17BEEC702B OPTICAL COMMUNICATION

OBJECTIVES:

- To facilitate the knowledge about optical fiber sources and transmission techniques
- To enrich the idea of optical fiber networks algorithm such as SONET/SDH.
- To explore the trends of optical fiber measurement systems.

INTENDED OUTCOMES:

Upon completion of the course, students will be able to:

- Discuss the various optical fiber modes, configurations and various signal degradation factors associated with optical fiber.
- Explain the various optical sources and optical detectors and their use in the optical communication system.
- Analyze the digital transmission and its associated parameters on system performance.

UNIT I INTRODUCTION TO OPTICAL FIBERS

Evolution of fiber optic system- Element of an Optical Fiber Transmission link- Ra y Optics-Optical Fiber Modes and Configurations –Mode theory of Circular Wave guides- Overview of Modes-Key Modal concepts- Linearly Polarized Modes –Single Mode Fibers-Graded Index fiber structure.

UNIT II SIGNAL DEGRADATION OPTICAL FIBERS

Attenuation – Absorption losses, scattering losses, Bending Losses, Core and Cladding losses, Signal Distortion in Optical Wave Guides-Information Capacity determination –Group Delay-Material Dispersion, Wave guide Dispersion, Signal distortion in SM fibers-Polarization Mode dispersion, Intermodal dispersion, Pulse Broadening in GI fibers-Mode Coupling –Design Optimization of SM fibers-RI profile and cut-off wavelength.

UNIT III FIBER OPTICAL SOURCES AND COUPLING

Direct and indirect Band gap materials-LED structures –Light source materials –Quantum efficiency and LED power, Modulation of a LED, lasers Diodes-Modes and Threshold condition –Rate equations –External Quantum efficiency –Resonant frequencies –Laser Diodes, Temperature effects, Introduction to Quantum laser, Fiber amplifiers- Power Launching and coupling, Lancing schemes, Fibre –to- Fibre joints, Fibre splicing – Energy efficiency of LAS ER.

UNIT IV FIBER OPTICAL RECEIVERS

PIN and APD diodes –Photo detector noise, SNR, Detector Response time, Avalanche Multiplication Noise –Comparison of Photo detectors –

Fundamental Receiver Operation – preamplifiers, Error Sources –Receiver Configuration –Probability of Error – Quantum Limit.

UNIT V DIGITAL TRANSMISSION SYSTEM

Point-to-Point links System considerations –Link Power budget –Rise - time budget –Noise Effects on System Performance-Operational Principles of WDM, Solutions-Erbium-doped Amplifiers. Basic on concepts of SONET/SDH Network.

TEXTBOOK:

S.NO.	Author(s) Name	Title of the book	Publisher	Year of the publication
1.	Gerd Keiser	Optical Fiber Communication 4th Edition	McGraw Hill International,	2010
2.	Senior.J	Optical Communication Principles and Practice 2nd Edition	Prentice Hall of India, New Delhi	2007

REFERENCES:

S.NO.	Author(s) Name	Title of the book	Publisher	Year of the publication
1.	Gower.J	Optical Communication System	Prentice Hall of India, NewDelhi	2001
2	Ramaswami, Sivarajan and Sasaki	Optical Networks	Morgan Kaufmann Publishers	2009

UNIT I INTRODUCTION TO OPTICAL FIBERS

An optical fiber is a cylindrical dielectric waveguide made of low-loss materials such as silica glass. It has a central core in which the light is guided, embedded in an outer **cladding** of slightly lower refractive index (Fig. 8.0-1). Light rays incident on the core—cladding boundary at angles greater than the critical angle undergo total internal reflection and are guided through the core without refraction. **Rays** of greater inclination to the fiber axis lose part of their power into the cladding at each reflection and are not guided.

As a result of recent technological advances in fabrication, light can be guided through 1 km of glass fiber with a loss as low as = 0.16 dB (= 3.6 %). Optical fibers are replacing copper coaxial cables as the preferred transmission medium for electromagnetic waves, thereby revolutionizing terrestrial communications. Applications range from long-distance telephone and data communications to computer communications in a local area network.

In this chapter we introduce the principles of light transmission in optical fibers. These principles are essentially the same as those that apply in planar dielectric waveguides (Chap. 7), except for the cylindrical geometry. In both types of waveguide light propagates in the form of modes. Each mode travels along the axis of the waveguide with a distinct propagation constant and group velocity, maintaining its transverse spatial distribution and its polarization. In planar waveguides, we found that each mode was the sum of the multiple reflections of a TEM wave bouncing within the slab in the direction of an optical ray at a certain bounce angle. This approach is approximately applicable to cylindrical waveguides as well. When the core diameter is small, only a single mode is permitted and the **fiber** is said to be **a single-mode fiber.** Fibers with large core diameters are **multimode fibers.**

One of the difficulties associated with light propagation in multimode fibers arises from the diflerences among the group velocities of the modes. This results in a variety of travel times so that light pulses are broadened as they travel through the fiber. This effect, called **modal dispersion,** limits the speed at which adjacent pulses can be sent without overlapping and therefore the speed at which a fiber-optic communication system can operate.

Modal dispersion can be reduced by grading the refractive index of the fiber core from a maximum value at its center to a minimum value at the core—cladding boundary. The fiber is then called a **graded-index** fiber, whereas conventional fibers

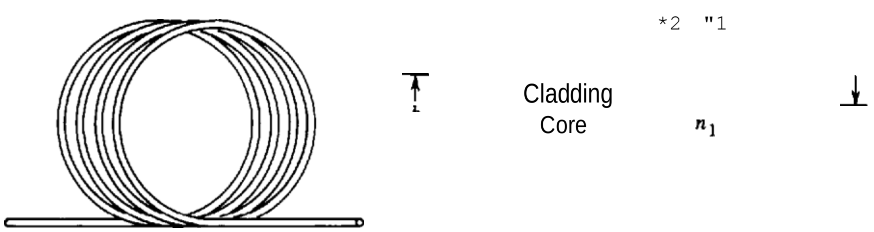


Figure 8.0-1 An optical fiber is a cylindrical dielectric waveguide.

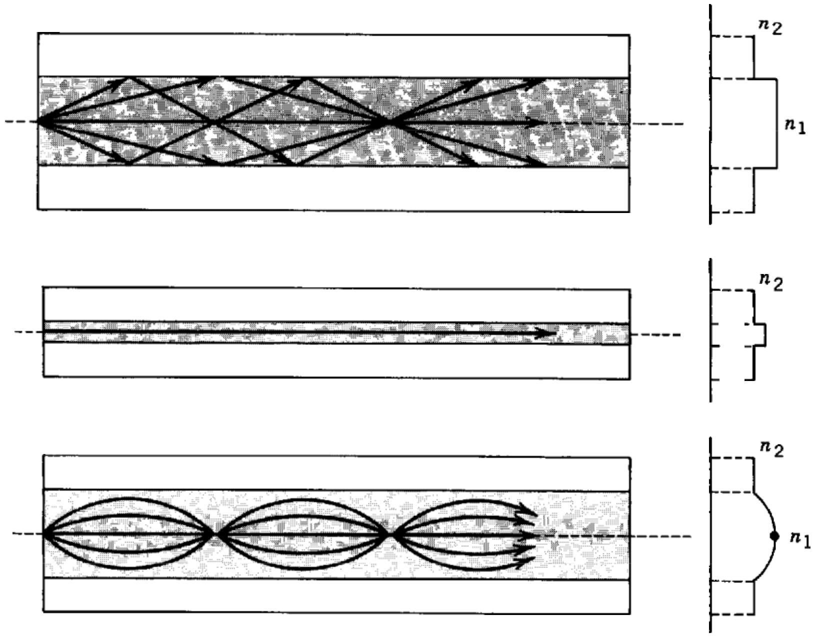


Figure 8.0•2 Geometry, refractive-index profile, and typical rays in: *a)* a multimode step-index fiber, *(b)* a single-mode step-index fiber, and (c) a multimode graded-index fiber.

with constant refractive indices in the core and the cladding are called step•index fibers. In a graded-index fiber the velocity increases with distance from the core axis (since the refractive index decreases). Although rays of greater inclination to the fiber axis must travel farther, they travel faster, so that the travel times of the different rays are equalized. Optical fibers are therefore classified as step-index or graded-index, and multimode or single-mode, as illustrated in Fig. 8.0-2.

This chapter emphasizes the nature of optical modes and their group velocities in step-index and graded-index fibers. These topics are presented in Secs. 8.1 and 8.2, respectively. The optical properties of the fiber material (which is usually fused silica), incluiling its attenuation and the effects tit material, modal, and waveguide dispersion on the transmission of light pulses, are discussed in Sec. 8.3. Optical fibers are revisited in Chap. 22, which is devoted to their use in lightwave communication systems.

8.1 STEP-INDEX FIBERS

A step-index fiber is a cylindrical dielectric waveguide specified by its core and cladding refractive indices, n, and n $_2$, and the radii *a* and *b* (see Fig. 8.0-1). Examples of standard core and cladding diameters 2 *a/ 2b* are 8/125, 50/125, 62.5/125, 85/125, 100/140 (units of pm). The refractive indices differ only slightly, so that the fractional refractive-index change

$$\Delta = \frac{n_1 - n_2}{} \tag{8.1-1}$$

is small (fi 1).

Almost all fibers currently used in optical communication systems are made of fused silica glass (SiO_2) of high chemical purity. Slight changes in the refractive index are made by the addition of low concentrations of doping materials (titanium, germanium, or boron, for example). The refractive index *n* is in the range from 1.44 to 1.46, depending on thc wavelength, and A typically lies between 0.001 and 0.02.

A. Guided Rays

An optical ray is guided by total internal reflections within the fiber core if its angle of incidence on the core—cladding boundary is greater than the critical angle 8 sin ^{I}(n t/n,), and remains so as the ray bounces.

Meridional Ray'z

The guiding condition is simple to see for meridional rays (rays in planes passing through the fiber axis), as illustrated in Fig. 8.1-1. These rays intersect the fiber axis and reflect in the same plane without changing their angle of incidence, as if they were in a planar waveguide. Meridional rays are guided if their angle 8 with the fiber axis is smaller than the complement of the critical angle 8 = w/2 — 8, = cos '(ii$_2$/n t). Since *n* = n $_2$, 8 is usually small and the guided rays are approximately paraxial.

Figure 8.I -1 The trajectory of a meridional ray lies in a plane passing through the fiber axis. The ray is guided if 8 < 8 - cos ’(ii /n $_2$).

Skewed nayc

An arbitrary ray is identified by its plane of incidence, a plane parallel to the fiber axis and passing through the ray, and by the angle with that axis, as illustrated in Fig. 8.1-2. The plane of incidence intersects the core—cladding cylindrical boundary at an angle d› with the normal to the boundary and lies at a distance ft from the fiber axis. The ray is identified by its angle 8 with the hber axis and by the angle d› of its plane. When d› 0 *R* 0) the ray is said to be skewed. For meridional rays d› = 0 and *R* — 0.

A skewed ray reflects repeatedly into planes that make the same angle d› with the core—cladding boundary, and follows a helical trajectory confined within a cylindrical shell of radii fi and *a*, as illustrated in Fig. 8.1-2. The projection of the trajectory onto the transverse (x—y) plane is a regular polygon, not necessarily closed. lt can be shown that the condition for a skewed ray to always undergo total internal reflection is that its angle 8 with the z axis be smaller than 8 .

6t*umericel Aperture*

A ray incident from air into the fiber becomes a guided ray if upon refraction into the core it makes an angle 8 with the fiber axis smaller than 8,. Applying Snell’s law at the air—core boundary, the angle 8, in air corresponding to 8, in the core is given by the relation 1 sin 8 = ri; sin 8,, from which (see Fig. 8.1-3 and Exercise 1.2-5) sin 8, = n (1 — cos^{2}8)"° = n,[1 — (n t/ri $_1$)^2]^1 = (n{ n$_2$)$^{l'}$ ^{2}. Therefore

$$8, = \sin^{i} NA, \tag{8.1-2}$$

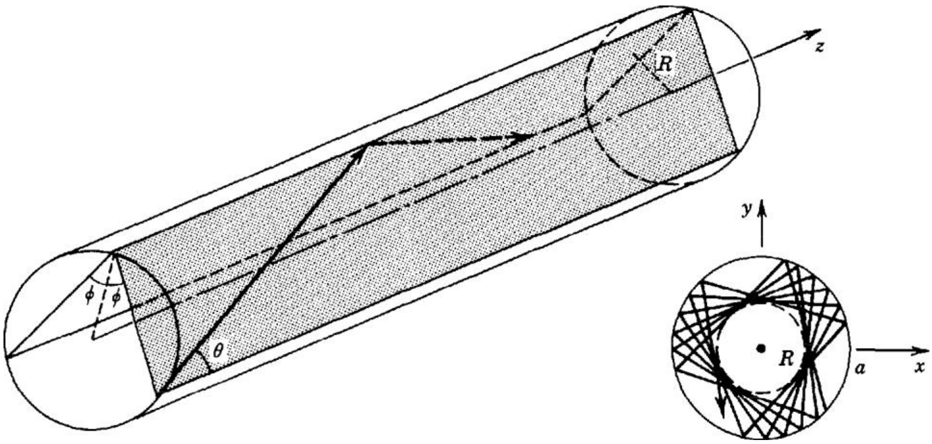


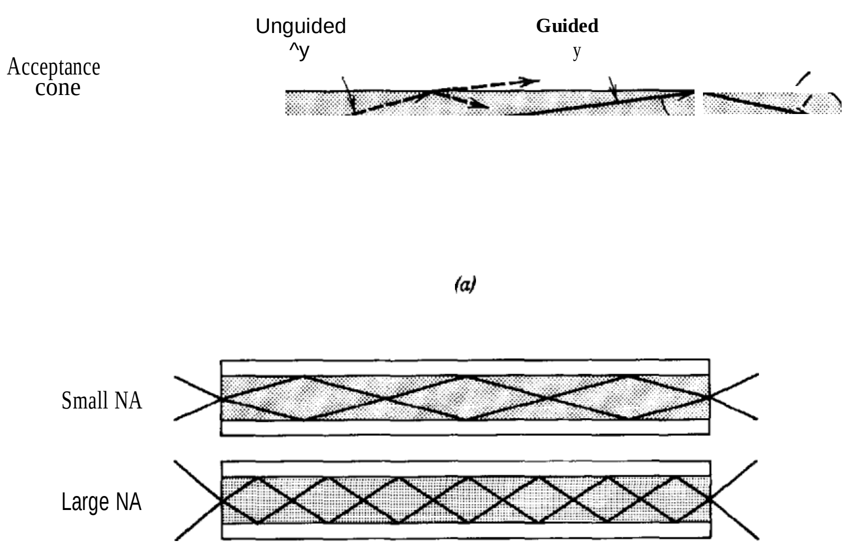
Figure 8.1-2 A skewed ray lies in a plane offset from the fiber axis by a distance fi. The ray is identified by the angles 8 and Q. It follows a helical trajectory confined within a cylindrical shell of radii *R* and a. The projection of the ray on the transverse plane is a regular polygon that is not necessarily closed.

where

NA – (2 2)” 2 = n (2 fi)” 2

(8.1-3)
Numerical Aperture

is the numerical aperture of the fiber. Thus 8, is the acceptance angle of the fiber. It



(b)

Figure 8.1•3 *a)* The acceptance angle 8, of a fiber. Rays within the acceptance cone are guided by total internal reflection. The numerical aperture NA = sin 8,. *tb)* The light-gathering capacity of a large NA fiber is greater than that of a small NA fiber. The angles 8, and 8 are typically quite small; they are exaggerated here for clarity.

determines the cone of external rays that are guided by the fiber. Rays incident at angles greater than 8 are retracted into the fiber but are guided only for a short distance. The numerical aperture therefore describes the light-gathering capacity of the fiber.

When the guided rays arrive at the other end of the fiber, they are refracted into a cone of angle 8,. Thus the acceptance angle is a crucial parameter for the design of systems for coupling light into or out of the fiber.

EXAMPLE 8.1-1. ***ci dded and Uncladded Fibers.*** **In a** silica glass fiber with ri = 1.46 and A = (n — n $_2$)/n = 0.01, the complementary critical angle 8 — cos *(n* $_2$/n t) - S.1°, and the acceptance angle 8, = 11.9°, corresponding to a numerical aperture NA = 0.206. By comparison, an uncladded silica glass fiber (n; = 1.46, •2 ' 1) has 8 = 46.8°, d, = 90°, and NA = 1. Rays incident from a// directions are guided by the uncladded fiber since they reflect within a cone of angle 8 = 46.8° inside the core. Although its light-gathering capacity is high, the uncladded fiber is not a suitable optical waveguide because of the large number of modes it supports, as will be shown subsequently.

B. Guided Waves

In this section we examine the propagation of monochromatic light in step-index fibers using electromagnetic thcory. We aim at dctermining the electric and magnetic fields of guided waves that satisfy Maxwell's equations and the boundary conditions imposed by the cylindrical dielectric core and cladding. As in all waveguides, there are certain special solutions, called modes (see Appendix C), each of which has a distinct propagation constant, a characteristic field distribution in thc transverse plane, and two independent polarization states.

Spatial Distributions

Each of the components of the electric and magnetic fields must satisfy the Helmholtz equation, *4 U + n k2 U* — 0, **where** n = n in the core *r < a)* and n — *n* $_2$ in the cladding *(r > a)* and *k* — 2r/â (see Sec. 5.3). **We assume** that the radius *b* of the cladding is sufficiently large that it can safely **be assumed** to be infinite when examining guided light in the core and near the core—cladding boundary. In a cylindrical coordinate system (see Fig. 8.1-4) the Helmholtz equation is

$$\frac{b^2U}{\partial r^2} + \frac{1}{r}\frac{EU}{br} + \frac{1}{r^2}\frac{d^2t/}{\partial \phi^2} + \frac{\tilde{n}^2 U}{\partial z^2} + {}^2\pounds\ t/ = 0, \qquad (8.1\text{-}4)$$

jbh2 14:27:59

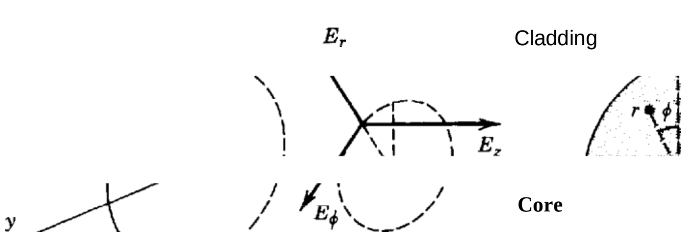


Figure 8.4 -4 Cylindrical coordinate system.

where **the complex amplitude** *U* — *U r, ‹f,* **z) represents any of** the Cartesian components of the electric or magnetic fields or the axial components *E* and *H* in cylindrical coordinates.

We are interested in solutions that take the form of waves traveling in the z direction with a propagation constant Q, so that the z dependence of *U* is of the form *e !!‘.* Since *U* must be a periodic function of the angle Q with period 2s, we assume that the dependence on d› is harmonic, *e°!'**, where I is an integer. Substituting

$$U(r,‹f, z) - u(r) e°!!"e°!!‘, \qquad 1 — 0, + 1, + 2, ... , \tag{8.1-5}$$

into (8.1-4), an ordinary differential equation for *u{r)* is obtained:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + \left(n^2 k_o^2 - Q - \frac{l'}{r^2}\right) ii = 0. \tag{8.1-6}$$

As in Sec. 7.2B, the wave is guided (or bound) if the propagation constant is smaller than the wavenumber in the core (JS < *n k p)* and greater than the wavenumber in the cladding (9 > ri 2 k,). It is therefore convenient to define

$$k_T^2 = n_1^2 k_o^2 - \beta^2 \tag{8.1-7a}$$

and

$$\gamma^2 = \beta^2 - n_2^2 k_o^2, \tag{8.1—7b}$$

so that for guided waves kJ and y 2 are positive and *k T* and y are real. Equation (8.1-6) may then be written in the core and cladding separately:

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} + \left(k_T^2 - \frac{l^2}{r^2}\right) u — 0, \qquad r < a \text{ (core)}, \tag{8.1-8a}$$

$$\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \left(\gamma^2 + \frac{l^2}{r^2}\right) u = 0, \qquad r > a \text{ (cladding)}. \tag{8.1-8b}$$

Equations (8.1-8) are well-known diflerential equations whose solutions are the family of Bessel functions. Excluding functions that approach at *r* — 0 in the core or at r —• in the cladding, we obtain the bounded solutions:

$$u(r) \propto \begin{cases} I(k_T r), & r < a \text{ (core)} \\ K(\gamma r), & r > a \text{ (cladding)}, \end{cases} \tag{8.1-9}$$

where / (z) is the Bessel function of the first kind and order *1,* and *K x)* is the modified Bessel function of the **second** kind and order /. The function It(z) oscillates like the sine or cosine functions but with a decaying amplitude. In the limit x » 1,

$$J_l(x) \approx \left(\frac{2}{\pi x}\right)^{1/2} \cos\left[z - (/ + ')\right], \qquad z >> t. \tag{8.1-10a}$$

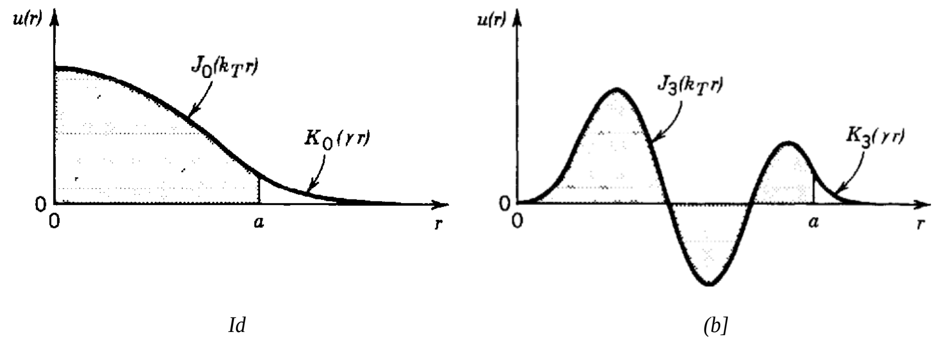


Figure 8.1-5 Examples of the radial distribution *u‹r 1* given by (8.1-9) for (a) I — 0 and *(b)* = 3. The shaded areas represent the fiber core and the unshaded areas the cladding. The parameters k_T and y and the two proportionality constants in (8.1-9) have been selected such that *u(r)* is continuous and has a continuous derivative at r = a. Larger values of *k z* and 2 lead to a greater number of oscillations in u(r).

In the same limit, *K (x)* decays with increasing z at an exponential rate,

$$K(x) - \left(\frac{\pi}{2}\right)^{1/2} \left(1 + \frac{4l^2 - 1}{8.}\right) \exp(—z), \qquad z >> 1. \qquad (8.1\text{-}10b)$$

Two examples of the radial distribution *u(r)* are shown in Fig. 8.1-5.

The parameters *k* and y determine the rate of change of *u(r)* in the core and in the cladding, respectively. A large value of *k* means faster oscillation of the radial distribution in the core. A large value of y means faster decay and smaller penetration of the wave into the cladding. As can be seen from (8.1-7), the sum of the squares of *k* and y is a constant,

$$k_I + y^2 = (n\$ — n_2^2)k_o^2 - NA^2 \qquad \textbf{(8.1-11)}$$

so that as *k* increases, y decreases and the field penetrates deeper into the cladding. As k_T exceeds NA *k p,* becomes imaginary and the wave ceases to be bound to the core.

the U **Prrameter**

It is convenient to normalize k_T f nd y by defining

$$X — k_T a, \qquad Y — pa. \qquad (8.1\text{-}12)$$

In view of (8.1-11),

$$X^2 + Y' - V^2, \qquad (8.1\text{-}13)$$

where *V* -- NA *k,a,* from which

$$\boxed{V = 2\pi \frac{}{\lambda} \text{NA.}} \qquad (8.1\text{-}14)$$

V' Parameter

As we shall see shortly, *V* is an important parameter that governs the number of modes of the fiber **and** their propagation constants. It is called the **fiber parameter** or V **parameter.** It is important to remember that for the wave to be guided, A must be smaller than *V.*

Modes

We now consider the boundary conditions. We begin by writing the axial components of the electric- and magnetic-field complex amplitudes £, and *H* in the form of (8.1-5). The condition that these components must be continuous at the core—cladding boundary r = *a* establishes a relation between the coefficients of proportionality in (8.1-9), so that we have only one unknown for 6, and one unknown for *H* . With the help of Maxwell's equations, yo e,ri ² E = V x H and —yniy,H = V x E, the remaining four components ñg, *H* , *E* , and H_r are determined **in terms** of 6, and ft,. Continuity of *E* and *H* at r = a yields two more equations. One equation relates the two unknown coefficients of proportionality in *E* and *H ;* the other equation gives a condition that the propagation constant Q must satisfy. This condition, called the **characteristic equation** or **dispersion relation, is** an equation for Q with the ratio n /âp And the fiber indices n t, ii 2 As known parameters.

For each azimuthal index /, the characteristic equation has multiple solutions yielding discrete propagation constants ;St,„ m — I , 2, ... , each solution representing a mode. The corresponding values of k_T and y, which govern the spatial distributions in the core and in the cladding, respectively, are determined by use of (8.1-7) and are denoted *k* and y p. A mode is therefore described by the indices / and *m* characterizing its azimuthal and radial distributions, respectively. The function *u(r)* depends on both / and bit; = 0 corresponds to meridional rays. There are two independent configurations of the E and H vectors for each mode, corresponding to two states of polarization. The classification and labeling of these configurations are generally quite involved (see specialized books in the reading list for more details).

C/iaracter/sf/c *Equation for the Weakly Guiding Fiber*

Most fibers are weakly guiding (i.e., $n = n_2$ or A in 1) so that the guided rays are paraxial (i.e., approximately parallel to the fiber axis). The longitudinal components of the electric and magnetic fields are then much weaker than the transverse components and the guided waves are approximately transverse electromagnetic (TEM). The linear polarization in the x and y directions then form orthogonal states of polarization. The linearly polarized (I, *m)* mode is usually denoted as the LPt„, mode. The two polarizations of mode *{1, m)* travel with the same propagation constant and have the same spatial distribution.

For weakly guiding fibers the characteristic equation obtained using the procedure outlined earlier turns out to be approximately equivalent to the conditions that the scalar function *u(r)* in (8.1-9) is continuous and has a continuous derivative at *r* — *a.* These two conditions are satisfied if

$$\frac{(k_T a)\, I'[\, k_T \bullet)}{I;(k_T a)} \qquad \frac{(ya)\, K'(ya)}{K(ya)} \qquad (8.1\text{-}15)$$

The derivatives 7/ and K/ of the Bessel functions satisfy the identities

$$J_l'(x) = \pm J_{l\mp 1}(x) \mp l \frac{J_l(x)}{x}$$

$$K_l'(x) = -K_{l\mp 1}(x) \mp l \frac{K_l(x)}{}$$

Substituting these identities into (8.1-15) and using the normalized parameters X — k a and Y— ya, we obtain the characteristic equation

$$X\frac{J_{l\pm1}(X)}{J_l(X)} = \pm Y\frac{K_{l\pm1}(Y)}{K_l(Y)} \quad (8.1\text{-}16)$$

Characteristic Equation

$$X^2 + Y^2 = V^2$$

Given V and *1,* the characteristic equation contains a single unknown variable X (since Y^2 - V^2 — X^2). Note that /_ (z) = (— 1)'/t(z) and *K_ (x)* - *K;{x),* so that if *1* is replaced with — *1,* the equation remains unchanged.

The characteristic equation may be solved graphically by plotting its right- and left-hand sides (RHS and LHS) versus X and finding the intersections. As illustrated in Fig. 8.1-6 for / = 0, the LHS has multiple branches and the RHS drops monotonically with increase of X until it vanishes at X — V (Y — 0). There are therefore multiple intersections in the intcrval 0 < X < V. Each intersection point corresponds to a fiber mode with a distinct value of X. These values are denoted X ,„„ m — 1, 2, ... , M *in* order of increasing X. Once the At,„ are found, the corresponding transverse propaga- tion constants k q, the decay parameters y ,„„ the propagation constants Qt,„, and the radial distribution functions *u;q(r)* may be readily determined by use of (8.1-12), (8.1-7), and (8.1-9). The graph in Fig. 8.1-6 is similar to that in Fig. 7.2-2, which governs the modes of a planar dielectric waveguide.

Each mode has a distinct radial distribution. The radial distributions *u{r)* shown in Fig. 8.1-5, for example, correspond to the LP_{01} mode (/ — 0, m — 1) in a fiber with V — 5; and the LP34 mode *(i* — 3, m = 4) in a fiber with V — 25. Since the (/, m) and (—/, m) modes have the same propagation constant, it is interesting to examine the spatial distribution of their superposition (with equal weights). The complex amplitude of the sum is proportional to u *,,,(r)* cos /d› exp(*—j9tpz).* The intensity, which is proportional to u p(r) $\cos^2$ /d›, is illustrated in Fig. 8.1-7 for the LP_{0} and $LP3_4$ modes (the same modes for which *o(r)* is shown in Fig. 8.1-5).

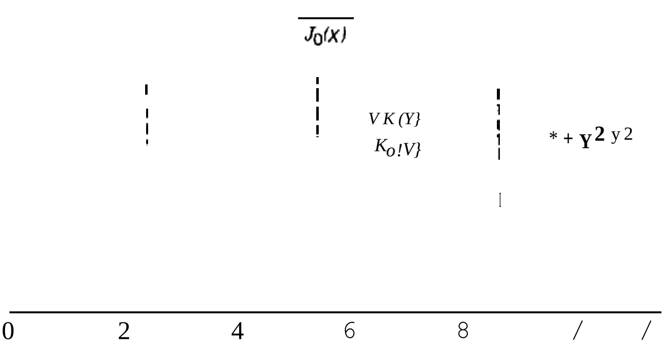
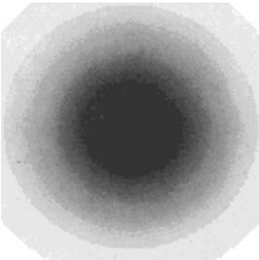


Figure 8.1•6 Graphical construction for solving the characteristic equation (8.1-16). The left- and right-hand sides are plotted as functions of A. The intersection points are the solutions. The LHS has multiple branches intersecting the abscissa at the roots of *I* (X). The RHS intersects each branch once and meets the abscissa at A — *V.* The number of modes therefore equals the number of roots of *I;*+(X) that are smaller than *V.* In this plot f = 0, *V* — 10, and either the or + signs in (8.1-16) may be taken.



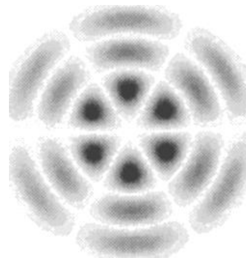


Figure 8.i-7 Distributions of the intensity of the *{a)* LP_0; and (fi) LPj modes in the transverse plane, assuming an azimuthal cos *ld›* dependence. The fundamental LPG, mode has a distribution similar to that of the Gaussian beam discussed in Chap. 3.

(b)

Mode Cutoff and Num6er of Modes

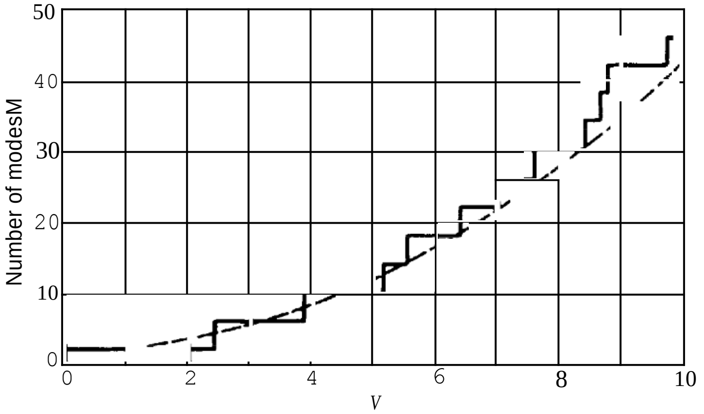
It is evident from the graphical construction in Fig. 8.1-6 that as *V* increases, the number of intersections (modes) increases since the LHS of the characteristic equation (8.1-16) is independent of U, whereas the RHS moves to the right as V increases. Considering the minus signs in the characteristic equation, branches of the LHS intersect the abscissa when *I _,{X)* = 0. These roots are denoted by *x q, m* — 1, 2, ...

The number of modes *M* is therefore equal to the number of roots of *I _ (X)* that are smaller than U. The (f, Art) mode is allowed if *V* > *x* . The mode reaches its cutoff point when *V* — *x* . As *V* decreases further, the (/, m – 1) mode also reaches its cutoff poiflt when a new root is reached, and so on. The smallest root of 7 _;(X) is x $_0$, = 0 for *1* — 0 and the next smallest is x = 2.405 for I = 1. When *V* < 2.405, all modes with the cxccption of thc fundamental LP_0 t mode are cut off. The fiber then operates as a single-mode waveguide. A plot of the number of modes *M* as a function of *V* is therefore a staircase function increasing by unity at each of the roots xt,,, of the Bessel function *I _ (X).* Some of these roots are listed in Table 8.1-1.

TABLE 8.1-1 Cutoff Y Parameter for the LP „, and LP „, Modes°

/	m:	1	2	3
0		0	3.832	7.016
1		2.405	5.520	8.654

°The cutoffs of the I = 0 modes occur at the roots of la $_1$(N) = — *I (X)*. The f = 1 modes are cut off at the roots of /p(A), and so on.



Fi9ure 8fi-8 Total number of modes M versus the fiber parameter *'V* — 2s(n/â)NA. Included in the count are two helical polarities for each mode with *1* > 0 and two polarizations per mode. For *V* < 2.405, there is only one mtide, the fundamental LPG; mode with two polarizations. The dashed curve is the relation *M* — 4V'/ v ^{2} + 2, which provides an approximate formula for the number of modss when *V* >> 1.

A composite count of the total number of modes *M* (for all /) is shown in Fig. 8.1-8 as a function of *V.* This is a staircase function with jumps at the roots of _ $_I$(z). Each root must be counted twice since for each mode of azimuthal index *1* > 0 there is a corresponding mode - *1* that is identical except for an opposite polarity of the angle d› (corresponding to rays with helical trajectories of opposite senses) as can be seen by using the plus signs in the characteristic equation. In addition, each mode has two states of polarization and must therefore be counted twice.

Number of Modes (Fibers with Large V Parameter)

For fibers with large V parameters, there are a large number of roots of 7t(X) in the interval 0 < X < V. Since *I (X)* is approximated by the sinusoidal function in (5.1-10a) when A >> 1, its roots xt„, are approximately given by *x;„, — (1 +* $_2$)(v/2) = (2m — 1)(w/2), i.e., *x* — (f + 2m — $_2$)w/ 2, so that the cutoff points of modes *(1,* m), which are the roots of *Jr + (X),* are

$$x_{lm} \approx \left[l + 2m - \frac{1}{2} + 1 - \frac{}{2} \right. \quad (\quad m \quad \pi \quad 0 \quad rn \qquad 8\ 1\ 17)$$

when m is large.

For a fixed f, these roots are spaced uniformly at a distance v, so that the number of roots *M;* satisfies (/ + 23f)w/2 - *V,* from which *M* -- ***V/w*** **//2.** Thus ***M;*** **drops** linearly with increasing /, beginning with *M* -- *'V/r* for / = 0 and ending at *M* -— 0 when f = /pp, where /pp = 2U/v, as illustrated in Fig. 8.1-9. Thus the total number of modes is *M* -- *E'* •*0 M* — L •0 (V/» —//2).

Since the number of terms in this sum is assumed large, it may be readily evaluated by approximating it as the area of the triangle in Fig. 8.1-9, *M* -- ı (2V/)(U,/v) = V ^{2}/r ^{2}. Allowing for two degrees of freedom for positive and negative *1* and two polarizations for each index (/, m), we obtain

$$V = \frac{4}{\ _2} V^2$$

(8.1-18)
Number of **Modes**
(V 1)

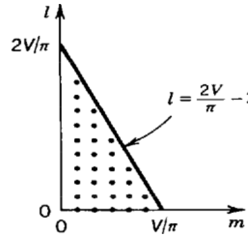


Figure 8.1-9 The indices or guided modes extend from m = 1 to m = *V/w* -- //2 and from *1* ---- 0 to = 2v/w.

This expression for *M* is analogous to that for the rectangular waveguide (7.3-3). Note that (8.1-18) is valid only for large *V*. This approximate number is compared to the exact number obtained from the characteristic equation in Fig. 8.1-8.

EXAMPLE 8.1-2. Approximate f¥umf›er *of* 3fodes. A silica fiber with n = 1.452 and A = 0.01 has a numerical aperture NA = (n2 -- n2)'*2 = n (2A)'*2 = 0.205. If â, - 0.85 pm and the core radius a = 25 pm, the *V* parameter is V = 2 w(n/â)NA = 37.9. There are therefore approximately *M* -- 4V 2/w 2 - 585 modes. If the cladding is stripped away so that the core is in direct contact with air, n_2 - 1 and NA = 1. The *V* parameter is then V = 184.8 and more than 13,800 modes are allowed.

Propagation Constants (Fibers with Large V *Parameter}*

As mentioned earlier, the propagation constants can be determined by solving the characteristic equation (8.1-16) for the *X* and using (8.1-7a) and (5.1-12) to obtain Qtp = ft2 *k2* -- Xt2/a 2)" 2. A number of approximate formulas for *X* applicable in certain limits are available in the literature, but there are no explicit exact formulas. If *V* >> 1, the crudest approximation is to assume that the *X* „, are equal to the cutoff values *K* „„. This is equivalent to assuming that the branches in Fig. 8.1-6 are approximately vertical lines, so that Xt„, = *x* . Since *V* >> 1, the majority of the roots would be large and the approximation in (8.1-17) may be used to obtain

$$|3 \quad -- n^2k^2 -- (I + 2m \quad)^2 \frac{p2}{4a^2} \quad 1/2 \tag{8.1-19}$$

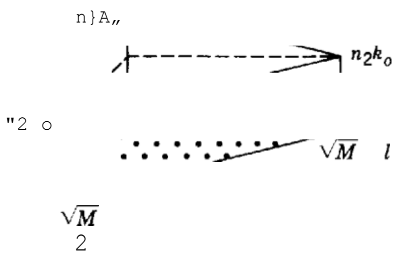
Since

$$M -- \frac{4}{,rr2} V^2 \quad \frac{4}{,rr\,2} NA^2 a^2 k_2 = \quad \frac{4}{\pi^2}(2n_1^2\Delta)k_o^2a^2 \tag{8.1-20}$$

(8.1-19) and (8.1-20) give

$$\beta_{lm} \approx n_1 k_o \left| 1 - 2 \frac{(l + 2\,in)^2 q}{M} \right. \quad ,,\ ^2 \tag{8.1-21}$$

Because ñ is small we use the approximation (1 + d)'*2 = 1 + d/2 for |6 | << 1, and



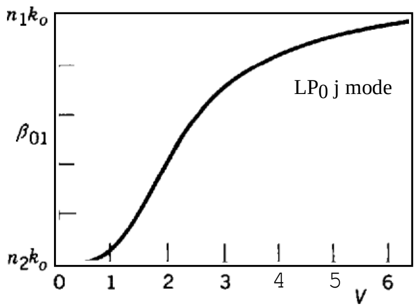


Figure 8.1-10 (a) Approximate propagation constants Q of the modes of a fiber with large *V* parameter as functions of the mode indices / and Sri. *tb)* Exact propagation constant IS_O of the fundamental LP_0 t modes as a function of the *V* parameter. For *V* m 1, Qb, - *n k* .

obtain

$$\beta_{lm} \approx n_1 k_o \left| 1 - \frac{(1 + 2m)}{M} \right.^{2}\$$$

(8.1-22)
Propagation Constants
l — 0,1,... ,*PM*
m - *) , 2, ... , (QM — lj /2*
[V >> 1)

Since *1 + 2m* varies between 2 and - *2V/r* —— *PM* (see Fig. 8.1-9), JStp varies approximately between *n k* and *n k* p(I — fi) = n $_2$ k,, as illustrated in Fig. 8.1-10.

‹s«up ***Velocities (Fi6ers with Large*** *V* ***Parameter)***

To determine the group velocity, u/„, *d‹»/d|3* „„ of the (/, lit)mode we express jS „, as an explicit function of ω by substituting $n_1 k_o = \omega / c_1$ and M $(4/\pi^2)(2n_1^2\Delta)k_o^2 a^2$ $(8/\pi^2)a^2\omega^2\Delta/c_1^2$ into (8.1-22) and assume that c_1 and Δ are independent of ω. The derivative *dui/d Q* ,„ gives

$$v_{lm} \approx c_1 \left| 1 + \frac{(l + 2m)^2}{M} \right. ,$$

Since A << 1, the approximate expansion (1 + 6) ' = 1 — 6 when |fi| << 1, gives

$$v_{lm} \approx c_1 \left| 1 - \frac{(l + 2\,m)^2}{M} \right.$$

(8.1-23)
Group Velocities
V 1)

Because the minimum and maximum values of *(/* + 2m) are 2 and *QM* , respectively, and since *M* 1, the group velocity varies approximately between c and c (1 — fi) = c ,(n $_2$/ f2 $_1$). Thus the group velocities of the low-order modes are approximately equal to the phase velocity of the core material, and those of the high-order modes are smaller.

The fractional group-velocity change between the fastest and the slowest mode is roughly equal to ñ, the fractional refractive index change of the fiber. Fibers with large A, although endowed with a large NA and therefore large light-gathering capacity, also have a large number of modes, large modal dispersion, and consequently high pulse spreading rates. These effects are particularly severe if the cladding is removed altogether.

C. Single-Mode Fibers

As discussed earlier, a fiber with core radius *a* and numerical aperture NA operates as a single-mode fiber in the fundamental LP_0, mode if V = 2s(o /âp)NA < 2.405 (see Table 8.1-1 on page 282). Single-mode operation is therefore achieved by using a small core diameter and small numerical aperture (making n_2 close to n), or by operating at a sufficiently long wavelength. The fundamental mode has a bell-shaped spatial distribution similar to the Gaussian distribution [see Figs. 8.1-5(a) and 8.1-7(a)] and a propagation constant Q that depends on *V* as illustrated in Fig. 8.1-10(b). This mode provides the highest confinement of light power within the corc.

EKAM LE a.1-3. Siripfe-Mode Operation. A silica glass fiber with nt = 1.447 and ñ = 0.01 (NA = 0.205) operates at â p = 1.3 pm as a single-mode fiber if U — 2'r-(a/X)NA < 2.405, i.e., if the core diameter 2a < 4.86 pm. If fi is reduced to 0.0025, single-mode operation requires a diameter 2a < 9.72 p m.

There are numerous advantages of using single-mode fibers in optical communication systems. As explained earlier, the modes of a multimode fiber travel at different group velocities and therefore undergo different time delays, so that a short-duration pulse of multimode light is delayed by different amounts and therefore spreads in time. Quantitative measures of modal dispersion are determined in Sec. 8.3B. In a single-mode fiber, on the other hand, there is only one mode with one group velocity, so that a short pulse of light arrives without delay distortion. As explained in Sec. 8.3B, other dispersion effects result in pulse spreading in single-mode fibers, but these are significantly smaller than modal dispersion.

As also shown in Sec. 8.3, the rate of power attenuation is lower in a single-mode fiber than in a multimode hber. This, together with the smaller pulse spreading rate, permits substantially higher data rates to be transmitted by single-mode fibers in comparison with the maximum rates feasible with multimode fibers. This topic is discussed in Chap. 22.

Another difficulty with multimode **fibers is** caused by the random interference of the modes. As a result of uncontrollable imperfections, strains, and temperature fluctuations, each mode undergoes a random phase **shift so** that the sum of the complex amplitudes of the modes has a random intensity. This randomness is a form of noise known as **modal noise** or **speckle.** This effect is similar to the fading of radio signals due to multiple-path transmission. In a single-mode fiber there is only one path and therefore no modal noise.

Because of their small size **and** small **numerical apertures,** single-mode fibers **are** more compatible with integrated-optics technology. However, such features make them more difficult to manufacture and work with because of the reduced allowable mechanical tolerances for **splicing** or joining with demountable connectors and for coupling optical power into the fiber.

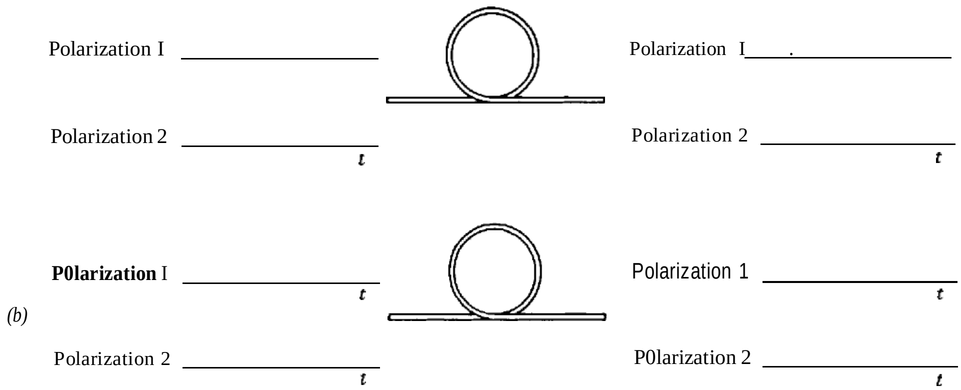


Figure 8.4-11 (a) Ideal polarization-maintaining fiber. *(b)* Random transfer of power between two polarizations.

Polarization-Maintaining Fibers

In a fiber with circular cross scction, cach mode has two independent states of polarization with the same propagation constant. Thus the fundamental LPG mode in a single-mode weakly guiding fiber may be polarized in the z or y direction with the two orthogonal polarizations having the same propagation constant and the same group velocity.

In principle, there is no exchange of power between the two polarization components. If the power of the light source is delivered into one polarization only, the power received remains in that polarization. In practice, however, slight random imperfections or uncontrollable strains in the fiber result in random power transfer between the two polarizations. This cou pling is facilitated since the two polarizations have the same propagation constant and their phases are therefore matched. Thus linearly polarized light at the fiber input is transformed into elliptically polarized light at the output. As a result of fluctuations of strain, temperature, or source wavelength, the ellipticity of the received light fluctuates randomly with time. Nevertheless, the total power remains fixed (Fig. 8.1-11). If we are interested only in transmitting light power, this randomization of the power division between the two polarization components poses no difficulty, provided that the total power is collected.

In many areas related to fiber optics, e.g., coherent optical communications, integrated-optic dcviccs, and optical sensors based on interferometric tcchniques, the fiber is used to transmit the complex amplitude of a specific polarization (magnitude and phase). For these applications, polarization-maintaining fibers are necessary. To make a polarization-maintaining fiber the circular symmetry of the conventional fiber must be removed, by using fibers with elliptical cross sections or stress-induced anisotropy of the refractive index, for example. This eliminates the polarization degeneracy, i.e., makes the propagation constants of the two polarizations different. The coupling efficiency is then reducecl us a result of the introduction of phase mismatch.

8.2 GRADED-INDEX FIBERS

Index grading is an ingenious method for reducing the pulse spreading caused by the differences in the group velocities of the modes of a multimode fiber. The core of a graded-index fiber has a varying refractive index, highest in thc center and decreasing gradually to its lowest value at the cladding. The phase velocity of light is therefore minimum at the center and increases gradually with the radial distance. Rays of the

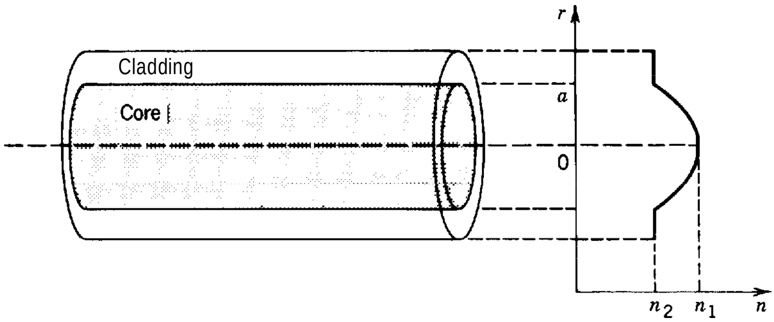


Figure 8.2-1 Geometry and refractive-index profile of a graded-index fiber.

most axial mode travel the shortest distance at the smallest phase velocity. Rays of the most oblique mode zigzag at a greater angle and travel a longer distance, mostly in a medium where the phase velocity is high. Thus the disparities in distances are compensated by opposite disparities in phase velocities. As a consequence, the difler-ences in the group velocities and the travel times are expected to be reduced. In this section we examine the propagation of light in graded-index fibers.

The core refractive index is a function *ntr)* of the radial position *r* and the cladding refractive index is a constant nt. The highest value of $n(r)$ is $n(0) = n_1$ and the lowest value occurs at the core radius $r = a$, $n(a) = n_2$, as illustrated in Fig. 8.2-1.

A versatile refractive-index profile is the power-law function

$$n^2(r) = n_1^2\left[1 - 2\Delta\left(\frac{r}{a}\right)^p\right], \qquad r \le a, \tag{8.2-1}$$

where

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1}, \tag{8.2-2}$$

and p, called the grade profile parameter, determines the steepness of the profile. This function drops from n_1 at $r = 0$ to n_2 at $r = a$. For $p = 1$, $n^2(r)$ is linear, and for $p = 2$ it is quadratic. As $p \to \infty$, $n^2(r)$ approaches a step function, as illustrated in Fig. 8.2-2. Thus the step-index fiber is a special case of the graded-index fiber with $p = \infty$.

Guided Pays

The transmission of light rays in a graded-index medium with parabolic-index profile was discussed in Sec. 1.3. **Rays** in meridional planes follow oscillatory planar trajecto-

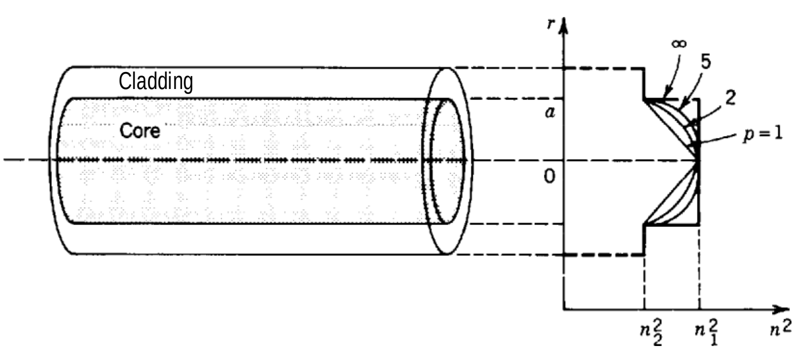


Figure 8.2-2 Power-law refractive-index profile $n^2(r)$ for different values of y.

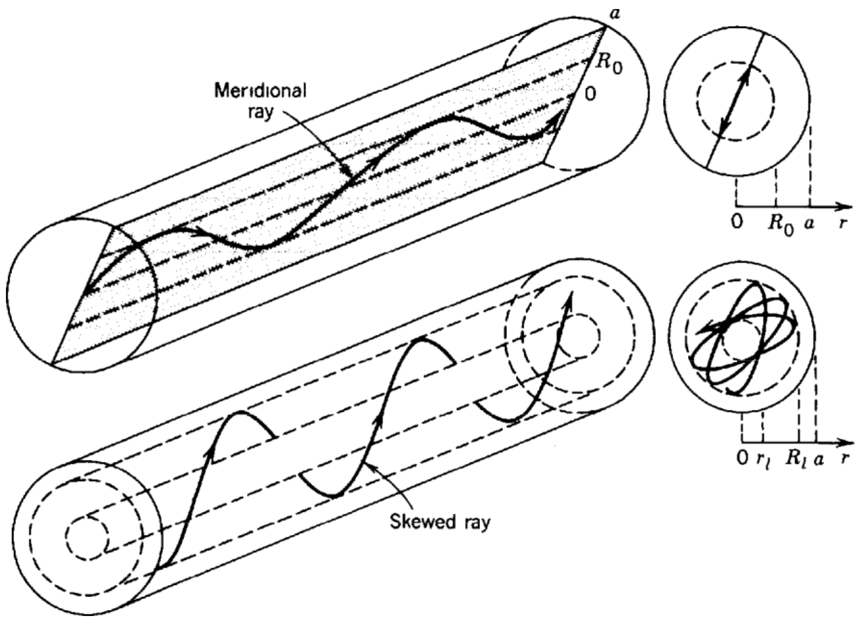


Figure 8.2-3 Guided rays in the core of a graded-index fiber. (a) A meridional ray confined to a meridional plane inside a cylinder of radius R_0. *(b)* A skewed ray follows a helical trajectory confined within two cylindrical shells of radii *r;* and fi .

ries, whereas skewed rays follow helical trajectories with the turning points forming cylindrical caustic surfaces, as illustrated in Fig. 8.2-3. Guided rays are confined within the core and do not reach the cladding.

A. Guided Waves

The modes of the graded-index fiber may be determined by writing the Helmholtz equation (8.1-4) with *n* — *n{r),* solving for the spatial distributions o1 the field components, and using Maxwell's equations and the boundary conditions to obtain the characteristic equation as was done in the step-index case. This procedure is in general difficult.

In this section we use instead an approximate approach based on picturing the field distribution as a quasi-plane wave traveling within the core, approximately along the trajectory of the optical ray. A quasi-plane wave is a wave that is locally identical to a plane **wave, but** changes its direction and amplitude slowly **as it travels. This** approach permits us to maintain the simplicity of rays optics but retain the phase associated with the wave, so that we can use the self-consistency condition to determine the propagation constants of the guided modes (as was done in the planar waveguide in Sec. 7.2). This approximate technique, called the WKB (Wentzel—Kramers—Brillouin) method, is applicable only to fibers with a large number of modes (large *V* parameter).

Quasi-Plane W'av'es

Consider a solution of the Helmholtz equation (8.1-4) in the form of a quasi-plane wave (see Sec. 2.3)

$$U(r) — (r) \quad cxp[\quad Jk \quad S(r)], \qquad (8.2\text{-}3)$$

where <(r) and S(r) are real functions of position that are slowly varying in comparison with the wavelength 3, = 2w/k,. We know from Sec. 2.3 that *S{)* approximately satisfies the eikonal equation *US* ^{2} - *n',* and that the rays travel in the direction of the gradient *US*. If we take *k „IS{r) - k s(r) + ld› + Qz,* where s(r) is a slowly varying function of *r,* the eikonal equation gives

$$\frac{ds}{dr}^{2} + Q^2 + g \; \frac{l^2}{2} = n^2(r)\, k^2. \qquad (8.2\text{-}4)$$

The local spatial frequency of the wave in the radial direction is the partial derivative of the phase k S(r) with respect to *r,*

$$k, — k\, g \frac{ds}{dr} \qquad (8.2\text{-}5)$$

so that (8.2-3) becomes

$$U(r) = a(r) \exp\left(-j \int_0^r k_r\, dr\right) e^{-jl\phi} e^{-j\beta z}, \qquad (8.2\text{-}6)$$

Ouasi-Plane Wave

and (8.2-4) gives

$$k_r^2 - n^2(r)\, k \quad ;S^2 \quad / \quad . \qquad \textbf{(8.2-7)}$$

Defining *k* — *1/r,* i.e., exp(*—j/d›)* = exp(*—jk$rdi),* and *k,* — |*3,* we find that (8.2-7) gives *k* + *k*2 + *k*2 — *n' r)k* 2 . The quasi-plane wave therefore has a local wavevector k with magnitude *n(r)k* and cylindrical-coordinate components *k,, k , k,).* Since *n{r)* and *k* are functions of r, *k,* is also generally position dependent. The direction of k changes slowly with r (see Fig. 8.2-4) following a helical trajectory similar to that of the skewed ray shown earlier in Fig. 8.2-3(b).

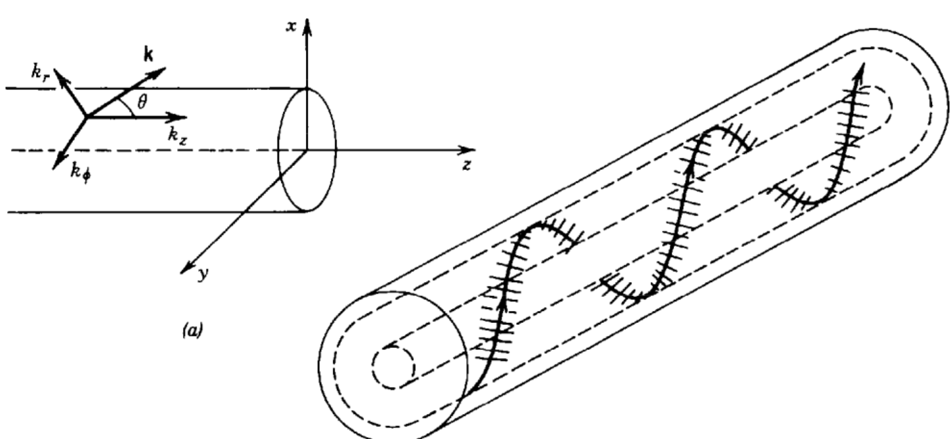
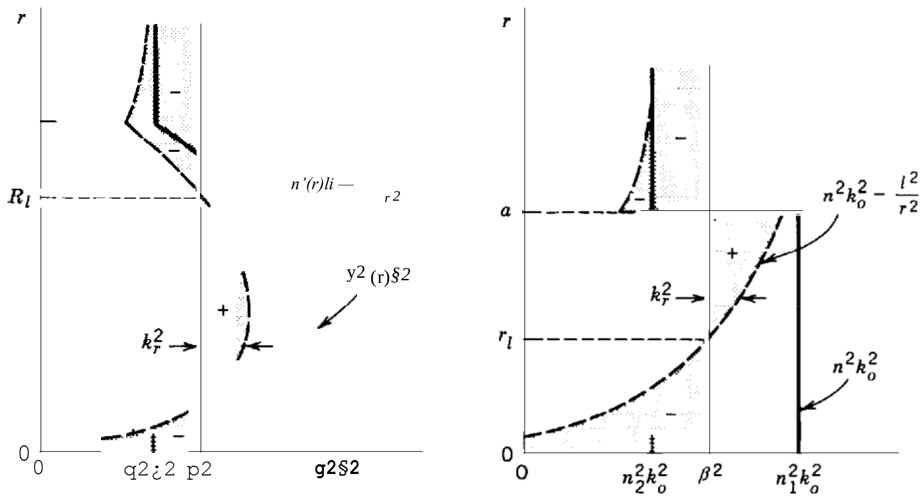


Figure 8.2-4 (a) The wavevector *k - k,, k , k)* in a cylindrical coordinate system. *(b)* Ouasi-plane wave following the direction of a ray.



fa) *(b)*

Figure 8.2-5 Dependence of n^2(r)k *2,* $\mathbf{n^2}$**(r)k**2 – I^2/r ^{2}, and *k* – *n'(r lk2* – J^2/**r** $\frac{2}{}$ Q^2 on the position *r.* At any r, *k2* is the width of the shaded area with the + and — signs denoting positive and negative *k2* . (a) Graded-index fiber; *k2* is positive in the region *r* < *r* < ft¿. *(b)* Step-index fiber; *k2* is positive in the region *r* < *r* < *a.*

To determine the region of the core within which the wave is bound, we determine the values of r for which *k,* is real, or *k2*> 0. For a given I and 9 we plot *k* - [n^2{r)k2— *1*^{2}*/r* ^{2} — Q ^{2}] as a function of *r.* The term n ^{2}(r)k2 is first plotted as a function of *r* [the thick continuous curve in Fig. 8.2-5(a)]. The term *1'/r'* is then subtracted, yielding the dashed curve. The value of Q is marked by the thin continuous vertical line. It follows that *k2* is represented by the difference between the dashed line and the thin continuous line, i.e., by the shaded area. Regions where k^2_r is positive or negative are indicated by the + or — **signs,** respectively. Thus *k* is real in the region *r* < *r* < *R* , where

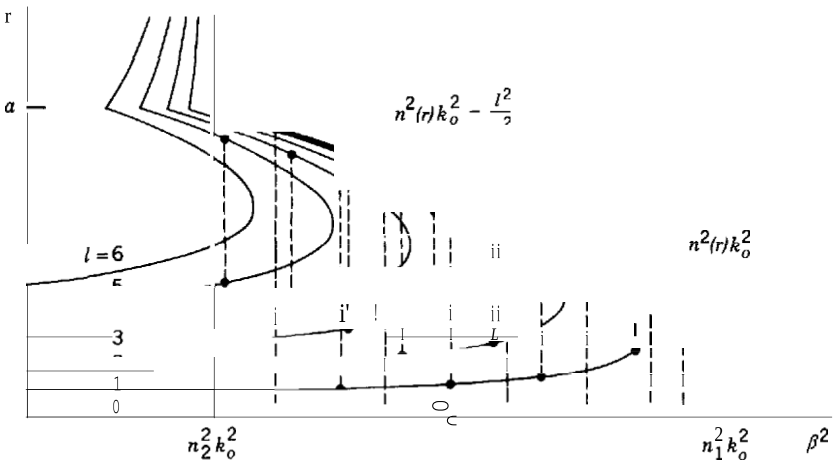
$$n^2(r)\,k_2 - \frac{l'}{{}_2} - JS^2 = 0, \qquad r - r \quad \text{and} \quad r - R\,. \tag{8.2-8}$$

It follows that the wave is basically confined within a cylindrical shell of radii *r;* and ftt just like the helical ray trajectory shown in Fig. 8.2-3(b).

These results are also applicable to the step-index fiber in which ri(r) = *n* for *r* < *a,* and n(r) = *n* $_2$ for *r* > *a.* In this case the quasi-plane wave is guided in the core by reflecting from the core—cladding boundary at *r* — *a.* As illustrated in Fig. 8.2-5(b), the region of confinement is *r* < *r* < *a,* where

$$n_1^2k_o^2 - \frac{2}{r_l^2} - \beta^2 = 0. \tag{8.2-9}$$

The wave bounces back and forth helically like the skewed ray shown in Fig. 8.1-2. In the cladding *(r a)* and near the center of the core (r < *r*), *k* is negative so that *k,* is imaginary, **and the wave** therefore **decays** exponentially. Note **that *r;*** depends on JS. For large J3 (or large *1), r* is large; i.e., the wave is confined to a thin cylindrical shell near the edge of the core.



Flgure 8.2-6 The propagation constants and confinement regions of the fiber modes. Each curve corresponds to an index f. In this plot f = 0, 1, ... , 6. Each mode (representing a certain value of m) is marked schematically by two dots connected by a dashed vertical line. The ordinates of the dots mark the radii *r;* and *R;* of the cylindrical shell within which the mode is confined. Values on the abscissa are the squared propagation constants Q ^{2} of the mode.

Modes

The modes of the fibcr are determined by imposing the self-consistency condition that the wave reproduce itself after one helical period of traveling between *r* and *R* and back. T'he azimuthal path length corresponding to an angle 2s must correspond to a multiple of 2s phase shift, i.e., kg2w r = 2s/; / = 0, + 1, + 2, ... This condition is evidently satisfied since *k* — *1/r.* In addition, the radial round-trip path length must correspond to a phase shift equal to an integer multiple of 2s,

$$2 \int_{r_l}^{R_l} k_r \, dr = 2\pi m, \qquad m = 1, 2, \ldots, M. \tag{8.2-10}$$

This condition, which is analogous to the self-consistency condition (7.2-2) for planar waveguides, provides the characteristic equation from which the propagation constants 9tp of the modes are determined. These values are marked schematically in Fig. 8.2-6; the mode m = 1 has the largest value of 9 (approximately n_1 k,) and in = *M* has the smallest value (approximately n_2 k,) .

/Vzzm6er *of Modes*

The total number of modes can be determined by adding the number of modes *M* for *1* — 0, 1, ... , fpp. We shall address this problem using a different procedure. We first determine the number *qq* of modes with propagation constants greater than a given value 9. For each *1,* the number of modes 3ft(J3) with propagation constant greater than jt3 is the number of multiples of 2s the integral in (8.2-10) yields, i.e.,

$$M_l(\beta) = \frac{1}{\pi}\int_{r_l}^{R_l} k_r \, dr = \frac{1}{\pi}\int_{r_l}^{R_l}\left[n^2(r)k_o^2 - \frac{l^2}{r^2} - \beta^2\right]^{1/2} dr, \tag{8.2-11}$$

where *r;* and fi are the radii of confinement corresponding to the propagation constant Q as given by (8.2-8). Clearly, *r;* and *R* depend on Q.

Thc total number of modes with propagation constant greater than Q is therefore

$$q_\beta = 4 \sum_{f=0}^{l_{\max}(\beta)} M_l(\beta), \tag{8.2-12}$$

where */qp(9)* is the maximum value of / that yields a bound mode with propagation constants greater than Q, i.e., for which the peak value of the function $n^2(r)k_2$ -- / ^{2}/r ^{2} is greater than 9 ^{2}. The grand total number of modes *M* is qp for 9 = *n* $_2$ k,. The factor of 4 in (8.2-12) accounts for the two possible polarizations and the two possible polarities of the angle d›, corresponding to positive or negative helical trajectories for each (f, m). lf the number of modes is sufficiently large, we can replace the summation in (8.2-12) by an integral,

$$q_\beta \approx 4\int_0^{l_{\max}(\beta)} M_l(\beta)\, dl. \tag{8.2-13}$$

For fibers with a power-law refractive-index profile, we substitute (8.2-1) into (8.2-11), and the result into (8.2-13), and evaluate the integral to obtain

$$q\,p \text{ -- } M \quad \frac{1 \quad (/n, k\ )^2}{28} \quad {}^{\prime}+^{\,2\,{}^{\prime\prime}} \tag{8.2-14}$$

where

$$M \approx \frac{p}{p+2} n_1^2 k_o^2 a^2 \Delta \qquad \frac{p}{p+2}\frac{V^2}{2} \tag{8.2-15}$$

Here fi - *n* -- *n2)/n* and *V* ---- 2w(a/X,)NA is the fiber *V* parameter. Since qq - *M* at ;S - *n k* , *M* is indeed the total number of modes.

For step-index fibers (p =),

$$\text{qp} = M \quad \frac{1 \text{ -- } (p/n \quad k\ )^2}{2A} \tag{8.2-16}$$

and

$$\boxed{M \text{ -- } \frac{y^{\;2}}{2}} \tag{8.2-17}$$

Number of Modes
(Step-Index Fiber)
V - 2s *a//*)NA

This expression for *M* is nearly the same as *M* -- 4 U ^{2}/ w ^{2} - 0.41U ^{2} in (8.1-18), which was obtained in Sec. 8.1 using a different approximation.

B. Propagation Constants and Velocities

Propagation Consfanfs

The propagation constant J3 of mode *q* is obtained by inverting (8.2-14),

$$\beta_q \approx n_1 k_o \left| 1 - 2 \left(\frac{q}{M} \right. \right. \quad ,‘ , \quad ‘ 2$$

q — 1 , 2, ... , *M*, (8.2-18)

where the index *qp* has been replaced by *q*, and 9 replaced by JSq. Since fi << 1, the approximation $(1 + \delta)^{1/2} \approx 1 + \frac{1}{2}\delta$ (when |fi| << 1) can be applied to (8.2-18), yielding

$$\beta_q \approx n_1 k_o \left| 1 - \left(\frac{q}{M} \right)^{p/(p+2)} \Delta \right.$$

(8.2-19)
Propagation Constants
q = 1, 2, ... , *M*

The propagation constant J3 therefore decreases from - *n k* (at q — 1) to ri $_2$ k, (at *q — M),* as illustrated in Fig. 8.2-7.

In the step-index fiber (p = m),

$$\beta_q \approx n_1 k_o \left(1 - \frac{q}{M} \Delta \right).$$

(8.2-20)
Propagation Constants
(Step-Index Fiber)
q - 1, 2, ... , M

This expression is identical to (8.1-22) if the index q = 1, 2, ... , *M* is replaced by *{1* + 2m)^2, where *1* — 0, 1, ... , *PM ,'m* – 1, 2, ... , M /2 – //2.

Group Velocities

To determine the group velocity *vg — dn›/d* |3*q*, we write Q as a function of w by substituting (8.2-15) into (8.2-19), substituting $n_1 k p$ — *ui/c* into the result, and evaluating *' {d /dw) '.* With the help of the approximation (1 + fi) - 1 — d when

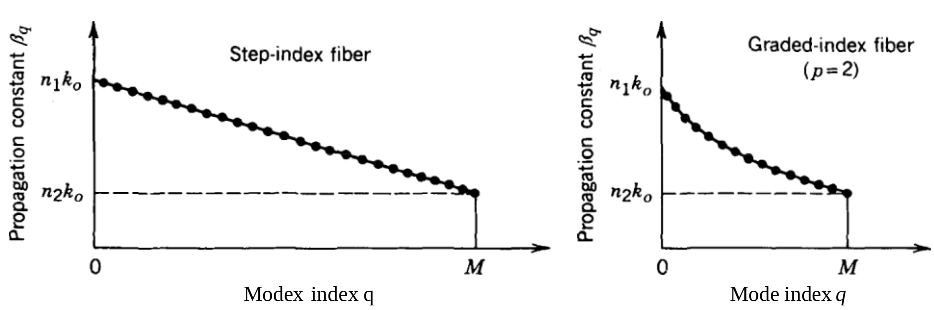


Figure 8.2-7 Dependence of the propagation constants Qq on the mode index q = 1, 2, ... , M.

|8| << 1, and assuming that c and fi are independent of ni (i.e., ignoring material dispersion), we obtain

$$v_q \approx c_1 \left[1 - \frac{p-2}{p+2}\left(\frac{q}{M}\right)^{2p/(p+2)} \Delta\right]$$

(8.2-21)
Group Velocities
q - 1, 2, ... , M

For the step-index fiber *(p* ——)

$$v_q \approx c_1 \left(1 - \frac{q}{M} \Delta\right)$$

(8.2-22)
Group Velocities
(Step-Index Fiber)
q - 1, 2, ... , M

The group velocity varies from approximately e to c;(1 A). This reproduces the result obtained in (8.1-23).

Optimal* index *Profile

Equation (5.2-21) indicates that the grade profile parameter p = 2 yields a group velocity r, - c, for all q, so that all modes travel at approximately the same velocity c_1. The advantage of the graded-index fiber for multimode transmission is now apparent.

To determine the group velocity with better accuracy, we repeat the derivation of *vq* from (8.2-18), taking three terms in the Taylor's expansion (1 + 6)" ^{2} = 1 + 6/2 fi ^{2}/8, instead of two. For p = 2, the result is

$$v_q = c_1 \left(1 - \frac{q}{M} \frac{\Delta^2}{2}\right).$$

(8.2-23)
Group Velocities
(- 2)
q = 1, ... , M

Thus the group velocities vary from approximately c at q = 1 to approximately c (1 — A^2/2) at q = *M.* In comparison with the step-index fiber, for which the group velocity ranges between c_1 and *c I*1 fi), the fractional velocity difference for the parabolically graded fiber is A^2/2 instead of A for the step-index fiber (Fig. 8.2-5). Under ideal conditions, the graded-index fiber therefore reduces the group velocity

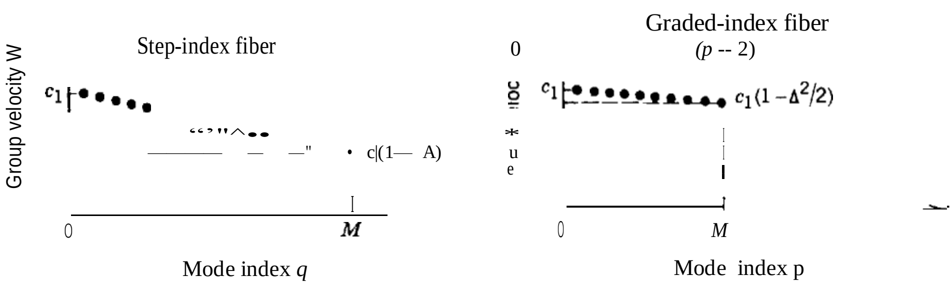


Figure 8.2-8 Group velocities c', of the modes of a step-index fiber (p = o) and an optimal graded-index hber *(p* — 2).

diflerence by a factor fi/2, thus realizing its intended purpose of equalizing the mode velocities. Since the analysis leading to (8.2-23) is based on a number of approximations, however, this improvement factor is only a rough estimate; indeed it is not fully attained in practice.

For p — 2, the number of modes M given by (8.2-15) becomes

$$M \approx \frac{V^2}{4}. \tag{8.2-24}$$

Number of Modes
(Graded-Index Fiber, p— 2)
V' — 2 (/J,)NA

Comparing this with (8.2-17), we see that the number of modes in an optimal graded-index fiber is approximately one-half the number of modes in a step-index fiber of the same parameters n , n_2, and n.

8.3 ATTENUATION AND DISPERSION

Attenuation and dispersion limit the performance of the optical-fiber medium as a data **transmission channel.** Attenuation limits the magnitude of the optical power **transmit**ted, whereas dispersion limits the rate at which data may be transmitted through the fiber, since it governs the tcmporal spreading of the optical pulses carrying the data.

A. Attenuation

The* Aneiirafion *Coetticient

Light traveling through an optical fiber exhibits a power that decreases exponentially with the distance as a result of absorption and scattering. The attenuation coefficient n is usually defined in units of dB/km,

‹i = 1 / 10 log 1 (8.3-1)

where - P $L)/$ P 0) is the power transmission ratio (ratio of transmitted to incident power) for a fiber of length L km. The relation between n and is illustrated in Fig. 8.3-1 for L = 1 km. A 3-dB attenuation, for example, corresponds to — 0.5, while 10 dB is equivalent to = 0.1 and 20 dB corresponds to = 0.01, and so on.

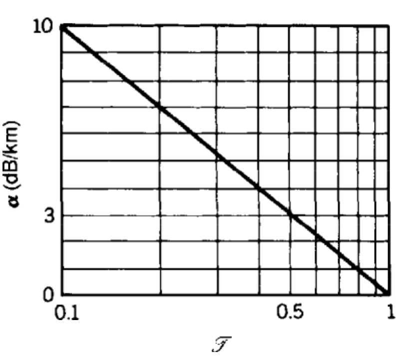


Figure 8.3-1 Relation between transmittance ,P and attenuation coefficient o in d B units.

Losses in dB units are additive, whereas the transmission ratios are multiplicative. Thus for a propagation distance of z kilometers, the loss is ‹cz decibels and the power transmission ratio is

$$\frac{P()}{P(0)} \quad 10 \ ''' \ ^{,0} = e^{\ 0 \wedge *} . \qquad (\text{n in dB/km}) \qquad (8.3\text{-}2)$$

Note that if the attenuation coefficient is measured in km units, instead of in dB/km, then

$$P(z)/P(0) = e^{\ \circ\prime} \qquad (8.3\text{-}3)$$

where a - 0.23a. Throughout this section ‹x is taken in d B/km units so that (8.3-2) applies. Elsewhere in the book, however, we use n to denote the attenuation coefficient (m ' or cm ’) in which case the power attenuation is described by (8.3-3).

Absorption

The attenuation coefficient of fused silica glass (SiO_2) is strongly dependent on wavelength, as illustrated in Fig. 8.3-2. This material has two strong absorption bands: a middle-infrared absorption band resulting from vibrational transitions and an ultraviolet absorption band due to electronic and molecular transitions. There is a window bounded by the tails of these bands in which there is essentially no intrinsic absorption. This window occupies the near-infrared region.

ScaRering

Rayleigh scattering is another intrinsic effect that contributes to the attenuation of light in glass. The random localized variations of the molecular positions in glass creatc random inhomogeneities of the refractive index that act as tiny scattering centers. The amplitude of the scattered field is proportional to $u›^2$.† The scattered intensity is therefore proportional to o^4 or to $1/X^4$, so that short wavelengths are scattered more than long wavelengths. Thus blue light is scattered more than red (a similar effect, the

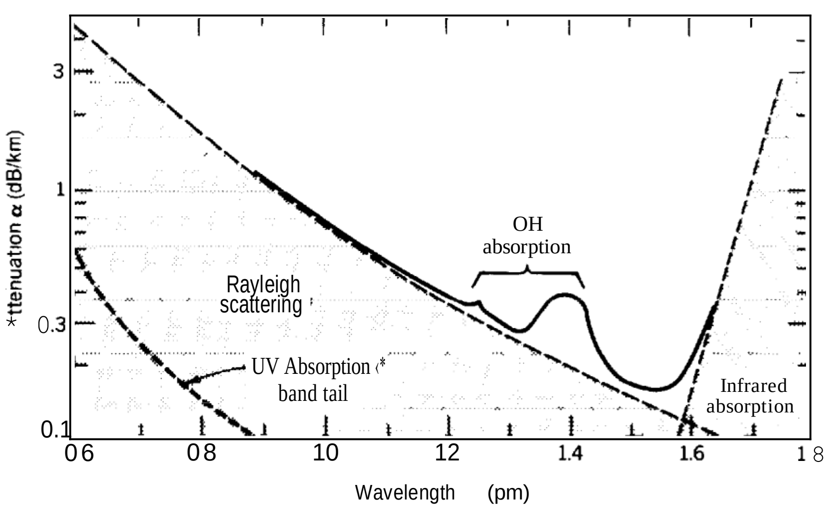


Figure 8.3-2 Dependence of the attenuation coefficient « of silica glass on the wavelength â,. There is a local minimum at 1.3 p m (o = 0.3 dB/km) and an absolute minimum at 1 55 p m (n = 0.16 dB/km).

The scattering medium creates a polarization density & which corresponds to a source of radiation proportional to $\hat{a}^2 P/dt^2$ = — ui^2 W; see (5.2-19).

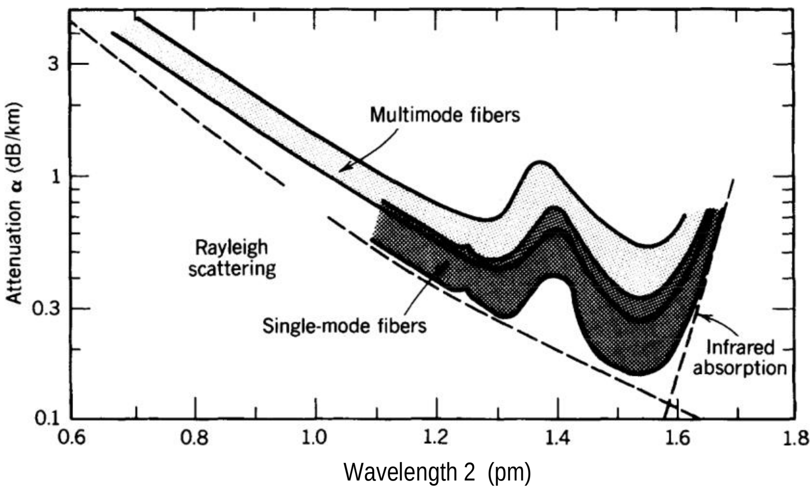


Figure 8.3-3 Ranges of attenuation coefficients of silica glass single-mode and multimode fibers.

scattering of sunlight from tiny atmospheric molecules, is the reason the sky appears blue). The attenuation caused by Rayleigh scattering therefore decreases with wavelength as $1/Z^4$,,a relation known as **Rayleigh's inverse fourth-power law. In** the visible band, Rayleigh scattering is more significant than the tail of the ultraviolet absorption band, but it becomes negligible in comparison with infrared absorption for wavelengths greater than 1.6 pm.

The transparent window in silica glass is therefore bounded by Rayleigh scattering on the short-wavelength side and by infrared absorption on the long-wavelength side (as indicated by the dashed lines in Fig. 8.3-2).

Extrinsic Effects

In addition to these intrinsic effects there are extrinsic absorption bands due to impurities, mainly OH vibrations associated with water vapor dissolved in the glass and metallic-ion impurities. Recent progress in the technology of fabricating glass fibers has made it possible to remove most metal impurities, but OH impurities are difficult to eliminate. Wavelengths at which glass fibers are used for optical communication are selected to avoid these absorption bands. Light-scattering losses may also be accentuated when dopants are added for the purpose of index grading, for example.

The attenuation coefficient of guided light in glass fibers depends on the absorption and scattering in the core and cladding materials. Since each mode has a diflerent penetration depth into the cladding so that rays travel different effective distances, the attenuation coefficient is mode dependent. It is generally higher for higher-order modes. Single-mode fibers therefore typically have smaller attenuation coefficients than multimode fibers (Fig. 8.3-3). Losses are also introduced by small random variations in the geometry of the fiber and by bends.

B. Dispersion

When a short pulse of light travels through an optical fiber its power is "dispersed" in time so that the pulse spreads into a wider time interval. There are four sources of dispersion in optical fibers: modal dispersion, material dispersion, waveguide dispersion, and nonlinear dispersion.

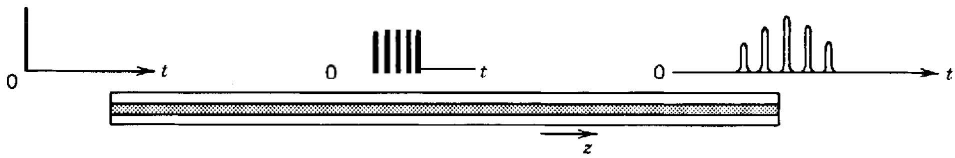


Figure 8.3-4 Pulsc spreading caused by modal dispersion.

Model Dicpersion

Modal dispersion occurs in multimode fibers as a result of the differences in the group velocities of the modes. A single impulse of light entering an M-mode fiber at z = 0 spreads into *M* pulses with the differential delay increasing as a function of z. For a fiber of length L, the time delays encountered by the different modes are *Tq L/* q = 1, ... , ñf, where u, is the group velocity of mode q. If rp,, and t'qp are the smallest and largest group velocities, the received pulse spreads over a time interval L/upp — L/r,,,p. Since the modes are generally not excited equally, the overall shape of the received pulse is a smooth profile, as illustrated in Fig. 8.3-4. An estimate of the overall rms pulse width is o-, = 2 *L/v q,g* L /cpp). This width represents the response time of the fiber.

In a step-index fiber with a large number of modes, c'pp c (1 — fi) and cpp *c* (see Sec. 8.1B and Fig. 8.2-8). Since (1 — fi) ' = 1 + ñ, the response time is

$$\approx \sigma_\tau \quad c_1 \quad 2 \text{ '} \tag{8.3-4}$$

Response Time
(Multimode Step-Index Fiber)

i.e., it is a fraction fi/2 of the delay time L /r t.

Modal dispersion is much smaller in graded-index fibers than in step-index fibers since the group velocities are equalized and the differences between the delay times r = *L/v* of the modes are reduced. It was shown in Sec. 8.2B and in Fig. 8.2-8 that in a graded-index fiber with a large number of modes and with an optimal index profile, cpp - c and rp, - c (1 — ñ ^{2}/2). The response time is therefore

$$w_{''} - \quad c \quad \frac{g2}{4} \tag{8.3-5}$$

Response Time
(Graded-Index Fiber)

which is a factor of ñ/2 smaller than that in a step-index fiber.

EXAMPLE 8.3•i. Multimode *Pulse* 8roadenfnp hare. In a step-index fiber with fi - 0.01 and n - 1.46, pulses spread at a rate of approximately o-,/L = fi/2c$_1$ = n;A/2cp - 24 ns/km. In a 100-km fiber, therefore, an impulse spreads to a width of - 2.4 ps. If the same fiber is optimally index graded, the pulse broadening rate is approximately n A^2/4c - 122 ps/km, which ix substantially reduced.

The pulse broadening arising from modal dispersion is proportional to the fiber length L in both step-index and graded-index fibers. This dependence, however, does not necessarily hold when the fibers are longer than a certain critical length because of mode coupling. Coupling occurs between modes of approximately the same propagation constants as a result of small imperfections in the fiber (random irregularities of the fiber surface, or inhomogeneities of the refractive index) which permit the optical power to be exchanged between the modes. Under certain conditions, the response time w, of mode-coupled fibers is proportional to L for small L and to L" [2] when a critical length is exceeded, so that pulses are broadened at a slower rate'.

gfater/a/ 0/spers/on

Glass is a dispersive medium; i.e, its refractive index is a function of wavelength. As discussed in Sec. 5.6, an optical pulse travels in a dispersive medium of refractive index n with a group velocity *u - c p/N,* where *N* — *n - 5 dn/dX p.* Since the pulse is a wavepacket, composed of a spectrum of components of different wavelengths each traveling at a di8erent group velocity, its width spreads. The temporal width of an optical impulse of spectral width ‹r (nm), after traveling a distance L, is w, = |*(d/dh,)(L/u)!c = (d/dX K LN/c p)* ‹r , from which

$$\sigma_\tau \tag{8.3-6}$$

Response Time (Material Dispersion)

where

$$D_\lambda = -\frac{\lambda_o}{c_o}\frac{d^2n}{d\lambda_o^2} \tag{8.3-z}$$

is the material dispersion coefficient [see (5.6-21)]. The response time increases linearly with the distance L. Usually, L is measured in km, ‹r, in ps, and e, in nm, so that D has units of ps/km-nm. This type of dispersion is called **material dispersion** (as opposed to modal dispersion).

The wavelength dependence of the dispersion coefficient D for silica glass is shown in Fig. 8.3-5. At wavelengths shorter than 1.3 pm the dispersion coefficient is negative,

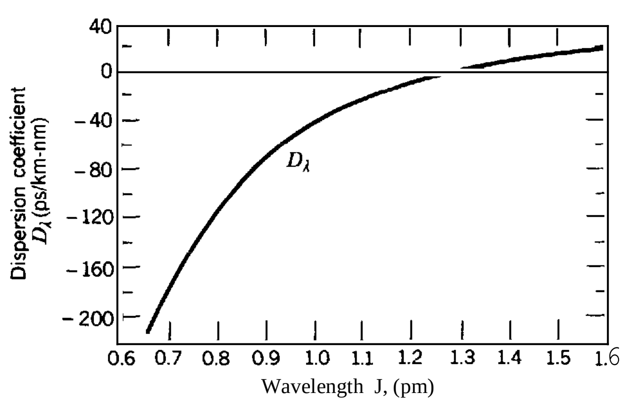


Figure 8.3-5 The dispersion coefficient D of silica glass as a function of wavelength â (see also Fig. 5.6-5).

*See, e.g., *J.* E. Midwinter, *Optical Fibers for Transmission,* Wiley, New York, 1979.

so that wavepackets of long wavelength travel faster than those of short wavelength. At a wavelength 3, = 0.87 pm, the dispersion coefficient *D* is approximately -- 80 ps/km-nm. At â, = 1.55 pm, *D* -- + 17 ps/km-nm. At â - 1.312 pm the dispersion coefficient vanishes, so that v, in (8.3-6) vanishes. A more precise expression for e, that incorporates the spread of the spcctral width o-, about â, = 1.312 pm yields a very small, but nonzero, width.

EXAMPLE 8.3-2. Ptdse Broadening *Associated with* Material OfSpersion. The dispersion coefficient *Dz* -- -- 80 ps/km-nm at â, = 0.87 p m. For a source of linewidth m = 50 nm (from an LED, for example) the pulse spreading rate in a single-mode fibcr with no other sources of dispersion is *D cr* = 4 ns/km. An impulse of light traveling a distance L -- 100 km in the fiber is therefore broadened to a width cr = *D¿|o L* ---- 0.4 ps. The response time of the fiber is then 0.4 ps. An impulse of narrower linewidth w, = 2 rim (from a laser diode, for example) operating near 1.3 pm, where the dispersion cocfficient is 1 ps/km-nm, spreads at a rate of only 2 ps/km. A 100-km fiber thus has a substantially shorter response time, w, - 0.2 ns.

Waveguide Dispersion

The group velocities of the modes depend on the wavelength even if material dispersion is negligible. This dependence, known as waveguide dispersion, results from the dependence of the field distribution in the fiber on the ratio between the core radius and the wavelength *(a/L).* If this ratio is altered, by altering â,, the relative portions of optical power in the core and cladding are modified. Since the phase velocities in the core and cladding are diflerent, the group velocity of the mode is altered. Waveguide dispersion is particularly important in single-mode fibers, where modal dispersion is not exhibited, and at wavelengths for which material dispersion is small (near â, = 1.3 pm in silica glass).

As discussed in Sec. 8.1B, the group velocity c - *(d p/d«i) '* and the propagation constant Q are determined from the characteristic equation, which is governed by the fiber *V* parameter *V* ---- 2s(a/â,)NA = (a - NA/c,)w. In the absence of material dispersion (i.e., when NA is independent of w), 7 is directly proportional to o, so that

$$\frac{1}{u} = \frac{dQ}{dn} = \frac{dQ}{d'}\frac{dV}{dn} = \frac{a\,\mathrm{NA}}{c_p}\frac{dQ}{dV} \qquad (8.3\text{-}8)$$

The pulse broadening associated with a source of spectral width ‹r, is related to the time delay *L/u* by, = *d/d5 pK L/v) m* . Thus

$$\sigma_\lambda \qquad (8.3\text{-}9)$$

where

$$D_p = \frac{d}{d5}\left(\frac{1}{v}\right) = -\frac{\omega}{\lambda_o}\frac{d}{d\omega}\left(\frac{1}{v}\right) \qquad (8.3\text{-}10)$$

is the waveguide dispersion coefficient. Substituting (8.3-8) into (8.3-10) we obtain

$$D_w = -\left(\frac{1}{2\pi c_o}\right)V^2 \frac{d'|3}{dV^2}, \qquad (8.3\text{-}11)$$

Thus the group velocity is inversely proportional to *d fl/dV* and the dispersion coefficient is proportional to *V' **d'|3/dV*** ^{2}**.** The dependence of Q on V is shown in Fig. 8.1-10(b) for the fundamental LP_0; mode. Since Q varies nonlinearly with *V,* the waveguide dispersion coefficient *Dq* is itself a function of *V* and is therefore also a function of the wavelength.' The dependence of *Dp* on â, may be controlled by altering the radius of thc corc or thc **indcx grading** profile for graded-index fibers.

Combined Materia/ *and Waveguide* dispersion

The combined effects of material dispersion and waveguide dispersion (referred to here as **chromatic dispersion)** may be determined by including the wavelength dependence of the refractive indices, n_I And n_2 and therefore NA, when determining *d Q/do* from the characteristic equation. Although generally smaller than material dispersion, waveguide dispersion does shift the wavelength at which the total chromatic dispersion is minimum.

Since chromatic dispersion limits the performance of single-mode fibers, more advanced fiber designs aim at reducing this effect by using graded-index cores with refractive-index profiles selected such that the wavclength at which waveguide dispersion compensates material dispersion is shifted to the wavelength at which the fiber is to be used. **Dispersion-shifted fibers** have been successfully made by using a linearly tapered core refractive index and a reduced core radius, as illustrated in Fig. 8.3-6 *a).* This technique can be used to shift the zero-chromatic-dispersion wavelength from 1.3 pm to 1.55 pm, where the fiber has its lowest attenuation. Note, however, that the process of index grading itself introduces losses since dopants are used. Other grading profiles have been developed for which the chromatic dispersion vanishes at two wavelengths and is reduced for wavelengths between. These fibers, called **dispersion-flattened,** have been implemented by using a quadruple-clad layered grading, as illustrated in Fig. 8.3-6(b).

Combined Material and Wodal Dispersion

The effect of material dispersion on pulse broadening in multimode fibers may be determined by returning to the original equations for the propagation constants 9, of the modes and determining the group velocities *' d /du›) '* with n_1 and nt being functions of ni. Consider, for example, the propagation constants of a graded-index fiber with a large number of modes, which are given by (8.2-19) and (8.2-15). Although *n* and tit ure dependent on o, it is rcasonablc to assumc that thc ratio fi — (n – n_2)/n is approximately independent of o. Using this approximation and evaluating u = *{d Q q/dui) ',* we obtain

$$v_q \approx \frac{c}{!} \; ' \; - \; \frac{p-2}{p+2M} \; q \, e/\wedge +2) \qquad (8.3\text{-}12)$$

where *N — (d/du›)(«i n) - n – X (dn /dX)* is the group index of the core material. Under this approximation, the earlier expression (8.2-21) for rq remains the same, except that the refractive index ii is replaced with the group index *N* . For a step-index fiber *(p* — m), the group velocities of the modes vary from *c p/N* to

* For more details on this topic, see the reading list, particularly the articles by Gloge.

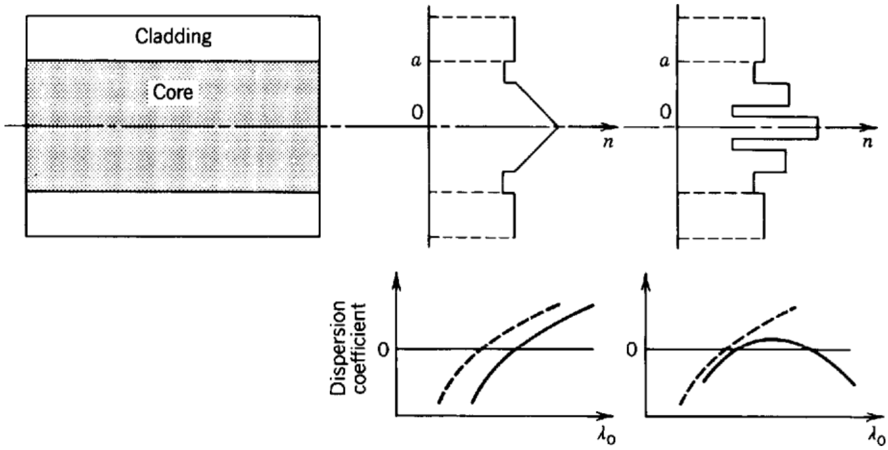


Figure 8.3-6 Refractive-index profiles and schematic wavelength dependences of the material dispersion coefficient (dashed curves) and the combined material and waveguide dispersion coefficients (solid curves) for (u) dispersion-shifted and *(b)* dispersion-fattened fibers.

(c /*N*)(1 — A), so that the response time is

$$\sigma_\tau \quad (c/N)" 2' \qquad (8.3\text{-}13)$$

Response Time
(Multimode Step-Index Fiber
with Material Dispersion)

This should be compared with (8.3-4) when there is no material dispersion.

EXERCISE 8.3-1

***Optimal* Crsde *Profile* Parameter.** Use (8.2-19) and (8.2-15) to derive the following expression for the group velocity rq when both n and A are wavelength dependent:

$$v_q \approx \frac{c_o}{N_1}\left[1 - \frac{p-2-p_s}{p+2}\left(\frac{q}{M}\right)^{Q/(Q+2)}\Delta\right], \qquad q = 1, 2, \ldots, V, \qquad (8.3\text{-}14)$$

where *p* — 2(n */N;)(x/A) db/d u›*. What is the optimal value of the grade profile parameter *p* for minimizing modal dispersion?

Nonlinear Dispersion

Yet another dispersion effect occurs when the intensity of light in the core is sufficiently high, since the refractive indices then become intensity dependent and the material exhibits nonlinear behavior. The high-intensity parts of an optical pulse undergo phase shifts different from the low-intensity parts, so that the frequency is shifted by different amounts. Because of material dispersion, the group velocities are modified, and consequently the pulse shape is altered. Under certain conditions, nonlinear dispersion can compensate material dispersion, so that the pulse travels **without** altering its temporal profile. The guided **wave is** then known as a solitary wave, or a soliton. Nonlinear optics is introduced in Chap. 19 and optical solitons are discussed in Sec. 19.8.

C. Pulse Propagation

As described in the previous sections, the propagation of pulses in optical fibers is governed by attenuation and several types of dispersion. The following is a summary and recapitulation of these effects, ignoring nonlinear dispersion.

An optical pulse of power rJ 'p(i / r $_0$) and short duration zq, where *p(I)* is a function which has unit duration and unit area, is transmitted through a multimode fiber of length L. The received optical power may be written in the form of a sum

$$P(i) \text{ a } \sum^{M} \exp(-0.23o,1) \; {*p} \frac{t-\tau_q}{} \qquad (8.3\text{-}15)$$

where *M* is the number of modes, the subscript q refers to mode q, n, is the attenuation coefficient (dB/km) ' /rq is the delay time, *Hq* S the group velocity, and ‹r > z$_0$ is the width of the pulse associated with mode *q*. In writing (8.3-15), we have implicitly assumed that the incident optical power is distributed equally among the *M* modes of the fiber. It has also been assumed that the pulse shape *p{t)* is not altered; it is only delayed by times and broadened to widths as a result of propagation. As was shown in Sec. 5.6, an initial pulse with a Gaussian profile is indeed broadened without altering its Gaussian nature.

The received pulse is thus composed of *M* pulses of widths ‹r, centered at time delays z,, as illustrated in Fig. 8.3-7. The composite pulse has an overall width e, which represents the overall response time of the fiber.

We therefore identify two **basic** types of dispersion: **intermodal** and **intramodal.** Intermodal, or **simply** modal, dispersion is the delay distortion caused by the disparity among the delay times ip Of the modes. The time difference $_2$ (zpp — zp) between the **longest and** shortest delay constitutes modal dispersion. It is given by (8.3-4) and (8.3-5) for step-index and graded-index fibers with a large number of modes, respectively. Material dispersion has some effect on modal dispersion since it affects the delay times. For example, (8.3-13) gives the modal dispersion of a multimode fiber with material dispersion. Modal dispersion is directly proportional to the fiber length L, except for long fibers, in which mode coupling plays a role, whereupon it becomes proportional to L^{I*2}.

Intramodal dispersion is the broadening of the pulses associated with the individual modes. It is caused by a combination of material dispersion and waveguide dispersion

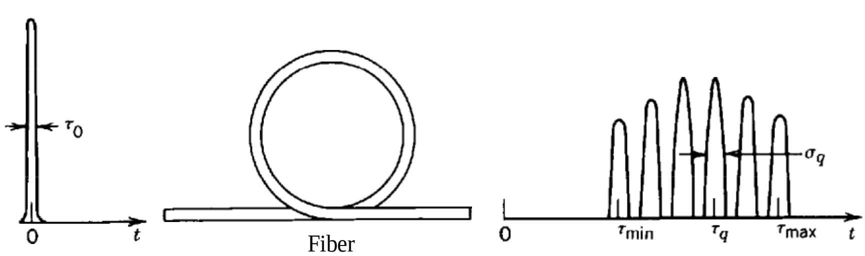


Figure e.3-7 Response of a multimode fiber to a single pulse.

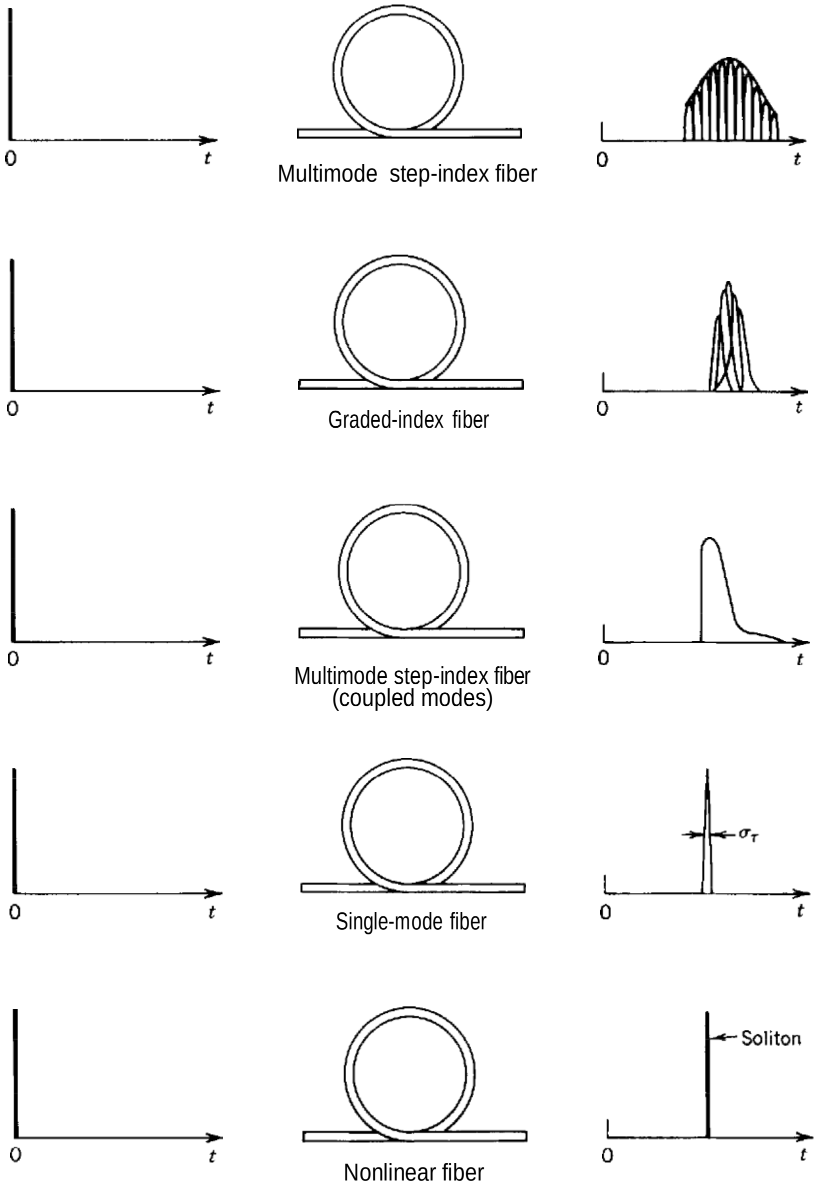


Figure 8.3-8 Broadening of a short optical pulse after transmission through different types of fibers. The width of the transmitted pulse is governed by modal dispersion in multimode (step-index and graded-index) fibers. In single-mode fibers the pulse width is determined by material dispersion and waveguide dispersion. Under certain conditions an intense pulse, called a soliton, can travel through a nonlinear fiber without broadening. This is a result of a balance between material dispersion and self-phase modulation (the dependence of the refractive index on the light intensity).

resulting from the finite spectral width of the initial optical pulse. The width e, is given by

$$\sigma_q^2 \approx \tau_0^2 + \left(D_q \sigma_\lambda L\right)^2, \qquad \textbf{(8.3-16)}$$

where *D* is a dispersion coefficient representing the combined effects of material and waveguide dispersion for mode *q*. Material dispersion is usually more significant. For a very short initial width zb, (8.3-16) gives

$$\sigma_q \approx D_q \sigma_\lambda L. \qquad \textbf{(8.3-17)}$$

Figure 8.3-8 is a schematic illustration in which the profiles of pulses traveling through different types of fibers are compared. In multimode step-index fibers, the modal dispersion }(z„,p — zq;,) is usually much greater than the material/waveguide d ispe rsion e„ so that intermodal dispersion dominates and m, = 1 (zpp — spin). In multimode graded-index fibers, 2 (rpp — sp in) may be comparable to „ so that thc overall pulse width involves all dispersion effects. In single-mode fibers, there is obviously no modal dispersion and the transmission of pulses is limited by material and waveguide dispersion. The lowest overall dispersion is achieved in a single-mode fiber operating at the wavelength for which the combined material—waveguide dispersion vanishes.

READING LIST

Books

See also the books on optical waveguides in Chapter 7.

P. K. Cheo, *Fiber Optics and Optoelectronics,* Prentice Hall, Englewood Cliffs, NJ, 1985, 2nd ed. **1990.**

F. C. Allard, *Fiber Gptics Handbook fur Engineers and Scientists,* McGraw-Hill, New York, 1990.

C. Yeh, *Handbook of Fiber Optics' Theury and Applications,* Academic Press, Orlando, FL, 1990.

L. B. Jeunhomme, *Single-Mode Fiber Optics,* Marcel Dekker, New York, 1983, 2nd ed. 1990.

P. W. France, ed., *Fluoride Glass Optical Fibers,* CRC Press, Boca Raton, FL, 1989.

P. Diament, *Wave Transmission rind Fiber Of›tic.s,* Macmillan, New York, 19b9.

W. B. Jones, Jr., *Introduction to Optical Fitter Communication Systems,* Holt, Rinehart and Winston, New York, l9Sh.

H. Murata, *Handbook of Optical Fibers and Cables,* Marcel Dekker, New York, 1988.

E. G. Neuman, *Single-Mode Fitters—Fundameninls,* Springer-Verlag, New Ytirk, 1988.

E. L. Salford, Jr. and J. A. McCann, *Fiberoptics and Laser Handbook,* Tab Books, Blue Ridge Summit, PA, 2nd ed. l9d8.

S. E. Miller and I. Kaminow, *Optical Fiber Telecommunications II,* Academic Press, Boston, MA, 1988.

J. Gowar, *Optical Communication Systems,* Prentice Hall, Englewood Cliffs, NJ, 1984.

R. G. Seippel, *Fiber Optics,* Reston Publishing, Reston, VA, 1984.

ANSI/IEEE Standards 812-1984, ***IEEE Slandurâ*** *Definitions uf Terms Relating tu Fiber Optics,* IEEE, New York, 1984.

A. H. Cherin, *An Introduction to Optical Fibers,* McGraw-Hill, New York, 1983.

G. E. Keiser, *Optical Fiber Communications,* McGraw-Hill, New York, 1983.

C. Hentschel, *Fiber Optics Handbook,* Hewlett-Packard, Palo Alto, CA, 1983.

Y. Suematsu and **K. 1ga,** *Introduction to Optical Fiber* ***Communications,*** Wiley, New York, 1982.

T. Okoshi, *Optical Fibers,* Academic Press, New York, 1982.

C. K. Kao, *Optical fiber Systems,* McGraw-Hill, New York, 1982.

E. A. Lacy, *Fiber Optics,* Prentice-Hall, Englewood Cliffs, NJ, 1982.

D. Marcuse, *Light Transmission Optics,* Van Nostrand Reinhold, New York, 1972, 2nd ed. 1982.

D. Marcuse, *Principles of Optical Fiber Measurements,* Academic Press, New York, 1981.

A. B. Sharma, S. J. Halme, and M. M. Butusov, *Optical Fiber Systems and Their Components,* Springer-Verlag, Berlin, 1951.

CSELT (Centro Studi e Laboratori Telecomunicazioni), *Optical Fibre Communications,* McGraw-Hill, New York, 1981.

M. K. Barnoski, ed., *Fundamentals of Optical Fiber Cammunications,* Academic Press, New York, 1976, 2nd ed. 1981.

C. P. Sandbank, ed., *Optical Ftbre Communication Systems,* Wiley, New York, 1980.

M. J. Howes and D. V. Morgan, eds., *Optical Fibre Communications,* Wiley, New York, 1980.

H. F. Wolf, ed., *Handbook of Fiber Optics,* Garland STPM Press, New York, 1979.

D. B. Ostrowsky, ed., *Fiber and Integrated Optics,* Plenum Press, New York, 1979.

J. E. Midwinter, *Optical Fibers for Transmission,* Wiley, New York, 1979.

S. E. Miller and A. G. Chynoweth, *Optical Fiber Telecommunications,* Academic Press, New York, 1979.

G. R. Elion and H. A. Elion, *Fiber Optics in Communication Systems,* Marcel Dekker, New York, 1978.

H. G. Unger, *Planar Optical Waveguides and Fibers,* Clarendon Press, Oxford, 1977.

J. A. Arnaud, *Beam and Fiber Optics,* Academic Press, New York, 1976.

W. B. Allan, *Fibre Optic.s: Theory and Practice,* Plenum Press, New York, 1973.

N. S. Kapany, *Fiber Optics: Principles and Applications,* Academic Press, New York, 1967.

Special Journal Issues

Special issue on fiber-optic sensors, *Journal of Lightwave Technology,* vol. LT-5, no. 7, 1957.

Special issue on fiber, cable, and splicing technology, *Journal of Lightwat!e Technology,* vol. LT-4, no. 8, 1986.

Special issue on low-loss fibers, *Journal of Lightwave Technology,* vol. LT-2, no. 10, 1984.

Special issue on fiber optics, *IEEE Transactions on Communications*, vol. COM-26, no. 7, 1975.

Articles

M. G. Drexhage and C. T. Moynihan, Infrared Optical Fibers, *Scientific American,* vol. 259, no. 5, pp. 110—114, 1988.

S. R. Nagel, Optical Fiber—the Expanding Medium, *IEEE Communications Magazine,* vol. 25, no. 4, pp. 33—43, 1987.

R. H. Stolen and R. P. DePaula, Single-Mode Fiber Components, *Proceedings of the IEEE,* vol. 75, pp. 1498— 1511, 1987.

P. S. Henry, Lightwave Primer, *IEEE Journal of Quantum Electronics,* vol. QE-21, pp. 1862—1579, 1985.

T. Li, Advances in Optical Fiber Communications: An Historical Perspective, *IEEE Journal on Selected Areas in Communications,* vol. SAC-1, pp. 356—372, 1983.

I. P. Kaminow, Polarization in Optical Fibers, *IEEE Journal of* Quantum *Electronics,* vol. **0E-17,** pp. 15-22, 1981.

P. J. B. Clarricoats, Optical Fibre Waveguides—A Review, in *Progress in Optics,* vol. 14, E. Wolf, ed., North-Holland, Amsterdam, 1977.

D. Gloge, Weakly Guiding Fibers, *Applied Optics,* vol. 10, pp. 2252-2258, 1971.

D. Gloge, Dispersion in Weakly Guiding Fibers, *Applied Optics,* vol. 10, pp. 2442—2445, 1971.

PROBLEMS

8.1-1 **Coupling Efficiency.** (a) A source emits light with optical power P and a distribution *It8) - 1/r)*P_0 *cos 8,* where /(B) is the power per unit solid angle in the direction making an angle 8 with the axis of a fiber. Show that the power collected

by the fiber is *P* — $(NA)^2$Pq, i.e., the coupling efficiency is NA^2 where NA is the numerical aperture of the fiber.
(b) If the source is a planar light-emitting diode of refractive index '!S bonded to the fiber, and assuming that the fiber cross-sectional area is larger than the LED emitting area, calculate the numerical aperture of the fiber and the coupling efficiency when n, = 1.46, n_2 = 1.455, and n, = 3.5.

8.1-2 **Modes.** A step-index fiber has radius a = 5 p m, core refractive index o, = 1.45, and fractional refractive-index change fi = 0.002. Determine the shortcst wavelength â for which the fiber is a single-mode waveguide. If the wavelength is changed to â,./2, identify the indices *1,* m) of all the guided modes.

8.1-3 Modal Dispersion. A step-index fibcr of numerical aperture NA = 0.16, core radius *a* —- 45 pm and core refractive index n_I = 1.45 is used at â = 1.3 m, where material dispersion is negligible. If a pulse of light of very short duration enters thc fiber at *t* — 0 and travels a distance of 1 km, sketch the shape of the receivcd pulse:
(a) Using ray optics and assuming that only meridional rays are allowed.
(b) Using wave optics and assuming that only meridional (/ = 0) modes are allowed.

8.1-4 Propagation Constants and **Group** Velocities. A step-index fiber with refractive indices ri, = 1.444 and n, = 1.443 operates at X, = 1.55 pm. Determine the core radius at which the fiber *V* parameter is 10. Use Fig. 8.1-6 to estimate the propagation constants of all the guided modes with *1* — 0. If the core radius is now changed so that *V* — 4, use Fig. 8.1-10(ti) to determine the propagation constant and the group velocity of the LP_0 mode. *Hint:* Derive an expression for the group velocity *v - dQ/du›) '* in terms of *d Q/dV* and use Fig. 8.1-10(b) to estimate *dQ/dV.* Ignore the effect of material dispersion.

8.21 Numerical **Aperture of a Graded-Index Fiber.** Compare the numerical apertures of a step-index fiber with *n* = 1.45 and fi = 0.01 and a graded-index fiber with n_1 = 1.45, fi = 0.01, and a parabolic refractive-index profile (p = 2). (See Exercise 1.3-2 on page 24.)

8.2-2 **Propagation Constants and Wavevector (Step-Index Fiber). A** step-index fiber of radius $a = 20$ μm and refractive indices $n_1 = 1.47$ and $n_2 = 1.46$ operates at â = 1.55 p m. Using the quasi-plane wave theory and considering only guided modes with azimuthal index f = 1:
(a) Determine the smallest and largest propagation constants.
(b) For the mode with the smallest propagation constant, determine the radii of the cylindrical shell within which the wave is confined, and the components of the wavevector k at r = 5 pm.

8.2-3 **Propagation Constants and Wavevector (Graded-Index Fiber).** Repeat Problem 8.2-2 for a graded-index fiber with parabolic refractive-index profile with *p* — 2.

8.3-1 **Scattering Loss.** At ñ = b20 nm the absorption loss of a fiber is 0.25 dB /km and the scattering loss is 2.25 dB km. If the fiber is used instead at â = 600 nm and calorimetric measurements of the heat generated by light absorption give a loss of 2 dB km, estimate the total attenuation at â — 600 nm.

8.3-2 Modal **Dispersion in Step-Index Fibers.** Dctermine the core radius of a multimode step-index fiber with a numerical aperture NA = 0.1 if the number of modes *M* — 5000 when the wavelength is 0.87 pm. If the core refractive index ri, = 1.445, the group index *N* — 1.456, and A is approximately independent of wavelength, determine the modal-dispersion response time u-, for a 2-km fiber.

8.3-3 Modal **Dispersion in Graded•Index Fibers.** Consider a graded-index fiber with n /3, = 10, n, = 1.45, A = 0.01, and a power-law profile with index *p.* Determine the number of modes *M*, and the modal-dispersion pulse-broadening rate *‹r /L* for *p* — 1.9, 2, 2.1, and .

8.3-4 **Pulse Propagation. A** pulse of initial width zq is transmitted through a graded-index fiber of length *L* kilometers and power-law refractive-index profile with profile index *p*. The peak refractive index *n* is wavelength-dependent with *D* — — *z /c p)d* ^{2}*n /dh'p, b* is approximately independent of wavelength, o-, is the source's spectral width, and â is the operating wavelength. Discuss the effect of increasing each of the following parameters on the width of the received pulse: L, rb, *p, D* |, ‹r , and â,.

UNIT – II
Signal Degradation in Optical Fibers

Signal Attenuation & Distortion in Optical Fibers

- What are the loss or signal attenuation mechanism in a fiber?
- Why & to what degree do optical signals get distorted as they propagate down a fiber?

- Signal attenuation (fiber loss) largely determines the maximum repeaterless separation between optical transmitter & receiver.
- Signal distortion cause that optical pulses to broaden as they travel along a fiber, the overlap between neighboring pulses, creating errors in the receiver output, resulting in the limitation of information-carrying capacity of a fiber.

Attenuation (fiber loss)

- Power loss along a fiber:

Z=0
P(0) mW

Z= l

$$P(l) = P(0)e^{-\alpha_p l} \quad \text{mw}$$

$$P(z) = P(0)e^{-\alpha_p z} \qquad [3\text{-}1]$$

- The parameter α_p is called fiber attenuation coefficient in a units of for example [1/km] or [nepers/km]. A more common unit is [dB/km] that is defined by:

$$\alpha[\text{dB/km}] = \frac{10}{l}\log\left[\frac{P(0)}{P(l)}\right] = 4.343\alpha_p \ [1/\text{km}] \qquad [3\text{-}2]$$

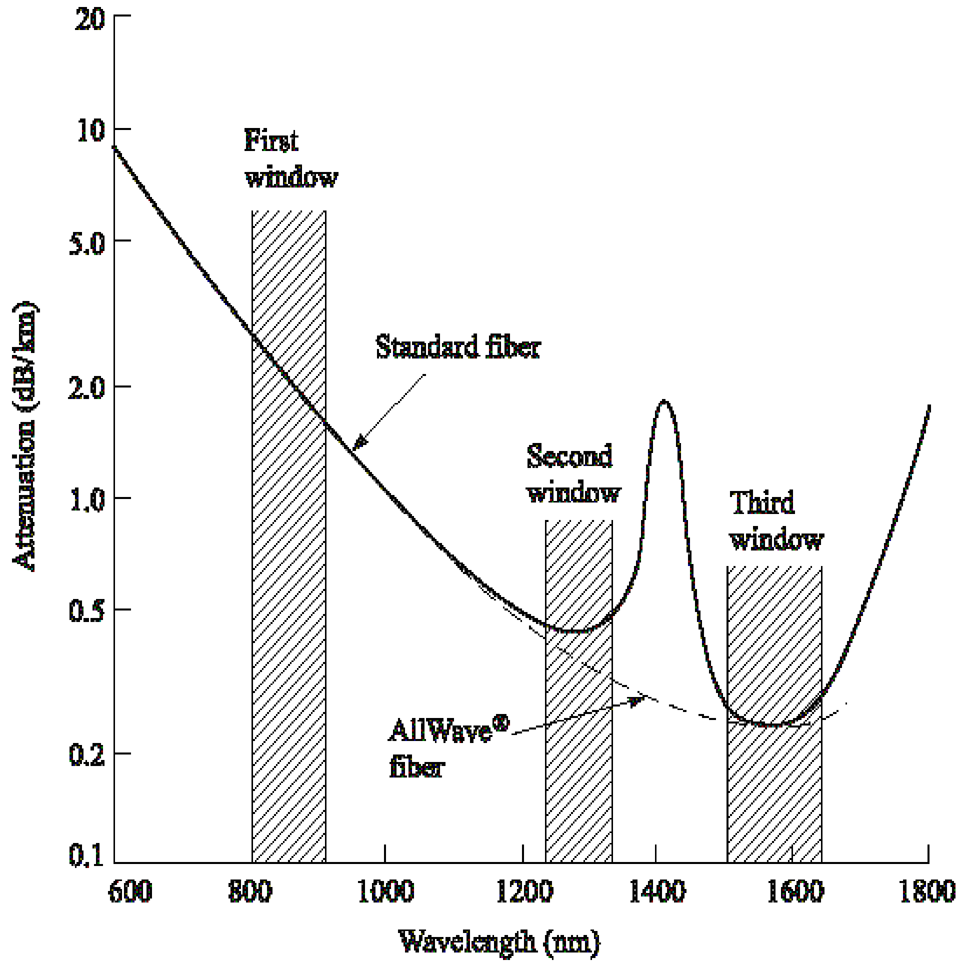
Fiber loss in dB/km

$P(0)[\mathrm{dBm}]$

$$P(l)[\mathrm{dBm}] = P(0)[\mathrm{dBm}] - \alpha[\mathrm{dB/km}] \times l[\mathrm{km}] \qquad [3\text{-}3]$$

- Where [dBm] or dB milliwat is 10log(*P* [mW]).

Optical fiber attenuation vs. wavelength



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Absorption

- Absorption is caused by three different mechanisms:

1- Impurities in fiber material: from transition metal ions (must be in order of ppb) & particularly from OH ions with absorption peaks at wavelengths 2700 nm, 400 nm, 950 nm & 725nm.

2- Intrinsic absorption (fundamental lower limit): electronic absorption band (UV region) & atomic bond vibration band (IR region) in basic SiO2.

3- Radiation defects

Scattering Loss

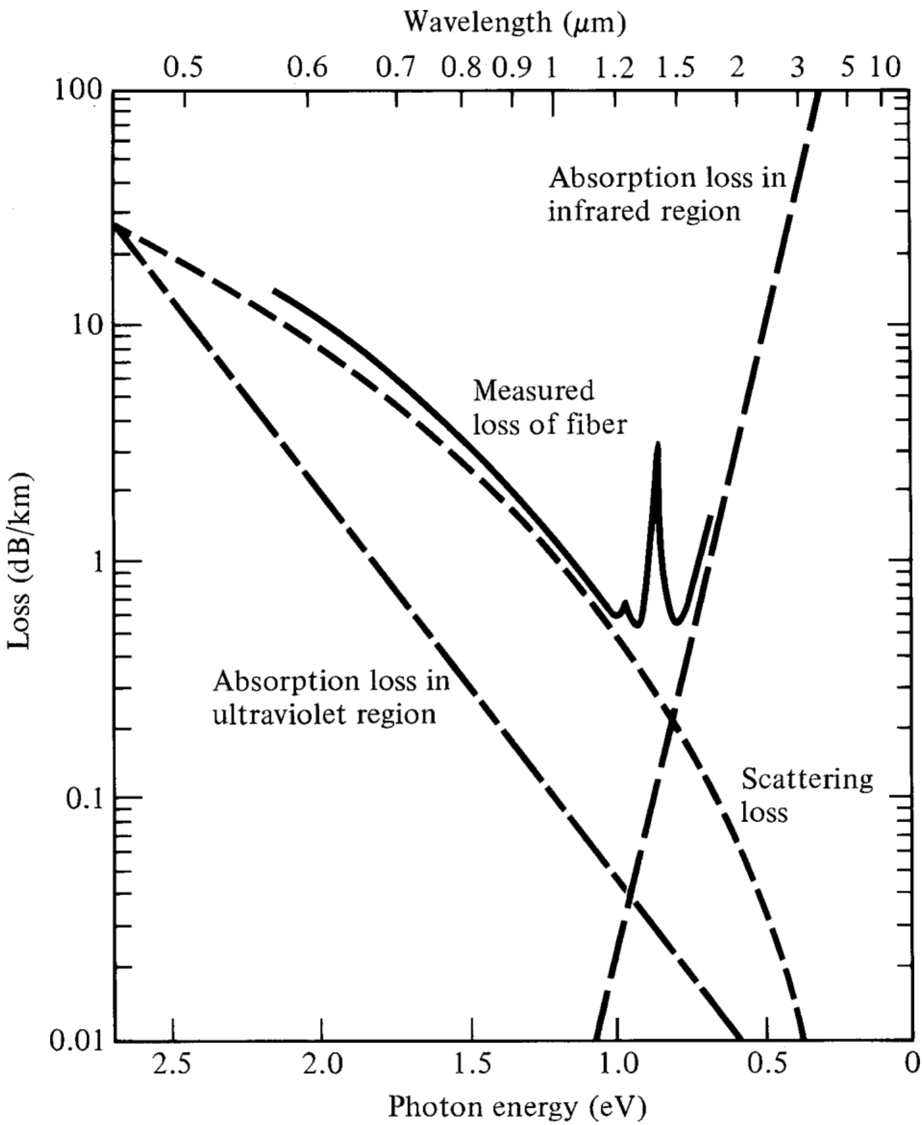
- Small (compared to wavelength) variation in material density, chemical composition, and structural inhomogeneity scatter light in other directions and absorb energy from guided optical wave.

- The essential mechanism is the Rayleigh scattering. Since the black body radiation classically is proportional to λ^{-4} (this is true for wavelength typically greater than 5 micrometer), the attenuation coefficient due to Rayleigh scattering is approximately proportional to λ^{-4}. This seems to me not precise, where the attenuation of fibers at 1.3 & 1.55 micrometer can be exactly predicted with Planck's formula & can not be described with Rayleigh-Jeans law. Therefore I believe that the more accurate formula for scattering loss is

$$\alpha_{scat} \propto \lambda^{-5}\left[\exp\left(\frac{hc}{\lambda k_B T}\right)\right]^{-1}$$

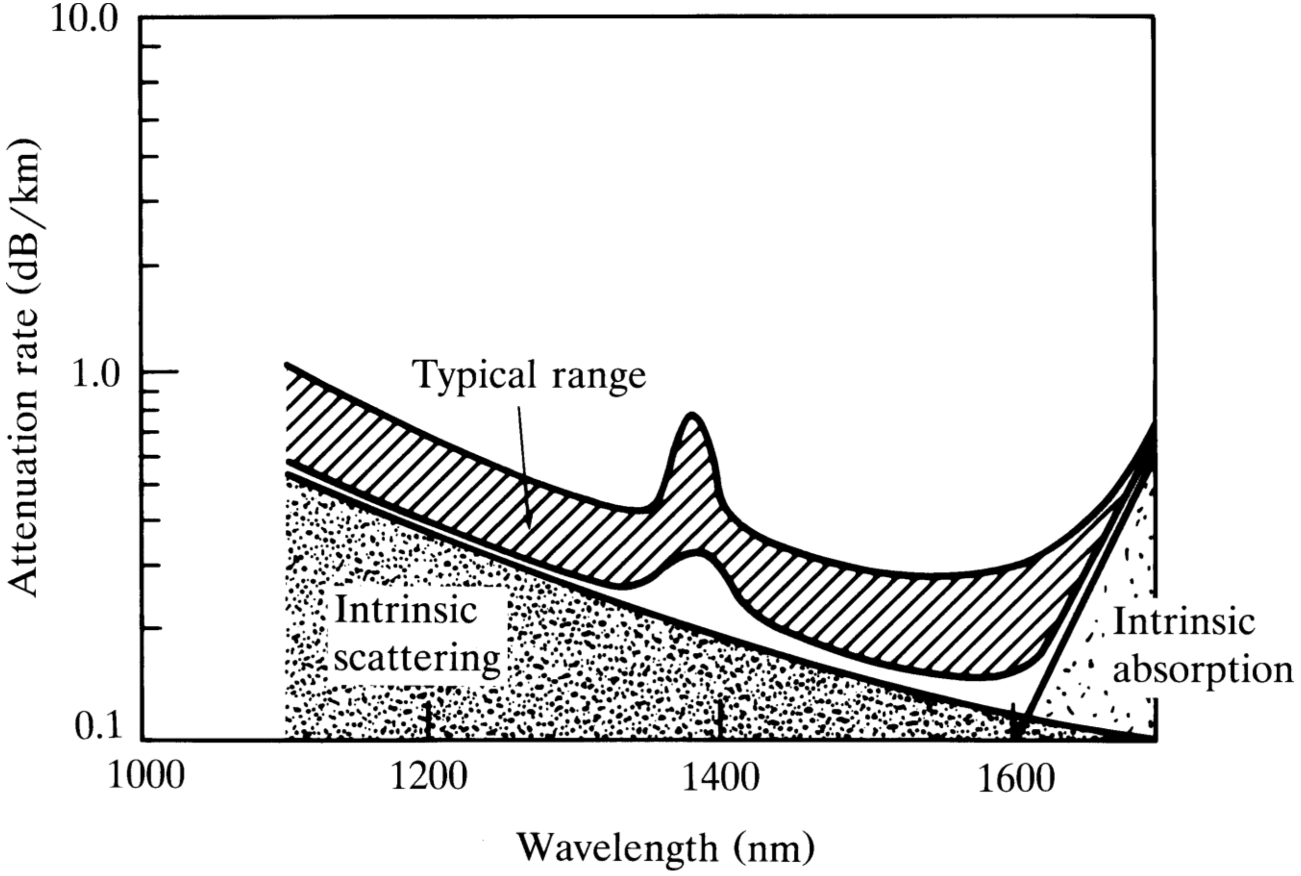
$$h = 6.626\times10^{-34}\ \text{Js}, \quad k_B = 1.3806\times10^{-23}\ \text{JK}^{-1},\ T : \text{Temperature}$$

Absorption & scattering losses in fibers



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

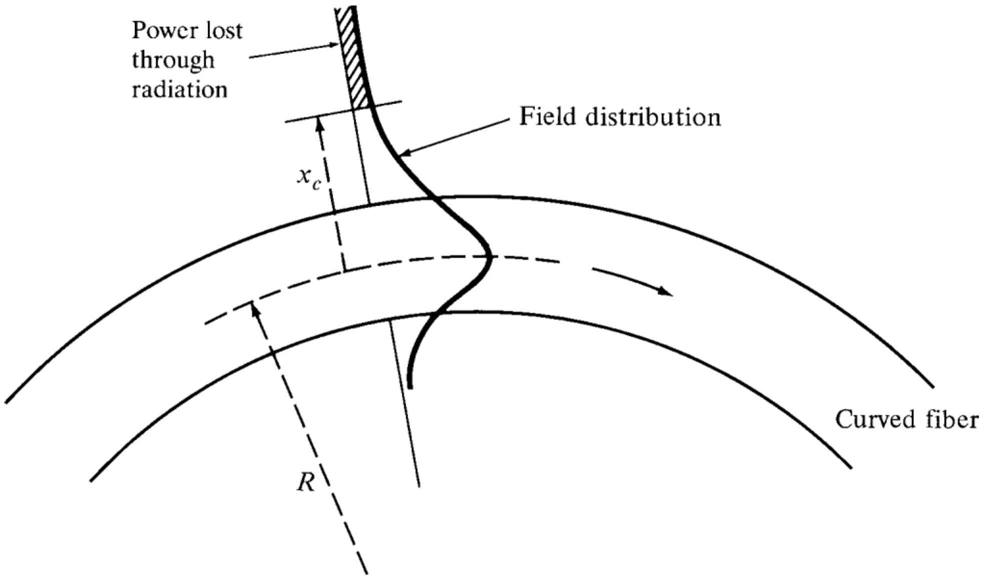
Typical spectral absorption & scattering attenuations for a single mode-fiber



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Bending Loss (Macrobending & Microbending)

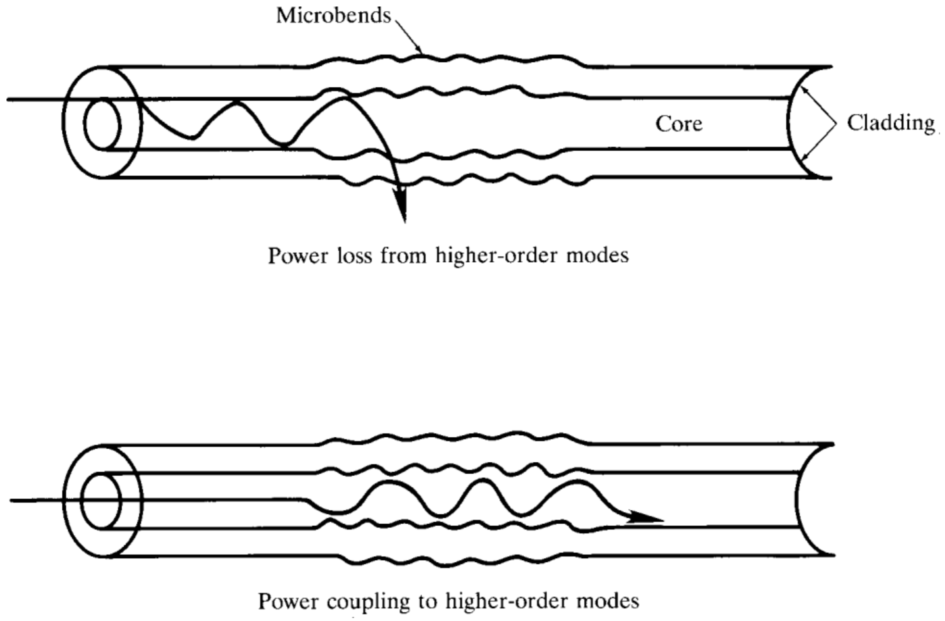
- **Macrobending Loss**: The curvature of the bend is much larger than fiber diameter. Lightwave suffers sever loss due to radiation of the evanescent field in the cladding region. As the radius of the curvature decreases, the loss increases exponentially until it reaches at a certain critical radius. For any radius a bit smaller than this point, the losses suddenly becomes extremely large. Higher order modes radiate away faster than lower order modes.



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Microbending Loss

- **Microbending Loss:** microscopic bends of the fiber axis that can arise when the fibers are incorporated into cables. The power is dissipated through the microbended fiber, because of the repetitive coupling of energy between guided modes & the leaky or radiation modes in the fiber.



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Dispersion in Optical Fibers

- **Dispersion**: Any phenomenon in which the velocity of propagation of any electromagnetic wave is wavelength dependent.

- In communication, dispersion is used to describe any process by which any electromagnetic signal propagating in a physical medium is degraded because the various wave characteristics (i.e., frequencies) of the signal have different propagation velocities within the physical medium.

- There are 3 dispersion types in the optical fibers, in general:

1- Material Dispersion
2- Waveguide Dispersion
3- Polarization-Mode Dispersion

Material & waveguide dispersions are main causes of Intramodal Dispersion.

Group Velocity

- Wave Velocities:
- 1- **Plane wave velocity**: For a plane wave propagating along *z*-axis in an unbounded homogeneous region of refractive index n_1, which is represented by $\exp(j\omega t - jk_1 z)$, the velocity of constant phase plane is:

$$v = \frac{\omega}{k_1} = \frac{c}{n_1} \qquad [3\text{-}4]$$

- 2- **Modal wave phase velocity**: For a modal wave propagating along *z*-axis represented by $\exp(j\omega t - j\beta z)$, the velocity of constant phase plane is:

$$v_p = \frac{\omega}{\beta} \qquad [3\text{-}5]$$

3- For transmission system operation the most important & useful type of velocity is the **group velocity,** V_g **.** This is the actual velocity which the signal information & energy is traveling down the fiber. It is always less than the speed of light in the medium. The observable delay experiences by the optical signal waveform & energy, when traveling a length of *l* along the fiber is commonly referred to as **group delay.**

Group Velocity & Group Delay

- The group velocity is given by:

$$V_g = \frac{d\omega}{d\beta} \qquad [3\text{-}6]$$

- The group delay is given by:

$$\tau_g = \frac{l}{V_g} = l\frac{d\beta}{d\omega} \qquad [3\text{-}7]$$

- It is important to note that all above quantities depend both on **frequency** & the **propagation mode**. In order to see the effect of these parameters on group velocity and delay, the following analysis would be helpful.

Input/Output signals in Fiber Transmission System

- The optical signal (complex) waveform at the input of fiber of length *l* is *f(t)*. The propagation constant of a particular modal wave carrying the signal is $\beta(\omega)$. Let us find the output signal waveform *g(t)*.

$\Delta\omega$ is the optical signal bandwidth.

$$f(t) = \int_{\omega_c - \Delta\omega}^{\omega_c + \Delta\omega} \tilde{f}(\omega) e^{j\omega t} d\omega \qquad [3\text{-}8]$$

$$g(t) = \int_{\omega_c - \Delta\omega}^{\omega_c + \Delta\omega} \tilde{f}(\omega) e^{j\omega t - j\beta(\omega) l} d\omega \qquad [3\text{-}9]$$

If $\Delta\omega << \omega_c$

$$\beta(\omega) = \beta(\omega_c) + \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c}(\omega-\omega_c) + \frac{1}{2}\left.\frac{d^2\beta}{d\omega^2}\right|_{\omega=\omega_c}(\omega-\omega_c)^2 + \ldots \qquad [3\text{-}10]$$

$$g(t) = \int_{\omega_c-\Delta\omega/2}^{\omega_c+\Delta\omega/2} \tilde{f}(\omega) e^{j\omega t - j\beta(\omega)l} d\omega \approx \int_{\omega_c-\Delta\omega/2}^{\omega_c+\Delta\omega/2} \tilde{f}(\omega) e^{j\omega t - j[\beta(\omega_c) + \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c}(\omega-\omega_c)]l} d\omega$$

$$\approx e^{-j\beta(\omega_c)l} \int_{\omega_c-\Delta\omega/2}^{\omega_c+\Delta\omega/2} \tilde{f}(\omega) e^{j\omega(t - l\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c})} d\omega$$

$$= e^{-j\beta(\omega_c)l} f(t - l\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c}) = e^{-j\beta(\omega_c)l} f(t-\tau_g) \qquad [3\text{-}11]$$

$$\tau_g = l\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c} = \frac{l}{V_g} \qquad [3\text{-}14]$$

Intramodal Dispersion

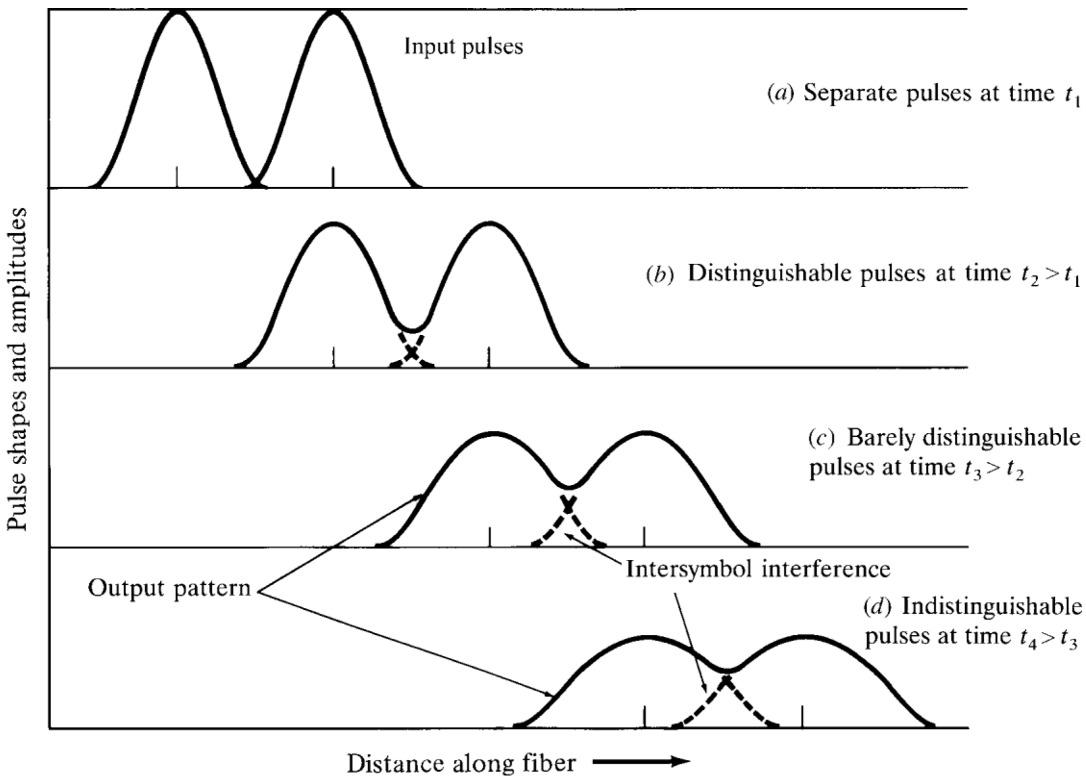
- As we have seen from Input/output signal relationship in optical fiber, the output is proportional to the delayed version of the input signal, and the delay is inversely proportional to the group velocity of the wave. Since the propagation constant, $\beta(\omega)$, is frequency dependent over band width $\Delta\omega$ sitting at the center frequency ω_c , at each frequency, we have one propagation constant resulting in a specific delay time. As the output signal is collectively represented by group velocity & group delay this phenomenon is called **intramodal dispersion or Group Velocity Dispersion (GVD)**. **This phenomenon arises due to a finite bandwidth of the optical source, dependency of refractive index on the wavelength and the modal dependency of the group velocity**.

- In the case of optical pulse propagation down the fiber, GVD causes pulse broadening, leading to Inter Symbol Interference (ISI).

Dispersion & ISI

A measure of information capacity of an optical fiber for digital transmission is usually specified by the **bandwidth distance product** $BW \times L$ **in GHz.km.**

For multi-mode step index fiber this quantity is about 20 MHz.km, for graded index fiber is about 2.5 GHz.km & for single mode fibers are higher than 10 GHz.km.



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

How to characterize dispersion?

- Group delay per unit length can be defined as:

$$\frac{\tau_g}{L} = \frac{d\beta}{d\omega} = \frac{1}{c}\frac{d\beta}{dk} = -\frac{\lambda^2}{2\pi c}\frac{d\beta}{d\lambda} \qquad \text{[3-15]}$$

- If the spectral width of the optical source is not too wide, then the delay difference per unit wavelength along the propagation path is approximately $\frac{d\tau_g}{d\lambda}$. For spectral components which are $\delta\lambda$ apart, symmetrical around center wavelength, the total delay difference $\delta\tau$ over a distance L is:

$$\delta\tau = \left|\frac{d\tau_g}{d\lambda}\right|\delta\lambda = -\frac{L}{2\pi c}\left(2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)\delta\lambda$$

$$= \left|\frac{d\tau}{d\omega}\right|\delta\omega = \frac{d}{d\omega}\left(\frac{L}{V_g}\right)\delta\omega = L\left(\frac{d^2\beta}{d\omega^2}\right)\delta\omega \qquad \text{[3-16]}$$

- $\beta_2 \equiv \frac{d^2\beta}{d\omega^2}$ is called **GVD parameter,** and shows how much a light pulse broadens as it travels along an optical fiber. The more common parameter is called **Dispersion,** and can be defined as the delay difference per unit length per unit wavelength as follows:

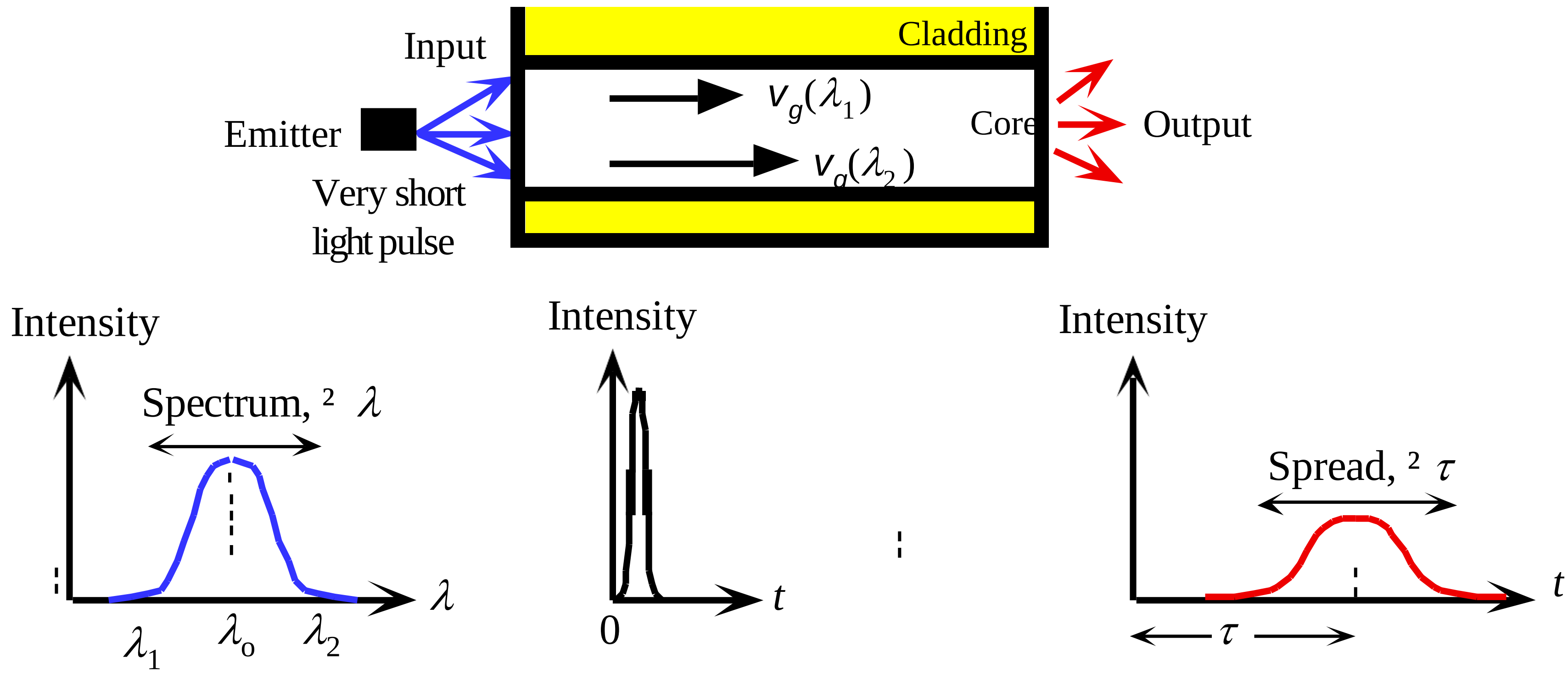
$$D = \frac{1}{L}\frac{d\tau_g}{d\lambda} = \frac{d}{d\lambda}\left(\frac{1}{V_g}\right) = -\frac{2\pi c}{\lambda^2}\beta_2 \qquad [3\text{-}17]$$

- In the case of optical pulse, if the spectral width of the optical source is characterized by its rms value of the Gaussian pulse σ_λ , the pulse spreading over the length of L, σ_g can be well approximated by:

$$\sigma_g \approx \left|\frac{d\tau_g}{d\lambda}\right|\sigma_\lambda = DL\sigma_\lambda \qquad [3\text{-}18]$$

- *D* has a typical unit of [ps/(nm.km)].

Material Dispersion



All excitation sources are inherently non-monochromatic and emit within a spectrum, $^2\ \lambda$, of wavelengths. Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of n_1. The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

Material Dispersion

- The refractive index of the material varies as a function of wavelength, $n(\lambda)$
- Material-induced dispersion for a plane wave propagation in homogeneous medium of refractive index *n*:

$$\tau_{mat} = L\frac{d\beta}{d\omega} = -\frac{\lambda^2}{2\pi c}L\frac{d\beta}{d\lambda} = -\frac{\lambda^2}{2\pi c}L\frac{d}{d\lambda}\left[\frac{2\pi\, n(\lambda)}{\lambda}\right]$$
$$= \frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right) \qquad [3\text{-}19]$$

- The pulse spread due to material dispersion is therefore:

$$\sigma_g \approx \left|\frac{d\tau_{mat}}{d\lambda}\right|\sigma_\lambda = \frac{L\sigma_\lambda}{c}\left|\lambda\frac{d^2 n}{d\lambda^2}\right| = L\sigma_\lambda\left|D_{mat}(\lambda)\right| \qquad [3\text{-}20]$$

$D_{mat}(\lambda)$ is material dispersion

Material Dispersion Diagrams

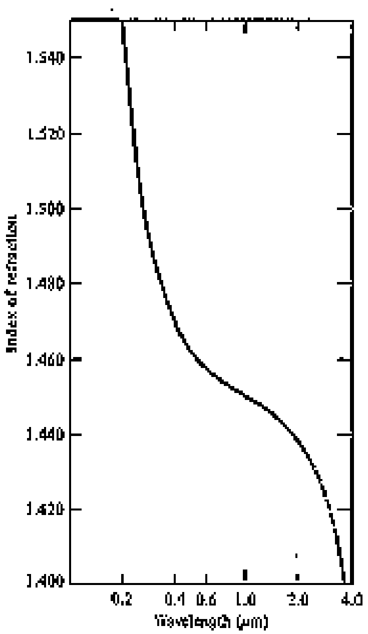


FIGURE 3-12
Variations in the index of refraction as a function of the optical wavelength for silica. (Reproduced with permission from I. H. Malitson, *J. Opt. Soc. Amer.*, vol. 55, pp. 1205–1209, Oct. 1965.)

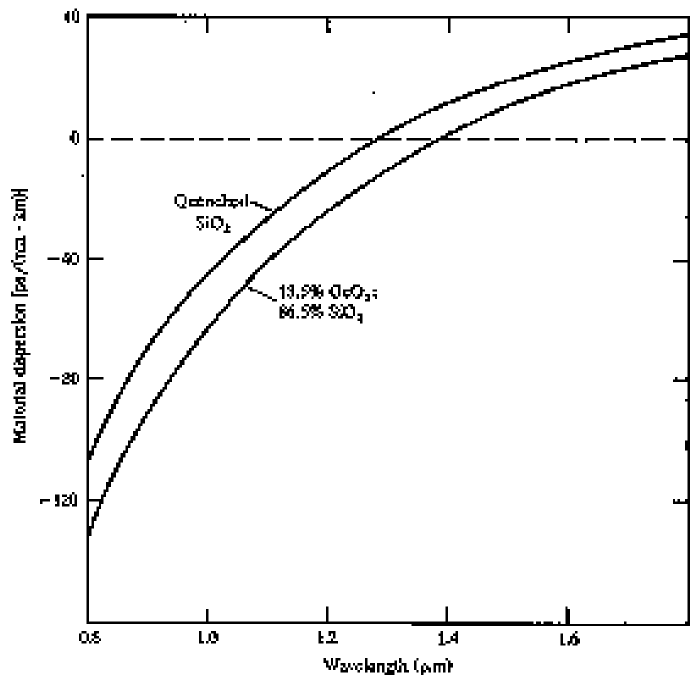


FIGURE 3-13
Material dispersion as a function of optical wavelength for pure silica and 13.5 percent GeO_2/86.5 percent SiO_2. (Reproduced with permission from J. W. Fleming, *Electron. Lett.*, vol. 14, pp. 326–328, May 1978.)

Waveguide Dispersion

- Waveguide dispersion is due to the dependency of the group velocity of the fundamental mode as well as other modes on the *V* number, (see Fig 2-18 of the textbook). In order to calculate waveguide dispersion, we consider that *n* is not dependent on wavelength. Defining the normalized propagation constant *b* as:

$$b = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2} \approx \frac{\beta/k - n_2}{n_1 - n_2} \qquad [3\text{-}21]$$

- solving for propagation constant:

$$\beta \approx n_2 k (1 + b\Delta) \qquad [3\text{-}22]$$

- Using *V* number:

$$V = ka(n_1^2 - n_2^2)^{1/2} \approx kan_2\sqrt{2\Delta} \qquad [3\text{-}23]$$

Waveguide Dispersion

- Delay time due to waveguide dispersion can then be expressed as:

$$\tau_{wg} = \frac{L}{c}\left[n_2 + n_2 \Delta \frac{d(Vb)}{dV} \right] \qquad [3\text{-}24]$$

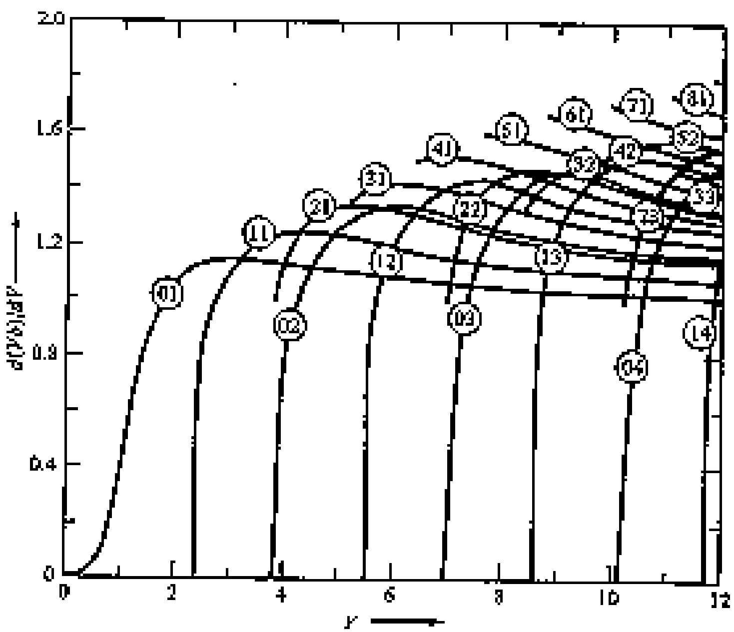


FIGURE 3-14
The group delay arising from waveguide dispersion as a function of the V number for a step-index optical fiber. The curve numbers jm designate the LP_{jm} modes. (Reproduced with permission from Gloge.[37])

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Waveguide dispersion in single mode fibers

- For single mode fibers, waveguide dispersion is in the same order of material dispersion. The pulse spread can be well approximated as:

$$\sigma_{wg} \approx \left|\frac{d\tau_{wg}}{d\lambda}\right|\sigma_{\lambda} = L\sigma_{\lambda}\left|D_{wg}(\lambda)\right| = \frac{n_2 L\Delta\sigma_{\lambda}}{c\lambda} V \frac{d^2(Vb)}{dV^2} \quad [3\text{-}25]$$

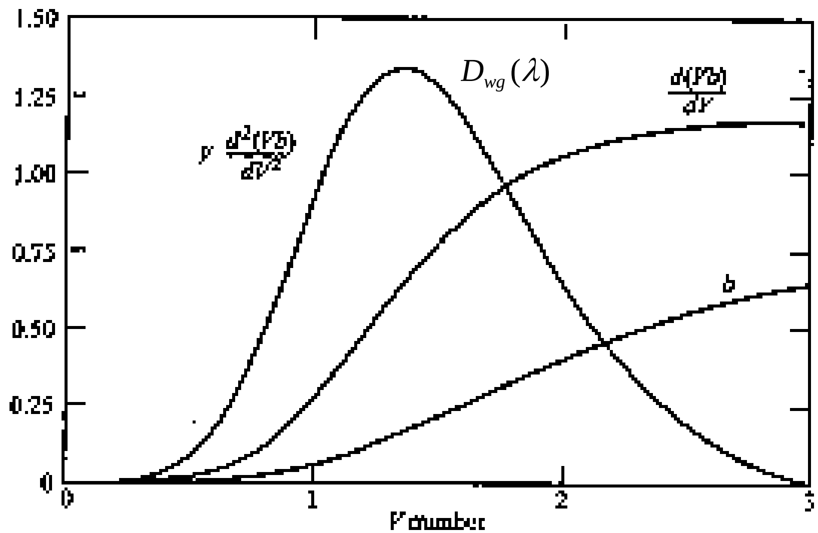
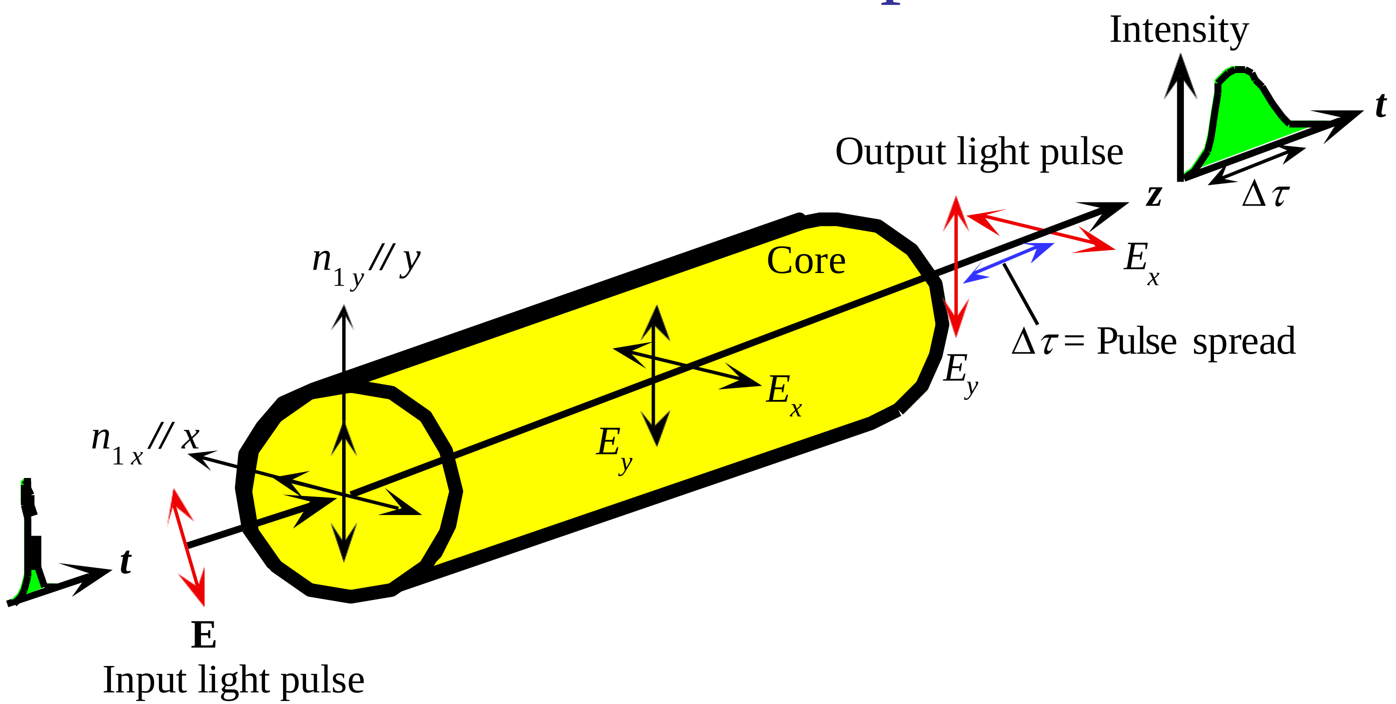


FIGURE 3-15
The waveguide parameter b and its derivatives $d(Vb)/dV$ and $V\,d^2(Vb)/dV^2$ plotted as a function of the V number for the HE_{11} mode.

Polarization Mode dispersion



Suppose that the core refractive index has different values along two orthogonal directions corresponding to electric field oscillation direction (polarizations). We can take x and y axes along these directions. An input light will travel along the fiber with E_x and E_y polarizations having different group velocities and hence arrive at the output at different times

Polarization Mode dispersion

- The effects of fiber-birefringence on the polarization states of an optical are another source of pulse broadening. **Polarization mode dispersion** (PMD) is due to slightly different velocity for each polarization mode because of the lack of perfectly symmetric & anisotropicity of the fiber. If the group velocities of two orthogonal polarization modes are v_{gx} and v_{gy} then the differential time delay $\Delta\tau_{pol}$ between these two polarization over a distance L is

$$\Delta\tau_{pol} = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right| \qquad [3\text{-}26]$$

- The rms value of the differential group delay can be approximated as:

$$\langle \Delta\tau_{pol} \rangle \approx D_{PMD}\sqrt{L} \qquad [3\text{-}27]$$

Chromatic & Total Dispersion

- Chromatic dispersion includes the material & waveguide dispersions.

$$D_{ch}(\lambda) \approx \left| D_{mat} + D_{wg} \right|$$

$$\sigma_{ch} = D_{ch}(\lambda) L \sigma_{\lambda}$$

[3-28]

- Total dispersion is the sum of chromatic , polarization dispersion and other dispersion types and the total rms pulse spreading can be approximately written as:

$$D_{total} \approx \left| D_{ch} + D_{pol} + \ldots \right|$$

$$\sigma_{total} = D_{total} L \sigma_{\lambda}$$

[3-29]

Total Dispersion, zero Dispersion

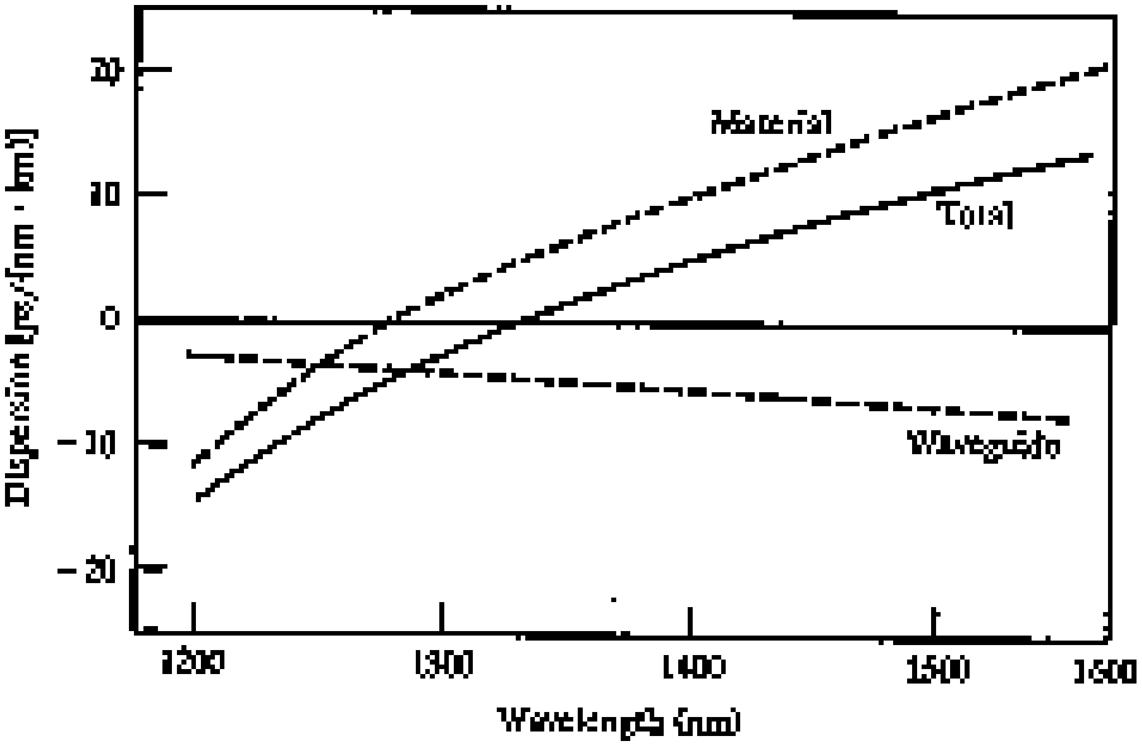


FIGURE 3-16 Examples of the magnitudes of material and waveguide dispersion as a function of optical wavelength for a single-mode fused-silica-core fiber. (Reproduced with permission from Keck,[16] © 1985, IEEE.)

Fact 1) Minimum distortion at wavelength about 1300 nm for single mode silica fiber.
Fact 2) Minimum attenuation is at 1550 nm for sinlge mode silica fiber.
Strategy: shifting the zero-dispersion to longer wavelength for minimum attenuation and dispersion.

Optimum single mode fiber & distortion/attenuation characteristics

Fact 1) Minimum distortion at wavelength about 1300 nm for single mode silica fiber.

Fact 2) Minimum attenuation is at 1550 nm for sinlge mode silica fiber.

Strategy: shifting the zero-dispersion to longer wavelength for minimum attenuation and dispersion by Modifying waveguide dispersion by changing from a simple step-index core profile to more complicated profiles. There are four major categories to do that:

1- 1300 nm optimized single mode step-fibers: matched cladding (mode diameter 9.6 micrometer) and depressed-cladding (mode diameter about 9 micrometer)

2- Dispersion shifted fibers.

3- Dispersion-flattened fibers.

4- Large-effective area (LEA) fibers (less nonlinearities for fiber optical amplifier applications, effective cross section areas are typically greater than $100\mu m^2$).

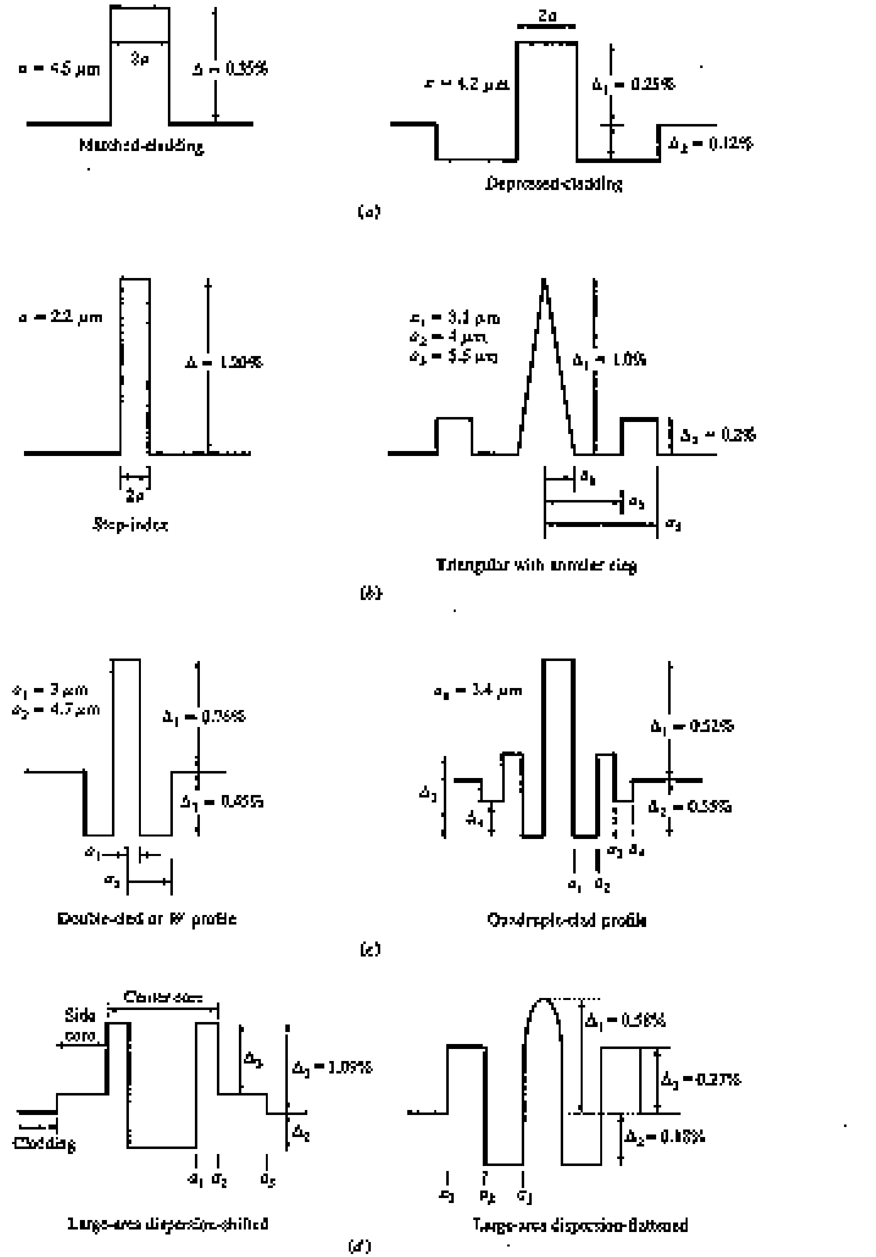
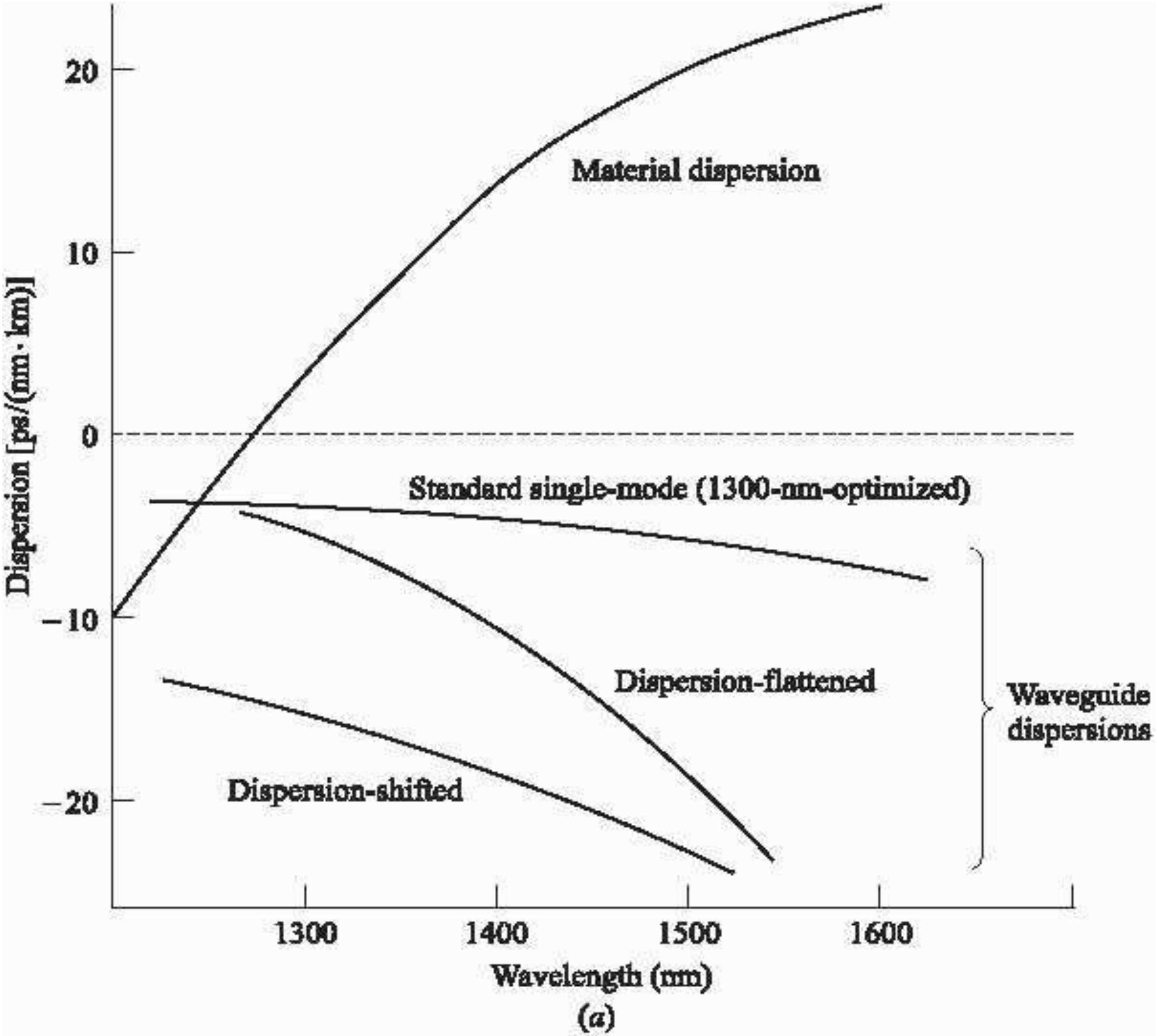


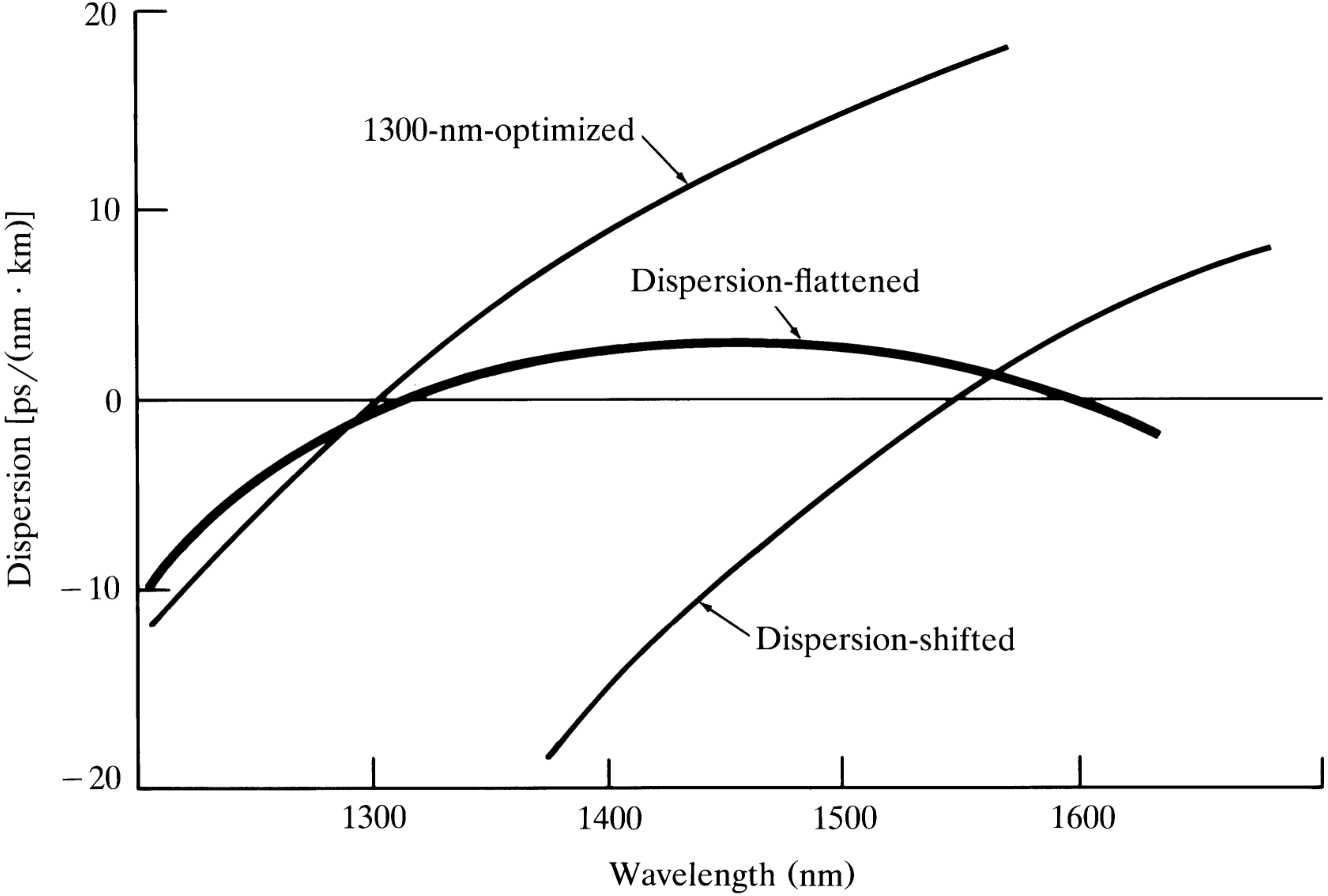
FIGURE 3-22
Representative cross sections of index profiles for (a) 1500-nm-optimized, (b) dispersion-shifted, (c) dispersion-flattened, and (d) large-effective-core-area fibers.

Single mode fiber dispersion



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Single mode fiber dispersion



(*b*)

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Single mode Cut-off wavelength & Dispersion

- Fundamental mode is HE_{11} or LP_{01} with V=2.405 and $\lambda_c = \frac{2\pi a}{V}\sqrt{n_1^2 - n_2^2}$ [3-30]
- Dispersion:

$$D(\lambda) = \frac{d\tau}{d\lambda} \approx D_{mat}(\lambda) + D_{wg}(\lambda)$$ [3-31]

$$o = D(\lambda) L \sigma_\lambda$$ [3-32]

- For non-dispersion-shifted fibers (1270 nm – 1340 nm)
- For dispersion shifted fibers (1500 nm- 1600 nm)

Dispersion for non-dispersion-shifted fibers (1270 nm – 1340 nm)

$$\tau(\lambda) = \tau_0 + \frac{S_0}{8}\left(\lambda - \frac{\lambda_0^2}{\lambda}\right)^2 \qquad [3\text{-}33]$$

- τ_0 is relative delay minimum at the zero-dispersion wavelength λ_0 , and S_0 is the value of the dispersion slope in $\text{ps/(nm}^2\text{.km)}$.

$$S_0 = S(\lambda_0) = \left.\frac{dD}{d\lambda}\right|_{\lambda=\lambda_0} \qquad [3\text{-}34]$$

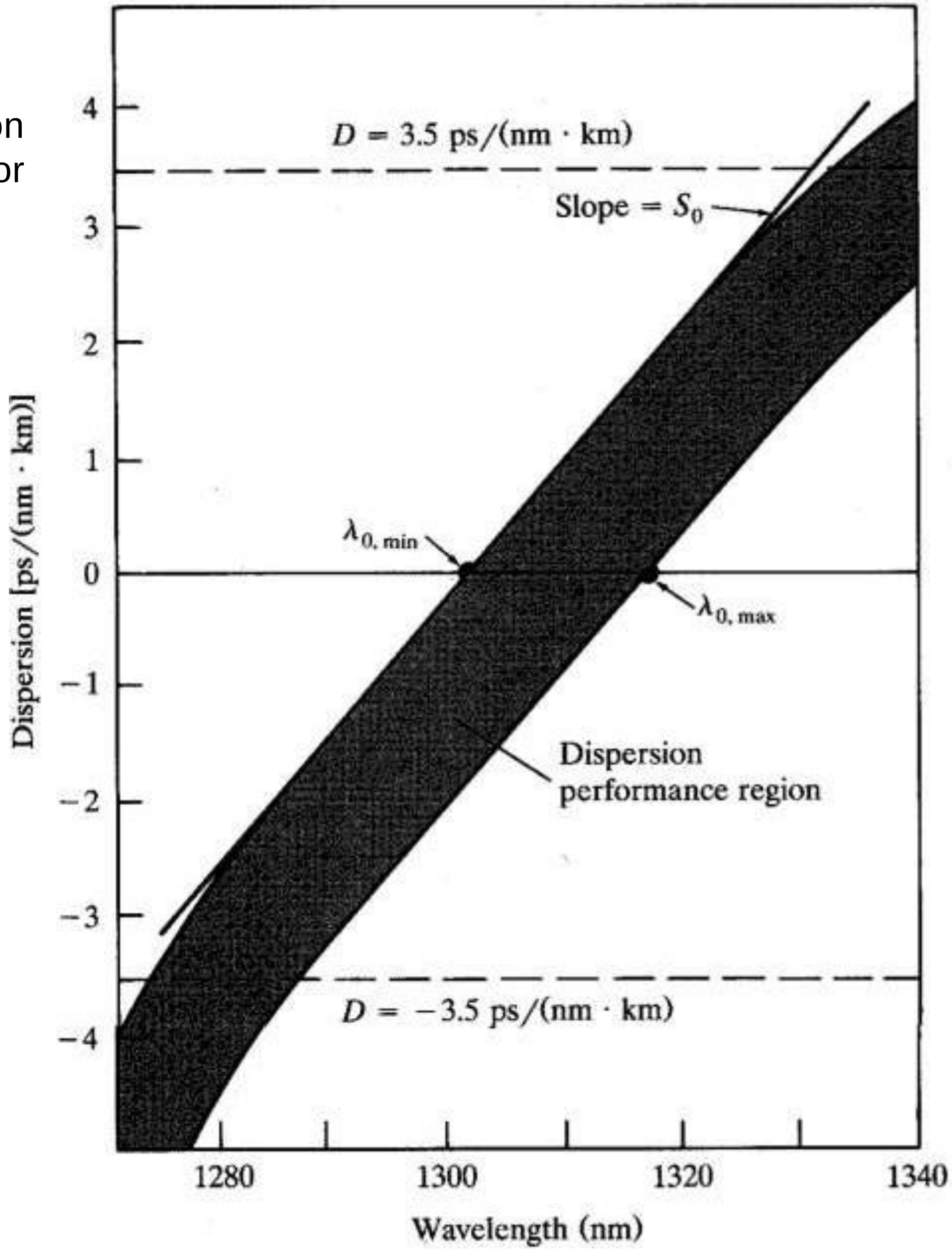
$$D(\lambda) = \frac{\lambda S_0}{4}\left[1 - \left(\frac{\lambda_0}{\lambda}\right)^4\right] \qquad [3\text{-}35]$$

Dispersion for dispersion shifted fibers (1500 nm- 1600 nm)

$$\tau(\lambda) = \tau_0 + \frac{S_0}{2}(\lambda - \lambda_0)^2$$ [3-36]

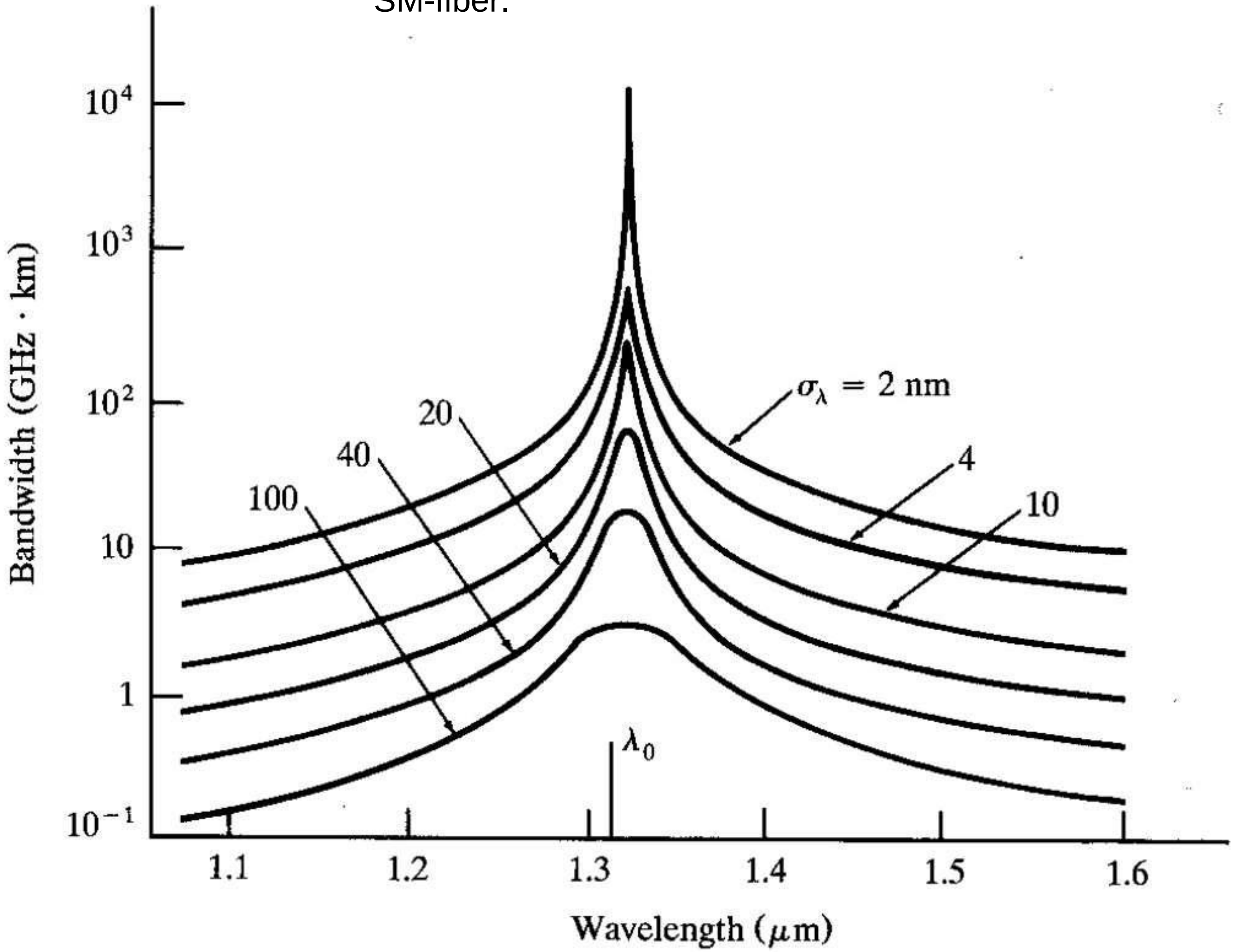
$$D(\lambda) = (\lambda - \lambda_0)S_0$$ [3-37]

Example of dispersion Performance curve for Set of SM-fiber



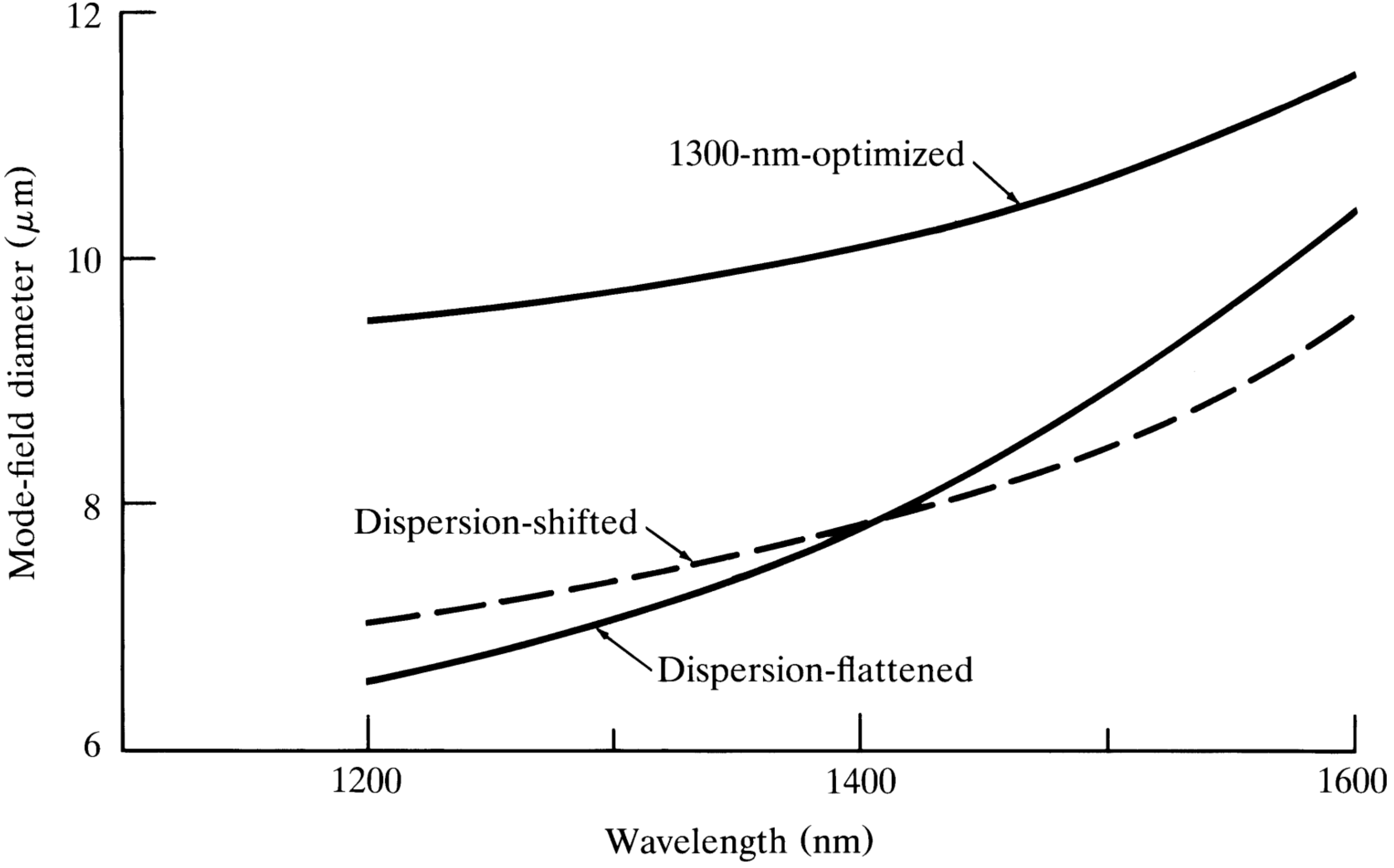
Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Example of BW vs wavelength for various optical sources for SM-fiber.



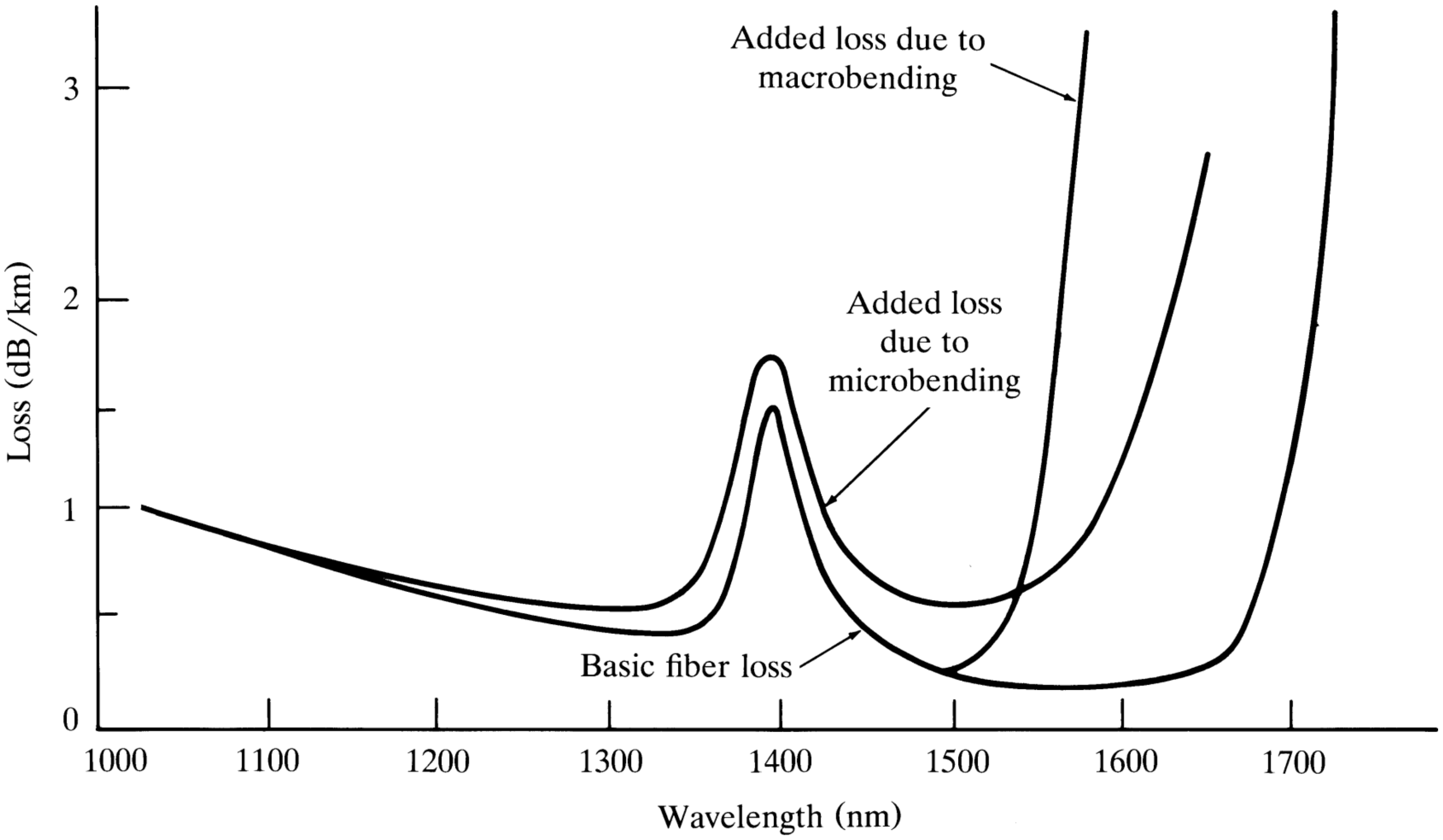
Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

MFD



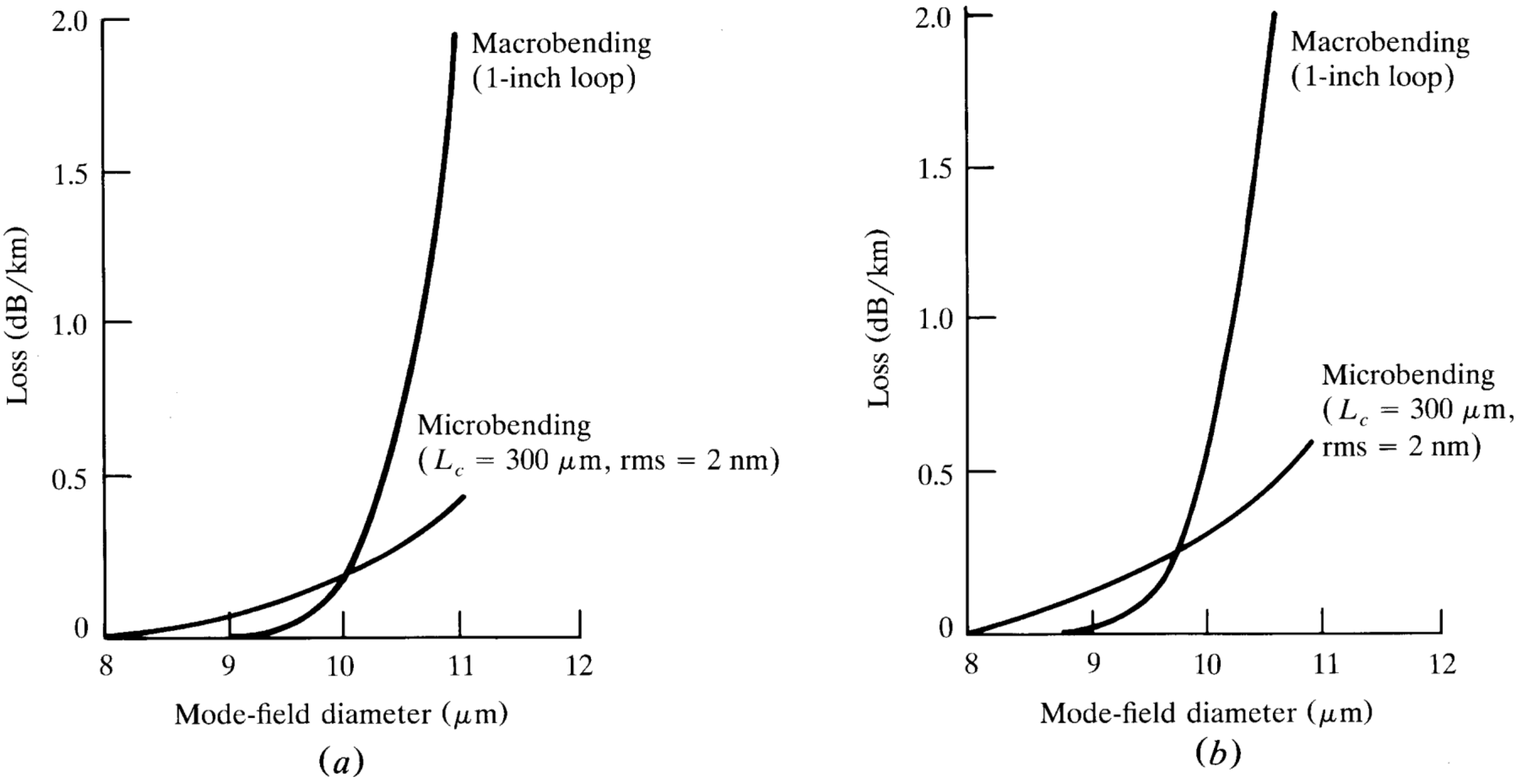
Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Bending Loss



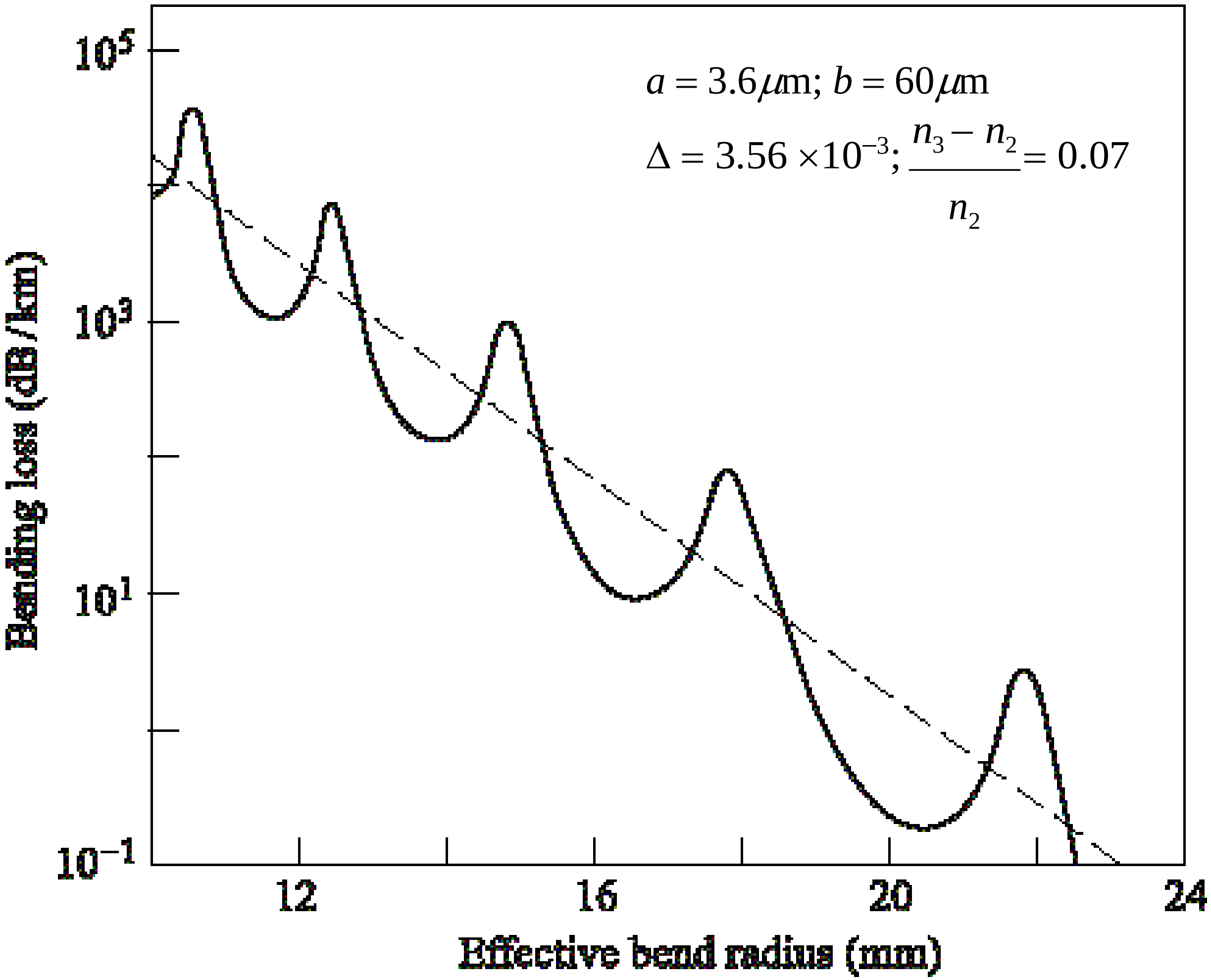
Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Bending effects on loss vs MFD



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

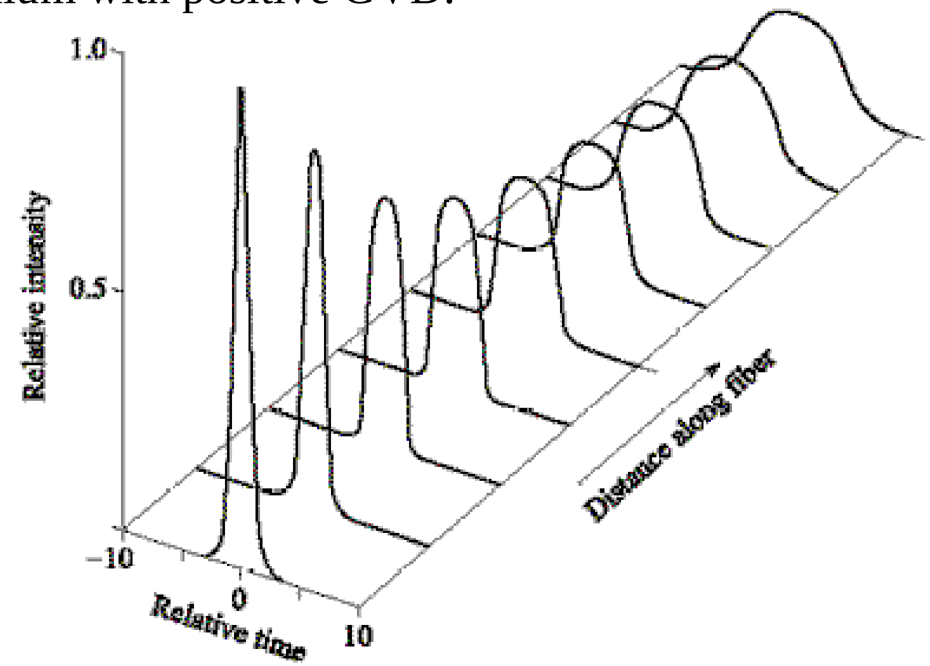
Bend loss versus bend radius



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Kerr effect

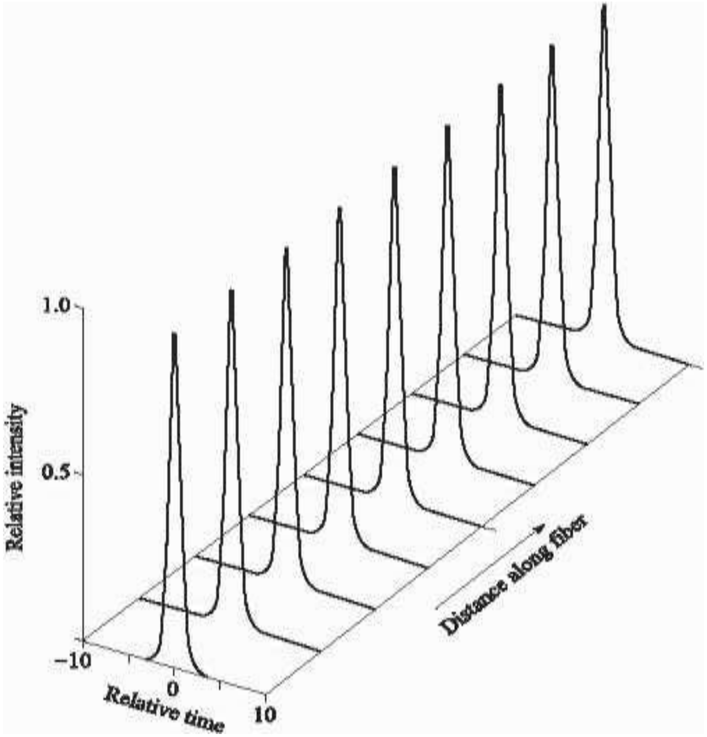
Temporal changes in a narrow optical pulse that is subjected to Kerr nonlinearity in
A dispersive medium with positive GVD.



$$n = n_0 + n_2 I$$

Kerr nonlinearity in fiber, where I is the intensity of
Optical wave.

First-order Soliton



Temporal changes in a medium with Kerr nonlinearity and negative GVD. Since dispersion tends to broaden the pulse, Kerr Nonlinearity tends to squeeze the pulse, resulting in a formation of **optical soliton.**

UNIT-III FIBER OPTICAL SOURCES AND COUPLING

Contents

- Review of Semiconductor Physics
- Light Emitting Diode (LED)

  - Structure, Material,Quantum efficiency, LED Power, Modulation
- Laser Diodes

  - structure, Modes, Rate Equation,Quantum efficiency, Resonant frequencies, Radiation pattern
- Single-Mode Lasers

  - DFB (Distributed-FeedBack) laser, Distributed-Bragg Reflector, Modulation

- Light-source Linearity
- Noise in Lasers

Considerations with Optical Sources

- Physical dimensions to suit the fiber
- Narrow radiation pattern (beam width)
- Linearity (output light power proportional to driving current)
- Ability to be directly modulated by varying driving current
- Fast response time (wide band)
- Adequate output power into the fiber

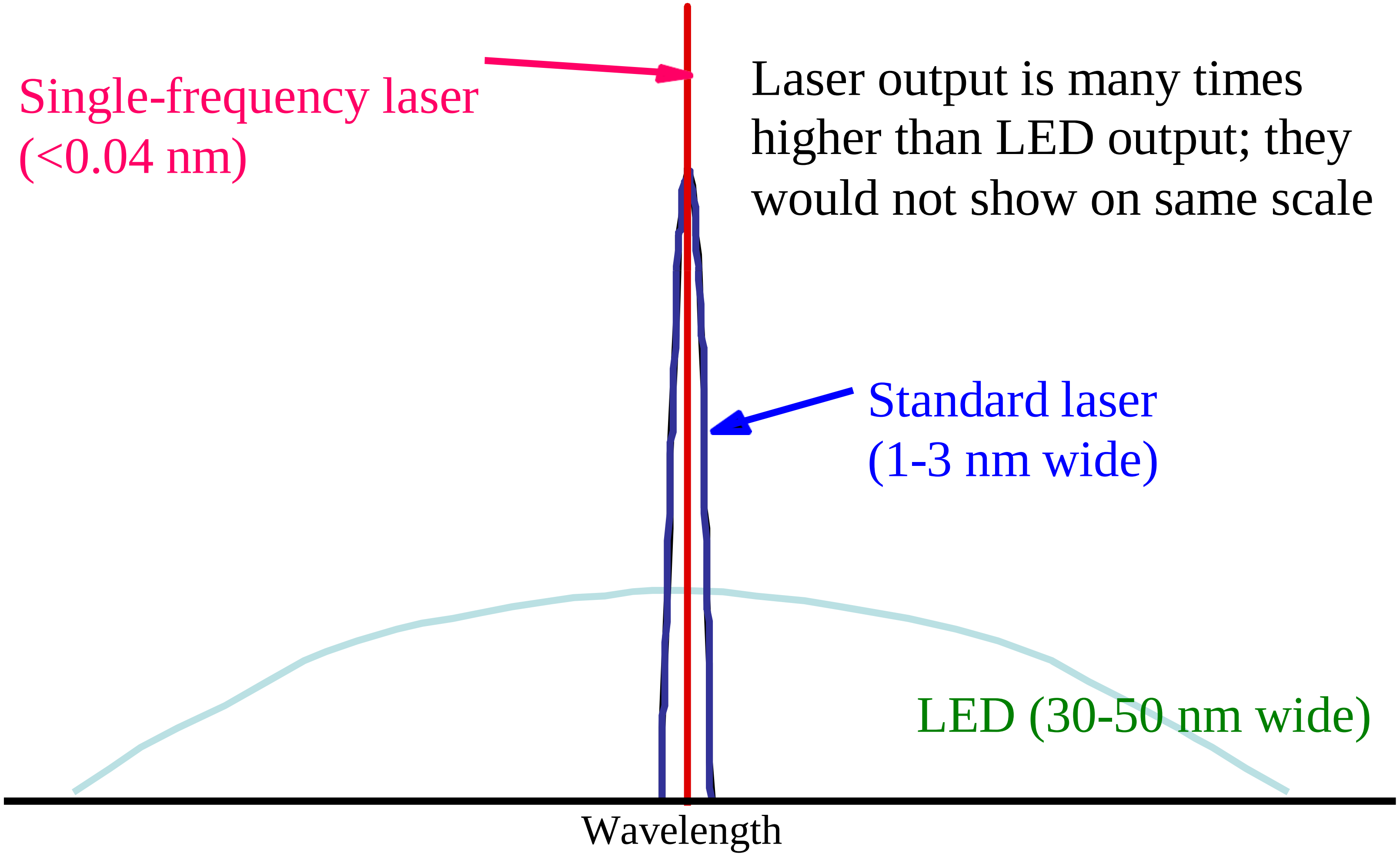
Considerations…

- Narrow spectral width (or line width)
- Stability and efficiency
- Driving circuit issues
- Reliability and cost

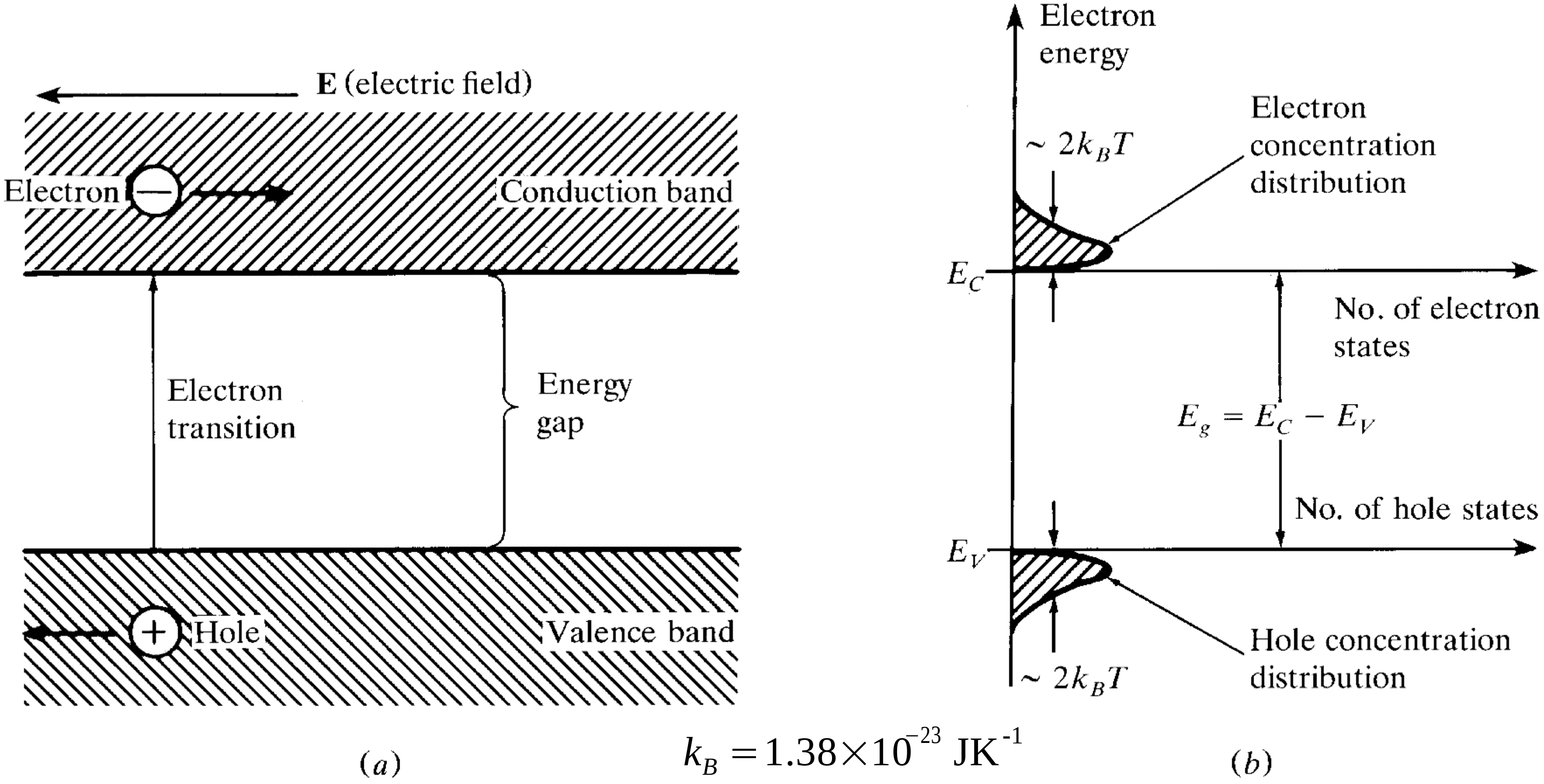
Semiconductor Light Sources

- A PN junction (that consists of direct band gap semiconductor materials) acts as the *active* or *recombination* region.
- When the PN junction is forward biased, electrons and holes recombine either *radiatively* (emitting photons) or *non-radiatively* (emitting heat). This is simple LED operation.
- In a LASER, the photon is further processed in a resonance cavity to achieve a *coherent, highly directional* optical beam with *narrow linewidth.*

LED vs. laser spectral width



Review of Semiconductor Physics

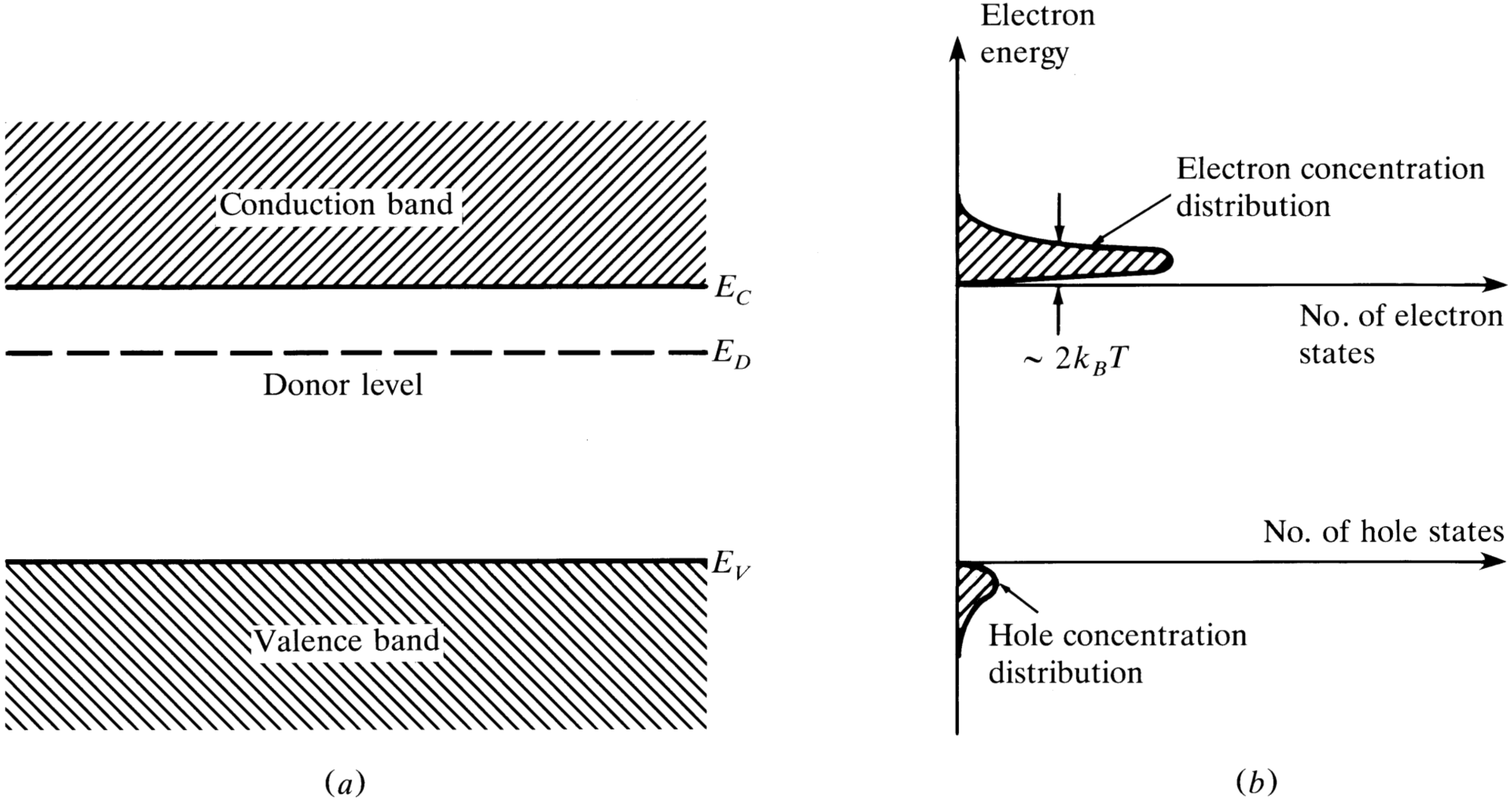


$$k_B = 1.38 \times 10^{-23}\ \mathrm{JK}^{-1}$$

a) Energy level diagrams showing the excitation of an electron from the valence band to the conduction band. The resultant free electron can freely move under the application of electric field.

b) Equal electron & hole concentrations in an intrinsic semiconductor created by the thermal excitation of electrons across the band gap

n-Type Semiconductor

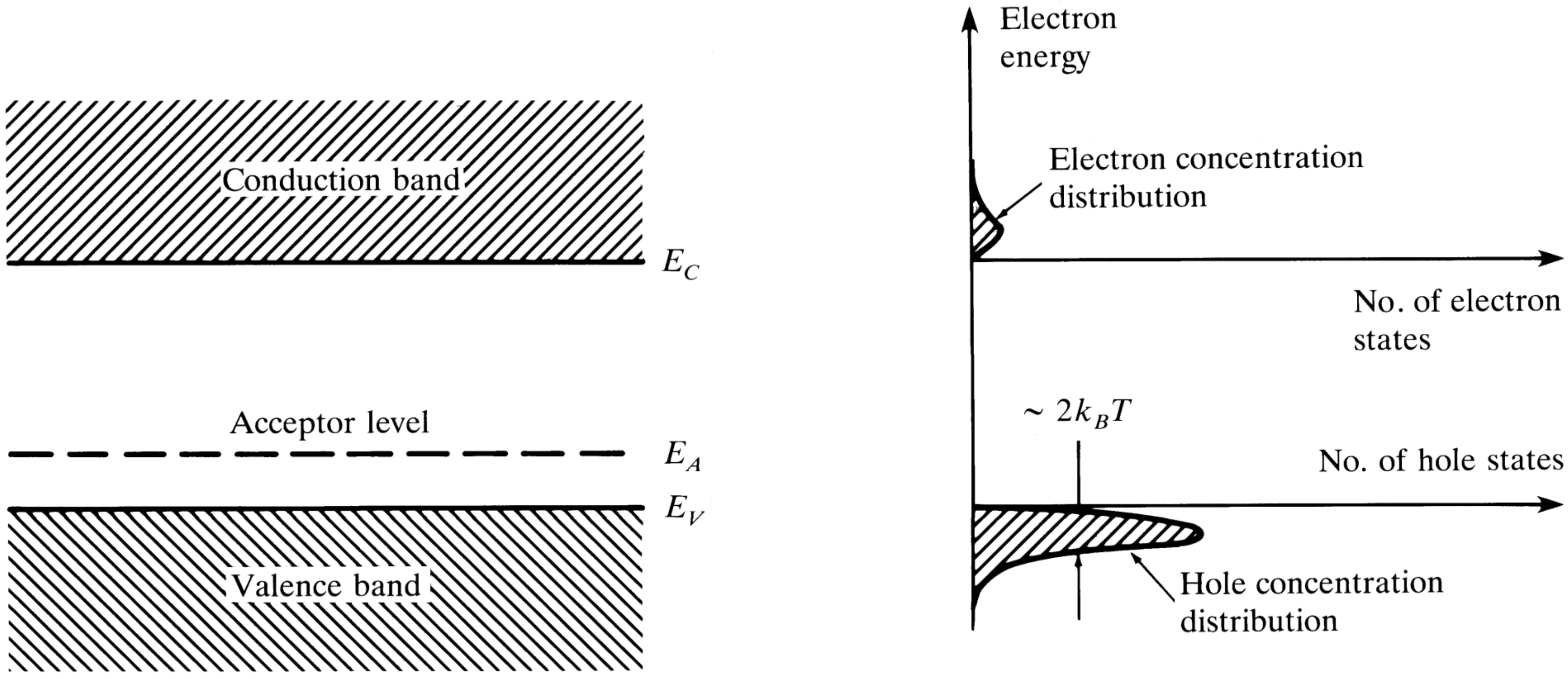


a) Donor level in an *n*-type semiconductor.

b) The ionization of donor impurities creates an increased electron concentration distribution.

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

p-Type Semiconductor



(*a*) (*b*)

a) Acceptor level in an *p*-type semiconductor.

b) The ionization of acceptor impurities creates an increased hole concentration distribution

Intrinsic & Extrinsic Materials

- Intrinsic material: A perfect material with no impurities.

$$n = p = n_i \propto \exp\left(-\frac{E_g}{2k_B T}\right) \quad [4\text{-}1]$$

n & p & n_i are the electron, hole & intrinsic concentrations respectively.

E_g is the gap energy, T is Temperature.

- Extrinsic material: donor or acceptor type semiconductors.

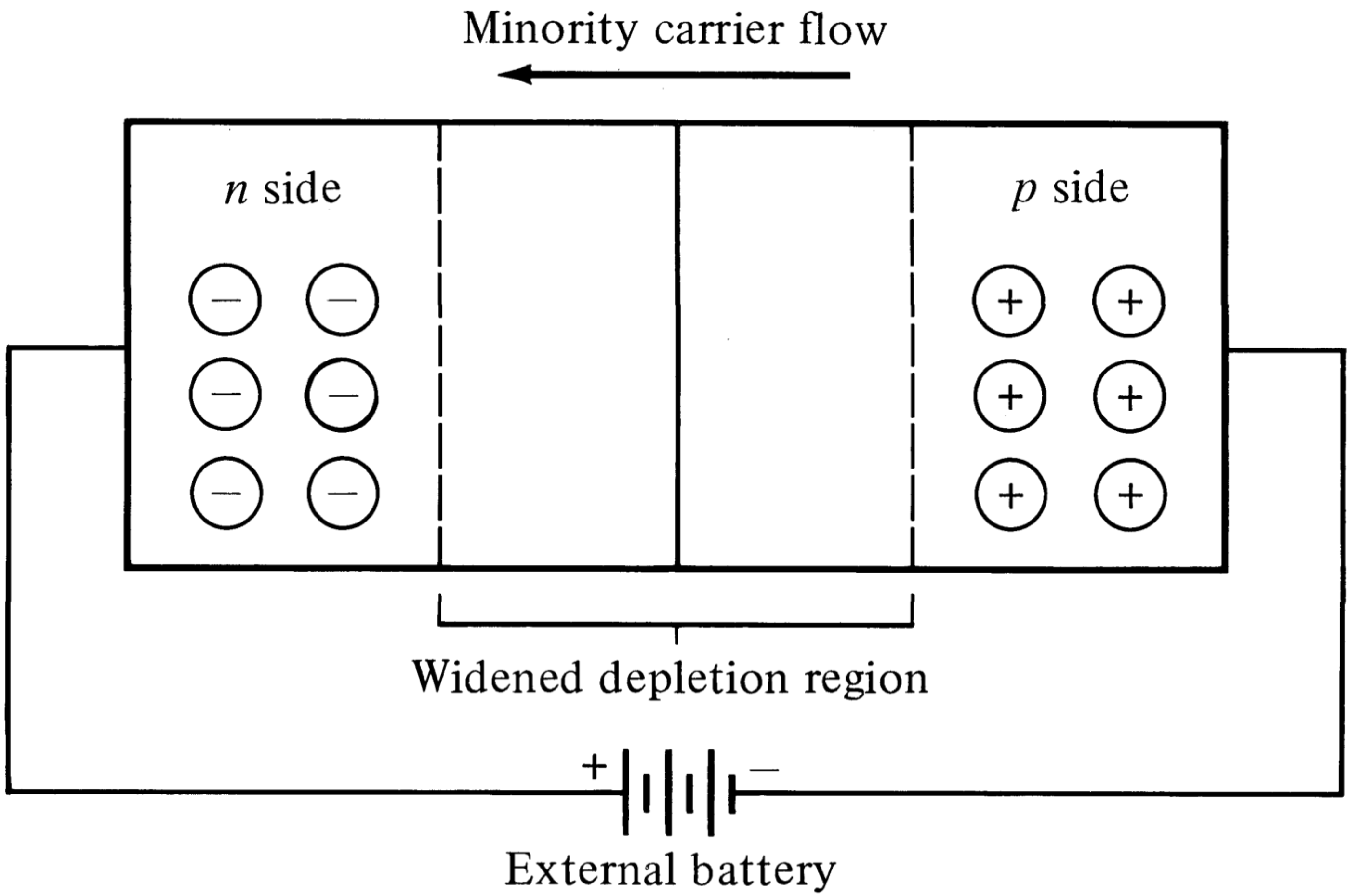
$$pn = n_i^2 \quad [4\text{-}2]$$

- Majority carriers: electrons in *n*-type or holes in *p*-type.
- Minority carriers: holes in *n*-type or electrons in *p*-type.
- The operation of semiconductor devices is essentially based on the **injection** and **extraction** of minority carriers.

The *pn* Junction

Electron diffusion across a pn junction creates a barrier potential (electric field) in the depletion region.

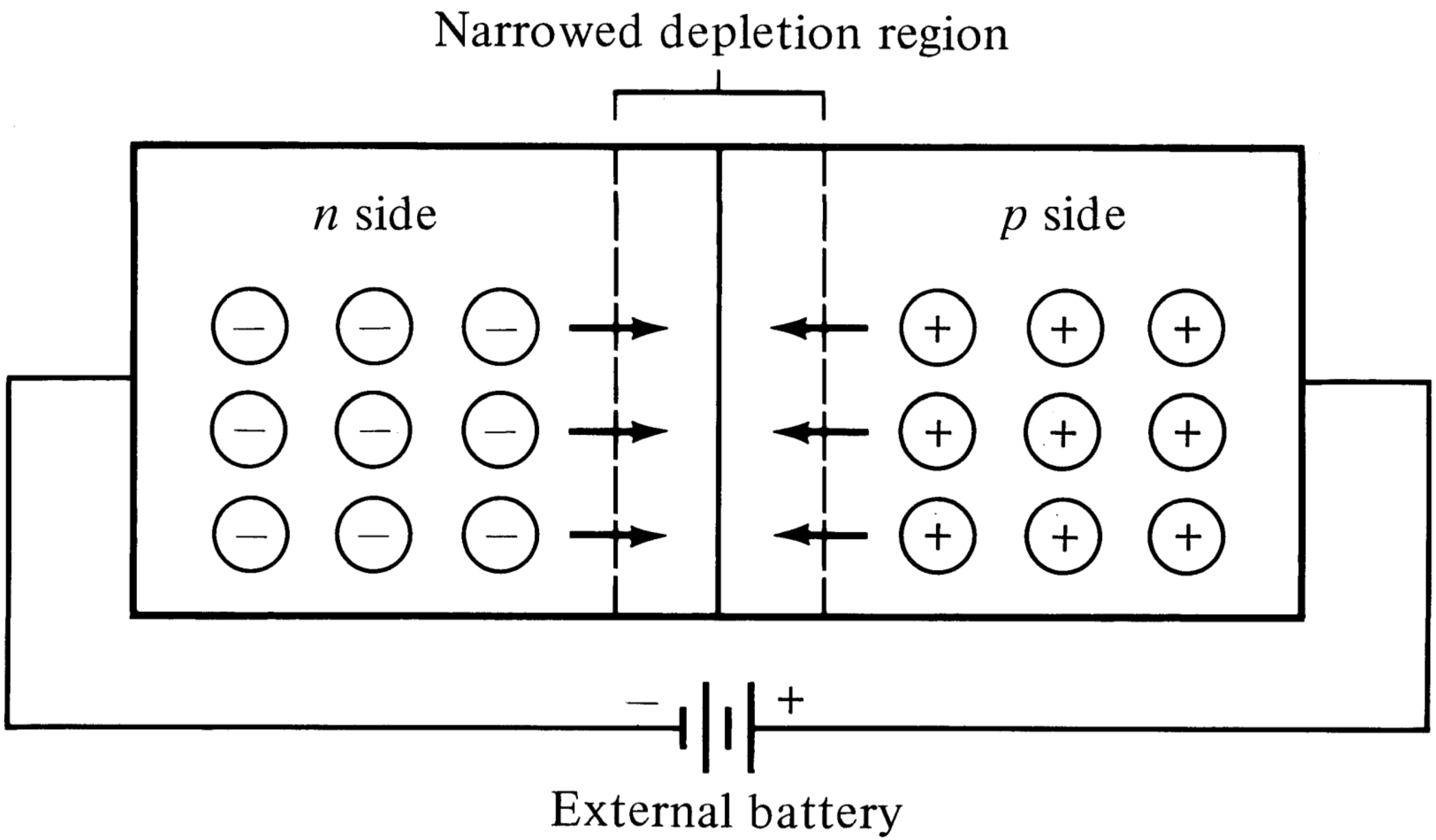
Reverse-biased *pn* Junction



A reverse bias widens the depletion region, but allows minority carriers to move freely with the applied field.

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

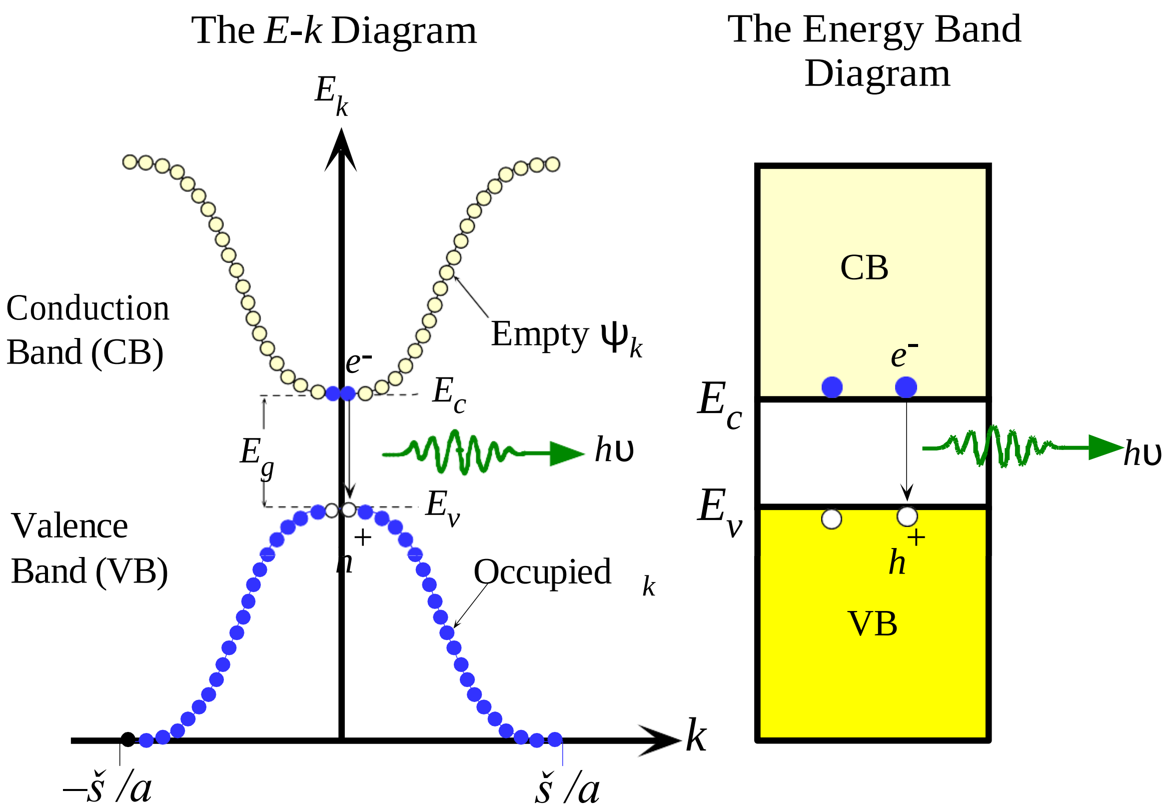
Forward-biased *pn* Junction



Lowering the barrier potential with a forward bias allows majority carriers to diffuse across the junction.

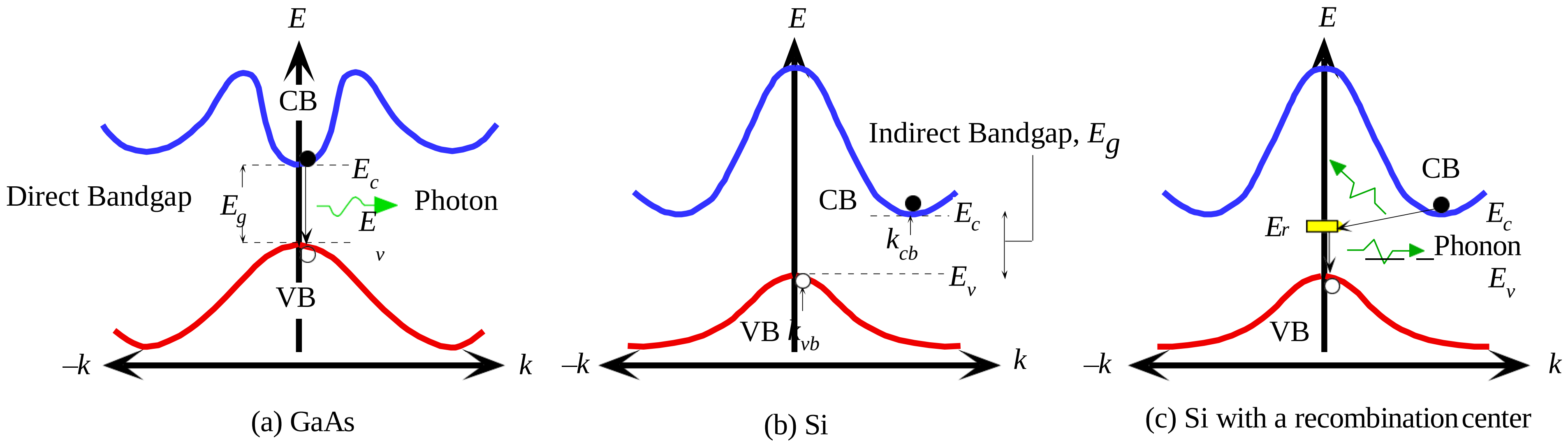
Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Direct Band Gap Semiconductors



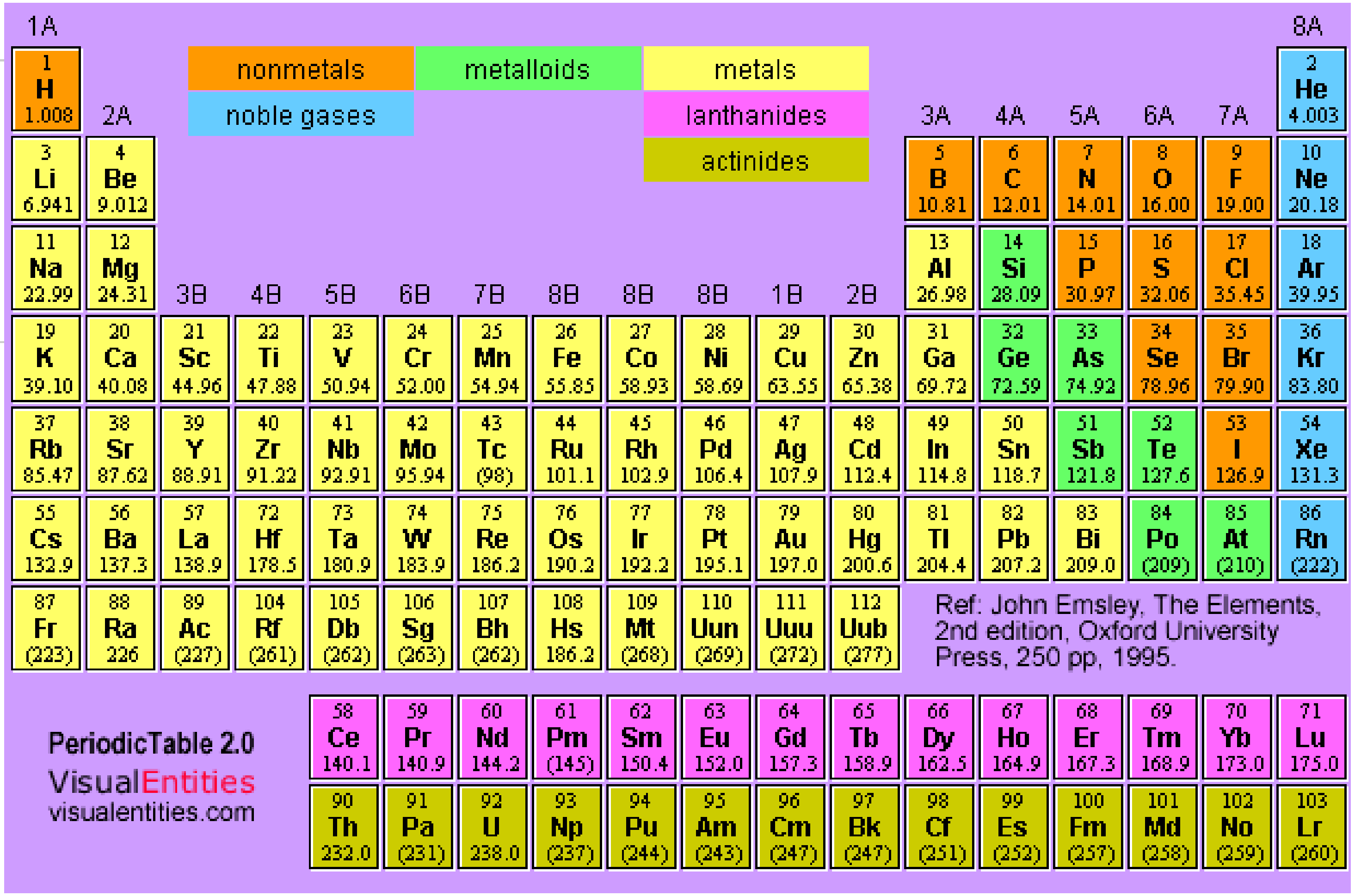
The E-k diagram of a direct bandgap semiconductor such as GaAs. The E-k curve consists of many discrete points with each point corresponding to a possible state, wavefunction $\psi_k(x)$, that is allowed to exist in the crystal. The points are so close that we normally draw the E-k relationship as a continuous curve. In the energy range E_v to E_c there are no points ($\psi_k(x)$ solutions).

Indirect Band Gap Semiconductors



(a) In GaAs the minimum of the CB is directly above the maximum of the VB. GaAs is therefore a direct bandgap semiconductor. (b) In Si, the minimum of the CB is displaced from the maximum of the VB and Si is an indirect bandgap semiconductor. (c) Recombination of an electron and a hole in Si involves a recombination center .

Periodic table



Light-Emitting Diodes (LEDs)

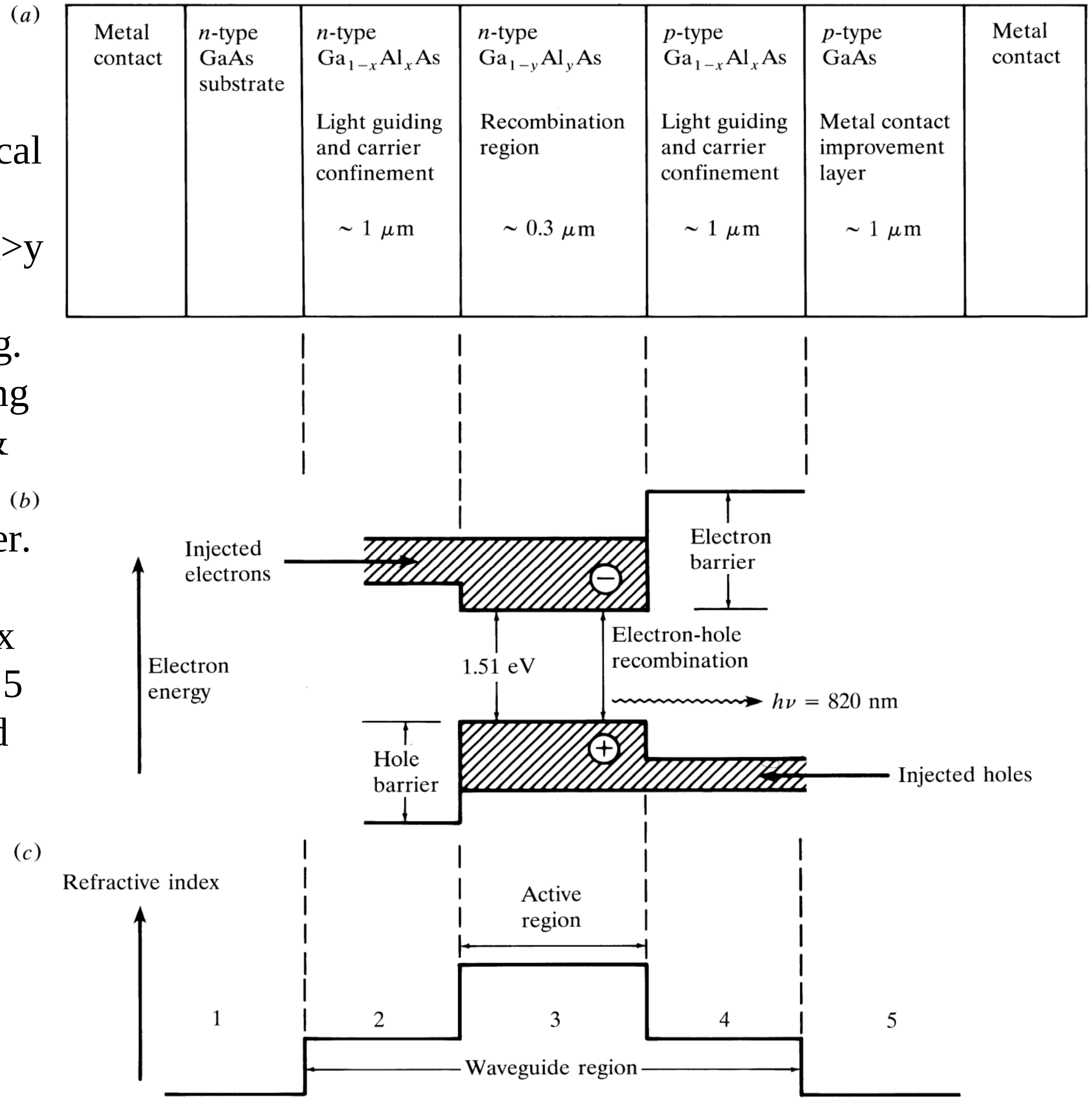
- For photonic communications requiring data rate 100-200 Mb/s with multimode fiber with tens of microwatts, LEDs are usually the best choice.
- LED configurations being used in photonic communications:

  1- Surface Emitters (Front Emitters)

  2- Edge Emitters

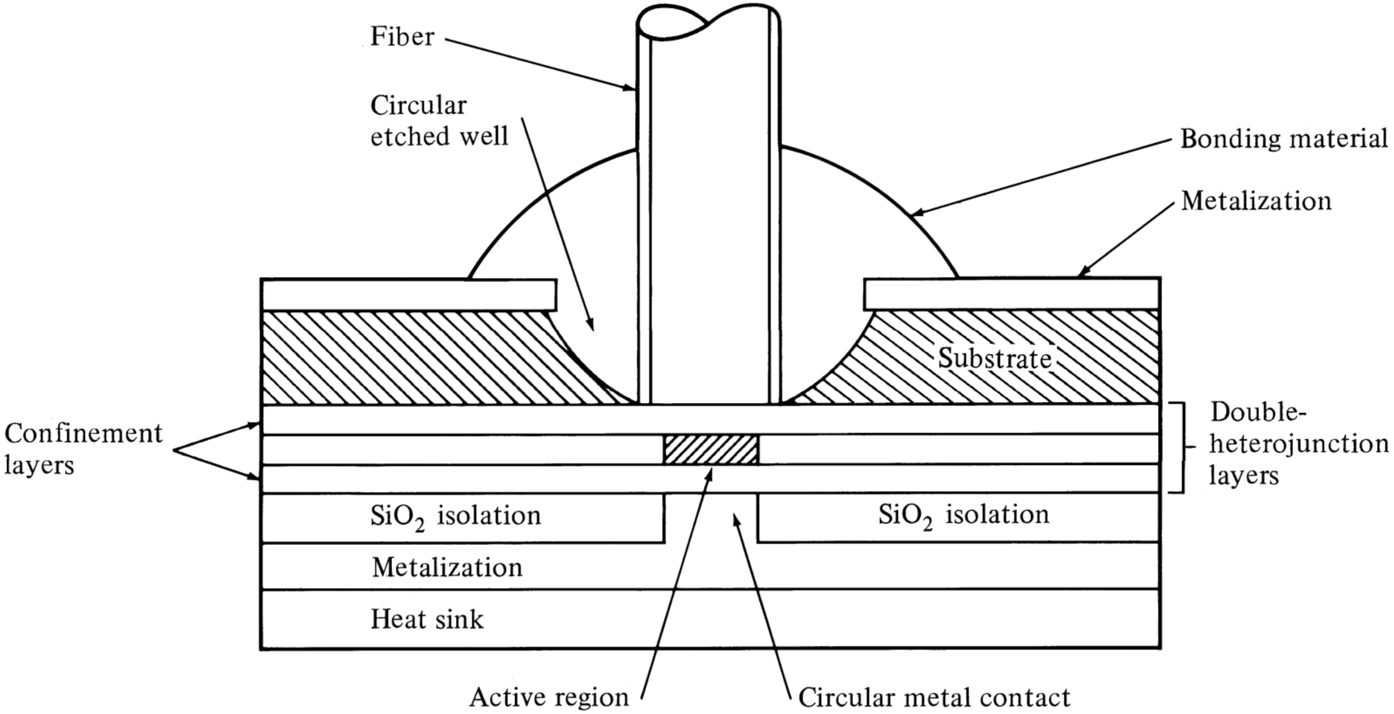
Cross-section drawing of a typical GaAlAs double heterostructure light emitter. In this structure, x>y to provide for both carrier confinement and optical guiding.
b) Energy-band diagram showing the active region, the electron & hole barriers which confine the charge carriers to the active layer.
c) Variations in the refractive index; the lower refractive index of the material in regions 1 and 5 creates an optical barrier around the waveguide because of the higher band-gap energy of this material.

$$\lambda(\mu m) = \frac{1.240}{E_g(eV)}$$



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

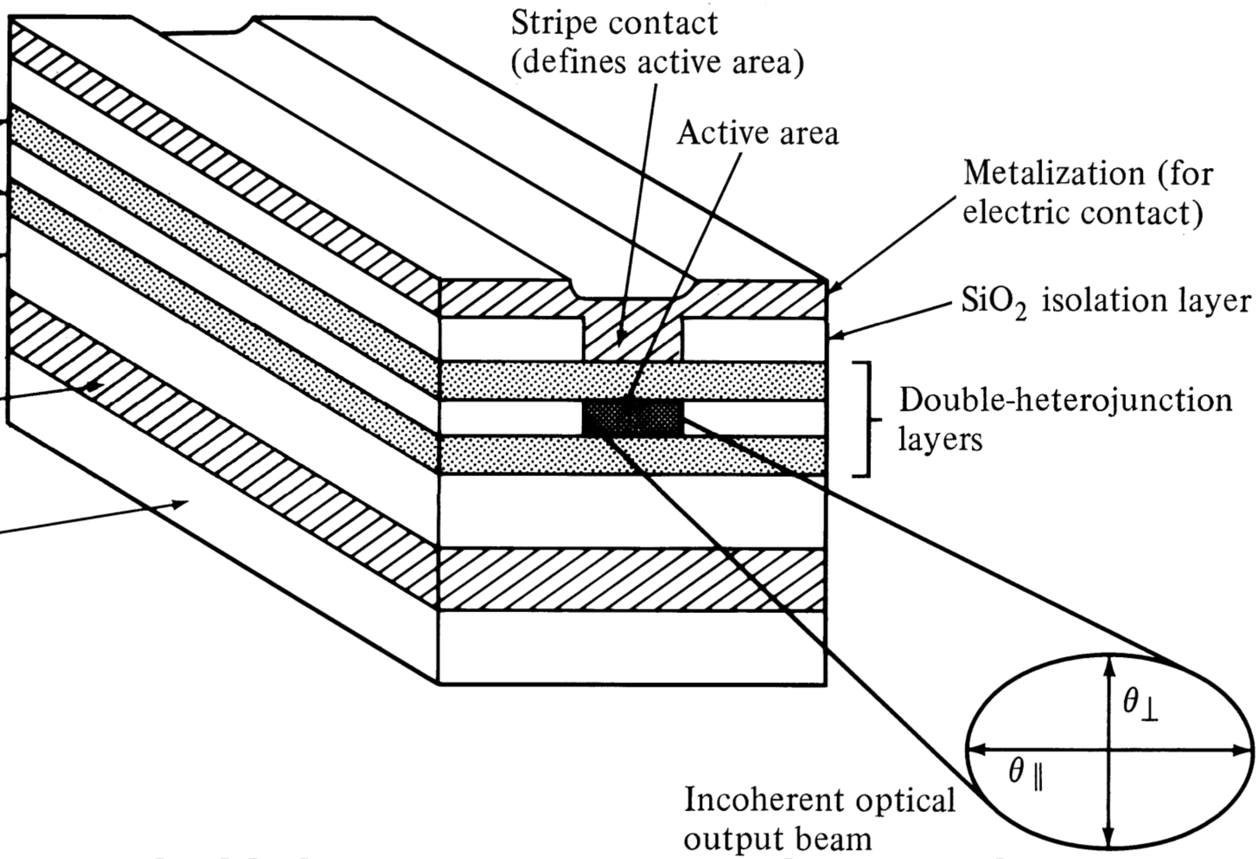
Surface-Emitting LED



Schematic of high-radiance surface-emitting LED. The active region is limitted to a circular cross section that has an area compatible with the fiber-core end face.

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Edge-Emitting LED

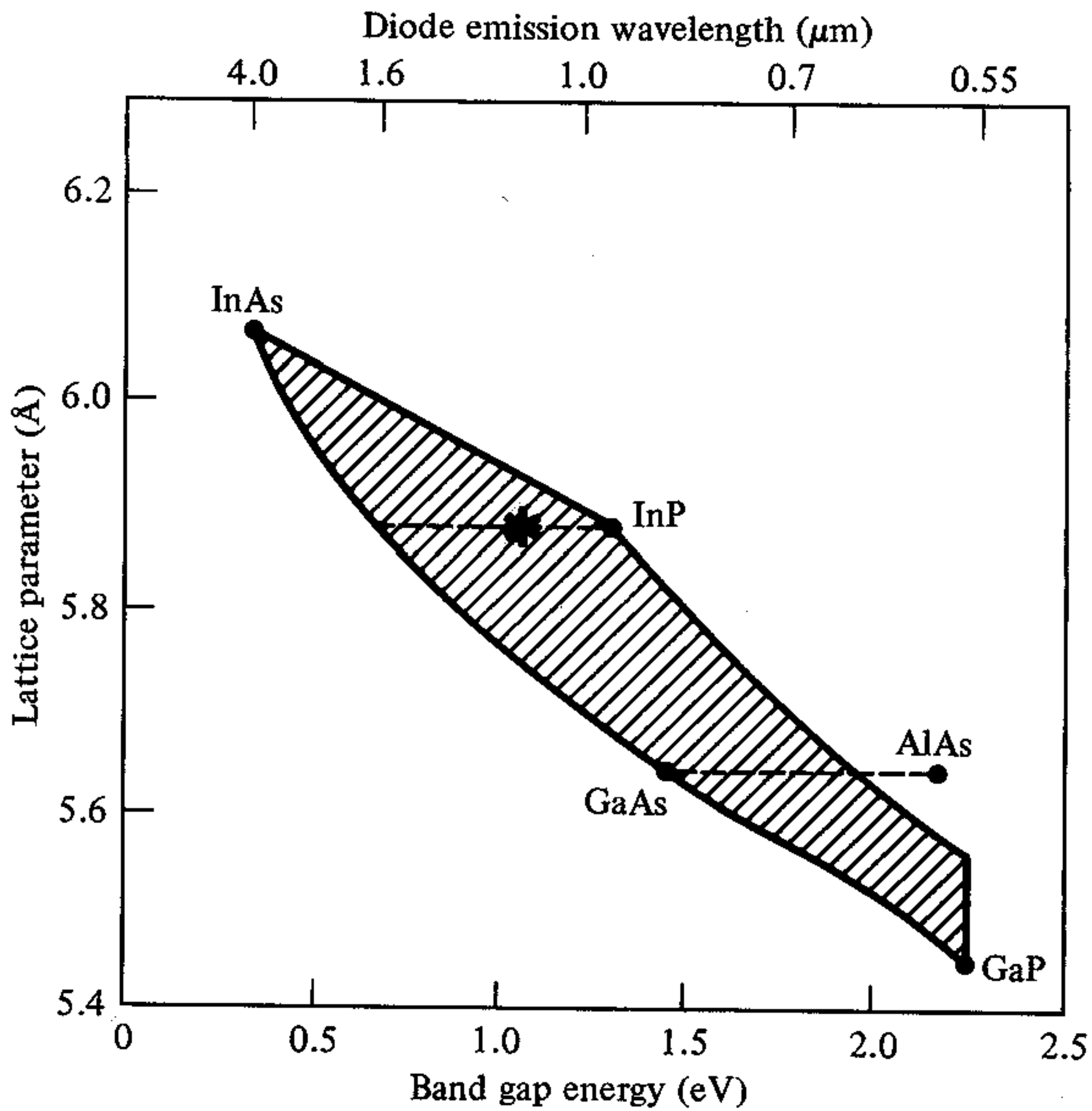


Schematic of an edge-emitting double heterojunction LED. The output beam is Lambertian in the plane of junction ($\theta_{||} = 120^o$) and highly directional perpendicular to pn junction ($\theta_{\perp} = 30^o$) . They have high quantum efficiency & fast response.

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

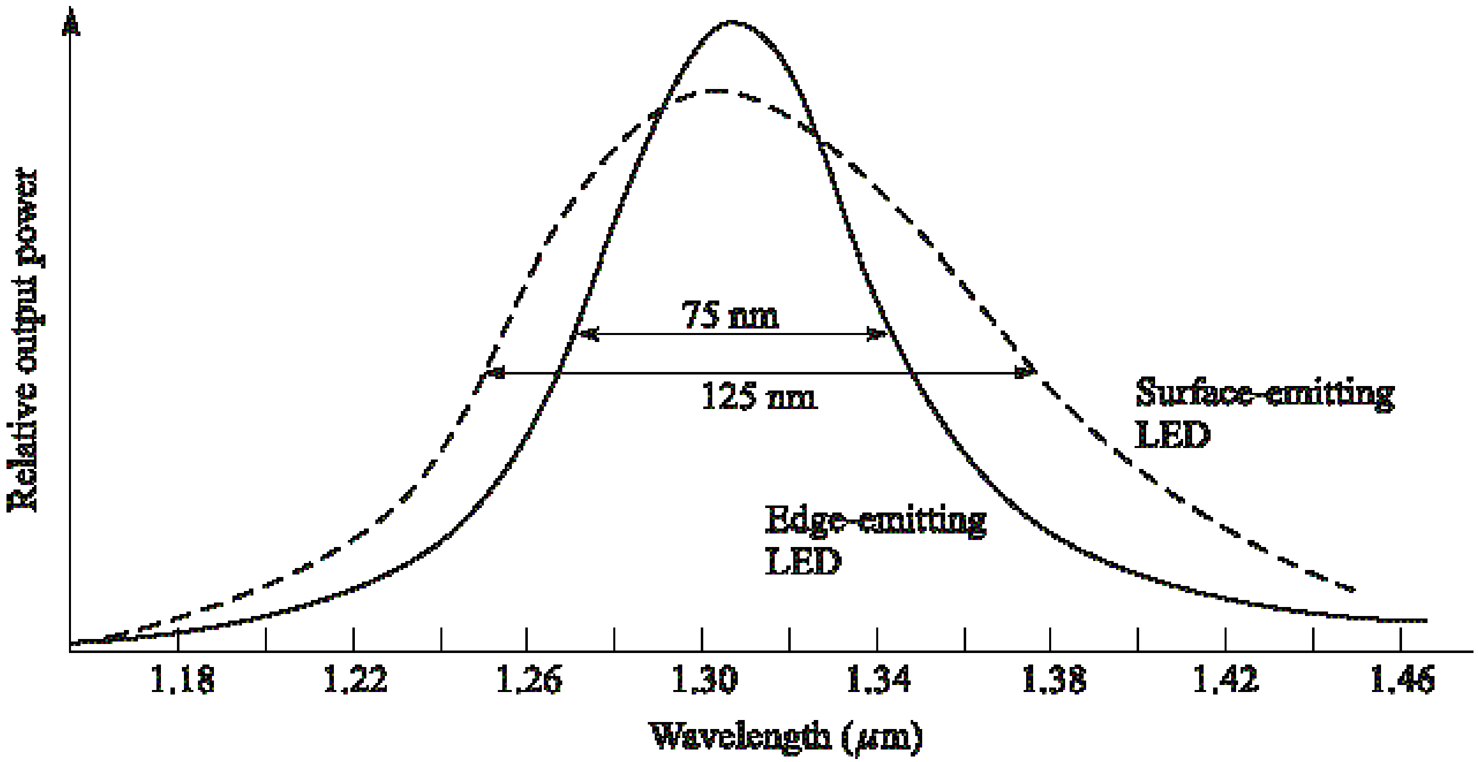
Light Source Material

- Most of the light sources contain III-V ternary & quaternary compounds.
- $Ga_{1-x}Al_xAs$ by varying x it is possible to control the band-gap energy and thereby the emission wavelength over the range of 800 nm to 900 nm. The spectral width is around 20 to 40 nm.
- $In_{1-x}Ga_xAs_yP_{1-y}$ By changing $0<x<0.47$; y is approximately 2.2x, the emission wavelength can be controlled over the range of 920 nm to 1600 nm. The spectral width varies from 70 nm to 180 nm when the wavelength changes from 1300 nm to 1600 nm. These materials are lattice matched.


Diode emission wavelength (μm)
4.0
1.6
1.0
0.7
0.55
6.2
6.0
5.8
5.6
5.4
Lattice parameter (Å)
InAs
InP
AlAs
GaAs
GaP
0
0.5
1.0
1.5
2.0
2.5
Band gap energy (eV)

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Spectral width of LED types



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Rate equations, Quantum Efficiency & Power of LEDs

- When there is no external carrier injection, the excess density decays exponentially due to electron-hole recombination.

$$n(t) = n_0 e^{-t/\tau} \quad [4\text{-}4]$$

- *n* is the excess carrier density,

n_0: initial injected excess electron density

τ: carrier lifetime.

- Bulk recombination rate *R*:

$$R = -\frac{dn}{dt} = \frac{n}{\tau} \quad [4\text{-}5]$$

- Bulk recombination rate (*R*)=Radiative recombination rate + nonradiative recombination rate

bulk recombination rate $(R = 1/\tau) =$
radiative recombination rate $(R_r = 1/\tau_r)$ + nonradiative recombination rate$(R_{nr} = 1/\tau_{nr})$

With an external supplied current density of J the rate equation for the electron-hole recombination is:

$$\frac{dn(t)}{dt} = \frac{J}{qd} - \frac{n}{\tau} \qquad [4\text{-}6]$$

q : charge of the electron; d : thickness of recombination region

In equilibrium condition: *dn/dt=0*

$$n = \frac{J\tau}{qd} \qquad [4\text{-}7]$$

Internal Quantum Efficiency & Optical Power

$$\eta_{int} = \frac{R_r}{R_r + R_{nr}} = \frac{\tau_{nr}}{\tau_r + \tau_{nr}} = \frac{\tau}{\tau_r} \qquad [4\text{-}8]$$

η_{int} : internal quantum efficiency in the active region

Optical power generated internally in the active region in the LED is:

$$P_{int} = \eta_{int} \frac{I}{q} h\nu = \eta_{int} \frac{hcI}{q\lambda} \qquad [4\text{-}9]$$

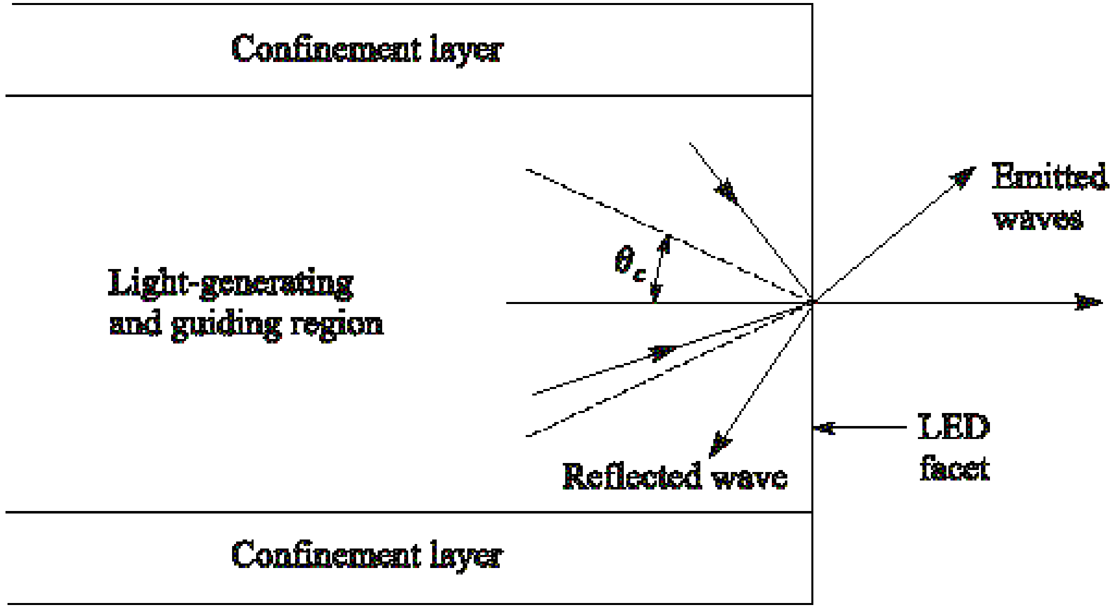
P_{int} : Internal optical power,

I : Injected current to active region

External Quantum Eficiency

$$\eta_{ext} = \frac{\#\text{ of photons emitted from LED}}{\#\text{ of LED internally generated photons}} \qquad [4\text{-}10]$$

- In order to calculate the external quantum efficiency, we need to consider the reflection effects at the surface of the LED. If we consider the LED structure as a simple 2D slab waveguide, only light falling within a cone defined by critical angle will be emitted from an LED.



$$\eta_{ext} = \frac{1}{4\pi}\int_0^{\phi_c} T(\phi)(2\pi \sin\phi)d\phi \qquad [4\text{-}11]$$

$$T(\phi): \text{Fresnel Transmission Coefficient} \approx T(0) = \frac{4n_1 n_2}{(n_1+n_2)^2} \qquad [4\text{-}12]$$

$$\text{If } n_2 = 1 \Rightarrow \eta_{ext} \approx \frac{1}{n_1(n_1+1)^2} \qquad [4\text{-}13]$$

$$\text{LED emitted optical power, } P = \eta_{ext} P_{int} \approx \frac{P_{int}}{n_1(n_1+1)^2} \qquad [4\text{-}14]$$

Modulation of LED

- The frequency response of an LED depends on:
  1- Doping level in the active region
  2- Injected carrier lifetime in the recombination region, τ_i .
  3- Parasitic capacitance of the LED
- If the drive current of an LED is modulated at a frequency of ω the output optical power of the device will vary as:

$$P(\omega) = \frac{P_0}{\sqrt{1+(\omega\tau_i)^2}} \quad [4\text{-}15]$$

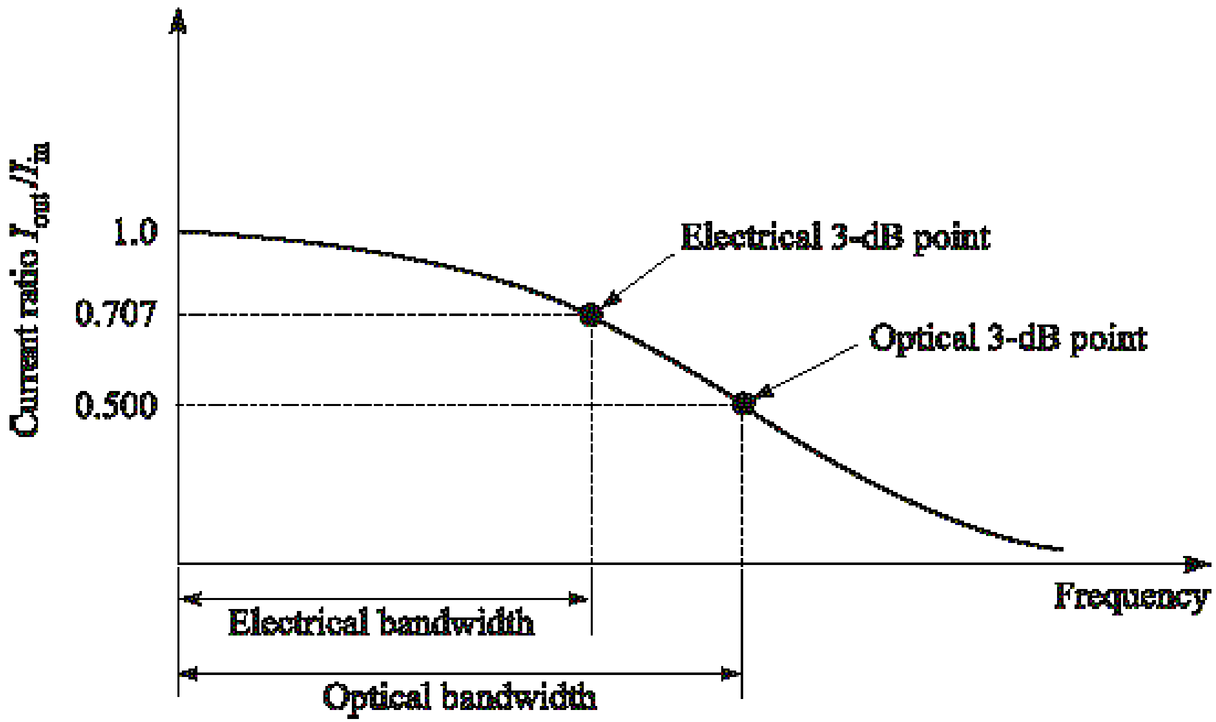
- Electrical current is directly proportional to the optical power, thus we can define electrical bandwidth and optical bandwidth, separately.

$$\text{Electrical BW} = 10\log\left(\frac{P(\omega)}{P(0)}\right) = 20\log\left(\frac{I(\omega)}{I(0)}\right) \quad [4\text{-}16]$$

P : electrical power, I : electrical current

$$\text{Optical BW} = 10\log\left(\frac{P(\omega)}{P(0)}\right) = 10\log\left(\frac{I(\omega)}{I(0)}\right) \qquad [4\text{-}17]$$

P : optical power, I : detected electric current, $I \propto P$



Optical Fiber communications, 3[rd] ed.,G.Keiser,McGrawHill, 2000

Drawbacks & Advantages of LED

Drawbacks

- Large line width (30-40 nm)
- Large beam width (Low coupling to the fiber)
- Low output power
- Low E/O conversion efficiency

Advantages

- Robust
- Linear

LASER

(Light Amplification by the Stimulated Emission of Radiation)

- Laser is an optical oscillator. It comprises a resonant optical amplifier whose output is fed back into its input with matching phase. Any oscillator contains:

  1- An amplifier with a gain-saturated mechanism

  2- A feedback system

  3- A frequency selection mechanism

  4- An output coupling scheme

- In laser, the amplifier is the pumped active medium, such as biased semiconductor region, feedback can be obtained by placing active medium in an optical resonator, such as Fabry-Perot structure, two mirrors separated by a prescribed distance. Frequency selection is achieved by resonant amplifier and by the resonators, which admits certain modes. Output coupling is accomplished by making one of the resonator mirrors partially transmitting.

Lasing in a pumped active medium

- In thermal equilibrium the stimulated emission is essentially negligible, since the density of electrons in the excited state is very small, and optical emission is mainly because of the spontaneous emission. Stimulated emission will exceed absorption only if the population of the excited states is greater than that of the ground state. This condition is known as **Population Inversion**. Population inversion is achieved by various **pumping** techniques.

- In a semiconductor laser, population inversion is accomplished by injecting electrons into the material to fill the lower energy states of the conduction band.

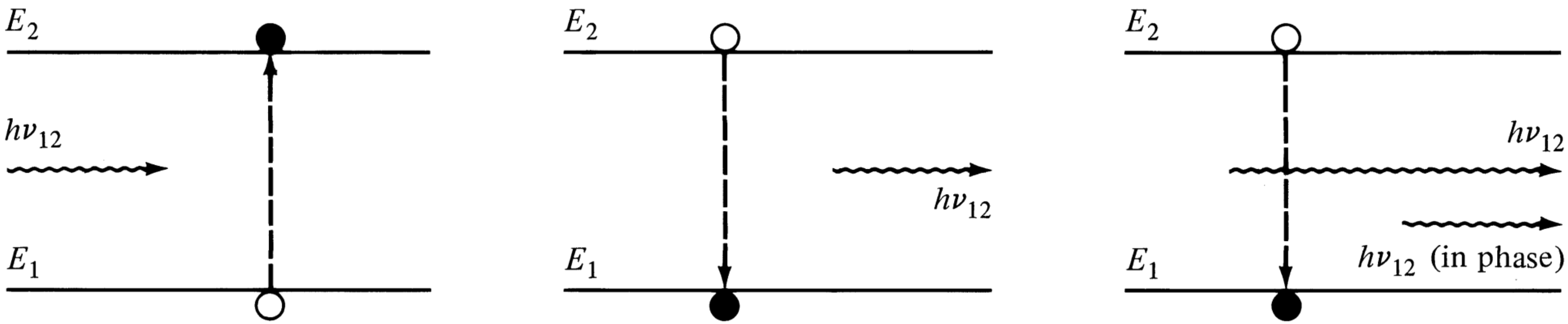
Pumped active medium

- Three main process for laser action:

  1- Photon absorption

  2- Spontaneous emission

  3- Stimulated emission



(*a*) Absorption (*b*) Spontaneous emission (*c*) Stimulated emission

Energy absorbed from the incoming photon

Random release of energy

Coherent release of energy

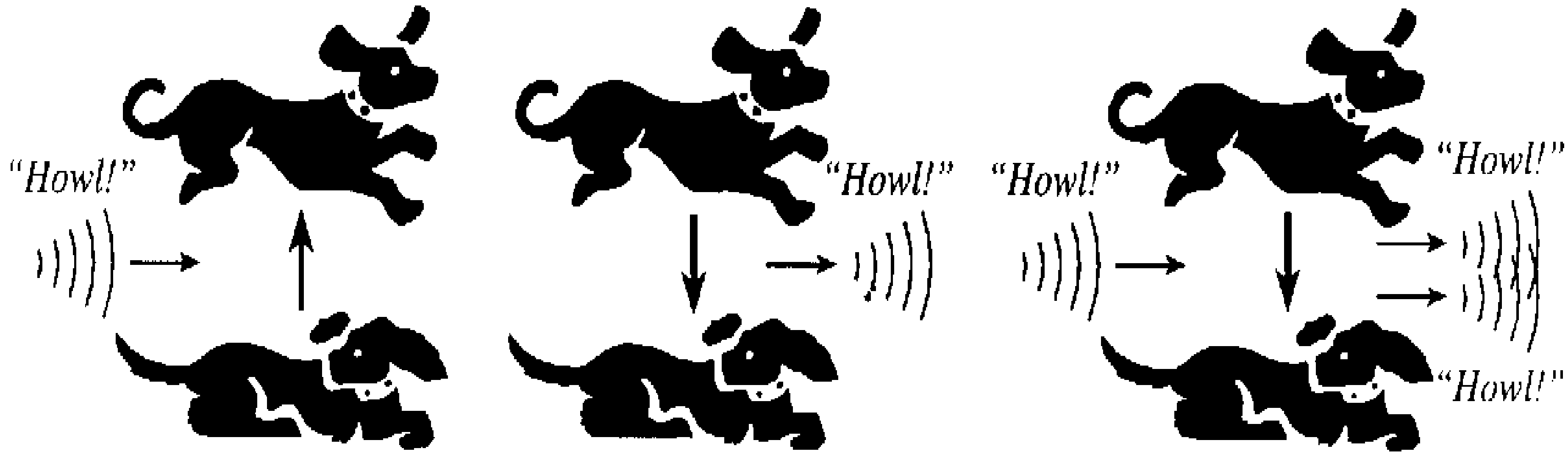
Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Howling Dog Analogy

1. absorption: a dog in the ground state might hear the howl from another dog and become excited, thus making a transition to the excited state.

2. spontaneous emission: a dog in the excited state might randomly let out a howl, which, through release of tension, enables him to relax to the ground state.

3. stimulated emission: a dog in the excited state might be stimulated to let out a howl when he hears the howl from another dog. The single howl becomes two howls voiced simultaneously, thus sounding like one howl with twice the intensity!

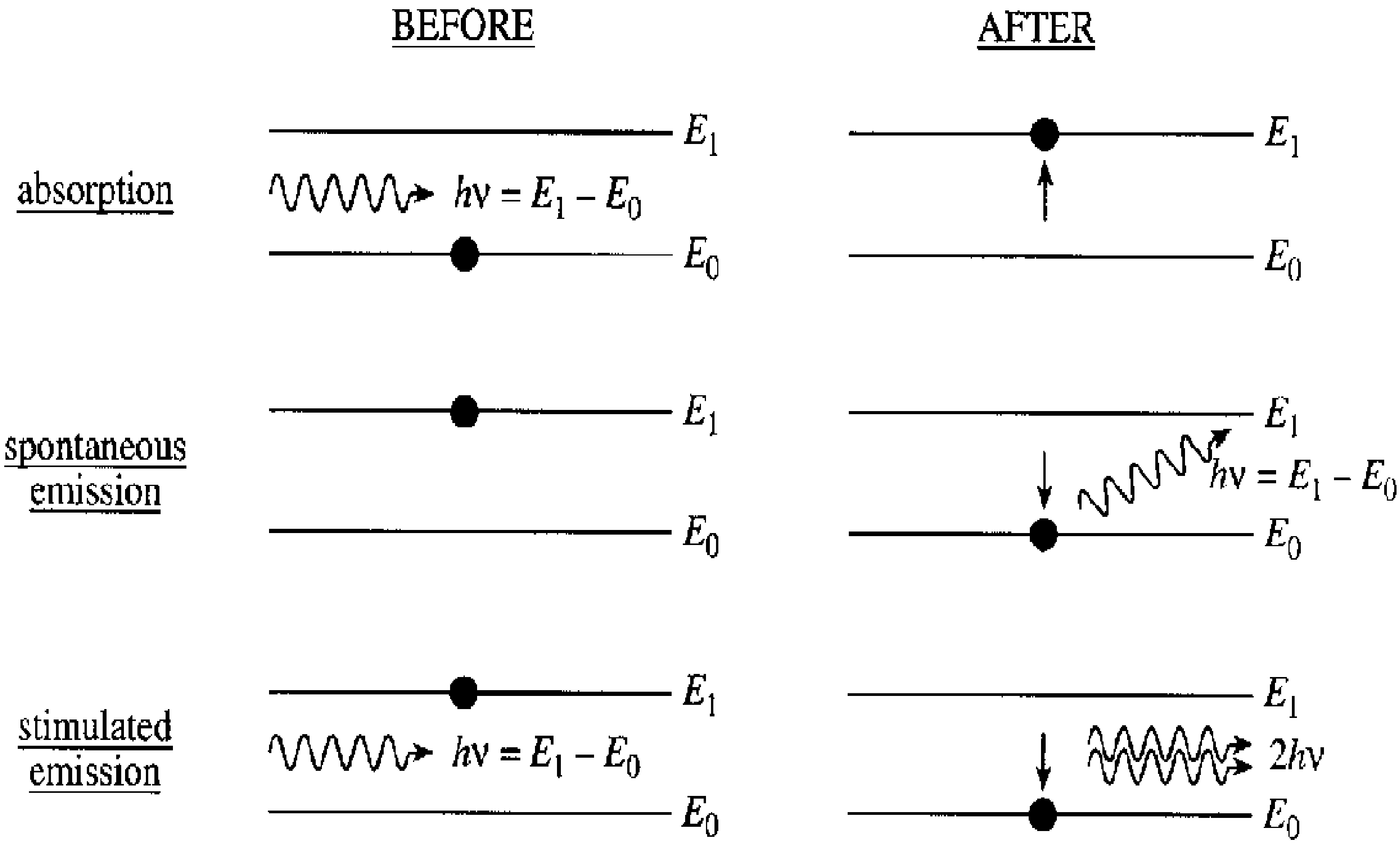


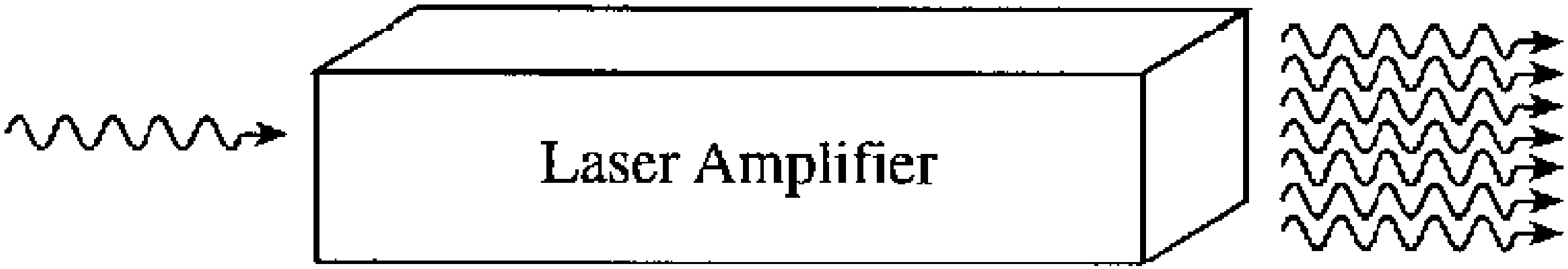
absorption spontaneous emission stimulated emission

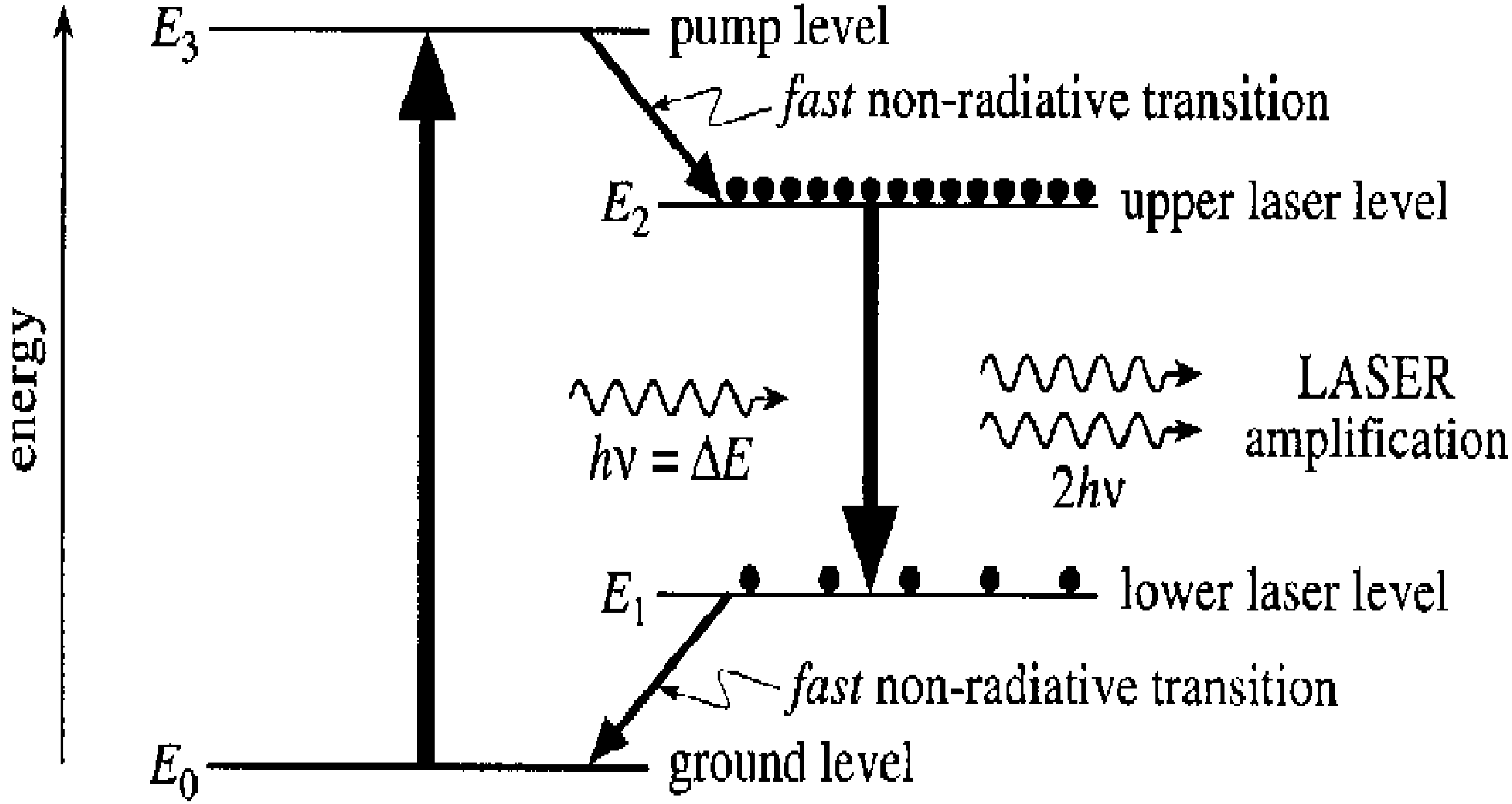
In Stimulated Emission incident and stimulated photons will have

- Identical energy € Identical wavelength € Narrow linewidth
- Identical direction € Narrow beam width
- Identical phase € Coherence and
- Identical polarization

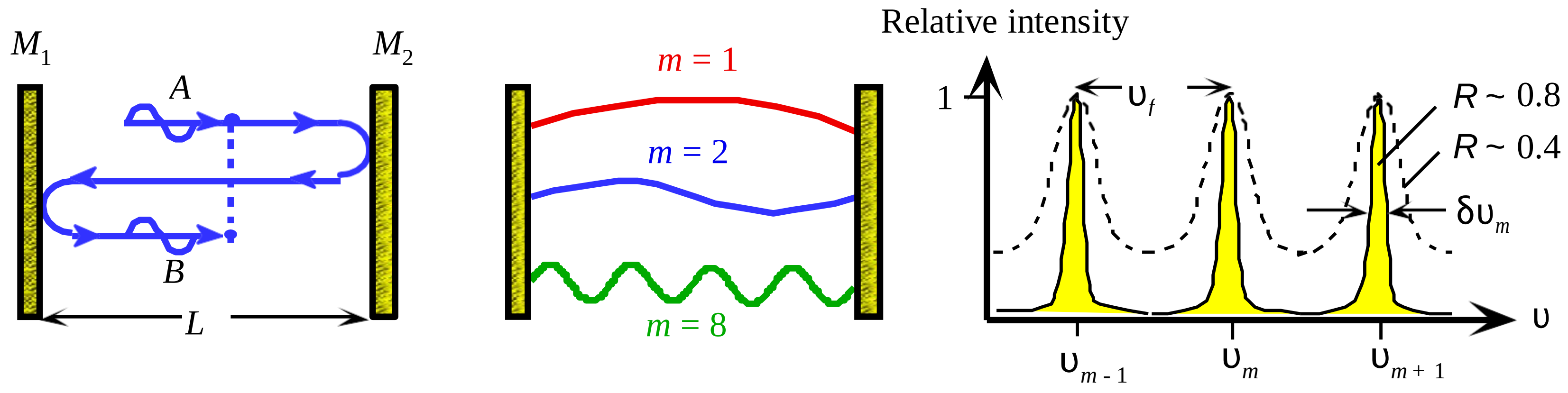
Stimulated Emission




Laser Amplifier


energy
E_3
pump level
fast non-radiative transition
E_2
upper laser level
$h\nu = \Delta E$
2$h\nu$
LASER
amplification
E_1
lower laser level
fast non-radiative transition
E_0
ground level

Fabry-Perot Resonator



(a) **(b)** **(c)**

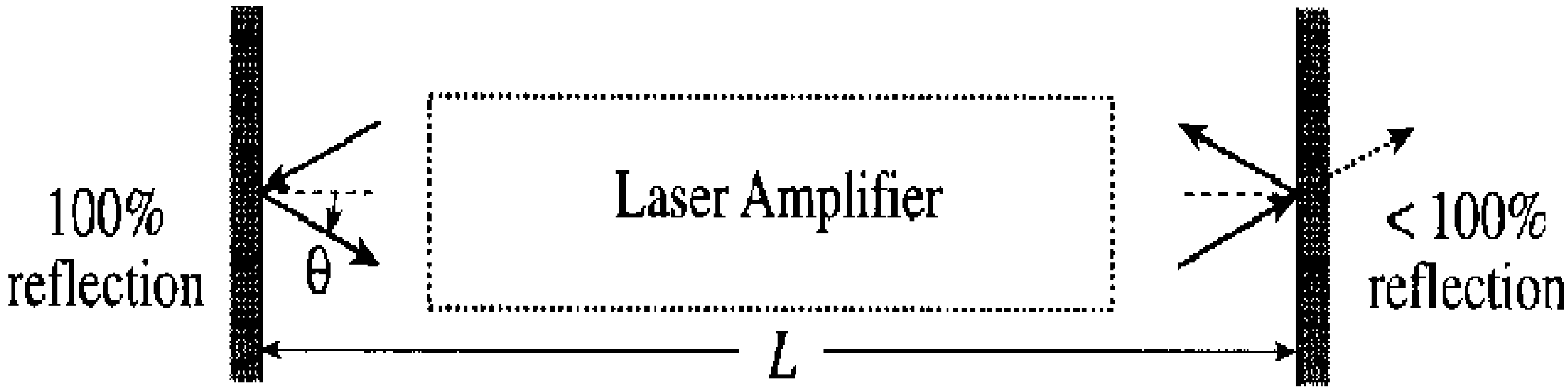
$$\text{Resonant modes}: kL = m \qquad m = 1,2,3,..$$

Schematic illustration of the Fabry-Perot optical cavity and its properties. (a) Reflected waves interfere. (b) Only standing EM waves, *modes*, of certain wavelengths are allowed in the cavity. (c) Intensity vs. frequency for various modes. *R* is mirror reflectance and lower *R* means higher loss from the cavity.

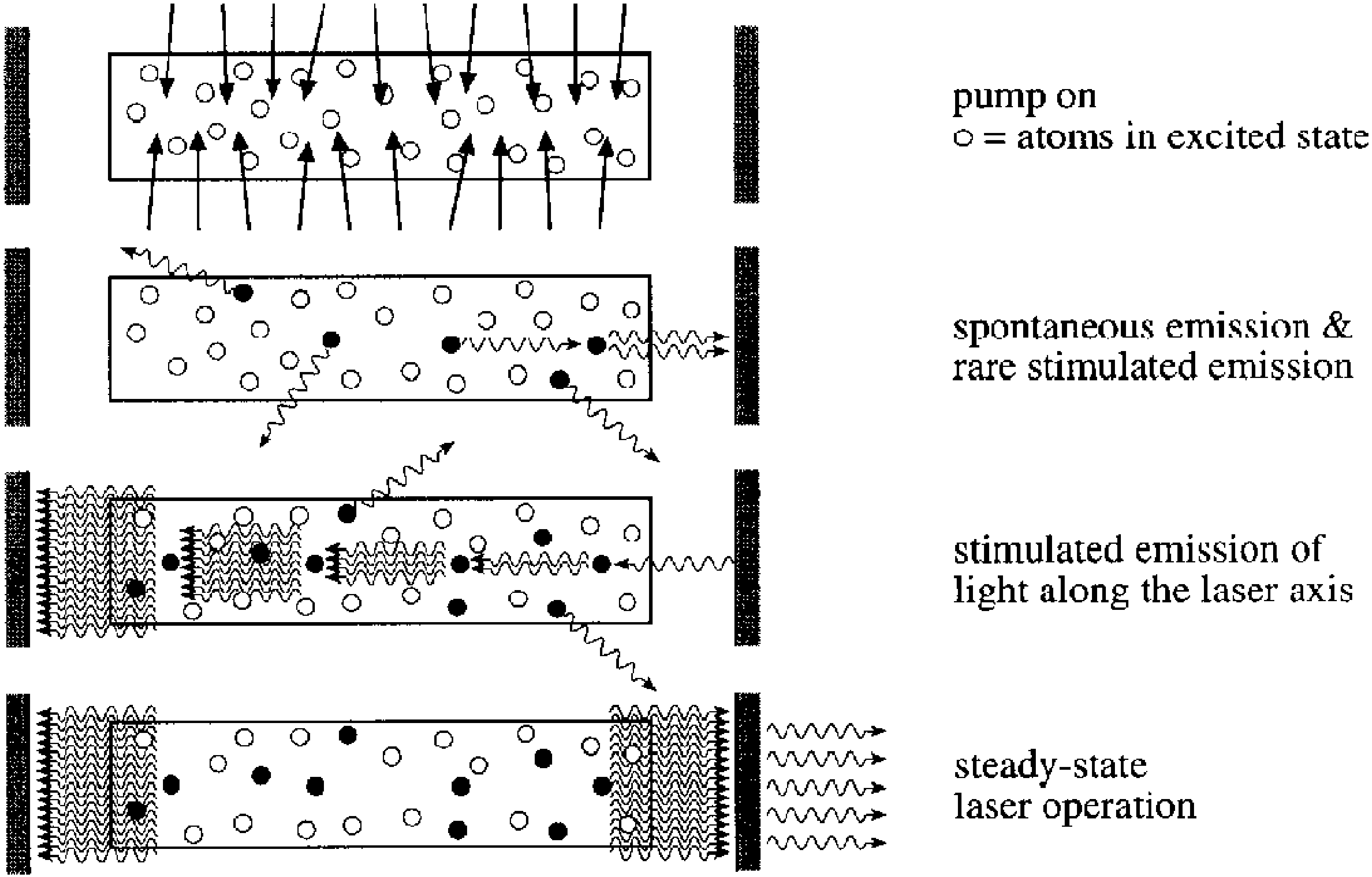
$$I_{trans} = I_{inc} \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2(kL)} \qquad [4\text{-}18]$$

R: reflectance of the optical intensity, *k*: optical wavenumber

Mirror Reflections

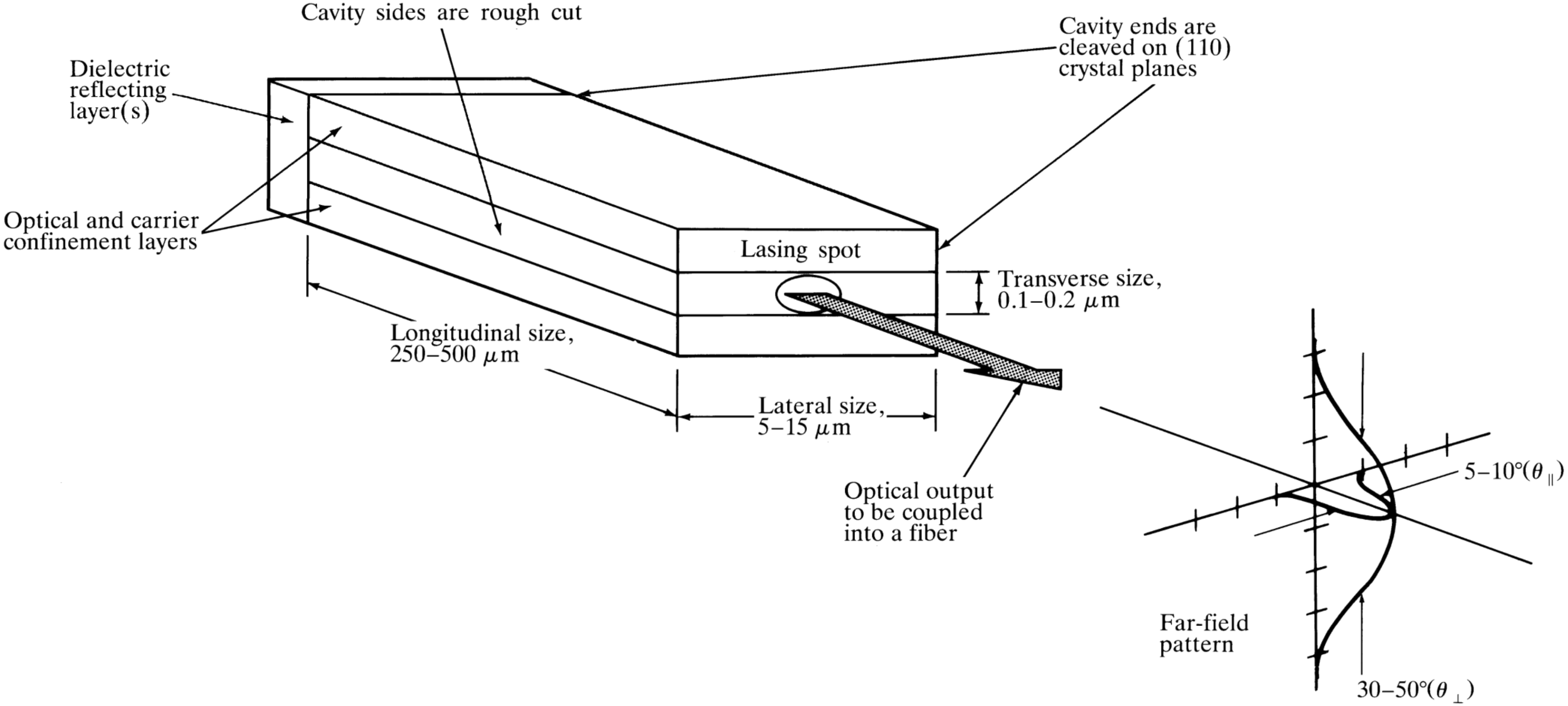


How a Laser Works



Laser Diode

- Laser diode is an improved LED, in the sense that uses stimulated emission in semiconductor from optical transitions between distribution energy states of the valence and conduction bands with optical resonator structure such as Fabry-Perot resonator with both optical and carrier confinements.



Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

Laser Diode Characteristics

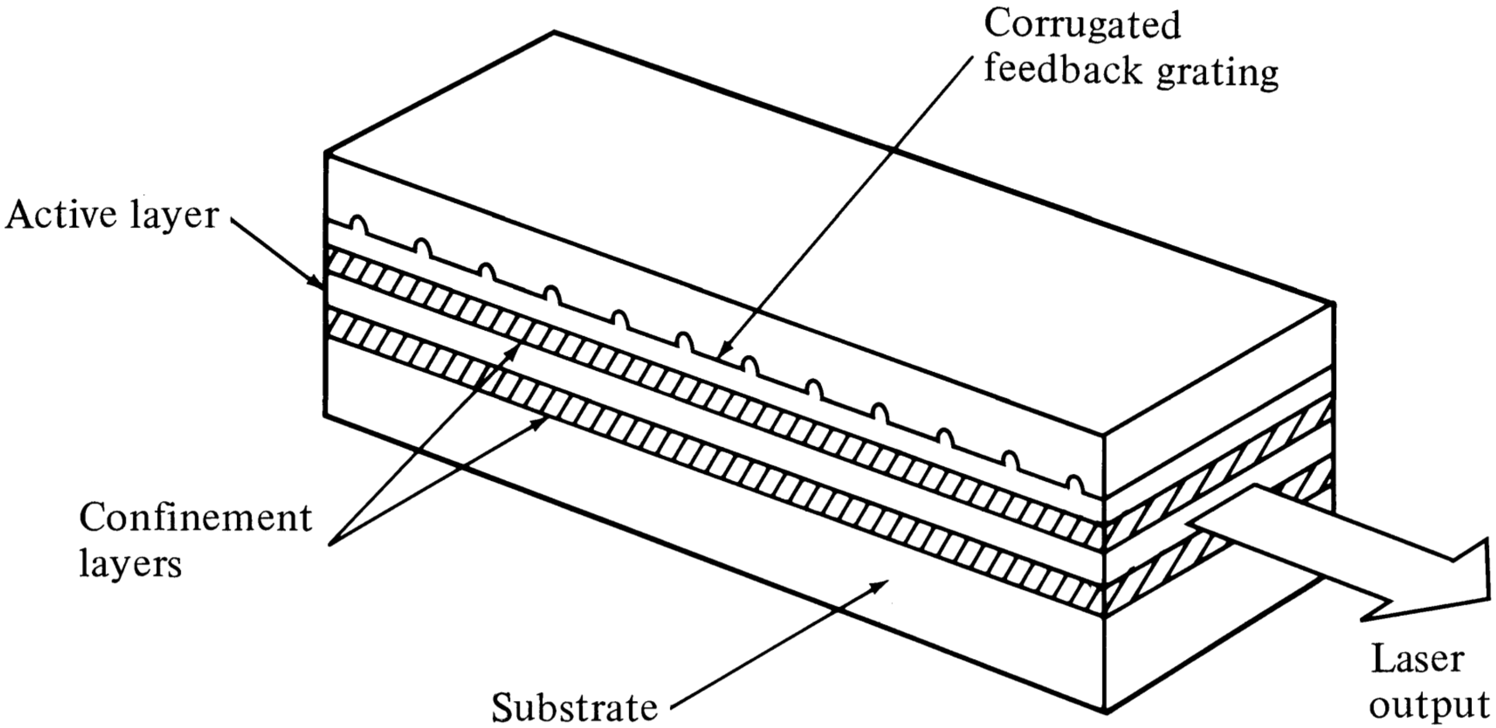
- Nanosecond & even picosecond response time (GHz BW)
- Spectral width of the order of nm or less
- High output power (tens of mW)
- Narrow beam (good coupling to single mode fibers)

- Laser diodes have three distinct radiation modes namely, longitudinal, lateral and transverse modes.

- In laser diodes, end mirrors provide strong optical feedback in longitudinal direction, so by roughening the edges and cleaving the facets, the radiation can be achieved in longitudinal direction rather than lateral direction.

DFB(Distributed FeedBack) Lasers

- In DFB lasers, the optical resonator structure is due to the incorporation of Bragg grating or periodic variations of the refractive index into multilayer structure along the length of the diode.



The optical feedback is provided by fiber Bragg Gratings
€ Only one wavelength get positive feedback

Optical Fiber communications, 3rd ed.,G.Keiser,McGrawHill, 2000

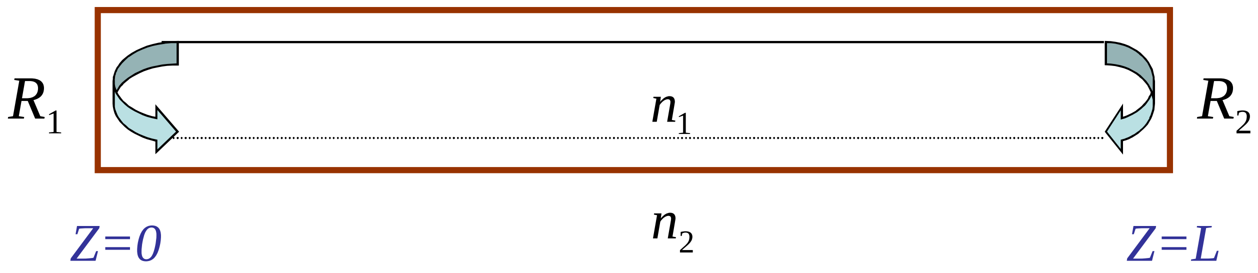
Laser Operation & Lasing Condition

- To determine the lasing condition and resonant frequencies, we should focus on the optical wave propagation along the longitudinal direction, *z*-axis. The optical field intensity, *I*, can be written as:

$$I(z,t) = I(z)e^{j(\omega t - \beta z)} \quad [4\text{-}19]$$

- Lasing is the condition at which light amplification becomes possible by virtue of population inversion. Then, stimulated emission rate into a given EM mode is proportional to the intensity of the optical radiation in that mode. In this case, the loss and gain of the optical field in the optical path determine the lasing condition. The radiation intensity of a photon at energy $h\nu$ varies exponentially with a distance z amplified by factor *g*, and attenuated by factor α according to the following relationship:

$$I(z) = I(0)\exp\left[\left(\Gamma g(h\nu) - \alpha(h\nu)\right)z\right] \qquad [4\text{-}20]$$



$$I(2L) = I(0)R_1R_2\exp\left(\Gamma g(h\nu) - \bar{\alpha}(h\nu)\right)(2L) \qquad [4\text{-}21]$$

Γ : Optical confinement factor, g : gain coefficient

$\bar{\alpha}$: effective absorption coefficient, $R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$

Lasing Conditions:

$$I(2L) = I(0)$$
$$\exp(-j2\beta L) = 1 \qquad [4\text{-}22]$$

Threshold gain & current density

$$\Gamma g_{th} = \bar{\alpha} + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) \qquad [4\text{-}23]$$

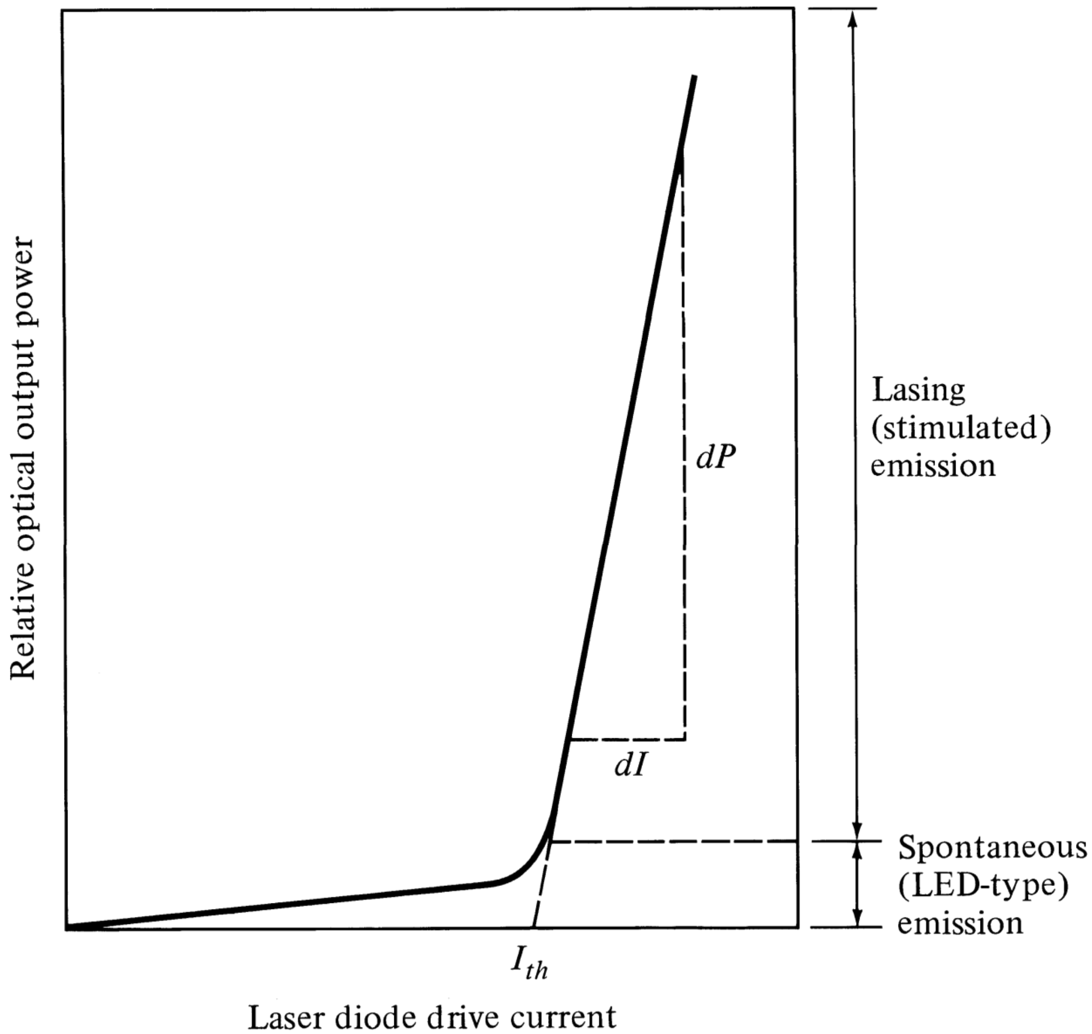
Laser starts to "lase" iff $: g \geq g_{th}$

For laser structure with strong carrier confinement, the threshold current Density for stimulated emission can be well approximated by:

$$g_{th} = \beta J_{th} \qquad [4\text{-}24]$$

β : constant depends on specific device construction

Optical output vs. drive current



Optical Fiber communications, 3[rd] ed.,G.Keiser,McGrawHill, 2000

Semiconductor laser rate equations

- Rate equations relate the optical output power, or # of photons per unit volume, Φ , to the diode drive current or # of injected electrons per unit volume, *n.* For active (carrier confinement) region of depth *d*, the rate equations are:

$$\frac{d\Phi}{dt} = Cn\Phi + R_{sp} - \frac{\Phi}{\tau_{ph}}$$

$$\text{Photon rate} = \text{stimulated emission} + \text{spontaneous emission} + \text{photon loss} \qquad [4\text{-}25]$$

$$\frac{dn}{dt} = \frac{J}{qd} - \frac{n}{\tau_{sp}} - Cn\Phi$$

$$\text{electron rate} = \text{injection} + \text{spontaneous recombination} + \text{stimulated emission}$$

C : Coefficient expressing the intensity of the optical emission & absorption process

R_{sp} : rate of spontaneous emission into the lasing mode

τ_{ph} : photon life time

J : Injection current density

Threshold current Density & excess electron density

- At the threshold of lasing: $\Phi \approx 0, d\Phi / dt \geq 0, R_{sp} \approx 0$

$$\text{from eq.}[4-25] \Rightarrow Cn\Phi - \Phi / \tau_{ph} \geq 0 \Rightarrow \boxed{n \geq \frac{1}{C\tau_{ph}} = n_{th}} \quad [4\text{-}26]$$

- The threshold current needed to maintain a steady state threshold concentration of the excess electron, is found from electron rate equation under steady state condition *dn/dt=0* when the laser is just about to lase:

$$0 = \frac{J_{th}}{qd} - \frac{n_{th}}{\tau_{sp}} \Rightarrow \boxed{J_{th} = qd\frac{n_{th}}{\tau_{sp}}} \quad [4\text{-}27]$$

Laser operation beyond the threshold

$$J > J_{th}$$

- The solution of the rate equations [4-25] gives the steady state photon density, resulting from stimulated emission and spontaneous emission as follows:

$$\Phi_s = \frac{\tau_{ph}}{qd}(J - J_{th}) + \tau_{ph} R_{sp} \qquad [4\text{-}28]$$

External quantum efficiency

- Number of photons emitted per radiative electron-hole pair recombination above threshold, gives us the external quantum efficiency.

$$\eta_{ext} = \frac{\eta_i (g_{th} - \alpha)}{g_{th}}$$

$$= \frac{q}{E_g} \frac{dP}{dI} = 0.8065\lambda[\mu\text{m}] \frac{dP(\text{mW})}{dI(\text{mA})} \qquad [4\text{-}29]$$

- Note that: $\eta_i \approx 60\% - 70\%; \qquad \eta_{ext} \approx 15\% - 40\%$

Laser Resonant Frequencies

- Lasing condition, namely eq. [4-22]:

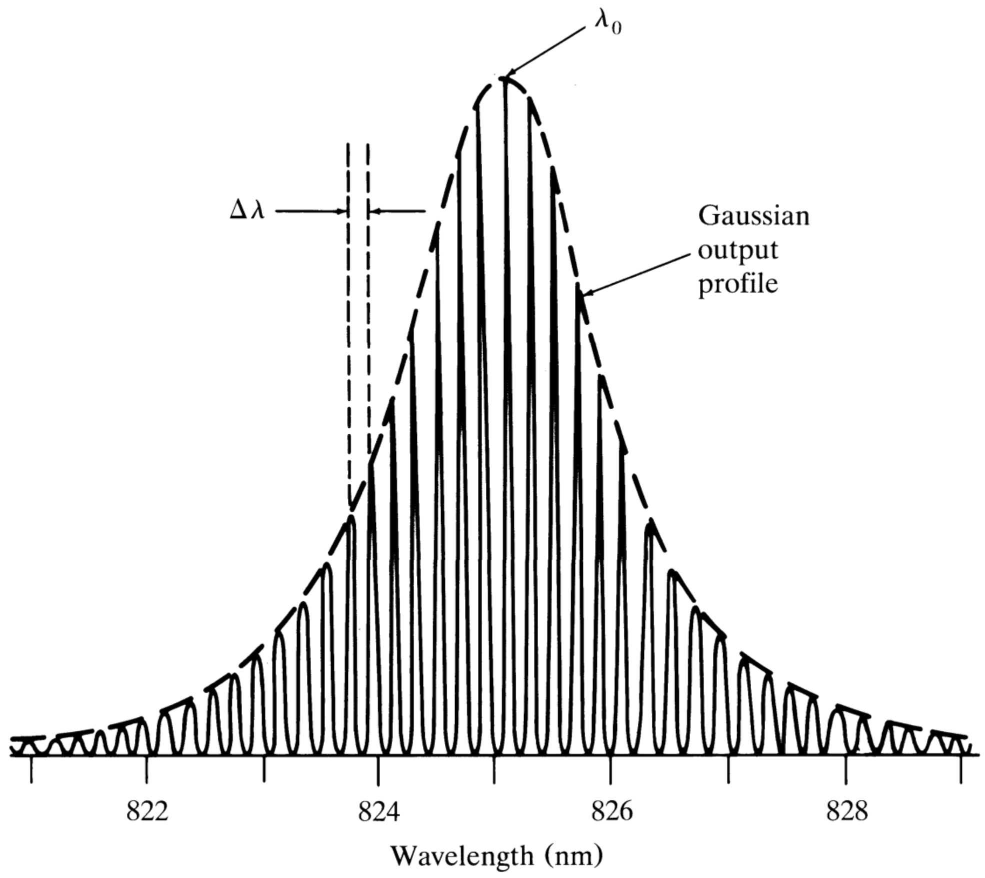
$$\exp(-j2\beta L) = 1 \Rightarrow \qquad 2\beta L = 2m\pi, \quad m = 1,2,3,...$$

- Assuming $\beta = \frac{2\pi n}{\lambda}$ the resonant frequency of the *m*th mode is:

$$\nu_m = \frac{mc}{2Ln} \qquad m = 1,2,3,... \qquad [4\text{-}30]$$

$$\Delta\nu = \nu_m - \nu_{m-1} = \frac{c}{2Ln} \Leftrightarrow \Delta\lambda = \frac{\lambda^2}{2Ln} \qquad [4\text{-}31]$$

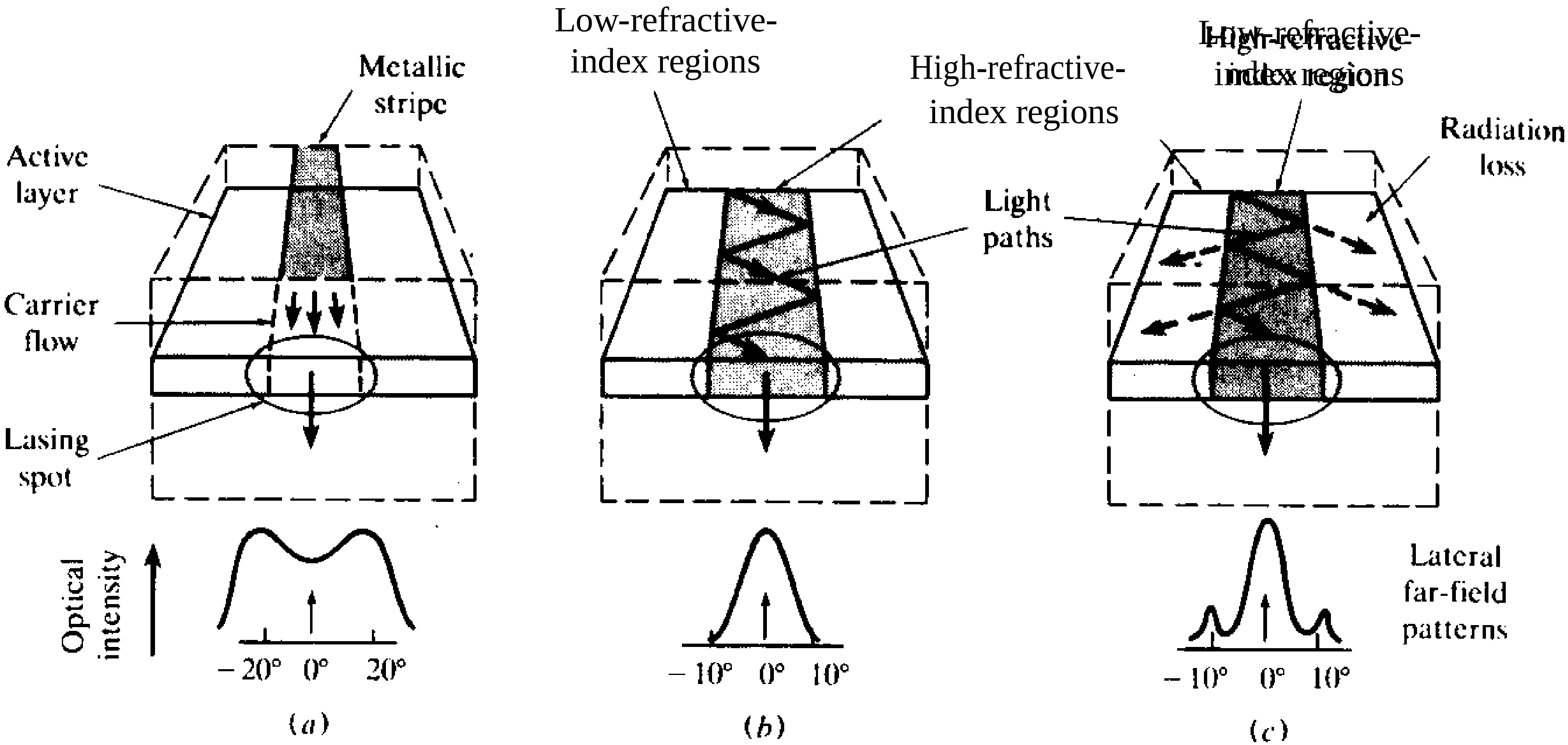
Spectrum from a Laser Diode



$$g(\lambda) = g(0)\exp\left[-\frac{(\lambda-\lambda_0)^2}{2\sigma^2}\right] \quad \sigma : \text{spectral width} \qquad [4\text{-}32]$$

Laser Diode Structure & Radiation Pattern

- Efficient operation of a laser diode requires reducing the # of lateral modes, stabilizing the gain for lateral modes as well as lowering the threshold current. These are met by structures that confine the optical wave, carrier concentration and current flow in the lateral direction. The important types of laser diodes are: **gain-induced, positive index guided, and negative index guided**.



(a) gain-induced guide (b)positive-index waveguide (c)negative-index waveguide

Unstable, two-peaked beam

Can made single-mode laser

Laser Diode with buried heterostructure (BH)

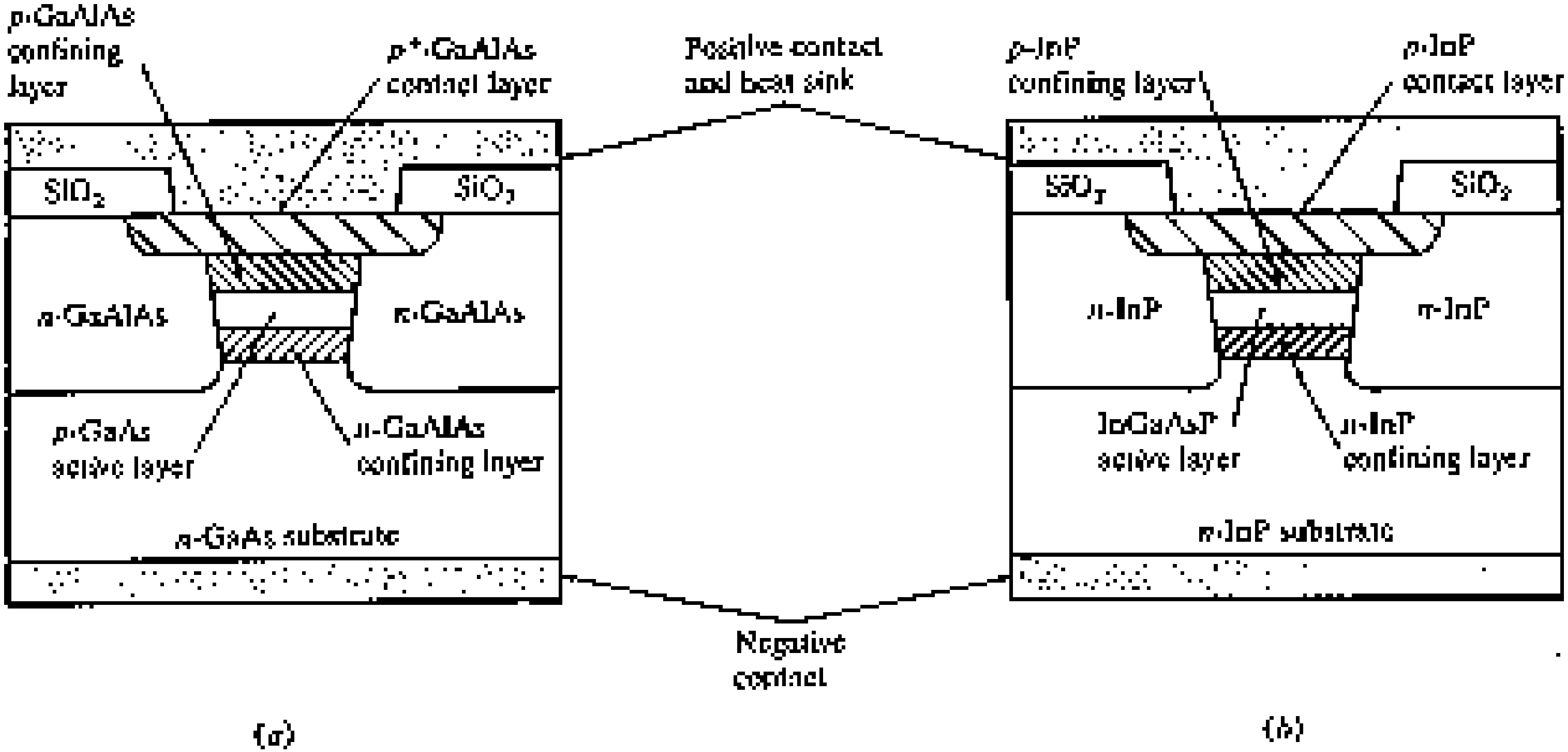


FIGURE 4-23
(a) Short-wavelength (800–900 nm) GaAlAs and (b) long-wavelengths (1300–1600 nm) InGaAsP buried-heterostructure laser diodes.

Single Mode Laser

- Single mode laser is mostly based on the index-guided structure that supports only the fundamental transverse mode and the fundamental longitudinal mode. In order to make single mode laser we have four options:

  1- Reducing the length of the cavity to the point where the frequency separation given in eq[4-31] of the adjacent modes is larger than the laser transition line width. This is hard to handle for fabrication and results in low output power.

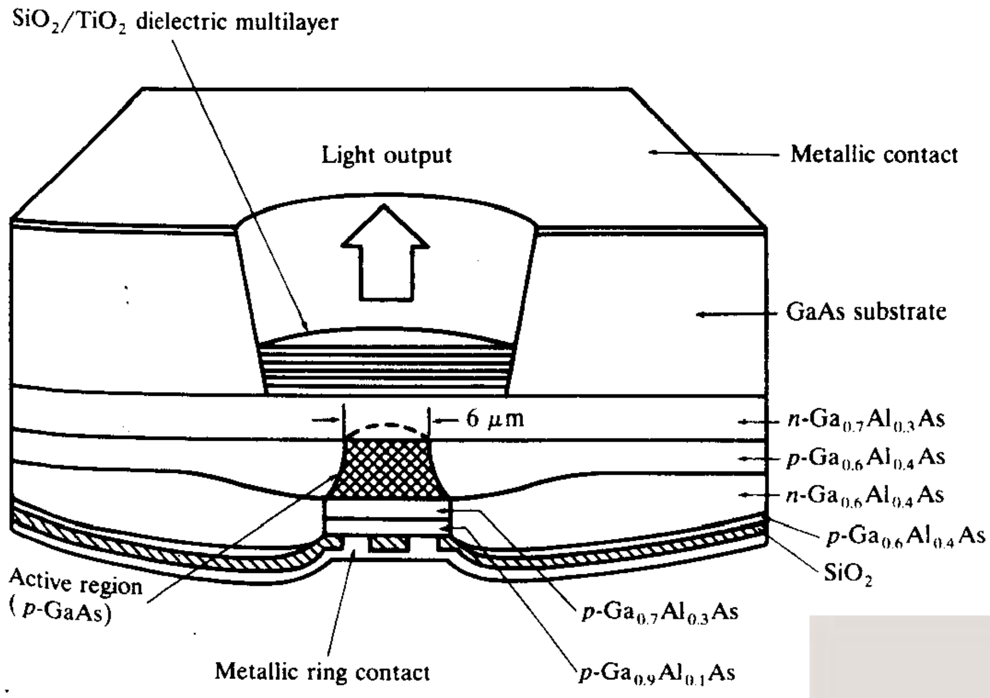
  2- **Vertical-Cavity Surface Emitting laser (VCSEL)**

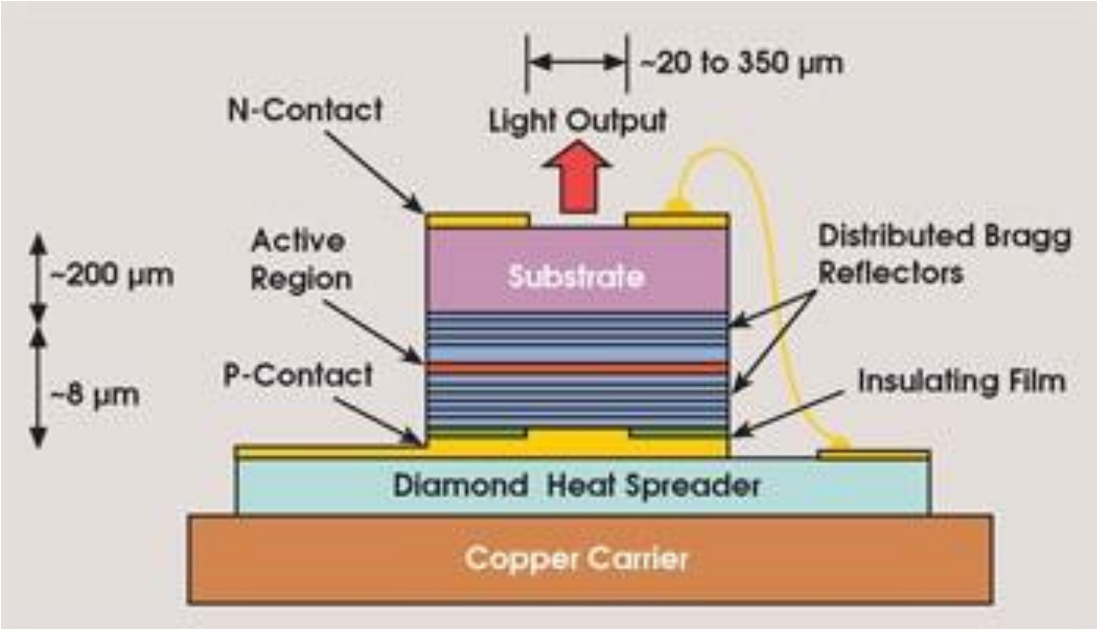
  3- Structures with built-in frequency selective grating

  4- tunable laser diodes

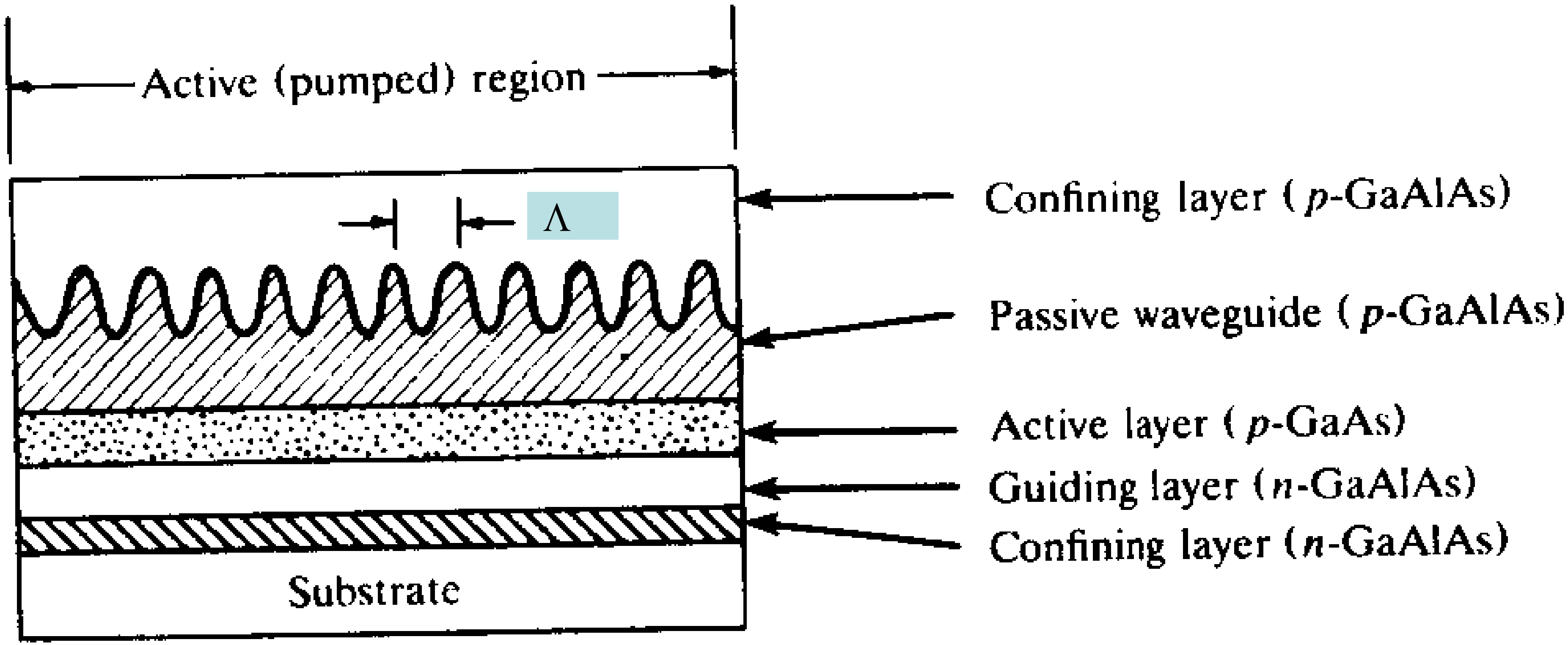
.

VCSEL


SiO2/TiO2 dielectric multilayer
Light output
Metallic contact
GaAs substrate
6 μm
n-Ga0.7Al0.3As
p-Ga0.6Al0.4As
n-Ga0.6Al0.4As
p-Ga0.6Al0.4As
SiO2
Active region
(p-GaAs)
Metallic ring contact
p-Ga0.7Al0.3As
p-Ga0.9Al0.1As

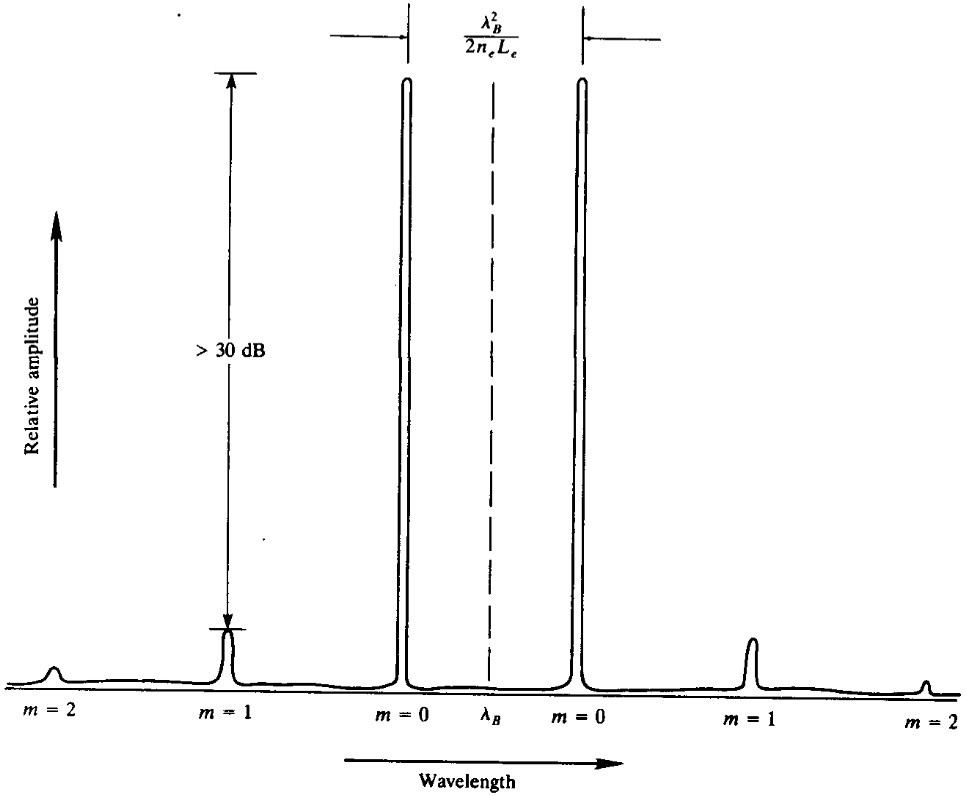

~20 to 350 μm
N-Contact
Light Output
~200 μm
Active
Region
Substrate
Distributed Bragg
Reflectors
~8 μm
P-Contact
Insulating Film
Diamond Heat Spreader
Copper Carrier

Frequency-Selective Laser Diodes: Distributed Feedback (DFB) Laser



Bragg wavelength $$\lambda_B = \frac{2 n_e \Lambda}{m}$$ [4-33]

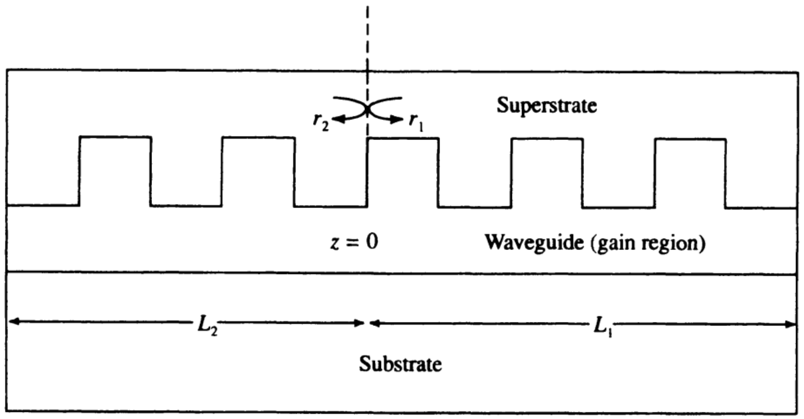
λ_B : effective refractive index; m : order of the grating

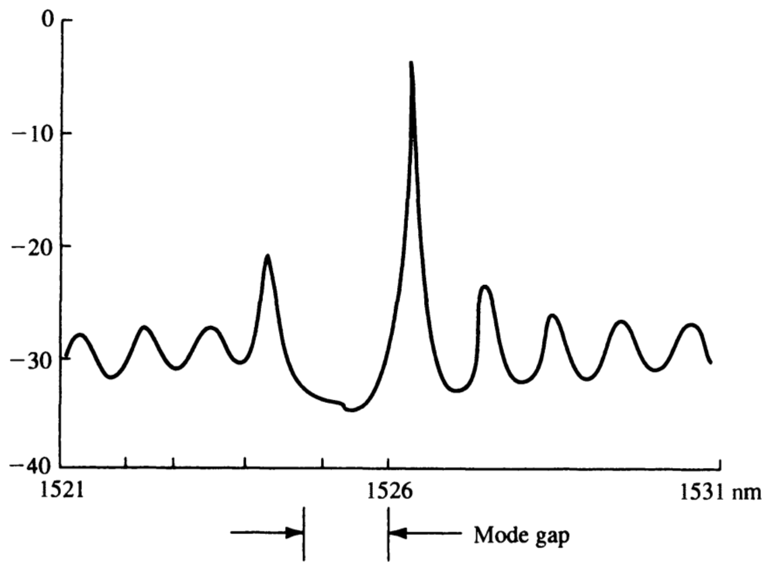


Output spectrum symmetrically distributed around Bragg wavelength
in an idealized DFB laser diode

$$\lambda = \lambda_B \pm \frac{\lambda_B^{\ 2}}{2n_e L_e}\left(m + \frac{1}{2}\right) \qquad [4\text{-}35]$$

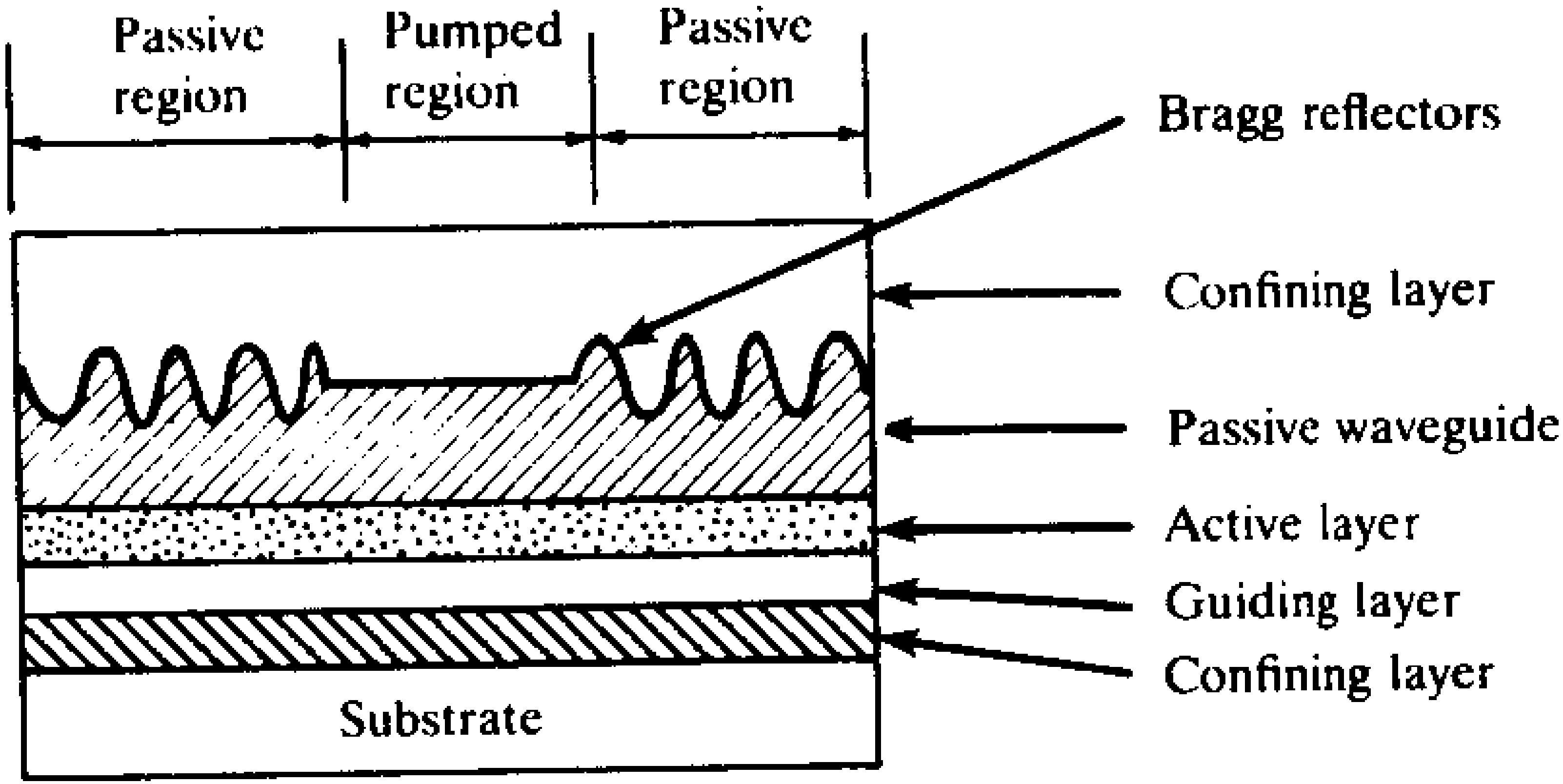
L_e : effective grating length;
m (=0,1,2) : mode order



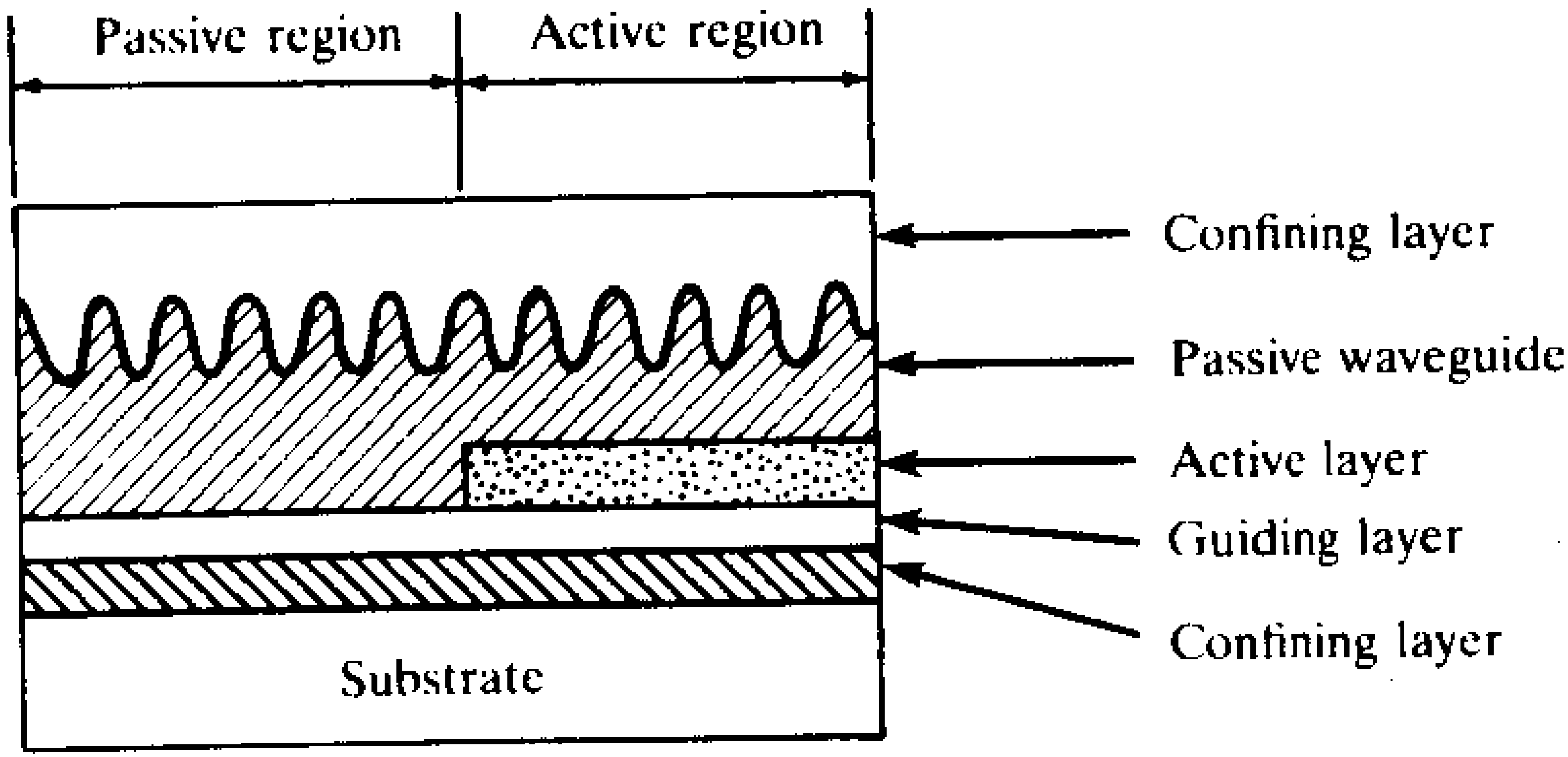


A. Yariv, P. Yeh, Photonics: Optical Electronics in Modern Communications, Oxford, 2007

Frequency-Selective laser Diodes: Distributed Feedback Reflector (DBR) laser



Frequency-Selective Laser Diodes: Distributed Reflector (DR) Laser



Modulation of Laser Diodes

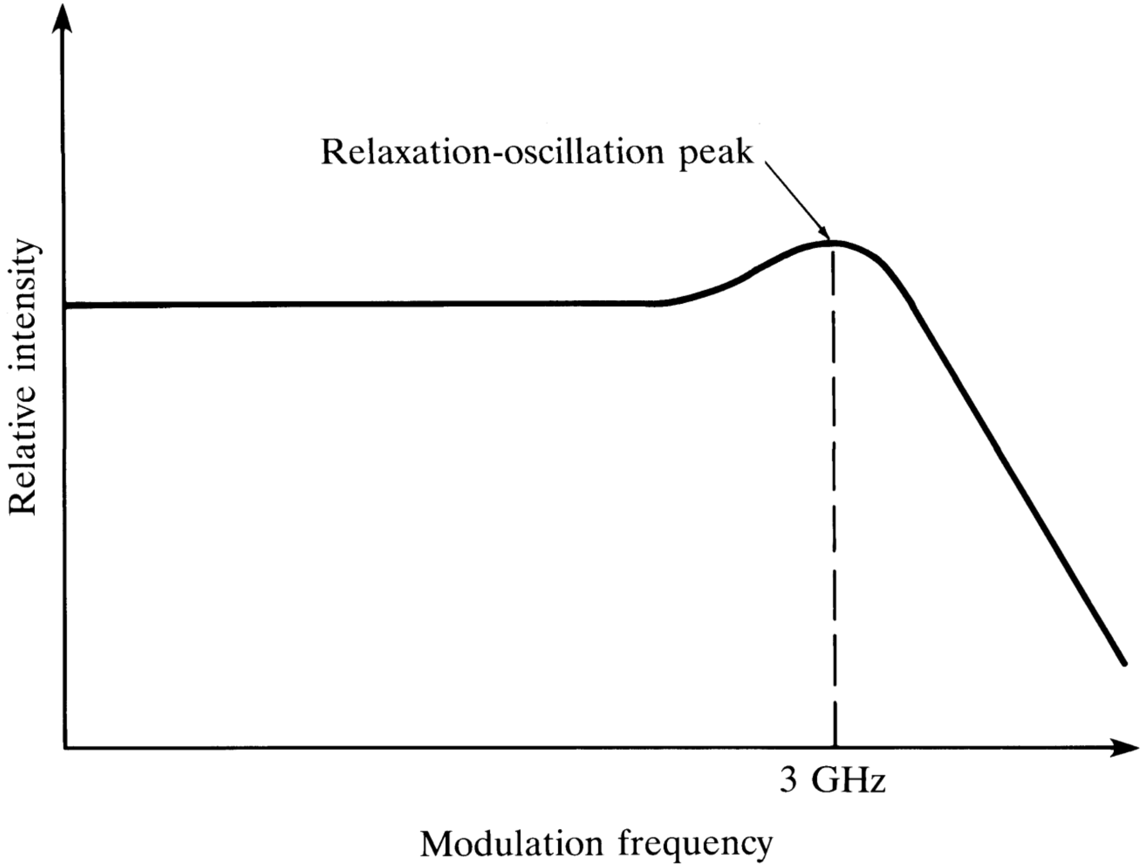
- Internal Modulation: Simple but suffers from non-linear effects.
- External Modulation: for rates greater than 2 Gb/s, more complex, higher performance.
- Most fundamental limit for the modulation rate is set by the photon life time in the laser cavity:

$$\frac{1}{\tau_{ph}} = \frac{c}{n}\left(\alpha + \frac{1}{2L}\ln\frac{1}{R_1 R_2}\right) = \frac{c}{n} g_{th} \qquad [4\text{-}36]$$

- Another fundamental limit on modulation frequency is the relaxation oscillation frequency given by:

$$f = \frac{1}{2\pi}\frac{1}{\sqrt{\tau_{sp}\tau_{ph}}}\left(\frac{I}{I_{th}} - 1\right)^{1/2} \qquad [4\text{-}37]$$

Relaxation oscillation peak



Pulse Modulated laser

- In a pulse modulated laser, if the laser is completely turned off after each pulse, after onset of the current pulse, a time delay, t_d given by:

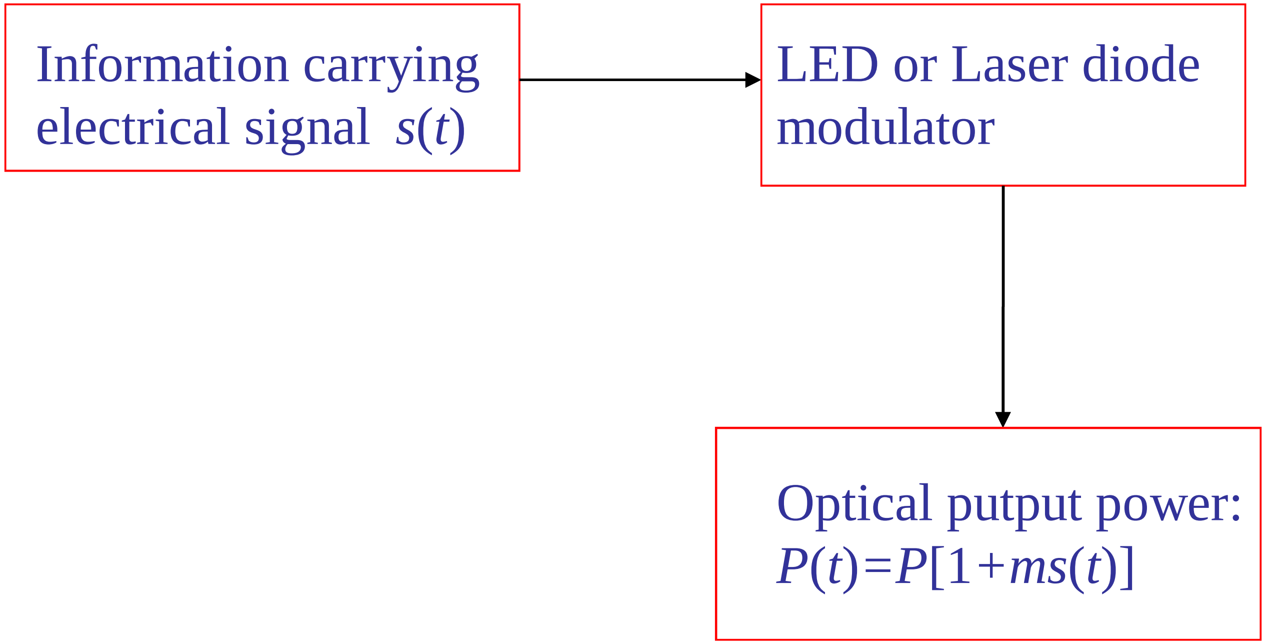
$$t_d = \tau \ln \left[\frac{I_p}{I_p + (I_B - I_{th})} \right] \quad [4\text{-}38]$$

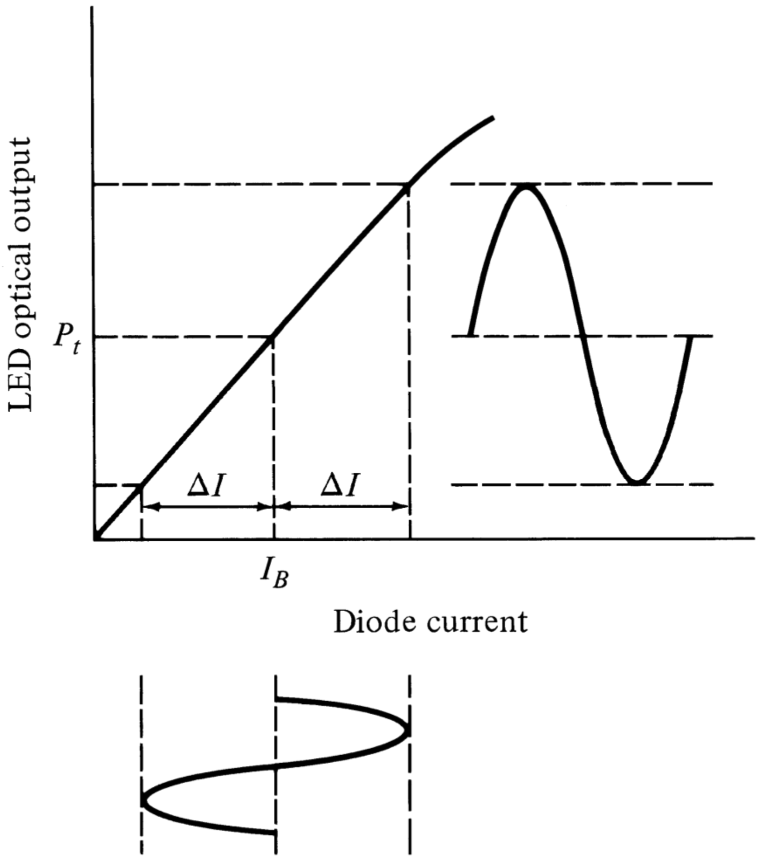
τ : carrier life time

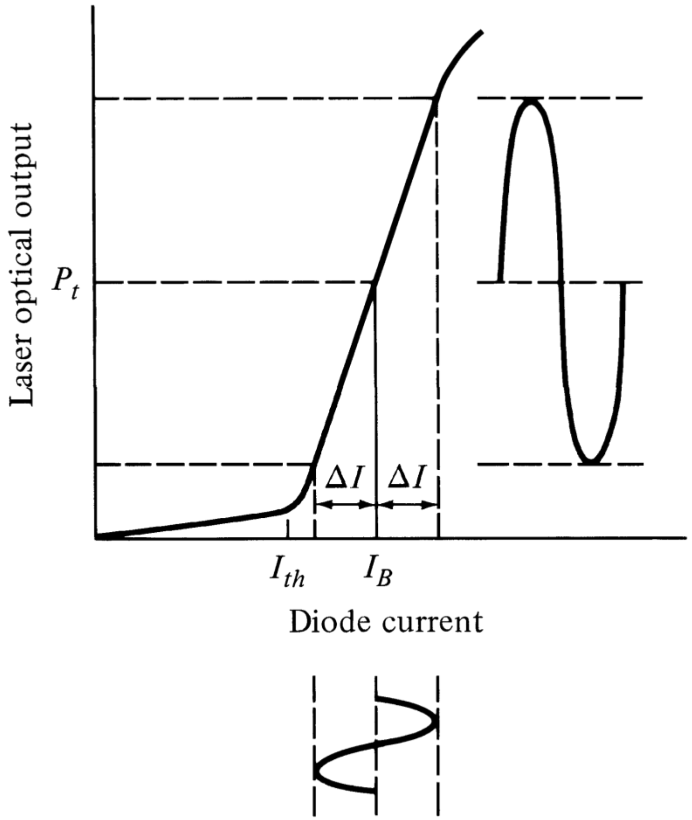
I_p : Current pulse amplitude

I_B : Bias current

Linearity of Laser




LED optical output
P_t
ΔI
ΔI
I_B
Diode current


Laser optical output
P_t
ΔI
ΔI
I_{th}
I_B
Diode current

Nonlinearity

$$x(t) = A\cos\omega t$$

$$y(t) = A_0 + A_1\cos\omega t + A_2\cos 2\omega t + \ldots$$

N^th order harmonic distortion:

$$20\log\left(\frac{A_n}{A_1}\right)$$

Intermodulation Distortion

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t \Rightarrow$$

$$y(t) = \sum_{m,n} B_{mn} \cos(m\omega_1 + n\omega_2)t \qquad m,n = 0, \pm 1, \pm 2, \ldots$$

Harmonics:

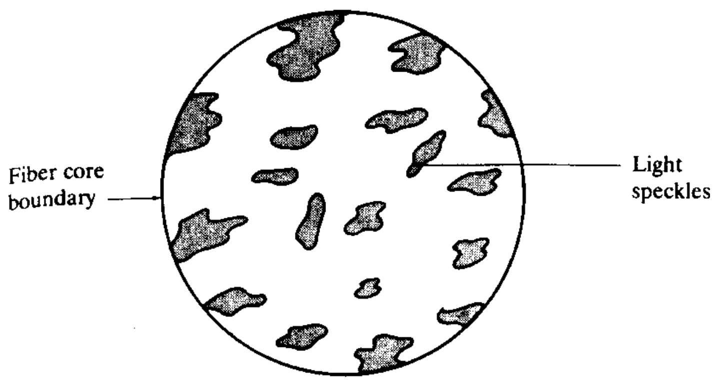
$$n\omega_1, m\omega_2$$

Intermodulated Terms:

$$\omega_1 \pm \omega_2, 2\omega_1 \pm \omega_2, \omega_1 \pm 2\omega_2, \ldots$$

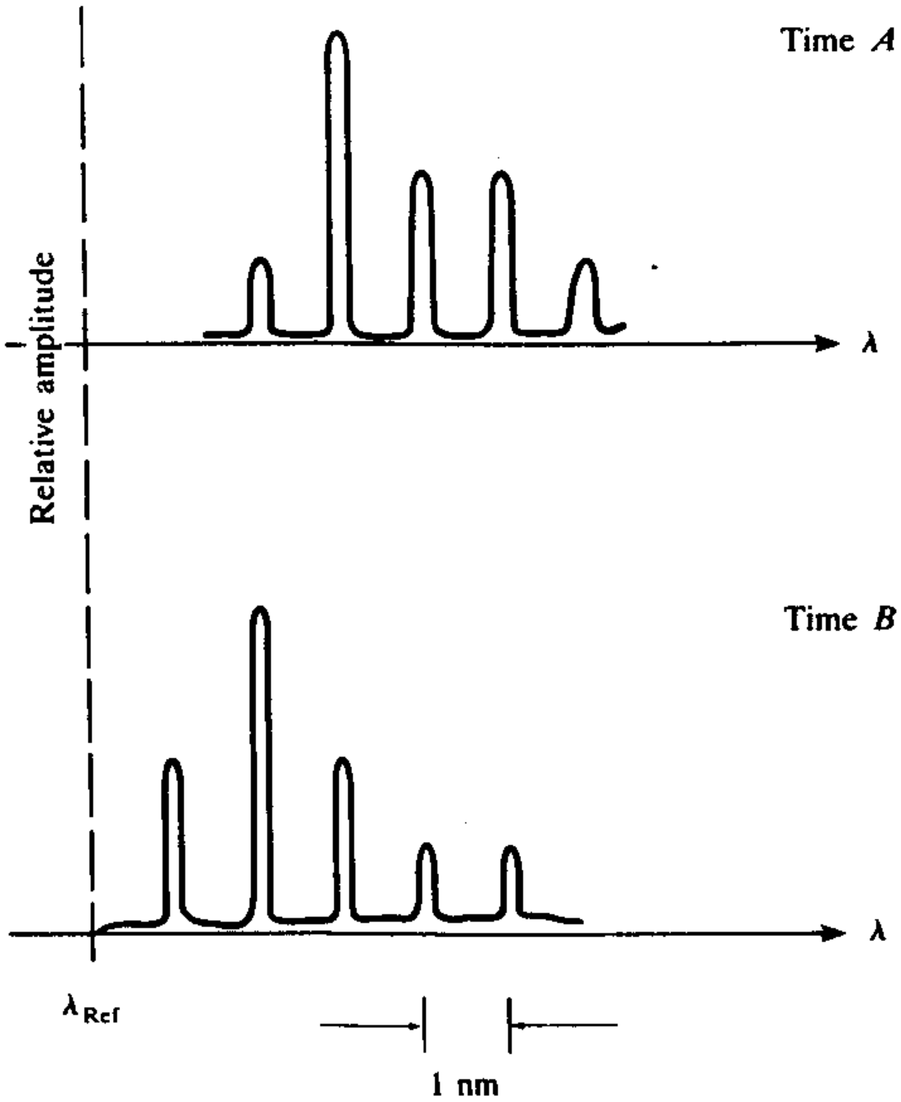
Laser Noise

- **Modal (speckle) Noise**: Fluctuations in the distribution of energy among various modes.
- **Mode partition Noise**: Intensity fluctuations in the longitudinal modes of a laser diode, main source of noise in single mode fiber systems.
- **Reflection Noise**: Light output gets reflected back from the fiber joints into the laser, couples with lasing modes, changing their phase, and generate noise peaks. Isolators & index matching fluids can eliminate these reflections.



A speckle pattern

Intensity Fluctuation



Different modes or groups of modes dominate the optical output at different times.

Modulation of Optical Sources

- Optical sources can be modulated either directly or externally.
- Direct modulation is done by modulating the driving current according to the message signal (digital or analog)
- In external modulation, the laser emits continuous wave (CW) light and the modulation is done in the fiber

Why Modulation

- A communication link is established by transmission of information reliably
- Optical modulation is embedding the information on the optical carrier for this purpose
- The information can be digital (1,0) or analog (a continuous waveform)
- The bit error rate (BER) is the performance measure in digital systems
- The signal to noise ratio (SNR) is the performance measure in analog systems

Parameters to characterize performance of optical modulation

- modulation depth
- bandwidth
- insertion loss
- degree of isolation
- power handling
- induced chirp

Important parameters used to characterize and compare different modulators

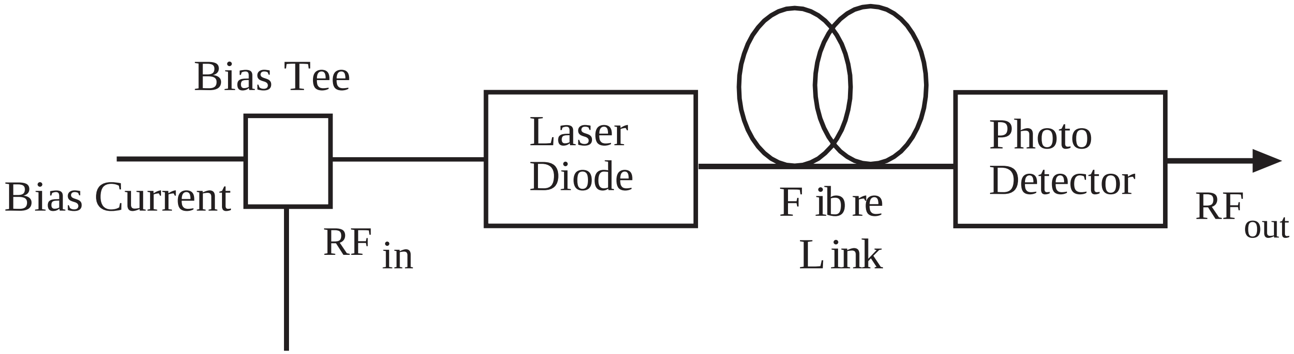
- ***Modulation efficiency***: **Defined differently depending on if we modulate intensity, phase or frequency. For intensity it is defined as $(I_{max} - I_{min})/I_{max}$.**
- ***Modulation depth***: **For intensity modulation it is defined in decibel by 10 log (I_{max}/I_{min}).**
- ***Modulation bandwidth***: **Defined as the high frequency at which the efficiency has fallen by 3dB.**
- ***Power consumption***: **Simply the power consumption per unit bandwidth needed for (intensity) modulation.**

Types of Optical Modulation

- Direct modulation is done by superimposing the modulating (message) signal on the driving current
- External modulation is done after the light is *generated;* the laser is driven by a dc current and the modulation is done after that separately
- Both these schemes can be done with either *digital* or *analog* modulating signals

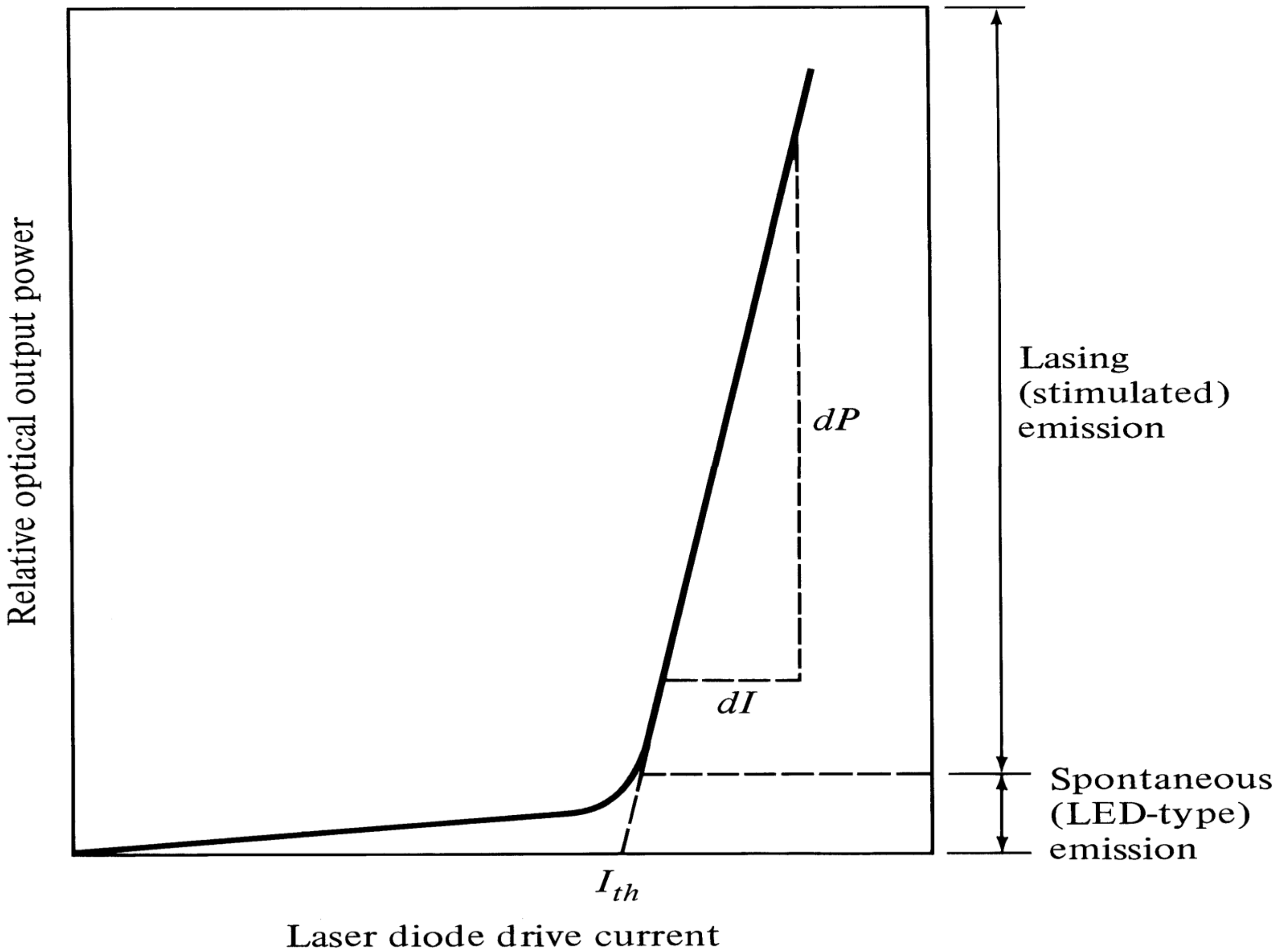
- Direct modulation of semiconductor lasers
  --- frequency response
  --- relaxation oscillation
  --- chirp
- external modulators:
  --- Electro-absorption modulators
  --- Mach-Zehnder interferometer
- New mechanisms for laser-diode modulation
- Short-pulse techniques

Direct Modulation



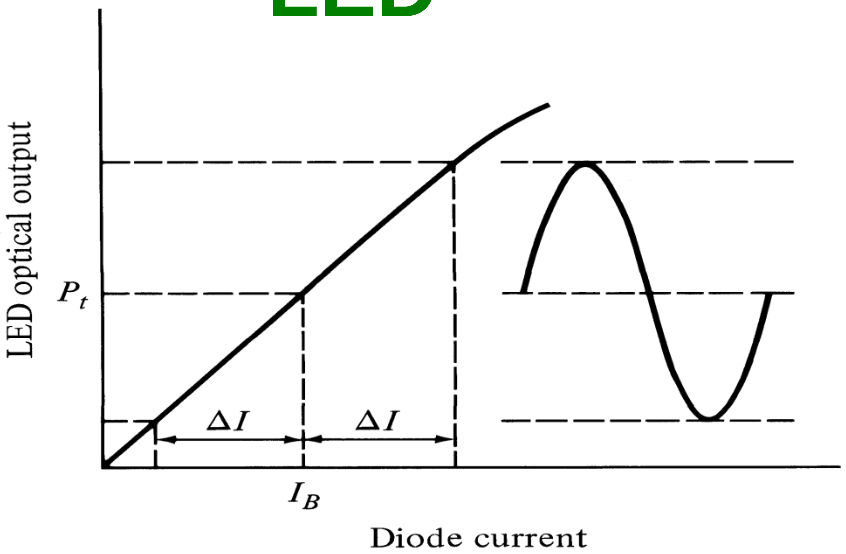
- The message signal (ac) is superimposed on the bias current (dc) which modulates the laser
- Robust and simple, hence widely used
- Issues: laser resonance frequency, chirp, turn on delay, clipping and laser nonlinearity

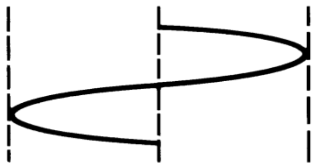
Optical Output vs. Drive Current of a Laser



Direct Analog Modulation

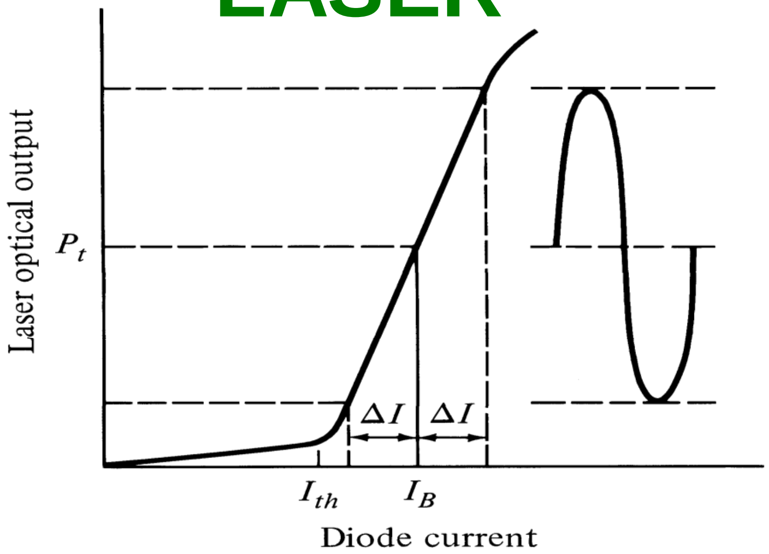
LED



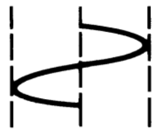


$$I_B' = I_B$$

LASER

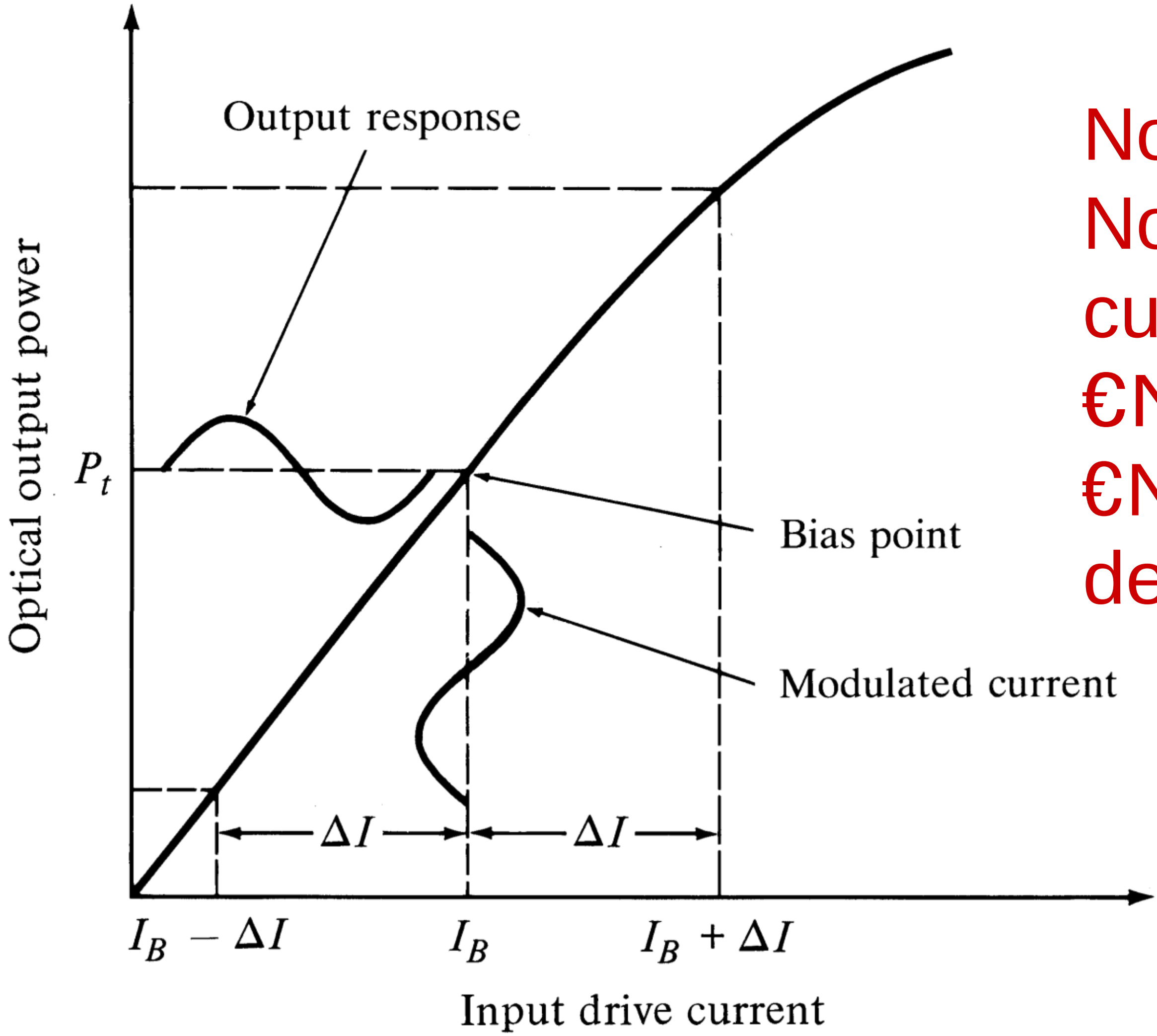


$$I_B' = I_B - I_{th}$$



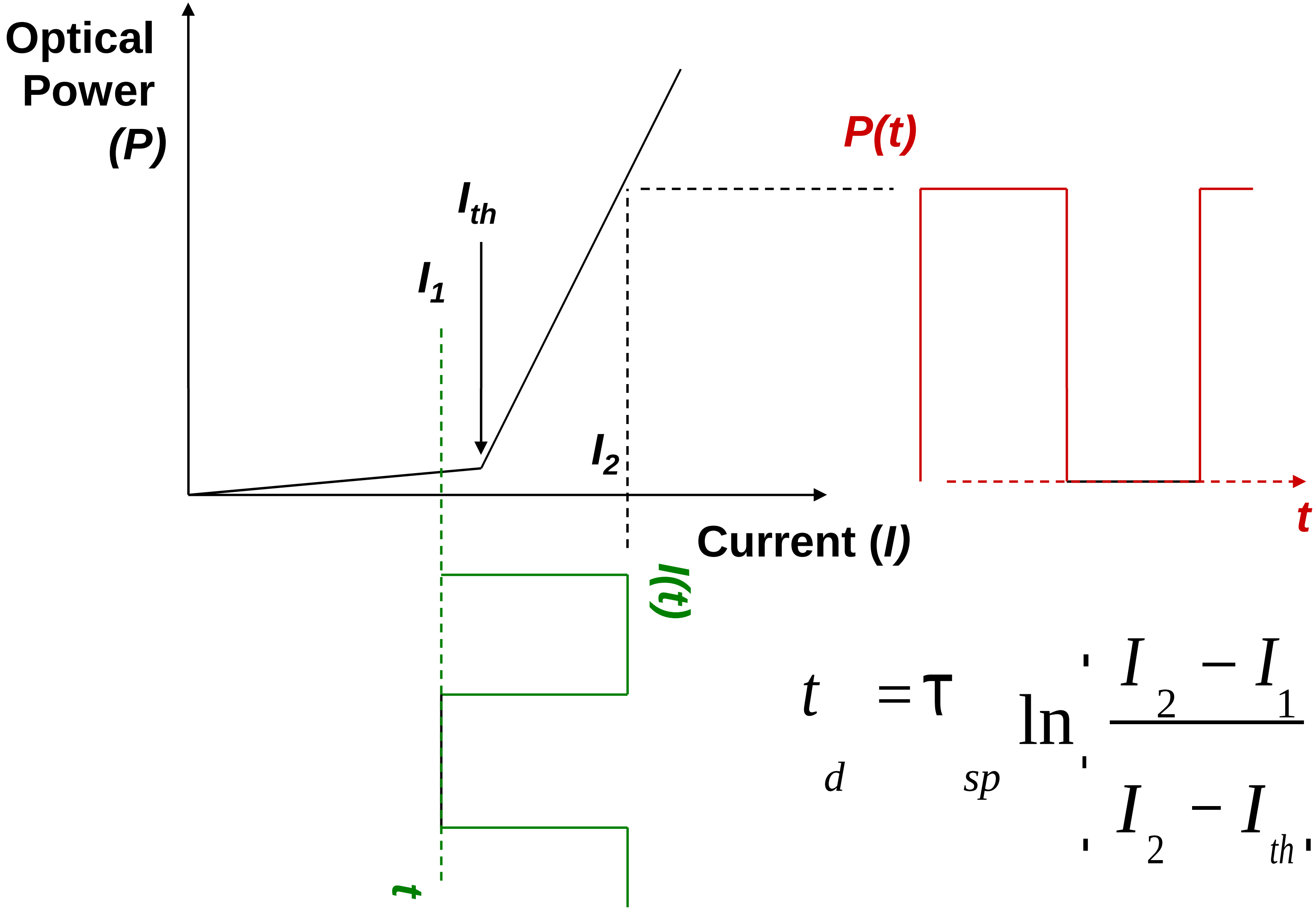
Modulation index (depth) $$m = \Delta I / I_B'$$

Analog LED Modulation



Note:
No threshold current
€No clipping
€No turn on delay

Laser Digital Modulation



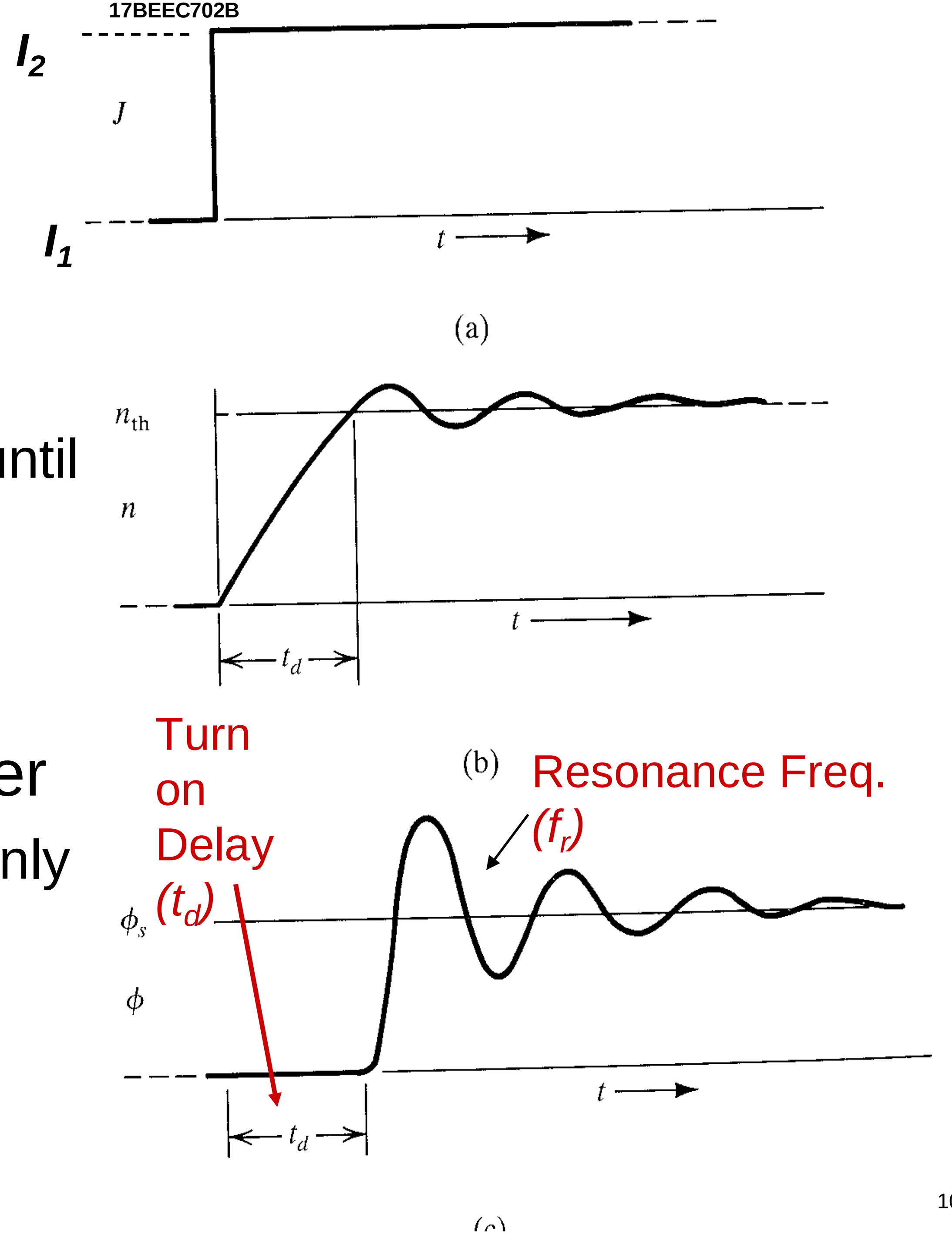
$$t_d = \tau_{sp} \ln\left[\frac{I_2 - I_1}{I_2 - I_{th}}\right]$$

Turn on Delay (lasers)

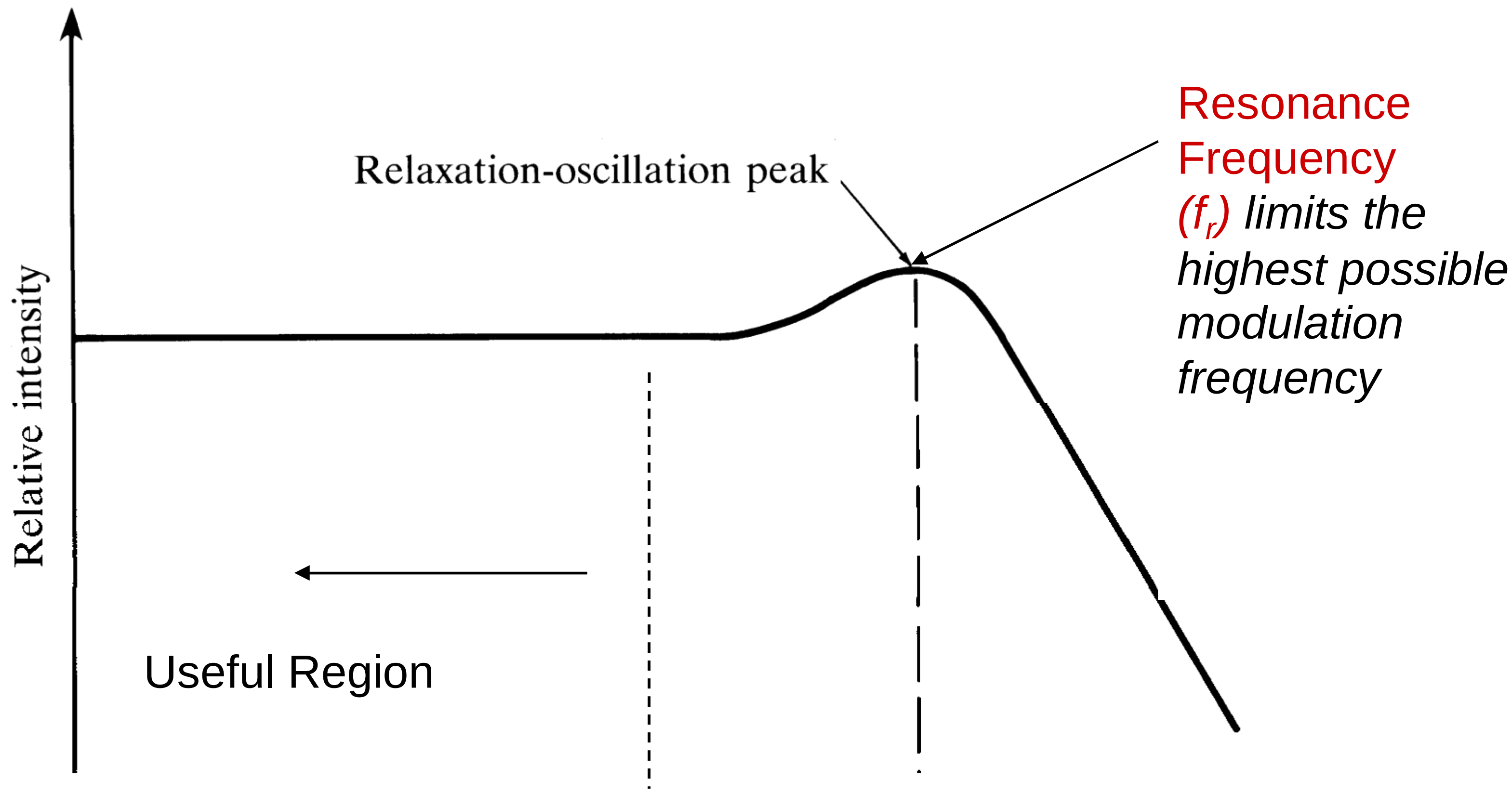
- When the driving current suddenly jumps from low ($I_1 < I_{th}$) to high ($I_2 > I_{th}$) , (step input), there is a finite time before the laser will turn on
- This delay limits bit rate in *digital systems*
- Can you think of any solution?

$$t_d = \tau_{sp} \ln \left(\frac{I_2 - I_1}{I_2 - I_{th}} \right)$$

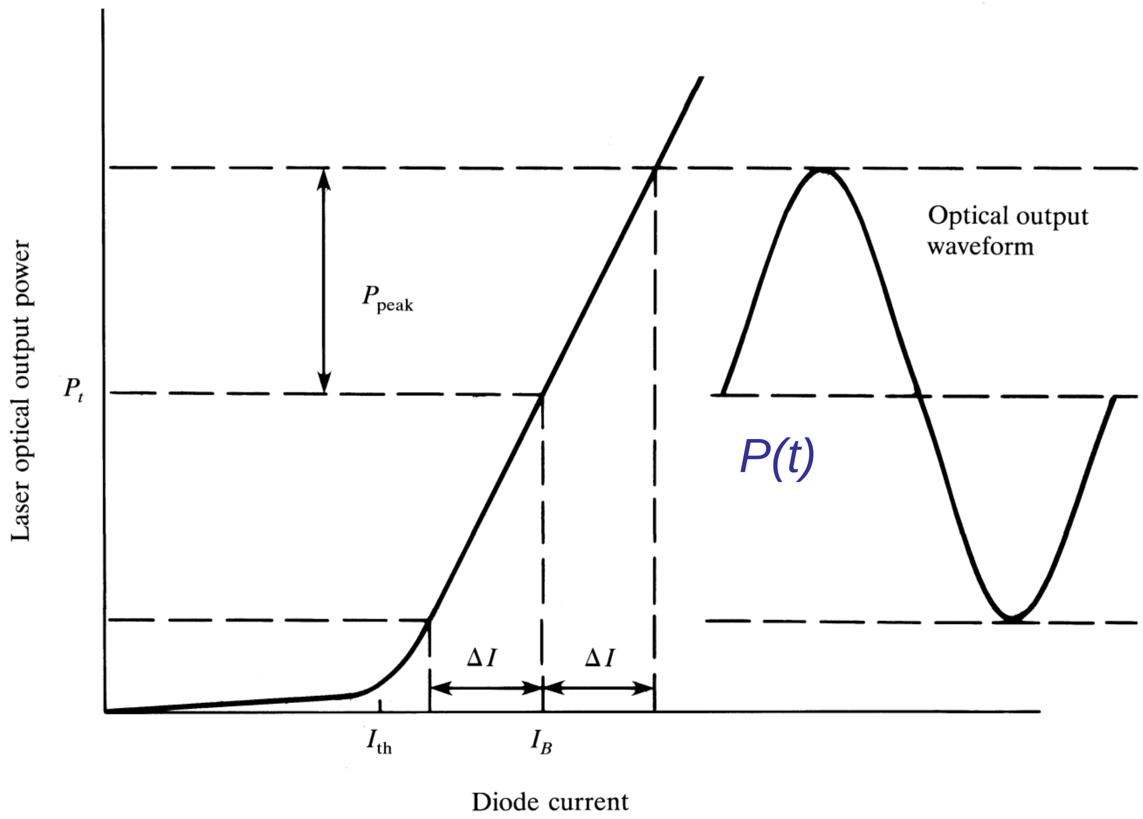
- Input current
  - Assume step input
- Electron density
  - steadily increases until threshold value is reached
- Output optical power
  - Starts to increase only after the electrons reach the threshold



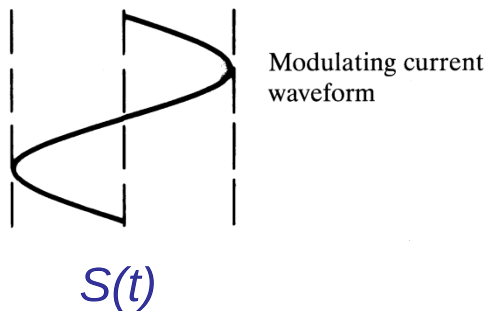
Frequency Response of a Laser



Laser Analog Modulation



$$P(t) = P_t[1 + ms(t)]$$

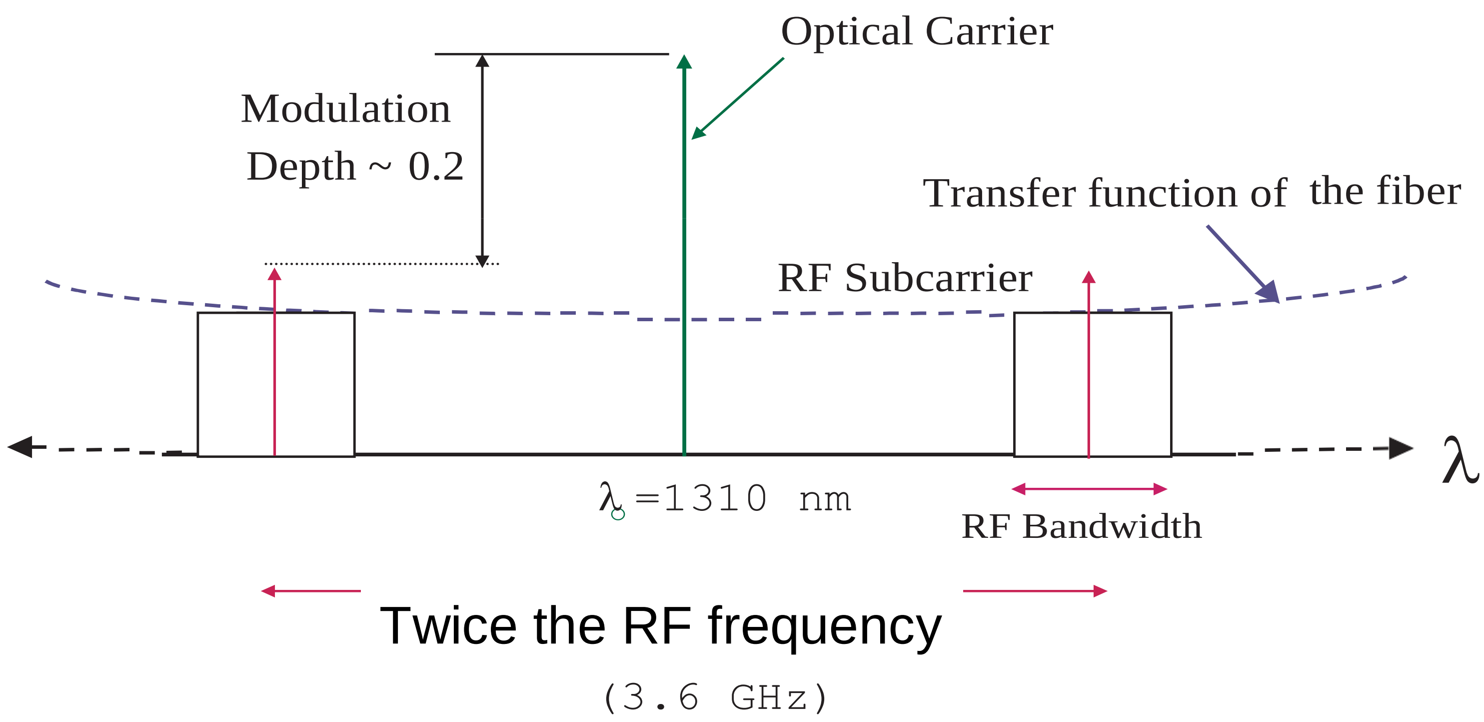


Here *s(t)* is the modulating signal,

P(t): output optical power

P_t: mean value

The modulated spectrum



Two sidebands each separated by modulating frequency

Limitations of Direct Modulation

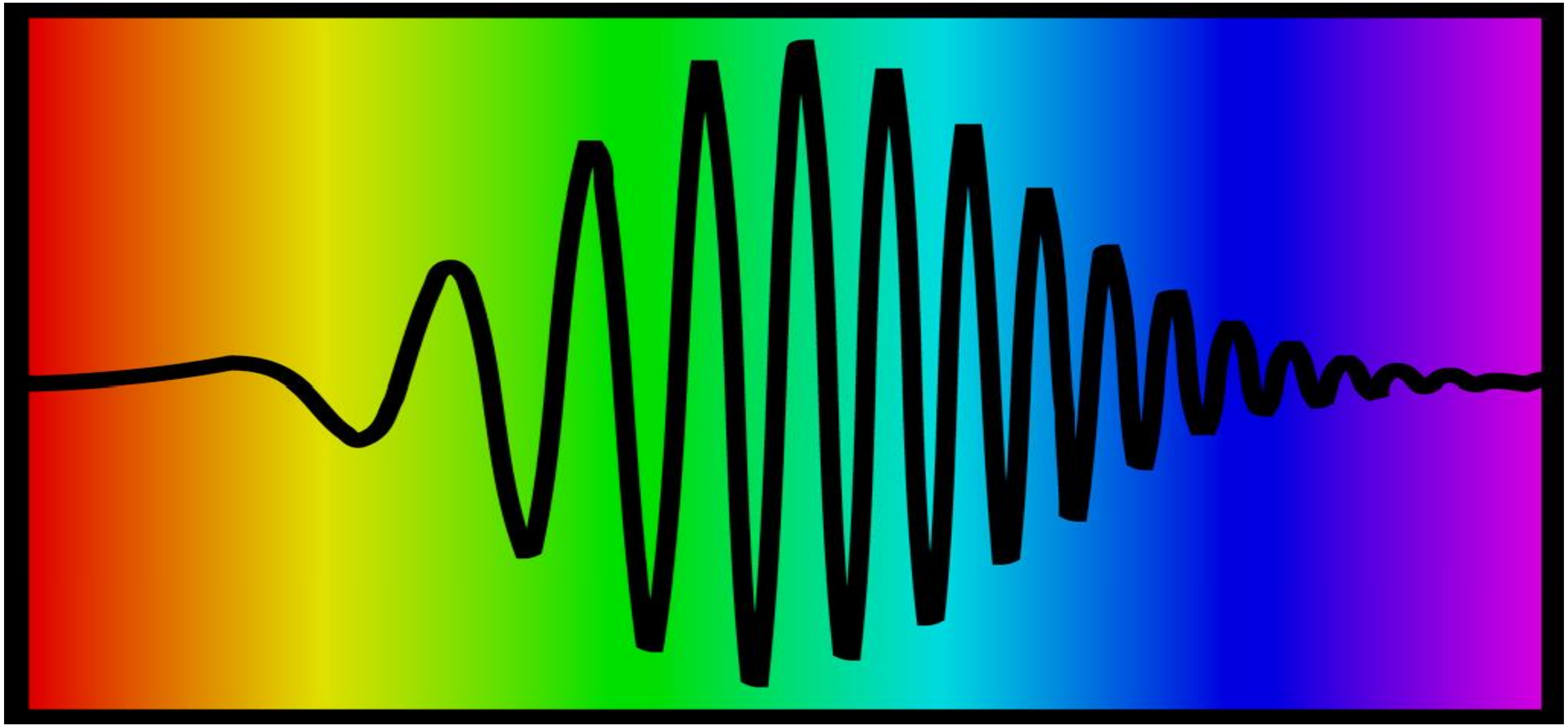
- Turn on delay and resonance frequency are the two major factors that limit the speed of digital laser modulation
- Saturation and clipping introduces nonlinear distortion with analog modulation (especially in multi carrier systems)
- Nonlinear distortions introduce second and third order intermodulation products
- Chirp: Laser output wavelength drift with modulating current is also another issue, resulting in line broadening.

Chirp

In laser diode, the refractive index varies with carrier density.

Modulation → vary current → vary carrier density
→ vary refractive index → index varies with time
→ phase delay varies with time → induces new frequency
frequency varies with time : chirp

- chirp results in broadening of a laser linewidth
- chirp magnitude is ~ 100MHz - GHz/mA, ~ 0.001% of center frequency



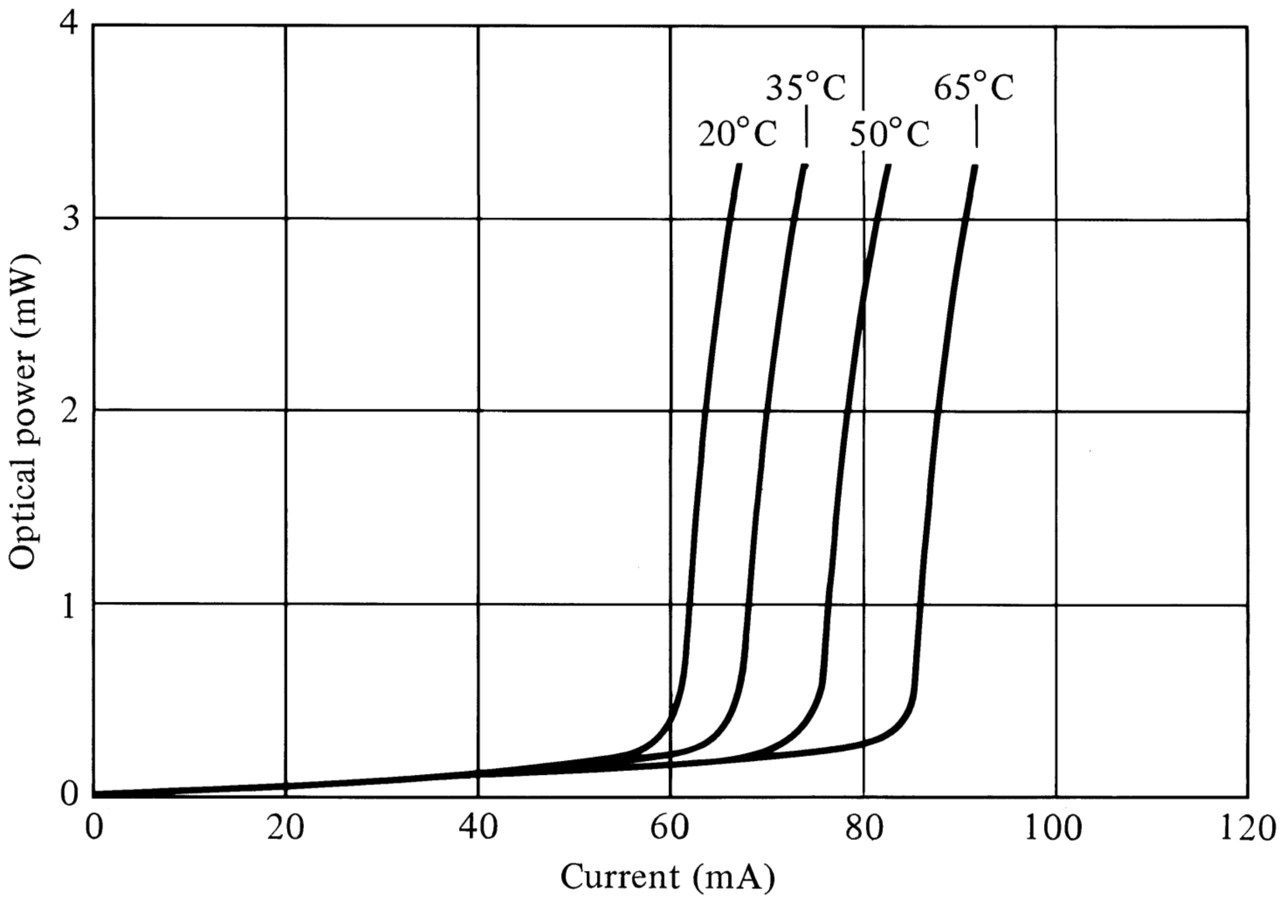
A pulse can have a frequency that varies in time.

This pulse increases its frequency linearly in time (from red to blue).

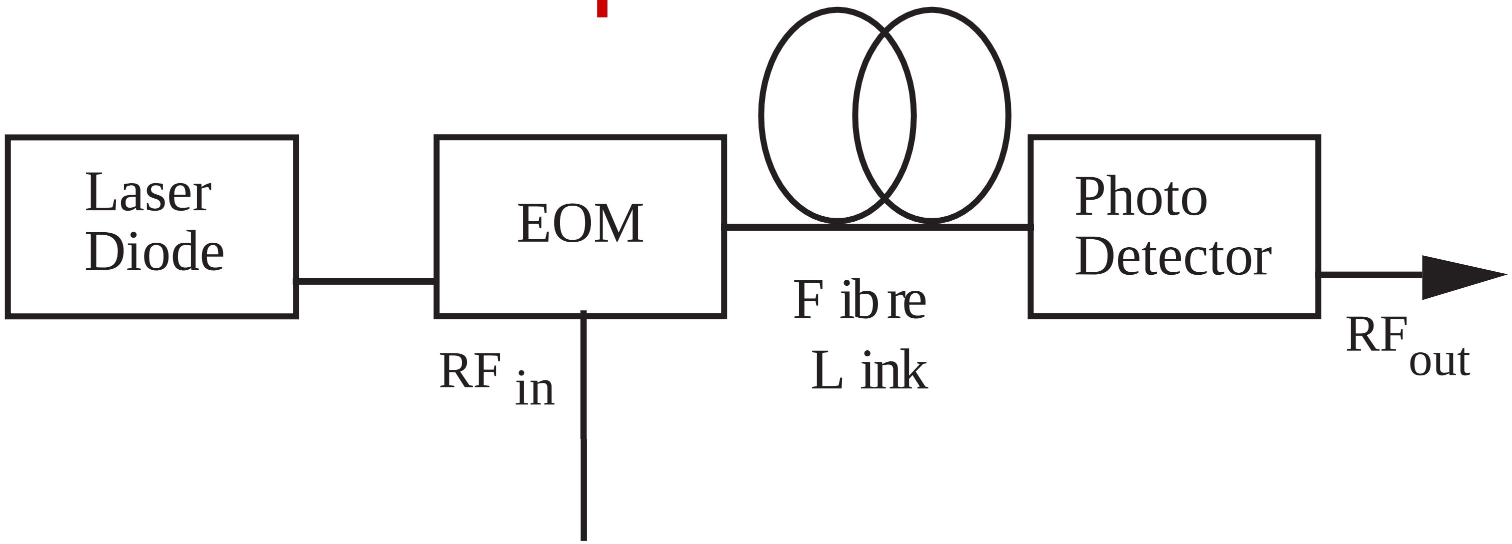
In analogy to bird sounds, this pulse is called a "chirped" pulse.

Temperature variation of the threshold current

$$I_{th}(T) = I_z e^{T/T_0}$$

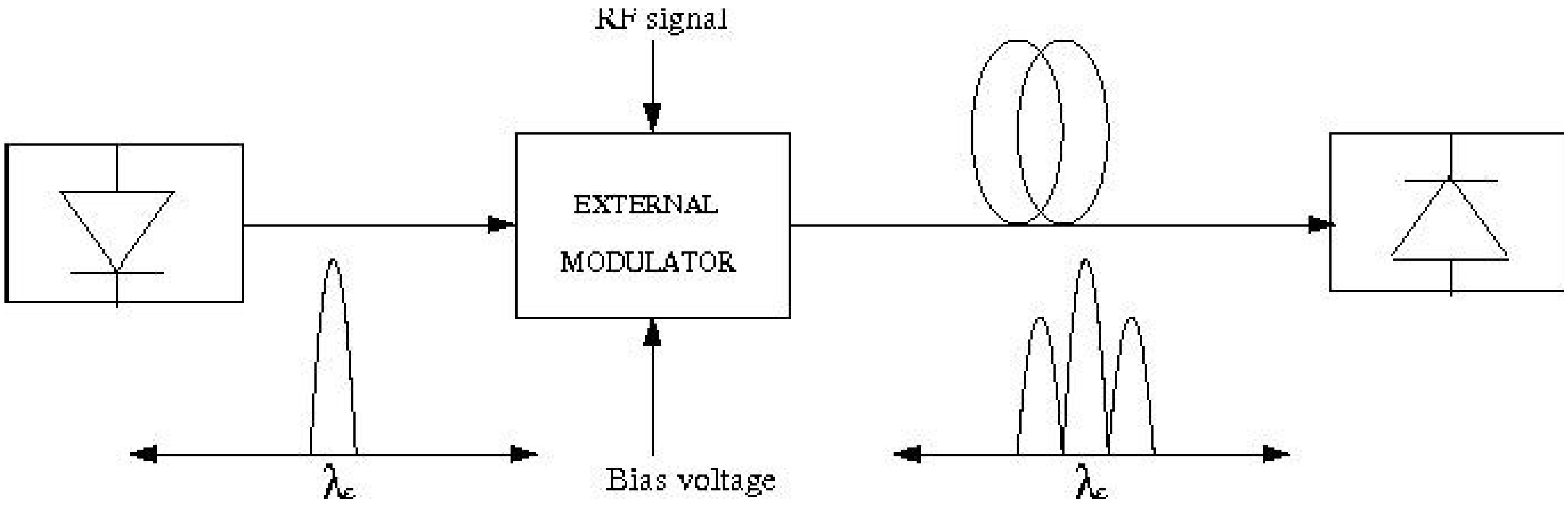


External Optical Modulation



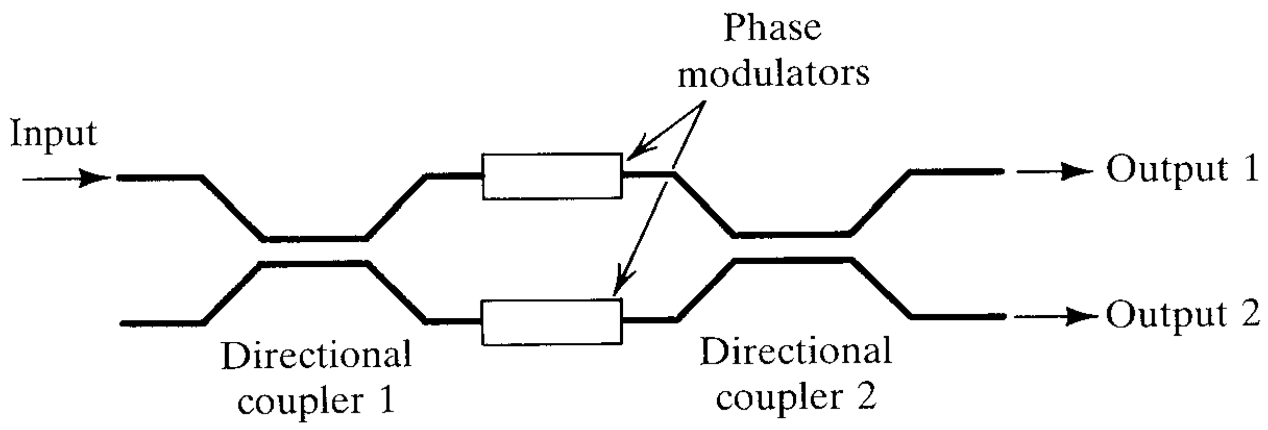
- Modulation and light generation are separated
- Offers much wider bandwidth € up to 60 GHz
- More expensive and complex
- Used in high end systems

External Modulated Spectrum

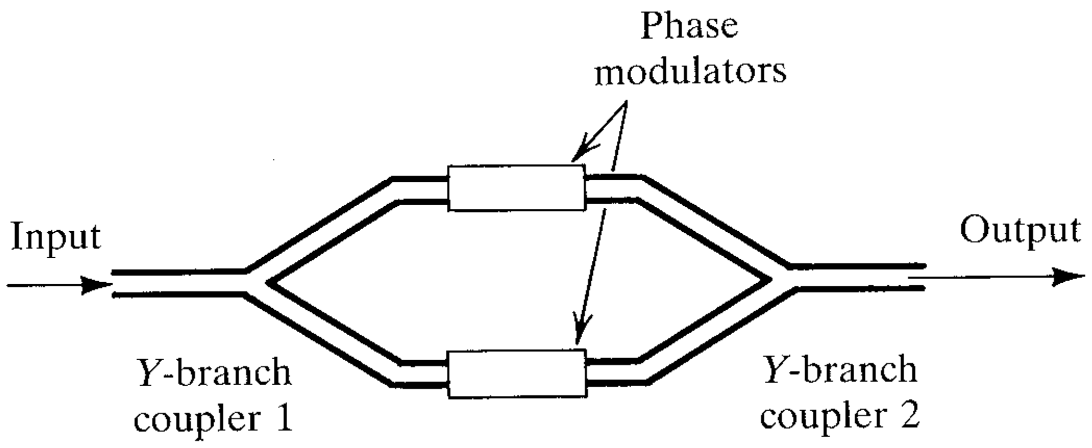


- Typical spectrum is double side band
- However, single side band is possible which is useful at extreme RF frequencies

Mach-Zehnder Interferometers

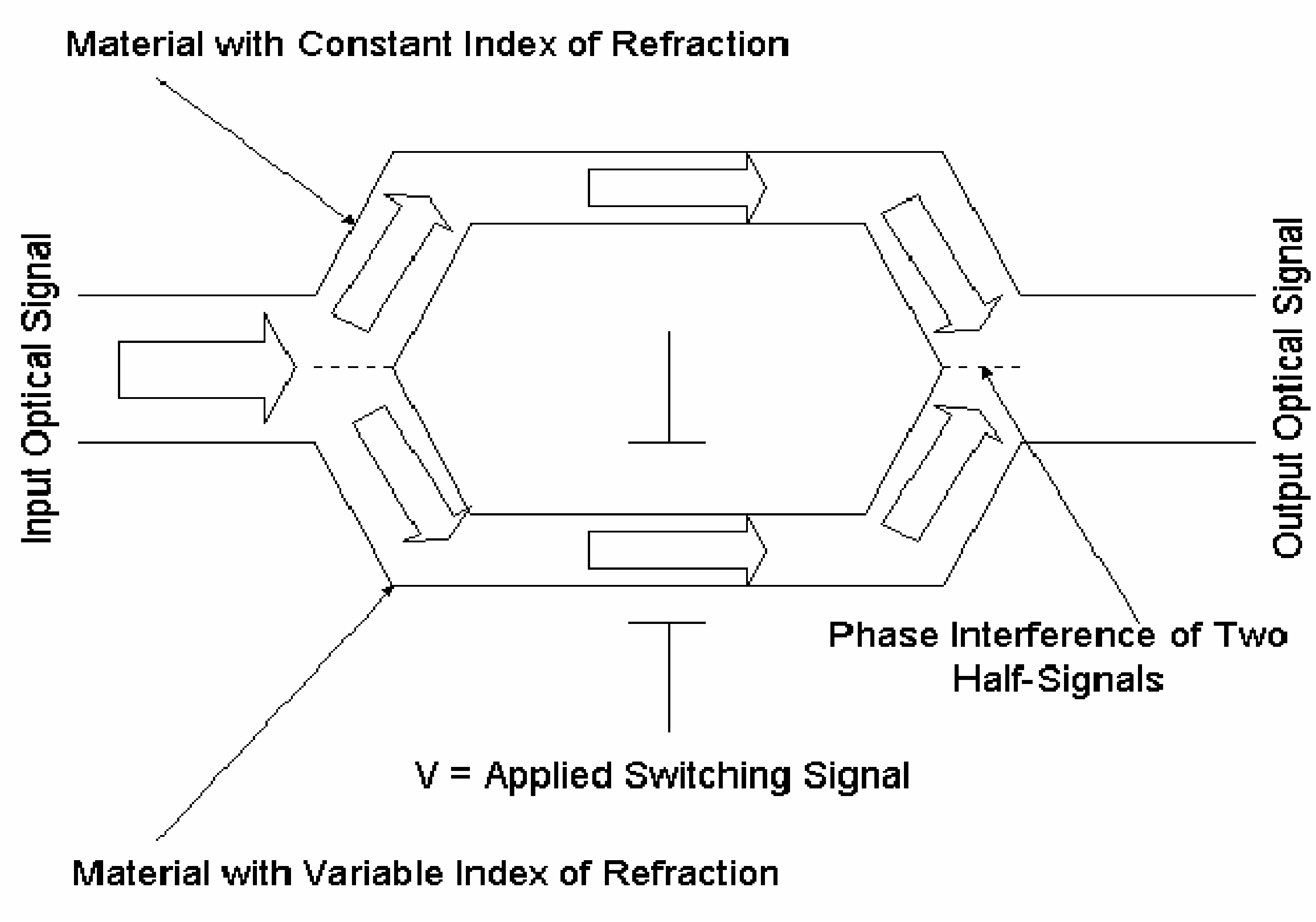


(a)

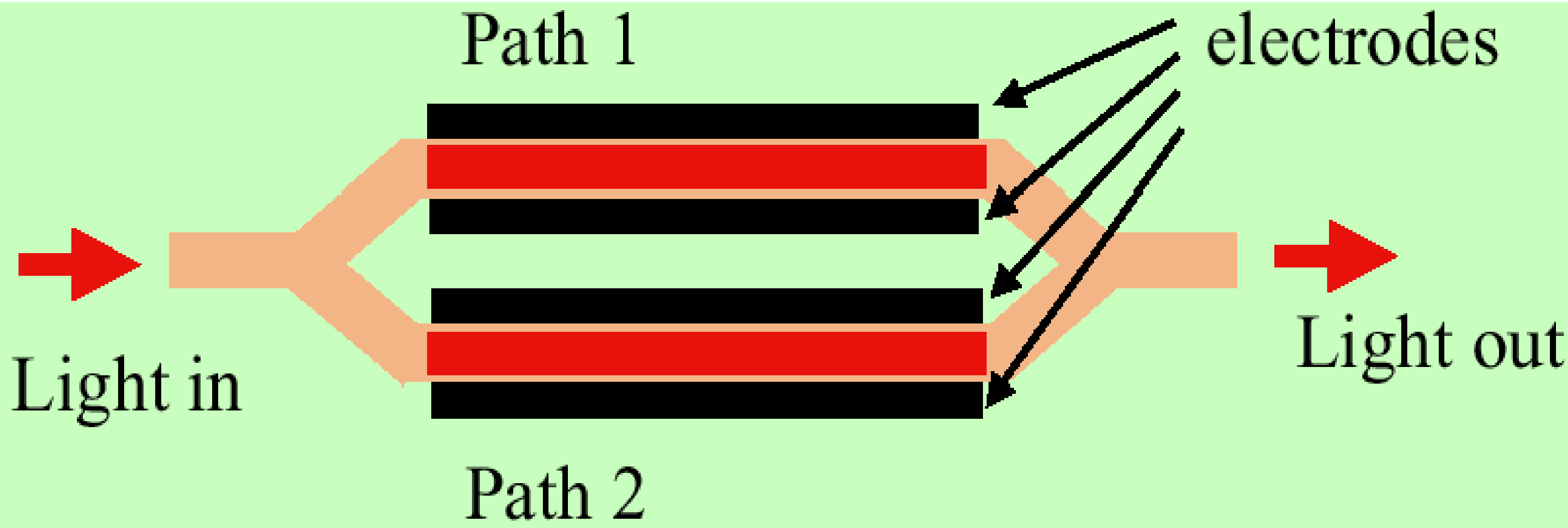


(b)

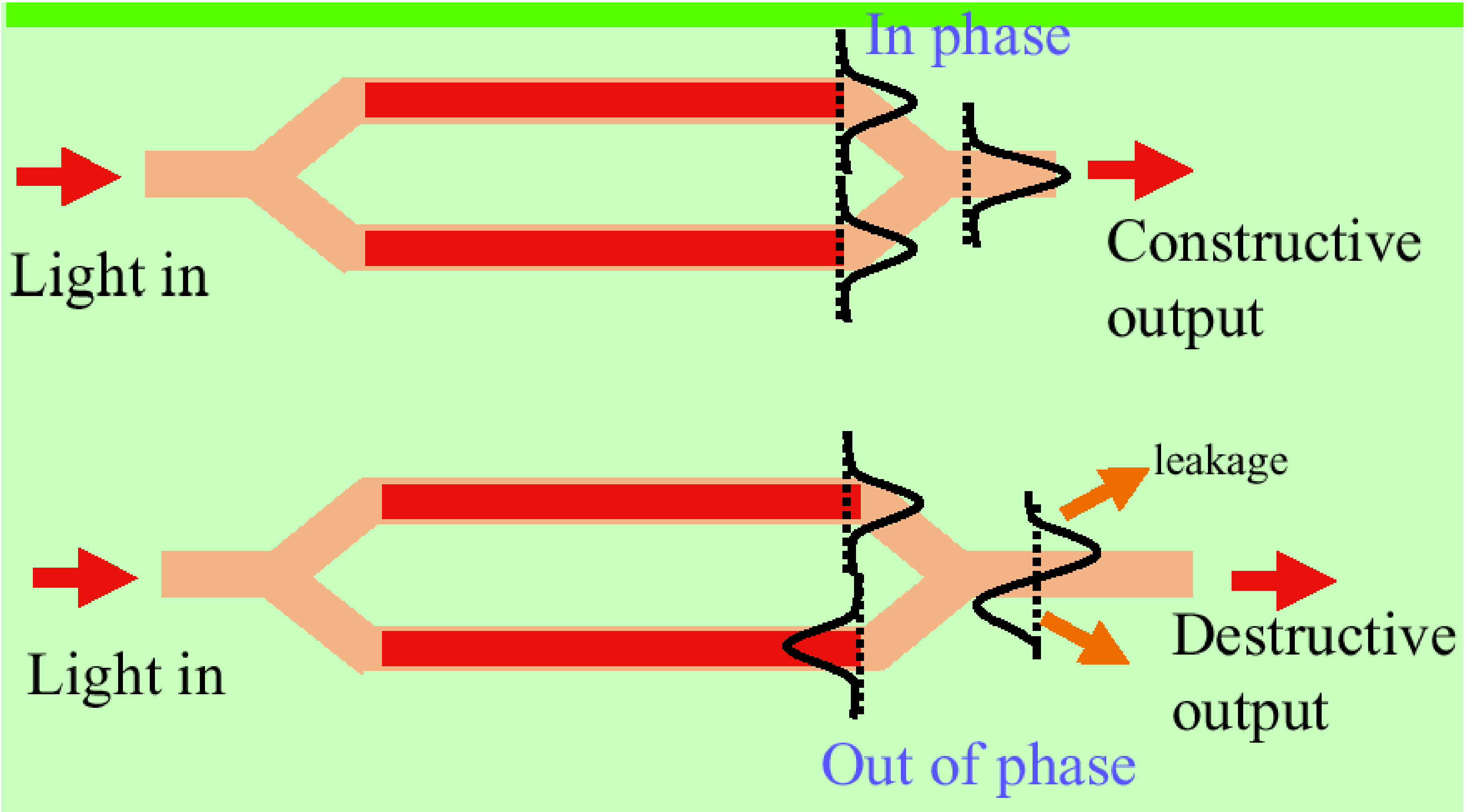
Mach- Zehnder modulator

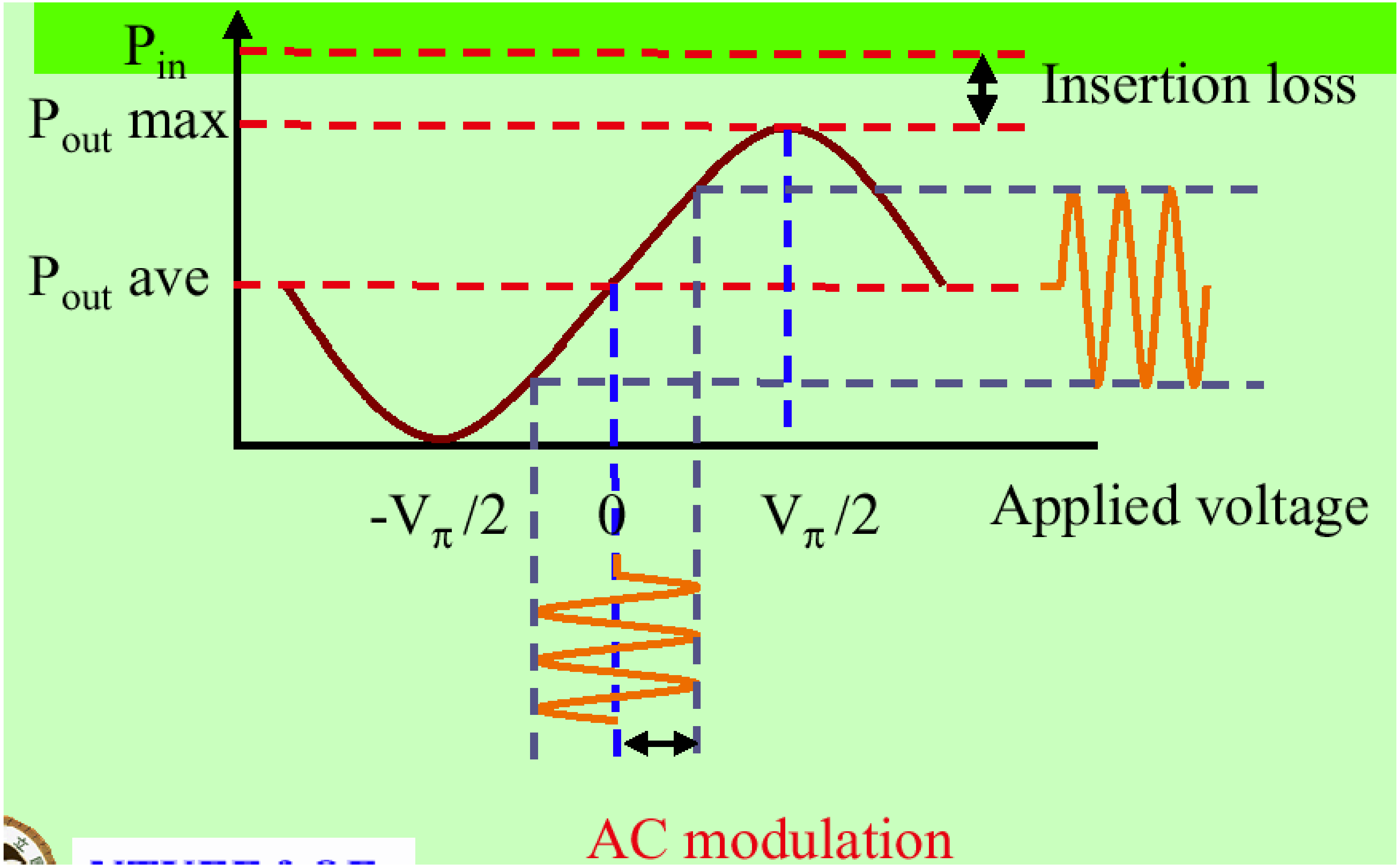


Mach- Zehnder modulator



- Applying voltages to electrodes to change the refractive indices of light paths 1 & 2.
- The optical paths of 1 & 2 vary with the applied voltage.
- In phase → strong output light; out of phase → weak output.
- Output light is then modulated by voltage signal.

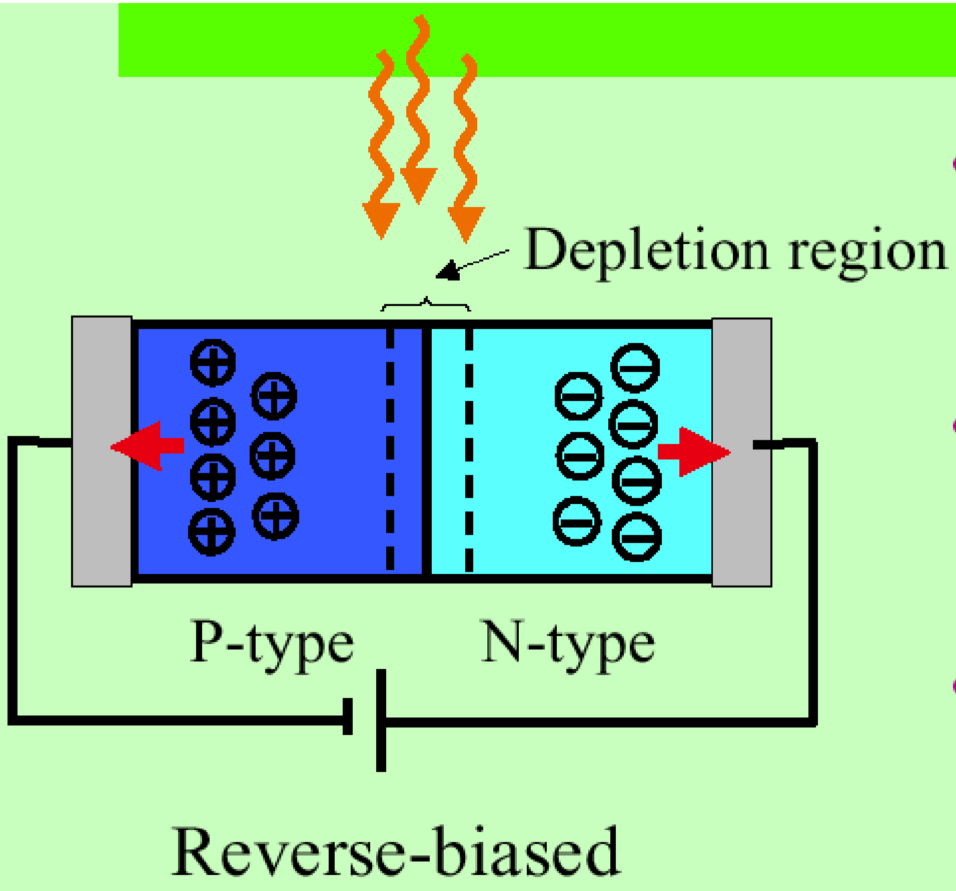

In phase
Light in
Constructive
output
leakage
Light in
Destructive
output
Out of phase


P_in
P_out max
P_out ave
Insertion loss
-V_π /2
0
V_π /2
Applied voltage
AC modulation

Characteristics of Mach-Zehnder modulator

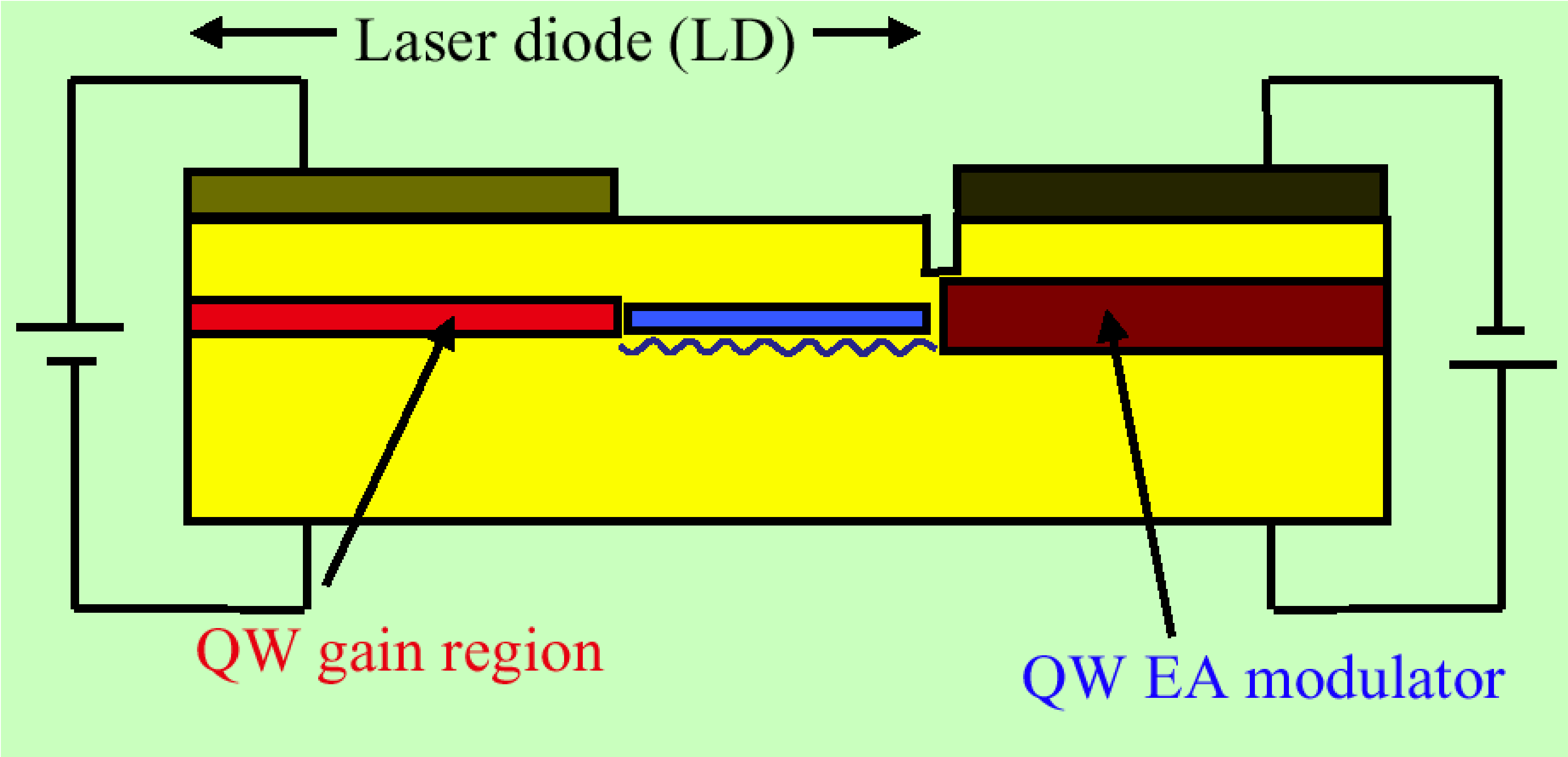
- material: $LiNbO_3$
- modulation depth: better than 20 dB
- bandwidth : could be 60 GHz
- insertion loss : ≥ 4 dB
- power handling : 200 mW
- induced chirp : negligible
- V_π: a few volts, depending on bandwidth

Electro- absorption (EA) modulator



- When the P-N structure in LED is reverse-biased, it becomes light absorption.
- At zero-bias, absorption is weak. Under strong reverse-biased, absorption is strong.
- Light intensity is then modulated by the voltage signal.

Integration of EA modulator with LD



Quantum well (QW) Laser: laser diode whose active region is so narrow that quantum confinement occurs

Characteristics of EA modulator

- material: semiconductor QWs
- modulation depth: better than 10 dB
- bandwidth : could be 40 GHz
- insertion loss : almost zero
- power handling : 1 mW
- induced chirp : negligible
- operation voltage: 2 V
- integrable with LD

UNIT-IV FIBER OPTICAL RECEIVERS

Content

- **Fundamental Receiver Operation**
  - Digital Signal Transmission
  - Error Sources
- **Digital Receiver Performance**
  - Probability of Error
  - Receiver Sensitivity
  - The Quantum Limit
- **Coherent Detection**

- # Analog Receiver

Optical Receiver Operation

Digital Signal Transmission

- **A typical digital fiber transmission link is shown in Fig. 7-1. The transmitted signal is a two-level binary data stream consisting of either a '0' or a '1' in a *bit period* T_b.**

- **The simplest technique for sending binary data is *amplitude-shift keying*, wherein a voltage level is switched between *on* or *off* values.**

- **The resultant signal wave thus consists of a voltage pulse of amplitude *V* when a binary 1 occurs and a zero-voltage-level space when a**

binary 0 occurs.

Digital Signal Transmission

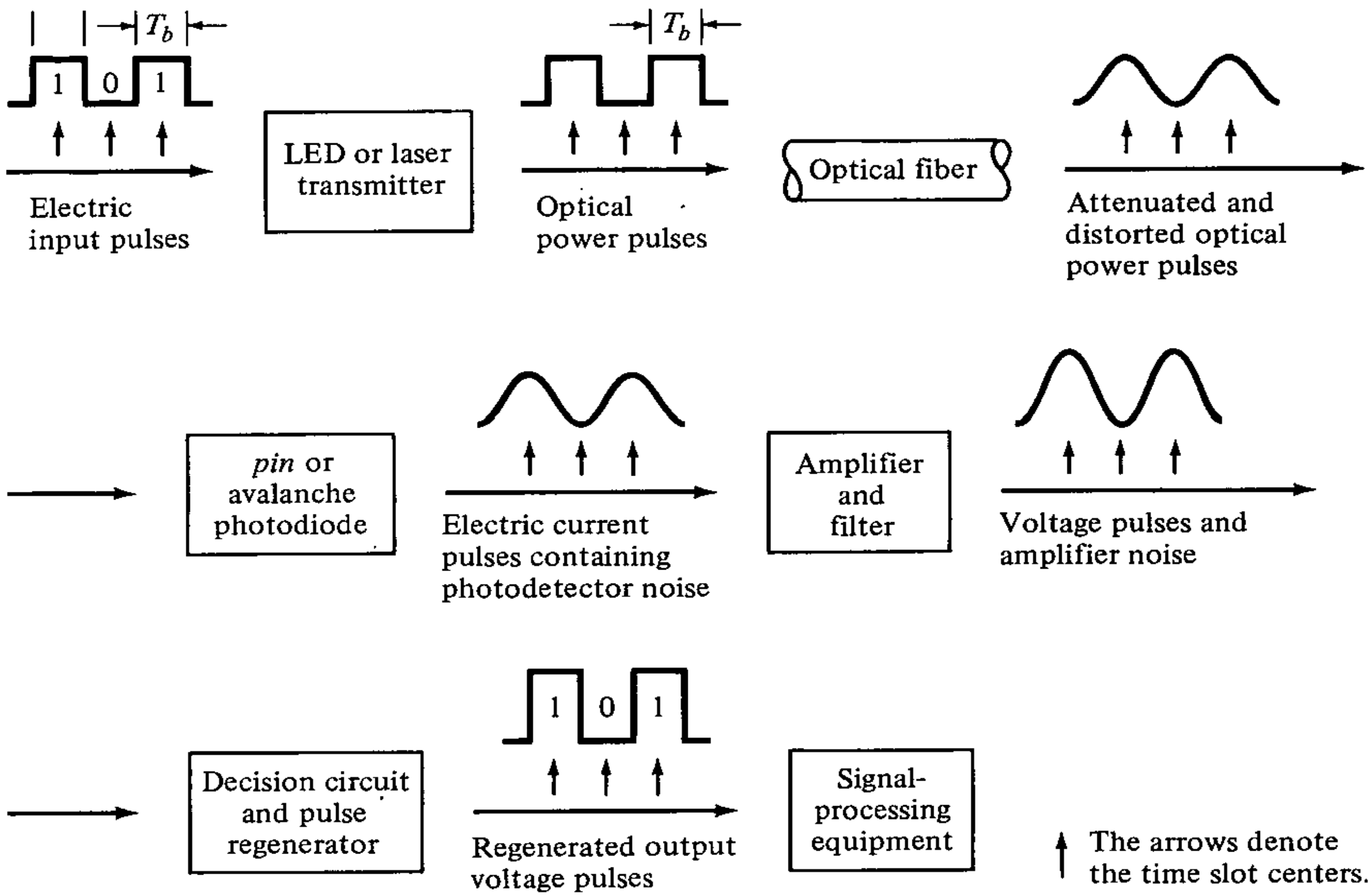


Fig. 7-1 Signal path through an optical data link.

Digital Signal Transmission (2)

- **An electric current $i(t)$ can be used to modulate directly an optical source to produce an optical output power $P(t)$.**

- **In the optical signal emerging from the transmitter, a '1' is represented by a light pulse of duration T_b, whereas a '0' is the absence of any light.**

- **The optical signal that gets coupled from the light source to the fiber becomes attenuated and distorted as it propagates along the fiber waveguide.**

Digital Signal Transmission (3)

- **Upon reaching the receiver, either a PIN or an APD converts the optical signal back to an electrical format.**

- **A decision circuit compares the amplified signal in each time slot with a *threshold level*.**

- **If the received signal level is greater than the threshold level, a '1' is said to have been received.**

- **If the voltage is below the threshold level, a '0' is assumed to have been received.**

Error Sources

- **Errors in the detection mechanism can arise from various noises and disturbances associates with the signal detection system.**
- **The two most common samples of the spontaneous fluctuations are shot noise and thermal noise.**
- **Shot noise arises in electronic devices because of the discrete nature of current flow in the device.**
- **Thermal noise arises from the random motion of electrons in a conductor.**

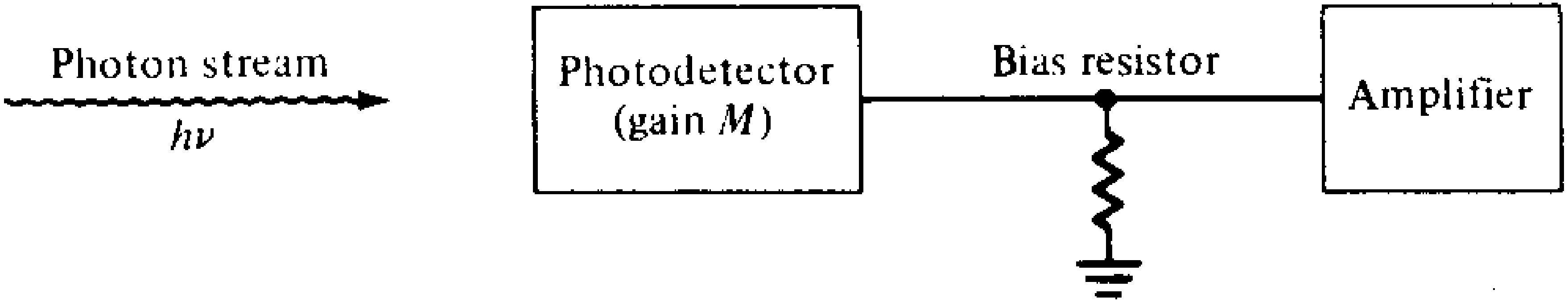
Error Sources (2)

- **The random arrival rate of signal photons produces a quantum (or shot) noise at the photodetector. This noise depends on the signal level.**

- **This noise is of particular importance for PIN receivers that have large optical input levels and for APD receivers.**

- **When using an APD, an additional shot noise arises from the statistical nature of the multiplication process. This noise level increases with increasing avalanche gain *M*.**

Error Sources (3)



- Photon detection quantum noise (Poisson fluctuation)

- Bulk dark current
- Surface leakage current
- Statistical gain fluctuation (for avalanche photodiodes)

- Thermal noise

- Amplifier noise

Noise sources and disturbances in the optical pulse detection mechanism.

Error Sources (4)

- **Thermal noises arising from the detector load resistor and from the amplifier electronics tend to dominate in applications with low SNR when a PIN photodiode is used.**

- **When an APD is used in low-optical-signal-level applications, the optimum avalanche gain is determined by a design tradeoff between the thermal noise and the gain-dependent quantum noise.**

Error Sources (5)

- **The primary photocurrent generated by the photodiode is a time-varying Poisson process.**
- **If the detector is illuminated by an optical signal *P*(*t*), then the average number of electron-hole pairs generated in a time τ is**

$$\bar{N} = \frac{\eta}{h\nu}\int_0^{\tau} P(t)dt = \frac{\eta E}{h\nu} \qquad \textbf{(7-1)}$$

where η is the detector quantum efficiency, $h\nu$ is the photon energy, and *E* is the energy received in a time interval .

Error Sources (6)

- **The actual number of electron-hole pairs *n* that are generated fluctuates from the average according to the Poisson distribution**

$$P_r(n) = \overline{N}^n \frac{e^{-\overline{N}}}{n!} \qquad \textbf{(7-2)}$$

where $P_r(n)$ is the probability that *n* electrons are emitted in an interval τ.

Error Sources (7)

- **For a detector with a mean avalanche gain *M* and an ionization rate ratio *k*, the excess noise factor *F*(*M*) for electron injection is**

$$F(M) = kM + \left(2 - \frac{1}{M}\right)(1 - k) \qquad \textbf{(7-3)}$$

or

$$F(M) \cong M^x \qquad \textbf{(7-4)}$$

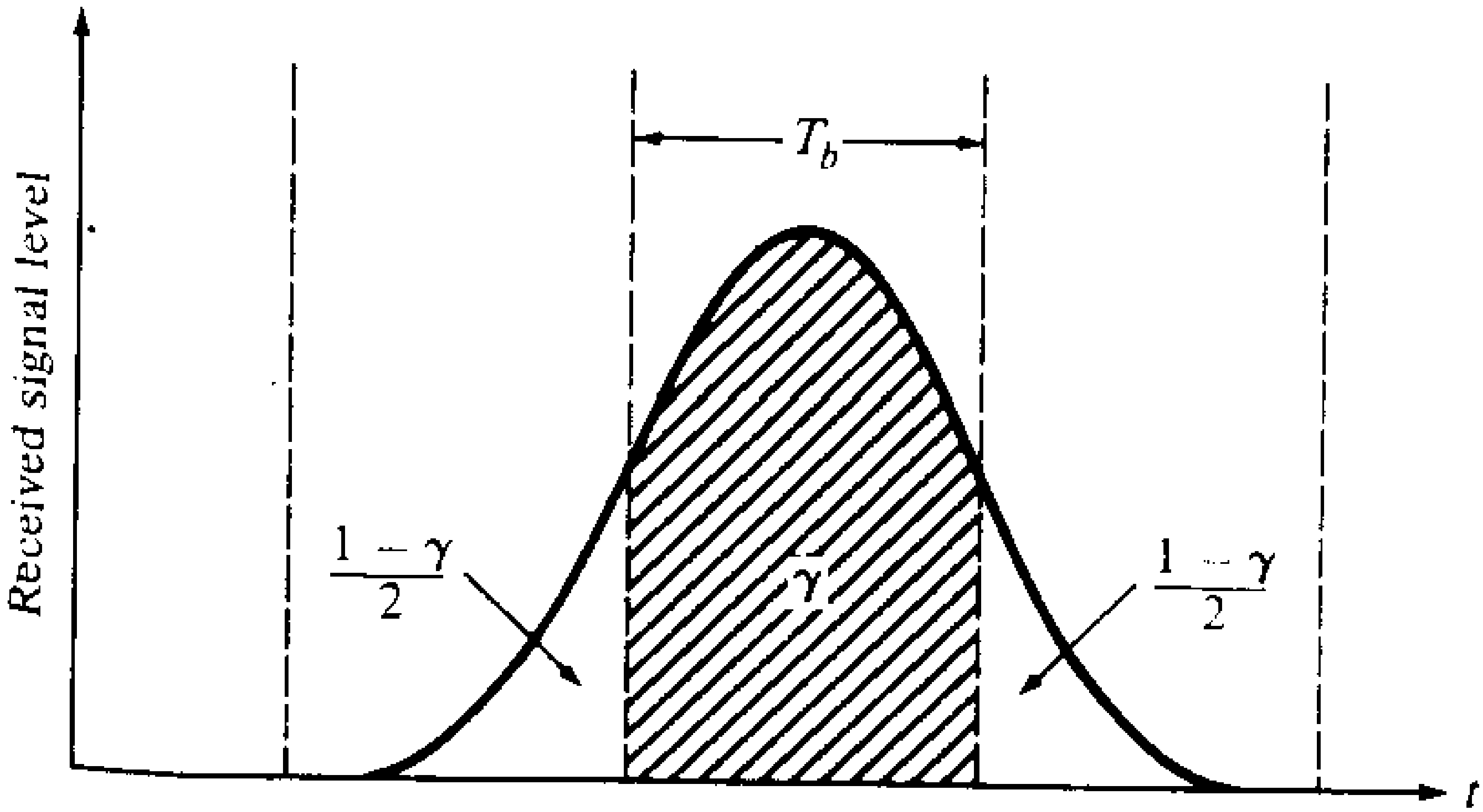
where the factor *x* ranges between 0 and 1.0 depending on the photodiode material.

Error Sources (8)

- A further error source is attributed to *intersymbol interference* (ISI), which results from pulse spreading in the optical fiber.

- The fraction of energy remaining in the appropriate time slot is designated by γ, so that 1-γ is the fraction of energy that has spread into adjacent time slots.

Error Sources (9)



Pulse spreading in an optical signal that leads to ISI.

Receiver Configuration

- A typical optical receiver is shown in Fig. 7-4.

  The three basic stages of the receiver are a photo-detector, an amplifier, and an equalizer.

- The photo-detector can be either an APD with a mean gain M or a PIN for which M=1.

- The photodiode has a quantum efficiency η and a capacitance C_d.

- The detector bias resistor has a resistance R_b which generates a thermal noise current $i_b(t)$.

Receiver Configuration (2)

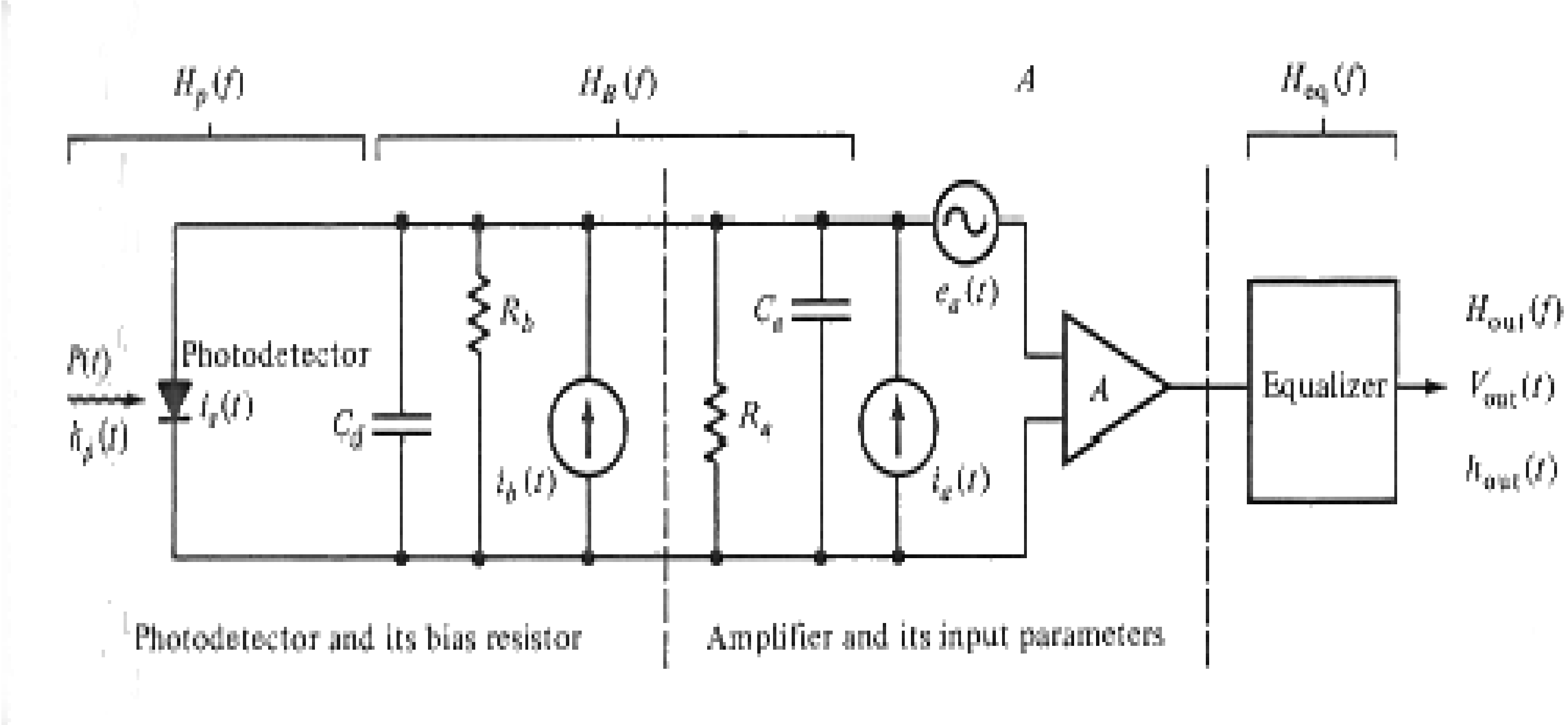


Figure 7-4. Schematic diagram of a typical optical receiver.

Receiver Configuration (3)

Amplifier Noise Sources:

- **The input noise current source $i_a(t)$ arises from the thermal noise of the amplifier input resistance R_a;**

- **The noise voltage source $e_a(t)$ represents the thermal noise of the amplifier channel.**

- **The noise sources are assumed to be Gaussian in statistics, flat in spectrum (which characterizes *white* noise), and uncorrelated (statistically independent).**

Receiver Configuration (4)

The Linear Equalizer:

- **The equalizer is normally a linear frequency-shaping filter that is used to mitigate the effects of signal distortion and intersymbol interference (ISI).**

- **The equalizer accepts the combined frequency response of the transmitter, the fiber, and the receiver, and transforms it into a signal response suitable for the following signal-processing electronics.**

Receiver Configuration (5)

- **The binary digital pulse train incident on the photo-detector can be described by**

$$P(t) = \sum_{n=-\infty}^{\infty} b_n h_p(t - nT_b)$$

- **Here, $P(t)$ is the received optical power,**

  T_b is the bit period,

  b_n is an amplitude parameter representing the n^{th} message digit,

  and $h_p(t)$ is the received pulse shape.

Receiver Configuration (6)

- Let the nonnegative photodiode input pulse $h_p(t)$ be normalized to have unit area

$$\int_{-\infty}^{\infty} h_p(t)\,dt = 1$$

then b_n represents the energy in the n^{th} pulse.

- The mean output current from the photodiode at time t resulting from the pulse train given previously is

$$\langle i(t) \rangle = \frac{\eta q}{h\nu} MP(t) = R_0 \sum_{n=-\infty}^{\infty} b_n h_p(t - nT_b)$$

where $R_o = \eta q / h\nu$ is the photodiode responsivity.

- The above current is then amplified and filtered to produce a mean voltage at the output of the equalizer.

Digital Receiver Performance

- **In a digital receiver the amplified and filtered signal emerging from the equalizer is compared with a threshold level once per time slot to determine whether or not a pulse is present at the photodetector in that time slot.**

- **Bit-error rate (BER) is defined as:**

$$\text{BER} = \frac{N_e}{N_t} = \frac{N_e}{Bt}$$

where B=1/T_b(bit rate). N_e,N_t : Number of errors, pulses.

- **To compute the BER at the receiver, we have to know the probability distribution of the signal at the equalizer output.**

Probability of Error (2)

The shapes of two signal pdf's are shown in Fig. 7.7.

- **These are**

$$P_1(v) = \int_{-\infty}^{v} p(y\,|1)dy \qquad \textbf{(7-6)}$$

which is the probability that the equalizer output voltage is less than *v* when a logical '1' pulse is sent, and

$$P_0(v) = \int_{v}^{\infty} p(y\,|\,0)dy \qquad \textbf{(7-7)}$$

which is the probability that the output voltage exceeds *v* when a logical '0' is transmitted.

Probability of Error (3)

- The different shapes of the two pdf's in Fig. 7-7 indicate that the noise power for a logical '0' is not the same as that for a logical '1'.

- The function $p(y|x)$ is the conditional probability that the output voltage is y, given that an x was transmitted.

Probability of Error (4)

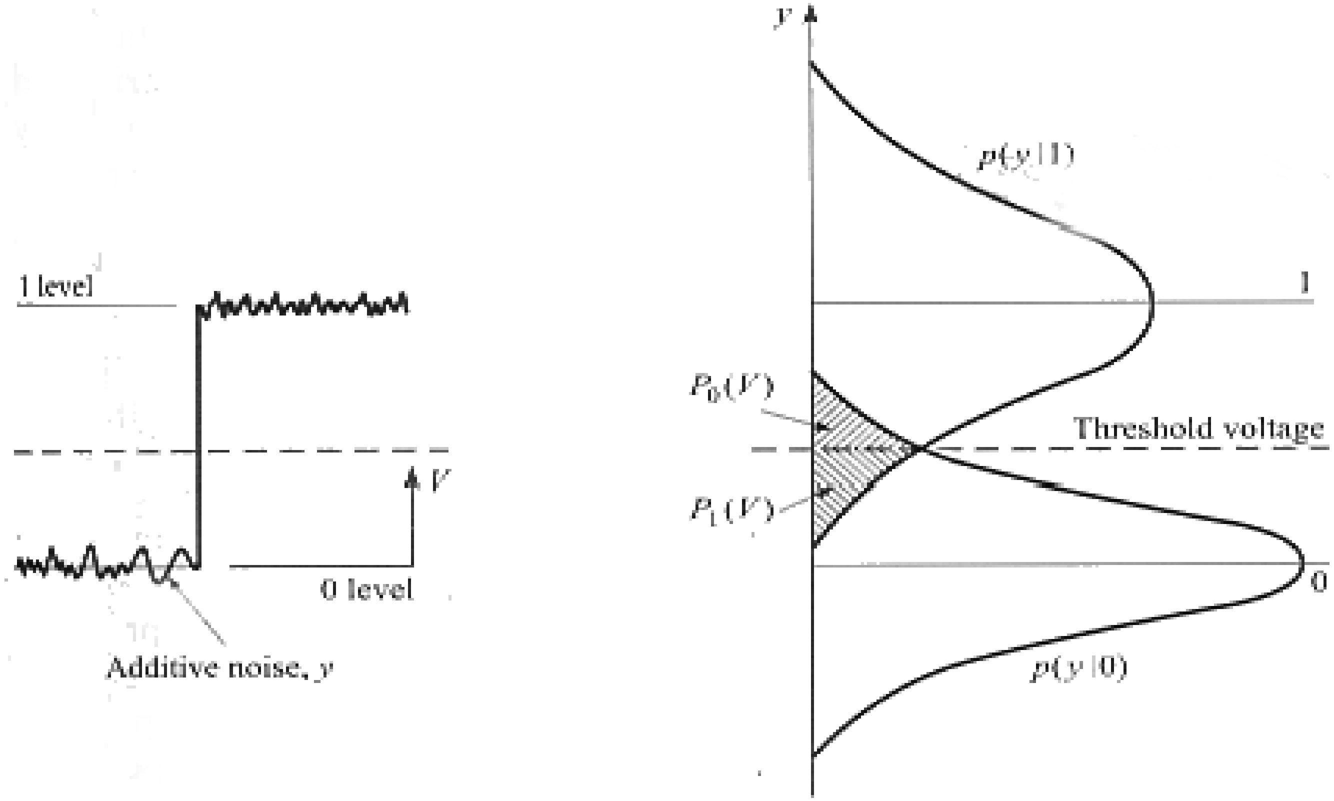


Figure 7-7. Probability distributions for received '0' and '1' signal pulses. Different widths of the two distributions are caused by various signal distortion effects.

Probability of Error (5)

- **If the threshold voltage is v_{th} then the error probability P_e is defined as**

$$P_e = aP_1(v_{th}) + bP_0(v_{th}) \quad \textbf{(7-8)}$$

- **The weighting factors *a* and *b* are determined by the a priori distribution of the data.**

- **For unbiased data with equal probability of '1' and '0' occurrences, $a = b = 0.5$.**

- **The problem to be solved is to select the decision threshold at that point where P_e is minimum.**

Probability of Error (6)

- **To calculate the error probability we require a knowledge of the mean-square noise voltage which is superimposed on the signal voltage at the decision time.**

- **It is assumed that the equalizer output voltage $v_{out}(t)$ is a Gaussian random variable.**

- **Thus, to calculate the error probability, we need only to know the mean and standard deviation of $v_{out}(t)$.**

Probability of Error (7)

- **Assume that a signal $s(t)$ has a Gaussian pdf $f(s)$ with a mean value m. The signal sample at any s to $s+ds$ is given by**

$$f(s)ds = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(s-m)^2/2\sigma^2} ds \qquad \textbf{(7-9)}$$

where σ^2 is the noise variance, and σ the *standard deviation*.

- **The quantity measures the full width of the probability distribution at the point where the amplitude is 1/*e* of the maximum.**

Probability of Error (8)

- **As shown in Fig. 7-8, the mean and variance of the Gaussian output for a '1' pulse are b_{on} and σ_{on}^2, whereas for a '0' pulse they are b_{off} and σ_{off}^2, respectively.**
- **The probability of error $P_0(v)$ is the chance that the equalizer output voltage $v(t)$ will fall somewhere between v_{th} and ∞.**
- **Using Eqs. (7-7) and (7-9), we have**

$$P_0(v_{th}) = \int_{v_{th}}^{\infty} p(y\,|\,0)dy = \int_{v_{th}}^{\infty} f_0(v)dv$$

$$= \frac{1}{\sqrt{2\pi}\sigma_{off}} \int_{v_{th}}^{\infty} \exp\left[-\frac{(v-b_{off})^2}{2\sigma_{off}^2} \right] dv \qquad \textbf{(7-10)}$$

where the subscript 0 denotes the presence of a '0' bit.

Probability of Error (9)

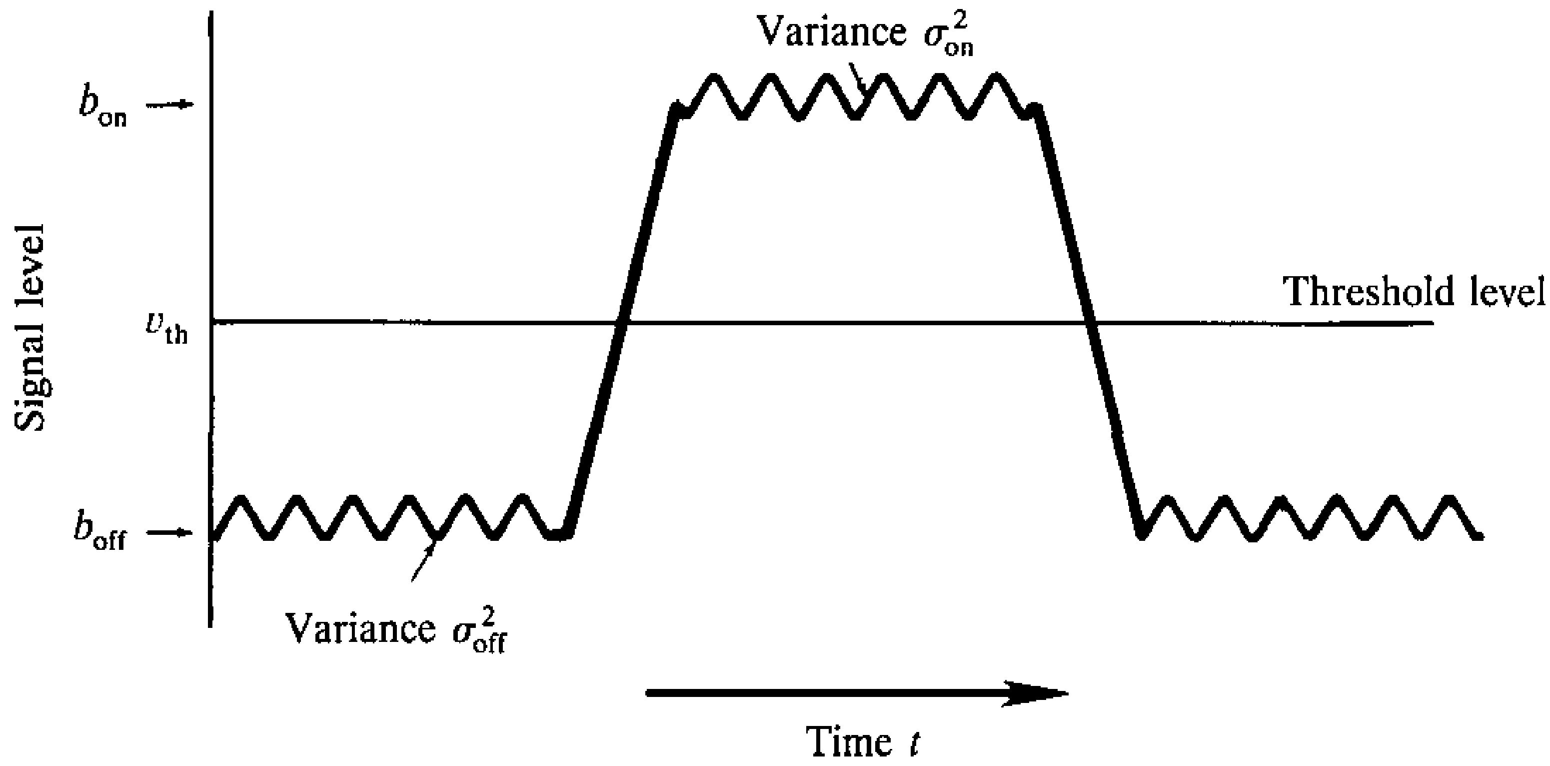


Figure 7-8. Gaussian noise statistics of a binary signal showing variances about the on and off signal levels.

Probability of Error (10)

- **Similarly, the error probability a transmitted '1' is misinterpreted as a '0' is the likelihood that the sampled signal-plus-noise pulse falls below v_{th}.**

- **From Eqs. (7-6) and (7-9), this is simply given by**

$$P_1(v_{th}) = \int_{-\infty}^{v_{th}} p(y|1)dy = \int_{-\infty}^{v_{th}} f_1(v)dv$$
$$= \frac{1}{\sqrt{2\pi}\sigma_{on}} \int_{-\infty}^{v_{th}} \exp\left[-\frac{(b_{on}-v)^2}{2\sigma_{on}^2}\right] dv \qquad \textbf{(7-11)}$$

where the subscript 1 denotes the presence of a '1' bit.

Probability of Error (11)

- **Assume that the '0' and '1' pulses are equally likely, then, using Eqs. (7-10) and (7-11), the BER or the error probability P_e given by Eq. (7-8) becomes**

$$\text{BER} = P_e(Q) = \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^{\infty} e^{-x^2} dx = \frac{1}{2}\left[1 - \text{erf}\left(\frac{Q}{\sqrt{2}}\right)\right] \approx \frac{1}{\sqrt{2\pi}} \frac{e^{-Q^2/2}}{Q} \qquad \textbf{(7-12)}$$

where the parameter Q is defined as

$$Q = \frac{v_{th} - b_{off}}{\sigma_{off}} = \frac{b_{on} - v_{th}}{\sigma_{on}} = \frac{b_{on} - b_{off}}{\sigma_{on} + \sigma_{off}} \qquad \textbf{(7-13)}$$

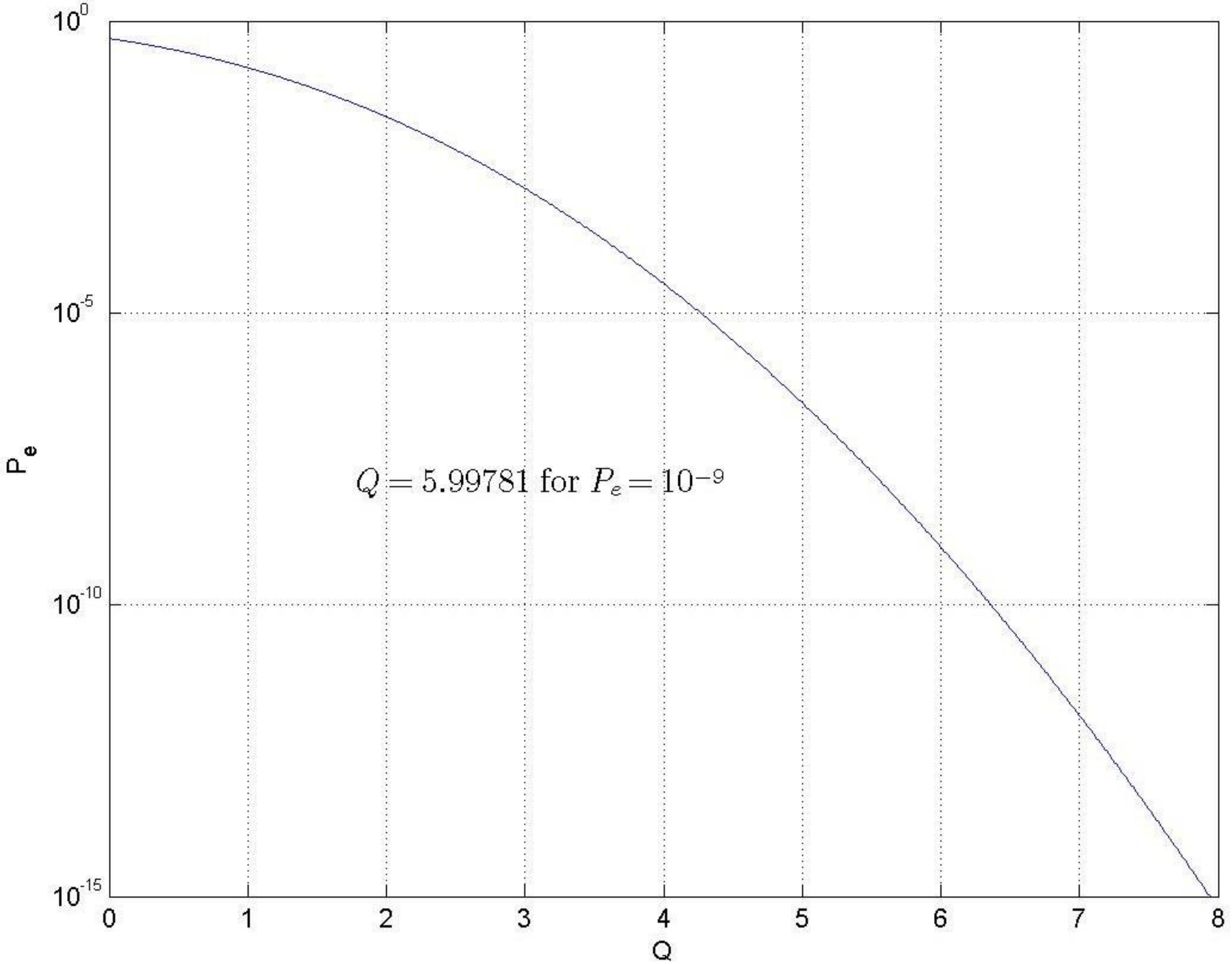
- **The approximation is obtained from the asymptotic expansion of *error function*** **.**

$$\text{erfc}(x) = 1 - \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-y^2} dy \xrightarrow{\text{large } x} \frac{e^{-x^2}}{x\sqrt{\pi}}$$

Probability of Error (12)

- Figure 7-9 shows how the BER varies with Q.
- The approximation for P_e given in Eq. (7-12) and shown by the dashed line in Fig. 7-9 is accurate to 1% for Q~3 and improves as Q increases.
- A commonly quoted Q value is 6, corresponding to a BER = 10^{-9}.

Probability of Error (13)



Plot of the BER (P_e) versus the factor *Q*.

Calculation Examples

Example 7.1: When there is little ISI, 1-γ is small, so that $\sigma_{on}^2 \cong \sigma_{off}^2$. Then, by letting b_{off} =0,

$$Q = \frac{b_{on}}{2\sigma_{on}} = \frac{1}{2}\frac{S}{N}$$

which is one-half SNR. In this case, $v_{th} = b_{on}/2$.

Example 7.2: For an error rate of 10^{-9},

$$P_e(Q) = 10^{-9} = \frac{1}{2}\left[1 - \text{erf}\left(\frac{Q}{\sqrt{2}}\right)\right] \rightarrow Q = 5.99781 \approx 6$$

The SNR then becomes 12 or 10.8 dB.

Probability of Error (14)

- **Consider the special case when**

  $\sigma_{off} = \sigma_{on} = \sigma$ **and** $b_{off} = 0$**, so that** $b_{on} = V$**.**

- **From Eq. (7-13) the threshold voltage is**

  $v_{th} = V/2$**, so that** $Q = V/2\sigma$**.**

- **Since σ is the *rms noise*, the ratio *V*/σ is the *peak- signal-to-rms-noise ratio*.**

- **In this case, Eq. (7-13) becomes**

$$P_e(\sigma_{on} = \sigma_{off}) = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{V}{2\sqrt{2}\sigma}\right)\right] \qquad \textbf{(7-16)}$$

Probability of Error (15)

<u>Example 7-3</u>:
Figure 7-10 shows a plot of the BER expression from Eq. (7-16) as a function of the SNR.

(a). For a SNR of 8.5 (18.6 dB) we have $P_e = 10^{-5}$. If this is the received signal level for a standard DS1 telephone rate of 1.544 Mb/s, the BER results in a misinterpreted bit every 0.065s, which is highly unsatisfactory.

However, by increasing the signal strength so that $V/\sigma = 12.0$ (21.6 dB), the BER decreases to $P_e = 10^{-9}$. For the DS1 case, this means that a bit is misinterpreted every 650s, which is tolerable.

(b). For high-speed SONET links, say the OC-12 rate which operates at 622 Mb/s, BERs of 10^{-11} or 10^{-12} are required. This means that we need to have at least $V/\sigma = 13.0$ (22.3 dB).

Probability of Error (16)

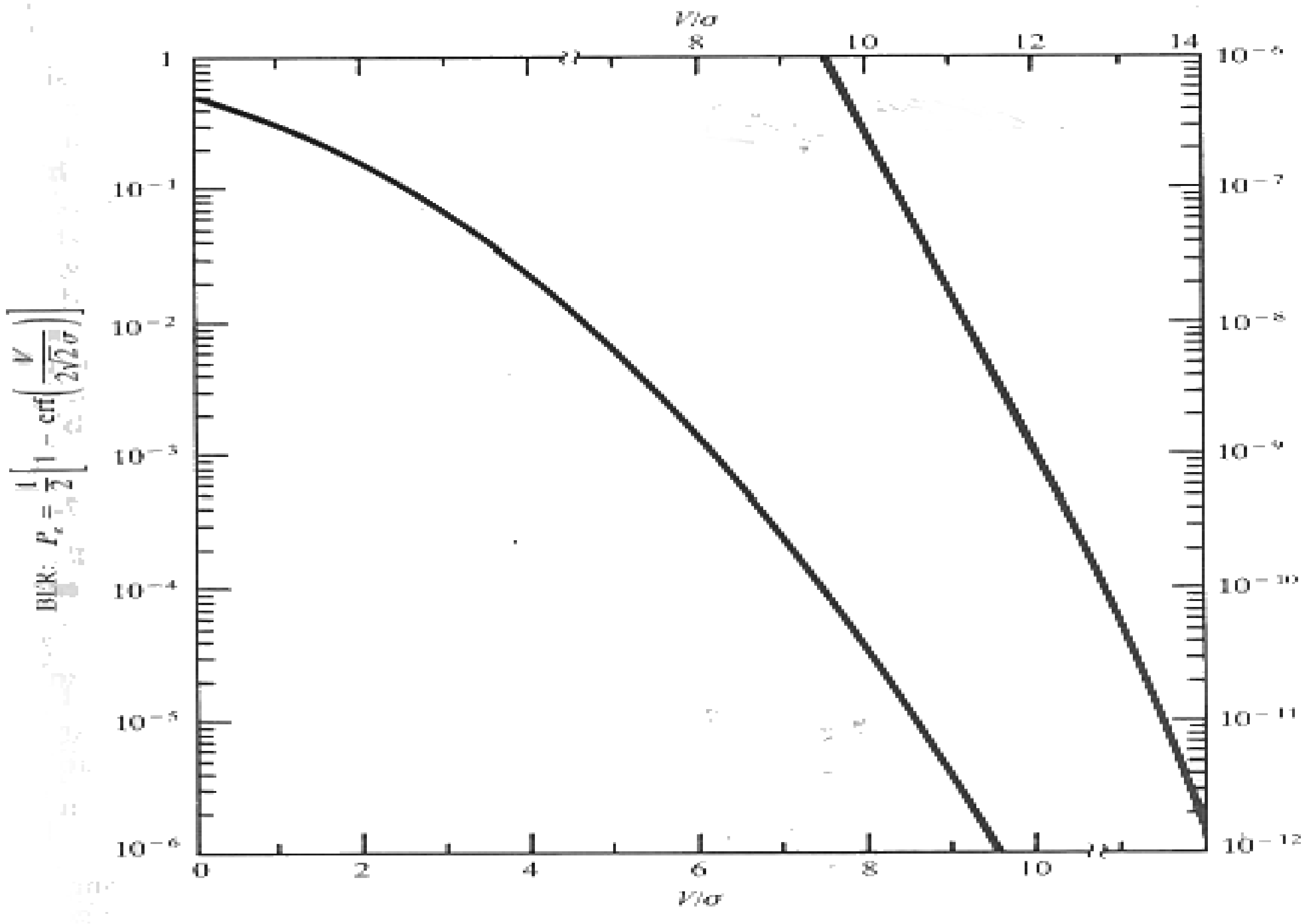


Figure 7-10. BER as a function of SNR when the standard deviations are equal ($\sigma_{on} = \sigma_{off}$) and $b_{off} = 0$.

The Quantum Limit

- **For an ideal photo-detector having unity quantum efficiency and producing no dark current, it is possible to find the minimum received optical power required for a specific BER performance in a digital system.**
- **This minimum received power level is known as the *quantum limit*.**
- **Assume that an optical pulse of energy *E* falls on the photo-detector in a time interval τ.**
- **This can be interpreted by the receiver as a '0' pulse if no electron-hole pairs are generated with the pulse present.**

The Quantum Limit (2)

- **From Eq. (7-2) the probability that *n* = 0 electrons are emitted in a time interval τ is**

$$P_r(0) = e^{-\overline{N}} \qquad \textbf{(7-23)}$$

where the average number of electron-hole pairs, N, is given by Eq. (7-1).

- **Thus, for a given error probability $P_r(0)$, we can find the minimum energy *E* required at a specific wavelength λ.**

The Quantum Limit (3)

<u>Example 7-4</u>:

A digital fiber optic link operating at 850-nm requires a maximum BER of 10^{-9}.

(a). From Eq. (7-16) the probability of error is

$$P_r(0) = e^{-\overline{N}} = 10^{-9}$$

Solving for $\overline{N}$ yields $\overline{N}$ = 9ln10 = 20.7 ~ 21.

Hence, an average of 21 photons per pulse is required for this BER.

Using Eq. (7-1) and solving for *E*, we get

$$E = 20.7h\nu/\eta.$$

The Quantum Limit (4)

(b). Now let us find the minimum incident optical power P_0 that must fall on the photodetector to achieve a 10^{-9} BER at a data rate of 10 Mb/s for a simple binary-level signaling scheme.

If the detector quantum efficiency $\eta = 1$, then

$$E = P_i \tau = 20.7h\nu = 20.7hc/\lambda,$$

where $1/\tau = B/2$, B being the data rate.

Solving for P_i, we have

$$P_i = 20.7hcB/2\lambda$$

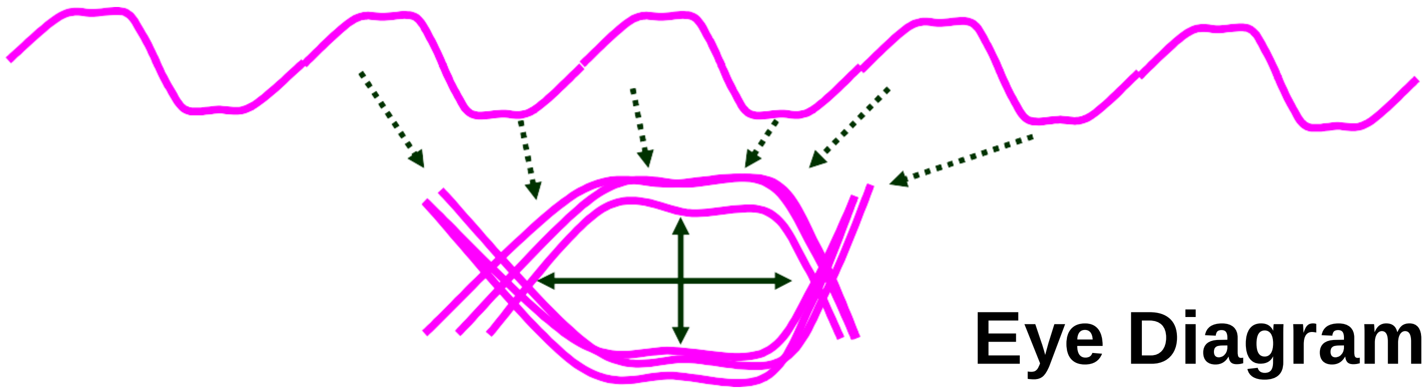
$$= \frac{20.7(6.626\times10^{-34}\text{J.s})(3\times10^{8}\text{m/s})(10\times10^{6}\text{bits/s})}{2(0.85\times10^{-6}\text{m})}$$

$$= 24.2\text{pW} = -76.2 \text{ dBm}.$$

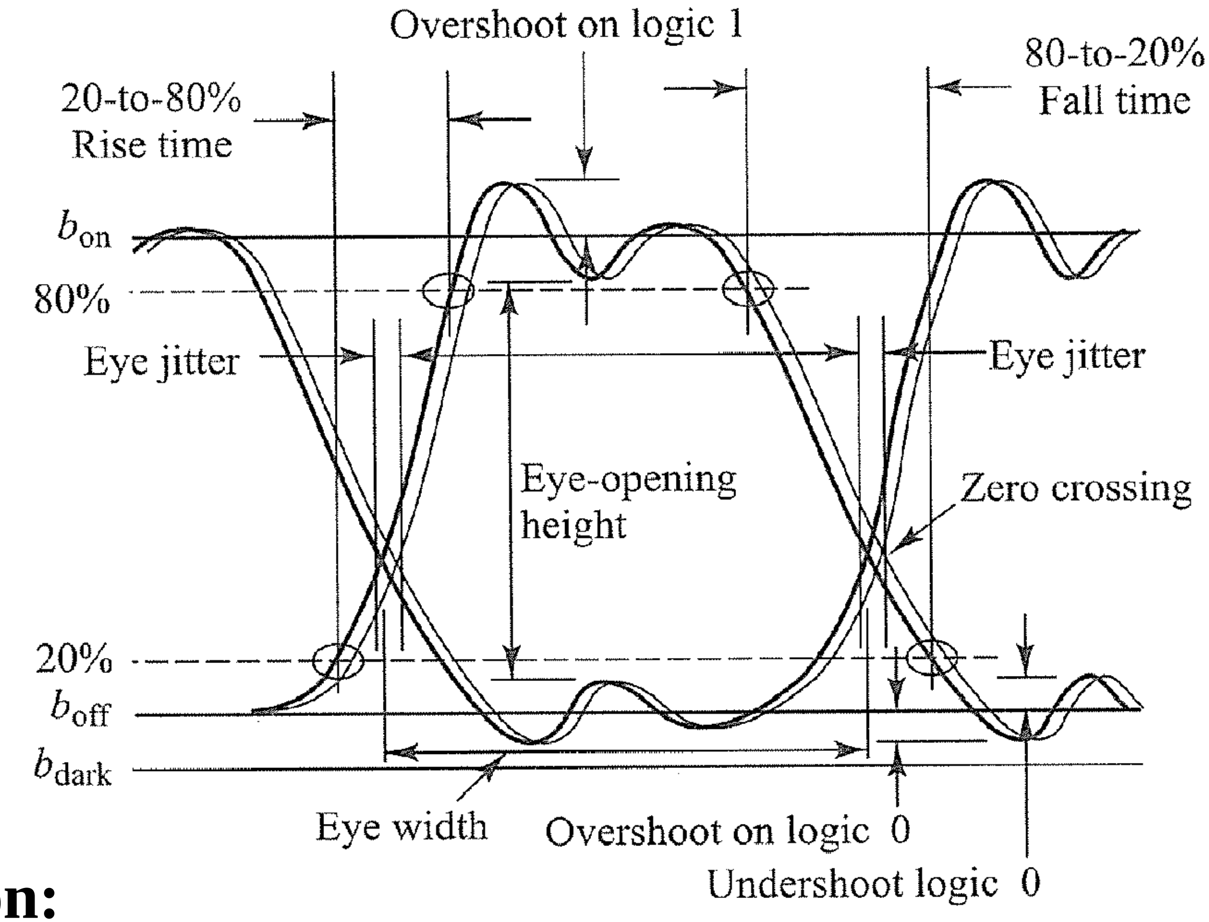
In practice, the sensitivity of most receivers is around 20 dB higher than the quantum limit because of various nonlinear distortions and noise effects in the transmission link.

Eye Diagram

- The eye diagram is a convenient way to represent what a receiver will see as well as specifying characteristics of a transmitter.
- The eye diagram maps all UI intervals on top of one and other. (UI = Unit Interval, i.e., signal duration time)
- The opening in eye diagram is measure of signal quality.
- This is the simplest type of eye diagram. The are other form which we will discuss later



Eye Diagram



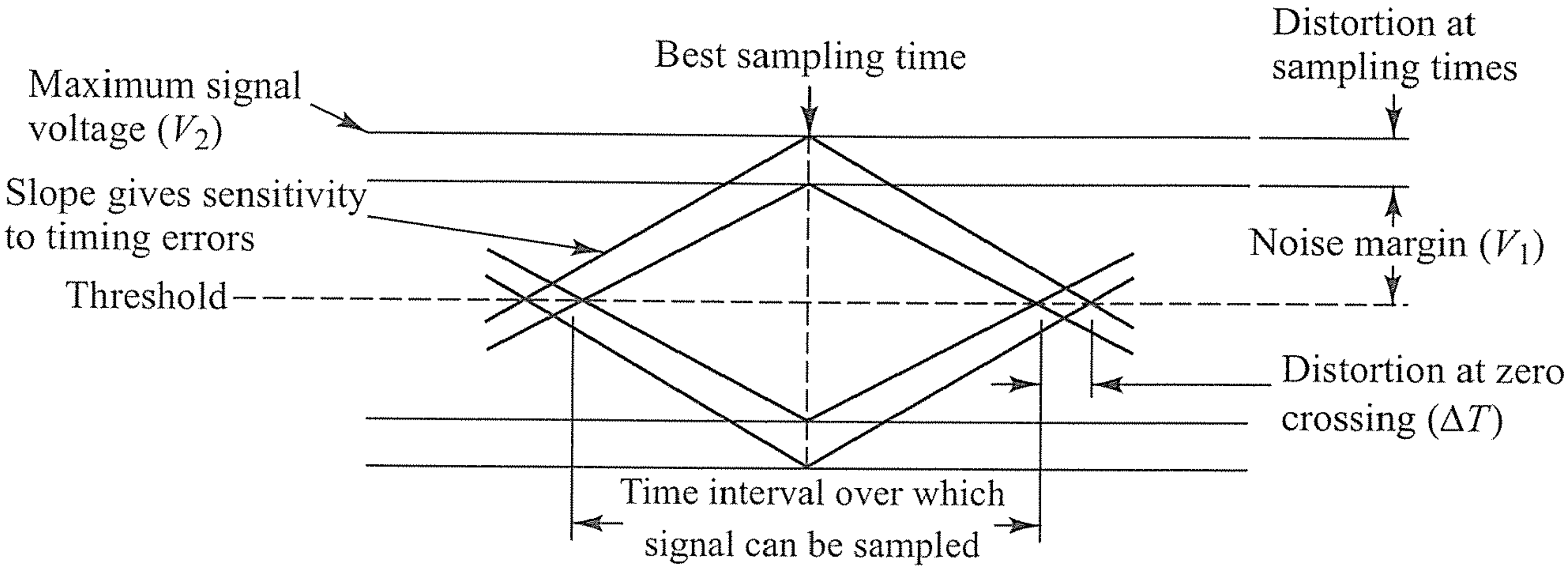
Information:

• Width of eye-opening : time interval over which the received signal can be sampled without ISI error.

• Best time to sample = when height of eye-opening is largest

• Rise time = time interval between 10% point and 90% point, can be approximated by

$$T_{10-90} = 1.25 \times T_{20-80}$$



Noise margin:

$$\text{Noise margin (percent)} = \frac{V_1}{V_2} \times 100\%$$

Timing jitter (edge jitter, phase distortion): due to noise in the receiver and pulse distortion in optical fiber.

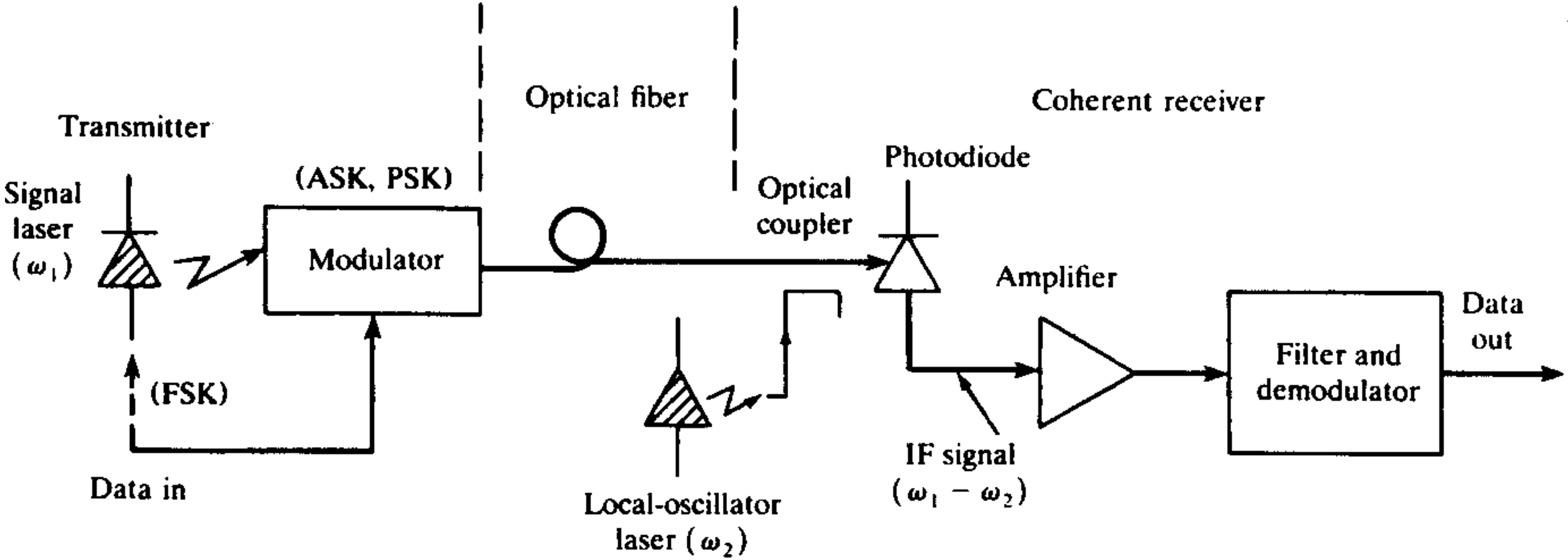
$$\text{Timing jitter (percent)} = \frac{\Delta T}{T_b} \times 100\%$$

Coherent Detection

- Primitive method : *intensity modulation with direct detection* (IM/DD)
- Since the improvement of semiconductor lasers around 70s, coherent optical communication systems become available.
- Coherent detection depends on *phase coherence* of the optical carrier.

Fundamental Concepts

- Key principle : mixing incoming signal with a locally generated continuous-wave (CW) optical field.
- Shot noise from local oscillator limits sensitivity.



Modulation Techniques

- Transmitted optical signal given by:

$$E_s = A_s \cos\left[\omega_s t + \phi_s(t)\right]$$

where A_s, ω_s, ϕ_s are amplitude, angular frequency, phase of the optical signal. Following techniques are possible:

1. *Amplitude shift keying* (ASK) or *on-off keying* (OOK) : ϕ_s is constant, and A_s have two levels for 0 or 1.
2. *Frequency shift keying* (FSK) : A_s is constant and $\phi_s(t) = \omega_1 t$, or $\omega_2 t$.
3. *Phase shift keying* (PSK) : A_s is constant and $\phi_s(t)$ differs by π.

Direct Detection

- The detected current is proportional to the intensity I_{DD} (the square of the electric field) of the optical signal, yielding

$$I_{DD} = E_s E_s^* = \frac{1}{2} A_s^2 \left[1 + \cos\left(2\omega_s t + 2\phi_s \right) \right]$$

- The double angle term gets eliminated since its frequency is beyond the response capability of the detector. Thus,

$$I_{DD} = E_s E_s^* \approx \frac{1}{2} A_s^2$$

Coherent Lightwave Systems

- Four basic demodulation formats:
  - How optical signal mixed with the local oscillator (*homodyne* or *heterodyne*)
  - How electrical signal is detected (*synchronous* or *asynchronous*)
- If the local-oscillator (LO) field is given by

$$E_{LO} = A_{LO} \cos\left[\omega_{LO} t + \phi_{LO}(t)\right]$$

- The detected current $I_{\text{coh}}(t) \propto$ square of the total electric field of the signal falling on the photodetector, i.e.,

$$I_{coh}(t) = (E_s + E_{LO})^2$$

$$= \frac{1}{2} A_s^2 + \frac{1}{2} A_{LO}^2 + A_s A_{LO} \cos\left[(\omega_s - \omega_{LO})t + \phi(t)\right] \cos\theta(t)$$

where $\phi(t) = \phi_s(t) - \phi_{LO}(t)$, and

$$\cos\theta(t) = \frac{\overline{\mathbf{E}}_s \cdot \overline{\mathbf{E}}_{LO}}{\left|\overline{\mathbf{E}}_s\right|\left|\overline{\mathbf{E}}_{LO}\right|}$$

represents the polarization misalignment between the signal wave and the LO wave.

The optical power then becomes

$$P(t) = P_s + P_{LO} + 2\sqrt{P_s P_{LO}}\cos\left[\omega_{IF} t + \phi(t)\right]\cos\theta(t); \omega_{IF} = \omega_s - \omega_{LO}$$

where P_s , P_{LO} : signal and LO optical powers with $P_{LO} >> P_s$. ω_{IF} is intermediate frequency. $\phi(t)$ represents the time-varying phase difference between signal and LO.

Homodyne Detection

- $\omega_s = \omega_{LO}$, i.e., $\omega_{IF} = 0$. The power becomes

$$P(t) = P_s + P_{LO} + 2\sqrt{P_s P_{LO}}\cos\phi(t)\cos\theta(t)$$

- The first term can be ignored (since $P_{LO} >> P_s$), the second term is constant, thus the last term contains transmitted information.
- Can be used for both OOK and PSK.
- Most sensitive receiver.
- Difficult to build; needs local oscillator controlled by optical PLL.
- Restrictions on optical sources for transmitter and LO.

Heterodyne Detection

- $\omega_s \neq \omega_{LO}$ and no need for PLL.
- Ignoring P_s, the detected current contains two terms:

$$i_{dc} = \frac{\eta q}{h\nu} P_{LO}$$

$$i_{IF}(t) = \frac{2 q}{h\nu} \sqrt{P_s P_{LO}} \cos[\omega_{IF} t + \phi(t)] \cos\theta(t)$$

- The dc term is filtered out and the information is recovered from the amplified IF term.
- Can be used for OOK, FSK and PSK.

BER : direct-detection OOK

Assume 1 and 0 occur with equal probability, $\overline{N}$ and 0 electron-holes pairs are created during 1 and 0 pulses, and unity quantum efficiency ($\eta = 1$), the average number of photons per bit is

$$\overline{N}_p = \frac{1}{2}\overline{N} + \frac{1}{2}(0) \rightarrow \overline{N} = 2\overline{N}_p$$

The probability of error becomes:

$$P_r(0) = e^{-2\overline{N}_p}$$

Taking quantum efficiency into account, then

$$\text{BER} = P_e = \frac{1}{2}P_r(0) = \frac{1}{2}e^{-2\eta\overline{N}_p}$$

BER : OOK homodyne system

When a 0 pulse of duration T is received, the average number $\overline{N}_0$ of electron-holes pairs created is the number generated by the local-oscillator, i.e., $N_0 = A_{LO}^2 T$

For a 1 pulse, the average number is

$$\overline{N}_1 = \left(A_s + A_{LO}\right)^2 T \approx \left(A_{LO}^2 + 2A_s A_{LO}\right) T$$

Since the LO output power is much higher than the received signal power, the voltage V seen by the receiver during a 1 pulse is

$$V = \overline{N}_1 - \overline{N}_0 = 2A_s A_{LO} T$$

and the associated rms noise is

$$\sigma \cong \sqrt{\overline{N}_1} \approx \sqrt{\overline{N}_0}$$

BER : OOK homodyne system (2)

BER becomes

$$\text{BER} = P_e = \frac{1}{2}\left[1-\text{erf}\left(\frac{V}{2\sqrt{2}\sigma}\right)\right] = \frac{1}{2}\text{erfc}\left(\frac{V}{2\sqrt{2}\sigma}\right) = \frac{1}{2}\text{erfc}\left(\frac{A_s\sqrt{T}}{\sqrt{2}}\right)$$

For example, to achieve BER=10^{-9}, $V/\sigma = 12$ and $A_s^2 T = 36$

which is the expected number of signal photons per pulse.

Assume 0 and 1 occur with same probability, then the average number of photons per bit is half the required number per pulse. Since $\overline{N}_p = A_s^2 T/2$ and taking quantum efficiency into account yields

$$\text{BER} = \frac{1}{2}\text{erfc}\left(\sqrt{\eta \overline{N}_p}\right) \xrightarrow{\eta \overline{N}_p \geq 5} \frac{e^{-\eta \overline{N}_p}}{\sqrt{\pi \eta \overline{N}_p}}$$

BER : PSK homodyne system

The average number of electron-holes pairs created during a 0 and 1 pulse are given by

$$\bar{N}_{\substack{1\\0}} = \left(A_{LO} \pm A_s\right)^2 T$$

assuming 1 and 0 pulses are in-phase and out-of-phase.

The voltage V seen by the receiver is

$$V = \bar{N}_1 - \bar{N}_0 = 4A_s A_{LO} T$$

and the associated rms noise is $\sigma = \sqrt{A_{LO}^2 T}$

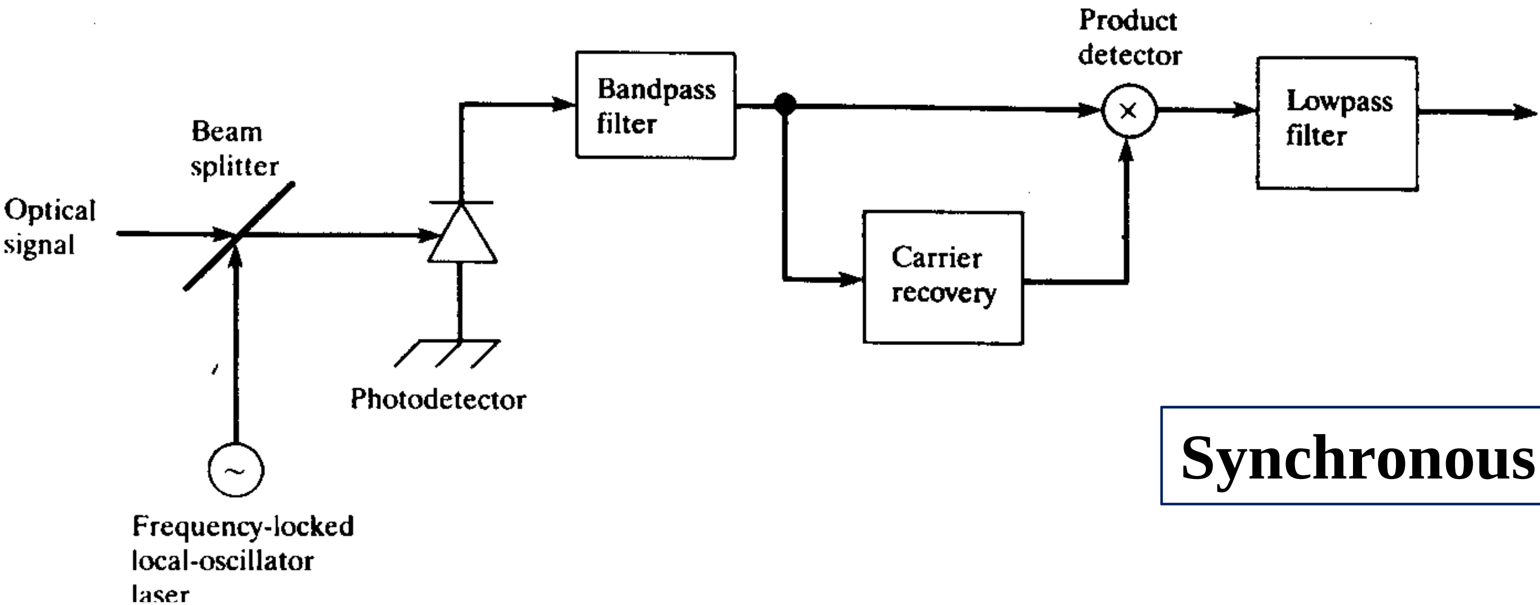
For example, to achieve BER=10^{-9}, $V/\sigma = 12$ and $A_s^2 T = 9$

Since $\bar{N}_p = A_s^2 T$

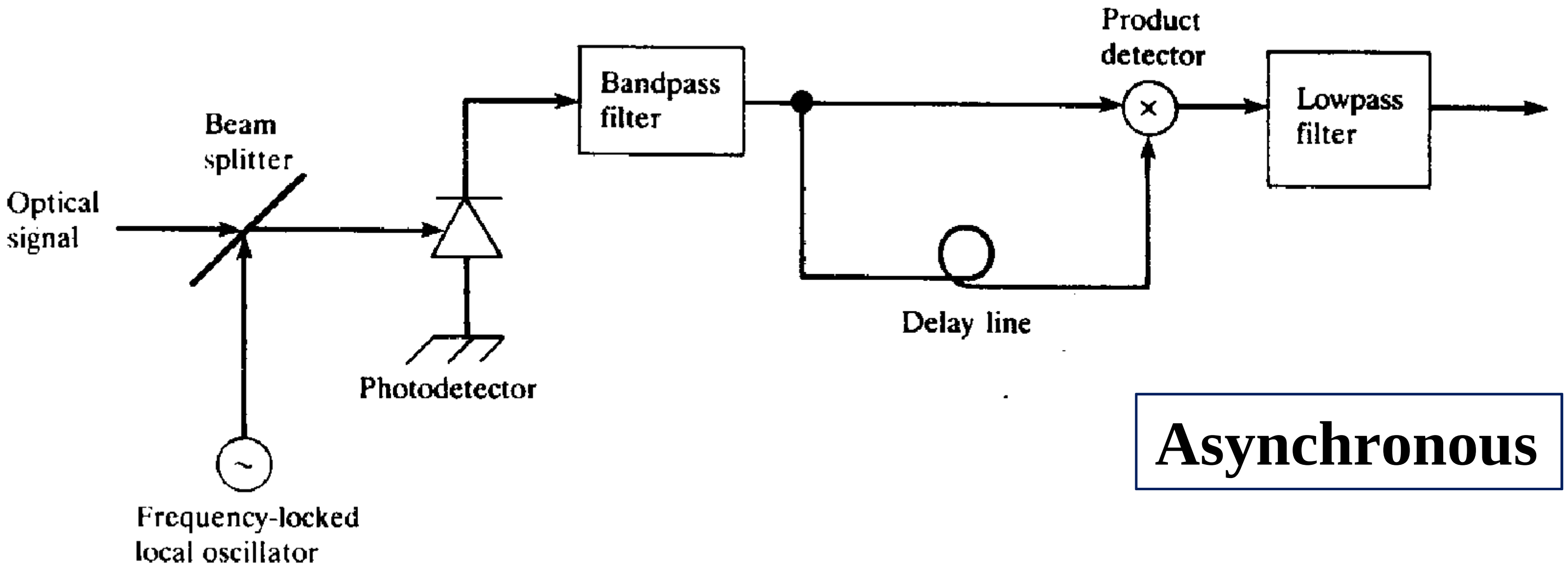
it follows that

$$\mathrm{BER} = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{2\eta \bar{N}_p}\right)$$

Heterodyne System



Synchronous



Asynchronous

BER : Summary

Modulation	Probability of error: Homodyne	Heterodyne: Synchronous detection	Heterodyne: Asynchronous detection	Direct detection
On-off keying (OOK)	$\frac{1}{2}\,\mathrm{erfc}\,(\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\,\mathrm{erfc}\,(\frac{1}{2}\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\exp(-\frac{1}{2}\eta \bar{N}_p)$	$\frac{1}{2}\exp(-2\eta \bar{N}_p)$
Phase-shift keying (PSK)	$\frac{1}{2}\,\mathrm{erfc}\,(2\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\,\mathrm{erfc}\,(\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\exp(-\eta \bar{N}_p)$	—
Frequency-shift keying (FSK)	—	$\frac{1}{2}\,\mathrm{erfc}\,(\frac{1}{2}\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\exp(-\frac{1}{2}\eta \bar{N}_p)$	—

BER $= 10^{-9}$

Modulation	Number of photons: Homodyne	Heterodyne: Synchronous detection	Heterodyne: Asynchronous detection	Direct detection
On-off keying (OOK)	18	36	40	10
Phase-shift keying (PSK)	9	18	20	—
Frequency-shift keying (FSK)	—	36	40	—

Analog Transmission System

- In photonic analog transmission system the performance of the system is mainly determined by signal-to-noise ratio at the output of the receiver.
- In case of amplitude modulation the transmitted optical power *P(t)* is in the form of:

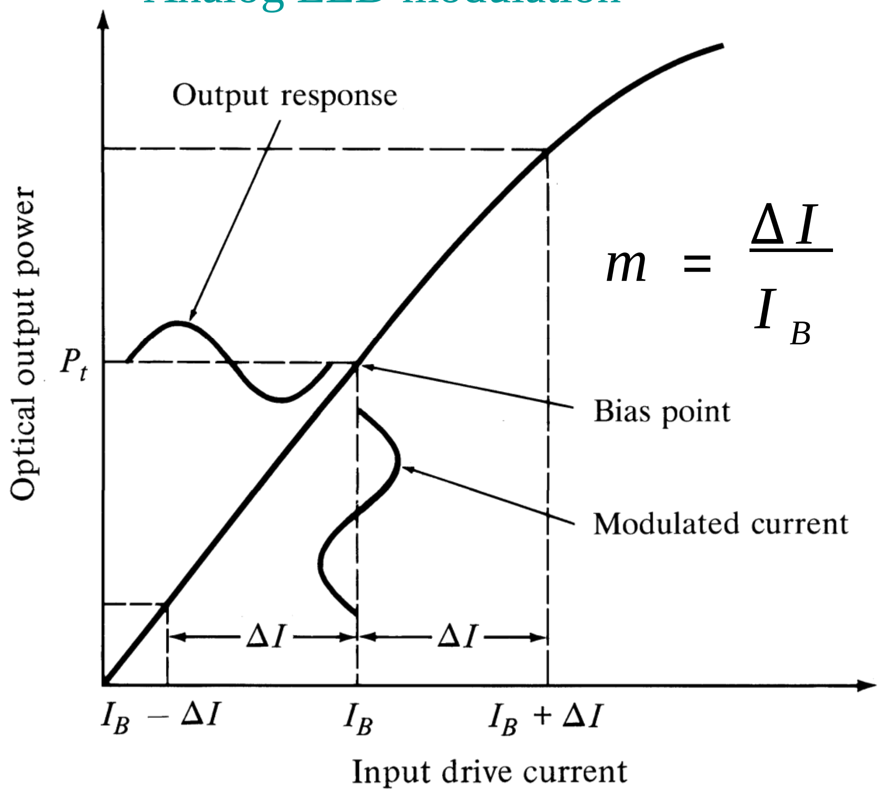
$$P(t) = P_t[1 + ms(t)]$$

where *m* is modulation index, and *s(t)* is analog modulation signal.

- The photocurrent at receiver can be expressed as:

$$i_s(t) = RMP_r[1 + ms(t)] = I_P M[1 + ms(t)]$$

Analog LED modulation



$$m = \frac{\Delta I}{I_B}$$

- By calculating mean square of the signal and mean square of the total noise, which consists of quantum, dark and surface leakage noise currents plus resistance thermal noise, the S/N can be written as:

$$\frac{S}{N} = \frac{\langle i_s^2 \rangle}{\langle i_N^2 \rangle} = \frac{(1/2)(\mathcal{R} M m P_r)^2}{2q(\mathcal{R} P_r + I_D) M^2 F(M) B + (4 k_B T B / R_{eq}) F_t}$$

$$= \frac{(1/2)(M m I_P)^2}{2q(I_P + I_D) M^2 F(M) B + (4 k_B T B / R_{eq}) F_t}$$

I_P : primary photocurrent $= \mathcal{R} P_r$; I_D : primary bulk dark current;

I_L : Surface - leakage current; $F(M)$: excess photodiode noise factor $\approx M^x$

B : effective noise bandwidth; R_{eq} : equivalent resistance of photodetector load and amplifier

F_t : noise figure of baseband amplifier; P_r : average received optical power

pin Photodiode *SNR*

- For *pin* photodiode, *M=1:*

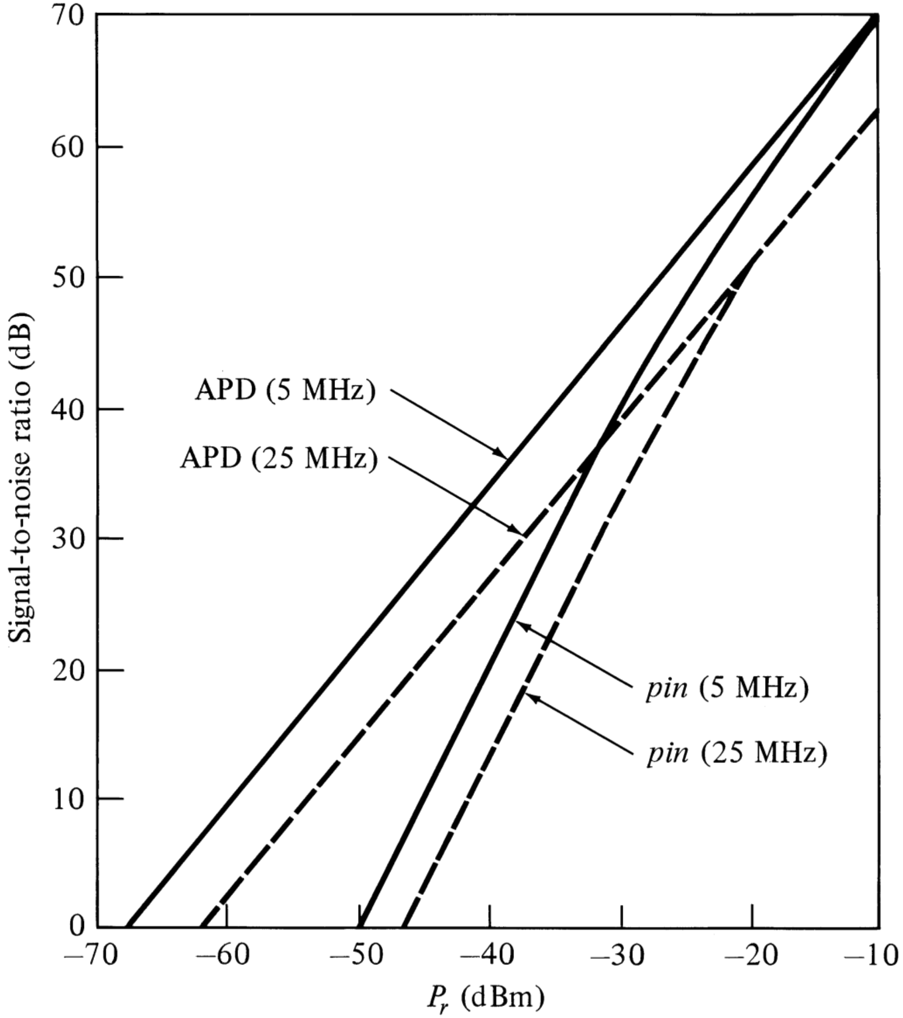
Low input signal level, thermal noise dominates:

$$\frac{S}{N} \cong \frac{(1/2)(I_P m)^2}{(4k_B TB/R_{eq})F_t} = \frac{(1/2)m^2 \mathcal{R}^2 P_r^2}{(4k_B TB/R_{eq})F}$$

Large signal level, shot noise dominates:

$$\frac{S}{N} \cong \frac{m^2 \mathcal{R} P_r}{4qB}$$

SNR vs. optical power for photodiodes



UNIT-V DIGITAL TRANSMISSION SYSTEM

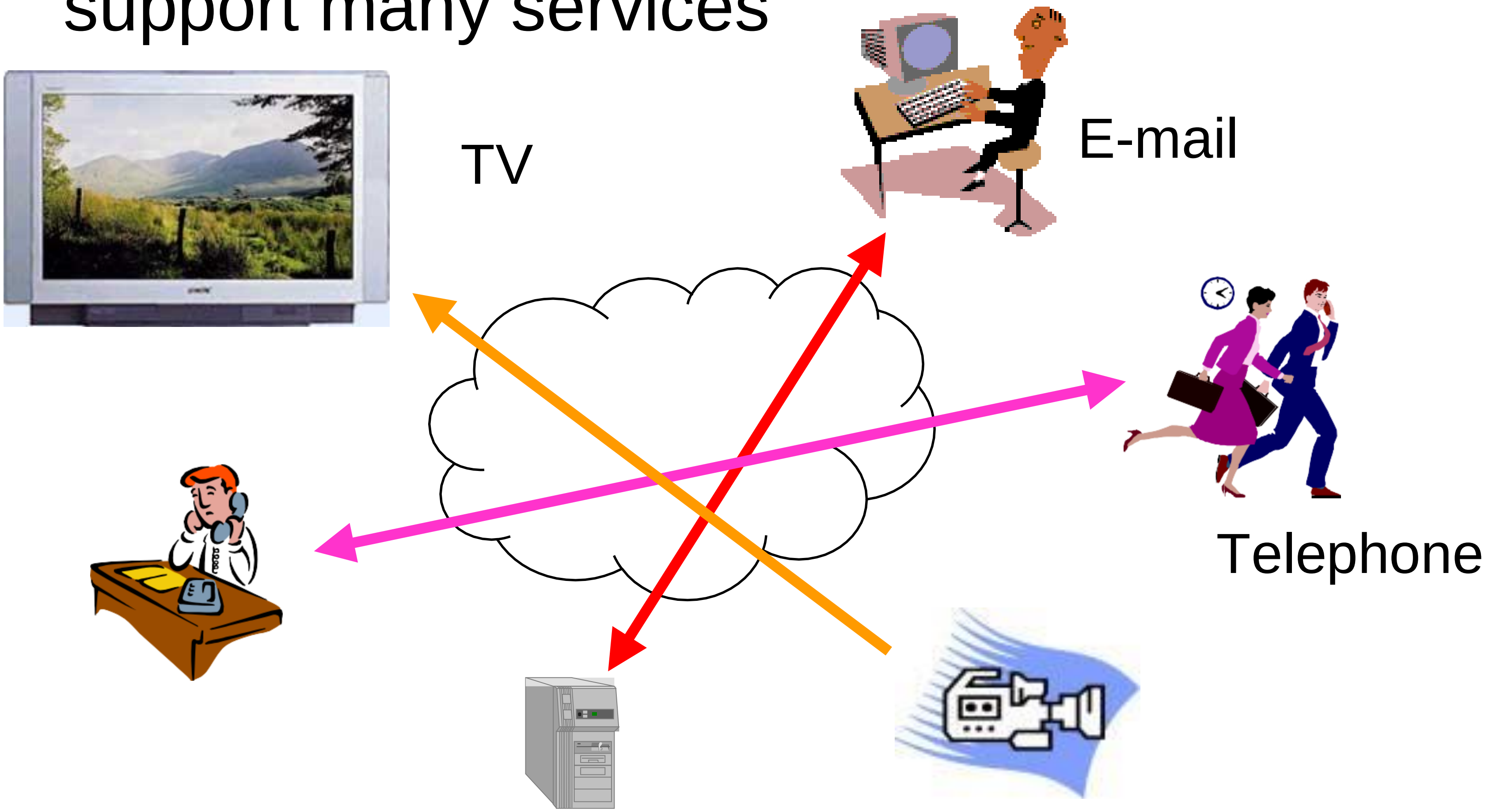
Digital Transmission Fundamentals

Digital Representation of Information
Why Digital Communications?
Digital Representation of Analog Signals
Characterization of Communication Channels
Fundamental Limits in Digital Transmission

Line Coding
Modems and Digital Modulation
Properties of Media and Digital Transmission Systems
Error Detection and Correction

Digital Networks

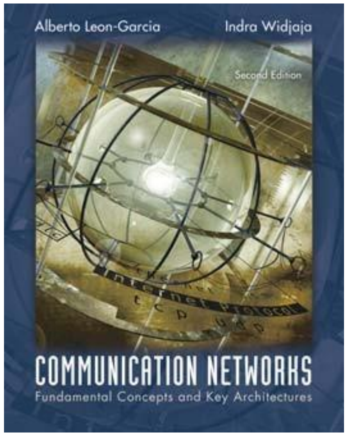
- Digital transmission enables networks to support many services



Questions of Interest

- How long will it take to transmit a message?
  - How many bits are in the message (text, image)?
  - How fast does the network/system transfer information?
- Can a network/system handle a voice (video) call?
  - How many bits/second does voice/video require? At what quality?
- How long will it take to transmit a message without errors?
  - How are errors introduced?
  - How are errors detected and corrected?
- What transmission speed is possible over radio, copper cables, fiber, infrared, …?

Chapter 3
Digital Transmission Fundamentals



Digital Representation of Information

Bits, numbers, information

- Bit: number with value 0 or 1
  - *n* bits: digital representation for 0, 1, … , 2^n
  - Byte or Octet, *n* = 8
  - Computer word, *n* = 16, 32, or 64
- *n* bits allows enumeration of 2^n possibilities
  - *n*-bit field in a header
  - *n*-bit representation of a voice sample
  - Message consisting of n bits
- *The number of bits required to represent a message is a measure of its information content*
  - More bits → More content

Block vs. Stream Information

Block

- Information that occurs in a single block
  - Text message
  - Data file
  - JPEG image
  - MPEG file
- Size = Bits / block

  or bytes/block
  - 1 kbyte = 2^{10} bytes
  - 1 Mbyte = 2^{20} bytes
  - 1 Gbyte = 2^{30} bytes

Stream

- Information that is produced & transmitted *continuously*
  - Real-time voice
  - Streaming video
- Bit rate = bits / second
  - 1 kbps = 10^3 bps
  - 1 Mbps = 10^6 bps
  - 1 Gbps = 10^9 bps

Transmission Delay

- *L* — number of bits in message
- *R* bps — speed of digital transmission system
- *L/R* — time to transmit the information
- t_{prop} — time for signal to propagate across medium
- *d* — distance in meters
- *c* — speed of light (3×10^8 m/s in vacuum)

$$Delay = t_{prop} + L/R = d/c + L/R \text{ seconds}$$

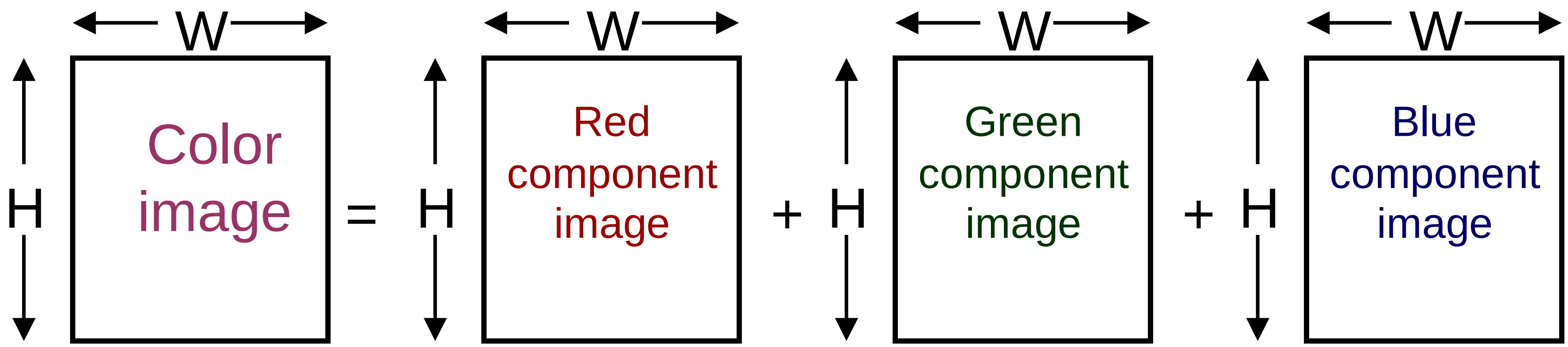
Use data compression to reduce L
Use higher speed modem to increase R
Place server closer to reduce d

Compression

- Information usually not represented efficiently
- Data compression algorithms
  - Represent the information using fewer bits
  - Noiseless: original information recovered exactly
    - E.g. `zip`, `compress`, GIF, fax
  - Noisy: recover information approximately
    - JPEG
    - Tradeoff: # bits vs. quality
- Compression Ratio

#bits (original file) / #bits (compressed file)

Color Image



Total bits = 3 × H × W pixels × B bits/pixel = 3HWB bits

Example: 8×10 inch picture at 400 × 400 pixels per inch^2

400 × 400 × 8 × 10 = 12.8 million pixels

8 bits/pixel/color

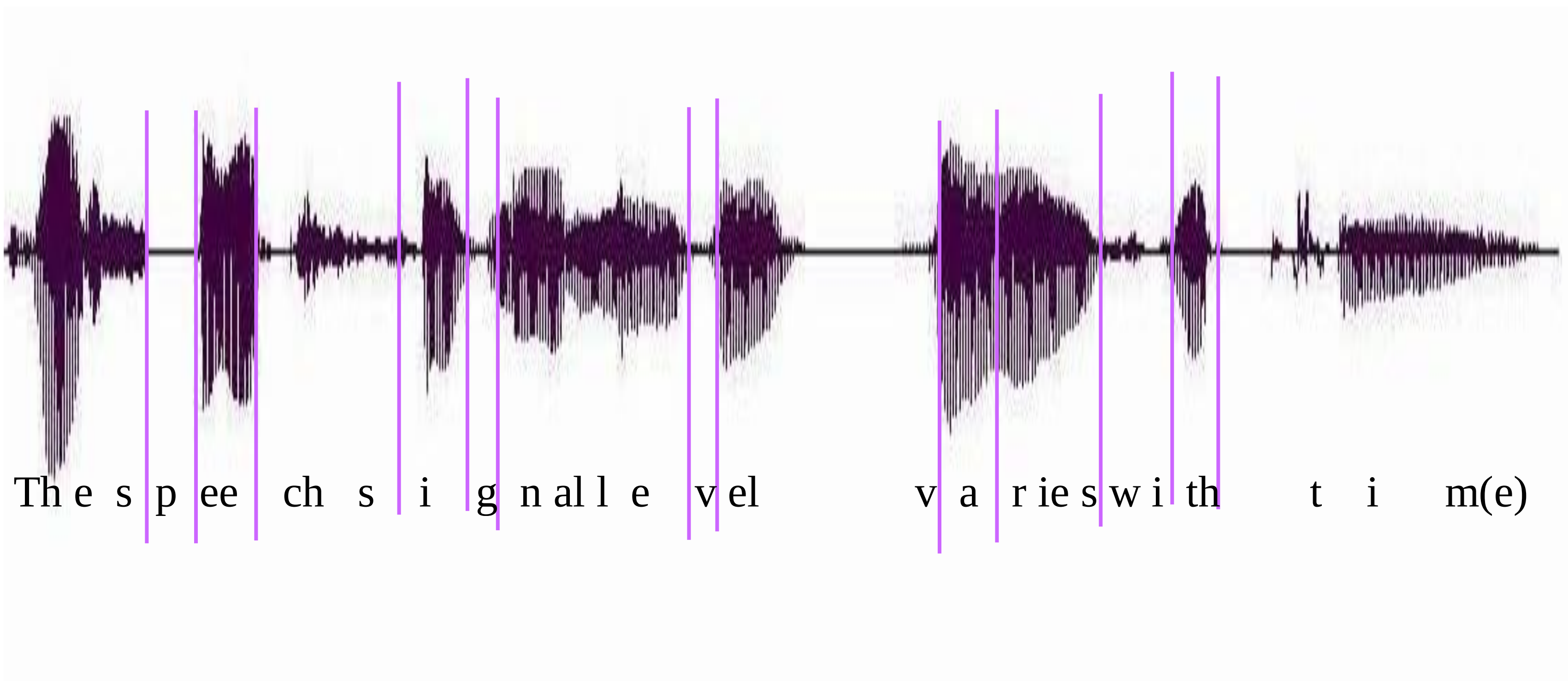
12.8 megapixels × 3 bytes/pixel = 38.4 megabytes

Examples of Block Information

Type	Method	Format	Original	Compressed (Ratio)
Text	Zip, compress	ASCII	Kbytes-Mbytes	(2-6)
Fax	CCITT Group 3	A4 page 200x100 pixels/in^2	256 kbytes	5-54 kbytes (5-50)
Color Image	JPEG	8x10 in^2 photo 400^2 pixels/in^2	38.4 Mbytes	1-8 Mbytes (5-30)

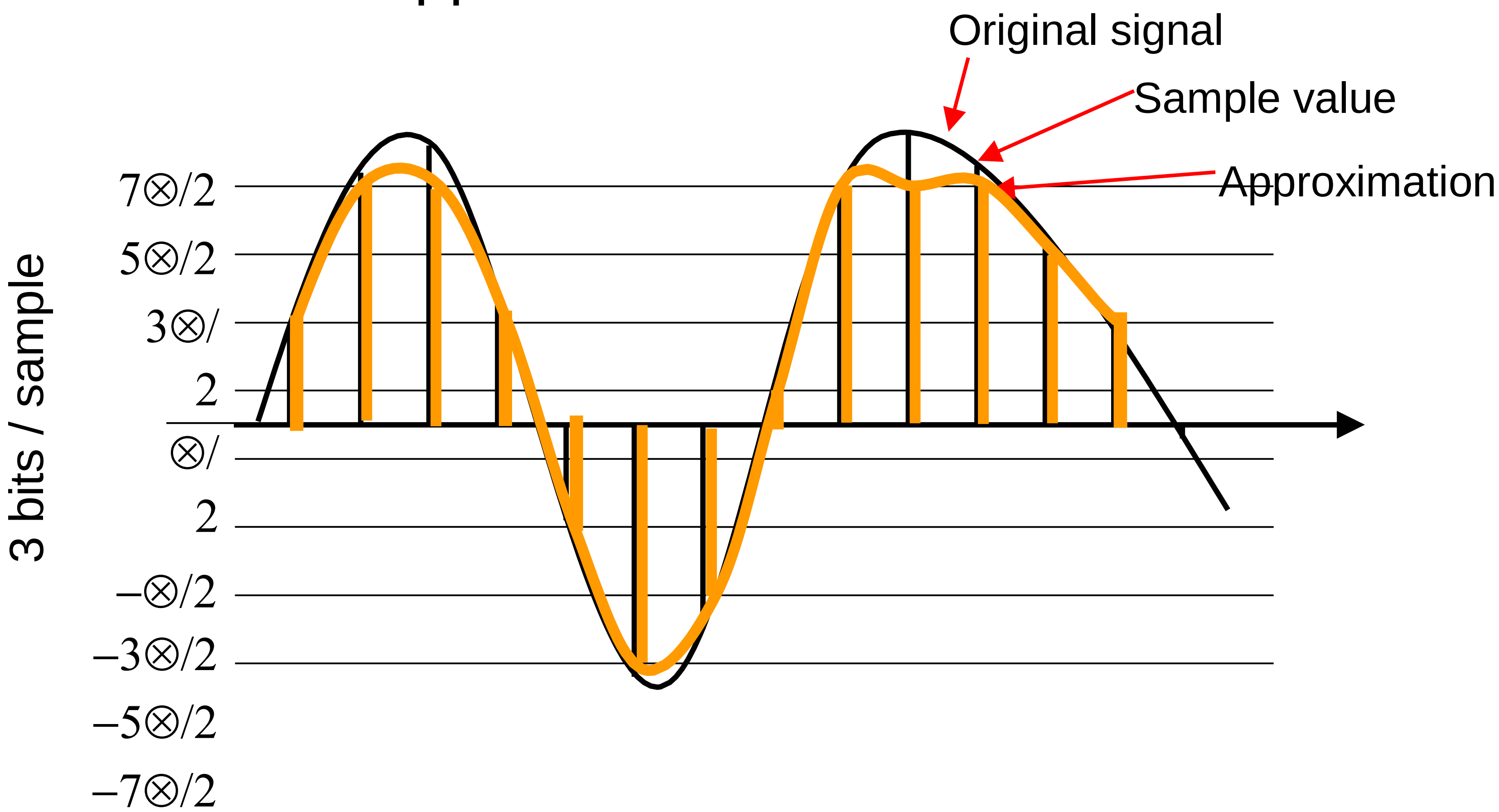
Stream Information

- A real-time voice signal must be digitized & transmitted as it is produced
- Analog signal level varies continuously in time



Digitization of Analog Signal

- Sample analog signal in time and amplitude
- Find closest approximation



R_s = Bit rate = # bits/sample x # samples/second

Digitization of Analog Signal

Bit Rate of Digitized Signal

- Bandwidth W_s Hertz: how fast the signal changes
  - Higher bandwidth → more frequent samples
  - Minimum sampling rate = 2 x W_s

- Representation accuracy: range of approximation error
  - Higher accuracy

  → smaller spacing between approximation values

  → more bits per sample

Example: Voice & Audio

Telephone voice

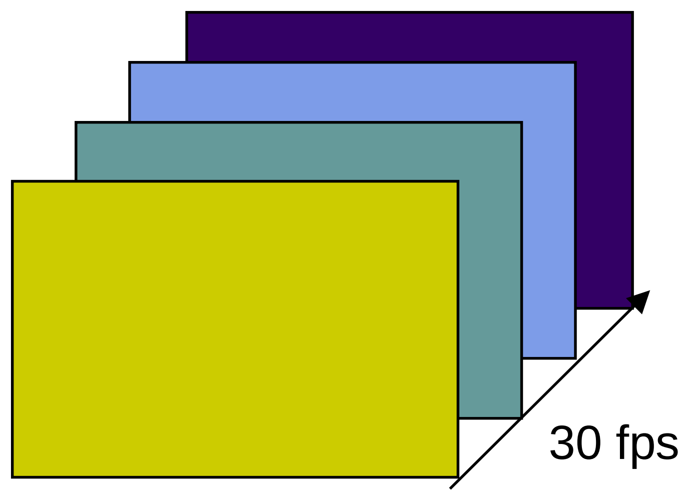
- W_s = 4 kHz → 8000 samples/sec
- 8 bits/sample
- R_s=8 x 8000 = 64 kbps
- Cellular phones use more powerful compression algorithms: 8-12 kbps

CD Audio

- W_s = 22 kHertz → 44000 samples/sec
- 16 bits/sample
- R_s=16 x 44000= 704 kbps per audio channel
- MP3 uses more powerful compression algorithms: 50 kbps per audio channel

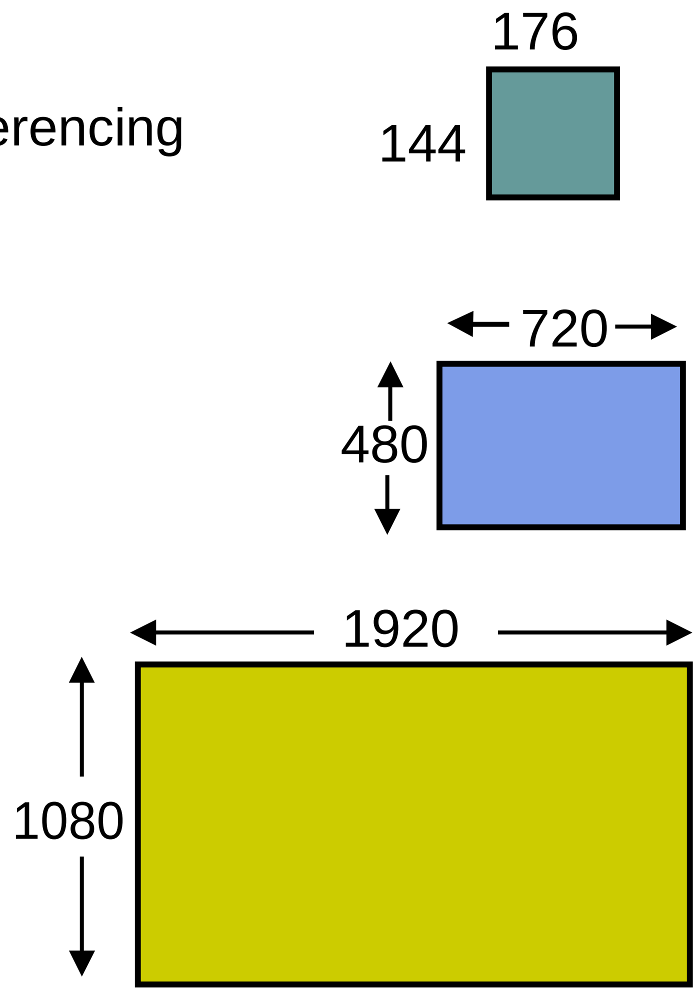
Video Signal

- Sequence of picture frames
  - Each picture digitized & compressed
- Frame repetition rate
  - 10-30-60 frames/second depending on quality
- Frame resolution
  - Small frames for videoconferencing
  - Standard frames for conventional broadcast TV
  - HDTV frames



Rate = M bits/pixel **x** (WxH) pixels/frame **x** F frames/second

Video Frames



QCIF videoconferencing

at 30 frames/sec =

760,000 pixels/sec

Broadcast TV

at 30 frames/sec =

10.4×10^6 pixels/sec

HDTV

at 30 frames/sec =

67×10^6 pixels/sec

Digital Video Signals

Type	Method	Format	Original	Compressed
Video Confer-ence	H.261	176x144 or 352x288 pix @10-30 fr/sec	2-36 Mbps	64-1544 kbps
Full Motion	MPEG 2	720x480 pix @30 fr/sec	249 Mbps	2-6 Mbps
HDTV	MPEG 2	1920x1080 @30 fr/sec	1.6 Gbps	19-38 Mbps

Transmission of Stream Information

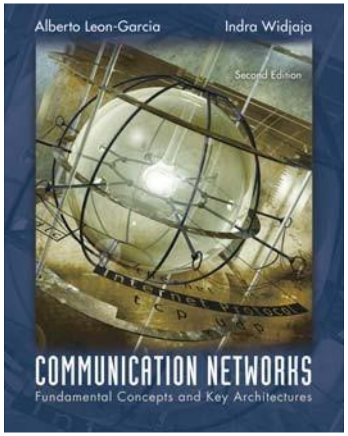
- Constant bit-rate
  - Signals such as digitized telephone voice produce a steady stream: e.g. 64 kbps
  - Network must support steady transfer of signal, e.g. 64 kbps circuit
- Variable bit-rate
  - Signals such as digitized video produce a stream that varies in bit rate, e.g. according to motion and detail in a scene
  - Network must support variable transfer rate of signal, e.g. packet switching or rate-smoothing with constant bit-rate circuit

Stream Service Quality Issues

Network Transmission Impairments

- Delay: Is information delivered in timely fashion?
- Jitter: Is information delivered in sufficiently smooth fashion?
- Loss: Is information delivered without loss? If loss occurs, is delivered signal quality acceptable?
- Applications & application layer protocols developed to deal with these impairments

Chapter 3
Communication Networks and Services



Why Digital Communications?

A Transmission System

Transmitter

- Converts information into *signal* suitable for transmission
- Injects energy into communications medium or channel
  - Telephone converts voice into electric current
  - Modem converts bits into tones

Receiver

- Receives energy from medium
- Converts received signal into form suitable for delivery to user
  - Telephone converts current into voice
  - Modem converts tones into bits

Transmission Impairments

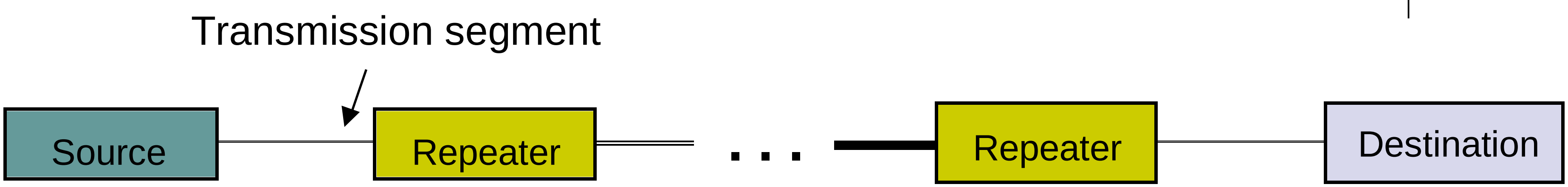
Communication Channel

- Pair of copper wires
- Coaxial cable
- Radio
- Light in optical fiber
- Light in air
- Infrared

Transmission Impairments

- Signal attenuation
- Signal distortion
- Spurious noise
- Interference from other signals

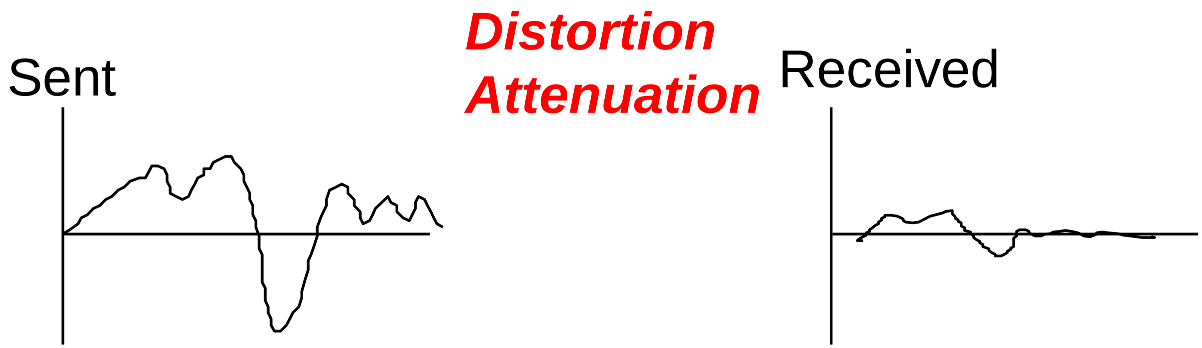
Analog Long-Distance Communications



- Each repeater attempts to restore analog signal to its original form
- Restoration is imperfect
  - Distortion is not completely eliminated
  - Noise & interference is only partially removed
- Signal quality decreases with # of repeaters
- Communications is distance-limited
- Still used in analog cable TV systems
- Analogy: Copy a song using a cassette recorder

Analog vs. Digital Transmission

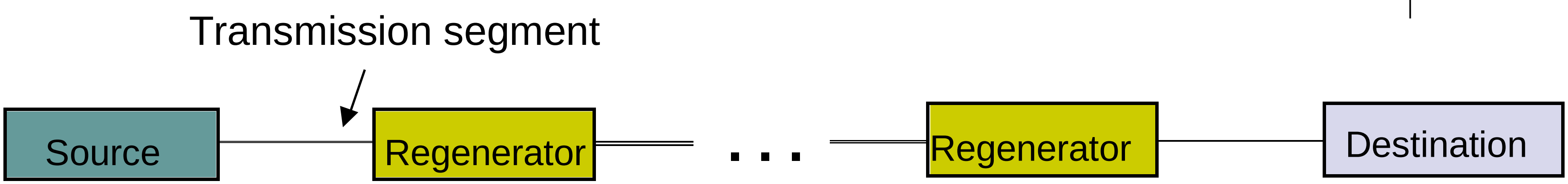
Analog transmission: all details must be reproduced accurately



Digital transmission: only discrete levels need to be reproduced

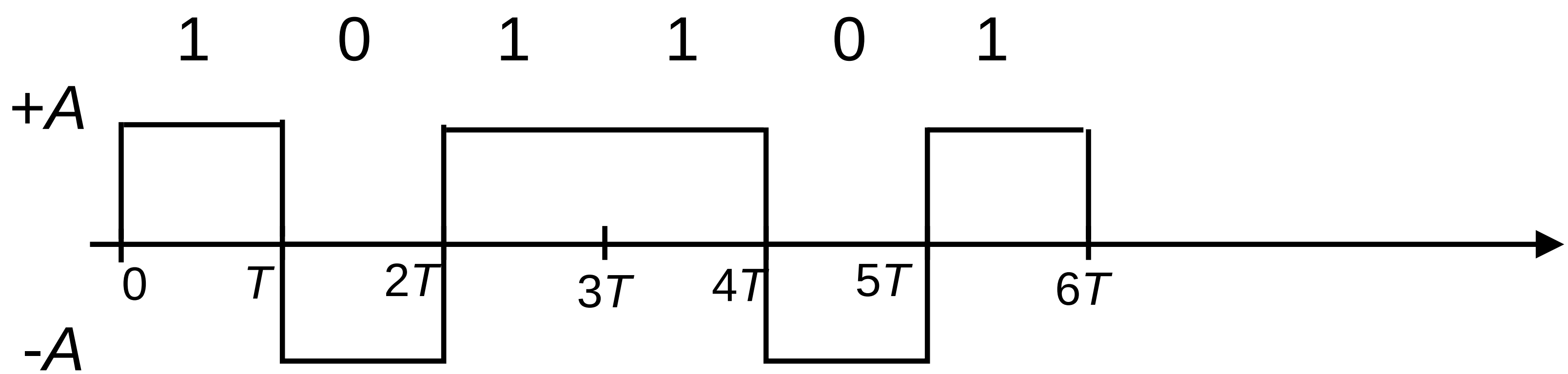
Simple Receiver: Was original pulse positive or negative?

Digital Long-Distance Communications



- Regenerator recovers original data sequence and retransmits on next segment
- Can design so error probability is very small
- Then each regeneration is like the first time!
- Analogy: copy an MP3 file
- Communications is possible over very long distances
- Digital systems vs. analog systems
  - Less power, longer distances, lower system cost
  - Monitoring, multiplexing, coding, encryption, protocols…

Digital Binary Signal



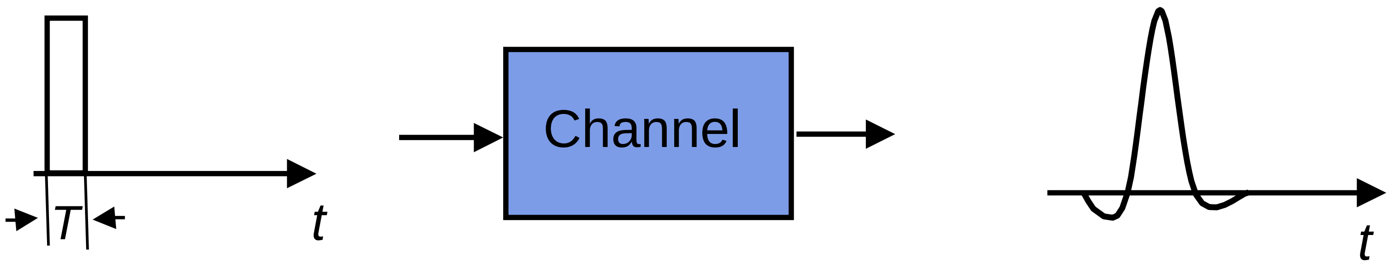
Bit rate = 1 bit / T seconds

For a given communications medium:

- How do we increase transmission speed?
- How do we achieve reliable communications?
- Are there limits to speed and reliability?

Pulse Transmission Rate

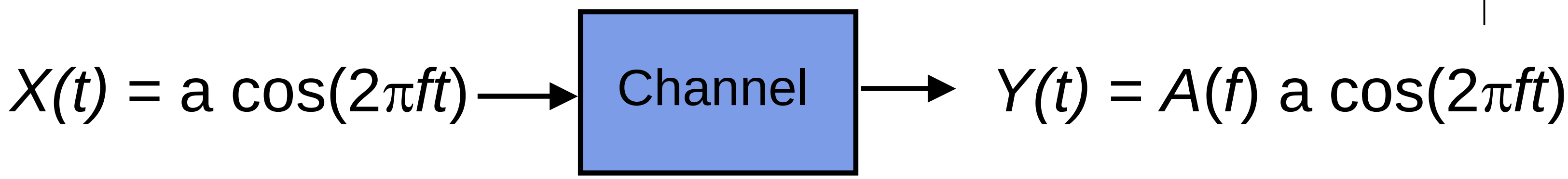
- Objective: Maximize pulse rate through a channel, that is, make *T* as small as possible



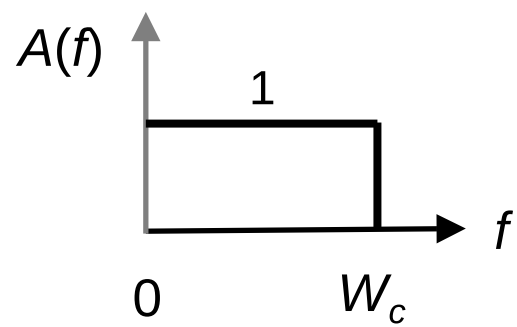
- If input is a narrow pulse, then typical output is a spread-out pulse with ringing
- Question: How frequently can these pulses be transmitted without interfering with each other?
- Answer: 2 x W_c pulses/second

where W_c is the bandwidth of the channel

Bandwidth of a Channel



- If input is sinusoid of frequency f, then
  - output is a sinusoid of same frequency f
  - Output is attenuated by an amount $A(f)$ that depends on f
  - $A(f) \approx 1$, then input signal passes readily
  - $A(f) \approx 0$, then input signal is blocked
- Bandwidth W_c is range of frequencies passed by channel



Ideal low-pass channel

Multilevel Pulse Transmission

- Assume channel of bandwidth W_c, and transmit 2 W_c pulses/sec (without interference)
- If pulses amplitudes are either $-A$ or $+A$, then each pulse conveys 1 bit, so

  Bit Rate = 1 bit/pulse x 2W_c pulses/sec = 2W_c bps
- If amplitudes are from $\{-A, -A/3, +A/3, +A\}$, then bit rate is 2 x 2W_c bps
- By going to $M = 2^m$ amplitude levels, we achieve

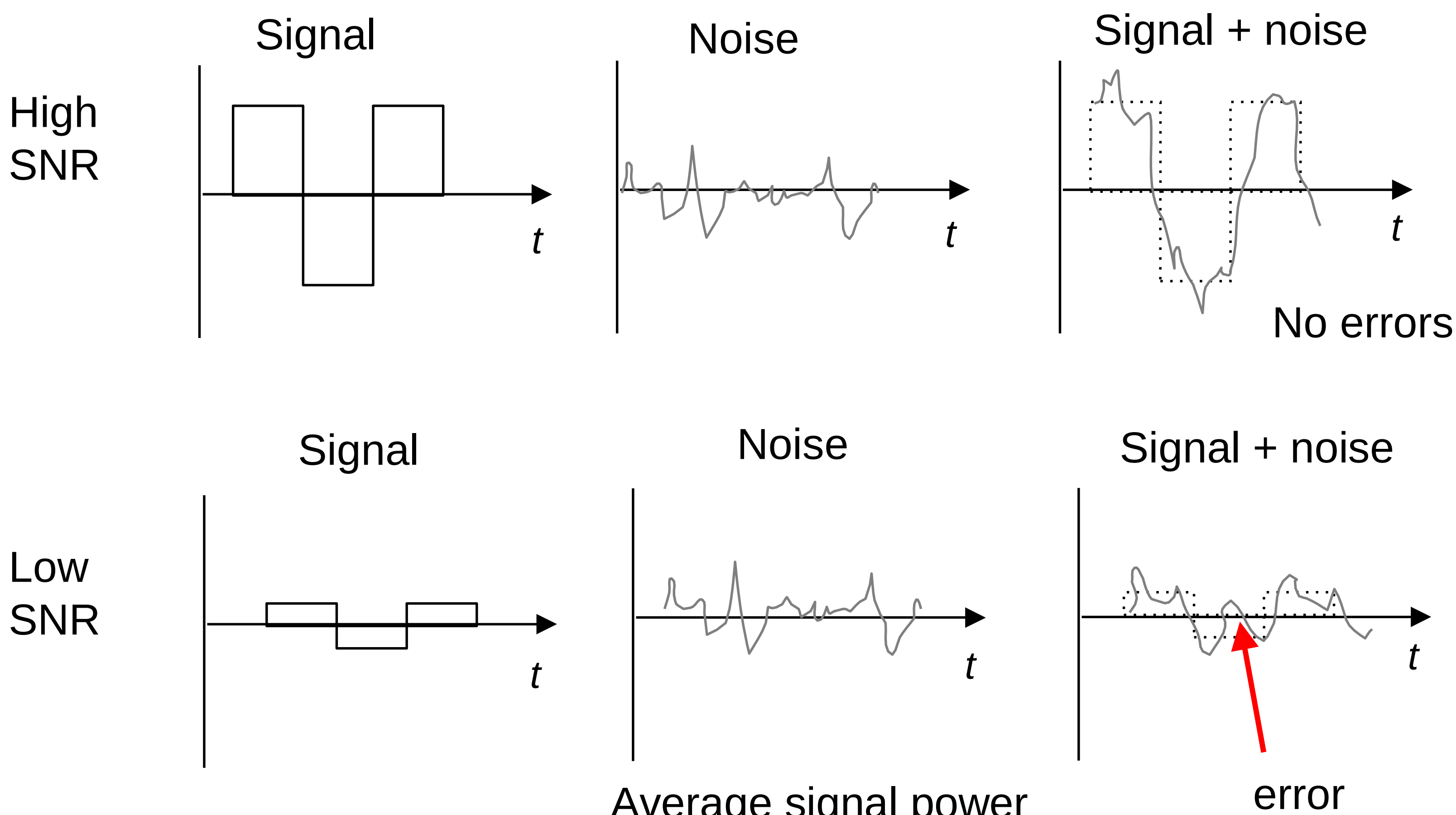
Bit Rate = m bits/pulse x 2W_c pulses/sec = 2mW_c bps

In the absence of noise, the bit rate can be increased without limit by increasing m

Noise & Reliable Communications

- All physical systems have noise
  - Electrons always vibrate at non-zero temperature
  - Motion of electrons induces noise
- Presence of noise limits accuracy of measurement of received signal amplitude
- Errors occur if signal separation is comparable to noise level
- Bit Error Rate (BER) increases with decreasing signal-to-noise ratio
- Noise places a limit on how many amplitude levels can be used in pulse transmission

Signal-to-Noise Ratio



$$\text{SNR} = \frac{\text{Average signal power}}{\text{Average noise power}}$$

$$\text{SNR (dB)} = 10 \log_{10} \text{SNR}$$

Shannon Channel Capacity

$$C = W_c \log_2(1 + SNR)\,\text{bps}$$

- Arbitrarily reliable communications is possible if the transmission rate $R < C$.
- If $R > C$, then arbitrarily reliable communications is not possible.
- "Arbitrarily reliable" means the BER can be made arbitrarily small through sufficiently complex coding.
- C can be used as a measure of how close a system design is to the best achievable performance.
- Bandwidth W_c & SNR determine C

Example

- Find the Shannon channel capacity for a telephone channel with $W_c = 3400$ Hz and $SNR = 10000$

$$C = 3400 \log_2 (1 + 10000)$$
$$= 3400 \log_{10} (10001)/\log_{10} 2 = 45200 \text{ bps}$$

Note that $SNR = 10000$ corresponds to

SNR (dB) $= 10 \log_{10}(10001) = 40$ dB

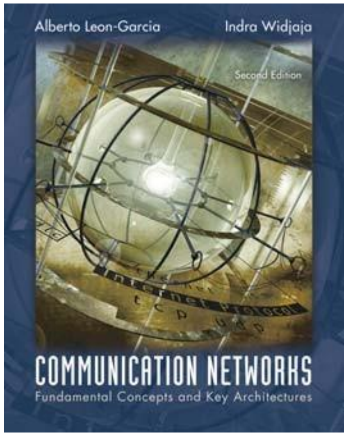
Bit Rates of Digital Transmission Systems

System	Bit Rate	Observations
Telephone twisted pair	33.6-56 kbps	4 kHz telephone channel
Ethernet twisted pair	10 Mbps, 100 Mbps	100 meters of unshielded twisted copper wire pair
Cable modem	500 kbps-4 Mbps	Shared CATV return channel
ADSL twisted pair	64-640 kbps in, 1.536-6.144 Mbps out	Coexists with analog telephone signal
2.4 GHz radio	2-11 Mbps	IEEE 802.11 wireless LAN
28 GHz radio	1.5-45 Mbps	5 km multipoint radio
Optical fiber	2.5-10 Gbps	1 wavelength
Optical fiber	>1600 Gbps	Many wavelengths

Examples of Channels

Channel	Bandwidth	Bit Rates
Telephone voice channel	3 kHz	33 kbps
Copper pair	1 MHz	1-6 Mbps
Coaxial cable	500 MHz (6 MHz channels)	30 Mbps/ channel
5 GHz radio (IEEE 802.11)	300 MHz (11 channels)	54 Mbps / channel
Optical fiber	Many TeraHertz	40 Gbps / wavelength

Chapter 3
Digital Transmission Fundamentals



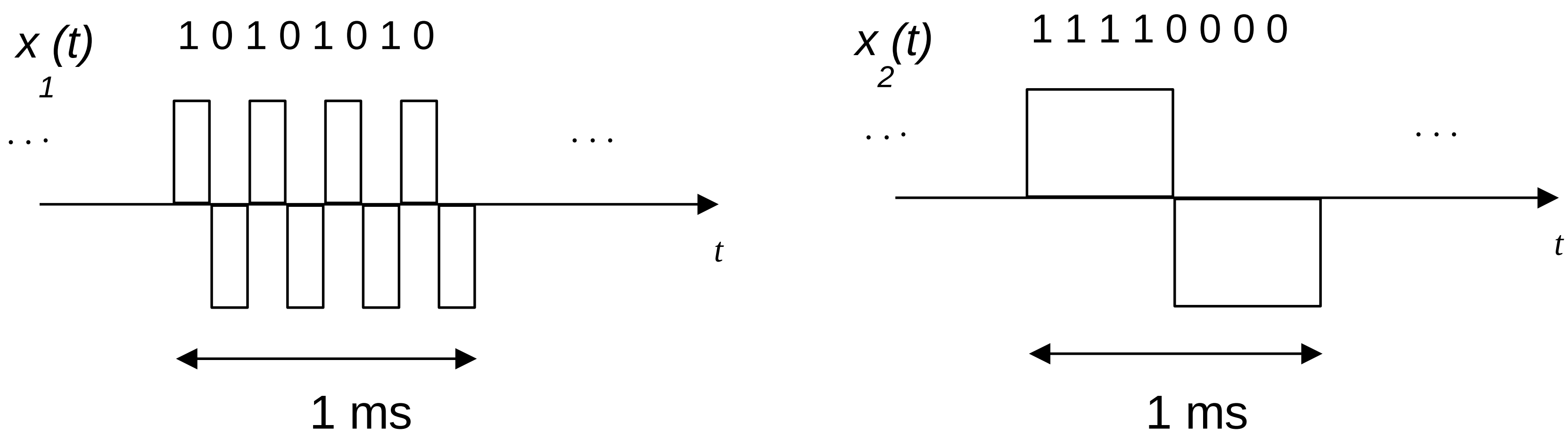
Digital Representation of Analog Signals

Digitization of Analog Signals

1. Sampling: obtain samples of *x(t)* at uniformly spaced time intervals
2. Quantization: map each sample into an approximation value of finite precision
   - Pulse Code Modulation: telephone speech
   - CD audio
3. Compression: to lower bit rate further, apply additional compression method
   - Differential coding: cellular telephone speech
   - Subband coding: MP3 audio
   - Compression discussed in Chapter 12

Sampling Rate and Bandwidth

- A signal that varies faster needs to be sampled more frequently
- *Bandwidth* measures how fast a signal varies



- What is the bandwidth of a signal?
- How is bandwidth related to sampling rate?

Periodic Signals

- A periodic signal with period T can be represented as sum of sinusoids using Fourier Series:

$$x(t) = a_0 + a_1\cos(2\pi f_0 t + \phi_1) + a_2\cos(2\pi 2 f_0 t + \phi_2) + \ldots$$
$$+ a_k\cos(2\pi k f_0 t + \phi_k) + \ldots$$

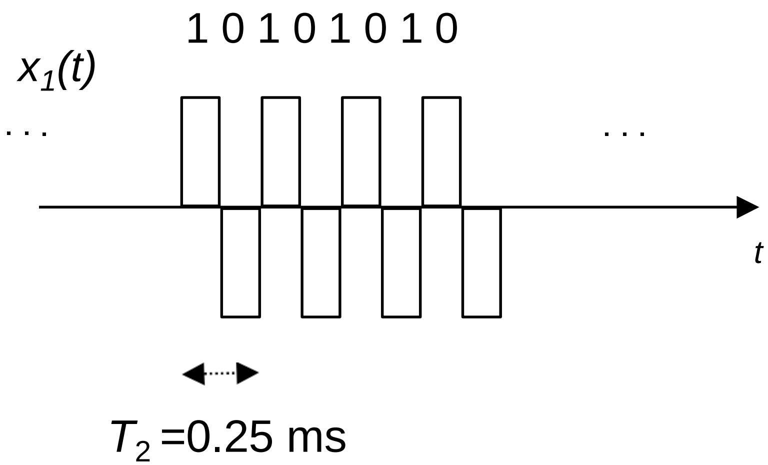
"DC" long-term average

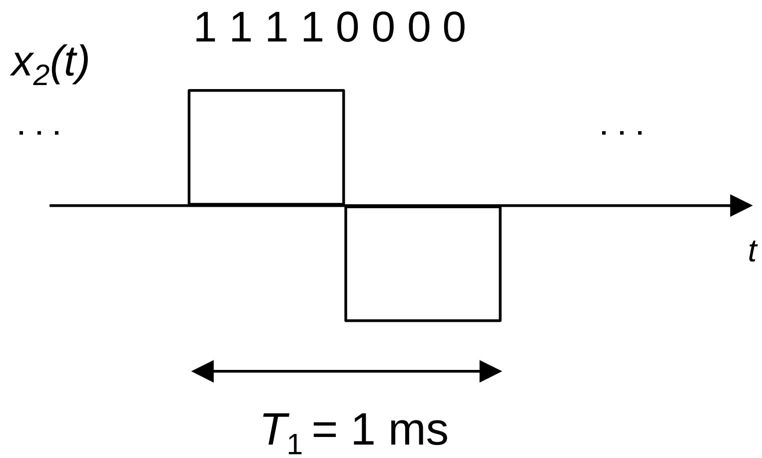
fundamental frequency $f_0=1/T$ first harmonic

kth harmonic

- $|a_k|$ determines amount of power in kth harmonic
- Amplitude specturm $|a_0|, |a_1|, |a_2|, \ldots$

Example Fourier Series





$$x_1(t) = 0 + \frac{4}{\pi}\cos(2\pi 4000t) + \frac{4}{3\pi}\cos(2\pi 3(4000)t) + \frac{4}{5\pi}\cos(2\pi 5(4000)t) + \ldots$$

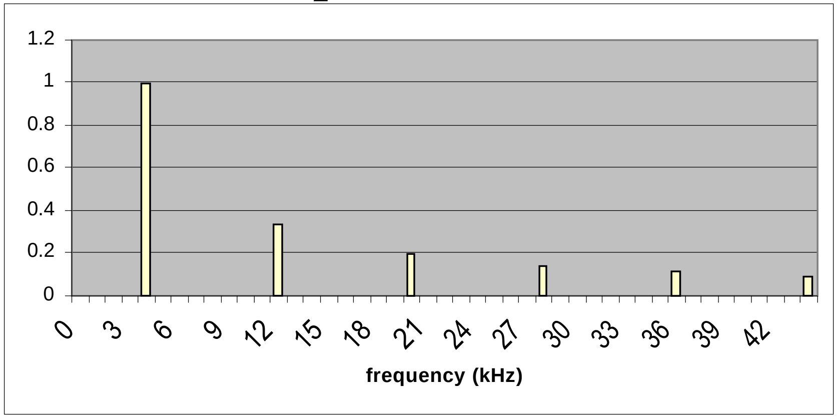
$$x_2(t) = 0 + \frac{4}{\pi}\cos(2\pi 1000t) + \frac{4}{3\pi}\cos(2\pi 3(1000)t) + \frac{4}{5\pi}\cos(2\pi 5(1000)t) + \ldots$$

Only odd harmonics have power

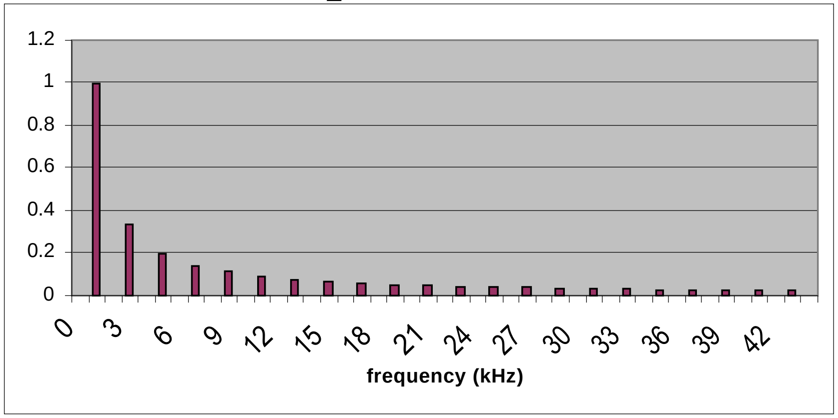
Spectra & Bandwidth

- Spectrum of a signal: magnitude of amplitudes as a function of frequency
- $x_1(t)$ varies faster in time & has more high frequency content than $x_2(t)$
- Bandwidth W_s is defined as range of frequencies where a signal has non-negligible power, e.g. range of band that contains 99% of total signal power

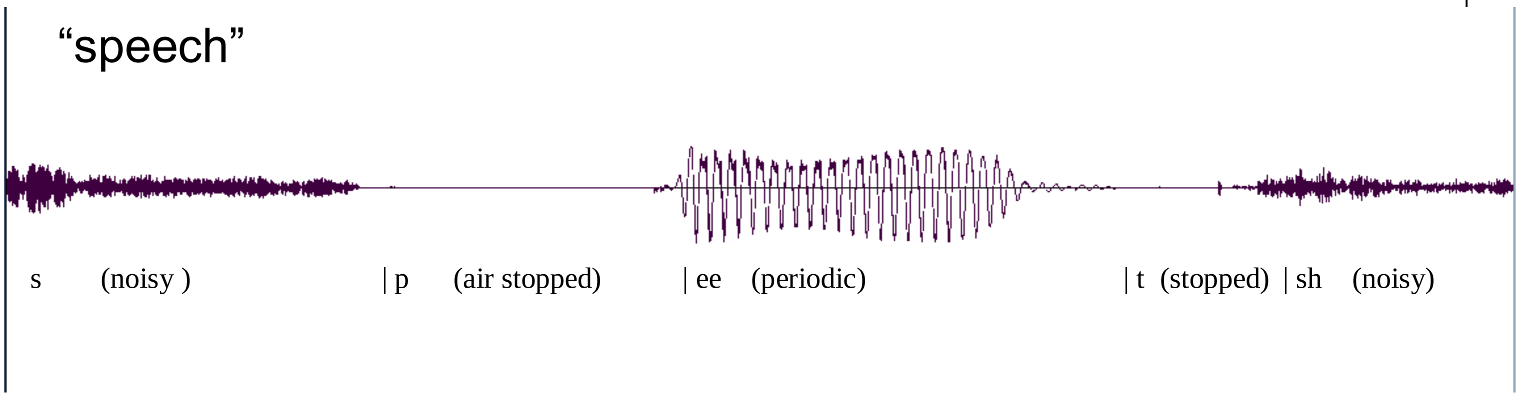
Spectrum of $x_1(t)$



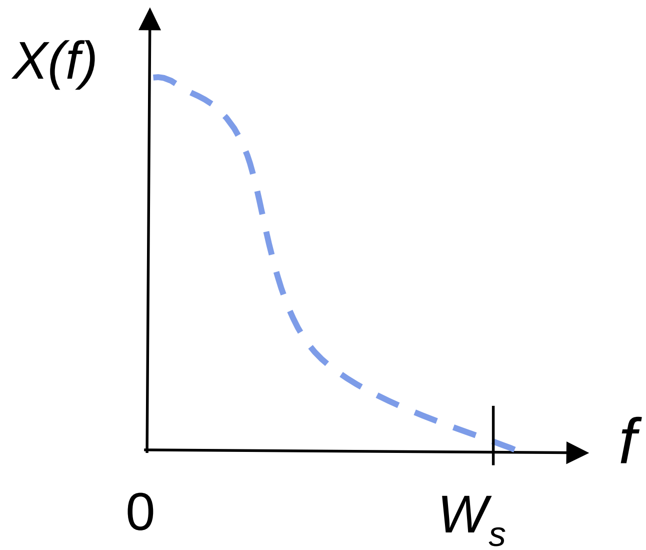
Spectrum of $x_2(t)$



Bandwidth of General Signals

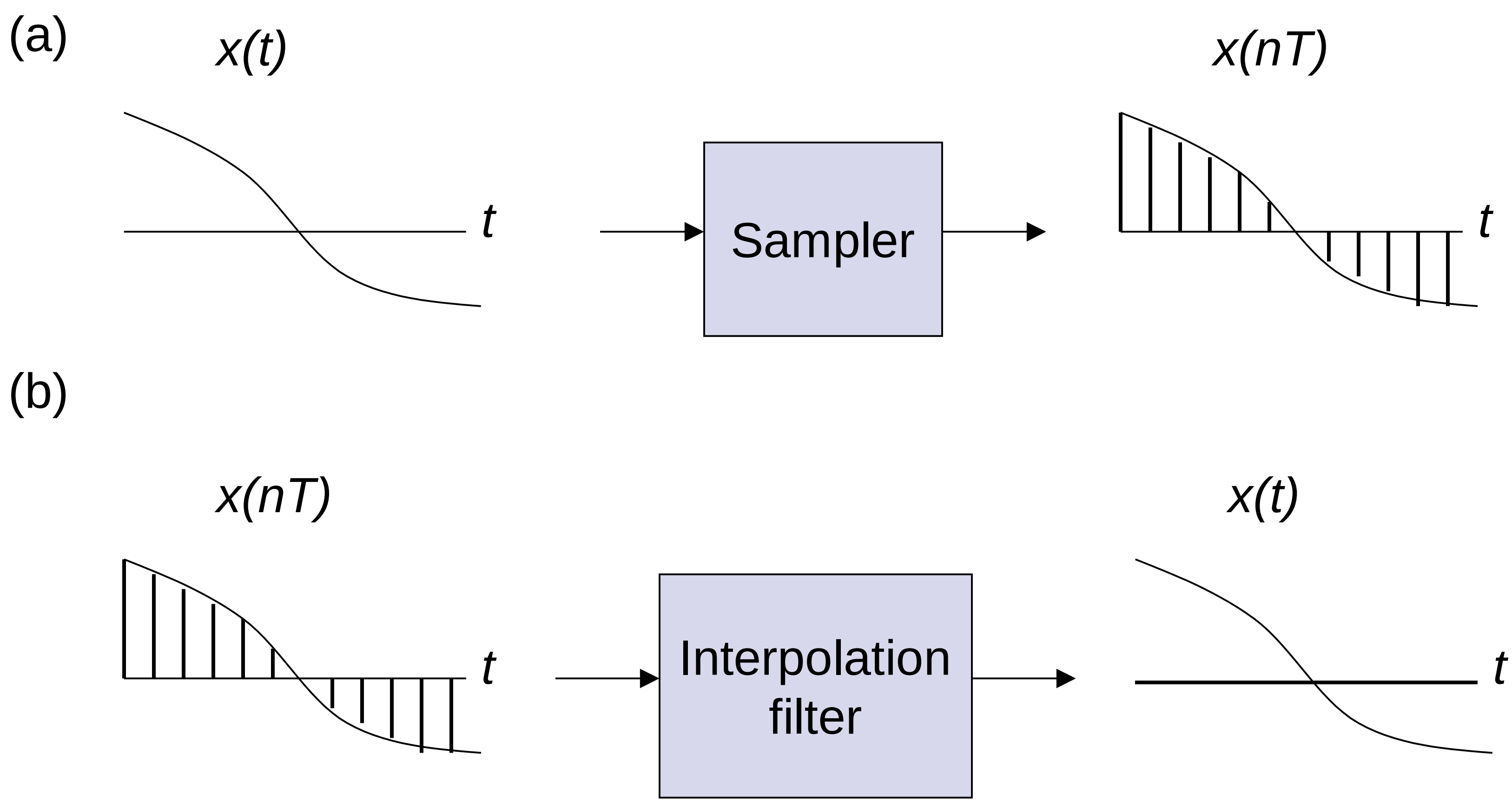


- Not all signals are periodic
  - E.g. voice signals varies according to sound
  - Vowels are periodic, “s” is noiselike
- Spectrum of long-term signal
  - Averages over many sounds, many speakers
  - Involves Fourier transform
- Telephone speech: 4 kHz
- CD Audio: 22 kHz

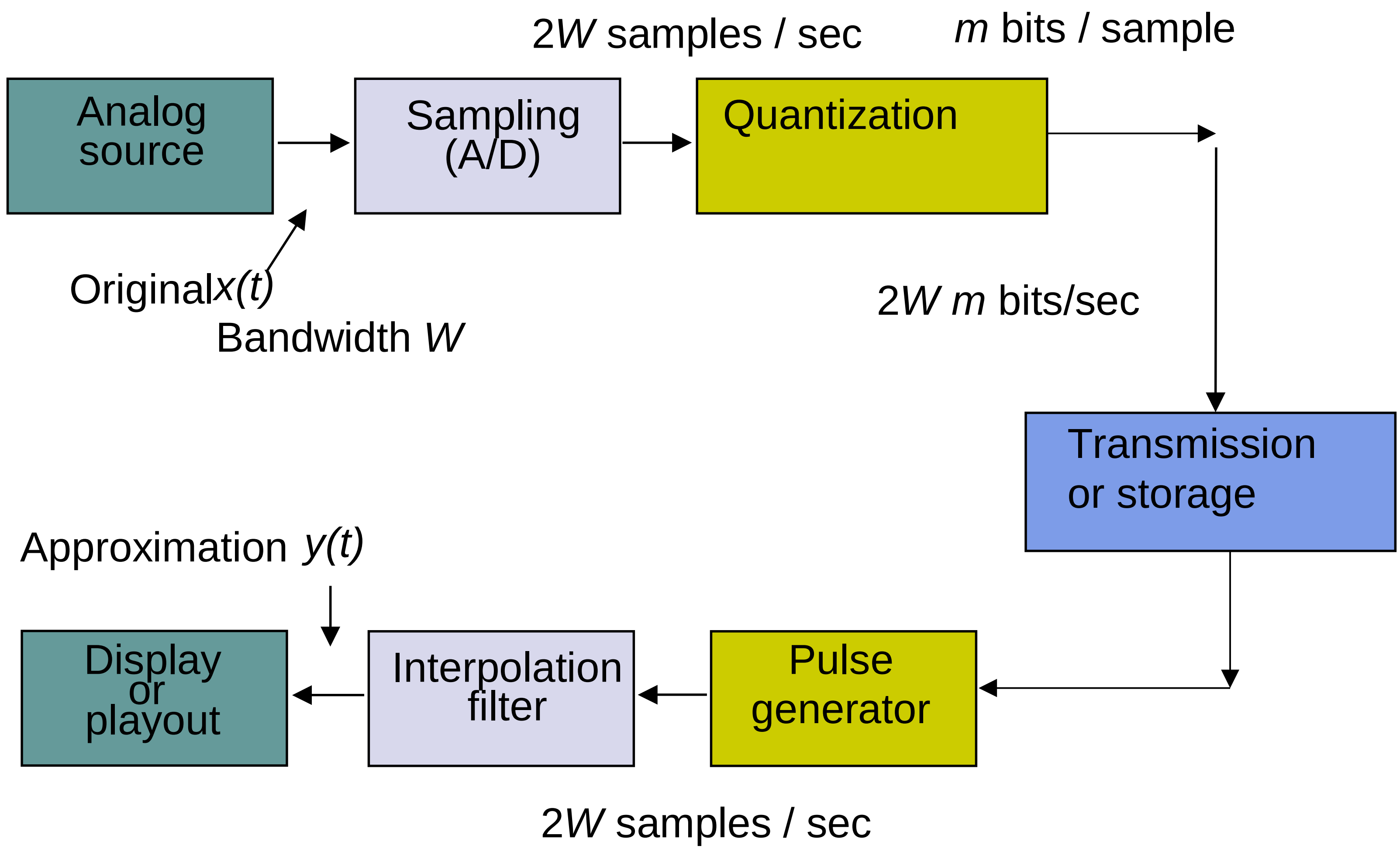


Sampling Theorem

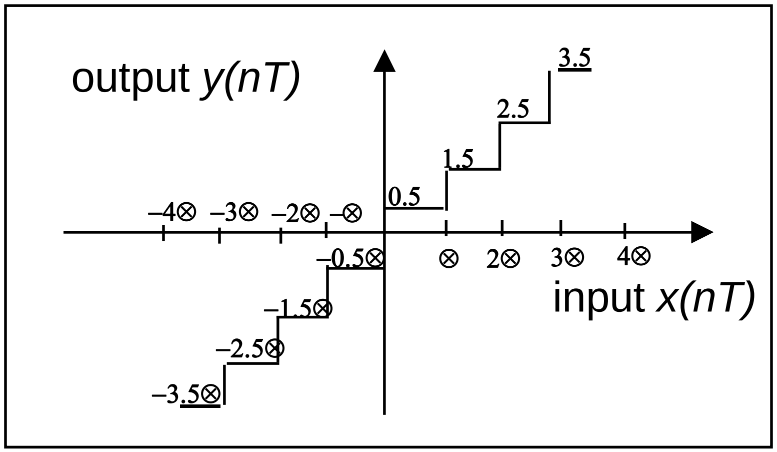
Nyquist: Perfect reconstruction if sampling rate $1/T > 2W_s$



Digital Transmission of Analog Information

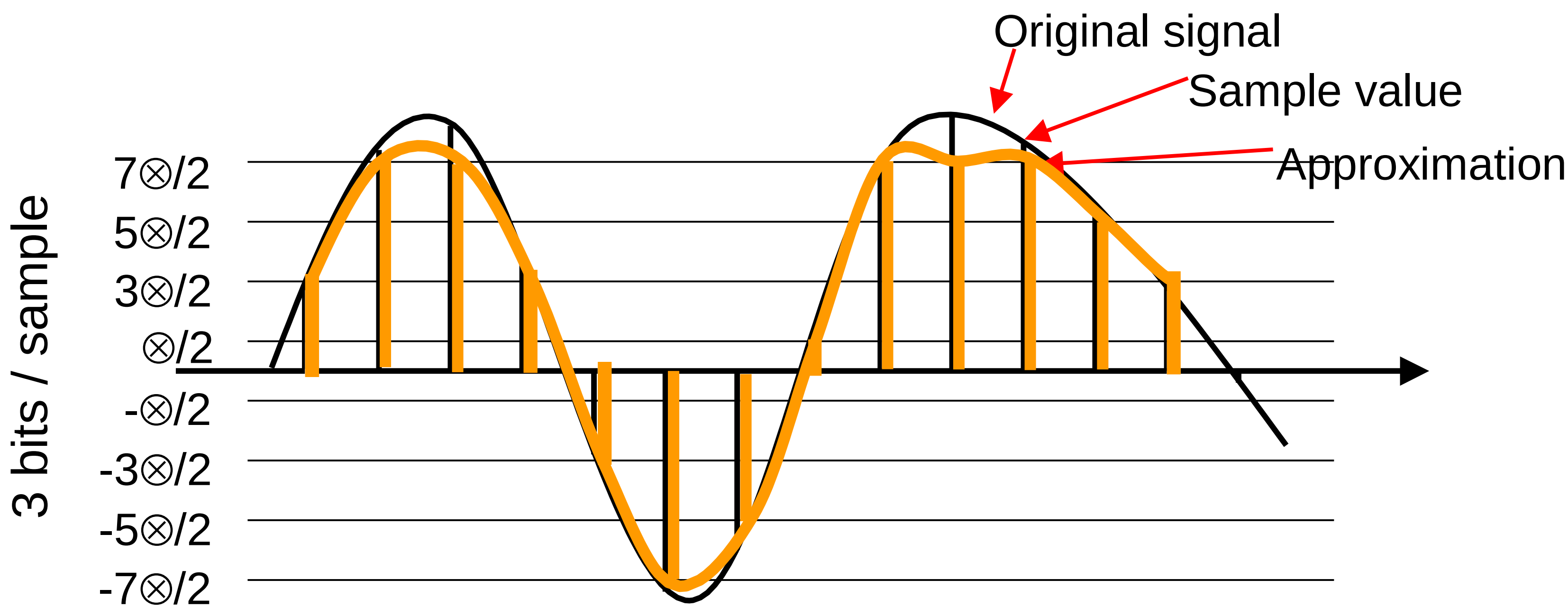


Quantization of Analog Samples



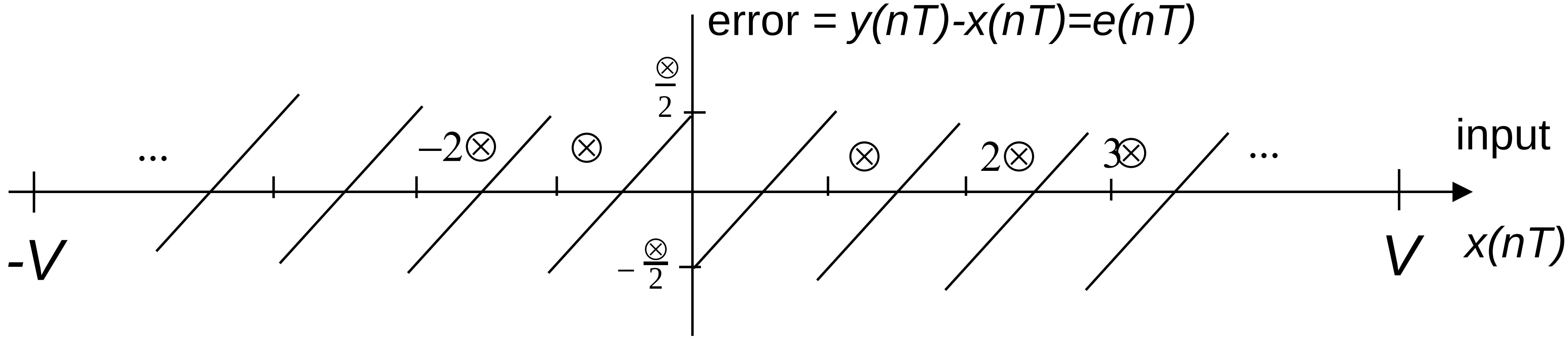
Quantizer maps input into closest of 2^m representation values

Quantization error:
"noise" = $x(nT) – y(nT)$



Quantizer Performance

$M = 2^m$ levels, Dynamic range($-V$, V) $\Delta = 2V/M$



If the number of levels M is large, then the error is approximately uniformly distributed between (-Δ/2, Δ2)

Average Noise Power = Mean Square Error:

$$\sigma_e^2 = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x^2 \frac{1}{\Delta}\, dx = \frac{\Delta^2}{12}$$

Quantizer Performance

Figure of Merit:

Signal-to-Noise Ratio = Avg signal power / Avg noise power
Let σ_x^2 be the signal power, then

$$SNR = \frac{\sigma_x^2}{\otimes^2/12} = \frac{12\sigma_x^2}{4V^2/M^2} = 3\left(\frac{\sigma_x}{V}\right)^2 M^2 = 3\left(\frac{\sigma_x}{V}\right)^2 2^{2m}$$

The ratio $V/\sigma_x \approx 4$

The SNR is usually stated in decibels:

SNR db = 10 $\log_{10} \sigma_x^2/\sigma_e^{2<}$ = 6 + 10 $\log_{10}$ $3\sigma_x^2/V^{2<}$

SNR db = 6*m* - 7.27 dB for $V/\sigma_x = 4$.

Example: Telephone Speech

W = 4KHz, so Nyquist sampling theorem

$\Rightarrow$ 2W = 8000 samples/second

Suppose error requirement $=$ 1% error

SNR = 10 log(1/.01)^2 = 40 dB

Assume $V/\sigma_x = 4$, then

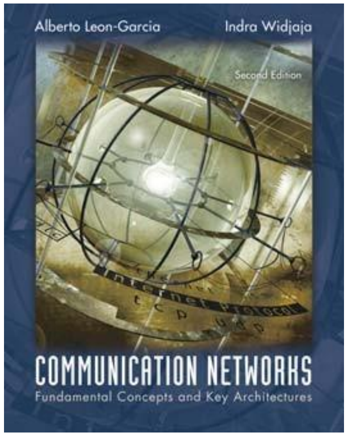
40 dB = 6m – 7

$\Rightarrow$ m = 8 bits/sample

PCM ("Pulse Code Modulation") Telephone Speech:

Bit rate= 8000 x 8 bits/sec= 64 kbps

Chapter 3
Digital Transmission Fundamentals



Characterization of Communication Channels

Communications Channels

- A *physical medium* is an inherent part of a communications system
  - Copper wires, radio medium, or optical fiber
- Communications system includes electronic or optical devices that are part of the path followed by a signal
  - Equalizers, amplifiers, signal conditioners
- By *communication channel* we refer to the combined end-to-end physical medium and attached devices
- Sometimes we use the term *filter* to refer to a channel especially in the context of a specific mathematical model for the channel

How good is a channel?

- Performance: What is the maximum reliable transmission speed?
  - Speed: Bit rate, *R* bps
  - Reliability: Bit error rate, BER=10^{-k}
  - Focus of this section
- Cost: What is the cost of alternatives at a given level of performance?
  - Wired vs. wireless?
  - Electronic vs. optical?
  - Standard A vs. standard B?

Communications Channel

Signal Bandwidth

- In order to transfer data faster, a signal has to vary more quickly.

Channel Bandwidth

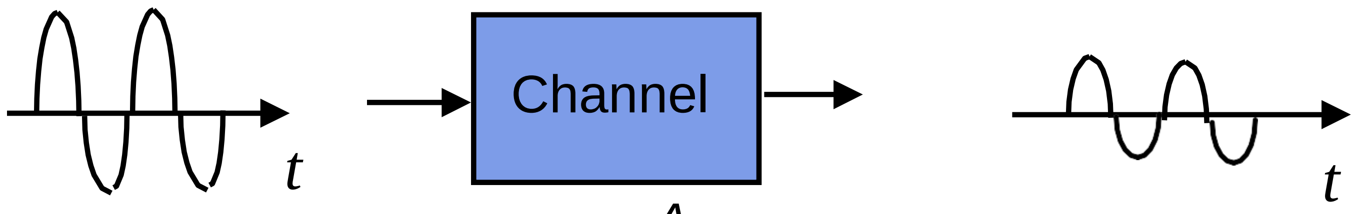
- A channel or medium has an inherent limit on how fast the signals it passes can vary
- *Limits how tightly input pulses can be packed*

Transmission Impairments

- Signal attenuation
- Signal distortion
- Spurious noise
- Interference from other signals
- *Limits accuracy of measurements on received signal*

Frequency Domain Channel Characterization

$x(t) = A_{in} \cos 2\pi f t$ $\qquad$ $y(t) = A_{out} \cos (2\pi f t + \varphi(f))$



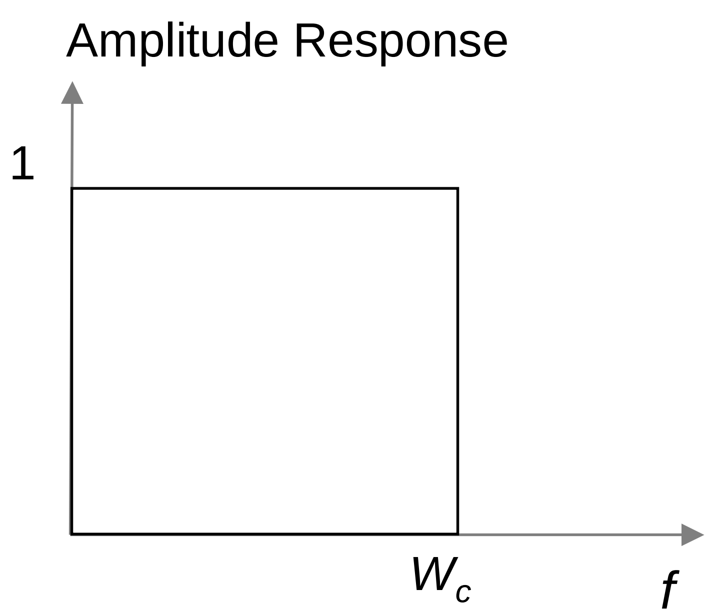
$$A(f) = \frac{A_{out}}{A_{in}}$$

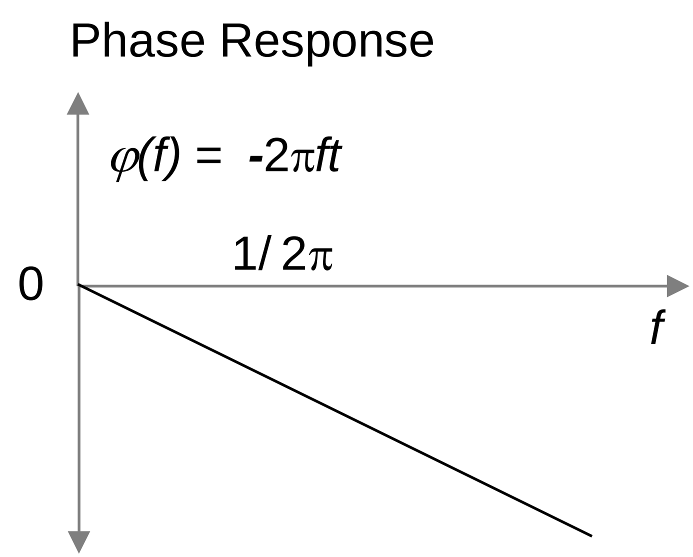
- Apply sinusoidal input at frequency *f*
  - Output is sinusoid at same frequency, but attenuated & phase-shifted
  - Measure amplitude of output sinusoid (of same frequency *f*)
  - Calculate amplitude response
    - $A(f)$ = ratio of output amplitude to input amplitude
  - If $A(f) \approx 1$, then input signal passes readily
  - If $A(f) \approx 0$, then input signal is blocked
- Bandwidth W_c is range of frequencies passed by channel

Ideal Low-Pass Filter

- Ideal filter: all sinusoids with frequency $f<W_c$ are passed without attenuation and delayed by τ seconds; sinusoids at other frequencies are blocked

$$y(t)=A_{in}\cos(2\pi ft - 2\pi f\tau) = A_{in}\cos(2\pi f(t-\tau)) = x(t-\tau)$$



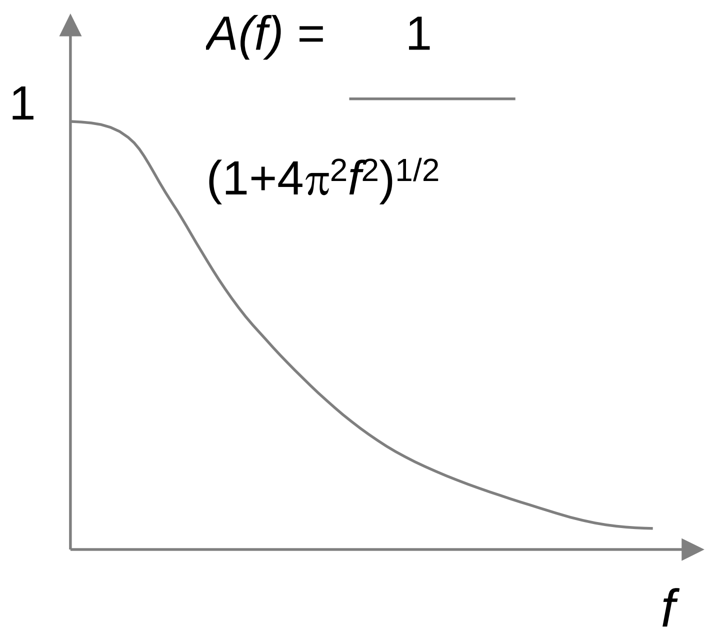


Example: Low-Pass Filter

- Simplest non-ideal circuit that provides low-pass filtering
  - Inputs at different frequencies are attenuated by different amounts
  - Inputs at different frequencies are delayed by different amounts

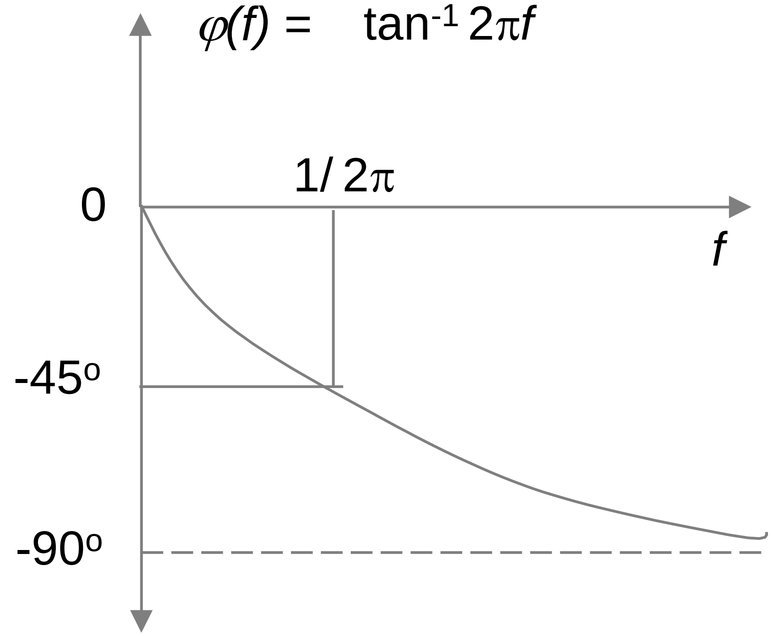
Amplitude Response

$$A(f) = \frac{1}{(1+4\pi^2 f^2)^{1/2}}$$

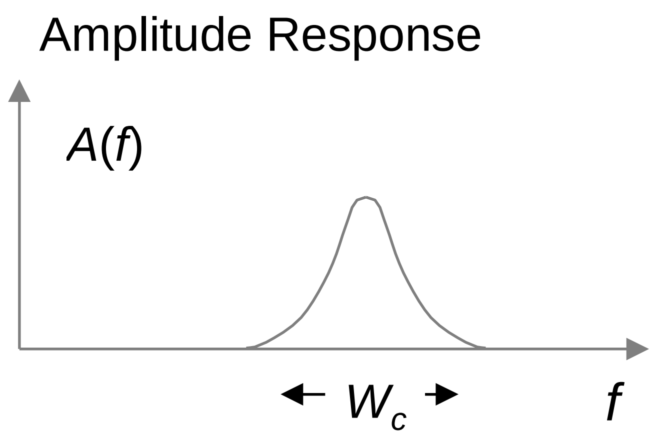


Phase Response

$$\varphi(f) = \tan^{-1} 2\pi f$$



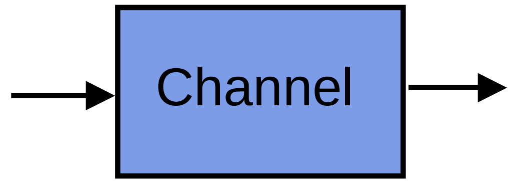
Example: Bandpass Channel



- Some channels pass signals within a band that excludes low frequencies
  - Telephone modems, radio systems, …
- *Channel bandwidth* is the width of the frequency band that passes non-negligible signal power

Channel Distortion

$x(t) = \sum a_k \cos(2\pi f_k t + \theta_k)$



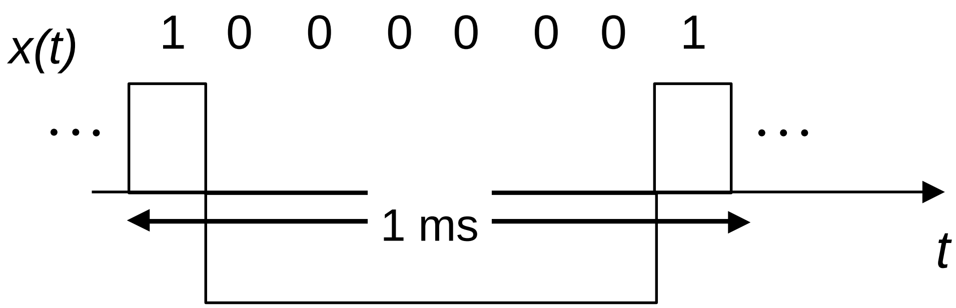
$y(t)$

- Let *x(t)* corresponds to a digital signal bearing data information
- How well does *y(t)* follow *x(t)*?

$$y(t) = \sum A(f_k)\, a_k \cos(2\pi f_k t + \theta_k + \Phi(f_k))$$

- Channel has two effects:
  - If amplitude response is not flat, then different frequency components of *x(t)* will be transferred by different amounts
  - If phase response is not flat, then different frequency components of *x(t)* will be delayed by different amounts
- In either case, the shape of *x(t)* is altered

Example: Amplitude Distortion

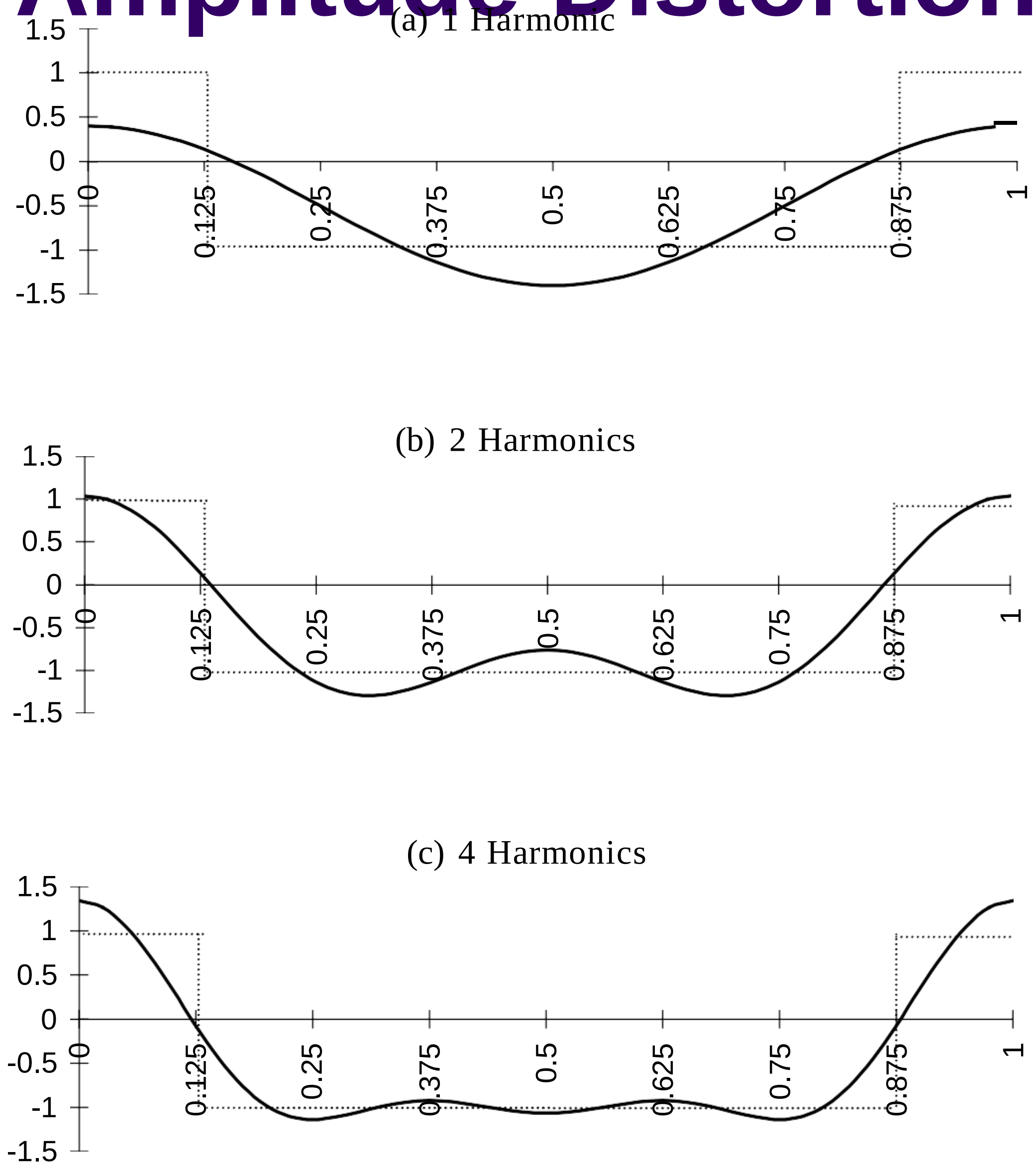


- Let *x(t)* input to ideal lowpass filter that has zero delay and W_c = 1.5 kHz, 2.5 kHz, or 4.5 kHz

$$x(t) = -0.5 + \frac{4}{\pi}\sin\left(\frac{\pi}{4}\right)\cos(2\pi 1000t) + \frac{4}{\pi}\sin\left(\frac{2\pi}{4}\right)\cos(2\pi 2000t) + \frac{4}{\pi}\sin\left(\frac{3\pi}{4}\right)\cos(2\pi 3000t) + \ldots$$

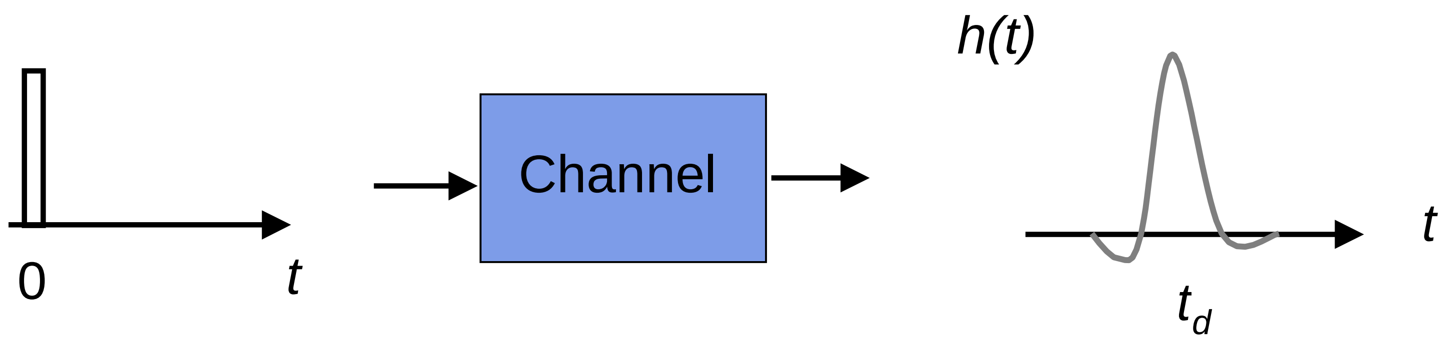
- W_c = 1.5 kHz passes only the first two terms
- W_c = 2.5 kHz passes the first three terms
- W_c = 4.5 kHz passes the first five terms

Amplitude Distortion



- As the channel bandwidth increases, the output of the channel resembles the input more closely

Time-domain Characterization

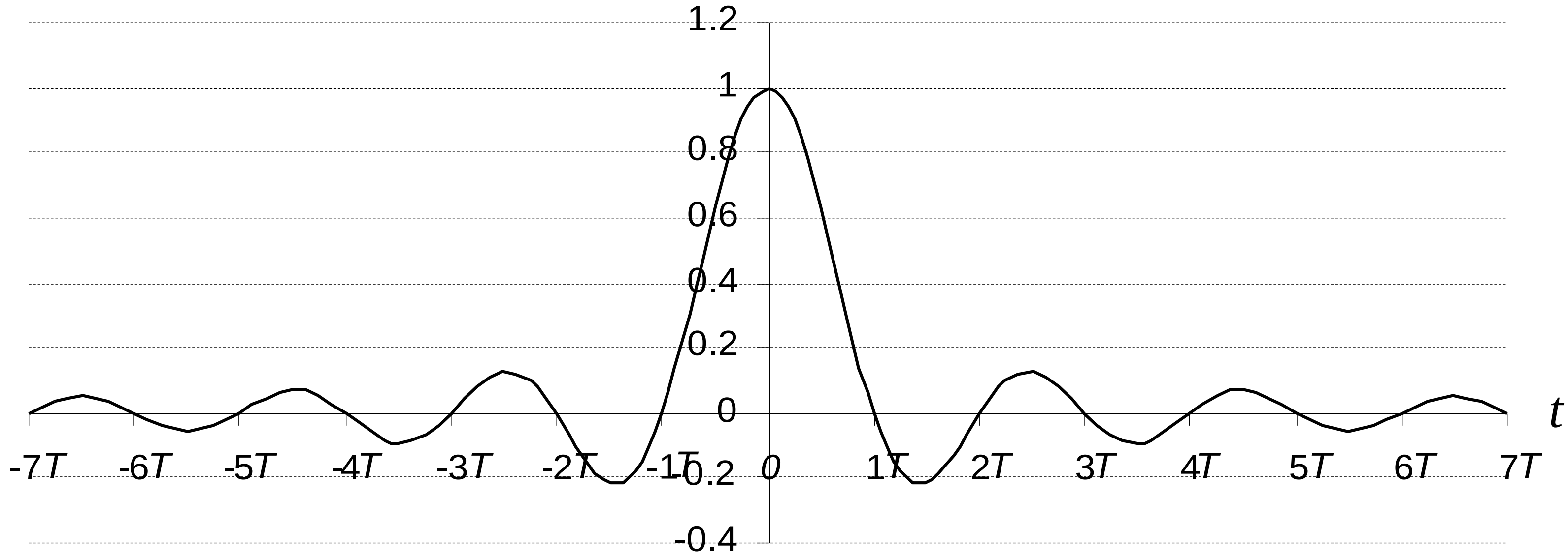


- Time-domain characterization of a channel requires finding the *impulse response h(t)*
- Apply a very narrow pulse to a channel and observe the channel output
  - *h(t)* typically a delayed pulse with ringing
- Interested in system designs with *h(t)* that can be packed closely without interfering with each other

Nyquist Pulse with Zero Intersymbol Interference

- For channel with ideal lowpass amplitude response of bandwidth W_c, the impulse response is a Nyquist pulse $h(t)=s(t-\tau)$, where $T = 1/2\,W_c$, and

$$s(t) = \sin(2\pi W_c\, t) / 2\pi W_c t$$



- $s(t)$ has zero crossings at $t = kT$, $k = \pm 1, \pm 2, \ldots$
- Pulses can be packed every T seconds with *zero interference*

Example of composite waveform

Three Nyquist pulses shown separately

- + *s(t)*
- + *s(t-T)*
- - *s(t-2T)*

Composite waveform

$r(t) = s(t)+s(t-T)-s(t-2T)$

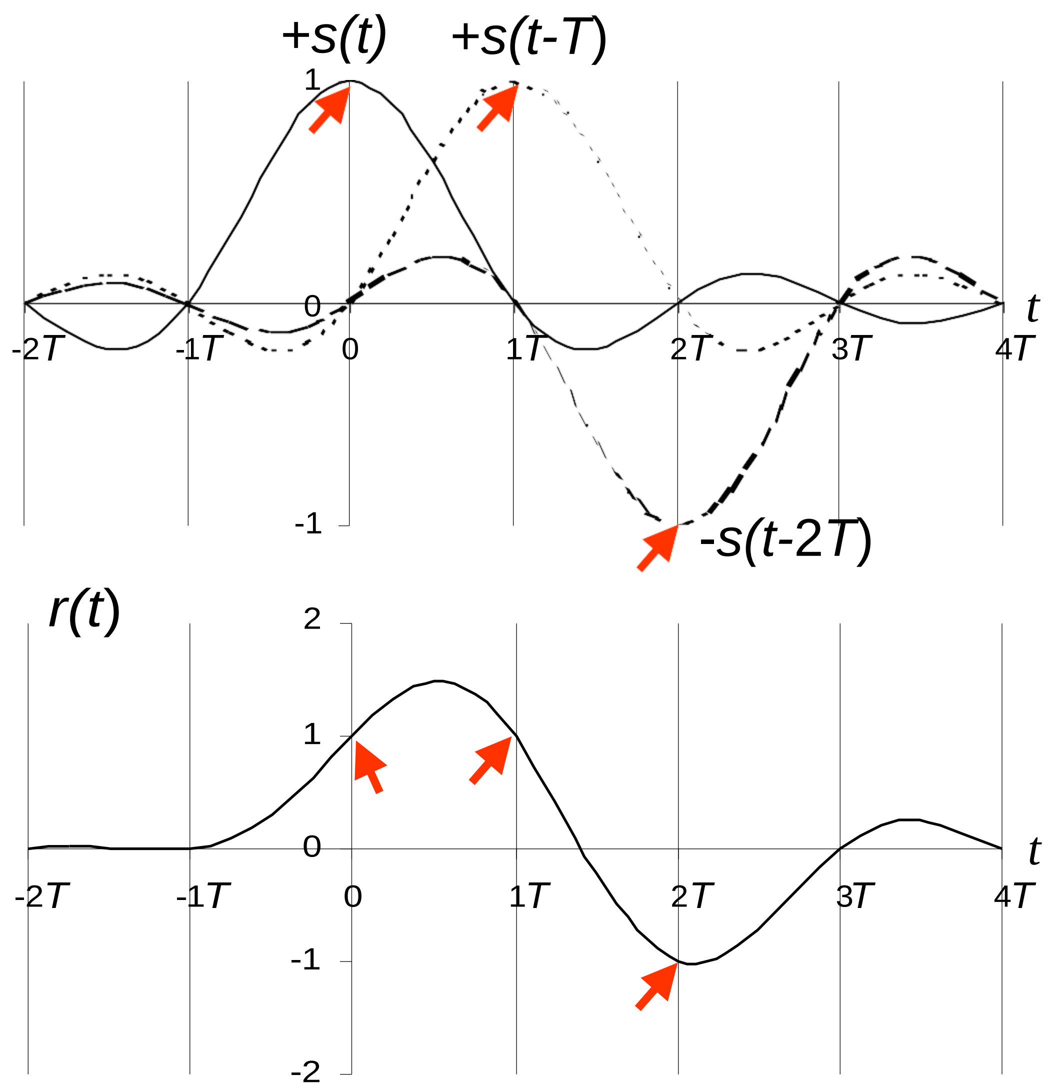
Samples at *kT*

$r(0)=s(0)+s(-T)-s(-2T)=+1$

$r(T)=s(T)+s(0)-s(-T)=+1$

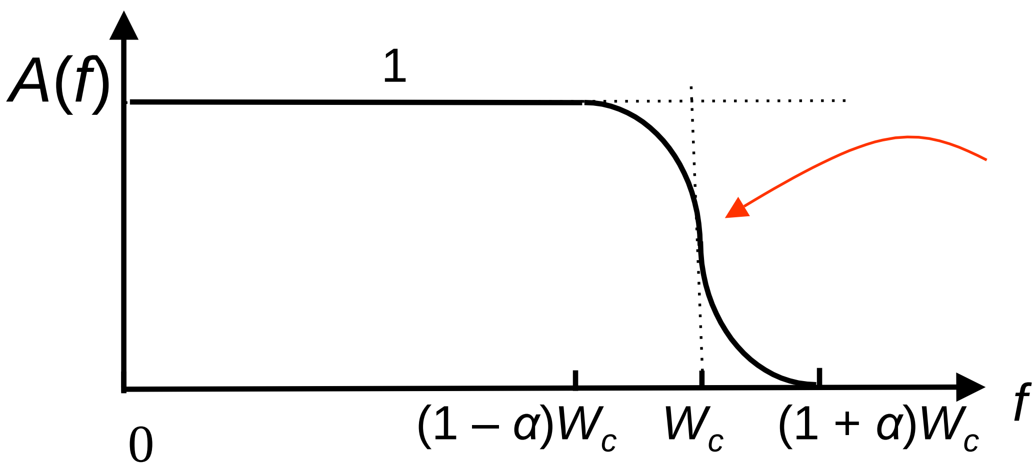
$r(2T)=s(2T)+s(T)-s(0)=-1$

Zero ISI at sampling times *kT*



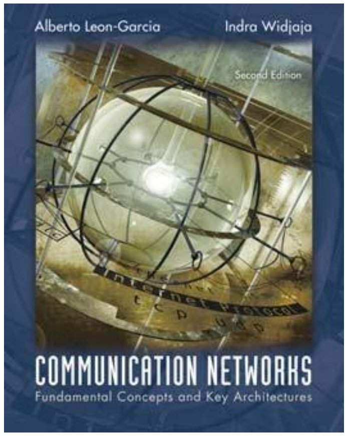
Nyquist pulse shapes

- ***If channel is ideal low pass with W_c, then pulses maximum rate pulses can be transmitted without ISI is* $T = 1/2W_c$ *sec.***
- *s(t)* is one example of class of Nyquist pulses with zero ISI
  - Problem: sidelobes in *s(t)* decay as $1/t$ which add up quickly when there are slight errors in timing
- Raised cosine pulse below has zero ISI
  - Requires slightly more bandwidth than W_c
  - Sidelobes decay as $1/t^3$, so more robust to timing errors



$$\frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\pi \alpha t/T)}{1-(2\alpha t/T)^2}$$

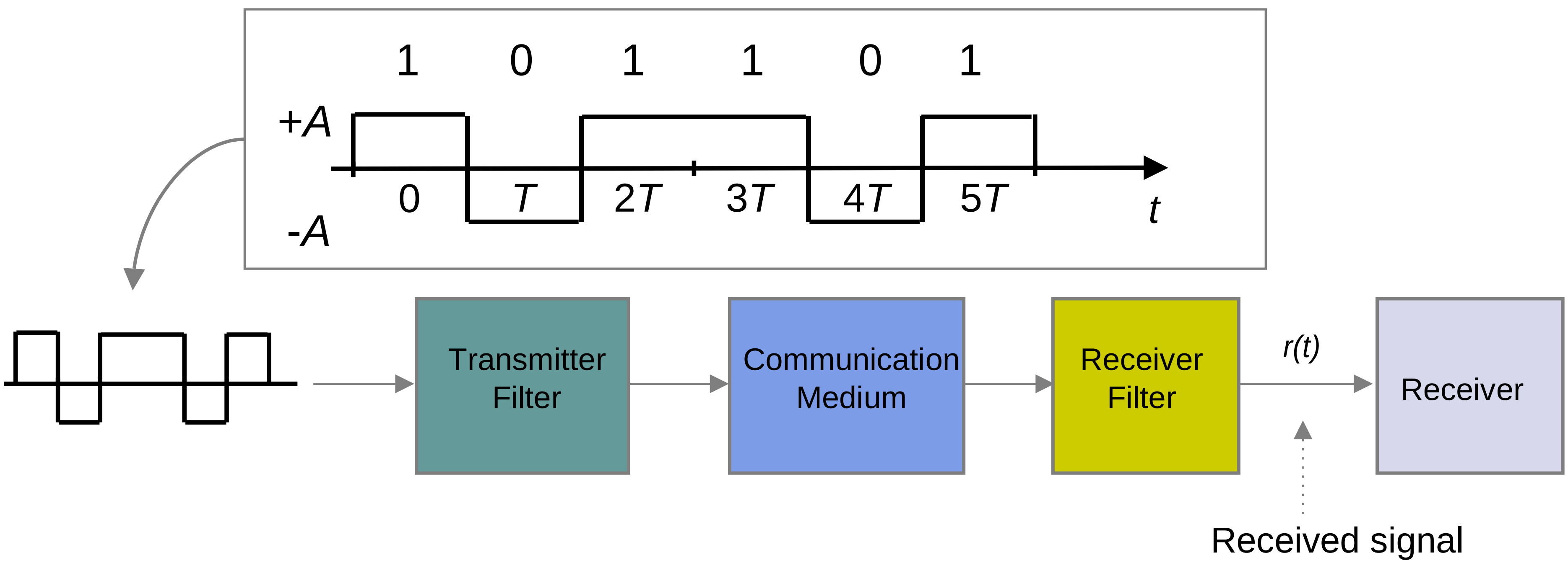
Chapter 3
Digital Transmission Fundamentals



Fundamental Limits in Digital Transmission

Signaling with Nyquist Pulses

- *p(t)* pulse at receiver in response to a single input pulse (takes into account pulse shape at input, transmitter & receiver filters, and communications medium)
- *r(t)* waveform that appears in response to sequence of pulses
- If *s(t)* is a Nyquist pulse, then *r(t)* has zero intersymbol interference (ISI) when sampled at multiples of *T*



Multilevel Signaling

- Nyquist pulses achieve the maximum signalling rate with zero ISI,

  $2W_c$ pulses per second or

  $2W_c$ pulses / W_c Hz = 2 pulses / Hz

- With two signal levels, each pulse carries one bit of information

  Bit rate = $2W_c$ bits/second

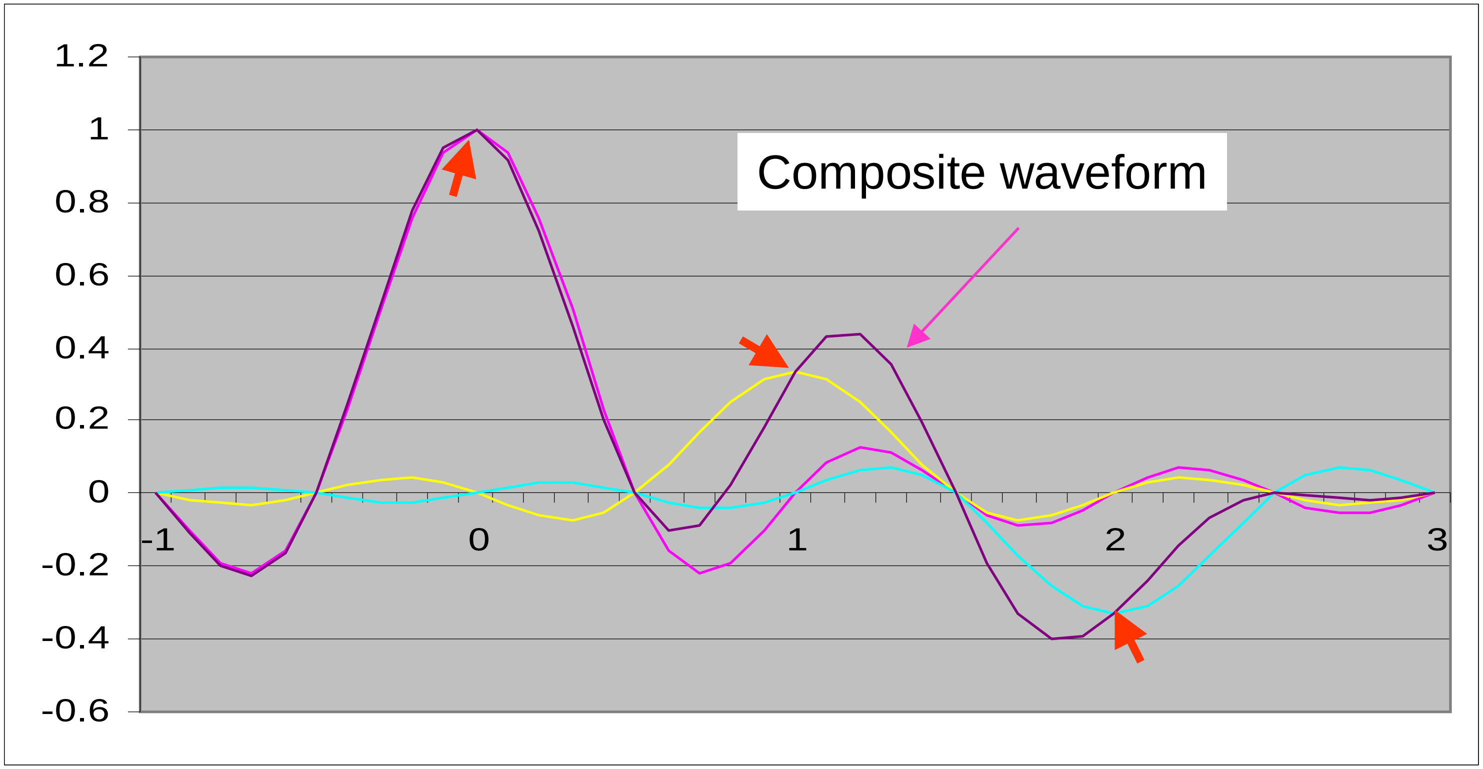
- With $M = 2^m$ signal levels, each pulse carries m bits

  Bit rate = $2W_c$ pulses/sec. * m bits/pulse = $2W_c m$ bps

- *Bit rate can be increased by increasing number of levels*
- *r(t) includes additive noise, that limits number of levels that can be used reliably.*

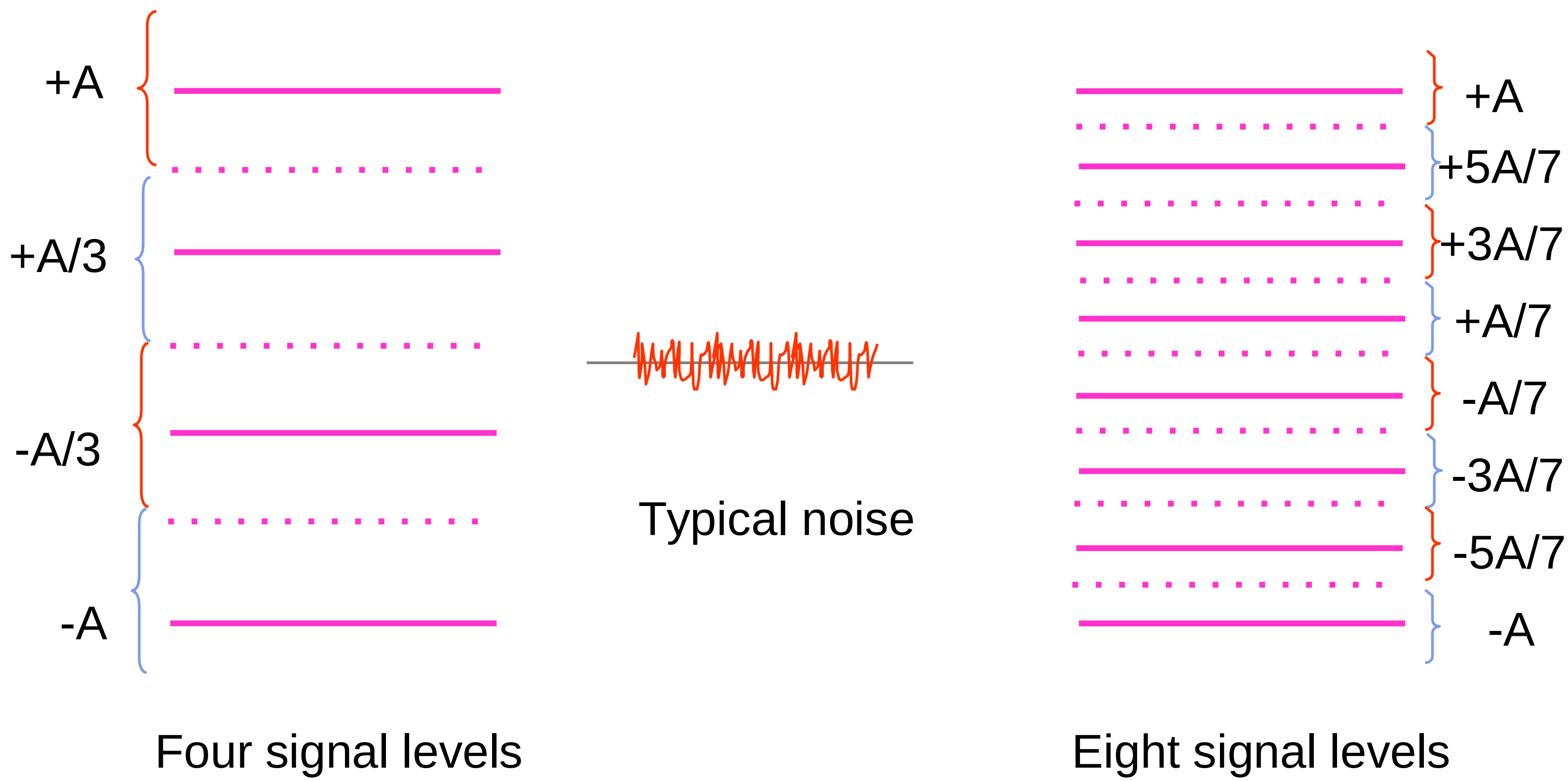
Example of Multilevel Signaling

- Four levels {-1, -1/3, 1/3, +1} for {00,01,10,11}
- Waveform for 11,10,01 sends +1, +1/3, -1/3
- Zero ISI at sampling instants



Noise Limits Accuracy

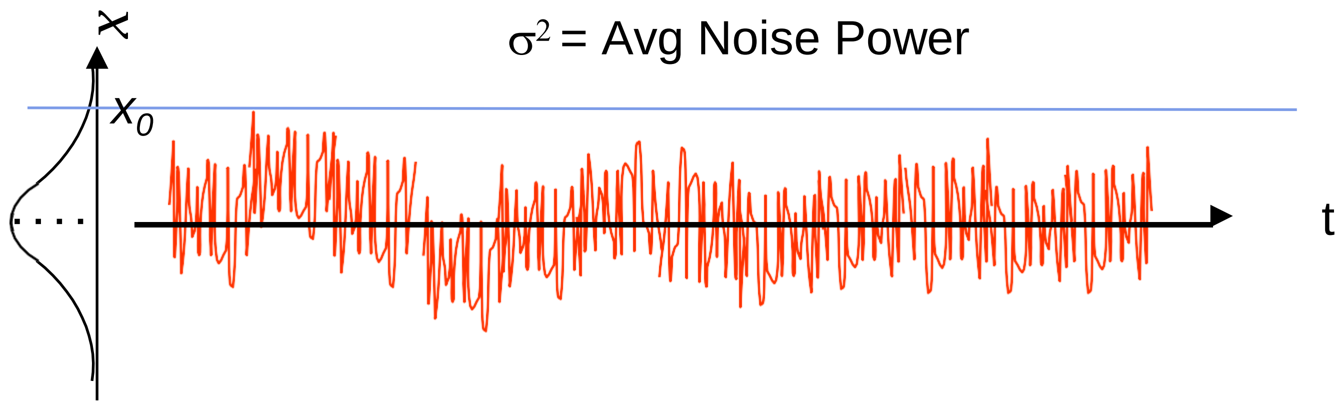
- Receiver makes decision based on transmitted pulse level + noise
- Error rate depends on relative value of noise amplitude and spacing between signal levels
- Large (positive or negative) noise values can cause wrong decision
- Noise level below impacts 8-level signaling more than 4-level signaling



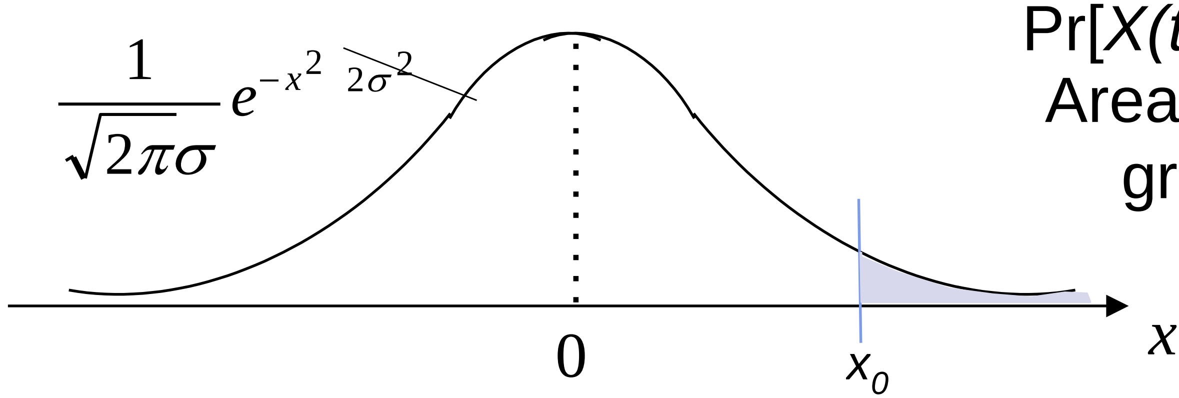
Noise distribution

- Noise is characterized by probability density of amplitude samples
- Likelihood that certain amplitude occurs
- Thermal electronic noise is inevitable (due to vibrations of electrons)
- Noise distribution is Gaussian (bell-shaped) as below

$\Pr[X(t) > x_0] = ?$



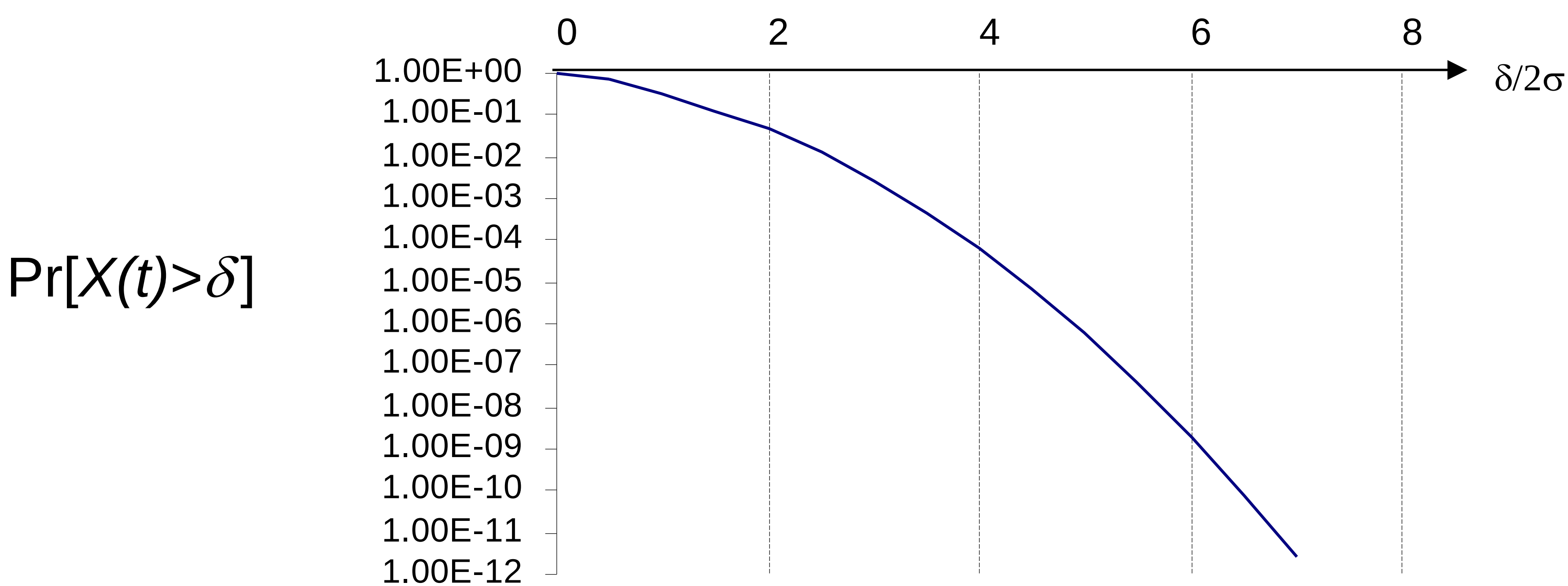
$$\frac{1}{\sqrt{2\pi\sigma}} e^{-x^2 / 2\sigma^2}$$



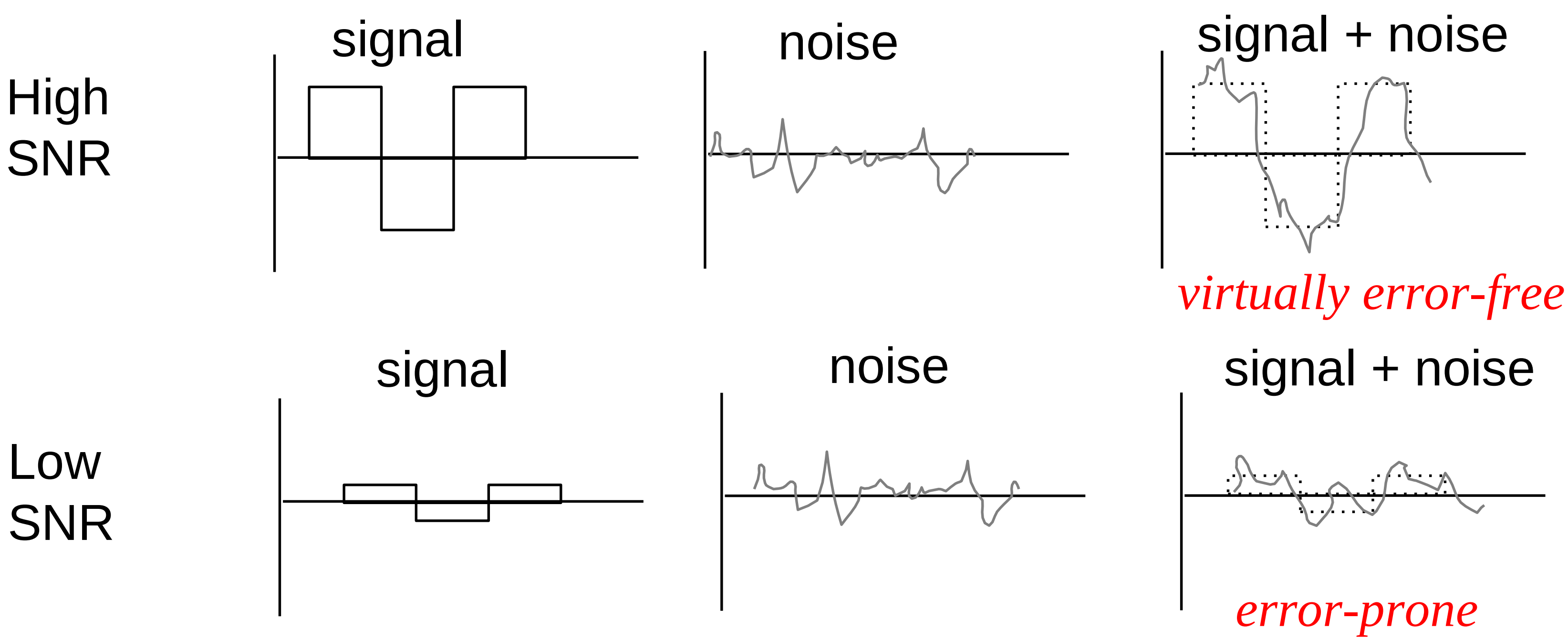
$\Pr[X(t) > x_0] =$ Area under graph

Probability of Error

- Error occurs if noise value exceeds certain magnitude
- Prob. of large values drops quickly with Gaussian noise
- Target probability of error achieved by designing system so separation between signal levels is appropriate relative to average noise power



Channel Noise affects Reliability



$$\text{SNR} = \frac{\text{Average Signal Power}}{\text{Average Noise Power}}$$

$$\text{SNR (dB)} = 10 \log_{10} \text{SNR}$$

Shannon Channel Capacity

- If transmitted power is limited, then as *M* increases spacing between levels decreases
- Presence of noise at receiver causes more frequent errors to occur as *M* is increased

Shannon Channel Capacity:

The maximum reliable transmission rate over an ideal channel with bandwidth *W* Hz, with Gaussian distributed noise, and with SNR *S/N* is

$$C = W \log_2 (1 + S/N) \text{ bits per second}$$

- Reliable means error rate can be made arbitrarily small by proper coding

Example

- Consider a 3 kHz channel with 8-level signaling. Compare bit rate to channel capacity at 20 dB SNR

- 3KHz telephone channel with 8 level signaling

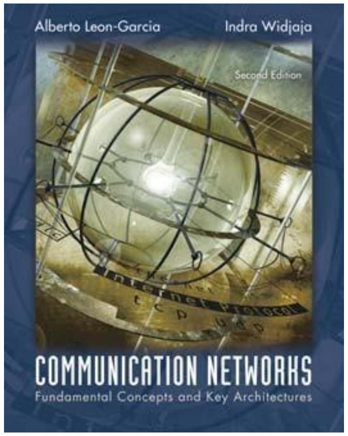
  Bit rate = 2*3000 pulses/sec * 3 bits/pulse = 18 kbps

- 20 dB SNR means $10 \log_{10} S/N = 20$

  Implies S/N = 100
- Shannon Channel Capacity is then

  C = 3000 log (1 + 100) = 19, 963 bits/second

Chapter 3
Digital Transmission Fundamentals

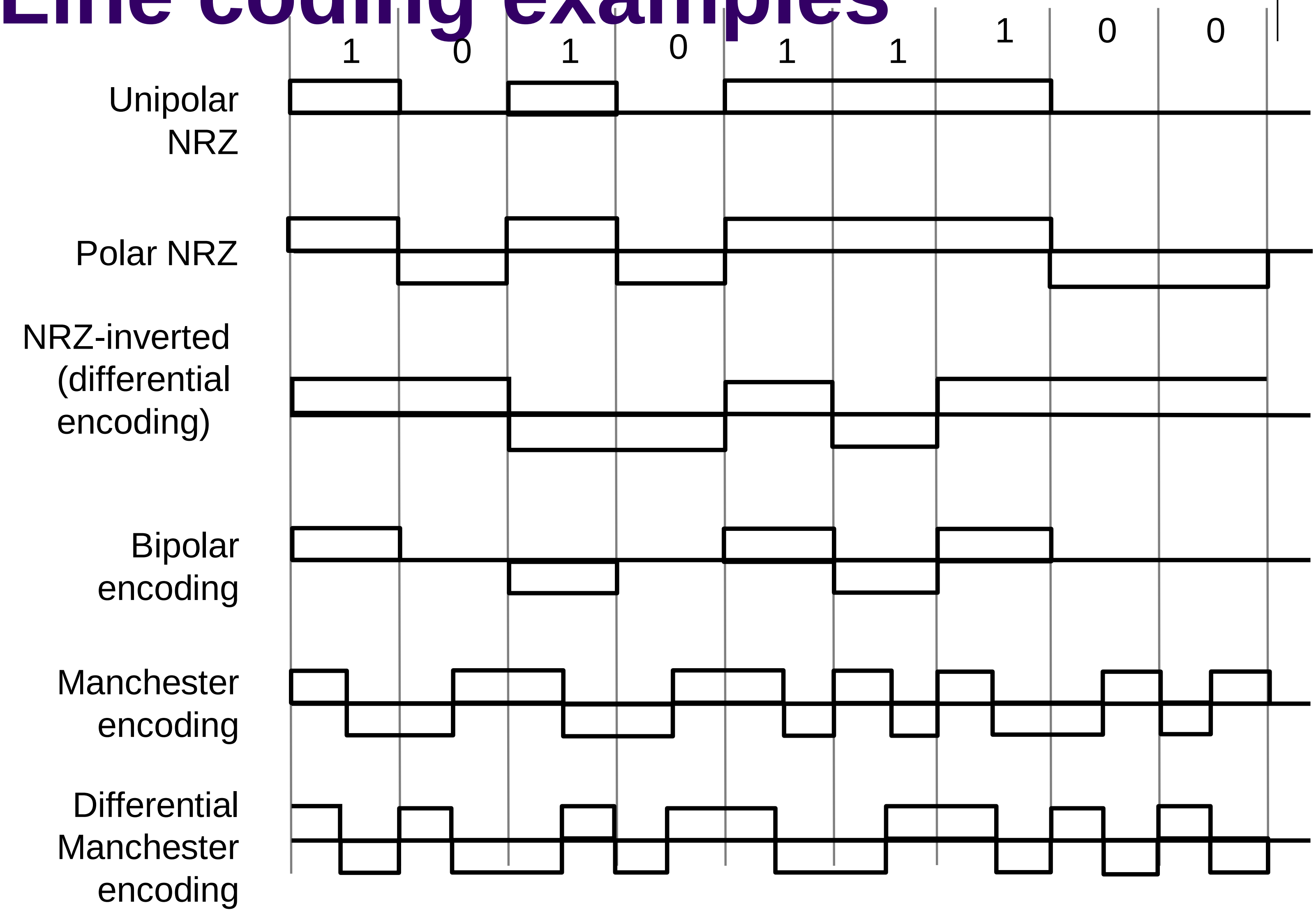


Line Coding

What is Line Coding?

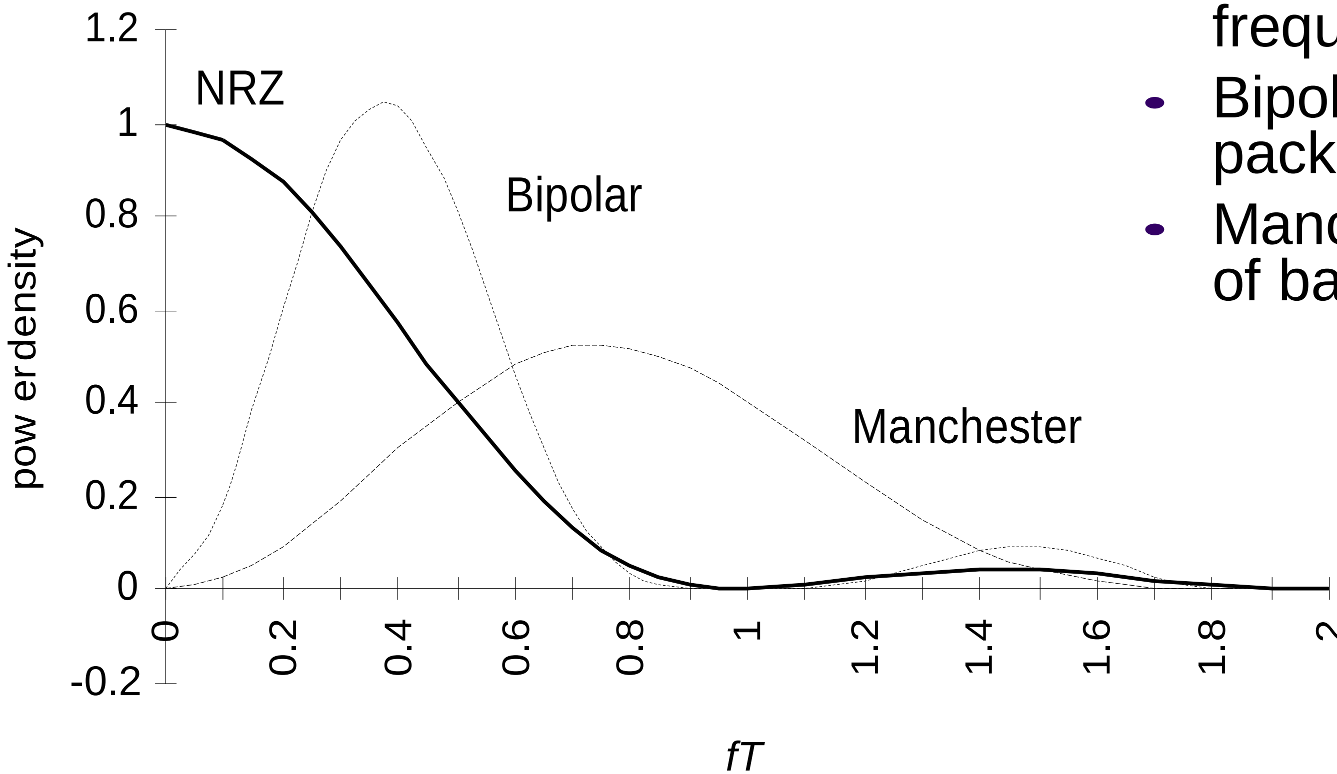
- Mapping of binary information sequence into the digital signal that enters the channel
  - Ex. “1” maps to +A square pulse; “0” to –A pulse
- Line code selected to meet system requirements:
  - *Transmitted power*: Power consumption = $
  - *Bit timing*: Transitions in signal help timing recovery
  - *Bandwidth efficiency*: Excessive transitions wastes bw
  - *Low frequency content*: Some channels block low frequencies
    - long periods of +A or of –A causes signal to “droop”
    - Waveform should not have low-frequency content
  - *Error detection*: Ability to detect errors helps
  - *Complexity/cost*: Is code implementable in chip at high speed?

Line coding examples



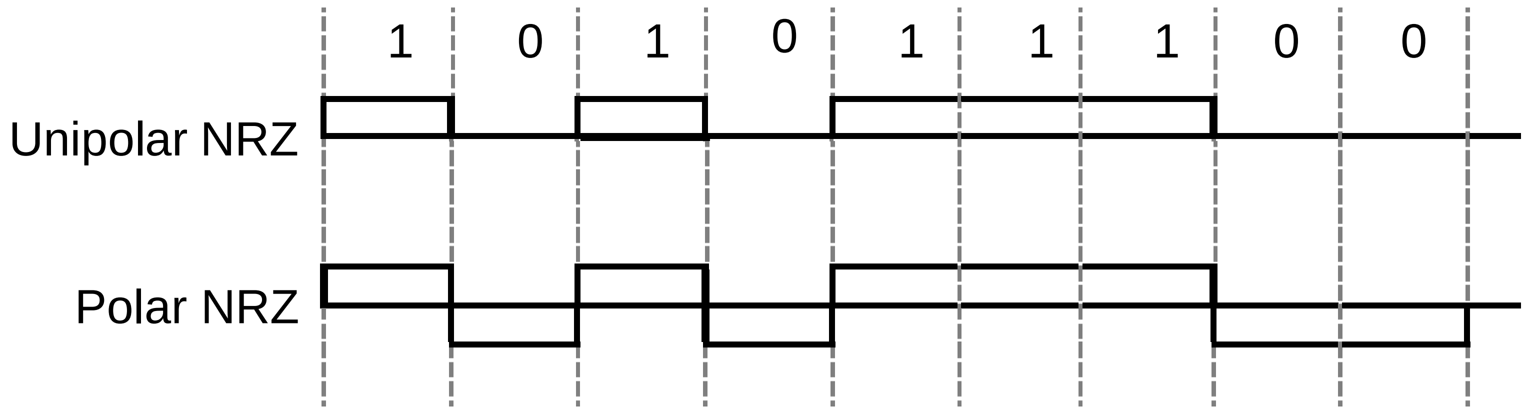
Spectrum of Line codes

- Assume 1s & 0s independent & equiprobable



- NRZ has high content at low frequencies
- Bipolar tightly packed around *T*/2
- Manchester wasteful of bandwidth

Unipolar & Polar Non-Return-to-Zero (NRZ)



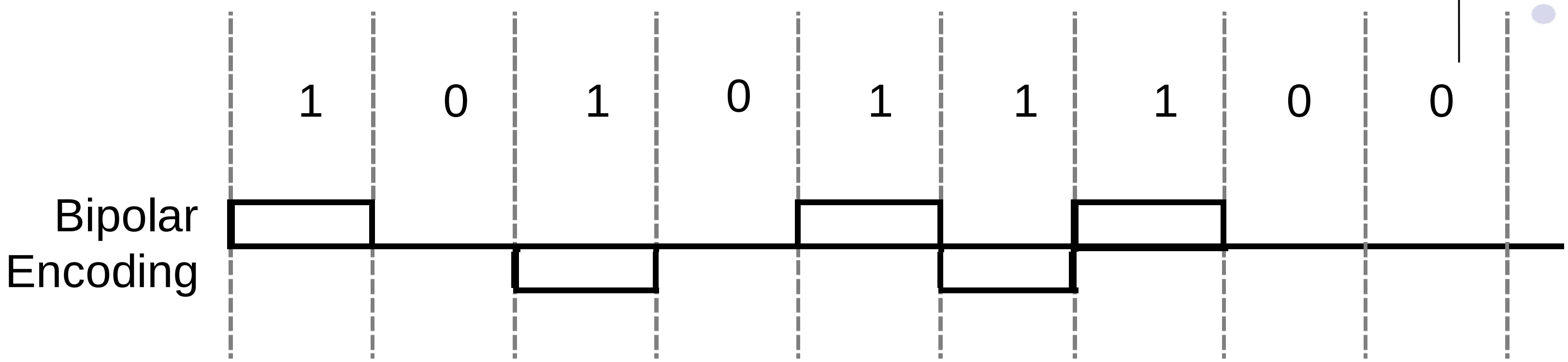
Unipolar NRZ

- "1" maps to +A pulse
- "0" maps to no pulse
- High Average Power
  $0.5*A^2 + 0.5*0^2 = A^2/2$
- Long strings of A or 0
  - Poor timing
  - Low-frequency content
- Simple

Polar NRZ

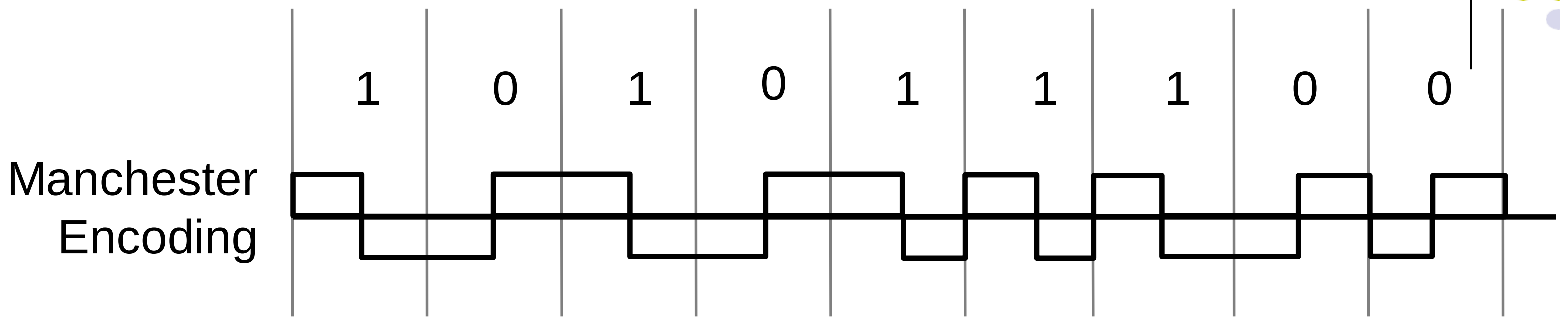
- "1" maps to +A/2 pulse
- "0" maps to –A/2 pulse
- Better Average Power
  $0.5*(A/2)^2 + 0.5*(-A/2)^2 = A^2/4$
- Long strings of +A/2 or –A/2
  - Poor timing
  - Low-frequency content
- Simple

Bipolar Code



- Three signal levels: {-A, 0, +A}
- "1" maps to +A or –A in alternation
- "0" maps to no pulse
  - Every +pulse matched by –pulse so little content at low frequencies
- String of 1s produces a square wave
  - Spectrum centered at *T*/2
- Long string of 0s causes receiver to lose synch
- Zero-substitution codes

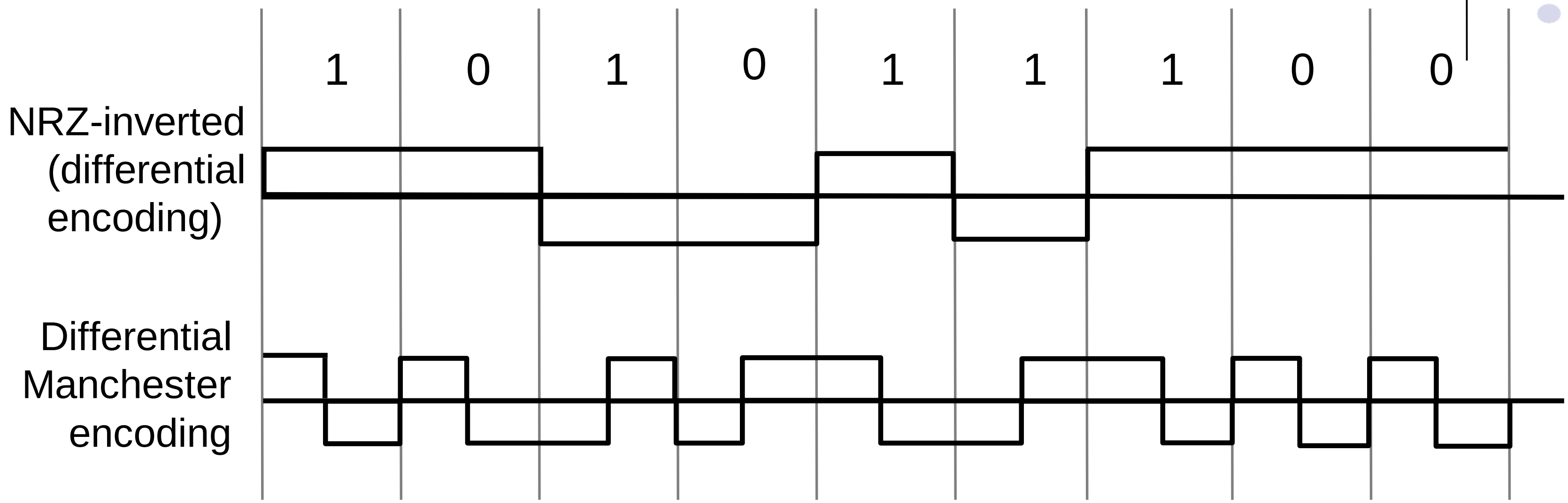
Manchester code & *m*B*n*B codes



- "1" maps into A/2 first *T*/2, -A/2 last T/2
- "0" maps into -A/2 first *T*/2, A/2 last *T*/2
- Every interval has transition in middle
  - Timing recovery easy
  - Uses double the minimum bandwidth
- Simple to implement
- Used in 10-Mbps Ethernet & other LAN standards

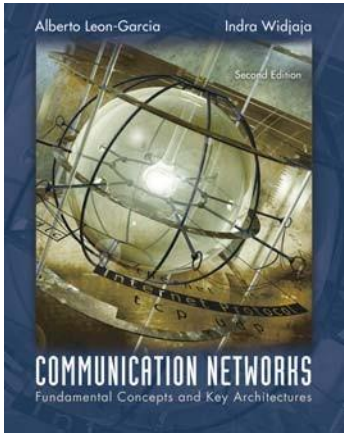
- *m*B*n*B line code
- Maps block of *m* bits into *n* bits
- Manchester code is 1B2B code
- 4B5B code used in FDDI LAN
- 8B10b code used in Gigabit Ethernet
- 64B66B code used in 10G Ethernet

Differential Coding



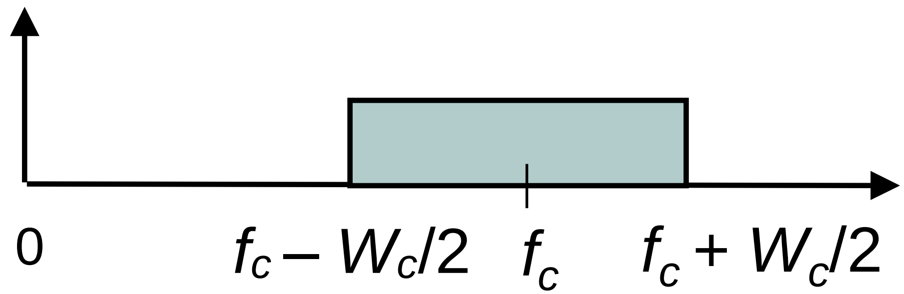
- Errors in some systems cause transposition in polarity, +A become –A and vice versa
  - All subsequent bits in Polar NRZ coding would be in error
- Differential line coding provides robustness to this type of error
- "1" mapped into transition in signal level
- "0" mapped into no transition in signal level
- Same spectrum as NRZ
- Errors occur in pairs
- Also used with Manchester coding

Chapter 3
Digital Transmission Fundamentals



Modems and Digital Modulation

Bandpass Channels

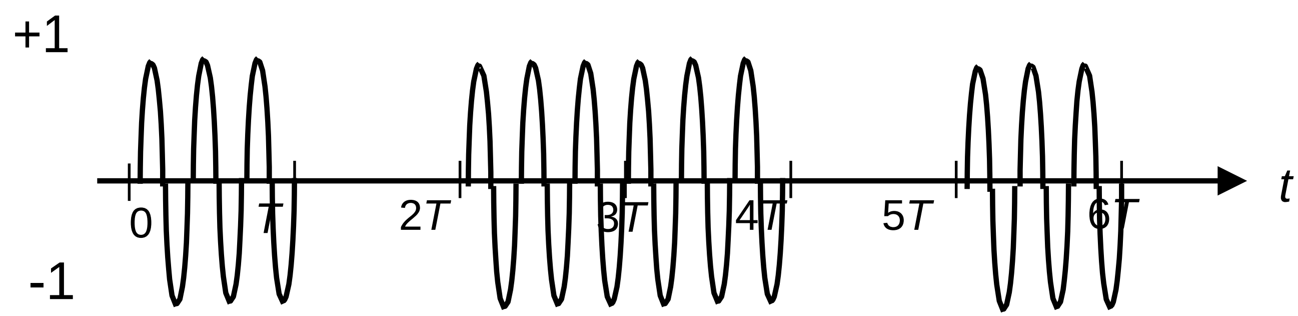


- Bandpass channels pass a range of frequencies around some center frequency f_c
  - Radio channels, telephone & DSL modems
- Digital modulators embed information into waveform with frequencies passed by bandpass channel
- Sinusoid of frequency f_c is centered in middle of bandpass channel
- Modulators embed information into a sinusoid

Amplitude Modulation and Frequency Modulation

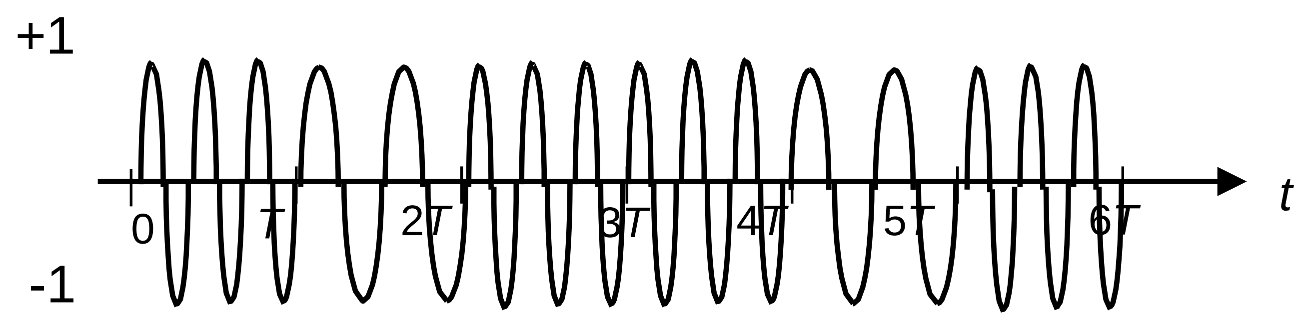
Information 1 0 1 1 0 1

Amplitude Shift Keying



Map bits into amplitude of sinusoid: "1" send sinusoid; "0" no sinusoid
Demodulator looks for signal vs. no signal

Frequency Shift Keying

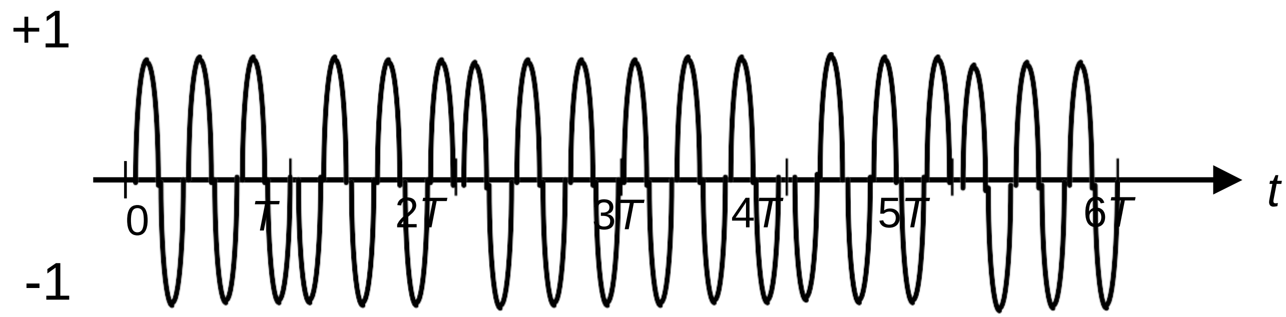


Map bits into frequency: "1" send frequency $f_c + \delta$; "0" send frequency $f_c - \delta$
Demodulator looks for power around $f_c + \delta$ or $f_c - \delta$

Phase Modulation

Information 1 0 1 1 0 1

Phase Shift Keying



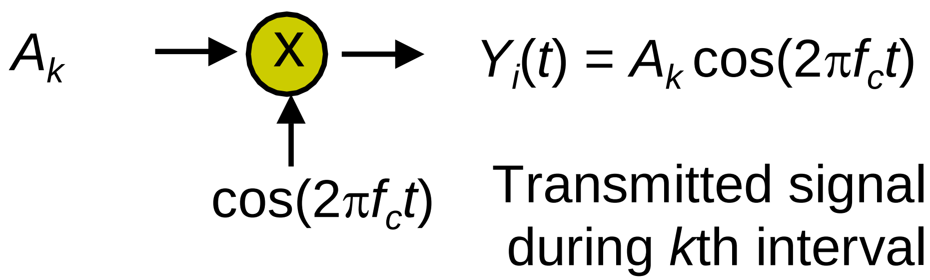
- Map bits into phase of sinusoid:
  - "1" send A $\cos(2\pi ft)$, i.e. phase is 0
  - "0" send A $\cos(2\pi ft+\pi)$, i.e. phase is π

- Equivalent to multiplying $\cos(2\pi ft)$ by +A or -A
  - "1" send A $\cos(2\pi ft)$, i.e. multiply by 1
  - "0" send A $\cos(2\pi ft+\pi) = -A\cos(2\pi ft)$, i.e. multiply by -1

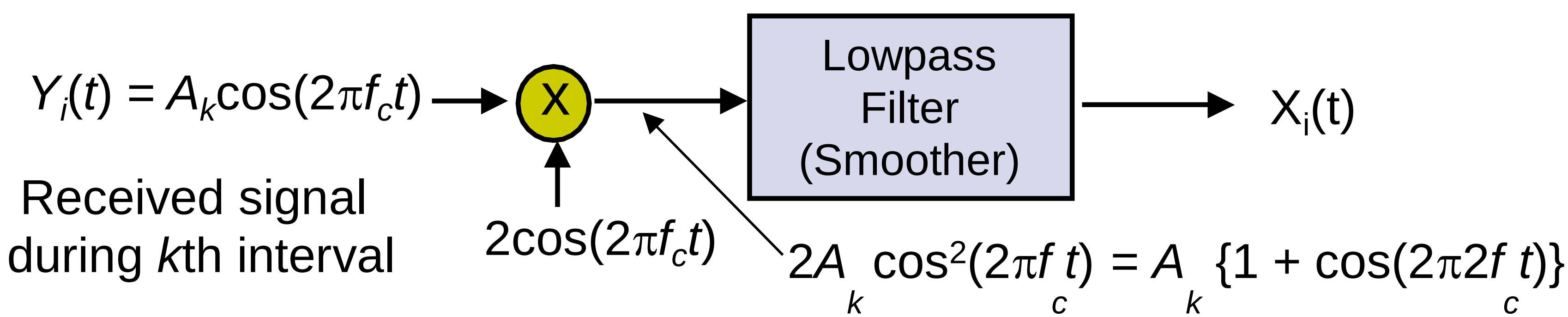
- We will focus on phase modulation

Modulator & Demodulator

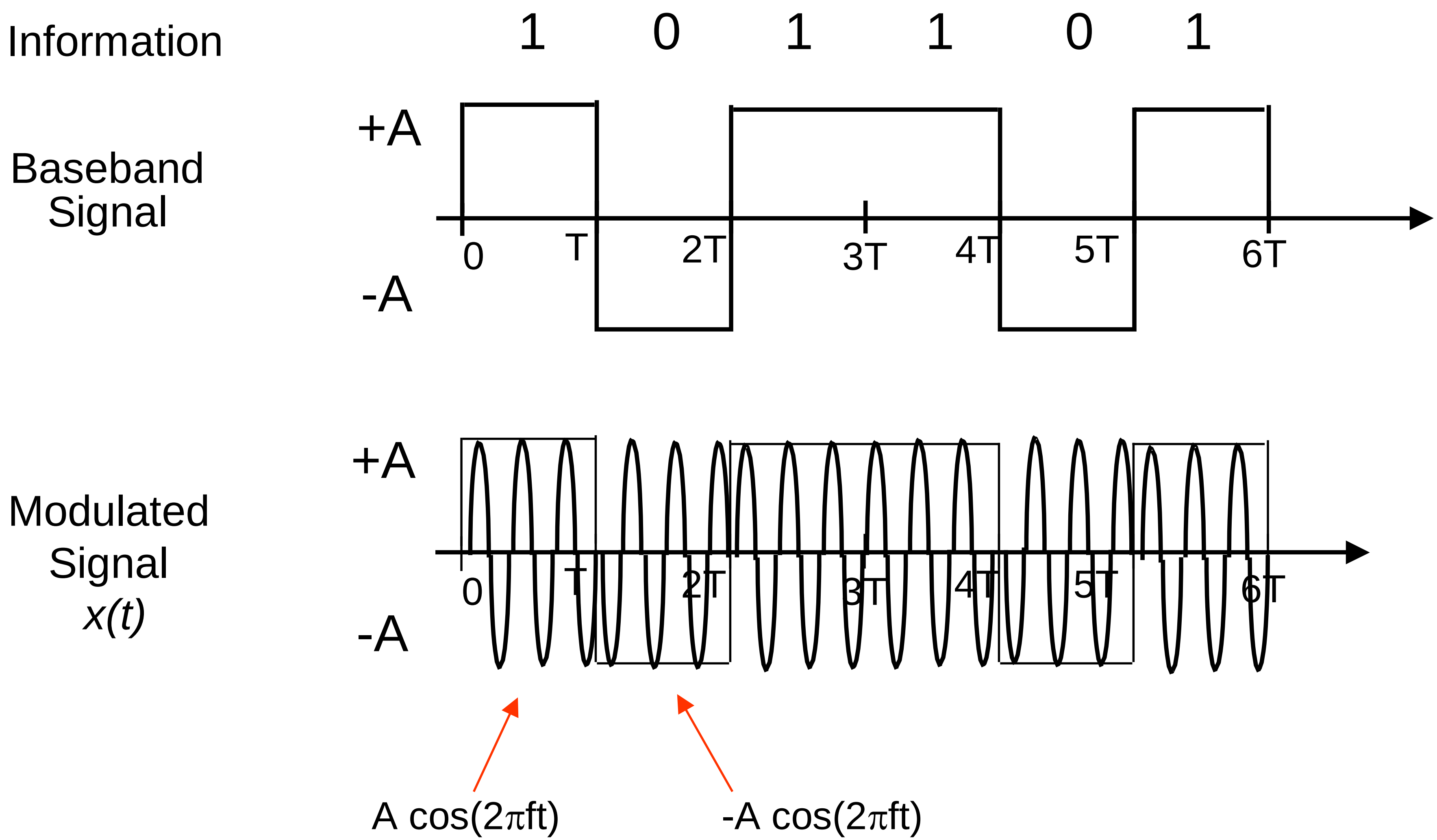
Modulate $\cos(2\pi f_c t)$ by multiplying by A_k for T seconds:



Demodulate (recover A_k) by multiplying by $2\cos(2\pi f_c t)$ for T seconds and lowpass filtering (smoothing):

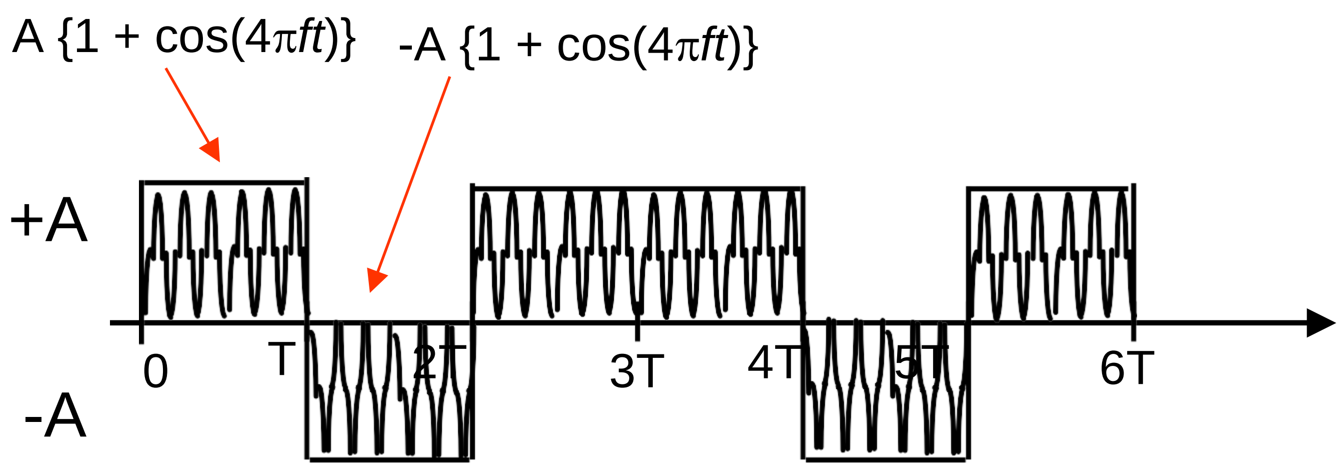


Example of Modulation

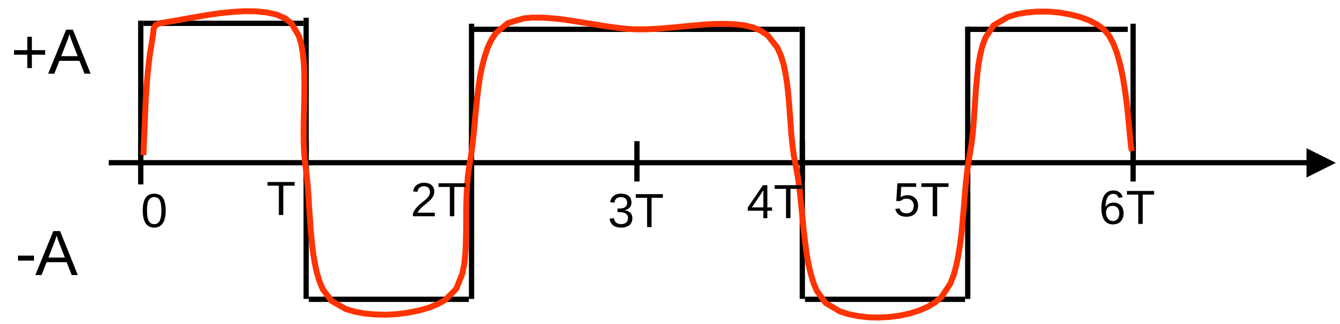


Example of Demodulation

After multiplication at receiver
$x(t)\cos(2\pi f_c t)$



Baseband signal discernable after smoothing



Recovered Information

1 0 1 1 0 1

Signaling rate and Transmission Bandwidth

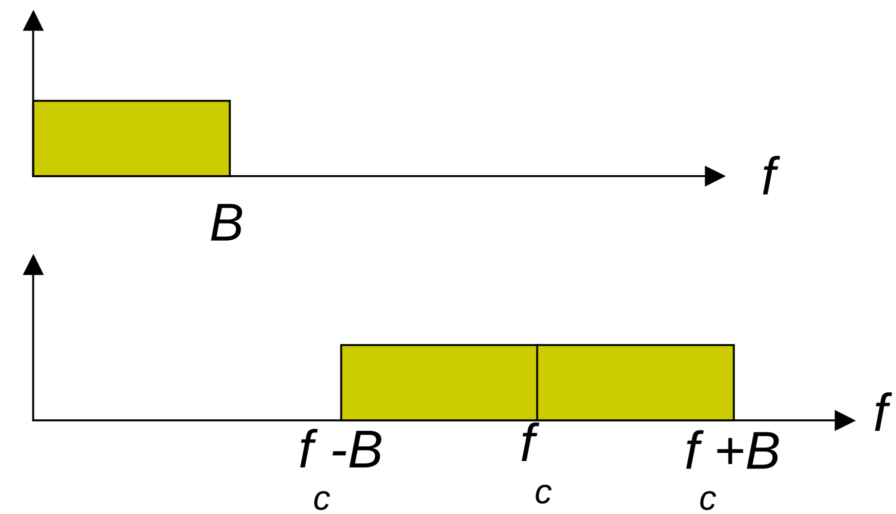
- Fact from modulation theory:

If

Baseband signal *x(t)* with bandwidth *B* Hz

then

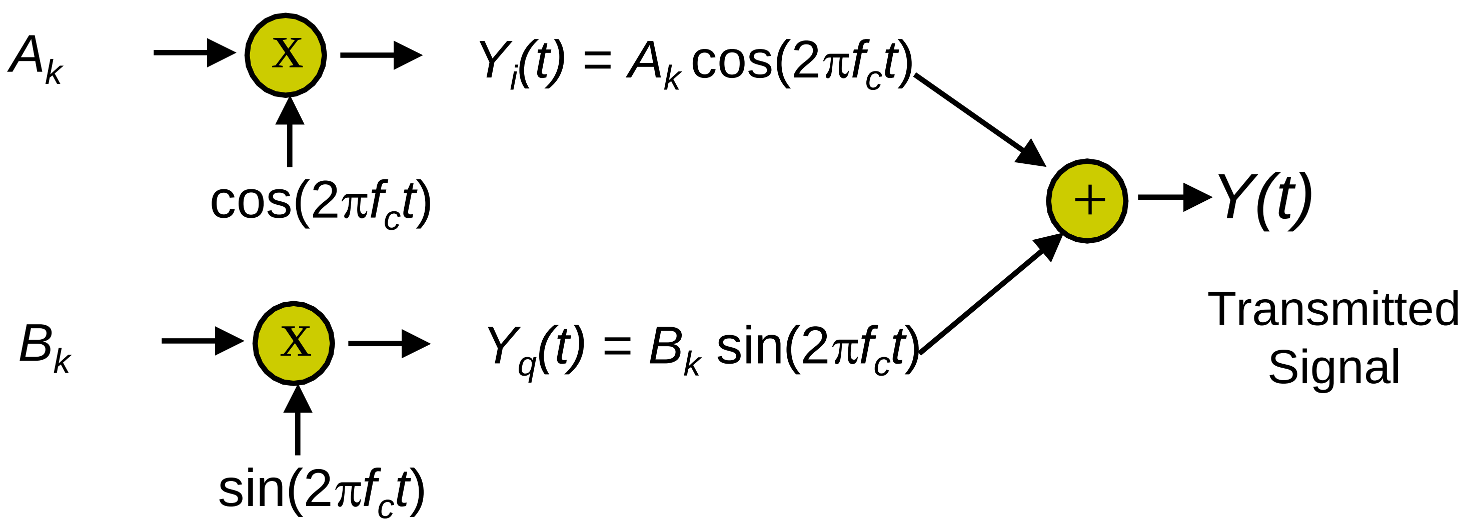
Modulated signal $x(t)\cos(2\pi f_c t)$ has bandwidth 2*B* Hz



- If bandpass channel has bandwidth W_c Hz,
  - Then baseband channel has $W_c/2$ Hz available, so
  - modulation system supports $W_c/2 \times 2 = W_c$ pulses/second
  - That is, W_c pulses/second per W_c Hz = 1 pulse/Hz
  - Recall baseband transmission system supports 2 pulses/Hz

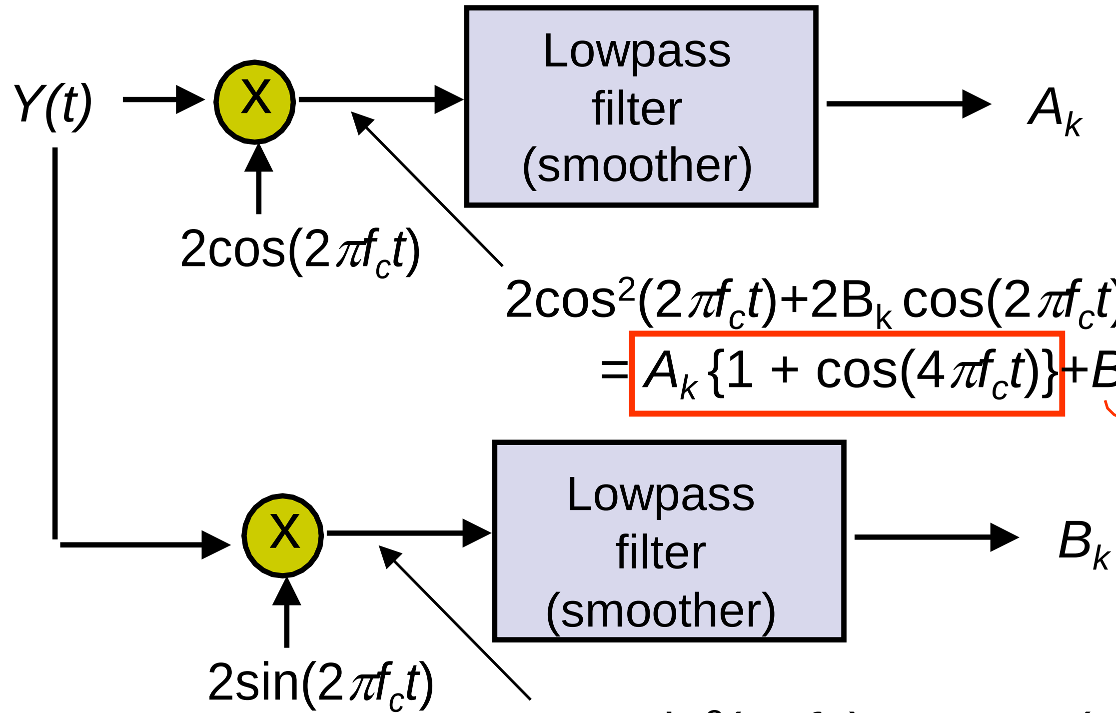
Quadrature Amplitude Modulation (QAM)

- QAM uses two-dimensional signaling
  - A_k modulates in-phase $\cos(2\pi f_c t)$
  - B_k modulates quadrature phase $\cos(2\pi f_c t + \pi/4) = \sin(2\pi f_c t)$
  - Transmit sum of inphase & quadrature phase components



- $Y_i(t)$ and $Y_q(t)$ both occupy the bandpass channel
- QAM sends 2 pulses/Hz

QAM Demodulation



$$2\cos^2(2\pi f_c t)+2B_k\cos(2\pi f_c t)\sin(2\pi f_c t) = \boxed{A_k\{1+\cos(4\pi f_c t)\}} + \underbrace{B_k\{0+\sin(4\pi f_c t)\}}_{\text{smoothed to zero}}$$

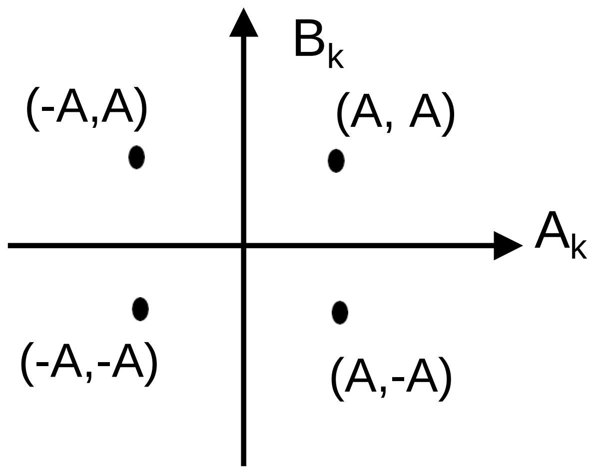
smoothed to zero

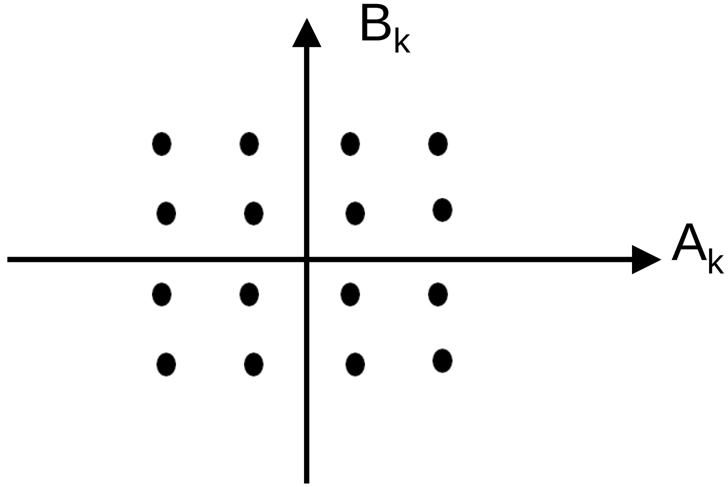
$$2B_k\sin^2(2\pi f_c t)+2A_k\cos(2\pi f_c t)\sin(2\pi f_c t) = \boxed{B_k\{1-\cos(4\pi f_c t)\}} + \underbrace{A_k\{0+\sin(4\pi f_c t)\}}_{\text{smoothed to zero}}$$

smoothed to zero

Signal Constellations

- Each pair (A_k, B_k) defines a point in the plane
- *Signal constellation* set of signaling points





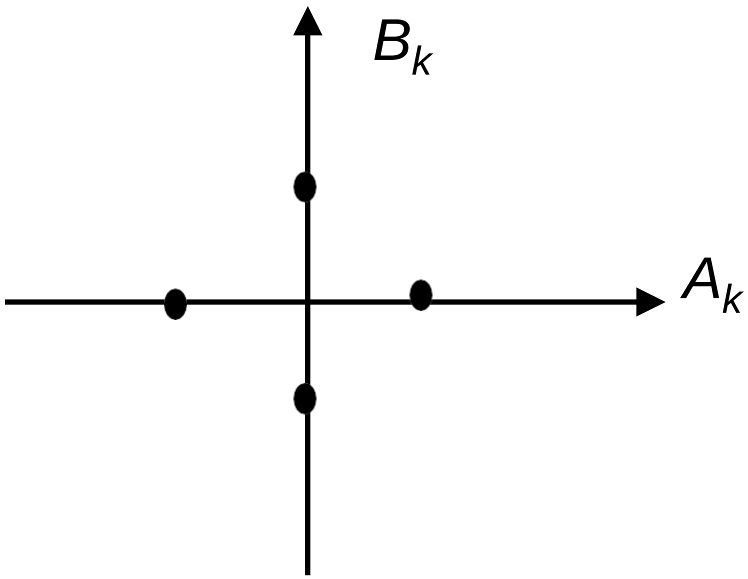
4 possible points per *T* sec.
2 bits / pulse

16 possible points per *T* sec.
4 bits / pulse

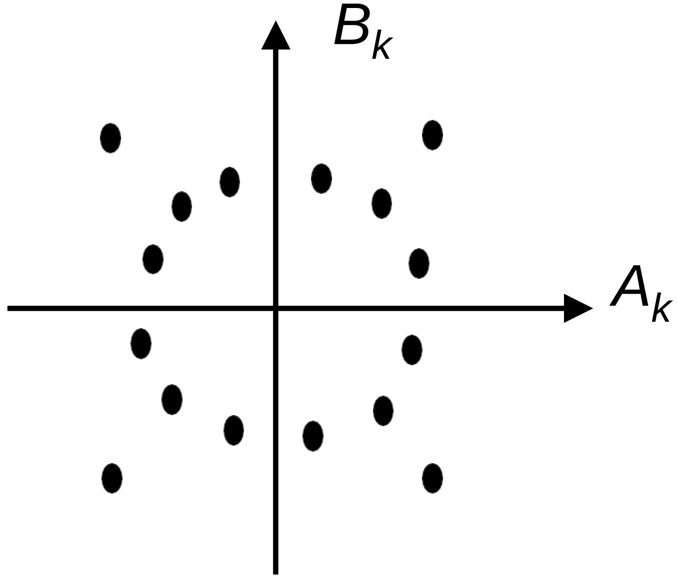
Other Signal Constellations

- Point selected by amplitude & phase

$$A_k \cos(2\pi f_c t) + B_k \sin(2\pi f_c t) = \sqrt{A_k^2 + B_k^2}\cos(2\pi f_c t + \tan^{-1}(B_k/A_k))$$



4 possible points per T sec.



16 possible points per T sec.

Telephone Modem Standards

Telephone Channel for modulation purposes has

W_c = 2400 Hz → 2400 pulses per second

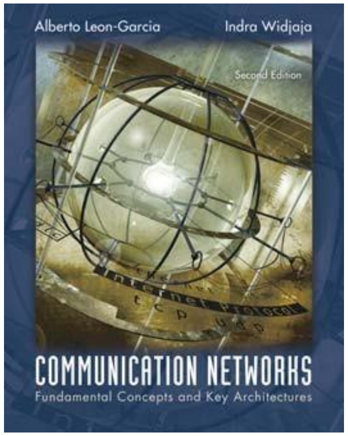
Modem Standard V.32bis

- Trellis modulation maps m bits into one of 2^{m+1} constellation points
- 14,400 bps Trellis 128 2400x6
- 9600 bps Trellis 32 2400x4
- 4800 bps QAM 4 2400x2

Modem Standard V.34 adjusts pulse rate to channel

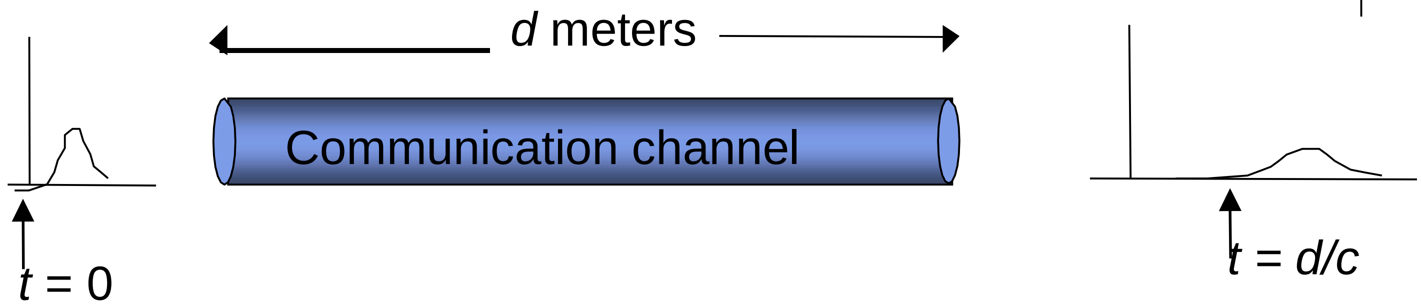
- 2400-33600 bps Trellis 960 2400-3429 pulses/sec

Chapter 3
Digital Transmission Fundamentals



Properties of Media and Digital Transmission Systems

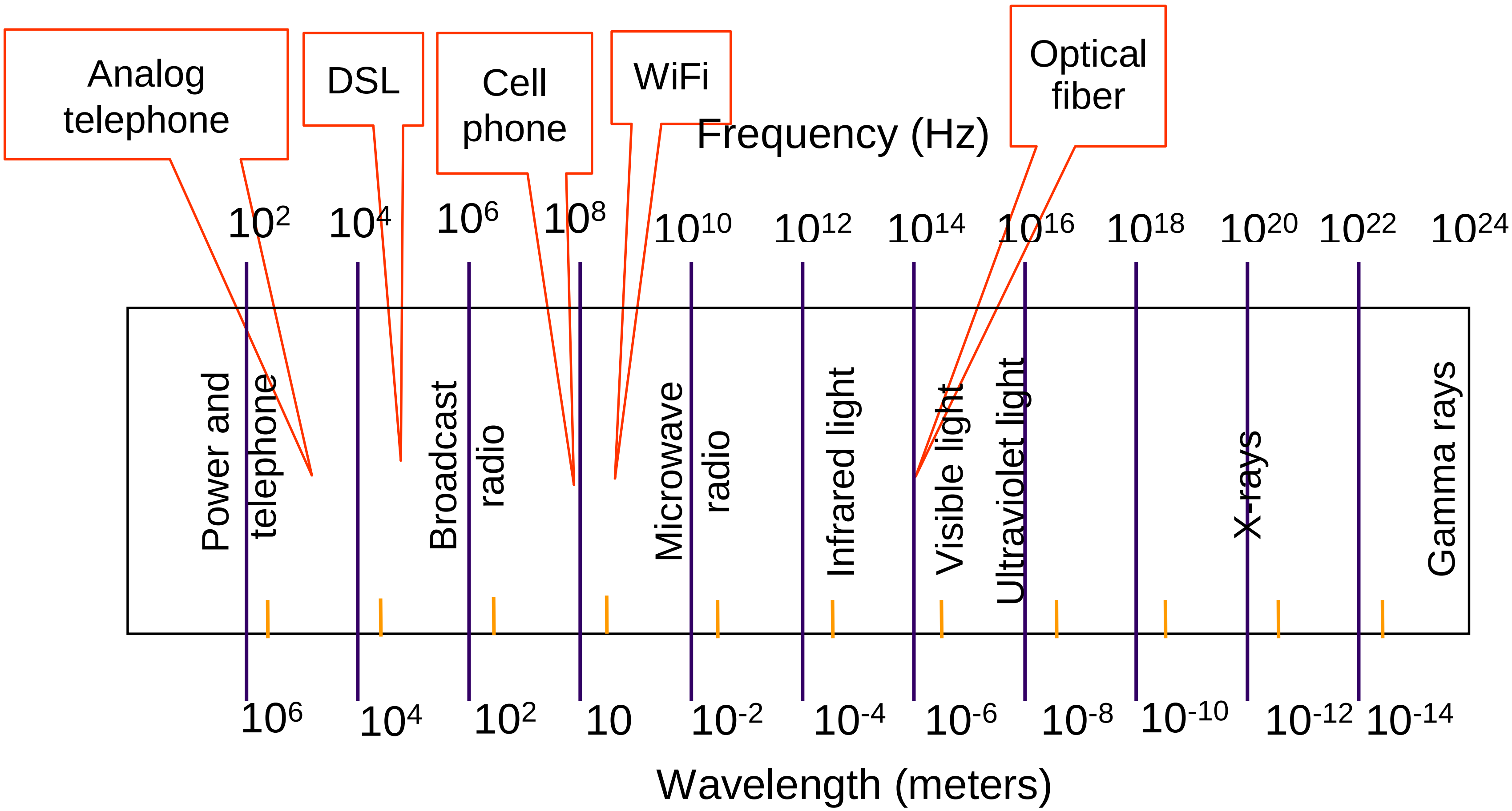
Fundamental Issues in Transmission Media



- Information bearing capacity
  - Amplitude response & bandwidth
    - dependence on distance
  - Susceptibility to noise & interference
    - Error rates & SNRs
- Propagation speed of signal
  - $c = 3 \times 10^8$ meters/second in vacuum
  - $\nu = c/\sqrt{\varepsilon}$ speed of light in medium where $\varepsilon > 1$ is the dielectric constant of the medium
  - $\nu = 2.3 \times 10^8$ m/sec in copper wire; $\nu = 2.0 \times 10^8$ m/sec in optical fiber

Communications systems & Electromagnetic Spectrum

- Frequency of communications signals



Wireless & Wired Media

Wireless Media

- Signal energy propagates in space, limited directionality
- Interference possible, so spectrum regulated
- Limited bandwidth
- Simple infrastructure: antennas & transmitters
- No physical connection between network & user
- Users can move

Wired Media

- Signal energy contained & guided within medium
- Spectrum can be re-used in separate media (wires or cables), more scalable
- Extremely high bandwidth
- Complex infrastructure: ducts, conduits, poles, right-of-way

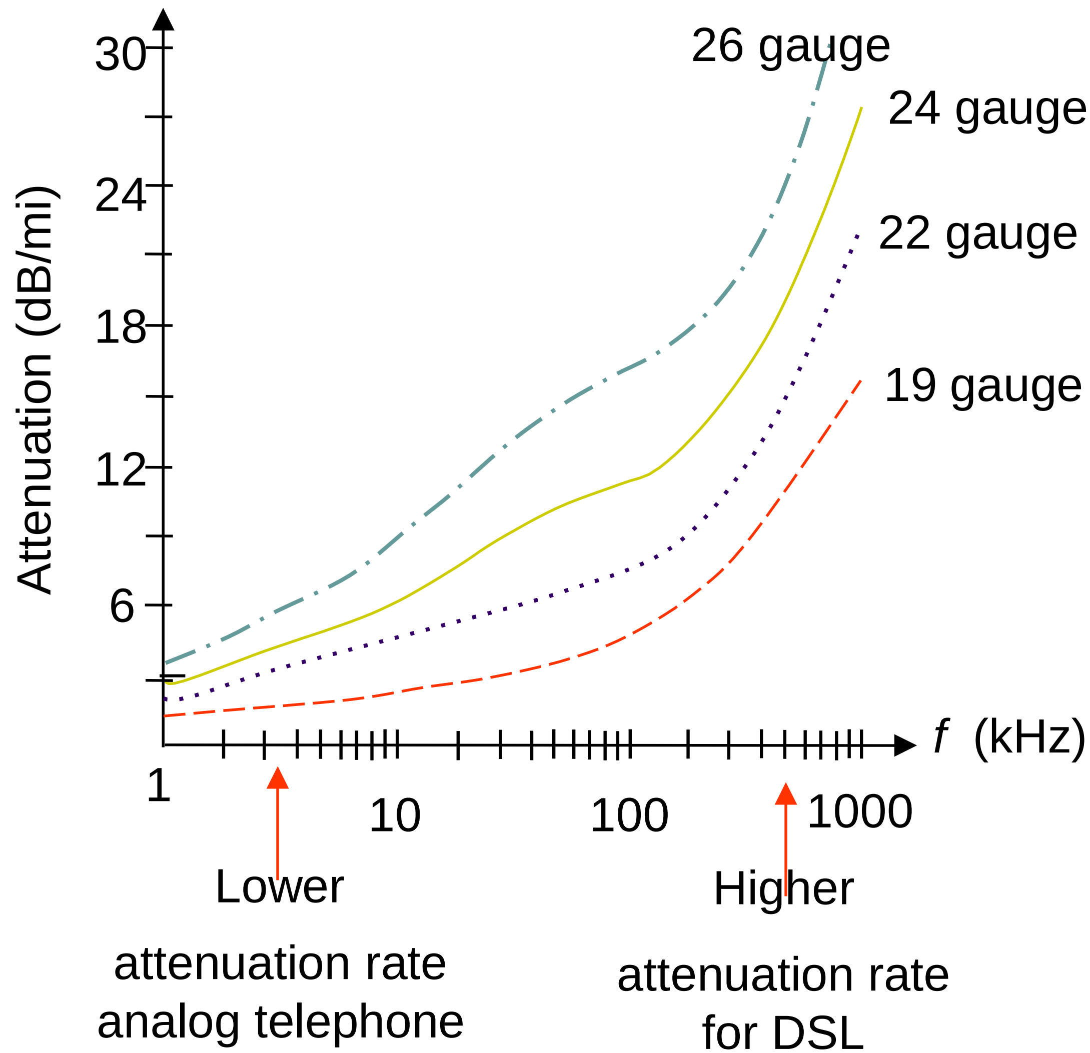
Attenuation

- Attenuation varies with media
  - Dependence on distance of central importance
- Wired media has exponential dependence
  - Received power at d meters proportional to 10^{-kd}
  - Attenuation in dB = *k d*, where *k* is dB/meter
- Wireless media has logarithmic dependence
  - Received power at d meters proportional to d^{-n}
  - Attenuation in dB = *n log d*, where *n* is path loss exponent; *n=2* in free space
  - Signal level maintained for much longer distances
  - Space communications possible

Twisted Pair

Twisted pair

- Two insulated copper wires arranged in a regular spiral pattern to minimize interference
- Various thicknesses, e.g. 0.016 inch (24 gauge)
- Low cost
- Telephone subscriber loop from customer to CO
- Old trunk plant connecting telephone COs
- Intra-building telephone from wiring closet to desktop
- In old installations, loading coils added to improve quality in 3 kHz band, but more attenuation at higher frequencies



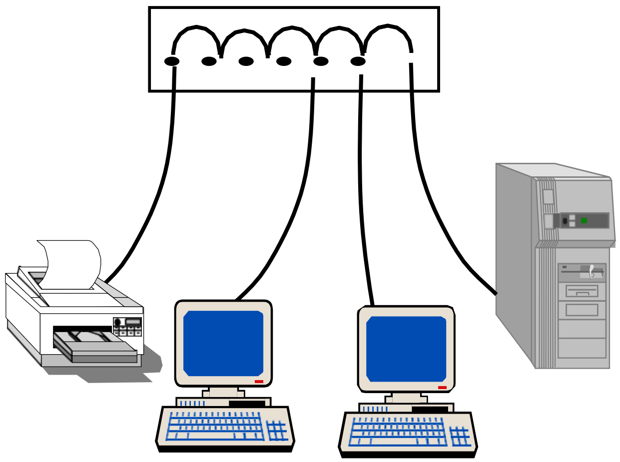
Twisted Pair Bit Rates

Table 3.5 Data rates of 24-gauge twisted pair

Standard	Data Rate	Distance
T-1	1.544 Mbps	18,000 feet, 5.5 km
DS2	6.312 Mbps	12,000 feet, 3.7 km
1/4 STS-1	12.960 Mbps	4500 feet, 1.4 km
1/2 STS-1	25.920 Mbps	3000 feet, 0.9 km
STS-1	51.840 Mbps	1000 feet, 300 m

- Twisted pairs can provide high bit rates at short distances
- Asymmetric Digital Subscriber Loop (ADSL)
  - High-speed Internet Access
  - Lower 3 kHz for voice
  - Upper band for data
  - 64 kbps inbound
  - 640 kbps outbound
- Much higher rates possible at shorter distances
  - Strategy for telephone companies is to bring fiber close to home & then twisted pair
  - Higher-speed access + video

Ethernet LANs

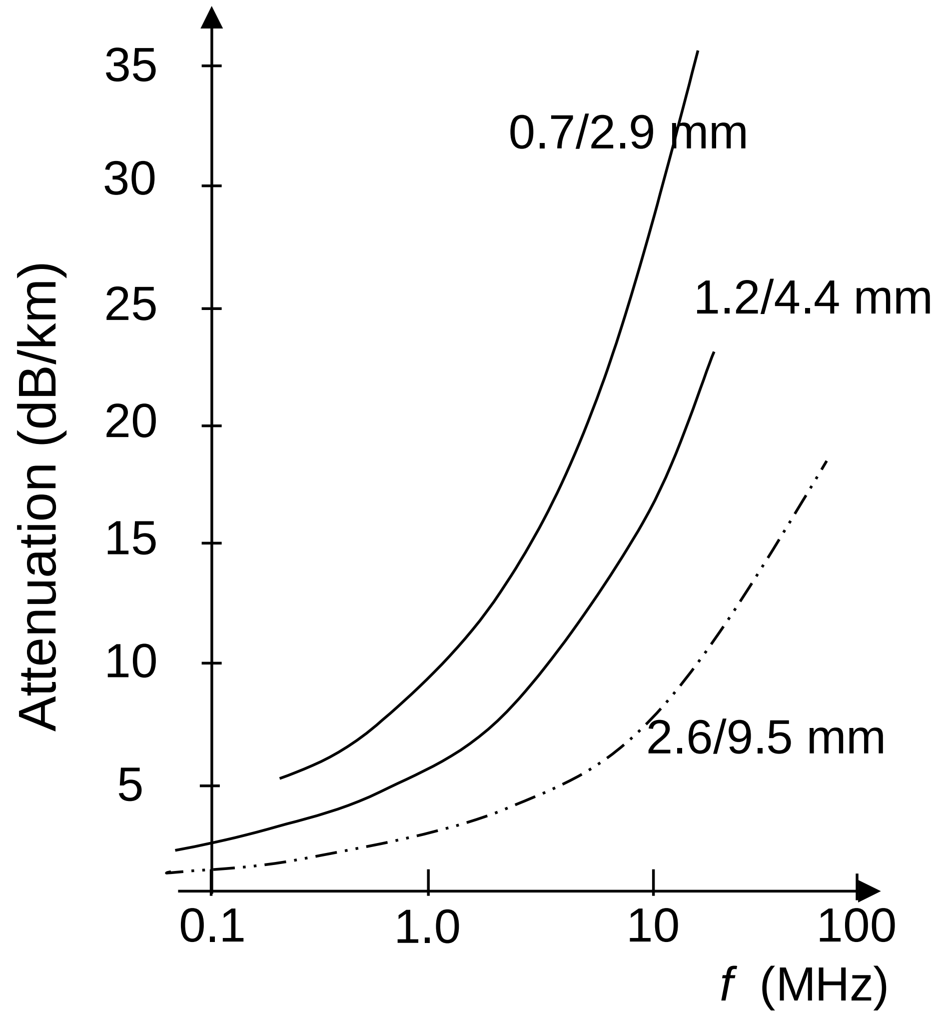


- Category 3 unshielded twisted pair (UTP): ordinary telephone wires
- Category 5 UTP: tighter twisting to improve signal quality
- Shielded twisted pair (STP): to minimize interference; costly
- 10BASE-T Ethernet
  - 10 Mbps, Baseband, Twisted pair
  - Two Cat3 pairs
  - Manchester coding, 100 meters
- 100BASE-T4 *Fast* Ethernet
  - 100 Mbps, Baseband, Twisted pair
  - Four Cat3 pairs
  - Three pairs for one direction at-a-time
  - 100/3 Mbps per pair;
  - 3B6T line code, 100 meters
- Cat5 & STP provide other options

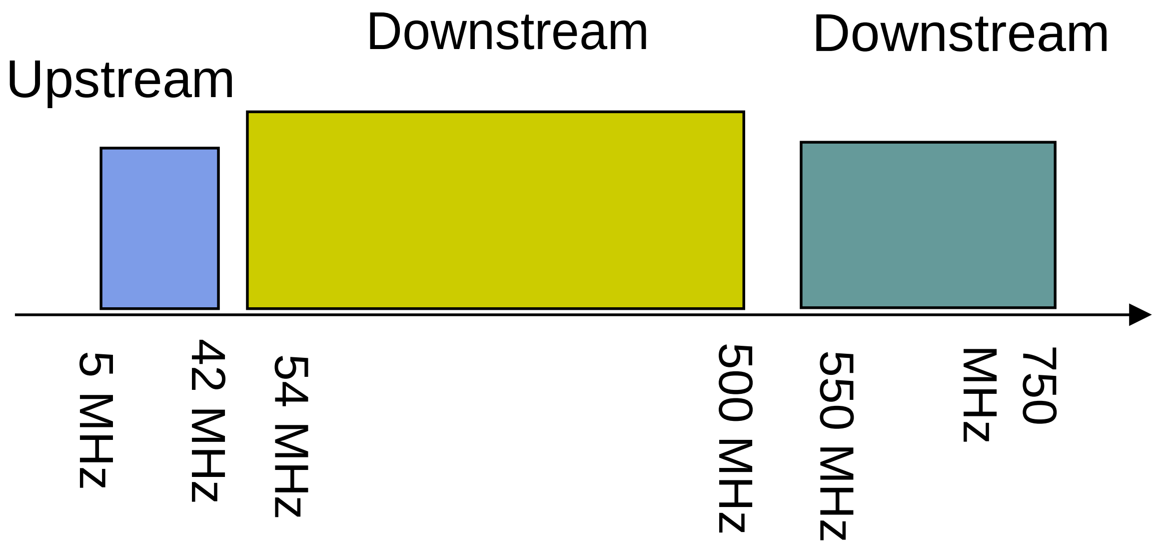
Coaxial Cable

Twisted pair

- Cylindrical braided outer conductor surrounds insulated inner wire conductor
- High interference immunity
- Higher bandwidth than twisted pair
- Hundreds of MHz
- Cable TV distribution
- Long distance telephone transmission
- Original Ethernet LAN medium

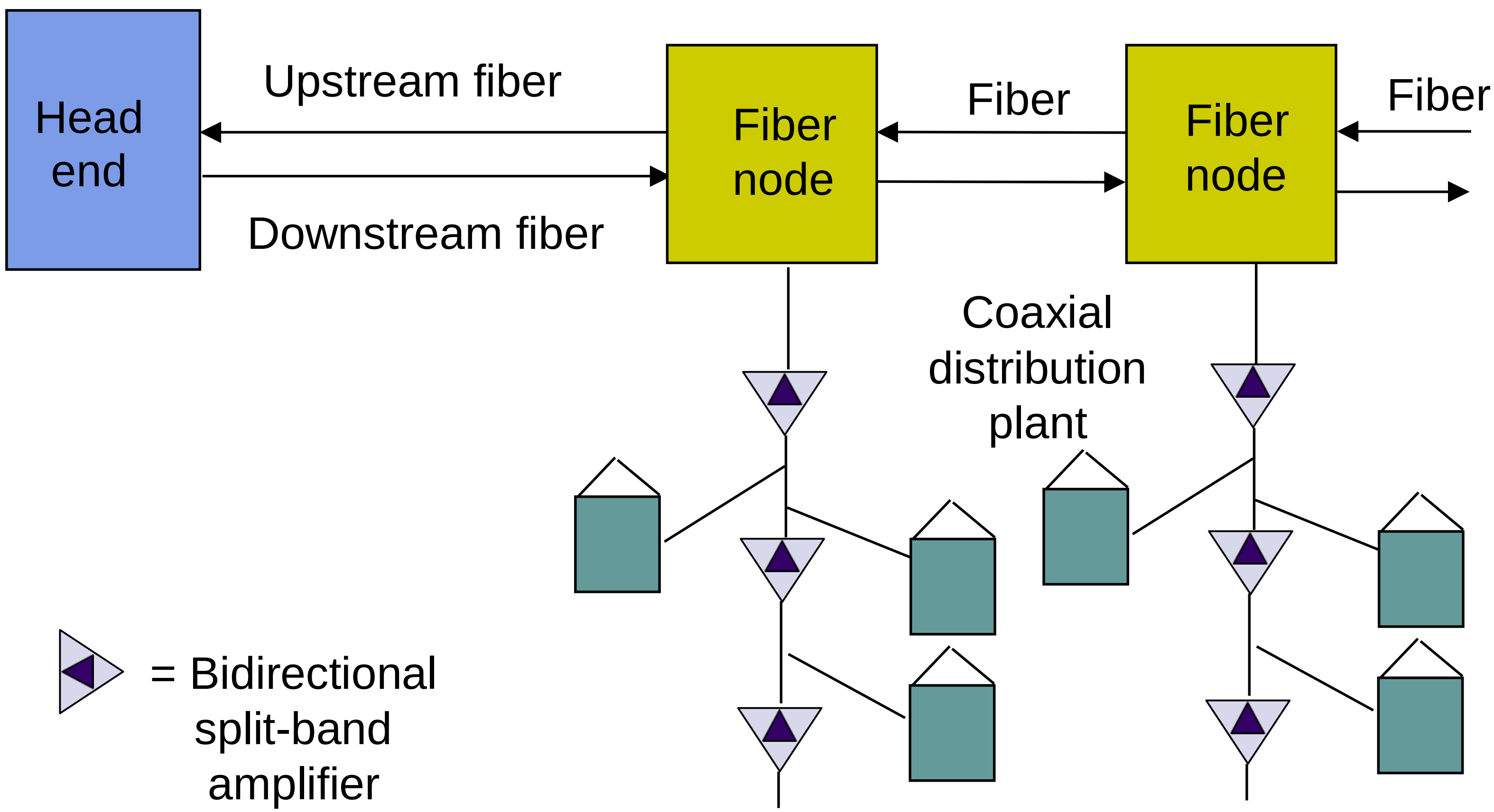


Cable Modem & TV Spectrum

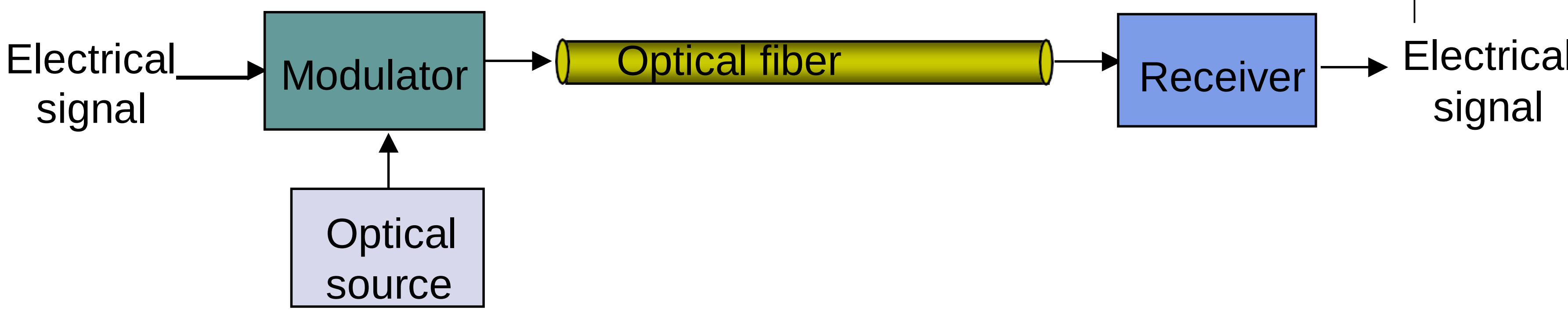


- Cable TV network originally unidirectional
- Cable plant needs upgrade to bidirectional
- 1 analog TV channel is 6 MHz, can support very high data rates
- Cable Modem: *shared* upstream & downstream
  - 5-42 MHz upstream into network; 2 MHz channels; 500 kbps to 4 Mbps
  - >550 MHz downstream from network; 6 MHz channels; 36 Mbps

Cable Network Topology



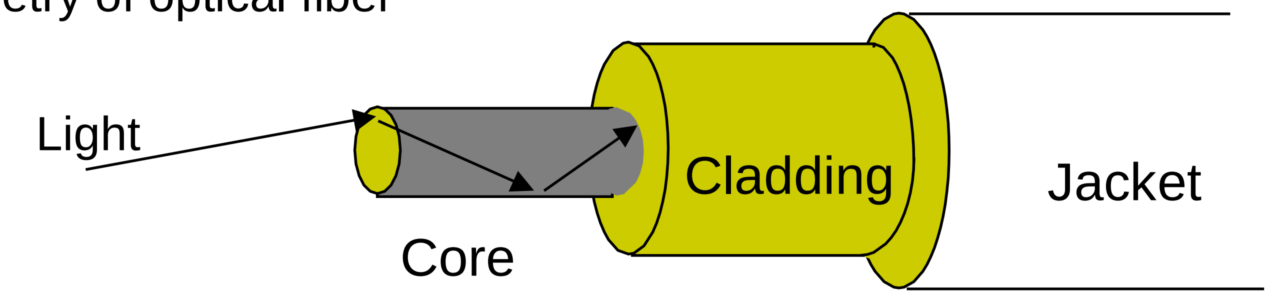
Optical Fiber



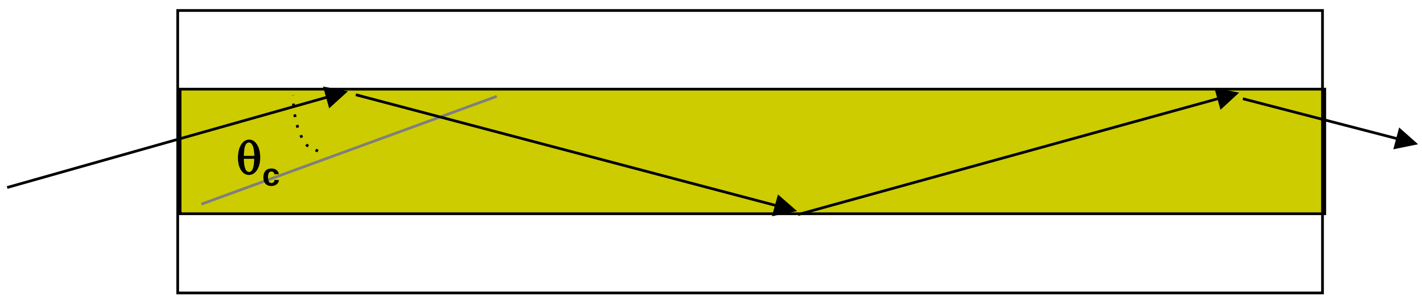
- Light sources (lasers, LEDs) generate pulses of light that are transmitted on optical fiber
  - Very long distances (>1000 km)
  - Very high speeds (>40 Gbps/wavelength)
  - Nearly error-free (BER of 10^{-15})
- Profound influence on network architecture
  - Dominates long distance transmission
  - Distance less of a cost factor in communications
  - Plentiful bandwidth for new services

Transmission in Optical Fiber

Geometry of optical fiber

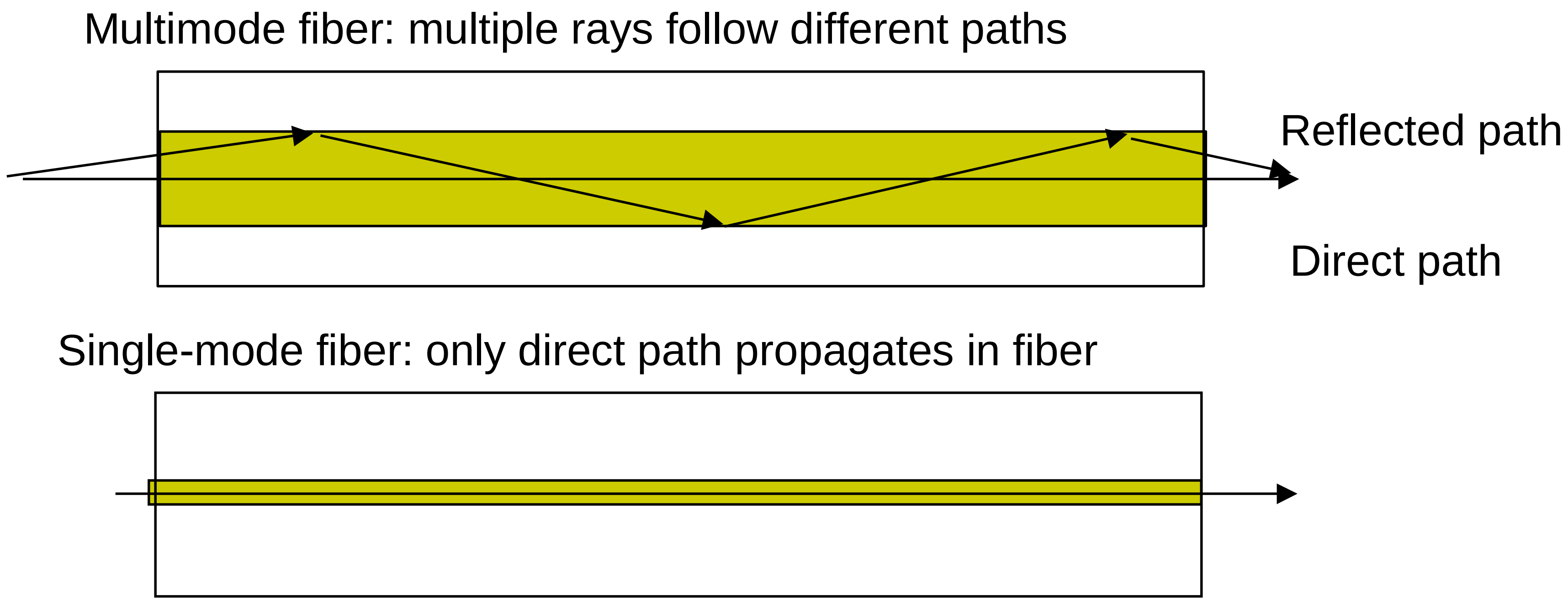


Total Internal Reflection in optical fiber



- Very fine glass cylindrical core surrounded by concentric layer of glass (cladding)
- Core has higher index of refraction than cladding
- Light rays incident at less than critical angle θ_c is completely reflected back into the core

Multimode & Single-mode Fiber



- Multimode: Thicker core, shorter reach
  - Rays on different paths interfere causing dispersion & limiting bit rate
- Single mode: Very thin core supports only one mode (path)
  - More expensive lasers, but achieves very high speeds

Optical Fiber Properties

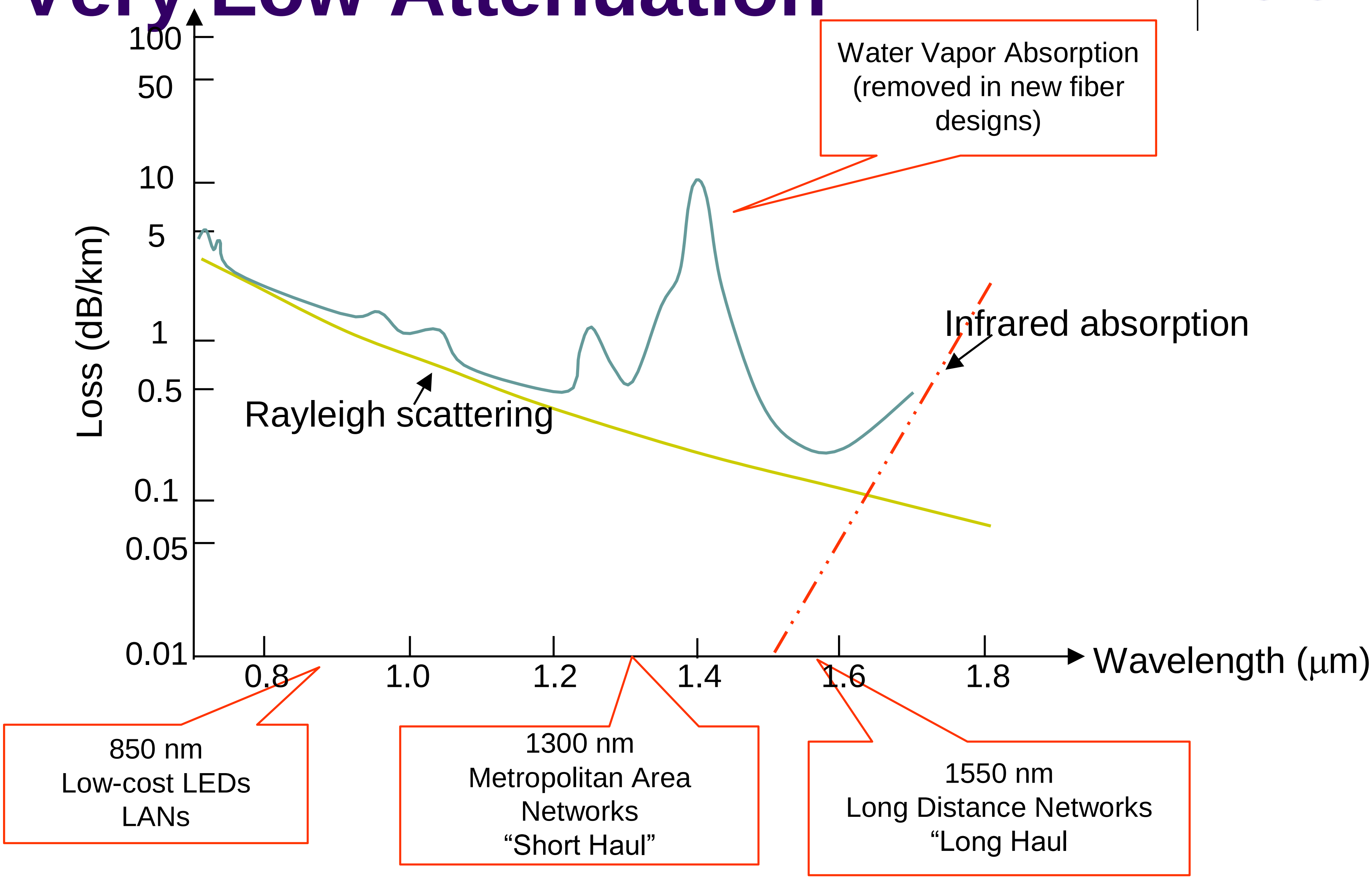
Advantages

- ***Very low attenuation***
- ***Noise immunity***
- ***Extremely high bandwidth***
- Security: Very difficult to tap without breaking
- No corrosion
- More compact & lighter than copper wire

Disadvantages

- New types of optical signal impairments & dispersion
  - Polarization dependence
  - Wavelength dependence
- Limited bend radius
  - If physical arc of cable too high, light lost or won't reflect
  - Will break
- Difficult to splice
- Mechanical vibration becomes signal noise

Very Low Attenuation



Huge Available Bandwidth

- Optical range from λ_1 to $\lambda_1 + \Delta\lambda$ contains bandwidth

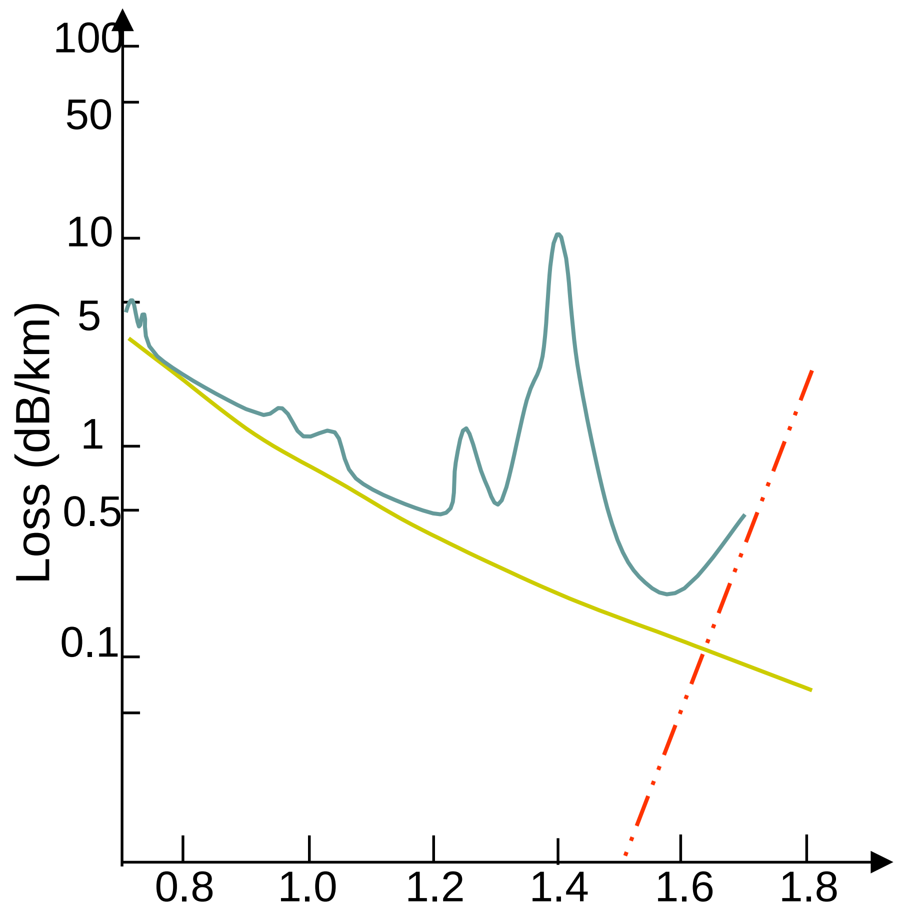
$$B = f_1 - f_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_1 + \Delta\lambda}$$

$$= \frac{v}{\lambda_1}\left\{\frac{\Delta\lambda / \lambda_1}{1 + \Delta\lambda / \lambda 1}\right\} \approx \frac{v\ \Delta\lambda}{\lambda_1^2}$$

- Example: $\lambda_1 = 1450$ nm

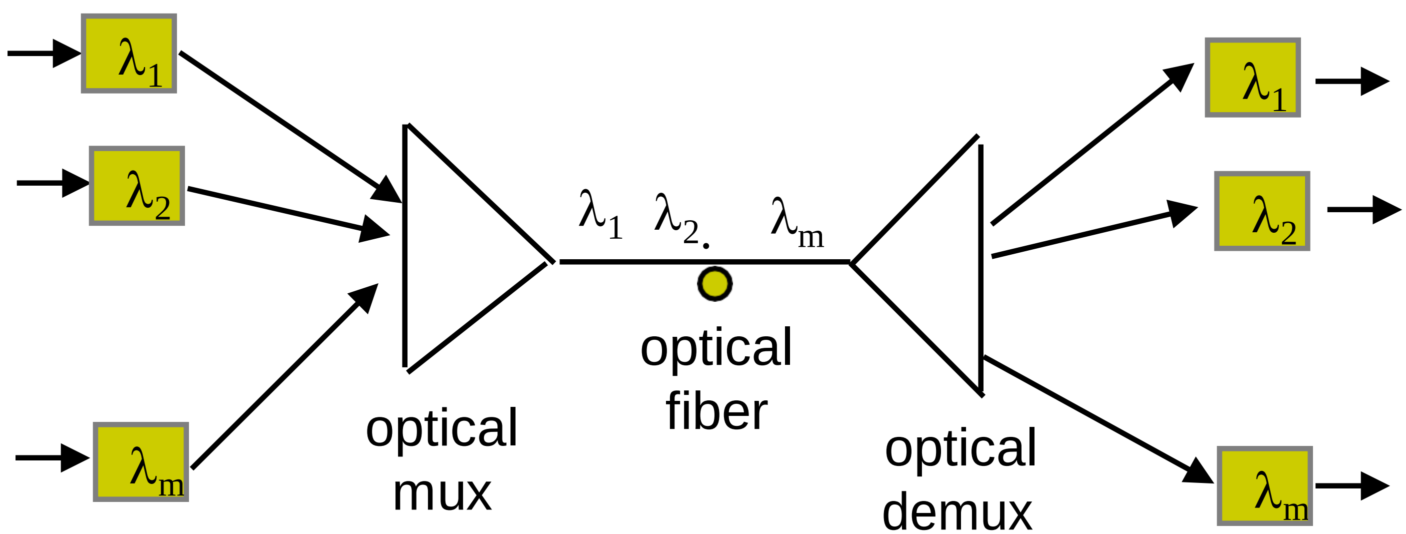
$\lambda_1 + \Delta\lambda = 1650$ nm:

$$B = \frac{2(10^8)\text{m/s}\ 200\text{nm}}{(1450\ \text{nm})^2} \approx 19\ \text{THz}$$

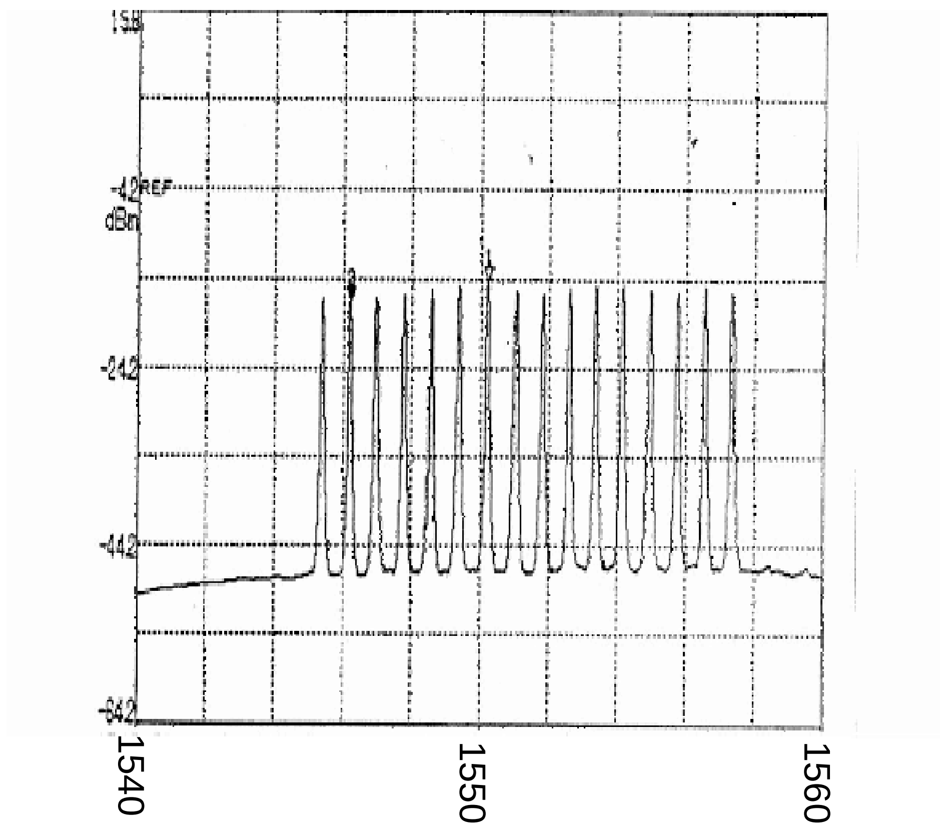


Wavelength-Division Multiplexing

- Different wavelengths carry separate signals
- Multiplex into shared optical fiber
- Each wavelength like a separate circuit
- A single fiber can carry 160 wavelengths, 10 Gbps per wavelength: 1.6 Tbps!



Coarse & Dense WDM



Coarse WDM

- Few wavelengths 4-8 with very wide spacing
- Low-cost, simple

Dense WDM

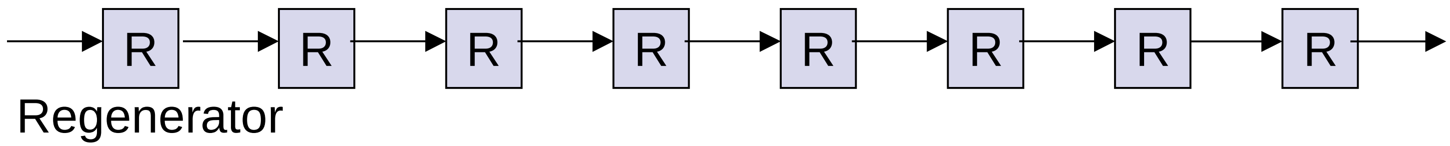
- Many tightly-packed wavelengths
- ITU Grid: 0.8 nm separation for 10Gbps signals
- 0.4 nm for 2.5 Gbps

Regenerators & Optical Amplifiers

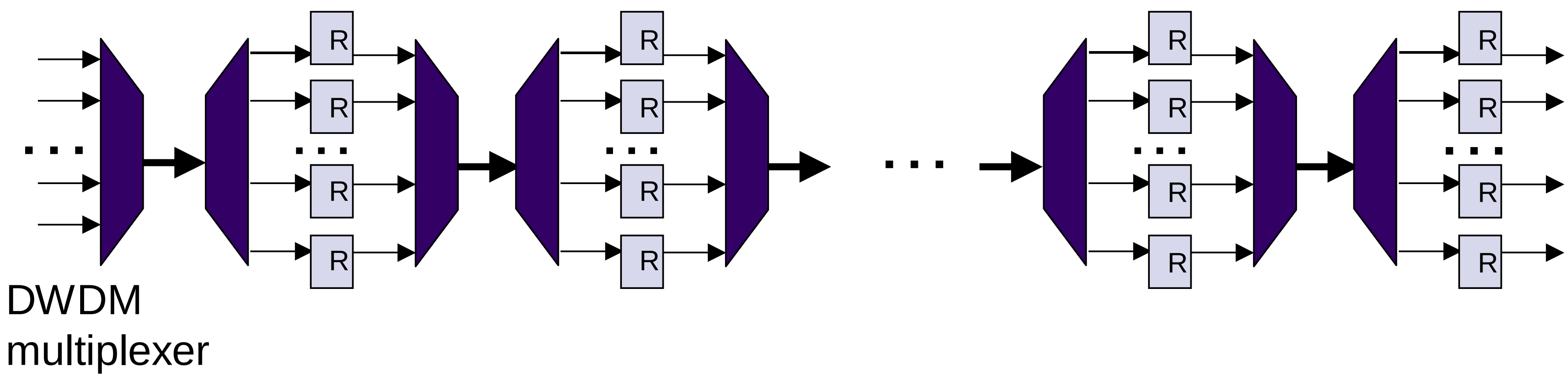
- The maximum span of an optical signal is determined by the available power & the attenuation:
  - Ex. If 30 dB power available,
  - then at 1550 nm, optical signal attenuates at 0.25 dB/km,
  - so max span = 30 dB/0.25 km/dB = 120 km
- Optical amplifiers amplify optical signal (no equalization, no regeneration)
- Impairments in optical amplification limit maximum number of optical amplifiers in a path
- Optical signal must be regenerated when this limit is reached
  - Requires optical-to-electrical (O-to-E) signal conversion, equalization, detection and retransmission (E-to-O)
  - Expensive
- Severe problem with WDM systems

DWDM & Regeneration

- Single signal per fiber requires 1 regenerator per span

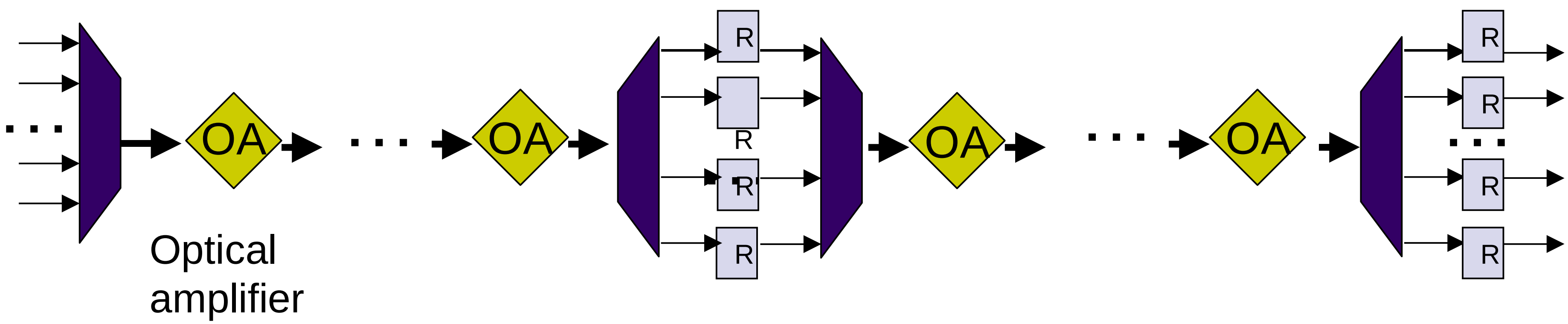


- DWDM system carries many signals in one fiber
- At each span, a separate regenerator required per signal
- Very expensive



Optical Amplifiers

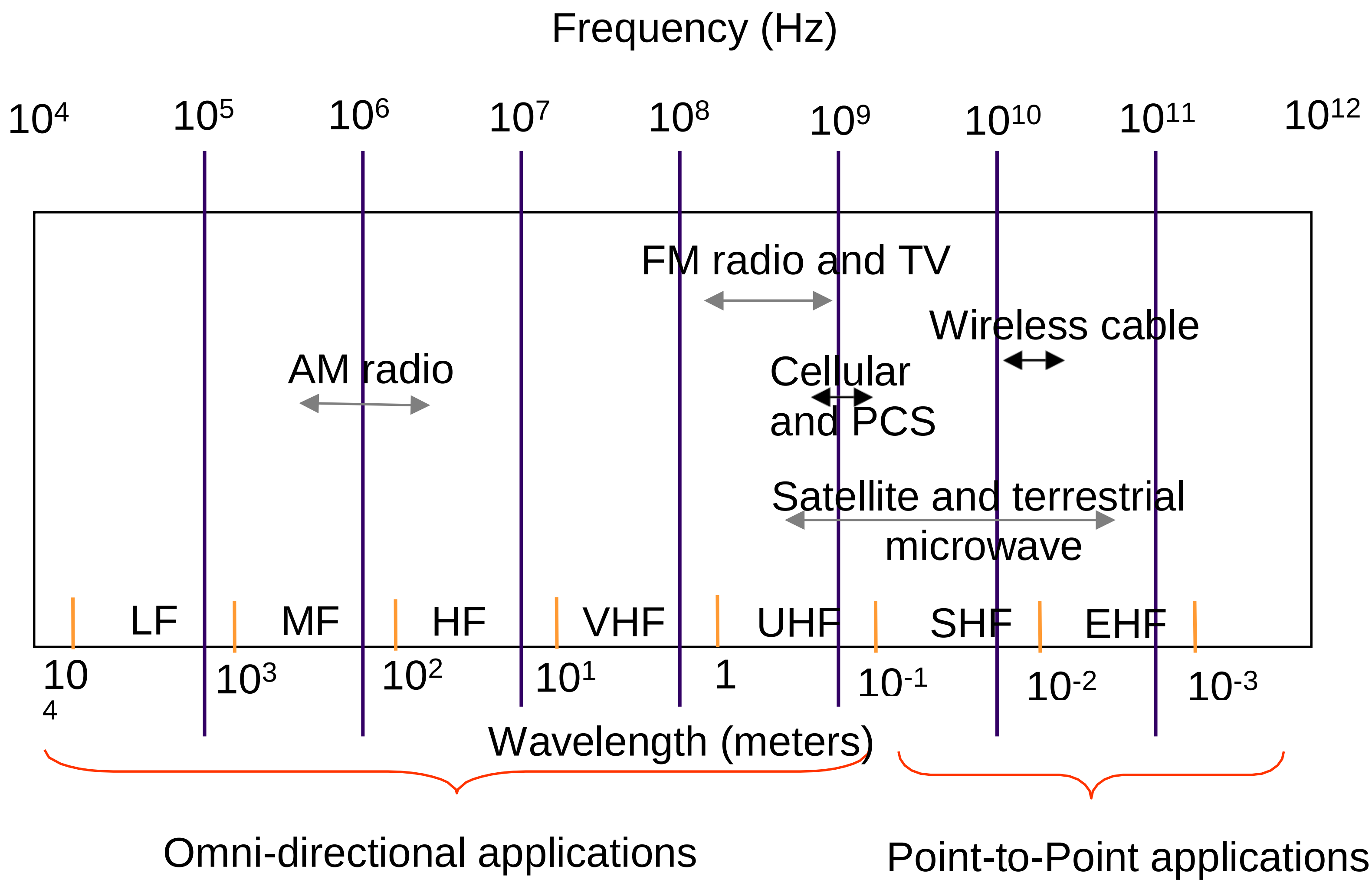
- Optical amplifiers can amplify the composite DWDM signal without demuxing or O-to-E conversion
- Erbium Doped Fiber Amplifiers (EDFAs) boost DWDM signals within 1530 to 1620 range
  - Spans between regeneration points >1000 km
  - Number of regenerators can be reduced dramatically
- Dramatic reduction in cost of long-distance communications



Radio Transmission

- Radio signals: antenna transmits sinusoidal signal ("carrier") that radiates in air/space
- Information embedded in carrier signal using modulation, e.g. QAM
- Communications without tethering
  - Cellular phones, satellite transmissions, Wireless LANs
- Multipath propagation causes fading
- Interference from other users
- Spectrum regulated by national & international regulatory organizations

Radio Spectrum



Examples

Cellular Phone

- Allocated spectrum
- First generation:
  - 800, 900 MHz
  - Initially analog voice
- Second generation:
  - 1800-1900 MHz
  - Digital voice, messaging

Wireless LAN

- Unlicenced ISM spectrum
  - Industrial, Scientific, Medical
  - 902-928 MHz, 2.400-2.4835 GHz, 5.725-5.850 GHz
- IEEE 802.11 LAN standard
  - 11-54 Mbps

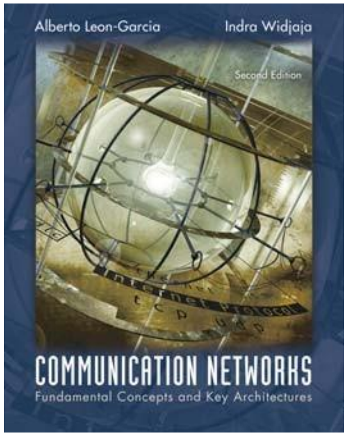
Point-to-Multipoint Systems

- Directional antennas at microwave frequencies
- High-speed digital communications between sites
- High-speed Internet Access Radio backbone links for rural areas

Satellite Communications

- Geostationary satellite @ 36000 km above equator
- Relays microwave signals from uplink frequency to downlink frequency
- Long distance telephone
- Satellite TV broadcast

Chapter 3
Digital Transmission Fundamentals



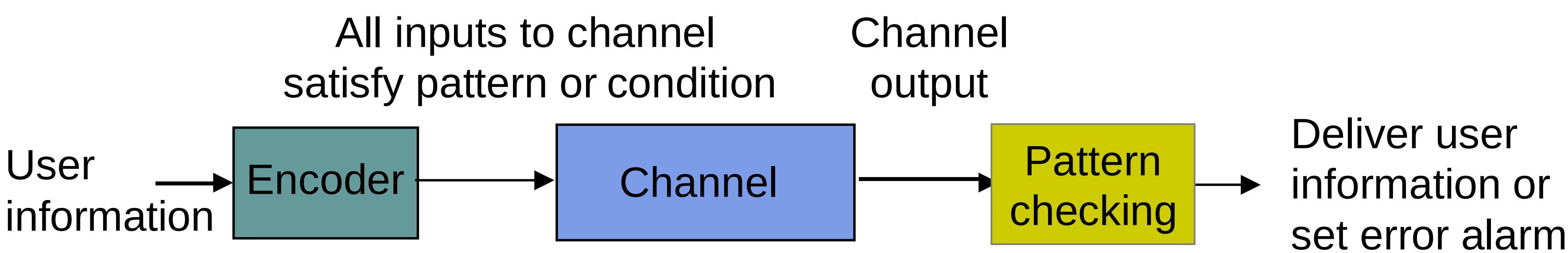
Error Detection and Correction

Error Control

- Digital transmission systems introduce errors
- Applications require certain reliability level
  - Data applications require error-free transfer
  - Voice & video applications tolerate some errors
- Error control used when transmission system does *not* meet application requirement
- Error control ensures a data stream is transmitted to a certain level of accuracy despite errors
- Two basic approaches:
  - Error ***detection*** & retransmission (ARQ)
  - Forward error ***correction*** (FEC)

Key Idea

- All transmitted data blocks ("codewords") satisfy a pattern
- If received block doesn't satisfy pattern, it is in error
- Redundancy: Only a subset of all possible blocks can be codewords
- Blindspot: when channel transforms a codeword into another codeword



Single Parity Check

- Append an overall parity check to *k* information bits

Info Bits: $b_1, b_2, b_3, \ldots, b_k$

Check Bit: $b_{k+1} = b_1 + b_2 + b_3 + \ldots + b_k$ modulo 2

Codeword: $(b_1, b_2, b_3, \ldots, b_{k,}, b_{k+!})$

- All codewords have even # of 1s
- Receiver checks to see if # of 1s is even
  - All error patterns that change an odd # of bits are detectable
  - All even-numbered patterns are undetectable
- Parity bit used in ASCII code

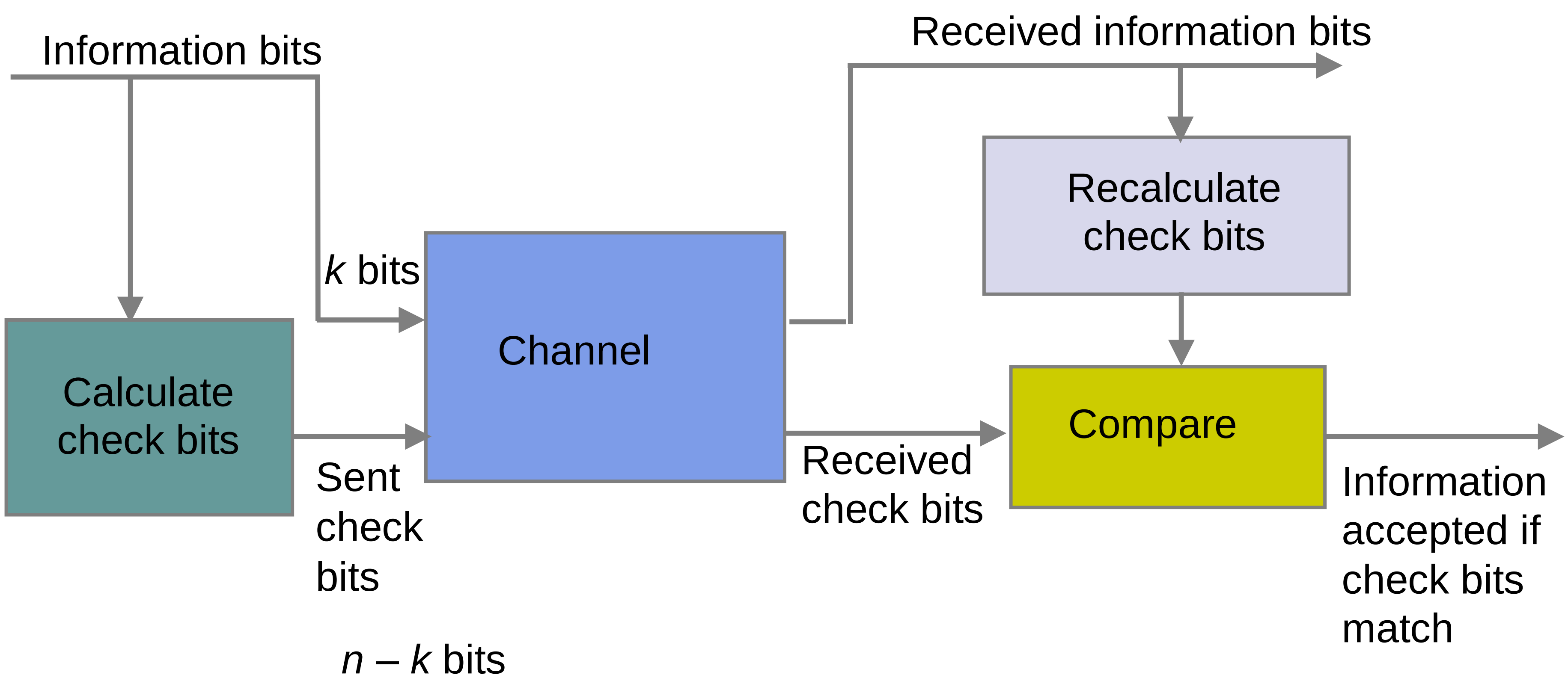
Example of Single Parity Code

- Information (7 bits): (0, 1, 0, 1, 1, 0, 0)
- Parity Bit: $b_8 = 0 + 1 + 0 + 1 + 1 + 0 = 1$
- Codeword (8 bits): (0, 1, 0, 1, 1, 0, 0, 1)

- If single error in bit 3 : (0, 1, 1, 1, 1, 0, 0, 1)
  - # of 1's =5, odd
  - Error detected

- If errors in bits 3 and 5: (0, 1, 1, 1, 0, 0, 0, 1)
  - # of 1's =4, even
  - Error not detected

Checkbits & Error Detection



How good is the single parity check code?

- *Redundancy*: Single parity check code adds 1 redundant bit per k information bits: overhead = $1/(k + 1)$
- *Coverage*: all error patterns with odd # of errors can be detected
  - An error patten is a binary $(k + 1)$-tuple with 1s where errors occur and 0's elsewhere
  - Of 2^{k+1} binary $(k + 1)$-tuples, ½ are odd, so 50% of error patterns can be detected
- Is it possible to detect more errors if we add more check bits?
- Yes, with the right codes

What if bit errors are random?

- Many transmission channels introduce bit errors at random, independently of each other, and with probability *p*
- Some error patterns are more probable than others:

$$P[10000000] = p(1-p)^7 = (1-p)^8 \left(\frac{p}{1-p}\right) \text{ and}$$

$$P[11000000] = p^2(1-p)^6 = (1-p)^8 \left(\frac{p}{1-p}\right)^2$$

- In any worthwhile channel $p < 0.5$, and so $(p/(1-p) < 1$
- It follows that patterns with 1 error are more likely than patterns with 2 errors and so forth
- What is the probability that an undetectable error pattern occurs?

Single parity check code with random bit errors

- Undetectable error pattern if even # of bit errors:

$$P[\text{error detection failure}] = P[\text{undetectable error pattern}]$$
$$= P[\text{error patterns with even number of 1s}]$$

$$= \binom{n}{2} p^2(1-p)^{n-2} + \binom{n}{4} p^4(1-p)^{n-4} + \ldots$$

- Example: Evaluate above for $n = 32$, $p = 10^{-3}$

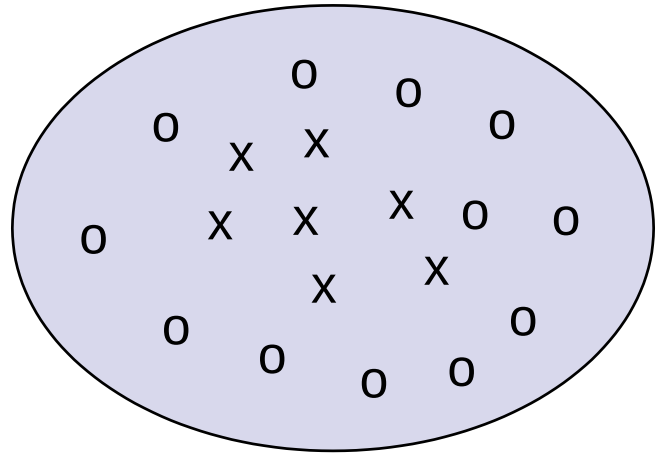
$$P[\text{undetectable error}] = \binom{32}{2} (10^{-3})^2 (1-10^{-3})^{30} + \binom{32}{4} (10^{-3})^4 (1-10^{-3})^{28}$$

$$\approx 496\,(10^{-6}) + 35960\,(10^{-12}) \approx 4.96\,(10^{-4})$$

- For this example, roughly 1 in 2000 error patterns is undetectable

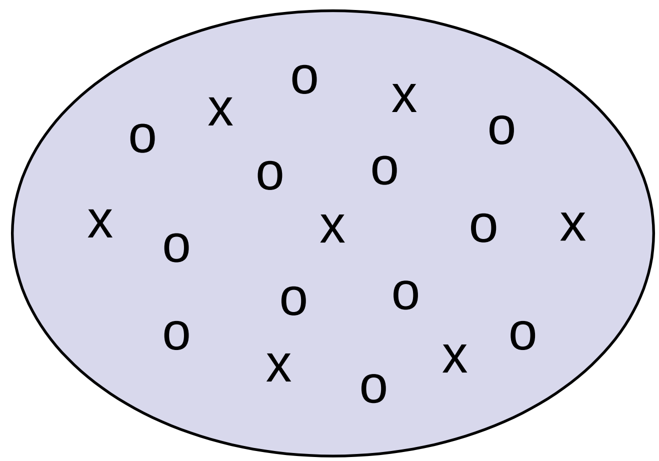
What is a good code?

- Many channels have preference for error patterns that have fewer # of errors
- These error patterns map transmitted codeword to nearby *n*-tuple
- If codewords close to each other then detection failures will occur
- Good codes should maximize separation between codewords



Poor distance properties

x = codewords
o = **noncodewords**



Good distance properties

Two-Dimensional Parity Check

- More parity bits to improve coverage
- Arrange information as columns
- Add single parity bit to each column
- Add a final "parity" column
- Used in early error control systems

1	0	0	1	0	0
0	1	0	0	0	1
1	0	0	1	0	0
1	1	0	1	1	0
1	0	0	1	1	1

Last column consists of check bits for each row

Bottom row consists of check bit for each column

Error-detecting capability

One error

1	0	0	1	0	0
0	**0**	0	0	0	1 ←
1	0	0	1	0	0
1	1	0	1	1	0
1	0	0	1	1	1
	↑				

Two errors

1	0	0	1	0	0
0	**0**	0	0	0	1 ←
1	0	0	1	0	0
1	**0**	0	1	1	0 ←
1	0	0	1	1	1

1, 2, or 3 errors can always be detected; Not all patterns >4 errors can be detected

Three errors

1	0	0	1	0	0
0	**0**	0	**1**	0	1
1	0	0	1	0	0
1	**0**	0	1	1	0 ←
1	0	0	1	1	1
			↑		

Four errors (undetectable)

1	0	0	1	0	0
0	**0**	0	**1**	0	1
1	0	0	1	0	0
1	**0**	0	**0**	1	0
1	0	0	1	1	1

Arrows indicate failed check bits

Other Error Detection Codes

- Many applications require very low error rate
- Need codes that detect the vast majority of errors
- Single parity check codes do not detect enough errors
- Two-dimensional codes require too many check bits
- The following error detecting codes used in practice:
  - Internet Check Sums
  - CRC Polynomial Codes

Internet Checksum

- Several Internet protocols (e.g. IP, TCP, UDP) use check bits to detect errors in the *IP header* (or in the header and data for TCP/UDP)
- A checksum is calculated for header contents and included in a special field.
- Checksum recalculated at every router, so algorithm selected for ease of implementation in software
- Let header consist of L, 16-bit words,

  $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{L-1}$
- The algorithm appends a 16-bit checksum $\mathbf{b}_L$

Checksum Calculation

The checksum $\mathbf{b}_L$ is calculated as follows:

- Treating each 16-bit word as an integer, find

$$\mathbf{x} = \mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \ldots + \mathbf{b}_{L-1} \text{ modulo } 2^{16}-1$$

- The checksum is then given by:

$$\mathbf{b}_L = -\,\mathbf{x} \quad \text{modulo } 2^{16}-1$$

Thus, the headers must satisfy the following ***pattern***:

$$\mathbf{0} = \mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \ldots + \mathbf{b}_{L-1} + \mathbf{b}_L \text{ modulo } 2^{16}-1$$

- The checksum calculation is carried out in software using one's complement arithmetic

Internet Checksum Example

Use Modulo Arithmetic

- Assume 4-bit words
- Use mod 2^4-1 arithmetic
- $\underline{b}_0$=1100 = 12
- $\underline{b}_1$=1010 = 10
- $\underline{b}_0$+$\underline{b}_1$=12+10=7 mod15
- $\underline{b}_2$ = -7 = 8 mod15
- Therefore
- $\underline{b}_2$=1000

Use Binary Arithmetic

- Note 16 =1 mod15
- So: 10000 = 0001 mod15
- leading bit wraps around

$b_0 + b_1$ = 1100+1010
=10110
=10000+0110
=0001+0110
=0111
=7

Take 1s complement

b_2 = -0111 =1000

Polynomial Codes

- Polynomials instead of vectors for codewords
- Polynomial arithmetic instead of check sums
- Implemented using shift-register circuits
- Also called *cyclic redundancy check (CRC)* codes
- Most data communications standards use polynomial codes for error detection
- Polynomial codes also basis for powerful error-correction methods

Binary Polynomial Arithmetic

- Binary vectors map to polynomials

$$(i_{k-1}, i_{k-2}, \ldots, i_2, i_1, i_0) \text{ ⓪ } i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \ldots + i_2x^2 + i_1x + i_0$$

Addition:

$$(x^7 + x^6 + 1) + (x^6 + x^5) = x^7 + x^6 + x^6 + x^5 + 1$$

$$= x^7 + (1+1)x^6 + x^5 + 1$$

$$= x^7 + x^5 + 1 \quad \text{since } 1+1=0 \text{ mod2}$$

Multiplication:

$$(x + 1)(x^2 + x + 1) = x(x^2 + x + 1) + 1(x^2 + x + 1)$$

$$= x^3 + x^2 + x) + (x^2 + x + 1)$$

$$= x^3 + 1$$

Binary Polynomial Division

- Division with Decimal Numbers

```
          34        ← quotient
    35 ) 1222       ← dividend
divisor   105
           172
           140
            32      ← remainder
```

dividend = quotient x divisor +remainder

1222 = 34 x 35 + 32

- Polynomial Division

$$
\begin{array}{rl}
 & x^3 + x^2 + x \qquad = q(x) \text{ quotient} \\
x^3 + x + 1 \,) & x^6 + x^5 \qquad \leftarrow \text{dividend} \\
\text{divisor} & x^6 + \quad x^4 + x^3 \\
 & x^5 + x^4 + x^3 \\
 & x^5 + \quad x^3 + x^2 \\
 & x^4 + \quad x^2 \\
 & x^4 + \quad x^2 + x \\
 & x \qquad = r(x) \text{ remainder}
\end{array}
$$

Note: Degree of r(x) is less than degree of divisor

Polynomial Coding

- Code has binary *generating polynomial* of degree $n–k$

$$g(x) = x^{n-k} + g_{n-k-1}x^{n-k-1} + \ldots + g_2x^2 + g_1x + 1$$

- *k information bits* define polynomial of degree $k – 1$

$$i(x) = i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \ldots + i_2x^2 + i_1x + i_0$$

- Find *remainder polynomial* of at most degree $n – k – 1$

$$\begin{array}{r} q(x) \\ g(x)\ \overline{)\ x^{n-k}i(x)} \\ r(x) \end{array}$$

$$x^{n-k}i(x) = q(x)g(x) + r(x)$$

- Define the *codeword polynomial* of degree $n – 1$

$$\underbrace{b(x)}_{n \text{ bits}} = \underbrace{x^{n-k}i(x)}_{k \text{ bits}} + \underbrace{r(x)}_{n-k \text{ bits}}$$

Polynomial example: $k = 4$, $n - k = 3$

Generator polynomial: $g(x) = x^3 + x + 1$

Information: (1,1,0,0) $i(x) = x^3 + x^2$

Encoding: $x^3 i(x) = x^6 + x^5$

$$
\begin{array}{r l}
 & x^3 + x^2 + x \\
\hline
x^3 + x + 1 \,) & x^6 + x^5 \\
 & x^6 + \quad x^4 + x^3 \\
\hline
 & \quad x^5 + x^4 + x^3 \\
 & \quad x^5 + \quad x^3 + x^2 \\
\hline
 & \qquad x^4 + \quad x^2 \\
 & \qquad x^4 + \quad x^2 + x \\
\hline
 & \qquad\qquad x
\end{array}
$$

$$
\begin{array}{r l}
 & 1110 \\
\hline
1011 \,) & 1100000 \\
 & 1011 \\
\hline
 & \;1110 \\
 & \;1011 \\
\hline
 & \;\;1010 \\
 & \;\;1011 \\
\hline
 & \;\;\;010
\end{array}
$$

Transmitted codeword:

$b(x) = x^6 + x^5 + x$

$\Longrightarrow$ $\underline{b} = (1,1,0,0,0,1,0)$

The *Pattern* in Polynomial Coding

- All codewords satisfy the following **pattern**:

$$b(x) = x^{n-k}i(x) + r(x) = q(x)g(x) + r(x) + r(x) = q(x)g(x)$$

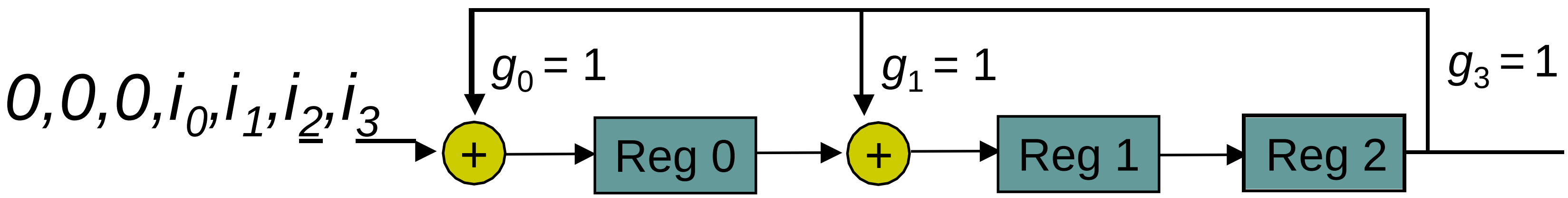
- All codewords are a multiple of *g(x)!*
- Receiver should divide received n-tuple by g(x) and check if remainder is zero
- If remainder is nonzero, then received n-tuple is not a codeword

Shift-Register Implementation

1. Accept information bits $i_{k-1}, i_{k-2}, \ldots, i_2, i_1, i_0$
2. Append $n - k$ zeros to information bits
3. Feed sequence to shift-register circuit that performs polynomial division
4. After n shifts, the shift register contains the remainder

Division Circuit

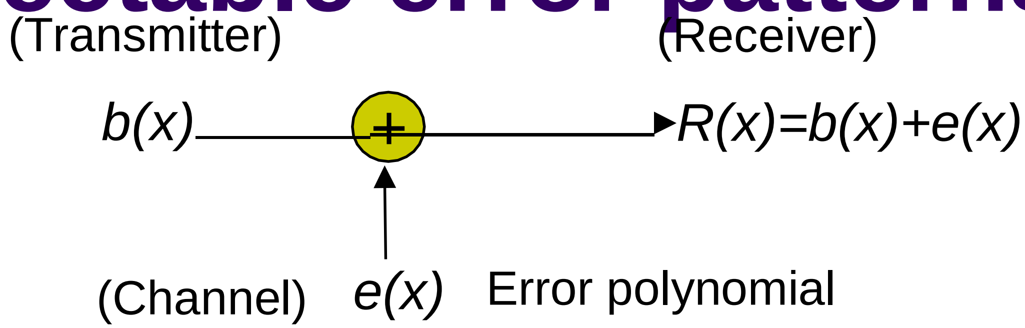
Encoder for $g(x) = x^3 + x + 1$



Clock	Input	Reg 0	Reg 1	Reg 2
0	-	0	0	0
1	$1 = i_3$	1	0	0
2	$1 = i_2$	1	1	0
3	$0 = i_1$	0	1	1
4	$0 = i_0$	1	1	1
5	0	1	0	1
6	0	1	0	0
7	0	**0**	**1**	**0**
	Check bits:	$\boldsymbol{r_0 = 0}$	$\boldsymbol{r_1 = 1}$	$\boldsymbol{r_2 = 0}$

$\Longrightarrow$ $\boldsymbol{r(x) = x}$

Undetectable error patterns



- *e(x)* has 1s in error locations & 0s elsewhere
- Receiver divides the received polynomial *R(x)* by *g(x)*
- Blindspot: If *e(x)* is a multiple of *g(x)*, that is, *e(x)* is a nonzero codeword, then

$$R(x) = b(x) + e(x) = q(x)g(x) + q'(x)g(x)$$

- *The set of undetectable error polynomials is the set of nonzero code polynomials*
- *Choose the generator polynomial so that selected error patterns can be detected.*

Designing good polynomial codes

- Select generator polynomial so that likely error patterns are not multiples of *g(x)*
- *Detecting Single Errors*
  - $e(x) = x^i$ for error in location $i + 1$
  - If *g(x)* has more than 1 term, it cannot divide x^i
- *Detecting Double Errors*
  - $e(x) = x^i + x^j = x^i(x^{j-i}+1)$ *where* $j>i$
  - If *g(x)* has more than 1 term, it cannot divide x^i
  - If g(x) is a *primitive* polynomial, it cannot divide x^m+1 for all $m<2^{n-k}-1$ (Need to keep codeword length less than $2^{n-k}-1$)
  - Primitive polynomials can be found by consulting coding theory books

Designing good polynomial codes

- *Detecting Odd Numbers of Errors*
  - Suppose all codeword polynomials have an even # of 1s, then all odd numbers of errors can be detected
  - As well, $b(x)$ evaluated at $x = 1$ is zero because $b(x)$ has an even number of 1s
  - This implies $x + 1$ must be a factor of all $b(x)$
  - Pick $g(x) = (x + 1)\ p(x)$ where $p(x)$ is primitive

Standard Generator Polynomials

CRC = cyclic redundancy check

- CRC-8:

  $= x^8 + x^2 + x + 1$ ATM

- CRC-16:

  $= x^{16} + x^{15} + x^2 + 1$ Bisync

  $= (x + 1)(x^{15} + x + 1)$

- CCITT-16:

  $= x^{16} + x^{12} + x^5 + 1$ HDLC, XMODEM, V.41

- CCITT-32: IEEE 802, DoD, V.42

  $= x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$

Hamming Codes

- Class of *error-correcting* codes
- Capable of correcting all *single-error* patterns
- For each $m \geq 2$, there is a Hamming code of length $n = 2^m - 1$ with $n - k = m$ parity check bits

Redundancy

m	$n = 2^m - 1$	$k = n - m$	m/n
3	7	4	3/7
4	15	11	4/15
5	31	26	5/31
6	63	57	6/63

m = 3 Hamming Code

- Information bits are b_1, b_2, b_3, b_4
- Equations for parity checks b_5, b_6, b_7

$$b_5 = b_1 + b_3 + b_4$$

$$b_6 = b_1 + b_2 + b_4$$

$$b_7 = + b_2 + b_3 + b_4$$

- There are $2^4 = 16$ codewords
- (0,0,0,0,0,0,0) is a codeword

Hamming (7,4) code

Information				Codeword							Weight
b_1	b_2	b_3	b_4	b_1	b_2	b_3	b_4	b_5	b_6	b_7	$w(\underline{b})$
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	1	1	1	4
0	0	1	0	0	0	1	0	1	0	1	3
0	0	1	1	0	0	1	1	0	1	0	3
0	1	0	0	0	1	0	0	0	1	1	3
0	1	0	1	0	1	0	1	1	0	0	3
0	1	1	0	0	1	1	0	1	1	0	4
0	1	1	1	0	1	1	1	0	0	1	4
1	0	0	0	1	0	0	0	1	1	0	3
1	0	0	1	1	0	0	1	0	0	1	3
1	0	1	0	1	0	1	0	0	1	1	4
1	0	1	1	1	0	1	1	1	0	0	4
1	1	0	0	1	1	0	0	1	0	1	4
1	1	0	1	1	1	0	1	0	1	0	4
1	1	1	0	1	1	1	0	0	0	0	3
1	1	1	1	1	1	1	1	1	1	1	7

Parity Check Equations

- Rearrange parity check equations:

$$0 = b_5 + b_5 = b_1 \quad + b_3 + b_4 + b_5$$

$$0 = b_6 + b_6 = b_1 + b_2 \quad + b_4 \quad + b_6$$

$$0 = b_7 + b_7 = \quad + b_2 + b_3 + b_4 \quad + b_7$$

- In matrix form:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{bmatrix} 1\,0\,1\,1\,1\,0\,0 \\ 1\,1\,0\,1\,0\,1\,0 \\ 0\,1\,1\,1\,0\,0\,1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} = \mathbf{H}\, b^t = \underline{0}$$

- All codewords must satisfy these equations
- Note: each nonzero 3-tuple appears once as a column in **check matrix H**

Error Detection with Hamming Code

$$\underline{s} = \mathbf{H}\,\underline{e} = \begin{bmatrix} 1&0&1&1&1&0&0 \\ 1&1&0&1&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix} \begin{bmatrix} 0\\0\\1\\0\\0\\0\\0 \end{bmatrix} = \begin{bmatrix} 1\\0\\1 \end{bmatrix}$$

Single error detected

$$\underline{s} = \mathbf{H}\,\underline{e} = \begin{bmatrix} 1&0&1&1&1&0&0 \\ 1&1&0&1&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix} \begin{bmatrix} 0\\1\\0\\0\\1\\0\\0 \end{bmatrix} = \begin{bmatrix} 0\\1\\1 \end{bmatrix} + \begin{bmatrix} 1\\0\\0 \end{bmatrix} = \begin{bmatrix} 1\\1\\1 \end{bmatrix}$$

Double error detected

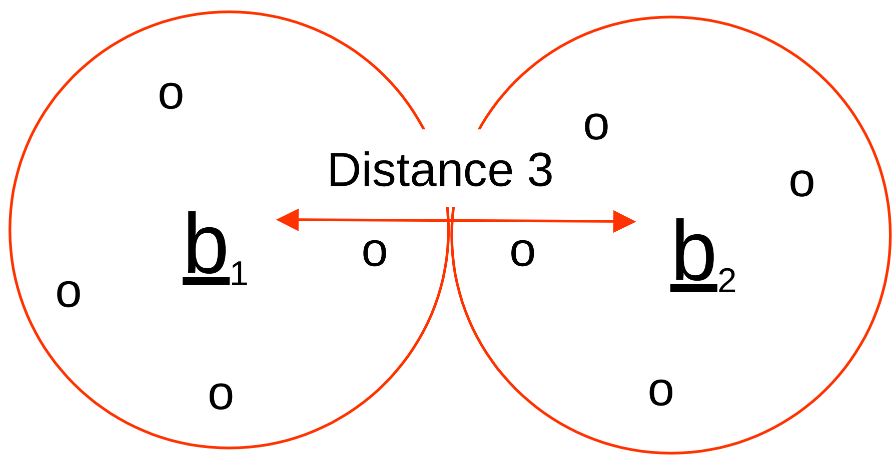
$$\underline{s} = \mathbf{H}\,\underline{e} = \begin{bmatrix} 1&0&1&1&1&0&0 \\ 1&1&0&1&0&1&0 \\ 0&1&1&1&0&0&1 \end{bmatrix} \begin{bmatrix} 1\\1\\1\\0\\0\\0\\0 \end{bmatrix} = \begin{bmatrix} 1\\1\\0 \end{bmatrix} + \begin{bmatrix} 0\\1\\1 \end{bmatrix} + \begin{bmatrix} 1\\0\\1 \end{bmatrix} = \underline{0}$$

Triple error not detected

Minimum distance of Hamming Code

- Previous slide shows that undetectable error pattern must have 3 or more bits
- At least 3 bits must be changed to convert one codeword into another codeword

Set of *n*-tuples within distance 1 of b_1



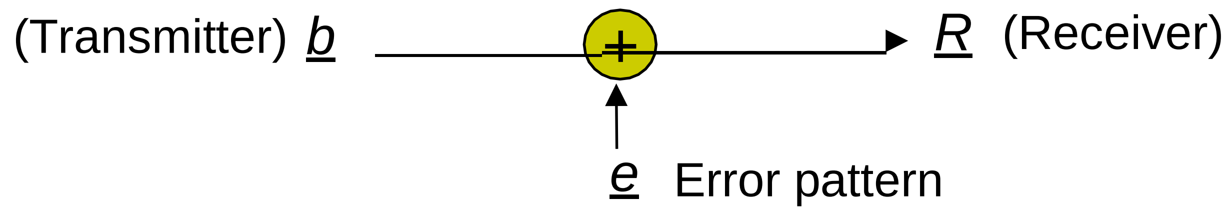
Set of *n*-tuples within distance 1 of b_2

- Spheres of distance 1 around each codeword do not overlap
- If a single error occurs, the resulting *n*-tuple will be in a unique sphere around the original codeword

General Hamming Codes

- For $m \geq 2$, the Hamming code is obtained through the check matrix H:
  - Each nonzero *m*-tuple appears once as a column of H
  - The resulting code corrects all single errors
- For each value of *m*, there is a polynomial code with $g(x)$ of degree *m* that is equivalent to a Hamming code and corrects all single errors
  - For $m = 3$, $g(x) = x^3+x+1$

Error-correction using Hamming Codes



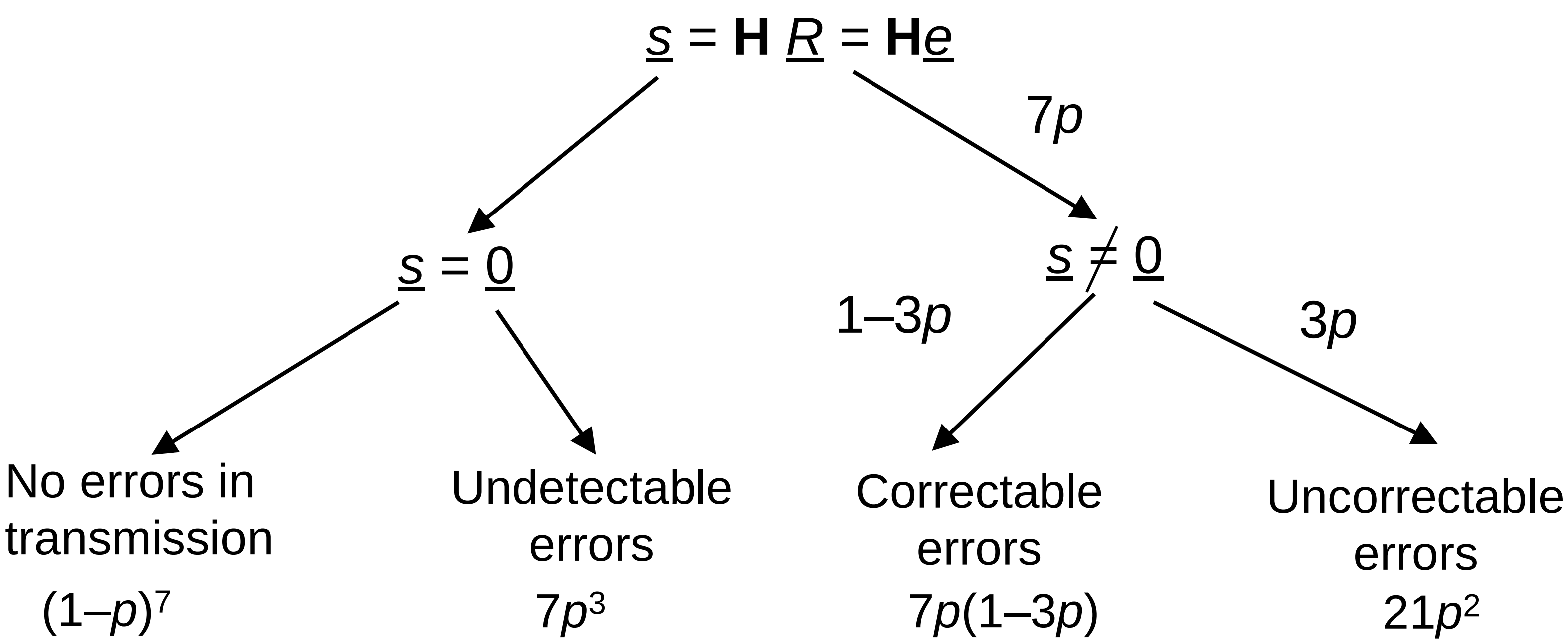
- The receiver first calculates the syndrome:

  $\underline{s} = H\underline{R} = H(\underline{b} + \underline{e}) = H\underline{b} + H\underline{e} = H\underline{e}$

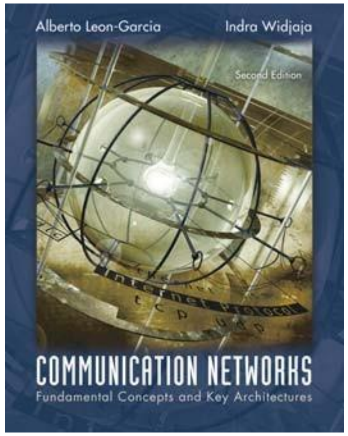
- If $\underline{s} = \underline{0}$, then the receiver accepts $\underline{R}$ as the transmitted codeword
- If $\underline{s}$ is nonzero, then an error is detected
  - Hamming decoder *assumes* a single error has occurred
  - Each single-bit error pattern has a unique syndrome
  - The receiver matches the syndrome to a single-bit error pattern and corrects the appropriate bit

Performance of Hamming Error-Correcting Code

- Assume bit errors occur independent of each other and with probability p



Chapter 3
Digital Transmission Fundamentals



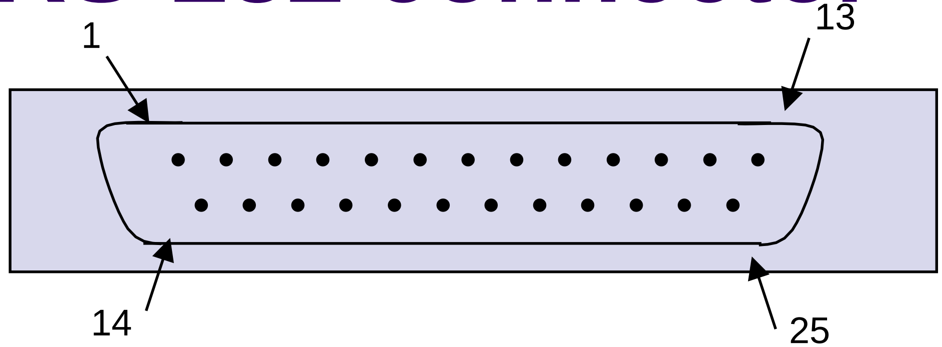
RS-232 Asynchronous Data Transmission

Recommended Standard (RS) 232

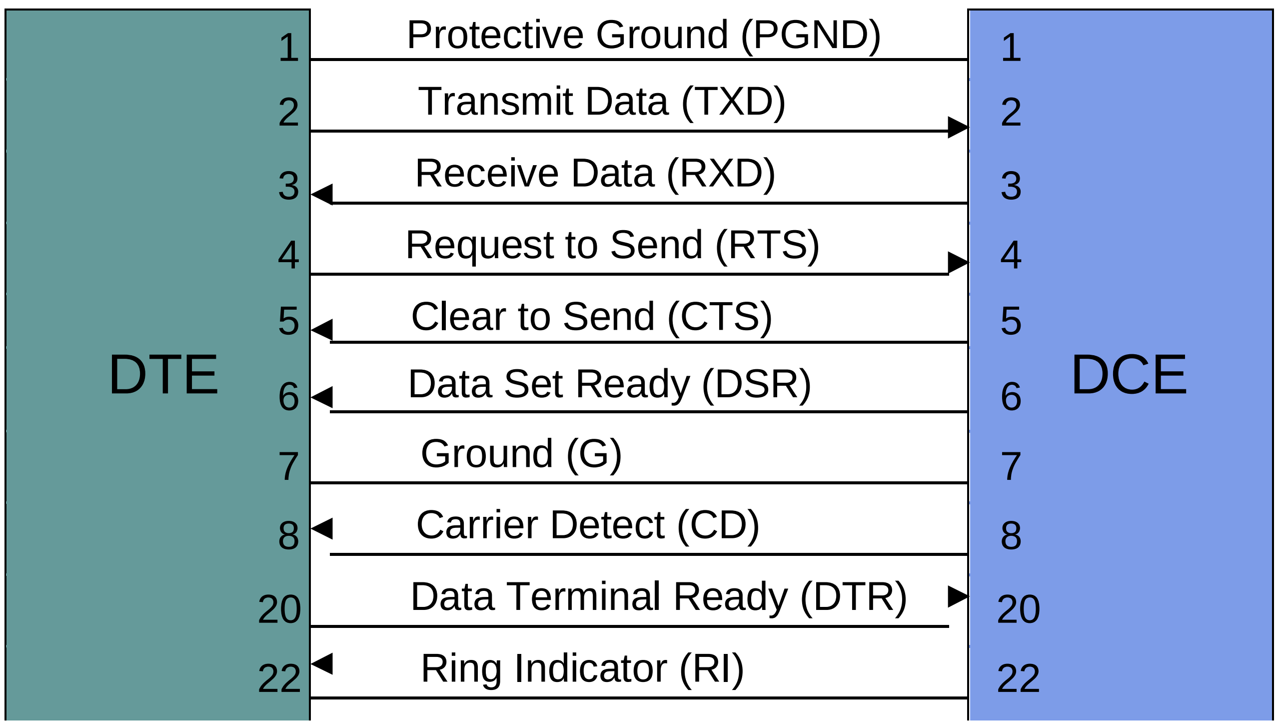
- Serial line interface between computer and modem or similar device
- Data Terminal Equipment (DTE): computer
- Data Communications Equipment (DCE): modem
- Mechanical and Electrical specification

Pins in RS-232 connector

(a)

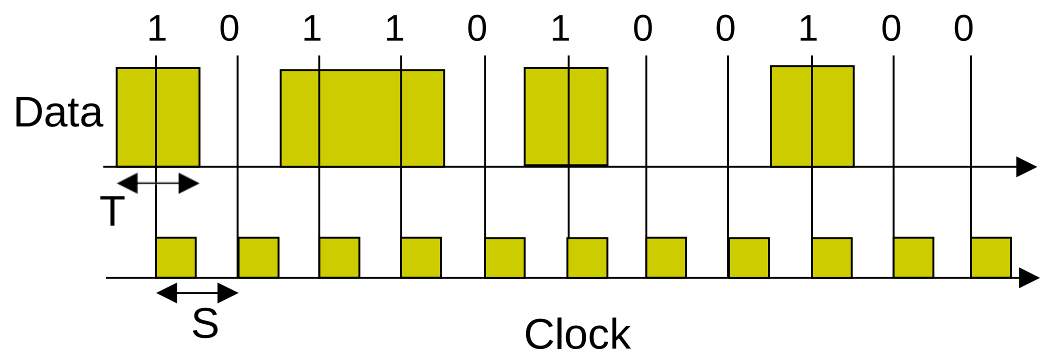


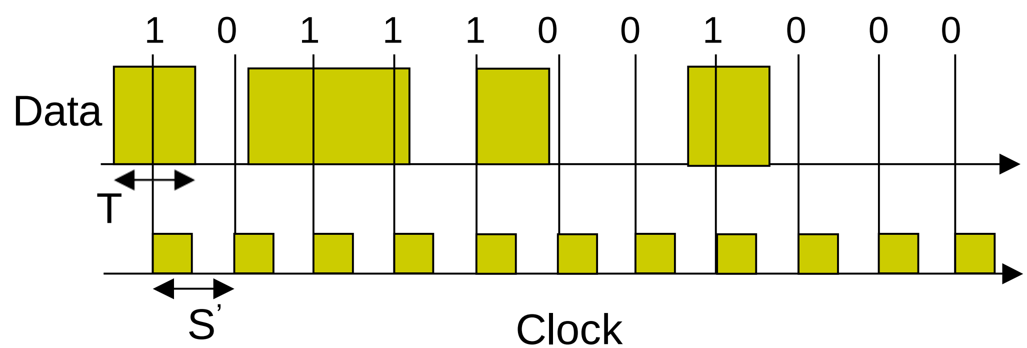
(b)



Synchronization

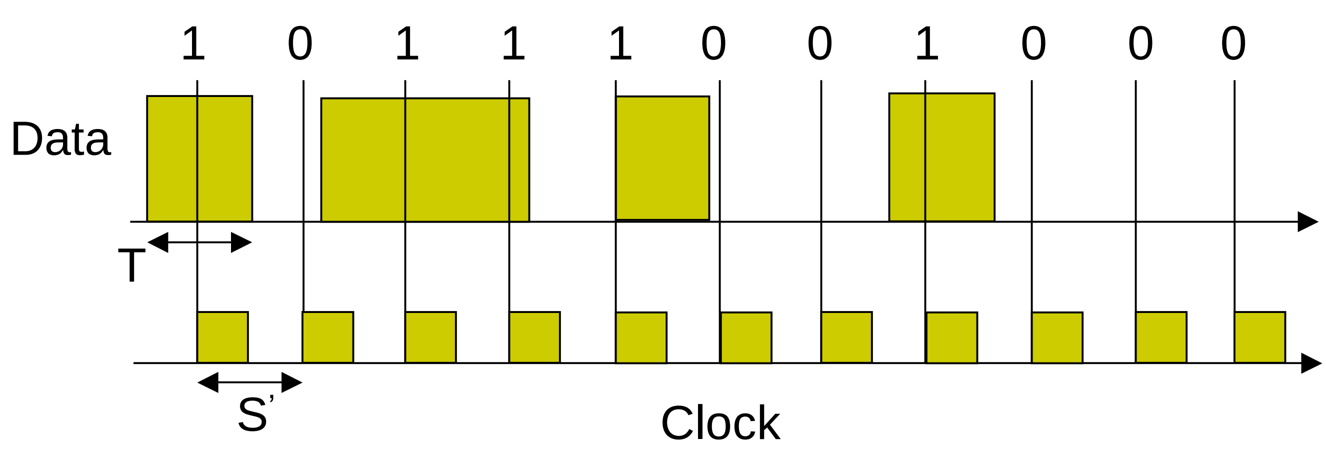
- Synchronization of clocks in transmitters and receivers.
  - clock drift causes a loss of synchronization
- Example: assume '1' and '0' are represented by V volts and 0 volts respectively
  - Correct reception
  - Incorrect reception due to incorrect clock (slower clock)





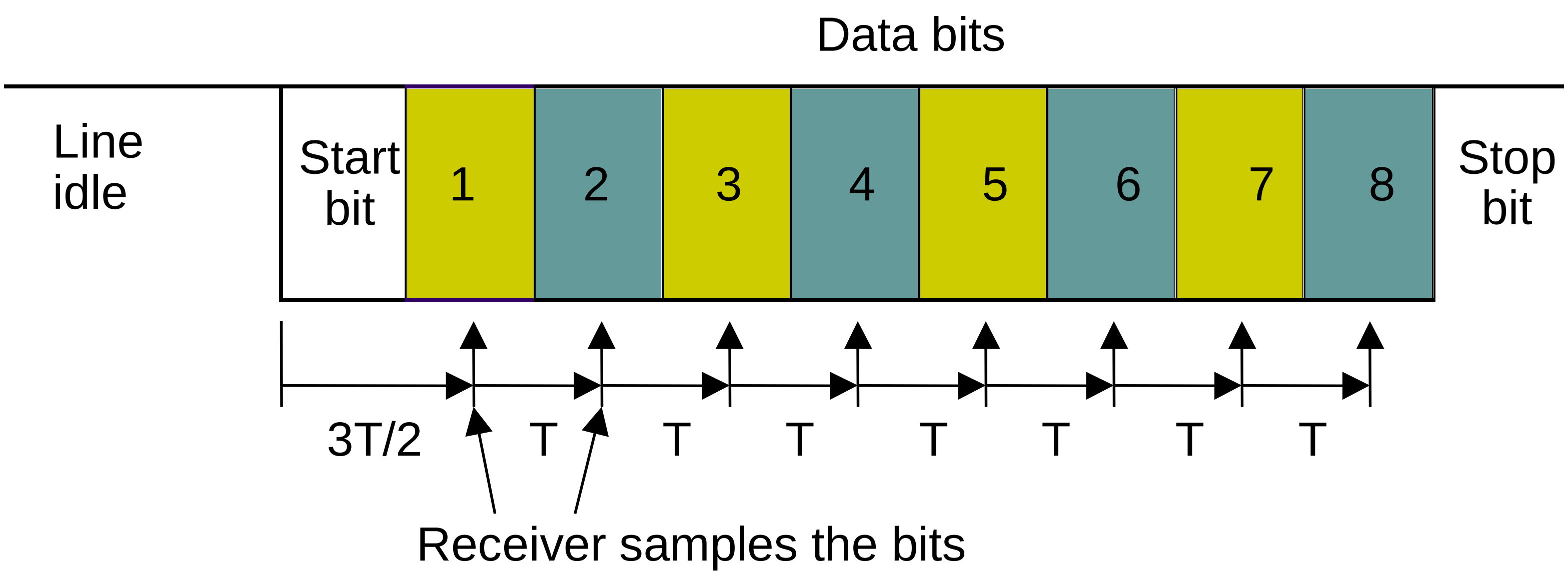
Synchronization (cont'd)

- Incorrect reception (faster clock)
- How to avoid a loss of synchronization?
  - Asynchronous transmission
  - Synchronous transmission



Asynchronous Transmission

- Avoids synchronization loss by specifying a short maximum length for the bit sequences and resetting the clock in the beginning of each bit sequence.
- Accuracy of the clock?



Synchronous Transmission

- Sequence contains data + clock information (line coding)
  - i.e. Manchester encoding, self-synchronizing codes, is used.
- R transition for *R* bits per second transmission
- R transition contains a sine wave with *R* Hz.
- *R* Hz sine wave is used to synch receiver clock to the transmitter's clock using PLL (phase-lock loop)