

FACULTY OF ENGINEERING
DEPARTMENT OF CIVIL ENGINEERING

B.E Civil Engineering

2017-2018

Semester-III		
16BECE302	Solid Mechanics-I	2H-2C
Instruction Hours/week: L: 2 T: 0 P: 0	Marks: Internal:40	External:60
Total: 100		

End Semester Exam:3 Hours

Course Objective

- To impart to the students the concepts of stresses and strains and Hooke's law.
- To enlighten the students about different types of truss analysis.
- To teach the students about the beam analysis
- To teach about thin cylindrical and spherical shell analysis when subjected to internal pressure
- To impart ideas of torsional stresses and how to evaluate it in circular sections and its applications in spring analysis.

Course Outcome

On completion of the course, the students will be able to:

- The concepts of stresses, strains and Hooke's law.
- Different types of truss analysis, beam behavior and analysis.
- Thin cylindrical and spherical shell analysis when subjected to internal pressure.
- Ideas of torsional stresses and how to evaluate it in circular sections and its applications in spring analysis.

UNIT-I: SIMPLE STRESSES AND STRAINS: Hook's Law-Principle of superposition-Composite Sections-Temperature Stress-Hoop Stress-Elastic Constants-Principal Stresses and Strains-Mohr's Circle-Strain Energy and impact loading-Stresses due to gradual, sudden and impact loading-Proof resilience-Shear resilience. **9**

UNIT-II: GEOMETRICAL PROPERTIES OF SECTIONS: Centroid-Centre of mass-Centre of gravity-Moment of inertia-Area moment of inertia-Mass Moment of inertia-Rectangular moment of inertia-Polar moments of inertia-Radius of gyration of an area-Perpendicular axis theorem-Parallel axis theorem-Moment of inertia. **9**

UNIT-III: BENDING OF BEAMS: Types of beams and loads - Theory of simple bending – B.M.D. and S.F.D for Cantilever, Simply Supported and Overhanging beams subjected to various types of loading –UDL, Point Load, UVL- point of contraflexure- Calculation of shear stress and bending stress. **9**

UNIT-IV: DEFLECTION OF BEAMS: Slope and Deflection at a point- Estimation of slope and deflection for Cantilever, Simply Supported and Overhanging beams subjected to various types of loading (Only application of formulae) -Mohr's theorem-Strain energy method. **9**

UNIT-V: TORSION OF SHAFTS: Assumptions-horse power transmitted by a shaft-Strength of solid shaft,Hollow shafts,composite shafts & stepped shafts -Torsional strain energy. Spring-Leaf spring-Helical springs-Strain energy stored in a spring. **9**

SUPPORTING MATERIALS

TEXT BOOKS:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Strength of Materials	S.Ramamrutham	DhanpatRai Publishing Company, New Delhi	2012

REFERENCE BOOKS:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Strength of Materials and Theory of Structures Vol.I	Dr.B.C.Punima	Laxmi Publication, New Delhi	2013
2	Strength of Materials (Mechanics of Solids)	Rajput R.K	S.Chand& Company Ltd., New Delhi	2012
3	Strength of Materials (Mechanics of Solids)	Khurmi R.S.	S.Chand& Company Ltd., New Delhi	2012

STAFF INCHARGE

(Ms.S.M.Leela Bharathi)

HOD (Department of Civil Engineering

DEAN (FOE)

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Under section 3 of UGC act 1956)
COIMBATORE-641021
FACULTY OF ENGINEERING
DEPARTMENT OF CIVIL ENGINEERING

16BECE302/ SOLID MECHANICS-I
LECTURE PLAN

Number of credits: 3

Contact hours: 3 hours per week

Lecturer: Ms.S.M.Leela Bharathi

Semester: III– (2017-2018)

Course Type: Core

Lecture	Hours	Topics to be Covered	Text / Reference	Page No
1	1	Conc Hook's Law and Principle of superposition	T1,R2	112, 194
2	1	Composite Sections, Temperature Stress	T1,R1	114, 197
3	1	Hoop Stress, Elastic Constants	T1,R2	116, 199
4	1	Principal Stresses and Strains	T2,R2	191, 201
5	1	Mohr's Circle-Strain Energy between strains and elastic moduli	T1, R1	134, 211
6	1	Impact loading	T1,R2	139, 220
7	1	Stresses due to gradual, sudden and impact loading	T1,R2	131, 228
8	1	Proof resilience	T1,R1	144, 222
9	1	Shear resilience	T1,R1	148, 218
Total	9 Hrs			
10	1	Centroid, Centre of mass	T1,R1	154, 226
11	1	Centre of gravity, Moment of inertia	T1,R2	156, 228
12	1	Area moment of inertia	T1,R2	151, 224
13	1	Mass Moment of inertia	T1,R1	162, 212
14	1	Rectangular moment of inertia	T1,R1	168, 229
15	1	Polar moments of inertia	T1,R2	167, 234
16	1	Radius of gyration of an area	T1,R2	175, 237
17	1	Perpendicular axis theorem-Parallel axis theorem	T1,R2	178, 239
18	1	Moment of inertia	T1,R1	176, 246
Total	9 Hrs			

19	1	Types of beams and loads	T1,R1	192, 259
20	1	Theory of simple bending	T1,R1	198, 261
21	1	B.M.D. and S.F.D for Cantilever	T1,R1	197, 264
22	1	Simply Supported and Overhanging beams subjected to various types of loading	T2,R2	163, 267
23	1	UDL	T1,R1	232, 269
24	1	Point Load	T1,R1	225, 285
25	1	UVL	T1,R2	234, 289
26	1	point of contraflexure	T1,R2	244, 300
27	1	Calculation of shear stress and bending stress	T1,R1	256, 301
Total	9 Hrs			
28	1	Slope at a point	T1,R1	261, 317
29	1	Deflection at a point	T2,R1	260, 320
30	1	Estimation of slope for Cantilever	T2,R1	265, 325
31	1	Estimation of deflection for Cantilever	T1,R1	264, 333
32	1	Simply Supported subjected to various types of loading	T1,R2	266, 334
33	1	Overhanging beams subjected to various types of loading	T1,R1	275, 338
34	1	Moh's theorem method	T2,R2	265, 347
35	1	Mohr's theorem concepts	T1,R1	279, 349
36	1	Strain energy method	T1,R1	281, 359
Total	9 Hrs			
37	1	Assumptions on Torsion of shafts	T1,R1	286, 360
38	1	Horse power transmitted by a shaft	T1,R2	294, 361
39	1	Strength of solid shaft	T2,R1	299, 372
40	1	Hollow shafts	T1,R2	311, 373
41	1	Composite shafts	T1,R1	323, 376
42	1	Stepped shafts	T1,R1	326, 381
43	1	Torsional strain energy	T1,R2	332, 391
44	1	Spring, Leaf spring, Helical springs	T1 ,R1	345, 392
45	1	Strain energy stored in a spring	T1,R2	356, 395
Total	9 Hrs			

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I. SIMPLE STRESS, STRAIN AND THERMAL STRESSES:-

Technical Terms:-

1. Strength of material

The internal resistance developed (or) offered by any Material which is loaded, is known as strength of material. Generally the external load is applied upto "limit of proportionality" where Hooke's law is valid (or) applicable. The Analysis of loaded Material is based on "Elastic Zone" only.

2. Engineering Stress :- (or) Conventional Stress:-

It is defined as The internal resistance offered by any material per unit of its original Cross-Sectional area. Its symbol is " σ " or " f ".

$$\sigma = f = \frac{P}{A_0}$$

Its unit is " N/m^2 " in S.I system.

kgf/m^2 in MKS system

$Dyne/cm^2$ in CGS

$$1 \text{ kgf} = 1 \text{ ms} \times 1 \text{ m} / \text{Sec}^2$$

$$= 9.81 \text{ kg} \cdot \text{m} / \text{Sec}^2$$

$$1 \text{ kgf} = 9.81 \text{ N}$$

In S.I system

$$1 \text{ kg} = 9.81 \text{ N}$$

$$1 \text{ kN} = 100 \text{ kg}$$

$$1 \text{ N/m}^2 = \therefore \text{pascal} = 1 \text{ Pa.}$$

$$1 \text{ Pascal} = 1 \text{ N/m}^2$$

$$= \frac{1 \text{ N}}{(1000 \text{ mm})^2}$$

$$10^6 \text{ Pa} = 1 \text{ N/mm}^2$$

$$1 \text{ MPa} = 1 \text{ N/mm}^2$$

Dimension of stress:-

$$\sigma = f = \frac{P_{\text{force}}}{A} = \frac{\text{mass} \times \text{acceleration}}{\text{Area}}$$

$$= \frac{M \times L T^{-2}}{L^2}$$

$$\text{SI unit of stress} = M L^{-1} T^{-2}$$

3. True stress:-

It is defined as the internal resistance (force) developed by any loaded material per unit of its instantaneous cross sectional area

$$\sigma_{\text{true}} = \frac{P}{A} \text{ - instantaneous area}$$

$$\sigma_{\text{true}} = \frac{P}{A} \times \frac{A_0}{A_0}$$

$$\sigma_{true} = \left(\frac{P}{A_0} \right) \times \left(\frac{A_0}{A} \right)$$

$$\sigma_{true} = \sigma_{Engg.} \times \left(\frac{A_0}{A} \right)$$

Volume of material before loading } = Volume of material after loading

$$\boxed{V_0 = V}$$

$$A_0 \times l_0 = A \times l$$

$$\frac{A_0}{A} = \frac{l}{l_0} = \frac{l_0 \pm \Delta l}{l_0}$$

$$= \frac{l_0}{l_0} \pm \frac{\Delta l}{l_0}$$

$$\left(A_c = \frac{A_0}{1 \pm \epsilon} \right)$$

$$\checkmark \quad \boxed{\frac{A_0}{A} = 1 \pm \epsilon}$$

$$\checkmark \quad \boxed{\sigma_{true} = \sigma_{Engg.} \times (1 \pm \epsilon)}$$

linear strain
 → Tensile force
 → compressive force.

4. Conventional Strain (or) Engg. strain!

It is defined as the change in length per unit of original length of a loaded material. Its Symbol ϵ (ϵ or e)

$$\epsilon = e = \frac{\int_{l_0}^l \delta l}{l_0} = \int_{l_0}^l \frac{dl}{l_0}$$

$$= \frac{1}{l_0} [l]_{l_0}^l$$

$$= \frac{1}{l_0} \times [l - l_0]$$

$$\boxed{e = \epsilon = \frac{\Delta l}{l_0}}$$

$$\text{Unit} \rightarrow \text{mm/mm} \rightarrow \text{M}^0 \text{L}^0 \text{T}^0$$

\rightarrow No dimension.

5. Natural strain :- (ϵ_{nat})

It is defined as the change in length per unit instantaneous length of a loaded material.

$$\epsilon_{nat} = \frac{\delta l}{l} = \frac{dl}{l}$$

$$= \int_{l_0}^l \frac{dl}{l}$$

$$= [\log_e l]_{l_0}^l$$

$$= \log_e l - \log_e l_0$$

$$= \log_e \left[\frac{l}{l_0} \right]$$

$$= \log_e \left[\frac{l_0 \pm \Delta l}{l_0} \right]$$

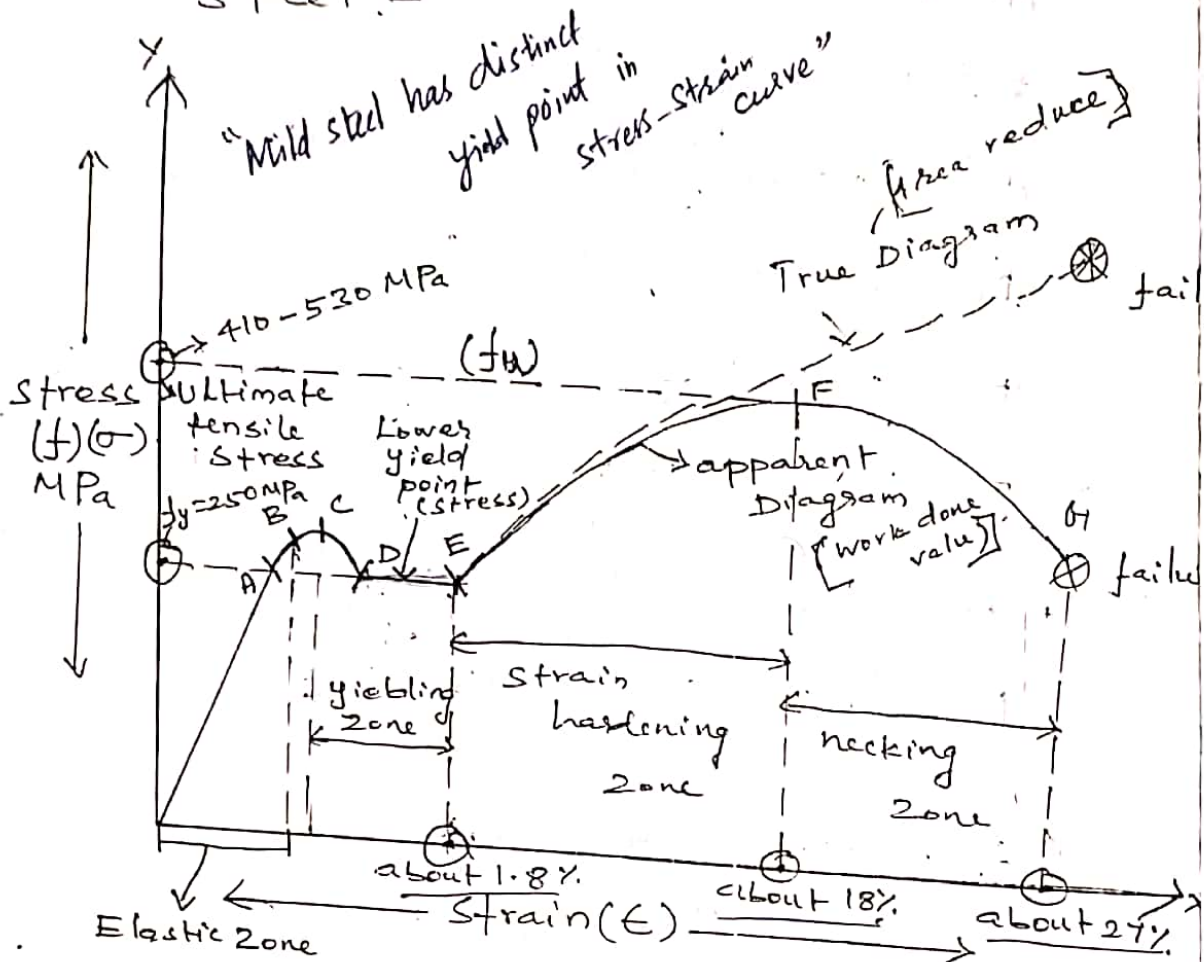
$$= \log_e \left[\frac{l_0}{l_0} \pm \frac{\Delta l}{l_0} \right]$$

$$\boxed{\epsilon_{nat} = \log_e [1 \pm e]}$$

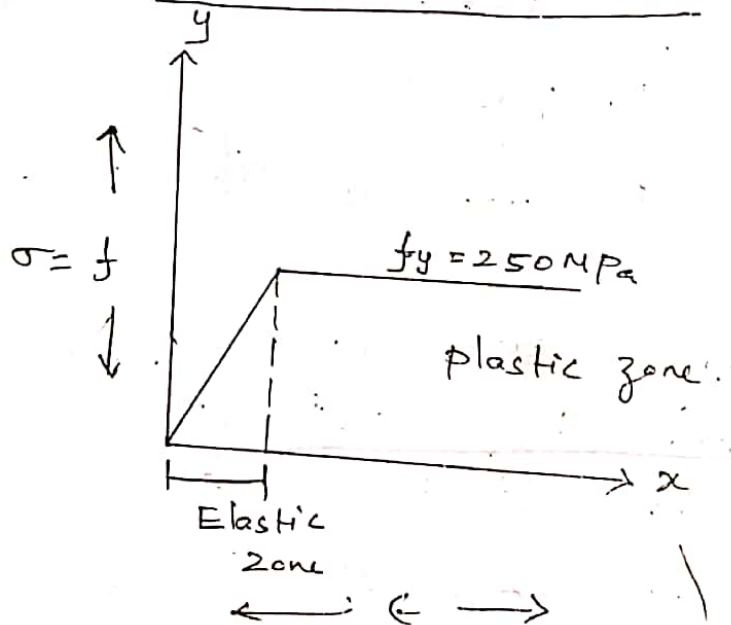
$$= \ln [1 \pm e]$$

Conventional strain

6. Stress - Strain curve of Mild steel:-



IDEALIZED DIAGRAM



A - Limit of proportionality.

B - Elastic limit

C - Upper yield point

D - Lower yield point

E - End of lower yield point

D E - yield stress $[f_y = 250 \text{ MPa} = \sigma_y]$

EF - Strain Hardening.

F - Ultimate stress point
(Tensile stress)

$[f_u = 410 \text{ to } 530 \text{ MPa}]$

(Tensile stress)

(Ultimate tensile stress)

FG - Necking Zone

G - Failure of mild steel.

Note! - * *

Any material which do not have

① a distinct yield point (D-E) then the yield stress is taken as "Residual stress" equal to 0.2% after removal of the load, also known as 0.2% proof stress.

② Generally it happens in case of Aluminium, Cast Iron and Higher strength steel.

7. Elasticity :-

It is defined as the property of a material due to which the material regains its original shape after removal of external load.

** 8. Poisson's Ratio :-

It is defined as the ratio of lateral strain to longitudinal (or) linear strain. its symbol is μ (or) $\frac{1}{m}$ (or) ν

$$\mu = \frac{1}{m} = \nu = \frac{\epsilon_{\text{lateral}}}{\epsilon_{\text{linear}}} = \frac{\frac{\delta b}{b}}{\frac{\delta l}{l}}$$

Unit - no dimension.

$\mu = 0.3 \rightarrow$ Mild steel in Elastic range

$\mu = 0.5 \rightarrow$ Mild steel in plastic range.

$\mu = 0.33 \rightarrow$ for Aluminium

$\mu = 0.2 \rightarrow$ Concrete

** 9. Hooke's Law :-

According to Hooke's law the linear strain developed in a material which is loaded upto "Limit of proportionality" is directly proportional to stress.

$$\epsilon \propto f \quad (\text{or}) \quad \epsilon \propto \sigma$$
$$\left| \epsilon = \frac{1}{k} \times f \right| \quad \epsilon = \frac{1}{k} \times \sigma$$

$$\epsilon = \frac{\Delta L}{L} \times f$$

$$\sigma = \boxed{f = \epsilon \times E}$$

$$\frac{P}{A_0} = \frac{\Delta L}{L} \times E$$

Change
in length
where,

$$\boxed{\Delta L = \frac{PL}{A_0 E}}$$

~~E~~ $E \rightarrow$ Modulus of Elasticity.

\rightarrow Young's Modulus of Elasticity

$\rightarrow 2 \times 10^5 \text{ N/mm}^2$ for Mild steel
(MPa)

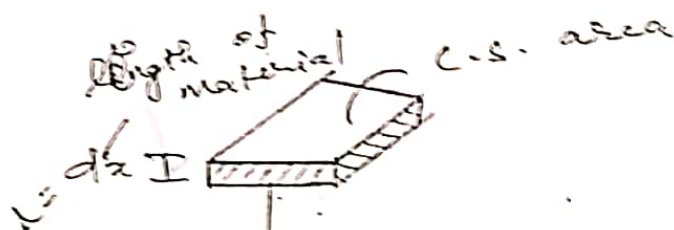
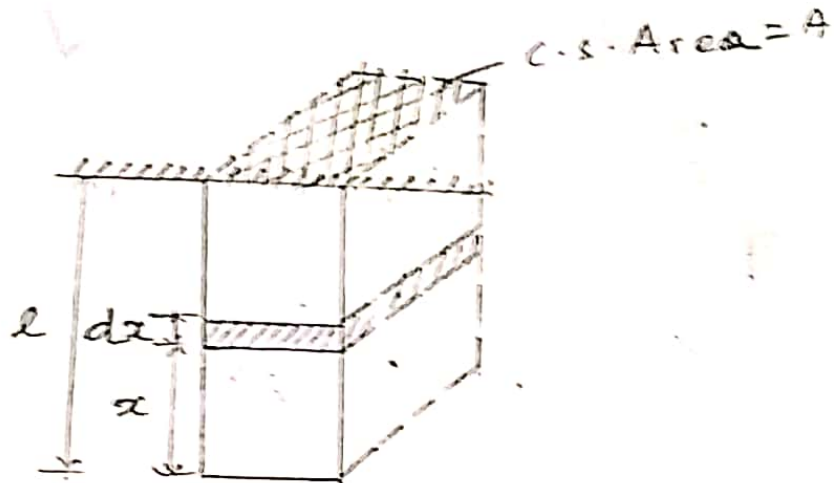
$AE \rightarrow$ Axial Rigidity.

\rightarrow Newton (SI)

$\rightarrow \text{MLT}^{-2}$ (Dimension)

==

① Derive an expression for Elongation due to Self weight:-



P = Self weight of hanging portion
 $= \rho \times \text{volume} = \rho \times A \times x$

For Elemental strip:-

$$\delta l_x = \frac{P \times dx}{A \times E}$$

δl_x length of elemental strip.

$$\delta l = \frac{(\rho \times A \times x) \times dx}{A \times E}$$

$$\delta l = \int_0^l \frac{\rho \times A}{A \times E} \times (x \times dx)$$

$$= \frac{\rho \times A}{A \times E} \left[\frac{x^2}{2} \right]_0^l$$

$$= \frac{\rho \times l}{A \times E} \times \left[\frac{l^2}{2} \right]$$

$$= \frac{[w^2 \times A \times l] \times l}{2AE}$$

$$\boxed{\delta l = \frac{w_{total} \times l}{2AE}} \checkmark$$

(or)

W - Load varies from 0 to w

$$\delta l = \frac{Pl}{AE}$$

take Avg Load

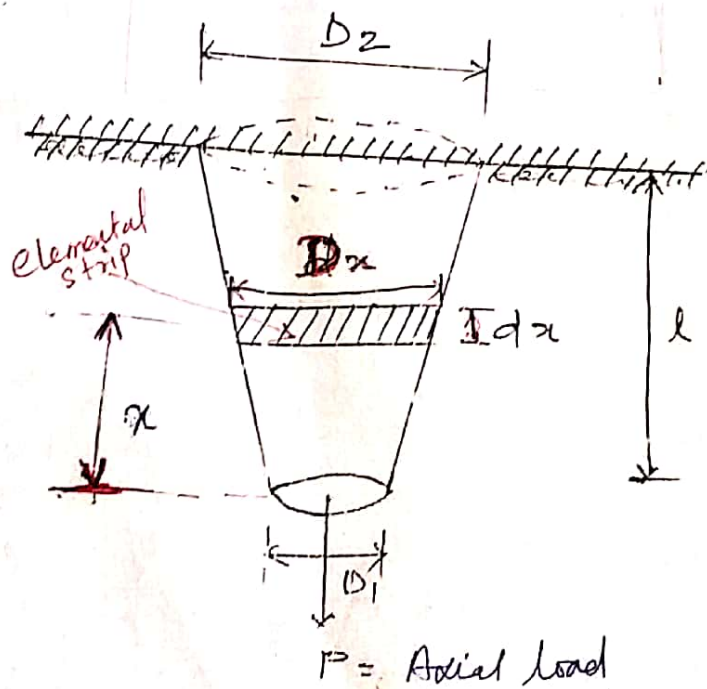
$$= \frac{(0+w) \times l}{2AE}$$

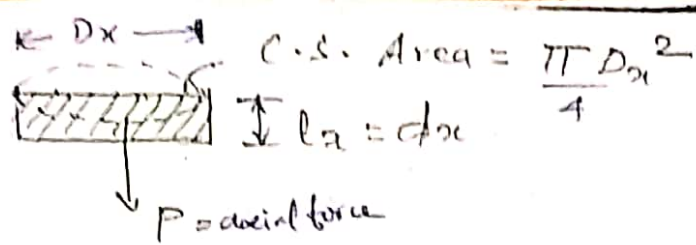
$$\boxed{\delta l = \frac{wl}{2AE}} \checkmark$$

* *

2. Expression for Elongation of a Circular Tapering Section Subjected to an axial load P .

Derivation





Dia of the elemental strip $\} D_x = D_1 + \left[\frac{D_2 - D_1}{l} \right] \times x$

Dia of the elemental strip $\} D_x = D_1 + kx \quad \text{Take } \left\{ k = \frac{D_2 - D_1}{l} \right\}$

For Elemental strip:-

$$\delta l = \frac{Pl}{AE}$$

$$\delta l_x = \frac{P_x dx}{A_x E_x}$$

$$\delta l_x = \frac{P_x dx}{\frac{\pi D_x^2}{4} E_x}$$

$$\delta l = \frac{P dx}{\frac{\pi}{4} (D_1 + kx)^2 E}$$

$$\begin{aligned} \delta l &= \int_0^l \frac{P dx}{\frac{\pi}{4} (D_1 + kx)^2 E} \\ &= \frac{4P}{\pi E} \int_0^l \frac{dx}{(D_1 + kx)^2} \\ &= \frac{4P}{\pi E} \int_0^l (D_1 + kx)^{-2} dx \\ &= \frac{4P}{\pi E} \left[\frac{(D_1 + kx)^{-1}}{(-1) \times (k)} \right]_0^l \\ &= -\frac{4P}{\pi E} \left[\frac{1}{k(D_1 + kx)} \right]_0^l \end{aligned}$$

$$= -\frac{4P}{k\pi E} \left[\frac{1}{D_1 + kl} - \frac{1}{D_1} \right]$$

$$= -\frac{4P}{k\pi E} \left[\frac{\cancel{D_1} - \cancel{D_1} - kl}{D_1 \times (D_1 + kl)} \right]$$

$$= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1 \times D_1 + D_1 kl}$$

$$= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1^2 + D_1 kl}$$

$$= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1^2 + D_1 \left[\cancel{D_1} \left(\frac{D_2 - \cancel{D_1}}{l} \right) \right]}$$

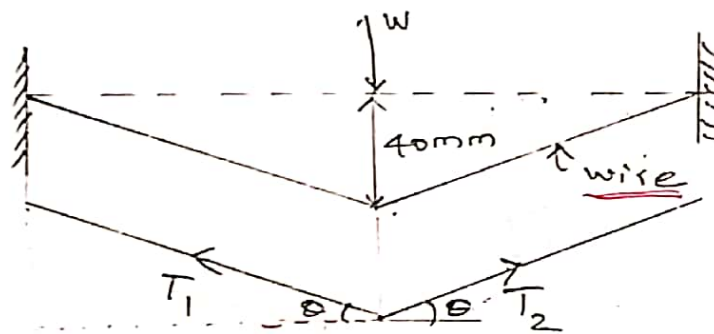
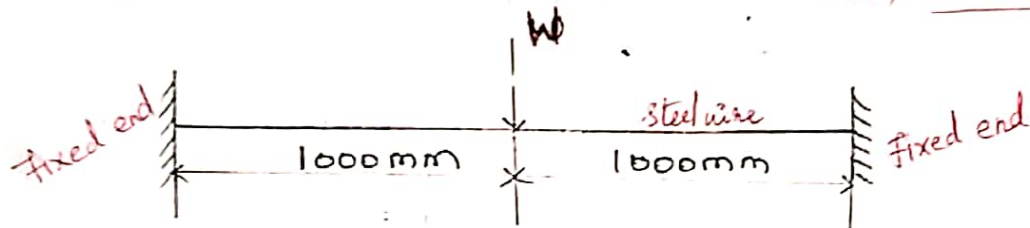
$$= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1^2 + D_1 D_2}$$

$$= \frac{4Pl}{\pi E} \cdot \frac{1}{D_1 (D_1 + D_2)}$$

$$= \frac{4Pl}{\pi E \times D_1} \cdot \frac{1}{\left[\cancel{D_1} + \frac{D_2 - \cancel{D_1}}{l} \times l \right]}$$

$$\delta l = \frac{4Pl}{\pi E D_1 D_2}$$

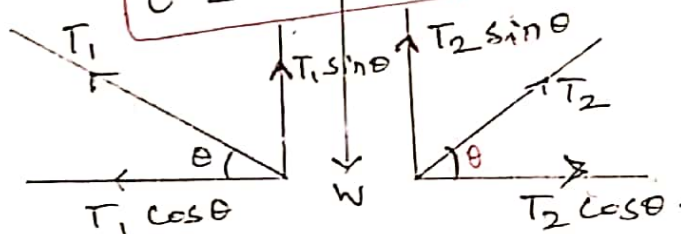
3. A steel wire 1mm in diameter & 2m long and fixed at the ends. If it is subjected to a central point load (W) such that the central deflection is 40mm. Determine the magnitude of central point load (W) and the stress developed in the wire.
 Take, $E = 2 \times 10^5 \text{ N/mm}^2$



$$\tan \theta = \frac{\text{Opp}}{\text{Adj}}$$

$$\tan \theta = \frac{40}{1000}$$

$$\theta = 2.291^\circ$$



Law of equilibrium equation:-

$$\sum H = 0 \quad T_1 \cos \theta = T_2 \cos \theta$$

$$T_1 = T_2 \quad \text{--- (1)}$$

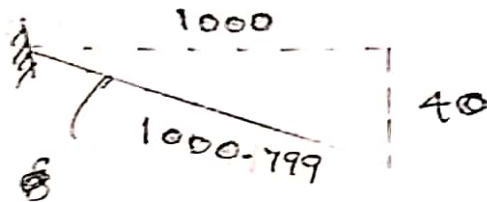
$$\sum V = 0$$

$$T_1 \sin \theta + T_2 \sin \theta = W$$

$$2T_1 = W / \sin \theta$$

$$T_1 = \frac{W}{2 \sin 2.291} \rightarrow (2)$$

According to Hooke's law



$$\begin{aligned} \Delta l &= 1000.799 - 1000 \\ &= 0.799 \text{ mm.} \end{aligned}$$

$$\frac{\sigma}{\epsilon} = E$$

$$\sigma = \epsilon E$$

$$\sigma = \frac{\Delta l}{l} \times E$$

$$\frac{F}{A} = \epsilon E$$

$$= \frac{\Delta l}{l} \times E$$

$$= \frac{0.799}{1000} \times E$$

$$= 7.99 \times 10^{-4} \times 2 \times 10^5$$

$$\sigma = \boxed{\frac{F}{A} = 159.8 \text{ N/mm}^2}$$

$$\frac{P}{A} = 159.8$$

$$\frac{P}{\pi \times (1)^2} = 159.8$$

$$\frac{P}{4}$$

$$T_1 = T_2 = \boxed{P = 125.51 \text{ N}} \text{ — Tensile force}$$

$$125.51 = \frac{W}{2 \sin 2.291}$$

$$\boxed{W = 10.03 \text{ N}}$$

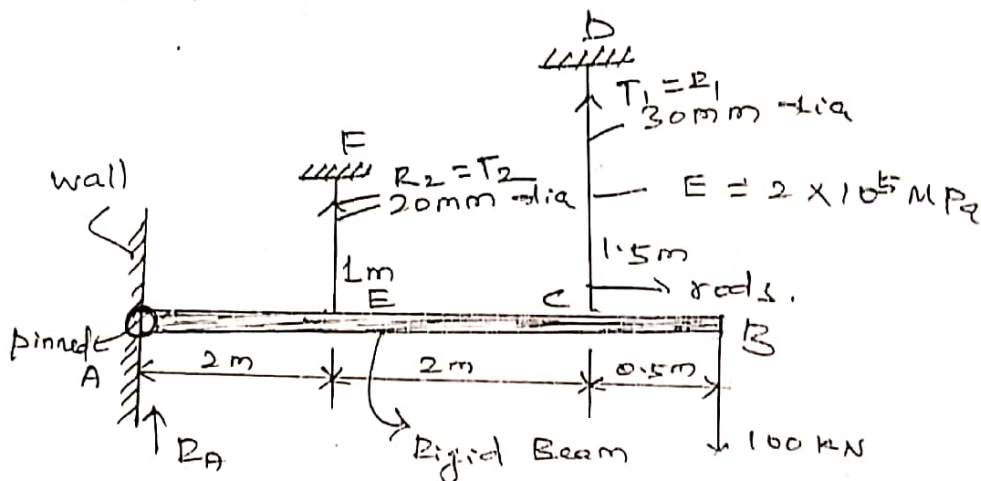
~~$$T_1 = \frac{125.51}{2 \sin 2.91}$$~~

~~$$T_1 = 15.86 \text{ N} \neq T_2$$~~

$$125.51 = \frac{W}{2 \sin 2.91}$$

$$W = 10.03 \text{ N}$$

4. A rigid beam is shown in figure below. Determine stresses developed in the rods supporting the rigid beam.



Law of moment;

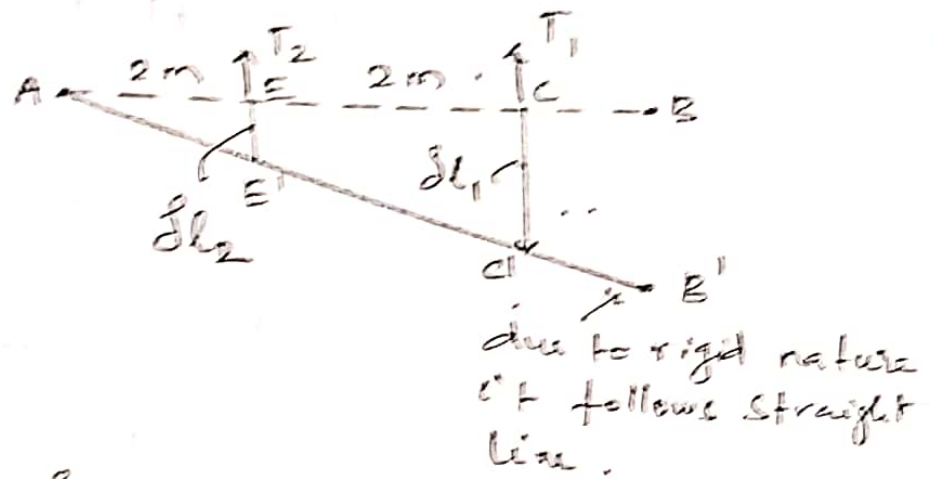
$$\sum M_A = 0 \text{ (B'cos of pinned support)}$$

$$R_1 \times 4 + R_2 \times 2 = 100 \times 4.5$$

$$4R_1 + 2R_2 = 450 \quad \text{--- (1)}$$

$$\sum V = 0$$

$$R_A + R_1 + R_2 = 100 \quad \text{--- (2)}$$



$$\frac{\delta l_1}{\delta l_2} = \frac{4}{2} = 2$$

$$\frac{\delta l_1}{\delta l_2} = 2$$

$$f_1 = E \times e_1 = E \times \frac{\delta l_1}{L_1}$$

$$f_2 = E \times e_2 = E \times \frac{\delta l_2}{L_2}$$

$$\delta l_1 = 2 \delta l_2$$

$$\frac{R_1 L_1}{A_1 E_1} = \frac{2 R_2 L_2}{A_2 E_2}$$

$$\frac{R_1 (1.5)}{\frac{\pi}{4} \times 30^2} = \frac{2 R_2 (1)}{\frac{\pi}{4} \times 20^2}$$

$$1.5 R_1 \times 20^2 = 2 R_2 \times 30^2$$

$$600 R_1 = 1800 R_2$$

$$R_1 = 3 R_2$$

$$4R_1 + 2R_2 = 450$$

$$4(3R_2) + 2R_2 = 450$$

$$14R_2 = 450$$

$$R_2 = 32.14 \text{ kN}$$

$$R_1 = 96.43 \text{ kN}$$

$$R_A = -28.57 \text{ kN}$$

$$\text{Stress developed in base} = 28.57 (\downarrow) \text{ kN.}$$

$$f_1 = \frac{R_1}{A}$$

$$= \frac{96.43}{\frac{\pi}{4} \times 30^2}$$

$$= 0.1364 \text{ kN/mm}^2$$

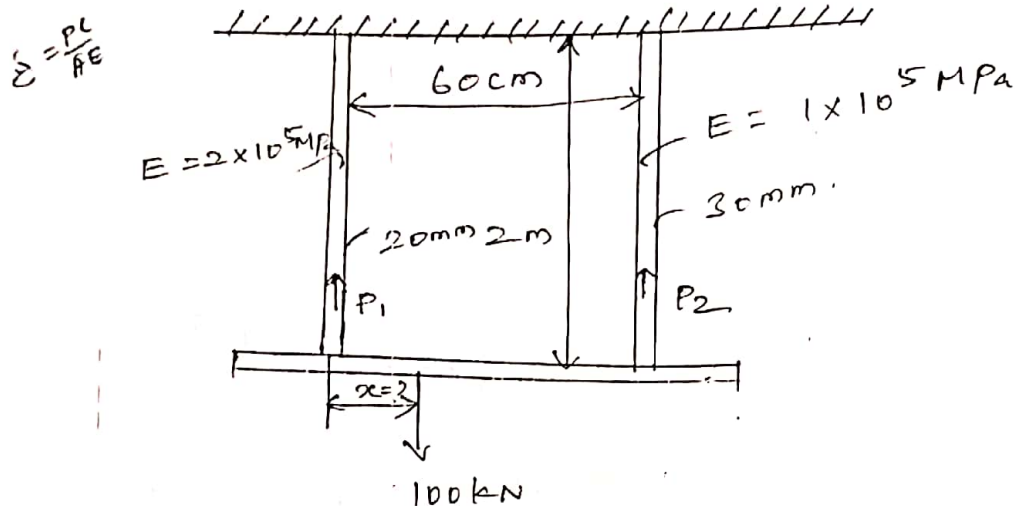
$$= 136.4 \text{ N/mm}^2$$

$$f_2 = \frac{R_2}{A}$$

$$= \frac{32.14}{\frac{\pi}{4} \times 20^2}$$

$$= 102.30 \text{ N/mm}^2$$

5. Two metallic bars are used to support a load as shown in figure below. Determine the position of load such that the bottom supporting member remains horizontal. Also find the stress in ~~bottom~~ bars.



$$\sum V = 0,$$

$$P_1 + P_2 = 100 \quad \text{--- (1) } \checkmark$$

Due to horizontal condition of the bottom support.

$$\delta l_1 = \delta l_2 \quad \checkmark$$

$$\frac{P_1 \cdot l_1}{A_1 E_1} = \frac{P_2 \cdot l_2}{A_2 E_2}$$

$$\frac{P_1 \times 2000}{\frac{\pi}{4} (20)^2 \times 2 \times 10^5} = \frac{P_2 \times 2000}{\frac{\pi}{4} (30)^2 \times 1 \times 10^5}$$

$$900 P_1 = 800 P_2$$

$$P_1 = \frac{8}{9} P_2 \quad \checkmark$$

$$8/9 P_2 + P_2 = 100$$

$$P_2 = 52.94 \text{ kN}$$

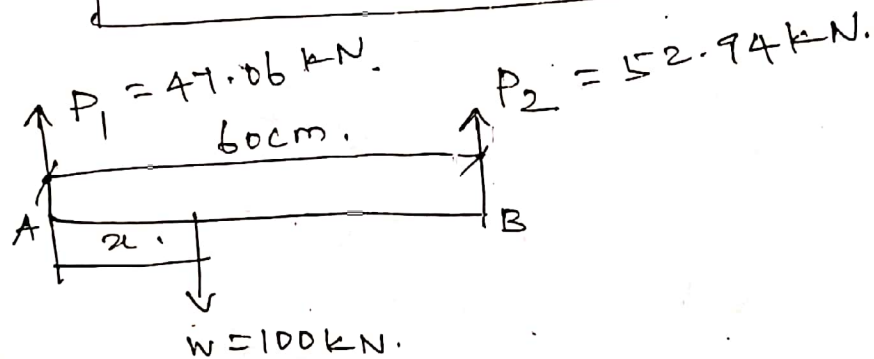
$$P_1 = 47.06 \text{ kN}$$

$$f_1 = \frac{P_1}{A_1} = \frac{47.06}{\frac{\pi}{4} \times 20^2}$$

$$f_1 = 149.79 \text{ N/mm}^2$$

$$f_2 = \frac{P_2}{A_2} = \frac{52.94}{\frac{\pi}{4} \times 30^2}$$

$$f_2 = 74.89 \text{ N/mm}^2$$



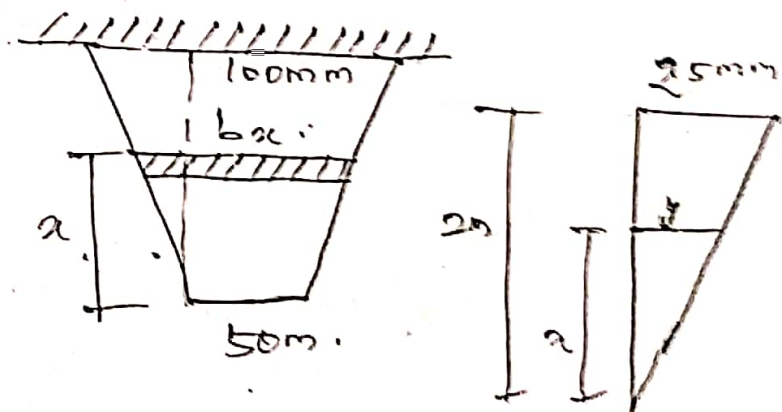
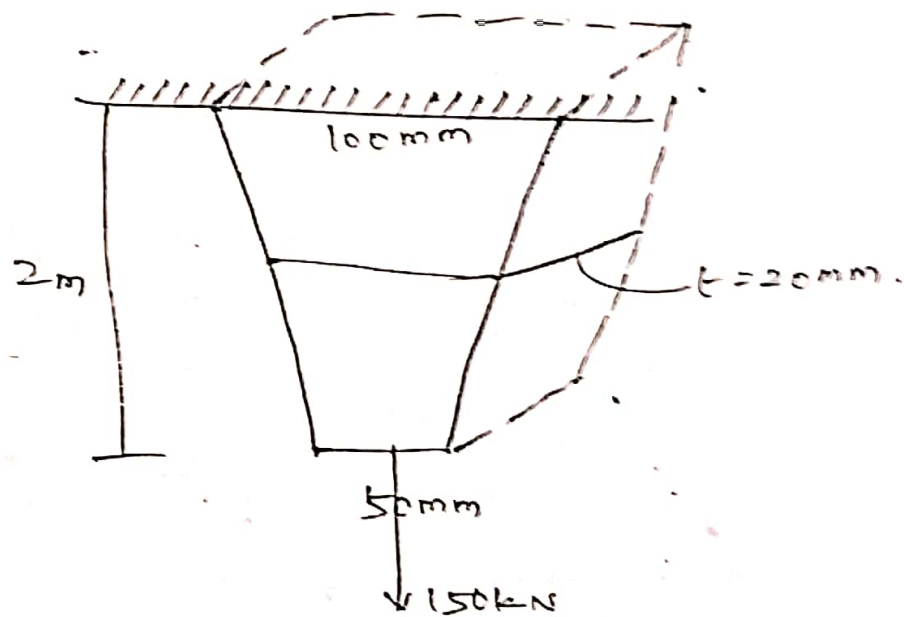
Law of moment,

$$\sum M_A = 0.$$

$$52.94 \times 60 = 100x.$$

$$x = 31.76 \text{ cm}$$

6. A metallic bar having length 2m is subjected to an axial tensile load of 150kN. The width of the bar varies (taper) from 100mm at the top to 50mm at the bottom. The thickness of the bar is 20mm (constant). Determine the elongation of the bar $E = 2 \times 10^5 \text{ MPa}$.



$$b_x = (50 + 75x)$$

$$b_x = 50 + \frac{50}{2000} x$$

$$= 50 + \frac{1}{40} x$$

$$\frac{x}{y} = \frac{2}{25}$$

$$y = \frac{25x}{2000}$$

$$\delta l = \int_0^L \frac{P}{AE} dx$$

$$\delta l = \int_0^{2000} \frac{15000}{\left(50 + \frac{1}{40}x\right) \times 20 \times 2 \times 10^5} dx$$

$$= \int_0^{2000} \frac{150000}{40 \times 10^5} \left[50 + \frac{1}{40}x\right]^{-1} dx$$

$$= \frac{150000}{40 \times 10^5} \left[\frac{\left(50 + \frac{1}{40}x\right)^0}{\left(\frac{1}{40}\right)(-1+1)} \right]_0^{2000}$$

$$= \frac{150000}{40 \times 10^5 \times 1} \left[\log\left(50 + \frac{1}{40}x\right) \right]_0^{2000}$$

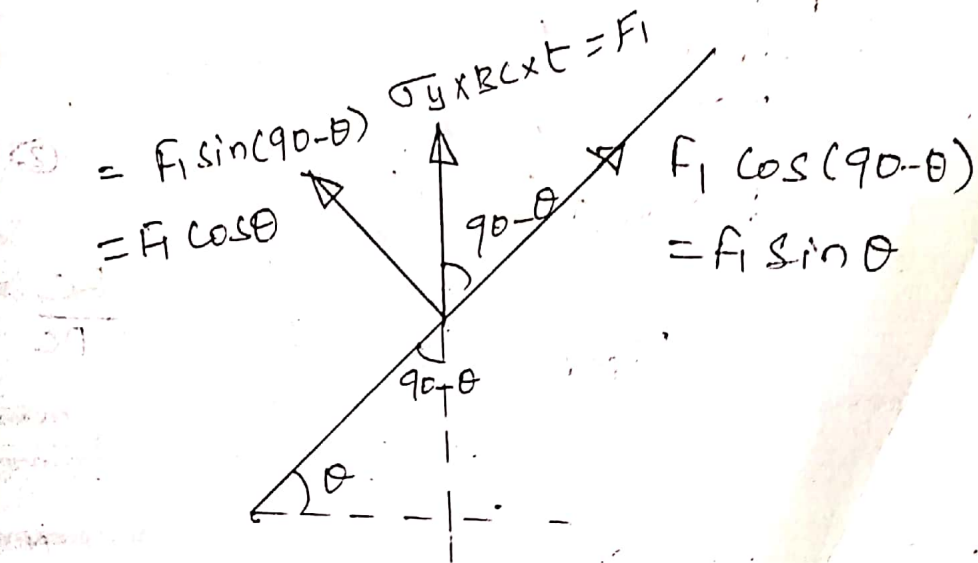
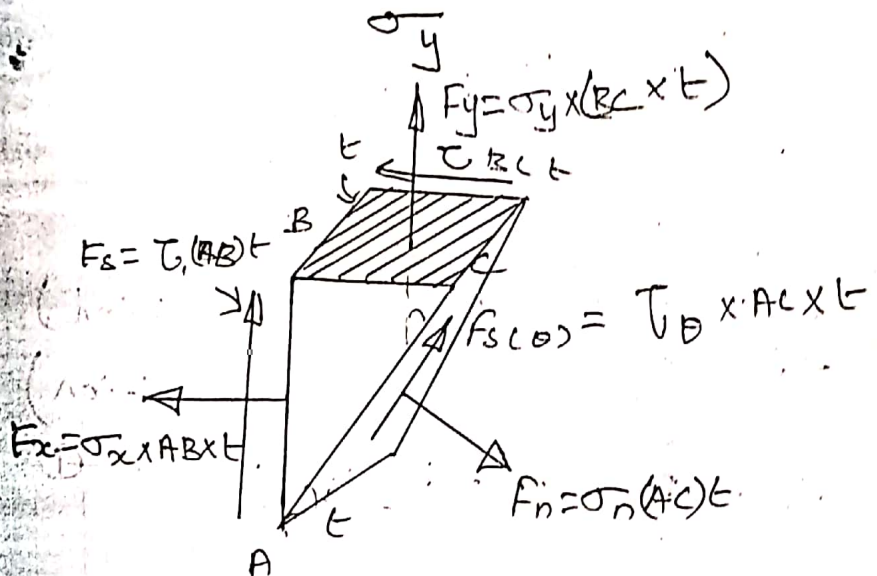
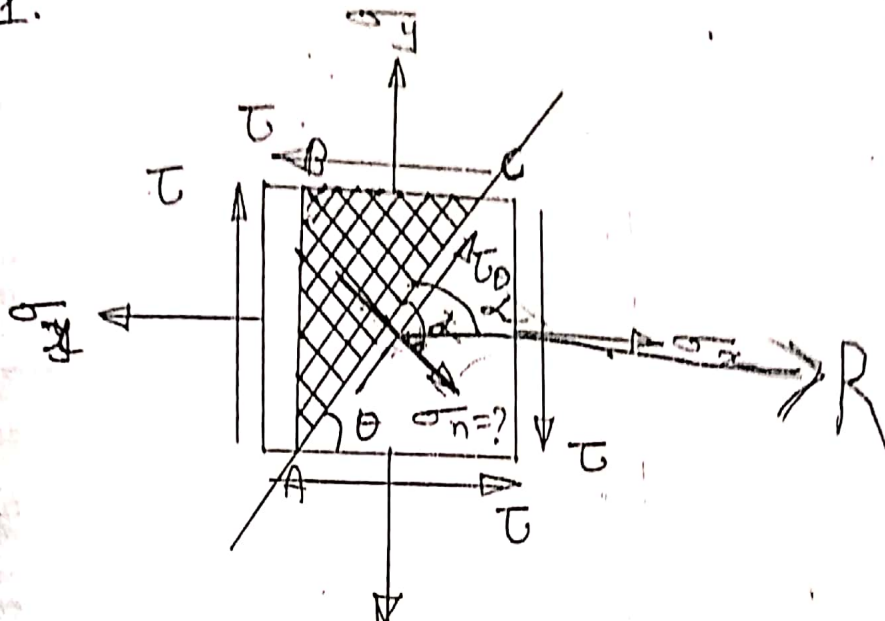
$$= 1.5 [4.605 - 3.912]$$

$$\boxed{\delta l = 1.0395 \text{ mm}}$$

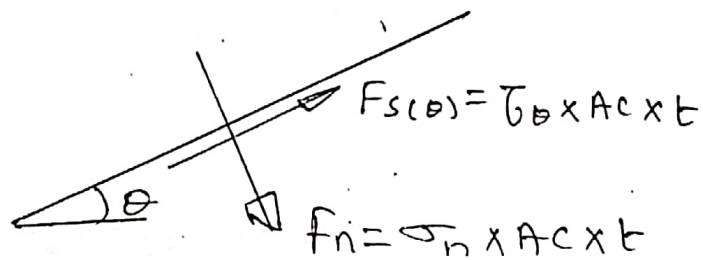
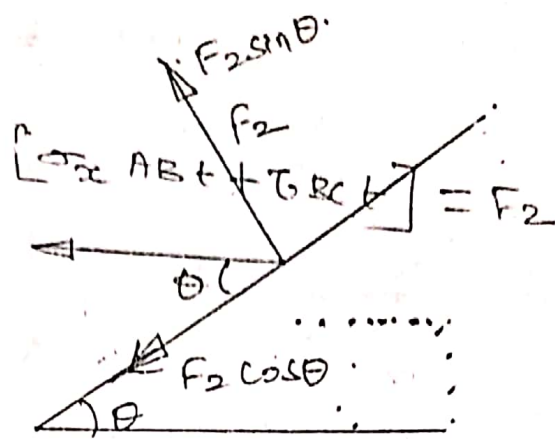
19/08/10

II. COMPLEX STRESS

1.



$$\sigma_y \times BC \times t + \tau \times AB \times t = F_1$$



$$\sum F_{\text{along the plane}} = 0 \quad (\text{For No vibration})$$

(or)
(stable condition)

$$F_1 \sin \theta - F_2 \cos \theta + F_s(\theta) = 0 \quad \text{--- ①}$$

$$\sum F_{\perp \text{ to plane}} = 0$$

$$F_1 \cos \theta + F_2 \sin \theta - F_n = 0 \quad \text{--- ②}$$

$$\sin \theta = \frac{AB}{AC}, \quad \cos \theta = \frac{BC}{AC}$$

$$\sigma_y BC t \sin \theta + \tau_{AB} t \sin \theta + \sigma_x AB t \cos \theta - \tau_{BC} t \cos \theta + \tau_{\theta AC} t = 0.$$

$$\begin{aligned} \tau(\theta) &= \sigma_x \left(\frac{AB}{AC} \right) \cos \theta - \sigma_y \left(\frac{BC}{AC} \right) \sin \theta \\ &\quad + \tau \left(\frac{BC}{AC} \right) \cos \theta - \tau \left(\frac{AB}{AC} \right) \sin \theta \\ &= \sigma_x \sin \theta \cos \theta - \sigma_y \sin \theta \cos \theta \\ &\quad + \tau \cos^2 \theta - \tau \sin^2 \theta. \end{aligned}$$

$$** = (\sigma_x - \sigma_y) \sin \theta \cos \theta + \tau (\cos^2 \theta - \sin^2 \theta)$$

$$\tau(\theta) = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$$

$$F_n = F_1 \cos \theta + F_2 \sin \theta. \quad \text{← Tangential stress}$$

$$\begin{aligned} \sigma_n \times AC \times t &= \sigma_y BC t \cos \theta + \tau_{AB} t \cos \theta \\ &\quad + \sigma_x AB t \sin \theta + \tau_{BC} t \sin \theta \end{aligned}$$

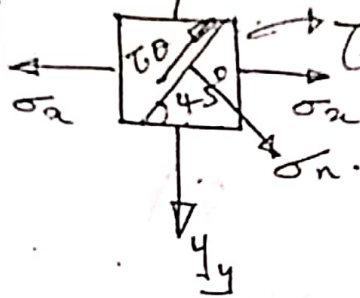
$$\begin{aligned} \sigma_n &= \sigma_y \cos^2 \theta + \tau \cos \theta \sin \theta \\ &\quad + \sigma_x \sin^2 \theta + \tau \cos \theta \sin \theta. \end{aligned}$$

$$** \quad \sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + \tau \sin 2\theta$$

$$\sigma_R = \sqrt{\sigma_n^2 + \tau_\theta^2}$$

$$\alpha = \tan^{-1} \left(\frac{\sigma_n}{\tau_\theta} \right)$$

For Maximum Value of τ_θ (Tangential stress)

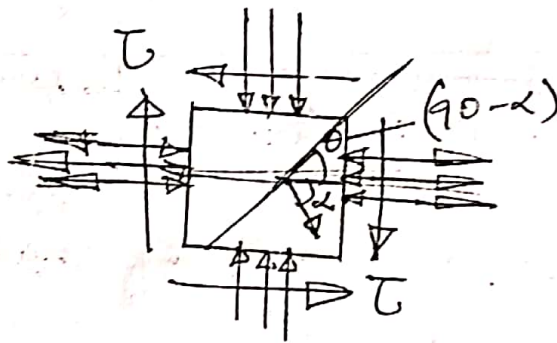
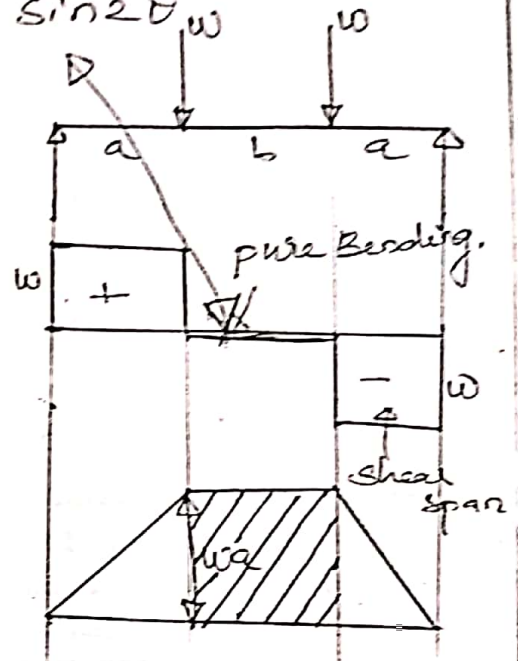


$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

$$\sin 2\theta = 1$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

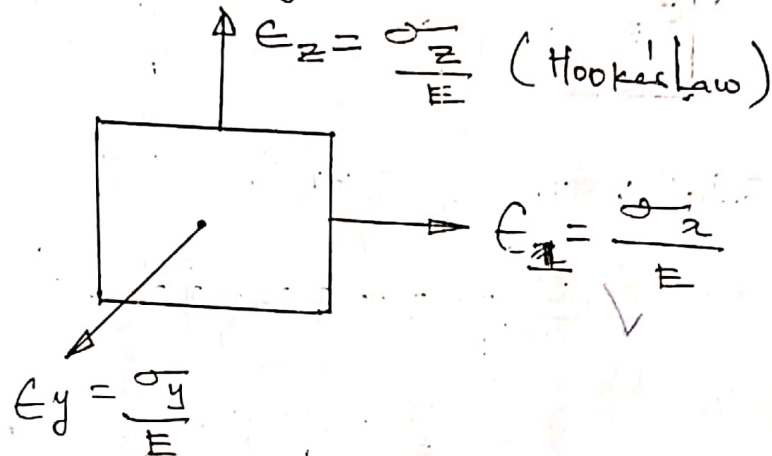
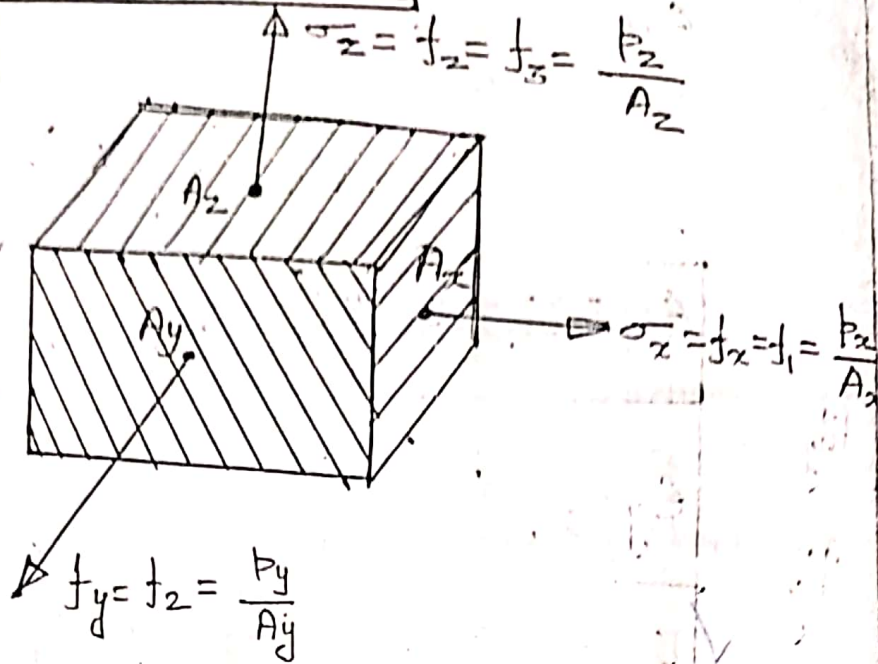


$$\sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + \tau \sin 2\theta$$

$$\sigma_n = \sigma_x \sin^2 \theta - \sigma_y \cos^2 \theta + \tau \sin 2\theta$$

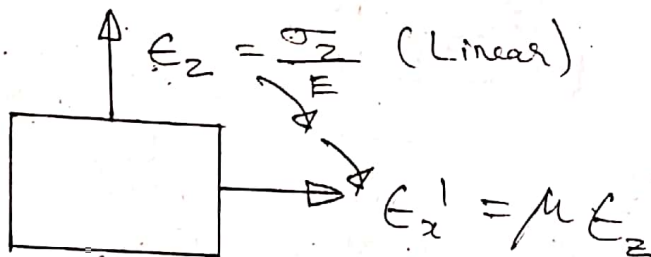
$$\tau_\theta = \frac{\sigma_x + \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$$

COMPLEX STRAIN

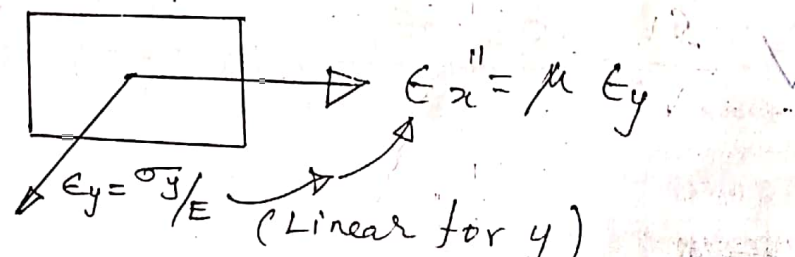


Poisson's Ratio,

$$\mu = \frac{\epsilon_{\text{lateral}}}{\epsilon_{\text{linear}}}$$



For z , x & y called Lateral.



Final strain in 'x' direction,

$$\epsilon_x = \epsilon_1 + \epsilon_1' + \epsilon_1''$$

$$= \frac{\sigma_x}{E} + \left(-\mu \frac{\sigma_z}{E}\right) + \left(-\mu \frac{\sigma_y}{E}\right)$$

$\epsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_z}{E} - \mu \frac{\sigma_y}{E} \quad \text{--- (1)}$
$\epsilon_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} \quad \text{--- (2)}$
$\epsilon_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} \quad \text{--- (3)}$

Total volumetric strain,

$$\sqrt{\epsilon_v} = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\sigma_x}{E} - \frac{\sigma_y}{E} \mu - \frac{\sigma_z}{E} \mu + \frac{\sigma_y}{E} - \frac{\sigma_x}{E} \mu - \frac{\sigma_z}{E} \mu + \frac{\sigma_x}{E} - \frac{\sigma_y}{E} \mu - \frac{\sigma_z}{E} \mu$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\mu}{E} (\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{(\sigma_x + \sigma_y + \sigma_z)}{E} [1 - 2\mu]$$

$$\sqrt{\frac{\Delta V}{V}} = \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu)$$

$$\delta v = \frac{v \times (\sigma_x + \sigma_y + \sigma_z)}{E} (1 - 2\mu)$$

Assumption

$$\forall \epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu)$$

if $\sigma_x = \sigma_y = \sigma_z = \sigma = p$

$$\epsilon_v = \frac{3p}{E} (1 - 2\mu)$$

Bulk modulus

$$E = 3 \left(\frac{p}{\epsilon_v} \right) (1 - 2\mu)$$

$$E = 3k (1 - 2\mu) \quad \checkmark$$

$E = \frac{f}{\epsilon_{\text{linear}}}$	$k = \frac{p}{\epsilon_{\text{vol}}}$	$G = C = N = \frac{\tau}{\phi}$
--	---------------------------------------	---------------------------------

$$\checkmark E = 3k (1 - 2\mu)$$

$$\checkmark E = 2G (1 + \mu)$$

$$\frac{E}{2G} = 1 + \mu \quad ; \quad \boxed{\mu = \frac{E}{2G} - 1}$$

$$E = 3k \left(1 - \frac{2E}{2G} + 2 \right)$$

$$E = 3k \left(\frac{2}{3} - \frac{E}{G} \right)$$

$$E = 3k - \frac{3kE}{G}$$

$$E + \frac{3kE}{4} = 9k$$

$$E(4 + 3k) = 9k4$$

$$\checkmark \quad E = \frac{9k4}{(3k + 4)}$$

$$E = k$$

$$E = 24(1 + \mu)$$

$$E = 3k(1 - 2\mu)$$

$$E = 3E(1 - 2\mu)$$

$$3 - 6\mu = 1$$

$$3/6\mu = 2$$

$$\mu = \frac{1}{3}$$

$$k = 4$$

$$3k(1 - 2\mu) = 24(1 + \mu)$$

$$34(1 - 2\mu) = 24(1 + \mu)$$

$$3 - 6\mu = 2 + 2\mu$$

$$8\mu = 1$$

$$\mu = \frac{1}{8}$$

$$\text{if } E = 24$$

$$E = 24(1 + \mu)$$

$$E = 24(1 + \mu)$$

$$\mu = -\frac{1}{2}$$

Important Formula : \rightarrow

17/09/10

$$1. \sigma = f = \frac{P}{A}$$

$$2. \sigma = f = E \times \epsilon$$

$$3. E = \frac{Sl}{\Delta l}$$

$$4. E = \frac{\text{Longitudinal stress}}{\text{Linear strain (Main strain) (Longitudinal strain)}}$$

$$5. \text{Poisson ratio } \left(\frac{1}{m}\right) (\mu),$$

$$\begin{aligned} \mu &= \frac{\text{Lateral strain or, Transverse strain}}{\text{Linear strain or, Longitudinal strain}} \\ &= \frac{\Delta b/b}{\Delta l/l} \end{aligned}$$

* * 6. The value of Poisson's Ratio (μ or $\frac{1}{m}$)

a) Concrete - 0.2 ✓

b) Cast iron - 0.27

c) Wrought iron - 0.278

d) Steel - 0.288

e) stainless steel - 0.3 ✓

* f) Aluminium - 0.33 ✓

* g) Brass - 0.34

* h) Bronze - 0.35

i) Copper - 0.355

$$\begin{aligned}
 7. f_{temp} &= \epsilon \times E \\
 &= \frac{\delta l}{l} \times E \\
 &= \frac{\alpha \Delta t}{1} \times E \\
 &= \alpha (\Delta t) \times E
 \end{aligned}$$

8. Young's Modulus (Modulus of Elasticity)

$$E = \frac{f_{linear}}{\epsilon_{linear}} \quad (N/mm^2)$$

$$E = \frac{P/A}{\delta l/l} \Rightarrow \delta l = \frac{Pl}{AE} \checkmark$$

9. Bulk Modulus = $\frac{\text{Change in pressure}}{\text{Volumetric strain}}$

$$K = \frac{P}{\epsilon_v} = \frac{P}{\Delta V/V} \quad (N/mm^2)$$

10. Modulus of Rigidity (shear modulus)

$$= \frac{\text{Shear stress}}{\text{Shear strain} \quad \text{Angular Deformation}}$$

$$C = G = N = \frac{\tau}{\phi} \quad (MPa)$$

11. Three Moduli: \Rightarrow

$$E, K, G$$

$$12. E = 3K(1 - 2/\mu)$$

$$= 3K(1 - \frac{2}{m})$$

$$13. E = 2G(1 + \mu) \\ = 2G\left(1 + \frac{1}{m}\right)$$

$$14. E = \frac{9KH}{3K + H}$$

15. Strain in 'x' - direction,

$$\epsilon_x = \frac{f_x}{E} - \frac{f_y}{mE} - \frac{f_z}{mE} \\ = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

** 16. Strain in 'y' - direction,

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

17. Strain in 'z' - direction,

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

18. If $\epsilon_x, \epsilon_y, \mu$ are known, then

$$\sigma_x = ?$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$$

$$\frac{\sigma_y}{E} = \epsilon_y + \frac{\mu\sigma_x}{E}$$

$$\sigma_y = \left(\epsilon_y + \frac{\mu\sigma_x}{E} \right) E$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu}{E} \left(\epsilon_y + \frac{\mu \sigma_x}{E} \right) E$$

$$\epsilon_x = \frac{\sigma_x}{E} - \mu \epsilon_y - \frac{\mu^2 \sigma_x}{E}$$

$$(\epsilon_x + \mu \epsilon_y) = \sigma_x \left(\frac{1}{E} - \frac{\mu^2}{E} \right)$$

$$\begin{aligned} \sigma_x &= \frac{\epsilon_x + \mu \epsilon_y}{\left(\frac{1}{E} - \frac{\mu^2}{E} \right)} \\ &= \frac{(\epsilon_x + \mu \epsilon_y) E}{1 - \mu^2} \end{aligned}$$

$$19. \epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z = \epsilon_1 + \epsilon_2 + \epsilon_3$$

$$\begin{aligned} &= \left(\frac{f_1}{E} - \frac{f_2}{mE} - \frac{f_3}{mE} \right) + \left(\frac{f_2}{E} - \frac{f_1}{mE} - \frac{f_3}{mE} \right) \\ &\quad + \left(\frac{f_3}{E} - \frac{f_1}{mE} - \frac{f_2}{mE} \right) \end{aligned}$$

$$= \frac{f_1 + f_2 + f_3}{E} - \frac{2}{mE} (f_1 + f_2 + f_3)$$

$$= \frac{f_1 + f_2 + f_3}{E} \left(1 - \frac{2}{m} \right)$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu)$$

where, $\sigma_x = + \frac{P_x}{A_x}$ or, $-\frac{P_x}{A_x}$ Tensile Compressive

$$\sigma_y = + \frac{P_y}{A_y} \text{ or, } - \frac{P_y}{A_y}$$

$$\sigma_z = + \frac{P_z}{A_z} \text{ or, } - \frac{P_z}{A_z}$$

20. change in volume,

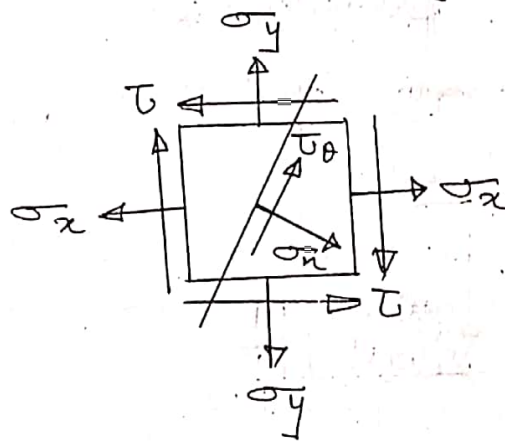
$$\epsilon_v = \frac{\delta v}{v}$$

$$\delta v = \epsilon_v \times v$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu) \times l \times b \times d$$

Normal stress,

21. $\sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + \tau \sin 2\theta$



$$\sigma_n = \sigma_x \left(\frac{1 - \cos 2\theta}{2} \right) + \sigma_y \left(\frac{1 + \cos 2\theta}{2} \right) + \tau \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau \sin 2\theta$$

22. Tangential stress (shear stress)

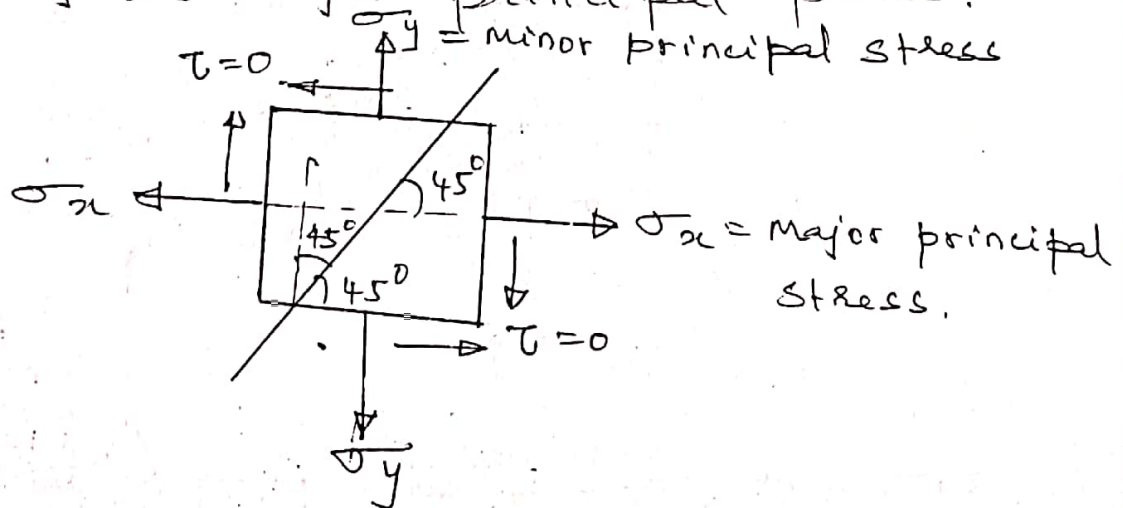
$$\tau_{\theta} = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$$

*** 23. Maximum shear stress,

$$\tau_{\max} = \frac{\sigma_x - \sigma_y}{2}$$

$$= \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$$

24. The plane on which maximum tangential stress ($\tau_{\max} = \tau_{\theta}$) takes place is at 45° from the direction of major principal stress or 45° from major principal plane.



25. Strain energy stored upto elastic limit is called resilience.

26. The maximum strain energy up to elastic limit is called proof Resilience

27. The strain energy per unit volume is called Modulus of Resilience ($\frac{f^2}{2E}$)

28. Strain energy due to axial load

$$U = \frac{P^2 l}{2AE} = \frac{f^2}{2E} \times \text{Volume.}$$

29. Strain energy due to shear stress,

$$U = \frac{\tau^2}{2G} \times \text{Volume}$$

30. Strain energy due to Volumetric strain,

$$U = \frac{\sigma^2}{2K} \times \text{Volume.}$$

31. Strain energy due to Torsion,

$$U = \frac{T^2 L}{2GJ} = \frac{\tau^2}{4G} \times \text{Volume}$$

32. Strain energy due to bending,

$$U = \int \frac{M^2 dx}{2EI}$$

33. $AE =$ Axial Rigidity

$$= kN, N$$

$$= MLT^{-2}$$

34. Strain energy due to shear force,
$$U = \int \frac{v^2 dx}{2 GA}$$

35. GA = shear rigidity
 $= N$
 $= MLT^{-2}$

36. GJ = Torsional rigidity
 $= N-m^2$
 $= ML^3T^{-2}$

37. EI = Flexural rigidity
 $= N-m^2$
 $= ML^3T^{-2}$

38. $\frac{AE}{L}$ = Axial stiffness
 $= N/m$
 $= ML^0T^{-2}$

39. $\frac{GA}{L}$ = shear stiffness
 $= N/m$
 $= ML^0T^{-2}$

40. $\frac{EI}{L}$ = Flexural stiffness
 $= N-m$
 $= ML^2T^{-2}$

41. $\frac{GJ}{L}$ = Torsional stiffness
 $= N-m$
 $= ML^2T^{-2}$

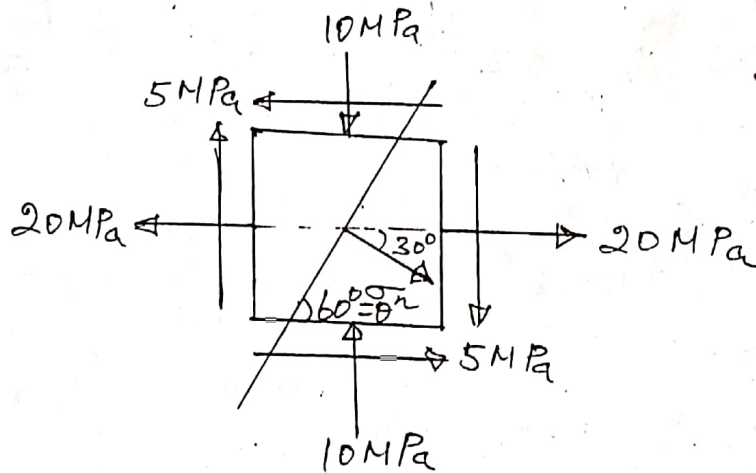
*

42. Effect due to impact load stress

$$= 2 \times \text{Effect due to static load}$$

Numericals: \rightarrow

1. Determine Normal & tangential stress along an inclined plane which is subjected to two \perp stresses 20 MPa tensile along x-direction and 10 MPa compressive along y-direction. The shear stress is 5 MPa. The Normal stress on the inclined plane makes an angle 30° with the direction of tensile stress & also determine resultant stress & its direction.



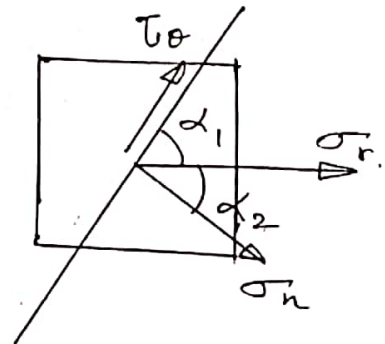
$$\begin{aligned}\sigma_n &= \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + \tau \sin 2\theta \\ &= +20 \sin^2 60^\circ + (-10) \cos^2 60^\circ + 5 \sin 2 \times 60^\circ \\ &= +16.83 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\tau_\theta &= \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta \\ &= \frac{20 - (-10)}{2} \sin 2 \times 60 + 5 \cos 2 \times 60 \\ &= +10.49 \text{ MPa}.\end{aligned}$$

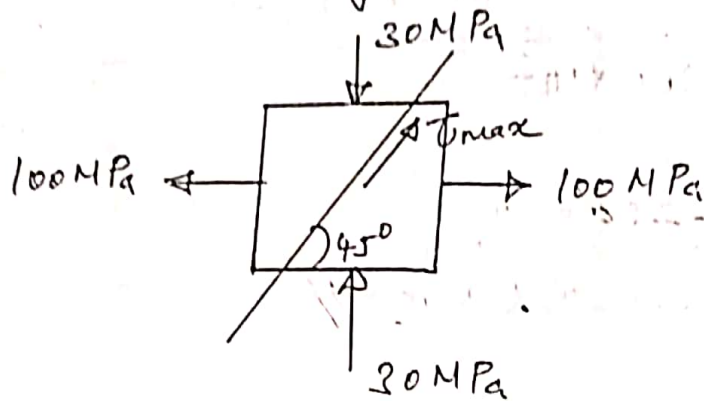
$$\begin{aligned}\sigma_r &= \sqrt{\sigma_n^2 + \tau_\theta^2} \\ &= 19.83 \text{ MPa}.\end{aligned}$$

$$\begin{aligned}\alpha_1 &= \tan^{-1} \left(\frac{\sigma_n}{\tau_\theta} \right) \\ &= 58.07^\circ\end{aligned}$$

$$\begin{aligned}\alpha_2 &= \tan^{-1} \left(\frac{\tau_\theta}{\sigma_n} \right) \\ &= 31.93^\circ.\end{aligned}$$

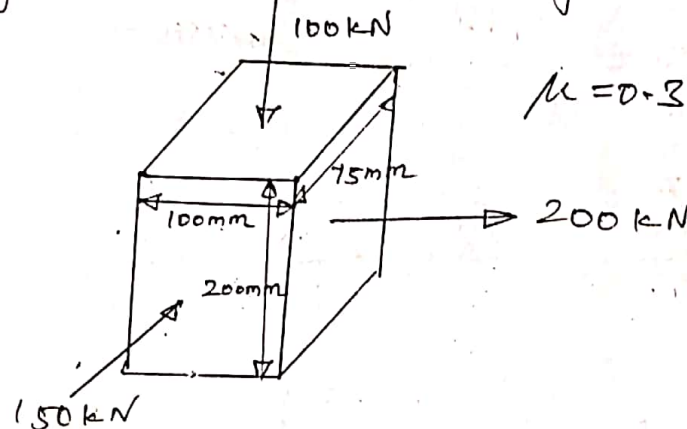


2. Determine maximum shear stress & its plane of a loaded member as shown in figure below.



$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2} = \frac{100 - (-30)}{2} = 65 \text{ MPa}$$

3. Determine change in volume of an object as shown in figure below.



$$\mu = 0.3, E = 200 \text{ GPa}$$

$$\epsilon_v = \frac{200}{75 \times 100}$$

$$\Delta V = \frac{(\sigma_x + \sigma_y + \sigma_z)}{E} (1 - 2\mu) \times l \times b \times d$$

$$\sigma_x = \frac{200 \times 10^3}{200 \times 25} = 13.33 \text{ MPa}$$

$$\sigma_y = -\frac{100 \times 10^3}{100 \times 25} = -13.33 \text{ MPa}$$

$$\sigma_z = \frac{150 \times 10^3}{100 \times 200} = -7.5 \text{ MPa.}$$

$$\Delta V = \frac{(13.33 - 13.33 - 7.5)}{200 \times 10^3} \times (1 - 2 \times 0.3) \times 100 \times 200 \times 75$$

$$= -2225 \text{ mm}^3$$

$$\Delta V = -22.5 \text{ mm}^3 //$$

4. Determine change in length in x -direction if $\sigma_x = 100 \text{ MPa}$ tensile & $\sigma_y = 50 \text{ MPa}$ compressive. take $\frac{1}{m} = 0.3$ & total length 700 mm .

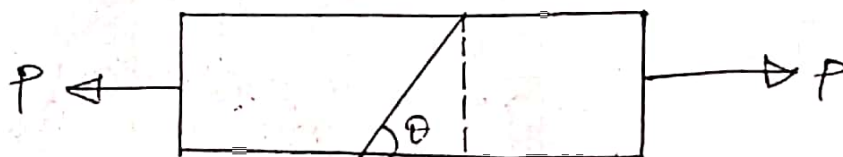
$$\epsilon_x = \frac{\sigma_x^{\text{tension}}}{E} - \frac{\sigma_y^{\text{compressive}}}{mE}$$

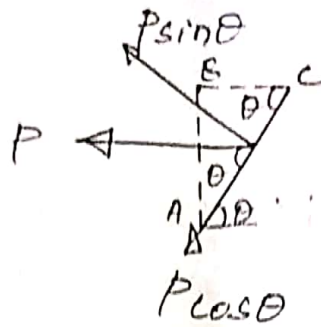
$$\frac{\delta l}{700} = \frac{100}{2 \times 10^5} + \frac{50}{2 \times 10^5} \times 0.3$$

$$\delta l = 0.4025 \text{ mm} //$$

$$\epsilon_x = 5.75 \times 10^{-4}$$

5. Q4 # Determine the Normal stress developed on a plane as shown in figure below.





$$F_n = P \sin \theta$$

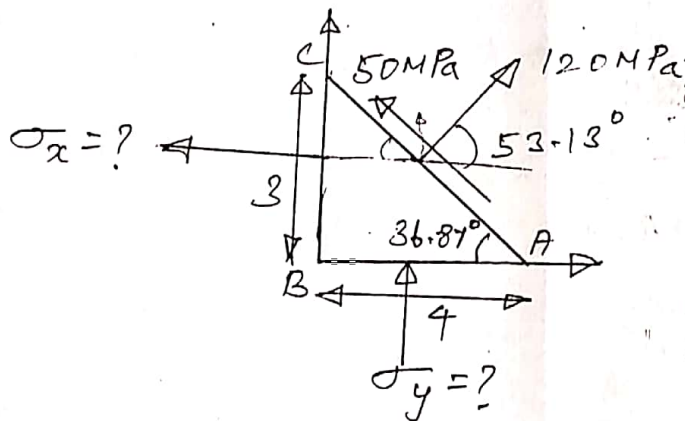
$$\sigma_n = \frac{F_n}{A} = \frac{P \sin \theta}{AC \times 1 \text{ unit thickness}}$$

$$= \frac{\sigma_x \times AB \times \sin \theta}{AC}$$

$$= \sigma_x \sin \theta \times \sin \theta$$

$$\boxed{\sigma_n = \sigma_x \sin^2 \theta}$$

6. Determine the Normal stresses on two L_r direction as shown in figure below.



$$\sum H = 0$$

$$\sigma_x \times 3 \times 1 + 50 \times 5 \times 1 \cos 36.87^\circ - 120 \times 5 \times 1 \cos 53.13^\circ = 0$$

$$3\sigma_x = -199.99 + 360.00$$

$$\sigma_x = 453.34 \text{ MPa (Tensile)}$$

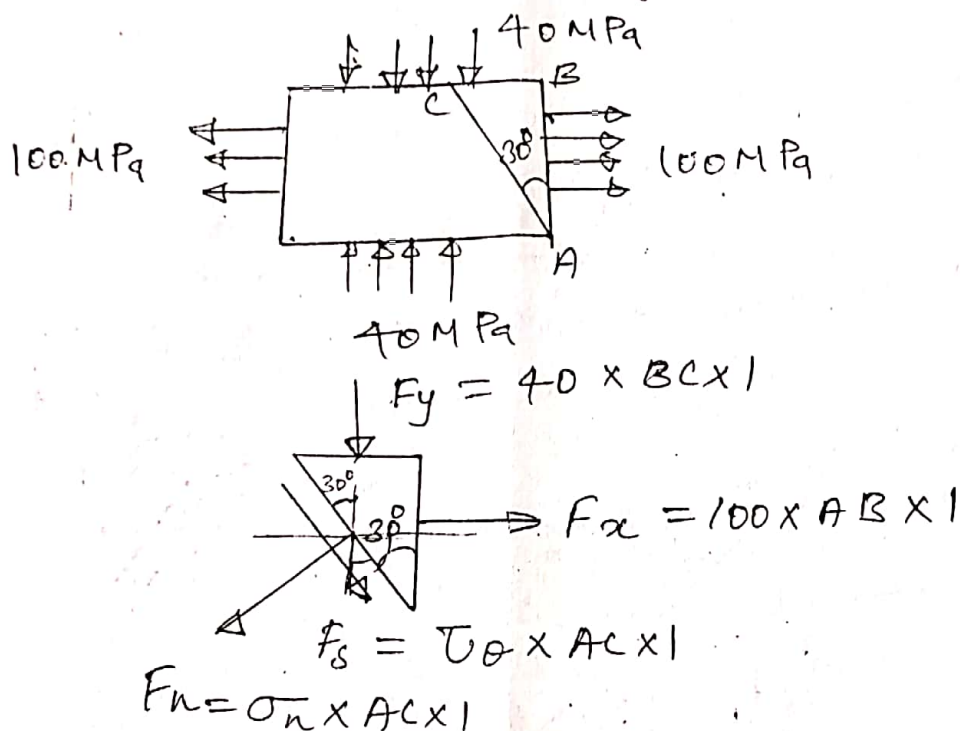
$$\sum V = 0,$$

$$\sigma_y \times 4 \times 1 + 50 \times 5 \times 1 \times \sin 36.87^\circ + 120 \times 5 \times 1 \times \sin 53.13^\circ = 0$$

$$4\sigma_y = -150.00 - 479.99$$

$$\sigma_y = -157.499 \text{ MPa (Tensile)}$$

7. Determine resultant stress on a given plane as shown in figure below.



$$\sum F_{\text{along plane}} = 0;$$

$$\tau_\theta \times AC \times 1 + 40 \times BC \times 1 \cos 30^\circ + 100 \times AB \times 1 \cos 60^\circ = 0.$$

$$AC \tau_\theta = -40 BC \cos 30^\circ - 100 AB \cos 60^\circ$$

$$\tau_\theta = -40 \cos 30^\circ \sin 30^\circ - 100 \cos 60^\circ \sin 30^\circ$$

$$= -17.32 - 48.30$$

$$\tau_\theta = -60.62 \text{ MPa (direction reversed)}$$

$$\sum F_{\perp \text{ to plane}} = 0,$$

$$F_n \times AC \times 1 = -40 \times BC \times 1 \times \sin 30^\circ + 100 \times AB \times 1 \times \sin 60^\circ$$

$$A \times F_n = -40 \frac{BC}{AC} \sin 30^\circ + 100 \frac{AB}{AC} \sin 60^\circ$$

$$= -40 \sin 30^\circ \times \sin 30^\circ + 100 \cos 30^\circ \sin 60^\circ$$

$$= -10 + 75$$

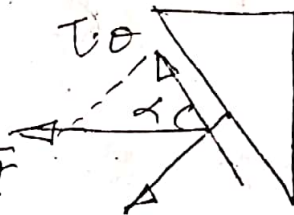
$$\sigma_n = 65 \text{ MPa}$$

$$\sigma_r = \sqrt{65^2 + 60.62^2}$$

$$\sigma_r = 88.88 \text{ MPa}$$

$$\alpha = \tan^{-1} \left(\frac{65}{60.62} \right)$$

$$= 46.99^\circ$$



- * 8. A metallic bar having size ~~200mm~~³⁰⁰ in length & 40mm^{square} in cross section is subjected an axial load of 160kN. The change in length is 0.12mm & change in breadth is 0.005mm. Determine,

- i) Young's modulus
- ii) Poisson ratio
- iii) Bulk modulus
- iv) Modulus of rigidity
- v) Volumetric strain
- vi) change in volume.

$$\mu = \frac{\Delta b/b}{\Delta l/l} = \frac{0.005/40}{0.12/\frac{6000}{300}} = \frac{0.2}{1.25 \times 10^{-4}} = \frac{2.08}{0.625} = 0.3125 //$$

$$f = \frac{P}{A} = \frac{160 \times 10^3}{40 \times 40} = 100 \text{ MPa} //$$

$$E = \frac{f}{\epsilon} = \frac{100}{0.12/\frac{6000}{300}} = \frac{2.5}{16.67} \times 10^5 \text{ MPa} //$$

$$E = 3k(1-2\mu) //$$

$$2.5 \times 10^5 = 3k(1-2 \times 0.3125)$$

$$k = 2.22 \times 10^5 \text{ MPa} //$$

$$E = 24(1+\mu)$$

$$2.5 \times 10^5 = 24(1+0.3125)$$

$$24 = 0.95 \times 10^5 \text{ MPa} //$$

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E} (1 - 2\mu)$$

$$= \frac{100 + 0 + 0}{2.5 \times 10^5} (1 - 2 \times 0.3125)$$

$$\epsilon_v = 1.5 \times 10^{-4} //$$

$$\epsilon_v = \frac{\Delta v}{v}$$

$$\Delta v = \epsilon_v \times v$$

$$= 1.5 \times 10^{-4} \times 40 \times 40 \times 300$$

$$= 720 \text{ mm}^3 // \text{ (increase in volume)}$$

III - PRINCIPAL STRESSES

2 THEORIES OF FA

1. The principal stress is the Normal Stress acting on any plane where Shear stress is zero.

2. The plane where only a Normal stress acts [~~a~~ Tangential stress = 0] is called the principal plane.

3. There are two principal planes! —

a) Major principal plane

b) Minor principal plane

4. The principal planes always meet at right angle to each other [90°]

** 5. The maximum shear stress is equal to
$$= \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$$

* 6. The plane along which maximum shear stress [Tangential stress] acts makes an angle 45° from the principal plane.

7. To determine the Location of principal plane, the tangential stress [shear stress] $[\tau_{\theta}]$ is made equal to zero. from where ' θ ' can be determined.

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

**

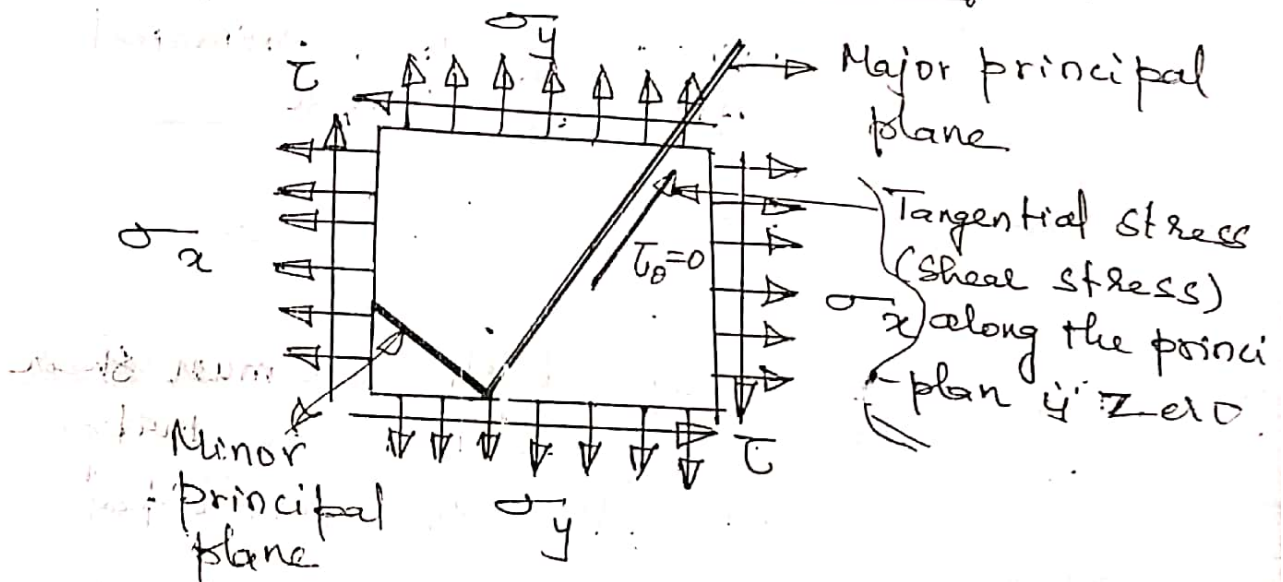
8. The major & minor principal stresses are given by formula:-

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$\frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$ - Radius of Mohr Circle.

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2} \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

9. Location of principal planes: →



$$\tau_{\theta} = 0$$

$$\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta = 0$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{-\tau}{(\sigma_x - \sigma_y)/2}$$

$$\tan 2\theta = \frac{-2\tau}{(\sigma_x - \sigma_y)}$$

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$180 - 2\theta = \tan^{-1}\left(\frac{2\tau}{\sigma_x - \sigma_y}\right)$$

$$\theta = 90 - \frac{1}{2} \tan^{-1}\left(\frac{2\tau}{\sigma_x - \sigma_y}\right)$$

10. a) If $\sigma_x > \sigma_y$, $\theta_{p.p} = 45^\circ$ to 90°
from σ_x -direction

b) If $\sigma_x = \sigma_y$, $\theta_{p.p} = 45^\circ$ from σ_x
direction

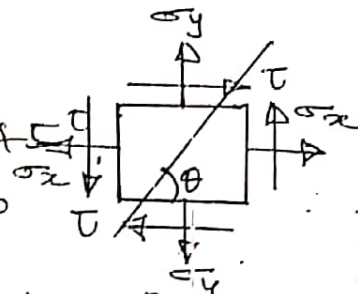
c) If $\sigma_x < \sigma_y$, $\theta_{p.p} = 0$ to 45° from
 σ_x -direction

If τ - Reverse direction,

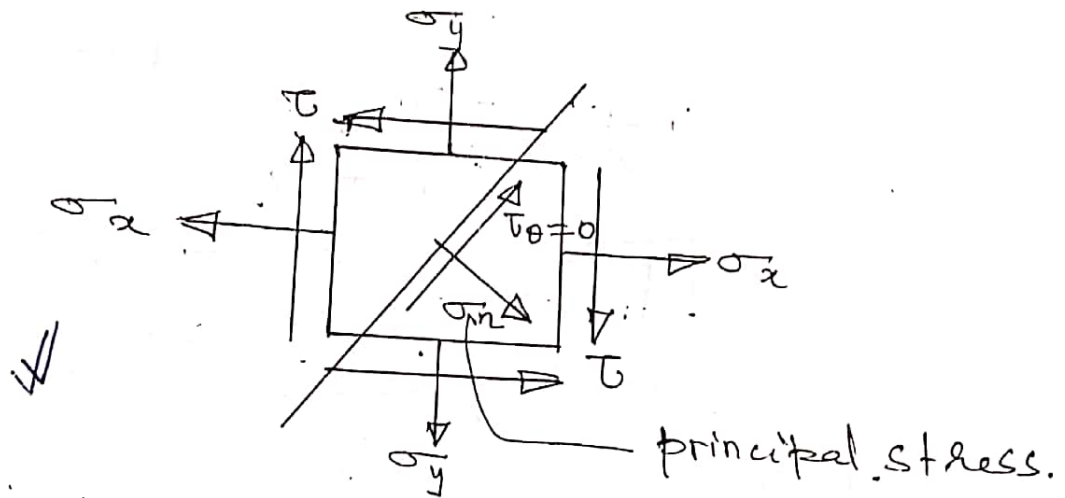
a) $\sigma_x > \sigma_y$, $\theta_{p.p} = 0$ to 45°

b) $\sigma_x = \sigma_y$, $\theta_{p.p} = 45^\circ$

c) $\sigma_x < \sigma_y$, $\theta_{p.p} = 45^\circ$ to 90°
from σ_x -direction



10. The Magnitude of principal stress [Analytically]



$$\begin{aligned}\sigma_n &= \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + \tau \sin 2\theta \\ &= \sigma_x \left[\frac{1 - \cos 2\theta}{2} \right] + \sigma_y \left[\frac{1 + \cos 2\theta}{2} \right] + \tau \sin 2\theta \\ &= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau \sin 2\theta\end{aligned}$$

$$\tan 2\theta = \frac{-2\tau}{(\sigma_x - \sigma_y)} = \frac{\text{opposite}}{\text{adjacent.}}$$

$$\sin 2\theta = \frac{-2\tau}{\pm \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

$$\begin{aligned}[\text{hyp}] &= \pm \sqrt{(-2\tau)^2 + (\sigma_x - \sigma_y)^2} \\ &= \pm \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}\end{aligned}$$

$$\cos 2\theta = \frac{(\sigma_x - \sigma_y)}{\pm \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2} \right) \times \frac{\sigma_x - \sigma_y}{\pm \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

$$+ \tau \times \frac{-2\tau}{\pm \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

Major principle stress,

$$\sigma_1 = \sigma_x + \sigma_y + \sigma_1$$

$$= \sigma_y + \frac{\sigma_x - \sigma_y}{2} + \frac{1}{2} \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

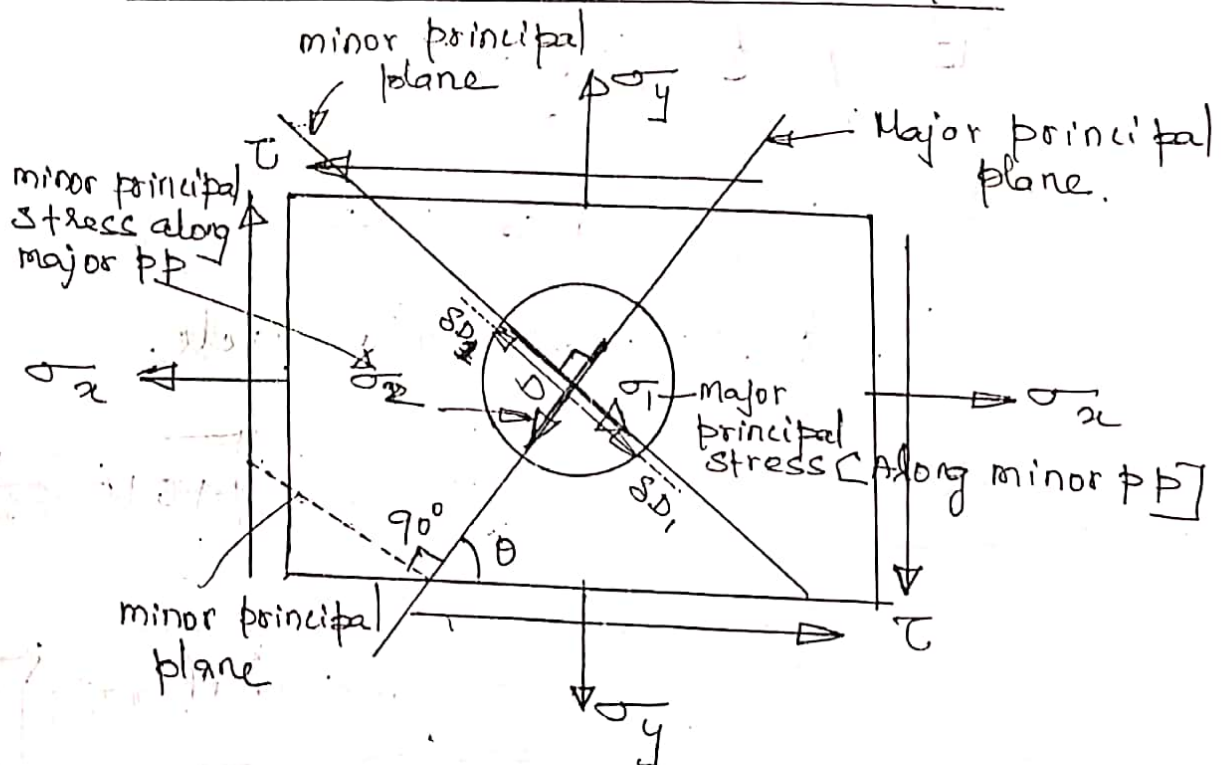
$\triangle O'CA$,

$$\tan(180-2\theta) = \frac{\tau}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

$$\tan(180-2\theta) = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

11-10-10

Circular Hole Become Ellipse:-



Major principal strain,

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$\frac{\Delta D_1}{D} = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$\Delta D_1 = D \left[\frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \right]$$

⇒ Along
minor
principal
plane

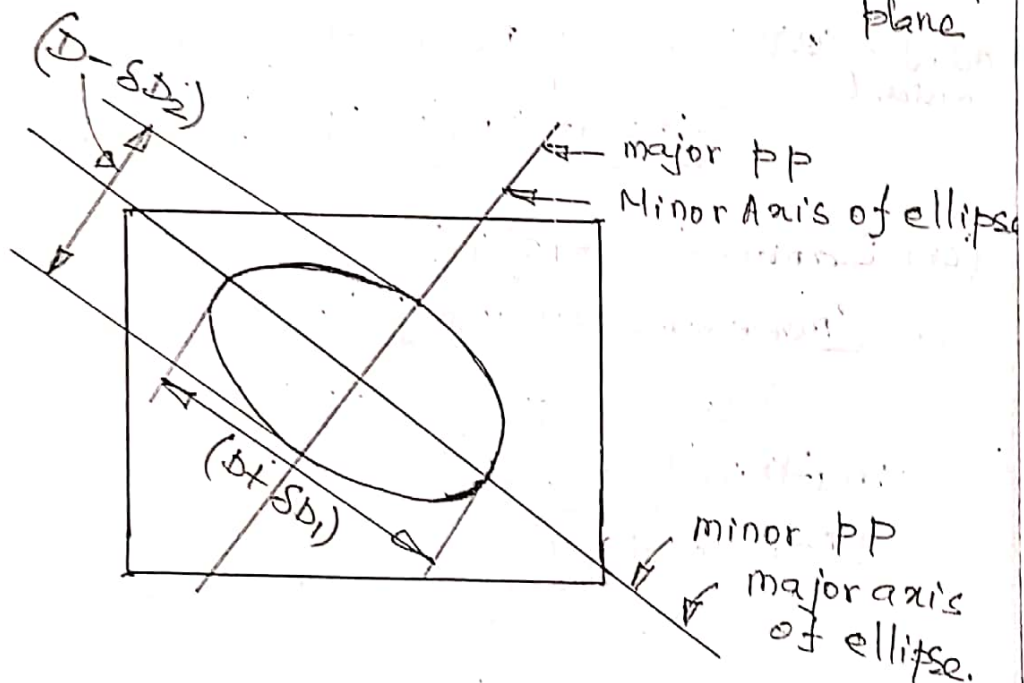
Minor principal strain,

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$\frac{\Delta D_2}{D} = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$\Delta D_2 = D \left[\frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} \right]$$

⇒ Along
major
principal
plane



THEORIES OF FAILURE →

There are five theories of failure:-
 [Based on Elastic condition of loading]
 [Hooke's Law is obeyed]

Brittle material (a) Rankine's theory
 [Maximum principal stress theory]

Brittle (b) Saint Venant's theory
 [Maximum principal strain theory]

Ductile material (c) Tresca or, Guest theory
 [Maximum shear stress theory]

Ductile (d) Haig's theory
 [Total strain energy theory]

General material (e) Von Mises theory
 [Shear-strain energy theory]

(a) Rankine's theory :- →
 [Maximum principal stress theory]

It is mainly applicable for Brittle material [$\epsilon_y = 0.002\%$] [Do not have distinct yield point]

$$\sigma_1 > \sigma_0$$

$$\sigma_1 > \frac{\sigma_y}{F.O.S} \rightarrow \text{for failure}$$

For no failure,

$$\sigma_1 \leq \sigma_0$$

$$\boxed{\frac{\sigma_x + \sigma_y}{2} + \frac{1}{2} \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2} \leq \frac{f_y}{F.O.S}}$$

(b) Saint Venant theory

[Maximum principal strain theory]

It is mainly applicable for general material [neither brittle nor ductile]

$\epsilon_1 > \epsilon_0$, for failure

For No failure,

$$\epsilon_1 \leq \epsilon_0 \text{ (permissible)}$$

$$\frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \leq \frac{\sigma_0}{E} \rightarrow \text{As per Hooke's law.}$$

$$\sigma_1 - \mu \sigma_2 \leq \sigma_0$$

$$\boxed{\sigma_1 - \mu \sigma_2 \leq \frac{f_y}{F.O.S}}$$

(c) Tresca or, Guest theory:-

[Maximum shear stress theory]

It is mainly applicable for Ductile material.

$$\tau_{max} > \tau_0 \text{ (perm)} - \text{for failure}$$

for safety, No failure,

$$\tau_{max} \leq \tau_o \text{ (perm)}$$

$$\frac{\sigma_1 - \sigma_2}{2} \leq \frac{\sigma_o}{2}$$

$$\sigma_1 - \sigma_2 \leq \sigma_o$$

$$\sigma_1 - \sigma_2 \leq \frac{f_y}{F.O.S}$$

(d) Haig theory:-

[Total strain energy theory]

It is applicable for Ductile material & thick cylinder.

Equivalent stress;

$$\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{2}{3}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)} \leq \left[\sigma_o = \frac{f_y}{F.O.S} \right]$$

For No failure

(e) Von mises theory:-

[Shear strain energy theory]

[For General material]

$$\sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \leq \left[\sigma_o = \frac{f_y}{F.O.S} \right]$$

For No failure

Numericals: →

1. A circular hole is drilled in a mild steel plate which is subjected to stresses $\sigma_x = 100 \text{ MPa}$ (tensile), $\sigma_y = 30 \text{ MPa}$ (tensile), $\tau = 40 \text{ MPa}$. The dia of hole is 300mm. Determine,

(i) Major principal stress & Minor principal stress.

(ii) Maximum principal strain

(iii) Principal plane location

(iv) Maximum shear stress

(v) Major axis of ellipse

take $\mu = 0.3$ & $E = 210 \text{ GPa}$.

(i)

$$\begin{aligned}\sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2} \\ &= \frac{100 + 30}{2} \pm \frac{1}{2} \sqrt{4 \times 40^2 + (100 - 30)^2} \\ &= 65 \pm 53.15\end{aligned}$$

$$\sigma_1 = 118.15 \text{ MPa (tensile)}$$

$$\sigma_2 = 11.85 \text{ MPa (tensile)}$$

ii) $\epsilon_{1/2} \neq \epsilon_x$

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$= \frac{2 \times 40}{100 - 30}$$

$$\theta_1 = 65.59^\circ$$

$$\theta_2 = 90^\circ + \theta_1$$

$$\theta_2 = 155.59^\circ$$

$$\text{iii) } \tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{118.15 - 11.85}{2}$$

$$\tau_{max} = 58.15 \text{ MPa}$$

[Radius of Mohr Circle]

$$\text{(iv) } \epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$= \frac{1}{2.1 \times 10^5} (118.15 - 0.3 \times 11.85)$$

$$\epsilon_1 = 5.457 \times 10^{-4}$$

$$\frac{\delta D_1}{D} = 5.457 \times 10^{-4}$$

$$\delta D_1 = 5.457 \times 10^{-4} \times 300$$

$$\delta D_1 = 0.164 \text{ mm}$$

Major axis of Ellipse,

$$\begin{aligned} D_1 &= D + \delta D_1 \\ &= 300 + 0.164 \\ &= 300.164 \text{ mm.} \end{aligned}$$

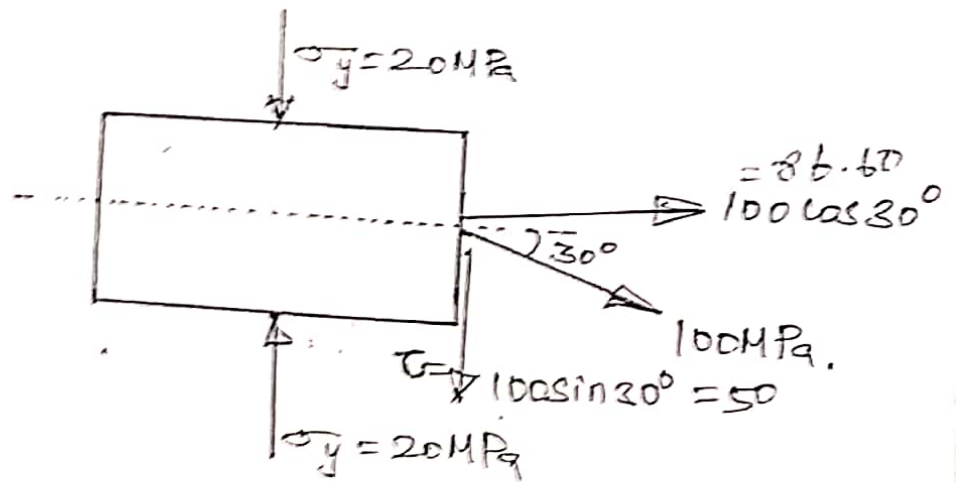
Minor axis of ellipse,

$$\begin{aligned} \epsilon_2 &= \left(11.85 - 0.2 \times 118.15 \right) \frac{1}{3.1 \times 10^5} \\ \frac{\delta D_2}{D} &= -1.124 \times 10^{-4} \\ \delta D_2 &= -1.124 \times 10^{-4} \times 300 \\ \delta D_2 &= -0.0337 \text{ mm.} \end{aligned}$$

Minor axis of Ellipse,

$$\begin{aligned} D_2 &= D + \delta D_2 \\ &= 300 + (-0.0337) \\ D_2 &= 299.966 \text{ mm} \end{aligned}$$

2. A steel material is subjected to direct stress at an angle 30° with horizontal / having tensile nature of value 100 MPa the material is subjected to compressive stress 20 MPa in \perp direction. Determine principal stresses, maximum shear stress & principal planes.



$$\sigma_{1,2} = \frac{100 \cos 30^\circ + 20}{2} \pm \frac{1}{2} \sqrt{4 \times (100 \sin 30^\circ)^2 + [100 \cos 30^\circ - (-20)]^2}$$

$$= 38.30 \pm 73.08$$

$$\sigma_1 = 106.38 \text{ MPa (tensile)}$$

$$\sigma_2 = -39.78 \text{ (compressive)}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = 73.08 \text{ MPa}$$

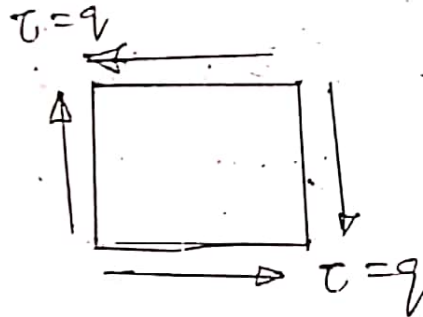
at 45° from ~~APP~~

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$= \frac{2 \times 50}{86.60 - (-20)}$$

$$\theta = 68.42^\circ$$

3. A steel material is subjected to ^{pure} shear [No Bending] 50 MPa. Determine principal stresses, principal plane & Dia of Mohr's circle,



$$\sigma_{1,2} = \frac{0+0}{2} \pm \frac{1}{2} \sqrt{4 \times 50^2 + (0-0)^2}$$

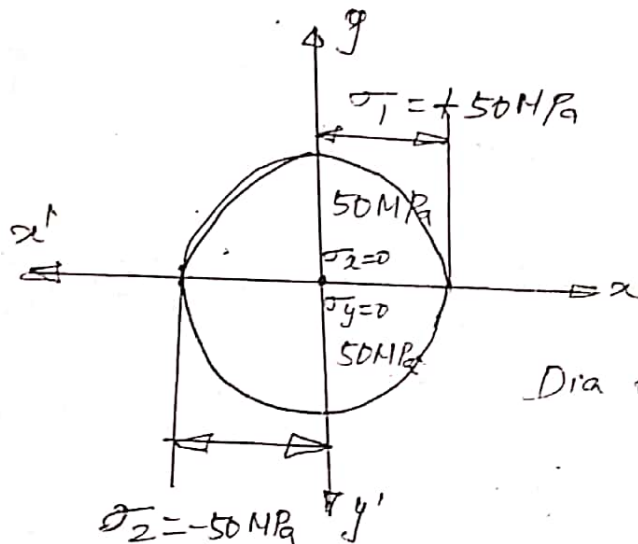
$$= 0 \pm 50$$

$$\sigma_1 = +50 \text{ MPa}$$

$$\sigma_2 = -50 \text{ MPa}$$

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2\tau}{0} = \infty$$

$$\theta = 45^\circ$$



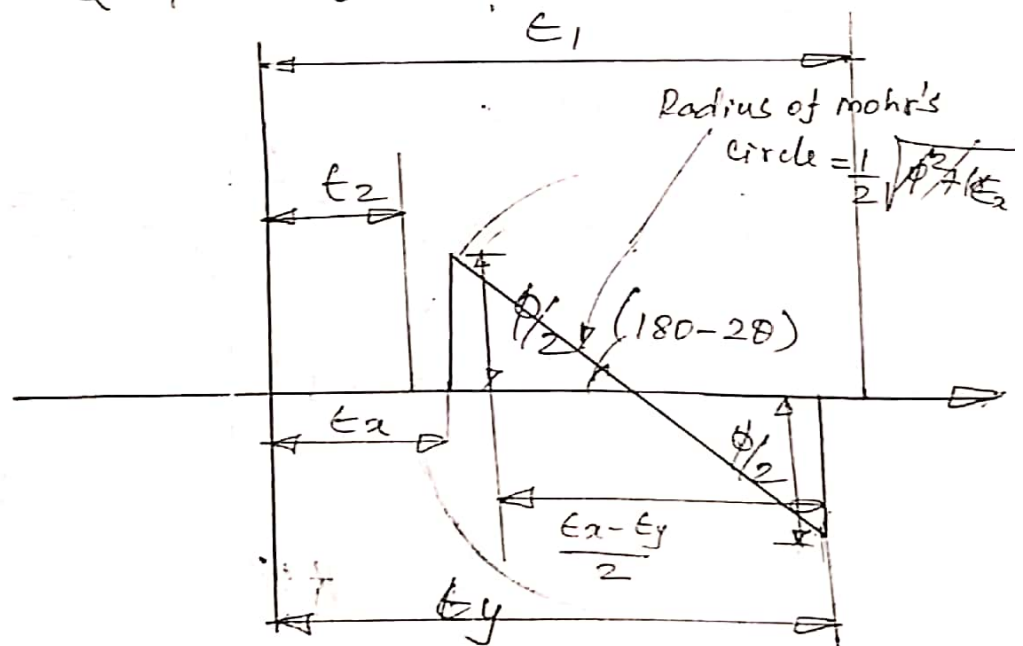
Dia of Mohr's circle = 100 MPa
 $= 2\tau_{max}$
 $= 2 \times \tau_{shear}$

4. Determine principal strains & Dia of Mohr circle of strain if $\epsilon_x = 2 \times 10^{-3}$, $\epsilon_y = 3 \times 10^{-4}$, $\phi_{xy} = 10^{-4}$.

$$\begin{aligned}\epsilon_{1,2} &= \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2} \sqrt{\phi_{xy}^2 + (\epsilon_x - \epsilon_y)^2} \\ &= \frac{2 \times 10^{-3} + 3 \times 10^{-4}}{2} \pm \frac{1}{2} \sqrt{(10^{-4})^2 + (2 \times 10^{-3} - 3 \times 10^{-4})^2} \\ &= 1.15 \times 10^{-3} \pm 8.515 \times 10^{-4}\end{aligned}$$

$$\epsilon_1 = 2.00 \times 10^{-3}$$

$$\epsilon_2 = 2.985 \times 10^{-4}$$



$$\begin{aligned}\text{Radius of Mohr's circle} &= \frac{1}{2} \sqrt{\phi^2 + (\epsilon_x - \epsilon_y)^2} \\ &= 8.515 \times 10^{-4}\end{aligned}$$

$$\begin{aligned}\text{Dia} &= \sqrt{\phi^2 + (\epsilon_x - \epsilon_y)^2} \\ &= 1.703 \times 10^{-3}\end{aligned}$$

$$\tan(180-2\theta) = \frac{\phi/2}{(\epsilon_x - \epsilon_y)/2}$$

$$\begin{aligned}\tan(180-2\theta) &= \frac{\phi}{(\epsilon_x - \epsilon_y)} \\ &= \frac{10^{-4}}{(2 \times 10^{-3} - 3 \times 10^{-4})}\end{aligned}$$

$$\theta = 88.32^\circ$$

5. A steel material is subjected to stresses, $\sigma_x = 90 \text{ MPa}$, (tensile), $\sigma_y = 40 \text{ MPa}$ (compressive) & $\tau = 30 \text{ MPa}$. Determine the P.O.S if yield stress 270 MPa using Haig theory. take $\mu = 0.3$

$$\begin{aligned}\sigma_{1,2} &= \frac{90 + (-40)}{2} \pm \frac{1}{2} \sqrt{30^2 \times 4 + (90 - (-40))^2} \\ &= 25 \pm 71.59\end{aligned}$$

$$\sigma_1 = 96.59 \text{ MPa}$$

$$\sigma_2 = -46.59 \text{ MPa}$$

Haig theory,

$$\sqrt{96.59^2 + (-46.59)^2} - \frac{2 \times 0.3}{1+0.3} (96.59 \times (-46.59)) \leq \frac{\sigma_y}{F.O.S.}$$

+ 0 + 0

$$\frac{119.17}{202.72} \leq \frac{270}{F.O.S.}$$

$$F.O.S. \leq 2.27$$

6. A steel material is subjected to principal stresses 80 MPa tensile & 30 MPa compressive,

(i) equivalent stress based on Saint Venant theory, & Tresca theory, also determine F.O.S. based on Rankine theory & Von Mises theory, take $\mu = 0.3$ & yield stress = 270 MPa.

Saint Venant,

$$\sigma_1 - \mu \sigma_2 \leq \frac{f_{yt}}{F.O.S.} \sigma_0$$

$$80 - 0.3 \times (-30) \leq \frac{270}{F.O.S.} \sigma_0$$

$$\sigma_0 \geq 89 \text{ MPa.}$$

Tresca,

$$\sigma_1 - \sigma_2 \leq \sigma_0$$

$$80 - (-30) \leq \sigma_0$$

$$\sigma_0 \geq 110 \text{ MPa.}$$

Rankine,

$$\sigma_1 \leq \sigma_o$$
$$+80 \leq \frac{270}{F}$$

$$F \leq 270/80$$

$$F.O.S \leq 3.375$$

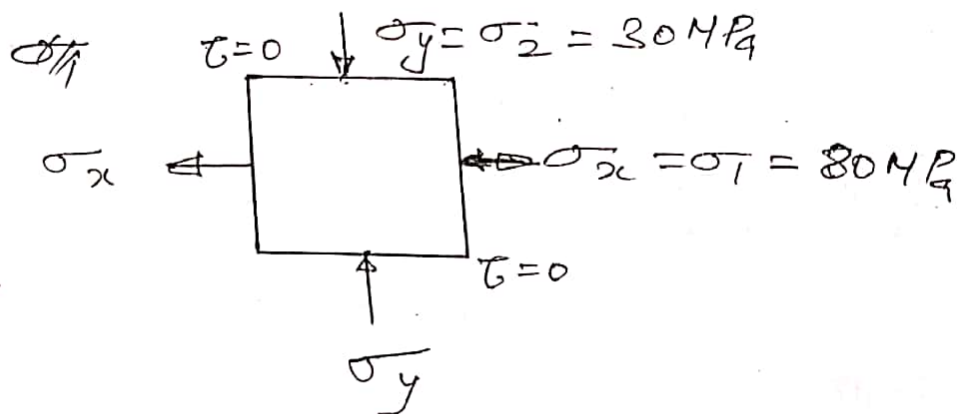
Von mises,

$$\sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \leq \frac{f_y}{F.O.S}$$
$$\sqrt{\frac{1}{2}[(80 - (-30))^2 + (-30 - 0)^2 + (0 - 80)^2]} \leq \frac{270}{F.O.S}$$

$$F.O.S \leq \frac{270}{98.49}$$

$$F.O.S \leq 2.741$$

7. If σ_2 is 30 MPa (compressive) & σ_1 is 80 MPa (tensile). Determine σ_x & σ_y .
If $\tau = 0$.

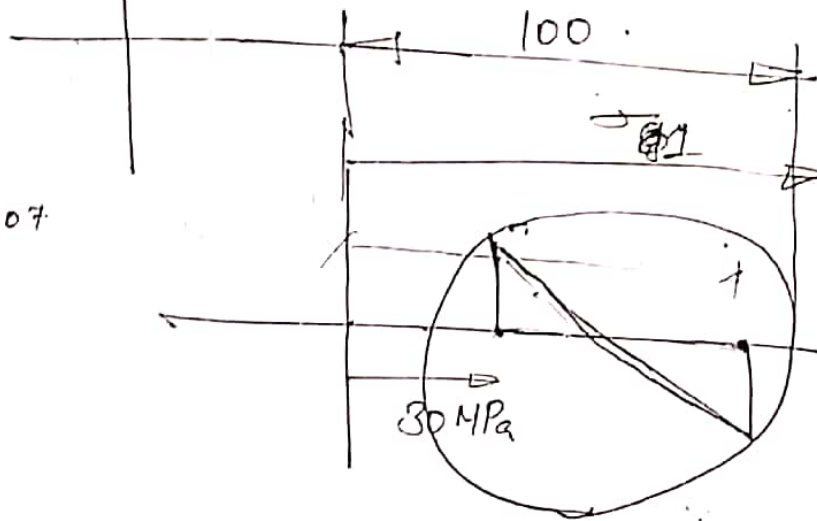


8. If $\sigma_x = 100 \text{ MPa}$ (tensile), $\sigma_y = 30 \text{ MPa}$ (Comp),
 $\tau = 20 \text{ MPa}$. Determine minor principal
 stress.

$$100 = \frac{\sigma_x + (-30)}{2} + \frac{1}{2} \sqrt{4 \times 20^2 + (\sigma_x - 30)^2}$$

$$= \frac{\sigma_x}{2} - 15 + \frac{1}{2} \sqrt{1600 + (\sigma_x - 30)^2}$$

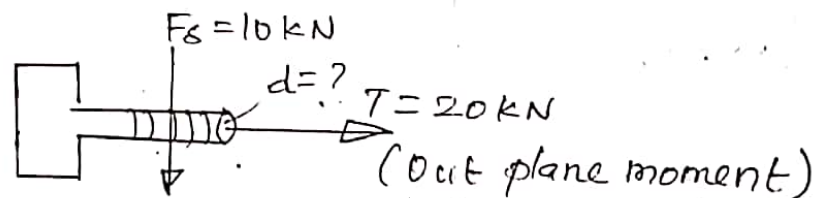
$$\sigma_2^{(-)} = 33.07$$



9. A mild bolt is subjected to transverse shear 10 kN & axial tension 20 kN. yield stress is 270 MPa & P.O.S = 3. Determine dia of bolt by using,

- 1) Rankine theory
- 2) Vincent theory
- 3) Tresca theory
- 4) Haig theory
- 5) Von Mises theory, $\mu = 0.3$

$$E = 210 \text{ GPa}$$



$$\tau = \frac{F_s}{A} = \frac{10 \times 10^3}{\pi/4 \times d^2} =$$

$$\sigma_x = \frac{T}{A} = \frac{20 \times 10^3}{\pi/4 \times d^2}$$

$$\sigma_y = 0$$

$$\left(\frac{\tau_{xy}}{\tau_d} \right)^2 + \left(\frac{\sigma_{xt}}{\sigma_d} \right)^2 \leq 1$$

$$\begin{aligned} \sigma_{1,2} &= \frac{20000}{\frac{\pi}{4} \times d^2} \times \frac{1}{2} \pm \frac{1}{2} \sqrt{4 \times \left(\frac{10000}{A} \right)^2 + \left(\frac{20000}{A} \right)^2} \\ &= \frac{10000}{A} \pm \frac{28284.27}{A} \end{aligned}$$

$$\sigma_1 = 24142.14 / A$$

$$\sigma_2 = -4142 / A$$

1) Rankine,

$$\sigma_1 \leq \sigma_o$$

$$\frac{24142}{\pi/4 d^2 A} \leq \frac{270}{f.o.s} = 3$$

$$d \geq 18.48 \text{ mm.}$$

2) Vianant theory,

$$(\sigma_1 - \mu \sigma_2) \leq (\sigma_o = f_y / f.o.s)$$

$$\left[\frac{24142}{\pi/4 d^2 A} - 0.3 \times \left(\frac{-4142}{A} \right) \right] \leq \frac{270}{3}$$

$$d \geq 18.95 \text{ mm.}$$

3) Tresca,

$$\sigma_1 - \sigma_2 \leq \sigma_o$$

$$\frac{24142}{A} - \left(\frac{-4142}{A} \right) \leq \frac{270}{3}$$

$$d \geq 20.00 \text{ mm}$$

4) Haig theory,

$$\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)} \leq \sigma_b$$

$$\sqrt{\left(\frac{24142}{A}\right)^2 + \left(\frac{-4142}{A}\right)^2 + 0 - 2 \times 0.3 \left(\frac{24142}{A}\right) \left(\frac{-4142}{A}\right) + 0 + 0} \leq \sigma_b$$

$$\frac{25690.27}{\pi/4 d^2} \leq \frac{270}{3}$$

$$d \geq 19.06 \text{ mm}$$

5) Vonmises theory,

$$\sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \leq \frac{f_y}{F}$$

$$\sqrt{\frac{1}{2}\left[\left(\frac{24142}{A} - \left(\frac{-4142}{A}\right)\right)^2 + \left(\frac{-4142}{A}\right)^2 + \left(\frac{-24142}{A}\right)^2\right]} \leq \frac{f_y}{3}$$

$$\frac{26457.3}{\pi/4 \times d^2} \leq \frac{270}{3}$$

$$d \geq 19.35 \text{ mm}$$

IV. TORSION & PRINCIPAL STRESSES

THEORY OF FAILURE

1. Torque is the moment applied about the axis of the member [about z-axis] this phenomenon is called Torsion.
2. The Torque develops shear stress.
3. The torque is represented by symbol 'T'.

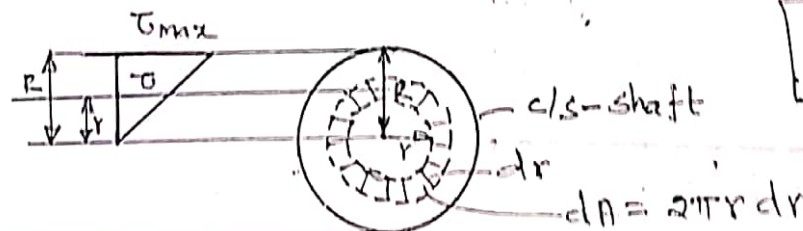
$$T = \text{Force} \times \text{radius}$$

$$T = F \times r$$

$$\text{Unit} = \text{N-m} \quad ; \quad \text{Dimension} = \text{ML}^2\text{T}^{-2}$$

4. Torsion is mainly used for shaft-drivers to transmit power on the shaft.

5. Expression for Torque:-



From IInd Δth

$$\left[\tau = \frac{\tau_{\max}}{R} \times r \right]$$

$$dT = dF \times r$$

$$= (\tau \times dA) \times r$$

$$= \tau \times 2\pi r \times dr \times r$$

$$dT = \tau_{\max} \times \frac{r}{R} \times 2\pi r dr \times r$$

$$T = \int_0^R \frac{\tau_{\max}}{R} \times 2\pi r^3 dr$$

$$= \frac{2\pi \tau_{\max}}{R} \left[\frac{r^4}{4} \right]_0^R = \frac{2\pi \tau_{\max} \times R^4}{4R}$$

$$\boxed{T = \frac{\pi \tau_{\max} R^3}{2}}$$

$$R = \frac{D}{2}$$

$$T = \frac{\pi \tau_{max}}{2} \left(\frac{D}{2}\right)^3$$

$$= \frac{\pi}{2} \tau_{max} \times \frac{D^3}{8} \times \frac{D}{D}$$

$$= \frac{\pi}{2} \times \tau_{max} \times \frac{D^4}{8 \times 2R}$$

$$= \frac{\pi}{32} \times \tau_{max} \times D^4 \times \frac{1}{R}$$

$$= \frac{\pi}{32} \times D^4 \times \tau_{max} \times \frac{1}{R} \quad \left[\frac{\pi}{32} D^4 = J \right]$$

$$T = \tau_{max} \times J \times \frac{1}{R}$$

$$\boxed{\frac{T}{J} = \frac{\tau_{max}}{R}} \quad ***$$

Polar M.I = $I_P = J = MI$ about z-z axis

$$= I_{xx} + I_{yy}$$

$$= \frac{\pi}{64} D^4 + \frac{\pi}{64} D^4$$

$$\boxed{J = I_P = \frac{\pi}{32} D^4} \quad \text{— solid shaft}$$

$$\boxed{J = I_P = \frac{\pi}{32} [D^4 - d^4]} \quad \text{— Hollow shaft}$$

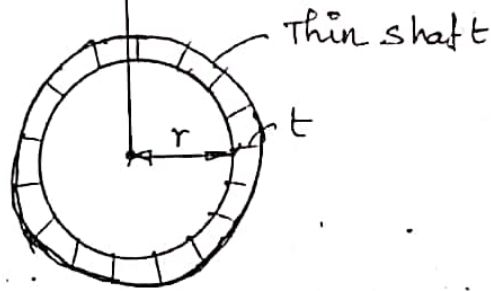
$$T = \frac{\tau_{max}}{R} \times J$$

$$= \tau_{max} \times \frac{\pi D^4}{32 \times 16}$$

$$\boxed{T = \frac{\pi D^3}{16} \tau_{max}}$$

$$\boxed{\tau_{max} = \frac{16T}{\pi D^3}}$$

For thin shaft →



Polar M.I = Moment of moment of Area

= Second moment of area

$$= (2\pi r t) \times r \times r$$

$$= 2\pi t r^3$$

$$= 2\pi t \times \left(\frac{D}{2}\right)^3$$

$$= 2\pi t \frac{D^3}{8}$$

$$I_{\text{polar}} = \frac{\pi D^3 t}{4}$$

For thin shaft:-

$$\frac{T}{J} = \frac{\tau_{\text{max}}}{R}$$

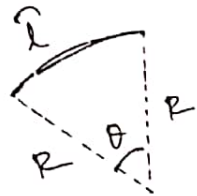
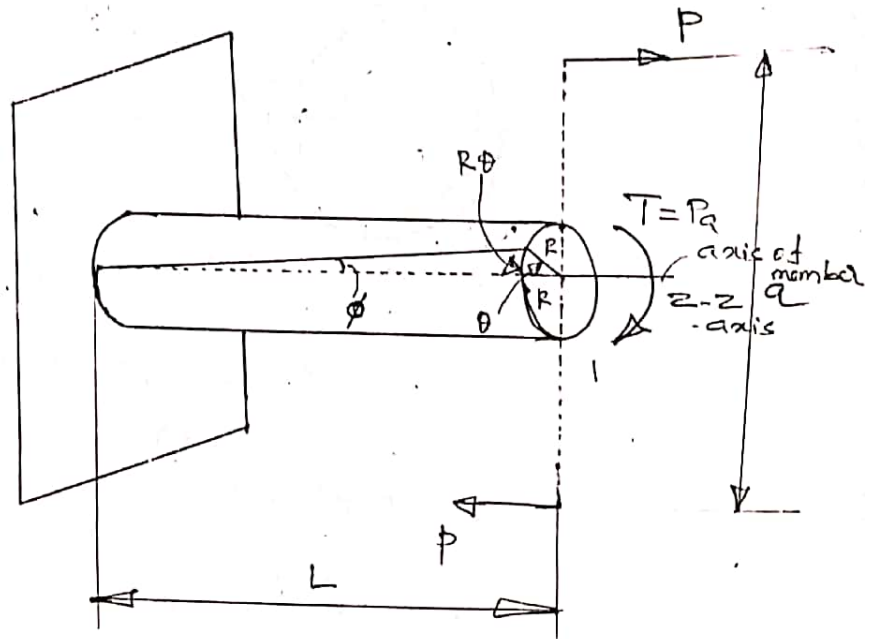
$$\frac{T}{\frac{\pi D^3 t}{4}} = \frac{\tau_{\text{max}}}{\frac{D}{2}}$$

$$\frac{T}{\frac{\pi D^3 t}{4}} = \frac{\tau_{\text{max}}}{\frac{D}{2}}$$

Note: →

In case of Hollow shaft, the shear stress distribution is non-uniform across the thickness. ~~But~~ therefore law of integration is required. But in case of thin shaft the shear stress distribution is almost uniform. Therefore there is no need of applying integration.

Expression for Angle of twist:-



$$\phi = \frac{l}{R}$$

$$l = R\theta$$

$$\tan \phi = \frac{R\theta}{L}$$

$$\left[\phi = \frac{R\theta}{L} \right] \because \left[\phi = \text{is very small} \right]$$

$$\left[\tan \phi \approx \phi \right]$$

ϕ = shear strain

∴ shear stress = shear strain × Modulus of Elasticity

$$\tau = \phi \times G$$

$$\phi = \frac{\tau}{G}$$

$$[C = G = N]$$

$$\frac{R\theta}{L} = \frac{\tau}{G}$$

$$** \quad \frac{G\theta}{L} = \frac{\tau}{R}$$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

$$\frac{T}{J} = \frac{G\theta}{L}$$

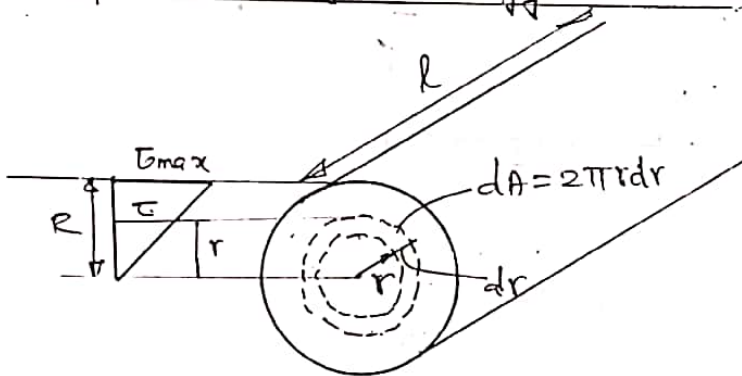
$$\boxed{\frac{TL}{\theta} = GJ} = \text{Torsional Rigidity.}$$

$$\boxed{\frac{T}{\frac{\pi}{32} D^4} = \frac{\tau_{\max}}{D/2} = \frac{G\theta}{L}}$$

$D = ?$

Adopt greater value of 'D'.

Expression for Energy due to Torsion:-



$$U = \frac{f^2}{2E} \times \text{volume}$$

$$dU = \frac{\tau^2}{2G} \times dV$$

$$= \frac{\tau^2}{2G} \times (dA \times l)$$

$$= \frac{\tau^2}{2G} \times 2\pi r dr \times l$$

$$dU = \left(\tau_{\max} \times \frac{r}{R} \right)^2 \times \frac{2\pi r dr \times l}{2G}$$

$$U = \int_0^R \frac{\tau_{\max}^2}{2GR^2} \times 2\pi r^3 l dr$$

$$= \frac{\tau_{\max}^2 \times \pi l}{GR^2} \int_0^R r^3 dr$$

$$= \frac{\pi l \tau_{max}^2}{4 R^2} \left[\frac{r^4}{4} \right]_0^R$$

$$U = \frac{\pi l \tau_{max}^2}{4 R^2} \times \frac{R^4}{4}$$

$$= \frac{\tau_{max}^2 \times \pi R^2 \times l}{4 b}$$

$$= \frac{\tau_{max}^2}{4 b} \times A \times l$$

$$U = \frac{\tau_{max}^2}{4 b} \times \text{Volume}$$

$$U = \frac{\tau_{max}^2}{4 b} \times \text{Volume}$$

Note!:-

Modulus of resilience

$$U = \frac{f^2}{2E} \times \text{Volume} - \text{Axially applied load}$$

$$= \frac{\tau^2}{4b} \times \text{Volume} - \text{Torsion Effect}$$

$$= \frac{\tau^2}{2b} \times \text{Volume} - \text{Maximally loaded member}$$

$$= \frac{p^2}{2k} \times \text{Volume} - \text{Shear stress (SF)}$$

$$= \int \frac{M^2 ds}{2EI} - \text{Volumetric change due to Hydrostatic force (Bulk mod)}$$

$$= \int \frac{V^2 ds}{2bA} - \text{Bending (flexural)}$$

$$= \int \frac{V^2 ds}{2bA} - \text{Shear force}$$

EI - Flexural rigidity.

GA - Shear rigidity.

GT - Torsional rigidity.

AE - Axial rigidity.

$$U = \frac{\tau_{max}^2}{4G} \times \text{Volume}$$

$$\left[\frac{\tau}{J} = \frac{\tau_{max}}{R} \right]$$

$$= \frac{\left[\frac{\tau}{J} \times R \right]^2}{4G} \times \text{Volume}$$

$$= \frac{\tau^2 R^2}{J^2 \times 4G} \times \pi R^2 \times L$$

$$= \frac{\tau^2 L}{J^2 \times 4G} \times \pi R^4$$

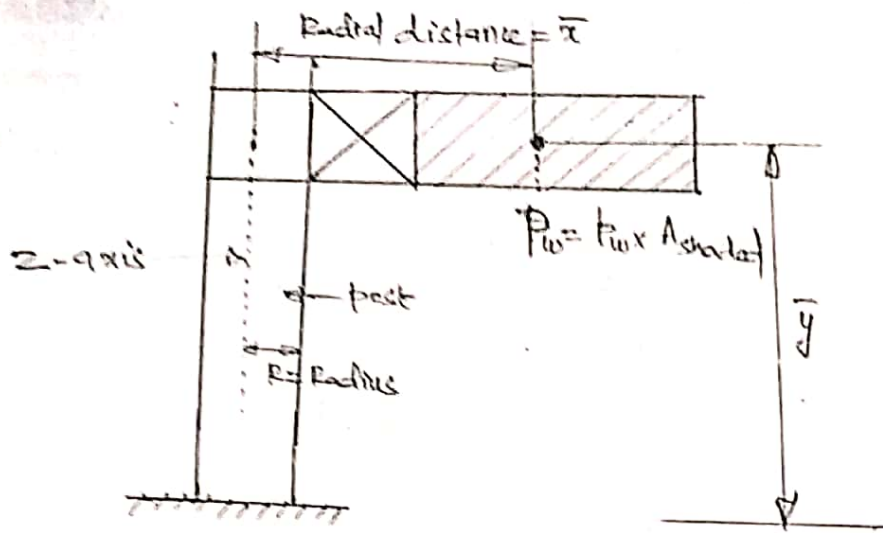
$$= \frac{\tau^2 L}{\left(\frac{\pi}{32} \times D^4 \right)^2 \times 4G} \times \pi \times \left(\frac{D}{2} \right)^4$$

$$= \frac{\tau^2 L}{J^2 \times 2G} \times \frac{\pi D^4}{32}$$

$$= \frac{\tau^2 L}{J^2 \times 2G} \times J$$

$$\boxed{U = \frac{\tau^2 L}{2GJ}}$$

Expression for Equivalent Torque & equivalent Bending moment due to combined effect of Torsion & Bending moment:-



$$P_w = p_w \times A_{\text{sheared}}$$

$$\text{Torque, } T = [P_w \times \bar{x}]$$

$$\text{BM, } M = P_w \times \bar{y}$$

Due to torque, shear stress develops,

$$\begin{aligned} \frac{T}{J} &= \frac{\tau}{R} \Rightarrow \tau = \frac{T}{J} \times R \\ &= \frac{T \times \frac{D}{2}}{\frac{\pi}{32} D^4} \end{aligned}$$

✓

$$\tau = \frac{16T}{\pi D^3}$$

Due to BM, Bending stress develops,

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$y = \frac{M}{I} \times y$$

$$\sigma_x = \frac{M}{\frac{\pi D^4}{64}} \times \frac{D}{2}$$

$$\sigma_x = \frac{32M}{\pi D^3}$$

✓ $\sigma_y = 0$
[other direction]
[fall down]

Principal stress,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$= \frac{\frac{32M}{\pi d^3} + 0}{2} \pm \frac{1}{2} \sqrt{\left(\frac{32M}{\pi d^3} - 0\right)^2 + 4 \times \left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{16M}{\pi d^3} \pm \frac{1}{2} \sqrt{\left(\frac{16}{\pi d^3}\right)^2 [(2M)^2 + 4 \times T^2]}$$

$$\checkmark \sigma_{1,2} = \frac{16M}{\pi d^3} [M \pm \sqrt{M^2 + T^2}]$$

Major principal stress,

$$\sigma_1 = \frac{16}{\pi d^3} [M + \sqrt{M^2 + T^2}]$$

= Resultant Bending stress
[Equivalent bending stress]

$$\frac{M}{I} = \frac{f}{y}$$

$$\frac{M_{equi}}{\frac{\pi}{64} d^4} = \frac{\sigma_1}{\frac{d}{2}}$$

$$\frac{32 M_{equi}}{\pi d^3} = \frac{16}{\pi d^3} [M + \sqrt{M^2 + T^2}]$$

$$\checkmark \boxed{M_{equi} = \frac{1}{2} [M + \sqrt{M^2 + T^2}]}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \text{Radius of Mohr's Circle}$$

$$= \left[\frac{16}{\pi d^3} [M + \sqrt{M^2 + T^2}] - \frac{16}{\pi d^3} [M - \sqrt{M^2 + T^2}] \right] \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{16}{\pi d^3} \left[2 \sqrt{M^2 + T^2} \right]$$

$$\tau_{max} = \frac{16}{\pi d^3} \left[\sqrt{M^2 + T^2} \right] \quad \text{--- ①}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{T_{equi}}{\frac{\pi d^4}{32}} = \frac{\tau_{max}}{\frac{d}{2}}$$

$$\tau_{max} = \frac{16 T_{equi}}{\pi d^3} \quad \text{--- ②}$$

$$\text{①} = \text{②} \Rightarrow$$

$$\frac{16 T_{equi}}{\pi d^3} = \frac{16}{\pi d^3} \left[\sqrt{M^2 + T^2} \right]$$

$$\checkmark \boxed{T_{equi} = \sqrt{M^2 + T^2}}$$

power transmitted by shaft:-

$$\text{Power} = \frac{W.D}{\text{time}} = \frac{F \times \text{distance}}{t}$$

$$= F \times \sqrt{r} (\text{linear})$$

$$= F \times r \omega$$

$$\text{power} = T \omega$$

$$\boxed{\text{power} = \frac{2\pi NT}{60}}$$

Theories of Failure:—

1. Rankine's theory :- [Maximum principal stress theory]
[Brittle material]

$$\sigma_1 \leq \sigma_0$$

$$\frac{16}{\pi D^3} [M + \sqrt{M^2 + T^2}] \leq \left[\sigma_0 = \frac{f_y}{FOS} \right]$$

2. Saint Venant theory :- [Maximum principal strain theory]

$$\epsilon_1 \leq \epsilon_0$$

$$\frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \leq \frac{\sigma_0}{E}$$

$$\sigma_1 - \mu \sigma_2 \leq \frac{f_y}{FOS}$$

3. Tresca, or, Guest Theory :- [Ductile material]
[Maximum shear stress theory]

$$\frac{\sigma_1 - \sigma_2}{2} \leq \frac{\sigma_0}{2}$$

$$\sigma_1 - \sigma_2 \leq \frac{f_y}{FOS}$$

4. Haig theory :- [Total strain energy theory]
[Ductile material] [thin cylinder]

$$\sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2)} \leq \sigma_0 = \frac{f_y}{FOS}$$

5. Von Mises theory :- [~~Max~~ shear strain energy]
[Distortion]

$$\sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \leq \left[\sigma_0 = \frac{f_y}{FOS} \right]$$

Numericals: →

1. A solid shaft has to transmit mean power of 1000 kW at 300 rpm. The maximum shear stress is 60 MPa. Determine the dia of shaft if the maximum torque is 25% more than the mean torque. Also determine % saving in material if hollow shaft having inner dia 0.6 times of outer is used.

$$\frac{T}{J} = \frac{\tau}{R} ; \quad P = \frac{2\pi NT_{\text{mean}}}{60}$$
$$1000 \times 10^3 = \frac{2\pi N T_{\text{mean}}}{60}$$

$$T_{\text{mean}} = 31830.99 \text{ N-m}$$

$$T_{\text{max}} = 0.25 \times T_{\text{mean}} + T_{\text{mean}}$$
$$= 0.25 \times 31830.99 + 31830.99$$
$$= 7957.75 + 31830.99$$
$$= 39788.74 \text{ N-m}$$

$$\frac{39788.74}{\frac{\pi \times D^4}{32 \times 16}} = \frac{60 \times 10^6}{2}$$

$$D = 0.15 \text{ m} = 150 \text{ mm}$$

Hollow shaft.

$$\frac{T}{J} = \frac{\tau}{R}$$
$$\frac{39788.74}{\frac{\pi (D^4 - d^4)}{32}} = \frac{60 \times 10^6}{2}$$

$$\frac{39738.74}{\frac{\pi}{16} (D^4 - (0.6D)^4)} = \frac{60 \times 10^6}{D}$$

$$D = 0.157 \text{ m}$$

$$D_H = 157 \text{ mm}$$

$$\% \text{ Saving in material} = \frac{\text{wt. of solid shaft} - \text{wt. of hollow shaft}}{\text{wt. of solid shaft}}$$

$$= \frac{W_S - W_H}{W_S} \times 100$$

$$= \frac{\gamma_S A_S L_S - \gamma_H A_H L_H}{\gamma_S A_S L_S}$$

$$= \frac{A_S - A_H}{A_S}$$

$$= \frac{\frac{\pi}{4} D_S^2 - \frac{\pi}{4} (D^2 - d^2)}{\frac{\pi}{4} D_S^2}$$

$$= \frac{D_S^2 - D^2 + (0.6D)^2}{D_S^2}$$

$$= 29.76\%$$

2. A thin cylinder having dia 300mm & thickness 10mm is subjected to external torque 100N-m & inner fluid pressure 1MPa. Determine,

- i) ~~Maximum~~ shear stress due to torque
- ii) Maximum principal stress
- iii) Maximum shear stress

IV) Absolute Maximum shear stress.

Soln:-

$$t = 10 \text{ mm}$$

$$T = 100 \times 1000 \text{ N-mm}$$

$$R = \frac{D}{2} = \frac{300}{2} = 150 \text{ mm}$$

$$\tau = ?$$

$$\frac{T}{\frac{\pi D^3 t}{4}} = \frac{\tau}{\frac{D}{2}}$$

$$\frac{100 \times 10^3}{\frac{\pi \times 300^3 \times 10}{4}} = \frac{\tau}{\frac{300}{2}}$$

$$\tau = 0.071 \text{ N/mm}^2$$

$$\sigma_1 = \frac{pD}{2t} \quad \sigma_2 = \frac{pD}{4t}$$

$$\sigma_2 = \text{Hoop stress} = \frac{pD}{2t} = \frac{1 \times 300}{2 \times 10} = 15 \text{ MPa}$$

$$\sigma_y = \text{Longitudinal stress} = \frac{pD}{4t} = 7.5 \text{ MPa}$$

$$\begin{aligned} \sigma_{1,2} &= \frac{\sigma_2 + \sigma_y}{2} \pm \frac{1}{2} \sqrt{4\tau^2 + (\sigma_2 - \sigma_y)^2} \\ &= \frac{15 + 7.5}{2} \pm \frac{1}{2} \sqrt{4 \times 0.071^2 + (15 - 7.5)^2} \\ &= 11.25 \pm 3.75 \end{aligned}$$

$$\sigma_1 = 15.00 \text{ MPa}$$

$$\sigma_2 = 7.50 \text{ MPa}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 3.75 \text{ MPa} \dots$$

$$\begin{aligned} \text{Absolute maximum shear} &= \frac{\sigma_1}{2} \\ &= \frac{15}{2} \\ &= 7.5 \text{ MPa} \dots \end{aligned}$$

V. SFD & BMD

1. ✓ Beam is a structural member which is subject to load \perp to the axis of the member [Transverse load] & it transfers load to the support through Bending only. Beam is a bending member which is design on the basis of the maximum bending moment & maximum shear force.

2. There are following types of load :-

a) point load [concentrated load] $[W]$ $[KN]$

b) Uniformly distributed load (UDL) (W) (KN/m) or Rectangular load.

c) Gradually varying load [GVL] or Triangular load $[0 \text{ to } w]$ $[KN/m]$

13. BM The algebraic sum of all moments consider from extreme end to any section of the beam is called Bending moment at that section

4. The graphical representation of Bending moment along with its nature (+ or -) called Bending moment diagram [BMD] which is very-very essential to find position of steel bars in RCC structures [Beams].

If the Bending moment is consider from left to right then clockwise moment is plus & Anticlockwise called minus.

But if the moment is consider from right to left then clockwise called minus & anti-clockwise called plus.

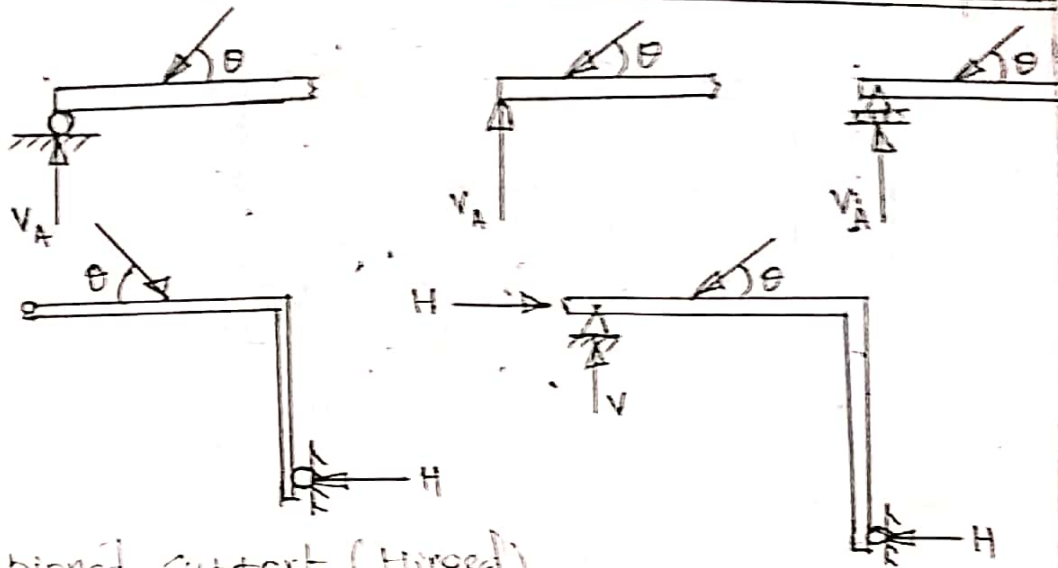
5. The algebraic sum of all forces consider from extreme end to any section of the beam is called shear force at that section. The graphical representation of shear force along with its nature is called shear force diagram (SFD) which is very, very essential to determine the position of stirrups in RCC beams.

If the force is upward, consider from left to right, is considered as plus & downward force is taken as minus but if it is consider from right to left then upward force is taken as minus & downward force is taken as plus.

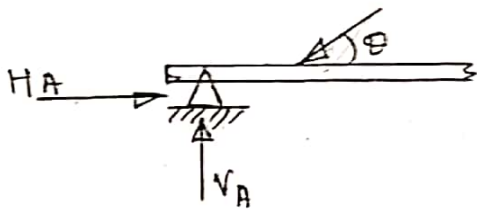
Any force developing clockwise moment from left to right then that force is consider as plus & if it develops anticlockwise moment [left to right] then that force is consider as negative.

b. Support reaction [Type of support]

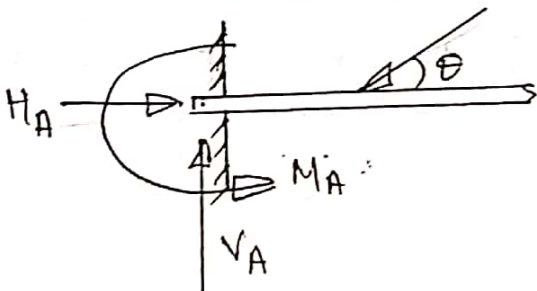
a) Roller support, simple support, Free support



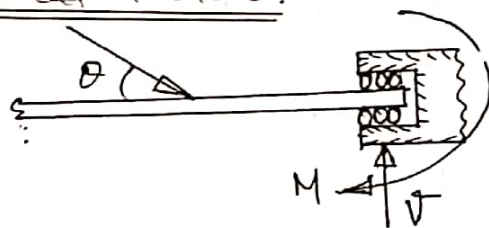
b) pinned support (Hinged)



c) Fixed support [Built-in] [Encastered]

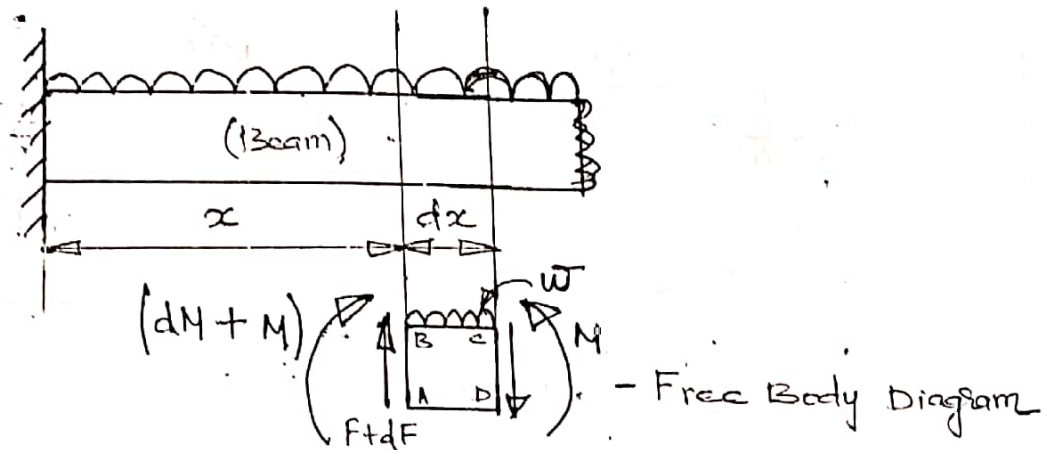


d) Guided Roller:-



7. Relation among shear force, Bending moment & Load:-

7. Relation among Shear force, Bending moment & Load:-



Law of Equilibrium;

$$\sum V = 0$$

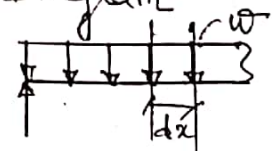
$$+(F+dF) - F - w \cdot dx = 0$$

$$dF = w \cdot dx$$

$$\boxed{\frac{dF}{dx} = w} \text{ - intensity of load}$$

$$dF = w \cdot dx$$

$$\boxed{F = \int w \cdot dx} \text{ - Area of loading Diagram}$$



$$\sum M_A = 0;$$

$$-F \cdot dx + M - w \cdot dx \cdot \frac{dx}{2} - (M+dM) = 0$$

$$-F \cdot dx - w \cdot \frac{(dx)^2}{2} - dM = 0$$

neglected $-dx$ - very small, $(dx)^2$ - very-very small.

$$dM = -F \cdot dx$$

$$\boxed{\frac{dM}{dx} = -F}$$

Rate of change of BM with respect to distance is called shear force

$$\boxed{\frac{dM}{dx} = S.F}$$

For maximum Bending moment,

$$\frac{dM}{dx} = 0.$$

$$\text{Shear force} = 0.$$

$$V_{xx} = 0 \rightarrow \text{for maximum BM}$$

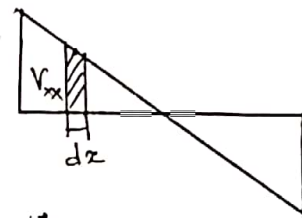
$$\frac{dM}{dx} = V_{xx}$$

$$dM = V_{xx} dx$$

$$\int dM = \int V_{xx} dx$$

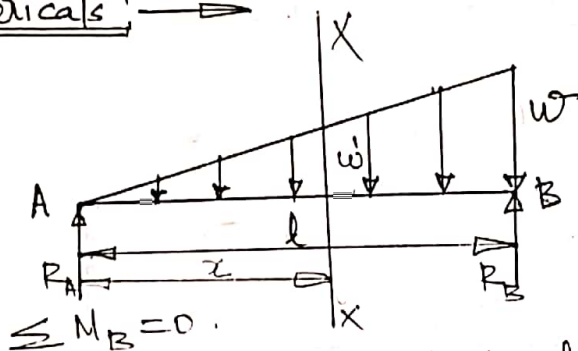
$$M = \int V_{xx} dx$$

$$\text{BM} = \text{Area of SFD}$$



Numericals: →

1.



(Triangular load)
Total load, $= \frac{1}{2} w \times l$

$$R_A \times l = \frac{1}{2} \times w \times l \times \frac{l}{3}$$

$$R_A = \frac{wl}{6} ; R_B = \frac{1}{2} \times w \times l - \frac{wl}{6} = \frac{wl}{3}$$

For Maximum BM,

$$V_{xx} = 0$$

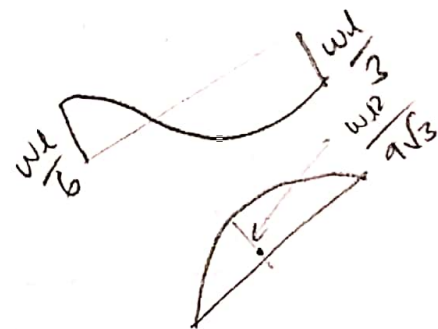
$$\frac{wl}{6} - \frac{1}{2} \times w' \times x = 0.$$

$$\frac{wl}{6} - \frac{1}{2} \times w \times \frac{x}{l} \times x = 0.$$

$$\frac{wx^2}{2l} = \frac{wl}{6.3}$$

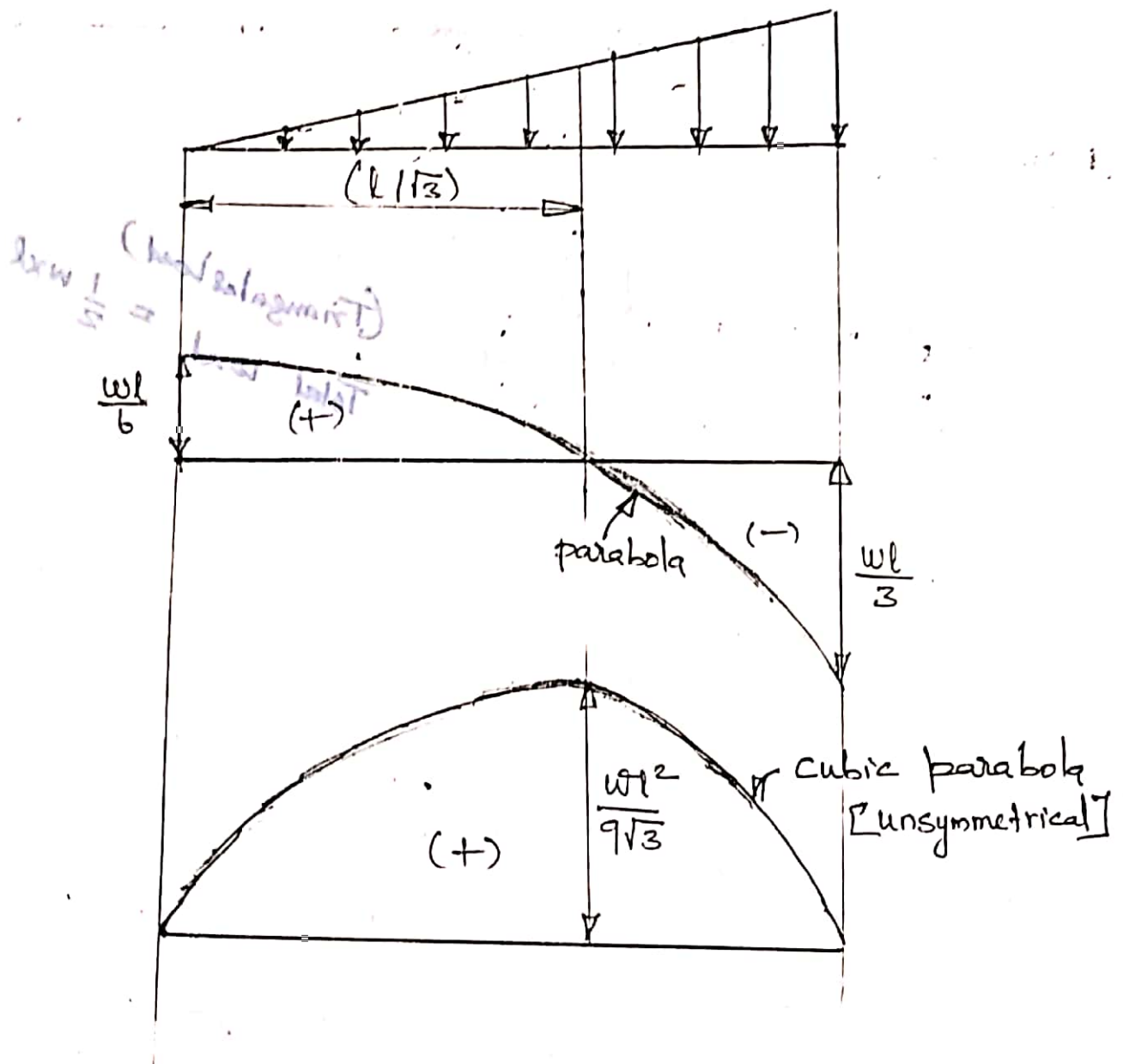
$$x^2 = \frac{l^2}{3}$$

$$x = \frac{l}{\sqrt{3}} = 0.577l$$



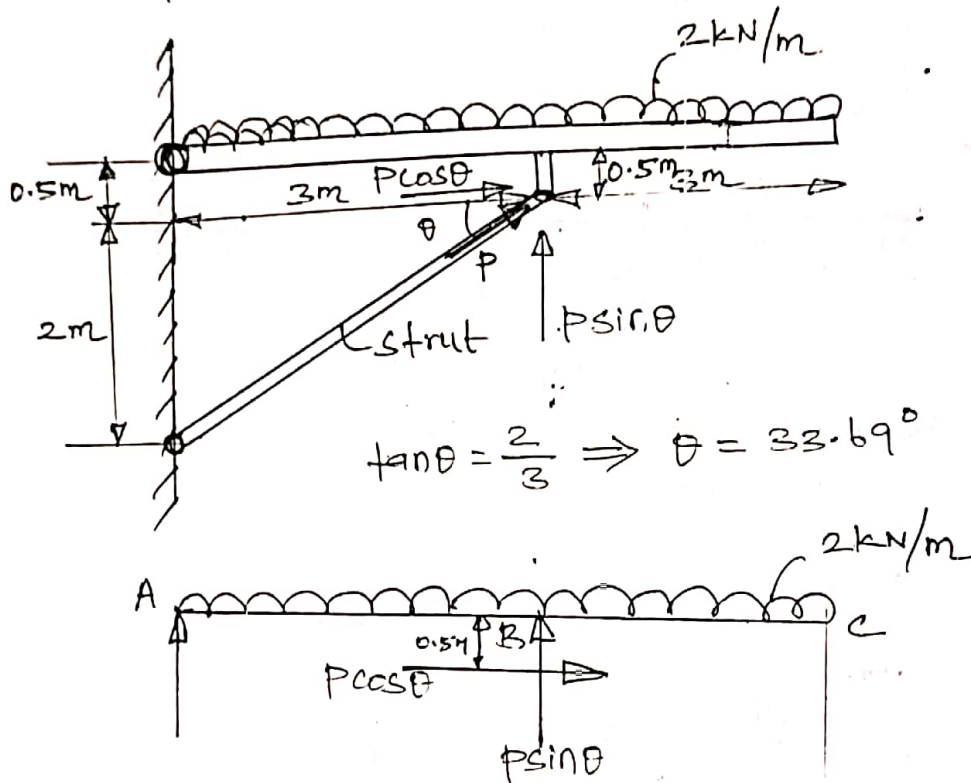
$$\begin{aligned}
 M_{xx} = M_{max} &= \frac{wl}{b} \times x - \frac{1}{2} \times \frac{wl}{l} \times x \times \frac{x}{2} \\
 &= \frac{wlx}{b} - \frac{wlx^2}{b \cdot l} \quad \text{Cubic parabolic BMD} \\
 &= \frac{wlx}{\sqrt{3} \times b} - \frac{w}{bl} \times \left(\frac{l}{\sqrt{3}} \right)^3 \\
 &= \frac{wl^2}{b\sqrt{3}} - \frac{wl^2}{b \times 3 \times \sqrt{3}}
 \end{aligned}$$

$$M_{max} = \frac{wl^2}{9\sqrt{3}}$$



2. A beam is supported by a strut as shown in figure below. Determine,

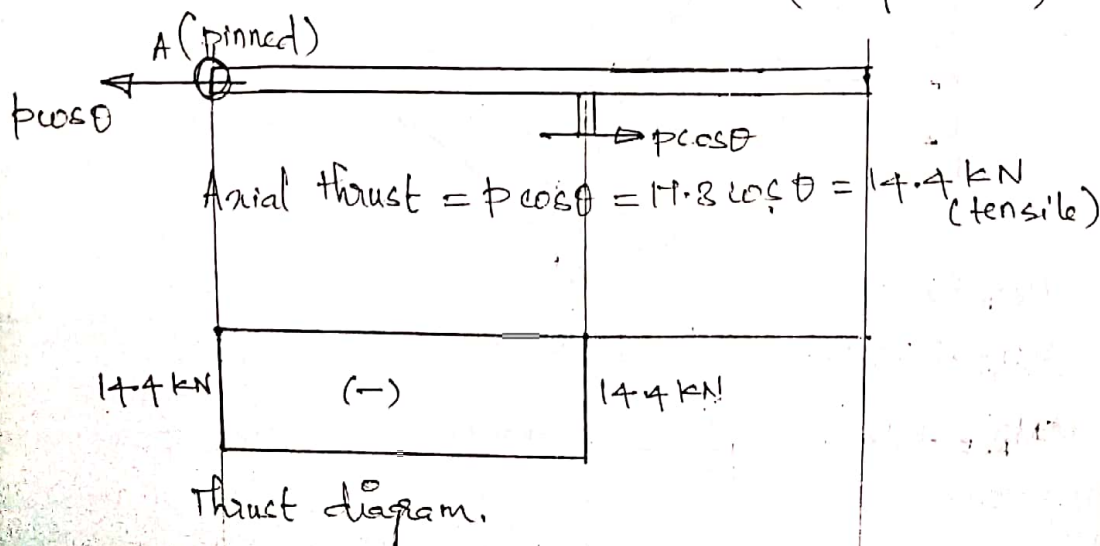
- (i) Force in the strut
- (ii) Thrust in the beam
- (iii) Maximum bending moment in the beam
- (iv) Maximum shear force in the beam.
- (v) SFD & BMD



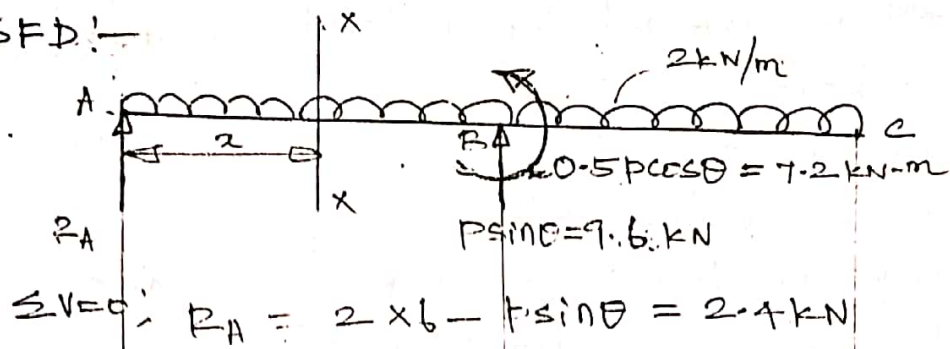
$$\sum M_A = 0;$$

$$- 2 \times \frac{6^2}{2} + P \cos \theta \times 0.5 + P \sin \theta \times 3 = 0$$

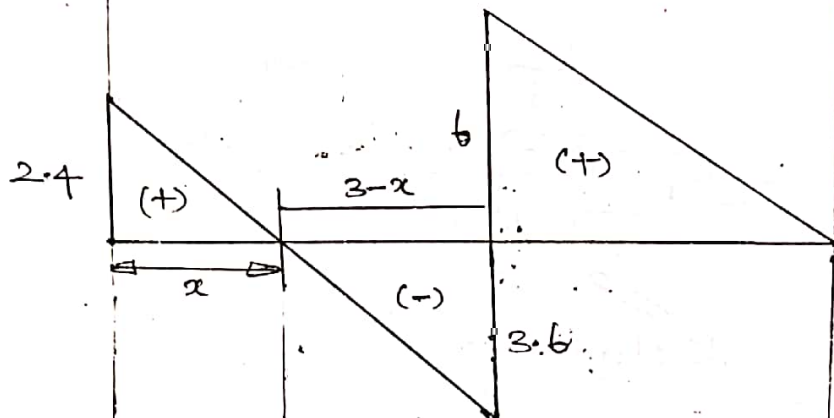
Force in the strut, $P = +17.30 \text{ kN}$
(compressive)



SFD:-



$$\sum V = 0; R_A = 2 \times 3 - 9.6 = 2.4 \text{ kN}$$



Maximum shear force = 1 kN

$$\frac{x}{2.4} = \frac{3-x}{3.6} \Rightarrow 3.6x = 7.2 - 2.4x$$

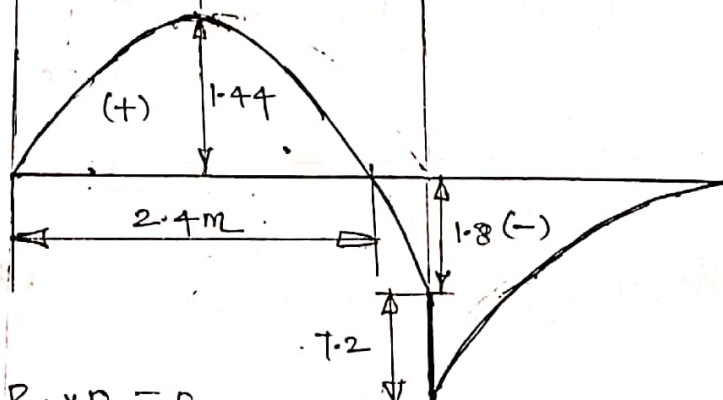
$$\boxed{x = 1.2 \text{ m}}$$

(or)

For max^m BM, $V_{xx} = 0; R_A - 2x = 0$

$$\boxed{x = 1.2 \text{ m}}$$

BMD:-



$$M_A = R_A \times 0 = 0$$

$$M_{xx} = R_A \times x - \frac{2x^2}{2} = 1.44 \text{ kN-m}$$

$$M_{A-B} = R_A \times 3 - \frac{2 \times 3^2}{2} = -1.8 \text{ kN-m}$$

$$M_B = -1.8 - 7.2 = -9 \text{ kN-m.}$$

$$M_C = 0$$

For zero BM, $M_{xx} = 0$

$$R_A x - \frac{2x^2}{2} = 0$$

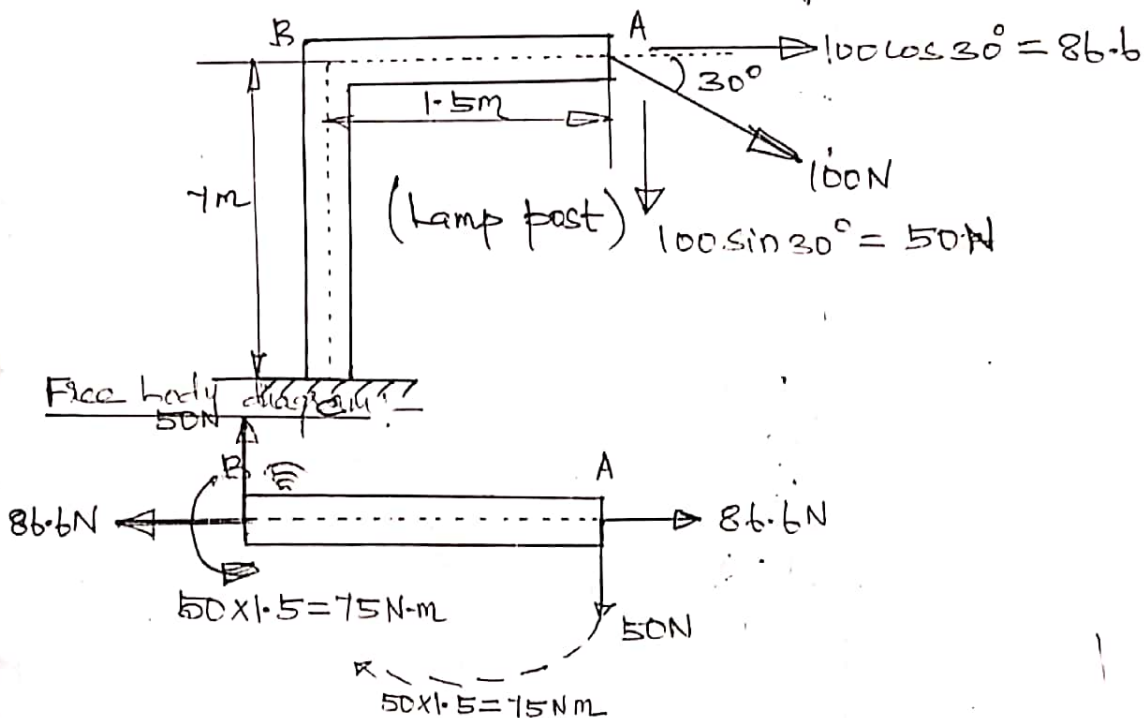
$$2.4x - x^2 = 0.$$

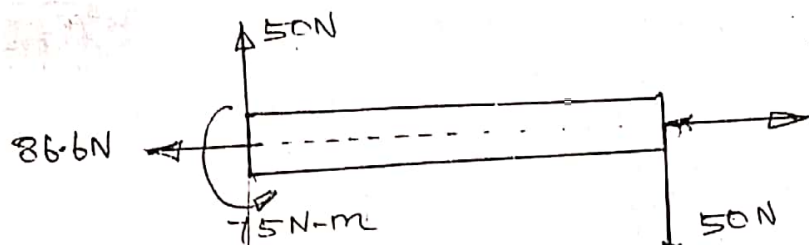
$$x = 0, 2.4 \text{ m.}$$

$x = 2.4 \text{ m}$ — point of inflexion.

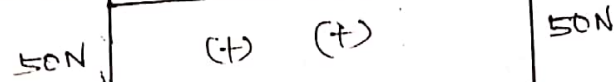
3. A structural member is shown in figure below, determine

- (i) Maximum shear force for the beam,
- (ii) Maximum BM for the beam,
- (iii) Maximum thrust in the beam,
- (iv) Maximum BM & thrust in the column,
- (v) SFD & BMD & thrust diagram.

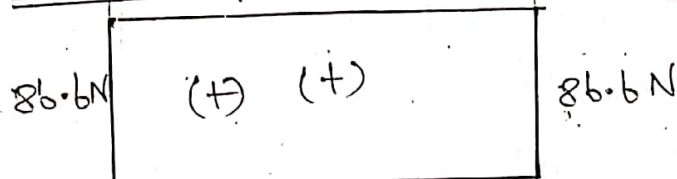




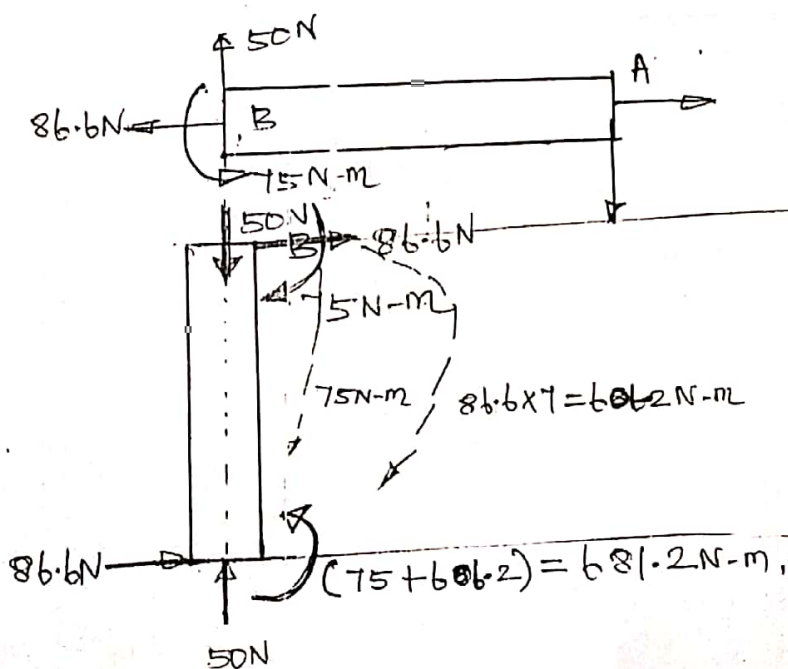
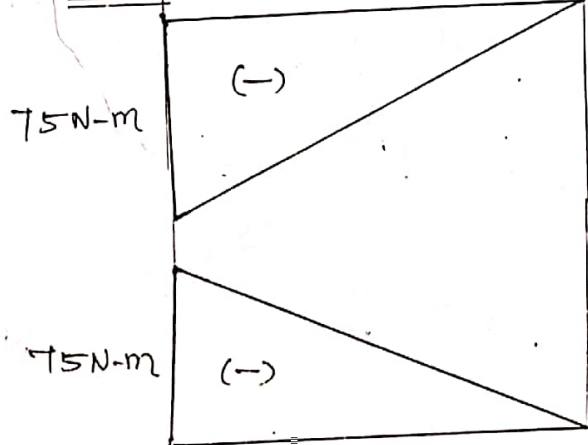
SFD:-

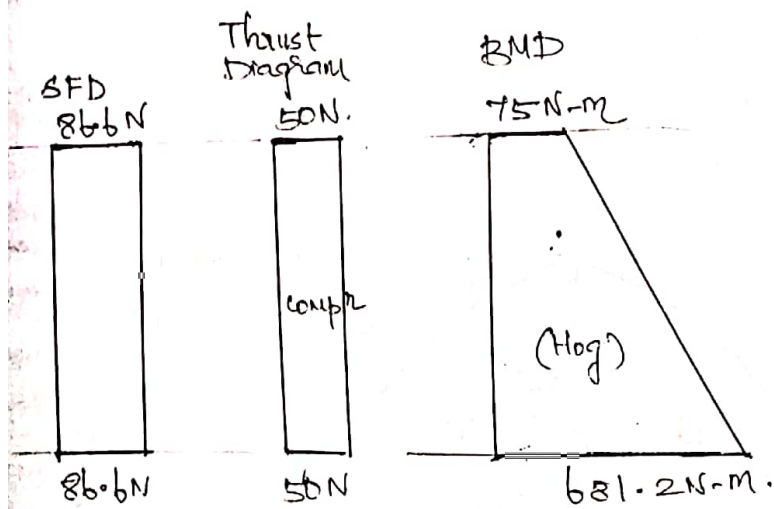


Thrust diagram:-

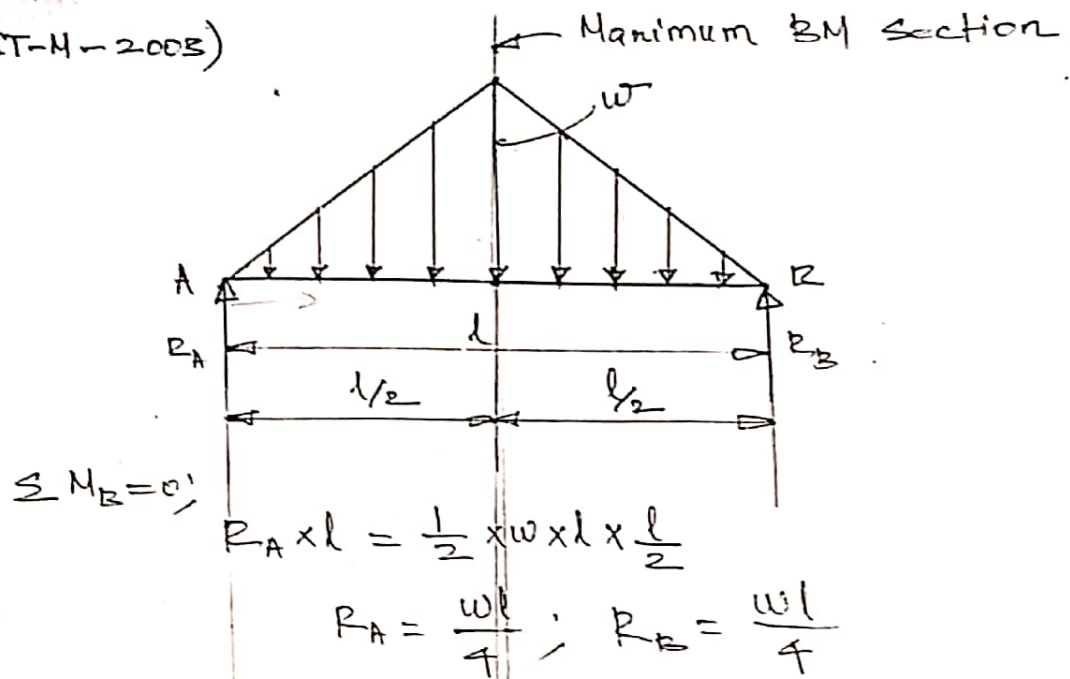


BMD:-

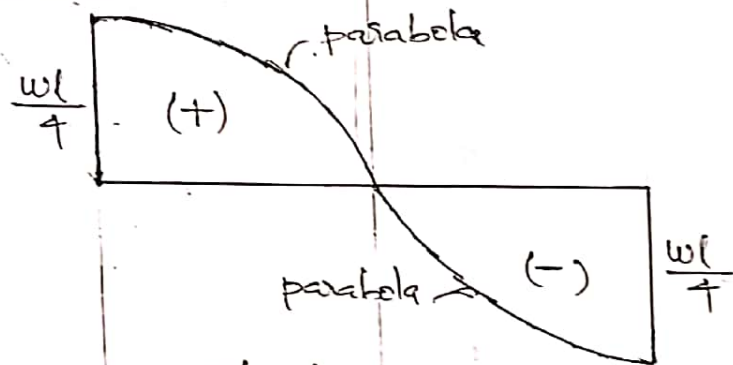




4. (IIT-M-2003)



SFD:-

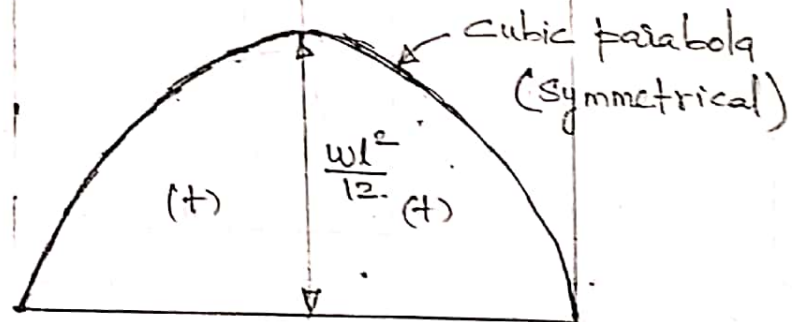


$$M_{max} = M_{mid} = \frac{wl}{4} \times \frac{l}{2} - \frac{1}{2} \times w \times \frac{l}{2} \times \frac{1}{3} \times \frac{l}{2}$$

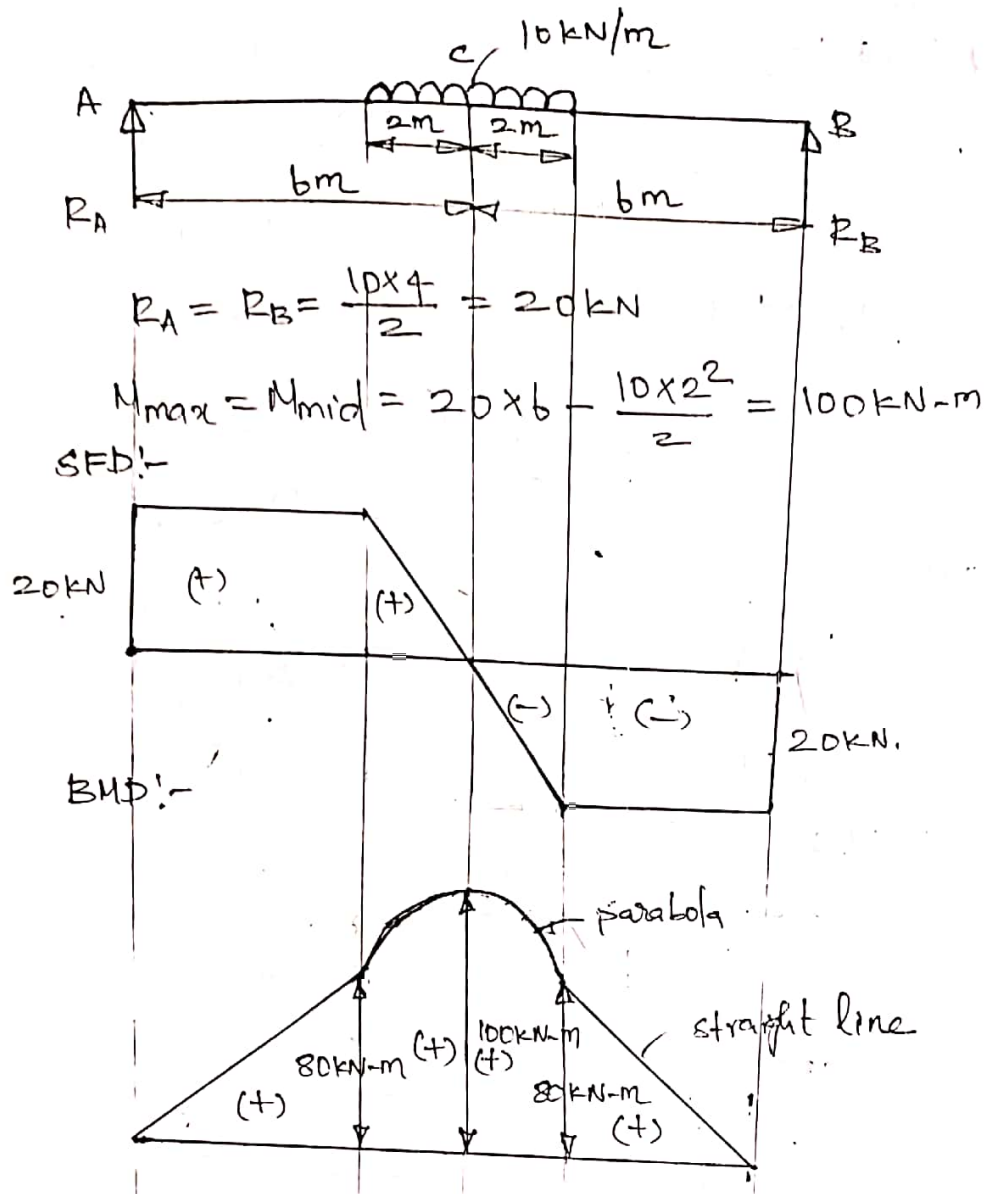
$$= \frac{wl^2}{8} - \frac{wl^2}{24}$$

$$M_{max} = \frac{wl^2}{12}$$

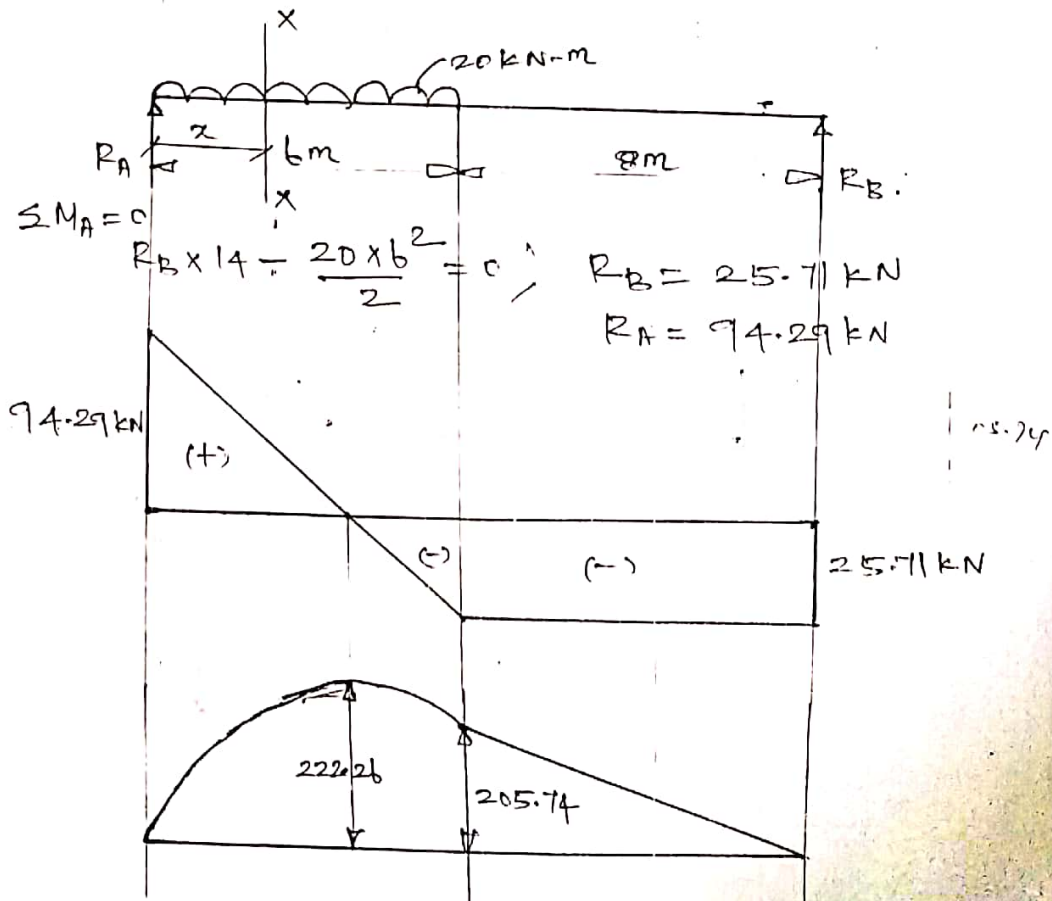
BMD:-



5.



6.



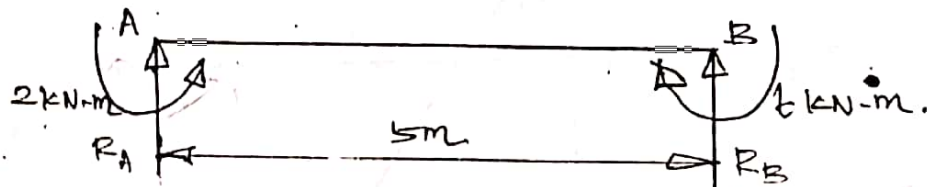
$$V_{xx} = 0, \quad R_A - 20x = 0$$

$$x = \frac{94.29}{20}$$

$$x = 4.71 \text{ m.}$$

$$\begin{aligned} M_{\max} = M_{x-x} &= 94.29 \times x - 20 \times \frac{x^2}{2} \\ &= 94.29 \times 4.71 - 20 \times \frac{4.71^2}{2} \\ &= 222.26 \text{ kN-m} \end{aligned}$$

7. (Hw)



$$R_A + R_B = 0,$$

$$\sum M_B = 0, \quad R_A \times 5 - 2 + 6 = 0.$$

$$R_A = -4/5 \text{ kN}$$

$$R_B = +4/5 \text{ kN}$$

SFD

4/5

(-)

(-)

4/5

BMD

2 kN-m

(-)

(-)

6 kN-m

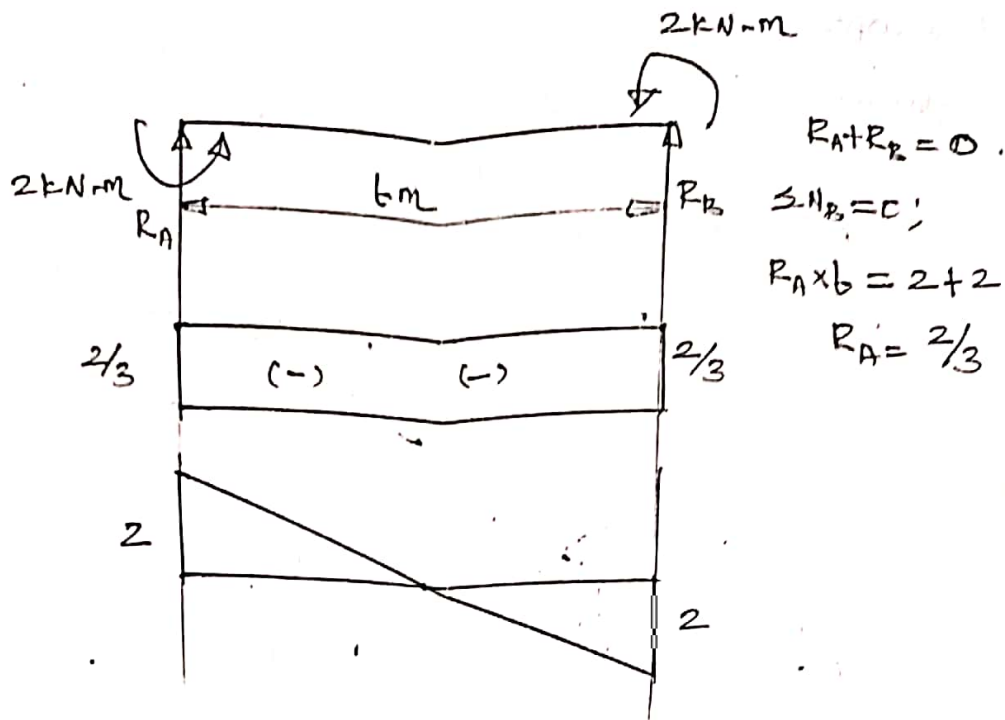
2 kN-m

(-)

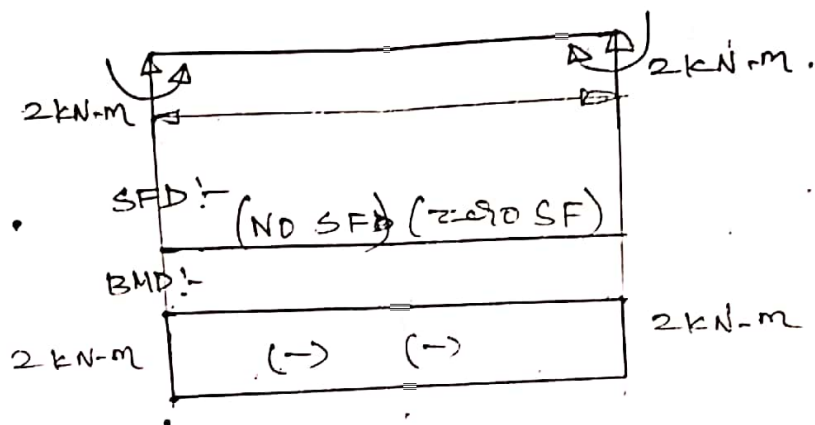
(-)

6 kN-m

8.



9.



10.

