17BEEC404 COMMUNICATION THEORY **L T P C**
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OBJECTIVES

- To provide various Amplitude modulation and demodulation systems.
- To provide various Angle modulation and demodulation systems.
- To provide some depth analysis in noise performance of various receiver.
- To study some basic information theory with some channel coding theorem.

INTENDED OUTCOMES:

Student will gain knowledge on

- Various Amplitude modulation and demodulation systems.
- Various Angle modulation and demodulation systems.
- Some depth analysis in noise performance of various receiver.
- Some basic information theory with some channel coding theorem.

UNIT-I AMPLITUDE MODULATION

Generation and demodulation of AM, DSB-SC, SSB-SC, VSB Signals, Filtering of sidebands, Comparison of Amplitude modulation systems, Frequency translation, Frequency Division multiplexing, AM transmitters – Super heterodyne receiver, AM receiver.

UNIT-II ANGLE MODULATION

Angle modulation, frequency modulation, Narrowband and wideband FM, transmission bandwidth of FM signals, Generation of FM signal – Direct FM – indirect FM, Demodulation of FM signals, FM stereo multiplexing, PLL – Nonlinear model and linear model of PLL, Non-linear effects in FM systems, FM Broadcast receivers, FM stereo receives.

UNIT-III RANDOM PROCESS

Random variables, Central limit theorem, Random process, Stationary processes, Mean, Correlation & Covariance functions, Power spectral density, Ergodic processes, Gaussian process, Transmission of a Random process through a LTI filter.

UNIT-IV NOISE CHARACTERIZATION

Noise sources and types, Noise figure and Noise temperature, Noise cascaded systems, Narrow band noise, PSD of in-phase and quadrature noise, Noise performance in AM systems, Noise performance in FM systems, Pre-emphasis and de-emphasis, Capture effect, Threshold effect.

UNIT-V INFORMATION THEORY

Uncertainty, Information and entropy, Source coding theorem, Data compaction, Discrete memory less channels, mutual information, channel capacity, channel coding theorem, differential entropy, and mutual information for continuous ensembles, information capacity theorem, implication of the information capacity theorem, rate distortion theory, Compression of information.

TEXT BOOKS:

S.No.	Author(s) Name	Title of the book	Publisher	Year of publication
1	Simon Haykin	Communication Systems	John Wiley & Sons, New Jersy	2011
2	Wayne Tomasi	Electronic Communication Theory Systems	Pearson Edition, New Jersy	2003
3	J.G.Proakis, M.Salehi	Fundamentals of Communication Systems	Pearson Eduction	2006

REFERENCES:

S.No.	Author(s) Name	Title of the book	Publisher	Year of publication
1	Roddy and Coolen	Electronic Communication	PHI, New Delhi	2003
2	Taub and Schilling	Principles of Communication Systems	TMH, New Delhi	2008
3	B.P.Lathi	Modern Digital and Analog Communication Systems 3rd Edition	Oxford University Press	2007

WEBSITES:

1. http:Williamson-labs.com/480_mod.htm
2. www.mit.edu
3. http://www.sfu.ca/~truax/fmtut,html
4. https://www.scribd.com/document/96140878/Comparison-of-Modulation-Methods

UNIT – I

AMPLITUDE MODULATION

INTRODUCTION TO COMMUNICATION SYSTEM

INTRODUCTION TO COMMUNICATION SYSTEMS

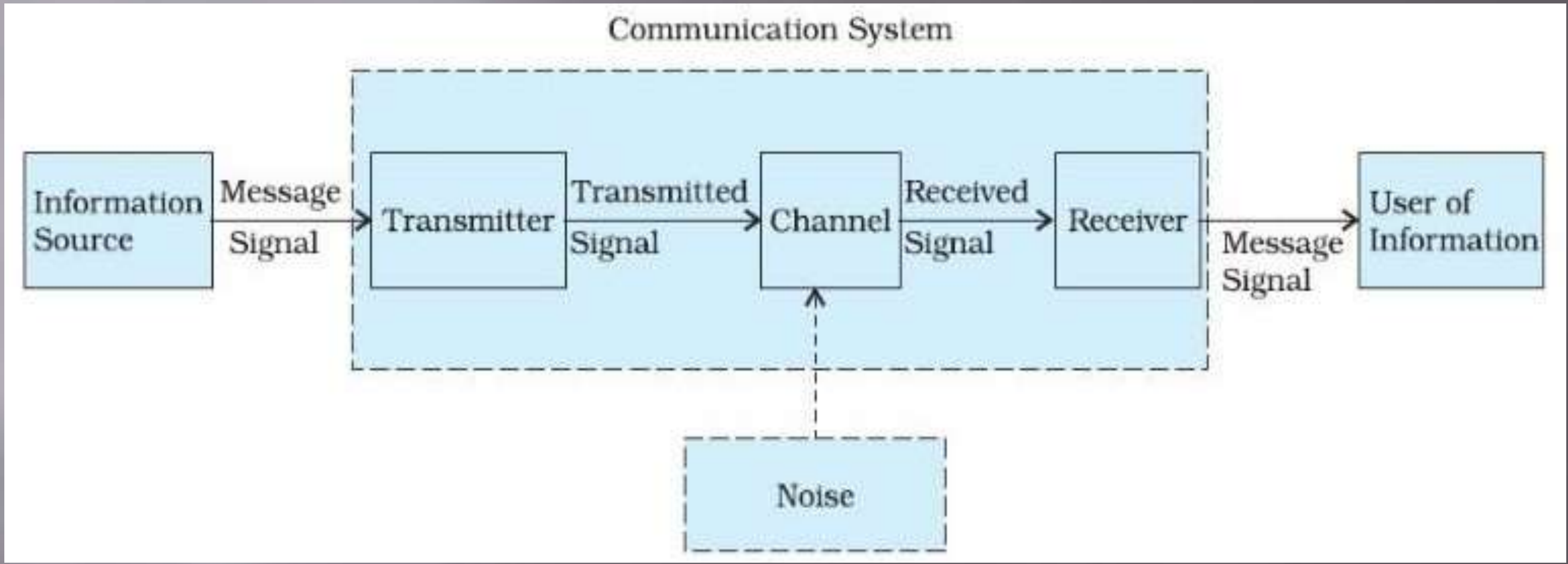
What is Communication ?

Communication is the process of conveying information from one point to another

What is the purpose of Communication system?

- The purpose of communication systems is to communicate information bearing signals from a source located at one point in space to a user destination located at another point.
- The three most common sources of information are: speech (or sound), video and data.
- Regardless of the source, the information that is transmitted and received in a communication system consists of a signal, encoding the information in some appropriate fashion

MODEL OF COMMUNICATION SYSTEMS

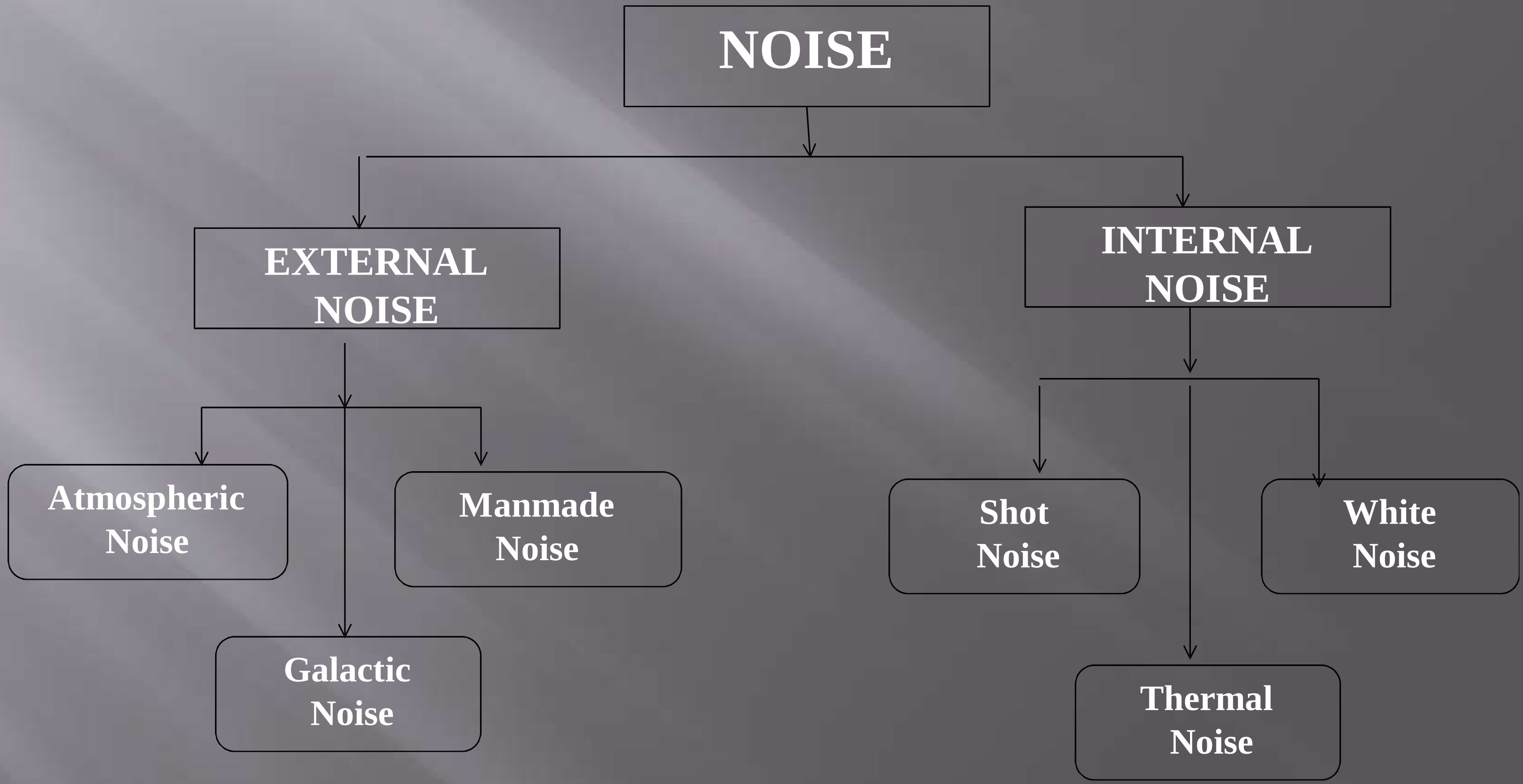


INTRODUCTION TO COMMUNICATION

- The purpose of communication systems is to communicate information
- Figure depicts the general layout of a communication :
- An **input transducer** (e.g., a microphone) converts the input message into a message signal (e.g., a time varying voltage)
- that is transmitted over a channel by means of a **transmitter** which performs a very important function on communication signals by encoding the signals in some fashion making use of a carrier signal , and converted by a **receiver** into an output signal.
- An **output transducer** (e.g., a loudspeaker) converts the received signal into an output message (e.g.: sound).The information is contained in a so- called modulating signal that modulates a carrier signal.
- **For example**, in FM radio the modulating signal consists of speech and music, and the carrier is a sinusoidal wave of pre-determined frequency, much higher than the modulating signal frequency.

NOISE

Noise can be defined as an unwanted signals that tends to disturb the transmission and processing of signals in communication system

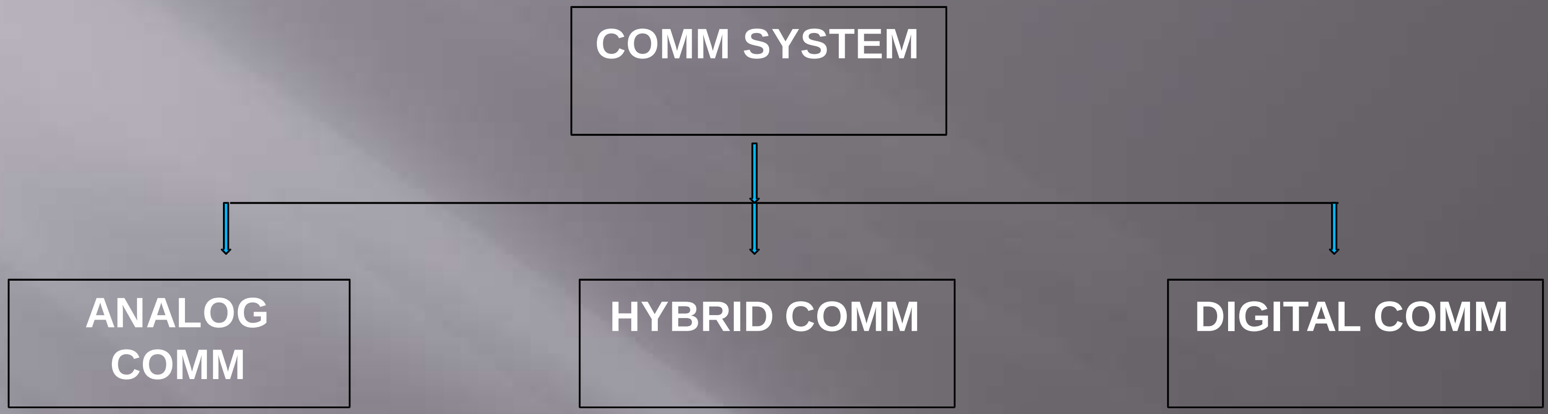


INFORMATION THEORY

- It provides quantitative measure of the information contained in message signal and allows us to determine the capacity of a communication system to transfer the information from source to destination

- The information theory is used for mathematical modelling and and analysis of the communication system

TYPES OF COMMUNICATION SYSTEM



Analog Communication System:

- **Analog communication** is that types of communication in which the message or information signal i.e transmitted is analog in nature.
- This means that in analog communication the modulating signal (i.e base-band signal) is an analog signal.
- This **analog message signal** may be obtained from sources such as speech, video shooting etc. Analog signals are continuous in both time and value.
- Analog signals are used in many systems, although the use of analog signals has declined with the advent of cheap digital signals.
- All natural signals are Analog in nature.

TYPES OF COMMUNICATION CONT…

Digital Communication System

In digital communication, the message signal to be transmitted is digital in nature.

This means that digital communication involves the transmission of information in digital form.

Digital signals are discrete in time and value.

Digital signals are signals that are represented by binary numbers, "1" or "0". The 1 and 0 values can correspond to different discrete voltage values, and any signal that *doesn't quite fit* into the scheme just gets rounded off.

Digital signals are sampled, quantized & encoded version of continuous time signals which they represent. In addition, some techniques also make the signal undergo encryption to make the system more tolerent to the channel.

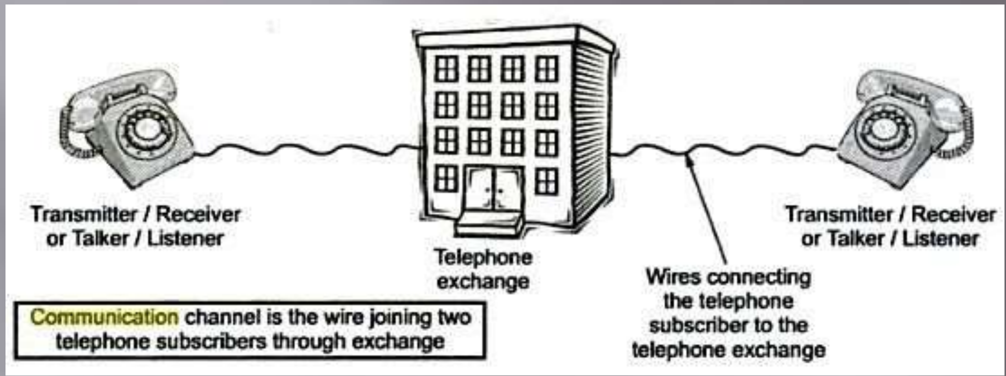
- **Advantage of digital communication over analog communication system**

- Increased immunity to channel noise and external interference.
- Flexible operation of the system.
- A common format for the transmission of different kinds of message signal (e.g voice signal, video signal , computer data).

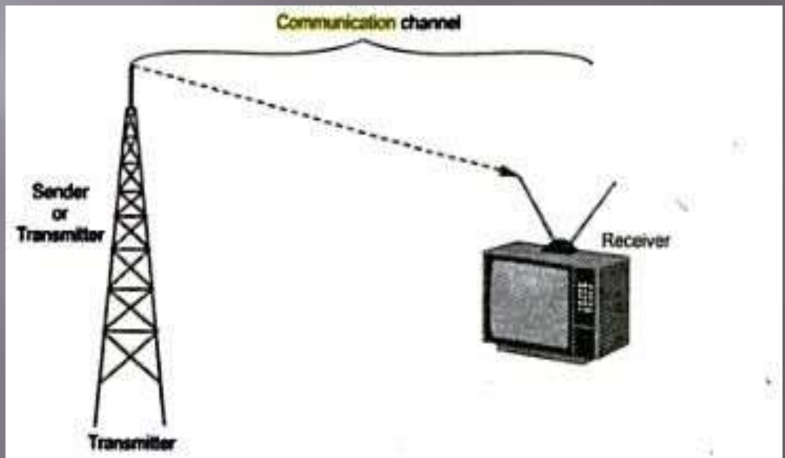
MAJOR CLASSIFICATION OF COMMUNICATION SYSTEM

BASED ON PHYSICAL STRUCTURE

LINE COMMUNICATION



RADIO COMMUNICATION



MAJOR CLASSIFICATION OF COMMUNICATION SYSTEM

BASED ON THE SIGNAL SPECIFICATIONS

- Nature of the baseband or information signal.
- Nature of the transmitted signal.

BASED ON THE NATURE OF THE BASEBAND SIGNAL

- Analog communication systems
- Digital communication systems

BASED ON THE NATURE OF TRANSMITTED SIGNAL:

The baseband signal can either be transmitted as it is without modulation, or through a carrier signal with modulation. The two systems can then be categorized as:

- Baseband communication system
- Carrier communication system

Therefore, **the four types of communication system** categorized based on signal specification are:

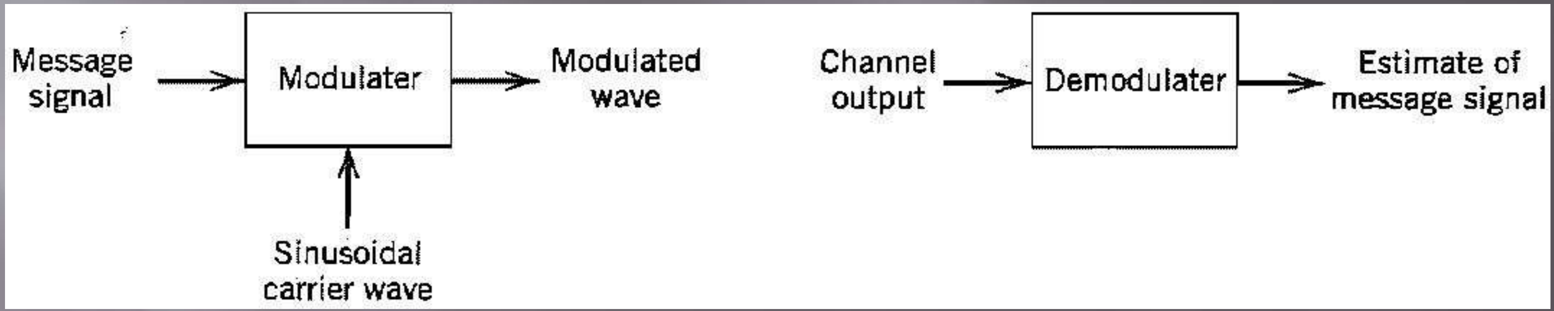
- Analog communication systems
- Digital communication systems
- Baseband communication systems
- Carrier communication systems

MODULATION

MODULATION

What is modulation?

- Modulation is performed at the transmitting end of the communication system.
- At the receiving end of the system we usually require the original baseband signal to be restored, this is usually accomplished by using a process known as demodulation which is the reverse process of the modulation



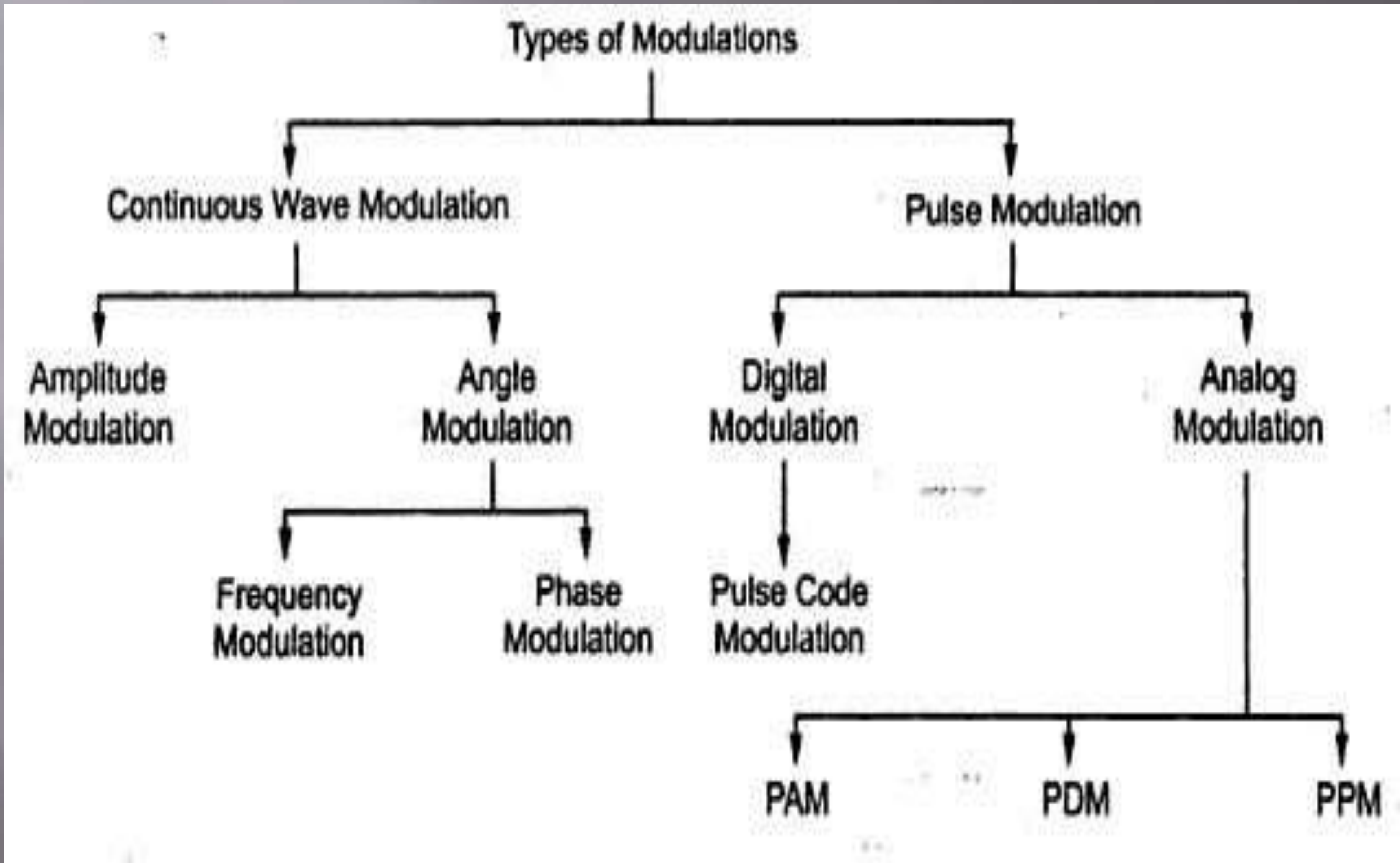
- In basic signal processing terms, we thus find that the transmitter of an analog communication system consists of a modulator and the receiver consists of a demodulator as

What are the reasons for modulation?

If modulation is not employed however, the system designer could confront the following problems:

- Antenna Height
- Narrow Banding
- Poor radiation and penetration
- Diffraction angle
- Multiplexing
- To overcome equipment limitations
- To reduce noise and interferences

What are the Different Modulation Methods?



AMPLITUDE MODULATION

What is Amplitude modulation?

Define Amplitude modulation ?

- A Sinusoidal carrier wave c(t) is given as

$$c(t) = A_c \cos(2\pi f_c t)$$

- Let m(t) denote the baseband signal that carries the specification of the message
- The source of carrier wave c(t) is physically independent of the source responsible for generating m(t)
- Amplitude modulation is defined as a process in which the amplitude of the carrier wave c(t) is varied about a mean value, linearly with the baseband signal m(t).
- An amplitude modulated wave may thus be described in its most general form as a function of time as follows

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

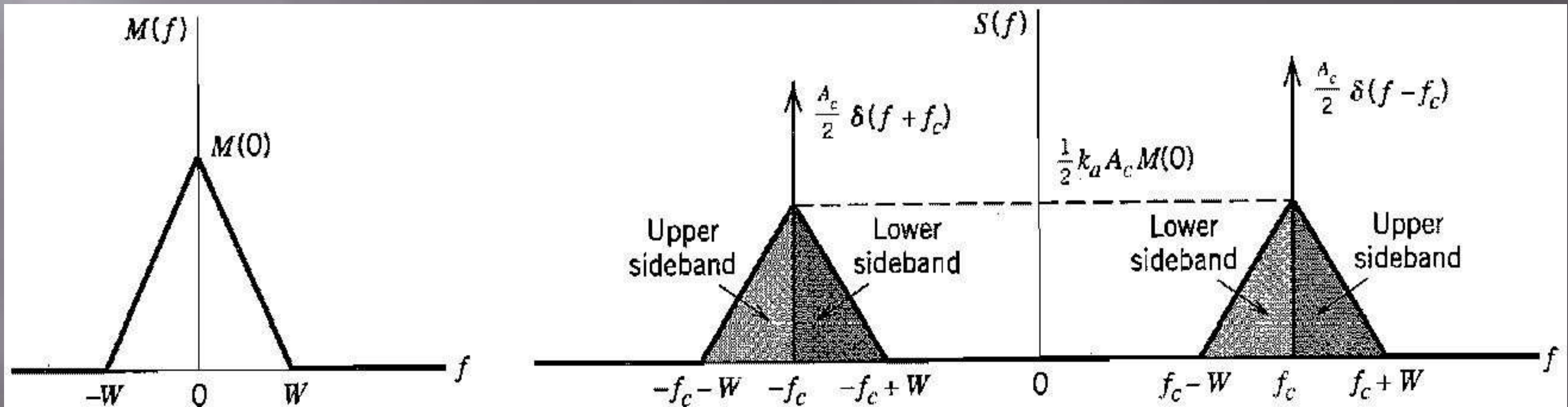
AMPLITUDE MODULATION

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

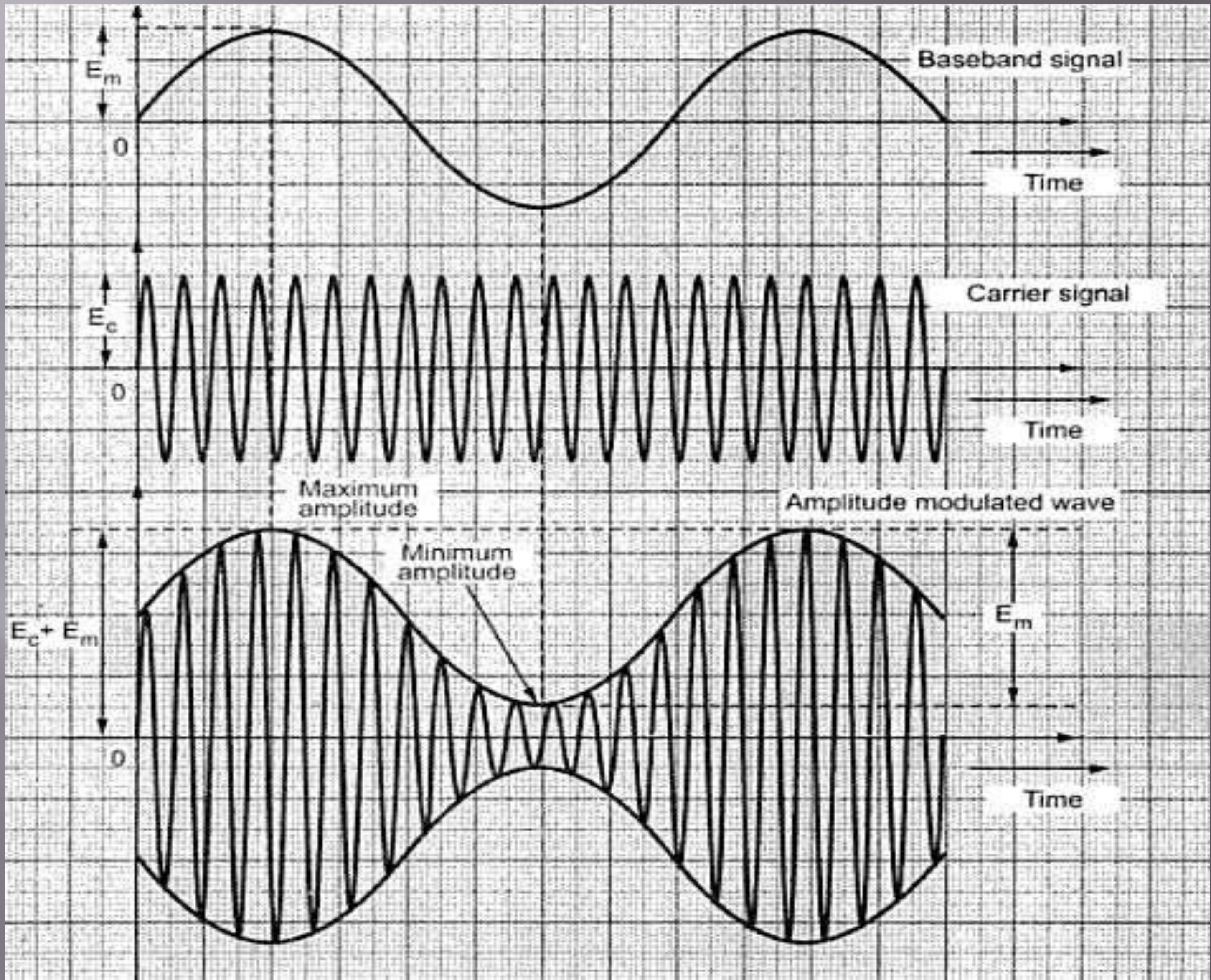
From the above eqn , we find that the fourier transform of the AM wave s(t) is given by

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$

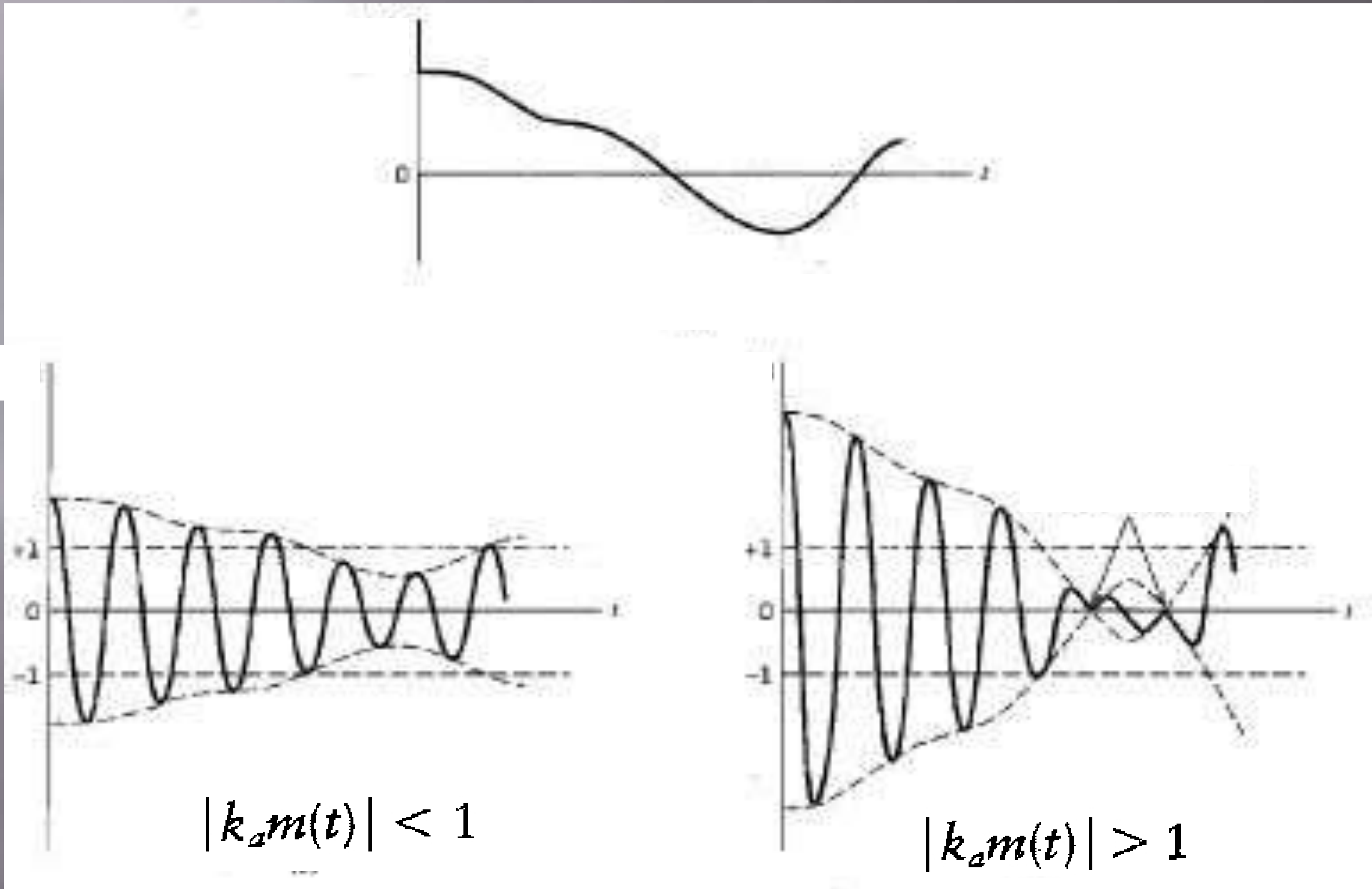
Spectrum of baseband signal **Spectrum of AM wave**



BASEBAND SIGNAL CARRIER SIGNAL & AMPLITUDE MODULATED WAVE



OVER MODULATION AND UNDER MODULATION



EQUATION OF AN AM WAVE

The instantaneous value of modulating signal and carrier signal can be represented as given below

Instantaneous value of modulating signal

$$e_m = E_m \sin \omega_m t$$

where e_m = instantaneous amplitude

E_m = maximum amplitude

$\omega_m = 2\pi f_m$ = angular frequency and

f_m = frequency of modulating signal

Instantaneous value of carrier signal

$$e_c = E_c \sin \omega_c t$$

where e_c = instantaneous amplitude

E_c = maximum amplitude

$\omega_c = 2\pi f_c$ = angular frequency and

f_c = frequency of carrier signal

Equation of an AM Wave cont…

- Instantaneous value of amplitude modulated signal
- Using the above mathematical expression for modulating and carrier signals, we can create a new mathematical expression for the complete modulated wave, as given below

$$E_{AM} = E_c + e_m \qquad \because e_m = E_m \sin \omega_m t$$

$$= E_c + E_m \sin \omega_m t$$

The instantaneous value of the amplitude modulated wave is given as

$$e_{AM} = E_{AM} \sin \theta$$

$$= E_{AM} \sin \omega_c t$$

$$= (E_c + E_m \sin \omega_m t) \sin \omega_c t$$

CALCULTION OF PERCENTAGE OF MODULATION

- Looking at the figure we can visualize that something unusual (distortion) will occur if E_m is greater than E_c
- Therefore the modulating signal voltage E_m must be less than the carrier voltage E_c for proper amplitude modulation
- This relationship between the amplitude of the modulating and carrier signals is important and it is expressed in terms of their ratio, commonly known as modulation index (m)
- It is also called **modulation factor, modulation coefficient or the degree of modulation**
- The m is the ratio of the modulating signal voltage to the carrier voltage

$$m = \frac{E_m}{E_c}$$

- The modulation index is a number lying between 0 and 1 and it is very often expressed as a percentage and called the percentage modulation

DEGREE OF MODULATION

Calculating the modulation index using AM wave

We know that $m = \frac{E_m}{E_c}$ with this relation we can calculate the modulation index from the modulated waveform . Hence we can write

$$E_m = \frac{E_{max} - E_{min}}{2}$$

$$E_c = E_{max} - E_m$$

By substituting 1st eqn in 2nd eqn we get

$$= E_{max} - \left(\frac{E_{max} - E_{min}}{2}\right) \quad = \frac{2E_{max} - E_{max} + E_{min}}{2}$$

By diving 1st and 3rd eqn we get

$$m = \frac{E_m}{E_c} = \frac{(E_{max} - E_{min})/2}{(E_{max} + E_{min})/2}$$

$$m = \frac{E_m}{E_c}$$

$$m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

FREQUENCY SPECTRUM AND BANDWIDTH OF A.M WAVE

The modulated carrier has new signals at different frequencies, called side frequencies or sidebands which occur in the frequency spectrum directly above and below the carrier frequency

$$f_{USB} = f_c + f_m$$

$$f_{LSB} = f_c - f_m$$

The expression for the instantaneous value of the amplitude modulated wave

$$e_{AM} = (E_c + E_m \sin \omega_m t) \sin \omega_c t \quad \dots (1)$$

We know that, $m = \frac{E_m}{E_c}$

$$E_m = m E_c$$

Substituting the value in eqn we get

$$e_{AM} = (E_c + m E_c \sin \omega_m t) \sin \omega_c t$$

$$= E_c (1 + m \sin \omega_m t) \sin \omega_c t \quad \dots (2)$$

$$= E_c \sin \omega_c t + m E_c \sin \omega_m t \sin \omega_c t \quad \dots (3)$$

Eqn 3 can be further expanded by means of the trignometric relation

$$\left[\sin a \sin b = \frac{1}{2}[\cos (a-b) - \cos (a+b)]\right]$$

FREQUENCY SPECTRUM AND BANDWIDTH OF A.M WAVE

$$e_{AM} = \underbrace{E_c \sin\omega_c t}_{\text{carrier}} + \underbrace{\frac{mE_c}{2}\cos(\omega_c - \omega_m)t}_{\text{Lower side band}} - \underbrace{\frac{mE_c}{2}\cos(\omega_c - \omega_m)t}_{\text{Upper side band}} \quad \ldots (4)$$

- Looking at eqn 4 we can say that 1st term represents unmodulated carrier and two additional terms represents two sidebands
- The frequency of the lower sideband (LSB) is fc – fm and the frequency of the upper sideband (USB) is fc+fm

BANDWIDTH OF AM WAVE

- We know bandwidth can be measured by subtracting lowest frequency of the signal from highest frequency of the signal
- For amplitude modulated wave it is given by

$$B_w = f_{USB} - f_{LSB}$$

$$= (f_c + f_m) - (f_c - f_m)$$

$$\boxed{B_w = 2 f_m}$$

- Therefore the bandwidth required for the amplitude modulation is twice the frequency of the modulating signal

POWER DISTRIBUTION IN AN A.M WAVE

We have seen that AM wave has three components :

- Unmodulated carrier
- Lower sideband
- Upper sideband

Therefore the total power of AM wave is the sum of the carrier power Pc and Power in the two sidebands Pusb and Plsb. It is given as

$$P_{Total} = P_c + P_{USB} + P_{LSB}$$

$$= \frac{E_{carr}^2}{R} + \frac{E_{LSB}^2}{R} + \frac{E_{LSB}^2}{R}$$

Where all three voltage represents r.m.s values and resistance R is a characteristic impedance of antenna in which the power is dissipated

POWER DISTRIBUTION IN AN A.M WAVE CONT…

- Carrier power
- Power in sideband
- Total power
- Modulation index in terms of Pc and Ptotal
- Transmission efficiency

VIRTUES AND LIMITATIONS OF AM MODULATION

Virtue: Its greatest virtue is its simplicity of implementation

What happens in the transmitter side?

What happens in the receiver side?

Limitations

Amplitude modulation is wasteful in power

Amplitude modulation is wasteful in bandwidth

How to overcome these limitations?

DOUBLE SIDE BAND-SUPPRESSED CARRIER

- What is DSB-SC?
- Power saving in DSB-SC signal
- Equation of DSB-SC signal

s(t)=c(t)m(t)

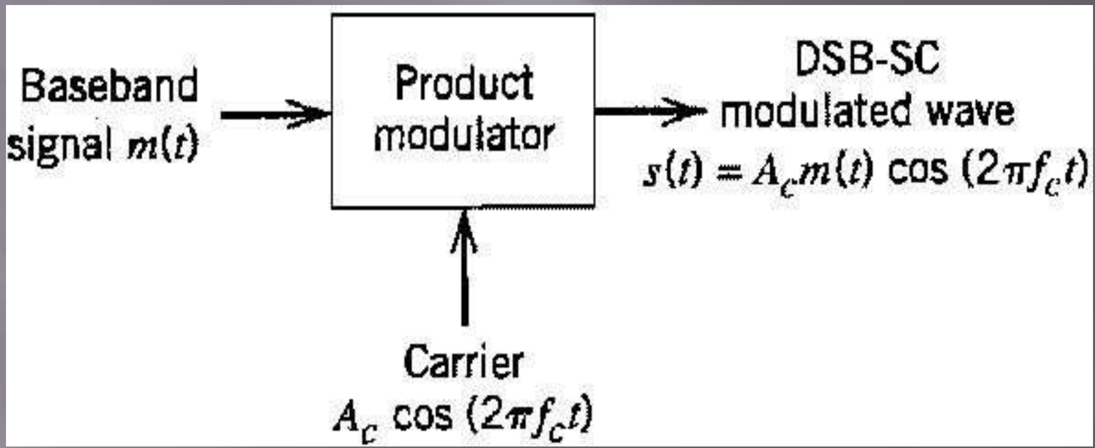
$$s(t) = A_c m(t) \cos(2\pi f_c t)$$

From the above eqn the Fourier transform of s(t) is

$$S(f) = \frac{1}{2} A_c[M(f - f_c) + M(f + f_c)]$$

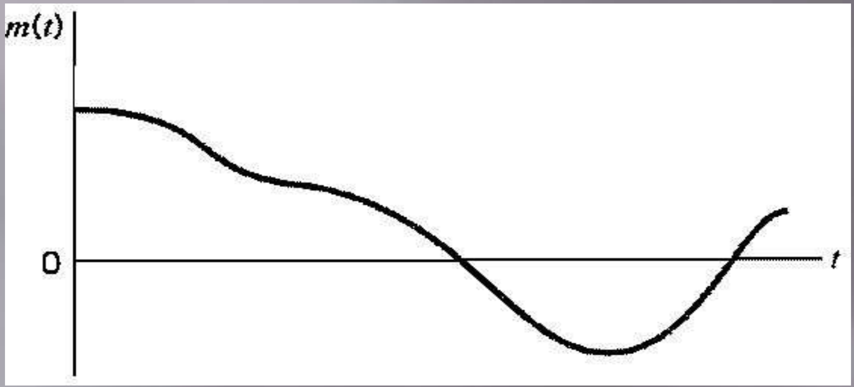
DOUBLE SIDE BAND-SUPPRESSED CARRIER

- It is a form of linear modulation where the signal is generated by simply multiplying a message signal along with a carrier wave

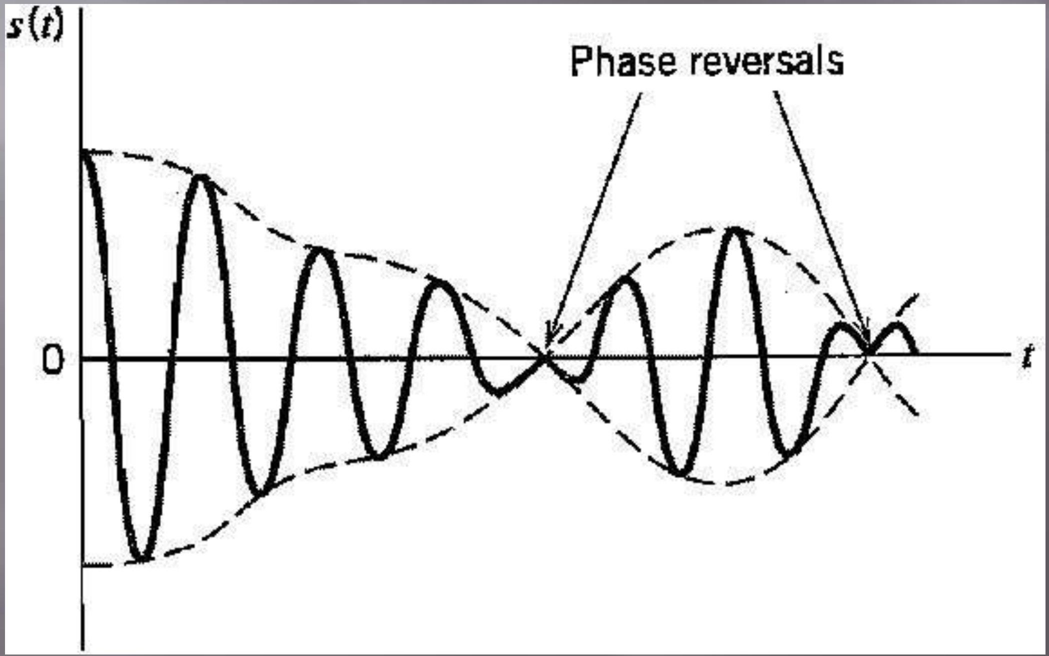


DOUBLE SIDE BAND- SUPPRESSED CARRIER

PHYSICAL APPEARANCE OF DSB-SC SIGNAL



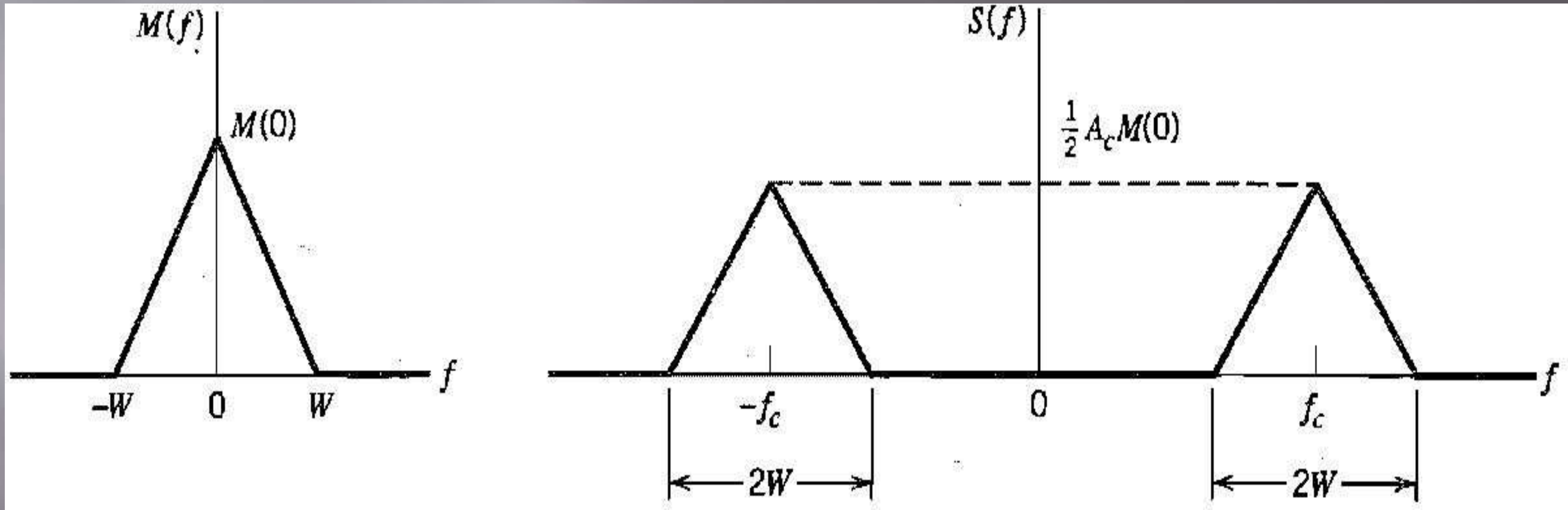
BASEBAND SIGNAL



DSB-SC MODULATED WAVE

DOUBLE SIDE BAND-SUPPRESSED CARRIER

FREQUENCY SPECTRUM OF DSB-SC SIGNAL



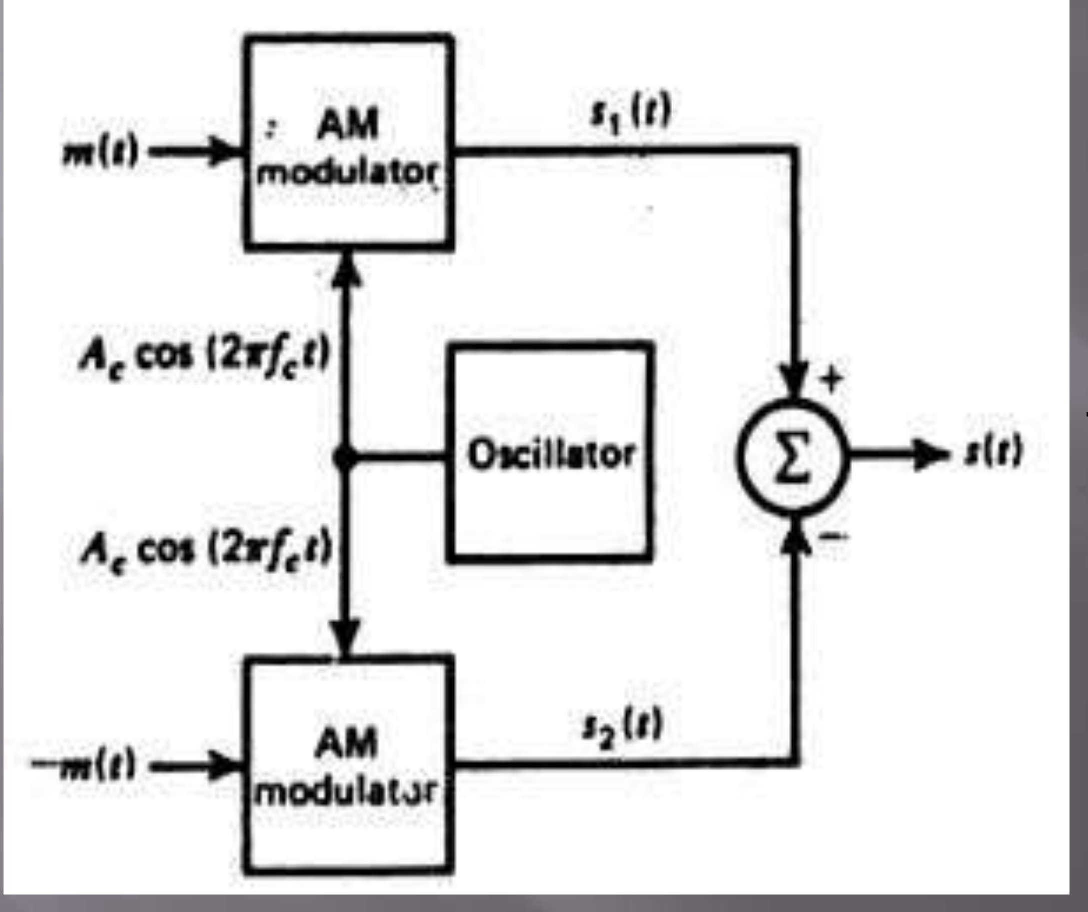
SPECTRUM OF BASEBAND SIGNAL **SPECTRUM OF DSB-SC MODULATED WAVE**

GENERATION OF DSB-SC

BALANCED MODULATOR

What is balanced modulator?

Operation of balanced modulator



BALANCED MODULATOR

GENERATION OF DSB-SC

EQUATION OF BALANCED MODULATOR

The outputs of the two AM modulators may be expressed as follows

$$s_1(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

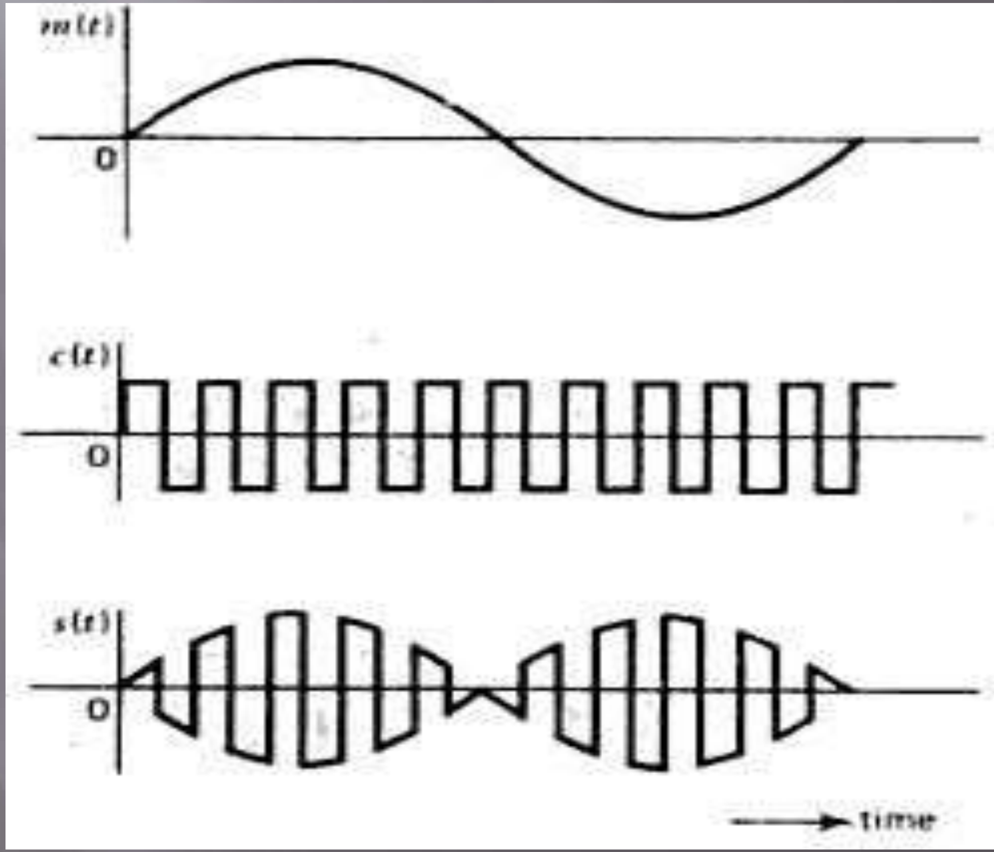
and $$s_2(t) = A_c[1 - k_a m(t)]\cos(2\pi f_c t)$$

Subtracting 2nd eqn from the 1st eqn we obtain

$$\begin{aligned} s(t) &= s_1(t) - s_2(t) \\ &= 2k_a A_c \cos(2\pi f_c t) m(t) \end{aligned}$$

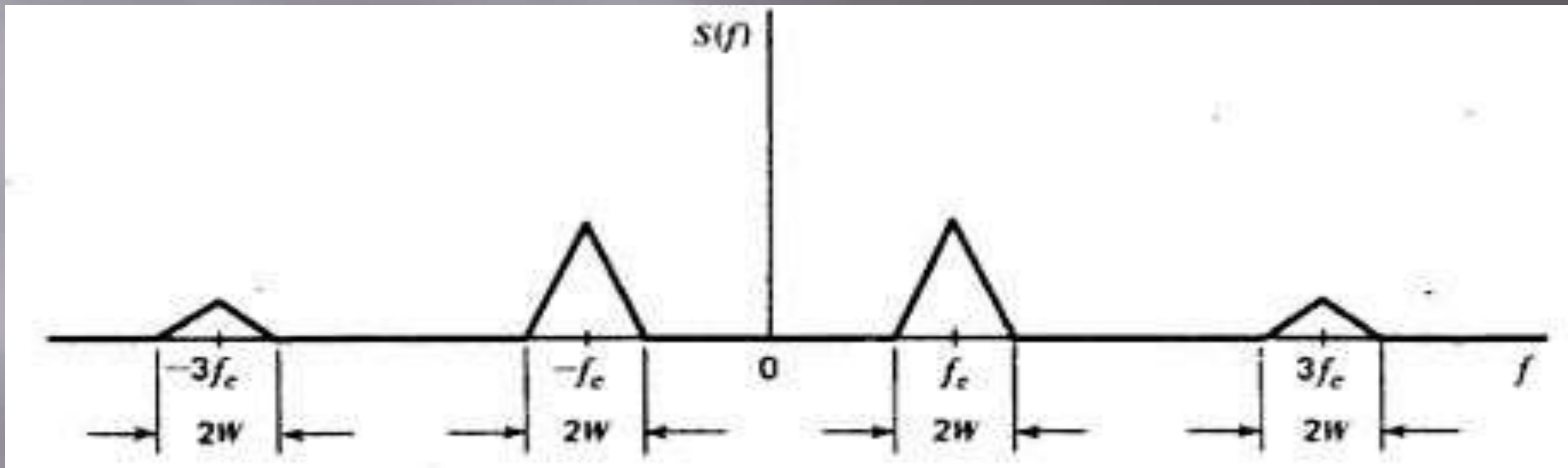
WAVEFORM ILLUSTRATING THE OPERATION OF THE RING MODULATOR FOR A SINUSOIDAL MODULATING WAVE

MODULATING WAVE



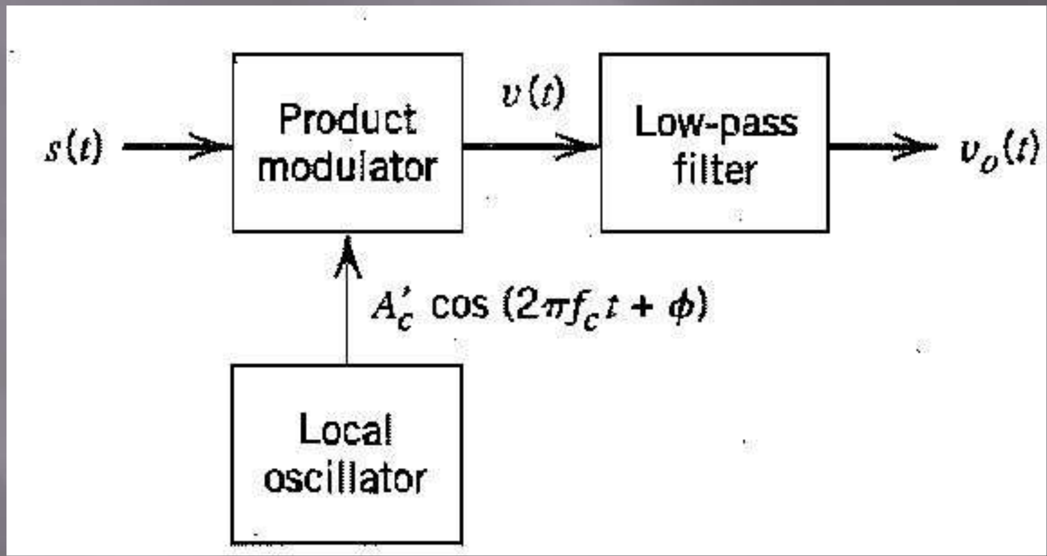
SPECTRUM OF RING MODULATOR OUTPUT

ILLUSTRATING THE SPECTRUM OF RING MODULATOR OUTPUT



DEMODULATION OF DSB-SC

- COHERENT DETECTION
- What is Coherent Detection?
- Operation of Coherent Detection



COHERENT DETECTOR FOR MODULATING DSB-SC

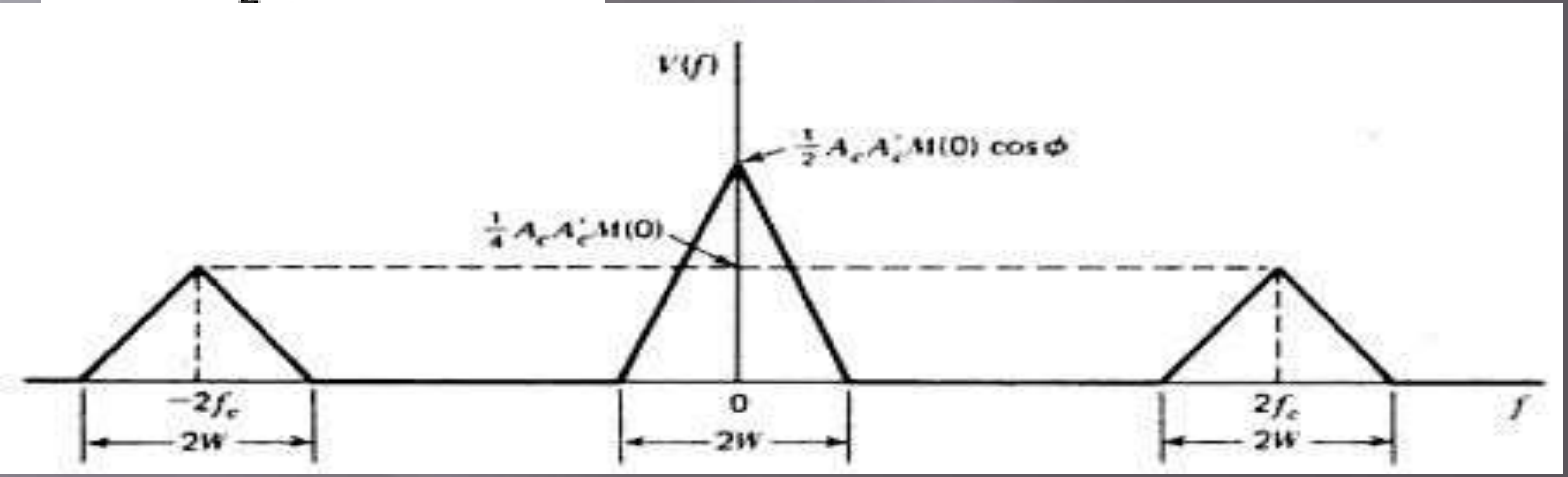
EQUATION OF COHERENT DETECTION OF DSBSC WAVES

The product modulator output is given as shown in the figure

$$v(t) = A'_c \cos(2\pi f_c t + \phi) s(t)$$
$$= A_c A'_c \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) m(t)$$
$$= \frac{1}{2} A_c A'_c \cos(4\pi f_c t + \phi) m(t) + \frac{1}{2} A_c A'_c \cos\phi \, m(t)$$

At the filter output we then obtain a signal given by

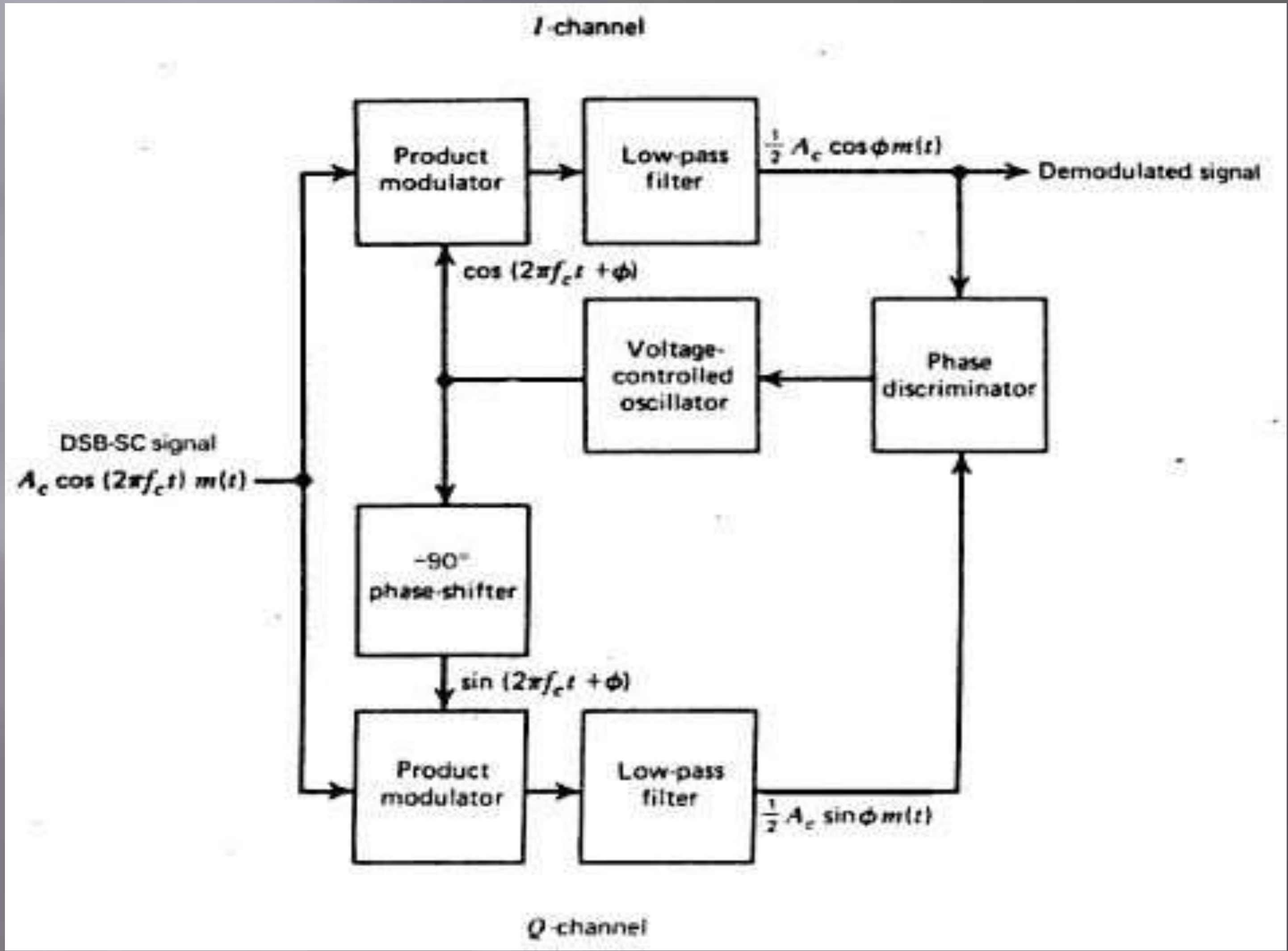
$$v_o(t) = \frac{1}{2} A_c A'_c \cos\phi \, m(t)$$



Illustrating the spectrum of product modulator o/p with a DSBSC wave as i/p

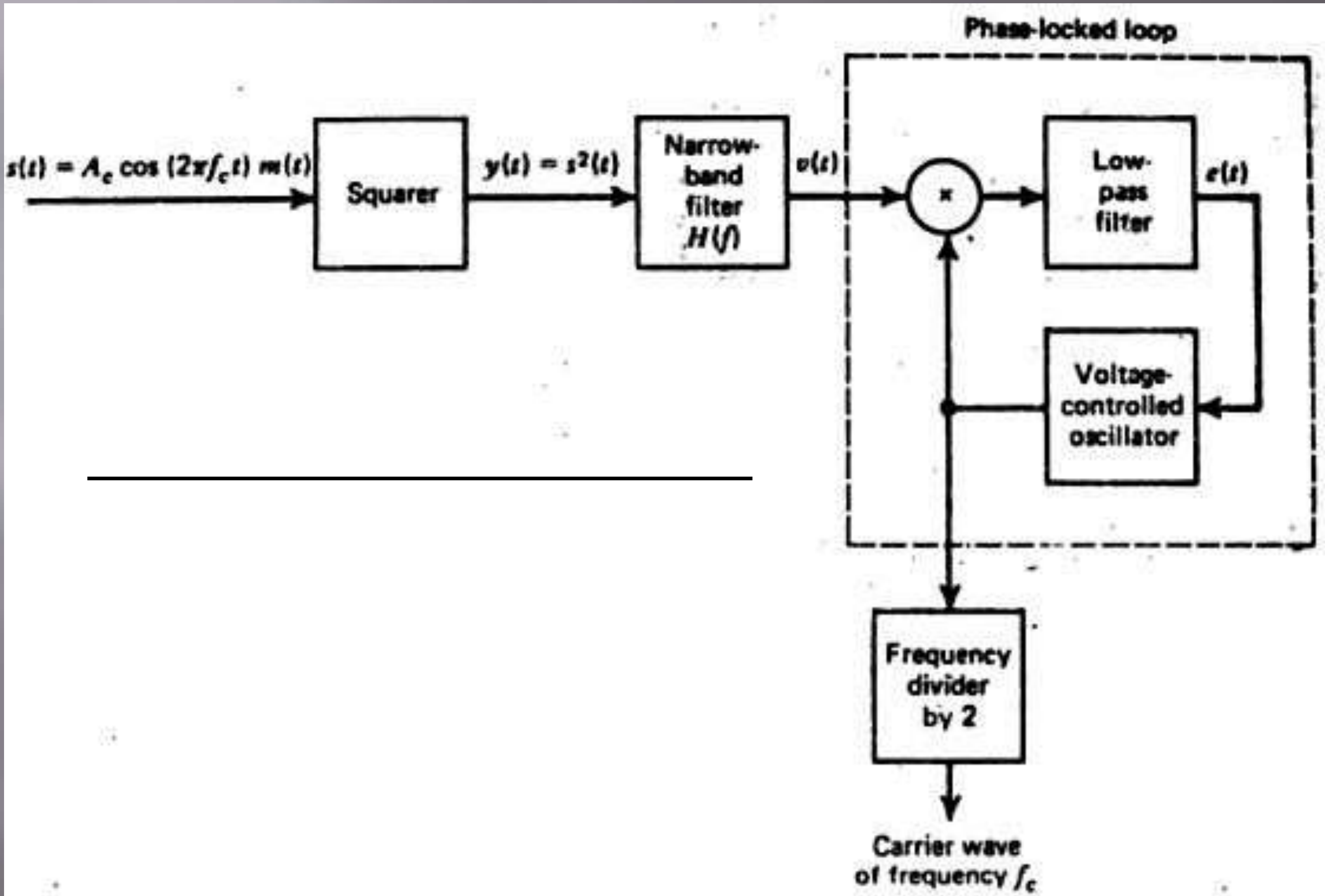
DEMODULATION OF DSB-SC

COSTAS RECEIVER



DEMODULATION OF DSB-SC

SQUARING LOOP



EQUATION OF SQUARING LOOP

The square law device is characterized by the relation

$$y(t) = s^2(t)$$

Therefore the DSBSC wave

$$s(t) = A_c \cos(2\pi f_c t) m(t)$$

Applied to the input of this square law device we obtain

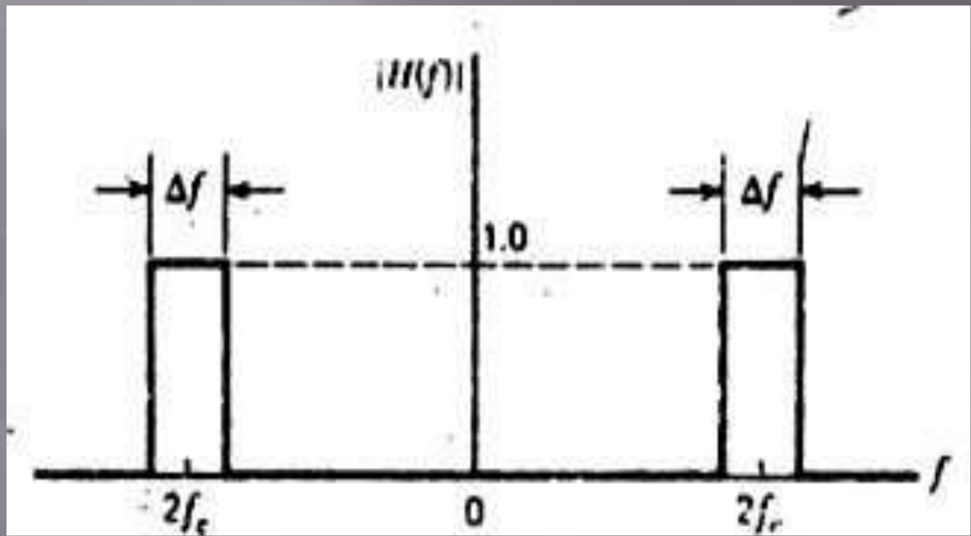
$$y(t) = A_c^2 \cos^2(2\pi f_c t) m^2(t)$$
$$= \frac{A_c^2}{2} m^2(t)[1 + \cos(4\pi f_c t)]$$

The o/p is approximately sinusoidal as shown

$$v(t) \simeq \frac{A_c^2}{2} E \, \Delta f \, \cos(4\pi f_c t)$$

SQUARING LOOP Cont..

Illustrating the Amplitude response of narrow band filter



SINGLE SIDE BAND

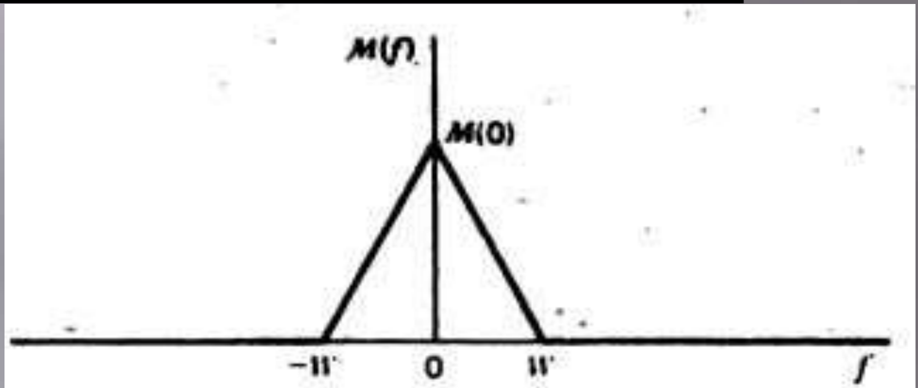
What are the limitations of DSB-SC ?

What is SSB signal?

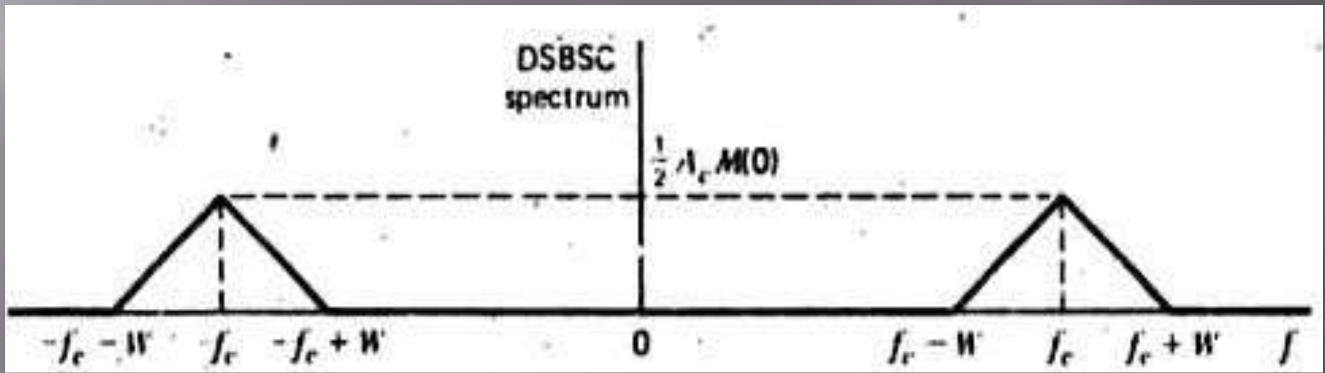
- If we consider the fact that two sidebands carry same information, DSB signal is redundant
- That is in DSB the basic information is transmitted twice once in each sideband
- Therefore there is absolutely no reason to transmit both sidebands in order to convey the information
- One sideband may be suppressed
- The resulting signal is a single sideband commonly referred to as single sideband suppressed carrier signal

SINGLE SIDE BAND

SPECTRUM OF BASEBAND SIGNAL

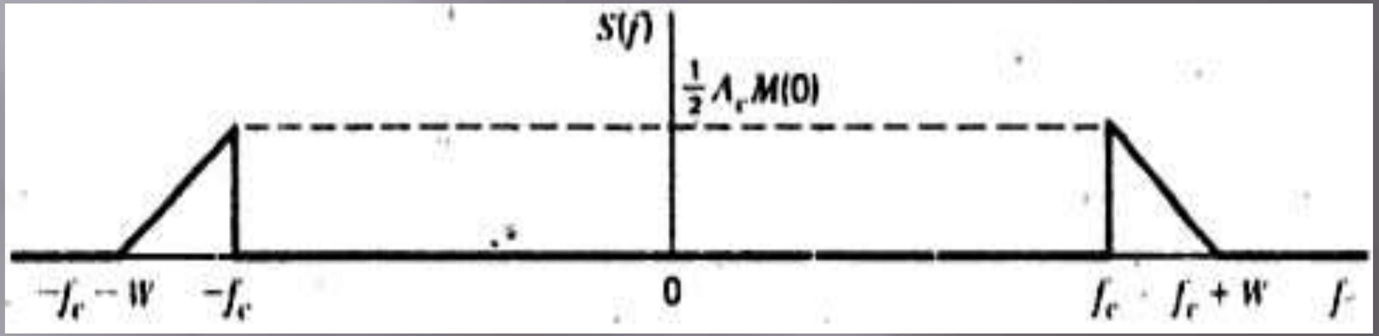


SPECTRUM OF DSBSC WAVE

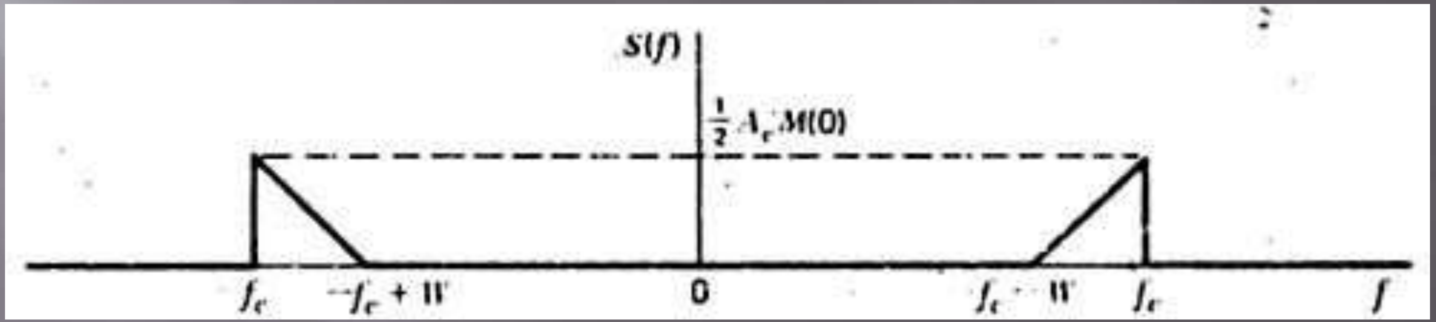


SINGLE SIDE BAND

SPECTRUM OF SSB WAVE WITH THE UPPER SIDEBAND TRANSMITTED



SPECTRUM OF SSB WAVE WITH THE LOWER SIDEBAND TRANSMITTED



EQUATION OF SSB SIGNAL

The time domain description of an SSB wave s(t) in the canonical form

$$s(t) = s_c(t)\cos(2\pi f_c t) - s_s(t)\sin(2\pi f_c t)$$

The fourier transform of $S_C(t)$ and $S_S(t)$ are related to that of the SSB wave s(t) as

$$S_c(f) = \begin{cases} S(f - f_c) + S(f + f_c), & -W \leq f \leq W \\ 0, & \text{elsewhere} \end{cases}$$

$$S_s(f) = \begin{cases} j[S(f - f_c) - S(f + f_c)], & -W \leq f \leq W \\ 0, & \text{elsewhere} \end{cases}$$

On the basis of the below figure we can write

$$S_c(f) = \tfrac{1}{2} A_c \bar{M}(f)$$

Accordingly the in phase component sc(t) is defined by

$$s_c(t) = \tfrac{1}{2} A_c m(t)$$

EQUATION OF SSB SIGNAL Cont…

On the basis of the fig we can write

$$S_s(f) = \begin{cases} -\frac{j}{2} A_c M(f), & f > 0 \\ 0, & f = 0 \\ \frac{j}{2} A_c M(f), & f < 0 \end{cases}$$

$$= -\frac{j}{2} A_c \operatorname{sgn}(f) M(f)$$

Where sgn(f) is the signum function equal to +1 for positive frequencies , zero for f=0 and -1 for –ve frequencies. However we note that

$$-j \operatorname{sgn}(f) M(f) = \hat{M}(f)$$

The hilbert transform of m(t) substituting 2nd eqn in the 1st eqn

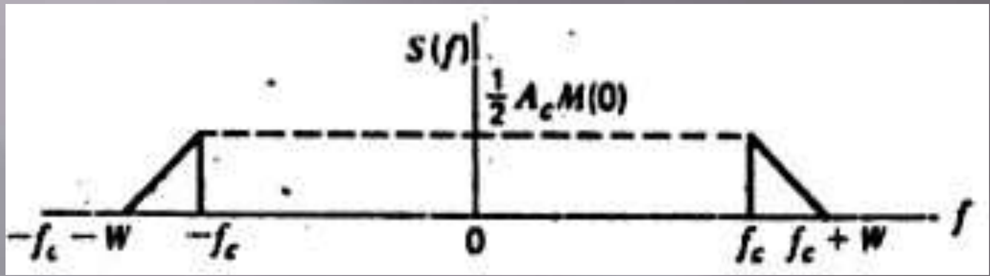
$$S_s(f) = \frac{1}{2} A_c \hat{M}(f)$$

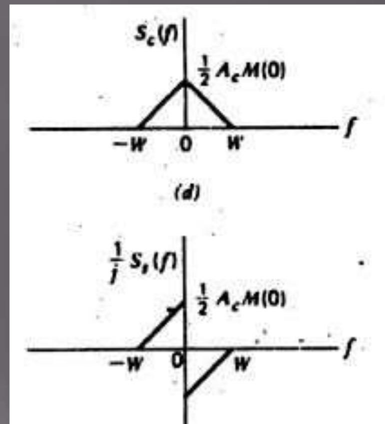
Which shows that the quadrature component $s_s(t)$ is defined by

$$s_s(t) = \frac{1}{2} A_c \hat{m}(t)$$

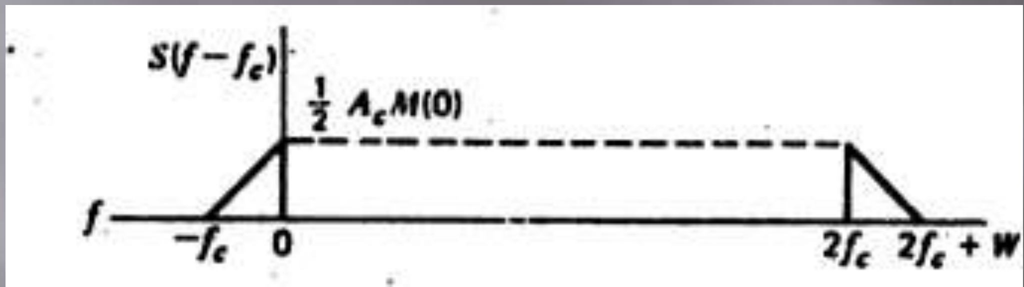
SPECTRUM OF SSB WAVE

SPECTRUM OF SSB WAVE

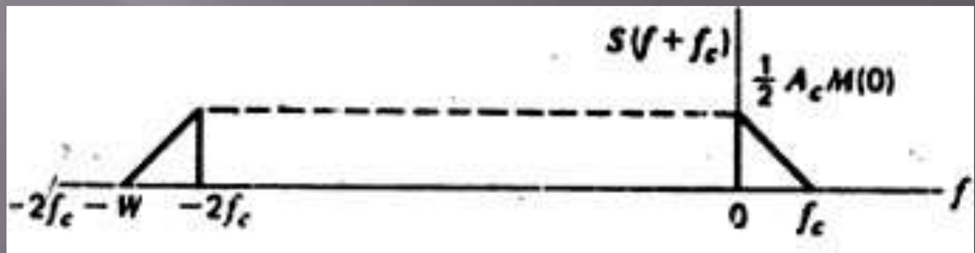




SPECTRUM OF SSB WAVE SHIFTED TO THE RIGHT BY f_c

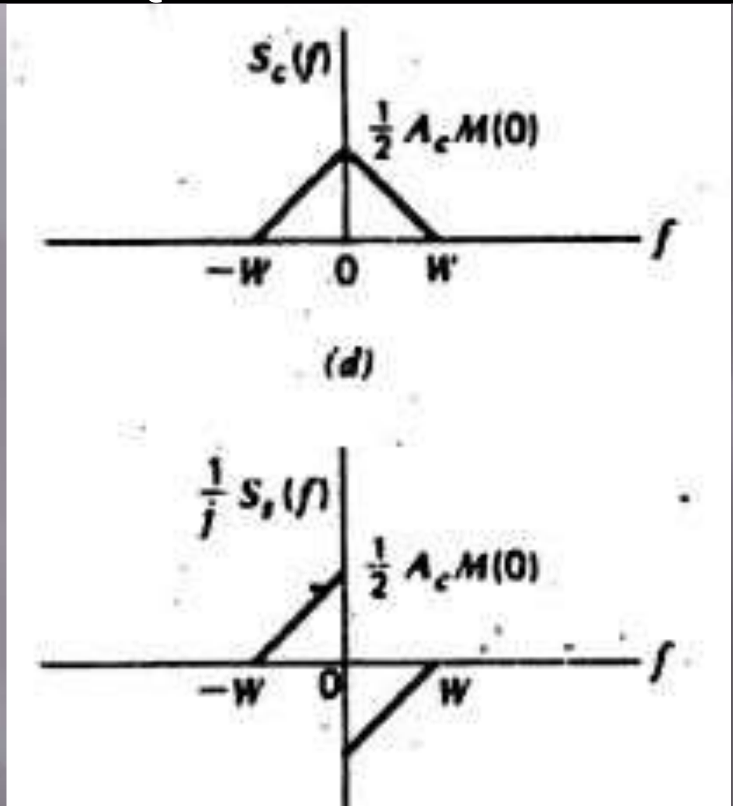


SPECTRUM OF IN PHASE COMPONENT



SPECTRUM Cont…

SPECTRUM OF QUADRATURE COMPONENT



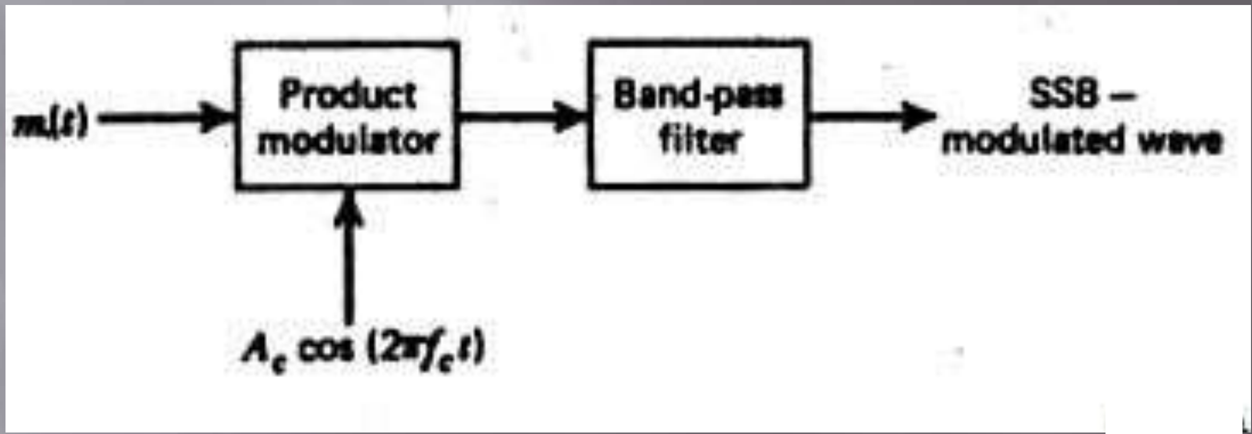
The cananonical representation of an SSB wave s(t) obtained bt transmitting only the upper sideband is as follows

$$s(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) - \frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

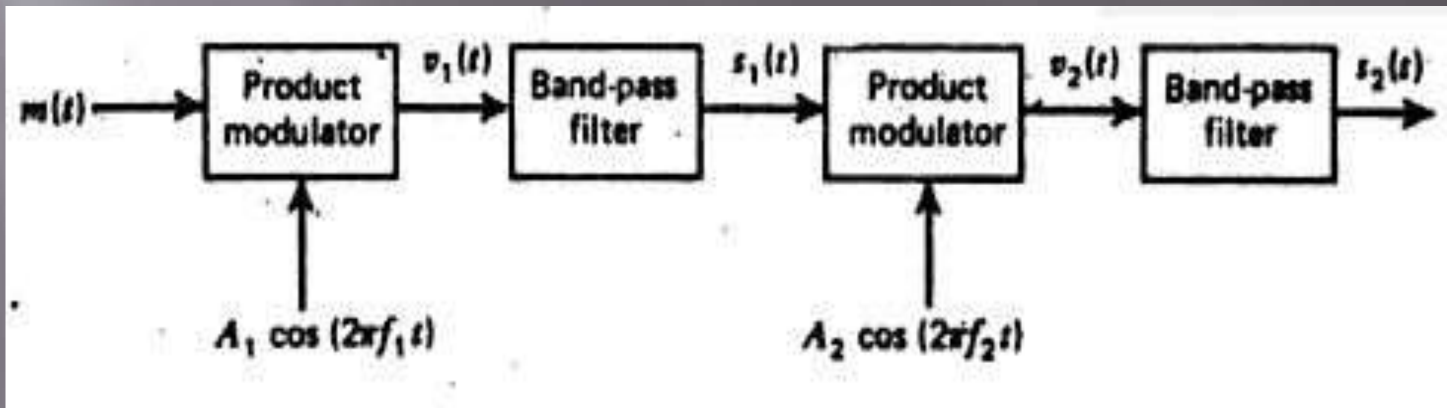
GENERATION OF SSB WAVES

FREQUENCY DISCRIMINATION METHOD

Block Diagram of the Frequency Discrimination Method for Generating SSB Waves

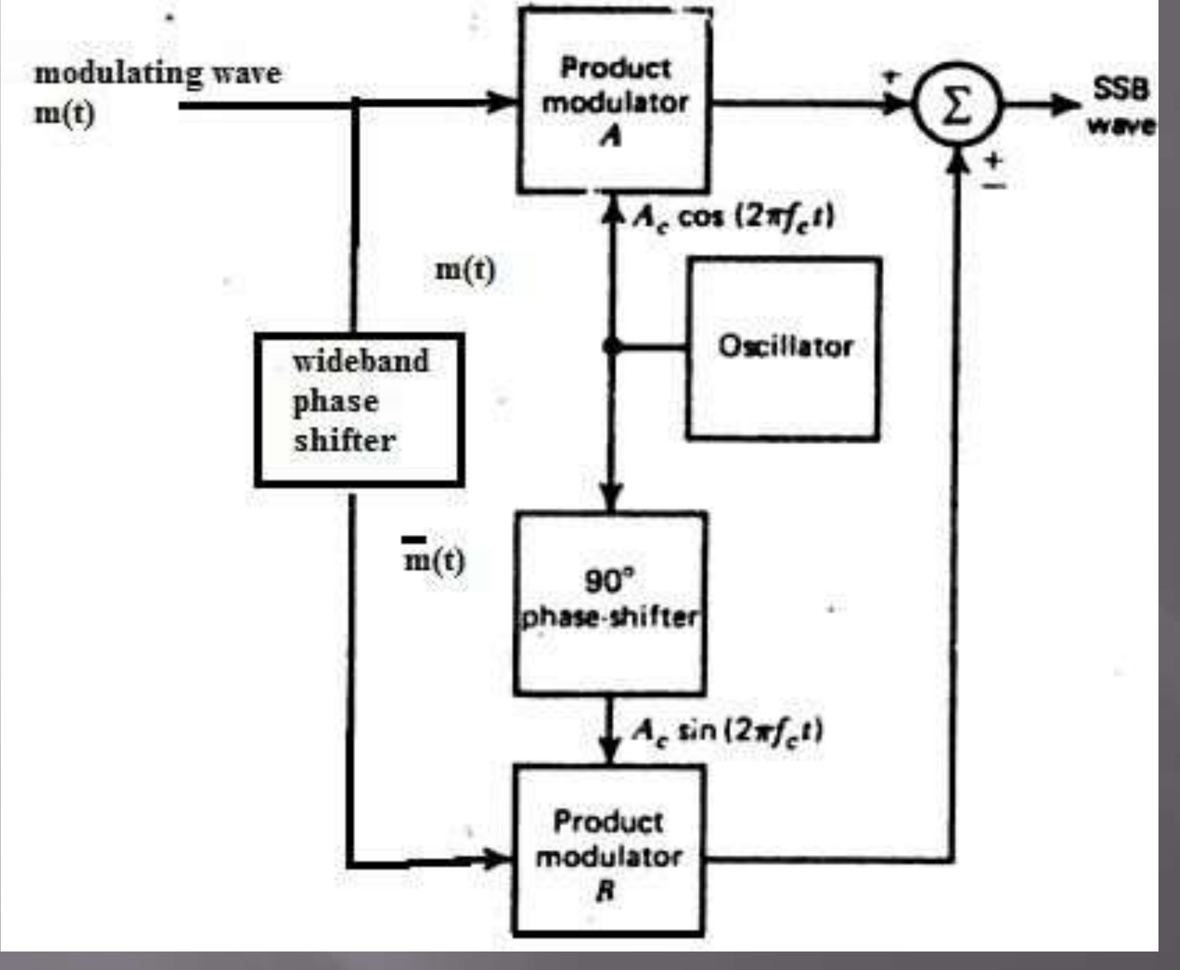


Block diagram of a two stage SSB modulator



GENERATION OF SSB WAVES

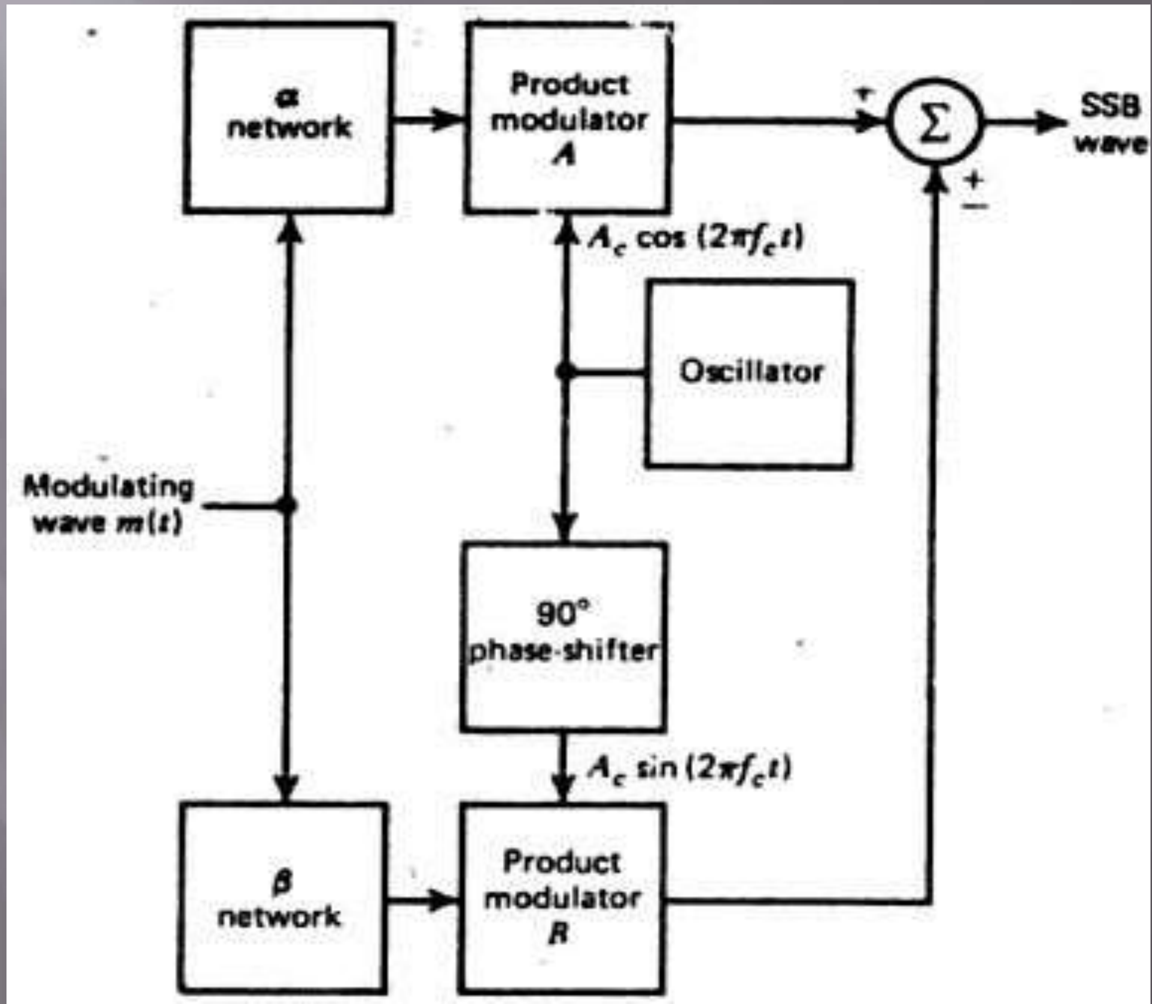
PHASE DISCRIMINATION METHOD



BLOCK DIAGRAM OF THE PHASE DESCRIPTION METHOD FOR GENERATING SSB WAVES

PHASE DISCRIMINATION METHOD

Block diagram of the phase discrimination method for generating SSB waves by using a pair of phase shifting network to realize a constant 90 degree phase difference

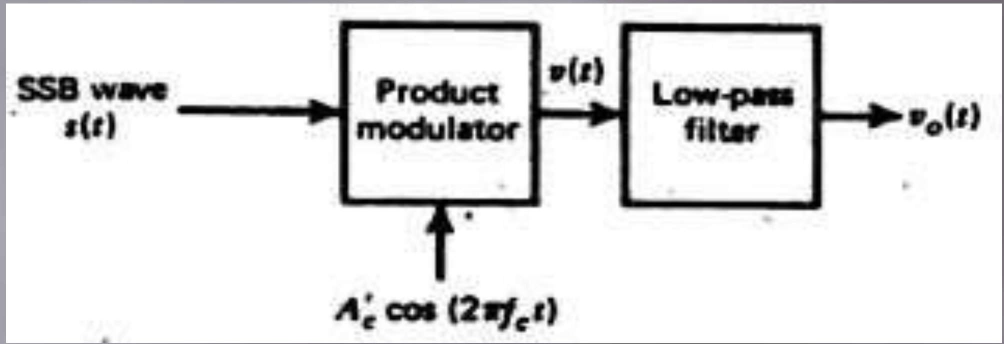


COMPARISON BETWEEN SSB SUPRESSION METHODS

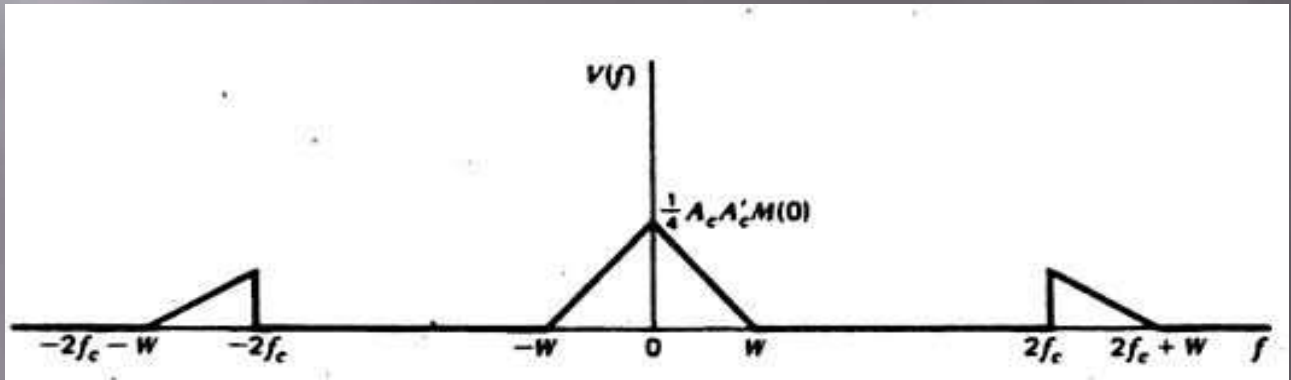
Sr. No.	Parameter	Filter Method	Phase shift method	Third method
1.	Method used	Filter is used to remove unwanted sideband.	Phase-shifting techniques is used to remove unwanted sideband.	Similar to phase-shift method, but carrier signal is phase shifted by 90°.
2.	90° phase shift	Not required	Requires complex phase shift network	Phase shift network is simple RC circuit
3.	Possible frequency range of SSB	Not possible to generate SSB at any frequency.	Possible to generate SSB at any frequency.	Possible to generate SSB at any frequency.
4.	Need for up-conversion	Required	Not required	Not required
5.	Complexity	Less	Medium	High
6.	Design aspects	Q of tuned circuit, Filter type, it size, weight and upper frequency limit.	Design of 90° phase shifter for entire modulating frequency range. Symmetry of balanced modulators.	Symmetry of balanced modulators.
7.	Bulkyness	Yes	No	No
8.	Switching ability	Not possible with existing circuit. Extra filter and switching network is necessary	Easily possible	Easily possible. But extra crystal is required

DEMODULATION OF SSB WAVE

COHERENT DETECTION OF SSB WAVE



COHERENT DETECTION OF AN SSB WAVE



SPECTRUM OF THE PRODUCT MODULATOR OUTPUT V(t)

EQUATION OF COHERENT DETECTION

The product modulator output is given by

$$v(t) = \frac{1}{2}A_c A_c' \cos(2\pi f_c t)[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)]$$
$$= \frac{1}{4}A_c A_c' m(t) + \frac{1}{4}A_c A_c'[m(t)\cos(4\pi f_c t) - \hat{m}(t)\sin(4\pi f_c t)]$$

The resulting demodulated signal is given by

$$v_o(t) = \frac{1}{4}A_c A_c'[m(t)\cos(2\pi \Delta f t) + \hat{m}(t)\sin(2\pi \Delta f t)]$$

$$v_o(t) = \frac{1}{4}A_c A_c'[m(t)\cos\phi + \hat{m}(t)\sin\phi]$$

The fourier transform of the above eqn is

$$V_o(f) = \frac{1}{4}A_c A_c'[M(f)\cos\phi + \hat{M}(f)\sin\phi]$$

From the definition of hilbert transform its given as

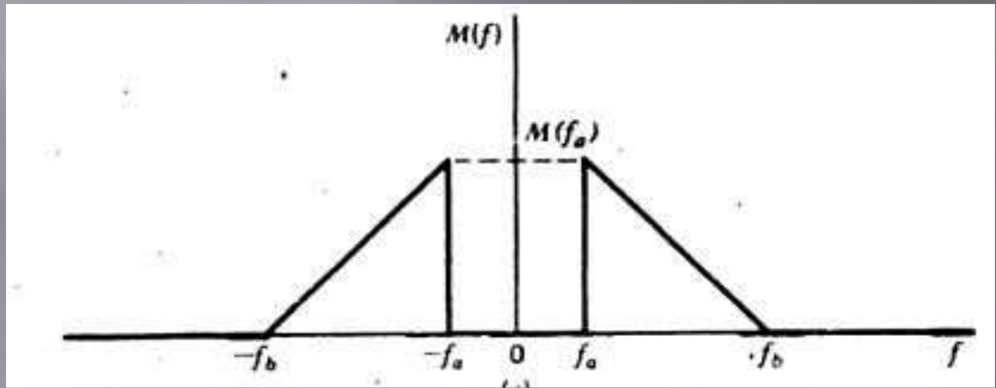
$$\hat{M}(f) = -j\,\mathrm{sgn}(f)M(f)$$

Substituting the eqn in the above eqn we get

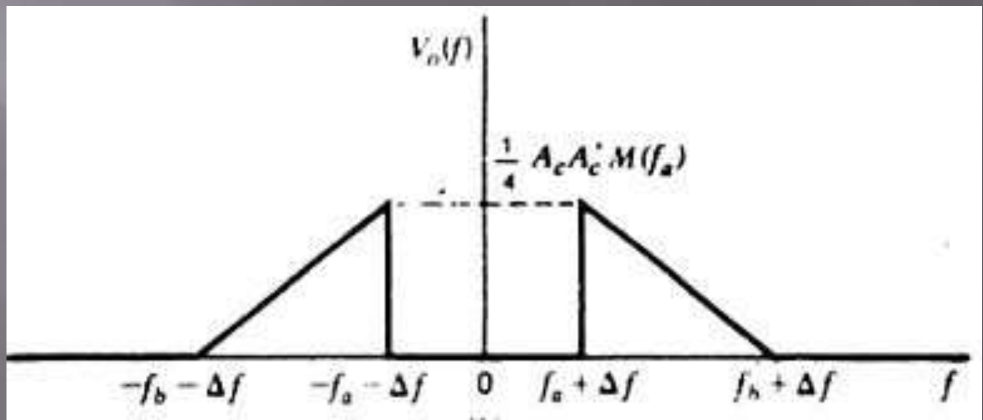
$$V_o(f) = \begin{cases} \frac{1}{4}A_c A_c' M(f)\exp(-j\phi) & f > 0 \\ \frac{1}{4}A_c A_c' M(f)\exp(+j\phi) & f < 0 \end{cases}$$

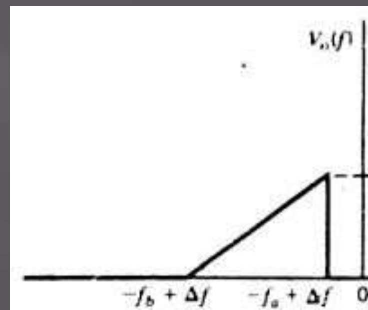
EFFECT OF THE FREQUENCY ERROR f ON THE O/P OF THE COHERENT DETECTOR WITH SSB WAVE s(t) AS I/P

Spectrum of Baseband Signal with Energy Gap in the Interval $-f_a<f<f_a$



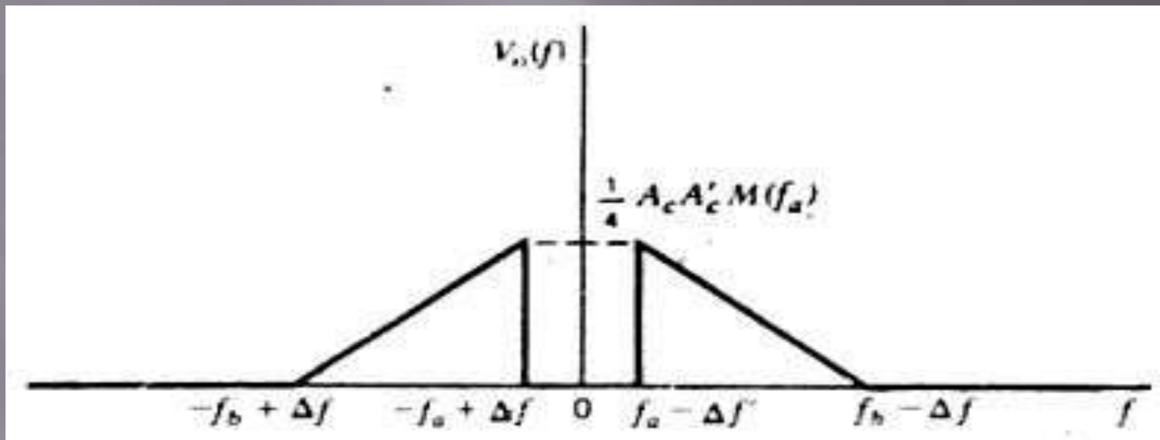
Spectrum of coherent detector o/p with s(t) containing lower side band and $\Delta f>0$ or with s(t) containing upper sideband and $\Delta f<0$





EFFECT OF THE FREQUENCY ERROR f ON THE O/P OF THE COHERENT DETECTOR WITH SSB WAVE s(t) AS I/P cont…

Spectrum of coherent detector output with s(t) containing upper sideband and $\Delta f>0$ or with s(t) containing lower sideband and $\Delta f<0$

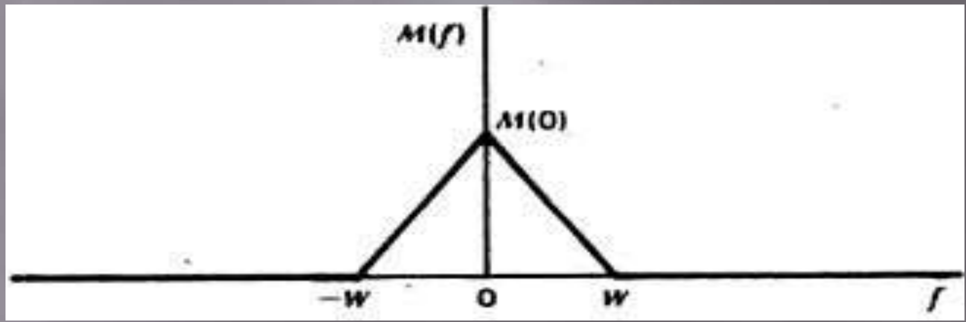


VESTIGIAL SIDE BAND

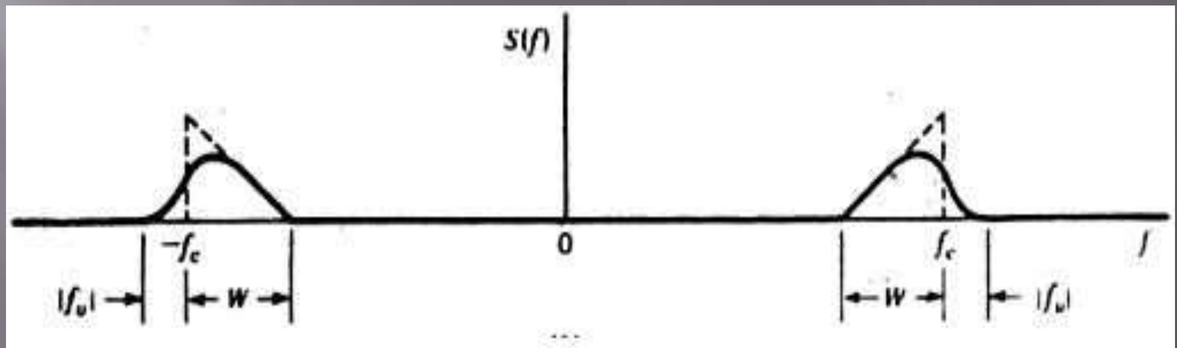
What are the limitations of SSB?

What is Vestigial sideband modulation & How is VSB wave generated?

SPECTRUM OF BASEBAND SIGNAL

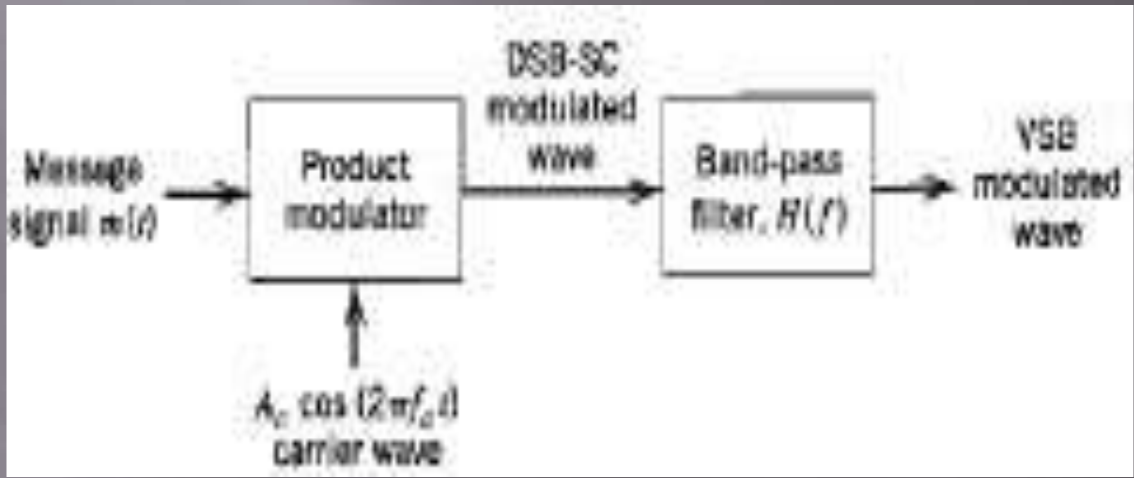


SPECTRUM OF VSB WAVE



SCHEME FOR THE GENERATION AND DEMODULATION OF A VSB WAVE

BLOCK DIAGRAM OF VSB MODULATOR



EQUATION OF VSB WAVE

The fourier transform of $s_c(t)$ is given by

$$S_c(f)=\frac{1}{2}A_cM(f)[H(f-f_c)+H(f+f_c)]$$

This eqn can be simplified as

$$S_c(f)=\frac{1}{2}A_cM(f)$$

This relation shows that the in-phase component of the VSB wave s(t) is defined by

$$s_c(t)=\frac{1}{2}A_cm(t)$$

To determine the quadrature component $s_s(t)$ of the VSB wave s(t) we first obtained the fourier transform of $s_c(t)$ as

$$S_s(f)=\frac{j}{2}A_cM(f)[H(f-f_c)-H(f+f_c)]$$

This eqn suggests that we may generate ss(t) except for a scaling factor by passing the message signal m(t) through a filter whose transfer function is defined by

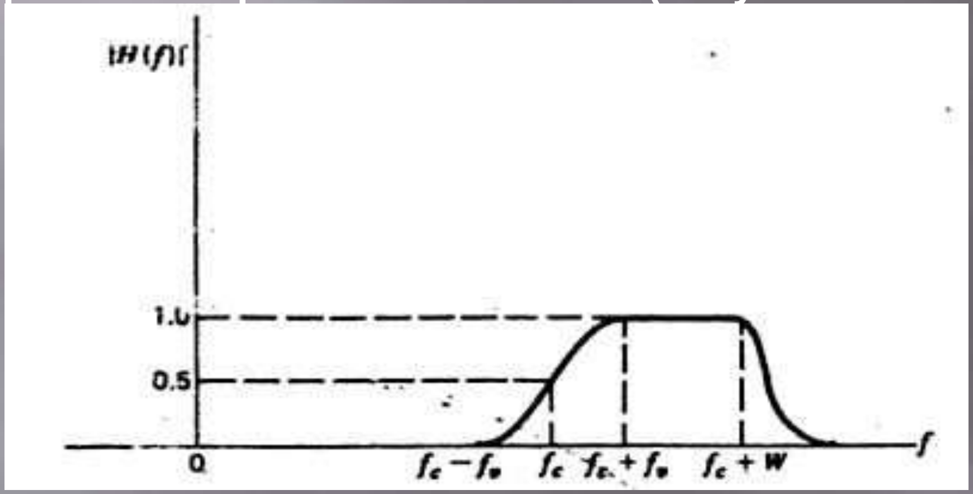
$$H_s(f)=j[H(f-f_c)-H(f+f_c)]$$

Thus the quadrature component of VSB wave is

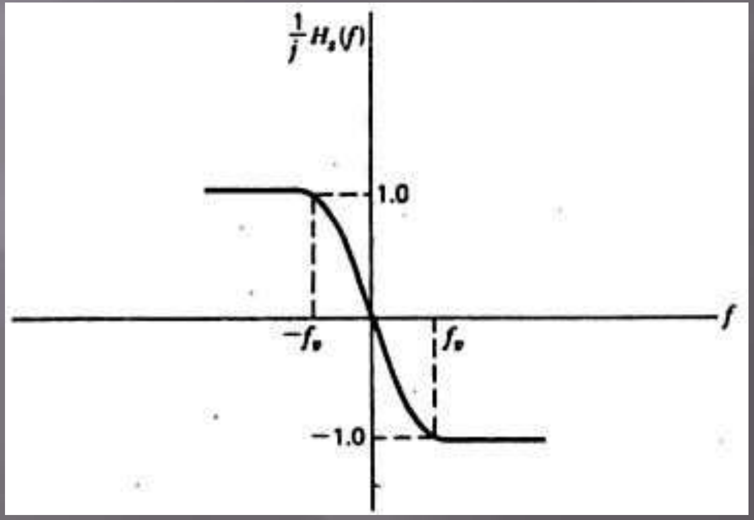
$s_s(t)=1/2\ A_cm_s(t)$

RESPONSE OF A VSB FILTER

Amplitude Response of VSB Filter (Only Positive Frequency Portion)



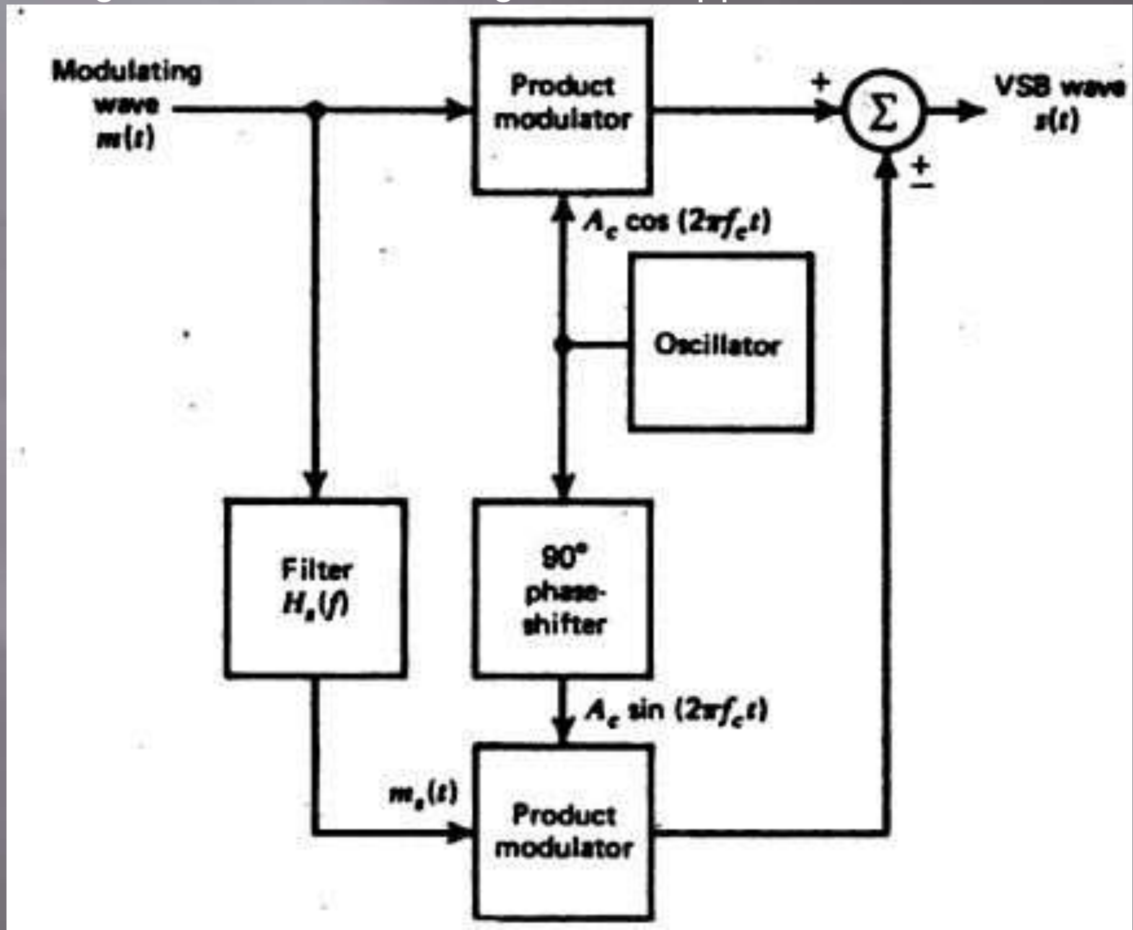
Frequency response of filter for producing the quadrature component of the VSB wave



GENERATION OF VSB WAVE

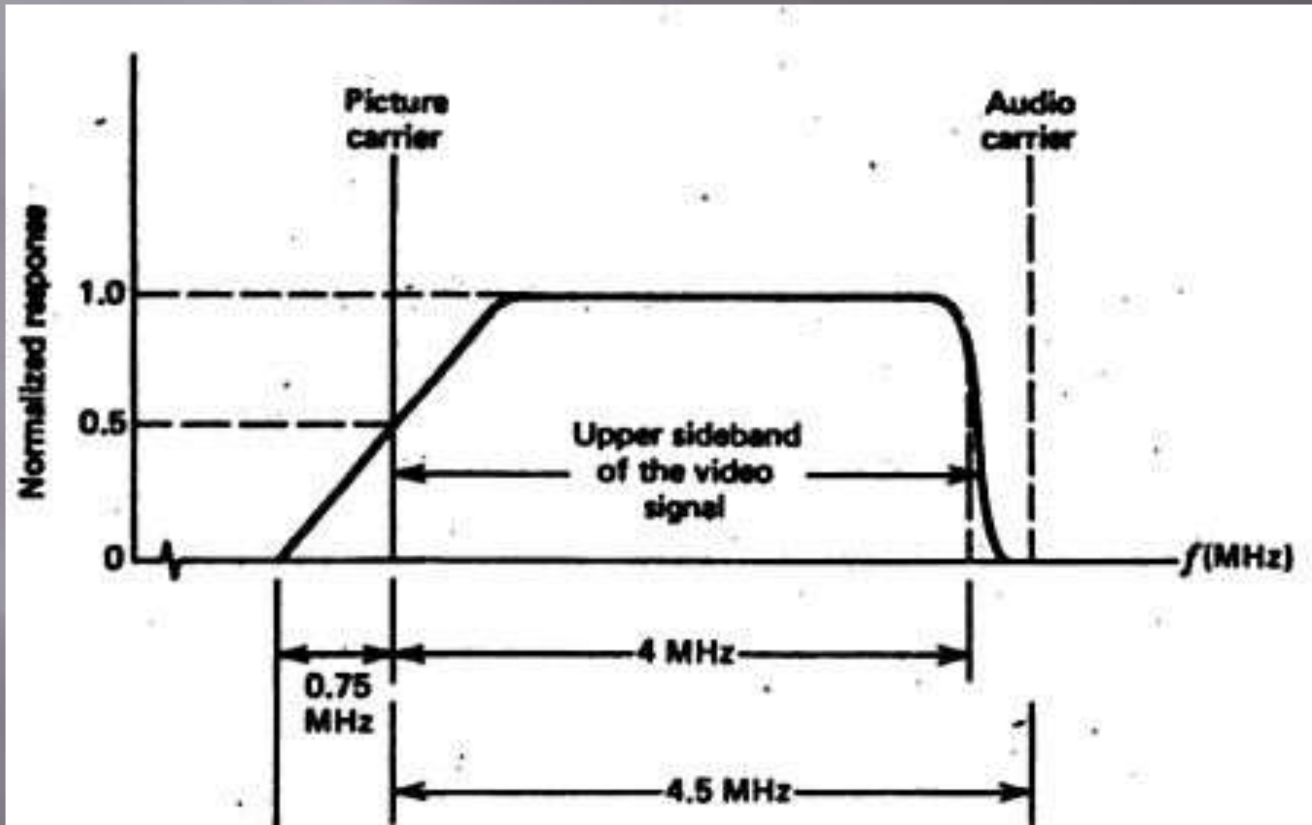
Block diagram of phase discrimination method for generating a VSB wave

- When the minus sign is selected a vestige of the lower sideband is transmitted
- When the plus sign is selected a vestige of the upper sideband is transmitted



RESPONSE OF VSB FILTER

Frequency response of a VSB filter used in TV receivers



ENVELOPE DETECTION OF A VSB WAVE PLUS CARRIER

The factor k_a modifies the modulated wave applied to the envelope detector input as

$$s(t) = A_c[1 + \tfrac{1}{2}k_a m(t)]\cos(2\pi f_c t) - \tfrac{1}{2}k_a A_c m_s(t)\sin(2\pi f_c t)$$

The envelope detector output denoted by a(t) is therefore

$$a(t) = A_c\{[1 + \tfrac{1}{2}k_a m(t)]^2 + [\tfrac{1}{2}k_a m_s(t)]^2\}^{1/2}$$

$$= A_c[1 + \tfrac{1}{2}k_a m(t)]\left\{1 + \left[\frac{\tfrac{1}{2}k_a m_s(t)}{1 + \tfrac{1}{2}k_a m(t)}\right]^2\right\}^{1/2}$$

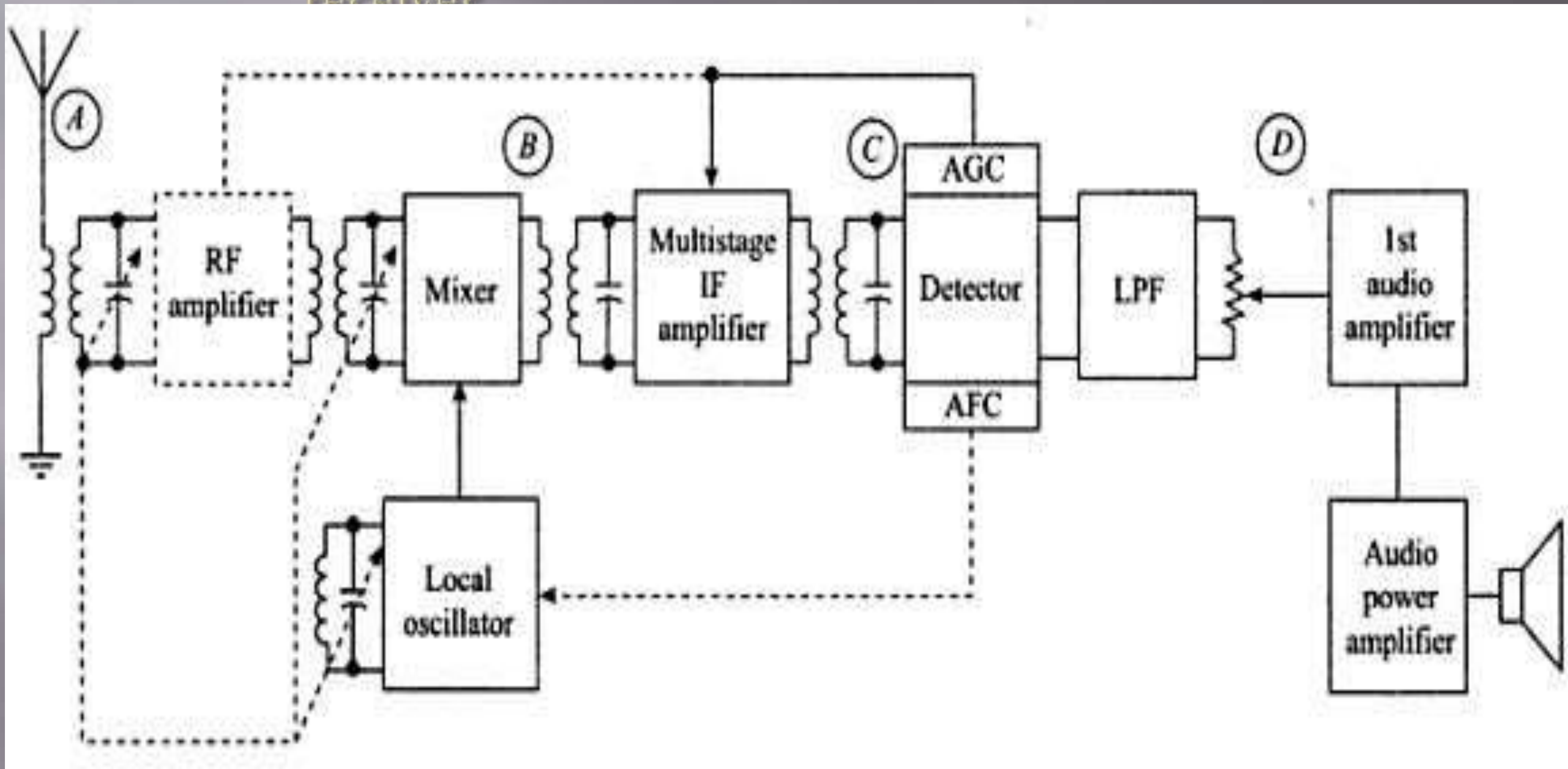
COMPARISON OF SSB ,DSB, VSB

Sr. No.	Parameter	SSB	DSB	ISB	VSB
1.	Power	less	Medium	High	High but less than ISB
2.	Bandwidth	f_m	$2\ f_m$	$f_{m1}+f_{m2}$	$f_m < Bw < 2f_m$
3.	Modulating inputs	1	1	2	1
4.	Use for	Radio communication	Radio communication	Telegraphy and telephony	Television
5.	Carrier suppression	Complete	Complete	Partly	No
6.	Sideband suppression	One sideband completely	No	One per channel	One sideband suppressed partially
7.	Transmission efficiency	Maximum	Moderate	Moderate	Moderate

SUPER HETRODYNE RECEIVER

What is superhetrodyne receiver?

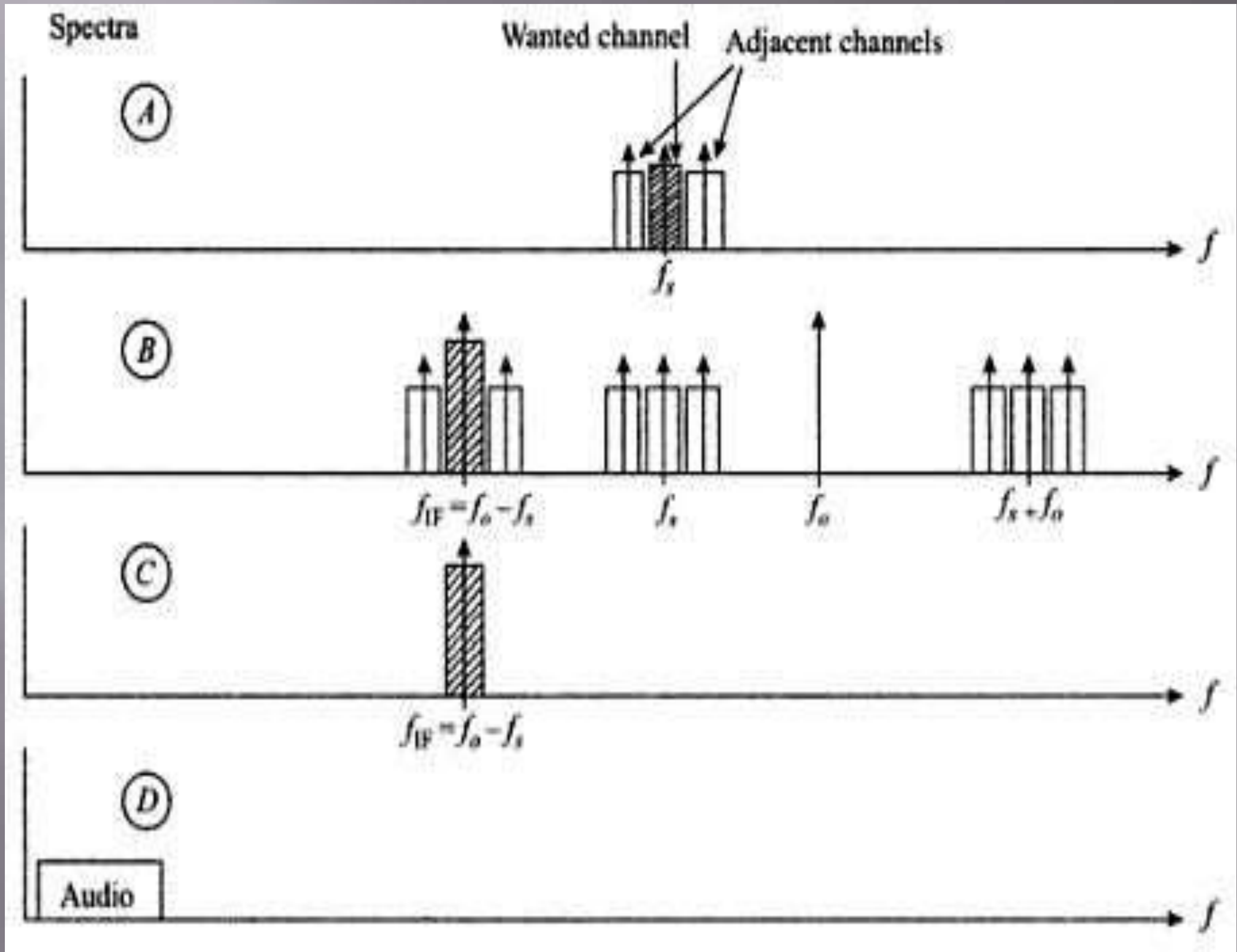
Operation of superhetrodyne receiver



Block diagram of superhetrodyne receiver

SUPER HETRODYNE RECEIVER

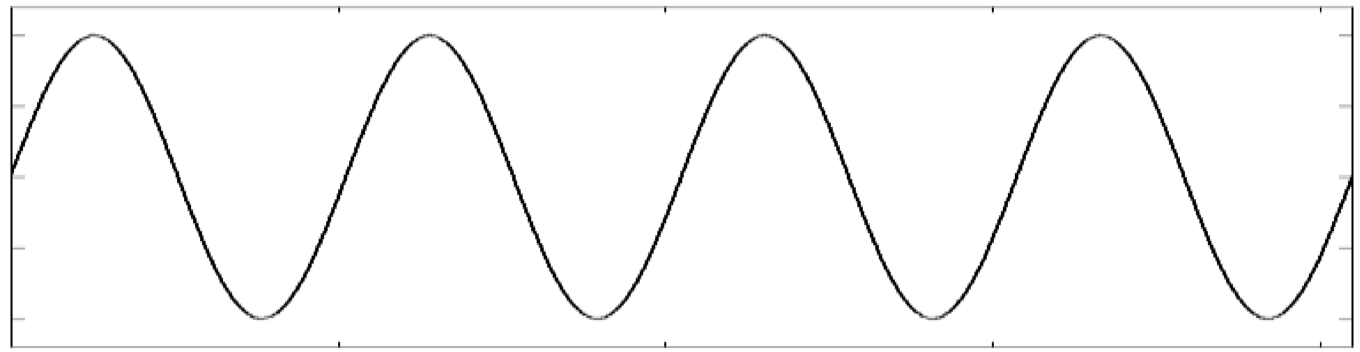
SPECTRUM OF SUPERHETRODYNE RECEIVER



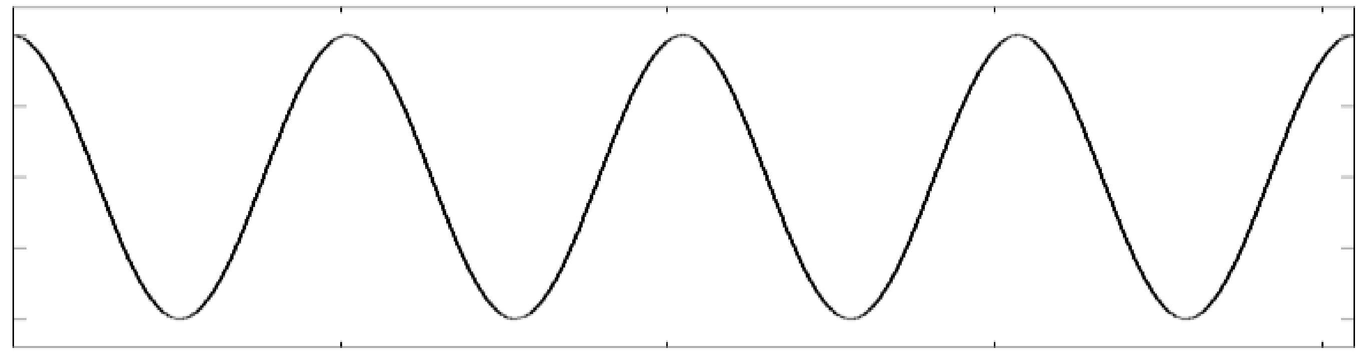
ANGLE MODULATION

Sinusoids

Sinusoid waveform

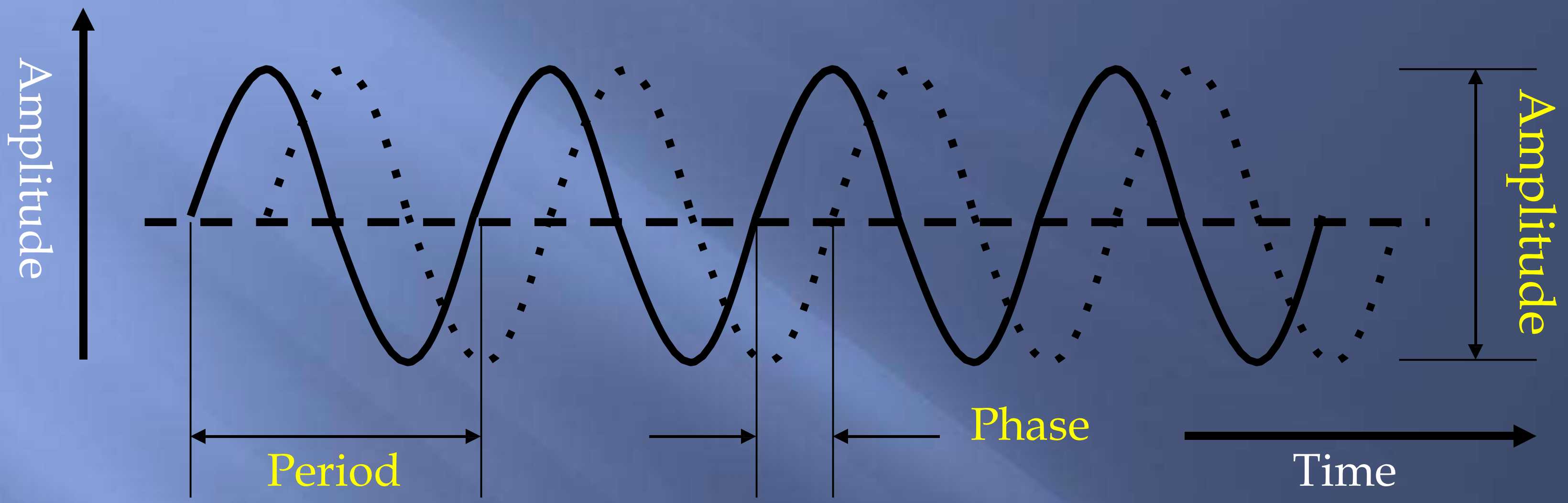


Cosinusoid waveform



A sinusoid, meaning a sine wave -or- a cosine wave, is the basic building block of all signals.

Sinewave Characteristics

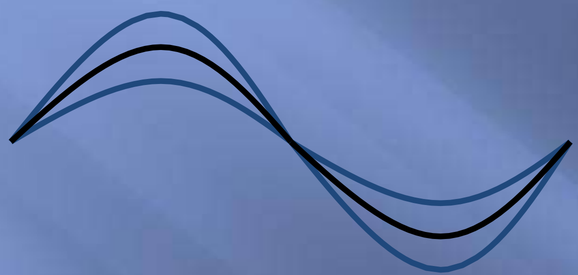


$$\text{Frequency} = \frac{1}{\text{Period}}$$

A sinusoid has three properties . These are its amplitude, period (or frequency), and phase.

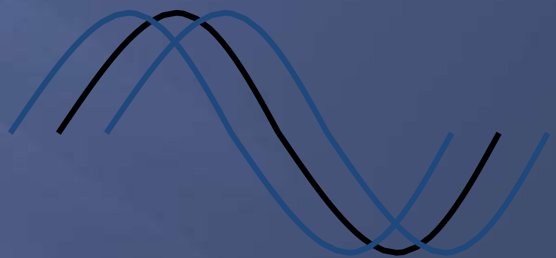
Types of Modulation

Amplitude Modulation



$$V \cdot \sin(\omega * t + \Phi)$$

Phase Modulation



$$V \cdot \sin(\omega * t + \Phi)$$

With very few exceptions, phase modulation is used for digital information.

Frequency Modulation

$$V \cdot \sin(\omega * t + \Phi)$$

Types of Modulation

Types of Information

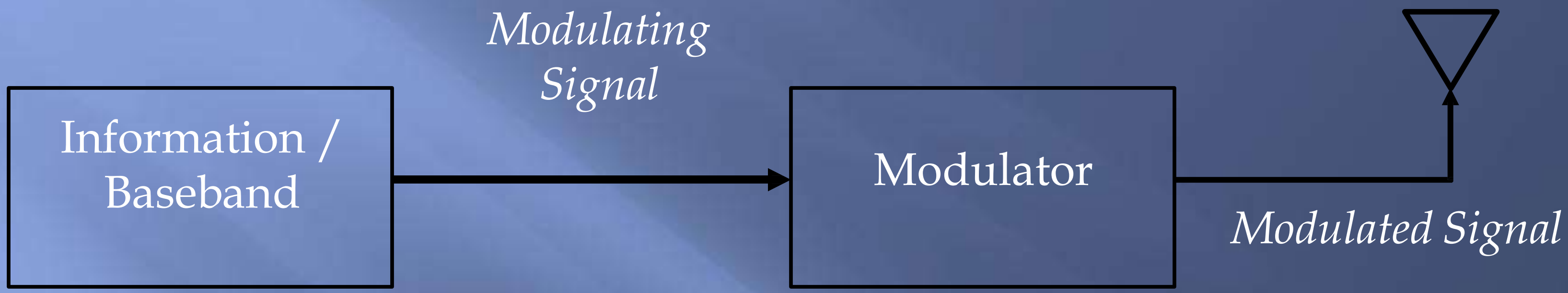
- Analog
- Digital

These two constitute angle modulation.

Carrier Variations

- Amplitude
- Frequency
- Phase

Modulation Process



The modulation general process is the same regardless of the how the carrier is modulated. For our purposes, modulation means the variation of a carrier wave in order to transfer information.

What is Angle Modulation?

$$v_c(t) = V \cdot \sin(2 \cdot \pi \cdot f_c \cdot t + \text{phase})$$

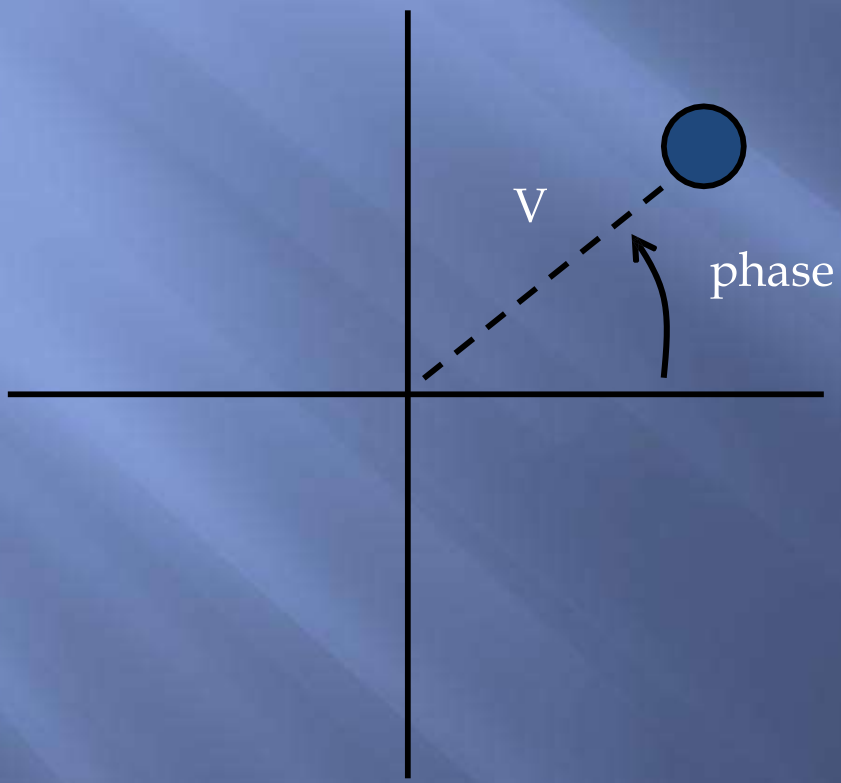
Amplitude

Frequency

Phase

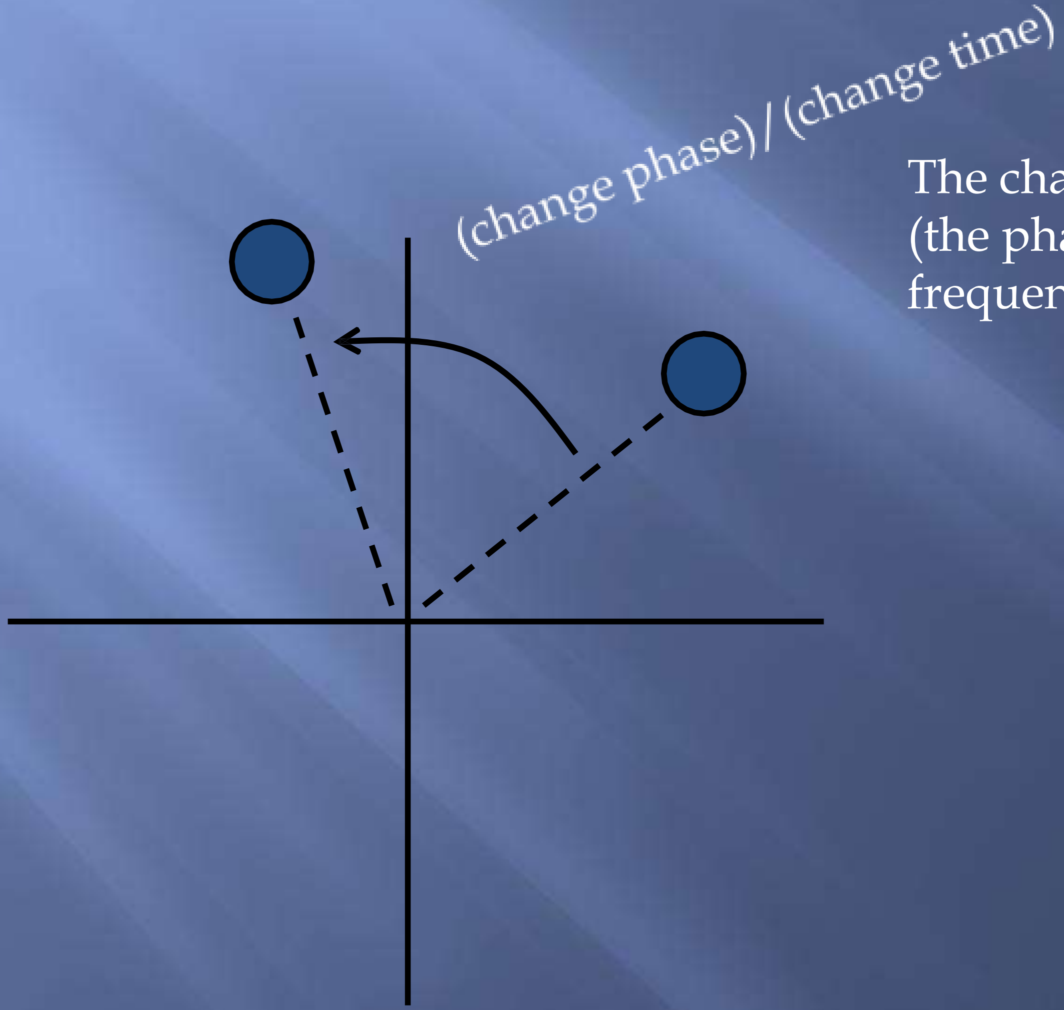
Angle modulation is a variation of one of these two parameters.

Understanding Phase vs. Amplitude



To understand the difference between phase and frequency, a signal can be thought of using a phasor diagram. The distance from the center is the signal's amplitude. The angle from the positive horizontal axis in phase.

Understanding Phase vs. Frequency



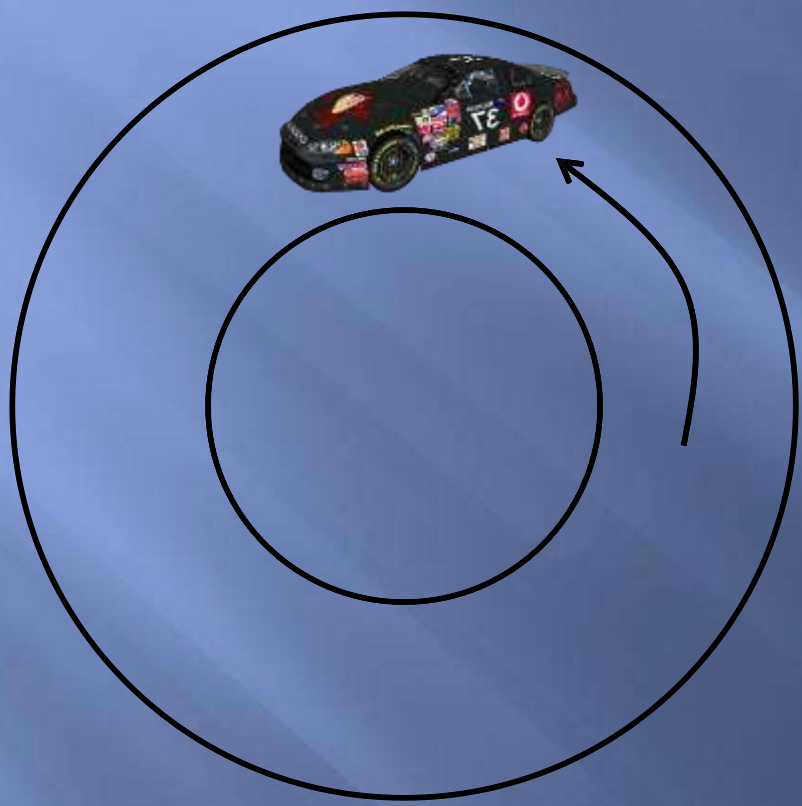
The change in the phase over time (the phase velocity) is the signal's frequency.

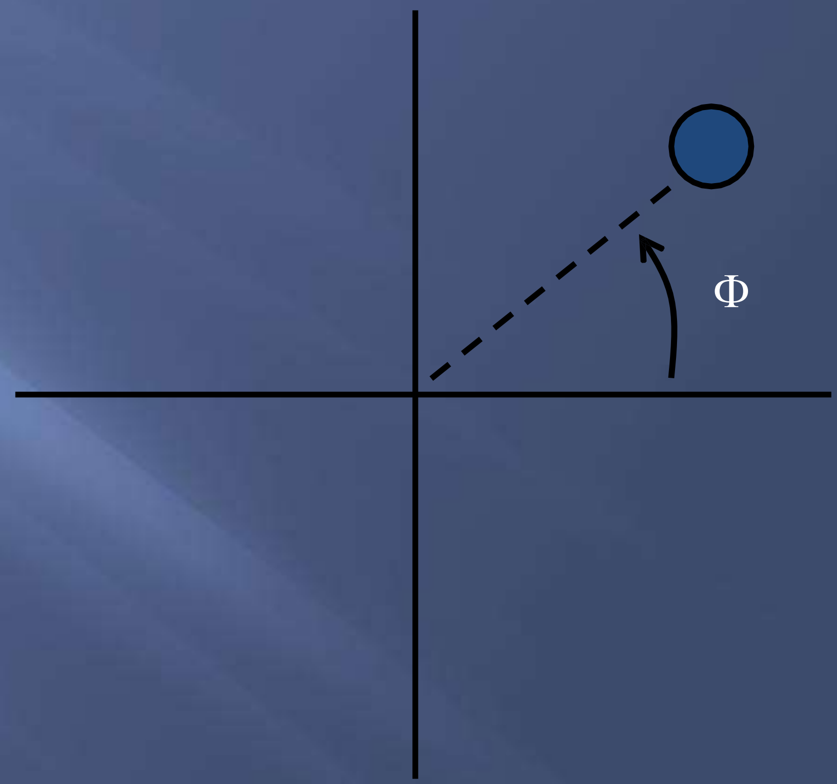
Understanding Phase vs. Frequency

$$\text{Phase} = \Phi$$

$$\text{Frequency} = \frac{\Delta\Phi}{\Delta t}$$

Signals vs Racing Car

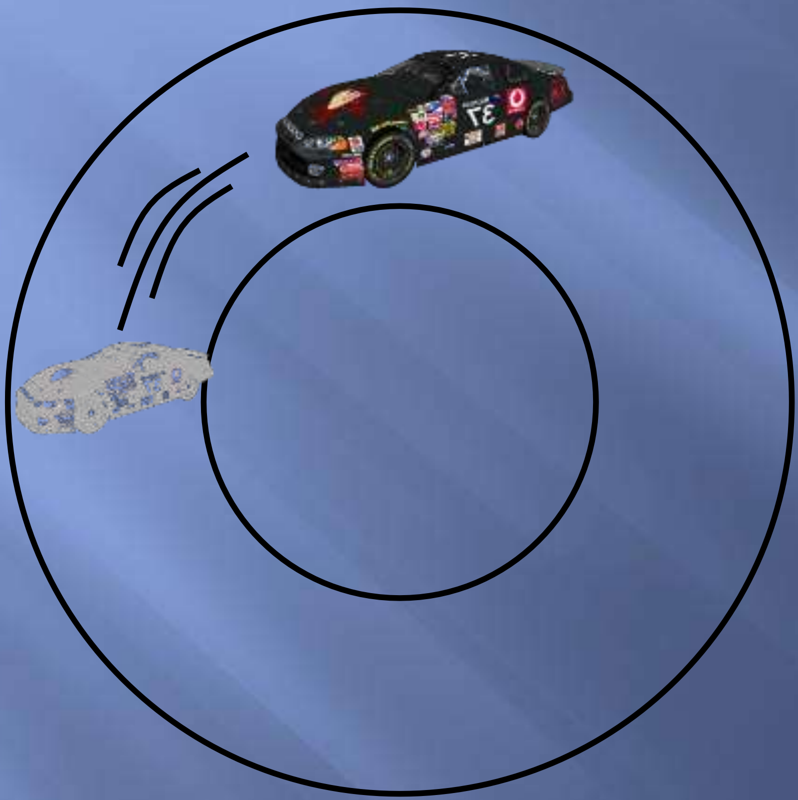




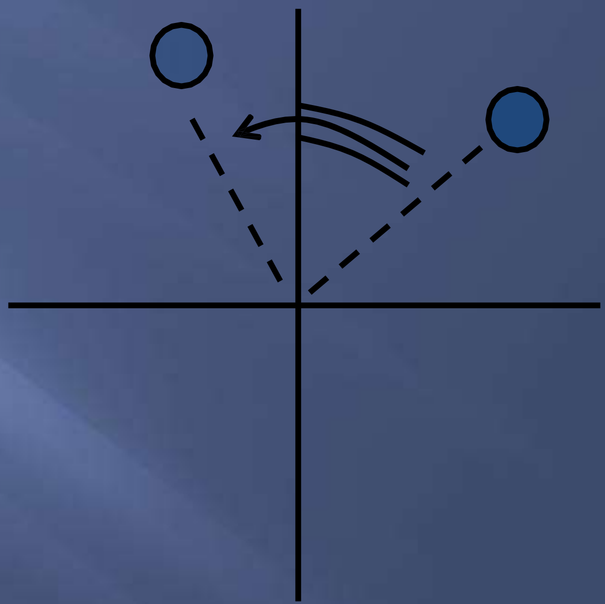
In Racing, we track each car by its position on the track.

In signals, we track the signal by its phase. This is its position on the phasor diagram.

Signals vs Racing Car



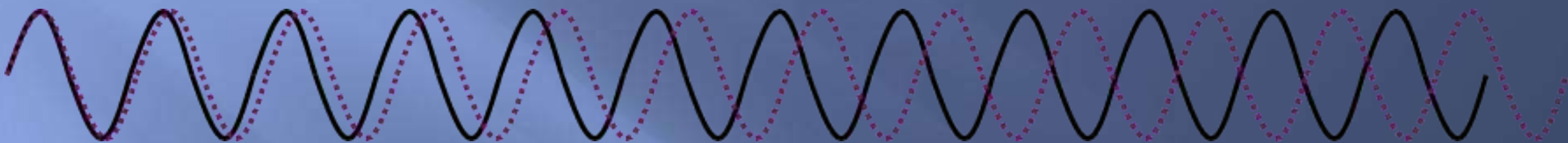
In Racing, we track a car's velocity by how fast it goes around the track.



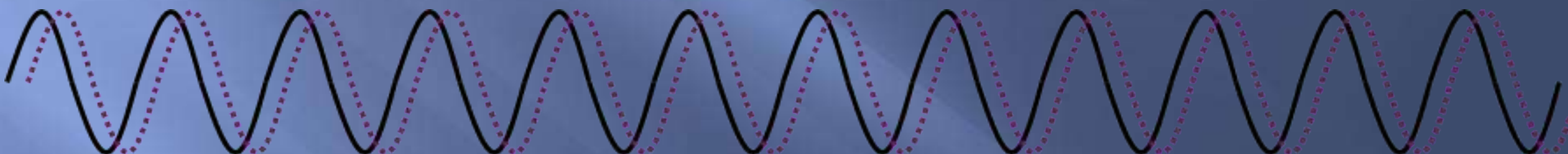
In signals, we track the signal's velocity by its frequency. This is how fast it goes around the phasor diagram.

Understanding Angle Modulation

Frequency Modulation



Phase Modulation



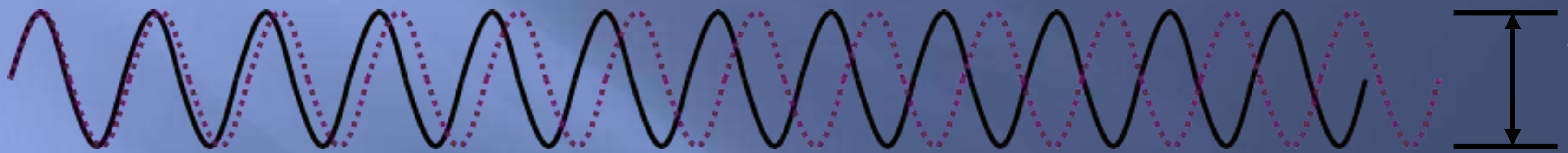
Angle modulation, either PM or FM, varies the frequency or phase of the carrier wave. Because of the practicalities of implementation, FM is predominant; analog PM is only used in rare cases.

$$V \cdot \sin(\omega * t + \Phi)$$

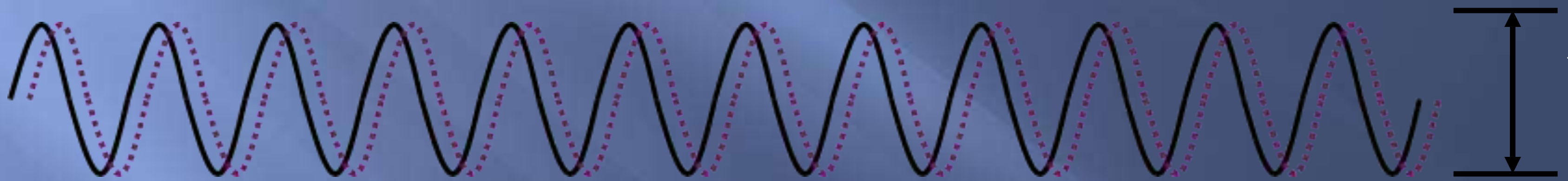
Vary one of these parameters

Understanding Angle Modulation

Frequency Modulation



Phase Modulation



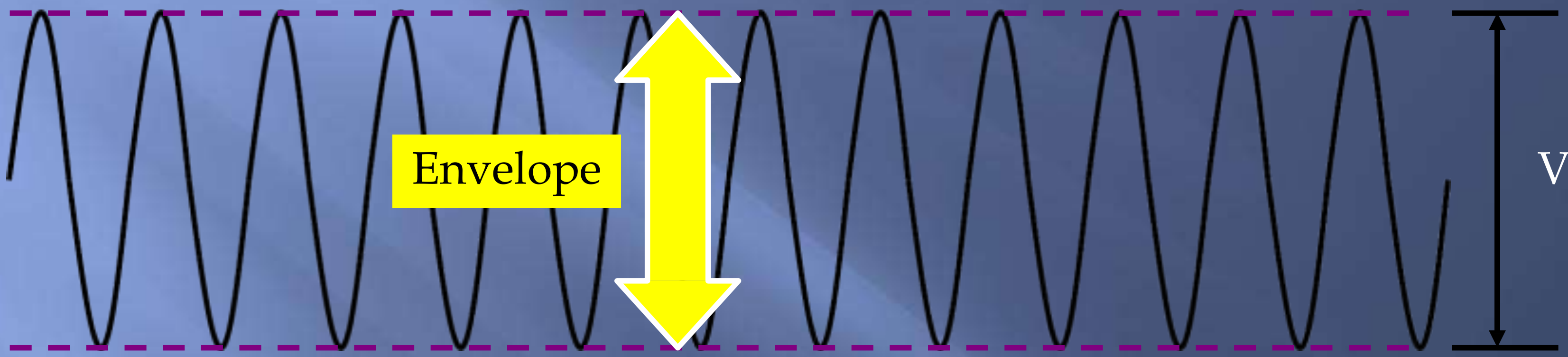
$$V \cdot \sin(\omega * t + \Phi)$$

This remains constant!

In either analog FM or PM, the amplitude remains constant.

Understanding Angle Modulation

Frequency Modulation



The envelope, meaning the difference between the maximum and minimum of the carrier, is constant in an FM signal. That's why FM is called a **constant envelope** signal. The power of an FM signal is shown at right. It does not depend upon the modulating signal or the amount of deviation (

$$\text{Power} = V^2$$

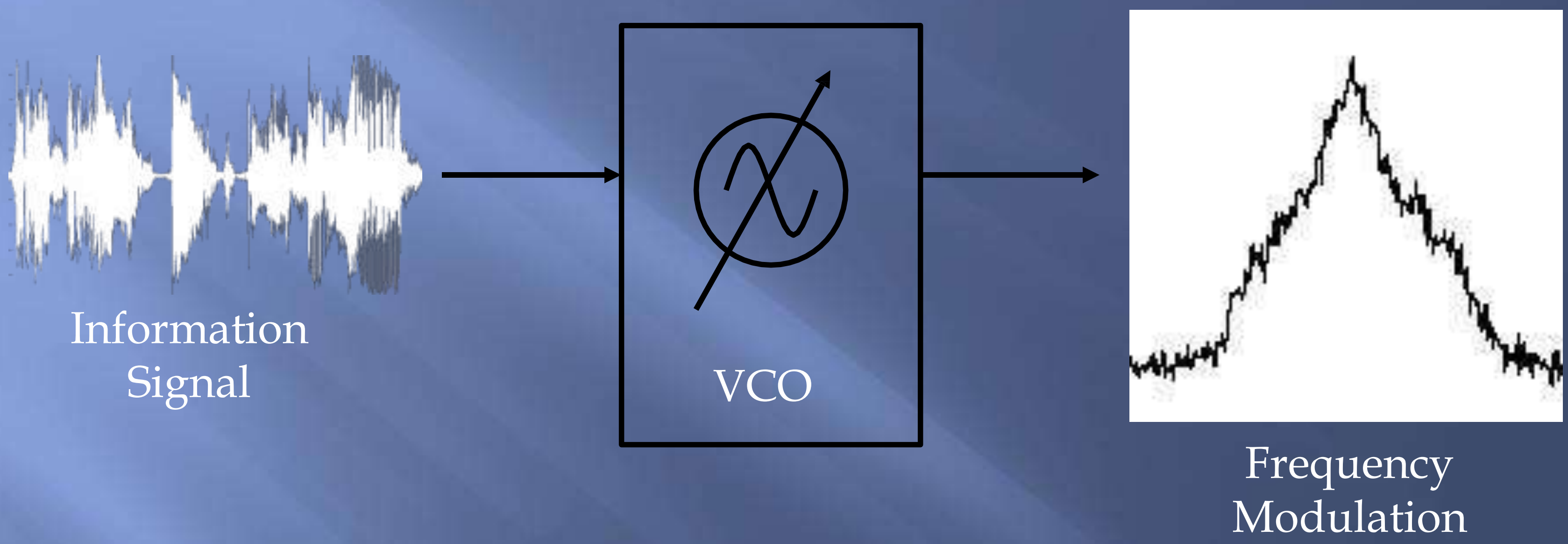
Calculating Total Power

$$v_c(t) = V \cdot \sin(2 \cdot \pi \cdot f_c \cdot t + \Phi(t))$$

$$\text{Total Power} = V^2$$

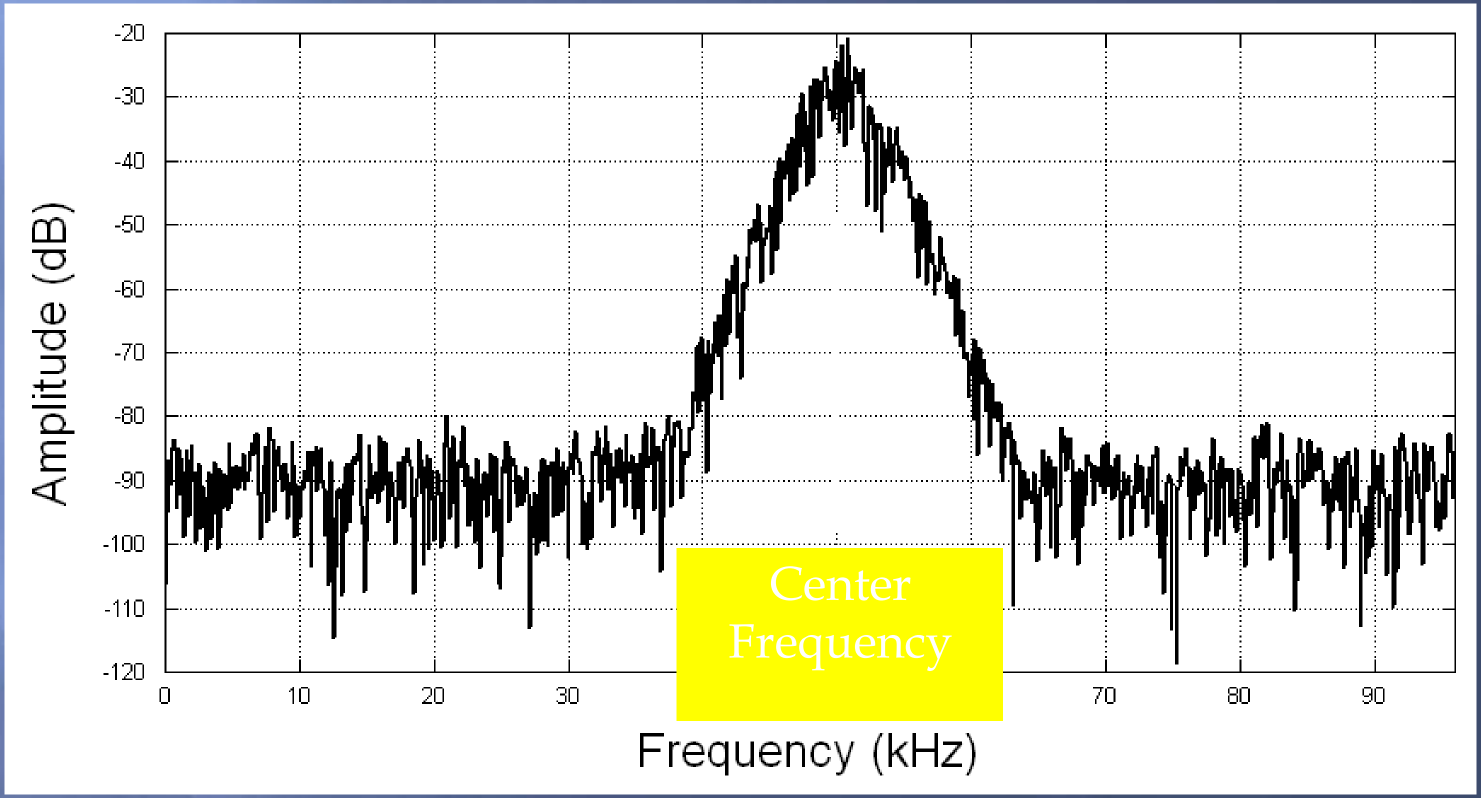
The total power of an FM signal is simply V^2. Therefore, the total power of an FM signal is the power of the carrier. Period. *This is regardless of the information or the deviation ratio.*

Varying the Frequency



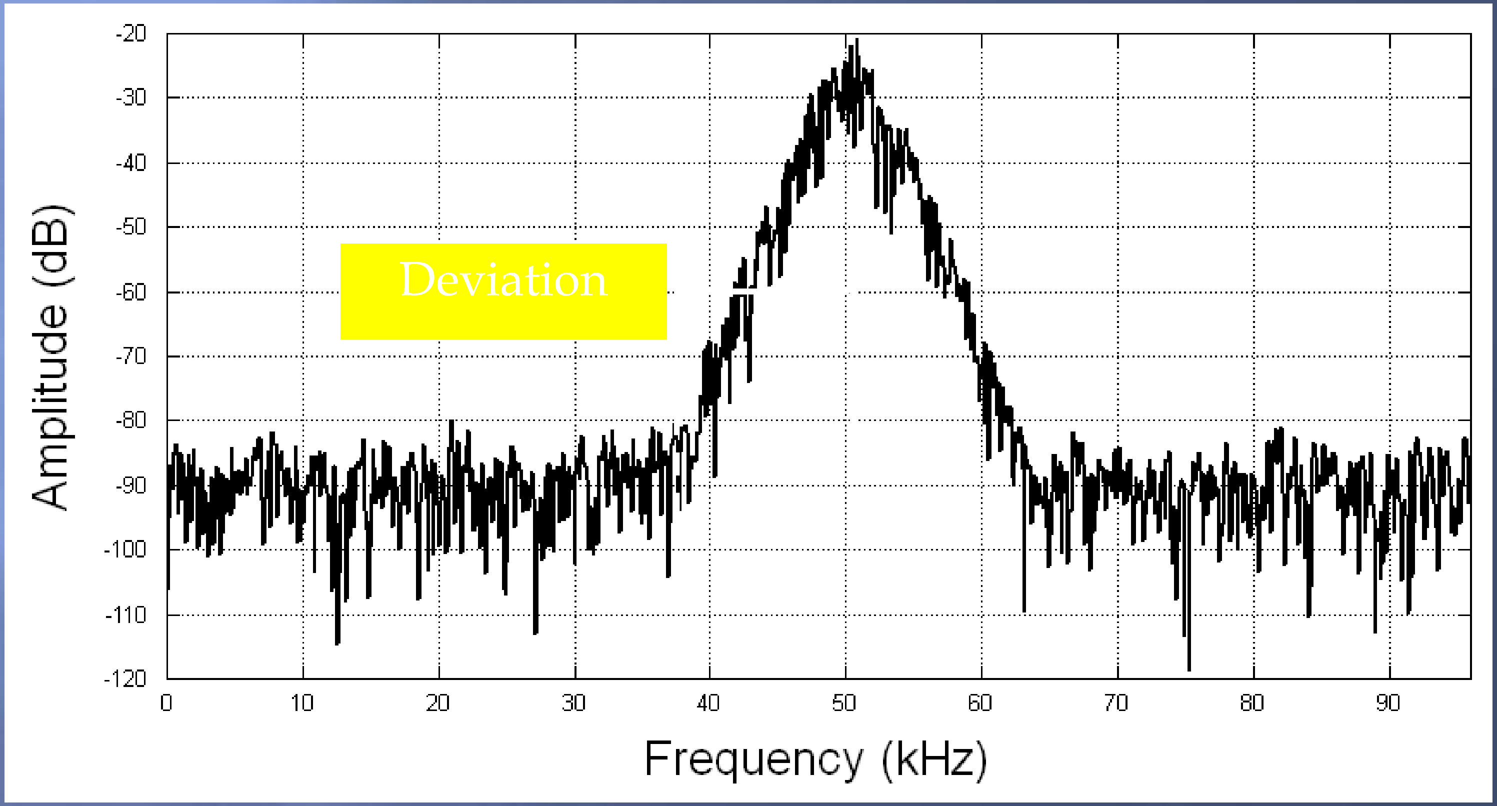
The voltage-controlled oscillator (VCO) is a device whose output frequency changes with the amplitude of the modulating signal. The amount of change, called its <u>**deviation constant**</u>, is dependent upon its design.

Understanding Terms



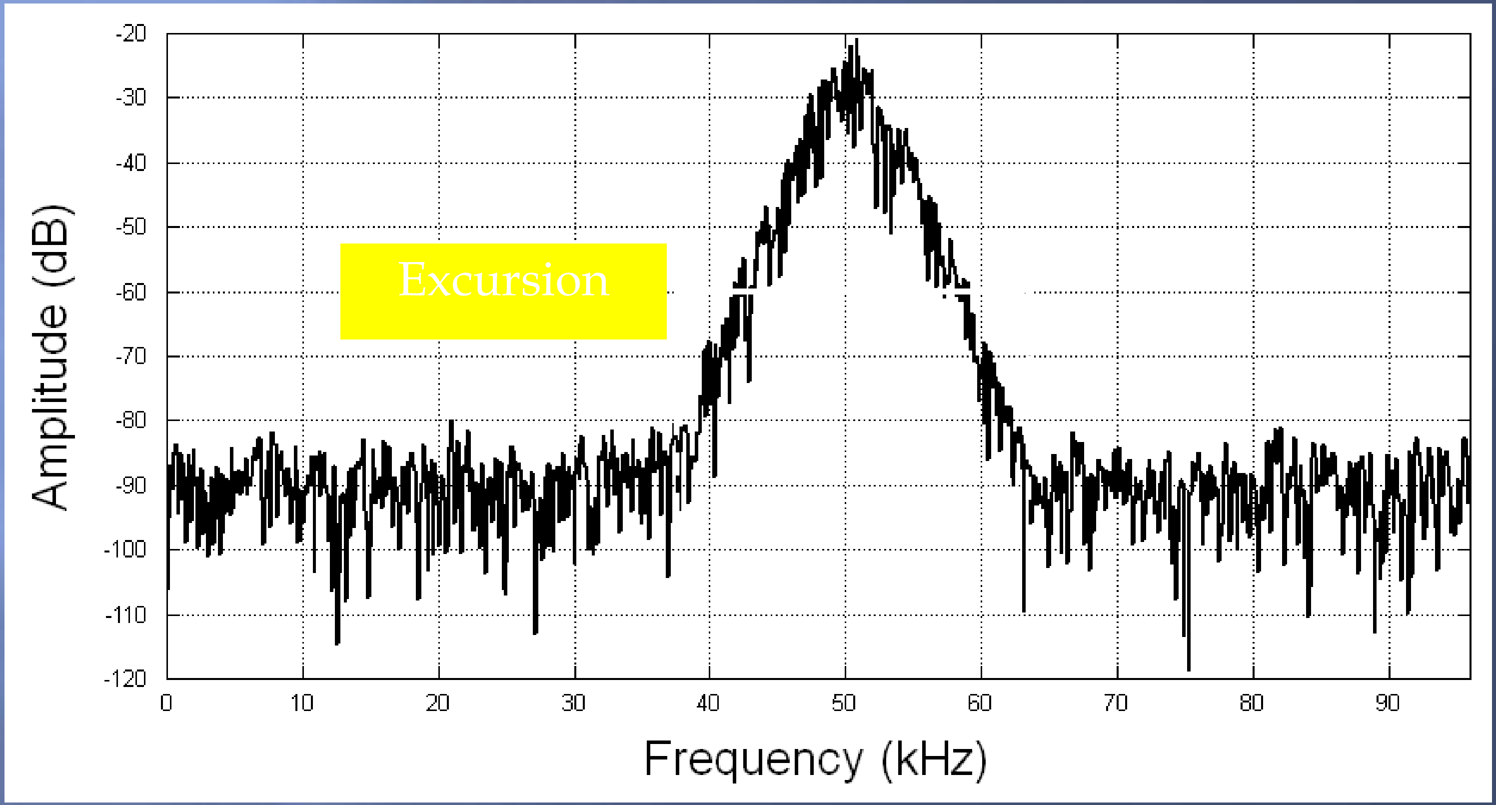
An FM signal has its energy spread over an infinite number of spectral components. It's center frequency is the average center of the energy.

Understanding Terms



The **deviation** is the maximum frequency change from the center frequency.

Understanding Terms



The **excursion** is the difference between the maximum and minimum frequency changes. This is also called the **maximum deviation** or **total deviation**.

Amplitude vs Angle Modulation

Amplitude Modulation

$$v_c(t) = V(t) \cdot \sin(2 \cdot \pi \cdot f_c \cdot t + \Phi)$$

Angle Modulation

$$v_c(t) = V \cdot \sin(2 \cdot \pi \cdot f_c \cdot t + \Phi(t))$$

Angle Modulation – Frequency Modulation

Consider again the general carrier

$$v_c(t) = V_c \cos(\omega_c t + \varphi_c)$$

$(\omega_c t + \varphi_c)$ represents the angle of the carrier.

There are two ways of varying the angle of the carrier.

a) By varying the frequency, ω_c – **Frequency Modulation.**
b) By varying the phase, ϕ_c – **Phase Modulation**

Frequency Modulation

In FM, the message signal $m(t)$ controls the frequency f_c of the carrier. Consider the carrier

$$v_c(t) = V_c \cos(\omega_c t)$$

then for FM we may write:

FM signal $v_s(t) = V_c \cos(2\pi(f_c + \text{frequency deviation})t)$,where the frequency deviation will depend on $m(t)$.

Given that the carrier frequency will change we may write for an instantaneous carrier signal

$$V_c \cos(\omega_i t) = V_c \cos(2\pi f_i t) = V_c \cos(\varphi_i)$$

where ϕ_i is the instantaneous angle = $\omega_i t = 2\pi f_i t$ and f_i is the instantaneous frequency.

Frequency Modulation

Since $\varphi_i = 2\pi f_i t$ then $\dfrac{d\varphi_i}{dt} = 2\pi f_i$ or $f_i = \dfrac{1}{2\pi}\dfrac{d\varphi_i}{dt}$

i.e. frequency is proportional to the rate of change of angle.

If f_c is the unmodulated carrier and f_m is the modulating frequency, then we may deduce that

$$f_i = f_c + \Delta f_c \cos(\omega_m t) = \frac{1}{2\pi}\frac{d\varphi_i}{dt}$$

Δf_c is the peak deviation of the carrier.

Hence, we have $\dfrac{1}{2\pi}\dfrac{d\varphi_i}{dt} = f_c + \Delta f_c \cos(\omega_m t)$,*i.e.* $\dfrac{d\varphi_i}{dt} = 2\pi f_c + 2\pi \Delta f_c \cos(\omega_m t)$

Frequency Modulation

After integration *i.e.* $\int \left(\omega_c + 2\pi \Delta f_c \cos(\omega_m t)\right) dt$

$$\varphi_i = \omega_c t + \frac{2\pi \Delta f_c \sin(\omega_m t)}{\omega_m}$$

$$\varphi_i = \omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)$$

Hence for the FM signal, $v_s(t) = V_c \cos(\varphi_i)$

$$v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)\right)$$

Frequency Modulation

The ratio $\frac{\Delta f_c}{f_m}$ is called the **Modulation Index** denoted by β *i.e.*

$$\beta = \frac{\text{Peak frequency deviation}}{\text{modulating frequency}}$$

Note – FM, as implicit in the above equation for $v_s(t)$, is a non-linear process – *i.e.* the principle of superposition does not apply. The FM signal for a message $m(t)$ as a band of signals is very complex. Hence, $m(t)$ is usually considered as a 'single tone modulating signal' of the form

$$m(t) = V_m \cos(\omega_m t)$$

Frequency Modulation

The equation $v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m}\sin(\omega_m t)\right)$ may be expressed as Bessel series (Bessel functions)

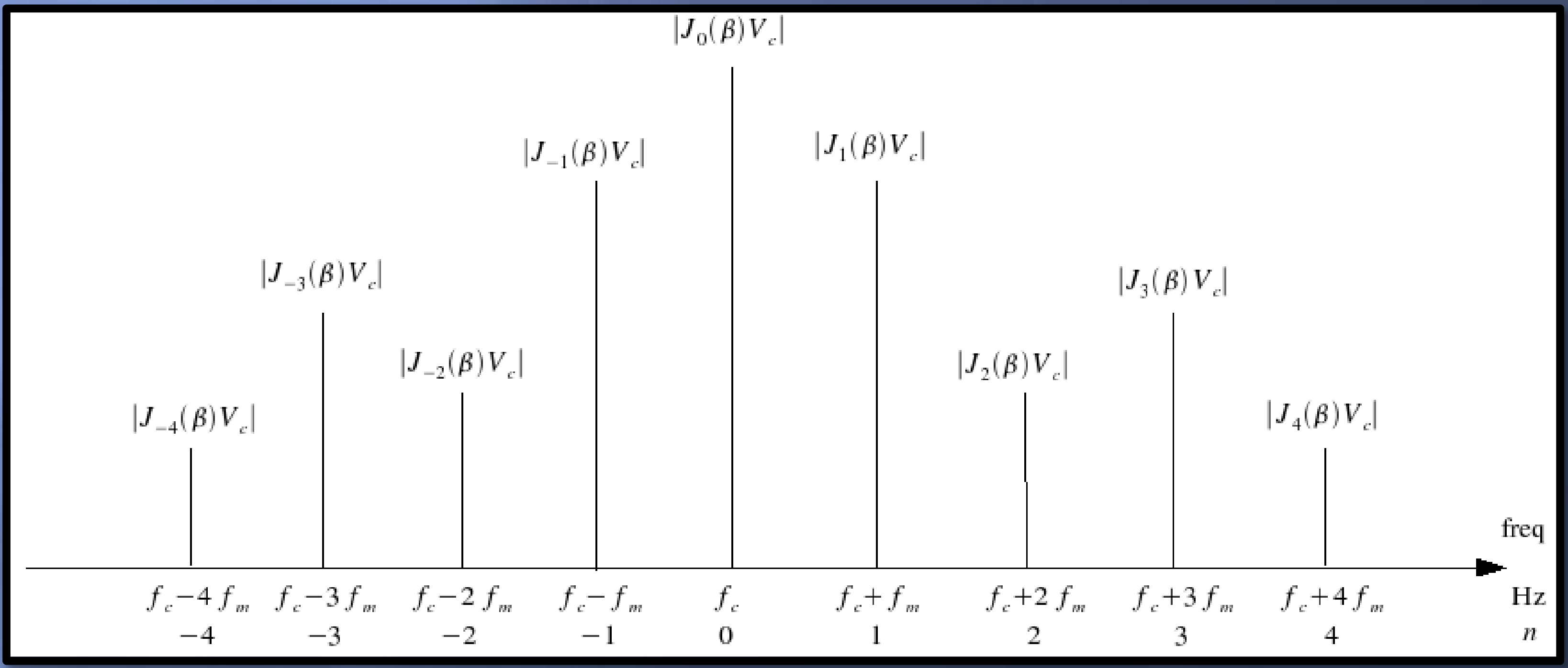
$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos(\omega_c + n\omega_m)t$$

where $J_n(\beta)$ are Bessel functions of the first kind. Expanding the equation for a few terms we have:

$$v_s(t) = \underbrace{V_c J_0(\beta)}_{\text{Amp}} \cos(\underbrace{\omega_c}_{f_c})t + \underbrace{V_c J_1(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c + \omega_m)}_{f_c + f_m}t + \underbrace{V_c J_{-1}(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c - \omega_m)t}_{f_c - f_m}$$

$$+ \underbrace{V_c J_2(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c + 2\omega_m)t}_{f_c + 2f_m} + \underbrace{V_c J_{-2}(\beta)}_{\text{Amp}} \cos\underbrace{(\omega_c - 2\omega_m)t}_{f_c - 2f_m} + \cdots$$

FM Signal Spectrum.



The amplitudes drawn are completely arbitrary, since we have not found any value for $J_n(\beta)$ – this sketch is only to illustrate the spectrum.

Calculating FM Bandwidth

The Fourier transform for an FM signal modulated by a real signal would be extremely difficult. Instead, engineers use the special case of an FM signal modulated by sinusoid, which boils down to:

$$\frac{1}{2\cdot\pi}\cdot\int_{-\pi}^{\pi} e^{-j(n\cdot x-\beta\cdot\sin(x))}\,dx$$

This integral cannot be solved in closed form. In order to figure out actual numerical answers, we use Bessel functions, specifically Bessel functions of the first kind of order **n** and argument **β**.

Calculating FM Bandwidth

Angle Modulation

$$v_c(t) = V \cdot \sin(2 \cdot \pi \cdot f_c \cdot t + \Phi(t))$$

For PM

$$\Phi(t) = \beta \cdot \sin(\omega_m \cdot t)$$

For FM

$$\Phi(t) = \int \beta \cdot \sin(\omega_m \cdot t)\, dt$$

Calculating FM Bandwidth

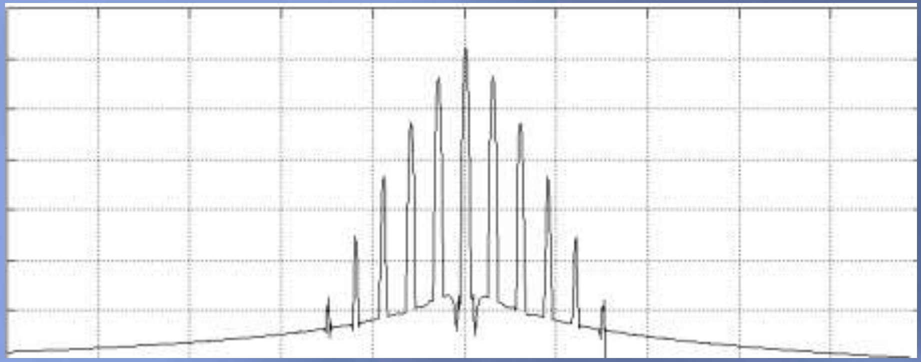
Angle Modulation

$$v_c(t) = V \cdot \sin(2 \cdot \pi \cdot f_c \cdot t + \Phi(t))$$

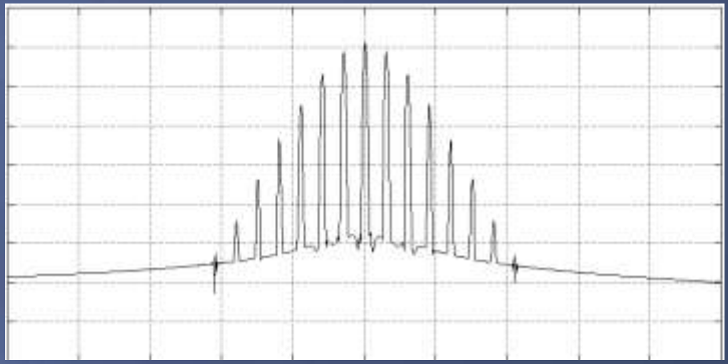
$$\beta = \frac{f_d \cdot V}{f_m}$$

The beta value, called the **modulation index**, is the ratio of the deviation of the modulator, f_d, multiplied by the amplitude of the modulating signal and divided by the modulating frequency, f_m *(Objectives 2, 3, 4b, 6)*.

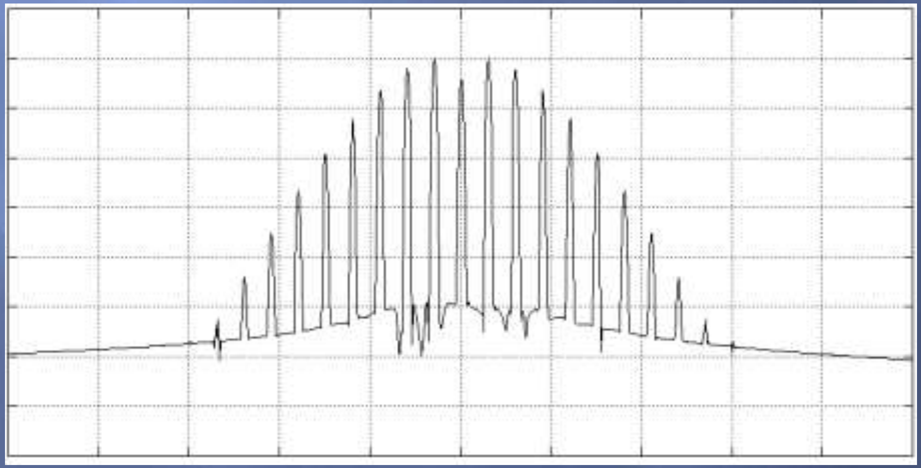
Understanding FM Bandwidth



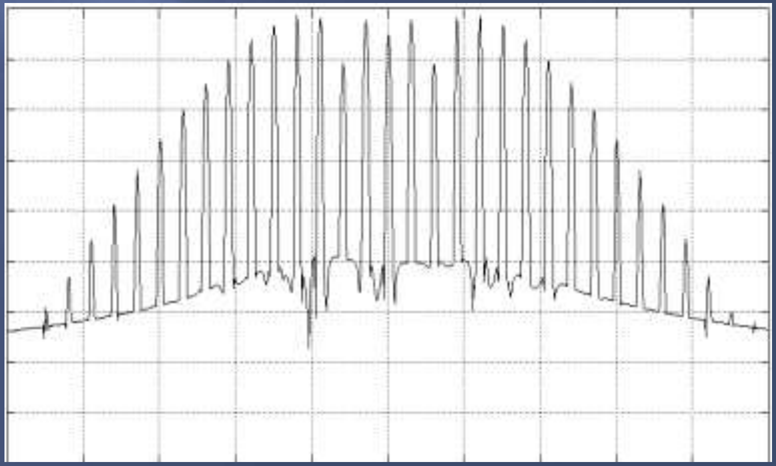
β=0.5



β=1



β=5



β=10

Calculating FM Bandwidth

Angle Modulation

$$v_c(t) = V \cdot \sin(2 \cdot \pi \cdot f_c \cdot t + \Phi(t))$$

$$D = \frac{f_d \cdot V}{W}$$

The designator when looking at real signals is the deviation ratio, D, which is the product of the modulator deviation, f_d, multiplied by the amplitude of the modulating signal, V, divided by the maximum frequency of the modulating signal, W *(Objectives 2, 3, 4b, 6)*.

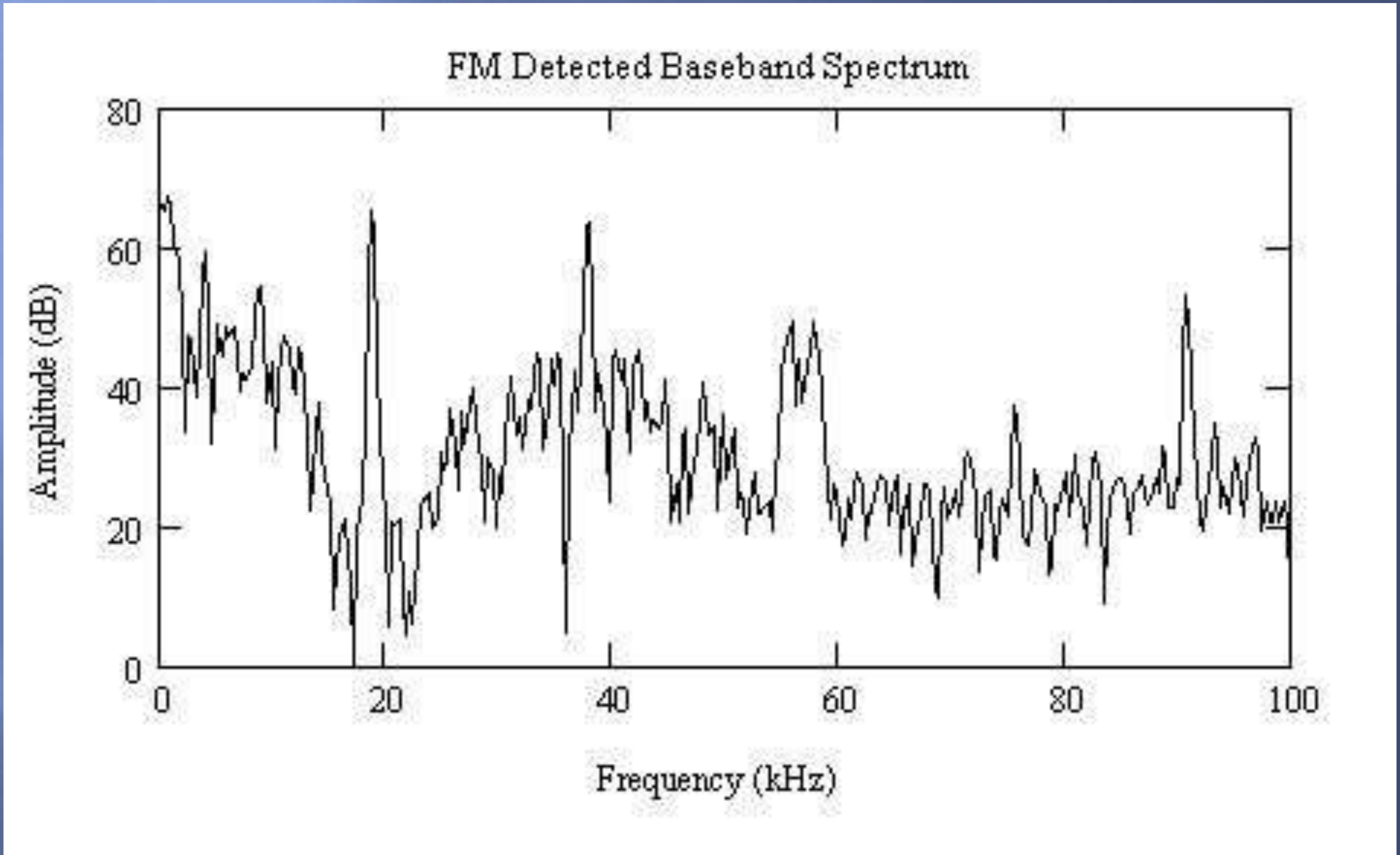
Carson's Rule

$$BW = 2 \cdot f_{max} \cdot (\beta + 1)$$

$$= 2 \cdot (f_{max} + f_{dev})$$

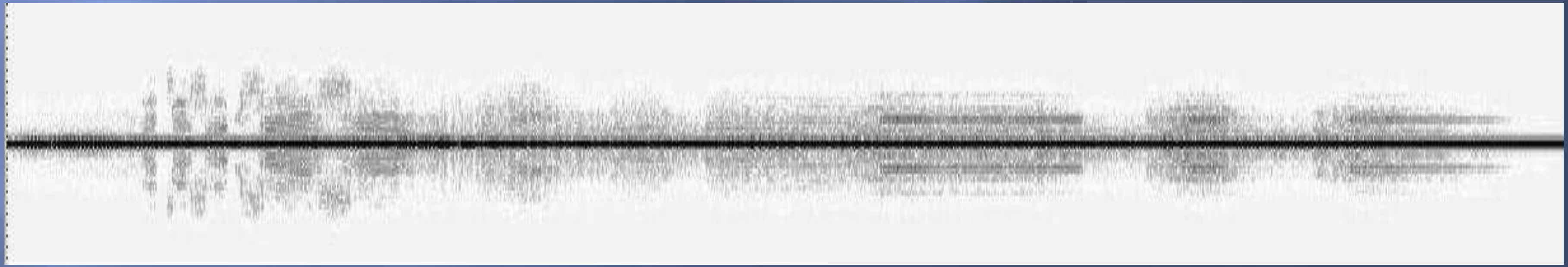
Carson's Rule, named after an engineer who did not think that FM would provide any improvement over AM, provides a rough calculation of the bandwidth of an FM signal based upon its design parameters and the parameters of the modulating signal *(Objectives 2, 3, 4b, 6)*.

FM Broadcast Signal



FM Spectrum

The spectrogram of an FM signal shows how the spectrum varies with time. Note how it is asymmetric.



This is the spectrum of an AM signal modulated with the same information as above. But it has a symmetric spectrum.

Why FM and not PM?

For practical implementation reasons, analog FM is easier to generate than analog PM, and FM provides better performance in most common environments. However, analog PM has been (and continues to be) used for a few, isolated systems.

- Broadcast analog television chrominance (color)
- spacecraft communications
- AM stereo

Random process

SAMPLE SPACE AND PROBABILITY

- ***Random experiment:*** its outcome, for some reason, cannot be predicted with certainty.
  - Examples: throwing a die, flipping a coin and drawing a card from a deck.
- ***Sample space:*** the set of all possible outcomes, denoted by S. Outcomes are denoted by E's and each E lies in S, i.e., $E \in S$.
- A sample space can be discrete or continuous.
- Events are subsets of the sample space for which measures of their occurrences, called probabilities, can be defined or determined.

THREE AXIOMS OF PROBABILITY

- For a discrete sample space S, define a probability measure P on as a set function that assigns nonnegative values to all events, denoted by E, in such that the following conditions are satisfied
- Axiom 1: $0 \le P(E) \le 1$ for all $E \in S$
- Axiom 2: $P(S) = 1$ (when an experiment is conducted there has to be an outcome).
- Axiom 3: For mutually exclusive events E1, E2, E3,. . . we have

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i).$$

PROBABILITY

- We observe or are told that event E1 has occurred but are actually interested in event E2: Knowledge that of E1 has occurred changes the probability of E2 occurring.
- If it was P(E2) before, it now becomes P(E2|E1), the probability of E2 occurring given that event E1 has occurred.
- This conditional probability is given by

$$P(E_2|E_1) = \begin{cases} \dfrac{P(E_2 \cap E_1)}{P(E_1)}, & \text{if } P(E_1) \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

- If P(E2|E1) = P(E2), or P(E2 ∩ E1) = P(E1)P(E2), then E1 and E2 are said to be statistically independent.
- Bayes' rule
  - P(E2|E1) = P(E1|E2)P(E2)/P(E1)

MATHEMATICAL MODEL FOR SIGNALS

- Mathematical models for representing signals
  - Deterministic
  - Stochastic
- Deterministic signal: No uncertainty with respect to the signal value at any time.
  - Deterministic signals or waveforms are modeled by explicit mathematical expressions, such as

$$x(t) = 5 \cos(10*t).$$

  - ***Inappropriate for real-world problems???***
- Stochastic/Random signal: Some degree of uncertainty in signal values before it actually occurs.
  - For a random waveform it is not possible to write such an explicit expression.
  - Random waveform/ random process, may exhibit certain regularities that can be described in terms of probabilities and statistical averages.
  - e.g. thermal noise in electronic circuits due to the random movement of electrons

ENERGY AND POWER SIGNALS

- The performance of a communication system depends on the received signal energy: higher energy signals are detected more reliably (with fewer errors) than are lower energy signals.
- An electrical signal can be represented as a voltage *v*(*t*) or a current *i*(*t*) with instantaneous power *p*(*t*) across a resistor defined by

$$p(t) = \frac{v^2(t)}{\Re}$$

OR

$$p(t) = i^2(t)\Re$$

ENERGY AND POWER SIGNALS

- In communication systems, power is often normalized by assuming R to be 1.
- The normalization convention allows us to express the instantaneous power as

$$p(t) = x^2(t)$$

where $x(t)$ is either a voltage or a current signal.

- The energy dissipated during the time interval (-T/2, T/2) by a real signal with instantaneous power expressed by Equation (1.4) can then be written as:

$$E_x^T = \int_{-T/2}^{T/2} x^2(t)\,dt$$

- The average power dissipated by the signal during the interval is:

$$P_x^T = \frac{1}{T} E_x^T = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

ENERGY AND POWER SIGNALS

- We classify $x(t)$ as an *energy signal* if, and only if, it has nonzero but finite energy ($0 < E_x < \infty$) for all time, where

$$E_x = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$
$$= \int_{-\infty}^{\infty} x^2(t)\, dt$$

- An energy signal has finite energy but *zero average power*
- Signals that are both deterministic and non-periodic are termed as Energy Signals

ENERGY AND POWER SIGNALS

- Power is the rate at which the energy is delivered
- We classify $x(t)$ as an *power signal* if, and only if, it has nonzero but finite energy ($0 < P_x < \infty$) for all time, where

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

  - A power signal has finite power but infinite energy

- Signals that are random or periodic termed as Power Signals

RANDOM VARIABLE

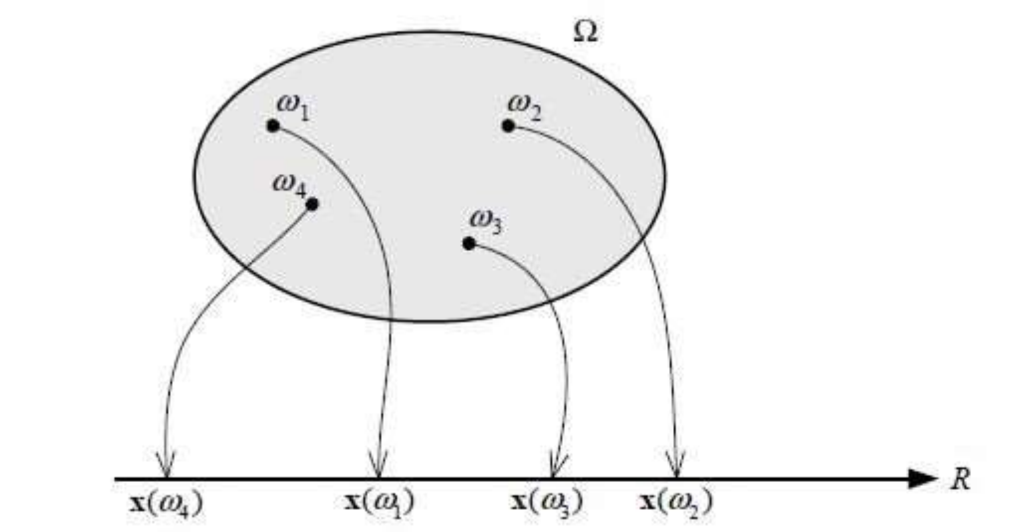
- Functions whose domain is a sample space and whose range is a some set of real numbers is called ***random variables.***
- Type of RV's
  - Discrete
    - E.g. outcomes of flipping a coin etc
  - Continuous
    - E.g. amplitude of a noise voltage at a particular instant of time

Random Variables

- All useful signals are random, i.e. the receiver does not know a priori what wave form is going to be sent by the transmitter
- Let a *random variable* $X(A)$ represent the functional relationship between a random event A and a real number.
- The *distribution function* $F_x(x)$ of the random variable X is given by $F_X(x) = P(X \leq x)$

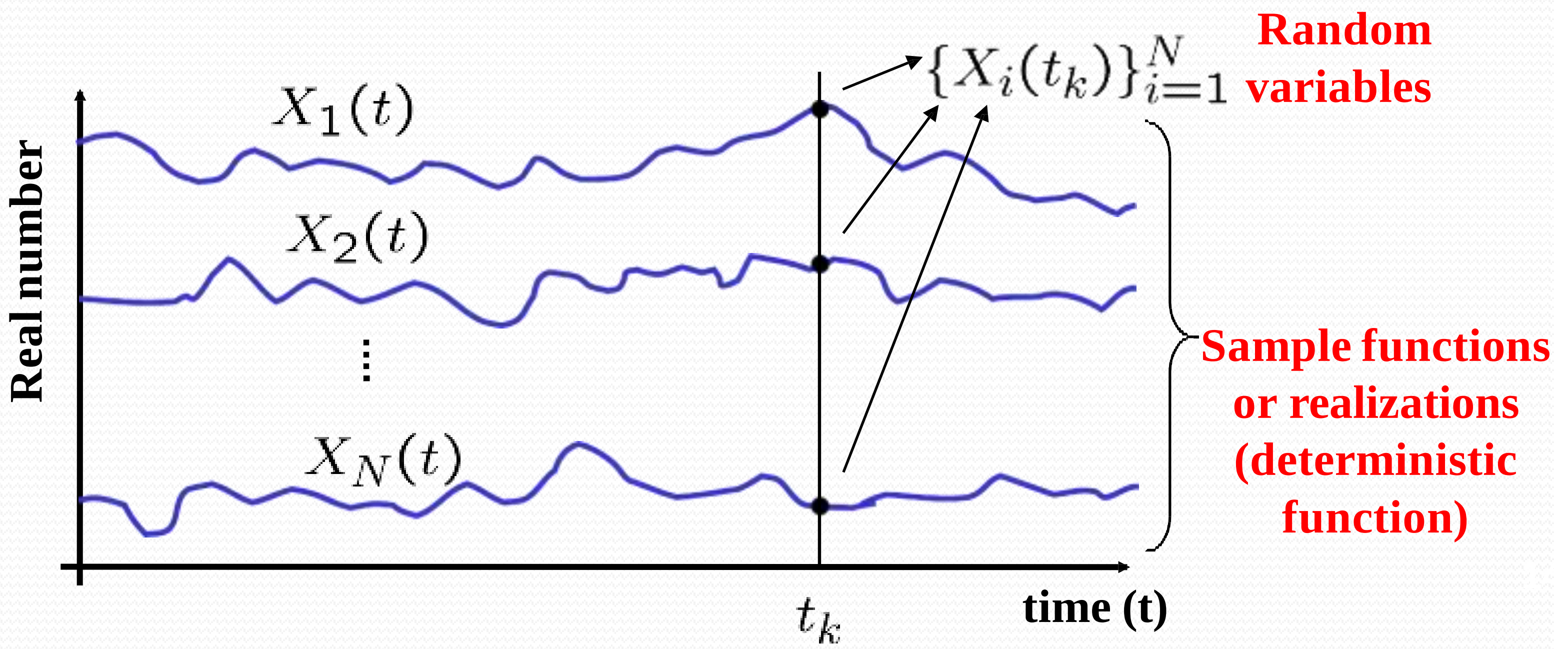
RANDOM VARIABLE

- A random variable is a mapping from the sample space to the set of real numbers.
- We shall denote random variables by boldface, i.e., **x, y,** etc., while individual or specific values of the mapping **x** are denoted by **x**(w).



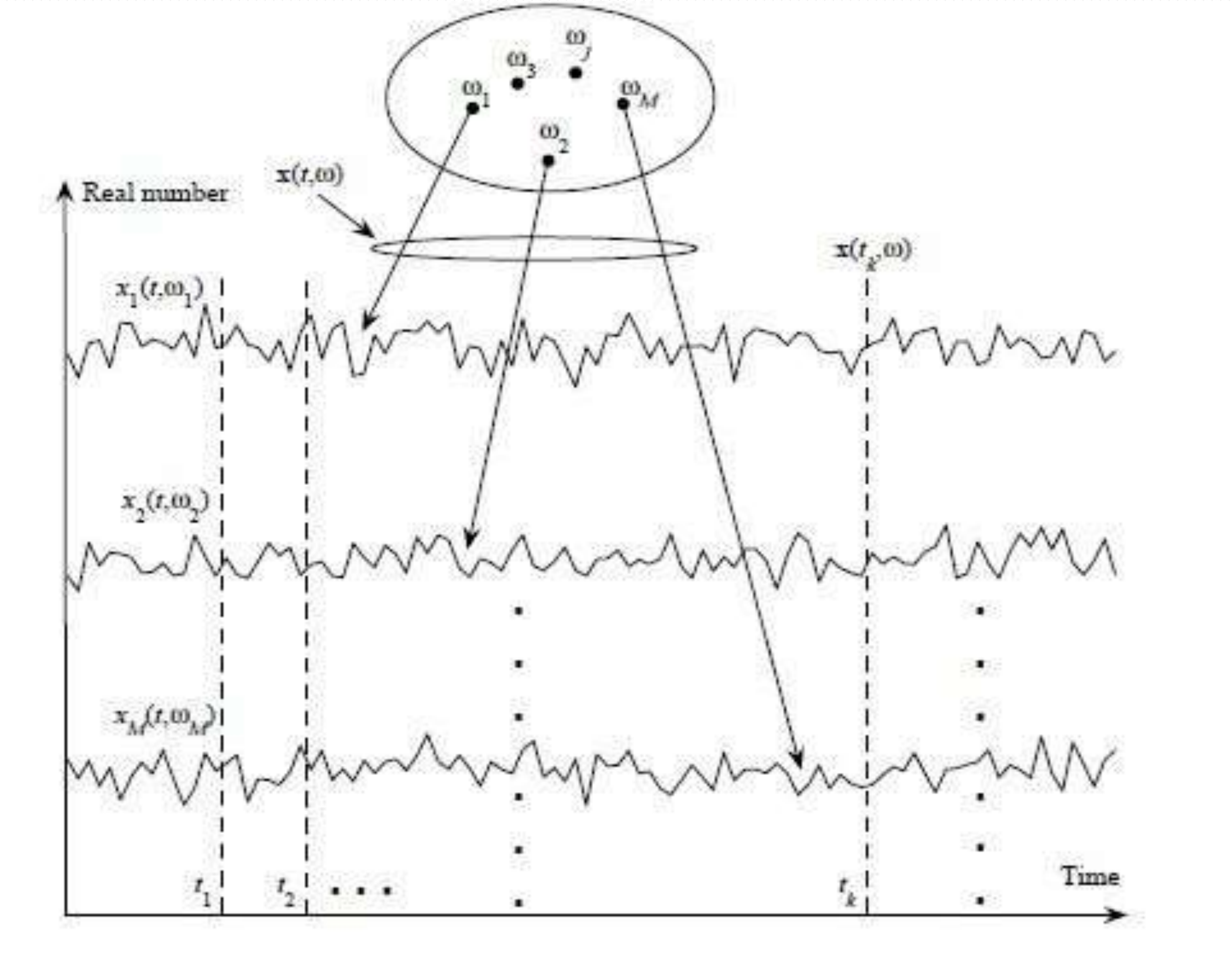
RANDOM PROCESS

- A random process is a collection of time functions, or signals, corresponding to various outcomes of a random experiment. For each outcome, there exists a deterministic function, which is called a sample function or a realization.



RANDOM PROCESS

- A mapping from a sample space to a set of time functions.



RANDOM PROCESS CONTD

- ***Ensemble***: The set of possible time functions that one sees.
- Denote this set by x(t), where the time functions x1(t, w1), x2(t, w2), x3(t, w3), . . . are specific members of the ensemble.
- At any time instant, t = tk, we have random variable x(tk).
- At any two time instants, say t1 and t2, we have two different random variables x(t1) and x(t2).
- Any relationship b/w any two random variables is called Joint PDF

CLASSIFICATION OF RANDOM PROCESSES

- Based on whether its statistics change with time: the process is non-stationary or stationary.
- Different levels of stationary:
  - Strictly stationary: the joint pdf of any order is independent of a shift in time.
  - Nth-order stationary: the joint pdf does not depend on the time shift, but depends on time spacing

PROBABILITY DENSITY FUNCTION

- The pdf is defined as the derivative of the cdf:

  fx(x) = d/dx Fx(x)

- It follows that:

$$P(x_1 \leq \mathbf{x} \leq x_2) = P(\mathbf{x} \leq x_2) - P(\mathbf{x} \leq x_1)$$
$$= F_{\mathbf{x}}(x_2) - F_{\mathbf{x}}(x_1) = \int_{x_1}^{x_2} f_{\mathbf{x}}(x)\mathrm{d}x.$$

- Note that, for all i, one has pi ≥ 0 and ∑pi = 1.

GAUSSIAN (OR NORMAL) RANDOM VARIABLE (PROCESS)

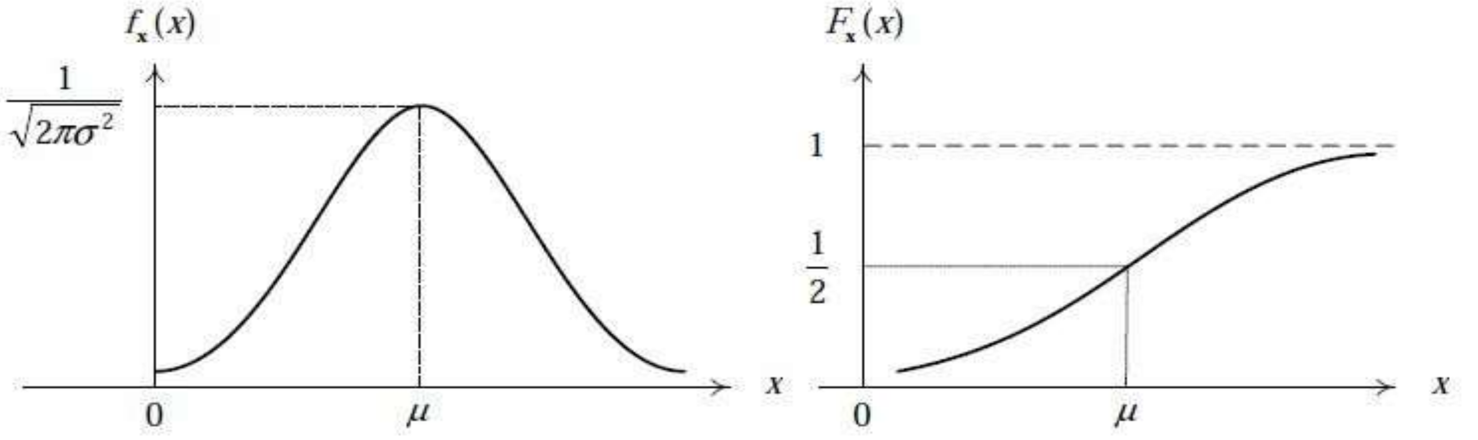
- A continuous random variable whose pdf is:

$$f_{\mathbf{x}}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\},$$

  μ and σ^2 are parameters. Usually denoted as

  N(μ, σ^2) .

- Most important and frequently encountered random variable in communications.



CENTRAL LIMIT THEOREM

- CLT provides justification for using Gaussian Process as a model based if
  - The random variables are statistically independent
  - The random variables have probability with same mean and variance

CLT

- The central limit theorem states that
  - “The probability distribution of Vn approaches a normalized Gaussian Distribution N(0, 1) in the limit as the number of random variables approach infinity”
- At times when N is finite it may provide a poor approximation of for the actual probability distribution

AUTOCORRELATION

Autocorrelation of Energy Signals

- Correlation is a matching process; *autocorrelation* refers to the matching of a signal with a delayed version of itself
- The autocorrelation function of a real-valued energy signal $x(t)$ is defined as:

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)\,dt \qquad \text{for} -\infty < \tau < \infty$$

- The autocorrelation function $R_x(\tau)$ provides a measure of how closely the signal matches a copy of itself as the copy is shifted τ units in time.
- $R_x(\tau)$ is not a function of time; it is only a function of the time difference τ between the waveform and its shifted copy.

AUTOCORRELATIO N

$$R_x(\tau) = R_x(-\tau)$$

$$R_x(\tau) \le R_x(0) \text{ for all } \tau$$

$$R_x(\tau) \leftrightarrow \psi_x(f)$$

$$R_x(0) = \int_{-\infty}^{\infty} x^2(t)\,dt$$

- symmetrical in τ about zero
- maximum value occurs at the origin
- autocorrelation and ESD form a Fourier transform pair, as designated by the double-headed arrows
- value at the origin is equal to the energy of the signal

AUTOCORRELATION OF A PERIODIC (POWER) SIGNAL

- The autocorrelation function of a real-valued power signal $x(t)$ is defined as:

$$R_x(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t+\tau)\,dt \qquad \text{for} -\infty < \tau < \infty$$

- When the power signal $x(t)$ is periodic with period T_0, the autocorrelation function can be expressed as:

$$R_x(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)x(t+\tau)\,dt \qquad \text{for} -\infty < \tau < \infty$$

AUTOCORRELATION OF POWER SIGNALS

The autocorrelation function of a real-valued *periodic* signal has properties similar to those of an energy signal:

$$R_x(\tau) = R_x(-\tau)$$

$$R_x(\tau) \le R_x(0) \quad \text{for all } \tau$$

$$R_x(\tau) \leftrightarrow G_x(f)$$

$$R_x(0) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\, dt$$

- symmetrical in τ about zero
- maximum value occurs at the origin
- autocorrelation and PSD form a Fourier transform pair, as designated by the double-headed arrows
- value at the origin is equal to the average power of the signal

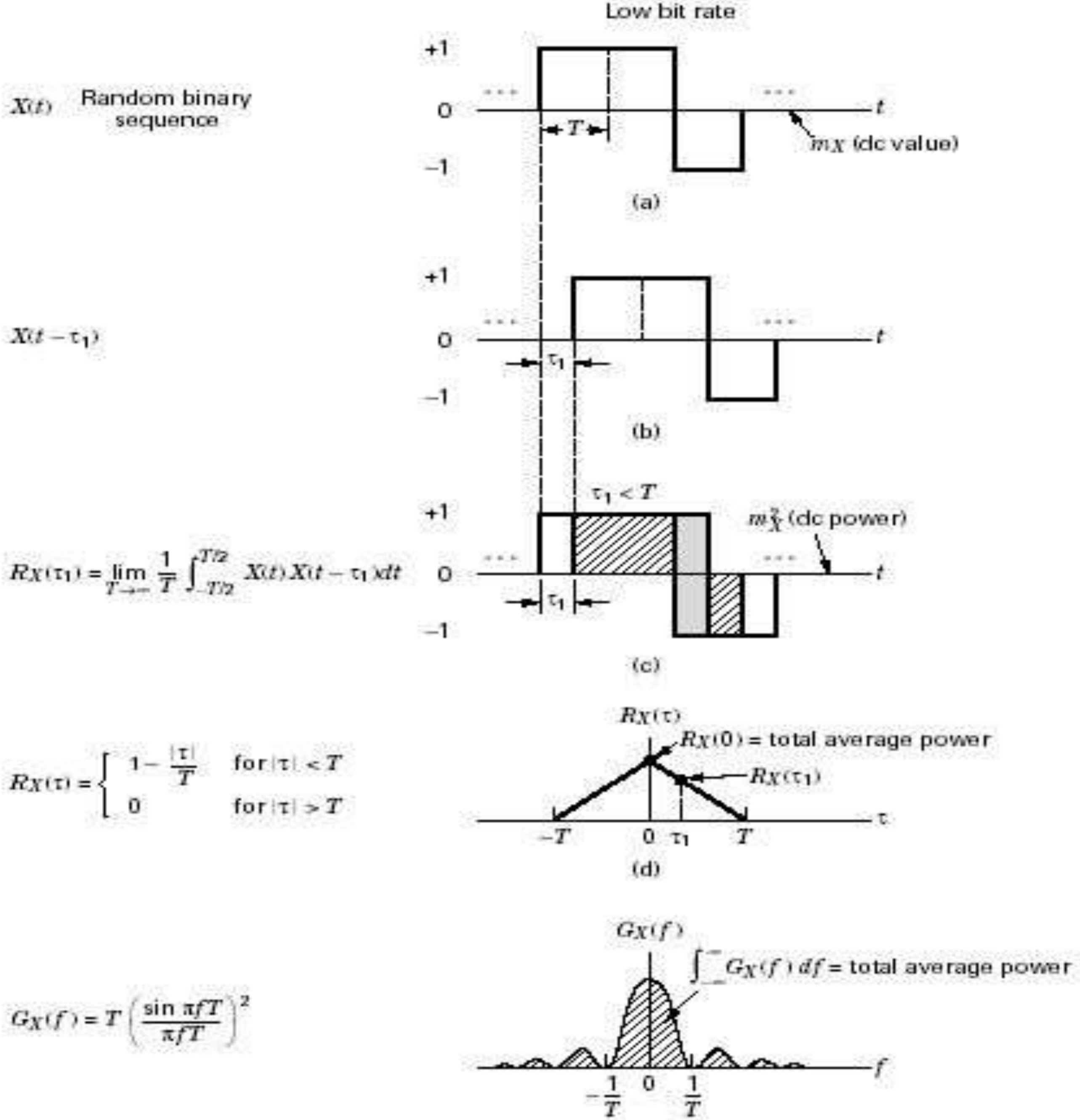


Figure 1.6 Autocorrelation and power spectral density.

$X(t)$ Random binary sequence

$X(t-\tau_1)$

$$R_X(\tau_1) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} X(t) X(t-\tau_1)\, dt$$

$$R_X(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T} & \text{for } |\tau| < T \\ 0 & \text{for } |\tau| > T \end{cases}$$

$$G_X(f) = T \left(\frac{\sin \pi f T}{\pi f T} \right)^2$$

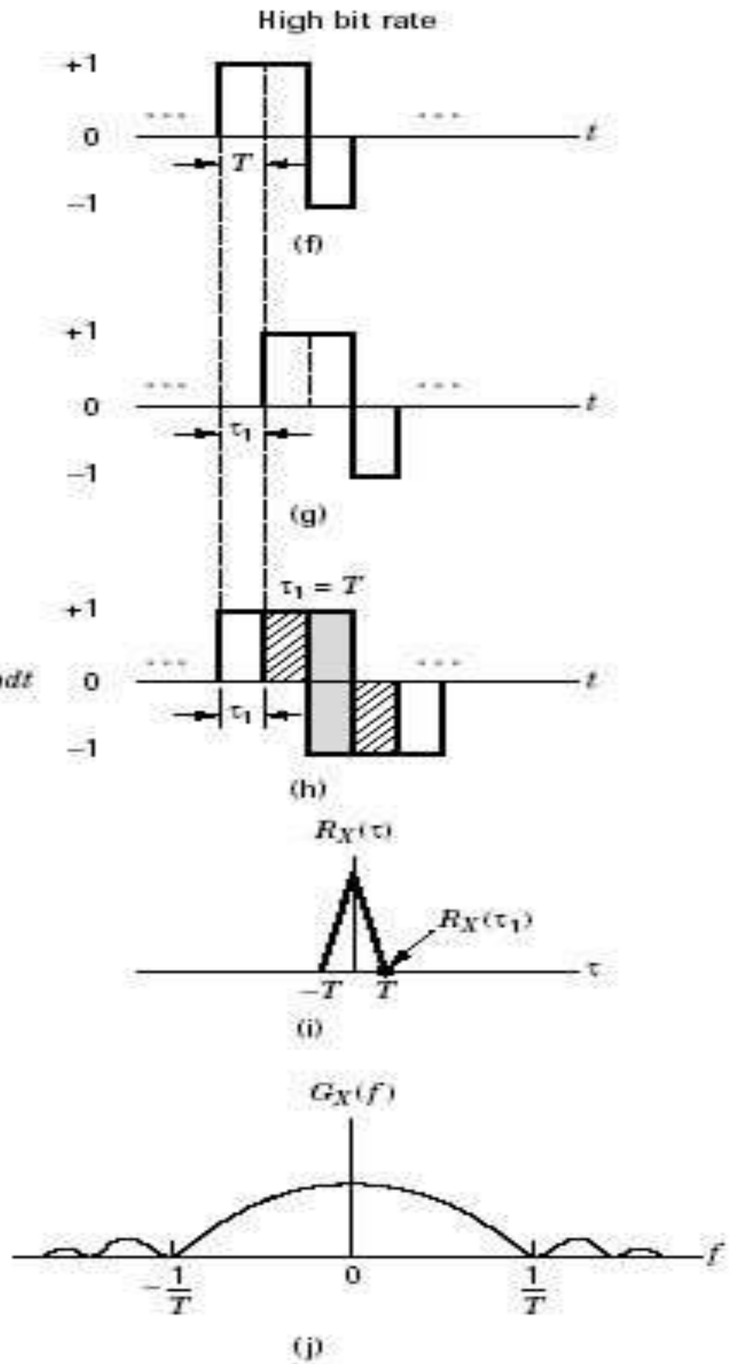


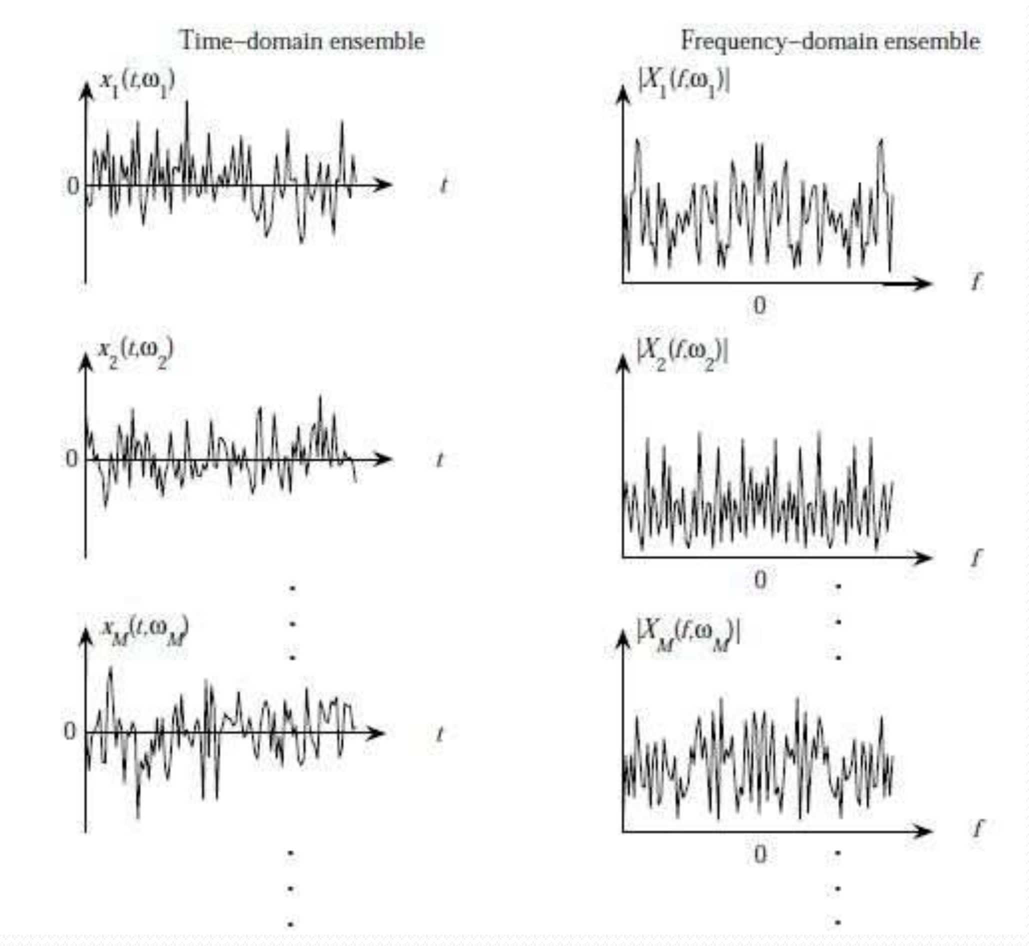
Figure 1.6 continued

SPECTRAL DENSITY

- The *spectral density* of a signal characterizes the distribution of the signal's energy or power, in the frequency domain
- This concept is particularly important when considering filtering in communication systems while evaluating the signal and noise at the filter output.
- The energy spectral density (ESD) or the power spectral density (PSD) is used in the evaluation.
- Need to determine how the average power or energy of the process is distributed in frequency.

SPECTRAL DENSITY

- Taking the Fourier transform of the random process does not work



ENERGY SPECTRAL DENSITY

- Energy spectral density describes the energy per unit bandwidth measured in joules/hertz
- Represented as φx(t), the squared magnitude ***spectrum***

  φx(t) =|x(f)|2

- According to Parseval's Relation $$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} |X(f)|^2\, df$$
- Therefore $$E_x = \int_{-\infty}^{\infty} \psi_x(f)\, df$$
- The Energy spectral density is symmetrical in frequency about origin and total energy of the signal x(t) can be expressed as

$$E_x = 2\int_{0}^{\infty} \psi_x(f)\, df$$

POWER SPECTRAL DENSITY

- The power spectral density (PSD) function $G_x(f)$ of the periodic signal x(t) is a real, even ad nonnegative function of frequency that gives the distribution of the power of x(t) in the frequency domain.
- PSD is represented as (Fourier Series):
- PSD of non-periodic signals: $$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \, \delta(f - nf_0)$$
- Whereas the average power of a periodic signal x(t) is represented as:

$$G_x(f) = \lim_{T \to \infty} \frac{1}{T} |X_T(f)|^2$$

$$P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\, dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

TIME AVERAGING AND ERGODICITY

- A process where any member of the ensemble exhibits the same statistical behavior as that of the whole ensemble.
- For an ergodic process: To measure various statistical averages, it is sufficient to look at only one realization of the process and find the corresponding time average.
- For a process to be ergodic it must be stationary. The converse is not true.

Ergodicity

- A random process is said to be ***ergodic*** if it is *ergodic in the mean* and *ergodic in correlation*:
  - Ergodic in the mean: $m_x = E\{x(t)\} = \langle x(t) \rangle$

    time average operator: $\langle g(t) \rangle = \lim_{t \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)\,dt$
  - Ergodic in the correlation:

$$\phi_x(\tau) = E\{x(t)x(t+\tau)\} = \langle x(t)x(t+\tau) \rangle$$

- In order for a random process to be ergodic, it must first be ***Wide Sense Stationary***.
- If a R.P. is ergodic, then we can compute ***power*** three different ways:
  - From any sample function: $P_x = \lim_{t \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\,dt = \langle |x(t)|^2 \rangle$
  - From the autocorrelation: $P_x = \phi_x(0)$
  - From the Power Spectral Density: $P_x = \int_{-\infty}^{\infty} \Phi_x(f)\,df$

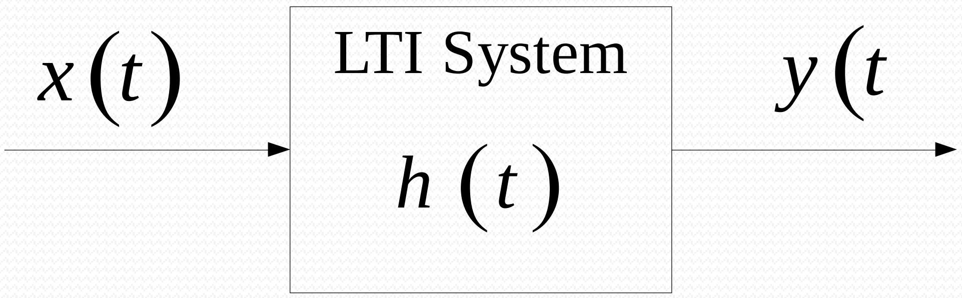
Stationarity

- A process is ***strict-sense stationary*** (SSS) if all its joint densities are invariant to a time shift:
  - $p_x(x(t)) = p_x(x(t+t_o))$
  - $p_x(x(t_1), x(t_2)) = p_x(x(t_1+t_o), x(t_2+t_0))$
  - $p_x(x(t_1), x(t_2), \ldots, x(t_N)) = p_x(x(t_1+t_o), x(t_2+t_0), \ldots, x(t_N+t_0))$
  - – in general, it is difficult to prove that a random process is strict sense stationary.
- A process is ***wide-sense stationary*** (WSS) if:
  - The mean is a constant:
    - $m_x(t) = m_x$
  - The autocorrelation is a function of time difference only: $\phi(t_1, t_2) = \phi(\tau)$ where $\tau = t_2 - t_1$

- If a process is *strict-sense stationary*, then it is also *wide-sense stationary*.

Transmission over LTI Systems 1/3

- Linear Time-Invariant (LTI) Systems



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} h(\tau) x(t-\tau) = h(t) * x(t)$$

$$Y(f) = X(f) \cdot H(f)$$

Transmission over LTI Systems 2/3

- Assumptions:

  x(t) and h(t) are real-valued and x(t) is WSS.

- The mean of the output y(t)

$$E\{y(t)\} = m_x \cdot \int_{-\infty}^{\infty} h(\tau)\, d\tau = m_x \cdot H(0)$$

- The cross-correlation function

$$\begin{cases} R_{YX}(\tau) = E\{Y(t)X(t+\tau)\} = h(-\tau) * R_{XX}(\tau) \\ R_{XY}(\tau) = E\{X(t)Y(t+\tau)\} = h(\tau) * R_{XX}(\tau) \end{cases}$$

Transmission over LTI Systems 3/3

- The A.F. of the output

$$R_{YY}(\tau) = E\left\{Y(t)Y(t+\tau)\right\}$$
$$= R_{YX}(\tau) * h(\tau)$$
$$= R_{XY}(\tau) * h(-\tau)$$
$$= h(-\tau) * R_{XX}(\tau) * h(\tau)$$
$$= R_{XX}(\tau) * h(\tau) * h(-\tau)$$

- The PSD of the output

$$S_{YY}(f) = S_{XX}(f) \cdot |H(f)|^2$$

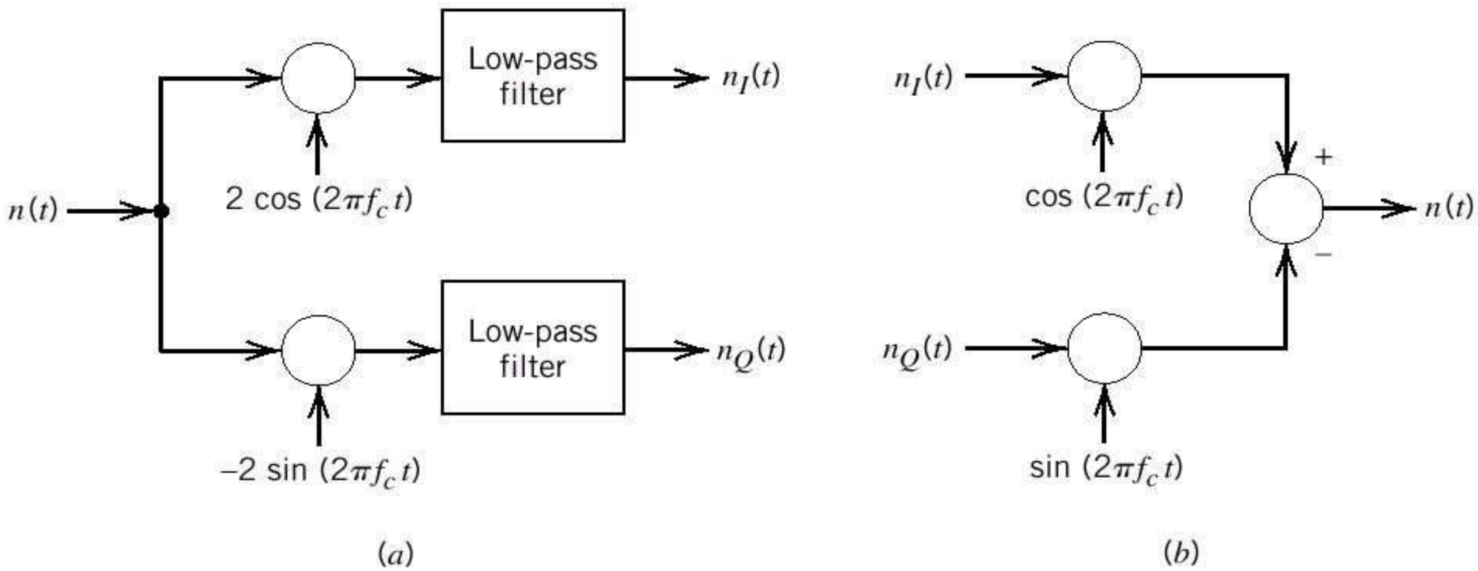
NOISE CHARACTERIZATION

UNIT IV

- Consider a narrowband noise $n(t)$ of bandwidth $2\boldsymbol{B}$ centered on frequency f_c ,as illustrated in Figure 1.18.
- We may represent n(t) in the canonical (standard) form:

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

where, $n_I(t)$is ***in-phase*** component of $n(t)$ and $n_Q(t)$is ***quadrature*** component of $n(t)$.



- $n_I(t)$ and $n_Q(t)$ have important properties:
  - $n_I(t)$ and $n_Q(t)$ have zero mean.
  - $n(t)$ is Gaussian, then $n_I(t)$ and $n_Q(t)$ are jointly Gaussian.
  - $n(t)$ is stationary, then $n_I(t)$ and $n_Q(t)$ are jointly stationary.
  - Both $n_I(t)$ and $n_Q(t)$ have the same power spectral density.

$$S_{N_I}(f) = S_{N_Q}(f) = \begin{cases} S_N(f - f_c) + S_N(f + f_c), & -B \le f \le B \\ 0, & otherwise \end{cases}$$

  - $n_I(t)$ and $n_Q(t)$ have the same variance as $n(t)$

- The cross-spectral density of the $n_I(t)$ and $n_Q(t)$ is purely imaginary:

$$S_{N_I N_Q}(f) = -S_{N_Q N_I}(f)$$

$$= \begin{cases} j[S_N(f+f_c) - S_N(f-f_c)], & -B \le f \le B \\ 0, & otherwise \end{cases}$$

- If $n(t)$ is Gaussian and its power spectral density $S_N(t)$ is symmetric about the mid-band frequency f_c ,

then $n_I(t)$ and $n_Q(t)$ are statistically independent.

- Here we represent $n(t)$ in terms of envelope and phase components:

$$n(t) = r(t)\cos[2\pi f_c t + \psi(t)]$$

where, $r(t) = [n_I^2(t) + n_Q^2(t)]^{1/2}$ and $\psi(t) = \tan^{-1}\left[\frac{n_Q(t)}{n_I(t)}\right]$

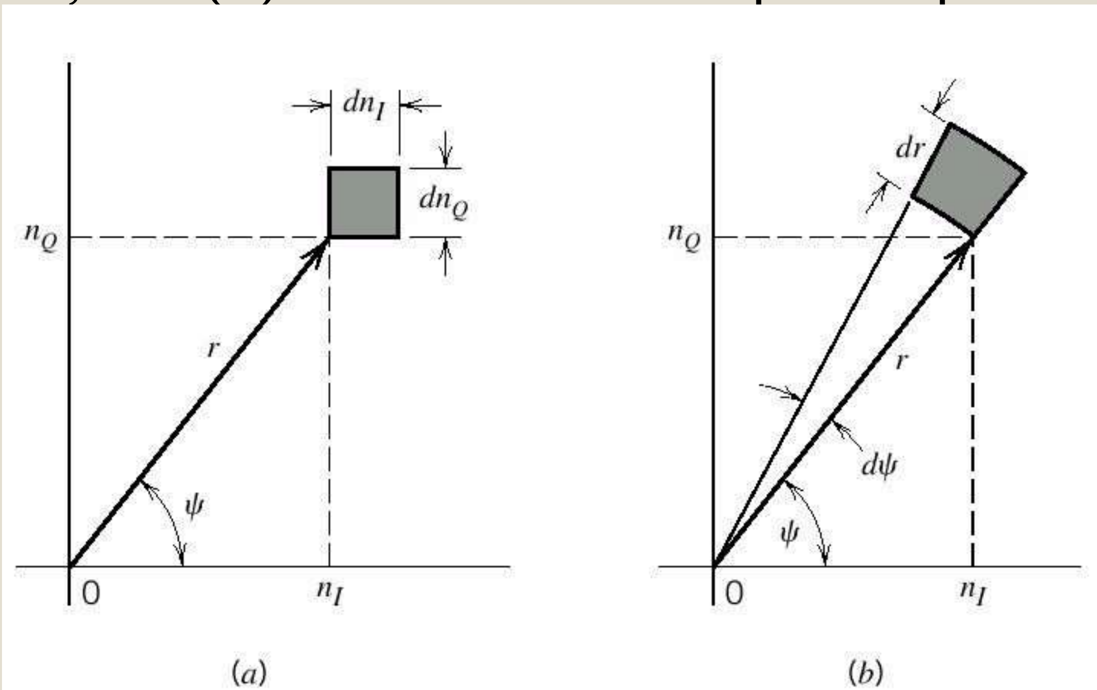
- $r(t)$ is called the ***envelope*** of $n(t)$, and the $\psi(t)$ is called the ***phase*** of $n(t)$.

- The probability distributions of $r(t)$ and $\psi(t)$ may be obtained from those of $n_I(t)$ and $n_Q(t)$ as follows.

- Let N_I and N_Q denote the random variables obtained by the sample functions $n_I(t)$ and $n_Q(t)$, respectively.
- Then, N_I and N_Q are independent Gaussian random variables of zero mean and variance σ^2.
- So, we may express their joint probability density function by

$$f_{N_I,N_Q}(n_I,n_Q)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{n_I^{\,2}+n_Q^{\,2}}{2\sigma^2}\right)$$

- **Figure 2.1** Illustrating the coordinate system for representation of narrowband noise: (*a*) in terms of in-phase and quadrature components, and (*b*) in terms of envelope and phase.



$$n_I = r\cos\psi$$

$$n_Q = r\sin\psi$$

$$dn_I dn_Q = r dr d\psi$$

- Now, let R and Ψ denote the random variables obtained by the sample functions $r(t)$ and $\psi(t)$, respectively.

- Then we find the joint probability density function of R and is Ψ

$$f_{R,\Psi}(r,\psi) = \frac{r}{2\pi\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) \qquad (1.113)$$

- From (1.113), the random variables R and Ψ are statistically independent.
- Therefore,

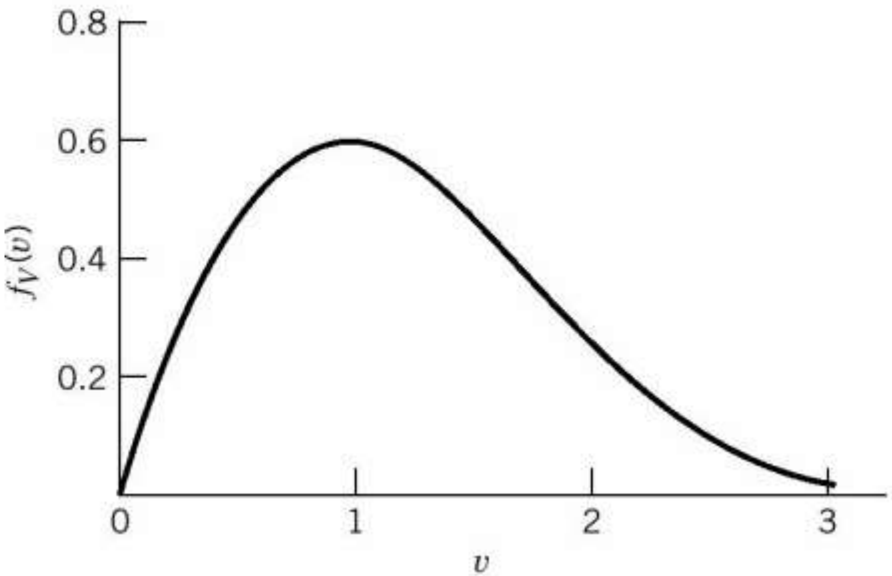
C o n t ..

$$f_{\Psi}(\psi) = \begin{cases} \frac{1}{2\pi}, & 0 \le \psi \le 2\pi \\ 0, & elsewhere \end{cases}$$

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right), & r \ge 0 \\ 0, & elsewhere \end{cases}$$

- ***Rayleigh distribution*** (Figure 1.22) : A random variable having the probability density function of Equation (1.115).
- Let, $\upsilon = r/\sigma$ then the normalized form is

$$f_V(v) = \begin{cases} v\exp\left(-\frac{v2}{2}\right), & v \geq 0 \\ 0, & elsewhere \end{cases}$$



- Add the sinusoidal wave $A\cos(2\pi f_c t)$ to the narrowband noise $n(t)$.

$$x(t) = A\cos(2\pi f_c t) + n(t)$$

- Use in-phase and quadrature components for $n(t)$

$$x(t) = n_I'(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

where, $n_I'(t) = A + n_I(t)$

- We assume that $n(t)$ is Gaussian with zero mean and variance σ^2, then we find that:

- Joint probability density function of N_I' and N_Q

$$f_{N_I',N_Q}(n_I',n_Q)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{(n_I'-A)^2+n_Q^{\ 2}}{2\sigma^2}\right)$$

- Joint probability density function of R and Ψ

$$f_{R,\Psi}(r,\psi)=\frac{r}{2\pi\sigma^2}\exp\left(-\frac{r^2+A^2-2Ar\cos\psi}{2\sigma^2}\right)$$

- Now we are interested in the probability density function of R

$$f_R(r) = \int_0^{2\pi} f_{R,\Psi}(r,\psi)d\psi$$

$$= \frac{r}{2\pi\sigma^2}\exp\left(-\frac{r^2+A^2}{2\sigma^2}\right)\int_0^{2\pi}\exp\left(\frac{Ar}{\sigma^2}\cos\psi\right)d\psi \qquad (1.126)$$

modified Bessel function of the first kind of zero order

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi}\exp(x\cos\psi)d\psi \quad ,x = Ar/\sigma^2$$

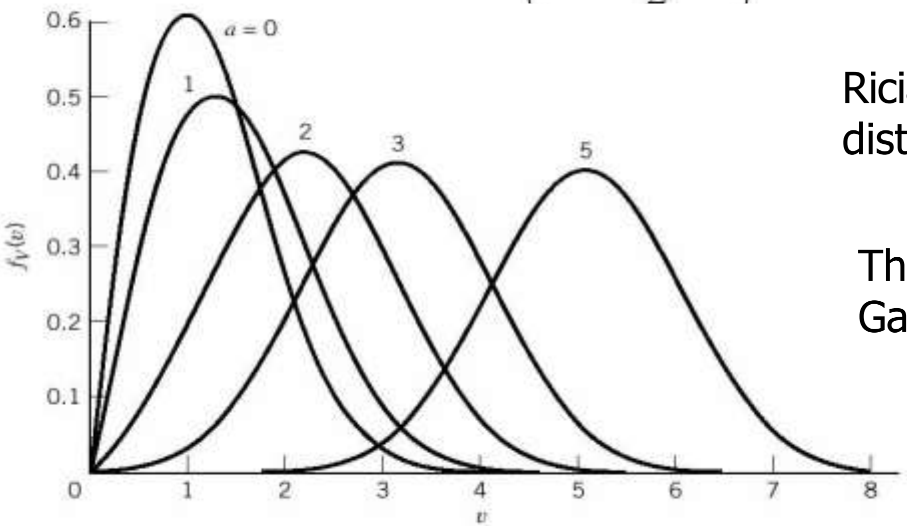
C o n t …

- Rewrite (1.126)

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right)$$

- This is called the ***Rician distribution***. (Figure 1.23)
- Let $v = r/\sigma,\ a = A/\sigma$, then the normalized form is

$$f_V(v) = v\exp\left(-\frac{v^2 + a^2}{2}\right) I_0(av)$$



Rician distribution reduced to the Rayleigh distribution

The envelope distribution is approximately Gaussian when a is large

- Noise can broadly be defined as any unknown signal that affects the recovery of the desired signal.

- The received signal is modeled as

$$r(t) = s(t) + w(t) \qquad (1)$$

- $s(t)$ is the transmitted signal

- $w(t)$ is the additive noise

Noise in Communication Systems

- The mean of the random process
  - Both noise and signal are generally assumed to have zero mean.
- The autocorrelation of the random process.
  - With white noise, samples at one instant in time are uncorrelated with those at another instant in time regardless of the separation. The autocorrelation of white noise is described by

$$R_w(\tau) = \frac{N_0}{2}\delta(\tau) \qquad (2)$$

- The spectrum of the random process. For additive white Gaussian noise the spectrum is flat and defined as

$$S_w(f) = \frac{N_0}{2} \qquad (3)$$

$$N = N_0 B_T \qquad (4)$$

- To compute noise power, we must measure the noise over a specified bandwidth.

Fig
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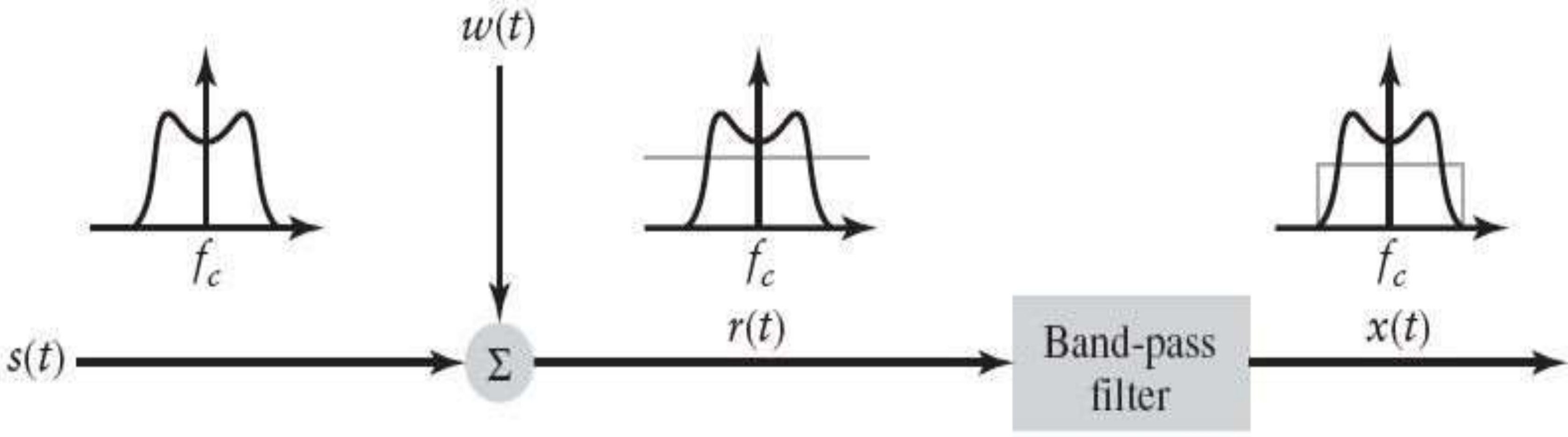


FIGURE Block diagram of signal plus noise before and after filtering, showing spectra at each point.

Signal-to-Noise Ratios

- The desired signal, $s(t)$, a narrowband noise signal, $n(t)$

$$x(t) = s(t) + n(t) \qquad (5)$$

- For zero-mean processes, a simple measure of the signal quality is the ratio of the variances of the desired and undesired signals.

- Signal-to-noise ratio is defined by

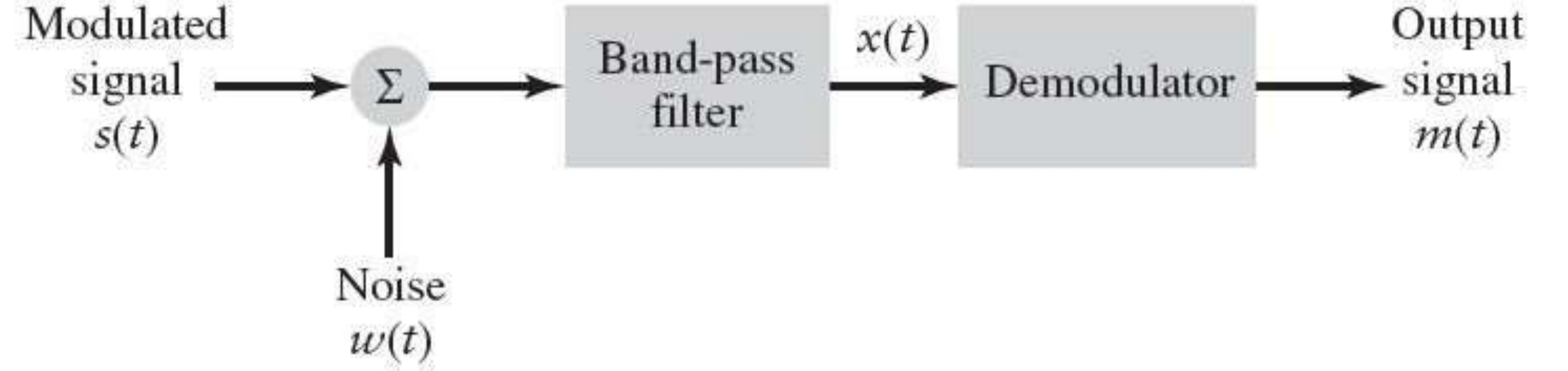
$$\mathrm{SNR} = \frac{\mathrm{E}[s^2(t)]}{\mathrm{E}[n^2(t)]} \qquad (6)$$

- The signal-to-noise ratio is often considered to be a ratio of the average signal power to the average noise power.

- If the signal-to-noise ratio is measured at the front-end of the receiver, then it is usually a measure of the quality of the transmission link and the receiver front-end.
- If the signal-to-noise ratio is measured at the output of the receiver, it is a measure of the quality of the recovered intormation-bearing signal whether it be audio, video, or otherwise.
- Reference transmission model
- This reference model is equivalent to transmitting the message at baseband.

Cont...

Fig.



High-level block diagram of a communications receiver.

1. The message power is the same as the modulated signal power of the modulation scheme under study.
2. The baseband low-pass filter passes the message signal and rejects out-of-band noise. Accordingly, we may define the reference signal-to-noise ratio, SNR_{ref}, as

$$\mathrm{SNR}_{\mathrm{ref}} = \frac{\text{average power of the modulated message signal}}{\text{average power of noise mesured in the message bandwidth}} \quad (11)$$

- A Figure of merit

$$\text{Figure of merit} = \frac{\text{post}-\text{detection SNR}}{\text{reference SNR}}$$

Cont...

Fig.

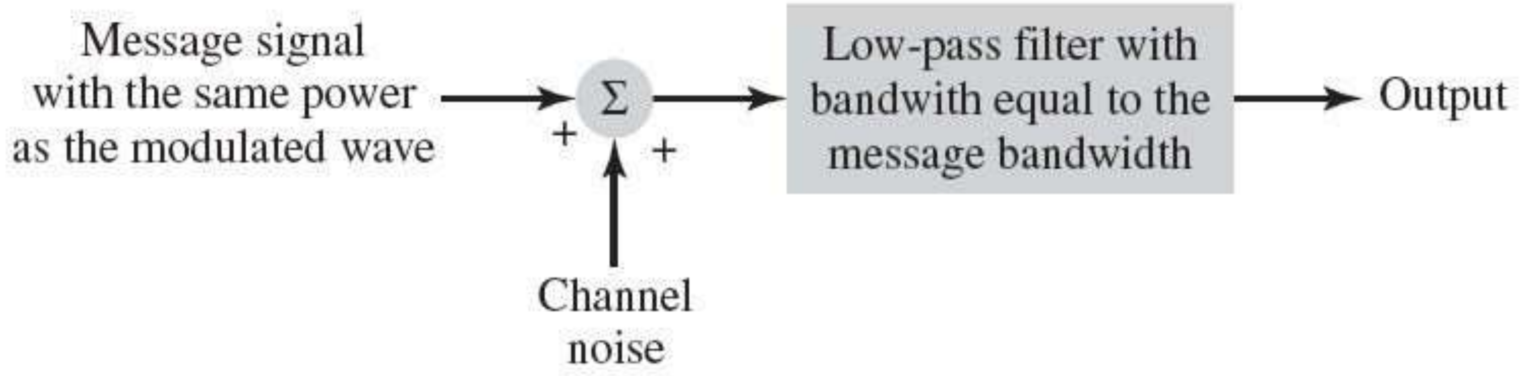


FIGURE Reference transmission model for analog communications.

- The higher the value that the figure of merit gas, the better the noise performance of the receiver will be.

- To summarize our consideration of signal-to-noise ratios:
  - The pre-detection SNR is measured before the signal is demodulated.
  - The post-detection SNR is measured after the signal is demodulated.
  - The reference SNR is defined on the basis of a baseband transmission model.
  - The figure of merit is a dimensionless metric for comparing sifferent analog modulation-demodulation schemes and is defined as the ratio of the post-detection and reference SNRs.

Cont…

Band-Pass Receiver Structures

- Fig. shows an example of a superheterodyne receiver
- AM radio transmissions
  - Common examples are AM radio transmissions, where the RF channels' frequencies lie in the range between 510 and 1600 kHz, and a common IF is 455 kHz
- FM radio
  - Another example is FM radio, where the RF channels are in the range from 88 to 108 MHz and the IF is typically 10.7 MHz.

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \qquad (12)$$

- The filter preceding the local oscillator is centered at a higher RF frequency and is usually much wider, wide enough to encompass all RF channels that the receiver is intended to handle.
- With the same FM receiver, the band-pass filter after the local oscillator would be approximately 200kHz wide; it is the effects of this narrower filter that are of most interest to us.

Fig.

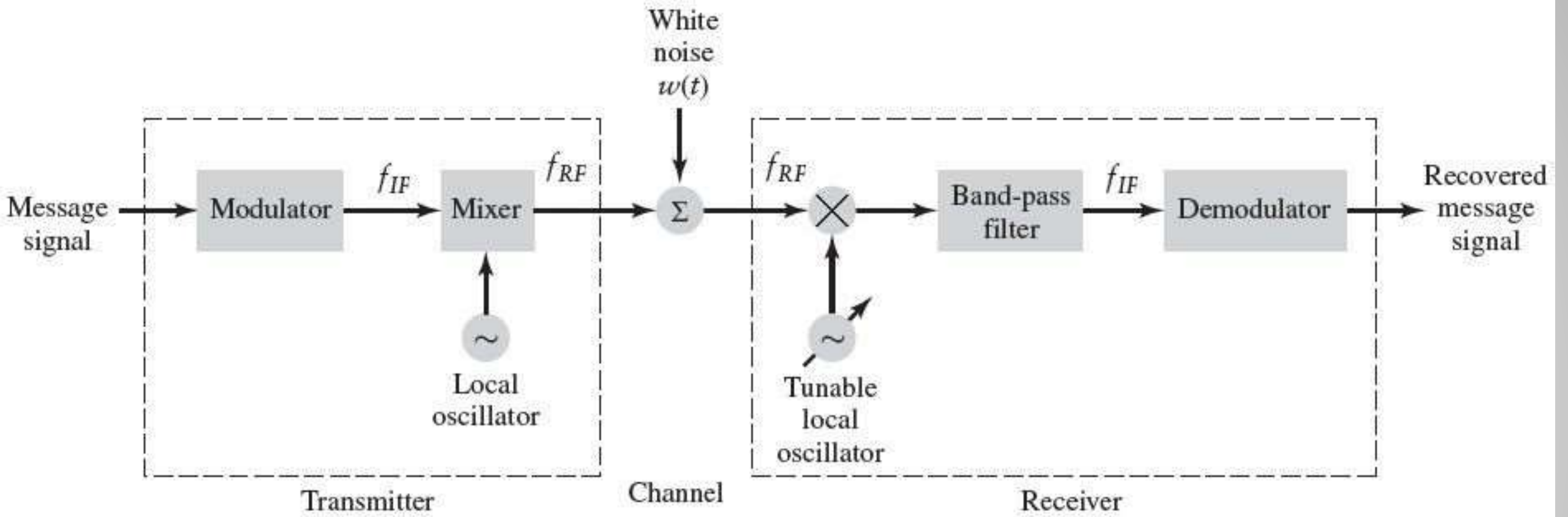


FIGURE Block diagram of band-pass transmission showing a superheterodyne receiver.

Noise in Linear Receivers Using Coherent *Detection*

- Double-sideband suppressed-carrier (DSB-SC) modulation, the modulated signal is represented as

$$s(t) = A_c m(t)\cos(2\pi f_c t + \theta) \quad (13)$$

- f_c is the carrier frequency
- $m(t)$ is the message signal
- The carrier phase $w(t)$
- In Fig. , the received RF signal is the sum of the modulated signal and white Gaussian noise
- After band-pass filtering, the resulting signal is

$$x(t) = s(t) + n(t) \quad (14)$$

Fig.

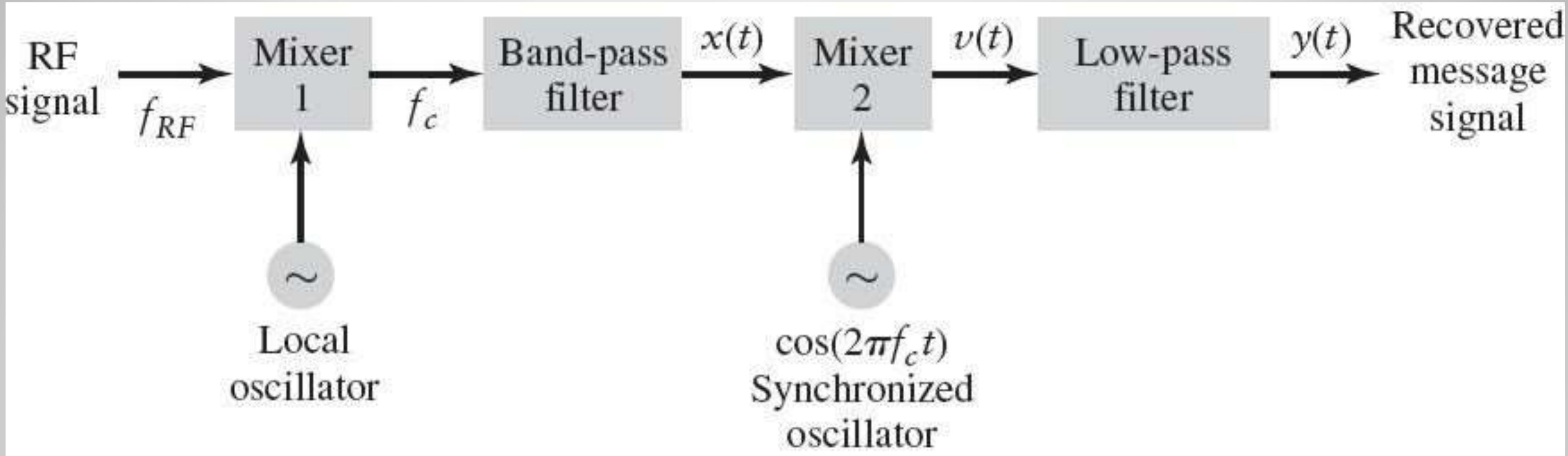


FIGURE A linear DSB-SC receiver using coherent demodulation.

- In Fig.
  - The assumed power spectral density of the band-pass noise is illustrated
- For the signal $s(t)$), the average power of the signal component is given by expected value of the squared magnitude.
- The carrier and modulating signal are independent

$$E[s^2(t)] = E[(A_c \cos(2\pi f_c t + \theta))^2] E[m^2(t)] \quad (15)$$

$$P = E[m^2(t)] \quad (16)$$

$$E[s^2(t)] = \frac{A_c^2 P}{2} \quad (17)$$

- Pre-detection signal-to-noise ratio of the DSB-SC system
  - A noise bandwidth B_T
  - The signal-to-noise ratio of the signal is $\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{DSB}} = \frac{A_c^2 P}{2N_0 B_T}$ (18)

Cont…

Fig.

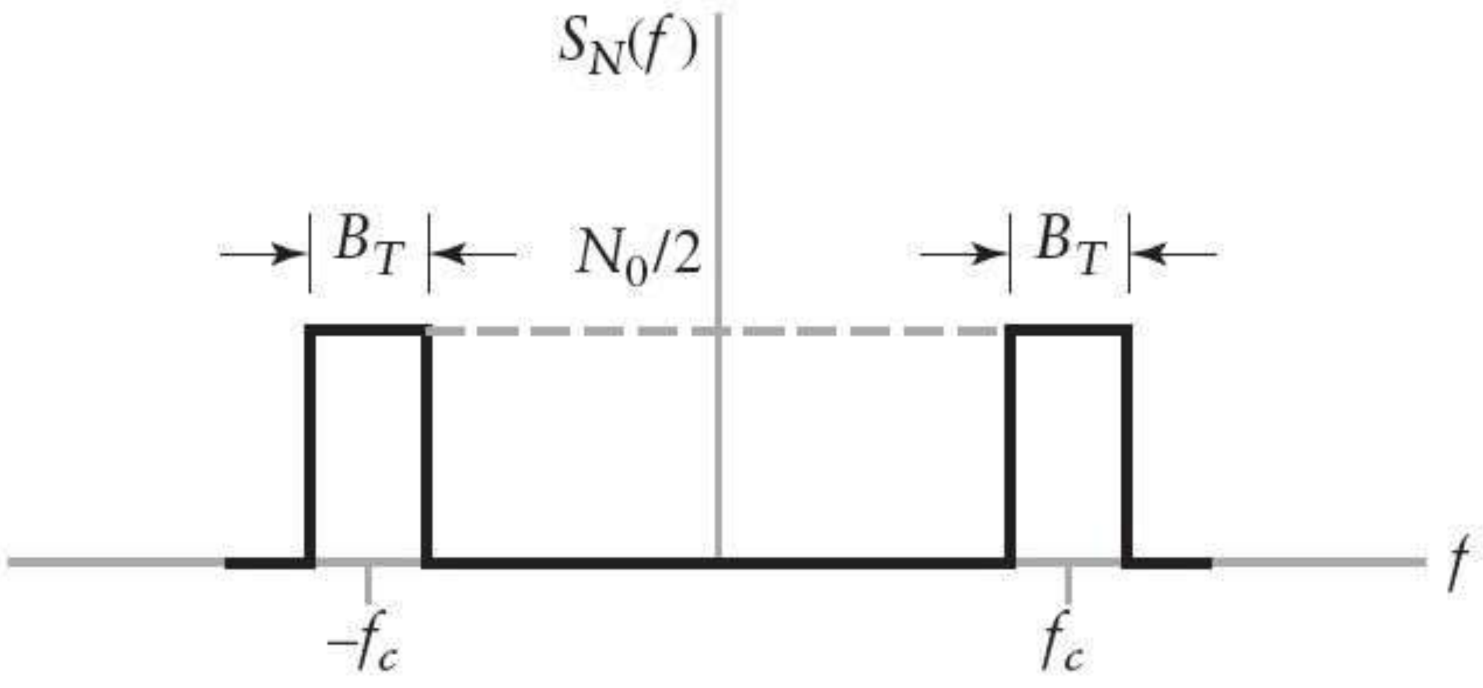


FIGURE Power spectral density of band-pass noise.

Cont…

- The signal at the input to the coherent detector of Fig.

$$x(t) = s(t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \quad (19)$$

$$v(t) = x(t)\cos(2\pi f_c t)$$
$$= \tfrac{1}{2}(A_c m(t) + n_I(t))$$
$$+ \tfrac{1}{2}(A_c m(t) + n_I(t))\cos(4\pi f_c t) - \tfrac{1}{2} n_Q(t)\sin(4\pi f_c t) \quad (20)$$

$$\cos A \cos A = \frac{1+\cos 2A}{2} \quad \text{and} \quad \sin A \cos A = \frac{\sin 2A}{2}$$

$$y(t) = \tfrac{1}{2}(A_c m(t) + n_I(t)) \quad (21)$$

- These high-frequency components are removed with a low-pass filter

- The message signal $m(t)$ and the in-phase component of the filtered noise $n_I(t)$ appear additively in the output.
- The quadrature component of the noise is completely rejected by the demodulator. Post-detection signal to noise ratio
- The message component is $\frac{1}{2}A_c m(t)$, so analogous to the computation of the predetection signal power, the post-detection signal power is $\frac{1}{4}A_c P$ where P is the average message power as defined in Eq. (9.16).
- The noise component is $\frac{1}{2}n_I(t)$ after low-pass filtering. As described in Section 8.11, the in-phase component has a noise spectral density of N_0 over the bandwidth from $-B_T/2 \; to \; B_T/2$ .If the low-pass filter has a noise bandwidth W, corresponding to the message bandwidth, which is less than or equal to $B_T/2$, then the output noise power is $E[n_I^2(t)] = \int_{-W}^{W} N_0 df$

$$= 2N_0 W \quad (22)$$

Cont...

- Post-detection SNR of

$$\text{SNR}_{\text{post}}^{\text{DSB}} = \frac{\frac{1}{4}(A_c^2)P}{\frac{1}{4}(2N_0 W)}$$

$$= \frac{A_c^2 P}{2N_0 W} \qquad (23)$$

- Post-detection SNR is twice pre-detection SNR.
- Figure of merit for this receiver is

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{DSB}}}{\text{SNR}_{\text{ref}}} = 1$$

- We lose nothing in performance by using a band-pass modulation scheme compared to the baseband modulation scheme, even though the bandwidth of the former is twice as wide.

Cont...

Noise in AM Receivers Using Envelope Detection

- The envelope-modulated signal

$$s(t) = A_c(1 + k_a m(t))\cos(2\pi f_c t) \qquad (24)$$

- The power in the modulated part of the signal is

$$\begin{aligned} \mathrm{E}[(1 + k_a m(t))^2] &= \mathrm{E}[1 + 2k_a m(t) + k_a^2 m^2(t)] \\ &= 1 + 2k_a \mathrm{E}[m(t)] + k_a^2 \mathrm{E}[m^2(t)] \\ &= 1 + k_a^2 P \end{aligned} \qquad (25)$$

- The pre-detection signal-to-noise ratio is given by

$$\mathrm{SNR}_{\mathrm{pre}}^{\mathrm{AM}} = \frac{A_c^2(1 + k_a^2 P)}{2N_0 \mathrm{B}_{\mathrm{T}}} \qquad (26)$$

Fig.

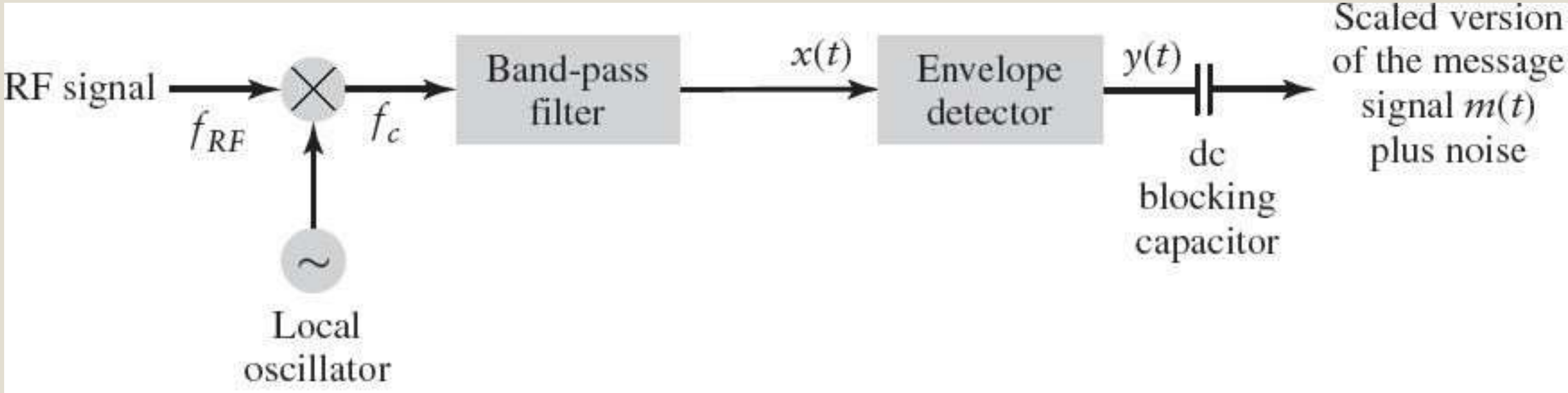


FIGURE Model of AM receiver using envelope detection.

- Model the input to the envelope detector as

$$x(t) = s(t) + n(t)$$
$$= [A_c + A_c k_a m(t) + n_I(t)]\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \quad (27)$$

- The output of the envelope detector is the amplitude of the phasor representing $x(t)$ and it is given by

$$y(t) = \text{envelope of } x(t)$$
$$= \{[A_c(1 + k_a m(t)) + n_I(t)]^2 + n_Q^2(t)\}^{1/2} \quad (28)$$

$$\sqrt{A^2 + B^2} \approx A \quad when \quad A >> B,$$

- Using the approximation

$$y(t) \approx A_c + A_c k_a m(t) + n_I(t) \quad (29)$$

Cont...

Cont…

- The post-detection SNR for the envelope detection of AM,

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}} = \frac{A_c^2 k_a^2 P}{2N_0 W} \quad (30)$$

- This evaluation of the output SNR is only valid under two conditions:
  - The SNR is high.
  - is adjusted for 100% modulation or less, so there is no distortion of the signal envelope.

- The figure of merit for this AM modulation-demodulation scheme is

$$\text{Figure of merit} = \frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{AM}}}{\mathrm{SNR}_{\mathrm{ref}}} = \frac{k_a^2 P}{1 + k_a^2 P} \quad (31)$$

Fig.

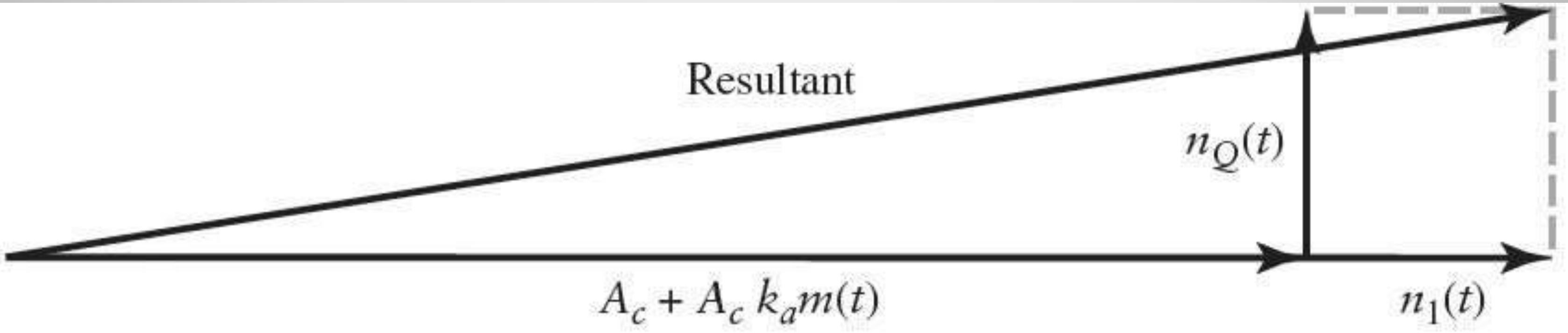


FIGURE Phasor diagram for AM wave plus narrowband noise.

- In the experiment, the message is a sinusoidal wave $m(t) = A\sin(2\pi f_m t)$,
- We compute the pre-detection and post-detection SNRs for samples of its signal. These two measures are plotted against one another in Fig. for $k_a = 0.3$.
- The post-detection SNR is computed as follows:
  - The output signal power is determined by passing a noiseless signal through the envelope detector and measuring the output power.
  - The output noise is computed by passing plus noise through the envelope detector and subtracting the output obtained form the clean signal only. With this approach, any distortion due to the product of noise and signal components is included as noise contribution.
- From Fig, there is close agreement between theory and experiment at high SNR values, which is to be expected. There are some minor discrepancies, but these can be attributed to the limitations of the discrete time simulation. At lower SNR there is some variation from theory as might also be expected.

Cont…

Fig.

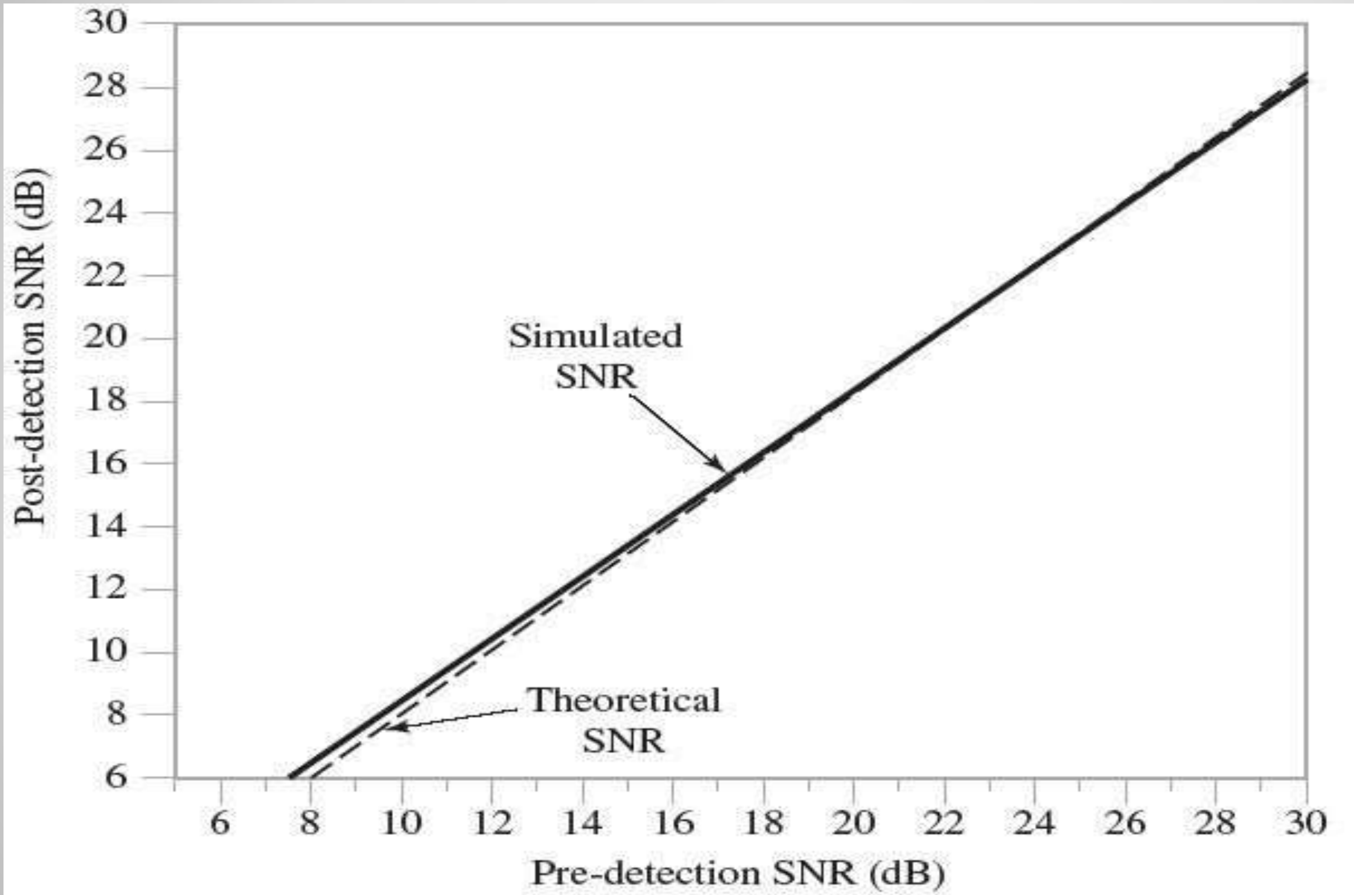


FIGURE Comparison of pre-detection and post-detection SNRs with simulated envelope detection of AM.

Noise in SSB Receivers

- The modulated wave as

$$s(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t)\sin(2\pi f_c t) \qquad (32)$$

- We may make the following observations concerning the in-phase and quadrature components of $s(t)$ in Eq. (32) :
  1. The two components $m(t)$ and $\hat{m}(t)$ are uncorrelated with each other. Therefore, their power spectral densities are additive.
  2. The Hilbert transform $\hat{m}(t)$ is obtained by passing $m(t)$ through a linear filter with transfer function $-j\,\text{sgn}(f)$. The squared f magnitude of this transfer function is equal to one for all p . Accordingly, $m(t)$ and $\hat{m}(t)$ have the same average power .

- The pre-detection signal-to-noise ratio of a coherent receiver with SSB modulation is

$$\mathrm{SNR}^{\mathrm{SSB}}_{\mathrm{pre}} = \frac{A_c^2 P}{4N_0 W} \qquad (33)$$

- The band-pass signal after multiplication with the synchronous oscillator output $\cos(2\pi f_c t)$ is

$$v(t) = x(t)\cos(2\pi f_c t)$$
$$= \tfrac{1}{2}\left(\frac{A_c}{2}m(t) + n_I(t)\right)$$
$$+ \tfrac{1}{2}\left(\frac{A_c}{2}m(t) + n_I(t)\right)\cos(4\pi f_c t) - \tfrac{1}{2}\left(\frac{A_c}{2}\hat{m}(t) + n_Q(t)\right)\sin(4\pi f_c t) \qquad (34)$$

After low-pass filtering the $v(t)$, we are left with

Cont…

$$y(t) = \tfrac{1}{2}\left(\frac{A_c}{2}m(t) + n_I(t)\right) \qquad (35)$$

- The spectrum of the in-phase component of the noise $n_I(t)$ is given by

$$s_{N_I}(f)=\begin{cases} S_N(f-f_c)+S_N(f+f_c), & -B\le f\le B \\ 0, & \text{otherwise} \end{cases} \qquad (36)$$

$$s_{N_I}(f)=\begin{cases} \frac{N_0}{2}, & -W\le f\le W \\ 0, & \text{otherwise} \end{cases} \qquad (37)$$

- The post-detection signal-to-noise ratio

$$\mathrm{SNR}_{\mathrm{post}}^{\mathrm{SSB}}=\frac{A_c^2 P}{4N_0 W} \qquad (38)$$

- The figure of merit for the SSB system

Cont...

$$\text{Figure of merit}=\frac{\mathrm{SNR}_{\mathrm{post}}^{\mathrm{SSB}}}{\mathrm{SNR}_{\mathrm{ref}}}=1 \qquad (39)$$

Cont…

- Comparing the results for the different amplitude modulation schemes
  - There are a number of design tradeoffs.
- Single-sideband modulation achieves the same SNR performance as the baseband reference model but only requires half the transmission bandwidth of the DSC-SC system.
- SSB requires more transmitter processing.

Detection of Frequency Modulation (FM)

- The frequency-modulated signal is given by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right] \quad (40)$$

- Pre-detection SNR
  - The pre-detection SNR in this case is simply the carrier power $A_C^2/2$ divided by the noise passed by the bandpass filter, ; namily, $N_0 B_T$

$$\text{SNR}_{\text{pre}}^{\text{AM}} = \frac{A_c^2}{2N_0 B_T}$$

1. A slope network or differentiator with a purely imaginary frequency response that varies linearly with frequency. It produces a hybrid-modulated wave in which both amplitude and frequency vary in accordance with the message signal.
2. An envelope detector that recovers the amplitude variation and reproduces the message signal.

Fig.

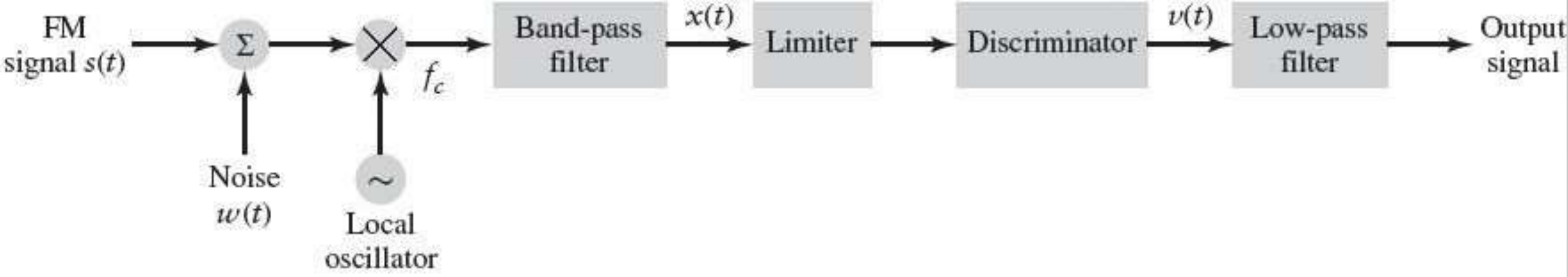


FIGURE Model of an FM receiver.

Cont..

- Post-detection SNR
  - The noisy FM signal after band-pass filtering may be represented as

$$x(t) = s(t) + n(t) \qquad (41)$$

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \qquad (42)$$

  - We may equivalently express $n(t)$ in terms of its envelope and phase as

$$n(t) = r(t)\cos[2\pi f_c t + \phi_n(t)] \qquad (43)$$

$$r(t) = [n_I^2(t) + n_Q^2(t)]^{1/2} \qquad (44)$$

  - Where the envelope is

$$\phi_n(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right) \qquad (45)$$

  - And the phase is

Fig.

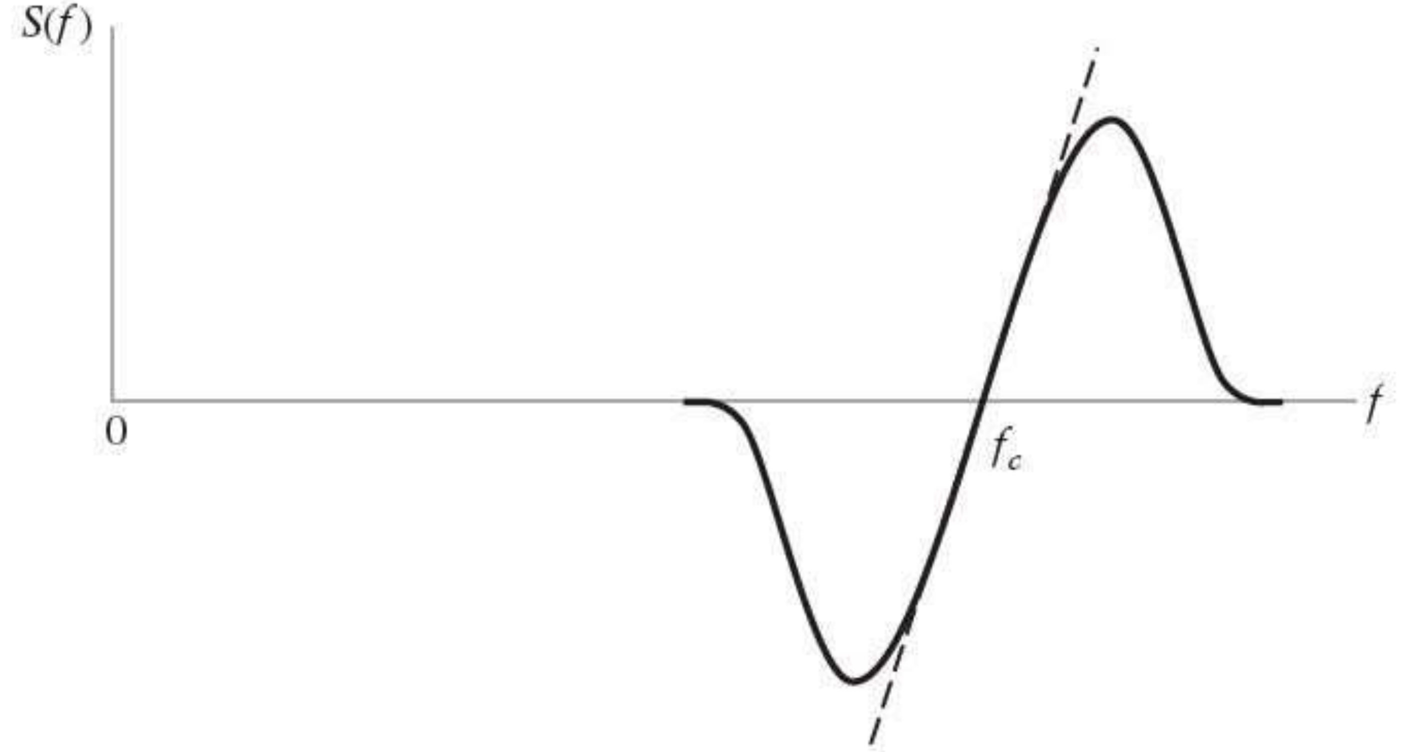


FIGURE Amplitude response of slope network used in FM discriminator.

- We note that the phase of $s(t)$ is

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)\,d\tau \qquad (46)$$

- The noisy signal at the output of the band-pass filter may be expressed as

$$\begin{aligned} x(t) &= s(t) + n(t) \\ &= A_c \cos[2\pi f_c t + \phi(t)] + r(t)\cos[2\pi f_c t + \psi(t)] \end{aligned} \qquad (47)$$

- The phase $\theta(t)$ of the resultant is given by

$$\theta(t) = \phi(t) + \tan^{-1}\left\{ \frac{r(t)\sin(\psi(t))}{A_c + r(t)\cos(\psi(t))} \right\} \qquad (48)$$

Cont...

Fig.

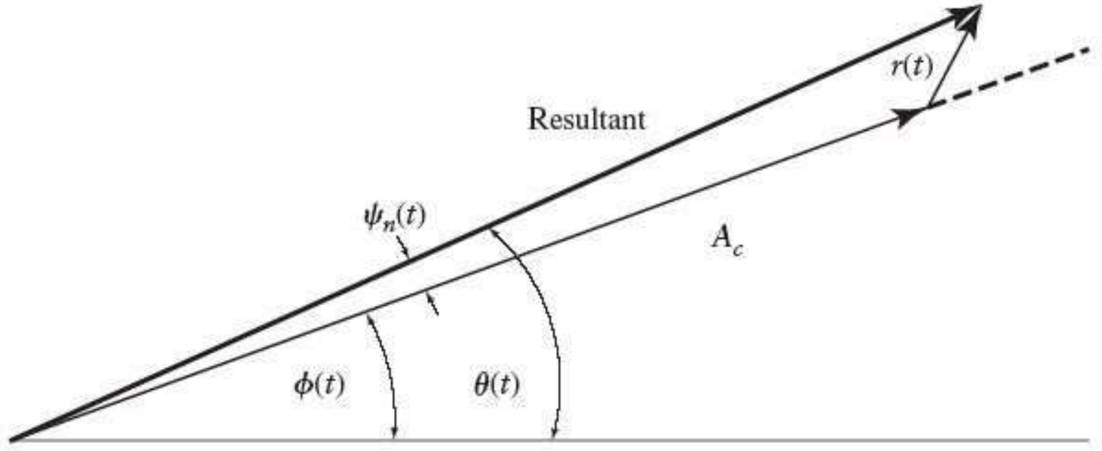


FIGURE Phasor diagram for FM signal plus narrowband noise assuming high carrier-to-noise ratio.

- Under this condition, and noting that $\tan^{-1}\xi \approx \xi$ since $\xi << 1$, the expression for the phase simplifies to

$$\theta(t) = \phi(t) + \frac{r(t)}{A_c}\sin[\psi(t)] \qquad (49)$$

- Then noting that the quadrature component of the noise is $n_Q(t) = r(t)\sin[\psi(t)]$, we may simplify Eq.(49) to

$$\theta(t) = \phi(t) + \frac{n_Q(t)}{A_c} \qquad (50)$$

$$\theta(t) \approx 2\pi k_f \int_0^t m(\tau)\,d\tau + \frac{n_Q(t)}{A_c} \qquad (51)$$

The ideal discriminator output $v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$

$$= k_f m(t) + n_d(t) \qquad (52)$$

Cont…

- The noise term $n_d(t)$ is defined by

$$n_d(t) = \frac{1}{2\pi A_c} \frac{dn_Q(t)}{dt} \qquad (53)$$

- The additive noise at the discriminator output is determined essentially by the quadrature component $n_Q(t)$ of the marrowband noise $n(t)$.

$$G(f) = \frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c} \qquad (54)$$

- The power spectral density $S_{N_Q}(f)$ of the quadrature noise component $n_Q(t)$ as follows;

$$S_{N_d}(f) = |G(f)|^2 S_{N_Q}(f)$$

$$= \frac{f^2}{A_c^2} S_{N_Q}(f) \qquad (55)$$

Cont...

- Power spectral density of the noise $n_d(t)$ is shown in Fig.16

$$S_{N_d}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2}, & |f| < \dfrac{B_T}{2} \\ 0, & \text{otherwise} \end{cases} \qquad (56)$$

- Therefore, the power spectral density $S_{N_0}(f)$ of the noise $n_0(t)$ appearing at the receiver output is defined by

$$S_{N_0}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2}, & |f| < \mathrm{W} \\ 0, & \text{otherwise} \end{cases} \qquad (57)$$

$$\text{Average post-detection noise power} = \frac{N_0}{A_c^2}\int_{-\mathrm{W}}^{\mathrm{W}} f^2 df$$

$$= \frac{2N_0\mathrm{W}^3}{3A_c^2} \qquad (58)$$

Cont...

Fig.

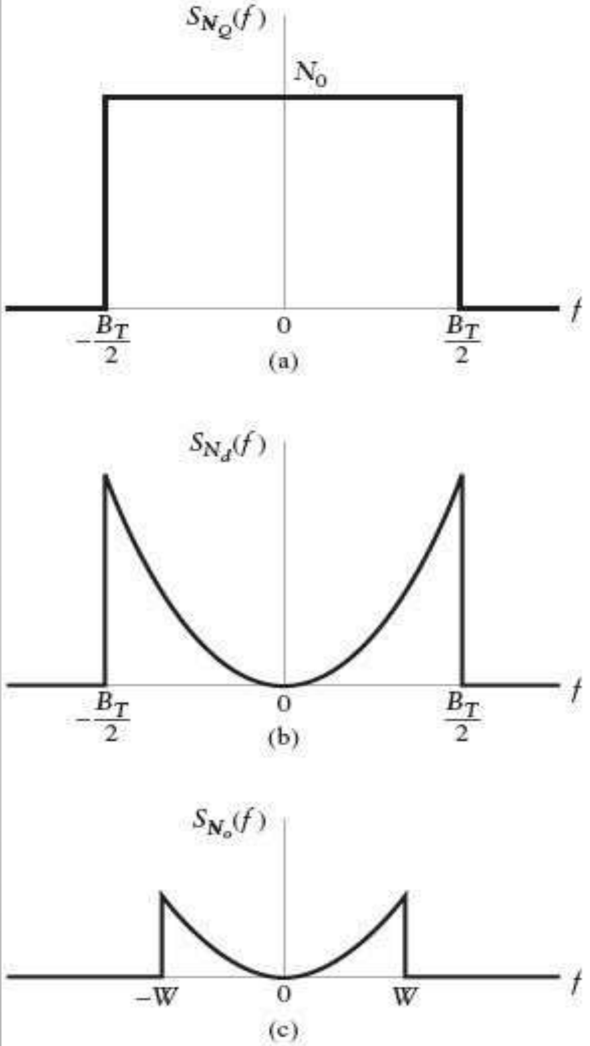


FIGURE Noise analysis of FM receiver. (*a*) Power spectral density of quadrature component $n_Q(t)$ of narrowband noise $n(t)$. (*b*) Power spectral density $n_d(t)$ at discriminator output. (*c*) Power spectral density of noise $n_o(t)$ at receiver output.

$$\text{SNR}_{\text{post}}^{\text{FM}} = \frac{3A_c^2 k_f^2 P}{2N_0 \text{W}^3} \qquad (59)$$

- Figure of merit

$$\text{Figure of merit} = \frac{\text{SNR}_{\text{post}}^{\text{FM}}}{\text{SNR}_{\text{ref}}} = \frac{\dfrac{3A_c^2 k_f^2 P}{2N_0 \text{W}^3}}{\dfrac{A_c^2}{2N_0 \text{W}}}$$

$$= 3\left(\frac{k_f^2 P}{\text{W}^2}\right)$$

$$= 3D^2 \qquad (60)$$

- The figure of merit for an FM system is approximately given by

$$\text{Figure of merit} \approx \frac{3}{4}\left(\frac{B_T}{\text{W}}\right)^2 \qquad ((61)$$

Cont...

- Thus, when the carrier to noise level is high, unlike an amplitude modulation system an FM system allows us to trade bandwidth for improved performance in accordance with square law.

Cont…

Cont…

- Threshold effect
  - At first, individual clicks are heard in the receiver output, and as the pre-detection SNR decreases further, the clicks merge to a crackling or sputtering sound. At and below this breakdown point, Eq.(59) fails to accurately predict the post-detection SNR.
- Computer experiment : Threshold effect with FM
  - Complex phasor of the FM signal is given by

$$s(t) = A_c \exp\left\{ j2\pi k_f \int_0^t m(\tau) d\tau \right.$$

- Similar to the AM computer experiment, we measure the pre-detection and post-detection SNRs of the signal and compare the results to the theory developed in this section.

FM Pre-emphasis and De-emphasis

- To compensate this distortion, we appropriately pre-distort or pre-emphasize the baseband signal at the transmitter, prior to FM modulation, using a filter with the frequency response

$$H_{pre}(f) = \frac{\square 1}{H_{de}(f)} \quad |f| < W \qquad (62)$$

- The de-emphasis filter is often a simple resistance-capacitance (RC) circuit with

$$H_{de}(f) = \frac{1}{1 + j\dfrac{f}{f_{3dB}}} \qquad (63)$$

- At the transmitting end, the pre-emphasis filter is

$$H_{pre}(f) = 1 + j\frac{f}{f_{3dB}} \qquad (64)$$

Cont…

- The modulated signal is approximately

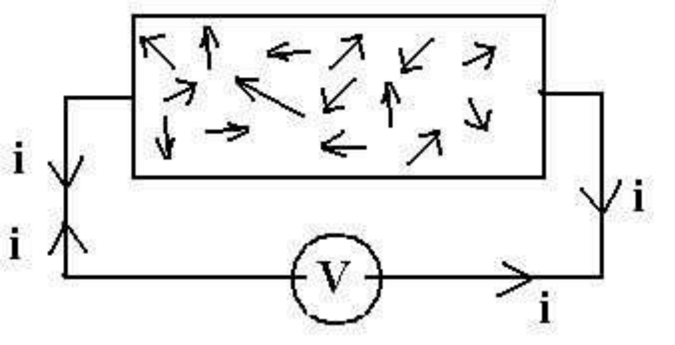
$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t \left(m(s) + \alpha \frac{dm(s)}{ds}\right) ds\right)$$

$$= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(s)\,ds + 2\pi k_f \alpha m(t)\right)$$

- Pre-emphasis can be used to advantage whenever portions of the message band are degraded relative to others.

Thermal Noise (Johnson Noise)

This type of noise is generated by all resistances (e.g. a resistor, semiconductor, the resistance of a resonant circuit, i.e. the real part of the impedance, cable etc).



Experimental results (by Johnson) and theoretical studies (by Nyquist) give the mean square noise voltage as

$$\bar{V}^2 = 4kTBR \ (volt^2)$$

Where k = Boltzmann's constant = 1.38 x 10^{-23} Joules per K

T = absolute temperature

B = bandwidth noise measured in (Hz)

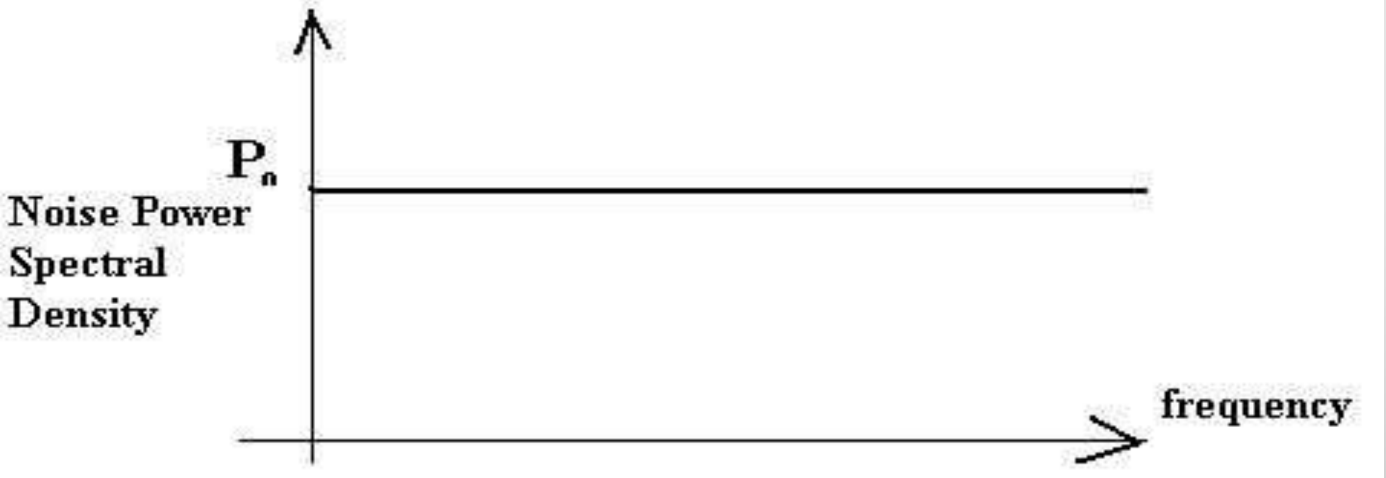
R = resistance (ohms)

Thermal Noise (Johnson Noise) (Cont'd)

The law relating noise power, N, to the temperature and bandwidth is

$$N = k\,TB \text{ watts}$$

Thermal noise is often referred to as 'white noise' because it has a uniform 'spectral density'.



Uniform Noise Power Spectral Density

Shot Noise

•Shot noise was originally used to describe noise due to random fluctuations in electron emission from cathodes in vacuum tubes (called shot noise by analogy with lead shot).

•Shot noise also occurs in semiconductors due to the liberation of charge carriers.

• For *pn* junctions the mean square shot noise current is

$$I_n^2 = 2\left(I_{DC} + 2I_o\right)q_e B \quad (amps)^2$$

Where

is the direct current as the *pn* junction (amps)

is the reverse saturation current (amps)

is the electron charge = 1.6 x 10-19 coulombs

B is the effective noise bandwidth (Hz)

Signal to Noise

The signal to noise ratio is given by

$$\frac{S}{N} = \frac{Signal Power}{Noise Power}$$

The signal to noise in dB is expressed by

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$ for S and N measured in mW.

Noise Factor- Noise Figure

Consider the network shown below,

Noise Factor- Noise Figure (Cont'd)

- The amount of noise added by the network is embodied in the Noise Factor F, which is defined by

$$\text{Noise factor } F = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}}$$

- F equals to 1 for noiseless network and in general F > 1. The noise figure in the noise factor quoted in dB

  i.e. Noise Figure F dB = 10 log10 F   F ≥ 0 dB

- The noise figure / factor is the measure of how much a network degrades the (S/N)IN, the lower the value of F, the better the network.

Noise Temperature

N_{IN} is the 'external' noise from the source i.e. $N_{IN} = kT_S B_n$

T_S is the equivalent noise temperature of the source (usually 290K).

We may also write $N_e = kT_e B_n$, where T_e is the equivalent noise temperature of the element i.e. with noise factor F and with source temperature T_S.

$$\text{i.e. } kT_e B_n = (F-1)\, kT_S B_n$$

$$\text{or } T_e = (F-1)T_S$$

UNIT –V

INFORMATION THEORY

FUNDAMENTAL LIMITS ON PERFORMANCE

- Given an information source, and a noisy channel

  1)Limit on the minimum number of bits per symbol

  2)Limit on the maximum rate for reliable communication

➔ <u>Shannon's 3 theorems</u>

UNCERTAINTY, INFORMATION AND ENTROPY

- Let the source alphabet,

$$S = \{s_0, s_1, .., s_{K-1}\}$$

with the prob. of occurrence

$$P(s = s_k) = p_k \,, \quad k = 0,1, .., K-1 \quad and \quad \sum_{k=0}^{K-1} p_k = 1$$

- Assume the discrete memoryless source (DMS)

What is the measure of information?

UNCERTAINTY, INFORMATION, AND ENTROPY (CONT')

- **Interrelations between info., uncertainty or surprise**

  No surprise ⟶ no information

$$(\approx Info. \propto \frac{1}{Prob.})$$

- **The amount of info may be related to the inverse of the prob. of occurrence.**

$$\therefore\ I(S_k) = \log(\frac{1}{p_k})$$

PROPERTY OF INFORMATION

1) $I(\mathrm{s}_\mathrm{k}) = 0 \text{ for } \mathrm{p}_\mathrm{k} = 1$

2) $I(s_k) \geq 0 \quad \text{for} \quad 0 \leq \mathrm{p}_\mathrm{k} \leq 1$

3) $I(s_k) > I(s_i) \quad \text{for} \quad \mathrm{p}_\mathrm{k} < \mathrm{p}_\mathrm{i}$

4) $I(s_k s_i) = I(s_k) + I(s_i), \quad \text{if } \mathrm{s}_\mathrm{k} \text{ and } \mathrm{s}_\mathrm{i} \text{ statist. indep.}$

* Custom is to use logarithm of base 2

ENTROPY

- Def. : measure of average information contents per source symbol

➡ The mean value of $I(s_k)$ over S,

$$H(S) = \mathrm{E}[I(s_k)] = \sum_{k=0}^{K-1} p_k I(s_k) = \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right)$$

➡ The property of H

$$0 \le H(S) \le \log_2 K, \ where\ K\ is\ radix\ (=\ \#\ of\ symbols)$$

1) H(S)=0,**iff** $p_k = 1$ **for some *k*, and all other** $p_i's = 0$

⟶ No Uncertainty

2) H(S)= $\log_2 K,\ iff\ p_k = \frac{1}{K}\ for\ all\ k$

⟶ Maximum Uncertainty

Channel Capacity

- transmission data rate of a channel (bps)

- Bandwidth
  - bandwidth of the transmitted signal (Hz)
- Noise
  - average noise over the channel
- Error rate
  - symbol alteration rate. i.e. 1-> 0

Channel Capacity

- if channel is noise free and of bandwidth W, then maximum rate of signal transmission is 2W
- This is due to intersymbol interface

Shannon's Channel Capacity Theorem

FOR BANDLIMITED, POWER LIMITED GAUSSIAN CHANNELS

$$C = B\log_2\left(1+\frac{P}{N}\right) \text{ (bits/s)}$$

The capacity of a channel of bandwidth B, perturbed by additive white gaussian noise of psd $N_0/2$, and limited in bandwidth to B,

P is the average transmitted power, and N is the noise ($N_o B$)

- It is not possible to transmit at rate higher than C reliability by any means.
- It does not say how to find coding and modulation to achieve maximum capacity, but it indicates that approaching this limit, the transmitted signal should have statistical property approximately to Gaussian noise.

Channel Capacity

- **For a dms with input X, output Y, &** $p(y_k \mid x_j)$**,**

$$I(X;Y)=\sum_{j=0}^{J-1}\sum_{k=0}^{K-1} p(x_j,y_k)\log_2\left[\frac{p(y_k \mid x_j)}{p(y_k)}\right]$$

where $p(x_j,y_k)=p(y_k \mid x_j)\,p(x_j)$, $p(y_k)=\sum_{j=0}^{J-1} p(y_k \mid x_j)\,p(x_j)$

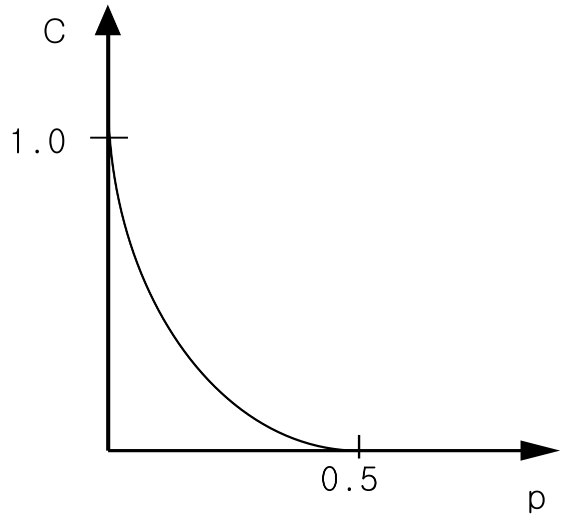
$\Rightarrow$ **I(X;Y) just depends upon** $\{p(x_j),\ j=0,1,2,\dots,J-1\}$ **, & channel. Since** $\{p(x_j)\}$ **is indep. of the channel, it is possible to maximize I(X;Y) w.r.t.** $\{p(x_j)\}$ **.**

- **Def. of Channel Capacity.**

$$C=\max_{\{p(x_j)\}} I(X;Y)$$ **(bits per channel use)**

$$C = \max I(X;Y) = I(X;Y)|_{p(x_0)=0.5}$$

$$\therefore C = 1 + p\log_2 p + (1-p)\log_2(1-p) = 1 - H(p)$$



- For reliable communication , needs channel encoding & decoding. "any coding scheme which gives the error as small as possible, and which is efficient enough that code rate is not too small?"

=> Shannon's second theorem (noisy coding theorem)

Let dms with alphabet X have entropy H(X) and produce symbols once every Ts, and dmc have capacity C and be used once every Tc . Then,

i) if $\frac{H(X)}{T_s} \leq \frac{C}{T_c}$, there exists a coding scheme.

ii) if $\frac{H(X)}{T_s} > \frac{C}{T_c}$, it is not possible to transmit with arbitrary small error.

$$p_0 = 0.5$$

The condition for reliable comm. ,

$$\frac{1}{T_s} \leq \frac{C}{T_c}$$

Let $\frac{T_c}{T_s}$ be r , then $r \leq C$

∴ for $r \leq C$, there exists a code (with code rate less than or equal to C) capable of achieving an arbitrary low probability of error.

" The code rate $r = \frac{k}{n}$ where k is k-bit input, and n is n-bit coded bits,."

Channel Capacity

- doubling bandwidth doubles the data rate

- doubling the number of bits per symbol also doubles the data rate (assuming an error free channel)

(S/N):-signal to noise ratio

$$(S/N)_{dB} = 10\log \frac{\text{signal power}}{\text{noise power}}$$

Source Coding Theorem

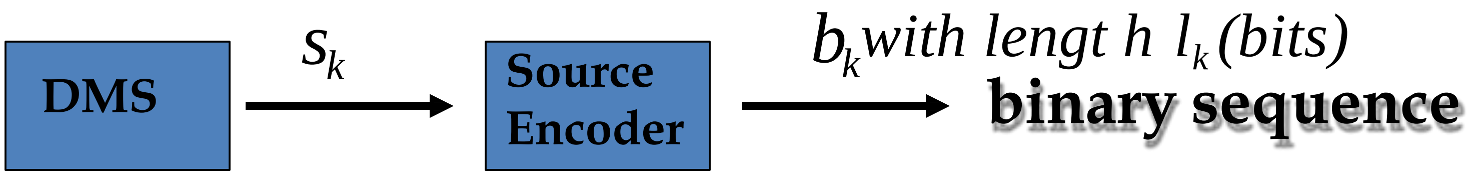
- Efficient representation of data → compaction
- Be uniquely decodable
- Need of statistics of the source

(There is an algorithm called “Lempel-Ziv” for unknown statistics of the source)

(Another frequent method is Run-Length code)

Source Coding Theorem (cont')

Variable length code ⟷ Fixed length code



➡ The average code-Length, $\bar{L}$, is

$$\bar{L} = \sum_{k=0}^{K-1} p_k l_k$$

➡ The coding efficiency, $\eta = \frac{L_{\min}}{\bar{L}}$

where $L_{\min}$ is the minimum possible value of $\bar{L}$

Shannon's first theorem :

Source-coding theorem

- Given a dms of entropy H(S), the average code-word length $\bar{L}$ for any source coding is

$$\bar{L} \geq H(S)$$

(i.e.) $L_{\min} = H(S)$ & $\eta = \frac{H(S)}{\bar{L}}$

Discrete Memoryless Channel

$$X\begin{cases} x_0 \\ x_1 \\ . \\ . \\ . \\ x_{J-1} \end{cases} \quad X \rightarrow P(y_k \mid x_j) \rightarrow Y \quad \left.\begin{matrix} y_0 \\ y_1 \\ . \\ . \\ . \\ y_{K-1} \end{matrix}\right\} Y$$

- Definition of DMC
  - Channel with input X & output Y which is noisy version of X.
  - Discrete when both of alphabets X & Y finite sizes.
  - Memoryless when no dependency between input symbols.

DISCRETE MEMORYLESS CHANNEL (CONT')

- Channel Matrix (Transition Probability Matrix)

$$P = \begin{bmatrix} p(y_0|x_0) & p(y_1|x_0) \ldots & p(y_{K-1}|x_0) \\ p(y_0|x_1) & \ldots\ldots & p(y_{K-1}|x_1) \\ . & & \\ . & & \\ p(y_0|x_{J-1}) & \ldots\ldots & p(y_{K-1}|x_{J-1}) \end{bmatrix}$$

➡ The size is J by K

$$\sum_{k=0}^{K-1} p(y_k | x_j) = 1 \quad \text{for all } j$$

➡ a priori prob. is :

$$P_k = p(x_j), \; j = 0, 1, .., J-1$$

Discrete Memoryless Channel (cont')

- Given a priori prob. $p(x_j)$, and the channel matrix, P then we can find the prob. of the various output symbols, $p(y_k)$ as
  - ➡ the joint prob. dist'n of X and Y

$$p(x_j, y_k) = p(X = x_j, Y = y_k) = p(Y = y_k / X = x_j)\, p(X = x_j)$$

$$= p(y_k / x_j)\, p(x_j)$$

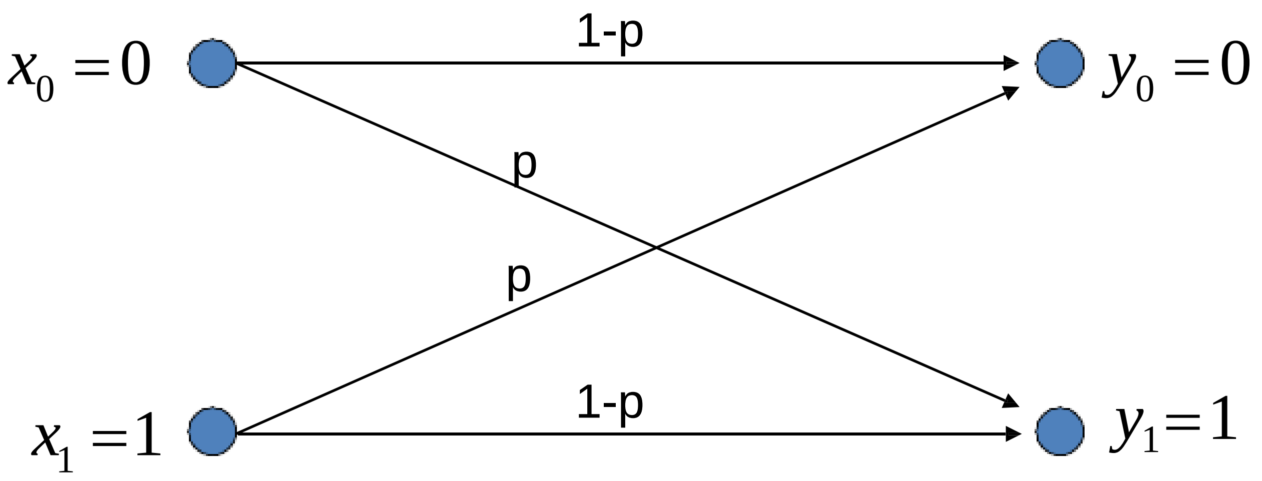
  - ➡ the marginal prob. dist'n of the output Y,

$$p(y_k) = p(Y = y_k) = \sum_{j=0}^{J-1} p(Y = y_k / X = x_j)\, p(X = x_j)$$

$J-1$

Discrete Memoryless Channel(cont')

- BSC (BINARY SYMMETRIC CHANNEL)



Redundancy vs. Efficiency

- 100% efficiency of encoding means that the average word length must be equal to the entropy of the original message ensemble:

$$\text{Efficiency} = \frac{H(X)}{\bar{L} \cdot \log D} \cdot 100\% \underset{\text{for} D=2}{=} \frac{H(X)}{\bar{L}}$$

if $D=2$ then $\log D=1$

- If the entropy of the original message ensemble is less than the length of the word over the original alphabet, this means that the original encoding is redundant and that the original information may be compressed by the efficient encoding.

Redundancy vs. Efficiency

- On the other hand, as we have seen, to be able to detect and to correct the errors, a code must be redundant, that is its efficiency must be lower that 100%: the average word length must be larger than the entropy of the original message ensemble:

$$\text{Efficiency} = \frac{H(X)}{\bar{L} \cdot \underbrace{\log D}_{\text{if } D=2 \text{ then } \log D=1}} \cdot 100\% \underset{\text{for} D=2}{=} \frac{H(X)}{\bar{L}}$$

Shannon-Fano Encoding

- **SOURCES WITHOUT MEMORY** ARE SUCH SOURCES OF INFORMATION, WHERE THE PROBABILITY OF THE NEXT TRANSMITTED SYMBOL (MESSAGE) DOES NOT DEPEND ON THE PROBABILITY OF THE PREVIOUS TRANSMITTED SYMBOL (MESSAGE).

- Separable codes are those codes for which the unique decipherability holds.
- Shannon-Fano encoding constructs reasonably efficient separable binary codes for sources without memory.

Shannon-Fano Encoding

- **SHANNON-FANO ENCODING** IS THE FIRST ESTABLISHED AND WIDELY USED ENCODING METHOD. THIS METHOD AND THE CORRESPONDING CODE WERE INVENTED SIMULTANEOUSLY AND INDEPENDENTLY OF EACH OTHER BY C. SHANNON AND R. FANO IN 1948.

Shannon-Fano Encoding

- Let us have the ensemble of the original messages to be transmitted with their corresponding probabilities:

$$[X] = [x_1, x_2, ..., x_n]; [P] = [p_1, p_2, ..., p_n]$$

- Our task is to associate a sequence C_k of binary numbers of unspecified length n_k to each message x_k such that:

Shannon-Fano Encoding

- No sequences of employed binary numbers C_k can be obtained from each other by adding more binary digits to the shorter sequence (prefix property).
- The transmission of the encoded message is "reasonably" efficient, that is, 1 and 0 appear independently and with "almost" equal probabilities. This ensures transmission of "almost" 1 bit of information per digit of the encoded messages.

Shannon-Fano Encoding

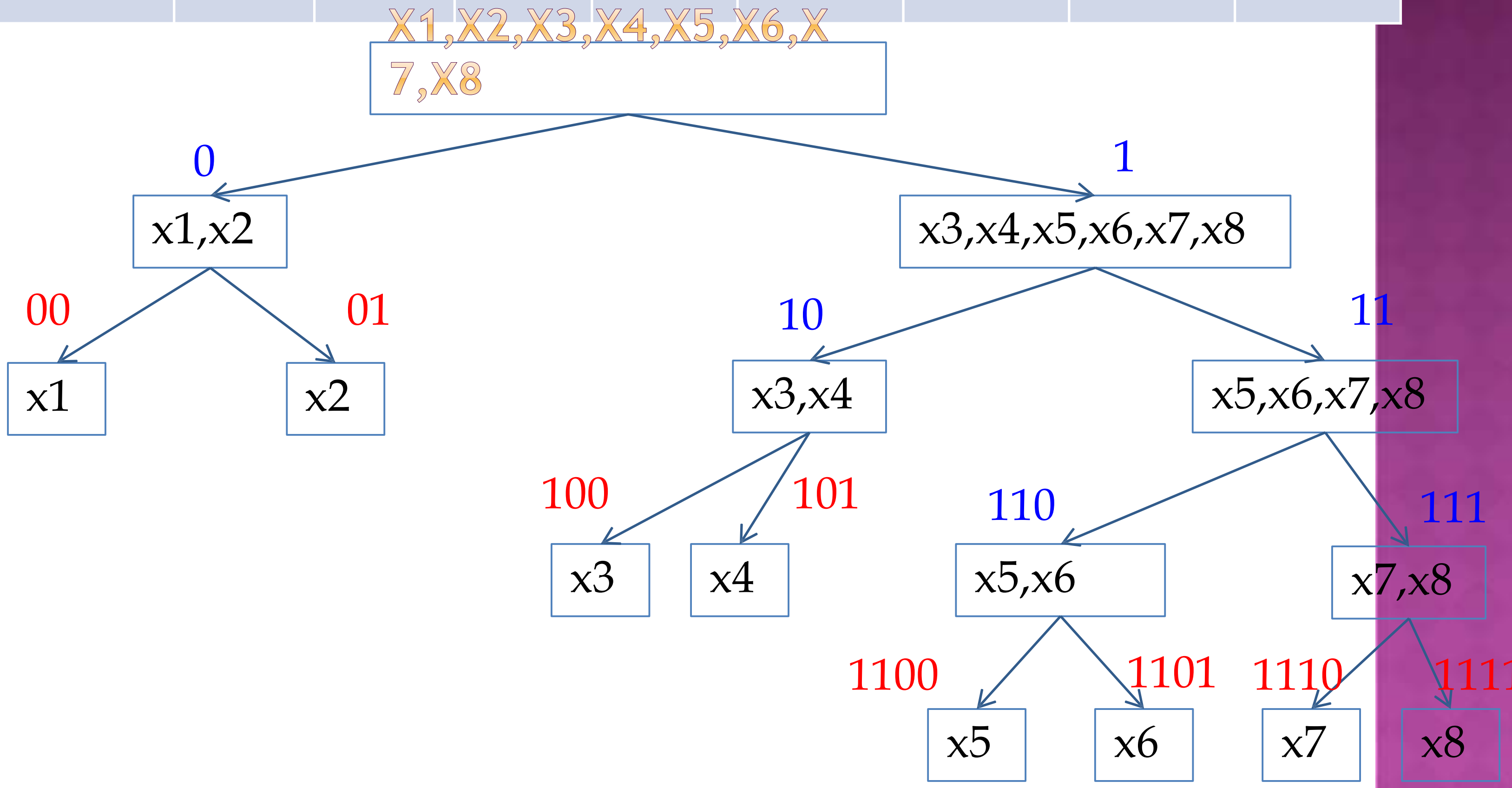
- Another important general consideration, which was taken into account by C. Shannon and R. Fano, is that (as we have already considered) a more frequent message has to be encoded by a shorter encoding vector (word) and a less frequent message has to be encoded by a longer encoding vector (word).

Shannon-Fano Encoding: Algorithm

- The letters (messages) of (over) the input alphabet must be arranged in order from most probable to least probable.
- Then the initial set of messages must be divided into two subsets whose total probabilities are as close as possible to being equal. All symbols then have the first digits of their codes assigned; symbols in the first set receive "0" and symbols in the second set receive "1".
- The same process is repeated on those subsets, to determine successive digits of their codes, as long as any sets with more than one member remain.
- When a subset has been reduced to one symbol, this means the symbol's code is complete.

Shannon-Fano Encoding: Example

Message	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Probability	0.25	0.25	0.125	0.125	0.0625	0.0625	0.0625	0.0625



Shannon-Fano Encoding: Example

Message	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Probability	0.25	0.25	0.125	0.125	0.0625	0.0625	0.0625	0.0625
Encoding vector	00	01	100	101	1100	1101	1110	1111

- Entropy

$$H = -\left(2\cdot\left(\frac{1}{4}\log\frac{1}{4}\right)+2\cdot\left(\frac{1}{8}\log\frac{1}{8}\right)+4\cdot\left(\frac{1}{16}\log\frac{1}{16}\right)\right)=2.75$$

- Average length of the encoding vector

$$\bar{L}=\sum P\{x_i\}n_i=\left(2\cdot\left(\frac{1}{4}\cdot 2\right)+2\cdot\left(\frac{1}{8}\cdot 3\right)+4\cdot\left(\frac{1}{16}\cdot 4\right)\right)=2.75$$

- The Shannon-Fano code gives 100% efficiency

Shannon-Fano Encoding: Example

Message	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Probability	0.25	0.25	0.125	0.125	0.0625	0.0625	0.0625	0.0625
Encoding vector	00	01	100	101	1100	1101	1110	1111

The Shannon-Fano code gives 100% efficiency. Since the average length of the encoding vector for this code is 2.75 bits, it gives the 0.25 bits/symbol compression, while the direct uniform binary encoding (3 bits/symbol) is redundant.

Huffman Encoding

- THIS ENCODING ALGORITHM HAS BEEN PROPOSED BY DAVID A. HUFFMAN IN 1952, AND IT IS STILL THE MAIN LOSS-LESS COMPRESSION BASIC ENCODING ALGORITHM.

- The Huffman encoding **ensures** constructing separable codes (the unique decipherability property holds) with **minimum redundancy** for a set of discrete messages (letters), that is, this encoding results in an optimum code.

Huffman Encoding: Background

- For an optimum encoding, the longer encoding vector (word) should correspond to a message (letter) with lower probability:

$$P\{x_1\} \geq P\{x_2\} \geq \ldots \geq P\{x_N\} \leftrightarrow L\{x_1\} \leq L\{x_2\} \leq \ldots \leq L\{x_N\}$$

- For an optimum encoding it is necessary that

$$L(x_{N-1}) = L(x_N)$$

otherwise the average length of the encoding vector will be unnecessarily increased.

- It is important to mention that not more than D (D is the number of letters in the encoding alphabet) encoding vectors could have equal length (for the binary encoding D=2)

Huffman Encoding: Background

- FOR AN OPTIMUM ENCODING WITH D=2 IT IS NECESSARY THAT THE LAST TWO ENCODING VECTORS ARE IDENTICAL EXCEPT FOR THE LAST DIGITS.

- For an optimum encoding it is necessary that each sequence of length $L(x_N)-1$ digits either must be used as an encoding vector or must have one of its prefixes used as an encoding vector.

Huffman Encoding: Algorithm

- The letters (messages) of (over) the input alphabet must be arranged in order from most probable to least probable.
- Two least probable messages (the last two messages) are merged into the composite message with a probability equal to the sum of their probabilities. This new message must be inserted into the sequence of the original messages instead of its "parents", accordingly with its probability.
- The previous step must be repeated until the last remaining two messages will compose a message, which will be the only member of the messages' sequence.
- The process may be utilized by constructing a binary tree – the Huffman tree.

Huffman Encoding: Algorithm

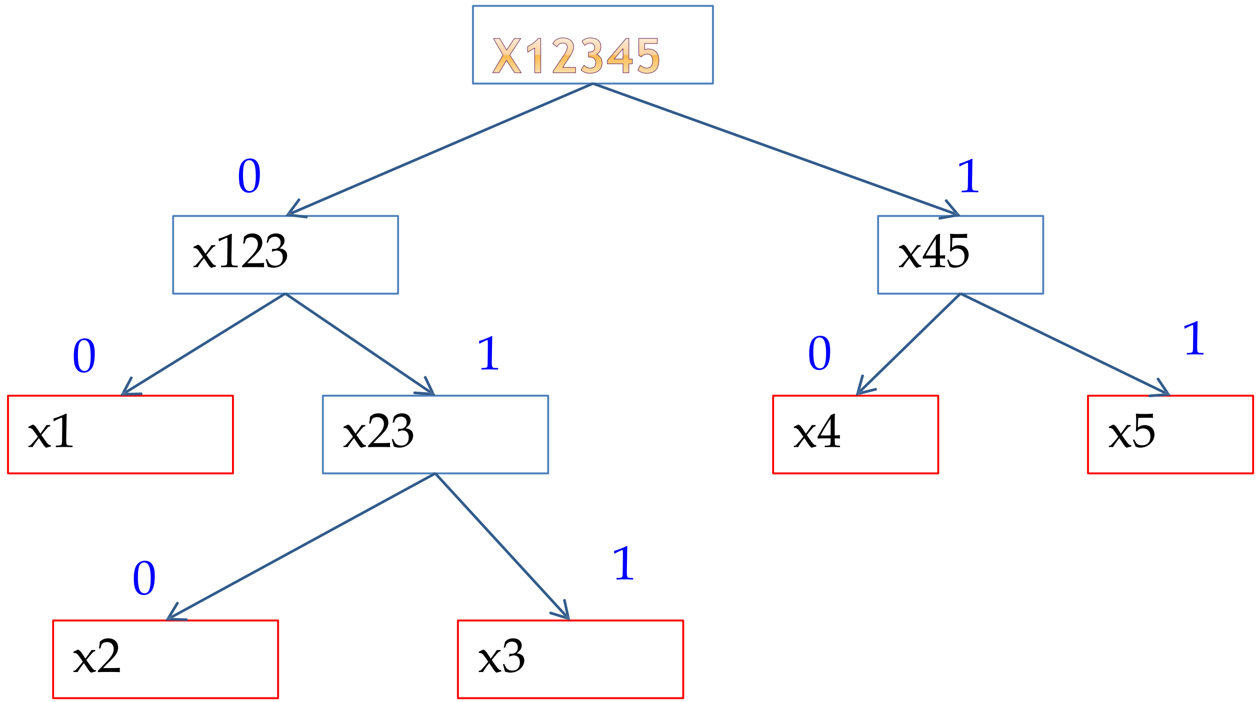
- The Huffman tree should be constructed as follows: 1)A root of the tree is a message from the last step with the probability 1; **2**) Its children are two messages that have composed the last message; **3**) The step 2 must be repeated until all leafs of the tree will be obtained. These leafs are the original messages.
- The siblings-nodes from the same level are given the numbers 0 (left) and 1 (right).
- The encoding vector for each message is obtained by passing a path from the root's child to the leave corresponding to this message and reading the numbers of nodes (root's child→intermidiates→leaf) that compose the encoding vector.

Huffman Encoding: Example

- Let us construct the Huffman code for the following set of messages: x1, x2, x3, x4, x5 with the probabilities p(x1)=…=p(x5)=0.2
- 1) x1 (p=0.2), x2 (p=0.2), x3 (p=0.2), x4 (p=0.2), x5 (p=0.2)

- 2) x4,x5→x45 (p=0.4)=> x45,x1,x2,x3
- 3) x2,x3→x23 (p=0.4)=>x45, x23, x1
- 4) x1,x23→x123(p=0.6)=> x123, x45
- 5) x123, 45→x12345 (p=1)

Huffman Encoding: Example



Encoding vectors: x1→(00); x2→(010); x3→(011); x4→(10); x5→(11)

Huffman Encoding: Example

- Entropy $H(X) = -5(0.2\log 0.2) = -5\left(\frac{1}{5}\log\frac{1}{5}\right) = -\log\frac{1}{5} = \log 5 \approx 2.32$
- Average length of the encoding vector

$$\bar{L} = 3\cdot\left(\frac{1}{5}\cdot 2\right) + 2\left(\frac{1}{5}\cdot 3\right) = \frac{12}{5} = 2.4$$

- The Huffman code gives (2.32/2.4)100% = 97% efficiency

Huffman Coding

ALGORITHM IS SHOWN BY ANOTHER EXAMPLE

Symbol	P_k(stage1)	Stage 2	Stage 3	stage4
S_0	0.4	0.4	0.4	0.6
S_1	0.2	0.2	0.4	0.4
S_2	0.2	0.2	0.2	
S_3	0.1	0.2		
S_4	0.1			

Huffman Coding (cont')

- The result is

Symbol	P_k	Code – word
S_0	0.4	0 0
S_1	0.2	1 0
S_2	0.2	1 1
S_3	0.1	0 1 0
S_4	0.1	0 1 1

⇨ Then, $\bar{L} = 2.2$

while, H(S) = 2.12193

- Huffman encoding is not unique.

1) 0 / 1 or 1 / 0 → trivial