode of response, require little space and can be operated simultaneously with others in a group. They give both visual and tactual indication of their state.
- Knobs for continuous control can be shape and colour coded effectively and with appropriate gearing. Provide a considerably flexible choice of adjustments.
- Levers are commonly used as gear shifts, throttles, or joysticks in positioning or tracking tasks.
- Cranks are used for high rates of adjustments over long distances position feedback is poor.
- Steering wheels (hand wheels) are useful where large rotary forces must be applied but they require more space.
- Pedals have characteristics, similar to hand wheels good for large force applications. They do not permit precision but are useful in distributing the work.

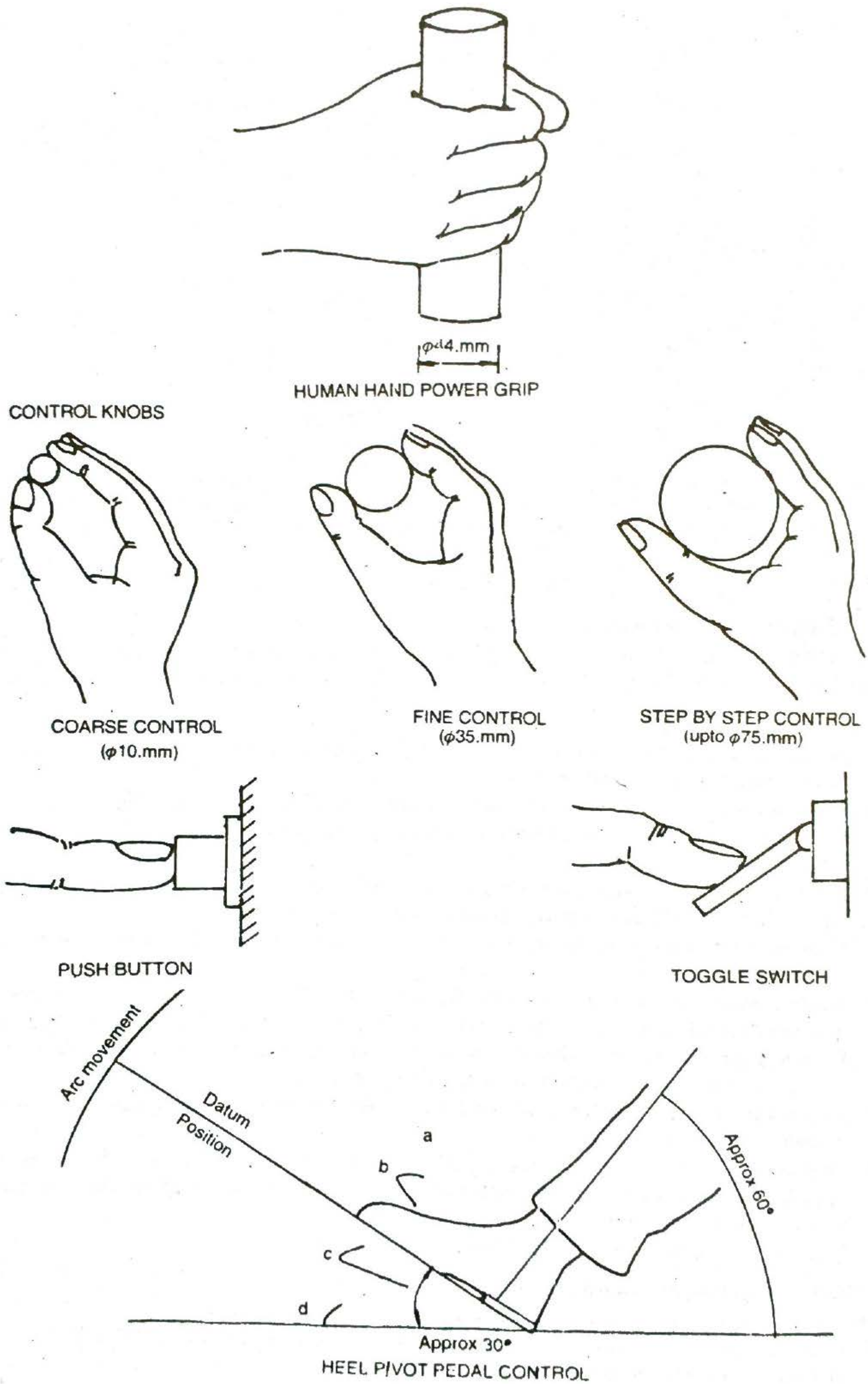


Figure 11 Optimum size for human grip

6.7 ENVIRONMENTAL FACTORS

The role that ergonomics plays in the environmental man-machine interface is essentially threefold.

- First, identifying the effects that the environment has on man's physiological and psychological process.
- Second, ensuring that work patterns, equipment and machine interfaces are designed to minimise the individual variation in performance.
- Third, ensuring that all the necessary protective systems are designed to take an account of physiological and psychological variations in man.

The environmental factors that affect the performance are:

6.7.1 ILLUMINATION

When human activities are carried out indoors or at night, it is necessary to provide some sort of artificial illumination the type of lighting or illumination depends upon the type of work being performed, the size of the objects, accuracy, speed and duration of the work, etc.

The lighting system should provide

- Sufficient brightness.
- Uniform illumination.
- A contrast between the brightness of the job and of background.
- No direct or reflected glare.

Factors affecting visual performance

- Normal performance depends upon the eye/brain system, system at its optimum efficiency and normal vision.
- Colour blindness.
- Visual adaptation to light levels is an important physiological mechanism.
- External factors affecting include luminance, contrast, quality and amount of illumination, time of observation, movement, size of objects and glare.

Principles involved in improving visual performance

- Since time is required for an eye to adjust, the design should avoid the need to identify visual information at widely different distances or in slightly different illumination levels.
- Minimise the extent of eye movement required to resolve visual information by the arrangement of displays rather than by reducing the size of the details.

6.7.2 NOISE

Noise has been defined as unwanted sound and it has been shown to have both short and long-term effects human performance, These effects may be internal and physiological in nature, resulting in the auditory system being unable to perceive sound. The amount of having a loss is related to the level of the noise to which the operator is exposed and also it depends upon the exposure time for high-frequency intensities. A sudden loud sound which is entirely unexpected can use an increase in blood pressure, sweating, heart rate, respiration and muscular contraction and repeated exposure may affect person's digestion.

Recommendations

- Where the problem is the talker, speech training may be necessary to correct those aspects that are at fault.
- Where noise is present and has been identified as a problem it should be reduced at source.
- Impact equipment and excessively noisy machines and equipment and operations should be isolated by constructing per enclosures that amount of noise transmits beyond the enclosure is reduced.
- The use of baffles, sound absorbers and acoustical treatment of walls, ceilings and floors can also reduce sound reflection.
- In server noise situations, workers should be provided with personal protective devices such as earplugs, earmuffs and helmets.

6.7.3 VIBRATION

Usually, vibrations of the air are detected as sound but air vibrations below 20 Hz are not heard but can be felt. Vibrations can affect the performance on target tracking. Ideally, vibrations should be minimised at source. Normally protection from residual vibration is achieved by reducing the forces transmitted, by converting vibration energy into thermal energy by using mechanical or hydraulic dampers and by altering body position and body support.

6.7.4 THERMAL CONDITIONS (TEMPERATURE, HUMIDITY AND AIR FLOW)

Poor heat and humid conditions produce thermal stresses in the workers which affect their efficiency, concentration and dexterity of their members of the body. The working temperature of 60–65 °F is considered normal but it varies according to the nature of work. Humidity and heat are related to each other both affect comfort and tolerance of the body to the heat. If humidity is high, evaporation of the sweet is reduced. Humidity as a general rule should not be allowed to exceed above 70%. The effect of heat can be minimised by:

- Shielding, isolating heat sources to reduce direct transmission by radiation of heat between body and the heat source.
- Installation and provision for adequate local ventilation to get rid of smokes, fumes, etc.
- Permit rest pauses in cool, extreme hot conditions.

6.7.5 VENTILATION

Ventilation is the process of displacement of state air of the building by fresh air to reduce the presence of bad odour, CO_2 concentration, humidity and temperature.

A good ventilation system provides fresh air.

Most common methods of ventilation are:

- Windows and ventilators provide natural ventilation.
- Exhaust fans extract stale air and create low-pressure area to be filled by fresh air.

7 COST CONCEPTS

Cost is the amount of resources sacrificed or given up to achieve a specific objective which may be the acquisition of goods or services. Costs are always expressed in money terms, e.g., a manufacturer incurs costs in buying materials and in hiring labour, etc.

7.1 COSTS OF PRODUCTION

The costs of production include:

- Purchase costs of raw materials, bought out components and subassemblies, procurement and transportation costs.
- Purchase costs of supplies such as oils, lubricants, tools of small value, fuel oil, machinery spares, cotton waste, etc.
- Wages and salaries paid to direct production workers, maintenance inspection, stores staff, supervisors and other staff.
- Costs paid to subcontractors for the orders placed on them.
- Cost of production line rejections, wastage, spoilage and rework.
- Expenses towards rent and insurance of factory buildings, insurance on plant and machinery, stores, etc.
- Interest on working capital to the extent it relates to inventory.
- Cost of procurement of capital assets like buildings, machinery, tooling, inspection equipment, furniture, etc., and the depreciation of these capital assets.

7.2 CLASSIFICATION OF COSTS

Classification of costs is based on the following:

- Natural characteristics (material, labour and overhead)
- Changes in activity or volume (fixed, variable, mixed)

7.2.1 NATURAL CLASSIFICATION OF COSTS

This classification refers to the basic physical characteristics of the cost. In a manufacturing Company, the total cost of a product includes the following four elements:

7.2.1.1 DIRECT MATERIAL

Direct material refers to the cost of materials which become a major part of the finished product. They are the raw materials that become an integral part of the finished product and are traceable to specific units of output.

Examples: Raw cotton in textiles, crude oil to make diesel, steel to make automobile parts. The following groups of materials come under direct material:

- All materials purchased for a particular job, process or product.
- All materials acquired from stores for production.
- Components or parts purchased or produced.
- Materials passing from one process to another process.

7.2.1.2 DIRECT LABOUR

Direct labour is defined as the labour associated with workers who are engaged in the production process. It is the labour costs for specific work performed on products that are traceable to end products.

Examples: Labour of machine operators, assembly operators, etc.

7.2.1.3 DIRECT EXPENSES

The expenditure incurred (other than direct material and direct labour) on a specific job or product is included in direct expenses. These are also called chargeable expenses.

Examples: Cost of special layout, design or drawings, hiring special machines for specific product manufacture, etc.

7.2.1.4 FACTORY OVERHEADS

These are also called manufacturing costs. These include the costs of indirect materials, indirect labour and indirect expenses.

- Indirect material refers to materials that are needed for the completion of the product but it is not possible to trace or identify it with an end product.
  Examples: cutting oil, lubricants, etc., cannot be charged to a specific product.
- Indirect labour refers to the labour hours expended which will not directly affect the composition or construction of the finished product.
  Examples: Foreman, shop clerks, material handlers, maintenance employees. Their labour is considered indirect because it is not economically possible to trace them with a specific product.
- Indirect expenses are the expenditure incurred by the manufacturing company from the beginning (start) of production to its completion and transfer to the finished goods store.

Direct costs and factory overheads together are called conversion costs.

7.2.1.5 DISTRIBUTION AND ADMINISTRATIVE OVERHEADS

Distribution overheads are also called marketing or selling overheads. These costs include advertising, salesmen salaries and commission, packaging, storage, transportation and sales administrative costs. Administrative overhead includes costs of planning and controlling of general business operations. All costs which are not charged to production and sales are included in administrative overheads.

Examples: Chairman's salary, fees of the board of directors, rent of administrative office, etc.

Costs of Manufacturing Company

Prime Cost = Direct Material + Direct Labour + Direct Expenses

Factory Overhead = Indirect Material + Indirect Labour + Indirect Expenses

Factory Cost = Prime Cost + Factory Overhead

Total Cost = Factory Cost + Distribution and Administrative Overhead

7.2.2 CLASSIFICATION BASED ON ACTIVITY OR VOLUME

7.2.2.1 FIXED COST

The costs which do not change for a given period in spite of the change in volume of production. This cost is independent of the volume of production.

Examples: Rent, taxes, salaries of supervisors, depreciation, insurance, etc.

Fixed costs are normally expressed in terms of time period, i.e., per day, per annum, etc.

Fixed cost does not mean that they never change. They are constant up to specific volume or range of volume.

7.2.2.2 VARIABLE COSTS

These vary directly and proportionately with the output. There is a constant ratio between the change in the cost and change in the level of output. Direct material cost and direct labour costs are generally variable costs. Variable costs results from the utilisation of raw materials and direct labour in production departments.

7.2.2.3 MIXED COSTS

Mixed costs are made up of fixed and variable costs. They are a combination of semi-variable and semi-fixed costs. Because of the variable component, they fluctuate with volume, because of the fixed component, they will not change in direct proportion to output. Semi-fixed costs are those costs which remain constant up to a certain level of output after which they become variable. Semi-variable cost is the cost which is basically variable but whose slope may change abruptly when a certain output level is reached.

8 MAKE OR BUY DECISION

Make or buy decision is an important management policy. A company can satisfy the demand of customers either by making the required products using the facilities which are available within the company or buying them from a subcontractor. Before deciding on the alternative to he followed to satisfy the demand, the company should analyze the costs and available capacities of the alternatives. Then, the alternative which involves the least cost should be recommended to the company for implementation. Low volume of usage favours buying, which entails little or no fixed costs. If the total cost of buying an item is more than or equal to the total cost of making that item, then a company can manufacture the item within the company. Otherwise, the item can be bought from a vendor.

8.1 POSSIBLE ALTERNATIVES WHILE STARTING FOR NEW PRODUCTS

When a company plans for new products, the following alternatives can be considered:

- Purchase the complete product from a contracted supplier.
- Purchase some components and materials and manufacture and assemble the balance in its own plants.
- Manufacture the product completely in its own plants, starting with the extractions of the basic raw materials.

While purchasing a product from outside vendors, which was earlier manufactured at the company's workshop, the following points are to be examined:

- What quantities are involved?
- Will drawing need modification?
- Should jigs, tools, gauges be loaned?
- Will demand be temporary or permanent?
- Will demand fluctuate?
- Are special manufacturing techniques involved?
- Is there any question of secrecy?
- Is there a likely market elsewhere?
- Are frequent design changes likely?
- Arrangement for inspection, sampling, etc.
- Retention of own production personnel.
- What notice of termination is required?

The factors which are to be considered in manufacturing a product within the company, if some of its components have been purchased presently are given below:

- Are copyrights involved?
- If so, what are the royalties?
- Have the best prices been obtained?
- Are the quantities optimized?
- Is the previously contracted firm already making something similar which could be added to the new item, thereby reducing the production cost?
- Is the raw material readily available?

8.2 CRITERIA FOR MAKE OR BUY

8.2.1 CRITERIA FOR MAKE

The following are the criteria for making:

The finished product can be made cheaper by the firm than by outside suppliers.

The finished product is being manufactured only by a limited number of outside firms which are unable to meet the demand.

The part has an importance for the firm and requires extremely strict quality control.

The part can be manufactured with the firm's existing facilities for other items in which the company has manufacturing experience.

8.2.2 CRITERIA FOR BUYING

The following are the criteria for buying:

- High investments on facilities which are already available at the supplier’s plant.
- The company does not have facilities to make it, and there are more profitable opportunities for investing in the company’s capital.
- Existing facilities can be used more economically to make other parts.
- The skill of personnel employed by the company cannot be readily utilized to make the part.
- Patent or other legal barriers prevent the company from making the part.

- Demand for the part is either temporary or seasonal.

8.3 APPROACHES FOR MAKE OR BUY DECISION

The types of approach followed in make or buy decision are as follows:

- Simple cost analysis.
- Economic analysis.
- Break-even analysis.

8.3.1 SIMPLE COST ANALYSIS

In this analysis, the cost of making a product and that of buying a product are calculated. Then, the alternative which involves the minimum cost is suggested for implementation.

8.3.2 BREAK-EVEN ANALYSIS

Actually in any business organization, for manufacturing a product, there are two major costs, namely, fixed cost and variable cost. The sum of these two costs is known as the total cost of the product. The fixed cost is constant irrespective of production volume of a product which is manufactured by the organization. But, the variable cost is a function of the production volume of the product. For a low volume of production, the total cost of buying will be less than the total cost of making and after reaching a certain level of production volume, it will be more than the total cost of making. The point at which the total cost of buying becomes equal to the total cost of making is known as the break-even point. At this point, there is no loss or gain to the organization. This analysis is termed as break-even analysis.

Total cost of making = Fixed cost + Variable cost/unit × Quantity

Total cost of buying = Purchase cost/unit × Quantity

At break-even point,

Total cost of making = Total cost of buying

Quantity = Break-even quantity/Break-even point (B.E.P.)

Fixed cost + Variable cost/unit × B.E.P. = Purchase cost/unit × B.E.P.

$$\text{B.E.P.} = \frac{\text{Fixed cost}}{\text{Purchase cost/unit} - \text{Variable cost/unit}}$$

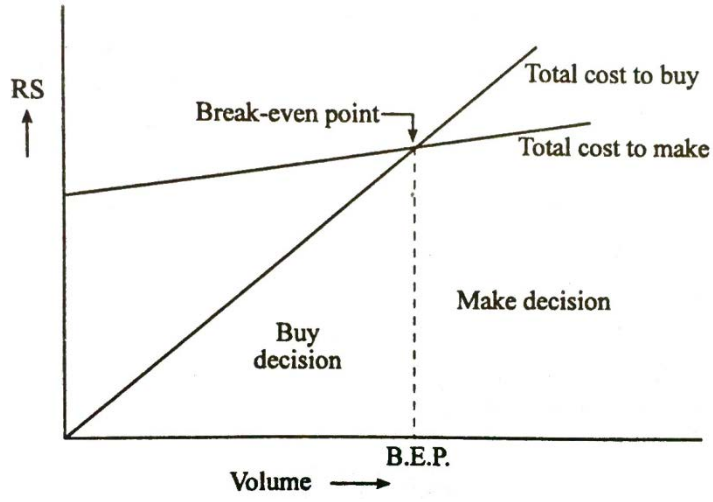


Figure 12 Break-even chart

9 SOLVED PROBLEMS

1 Alpha Company has the following demand pattern. Fit a straight line and forecast the demand for the year 2019 and 2020:

(i) using the entire data, and
(ii) using the recent 8 years data.

Year	2010	2011	2012	2013	2014	2015	2016	2017	2018
Demand (in thousands)	13	20	20	28	30	32	33	38	43

SOLUTION:

(i) Entire data

Year (X)	Demand (Y)	x = X – 5	x^2	xY
1	13	– 4	16	– 52
2	20	– 3	9	– 60
3	20	– 2	4	– 40
4	28	– 1	1	– 28
5	30	0	0	0
6	32	1	1	32
7	33	2	4	66
8	38	3	9	114
9	43	4	16	172
Total (Σ)	257	0	60	204

$$a = \frac{\Sigma Y}{n}$$

$$= \frac{257}{9}$$

$$= 28.5556$$

$$b = \frac{\Sigma xY}{\Sigma x^2}$$

$$= \frac{204}{60}$$

$$= 3.4$$

The equation of the straight line of best fit is

$$Y = a + bx$$

$$Y = 28.5556 + 3.4\,(X - 5)$$

Demand forecast for the year 2019

$$Y = 28.5556 + 3.4\,(10 - 5)$$

$= 45.5556$ thousands

$= 45555.6$

Demand forecast for the year 2020

$\mathrm{Y} = 28.5556 + 3.4\ (11 - 5)$

$= 48.9556$ thousands

$= 48955.6$

(ii) Recent 8 years data

Year (X)	Demand (Y)	x = X – 4.5	x^2	xY
1	20	– 3.5	12.25	– 70
2	20	– 2.5	6.25	– 50
3	28	– 1.5	2.25	– 42
4	30	– 0.5	0.25	– 15
5	32	0.5	0.25	16
6	33	1.5	2.25	49.5
7	38	2.5	6.25	95
8	43	3.5	12.25	150.5
Total (Σ)	244	0	42	134

$\mathrm{a} = \frac{\Sigma \mathrm{Y}}{\mathrm{n}}$

$= \frac{244}{8}$

$= 30.5$

$\mathrm{b} = \frac{\Sigma \mathrm{xY}}{\Sigma \mathrm{x}^2}$

$= \frac{134}{42}$

$= 3.1905$

The equation of the straight line of best fit is

$\mathrm{Y} = \mathrm{a} + \mathrm{bx}$

$\mathrm{Y} = 30.5 + 3.1905\ (\mathrm{X} - 4.5)$

Demand forecast for the year 2019

$\mathrm{Y} = 30.5 + 3.1905\ (9 - 4.5)$

$= 44.8571$ thousands

$= 44857.1$

Demand forecast for the year 2020

$\mathrm{Y} = 30.5 + 3.1905\ (10 - 4.5)$

$= 48.0476$ thousands

$= 48047.6$

2 The data given below represents demand figures of ABC Company for the 11 months of the year 2018.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov
Demand (in lakhs)	4.0	4.9	5.7	5.0	6.4	6.8	7.1	8.0	8.2	9.1	8.6

(i) Compute the three months moving average, forecast and error.
(ii) Compute the weighted moving average with a weight of 0.5 to the most recent demand value, a weight of 0.3 to the next most recent demand value and a weight of 0.2 to the oldest demand value.
(iii) Forecast the demand for Dec 2018.
(iv) If the actual demand for Dec 2018 is 9.5 lakh units, what should be the forecast for the month of Jan 2019?

SOLUTION:

Simple Moving Average Method

Month (t)	Demand D(t)	Moving Average M(t)	Forecast F(t)	Error e(t) = D(t) – F(t)
1	4.0			
2	4.9			
3	5.7	4.8667		
4	5.0	5.2	4.8667	0.1333
5	6.4	5.7	5.2	1.2
6	6.8	6.0667	5.7	1.1
7	7.1	6.7667	6.0667	1.0333
8	8.0	7.3	6.7667	1.2333
9	8.2	7.7667	7.3	0.9
10	9.1	8.4333	7.7667	1.3333
11	8.6	8.6333	8.4333	0.1667
12	9.5	9.0667	8.6333	0.8667

For *n* months moving average,

$$\mathrm{M(t)} = \frac{\mathrm{D(t) + D(t-1) + D(t-2) + \cdots + D\big(t-(n-1)\big)}}{\mathrm{n}}$$

For three months moving average,

$$\mathrm{M(t)} = \frac{\mathrm{D(t) + D(t-1) + D(t-2)}}{3}$$

$$\mathrm{M(3)} = \frac{\mathrm{D(3) + D(3-1) + D(3-2)}}{3}$$

$$= \frac{\mathrm{D(3) + D(2) + D(1)}}{3}$$

$$= \frac{4.0 + 4.9 + 5.7}{3}$$

$= 4.8667$

$F(t) = M(t-1)$

$F(4) = M(4-1) = M(3) = 4.8667$

Forecast for the month of Dec 2018

$t = 12$

$F(t) = M(t-1)$

$F(12) = M(12-1) = M(11) = 8.6333$

Forecast for the month of Dec 2018 is 8.6333 lakhs.

Forecast for the month of Jan 2019

$t = 13$

$F(t) = M(t-1)$

$F(13) = M(13-1) = M(12) = 9.0667$

Forecast for the month of Jan 2019 is 9.0667 lakhs.

Weighted Moving Average Method

Month (t)	Demand D(t)	Moving Average M(t)	Forecast F(t)	Error e(t) = D(t) – F(t)
1	4.0			
2	4.9			
3	5.7	5.12		
4	5.0	5.19	5.12	– 0.12
5	6.4	5.84	5.19	1.21
6	6.8	6.32	5.84	0.96
7	7.1	6.87	6.32	0.78
8	8.0	7.49	6.87	1.13
9	8.2	7.92	7.49	0.71
10	9.1	8.61	7.92	1.18
11	8.6	8.67	8.61	– 0.01
12	9.5	9.15	8.67	0.83

$$M(t) = \sum_{i=1}^{n} W_i D(t-i+1)$$

$M(3) = W_1 D(3-1+1) + W_2 D(3-2+1) + W_3 D(3-3+1)$

$= W_1 D(3) + W_2 D(2) + W_3 D(1)$

$= 0.5 \times 5.7 + 0.3 \times 4.9 + 0.2 \times 4.0$

$= 5.12$

Forecast for the month of Dec 2018

$\mathrm{t} = 12$

$\mathrm{F(t)} = \mathrm{M(t-1)}$

$\mathrm{F(12)} = \mathrm{M(12-1)} = \mathrm{M(11)} = 8.67$

Forecast for the month of Dec 2018 is 8.67 lakhs.

Forecast for the month of Jan 2019

$\mathrm{t} = 13$

$\mathrm{F(t)} = \mathrm{M(t-1)}$

$\mathrm{F(13)} = \mathrm{M(13-1)} = \mathrm{M(12)} = 9.15$

Forecast for the month of Jan 2019 is 9.15 lakhs.

3 The Super Snow paint shop has recorded the demand for a particular colour during the past 6 weeks as shown below.

Week	1	2	3	4	5	6
Demand (in litre)	19	17	22	27	29	33

(i) Compute a three months moving average for the data to forecast demand for the next week and error.

(ii) Compute a weighted average forecast for the data, using a weight of 0.6 for the most recent data and weights of 0.3 and 0.1 for successive older data.

SOLUTION:

Simple Moving Average Method

Month (t)	Demand D(t)	Moving Average M(t)	Forecast F(t)	Error e(t) = D(t) – F(t)
1	19			
2	17			
3	22	19.3333		
4	27	22	19.3333	7.6667
5	29	26	22	7
6	33	29.6667	26	7

For *n* months moving average,

$$\mathrm{M(t)} = \frac{\mathrm{D(t) + D(t-1) + D(t-2) + \cdots + D\big(t-(n-1)\big)}}{\mathrm{n}}$$

For three months moving average,

$$\mathrm{M(t)} = \frac{\mathrm{D(t) + D(t-1) + D(t-2)}}{3}$$

$$\mathrm{M(3)} = \frac{\mathrm{D(3) + D(3-1) + D(3-2)}}{3}$$

$$= \frac{\mathrm{D(3) + D(2) + D(1)}}{3}$$

$$= \frac{22 + 17 + 19}{3}$$

$$= 19.3333$$

$$\mathrm{F(t) = M(t\text{–}1)}$$

$$\mathrm{F(4) = M(4\text{–}1) = M(3)} = 19.3333$$

Forecast for the week 7

$$\mathrm{F(t) = M(t\text{–}1)}$$

$$\mathrm{F(7) = M(7\text{–}1) = M(6)} = 29.6667$$

Forecast for the week 7 is 29.6667 litre.

Weighted Moving Average Method

Month (t)	Demand D(t)	Moving Average M(t)	Forecast F(t)	Error e(t) = D(t) – F(t)
1	19			
2	17			
3	22	20.2		
4	27	24.5	20.2	6.8
5	29	27.7	24.5	4.9
6	33	31.2	27.7	5.3

$$M(t) = \sum_{i=1}^{n} W_i D(t - i + 1)$$

$$M(3) = W_1 D(3 - 1 + 1) + W_2 D(3 - 2 + 1) + W_3 D(3 - 3 + 1)$$

$$= W_1 D(3) + W_2 D(2) + W_3 D(1)$$

$$= 0.6 \times 22 + 0.3 \times 17 + 0.1 \times 19$$

$$= 20.2$$

Forecast for the week 7

$$F(t) = M(t-1)$$

$$F(7) = M(7-1) = M(6) = 31.2$$

Forecast for the week 7 is 31.2 litre.

4 Estimate the demand forecast for the year 2019 using simple exponential smoothing. Take α = 0.5 and forecast for the year 2014 as 160 lakh units. Compare the forecast with the least square method.

Year	2014	2015	2016	2017	2018
Demand (in lakhs)	180	168	159	170	188

SOLUTION:

Exponential Smoothing Method

$$\mathrm{F(t)} = \alpha\,\mathrm{D(t-1)} + (1-\alpha)\mathrm{F(t-1)}$$

$$\mathrm{F(2)} = 0.5\,\mathrm{D(2-1)} + (1-0.5)\mathrm{F(2-1)}$$

$$= 0.5\,\mathrm{D(1)} + (1-0.5)\mathrm{F(1)}$$

$$= 0.5\ \times 180 + (1-0.5) \times 160$$

$$= 170$$

Year (t)	Demand D(t)	Forecast F(t)
1	180	160
2	168	170
3	159	169
4	170	164
5	188	167
6		177.5

The demand forecast for the year 2019 is 177.5 lakhs.

Least Square Method

Year (X)	Demand (Y)	x = X – 3	x^2	xY
1	180	– 2	4	– 360
2	168	– 1	1	– 168
3	159	0	0	0
4	170	1	1	170
5	188	2	4	376
Total (Σ)	865	0	10	18

$$\mathrm{a} = \frac{\Sigma\mathrm{Y}}{\mathrm{n}}$$

$$= \frac{865}{5}$$

$$= 173$$

$$\mathrm{b} = \frac{\Sigma\mathrm{xY}}{\Sigma\mathrm{x}^2}$$

$$= \frac{18}{10}$$

$= 1.8$

The equation of the straight line of best fit is

$\mathrm{Y = a + bx}$

$\mathrm{Y} = 173 + 1.8\ (\mathrm{X} - 3)$

Demand forecast for the year 2019

$\mathrm{Y} = 173 + 1.8\ (6 - 3)$

$= 178.4$ lakhs

5 An automobile company has extra capacity that can be used to produce gears that the company has been buying for ₹ 300 each. If the company makes the gears, it will incur materials cost of ₹ 90 per unit, labour cost of ₹ 120 per unit and variable overhead cost of ₹ 30 per unit. The annual fixed cost associated with the unused capacity is ₹ 240,000. Demand over the next year is estimated at 4000 units.

(i) Would it be profitable for the company to make the gears?
(ii) Suppose the capacity could be used by another department for the production of some agricultural equipment that would cover its fixed and variable cost and contribute ₹ 90,000 to profit. Which would be more advantageous, gear production or agricultural equipment production?

SOLUTION:

(i)

Cost to make:

Variable cost/unit = Material cost/unit + labour cost/unit + overhead cost/unit

= ₹ 90 + ₹ 120 + ₹ 30

= ₹ 240

Total variable cost = Variable cost/unit × Quantity

= ₹ 240 × 4000

= ₹ 960,000

Fixed cost = ₹ 240,000

Total cost = Fixed cost + Total variable cost

= ₹ 240,000 + ₹ 960,000

= ₹ 1,200,000

Cost to buy:

Purchase cost = Purchase cost/unit × Quantity

= ₹ 300 × 4000

= ₹ 1,200,000

Fixed cost = ₹ 240,000

Total cost = Fixed cost + Purchase cost

= ₹ 240,000 + ₹ 1,200,000

= ₹ 1,440,000

The cost of making the gears is less than that of buying gears from outside. Hence, making the gears is advantageous.

(ii) The cost calculation for each of the alternatives is summarized in the following table.

	Make Gears	Purchase Gears and Make Agricultural Equipment
Total variable cost (₹)	960,000	1,200,000
Fixed cost (₹)	240,000	0
Total cost (₹)	1,200,000	1,200,000
Contribution to profit (₹)	0	90,000
Net relevant cost (₹)	1,200,000	1,110,000

From the above table it is clear that the net cost of the second alternative is less than that of the first alternative. Hence, it is advisable to produce agricultural equipment using the existing capacity of the company and buy the gears from a supplier.

6 There are three alternatives available to meet the demand for a particular product. They are as follows:

1. Making the product using process A.
2. Making the product using process B.
3. Buying the product.

The details are as follows:

Cost Elements	Making using Process A	Making using Process B	Buying
Fixed cost (₹) / year	100,000	300,000	–
Variable cost (₹) / unit	75	70	–
Purchase price (₹) / unit	–	–	80

The annual demand for the product is 10,000 units.

(i) Should the company make the product using process A or process B, or buy it?
(ii) At what annual volume should the company switch from buying to making using process A?
(iii) At what annual volume should the company switch from process A to B?

SOLUTION:

(i) Compute the annual cost for each alternative.

Annual cost of process A = Fixed cost + Variable cost/unit × Quantity

= 100,000 + 75 × 10,000

= ₹ 850,000

Annual cost of process B = Fixed cost + Variable cost/unit × Quantity

= 300,000 + 70 × 10,000

= ₹ 1,000,000

Annual cost of buying = Purchase price/unit × Quantity

= 80 × 10,000

= ₹ 800,000

Since, the annual cost of buy option is minimum among all the alternative costs, the company should buy the product.

(ii) Let Q be the quantity at which the company switches from buying to making, using process A.

Hence, total annual cost of process A ≤ Total annual cost of buying

100,000 + 75 × Q ≤ 80 × Q

100,000 ≤ 5 Q

$$Q \geq \frac{100{,}000}{5}$$

$$Q \geq 20{,}000$$

Thus, if the quantity of production is more than 20,000 units the company should switch from buying to making option using process A.

(ii) Let Q be the quantity at which the shift making using process A to making using process B is preferable.

Total annual cost of process A $\geq$ Total annual cost of process B

$100{,}000 + 75 \times \mathrm{Q} \geq 300{,}000 + 70 \times \mathrm{Q}$

$5\mathrm{Q} \geq 200{,}000$

$\mathrm{Q} \geq \frac{200{,}000}{5}$

$\mathrm{Q} \geq 40{,}000$

Thus, if the production quantity is more than 40,000 units, the company can shift from process A to Process B.

7 The ABC Company is investigating the decision whether to make or buy a plastic packaging which is currently being purchased at ₹ 7 each. The demand estimates are shown below:

Demand (Units)	20,000	30,000	40,000	50,000	60,000
Chance (%)	10	30	40	15	5

The decision to manufacture in-house costs the company an annual fixed cost of ₹ 80,000 towards renovation and conditioning, and variable costs are estimated at ₹ 5 per unit.
Give your decision whether to make or buy. At what quantity it is profitable to produce rather than buy.

SOLUTION:

Make or buy decision

The expected demand is determined treating percentage chance as probability.

Demand (D)	Probability (P)	D × P
20,000	0.10	2,000
30,000	0.30	9,000
40,000	0.40	16,000
50,000	0.15	7,500
60,000	0.05	3,000
Total		37,500

Total cost of making = Fixed cost + Variable cost/unit × Quantity

= 80,000 + 5 × 37,500

= ₹ 267,500

Total cost of buying = Purchase price/unit × Quantity

= 7 × 37,500

= ₹ 262,500

Since, the total cost of buying is less than the total cost of making, the company should buy the product.

Break-Even Point

Let Q be the quantity at which the company switches from buying to making.

Hence, total annual cost of making ≤ Total annual cost of buying

$80{,}000 + 5 \times Q \leq 7 \times Q$

$80{,}000 \leq 2\,Q$

$$Q \geq \frac{80{,}000}{2}$$

$$Q \geq 40{,}000$$

Thus, if the quantity of production is more than 40,000 units the company should switch from buying to making option.

8 An item which is required by the company can be manufactured on any of the three following machines and also it can be purchased at a price of ₹ 1.2 per component.

Machine	Fixed Cost (₹)	Variable Cost (₹ / Unit)
A	9,000	0.75
B	3,500	0.50
C	92,000	0.05

Suggest the best option if the requirement is 120,000 units and give the decision rules. Also plot the quantity vs. cost.

SOLUTION:

Best option if the requirement is 120,000 units

Compute the annual cost for each alternative.

Annual cost of machine A = Fixed cost + Variable cost/unit × Quantity

= 9,000 + 0.75 × Q

= 9,000 + 0.75 × 120,000

= ₹ 99,000

Annual cost of machine B = Fixed cost + Variable cost/unit × Quantity

= 3,500 + 0.50 × Q

= 3,500 + 0.50 × 120,000

= ₹ 63,500

Annual cost of machine C = Fixed cost + Variable cost/unit × Quantity

= 92,000 + 0.05 × Q

= 92,000 + 0.05 × 120,000

= ₹ 98,000

Annual cost of buying = Purchase price/unit × Quantity

= 1.2 × Q

= 1.2 × 120,000

= ₹ 144,000

The best option is to make on machine B which results in lowest cost.

Quantity vs. cost plot

Quantity (Q)	Total Annual Cost			
	Machine A = 9000 + 0.75 Q	Machine B = 3500 + 0.50 Q	Machine C = 92000 + 0.05 Q	Buying = 1.2 Q
0	9,000	3,500	92,000	0
25,000	27,750	16,000	93,250	30,000
50,000	46,500	28,500	94,500	60,000
75,000	65,250	41,000	95,750	90,000
100,000	84,000	53,500	97,000	120,000
125,000	102,750	66,000	98,250	150,000
150,000	121,500	78,500	99,500	180,000
175,000	140,250	91,000	100,750	210,000
200,000	159,000	103,500	102,000	240,000
225,000	177,750	116,000	103,250	270,000

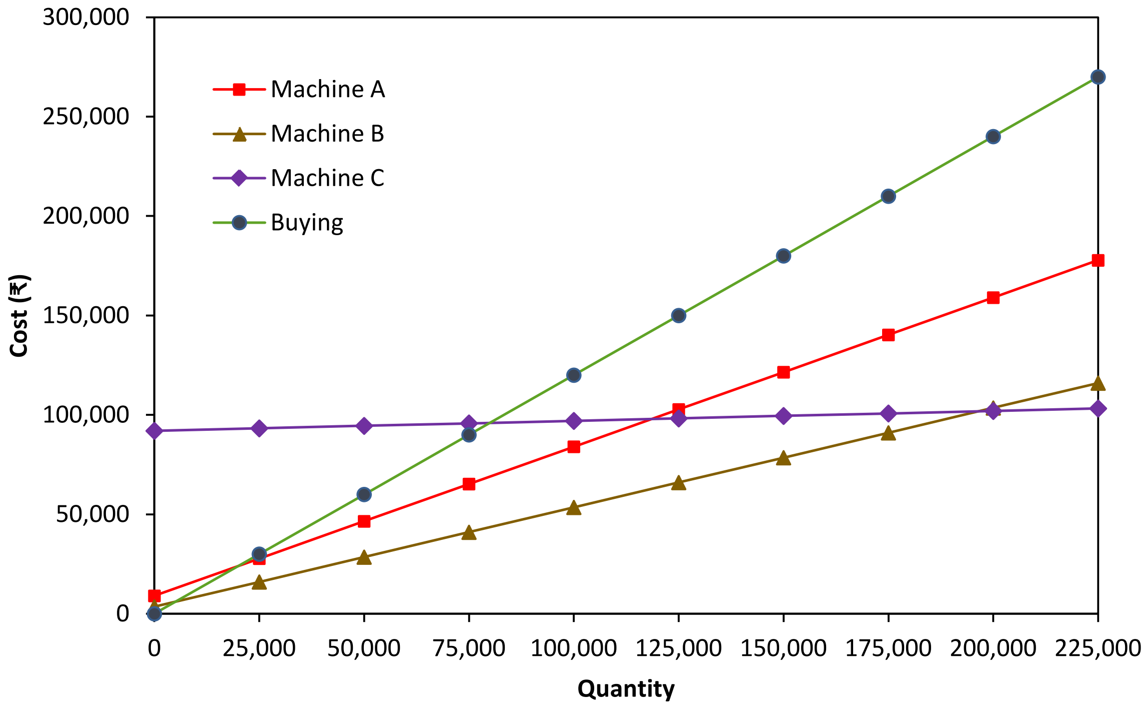


Figure 13 Break Even Chart

9 A company is buying one of the components that go into their product at a total cost of ₹ 20 per unit. Their annual requirement of the component is 6000 units. One of the supervisors had put up a proposal for making the component in the shop itself because the variable cost per unit comes to ₹ 2 only. It was found that it requires sophisticated machines and the accounts department had worked out and estimated the annual fixed cost of ₹ 140,000. Should the proposal be accepted? Draw the break even chart and find out the minimum level of annual requirement above which making the component would be profitable.

SOLUTION:

(i) Compute the annual cost for each alternative.

Annual cost of making = Fixed cost + Variable cost/unit × Quantity

= 140,000 + 2 × 6,000

= ₹ 152,000

Annual cost of buying = Purchase price/unit × Quantity

= 20 × 6,000

= ₹ 120,000

Since, the annual cost of making is more than the cost of buying, the proposal for making the component for annual requirement of 600 units is unacceptable.

(ii)

Quantity (Q)	Total Annual Cost	
	Making = 140000 + 2 Q	Buying = 20 Q
0	140,000	0
1,000	142,000	20,000
2,000	144,000	40,000
3,000	146,000	60,000
4,000	148,000	80,000
5,000	150,000	100,000
6,000	152,000	120,000
7,000	154,000	140,000
8,000	156,000	160,000
9,000	158,000	180,000

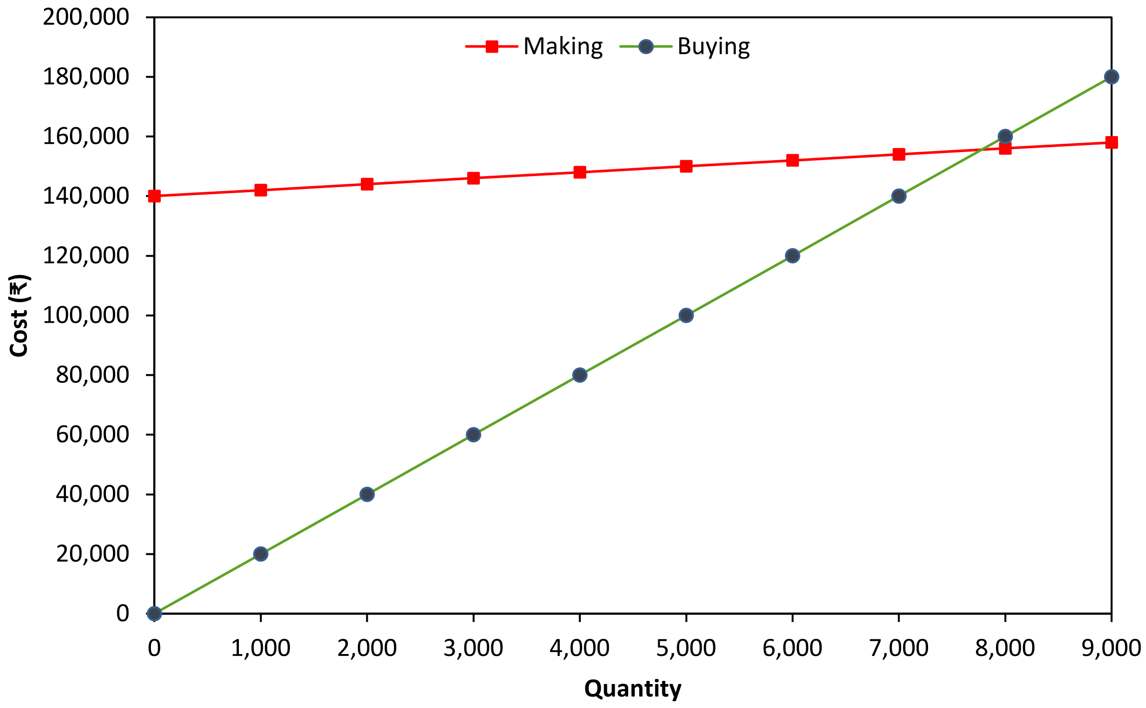


Figure 14 Break Even Chart

Let Q be the quantity at which the company switches from buying to making.

Hence, total annual cost of making $\leq$ Total annual cost of buying

$140{,}000 + 2 \times Q \leq 20 \times Q$

$140{,}000 \leq 18\,Q$

$$Q \geq \frac{140{,}000}{18}$$

$Q \geq 7777.7778$

Thus, if the quantity of production is more than 7,778 units the company should switch from buying to making option.

10 TWO MARKS QUESTIONS AND ANSWERS

1. What do you mean by PPC?

Production planning and control can be defined as the "direction and coordination a firms resources towards attaining the prefixed goals."

2. Define plant layout.

Plant layout is the arrangement of buildings, machinery, equipments, workplaces and other facilities of production to manufacture products in most efficient manner.

3. What are the various types of plant layout?

- Process layout
- Product layout
- Group layout (Combination layout)
- Fixed position layout

4. Define forecasting.

Forecasting is an estimate of future event achieved by systematically combining and casting forward in a predetermined way about the past data.

5. What are the factors affecting forecasting?

- Business cycle
- Random variation
- Customer's plan
- Product's life cycle
- Competition's efforts and prices
- Customer's confidence and attitude
- Quality
- Credit policy
- Design of goods or services
- Reputation for service
- Sales effort
- Advertising

6. What is Gantt chart?

Gantt chart is a graphic representation of scheduling. The chart is drawn to time scale. This is used to know whether actual production is keeping with schedule or falling behind.

7. What are the principles of scheduling?

- Principle of optimum task size
- Principle of optimum production plan
- Principle of optimum sequence

8. What are the various scheduling strategies?

- Forward scheduling
- Backward scheduling
- Finite loading
- Critical ratio scheduling

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FACULTY OF ENGINEERING

DEPARTMENT OF AUTOMOBILE ENGINEERING

INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH

UNIT-III

LINEAR PROGRAMMING TECHNIQUES

CONTENTS

1 BASICS OF OPERATIONS RESEARCH

1.1 INTRODUCTION

The term 'operations research' was coined in 1940 by Joseph F. McCloskey and Florence N. Trefethen in a small town of Bawdsey in England. It is a science that came into existence in a military context. During world war II, the military management of UK called on scientists from various disciplines and organized them into teams to assist it in solving strategic and tactical problems relating to air and land defence of the country. They were required to formulate specific proposals and plans for aiding the military commands to arrive at decisions on optimal utilization of scarce military resources and efforts and also to implement the decisions effectively. This new approach to the systematic and scientific study of the operations of the system was called Operations Research (OR), or operational research. Hence OR can be termed as 'an art of winning the war without actually fighting it.'

1.1.1 DEFINITION

There are many definitions of Operations Research. According to one such definition:

"Operations Research is the application of scientific methods to complex problems arising from operations involving large systems of men, machines, materials and money in the industry, business, government and defence."

1.2 SCOPE OF OPERATIONS RESEARCH

There is a great scope for economists, statisticians, administrators and technicians working as a team to solve problems of defence by using the OR approach. Besides this, OR is useful in various other important fields like:

- Agriculture
- Finance
- Industry
- Marketing
- Personnel management
- Production management
- Research and development

1.3 PHASES OF OPERATIONS RESEARCH

The procedure to be followed in the study of OR generally involves the following major phases:

1. Formulating the problem
2. Constructing a mathematical model
3. Deriving the solution from the model
4. Testing the model and its solution (updating the model)
5. Controlling the solution
6. Implementation

1.4 MODELS IN OPERATIONS RESEARCH

A model in OR is a simplified representation of an operation or is a process in which only the basic aspects or the most important features of a typical problem under investigation are considered. The objective of a model is to identify significant factors and interrelationships. The reliability of the solution obtained from a model depends on the validity of the model representing the real system.

A good model must possess the following characteristics:

- It should be capable of taking into account, the new formulation without having any change in its frame.
- Assumptions made in the model should be as few as possible.
- Variables used in the model must be less in number ensuring that it is simple and coherent.
- It should be open to the parametric type of treatment.
- It should not take much time in its construction for any problem.

1.4.1 ADVANTAGES OF A MODEL

There are certain significant advantages of using a model. These are:

- Problems under consideration become controllable through a model.
- A model provides a logical and systematic approach to the problem.
- A model clearly shows the limitations and scope of an activity.
- It helps in finding useful tools that eliminate duplication of methods applied to solve problems.
- It helps in finding solutions for research and improvement in a system.
- It provides an economic description and explanation of either the operation or the systems it represents.

1.4.2 CHARACTERISTICS OF A GOOD MODEL

- The number of variables used should be as few as possible.
- The number of assumptions should be as few as possible.
- It should be easy and economical to construct.
- It should assimilate the system environmental changes without a change in the framework.
- It should be adaptable to the parametric type of treatment.

1.5 CLASSIFICATION OF MODELS

Classification of models is a subjective problem. Models may be distinguished:

- by the degree of abstraction
- by function
- by structure
- by nature of an environment
- by the extent of generality

1.5.1 MODELS BY FUNCTION

These models can further be classified as

- Descriptive models,
- Predictive models, and
- Normative models.

1.5.1.1 DESCRIPTIVE MODELS

They describe and predict facts and relationships among the various activities of the problem. They do not have an objective function as a part of the model to evaluate decision alternatives. Through them, it is possible to get information on how one or more factors change as a result of changes in other factors.

1.5.1.2 NORMATIVE OR OPTIMIZATION MODELS

They are prescriptive in nature and develop objective decision rule for optimum solutions.

1.5.2 MODELS BY STRUCTURE

These models are represented by

- Iconic models,
- Analogue models, and
- Symbolic models.

1.5.2.1 ICONIC OR PHYSICAL MODELS

These are pictorial representations of real systems and have the appearance of the real thing. An iconic model is said to be scaled down or scaled up according to the dimensions of the model, which may be smaller or greater than that of the real item, e.g., city maps, blueprints of houses, globe and so on. These models are easy to observe and describe, but are difficult to manipulate and are not very useful for the purposes of prediction.

1.5.2.2 ANALOGUE MODELS

They are more abstract than the iconic model as there is no similarity between these models and real-life items. The models in which one set of properties is used to represent another set of properties are called analogue models. After the problem is solved, the solution is reinterpreted in terms of the original system. These models are less specific and concrete, but easier to manipulate than iconic models.

1.5.2.3 MATHEMATIC OR SYMBOLIC MODELS

They are most abstract in nature and employ a set of mathematical symbols to represent the components of the real system. These variables are related together by means of mathematical equations to describe the behaviour of the system. The solution to the problem is then obtained by applying well developed mathematical techniques to the model.

The symbolic model is usually the easiest to manipulate experimentally and it is also the most general and abstract. Its function is more explanatory than descriptive.

1.5.3 MODELS BY NATURE OF AN ENVIRONMENT

These models can be classified into

- Deterministic models, and
- Probabilistic models.

1.5.3.1 DETERMINISTIC MODELS

In these models, all parameters and functional relationships are assumed to be known with certainty when the decision is to be made. Linear programming and break-even model are good examples of deterministic models.

1.5.3.2 PROBABILISTIC OR STOCHASTIC MODELS

These models have at least one parameter or decision variable as a random variable. These models reflect some extent the complexity of the real world and the uncertainty surrounding it.

1.5.4 MODELS BY THE EXTENT OF GENERALITY

These models can be categorized as

- Specific models, and
- General models.

When a model presents a system at some specific time, it is known as a *specific model*. In these models, if the time factor is not considered, they are termed as *static models*. An inventory problem of determining economic order quantity for the next period assuming that the demand in planning period would remain the same as that of today is an example of a static model. Dynamic programming may be considered as an example of a *dynamic model*.

Simulation and heuristic models fall under the category of *general models*. These models are used to explore alternative strategies which have been overlooked previously.

1.6 USES AND LIMITATIONS OF OPERATIONS RESEARCH

1.6.1 USES

- It provides a logical and systematic approach to the problem.
- It allows modification of mathematical solutions before they are put to use.
- Suggests all the alternate courses of action for the same management.
- Helps in finding avenues for new research and improvement in the system.
- Facilitates improved quality of decision.
- Leads to optimum use of managers' production factor.
- It makes the overall structure of the production problem more comprehensible and helps in dealing with the problem as a whole.
- Aids in preparation of future managers by improving their knowledge and skill.
- Indicates the scope as well as limitation of a problem.

1.6.2 LIMITATIONS

Models are only idealized representations of reality and cannot be regarded as absolute in any case.

The validity of a model, for a particular situation, can be ascertained only by conducting experiments on it.

Mathematical models are applicable to only specific categories of problems as they do not take qualitative factors into account. All influencing factors, which cannot be quantified, find no place in mathematical models.

Operations Research requires huge calculations which cannot be handled manually and requires computers, resulting in heavy costs.

As it is a new field, there is a resistance from the employees to the new proposals.

The implementation of OR mainly depends on the person who provides the solution, and the person (manager) who uses the solution.

1.7 OPERATIONS RESEARCH AND DECISION-MAKING

Operations research or management science, as the name suggests, is the science of managing, which most of the time is about making decisions. It is thus a decision science that helps the management to make 'better decisions', a pivotal word in managing.

Decision-making can be improved and in fact, there is a wide scope for such improvements. The essential characteristics of all decisions are:

- Objectives
- Alternatives
- Influencing factors (constraints)

Once these characteristics are known, one can work towards improving the decisions.

In OR, scientific quantification is used in order to make better management decisions.

Thus, in OR, the essential features of decisions, namely, objectives, alternatives and influencing factors are expressed in terms of scientific quantifications or mathematical equations.

Operations research helps to overcome the complexity of the decision-making mode as it provides the management with the much needed tools for improving their decisions.

2 LINEAR PROGRAMMING

2.1 INTRODUCTION

Linear programming deals with the optimization (maximization or minimization) of a function of variables known as objective functions. It is subject to a set of linear equalities and/or inequalities known as constraints. Linear programming is a mathematical technique which involves the allocation of limited resources in an optimal manner, on the basis of a given criterion of optimality.

The graphical method of solving a Linear Programming Problems (LPP) is applicable where two variables are involved. The most widely used method for solving LPP problems consisting of any number of variables is called *simplex method*, developed by G. Dantzig in 1947 and made generally available in 1951.

2.2 FORMULATION OF LP PROBLEMS

The procedure for mathematical formulation of a LPP consists of the following steps:

1. To write down the decision variables of the problem.
2. To formulate the objective function to be optimized (maximized or minimized) as a linear function of the decision variables.
3. To formulate the other conditions of the problem such as resource limitation, market constraints, interrelations between variables etc., as linear inequations or equations in terms of the decision variables.
4. To add the non-negativity constraint from the considerations so that the negative values of the decision variables do not have any valid physical interpretation.

The objective function, the set of constraints and the non-negative restrictions together form a Linear Programming Problem (LPP).

2.2.1 GENERAL FORMULATION OF LPP

The general formulation of the LPP can be stated as follows:

In order to find the values of n decision variables $x_1, x_2, \dots x_n$ to maximize or minimize the objective function.

$$z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

and also satisfy *m* constraints

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_i$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

where constraints may be in the form of inequality ≤ or ≥ or even in the form of an equation (=) and finally satisfy the non-negative restrictions.

$$x_1 \geq 0, x_2 \geq 0, \dots x_n \geq$$

2.2.2 MATRIX FORM OF LP PROBLEM

The LPP can be expressed in the matrix as follows:

Maximize or minimize $z = cx$ Objective function

Subject to, $Ax\ (\leq = \geq)\ b$ Constraint equation

$b \geq 0, x \geq 0$ Non-negativity restrictions

where

$$x = (x_1\ x_2\ \dots\ x_n)$$

$$c = (c_1\ c_2\ \dots\ c_n)$$

$$b = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}$$

$$A = \begin{pmatrix} a_{11}\ a_{12}\ \dots\ a_{1n} \\ a_{21}\ a_{22}\ \dots\ a_{2n} \\ \dots \\ a_{m1}\ a_{m2}\ \dots\ a_{mn} \end{pmatrix}$$

3 GRAPHICAL METHOD

Simple linear programming problems with two decision variables can be easily solved by graphical method.

3.1 PROCEDURE FOR SOLVING LPP BY GRAPHICAL METHOD

The steps involved in the graphical method are as follows:

1. Consider each inequality constraint as an equation.
2. Plot each equation on the graph, as each will geometrically represent a straight line.
3. Mark the region. If the inequality constraint corresponding to that line is $\leq$, then the region below the line lying in the first quadrant (due to the non-negativity of variables) is shaded. For the inequality constraint $\geq$ sign, the region above the line in the first quadrant is shaded. The points lying in the common region will satisfy all the constraints simultaneously. The common region thus obtained is called the '*feasible region*'.
4. Assign an arbitrary value, say zero, to the objective function.
5. Draw the straight line to represent the objective function with the arbitrary value (i.e., a straight line through the origin).
6. Stretch the objective function line till the extreme points of the feasible region. In the maximization case, this line will stop farthest from the origin, passing through at least one corner of the feasible region. In the minimization case, this line will stop nearest to the origin, passing through at least one corner of the feasible region.
7. Find the coordinates of the extreme points selected in step 6 and find the maximum or minimum value of z.

Note: As the optimal values occur at the corner points of the feasible region, it is enough to calculate the value of the objective function of the corner points of the feasible region and select the one that gives the optimal solution. That is, in the case of maximization problem, the optimal point corresponds to the corner point at which the objective function has a maximum value, and in the case of minimization, the optional solution is the corner point which gives the objective function the minimum value for the objective function.

3.2 SOME MORE CASES

There are some linear programming problems which may have,

- a unique optimal solution,
- an infinite number of optimal solutions,
- an unbounded solution, and

- no solution.

3.3 CANONICAL OR STANDARD FORMS OF LPP

The general LPP can be classified as canonical or standard forms.

In *standard form*, irrespective of the objective function, namely, maximize or minimize, all the constraints are expressed as equations. Moreover, RHS of each constraint and all variables are non-negative.

3.3.1 CHARACTERISTICS OF THE STANDARD FORM

- The objective function is of maximization type.
- All constraints are expressed as equations.
- Right-hand side of each constraint is non-negative.
- All variables are non-negative.

In *canonical form*, if the objective function is of maximization, all the constraints other than non-negative conditions are '≤' type. If the objective function is of minimization, all the constraints other than non-negative condition are '≥' type.

3.3.2 CHARACTERISTICS OF THE CANONICAL FORM

- The objective function is of maximization type.
- All constraints are of ≤ type.
- All variables x_i are non-negative.

Note:

- Minimization of a function z is equivalent to maximization of the negative expression of this function, i.e., Min z = – Max $(-z)$
- An inequality in one direction can be converted into inequality in the opposite direction by multiplying both sides by –1.
- Suppose we have the constraint equation,
  $$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
  This equation can be replaced by two weak inequalities in opposite directions,
  $$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$
  and
  $$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq b_1$$
- If a variable is unrestricted in sign, then it can be expressed as a difference of two non-negative variables, i.e., if x_1 is unrestricted in sign, then $x_1 = x_1' - x_1''$, where x_1', x_1'' are ≥ 0.
- In standard form, all the constraints are expressed in an equation, which is possible by introducing some additional variables called 'slack variables' and 'surplus variables' so that a system of simultaneous linear equations is obtained. The necessary transformation will be made to ensure that $b_i \geq 0$.

3.4 DEFINITIONS

- A set of values x_1, x_2, … x_n that satisfies the constraints of the LPP is called its *solution*.
- Any solution to a LPP, which satisfies the non-negativity restrictions of the LPP is called its *feasible solution*.

- Any feasible solution, which optimizes (minimizes or maximizes) the objective function of the LPP is called its *optimum solution*.
- Given a system of *m* linear equations with *n* variables (*m* < *n*), any solution that is obtained by solving for *m* variables keeping the remaining *n* – *m* variables zero is called a *basic solution*. Such *m* variables are called *basic variables* and the remaining are called *non-basic variables*.
- The number of basic solutions
$$\leq \frac{n!}{m!\,(n-m)!}$$
- A *basic feasible solution* is a basic solution which also satisfies all basic variables are non-negative. Basic feasible solutions are of two types:
  Non-degenerate: A non-degenerate basic feasible solution is the basic feasible solution that has exactly *m* positive x_i (*i* = 1, 2, … *m*) i.e., none of the basic variables are zero.
  Degenerate: A basic feasible solution is said to degenerate if one or more basic variables are zero.
- If the value of the objective function z can be increased or decreased indefinitely, such solutions are called *unbounded solutions*.
- If the constraints of a general LPP be
$$\sum_{j=1}^{n} a_{ij}\,x_j \leq b_i\ (i = 1, 2, \dots m)$$
  Then the non-negative variables s_i, which are introduced to convert the inequalities ≤ to the equalities,
$$\sum_{j=1}^{n} a_{ij}\,x_j + s_i = b_i\ (i = 1, 2, \dots m)$$
  are called '*slack variables*'.
  Slack variables are also defined as the non-negative variables that are added in the LHS of the constraint to convert the inequality '≤' into an equation.
- If the constraints of a general LPP be
$$\sum_{j=1}^{n} a_{ij}\,x_j \geq b_i\ (i = 1, 2, \dots m)$$
  Then the non-negative variables s_i, which are introduced to convert the inequalities ≥ to the equalities,
$$\sum_{j=1}^{n} a_{ij}\,x_j - s_i = b_i\ (i = 1, 2, \dots m)$$
  are called '*surplus variables*'.
  Surplus variables are also defined as the non-negative variables that are removed from the LHS of the constraint to convert the inequality '≥' into an equation.

4 SIMPLEX METHOD

4.1 INTRODUCTION

Simplex method is an iterative procedure for solving LPP in a finite number of steps. It provides an algorithm, which consists of moving from one vertex of the region of a feasible solution to another in a manner that the value of the objective function at the succeeding vertex is less or more, as the case may be, than at the previous vertex. This procedure is repeated and since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or indicates the existence of an unbounded solution.

4.2 SIMPLEX ALGORITHM

For the solution of any LPP by simplex algorithm, the existence of an initial basic feasible solution is always assumed. The steps for the computation of an optimum solution are as follows:

Consider the following general LPP.

Maximize $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to,

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq \text{(or)} \geq \text{(or)} = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq \text{(or)} \geq \text{(or)} = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq \text{(or)} \geq \text{(or)} = b_m$$

$$x_i \geq 0, i = 1, 2, \ldots n$$

1. Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then convert it into a problem of maximization by

$$\text{Min } z = -\text{Max } (-z)$$

2. Check whether all b_i (*i* = 1, 2, … *m*) are positive. If any b_i is negative then multiply the inequation of the constraint by –1 so as to get all b_i to be positive.
3. Express the problem in standard form by introducing slack variables (for ≤ type) or surplus variables (for ≥ type) to convert the inequality constraints into equations.

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + s_1 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + s_2 = b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + s_m = b_m$$

4. Find the initial basic solution by setting $x_i = 0, i = 1, 2, \ldots n$.

$$\therefore s_1 = b_1, s_2 = b_2, \ldots s_m = b_m$$

5. Write the objective function in terms of the slack (or surplus) variables with zero coefficients.

$$z = c_1x_1 + c_2x_2 + \cdots + c_nx_n + 0s_1 + 0s_2 + \cdots + 0s_m$$

$$z - c_1x_1 - c_2x_2 - \cdots - c_nx_n + 0s_1 + 0s_2 + \cdots + 0s_m = 0$$

6. Making use of the above steps, form the initial simplex table. The first row represents the objective function row. The next rows correspond to the constraint conditions. Put the slack (or surplus) variables under the column basis. Write the values of b_i, *i* = 1, 2, … *m* under the column RHS.

Initial Simplex table

Basis	Coefficient of									RHS
	z	x_1	x_2	…	x_n	s_1	s_2	…	s_m	
–	1	$-c_1$	$-c_2$	…	$-c_n$	0	0	…	0	0
s_1	0	a_{11}	a_{12}	…	a_{1n}	1	0	…	0	b_1
s_2	0	a_{21}	a_{22}	…	a_{2n}	0	1	…	0	b_2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
s_m	0	a_{m1}	a_{m2}	…	a_{mn}	0	0	…	1	b_m

7. If all the coefficients of the variables in the objective row are positive, the initial basic feasible solution is an optimum basic feasible solution. If at least one of them is negative go to the next step.
8. Choose the most –ve coefficient in the objective row. Let it be $-c_r$. Then the corresponding column is the *key column* and x_r will enter the basis. If there are more than one negative coefficients having the same most negative coefficient, then any one of them can be selected arbitrarily as the entering variable.
   If all $a_{ir} \leq 0$ (*i* = 1, 2, … *m*) then there is an unbounded solution to the given problem.
   If at least one $a_{ir} \leq 0$ (*i* = 1, 2, … *m*) then the corresponding x_r will enter the basis.
9. The next step is to determine which variable should be replaced in the basis. For this find the ratios by dividing RHS of each row (except objective row) by the corresponding element of the key column i.e., calculate
   $$\frac{b_1}{a_{1r}}, \frac{b_2}{a_{2r}}, \dots \frac{b_m}{a_{mr}}$$
   Choose the minimum non-negative ratio. Let it be b_k/a_{kr}. The row corresponds to the ratio is the *key row* and the corresponding variable is the leaving basic variable.
   The element at the intersection of the key column and key row is called the *pivot element* or *key element*. a_{kr} is the key element.
10. If the key element is not one, it can be made one by dividing the same row suitably.
11. Eliminate all the elements of key column except the key element by making use of elementary row operations.
12. Go to step 7 and repeat the procedure until either an optimum solution is obtained or there is an indication of the unbounded solution.

5 ARTIFICIAL VARIABLES TECHNIQUE

5.1 INTRODUCTION

LPP in which constraints may also have $\geq$ and $=$ signs after ensuring that all $b_i \geq 0$ are considered here. In such cases basis matrix cannot be obtained as an identify matrix in the starting simplex table, therefore introduce a new type of variable called the *artificial variable*. These variables are fictitious and cannot have any physical meaning. The artificial variable technique is merely a device to get the starting basic feasible solution, so that simplex procedure may be adopted as usual until the optimal solution is obtained. To solve such LPP there are two methods.

- The Charne's big *M* method or method of penalties
- The two-phase simplex method

5.2 THE CHARNE'S BIG *M* METHOD

The following steps are involved in solving an LPP using the big *M* method.

1. Express the problem in the standard form.

2. Add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type $\geq$ or $=$. However, the addition of these artificial variables causes a violation of the corresponding constraints. Therefore, we would like to get rid of these variables and not allow them to appear in the final solution. This is achieved by assigning a very large penalty (–*M* for maximization and *M* for minimization) in the objective function.

3. Solve the modified LPP by simplex method, until any one of the three cases may arise.
   (i) If no artificial variable appears in the basis and the optimality conditions are satisfied, then the current solution is an optimal basic feasible solution.
   (ii) If at least one artificial variable in the basis at zero level and the optimality condition is satisfied, then the current solution is an optimal basic feasible solution (though degenerated).
   (iii) If at least one artificial variable appears in the basis at positive level and the optimality condition is satisfied, then the original problem has no feasible solution. The solution satisfies the constraints but does not optimize the objective function, since it contains a very large penalty *M* and is called *pseudo optimal solution*.

Note: While applying simplex method, whenever an artificial variable happens to leave the basis, drop that artificial variable and omit all the entries corresponding to its column from the simplex table.

5.3 THE TWO-PHASE SIMPLEX METHOD

The two-phase simplex method is another method to solve a given LPP involving some artificial variables. The solution is obtained in two phases.

5.3.1 PHASE I

In this phase, construct an auxiliary LPP leading to a final simplex table containing a basic feasible solution to the original problem.

1. Assign a cost –1 to each artificial variable and a cost 0 to all other variables and get a new objective function $z^* = -A_1 - A_2 - A_3$ ... where A_i are artificial variables.
2. Write down the auxiliary LPP in which the new objective function is to be maximized, subject to the given set of constraints.
3. Solve the auxiliary LPP by simplex method until either of the following three cases arise:
   (i) Max $z^* < 0$ and at least one artificial variable appears in the optimum basis at positive level.
   (ii) Max $z^* = 0$ and at least one artificial variable appears in the optimum basis at zero level.
   (iii) Max $z^* = 0$ and no artificial variable appears in the optimum basis.

In case (i), given LPP does not possess any feasible solution, whereas in cases (ii) and (iii), go to phase II.

5.3.2 PHASE II

Use the optimum basic feasible solution of phase I as a starting solution for the original LPP. Assign the actual costs to the variable in the objective function and a zero cost to every artificial variable in the basis at zero level. Delete the artificial variable column that is eliminated from the basis in phase I from the table. Apply simplex method to the modified simplex table obtained at the end of phase I till an optimum basic feasible solution is obtained or till there is an indication of an unbounded solution.

5.4 DEGENERACY

The phenomenon of obtaining a degenerate basic feasible solution in a LPP is known as degeneracy. Degeneracy in LPP may arise

(i) at the initial stage
(ii) at any subsequent iteration stage.

In case of (i), at least one of the basic variables should be zero in the initial basic feasible solution. Whereas in case of (ii) at any iteration of the simplex method more than one variable is eligible to leave the basis, and hence the next simplex iteration produces a degenerate solution in which at least one basic variable is zero, i.e., the subsequent iteration may not produce improvements in the value of the objective function. As a result, it is possible to repeat the same sequence of simplex iteration endlessly without improving the solution. This concept is known as *cycling* (tie).

5.4.1 METHODS TO RESOLVE DEGENERACY

The following systematic procedure can be utilized to avoid cycling due to degeneracy in LPP.

1. First, find out the rows for which the minimum non-negative ratio is the same (tie); suppose there is a tie between first and third row.
2. Now rearrange the columns of the usual simplex table so that the columns forming the original unit matrix come first in proper order.
3. Find the minimum of the ratio,
$$\frac{\text{Elements of the first column of the unit matrix}}{\text{Corresponding elements of the key column}}$$
only for the tied rows, i.e., for the first and third rows.
   (i) If the third row has the minimum ratio then this row will be the key row and the key element can be determined by intersecting the key row with key column.
   (ii) If this minimum is also not unique, then go to the next step.
4. Now find the minimum of the ratio, only for the tied rows. If this minimum ratio is unique for the first row, then this row will be the key row for determining the key element by intersecting with the key column.
$$\frac{\text{Elements of the second column of the unit matrix}}{\text{Corresponding elements of the key column}}$$
If this minimum is also not unique, then go, to the next step.
5. Find the minimum of the ratio. The above step is repeated till the minimum ratio is obtained so as to resolve the degeneracy. After the resolution of this tie, simplex method is applied to obtain the optimum solution.
$$\frac{\text{Elements of the third column of the unit matrix}}{\text{Corresponding elements of the key column}}$$

6 DUALITY IN LINEAR PROGRAMMING

6.1 INTRODUCTION

Every LPP (called the primal) is associated with another LPP (called its dual). Either problem can be considered as primal and the other one as dual.

The importance of the duality concept is because of two main reasons:

- If the primal contains a large number of constraints and a smaller number of variables, the labour of computation can be considerably reduced by converting it into the dual problem and then solving it.
- The interpretation of the dual variables from the cost or economic point of view proves extremely useful in making future decisions in the activities being programmed.

6.2 FORMATION OF DUAL PROBLEMS

For formulating a dual problem, first, bring the problem in the canonical form. The following changes are used in formulating the dual problem:

1. Change the objective function of maximization in the primal into minimization in the dual and vice versa.
2. The number of variables in the primal will be the number of constraints in the dual and vice versa.
3. The cost coefficients $c_1, c_2, \ldots c_n$ in the objective function of the primal will be the RHS constant of the constraints in the dual and vice versa.
4. In forming the constraints for the dual, consider the transpose of the body matrix of the primal problem.
5. The variables in both problems are non-negative.
6. If a variable in the primal is unrestricted in sign, then the corresponding constraint in the dual will be an equation and vice versa.

6.3 DEFINITION OF THE DUAL PROBLEM

Let the primal problem be

Maximize $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

Subject to,

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

$$x_1, x_2, \ldots x_n \geq 0$$

The dual problem is defined as,

Minimize $z' = b_1w_1 + b_2w_2 + \cdots + b_mw_m$

Subject to,

$$a_{11}w_1 + a_{21}w_2 + \cdots + a_{m1}w_m \leq c_1$$

$$a_{12}w_1 + a_{22}w_2 + \cdots + a_{m2}w_m \leq c_2$$

$$\vdots$$

$$a_{1n}w_1 + a_{2n}w_2 + \cdots + a_{mn}w_m \leq c_n$$

$$w_1, w_2, \ldots w_m \geq 0$$

where w_1, w_2, … w_m are called *dual variables*.

6.4 IMPORTANT RESULTS IN DUALITY

- The dual of the dual is primal.
- If one is a maximization problem, then the other is of minimization.
- The necessary and sufficient condition of any LPP and its dual to have an optimal solution is that both must have a feasible solution.
- Fundamental duality theorem states, if either the primal or dual problem has a finite optimal solution, then the other problem also has a finite optimal solution and also the optimal values of the objective function in both the problems are the same, i.e., Max z = Min z'.
- Existence theorem states that, if either problem has an unbounded solution then the other problem has no feasible solution.
- Complementary slackness theorem states that:
    - If a primal variable is positive, then the corresponding dual constraint is an equation at the optimum and vice versa.
    - If a primal constraint is a strict inequality then the corresponding dual variable is zero at the optimum and vice versa.

6.5 DUAL SIMPLEX METHOD

The dual simplex method is very similar to the regular simplex method. The only difference lies in the criterion used for selecting a variable to enter and leave the basis. In dual simplex method, first, select the variable to leave the basis and then the variable to enter the basis. This method yields an optimal solution to the given LPP in a finite number of steps, provided no basis is repeated.

The dual simplex method is used to solve problems which start dual feasible (i.e., whose primal is optimal but infeasible). In this method, the solution starts optimum, but infeasible and remains infeasible until the true optimum is reached, at which the solution becomes feasible. The advantage of this method lies in its avoiding the artificial variables introduced in the constraints along with the surplus variables as all '≥' constraints are converted into '≤' type.

7 SOLVED PROBLEMS

1 A company produces two types of hats. Every hat *A* requires twice as much labour time as the second hat *B*. If the company produces only hat *B* then it can produce a total of 500 hats a day. The market limits daily sales of hat *A* and *B* to 150 and 250 respectively. The profits on hat *A* and *B* are ₹ 8 and ₹ 5 respectively. Solve graphically to get the optimal solution.

SOLUTION:

Decision variables:

Let x and y be the number of units of type *A* and *B* hats respectively.

Objective function:

Since the profit for the type *A* and *B* hats are given, the objective function is to maximize the profit.

$$\text{Maximize } z = 8x + 5y$$

Constraints:

Since the company can produce at the most 500 hats of type *B* and type *A* hat requires twice as much labour time as that of type *B* hat, production restriction is given by

$$2x + y \leq 500$$

Since the market limits daily sales of hat *A* and *B* to 150 and 250 respectively,

$$x \leq 150$$

$$y \leq 250$$

Non-negativity constraints:

Since the number of hats produced by the company cannot be negative,

$$x \geq 0$$

$$y \geq 0$$

Maximize $z = 8x + 5y$

Subject to, $2x + y \leq 500$

$x \leq 150$

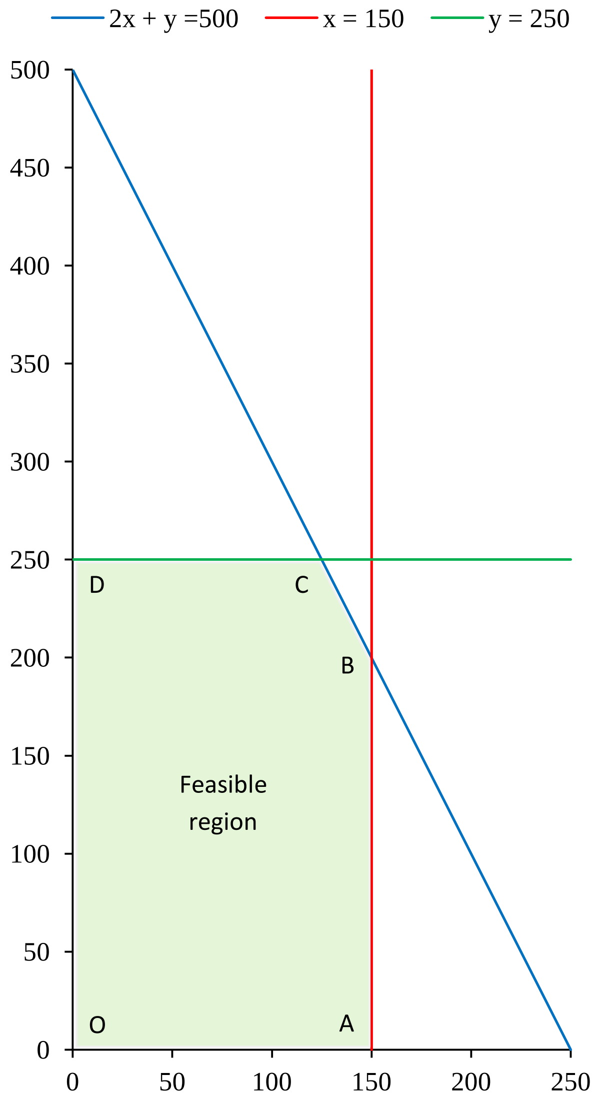
$y \leq 250$

and $x, y \geq 0$

Replace all the inequalities of the constraints by equation.

Sl. No.	Equation of Line	Passes through
1	$2x + y = 500$	(0, 500), (250, 0)
2	$x = 150$	(150, 0)
3	$y = 250$	(0, 250)

Plot each equation on the graph. Mark the regions below the lines lying in the first quadrant as the inequality of the constraints are $\leq$.



The feasible region is *OABCD*.

B is point of intersection of lines $2x + y = 500$ and $x = 150$.

Substitute $x = 150$ in equation $2x + y = 500$,

$2 \times 150 + y = 500$

$y = 200$

Point *B* passes through (150, 200).

C is point of intersection of lines $2x + y = 500$ and $y = 250$.

Substitute $y = 250$ in equation $2x + y = 500$,

$2x + 250 = 500$

$x = 125$

Point *C* passes through (125, 250).

Corner point	x	y	$z = 8x + 5y$ (₹)
O	0	0	0
A	150	0	1,200
B	150	200	2,200
C	125	250	2,250
D	0	250	1,250

The maximum value of z is attained at point *C* (125, 250).

Hence, the optimal solution is $x = 125$ and $y = 250$.

i.e., The company should produce 125 hats of type A and 250 hats of type B in order to get the maximum profit of ₹ 2,250.

2 Solve the following LPP by graphical method.

Minimize $z = 20x + 10y$

Subject to, $x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

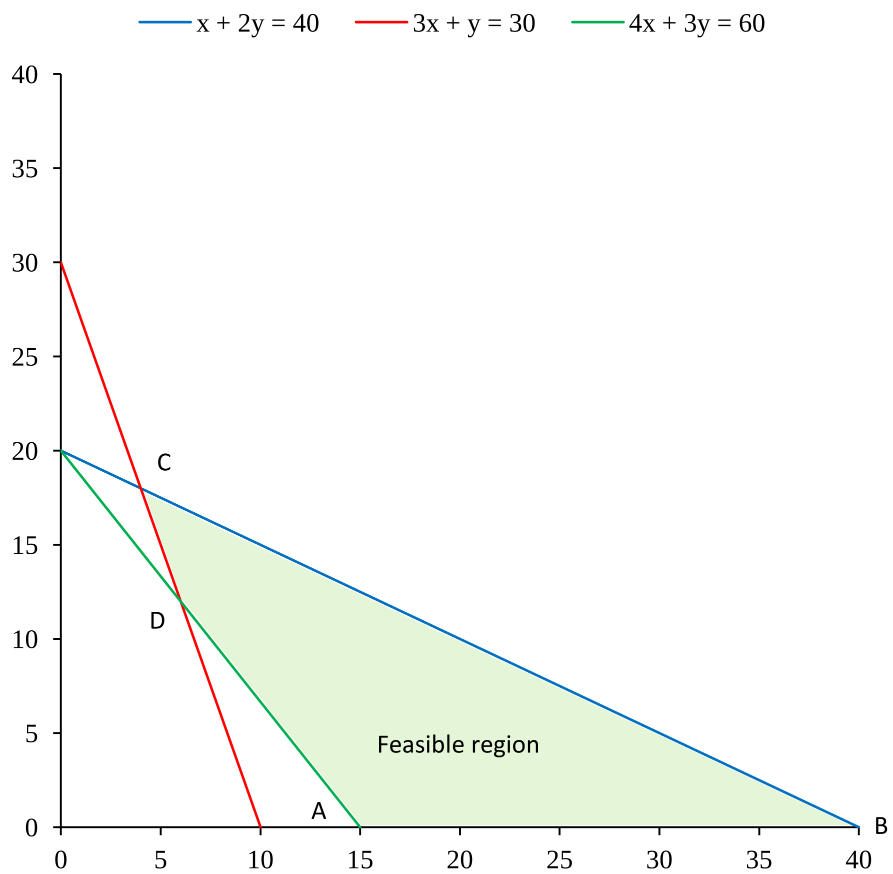
and $x, y \geq 0$

SOLUTION:

Replace all the inequalities of the constraints by equation.

Sl. No.	Equation of Line	Passes through
1	$x + 2y = 40$	(0, 20), (40, 0)
2	$3x + y = 30$	(0, 30), (40, 0)
3	$4x + 3y = 60$	(0, 20), (15, 0)

Plot each equation on the graph.



The feasible region is *ABCD*.

C is point of intersection of lines (1) and (2).

$3x + y = 30$

$y = 30 - 3x$

Substitute the above in equation $x + 2y = 40$,

$x + 2(30 - 3x) = 40$

$x + 60 - 6x = 40$

$5x = 20$

$x = 4$

Substitute the above in equation $x + 2y = 40$,

$4 + 2y = 40$

$2y = 36$

$y = 18$

Point *C* passes through (4, 18).

D is point of intersection of lines (2) and (3).

$3x + y = 30$

$y = 30 - 3x$

Substitute the above in equation $4x + 3y = 60$,

$4x + 3(30 - 3x) = 60$

$4x + 90 - 9x = 60$

$5x = 30$

$x = 6$

Substitute the above in equation $4x + 3y = 60$,

$4 \times 6 + 3y = 60$

$3y = 36$

$y = 12$

Point *D* passes through (6, 12).

Corner point	x	y	$z = 20x + 10y$
A	15	0	300
B	40	0	800
C	4	18	260
D	6	12	240

The minimum value of z is attained at point *D* (6, 12).

Hence, the optimal solution is $x = 6$ and $y = 12$.

3 Use simplex method to solve the LPP.

Maximize $z = 3x + 5y$

Subject to, $x + y \leq 60$

$x \leq 40$

$y \leq 30$

and $x, y \geq 0$

SOLUTION:

Convert the inequality constraints into equations by introducing slack variables.

$$x + y + s_1 = 60$$

$$x + s_2 = 40$$

$$y + s_3 = 30$$

$$x, y, s_1, s_2, s_3 \geq 0$$

The objective function can be written as

$$z - 3x - 5y + 0s_1 + 0s_2 + 0s_3 = 0$$

Initial simplex table

Basis	Coefficient of						RHS
	z	x	y	s_1	s_2	s_3	
–	1	−3	−5	0	0	0	0
s_1	0	1	1	1	0	0	60
s_2	0	1	0	0	1	0	40
s_3	0	0	1	0	0	1	30

Initial basic feasible solution

$x = y = 0,\ s_1 = 60,\ s_2 = 40,\ s_3 = 30,\ z = 0$

Entering basic variable

Choose the variable with the most negative coefficient in the objective row of the table. The column corresponding to the *most negative coefficient* is the *key column* and the corresponding variable is the entering basic variable.

Basis	Coefficient of						RHS
	z	x	y	s_1	s_2	s_3	
–	1	–3	–5	0	0	0	0
s_1	0	1	1	1	0	0	60
s_2	0	1	0	0	1	0	40
s_3	0	0	1	0	0	1	30

Here – 5 is the most negative coefficient and y is the entering basic variable.

Leaving basic variable

Find the ratio by dividing the RHS of each row (except the objective row) by the corresponding element of the key column. The row corresponding to the *smallest non negative ratio* is the *key row* and the corresponding variable is the leaving basic variable.

Here 30 is the least positive ratio and s_3 is the leaving basic variable.

Basis	Coefficient of						RHS	Ratio
	z	x	y	s_1	s_2	s_3		
–	1	–3	–5	0	0	0	0	
s_1	0	1	1	1	0	0	60	$\frac{60}{1}$ = 60
s_2	0	1	0	0	1	0	40	$\frac{40}{0}$ = ∞
s_3	0	0	1	0	0	1	30	$\frac{30}{1}$ = 30

The element at the intersection of key column and key row is the *pivot number* or *key number*. If the pivot number is not 1, make it unity by dividing the same row suitably.

Eliminate all the elements in the key column except the pivot element.

First iteration

Basis	Coefficient of						RHS	
	z	x	y	s_1	s_2	s_3		
–	1 + 5×0	–3 + 5×0	–5 + 5×1	0 + 5×0	0 + 5×0	0 + 5×1	0 + 5×30	$E_{1i} + 5E_{ki}$
s_1	0 – 0	1 – 0	1 – 1	1 – 0	0 – 0	0 – 1	60 – 30	$E_{2i} - E_{ki}$
s_2	0	1	0	0	1	0	40	
y	0	0	1	0	0	1	30	

Basis	Coefficient of						RHS
	z	x	y	s_1	s_2	s_3	
–	1	–3	0	0	0	5	150
s_1	0	1	0	1	0	–1	30
s_2	0	1	0	0	1	0	40
y	0	0	1	0	0	1	30

Basis	Coefficient of						RHS	Ratio
	z	x	y	s_1	s_2	s_3		
–	1	–3	0	0	0	5	150	
s_1	0	1	0	1	0	–1	30	$\frac{30}{1}$ = 30
s_2	0	1	0	0	1	0	40	$\frac{40}{1}$ = 40
y	0	0	1	0	0	1	30	$\frac{30}{0}$ = ∞

Second iteration

Basis	Coefficient of						RHS	
	z	x	y	s_1	s_2	s_3		
–	1 + 3×0	–3 + 3×1	0 + 3×0	0 + 3×1	0 + 3×0	5 + 3×(–1)	150 + 3×30	$E_{1i} + 3E_{ki}$
x	0	1	0	1	0	–1	30	
s_2	0 – 1	1 – 1	0 – 0	0 –1	1 – 0	0 – (–1)	40 – 30	$E_{3i} - E_{ki}$
y	0	0	1	0	0	1	30	

Basis	Coefficient of						RHS
	z	x	y	s_1	s_2	s_3	
–	1	0	0	3	0	2	240
x	0	1	0	1	0	–1	30
s_2	–1	0	0	–1	1	1	10
y	0	0	1	0	0	1	30

Since all the coefficients of the variables in the objective row are non-negative, the solution is optimum and it is given by

$x = 30$

$y = 30$

Max $z = 240$

4 Apply the simplex method to solve the LPP.

Maximize $z = 100x_1 + 200x_2 + 50x_3$

Subject to, $5x_1 + 5x_2 + 10x_3 \leq 1000$

$10x_1 + 8x_2 + 5x_3 \leq 2000$

$10x_1 + 5x_2 \leq 500$

and $x_1, x_2, x_3 \geq 0$

SOLUTION:

Convert the inequality constraints into equations by introducing slack variables.

$$5x_1 + 5x_2 + 10x_3 + s_1 = 1000$$

$$10x_1 + 8x_2 + 5x_3 + s_2 = 2000$$

$$10x_1 + 5x_2 + s_3 = 500$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

The objective function can be written as

$$z - 100x_1 - 200x_2 - 50x_3 + 0s_1 + 0s_2 + 0s_3 = 0$$

Initial simplex table

Basis	Coefficient of							RHS
	z	x_1	x_2	x_3	s_1	s_2	s_3	
–	1	–100	–200	–50	0	0	0	0
s_1	0	5	5	10	1	0	0	1000
s_2	0	10	8	5	0	1	0	2000
s_3	0	10	5	0	0	0	1	500

Initial basic feasible solution

$x_1 = x_2 = x_3 = 0$, $s_1 = 1000$, $s_2 = 2000$, $s_3 = 500$, $z = 0$

Entering basic variable

The most negative coefficient in the objective row (i.e., –200) of the table corresponds to the variable x_2. So x_2 is the entering basic variable and the corresponding column is the key column.

Leaving basic variable

Find the ratio by dividing the RHS of each row (except the objective row) by the corresponding element of the key column. The row corresponding to the smallest non negative ratio (i.e., 100) is the key row and the corresponding variable (i.e., s_3) is the leaving basic variable.

The element at the intersection of key column and key row is the pivot number or key number (i.e., 5). Make it unity.

Basis	Coefficient of							RHS	Ratio
	z	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	–100	–200	–50	0	0	0	0	
s_1	0	5	5	10	1	0	0	1000	$\frac{1000}{5}$ = 200
s_2	0	10	8	5	0	1	0	2000	$\frac{2000}{8}$ = 250
s_3	$\frac{0}{5}$	$\frac{10}{5}$	$\frac{5}{5}$	$\frac{0}{5}$	$\frac{0}{5}$	$\frac{0}{5}$	$\frac{1}{5}$	$\frac{500}{5}$	$\frac{500}{5}$ = 100

Eliminate all the elements in the key column except the pivot element.

First iteration

Basis	Coefficient of							RHS	
	z	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	300	0	–50	0	0	40	20000	$E_{1i} + 200E_{ki}$
s_1	0	–5	0	10	1	0	–1	500	$E_{2i} - 5E_{ki}$
s_2	0	–6	0	5	0	1	$-\frac{8}{5}$	1200	$E_{3i} - 8E_{ki}$
x_2	0	2	1	0	0	0	$\frac{1}{5}$	100	

Basis	Coefficient of							RHS	Ratio
	z	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	300	0	–50	0	0	40	20000	
s_1	$\frac{0}{10}$	$-\frac{5}{10}$	$\frac{0}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{0}{10}$	$-\frac{1}{10}$	$\frac{500}{10}$	$\frac{500}{10}$ = 50
s_2	0	–6	0	5	0	1	$-\frac{8}{5}$	1200	$\frac{1200}{5}$ = 240
x_2	0	2	1	0	0	0	$\frac{1}{5}$	100	$\frac{100}{0}$ = ∞

Second iteration

Basis	Coefficient of							RHS	
	z	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	275	0	0	5	0	35	22500	$E_{1i} + 50E_{ki}$
x_3	0	$-\frac{1}{2}$	0	1	$\frac{1}{10}$	0	$-\frac{1}{10}$	50	
s_2	0	$-\frac{7}{2}$	0	0	$-\frac{1}{2}$	1	$-\frac{11}{10}$	950	$E_{3i} - 5E_{ki}$
x_2	0	2	1	0	0	0	$\frac{1}{5}$	100	

Since all the coefficients of the variables in the objective row are non-negative, the solution is optimum and it is given by

$x_1 = 0$

$x_2 = 100$

$x_3 = 50$

Max $z = 22250$

5 Solve the following LPP using simplex method.

Minimize $z = 5x_1 - 8x_2 - 3x_3$

Subject to, $2x_1 + 5x_2 - x_3 \leq 1$

$-3x_1 - 8x_2 + 2x_3 \leq 4$

$-2x_1 - 12x_2 + 3x_3 \leq 9$

and $x_1, x_2, x_3 \geq 0$

SOLUTION:

Since the given objective function is of minimization convert it into maximization using

Minimize z = – Maximize $(-z)$ = – Maximize z^*

Minimize $z^* = -5x_1 + 8x_2 + 3x_3$

Subject to, $2x_1 + 5x_2 - x_3 \leq 1$

$-3x_1 - 8x_2 + 2x_3 \leq 4$

$-2x_1 - 12x_2 + 3x_3 \leq 9$

Convert the inequality constraints into equations by introducing slack variables.

$2x_1 + 5x_2 - x_3 + s_1 = 1$

$-3x_1 - 8x_2 + 2x_3 + s_2 = 4$

$-2x_1 - 12x_2 + 3x_3 + s_3 = 9$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

The objective function can be written as

$$z^* + 5x_1 - 8x_2 - 3x_3 + 0s_1 + 0s_2 + 0s_3 = 0$$

Initial simplex table

Basis	Coefficient of							RHS
	z^*	x_1	x_2	x_3	s_1	s_2	s_3	
–	1	5	–8	–3	0	0	0	0
s_1	0	2	5	–1	1	0	0	1
s_2	0	–3	–8	2	0	1	0	4
s_3	0	–2	–12	3	0	0	1	9

Initial basic feasible solution

$x_1 = x_2 = x_3 = 0$, $s_1 = 1$, $s_2 = 4$, $s_3 = 9$, $z = 0$

Entering basic variable

The most negative coefficient in the objective row (i.e., –8) of the table corresponds to the variable x_2. So x_2 is the entering basic variable and the corresponding column is the key column.

Leaving basic variable

Find the ratio by dividing the RHS of each row (except the objective row) by the corresponding element of the key column. The row corresponding to the smallest non-negative ratio (i.e., 1/5) is the key row and the corresponding variable (i.e., s_1) is the leaving basic variable.

The element at the intersection of key column and key row is the pivot number or key number (i.e., 5). Make it unity.

Basis	Coefficient of							RHS	Ratio
	z^*	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	5	–8	–3	0	0	0	0	
s_1	$\frac{0}{5}$	$\frac{2}{5}$	$\frac{5}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{0}{5}$	$\frac{0}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
s_2	0	–3	–8	2	0	1	0	4	$-\frac{4}{8}$
s_3	0	–2	–12	3	0	0	1	9	$-\frac{9}{12}$

Eliminate all the elements in the key column except the pivot element.

First iteration

Basis	Coefficient of							RHS	
	z^*	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	$\frac{41}{5}$	0	$-\frac{23}{5}$	$\frac{8}{5}$	0	0	$\frac{8}{5}$	$E_{1i} + 8E_{ki}$
x_2	0	$\frac{2}{5}$	1	$-\frac{1}{5}$	$\frac{1}{5}$	0	0	$\frac{1}{5}$	
s_2	0	$\frac{1}{5}$	0	$\frac{2}{5}$	$\frac{8}{5}$	1	0	$\frac{28}{5}$	$E_{3i} + 8E_{ki}$
s_3	0	$\frac{14}{5}$	0	$\frac{3}{5}$	$\frac{12}{5}$	0	1	$\frac{57}{5}$	$E_{4i} + 12E_{ki}$

Basis	Coefficient of							RHS	Ratio
	z^*	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	$\frac{41}{5}$	0	$-\frac{23}{5}$	$\frac{8}{5}$	0	0	$\frac{8}{5}$	
x_2	0	$\frac{2}{5}$	1	$-\frac{1}{5}$	$\frac{1}{5}$	0	0	$\frac{1}{5}$	$-\frac{1/5}{1/5} = -1$
s_2	$\frac{0}{2/5}$	$\frac{1/5}{2/5}$	$\frac{0}{2/5}$	$\frac{2/5}{2/5}$	$\frac{8/5}{2/5}$	$\frac{1}{2/5}$	$\frac{0}{2/5}$	$\frac{28/5}{2/5}$	$\frac{28/5}{2/5} = 14$
s_3	0	$\frac{14}{5}$	0	$\frac{3}{5}$	$\frac{12}{5}$	0	1	$\frac{57}{5}$	$\frac{57/5}{3/5} = 19$

Second iteration

Basis	Coefficient of							RHS	
	z^*	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	$\frac{21}{2}$	0	0	20	$\frac{23}{2}$	0	66	$E_{1i} + \frac{23}{5}E_{ki}$
x_2	0	$\frac{1}{2}$	1	0	1	$\frac{1}{2}$	0	3	$E_{2i} + \frac{1}{5}E_{ki}$
x_3	0	$\frac{1}{2}$	0	1	4	$\frac{5}{2}$	0	14	
s_3	0	$\frac{5}{2}$	0	0	0	$-\frac{3}{2}$	1	3	$E_{4i} - \frac{3}{5}E_{ki}$

Since all the coefficients of the variables in the objective row are non-negative, the solution is optimum and it is given by

$x_1 = 0$

$x_2 = 3$

$x_3 = 14$

Max $z^* = 66$

Min $z = -$ Max $z^* = -66$

6 Use simplex method to solve the LPP.

Minimize $z = x_1 - 3x_2 + 2x_3$

Subject to, $3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

and $x_1, x_2, x_3 \geq 0$

SOLUTION:

Since the given objective function is of minimization convert it into maximization using

Minimize z = – Maximize $(-z)$ = – Maximize z^*

Minimize $z^* = -x_1 + 3x_2 - 2x_3$

Subject to, $3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

Convert the inequality constraints into equations by introducing slack variables.

$3x_1 - x_2 + 2x_3 + s_1 = 7$

$-2x_1 + x_2 + s_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + s_3 = 10$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

The objective function can be written as

$$z^* + x_1 - 3x_2 + 2x_3 + 0s_1 + 0s_2 + 0s_3 = 0$$

Initial simplex table

Basis	Coefficient of							RHS
	z^*	x_1	x_2	x_3	s_1	s_2	s_3	
–	1	1	–3	2	0	0	0	0
s_1	0	3	–1	2	1	0	0	7
s_2	0	–2	1	0	0	1	0	12
s_3	0	–4	3	8	0	0	1	10

Initial basic feasible solution

$x_1 = x_2 = x_3 = 0$, $s_1 = 7$, $s_2 = 12$, $s_3 = 10$, $z = 0$

Entering basic variable

The most negative coefficient in the objective row (i.e., −3) of the table corresponds to the variable x_2. So x_2 is the entering basic variable and the corresponding column is the key column.

Leaving basic variable

Find the ratio by dividing the RHS of each row (except the objective row) by the corresponding element of the key column. The row corresponding to the smallest non-negative ratio (i.e., 3.33) is the key row and the corresponding variable (i.e., s_3) is the leaving basic variable.

The element at the intersection of key column and key row is the pivot number or key number (i.e., 3). Make it unity.

Basis	Coefficient of							RHS	Ratio
	z^*	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	1	−3	2	0	0	0	0	
s_1	0	3	−1	2	1	0	0	7	$-\frac{7}{1} = -7$
s_2	0	−2	1	0	0	1	0	12	$\frac{12}{1} = 12$
s_3	$\frac{0}{3}$	$-\frac{4}{3}$	$\frac{3}{3}$	$\frac{8}{3}$	$\frac{0}{3}$	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{10}{3}$	$\frac{10}{3} = 3.33$

Eliminate all the elements in the key column except the pivot element.

First iteration

Basis	Coefficient of							RHS	
	z^*	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	−3	0	10	0	0	1	10	$E_{1i} + 3E_{ki}$
s_1	0	$\frac{5}{3}$	0	$\frac{14}{3}$	1	0	$\frac{1}{3}$	$\frac{31}{3}$	$E_{2i} + E_{ki}$
s_2	0	$-\frac{2}{3}$	0	$-\frac{8}{3}$	0	1	$-\frac{1}{3}$	$\frac{26}{3}$	$E_{3i} - E_{ki}$
x_2	0	$-\frac{4}{3}$	1	$\frac{8}{3}$	0	0	$\frac{1}{3}$	$\frac{10}{3}$	

Basis	Coefficient of							RHS	Ratio
	z^*	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	−3	0	10	0	0	1	10	
s_1	$\frac{0}{5/3}$	$\frac{5/3}{5/3}$	$\frac{0}{5/3}$	$\frac{14/3}{5/3}$	$\frac{1}{5/3}$	$\frac{0}{5/3}$	$\frac{1/3}{5/3}$	$\frac{31/3}{5/3}$	$\frac{31/3}{5/3} = \frac{31}{5}$
s_2	0	$-\frac{2}{3}$	0	$-\frac{8}{3}$	0	1	$-\frac{1}{3}$	$\frac{26}{3}$	$-\frac{26/3}{2/3} = -13$
x_2	0	$-\frac{4}{3}$	1	$\frac{8}{3}$	0	0	$\frac{1}{3}$	$\frac{10}{3}$	$-\frac{10/3}{4/3} = \frac{-10}{4}$

Second iteration

Basis	Coefficient of							RHS	
	z^*	x_1	x_2	x_3	s_1	s_2	s_3		
–	1	0	0	$\frac{92}{5}$	$\frac{9}{5}$	0	$\frac{8}{5}$	$\frac{143}{5}$	$E_{1i} + 3E_{ki}$
x_1	0	1	0	$\frac{14}{5}$	$\frac{3}{5}$	0	$\frac{1}{5}$	$\frac{31}{5}$	
s_2	0	0	0	$-\frac{4}{5}$	$\frac{2}{5}$	1	$-\frac{1}{5}$	$\frac{64}{5}$	$E_{3i} + \frac{2}{3}E_{ki}$
x_2	0	0	1	$\frac{32}{5}$	$\frac{4}{5}$	0	$\frac{3}{5}$	$\frac{58}{5}$	$E_{4i} + \frac{4}{3}E_{ki}$

Since all the coefficients of the variables in the objective row are non-negative, the solution is optimum and it is given by

$$x_1 = \frac{31}{5}$$

$$x_2 = \frac{58}{5}$$

$$x_3 = 0$$

$$\text{Max } z^* = \frac{143}{5}$$

$$\text{Min } z = -\text{Max } z^* = -\frac{143}{5}$$

7 Solve by big M method.

Maximize $z = 2x_1 + 5x_2$

Subject to the constraints, $x_1 \leq 40$

$x_2 \leq 30$

$x_1 + x_2 \geq 60$

and $x_1, x_2 \geq 0$

SOLUTION:

Convert the inequality constraints into equations by introducing slack variables s_1, $s_2 \geq 0$ and surplus variable $s_3 \geq 0$.

$$x_1 + s_1 = 40$$

$$x_2 + s_2 = 30$$

$$x_1 + x_2 - s_3 = 60$$

The objective function can be written as

$$z - 2x_1 - 5x_2 + 0s_1 + 0s_2 + 0s_3 = 0$$

Initial basic feasible solution

$x_1 = x_2 = 0$, $s_1 = 40$, $s_2 = 30$, $s_3 = -60$, $z = 0$

Since $s_3 = -60$ is infeasible, introduce artificial variable $A_1 \geq 0$ in the third constraint.

$$x_1 + s_1 = 40$$

$$x_2 + s_2 = 30$$

$$x_1 + x_2 - s_3 + A_1 = 60$$

Modified objective function

For the maximization problem add $-MA_1$ to the RHS of the objective function, where M is a very large positive quantity.

$$z - 2x_1 - 5x_2 = -MA_1$$

$$z - 2x_1 - 5x_2 + MA_1 = 0$$

Revised modified objective function = Modified objective function + $(-M)$ A_1 equation

$(z - 2x_1 - 5x_2 + MA_1 = 0) + (-M)(x_1 + x_2 - s_3 + A_1 - 60 = 0)$

$z - 2x_1 - 5x_2 + MA_1 - Mx_1 - Mx_2 + Ms_3 - MA_1 + 60M = 0$

$z - (2 + M)x_1 - (5 + M)x_2 + Ms_3 = -60M$

Initial simplex table

Basis	Coefficient of							RHS
	z	x_1	x_2	s_1	s_2	s_3	A_1	
–	1	–(2+*M*)	–(5+*M*)	0	0	*M*	0	–60*M*
s_1	0	1	0	1	0	0	0	40
s_2	0	0	1	0	1	0	0	30
A_1	0	1	1	0	0	–1	1	60

Entering basic variable

Since *M* is a very large positive quantity, –(5+*M*) is the most negative coefficient in the objective row. So x_2 is the entering basic variable and the corresponding column is the key column.

Leaving basic variable

Find the ratio by dividing the RHS of each row (except the objective row) by the corresponding element of the key column. The row corresponding to the smallest non-negative ratio (i.e., 30) is the key row and the corresponding variable (i.e., s_2) is the leaving basic variable.

The element at the intersection of key column and key row is the pivot number or key number (i.e., 1).

Basis	Coefficient of							RHS	Ratio
	z	x_1	x_2	s_1	s_2	s_3	A_1		
–	1	–(2+*M*)	–(5+*M*)	0	0	*M*	0	–60*M*	
s_1	0	1	0	1	0	0	0	40	$\frac{40}{0} = \infty$
s_2	0	0	1	0	1	0	0	30	$\frac{30}{1} = 30$
A_1	0	1	1	0	0	–1	1	60	$\frac{60}{1} = 60$

Eliminate all the elements in the key column except the pivot element.

First iteration

Basis	Coefficient of							RHS	
	z	x_1	x_2	s_1	s_2	s_3	A_1		
–	1	–(2+*M*)	0	0	5+*M*	*M*	0	150–30*M*	$E_{1i} + (5 + M)E_{ki}$
s_1	0	1	0	1	0	0	0	40	
x_2	0	0	1	0	1	0	0	30	
A_1	0	1	0	0	–1	–1	1	30	$E_{4i} - E_{ki}$

Basis	Coefficient of							RHS	Ratio
	z	x_1	x_2	s_1	s_2	s_3	A_1		
–	1	−(2+M)	0	0	5+M	M	0	150–30M	
s_1	0	1	0	1	0	0	0	40	$\frac{40}{1}=40$
x_2	0	0	1	0	1	0	0	30	$\frac{30}{0}=\infty$
A_1	0	1	0	0	−1	−1	1	30	$\frac{30}{1}=30$

Second iteration

Basis	Coefficient of							RHS	
	z	x_1	x_2	s_1	s_2	s_3	A_1		
–	1	0	0	0	3	−2	2+M	210	$E_{1i}+(2+M)E_{ki}$
s_1	0	0	0	1	1	1	−1	10	$E_{2i}-E_{ki}$
x_2	0	0	1	0	1	0	0	30	
x_1	0	1	0	0	−1	−1	1	30	

Basis	Coefficient of							RHS	Ratio
	z	x_1	x_2	s_1	s_2	s_3	A_1		
–	1	0	0	0	3	−2	2+M	210	
s_1	0	0	0	1	1	1	−1	10	$\frac{10}{1}=10$
x_2	0	0	1	0	1	0	0	30	$\frac{30}{0}=\infty$
x_1	0	1	0	0	−1	−1	1	30	$-\frac{30}{1}=-1$

Third iteration

Basis	Coefficient of							RHS	
	z	x_1	x_2	s_1	s_2	s_3	A_1		
–	1	0	0	2	5	0	M	230	$E_{1i}+2E_{ki}$
s_3	0	0	0	1	1	1	−1	10	
x_2	0	0	1	0	1	0	0	30	
x_1	0	1	0	1	0	0	0	40	$E_{4i}+E_{ki}$

Since all the coefficients of the variables in the objective row are non-negative, the solution is optimum and it is given by

$x_1 = 40$

$x_2 = 30$

$x_3 = 0$

Max $z = 230$

8 Use penalty method to solve the LPP.

Minimize $z = 4x + 3y$

Subject to, $2x + y \geq 10$

$-3x + 2y \leq 6$

$x + y \geq 6$

and $x, y \geq 0$

SOLUTION:

Since the given objective function is of minimization convert it into maximization using

Minimize z = – Maximize $(-z)$ = – Maximize z^*

Maximize $z^* = -4x - 3y$

Subject to, $3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

Convert the inequality constraints into equations by introducing surplus variables $s_1, s_3 \geq 0$ and slack variable $s_2 \geq 0$.

$$2x + y - s_1 = 10$$

$$-3x + 2y + s_2 = 6$$

$$x + y - s_3 = 6$$

The objective function can be written as

$$z^* + 4x + 3y + 0s_1 + 0s_2 + 0s_3 = 0$$

Initial basic feasible solution

$x = y = 0, s_1 = -10, s_2 = 6, s_3 = -6, z = 0$

Since s_1 and s_3 are infeasible, introduce artificial variables $A_1 \geq 0$ and $A_3 \geq 0$ in the first and third constraints respectively.

$$2x + y - s_1 + A_1 = 10$$

$$-3x + 2y + s_2 = 6$$

$$x + y - s_3 + A_2 = 6$$

Modified objective function

For the maximization problem, add $-MA_1$ and $-MA_2$ to the RHS of the objective function, where M is a very large positive quantity.

$$z^* + 4x + 3y = -MA_1 - MA_2$$

$$z^* + 4x + 3y + MA_1 + MA_2 = 0$$

Revised modified objective function = Modified objective function + $(-M)$ A_1 equation + $(-M)$ A_2 equation

$(z^* + 4x + 3y + MA_1 + MA_2) + (-M)(2x + y - s_1 + A_1 - 10 = 0) + (-M)(x + y - s_3 + A_2 - 6 = 0)$

$z^* + 4x + 3y + MA_1 + MA_2 - 2Mx - My + Ms_1 - MA_1 + 10M - Mx - My + Ms_2 - MA_2 + 6M = 0$

$z^* + (4 - 3M)x + (3 - 2M)y + Ms_1 + Ms_2 = -16M$

Initial simplex table

Basis	Coefficient of								RHS
	z^*	x	y	s_1	s_2	s_3	A_1	A_2	
–	1	4–3M	3–2M	M	0	M	0	0	–16M
A_1	0	2	1	–1	0	0	1	0	10
s_2	0	–3	2	0	1	0	0	0	6
A_2	0	1	1	0	0	–1	0	1	6

Entering basic variable

Since M is a very large positive quantity, 4–3M is the most negative coefficient in the objective row. So x is the entering basic variable and the corresponding column is the key column.

Leaving basic variable

Find the ratio by dividing the RHS of each row (except the objective row) by the corresponding element of the key column. The row corresponding to the smallest non-negative ratio (i.e., 30) is the key row and the corresponding variable (i.e., s_2) is the leaving basic variable.

The element at the intersection of key column and key row is the pivot number or key number (i.e., 1).

Basis	Coefficient of								RHS	Ratio
	z^*	x	y	s_1	s_2	s_3	A_1	A_2		
–	1	4–3M	3–2M	M	0	M	0	0	–16M	
A_1	$\frac{0}{2}$	$\frac{2}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{0}{2}$	$\frac{0}{2}$	$\frac{1}{2}$	$\frac{0}{2}$	$\frac{10}{2}$	$\frac{10}{2}$ = 5
s_2	0	–3	2	0	1	0	0	0	6	$-\frac{6}{3}$ = –2
A_2	0	1	1	0	0	–1	0	1	6	$\frac{6}{1}$ = 6

Eliminate all the elements in the key column except the pivot element.

First iteration

Basis	Coefficient of								RHS	
	z^*	x	y	s_1	s_2	s_3	A_1	A_2		
–	1	0	$1-\frac{M}{2}$	$2-\frac{M}{2}$	0	M	$\frac{3M}{2}-2$	0	$-M-20$	$E_{1i} - (4-3M)\,E_{ki}$
x	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	5	
s_2	0	0	$\frac{7}{2}$	$-\frac{3}{2}$	1	0	$\frac{3}{2}$	0	21	$E_{3i}+3E_{ki}$
A_2	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	-1	$-\frac{1}{2}$	1	1	$E_{4i}-E_{ki}$

Basis	Coefficient of								RHS	Ratio
	z^*	x	y	s_1	s_2	s_3	A_1	A_2		
–	1	0	$1-\frac{M}{2}$	$2-\frac{M}{2}$	0	M	$\frac{3M}{2}-2$	0	$-M-20$	
x	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	5	$\frac{5}{1/2}=10$
s_2	0	0	$\frac{7}{2}$	$-\frac{3}{2}$	1	0	$\frac{3}{2}$	0	21	$\frac{21}{7/2}=6$
A_2	$\frac{0}{1/2}$	$\frac{0}{1/2}$	$\frac{1/2}{1/2}$	$\frac{1/2}{1/2}$	$\frac{0}{1/2}$	$-\frac{1}{1/2}$	$-\frac{1/2}{1/2}$	$\frac{1}{1/2}$	$\frac{1}{1/2}$	$\frac{1}{1/2}=2$

Second iteration

Basis	Coefficient of								RHS	
	z^*	x	y	s_1	s_2	s_3	A_1	A_2		
–	1	0	0	1	0	2	$M-1$	$M-2$	-22	$E_{1i} - (1-\frac{M}{2})\,E_{ki}$
x	0	1	0	-1	0	1	1	-1	4	$E_{2i}-\frac{1}{2}E_{ki}$
s_2	0	0	0	-5	1	7	5	-7	14	$E_{3i}-\frac{7}{2}E_{ki}$
y	0	0	1	1	0	-2	-1	2	2	

Since all the coefficients of the variables in the objective row are non-negative, the solution is optimum and it is given by

$x = 4$

$y = 2$

$\text{Max } z^* = -22$

$\text{Min } z = -\text{Max } z^* = -(-22) = 22$

8 TWO MARKS QUESTIONS AND ANSWERS

1. **What is Operations Research?**

   There are many definitions of Operations Research. According to one such definition:

   "Operations Research is the application of scientific methods to complex problems arising from operations involving large systems of men, machines, materials and money in industry, business, government and defence."

2. **What are the various types of models?**

   The various types of models are

   - Iconic or physical model
   - Analogue or schematic model
   - Symbolic or mathematical models.

3. **What is an analogue model?**

   Analogue model can represent dynamic situations. They are analogous to the characteristic of the system under study. They use one set of properties to represent some other set of properties of the system. After the model is solved, the solution is reinterpreted in terms of the original system.

4. **What is an iconic model?**

   Iconic models are pictorial representations of real systems and have the appearance of the real structure. Examples of such models are city maps, houses, blueprints, etc.

5. **What is a symbolic model?**

   Symbolic model is one which employs a set of mathematical symbols to represent the decision variables of the system. These variables are related together by mathematical equations which describe the properties of the system.

6. **Name some characteristics of a good model.**

   - It should be simple and coherent.
   - It should be open to parametric type of treatment.
   - There should be less number of variables.
   - Assumptions made in the model should be clearly mentioned and should be as small as possible.

7. **What are the main characteristics of operations research?**

   Some of the main characteristics of operations research are:

   - Its system orientation.
   - The use of inter-disciplinary forms.
   - Application of scientific method.
   - Uncovering of new problems.

8. State any four applications of operations research.

- Assignment of jobs to applicants to maximize total profit or minimize total costs.
- Replacement techniques are used to replace the old machines with new ones.
- Inventory control techniques are used in industries to purchase optimum quantity of raw materials.
- Before executing a project, activities are sequenced and scheduled using the PERT chart.

9. What are the methods used for solving operations research models?

- Analytic procedure
- Iterative procedure
- Monte-Carlo technique.

10. Explain the principles of modelling?

- Models should be validated prior to implementation.
- Models are the only aids in decision-making.
- Models should not be complicated. They should be as simple as possible.
- Models should be accurate as possible.

11. Define a feasible region.

A region in which all the constraints are satisfied simultaneously is called a feasible region.

12. Define a feasible solution.

Any solution to a LPP which satisfies the non-negativity restrictions of the LPP is called its feasible solution.

13. What is a redundant constraint?

A constraint that does not form boundary of feasible region and has impact on the solution of the problem, remodel of which does not alter the solution is called a redundant constraint.

14. Define optimal solution.

Any feasible solution which optimizes (minimizes or maximizes) the objective function is called its optimal solution.

15. What is the difference between feasible solution and basic feasible solution?

The solution of m basic variables when each of the $(n – m)$ non-basic variable is set to zero is called basic solution.

A basic solution in which all the basic variables are zero is called a basic feasible solution.

16. Define the following: (i) Basic solution (ii) non-degenerate solution (iii) degenerate solution.

Basic solution

Given a system of m linear equations with n variables, any solution which is obtained by solving for m variables keeping the remaining $n – m$ variables as zero is called a basic solution.

Such *m* variables are called basic variables and *n* – *m* variables are called non-basic variables.

Non-degenerate solution

A non-degenerate basic feasible solution is the basic feasible solution which has exactly *m* positive x_i (*i* = 1, 2, ... *m*), i.e., none of the basic variables are zero.

Degenerate solution

A basic feasible solution is said to be degenerate if one or more basic variables are zero.

17. Define unbounded solution.

If the value of the objective function z can be increased or decreased indefinitely, such solutions are called unbounded solutions.

18. What are the two forms of a LPP?

The two forms of LPP are

- Standard form and
- Canonical form.

19. What do you mean by canonical form of a LPP?

In canonical form, if the objective function is of maximization, then all the constraints other than non-negativity conditions are '≤' type. Similarly, if the objective function is of minimization, then all the constraints are '≥' type.

20. What do you mean by standard form of LPP?

In standard form, irrespective of the objective function namely maximize or minimize, all the constraints are expressed as equations, also right hand side constants are non-negative, i.e. all the variables are non-negative.

21. State the characteristics of canonical form and write the canonical form of LPP in matrix form.

Characteristics of canonical form

The objective function is of maximization type.

All constraints are '≤' type.

All variables x_i are non-negative.

Matrix form

$$\begin{array}{lcl} \text{Max } z = cx & & \text{Min } z = cx \\ \text{Subject to, } Ax \le b & \text{Or} & \text{Subject to, } Ax \ge b \\ \qquad\qquad x \ge 0 & & \qquad\qquad x \ge 0 \end{array}$$

22. State the characteristics of standard form and write the standard form of LPP in matrix form.

Characteristics of standard form

- The objective function is of maximization type.
- All constraints are expressed as equations.
- RHS of each constraint is non-negative.
- All variables are non-negative.

Matrix form

Max $z = cx$

Subject to, $Ax = b$

$x \geq 0$

23. What are the limitations of LPP?

- For larger problems having many limitations and constraints, the computational difficulties are enormous even when computers are used.
- Many times it is not possible to express both the objective function and constraints in linear form.
- The solution variables may have any values. Sometimes the solution variables are restricted to take only integer values.
- This method does not take into account the effect of time.

24. What are slack and surplus variables?

The non-negative variable which is added to LHS of the constraint to convert the inequality $\leq$ into an equation is called slack variable.

The non-negative variable which is removed from the LHS of the constraint to convert the inequality into an equation is called a surplus variable.

25. What are decision variables in the construction of operation research problems?

While making mathematical modelling of operations research problems, the variables which are used and the value of which gives the solution are the decision variables.

26. How many basic feasible solutions are there to a given system of 4 simultaneous equations in 5 unknowns?

$5C_4 = 5$

27. What is the test of optimality in the simplex method?

Compute the net evaluation $z_j - c_j$ (j = 1, 2, ... n). If all $z_j - c_j \geq 0$, then the current feasible solution is optimal, which is the test of optimality.

28. What is key column and how is it selected?

Key column is the column which gives the entering variable column and is selected by finding the most negative value of $z_j - c_j$.

29. What is key row and how is it selected?

The leaving variable row is called the key row and is selected by finding the ratio between the solution column and the entering variable column by considering only the positive denominator.

30. When does the simplex method indicate that the LPP has unbounded solution?

The indication of unbounded solution of LPP can be obtained if all the variables in the key column are negative.

31. What is meant by optimality?

By performing optimality test we can find whether the current feasible solution can be improved or not, which is possible by finding the $z_j - c_j$ row.

32. How will you find whether a LPP has got an alternative optimal solution or not, from the optimal simplex table?

In optimal simplex table, in $z_j - c_j$ row if zero occurs for non-basic variables, it indicates that LPP has an alternate solution.

33. What are the methods used to solve an LPP involving artificial variables?

- Big *M* method or penalty cost method
- Two-phase simplex method

34. Define artificial variable.

Any non-negative variable which is introduced in the constraint in order to get the initial basic feasible solution is called artificial variable.

35. When does an LPP possess a pseudo-optimal solution?

An LPP possesses a pseudo-optimal solution if at least one artificial variable is in the basis at positive level even though the optimality conditions are satisfied.

36. What are the disadvantages of big *M* method over two-phase method?

Although big *M* method can always be used to check the existence of a feasible solution, it may be computationally inconvenient because of the manipulation of the constant *M*. Also when the problem is to be solved on a digital computer, *M* must be assigned some numerical value which is greater than c_1, c_2, ... in the objective function. But a computer has only a fixed number of digits. In two-phase method, these difficulties are overcome as it eliminates the constant *M* from calculations.

37. What is degeneracy?

The concept of obtaining a degenerate basic feasible solution in a LPP is known as degeneracy.

38. Define the phenomenon of cycling.

The phenomenon of repeating the same sequence of simplex iterations endlessly without improving the value of the objective function is known as cycling.

39. How can we resolve degeneracy in a LPP?

(i) Divide each element of the rows by the positive coefficients of the key column in that row.
(ii) Compare the resulting ratios, column by column, first in the identity and then in the body from left to right.
(iii) The row which first contains the smallest ratio contains the leaving variable.

40. Define dual of LPP.

For every LPP, there is a unique LPP associated with it involving the same data and closely related optimal solution. The original problem is then called the primal problem while the other is called its dual problem.

41. What are the advantages of duality?

- If primal contains a large number of constraints and a smaller number of variables, then the process of computations can be considerably reduced by converting it into the dual problem.
- Since the optimal solution to the objective function is the same for both primal and dual, a dual solution can be used to check the accuracy of the primal solution.

42. State the fundamental theorem of duality.

If either the primal or the dual has a finite optimal solution, then the other problem also has finite optimal solution and the values of the objective function are equal, i.e., Max z = Min z'. The solution of the other problem can be obtained from $z_j - c_j$ row of the optimal simplex table below the slack and surplus variables.

43. What is the difference between regular simplex method and dual simplex method?

In regular simplex method, we first determine the entering variable and then the leaving variable while in the case of dual simplex method, we first determine the leaving variable and then the entering variable.

44. What is the advantage of dual simplex method?

The advantage of dual simplex method is to avoid introducing the artificial variables along with the surplus variable as the '$\geq$' type constraint is converted into '$\leq$' type.

45. State the existence theorem of duality.

If either the primal or the dual problem has an unbounded solution, then the other problem has no feasible solution.

46. State the complementary slackness theorem of duality.

- If a primal variable is positive, then the corresponding dual constraint is an equation at the optimum and vice versa.
- If a primal constraint is a strict inequality then the corresponding dual variable is zero at the optimum and vice versa.

47. What are the two important forms of primal-dual pairs?

The two important forms of primal-dual pairs are

- symmetric form, and
- non-symmetric form.

48. What are the advantages of revised simplex method?

There is less accumulation of round off errors, since no calculation is done on a column unless it is ready to enter the basis.

The data can be stored more accurately and compactly, since the revised simplex table works only with the original data.

This method is more economical on the computer as it computes and stores only the relevant information needed currently for testing and/or updating the current solution.

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FACULTY OF ENGINEERING

DEPARTMENT OF AUTOMOBILE ENGINEERING

INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH

UNIT-IV

DISTRIBUTION METHODS AND ASSIGNMENT MODELS

CONTENTS

1 TRANSPORTATION PROBLEM

1.1 INTRODUCTION

The transportation problem is one of the subclasses of LPPs. Here the objective is to transport various quantities of a single homogeneous commodity that are initially stored at various origins to different destinations in such a way that the transportation cost is minimum. To achieve this we must know the amount and location of available supplies and the quantities demanded. In addition, we must know the costs that result from transporting one unit of the commodity from various origins to various destinations.

1.2 MATHEMATICAL FORMULATION

Consider a transportation problem with m origins (rows) and n destinations (columns). Let c_{ij} be the cost of transporting one unit of the product from the i^{th} origin to j^{th} destination. a_i the quantity of commodity available at origin i, b_j the quantity of commodity needed at destination j. x_{ij} is the quantity transported from i^{th} origin to j^{th} destination. The above transportation problem can be stated in the following tabular form.

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="4">Destination</td><td rowspan="2">Capacity</td></tr>
<tr><td>1</td><td>2</td><td>3</td><td>n</td></tr>
<tr><td rowspan="4">Origin</td><td>1</td><td>c_{11} x_{11}</td><td>c_{12} x_{12}</td><td>c_{13} x_{13}</td><td>c_{1n} x_{1n}</td><td>a_1</td></tr>
<tr><td>2</td><td>c_{21} x_{21}</td><td>c_{22} x_{22}</td><td>c_{23} x_{23}</td><td>c_{2n} x_{2n}</td><td>a_2</td></tr>
<tr><td>3</td><td>c_{31} x_{31}</td><td>c_{32} x_{32}</td><td>c_{33} x_{33}</td><td>c_{3n} x_{3n}</td><td>a_3</td></tr>
<tr><td>m</td><td>c_{m1} x_{m1}</td><td>c_{m2} x_{m2}</td><td>c_{m3} x_{m3}</td><td>c_{mn} x_{mn}</td><td>a_m</td></tr>
<tr><td colspan="2">Demand</td><td>b_1</td><td>b_2</td><td>b_3</td><td>b_n</td><td>$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$</td></tr>
</table>

The linear programming model representing the transportation problem is given by,

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subject to the constraints,

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, \dots m$$

(Row sum)

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \dots n$$

(Column sum)

$$x_{ij} \geq 0 \text{ for all } i \text{ and } j$$

The given transportation problem is said to be balanced if

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

i.e., if the total supply is equal to the total demand.

1.3 DEFINITIONS

1.3.1 FEASIBLE SOLUTION

Any set of non-negative allocations ($x_{ij} \geq 0$) which satisfies the row and column sum (rim requirement) is called a 'feasible solution'.

1.3.2 BASIC FEASIBLE SOLUTION

A feasible solution is called a 'basic feasible solution' if the number of non-negative allocations is equal to $m + n - 1$, where m is the number of rows and n is the number of columns in a transportation table.

1.3.3 NON-DEGENERATE BASIC FEASIBLE SOLUTION

Any feasible solution to a transportation problem containing m origins and n destinations is said to be 'non-degenerate' if it contains $m + n - 1$ occupied cells and each allocation is in an independent position.

The allocations are said to be in independent positions if it is impossible to form a closed path.

A path which is formed by allowing horizontal and vertical lines and all the corner cells of which are occupied is called a closed path.

*	*	
*	*	

*		*
*		*

	*	*	
	*		
	*	*	

Figure 1 Allocations are not in independent positions

*	*	
*		
*		

	*	
*	*	*
*		

*	*		
	*		*
		*	*

Figure 2 Allocations are in independent positions

1.3.4 DEGENERATE BASIC FEASIBLE SOLUTION

If a basic feasible solution contains less than $m + n - 1$ non-negative allocation, it is said to be 'degenerate'.

1.4 OPTIMAL SOLUTION

An optimal solution is a feasible solution (not necessarily basic), which minimizes the total cost.

The solution of a transportation problem can be obtained in two stages, namely initial and optimum solution.

An initial solution can be obtained by using any one of the three methods, viz.,

- North-West Corner Rule (NWCR)
- Least Cost Method (LCM) or Matrix Minima Method
- Vogel's Approximation Method (VAM)

VAM is preferred over the other two methods since the initial basic feasible solution obtained by this method is either optimal or very close to the optimal solution.

The cells in the transportation table can be classified as occupied and unoccupied cells. The allocated cells in the transportation table are called **occupied cells** and the empty ones are called **unoccupied cells**.

The improved solution of the initial basic feasible solution is called 'optimal solution', which is the second stage of the solution and can be obtained by MODI (modified distribution method).

1.4.1 NORTH-WEST CORNER RULE (NWCR)

1. Starting with the cell at the upper left corner (north-west) of the transportation matrix, allocate as much as possible so that either the capacity of the first row is exhausted or the destination requirement of the first column is satisfied, i.e., x_{11} = min (a_1, b_1).

2. If $b_1 > a_1$, move down vertically to the second row and make the second allocation of magnitude x_{21} = min (a_2, $b_1 - x_{11}$) in the cell (2, 1).

   If $b_1 < a_1$, move right horizontally to the second column and make the second allocation of magnitude x_{12} = min ($a_1 - x_{11}, b_2$) in the cell (1, 2).

   If $b_1 = a_1$, move diagonally to the next cell.

3. Repeat steps l and 2, moving down towards the lower right corner of the transportation table until all the rim requirements are satisfied.

1.4.2 LEAST COST METHOD (LCM) OR MATRIX MINIMA METHOD

1. Determine the smallest cost in the cost matrix of the transportation table. Let it be c_{ij}. Allocate x_{ij} = min (a_i, b_j) in the cell (i, j)

2. If $x_{ij} = a_i$, cross off the i^{th} row of the transportation table and decrease b_j by a_i. Then go to step 3.

   If $x_{ij} = b_j$, cross off the j^{th} row of the transportation table and decrease a_i by b_j. Then go to step 3.

   If $x_{ij} = a_i = b_j$, cross off either the i^{th} row or the j^{th} column but not both.

3. Repeat steps 1 and 2 for the resulting reduced transportation table until all the rim requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

1.4.3 VOGEL'S APPROXIMATION METHOD (VAM)

The steps involved in this method for finding the initial solution are as follows.

1. Find the penalty cost, namely the difference between the smallest and the next smallest costs in each row and column.
2. Among the penalties as found in step 1, choose the maximum penalty. If this maximum penalty is more than one (i.e., if there is a tie), choose any one arbitrarily.
3. In the selected row or column as by step 2, find out the cell having the least cost. Allocate to this cell as much as possible, depending on the capacity and requirements.
4. Delete the row or column that is fully exhausted. Again compute the column and row penalties for the reduced transportation table and then go to step 2. Repeat the procedure until all the rim requirements are satisfied.

Note: If the column is exhausted, then there is a change in row penalty and vice versa.

1.5 OPTIMALITY TEST

Once the initial basic feasible solution has been computed, the next step in the problem is to determine whether the solution obtained is optimum or not.

Optimality test can be conducted on an initial basic feasible solution of a transportation problem provided such an allocation has exactly $m + n - 1$, non-negative allocations. Where m is the number of origins and n is the number of destinations. Also, these allocations must be in independent positions.

To perform this optimality test, modified distribution method (MODI) can be used. The various steps involved in MODI method for performing the optimality test are given below.

1.5.1 MODI METHOD

1. Find the initial basic feasible solution of a transportation problem by using any one of the three methods.
2. Find out a set of numbers u_i and v_j for each row and column satisfying $u_i + v_j = c_{ij}$ for each occupied cell. To start with, assign a number '0' to any row or column having the maximum number of allocations. If this maximum number of allocations is more than one, choose any one arbitrarily.
3. For each empty (unoccupied) cell, find the sum u_i and v_j written in the bottom left corner of that cell.
4. Find out for each empty cell the net evaluation value $\Delta_{ij} = c_{ij} - (u_i + v_j)$, which is written at the bottom right corner of that cell. This step gives the optimality conclusion.

    (i) If all $\Delta_{ij} > 0$, the solution is optimum and a **unique solution** exists.

    (ii) If $\Delta_{ij} \geq 0$, then the solution is optimum, but an **alternate solution** exists.

(iii) If at least one $\Delta_{ij} < 0$, the solution is not optimum. In this case, go to the next step, to improve the total transportation cost.

5. Select the empty cell having the most negative value of Δ_{ij}. From this cell draw a closed path by drawing horizontal and vertical lines with the corner cells occupied. Assign sign + and – alternately and find the minimum allocation from the cell having a negative sign. This allocation should be added to the allocation having a positive sign and subtracted from the allocation having a negative sign.

6. The above step yields a better solution by making one (or more) occupied cell as empty and one empty cell as occupied. For this new set of basic feasible allocations repeat from step 2 onwards, till an optimum basic feasible solution is obtained.

1.6 DEGENERACY IN TRANSPORTATION PROBLEM

In a transportation problem, if the number of non-negative independent allocations is less than in *m* + *n* – 1, where *m* is the number of origins (rows) and *n* is the number of destinations (columns), there exists a degeneracy. This may occur either at the initial stage or at subsequent iteration.

To resolve this degeneracy, adopt the following steps:

1. Among the empty cells, choose an empty cell having the least cost, which is of an independent position. If such cells are more than one, choose any one arbitrarily.

2. To the cell as chosen in step 1, allocate a small positive quantity $\varepsilon > 0$.

The cells containing ε are treated like other occupied cells and degeneracy is removed by adding one (more) accordingly. For this modified solution, we adopt the steps involved in MODI method till an optimum solution is obtained.

1.7 UNBALANCED TRANSPORTATION PROBLEM

The given transportation problem is said to be unbalanced if $\Sigma a_i \neq \Sigma b_j$, i.e. if the total supply is not equal to the total demand. There are two possible cases.

1.7.1 CASE I

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$$

If the total supply is less than the total demand, a dummy source (row) is included in the cost matrix with zero cost, the excess demand is entered as a rim requirement for this dummy source (origin). Hence, the unbalanced transportation problem can be converted into a balanced transportation problem.

1.7.2 CASE II

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$$

If the total supply is greater than the total demand, the unbalanced transportation problem can be converted into a balanced transportation problem by adding a dummy destination (column) with zero cost. The excess supply is entered as a rim requirement for the dummy destination.

1.8 MAXIMIZATION CASE IN TRANSPORTATION PROBLEM

Here the objective is to maximize the total profit for which the profit matrix is given. For this, first, convert the maximization problem into minimization by subtracting all the elements from the highest element in the given transportation table. This modified minimization problem can be solved in the usual manner.

2 ASSIGNMENT PROBLEM

2.1 DEFINITION

Suppose there are *n* jobs to be performed and *n* persons are available for doing these jobs. Assume that each person can do each job at a time, though with varying degrees of efficiency. Let c_{ij} be the cost if the i^{th} person is assigned to the j^{th} job. The problem is to find an assignment (which job should be assigned to which person, on a one to one basis) so that the total cost of performing all the jobs is minimum. Problems of this kind are known as assignment problems.

An assignment problem can be stated in the form of $n \times n$ cost matrix (c_{ij}) of real numbers as given in the following table.

		Jobs						
		1	2	3	…	*j*	…	*n*
Persons	1	c_{11}	c_{12}	c_{13}		c_{1j}		c_{1n}
	2	c_{21}	c_{22}	c_{23}		c_{2j}		c_{2n}
	3	c_{31}	c_{32}	c_{33}		c_{3j}		c_{3n}
	…							
	i	c_{i1}	c_{i2}	c_{i3}		c_{ij}		c_{in}
	…							
	n	c_{n1}	c_{n2}	c_{n3}		c_{nj}		c_{nn}

2.2 MATHEMATICAL FORMULATION OF AN ASSIGNMENT PROBLEM

Mathematically, an assignment problem can be stated as,

$$\text{Minimize } z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

where *i* = 1, 2, … *n* and *j* = 1, 2, … *n*

Subjected to the restrictions

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th} \text{ person is assigned to the } j^{th} \text{job} \\ 0 \text{ if not} \end{cases}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ (one job is done by the } i^{\text{th}} \text{ person)}$$

and

$$\sum_{j=1}^{n} x_{ij} = 1 \text{ (only one person should be assigned to } j^{\text{th}} \text{ job)}$$

where x_{ij} denotes that the j^{th} job is to be assigned to the i^{th} person.

2.3 DIFFERENCE BETWEEN TRANSPORTATION AND ASSIGNMENT PROBLEMS

Sl. No.	Transportation problem	Assignment problem
1	The number of sources and destinations need not be equal. Hence, the cost matrix is not necessarily a square matrix.	Since the assignment is done on a one to one basis, the number of sources and destinations are equal. Hence, the cost matrix must be a square matrix.
2	x_{ij}, the quantity to be transported from i^{th} origin to j^{th} destination can take any possible positive value, and it satisfies the rim requirements.	x_{ij}, the j^{th} job is to be assigned to the i^{th} person and can take either the value 1 or 0.
3	The capacity and the requirement value is equal to a_i and b_j for the i^{th} source and j^{th} destination (i = 1, 2, ... n and j = 1, 2, ... n).	The capacity and the requirement value is exactly one, i.e., for each source of each destination, the capacity and the requirement value is exactly one.
4	The problem is unbalanced if the total supply and total demand are not equal.	The problem is unbalanced if the cost matrix is not a square matrix.

2.4 HUNGARIAN METHOD PROCEDURE

The solution of an assignment problem can be arrived at by using the Hungarian method. The steps involved in this method are as follows.

1. Prepare a cost matrix. If the cost matrix is not a square matrix then add a dummy row (column) with zero cost elements.

2. Subtract the minimum element in each row from all the elements of the respective rows.

3. Further, modify the resulting matrix by subtracting the minimum element of each column from all the elements of the respective columns. Thus, obtain the modified matrix.

4. Then, draw the minimum number of horizontal and vertical lines to cover all zeros in the resulting matrix. Let the minimum number of lines be N. Now there are two possible cases.
   Case I: If $N = n$, where n is the order of the matrix, then an optimal assignment can be made. So make the assignment to get the required solution.
   Case II: If $N < n$, then proceed to step 5.

5. Determine the smallest uncovered element in the matrix (element not covered by N lines). Subtract this minimum element from all uncovered elements and add the same element at the intersection of horizontal and vertical lines. Thus, the second modified matrix is obtained.

6. Repeat steps 3 and 4 until getting the case *I* of Step 4.

7. (To make zero assignment) Examine the rows successively until a row-wise exactly single zero is found. Circle (O) this zero to make the assignment. Then mark a cross (×) over all zeros if lying in the column of the circled zero, showing that they cannot be considered for future assignment. Continue in this manner until all the zeros have been examined. Repeat the same procedure for columns also.

8. Repeat step 6 successively until one of the following situations arises:

   (i) If no unmarked zero is left, then the process ends or

   (ii) If there lies more than one unmarked zero in any column or row, circle one of the unmarked zeros arbitrarily and mark a cross in the cells of remaining zeros in its row or column. Repeal the process until no unmarked zero is left in the matrix.

9. Thus, exactly one marked circled zero in each row and each column of the matrix is obtained. The assignment corresponding to these marked circled zeros will give the optimal assignment.

2.5 UNBALANCED ASSIGNMENT PROBLEM

Any assignment problem is said to be unbalanced if the cost matrix is not a square matrix, i.e., the number of rows and columns are not equal. To make it balanced, add a dummy row or dummy column with all the entries as zero.

2.6 MAXIMIZATION IN ASSIGNMENT PROBLEM

In this, the objective is to maximize the profit. To solve this, first convert the given profit matrix into the loss matrix by subtracting all the elements from the highest element. For this converted loss matrix, apply the steps in the Hungarian method to get the optimum assignment.

3 THE TRAVELLING SALESMAN PROBLEM

Assuming a salesman has to visit *n* cities. He wishes to start from a particular city, visit each city once and then return to his starting point. His objective is to select the sequence in which the cities are visited in such a way that his total travelling time is minimized.

To visit 2 cities (*A* and *B*), there is no choice. To visit 3 cities there are 2 possible routes. For 4 cities there are 3 possible routes. In general, to visit *n* cities there are $(n-1)!$ possible routes.

3.1 MATHEMATICAL FORMULATION

Let c_{ij} be the distance or time or cost of going from city *i* to city *j*. Let the decision variable x_{ij} be 1, if the salesman travels from city *i* to city *j*, otherwise let it be 0.

The objective is to minimize the travelling time.

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

Subjected to the constraints,

$$\sum_{j=1}^{n} x_{ij} = 1, i = 2, 3, \dots n$$

$$\sum_{i=1}^{n} x_{ij} = 1, j = 2, 3, \dots n$$

and subject to the additional constraint that x_{ij} is so chosen that, no city is visited twice before all the cities are visited.

In particular, going from *i* directly to *i* is not permitted. This means $c_{ij} = \infty$, when *i* = *j*.

In the travelling salesman problem, we cannot choose the element along the diagonal and this can be avoided by filling the diagonal with infinitely large elements.

The travelling salesman problem is very similar to the assignment problem except that in the former case, there is an additional restriction, x_{ij} is so chosen that no city is visited twice before the tour of all the cities is completed.

4 SEQUENCING PROBLEMS

4.1 INTRODUCTION

4.1.1 DEFINITION

Suppose there are *n* jobs (1, 2, ... *n*), each of which has to be processed one at a time at *m* machines (*A*, *B*, *C*, ...) . The order of processing each job through each machine is given. The problem is to find a sequence among ($n!^m$) number of all possible sequences for processing the jobs so that the total elapsed time for all the jobs will be minimum.

4.1.2 TERMINOLOGY AND NOTATIONS

Number of machines: It means the service facilities through which a job must pass before it is completed.

Processing order: It refers to the order in which various machines are required for completing the job.

Processing time: It means the time required for each job on each machine.

Idle time on a machine: This is the time for which a machine remains idle during the total elapsed time. The notation x_{ij} is used to denote the idle time of a machine *j* between the end of the $(i-1)^{\text{th}}$ job and the start of the i^{th} job.

Total elapsed time: This is the time between starting the first job and completing the last job, which also includes the idle time if present.

No passing rule: It means, passing is not allowed, i.e., maintaining the same order of jobs over each machine. If each of *N*-jobs is to be processed through 2 machines M_1 and M_2 in the order M_1M_2, then this rule will mean that each job will go to machine M_1 first and then to M_2. If a job is finished on M_1, it goes directly to machine M_2 if it is free, otherwise, it starts a waiting line or joins the end of the waiting line if one already exists. Jobs that form a waiting line are processed on machine M_2 when it becomes free.

4.1.3 PRINCIPAL ASSUMPTIONS

- No machine can process more than one operation at a time.
- Each operation once started must be performed till completion.
- Each operation must be completed before starting any other operation.
- Time intervals for processing are independent of the order in which operations are performed.
- There is only one machine of each type.
- A job is processed as soon as possible, subject to the ordering requirements.
- All jobs are known and are ready for processing before the period under consideration begins.
- The time required to transfer jobs between machines is negligible.

4.2 TYPE I: PROCESSING N JOBS THROUGH TWO MACHINES

The algorithm, which is used to optimize the total elapsed time for processing *n* jobs through two is called '**Johnson's algorithm**' and has the following steps.

Consider *n* jobs (1, 2, 3, ... *n*) processing on two machines A and B in the order AB. The processing periods (time) are A_1, A_2, ... A_n and B_1, B_2, ... B_n as given in the following table.

Machine/Job	1	2	3	...	*n*
A	A_1	A_2	A_3	...	A_n
B	B_1	B_2	B_3	...	B_n

The problem is to sequence the jobs so as to minimize the total elapsed time.

The solution procedure adopted by Johnson is given below.

1. Select the least processing time occurring in the list A_1, A_2, ... A_n and B_1, B_2, ... B_n. Let this minimum processing time occur for a job *K*.

2. If the shortest processing is for machine A, process the K^{th} job first and place it at the beginning of the sequence. If it is for machine B, process the K^{th} job last and place it at the end of the sequence.

3. When there is a tie in selecting the minimum processing time, then there may be three solutions.

   (i) If the equal minimum values occur only for machine A, select the job with larger processing time in B to be placed first in the job sequence.

   (ii) If the equal minimum values occur only for machine B, select the job with larger processing time in A to be placed last in the job sequence.

   (iii) If there are equal minimum values, one for each machine, then place the job in machine A first and the one in machine B last.

4. Delete the jobs already sequenced. If all the jobs have been sequenced, go to the next step. Otherwise, repeat step 1 to 3.

5. In this step, determine the overall or total elapsed time and also the idle time on machines A and B as follows.

Total elapsed time is the time between starting the first job in the optimal sequence on machine A and completing the last job in the optimal sequence on machine B.

Idle time on A = (Time when the last job in the optimal sequence is completed on machine B) – (Time when the last job in the optimal sequence is completed on machine A)

Idle time on B = when the first job in the optimal sequence starts on machine B + $\sum_{K=2}^{n}$[time K^{th} job starts on machine B + time $(K-1)^{\text{th}}$ job finished on machine B].

4.3 TYPE II: PROCESSING N JOBS THROUGH THREE MACHINES A, B, C

Consider *n* jobs (1, 2, ... *n*) processing on three machines *A*, *B*, *C* in the order *ABC*. The optimal sequence can be obtained by converting the problem into a two-machine problem. From this, get the optimum sequence using Johnson's algorithm.

The following steps are used to convert the given problem into a two-machine problem.

1. Find the minimum processing time for the jobs on the first and last machine and the maximum processing time for the second machine i.e., find Min (A_i, C_i) and Max (B_i), *i* = 1, 2, ... *n*.
2. Check the following inequalities.

$$\text{Min } (A_i) \geq \text{Max } (B_i)$$

$$\text{Min } (C_i) \geq \text{Max } (B_i)$$

3. If none of the inequalities in step 2 are satisfied, this method cannot be applied.
4. If at least one of the inequalities in step 2 is satisfied, define two machines *G* and *H*, such that the processing time on *G* and *H* are given by,

$$G_i = A_i + B_i \qquad i = 1, 2, \dots n$$

$$H_i = B_i + C_i \qquad i = 1, 2, \dots n$$

5. For the converted machines *G* and *H*, obtain the optimum sequence using the two-machine algorithm.

4.4 TYPE III: PROCESSING 2 JOBS THROUGH K MACHINES

Consider two jobs, each of which is to be processed on *K* machines M_1, M_2, ... M_K in two different orders. The ordering of each of the two jobs through *K* machines is known in advance. Such ordering may not be the same for both the jobs. The exact or expected processing times on all the given machines are known.

Each machine can perform only one job at a time The objective is to determine the optimal sequence of pressing the jobs so as to minimize total elapsed time.

The optimal sequence, in this case, can be obtained by making use of the graph.

The procedure is given in the following steps.

1. First draw a set of axes, where the horizontal axis represents processing time on job 1 and the vertical axis represents processing time on job 2.

2. Mark the processing time for job 1 and job 2 on the horizontal and vertical lines respectively, according to the given order of machines.

3. Construct various blocks starting from the origin (starting point), by pairing the same machines until the end point.

4. Draw the line starting from the origin to the end point by moving horizontally, vertically and diagonally along a line which makes an angle of 45° with the horizontal line (base). The horizontal segment of this line indicates that the first job is under process while the second job is idle. Similarly, the vertical line indicates that the second job is under process while the first job is idle. The diagonal segment of the line shows that the jobs are under process simultaneously.

5. An optimum path is one that minimizes the idle time for both the jobs. Thus, choose the path on which diagonal movement is maximum.

6. The total elapsed time is obtained by adding the idle time for either job to the processing time for that job.

5 SOLVED PROBLEMS

1 Solve the following transportation problem.

		Destination				Supply
		D_1	D_2	D_3	D_4	
Source	S_1	21	16	25	13	11
	S_2	17	18	14	23	13
	S_3	32	17	18	41	19
Demand		6	10	12	15	

SOLUTION:

Check for balance

Σ Supply = 11 + 13 + 19 = 43

Σ Demand = 6 + 10 + 12 + 15 = 43

Since Σ Supply = Σ Demand, the given transportation problem is a balanced one and there exists a feasible solution to the problem.

Initial basic feasible solution by VAM

Find the penalty cost, the difference between the smallest and next smallest cost in each row and column.

The maximum penalty is 10, which is under column D_4. In this column, choose the cell having the least cost i.e., 13.

Allocate to this cell with minimum of supply and demand i.e., min (11, 15) = 11. This exhausts the first row.

Delete this row. Since the row is deleted, there is a change in column penalty and row penalty remains the same. Repeat the procedure until all the rows and columns have been deleted.

I allocation

	D_1	D_2	D_3	D_4	Supply	P_I
S_1	21	16	25	13 11	11/0	3
S_2	17	18	14	23	13	3
S_3	32	17	18	41	19	1
Demand	6	10	12	15/4		
P_I	4	1	4	↑ 10		

II allocation

	D_1	D_2	D_3	D_4	Supply	P_{II}
S_2	17	18	14	23 4	13/9	3
S_3	32	17	18	41	19	1
Demand	6	10	12	4/0		
P_{II}	15	1	4	↑ 18		

III allocation

	D_1	D_2	D_3	Supply	P_{III}
S_2	17 6	18	14	9/3	3
S_3	32	17	18	19	1
Demand	6/0	10	12		
P_{III}	↑ 15	1	4		

IV allocation

	D_2	D_3	Supply	P_{IV}
S_2	18	14 3	3/0	4 ←
S_3	17	18	19	1
Demand	10	12/9		
P_{IV}	1	4		

V allocation

	D_2	D_3	Supply	P_V
S_3	17 10	18 9	19/9/0	1
Demand	10/0	9/0		
P_V	↑ *	↑ *		

Initial basic feasible solution

	D_1	D_2	D_3	D_4	Supply
S_1	21	16	25	13 11	11
S_2	17 6	18	14 3	23 4	13
S_3	32	17 10	18 9	41	19
Demand	6	10	12	15	

Check for degeneracy

Number of allocations = 6

Number of rows + Number of columns – 1 = $m + n - 1 = 3 + 4 - 1 = 6$

Since the number of allocations in the VAM table is equal to $m + n - 1$, the solution is non-degenerate basic feasible.

The initial transportation cost = 11 × 13 + 6 × 17 + 3 × 14 + 4 × 23 + 10 × 17 + 9 × 18

= 711

Optimal solution by MODI method

For occupied cells, cost of transportation from source to destination,

$$c_{ij} = u_i + v_j$$

$$c_{14} = u_1 + v_4$$

$$c_{21} = u_2 + v_1$$

$$c_{23} = u_2 + v_3$$

$$c_{24} = u_2 + v_4$$

$$c_{32} = u_3 + v_2$$

$$c_{33} = u_3 + v_3$$

Assume $u_2 = 0$ as second row has the maximum number of allocation.

$$c_{21} = u_2 + v_1$$

$$17 = 0 + v_1$$

$$v_1 = 17$$

For unoccupied cells, net evaluation value,

$$\Delta_{ij} = c_{ij} - (u_i + v_j)$$

$$\Delta_{11}= c_{11} - (u_1 + v_1)$$

$$\Delta_{11}= 21 - (-10 + 17)$$

$$\Delta_{11}= 14$$

$$\Delta_{12}= c_{12} - (u_1 + v_2)$$

$$\Delta_{13}= c_{13} - (u_1 + v_3)$$

$$\Delta_{22}= c_{22} - (u_2 + v_2)$$

$$\Delta_{31}= c_{31} - (u_3 + v_1)$$

$$\Delta_{34}= c_{34} - (u_3 + v_4)$$

	D_1		D_2		D_3		D_4		u_i
S_1		21		16		25		13	$u_1 = -10$
		14		13		21	11		
S_2		17		18		14		23	$u_2 = 0$
	6			5	3		4		
S_3		32		17		18		41	$u_3 = 4$
		11	10		9			14	
v_j	$v_1 = 17$		$v_2 = 13$		$v_3 = 14$		$v_4 = 23$		

Since all $\Delta_{ij} \geq 0$, the solution is optimal and unique.

The minimum transportation cost = 11 × 13 + 6 × 17 + 3 × 14 + 4 × 23 + 10 × 17 + 9 × 18

= 711

2 Solve the following transportation problem starting with the initial solution obtained by VAM.

		Destination				Supply
		D_1	D_2	D_3	D_4	
Origin	O_1	2	2	2	1	3
	O_2	10	8	5	4	7
	O_3	7	6	6	8	5
Demand		4	3	4	4	

SOLUTION:

Check for balance

Σ Supply = 3 + 7 + 5 = 15

Σ Demand = 4 + 3 + 4 + 4 = 15

Since Σ Supply = Σ Demand, the given transportation problem is a balanced one and there exists a feasible solution to the problem.

Initial basic feasible solution by VAM

I allocation

	D_1	D_2	D_3	D_4	Supply	P_I
O_1	2 3	2	2	1	3/0	1
O_2	10	8	5	4	7	1
O_3	7	6	6	8	5	1
Demand	4/1	3	4	4		
P_I	↑ 5	4	3	3		

II allocation

	D_1	D_2	D_3	D_4	Supply	P_{II}
O_2	10	8	5	4 4	7/3	1
O_3	7	6	6	8	5	1
Demand	1	3	4	4/0		
P_{II}	3	2	1	↑ 4		

III allocation

	D_1	D_2	D_3	Supply	P_{III}
O_2	10	8	5 3	3/3	3 ←
O_3	7	6	6	5	1
Demand	1	3	4/1		
P_{III}	3	2	1		

IV allocation

	D_1	D_2	D_3	Supply	P_{IV}
O_3	7 1	6 3	6 1	5/2/1/0	1
Demand	1/0	3/0	1/0		
P_{IV}	↑ *	↑ *	↑ *		

Initial basic feasible solution

	D_1	D_2	D_3	D_4	Supply
O_1	2 3	2	2	1	3
O_2	10	8	5 3	4 4	7
O_3	7 1	6 3	6 1	8	5
Demand	4	3	4	4	

Check for degeneracy

Number of allocations = 6

Number of rows + Number of columns – 1 = $m + n - 1 = 3 + 4 - 1 = 6$

Since the number of allocations in the VAM table is equal to $m + n - 1$, the solution is non-degenerate basic feasible.

The initial transportation cost = 3 × 2 + 3 × 5 + 4 × 4 + 1 × 7 + 3 × 6 + 1 × 6

= 68

Optimal solution by MODI method

For occupied cells, cost of transportation from source to destination,

$$c_{ij} = u_i + v_j$$

$$c_{11} = u_1 + v_1$$

$$c_{23} = u_2 + v_3$$

$$c_{24} = u_2 + v_4$$

$$c_{31} = u_3 + v_1$$

$$c_{32} = u_3 + v_2$$

$$c_{33} = u_3 + v_3$$

Assume $u_3 = 0$ as third row has the maximum number of allocation.

$$c_{31} = u_3 + v_1$$

$$7 = 0 + v_1$$

$$v_1 = 7$$

For unoccupied cells, net evaluation value,

$$\Delta_{ij} = c_{ij} - (u_i + v_j)$$

$$\Delta_{12} = c_{12} - (u_1 + v_2)$$

$$\Delta_{12} = 2 - (-5 + 6)$$

$$\Delta_{12} = 1$$

	D_1	D_2	D_3	D_4	u_i
O_1	2 3	2 1	2 1	1 1	$u_1 = -5$
O_2	10 4	8 3	5 3	4 4	$u_2 = -1$
O_3	7 1	6 3	6 1	8 3	$u_3 = 0$
v_j	$v_1 = 7$	$v_2 = 6$	$v_3 = 6$	$v_4 = 5$	

Since all $\Delta_{ij} \geq 0$, the solution is optimal and unique.

The minimum transportation cost = 3 × 2 + 3 × 5 + 4 × 4 + 1 × 7 + 3 × 6 + 1 × 6

= 68

3 Solve the transportation problem for minimization.

		Destination				Supply
		D_1	D_2	D_3	D_4	
Source	S_1	14	56	48	27	70
	S_2	82	35	21	81	47
	S_3	99	31	71	63	93
Demand		70	35	45	60	

Solution:

Check for balance

Σ Supply = 70 + 47 + 93 = 210

Σ Demand = 70 + 35 + 45 + 60 = 210

Since Σ Supply = Σ Demand, the given transportation problem is a balanced one and there exists a feasible solution to the problem.

Initial basic feasible solution by VAM

I allocation

	D_1	D_2	D_3	D_4	Supply	P_{I}
S_1	14 70	56	48	27	70/0	13
S_2	82	35	21	81	47	14
S_3	99	31	71	63	93	32
Demand	70/0	35	45	60		
P_{I}	↑ 68	4	27	36		

II allocation

	D_2	D_3	D_4	Supply	P_{II}
S_2	35	21 45	81	47/2	14
S_3	31	71	63	93	32
Demand	35	45/0	60		
P_{II}	4	↑ 50	18		

III allocation

	D_2	D_4	Supply	P_{III}
S_2	35 2	81	2/0	46 ←
S_3	31	63	93	32
Demand	35/33	60		
P_{III}	4	18		

IV allocation

	D_2	D_4	Supply	P_{IV}
S_3	31 33	63 60	93/60/0	32
Demand	33/0	60/0		
P_{IV}	↑ *	↑ *		

Initial basic feasible solution

	D_1	D_2	D_3	D_4	Supply
S_1	14 70	56	48	27	70
S_2	82	35 2	21 45	81	47
S_3	99	31 33	71	63 60	93
Demand	70	35	45	60	

Check for degeneracy

Number of allocations = 5

Number of rows + Number of columns – 1 = $m + n - 1 = 3 + 4 - 1 = 6$

Since the number of allocations in the VAM table is not equal to $m + n - 1$, the solution is degenerate basic feasible.

Choose the least cost empty cell which is of independent position and give a small quantity $\varepsilon > 0$. This removes degeneracy.

Optimal solution by MODI method

For occupied cells, cost of transportation from source to destination,

$$c_{ij} = u_i + v_j$$

$$c_{11} = u_1 + v_1$$

$$c_{14} = u_1 + v_4$$

$$c_{22} = u_2 + v_2$$

$$c_{23} = u_2 + v_3$$

$$c_{32} = u_3 + v_2$$

$$c_{34} = u_3 + v_4$$

Assume $u_1 = 0$.

$$c_{11} = u_1 + v_1$$

$$14 = 0 + v_1$$

$$v_1 = 14$$

For unoccupied cells, net evaluation value,

$$\Delta_{ij} = c_{ij} - (u_i + v_j)$$

$$\Delta_{12} = c_{12} - (u_1 + v_2)$$

$$\Delta_{12} = 56 - (0 - 5)$$

$$\Delta_{12} = 61$$

	D_1	D_2	D_3	D_4	u_i
S_1	14 70	56 61	48 67	27 ε	$u_1 = 0$
S_2	82 28	35 2	21 45	81 14	$u_2 = 40$
S_3	99 49	31 33	71 54	63 60	$u_3 = 36$
v_j	$v_1 = 14$	$v_2 = -5$	$v_3 = -19$	$v_4 = 27$	

Since all $\Delta_{ij} \geq 0$, the solution is optimal and unique.

The optimum transportation cost = 70 × 14 + ε × 27 + 2 × 35 + 45 × 21 + 33 × 31 + 60 × 63

= 6798 + 27ε

= 6798 as ε → 0

4 Solve the transportation problem when the unit transportation costs, demands and supplies are as given below:

		Destination				Supply
		D_1	D_2	D_3	D_4	
Source	S_1	6	1	9	3	70
	S_2	11	5	2	8	55
	S_3	10	12	4	7	70
Demand		85	35	50	45	

SOLUTION:

Check for balance

Σ Supply = 70 + 55 + 70 = 195

Σ Demand = 85 + 35 + 50 = 215

Since Σ Supply $\neq$ Σ Demand, the given transportation problem is a unbalanced one.

Convert it into a balanced one by adding a dummy source S_4 with cost zero and supply equal to Σ Demand – Σ Supply = 215 – 195 = 20

		Destination				Supply
		D_1	D_2	D_3	D_4	
Source	S_1	4	1	7	0	80
	S_2	3	2	2	0	20
	S_3	5	3	4	0	50
	S_4	0	0	0	0	20
Demand		70	60	40	15	

Initial basic feasible solution by VAM

I allocation

	D_1	D_2	D_3	D_4	Supply	P_I
S_1	6	1	9	3	70	2
S_2	11	5	2	8	55	3
S_3	10	12	4	7	70	3
S_4	0 20	0	0	0	20/0	0
Demand	85/65	35	50	45		
P_I	↑ 6	1	2	3		

II allocation

	D_1	D_2	D_3	D_4	Supply	P_{II}
S_1	6 65	1	9	3	70/5	2
S_2	11	5	2	8	55	3
S_3	10	12	4	7	70	3
Demand	65/0	35	50	45		
P_{II}	↑ 4	4	2	4		

III allocation

	D_2	D_3	D_4	Supply	P_{III}
S_1	1 5	9	3	5/0	2
S_2	5	2	8	55	3
S_3	12	4	7	70	3
Demand	35/30	50	45		
P_{III}	↑ 4	2	4		

IV allocation

	D_2	D_3	D_4	Supply	P_{IV}
S_2	5 30	2	8	55/25	3
S_3	12	4	7	70	3
Demand	30/0	50	45		
P_{IV}	↑ 7	2	1		

V allocation

	D_3	D_4	Supply	P_V
S_2	2 25	8	25	6 ←
S_3	4	7	70	3
Demand	50/25	45		
P_V	2	1		

VI allocation

	D_3	D_4	Supply	P_{VI}
S_3	4 25	7 45	70/45/0	3
Demand	25/0	45/0		
P_{VI}	↑ *	↑ *		

Initial basic feasible solution

	D_1	D_2	D_3	D_4	Supply
S_1	6 65	1 5	9	3	70
S_2	11	5 30	2 25	8	55
S_3	10	12	4 25	7 45	70
S_4	0 20	0	0	0	20
Demand	85	35	50	45	

Check for degeneracy

Number of allocations = 7

Number of rows + Number of columns – 1 = m + n – 1 = 4 + 4 – 1 = 7

Since the number of allocations in the VAM table is equal to $m + n - 1$, the solution is non-degenerate basic feasible.

The total transportation cost = 65 × 6 + 5 × 1 + 30 × 5 + 25 × 2 + 25 × 4 + 45 × 7 + 20 × 0

= 1010

Optimal solution by MODI method

For occupied cells, cost of transportation from source to destination,

$$c_{ij} = u_i + v_j$$

$$c_{11} = u_1 + v_1$$

$$c_{12} = u_1 + v_2$$

$$c_{22} = u_2 + v_2$$

$$c_{23} = u_2 + v_3$$

$$c_{33} = u_3 + v_3$$

$$c_{34} = u_3 + v_4$$

$$c_{41} = u_4 + v_1$$

Assume $u_1 = 0$.

$$c_{11} = u_1 + v_1$$

$$6 = 0 + v_1$$

$$v_1 = 6$$

For unoccupied cells, net evaluation value,

$$\Delta_{ij} = c_{ij} - (u_i + v_j)$$

$$\Delta_{13} = c_{13} - (u_1 + v_3)$$

$$\Delta_{13} = 9 - (0 - 2)$$

$$\Delta_{13} = 11$$

Initial table

	D_1	D_2	D_3	D_4	u_i
S_1	6 65–	1 5+	9 11	3 2	$u_1 = 0$
S_2	11 1	5 30–	2 25+	8 3	$u_2 = 4$
S_3	10 ✓+ –2	12 5	4 25–	7 45	$u_3 = 6$
S_4	0 20	0 0	0 3	0 0	$u_4 = -1$
v_j	$v_1 = 6$	$v_2 = 1$	$v_3 = -2$	$v_4 = 1$	

Since there is one $\Delta_{ij} < 0$, the solution is not optimum. Choose the most negative Δ_{ij}. From this cell draw a closed path and assign + and – signs alternatively and find the minimum allocation from the cell having – sign i.e., min (65, 30, 25) = 25. Modify the solution by adding and subtracting the minimum allocation.

Modified basic feasible solution

	D_1	D_2	D_3	D_4	Supply
S_1	6 40	1 30	9	3	70
S_2	11	5 5	2 50	8	55
S_3	10 25	12	4	7 45	70
S_4	0 20	0	0	0	20
Demand	85	35	50	45	

As the number of independent allocations is equal to *m* + *n* –1, check for optimality.

I iteration table

	D_1	D_2	D_3	D_4	u_i
S_1	6 40	1 30	9 11	3 0	$u_1 = 0$
S_2	11 1	5 5	2 50	8 1	$u_2 = 4$
S_3	10 25	12 7	4 2	7 45	$u_3 = 4$
S_4	0 20	0 5	0 8	0 3	$u_4 = -6$
v_j	$v_1 = 6$	$v_2 = 1$	$v_3 = -2$	$v_4 = 3$	

Since all $\Delta_{ij} \geq 0$, the solution is optimal and an alternate solution exists as $\Delta_{14} = 0$.

The optimum transportation cost = 40 × 6 + 30 × 1 + 5 × 5 + 50 × 2 + 25 × 10 + 45 × 7 + 20 × 0

= 960

5 Solve the following transportation problem to maximize the profit.

		Destination				Supply
		D_1	D_2	D_3	D_4	
Origin	O_1	15	51	42	33	23
	O_2	80	42	26	81	44
	O_3	90	40	66	60	33
Demand		23	31	16	30	

SOLUTION:

Conversion to minimization problem

Since the given problem is to maximize the profit, convert this into loss matrix and minimize it. For converting it into minimization type, subtract all the elements from the highest element i.e., 90.

		Destination				Supply
		D_1	D_2	D_3	D_4	
Origin	O_1	75	39	48	57	23
	O_2	10	48	64	9	44
	O_3	0	50	24	30	33
Demand		23	31	16	30	

Check for balance

Σ Supply = 23 + 44 + 33 = 100

Σ Demand = 23 + 31 + 16 + 30 = 100

Since Σ Supply = Σ Demand, the given transportation problem is a balanced one and there exists a feasible solution to the problem.

Initial basic feasible solution by VAM

I allocation

	D_1	D_2	D_3	D_4	Supply	P_I
O_1	75	39	48	57	23	9
O_2	10	48	64	9	44	1
O_3	0	50	24 16	30	33/17	24
Demand	23	31	16/0	30		
P_I	10	9	↑ 24	21		

II allocation

	D_1	D_2	D_4	Supply	P_{II}
O_1	75	39	57	23	18
O_2	10	48	9	44	1
O_3	0 17	50	30	17/0	30 ←
Demand	23/6	31	30		
P_{II}	10	9	21		

III allocation

	D_1	D_2	D_4	Supply	P_{II}
O_1	75	39	57	23	18
O_2	10 6	48	9	44/38	1
Demand	6/0	31	30		
P_{II}	↑ 65	9	48		

IV allocation

	D_2	D_4	Supply	P_{II}
O_1	39	57	23	18
O_2	48	9 30	38/8	39
Demand	31	30/0		
P_{II}	9	↑ 48		

V allocation

	D_2	Supply	P_{II}
O_1	39 23	23/0	* ←
O_2	48 8	8/0	* ←
Demand	31/8/0		
P_{II}	9		

Initial basic feasible solution

	D_1	D_2	D_3	D_4	Supply
O_1	75	39 23	48	57	23
O_2	10 6	48 8	64	9 30	44
O_3	0 17	50	24 16	30	33
Demand	23	31	16	30	

Check for degeneracy

Number of allocations = 6

Number of rows + Number of columns – 1 = *m* + *n* – 1 = 3 + 4 – 1 = 6

Since the number of allocations in the VAM table is equal to *m* + *n* – 1, the solution is non-degenerate basic feasible.

Optimal solution by MODI method

For occupied cells, cost of transportation from source to destination,

$$c_{ij} = u_i + v_j$$

$$c_{12} = u_1 + v_2$$

$$c_{21} = u_2 + v_1$$

$$c_{22} = u_2 + v_2$$

$$c_{24} = u_2 + v_4$$

$$c_{31} = u_3 + v_1$$

$$c_{33} = u_3 + v_3$$

Assume $u_2 = 0$ as second row has the maximum number of allocation.

$$c_{21} = u_2 + v_1$$

$$10 = 0 + v_1$$

$$v_1 = 10$$

For unoccupied cells, net evaluation value,

$$\Delta_{ij} = c_{ij} - (u_i + v_j)$$

$$\Delta_{11} = c_{11} - (u_1 + v_1)$$

$$\Delta_{11} = 75 - (-9 + 10)$$

$$\Delta_{12} = 74$$

	D_1	D_2	D_3	D_4	u_i
O_1	75 74	39 23	48 43	57 57	$u_1 = -9$
O_2	10 6	48 8	64 50	9 30	$u_2 = 0$
O_3	0 17	50 12	24 16	30 31	$u_3 = -10$
v_j	$v_1 = 10$	$v_2 = 48$	$v_3 = 14$	$v_4 = 9$	

Since all $\Delta_{ij} \geq 0$, the solution is optimal and unique.

The optimum profit = 23 × 51 + 6 × 80 + 8 × 42 + 30 × 81 + 17 × 90 + 16 × 66

= 7005

6 A company has five jobs to be done on five machines. Any job can be done on any machine. The costs of doing the jobs on different machines are given below. Assign the jobs for different machines so as to minimize the total cost.

Job	Machine				
	A	B	C	D	E
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

SOLUTION:

Since the cost matrix is a square matrix, the problem is balanced.

Select the smallest element from each row and subtract it from all other elements from that row. Then select the smallest element from each column and subtract it from all other elements from that column.

Modified matrix

Job	Machine				
	A	B	C	D	E
1	13–8	8–8	16–8	18–8	19–8
2	9–9	15–9	24–9	9–9	12–9
3	12–4	9–4	4–4	4–4	4–4
4	6–6	12–6	10–6	8–6	13–6
5	15–6	17–6	18–6	12–6	20–6

Job	Machine				
	A	B	C	D	E
1	5	0	8	10	11
2	0	6	13	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

See row wise, allocate if a single zero exists and cancel the column containing the allocated zero.

Job	Machine				
	A	B	C	D	E
1	5	0	8	10	11
2	0	6	13	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

Job	Machine				
	A	B	C	D	E
1	5	0	8	10	11
2	0	6	13	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

Job	Machine				
	A	B	C	D	E
1	5	0	8	10	11
2	0	6	13	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

See column wise, allocate if a single zero exists and cancel the row containing the allocated zero.

Job	Machine				
	A	B	C	D	E
1	5	0	8	10	11
2	0	6	13	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

Since the number of assignments i.e., 4 is less than the order of matrix i.e., 5, form the second modified matrix.

Job	Machine				
	A	B	C	D	E
1	5	0	8	10	11
2	0	6	13	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

Select the smallest uncovered element i.e., 3, add this element to elements at intersections and subtract this element from uncovered elements.

Job	Machine				
	A	B	C	D	E
1	5	0	5	10	8
2	0	6	10	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

Repeat the allocation procedure.

Job	Machine				
	A	B	C	D	E
1	5	0	5	10	8
2	0	6	10	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

Job	Machine				
	A	B	C	D	E
1	5	0	5	10	8
2	0	6	10	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

Job	Machine				
	A	B	C	D	E
1	5	0	5	10	8
2	0	6	10	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

Job	Machine				
	A	B	C	D	E
1	5	0	5	10	8
2	0	6	10	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

Job	Machine				
	A	B	C	D	E
1	5	0	5	10	8
2	0	6	10	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

Since the number of assignments i.e., 5 equal to the order of matrix i.e., 5, optimum assignment is made.

Optimal assignment and optimum cost of assignment

Job	Machine	Cost
1	B	8
2	E	12
3	C	4
4	A	6
5	D	12
Total		42

Minimum total cost = 42

7 There are four jobs to be assigned to five machines. Only one job can be assigned to one machine. The amount of time in hours required for the jobs per machine are given in the following table.

Job	Machine				
	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6

Find an optimum assignment of jobs to the machines to minimize the total processing time and also find out for which machine no job is assigned. What is the total processing time to complete all the jobs?

SOLUTION:

Since the cost matrix is not a square matrix, the problem is unbalanced. Add a dummy job 5 with corresponding entries with zero.

Job	Machine				
	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6
5	0	0	0	0	0

Select the smallest element from each row and subtract it from all other elements from that row. Then select the smallest element from each column and subtract it from all other elements from that column.

Modified matrix

Job	Machine				
	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

See row wise, allocate if a single zero exists and cancel the column containing the allocated zero.

Job	Machine				
	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

Job	Machine				
	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

See column wise, allocate if a single zero exists and cancel the row containing the allocated zero.

Job	Machine				
	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

Job	Machine				
	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

Since the number of assignments i.e., 4 is less than the order of matrix i.e., 5, form the second modified matrix.

Job	Machine				
	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

Select the smallest uncovered element i.e., 1, add this element to elements at intersections and subtract this element from uncovered elements.

Job	Machine				
	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Repeat the allocation procedure.

Job	Machine				
	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Job	Machine				
	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Job	Machine				
	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Job	Machine				
	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Job	Machine				
	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Since the number of assignments i.e., 5 equal to the order of matrix i.e., 5, optimum assignment is made.

Optimal assignment and optimum time of assignment

Job	Machine	Time (hours)
1	B	3
2	A	10
3	D	1
4	C	6
5	D	0
Total		20

Since job 5 is dummy, no job is assigned for machine D.

8 A marketing manager has five salesmen and there are five sales districts. Considering the capabilities of the salesmen and the nature of districts, the estimates made by the marketing manager for the sales per week (in 1,000 rupees) for each salesman in each district would be as follows:

Salesman	District				
	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment of salesmen to the districts that will result in the maximum sales.

SOLUTION:

To maximize the profit, first convert it into loss matrix, which can be minimized. To convert it into the loss matrix, subtract all the elements from the highest element i.e., 41.

Loss matrix

Salesman	District				
	A	B	C	D	E
1	9	3	1	13	1
2	1	17	13	20	5
3	0	14	8	11	4
4	19	3	0	5	5
5	12	8	1	6	2

Since the cost matrix is a square matrix, the problem is balanced.

Select the smallest element from each row and subtract it from all other elements from that row. Modified matrix

Salesman	District				
	A	B	C	D	E
1	8	2	0	12	0
2	0	16	12	19	4
3	0	14	8	11	4
4	19	3	0	5	5
5	11	7	0	5	1

Select the smallest element from each column and subtract it from all other elements from that column.

Salesman	District				
	A	B	C	D	E
1	8	0	0	7	0
2	0	14	12	14	4
3	0	12	8	6	4
4	19	1	0	0	5
5	11	5	0	0	1

See row wise, allocate if a single zero exists and cancel the column containing the allocated zero.

Salesman	District				
	A	B	C	D	E
1	8	0	0	7	0
2	0	14	12	14	4
3	0	12	8	6	4
4	19	1	0	0	5
5	11	5	0	0	1

See column wise, allocate if a single zero exists and cancel the row containing the allocated zero.

Salesman	District				
	A	B	C	D	E
1	8	0	0	7	0
2	0	14	12	14	4
3	0	12	8	6	4
4	19	1	0	0	5
5	11	5	0	0	1

Salesman	District				
	A	B	C	D	E
1	8	0	0	7	0
2	0	14	12	14	4
3	0	12	8	6	4
4	19	1	0	0	5
5	11	5	0	0	1

Salesman	District				
	A	B	C	D	E
1	8	0	0	7	0
2	0	14	12	14	4
3	0	12	8	6	4
4	19	1	0	0	5
5	11	5	0	0	1

Since the number of assignments i.e., 4 is less than the order of matrix i.e., 5, form the second modified matrix.

Salesman	District				
	A	B	C	D	E
1	8	0	0	7	0
2	0	14	12	14	4
3	0	12	8	6	4
4	19	1	0	0	5
5	11	5	0	0	1

Select the smallest uncovered element i.e., 4, add this element to elements at intersections and subtract this element from uncovered elements.

Salesman	District				
	A	B	C	D	E
1	12	0	0	7	0
2	0	10	8	10	0
3	0	8	4	2	0
4	23	1	0	0	5
5	15	5	0	0	1

Repeat the allocation procedure.

Salesman	District				
	A	B	C	D	E
1	12	0	0	7	0
2	0	10	8	10	0
3	0	8	4	2	0
4	23	1	0	0	5
5	15	5	0	0	1

Salesman	District				
	A	B	C	D	E
1	12	0	0	7	0
2	0	10	8	10	0
3	0	8	4	2	0
4	23	1	0	0	5
5	15	5	0	0	1

Salesman	District				
	A	B	C	D	E
1	12	0	0	7	0
2	0	10	8	10	0
3	0	8	4	2	0
4	23	1	0	0	5
5	15	5	0	0	1

Salesman	District				
	A	B	C	D	E
1	12	0	0	7	0
2	0	10	8	10	0
3	0	8	4	2	0
4	23	1	0	0	5
5	15	5	0	0	1

Salesman	District				
	A	B	C	D	E
1	12	0	0	7	0
2	0	10	8	10	0
3	0	8	4	2	0
4	23	1	0	0	5
5	15	5	0	0	1

Since the number of assignments i.e., 5 equal to the order of matrix i.e., 5, optimum assignment is made.

Optimal assignment and optimum cost of assignment

Salesman	District	Sales (in 1,000 ₹)
1	B	38
2	A	40
3	E	37
4	C	41
5	D	35
Total		191

9 A travelling salesman has to visit five cities. He wishes to start from a particular city once and then return to his starting point. Cost (in rupees) of going from one city to another is shown below. Find the least cost route.

From City	To City				
	A	B	C	D	E
A	–	40	100	140	20
B	120	–	60	100	40
C	160	140	–	80	140
D	240	80	120	–	100
E	20	60	40	160	–

SOLUTION:

First solve this problem as an assignment problem.

Select the smallest element from each row and subtract it from all other elements from that row. Modified matrix

From City	To City				
	A	B	C	D	E
A	∞	20	80	120	0
B	80	∞	20	60	0
C	80	60	∞	0	60
D	160	0	40	∞	20
E	0	40	20	140	∞

Select the smallest element from each column and subtract it from all other elements from that column.

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

See row wise, allocate if a single zero exists and cancel the column containing the allocated zero.

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

Since the number of assignments i.e., 5 equal to the order of matrix i.e., 5, optimum assignment is made for assignment problem.

A → E, B → C, C → D, D → B, E → A

The salesman should go from A to E and then come back to A without covering B, C and D. But this is contradicting the constraint that no city is visited twice before all the cities are visited.

The next best solution can be obtained by bringing next minimum non-zero element i.e., 20.

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

From City	To City				
	A	B	C	D	E
A	∞	20	60	120	0
B	80	∞	0	60	0
C	80	60	∞	0	60
D	160	0	20	∞	20
E	0	40	0	140	∞

A → B, B → C, C → D, D → E, E → A

Since all the cities have been visited and no city is visited before completing the tour of all the cities, we have an optimal solution for the travelling salesman.

The least cost route is A → B → C → D → E → A.

Total cost = 40 + 60 + 80 + 100 + 20 = ₹ 300

10 A salesman has to visit five cities. The distance (in hundred kilometers) between the five cities is as follows:

From City	To City				
	A	B	C	D	E
A	–	4	7	3	4
B	4	–	6	3	4
C	7	6	–	7	5
D	3	3	7	–	7
E	4	4	5	7	–

If the salesman starts from city A and has to come back to his starting point, which route should he select so that the total distance travelled is minimum?

SOLUTION:

First solve this problem as an assignment problem.

Select the smallest element from each row and subtract it from all other elements from that row.

Modified matrix

From City	To City				
	A	B	C	D	E
A	∞	1	4	0	1
B	1	∞	3	0	1
C	2	1	∞	2	0
D	0	0	4	∞	4
E	0	0	1	3	∞

Select the smallest element from each column and subtract it from all other elements from that column.

From City	To City				
	A	B	C	D	E
A	∞	1	3	0	1
B	1	∞	2	0	1
C	2	1	∞	2	0
D	0	0	3	∞	4
E	0	0	0	3	∞

See row wise, allocate if a single zero exists and cancel the column containing the allocated zero.

From City	To City				
	A	B	C	D	E
A	∞	1	3	0	1
B	1	∞	2	0	1
C	2	1	∞	2	0
D	0	0	3	∞	4
E	0	0	0	3	∞

From City	To City				
	A	B	C	D	E
A	∞	1	3	0	1
B	1	∞	2	0	1
C	2	1	∞	2	0
D	0	0	3	∞	4
E	0	0	0	3	∞

From City	To City				
	A	B	C	D	E
A	∞	1	3	0	1
B	1	∞	2	0	1
C	2	1	∞	2	0
D	0	0	3	∞	4
E	0	0	0	3	∞

From City	To City				
	A	B	C	D	E
A	∞	1	3	0	1
B	1	∞	2	0	1
C	2	1	∞	2	0
D	0	0	3	∞	4
E	0	0	0	3	∞

Since the number of assignments i.e., 4 is less than the order of matrix i.e., 5, form the second modified matrix.

From City	To City				
	A	B	C	D	E
A	∞	1	3	0	1
B	1	∞	2	0	1
C	2	1	∞	2	0
D	0	0	3	∞	4
E	0	0	0	3	∞

Select the smallest uncovered element i.e., 1, add this element to elements at intersections and subtract this element from uncovered elements.

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

Repeat the allocation procedure.

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

Since the number of assignments i.e., 5 equal to the order of matrix i.e., 5, optimum assignment is made for assignment problem.

A $\rightarrow$ B, B $\rightarrow$ D, C $\rightarrow$ E, D $\rightarrow$ A, E $\rightarrow$ C

The salesman should go from A to B, B to D and then come back to A without covering C and E. But this is contradicting the constraint that no city is visited twice before all the cities are visited.

The next best solution can be obtained by bringing next minimum non-zero element i.e., 1.

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

From City	To City				
	A	B	C	D	E
A	∞	0	2	0	1
B	0	∞	1	0	1
C	1	0	∞	2	0
D	0	0	3	∞	5
E	0	0	0	4	∞

A → E, B → D, C → B, D → A, E → C

Since all the cities have been visited and no city is visited before completing the tour of all the cities, we have an optimal solution for the travelling salesman.

The least distance route is A → E → C → B → D → A.

Total distance = 4 + 5 + 6 + 3 + 3 = 21 hundred kilometers

1.11 A company has nine jobs, *A* to *I*. All the jobs have to go through two machines 1 and 2. The time required for the jobs on each machine in hours is given below. Find the optimum sequence that minimizes the total elapsed time. Also, find the idle time for each machine.

Job	A	B	C	D	E	F	G	H	I
Machine 1	2	5	4	9	6	8	7	5	4
Machine 2	6	8	7	4	3	9	3	8	11

SOLUTION:

Job	A	B	C	D	E	F	G	H	I
Machine 1	2	5	4	9	6	8	7	5	4
Machine 2	6	8	7	4	3	9	3	8	11

The shortest processing time is 2 on machine 1 for job *A*. Hence, process this job first.

A								

Deleting task *A*, reduced list of processing time is

Job	B	C	D	E	F	G	H	I
Machine 1	5	4	9	6	8	7	5	4
Machine 2	8	7	4	3	9	3	8	11

The next minimum processing time is same for jobs *E* and *G* on machine 2. The corresponding processing time on machine 1 for this job is 6 and 7. So sequence job *G* at the end and *E* next to it.

A							E	G

Deleting jobs *E* and *G*, reduced list of processing time is

Job	B	C	D	F	H	I
Machine 1	5	4	9	8	5	4
Machine 2	8	7	4	9	8	11

The minimum processing time is 4 for job *C*, *I* and *D*. For jobs *C* and *I*, it is on machine 1 and for job *D*, it is on machine 2. There is a tie in sequencing jobs *C* and *I*. To break this, consider the corresponding time on machine 2, the longest time is 11. Hence, sequence job I in the beginning followed by job *C*. For job *D*, as it is on machine 2, sequence it last.

A	I	C				D	E	G

Deleting the jobs that are sequenced, the reducing processing list is,

Job	B	F	H
Machine 1	5	8	5
Machine 2	8	9	8

The next minimum processing time is 5 on machine 1 for jobs *B* and *H*, which is again a tie. To break this, consider the corresponding longest time on the machine 2 and sequence job *B* or *H* first. Finally, job *F* is sequenced.

The optimal sequence is,

A	I	C	B	H	F	D	E	G

Total elapsed time and idle time for both the machines

Job	Machine 1				Machine 2			
	In Time	Time on m/c	Out Time	Idle Time	In Time	Time on m/c	Out Time	Idle Time
A	0	2	2	0	2	6	8	2
I	2	4	6	0	8	11	19	0
C	6	4	10	0	19	7	26	0
B	10	5	15	0	26	8	34	0
H	15	5	20	0	34	8	42	0
F	20	8	28	0	42	9	51	0
D	28	9	37	0	51	4	55	0
E	37	6	43	0	55	3	58	0
G	43	7	50	0	58	3	61	0
				61–50 = 11				
Total				11				2

Total elapsed time = 61 hours

Idle time for machine 1 = 11 hours

Idle time for machine 2 = 2 hours

12 There are six jobs, each of which must go through machines *A*, *B* and *C* in the order *ABC*. The time taken (in minutes) on each of the machines is given below.

Job	1	2	3	4	5	6
Machine A	3	12	5	2	9	11
Machine B	8	6	4	6	3	1
Machine C	13	14	9	12	8	13

Determine the sequence that will minimize the total elapsed time. Also, find the idle time for each machine.

SOLUTION:

Minimum processing time on machine *A* = 2

Minimum processing time on machine *C* = 8

Maximum processing time on machine *B* = 8

Min *A* ≥ Max *B* is not satisfied

Min *C* ≥ Max *B* is satisfied

Convert the problem into a two-machine problem by defining two machines *G* and *H*, such that the processing time on *G* and *H* are given by

$$G_i = A_i + B_i$$

$$H_i = B_i + C_i$$

Job	1	2	3	4	5	6
Machine G	3 + 8 = 11	12 + 6 = 18	5 + 4 = 9	2 + 6 = 8	9 + 3 = 12	11 + 1 = 12
Machine H	8 + 13 = 21	6 + 14 = 20	4 + 9 = 13	6 + 12 = 18	3 + 8 = 11	1 + 13 = 14

Job	1	2	3	4	5	6
Machine G	11	18	9	8	12	12
Machine H	21	20	13	18	11	14

The shortest processing time is 8 on machine *G* for job 4. Hence, process this job first.

4					

Deleting job 4, reduced list of processing time is

Job	1	2	3	5	6
Machine G	11	18	9	12	12
Machine H	21	20	13	11	14

The next minimum processing time is 9 on machine G for job 3. Hence, sequence job 3 next to job 4.

4	3				

Deleting job 3, reduced list of processing time is

Job	1	2	5	6
Machine G	11	18	12	12
Machine H	21	20	11	14

The next minimum processing time is same for jobs 1 and 5 on machine *G* and *H* respectively. So, sequence job 1 next to job 3 and 5 at the end.

4	3	1			5

Deleting the jobs that are sequenced, the reducing processing list is,

Job	2	6
Machine G	18	12
Machine H	20	14

The next minimum processing time is 12 on machine *G* for job 6. Hence, sequence job 6 next to job 1. Finally job 2 is sequenced.

The optimal sequence is,

4	3	1	6	2	5

Total elapsed time and idle time for both the machines

Job	Machine A				Machine B				Machine C			
	In Time	Time on m/c	Out Time	Idle Time	In Time	Time on m/c	Out Time	Idle Time	In Time	Time on m/c	Out Time	Idle Time
4	0	2	2	0	2	6	8	2	8	12	20	8
3	2	5	7	0	8	4	12	0	20	9	29	0
1	7	3	10	0	12	8	20	0	29	13	42	0
6	10	11	21	0	21	1	22	1	42	13	55	0
2	21	12	33	0	33	6	39	11	55	14	69	0
5	33	9	42	0	42	3	45	3	69	8	77	0
				77–42 = 35				77–45 = 32				
Total				35				49				8

Total elapsed time = 61 minutes

Idle time for machine *A* = 35 minutes

Idle time for machine *B* = 49 minutes

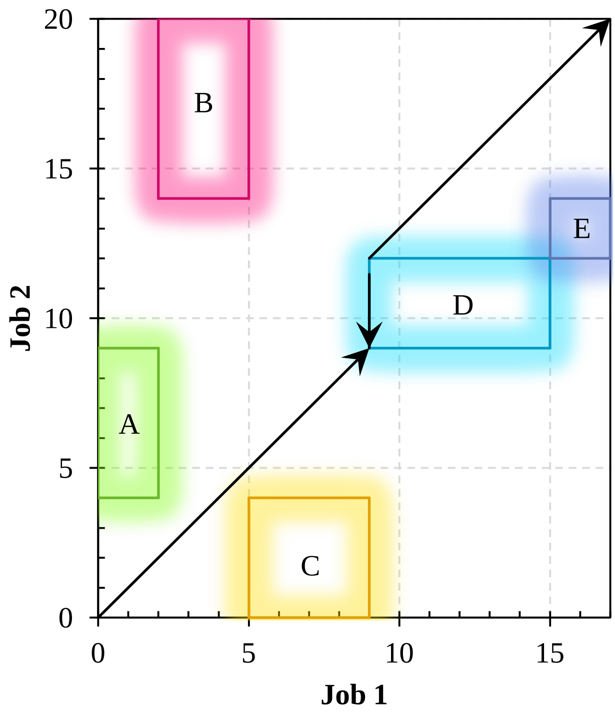
Idle time for machine C = 8 minutes

13 Use graphical method to minimize the total time needed to process the following jobs on the machines shown below, i.e., for each machine find the job that should be done first. Also calculate the total time needed to complete both the jobs.

Job 1	Sequence of machine	A	B	C	D	E
	Time	2	3	4	6	2
Job 2	Sequence of machine	C	A	D	E	B
	Time	4	5	3	2	6

SOLUTION:

The given information is shown in the figure. The blocks represent the overlaps that are to be avoided.



An optimal path is one that minimizes the idle time for job 1 (horizontal movement). Similarly, an optimal path is one that minimizes the idle time for job 2 (vertical movement).

Idle time for job 1 = 3 hours

Idle time for job 2 = 0 hours

Total elapsed time = Processing time of job 1 + Idle time for job 1 = 17 + 3 = 20 hours

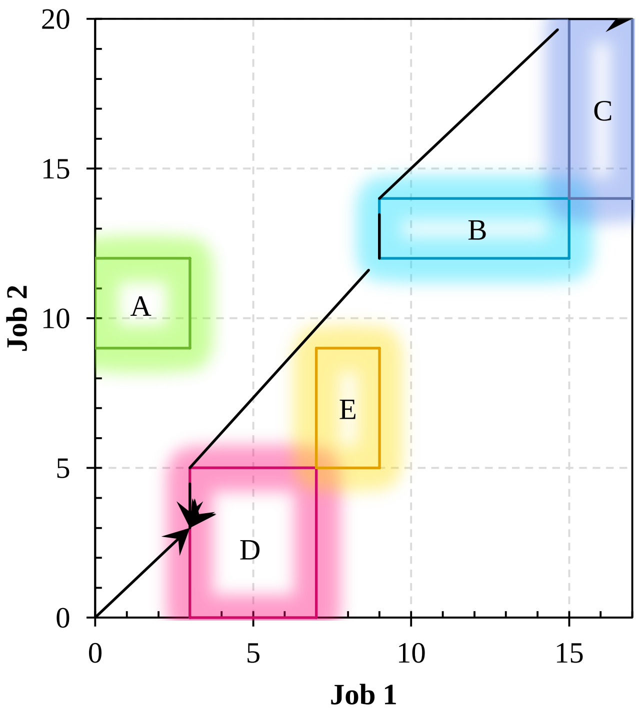
= Processing time of job 2 + Idle time for job 2 = 20 + 0 = 20 hours

14 Use the graphical method to minimize the time needed to process the following jobs on the machines shown, i.e., for each machine find the job that should be done first. Also calculate the total elapsed time to complete both the jobs.

Job 1	Sequence of machine	A	B	C	D	E
	Time	3	4	2	6	2
Job 2	Sequence of machine	B	C	A	D	E
	Time	5	4	3	2	6

SOLUTION:

The given information is shown in the figure. The blocks represent the overlaps that are to be avoided.



An optimal path is one that minimizes the idle time for job 1 (horizontal movement). Similarly, an optimal path is one that minimizes the idle time for job 2 (vertical movement).

Idle time for job 1 = 2 + 3 = 5 hours

Idle time for job 2 = 2 hours

Total elapsed time = Processing time of job 1 + Idle time for job 1 = 17 + 5 = 22 hours

= Processing time of job 2 + Idle time for job 2 = 20 + 2 = 22 hours

6 TWO MARKS QUESTIONS AND ANSWERS

1. **What do you understand by transportation problem?**

Transportation problem is a special class of linear programming problem in which we transport it commodity (single product) from the source to a destination in such a way that the total transportation cost is minimum.

2. **Define feasible, basic feasible, non-degenerate solution of a transportation problem.**

Any set of non-negative allocations ($x_{ij} \geq 0$) which satisfies the row and column sum (rim requirement) is called a 'feasible solution'.

A feasible solution is called a 'basic feasible solution' if the number of non-negative allocations is equal to *m* + *n* – 1, where *m* is the number of rows and *n* is the number of columns in a transportation table.

Any feasible solution to a transportation problem containing *m* origins and *n* destinations is said to be 'non-degenerate' if it contains *m* + *n* – 1 occupied cells and each allocation is in an independent position.

3. **Give reasons as to why the LPP solution techniques are not made use of while solving a transportation problem.**

As there are *m* + *n* – 1 equations in a transportation problem with *m* origins and *n* destinations, by adding an artificial variable to each equation, a large number of variables are involved.

(i) If the problem has *m* sources and *n* destinations and *m* + *n* – 1 equations can be formed. Hence, computation may exceed the capacity of the computer. So LPP technique is not made use of while solving a transportation problem.

(ii) The coefficient x_{ij} in the constraints are all in unity. For such a technique, transportation technique is easier than simplex method.

(iii) Transportation problem is minimization of objective function, whereas, simplex method is suitable for maximization problem.

4. **List any three approaches used with transportation problem for determining the starting solution.**

- North-west corner rule
- Least cost method (Matrix minima method)
- Vogel's approximation method

5. **Define the optimal solution to a transportation problem.**

The basic feasible solution to a transportation problem is said to be optimal if it minimizes the total transportation cost.

6. **State the necessary and sufficient condition for the existence of a feasible solution to a transportation problem.**

The necessary and sufficient condition for the existence of a feasible solution is a solution that satisfies all conditions of supply and demand.

7. **What is the purpose of MODI method?**

The purpose of MODI method is to get the optimal solution of a transportation problem.

8. When does a transportation problem have a unique solution?

A transportation problem has a unique solution, if the net evaluation given by $\Delta_{ij} = c_{ij} - (u_i + v_j)$ of all the empty cells are positive, i.e., of all $\Delta_{ij} > 0$.

9. What do you mean by degeneracy in a transportation problem?

If the number of occupied cells in a $m \times n$ transportation problem is less than $m + n - 1$, then it is called a degeneracy in a transportation problem.

10. Explain how degeneracy in a transportation problem may be resolved?

The degeneracy in a transportation problem can be resolved by adding one (more) empty cell having the least cost and is of independent position with a non-negative allocation ($\varepsilon > 0$).

11. What do you mean by an unbalanced transportation problem?

Any transportation problem is said to be unbalanced if the total supply is not equal to the total demand.

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

12. How do you convert an unbalanced transportation problem into a balanced one?

The unbalanced transportation problem can be converted into a balanced one by adding a dummy row (source) with cost zero and the excess demand is entered as a rim requirement if total supply < total demand. On the other hand, if the total supply > total demand, introduce a dummy column (destination) with cost zero and the excess supply is entered as a rim requirement for the dummy destination.

13. List the merits and limitations of using north-west corner rule.

Merits: This method is easy to follow because we need not to consider the transportation cost.

Limitations: The solution obtained may not be the best solution, as the allocations have been made without considering the cost of transportation. While performing optimality test, it may need more iterations to get the optimal solution.

14. Vogel's approximation method results in the most economical initial basic feasible solution. Why?

In this method, we take into account not only the least cost c_{ij} but also the costs that just exceed c_{ij}. This method considerably reduces the number of iteration required to arrive at the optimal solution. Also, it gives near optimal solution that may, at times, be the optimal solution.

15. How will you identify that a transportation problem has got an alternate optimal solution?

While performing optimality test. if some of Δ_{ij} value, where $\Delta_{ij} = c_{ij} - (u_i + v_j)$ for empty (non-basic) cell is zero, then it is the indication of an alternate solution.

16. What is an assignment problem? Give two applications.

The problem of assigning the number of jobs to equal number of facilities (machines or persons or destinations) at a minimum cost or maximum profit is called an assignment problem.

Applications:

- If *n* jobs have to be assigned to *n* workers or machines with unit cost or unit time of performing the job, we can use assignment model to get minimum cost.
- Travelling salesmen problem, i.e., a salesman has to visit a number of cities, not visiting the same city twice and return to the starting place.

17. What do you mean by an unbalanced assignment problem?

If the number of rows is not equal to the number of columns in the cost matrix of the assignment problem or if the cost matrix of the given assignment problem is not a square matrix, then the given assignment problem is said to be unbalanced.

18. Why can the transportation technique or the simplex method not be used to solve the assignment problem?

The transportation technique or simplex method cannot he used to solve the assignment problem because of degeneracy.

19. State the difference between the transportation problem and the assignment problem.

The major differences between transportation problem and assignment problem are:

The cost matrix in transportation problem is not necessarily a square matrix, whereas in assignment problem it is a square matrix.

Supply and demand at any source and at any destination may be positive quantity a_i , b_j in transportation problem whereas in assignment problem it will be 1, i.e., $a_i = b_j = 1$.

The allocations x_{ij} in the case of transportation problem can take any positive values satisfying the rim requirements, whereas, in assignment problem, x_{ij} will take only two possible values 1 or 0.

20. How is the presence of an alternate optimal solution established?

If the final cost matrix contains more than a required number of zeros at independent positions, then it indicates the presence of an alternate optimal solution.

21. What is the objective of the travelling salesman problem?

The objective of the travelling salesman problem is that the salesman has to visit various cities, not visiting the same place twice and return to the starting place by spending minimum transportation cost.

22. How do you convert the maximization assignment problem into a minimization one?

The maximization assignment problem can be converted into minimization assignment problem by subtracting all the elements in the given profit matrix from the highest element in that matrix.

23. If each entry is increased by 3 in a 4 × 4 assignment problem, what is the effect on the optimal value?

The effect in the optimal value when each entry is increased by 3 is given by

New optimal value = Original optimal value + 3 × 4, where 4 is the order of matrix.

24. Give the linear programming form of the assignment problem.

The assignment problem can be expressed as

$$\text{Minimize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

where c_{ij} is the cost of assigning i^{th} machine to the j^{th} job, subject to the constraints

$$x_{ij} = \begin{cases} 1 \text{ if } i^{\text{th}} \text{ machine is assigned to the } j^{\text{th}} \text{job} \\ 0 \text{ if not} \end{cases}$$

25. Why is assignment problem a completely degenerate form of a transportation problem?

Since the units available at each source and units demanded at each destination are equal, we get exactly one occupied cell in each row and each column. Hence, we get only *n* occupied cells in the place in the required $n + n - 1 = 2n - 1$ occupied cells. Hence, an assignment problem is a completely degenerate form of a transportation problem.

26. What is the name of the method used in getting the optimum assignment?

Hungarian method.

27. When is an assignment problem said to be unbalanced? How do you make it a balanced one?

If the cost matrix or profit matrix is not a square matrix, then the problem is said to be unbalanced. To make it balanced, we add a row or column accordingly with all the entries zero.

28. How do you solve an assignment problem if the profit is to be maximised?

The given profit matrix can be converted into a loss matrix or minimization type by subtracting all the elements from the highest element of the given matrix. For this minimization problem, apply steps of the Hungarian method to get an optimal assignment.

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KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University)

(Established Under Section 3 of UGC Act, 1956)

Pollachi Main Road, Eachanari Post, Coimbatore – 641021

FACULTY OF ENGINEERING

DEPARTMENT OF AUTOMOBILE ENGINEERING

INDUSTRIAL ENGINEERING AND OPERATIONS RESEARCH

UNIT-V

INVENTORY CONTROL

CONTENTS

1 INTRODUCTION

Inventory is defined as any idle resource of an enterprise. It is a physical stock of goods kept for future use. In a factory, the inventory may be in the form of raw materials, parts, semi-finished goods, etc. An inventory may also include furniture, machinery, etc.

2 REASONS FOR MAINTAINING INVENTORIES

The need of the management to make decisions regarding an inventory arises because of the various alternative courses of action available with the enterprise. It is essential for an enterprise to have an inventory, due to the following reasons.

- It helps in smooth and efficient running of the business.
- It provides adequate service to the customers.
- It reduces the possibility of duplication of orders.
- It helps in maintaining a balance in the economy by absorbing some of the fluctuations, when the demand of an item fluctuates or is seasonal.
- It helps in minimizing the loss due to deterioration, obsolescence, damage, etc.
- It acts as a buffer stock when raw materials are received late and shop rejections are too many.
- Takes advantages of price discounts by bulk purchasing.

Though inventories are essential and provide an alternative to production/purchase in the future, they also lock up the capital of the enterprise. This includes the expenses of stores, equipment, personnel, insurance, etc., therefore, excessive inventories are undesirable. Larger inventories do not necessarily lead to a high volume of output; instead, they might hamper the production.

Our problem is to strike a balance between the advantages of having inventories and the cost of carrying them, to arrive at an optimal level of inventories, minimizing the total inventory cost. This calls for controlling the inventories in the most profitable way. The basic objective of inventory control is to release capital for more productive use.

3 TYPES OF INVENTORY

There are five types of inventories, namely,

- Transportation inventories
- Buffer inventories
- Anticipation inventories
- De-coupling inventories
- Lot-size inventories

3.1 TRANSPORTATION INVENTORIES

They arise due to the transportation of inventory items to the various distribution centres and customers from the various production centres. The amount of transportation inventory depends on the time consumed in transportation and the nature of the demand.

3.2 BUFFER INVENTORIES

These are maintained to meet the uncertainty of demand and supply.

3.3 ANTICIPATION INVENTORIES

These are built in advance by anticipating or foreseeing the future demand. For example, production of crackers before the Diwali festival; electric fans or coolers before the onset of the summer season.

3.4 DE-COUPLING INVENTORIES

The inventories used to reduce the interdependence of the various stages of a production system are known as de-coupling inventories.

3.5 LOT-SIZE INVENTORIES

Generally, the rate of consumption is different from the rate of production or purchasing. Therefore, items are produced in larger quantities, which result in lot-size, also known as cycle inventories.

4 INVENTORY COSTS

There are four categories of inventory cost associated with maintaining inventories. They are,

- Item (production or purchase) cost
- Ordering or set-up cost
- Carrying or holding cost
- Shortage or stock out cost

4.1 ITEM COST (C_1)

It refers to the cost associated with an item, whether it is manufactured or purchased. The purchase price will be considered when discounts are allowed for any purchase above a certain quantity.

4.2 ORDERING COST OR SET-UP COST (C_2)

These costs include the fixed cost associated with obtaining the goods through placing of an order and purchasing, manufacturing or setting up a machinery before starting the production. They include the costs of purchase, requisition, follow-up, receiving the goods, quality control, etc. These are also called order costs or replenishment costs, usually denoted by C_2, per production run (cycle). They are assumed to be independent of the quantity ordered or produced.

4.3 CARRYING OR HOLDING COST (C_3)

The cost associated with carrying or holding the goods in stock is known as holding or carrying cost, which is denoted by C_3, per unit of goods for a unit of time. Holding cost is assumed to vary directly with the size of inventory as well as the time the item is held in stock. The following components constitute the holding cost.

- Invested capital cost: This is the interest charged over the capital invested.
- Record keeping and administrative cost.
- Handling cost: These include costs associated with movement of stock such as cost of labour, etc.
- Storage costs.

- Depreciation costs.
- Taxes and insurance, etc.

If *P* is the purchase price of an item, *I* is the stock holding cost per unit time as a fraction of stock value, then the holding cost $C_3 = IP$.

4.4 SHORTAGE COST OR STOCK OUT COST (C_4)

The penalty costs that are incurred as a result of running out of stock (i.e., shortage) are known as shortage or stock out costs. These are denoted by C_4 per unit of goods for a specified period.

If the unfilled demand for the goods can be satisfied at a later date (backlog case), these costs are assumed to vary directly with the shortage quantity and the delaying time. If the unfilled demand is lost (no backlog case), shortage cost becomes proportional to shortage quantity.

5 VARIABLES IN THE INVENTORY PROBLEM

The variables involved in the inventory model are of two types:

- Controlled variables and
- Uncontrolled variables

Controlled variables

These include three basic questions, namely,

- How much quantity of an item should be ordered?
- When should the order be placed? That is, the frequency or timing of acquisition.
- The completion stage of stocked items.

Uncontrolled variables

These include holding, shortage and set-up costs.

Note: Total inventory cost = Purchase cost of inventory items + Ordering cost + Carrying cost + Shortage costs

6 OTHER FACTORS INVOLVED IN INVENTORY ANALYSIS

6.1 DEMAND

Demand refers to the number of items required per period. It may be known exactly in terms of probabilities or may be completely unknown.

The demand pattern of items may be either deterministic or probabilistic. Problems in which the demand is known and fixed are called deterministic problem. Whereas, those problems in which the demand is assumed to be a random variable are called stochastic or probabilistic problems.

In case of deterministic demands, it is assumed that the quantities needed over subsequent periods of time are known exactly. However, the known demand may be fixed or variable with time. Such demands are respectively called static or dynamic demands.

Probabilistic demand occurs when the demand over a certain period of time is not known with certainty; but it is described by a known probability distribution. A probabilistic demand may be either stationary or non-stationary over a period of time.

6.2 LEAD TIME

The time gap between the placing of an order and the actual arrival of the inventory is known as lead time. If the lead time is known and is not equal to zero; and if the demand is deterministic, all that one requires to do is to order in advance, by the time equal to the lead time. If the lead time is zero, there is no need to order in advance.

In case the lead time is a variable, which is known only probabilistically, then the question of when to order is more difficult. The amount and timing of replenishment is found by considering the expected costs of holding and shortage, over the lead time required.

6.3 AMOUNT DELIVERED (SUPPLY OF GOODS)

The supply of goods may be instantaneous or spread over a period of time. If a quantity q is ordered, purchased or produced, the amount delivered may vary around q with a known probability density function.

6.4 ORDER CYCLE

The time period between the placement of two successive orders is referred to as an order cycle. It may be created on the basis of the following two types of inventory review systems.

- **Continuous review:** The record of the inventory level is checked continuously until a certain lower limit (known as recorder level) is reached before a new order is placed. This is often known as a two-bin system.
- **Periodic review:** In this, the inventory levels are reviewed at equal time intervals and are placed at such intervals. The quantity ordered each time depends on the available inventory level at the time of review.

6.5 TIME HORIZON

The time period over which the inventory level will be controlled is known as time horizon.

6.6 RECORDER LEVEL

The level between the maximum and the minimum stock at which the purchasing (manufacturing) activities for replenishment must begin, is known as recorder level.

The inventory model can be classified into two categories.

- Deterministic inventory model
- Probabilistic inventory model

7 DETERMINISTIC INVENTORY MODEL

In this model, the demand is assumed to be fixed and completely predetermined, i.e., **static demand**. Such models are referred to as economic lot size models.

There are four types of models under this category, namely,

(i) Purchasing model with no shortages
(ii) Manufacturing model with no shortages
(iii) Purchasing model with shortages
(iv) Manufacturing model with shortages

7.1 EOQ MODELS WITHOUT SHORTAGES

7.1.1 MODEL I: PURCHASING MODEL WITH NO SHORTAGES

(a) The Economic lot size system with uniform demand

In this model, we have to derive an economic lot size formula for the optimum production quatitity cycle of a single product, so as to minimize the total average variable cost per unit time.

The assumptions for this model are as follows.

- Demand rate is uniform.
- Lead time is zero:
- Production rate is infinite, i.e., production is instantaneous.
- Shortages are not allowed.
- Holding cost is rupees C_3 per quantity unit, per unit time.
- Set-up cost is rupees C_2 per time set-up.

8 SOLVED PROBLEMS

1 The annual demand of an item is 3,200 units. The unit cost is ₹ 6 and inventory carrying charges are 25% per annum. If the cost of one procurement is ₹ 150, determine the

(i) economic order quantity,
(ii) number of orders per year,
(iii) time between two consecutive orders, and
(iv) optimal cost.

GIVEN:

Demand rate, D = 3,200/year

Item cost, C_1 = ₹ 6/unit

Ordering cost, C_2 = ₹ 150/order

Carrying cost, C_3 = 0.25 C_1/unit/year = 0.25 × 6 = ₹ 1.5/unit/year

TO FIND:

Economic order quantity

Number of orders per year

Time between two consecutive orders

Optimal cost

SOLUTION:

Economic order quantity

$$Q^* = \sqrt{\frac{2DC_2}{C_3}}$$

$$= \sqrt{\frac{2 \times 3200 \times 150}{1.5}}$$

$$= 800 \text{ units}$$

Number of orders per year

$$N^* = \frac{D}{Q^*}$$

$$= \frac{3200}{800}$$

$$= 4$$

Time between two consecutive orders

$$T^* = \frac{1}{N^*}$$

$$= \frac{1}{4} \text{ year}$$

$$= \frac{12}{4} \text{ months}$$

$$= 3 \text{ months}$$

The optimal cost

$= C_1D + \sqrt{2DC_2C_3}$

$= 6 \times 3200 + \sqrt{2 \times 3200 \times 150 \times 1.5}$

$=$ ₹ $20{,}400$

2 A company purchases 9,000 parts of a machine for its annual requirements, ordering one month's usage at a time. Each part costs ₹ 20. The ordering cost per order is ₹ 15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it save the company per year?

GIVEN:

Demand rate, D = 9,000/year

Item cost, C_1 = ₹ 20/part

Ordering cost, C_2 = ₹ 15/order

Carrying cost, C_3 = 15% of the average inventory per year = 0.15 × 20 = ₹ 3/part/year

TO FIND:

Suggest a more economical purchasing policy for the company.

How much it will save the company per year?

SOLUTION:

Economic order quantity

$$Q^* = \sqrt{\frac{2DC_2}{C_3}}$$

$$= \sqrt{\frac{2 \times 9000 \times 15}{3}}$$

$$= 300 \text{ parts}$$

Number of orders per year

$$N^* = \frac{D}{Q^*}$$

$$= \frac{9000}{300}$$

$$= 30$$

Time between two consecutive orders

$$T^* = \frac{1}{N^*}$$

$$= \frac{1}{30} \text{ year}$$

$$= \frac{365}{30} \text{ days}$$

$$= 12.1667 \text{ days}$$

Minimum annual inventory cost

$$C^* = \sqrt{2DC_2C_3}$$

$$= \sqrt{2 \times 9000 \times 15 \times 3}$$

$$= ₹\ 900$$

If the company follows the policy of ordering every month, then

Annual ordering cost = 12 × 15 = ₹ 180

Lot size of inventory for each month,

$$Q = \frac{9000}{12}$$

$$= 750$$

Average inventory at any time

$$= \frac{750}{2}$$

$$= 375$$

Storage cost at any time

= Average inventory at any time $\times$ C_3

= 375 $\times$ 3

= ₹ 1125

Total annual cost

= Storage cost + Ordering cost

= 1125 + 180

= ₹ 1305

If the company purchases 300 parts at time intervals of 12.1667 days instead of ordering 750 parts each month, there will be net savings of 1305 – 900 = ₹ 405/year.

3 The annual requirement for a product is 3,000 units. The ordering cost is ₹ 100 per order. The cost per unit is ₹ 10. The carrying cost per unit per year is 30% of the unit cost. Find the economic order quantity.

By using better organizational methods, the ordering cost per order can be brought down to ₹ 80 per order, but the same quantity as determined above has to be ordered. If a new economic order quantity is found by using the ordering cost as ₹ 80, what would be the further savings in cost?

GIVEN:

Demand rate, D = 3,000/year

Item cost, C_1 = ₹ 10/unit

Ordering cost, C_2 = ₹ 100/order

Carrying cost, C_3 = 30% of the unit cost/unit/year = 0.30 × 10 = ₹ 3/unit/year

New Ordering cost, $C_{2,N}$ = ₹ 80/order

TO FIND:

Economic order quantity

Savings in cost when ordering cost is ₹ 80

SOLUTION:

Case I: Ordering cost is ₹ 100/order

Economic order quantity

$$(Q^*)_I = \sqrt{\frac{2DC_2}{C_3}}$$

$$= \sqrt{\frac{2 \times 3000 \times 100}{3}}$$

$$= 447.2136 \text{ units}$$

$$\cong 447 \text{ units}$$

Total inventory cost

$$(C^*)_I = \sqrt{2DC_2C_3}$$

$$= \sqrt{2 \times 3000 \times 100 \times 3}$$

$$= ₹\ 1341.64$$

Case II: Ordering cost is ₹ 80/order

Economic order quantity

$$(Q^*)_{II} = \sqrt{\frac{2DC_{2,N}}{C_3}}$$

$$= \sqrt{\frac{2 \times 3000 \times 80}{3}}$$

$$= 400 \text{ units}$$

Total inventory cost

$$(C^*)_{II} = \sqrt{2DC_{2,N}C_3}$$

$= \sqrt{2 \times 3000 \times 80 \times 3}$

$= ₹\ 1200$

Savings in cost = $(C^*)_I - (C^*)_{II}$

= 1341.64 – 1200

= ₹ 141.64

4 The following table gives the annual demand and unit price of four items.

Item	A	B	C	D
Annual demand (units)	800	400	392	13800
Unit price (₹)	0.02	1.00	8.00	0.20

Order cost is ₹ 5 per order and holding cost is 10% of the unit price.

(i) Determine the EOQ in units
(ii) Calculate total variable cost
(iii) Compute EOQ in ₹
(iv) Compute EOQ in years of supply
(v) Determine the number of orders per year.

GIVEN:

Item	A	B	C	D
Annual demand, D	800	400	392	13800
Item cost, $\boldsymbol{C_1}$	₹ 0.02	₹ 1.00	₹ 8.00	₹ 0.20
Ordering cost, $\boldsymbol{C_2}$	₹ 5	₹ 5	₹ 5	₹ 5
Holding cost, $\boldsymbol{C_3 = 0.10\ C_1}$	₹ 0.002	₹ 0.10	₹ 0.80	₹ 0.02

TO FIND:

(i) EOQ in units
(ii) Total variable cost
(iii) EOQ in ₹
(iv) EOQ in years of supply
(v) Number of orders per year.

SOLUTION:

Item	Formula	A	B	C	D
Annual demand, D	-	800	400	392	13800
Item cost, C_1	-	₹ 0.02	₹ 1.00	₹ 8.00	₹ 0.20
Ordering cost, C_2	-	₹ 5	₹ 5	₹ 5	₹ 5
Holding cost, C_3	-	₹ 0.002	₹ 0.10	₹ 0.80	₹ 0.02
EOQ, Q^*	$\sqrt{\frac{2DC_2}{C_3}}$	2000	200	70	2627
Total variable cost, C^*	$\sqrt{2DC_2C_3}$	₹ 4	₹ 20	₹ 56	₹ 52.54
EOQ in ₹	$Q^* \times C_1$	₹ 40	₹ 200	₹ 560	₹ 525.40
EOQ in years of supply	$\frac{Q^*}{D}$	2.5 years	0.5 year	0.1786 year	0.1904 year
Number of orders per year, N^*	$\frac{D}{Q^*}$	0.4	2	5.6	5.2531

5 The demand rate for an item in a company is 18,000 units per year. The company can produce at the rate of 3,000 units per month. The set-up cost is ₹ 500 per order and the holding cost is ₹ 0.15 per unit per month. Calculate,

(i) Optimum manufacturing quantity
(ii) The maximum inventory
(iii) Time between orders
(iv) The number of orders per year
(v) The time of manufacture
(vi) The optimum annual cost, if the cost of an item is ₹ 2 per unit.

GIVEN:

Demand rate, D = 18,000 units/year = 18000/12 = 1,500 units/month

Production rate, R = 3,000 units/month

Item cost, C_1 = ₹ 2/unit

Set-up cost, C_2 = ₹ 500/order

Carrying cost, C_3 = ₹ 0.15/unit/month

TO FIND:

(i) Optimum manufacturing quantity
(ii) The maximum inventory
(iii) Time between orders
(iv) The number of orders per year
(v) The time of manufacture
(vi) The optimum annual cost

SOLUTION:

Optimum manufacturing quantity

$$Q^* = \sqrt{\frac{2DC_2}{C_3}\frac{R}{(R-D)}}$$

$$= \sqrt{\frac{2 \times 1500 \times 500}{0.15}\frac{3000}{(3000-1500)}}$$

$$= 4472 \text{ units}$$

The maximum inventory

$$= \frac{Q^*}{R}(R-D)$$

$$= \frac{4472}{3000}(3000-1500)$$

$$= 2236 \text{ units}$$

The number of orders per year

$$N^* = \frac{D}{Q^*}$$

$$= \frac{1500}{4472}$$

$$= 0.3354/\text{month}$$

$$= 0.3354 \times 12$$

$= 4.0250/\text{year}$

Time between two consecutive orders

$$T^* = \frac{1}{N^*}$$

$$= \frac{1}{0.3354} \text{ month}$$

$$= 2.9813 \text{ months}$$

The time of manufacture

$$= \frac{Q^*}{R}$$

$$= \frac{4472}{3000}$$

$$= 1.4907 \text{ months}$$

The optimum annual cost

$$= C_1 D + \sqrt{2DC_2C_3\left(\frac{R-D}{R}\right)}$$

$$= 2 \times 1500 + \sqrt{2 \times 1500 \times 300 \times 0.15 \times \frac{(3000-1500)}{3000}}$$

$$= ₹\ 3{,}259.81/\text{month}$$

$$= 3{,}259.81 \times 12$$

$$= ₹\ 39{,}117.69/\text{year}$$

6 The demand for an item is deterministic and constant over time and is equal to 600 units per year. The unit cost of the item is ₹ 50, while the cost of placing an order is ₹ 5. The inventory carrying cost is 20% of the cost of inventory per annum and the cost of shortage is ₹ 1 per unit per month. Find the optimal ordering quantity when stock-outs are permitted. If stock-outs are not permitted, what would be the loss to the company?

GIVEN:

Demand rate, D = 600 units/year

Item cost, C_1 = ₹ 50/unit

Ordering cost, C_2 = ₹ 5/order

Carrying cost, C_3 = 20% of cost of inventory = 0.2 × 50 = ₹ 10/unit/year

Shortage cost, C_4 = ₹ 1/unit/month = ₹ 12/unit/year

TO FIND:

Optimal ordering quantity when stock-outs are permitted

Loss to the company if stock-outs are not permitted

SOLUTION:

Case I: Stock-outs are permitted

Optimal ordering quantity

$$Q^* = \sqrt{\frac{2DC_2}{C_3}\frac{(C_3 + C_4)}{C_4}}$$

$$= \sqrt{\frac{2 \times 600 \times 5}{10}\frac{(10 + 12)}{12}}$$

$$= 33 \text{ units}$$

Total annual inventory cost

$$C^* = \sqrt{2DC_2C_3\frac{C_4}{(C_3 + C_4)}}$$

$$= \sqrt{2 \times 600 \times 5 \times 10 \times \frac{12}{(10 + 12)}}$$

$$= ₹\, 180.91$$

Case II: Stock-outs are not permitted

Optimal ordering quantity

$$Q^* = \sqrt{\frac{2DC_2}{C_3}}$$

$$= \sqrt{\frac{2 \times 600 \times 5}{10}}$$

$$= 24.50 \text{ units}$$

Total annual inventory cost

$C^* = \sqrt{2DC_2C_3}$

$= \sqrt{2 \times 600 \times 5 \times 10}$

$= ₹\ 244.95$

Loss to the company if stock-outs are not permitted = 244.95 – 180.91 = ₹ 64.04

7 The demand for an item in a company is 18,000 units per year. The company can produce the items at a rate of 3,000 per month. The cost of one set-up is ₹ 500 and the holding cost of one unit per month is ₹ 0.15. The shortage cost of one unit is ₹ 20 per year. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and the time between set-ups.

GIVEN:

Demand rate, D = 18,000 units/year = 18000/12 = 1,500 units/month

Production rate, R = 3,000 units/month

Item cost, C_1 = ₹ 2/unit

Set-up cost, C_2 = ₹ 500/order

Carrying cost, C_3 = ₹ 0.15/unit/month

Shortage cost, C_4 = ₹ 20/unit/year = ₹ 1.67/unit/month

TO FIND:

Optimum manufacturing quantity

Number of shortages

The manufacturing time

Time between set-ups

SOLUTION:

Optimum manufacturing quantity

$$Q^* = \sqrt{\frac{2DC_2}{C_3}\frac{R}{(R-D)}\frac{(C_3+C_4)}{C_4}}$$

$$= \sqrt{\frac{2\times 1500\times 500}{0.15}\frac{3000}{(3000-1500)}\frac{(0.15+1.67)}{1.67}}$$

$$= 4669.0470 \text{ units}$$

$$\cong 4669 \text{ units}$$

Number of shortages

$$S^* = \sqrt{\frac{2DC_2}{C_4}\frac{(R-D)}{R}\frac{C_3}{(C_3+C_4)}}$$

$$= \sqrt{\frac{2\times 1500\times 500}{1.67}\frac{(3000-1500)}{3000}\frac{0.15}{(0.15+1.67)}}$$

$$= 192.7588 \text{ units}$$

$$\cong 193 \text{ units}$$

The manufacturing time

$$= \frac{Q^*}{R}$$

$$= \frac{4669}{3000}$$

$$= 1.5563 \text{ months}$$

Time between set-ups

$= \frac{Q^*}{D}$

$= \frac{4669}{1500}$

$= 3.1127$ months

8 The demand for an item is 12,000 per year and shortage is allowed. If the unit cost is ₹ 15 and the holding cost is ₹ 20 per year per unit, determine the optimum total yearly cost. The cost of placing one order is ₹ 6,000 and the cost of one shortage is ₹ 100 per year.

GIVEN:

Demand rate, D = 12,000 units/year

Item cost, C_1 = ₹ 15/unit

Ordering cost, C_2 = ₹ 6,000/order

Carrying cost, C_3 = ₹ 20/unit/year

Shortage cost, C_4 = ₹ 100/unit/year

TO FIND:

Optimum total yearly cost

SOLUTION:

Optimum total yearly cost

$$= C_1 D + \sqrt{2DC_2C_3 \frac{C_4}{(C_3 + C_4)}}$$

$$= 15 \times 12000 + \sqrt{2 \times 12000 \times 6000 \times 20 \times \frac{100}{(20 + 100)}}$$

$$= ₹\ 2{,}28{,}989.79$$

9 The annual consumption of an item is 2,000 units. The ordering cost is ₹ 100 per order. The carrying cost is ₹ 0.80 per unit per year. Assuming working days as 200, lead time as 20 days, and safety stock as 100 units, calculate

(i) Economic order quantity
(ii) The number of orders per year
(iii) Re-order level
(iv) The total annual ordering and carrying costs.

GIVEN:

Demand rate, D	= 2,000 units/year
Ordering cost, C_2	= ₹ 100/order
Carrying cost, C_3	= ₹ 0.80/unit/year
Working days	= 200
Lead time	= 20
Safety stock	= 100 units

TO FIND:

(i) Economic order quantity
(ii) The number of orders per year
(iii) Re-order level
(iv) The total annual ordering and carrying costs.

SOLUTION:

Economic order quantity

$$Q^* = \sqrt{\frac{2DC_2}{C_3}}$$

$$= \sqrt{\frac{2 \times 2000 \times 100}{0.80}}$$

$$= 707.1068 \text{ units}$$

$$\cong 707 \text{ units}$$

The number of orders per year

$$N^* = \frac{D}{Q^*}$$

$$= \frac{2000}{707}$$

$$= 2.8289$$

$$\text{Re} - \text{order level} = \text{Lead time demand} + \text{Safety stock}$$

$$= \text{Lead time} \times \frac{\text{Demand rate}}{\text{Working days}} + \text{Safety stock}$$

$$= 20 \times \frac{2000}{200} + 100$$

$$= 300 \text{ units}$$

The total annual ordering and carrying costs

$C^* = \sqrt{2DC_2C_3}$

$= \sqrt{2 \times 2000 \times 100 \times 0.80}$

$= ₹\ 565.69$

10 A newspaper boy buys papers for ₹ 3.50 each and sells them for ₹ 5. He cannot return unsold newspapers. Daily demand has the following distribution.

No. of customers	123	124	125	126	127	128	129	130	131	132
Probability	0.01	0.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	0.05

If each day's demand is independent of the previous days, how many papers should he order each day?

SOLUTION:

Let *Q* be the number of newspapers ordered per day and *D* be the demand for it i.e., the number of newspapers actually sold per day.

Unit cost of over ordering, C_5 = Unit purchase price – Carrying cost per unit of item left unsold – Salvage value for excess units

Unit cost of over ordering, C_5 = 3.50 – 0 – 0 = ₹ 3.50

Unit cost of under ordering, C_6 = Unit selling price – Unit purchase price – Carrying cost per unit of item left unsold/2 + Shortage cost for under stocking an item

Unit cost of under ordering, C_6 = 5 – 3.50 – 0 + 0 = ₹ 1.50

The cumulative probability distribution of daily demand is as follows:

No. of customers	123	124	125	126	127	128	129	130	131	132
Probability	0.01	0.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	0.05
Cumulative Probability	0.01	0.04	0.10	0.20	0.40	0.65	0.80	0.90	0.95	1.00

$$\frac{C_6}{C_5 + C_6} = \frac{1.50}{3.50 + 1.50} = 0.3$$

Here, 0.3 lies between 0.20 and 0.40, corresponding to 126 and 127 customers.

Number of papers to be ordered each day = 127

11 An ice-cream company sells one of its ice-creams by weight. If the product is not sold on the day it is prepared, it can be sold for a loss of ₹ 0.50 per kg. But there is an unlimited market for one day old ice-creams. On the other hand, the company makes a profit of ₹ 3.20 on every kg of ice cream sold on the day it is prepared. If daily orders form a distribution with $(x) = 0.02 - 0.0002x\ 0 \leq x \leq 100$, how many kg of ice-creams should the company prepare every day?

SOLUTION:

C_5 = 0.50

C_6 = 3.20

$$\frac{C_6}{C_5 + C_6} = \frac{3.20}{0.50 + 3.20} = 0.8649$$

Let Q be the amount of ice-cream prepared every day.

$$\int_0^Q f(x)dx = \frac{C_6}{C_5 + C_6}$$

$$\int_0^Q (0.02 - 0.0002x)dx = 0.8649$$

$$\left(0.02x - \frac{0.0002x^2}{2}\right)\begin{matrix} Q \\ 0 \end{matrix} = 0.8649$$

$$0.02Q - 0.0001Q^2 - 0.8649 = 0$$

$$0.0001Q^2 - 0.02Q + 0.8649 = 0$$

$$Q = \frac{-(-0.02) \pm \sqrt{(-0.02)^2 - (4 \times 0.0001 \times 0.8649)}}{2 \times 0.0001}$$

$$= \frac{0.02 \pm 0.0073}{0.0002}$$

$$= 136.7607 \text{ and } 63.2393$$

$$= \ 63.2393 \text{ kg } (0 \leq x \leq 100)$$

The company should prepare 63.2393 kg of ice-creams.

9 TWO MARKS QUESTIONS AND ANSWERS

1. What is meant by inventory?

Inventory is defined as idle resources of an enterprise. It is the physical stock of goods kept for future use.

2. What are the main objectives of an inventory model?

- It provides adequate service to the customers.
- It reduces the possibility of duplication of orders.
- It helps in minimizing the loss due to deterioration, obsolescence, damage, etc.
- It optimizes the cost associated with inventory.
- It helps in deciding whether to avail price discount of bulk purchases.

3. What are the different types of inventories?

- Transportation inventories,
- Buffer inventories,
- Anticipation inventories,
- Decoupling inventories, and
- Lot-size inventories.

4. What are the different costs that are involved in the inventory problem?

- Item (purchase or production) cost,
- Ordering or set-up cost,
- Carrying or holding cost, and
- Shortage or stock out cost.

5. Define holding cost and set-up cost.

The cost associated with carrying or holding the goods in stock is known as holding cost or carrying cost. Examples are invested capital cost, record keeping cost, taxes and insurance, etc.

The cost which includes the fixed cost associated with obtaining goods through placing of an order or purchasing or manufacturing or setting up machinery before starting the production is called set-up cost.

6. Define shortage cost.

The penalty costs that are incurred as a result of running out of stock (i.e. shortage) are known as shortage costs.

7. What are the two types of variables in the inventory?

- Controlled variable, and
- Uncontrolled variables.

8. Define lead time.

The time gap between placing of an order and its actual arrival in the inventory is known as lead time.

9. Define order cycle.

The time period between placements of two successive orders is referred to as an order cycle.

10. Define time horizon.

The time period over which the inventory level will be controlled is known as time horizon.

11. Define re-order level.

The level between maximum and minimum stock at which purchasing activities must start for replenishment is known as re-order level.

Re-order level = Lead time × Demand rate

12. Define buffer stock or safety stock.

Buffer stock means the extra inventory maintained in addition to the inventory required corresponding to normal consumption levels.

Optimum buffer stock = (Max. lead time – Min. lead time) × Demand rate

13. What is total inventory cost?

Total inventory cost = Purchase cost of inventory items + Ordering cost + Carrying cost + Shortage cost

14. What is economic order quantity?

It is that size of order which minimizes total annual cost of carrying inventory and cost of ordering under the assumed conditions of certainty and the annual demands are constant.

15. What are the different classifications of inventory model?

- Deterministic inventory model, and
- Probabilistic inventory model

16. Write the EOQ formula under purchasing model without shortages.

$$Q^* = \sqrt{\frac{2DC_2}{C_3}}$$

D – Demand rate

C_2 – Ordering cost per order

C_3 – Holding cost per unit per unit time

17. Write the formula for finding the minimum inventory cost under the purchasing model without shortages.

$$C^* = \sqrt{2DC_2C_3}$$

D – Demand rate

C_2 – Ordering cost per order

C_3 – Holding cost per unit per unit time

18. Write the EOQ formula under deterministic demand with shortages where lead time is zero.

$$Q^* = \sqrt{\frac{2DC_2}{C_3}\frac{(C_3 + C_4)}{C_4}}$$

D – Demand rate

C_2 – Ordering cost per order

C_3 – Holding cost per unit per unit time

C_4 – Shortage cost per unit per unit time

19. State the formula for EOQ under manufacturing model where (i) shortages are allowed, and (ii) shortages are not allowed.

(i) Shortages are allowed

$$Q^* = \sqrt{\frac{2DC_2}{C_3}\frac{R}{(R-D)}\frac{(C_3 + C_4)}{C_4}}$$

(i) Shortages are not allowed

$$Q^* = \sqrt{\frac{2DC_2}{C_3}\frac{R}{(R-D)}}$$

D – Demand rate

R – Production rate

C_2 – Ordering cost per order

C_3 – Holding cost per unit per unit time

C_4 – Shortage cost per unit per unit time

20. Distinguish between deterministic model and probabilistic model.

Deterministic model	Probabilistic model
Demand is either static or dynamic.	Demand is stationary or non-stationary.
Lead time is constant.	Lead time is not constant.
Lead time demand is known and fixed.	Lead time demand is assumed to follow normal distribution.

21. Briefly explain probabilistic inventory model.

Demand can be classified into stationary demand, i.e., single period model and non-stationary demand, i.e., multi-period with variable lead time model. Stationary demand can be further classified into:

- Model with instantaneous demand, no set-up cost,
- Model with continuous demand, no set-up cost, and
- Model with instantaneous demand and set-up cost.

22. What are the two main decisions to be made in inventory control?

- Size of the order, and
- The time of placing an order.

23. Write the formula for stochastic inventory model with demand as continuous variable.

$$\int_{0}^{Q} f(x)dx = \frac{C_6}{C_5 + C_6}$$

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