17BTBT102 **MATHEMATICS I** **3 2 0 4**

OBJECTIVES:

To impart analytical ability in solving mathematical problems of Physical or Engineering models.
To understand the concepts of Matrices, Theory of Equations, Differential Calculus and its application, Integral Calculus and its application, Ordinary differential equations**.**

INTENDED OUTCOMES:

This course equips students to have basic knowledge and understanding in the field of matrices, integral and differential calculus.
The students acquire the knowledge of techniques in solving ordinary differential equations that model engineering problems.

UNIT I MATRICES **(12)**
Fundamentals of Matrix- Inverse of a matrix- Rank of a Matrix – Consistency and Inconsistency of a system of 'm' linear equations in 'n' unknowns – Eigenvalues and Eigenvectors of a real matrix.

UNIT II THEORY OF EQUATIONS **(12)**
Relations between coefficients and roots: Irrational and imaginary roots – symmetric functions of the roots – transformation of equations – reciprocal equations and formation of equations whose roots are given.

UNIT III DIFFERENTIAL CALCULUS AND ITS APPLICATION **(12)**
Differentiation and Derivatives of simple functions – Successive Differentiation – Tangent and Normal-Radius of curvature – Velocity and acceleration.

UNIT IV INTEGRAL CALCULUS AND ITS APPLICATIONS **(12)**
Various types of integration - Reduction formula for $e^{ax} x^n$, $\sin^n x$, $\cos^n x$ $\sin^n x \cos^m x$
, (Statement only). – Length, Area and Volume of solid revolution.

UNIT V ORDINARY DIFFERENTIAL EQUATIONS **(12)**
Differential equations of first order and higher degree – higher order differential equations with constant coefficients- Euler's form of Differential equations.

Total: 60

TEXT BOOKS:

S. NO.	AUTHOR(S) NAME	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1	Grewal. B.S	Higher Engineering Mathematics	Khanna Publications, Delhi.	2013
2	B.V.Ramana	Higher Engineering Mathematics	Tata McGraw Hill EducationPvt.Ltd, New Delhi.	2010

REFERENCES:

S. NO.	AUTHOR(S) NAME	TITLE OF THE BOOK	PUBLISHER	YEAR OF PUBLICATION
1	Dass H.K.	Engineering Mathematics	S.Chand& Co., New Delhi.	2008
2	Bali N.P., Manish Goyal	A text book of Engineering Mathematics	Laxmi publications Pvt. Ltd, New Delhi.	2014
3	Michael D. Greenberg	Advanced Engineering Mathematics	Pearson Education, India	2006

WEBSITES:

www.intmath.com
www.efunda.com
www.mathcentre.ac.uk

KARPAGAM ACADEMY OF HIGHER EDUCATION

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COIMBATORE-641 021

DEPARTMENT OF SCIENCE AND HUMANITIES

FACULTY OF ENGINEERING

B.Tech - Biotechnology - (Regular) - II Semester

LESSON PLAN

Name: B.DEEPA

Subject: MATHEMATICS-II **Subject Code: 17BTBT202**

S.NO	Topics covered	No. of hours
	UNIT- I MULTIPLE INTEGRALS	
1.	**Introduction of integration and basic formulas**	1
2.	Double integration in Cartesian coordinates	1
3.	Problems based on double integration in Cartesian coordinates	1
4.	Change of order of integration	1
5.	Problems based on change of order of integration	1
6.	Tutorial 1: Change of order of integration problems	1
7.	Area as a double integral	1
8.	Problems based on area as a double integral	1
9.	Tutorial 2: Area as a double integral problems	1
10.	Triple integration in Cartesian coordinates	1
11.	Problems based on Triple integration in Cartesian coordinates	1
12.	Tutorial 3: Triple integration in Cartesian coordinates.	1
	Total	12
	UNIT II FUNCTIONS OF SEVERAL VARIABLES	
13.	Introduction of functions of several variables	**1**
14.	Problems based on functions of several variables	1
15.	Taylor's expansion	1
16.	Problems based on Taylor's expansion	1
17.	Tutorial 4: Problems based on Taylor's expansion	1
18.	Concept of maxima and minima	1
19.	Problems based on maxima and minima	1
20.	Constrained maxima and minima by Lagrangian multiplier method	1
21.	Problems based on maxima and minima by Lagrangian multiplier method	1
22.	Tutorial 5: Maxima and minima by Lagrangian multiplier method problems	1
23.	Introduction of Jacobians and problems	1
24.	Tutorial 6: Problems on Jacobians	1
	Total	12
	UNIT III FOURIER SERIES	
25.	Introduction to basic integration and Bernoulli's integration	1
26.	Problems based on Bernoulli's integration	1
27.	Periodic function - Dirchlet's conditions and Statement of Fourier theorem	1
28.	Fourier coefficients and solving problems	1
29.	Full range series in the interval $(-\pi,\pi)$ and $(0,2\pi)$	1
30.	Problems based on full range series in the interval $(-\pi,\pi)$ and $(0,2\pi)$	1

31.	Tutorial:7 Problems based on full range series in the interval $(-\pi, \pi)$ and $(0, 2\pi)$	1
32.	Concept of change of scale and Half range series	1
33.	Problems based on half range series in the interval $(0, \pi)$	1
34.	Tutorial:8 Problems based on half range series in the interval $(0, \pi)$	1
35.	Harmonic Analysis	1
36.	Tutorial: 9 Problems on Harmonic Analysis	1
	Total	12
	UNIT IV BOUNDARY VALUE PROBLEMS	
37.	Introduction with application of partial differential equations	1
38.	Classification of second order quasi linear PDE	1
39.	Method of separation of variables	1
40.	Tutorial : 10 Problems on method of separation of variables	1
41.	Solution of One dimensional wave equation	1
42.	Problems on One dimensional wave equation	1
43.	Tutorial :11 Problems on One dimensional wave equation	1
44.	Solution of One dimensional heat equation	1
45.	Problems on One dimensional heat equation	1
46.	Steady state solution of two dimensional heat equations	1
47.	Problems based on zero boundary conditions	1
48.	Tutorial :12 Problems based on zero boundary conditions	1
	Total	12
	UNIT V STATISTICS	
49.	Introduction of Statistics	1
50.	Concept of measures of central tendency	1
51.	Concept of Mean, Median, Mode, Standard deviation	1
52.	Problems on Mean, Median, Mode, Standard deviation	1
53.	Problems on Mean, Median, Mode, Standard deviation	1
54.	Tutorial:13 Problems on Mean, Median, Mode, Standard deviation	1
55.	Moments – skewness and kurtosis	1
56.	Tutorial: 14 Problems based on moments – skewness and kurtosis	1
57.	Correlation – Types of correlation and formulas	1
58.	Concept of Rank correlation	1
59.	Problems based on rank correlation	1
60.	Tutorial:15 Problems based on rank correlation	1
	Total	12
	TOTAL	**60**

Staff In charge **HoD**

Unit-1

Matrices

Definition of a Matrices.

A system of any mn numbers arranged in a rectangular array of m-rows and n-columns is called Matrices of order $m \times n$ and is denoted by

$$A = (a_{ij})_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

where a_{ij}'s are called the entries or elements of the Matrices.

Types of Matrices.

1) Row's Matrices:-

A Matrices having only one row is called as row Matrices

eg: $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$.

2) Column's matrices:-

A Matrices having only one column is called as Column matrices.

eg: $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$.

3) Square Matrix.

A Matrix having equal number of rows and columns is called square matrix

eg: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

4) Diagonal Matrix.

A Square Matrix having its entries along the leading diagonal and all other entries are zero is called diagonal matrices.

eg: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$.

5) Scalar Matrix.

A diagonal Matrix whose leading diagonals are all Same is called Scalar Matrix

eg: $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$

6) Unit Matrix:-

A Scalar Matrix whose diagonal elements are one is called a Unit Matrix

eg: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$.

7) Triangular Matrix.

Types: Upper triangular Matrix.

A Square Matrix in which all the elements below the leading diagonal are zero is called Upper triangular Matrix.

eg:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}_{3\times3}$.

Lower triangular Matrix.

A Square Matrix in which all the elements above the leading diagonal are zero is called a lower triangular Matrix

eg: $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}_{3\times3}$.

8) Transpose of a Matrix.

The Matrix got from a given Matrix by interchanging its rows and columns is called Transpose of that Matrix.

ie:- if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$.

Then $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}_{3\times3}$.

9) Symmetric Matrix

A Square Matrix "A" is said to be Symmetric if $A=A^T$ and Skew Symmetric if $A=-A^T$.

eg: Here

$$A=\begin{bmatrix}1&4&5\\4&2&6\\5&6&3\end{bmatrix}_{3\times3}.$$

Thn

$$A^T=\begin{bmatrix}1&4&5\\4&2&6\\5&6&3\end{bmatrix}_{3\times3}.$$

$\therefore A=A^T$

$\therefore$ A is an Symmetric Matrix.

10) Conjugate of a Matrix.

A Matrix a obtained by replacing each element of A by its complex conjugate is called Conjugate of A and is denoted by $\bar{A}$

eg: if

$$A=\begin{bmatrix}1+i&2&3-i\\4&5+i&i\\7&8+3i&9\end{bmatrix}_{3\times3}.$$

Thn.

$$\bar{A}=\begin{bmatrix}1-i&2&3+i\\4&5-i&-i\\7&8-3i&9\end{bmatrix}_{3\times3}.$$

11) Hermitian Matrices and Skew Hermitian Matrices

A Square Matrix A is said to be Hermitian if $A=\overline{A^T}$.

A Square Matrix A is said to be Skew Hermitian if $A=-\overline{A^T}$.

Eg: $A=\begin{bmatrix}1&1-4i\\1+4i&2\end{bmatrix}_{2\times2}.$

Thn $A^T=\begin{bmatrix}1&1+4i\\1-4i&2\end{bmatrix}_{2\times2}$ $\overline{A^T}=\begin{bmatrix}1&1-4i\\1+4i&2\end{bmatrix}_{2\times2}$

$A=\overline{A^T}$.

$\therefore$ A is Hermitian.

Eg: $B=\begin{bmatrix}3i&2+i\\-2+i&i\end{bmatrix}_{2\times2}$

Thn. $B^T=\begin{bmatrix}3i&-2+i\\2+i&i\end{bmatrix}_{2\times2}.$ $\overline{B^T}=\begin{bmatrix}-3i&-2-i\\2-i&-i\end{bmatrix}_{2\times2}$

$\overline{B^T}=-\begin{bmatrix}3i&2+i\\-2+i&i\end{bmatrix}_{2\times2}.$

$B=-\overline{B^T}$.

$\therefore$ B is skew Hermitian.

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12) Trace of a Matrix

The sum of the main diagonal elements of a square matrix A is called trace of A and is denoted by trace (A) or tr(A).

if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

Then trace (A) = 1+5+9

tr(A) = 15.

13) determinant of a Matrix.

Determinant is Calculating the numerical value of a Matrix it is denoted by |A| or det(A) or Δ(A).

eg: if

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}_{3\times3}$

Then

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{vmatrix}_{3\times3}$.

$|A| = 1[6-3] -1[3+6] +1[-1-4]$

$= 3-9-5 = -11$

14) Singular and non Singular Matrices

Singular Matrices

If determinant of A is zero ie, |A|=0 then A is said to be a singular Matrix.

non singular Matrices.

If |A| ≠ 0 then A is said to be a non-singular Matrix.

15) Equal Matrix.

Two matrices A and B are said to be equal if

i) A & B have are of same order

ii) Each element of A is equal to the corresponding element of B.

eg: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$.

Then A = B.

16) Sub Matrix:

A Matrix are obtained from A by elementary transmation is called the sub matrix of A.

Properties of Determinants

Addition properties

Addition of two matrices.

i) Two Matrices A and B can be added if and only if A and B are of same order.

ii) Each element of A is added with the Corresponding element of B.

Eg: If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3}$$

Then

$$A+B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}_{3\times 3}$$

Types:

i) Commutative (ie) $A+B = B+A$.

ii) Associative (ie) $(A+B)+C = A+(B+C)$.

iii) Scalar Multiplication (ie) $(\alpha+\beta)A = \alpha A + \beta A$.

Multiplications of two Matrices:

Two Matrices A and B can be multiplied only if the number of Columns of the first Matrices is equal to the number of row's of the second matrices.

(ie) eg: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$

$$B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2}$$

1
16
25
36
77

Then

$2\times\boxed{3\ 3}\times 2$

2×2

$$AB = \begin{bmatrix} 1+4+9 & 4+10+18 \\ 4+10+18 & 16+25+36 \end{bmatrix}_{2\times 2}$$

$$= \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}_{2\times 2}.$$

Properties:-

i) Commutative (ie) $AB \neq BA$.

ii) Associative (ie) $(AB)C = A(BC)$

iii) Scalar Multiplication (ie) $\alpha(A) = \alpha A$.

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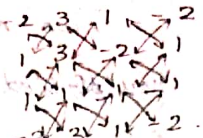
Finding adj adjoint of a Matrix.

Given:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ -2 & 1 & 3 \end{bmatrix}$$



The Co-factor of A is

$$A_{ij} = \begin{bmatrix} +\begin{vmatrix} -2 & 3 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -6-3 & -(3+6) & +(1-4) \\ -(3-1) & +(3+2) & -(1+2) \\ +(3+2) & -(3-1) & +(-2-1) \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 & -3 \\ -2 & 5 & -3 \\ 5 & -2 & -3 \end{bmatrix}$$

$$\text{Adjoint } A\,(a_{ij})^{T} = \begin{bmatrix} -9 & -2 & 5 \\ -9 & 5 & -2 \\ -3 & -3 & -3 \end{bmatrix}$$

Inverse of Matrix.

⇒ Inverse Matrix is also known as reciprocal of Matrix.

⇒ Inverse of a matrix can be obtained only for a square Matrix.

⇒ Inverse of a Matrix is denoted by A^{-1} and defined as

$$A^{-1} = \frac{1}{|A|}(\text{adj } A).$$

Find the Problem. Inverse of the matrix

1) $A = \begin{bmatrix} 1 & 2 \\ -1 & -4 \end{bmatrix}$.

$$\text{Adj } A = \begin{bmatrix} -4 & -2 \\ 1 & 1 \end{bmatrix}.$$

$$|A| = (-4+2) = -2.$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-2}\begin{bmatrix} -4 & -2 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 1 \\ -1/2 & -1/2 \end{bmatrix}.$$

ii)

2) $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$.

$$\text{Adj } A = \begin{bmatrix} -2 & -2 \\ -2 & 1 \end{bmatrix}.$$

$$|A| = -2 - 4$$
$$= -6.$$

$$A^{-1} = \frac{1}{|A|} adj(A).$$

$$= \frac{1}{-6} \begin{bmatrix} -2 & -2 \\ -2 & \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

$$\frac{(adj\, A)}{|A|} = A^{-1}$$

2) $$A = \begin{bmatrix} 8 & 3 \\ 2 & 3 \end{bmatrix}.$$

$$Adj\ A = \begin{bmatrix} 3 & -3 \\ -2 & 3 \end{bmatrix}$$

$$|A| = 9 - 6$$
$$= 3$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 3 & -3 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -2/3 & 1 \end{bmatrix}$$

3) $$A = \begin{bmatrix} 3 & -2 \\ 7 & 8 \end{bmatrix}.$$

$$Adj\ A = \begin{bmatrix} 8 & +2 \\ -7 & 3 \end{bmatrix}$$

$$|A| = 24 + 14.$$
$$= 38.$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$A^{-1} = \frac{1}{38} \begin{bmatrix} 8 & +2 \\ -7 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4/19 & 1/19 \\ -7/38 & 3/38 \end{bmatrix}.$$

$\frac{\cancel{8}\,4}{\cancel{38}\,19}$ $\frac{2\,1}{\cancel{38}\,19}$.

$7/38$ $3/38$

4) $$A = \begin{bmatrix} 1 & 0 & -4 \\ -2 & 2 & 5 \\ 3 & -1 & 2 \end{bmatrix}.$$

$$|A| = 1 \begin{vmatrix} 2 & 5 \\ -1 & 2 \end{vmatrix} - 4 \begin{vmatrix} -2 & 2 \\ 3 & -1 \end{vmatrix}$$

$$= 1(4+5) - 4(2-6)$$
$$= 1(9) - 4(-4)$$
$$= 9 + 16$$
$$= 25.$$

$$adj(A) = \begin{vmatrix} 1 & 0 & -4 \\ -2 & 2 & 5 \\ 3 & -1 & 2 \end{vmatrix}.$$

$$= \begin{bmatrix} +\begin{vmatrix} 2 & 5 \\ -1 & 2 \end{vmatrix} & -\begin{vmatrix} -2 & 5 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} -2 & 2 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} 0 & -4 \\ -1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & -4 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} \\ +\begin{vmatrix} 0 & -4 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & -4 \\ -2 & 5 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ -2 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (4+5) & -(-4-15) & +(2-6) \\ -(-4) & +(2+12) & -(-1) \\ (8) & -(5-8) & +(2) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$$

$$adj(A) \Rightarrow (a_{ij})^T = \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$= \frac{1}{25} \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 9/25 & 4/25 & 8/25 \\ 19/25 & 14/25 & 3/25 \\ -4/25 & 1/25 & 2/25 \end{bmatrix}$$

5) $A = \begin{bmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{bmatrix}$.

$$|A| = 5\begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} + 6\begin{vmatrix} 7 & -3 \\ 2 & 6 \end{vmatrix} + 4\begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix}$$

$= 5(24+3) + 6(42+6) + 4(7-8)$

$= 5(27) + 6(48) + 4(-1)$

$= 135 + 288 - 4$

$= 419.$

$288 + 135 = 423$ $27 \times 5 = 135$ $48 \times 6 = 288$

$$A_{ij} = \begin{bmatrix} +\begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} & -\begin{vmatrix} 7 & -3 \\ 2 & 6 \end{vmatrix} & +\begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} -6 & 4 \\ 1 & 6 \end{vmatrix} & +\begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix} & -\begin{vmatrix} 5 & -6 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} -6 & 4 \\ 4 & -3 \end{vmatrix} & -\begin{vmatrix} 5 & 4 \\ 7 & -3 \end{vmatrix} & +\begin{vmatrix} 5 & -6 \\ 7 & 4 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 27 & -48 & -1 \\ +40 & 22 & -17 \\ +2 & 43 & 62 \end{bmatrix}$$

$24+3$

$42+6$

$7-8$

$-36-4$

$30-8$

$5+12$

17

$18-16$

$-15-28$

$28 + 15 = 43$

$20+42$

$$adj\ A = \begin{bmatrix} 27 & 40 & 2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}.$$

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$$A^{-1} = \frac{1}{|A|}(\text{adj}(A))$$

$$= \frac{1}{419}\begin{bmatrix} 27 & 40 & 2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} \frac{27}{419} & \frac{40}{419} & \frac{2}{419} \\ \frac{-48}{419} & \frac{22}{419} & \frac{43}{419} \\ \frac{-1}{419} & \frac{-17}{419} & \frac{62}{419} \end{bmatrix}$$

Rank of the Matrix:

⇒ The Rank of the Matrix ~~used~~ is the order to ~~find~~ of any highest ~~the degree of the~~ non-vanishing minor of the Matrix.

⇒ Eg: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$.

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1. \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} R_3 \to R_2 + R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$\rho(A) = 3$.

1) $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 1 & -4 \\ 1 & -7 & -1 \end{bmatrix}$.

$$\sim \begin{bmatrix} 1 & -7 & -1 \\ 2 & 1 & -4 \\ 4 & 3 & 2 \end{bmatrix} R_1 \leftrightarrow R_3$$

2 1 -4
-2 +14 +2
0 15 -2

$$\sim \begin{bmatrix} 1 & -7 & -1 \\ 0 & 15 & -2 \\ 0 & 1 & 6 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_2 \end{array}$$

4 3 2
-2 8
0 1 6

$$\sim \begin{bmatrix} 1 & -7 & -1 \\ 0 & 15 & -2 \\ 0 & 0 & 88 \end{bmatrix} R_3 \to R_2 - 15R_3$$

0 15 -2
0 -15 90
0 0 88

$\rho(A) = 3$.

Consistency and Inconsistency of the Matrix

Consistent	$\rho(A) = \rho(A \cdot B) = n$	Unique solution
	$\rho(A) = \rho(AB) < n$	Infinite many soln
Inconsistent	$\rho(A) \neq \rho(AB)$	No solution.

1) $x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=\mu$

$$A=\begin{bmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{bmatrix},\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}6\\10\\\mu\end{bmatrix}$$

$AX=B$

$$\begin{bmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\10\\\mu\end{bmatrix}$$

$$(A,B)=\begin{bmatrix}1&1&1&6\\1&2&3&10\\1&2&\lambda&\mu\end{bmatrix}$$

$$\sim\begin{bmatrix}1&1&1&6\\0&1&2&4\\0&0&\lambda-3&\mu-10\end{bmatrix}\begin{matrix}\\R_2\to R_2-R_1\\R_3\to R_3-R_2\end{matrix}$$

Case (i) = $\lambda=3$ $\mu\neq 10$

$\rho(A)=2$ $\rho(A,B)=3$

$\rho(A)\neq\rho(A,B)$

inconsistent and has no solution.

Case (ii) = $\lambda\neq 3$ $\mu=10$

$\rho(A)=\rho(A,B)=3$ if $n=3$

Consistent & has unique soln.

Case (iii) $\lambda=3$, $\mu=10$

$\rho(A)=2$ $\rho(A,B)=2$ $\rho(A)=\rho(A,B)=2<\boxed{n=3}$

consistent and has infinitely many soln.

2) Test for consistency and solve if consistent

1) $4x+3y+2z+7=0$ $\qquad 4x+3y+2z=-7$

$2x+y-4z+1=0$ $\qquad 2x+y-4z=-1$

$x-7y-z=0$ $\qquad x-7y-z=0$

$$A=\begin{bmatrix}4&3&2\\2&1&-4\\1&-7&-1\end{bmatrix},\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}-7\\-1\\0\end{bmatrix}$$

$AX=B$.

where,

$$\begin{bmatrix}4&3&2\\2&1&-4\\1&-7&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-7\\-1\\0\end{bmatrix}.$$

$$(A,B)=\begin{bmatrix}4&3&2&-7\\2&1&-4&-1\\1&-7&-1&0\end{bmatrix}$$

$$\sim\begin{bmatrix}1&-7&-1&0\\2&1&-4&-1\\4&3&2&-7\end{bmatrix}R_1\leftrightarrow R_3$$

$$\sim\begin{bmatrix}1&-7&-1&0\\0&15&-2&-1\\0&31&6&-7\end{bmatrix}\begin{matrix}\\R_2\to R_2-2R_1\\R_3\to R_3-4R_1\end{matrix}$$

$$\sim\begin{bmatrix}1&-7&-1&0\\0&15&-2&-1\\0&1&10&-5\end{bmatrix}R_3\to R_3-2R_2$$

$$\sim\begin{bmatrix}1&-7&-1&0\\0&15&-2&-1\\0&0&152&-74\end{bmatrix}R_3\to 15R_3-R_2$$

$\rho(A) = 3$

$\rho(A, B) = 3$

$\rho(A) = \rho(A, B) = n$

The soln is consistent and it has unique soln.

2) $4x+3y+2z+7=0$

$2x+y-4z+1=0$

$x-7y-z=0$.

$4x+3y+2z=-7$

$2x+y-4z=-1$

$x-7y-z=0$

$AX = B$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 1 & -4 \\ 1 & -7 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -7 \\ -1 \\ 0 \end{bmatrix}.$$

$$[A, B] = \begin{bmatrix} 4 & 3 & 2 & -1 \\ 2 & 1 & -4 & -1 \\ 1 & -7 & -1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 4 & 3 & 2 & -7 \\ 0 & -2 & -20 & 10 \\ 0 & -31 & -6 & 7 \end{bmatrix} \begin{array}{l} \\ R_2 \to 4R_2 - 2R_1 \\ R_3 \to 4R_3 - 1R_1 \end{array}$$

$$\sim \begin{bmatrix} 4 & 3 & 2 & -7 \\ 0 & -2 & -20 & +10 \\ 0 & 0 & -608 & 296 \end{bmatrix} \begin{array}{l} \\ \\ R_3 \to -2R_3 + 31R_2 \end{array}$$

$\rho(A) = 3$,

$\rho(A, B) = 3$

$\rho(A) = \rho(A, B) = n = 3$.

The system of equation is consistent & has a unique soln.

$-608z = 296$

$z = \frac{-296}{608}$

$-2y - 20z = 10$

$-2y - 20\left(\frac{-37}{76}\right) = 10$

$-2y + \frac{185}{19} = 10$

$y = \left(10 - \frac{185}{19}\right)\left(\frac{-1}{2}\right)$

$y = \frac{-5}{38}$

$4x + 3y + 2z = -7$

$4x + 3\left(\frac{-5}{38}\right) + 2\left(\frac{-37}{76}\right) = -7$

$x = \frac{-107}{76}$

2) $x+y+z = 3$

$x+y-z=1$

$3x+3y-5z=1$

Given $x+y+z=3$

$x+y-z=1$

$3x+3y-5z=1$

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where $AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 3 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

$$(A, B) = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 1 & -1 & 1 \\ 3 & 3 & -5 & 1 \end{bmatrix}.$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 0 & -2 & -2 \\ 0 & 0 & -8 & -8 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 1R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$\rho(A) = 3$ $\rho(A, B) = 3$

$\rho(A) = \rho(A, B) = n = 3.$

The system is consistent and has unique solution.

$-8z = -8$

$\boxed{z = 1}$.

$\boxed{y=0}$. Take $\boxed{y = k}$.

$x + y + z = 3$

$x + 0 + 1 = 3$

$x + 1 = 3$

$x = 3 - 1$

$\boxed{x = 2}$.

$x + y + z = 3$

$x + k + 1 = 3$

$x + k = 2$

$\boxed{x = 2 - k}$

$x =$

$\boxed{x = 2 - k, \ y = k, \ z = 1}$.

3) $2x + 3y - z = 9$

$x + y + z = 9$

$3x - y - z = -1$

$AX = B$

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}.$$

$$[A, B] = \begin{bmatrix} 2 & 3 & -1 & 9 \\ 1 & 1 & 1 & 9 \\ 3 & -1 & -1 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 3 & -1 & 9 \\ 0 & -1 & 3 & 9 \\ 0 & -11 & 1 & -29 \end{bmatrix} \begin{matrix} R_2 \to 2R_2 - R_1 \\ R_3 \to 2R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 3 & -1 & 9 \\ 0 & -1 & 3 & 9 \\ 0 & 0 & -32 & -128 \end{bmatrix} R_3 \to R_3 - 11R_2$$

$\rho(A) = 3$

$\rho(A, B) = 3$

$\rho(A) = \rho(A, B) = n = 3$

The system is consistent and has unique soln.

$-32z = -128$

$z = \frac{+128}{+32}$

$\boxed{z = 4}$.

$-1y + 3(z) = 9$

$-1y + 3(4) = 9$

$-1y + 12 = 9$

$-1y = 9 - 12$

$-1y = -3$

$\boxed{y = 3}$

$2x + 3y - 1z = 9$

$2x + 3(3) - 1(4) = 9$

$2x + 9 - 4 = 9$

$2x = 9 - 9 + 4$

$2x = 4$

$\boxed{x = 2}$.

4) $x + y + z = 6$

$x + 2y - 2z = -3$

$2x + 3y + z = 11$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \\ 2 & 3 & 1 \end{bmatrix}, \; x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \; B = \begin{bmatrix} 6 \\ -3 \\ 11 \end{bmatrix}$$

$Ax = B$

$$[A, B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & -2 & -3 \\ 2 & 3 & 1 & 11 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -3 & -9 \\ 0 & 1 & -1 & -1 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -3 & -9 \\ 0 & 0 & -2 & 8 \end{bmatrix} R_3 \to R_3 - R_2.$$

$\rho(A) = 3$. $\rho(A, B) = 3$

$\rho(A) = \rho(A, B) = 3$

The System is Consistent and has Unique Solution

$-2z = 8$

$\boxed{z = -4}$.

$1y - 3z = -9$

$1y - 3(-4) = -9$

$y + 12 = -9$

$y = -9 - 12$

$\boxed{y = -21}$.

$x + y + z = 6$

$x - 21 - 4 = 6$

$x - 25 = 6$

$x = 6 + 25$

$\boxed{x = 31}$.

Finding the ~~[illegible]~~ Eigen Values and ~~[illegible]~~ Eigen Vector

$A =$ [illegible]

The Characteristic eqn $|A - \lambda I| = 0$

In other words

For. 2×2 matrix

C.E $\lambda^2 - S_1\lambda + S_2 = 0$

where S_1 - Trace of the Matrix

$S_2 = |A|$.

For 3×3 Matrix

CE $\rightarrow$ $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

where

S_1 - Trace of the Matrix.

S_2 = sum of the minors of the main diagonal elements of A.

$S_3 = |A|$.

To Find the Eigen Values

Find the roots of the CE.

To Find Eigen vector:

$(A - \lambda I)x = 0$

where $x \neq 0$.

To find the roots:

eg: ①

$\lambda^2 + 4\lambda + 4 = 0$

$(\lambda+2)(\lambda+2) = 0$

$\lambda = -2, -2$.

eg: ②.

$2\lambda^2 + 4\lambda + 4 = 0$.

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 4(2)(4)}}{2 \times 2}$

$= \frac{-4 \pm \sqrt{-16}}{4}$

$= \frac{-4 \pm 4\sqrt{-1}}{4}$

$= \frac{-4 \pm 4i}{4}$

$= -1 \pm i$

$$\boxed{\begin{array}{l} \lambda_1 = -1 + i \\ \lambda_2 = -1 - i \end{array}}$$

eg: $\lambda^3 + \lambda^2 + 2\lambda - 4 = 0$

1	0 1	2	-4	
	0 1	2	4	
	1 2	4		0

$\lambda^2 + 2\lambda + 4 = 0$.

$$\lambda = \frac{-2 \pm \sqrt{4-4(1)(4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-16}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}}{2}$$

$$\lambda = -1 \pm \sqrt{3}\, i$$

$$\boxed{\begin{array}{l} \lambda_1 = -1 + \sqrt{3}\, i \\ \lambda_2 = -1 - \sqrt{3}\, i \end{array}}$$

Find the Eigen values and Eigen vectors of the matrices ~~to~~:

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

Soln.

Let
$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

The characteristic eqn of A is $|A - \lambda I| = 0$

i.e, $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

where

S_1 = Trace of A

$= 2+3+2$

$= 7$

S_2 = Sum of the minors of the main diagonals element of A.

$$= \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$$

$= (6-2) + (4-1) + (6-2)$

$= 4+3+4 = 11$.

$S_3 = |A|$

$$= \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$= 2(4) - 2(1) + 1(-1) = 5$

The characteristic eqn is

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

To find the Eigen Values.

$$\begin{array}{c|cccc} 1 & 1 & -7 & 11 & -5 \\ & 0 & 1 & -6 & 5 \\ \hline & 1 & -6 & 5 & \boxed{0} \end{array}$$

$\boxed{\lambda_1 = 1}$.

$\lambda^2 - 6\lambda + 5 = 0$

$(\lambda-1)(\lambda-5) = 0$

$\lambda = 1, 5$.

$\boxed{\lambda_2 = 1}$

$\boxed{\lambda_3 = 5}$.

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∴ The ~~The~~ Eigen values are

$\boxed{\lambda_1=1}$, $\boxed{\lambda_2=1}$, $\boxed{\lambda_3=5}$ ...

To find the Eigen vectors.

$|A-\lambda I|x=0$, where $x\neq 0$.

ie,

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

Case(i) $\lambda=1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$x_1+2x_2+x_3=0$

$x_1+2x_2+x_3=0$

$x_1+2x_2+x_3=0$

put $x_1=0$

$2x_2+x_3=0$

$2x_2=-x_3$

$\frac{x_2}{-1}=\frac{x_3}{2}$

$$x_1=\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

Case(ii) $\lambda=1$

put $x_2=0$

$x_1+x_3=0$

$x_1=-x_3$

$\frac{x_1}{-1}=\frac{x_3}{1}$

$$x_2=\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Case(iii) $\lambda=5$

$$\begin{bmatrix} 2-5 & 2 & 1 \\ 1 & 3-5 & 1 \\ 1 & 2 & 2-5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=0$$

$$\left.\begin{aligned} -3x_1+2x_2+x_3&=0 \\ x_1-2x_2+x_3&=0 \\ x_1+2x_2-3x_3&=0 \end{aligned}\right\} \rightarrow ②$$

$\frac{x_1}{6-2}=\frac{-x_2}{-3-1}=\frac{x_3}{2+2}$

$\frac{x_1}{4}=\frac{-x_2}{-4}=\frac{x_3}{4}$

$\div 4$ $\quad \frac{x_1}{1}=\frac{x_2}{1}=\frac{x_3}{1}$

$$x_3=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

The eigen vectors corresponding to the Eigen values $\lambda=1,1,5$ are $X_1=\begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$, $X_2=\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $X_3=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. respectively.

Find the Eigen values and Eigen vectors of the Matrix :-

2. $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

Soln:-

Let

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

The characteristic eqn of A is $|A-\lambda I| = 0$

i,e, $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$.

where,

S_1 = Trace of A.

$= 3+2+3 = 8$.

S_2 = Sum of the minors of the main diagonals element of A.

$$= \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix}$$

$= (6-1) + (9-0) + (6-1)$

$= 5 + 9 + 5$

$= 19$.

$S_3 = |A|$

$$= \begin{vmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{vmatrix}$$

$= 3(6-1) + 1(-3+0) + 0$

$= 15 - 3$

$= 12$.

The characteristic eqn is

$\lambda^3 - 8\lambda^2 + 19\lambda - 12 = 0$

To find the Eigen value

$$\begin{array}{c|cccc} 1 & 1 & -8 & 19 & -12 \\ & 0 & 1 & -7 & 12 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$

$\boxed{\lambda_1 = 1}$

$\lambda^2 - 7\lambda + 12 = 0$

$\cancel{\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

$\cancel{= \frac{7 \pm \sqrt{49 - 4(1)(12)}}{2}}$

$\cancel{= \frac{7 \pm \sqrt{1 - 48}}{2}}$

$\cancel{= \frac{7 \pm \sqrt{-24}}{2}}$

$\cancel{= 7 \pm}$

$(\lambda-4)(\lambda-3) = 0$

$\boxed{\lambda_2 = 4}$ $\boxed{\lambda_3 = 3}$.

The Eigen values are

$\lambda_1 = 1$, $\lambda_2 = 4$, $\lambda_3 = 3$

To find the Eigen vectors

$|A-\lambda I|\, x = 0$ where $x \neq 0$

ie
$$\begin{bmatrix} 3-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Case (i) $\lambda = 1$

$$\begin{bmatrix} 3-1 & -1 & 0 \\ -1 & 2-1 & -1 \\ 0 & -1 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$2x_1 - x_2 = 0$

$-x_1 + x_2 - x_3 = 0$

$-x_2 + 2x_3 = 0$

$\boxed{\begin{matrix} 2x_1 - x_2 + 0 = 0 \\ 0 - x_2 + 2x_3 = 0 \end{matrix}}$

$$\frac{-x_1}{-2-0} = \frac{x_2}{4-0} = \frac{-x_3}{-2-0} = 0$$

$$\frac{-x_1}{-2} = \frac{x_2}{4} = \frac{-x_3}{+2}$$

$$\frac{x_1}{2} = \frac{x_2}{4} = \frac{x_3}{2}$$

÷ by 2

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{1}$$

$$X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\lambda_2 = 4$

$$\begin{bmatrix} 3-4 & -1 & 0 \\ -1 & 2-4 & -1 \\ 0 & -1 & 3-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_1 - x_2 = 0$

$-x_1 - 2x_2 - x_3 = 0$

$-x_2 - x_3 = 0$

$$\frac{-x_1}{1-0} = \frac{-2x_2}{-1-0} = \frac{-x_3}{1-0}$$

$$\frac{-x_1}{1} = \frac{-2x_2}{-1} = \frac{-x_3}{-1}$$

$$X_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\frac{x_1}{1} = \frac{-x_2}{1} = \frac{x_3}{1}$$

Case 3 $\lambda_3 = 3$

$$\begin{bmatrix} 3-3 & -1 & 0 \\ -1 & 2-3 & -1 \\ 0 & -1 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$0 - x_2 + 0 = 0$

$-x_1 - x_2 - x_3 = 0$

$0 - x_2 + 0 = 0$ $\quad x_3 = 0$

$-x_1 - x_2 - 0 = 0$

$$\frac{0}{0-1} = \frac{-x_2}{0-0} = \frac{0}{0}$$

$-x_1 = x_2$

$$\frac{x_1}{1} = \frac{x_2}{-1}$$

$$x_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

The eigen Vectors Corresponding to the Eigen value $\lambda = 1, 4, 3$ are

The Eigen $X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$,

$$x_3 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

1) $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

Soln:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

The characteristic eqn of A is $|A - \lambda I| = 0$

ie, $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

S_1 = Trace of A.

$= 2 + 1 + 2$

$= 5$

S_2 = Sum of the minors of the main diagonals element of A

$$= \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$$

$= (2-0) + (4-1) + (2-0)$

$= 2 + 3 + 2$

$= 7.$

$S_3 = |A|$

$$= \begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix}$$

$= 2(2-0) - 1(0-0) + 1(0-1)$

$= 4 - 1$

$= 3.$

Arthi

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

To find Eigen:

$$\begin{array}{c|cccc} 1 & 1 & -5 & 7 & -3 \\ & 0 & 1 & -4 & 3 \\ \hline & 1 & -4 & 3 & \boxed{0} \end{array} \qquad \boxed{\lambda = 1}$$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\boxed{\lambda_2 = 3}$ $\boxed{\lambda_3 = 1}$.

The Eigen Values $\boxed{\lambda_1 = 1}$ $\boxed{\lambda_2 = 3}$ $\boxed{\lambda_3 = 1}$

To find the Eigen Vectors:

Let

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Case (i) $\lambda = 1$

$$\begin{bmatrix} 2-1 & 1 & 1 \\ 0 & 1-1 & 0 \\ 1 & 1 & 2-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_1 + x_2 + x_3 = 0$

$x_1 + x_2 + x_3 = 0$

$x_1 = 0$

$x_2 + x_3 = 0$

$x_2 = -x_3$

$\frac{x_2}{-1} = \frac{x_3}{1}$

$$X_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}.$$

Case (ii) $\lambda_2 = 3$

$$\begin{bmatrix} 2-3 & 1 & 1 \\ 0 & 1-3 & 0 \\ 1 & 1 & 2-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$-x_1 + x_2 + x_3 = 0$

$-2x_2 = 0$

$x_1 + x_2 - x_3 = 0$

$$\frac{x_1}{-1-1} = \frac{-x_2}{1-1} = \frac{x_3}{-1-1}$$

$$\frac{x_1}{-2} = \frac{-x_2}{0} = \frac{x_3}{-2}$$

$$X_2 = \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}$$

case: 3 $\lambda = 1$

consider $x_1 + x_2 + x_3 = 0$

sub: $x_2 = 0$.

$x_1 = -x_3$

$$\frac{x_1}{-1} = \frac{x_3}{1}$$

$$X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

The Eigen values and Eigen vectors are $\lambda_1 = 1$, $\lambda_2 = 3$, $\lambda_3 = 1$ and vectors

$$X_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}, X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

2) Application of Matrix.

1. Physical related application:

i) Electrical Circuits, condom mechanical optics.

ii) Calculation of battery power outputs, Conversion of electrical energy to Useful energy of resistors

iii) solving problems of Kirchoff's law (voltage and current).

2) Computer related application:-

i) Projection of 3D image to 2D, Creating realistic screening motion.

~~Sotastic mati~~

ii) Stochastic matrices and Eigen vectors solvers are used in page rank algorithms, ie, used in ranking of web pages in Google search.

3) Matrices Calculus:-

i) Generalization of anatical notions like ~~explants~~ exponationals to higher Dimensions.

ii) Coding are entrupting the message.

4) Geology:-

i) Seismic servays

ii) to plot graphs.

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iii) statics and do Scientific Studies.
iv) robotic and automotions.

i) Used in Graph theory, Quantum Mechanics, Computer Graphics, Solving linear equation, Cytography.

(2) Properties of Eigen values:

1) Sum of the Eigen Values is equal to Trace of the Matrix.

2) Product of the Eigen Values = $|A|$.

3) If λ is the Eigen Values of A then $\frac{1}{\lambda}$ is the Eigen value of A^{-1}.

4) If λ is of eigen value ~~the~~ of A the λ^m is the eigen value of A^m.

5) If λ is the eigen value of A then $k\lambda$ is the eigen value of kA.

6) A and A^T have the same eigen values.

7) The eigen values of a real Symmetric Matrix is all real numbers.

8) The eigen values of a triangular Matrix are its elements in the main diagonal.

9) Similar Matrices have the same Eigen values.

Problems:

1) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and $\lambda_1 = 1$, $\lambda_2 = 3$ find. then, $\lambda_3 = ?$

Soln:

W.K.T:

Sum of the Eigen Values = Trace of the Matrix A.

$$1+3+\lambda_3 = 2+1+2$$
$$4+\lambda_3 = 5$$
$$\lambda_3 = 5-4$$
$$\boxed{\lambda_3 = 1}$$

$$\therefore \boxed{\lambda_3 = 1}$$

2) If the Eigen Values of a 3x3 matrix are 1, 3 and 1 find the determinant of $|A|$ ~~without expanding~~.

Soln:

Product of the Eigen Values = $|A|$.

W.K.T: $1 \times 3 \times 1 = |A|$ ie, $\lambda_1 \lambda_2 \lambda_3 = |A|$

$$3 = |A|$$

$$\therefore \boxed{|A| = 3}$$

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3) Similar Matrix

$A \sim B$

ie, $A = B^{-1}AB$.

Properties of Eigen vectors:-

1) Eigen vectors of a Matrix A is not Unique

2) Two Eigen vectors X_1, X_2 are called Orthogonal if $X_1^T X_2 = 0$

3) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are distinct Eigen values of a $n \times n$ matrix then the Corresponding Eigen Vectors $X_1, X_2, \ldots, X_n$ form a linearly independent set.

30/6/17.

Unit: 2:- Theory of Equations:-

Relations between Co-efficient and roots.

i) Equations with rationals Co-efficient and irrationals roots.

⇒ Equations with rationals Co-efficient and irrational roots will occurs in pairs.

⇒ Let $f(x) = 0$ be a equation and Suppose that $a+\sqrt{b}$ is a root of the equation where a & b are rational and $\sqrt{b}$ is irrational, then $a-\sqrt{b}$ is also a root.

Problems:

Solve the equation

$x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$

Given that one of the root is

$1-\sqrt{5}$.

Soln:-

Given the

we know that if $1-\sqrt{5}$ is a root then $1+\sqrt{5}$ is also an root.

Therefore, The factors are

$[x-(1-\sqrt{5})] \cdot [x-(1+\sqrt{5})]$

~~$[x+2\sqrt{5}]$ [x ...] = 0~~

$$x^2 - x(1+\sqrt{5}) - (1-\sqrt{5})x + (1-\sqrt{5})(1+\sqrt{5}) = 0$$
$$x^2 - x[1+\sqrt{5}+1-\sqrt{5}] + [1-5] = 0$$
$$x^2 - 2x - 4 = 0$$

$1-\sqrt{5}$ & $1+\sqrt{5}$

Sum of the roots $= 1-\sqrt{5}+1+\sqrt{5} = 2$

Product of the roots $= (1-\sqrt{5})(1+\sqrt{5}) = 1-5 = -4$.

$x^2 - [\text{Sum of the roots}]x + \text{product of the roots} = 0$

$$x^2 - 2x - 4 = 0.$$

$$x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$$

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x^2 - 2x - 4 & x^4 - 5x^3 + 4x^2 + 8x - 8 \\
 & x^4 - 2x^3 - 4x^2 \\
\hline
 & -3x^3 + 8x^2 + 8x \\
 & -3x^3 + 6x^2 + 12x \\
\hline
 & 2x^2 - 4x - 8 \\
 & 2x^2 - 4x - 8 \\
\hline
 & 0
\end{array}
$$

$\therefore$ $x^2 - 3x + 2$ is also a factor.

$\therefore$ $x^4 - 5x^3 + 4x^2 + 8x - 8 = (x^2 - 2x - 4)(x^2 - 3x + 2) = 0$

$[x - (1-\sqrt{5})][x - (1+\sqrt{5})]\ [(x-2)(x-1)] = 0$

$1-\sqrt{5}$, $1+\sqrt{5}$, 2, 1 are the roots of the eqn.

2) Solve the eqn $3x^3 - 23x^2 + 72x - 70 = 0$ having $(3+\sqrt{-5})$ as a root.

Soln:

W.K.T.

If $(3+\sqrt{-5})$ is a root then $(3-\sqrt{-5})$ is also a root.

Sum of the roots $= 3+\sqrt{-5} + 3 - \sqrt{-5}$
$= 6$

Product of the roots $= (3+\sqrt{-5})(3-\sqrt{-5}) = 9 - (-5) = 14$

$x^2 - [\text{Sum of the roots}]x + \text{product of the roots} = 0$

$$x^2 - 6x + 14 = 0.$$

$$3x^3 - 23x^2 + 72x - 70 = 0$$

$$
\begin{array}{r|l}
 & 3x - 5 \\
\hline
x^2 - 6x + 14 & 3x^3 - 23x^2 + 72x - 70 \\
 & 3x^3 - 18x^2 + 42x \\
\hline
 & -5x^2 + 30x - 70 \\
 & -5x^2 + 30x - 70 \\
\hline
 & 0
\end{array}
$$

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$3x^3-23x^2+72x-70=0$

$\Rightarrow (x^2-6x+14)(3x-5)=0$

$\Rightarrow (3+\sqrt{5}),(3-\sqrt{5}), 5/3$ are the roots of the above eqn:-

H/w

3) Find the eqn whose roots are.

$4\sqrt{3}$, $5+2\sqrt{-1}$.

Soln:-

The roots are $4\sqrt{3}, -4\sqrt{3}, 5+2\sqrt{-1}, 5-2\sqrt{-1}$

The factor is

$4\sqrt{3}, -4\sqrt{3}$. $(4\sqrt{3})^2$ 16×3 48

Sum of the roots $=4\sqrt{3}-4\sqrt{3}=0$

product of the roots $=(4\sqrt{3})(-4\sqrt{3})=-48$.

x^2-[Sum of the roots]$x+$ product of the roots $=0$

~~The~~ $x^2-48=0$. $\rightarrow$ ①

The factor is

$5+2\sqrt{-1}, 5-2\sqrt{-1}$.

Sum of the roots $=5+2\sqrt{-1}+5-2\sqrt{-1}=0$
$=10$

Product of the roots $=(5+2\sqrt{-1})(5-2\sqrt{-1})$
$=29$

x^2-[Sum of the roots]$x+$product of the roots $=0$

$x^2-10x+29=0 \rightarrow$ ② $25-4$ $5^2-25-21$

The eqn is 48×29

① × ②

$(x^2-48)(x^2-10x+29)=0$. $(5-2\sqrt{-1})$ $25+4(-1)$ $25-4$

$x^4-10x^3+29x^2-48x^2+480x-1392=0$

$x^4-10x^3-19x^2+480x-1392=0$.

Find the eqn with rational Co-efficients whose roots are $1+5\sqrt{-1}$ and $5-\sqrt{-1}$.

Soln:

The roots are ~~1+5√-1~~

$1+5\sqrt{-1}, 1-5\sqrt{-1}, 5-\sqrt{-1}, 5+\sqrt{-1}$.

The factor is $(1-5\sqrt{-1})^2$

$1+5\sqrt{-1}, 1-5\sqrt{-1}$ $1-25(-1)$

Sum of the roots $=1+5\sqrt{-1}+1-5\sqrt{-1}$
$=2$ $1+25$ 1^2+25 26

Product of the roots $=(1+5\sqrt{-1})(1-5\sqrt{-1})$
$=26$

x^2-[Sum of the roots]$x+$ Product of the roots $=0$

$x^2-2x+26=0 \rightarrow$ ①.

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The factor is

$5-\sqrt{-1},\ 5+\sqrt{-1}$.

Sum of the root $= 5-\sqrt{-1}+5+\sqrt{-1}$.

$= 10$

Product of the roots $= (5-\sqrt{-1})(5+\sqrt{-1})$ $\quad 25+1$

$= 26$

$x^2 - [\text{Sum of the roots}]x + \left.\begin{matrix}\text{Product of} \\ \text{the roots}\end{matrix}\right\} = 0$

$x^2 - 10x + 26 = 0 \longrightarrow (2)$

The eq^n is,

(1) & (2).

$(x^2-2x+26)(x^2-10x+26)=0$

$$\begin{array}{r} 26\times 2 \\ \hline 52 \end{array} \qquad \begin{array}{r} 260 \\ 52 \\ \hline 312 \end{array} \qquad \begin{array}{r} 26\times 26 \\ 156 \\ 52 \\ \hline 676 \end{array}$$

$x^4 - 10x^3 + 26x^2 - 2x^3 + 20x^2 - 52x + 26x^2$

$- 260x + 676 = 0$

$x^4 - 12x^3 + 72x^2 - 312x + 676 = 0$.

Relations between roots and Co-efficients of eqn.

Let the eqn be

$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \dots + P_{n-1}x + P_n = 0$.

If the eqn has roots

$\alpha_1, \alpha_2, \dots \alpha_n$.

then

$\sum\alpha_1 = \text{Sum of the roots} = (-1)P_1 : (-1)P_1 - P$

$\sum\alpha_1\alpha_2 = \left.\begin{matrix}\text{Sum of the the roots} \\ \text{taken 2 at a time}\end{matrix}\right\} = (-1)^2 P_2 \quad Q$

$= P_2$

$\sum\alpha_1\alpha_2\alpha_3 = \left.\begin{matrix}\text{Sum of the the roots} \\ \text{taken 3 at a time}\end{matrix}\right\} = (-1)^3 P_3 \quad -R$

$\alpha\beta\gamma$

$\alpha_1\alpha_2 \dots \alpha_n = \text{Product of the roots} = (-1)^n P_n$.

$x^3 + P_1 x^2 + P_2 x + P_3 = 0$

$\alpha_1 \quad \alpha_2 \quad \alpha_3$

$\alpha_1\alpha_2 + \alpha_1\alpha_3$

$\alpha\beta + \alpha\gamma + \beta\gamma = Q$.

$\sum\alpha_1 = \alpha_1 + \alpha_2 + \alpha_3 = 6$

$\sum\alpha_1\alpha_2 = \alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2\alpha_3 = 1\times 2 + 1\times 3 + 2\times 3 = 11$

$\alpha_1\alpha_2\alpha_3 = 1\times 2\times 3 = 6$.

$x^3 - 6x^2 + 11x - 6 = 0$

If α, β, γ are the roots of the eqn.

$x^3 + px^2 + qx + r = 0$, express the Value

$\sum\alpha^2,\ \sum\frac{1}{\alpha},\ \sum\frac{1}{\alpha\beta},\ \sum\alpha^2\beta$

$\alpha + \beta + \gamma = (-1)P = -P$

$\alpha\beta + \quad = Q$

$\alpha\beta\gamma \quad = R$

Soln:

The Given eqn.

$$x^3+px^2+Qx+R=0$$

The roots are α, β, γ.

$\Sigma\alpha = \alpha+\beta+\gamma = -P$.

$\Sigma\alpha\beta = \alpha\beta+\beta\gamma+\alpha\gamma = Q$

$\alpha\beta\gamma = -R$

i) $\Sigma\alpha^2$.

$\Sigma\alpha^2 = \alpha^2+\beta^2+\gamma^2$.

$(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\alpha\gamma+2\beta\gamma$.

$\therefore = (\alpha+\beta+\gamma)^2 - 2\alpha\beta - 2\beta\gamma - 2\alpha\gamma$.

$= (-p)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$

$= p^2 - 2(Q)$.

$= p^2 - 2Q$.

ii) $\Sigma\frac{1}{\alpha}$

$\Sigma\frac{1}{\alpha} = \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$= \frac{\gamma\beta+\alpha\gamma+\beta\alpha}{\alpha\beta\gamma}$

$= -\frac{Q}{R}$.

iii) $\Sigma\frac{1}{\alpha\beta}$.

$\Sigma\frac{1}{\alpha\beta} = \frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$

$= \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma}$

$= \frac{-P}{-R}$

$= P/R$

iv) $\Sigma\alpha^2\beta$.

$\Sigma\alpha^2\beta = \alpha^2\beta+\alpha^2\gamma+\beta^2\alpha+\beta^2\gamma+\gamma^2\alpha+\gamma^2\beta$

$= \alpha\beta[\alpha+\beta]+\alpha\gamma[\alpha+\gamma]+\beta\gamma[\beta+\gamma]$

$= \alpha\beta[\alpha+\beta+\gamma-\gamma]+\alpha\gamma[\alpha+\gamma+\beta-\beta]+\beta\gamma[\beta+\gamma+\alpha-\alpha]$

$= \alpha\beta[\alpha+\beta+\gamma]-\alpha\beta\gamma+\alpha\gamma[\alpha+\beta+\gamma]-\alpha\beta\gamma+\beta\gamma[\alpha+\beta+\gamma]-\alpha\beta\gamma$

$= [\alpha+\beta+\gamma][\alpha\beta+\alpha\gamma+\beta\gamma]-3\alpha\beta\gamma$

$= -PQ+3R$.

2. If α, β, γ be the roots of the eqn $x^3-px^2+Qx-R=0$. Then find.

i) $\Sigma\alpha^2$, $\Sigma\frac{1}{\alpha}$, $\Sigma\frac{1}{\alpha\beta}$, $\Sigma\alpha^2\beta^2$, $\Sigma\alpha^3$.

$\alpha+\beta+\gamma = (-1)(-p) = p$

$\alpha\beta+\beta\gamma+\gamma\alpha = (-1)^2 Q = Q.$

$\alpha\beta\gamma = (-1)^3(-R) = R.$

i) $\sum \alpha^2$

$= \alpha^2+\beta^2+\gamma^2$

$= (\alpha+\beta+\gamma)^2 - 2\alpha\beta - 2\alpha\gamma - 2\beta\gamma$

$= (\alpha+\beta+\gamma)^2 - 2[\alpha\beta+\alpha\gamma+\beta\gamma]$

$= P^2 - 2Q.$

ii) $\sum \frac{1}{\alpha}.$

$= \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$= \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$

$= \frac{Q}{R}.$

iii) $\sum \frac{1}{\alpha\beta}.$

$= \frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$

$= \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma}$

$= P/R$

iv] $\sum \alpha^2\beta^2$

$= \alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2.$

$= [\alpha\beta+\beta\gamma+\alpha\gamma]^2 - 2(\alpha\beta)(\beta\gamma) - 2(\alpha\beta)(\alpha\gamma)$

$- 2(\beta\gamma)(\alpha\gamma)$

$= [\alpha\beta+\beta\gamma+\alpha\gamma]^2 - 2[\alpha\beta^2\gamma+\alpha^2\beta\gamma+\gamma^2\alpha\beta]$

$= [\alpha\beta+\beta\gamma+\alpha\gamma]^2 - 2[\beta(\alpha\beta\gamma)+\alpha(\alpha\beta\gamma)+\gamma(\alpha\beta\gamma)]$

$= Q^2 - 2[(\alpha\beta\gamma)[\alpha+\beta+\gamma]]$

$= Q^2 - 2RP.$

v) $\sum \alpha^3.$

$= \alpha^3+\beta^3+\gamma^3$

$= (\alpha+\beta+\gamma)^3$

$(a+b+c)^3 = a^3+b^3+c^3 - 3[(a+b+c)(ab+bc+ca)]$
$+3abc.$

$(\alpha+\beta+\gamma)^3 = \alpha^3+\beta^3+\gamma^3 - 3\{[\alpha+\beta+\gamma][\alpha\beta+\gamma\beta+\alpha\gamma]\} + 3\alpha\beta\gamma$

$(\alpha+\beta+\gamma)^3 - 3[PQ] + 3R$

$= P^3 - 3PQ + 3R$

$= P^3 - 3PQ + 3R.$

3) If $\alpha+\beta+\gamma$ be the roots of the eqn $x^3+px^2+Qx+R=0$ find the value of

$$\frac{(\beta^2+\gamma^2)}{\beta\gamma}+\frac{(\gamma^2+\alpha^2)}{\alpha\gamma}+\frac{(\alpha^2+\beta^2)^2}{\alpha\beta}.$$

$\alpha+\beta+\gamma=(-1)P=-P$

$\alpha\beta+\beta\gamma+\gamma\alpha=(-1)^2Q=Q$

$\alpha\beta\gamma=(-1)^3(R)=-R.$

$\alpha+\beta+\gamma=-P$
$\alpha\beta+\beta\gamma+\gamma\alpha=Q$
$\alpha\beta\gamma=-R.$

$$\frac{\beta^2+\gamma^2)}{\beta\gamma}+\frac{(\gamma^2+\alpha^2)}{\alpha\gamma}+\frac{(\alpha^2+\beta^2)}{\alpha\beta}$$

$$=\frac{(\alpha^2+\beta^2+\gamma^2-\alpha^2)}{\beta\gamma}+\frac{(\alpha^2+\beta^2+\gamma^2-\beta^2)}{\alpha\gamma}+\frac{(\alpha^2+\beta^2+\gamma^2-\gamma^2)}{\alpha\beta}$$

$$=\frac{(\alpha^2+\beta^2+\gamma^2)}{\beta\gamma}-\frac{\alpha^2}{\beta\gamma}+\frac{(\alpha^2+\beta^2+\gamma^2)}{\alpha\gamma}-\frac{\beta^2}{\alpha\gamma}+\frac{(\alpha^2+\beta^2+\gamma^2)}{\alpha\beta}-\frac{\gamma^2}{\alpha\beta}$$

$$=(\alpha^2+\beta^2+\gamma^2)\left[\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}+\frac{1}{\alpha\beta}\right]-\left[\frac{\alpha^2}{\beta\gamma}+\frac{\beta^2}{\alpha\gamma}+\frac{\gamma^2}{\alpha\beta}\right]$$

$$=\left[(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)\right]\left[\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}\right]-\left[\frac{\alpha^3+\beta^3+\gamma^3}{\alpha\beta\gamma}\right]$$

$$=\left[(-P)^2-2Q\right]\left[\frac{-P}{-R}\right]-\left[\frac{1}{-R}(\alpha^3+\beta^3+\gamma^3)\right]$$

Consider :-

$$\alpha^3+\beta^3+\gamma^3=(\alpha+\beta+\gamma)^3+3\alpha\beta\gamma-3(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)$$

$$=(-P)^3+3(-R)-3(-P)(Q)$$

$$=(-P)^3-3R+3PQ.$$

Let

$$=\left[P^2-2Q\right]\left[\frac{P}{R}\right]+\frac{(-P^3-3R+3PQ)}{R}$$

$$=\frac{P^3-2PQ-P^3-3R+3PQ}{R}$$

$$=\frac{PQ-3R}{R}$$

4) If α,β and γ be the roots of the eqn $x^3+px^2+qx+R=0$. find the value of $\sum\alpha^2+1$.

Soln:

We know that.

$\alpha+\beta+\gamma=-P$

$\alpha\beta+\beta\gamma+\gamma\alpha=Q$

$\alpha\beta\gamma=-R.$

$$\sum\alpha^2+1=\alpha^2+1+\beta^2+1+\gamma^2+1$$

$$=\alpha^2+\beta^2+\gamma^2+3.$$

$$=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)+3$$

$$(-P)^2-2(Q)+3$$

$$P^2-2(Q)+3$$

Reciprocal Equation.

Conditions:

Let the eqn be.

$$x^n + p_1 x^{n-1} + p_2 x^{n-2} + \dots + p_{n-1} x + p_n = 0.$$

1) If the Co-efficients have all like Sign.

ii) Then (-1) is a root.

If (-1) is a root then (x+1) is a factor.

iii) If the Co-efficients of the term equi distance from the first and the last have opposite sign. Then (+1) is a root. then (x-1) is a factor.

(i) If the equation is of odd degree then (x-1) is a factor.

(ii) If the equation is of even degree then (alternate for -) then (x^2-1) is a factor.

Problems:

1) Find the roots of the equation.

$$x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0.$$

Soln:

All the Co-efficients are positive

∴ WKT

(-1) is the root.

(i.e) (x+1) is a factor.

$$x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0.$$

$$x^5 + x^4 + 3x^4 + 3x^3 + 3x^2 + 3x + x + 1 = 0$$

$$x^4(x+1) + 3x^3(x+1) + 3x(x+1) + (x+1) = 0$$

$$(x+1)\,(x^4 + 3x^3 + 3x + 1) = 0$$

$(x+1) \Rightarrow x = -1$ is a root.

The eqn is divide by x^2.

$$x^2 \left[\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}\right] = 0$$

$$\left[x^2 + 3x + 3/x + \frac{1}{x^2}\right] = 0.$$

$$\left[x^2 + \frac{1}{x^2} + 3\left(x + \frac{1}{x}\right)\right] = 0 \rightarrow (1).$$

Let put $\boxed{x + \frac{1}{x} = z}$

$$\left(x + \frac{1}{x}\right)^2 = z^2.$$

$x + \frac{1}{x} = z$

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$$\left(x+\frac{1}{x}\right)=z^2$$

$$x^2+2x\frac{1}{x}+\frac{1}{x^2}=z^2$$

$$x^2+2+\frac{1}{x^2}=z^2$$

$$\boxed{x^2+\frac{1}{x^2}=z^2-2}$$

(1) $\Rightarrow$ $z^2-2+3z=0$

$$z^2+3z-2=0$$

$$z=\frac{-3\pm\sqrt{9-4(1)(-2)}}{2(1)}$$

$$=\frac{-3\pm\sqrt{9+8}}{2}$$

$$\boxed{z=\frac{-3\pm\sqrt{17}}{2}}$$

We know.

$$z=x+\frac{1}{x}$$

$$\frac{-3\pm\sqrt{17}}{2}=x+\frac{1}{x}$$

$$\frac{-3\pm\sqrt{17}}{2}=\frac{x^2+1}{x}$$

$$(-3\pm\sqrt{17})x=(x^2+1)2$$

$$2x^2-(-3\pm\sqrt{17})x+2=0$$

$$x=\frac{(-3\pm\sqrt{17})\pm\sqrt{(-3\pm\sqrt{17})^2-4(2)(2)}}{2(2)}$$

$$x=\frac{-3\pm\sqrt{17}\pm\sqrt{(-3\pm\sqrt{17})^2-16}}{4}$$

$\& x=-1$ are the roots of the above equation.

2). Solve the eqn $6x^5-x^4-43x^3+43x^2+x-6=0$

Soln:

WKT:

$(x-1)$ is the factor.

Therefore $x=1$ is a root.

$$6x^5-x^4-43x^3+43x^2+x-6=0$$

$$(6x^5-6x^4+5x^4-5x^3-38x^3+38x^2+5x^2-5x+6x-6.$$

$$6x^4(x-1)+5x^3(x-1)-38x^2(x-1)+5x(x-1)+6(x-1)=0$$

$$(x-1)(6x^4+5x^3-38x^2+5x+6)=0$$

$(x-1)\Rightarrow x=1$ is a root.

The eqn is divide by x^2.

$$\left[\frac{6x^4}{x^2}+\frac{5x^3}{x^2}-\frac{38x^2}{x^2}+\frac{5x}{x^2}+\frac{6}{x^2}\right]=0$$

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$$\left[6x^2+5x-38+\frac{5}{x}+\frac{6}{x^2}\right]=0$$

$$6\left(x^2+\frac{1}{x^2}\right)+5\left(x+\frac{1}{x}\right)-38=0 \rightarrow ①$$

Let $z=x+\frac{1}{x}$

$$z^2-2=x^2+\frac{1}{x^2}$$

Sub in ①.

$$6(z^2-2)+5z-38=0$$

$$6z^2+5z-12-38=0$$

$$6z^2+5z-50=0$$

$$z=\frac{-5\pm\sqrt{25-4(6)(-50)}}{2(6)}$$

$$=\frac{-5\pm\sqrt{25+1200}}{12}$$

$$=\frac{-5\pm\sqrt{1225}}{12}$$

$$=\frac{-5\pm35}{12}$$

$$=\frac{30}{12},\ \frac{-40}{12}$$

$$=\frac{5}{2},\ \frac{-10}{3}$$

WKT:

$$z=x+\frac{1}{x}$$

$$xz=x^2+1$$

$$x^2-xz+1=0$$

Case(i) put $z=\frac{5}{2}$.

$$x^2-\frac{5}{2}x+1=0$$

$$x=\frac{+\left(\frac{5}{2}\right)\pm\sqrt{\frac{25}{4}-4(1)(1)}}{2(1)}$$

$$=\frac{\frac{5}{2}\pm\frac{3}{2}}{2}=\frac{8}{2\times2},\ \frac{2}{2\times2}$$

$$=2,\ \frac{1}{2}$$

Case(ii) put $z=-\frac{10}{3}$.

$$x^2+\frac{10}{3}x+1=0$$

$$x=\frac{-\frac{10}{3}\pm\sqrt{\frac{100}{9}-4(1)(1)}}{2}$$

$$=\frac{-\frac{10}{3}\pm\frac{8}{3}}{2}=\frac{-2}{6},\ \frac{-18}{6}$$

$$=-\frac{1}{3},\ -3.$$

The required roots are,

$$x=1,2,\frac{1}{2},-\frac{1}{3},-3.$$

3) Solve: $6x^6-35x^5+56x^4-56x^2+35x-6=0$.

Soln:

WKT: (x^2-1) is the factor

$\therefore\ x=\pm1$ is a root.

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$6x^6-6-35x^5+35x+56x^4-56x^2=0$

$6(x^6-1)-35x(x^4-1)+56x^2(x^2-1)=0$

Consider.

$x^6-1=(x^2)^3-1^3$.

$=(x^2-1)(x^4+x^2+1)$.

$x^4-1=(x^2)^2-1^2$

$=(x^2+1)(x^2-1)$

$6[(x^2-1)][x^4+x^2+1)]-35x((x^2+1)(x^2-1)$
$+56x^2(x^2-1)=0$

$(x^2-1)[6(x^4+x^2+1)-35x(x^2+1)+56x^2=0$

$x\pm1$
is a root.

$6x^4+6x^2+6-35x^3-35x+56x^2=0$.

$6x^4-35x^3+62x^2-35x+6=0$.

$\div x^2$ $\quad \frac{6x^4}{x^2}-\frac{35x^3}{x^2}+\frac{62x^2}{x^2}+\frac{6}{x^2}=0$

$6x^2-35x+62+\frac{6}{x^2}=0$.

$6\left(x^2+\frac{1}{x^2}\right)-35x+62=0$.

Let $z=x+\frac{1}{x}$

$\cancel{x+1/x}$ $\quad z^2-2=x^2+\frac{1}{x^2}$.

$\cancel{6(z^2-2)-35x+62=0}$.

$\cancel{6z^2-35x-12+62=0}$

$6x^4-35x^3+62x^2-35x+6=0$.

$\div x^2$ $\quad \frac{6x^4}{x^2}-\frac{35x^3}{x^2}+\frac{62x^2}{x^2}-\frac{35x}{x^2}+\frac{6}{x^2}=0$

$6x^2-35x+62-\frac{35}{x}+\frac{6}{x^2}=0$

$6\left(x+\frac{1}{x^2}\right)-35\left(x+\frac{1}{x}\right)+62=0$.

Let $z=x+\frac{1}{x}$.

$z^2-2=x^2+\frac{1}{x^2}$.

$6(z^2-2)-35(z)+62=0$

$6z^2-12-35z+62=0$

$6z^2-35z+50=0$.

$z=\frac{35\pm\sqrt{1225-4(6)(50)}}{2(6)}$.

$=\frac{35\pm\sqrt{1225-1200}}{12}$.

$=\frac{35\pm\sqrt{25}}{12}$

$=\frac{35\pm5}{12}$

$z=\frac{40}{12},\frac{30}{12}$.

$\boxed{z=\frac{10}{3},\frac{5}{2}}$

35×35
175

200×6
1200

W.K.T.

$z = x + \frac{1}{x}$

$xz = x^2 + 1$

$x^2 - xz + 1 = 0$

Case (i) $z = \frac{10}{3}$

$\cancel{\frac{100}{9} - \frac{40}{3}z + 1 = 0}$

$x^2 - \frac{10}{3}z + 1 = 0$

$x = \frac{\frac{10}{3} \pm \sqrt{\frac{100}{9} - 4(1)(1)}}{2(1)}$

$= \frac{\frac{10}{3} \pm \sqrt{\frac{64}{9}}}{2}$

$= \frac{\frac{10}{3} \pm \frac{8}{3}}{2}$

$x = \frac{18}{3} \times \frac{1}{2} = \frac{18}{6}, \quad \frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$

$\boxed{x = 3, \frac{1}{3}}$

Case ii $z = \frac{5}{2}$.

$x^2 - \frac{5}{2}x + 1 = 0$

$x = \frac{\frac{5}{2} \pm \sqrt{\frac{25}{4} - (4)(1)(1)}}{2}$

$= \frac{5}{2} \pm \sqrt{\frac{9}{4}}$

$= \frac{\frac{5}{2} \pm \frac{3}{2}}{2}$

$x = \frac{8}{4}, \frac{2}{4}$.

$\boxed{x = 2, \frac{1}{2}}$.

The required roots $\pm 1, 2, 3, \frac{1}{3}, \frac{1}{2}$.

3) $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$.

$6(x^6 - 1) - 25x(x^4 - 1) + 31x^2(x^2 - 1) = 0$.

Consider.

$x^6 - 1 = (x^2)^3 - 1^3$.

$= (x^2 - 1)(x^4 + x^2 + 1)$.

$x^4 - 1 = (x^2)^2 - 1$.

$= (x^2 + 1)(x^2 - 1)$.

$6(x^2 - 1)(x^4 + x^2 + 1) - 25x(x^2 + 1)(x^2 - 1) + 31x^2(x^2 - 1) = 0$

$(x^2 - 1)\left[6(x^4 + x^2 + 1) - 25x(x^2 + 1) + 31x^2\right] = 0$

$\therefore$ $x = \pm 1$ are the roots.

$6x^4 + 6x^2 + 6 - 25x^3 - 25x + 31x^2 = 0$

$\cancel{\div x^2}$

$\frac{6x^4}{x^2} + \frac{6x^2}{x^2} + \frac{6}{x}$

$6\cancel{x^4 + 6x}$

$$6x^4 - 25x^3 + 37x^2 - 25x + 6 = 0$$

$\div x^2$.

$$\frac{6x^4}{x^2} - \frac{25x^3}{x^2} + \frac{37x^2}{x^2} - \frac{25x}{x^2} + \frac{6}{x^2} = 0$$

$$6x^2 - 25x + 37 - \frac{25}{x} + \frac{6}{x^2} = 0$$

$$6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x + \frac{1}{x}\right) + 37 = 0$$

Let $z = x + \frac{1}{x}$.

$$\begin{array}{r} 37 \\ 12 \\ \hline 25 \end{array}$$

$$z^2 - 2 = x^2 + \frac{1}{x^2}.$$

$$6z^2 - 12 - 25z + 37 = 0$$

100×6

$$6z^2 - 25z + 25 = 0.$$

$$z = \frac{25 \pm \sqrt{225 - 4(6)(25)}}{2(6)}$$

$$= \frac{25 \pm \sqrt{625 - 600}}{\cancel{50}\,12}$$

$$= \frac{25 \pm \sqrt{25}}{\cancel{50}\,12}$$

$$z = \frac{25 \pm 5}{\cancel{50}\,12}$$

$$\cancel{z = \frac{30}{100}, \frac{20}{100}} \quad z = \frac{30}{12}, \frac{20}{12}$$

$$\cancel{= \frac{3}{5}, \frac{2}{5}} \quad \boxed{z = \frac{5}{3}, \frac{5}{2}}$$

Let $z = x + \frac{1}{x}$.

$$zx = x^2 + 1$$

$$x^2 - zx + 1 = 0$$

Case (i) $z = \frac{5}{3}$.

$$x^2 - \frac{5}{3}x + 1 = 0$$

$$x = \frac{\frac{5}{3} \pm \sqrt{\frac{25}{9} - 4(1)(1)}}{2}$$

$$\begin{array}{r} 36 \\ 25 \\ \hline 11 \end{array}$$

$$= \frac{\frac{5}{3} \pm \sqrt{\frac{-11}{9}}}{2}$$

$$= \frac{\frac{5}{3} \pm \frac{1}{3}\sqrt{-11}}{2}$$

$$x = \frac{3}{2}, \frac{1}{6}$$

$$\boxed{x = \frac{3}{2}, \frac{1}{6}}$$

Case (ii) $z = \frac{5}{2}$.

$$x^2 - \frac{5}{2}x + 1 = 0$$

$$x = \frac{\frac{5}{2} \pm \sqrt{\frac{25}{4} - 4(1)(1)}}{2}$$

$$= \frac{\frac{5}{2} \pm \sqrt{\frac{9}{4}}}{2}$$

$$= \frac{\frac{5}{2} \pm \frac{3}{2}}{2}$$

$$= \frac{8}{4}, \frac{2}{4}.$$

$$\boxed{x = 2, \frac{1}{2}}$$

Case(ii) $x = 5/3$

$$x^2 - \frac{5}{3}x + 1 = 0.$$

$$x = \frac{5/3 \pm \sqrt{\frac{25}{9} - 4(1)(1)}}{2}$$

$$= \frac{5/3 \pm \sqrt{\frac{25-36}{9}}}{2}$$

$$= \frac{5/3 \pm \sqrt{\frac{-11}{9}}}{2}$$

$$= \frac{\frac{5}{3} \pm \frac{1}{3}\sqrt{-11}}{2}$$

$$= \frac{1}{3} \times \frac{5 \pm \sqrt{-11}}{2}$$

$$= \frac{5 \pm \sqrt{-11}}{6}$$

$$x = \frac{5 + \sqrt{-11}}{6}, \quad \frac{5 - \sqrt{-11}}{6}.$$

$\therefore$ The required roots of the eqn are $\frac{5+\sqrt{-11}}{6}, \frac{5-\sqrt{-11}}{6}, 2, \frac{1}{2}, \pm 1.$

5) solve $x^{10} - 3x^8 + 5x^6 - 5x^4 + 3x^2 - 1 = 0$

$$x^{10} - 3x^8 + 5x^6 - 5x^4 + 3x^2 - 1 = 0$$

(x^2-1) is a factor.

$$x^{10} - x^8 - 2x^8 + 2x^6 + 3x^6 - 3x^4 - 2x^4 + 2x^2 + x^2 - 1 = 0$$

$$x^8(x^2-1) - 2x^6(x^2-1) + 3x^4(x^2-1) - 2x^2(x^2-1) + (x^2-1) = 0$$

$$(x^2-1)(x^8 - 2x^6 + 3x^4 - 2x^2 + 1) = 0.$$

(x^2-1) is a factor.

$x^2 - 1 \Rightarrow x = \pm 1$ is a root.

The eqn divide by x^4.

$$x^8 - 2x^6 + 3x^4 - 2x^2 + 1 = 0.$$

$$\frac{x^8}{x^4} - \frac{2x^6}{x^4} + \frac{3x^4}{x^4} - \frac{2x^2}{x^4} + \frac{1}{x^4} = 0$$

$$x^4 - 2x^2 + 3 - \frac{2}{x^2} + \frac{1}{x^4} = 0.$$

$$\left(x^4 + \frac{1}{x^4}\right) - 2\left(x^2 + \frac{1}{x^2}\right) + 3 = 0.$$

$z = x^2 + \frac{1}{x^2}$ $\boxed{1 = z} = \frac{x^4+1}{x^2} =$

$\cancel{x^4 + \frac{1}{x^4} = z^2}$

$z =$

13/9/17

Differential Calculus:

And its Application:-

Differentiation and derivatives of Simple functions.

The rate of change in y with respect to x can be measured using the derivatives.

$\frac{dy}{dx}$ in physics velocity is equal to $\frac{dx}{dt}$.

The method of finding the derivative of a function is called differentiation.

Basic diffentiation formula:

1) $\frac{d}{dx} c = 0$,

2) $\frac{d}{dx} x^n = nx^{n-1}$

3) $\frac{d}{dx} e^{ax} = ae^{ax}$.

4) $\frac{d}{dx} \sin ax = a \cos ax$..

DSC

5) $\frac{d}{dx} \cos ax = -a \sin ax$

6) $\frac{d}{dx} \tan ax = a \sec^2 ax$.

7) $\frac{d}{dx} \sec ax = a \sec ax \tan ax$.

8) $\frac{d}{dx} \operatorname{cosec} ax = -a \operatorname{cosec} ax \cot ax$.

9) $\frac{d}{dx} \cot ax = -a \operatorname{cosec}^2 ax$.

10) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$.

11) $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$.

12) $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$.

13) $\frac{d}{dx} (u \pm v) = \frac{d}{dx} u \pm \frac{dv}{dx}$.

14) $\frac{d}{dx} (uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

$= uv' + vu'$.

15) $\frac{d}{dx} \left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$.

16) $\frac{d}{dx} cu = c\frac{d}{dx} u = cu'$.

$e^{x^2} = e$

$d(e^{x^2})$

put $u = x^2$

$\boxed{du = 2x\,dx}$.

$\frac{u}{v} = \frac{vu' - uv'}{v^2}$

1) Find the derivative of the following:-

i) $\frac{d}{dx}[4x^3]$

$= 4[3(x)^{3-1}]$

$= 12x^2$.

2) $\frac{d}{dx}\{4x^3+5\sin x\}$.

$\frac{d}{dx}[4x^3+5\sin x] = 4(3x^{3-1}) + 5\cos x$.

$= 12x^2 + 5\cos x$.

3) $\frac{d}{dx}[e^{7x} + 8\cos 3x]$

$\frac{d}{dx}[e^{7x}+8\cos 3x] = 7e^{7x} - 8[3\sin 3x]$

$= 7e^{7x} - 24\sin 3x$.

4) $\frac{d}{dx}[5 + x^6 + \cos 5x + 7e^{4x}]$

$\frac{d}{dx}[5+x^6+\cos 5x+7e^{4x}] = 0 + 6x^5 - 5\sin 5x + 7\cdot 4e^{4x}$.

$= 6x^5 - 5\sin 5x + 28e^{4x}$

5) $\frac{d}{dx}[5\cos 2x + 6\sin 2x + 7\sec 4x]$.

$\frac{d}{dx}[5\cos 2x + 6\sin 2x + 7\sec 4x] = 5(-2\sin 2x) + 6(2\cos 2x) + 7(4\sec 4x\tan 4x)$

$= -10\sin 2x + 12\cos 2x + 28\sec 4x\tan 4x$

II Find the derivative of the following:-

1). $\frac{d}{dx}[4\cot 5x + 6\tan 3x]$

$\frac{d}{dx}[4\cot 5x + 6\tan 3x] = -20\,\mathrm{cosec}^2 5x + 18\sec^2 3x$.

2) $\frac{d}{dx}[3 + 7\sec 8x + 4\,\mathrm{cosec}\,x] = 0 + 7[8\sec 8x\tan 8x] - 4[\mathrm{cosec}\,x\cot x]$

$= 56\sec 8x\tan 8x - 4\,\mathrm{cosec}\,x\cot x$

3) $\frac{d}{dx}[8e^{4x} + 9x^9 + 6e^{8x}] = 8(4e^{4x}) + 9(9x^{9-1}) + 6(8e^{8x})$

$= 32e^{4x} + 81x^8 + 48e^{8x}$.

4) $\frac{d}{dx}[8\cos^{-1}x + 4\sin^{-1}x] = 8\frac{-1}{\sqrt{1-x^2}} + 4\frac{1}{\sqrt{1-x^2}}$.

$= \frac{-8+4}{\sqrt{1-x^2}}$

$= \frac{-4}{\sqrt{1-x^2}}$.

$\sin^{-1}: \frac{1}{\sqrt{1-x^2}}$

$\cos^{-1}: \frac{-1}{\sqrt{1-x^2}}$

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$\frac{d}{dx}$

$= 4\cos 4x + \frac{1}{1+x^2}$.

$= \cancel{\frac{4\cos 4x}{1+x^2}} = 4\cos 4x + \frac{1}{1+x^2}$.

$\log_a 2x$

I)

$\frac{1}{2x}$

1) $\frac{d}{dx} e^{\sin x^2}$

$= \sin x^2 e^{\sin x^2} \cos x^2 (2x)$.

2) $\frac{d}{dx}(x^2+7x+2)(e^x-\log x)$.

$= (x^2+7x+2)(e^x-\frac{1}{x}) - (e^x-\log x)(2x+7)$

$=$

I) $y = x+\cos x$.

$\frac{dy}{dx} = 1-\sin x$

$\frac{d^2y}{dx^2} = -\cos x$.

2) $y = xe^{-x}+be^{x}$.

$\frac{dy}{dx} = x(-e^{-x}) + e^{-x}(1) + b(e^{x}) + e^{x}$.

$= -xe^{-x} + e^{-x} + be^{x}$

$\frac{d^2y}{dx^2} = -x(-e^{-x}) + e^{-x}(-1) + (-e^{-x}) + b(e^{x}) + 0$

$= xe^{-x} - e^{-x} - e^{-x} + be^{x}$.

$= xe^{-x} - 2e^{-x} + be^{x}$.

I. If $y = x + 1/x$ show that $x^2y_2 + xy_1 - y = 0$

$y = x + 1/x$.

$y_1 = \frac{dy}{dx} = (1) - \frac{1}{x^2}$.

$y_2 = \frac{d^2y}{dx^2} = 2/x^3$.

Consisder

$x^2y_2 + xy_1 - y = x^2 \cdot 2/x^3 + x(1-1/x^2) - (x+1/x)$

$= 2/x + x - 1/x - x - 1/x$

$= 2/x - 2/x = 0$.

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2) If $y = e^x \sin x$ PT $y_2 - 2y_1 + 2y = 0$

$y_1 \Rightarrow \frac{dy}{dx} = e^x \cos x + \sin x\, e^x$.

$y_2 \Rightarrow \frac{d^2y}{dx^2} = e^x(-\sin x) + \cos x\, e^x + \sin x\, e^x + \cos x\, e^x$

$= -\sin x\, e^x + \cos x\, e^x + \sin x\, e^x + \cos x\, e^x$

$= 2\cos x\, e^x$.

$y_2 - 2y_1 + 2y = 2\cos x\, e^x - 2(e^x \cos x + \sin x\, e^x) + 2e^x \sin x$

$= 2\cos x\, e^x - 2e^x \cos x - 2\sin x\, e^x + 2e^x \sin x$

$= 0$

$e^x \cos x + \sin x\, e^x$

3) If $y = e^{ax} \sin bx$. PT $y_2 - 2ay_1 + (a^2+b^2)y = 0$

$y_1 = \frac{dy}{dx} = e^{ax}(b\cos bx) + \sin bx\, ae^{ax}$

$= e^{ax} b\cos bx + \sin bx\, a e^{ax}$.

$y_2 = \frac{d^2y}{dx^2} = e^{ax}(-b^2 \sin bx) + b\cos bx\, ae^{ax} + \sin bx\, a^2 e^{ax}$

$+ ae^{ax} b\cos bx$

$= -b^2 e^{ax} \sin bx + b\cos bx\, ae^{ax} + \sin bx\, a^2 e^{ax}$

$+ ae^{ax} b\cos bx$

$y_2 - 2ay_1 + (a^2+b^2)y = -b^2 e^{ax} \sin bx + b\cos bx\, ae^{ax} +$

$\sin bx\, a^2 e^{ax} + a e^{ax} b\cos bx$

$-2ae^{ax} b\cos bx - 2a \sin bx\, a e^{ax}$

$+ a^2 e^{ax} \sin bx + b^2 e^{ax} \sin bx$

$= 0$ //

4) $y = x^3 - 1$ PT $x^2 y_3 - 2xy_2 + 2y_1 = 0$

$y_1 = \frac{dy}{dx} = 3x^2 - 0 = 3x^2$

$y_2 = \frac{d^2y}{dx^2} = 3(2x) = 6x$

$y_3 = \frac{d^3y}{dx^3} = 6(1) = 6$

$x^2 y_3 - 2xy_2 + 2y_1 = x^2(6) - 2x(6x) + 2(3x^2)$

$= 6x^2 - 12x^2 + 6x^2$.

$= 0$ //.

7) $y = (x + \sqrt{x^2-1})^m$ PT $(x^2-1)y_2 + xy_1 - m^2 y = 0$.

$y_1 = \frac{dy}{dx} = m(x + \sqrt{x^2-1})^{m-1} \left(1 + \frac{1}{2}(x^2-1)^{-1/2}\right)(2x)$

~~$= m(x + (x^2-1)^{1/2})^{m-1} \left(1 + \frac{1}{\sqrt{x^2-1}}\right)$~~

$= m(x + (x^2-1)^{1/2})^{m-1} \left[1 + \frac{1}{(x^2-1)^{1/2}}\right]$

$y_2 = \frac{d^2y}{dx^2} = m(m-1)(x + (x^2-1)^{1/2})\left(1 + \frac{1}{2}(x^2-1)^{-1/2}\right)$

$\left[1 + \frac{1}{(x^2-1)^{1/2}}\right] + m(x + (x^2-1)^{1/2})^{m-1}$

$\left[\frac{(x^2-1)^{1/2}(1) - x\frac{1}{2}(x^2-1)^{-1/2}\, 2x}{x^2-1}\right]$

$(x^2-1)y_2 + xy_1 - m^2 y = 0$.

x in $(m-1)$

Application of differential.

~~Finding~~ the Curve

Radius of curvature.

radius of curvature $\rho = \frac{[1+y_1^2]^{3/2}}{y_2}$

where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$

1) Find the radius of curvature of the parabola $y^2 = 4ax$

Soln:

Given

$y^2 = 4ax$ at (a,a)

$2yy_1 = 4a$

$y_1 = \frac{4a}{2y} = \frac{2a}{y}$, $y_1(a,a) = \frac{2a}{a} = 2$.

$y_2 = \frac{-2a}{y^2}\frac{dy}{dx}$

$y_2 = \frac{-2a}{y^2} y_1$

$y_2 = \frac{-2a}{y^2} \times \frac{2a}{y}$

$y_2 = -\frac{4a^2}{y^3}$

at $y_2(a,a) = \frac{-4a^2}{a^3} = \frac{-4}{a}$

Radius of curvature $\rho = \frac{[1+y_1^2]^{3/2}}{y_2}$

$= \frac{[1+4]^{3/2}}{-4/a}$

$= -\frac{5^{3/2}a}{4}$

Radius cannot be in negative

Radius of $\rho = \frac{a5^{3/2}}{4}$

2) Find the radius of curvature at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ on the curve $\sqrt{x}+\sqrt{y} = 1$.

Given $\sqrt{x}+\sqrt{y} = 1$ at $(1/4, 1/4)$

i.e $x^{1/2}+y^{1/2} = 1$

$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y_1 = 0$

$y_1 = -\frac{1}{2}x^{-1/2} \cdot 2y^{1/2} = \frac{-y^{1/2}}{x^{1/2}}$

$\rho = \frac{[1+y_1^2]^{3/2}}{y_2}$

$y_1(1/4, 1/4) = \frac{-(1/4)^{1/2}}{(1/4)^{1/2}} = -1$

$y_2 = \frac{-\left[x^{1/2}\left(-\frac{1}{2}y^{-1/2}\right)y_1 - y^{1/2}\left(-\frac{1}{2}\right)x^{-1/2}\right]}{(x^{1/2})^2}$

$= \frac{-\left[(1/4)^{1/2}\left(\frac{+1}{2(1/4)^{1/2}}\right)(-1) - (1/4)^{1/2}(-1/2)\frac{1}{(1/4)^{1/2}}\right]}{1/4}$

$= \frac{-[1/2 + 1/2]}{1/4} = \frac{+1}{1/4} = 4$.

$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{[1+(-1)^2]^{3/2}}{+4} = \frac{2^{3/2}}{4} = \frac{\sqrt{2\times2\times2}}{4} = \frac{\sqrt{2}}{2}$

$= 1/\sqrt{2}$

radius of curvature $\rho = \frac{1}{\sqrt{2}}$

2) Find the radius of curvature $xy = c$ at $(1,1)$.

$xy = c$ at $(1,1)$.

$x\frac{dy}{dx} + y(1) = 0$

$xy_1 + y = 0$

$y_1 = -y/x$ at

$y_1(1,1) = \frac{-1}{1} = -1$

$\frac{vu' - uv'}{v^2}$

$y_2 = \frac{x \cdot (\frac{dy}{dx}) - y(1)}{x^2}$

$y_2 = \frac{-xy_1 + y}{x^2}$

$y_2 = \frac{-x \cdot \frac{-y}{x} + y}{x^2}$

$= \frac{y+y}{x^2} = \frac{2y}{x^2} = 2.$

$\rho = \frac{(1+(y_1)^2)^{3/2}}{y_2}$

$= \frac{(1+1)^{3/2}}{2} = \frac{(2)^{3/2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

radius of curvature $\rho = \sqrt{2}$.

1) Find the radius of curvature of the $xy^2 = a^3 - x^3$ at (a,a).

$2xyy_1 + y^2 = -3x^2$.

$y_1 = \frac{-3x^2 - y^2}{2xy}$.

$y_1(a,a) = \frac{-3(a^2) - (a)^2}{2(a)(a)}$

$= \frac{-4a^2}{2a^2}$

$y_1 = -2.$

$y_2 = \frac{2xy[-6x - 2yy_1] - [-3x^2 - y^2]2(xy_1 + y)}{4x^2y^2}$

$y_2(a,a) = \frac{2a^2[-6(a) - 2(a)(-2)] - [-3(a)^2 - a^2]\,2(a(-2) + a)}{4a^4}$

$= \frac{2a^2[-2a] + 4a^2[-2a]}{4a^4}$

$= \frac{2a^2[-2a] - 8a^3}{4a^4}$

$= \frac{-4a^3 - 8a^3}{4a^4} = \frac{-12a^3}{4a^4} = -3/a$

$\rho = \frac{[1+(y_1)^2]^{3/2}}{y_2}$

$= \frac{[1+4]^{3/2}}{-3/a} = -\frac{(5)^{3/2}a}{3} = \frac{-5\sqrt{5}a}{3}$

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Since radius cannot be in negative

So Radius of Curvature $\rho = \frac{5\sqrt{5}a}{3}$

5) Find the radius of curvature of the parabola $x = at^2$, $y = 2at$, at T.

$x = at^2$ | $y = 2at$

$\frac{dx}{dt} = 2at$ | $\frac{dy}{dt} = 2a$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{1}{t}\right)\frac{1}{2at}$$

$$= -\frac{1}{t^2}\frac{1}{2at}$$

$$y_2 = \frac{-1}{2at^3}$$

$$\rho = \frac{[1+(y_1)^2]^{3/2}}{y_2}$$

$$= \frac{\left[1+\frac{1}{t^2}\right]^{3/2}}{-\frac{1}{2at^3}}$$

$$= -\left[\frac{t^2+1}{t^2}\right]^{3/2} \cdot 2at^3$$

$$= \frac{-(t^2+1)^{3/2} \cdot 2at^3}{t^3}$$

$$= -(t^2+1)^{3/2} \cdot 2a$$

The radius cannot be in negative

So radius of curvature $\rho = (t^2+1)^{3/2} \cdot 2a$.

6) Find the radius of curvature at $(a,0)$ on the curve $xy^2 = a^3 - x^3$.

Given $xy^2 = a^3 - x^3$

$$y^2 = \frac{a^3 - x^3}{x}$$

$$2yy_1 = \frac{x(-3x^2) - (a^3 - x^3)}{x^2}$$

$$2yy_1 = \frac{-3x^3 - a^3 + x^3}{x^2}$$

$$2yy_1 = \frac{-2x^3 - a^3}{x^2} = \frac{-(2x^3 + a^3)}{x^2}$$

$$y_1 = \frac{-2x^3 + a^3}{2yx^2}$$

$$y_1(a,0) = \frac{-2a^3 + a^3}{2(0)x^3}$$

$$= \frac{a^3}{0} =$$

Velocity and Acceleration:-

Velocity:

If ~~s~~ is the distance travelled by the particle in time t (sec), then the rate of ~~dansife~~ displacement is given by ds/dt, it is denoted by V (velocity). ie,

$$V = \frac{ds}{dt}$$

Acceleration:-

The rate of change of velocity is the Acceleration and is given by

$$\text{acceleration} = \frac{dv}{dt} = \frac{d^2s}{dt^2} = a.$$

Note:

i) Initial Velocity $V = \frac{ds}{dt}$ at time $t = 0$

ii) Initial acceleration $a = \frac{d^2s}{dt^2}$ at time $t = 0$.

Problems:-

1) The distance-time formula of moving particles is $S = 2t^3 + 3t^2 - 72t + 1$ find

i) Velocity at $t = 3$ sec.

ii) Initial velocity.

iii) Initial acceleration.

iv) acceleration after 4 sec.

Soln:-

Given : $S = 2t^3 + 3t^2 - 72t + 1$.

$$V = \frac{ds}{dt} = 6t^2 + 6t - 72.$$

$$a = \frac{d^2s}{dt^2} = \frac{dv}{dt} = 12t + 6.$$

i) $V_{(t=3\,sec)} = 6(3)^2 + 6(3) - 72$

$= 0$

(ii) $V_{(t=0)} = 6(0) + 6(0) - 72 = -72$ units/sec.

iii) $a_{(t=0)} = 12(0) + 6 = 6.$ units/sec^2.

iv) $a_{(t=4)} = 12(4) + 6$

$= 54.$ units/sec^2.

2) The distance-time formula of a moving particle is $S = \frac{t^3}{3} - \frac{7}{2}t^2 + 6t - 10$. find.

i) Velocity at $t = 3$ sec.

soln:

Given $\Rightarrow S = \frac{t^3}{3} - \frac{7}{2}t^2 + 6t - 10$

$$V = \frac{ds}{dt} = \frac{3t^2}{3} - \frac{14t}{2} + 6$$

$$\frac{ds}{dt} = t^2 - 7t + 6.$$

$a = \frac{d^2s}{dt^2} = 2t - 7$
$= 2t - 7$

i) velocity at t=3 sec

$V_{(t=3sec)} = 3^2 - 7(3) + 6$
$= 9 - 21 + 6$
$= -6$ Units / sec.

ii) velocity Initial

$V_{(t=0)} = 0 - 0 + 6$
$= 6.$ Units / sec.

iii) acceleration Initial.

$a_{(t=0)} = 0 - 7$
$= -7$ Units / sec^2

iv) acceleration at t=4 sec.

$a_{(t=4sec)} = 8 - 7$
$= 1$ Units / sec^2.

8) The distance time formula of a moving particle is $s = \frac{t^3}{6} - 5t^2 + 6t - 10$. Find.

i) Initial velocity.

$v = \frac{ds}{dt} = \frac{3t^2}{6_2} - 5(2t) + 6$
$= \frac{t^2}{2} - 10t + 6.$

$a = \frac{d^2s}{dt^2} = \frac{2t}{2} - 10$
$= t - 10.$

i) Initial Velocity.

$V_{(t=0)} = \frac{0^2}{2} - 0 + 6$
$= 6$ Units / sec.

ii) Velocity at t = 3 sec..

$V_{(t=3sec)} = \frac{9}{2} - 10(3) + 6$
$= \frac{9}{2} - 30 + 6.$
$= \frac{9}{2} - 24 = \frac{9-48}{2} = \frac{-39}{2}$ Units / sec

iii) Initial acceleration.

$a_{(t=0)} = 0 - 10 = -10$ Units / sec^2.

iv) acceleration at t = 4 sec.

$a_{(t=4)} = 4 - 10 = -6$ Unit / sec^2.

Tangents and Normal:

Tangent:

The straight line which touches the curve at a point is called the tangent.

Normal:

Normal is the straight line which is perpendicular to the tangent and passing through the point at which the tangent touches the curve.

Note:-

i) slope of a tangent $= \frac{dy}{dx} = m$.

ii) slope of a Normal $= \frac{-1}{m}$.

iii) Eqn of a tangent $\Rightarrow y-y_1 = m(x-x_1)$

iv) Eqn of a normal $\Rightarrow y-y_1 = \frac{-1}{m}(x-x_1)$

Problems:-

i) Find the eqn of tangent and normal to the curve $2y = 3-x^2$ at $(1,1)$.

Given $2y = 3-x^2$

$$2\frac{dy}{dx} = 0-2x.$$

$$\cancel{2}\frac{dy}{dx} = -\cancel{2}x$$

$$\frac{dy}{dx} = -x.$$

$$\frac{dy}{dx}_{(1,1)} = -1.$$

Slope of a tangent $m = -1$

Slope of a normal $\frac{-1}{m} = 1$.

Eqn of a tangent $\Rightarrow y-1 = -1(x-1)$

$$y-1 = -x+1$$

$$\boxed{x+y-2=0}$$

Eqn of a normal $\Rightarrow y-1 = 1(x-1)$

$$x-y=0$$

$$\boxed{x-y=0}$$

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2) Find the eqn of the Tangent and Normal to the Curve $y = 5 - 2x - 3x^2$ at $(2, -11)$

Given.

$y = 5 - 2x - 3x^2$.

$\frac{dy}{dx} = 0 - 2 - 3(2x)$

$= -2 - 6x$.

$\frac{dy}{dx} = -6x - 2$.

$\frac{dy}{dx}(2,-11) = -6(2) - 2$

$= -12 - 2$

$= -14$.

$\therefore$ Slope of the tangent $m = -14$.

$\therefore$ Slope of the normal $\frac{-1}{m} = \frac{-1}{-14} = \frac{1}{14}$.

Eqn of the tangent $y - y_1 = m(x - x_1)$

$y + 11 = -14(x - 2)$

$y + 11 = -14x + 28$.

$14x + y + 11 - 28 = 0$

$14x + y - 17 = 0$.

Eqn of the Normal $y - y_1 = \frac{-1}{m}(x - x_1)$

$y + 11 = \frac{1}{14}(x - 2)$

$\frac{14 \times 11}{14}$

$\frac{14}{154}$

$14y + 154 = x - 2$

$x - 14y - 156 = 0$

$x - 14y - 156 = 0$.

3) Find the eqn of the tangent and Normal to the Curve $y = \frac{5x^2}{1+x^2}$ at $(2, 4)$

Given $\Rightarrow$ $y = \frac{5x^2}{1+x^2}$.

$\frac{dy}{dx} = \frac{(1+x^2)(10x) - 5x^2(0+2x)}{(1+x^2)^2}$.

$= \frac{1+x^2(10x) - 5x^2(2x)}{(1+x^2)^2}$

$= \frac{10x + 10x^3 - 5 \times 10x^3}{(1+x^2)^2}$

$\frac{dy}{dx} = \frac{10x}{(1+x^2)^2}$.

$\frac{dy}{dx} = \frac{10x}{(1+(2)^2)^2} = \frac{10(2)}{(5)^2} = \frac{20}{25} = \frac{4}{5}$.

slope of the tangent $m = \frac{4}{5}$

slope of the normal $\frac{-1}{m} = -\frac{5}{4}$.

Eqn of the tangent $y - y_1 = m(x - x_1)$

$$y - 4 = \frac{4}{5}(x - 2)$$

$$5y - 20 = 4x - 8$$

$$4x - 5y + 12 = 0.$$

Eqn of the Normal $\Rightarrow y - y_1 = \frac{-1}{m}(x - x_1)$

$$y - 4 = \frac{-5}{4}(x - 2)$$

$$4y - 16 = -5x + 10$$

$$5x + 4y - 10 - 16 = 0$$

$$5x + 4y - 26 = 0$$

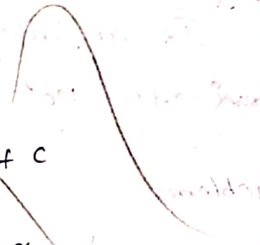
Intergration:-

Intergration is reciprocal of differentiation

Basic formula:-

1) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

2) $\int e^{ax} \, dx = \frac{e^{ax}}{a} + c$



3) $\int \cos ax \, dx = \frac{\sin ax}{a} + c$

4) $\int \sin ax \, dx = \frac{-\cos ax}{a} + c$

5) $\int \frac{1}{x} \, dx = \log x + c$

6) $\int \tan ax \, dx = \frac{\log(\sec ax)}{a} + c$

7) $\int \sec ax \, dx = \frac{\log(\tan ax + \sec ax)}{a} + c$

8) $\int \sec^2 ax \, dz = \frac{\tan ax}{a} + c$

9) $\int \operatorname{cosec}^2 ax \, dx = \frac{-\cot ax}{a} + c$

10) $\int uv \, dx = vu_1 - u'v_2 + u''v_3 - \ldots$

11) $\int u \, dv = uv - \int v \, du$.

$\cos ax \, dx = \frac{\sin ax}{a} + c$

$\sin ax \, dx = \frac{-\cos ax}{a} + c$

$\frac{1}{x} = \log x + c$

$\tan ax = \log(\sec ax) + c$

$\sec ax = \log(\tan ax + \sec ax)$

12) $\int c\, dx = cx + c$.

definite integral:-

Is an integral where the limits are given.

$(0,5) \to 0 < x < 5$

$[0,5] : 0 \le x \le 5$

$[0,5) : 0 \le x < 5$

$(0,5] : 0 < x \le 5$

Indefinite integral:-

Is an integral where the limits are not given.

1) Problems

1) $\int (5e^{3x} + \cos 4x + \frac{1}{x+3} + \sec^2 4x + 3)\, dx$

$= \frac{5e^{3x}}{3} + \frac{\sin 4x}{4} + \log(x+3) + \frac{\tan 4x}{4} + 3x + c.$

2) $\int x^2 e^{5x}\, dx$

$u = x^2$ $v = e^{5x}$

$u' = 2x$ $v_1 = \frac{e^{5x}}{5}$

$u'' = 2$ $v_2 = \frac{e^{5x}}{25}$

$u''' = 0$ $v_3 = \frac{e^{5x}}{125}$

$\int uv\, dx = uv_1 - u'v_2 + u''v_3 - \ldots$

$\int x^2 e^{5x} dx = \frac{x^2 e^{5x}}{5} - \frac{2x e^{5x}}{25} + \frac{2e^{5x}}{125}.$

3) $\int x^2 \log x\, dx$.

$~~u = x^2~~$ $~~v = \log x~~$

$~~u' = 2x~~$ $~~v_1 =~~$

$~~u'' = 2~~$

$~~u''' = 0~~$

$u = \log x$ $v = x^2$.

$u' = \frac{1}{x}$ $v_1 = \frac{x^3}{3}$

$u'' = -\frac{1}{x^2}$ $v_2 = \frac{x^4}{12}$

$u''' = +\frac{2}{x^3}$ $v_3 = \frac{x^5}{60}$.

$\frac{12 \times 5}{60}$

$\int uv\, dx = uv_1 - u'v_2 + u''v_3 - \ldots$

$= \log x \frac{x^3}{3} - \frac{1}{x}\frac{x^4}{12} - \frac{1}{x^2}\frac{x^5}{60} \ldots + c$

$= \frac{\log x\, x^3}{3} - \frac{x^4}{12x} - \frac{x^5}{60x^2} \ldots + c$

$= \frac{\log x\, x^3}{3} - \frac{x^3}{12} - \frac{x^3}{60} \ldots + c.$

3) $\int x \tan^2 x \, dx$

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

$\int x \{\sec^2 x - 1\} \, dx$

$\int [x \sec^2 x - x] \, dx$

$\sin^3\theta = \left[\frac{3\sin\theta - \sin 3\theta}{4}\right]$

$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$

$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

$u = x$ $\quad v = \sec^2 x$

$u' = 1$ $\quad v_1 = \tan x$

$u'' = 0$ $\quad v_2 = \log \sec x$

$\int uv \, dx = uv_1 - u'v_2 + u''v_3 - \dots$

$\int x \tan^2 x \, dx = x \tan x - \log \sec x - \frac{x^2}{2}$

H.W.

$\sin^n x = 0$

$\cos^n x =$

1) $\int e^{5x} x^5 \, dx$

2) $\int \sin 4x \, e^{3x} \, dx$

3) $\int x^3 \sin^3 x \, dx$

4) $\int x^4 \sin^2 x \, dx$

5) $\int x^3 \cos^2 4x \, dx$

Area.

Area bounded by a closed curve

The definite integral

$$\int_a^b y \, dx = \int_a^b x \, dy$$

gives the area of the region which is bounded by the curve $y = f(x)$, the axis of x and the two ordinates $x = a$ and $x = b$.

1) Find the area bounded by the curve $y = x^2 + x$ from $x = 1$ and $x = 3$.

$$\int_a^b y \, dx = \int_a^b x \, dy$$

$a = 1$; $b = 3$.

$y = x^2 + x$.

$$\int_a^b y \, dx = \int_1^3 x^2 + x$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_1^3$$

$= \left[\frac{27}{3}+\frac{9}{2}\right] - \left[\frac{1}{3}+\frac{1}{2}\right]$

$= \left[\frac{54+\cancel{18}\,27}{6}\right] - \left[\frac{2+3}{6}\right]$

$= \left[\frac{81}{6}\right] - \left[\frac{5}{6}\right]$

$= \frac{67}{6} \cdot \frac{76}{6}$

27, 27, 54

54, 18, 72

72, 5, 67

2, 54, 27, 81

H/W) 1)

$\int e^{5x} x^5 \, dx$

$u = x^5 \qquad v = e^{5x}$

$u' = 5x \qquad v_1 = \frac{e^{5x}}{5}$

$u'' = 5 \qquad v_2 = \frac{e^{5x}}{25}$

$u''' = 0 \qquad v_3 = \frac{e^{5x}}{125}$

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \ldots$

$\int e^{5x} x^5 dx = x^5 \frac{e^{5x}}{5} - \frac{5x e^{5x}}{25} + \frac{5e^{5x}}{125}.$

2) $\int \sin 4x \, e^{3x} \, dx$

$\int \sin 4x \, e^{3x} \, dx = e^{3x} \sin 4x - \cos 3e^{3x} - \frac{\cos 4}{4} + 9e^{3x} \frac{\cos 4x}{16} + 27e^{3x} - \frac{\cos 4x}{64}$

3) $\int x^3 \sin^3 x \, dx$

$= \int x^3 \left[\frac{3\sin\theta - \sin 3\theta}{4}\right] dx$

$= \frac{3}{4}\int x^3 \sin x \, dx - \frac{1}{4}\int x^3 \sin 3x \, dx$

$= \frac{3}{4}\int x^3(-\cos x - 3x^2(-\sin x) + 6x(\cos x) - 6\sin x)$

$= \frac{1}{4}\left(x^3\left(\frac{-\cos 3x}{3}\right) - 3x^2\left(\frac{-\sin 3x}{3^2}\right)\right.$

$6x\left(\frac{\cos^2 x}{3^2}\right)$

$\left. - 6\left(\frac{\sin 3x}{3^4}\right)\right).$

c) $\int x^3 \cos^2 4x \, dx$

$u = x^3$ | $v = \cos^2 4x$

$u' = 3x^2$ | $v_1 = \frac{1 - \cos^2 2x}{2} - \frac{x^4 \cos 2x}{16}$

$u'' = 6x$

$u''' = 6$ | $v_2 = \frac{\frac{x^5}{20} \cdot \frac{\sin 2x}{4}}{2}$

$u^{iv} = 0$

$\int x^3 \cos^2 4x \, dx = \frac{x^2}{2} \frac{x^3 \cos 2x}{2} + \frac{x^4}{8} - 3x^4 \frac{\cos 2x}{16}$

$+ \frac{x^5}{10}\left(\frac{x^5}{20}\right) \cdot \frac{\sin 2x}{4 \times 2}$

4) $\int x^4 \sin^2 x \, dx$

$u = x^4$

$u' = 4x^3$

$u'' = 4(3x) = 12x$

$u''' = 12$

$u^{iv} = 0$

$v = \sin^2 x$:

$v_1 = \frac{x - \sin 2x}{4}$

$= \frac{x^2 + \cos 2x}{4}$

$v_2 = x^3 + \frac{\cos 2x}{16}$

$= \frac{x^3 + \sin 2x}{966}$

$v = \sin^2 x$

$v_1 = \frac{1 - \cos^2 2x}{2 \times 2}$

$v_1 = \frac{x - \sin 2x}{4 \times 2 \times 2}$

$v_2 = \frac{x^3 + \cos 2x}{16}$

$v_2 = x^3.$

$= x^4\left(\frac{1 - \cos 2x}{2}\right) - 4x^3\left(\frac{x - \sin 2x}{4}\right)$

$+ 12\left(\frac{x^3 + \sin 2x}{96}\right) + \ldots$

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1) find the area bounded by the Curve $y = x^2+1$ from $x = 1$, to 3.

$$\int_a^b y\,dx = \int_1^3 x^2+1$$

$$= \left[\frac{x^3}{3}+x\right]_1^3$$

$$= \left[\frac{x^3+3x}{3}\right]_1^3$$

$$= \left[\frac{27+9}{3}\right] - \left[\frac{1+3}{3}\right]$$

$$= \frac{36}{3} - \frac{4}{3}$$

$$= \frac{32}{3}$$

2) find the area bounded by the Curve $y = x^3$. $x = 1$ to 4.

$$\int_a^b y\,dx = \int_1^4 x^3\,dx$$

$$= \left[\frac{x^4}{4}\right]_1^4$$

$$= \frac{4^4}{4} - \frac{1}{4}$$

$$= \frac{256}{4} - \frac{1}{4} = \frac{255}{4}$$

3) find the area bounded by the Curve, $y = x^2$ from $x = 0$ to 1.

$$\int_a^b y\,dx = \int_0^1 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{3} - \frac{0}{3}$$

$$= \frac{1}{3}$$

$\sin 0 = 0$

$\sin 90 = 1$

$\sin \pi/2 = 1$

$\pi = 180$

4) find the area bounded by the Curve $y = 2x^2$ from $x = 0$ to 2

$$\int_a^b y\,dx = \int_0^2 2x^2\,dx.$$

$$= \left[\frac{2x^3}{3}\right]_0^2$$

$$= \frac{16}{3} - \frac{0}{3}$$

$$= \frac{16}{3}$$

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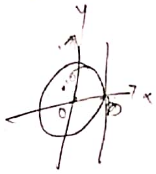
2) Find the area of the circle of radius b using integration method

The eqn of the circle with centre of the origin & radius 'b' is

$x^2+y^2=b^2$

Here x varies from $x=0$ to $x=b$ and from $x^2+y^2=b^2$



$\Rightarrow y^2 = b^2-x^2$

$\Rightarrow y = \pm\sqrt{b^2-x^2}$.

$\therefore$ Area of the circle $=4\int_0^b \sqrt{b^2-x^2}\,dx$.

$$=4\left[\frac{x}{b}\sqrt{b^2-x^2}+\frac{b^2}{2}\sin^{-1}\frac{x}{b}\right]_0^b.$$

$$=4\left[\frac{b^2}{2}\sin^{-1}1 - 0\right]$$

$$=4\,\frac{b^2}{2}\,\frac{\pi}{2} = \frac{4b^2\pi}{4} = b^2\pi.$$

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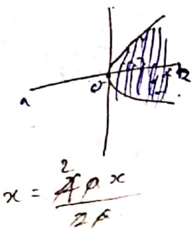
Volume:-

The volume of the solid obtained by rotating the area bounded by the curve $y=f(x)$ and x-axis between $x=a$ and $x=b$ about the x axis is integral $\int_a^b \pi y^2 dx$ is equal to $\int_a^b \pi (f(x))^2 dx$.

The volume of when the area bounded by the curve $y=f(x)$ and y axis between is revolved about the y axis between $y=a$, $y=b$ is integral

$\int_a^b \pi x^2 dx$.

1] Area: Parabola $\rightarrow y^2=4ax$,



$y^2 = 4ax$ $\quad x=\frac{y^2}{4a}$

$y = 2\sqrt{a}\sqrt{x}$.

limit $x=0$, and $x=b$.

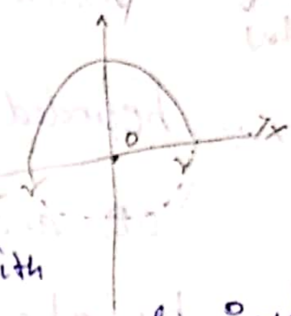
$y = 2\sqrt{a}\sqrt{x}$

Parabola $= 2\int_0^b 2\sqrt{a}\sqrt{x}\,dx$.

find the Volume of the Sphere of radius of r using integration.

Soln:

Volume of the Sphare is Obtained when the area bounded by the Semi circle & $x^2+y^2=r^2$ and the x axis when it is rotated about the x-axis



we known That $x^2+y^2=r^2$ is a Circle when center at the origin with radius r, when we Consisted a Semicircle, x, varies from $-r$ to r and $y^2=r^2-x^2$.

$\therefore$ Volume of the sphere is $\int_a^b \pi y^2 dx$

$\int y^2 dx$

$$\int_a^b \pi y^2 dx = \int_{-r}^{r} \pi (r^2-x^2)\, dx$$

$$= 2\pi \int_0^r (r^2-x^2)\, dx$$

$$= 2\pi \left[r^2 x - \frac{x^3}{3} \right]_0^r$$

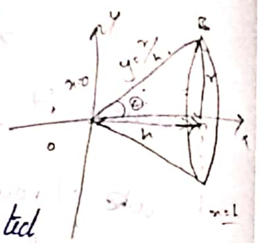
$= 2\pi \left[r^3 - \frac{r^3}{3} \right]$

$= \frac{4\pi r^3}{3}$.

2) Find the volume of the right circular cone of base radius 'r'.

Soln:

When the area of AOB bounded by the line OB and x-axis is rotated by about the x-axis, the solid cone obtained:



$\therefore$ Here x varies from 0 to h

and $y = \frac{r}{h} x$, $y = mx$

$\Rightarrow y = \frac{r}{h} x$

where $m = \frac{r}{h}$.

$$\int_a^b \pi y^2 \, dx = \int_0^h \pi \frac{r^2}{h^2} x^2 \, dx$$

$$= \frac{\pi r^2}{h^2} \int_0^h x^2 \, dx$$

$$= \frac{\pi r^2}{h^2} \left[\frac{x^3}{3} \right]_0^h$$

$$= \frac{\pi r^2 h^3}{3h^2}$$

$$= \frac{\pi r^2 h}{3}$$

3) **Reduction formula:**

1) $$\int \sin^m x \cos^n x = \frac{-\cos^{n-1} x \sin^{m-1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x$$

$$= \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x$$

2) $$\int_0^{\pi/2} \sin^m x \cos^n x$$

Case(i) when m is odd (n = odd or even)

$$I_{m,n} = \frac{m-1}{m+n} \cdot \frac{m-3}{m+n-2} \cdots \frac{2}{3+n} \cdot \frac{1}{n+1}$$

Case(ii) when m is even & n is odd

$$I_{m,n} = \frac{(m-1)}{m+n} \cdot \frac{(m-3)}{m+n-2} \cdots \frac{1}{2+n} \cdot \frac{(n-1)}{n} \frac{(n-3)}{n-2} \cdots$$

Case (iii) :

when m is even & n is even.

$$I_{m,n} = \frac{m-1}{m+n} \cdot \frac{(m-3)}{m+n-2} \cdots \frac{1}{2+n} \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2}.$$

$$I_n = \int \sin^n x \, dx.$$

$$nI_n = -\sin^{n-1} x \cos x + (n-1) I_{n-2}.$$

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \quad (n \text{ is odd})$$

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2} \quad (n \text{ is even})$$

$$I_n = \int \cos^n x \, dx.$$

$$nI_n = \cos^{n-1} x \sin x + (n-1) I_{n-2}$$

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \quad (n \text{ is odd})$$

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2} \quad (n \text{ is even})$$

Evaluate integral $\int \cos^3 x \, dx$ using

i) Usual integration method

ii) Reduction formula.

iii) find $\int_0^{\pi/2} \cos^3 x \, dx$.

Soln:

i) Usual integration method:

$$\int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx \quad \text{put } y = \sin x, \; dy = \cos x \, dx$$

$$= \int (1 - \sin^2 x) \cos x \, dx$$

$$= \int (1 - y^2) \, dy$$

$$= \left[y - \frac{y^3}{3} \right] + c$$

$$= \left[\sin x - \frac{\sin^3 x}{3} \right] + c$$

ii) Using Reduction formula

$$I_n = \int \cos^n x \, dx.$$

$$nI_n = \cos^{n-1} x \sin x + (n-1) I_{n-2}.$$

$n = 3$

$$\int \cos^3 x \, dx =$$

$$3I_3 = \cos^2 x \sin x + 2 I_{3-2}$$

$$3I_3 = \cos^2 x \sin x + 2I_1$$

$$I_3 = \frac{\cos^2 x \sin x + 2I_1}{3}$$

$$= \frac{1}{3} \left[\cos^2 x \sin x + 2 \int \cos x \, dx \right]$$

$$= \frac{1}{3} \left[\cos^2 x \sin x + 2 \sin x \right] + c$$

iii) $\int_0^{\pi/2} \cos^3 x\, dx$.

formula

$$\boxed{\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a}}$$

$$\int_0^{\pi/2} \cos^3 x\, dx.$$

Here $n=3$ (odd)

$$\int_0^{\pi/2} \cos^n x\, dx = \frac{(n-1)(n-3)(n-5)}{n(n-2)(n-4)} \cdots \frac{2}{3},$$

$$\int_0^{\pi/2} \cos^3 x\, dx = \frac{2}{3}.$$

2) Evaluate ~~integral~~ $\int \sin^5 x\, dx$

i) Using Usual substitutional method

2) Reduction formula.

3) $\int_0^{\pi/2} \sin^5 x\, dx$. find.

Soln:

i) Usual Substitution method.

$$\int \sin^5 x\, dx = \int \sin^4 x \sin x\, dx$$

Let $y = \cos x$

$dy = -\sin x\, dx$

$-dy = \sin x\, dx$.

$$= \int (\sin^2 x)^2 \sin x\, dx$$

$$= \int (1-\cos^2 x)\sin x\, dx$$

$$= -\int (1-y^2)^2 dy = -\int (1-2y^2+y^4) dy$$

$$= -\left[y - \frac{2y^3}{3} + \frac{y^5}{5}\right] = -\cos x + \frac{2}{3}\cos^3 x - \frac{\cos^5 x}{5}$$

ii) Reduction method.

$$I_n = \int \sin^n x\, dx.$$

$$I_n = \frac{1}{n}\left[-\sin^{n-1} x \cos x + (n-1) I_{n-2}\right]$$

Here $n=5$

$$I_5 = \frac{1}{5}\left[-\sin^4 x \cos x + 4 I_3\right]$$

$$= \frac{1}{5}\left[-\sin^4 x \cos x + \frac{4}{3}\left[-\sin^2 x \cos x + 2 I_1\right)\right.$$

$$= \frac{1}{5}\left[-\sin^4 x \cos x + \frac{4}{3}\left[-\sin^2 x \cos x + 2\int \sin x\, dx\right]\right.$$

$$= \frac{1}{2}\left[-\sin^4 x \cos x + \frac{4}{3}\left[-\sin^2 x \cos x - 2\cos x\right]\right] + C.$$

iii) $\int_0^{\pi/2} \sin^5 x\, dx$

Here $n=5$ (odd)

$$\int_0^{\pi/2} \sin^5 x\, dx = \frac{(n-1)(n-3)(n-5)}{n\cdot(n-2)(n-4)} \cdots \frac{2}{3}$$

$$= \frac{4}{5}\cdot\frac{2}{3}$$

$$= \frac{8}{15}.$$

3) $\int \sin^4 x \, dx$.

i) Usual Integration Method.

$$\int \sin^4 x \, dx =$$

i) Using Reduction method.

$$\int \sin^4 x \, dx = \frac{1}{4}\left[-\sin^3 x \cos x + 3 I_2\right]$$

$$= \frac{1}{4}\left[-\sin^3 x \cos x + \frac{3}{2}\left[-\sin x \cos x + I_0\right]\right.$$

$$= \frac{1}{4}\left[-\sin^3 x \cos x + \frac{3}{2}\left[-\sin x \cos x + I_0 + c\right]\right]$$

(ii) $\int_0^{\pi/2} \sin^4 x \, dx$.

$$\int_0^{\pi/2} \sin^4 x \, dx = \frac{(n-1)(n-3)(n-5)}{n \cdot (n-2)(n-4)} \cdots \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \frac{3\pi}{16}.$$

4) (i) $\int \cos^6 x \, dx$

$$\int \cos^n x \, dx = \frac{1}{n}\left[\cos^{n-1} x \sin x + (n-1) I_{n-2}\right]$$

$$= \frac{1}{6}\left[\cos^5 x \sin x + 5 I_3\right]$$

$$= \frac{1}{6}\left[\cos^5 x \sin x + \frac{5}{3}\left[\cos^2 x \sin x + 2 I_1\right]\right]$$

$$= \frac{1}{6}\left[\cos^5 x \sin x + \frac{5}{3}\left[\cos^2 x \sin x + 2 \sin x\right] + C\right]$$

(ii) $$\int_0^{\pi/2} \cos^6 x \, dx = \frac{(n-1)(n-3)(n-5)}{n \cdot (n-2)(n-4)} \cdots \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \frac{5\pi}{32}.$$

5) Evaluate:

$$\int \sin^5 x \cos^7 x \, dx$$

Using (i) Integration method

(ii) $\int_0^{\pi/4} \sin^5 x \cos^7 x \, dx$.

Soln!

i) Integral method.

$$\int \sin^5 x \cos^7 x \, dx.$$

$$\int \sin^m x \cos^n x \, dx = -\cos^{n+1} x \sin^{m-1} x + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x \, dx$$

$$\int \sin^5 x \cos^7 x \, dx = -\cos^8 x \sin^4 x + \frac{4}{12}\int \sin^3 x \cos^7 x \, dx.$$

$$= -\cos^8 x \sin^4 x + \frac{1}{3}\left[-\cos^8 x \sin^2 x + \frac{2}{10}\int \sin x \cos^7 x \, dx\right]$$

$$= -\cos^8 x \sin^4 x + \frac{1}{3}\left[-\cos^8 x \sin^2 x + \frac{1}{5}\left[-\cos^8 x\right]\right] + C$$

(ii) $\int_0^{\pi/2} \sin^5 x \cos^7 x \, dx.$

$m=5, \quad n=7.$

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{(m-1)(m-3)\cdots(n-1)(n-3)\cdots 2}{(m+n)(m+n-2)(m+n-4)\cdots 2}$$

$$= \frac{4\cdot 2\cdot 6\cdot 4\cdot 2}{12\cdot 10\cdot 8\cdot 6\cdot 4\cdot 2}$$

$$= \frac{2}{12\cdot 5\cdot 3} = \frac{1}{30}$$

1) $\int \sin^n x \, dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{(n-1)}{n} I_{n-2}$

2) $\int \cos^n x \, dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{(n-1)}{n} I_{n-2}$

1) Eg. $\int \sin^4 x \, dx = -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4} I_2 \ (n=4)$

$$= -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4}\left[-\frac{1}{2}\sin x \cos x + \frac{1}{2} I_0\right]$$

$$= -\frac{1}{4}\sin^3 x \cos x - \frac{3}{8}\sin x \cos x + C$$

Eg: $\int \cos^7 x \, dx = \frac{1}{7}\cos^6 x \sin x + \frac{6}{7} I_5 \ (n=7)$

$$= \frac{1}{7}\cos^6 x \sin x + \frac{6}{7}\left[\frac{1}{5}\cos^4 x \sin x + \frac{4}{5} I_3\right] \quad n=5$$

$$= \frac{1}{7}\cos^6 x \sin x + \frac{6}{7}\left[\frac{1}{5}\cos^4 x \sin x + \frac{4}{5}\left[\frac{1}{3}\cos^2 x \sin x + \frac{2}{3} I_1\right]\right] \ (n=3)$$

$$= \frac{1}{7}\cos^6 x \sin x + \frac{6}{7}\left[\frac{1}{5}\cos^4 x \sin x + \frac{4}{5}\left[\frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x\right]\right] \ (n=1).$$

$$\int_0^{\pi/2} \cos^n x \, dx = \int_0^{\pi/2} \sin^n x \, dx$$

$$= \frac{(n-1)(n-3)(n-5)\cdots}{n(n-2)(n-4)\cdots} \quad \text{(when } n \text{ is}$$

$$= \frac{(n-1)(n-3)(n-5)\cdots}{n(n-2)(n-4)\cdots}\cdot\frac{\pi}{2} \quad \text{(when } n \text{ is even}$$

Eg: $\int_0^{\pi/2} \sin^4 x\, dx = \frac{3\cdot 1}{4\cdot 2}\frac{\pi}{2} = \frac{3\pi}{16}$

Eg: $\int_0^{\pi/2} \cos^7 x\, dx = \frac{6\cdot 4\cdot 2}{7\,5\,3} = \frac{16}{35}$

① $\int \sin^m x \cos^n x\, dx = \frac{-1}{m+n}\cos^{n+1}x \sin^{m-1}x + \frac{m-1}{m+n}\int \sin^{m-2}x \cos^n x\, dx$.

Eg: $\int \sin^5 x \cos^7 x\, dx = \frac{-1}{12}\cos^8 x \sin^4 x + \frac{4}{12}\int \sin^3 x \cos^7 x\, dx.$

$= \frac{-1}{12}\cos^8 x \sin^4 x + \frac{1}{3}\left[\frac{-1}{10}\cos^8 x \sin^2 x + \frac{2}{10}\int \sin x \cos^7 x\, dx\right]$

$= \frac{-1}{12}\cos^8 x \sin^4 x + \frac{1}{3}\left[\frac{-1}{10}\cos^8 x \sin^2 x + \frac{1}{5}\left[\frac{-1}{8}\cos^8 x\right]\right].$

1) $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$.

$= \frac{(m-1)(m-3)(m-5)\ldots\ldots(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)\ldots\ldots}$

(when anyone value is odd).

$= \frac{(m-1)(m-3)(m-5)\ldots\ldots(n-1)(n-3)(n-5)\ldots}{(m+n)(m+n-2)(m+n-4)\ldots\ldots}\cdot\frac{\pi}{2}$

(when both m & n is even).

Eg: $\int_0^{\pi/2} \sin^5 x \cos^7 x\, dx = \frac{4\cdot 2\cdot 6\cdot 4\cdot 2}{12\cdot 10\cdot 8\cdot 6\cdot 4\cdot 2} = \frac{1}{120}$.

Eg: $\int_0^{\pi/2} \sin^4 x \cos^3 x\, dx = \frac{3\cdot 1\cdot 2}{7\,5\,3\,1} = \frac{2}{35}$

Eg: $\int_0^{\pi/2} \sin^6 x \cos^8 x\, dx = \frac{5\cdot 3\cdot 1\cdot 7\cdot 5\cdot 3\cdot 1}{14\cdot 12\cdot 10\cdot 8\cdot 6\cdot 4\cdot 2}\left(\frac{\pi}{2}\right)$

$= \frac{5\pi}{2048}$.

64×4
256×8
2048

1) $\int_0^{\pi/2} \sin^7 x \cos^3 x\, dx$.

2) $\int_0^{\pi/2} \sin^4 x \cos^4 x\, dx$

3) $\int_0^{\pi/2} \sin^6 x \cos^5 x\, dx$

4) $\int_0^{\pi/2} \sin^3 x \cos^3 x\, dx$.

1) $\int 5x^4 dx = \frac{5x^5}{5} = x^5$

2) $\int \sin 8x\, dx = \frac{-\cos 8x}{8}$

3) $\int \cos x\, dx = \sin x$

4) $\int \frac{1}{x} dx = \log x$

5) $\int 0\, dx = c$

6) $\int 5\, dx = 5\int dx = 5x$

7) $\int \frac{1}{x+2} dx = \log(x+2)$

8) $\int \frac{x}{x^2+3} dx = \frac{2}{2}\int \frac{x}{x^2+3} dx = \frac{1}{2}\int \frac{2x}{x^2+3} dx = \frac{1}{2}\log(x^2+3)$.

$\int \sqrt{5^2-x^2}\, dx = \frac{x}{2}\sqrt{5^2-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}$

$\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\left(\sin^{-1}(x/a)\right)$

$\int \frac{dx}{2^2-x^2} = \frac{1}{2(2)}\log\left(\frac{2+x}{2+x}\right)+c$

$\int (5-2x)^5 dx = \frac{(5-2x)^6}{6(-2)} = \frac{(5-2x)^6}{-12}$.

$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$

$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)(a)}$

3850

1) $\int x \log x\, dx$.

2) $\int x \tan^2 x\, dx$.

1) Area of the region bounded by $x=a$, $x=b$ and a curve, lies above the x axis is $= \int_a^b y\, dx$.

2) Area of the region bounded by a curve, $x=a$, $x=b$ & lies below the x axis $= \int_a^b -y\, dx$.

3) Area of the region bounded by a curve, $y=a$; $y=b$ & lies right to y-axis is $= \int_a^b x\, dy$.

4) Area of the region bounded by a curve $y=a$, $y=b$ & lies left to y-axis is $= \int_a^b -x\, dy$.

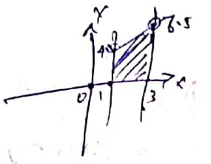
5) Area of the region bounded above and below the X-axis & $x=a$ & $x=b$ $= \int_a^c y\, dx + \int_c^b$

6) Area of the region bounded right and left of the y-axis & $y=a$ & $y=b$ $= \int_a^c x\, dy + \int_c^b -x$

Scanned with CamScanner

1) Find the area of the region bounded by line $3x-2y+6=0$ and $x=1$, $x=3$ and x axis.

Soln!



Given: $3x-2y+6=0$, $x=1$, $x=3$ and x-axis.

When $x=1$, $y=4.5$

when $x=3$, $y=7.5$

$\therefore y = \frac{3x+6}{2}$ $\therefore$ Area $= \int_a^b y\,dx$

Area $= \int_1^3 \frac{3x+6}{2}\,dx$.

$\int_1^3 \frac{3x+6}{2}\,dx = \frac{1}{2}\int_1^3 (3x+6)\,dx$

$= \frac{1}{2}\left[\frac{3x^2}{2}+6x\right]_1^3$

$= \frac{1}{2}\left[\left(\frac{3\times 9}{2}+6\times 3\right)-\left(\frac{3}{2}+6\right)\right]$

$= \frac{1}{2}\left[\frac{27}{2}+18-\frac{3}{2}-6\right]$

$= \frac{1}{2}\left[\frac{24}{2}+12\right]$

$= \frac{1}{2}\left[\frac{24+24}{2}\right]$

$= \frac{1}{2}\left[\frac{48}{2}\right]$

$= 12$ sq. units.

2) Find the area of the region bounded by the Curve

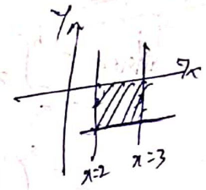
Soln! Given:

$y=x^2-5x+4$, $x=2$, $x=3$ & x-axis. $4-10+4$

$8-10$

when $x=2$, $y=-2$ $9-15+4$

when $x=3$ $y=-2$ $13-15$



Soln!

Area $= \int_a^b -y\,dx$.

$= \int_2^3 -(x^2-5x+4)\,dx$.

$= -\left[\frac{x^3}{3}-\frac{5x^2}{2}+4x\right]_2^3$.

$= -\left[\left(\frac{27}{3}-\frac{45}{2}+12\right)-\left(\frac{8}{3}-\frac{20}{2}+8\right)\right]$

$= -[9-22.5+12-2.6+10-8]$

$= 2.1$ sq. units.

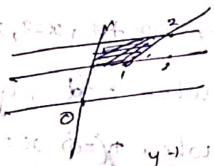
1) $y=2x+1$, $y=3$, $y=5$ & y= axis. (right)

2) $y=2x+7$, $y=1$, $y=3$ & y=axis (left).

3) $y = 2x+1$

$y=3$, $x=1$

$y=5$ $x=2$.



$\frac{y-1}{2}$

Area $= \int_a^b x\,dy$.

$= \int_3^5 \left(\frac{y-1}{2}\right) dy$

$= \frac{1}{2}\left[\frac{y^2}{2} - y\right]_3^5$

$= \frac{1}{2}\left[\frac{y^2-2y}{2}\right]_3^5$

$= \frac{1}{2}\left[\left[\frac{25-10}{2}\right] - \left[\frac{9-6}{2}\right]\right]$

$= \frac{1}{2}\left[\frac{+15}{2} - \frac{(3)}{2}\right]$

$= \frac{1}{2}\left[\frac{15-3}{2}\right]$

$= \frac{1}{2}\left[\frac{12}{2}\right]$

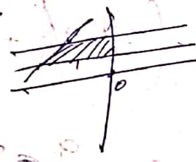
$= 3$ Sq. Units..

4). $y = 2x+7$.

$\frac{y-7}{2}$

$y=1$, $x = -3/2$ $= x = -1.5$

$y=3$, $x = -1/2$ $= x = -0.5$



Area $= \int_a^b -x\,dy$

$= \int_1^3 -\left(\frac{y-7}{2}\right) dy$.

$= -\frac{1}{2}\left[\frac{y^2}{2} - 7y\right]_1^3$ $\frac{-15}{2} = (-7/2)$

$\frac{-15+7}{2}$

$= \frac{-1}{2}\left[\left[\frac{9}{2} - 21\right] - \left[\frac{1}{2} - 7\right]\right]$

$= -\frac{1}{2}\left[\left[\frac{9-21}{2}\right] - \left[\frac{1-8}{2}\right]\right]$

$= +\frac{1}{2}\left[+\frac{8}{2}\right]$

$= 2$ Sq. Unit.

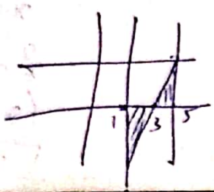
5) Find the area bounded by the Curve $y+3=x$, $x=1$, $x=5$.

$x=1$, $y=-2$ | $x=5$, $y=2$

$\int_1^3 -y\,dx + \int_3^5 y\,dx$.

$\Rightarrow -\int_1^3 (x-3)dx + \int_3^5 (x-3)dx$.

$= -\left[\frac{x^2}{2} - 3x\right]_1^3 + \left[x^2/2 - 3x\right]_3^5$



$$-\left(\left(\frac{9-18}{2}\right)-\left(\frac{1-6}{2}\right)\right)+\left(\frac{25-30}{2}\right)-\left(\frac{9-18}{2}\right)$$

$$= -\left(-\frac{9}{2}+\frac{5}{2}+-\frac{5}{2}+\frac{9}{2}\right)$$

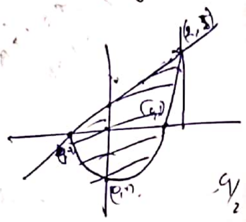
$$= \frac{9}{2} - \frac{5}{2} - \frac{5}{2} + \frac{9}{2}$$

$$= \frac{18}{2} - \frac{10}{2} = \frac{8}{2}$$

$= 4$ Sq. Units.

Formula: Area enclosed between two curves $R = \int_a^b (f-g)\,dx$

6) Find the area between the lines $y=x+1$ and $y=x^2-1$



$y=x+1$	$y=x^2-1$
$x=0,\ y=1$	$x=0,\ y=-1$
$y=0,\ x=-1$	$y=0,\ x=\pm 1$

From ① & ②

$x+1=x^2-1$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$\boxed{x=2,-1}$

when $x=2$ then $y=3$

$$= \int_a^b (f(x)-g(x))\,dx$$

$$= \int_{-1}^{2} \left[(x+1)-(x^2-1)\right] dx$$

$$= \left[\left[\frac{x^2}{2}+x\right]-\left[\frac{x^3}{3}-x\right]\right]_{-1}^{2}$$

$$= \left[\frac{x^2}{2}+x-\frac{x^3}{3}+x\right]_{-1}^{2}$$

$$= \left[\frac{x^2}{2}-\frac{x^3}{3}+2x\right]_{-1}^{2}$$

$$= \left[\frac{4}{2}-\frac{8}{3}+4\right]-\left[\frac{1}{2}+\frac{1}{3}-2\right]$$

$$= \left[\frac{12-16+24}{6}\right]-\left[\frac{3+2-12}{6}\right]$$

$$= \frac{20}{6}+\frac{7}{6}$$

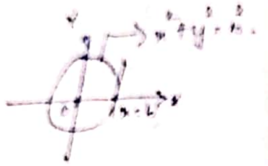
$$= \frac{27}{6}$$

15 12

36 − 16 = 20

1) Area of Circle with radius 'b' is

$x^2+y^2=b^2$



$y^2=b^2-x^2$

$y=\sqrt{b^2-x^2}$

$x=0$ to b.

$$\text{area} = 4\int_0^b y\,dx.$$

$$4\int_0^b y\,dx = 4\int_0^b \left(\sqrt{b^2-x^2}\right)dx$$

$$= 4\left[\frac{x}{2}\sqrt{b^2-x^2} + \frac{b^2}{2}\sin^{-1}\frac{x}{b}\right]_0^b.$$

$$= 4\left[\frac{b^2}{2}\sin^{-1}1 + 0\right]$$

$$= 4\left[\frac{b^2}{2}\cdot\frac{\pi}{2}\right]$$

$$= 4\frac{b^2\pi}{4} \qquad = b^2\pi \text{ sq. Units}.$$

2) Area of ~~ep~~ ellipse with radius 'b' is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$

$y^2=b^2\left(1-\frac{x^2}{a^2}\right)$

$y=b\sqrt{1-\frac{x^2}{a^2}}.$ $\qquad x=0$ to a.

$$\text{Area} = 4\int_0^a y\,dx$$

$$= 4\int_0^a b\left(\sqrt{1-\frac{x^2}{a^2}}\right)dx$$

$$= \frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx.$$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{4b}{a}\left[\frac{a^2}{2}\sin^{-1}1\right]$$

$$= \frac{4b}{a}\cdot\frac{a^2}{2}\cdot\frac{\pi}{2}$$

$$= ba\pi \text{ Sq. Units}$$

Volume:

1) Volume generated about x-axis

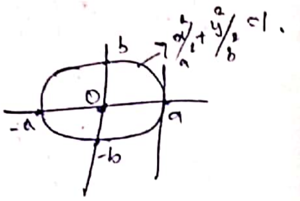
$$V = \int_a^b \pi y^2\,dx \text{ (cubic Units)}$$

2) Volume generated about y-axis

$$V = \int_a^b \pi x^2\,dy \text{ (cubic Units)}.$$

1) Find the volume of the solid generated by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolve about the x axis.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$y^2=b^2\left(1-\frac{x^2}{a^2}\right).$$

limit $x=-a$ to a.

$$\text{volume}=\int_{-a}^{a}\pi\, y^2\,dx$$

$$=2\int_{0}^{a}\pi\, b^2\left(1-\frac{x^2}{a^2}\right)dx.$$

$$=\frac{2\pi b^2}{a^2}\int_{0}^{a}(a^2-x^2)\,dx.$$

$$=\frac{2\pi b^2}{a^2}\left(a^2x-\frac{x^3}{3}\right)_0^a.$$

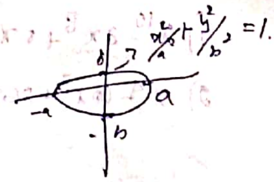
$$=\frac{2\pi b^2}{a^2}\left(a^3-\frac{a^3}{3}\right).$$

$$=\frac{2\pi b^2\, 2a^3}{a^2\ 3}$$

$$=\frac{4\pi b^2 a}{3}\text{ Cubic Units.}$$

2) Find the volume of the solid generated by the ellispe $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolve about the y-axis. (minor) axis)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



$$\frac{x^2}{a^2}=1-\frac{y^2}{b^2}$$

$$x^2=a^2\left(1-\frac{y^2}{b^2}\right).$$

limit $y=-b$ to b.

$$\text{volume } V=\int_{-b}^{b}\pi x^2\,dy$$

$$=2\pi\int_0^b x^2\,dy.$$

$$=2\pi\int_0^b a^2\left(1-\frac{y^2}{b^2}\right)dy.$$

$$=\frac{2\pi a^2}{b^2}\int_0^b(b^2-y^2)\,dy$$

$$=\frac{2\pi a^2}{b^2}\left[b^2y-\frac{y^3}{3}\right]_0^b$$

$$=\frac{2\pi a^2}{b^2}\left[b^3-\frac{b^3}{3}\right]$$

$$=\frac{2\pi a^2}{b^2}\left[\frac{2b^3}{3}\right]$$

$$=\frac{4}{3}\pi b a^2\text{ Cubic Units}$$

MATHEMATICS I
I - B.TECH - BIOTECHNOLOGY
ONLINE QUESTIONS WITH ANSWERS
UNIT I MATRICES
Objective type questions

	opt1	opt2	opt3	opt4	Answer
The sum of the main diagonal elements of a matrix is called------	trace of a matrix	quadratic form	eigen value	canonical form	trace of a matrix
If $\lambda_1, \lambda_2, \lambda_3,\ldots\ldots\ldots \lambda_n$ are the eigen values of A ,then $k\lambda_1, k\lambda_2, k\lambda_3,\ldots\ldots\ldots k\lambda_n$ are the eigen values of --------------	kA	kA^2	kA^-1	A^-1	kA
If atleast one of the eigen values of A is zero, then det A = -----	0	1	10	5	0
The equation det (A-λI) = 0 is used to find ----------	characteristic polynomial	characteristic equation	eigen values	eigen vectors	characteristic equation
det (A-λI) represents------	characteristic polynomial	characteristic equation	quadratic form	canonical form	characteristic polynomial
If $\lambda_1, \lambda_2, \lambda_3,\ldots\ldots\ldots \lambda_n$ are the eigen values of A ,then $1/\lambda_1, 1/\lambda_2, 1/\lambda_3,\ldots\ldots\ldots 1/\lambda_n$ are the eigen values of --------	A^-1	A	A^n	2A	A^-1
If $\lambda_1, \lambda_2, \lambda_3,\ldots\ldots\ldots \lambda_n$ are the eigen values of A ,then $\lambda_1^p, \lambda_2^p, \lambda_3^p,\ldots\ldots\ldots \lambda_n^p$ are the eigen values of	A^-1	A^2	A^-p	A^p	A^p
If all the eigen values of a matrix are distinct, then the corresponding eigen vectors are----------	linearly dependent	unique	not unique	linearly independent	linearly independent
The eigen values of a ---------------------- matrix are its diagonal elements	diagonal	symmetric	skew-matrix	triangular	triangular
If the sum of two eigen values and trace of a 3x3 matrix A are equal, then det A = --------	$\lambda_1 \lambda_2 \lambda_3$	0	1	2	0
If the characteristic equation of a matrix A is $\lambda^2 - 2 = 0$, then the eigen values are -------	2,2	(-2,-2)	(2^(1/2),-2^(1/2))	(2i,-2i)	(2^(1/2),-2^(1/2))
If 1,5 are the eigen values of a matrix A, then det A = -------	5	0	25	6	5
The eigen vector is also known as-------	latent vector	row vector	column vector	latent square	latent vector
If 1,3,7 are the eigen values of A, then the eigen values of 2A are ------------	1,3,7	1,9,21	2,6,14	1,9,49	2,6,14
To multiply a matrix by scalar k, multiply	Any row by k	every element by k	any column by k	diagonal element by k	every element by k

A system of equation is said to be inconsistent if they have	one solution	one or more solution	no solution	infinite solution	no solution
Eigen value of the characteristic equation $\lambda^2-4 = 0$ is	2, 4	2, -4	2, -2	2, 2	2,-2
Eigen value of the characteristic equation $\lambda^3-6\lambda^2+11\lambda-6 = 0$ is	1,2,3	1, -2,3	1,2,-3	1,-2,-3	1,2,3
Largest Eigen value of the characteristic equation $\lambda^3-3\lambda^2+2\lambda = 0$ is	1	0	2	4	2
Smallest Eigen value of the characteristic equation $\lambda^3-7\lambda^2+36 = 0$ is	-3	3	-2	6	-2
Sum of the principal diagonal elements =	product of eigen values	product of eigen vectors	sum of eigen values	sum of eigen vectors	sum of eigen values
Product of the eigen values =	(-|A|)	1/|A|	(-1/|A|)	|A|	|A|
A square matrix A its transpose A^T have the	same eigen vectors	different eigen vectors	same eigen values	different eigen values	same eigen values
If 1 and 2 are the eigen values of a 2X2 matrix A, then the eigen values of A^2 is	2, 4	3,4	5,6	1, 4	1, 4
If 1 and 2 are the eigen values of a 2X2 matrix A, then the eigen values of A^-1is	2,1/2	1,1/2	1,2	4,1/2	1,1/2
If a real symmetric matrix of order 2 has ---------then the matrix is a scalar matrix.	equal eigen vectors	different eigen vectors	equal eigen values	different eigen values	equal eigen values
If A and B are nxn matrices and B is a non singular matrix then A and B^-1AB have	same eigen vectors	different eigen vectors	same eigen values	different eigen values	same eigen values
If the eigen values of A are 2,3,4, then the eigen values of Adj A is	1/2,1/3,1/4	1/2,-1/3,1/4	2,5,6	2,5,-6	1/2,1/3,1/4
The maximum value of the rank of a 4x5 matrix is	1	5	4	3	4
A square matrix A which satisfies the relation A^2 = A is called	nilpotient	idempotient	Hermitian	Skew - Hermitian	idempotient
A matrix is idempotient if	A^3 = A	A^2 = 0	A^1 =A	A^2 = A	A^2 = A
If the rank of A is 2, then the rank of A^ -1 is	3	2	4	1	2
If sum of two eigen values of 3x3 matrix A are equal to the trace of the matrix, then the determinant of A is	1	2	0	5	0
If a matrix A is equal to A^T then A is a -------- matrix.	symmetric	non symmetric	skew-symmetric	singular	symmetric
If a matrix A is equal to -A^T then A is a -------- matrix.	symmetric	non symmetric	skew-symmetric	singular	skew-symmetric
A square matrix A is said to be ----if the determinant value of A is zero.	singular	non singular	symmetric	non symmetric	singular
A square matrix A is said to be ----if the determinant value of A is not equal to zero.	singular	non singular	symmetric	non symmetric	non singular
A square matrix A is said to be singular if the determinant value of A is ----.	1	2	non zero	zero	zero
A square matrix A is said to be non singular if the determinant value of A is ----.	1	2	non zero	zero	non zero
A square matrix in which all the elements below the leading diagonal are zeros,it is called an -----matrix.	upper triangular	lower triangular	symmetric	non symmetric	upper triangular
A square matrix in which all the elements above the leading diagonal are zeros,it is called an -----matrix.	upper triangular	lower triangular	symmetric	non symmetric	lower triangular
A unit matrix is a ----matrix.	scalar	lower triangular	symmetric	non symmetric	scalar
A system of equation is said to be consistent if they have	one solution	one or more solution	no solution	infinite solution	one or more solution
If rank of A is equal to the rank of [AB] then the system of equations is ------	Consistent	inconsistent	symmetric	non symmetric	Consistent
If rank of A is not equal to the rank of [AB] then the system of equations is ------	Consistent	inconsistent	symmetric	non symmetric	inconsistent

B^2

$$\phi(x - y, y - z)$$

UNIT II THEORY OF EQUATIONS

Objective type questions	opt1	opt2	opt3	opt4	Answer
If α, β,γ are the roots of the equation x^3-px+q=0, then Σ1/α =	pq	p+q	p-q	p/q	p/q
If α, β,γ are the roots of the equation x^3 = 7, then Σα^3 is	10	21	34	14	21
A root of x^3-3x^2+2.5 = 0 lies between	1.5 and 2	1.2 and 1.8	1 and 2	1.1 and 1.2	1.1 and 1.2
In an equation with real coefficients, imaginary roots must occur in	non conjugate pairs	conjugate pairs	real pairs	imaginary pairs	conjugate pairs
If f(α) and f(β) are of opposite signs, then f(x)=0 has atleast one root between α and βprovided	f(x) is continuous in (a,b)	f(x) is discontinuous in (a,b)	f'(x) is continuous in (a,b)	f(x) is continuous in (-a,-b)	f(x) is continuous in (a,b)
If α, β,γ are the roots of the equation x^3+2x+3=0, then α+3, β+3, γ +3 are the roots of the equation	x^3+9x^2+29x-24=0	x^3-9x^2+29x-24=0	x^3-9x^2+29x+24=0	x^3-9x^2+29x-20=0	x^3-9x^2+29x-24=0
If one root is double of another in x^3-7x^2+36=0, then its roots are	3,4,-2	3,6,5	4,6,-2	3,6,-2	3,6,-2
The equation whose roots are 10 times those of x^3-2x-7 = 0 is	x^3+200x-7000=0	x^3-200x-7000=0	x^3-200x+7000=0	x^3+200x+7000=0	x^3-200x-7000=0
If α, β,γ are the roots of the equation x^3+px^2+qx+r=0, then Σ(1/αβ) =	pr	p+r	p-r	p/r	p/r
√3 and -1+i are the roots of the biquadratic equation..	x^4+2x^3-x^2-6x-6=0	x^4-2x^3-x^2-6x-6=0	x^4+2x^3+x^2-6x-6=0	x^4+2x^3-x^2+6x-6=0	x^4+2x^3-x^2-6x-6=0
If α, β,γ are the roots of the equation x^3 -3x+2=0, then the value of α^2+ β^2+γ^2 is	4	6	8	2	6
If there is a root of f(x) = 0in the interval [a,b], then sign of f(a)/f(b) is	minus	plus	minus or plus	minus and plus	minus
If α, β,γ are the roots of the equation x^3 +px^2+qx+r=0, then the condition for α+ β+γ is	p+q=r	pq=r	p-q=r	p/q=r	pq=r
The three roots of x^3 = 1 are	1,1/2(-2±√3i)	1, 1+i	1,1/2(-1±√3i)	1, 1-i	1,1/2(-1±√3i)
One real root of the equation x^3 +x -5 = 0 lies in the interval	(2,3)	(3,4)	(1,2)	(-3,-2)	(1,2)
If two roots of x^3 -3x^2+2=0 are equal, then its roots are	1,1,-2	1,2,1	1,-1,2	2,1,-1	1,1,-2
The cubic equation whose two roots are 5 and 1-I is	x^3+7x^2+12x-10=0	x^3-7x^2-12x-10=0	x^3-7x^2+12x+10=0	x^3-7x^2+12x-10=0	x^3-7x^2+12x-10=0
The sum and product of the roots of the equation x^5 = 2 are	0 and 1	2 and 3	0 and 2	1 and 2	0 and 2
One real root of the equation x^3 +2x^2+5 = 0 lies between	3 and -2	(- 3 and - 2)	(5 and 2)	(- 5 and - 2)	(- 3 and - 2)
If the roots of the equation x^4+2x^3-αx^2-22x+40=0 are -5,-2, 1 and 4, then α =	11	15	20	21	21
If for the equation x^3-3x^2+kx+3=0 one root is the negative of another, then the value of k is	3	-3	1	-1	-1
If α, β,γare the roots of 2x^3-3x^2+6x+1=0, α^2+ β^2+γ^2 is	(15/4)	-3	(-15/4)	(33/4)	(-15/4)
X+2 is a factor of	x^4+2	x^4-x^2+12	x^4-2x^3-x+2	x^4+2x^3-x-2	x^4-2x^3-x+2
If α+ β+γ=5; αβ+ βγ+γ α=7; α βγ =3 then whose roots are α, β,γ is	$x^3-7=0$	$x^3-7x^2+3=0$	$x^3-5x^2+7x-3=0$	$x^3+7x^2-3=0$	$x^3-5x^2+7x-3=0$
If one of the roots of the equation x^3-6x^2+11x-6=0 is 2, then the other two roots are	1 and 3	0 and 4	-1 and 5	-2 and 6	1 and 3
Any value of x, for which the equation is satisfied, is known as the -------- of the equation.	factor $-\frac{a_2}{a_0}$	solution	function	coefficient	solution

In algebraic equations, solutions are known as ------- of the equation.	Roots or zeros	function	degree	order	Roots or zeros
The roots of the cubic equation can be obtained by ------ method.	Ferrari's	lagrange's	Horner's	cardano's	cardano's
The one of the relation between roots and coefficients of the equation is -(a_2/a_0)	(-a1/a0 = sum of the roots)	(-a1 = sum of the roots)	(a0/a1 = sum of the roots)	(a1/a0 = sum of the roots)	(-a1/a0 = sum of the roots)
The one of the relation between roots and coefficients of the equation is =	Sum of the products of the	Sum of the products of	Sum of the products of	Sum of the	Sum of the products of the
The equation whose roots are the reciprocals of the roots of is	$(x^3+1)/p.(x^2+1)/r=0$	$1/r.(x^3+1)/p(x+1)=0$	$rx^3+px^2+1=0$	$rx^3+px+1=0$	$rx^3+px+1=0$
If the roots of $x^3-3x^2+px+1=0$, are in arithmetic progression then the sum of squares of the largest and the smallest roots is	3	5	6	10	6
A root of $x^3-8x^2+px+q=0$ where p and q are real numbers is . The real root is	2	6	9	12	2
If a real root of f(x) = 0 lies in [a, b], then the sign of f(a) . f(b) is	Minus	plus	plus or minus	none	Minus
Theory of equations consists of methods of obtaining ------- of equations.	coefficients	functions	solutions	factor	solutions
For the linear equation ax + b = 0, the solution is ----------, a ≠ 0.	a/b	–b/a	a	b/a	–b/a
The roots of quartic equation are obtained by --------- method.	cardano's	Ferrari's	lagrange's	Newton's	Ferrari's
No literal equation exist for finding the solution of algebraic equation of degree	n > 2	n > 3	n>=4	n>=5	n>=5
The one of the relation between roots and coefficients of the equation is $–a_3/a_0$ =	sum of the products of the roots taken two	sum of the products of the roots taken three	sum of the products of the roots taken four	sum of the products of the roots taken five.	sum of the products of the roots taken three
The one of the relation between roots and coefficients of the equation is $(-1)^n a_n/a_0$ =	product of coefficient	product of function	product of roots	sum of the roots	product of roots
Atleast one root of the equation lies between -------- if f(a) and f(b) are of different (opposite) sign.	–a and –b	–a and b	a and b	a and –b	a and b
The common solutions of the equation $z^4 +1 = 0$, $z^6 – i = 0$ are	(-1+i)/√2, (1-i)/√2	(-1-i)/√2, (1-i)/√2	(-1+i)/√2, (-1-i)/√2	(-1+i)/√3,(1-2i)/√3	(-1+i)/√2, (1-i)/√2
If the equation $x^4 –4x^3 +ax^2+bx+1 = 0$ has four positive roots then a = ? and b = ?	6,–4	−6, 4	6, 4	−6,−4	6,–4
Every equation of the odd degree has atleast -------one real root.	one	two	three	four	one
If an equation remains unaltered on changing x to 1/x it is called a ------- equation.	quadratic	cubic	reciprocal	polynomial	reciprocal
If two roots of x^3 -3x^2+5x+k=0 are equal, but opposite in sign, then what is the value of k?	-14	-15	16	20	-15
A polynomial equation whose roots are 3 times those of the equation 2x^3 -5x^2+ 7= 0 is:	3x^3 -15x^2 + 21 = 0	2x^3 -15x^2 +189 = 0	2x^3 +15x^2 -189 = 0	3x^3 +15x^2 + 21 = 0	2x^3 -15x^2 +189 = 0
The number of real zeros of the polynomial function x^2 +1 is:	1	0	2	4	0
If α is an r -multiple root of f (x) = 0 , then which of the following polynomial has α as an(r -1) - multiple root ?	f ^2 (x) = 0	f ' (x) = 0	f (x) = 0	f (-x) = 0	f ' (x) = 0

If - 2 + 3i is a root of the polynomial equation p(x) = 0 , then another root is:	2 + 3i	2 -3i	-2 - 3i	3- 2 i	-2 - 3i
A zero of the polynomial x^3 + 2x - i equals:	- (i)	1	1-i	1+i	- (i)
If α, β are the roots of ax^2 -bx -c = 0 , then α + β equals:	(- b / a)	(- c / a)	(a / b)	(b / a)	(b / a)
If α, β,γ are the roots of the equation x^3 +px^2+qx+r=0, then the for αβ+βγ+γα equals	(-p /q)	(-p)	q	(-q)	q
If α, β,γ are the roots of the equation x^3 +px^2+qx+r=0, then (1/ α)+(1/β)+(1/γ) is	(-q/r)	(-p/r)	(q/r)	(-q/p)	(-q/r)
If α, β,γ are the roots of the equation x^3 +px^2+qx+r=0, then 1/ βγ +1/ γα +1/α β equals	(-p/q)	(p/r)	(-p/r)	(-q/r)	(p/r)
If α is a root of a reciprocal equation f (x) = 0 , then another root of f (x) = 0 is:	(-1/α)	1/α^2	√α	(1/α)	(1/α)
The equation x^3 + 2x + 3 = 0 has:	one positive real root	one negative real root	three real roots	four real roots	one negative real root
Greatest possible number of real roots of x^10 -10x^6 - 5x^3 + x + 4 = 0 is :	6	5	10	8	6
How many real roots are there for the equation x^5 - 6x^2 - 4x + 5 = 0 ?	5	1	3	0	3
If 3 is a double root of the equation 8x^3 - 47x^2 + 66x + 9 = 0 , the third root is:	(-1/8)	(1/8)	8	-8	(-1/8)

UNIT III DIFFERENTIAL CALCULUS AND ITS APPLICATION

Objective type questions	opt1	opt2	opt3	opt4	Answer
The derivative of x^n is ……………..	(n-1) x^(n-1)	n x^(n-1)	(n-1) x^n	n x^(n+1)	n x^(n-1)
The derivative of sinx is ……………..	Sinx	cosx	–sinx	cosx	cosx
The derivative of a constant is…………….	0	1	-1	∞	0
The derivative of a^x , a>0 , a≠1 is……………	a^x	x a^(x-1)	a^x loga	loga	a^x loga
The derivative of e^(-x) is…………….	e^x	x e^x	e^(-x)	-e^(-x)	-e^(-x)
The second order derivative of x^3-5x^2+3x+4 is............	3x^2 -10x+3	6	6x-10	3x^2-10x	6x-10
The second order derivative ofx^3 + tanx is............	6x- 2 (secx)^2 tanx	6x+ 2 (secx)^2 tanx	6x+ 2 secx tanx	6x-2 secx tanx	6x+ 2 (secx)^2 tanx
The second order derivative of logx is............	1/x	1	0	∞	1/x
The second order derivative of x^3-5x^2+3x+4 is............	3x^2 -10x+3	6	6x-10	3x^2-10x	6x-10
The derivative of x cosx is...........	(-xsinx+ cosx)	(xsinx+ cosx)	(-xsinx-cosx)	(xsinx-cosx)	(-xsinx+ cosx)
The derivative of x sinx is...........	(xcosx-sinx)	(-xcosx+sinx)	(xcosx+sinx)	(-xcosx-sinx)	(xcosx+sinx)
The derivative of x e^x is...........	e^(x) -x e^(x)	e^(x) +x e^(x)	(-e^(x)) +x e^(x)	(-e^(x))-x e^(x)	e^(x) +x e^(x)
The derivative of x e^(-x) is...........	e^(x) +x e^(x)	e^(x) -x e^(x)	(-e^(x)) +x e^(x)	(-e^(x))-x e^(x)	e^(x) -x e^(x)
If f(x)= √2x then f'(2)=.............	(1/2)	(-1/2)	-2	1	(1/2)
The derivative of x logx is...........	logx + 1	logx - 1	1/x	(-1/x)	logx + 1
The second order derivative of sin4x is............	16 sin4x	(-16 sin4x)	16 cos4x	(-16 cos4x)	16 sin4x
The second order derivative of cos4x is............	16 sin4x	(-16 sin4x)	16 cos4x	(-16 cos4x)	(-16 cos4x)
The second order derivative ofe^(x) is............	(-e^(x))	(-e^(-x))	e^(x)	e^(-x)	e^(x)
The second order derivative ofe^(-x) is............	(-e^(x))	e^(x)	(-e^(-x))	e^(-x)	e^(x)
The second order derivative ofe^(-2x) is............	(-4e^(2x))	(-4e^(-2x))	4e^(-2x)	4e^(-2x)	4e^(-2x)
The second order derivative of x^4+4x^3+x^2+3x+4 is............	4x^3+12x^2+2x+3	12x^2+24x+2	24x+24	24	12x^2+24x+2
The second order derivative of x^(n) is............	n x^ (n-1)	n (n-1) x^(n-1)	n(n-1) x^(n-2)	n (n-1)x^n	n(n-1) x^(n-2)
The second derivative of (1-x^(2)) is...........	2x	(-2x)	2	(-2)	(-2x)
If x= a t^2 , y=2at then dy/dx=..............	(-1/t)	1	1/t	-1	1/t
If x=a sin^(3) t, y= cos^(3) t then dy/dx=.........	cot t	tant	(-cot t)	(- tan t)	(-cot t)
If x= a(1+ sin θ), y= a(1+cosθ) then dy/dx=............	cot t	tant	(-cot t)	(- tan t)	(-cot t)
If xy=c^2 then dy/dx=............	y/x	(-y/x)	x/y	(-x/y)	(-y/x)

The slope of the tangent to the hyperbola $x^2 - y^2 = 12$ at (4,2) is............	4	2	(1/2)	(1/4)	2
The slope of the tangent to the curve y= x sinx at (π/2, π/2) is.............	-1	-2	2	1	1
The slope of the tangent to the curve $x^{(2)} = 4y$ at the point x=-2 is.............	-1	-2	2	1	-1
The gradient of the tangent to the curve at the point x=2 to the curve $y= 4x^{(3)}-15x^{(2)}$ is........	4	-12	-18	-24	-12
The gradient of the tangent to the curve at the point x=3 to the curve $y= 3x^{(2)}-7x-2$ is........	3	4	9	11	11
The slope of the normal to the hyperbola $x^2 - y^2 = 12$ at (4,2) is............	2	(1/2)	(-1/2)	(-1/4)	(1/2)
The slope of the normal to the curve y= x sinx at (π/2, π/2) is.............	-1	-2	2	1	-1
The slope of the normal to the curve $x^{(2)} = 4y$ at the point x=-2 is.............	-1	-2	2	1	1
The slope of the normal to the curve at the point x=2 to the curve $y= 4x^{(3)}-15x^{(2)}$ is........	(-1/12)	-12	(1/12)	12	(1/12)
The slope of the normal to the curve at the point x=3 to the curve $y= 3x^{(2)}-7x-2$ is........	-11	(-1/11)	11	(1/11)	(-1/11)
If f(x)= sinx then f'(0)=.............	0	-1	1	-2	1
If f(x)= cosx then f'(0)=.............	2	-1	-2	1	-1
If f(x)= logx then f'(1)=.............	2	-1	-2	1	1

UNIT-IV INTEGRAL CALCULUS AND ITS APPLICATION

Objective type questions	Opt 1	Opt2	Opt3	Opt4
∫ x^n dx=.............	x^(n+1)/ (n+1)+ C	x^(n-1)/ (n-1)+ C	nx^ (n-1)+ C	(n+1) x^ (n+1)+ C
∫ cosx dx=.............	sinx + C	cosx + C	(-cosx)+C	(-sinx)+C
∫ sinx dx=.............	sinx + C	cosx + C	(-cosx)+C	(-sinx)+C
∫ e^(x) dx=.............	(-e^x)+ C	e^(-x) + C	(-e^(-x))+C	e^x + C
∫ e^(-x) dx=.............	(-e^x)+ C	e^(-x) + C	(-e^(-x))+C	e^x + C
If u and v are differentiable functions then ∫ u dv =..........	uv+ ∫ v du	uv+ ∫ v du	(-uv)+ ∫ v du	(-uv)- ∫ v du
∫ cos^(4) x dx (from 0 to π/2) =.................	3π/16	5π/16	7π/16	9π/16
∫ cos^(6) x dx (from 0 to π/2) =................	3π/16	5π/16	7π/16	9π/16
∫ cos^(9) x dx (from 0 to π/2) =...................	3π/16	5π/16	7π/16	9π/16
∫ sin^(5) x dx (from 0 to π/2) =.................	π/15	π/15	8π/15	8π/15
∫ sin^(7) x dx (from 0 to π/2) =.....................	π/15	1/15	16π/35	16/35
∫ cos2x dx=.............	(sin2x)/2 + C	(cos2x)/2 + C	(-cosx)/2+C	(-sinx)/2+C
∫ sin3x dx=.............	(sin3x)/3 + C	(cos3x)/3 + C	(-cos3x)/3+C	(-sin3x)/3+C
∫ (1/x) dx=.............	1+ C	log x+C	(-1)+C	(-log x)+ C
The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=a^2 about its diameter is..........	(4/3)πa^3	(2/3)πa^3	(1/3)πa^3	πa^3
The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=2^2 about its diameter is..........	(32/3)π	(1/3)π	(2/3)π	π
The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=3^2 about its diameter is..........	16π	9π	36π	π
The Volume of a sphere of radius 'a' is.................	2/3 π a^3	4/3 π a^3	1/3 π a^3	π a^3
The surface are of the sphere of radius 'a' is.................	4πa^2	πa^2	3πa^2	2πa^2
∫ x e^(x) dx=.............	(-x)e^(x)-e^(x)+c	xe^(x)+e^(x)+c	(-x)e^(x)+e^(x)+c	xe^(x)-e^(x)+c
∫ cosmx dx=.............	(sinmx)/m + C	(cosmx)/m + C	(-cosmx)/m+C	(-sinmx)/m+C
∫ sinnx dx=.............	(sinnx)/n + C	(cosnx)/n + C	(-cosnx)/n+C	(-sinnx)/n+C

∫ dx=.............	x+C	1	0	x^2
∫ 5dx=.............	x+C	5x+C	x^2+C	5+C
∫ 3x^(2) dx=.............	3x^(2)+C	x+C	x^2+C	x^(3) +C
∫ Sec ^ (2) x dx=.............	secx.tanx+C	tanx+C	tan^(2) x +C	Secx+C
∫ Secx. tanx dx=.............	secx.tanx+C	tanx+C	tan^(2) x +C	Secx+C
∫ e^(2x) dx=.............	(-e^2x)/2+ C	e^(-2x)/2 + C	(-e^(-2x))/2+C	e^2x/2+ C
∫ e^(-2x) dx=.............	(-e^(-2x))/2+ C	e^(-2x)/2 + C	(-e^(-2x))/2+C	e^(-2x)/2+ C
The Volume of a sphere of radius '2' is..................	16/3 π	32/3 π	8/3 π	8 π
The surface area of the sphere of radius '3' is..................	36π	9π	27π	18π
∫ x^ (2) dx=.............	(x^(2)/2)+C	(x^(3)/3)+C	x+C	2x+C
∫ x logx dx=……….	1-logx+C	logx+C	0	1
∫cosec^(2) x dx=……….	cotx+C	tanx+C	(-tanx)+C	(-cotx)+C
∫sec^(2) x dx=……….	cotx+C	tanx+C	(-tanx)+C	(-cotx)+C

Answer
x^(n+1)/ (n+1)+ C
sinx + C
(-cosx)+C
e^x + C
(-e^(-x))+C
uv+ ∫ v du
3π/16
5π/16
5π/16
8/15
16/35
(sin2x)/2 + C
(-cos3x)/3+C
log x+C
(4/3)πa^3
(32/3)π
36π
4/3 π a^3
4πa^2
xe^(x)-e^(x)+c
(sinmx)/m+ C
(-cosnx)/n+C

$x+C$
$5x+C$
$x^{(3)} +C$
$\tan x+C$
$\sec x+C$
$e^{2x}/2 + C$
$e^{2x}/2 + C$
$32/3\ \pi$
36π
$(x^{(3)}/3)+C$
$1-\log x+C$
$(-\cot x)+C$
$\tan x+C$

UNIT - V ORDINARY DIFFERENTIAL EQUATIONS

Objective type questions	opt1	opt2	opt3	opt4	Answer
The solution of the differential equation (D^2 + 5D+6)y=0 is.............	A e^ (-2x)+ B e^ (-3x)	A e^ (2x)+ B e^ (3x)	A e^ (-2x)+ B e^ (3x)	A e^ (2x)+ B e^ (-3x)	A e^ (-2x)+ B e^ (-3x)
The solution of the differential equation (D^2 + 6D+9)y=0 is.............	(A+Bx) e^ (3x)	(A+Bx) e^ (x)	(A+Bx) e^ (-2x)	(A+Bx) e^ (-3x)	(A+Bx) e^ (-3x)
The solution of the differential equation (D^2 -4D+4)y=0 is.............	(A+Bx) e^ (3x)	(A+Bx) e^ (-2x)	(A+Bx) e^ (-3x)	(A+Bx) e^ (2x)	(A+Bx) e^ (2x)
The particular integral of (D^2 -3D+2)y=12 is.................	(1/5)	(1/6)	(1/4)	(1/3)	(1/6)
The complementary function of (D^2 -2D+1)y=x sinx is.............	(A+Bx) e^ (-x)	(A+Bx) e^ (x)	(A+Bx) e^ (-2x)	(A+Bx) e^ (2x)	(A+Bx) e^ (x)
If f(D)= D^ (2)- 2, 1/f(D) e^(-2x) is............	0.5 e^ (2x)	-0.5 e^ (2x)	0.5 e^ (-2x)	0.5 e^ (3x)	0.5 e^ (2x)
The particular integral of (D^2+4) y= cos2x is	(x cos2x)/2	(sin2x)/2	(sin2x)/2	(x sin2x)/4	(x sin2x)/4
If (D^2 +4)y=0 is a linear differential equation then general solution is	A cos2x+ B sin4x	Acos2x+Bsin2x	Asin2x+Bcos4x	Asin4x+Bsin4x	Acos2x+Bsin2x
If (D^2 − 6D+13) y = 0 is a linear differential equation then G.S. is -------	e^(3x) (A cos2x+ B sin2x)	e^(3x) (A cos4x+ B sin4x)	e^(3x) (A cos2x+ B sin2x)	e^(2x) (A cos2x+ B sin2x)	e^(3x) (A cos2x+ B sin2x)
The solution of the differential equation (D^2 -4D+3)y=0 is.............	A e^ (x)+ B e^ (3x)	A e^ (-x)+ B e^ (3x)	A e^ (x)+ B e^ (-3x)	A e^ (2x)+ B e^ (-3x)	A e^ (x)+ B e^ (3x)
The solution of the differential equation (D^2 +3D+2)y=0 is.............	A e^ (x)+ B e^ (2x)	A e^ (-x)+ B e^ (2x)	A e^ (-x)+ B e^ (x)	A e^ (-x)+ B e^ (-2x)	A e^ (-x)+ B e^ (-2x)
The particular integral of (D^2 +3D+2)y= 2 e^(x) is.............	e^(x)/3	(-e^(x))/3	e^(x)/6	(-e^(x))/6	e^(x)/3
The particular integral of (D^2+4) y= e^(x) is	1/5* e^(x)	1/5* e^(-x)	1/6* e^(x)	1/6* e^(x)	1/5* e^(-x)
If the roots of the auxilliary equation are real and distinct then the C.F is...	Ae^(m1x)+Be^(m2x)	(A+Bx) e^ (m1x)	e^(αx) (Acosβx+Bsinβx)	(A+Bx) e^ (m2x)	Ae^(m1x)+Be^(m2x)
If the roots of the auxilliary equation are real and equal then the C.F is...	Ae^(m1x)+Be^(m2x)	e^(αx) (Acosβx+Bsinβx)	(A+Bx) e^ (mx)	(A+Bx) e^ (-mx)	(A+Bx) e^ (mx)
If the roots of the auxilliary equation are complex then the C.F is...	Ae^(m1x)+Be^(m2x)	e^(-αx) (Acosβx+Bsinβx)	(A+Bx) e^ (mx)	e^(αx) (Acosβx+Bsinβx)	e^(αx) (Acosβx+Bsinβx)
The particular integral of (D^2 +10D+24)y= e^(-x) is.................	(1/35) e^(-x)	(-1/35)e^(-x)	(-1/25)e^(-x)	(1/25)e^(-x)	(1/25)e^(-x)
The particular integral of (D^2+9) y= cos2x is	cos2x/13	(-cos2x)/13	(-cos2x)/5	cos2x/5	cos2x/5
The particular integral of (D^2+9) y= cos3x is	x cos3x/2	(-x cos3x)/2	(xcos3x)/6	(-xcos3x)/6	(xcos3x)/6
The particular integral of (D^2 +12D+27)y= e^(-x) is..........	(1/16) e^ (-x)	(-1/16) e^ (-x)	(1/16) e^ (x)	(-1/16) e^ (x)	(1/16) e^ (-x)

The solution of the differential equation (D^2 +19D+60)y=0 is.............	A e^ (15x)+ B e^ (4x)	A e^ (-15x)+ B e^ (4x)	A e^ (15x)+ B e^ (-4x)	A e^ (-15x)+ B e^ (-4x)	A e^ (-15x)+ B e^ (-4x)
The solution of the differential equation (D^2 +13D+40)y=0 is.............	A e^ (5x)+ B e^ (8x)	A e^ (5x)+ B e^ (-8x)	A e^ (-5x)+ B e^ (-8x)	A e^ (-5x)+ B e^ (8x)	A e^ (-5x)+ B e^ (-8x)
The solution of the differential equation (D^2 -9D+20)y=0 is.............	A e^ (-5x)+ B e^ (4x)	A e^ (5x)+ B e^ (-4x)	A e^ (5x)+ B e^ (4x)	A e^ (-5x)+ B e^ (-4x)	A e^ (5x)+ B e^ (4x)
The solution of the differential equation (D^2 +D-72)y=0 is.............	A e^ (-8x)+ B e^ (-9x)	A e^ (-8x)+ B e^ (9x)	A e^ (8x)+ B e^ (9x)	A e^ (8x)+ B e^ (-9x)	A e^ (8x)+ B e^ (-9x)
The solution of the differential equation (D^2- 11D-42)y=0 is.............	A e^ (14x)+ B e^ (-3x)	A e^ (-14x)+ B e^ (-3x)	A e^ (-14x)+ B e^ (3x)	A e^ (14x)+ B e^ (3x)	A e^ (14x)+ B e^ (-3x)
The solution of the differential equation (D^2- 12D-45)y=0 is.............	A e^ (15x)+ B e^ (3x)	A e^ (-15x)+ B e^ (3x)	A e^ (15x)+ B e^ (-3x)	A e^ (-15x)+ B e^ (-3x)	A e^ (15x)+ B e^ (-3x)
The solution of the differential equation (D^2- 7D-30)y=0 is.............	A e^ (-10x)+ B e^ (-3x)	A e^ (10x)+ B e^ (-3x)	A e^ (10x)+ B e^ (3x)	A e^ (-10x)+ B e^ (3x)	A e^ (10x)+ B e^ (-3x)
The particular integral of (D^2 +19D+60)y= e^x is.............	(-e^(-x))/80	(e^(-x))/80	(e^x)/80	(-e^x)/80	(e^x)/80
The particular integral of (D^2+25) y= cosx is	(cosx)/24	(cosx)/25	(-cosx)/24	(-cosx)/25	cosx/24
The particular integral of (D^2+25) y= sin4x is	(-sin4x)/9	(sin4x)/9	(sin4x)/41	(-sin4x)/41	(sin4x)/9
The particular integral of (D^2+4) y= sin2x is	(-xsin2x)/4	xsin2x/4	(-xcos2x)/4	xcos2x/4	(-xcos2x)/4
The particular integral of (D^2+1) y= sinx is	xcosx/2	(-xcosx)/2	(-xsinx)/2	xsinx/2	(-xcosx)/2
The particular integral of (D^2 -9D+20)y=e^(2x) is.............	e^(2x) /6	e^(2x) /(-6)	e^(2x) /12	e^(2x) /(-12)	e ^ (2x) /6
The particular integral of (D^2 +D-72)y= e^(7x) is	e^(7x)/16	e^(-7x)/16	e^(7x)/(-16)	e^(-7x)/(-16)	e^(7x)/(-16)
The particular integral of (D^2-1) y= sin2x is	(-sin2x)/5	sin2x/5	sin2x/3	(-sin2x)/3	(-sin2x)/5
The particular integral of (D^2+2) y= cosx is	(-cosx)	(-sinx)	cosx	sinx	cosx
The particular integral of (D^2- 7D-30)y= 5 is.............	(1/30)	(-1/30)	(1/6)	(-1/6)	(-1/6)
The particular integral of (D^2- 12D-45)y= -9 is.............	(-1/5)	(1/5)	(1/45)	(-1/45)	(1/5)
The solution of the differential equation (D^2- 11D-42)y=21 is.............	(-1/42)	(1/42)	(1/2)	(-1/2)	A e^ (14x)+ B e^ (-3x)
The particular integral of (D^2+1) y= 2 is	1	2	-1	-2	2