KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University)
(Established Under Section 3 of UGC Act 1956)
Pollachi Main Road, Eachanari (Po),
COIMBATORE – 21

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

SUBJECT NAME: GAS DYNAMICS AND JET PROPULSION	SUBJECT CODE : 17BEME704	L	T	P	C
		3	2	0	4

INTENDED OUTCOMES:

- To understand the principles involved in gas dynamics related with energy and momentum
- To learn the underlying theories of rocket and jet propulsion

UNIT I BASIC CONCEPTS AND ISENTROPIC FLOWS

Energy and momentum equations of compressible fluid flows – Stagnation states, Mach waves and Mach cone –Effect of Mach number on compressibility – Isentropic flow through variable area ducts – Nozzle and Diffusers –area ratio as a function of Mach number, mass flow rate through nozzles and diffusers, effect of friction in flow through nozzles. Use of Gas tables.

UNIT II FLOW THROUGH DUCTS

Flow through constant area ducts with heat transfer (Rayleigh flow) and Friction (Fanno flow) – Variation of flow properties – Isothermal flow with friction in constant area ducts –Use of tables and charts – Generalised gas dynamics.

UNIT III NORMAL AND OBLIQUE SHOCKS

Governing equations – Variation of flow parameters across the normal and oblique shocks – Prandtl – Meyer relations – Use of table and charts – Applications.

UNIT IV JET PROPULSION

Theory of jet propulsion – Thrust equation – Thrust power and propulsive efficiency – Operation principle, cycle analysis and use of stagnation state performance of ram jet, turbojet, turbofan and turbo prop engines – Aircraft combustors.

UNIT V ROCKET PROPULSION

Types of rocket engines – Propellants – Ignition and combustion – Theory of rocket propulsion – solid and liquid propellants, comparison of different propulsion systems .Performance study – Staging – Terminal and characteristic velocity – Applications – Space flights.

TEXT BOOK:

S.NO	AUTHOR(S) NAME	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1	Yahya.S.M	Fundamentals of Compressible flow	New Age International (P) Ltd., New Delhi.	2009
2.	Rathakrishnan.E	Gas Dynamics	Prentice Hall of India, New Delhi.	2010

REFERENCES:

S.NO	AUTHOR(S) NAME	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1	Patrich.H.Oosthvizen, Willam E.Carscallen	Compressible fluid flow	McGraw–Hill,	1997
2	Zucker,R.D. and Biblarz,O.	Fundamentals of Gas Dynamics	John Willey,	2002
3	Ganesan .V.	Gas Turbines	Tata McGraw–Hill, New Delhi.	2010
4	P.Hill and C. Peterson	Mechanics and Thermodynamics of Propulsion	Addison – Wesley Publishing Company	1992
5	Zucrow, M.J. and Anderson, J.D.	Elements of gas dynamics	McGraw–Hill Book Co., New York	1989
6	N.J. Zucrow	Principles of Jet Propulsion and Gas Turbines	John Wiley, New York	1970

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4. http://soliton.ae.gatech.edu/people/jseitzma/classes/ae3450/StudyProblems.pdf
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FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

LESSON PLAN

Subject Name : **GAS DYNAMICS AND JET PROPULSION** **Subject Code** : **14BEME704**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT I	**BASIC CONCEPTS AND ISENTROPIC FLOWS**		
1.	1	• Fundamental of thermodynamics	
2.	1	• Introduction of compressible flow and properties of the fluid	T [1] pg no: 3-35
3.	1	• Stagnation and static states of the fluid	T [1] pg no: 39-42, 44-46
4.	1	• Flow regions of compressible fluid flow, Crocco number and their relation	T [1] pg no: 21,42 43,47,107,108,
5.	1	• Problems in related to the stagnation properties	T [1] pg no: 90-93
6.	1	• Tutorial 1 (Problems in related to the stagnation properties)	
7.	1	• Reference velocities, types of waves, Mach angle and Mach cone, Effect of Mach number on Compressibility	T [1] pg no: 45-46, 49-50, 107-108
8.	1	• Introduction to nozzles and diffusers • Types of nozzles	T [1] pg no: 69-72
9.	1	• Relation between change in area to Mach number and area ratio	T [1] pg no: 76-78
10.	1	• mass flow rate through nozzles and diffusers, effect of friction in flow through nozzles	T [1] pg no: 78-88
11.	1	• Use of gas tables with applications • Problems in related to the nozzles and diffusers	T [1] pg no: 124-132
12.	1	• Tutorial 2 (Problems in related to the nozzles and diffusers)	
13.	1	• **Discussion of previous year questions**	**KU – QUESTION PAPERS**
Total no. of Hours planned for unit - I			**13**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT II FLOW THROUGH DUCTS			
14.	1	• Types of flows and Description of Fanno and Rayeligh's flows	T [1] pg no: 211-213, 224-232, 247-249
15.	1	• Fanno flow equation and curve	T [1] pg no: 211-216
16.	1	• Expressions for Fanno flow	T [1] pg no: 216-218
17.	1	• Variation of flow properties and mach number	T [1] pg no: 218-223
18.	1	• Problems in related to the fanno flows	T [1] pg no: 233-235
19.	1	• Problems in related to the fanno flows	T [1] pg no: 236-244
20.	1	• Tutorial 3 (Problems in related to the fanno flows)	
21.	1	• Introduction of Rayeligh's flows and curve	T [1] pg no: 247-255
22.	1	• Expressions in Rayeligh's flows	T [1] pg no: 256-258
23.	1	• Variations of flow properties in Rayeligh's flows and heat transfer	T [1] pg no: 256-258
24.	1	• Problems in related to the Rayeligh's flows	T [1] pg no: 261-262
25.	1	• Tutorial 4 (Problems in related to the Rayeligh's flows)	
26.	1	• **Discussion of previous year questions**	**KU – QUESTION PAPERS**
Total no. of Hours planned for unit - II			**13 hrs**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT III NORMAL AND OBLIQUE SHOCKS			
27.	1	• Introduction to shock waves and their types Conditions for shock	T [1] pg no: 134-139
28.	1	• Prandtl-Meyer equation	T [1] pg no: 139-141
29.	1	• Down stream mach number across the shock waves	T [1] pg no: 142-143
30.	1	• Static properties across the shock waves, Stagnation properties across the shock waves	T [1] pg no: 142-143 T [1] pg no: 144-147
31.	1	• Normal shock waves with the flows	T [1] pg no: 137-138
32.	1	• Tutorial 5 (Problems in related to the Normal shock waves)	

33.	1	• Oblique shock waves with the flows	T [1] pg no: 174-177
34.	1	• Problems in related to the shock waves	T [1] pg no: 161-163
35.	1	• Problems in related to the shock waves	T [1] pg no: 168-171, 174-177
36.	1	• Tutorial 6 (Problems in related to the Oblique shock waves)	
37.	1	• **Discussion of previous year questions**	**KU – QUESTION PAPERS**
Total no. of Hours planned for unit - III			**11 hrs**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT IV JET PROPULSION			
38	1	• Introduction to Jet propulsion and types of jet propulsion systems	T [1] pg no: 357-359
39	1	• Thrust equations, power and propulsive efficiency	T [1] pg no: 336-380
40	1	• Types of jet engine, operations of ram jet engine	T [1] pg no: 360, 380-383
41	1	• Operations of turbojet and turbo prop engines , turbofan	T [1] pg no: 360-373
42	1	• Operation of aircraft combustion system, Use of stagnation state performance of jet engines (ram jet	T [1] pg no: 380-385
43	1	• Tutorial 7 (stagnation state performance of jet engines)	
44	1	• Use of stagnation state performance of jet engines (turbojet and turbo prop engines)	T [1] pg no: 376-379
45	1	• Problems solving related to performance of the jet engine	T [1] pg no: 390-394
46	1	• Problems solving related to performance of the jet engine	
47	1	• Tutorial 8 (Problems solving related to performance of the jet engine)	
48	1	• **Discussion of previous year questions**	**KU – QUESTION PAPERS**
Total no. of Hours planned for unit – IV			**11 hrs**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT V ROCKET PROPULSION			
49	1	• Introduction to rocket propulsion and types of rocket engines	T [1] pg no: 396-404
50	1	• Energy equations, power and efficiencies	T [1] pg no: 399-400
51	1	• Propellants, types and their comparisons	
52	1	• Ignition and combustion study of the rocket engines	T [1] pg no: 421-429
53	1	• Performance evaluation of the rocket engines	T [1] pg no: 421-423
54	1	• Staging, Terminal and characteristic velocity of the engines	T [1] pg no: 424-431
55	1	• Applications and problems solving related to rocket engines	
56	1	• Problems solving related to rocket engines	T [1] pg no: 450-452
57	1	• Problems solving related to rocket engines	
58	1	• Tutorial 9 Problems solving related to rocket engines	
59	1	• **Discussion of previous year questions**	**KU – QUESTION PAPERS**
Total no. of Hours planned for unit - V			**11 hrs**

Text Books : [1] Yahya.S.M., 2005 "Fundamentals of Compressible flow" New Age International (P) Ltd., New Delhi.

[2] Rathakrishnan.E., 2001 "Gas Dynamics" Prentice Hall of India, New Delhi.

Reference Books : [3] Patrich.H.Oosthvizen, Willam E.Carscallen., 1997 "Compressible fluid flow" McGraw-Hill,

[4] Ganesan .V ., 2010 "Gas Turbines" Tata McGraw–Hill, New Delhi.

[5] N.J. Zucrow., 1970 "Principles of Jet Propulsion and Gas Turbines" John Wiley, New York

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2. http://www.grc.nasa.gov/WWW/k–12/airplane/isndrv.html
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4. http://soliton.ae.gatech.edu/people/jseitzma/classes/ae3450/StudyProblems.pdf
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TOTAL NUMBER OF COURSE HOURS : 59 Hrs

UNIT – I

BASIC CONCEPTS & ISENTROPIC FLOW

1.1 Introduction:

Solid, liquid and gas are three main states of the matter. Solids have definite shape and have highest cohesive or intermolecular forces. These attraction forces are lesser in magnitudes for liquids. Liquids take the shape of the container and exhibit free surface. Gases have least intermolecular attraction and hence occupy the container completely.

1.2 System:

A system is a finite quantity of matter or a prescribed region of space.

1.3 Boundary:

The actual or hypothetical envelope enclosing the system is the boundary of the system. The boundary may be fixed or it may move, as and when a system containing a gas is compressed or expanded. The boundary may be real or imaginary

1.4 Closed System:

If the boundary of the system is impervious to the flow of matter, it is called a closed system. An example of this system is mass of gas or vapour contained in an engine cylinder, the boundary of which is drawn by the cylinder walls, the cylinder head and piston crown

1.5 Open System

An open system is one in which matter flows into or out of the system. Most of the engineering systems are open.

1.6 Isolated System

An isolated system is that system which exchanges neither energy nor matter with any other system or with environment.

1.7 Adiabatic System

An adiabatic system is one which is thermally insulated from its surroundings. It can, however, exchange work with its surroundings. If it does not, it becomes an isolated system.

1.8 Phase

A phase is a quantity of matter which is homogeneous throughout in chemical composition and physical structure.

1.9 Homogeneous System

A system which consists of a single phase is termed as homogeneous system. Examples: Mixture of air and water vapour, water plus nitric acid and octane plus heptane.

1.10 Heterogeneous System

A system which consists of two or more phases is called a heterogeneous system. Examples: Water plus steam, ice plus water and water plus oil.

1.11 State:

State is the condition of the system at an instant of time as described or measured by its properties. Or each unique condition of a system is called a state.

1.12 Process:

A process occurs when the system undergoes a change in a state or an energy transfer at a steady state.

A process may be non-flow in which a fixed mass within the defined boundary is undergoing a change of state. Example: A substance which is being heated in a closed cylinder undergoes a non-flow process .Closed systems undergo non-flow processes.

A process may be a flow process in which mass is entering and leaving through the boundary of an open system. In a steady flow process mass is crossing the boundary from surroundings at entry, and an equal mass is crossing the boundary at the exit so that the total mass of the system remains constant.

1.13 Quasi-static process:

Quasi means ‘almost’. A quasi-static process is also called a reversible process. This process is a succession of equilibrium states and infinite slowness is its characteristic feature.

1.14 Cycle:
Any process or series of processes whose end states are identical is termed a cycle.

1.15 Properties of Systems:

A property of a system is a characteristic of the system which depends upon its state, but not upon how the state is reached. There are two sorts of property:
1. Intensive properties. These properties do not depend on the mass of the system. Examples: Temperature and pressure.
2. Extensive properties. These properties depend on the mass of the system. Example: Volume.

Extensive properties are often divided by mass associated with them to obtain the intensive properties. For example, if the volume of a system of mass m is V, then the specific volume of matter within the system is V/m = v which is an intensive property

1.16 Pure Substance:
A pure substance is one that has a homogeneous and invariable chemical composition even though there is a change of phase. In other words, it is a system which is (a) homogeneous in composition, (b) homogeneous in chemical

aggregation. Examples: Liquid, water, mixture of liquid water and steam, mixture of ice and water. The mixture of liquid air and gaseous air is not a pure substance.

1.17 Temperature:

The temperature is a thermal state of a body which distinguishes a hot body from a cold body. The temperature of a body is proportional to the stored molecular energy i.e., the average molecular kinetic energy of the molecules in a system.

1.18 Pressure:

Pressure is defined as a force per unit area. Pressures are exerted by gases, vapours and liquids.

Pressure measuring devices indicate the pressure either above or below that of the atmosphere. When it is above the atmospheric pressure, it is termed gauge pressure and is positive.

When it is below atmospheric, it is negative and is known as vacuum. Vacuum readings are given in millimetres of mercury or millimetres of water below the atmosphere.

A pressure of absolute zero can exist only in complete vacuum. Any pressure measured above the absolute zero of pressure is termed an 'absolute pressure'.

1.19 Thermodynamic Equilibrium:

A system is in thermodynamic equilibrium if the temperature and pressure at all points are same; there should be no velocity gradient; the chemical equilibrium is also necessary.

Systems under temperature and pressure equilibrium but not under chemical equilibrium are sometimes said to be in metastable equilibrium conditions. It is only under thermodynamic equilibrium conditions that the properties of a system can be fixed.

Thus for attaining a state of thermodynamic equilibrium the following three types of equilibrium states must be achieved:

1. Thermal equilibrium. The temperature of the system does not change with time and has same value at all points of the system.
2. Mechanical equilibrium. There are no unbalanced forces within the system or between the surroundings. The pressure in the system is same at all points and does not change with respect to time.
3. Chemical equilibrium. No chemical reaction takes place in the system and the chemical composition which is same throughout the system does not vary with time.

1.20 Energy:

Energy is a general term embracing energy in transition and stored energy. The stored energy of a substance may be in the forms of mechanical energy and internal energy (other forms of stored energy may be chemical energy and electrical energy).

Part of the stored energy may take the form of either potential energy (which is the gravitational energy due to height above a chosen datum line) or kinetic energy due to velocity. The balance part of the energy is known as internal energy

Heat and work are the forms of energy in transition. These are the only forms in which energy can cross the boundaries of a system. Neither heat nor work can exist as stored energy.

1.21 Work:

Work is said to be done when a force moves through a distance. If a part of the boundary of a system undergoes a displacement under the action of a pressure, the work done W is the product of the force (pressure × area), and the distance it moves in the direction of the force

1.21.1 Sign convention:

- If the work is done by the system on the surroundings, e.g., when a fluid expands pushing a piston outwards, the work is said to be positive. i.e., Work output of the system = + W
- If the work is done on the system by the surroundings, e.g., when a force is applied to a rotating handle, or to a piston to compress a fluid, the work is said to be negative. i.e., Work input to system = – W

1.22 Heat:

Heat (denoted by the symbol Q), may be, defined in an analogous way to work as follows:
"Heat is 'something' which appears at the boundary when a system changes its state due to a difference in temperature between the system and its surroundings".
Heat, like work, is a transient quantity which only appears at the boundary while a change is taking place within the system

1.22.1 Sign convention:

- If the heat flows into a system from the surroundings, the quantity is said to be positive and, conversely, if heat flows from the system to the surroundings it is said to be negative. In other words:
- Heat received by the system = + Q,
- Heat rejected or given up by the system = – Q.

1.23 Internal Energy:

It is the heat energy stored in a gas. If a certain amount of heat is supplied to a gas the result is that temperature of gas may increase or volume of gas may increase thereby doing some external work or both temperature and volume may increase; but it will be decided by the conditions under which the gas is supplied heat. If during heating of the gas the temperature increases its internal energy will also increase.

1.24 Specific Heats:

The specific heat of a solid or liquid is usually defined as the heat required raising unit mass through one degree temperature rise.
For small quantities, we have

$$dQ = m\,c\,dT$$

Where
m = mass, c = specific heat, and dT = temperature rise.

1.25 Enthalpy:

One of the fundamental quantities which occur invariably in thermodynamics is the sum of internal energy (u) and pressure volume product (pv). This sum is called Enthalpy (h).

$$\text{i.e., } h = u + pv$$

The enthalpy of a fluid is the property of the fluid, since it consists of the sum of a property and the product of the two properties. Since enthalpy is a property like internal energy, pressure, specific volume and temperature, it can be introduced into any problem whether the process is a flow or a non-flow process.

The total enthalpy of mass, m, of a fluid can be

$$H = U + PV,$$

Where, H = m h.

1.26 Ratio of Specific Heats:

The ratio of specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma).

$$\text{i.e., } \gamma = c_p/c_v$$

Since

$$c_p = c_v + R$$

It is clear that c_p must be greater than c_v for any perfect gas. It follows, therefore, that the ratio, $c_p/c_v = \gamma$ is always greater than unity.

1.27 The Characteristic Equation of State:

At temperatures that are considerably in excess of critical temperature of a fluid, and also at very low pressure, the vapour of fluid tends to obey the equation

$$Pv / T = \text{constant} = R$$

In practice, no gas obeys this law rigidly, but many gases tend towards it.

An imaginary ideal gas which obeys this law is called a perfect gas and the equation $pv/T = R$, is called the characteristic equation of a state of a perfect gas.

The constant R is called the gas constant. Each perfect gas has a different gas constant. Units of R are Nm/kg K or kJ/kg K.

Usually, the characteristic equation is written as $pv = RT$ or for m kg, occupying V m^3

$$pV = mRT$$

The characteristic equation in another form can be derived by using kilogram-mole as a unit.

1.28 Ideal Gas:

From experimental observations it has been established that an ideal gas (to a good approximation) behaves according to the simple equation

$$pV = mRT$$

where p, V and T are the pressure, volume and temperature of gas having mass m and R is a constant for the gas known as its gas constant.

$$pv = RT$$

(Where v = V/m)

In reality there is no gas which can be qualified as an ideal or perfect gas. However all gases tend to ideal or perfect gas behavior at all temperatures as their pressure approaches zero pressure.

1.29 Real Gases:

It has been observed that when experiments are performed at relatively low pressures and temperatures most of the real gases obey Boyle's and Charle's laws quite closely. But the actual behavior of real gases at elevated pressures and at low temperatures deviates considerably.

The ideal gas equation pv = RT can be derived analytically using the kinetic theory of gases by making the following assumptions:

(i) A finite volume of gas contains large number of molecules.

(ii) The collision of molecules with one another and with the walls of the container is perfectly elastic.

(iii) The molecules are separated by large distances compared to their own dimensions.

(iv) The molecules do not exert forces on one another except when they collide. As long as the above assumptions are valid the behaviour of a real gas approaches closely that of an ideal gas.

1.30 Laws of thermodynamics

1.30.1 Zeroth law of thermodynamics:

This law states that 'when system A is in thermal equilibrium with system B and system B is separately in thermal equilibrium with system C then system A and C are also in thermal equilibrium'.

This law portrays temperature as a property of the system and gives basis of temperature measurement.

1.30.2 First law of thermodynamics:

It states the energy conservation principle, 'energy can neither be created nor be destroyed but one form of the energy can be converted to other.

1.30.3 Second law of thermodynamics:

There are two statements of second law of thermodynamics.

Clausius statement:

It is impossible to construct a system which will operate in a cycle, transfers heat from the low temperature reservoir (or object) to the high temperature reservoir (or object) without any external effect or work interaction with surrounding.

Kelvin Plank statement:

It is impossible to construct a heat engine which produces work in a cycle while interacting with only one reservoir.

Kelvin Plank statement necessarily states that Perpetual Motion Machine of second kind is impossible. These statements introduce a new property termed as entropy which is the measure of disorder.

There are some corollaries of second law of thermodynamics

All the reversible heat engines working between same temperature limits have same efficiency.

Irreversible heat engine working in the same temperature limit as the reversible heat engine will have lower efficiency.

1.31 Continuum:

Fluid matter is made up of molecules or atoms. Continuous presence of the matter is called as continuum. This is the assumption which we will be using for most of the derivations of this course. This assumption helps us for calculation of gradient and flow variables smoothly.

1.32 Transport phenomenon:

Diffusion of mass, momentum and heat always take place from the region of higher concentration to the region of lower concentration.
Concentration gradient of mass leads to mass transfer. Fick's law gives the quantification of mass transport through the linear relation between mass flux and gradient of concentration.

Temperature variation leads to heat transfer. Fourier's law gives the relation between heat flux and gradient for temperature. The proportionality constant is the material property (thermal conductivity) and depends mainly on temperature of the material.

Newton's law of momentum diffusion states that momentum flux is proportional to the gradient of velocity. The proportionality constant of this relation is the fluid property (viscosity) which depends mainly on temperature of the fluid. Presence of boundary layer near the wall, in case of fluid flow over the same, is the example of momentum diffusion.

1.33 Compressibility of fluid and flow:

If application of pressure changes volume or density of the fluid then fluid is said to be compressible.

$$\text{Compressibility } (\tau) = -\frac{1}{\vartheta}\frac{\partial \vartheta}{\partial P} = \frac{1}{\rho}\frac{\partial \rho}{\partial P}$$

Compressibility is thus inverse of bulk modulus. Hence compressibility can be defined as the incurred volumetric strain for unit change in pressure. Negative sign in the above expression is the fact that volume decreases with increase in applied pressure. For example, air is more compressible than water. Since definition of compressibility involves change in volume due to change in pressure, hence compressibility can be isothermal, where volume change takes place at constant temperature or isentropic where volume change takes place at constant entropy.

$$\text{Isothermal compressibility } (\tau) = -\frac{1}{\vartheta}\left(\frac{\partial \vartheta}{\partial P}\right)_{T=\text{constant}} = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_{T=\text{constant}}$$

$$\text{Isentropic compressibility } (\tau) = -\frac{1}{\vartheta}\left(\frac{\partial \vartheta}{\partial P}\right)_{s=\text{constant}} = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_{s=\text{constant}}$$

Also, for isothermal compressibility we know,

$$\tau = -\frac{1}{\vartheta}\left(\frac{\partial \vartheta}{\partial P}\right)_{T=\text{constant}}$$

Since $Pv = RT$ for ideal gas, we have,

$$\left(\frac{\partial \vartheta}{\partial P}\right)_{T=\text{constant}} = -\frac{\vartheta}{P}$$

Hence, isothermal compressibility is

$$\tau = \frac{1}{P}$$

For isentropic compressibility we know,

$$\tau = -\frac{1}{\vartheta}\left(\frac{\partial \vartheta}{\partial P}\right)_{s=\text{constant}}$$

Since, Pv^{γ} = constant for isentropic process for an ideal gas, we have,

$$\left(\frac{\partial \vartheta}{\partial P}\right)_{s=\text{constant}} = -\frac{\vartheta}{\gamma P}$$

Hence, isentropic compressibility is

$$\tau = \frac{1}{\gamma P}$$

Comparing the above equations we can see that, isothermal compressibility is always higher than isentropic compressibility of gas since specific heat ratio is always greater than one.

1.34 Energy Equation from First law of Thermodynamics:

Consider the flow of fluid in and out of a control volume (open system) including the heat transferred and the work transfer. For such a system, the energy equation or energy balance for equilibrium condition is given by first law of thermodynamics as

$$Q = W + \Delta E$$

Where,

Q - Heat transferred in or out of the system, W

W - Work transfer, W

ΔE - Change in energy of the system, W

The energy of the system may exist in different forms like gravitational potential energy, kinetic energy, internal energy, strain energy, nuclear energy, chemical energy, magnetic energy etc.

1.35 Energy Equation for a non flow and flow process

For processes like expansion and compression process, when compared to change in internal energy, the change in K.E. and P.E. are negligible, hence the energy equation can be written as

$$Q = W + \Delta U$$

The work done is equal to pressure volume work done, (p dV work) and the change in internal energy is given as $mc_v dT$.

Hence for a process 1-2

$$Q = \int_1^2 p\, dV + m\, c_v\, (T_2 - T_1)$$

The work done changes with respect to the type of process the system undergoes

For a case of control volume, the change in K.E. and P.E. cannot be neglected; the energy equation for a steady flow process is called as SFEE (Steady flow energy equation)

$$H_1 + m\,g\,Z_1 + \frac{1}{2}\,{C_1}^2 + Q_{1-2} = H_2 + m\,g\,Z_2 + \frac{1}{2}\,{C_2}^2 + W_{1-2}$$

Where,

H	- Enthalpy (H = U + pV)
Z	- Elevation from datum, m
C	- Velocity, m/s

1.36 Adiabatic Energy Equation:

Adiabatic energy equation is given as

$$h_1 + g\,Z_1 + \frac{1}{2}\,{C_1}^2 = h_2 + g\,Z_2 + \frac{1}{2}\,{C_2}^2 + w$$

$$h_1 + \frac{1}{2}\,{C_1}^2 = h_2 + \frac{1}{2}\,{C_2}^2 + w$$

1.36.1 Adiabatic Energy Transformation:

Some adiabatic processes involve only energy transformation example, expansion of gases in nozzles and their compression in diffusers. In these processes the shaft work is absent and the adiabatic energy equation is modified as

$$h_1 + g\,Z_1 + \frac{1}{2}\,{C_1}^2 = h_2 + g\,Z_2 + \frac{1}{2}\,{C_2}^2$$

$$h_1 + \frac{1}{2}\,{C_1}^2 = h_2 + \frac{1}{2}\,{C_2}^2$$

1.36.2 Stagnation Enthalpy, h_o

Stagnation enthalpy of a gas or vapour is defined as the enthalpy of the same when the gas or vapour is decelerated to zero velocity at zero elevation.

$$h_o = h + \frac{1}{2}\, c^2$$

1.36.3 Stagnation Temperature, T_o

Stagnation temperature is the temperature of the gas when it is adiabatically decelerated to zero velocity at zero elevation.

We know that, h = c_pT, therefore

$$h_o = h + \frac{1}{2}\, c^2 \quad \leftrightarrow \quad c_p\, T_o = c_p\, T + \frac{1}{2}\, c^2$$

$$T_o = T + \frac{c^2}{2\, c_p}$$

The quantity $c^2/2c_p$ is known as velocity temperature

$$T_o = T + T_c$$

$$c_p = \frac{\gamma R}{\gamma - 1}, velcoity\ of\ sound, a = \sqrt{\gamma R T}\ \ and\ Mach\ number, M = \frac{c}{a}$$

On substituting the above values in the stagnation temperature equation and on simplification, we get

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2}\, M^2$$

1.36.4 Stagnation Velocity of sound, a_o

It is the velocity of sound at given value of stagnation temperature, T_o

$$a_o = \sqrt{\gamma R T_o} = \sqrt{(\gamma - 1)\, h_o}\,, \qquad since\ c_p = \frac{\gamma R}{\gamma - 1} = \frac{h_o}{T_o}$$

1.36.5 Stagnation pressure, p_o

It is defined as the pressure of the fluid when it is decelerated to zero velocity at zero elevation.

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2}\, M^2\right)^{\frac{\gamma}{\gamma-1}}$$

1.36.6 Stagnation Density, ρ_o

The stagnation density for given value of stagnation pressure and temperature for an ideal gas is given as

$$\rho_o = \frac{p_o}{R\,T_o}$$

For isentropic relations,

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2}\,M^2\right)^{\frac{1}{\gamma-1}}$$

1.36.7 Stagnation State

The stagnation state is referred to the state of the system when it is isentropically decelerated to zero velocity at zero elevation and it is fully defined by its stagnation enthalpy, temperature and pressure.

1.37 Regions of flow:

Based on the adiabatic energy equation, the graph plotted between velocity of sound, a and velocity of flow, c gives a elliptical curve, which distinguishes the flow into five different types

1.37.1 Subsonic Flow:

When the fluid velocity is lower than the acoustic speed (M<1) then the fluid flow is called as subsonic. However Mach number of the flow changes while passing over an object or through a duct. Hence for simplicity, flow is considered as subsonic if Mach number is in the range of 0-0.8. All small amplitude disturbances travel with acoustic speed and speed of the flow in the subsonic regime is less than acoustic speed hence presence of the disturbance is felt by the whole fluid domain. Therefore subsonic flow is pre-warned or prepared to face the disturbance.

1.37.2 Transonic flow:

When the flow Mach number is in the range 0.8-1.2 it is called transonic flow. Highly unstable and mixed subsonic and supersonic flows are the main features of this regime.

1.37.3 Sonic flow:

When flow Mach number is 1 it is called sonic flow.

1.37.4 Supersonic Flow:

When the flow Mach number is more then everywhere in the domain then it is called as supersonic flow. This flow is not pre-warned since the fluid speed is more than the speed of sound.

1.37.5 Hypersonic Flow:

As per the thumb rule, when the flow Mach number is more than 5 then it is called as hypersonic flows. This is not the fixed definition for hypersonic flow since hypersonic flow is defined by certain characteristics of flow.

Adiabatic energy equation is given as

$$h_o = h + \frac{1}{2}\, c^2 = Constant$$

$$h_o = \frac{a^2}{\gamma - 1} + \frac{1}{2}\, c^2 = \frac{1}{2}\, c^2{}_{max} = \frac{{a_o}^2}{\gamma - 1}$$

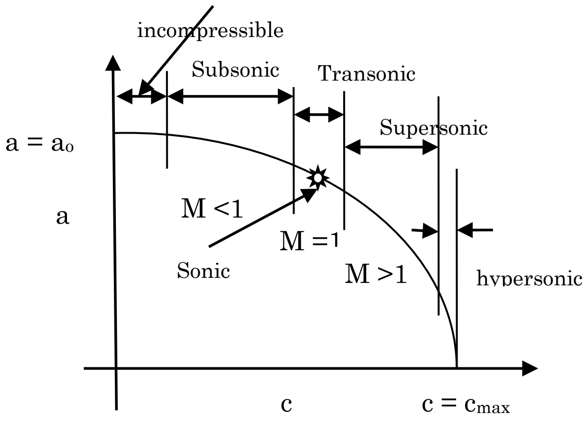


Fig. 1.1 Regions of Flow

1.38 Mach number, M

It is the ratio of the fluid velocity and the local velocity of sound,

$$M = \frac{c}{a} = \frac{c}{\sqrt{\gamma\, R\, T}}$$

1.39 Maximum fluid velocity, c_{max}

It is the velocity corresponding to the fluid velocity when it is accelerated to absolute zero temperature in an imaginary adiabatic expansion process.

$$c_{max} = \sqrt{2h_o} = \sqrt{\frac{2\, a_o{}^2}{\gamma - 1}}$$

$$\frac{c_{max}}{a_o} = \sqrt{\frac{2}{\gamma - 1}} = 2.24 \ \ (when, \gamma = 1.4)$$

1.40 Critical velocity

When Mach number becomes unity, this condition is achieved when the velocity of fluid becomes equal to velocity of sound.

$$M_{critical} = \frac{c^*}{a^*} = 1 , \qquad and\ c^* = a^* = \sqrt{\gamma\, R\, T^*}$$

The Mach number M* is given as the ratio of fluid velocity to critical fluid velocity,

$$M^* = \frac{c}{c^*} = \frac{a}{a^*}$$

And

$$M^{*2} = \frac{\frac{1}{2}\,(\gamma+1)\,M^2}{1+\ \frac{1}{2}\,(\gamma-1)\,M^2}$$

$$M^* = 0\ at\ M = 0$$
$$M^* = 1\ at\ M = 1$$

For M = ∞

$$M^*{}_{max} = \frac{c_{max}}{c^*} = \sqrt{\frac{\gamma+1}{\gamma-1}} = 2.45\ , \qquad for\ \gamma = 1.4$$

1.41 Crocco Number, C_r

Crocco number is the non dimensional fluid velocity which can be defined as the ratio of local fluid velocity to the maximum fluid velocity

$$C_r = \frac{c}{c_{max}} = \frac{c}{c^*}\,\frac{c^*}{c_{max}} = M^*\sqrt{\frac{\gamma-1}{\gamma+1}} = \sqrt{\frac{\frac{1}{2}\,(\gamma+1)\,M^2}{1+\ \frac{1}{2}\,(\gamma-1)\,M^2}}$$

And

$$M = \sqrt{\frac{2\,C_r^{\,2}}{(\gamma - 1)\left(1 - C_r^{\,2}\right)}}\ , \qquad and\ \frac{T_o}{T} = \frac{1}{1 - C_r^{\,2}}$$

1.42 Bernoulli Equation

Bernoulli Equation for isentropic compressible flow is given as

$$\frac{\gamma}{\gamma - 1}\,\frac{p_o}{\rho_o}\left(\frac{p}{p_o}\right)^{\frac{\gamma-1}{\gamma}} + \frac{1}{2}\,c^2 = \frac{\gamma}{\gamma - 1}\,\frac{p_o}{\rho_o}$$

1.43 Effect of Mach number on compressibility

The fluids are classified based on its compressibility and on view to it, for an incompressible fluid, the value of the pressure coefficient obtained from the Bernoulli equation is equal to unity.

$$\frac{p_o - p}{\frac{1}{2}\,\rho\,c^2} = 1$$

Whereas for compressible fluid the above value varies from unity, its magnitude increases with the Mach number of the flow
For an isentropic compressible flow the ratio of the stagnation and static pressures is given by

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2}\,M^2\right)^{\frac{\gamma}{\gamma-1}}$$

This can be expanded by the following series,

$$(1 + x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots$$

Here

$$x = \frac{\gamma - 1}{2}\,M^2, \qquad and\ n = \frac{\gamma}{\gamma - 1}$$

Therefore

$$\frac{p_o}{p} = 1 + \frac{\gamma}{2} M^2 + \frac{\gamma}{8} M^4 + \frac{\gamma\,(2-\gamma)}{48} M^6 + \ldots\ldots\ldots$$

$$\frac{p_o - p}{p\left(\frac{\gamma}{2} M^2\right)} = 1 + \frac{M^2}{4} + \frac{(2-\gamma)}{24} M^4 + \ldots\ldots\ldots$$

But p = ρRT and

$$\frac{\gamma}{2} M^2 = \frac{\gamma}{2}\frac{c^2}{a^2} = \frac{c^2}{2\,RT}$$

$$\therefore\ p\left(\frac{\gamma}{2} M^2\right) = \rho RT\left(\frac{c^2}{2\,RT}\right) = \frac{1}{2}\rho\, c^2$$

$$\frac{p_o - p}{\frac{1}{2}\rho\, c^2} = 1 + \frac{M^2}{4} + \frac{(2-\gamma)}{24} M^4 + \ldots\ldots\ldots$$

For $\gamma = 1.4$

$$\boldsymbol{\frac{p_o - p}{\frac{1}{2}\rho\, c^2} = 1 + \frac{M^2}{4} + \frac{M^4}{40} + \ldots\ldots\ldots}$$

The above equation sohws the variation of pressure coefficient from the incompressible flow value with Mach number M

1.44 Speed of sound

Consider an acoustic wave moving in a stationary fluid with speed ‘a’. Properties of fluid change due in the presence of the acoustic wave. These property variations can be predicted using 1D conservation equations. For simplicity we can assume the acoustic wave to be stationary and the fluid to be passing across the wave with velocity ‘a’. Consider the control volume shown in Fig. 3.1. for understanding, central hatched portion can be expressed as the acoustic wave. Let P, ρ and a be pressure, density and velocity ahead the acoustic wave respectively. Acoustic wave being a small amplitude disturbance, induces small change properties while fluid passing across it. Hence the properties behind the acoustic wave are P+dP, in ρ+dρ and a+da pressure, density and velocity respectively. Application of mass conservation (5.1) and momentum conservation (5.2) equations beween inlet and exit stations of control volume, we get,

$$\rho a = (\rho + d\rho)(a + da)$$
$$P + \rho u^2 = (P + dP)+(\rho + d\rho)(a + da)^2$$

From mass equation $\rho a = \rho a + \rho da + a d\rho + da d\rho$

We will neglect $dad\rho$ since both are small quantities. Hence their product will be even smaller.
Therefore $\rho da + ad\rho = 0$ and

$$\frac{d\rho}{da} = -\frac{\rho}{a}$$

From momentum equations we get,

$$p + \rho a^2 = (p + dp)+(\rho + d\rho)(a^2 + 2ada + da^2)$$

Neglecting da^2

$$p + \rho a^2 = (p + dp)+(\rho + d\rho)(a^2 + 2ada)$$
$$p + \rho a^2 = (p + dp)+(\rho a^2 + 2a\rho da + a^2 d\rho + 2adad\rho)$$

Neglecting 2adads,

$$p + \rho a^2 = (p + dp)+(\rho a^2 + 2a\rho da + a^2 d\rho)$$
$$0 = dp + 2a\rho da + a^2 d\rho$$
$$\frac{dp}{d\rho} + 2a\rho\frac{da}{d\rho} + a^2 = 0$$

Incorporating in above equation, we get,

$$\frac{dP}{d\rho} + 2a\rho - \frac{\rho}{a} + a^2 = 0$$

$$\frac{dP}{d\rho} - a^2 = 0$$

$$a^2 = \frac{dP}{d\rho} \quad \text{or } a = \sqrt{\frac{dP}{d\rho}}$$

This is the general formula for acoustic speed or speed of sound. We can express the same in terms of bulk modulus or compressibility using the definition of the compressibility (τ).

$$d\rho = \rho \tau dp$$

$$\frac{dp}{d\rho} = \frac{1}{\tau\rho}$$

$$a = \sqrt{\frac{1}{\tau\rho}}$$

Now this τ can be isothermal or adiabatic compressibility. However, changes in properties across sound wave are small and we have also not considered any dissipative effect like viscous effects, therefore we can treat the compressibility as the isentropic one. This proves that acoustic wave is isentropic (adiabatic reversible) in nature. Both the formulas derived for acoustic speed are valid for any state of matter. But if we consider gas then we can further simplify the expression as below.

$$a = \sqrt{\left(\frac{dp}{d\rho}\right)_{S-\text{constant}}}$$

Since the flow is adiabatic

$$\frac{p}{\rho^{\gamma}} = \text{constant} = K$$

$$\therefore p = K\rho^{\gamma}$$

$$\therefore \frac{dp}{d\rho} = K\gamma\rho^{\gamma-1} \text{ but } K = {}^{p}\!/\!{}_{\rho^{\gamma}}$$

$$\therefore \frac{dp}{d\rho} = K\gamma\rho^{\gamma-1} = \left({}^{p}\!/\!{}_{\rho^{\gamma}}\right)\gamma\rho^{\gamma-1} = \frac{\gamma p}{\rho} = \gamma RT \text{ Since } p = \rho RT$$

Therefore,

$$a = \sqrt{\left(\frac{\gamma p}{\rho}\right)} = \sqrt{\gamma RT}$$

1.45 Mach cone and Mach angle:

Consider a point source emanating infinitesimal pressure disturbances in a still fluid, in which the speed of sound is "a". If the point disturbance, is stationary then the wave fronts are concentric spheres. As shown in Fig. wave fronts are present at intervals of Δt.

Now suppose that source moves to the left at speed U < a. Figure shows four locations of the source, 1 to 4, at equal intervals of time Δt, with point 4 being the current location of the source.

At point 1, the source emanated a wave which has spherically expanded to a radius 3aΔt in an interval of time 3Δt.

During this time the source has moved to the location 4 at a distance of 3uΔt from point 1. The figure also shows the locations of the wave fronts emitted while the source was at points 2 and 3, respectively.

When the source speed is supersonic U > a (Fig. 39.2(c)), the point source is ahead of the disturbance and an observer in the downstream location is unaware of the approaching source. The disturbance emitted at different

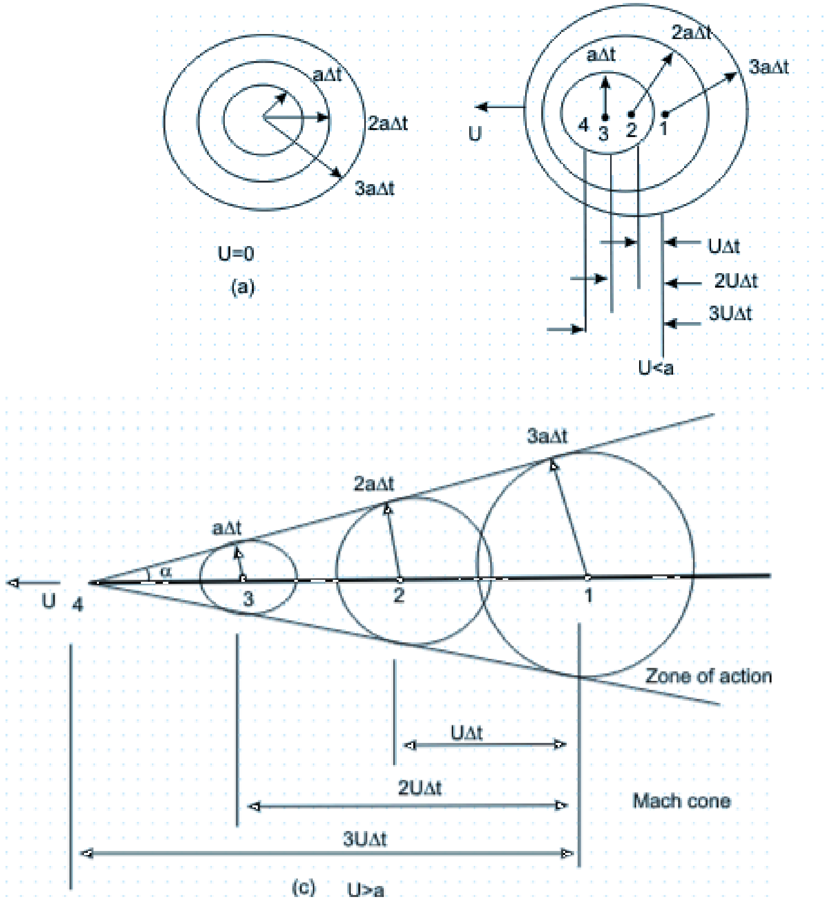


Fig.1.2 Wave fronts emitted from a point source in a still fluid

When the source speed is (a) U = 0 (still Source) (b) U < a (Subsonic) (c) U > a (Supersonic)

Points of time are enveloped by an imaginary conical surface known as "Mach Cone". The half angle of the cone α, is known as Mach angle and given by

$$\sin\alpha = \frac{a\Delta t}{U\Delta t} = \frac{1}{Ma}$$

$$\alpha = \sin^{-1}(1/Ma)$$

Since the disturbances are confined to the cone, the area within the cone is known as zone of action and the area outside the cone is zone of silence.
An observer does not feel the effects of the moving source till the Mach Cone covers his position.

1.46 Isentropic relations:

Isentropic relations are the relations between thermodynamic properties if the system undergoes isentropic process.

Consider a closed system interacting dQ amount of energy with the surrounding. If dU is the change in internal energy of the system and pdV is work done by the system against pressure p due to volume change dV. According to First Law of Thermodynamics we know that,

$$dQ = dU + pdV$$

From Second law of Thermodynamics,

$$\frac{dQ}{T} = dS$$

$$dQ = TdS$$

Here dS is the entropy chage due to reversible heat interaction dQ.

Therefore, combining First and Second Laws of Thermodynamics,

$$TdS = dU + pdV$$

However, we know that, if H is enthalpy of the system then,

$$H = U + pV$$

$$dH = dU + pdV + VdP$$

$$dH - Vdp = dU + pdV$$

Combining above equations we get,

$$TdS = dH - Vdp$$

For system with unit mass of matter, above equation can be written as,

$$Tds = dh - vdp$$

Here s, h and v are specific entropy, enthalpy and volume respectively.

Speciality of above equation is, its usefulness to calculate entropy change of any reversible process as below,

$$Tds = dh - vdp$$

$$ds = \frac{dh}{T} - \frac{v}{T}dp$$

$$ds = C_p\frac{dT}{T} - R\frac{dp}{P}$$

where, $dh = C_p\,dT$ and for calorifically perfect gas C_p (Specific heat at constant pressure) is assumed to be constant, Integrating above equation from starting state 1 to end state 2 of the process which system has undergone

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

For the special case, if system undergoes reversible adiabatic or isentropic process then, entropy change of the system (dS) is zero. Therefore the above equation can be written as,

$$C_pdT = vdp$$

From ideal gas relation, v = RT/P, above equations becomes

$$C_p dT = \frac{RT}{P} dp$$

$$\frac{C_p dT}{RT} = \frac{dp}{P}$$

We know the relations between specific heats as

$$\therefore \; C_p - C_v = R$$

$$\therefore \; C_p - \frac{C_p}{\gamma} = R$$

$$C_p = \frac{\gamma R}{\gamma - 1} \qquad C_v = \frac{R}{\gamma - 1}$$

Substituting above expression for Cp in above equation , we get

$$\frac{\gamma}{\gamma - 1} \frac{dT}{T} = \frac{dp}{P}$$

Integrating above equation from state 1 to state 2 for the isentropic process of the system, we get,

$$\frac{\gamma}{\gamma - 1} \ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{P_2}{P_1}\right)$$

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$$

Since the system is undergoing adiabatic process from state 1 to state 2,

$$\frac{P}{\rho^{\gamma}} = Const.$$

$$\left(\frac{\rho_1}{\rho_2}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

Here the above relations are called as isentropic equations.

1.47 Continuity Equation

The conservation of mass states that mass can neither be created nor destroyed, this leads to the continuity equation.

$$\dot{m} = \rho_1 c_1 A_1 = \rho_2 c_2 A_2 = constant$$

The above equation represents the continuity equation for the one dimensional steady flow.

1.48 Momentum Equation:

Considering the control volume and by taking the Newton's second law of motion, which states that the rate of change of momentum in a given direction is produced by the algebraic sum of the forces acting on the control surface in that direction.

$$\sum F = \frac{\partial}{\partial t}\int_{CV} \rho\, c\, dV + \oint_{cs} \rho\,(c dA)\, c$$

The term $\sum$F includes the body forces like the inertia, gravitational and the electromagnetic forces and the surface forces like friction, pressure and surface tension forces.

$$for\ steady\ flow, \frac{\partial}{\partial t}\int_{CV} \rho\, c\, dV = 0$$

$$\therefore \sum F = \oint \rho\,(c dA)\, c$$

For steady flow one-dimensional flow

$$\sum F = \rho_2 A_2 {c_2}^2 - \rho_1 A_1 {c_1}^2$$

Energy Equation:

$$\frac{dQ}{dt} = \frac{\partial}{\partial t}\int_{CV} \rho\, e\, dV + \oint_{cs}\left(h + \frac{1}{2}\, c^2 + gZ\right)\ \rho\,(c dA) + \frac{d\,W_s}{dt}$$

For steady flow

$$\left(\frac{\partial E}{\partial t}\right)_{cv} = 0$$

$$\therefore\ \frac{dQ}{dt} = \int_{out}\left(h + \frac{1}{2}\, c^2 + gZ\right) d\dot{m} - \int_{in}\left(h + \frac{1}{2}\, c^2 + gZ\right) d\dot{m} + \frac{d\,W_s}{dt}$$

For adiabatic flow

$$\frac{dQ}{dt} = 0$$

$$\frac{d\,W_s}{dt} = \int_{in}\left(h + \frac{1}{2}\, c^2 + gZ\right) d\dot{m} - \int_{out}\left(h + \frac{1}{2}\, c^2 + gZ\right) d\dot{m}$$

For one-dimensional and steady adiabatic flow

$$\frac{d\,W_s}{dt} = \dot{m}\left\{\left(h_1 + g\,Z_1 + \frac{1}{2}\,{c_1}^2\right) - \left(h_2 + g\,Z_2 + \frac{1}{2}\,{c_2}^2\right)\right\}$$

For gases $Z_1 - Z_2 \approx 0$ and

$$h_o = h + \frac{1}{2}\,c^2$$

$$\frac{d\,W_s}{dt} = \dot{m}\,\{h_{o1} - h_{o2}\}$$

If shaft work is absent,

$$h_{o1} = h_{o2}$$

1.49 Isentropic Flow with Variable duct:

To get rough idea on the variations on the various flow parameters along the flow directions and in many cases one dimensional flow is assumed to simplify such complex problems. And further this can be simplified by assuming the flow as isentropic.

Such assumptions do not go with the real cases, but serves as a standard for comparison with the actual processes.

Considering the adiabatic, the isentropic expansion and compression process of a perfect gas between two states 1 and 2. With the initial conditions stagnation pressure p_{o1}, kinetic energy ½ C_1^2, static temperature T_1 and stagnation temperature T_{o1}.

The final conditions for isentropic process are T_{2s} and ½ C_{2s}^2. But for an adiabatic process with same initial conditions shows an appreciable increase in the entropy. The stagnation pressure p_{o2a} is lower than its initial value p_{o1s}. Also the K.E is lower than its corresponding isentropic values. The below T-s and p-v diagrams explains this clearly.

The Mach number variation is given by the following equation and this equation is applied for both accelerating and decelerating processes for different values of Mach number.

$$\frac{dA}{A} = \frac{d\rho}{\rho\,c^2}\left(1 - \frac{c^2}{a^2}\right) = \frac{d\rho}{\rho\,c^2}\,(1 - M^2)$$

For accelerating devices like nozzles,
(i) For M<1, dA = - ve, this shows the nozzles area decreases in the range M =0 to M = 1 giving a convergent passage.
(ii) For M =1, dA = 0, this shows no change in the nozzles passage area, this section is called as throat of the nozzle where M = 1.
(iii) For M>1, dA = + ve, this shows an increase in the area of the nozzle continuously giving a divergent passage.
For decelerating devices like diffusers
(i) For M<1, dA = + ve, this shows the diffusers area increases in the range M =1 to M = 0 giving a divergent passage.
(ii) For M =1, dA = 0, this shows no change in the diffusers passage area, this section is called as throat of the diffuser where M = 1.
(iii) For M>1, dA = - ve, this shows an decrease in the area of the diffuser continuously giving a convergent passage.

Nozzles	M < 1 (subsonic flow)	M =1 (Sonic flow	M > 1 (Supersonic flow)
A	Decreases	A^*	Increases
p	Decreases	p^*	Decreases
c	Increases	$c^* = a^*$	Increases
Diffusers	M < 1 (subsonic flow)	M =1 (Sonic flow	M > 1 (Supersonic flow)
A	Increases	A^*	Decreases
p	Increases	p^*	Increases
c	Decreases	$c^* = a^*$	Decreases

Table 1.1

For isentropic flow

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}}$$

By considering the initial stagnation temperature and pressure for both the isentropic and the adiabatic processes same.
And also assuming that the exit temperature for both cases as same. The critical temperature ratio is given as

$$c_s{}^* = c_a{}^* = a_s{}^* = a_a{}^* = \sqrt{\gamma\, R\, T^*}$$

For critical state (M =1) in isentropic flow with $\gamma = 1.4$

$$\frac{T^*}{T_o} = \frac{2}{\gamma+1} = 0.833$$

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} = 0.634$$

On substituting the above values in the isentropic flow relations we obtain

$$\frac{T^*}{T} = \frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}\ M^2$$

$$\frac{p^*}{p} = \left(\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}\ M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho^*}{\rho} = \left(\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}\ M^2\right)^{\frac{1}{\gamma-1}}$$

The area ratio as a function of Mach number is given as

$$\frac{A}{A^*}\frac{p}{p_o} = \frac{\frac{1}{M}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2\,(\gamma-1)}}}{\left(1 + \frac{\gamma-1}{2}\ M^2\right)^{1/2}}$$

The impulse function or the wall force function is an important gas dynamic parameter, which refers to the quantities pA and ρ A c^2. The frequency of these quantities is found in compressible flow problems.

The impulse function is expressed as

$$\frac{F}{p_o\ A^*} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2\,(\gamma-1)}} \frac{1+\ \gamma\ M^2}{M\ \sqrt{1+\dfrac{\gamma-1}{2}\ M^2}}$$

1.49 Flow through Nozzles

A nozzle is a duct that increases the velocity of the flowing fluid at the expense of pressure drop.

A duct which decreases the velocity of a fluid and causes a corresponding increase in pressure is a diffuser. The same duct may be either a nozzle or a diffuser depending upon the end conditions across it.

If the cross-section of a duct decreases gradually from inlet to exit, the duct is said to be convergent.

Conversely if the cross section increases gradually from the inlet to exit, the duct is said to be divergent.

If the cross-section initially decreases and then increases, the duct is called a convergent-divergent nozzle.

The minimum cross-section of such ducts is known as throat.

A fluid is said to be compressible if its density changes with the change in pressure brought about by the flow.

If the density does not changes or changes very little, the fluid is said to be incompressible.

Usually the gases and vapors are compressible, whereas liquids are incompressible

1.49.1 Stagnation, Sonic Properties and Isentropic Expansion in Nozzle

The stagnation values are useful reference conditions in a compressible flow. Suppose the properties of a flow (such as T, p, ρ etc.) are known at a point. The stagnation properties at a point are defined as those which are to be obtained if the local flow were imagined to cease to zero velocity isentropically. The stagnation values are denoted by a subscript zero. Thus, the stagnation enthalpy is defined as

$$h_0 = h + \frac{1}{2}V^2$$

For a calorically perfect gas, this yields,

$$c_p T_0 = c_p T + \frac{1}{2}V^2 \qquad \text{--- (1.49.1)}$$

This defines the stagnation temperature. It is meaningful to express the ratio of T_o/T in the form

$$\frac{T_0}{T} = 1 + \frac{V^2}{2c_pT} = 1 + \frac{\gamma - 1}{2} \cdot \frac{V^2}{\gamma RT}$$

or

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} Ma^2 \qquad \text{-- (1.49.2)}$$

If we know the local temperature (T) and Mach number (Ma), we can fine out the stagnation temperature T_o. Consequently, isentropic relations can be used to obtain stagnation pressure and stagnation density as.

$$\frac{P_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \frac{\gamma - 1}{2} Ma^2\right]^{\frac{\gamma}{\gamma\ 1}} \qquad \text{---- (1.49.3)}$$

$$\frac{P_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \left[1 + \frac{\gamma - 1}{2} Ma^2\right]^{\frac{\gamma}{\gamma\ 1}} \qquad \text{--- (1.49.4)}$$

In general, the stagnation properties can vary throughout the flow field.

However, if the flow is adiabatic, then $h + \frac{V^2}{2}$ is constant throughout the flow. It follows that the h_o, T_o and a_o are constant throughout an adiabatic flow, even in the presence of friction. Here 'a' is the speed of sound and the suffix signifies the stagnation condition. It is understood that all stagnation properties are constant along an isentropic flow. If such a flow starts from a large reservoir where the fluid is practically at rest, then the properties in the reservoir are equal to the stagnation properties everywhere in the flow (Fig 1.3).

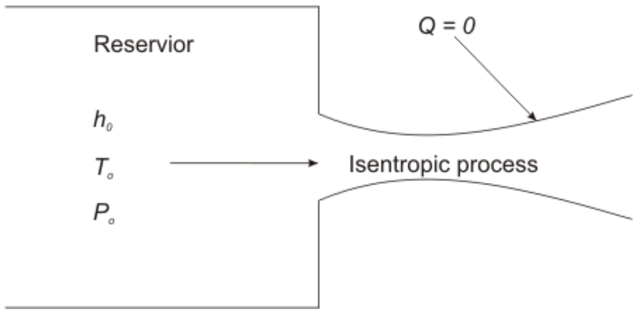


Fig 1.3 an isentropic process starting from a reservoir

There is another set of conditions of comparable usefulness where the flow is sonic, M_a=1.0. These sonic or critical properties are denoted by asterisks: $p^*, \rho^*, a^*,$ and. T^*. These properties are attained if the local fluid is imagined to expand or compress isentropically until it reaches M_a=1.

We have already discussed that the total enthalpy, hence T_o, is conserved so long the process is adiabatic, irrespective of frictional effects. In contrast, the stagnation pressure p_o and ρ_o density decrease if there is friction.
From Eq. (1.49.1), we note that

$$V^2 = 2c_p(T_0 - T)$$

Or

$$V = [\frac{2\gamma R}{\gamma - 1}(T_0 - T)]^{\frac{1}{2}} \quad \text{--- (1.49.5)}$$

is the relationship between the fluid velocity and local temperature (T), in an adiabatic flow. The flow can attain a maximum velocity of

$$V_{max} = [\frac{2\gamma R T_0}{\gamma - 1}]^{\frac{1}{2}} \quad \text{---- (1.49.6)}$$

As it has already been stated, the unity Mach number, M_a=1, condition is of special significance in compressible flow, and we can now write from Eq.(1.49.2), (1.49.3) and (1.49.4).

$$\frac{T_0}{T^*} = \frac{1+\gamma}{2} \quad \text{--- (1.49.7a)}$$

$$\frac{p_0}{p^*} = (\frac{1+\gamma}{2})^{\frac{\gamma}{\gamma-1}} \quad \text{---- (1.49.7b)}$$

$$\frac{\rho_0}{\rho^*} = (\frac{1+\gamma}{2})^{\frac{\gamma}{\gamma-1}} \quad \text{---- (1.49.7c)}$$

For diatomic gases, like air γ = 1.4, the numerical values are

$$\frac{T^*}{T_0} = 0.8333, \quad \frac{p^*}{p_0} = 0.5282, \quad and \quad \frac{\rho^*}{\rho_0} = 0.6339$$

The fluid velocity and acoustic speed are equal at sonic condition and is

$$V^* = a^* = [\gamma R T^*]^{1/2} \quad \text{----- (1.49.8a)}$$

or

$$V^* = [\frac{2\gamma}{\gamma+1} R T_0]^{\frac{1}{2}} \quad \text{----- (1.49.8b)}$$

We shall employ both stagnation conditions and critical conditions as reference conditions in a variety of one dimensional compressible flow.

1.49.2 Effect of Area Variation on Flow Properties in Isentropic Flow

In considering the effect of area variation on flow properties in isentropic flow, we shall concern ourselves primarily with the velocity and pressure. We shall determine the effect of change in area, *A,* on the velocity *V*, and the pressure *p*.

From Bernoulli's equation, we can write

$$\frac{dp}{\rho} + d(\frac{V^2}{2}) = 0 \quad \text{or} \quad dp = -\rho V\, dV$$

Dividing by $\rho\ V^2$, we obtain

$$\frac{dp}{\rho V^2} = -\frac{dV}{V} \quad \text{---- (1.49.9)}$$

A convenient differential form of the continuity equation can be written as

$$\frac{dA}{A} = -\frac{dV}{V} - \frac{d\rho}{\rho}$$

Substituting from Eq. (1.49.9),

$$\frac{dA}{A} = \frac{dp}{\rho V^2} - \frac{d\rho}{\rho} \quad \text{or} \quad \frac{dA}{A} = \frac{dp}{\rho V^2}[1 - \frac{V^2}{dp/d\rho}] \quad \text{--- (1.49.10)}$$

Invoking the relation

$$a^2 = \frac{dp}{d\rho}$$

For isentropic process in Eq. (1.49.10), we get

$$\frac{dA}{A} = \frac{dp}{\rho V^2}[1 - \frac{V^2}{\alpha^2}] = \frac{dp}{\rho V^2}[1 - Ma^2] \quad \text{---- (1.49.11)}$$

From Eq. (1.49.11), we see that for $M_a<1$ an area change causes a pressure change of the same sign, i.e. positive dA means positive dp for $M_a<1$. For $M_a>1$, an area change causes a pressure change of opposite sign.

Again, substituting from Eq.(1.49.9) into Eq. (1.49.11), we obtain

$$\frac{dA}{A} = -\frac{dV}{V}[1 - Ma^2] \quad \text{---- (1.49.12)}$$

From Eq. (1.49.12), we see that $M_a<1$ an area change causes a velocity change of opposite sign, i.e. positive dA means negative dV for Ma<1. For Ma>1, an area change causes a velocity change of same sign.

These results are summarized in Fig 1.4, and the relations (1.49.11) and (1.49.12) lead to the following important conclusions about compressible flows:

1. At subsonic speeds ($M_a<1$) a decrease in area increases the speed of flow. A subsonic nozzle should have a convergent profile and a subsonic diffuser should possess a divergent profile. The flow behavior in the regime of $M_a<1$ is therefore qualitatively the same as in incompressible flows.
2. In supersonic flows ($M_a>1$), the effect of area changes are different. According to Eq. (1.49.12), a supersonic nozzle must be built with an increasing area in the flow direction. A supersonic diffuser must be a converging channel. Divergent nozzles are used to produce supersonic flow in missiles and launch vehicles.

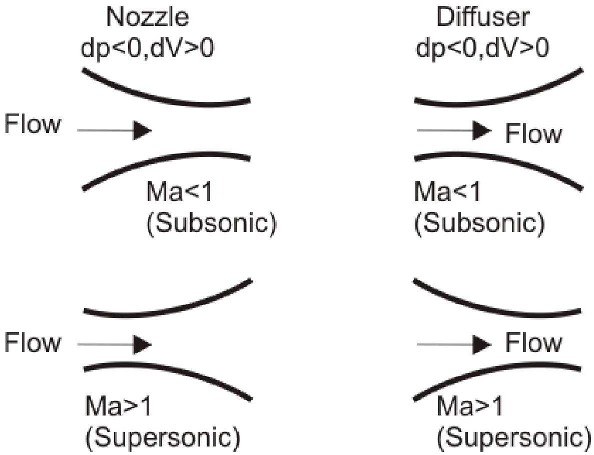


Fig 1.4. Shapes of nozzles and diffusers in subsonic and supersonic regimes

1.50 Convergent - Divergent Nozzle:

Suppose a nozzle is used to obtain a supersonic stream starting from low speeds at the inlet (Fig. 40.3). Then the Mach number should increase from Ma=0 near the inlet to Ma>1 at the exit. It is clear that the nozzle must converge in the subsonic portion and diverge in the supersonic portion. Such a nozzle is called a convergent-divergent nozzle. A convergent-divergent nozzle is also called a de Laval nozzle, after Carl G.P. de Laval who first used such a configuration in his steam turbines in late nineteenth century.

From Fig. 1.5 it is clear that the Mach number must be unity at the throat, where the area is neither increasing nor decreasing. This is consistent with Equation above which shows that dV can be nonzero at the throat only if Ma =1. It also follows that the sonic velocity can be achieved only at the throat of a nozzle or a diffuser.

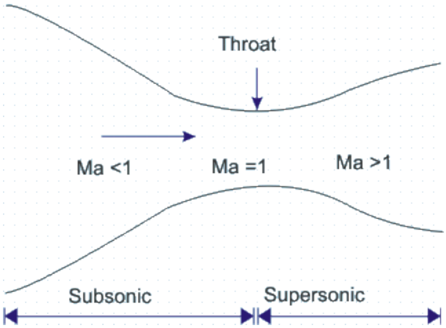


Fig 1.5 Convergent-Divergent Nozzle

The condition, however, does not restrict that Ma must necessarily be unity at the throat. According to Eq, a situation is possible where Ma ≠ 1 at the throat if dV = 0 there. For an example, the flow in a convergent-divergent duct may be subsonic everywhere with Ma increasing in the convergent portion and decreasing in the divergent portion with Ma ≠ 1 at the throat (see Fig. 1.5).

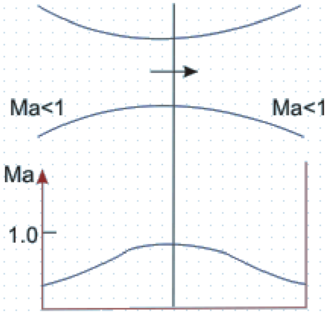


Fig 1.6 Convergent-Divergent duct with Ma ≠ 1 at throat

The first part of the duct is acting as a nozzle, whereas the second part is acting as a diffuser. Alternatively, we may have a convergent divergent duct in which the flow is supersonic everywhere with M_a decreasing in the convergent part and increasing in the divergent part and again Ma ≠ 1 at the throat (see Fig. 1.6 below)

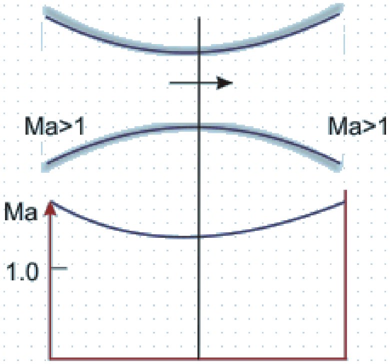


Fig. 1.5 Convergent-Divergent duct with Ma ≠ 1 at throat

Important Formulas:

1. Mach Number

$$M = \frac{c}{a}$$

2. Velocity of sound

$$a = \sqrt{\gamma R T}, \qquad m/s$$

3. Critical velocity

$$c^* = a^* = \sqrt{\gamma R T^*}, \qquad m/s$$

4. Maximum velocity

$$c_{max} = \sqrt{2h_o} = \sqrt{\frac{2\, a_o{}^2}{\gamma - 1}}$$

5. Stagnation Temperature

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

6. Stagnation pressure

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

7. Stagnation Density

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}}$$

8. Static enthalpy

$$h = c_p T$$

9. Stagnation enthalpy

$$h_o = c_p T_o$$

10. Stagnation enthalpy equation

$$h_o = \frac{a^2}{\gamma - 1} + \frac{1}{2} c^2 = \frac{1}{2} c^2{}_{max} = \frac{a_o{}^2}{\gamma - 1}$$

11. Crocco Number

$$C_r = \frac{c}{c_{max}}$$

12. Mass Flow Rate

$$m = \rho\, A\, c = \rho_1 A_1 c_1 = \rho_2 A_2 c_2$$

13. Critical temperature

$$T^* = \frac{2\, T_o}{\gamma + 1}$$

14. Gas constant

$$R = \left[\frac{\gamma - 1}{\gamma}\right] c_p$$

For air $\gamma = 1.4$ & $c_p = 1005$ J/kgK and R = 287 J/kgK

15. Mach angle

$$\alpha = \sin^{-1}\left(\frac{1}{M}\right)$$

16. Stagnation pressure from Bernoulli equation

$$p_o = p + \frac{1}{2}\, \rho\, c^2$$

17. For isentropic flow

Stagnation temperature remains constant, $T = T_{o1} = T_{o2}$
Stagnation pressure remains constant, $p_o = p_{o1} = p_{02}$

18. Characteristic Mach number

$$M^{*2} = \frac{\left(\frac{\gamma + 1}{2}\right) M^2}{1 + \left(\frac{\gamma - 1}{2}\right) M^2}$$

19. Maximum mass flow rate

$$\frac{m_{max}}{A^*}\frac{\sqrt{T_o}}{p_o} \times \sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2\,(\gamma-1)}}$$

20. Power required

$$P = m\, c_p\, (T_o - T_1), \qquad watt$$

21. Critical state

$$\frac{T^*}{T_o} = \frac{2}{\gamma + 1}$$

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho^*}{\rho_o} = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma-1}}$$

1. What is the basic difference between compressible and incompressible fluid flow?

Compressible
1. Fluid velocities are appreciable compared with the velocity of sound
2. Density is not constant
3. Compressibility factor is greater than one.

Incompressible
1. Fluid velocities are small compared with the velocity of
Sound
2. Density is constant
3. Compressibility factor is one.

2. Define Stagnation Pressure.

It is defined as the pressure of the fluid when it is decelerated to zero velocity at zero elevation.

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$$

3. Define Stagnation Enthalpy.

Stagnation enthalpy of a gas or vapour is defined as the enthalpy of the same when the gas or vapour is decelerated to zero velocity at zero elevation.

$$h_o = h + \frac{1}{2} c^2$$

4. Define Stagnation temperature.

Stagnation temperature is the temperature of the gas when it is adiabatically decelerated to zero velocity at zero elevation.

$$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2} M^2$$

5. Define Mach number.

It is the ratio of the fluid velocity and the local velocity of sound,

$$M = \frac{c}{a} = \frac{c}{\sqrt{\gamma\, R\, T}}$$

6. Define Stagnation Density.

The stagnation density for given value of stagnation pressure and temperature for an ideal gas is given as

$$\rho_o = \frac{p_o}{R\, T_o}$$

For isentropic relations,

$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2}\, M^2\right)^{\frac{1}{\gamma-1}}$$

7. Write adiabatic energy equation.

$$h_1 + \frac{1}{2}\, {C_1}^2 = h_2 + \frac{1}{2}\, {C_2}^2 + w$$

8. Define Crocco number.

Crocco number is the non dimensional fluid velocity which can be defined as the ratio of local fluid velocity to the maximum fluid velocity

$$C_r = \frac{c}{c_{max}}$$

9. Define characteristic Mach number.

The Mach number M^* is given as the ratio of fluid velocity to critical fluid velocity,

$$M^* = \frac{c}{c^*} = \frac{a}{a^*}$$

10. Define stagnation Velocity of sound.

It is the velocity of sound at given value of stagnation temperature, T_o

$$a_o = \sqrt{\gamma\, R\, T_o}$$

11. What is meant by transonic flow?

When the flow Mach number is in the range 0.8-1.2 it is called transonic flow. Highly unstable and mixed subsonic and supersonic flows are the main features of this regime

12. What is meant by hypersonic flow?

As per the thumb rule, when the flow Mach number is more than 5 then it is called as hypersonic flows. This is not the fixed definition for hypersonic flow since hypersonic flow is defined by certain characteristics of flow.

13. Define Maximum fluid velocity.

It is the velocity corresponding to the fluid velocity when it is accelerated to absolute zero temperature in an imaginary adiabatic expansion process.

$$c_{max} = \sqrt{2h_o} = \sqrt{\frac{2\, {a_o}^2}{\gamma - 1}}$$

14. Define critical velocity of sound or critical velocity of fluid.

When Mach number becomes unity, this condition is achieved when the velocity of fluid becomes equal to velocity of sound.

$$M_{critical} = \frac{c^*}{a^*} = 1\,, \qquad and\; c^* = a^* = \sqrt{\gamma\, R\, T^*}$$

15. Define velocity of sound (a).

    It is the velocity with which the sound wave propagates in a medium. Sound waves are generated due to infinitesimally small pressure disturbance.

16. Name the different regions of compressible fluid flow.

    Incompressible flow region, subsonic flow region, sonic flow region, transonic flow region, supersonic flow region, hypersonic flow region.

17. Explain Mach cone and Mach angle.

    The tangents drawn from the source points on the spheres define a conical surface referred as mach cone.
    The angle between the Mach line and the direction of motion of the body (flow direction) is known as Mach angle.

18. Define adiabatic process.

    In an adiabatic process there is no heat transfer between the system and the surrounding.

    i.e. Q=0

19. What is meant by isentropic flow with variable area?

    A steady one dimensional isentropic flow in a variable area passages is called "variable area flow". The heat transfer is negligible and there are no other irreversibility's due to fluid friction, etc.

20. Find the sonic velocity in oxygen when it is at 110° C, γ =1.4 and molecular weight 32.

$$a = \sqrt{\gamma\, R\, T} = \sqrt{1.4 \times 259.81 \times 383} = 373.244\ m/s$$

21. What is impulse function

    The sum of pressure force and impyusle force gives impulse function

$$F = pA + \rho A c^2$$

22. State the expression for dA/A as a function of Mach number.

$$\frac{dA}{A} = \frac{dp}{\rho c^2}\,[1 - M^2]$$

23. What are the types of nozzle?

    (i) Convergent nozzle
    (ii) Divergent nozzle
    (iii) Convergent Divergent nozzle.

24. Give the expression for T/T_o and T/T^* for isentropic flow through variable area in terms of Mach number.

$$\frac{T}{T_o} = \frac{1}{1 + \frac{\gamma - 1}{2}\, M^2}$$

$$\frac{T}{T^*} = \frac{1}{\frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1}\, M^2}$$

25. Give the expression for p/p_o and p^*/p_o for isentropic flow.

$$\frac{p}{p_o} = \frac{1}{\left(1 + \frac{\gamma - 1}{2}\, M^2\right)^{\frac{\gamma}{\gamma - 1}}}$$

and

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}}$$

26. Give the expression for A/A* in terms of Mach number

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}\;M^2\right)^{\frac{\gamma+1}{2\,(\gamma-1)}}$$

27. For isentropic flow write the expression for the density ratio between two sections in terms of Mach number.

$$\frac{\rho_o}{\rho} = \left(1+\frac{\gamma-1}{2}\;M^2\right)^{\frac{1}{\gamma-1}}$$

28. What is the critical pressure ratio of a nozzle in terms of specific heat?

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

29. Find the critical pressure ratio p^*/p_o for a gas with $\gamma = 1.13$.

$$\frac{p^*}{p_o} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{2}{1.13+1}\right)^{\frac{1.13}{1.13-1}} = 0.578$$

30. What is choked flow?

When the back pressure is reduced in a nozzle, the mass flow rate will increase. The maximum mass flow conditions are reached hen the back pressure is equal to the critical pressure. When the back pressure is reduced further, the mass flow rate will not change and is constant. The condition of flow is called “chocked flow”.

MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	answer
In a nozzle, the effect of supersaturation is to	decrease dryness fraction of steam	decrease specific volume of steam	increase the entropy	increase the entropy	increase the entropy
The density of supersaturated about that of the ordinary saturated vapour at the corresponding pressure	same as	2 times	4 times	8 times	8 times
The discharge at critical pressure is	zero	minimum	maximum	none of these	maximum
Thermal equilibrium means that the flow of steam is	isothermal	adiabatic	hyperbolic	polytropic	adiabatic
When the back pressure of a nozzle is below the designed value of pressure at exit of nozzle is said to be	choked	underdamping	overdamping	none of these	underdamping
When the nozzle operates with maximum mass flow the nozzle is said to be	choked	underdamping	overdamping	none of these	choked
The action of steam in a steam turbine is	static	dynamic	static and dynamic	neither static nor dynamic	dynamic
The turbine blades are	straight	circular	curved	none of these	curved
De-level turbine is a	simple impulse turbine	simple reaction turbine	impulse reaction turbine	none of these	simple impulse turbine
In turbines,the fluid undergoes a continuous steady flow process and the speed of the flow is	low	very low	high	very high	very high
The ratio of the workdone on the blades to the energy supplied to the blades is called	blading or diagram efficiency	nozzle efficiency	gross or stage efficiency	mechanical efficiency	blading or diagram efficiency
The ratio of the workdone on the blades per kg of steam to the total energy supplied per stage per kg of steam is called	blading or diagram efficiency	nozzle efficiency	gross or stage efficiency	mechanical efficiency	gross or stage efficiency
Reheating of steam in a turbine	increases the workdone through the turbine	increases the efficiency of the turbine	reduces the wearof the turbine	all of these	all of these

The reheat factor depends upon	intial pressure and super heat	exit pressure	turbine stage efficiency	all of these	all of these
A closed cycle gas turbine gives the efficiency as compared to an open cycle gas turbine is	same	lesser	higher	none of these	higher
Reheating in gas turbine	increases the thermal efficiency	increases the compressor work	increases the turbine work	decreases the thermal efficiency	increases the thermal efficiency
What is the basic property of incompressible fluid	Compressibility factor is equal to one	Compressibility factor is grater then one	Compressibility factor is less than one	is unpredictable	Compressibility factor is grater then one
Subsonic flow means	M = 1	M < 1	M > 1	M >> 1	M < 1
Supersonic flow means	M = 1	M < 1	M > 1	M >> 1	M > 1
Hypersonic flow means	M = 1	M < 1	M > 1	M >> 1	M >> 1
Mach angle is formed, when an object is moving with ___________ speed.	Sonic	Subsonic	Supersonic	Hypersonic	Supersonic
Sonic flow means	M = 1	M < 1	M > 1	M >> 1	M = 1
The pressure at which the steam leaves the nozzle is known as	back pressure	stagnation pressure	maximum pressure	critical pressure	back pressure
The discharge of steam in a convergent-divergent nozzle __________ after the throat (i.e. in the divergent portion of the nozzle)	remains constant	decreases	increases	is unpredictable	remains constant
The rate of discharge through the nozzle __________ when the exit pressure is gradually reduced.	remains constant	decreases	increases	is unpredictable	increases
In a nozzle, whole frictional loss is assumed to occur between	inlet and throat	inlet and outlet	throat and exit	is unpredictable	throat and exit
In a nozzle, the discharge is __________ at critical pressure.	Zero	Minimum	Maximum	is unpredictable	Maximum
The supersaturated flow of steam through a nozzle as compared to a stable flow, the available heat drop	remains the same	increases	decreases	is unpredictable	decreases

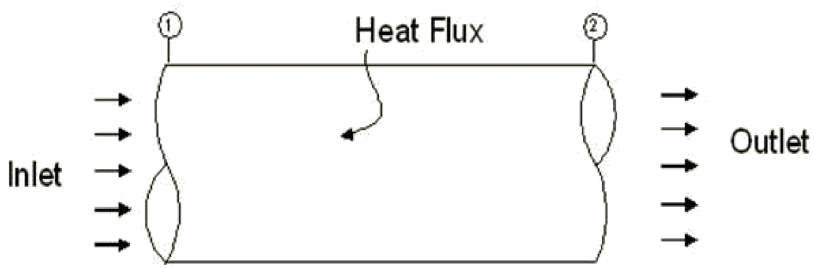
The steam leaves the nozzle at a	high pressure and a low velocity	\ high pressure and a high velocity	low pressure and a low velocity	low pressure and a high velocity	low pressure and a high velocity
The steam leaves the diffuser at a	high pressure and a low velocity	\ high pressure and a high velocity	low pressure and a low velocity	low pressure and a high velocity	high pressure and a low velocity
The critical pressure gives the velocity of steam at the throat	equal to the velocity of sound	less than the velocity of sound	more than the velocity of sound	is unpredictable	equal to the velocity of sound
Supersonic flow occurs in_______.	Convergent nozzle	Divergent nozzle	Under expanding nozzle	Choked nozzle	Under expanding nozzle
Equation for state for perfect gas is _______.	Pv=RT	P/v=RT	P+v=R+T	P-v=R-T	Pv=RT

UNIT – II

FLOW THROUGH DUCTS

Flow with friction:

Consider a flow through constant area pipe as shown in Fig. A subsonic or supersonic flow enters in the pipe at section 1 and leaves at section 2. Thermodynamic properties along with the velocity of the flow change from their initial value at station 1 to the station 2 in the presence of friction force. This 1D flow with friction is called as Fanno flow. Analysis of this flow would lead to prediction of properties of the flow at the exit for known inlet conditions and pipe configuration.



Here we will be considering the effect of friction between pipe wall and fluid. However this assumption will be used only in momentum equation. Hence total temperature can be considered to be constant in the flow process. The 1D governing equations for this flow are,

$\rho 1u1 = \rho 2u2$ (mass conservation)

And

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$ (energy conservation)

The main change takes place in momentum equation. Therefore consider the integral form of momentum equation for 1D flow.

$$\frac{\partial}{\partial t}\oint_{v} \rho du + \oint_{s} (\rho u ds) u = -\oint_{s} p ds + \oint_{s} \tau ds$$

Let's assume the flow to be steady through the pipe. Hence,

$$\oint_{s} (\rho u ds) u = -\oint_{s} p ds + \oint_{s} \tau ds$$

For integration of pressure and momentum terms, area is the cross sectional area while for the shear stress term area s the circumferential area of the pipe. Therefore the wetted area for shear stress includes diameter and length of the pipe. Negative sign should be associated with the shear stress term since shear acts in the direction opposite to the flow. Hence the momentum equation is,

p1 + ρu12 = p1 + ρu12 + F

where term F corresponds the frictional force and can be expressed in terms of pipe dimensions and friction coefficient.

Let's try to express the change in static and total properties of the flow from station 1 and 2 of the pipe due to consideration of wall shear or friction.

Since total temperature is constant, we can express the static temperature ratio as

$$\frac{T_2}{T_1} = \frac{T_0/T_1}{T_0/T_2} = \frac{\left(1+\frac{\gamma-1}{2}M_1^{\,2}\right)}{\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)}$$

Hence,

$$\therefore \frac{T_2}{T_1} = \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}$$

From, mass conservation equation, we can get,

ρ1u1 = ρ2u2

But,

$$a^2 = \frac{\gamma p}{\rho}$$

$$\therefore \rho = \frac{\gamma p}{a^2}$$

Substituting this expression in mass conservation equation, we get,

$$\therefore \left(\frac{\gamma p_1}{a_1^2}\right)u_1 = \left(\frac{\gamma p_2}{a_2^2}\right)u_2$$

$$\therefore \frac{p_1 M_1}{a_1} = \frac{p_2 M_2}{a_2}$$

$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\frac{a_2}{a_1} = \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}$$

$$\therefore \frac{p_2}{p_1} = \frac{M_1}{M_2}\left[\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right]^{0.5}$$

We can use ideal gas equation to calculate the density ratio from pressure and temperature ratio.

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}\frac{T_1}{T_2}$$

$$\therefore \frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\sqrt{\frac{T_2}{T_1}}\frac{T_1}{T_2} = \frac{M_1}{M_2}\sqrt{\frac{T_1}{T_2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{2+(\gamma-1)M_1^{\;2}}{2+(\gamma-1)M_2^{\;2}}\right]^{-0.5}$$

The earlier equations are for static property ratios. For total property ratios between two stations we have,

$$\frac{T_{0_2}}{T_{0_1}} = 1$$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left(p_{0_2}/p_2\right)p_2}{\left(p_{0_1}/p_1\right)p_1} = \frac{\left(1+\frac{\gamma-1}{2}M_2^{\;2}\right)^{\gamma/\gamma-1}}{\left(1+\frac{\gamma-1}{2}M_1^{\;2}\right)^{\gamma/\gamma-1}}\frac{p_2}{p_1}$$

$$\frac{p_{0_2}}{p_{0_1}} = \left[\frac{2+(\gamma-1)M_2^{\;2}}{2+(\gamma-1)M_1^{\;2}}\right]^{\gamma+1/2(\gamma-1)}\frac{M_1}{M_2}$$

$$\frac{\rho_{0_2}}{\rho_{0_1}} = \frac{p_{0_2}}{p_{0_1}}\frac{T_{0_1}}{T_{0_2}} = \frac{p_{0_2}}{p_{0_1}}$$

These expressions provide the ratios of thermodynamic properties for the known Mach number at station 1 and 2.

Reference conditions for Fanno flow:

If the inlet flow, either subsonic or supersonic, attains Mach number equal to 1 or sonic condition, at the station 2, then such a condition is taken as reference for calculations of Fanno flow. The corresponding length of the pipe is terms as critical length of the pipe. We can use the reference conditions for frictional pipe flow analysis. The expressions for property ratios are then given as,

$$\therefore \frac{T}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_\infty^{\ 2}}$$

$$\frac{p}{p^*} = \frac{1}{M_\infty}\left(\frac{\gamma+1}{2+(\gamma-1)M_\infty^{\ 2}}\right)^{1/2}$$

$$\frac{\rho}{\rho^*} = \frac{1}{M_\infty}\left(\frac{\gamma+1}{2+(\gamma-1)M_\infty^{\ 2}}\right)^{-1/2}$$

$$\frac{p_0}{p_0^*} = \frac{1}{M}\left[\frac{2+(\gamma-1)M_\infty^{\ 2}}{\gamma+1}\right]^{\gamma+1/2(\gamma-1)}$$

Since M2 is equal to 1, the Mach number at station 1 (M1) is the free stream Mach number (M∞).

Differential relations and analysis of flow with friction:

We know the mass, momentum and energy equations for 1D flow with friction. Herewith we will try to derive the differential form for the same.

$\rho u = Constant$ mass conservation

$d(\rho u) = 0$

$\frac{d\rho}{\rho} + \frac{du}{u} = 0$ Differential form of conservation equation

$h + \frac{u^2}{2} = constant$ Energy equation

Differential form of energy equation

$dh + udu = 0$

For momentum equation, consider the control volume shown in Fig. 2.1 and initially consider the steady integral form of the momentum equation

$$\oint_s (\rho u ds) u = -\oint_s p ds + \oint_s \tau ds$$

We know that, for integration of wall shear, the area of to be considered is the circumferential area. For integration of pressure and momentum term, the area of to be considered is cross-sectional area,

$$\frac{\pi}{4} D^2 ,$$

Hence

$$\therefore \oint_s \left(p + (\rho u ds) u\right) ds = -\oint_s \tau ds = -\int_0^l \tau D dx \pi = -\pi D \int_0^l \tau dx$$

If we consider the distance between two stations to be infinitesimal (dx) hence the differential form of the momentum equation is as,

$$\therefore (p+dp)A + A(\rho+d\rho)(u+du)^2 - (p+\rho u^2)A = -\tau A_5$$

$$(pA+dpA) + A(\rho+d\rho)(u^2+2udu+du^2) - (pA+A\rho u^2) = -\tau A_5$$

$$Adp + A\rho(u^2+2udu+du^2) + Adp(u^2+2udu+du^2) - A\rho u^2 = -\tau A_5$$

$$Adp + 2\rho udu + Ad\rho u^2 = -\tau A_5$$

We can use the mass conservation equation and simplify the above equation,

$$\frac{d\rho}{\rho} + \frac{du}{u} = 0$$

$$\frac{d\rho}{\rho} = -\frac{du}{u}$$

$$d\rho = -\frac{du}{u}\rho$$

Using above equation, the following equation can be simplified as,

$$Adp + 2\rho uduA + Au(-\rho du) = -\tau A_5$$

$$Adp + \rho uduA = -\tau A_5$$

$$\frac{dp}{\rho u^2} + \frac{du}{u} = -\frac{\tau}{\rho u^2}\frac{A_5}{A}$$

As we know,

$$A = \frac{\pi}{4}D^2$$

$$A_5 = \pi D dx$$

$$\frac{A_5}{A} = \frac{4}{D}dx$$

Using this equation we can re-write the Eq (16.2) as,

$$\frac{dp}{\rho u^2}+\frac{du}{u}=-\frac{\tau}{\rho u^2}\frac{4dx}{D}$$

We can use the definition of skin friction coefficient as,

$$C_f=\frac{\tau}{\rho u^2/2}$$

$$\frac{C_f}{2}=\frac{\tau}{\rho u^2}$$

Hence differential form of momentum equation becomes,

$$\frac{dp}{\rho u^2}+\frac{du}{u}=-\frac{C_f}{2}\frac{4dx}{D}$$

From further simplification, we can replace the dynamic pressure, as,

$$\rho u^2=\rho u^2\frac{\gamma p}{\gamma p}=u^2\frac{\rho}{\gamma p}\gamma p=\frac{u^2}{a^2}\gamma p=M^2\gamma p$$

Hence,

$$\frac{1}{M^2\gamma}\frac{dp}{p}+\frac{du}{u}=-\frac{C_f}{2}\frac{4dx}{D}$$

Above equation represents the differential form of momentum equation.

We can use the Eq. for further analysis of Fanno flow. For this purpose lets replace dp/p and du/u of this equation. We can use ideal gas equation for replacing dp/p as,

$$p = \rho RT$$ (Differential form of ideal gas equation)

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

using mass differential form of mass conservation equation,
we get,

$$\frac{dp}{p} = -\frac{du}{u} + \frac{dT}{T}$$ (Modified differential form of ideal gas equation)

For this purpose lets re-express the terms du/u and dT/T.

As we know

$$dh + udu = 0$$

$$c_p dT + udu = 0$$

$$\frac{dT}{T} + \frac{udu}{c_p T} = 0$$

But

$$C_p = \frac{\gamma}{\gamma - 1} R$$

$$\frac{dT}{T} + \frac{udu}{\frac{\gamma}{\gamma - 1} RT} = 0$$

$$\frac{dT}{T} + \frac{(\gamma - 1) udu}{\gamma RT} = 0$$

$$\frac{dT}{T} + \frac{(\gamma - 1)udu}{a^2} = 0$$

$$\frac{dT}{T} + \frac{(\gamma - 1)u^2}{a^2}\frac{du}{u} = 0$$

$$\frac{dT}{T} + (\gamma - 1)M^2\frac{du}{u} = 0$$

$$\frac{dT}{T} = -(\gamma - 1)M^2\frac{du}{u}$$

$$\frac{du}{u} = \frac{-1}{(\gamma - 1)M^2}\frac{dT}{T}$$

We can use this equation for replacing the term du/u of Eq. 16.3. However we can further simplify this equation by replacing dT/T. As,

$$M = \frac{u}{a} = \frac{u}{(\gamma RT)^{0.5}}$$

$$M(\gamma RT)^{0.5} = u$$

$$dM.(\gamma RT)^{0.5} + \frac{1}{2}M.(\gamma R)^{0.5}(T)^{-0.5}dT = du$$

Dividing by M $(\gamma RT)^{0.5}$ on both the sides

$$\frac{dM}{M} + \frac{dT}{2T} = \frac{du}{u}$$

$$\frac{dM}{M} + \frac{dT}{2T} = \frac{du}{u}$$

We can use the Eq 4.16 to replace the term dT/T

$$\frac{dM}{M} = \left(1 + \frac{(\gamma - 1)M^2}{2}\right)\frac{du}{u}$$

$$\frac{du}{u} = \left(1 + \frac{(\gamma - 1)M^2}{2}\right)^{-1}\frac{dM}{M}$$

This is the simplest form of du/u term expressed in terms of Mach number and specific heat ratio.

Analysis of Fanno Flow:

We can use the Eq. for critical analysis of Fanno flow. For this purpose let's replace dp/p and du/u of this equation.

Let again consider Eq again and replace the term du/d, we get,

$$\frac{dM}{M} + \frac{dT}{2T} = \frac{-1}{(\gamma - 1)M^2}\frac{dT}{T}$$

$$\frac{dM}{M} = -\frac{dT}{T}\left(\frac{1}{(\gamma - 1)M^2} + \frac{1}{2}\right)$$

$$\frac{dM}{M} = -\frac{dT}{T}\left(\frac{2 + (\gamma - 1)M^2}{2(\gamma - 1)M^2}\right)$$

$$\frac{dT}{T} = -\left(\frac{2(\gamma - 1)M^2}{2 + (\gamma - 1)M^2}\right)\frac{dM}{M}$$

Modified differential form of ideal gas equation can be re-written using equation as

$$\frac{dp}{p} = -\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M} - \left(\frac{(\gamma-1)M^2}{1+\left(\frac{(\gamma-1)}{2}\right)M^2}\right)\frac{dM}{M}$$

$$\frac{dp}{p} = -\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(1+(\gamma-1)M^2\right)$$

Now lets consider the differential form of momentum Eq. and replace the term du/u and dp/p using Equations above, we get,

$$\frac{-1}{M^2\gamma}\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(1+(\gamma-1)M^2\right)+\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M} = -\frac{C_f}{2}\frac{4dx}{D}$$

$$\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(\frac{-1-\gamma M^2+M^2}{M^2\gamma}\right)+\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M} = -\frac{C_f}{2}\frac{4dx}{D}$$

$$\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(\frac{-1-M^2(\gamma-1)}{M^2\gamma}\right)+\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M} = -\frac{C_f}{2}\frac{4dx}{D}$$

$$\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(\frac{-1}{M^2\gamma}-\frac{(\gamma-1)}{\gamma}\right)+\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M} = -\frac{C_f}{2}\frac{4dx}{D}$$

$$\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(\frac{-1}{M^2\gamma}-\frac{(\gamma-1)}{\gamma}+1\right) = -\frac{C_f}{2}\frac{4dx}{D}$$

$$\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(\frac{M^2-1}{M^2\gamma}\right) = -\frac{C_f}{2}\frac{4dx}{D}$$

$$C_f\frac{4dx}{D} = 2\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(\frac{1-M^2}{M^2\gamma}\right)$$

$$\frac{dM}{dx} = C_f\frac{2}{D}\left(1+\frac{(\gamma-1)M^2}{2}\right)\left(\frac{M^3\gamma}{1-M^2}\right)$$

We can clearly observe that $\frac{dM}{dx} > 0$ if M<1 and $\frac{dM}{dx} < 0$ for M>1.

Hence from above Eq., we can clearly state that, for subsonic flow Mach number increases in the presence of friction, while Mach number for supersonic flow decreases in the presence of friction.

From above Eq., it becomes clear that, for increases in Mach number, velocity of the subsonic flow increases while that of supersonic flow decreases due to decrease in Mach number.

From Eq., it becomes clear that temperature of the subsonic flow decreases due to increase in Mach number in the presence of friction while temperature increases for supersonic flow with friction.

We can prove from Eq. that pressure decreases for subsonic flow while pressure increases for supersonic flow with friction. For entropy change we can arrive at the expression as,

$$Tds = dh - vdp$$

$$ds = cp\frac{dT}{T} - R\frac{dp}{p}$$

$$ds = -c_p\left(\frac{(\gamma-1)M^2}{1+\left((\gamma-1)/2\right)M^2}\right)\frac{dM}{M} + R\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(1+(\gamma-1)M^2\right)$$

$$ds = -\frac{\gamma R M^2}{1+\left((\gamma-1)/2\right)M^2}\frac{dM}{M} + R\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(1+(\gamma-1)M^2\right)$$

$$ds = R\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(-\gamma M^2+1+(\gamma-1)M^2\right)$$

$$ds = R\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(-\gamma M^2+1+\gamma M^2-M^2\right)$$

$$ds = R\left(1+\frac{(\gamma-1)M^2}{2}\right)^{-1}\frac{dM}{M}\left(1-M^2\right)$$

For supersonic flow dM is negative however 1 - M2 is also negative, therefore ds is positive for supersonic flow.

For subsonic flow dM is positive however 1 - M2 is also positive, therefore ds is positive for supersonic flow.
This expression clearly proves that entropy increases for both subsonic and supersonic flows while sonic flow is isentropic.

Fanno line or curve:

Typical Fanno line for a particular mass flow rate is shown in Fig. Previously proved facts are clearly evident in this figure. For a subsonic flow through a frictional pipe, enthalpy decreases and entropy increases. With increase in length of pipe, more expansion of the flow takes place for the subsonic flow due to increase in Mach number, decrease in pressure and increase in velocity.

For a particular length of pipe flow, inlet subsonic flow attains sonic state at the exit. Corresponding length of the pipe is called as critical length.

Further increase in length of pipe doesn't change state at the exit however inlet conditions change and subsonic flow becomes lower subsonic. Hence the flow condition for the critical pipe length is called as choked flow.

For supersonic flow at the entry to a constant area pipe, deceleration of flow takes place and flow attains lower supersonic conditions. With increase in length of the pipe, Mach number at the exit decreases due to deceleration and at a particular pipe length flow becomes sonic at the exit.

Further increase length of the pipe doesn't change exit conditions while inlet conditions become subsonic. Therefore choked conditions can be said to be attained for critical length of pipe for which entry is supersonic.

This curve shows that there is only one critical point for flow with friction. This critical point corresponds to maximum entropy and hence the sonic state.

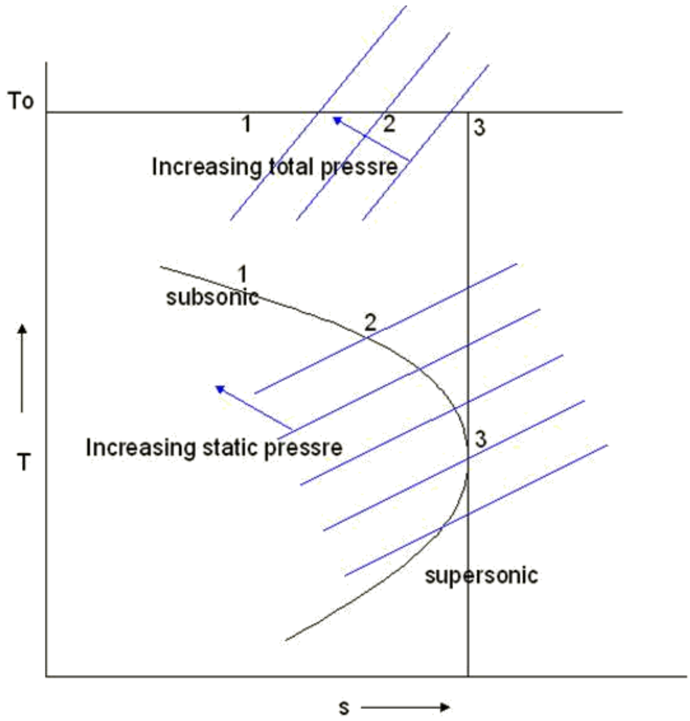


Fig. h-s diagram for Fanno flow.

Like Rayleigh curve, h-s diagram for Fanno flow shown in Fig. corresponds to a particular value of mass flow rate. Fanno curve for various mass flow rate conditions is given in Fig.

Explanation for Fanno curve for various mass flow rates can be obtained on similar line as it has been obtained for Rayleigh flow for various mass flow rates where instead of changing the applied heat flux we have to change the pipe length. Fanno curve for increased mass flow rate is also seen to be shrunk like Rayleigh curve.

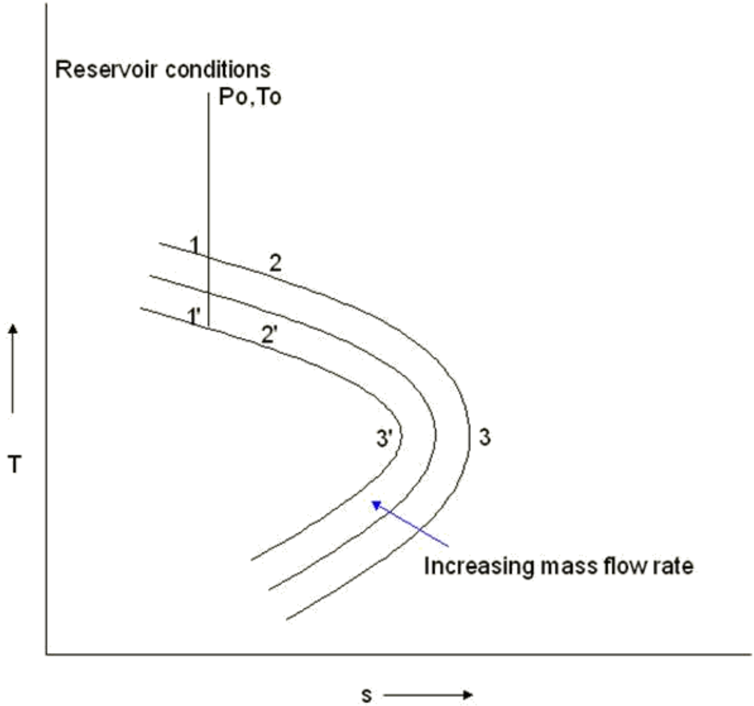


Fig. h-s diagram for Fanno flow for various mass flow rates.

One dimensional flow with heat addition (Rayleigh Flow)

Consider the control volume as shown in Fig. for 1D flow with heat addition. The fluid flow of this kind is called as Rayleigh flow. Here station 1 is representative station before heat addition while station 2 is representative station after heat addition. This control volume is necessarily a constant cross-section pipe hence variation is the inviscid flow properties is expected in the direction of the flow due to addition of heat.

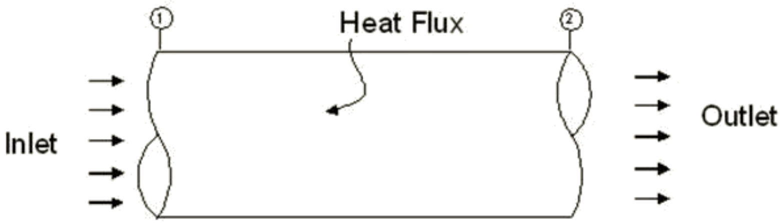


Fig. Typical Control volume for 1D flow with heat addition.

Assume the flow to be inviscid and steady between these two stations. Therefore the mass and momentum conservation equations (5.1 and 5.2) remain unaltered from the normal shock case but energy equation will have a term corresponding to external heat addition in comparison with equation (5.3). Hence the 1D conservation equations for flow with heat addition are as follows.

$$\rho_1 u_1 = \rho_2 u_2$$
$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$
$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

Here 'q' is amount of heat added per unit mass. Hence,

$$q = (h_1 - h_2) + \left(\frac{v_2^2}{2} - \frac{v_1^2}{2}\right)$$

However, we know that

$$h + \frac{v^2}{2} = ho$$

q = ho2 - ho1 = cp(To2 - To1)

This equation suggests that change in total temperature takes place due to heat addition between two stations.

Lets represent the ratios of static and total properties in terms of upstream (station 1) and downstream (station 2) Mach number and specific heat ratio. Lets consider the momentum equation,

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$p_1 + \gamma p_1 \frac{\rho_1}{\gamma p_1} u_1^2 = p_2 + \gamma p_2 \frac{\rho_2}{\gamma p_2} u_2^2$$

$$p_1 + \gamma p_1 \frac{u_1^2}{a_1^2} = p_2 + \gamma p_2 \frac{u_2^2}{a_2^2}$$

$$p_1 + \gamma p_1 M_1^2 = p_2 + \gamma p_2 M_2^2$$

$$p_1\left(1+\gamma M_1^2\right) = p_2\left(1+\gamma M_2^2\right)$$

$$\frac{p_2}{p_1} = \frac{\left(1+\gamma M_1^2\right)}{\left(1+\gamma M_2^2\right)}$$

Also from ideal gas assumption

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_2}{\rho_1}$$

But ρ1u1 = ρ2u2

$$\frac{\rho_1}{\rho_2} = \frac{u_2}{u_1}$$

, ,

Therefore,

$$\frac{T_2}{T_1} = \frac{\rho_2}{\rho_1}\frac{u_2}{u_1}$$

$$\frac{T_2}{T_1} = \frac{\rho_2}{\rho_1}\frac{M_2}{M_1}\frac{a_2}{a_1}$$

$$\frac{T_2}{T_1} = \frac{\rho_2}{\rho_1}\frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}$$

$$\sqrt{\frac{T_2}{T_1}} = \frac{\rho_2}{\rho_1}\frac{M_2}{M_1}$$

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^2\left(\frac{M_2}{M_1}\right)^2$$

Hence from above equations we get,

$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2\left(\frac{M_2}{M_1}\right)^2$$

Therefore,

$$\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}\frac{T_1}{T_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{\left(\dfrac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)}{\left(\dfrac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2\left(\dfrac{M_2}{M_1}\right)^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{1}{\left(\dfrac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)\left(\dfrac{M_2}{M_1}\right)^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{1+\gamma M_2^2}{1+\gamma M_1^2}\left(\frac{M_1}{M_2}\right)^2$$

For ratio of total properties,

$$\frac{p_{0_2}}{p_{0_1}} = \left(\frac{p_{0_2}/p_2}{p_{0_1}/p_1}\right)\frac{p_2}{p_1}$$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left(1+\dfrac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1}}{\left(1+\dfrac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1}}\frac{1+\gamma M_1^2}{1+\gamma M_2^2}$$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\left[\frac{\left(1+\dfrac{\gamma-1}{2}M_2^2\right)}{\left(1+\dfrac{\gamma-1}{2}M_1^2\right)}\right]^{\gamma/\gamma-1}$$

Similarly

$$\frac{T_{0_2}}{T_{0_1}} = \left(\frac{T_{0_2}/T_2}{T_{0_1}/T_1} \right) \frac{T_2}{T_1}$$

$$\frac{T_{0_2}}{T_{0_1}} = \left[\frac{\left(1+\frac{\gamma-1}{2}M_2^{\,2}\right)}{\left(1+\frac{\gamma-1}{2}M_1^{\,2}\right)} \right] \left(\frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}} \right)^2 \left(\frac{M_2}{M_1} \right)^2$$

From these two ratios we can find out ρ_{o2}/ρ_{o1} as

$$\frac{\rho_{0_2}}{\rho_{0_1}} = \frac{p_{0_2}}{p_{0_1}} \frac{T_{0_2}}{T_{0_1}}$$

Reference conditions for flows with heat addition

We have represented all the ratios in terms of upstream and down stream Mach numbers. If we consider a particular case where heat addition leads to downstream Mach number equal to one or post heat addition Mach number is unity, then equations (11.2) to (11.6) can be written as,

$$\frac{p_2}{p_1} = \frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}$$

Since $M2 = 1$ & $p2 = p^*$ & $p1 = p\infty$ & $M1 = M\infty$. Here flow properties after heat addition are the stared quantities due to unity of the local Mach number. Hence these quantities are of very much of importance since can be used as reference quantities.

$$\frac{p^*}{p_\infty} = \frac{1+\gamma M_\infty^{\ 2}}{1+\gamma}$$

$$\frac{p_\infty}{p^*} = \frac{1+\gamma}{1+\gamma M_\infty^{\ 2}}$$

Similarly

$$\frac{T_\infty}{T^*} = M_\infty^{\ 2}\left(\frac{1+\gamma}{1+\gamma M_\infty^{\ 2}}\right)^2$$

$$\frac{\rho_\infty}{\rho^*} = \frac{1}{M_\infty^{\ 2}}\left(\frac{1+\gamma M_\infty^{\ 2}}{1+\gamma}\right)$$

$$\frac{p_0}{p_0^*} = \frac{1+\gamma}{1+\gamma M_\infty^{\ 2}}\left[\frac{2+(\gamma-1)M_\infty^{\ 2}}{1+\gamma}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_0}{T_0^*} = \frac{(1+\gamma)M_\infty^{\ 2}}{1+\gamma M_\infty^{\ 2}}\left[2+(\gamma-1)M_\infty^{\ 2}\right]$$

From all the ratios for 1D flow with heat addition, following conclusions can be drawn for supersonic and subsonic flows.

1. Addition of heat in supersonic flows

    Decreases Mach number
    Increases static pressure
    Increases static temperature
    Decreases total pressure
    Increases total temperature
    Decreases velocity

2. Addition of heat in subsonic flow

Increases Mach number

Decreases static pressure

Increases static temperature

If $M_\infty < \gamma^{-1/2}$ and decreases if $M_\infty > \gamma^{-1/2}$

Decreases total pressure

Increases total temperature

Increases velocity

Hence supersonic flow decelerates towards sonic value while subsonic flow accelerates towards the same due to heat addition. In a way, having a subsonic flow to start with we can keep on adding heat to reach sonic and then remove heat to attain required supersonic flow conditions. Exactly reverse procedure needs to be followed if given flow is supersonic and the target is to attain specified subsonic condition.

It had been shown that addition of heat in subsonic flow increases the static temperature till

$$M_\infty < \gamma^{-1/2}$$

And decrease afterwards.

Main reason for this phenomenon is explained herewith. Addition of heat to subsonic flow increases the velocity and temperature initially for small amount of heat addition. Therefore both Kinetic energy (KE) and Internal energy (IE) of the flow increase due to externally added heat in subsonic flow. However for a given mass flow rate, rate of increase of KE is more than IE for given amount of heat addition. Therefore after particular amount of addition of external heat, it becomes impossible to increase IE (hence temperature) and KE (hence velocity) both keeping mass flow rate same. As a result beyond particular amount of heat addition for a given mass flow rate condition, temperature (hence IE) decreases but velocity (hence KE) continues to increase. We can as well interpret the same phenomenon as, after certain critical amount of heat addition in subsonic flow added external heat becomes insufficient to increase the velocity (hence KE) while keeping the mass flow rate same, hence required extra energy is supplied by the flow itself from its internal energy, by virtue of which temperature decrease though we add heat in subsonic flow.

From the above mentioned formulae for sonic conditions or 'star' properties, we can calculate total temperature of the sonic flow after heat addition from any given initial conditions and hence the amount of heat required to be added to reach sonic condition from any given initial conditions. The properties at this sonic conditions for a given mass flow rate remain independent of upstream or free stream Mach number. Therefore, we can use this concept or these properties as reference properties for handling 1D flow with heat addition.
If the amount of heat added in the flow is more than the critical heat required to reach sonic condition, then flow cannot accommodate this heat. The main reason for this fact is the anchoring of conditions after heat addition to sonic point. Hence to accommodate the added extra heat, upstream conditions of the flow change from supersonic to subsonic or from subsonic to lower subsonic for which the externally added heat is the heat required to reach sonic condition.

Rayleigh curve:

As we know, Rayleigh flow is called as the flow with heat addition. The curve or plot or state chart dealing with heat addition is called Rayleigh curve. Lets derive the expression for this curve in p-v and h-s chart.

$\rho v = const = k$ Mass Conservation

$p + \rho v^2 = const \text{ or } p + \dfrac{k^2}{\rho} = const$ Momentum Conservation

For 1D, it can be expressed as pA + ρv2A = const, called as Impulse function or Thrust function.

$$h_1 + \frac{v_1^2}{2} + q = h_2 + \frac{v_2^2}{2}$$ Energy Conservation

We can draw a p-v diagram for flow with heat addition or Rayleigh flow like Hugoniot curve, using momentum equation as,

$$p + \frac{k_1^2}{\rho} = const \quad \text{Or} \quad p + \vartheta k^2 = const$$

This equation is the equation for straight line on p-v diagram where k_2 corresponds to slope which eventually is the mass flow rate. Hence, slope of the line joining any point, corresponding to initial state and final state on Rayleigh curve, represents mass fluxes or mass flow rates. This fact is same as that observed for Hugoniot curve.

Slope of this Rayleigh line can be calculated as

p1 + v1k2 = p2 + v2k2

$$\frac{p_2 - p_1}{v_2 - v_1} = -k^2$$

Such curve represented by points 1, 2, 3 and 4 for flow with heat addition is as shown in Figure along with the isentropic and isothermal line on p-v chart

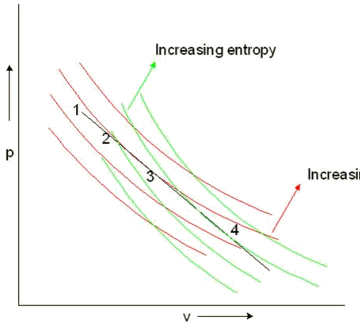


Fig. P-V diagram for the heat addition process

For better understanding, consider the process of heat addition in subsonic flow. Suppose given conditions are described by point 1 in Fig.. Change in thermodynamic states of the flow in the process of heat addition is shown in the figure by a straight line. Slope of this straight line is proportional to the mass flow rate and is given by Eq.. Here point 2 represents the conditions after certain amount of heat addition. Point 2 essentially has lesser pressure and higher specific volume as that of point 1 since expansion of the subsonic flow takes place due to heat addition. Increase in temperature can also be observed here for the subsonic flow. Sufficient amount of heat addition would lead to reach point 3 from initial conditions 1.

We can clearly see in this figure that the Rayleigh line is tangent to an isotherm at point 3, hence the temperature given by the corresponding isotherm is the maximum attainable temperature by adding heat in the given subsonic flow of initial conditions 1. Conditions represented by point 4 become possible by further addition of heat.

It can also be seen here that Rayleigh line is tangent to an isentropic at point 4, hence point 4 represents the maximum entropy point or sonic point. Reduction is temperature in the process 3-4 is clearly evident in the presence of heat addition.

Therefore there are two critical points in Rayleigh curve, one of which corresponds to maximum enthalpy or temperature and other corresponds to maximum entropy or total temperature or total enthalpy.

We can as well use h-s diagram to explain the heat addition process in the same subsonic flow as shown in Fig.

Analysis of Critical Points of Rayleigh Curve:

The same conservation equations for 1D heat addition can be re-written in differential form as, For mass conservation equation

ρu = const = k

d(ρu) = 0 Mass Conservation

$$d\rho = -\frac{\rho}{u}\,du \quad \text{or} \quad du = -\frac{u}{\rho}\,d\rho$$

For momentum conservation equation

P + ρu2 = const

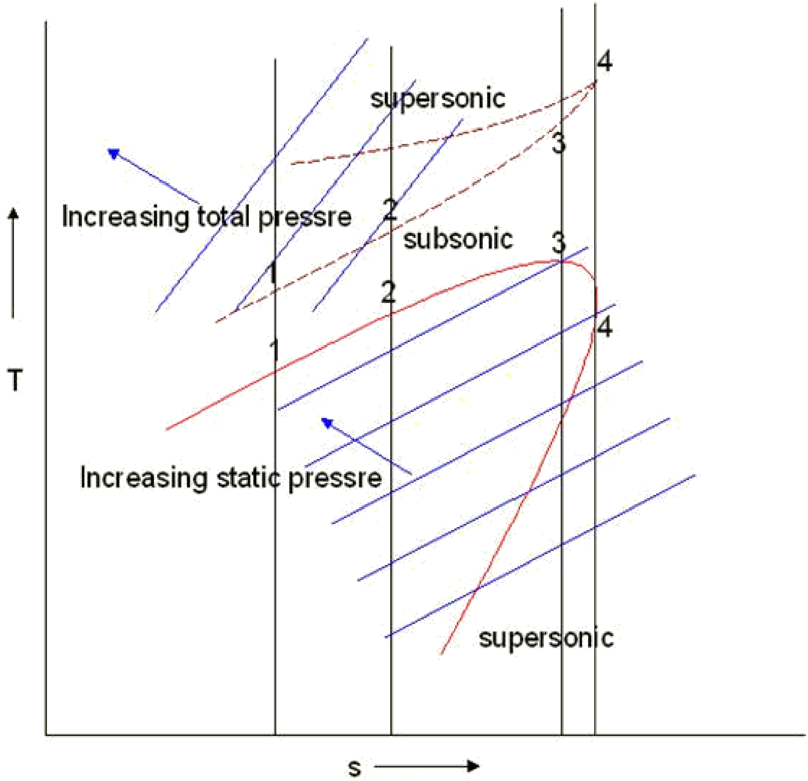


Fig. Rankine curve in h-s chart

$$dp - \frac{k^2}{\rho^2} d\rho = 0$$

Momentum Conservation

For energy conservation equation

$$(h_2 - h_1) + \left(\frac{v_2^{\,2}}{2} - \frac{v_1^{\,2}}{2}\right) = q$$

dh + udu = dq Energy Conservation

$P + vk^2 = \text{const}$

From laws of thermodynamics,

Tds = dh – vdp

However for point 3 (where Rankine line is tangent to an isotherm) dh=0 or dT=0, hence.

Tds = - vdp

From energy equation, for the same point we have,

Tds = udu

Hence,

- vdp = udu

Using differential form of mass conservation equation, we can replace du and we get,

$$-vdp = u\left(-\frac{u}{\rho}d\rho\right)$$

$$\frac{dp}{d\rho} = v^2$$

However, we know that point 3 also lies on an the isotherm, we can calulcate the slope of the isotherm to obtain another expression

for $dp/d\rho$ as,

$$p = \rho RT \qquad dp = d\rho RT \qquad \frac{dp}{d\rho} = RT$$

Hence we can put this $dp/d\rho$ in above equation

$$RT = u^2$$

$$\frac{\gamma RT}{\gamma} = u^2$$

$$\frac{a^2}{\gamma} = u^2$$

$$\frac{1}{\gamma} = \frac{u^2}{a^2}$$

$$M^2 = \gamma^{-1}$$

$$M = \gamma^{-1/2}$$

This procedure provides a formal proof for the Mach number at the point which corresponds to maximum temperature on Rayleigh curve. This expression also proves the fact that upper branch of Rankine curve (Fig.) in h-s diagram corresponds to subsonic flow.

Similarly we can also prove that the sonic point (M=1), represented by point 4, corresponds to maximum entropy.

We know the momentum conservation equation as

$$p + vk^2 = \text{const}$$

The differential form of the same can be written as

$$dp + k^2 dv = \text{const}$$

Hence slope of Rankine line on p-v diagram is

$$\frac{dp}{dv} = -k^2$$

We know that, at the point of maximum entropy, Rankine line is tangent to the isentropic. Hence slope of the isentropic at point 4 is

$$pv\gamma = \text{const}$$

Differentiating this equation we get

$$v\gamma dp + p\gamma v\gamma\text{-}1 dv = 0$$

$$\frac{dp}{dv} = -\frac{p\gamma}{v}$$

Equating the slopes we get,

$$\frac{dp}{dv} = -\frac{p\gamma}{v} = -k^2 = -(\rho u)^2$$

$$\frac{p\gamma}{v} = (\rho u)^2$$

$$\frac{p\gamma}{\rho} = u^2$$

$$a^2 = u^2$$

$$M = 1$$

This expression proves the fact that maximum entropy point on Rayleigh curve corresponds to Mach number 1 or sonic condition. We can extend our analysis about heat addition in subsonic flow and attainment of maximum entropy as follows.

We know that addition of heat increases the total temperature and hence total enthalpy of the flow.

$$h_0 = h + \frac{v^2}{2}$$

Hence the differential form of this equation is,

dh0 = dh + udu

However added heat gives rise to this change in total enthalpy, hence

dq = dh0

But $dq = Tds$. Therefore

$$Tds = dh0 = dh + udu$$

However at the maximum entropy point, ds = 0, therefore dh0 = 0. Hence we can conclude that, maximum entropy point also corresponds to maximum total enthalpy and hence total temperature point. Therefore further addition of heat is not possible for the given mass flow rate. Hence point 4 also represents the chocking condition. Change in upstream conditions take place if we add heat more that the heat required to chock the flow.

Choking of the flow with Heat Addition

It had been observed that the point of maximum entropy or maximum total temperature represents sonic condition. This condition also represents the maximum possible heat addition in a flow. However further increase in amount of heat addition in the flow decreases the mass flow rate through change in upstream or inlet conditions. Expected streamline pattern for un-choked and choked flow is shown in Fig. respectively.

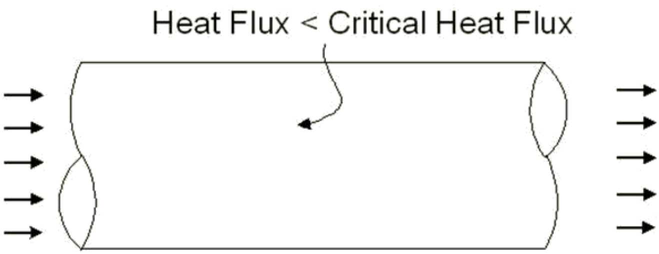


Fig. Streamline pattern for unchocked condition.

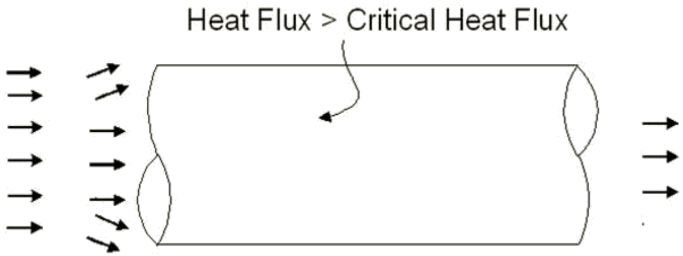


Fig. Streamline pattern for chocked condition.

Total Property Line of Rayleigh Curve:

Total enthalpy and entropy for 1D flow with heat addition has also been plotted in Fig. Slope of this total enthalpy curve can be obtained from the above equation as,

$$\frac{dh_0}{ds} = T$$

Value of the local temperature on h0-s diagram gives the slope of the curve. The Fig has been plotted for the single value of mass flow rate and we have already proved that upper branch of h-s diagram of Rayleigh line corresponds to subsonic flow.

Hence bottom branch showing monotonic h-s curve for lower static enthalpy at the same mass flow rate corresponds to supersonic flow. Since subsonic flow has lesser kinetic energy, it has higher static enthalpy in turn higher static temperature in comparison with the supersonic flow for the same mass flow rate.

Hence total enthalpy line for supersonic flow has lower slope than subsonic flow total enthalpy line. However both the lines should meet at highest total

enthalpy point or highest entropy or sonic point for this mass flux condition. Hence, supersonic flow total enthalpy line should be at the top of subsonic total enthalpy line which is unlike the static enthalpy entropy lines.
This fact can also be interpreted from the definition of total enthalpy where velocity appears in square. Since, mass flow rate is same and velocity of supersonic flow is higher than the velocity of subsonic flow, total enthalpy of supersonic flow will be higher than that of subsonic flow.

Both static and total enthalpy lines cut the constant pressure lines on the corresponding charts which represent static and total pressures respectively.

We can clearly see from Fig. that static pressure increases due to heat addition in supersonic flow and the same decreases for subsonic flow. It is also clear from this figure that total pressure decreases for both supersonic and subsonic flows due to heat addition.

Understanding of process of heat addition in supersonic flow is much simpler than that for subsonic flow. It is also evident from the h-s diagram which represents monotonic curve for supersonic flow.

It is clearly evident from this figure that static enthalpy, entropy and pressure increase due to heat addition for supersonic flow.

From equation of Rayleigh curve for p-v chart, we can also know that increase in pressure leads to decrease in specific volume and hence increase in density for supersonic flow with heat addition.
Highest amount of heat addition in the given supersonic flow leads to choking, maximum entropy and maximum temperature. Therefore point 4 represents the choking of supersonic flow as it represented for the subsonic flow of same mass flow rate.

It can be clearly concluded that the process of heat addition can lead to supersonic flow to sonic state and further heat rejection to lower subsonic state. Moreover, heat addition in subsonic flow can fetch the sonic conditions and further heat rejection leads to supersonic condition.

Hence possibility of reversible heat interaction eventually leads to trace the complete h-s plot of Rayleigh curve for a given mass flow rate.

Effect of Change in Mass Flow Rate on T-S diagram for Rayleigh flow:

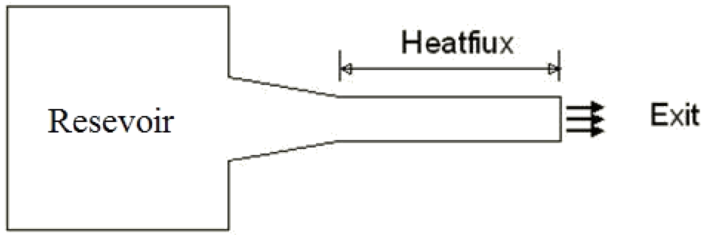


Fig A proposed experimental set up for 1D heat addition studies.

The h-s plot shown for heat addition process in Fig. belongs to a particular value of mass flow rate. If we change the mass flow rate then the curve changes. Hence we need to find out the new curve in case of decreased or increased mass flow rate conditions.

Consider the proposed experimental set up shown in Fig. for the Rayleigh flow studies. Here we can change the mass flow rate by changing the reservoir conditions (pressure and temperature) and also the exit pressure. During the experiment, gas stored in the reservoir expands through the nozzle and achieves certain mass flow rate while flowing through the constant area duct where heat addition takes place.

We are going to learn the topic "flow through nozzle" in detail in the coming lectures, however for now, we can assume that, mass flow rate for subsonic flow can be increased by decreasing the exit pressure or increasing the reservoir pressure. Similarly, we can increase the mass flow rate of supersonic flow by increase in reservoir pressure or by decreasing the reservoir temperature.

Consider the case of subsonic flow with increased mass flow rate. Initial isentropic expansion in the nozzle is given by the vertical line joining reservoir conditions and point 1 in Fig.

If we add heat in the flow through constant area duct then the process of heat addition is represented by the dotted curve in the same figure.

Suppose we increase the mass flow rate by decreasing exit pressure, then static enthalpy or static temperature at the entry to the heating duct decreases. Hence length of the initial vertical line increases and the nozzle exit conditions for subsonic flow are represented by 1'.

Further process of heat addition is shown by the thick line in the Fig. Hence increase in mass flow rate of subsonic flow shifts the Rayleigh curve downwards. Now consider the process of heat addition in supersonic flow. We will have to replace the nozzle connecting the reservoir and heat addition section in Fig. by corresponding supersonic nozzle to attain required supersonic Mach number at the entry to the constant cross-section heat addition section.

Dotted Rayleigh curve given in Fig. represents the process of heat addition in the initial mass flow rate of supersonic flow. Increase in this mass flow rate can be achieved by increasing the reservoir pressure which is necessarily the total pressure of the flow.

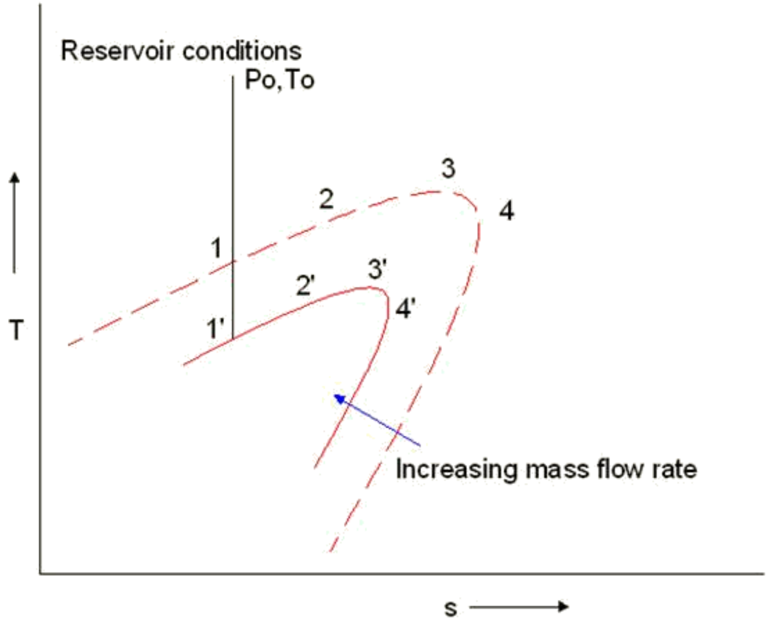


Fig. Rayleigh for increase in mass flow rate conditions

Since we have to increase the total pressure to increase the mass flow rate of supersonic flow, the static pressure, and enthalpy and hence temperature increases at the entry to the heating duct. Further process of heat addition is given by the thick line in fig. Hence increase in mass flow rate of supersonic flow shifts the Rayleigh curve upwards.

Two Rayleigh curves drawn for two mass flow rates can not cut each other since, in such a case, the point of intersection of two curves will represent two different values of a thermodynamic property like density which is impossible.

Therefore complete shrinking of the Rayleigh curve is necessary to represent the increased mass flow rate. This proves that maximum entropy attained by addition of heat decreases with increase in mass flow rate.

This understanding helps in accessing the situation where addition of heat is more than the required to reach sonic state for subsonic or supersonic flow. In both the situations, upstream mass flow rate at the heat addition station decreases.

Hence entering subsonic flow at the heating station becomes lower subsonic or entering supersonic flow becomes subsonic in the presence of a shock to accommodate the extra heat.

Differential form of equations for 1D heat addition:

We know the mass, momentum and energy equations for 1D heat addition process.

$$\rho u = const$$ (mass conservation)

$$\frac{d\rho}{\rho} = -\frac{du}{u}$$ (differential form of Mass conservation)

$$p + \rho u^2 = const$$ (momentum conservation)

$$dp + \rho u du = 0$$ (differential form of Momentum conservation)

$$dq = h_{0_2} - h_{0_1} = C_p T_{0_2} - C_p T_{0_1} = C_p dT_0$$ (energy conservation)

But we know that

$$h_0 = h + \frac{u^2}{2}$$

$$C_p T_0 = C_p T + \frac{u^2}{2}$$

Differential form of this equation is

$$dT_0 = dT + \frac{udu}{C_p}$$

Putting above equation in energy equation we get,

$$\frac{dq}{C_p} = dT + \frac{udu}{C_p}$$

Also,

$$\frac{dq}{C_p} = C_p \frac{dT}{du} + u \qquad \text{(differential form of energy equation)}$$

We also know the ideal gas equation,

$$p = \rho RT \qquad \text{(ideal gas equation)}$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} \qquad \text{(differential form of ideal gas equation)}$$

We can use all the above mentioned differential forms for better understanding of the flow with heat addition.

We will initially consider the differential form of energy equation to replace the term dT/du.

In this process, we can use differential form of mass conservation and ideal gas equation as,

$$\frac{dp}{p} = -\frac{du}{u} + \frac{dT}{T}$$

Using differential form of momentum conservation we can replace dP of the above equation as,

$$-\frac{\rho u du}{p} = -\frac{du}{u} + \frac{dT}{T}$$

Hence,

$$-\frac{\rho u}{p} + \frac{1}{u} = \frac{dT}{du}\frac{1}{T}$$

$$\frac{dT}{du} = \frac{T}{u} - \frac{\rho u T}{p} = \frac{T}{u} - \frac{u}{R}$$

Now we can use above equation to replace dT/du from differential form of energy equation. Hence differential form of energy equation becomes,

$$\frac{dq}{du} = \left(\frac{T}{u} - \frac{u}{R}\right)C_p + u$$

$$\frac{dq}{du} = C_p\frac{T}{u} - u\left(\frac{C_p}{R} - 1\right)$$

But we know that,

$$C_p = \frac{\gamma R}{\gamma - 1} \text{ or } \frac{C_p}{R} = \frac{\gamma}{\gamma - 1}$$

Therefore,

$$\frac{dq}{du} = C_p \frac{T}{u} - \left[\frac{\gamma}{\gamma - 1} - 1\right] u$$

$$\frac{dq}{du} = C_p \frac{T}{u} - \frac{u}{\gamma - 1}$$

Therefore, there are three cases viz. dq/du to be zero, positive or negative. Consider first case to start with,

$$C_p \frac{T}{v} = \frac{\gamma}{\gamma - 1}$$

$$u^2 = C_p(\gamma - 1)T = \gamma RT = a^2$$

$$M = 1$$

This particular case can be interpreted as further change in velocity is impossible with heat addition. Hence, dq/du to be zero represents maximum entropy or choking or sonic condition.

Now consider dq/du to be positive,

$$C_p \frac{T}{u} > \frac{u}{\gamma - 1}$$

$$\therefore a^2 > u^2$$

This particular case belongs to subsonic flow. Therefore for subsonic flow, dq/du is positive which implicitly means velocity of the subsonic flow increases (du > 0) if heat is added in it (dq > 0). At the same time, velocity of the subsonic flow decreases (du < 0) if heat is rejected from it (dq < 0).

Now consider dq/du to be negative,

$$C_p \frac{T}{u} < \frac{u}{\gamma - 1}$$

$$\therefore\ a^2 < u^2$$

This particular case belongs to supersonic flow. Therefore for supersonic flow, dq/du is negative which implicitly means velocity of the supersonic flow decreases (du < 0) if heat is added in it (dq > 0). At the same time, velocity of the supersonic flow increases (du > 0) if heat is rejected from it (dq < 0).
Now consider Eq.

$$\frac{dT}{du} = \frac{T}{u} - \frac{\rho u T}{p} = \frac{T}{u} - \frac{u}{R}$$

We can clearly see the existence of three cases here also, in which dT/du can be either zero, positive, or negative. Consider dT/du to be zero so,

$$\frac{T}{u} = \frac{u}{R}$$

$$\therefore\ u^2 = RT = \frac{\gamma RT}{\gamma} = \frac{a^2}{\gamma}$$

$$M = \gamma^{-1/2}$$

Hence for this condition we will have dT/du zero means dT = 0. This condition corresponds to maximum temperature attained by heat addition. From M = $\gamma^{-1/2}$, it becomes clear that, this condition belongs to subsonic flow where further increase in temperature is not possible with heat addition.

Now consider the case where, dT/du is positive. Hence

$$\frac{T}{u} > \frac{u}{R}$$

$$M^2 < \gamma^{-1/2}$$

This means that for subsonic flow till $M^2 < \gamma^{-1/2}$, dT/du is positive. We have already seen that for subsonic flow du is positive.

This proves that, for subsonic flow, dT is positive till $M^2 > \gamma^{-1/2}$

Now for the third case with dT/du is negative, we will get, $M > \gamma^{-1/2}$. However for subsonic flow du is always positive, hence for any subsonic Mach number which is $M > \gamma^{-1/2}$ till $M = 1$, dT is negative. Therefore in this range of subsonic Mach number added heat decreases temperature and increases the velocity.

However from present expression, dT/du is negative for all $M > \gamma^{-1/2}$. Hence for supersonic flows also dT/du is negative. But we have already seen that du is negative for supersonic flow with heat addition, hence dT is positive for supersonic flow. Hence externally added heat in supersonic flow decreases velocity and increases temperature.

We can modify the right hand side of Eq. as

$$\frac{TR - u^2}{Ru} = \frac{(\gamma RT)/u^2}{Ru} = \frac{a^2/\gamma - u^2}{Ru}$$

Hence, Eq. can be written as,

$$\frac{dT}{du} = \frac{a^2/\gamma - u^2}{Ru} = \frac{1 - M^2\gamma}{Ru}$$

The proof about all the critical points can be achieved from this expression as well.

TWO MARK QUESTIONS WITH ANSWERS

1. What is Rayleigh flow?

   Flow is a constant area duct with heat transfer and without friction is known as Rayleigh flow.

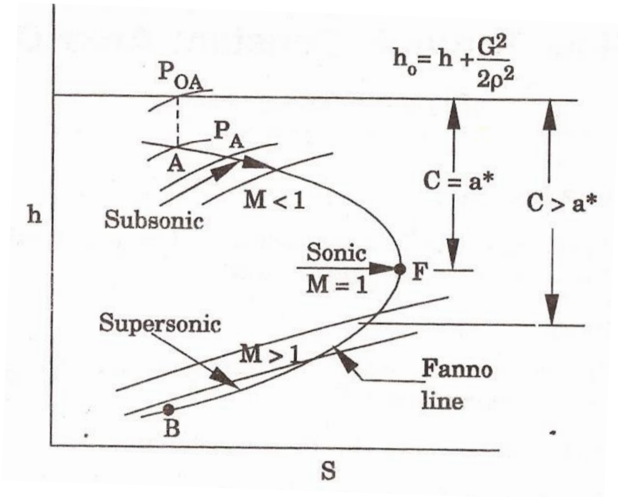
2. Define the term "Fanno flow".

   A steady one-dimensional flow in a constant area duct with friction in the absence of work and heat transfer is known as "fanno flow".

3. Define Fanno line.

   The locus of the state which satisfies the continuity and energy equation for a frictional flow is known as "fanno line".

4. Give fanno line in h – s diagram with isentropic stagnation line and show various Mach number regions.



5. Explain briefly the chocking in fanno flow.

   In a fanno line, any heating process (both subsonic and supersonic) will increase the enthalpy, entropy and mass flow rate. This will go upto the limiting state where Mach number M* = 1. Further heating is not possible, because the entropy change will be negative which violates the second law of thermodynamics. Hence the mass flow rate is maximum at the critical state and is constant afterwards, then the flow is said to be "chocked flow".

6. Give two practical examples where the fanno flow occurs.

   Flow occurs in gas ducts of aircraft propulsion engines, flow in air-conditioning ducts and flow of oil in long pipes etc.

7. Give the effect of increasing the flow length after reaching critical condition in a fanno flow.

   The mass flow rate will increase only upto the critical condition and is constant afterwards. Therefore, if the length of pipe is increased afterwards will not give any effect.

8. Write down the expression for the length of duct in terms of the two mach numbers M1 and M2 for a flow through a constant area duct with the influence of friction.

$$\frac{4fL}{D} = \left(\frac{4fL_{max}}{D}\right)_{M_1} - \left(\frac{4fL_{max}}{D}\right)_{M_2}$$

9. Define isothermal flow with friction. Give the applications.
   A steady one dimensional flow with friction and heat transfer in a constant area duct is called isothermal flow with friction. Such a flow occurs in long ducts where sufficient time is available for the heat transfer

to occur and therefore the temperature may remains constant. Hence the friction factor may be assumed constant along the duct. The applications of isothermal flow are oil or water flow in buried pipe.

10. State assumptions made to derive the equations for isothermal flow.

    1. One dimensional flow with friction and heat transfer.
    2. Constant area duct
    3. Perfect gas with constant specific heats and molecular weights
    4. Isothermal flow i.e., the temperature is constant
    5. On account of constant temperature the friction factor may be assumed constant along the duct.

11. Differentiate between isothermal flow and fanno flow.

    ISOTHERMAL FLOW
    a) Static temperature is constant
    b) With heat transfer.
    c) Flow occurs in long ducts where sufficient time is required for heat transfer.
    d) On account of constant temperature, the friction factor is assumed as constant.

    FANNO FLOW
    a) Static temperature is not constant
    b) Without heat transfer.
    c) Long ducts are not required.
    d) Friction factor is constant.

12. Define Rayleigh line.

    The locus of the points of properties during a constant area frictionless flow with heat exchange is called "Rayleigh line".

13. What is diabatic flow?

It is the flow which deals with the exchange of heat from the system in the absence of friction (Rayleigh flow).

14. Give the assumptions made in Rayleigh flow.

i. Perfect gas with constant specific heats and molecular weight.
ii. Constant area duct,
iii. One dimensional, steady frictionless flow with heat transfer.
iv. Absence of body forces.

15. What do you understand by chocking in Rayleigh flow?

When the fluid is heated in a subsonic region, the entropy increases and the mach number and fluid properties move to the right until the maximum entropy is reached where M* = 1. When the fluid is heated in a supersonic region, the entropy increases and the mach number and the fluid properties move to the right until the maximum entropy is reached where M* = 1. Further heating is not possible because, if it is heated the change in entropy is negative which violates the second law of thermodynamics. Therefore, the type of flow when the limiting condition M* = 1 is called “chocked flow”.

16. Differentiate between Fanno flow and Rayleigh flow.

FANNO FLOW
a) One dimensional steady frictional flow.
b) Stagnation temperature is constant.
c) Because of considering the wall friction forces it is accurate.
d) Without heat transfer.

RAYLEIGH FLOW

a) One dimensional steady frictionless flow.
b) Stagnation temperature is not constant
c) Less accurate.
d) With heat transfer.

17. Give two practical examples for Rayleigh flow.
    i) Flow in combustion chamber
    ii) Flow in regenerators
    iii) Flow in heat exchangers
    iv) Flow in intercoolers

18. Write down the expression for the pressure ratio of two sctions in terms of Mach number in Rayleigh flow.

$$\frac{p_2}{p_1} = \frac{1 + \gamma {M_1}^2}{1 + \gamma {M_2}^2}$$

19. State the stagnation pressure expression for Rayleigh process.

$$\frac{p_{02}}{p_{01}} = \frac{1 + \gamma {M_1}^2}{1 + \gamma {M_2}^2} \times \frac{\left[1 + \frac{\gamma - 1}{2} {M_2}^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} {M_1}^2\right]^{\frac{\gamma}{\gamma-1}}}$$

20. What is the value of Mach number of air at the maximum point in Rayleigh heating process?

    At maximum point in Rayleigh curve, the value of Mach number is one.

21. State the stagnation temperature expression for Rayleigh process.

$$\frac{T_{02}}{T_{01}} = \frac{{M_2}^2}{{M_1}^2} \times \left[\frac{1 + \gamma {M_1}^2}{1 + \gamma {M_2}^2}\right]^2 \times \left[\frac{1 + \frac{\gamma - 1}{2} {M_2}^2}{1 + \frac{\gamma - 1}{2} {M_1}^2}\right]$$

22. Write down the ratio of velocities between any two sections in terms of their Mach numbers in a Fanno flow.

$$\frac{c_2}{c_1} = \frac{M_1}{M_2} \times \left[\frac{1 + \frac{\gamma - 1}{2} {M_1}^2}{1 + \frac{\gamma - 1}{2} {M_2}^2}\right]^{1/2}$$

23. Write down the ratio of density between any two sections in terms of their Mach numbers in a Fanno flow.

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2} \times \left[\frac{1 + \frac{\gamma - 1}{2} {M_2}^2}{1 + \frac{\gamma - 1}{2} {M_1}^2}\right]^{1/2}$$

24. Write down the ratio of pressures between any two sections in terms of their Mach numbers in a Fanno flow.

$$\frac{p_2}{p_1} = \frac{M_1}{M_2} \times \left[\frac{1 + \frac{\gamma - 1}{2} {M_1}^2}{1 + \frac{\gamma - 1}{2} {M_2}^2}\right]^{1/2}$$

25. Write down the ratio of temperature between any two sections in terms of their Mach numbers in a Fanno flow.

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} {M_1}^2}{1 + \frac{\gamma - 1}{2} {M_2}^2}$$

26. Define Fanning's coefficient of skin friction.

It is defined as the ratio between wall shear stress τ and dynamic head.

27. Give the expression to find the increase in entropy for Fanno flow.

$$\frac{s_2 - s_1}{R} = \ln\left(\frac{M_1}{M_2}\right) \times \left[\frac{1 + \frac{\gamma - 1}{2} {M_1}^2}{1 + \frac{\gamma - 1}{2} {M_2}^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

EXERCISE PROBLEMS

1. The pressure, temperature and Mach number of air in a combustion chamber are 4 bar, 100°C and 0.2 respectively. The stagnation temperature of air in a combustion chamber is increased 3 times its initial value. Calculate
    i. The mach number, pressure and temperature at the exit
    ii. Stagnation pressure loss
    iii. Heat supplied per kg of air
2. Air enters 55cm diameter duct at pressure of 680 m bar, temperature of 300K and a Mach number of 3.0. The flow passes through a normal shock wave at a section L_1 meters downstream of the entry where the Mach number is 2.6. The Mach number at the exit (at a distance L_2 meters downstream of the shock) is 0.78. The mean coefficient of the friction is0.004. Determine
    i. The length L_1 and L_2
    ii. State of air at exit
    iii. Mass flow rate
3. The pressure, temperature and Mach number of the gas at exit are 2bar, 1200°C and 0.7 respectively. The ratio of stagnation temperature at exit to entry 3.85. Calculate the following.
    i. The mach number, pressure and temperature of the gas at entry
    ii. The heat supplied per kg of gas
    iii. The maximum heat supplied
    iv. Is it a cooling or heating process

    Take $\gamma = 1.3$ c_p=1.22KJ/kgK

4. Air enters 20mm diameter, 11m long pipe at a Mach number of 0.24, pressure of 2 bar and temperature of 300K. If the friction factor is 0.003. Determine the following
   i. Mass flow rate
   ii. Exit pressure
   iii. Exit temperature
   iv. Exit mach number
5. A long pipe of 25.4 mm diameter has a mean coefficient friction of 0.003. air enters the pipe at a mach number of 2.5, stagnation temperature of 310 K and static pressure of 0.507 bar. Determine for a sectionat which the mach number reaches 1.2
   i. Static pressure and temperature
   ii. Stagnation pressure and temperature
   iii. Velocity of air
   iv. Distance of this section from inlet (L)
   v. Mass flow rate
6. The friction factor for a 50mm diameter steel pipe is 0.005. at the inlet to the pipe the velocity is 70 m/s, temperature is 80°C and the pressure is 10 bar. Find the temperature, pressure and mach number at exit if the pipe is 25m long. Also determine the maximum possible length.
7. Air is heated in a constant area duct from a mach number of 0.2 to 0.8. The inlet stagnation conditions are 2 bar and 93°C. Determine the stagnation conditions of air at exit, the amount of heat transferred per unit flow and change in entropy.

MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	answer
The variation of steam pressure in the nozzle depends upon	velocity of steam	specific volume of steam	dryness fraction of steam	all of these	all of these
The steam enters the nozzle at a	high pressure and a low velocity	high pressure and a high velocity	low pressure and a low velocity	low pressure and a high velocity	high pressure and a low velocity
The ratio of the useful heat drop to the isentropic heat drop is called	condenser efficiency nozzle efficiency	nozzle efficiency	boiler efficiency	vacuum efficiency	nozzle efficiency
The critical pressure ratio is given by	p1p2	p1/p2	p2/p1	none of these	p2/p1
The critical pressure ratio for initially wet steam is	0.546	0.577	0.582	0.601	0.582
The critical pressure ratio for initially dry saturated steam is	0.546	0.577	0.582	0.601	0.577
The flow of steam is supersonic	at the entrance to the nozzle	at the throat of the nozzle	in the convergent portion of the nozzle	in the divergent portion of the nozzle	in the divergent portion of the nozzle
If the critical pressure ratio for steam is 0.546,then the steam is initially	wet steam	dry saturated steam	super saturated steam	none of these	super saturated steam
The difference of supersaturated temperature and saturation temperature at that pressure is called	degree of supersaturation	degree of super heat	degree of under cooling	none of these	degree of under cooling
Evaporation of water is a spontaneous process although it	is an exothermic reaction	is an endothermic reaction	is a photo chemical reaction	proceed without heat loss or heat gain	is an endothermic reaction
Identify the intensive property from the following	enthalpy	temperature	volume	refractive index	temperature, refractive index

The dissociation energy of CH_4 and C_2H_6 are respectively 360 & 620 k. cal/mole. The bond energy of C–C is	260 kcal/mole	180 kcal/mole	130 kcal/mole	80 kcal/mole	80 kcal/mole
what will happen if the air flowing through a nozzle	velocity increases, pressure decreases	velocity decreases, pressure increases	velocity constant, pressure decreases	velocity increases, pressure constant	velocity increases, pressure decreases
what will happen if the air flowing through a diffuser	velocity increases, pressure decreases	velocity decreases, pressure increases	velocity constant, pressure decreases	velocity increases, pressure constant	velocity decreases, pressure increases
Area of a nozzle section is _____ from start to end point.	is unpredictable	increases	decreases	remains constant	increases
Area of a diffuser section is _____ from start to end point.	is unpredictable	increases	decreases	remains constant	decreases
What happen to Mach number if flow through nozzle	is unpredictable	increases	decreases	remains constant	is unpredictable
What happen to Mach number if flow through diffuser	is unpredictable	increases	decreases	remains constant	decreases
What will happen if the air flowing through a nozzle is heated	velocity increases	pressure decreases	velocity decreases	pressure increases	pressure increases
What will happen if the air flowing through a diffuser is heated	velocity increases	pressure decreases	velocity decreases	pressure increases	pressure decreases
Nozzle exit pressure ratio is equal to the critical pressure ratio where the Mach number	M = 1	M = 0	M > 1	M < 1	M = 1
Which of the following statement is correct?	The expansion of steam in a nozzle follows Rankine cycle.	The friction in the nozzle decreases the dryness fraction of steam	The pressure of steam at throat is called stagnation pressure	The pressure at which the steam leaves the nozzle is known as critical pressure	The expansion of steam in a nozzle follows Rankine cycle.

The flow of steam is super-sonic	at the entrance to the nozzle	at the throat of the nozzle	in the convergent portion of the nozzle	in the divergent portion of the nozzle	in the divergent portion of the nozzle
The effect of supersaturation is that the	mass of the steam discharged increases	entropy and specific volume of the steam increases	exit velocity of steam reduces	all of these	all of these
The friction in the nozzle ___________ exit velocity of steam	has no effect on	decreases	increases	is unpredictable	decreases
The maximum discharge of steam through a convergent-divergent nozzle depends upon	area of nozzle at throat	initial pressure and volume of steam	final pressure of steam leaving the nozzle	both (a) and (b)	both (a) and (b)
The expansion of steam in a nozzle follows	Carnot cycle	Rankine cycle	Joule cycle	Stirling cycle	Rankine cycle
In Rayleigh flow, the velocity of gas at the maximum entropy point is	Sonic	Subsonic	Supersonic	Hypersonic	Sonic
Define Rayleigh flow	Flow through constant area ducts with heat transfer and friction	Flow through constant area ducts with heat transfer and without friction	Flow through constant area ducts friction with and without heat transfer	none of these	Flow through constant area ducts friction with and without heat transfer
The limiting point of the heating processes on both the subsonic and supersonic branches of the Rayleigh line is	Maximum entropy point	Maximum enthalpy point	Both (a) & (b)	none of these	Maximum entropy point
In Rayleigh flow, the Mach number is _______ by heating and _______ by cooling, at supersonic speeds	Increased, Increased	Decreased, Decreased	Increased, Decreased	Decreased, Increased	Decreased, Increased
The impulse function in case of Rayleigh flow	Remains constant	decreases	increases	is unpredictable	Remains constant
Which of the following statements is incorrect	The frictionless flow process with heat transfer in a constant area duct	In Rayleigh flow, the states of a gas moves towards the limiting point	The heat addition should correspond to entropy increase	In Rayleigh flow, the stagnation temperature	In Rayleigh flow, the stagnation temperature of the gas stream is remains constant.

	is referred as Rayleigh flow	during heating and away from it during cooling.	and heat rejection must correspond to an entropy decrease.	of the gas stream is remains constant.	

UNIT – III

NORMAL AND OBLIQUE SHOCKS

3.1. Introduction to Normal Shocks:

Shock waves are highly localized irreversibility's in the flow. Within the distance of a mean free path, the flow passes from a supersonic to a subsonic state, the velocity decreases suddenly and the pressure rises sharply. A shock is said to have occurred if there is an abrupt reduction of velocity in the downstream in course of a supersonic flow in a passage or around a body.

Normal shocks are substantially perpendicular to the flow and oblique shocks are inclined at any angle. Shock formation is possible for confined flows as well as for external flows. Normal shock and oblique shock may mutually interact to make another shock pattern.

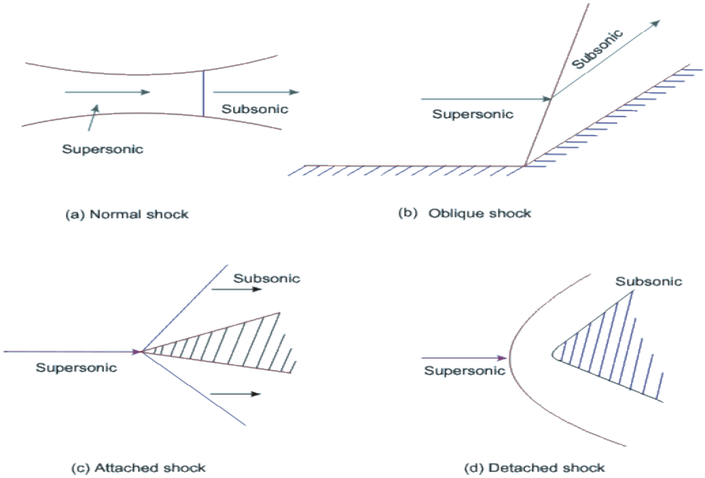


Fig. 3.1. Different type of Shocks

Figure below shows a control surface that includes a normal shock.

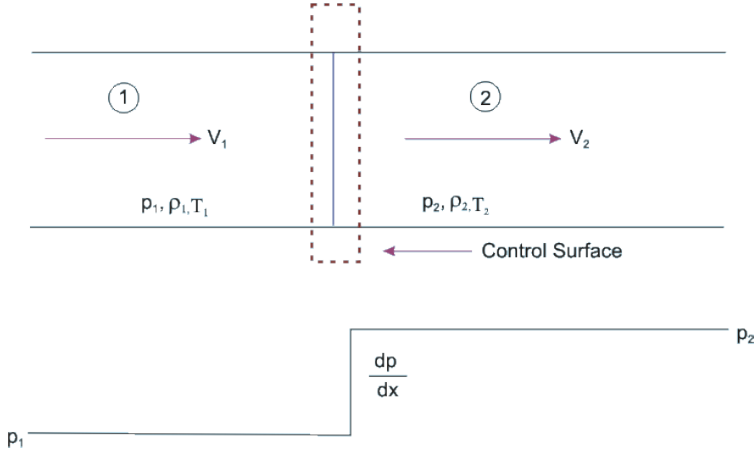


Fig .3.2.One Dimensional Normal Shock

The fluid is assumed to be in thermodynamic equilibrium upstream and downstream of the shock, the properties of which are designated by the subscripts 1 and 2, respectively.

Continuity equation can be written as

$$\frac{\dot{m}}{A} = \rho_1 V_1 = \rho_2 V_2 = G$$

Where G is the mass velocity kg/ m² s, and $\dot{m}$ is mass flow rate

From momentum equation, we can write

$$p_1 - p_2 = \frac{\dot{m}}{A}(V_2 - V_1) = \rho_2 V_2^2 - \rho_1 V_1^2$$

$$\Rightarrow p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

$$\Rightarrow \qquad F_1 = F_2$$

Where $p + \rho V^2$ is termed as Impulse Function

The energy equation is written as

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = h_{01} = h_{02} = h_0$$

where h_0 is stagnation enthalpy.

From the second law of thermodynamics, we know

$$s_2 - s_1 \geq 0$$

To calculate the entropy change, we have

$$T ds = dh - \forall dp$$

For an ideal gas

$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}$$

For an ideal gas the equation of state can be written as

$$p = \rho R T$$

For constant specific heat, the above equation can be integrated to give

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Equations above are the governing equations for the flow of an ideal gas through normal shock.

If all the properties at state 1 (upstream of the shock) are known, then we have six unknowns $T_2, p_2, \rho_2, V_2, h_2, s_2$ in these five equations.

We know relationship between h and T for an ideal gas, $dh = c_p\, dT$. For an ideal gas with constant specific heats,

$$\Delta h = h_2 - h_1 = c_p (T_2 - T_1)$$

Thus, we have the situation of six equations and six unknowns.

3.2. Normal shock relations:

It had already been discussed that the subsonic flow is pre-warned and supersonic flow is not. The reason behind this fact is that, any small amplitude disturbance travels with acoustic speed, however speed of fluid particle is more than the speed of sound in case of supersonic flows. Therefore the message of presence of the obstacle can not propagate upstream. Hence a messenger gets developed in front of the obstacle to warn the flow in order to avoid its direct collision with the obstacle. This messenger is called as shock.

In the presence of normal shock, fluid velocity decreases to the extent where flow Mach number behind the shock attains value below one. Due to this subsonic speed attainment of the flow, it becomes aware about the presence of the obstacle well in advance in the narrow space between shock and obstacle. Herewith we will deal for computation of flow properties behind the normal shock.
In the presence of a general obstacle the shock pattern is shown here in Fig.

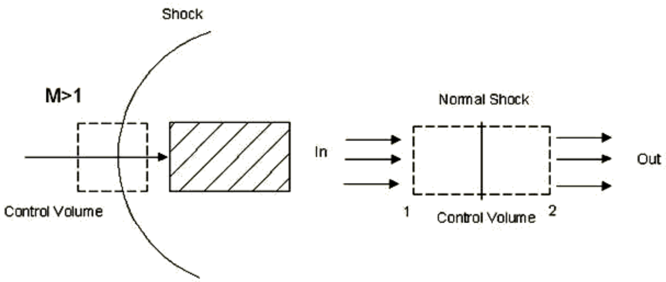


Fig.3.3. Shock pattern for a blunt or bluff obstacle

The shock for the stagnation streamline can be considered as normal to it. Therefore we can use the earlier derived 1D flow relations along with the assumptions of flow steady, adiabatic and inviscid flow. Consider a small control volume around normal shock for application of these relations between two stations of the control volume, mainly, inlet and outlet as shown in Fig.

$$\rho_1 u_1 = \rho_2 u_2$$
$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$
$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

Lets us examine the reference star properties of the flow in the process to calculate the flow properties behind the normal shock from the known inlet conditions. We can take the advantage of using stared temperature since the flow is adiabatic in nature.

Imagine that flow is adiabatically brought to Mach number one on either sides of the shock independently. In this case, we should get same stared temperature on either sides of shock. We can also show that total temperature is also same on either side.

The explicit formulation using the star temperature and concerned acoustic speed before the normal shock is,

$$h_1 + \frac{u_1^2}{2} = h_1^* + \frac{u_1^*}{2}$$
$$h_1 + \frac{u_1^2}{2} = C_p T^* + \frac{a^*}{2}$$
$$h_1 + \frac{u_1^2}{2} = C_p T^* + \frac{\gamma R T^*}{2} = \frac{\gamma R T^*}{\gamma - 1} + \frac{\gamma R T^*}{2} \left[\text{Q}\, C_p = \frac{\gamma R}{\gamma - 1} \right]$$
$$h_1 + \frac{u_1^2}{2} = \gamma \left[\frac{1}{\gamma - 1} + \frac{1}{2} \right] R T^* = \frac{\gamma(\gamma + 1)}{2(\gamma - 1)} R T^* = \frac{(\gamma + 1)}{2(\gamma - 1)} a^{*2}$$

Applying same strategy at the outlet we get,

$$h_2 + \frac{u_2^2}{2} = \frac{(\gamma+1)}{2(\gamma-1)} a^{*2}$$

However, we can write static enthalpy in terms of acoustic speed as,

$$h = C_p T = \frac{\gamma R}{\gamma - 1} T = \frac{\gamma R T}{\gamma - 1} = \frac{a^2}{\gamma - 1}$$

Therefore, the energy equation at the inlet becomes,

$$a_1^2 = \frac{(\gamma+1)}{2} a^{*2} - \frac{(\gamma-1)}{2} u_1^2$$

Similarly for the outlet station we have

$$a_2^2 = \frac{(\gamma+1)}{2} a^{*2} - \frac{(\gamma-1)}{2} u_2^2$$

Let's obtain the expression for velocity using mass and momentum equations to replace the acoustic speed term from equations.
From 1D mass and momentum conservation equations we have

$$\rho_1 u_1 = \rho_2 u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

Therefore,

$$\frac{p_1 + \rho_1 u_1^2}{\rho_1 u_1} = \frac{p_2 + \rho_2 u_2^2}{\rho_2 u_2}$$

$$\frac{p_1}{\rho_1 u_1} + u_1 = \frac{p_2}{\rho_2 u_2} + u_2 \left[\text{Q } \rho_1 u_1 = \rho_2 u_2 \text{ mass conservation} \right]$$

$$\frac{a_1^2}{\gamma u_1} + u_1 = \frac{a_2^2}{\gamma u_2} + u_2$$

$$u_2 - u_1 = \frac{a_1^2}{\gamma u_1} - \frac{a_2^2}{\gamma u_2}$$

Using the above equations, we get the following equation

$$u_2 - u_1 = \frac{1}{\gamma u_1}\left[\frac{(\gamma+1)}{2}a^{*2} - \frac{(\gamma-1)}{2}u_1^2\right] - \frac{1}{\gamma u_2}\left[\frac{(\gamma+1)}{2}a^{*2} - \frac{(\gamma-1)}{2}u_2^2\right]$$

Rearranging the terms of above equation, we get

$$u_2 - u_1 = \frac{1}{\gamma u_1}\frac{(\gamma+1)}{2}a^{*2} - \frac{1}{\gamma u_1}\frac{(\gamma-1)}{2}u_1^2 - \frac{1}{\gamma u_2}\frac{(\gamma+1)}{2}a^{*2} + \frac{1}{\gamma u_2}\frac{(\gamma-1)}{2}u_2^2$$

$$u_2 - u_1 = \frac{1}{\gamma u_1}\frac{(\gamma+1)}{2}a^{*2} - \frac{1}{\gamma u_2}\frac{(\gamma+1)}{2}a^{*2} + \frac{1}{\gamma u_2}\frac{(\gamma-1)}{2}u_2^2 - \frac{1}{\gamma u_1}\frac{(\gamma-1)}{2}u_1^2$$

Further rearrangements give

$$u_2 - u_1 = \frac{(\gamma+1)}{2\gamma}a^{*2}\left[\frac{u_2 - u_1}{u_1 u_2}\right] + \frac{(\gamma-1)}{2\gamma}[u_2 - u_1]$$

$$u_2 - u_1 = \frac{(\gamma+1)}{2\gamma}a^{*2}\left[\frac{1}{u_1} - \frac{1}{u_2}\right] + \frac{(\gamma-1)}{2\gamma}[u_2 - u_1]$$

$$u_2 - u_1 = \frac{(\gamma+1)}{2\gamma}\left[\frac{a^{*2}}{u_1 u_2}\right] + \frac{(\gamma-1)}{2\gamma}$$

Necessary rearrangement for the above equation is as given,

$$\left[\frac{a^{*2}(\gamma+1)}{u_1 u_2}\right]\frac{1}{2\gamma} = \frac{2\gamma - \gamma + 1}{2\gamma}$$

$$a^{*2} = u_1 u_2$$

$$\frac{u_1}{a^{*2}} = \frac{u_2}{a^{*2}} \Rightarrow M_1^{*2} = \frac{1}{M_2^{*2}}$$

This expression shows that, M_1^{*2} and M_2^{*2} are reciprocal of each other for a normal shock. This equation is called as Prandtl's relation for normal shock which can be used to prove that Mach number becomes subsonic behind the normal shock

3.3. Deviation of Mach number based on starred quantities:

The Prandtl's expression gives the relation between the starred acoustic speed based Mach numbers ahead and behind the shock. Using this expression, we can prove that $M_1^{*2} > 1$ which leads to $M_2^{*2} < 1$. Let’s derive the expression initially for M*

$$a^2 = \frac{(\gamma+1)}{2(\gamma-1)}a^{*2} - \frac{(\gamma-1)}{2}u^2$$

$$\frac{a^2}{\gamma-1} = \frac{\gamma+1}{(\gamma-1)2}a^{*2} - \frac{u^2}{2}$$

Dividing by square of velocity,

$$\therefore \frac{a^2/u^2}{\gamma-1} = \frac{\gamma+1}{(\gamma-1)2}\frac{a^{*2}}{u^2} - \frac{1}{2}$$

$$\therefore \frac{1/M^2}{\gamma-1} = \frac{\gamma+1}{(\gamma-1)2}\frac{1}{M^{*2}} - \frac{1}{2}$$

$$\therefore M^2 = \frac{2}{\left[\gamma+1/M^{*2}\right] - (\gamma-1)}$$

$$\therefore M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

Which clearly proves that, for M > 1, M* > 1 for M = 1, M* = 1 and M < 1, M* < 1

Therefore, from Prandtl’s relation becomes

$$\frac{1}{M_1^{*2}} = M_2^{*2}$$

Where M1 > 1 ⇒ M1*2 > 1 & M2 < 1 ⇒ M2*2 < 1

This clearly proves that, any supersonic flow while pass through normal shock attains subsonic speed behind it.

From known free stream Mach number or the Mach number ahead of the shock we can calculate the Mach number behind the shock using Prandtl's relation.

$$\frac{1}{M_1^{*2}} = M_2^{*2}$$

$$\therefore \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} = \frac{1}{\dfrac{(\gamma+1)M_2^2}{2+(\gamma+1)M_2^2}}$$

$$\therefore M_2^2 = \frac{1+\left[{(\gamma+1)}/{2}\right]M_1^2}{\gamma M_1^2 - {(\gamma-1)}/{2}}$$

We can derive the expression for the properties behind the shock wave using 1D conservation equations and know properties ahead the shock.

$$\rho_1 u_1 = \rho_2 u_2 \qquad \text{Mass Conservation Equation}$$

$$\therefore \rho_1 u_1^2 = \rho_2 u_1 u_2 \qquad \text{Multipling both the sides by } u_1$$

$$\therefore \rho_1 u_1^2 = \rho_2 a^{*2} \qquad \text{Using Prandtl's relation}$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1^2}{a^{*2}} = M_1^{*2}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} \qquad \text{Using relation between } M_1^2 \;\&\; M_1^{*2}$$

This equation gives the density ratio which is function of free stream Mach number and the specific heat ratio. We can find out the velocity ratio from this density ratio as,

$$\rho_1 u_1 = \rho_2 u_2 \qquad \text{Mass Conservation Equation}$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2}$$

$$\frac{u_1}{u_2} = \frac{(\gamma+1)M_1^{\ 2}}{2+(\gamma-1)M_1^{\ 2}}$$

Let's derive the expression for static pressure ratio. For simplicity of derivation, initially representation of dynamic pressure is necessary and can be expressed as follows

$$\rho_1 u_1^2 = \frac{\rho_1}{\gamma p_1} u_1^2 \gamma p_1 = \gamma p_1 \frac{u_1^2}{a_1^2} = \gamma p_1 M_1^2$$

From definition of Mach number

$$M = \frac{\gamma p}{\rho}$$

We know the 1D momentum conservation equation as

p1 + ρ1u12 = p2 + ρ2u22

Replacing the dynamic pressure from either side and rearranging we get

$$p_2 - p_1 = \gamma p_1 M_1^2 \left[1 - \frac{u_2}{u_1}\right]$$

$$\frac{p_2 - p_1}{p_1} = \gamma M_1^2 \left[1 - \frac{u_2}{u_1}\right]$$

$$\frac{p_2}{p_1} - 1 = \gamma M_1^2 \left[1 - \frac{u_2}{u_1} \right]$$

$$\text{But } \frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^{\ 2}}{2+(\gamma-1)M_1^{\ 2}}$$

$$\therefore \frac{p_2}{p_1} - 1 = \gamma M_1^2 \left[1 - \frac{2+(\gamma-1)M_1^{\ 2}}{(\gamma+1)M_1^{\ 2}} \right]$$

$$\therefore \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

This equation gives the static pressure ratio which is again function of free stream Mach number and the specific heat ratio. Temperature ratio can be obtained from the pressure and density ratio as,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{\rho_2}{\rho_1}\right)$$

$$\frac{T_2}{T_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_1^{\ 2} - 1\right) \right] \left[\frac{2+(\gamma-1)M_1^{\ 2}}{(\gamma+1)M_1^{\ 2}} \right]$$

Expression for static enthalpy ratio across the shock will be same as the static temperature ratio given by above equation.

Following things are clear from the property ratios, All the ratios are dependant on free stream Mach number and specific heat ratio (γ), where Mach number is flow property and specific heat ratio is fluid property.

Static pressure ratio, P_2/P_1 increases with increase in γ for same free stream Mach number.

Hence, P_2/P_1 Will be more for air than carbon-dioxide for the same Mach number since γ for carbon-dioxide is 1.28 while for air is 1.4. The variation in γ

can occur for the same gas due to dissociation, ionization or vibrational excitation, combustion, mixing of two gasses of different specific heat ratios etc.

P_2/P_1 increases with increase in Mach number for same γ (for same gas). ρ_2/ρ_1 ratio increases with increase in Mach number for same γ (for same gas). Density ratio, ρ_2/ρ_1, increases with decrease in γ, so for carbon-dioxide although it has lower P_2/P_1 for a given Mach number and γ, it will have higher ρ_2/ρ_1.

T_2/T_1 has the same behaviour as P_2/P_1 where it increases with increase in M and γ.

Each P_2/P_1 and $T_2/T_1 \to \infty$ as $M \to \infty$. However ρ_2/ρ_1 attains a constant value,

$$\frac{\gamma+1}{\gamma-1},$$

as $M\to \infty$. For air (($\gamma = 1.4$) that constant value is 6.

Mach number behind shock decreases as free stream Mach number increases. However, as $M1\to \infty$, M2 attains a constant value

$$\sqrt{\frac{\gamma-1}{2\gamma}}$$

Which is 0.378 for air ($\gamma = 1.4$)
Mach number behind the normal shock (M_2) increases as γ increases for a given free stream Mach number.
We know that entropy change in any process can be estimated as,

$$S_2 - S_1 = C_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

We can express the static temperature and pressure ratios in terms of free stream Mach number, Hence,

$$S_2 - S_1 = C_p \ln\left\{\left[1+\frac{2\gamma}{\gamma+1}\left(M_1^{\,2}-1\right)\right]\left[\frac{2+(\gamma-1)M_1^{\,2}}{(\gamma+1)M_1^{\,2}}\right]\right\} - R\ln\left[1+\frac{2\gamma}{\gamma+1}\left(M_1^{\,2}-1\right)\right]$$

From this expression it is clear that the entropy change would be positive if and only if M1 > 1. That means shock is present only for supersonic flows and not for subsonic flows according to second law of thermodynamics.

We know that 1D energy conservation equation is

$$h_1 + \frac{u_1^{\,2}}{2} = h_2 + \frac{u_2^{\,2}}{2}$$

If we image that the gas is isentropically brought to zero velocity on either side of the shock then the enthalpies would be stagnation enthalpies.

$$\therefore h_{01} = h_{02}$$

$$\therefore C_pT_{01} = C_pT_{02}$$

$$\therefore T_{01} = T_{02}$$

This expression re-asserts that flow field is adiabatic since total temperature of the gas remains constant across the shock. Hence compression through shock is adiabatic irreversible.

We can use already derived isentropic relation of total density to static density, equation, to obtain the total density ratio across the shock.

$$\frac{\frac{\rho_{o1}}{\rho_1}}{\frac{\rho_{o2}}{\rho_2}} = \frac{\rho_{o1}}{\rho_{o2}}\frac{\rho_2}{\rho_1} = \frac{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{1}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{1}{\gamma-1}}}$$

$$\therefore \frac{\rho_{o2}}{\rho_{o1}} = \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{1}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{1}{\gamma-1}}}\frac{\rho_2}{\rho_1}$$

$$\therefore \frac{\rho_{o2}}{\rho_{o1}} = \left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{1}{\gamma-1}}\left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]$$

Similarly we can find out the total pressure ratio across the shock using the isentropic relation given by

$$\frac{P_{o2}}{P_{o1}} = \frac{\frac{P_{o2}}{P_2}}{\frac{P_{o1}}{P_1}}\frac{P_2}{P_1} = \frac{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}\left[1+\frac{2\gamma}{\gamma+1}\left(M_1^2-1\right)\right]$$

Total pressure and total density ratios are of same magnitude for the ideal gas conditions, since total temperature remains constant across the shock

This can be proved using ideal gas equation.

$$P = \rho RT$$

$$\therefore \frac{P}{\rho} = RT$$

$$\therefore \frac{P_{o1}}{\rho_{o1}} = RT_{o1}$$

$$\therefore \frac{P_{o2}}{\rho_{o2}} = RT_{o2}$$

$$\therefore \frac{P_{o1}}{\rho_{o1}} = \frac{P_{o2}}{\rho_{o2}}$$

$$\therefore \frac{P_{o2}}{P_{o1}} = \frac{\rho_{o2}}{\rho_{o1}}$$

We have derived all the static and total property ratios as function of M_1 & γ, however, we can always express these relations as function of M_2 & γ, since M_2 is also function of M_1.

We can calculate the entropy change across the shock given by using the presently derived ratios,

$$S_2 - S_1 = C_P \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

If we imagine that the gas element is isentropically brought to zero on either sides of shock then

$$S_1 = S_{o1},\ S_2 = S_{o2}$$

$$\therefore S_2 - S_1 = S_{o2} - S_{o1}$$

$$\therefore S_2 - S_1 = C_P \ln\frac{T_{o2}}{T_{01}} - R\ln\frac{P_{o2}}{P_{o1}}$$

$$\therefore S_2 - S_1 = -R\ln\frac{P_{o2}}{P_{o1}}$$

$$\frac{P_{01}}{P_{02}} = e^{-\frac{\Delta S}{R}}$$

This proves that P_{o2}/P_{o1} is less than 1 since Δs is positive for shocked flow. This expression suggests that the total pressure decreases across the normal shock.

3.4. Hugoniot equation:

Shock wave compression is an adiabatic and irreversible process; hence shock waves can be treated as irreversible adiabatic compressors. Let's calculate the work required for this compression process.

$$\rho_1 u_1 = \rho_2 u_2$$

$$u_1 = \left(\frac{\rho_2}{\rho_1}\right) u_2$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$p_1 + \frac{\rho_2^2 u_2^2}{\rho_1} = p_2 + \rho_2 u_2^2$$

$$u_2^2 \left(\frac{\rho_2^2}{\rho_1} - \rho_2 \right) = p_2 - p_1$$

$$u_2^2 \left(\frac{\rho_2^2 - \rho_1 \rho_2}{\rho_1} \right) = p_2 - p_1$$

$$u_2^2 \frac{\rho_2}{\rho_1} (\rho_2 - \rho_1) = p_2 - p_1$$

$$u_2^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_1}{\rho_2} \right)$$

Similarly

$$u_1^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_2}{\rho_1} \right)$$

We know from the energy equation that,

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \qquad \text{Energy Conservation}$$

$$\frac{p_1}{\rho_1} + e_1 + \frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_2}{\rho_1} \right) \frac{1}{2} = \frac{p_2}{\rho_2} + e_2 + \frac{p_2 - p_1}{\rho_2 - \rho_1} \left(\frac{\rho_1}{\rho_2} \right) \frac{1}{2}$$

Putting the expression for u1 and u2

$$e_2 - e_1 = \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} + \frac{1}{2}\frac{(p_2 - p_1)}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right) - \frac{1}{2}\frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right)$$

$$e_2 - e_1 = \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} + \frac{\rho_2^2(p_2 - p_1) - \rho_1^2(p_2 - p_1)}{2(\rho_2 - \rho_1)\rho_1\rho_2}$$

$$e_2 - e_1 = \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} + \frac{p_1(\rho_1^2 - \rho_2^2) - p_2(\rho_1^2 - \rho_2^2)}{2(\rho_2 - \rho_1)\rho_1\rho_2}$$

$$e_2 - e_1 = \frac{2(\rho_2 - \rho_1)\rho_2 p_1 - 2(\rho_2 - \rho_1)\rho_1 p_2 + p_1(\rho_1^2 - \rho_2^2) - p_2(\rho_1^2 - \rho_2^2)}{2(\rho_2 - \rho_1)\rho_1\rho_2}$$

$$e_2 - e_1 = \frac{2(\rho_2 - \rho_1)\rho_2 p_1 - 2(\rho_2 - \rho_1)\rho_1 p_2 + p_1(\rho_1^2 - \rho_2^2) - p_2(\rho_1^2 - \rho_2^2)}{2(\rho_2 - \rho_1)\rho_1\rho_2}$$

$$e_2 - e_1 = \frac{p_1\rho_2^2 - 2p_1\rho_1\rho_2 + p_1\rho_1^2 + P_2\rho_2^2 - 2p_2\rho_1\rho_2 + p_2\rho_1^2}{2(\rho_2 - \rho_1)\rho_1\rho_2}$$

$$e_2 - e_1 = \frac{p_1(\rho_2 - \rho_1)^2 + p_2(\rho_2 - \rho_1)^2}{2(\rho_2 - \rho_1)\rho_1\rho_2}$$

$$e_2 - e_1 = \left(\frac{p_2 + p_1}{2}\right)\left[\frac{(\rho_2 - \rho_1)}{\rho_1\rho_2}\right]$$

$$e_2 - e_1 = \left(\frac{p_2 + p_1}{2}\right)\left[\frac{1}{\rho_1} - \frac{1}{\rho_2}\right]$$

The above equation is called as the Hugoniot equation which relates thermodynamic quantities across the normal shock. This relation is same as the first law of thermodynamics which states that internal energy change for an adiabatic process is equal to work done or pressure (average pressure in the present case) times change in specific volume.

We can plot the Hugoniot curve for the known initial pressure-volume conditions for various pressure-volume conditions after normal shock. Hence

Hugoniot curve, in principle, joins all the possible points on p-v plane starting from a known point on p-v plane. To generate such a plot we have to modify the Hugoniot equation by expressing the internal energy in terms of the pressure and volume as,

$$e = c_v T = \frac{R}{\gamma - 1} T = \frac{pv}{\gamma - 1}$$

$$e_2 - e_1 = \left(\frac{p_2 + p_1}{2} \right) [v_1 - v_2]$$

$$\left(\frac{p_2 v_2 - P_1 v_1}{\gamma - 1} \right) = \left(\frac{p_2 + p_1}{2} \right) [v_1] - \left(\frac{p_2 + p_1}{2} \right) [v_2]$$

$$v_2 \left[\frac{p_2}{\gamma - 1} + \frac{p_2 + p_1}{2} \right] = v_1 \left[\frac{p_1}{\gamma - 1} + \frac{p_2 + p_1}{2} \right]$$

$$v_2 = v_1 \frac{\left[\frac{p_1}{\gamma - 1} + \frac{p_2 + p_1}{2} \right]}{\left[\frac{p_2}{\gamma - 1} + \frac{p_2 + p_1}{2} \right]}$$

Using this expression, we can get all the possible values of v_2 for given values P_1, P_2 and υ_1 and hence we can plot the p-v diagram or Hugoniot curve for normal shock. The line joining initial point and any point on the curve specifies particular mass flow rate and hence a particular free stream Mach number.

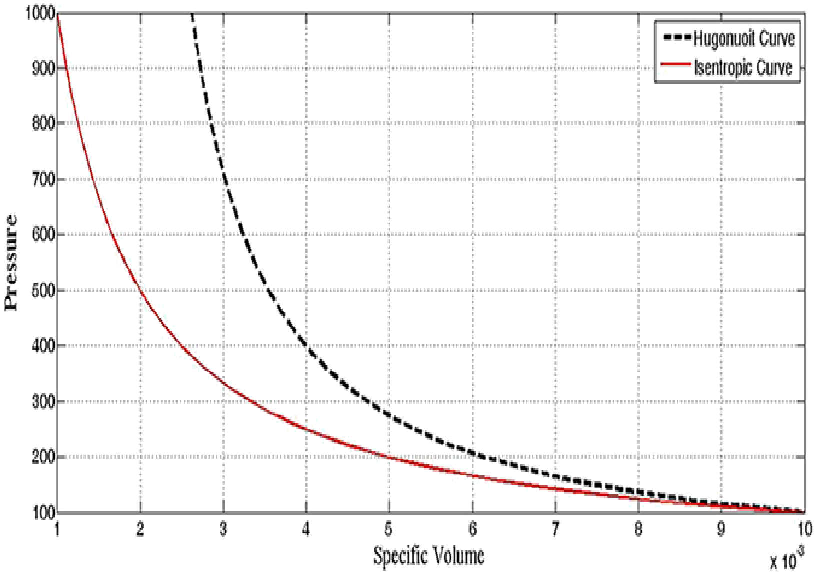


Fig.3.4

Now let's try to understand the basics of normal shock in view of this Hugoniot relation. The expression for specific volume is a consolidated equation for normal shock conditions. If we have a normal shock condition due to supersonic velocity u_1, then as we know from equation above

$$u_1^{\,2} = \frac{P_2 - P_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right) \text{Since } \rho = \frac{1}{v}$$

$$u_1^{\,2} = \frac{P_2 - P_1}{\frac{1}{v_2} - \frac{1}{v_1}}\left(\frac{v_1}{v_2}\right)$$

$$\frac{P_2 - P_1}{v_2 - v_1} = -\left(\frac{u_1}{v_1}\right)^2$$

This expression gives the slope of a straight line joining two points of Hugoniot curve, mainly initial point and any other point on the Hugoniot curve on p-v diagram. These two points necessarily define the upstream and downstream locations of the normal shock respectively. Since this expression is for slope and is comprised of velocity and density (inverse of specific volume), this line necessarily corresponds to a particular mass flow rate.

$$\frac{P_2 - P_1}{v_2 - v_1} = -\left(\rho_1 \ u_1 \right)^2 = -\left(\text{Massflux} \right)^2$$

Therefore if we know the mass flow rate and initial conditions we can easily find out the post shock conditions. These conditions are given by point of intersection of Hugoniot curve and the straight line drawn from initial conditions with slope equation.

In the same figure Hugoniot curve is plotted along with isentropic curve for compression. These curves originate from the same point. Slope of curve representing isentropic compression can be calculated as,

$$pv^{\gamma} = const$$

$$\frac{dp}{dv} v^{\gamma} + p\gamma v^{\gamma-1} = 0$$

$$\frac{dp}{dv} = -\frac{p\gamma}{v} = -p\gamma\rho = -\frac{p\gamma\rho^2}{\rho} = -a^2\rho^2$$

$$\frac{dp}{dv} = -\left(\frac{a^2}{v^2}\right)$$

This suggests that, at the initial condition is same for isentropic and Hugoniot curves. This means that the slope of both the curves is same at that point. This proves that, we have u1 = a1, M1 = 1 at the initial or starting point. Therefore for sonic flow there is no change in properties across the shock. As free stream Mach number increases u1 becomes greater than a1 and slope

$$(\tan\alpha) = -\left[\frac{u_1}{\upsilon_1}\right]^2_{\text{Hugoniot}} < -\left[\frac{a_1}{\upsilon_1}\right]^2_{\text{isentrop}} \quad \text{so} \quad \left|\left[\frac{u_1}{\upsilon_1}\right]^2\right| > \left|\left[\frac{a_1}{\upsilon_1}\right]^2\right|$$

Therefore slope of Hugoniot curve is more than the slope for isentropic curve for supersonic flows. Hence the graph herewith provides an evidence for the fact that, although isentropic compression is efficient, compression by normal shock is more effective since compression by normal shock gives higher pressure rise for same change in specific volume. Hence shock can also be called as thermodynamic compressor.

3.5. Hugoniot Curve:

We have understood the typical Hugoniot curve in Figure 3.4. As per the Hugoniot curve, there are many possible states behind the shock for a given initial condition. Therefore, knowledge of mass flux or Mach number is necessary to arrive at a particular state.

A straight line of the slope proportional to the mass flux or Mach number when drawn along with Hugoniot curve, gives a fixed state behind the normal shock for known upstream state properties.

The point at which this straight line cuts the Hugoniot curve corresponds to the state behind the shock. Possibility of such a state is still unclear since the procedure for finding the post shock condition is based on Hugoniot relation which in turn is based on mass, momentum equations and first law of thermodynamics.

The straight line equation is also based on mass and momentum equations only. Therefore to select a possible state for a given initial conditions and known mass flux, we have to use second law of thermodynamics.

Here we can prove the impossibility of expansion shock and presence of shock in supersonic and hypersonic flows.

We know that the Hugoniot equation is

$$e_2 - e_1 = u_2 - u_1 = \frac{p_1 + p_2}{2}\left[v_1 - v_2\right]$$

Since we know properties at station 1 therefore these can be treated as constants in this equation.

$$e_2 - e_1 = u_2 - u_1 = \frac{p_1 + p_2}{2} v_1 - \frac{p_1 + p_2}{2} v_2$$

Differentiating above equation

$$du = \frac{p_1 + p_2}{2}(0 - dv) + \frac{v_1 - v_2}{2}(0 + dp)$$

$$du = \frac{p_1 + p_2}{2}(-dv) + \frac{v_1 - v_2}{2}(dp)$$

We can assume that there is a reversible heat addition process in a closed system for which we will have same initial and final states as that of the normal shock. Therefore the internal energy and entropy change for assumed process will be same as that of the normal shock case.

Lets apply laws of thermodynamics to the assumed process.

dQ = du + pdv

Tds = du + pdv

du = Tds - pdv

Since change in internal energy is same for both the processes, let's equate the change in internal energy after removing subscript 2 we get,

$$Tds - pdv = \frac{p_1 + p}{2}(-dv) + \frac{v_1 - v}{2}(dp)$$

$$Tds - pdv = -\frac{1}{2}p_1dv - \frac{1}{2}pdv + \frac{1}{2}v_1dp - \frac{1}{2}vdp$$

$$Tds = -\frac{1}{2}p_1dv + \frac{1}{2}pdv + \frac{1}{2}v_1dp - \frac{1}{2}vdp$$

$$Tds = \frac{1}{2}dv(p - p_1) - \frac{1}{2}dp(v - v_1)$$

$$2Tds = dv(p - p_1) - dp(v - v_1)$$

$$2T\frac{ds}{dp} = (v_1 - v)\left[1 - \frac{(p_1 - p)}{v_1 - v}\frac{dv}{dp}\right]$$

Here T is the temperature at which heat is added reversibly to the assumed process. But we are overlapping the normal shock with the assumed reversible heat addition process. However, from combination of mass and momentum equation we have already derived the slope for normal shock in terms of mass flow rate. The same thing is mentioned herewith in short.

Mass flux = pv = p1v1 = p2v2 = constant = k

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$p_1 + \frac{k^2}{\rho_1} = p_2 + \frac{k^2}{\rho_2}$$

$$p_1 + v_1 k^2 = p_2 + v_2 k^2$$

$$k^2 = -\frac{p_1 - p_2}{v_1 - v_2}$$

This is the slope of the straight line drawn to intersect Hugoniot curve. Using above expression, we can re-write equation as,

$$2T\frac{ds}{dp} = (v_1 - v)\left[1 + \frac{k^2}{dp/dv}\right]$$

Since derivatives in the above expression are of the properties of system which can belong to any process (normal shock or reversible heat addition).

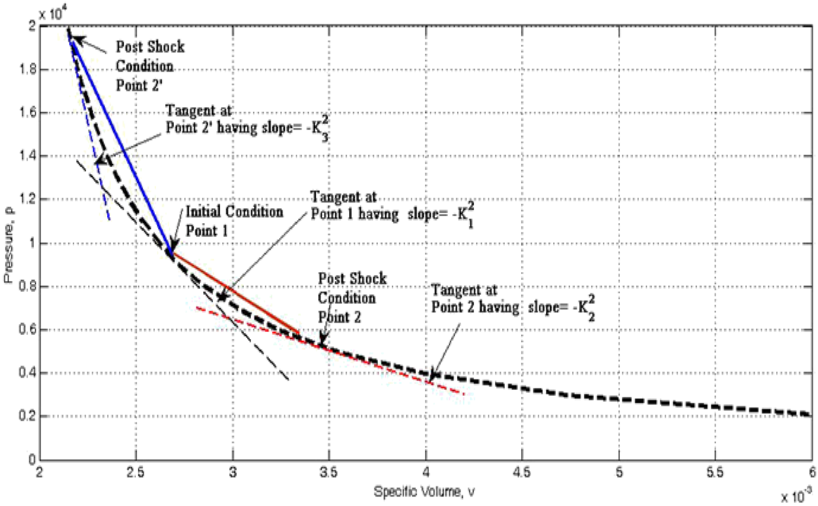


Fig 3.5: Analysis of normal shock using Hugoniot Curve

The above figure 3.5 will help us in finding most probable post shock condition. In this figure, along with Hugoniot curve, three different lines are drawn to get various possible post shock conditions. These lines correspond to different mass flow rates or free stream conditions forming the normal shock. One of those lines is tangent to Hugoniot curve at point 1 which represents the initial condition or pre-shock conditions. Hence the straight line at point 1 has slope

(-k_{12}) which is same as the Hugoniot curve at that point, *dp/dv*. Therefore term in bracket of

$$\left[1+\frac{k^2}{dp/dv}\right]$$

Becomes 2. However, (v1 - v) is equal to zero since there is no change in specific volume. Therefore *ds/dp* is equal to zero intern dS is equal to zero which makes this process possible.

Now consider the line with slope k_{22} intersecting at point 2 to Hugoniot curve. For this case right hand side of eq. (9.3) can be written as,

$$\left[1+\frac{k^2}{dp/dv}\right]=1-\frac{-k_2^{\,2}}{\left[dp/dv\right]_2}=1-\ \frac{\text{Solpe of straightline joining 1\&2}}{\text{Slope of tangent at point 2}}$$

for the corresponding line can be written as,

$$2T\left(\frac{\partial s}{\partial p}\right)_{at2}=\left(v_1-v_2\right)\left[1+\frac{k_2^{\,2}}{\left(\partial p/\partial v\right)_{at2}}\right]$$

3.6 Oblique Shock:

The discontinuities in supersonic flows do not always exist as normal to the flow direction. There are oblique shocks which are inclined with respect to the flow direction. Refer to the shock structure on an obstacle, as depicted qualitatively in Fig.1.6.

The segment of the shock immediately in front of the body behaves like a normal shock. Oblique shock can be observed in following cases- Oblique shock formed as a consequence of the bending of the shock in the free-stream direction (shown in Fig.1.6)

In a supersonic flow through a duct, viscous effects cause the shock to be oblique near the walls, the shock being normal only in the core region. The shock is also oblique when a supersonic flow is made to change direction near a sharp corner

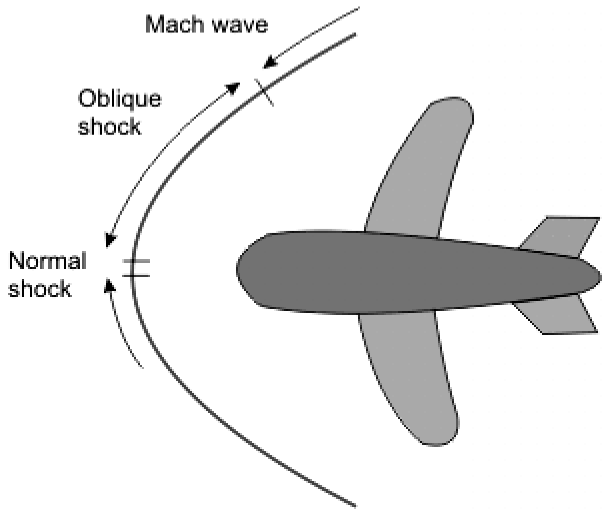


Fig 3.6 Normal and oblique Shock in front of an Obstacle

The relationships derived earlier for the normal shock are valid for the velocity components normal to the oblique shock. The oblique shock continues to bend in the downstream direction until the Mach number of the velocity component normal to the wave is unity. At that instant, the oblique shock degenerates into a so called Mach wave across which changes in flow properties are infinitesimal.

Let us now consider a two-dimensional oblique shock as shown in Fig.1.7 below

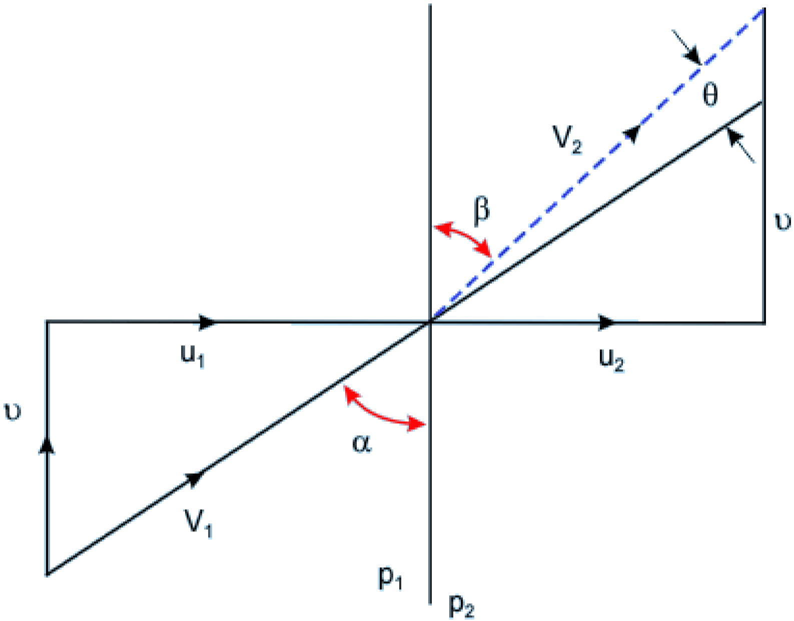


Fig 3.7 Two dimensional Oblique Shock

1.51 Oblique Shock Wave Relations:

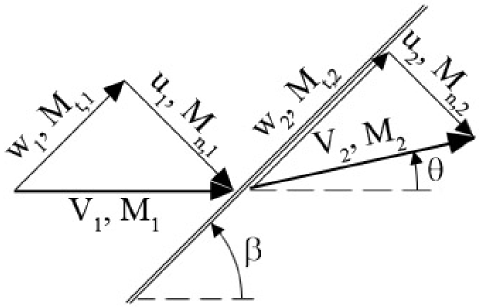


Fig 3.8 Properties of oblique shock wave

We know that the velocity V_1 before the shock wave is directed horizontally. We examine two components of this velocity: The component normal to the shock wave u_1 and the component tangential to the shock wave w_1. Corresponding are the Mach number normal to the shock wave $M_{n,1}$ and the Mach number tangential to the shock wave $M_{t,1}$.

We can do the same for the velocities after the shock wave (but now with subscript 2). All the properties have been visualized in figure 1.8. Also note the deflection angle θ.

Using the variables described above, we can derive some relations. It turns out that these relations are virtually the same as for a normal shock wave. There's only one fundamental difference. Instead of using the total velocity, we only need to consider the component of the velocity normal to the shock wave (being u). We then get

$$\rho_1 u_1 = \rho_2 u_2,$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2,$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}.$$

To find the tangential component of the velocity w by using the momentum equation, we can derive the simple relation

$$w_1 = w_2$$

So now we have used the continuity equation, the momentum equation and the energy equation. In the previous chapter we now continued to express ratios like p_2/p_1 as a function of the Mach number.

We can do the same again. However, this time we express everything in the component of the Mach number normal to the flow, being

$$M_{n,1} = M_1 \sin\beta.$$

Going through a lot of derivations, we can find that

$$M_{n,2}^2 = \frac{2 + (\gamma - 1) M_{n,1}^2}{2\gamma M_{n,1}^2 - (\gamma - 1)} \qquad \text{with} \qquad M_2 = \frac{M_{n,2}}{\sin(\beta - \theta)},$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma + 1) M_{n,1}^2}{2 + (\gamma - 1) M_{n,1}^2},$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}\left(M_{n,1}^2 - 1\right),$$

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} = \left(1 + \frac{2\gamma}{\gamma + 1}\left(M_{n,1}^2 - 1\right)\right)\left(\frac{2 + (\gamma - 1) M_{n,1}^2}{(\gamma + 1) M_{n,1}^2}\right).$$

We can once more see that these equations are virtually the same as for a normal shock wave. The only difference is that we now need to take the component of the Mach number normal to the flow.

Important Formulas:

1. Down stream Mach number or Mach number after the shock

$$M_y^{\,2} = \frac{\frac{2}{\gamma - 1} + M_x^{\,2}}{\frac{2\gamma}{\gamma - 1} M_x^{\,2} - 1}$$

2. Static pressure ratio across the shock

$$\frac{p_y}{p_x} = \frac{2\,\gamma}{\gamma + 1}\; M_x{}^2 - \frac{\gamma - 1}{\gamma + 1}$$

3. Static temperature ratio across the shock

$$\frac{T_y}{T_x} = \frac{\left[1 + \frac{\gamma - 1}{2}\; M_x{}^2\right]\left[\frac{2\,\gamma}{\gamma - 1}\; M_x{}^2 - 1\right]}{\frac{1}{2}\,\frac{(\gamma + 1)^2}{\gamma - 1}\; M_x{}^2}$$

4. Density ratio across the shock

$$\frac{\rho_y}{\rho_x} = \frac{1 + \frac{\gamma + 1}{\gamma - 1} \times \frac{p_y}{p_x}}{\frac{\gamma + 1}{\gamma - 1} + \frac{p_y}{p_x}}$$

5. Stagnation pressure loss

$$\Delta p_o = p_{ox} - p_{oy}$$

6. Percentage of stagnation pressure loss

$$\Delta p_o = \frac{p_{ox} - p_{oy}}{p_{ox}} \times 100$$

7. Increase in entropy

$$\Delta s = R\;\ln\left(\frac{p_{ox}}{p_{oy}}\right)$$

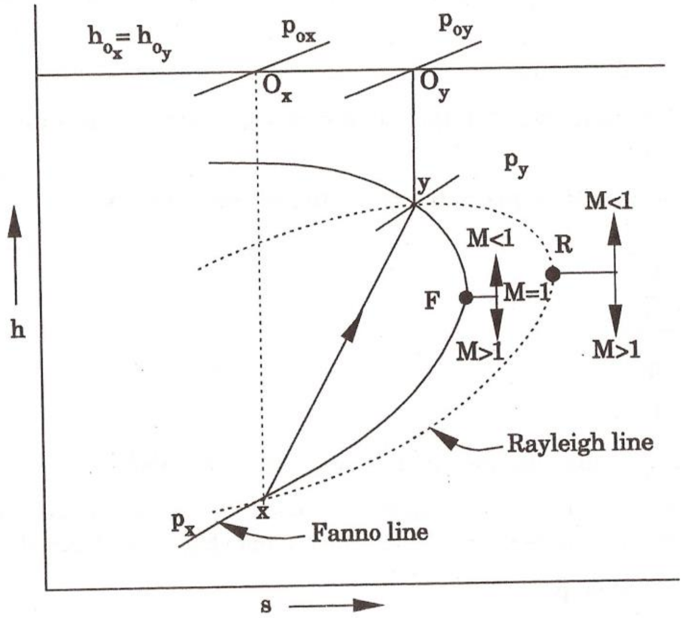
8. Diffuser Efficiency

$$\eta_D = \frac{\frac{T_{o1}}{T_1}\left(\frac{p_{ox}}{p_{oy}}\right)^{\gamma-1/\gamma} - 1}{\frac{\gamma - 1}{2}\; M_1{}^2}$$

Questions and Answers

1. What is the normal shock?
   When the shock waves are right angles to the direction of flow and the rise in pressure is abrupt are called normal shock waves.
2. What is meant by normal shock as applied to compressible flow?
   Compression wave front being normal to the direction of compressible fluid flow. It occurs when the flow is decelerating from supersonic flow. The fluid properties jump across the normal shock.

3. Shock waves cannot develop in subsonic flow? State the reason.
Shocks are introduced to increase the pressure and hence it is a deceleration process. Therefore, shocks are possible only when the fluid velocity is maximum. In a subsonic flow, the velocity of fluid is less then the critical velocity and hence deceleration is not possible. Thus, shock waves cannot develop in subsonic flow.
4. Define strength of a shock wave.
Strength of a shock wave is defined as the ratio of increase in static pressure across the shock to the inlet static pressure.
5. Define oblique shock where it occurs.
The shock wave which is inclined at an angle to the two dimensional flow direction is called as oblique shock. When the flow is supersonic, the oblique shock occurs at the corner due to the turning of supersonic flow.
6. Give the difference between normal and oblique shock.
NORMAL SHOCK
(a) The shock waves are right angles to the direction of flow.
(b) May be treated as one dimensional analysis.
OBLIQUE SHOCK
(a) The shock waves are inclined at an angle to the direction of flow.
(b) Oblique shock is two dimensional analysis.
7. What is Prandtl-Meyer relation? What its significance?
The fundamental relation between gas velocities before and after the normal shock and the critical velocity of sound is known as Prandtl-Meyer relation. i.e., (i) c_x x $c_y = a^{*2}$ and (ii) M^*_x x $M^*_y = 1$. It signifies the velocities (before and after the shock) with the critical velocity of sound and the product of mach numbers before and after the shock is unity.
8. Show a normal shock in h-s diagram with the help of Rayleigh line and Fanno line.



9. How the Mach number before and after the occurrence of a normal shock are related?

$$M_y^{\,2} = \frac{\frac{2}{\gamma - 1} + M_x^{\,2}}{\frac{2\,\gamma}{\gamma - 1}\,M_x^{\,2} - 1}$$

10. Give the expression for T_y/T_x across a normal shock.

$$\frac{T_y}{T_x} = \frac{\left[1 + \frac{\gamma - 1}{2}\,M_x^{\,2}\right]\left[\frac{2\,\gamma}{\gamma - 1}\,M_x^{\,2} - 1\right]}{\frac{1}{2}\,\frac{(\gamma + 1)^2}{\gamma - 1}\,M_x^{\,2}}$$

11. Write the equation to find efficiency of the diffuser.

$$\eta_D = \frac{\frac{T_{o1}}{T_1}\left(\frac{p_{ox}}{p_{oy}}\right)^{\gamma-1/\gamma} - 1}{\frac{\gamma-1}{2}\ {M_1}^2}$$

12. Write down the static pressure ratio expression for a normal shock.

$$\frac{p_y}{p_x} = \frac{2\,\gamma}{\gamma+1}\ {M_x}^2 - \frac{\gamma-1}{\gamma+1}$$

13. Write the Rankine-Hugoniot equation

$$\frac{\rho_y}{\rho_x} = \frac{1+\frac{\gamma+1}{\gamma-1}\times\frac{p_y}{p_x}}{\frac{\gamma+1}{\gamma-1}+\frac{p_y}{p_x}}$$

14. What are the properties which changes across the normal shock?

    Stagnation pressure decreases, stagnation temperature remains constant and the static temperature and static pressure increases.

15. Give some applications for moving shock wave.

    Jet engines, shock tubes, supersonic wind tunnel and practical admission turbines.

16. Shock waves cannot develop in subsonic flow, why?

    In subsonic flow, the velocity of fluid is less than the velocity of sound. Due to this reason, deceleration is not possible in subsonic flow. So shock waves cannot be developed in subsonic flow.

17. Define compression and rarefaction shock waves.

    A shock wave which is at a higher pressure than the fluid into which it is moving is called as compression shock wave.
    A shock wave which is at a lower pressure than the fluid into which it is moving is called as expansion or rarefaction shock wave.

18. State the necessary conditions for a normal shock to occur in compressible flow.

    The compression wave is to be at right agle to the compressible flow and the flow should be supersonic.

19. Is the flow thorugh a normal shock an equilibrium one.

    No, since the fluid properties like pressure, temperature and density are changed during normal shock.

20. Calculate the strength of shock wave when normal shock appears at M =2.

$$\xi = \frac{p_y - p_x}{p_x} = \frac{p_y}{p_x} - 1$$

Referring normal shock table for $M_x = 2$ and $\gamma = 1.4$

$$\frac{p_y}{p_x} = 4.5$$

$$\therefore \ \xi = 4.5 - 1 = 3.5$$

MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	answer
Normal shock occurs only from_______.	V2=V1	V2>V1	V1=V2=1	V2<V1	V2<V1
Subsonic flow over the wedge body produces ________ shock.	Oblique	normal	expansion	No	Oblique
When the oblique shock is reflected from the solid surface,it will produce ________ shock.	Oblique	normal	expansion	No	Oblique
Shock will always occur in flow with_________.	M = 0	M = 1	M > 1	M < 1	M > 1
When flow cross the normal shock wave, total pressure will be _________.	decreased	increased	zero	Constant	decreased
Detached shock wave occurs in a flow over _______ body.	blunt	sharp	wedge	Constant	blunt
The gas in cooling chamber of a closed of a closed cycle gas turbine is cooled at	constant volume	constant temperarure	constant pressure	none of these	constant pressure
Intercooling in gas turbines	decreases net output but increases thermal efficiency	increases net output but increases thermal efficiency	decreases net output and thermal efficiency both	increases net output and thermal efficiency both	decreases net output but increases thermal efficiency
The maximum temperature in gas turbine is	200ºC	500ºC	700ºC	1000ºC	700ºC
Calculate the enthalpy change when 50 ml of 0.01 M $Ca(OH)_2$ reacts with 25 ml of 0.01 M HCl. Given that ΔH^0_{neut} of a strong acid and strong base is 140 cal/ equivalent	14.0 cal	35 cal	10.0 cal	7.5 cal	35 cal
The factor that does not influence the heat of reaction	the physical state of	the temperature	the pressure or volume	the method by which the final	the method by which the final products are obtained

	reactants and products			products are obtained	
An endothermic reaction is spontaneous only if	the entropy of the surrounding increases	entropy of the system increases	total entropy decreases	none	entropy of the system increases
A gas turbine is also know as ________________	IC Engines	Jet Engine	External Combustion Engine	Turbine	IC Engines
The Aircraft Gas turbines work on	Otto Cycle	Rankine Cycle	Kelvin Cycle	Brayton Cycle	Brayton Cycle
In an ideal Brayton cycle Combustion take place at	Constant Pressure	Constant Volume	Cant Temperature	Constant Internal Energy	Constant Pressure
As Specific Fuel Consumption of the engine decreases, the Range of the Aircraft	Decreaseonsts	Increases	Remains Constant	None of the given	Increases
According to first law of thermodynamics	Total Energy is constant	Total Entropy is constant	Work done is constant	Both a and b	Total Energy is constant
Chocking condition in nozzle	M>1	M>1	M=1	All the given options	M=1
Nozzle is a device designed to control the direction and characteristics of a fluid flow Especially to----------	increase the pressure	decrease the velocity	increase the velocity	decrease the pressure	increase the velocity
De Laval nozzle is also called as	convergent divergent nozzle	Subsonic nozzle	Supersonic nozzle	hypersonic nozzle	convergent divergent nozzle
The rear third of the combustion liner is the ________.	Transition section	Liner wall	Flame stabilization zone	Can	Flame stabilization zone
Dump diffuser in the combustion chamber is used to ________.	Increase the velocity abruptly	Decrease the velocity quickly	Increase the pressure	Decrease the pressure	Decrease the velocity quickly
.____________compressors are used in medium to large thrust jet engines	axial	centrifugal	mixed flow	none of these	axial
Pressure rise in a ramjet is achieved by a-------------	diffuser	centrifugal compressor	axial compressor	none of these	diffuser

Normal shocks are always________.	one-dimensional	two-dimensional	three-dimensional	All of these	one-dimensional
An incident shock gets reflected as a shock from a solid boundary is called _________.	like reflections	simple regions	unlike reflections	non-simple regions	like reflections
Oblique shock is ____________ to flow direction	parallel	perpendicular	Inclined	All of these	Inclined
The three-dimensional Shock is known as _________.	Normal shock	Oblique shock	Expansion wave	Mach wave	Expansion wave
Flow across the normal shock wave is _________.	isentropic	iso baric	non isentropic	reversible adiabatic	non isentropic
When the Mach number ahead of a normal shock is infinity the Mach number behind the normal shock is	infinity	high supersonic	Zero	low subsonic	low subsonic
The lowest value of shock angle for oblique shocks is	Zero	12.5 deg	Mach angle	15 deg	Mach angle
is a pressure drag resulting from static pressure components located to either side of compression or shock waves	pressure drag	Wave Drag	skinfr	all of these	Wave Drag
Assumptions made to derive Normal Shock relation	Flow is 1-D, invisid	Flow is 2-D, invisid	Flow is 2-D, visicous	Flow is 3-D, invisid	Flow is 1-D, invisid
is the weak limit of an oblique shock wave	Mach wave	normal shock	Mach angle	shock	Mach wave

UNIT IV

JET PROPULSION

4.1. Basic Theory of Jet Propulsion:

Jet propulsion is the propelling force generated in the direction opposite to the flow of a mass of gas or liquid under pressure. The mass escapes through a hole or opening called a jet nozzle.

A familiar example is the nozzle at the end of a fire hose. The nozzle forms a smaller passageway through which the water must flow. The nozzle increases the velocity of the water, giving the term, “a jet of water.”

4.2. Principle:

There are many everyday examples of jet propulsion. A blown-up toy balloon with its neck closed shows no tendency to move because the air inside is pressing equally in all directions. If the neck is opened suddenly, the balloon shoots away. The escaping air relieves pressure at the neck, and there is a reaction from the air opposite the neck. It is not the air rushing out of the neck and pushing against the outside air, however, that drives the balloon ahead. It is the air pushing against the inside front wall of the balloon that propels it forward.

In fact, a jet would operate more efficiently in a vacuum because there would be no air to obstruct the escaping gases. The recoil of a rifle also illustrates action and reaction. Expanding gases propel the bullet out of the barrel at high velocity. The rifle in response to the force of the gases "kicks back." Another example of jet action is the garden hose whose nozzle jumps back when the water is suddenly turned on full force.

4.3. Types:

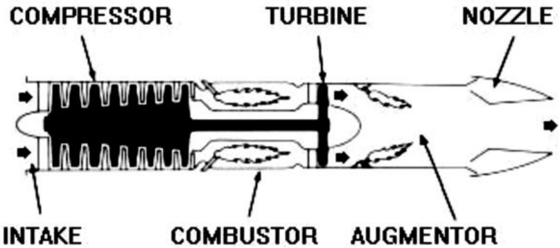
There are two general types of jet propulsion air-breathing and non air-breathing engines. Air-breathing engines use oxygen from the atmosphere in the combustion of fuel. They include the turbojet, turboprop, ramjet, and pulse-jet. The term jet is generally used only in reference to air-breathing engines.

Non air-breathing engines carry an oxygen supply. They can be used both in the atmosphere and in outer space. They are commonly called rockets and are of two kinds' liquid-propellant and solid-propellant.

Air-breathing engines may be further divided into two groups, based on the way in which they compress air for combustion. The turbojet and turboprop each has a compressor, usually turbine-driven, to take in air. They are called gas-turbine engines. The ramjet and the pulse-jet do not have compressors.

4.4. The Turbojet Engine:

The turbojet engine consists of diffuser which shows down the entrance air and thereby compresses it and slows down the entrance air and thereby compresses it, a simple open cycle gas turbine and an exist gas into kinetic energy. The increased velocity, of air thereby produces thrust.
Figure 4.1 & 4.2 shows the basic arrangement of the diffuser, compressor, combustion chamber, turbine and the exhaust nozzle of a turbojet engine. Of the total pressure rise of air, a part is obtained by the rain compression in the diffuser and rest in the compressor. The diffuser converts kinetic energy of the air into pressure energy. In the ideal diffuser, the air is diffused isentropically down to zero velocity. In the actual diffuser the process is irreversible adiabatic and the air leaves the diffuser at a velocity between 60 and 120 m/s.



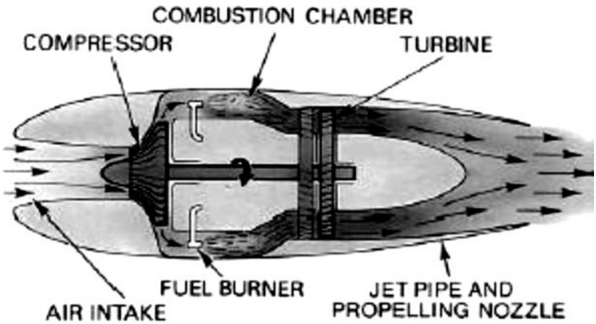


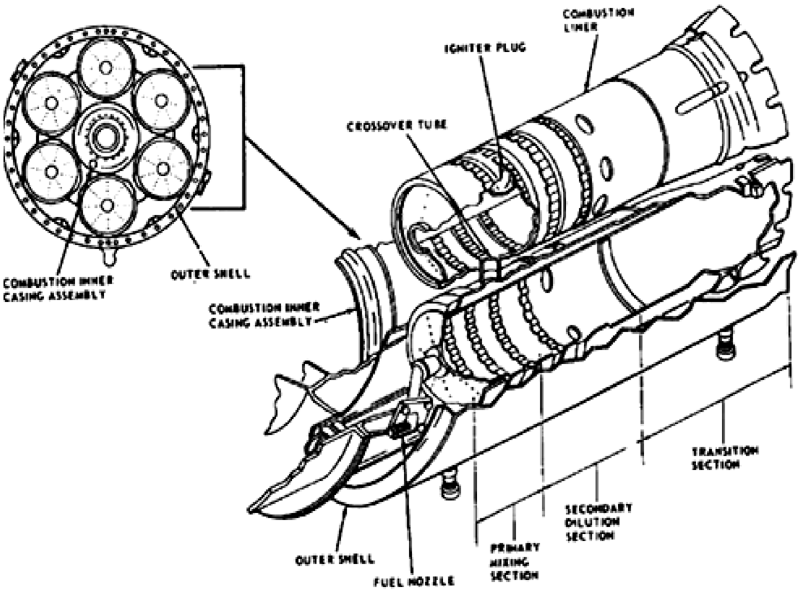
Fig. 4.1. & 4.2. Turbo Jet Engine

The composer used in a turbojet can be either centrifugal type or axial flow type. The centrifugal compressor gives a pressure ratio of about 4:1 to 5:1 in a single stage and usually a double-sided rotor is used. The turbojet using centrifugal compressor has a short and sturdy appearance. The advantages of centrifugal compressor are high durability, ease of manufacture and low cost and good operation under adverse conditions such as icing and when sand and small foreign particles are inhaled in the inlet duct. The primary disadvantage is the lack of straight-through airflow. Air leaves compressor in radial direction and ducting with the attendant pressure losses is necessary to change the direction. The axial flow is more efficient than the centrifugal type and gives the turbojet a long slim, streamlined appearance.

The engine diameter is reduced which results in low aircraft drag. A multistage axial flow compressor can develop a pressure ratio as high as 6:1 or more. The air handled by it is more than that handled by a centrifugal compressor of the same diameter. A variation of the axial compressor, the twin-spool (dual spool, split spool or coaxial) compressor has two or more sections, each revolving at or near the optimum speed for its pressure ratio and volume of air. A very high-pressure ratio of about 9:1 to 13:1 is obtained. The use of high-pressure ratio gives very good specific fuel consumption and is necessary for thrust ratings in the region of 50000 N or greater. In the combustion chamber heat is added to the compressed air nearly at constant pressure. The three types being ‘can’, ‘annular’ and ‘can-annular’.

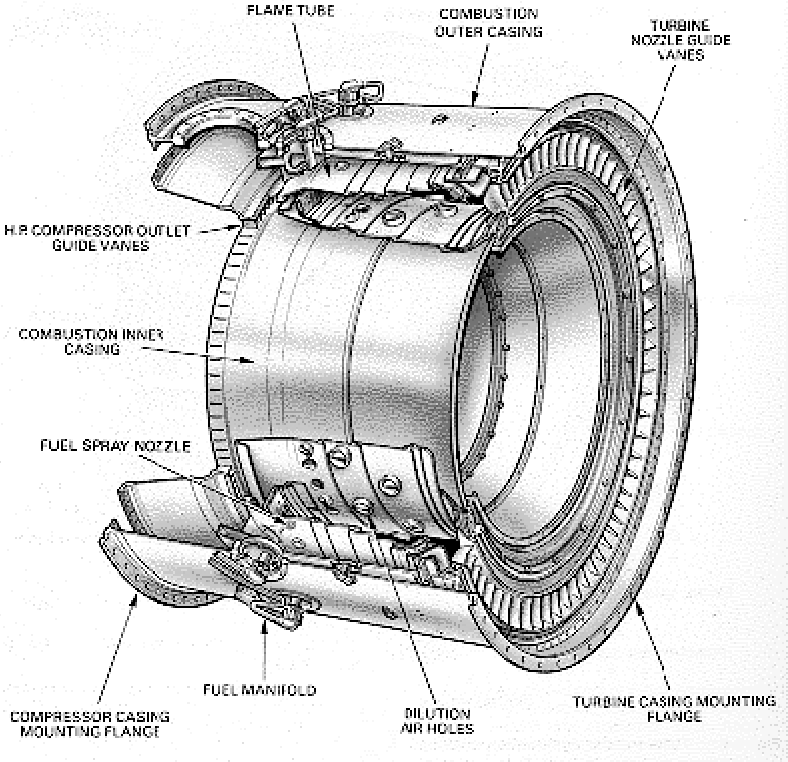
4.4.1. Can Type burner:

In the can type individual burners, or cans, are mounted in a circle around the engine axis with each one receiving air through its own cylindrical shroud. One of the main disadvantages of can type burners is that they do not make the best use of available space and this results in a large diameter engine. On the other hand, the burners are individually removable for inspection and air-fuel patterns are easier to control than in annular designs.



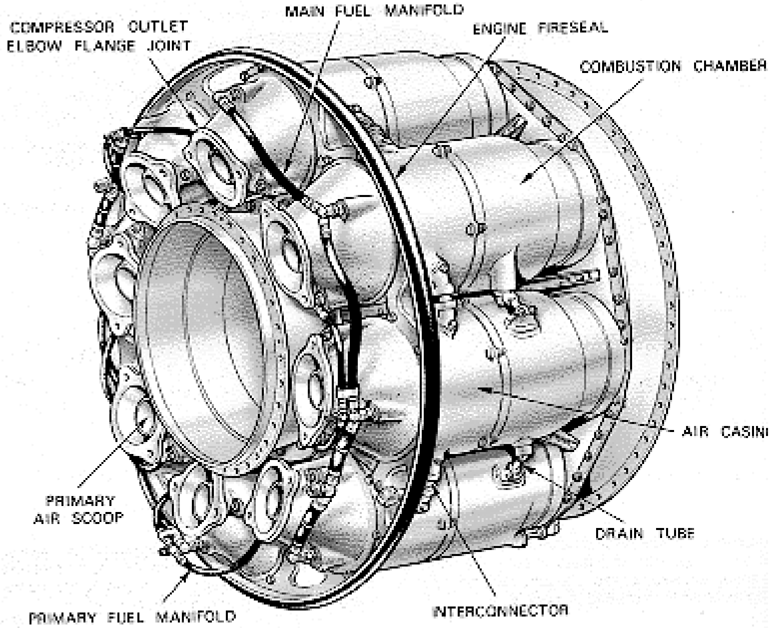
4.4.2. Annular Type burner:

The annular burner is essentially a single chamber made of concentric cylinders mounted co-axially about the engine axis. This arrangement makes more complete use of available space, has low pressure loss, fits well with the axial compressor and turbine and form a technical viewpoint has the highest efficiency, but has a disadvantage in that structural problems may arise due to the large diameter, thin-wall cylinder required with this type of chamber. The problem is more severe for larger engines. There is also some disadvantage in that the entire combustor must be removable from the engine for inspection and repairs.



4.4.3. Can - Annular Type burner:

The can annular design also makes good use of available space, but employs a number of individually replaceable cylindrical inner liners that receive air through a common annular housing for good control of fuel and air flow patterns. The can-annular arrangement has the added advantage of greater structural stability and lower pressure loss than that of the can type.



The heated air then expands through the turbine thereby increasing its velocity while losing pressure. The turbine extracts enough energy to drive the compressor and the necessary auxiliary equipments. Turbines of the impulse, reaction and a combination of both types are used.

In general, it may be stated that those engines of relatively low thrust and simple design employ the impulse type, while those of large thrust employ the reaction and combination types. The hot gas is then expanded through the exit nozzle and the energy of the hot gas is converted into as much kinetic energy as is possible. This change in velocity of the air passing through the engine multiplied by the mass flow of the air is the change of momentum, which produces thrust.

The nozzle can be a fixed jet or a variable area nozzle. The variable area nozzle permits the turbojet to operate at maximum efficiency over a wide range of power output. Clamshell, Finger or Iris, Centre plug with movable shroud, annular ring, annular ring with movable shroud are the various types of variable area nozzle for turbojet engines. The advantage of variable area nozzle is the increased cost, weight and complexity of the exhaust system.

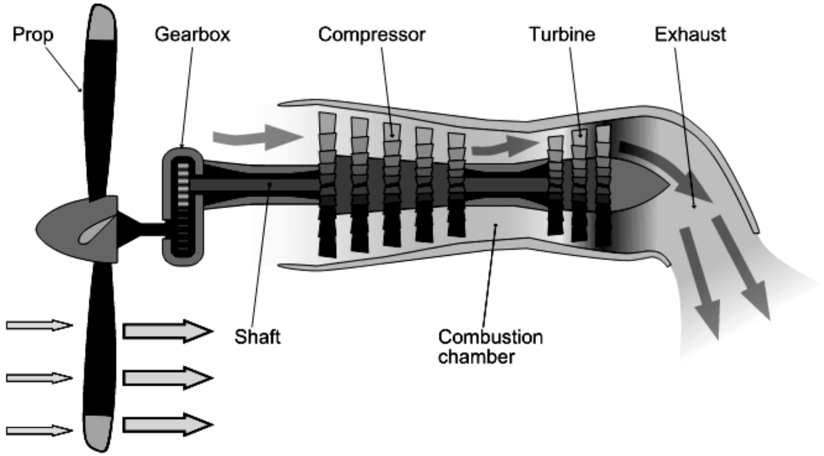
The needs and demands being fulfilled by the turbojet engine are

1. Low specific weight – ¼ to ½ of the reciprocating engine
2. Relative simplicity – no unbalanced forces or reciprocating engine
3. Small frontal area, reduced air cooling problem- less than ¼th the frontal area of the reciprocating engine giving a large decrease in nacelle drag and consequently giving a greater available excess thrust or power, particularly at high speeds.
4. Not restricted in power output - engines can be built with greatly increased power output over that of the reciprocating engine without the accompanying disadvantages.
5. Higher speeds can be obtained – not restricted by a propeller to speeds below 800 km/h.

4.5. Turboprop Engine

It is also known as Propeller turbine, turbo-propeller, prop jet, turbo-prop. For relatively high take-off thrust or for low-speed cruise applications, turboprop engines are employed to accelerate a secondary propellant stream, which is much larger than the primary flow through the engine. The relatively low work input per unit mass of secondary air can be adequately transmitted by a propeller.

Though a ducted fan could also be used for this purpose, a propeller is generally lighter compared to ducted fan could also be used for this purpose, a propeller is generally lighter compared to ducted fan engine and with variable pitch, it is capable of a wider range of satisfactory performance.



In general, the turbine section of a turboprop engine is very similar to that of a turbojet engine. The main difference is the design and arrangement of the turbines.

In the turbojet engine the turbine is designed to extract only enough power from the high velocity gases to drive the compressor, leaving the exhaust gases with sufficient velocity to produce the thrust required of the engine. The turbine of the turboprop engine extracts enough power from the gases to drive both the compressor and the propeller. Only a small amount of power is left as thrust. Usually a turboprop engine has two or more turbine wheels. Each wheel takes additional power from the jet stream, with the result that the velocity of the jet is decreased substantially.

Figure shows a schematic diagram of a turboprop engine. The air enters the diffuser as in a turbojet and is compressed in a compressor before passing to the combustion chamber. The compressor in the turboprop is essentially an axial flow compressor. The products of combustion expand in a two-stage or multistage turbine.

One stage of the turbine drives the compressor and the other drives the propeller. Thus the turbine expansion is used to drive both compressor as well as propeller and less energy is available for expansion in the nozzle. Due to lower speeds of propeller a reduction gear is necessary between turbine and the propeller. About 80 to 90% of the available energy in exhaust is extracted by the turbine while rest, about 10 to 20%, contributes the thrust by increasing the exhaust jet velocity.

Total thrust = jet thrust + propeller thrust

Turboprop engines combine in them the high take-off thrust and good propeller efficiency of the propeller engines at speeds lower than 800 km/h and the small weight, lower frontal area and reduced vibration and noise of the pure turbojet engine. Its operational range is between that of the propeller engines and turbojets though it can operate in any speed up to 800 km/h.

The power developed by the turboprop remains almost same at high altitudes and high speeds as that under sea-level and take-off conditions because as speed increases ram effect also increases. The specific fuel consumption increases with increase in speed and altitude. The thrust developed is high at take-off and reduces at increased speed.

4.5.1. Advantages:

1. Turboprop engines have a higher thrust at take-off and better fuel economy.
2. The frontal area is less than air screw so that drag is reduced.
3. The turboprop can operate economically over a wide range of speeds ranging from low speeds, where pure jet engine is uneconomical, to speeds of about 800 km/h where the propeller engine efficiency is low.
4. It is easy to maintain and has lower vibrations and noise.

5. The power output is not limited as in the case of propeller engines (air screw).
6. The multicast arrangement allows a great flexibility of operation over a wide range of speeds.

4.5.2. Disadvantages:

1. The main disadvantage is that at high speeds due to shocks and flow separation, the propeller efficiency decreases rapidly, thereby, putting up a maximum speed limit on the engine.
2. It requires a reduction gear which increases the cost and also consumes certain energy developed by the turbine in addition to requiring more space.

4.6. The Turbofan Engine

The turboprop is limited to Mach number of about 0.7 because of the sharp decrease in propeller efficiency encountered above that Mach number. However, the turboprop concept of increasing mass flow rate without producing an excessive increment in exhaust velocity is valid at any Mach number and the use of a ducted fan combined with a jet turbine provides more economical operation at Mach numbers close to unity than does the simple jet turbine. If a duct or shroud is placed around a jet engine and air is pumped through the annular passage by means of one or more sets of compressor blades, the resulting engine is called a turbofan, and is capable of producing (under proper conditions) somewhat better thrust specific fuel consumption characteristics than the turbojet itself.

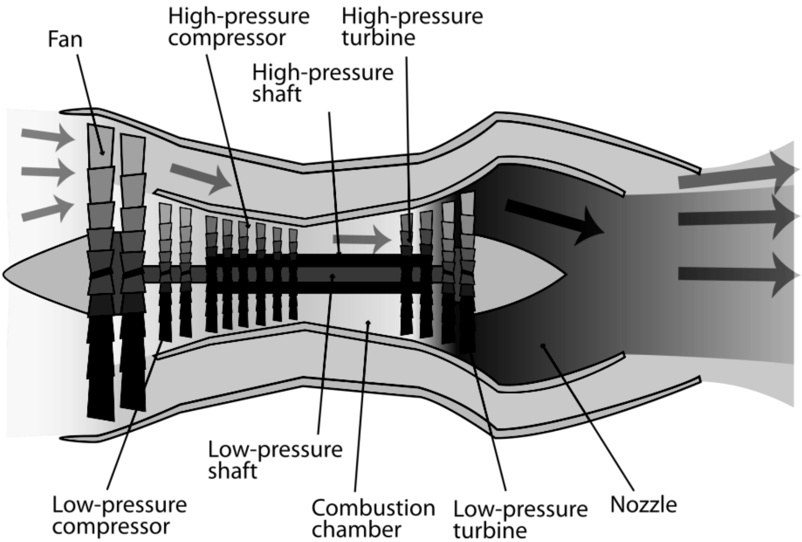


Fig. 4.3. Turbofan Engine

Basically, the air passing through the fan bypasses the combustion process but has energy added to it by the compressor fan, so that a sizable mass flow can be shunted through the fan. The air which bypasses the combustion process leaves the engine with a lower amount of internal energy and a lower exhaust speed than the jet exhaust. Yet, the thrust is not decreased since the turbofan can pump more air per unit time than a conventional jet at subsonic speeds.

Accordingly, the average exhausts velocity of the turbofan (averaging the turbine flow and the bypass flow) can be made smaller at a given flight speed than that of a comparable turbojet and greater efficiency can be obtained. In turbofan engine the fan cannot be designed for all compressor ratios which is efficient at all mach numbers, thus, the turbofan is efficient over a rather limited range of speeds. Within this speed range, however, its improved cruise economy makes it a desirable unit for jet transport aircraft.

The turbofan engine has a duct enclosed fan mounted at the front or rear of the engine and driven either mechanically geared down or at the same speed as the compressor, or by an independent turbine located to the rear of the compressor drive turbine. There are two methods of handling the fan air. Either the fan can exit separately from the primary engine air, or it can be ducted back to mix with the primary engine's air at the rear. If the fan air is ducted to the rear, the total fan pressure must be higher than the static pressure in the primary engine's exhaust, or air will not flow. Similarly, the static fan discharge pressure must be less than the total pressure the primary engine's exhaust, or the turbine will not be able to extract the energy required to drive the compressor and fan. By closing down the area of flow of the fan duct, the static pressure can be reduced and the dynamic pressure is increased.

The efficiency of the fan engine is increased over that of the pure jet by converting more of the fuel energy into pressure energy rather than the kinetic energy of a high velocity exhaust gas stream. The fan produces additional force or thrust without increasing fuel flow.

As in the turboprop primary engine exhaust gas velocities and pressures are low because of the extra turbine stages needed to drive the fan, and as a result this makes the turbofan engine much quieter.

One fundamental difference between the turbofan and the turboprop engine is that the air flow through the fan is controlled by design so that the air velocity relative to the fan blades is unaffected by the aircraft's speed. This eliminates the loss in operational efficiency at high air speeds which limits the maximum air speed of propeller driven aircraft.

Fan engines show a definite superiority over the pure jet engines at speeds below Mach 1. The increased frontal area of the fan presents a problem for high speed aircraft which, of course require small frontal areas. At high speeds air can be offset at least partially by burning fuel in the fan discharge air. This would expand the gas, and in order to keep the fan discharge air at the same pressure, the area of the fan jet nozzle is increased. This action results in an increase in gross thrust due to an increase in pressure times an area (PA), and an increase in gross thrust specific fuel consumption.

4.6. The Ramjet Engine:

The ramjet engine is an air breathing engine which operates on the same principle as the turbojet engine. Its basic operating cycle is similar to that of the turbojet. It compresses the incoming air by ram pressure, adds the heat energy to velocity and produces thrust. By converting kinetic energy of the incoming air into pressure, the ramjet is able to operate without a mechanical compressor. Therefore the engine requires no moving parts and is mechanically the simplest type of jet engine which has been devised. Since it depends on the velocity of the incoming air for the needed compression, the ramjet will not operate statistically. For this reason it requires a turbojet or rocket assist to accelerate it to operating speed.

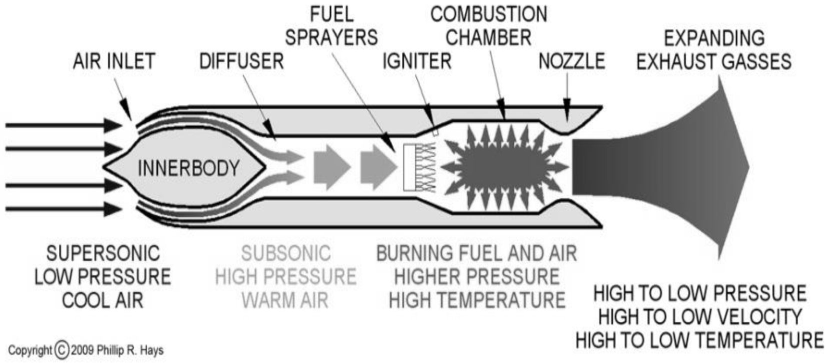


Fig. 4.4. Ramjet Engine

At supersonic speeds the ramjet engine is capable of producing very high thrust with high efficiency. This characteristic makes it quite useful on high speed aircraft and missiles, where its great power and low weight make flight possible in regions where it would be impossible with any other power plant except the rocket.

Ramjets have also been used at subsonic speeds where their low cost and light weight could be used to advantage. Principle of Operation: The ramjet consists of a diffuser, fuel injector, flame holder, combustion chamber and exit nozzle. The air taken in by the diffuser is compressed in two stages.

The external compression takes place takes place because the bulk of the approaching engine forces the air to change its course. Further compression is accomplished in the diverging section of the ramjet diffuser. Fuel is injected into and mixed with air in the diffuser. The flame holder provides a low velocity region favourable to flame propagation, and the fuel-air mixture recirculates within this sheltered area and ignites the fresh charge as it passes the edge of the flame holder.

The burning gases then pass through the combustion chamber, increasing in temperature and therefore in volume. Because the volume of air increases, it must speed up to get out of the way off the fresh charge following behind it, and a further increase in velocity occurs as the air is squeezed out through the exit nozzle. The thrust produced by the engine is proportional to this increase in velocity.

4.6.1 Advantages:

1. Ramjet is very simple and does not have any moving part. It is very cheap and requires almost no maintenance.
2. Since turbine is not used the maximum temperature which can be allowed in ramjet is very high, about 20000 C as compared to about 10000 C in turbojets. This allows a greater thrust to be obtained by burning fuel at A/F ratio of about 15.1 which gives higher temperatures.
3. The SFC is better than turbojet engines at high speed and high altitudes.
4. There seems to be no upper limit to the flight speed of the ramjet.

4.6.2 Disadvantages:

1. Since the compression of air is obtained by virtue of its speed relative to the engine, the take-off thrust is zero and it is not possible to start a ramjet without an external launching device.
2. The engine heavily relies on the diffuser and it is very difficult to design a diffuser which will give good pressure recovery over a wide range of speeds.
3. Due to high air speed, the combustion chamber requires flame holder to stabilize the combustion.
4. At very high temperature of about 20000 C dissociation of products of combustion occurs which will reduce the efficiency of the plant if not recovered in nozzle during expansion.

4.6.3 Application:

Due to its high thrust at high operational speed, it is widely used in high speed aircrafts and missiles. Subsonic ramjets are used in target weapons, in conjunction with turbojets or rockets for getting the starting torque.

4.7. Pulse Jet Engine:

The pulse jet engine is an intermittent, compressor less aerodynamic power plant, with few or none of the mechanical features of conventional aviation power plants.

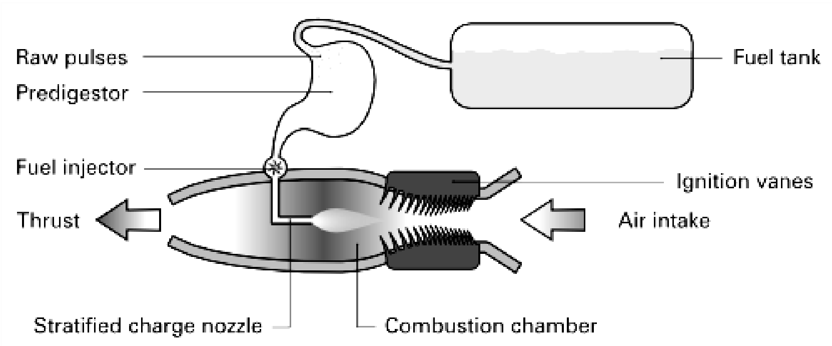


Fig. 4.5. Pulse Jet Engine

In its simplest form, the operation of the pulse jet depends only on the properties of atmospheric air, a fuel, a shaped tube and some type of flow-check valve, and not on the interposition of pistons, impellers, blades or other mechanical part whose geometry and motion are controllable. The pulse jet differs from other types of air breathing engines, in that the air flow through it is intermittent. It can produce static thrust.

4.7.1 Operations:

During starting compressed air is forced into the inlet which opens the spring loaded flapper valves. In practice this may done by blowing compressed air though the valve box or by the motion of the engine through the air.

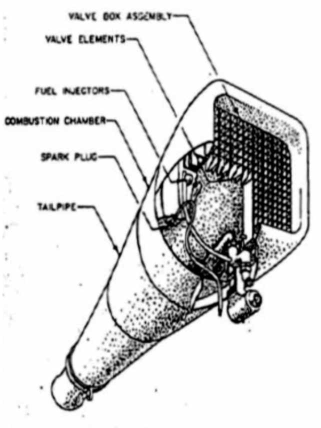


Figure ·10 Typical pulsejet engine.

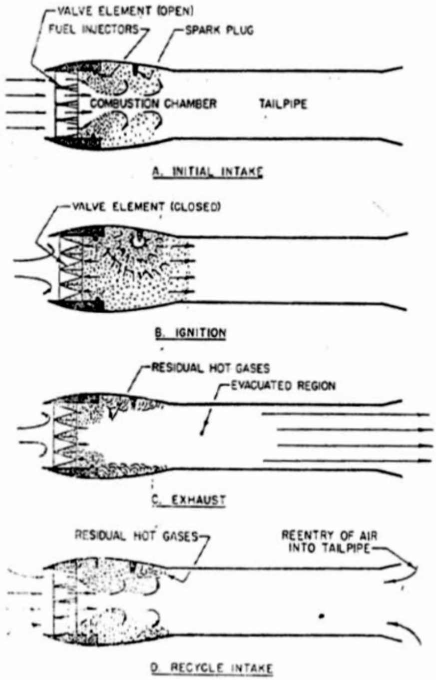


Figure ·10 Pulsejet-engine operating cycle.

Fig.4.6. Stages of Operation

The air that enters the engine passes by the fuel injector and is mixed with the fuel. When the fuel-air mixture reaches the proper proportion to burn, it is ignited by a spark plug. The burning takes place with explosive force, thus causing a very rapid rise in pressure, the increase in pressure forces the flapper valves shut and propels the charge of burned gases out of the tail pipe, as in B of the figure 4.6.

The momentum of the gases leaving the tailpipe causes the air to continue the flow out even after the pressure within the engine has reached atmospheric pressure. The pressure within the engine is therefore evacuated to below atmosphere, part C in figure 4.6.

After the pressure has reached its lowest point, atmospheric pressure (and the ram pressure if the engine is in flight) forces air into the engine through the flapper valves.

At the same time, air will also be drawn in through tailpipe, since the pressure within the tailpipe is low and has nothing to prevent the entry of air, At this point, part D in figure 4.6; the engine is ready to begin another cycle. The frequency of cycles depends upon the duct shape and working temperature in V-1 rocket it was about 40 c/s which corresponds to about 2400 rpm of a two stroke reciprocating engine.

Once the engine operation has become established, the spark plug is no longer necessary. The reignition between each cycle is accomplished when the fresh charge of fuel and air is ignited by some residual flame which is left over from the preceding cycle. The air flow which reenters the tailpipe is important from both the engine operation and thrust standpoints.

Experiments have shown that the amount of air which flows into the tailpipe can be several times as much as that which flows into the inlet. This mass flow of air increases the thrust of the engine by providing additional mass for the explosion pressure to work on. It also increases the pressure within the engine at the beginning of each explosion cycle, resulting in a more efficient burning process.

Re-entry of air into the tailpipe is made more difficult as the airspeed surrounding the engine increases. The thrust of the engine, therefore, tends to decrease with speed. As the speed increases, the amount of re-entering air flow decreases to the point where the internal pressure is eventually too low to support combustion and the engine will no longer operate.

4.7.2 Characteristics:

The chief advantages of the pulse jet are its simplicity, light weight, low cost and good zero speed thrust characteristic. Its particular disadvantages are its 650-800 km/h. operating speed limit, rather limited altitude range and somewhat unpredictable valve life.

One interesting and sometimes objectionable, feature of the pulse jet engine is the sound it makes when in operation. The sound is a series of loud reports caused by the firing of the individual charges of fuel and air in the combustion chamber. The frequency of the reports depends upon the length of the engine form the inlet valves to the end of the tailpipe and upon the temperature of the gases within the engine.

The resulting sound is a continuous, loud, and vibratory note that can usually be heard for several kilometres. The pulse jet has low thermal efficiency. In early designs the efficiency obtained was about 2 to 3% with a total flight life of 30 to 60 minutes.

The maximum operating speed is seriously limited by two factors: (i) It is possible to design a good diffuser at high speeds.

(ii) The flapper valves, the only mechanical part in the pulse jet, also have certain natural frequency and if resonance with the cycle frequency occurs then the valve may remain open and no compression will take place. Also, as the speed increases it is difficult for air to flow back. This reduces total compression pressure as well as the mass flow of air which results in inefficient combustion and lower thrust. The reduction in thrust and efficiency is quite sharp as the speed increases.

4.7.3 Advantages:

1. This is very simple device only next to ramjet and is light in weight. It requires very small and occasional maintenance.
2. Unlike ramjet, it has static thrust because of the compressed air starting, thus it does not need a device for initial propulsion. The static thrust is even more than the cruise thrust.
3. It can run on an almost any type of liquid fuel without much effect on the performance. It can also operate on gaseous fuel with little modifications.
4. Pulse jet is relatively cheap.

4.7.4 Disadvantages:

1. The biggest disadvantage is very short life of flapper valve and high rates of fuel consumption. The SFC is as high as that of ramjet.
2. The speed of the pulse jet is limited within a very narrow range of about 650-800 km/h because of the limitations in the aerodynamic design of an efficient diffuser suitable for a wide range.
3. The high degree of vibrations due to intermittent nature of the cycle and the buzzing noise has made it suitable for pilotless crafts only.
4. It has lower propulsive efficiency that turbojet engine.
5. The operational range of the pulse jet is limited in altitude range.

4.7.5 Applications:

German V-1 buzz bomb, American Helicopter Company's Jet Jeep Helicopter, Auxiliary power plant for sail planes.

4.8 Efficiency of the components of gas turbine power plant

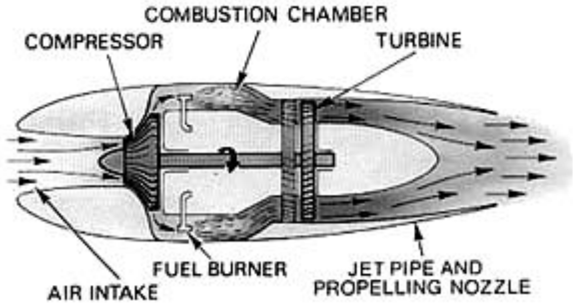


Fig. 4.7 gas turbine

From the above figure 4.7 the components inlet diffuser, compressor, combustion chamber, turbine and exit nozzle are considered to find their efficiency.

4.8.1 Diffuser efficiency:

The efficiency of the diffuser is taken as the ratio of static pressure rise in the actual process to the static pressure rise in the isentropic process for a small pressure rise and for the case of high pressure rise, it is taken as the ratio of the enthalpy change in isentropic diffusion to the enthalpy change in actual diffusion for the same exit pressure.

$$\eta_D = \frac{p_1 - p_i}{p_{is} - p_i} \; (or) \; \frac{p_1 - p_i}{\frac{1}{2}\,(c_i{}^2 - c_1{}^2)} \,, \qquad for\ small\ pressure\ rise$$

$$\eta_D = \frac{h_{is} - h_i}{h_1 - h_i} \ (or) \ \frac{\left(\frac{p_1}{p_i}\right)^{\gamma-1/\gamma} - 1}{\frac{\gamma - 1}{2} \ M_i^2} , \qquad for\ large\ pressure\ rise$$

4.8.2 Compressor:

The efficiency of the compressor is taken as the ratio of isentropic work done to the actual work done by the compressor.

$$\eta_C = \frac{T_{o1} \left[\left(\frac{p_{02}}{p_{01}}\right)^{\gamma-1/\gamma} - 1\right]}{T_{o2} - T_{o1}}$$

4.8.3 Combustion Chamber:

The efficiency of the combustion is taken as the ratio of the increase in the enthalpy of gases to the energy supplied by the fuel.

$$\eta_B = \frac{\frac{\dot{m}_a}{\dot{m}_f} \left(c_p T_{03} - c_p T_{02}\right) + c_p T_{03}}{C.V}$$

4.8.4 Turbine:

The efficiency of the gas turbine is the ratio of actual work done by the isentropic work done.

$$\eta_T = \frac{T_{o3} - T_{o4}}{T_{o3} \left[1 - \frac{1}{\left(\frac{p_{04}}{p_{03}}\right)^{\gamma-1/\gamma}}\right]}$$

4.8.5 Nozzle:

The efficiency of the nozzle is taken as the ratio of the actual enthalpy drop to the isentropic enthalpy drop.

$$\eta_N = \frac{T_{o4} - T_e}{T_{o4}\left[1 - \frac{1}{\left(\frac{p_e}{p_{04}}\right)^{\gamma-1/\gamma}}\right]}$$

Based on the efficiency of the nozzle, the exit velocity c_e is given as

$$\eta_N = \frac{h_{o4} - h_e}{h_{o4} - h_{es}}$$

And applying the adiabatic energy equation based on enthalpy

$$h_{o4} - h_e = \frac{1}{2}\, {c_e}^2$$

The exit velocity of the nozzle is given as

$$c_e = \sqrt{2\,\eta_N\, c_p\, T_{o4}\left[1 - \left(\frac{p_e}{p_{04}}\right)^{\gamma-1/\gamma}\right]}$$

4.9 Thrust

The force which propels the aircraft forward at a given speed is called as thrust or propulsive force. This mainly depends on the velocity f gases at the exit of the nozzle.

4.9.1 Jet Thrust:

The net thrust for a turbojet engine is given as

$$F = \left(\dot{m}_a + \dot{m}_f\right) c_e - \dot{m}_a \times u, \qquad N$$

Where

$\dot{m}_a$ = Mass of the air, kg/s
$\dot{m}_f$ = mass of the fuel, kg/s
c_e or c_j = exit velocity of nozzle or jet velocity for complete expansion, m/s
u = Flight speed, m/s

4.9.2 Propeller thrust:

For complete expansion

$$F = \dot{m}_a\left(c_j - u\right), \qquad N$$

4.9.3 Effective speed ratio:

It is defined as the ratio of the flight speed to the jet velocity.

$$\sigma = \frac{u}{c_j}$$

Based on this ratio, the thrust force

$$F = \dot{m}_a c_j (1 - \sigma), \qquad N$$

4.9.4 Specific Thrust:

It is defined as the thrust developed per unit mass flow rate.

$$F_{sp} = \frac{F}{\dot{m}}$$

4.9.5 Thrust specific fuel consumption

It is defined as the ratio of fuel consumption rate to the thrust force or simply fuel consumption rate per unit thrust force.

$$TSFC = \frac{\dot{m}_f}{F}$$

4.9.6 Specific Impulse:

The thrust developed per unit weight flow rate is known as specific impulse.

$$I_{sp} = \frac{F}{W} = \frac{\dot{m}(c_j - u)}{\dot{m}\, g} = \frac{c_j - u}{g}$$

$$I_{sp} = \frac{u}{g}\left(\frac{1}{\sigma} - 1\right)$$

4.9.7 Propulsive Efficiency:

It is defined as the ratio of propulsive power or thrust power to the power output of the engine.

$$\eta_p = \frac{Propulsive\ power}{power\ output\ of\ engine} = \frac{\dot{m}(c_j - u) \times u}{\frac{1}{2}\,\dot{m}\,(c_j{}^2 - u^2)}$$

$$\eta_p = \frac{2\,u}{c_j + u} = \frac{2\,\sigma}{1 + \sigma}$$

4.9.8 Thermal Efficiency:

It is defined as the ratio of the power output of the engine to the power input to the engine

$$\eta_t = \frac{\frac{1}{2}\,\dot{m}\,(c_j{}^2 - u^2)}{\dot{m}_f \times C.V}$$

If efficiency of combustion is considered

$$\eta_t = \frac{\frac{1}{2}\,\dot{m}\,(c_j{}^2 - u^2)}{\eta_B \times \dot{m}_f \times C.V}$$

4.9.9 Overall Efficiency:

It is defined as the ratio of propulsive power to the power input to the engine.

$$\eta_o = \frac{\dot{m}(c_j - u) \times u}{\dot{m}_f \times C.V}$$

$$\eta_o = \eta_p \times \eta_t$$

Important Formulas:

1. Diffuser efficiency

$$\eta_D = \frac{\left(\frac{p_1}{p_i}\right)^{\gamma-1/\gamma} - 1}{\frac{\gamma - 1}{2} M_i^{\,2}}$$

2. Compressor efficiency

$$\eta_C = \frac{T_{o1}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\gamma-1/\gamma} - 1\right]}{T_{o2} - T_{o1}}$$

3. Combustion efficiency

$$\eta_B = \frac{\frac{\dot{m}_a}{\dot{m}_f}\left(c_p T_{03} - c_p T_{02}\right) + c_p T_{03}}{C.V}$$

4. Turbine efficiency

$$\eta_T = \frac{T_{o3} - T_{o4}}{T_{o3}\left[1 - \frac{1}{\left(\frac{p_{04}}{p_{03}}\right)^{\gamma-1/\gamma}}\right]}$$

5. Nozzle efficiency

$$\eta_N = \frac{T_{o4} - T_e}{T_{o4}\left[1 - \frac{1}{\left(\frac{p_e}{p_{04}}\right)^{\gamma-1/\gamma}}\right]}$$

6. Exit velocity of nozzle or jet velocity

$$c_e = \sqrt{2\,\eta_N\,c_p\,T_{o4}\left[1-\left(\frac{p_e}{p_{04}}\right)^{\gamma-1/\gamma}\right]}$$

7. Thrust force

$$F = \left(\dot{m}_a + \dot{m}_f\right) c_e - \dot{m}_a \times u, \qquad N$$

8. Effective speed ratio

$$\sigma = \frac{u}{c_j}$$

9. Specific thrust

$$F_{sp} = \frac{F}{\dot{m}}$$

10. Thrust specific fuel consumption

$$TSFC = \frac{\dot{m}_f}{F}$$

11. Specific impulse

$$I_{sp} = \frac{F}{W} = \frac{\dot{m}\left(c_j - u\right)}{\dot{m}\,g} = \frac{c_j - u}{g}$$

$$I_{sp} = \frac{u}{g}\left(\frac{1}{\sigma} - 1\right)$$

12. Thrust power

$$\dot{m}\left(c_j - u\right) \times u$$

13. Propulsive efficiency

$$\eta_p = \frac{2\,u}{c_j + u} = \frac{2\,\sigma}{1+\sigma}$$

14. Thermal efficiency

$$\eta_t = \frac{\frac{1}{2}\,\dot{m}\left({c_j}^2 - u^2\right)}{\dot{m}_f \times C.V}$$

If efficiency of combustion is considered

$$\eta_t = \frac{\frac{1}{2}\,\dot{m}\left({c_j}^2 - u^2\right)}{\eta_B \times \dot{m}_f \times C.V}$$

15. Overall efficiency

$$\eta_o = \frac{\dot{m}\left(c_j - u\right) \times u}{\dot{m}_f \times C.V}$$

$$= \eta_p \times \eta_t$$

16. Air flow velocity at the propeller

$$\mathrm{c} = \frac{1}{2}\left(\mathrm{u} + c_j\right)$$

17. Air standard efficiency

$$\eta_a = 1 - \frac{1}{\left(\frac{p_{02}}{p_{01}}\right)^{\gamma-1/\gamma}}$$

18. Mass flow rate

$$\dot{m} = \rho\, A_1\, u = \rho\, A_j\, c_j$$

19. Stagnation temperature

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$T_o = T + \frac{c^2}{2\, c_p}$$

20. Mach number at entry

$$M_i = \frac{u}{a_i}$$

$$a_i = \sqrt{\gamma\, R\, T_i}\,, \qquad m/s$$

21. Mach number at exit

$$M_e = \frac{c_e \; or \; c_j}{a_e}$$

$$a_e = \sqrt{\gamma\, R\, T_e}\,, \qquad m/s$$

22. Isentropic relation

$$\frac{T_{o2}}{T_{o1}} = \left(\frac{P_{o2}}{p_{o1}}\right)^{\frac{\gamma-1}{\gamma}}$$

23. Absolute velocity

$$c_{abs} = c_j - u$$

For Ramjet Engine:

1. Efficiency of Ideal Cycle

$$\eta_I = \frac{1}{1 + \frac{2}{\gamma - 1} \times \frac{1}{{M_1}^2}}$$

2. Efficiency of Diffuser

$$\eta_D = \frac{\left(\frac{p_{02}}{p_1}\right)^{\gamma-1/\gamma} - 1}{\frac{\gamma - 1}{2}\, {M_1}^2}$$

3. Combustion efficiency

$$\eta_B = \frac{\dot{m}_a\, c_p (T_{03} - T_{02})}{\dot{m}_f \times C.V}$$

TWO MARK QUESTIONS WITH ANSWERS

1. What is meant by a jet propulsion system?

   It is the propulsion of a jet aircraft (or) other missiles by the reaction of jet coming out with high velocity. The jet propulsion in used when the oxygen is obtained from the surrounding atmosphere.

2. How will you classify propulsive engines?

   The jet propulsion engines are classified into
   - Air breathing engines and
   - Rocket engines which do not use atmospheric air.

3. What is the difference between shaft propulsion and jet propulsion?

   Shaft Propulsion
   a) The power to the propeller is transmitted through a reduction gear
   b) At higher altitude, the performance is poor. Hence it is suitable for lower altitudes.
   c) With increasing speeds and size of the aircrafts, the shaft propulsion engine becomes too complicated.
   d) Propulsive efficiency is less.
   Jet Propulsion
   a) There is no reduction gear.
   b) Suitable for higher altitudes.
   c) Construction is simpler.
   d) More.
4. List the different types of jet engines.
   i. Turbo-jet
   ii. Turpo-prop engine,
   iii. Ram jet engine,
   iv. Pulse jet engines.

5. Define the principle of Ram jet engine.

The principle of jet engine is obtained from the application of Newton's law of motion. We know that when a fluid is accelerated, a force is required to produce this acceleration is the fluid and at the same time, there is an equal and opposite reaction force of the fluid on the engine is known as the thrust, and therefore the principle of jet propulsion is based on the reaction principle.

6. Give the components of a turbo jet.

i. Diffuser
ii. Mechanical compressor,
iii. Combustion chamber,
iv. Turbine and
v. Exhaust nozzle.

7. Give the difference between pulse jet and ram jet engine.

Pulse Jet
a) Mechanical valve arrangements are used during combustion.
b) The stagnation temperature at the diffuser exit is comparatively less.
Ram Jet
a) Works without the aid of any mechanical device and needs no moving parts.
b) Since the mach number in Ram jet engine is supersonic, the stagnation temperature is very high.

8. Give the difference between turbojet and ram jet engine.
<u>Turbo Jet</u>
a) Compressor and turbine are used.
b) Lower thrust and propulsive efficiency at lower speeds.
c) Construction cost is more.

Ram Jet

a) Compressor and turbine are not used but diffuser and nozzle are used.
b) It provides high thrust per unit weight.
c) In the absence of rotating machines, the construction is simple and cheap

9. Give the difference between Jet propulsion and Rocket propulsion.

Jet Propulsion

a) Oxygen is obtained from the surrounding atmosphere for combustion purposes.
b) The jet consists of air plus combustion products.
c) Mechanical devices are also used.

Rocket Propulsion

a) The propulsion unit consists of its own oxygen supply for combustion purposes.
b) Jet consists of the exhaust gases only.
c) Mechanical devices are not used.

10. What is the difference between turbo prop engine and turbo jet engine.

Turbo – Prop

a) The specific fuel consumption based on thrust is low.
b) Propulsive efficiency within the range of operation is higher.
c) On account of higher thrust at low speeds the take-off role is short and requiring shorter runway.
d) Use of centrifugal compressor stages increases the frontal area.
e) Higher weight per unit thrust.

Turbo - Jet

a) TSFC is comparatively higher at lower speeds and altitudes.
b) Propulsive efficiency is low.
c) Take – off role is longer and requiring longer run way.
d) Lower Frontal area.
e) Lower weight per unit thrust.

11. What is ram effect?

    When an aircraft flies with high velocity, the incoming air is compressed to high pressure without external work at the expense of velocity energy is known as "ram effect".

12. What is "thrust augmentation"?

    To achieve better take-off performance, higher rates of climb and increased performance at altitude during combat manoeuvres, there has been a demand for increasing the thrust output of aircraft for short intervals of time. This is achieved by during additional fuel in the tail pipe between the turbine exhaust and entrance section of the exhaust nozzle. This method of thrust increases the jet velocity is called "Thrust Augmentation".

13. Why after burners are used in turbojet engine?

    Exhaust gases from the turbine have large quantity of oxygen, which can support the combustion of additional fuel. Thus if a suitable burner is installed between the turbine and exhaust nozzle, a considerable amount of fuel can be burned in this section to produce temperatures entering the nozzle as high as 1900°C. The increased temperature greatly augments the exhaust gas velocity, and hence provides the thrust increase.

14. Why a ram jet engine does not require a compressor and a turbine?

    In general, the speed of a ram jet engine is supersonic (the range of Mach number) is very high. At this flight speed the contribution of the compressor to the total static pressure rise is insignificant. Hence, arm jet engine does not require compressor and turbine.

15. Define Thrust.

The force which propels the aircraft forward at a given speed is called as thrust or propulsive force. This mainly depends on the velocity f gases at the exit of the nozzle.

16. Define Jet Thrust.

The net thrust for a turbojet engine is given as

$$F = (\dot{m}_a + \dot{m}_f)\, c_e - \dot{m}_a \times u, \qquad N$$

Where

$\dot{m}_a$ = Mass of the air, kg/s

$\dot{m}_f$ = mass of the fuel, kg/s

ce or cj = exit velocity of nozzle or jet velocity for complete expansion, m/s

u = Flight speed, m/s

17. What is meant by Propeller thrust?

For complete expansion

$$F = \dot{m}_a(c_j - u), \qquad N$$

18. Define Effective speed ratio.

It is defined as the ratio of the flight speed to the jet velocity.

$$\sigma = \frac{u}{c_j}$$

Based on this ratio, the thrust force

$$F = \dot{m}_a c_j (1 - \sigma), \qquad N$$

19. Define Specific Thrust.

It is defined as the thrust developed per unit mass flow rate.

$$F_{sp} = \frac{F}{\dot{m}}$$

20. Define Thrust specific fuel consumption.

It is defined as the ratio of fuel consumption rate to the thrust force or simply fuel consumption rate per unit thrust force.

$$TSFC = \frac{\dot{m}_f}{F}$$

21. Define Specific Impulse.

The thrust developed per unit weight flow rate is known as specific impulse.

$$I_{sp} = \frac{F}{W} = \frac{\dot{m}(c_j - u)}{\dot{m}\, g} = \frac{c_j - u}{g}$$

$$I_{sp} = \frac{u}{g}\left(\frac{1}{\sigma} - 1\right)$$

22. Define Propulsive Efficiency.

It is defined as the ratio of propulsive power or thrust power to the power output of the engine.

$$\eta_p = \frac{\text{Propulsive power}}{\text{power output of engine}} = \frac{\dot{m}(c_j - u) \times u}{\frac{1}{2}\, \dot{m}\, (c_j^2 - u^2)}$$

$$\eta_p = \frac{2\, u}{c_j + u} = \frac{2\, \sigma}{1 + \sigma}$$

23. Define Thermal Efficiency.

It is defined as the ratio of the power output of the engine to the power input to the engine

$$\eta_t = \frac{\frac{1}{2}\,\dot{m}\left(c_j^{\,2} - u^2\right)}{\dot{m}_f \times C.V}$$

If efficiency of combustion is considered

$$\eta_t = \frac{\frac{1}{2}\,\dot{m}\left(c_j^{\,2} - u^2\right)}{\eta_B \times \dot{m}_f \times C.V}$$

24. Define Overall Efficiency.

It is defined as the ratio of propulsive power to the power input to the engine.

$$\eta_o = \frac{\dot{m}\left(c_j - u\right) \times u}{\dot{m}_f \times C.V}$$

$$\eta_o = \eta_p \times \eta_t$$

25. Define Thrust power or propulsive power.

It is defined as the product of thrust force F and flight speed u

$$\text{Thrust power} = \dot{m}\left(c_j - u\right) \times u$$

26. What is scram jet?

A supersonic combustion ramjet engine is known as scram jet. In this the flow enters the combustor at supersonic velocity and comparatively lower temperature. The static pressure is high enough to provide the required expansion in nozzle.

EXERCISE PROBLEMS

1. An aircraft propeller flies at a speed of 440 kmph. The diameter of the propeller is 4.1m and the speed ratio is 0.8. The ambient conditions of air at the flight altitude are T=255K and p=0.55 bar.

   Find the following:

   i. Thrust
   ii. Thrust power
   iii. Propulsive efficiency
2. An aircraft takes 45 kg/s of air from the atmosphere and files at a speed of 950 kmph. The air fuel ratio is 50 and the calorific value of the fuel is 42 MJ/kg. for maximum thrust power, find:
   i. Jet velocity
   ii. Thrust
   iii. Specific thrust
   iv. Thrust power
   v. Propulsive efficiency
   vi. Thermal efficiency
   vii. Overall efficiency
   viii. Thrust specific fuel consumption (TSFC)
3. The flight peed of a turbojet is 800 km/h at an ambient pressure of 1.1 bar. The mass flow rate of air is 15kg/s. the pressure of the gas entering the nozzle is 4 bar and the temperature is 300°C. Determine the following.
   i. Thrust
   ii. Thrust power
   iii. Propulsive efficiency

   Take $\gamma = 1.4$ and R = 287 J/kgK

4. A turbojet propels an aircraft at a speed of 900 km/h while taking 3000kg of air per minute. The isentropic enthalpy drop in the nozzle is 200kJ/kg and the nozzle efficiency is 90%. The air fuel ratio is 85 and the combustion efficiency is 95%. The calorific value of the fuel is 42,000kJ/kg. calculate
    i. Propulsive power or thrust power
    ii. Thermal efficiency
    iii. Propulsive efficiency
5. The fight speed of a turbojet is 800 km/h at 10,000m attitude. The density of air at that altitude is 0.17kg/m3. The drag for the plane is 6.8kN. The propulsive efficiency of the jet is 60%. Calculate the SFC, Air Fuel ratio, Jet Velocity. Assume the calorific value of the fuel is 45000kJ/kg and overall efficiency of the turbojet plane is 18%.

6. A turbojet has a speed of 750km/h while flying at an altitude of 10000m. The propulsive efficiency of the jet is 50% and the overall efficiency of the turbine plant is 16%. The density of air at 10,000 m altitude is 0.173kg/m3. The drag on the plane is 6250 N. calorific value of the fuel is 48000kJ/kg. calculate :
    i. Absolute velocities of the jet
    ii. Diameter of the jet
    iii. Power output of the unit in kW

MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	answer
The principle of jet propulsion is obtained from the application of newton's which law?	newton's first law	newton's second law	newton's third law	newton's law of inertia	newton's third law
Which of the following is a type of air breathing engine	ramjet engine	turbofan engine	pulse jet engine	pulse jet engine	pulse jet engine
The ratio of thrust power to power input of engine is defined as	thermal efficiency	overall efficiency	engine efficiency	none of these	overall efficiency
. Which type of compressor is used in turbojet	reciprocating compressor	rotary compressor	axial piston compressor	screw compressor	rotary compressor
For which of the following engine gear arrangement is not necessary	turbo fan engine	turbo prop engine	both a and b	none of these	turbo fan engine
The product of thrust and flight speed gives which of the following	nozzle thrust	engine power	thrust power	jet power	thrust power
The ratio of fuel consumption rate per unit thrust is said to be	specific thrust	thrust	thrust specific fuel consumption	relative thrust	thrust specific fuel consumption
Thrust developed per unit weight flow rate is said to be	specific thrust	specific impulse	relative thrust	relative impulse	specific impulse
In ramjet engine the conversion of kinetic energy of the entering air into pressure energy is called	ram conversion	energy conversion	ram effect	pulse effect	ram effect
Turbojet engine works on which of the following cycle	otto cycle	diesel cycle	brayton cycle	dual cycle	brayton cycle
Ramjet engine works on which of the following cycle	brayton cycle	otto cycle	diesel cycle	dual cycle	brayton cycle
The ratio of flight speed to jet velocity is said to be	effective speed	effective speed ration	relative speed	flight speed	effective speed ration
Ramjet Engines are highly suited for	subsonic application	supersonic application	hypersonic application	none of these	supersonic application
Which of the following is not an type of air breathing engine	ramjet engine	turbo prop engine	turbo fan	rocket engine	rocket engine
Which of the following engine is best suited for piloted-air-crafts	ramjet engine	turbojet engine	pulse jet engine	turbo prop engine	turbojet engine

The maximum temperature in a gas turbine is	200°C	500°C	700°C	1000°C	700°C
The stagnation pressure rise in a centrifugal compressor takes place	in the diffuser only	in the impeller only	in the diffuser and impeller	in the inlet guide vanes only	in the diffuser and impeller
The efficiency of a jet engine is higher at	in the diffuser only	in the impeller only	in the diffuser and impeller	in the inlet guide vanes only	in the diffuser and impeller
The efficiency of a jet engine is higher at	low speeds	high speeds	low altitudes	high altitudes	high altitudes
In a centrifugal compressor, the highest Mach number leading to shock wave in the fluid flow occurs at	diffuser inlet radius	diffuser outlet radius	impeller inlet radius	impeller outlet radius	diffuser outlet radius
In a jet engine, the compression varies as the square of the speed.	two times	three times	four times	six times	two times
The ratio of the net work obtained from the gas turbine plant to the turbine work is known as	compression ratio	work ratio	pressure ratio	none of these	work ratio
In jet propulsion power unit, the inlet duct of diverging shape is used for converting _____ energy of air into _____ energy	Kinetic, pressure	Pressure, Kinetic	Kinetic, potential	Potential, Pressure	Kinetic, pressure
In open cycle turbo-jet engines used in military aircraft, reheating the exhaust gas from the turbine by burning, more fuel is used to increase the	thrust and range of aircraft	efficiency of the engine	power of the engine	energy of the engine	thrust and range of aircraft
A closed cycle gas turbine works on	Carnot cycle	Rankine cycle	Ericsson cycle	Joule cycle	Joule cycle
A Turbojet engine has	no propeller	propeller in front	propeller at back	propeller on the top	no propeller

A turbo-prop is preferred to turbo-jet because	it has high propulsive efficiency at high speeds	it can fly at super sonic speeds	it can fly at high elevations	it has high power for take off	it has high power for take off
A simple turbo-jet engine is basically a ______engine equipped with a propulsive nozzle and diffuser	Turbo-prop	gas turbine	jet	rocket	gas turbine
In a jet propulsion unit, the products of combustion after passing through the gas turbine are discharged into	Atmosphere	Vacuum	discharge nozzle	back to the compressor	discharge nozzle
In a jet engine, the air-fuel ratio is	30:1	40:1	50:1	60:1	60:1
Gas turbines use following type of air compressor	centrifugal type	reciprocating type	lobe type	axial flow type	axial flow type
The maximum combustion pressure in gas turbine as compared to I.C. engine is	more	less	same	depends on other factors	less
Water is injected in gas turbine cycle to	control temperature	control output of turbine	control fire hazards	increase efficiency	control output of turbine

UNIT V

ROCKET PROPULSION

5.1 Theory of Rocket Propulsion:

A rocket is a machine that develops thrust by the rapid expulsion of matter. The major components of a chemical rocket assembly are a rocket motor or engine, propellant consisting of fuel and an oxidizer, a frame to hold the components, control systems and a cargo such as a satellite. A rocket differs from other engines in that it carries its fuel and oxidizer internally, therefore it will burn in the vacuum of space as well as within the Earth's atmosphere. The cargo is commonly referred to as the payload. A rocket is called a launch vehicle when it is used to launch a satellite or other payload into space. A rocket becomes a missile when the payload is a warhead and it is used as a weapon. At present, rockets are the only means capable of achieving the altitude and velocity necessary to put a payload into orbit.

There are a number of terms used to describe the power generated by a rocket.

(i) Thrust:

It is the force generated, measured in pounds or kilograms. Thrust generated by the first stage must be greater than the weight of the complete launch vehicle while standing on the launch pad in order to get it moving. Once moving upward, thrust must continue to be generated to accelerate the launch vehicle against the force of the Earth's gravity.

$$F = \dot{m}_p\, c_e + (p_e - p_a)\, A_e, \qquad N$$

When p_e (exit Pressure of nozzle) = p_a (ambient pressure)

$$F = \dot{m}_p\, c_e = \dot{m}_p\, c_j, \qquad N$$

To place a satellite into orbit around the Earth, thrust must continue until the minimum altitude and orbital velocity have been attained or the launch vehicle will fall back to the Earth. Minimum altitude is rarely desirable; therefore thrust must continue to be generated to gain additional orbital altitude.

(ii) Specific Impulse:

The impulse, sometimes called total impulse, is the product of thrust and the effective firing duration. A shoulder fired rocket such as the LAW has an average thrust of 272.2 kg and a firing duration of 0.2 seconds for an impulse of 533.8 Ns.

$$I_{sp} = \frac{F}{W_p} = \frac{\dot{m}_p \times c_j}{\dot{m}_p \times g}$$

$$I_{sp} = \frac{c_j}{g}$$

(iii) Specific Propellant Consumption:

It is the propellant consumption rate per unit thrust

$$SPC = \frac{W_p}{F} = \frac{1}{I_{sp}}$$

$$SPC = \frac{g}{c_j}$$

(iv) Weight flow coefficient or propellant weight flow coefficient:

It is the ratio of propellant flow rate to the throat force

$$c_w = \frac{W_p}{F^*} = \frac{W_p}{p_o\, A^*}$$

(v) Thrust coefficient:

It is the ratio of thrust to the throat force.

$$c_F = \frac{F}{F^*} = \frac{F}{p_o\, A^*}$$

This show,

$$F = c_F \times p_o A^*$$

Also,

$$W_p = c_w \times p_o A^*$$

For specific impulse

$$I_{sp} = \frac{c_F}{c_w}$$

We know that,

$$SPC = \frac{1}{I_{sp}} = \frac{c_w}{c_F}$$

(vi) Impulse to weight ratio (IWR)

It is the ratio of total impulse of the rocket to the total weight of the rocket.

$$IWR = \frac{I_{total}}{W_{total}}$$

Where

$$I_{total} = I_{sp} \times W_p$$

(vii) Characteristic Velocity

It is the ratio of effective jet velocity and thrust coefficient.

$$c^* = \frac{c_j}{c_F}$$

(viii) Propulsive Efficiency:

It is defined as the ratio of propulsive power or thrust power to the power output of the engine.

$$\eta_p = \frac{Propulsive\ power}{power\ output\ of\ engine} = \frac{\dot{m}_p \times c_j \times u}{\frac{1}{2}\,\dot{m}_p \left(c_j{}^2 - u^2\right)}$$

$$\eta_p = \frac{2\,\sigma}{1 + \sigma^2}$$

(ix) Thermal Efficiency:

It is defined as the ratio of the power output of the engine to the power input to the engine

$$\eta_t = \frac{\frac{1}{2}\,\dot{m}_p\left(c_j{}^2 + u^2\right)}{\dot{m}_p \times C.V}$$

$$\eta_t = \frac{c_j{}^2 + u^2}{2 \times C.V}$$

(x) Overall Efficiency:

It is defined as the ratio of propulsive power to the power input to the engine.

$$\eta_o = \frac{\dot{m}_p \times c_j \times u}{\dot{m}_p \times C.V}$$

$$\eta_o = \frac{c_j \times u}{C.V} = \eta_p \times \eta_t$$

A rocket's mass ratio is defined as the total mass at liftoff divided by the mass remaining after all the propellant has been consumed. A high mass ratio means that more propellant is pushing less launch vehicle and payload mass, resulting in higher velocity. A high mass ratio is necessary to achieve the high velocities needed to put a payload into orbit.

5.2 Rocket Engines:

Many different types of rocket engines have been designed or proposed. Currently, the most powerful are the chemical propellant rocket engines. Other types being designed or that are propose dare ion rockets, photon rockets, magnetohydrodynamic drives and nuclear fission rockets; however, they are generally more suitable for providing long term thrust in space rather than launching a rocket and its payload from the Earth's surface into space.

5.3 Categories of Chemical Propellants:

There are three categories of chemical propellants for rocket engines: liquid propellant, solid propellant, and hybrid propellant. The propellant for a chemical rocket engine usually consists of a fuel and an oxidizer. Sometimes a catalyst is added to enhance the chemical reaction between the fuel and the oxidizer. Each category has advantages and disadvantages that make them best for certain applications and unsuitable for others.

5.4 Solid Propellants:

Solid propellant rockets are basically combustion chamber tubes packed with a propellant that contains both fuel and oxidizer blended together uniformly. Gas generated in a sold propellant is produced by reaction of reducing and oxidizing agent in the combustion chamber. The oxidizing agent is usually an inorganic slat or one or more organic nitro compounds. The reducing agent is usually a polymeric organic binder compound of carbon hydrogen and sometimes sulphur. The reducing agent imparts mechanical strength to the propellant.

1. Heterogeneous or composite propellants
2. Homogeneous propellant

In heterogeneous solid propellants, plastic polymers and polyvinyl chlorides are used as fuels. Nitrates and per-chlorates are used as oxidizers. In homogeneous solid propellants, nitroglycerine and nitrocellouse are used. It combines the properties of fuel and oxidizers.

5.4.1 Properties of Solid Propellants:

- Should release large amount of heat during combustion
- Physical & chemical properties should not change during processing.
- It should have high density
- Should not be poisonous and hazardous
- Should be cheap and easily available
- Should be noncorrosive and nonreactive with the components of the engine.
- Storage and handling should be easy.

5.4.2 Advantages:

1. The principal advantage is that a solid propellant is relatively stable therefore it can be manufactured and stored for future use. Solid propellants have a high density and can burn very fast.
2. They are relatively insensitive to shock, vibration and acceleration.
3. No propellant pumps are required thus the rocket engines are less complicated.

5.4.3 Disadvantages:

1. Once ignited, solid propellants cannot be throttled, turned off and then restarted because they burn until all the propellant is used.
2. The surface area of the burning propellant is critical in determining the amount of thrust being generated.
3. Cracks in the solid propellant increase the exposed surface area, thus the propellant burns faster than planned.
4. If too many cracks develop, pressure inside the engine rises significantly and the rocket engine may explode.
5. Manufacture of a solid propellant is an expensive, precision operation.

5.5 Solid Propellant Rocket Engine:

5.5.1 Construction:

The construction of solid propellant rocket engine is shown in figure:

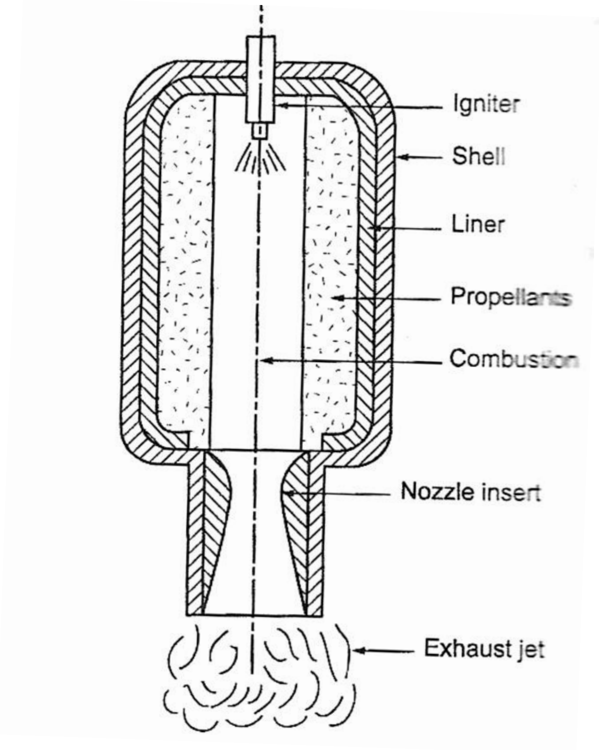


Fig. 5.1 Solid Propellant Rocket

1. Solid propellant is the combination of solid fuel (plastic or resin material) and oxidizer (Nitrates & per-chlorates).
2. Solid fuel & oxidizer is homogeneously mixed & packed inside the shell.
3. A liner is provided between the shell and the propellant. The purpose of the liner is to protect the shell as high temperature will be generated during combustion process.

5.5.2 Working:

1. The igniters are located at the top and ignite the spark. So combustion takes place.
2. When the combustion take place in the combustion chamber, very high pressure and temperature gases are produced
3. The highly heated products of combustion gases are then allowed to expand in the nozzle section.
4. In the nozzle pressure energy of the gas is converted into kinetic energy. So the gases come out from the unit with very high velocities.
5. Due to high velocities of gases coming out from the unit, a force or thrust is produced in opposite direction. This thrust propels the rocket.

5.5.3 Advantages:

1. Simple in design and construction
2. They do not require feed system. So they are free from the problems of moving parts such as pumps, valves etc
3. Less vibration due to absence of moving parts.
4. Less maintenance
5. Suitable for short range applications.
6. Problems arising from the sudden emptying of propellant tanks are absent.

5.5.4 Disadvantages:

1. In case of emergency it is difficult to stop the engine in the midway.
2. Decrease of speed is not possible
3. Low specific impulse
4. At the end of an operation, the burnt up debris cannot be reused. So it is uneconomical.
5. Nozzle cooling is not possible.
6. Nozzle erosion is unavoidable due to the presence of solid particles in the high temperature and high speed gases.
7. Transportation and handling of these rockets before firing require due to the presence of propellant throughout.

5.5.5 Examples:

Solid propellant rockets range in size from the Light Antitank Weapon to the 100 foot long Solid Rocket Boosters (SRBs) used on the side of the main fuel tank of the Space Shuttle. Other examples include air defense missiles such as Patriot and Hawk, air to air missiles such as Sidewinder, Sparrow, antitank missiles such as the TOW and Hellfire, and Intercontinental Ballistic Missiles (ICBMs).

5.6 Liquid Propellants:

Liquid propellant rocket engines burn two separately stored liquid chemicals, a fuel and an oxidizer, to produce thrust.

Typical Liquid Propellant Fuel/Oxidizer Combinations		
Fuel	**Oxidizer**	**Type**
Liquid Hydrogen	Liquid Oxygen	Cryogenic
RP-1 Kerosene	Liquid Oxygen	Liquid/Cryogenic
Aniline	Nitric Acid	Hypergolic
Hydrazine	Monopropellant	

Table.5.1

5.6.1 Cryogenic Propellant:

A cryogenic propellant is one that uses very cold, liquefied gases as the fuel and the oxidizer. Liquid oxygen boils at 147 °C and liquid hydrogen boils at 217 °C. Cryogenic propellants require special insulated containers and vents to allow gas from the evaporating liquids to escape. The liquid fuel and oxidizer are pumped from the storage tanks to an expansion chamber and injected into the combustion chamber where they are mixed and ignited by a flame or spark. The fuel expands as it burns and the hot exhaust gases are directed out of the nozzle to provide thrust.

5.6.2 Hypergolic Propellant:

A hypergolic propellant is composed of a fuel and oxidizer that ignite when they come into contact with each other. No spark is needed. Hypergolic propellants are typically corrosive so storage requires special containers and safety facilities.

5.6.3 Mono-propellants:

Monopropellants combine the properties of fuel and oxidizer in one chemical. By their nature, monopropellants are unstable and dangerous. Monopropellants are typically used in adjusting or vernier rockets to provide thrust for making changes to orbits once the payload is in orbit.

5.6.4 Advantages and Disadvantages of Liquid Propellants:

Advantages of liquid propellant rockets include the highest energy per unit of fuel mass, variable thrust, and a restart capability. Raw materials, such as oxygen and hydrogen are in abundant supply and a relatively easy to manufacture. Disadvantages of liquid propellant rockets include requirements for complex storage containers, complex plumbing, precise fuel and oxidizer injection metering, high speed/high capacity pumps, and difficulty in storing fuelled rockets.

5.6.5 Properties of Liquid Propellants:

1. Propellant should have high calorific value
2. Its density should be high
3. It should have low values of vapour pressure and viscosity

4. It should have higher specific heat and thermal conductivity
5. Products of combustion should have low molecular weight to produce high jet velocity
6. It should be non corrosive and non reactive with components of the engine
7. It should not be poisonous and hazardous
8. It should be cheap and easily available
9. Energy released during combustion per unit mass of the propellant combination should be high
10. It should be easily ignitable

5.7 Liquid Propellant Rocket Engine:

5.7.1 Construction:

The construction of liquid propellant rocket engine is shown in figure 5.2

Liquid fuel (refined petrol, liquid hydrogen, hydrazine etc) and liquid oxygen are used in this engine. Liquid fuel and liquid oxygen are stored separately in two different tanks. Preheater is used to heat the fuel and oxidizer. Nozzle is used to increase the velocity and decrease the pressure of the gases

5.7.2 Working:

1. Liquid fuel and liquid oxygen are pumped separately into a combustion chamber through control valves.
2. Since the liquid fuel and liquid oxygen are stored at very low temperature, they are preheated in the preheated to a suitable temperature
3. The preheater fuel-oxidizer mixture is injected into the combustion chamber through suitable injector and combustion takes place.

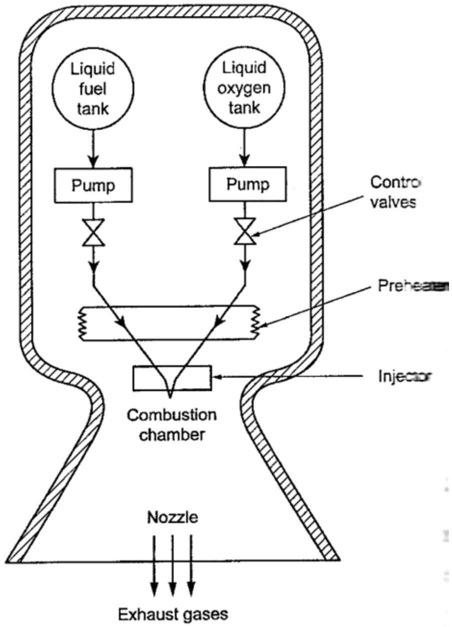


Fig. 5.2 Liquid Propellant Rocket Engine

4. When the combustion takes place in the combustion chamber, very high pressure and temperature gases are produced
5. The highly heated products of combustion gases are then allowed to expand in nozzle section.
6. In the nozzle, pressure energy of the gas is converted into kinetic energy. So the gases coming out from the unit with the very high velocity.
7. Due to high velocity of gases coming out from the unit, a force or thrust is produced in the opposite direction. This thrust propels the rocket.

5.7.3 Advantages:

1. Liquid propellant engines can be reused after recovery. So it is economical
2. Combustion process is controllable.

3. Speed regulation is possible
4. High specific impulse
5. More economical for long range operation
6. Malfunctions and accidents can be rectified at any stage

5.7.4 Disadvantages:

1. Its construction is more complicated compared to solid propellant rock
2. There are additional handling and safety problems if the propellants are poisonous and corrosive
3. Manufacturing cost is high
4. High vibration
5. Size and weight of the engine is more compared to solid propellant rockets
6. Any liquid propellants can exist in liquid state at very low temperature. So proper insulation is needed

5.8 Solid Propellant Rockets (SPR)

In solid propellant rockets the propellant to be burned is contained within the combustion chamber of case. The propellant charge is called the grain and it contains all the chemical elements for complete burning. Once ignited, it usually burns smoothly at nearly constant rate on the exposed surface of the charge. Because there are no feed systems or valves, such as there in liquid units, solid propellant rockets are usually relatively simple in construction.

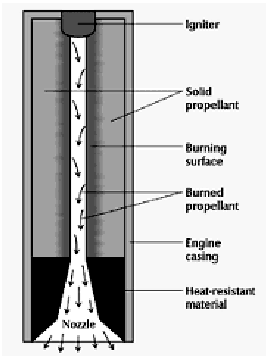


Fig. 5.3 Solid Propellant Rocket

5.8.1 Advantages of SPR over LPR

1. Simpler in construction and design - Fuel and oxidizer are in one mass inside the combustion chamber and there are no moving parts like pumps as in the case of LPR.
2. SPR can be handled with greater ease in the field - there are no special storage problems, no deterioration during storage and less danger due to explosion.
3. Lower initial cost - even though solid propellants are costlier the overall cost is less.
4. Excellent reliability - chances of malfunctioning are less.
5. Low servicing problems
6. For low total impulse applications it is lighter.
7. Much easier in achieving multistage or clustering with SPR.

5.8.2 Disadvantages:

1. Cooling of the combustion chamber and the throat of the nozzle is a big problem.
2. On off control is not possible - difficult to extinguish and reignite, since the control of propellant is not in our hand.
3. Specific impulse is lower than LPR engines.
4. For long duration or high thrust it becomes heavier.
5. Thrust cannot be varied readily.
6. Do not have rapid thrust vector control. The direction of flow from the nozzle is not easy to deflect.

5.9 Liquid Propellant Rockets (LPR):

Liquid propellant rocket use liquid propellants that are fed under pressure from tanks into a thrust chamber. In the thrust chamber the propellant react to form hot gases, which in turn are accelerated and ejected at a high velocity through a supersonic nozzle, thereby imparting momentum to the system. A liquid rocket unit usually permits repetitive operation and can be started and shut-off at will. If the thrust chamber is provided with adequate cooling capacity, it is possible t run liquid rockets for periods extending one hour, dependent only on the propellant supply. A liquid propellant rocket system, is however, relatively complicated; it requires several precision valves and a complex feed mechanism which often includes propellant pumps, turbines or a propellant pressurizing device, and a relatively intricate combustion or thrust chamber.

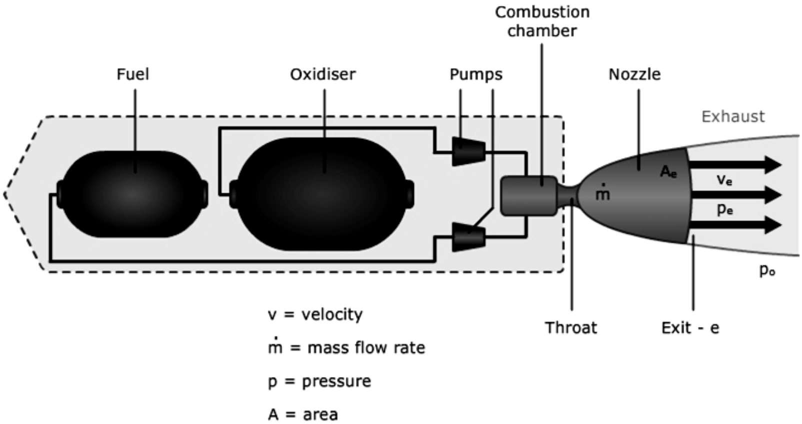


Fig. 5.4 Liquid Propellant Rocket

5.9.1 Pressure Feed System:

One of the simplest and most common means of pressurizing the propellant is to force them out of their respective tanks by displacing them with high pressure gas. The gas is fed into the propellant tanks at a controlled pressure, thereby giving a controlled propellant discharge. For low thrust and/or short duration, such as for space vehicles or anti-aircraft rockets, a feed system of this type is preferred. The rocket engines with pressurized feed system can be very reliable because of their simplicity.

A simple pressurized feed system is shown schematically in Figure. It consists essentially of a high pressure gas tank, a gas shut-off and starting valve, a pressure regulator, propellant tanks, propellant valves and feed lines. Additional components such as filling and draining provisions, check valves and filters are also often incorporated.

After all tanks are filled, the high-pressure air valve is remotely actuated and admits air through the pressure regulator at a constant pressure to the propellant tanks. The purpose of the check valves is to prevent mixing of the oxidizer with the fuel when the unit is not in the upright position. The propellants are fed to the thrust chamber by opening valves. When the propellants are completely consumed, the pressuring air serves also as a scavenging agent and cleans lines and valves of liquid propellant residue.

5.9.2 Turbo Pump Feed System:

The turbo pump rocket feed system pressurizes the propellant by means of pumps, which in turn are driven by turbines. The turbines derive their power from the expansion of hot gases. Turbo pump rocket systems are usually used on high thrust and long duration rocket units; they are lighter than other types for these applications. Their engine weight is essentially independent of thrust.

5.9.3 Advantages of LPR over SPR:

1. Duration of operation can be controlled.
2. Size of the combustion chamber is small as whole of the fuel need not be stored in it. (The fuel and oxidizer are stored outside in separate tanks)
3. On and off control is possible.
4. Use of cooling allows the thrust chamber walls t maintain their strength, use of less expensive or non-critical materials and preheating of the fuel.
5. Control of LPR is easier than control of SPR.

5.10 Hybrid Rockets:

Hybrid rockets make use of various combinations of solid and liquid propellants. Most common is the liquid oxidizer-solid fuel concept shown in figure. The oxidizer can be either a storable of a cryogenic liquid depending on the specific impulse or other requirements of the application. RFNA (Red fuming nitric acid) and a fuel grain of polybutandiene-polymethyl methacrylate) are typical low cost hybrid propellants. Actual specific impulse valves are between 170 and 220s for storable oxidizers and plastic fuels

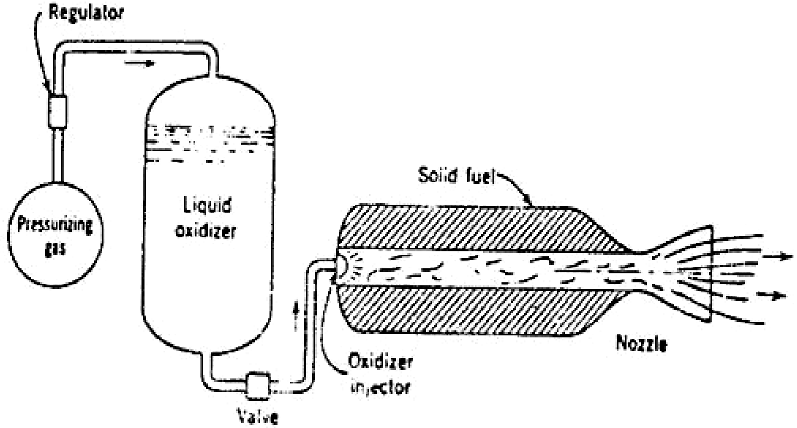


Fig. 5.5 Hybrid Rocket

The main advantages of a hybrid rocket are: (a) low cost for applications where economy is essential and low performance is acceptable. (b) Simplicity of stored grain fuel. (c) a liquid for nozzle cooling and thrust modulation. (d) start-up-restart capabilities. (e) Good storability traits and (f) safety during storage or operation.

5.11 Integral rocket-ramjet:

The principles of rocket and ramjet can be combined so that the two propulsion systems operate in sequence and in tandem and yet utilize a common combustion chamber volume as shown in Figure. The low-volume configuration, known as an integral rocket-ramjet, is particularly attractive in air launched missiles using ramjet Propulsion. The transition from the rocket to the ramjet requires enlarging the exhaust nozzle (usually by ejecting rocket nozzle parts), opening the ramjet air inlet combustion chamber interface, and following these two events with the normal ramjet starting sequence.

5.12 Application of Rockets:

1. Airplane power plants:
    a. Primary power plants
       German Mel 163 fighter used in II World war
       X- 1 Research engine (first to break the sonic barrier)
       X - 15 supersonic research aircraft
    b. Auxiliary power plants for super performance or improving the performance (speed, rate of climb), assisted take-off.
    c.
2. Weapons
    a. Rocket projectiles (unguided missile) - not accurate, payloads explosive charges, smoke charge, other military payloads (or mail carriers to reach remote villages in mountain areas)
    b. Guided missiles: - similar to rocket projectiles but bigger in size and the trajectory is controlled - ground to ground, ground to air (aircraft), air to air, air to ground, ship to air and ship to ship missiles - payloads are atomic weapons.
3. Space vehicles
    a. military purposes – reconnaissance
    b. commercial purposes - communication and weather studies
    c. scientific purposes - lunar, space and planets
    d. Manned and unmanned space vehicles
    e. Near earth, longer, interplanetary, trains- solar systems

5.13 Rocket engines are used to

1. Take-off from earth, ascend and achievement of orbit
2. Altitude control, trajectory connections, re-entry, attainment of lunar and planetary orbits and landing, separation of vehicle stages, manoeuvres etc.
3. Sounding rockets – rockets which carry instruments to measure meteorological and scientific data at high altitudes, may be guided or unguided.
4. Other applications – for throwing life line to ships, signal rockets, antitank rockets, under water rockets etc.
5. Miscellaneous – ejection or crew escape capsules and stores personnel "propulsion belts", Propulsion for target drones, signal rockets, decoy rockets, spin rockets, vernier rockets, under water rockets for torpedoes and missiles, the throwing of life lines to ships and "Fourth of July rockets"

5.14 Comparison of Various propulsion systems:

Type	Uses	Advantages	Disadvantages
Solid fuel chemical propulsion	main booster	simple, reliable, few moving parts, lots of thrust	not restartable
Liquid fuel chemical propulsion	main booster, small control	restartable, controllable, lots of thrust	complex
Cold-gas chemical propulsion	small control	restartable, controllable	low thrust
Ion	in space booster	restartable, controllable, high specific impulse	complex

Table 5.2.

The solid motor is used mainly as a booster for launch vehicles. Solid motors are almost never used in space because they are not controllable. The boosters are lit and then they fire until all the propellant has burned. Their main benefits are simplicity, a shelf life which can extend to years as in the case of missiles, and high reliability.

Liquid motors come in many shapes and sizes: Most of them are controllable (can be throttled up and down), re startable, are often used as control and manoeuvring thrusters. Liquid thrusters can be broken into three main types: monopropellant, bipropellant, and cryogenic thrusters. Monopropellants only use one propellant such as hydrazine. Bipropellants use a fuel and an oxidizer such as RP-1 and H_2O_2. Cryogenic systems use liquefied gases such as LiH and LOX (liquid hydrogen and liquid oxygen). Cryogenic means super-cooled. You would have to super-cool hydrogen and oxygen to make them liquids. With each step from monopropellant to bipropellant to cryogenic the thrusters' complexity goes up but the performance also goes up.

Cold-gas motors have controllability similar to liquids but are the simpler and lighter. They are basically a high pressure tank with switches which flip between the open and shut state. They function a little like spray paint, with the contents under pressure inside, and when the valve is opened, they stream out.

Ion engines are vastly different from chemical (solid, liquid) engines in that they are low thrust engines which can run for extended periods of time. The length of use of chemical engines is usually from seconds to days while the length of use of ion engines can be anywhere from days to months.

Important Formulas:

1. Thrust:

$$F = \dot{m}_p\, c_e + (p_e - p_a)\, A_e, \qquad N$$

2. Specific Impulse:

$$I_{sp} = \frac{F}{W_p} = \frac{\dot{m}_p \times c_j}{\dot{m}_p \times g}$$

3. Specific Propellant Consumption:

$$SPC = \frac{W_p}{F} = \frac{1}{I_{sp}}$$

$$SPC = \frac{g}{c_j}$$

4. Effective speed ratio

$$\sigma = \frac{u}{c_j}$$

5. Weight flow coefficient or propellant weight flow coefficient:

$$c_w = \frac{W_p}{F^*} = \frac{W_p}{p_o\, A^*}$$

6. Thrust coefficient:

$$c_F = \frac{F}{F^*} = \frac{F}{p_o\, A^*}$$

7. Characteristic Velocity

$$c^* = \frac{c_j}{c_F}$$

8. Propulsive Efficiency:

$$\eta_p = \frac{Propulsive\ power}{power\ output\ of\ engine}$$

$$\eta_p = \frac{2\,\sigma}{1 + \sigma^2}$$

9. Thermal Efficiency:

$$\eta_t = \frac{{c_j}^2 + u^2}{2 \times C.V}$$

10. Overall Efficiency:

$$\eta_o = \frac{c_j \times u}{C.V} = \eta_p \times \eta_t$$

11. Propellant Flow rate:

$$\dot{m}_p = \rho_e \times A_e \times c_e$$

TWO MARK QUESTIONS WITH ANSWERS

1. Define Rocket propulsion.

   If the propulsion unit contains its own oxygen supply for combustion purposes, the system is known as "Rocket propulsion".

2. Define thrust for a rocket engine and how it is produced.

   The force that propels the rocket at a given velocity is known as thrust. This is produced due to the change in momentum flux of the outgoing gases as well as the difference between the nozzle exit pressure and the ambient pressure.

3. What are the types of rocket engines?

   Rocket engines are classified in the following manner.
   a) On the basis of source of energy employed
      i. Chemical rockets,
      ii. Solar rockets
      iii. Nuclear rockets and
      iv. Electrical rockets
   b) On the basis of propellants used
      i. Liquid propellant
      ii. Solid propellant
      iii. Hybrid propellant rockets.

4. Compare solid and liquid propellant rockets.
   Solid Propellant
      a) Solid fuels and oxidizers are used in rocket engines
      b) Generally stored in combustion chamber (both oxidizer and fuel).
      c) Burning in the combustion chamber is uncontrolled rate.

Liquid Propellant

a) Liquid fuels and oxidizers are used.
b) Separate oxidizer and fuel tanks are used for storing purposes.
c) Controlled rate.

5. What are the types of liquid propellants used in rocket engines?
   i. Mono propellants
   ii. Bi – propellants

6. Give two liquid propellants.

Liquid fuels	: Liquid hydrogen, UDMH, hydrazine
Solid fuels	: Polymers, plastics and resin material

7. What are mono-propellants? Give example.

A liquid propellant which contains both the fuel and oxidizer in a single chemical is known as “mono propellant”. e.g.
i. Hydrogen peroxide
ii. Hydrazine
iii. Nitro-glycerine and
iv. Nitro methane, etc.

8. What is bi-propellant? Give Example.
If the fuel and oxidizer are different from each other in its chemical nature, then the propellant is called bipropellant. Example: liquid oxygen – gasoline & hydrogen peroxide – hydrazine.

9. Classify the rocket engines based on source of energy employed.

Chemical rocket engine, Solar rocket engine, Nuclear rocket engine and Electrical rocket engine.

10. Name some oxidizers used in rockets.

A liquid propellant which contains the fuel and oxidizer in separate units is known as bi-propellant. The commonly used bi-propellant combinations are:

Oxidizer	Fuel
a) Liquid oxygen b) Hydrogen peroxide c) Nitrogen tetroxide d) Nitric acid	a) Gasoline b) Liquid hydrogen c) UDMH d) Alcohol, ethanol

11. Name few advantages of liquid propellant rockets over solid propellant rockets.

a) Liquid propellant can be reused or recharged. Hence it is economical.
b) Increase or decrease of speed is possible when it is in operation.
c) Storing and transportation is easy as the fuel and oxidizer are kept separately.
d) Specific impulse is very high.

12. What are inhibitors?

Inhibitors are used to regulate (or prevent) the burning of propellant at some sections.

13. Give the important requirements of rocket engine fuels.

It must be able to produce a high chamber temperature. It should have a high calorific value per unit of propellant.
It should not chemically react with motor system including tanks, piping, valves and injection nozzles.

14. What is meant by restricted burning in rockets?

In this case, the inhibition material (or) restrictions prevents the propellant grain from burning in all directions. The propellant grain burns only at some surfaces while other surfaces are prevented from burning.

15. Define Thrust?

It is the force generated, measured in pounds or kilograms. Thrust generated by the first stage must be greater than the weight of the complete launch vehicle while standing on the launch pad in order to get it moving. Once moving upward, thrust must continue to be generated to accelerate the launch vehicle against the force of the Earth's gravity.

$$F = \dot{m}_p\, c_e + (p_e - p_a)\, A_e, \qquad N$$

When p_e (exit Pressure of nozzle) = p_a (ambient pressure)

$$F = \dot{m}_p\, c_e = \dot{m}_p\, c_j, \qquad N$$

16. Define Specific Impulse.

The impulse, sometimes called total impulse, is the product of thrust and the effective firing duration. A shoulder fired rocket such as the LAW has an average thrust of 272.2 kg and a firing duration of 0.2 seconds for an impulse of 533.8 Ns.

$$I_{sp} = \frac{F}{W_p} = \frac{\dot{m}_p \times c_j}{\dot{m}_p \times g}$$

$$I_{sp} = \frac{c_j}{g}$$

17. Define Specific Propellant Consumption.

It is the propellant consumption rate per unit thrust

$$SPC = \frac{W_p}{F} = \frac{1}{I_{sp}}$$

$$SPC = \frac{g}{c_j}$$

18. Define Weight flow coefficient or propellant weight flow coefficient.

    It is the ratio of propellant flow rate to the throat force

$$c_w = \frac{W_p}{F^*} = \frac{W_p}{p_o\ A^*}$$

19. Define Thrust coefficient:

    It is the ratio of thrust to the throat force.

$$c_F = \frac{F}{F^*} = \frac{F}{p_o\ A^*}$$

    This show,

$$F = c_F \times \ p_o\ A^*$$

20. Define Impulse to weight ratio (IWR).

    It is the ratio of total impulse of the rocket to the total weight of the rocket.

$$IWR = \frac{I_{total}}{W_{total}}$$

    Where

$$I_{total} = I_{sp} \ \times W_p$$

21. Define Characteristic Velocity.

    It is the ratio of effective jet velocity and thrust coefficient.

$$c^* = \frac{c_j}{c_F}$$

22. Define Propulsive Efficiency.

    It is defined as the ratio of propulsive power or thrust power to the power output of the engine.

$$\eta_p = \frac{Propulsive\ power}{power\ output\ of\ engine} = \frac{\dot{m}_p \times\ c_j\ \times u}{\frac{1}{2}\ \dot{m}_p\ \left(c_j{}^2 -\ u^2\right)}$$

$$\eta_p = \frac{2\ \sigma}{1+\sigma^2}$$

23. Define Thermal Efficiency.

It is defined as the ratio of the power output of the engine to the power input to the engine

$$\eta_t = \frac{\frac{1}{2}\ \dot{m}_p\ \left(c_j{}^2 +\ u^2\right)}{\dot{m}_p\ \times C.V}$$

$$\eta_t = \frac{c_j{}^2 +\ u^2}{2\ \times C.V}$$

24. Define Overall Efficiency.

It is defined as the ratio of propulsive power to the power input to the engine.

$$\eta_o = \frac{\dot{m}_p \times\ c_j\ \times u}{\dot{m}_p\ \times C.V}$$

$$\eta_o = \frac{c_j\ \times u}{C.V} = \eta_p\ \times\ \eta_t$$

EXERCISE PROBLEMS

1. Explain the working principle of hybrid propellant rocket engine with merits and demerits.
2. Explain the working principle of arc plasma rocket engine with neat sketch.
3. Explain the working principle of ion rocket enginewith neat sketch.

4. A rocket flies at a speed of 10,000 km/hr with an effective exhaust jet velocity of 1350 m/s and the heat produced by the propellant is 6600kJ/kg. if the propellant flow rate is 4.8 kg/s, Determine:
   i. Propulsive efficiency
   ii. Propulsive power
   iii. Engine output
   iv. Thermal efficiency
   v. Overall efficiency

5. A rocket engine has the following data:

Effective jet velocity	=	1200 m/s
Flight to jet speed ratio	=	0.82
Oxidizer flow rate	=	3.4 kg/s
Fuel flow rate	=	1.2 kg/s
Heat of reaction per kg of the exhaust gases	=	2520 kJ/kg

Calculate the followings:

   i. Thrust
   ii. Specific impulse
   iii. Propulsive efficiency
   iv. Thermal efficiency
   v. Overall efficiency

6. A rocket has the following data:

Propellant flow rate	=	5 kg/s
Nozzle exit diameter	=	10 cm
Nozzle exit pressure	=	1.02 bar
Ambient pressure	=	1.013 bar
Thrust chamber pressure	=	20 bar
Thrust	=	7 Kn

Determine the followings:

i. Effective jet velocity
ii. Actual jet velocity
iii. Specific impulse
iv. Specific propellant consumption

7. The effective jet velocity from a rocket is 2700 m/s. the forward flight velocity is 1350 m/s and the propellant consumption is 78.6 kg/s. calculate the thrust, thrust power and propulsive efficiency.

8. The specific impulse of a rocket is 125 s and the flow rate of propellant is 44 kg/s. the nozzle throat area is 18 cm2 and the pressure in the combustor is 25 bar. Determine the thrust coefficient, propellant flow coefficient, specific propellant consumption and characteristic velocity.

MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	answer
The main difference between rocket propulsion and jet propulsion is	the former need more fuel to start	for combustion, oxygen is taken form the atmosphere for the latter propulsion	for the former propulsion, oxygen is taken from the atmosphere for combustion	none of these	for the former propulsion, oxygen is taken from the atmosphere for combustion
If the fuel and oxidized are different from each other in its chemical nature, then the propellant is called	monopropellant	homopropellant	heterogenous propellant	bipropellant	bipropellant
Mostly used fuel with liquid oxygen oxidizer is	alcohol	methane	aniline	ethane	methane
The best oxidizer used with aniline fuel is	liquid oxygen	white fuming nitric acid	hydrogen peloxide	red fuming nitric acid	red fuming nitric acid
Which of the following is the basic processor in combustion	injection	atomization	mixing	all of these	all of these
Which of the following engine is difficult to stop in the mid-way	ramjet engine	turbojet engine	solid propellant engine	liquid propellant engine	solid propellant engine
Thrust developed per unit weight flow rate of the propellant is known are	thrust	specific thrust	specific impulse	relative impulse	specific impulse
Specific propellant consumption is given by the formula	W_p/F	F/W_p	FxW_P	none of these	W_p/F
Expansion of IWR is	intense weight ration	inertia weight ration	impulse to weight ratio	none of these	impulse to weight ratio

Expansion of JATO is	jet assisted take off	jet applied take off	jet accelerated take off	none of these	jet assisted take off
The ratio of propellant flow to throat force is given by	specific impulse	characteristic velocity	flow coefficient	weight flow coefficient	flow coefficient
Which of the following is classification of rocket engine based on source of energy	nuclear rocket engine	electrical rocket engine	chemical rocket engine	All of these	All of these
The ratio of power output of engine to input power through fuel	thrust coefficient	propulsive efficiency	thermal efficiency	thrust	thermal efficiency
A liquid propellant which contains both fuel and oxidizer in a single chemical is called as	heteropropellant	monopropellant	bipropellant	homopropellant	monopropellant
Propulsive efficiency is also known as	thermal efficiency	relative efficiency	overall efficiency	none of these	none of these
Characteristic velocity is given by the formula	Cj/Cf	CjxCf	Cf/Cj	1/CfxCj	Cj/Cf
It is difficult to build large solar powered rockets because	large surface area is required to capture solar energy	cost is high	thrust produced is less	none of these	large surface area is required to capture solar energy
In which propellants plastics, polymers and PVC are used as fuels	heterogeneous propellant	monopropellant	bipropellant	both b and c	heterogeneous propellant
Which of the following is used as homogenous propellant	nitroglyceline	mitrocellulose	both a and b	none of these	both a and b

In which of the following engine decrease of speed is not possible	ramjet engine	liquid propellant engine	solid propellant engine	none of these	solid propellant engine
A rocket works with maximum overall efficiency when air-craft velocity is ___________ the jet velocity.	equal to	one-half	double	one- third	one-half
In the aircraft propellers	the propulsive matter is ejected from within the propelled body	the propulsive matter is caused to flow around the propelled body	its functioning does not depend upon the presence of air	none of these	the propulsive matter is ejected from within the propelled body
Only rocket engines can be propelled to space because	they can generate very high thrust	they have high propulsion efficiency	these engines can work on several fuels	they are not air-breathing engines	they are not air-breathing engines
The type of rotary compressor used in aeroplanes, is of	centrifugal type	axial flow type	radial flow type	none of these	axial flow type
Which of the following statement is correct relating to rocket engines	The combustion chamber in a rocket engine is directly analogous to the reservoir of a super sonic wind tunnel	The stagnation conditions exist at the combustion chamber	The exit velocities of exhaust gases are much higher than those in jet engine	all of these	all of these
A rocket engine uses ___________ for the combustion of its fuel.	surrounding air	compressed atmospheric air	its own oxygen	none of these	its own oxygen
A compressor at high altitudes will require ___________ power	Same	Less	More	Very high	Less

The thrust of a jet propulsion power unit can be increased by	injecting water into the compressor	burning fuel after gas turbine	injecting ammonia into the combustion chamber	all of these	all of these
The basic principle of the rocket propulsion is the ______ as that of jet propulsion	Same	Different	Advanced	Complicated	Same
Standard air is the air at __________ and relative humidity of 36 percent	0.1 bar and 20° C	1 bar and 20° C	0.1 bar and 40° C	1 bar and 40° C	1 bar and 20° C
The velocity of air entering in a rocket is __________ as compared to an aircraft.	Zero	Less	More	Same	Zero
The material commonly used for air craft gas turbine is	The material commonly used for air craft gas turbine is	high alloy' steel	duralumin	Timken, Haste and Colonel alloys	Timken, Haste and Colonel alloys
It is not possible to use closed gas turbine cycle in aeronautical engines because	it is inefficient	it is bulky	t requires cooling water for its operation	it is complicated	t requires cooling water for its operation
A gas turbine used in air craft should have	high h.p. and low weight	low weight and small frontal area	small frontal area and high h.p.	high speed and high h.p.	low weight and small frontal area