KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE – 21

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

14BEME-E04 **QUALITY CONTROL AND RELIABILITY ENGINEERING** **3 0 0 3 100**

OBJECTIVES

1. To introduce the concept of SQC
2. To understand process control and acceptance sampling procedure and their application.
3. To learn the concept of reliability

UNIT I INTRODUCTION AND PROCESS CONTROL FOR VARIABLES 9

Introduction, definition of quality, basic concept of quality, definition of SQC, benefits and limitation of SQC, Quality assurance, Quality cost–Variation in process– factors – process capability – process capability studies and simple problems – Theory of control chart– uses of control chart – Control chart for variables – X chart, R chart and σ chart.

UNIT II PROCESS CONTROL FOR ATTRIBUTES 9

Control chart for attributes –control chart for proportion or fraction defectives – P chart and NP chart – control chart for defects – C and U charts, State of control and process out of control identification in charts.

UNIT III ACCEPTANCE SAMPLING 9

Lot by lot sampling – Types – probability of acceptance in single, double, multiple sampling techniques – O.C. curves – producer's Risk and consumer's Risk. AQL, LTPD, AOQL concepts–standard sampling plans for AQL and LTPD– uses of standard sampling plans.

UNIT IV LIFE TESTING – RELIABILITY 9

Life testing – objective: – failure data analysis, Mean failure rate, mean time to failure, mean time between failure, hazard rate, system reliability, series, parallel and mixed configuration – simple problems. Maintainability and availability – simple problems. Acceptance sampling based on reliability test – O.C Curves.

UNIT V QUALITY AND RELIABLITY 9

Reliability improvements – techniques– use of Pareto analysis – design for reliability – redundancy unit and standby redundancy – Optimization in reliability – Product design – Product analysis – Product development – Product life cycles.

TOTAL 45 PERIODS

Note: Permitted to use approved statistical table in the examination.

TEXT BOOKS

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Grant. Eugene .L	Statistical Quality Control	McGraw–Hill, New Delhi	2008
2	Srinath L.S	Reliability Engineering	Affiliated East west press New Delhi	1991

REFERENCES

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Manohar Mahajan	Statistical Quality Control	Dhanpat Rai and Sons, New Delhi	2001
2	Besterfield D.H	Quality Control	Prentice Hall, New Delhi	1993
3	Danny Samson	Manufacturing and Operations Strategy	Prentice Hall, New Delhi	1991

4	Connor P.D.T.O	Practical Reliability Engineering	John Wiley, New Delhi	1993

WEB REFERENCES

1. http://www.statsoft.com/textbook/stquacon.html
2. http://www.isixsigma.com/library/content/c010806a.asp
3. http://www.statgraphics.com/control_charts.htm
4. http://www.sqconline.com/sampling–plans.html
5. http://reliability.sandia.gov/Maintenance/Data_Failure_Analysis/data_failure_analysis.html
6. http://www.designinindia.net/everywhere/disciplines/product–design/index.html

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE – 21

FACULTY OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE PLAN

Subject Name : QUALITY CONTROL AND RELIABILITYENGINEERING
Subject Code : 14BEME-E04 (Credits - 3)
Name of the Faculty : P. M. GOPAL
Designation : Assistant Professor
Year/Semester/Section : IV YEAR / VII
Branch : MECHANICAL ENGINEERING

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT I		**INTRODUCTION AND PROCESS CONTROL FOR VARIABLES**	
1.	1	Introduction to QCRE	-
2.	1	Introduction to quality engineering	T[1]
3.	1	Definition of quality-Basic concept of quality	T[1]
4.	1	Definition of SQC- Benefits and limitation of SQC	T[1]
5.	1	Quality assurance	T[1]
6.	1	Quality cost	T[1]
7.	1	Variation in process- factors	T[1]
8.	1	Process capability -Studies	T[1]
9.	1	Process capability and Simple problems	T[1]
10.	1	Theory of control chart-Uses of control chart	T[1]
11.	1	Control chart for variables – X, R and σ chart.	T[1]
Total no. of Hours planned for unit - I			**11**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT II PROCESS CONTROL FOR ATTRIBUTES			
12.	1	Control chart for attributes	W[2]
13.	1	Control chart for proportion or fraction defectives	T[1]
14.	1	P chart	T[1]
15.	1	Problems on P chart	T[1]
16.	1	NP chart	T[1]
17.	1	Problems on NP chart	T[1]
18.	1	Control chart for defects	T[1]
19.	1	C charts	T[1]
20.	1	U charts	T[1]
21.	1	State of control and process out of control identification in charts.	T[1]
Total no. of Hours planned for unit - II			**10**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT III ACCEPTANCE SAMPLING			
22.	1	Lot by lot sampling– types	**W[2]**
23.	1	Probability of acceptance in Single Sampling techniques	**W[2]**
24.	1	Probability of acceptance in Double Sampling techniques	**W[2]**
25.	1	Probability of acceptance in Multiple sampling techniques	**W[2]**
26.	1	O.C.Curves	**W[1]**
27.	1	Producer's Risk and consumer's Risk	**W[2]**
28.	1	AQL, LTPD concepts	**W[2]**
29.	1	AOQL concepts	**W[2]**
30.	1	Standard sampling plans for AQL and LTPD	**W[2]**
31.	1	Uses of standard sampling plans	**W[2]**
Total no. of Hours planned for unit - III			**10**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT IV LIFE TESTING – RELIABILITY			
32.	1	Life testing-objective	**T[2]**
33.	1	Failure data analysis	**T[2]**
34.	1	Mean failure rate, Mean time to failure	**T[2]**
35.	1	Mean time between failure, Hazard rate	**T[2]**
36.	1	System reliability- Series & parallel configuration	**T[2]**
37.	1	System reliability- mixed configuration	**T[2]**
38.	1	Simple problems	**T[2]**
39.	1	Maintainability and availability-simple problems	**T[2]**
40.	1	Acceptance sampling based on reliability test	**T[2]**
41.	1	O.C Curves	**T[2]**
Total no. of Hours planned for unit - IV			**10**

Sl. No.	Lecture Duration (Hr)	Topics to be Covered	Support Materials
UNIT V QUALITY AND RELIABLITY			
42.	1	Reliability improvements - Techniques	**T[2]**
43.	1	Use of Pareto analysis	**T[2]**
44.	1	Design for reliability	**T[2]**
45.	1	Redundancy and Standby redundancy	**T[2]**
46.	1	Optimization in reliability	**T[2]**
47.	1	Product design	**T[2]**
48.	1	Product Analysis	**T[2]**
49.	1	Product development	**T[2]**
50.	1	Product life cycles	**T[2]**
51.	1	Maintenance	**T[2]**
52.	1	**Discussion on Competitive Examination related Questions / University previous year questions**	**Question Bank**
Total no. of Hours planned for unit - V			**11**

TOTAL PERIODS : 52 hrs

TEXT BOOKS

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Grant. Eugene .L	Statistical Quality Control	McGraw–Hill, New Delhi	2008
2	Srinath L.S	Reliability Engineering	Affiliated East west press New Delhi	2002

REFERENCES

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Manohar Mahajan	Statistical Quality Control	Dhanpat Rai & Sons, New Delhi	2003
2	Besterfield D.H	Quality Control	Prentice Hall, New Delhi	1993
3	Danny Samson	Manufacturing and Operations Strategy	Prentice Hall, New Delhi	1991
4	Connor P.D.T.O	Practical Reliability Engineering	John Wiley, New Delhi	2011

WEB REFERENCES

1. http://www.statsoft.com/textbook/stquacon.html
2. http://www.sqconline.com/sampling–plans.html

UNIT	Total No. of Periods Planned	Lecture Periods	Tutorial Periods
I	11	11	0
II	10	10	0
III	10	10	0
IV	10	10	0
V	11	11	0
TOTAL	**52**	**52**	**0**

I. CONTINUOUS INTERNAL ASSESSMENT : 40 Marks

(Internal Assessment Tests: 30, Attendance: 5, Assignment/Seminar: 5)

II. END SEMESTER EXAMINATION : 60 Marks

TOTAL : 100 Marks

STAFF INCHARGE **HOD / MECH** **DEAN / FOE**

UNIT I INTRODUCTION AND PROCESS CONTROL FOR VARIABLES

DEFINITION OF QUALITY:

- The meaning of "Quality" is closely allied to cost and customer needs. "Quality" may simply be defined as fitness for purpose at lowest cost.
  - ✓ The component is said to possess good quality, if it works well in the equipment for which it is meant. Quality is thus defined as fitness for purpose.
- Quality is the 'totality of features and characteristics' both for the products and services that can satisfy both the explicit and implicit needs of the customers.
- "Quality" of any product is regarded as the degree to which it fulfills the requirements of the customer.
- "Quality" means degree of perfection. Quality is not absolute but it can only be judged or realized by comparing with standards. It can be determined by some characteristics namely, design, size, material, chemical composition, mechanical functioning, workmanship, finish and other properties.

MEANING OF CONTROL

Control is a system for measuring and checking (inspecting) a phenomenon. It suggests when to inspect, how often to inspect and how much to inspect. In addition, it incorporates a feedback mechanism which explores the causes of poor quality and takes corrective action.

Control differs from 'inspection', as it ascertains quality characteristics of an item, compares the same with prescribed quality standards and separates defective items from non-defective ones. Inspection, however, does not involve any mechanism to take corrective action.

MEANING OF QUALITY CONTROL

Quality Control is a systematic control of various factors that affect the quality of the product. The various factors include material, tools, machines, type of labour, working conditions, measuring instruments, etc.

Quality Control can be defined as the entire collection of activities which ensures that the operation will produce the optimum Quality products at minimum cost.

As per A.Y. Feigorbaum Total Quality Control is: "***An effective system for integrating the quality development, Quality maintenance and Quality improvement efforts of the various groups in an***

organization, so as to enable production and services at the most economical levels which allow full cu stom er satisfacti on "

In the words of Alford and Beatly, ***"Qual it y C on trol " m ay be broadl y defined as th at "I n du str ial m an agem ent tech n iqu e m ean s of w hich produ cts of u n iform accepted qu ali ty are m an u factu red." Qual it y Control is concerned with making things right rather than discovering and rejecting those made wrong.***

In short, we can say that quality control is a technique of management for achieving required standards of products.

FACTORS AFFECTING QUALITY

In addition to **men, materials, machines and manufacturing conditions** there are some other factors which affect the product quality. These are:

- Market Research i.e. indepth into demands of purchaser.
- Money i.e. capability to invest.
- Management i.e. Management policies for quality level.
- Production methods and product design.

Modern quality control begins with an evaluation of the customer's requirements and has a part to play at every stage from goods manufactured right through sales to a customer, who remains satisfied.

OBJECTIVES OF QUALITY CONTROL

- To decide about the standard of quality of a product that is easily acceptable to the customer and at the same time this standard should be economical to maintain.
- To take different measures to improve the standard of quality of product.
- To take various steps to solve any kind of deviations in the quality of the product during manufacturing.

FUNCTIONS OF QUALITY CONTROL DEPARTMENT

- Only the products of uniform and standard quality are allowed to be sold.
- To suggest method and ways to prevent the manufacturing difficulties.
- To reject the defective goods so that the products of poor quality may not reach to the customers.
- To find out the points where the control is breaking down and to investigate the causes of it.
- To correct the rejected goods, if it is possible. This procedure is known as rehabilitation of defective goods.

ADVANTAGES OF QUALITY CONTROL

- Quality of product is improved which in turn increases sales.
- Scrap rejection and rework are minimized thus reducing wastage. So the cost of manufacturing reduces.
- Good quality product improves reputation.
- Inspection cost reduces to a great extent.
- Uniformity in quality can be achieved.
- Improvement in manufacturer and consumer relations.

STATISTICAL QUALITY CONTROL (S.Q.C):

- **Statistics**: Statistics means data, a good amount of data to obtain reliable results. The science of statistics handles this data in order to draw certain conclusions.
- **S.Q.C:** This is a quality control system employing the statistical techniques to control quality by performing inspection, testing and analysis to conclude whether the quality of the product is as per the laid quality standards.

Using statistical techniques, S.Q.C. collects and analyses data in assessing and controlling product quality. The technique of S.Q.C. was though developed in 1924 by Dr.WalterA.Shewartan American scientist; it got recognition in industry only second world war. The technique permits a more fundamental control.

"S tat istical qu ali ty con trol can be sim ply defined as an economic & effective system of maintaining & improving the quality of outputs throughout the whole operating process of specification, production & i n specti on based on con ti nu ou s testin g wi th ran dom sam ples." ***-YA LUN CHOU***

"S tat istical qu ali ty control should be viewed as a kit of tools which may influence decisions to the functions of specification, production or inspection. ***-EUGENE L. GRANT***

The fundamental basis of S.Q.C. is the theory of probability. According to the theories of probability, the dimensions of the components made on the same machine and in one batch (if measured accurately) vary from component to component. This may be due to inherent machine characteristics or the environmental conditions. The chance or condition that a sample will represent the entire batch or population is developed from the theory of probability.

Relying itself on the probability theory, S.Q.C. evaluates batch quality and controls the quality of processes and products. S.Q.C. uses three scientific techniques, namely;

- Sampling inspection
- Analysis of the data, and
- Control charting

ADVANTAGES OF S.Q.C

S.Q.C is one of the tool for scientific management, and has following main advantages over 100 percent inspection:

- **Reduction in cost:** Since only a fractional output is inspected, hence cost of inspection is greatly reduced.
- **Greater efficiency:** It requires lesser time and boredom as compared to the 100 percent inspection and hence the efficiency increases.
- **Easy to apply:** Once the S.Q.C plan is established, it is easy to apply even by man who does not have extensive specialized training.
- **Accurate prediction**: Specifications can easily be predicted for the future, which is not possible even with 100 percent inspection.
- **Can be used where inspection is needs destruction of items:** In cases where destruction of product is necessary for inspecting it, 100 percent inspection is not possible (which will spoil all the products), sampling inspection is resorted to.
- **Early detection of faults:** The moment a sample point falls outside the control limits, it is taken as a danger signal and necessary corrective measures are taken. Whereas in 100 percent inspection, unwanted variations in quality may be detected after large number of defective items have already been produced. Thus by using the control charts, we can know from graphic picture that how the production is proceeding and where corrective action is required and where it is not required.

PROCESS CONTROL

Under this the quality of the products is controlled while the products are in the process of production.

The process control is secured with the technique of control charts. Control charts are also used in the field of advertising, packing etc. They ensure that whether the products confirm to the specified quality standard or not.

Process Control consists of the systems and tools used to ensure that processes are well defined, performed correctly, and maintained so that the completed product conforms to established requirements. Process Control is an essential element of managing risk to ensure the safety and reliability of the Space Shuttle Program. It is recognized that strict process control practices will aid in the prevention of process escapes that may result in or contribute to in-flight anomalies, mishaps, incidents and non-conformances.

The five elements of a process are:

- People – skilled individuals who understand the importance of process and change control
- Methods/Instructions – documented techniques used to define and perform a process
- Equipment – tools, fixtures, facilities required to make products that meet requirements
- Material – both product and process materials used to manufacture and test products
- Environment – environmental conditions required to properly manufacture and test products

PROCESS CONTROL SYSTEMS FORMS

Process control systems can be characterized as one or more of the following forms:

- **Discrete** – Found in many manufacturing, motion and packaging applications. Robotic assembly, such as that found in automotive production, can be characterized as discrete process control. Most discrete manufacturing involves the production of discrete pieces of product, such as metal stamping.
- **Batch** – Some applications require that specific quantities of raw materials be combined in specific ways for particular durations to produce an intermediate or end result. One example is the production of adhesives and glues, which normally require the mixing of raw materials in a heated vessel for a period of time to form a quantity of end product. Other important examples are the production of food, beverages and medicine. Batch processes are generally used to produce a relatively low to intermediate quantity of product per year (a few pounds to millions of pounds).
- **Continuous** – Often, a physical system is represented through variables that are smooth and uninterrupted in time. The control of the water temperature in a heating jacket, for example, is an example of continuous process control. Some important continuous processes are the production of fuels, chemicals and plastics. Continuous processes in manufacturing are used to produce very large quantities of product per year (millions to billions of pounds).

STATISTICAL PROCESS CONTROL (SPC)

SPC is an effective method of monitoring a process through the use of control charts. Much of its power lies in the ability to monitor both process center and its variation about that center. By collecting data from samples at various points within the process, variations in the process that may affect the quality of the end product or service can be detected and corrected, thus reducing waste as well as the likelihood that problems will be passed on to the customer. It has an emphasis on early detection and prevention of problems.

CONTROL CHARTS

Since variations in manufacturing process are unavoidable, the control chart tells when to leave a process alone and thus prevent unnecessary frequent adjustments. Control charts are graphical representation and are based on statistical sampling theory, according to which an adequate sized random sample is drawn from each lot. Control charts detect variations in the processing and warn if there is any departure from the specified tolerance limits. These control charts immediately tell the undesired variations and help in detecting the cause and its removal.

In control charts, where both upper and lower values are specified for a quality characteristic, as soon as some products show variation outside the tolerances, a review of situation is taken and corrective step is immediately taken.

If analysis of the control chart indicates that the process is currently under control (i.e. is stable, with variation only coming from sources common to the process) then data from the process can be used to predict the future performance of the process. If the chart indicates that the process being monitored is not in control, analysis of the chart can help determine the sources of variation, which can then be eliminated to bring the process back into control. A control chart is a specific kind of run chart that allows significant change to be differentiated from the natural variability of the process.

The control chart can be seen as part of an objective and disciplined approach that enables correct decisions regarding control of the process, including whether or not to change process control parameters. Process parameters should never be adjusted for a process that is in control, as this will result in degraded process performance.

In other words, control chart is:

- A device which specifies the state of statistical control,
- A device for attaining statistical control,
- A device to judge whether statistical control has been attained or not.

PURPOSE AND ADVANTAGES:

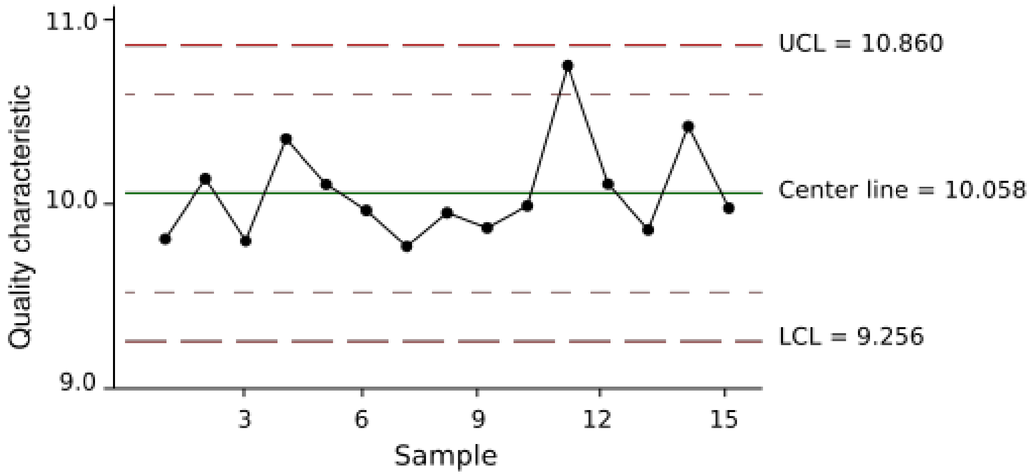
1. A control charts indicates whether the process is in control or out of control.
2. It determines process variability and detects unusual variations taking place in a process.
3. It ensures product quality level.
4. It warns in time, and if the process is rectified at that time, scrap or percentage rejection can be reduced.
5. It provides information about the selection of process and setting of tolerance limits.
6. Control charts build up the reputation of the organization through customer’s satisfaction.

A control chart consists of:

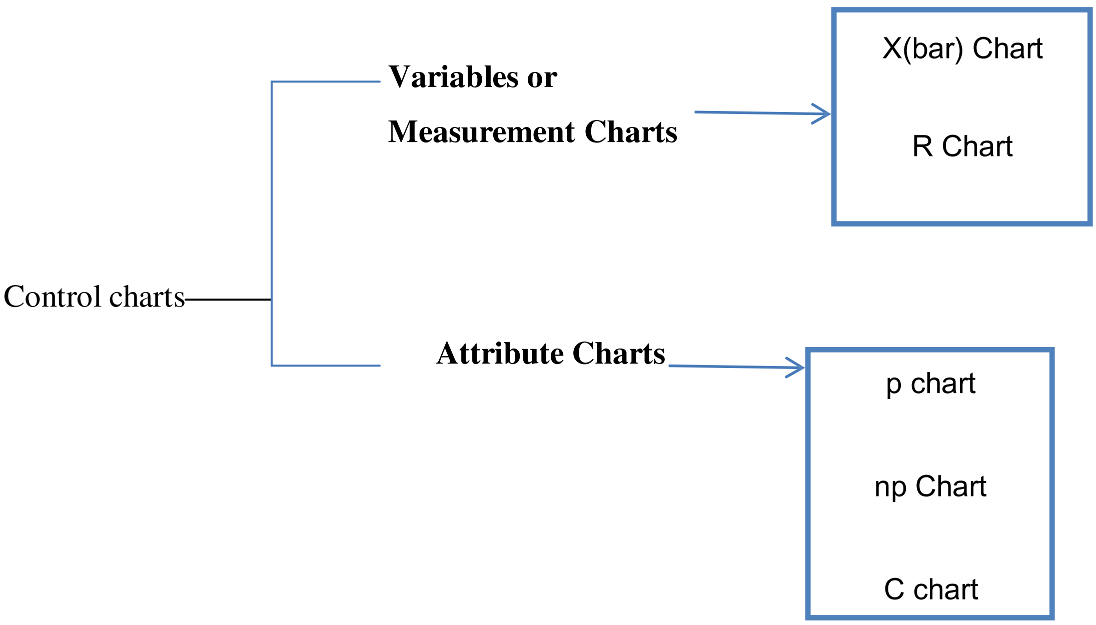
- Points representing a statistic (e.g., a mean, range, proportion) of measurements of a quality characteristic in samples taken from the process at different times [the data]
- The mean of this statistic using all the samples is calculated (e.g., the mean of the means, mean of the ranges, mean of the proportions)
- A center line is drawn at the value of the mean of the statistic
- The standard error (e.g., standard deviation/sqrt(n) for the mean) of the statistic is also calculated using all the samples
- Upper and lower control limits (sometimes called "natural process limits") that indicate the threshold at which the process output is considered statistically 'unlikely' are drawn typically at 3 standard errors from the center line

The chart may have other optional features, including:

- Upper and lower warning limits, drawn as separate lines, typically two standard errors above and below the center line
- Division into zones, with the addition of rules governing frequencies of observations in each zone
- Annotation with events of interest, as determined by the Quality Engineer in charge of the process's quality



TYPES OF CONTROL CHARTS



Control charts can be used to measure any characteristic of a product, such as the weight of a cereal box, the number of chocolates in a box, or the volume of bottled water. The different characteristics that can be measured by control charts can be divided into two groups: **variables** and **attributes.**

- A ***control chart for variables*** is used to monitor characteristics that can be measured and have a continuum of values, such as height, weight, or volume. A soft drink bottling operation is an example of a variable measure, since the amount of liquid in the bottles is measured and can take on a number of different values. Other examples are the weight of a bag of sugar, the temperature of a baking oven, or the diameter of plastic tubing.
- A ***control chart for attributes***, on the other hand, is used to monitor characteristics that have discrete values and can be counted. Often they can be evaluated with a simple yes or no decision. Examples include color, taste, or smell. The monitoring of attributes usually takes less time than that of variables because a variable needs to be measured (e.g., the bottle of soft drink contains 15.9 ounces of liquid). An attribute requires only a single decision, such as yes or no, good or bad, acceptable or unacceptable (e.g., the apple is good or rotten, the meat is good or stale, the shoes have a defect or do not have a defect, the lightbulb works or it does not work) or counting the number of defects (e.g., the number of broken cookies in the box, the number of dents in the car, the number of barnacles on the bottom of a boat).

CONTROL CHARTS FOR VARIABLES VS. CHARTS FOR ATTRIBUTES

A comparison of variable control charts and attribute control charts are given below:

- Variables charts involve the measurement of the job dimensions and an item is accepted or rejected if its dimensions are within or beyond the fixed tolerance limits; whereas as attribute chart only differentiates between a defective item and a non-defective item without going into the measurement of its dimensions.

- Variables charts are more detailed and contain more information as compared to attribute charts.
- Attribute charts, being based upon go and no go data (which is less effective as compared to measured values) require comparatively bigger sample size.
- Variables charts are relatively expensive because of the greater cost of collecting measured data.
- Attribute charts are the only way to control quality in those cases where measurement of quality characteristics is either not possible or it is very complicated and costly to do so—as in the case of checking colour or finish of a product, or determining whether a casting contains cracks or not. In such cases the answer is either yes or no.

ADVANTAGES OF ATTRIBUTE CONTROL CHARTS

Attribute control charts have the advantage of allowing for quick summaries of various aspects of the quality of a product, that is, the engineer may simply classify products as acceptable or unacceptable, based on various quality criteria. Thus, attribute charts sometimes bypass the need for expensive, precise devices and time-consuming measurement procedures. Also, this type of chart tends to be more easily understood by managers unfamiliar with quality control procedures; therefore, it may provide more persuasive (to management) evidence of quality problems.

ADVANTAGES OF VARIABLE CONTROL CHARTS

Variable control charts are more sensitive than attribute control charts. Therefore, variable control charts may alert us to quality problems before any actual "unacceptables" (as detected by the attribute chart) will occur. Montgomery (1985) calls the variable control charts *leading indicators* of trouble that will sound an alarm before the number of rejects (scrap) increases in the production process.

COMMONLY USED CHARTS

1. (X-Bar) and R charts, for process control.
2. P chart, for analysis of fraction defectives
3. C chart, for control of number of defects per unit.

- **Mean (x-Bar) ($\bar{x}$) Charts**

A mean control chart is often referred to as an ***x-bar chart.*** It is used to monitor changes in the mean of a process. To construct a mean chart we first need to construct the center line of the chart. To do this we take multiple samples and compute their means. Usually these samples are small, with about four or five observations. Each sample has its own mean. The center line of the chart is then computed as the mean of all sample means, where _ is the number of samples:

1. It shows changes in process average and is affected by changes in process variability.
2. It is a chart for the measure of central tendency.
3. It shows erratic or cyclic shifts in the process.
4. It detects steady progress changes, like tool wear.
5. It is the most commonly used variables chart.
6. When used along with R chart:
    a. It tells when to leave the process alone and when to chase and go for the causes leading to variation;
    b. It secures information in establishing or modifying processes, specifications or inspection procedures;
    c. It controls the quality of incoming material.
7. X-Bar and R charts when used together form a powerful instrument for diagnosing quality problems.

Range (R) charts

These are another type of control chart for variables. Whereas x-bar charts measure shift in the central tendency of the process, range charts monitor the dispersion or variability of the process. The method for developing and using R-charts are the same as that for x-bar charts. The center line of the control chart is the average range, and the upper and lower control limits are computed. The R chart is used to monitor process variability when sample sizes are small (n<10), or to simplify the calculations made by process operators. This chart is called the R chart because the statistic being plotted is the sample range.

1. It controls general variability of the process and is affected by changes in process variability.
2. It is a chart for measure of spread.
3. It is generally used along with X-bar chart.

Plotting of $\bar{X}$ and R charts:

A number of samples of component coming out of the process are taken over a period of time. Each sample must be taken at random and the size of sample is generally kept as 5 but 10 to15 units can be taken for sensitive control charts. For each sample, the average value $\bar{X}$ of all the measurements and the range R are calculated. The grand average $\bar{\bar{X}}$ (equal to the average value of all the average $\bar{X}$) and $\bar{R}$ ($\bar{R}$ is equal to the average of all the ranges R) are found and from these we can calculate the control limits for the $\bar{X}$ and R charts. Therefore,

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m}$$

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m}$$

Variables Data ($\bar{x}$ and R Control Charts)

$\bar{x}$ Control Chart

UCL = $\bar{\bar{x}} + A_2\bar{R}$

LCL = $\bar{\bar{x}} - A_2\bar{R}$

CL = $\bar{\bar{x}}$

R Control Chart

UCL = $\bar{R} D_4$

LCL = $\bar{R} D_3$

CL = $\bar{R}$

Here the factors A_2 , D_4 and D_3 depend on the number of units per sample. Larger the number, the close the limits. The value of the factors A_2, D_4 and D_3 can be obtained from S.Q.C tables. However for ready reference these are given below in tabular form:

n	A_2	D_3	D_4	d_2
2	1.880	0.000	3.267	1.128
3	1.023	0.000	2.574	1.693
4	0.729	0.000	2.282	2.059
5	0.577	0.000	2.114	2.326
6	0.483	0.000	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

Notation:

n or m= sample size

Example

Piston for automotive engine are produced by a forging process. We wish to establish statistical control of inside diameter of the ring manufactured by this process using *x* and *R* charts.

Twenty-five samples, each of size five, have been taken when we think the process is in control. The inside diameter measurement data from these samples are shown in table.

Sample Number	Observations					$\bar{x}_i$	R_i
1	74.030	74.002	74.019	73.992	74.008	74.010	0.038
2	73.995	73.992	74.001	74.011	74.004	74.001	0.019
3	73.988	74.024	74.021	74.005	74.002	74.008	0.036
4	74.002	73.996	73.993	74.015	74.009	74.003	0.022
5	73.992	74.007	74.015	73.989	74.014	74.003	0.026
6	74.009	73.994	73.997	73.985	73.993	73.996	0.024
7	73.995	74.006	73.994	74.000	74.005	74.000	0.012
8	73.985	74.003	73.993	74.015	73.988	73.997	0.030
9	74.008	73.995	74.009	74.005	74.004	74.004	0.014
10	73.998	74.000	73.990	74.007	73.995	73.998	0.017
11	73.994	73.998	73.994	73.995	73.990	73.994	0.008
12	74.004	74.000	74.007	74.000	73.996	74.001	0.011
13	73.983	74.002	73.998	73.997	74.012	73.998	0.029
14	74.006	73.967	73.994	74.000	73.984	73.990	0.039
15	74.012	74.014	73.998	73.999	74.007	74.006	0.016
16	74.000	73.984	74.005	73.998	73.996	73.997	0.021
17	73.994	74.012	73.986	74.005	74.007	74.001	0.026
18	74.006	74.010	74.018	74.003	74.000	74.007	0.018
19	73.984	74.002	74.003	74.005	73.997	73.998	0.021
20	74.000	74.010	74.013	74.020	74.003	74.009	0.020
21	73.982	74.001	74.015	74.005	73.996	74.000	0.033
22	74.004	73.999	73.990	74.006	74.009	74.002	0.019
23	74.010	73.989	73.990	74.009	74.014	74.002	0.025
24	74.015	74.008	73.993	74.000	74.010	74.005	0.022
25	73.982	73.984	73.995	74.017	74.013	73.998	0.035
						$\Sigma = 1850.028$	0.581
						$\bar{\bar{x}} = 74.001$	$\bar{R} = 0.023$

So,

$$\bar{\bar{X}} = 74.001$$

$$\bar{R} = 0.023$$

From S.Q.C tables (Fig.3) for sample size 5

A_2=0.58, D_4=2.11 and D_3= 0

UCL $\bar{X}$ = $\bar{\bar{X}} + A_2 \bar{R}$

= 74.001+ 0.58(0.023)

= 74.01434

LCL $\bar{X}$ = $\bar{\bar{X}}$ -- $A_2 \bar{R}$

= 74.001- 0.58(0.023)

= 73.98766

UCL (R chart) = $D_4 \bar{R}$

= 2.11*0.023

= 0.04853

LCL (R chart) = $D_3 \bar{R}$

= 0*0.023

= 0

Now $\bar{X}$ and R charts are plotted on the plot as shown in Fig.1 and Fig.2

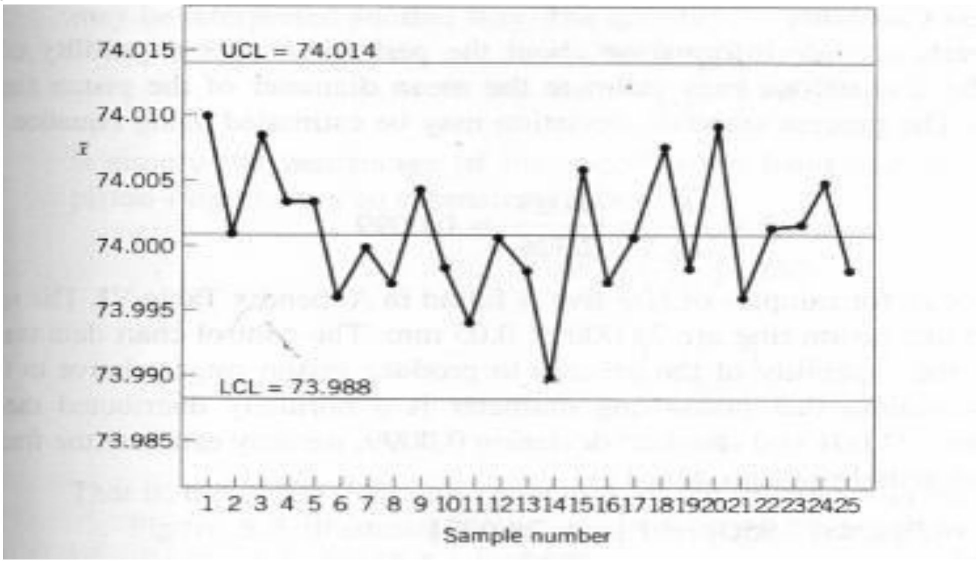


Fig.1: $\overline{X}$ Chart

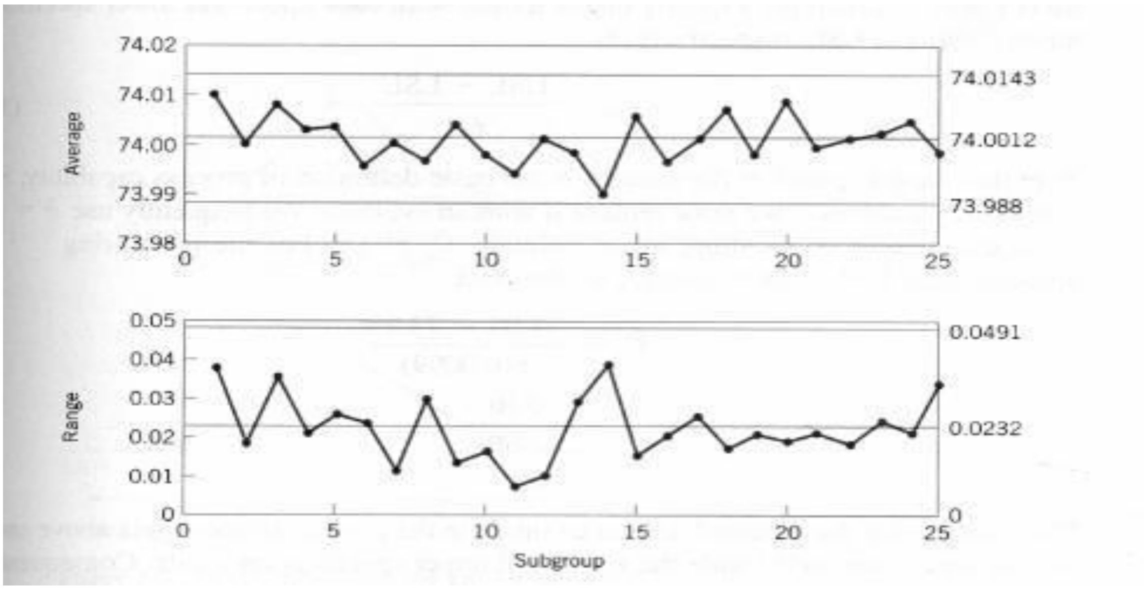


Fig.2: R Chart

Inference:

In the $\overline{X}$ chart, all of the time the plotted points representing average are well within the control limits but if some samples fall outside the control limits then it means something has probably gone wrong or is about to go wrong with the process and a check is needed to prevent the appearance of defective products.

	Chart for Averages			Chart for Standard Deviations						Chart for Ranges						
Observations in Sample, n	Factors for Control Limits			Factors for Center Line		Factors for Control Limits				Factors for Center Line		Factors for Control Limits				
	A	A_2	A_3	c_4	$1/c_4$	B_3	B_4	B_5	B_6	d_2	$1/d_2$	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$.

$$A = \frac{3}{\sqrt{n}} \quad A_3 = \frac{3}{c_4\sqrt{n}} \quad c_4 \cong \frac{4(n-1)}{4n-3}$$

$$B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}} \quad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}}$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}} \quad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}$$

Fig.3

PROCESS OUT OF CONTROL

After computing the control limits, the next step is to determine whether the process is in statistical control or not. If not, it means there is an external cause that throws the process out of control. This cause must be traced or removed so that the process may return to operate under stable statistical conditions. The various reasons for the process being out of control may be:

1. Faulty tools
2. Sudden significant change in properties of new materials in a new consignment
3. Breakout of lubrication system
4. Faults in timing of speed mechanisms.

PROCESS IN CONTROL

If the process is found to be in statistical control, a comparison between the required specifications and the process capability may be carried out to determine whether the two are compatible.

Conclusions:

When the process is not in control then then the point fall outside the control limits on either $\bar{x}$ or R charts. It means assignable causes (human controlled causes) are present in the process. When all the points are inside the control limits even then we cannot definitely say that no assignable cause is present but it is not economical to trace the cause. No statistical test can be applied. Even in the best manufacturing process, certain errors may develop and that constitute the assignable causes but no statistical action can be taken.

MCQ

QUESTIONS	**Option 1**	**Option 2**	**Option 3**	**Option 4**	**Answer**
Quality means	Degree of conformance to set standard.	Fit to function	Six sigma	Zero defect	Degree of conformance to set standard.
Objective of Statistical process control	To monitor the various stages of manufacturing	To reduce the final stage rejection.	To increase the cost of quality	To control the non conformity	To control the non conformity
If a sample of items is measured and the mean of the sample is outside the control limits the process is	out of control and the cause should be established	in control, but not capable of producing within the established control limits	within the established control limits with only natural causes of variation	monitored closely to see if the next sample mean will also fall outside the control limits	out of control and the cause should be established
Assignable causes	are not as important as natural causes	are within the limits of a control chart	depend on the inspector assigned to the job	are causes of variation that can be identified and removed	are causes of variation that can be identified and removed
Control charts for variables are based on data that comes from	acceptance sampling	individual items	averages of small samples	averages of large samples	averages of small samples
The purpose of an x-bar chart is to determine whether there has been a	gain or loss in uniformity	change in the percent defective in a sample	change in the central tendency of the process output	change in the number of defects in a sample	change in the central tendency of the process output
A run test is used	to examine variability in acceptance sampling plans	in acceptance sampling to establish control	to examine points in a control chart to check for natural variability	to examine points in a control chart to check for nonrandom variability	to examine points in a control chart to check for nonrandom variability

Statistical Process Control charts	display the measurements on every item being produced	display upper and lower limits for process variables or attributes, and signal when a process is no longer in control	indicate to the process operator the average outgoing quality of each lot	indicate to the operator the true quality of material leaving the process	display upper and lower limits for process variables or attributes, and signal when a process is no longer in control
Up to three standard deviations above or below centerline is the amount of variation that statistical process control allows for	Type I errors	about 95.5% variation	natural variation	assignable variation	natural variation
X- bar and R charts are used to find out	Production control	Cost control	Process control	Material control	Process control
Quality assurance is _______________	Proactive process, where controls are enforced on the process.	Reactive process , where controls the acceptance of end finished product	Identifying the defects after the product development	Coming under SQC technique.	Proactive process, where controls are enforced on the process.
Which of the following is true regarding the process capability index Cpk?	Cpk index value of 1 is ideal, meaning all units meet specifications.	The larger the Cpk the more units meet specifications.	The Cpk index can only be used when the process centerline is also the specification centerline.	Positive values of the Cpk index are good, negative values are bad.	The larger the Cpk the more units meet specifications.
CS 447- SE 465 standard refers to	Software testing and quality assurance	Hardware testing and quality control	Aerospace testing and flying approval	Marine boiler operational approval	Software testing and quality assurance
X bar ‘charts represents	Average of the values in sub groups.	Maximum of the values in sub groups	Minimum of the values in sub groups	Range of the values in sub groups	Average of the values in sub groups.

Improvement of process capability can be achieved by	Reducing common causes.	Eliminating odd data values.	Improvement in incoming quality	Achieving better OEE in assembly line	Reducing common causes.
The x-bar chart tells us whether there has been a	gain or loss in dispersion	change in the percent defective in a sample	change in the central tendency of the process output	change in the number of defects in a sample	change in the central tendency of the process output
The mean and standard deviation for a process for which we have a substantial history are x=120 and σ = 2. For the variable control chart, a sample size of 16 will be used. What is the mean of the sampling distribution?	1/8(0.125)	0.5	2	none of the above	none of the above
Jars of pickles are sampled and weighed. Sample measures are plotted on control charts. The ideal weight should be precisely 11 oz. Which type of chart(s) would you recommend?	p-charts	c-charts	x-bar and R charts	x-bar, but not R charts	x-bar and R charts
If x-double bar = 23 ounces, σ = 0.4 ounces, and n = 16, the ±3 σ (i.e. 3-standard deviation) control limits will be:	21.8 to 24.2 ounces	23 ounces	22.70 to 23.30 ounces	22.25 to 23.75 ounces	22.70 to 23.30 ounces
The purpose of an x-bar chart is to determine whether there has been a	gain or loss in uniformity	change in the percent defective in a sample	change in the central tendency of the process output	change in the number of defects in a sample	change in the central tendency of the process output
According to the text, the most common choice for limits for control charts is usually	±1 standard deviation	± 2 standard deviations	±3 standard deviations	±3 standard deviation for means and ±2 standard deviations for ranges	±3 standard deviations

The usual purpose of an R-chart is to signal whether there has been a	gain or loss in dispersion	hange in the percent defective in a sample	change in the central tendency of the process output	change in the number of defects in a sample	gain or loss in dispersion
Plots of sample ranges indicate that the most recent value is below the lower control limit. What course of action would you recommend?	Since there is no obvious pattern in the measurements, variability is in control.	One value outside the control limits is insufficient to warrant any action.	Lower than expected dispersion is a desirable condition; there is no reason to investigate.	Variation is not in control; investigate what created this condition.	Variation is not in control; investigate what created this condition.
To set x -chart upper and lower control limits, one must know the process central line, which is the	average of the sample means	total number of defects in the population	percent defects in the population	size of the population	average of the sample means
The variation of the average of the sample is shown by	X-bar chart	U- chart	R-chart	P-chart	X-bar chart
A fundamental attribute of TQM is	Drawing control charts	Having team meetings	Top management's direct involvement	Meeting ISO 9000 audit	Having team meetings
Process capability =1 indicates that	Suppliers can be trusted	Process is in control	There are no random variations	Some fraction of production is outside specs	Some fraction of production is outside specs
SPC helps determine	If assignable causes are disturbing the process	If vendor performance is falling	If customers are happy	If customers are motivated	If assignable causes are disturbing the process
Inspection assures that	The process is in control	Workers are motivated	Product meets specification	Quality problems are solved	Product meets specification

A control chart displays	Whether workers are motivated	Top management takes interest in quality	· Inspectors are doing their job	Process variability	Process variability
Quality is wanting generally because	Workers lack team spirit	No competition exists	People don't know statistics	Supplier quality is acceptable	No competition exists
An assignable cause is generally known to	Vendors	Top management	Product designer	Workers	Workers
Flow charts indicate	Causes of process variation	The kind of forms to fill out	Who reports to whom	How inputs get processed into outputs	How inputs get processed into outputs
Systematic problem solving requires	Motivating the worker	Defining the problem to be solved	Drawing control charts	Keeping management informed	Defining the problem to be solved
ISO 9000 determines	If the company practices its written procedures	If vendors are performing well	Process capability	Random causes of variation	If the company practices its written procedures
An example of a random cause is	Absenteeism	Shortage of material supplies	Photocopy machine failure	Small vibrations in the equipment	Small vibrations in the equipment
Control charts help in	Reaching six sigma	Rejecting parts supplied by vendors	Keeping workers motivated	Deciding when to investigate the process	Deciding when to investigate the process
Seven tools include	Team meetings	Management meeting regularly with workers	Workers' toolkit	Histogram	Histogram

Problem identification requires	Flow charting the process	Monitoring customer complaints	Team meetings	Maintaining clean cafeterias	Flow charting the process
Problem solving begins with	SPC	Design of experiments	Problem identification	Punching time clock	Problem identification
Individuals who have no role in quality management	Teachers in universities	Government regulators	Workers	ISO 9000 trainers	Government regulators
Quality management requires	Workers not working overtime	Printing promotional brochures	Keeping internal customers satisfied	Keeping oil off the floor	Keeping internal customers satisfied
Cost of quality is really	A way to prioritize actions	Cost of production	Cost of sales	Cost of sales	A way to prioritize actions
Six sigma is	Latest Japanese Quality Theory	A BMW	Cpk = 2.0	Cost of sales	Cpk = 2.0
Six Sigma implies	A statistical method	A trouble-shooting method	Teams are effective	3 defects per million in output	3 defects per million in output
Quality control does not apply to	Drawing flow charts	Drawing control charts	Driving	Idea generation	Idea generation
SPC implies	Statistical process control	Use of control charts	· Fixing assignable causes	All above	All above

The quantity sigma (s) indicates	Trend in the process	Dispersion in the data	Lack of attention by workers	Average	Dispersion in the data
Quality is a problem because	Modern processes are too complex	Workers don't do the job	It is expensive to control	All processes have some variation	All processes have some variation
The word target in quality means	The specification	The control limits	Xbar points on the chart	The ideal quality requirement	The ideal quality requirement

2 MARKS

1. Define Quality.
2. Explain the Aspects of Quality cost.
3. What is appraisal cost ?
4. What is Gap analysis in process control ?
5. What is continuous process improvement ?
6. Define standard deviation .
7. What do you mean by Variable ?
8. What do you mean by SPC ?
9. Explain the Significance of Control charts.
10. What do you meant by RUN chart ?
11. Explain the control charts for Variable briefly .
12. Define Process capability.

14 MARKS

1. Explain Process Capability in detail.

2. The percent of water absorption is an important characteristic of common building brick. A certain company occasionally measured this characteristic of its product but records were never kept. It was decided to analyze the process with control chart. Twenty-five samples of four bricks each yielded these results.

Sample number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
X – bar	15.01	12.3	7.4	8.7	8.8	11.7	10.2	11.5	11.2	10.2	9.6	7.6
R	9.1	9.9	9.7	6.7	7.1	9.1	12.1	10.8	13.5	6.9	5	8.2

Sample number	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
X – bar	7.6	9.8	8.8	8.1	6.3	10.5	9.7	11.7	13.2	12.5	7.5	8.8	8.0
R	5.4	18	11	4.4	4.1	5.7	6.4	4.6	7.2	8.3	6.4	6.9	6.4

Estimate the control limits for **X-bar and R chart.** If any point lies out of the control limits estimate the revised control limits.
(14)

3 a) Explain the four categories of Quality cost.

b) Describe the’ process capability ’& find the process capability index (Cpk) for the data given. Observed dimension of piston ring diameter: UCL = 74.05 mm

,LCL=73.95 mm , Standard deviation=.0099

4. The percent of water absorption is an important characteristic of common building brick. A certain company occasionally measured this characteristic of its product but records were never kept. It was decided to analyze the process with control chart. Twenty-five samples of four bricks each yielded these results.

Sample number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
X – bar	15.01	12.30	7.40	8.70	8.80	11.70	10.20	11.50	11.20	10.20	9.60	7.60
R	9.1	9.9	9.7	6.7	7.1	9.1	12.1	10.8	13.5	6.9	5	8.2

Sample number	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
X – bar	7.60	9.80	8.80	8.10	6.30	10.50	9.70	11.70	13.20	12.50	7.50	8.80	8.00
R	5.4	18	11	4.4	4.1	5.7	6.4	4.6	7.2	8.3	6.4	6.9	6.4

Estimate the control limits for **X and R chart.** If any point lies out of the control limits, estimate the revised control limits.

5. Telstar Appliance Company uses a process to paint refrigerators with a coat of enamel. During each shift, a sample of 5 refrigerators is selected (1.4 hours apart) and the thickness of the paint (in mm) is determined. If the enamel is too thin, it will not provide enough protection. If it's too thick, it will result in an uneven appearance with running and wasted paint. The measurements from 10 consecutive shifts listed below in Table (15). A sample of 5 from the same shift is a subgroup and we have 10 subgroups. Provide an **X-bar, R and S chart.**

Shift no.	**Thickness (in mm)**					**Mean**
1	3	2	3	2	3	2.54
2	3	2	3	2	3	2.54
3	2	2	2	3	2	2.38
4	3	2	2	3	3	2.56
5	3	3	3	2	3	2.52
6	2	2	3	2	3	2.4
7	2	3	2	2	2	2.3
8	3	3	3	3	3	2.64

9	2	3	2	2	2	2.42
10	3	2	2	3	2	2.36

3. A firm that produces bolts is investigated by P-chart. Ten samples are selected with different sample size.

Sample	1	2	3	4	5	6	7	8	9	10
Nonconforming	19	10	4	6	8	9	3	1	0	4

Sample number	1	2	3	4	5	6	7	8	9	10
Sample Size (n)	15	20	30	18	17	32	27	19	21	24
Non conformance	2	4	4	3	2	5	4	3	3	3

Construct a p-chart for this process using control limits. Is the process in-control?

UNIT II PROCESS CONTROL FOR ATTRIBUTES

CONTROL CHARTS FOR ATTRIBUTES

Control charts for attributes are used to measure quality characteristics that are counted rather than measured. Attributes are discrete in nature and entail simple yes-or-no decisions. For example, this could be the number of nonfunctioning lightbulbs, the proportion of broken eggs in a carton, the number of rotten apples, the number of scratches on a tile, or the number of complaints issued. Two of the most common types of control charts for attributes are p-charts and c-charts.

- **P-charts** are used to measure the proportion of items in a sample that are defective. Examples are the proportion of broken cookies in a batch and the proportion of cars produced with a misaligned fender. P-charts are appropriate when both the number of defectives measured and the size of the total sample can be counted. A proportion can then be computed and used as the statistic of measurement.

1. It can be a fraction defective chart.
2. Each item is classified as good (non-defective) or bad (defective).
3. This chart is used to control the general quality of the component parts and it checks if the fluctuations in product quality (level) are due to chance alone.

Plotting of P-charts: By calculating, first, the fraction defective and then the control limits.

The process is said to be in control if fraction defective values fall within the control limits. In case the process is out of control an investigation to hunt for the cause becomes necessary.

The mean proportion defective ($\overline{p}$):

$$\overline{p} = \frac{\text{Total Number of Defectives}}{\text{Total Number Inspected}}$$

The standard deviation of p:

$$\sigma_{\overline{p}} = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

where n = sample size.

Control Limits are:

$$UCL = \overline{p} + Z^* \sigma_{\overline{p}} \qquad LCL = \overline{p} - Z^* \sigma_{\overline{p}}$$

or

$$UCL = \overline{p} + Z^* \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \qquad LCL = \overline{p} - Z^* \sqrt{\frac{\overline{p}(1-\overline{p})}{n}}$$

Usually the Z value is equal to 3 (as was used in the X and R charts), since the variations within three standard deviations are considered as natural variations. However, the choice of the value of Z depends on the environment in which the chart is being used, and on managerial judgment.

- **C-charts** count the actual number of defects. For example, we can count the number of complaints from customers in a month, the number of bacteria on a petri dish, or the number of barnacles on the bottom of a boat. However, we cannot compute the proportion of complaints from customers, the proportion of bacteria on a petri dish, or the proportion of barnacles on the bottom of a boat.

Defective items vs individual defects

The literature differentiates between *defect* and *defective*, which is the same as differentiating between *nonconformity* and *nonconforming units*. This may sound like splitting hairs, but in the interest of clarity let's try to unravel this man-made mystery.

Consider a wafer with a number of chips on it. The wafer is referred to as an "item of a product". The chip may be referred to as "a specific point". There exist certain specifications for the wafers. When a particular wafer (e.g., the item of the product) does not meet at least one of the specifications, it is classified as a nonconforming item. Furthermore, each chip, (e.g., the specific point) at which a specification is not met becomes a defect or nonconformity.

So, a nonconforming or defective item contains at least one defect or nonconformity. It should be pointed out that a wafer can contain several defects but still be classified as conforming. For example, the defects may be located at noncritical positions on the wafer. If, on the other hand, the number of the so-called "unimportant" defects becomes alarmingly large, an investigation of the production of these wafers is warranted.

Control charts involving counts can be either for the *total number* of nonconformities (defects) for the sample of inspected units, or for the *average number* of defects per inspection unit.

Defect vs. Defective

- 'Defect' – a single nonconforming quality characteristic.
- 'Defective' – items having one or more defects.

C charts can be plotted by using the following formulas:

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$\bar{c} = \frac{\text{total number of defects}}{\text{total number of samples}}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

THE PRIMARY DIFFERENCE BETWEEN USING A P-CHART AND A C-CHART IS AS FOLLOWS.

A P-chart is used when both the total sample size and the number of defects can be computed.

A C-chart is used when we can compute *only* the number of defects but cannot compute the proportion that is defective.

MCQ

QUESTIONS	Option 1	Option 2	Option 3	Option 4	Answer
Which of the following is true of a p-chart?	The lower control limit is found by subtracting a fraction from the average number of defects.	The lower control limit indicates the minimum acceptable number of defects.	The lower control limit may be below zero.	The lower control limit may be at zero.	The lower control limit may be below zero.
The statistical process chart used to control the number of defects per unit of output is the	X-bar chart	R-chart	p-chart	c-chart	c-chart
The c-chart signals whether there has been a	gain or loss in uniformity	change in the number of defects per unit	change in the central tendency of the process output	change in the percent defective in a sample	change in the number of defects per unit
The following is an attribute chart:	X- bar chart	R chart	σ - chart	np chart	np chart
A nationwide parcel delivery service keeps track of the number of late deliveries (more than 30 minutes past the time promised to clients) per day. They plan on using a control chart to plot their results. Which type of control chart(s) would you recommend?	x – bar and R-charts	p-charts	c-chart	x - bar, but not R-charts	c-chart
The type of inspection that classifies items as being either good or defective is	variable inspection	attribute inspection	fixed inspection	all of the above	attribute inspection
A manufacturer uses statistical process control to control the quality of the firm's products. Samples of 50 of Product A are taken, and a defective/acceptable decision is made on each unit sampled. For Product B, the number of flaws per unit is counted. What type(s) of control charts should be used?	p-charts for A and B	p-chart for A, c-chart for B	c-charts for both A and B	p-chart for A, mean and range charts for B	p-chart for A, c-chart for B
Which of the following is true of a p-chart?	The lower control	The lower	The lower	The upper control	The lower

	limit is found by subtracting a fraction from the average number of defects.	control limit indicates the minimum acceptable number of defects.	control limit may be below zero.	limit may be zero.	control limit may be below zero.
The normal application of a p-chart is in	process sampling by variables	acceptance sampling by variables	process sampling by attributes	acceptance sampling by attributes	rocess sampling by attributes
The c-chart signals whether there has been a	gain or loss in uniformity	change in the number of defects per unit	change in the central tendency of the process output	change in the percent defective in a sample	change in the number of defects per unit
The local newspaper receives several complaints per day about typographic errors. Over a seven-day period, the publisher has received calls from readers reporting the following number of errors: 4, 3, 2, 6, 7, 3, and 9. based on these data alone, what type of control chart(s) should the publisher use?	p-chart	c-chart	u-chart	R-chart	c-chart
Which of the following is true regarding the process capability index Cpk?	Cpk index value of 1 is ideal, meaning all units meet specifications.	The larger the Cpk the more units meet specifications.	The Cpk index can only be used when the process centerline is also the specification centerline.	Positive values of the Cpk index are good, negative values are bad.	The larger the Cpk the more units meet specifications.
Acceptance sampling procedures the objective for variables parameter to be referred to	Skip- lot sampling	Dodge – Roming plan	LPTD – Plans and hypothesis testing	Chain sampling	LPTD – Plans and hypothesis testing
The type of inspection that classifies items as being either good or defective is	Variable inspection	attribute inspection	fixed inspection	all of the above	attribute inspection
The chart involving the raw values of the variable of interest is called the ______ chart.	x-bar	X	R	S	X

The chart which involves the number of defects per item, is called the __________ chart	x-bar	X	R	C	C
The most important parameter in statistical quality control is the	mean	range	variance	proportion	variance
The statistician who invented the control chart for use in industrial quality control was:	Shewhart	Deming	Taguchi	Pareto	Shewhart
The chart which aggregates poor quality outcomes to show management which are the most important problems is the:	Pareto chart	s chart	R chart	Taguchi chart	Pareto chart
Which of the following control charts should be used first in an actual statistical quality control program	Pareto chart	x-bar chart	c chart	s chart	s chart
The standard deviation is	always normal	skewed	approximately normal	either Poisson or binomial	skewed
The lower control limit for p charts is	sometimes zero	always zero	never zero	All these above	sometimes zero
One of Deming's main theories on quality control and management involves	TQM	Taguci methods	Pareto diagrams	control charts	TQM
In an actual quality control problem, the first test would be on the	mean	variation	number of defects	All these above	variation
Based on the c chart, is this process in control	it may be out of control, based on one observation outside the limits	it appears to be in control	several observations are outside the limits	insufficient information to determine	it may be out of control, based on one observation outside the limits
The purpose of control charts is to:	estimate the proportion of output that is acceptable	weed out defective items	determine if the output is within tolerances/specifications	distinguish between random variation and non-random variation in the process	distinguish between random variation and non-random variation in the process
In statistical process control, p-charts are used to	sample size	number of	proportion of	useful life period	useful life

signal assignable variation that is affecting the ___________:		defects in a typical unit of output	defective units being produced	of units produced	period of units produced
Is this process in control, based on the x-bar chart?	all group means are above the LCL, but not all are below the UCL	all groups are in control except the third group	no, the process is not in control	yes, this process is in control	yes, this process is in control
Compute the standard error of p-bar.	0.0456	0.0841	0.09375	0.291	0.0456
Based on the p chart, is this process in control?	no, two sample proportions are outside the limits	no, since all but one sample proportion are outside the limits	no, exactly one sample proportion is outside the limits	yes, it is in control	no, exactly one sample proportion is outside the limits
In the c chart, c is estimated by c-bar which is the:	average range of each group	average number of defects per item	average number of defective items in the entire population	standard deviation of the number of defectives	average number of defects per item
Based on the c chart, is this process in control?	it may be out of control, based on one observation outside the limits	it appears to be in control	several observations are outside the limits	insufficient information to determine	it may be out of control, based on one observation outside the limits
The lower control limit for p charts is:	sometimes zero	always zero	never zero	p-bar - [(p-bar)(1 - (p-bar))]	sometimes zero
The purpose of control charts is to:	estimate the proportion of output that is acceptable	weed out defective items	determine if the output is within tolerances/specifications	distinguish between random variation and non-random variation in the process	distinguish between random variation and non-random variation in the process

In statistical process control, p-charts are used to signal assignable variation that is affecting the ___________:	sample size	number of defects in a typical unit of output	proportion of defective units being produced	useful life period of units produced	proportion of defective units being produced
One of Deming's main theories on quality control and management involves:	TQM	Taguci methods	Pareto diagrams	control charts	TQM

2 MARKS

1. Give the three parameters of Quality Characteristics ?
2. What are the of 4 types 0f Quality cost.
3. Explain the Normal distribution curve .
4. Illustrate the SEVEN QC TOOLS.
5. What is continuous process improvement ?
6. Define fraction non conforming ?
7. Quote the PROCESS CAPABILITY ANALYSIS.
8. Classify the types of three variations ?
9. When will the assignable cause would occur ?
10. Describe the formula to find UCL & LCL values ,while plotting U chart.
11. Tabulate the PPM Defect can only be achieved during 2 ,4,6 Sigma level of Quality.
12. Differentiate between Attribute and Variable with suitable example.

14 MARKS

1 A firm that produces bolts is investigated by P-chart. Ten samples are selected with different sample size.

Sample number	1	2	3	4	5	6	7	8	9	10
Sample Size (n)	15	20	30	18	17	32	27	19	21	24
Non conformance	2	4	4	3	2	5	4	3	3	3

Construct a p-chart for this process using control limits. Is the process in-control?

2. In a manufacturing process, the numbers of defectives found in the inspection of 15 lots of 400 items each are given below.

Lot No.	No. of defective	Lot No.	No. of defective
1	2	9	18
2	5	10	8
3	0	11	6
4	14	12	0
5	3	13	3
6	0	14	0
7	1	15	6
8	0		

Determine the trial control limits for **np-chart** and state whether the process is in control.

3. A company manufactures valves for industrial use. Ten samples of 15 valves (sample size) each were taken from the production line and tested. The results are reported below.

Sample	1	2	3	4	5	6	7	8	9	10
Non conformance	3	1	0	0	0	2	0	3	1	0

(i) Compute the fraction defective for each sample.
(ii) Construct a p-chart for this process using control limits. Is the process in-control?

4. Repeated samples of 150 coffee cans are inspected to determine whether a can is out oround or whether it contains leaks due to improper construction. Such a can is said to be nonconforming. Following is the data. Construct **p-chart** and check whether the process is in statistical control.

5. UPS sets were manufactured through a series of processes ,and inspected for defects. If any defect found then it should be scrapped. The inspector goes for tracking the number of defective units produced. The UPS sets are produced in batches of 100. The results are tabulated for 10 Batches as follows:

Batch	1	2	3	4	5	6	7	8	9	10
Number Nonconforming	20	32	18	10	12	25	30	16	19	27

Construct **nP-chart** and check whether the process is in statistical control.

6. In a manufacturing process, the numbers of defectives found in the inspection of 15 lots of 400 items each are given below.

Lot No.	No. of defective	Lot No.	No. of defective
1	2	9	18
2	5	10	8
3	0	11	6
4	14	12	0
5	3	13	3
6	0	14	0
7	1	15	6
8	0		

Determine the trial control limits for **np-chart** and state whether the process is in control.

UNIT III ACCEPTANCE SAMPLING

ACCEPTANCE SAMPLING

"Acceptance Sampling is concerned with the decision to accept a mass of manufactured items as conforming to standards of quality or to reject the mass as non-conforming to quality. The decision is reached through sampling." - SIMPSON AND KAFKA

Acceptance sampling uses statistical sampling to determine whether to accept or reject a production lot of material. It has been a common quality control technique used in industry and particularly the military for contracts and procurement. It is usually done as products leave the factory, or in some cases even within the factory. Most often a producer supplies a consumer a number of items and decision to accept or reject the lot is made by determining the number of defective items in a sample from the lot. The lot is accepted if the number of defects falls below where the acceptance number or otherwise the lot is rejected

For the purpose of acceptance, inspection is carried out at many stages in the process of manufacturing. These stages may be: inspection of incoming materials and parts, process inspection at various points in the manufacturing operations, final inspection by a manufacturer of his own product and finally inspection of the finished product by the purchaser.

Inspection for acceptance is generally carried out on a sampling basis. The use of sampling inspection to decide whether or not to accept the lot is known as Acceptance Sampling. A sample from the inspection lot is inspected, and if the number of defective items is more than the stated number known as acceptance number, the whole lot is rejected.

The purpose of Acceptance Sampling is, therefore a method used to make a decision as to whether to accept or to reject lots based on inspection of sample(s).

Incoming Inspection Accepted Lot Outgoing Quality

Rejected lot
Subjected to cent
Percent inspection

Replacement of substandard items by good ones from assemblies

and rejection of individual defective item

Acceptance sampling is the process of randomly inspecting a sample of goods and deciding whether to accept the entire lot based on the results. Acceptance sampling determines whether a batch of goods should be accepted or rejected.

Acceptance Sampling is very widely used in practice due to the following merits:

1. Acceptance Sampling is much less expensive than 100 percent inspection.
2. It is general experience that 100 percent inspection removes only 82 to 95 percent of defective material. Very good 100 percent inspection may remove at the most 99 percent of the defectives, but still cannot reach the level of 100 percent. Due to the effect of inspection fatigue involved in 100 percent inspection, a good sampling plan may actually give better results than that achieved by 100 percent inspection.
3. Because of its economy, it is possible to carry out sample inspection at various stages.

Inspection provides a means for monitoring quality. For example, inspection may be performed on incoming raw material, to decide whether to keep it or return it to the vendor if the quality level is not what was agreed on. Similarly, inspection can also be done on finished goods before deciding whether to make the shipment to the customer or not. However, performing 100% inspection is generally not economical or practical, therefore, sampling is used instead.

Acceptance Sampling is therefore a method used to make a decision as to whether to accept or to reject lots based on inspection of sample(s). The objective is not to control or estimate the quality of lots, only to pass a judgment on lots.

Using sampling rather than 100% inspection of the lots brings some risks both to the consumer and to the producer, which are called the consumer's and the producer's risks, respectively. We encounter making decisions on sampling in our daily affairs.

Operating Characteristic Curve

The Operating Characteristic Curve (OC Curve) shows you the probability that you will accept lots with various levels of quality. It is the working plan of acceptance sampling.

AQL – Acceptance Quality Level

The AQL (Acceptance Quality Level), the maximum % defective that can be considered satisfactory as a process average for sampling inspection

RQL – Rejectable Quality Level

The RQL (Rejectable Quality Level) is the % defective. t is also known as the Lot Tolerance Percent Defective (LTPD).

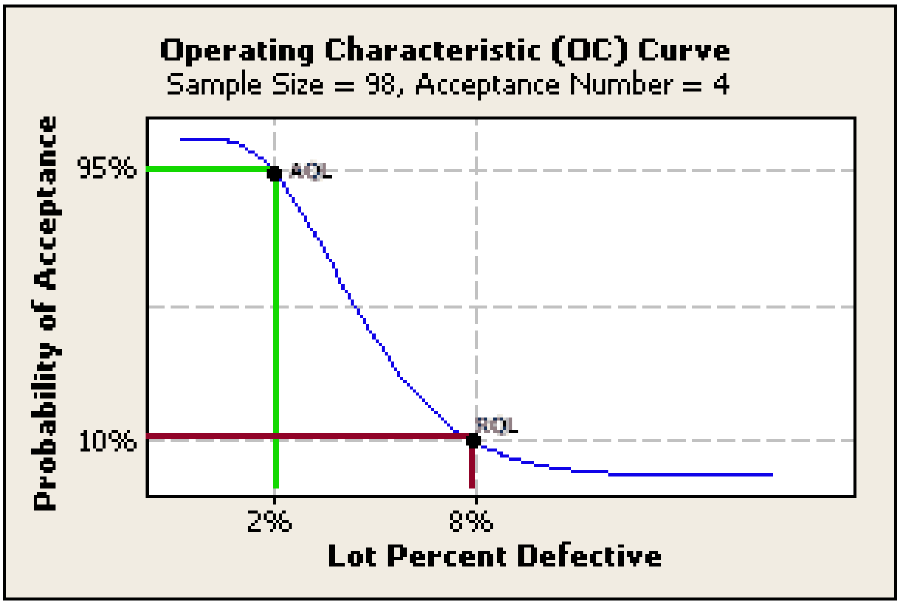
LTPD – Lot Tolerance Percent Defective

The LTPD of a sampling plan is a level of quality routinely rejected by the sampling plan. It is generally defined as that level of quality (percent defective, defects per hundred units, etc.) which the sampling plan will accept 10% of the time.

Risks in Acceptance sampling

1. P roducer's risk -: Sometimes inspite of good quality, the sample taken may show defective units as such the lot will be rejected, such type of risk is known as producer's risk.

2. C onsum er's R isk -: Sometimes the quality of the lot is not good but the sample results show good quality units as such the consumer has to accept a defective lot, such a risk is known as consumer's risk.



ACCEPTANCE SAMPLING PLANS

A **sampling plan** is a plan for acceptance sampling that precisely specifies the parameters of the sampling process and the acceptance/rejection criteria. The variables to be specified include the size of the lot (*N*), the size of the sample inspected from the lot (*n*), the number of defects above which a lot is rejected (*c*), and the number of samples that will be taken.

There are different types of sampling plans.

- Single Sampling (Inference made on the basis of only one sample)
- Double Sampling (Inference made on the basis of one or two samples)
- Sequential Sampling (Additional samples are drawn until an inference can be made) etc.
-

Single Sampling Plan

In single sampling plan, the decision regarding the acceptance or rejection is made after drawing a sample from a bigger lot. Inspection is done and if the defectives exceed a certain number the lot is rejected. Otherwise, the lot is accepted when the number of defectives is less than the acceptance number.

Double Sampling Plan

In this, a small sample is first drawn. If the number of defectives is less than or equal to the acceptance number (C1) the lot is accepted. If the number of defectives is more than another acceptance number (C2) which is higher, then C1 then the lot is rejected. If in case, the number in the inspection lies between C2 and C1, then a second sample is drawn. The entire lot is accepted or rejected on the basis of outcome of second inspection.

Sequential Sampling Plan

Sequential sampling plan is used when three or more samples of stated size are permitted and when the decision on acceptance or rejection must be reached after a stated number of samples.

A first sample of n1 is drawn, the lot is accepted if there are no more than c1 defectives, the lot is rejected if there are more than r1 defectives. Otherwise a second sample of n2 is drawn. The lot is accepted if there are no more than c2 defectives in the combined sample of n1 + n2. The lot is rejected if there are more than r2 defectives in the combined sample of n1 + n2. The procedure is continued in accordance with the table below.

Sample	***Sample Size***	***Size***	***Acceptance Number***	***Rejection Number***
First	n_1	n_1	c_1	r_1
Second	n_2	$n_1 + n_2$	c_2	r_2
Third	n_3	$n_1 + n_2 + n_3$	c_3	r_3
Fourth	n_4	$n_1 + n_2 + n_3 + n_4$	c_4	r_4
Fifth	n_5	$n_1 + n_2 + n_3 + n_4 + n_5$	c_5	$c_5 + 1$

If by the end of fourth sample, the lot is neither accepted nor rejected, a sample n5 is drawn. The lot is accepted if the number of defectives in the combined sample of n1 + n2 + n3 + n4 + n5 does not exceed c5. Otherwise the lot is rejected.

A sequential sampling plan involves higher administrative costs and use of experienced inspectors

MCQ

QUESTIONS	Option 1	Option 2	Option 3	Option 4	ANSWERS
Acceptance sampling is mainly applicable in	Final product inspection areas	Incoming goods inspection areas	Design and application engineering area.	Both (a) & (b)	Both (a) & (b)
Even though parts are of the same quality, acceptance sampling plans	Improve the quality	Increase the productivity	Do not improve the quality	Improve reliability	Do not improve the quality
Lots sentencing decision refers to	Rejected lots to returned to supplier	Accepted lots put into production	Send the sample for reliability testing	Submission of lot for material analysis	Accepted lots put into production
Lot may actually contained more defect than it should, but it accepted, this is called -----------	Producer's risk	Sub-contractor's risk	Consumer's risk	Management risk	Consumer's risk
Acceptance sampling is useful when product testing is ----------------	Destructive and time consuming	Proto typing the product	Production part approval process	Less time consuming	Destructive and time consuming
Acceptance sampling procedures the objective for variables parameter to be referred to	Skip- lot sampling	Dodge – Roming plan	LPTD – Plans and hypothesis testing	Chain sampling	LPTD – Plans and hypothesis testing
Average sample number curve is drawn as	ASN VS AQL Value	LTPD VS AOQL	ASN VS Lot fraction defective 'p'	Probability of acceptance Vs % of defective parts	ASN VS Lot fraction defective 'p'
Sampling procedure for inspection by attributes developed during world war II ,	DRDO Standard 109 E (ISO 2958)	Military standard 105 E (ISO 2859)	Defense standard 105 E (ISO 14001)	Military standard 101E (ISO 9004)	Military standard 105 E (ISO 2859)
Accepted sampling based Product lifecycle OC	Probability of	Probability of	Probability of	Probability	Probability

curve constructed between	acceptance V s Mean life	defective V s average life	average life V s reliability number	of acceptance V s fraction defective	of acceptance V s fraction defective
Acceptance sampling is useful when product testing is ----------------	Destructive and time consuming	Proto typing the product	Production part approval process	Less time consuming	Destructive and time consuming
Operating characteristic (OC) Curve is constructed between	AOQ vs AQL	Probability of rejection vs Acceptance number	True process average vs Number of sample pieces	Probability of acceptance vs Fraction defective	Probability of acceptance vs Fraction defective
AOQL – (average outgoing quality limit) refers to	Limiting quality level of accepted lots	Average total inspection in a sample stream of lots	Reject able quality lots sample lots	Average level of quality across a large stream of lots	Average level of quality across a large stream of lots
A sampling plan helps in	Keeping the process in control	Keeping workers motivated	Adjusting ovens in the kitchen	Rejecting lots that are of unacceptable quality	Rejecting lots that are of unacceptable quality
The AQL indicates:	acceptable quality level	average quality level	actual quality level	aggregate quality level	acceptable quality level
A Type II (beta) error occurs when:	A bad lot is accepted.	A good lot is rejected.	A bad lot is rejected.	A good lot is accepted.	A bad lot is accepted.
. A Type I (alpha) error occurs when:	A bad lot is accepted.	.A good lot is rejected.	A bad lot is rejected.	A good lot is accepted.	.A good lot is rejected.
In acceptance sampling, the level of inspection automatically adjusts to the quality of lots being inspected, assuming that	A single-sampling plan is used.	Double-sampling plans are used.	Rejected lots are subjected to 100 percent inspection	A good lot is accepted.	Rejected lots are subjected to 100 percent inspection
An AOQ curve shows:	he maximum quality	average outgoing quality relative to incoming quality	how well a sampling plan can discriminate between good and bad lots	he actual quality level	average outgoing quality relative to incoming

					quality
The ability of a sampling plan to discriminate between lots of high quality and lots of low quality is described by:	a Gantt chart	an operating characteristic curve	an average outgoing quality curve	a process control chart	an operating characteristic curve
An OC curve shows:	average outgoing quality	probability of acceptance versus lot quality	operating control	the average outgoing quality limit	probability of acceptance versus lot quality
A lot can be "accepted" or "rejected" in a multiple-sampling plan:	after one sample is taken	after two samples are taken	after three or more samples are taken	all of the above	all of the above
A lot can be "accepted" or "rejected" in a double-sampling plan:	after one sample is taken	after two samples are taken	Only a and b are correct.	only after two samples are taken	Only a and b are correct.
Sampling plans typically specify:	. lot size	sample size	number of samples to be taken	all of the above	all of the above
Acceptance sampling plans might call for selection of:	one or more samples	a variable number of samples based on actual results	.random inspections	outside audits by testing agencies	one or more samples
The purpose of acceptance sampling is to:	. estimate process quality	estimate lot quality	detect and eliminate defectives	decide if a lot meets predetermined standards	decide if a lot meets predetermined standards
If he uses an acceptance sampling plan of n=5 and c=2, what will be the average outgoing quality (AOQ) for batches with 40% defectives?	0.1270	0.273	0.3174	0.6826	0.273
If, based on his acceptance sample, he decides to ship this batch without inspection, which type error would be possible?	none	. producer's error	Type I error	Type II error	Type II error
Which one of the following would <u>not</u> be a reason	high cost of	large number	. destructive	low cost of	high cost of

for using acceptance sampling?	passing defectives	of items	testing	passing defectives.	passing defectives
In acceptance sampling, the level of inspection automatically adjusts to the quality of lots being inspected, assuming that	A single-sampling plan is used.	Double-sampling plans are used.	Multiple-sampling plans are used.	Rejected lots are subjected to 100 percent inspection	Rejected lots are subjected to 100 percent inspection
Operating characteristic (OC) Curve is constructed between	AOQ vs AQL	Probability of rejection vs Acceptance number	True process average vs Number of sample pieces	Probability of acceptance vs Fraction defective	Probability of acceptance vs Fraction defective

2 MARKS

1. Identify any four situations where the Acceptance sampling is most likely to be useful.
2. List out the important considerations in for lot formation.
3. Define Single sampling plan .
4. Explain the purpose of OC Curve.
5. What is the effect of sample size on the OC curve?
6. Classify the types of three variations.
7. Predict the terms ‘AQL ‘ & ‘LPTD in the context of Sampling plan.
8. Draw the Type A & Type B Curves ,When N = 500,2000 ; n = 50 ; c =1.
9. Give the three parameters of Quality Characteristics?
10. List out the important considerations in for lot formation.

14 MARKS

1 What is meant by Acceptance Sampling? List the advantages, limitations and its application.

2 i) Explain the step-by-step procedure for constructing the OC curve for a single sampling plan.
ii) Draft the OC curve for single sampling plan with a sample size, n=100 and acceptance number, c = 2. Acceptance Quality level is 2 %.

3. Explain the Producer’s risk and Consumer’s risk with the help of OC curve.

4. Describe Lot by Lot sampling and Single sampling techniques in detail?

5. What is acceptance sampling? Compare acceptance sampling by variables with attributes and state the advantages and limitations of sampling inspection.

6.i) Explain the step-by-step procedure for constructing the OC curve for a single sampling plan.

ii) Draft the OC curve for single sampling plan with a sample size, n=100 and acceptance number, c = 2. Acceptance Quality level is 2 %.

7. Explain OC Curve with reference to sampling inspection and meaning of the terms :

i) AQL ii)IQL iii) LTPD iv) AOQ

8. Explain about Single Sampling plan and Double Sampling Plan with a flowchart.

UNIT IV LIFE TESTING – RELIABILITY

"A system is a collection of components, subsystems and/assemblies arranged to a specific design in order to achieve the desired functions with acceptable performance and reliability". The types of components, their qualities, their quantities and the way in which they are set inside the system have direct effect on system reliability. Malfunction of a component or part may lead to the breakdown of the whole system in few cases and may not in others, depending upon the functional relationship among the components. This necessitates a cautious study of component failures and failure modes and their functions and also their failure models.

FAILURES AND FAILURE MODES

"Failure is defined as Non-conformance to some defined performance criterion". Some products have well defined failures while others do not. For example, electric bulbs and switches have well distinct failures. Either they are working or nonworking (failed). Such products are known as two state products. Some products like voltage – stabilizers, resistors etc. work in a range. For example, the output voltage of a stabilizer may lie inside the limits of V -□ and V +□. When the output voltage crosses these limits only, the device is supposed to have failed. For evaluating the quantitative reliability of a device, the concept of failure and their details is to be used.

Several years of knowledge of failure data of different devices has revealed that based upon the nature of failure, the failures can be grouped into different kinds. When a large collection of elements are put into operation, it is possible that there are a huge number of failures initially which are called *initial failures or infant mortality.* These initial failures are due to production defects, such as weak parts, poor fit, poor insulation, bad assembly etc. This period is known as the *burn-in or debugging period, a*s the malfunctioning units are eliminated in the initial failure periods.

"The failures which occur after initial failures, due to the sharp change in parameters determining the performance of the units, either as a result of the change in the working stresses or environment conditions are called *random failures or catastrophic failure".* Random failures are few in long period of operation during which it is difficult to predict the time of stress occurrence and their amplitude.

With the passage of time, the units get worn out and begin to weaken. A gradual decrease in the values of the parameters affects the performance of the product and when the parameters exceed the limits of tolerance, the product fails. This area is called the *wear-out region* where the failure rate increases and the prediction of wear-out failure is very complicated.

The characteristic curve depicting the above modes of behavior which if often known as *bath-tub curve* is shown in the Fig.3.1. Manufacturers who manufacture high-reliability products subject their products to an initial burn-in-period in order to reduce manufacturing defects. The useful life time of the product is the period $t_2 - t_1$. After time 't_2', the product is changed with a pre-tested product.

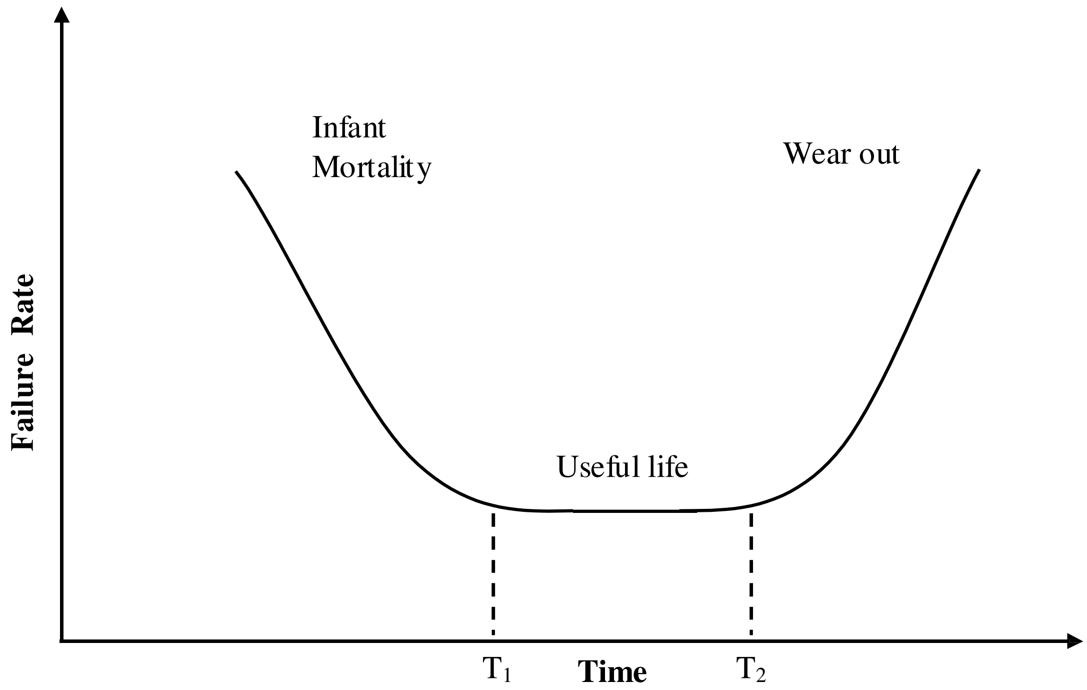


Fig. 3.1 Bath-tub curve

A curve is plotted with the failure rate on the y-axis and with the product life on the x-axis. The life can be in cycles, actuations, years, hours, minutes, or any other units of time or use, which are quantifiable. Failures among surviving units per time unit is known as failure rate. From this plot, it can be shown that many products begin their lives with a higher failure rate due to poor workmanship, poor quality control of incoming parts, manufacturing defects, etc. and exhibit a declining failure rate. The failure rate then generally stabilizes to more or less constant rate where the failures observed are a chance failure which is known as useful life region. With usage, the products experience more wear and tear, the failure rate begins to grow and the products failure begins to occur related to wear-out. The mortality rate is more during the first year or so of life, then drops to a low constant level during teenage and early adult life and then rises as we progress in years in case of human mortality. Infant mortality occurs in early age is characterized by a decreasing failure rate. Occurrence of failures during this period is not random in time but rather the result of a few components with gross defects and the lack of adequate quality assurance controls in the manufacturing process.

Causes for increase in failure rate /wear out failures are:

- Owing to oxidation, friction wear, shrinkage, corrosion, atomic migration, breakdown of insulation, fatigue, etc.
- Populations of substandard items owing to microscopic flaws where the components fail when random fluctuations (transients) of stress go beyond the item strength.
- Usually related to quality assurance and manufacture, e.g. joints, connections, welds, cracks, wrap, impurities, dirt, insulation or coating flaws, incorrect positioning or adjustment.

CAUSES OF FAILURES AND UNRELIABILITY

The causes for failures of component and equipments in a system can be many. Some are known and others are unknown due to the complexity of the system and its environment. A few of them are listed below:

1. Poor component design or system design

2. Wrong method of production-technique
3. Lack of complete knowledge and skill
4. High complexity of equipment
5. Very poor maintenance policies,
6. Organizational inflexibility and complexity
7. Human errors
8. Failure resulting from environmental factors ,software elements and human factors
9. Common mode failure in a redundant system where all replicated units fail by a common factor.

Component hazard data can be obtained in the following two ways:

a) It is not reasonable to ascertain failure rates of products for all working conditions from part-failure data obtained from either failure reports of consumers or from life-tests. It is not impossible through interpolation or extrapolation to forecast reliability under prescribed stress conditions from the data available.
b) From the basic failure rates and working stresses applying stress models.

RELIABILITY OF A PRODUCT FROM TEST DATA

Let us consider a population comprising of a group of *N* items subjected to operation while time $t = 0$. As time grows, the products fail. Let us assume that $N_s(t)$ denote the number of surviving products and $N_f(t)$ *indicate* the products that have failed after time t . Each item independently fails with the probability of failure.

$$F(t) = 1 - R(t) \quad \text{..... (3.1)}$$

Where *R(t) represents* the probability of survival.

Then

$$R(t) = \frac{N_S(t)}{N} = 1 - \frac{N_f(t)}{N} \quad \text{..... (3.2)}$$

"The hazard rate regarded as a measure of *instantaneous speed of failure* is defined as

$$Z(t) = \lim_{t \to 0} \frac{N_s(t) - N_s(t + \Delta t)}{N_s(t)\ \Delta t} = \frac{1}{N_s(t)} \frac{d\,N_s(t)}{dt} \quad \text{....(3.3)}$$

Differentiating *R(t)* with respect *t*,

$$\frac{dR(t)}{dt} = \frac{1}{N} \frac{dN_S(t)}{dt}$$

substituting in terms of the hazard rate,

$$\frac{dR(t)}{dt} = -\frac{N_S(t)z(t)}{N} = -R(t)z(t)$$

Then,

$$z(t) = -\frac{1}{R(t)} \frac{dR(t)}{dt} \quad \text{...(3.4)}$$

Integrating on both the sides,

$$\int_0^t z(x)dx = -\log(R(t))$$

or

$$R(t) = \exp\left[-\int_0^t z(x)dx\right] \quad \text{..... (3.5)}$$

then,

$$F(t) = 1 - \exp\left[-\int_0^t z(x)dx \right]$$

$$f(t) = \frac{dF(t)}{dt}$$

$$= z(t)\exp\left[-\int_0^t z(x)dx \right]$$

$$= z(t)R(t)$$

Therefore,

$$z(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)} \qquad \text{.... (3.6)}$$

MEAN TIME TO FAILURE (MTTF)

The probable time to failure for a *non-repairable system is* described as *Mean* time to failure. "MTTF is an estimate of the average, or mean time until component's first failure or disruption in the operation of the product, process, procedure, or design occurs". MTTF assumes that the product cannot resume any of its normal operations as it cannot be repaired. "It is frequently used to describe the system or equipment reliability, of use when estimating maintenance costs and is significant even if there is no constant failure rate". MTTF is properly used only for components that can be repaired and returned to service.

The knowledge about the mean time to failure of a product is important rather than the complete failure details. MTTF for all the products which are identical in their design and operate under identical conditions is assumed to be same. If we have life-tests data on the population of N items with failure times t_1, t_2,........tn then the MTTF can be mathematically expressed as

$$MTTF = \frac{1}{N}\sum_{i=1}^{n} ti \qquad \text{....... (3.7)}$$

When a component is described by hazard model and its reliability function, then the MTTF is a function of the random variable T relating the time to failure of the component. MTTF can be defined as

$$MTTF = E[T] = \int_0^{\infty} tf(t)dt \qquad \text{.......(3.8)}$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

Hence, $$MTTF = -\int_0^{\infty} tdT(t)$$

$$= -tR(t)\Big|_0^{\infty} + \int_0^{\infty} R(x)dt$$

$$= \int_0^{\infty} R(t)dt \qquad \text{.........(3.9)}$$

The MTTF can also be evaluated as the Laplace - transform of $R(t)$ i.e.,

$$\text{MTTF} = \int_0^{\infty} t(R)dt = \lim_{t\to 0}\int_0^{t} R(x)\,dx$$

However, $$\lim_{t\to\infty}\int_0^{t} R(x)\,dx = \lim_{s\to 0} R(s)$$

Where $R(s)$ is the Laplace transform of $R(t)$

Thus, $$MTTF = \lim_{S\to 0} R(s) \qquad \text{......... (3.10)}$$

TIME DEPENDENT HAZARD MODELS

Field –Data Curves

From the failure analysis data, hazard rate of a component can be computed by using the formula,

$$z(t) = \frac{N_S(t + \Delta t)}{N_S(t)\Delta t} \quad \text{..... (3.11)}$$

for various time intervals. A general hazard-rate curve is shown in Fig.3.2. It is assumed that the time intervals are identical. The time intervals need not be equal.

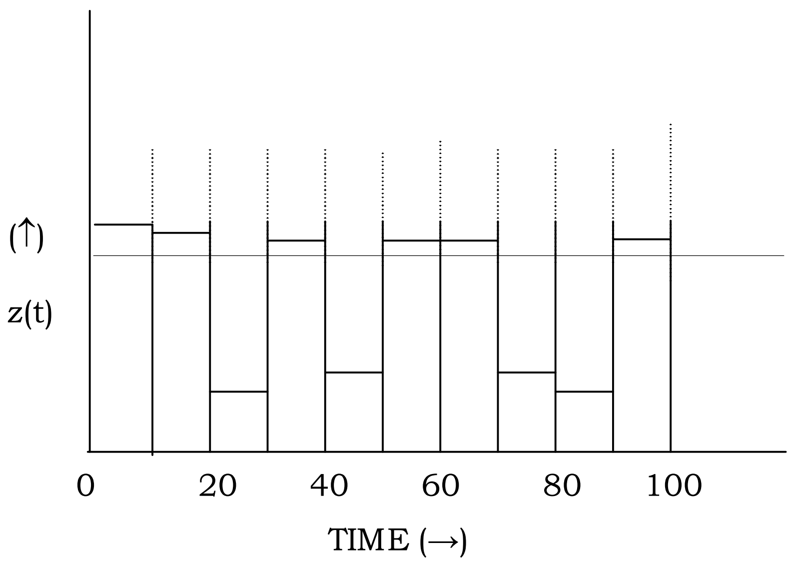


Fig. 3.2 Hazard-rate curve

When the data happens to be large and the time interval approaches zero, the piecewise hazard-rate function will tend to be the continuous hazard-rate function. Using such failure curves, the failure rate of other identical components operating in identical conditions can be predicted. But this requires a few mathematical models which approximately describe the failure behavior. A few such models which depict various shapes of failure curves will be discussed in the subsequent sections.

Constant-Hazard Model

This model assumes that hazard rate is constant which does not significantly increase with component age. When a product is functioning during its useful lifetime constant-hazard model is precisely suitable.

Reliability of a constant-hazard model:

$$R(t) = 1 - F(t) = exp(-\lambda t) \quad \text{..... (3.12)}$$

Let us consider constant failure rate and an exponential probability distribution;

Failure rate; $f(t) = \lambda e^{-\lambda t}$ .. (3.13)

Reliability function, $R(t) = e^{-\lambda t}$

Then, hazard function,

$$h(t) = f(t)/R(t) = \lambda e^{-\lambda t} / e^{-\lambda t} \quad \text{.... (3.14)}$$

When the hazard rate is a constant and is equal to the failure rate, the ‘constant-hazard model’ takes the form

$$Z(t) = \lambda \quad \text{.......(3.15)}$$

Where λ is a constant. This attribute is being shown by many components, particularly, electronic products. For many years, this model has been used in reliability studies.

An item with constant hazard rate will have the following reliability and unreliability functions.

$$f(t) = \lambda e - \lambda^{t} \quad \text{........ (3.16)}$$

$$R(t) = e^{-\lambda t} \quad \text{........ (3.17)}$$

$$F(t) = 1 - e^{-\square t} \qquad \text{........ (3.18)}$$

The mean time to failure of the item is:

$$MTTF = \int_0^{\infty} e^{-\square} dt = \frac{1}{} \qquad \text{... (3.19)}$$

□

Linear – Hazard Model

Several components which are under mechanical stresses will fail due to deterioration or wear-out. The hazard rate increases with time for such components. The linear-hazard model which is the simplest time-dependent model has the form

$$Z(t) = bt, \quad t > 0 \qquad \text{..... (3.20)}$$

where *b* is a constant

$$R(t) = \exp\left[-\int_0^{\infty} bt \quad dt\right] \qquad \text{..... (3.21)}$$

$$= \exp(-bt^2/2)$$

$$f(t) = bt \quad \exp(bt^2/2) \qquad \text{..... (3.22)}$$

where f *(t)* is a Rayleigh-density function. The portion beyond the useful period in the bath-tub curve, might follow this model for few cases. The mean time to failure is given by

$$MTTF = \int_0^{\infty} e^{-bt^2/2} dt$$

$$= \frac{T(1/2)}{2\sqrt{b/2}} = \sqrt{\frac{\square}{2b}} \qquad \text{..... (3.23)}$$

Non-Linear Hazard Model

The hazard rate which is not always a linearly increasing function of time has a more general form of the hazard model:

$$Z(t) = at^b \qquad \text{..... (3.24)}$$

Where *a* and *b* are constant

This gives us

$$R(t) = \exp\left[-at^{b+1}/(b+1)\right] \qquad \text{..... (3.25)}$$

$$f(t) = at^b \exp\left[-at^{b+1}/b+1)\right] \qquad \text{...... (3.26)}$$

The above general form is called the Weibull model which generates a wide range of curves for various values of *a* and *b.* This model represents the constant-hazard model which includes both the previously discussed models when b=*0* and represents linearly increasing-hazard model when *b=1*. The parameter *b* the shape of *z(t)* and *a* affects the amplitude and so they are called *shape* and *scale* parameters respectively.

For the above model,

$$MTTF = \frac{\Gamma\left(\frac{1}{b+1}\right)}{(b+1)\left[\frac{a}{(b+1)}\right]^{1/(b+1)}} \qquad \text{..... (3.27)}$$

$$= \frac{dT(d)}{(ad)^a}$$

Where $d = \dfrac{1}{b+1}$

Gamma Model

As the gamma is a variable life distribution model, it may offer a good fit to a few sets of failure data. However, it is not widely used as a life distribution model for the common failure mechanisms.

The hazard model and it's associated functions are

$$z(t) = \frac{\lambda (\lambda t)^{a-1}}{(a-1)!} \frac{e^{-\lambda t}}{R(t)} \quad \text{............ (3.28)}$$

$$R(t) = \sum_{j=0}^{n-1} \frac{(\lambda t)^{j}}{j!} e^{-\lambda t} \quad \text{......... (3.29)}$$

$$f(t) = \frac{\lambda}{(a-1)!} (\lambda t)^{a-1} e^{-\lambda t} \quad \text{............ (3.30)}$$

Where α is a positive integer and λ is a positive constant

$MTTF = a / \lambda$

For a > 1, *z(t)* increases and for a=1, it becomes a constant-hazard rate model. The above functions are applicable to a component which is replaced (a-1) times by identical components with practically zero replacement time. In effect a total of 'a' components were used in sequence to accomplish the task. The case a = 1 represents that there is only one component.

Other Models

In statistics and probability theory, the exponential distribution is a set of continuous probability distribution which describes the time elapsed between trials in a Poisson process, i.e. a process in which trials occur independently and continuously at a steady average rate. It may be noted that the exponential distribution is not the identical to the class of exponential sets of distributions, which is a large class of probability distributions that includes the exponential distribution as one of its members, but also includes the Poisson distribution, binomial-distribution, normal distribution, gamma distribution, and many others.

There are few more models that are applied sometimes to describe the failure curves which do not fit in with the earlier discussed models. The shifter model takes the form

$$Z(t) = a(t-t_0)^b \text{ where } t > t \qquad \text{..... (3.31)}$$

This is used when the initial hazard rate is almost zero for some time. This model is called the three-parameter Weibull model. This model roughly combines the constant and linear models to describe a failure curve that is initially constant and then increases.

The following model also can serve this purpose to a limited extent.

$$Z(t) = ae^{ct} \qquad \text{..... (3.32)}$$

Where *a* and *c* are constants

For this,

$$R(t) = \exp\left[-(a/c)\left(e^{a} - 1\right)\right] \qquad \text{..... (3.33)}$$

This model is generally known as the exponential-hazard model.

STRESS-DEPENDENT HAZARD MODELS

Mostly, the reliability of a component is defined under stated operating and environmental conditions which implies that any variation in these situations can affect the failure rate of the component and thus its reliability. For almost all components the failure rate is stress-dependent. A component reliability which is influenced by more than one kind of stress, a power function model of the form below is used.

$$h(t) = z(t)\square_1^{a_1}\square_2^{a_2} \qquad \text{..... (3.34)}$$

σ_1 and σ_2 are stress conditions, a_1, a_2 are positive constants. For instance, for an electrical item, the stress may be

$$\sigma_1 = \frac{Operating\ Voltage}{rated\ \ voltage}$$

$$\sigma_2 = \frac{Operating\ \ current}{rated\ \ current}$$

The above models are applied for accelerated testing of products. Statistics shows that the variable *'a'* varies from 1 to 8 depending upon the variety of component. The other factors like quality factors, application factor, complexity factor, etc will also be used for the accurate estimation of failure rates.

COMPUTATION OF RELIABILITY FUNCTION USING MARKOV MODEL

This method of estimating the reliability of a part can be used if the hazard rate of the part is known. The part is assumed to exist in only two states:

State 0: the part is good

State 1: the part is failed

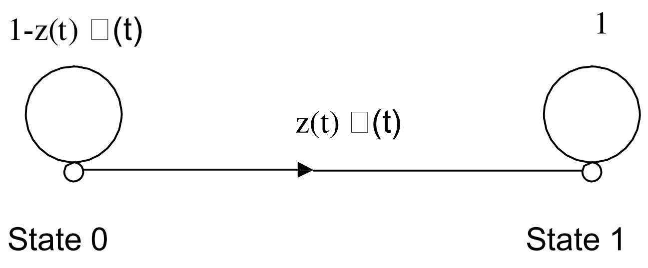


Fig. 3.3 Markov Graph

If a part is in state 0 at time *t*, then the probability of component failure and reach state 1 in the time interval Δt is *z* (t) Δt, where *z(t)* = the hazard rate of the component

If $P_0(t)$ = the reliability that the component will not fail in time *t;*

$P_1(t)$ = the probability that the component will fail in time *t;*

Then, the probability that the component will stay in state 0 at time *t* + *t* is given by

$$P_1(t+\Delta t) = z(t)\Delta t P_0(t) + P_1(t) \quad \text{..... (3.35)}$$

When the component fails, the probability of its remaining in state 1 (failed state) is unity because the component is not repaired. The probability of the component being in state 1 at time *t* + *t* is,

$$\frac{P_0(t+\Delta t) - P_0(t)}{\Delta t} = -z(t)P_0(t) \quad \text{..... (3.36)}$$

The probability equations can be written as

$$\frac{P_1(t+\Delta t) - P_1(t)}{\Delta t} = z(t)P_0(t) \quad \text{..... (3.37)}$$

$As\ \Delta t \to 0,$ We have

$$\frac{dP_0(t)}{dt} = -z(t)P_0(t)$$

The solution of this equation yields

$$\log P_0(t) = -\int_0^t z(x)dx + c_1$$

$$P_0(t) = \exp\left[-\int_0^t z(x)dx + c_1\right]$$

$$= c_2 \exp\left[-\int_0^t z(x)dx\right] \quad \text{..... (3.38)}$$

Where c_1 and c_2 are constants of ingtegration Initially when $t=0$ the component is in state 0 and therefore,

$$P_0(0) = 1 = c_2$$

Thus the reliability of the component is

$$R(t) = P_0(t) = \exp\left[-\int_0^t z(x)dx\right] \quad \text{..... (3.39)}$$

SYSTEM RELIABILITY MODELS

The main difficult task of a systems engineer is to evaluate various reliability parameters of the systems he deals with. The system configuration may differ from simple (consisting of one or two elements) to complex (involving thousands of elements). One method for analyzing such systems is to disintegrate them into subsystems of suitable size, each representing a precise function. Reliabilities of all the subsystems are evaluated and then combined to find the reliability of the complete system using certain probability laws. However, this approach requires total information about the physical structure of the system and the kind of its functions to evaluatefind the behavior of the system whenever a subsystem fails.

The task of a system engineer is to evaluate reliability of various systems. The system configurations vary from a simple one consisting of one or two components to a complex system consisting of plenty of components. Such systems can be analyzed by decomposing the system into subsystems of suitable size, each performing a specific function. After decomposition of the system, the reliabilities of subsystems are evaluated and combined to found out the reliability of the whole system using certain probability laws. This technique requires a comprehensive knowledge of the physical structure and to evaluate adequately well to the behavior of the entire system. The subsystem may consist of one or more components whose reliabilities are known.

The reliability models for various kinds of subsystems (or systems) are developed in this section. For all models the assumption is that each component fails independently of other, i.e. the failure of any component does not alter the failure of rest of the components.

3.8.1Series Systems

In a series system where components are connected serially, the failure of one of its components leads to system failure.

Assume a system is having a total of 'n' components. When the functional diagram suggests that for the successful operation of the system, the accurate operation of all the 'n' components, then we call that the system arrangement is a series type. Such systems are represented as shown in Fig.3.4 for the purpose of reliability estimation. The information from the IN end will reach the OUT end only if all the 'n' components function properly. Many complex systems can be reduced to a simple structure.

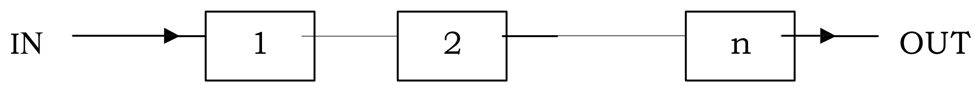


Fig. 3.4 Series System

Let 'E_i' represent the event that component 'i' is working state and '$\overline{E_i}$' be the event that component 'i' is in failure state. The intersection of E1, E2, E3,……, En represents the event of success of the system.

λ_1, λ_2, λ_3, …… λ_i,…… λ_n are the components hazard rates of 1, 2, 3, …..i,…..n respectively.

$P_i(t)$ be the probability that component 'i' is working at time 't'

Reliability of a series system,

R_s = the probability that all the units are in good condition

$$Rs = Pr(E_1 \cap E_2 \cap E_3 \ldots\ldots \cap E_n)$$

$$= Pr(E_1).Pr(E_2 / E_1).Pr(E_3 / E_2.E_1) \qquad \ldots\ldots(3.40)$$

$$= P_1(t).P_2(t).P_3(t)\ldots\ldots Pn(t)$$

$$= e^{-\lambda_1 t}.e^{-\lambda_2 t}.e^{-\lambda_3 t}\ldots..e^{-\lambda_n t} \qquad \ldots\ldots(3.41)$$

= Product of reliabilities of the components

i.e Reliability of a series system is the product of its component reliabilities.

3.8.2Systems with Parallel Components

"Parallel system is a system in which components are connected in parallel and the system does not fail, even one component is in good working condition i.e the system fails only when all components have failed".

Let, the event that component 'i' is in good working state be 'E_i'. The event that component 'i' is in failure state be '$\overline{E_i}$' the number of

67

components in the system be ' n' the hazard rates of components 1, 2,3,..i,.....n are λ1, λ2, λ3,... λn respectively.

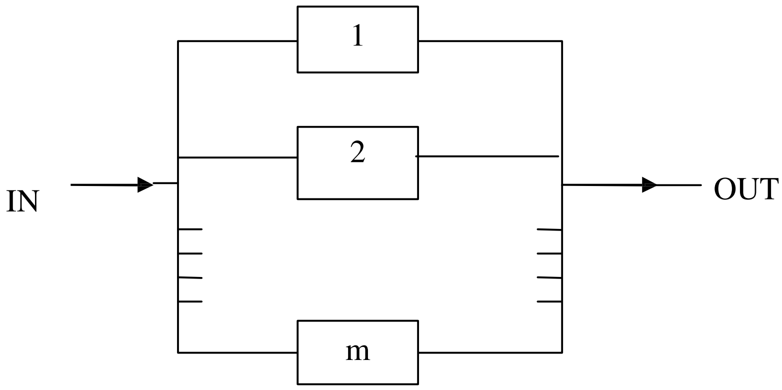


Fig. 3.5 Parallel System

Let the probability that component 'i' is functioning at time 't' be $P_i(t)$ System unreliability,

$$= P_r(\overline{E_1} \cap \overline{E_2} \cap \overline{E_3} \ldots\ldots \cap \overline{E_n}) \qquad \ldots\ldots (3.42)$$

$$= P_r(\overline{E_1}).P_r(\overline{E_2}/\overline{E_1}).P_r(\overline{E_3}/\overline{E_2}.\overline{E_1})..P_r(\overline{E_m})$$

$$= P_1(t) \,.\, P_2(t) \,.\, P_3(t) \ldots\ldots P_n(t)$$

$$= \left(1 - e^{-\lambda_1 t}\right)\left(1 - e^{-\lambda_2 t}\right)\left(1 - e^{\lambda_3 t}\right)\ldots\left(1 - e^{-\lambda_n t}\right) \qquad \ldots\ldots (3.43)$$

= Product of unreliability of components.

System reliability,

R_P = 1- Unreliability of the system

Reliability of a parallel system =1-the product of its component unreliability.

By adding one more unit to the existing system which is having already m units, the system reliability will increases to

$$R_{m+1}(t) = 1 - [1 - p(t)]^{m+1} \qquad \ldots\ldots (3.44)$$

and the increase in reliability is given by

$$\Delta R_{m+1}(t) = 1 - [1 - p(t)]^{m+1} \quad \text{..... (3.45)}$$

$$= [1 - p(t)]^{m} - [1 - p(t)]^{m+1}$$

$$= p(t)[1 - p(t)]^{m}$$

Hence the following recursive formula is available for estimating the reliability

$$R_1(t) = P^{(t)}$$

$$R_m(t) = R_{m-1}(t) + \Delta R_{m-1}(t), \geq 2 \quad \text{.. (3.46)}$$

If the failure rates are constant

$$R(t) = 1 - [1 - e^{-\square}]^{m}$$

The mean time to fail (MTTF) of the system is

$$MTTF = \int_0^{\infty} \{ - [1 - e^{-\square}]^{m} \} dt$$

$$Putting \quad (1 - e^{-\square}) = x, we \ get$$

$$MTTF = \frac{1}{\square} \int_0^1 \left[\frac{1 - x^m}{1 - x} \right] dx$$

$$= \frac{1}{\square} \int_0^1 (1 + x + x^2 + ...x^{m-1}) dx$$

$$= \frac{1}{\square} \left(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{m} \right)$$

$$= \frac{1}{\square} \sum_{i=1}^{m} \frac{1}{i} \quad \text{..... (3.47)}$$

For higher values of 'm',

$$MTTF = \frac{1}{\square} [In(m) + 0.577 + 1/2m] \quad \text{..... (3.48)}$$

$$= \int_0^{\infty} [\exp(-\square_1 t) + \exp(-\square_2 t) + ...\exp(\square_m t)$$

It is observed that the enhancement in the mean life with redundancy is logarithmic. If the component reliabilities are not equal, the problem becomes complex.

k-out-Of-m Systems

One of the important practical systems is where more than one of its parallel components is required to meet the demand. For example, in a power generating station where there are four generators, two generators are sufficient to provide the necessary power to the consumers. The other two generators are added to improve the supply reliability. One more instance is a four-engine aircraft where two engines are required for successful operation and two are kept as standby units. Many such systems are present in industries and other applications. The models which are developed for a simple parallel system cannot be applied to these systems.

The binomial distribution can be used to estimate the reliability for systems which have identical and statistically independent components. If the probability of survival of each component is p, then the probability that precisely x out of m components surviving is given by

$$P(M,x) = B(m,x)p^x(1-p)^{m-x} \qquad \text{..... (3.49)}$$

Where

$$B(m,x) = \binom{m}{x}$$

is the binomial coefficient. If the least number of components required for operation of the system without any failure is k , then the system will function if k, $k+1$, $k+2$,.... Or m components are functioning. The system reliability is the total of binomial probabilities, x varying from k to m, i.e.,

$$R = \sum_{i=k}^{m} B(m,i)\, p^{i} (1-p)^{m-i} \qquad \text{..... (3.50)}$$

When $k=m$ the system becomes a series-system and when $k=1$ the system becomes a parallel system.

In a k-out-of-m system, the reliability of the system is improved by the addition of m-k units. Such systems sometimes are called partial-redundant systems to represent that $k>1$. The k units are called basic units whose functioning is essential for the success of the system. If the demand on the system increases, the number of basic units required will increase and hence the redundant units decrease.

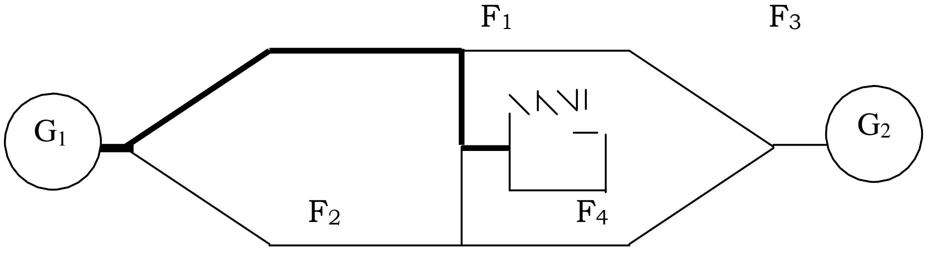


Fig. 3.6 Power distribution system for a chemical Processing industry

For example, a 4-unit power generating system where each unit with a capacity of 65 MW and can supply a demand of up to 260

MW. If the capacity is higher than the demand, the system can meet the demand even if there is a failure of any unit .When there is demand for less than 130 MW, the system will function as a 2-out-of-4 system. There is a reserve of another 130 MW. We can safely increase the demand up to a further 130 MW without installing any additional unit but increase is done at the cost of reliability. A total demand of, say 195 M.W would mean that the system works as a 3-out-of-4 system with a reduced reliability which is given by $\Delta R = 6p^2(1-p)^2$

There is a corresponding reduction in the mean life of the system. The reduction in case of constant failure rates is $\Delta T = \frac{1}{2\square}$

Using the binomial failure probabilities, the unreliability of a *k-out-of-m* system can be stated as

$$Q(k,m) = \sum_{i=m-k+1}^{m} \binom{m}{i} (1-p)^i p^{m-i} \qquad \text{..... (3.51)}$$

In case of high reliable components with constant hazard rates

$$Q(k,m) \approx B(m,k-1)\ (\square t)^{m-K+1} \qquad \text{..... (3.52)}$$

If we substitute k=1 in the above equation, it reduces to

$$Q(k,m) = (\square t)^m \qquad \text{..... (3.53)}$$

If the values of *m* and *k* are large, the approximate formula for mean life is

$$MTTF = \frac{1}{\square} \ln\left[\frac{m+\frac{1}{2}}{k-\frac{1}{2}}\right] \qquad \text{..... (3.54)}$$

Non Series – Parallel Systems

A complex system when simplified can produce a non – series parallel configuration. Bridge configuration comes under this type. The reliability of this type of configuration can be estimated using appropriate probability rules and logic diagram. A simple non series-parallel structure which is a bridge configuration is shown in Fig.

S_1, S_2 ...... are the subsystems and direction of arrows show the flow process. Such cases can be analyzed by using another approach called the logic diagram technique.

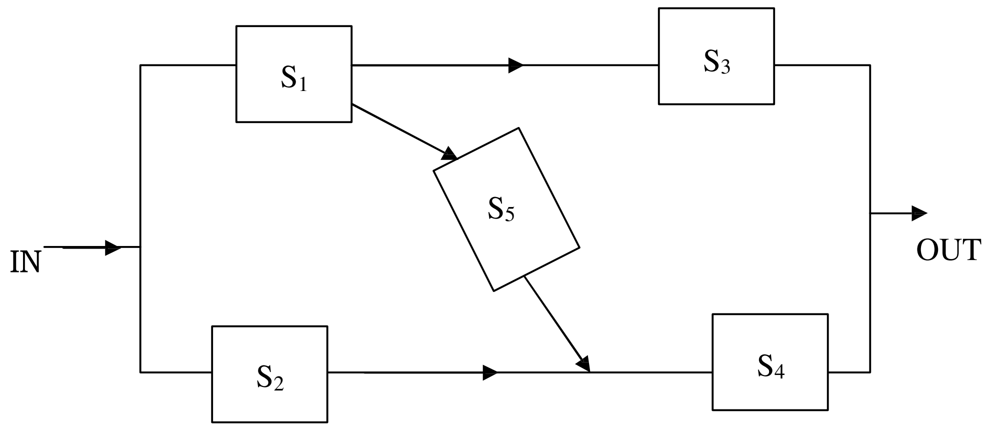


Fig. 3.7 Bridge Network

The system diagram is converted into a logic diagram that consists of many simple parallel paths between IN and OUT terminals by using the logic-diagram approach. The successful functioning of the system depends on the successful operation of the elements in different paths. If there is at least one continuous way between IN and OUT terminals, the system will function successfully. The logic diagram for the system shown in Fig3.7 is represented in Fig.3.8. The subsystem S_5 is unidirectional like any other subsystems.

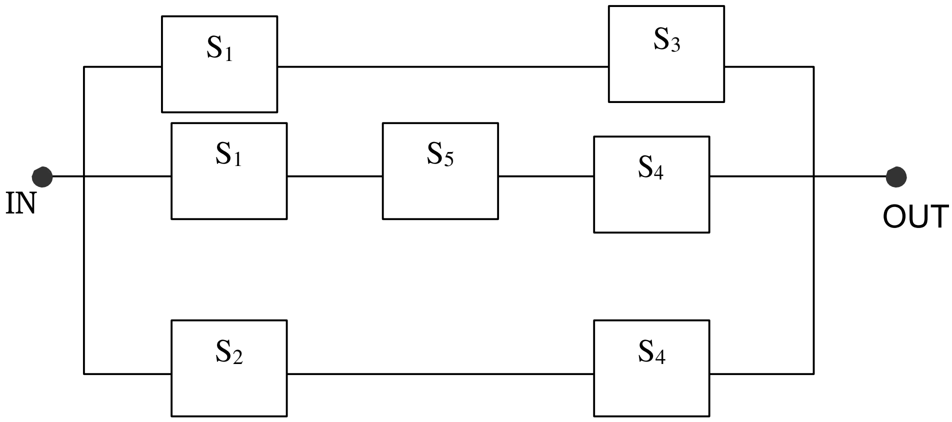


Fig. 3.8 Logic Diagram for Fig 3.7

The logic diagram is a plain series-parallel system whose reliability can be estimated by combining the models developed in sections 2 and 3 but care has to be exercised to recognize interdependency of paths. The failure of each subsystem is assumed to be independent of the failure of the paths is not independent since some subsystems find place in more than one path. The use of the following rule takes care of this problem.

If $P_r(E_1) = p_1p_2$ *and* $pr(E_2) = p_2p_3$ then

$$Pr(E_1) * Pr(E_2) = p_1p_2p_3 \quad \text{..... (3.55)}$$

That is $pi * pi = pi$ for any value of i.

Let the reliability of the subsystem, $S_i = P_i$ and the path i reliability *be* R_i.

Let the reliability of the path $i = R_i$ and the subsystem S_i reliability be P_i and.

$$R_1 = p_1p_3$$

$$R_2 = p_2p_5p_4$$

$$R_3 = p_2p_4$$

Then the entire structure reliability is

$$R=1-(1-R_1)(1-R_2)(1-R_3)$$

$$= R_1 + R_2 + R_3 - R_1R_2 - R_2R_3 - R_1R_3 + R_1R_2R_3$$

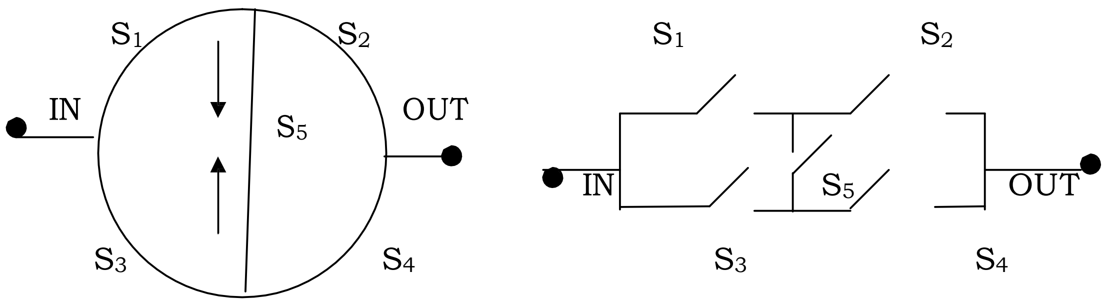
On simplification, we get

$$R=p_1p_3+p_1p_5p_4+p_2p_4-p_1p_3p_4p_5-p_1p_2p_4p_5-p_1p_2p_3p_4+p_1p_2p_3p_4p_5 \quad ....(3.56)$$

For $p_1=p_2= \ldots =p_5=p$

$$R=2p^2 +p^3 - 3p^4 + p^5 \quad ....(3.57)$$

There are some cases where the subsystem S_5 can be a bidirectional element. For instance, in the systems shown in Fig. 3.9, the subsystem S_5 transmits signals in either direction. The logic diagram showing the paths for these systems is illustrated in fig. 3.10.



(a) Power distribution system (b) Relay network

Fig. 3.9 Bridge networks

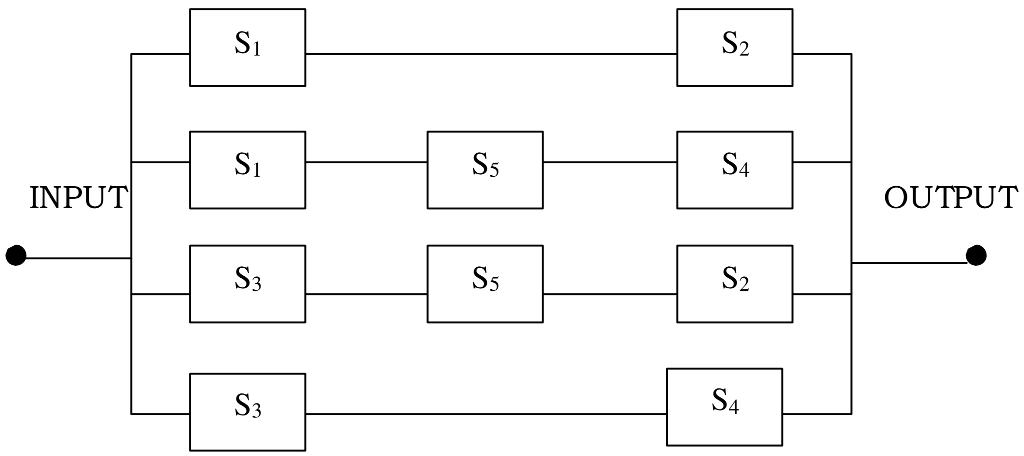


Fig. 3.10 Logic diagram of systems in Fig.3.9

The reliability of bridge networks shown in Fig. 3.9 is given by

$R=1-(1-p_1p_2)(1-p_3p_4)(1-p_1p_5p_4)(1-p_3p_5p_2)$

$= p_1p_2 + p_3p_4 + p_1p_5p_4 + p_3p_5p_2 - p_1p_2p_3p_4 - p_2p_3p_4p_5 - p_3p_4p_5p_1 - p_4p_5p_1p_2 -$

$p_5p_1p2p_3 + 2p_1p_2p_3p_4p_5$ ....(3.58)

For the case $p_1 = p$ for all i

$R = 2p^2 + 2p^3 - 5p^4 + 2p^5$ ....(3.59)

For a value *p=0.8 Eqs. (3.57) yields R=0.890 and Eq. (3.59) yields*

R=0.91

If we put $p_5 = o$ in Eqs. (3.57) and (3.58) the results would be

$R = p_1p_3 + p_2p_4 - p_1p_2p_3p_4$ (from Eq. (3.56)

$R = p_1p_2 + p_3p_4 - p_1p_2p_3p_4$ ....(3.60)

The slight difference in the positions of p_2 and p_3 is due to the difference in the physical location of S_2 and S_3.

The condition $p_s=0$ means that the subsystem S_5 is no more in operation and the system configuration is as shown in Fig. 3.11. In this case the reliability is given by Eq. (3.61)

$R=1-(1-p_1p_2)(1-p_3p_4)$ ....(3.61)

It may be noted that Eqs. (3.36) and (3.37) are identical. Similarly, if we assume $p_5=1$, Eq. (3.34) yields

$R= p_1p_2 + p_3p_4 + p_1p_4 + p_3p_2 - p_2p_3p_4 - p_3p_4p_1 - p_1p_2p_4 - p_1p_2p_3 + p_1p_2p_3p_4p_5$ ....(3.62)

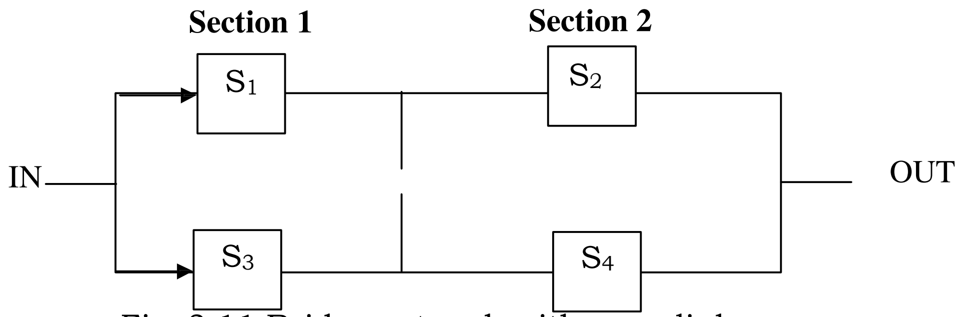


Fig. 3.11 Bridge network with open link

The assumption of $p5=1$ implies that the subsystem is replaced by a direct link of unit reliability and the system can be represented as shown in Fig.3.12. The system reliability is the product of the reliabilities of sections 1 and 2.

Further, we know that

$R(Section\ 1) = 1-(1-p_1)(1-p_3)$

$=p_1+p_3-p_1p_3$

$R(Section\ 2) = 1-(1-p_2)(1-p_4) = p_2+p_4-p_2p_4$

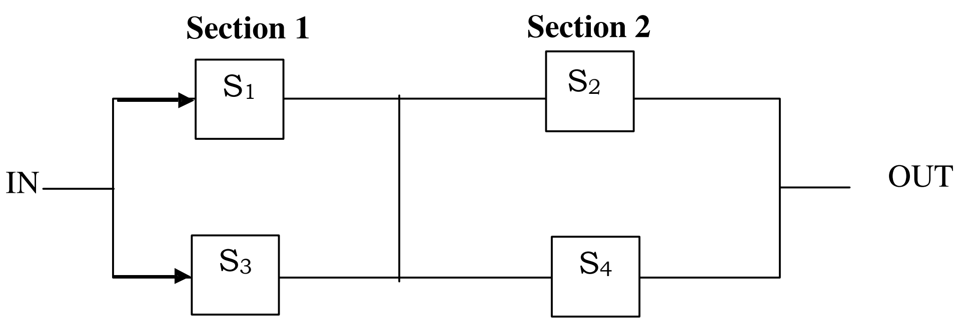


Fig. 3.12 Bridge network with a link of unit reliability

then, $R=(p_1+p_3-p_1p_3)(p_2+p_4-p_2p_4)$

$$=p_1p_2+p_2p_3+p_3p_4+p_4p_1-p_1p_2p_3-p_2p_3p_4-p_3p_4p_1-p_4p_1p_2+p_1p_2p_3p_4 \quad ....(3.63)$$

It may be noted that Eq. (3.63) is the same as that of Eq.(3.62) for equal Ps, Eq. (3.63) becomes

$$R=4p^2-4p^3+p^4$$

$= 0.9216$ for p 0.8

Systems With Mixed-Mode Failures

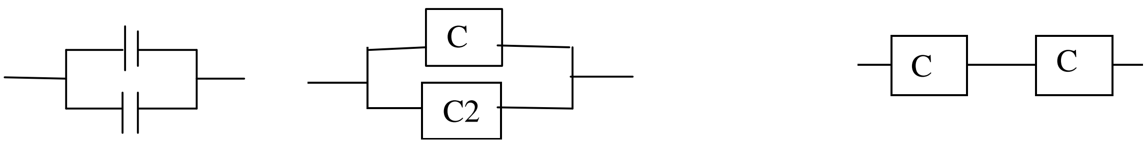
Until now it is assumed that when there is an open circuit relating the OUT and IN terminals of a system, it is considered to have failed. Such type of failure is known as "open-mode failure". But there are some exceptions. For example, an electric appliance will fail to function when there is an electric short-circuit between OUT and IN terminals. A relay is declared to be failed if it fails to open when it is required to open. Such failures are called "short-mode failures". When a system possess these type of components, both type of failures with their failure probabilities should be considered in estimation of system-reliability.

The reliability of a component subject to both failures, is given by

$$p=1-(q_0+q_s) \quad -----(3.64)$$

Where q_0 and q_s are the failure probabilities due to open-mode and short-mode respectively. Consider a system having two capacitors which are connected in parallel as shown in Fig. 3.13a. If there is an open-circuit in one of the capacitors, it does not cause a system to fail. If there is a short-circuit across any one of them, it will cause the system to fail. The capacitors are considered to be in series for the short-circuit type in the reliability sense.

For evaluating probabilistic failures the logic diagrams are shown in Fig. 3.13(b)



(i) Open mode (ii) short mode

(a) System mode (b) logic mode

Fig. 3.13 Systems with mixed mode failures

3.8.6 Fault-Tree Technique

Fault tree analysis technique was developed in the early 1960's. Since then they have been adopted for a broad variety of engineering disciplines for performing reliability and safety analysis. They graphically signify the relationship of failures and other events within a system. Basic events at the base of the fault tree are connected via logic symbols (known as gates) to one or more top events. These peak events represent known hazards or system failure modes for which reliability is predicted. Crucial events at the base of the fault tree usually represent human and component faults for which statistical failures and repair data is available.

Common basic events are

Switch fails closed

Pump failure

Operator does not respond

Temperature controller failure

Fault trees can be used to investigate large and complex systems. Fault trees are particularly capable of representing and analyzing redundancy arrangements. Further common cause events are easily handled.

The fault-tree approach to reliability analysis of complex systems is based on the events that will cause the system to fail. “A fault tree (FT) is a diagrammatic representation of all possible fault events, their logical combinations, and their relationship to the system failure”. The faults which are at the lowest level of the system are usually represented at the base of the tree and the system fault at the top. The events which are at the lowest level are known as “basic events”. “The events resulting from combinations of basic events which may or may not be of interest and such events are represented as intermediate events”. The system failure is found out by combining the failure probabilities of basic events to obtain the failure probabilities of intermediate events and lastly the top event.

For simplicity sake, a 3-unit system as shown in Fig. 3.14 is considered. This system will fail when the component C_1 fails, or by the failure of components C_2 and C_3 combined, or by the failure of all the three components. The fault tree diagram of this system is shown in Fig. 3.15. For combining two or more events the logical OR and AND gates are used.

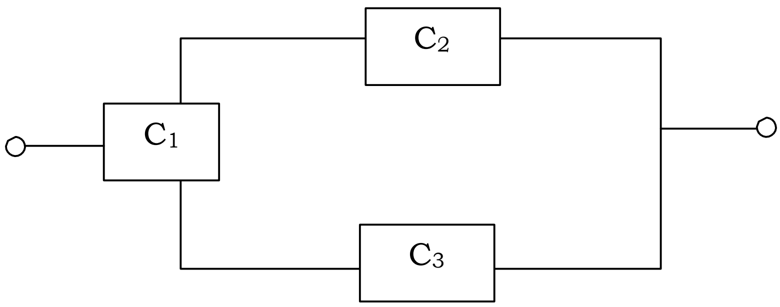


Fig. 3.14 A three unit system

An AND gate is a multi-input mechanism which produces an output signal only when all its inputs function. If *Y* represents the output and $X_1, X_2, \ldots\ldots.$ are the input events, then

$$Y = X_1 \cap X_2 \cap X_3 \cap \ldots\ldots.. \cap X_n$$

An OR gate, on the other hand, produces an output even when one of its inputs is success. In this case,

$$Y = X_1 \cup X_2 \cup X_3 \cup \ldots\ldots. \cup X_n$$

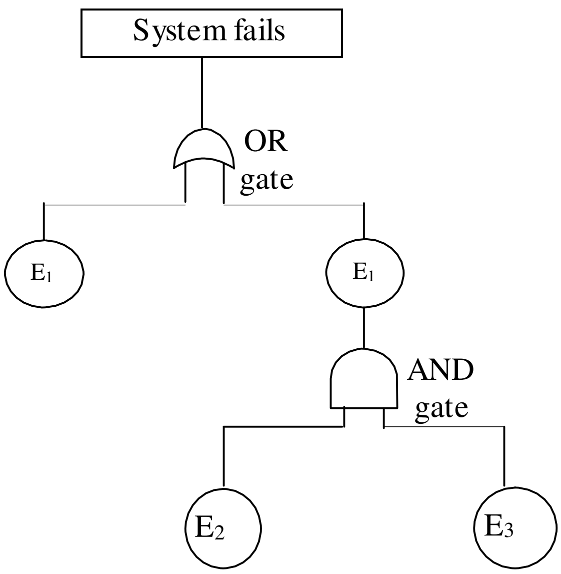


Fig. 3.15 Fault-tree diagram for the system in Fig. 3.14

MCQ

QUESTIONS	Option 1	Option 2	Option 3	Option 4	ANSWERS
Reliability is also known as	Probability of failure	Mean failure rate	Probability of survival	Mean Time to Failure	Probability of survival
The initial early failure period in "Bathtub curve" is ___________________.	Pre testing period	Post testing period	Burn-in or debugging period	Reliability period	Burn-in or debugging period
Technically MTBF should be used only in reference to a ______ item	Non-repairable item	trial item	development item	repairable item	repairable item
Reliability is expressed as a probability.	FALSE	TRUE	Not certain	Never certain	TRUE
Instantaneous failure rate is also called as	Failure density	Hazard rate	Reliability	Hypothesis testing	Hazard rate
._______ is the probability of an event occurring after the experiment is over.	Failure density	Failure rate	Posteriori	Poisson's ratio	Posteriori
Hard failure occurs when there is __________ damage made to the product.	Destructive	normal	non-destructive	permanent	permanent
Parallel configuration consists of several signal paths performing ___________ operation	parallel	series	mixed	same	parallel
MTBF is	Medium Time Between Follow up	Mean Time Between Failures	Maximum Time Before First production	Minimum Time between Failures.	Mean Time Between Failures
The failure rate is plotted between	fd vs time interval.	Z vs time interval	R vs time interval	Cpk vs time interval.	Z vs time interval
Failure rate is the ratio of the number of failures during a particular unit interval to the ______ population.	Initial	Final average	Average	Weighted average	Average
Failure density is calculated as a ratio of the number of failures during a given unit interval of time to the total number of items at the l number of items at the	End of the test	In between any time	Middle of the process time	Very beginning of the test	Very beginning of the test
The increasing complexity of present day equipment has brought into focus two other	Reliability and Probability	Maintability and Availability	Availability and Probability	Failure rate and Availability	Maintability and Availability

aspects known as ________					
Soft failure occurs when there is a damage made but the product returns to its _____ state	destructive	normal	non-destructive	non-destructive	normal
The reliability factor between t = 0 and t = t_1 , the unreliability factor for the same period Will be	1-R(t1)	1+R(t1)	R(t1)-1	R(t1)	1-R(t1)
The failure density is plotted between	fd vs time interval	Z vs time interval	R vs time interval	Cpk vs time interval	fd vs time interval
The difference between hard and soft failures with ______ margins increase the reliability and life of the product	increased	minimized	maximized	decreased	maximized
Mean Time Between Failures refers to the	The average time of breakdown until the device is beyond repair	The total time of breakdown until the device is beyond repair	The average working time of the device	The time to service the device	The average time of breakdown until the device is beyond repair
Which of the following methods will most improve the reliability of a product?	Reducing confidence levels	Reducing variation	Increasing sample size	Increasing test time	Increasing test time
A qualification test is planned to establish whether a unit meets required minimum mean time between failures (MTBF). Which of the following tools can be used to estimate the chance that the unit will pass the test even if its true MTBF is below the required level?	Operating characteristic curve	Block diagram	Fault tree	Transition state matrix	Operating characteristic curve
Which of the following stress-related characteristics should be considered by the reliability engineer during the material selection stage?	Activation energy	Flexural energy	Tensile strength	Compression strength	Activation energy
Which of the following test techniques would allow the reliability engineer to evaluate the material quickly?	Ambient temperature test	Step-stress test	Full system test	Durability test	Ambient temperature test
Which of the following approaches should be used for a field validation test of the new material?	Implement the material change and monitor field	Field test a sample of only the new material	Field test samples of both old and new	Run lab tests on two systems.	Field test samples of both old and new material in

	performance.	in the system.	material in the system.		the system.
Which of the following probability distributions best satisfies the inequality P(x < 0) > 0 ?	Two-parameter Weibull	One-parameter exponential	Normal	Lognormal	Normal
Which of the following is considered the most valuable source of information on actual failure modes and mechanisms?	Field data	Qualification tests	FMEAs	User profiles	Field data
The influence of a failure mode and effects analysis (FMEA) on reliability is maximized at which of the following stages of development?	Design	Prototype	Test	Operation	Design
A system has an availability of 95% when the MTBF is 500 hours and the mean time to repair is	22 hours	26 hours	133 hours	167 hours	26 hours
Accelerated life testing is most beneficial when performed on	dead-on-arrival products	products released to manufacturing	products under development	products returned from the field	products under development
When the plot of the cumulative MTBF and the cumulative operating hours on log-log paper follows a straight line, the plot is known as	a Weibull plot	a normal plot	a Duane plot	an exponential plot	an exponential plot
Which of the following calculations is used to determine the overall tolerance for a serial combination of components?	The average of the tolerances	The maximum of the tolerances	The sum of the tolerances	The square root of the sum of the squares of the tolerances	The square root of the sum of the squares of the tolerances
Which of the following statements is true about maintainability?	It should be initiated during the logistics review.	It should be initiated during the design stage.	It is primarily a field service issue.	It is primarily a contractual requirement.	It should be initiated during the design stage.
A fault tree analysis (FTA) differs from a failure mode and effects analysis (FMEA) in that an FTA	starts by considering individual or combined lower-level failures first	starts by considering system failure effects or top events first	does not take into account human factors such as incorrect operation	is typically used in conjunction with a cause and effect diagram	starts by considering system failure effects or top events first

Which of the following practices represents the principle of preventive maintenance?	Inspect periodically and repair or replace parts as necessary	When parts fail, replace with good ones.	When parts fail, repair the failed parts.	Replace all parts on a regularly scheduled basis.	Replace all parts on a regularly scheduled basis.
One hundred units are subjected to a reliability test with duration 500 clock hours. During the test, 2 failures occur, at T_1 = 110 hours and at T_2 = 300 hours. The failed units are not replaced. On the basis of this sample, the one-sided 95% lower confidence limit of reliability of these units for a mission of 600 hours is	0.858	0.926	0.976	0.988	0.858
Which of the following types of data is best to estimate the life of a product?	Time-to-failure	Time-to-repair	Failure modes	Failure criticality	Time-to-failure

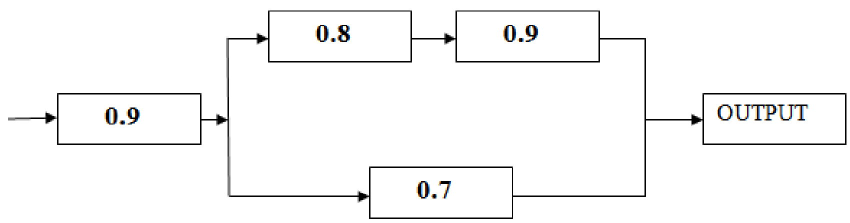
2 MARKS

1. What is the objective of life testing?
2. Estimate the three outcome data based on failure data analysis.
3. Classify the two other equivalent terminologies for Failure rate.
4. Examine the term Probability of Survival.
5. Formulate the term MTBF in Failure data analysis.
6. Formulate the term MTBF in Failure data analysis.
7. Differentiate between MTTF and MTBF.
8. What is meant by parallel configuration?

14 MARKS

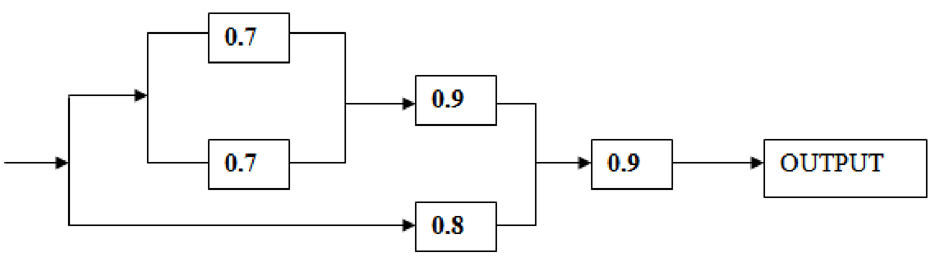
1.What is meant by Reliability? State and explain the factors that focus Reliability.

2.Four elements of a system are connected as shown in figure, which also indicates the reliability of each element. Calculate the system reliability.



3. Explain in detail MTBF and MTTF in reliability?

4. Five elements of a system are connected as shown in figure, which also indicates the reliability of each element. Calculate the system reliability.



5. In a survival test conducted on 100 card board boxes for their strength under impact loading, the following results were obtained given below For this case how will you define f_d, failure rate and reliability? Tabulate these quantities.

No. of impacts	0	20	22	24	26	29	32	35	37	40
No. of boxes failed	0	7	10	15	14	15	13	13	8	5

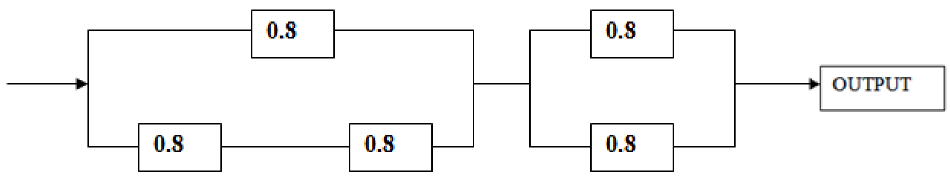
6. Explain the different types of availability depending on the time elements.

7.What is meant by System Reliability? Explain Series, Parallel and Mixed configuration.

8. Write short notes on the following.

MTTR, MTBF, Maintainability, Availability

9. Five elements of a system are connected as shown in figure 1, which also indicates the reliability of each element. Calculate the system reliability.



UNIT V QUALITY AND RELIABLITY

REDUNDANCY TECHNIQUES IN SYSTEM DESIGN

"Redundancy is the provision of alternative means or parallel paths in a system for accomplishing a given task such that all means must fail before causing a system failure". System reliability and mean life can be increased by additional means by applying redundancy at various levels. The different approaches of introducing redundancy in the system are:

1. A duplicate path is provided for the entire system itself which is known as *system or unit redundancy.*
2. A redundant path is provided for each component individually which is called *component redundancy.*
3. In the third approach, the weak components should be identified and strengthened for reliability.
4. In the last approach, a mix of the above techniques is used depending upon the reliability requirements and configuration of the system which is known as *mixed redundancy*.

The application of a particular technique depends upon many factors, for example the weight, size, initial cost and operating characteristics of components or systems. Particularly in electrical and electronic systems redundancy use at the component level introduces certain deviation in operating characteristics of the main systems. Particular attention should be given to such systems.

Redundancy can either be 'active' in which case all redundant elements operate simultaneously in performing the same function or 'standby' in which the duplicate element is switched into service when a primary element falls.

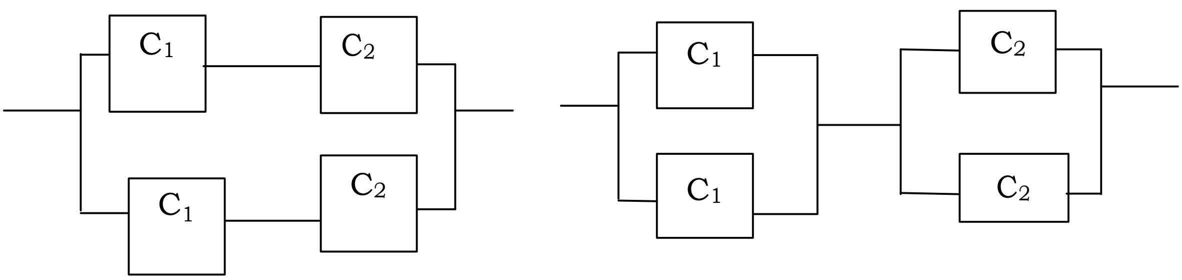
Component Versus Unit Redundancy

Duplication at the unit level is easy rather than at the component level. But higher reliability is achieved through component redundancy than unit redundancy.

Let us consider a two-component series system. Redundancy can be applied in two ways as shown in Fig. 3.15. Assuming that the units are statistically independent and identical at each level, the reliability of the system with unit-redundancy is

$$R_u = 1-(1-p_1p_2)(1-p_1p_2)$$

$$= 2p_1p_2 - p_1^2\, p_2^2 \qquad \text{.........(3.65)}$$



(a) Unit redundancy (b) Component redundancy

Fig. 3.16 Two-element active redundant systems:

Where the reliabilities of components C_1 and C_2 are p_1 and p_2. In case of component redundancy, the reliability is

$$R_c = [1-(1-p_1)^2]\ [1-(1-p_2)^2]$$

$$= 4p_1p_2 + p_1^2 p_2^2 - 2p_1^2 p_2^2 \qquad \text{........(3.66)}$$

If we assume $p_1 = p_2$ for simplicity sake , we obtain

$$R_u = 2p^2 - p^4$$

$$R_c = p^2(2-p)^2 \qquad \text{........(3.67)}$$

Then,

$$R_c - R_u = p^2[(2-p)^2 - (2-p^2)]$$

$$= p^2(2-4p+2p^2)$$

$$=2p^2(1-P)^2 \qquad \text{........ (3.68)}$$

It is obvious from Equation (6.1) that R_c - $R_u > 0$ for 0 < p < 1 and R_c - $R_u = 0$ *for* $p = 1$ which proves that the redundancy at the component level is more enhanced than redundancy at the unit level to the level as far as reliability is concerned. This is also correct even if the primary components of the system are nonidentical.

This analysis can be extended to a more wider case where the unit consists of *n* components in series. Suppose that *m-1* components are arranged in parallel at each stage, the system reliability would be

$$R_c = [1-(1-p)^m]^n \qquad \text{...... (3.70)}$$

In unit redundancy, suppose *m-1* units are added across the primary unit, the reliability is

$$R_u = 1-[1-p^n]^m \qquad \text{........ (3.71)}$$

It shows that a series system which has five redundant components has a greater reliability than a series system which has three unit redundant components.

Another important redundancy technique is to use *partial redundancy* popularly known as *k-out-of-m* system becomes a series structure when *k=m* and a parallel structure when *k=1*. Consider a simple 2-out-of-3 system. This system has, physically three components in parallel and the system is successful as long any two of them are working. In this case, component redundancy means duplication of each component.

The reliability of system with unit redundancy or no component is

$$R=p^2(3-2p)$$

$$=0.648 \text{ for } p=0.6$$

with unit redundancy this would become

$$R_u = 1-(1-0.648)^2 = 0.876$$

It may be noted that component redundancy change the system into a "2-out-of-6-system" and the net reliability would be

$$R_c = \sum_{i=2}^{6} \binom{6}{i} p^i (1-p)^{6-i}$$

$$=15p^2(1-p)^4 + 20p^3(1-p)^3 + 15p^4(1-p)^2 + 6p^5(1-p) + p^6$$

$=0.959$ for $p=0.6$

Again component redundancy is higher than unit redundancy.

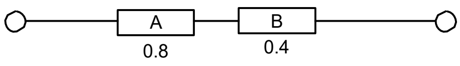
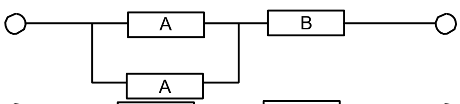
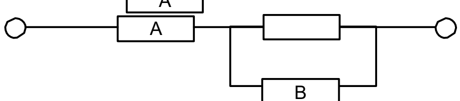
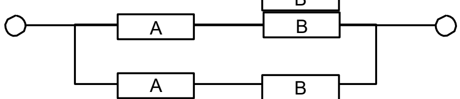
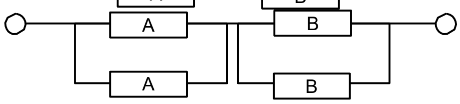
Weakest-Link Technique

For a series structure, reliability is at the most the same as the reliability of the weakest component in the structure. Assume a simple system having two components *A* and *B* in series. Their reliabilities are 0.8 and 0.4 respectively. This system will have a reliability of 0.32 which is much less than 0.4, the reliability of the weakest component. The system reliability can be enhanced by one of the following methods:-

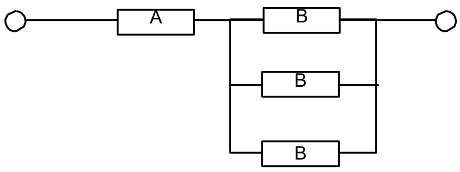
1. Apply redundancy across *B* only
2. Apply redundancy across *A* only.
3. Apply redundancy across both *A* and *B* individually (component redundancy).

Various system configurations and their resultant reliabilities are presented in table 3.1. Types (b) and (c) shows the use of one additional component to improve the reliability. Their reliabilities show that the application of redundancy across weaker equipment results in higher reliability when compared to the redundancy across stronger equipment. When an attempt is made to increase the reliabilities of various configurations of a series structure, it is seen the overall structure reliability is less than the weakest section reliability. So the pay off will not be much when investment is done on a section except the weakest one. If the costs of equipments *A* and *B* are same, then the reliability-cost ratio of type (b) is lower than that of type (c).

Table 3.1 Redundancy Techniques

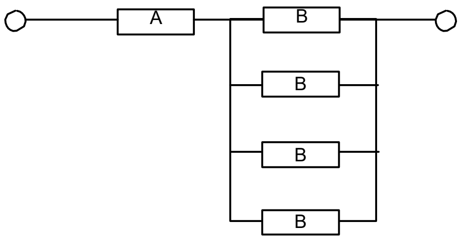
Type	System Configuration $(p_a = 0.8 \;\; and \;\; p_b = 0.4)$	System Reliability
A		$R_a = p_a p_b = 0.32$
B		$R_b = \left[1-(1-p_a)^2\right]p_b = 0.384$
C		$R_c = p_a\left[1-(1-p_b)^2\right] = 0.512$
D		$R_d = 2p_a p_b - p_a^2 p_b^2 = 0.5376$
E		$R_e = p_a p_b (2-p_a)(2-p_b) = 0.6282$

F



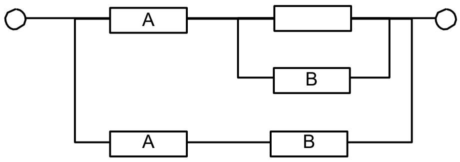
$$R_f = p_a\left[1-(1-p_b)^3\right] = 0.6272$$

G



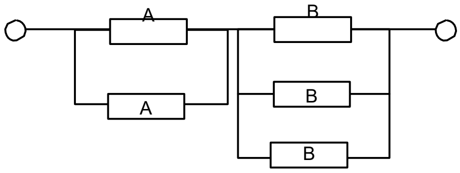
$$R_g = p_a\left[1-(1-p_b)^4\right] = 0.6963$$

H



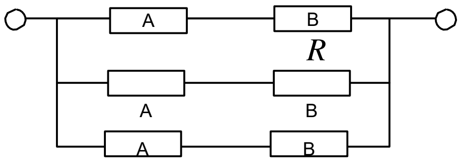
$$R_h = 1-\left[1-p_a p_b(2-p_b)\right]\left[1-p_a p_b\right]$$
$$= 0.6674$$

I



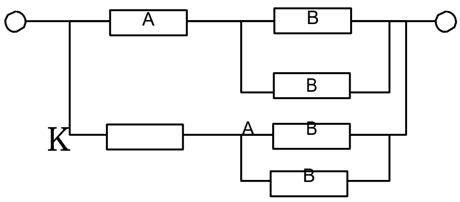
$$R_i = \left[1-(1-p_a)^2\right]\left[1-(1-p_b)^3\right]$$
$$= 0.7526$$

J



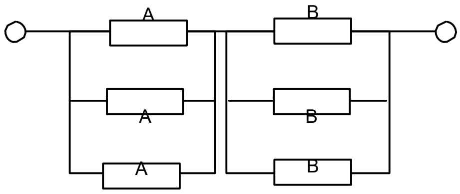
$$R_j = 1-(1-p_a p_b)^3 = 0.6855$$

K



$$R_k = 1-\left[1-p_a p_b(2-p_b)\right]^2$$
$$= 0.7618$$

L



$$R_l = \left[1-(1-p_a)^3\right]\left[1-(1-p_b)^3\right]$$
$$= 0.7777$$

Now the configurations (c), (f) and (g) are considered in which the redundancy is provided across the component *B* only. In (c) the reliability of *B* is increased to 0.512 which is still less than the reliability of *A*. Therefore, it is advisable to improve the reliability of *B* further as shown in (f). It may be noted that the reliability of type (e) is higher than the reliability of type (f) even though both of them employ the same number of equipments. In (f) the reliability of *B* is improved to 0.904 which is now higher than 0.8. Any further improvement should be done in *A.* Compare the reliabilities of types (g) and (i) both having the same number of equipments. Type (h) is superior to type (g).

Mixed Redundancy

Component and unit redundancies are easy to design and simple to implement. But they may not be the best configurations and there might be scope for further enhancement in their reliability-cost ratios. A comparison of types (j), (k) and (l) will demonstrate this. In configuration (k), weak component reliability is enhanced first and then the unit redundancy is applied. Its reliability is more than the reliability of the component redundancy shown in (I). The reliability of (k) can be further increased by providing a link in between two sections.

Stand by Redundancy

All the components and equipments are not suitable for "active redundancy". For example, resistors and capacitors generate design problems when they are placed actively in parallel. If one of two such components which are operating in parallel fails, there will be a change in the circuit constants. Similarly, two electric generators which are operating at different frequencies cannot be placed together in parallel. *Standby redundancy* is used to increase the reliability of such systems.

The failed component or equipment is replaced automatically or physically by its "equivalent" In standby redundancy. The reliability of the operator or sensing and switching mechanism must be considered for such cases.

Suppose in a system, a switch (contact) activated by a feedback sensing and control device is used for that replacement of factory Unit-1 by a standby Unit-2, then the system reliability is given by

$$R = p_1 p_2 p_c + (1 - p_1) p_2 p_c p_s + p_1 (1 - p_2) p_1 p_c \qquad \text{......... (3.72)}$$

Where

p_1 = Unit-1 reliability

p_2 = Unit-2 reliability

p_c = switch reliability

p_s = sensing and control device reliability

p_t = probability that a chance and premature switching may not occur.

REDUNDANCY OPTIMIZATION

Using redundancy, adequately reliable systems can be constructed using less reliable elements. By providing parallel elements at each stage the reliability of a system can be enhanced but the complexity, weight, size, cost etc. of the system increase substantially. So it is essential to maintain the cost and weight of the system to a least and reliability to a highest level when designing a redundant system. For simple parallel systems, the cost and reliability of can be expressed as

C = F(c, m, method of redundancy)

R = G (p, m, method of redundancy).

Where 'p' and 'c' are the reliability and cost and of each element and 'm' is the number of parallel elements.

Thus, the optimal redundancy problem is reduced to the determining the values of m for maximum redundancy for a given 'C' or minimum 'C' for a given 'R'.

The redundancy optimization problem can be solved by using a simple method with cost and volume constraints. The assumptions made in solving the problem are as follows.

(i) The number of redundant components in each stage is a continuous variable and the final solution is obtained by rounding off the solution to integer values.

(ii) The cost, volume/weight of any stage raise linearly with the number of components in that stage.

(iii) Active redundancy is considered.

(iv) The failure of elements is independent of others

The probability of failure of elements because of short circuit is zero.

Problem Formulation

At least one element in each stage must function for the successful operation of a series system. If 'm_i' elements each with unreliability 'q_i' are used at stage 'i' then the reliability of the entire system having 'n' stages is :

$$R=\prod_{i=1}^{n} R_i(m_i) \quad = \prod_{i=1}^{n}\{1-Q_i(m_i)\}$$

$$R=\prod_{i=1}^{n} 1-{q_i}^{m_i} \qquad ....(3.73)$$

and the unreliability is

$$Q=1-\prod_{i=1}^{n} 1-{q_i}^{m_i} \qquad ......(3.74)$$

When ${q_i}^{mi}$ is small, their product may be neglected.

Then

$$R=1-\sum_{i=1}^{n} {q_i}^{m_i} \qquad .... (3.75)$$

and $Q=\sum_{i=1}^{n} {q_i}^{m_i}$ ....(3.76)

The total reliability in a multi-constraint problem is limited by the following constraints.

$$\sum_{i=1}^{n} c_{ij} m_i \leq C_j \quad j=1,2,........K \qquad ....(3.77)$$

Where 'C_{ij}' is the j^{th} resource used at i^{th} stage. The constraints might represent total cost (C_1), total volume (C_2), etc. Equation (3.75) and (3.76) represent the objective functions and the equation (3.77) represent the constraint function.

Now the problem can be stated as

Find Vector M = (m_1, m_2........ m_n) which will

Maximize $\quad R=\prod_{i=1}^{n} 1-{q_i}^{m_i}$

Subject to $m_i > 1 \quad i = 1, 2, ... n$

$C_{ij} > 0 \quad j = 1, 2, ... k$

$\Sigma \quad c_{ij}\, m_i < C_j$

Computational Procedure

Since m_i is assumed to be a continuous variable, the maximum R can be found by differentiating equation (1) with respect to m_i and equality to zero.

From (1) $\log R = \sum_{i=1}^{n} \log (1 - q_i^{m_i})$

log R= $\prod_{i=1}^{n} \log (1 - q_i^{m_i})$ Differentiating, we get $\frac{1}{R}\frac{dR}{dm_i}$ = $(q_i^{mi} \log q_i)/(1 - q_i^{mi})$

$$\frac{dR}{dm_i} = (- q_i^{mi}\log(q_i))/(1 - q_i^{m_i}) * \prod_{i=1}^{n} 1 - q_i^{m_i} \qquad (3.78)$$

Neglecting the products of q^m, equation (3.78) can be written as

$$\frac{dR}{dm_i} = - q_i^{mi} \log (q_i) \quad i = 1, 2 \ldots n \qquad \ldots\ldots (3.79)$$

The optimal value of $m_i = m_i^*$ i s the solution of

$$\frac{dR}{dm_i} = - q_i^{mi} \log (q_i) = 0$$

$$q_i^{mi\,*} \log (q_i) = q_2^{mi\,*}\log (q_i) = q_n^{\,m_n^*} \log (q_i) \qquad \ldots\ldots(3.80)$$

Assuming that m_1 * is known, other values m_2 *, m_3 *.....m_n * can be determined by equation (3.80)

$$m_I^* = \frac{\log \frac{E * H}{G}}{G} \qquad i=2,3,\ldots\ldots..n \qquad \ldots.(3.81)$$

Where $E = q_1^{\,mi}$

$H = \log (q_1)$

$G = \log (q_i)$

The detailed procedure for computing the vector 'M' is given below:

Step 1: Choose the initial value m_1 *

Step 2: Find m_1 * i = 2, 3, ……….n

Step 3: Check cost and volume constraints. If they are not violated go to otherwise go to step 5.

Step 4: Give a small positive increment ‘dm’ to m_1^*, go to step 2

Step 5: Round of m_i^* such that the constraints are not violated.

Step 6: Calculate the system reliability, cost and volume of this systems.

3.10 Conclusions

The basic analytical functions in reliability engineering along with construction of reliability Bathtub curve are presented in this chapter. Various types of system configuration and their reliability functions are provided. The problem formulation and computational procedure for redundancy optimization of series- parallel system with cost and volume/weight constraints is presented.

MCQ

QUESTIONS	Option 1	Option 2	Option 3	Option 4	ANSWERS
Pareto Chart is plotted between	Number Failures vs time	Frequency of failures vs causative factors	Probability of survival vs time	Probability of failure vs time	Frequency of failures vs causative factors
Which of the following is not an assumption underlying testing and measurement?	Various approaches to measuring aspects of the same thing can be useful	Error is rarely present in the measurement process	Present-day behavior predicts future behavior	Testing and assessment benefit society	Error is rarely present in the measurement process
Systematic error is associated with	Reliability	Validity	fludity	quality	Validity
Which of the following is a type of criterion–related validity evidence?	Concurrent evidence	Predictive evidence	Internal consistency	Both a and b are correct answers	Both a and b are correct answers
If a test measures a single construct then	The items should correlate with the total score	The items should not correlate with the total score	The test should not correlate with other measures of the same construct	There must be a reliable alternative form	The items should correlate with the total score
Which scale is the simplest form of measurement	Nominal	Ordinal	Interval	Ratio	Nominal
Which is the process of gathering evidence supporting inferences based test scores	Validation	Validity	Reliability	Prediction	Validation
Which of the following is not a type of reliability	Test-retest	Split-half	Content	Internal consistency	Content
16. Which of the following types of reliability refers to the consistency of test scores over time? reliability	Equivalent forms	Split-half reliability	Test-retest reliability	Inter-scorer reliability	Test-retest reliability
Reliability is most simply known as which of the following	Consistency or stability	Appropriateness of interpretations on the basis of test scores	Ways in which people are the same	A rank order of participants on some characteristic	Consistency or stability
__________ refers to how well the	Construct validity	Criterion-related	Content validity	Face validity	Content validity

particular sample of behaviors used to measure a characteristic reflects the entire domain of behaviors that constitutes that characteristic	evidence	validity evidence	evidence	evidence	evidence
An ordinal scale is:	The simplest form of measurement	A rank-order scale of measurement	A scale with equal intervals between adjacent numbers	A scale with an absolute zero point	A rank-order scale of measurement
If a baseball coach calculates batting averages, what scale would be used	Interval scale	Ratio scale	Nominal scale	Ordinal scale	Ratio scale
______ tests focus on information acquired through the informal learning that goes on in life.	Personality	Achievement	Aptitude	Intelligence	Aptitude
Which of the following is most clearly an example of a psychological trait	Anxiety enduring for months or years	Anxiety over just seeing a spider	Shyness when meeting a stranger for the first time	Depression caused by the loss of a ball game	Anxiety enduring for months or years
Process control is carried out	before production	during production	after production control	All of the above	during production
Low cost, higher volume items requires	no inspection	little inspection	intensive inspection	100% inspection	little inspection
High cost, low volume items requires	no inspection	little inspection	intensive inspection	100% inspection	intensive inspection
The mean of sampling distribution is	less than mean of process distribution	more than mean of process distribution	equal to mean of process distribution	any of the above	equal to mean of process distribution
The percent of the sample means will have values that are within ±3 standard deviations of the distribution mean is	99.7	96.5	96.5	95.5	99.7
The dividing lines between random and non random deviations from mean of the distribution are known as	upper control limit	lower control limit	control limits	two sigma limits	control limits
The chart used to monitor variable is	Range Chart	P-chart	C-Chart	All of the above	Range chart

The chart used to monitor attributes is	Range Chart	P-chart	C-Chart	All of the above	P-chart
Central tendency of a process is monitored in	Range Chart	P-chart	C-Chart	Mean chart	Mean chart
Dispersion of a process in monitored in	Range Chart	P-chart	C-Chart	Mean chart	Range chart
The control chart used for the fraction of defective items in a sample is	Range Chart	P-chart	C-Chart	Mean chart	P-chart
The process capability is calculated as	(USL-LSL)/3σ	(USL+LSL)/3σ	(USL-LSL)/6σ	(USL+LSL)/6σ	(USL-LSL)/6σ
A six sigma process has defect level below ______ defects per million opportunities	3.4	4.5	5.6	6.7	3.4
The control chart used for the number of defects per unit is	Range Chart	P-chart	C-Chart	Mean chart	C-Chart
When evaluating tests and assessments, “reliability” refers to asking ourselves which of the following questions?	Does it measure what it is supposed to measure?	Are there ways to avoid subjective judgments when measuring something	Does it give consistent results	Does it give consistent results	Does it give consistent results

2 MARKS

1. Outline the merits of Pareto analysis
2. What is the objective of product data analysis?
3. What is meant by series configuration?
4. List out the Reliability improvements techniques–
5. List out the use of Pareto analysis
6. What is mean by design for reliability
7. Define redundancy unit
8. Define standby redundancy
9. Define optimization in reliability
10. Define Maintenance.

14 MARKS

1. What is meant by Redundancy? Explain Element Redundancy in detail.
2. Discuss about Optimization in Reliability with a suitable example.
3. What is meant by Redundancy? Explain Unit Redundancy in detail.
4. What is Reliability engineering? How does reliability engineering impact product quality?
5. Discuss about the five approaches for Effective Product optimization.
6. Explain the different stages that involved in Product Life Cycle.
7. Discuss on Pareto analysis and its use in reliability Improvements.