KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Under section 3 of UGC act 1956)
COIMBATORE-641021

FACULTY OF ENGINEERING
DEPARMENT OF CIVIL ENGINEERING

B.E Civil Engineering **2017-2018**

Semester-IV

16BECE302 Solid Mechanics - I 2H-2C

Instruction Hours/week: L: 2 T: 0 P: 0 **Marks:** Internal:**40** External:**60** Total:**100**

End Semester Exam:3 Hours

Course Objective

- To introduce to continuum mechanics and material modeling of engineering materials based on first energy principles: deformation and strain; momentum balance, stress and stress states; elasticity and elasticity bounds; plasticity and yield design.
- The overarching theme is a unified mechanistic language using thermodynamics, which allows understanding, modelling and design of a large range of engineering materials.
- The subject of mechanics of materials involves analytical methods for determining the strength, stiffness (deformation characteristics), and stability of the various members in a structural system.

Course Outcome

1. Describe the concepts and principles, understand the theory of elasticity including strain/displacement and Hooke's law relationships; and perform calculations, relative to the strength and stability of structures and mechanicalcomponents;
2. Define the characteristics and calculate the magnitude of combined stresses in individual members and complete structures; analyze solid mechanics problems using classical methods and energymethods;
3. Analyse various situations involving structural members subjected to combined stresses by application of Mohr's circle of stress; locate the shear center of thin wallbeams;and
4. Calculate the deflection at any point on a beam subjected to a combination of loads; solve for stresses and deflections of beams under unsymmetrical loading; apply various failure criteria for general stress states at points; solve torsion problems in bars and thin walledmembers;

Proposed Syllabus

UNIT-I: Simple Stresses and Strains- Concept of stress and strain, St. Venant's principle, stress and strain diagram, Elasticity and plasticity – Types of stresses and strains, Hooke's law -stress – strain diagram for mild steel – Working stress – Factor of safety – Lateral strain, Poisson's ratio and volumetric strain – Elastic moduli and the relationship between them – Barsofvaryingsection–compositebars–Temperaturestresses.StrainEnergy–Resilience-Gradual, sudden, impact and shock loadings – simpleapplications.

UNIT-II:Compound Stresses and Strains- Two dimensional system, stress at a point on a plane, principal stresses and principal planes, Mohr circle of stress, ellipse of stress and their applications. Two dimensional stress-strain system, principal strains and principal axis of strain, circle of strain and ellipse of strain. Relationship between elastic constants.

UNIT-III:Bending moment and Shear Force Diagrams- Bending moment (BM) and shear force (SF) diagrams.BM and SF diagrams for cantilevers simply supported and fixed beams with or without overhangs. Calculation of maximum BM and SF and the point of contra flexure under concentrated loads, uniformly distributed loads over the whole span or part of span, combination of concentrated loads (two or three) and uniformly distributed loads, uniformly varying loads, application of moments.

UNIT-IV: Flexural Stresses-Theory of simple bending – Assumptions – Derivation of bending equation: M/I = f/y = E/R - Neutral axis – Determination of bending stresses – Section modulus of rectangular and circular sections (Solid and Hollow), I,T, Angle and Channel sections – Design of simple beam sections.
Shear Stresses- Derivation of formula – Shear stress distribution across various beam sections like rectangular, circular, triangular, I, T angle sections.
Slope and deflection- Relationship between moment, slope and deflection, Moment area method, Macaulay's method. Use of these methods to calculate slope and deflection for determinant beams.

UNIT-V:Torsion- Derivation of torsion equation and its assumptions. Applications of the equation of the hollow and solid circular shafts, torsional rigidity, Combined torsion and bending of circular shafts, principal stress and maximum shear stresses under combined loading of bending and torsion. Analysis of close-coiled-helical springs.
Thin Cylinders and Spheres- Derivation of formulae and calculations of hoop stress, longitudinal stress in a cylinder, and sphere subjected to internal pressures.

SUPPORTING MATERIALS

TEXT BOOKS:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Elements of Strength of Materials (T1)	Timoshenko, S and Young, D.H	DVNC Publication, New York, USA	2012
2	Solid Mechanics (T2)	Kazmi, S.M.A	TMH Publication, Delhi, India	2009

REFERENCE BOOKS:

Sl.No	Title of Book	Author of Book	Publisher	Year of Publishing
1	Mechanics of Materials (R1)	Hibbeler, R.C	6th Ed. East Rutherford, NJ: Pearson Prentice Hall	2004
2	Strength of Materials (R2)	Subramanian, R	Oxford University Press, New Delhi	2010

STAFF INCHARGE

(**Ms.S.M.Leela Bharathi**)

HOD (Department of Civil Engineering

DEAN (FOE)

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16BECE302 / Solid Mechanics - I
LECTURE PLAN

Number of credits: 3
Contact hours: 3 hours per week
Lecturer: Ms.S.M.Leela Bharathi
Semester: IV– (2017-2018)
Course Type: Core

Lecture	Hours	Topics to be Covered	Text / Reference	Page No
1	1	Concept of stresses and strains	**T1,R1**	**101, 189**
2	1	Stress and strain diagram, Elasticity and Plasticity, Hooke's Law	**T1,R1**	**112, 191**
3	1	Stress strain diagram for mild steel, Working stress, Factor of Safety	**T1,R1**	**113, 193**
4	1	Lateral stain, Poisson's ratio, Volumetric strain	**T2,R1**	**151, 195**
5	1	Elastic moduli and the relationship between strains and elastic moduli	**T1,T2, R1**	**118, 160, 196**
6	1	Bars of varying section, composite bars, Temperature stresses	**T1,R2**	**119, 219**
7	1	Strain energy, Resilience	**T1,R2**	**121, 220**
8	1	Gradual, sudden, Impact and Shock loadings	**T1,R1**	**124, 197**
9	1	Simple applications and related problems	**T1,R1**	**128, 199**
Total	**9 Hrs**			
10	1	Bending moment (BM) and Shear force (SF) basic concepts	**T1,R1**	**134, 210**
11	1	BM and SF diagrams	**T1,R1**	**136, 212**
12	1	BM and SF diagrams for cantilever, simply supported, fixed beams	**T1,R2**	**141, 220**
13	1	Calculation of maximum BM and SF	**T1,R1**	**152, 218**
14	1	Point of contra flexure under concentrated loads and uniformly distributed loads	**T1,R1**	**155, 223**
15	1	Combination of concentrated loads (two/ three) and distributed loads	**T1,R2**	**160, 228**
16	1	Uniformly varying loads	**T1,R1**	**165, 227**
17	1	Application of moments	**T1,R1**	**168, 229**
18	1	The relationships between moments and shear	**T1,R1**	**175, 240**
Total	**9 Hrs**			

19	1	Two dimensional stress system	**T1,R1**	**182, 255**
20	1	Principal stresses and strains	**T1,R1**	**192, 256**
21	1	Mohr circle of stress	**T1,R1**	**192, 257**
22	1	Ellipse of stress and their applications	**T2,R1**	**173, 259**
23	1	Two dimensional stress-strain system	**T1,R1**	**212, 264**
24	1	Principal strains	**T1,R1**	**218, 270**
25	1	Principal axis of strain	**T1,T2, R1**	**219, 185, 274**
26	1	Circle of strain and ellipse of strain	**T1,R1**	**224, 290**
27	1	Relationship between elastic constants	**T1,R1**	**226, 306**
Total	**9 Hrs**			
28	1	Theory of simple bending, Assumptions of simple bending	**T1,R1**	**245, 318**
29	1	Derivation of bending equation	**T2,R1**	**210, 319**
30	1	Neutral axis, Determination of bending stresses	**T2,R1**	**215, 324**
31	1	Section modulus of rectangle and circular bars	**T1,R1**	**254, 331**
32	1	Design of simple beam, shear, stresses and derivations	**T1,R1**	**261, 334**
33	1	Shear stress distribution across various beam sections	**T1,R1**	**265, 339**
34	1	Slope and deflection	**T2,R2**	**245, 329**
35	1	Moment area and Mecaulay’s method	**T1,R1**	**269, 341**
36	1	Slope and deflection for determinant beams	**T1,R1**	**271, 348**
Total	**9 Hrs**			
37	1	Derivation of torsion equation and its assumptions	**T1,R1**	**286, 356**
38	1	Applications of torsion equation	**T1,R2**	**289, 359**
39	1	Combined torsion and bending of circular section	**T2,R1**	**295, 371**
40	1	Principal stress and maximum shear stresses	**T1,R1**	**300, 372**
41	1	Analysis of closed-coiled-helical springs	**T1,R1**	**312, 379**
42	1	Thin cylinders	**T1,R1**	**313, 380**
43	1	Spheres, derivation of formula and calculations of hoop stress	**T1,R2**	**312, 396**
44	1	Longitudinal stress ina cylinder	**T1 ,R1**	**323, 399**
45	1	Sphere subjected to internal stresses	**T1,R1**	**325, 412**
Total	**9 Hrs**			

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TEXT BOOKS:

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HOD (Department of Civil Engineering) **DEAN** (FOE)

I. SIMPLE STRESS, STRAIN AND THERMAL STRESSES :-

Technical Terms:-

1. Strength of material

The internal resistance developed (or) offered by any Material which is loaded, is know as strength of Material. Generally the external load is applied upto "limit of proportionality" where Hooke's law is valid (or) applicable. The Analysis of loaded Material is based on "Elastic Zone" only.

2. Engineering Stress :- (or) Conventional Stress :-

It is defined as The internal resistence offered by any material per unit of its original Cross-Sectional area. Its symbol is "σ or f"

$$\boxed{\sigma = f = \frac{P}{A_0}}$$

Its unit is "N/m^2" in S.I system.

kgf/m^2 in MKS System

$Dyne/cm^2$ - in CGS

$$1\ kgf = 1\ msl \times 1\ m/sec^2$$

$$= 9.81\ kg\text{-}m/sec^2$$

$$\boxed{1\ kgf = 9.81\ N}$$

In S.I system

$$1\ N/m^2 = \text{(1 pascal)} = 1\ pa.$$

$1 kg = 9.81 N$

$1 KN = 100 Kg$

$$1\ Pascal = 1\ N/m^2$$

$$= 1 \frac{N}{(1000mm)^2}$$

$$10^6\ Pa = 1\ N/mm^2$$

$$\boxed{1\ Mpa = 1\ N/mm^2}$$

Dimension of stress :-

$$\sigma = f = \frac{P}{A} \overset{\text{force}}{} = \frac{\text{mass} \times \text{acceleration}}{\text{Area}}$$

$$= \frac{M \times LT^{-2}}{L^2}$$

$$\boxed{\text{SI unit of stress} = ML^{-1}T^{-2}}$$

3. True stress :-

It is defined as the internal resistence (force) developed by any loaded material per unit of its instantaneous cross sectional area

$$\boxed{\sigma_{true} = \frac{P}{A}}$$ A – instantaneous area

$$\sigma_{true} = \frac{P}{A} \times \frac{A_0}{A_0}$$

$$\sigma_{true} = \left(\frac{P}{A_0}\right) \times \left(\frac{A_0}{A}\right)$$

$$\sigma_{true} = \sigma_{Engg.} \times \left(\frac{A_0}{A}\right)$$

Volume of material before loading $\Big\}$ = Volume of material after loading

$$\boxed{V_0 = V}$$

$$A_0 \times l_0 = A \times l.$$

$$\frac{A_0}{A} = \frac{l}{l_0} = \frac{l_0 \pm \Delta l}{l_0}$$

$$= \frac{l_0}{l_0} \pm \frac{\Delta l}{l_0}$$

$A_c = \frac{A_0}{1 \pm \epsilon}$

✓ $$\boxed{\frac{A_0}{A} = 1 \pm \underbrace{\epsilon}_{\text{linear strain}}}$$

✓ $$\boxed{\sigma_{True} = \sigma_{Engg.} \times (1 \pm \epsilon)}$$

$+ \to$ Tensile force

$- \to$ compressive force.

4. <u>Conventional strain</u> (or) <u>Engg. strain</u>: It is defined as the change in length per unit of original length of a loaded material. Its symbol is (ϵ or e)

$$\epsilon = e = \int_{l_0}^{l} \frac{\delta l}{l_0} = \int_{l_0}^{l} \frac{dl}{l_0}$$

$$= \frac{1}{l_0}[l]_{l_0}^{l}$$

$$= \frac{1}{l_0} \times [l - l_0]$$

$$\boxed{e = \epsilon = \frac{\Delta l}{l_0}}$$

Unit → mm/mm → $M^0 L^0 T^0$

→ No dimension.

5. Natural strain :- (ϵ_{nat})

It is defined as the change in length per unit instantaneous length of a loaded material.

$$\epsilon_{nat} = \frac{\delta l}{l} = \frac{dl}{l}$$

$$= \int_{l_0}^{l} \frac{dl}{l}$$

$$= [\log_e l]_{l_0}^{l}$$

$$= \log_e l - \log_e l_0$$

$$= \log_e \left[\frac{l}{l_0}\right]$$

$$= \log_e \left[\frac{l_0 \pm \Delta l}{l_0}\right]$$

$$= \log_e \left[\frac{l_0}{l_0} \pm \frac{\Delta l}{l_0}\right]$$

$$\boxed{\epsilon_{nat} = \log_e [1 \pm e]}$$

$$= \ln [1 \pm \epsilon]$$

Conventional strain

6. Stress - Strain curve of Mild steel :-
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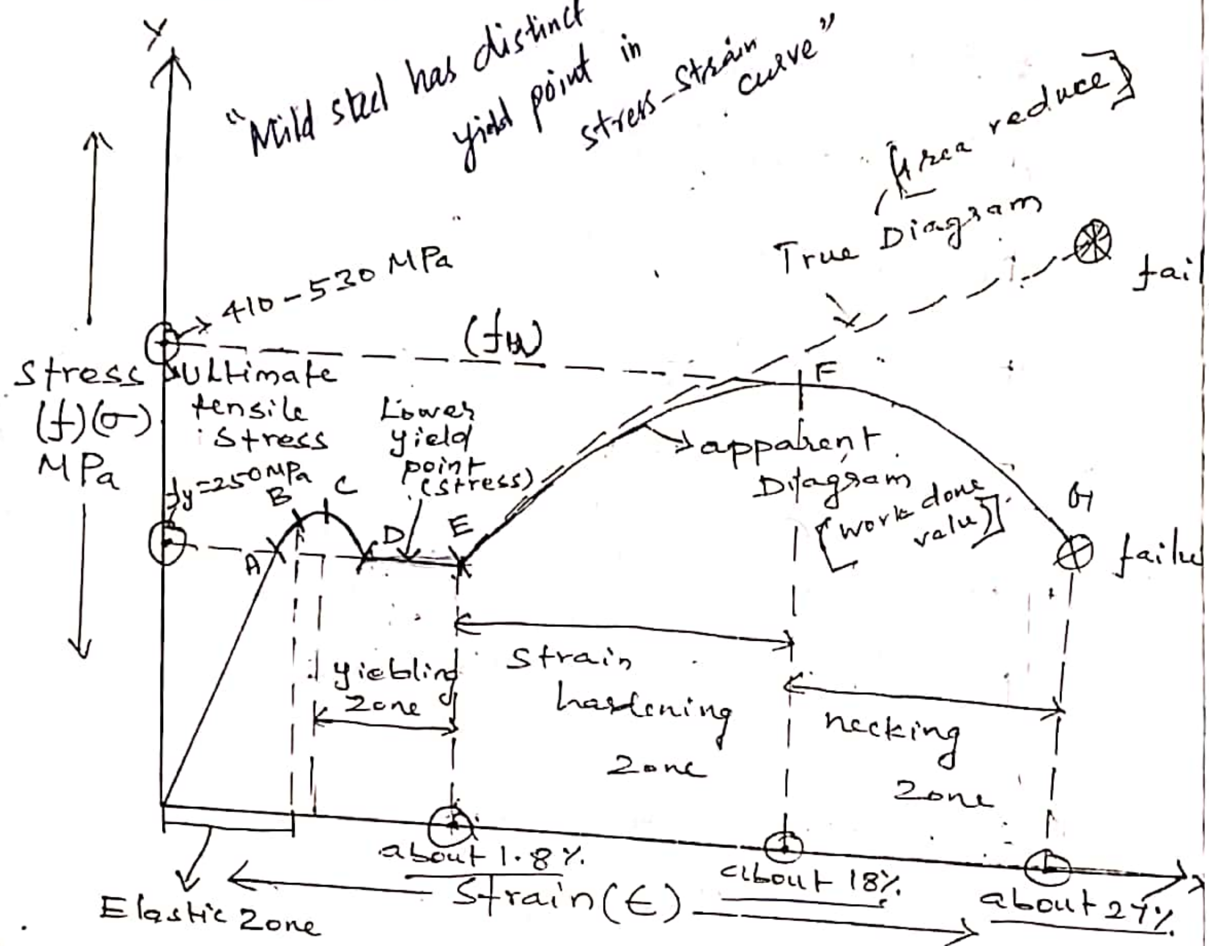
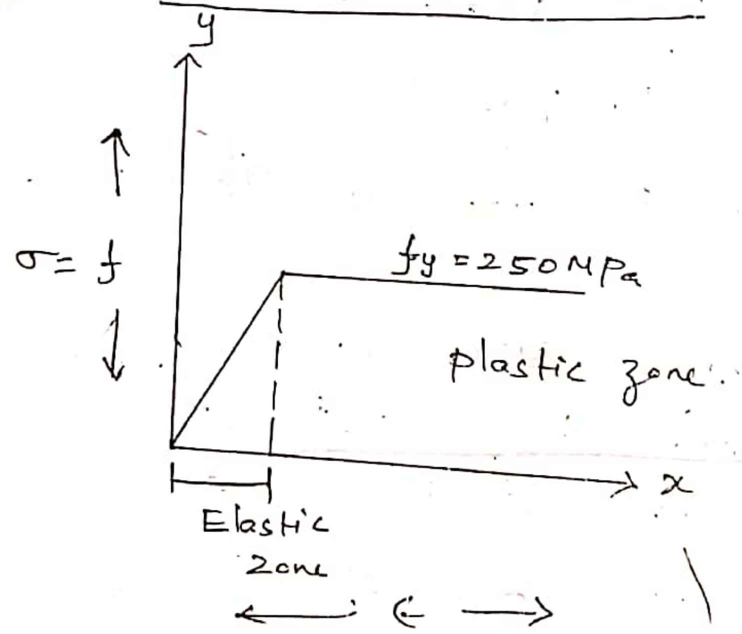


IDEALIZED DIAGRAM



A- Limit of proportionality.

B- Elastic limit

C- Upper yield point

D- Lower yield point

E- End of lower yield point

DE- Yield stress [$f_y = 250 Mpa = \sigma_y$]

EF- Strain Hardening.

F- Ultimate stress point
(Tensile stress)

[f_u = 410 to 530 Mpa
(Tensile stress)
(Ultimate tensile stress)]

FG- Necking Zone

G- Failure of Mild Steel.

Note!- * *

Any Material which do not have
* a distinct yield point (D-E) (—) than the *
yield stress is taken as "Residual strain"
equal to 0.2% after removal of the Lim
load, also known as 0.2% proof stress
Generally it happens in case of "Alumini
* -um, Cast Iron and Higher strength steel.

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7. Elasticity :-

It is defined as the property of a material due to which the material regains its original shape after removal of External load.

** 8. Poisson's Ratio :-

It is defined as the ratio of lateral strain to longitudinal (or) linear strain. its symbol is μ (or) $\frac{1}{m}$ (or) ν

$$\boxed{\mu = \frac{1}{m} = \nu = \frac{\epsilon_{lateral}}{\epsilon_{linear}} = \frac{\frac{\delta b}{b}}{\frac{\delta l}{l}}}$$

Unit - No dimension.

$\mu = 0.3 \rightarrow$ Mild steel in Elastic range

$\mu = 0.5 \rightarrow$ Mild steel in plastic range.

$\mu = 0.33 \rightarrow$ for Aluminium

$\mu = 0.2 \rightarrow$ Concrete

** 9. Hooke's Law :-

According to Hooke's law the linear strain developed in a material which is loaded upto "Limit of proportionality" is directly proportional to stress.

$\epsilon \propto f$ (or) $\epsilon \propto \sigma$

$\boxed{\epsilon = \frac{1}{K} \times f}$ $\quad \epsilon = \frac{1}{K} \times \sigma$

$$\epsilon = \frac{1}{E} \times f$$

$$\sigma = \boxed{f = \epsilon \times E}$$

$$\frac{P}{A_0} = \frac{\delta l}{l} \times E$$

Change in length

$$\boxed{\delta l = \frac{Pl}{A_0 E}}$$

where,

E → Modulus of Elasticity.

→ Young's Modulus of Elasticity

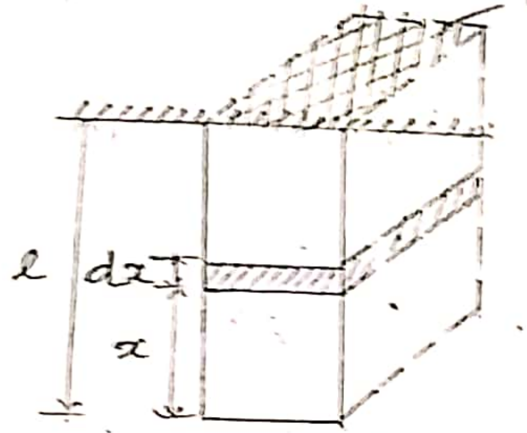
→ $2 \times 10^5 \, N/mm^2$ for Mild steel (MPa)

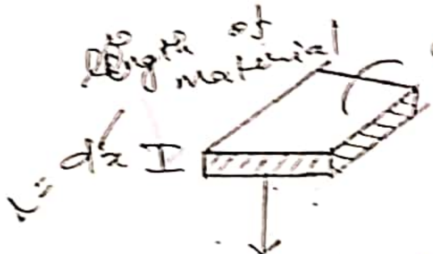
AE → Axial Rigidity.

→ Newton (SI)

→ MLT^{-2} (Dimension)

1. Derive an expression for Elongation due to Self weight :-





$\gamma = \frac{W}{V}$

$W = \gamma \times V$

$W = \gamma \times A \times l$

P = Self weight of hanging portion

$= \gamma \times$ volume $= \gamma \times A \times x$.

For Elemental strip :-

$$\boxed{\delta l_{(x)} = \frac{P_x \, l_x}{A_x \times E_x}}$$

$l_x \to$ length of the elemental strip.

$$\delta l = \frac{(\gamma \times A \times x) \times dx}{A \times E}$$

$$\delta l = \int_0^l \frac{\gamma A}{AE} \times (x \, dx)$$

$$= \frac{\gamma A}{AE} \left[\frac{x^2}{2} \right]_0^l$$

$$= \frac{\gamma \times A}{AE} \times \left[\frac{L^2}{2} \right]$$

$$= \frac{[\gamma \times A \times l] \times l}{2AE}$$

$$\boxed{\delta l = \frac{W_{total} \times l}{2AE}}$$ ✓

(or)

W - Load varies from 0 to W

$$\delta l = \frac{Pl}{AE}$$

take Avg load

$$= \frac{\frac{(0+W)\times l}{2}}{AE}$$

$$\boxed{\delta l = \frac{Wl}{2AE}}$$ ✓

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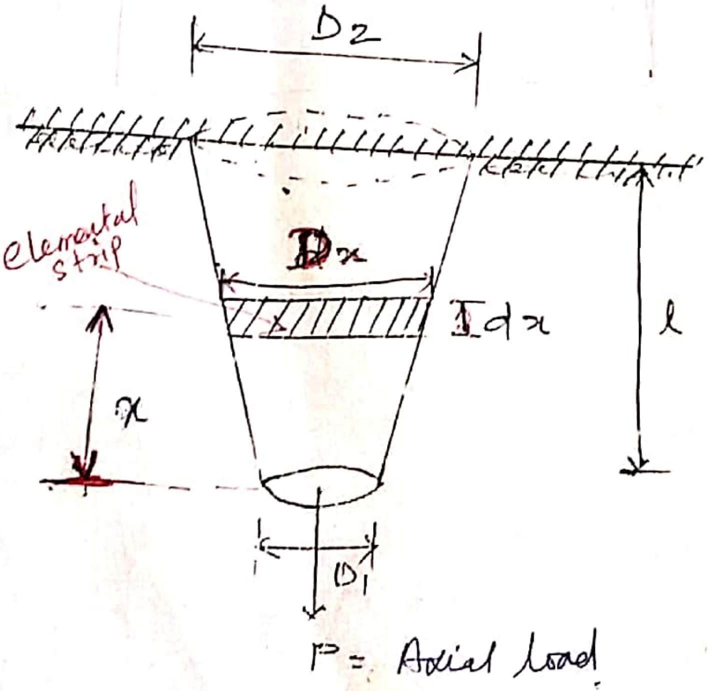
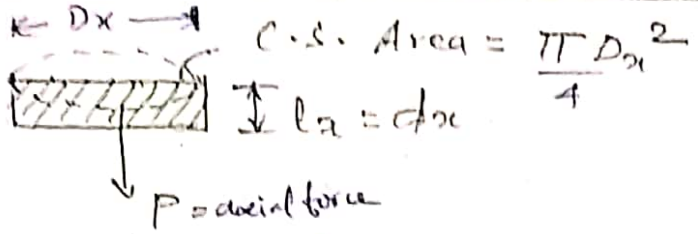
* * ② Expression for Elongation of a Circular tapping Section Subjected to an axial load 'P'.

Derivation





Dia of the elemental strip } $D_x = D_1 + \left[\frac{D_2 - D_1}{l}\right] \times x$

Dia of the elemental strip } $\boxed{D_x = D_1 + kx}$ Take $\left\{ k = \frac{D_2 - D_1}{l} \right\}$

For Elemental strip:-

$\boxed{\delta l = \frac{Pl}{AE}}$

$$\boxed{\delta l_x = \frac{P_x l_x}{A_x E_x}}$$

$$\delta l_x = \frac{P_x l_x}{\frac{\pi D_x^2}{4} E_x}$$

$$\int \delta l = \frac{P\,dx}{\frac{\pi}{4}(D_1 + kx)^2 E}$$

$$\delta l = \int_0^l \frac{P\,dx}{\frac{\pi}{4}(D_1 + kx)^2 E}$$

$$= \frac{4P}{\pi E} \int_0^l \frac{dx}{(D_1 + kx)^2}$$

$$= \frac{4P}{\pi E} \int_0^l (D_1 + kx)^{-2} \, dx$$

$$= \frac{4P}{\pi E} \left[\frac{(D_1 + kx)^{-1}}{(-1) \times (k)} \right]_0^l$$

$$= -\frac{4P}{\pi E} \left[\frac{1}{k(D_1 + kx)} \right]_0^l$$

$$= -\frac{4P}{k\pi E}\left[\frac{1}{D_1+kl} - \frac{1}{D_1}\right]$$

$$= -\frac{4P}{\cancel{k}\pi E}\left[\frac{\cancel{D_1} - \cancel{D_1} - kl}{D_1 \times (D_1+kl)}\right]$$

$$= \frac{4Pl}{\pi E}\cdot\frac{1}{D_1^2 + D_1 kl}$$

$$= \frac{4Pl}{\pi E}\cdot\frac{1}{D_1^2 + D_1 kl}$$

$$= \frac{4Pl}{\pi E}\cdot\frac{1}{D_1^2 + D_1\left[\left(\frac{D_2-\cancel{D_1}}{l}\right)\cancel{l}\right]}$$

$$= \frac{4Pl}{\pi E}\cdot\frac{1}{D_1^2 + \cancel{D_1}D_2}$$

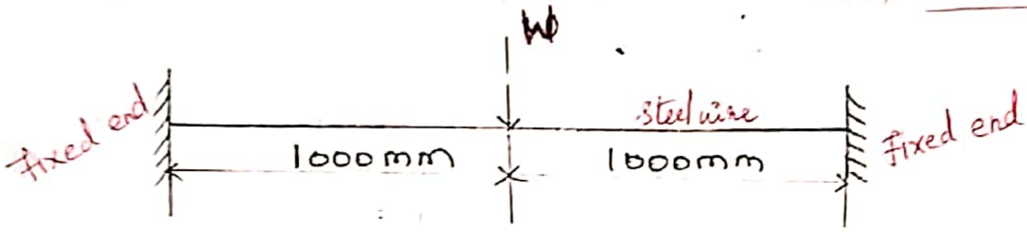
$$= \frac{4Pl}{\pi E}\cdot\frac{1}{D_1(D_1+kl)}$$

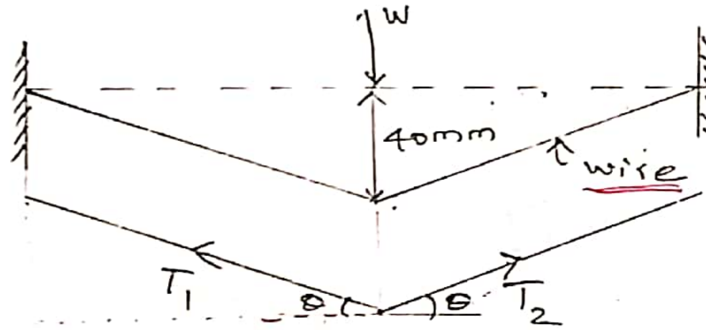
$$= \frac{4Pl}{\pi E\times D_1}\cdot\frac{1}{\left[\cancel{D_1} + \frac{D_2-\cancel{D_1}}{\cancel{l}}\times \cancel{l}\right]}$$

$$\boxed{\delta l = \frac{4Pl}{\pi E D_1 \times D_2}}$$

3. A steel wire 1mm in diameter is 2m long and fixed at the ends. If it is subjected to a central point load (W) such that the central deflection is 40mm. Determine the magnitude of central point load (W) and the stress developed in the wire.

Take, $E = 2 \times 10^5 \ N/mm^2$

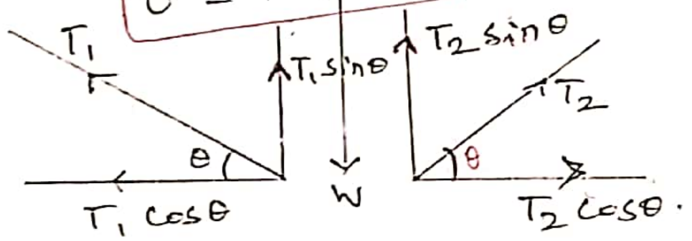




$\tan\theta = \frac{Opp}{Adj}$

$$\tan\theta = \frac{40}{1000}$$

$$\boxed{\theta = 2.291^\circ}$$



Law of equilibrium equation:-

$\boxed{\Sigma H = 0}$ $T_1 \cos\theta = T_2 \cos\theta$.

$$\boxed{T_1 = T_2} \longrightarrow (1)$$

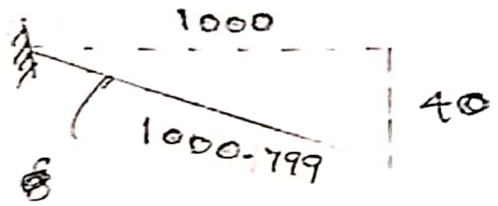
$\boxed{\Sigma V = 0}$

$$T_1 \sin\theta + T_2 \sin\theta = W$$

$$2T_1 = W/\sin\theta$$

$$T_1 = \frac{W}{2\sin 2.291} \rightarrow (2)$$

According to Hooke's law



$$\delta l_1 = 1000.799 - 1000$$
$$= 0.799 \text{ mm.}$$

$\frac{\sigma}{\varepsilon} = E$

$\sigma = \varepsilon E$

$\sigma = \frac{\delta l}{l} \times E$

$$f = \epsilon E$$
$$= \frac{\delta l}{l} \times E$$
$$= \frac{0.799}{1000} \times E$$
$$= 7.99 \times 10^{-4} \times 2 \times 10^5$$

$$\sigma = \boxed{f = 159.8 \text{ N/mm}^2}$$

$$\frac{P}{A} = 159.8$$

$$\frac{P}{\frac{\pi \times (1)^2}{4}} = 159.8$$

$$T_1 = T_2 = \boxed{P = 125.51 \text{ N}}$$ — Tensile force

$$125.51 = \frac{W}{2\sin 2.91}$$

$$\boxed{W = 10.03 \text{ N}}$$ //

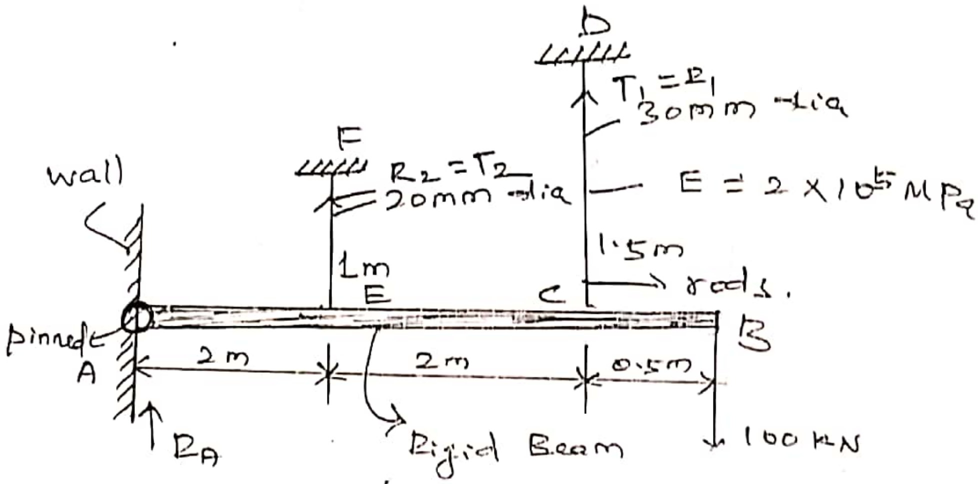
$$~~T_1 = \frac{125.51}{2\sin 2.91}~~$$

$$~~T_1 = 1589.86\ N = T_2~~$$

$$125.51 = \frac{W}{2\sin 2.91}$$

$$\boxed{W = 10.03N}$$

4. A rigid beam is shown in figure below. Determine stresses developed in the rods supporting the rigid beam.



Laws of moment;

$\Sigma M_A = 0$ (B'cos of pinned support)

$$R_1 \times 4 + R_2 \times 2 = 100 \times 4.5$$

$$4R_1 + 2R_2 = 450 \quad \text{—— (1)}$$

$\Sigma V = 0$

$$R_A + R_1 + R_2 = 100. \quad \text{—— (2)}$$

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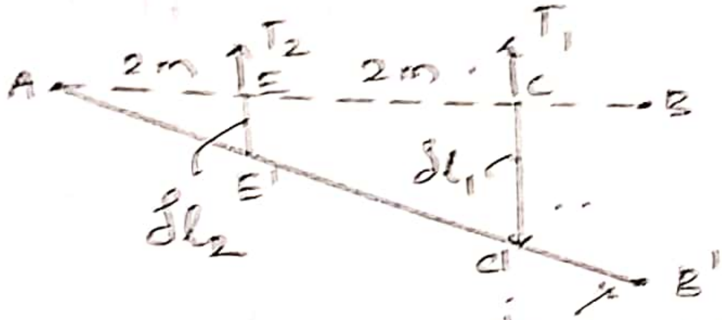


due to rigid nature it follows straight line.

$$\frac{\delta l_1}{\delta l_2} = \frac{4}{2} = 2$$

$$\frac{\delta l_1}{\delta l_2} = 2$$

$$f_1 = E \times \epsilon_1 = E \times \frac{\delta l_1}{L_1}$$

$$f_2 = E \times \epsilon_2 = E \times \frac{\delta l_2}{l_2}$$

$$\delta l_1 = 2\,\delta l_2$$

$$\frac{R_1 l_1}{A_1 \cancel{E_1}} = \frac{2 R_2 l_2}{A_2 E_2}$$

$$\frac{R_1 (1.5)}{\frac{\pi}{4} \times 30^2 \cancel{E}} = \frac{2 R_2 (1)}{\pi/4 \times 20^2 \cancel{E}}$$

$$1.5 R_1 \times 20^2 = 2 R_2 \times 30^2$$

$$600 R_1 = 1800 R_2$$

$$R_1 = 3 R_2$$

$$4R_1 + 2R_2 = 450$$

$$4(3R_2) + 2R_2 = 450$$

$$14R_2 = 450$$

$$\boxed{R_2 = 32.14\ kN}$$

$$\boxed{R_1 = 96.43\ kN}$$

$$R_A = -28.57\ kN$$

$$= 28.57 (\downarrow)\ kN.$$

Stress developed in bars

$$f_1 = \frac{R_1}{A}$$

$$= \frac{96.43}{\frac{\pi}{4} \times 30^2} =$$

$$= 0.1364\ kN/mm^2$$

$$= 136.4\ N/mm^2$$

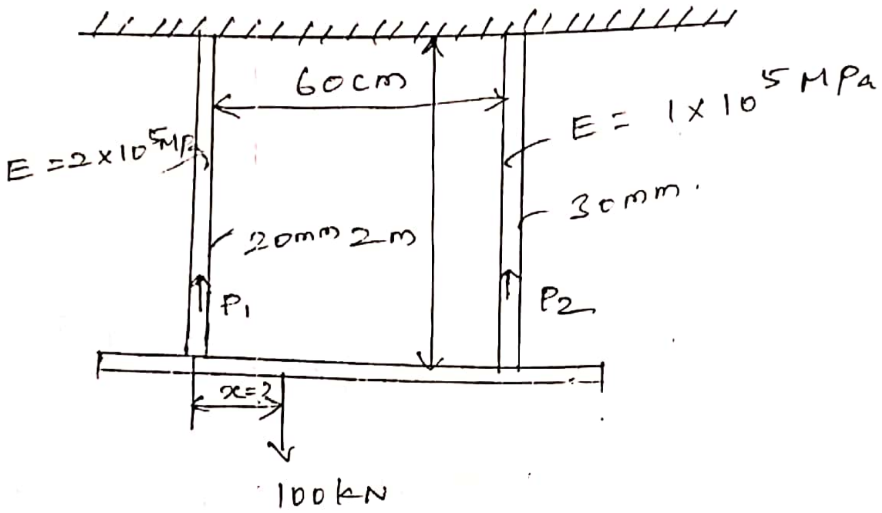
$$f_2 = \frac{R_2}{A}$$

$$= \frac{32.14}{\frac{\pi}{4} \times 20^2}$$

$$= 102.30\ N/mm^2$$

5. Two metallic bars are used to support a load as shown in figure below. Determine the position of load such that the bottom supporting member remains horizontal. Also find the stress in ~~bottom~~ bars.

$\delta = \frac{Pl}{AE}$



$\Sigma V = 0,$

$$P_1 + P_2 = 100 \quad \text{--- (1)}$$

Due to horizontal condition of the bottom support.

$$\delta l_1 = \delta l_2$$

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2}$$

$$\frac{P_1 \times 2000}{\frac{\pi}{4}(20)^2 \times 2 \times 10^5} = \frac{P_2 \times 2000}{\frac{\pi}{4}(30)^2 \times 1 \times 10^5}$$

$$900 P_1 = 800 P_2$$

$$P_1 = 8/9 \, P_2$$

$$\frac{8}{9} P_2 + P_2 = 100$$

$$\boxed{P_2 = 52.94 \text{ kN}}$$

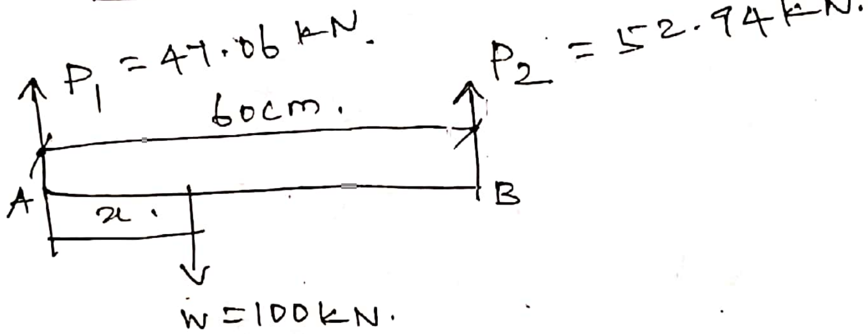
$$\boxed{P_1 = 47.06 \text{ kN}}$$

$$f_1 = \frac{P_1}{A_1} = \frac{47.06}{\frac{\pi}{4} \times 20^2}$$

$$\boxed{f_1 = 149.79 \text{ N/mm}^2}$$

$$f_2 = \frac{P_2}{A_2} = \frac{52.94}{\frac{\pi}{4} \times 30^2}$$

$$\boxed{f_2 = 74.89 \text{ N/mm}^2}$$



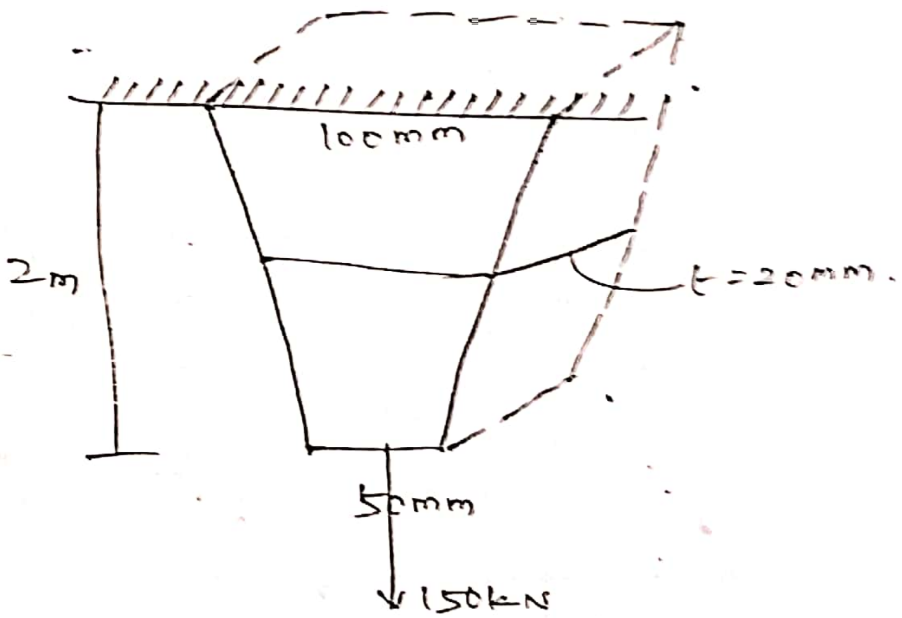
Law of moment,

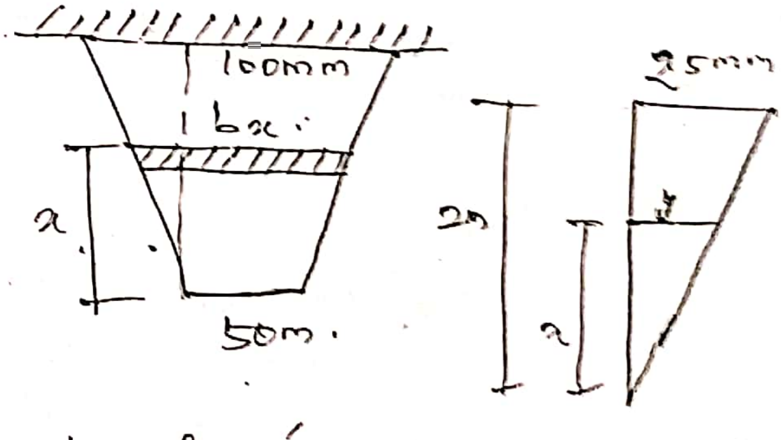
$$\Sigma M_A = 0.$$

$$52.94 \times 60 = 100x.$$

$$\boxed{x = 31.76 \text{ cm}}$$

6. A metallic bar having length 2m is subjected to an axial tensile load of 150kN. The width of the bar varies (tapers) from 100mm at the top to 50mm at the bottom. The thickness of the bar is 20mm (constant). Determine the elongation of the bar $E = 2 \times 10^5$ MPa.





$\cancel{b_x = (50 + 75x)}$

$\cancel{\frac{x}{y} = \frac{2}{25}}$

$b_x = 50 + \frac{50}{2000} x$ $\quad y = \frac{25x}{2000}$

$= 50 + \frac{1}{40} x$

$$\delta l_x = \int_0^{l} \frac{Pl}{AE}$$

$$\delta l = \int_0^{2000} \frac{150 \times dx}{(50 + \frac{1}{40}x) \times 20 \times 2 \times 10^5}$$

$$= \frac{150 \times 1000}{40 \times 10^5} \int_0^{2} \left[50 + \frac{1}{40}x\right]^{-1} dx$$

$$= \frac{150000}{40 \times 10^5} \left[\frac{(50 + \frac{1}{40}x)^0}{(\frac{1}{40})(-1+1)}\right]_0^{2000}$$

$$= \frac{150 \times 1000}{40 \times 10^5 \times \frac{1}{40}} \left[\log_e (50 + \frac{1}{40}x)\right]_0^{2000}$$

$$= 1.5 \left[4.605 - 3.912\right]$$

$$\boxed{\delta l = 1.0395 \text{ mm}}$$

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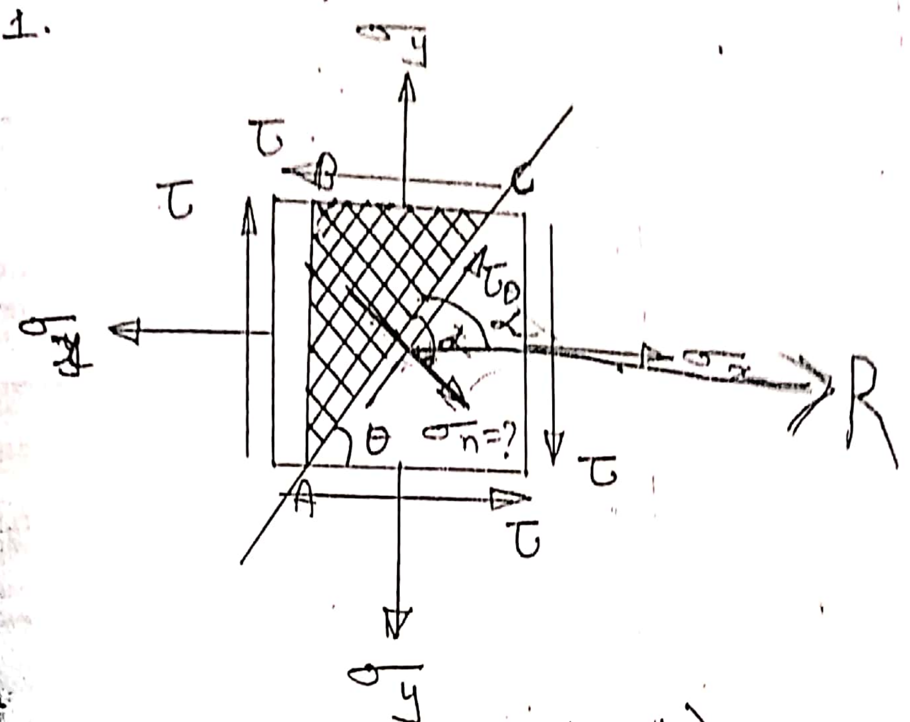
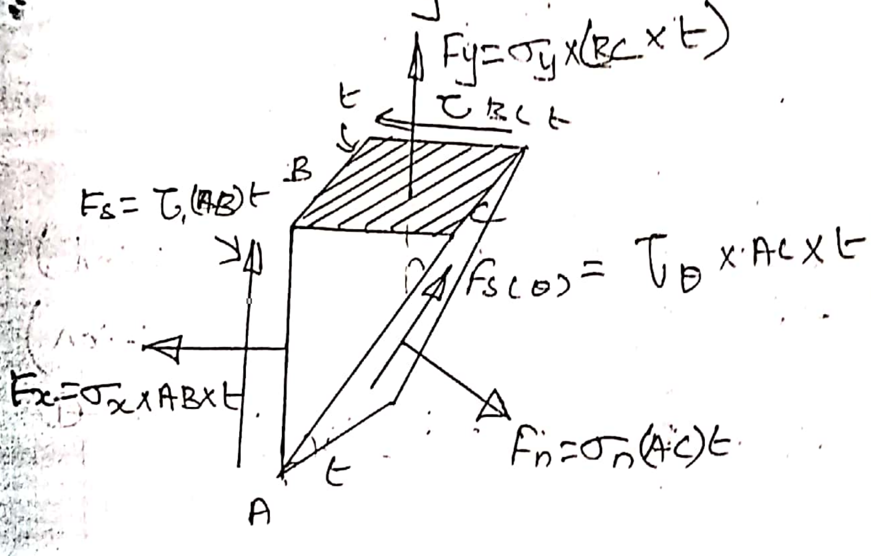
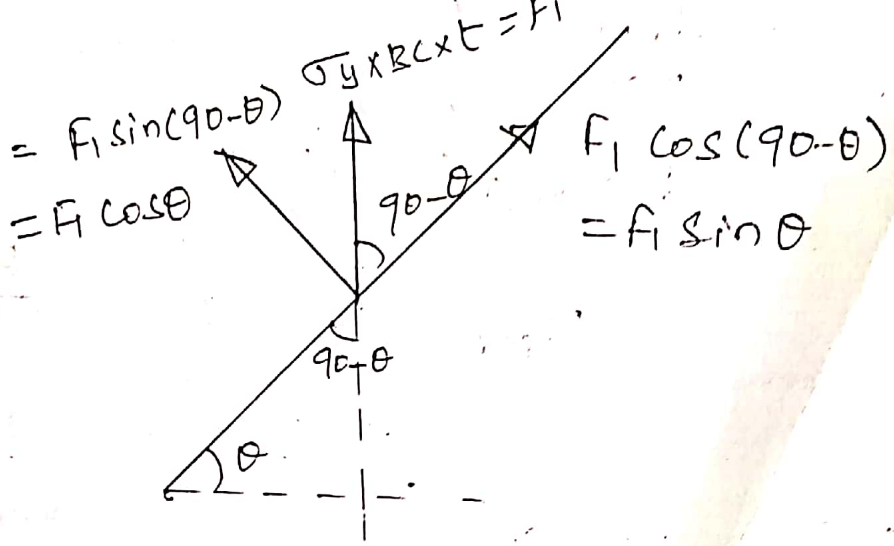
19/08/10

II. Complex Stress

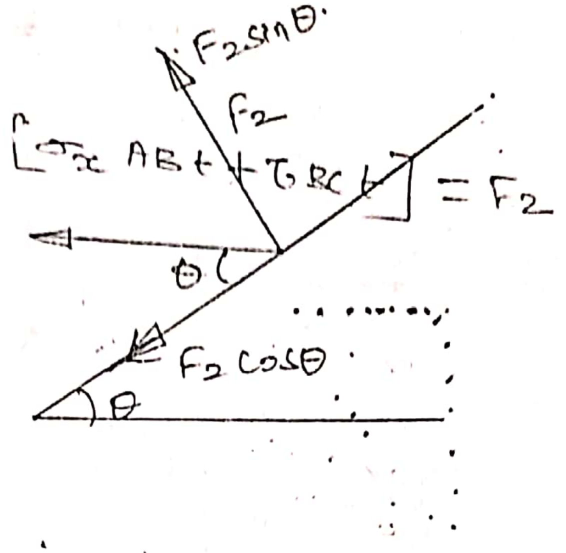
1.

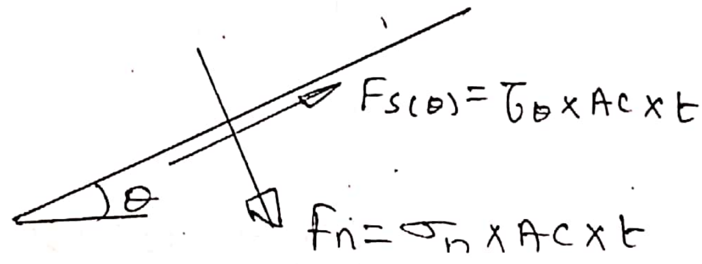






$$\sigma_y \times BC \times t + \tau \times AB \times t = F_1$$





$\sum F_{\text{along the plane}} = 0$ (For No vibration)
(or)
(stable condition)

$F_1 \sin\theta - F_2 \cos\theta + F_{s(\theta)} = 0$ —— (1)

$\sum F_{\text{to plane}} = 0.$

$F_1 \cos\theta + F_2 \sin\theta - F_n = 0$ —— (2)

$\sin\theta = \frac{AB}{AC}, \quad \cos\theta = \frac{BC}{AC}$

$\sigma_y BC\, t \sin\theta + \tau AB\, t \sin\theta + \sigma_x AB\, t\cos\theta$

$- \tau BC\, t \cos\theta + \tau_\theta AC\, t = 0.$

$$\tau_{(\theta)} = \sigma_x \left(\frac{AB}{AC}\right)\cos\theta - \sigma_y\left(\frac{BC}{AC}\right)\sin\theta$$

$$+ \tau\left(\frac{BC}{AC}\right)\cos\theta - \tau\left(\frac{AB}{AC}\right)\sin\theta$$

$$= \sigma_x \sin\theta\cos\theta - \sigma_y \sin\theta\cos\theta$$

$$+ \tau\cos^2\theta - \tau\sin^2\theta.$$

$$= (\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau(\cos^2\theta - \sin^2\theta)$$

**

$$\boxed{\tau_{(\theta)} = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau\cos 2\theta}$$

↳ tangential stress

$$F_n = F_1\cos\theta + F_2\sin\theta.$$

$$\sigma_n \times AC \times t = \sigma_y BC\, t\cos\theta + \tau AB\, t\cos\theta$$

$$+ \sigma_x AB\, t\sin\theta + \tau BC\, t\sin\theta$$

$$\sigma_n = \sigma_y\cos^2\theta + \tau\cos\theta\sin\theta$$

$$+ \sigma_x\sin^2\theta + \tau\cos\theta\sin\theta.$$

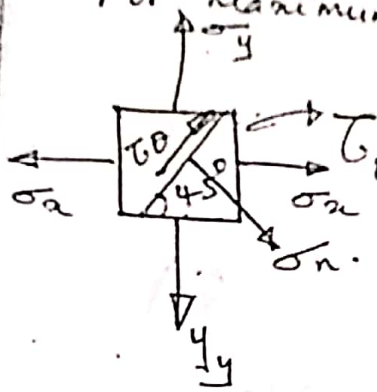
**

$$\boxed{\sigma_n = \sigma_x\sin^2\theta + \sigma_y\cos^2\theta + \frac{\tau}{\cancel{2}}\sin 2\theta}$$

$$\sigma_R = \sqrt{\sigma_n^2 + \tau_\theta^2}$$

$$\alpha = \tan^{-1}\left(\frac{\sigma_n}{\tau_\theta}\right)$$

For Maximum Value of τ_θ (Tangential stress)

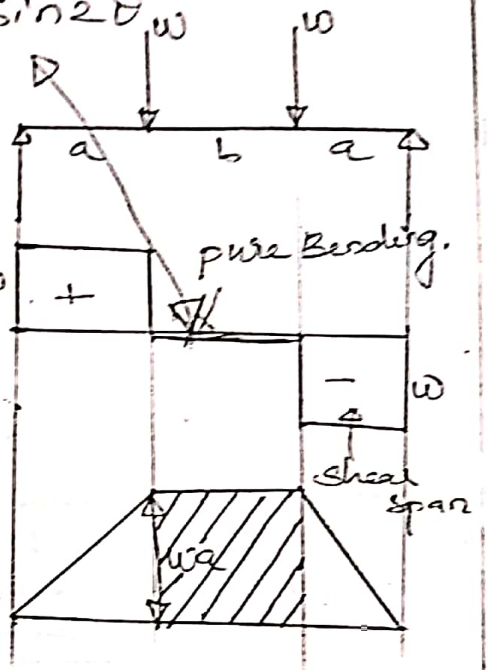


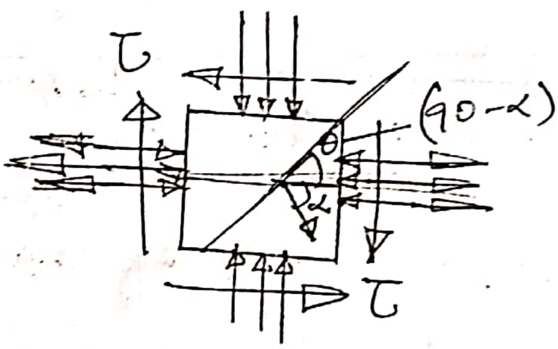
$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

$$\sin 2\theta = 1$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$





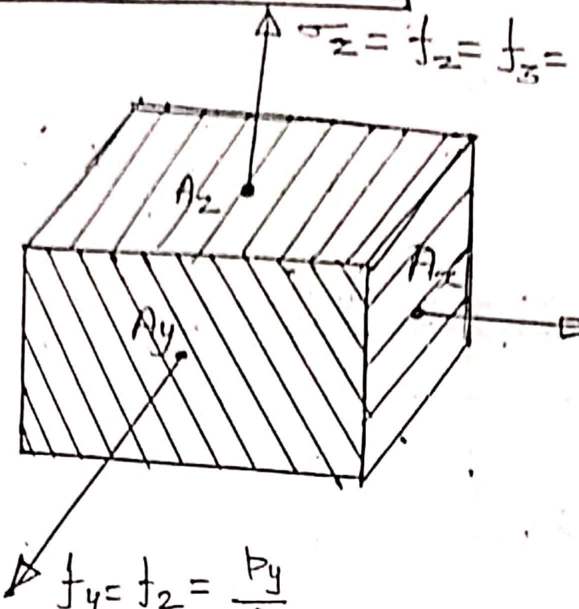
$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \, \overset{\sin 2\theta}{\cancel{\cos}}$$

$$\sigma_n = \sigma_x \sin^2\theta - \sigma_y \cos^2\theta + \tau \sin 2\theta$$

$$\tau_\theta = \frac{\sigma_x + \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta.$$

COMPLEX STRAIN

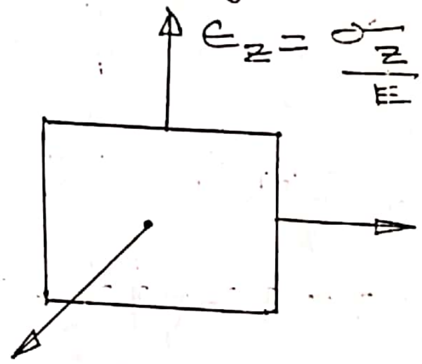
$\sigma_z = f_z = f_3 = \frac{P_z}{A_z}$



$\sigma_x = f_x = f_1 = \frac{P_x}{A_x}$

$f_y = f_2 = \frac{P_y}{A_y}$

$\epsilon_z = \frac{\sigma_z}{E}$ (Hooke's Law)



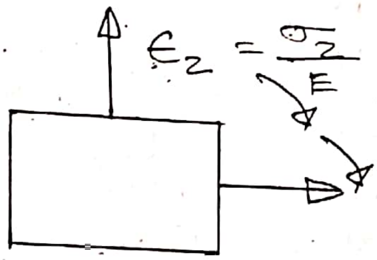
$\epsilon_x = \frac{\sigma_x}{E}$

$\epsilon_y = \frac{\sigma_y}{E}$

Poisson's Ratio,

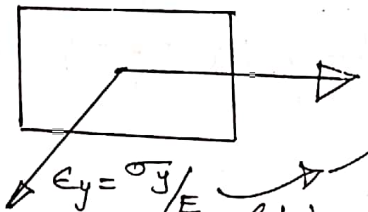
$$\mu = \frac{\epsilon_{lateral}}{\epsilon_{linear}}$$

$\epsilon_z = \frac{\sigma_z}{E}$ (Linear)



$\epsilon_x' = \mu \epsilon_z$

For z, x is called Lateral.



$\epsilon_x'' = \mu \epsilon_y$

$\epsilon_y = \sigma_y / E$ (Linear for y)

Final strain in 'X'- direction,

$$\epsilon_x = \epsilon_1 + \epsilon_1' + \epsilon_1''$$

$$= \frac{\sigma_x}{E} + \left(-\mu\frac{\sigma_z}{E}\right) + \left(-\mu\frac{\sigma_y}{E}\right)$$

$$\boxed{\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E}} \quad \text{— ①}$$

|||ly

$$\boxed{\epsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E}} \quad \text{— ②}$$

|||ly

✓ $$\boxed{\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}} \quad \text{— ③}$$

Total volumetric strain,

✓ $$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$$

$$= \frac{\sigma_x}{E} - \frac{\sigma_y}{E}\mu - \frac{\sigma_z}{E}\mu + \frac{\sigma_y}{E} - \frac{\sigma_x}{E}\mu - \frac{\sigma_z}{E}\mu + \frac{\sigma_z}{E} - \frac{\sigma_x}{E}\mu - \frac{\sigma_y}{E}\mu$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\mu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{(\sigma_x + \sigma_y + \sigma_z)}{E}\left[1 - 2\mu\right]$$

✓ $$\frac{\delta V}{V} = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\mu)$$

$$\boxed{\delta V = \frac{V \times (\sigma_x + \sigma_y + \sigma_z)}{E}(1-2\mu)}$$

derivation

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

if $\sigma_x = \sigma_y = \sigma_z = \sigma = p$

$$\epsilon_v = \frac{3p}{E}(1-2\mu)$$

Bulk modulus

$$E = 3\frac{p}{\epsilon_v}(1-2\mu)$$

$$\boxed{E = 3K(1-2\mu)}$$

$E = \frac{f}{\epsilon_{linear}}$	$K = \frac{p}{\epsilon_{vol}}$	$G = C = N = \frac{\tau}{\phi}$

✓ $E = 3K(1-2\mu)$

✓ $E = 2G(1+\mu)$

$$\frac{E}{2G} = 1+\mu \; ; \quad \boxed{\mu = \frac{E}{2G} - 1}$$

$$E = 3K\left(1 - \frac{2E}{2G} + 2\right)$$

$$E = 3K\left(3 - \frac{E}{G}\right)$$

$$E = 9K - \frac{3KE}{G}$$

$$E + \frac{3kE}{G} = 9k.$$

$$E(G + 3k) = 9kG$$

✓ $$\boxed{E = \frac{9kG}{(3k + G)}}$$

$E = k$,

$E = 2G(1+\mu)$

$E = 3k(1-2\mu)$.

$\cancel{E} = 3\cancel{E}(1-2\mu)$

$3 - 6\mu = 1$

$3\cancel{6}\mu = 2$

$$\boxed{\mu = 1/3}$$

$k = G$.

$3k(1-2\mu) = 2G(1+\mu)$.

$3\cancel{G}(1-2\mu) = 2\cancel{G}(1+\mu)$.

$3 - 6\mu = 2 + 2\mu$.

$8\mu = 1$

$$\boxed{\mu = 1/8}$$

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if $E = G$

$$E = 2G(1+\mu)$$

$$\cancel{E} = 2\cancel{E}(1+\mu)$$

$$\boxed{\mu = -1/2}$$

Important Formula :→

17/09/10

1. $\sigma = f = \frac{P}{A}$
2. $\sigma = f = \epsilon \times E$
3. $\epsilon = \frac{\delta l}{l}$
4. $E = \frac{\text{Longitudinal stress}}{\text{Linear strain (Main strain) (Longitudinal strain)}}$
5. Poisson ratio $(\frac{1}{m})$ (μ),

$$\checkmark \mu = \frac{\text{Lateral strain or, Transverse strain}}{\text{Linear strain or, Longitudinal strain}}$$

$$= \frac{\delta b / b}{\delta l / l}$$

** 6. The value of poisson's Ratio $(\mu \text{ or } \frac{1}{m})$

a) Concrete – 0.2 ✓

b) Cast iron – 0.27

c) Wrought iron – 0.278

d) Steel – 0.288

e) stainless steel – 0.3 ✓

* f) Aluminium – 0.33 ✓

*g) Brass – 0.34

*h) Bronze – 0.35

i) Copper – 0.355

7. $f_{temp} = \epsilon \times E$

$$= \frac{\delta l}{l} \times E$$

$$= \frac{l \alpha \Delta t}{l} \times E$$

$$= \alpha (\Delta t) \times E$$

8. Young's Modulus (Modulus of Elasticity)

$$E = \frac{f_{linear}}{\epsilon_{linear}} \quad (N/mm^2)$$

$$E = \frac{P/A}{\delta l / l} \Rightarrow \delta l = \frac{Pl}{AE}$$

9. Bulk Modulus = $\frac{\text{Change in pressure}}{\text{Volumetric strain}}$

$$K = \frac{p}{\epsilon_v} = \frac{p}{\Delta V / V} \quad (N/mm^2)$$

10. Modulus of Rigidity (shear modulus)

$$= \frac{\text{shear stress}}{\text{shear strain}}$$ — Angular Deformation

$$C = G = N = \frac{\tau}{\phi} \quad (MPa)$$

11. Three Moduli ! →

E, K, G

12. $E = 3K(1 - 2\mu)$

$$= 3K\left(1 - \frac{2}{m}\right)$$

13. $E = 2G(1+\mu)$

$= 2G\left(1+\frac{1}{m}\right)$

14. $E = \frac{9KG}{3K+G}$

15. strain in 'X'- direction,

$$\epsilon_x = \frac{f_x}{E} - \frac{f_y}{mE} - \frac{f_z}{mE}$$

$$= \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

** 16. Strain in 'Y'- direction,

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

17. strain in 'Z'-direction,

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

18. If $\epsilon_x, \epsilon_y, \mu$ are known, then

$\sigma_x = ?$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E}$$

$$\frac{\sigma_y}{E} = \epsilon_y + \frac{\mu\sigma_x}{E}$$

$$\sigma_y = \left(\epsilon_y + \frac{\mu\sigma_x}{E}\right)E$$

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\mu}{E}\left(\epsilon_y + \frac{\mu \sigma_x}{E}\right)\cancel{E}$$

$$\epsilon_x = \frac{\sigma_x}{E} - \mu \epsilon_y \cancel{E} - \frac{\mu^2 \sigma_x}{E}$$

$$(\epsilon_x + \mu \epsilon_y \cancel{E}) = \sigma_x \left(\frac{1}{E} - \frac{\mu^2}{E}\right)$$

$$\sigma_x = \frac{\epsilon_x + \mu \epsilon_y \cancel{E}}{\left(\frac{1}{E} - \frac{\mu^2}{E}\right)}$$

$$= \frac{(\epsilon_x + \mu \epsilon_y) E}{1 - \mu^2}$$

19. $\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z = \epsilon_1 + \epsilon_2 + \epsilon_3$

$$= \left(\frac{f_1}{E} - \frac{f_2}{mE} - \frac{f_3}{mE}\right) + \left(\frac{f_2}{E} - \frac{f_1}{mE} - \frac{f_3}{mE}\right)$$

$$+ \left(\frac{f_3}{E} - \frac{f_1}{mE} - \frac{f_3}{mE}\right)$$

$$= \frac{f_1 + f_2 + f_3}{E} - \frac{2}{mE}(f_1 + f_2 + f_3)$$

$$= \frac{f_1 + f_2 + f_3}{E}\left(1 - \frac{2}{m}\right)$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\mu)$$

where,

Tensile / Compressive

$$\sigma_x = +\frac{P_x}{A_x} \text{ or, } -\frac{P_x}{A_x}$$

$$\sigma_y = +\frac{P_y}{A_y} \text{ or, } -\frac{P_y}{A_y}$$

$$\sigma_z = +\frac{P_z}{A_z} \text{ or, } -\frac{P_z}{A_z}$$

20. Change in volume,

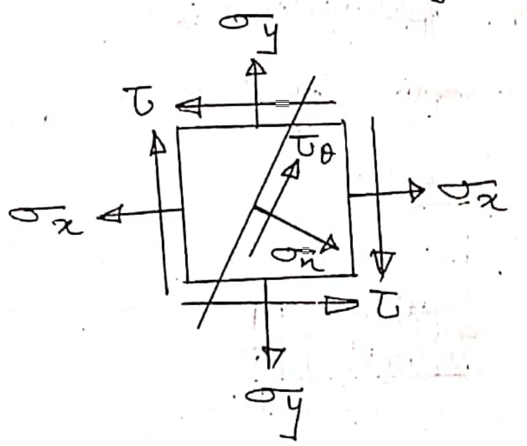
$$\epsilon_v = \frac{\delta v}{v}$$

$$\delta v = \epsilon_v \times v$$

$$= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu) \times l \times b \times d$$

Normal stress,

21. $\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \sin 2\theta$



$$\sigma_n = \sigma_x\left(\frac{1-\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1+\cos 2\theta}{2}\right) + \tau \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau \sin 2\theta$$

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22. Tangential stress (shear stress)

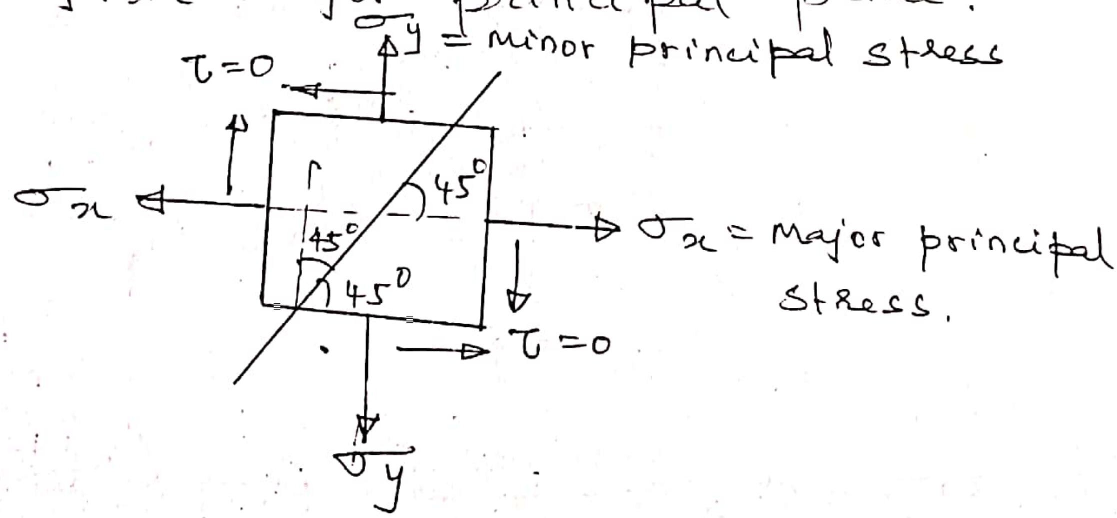
$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$$

*** 23. Maximum shear stress,

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$$

$$= \frac{\text{Major principal stress} - \text{Minor principal stress}}{2}$$

24. The plane on which maximum tangential stress ($\tau_{max} = \tau_\theta$) takes place is at 45° from the direction of major principal stress or 45° from major principal plane.



25. Strain energy stored upto elastic limit is called resilience.

26. The maximum strain energy upto elastic Limit is called proof Resilien

27. The strain energy per unit volume is called Modulus of Resilience $\left(\frac{f^2}{2E}\right)$

28. Strain energy due to axial load

$$U = \frac{P^2 l}{2AE} = \frac{f^2}{2E} \times \text{Volume}.$$

29. Strain energy due to shear stress,

$$U = \frac{\tau^2}{2G} \times \text{volume}$$

30. Strain energy due to Volumetric Strain,

$$U = \frac{\sigma^2}{2K} \times \text{volume}.$$

31. Strain energy due to Torsion,

$$U = \frac{T^2 L}{2GJ} = \frac{\tau^2}{4G} \times \text{volume}$$

32. Strain energy due to bending,

$$U = \int \frac{M^2 dx}{2EI}$$

33. AE = Axial Rigidity

= kN, N

$= MLT^{-2}$

34. Strain energy due to shear force,

$$U = \int \frac{V^2 dx}{2GA}$$

35. GA = shear rigidity

$= N$

$= MLT^{-2}$

36. GJ = Torsional rigidity

$= N\text{-}m^2$

$= ML^3T^{-2}$

37. EI = Flextural rigidity

$= N\text{-}m^2$

$= ML^3T^{-2}$

38. $\frac{AE}{L}$ = Axial stiffness

$= N/m$

$= ML^0T^{-2}$

39. $\frac{GA}{L}$ = shear stiffness

$= N/m$

$= ML^0T^{-2}$

40. $\frac{EI}{L}$ = Flextural stiffness

$= N\text{-}m$

$= ML^2T^{-2}$

41. $\frac{GJ}{L}$ = Torsional stiffness

$= N\text{-}m$

$= ML^2T^{-2}$

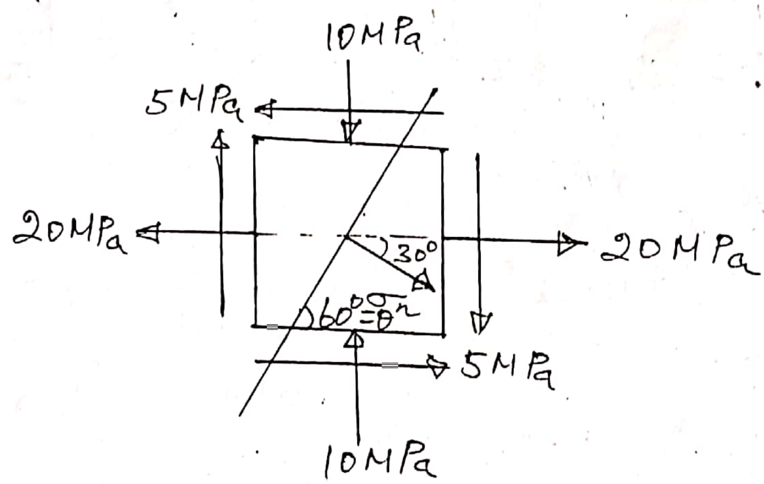
*

42. Effect due to impact loa[d] stress

= 2 × Effect due to st[atic load]

Numericals :→

1. Determine Normal & tangential stress along an inclined plane which is subjected to two tr stresses 20 MPa tensile along x-direction and 10 MPa compressive along y-direction. The shear stress is 5 MPa. The Normal stress on the inclined plane makes an angle 30° with the direction of tensile stress & also determine resultant stress & its direction.



$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \sin 2\theta$$

$$= +20 \underset{\cdot 5}{\sin^2 60^\circ} + (-10) \underset{2.5}{\cos^2 60^\circ} + 5 \sin 2\times 60^\circ$$

$$= +16.83 \text{ MPa}$$

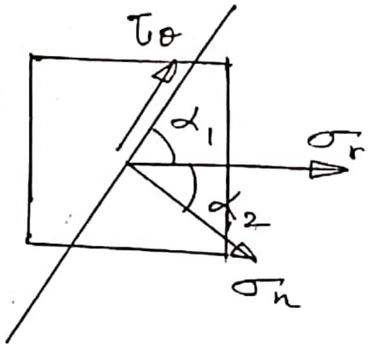
$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta$$

$$= \frac{20-(-10)}{2} \sin 2\times 60 + 5 \cos 2\times 60$$

$$= +10.49 \text{ MPa.}$$

$$\sigma_r = \sqrt{\sigma_n^2 + \tau_\theta^2}$$

$$= 19.83 \text{ MPa.}$$



$$\alpha_1 = \tan^{-1}\left(\frac{\sigma_n}{\tau_\theta}\right)$$

$$= 58.07^\circ.$$

$$\alpha_2 = \tan^{-1}\left(\frac{\tau_\theta}{\sigma_n}\right)$$

$$= 31.93^\circ.$$

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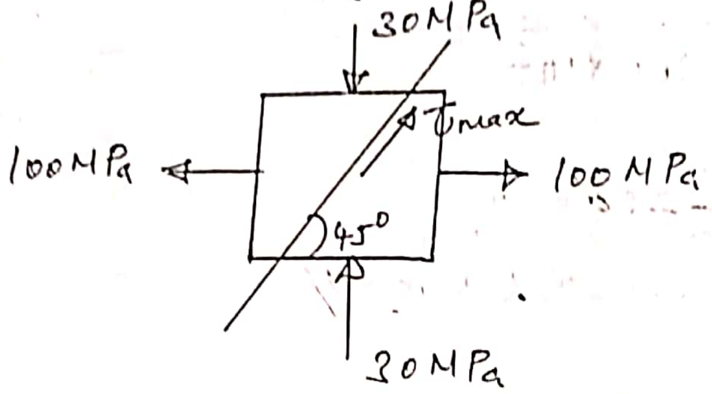
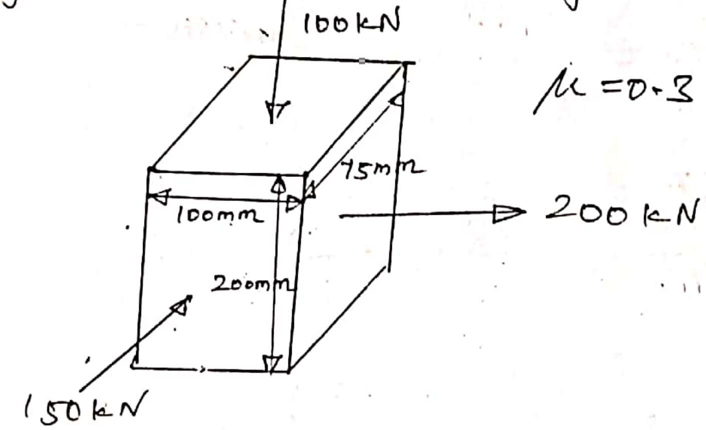
2. Determine maximum shear stress & its plane of a loaded member as shown in figure below.



$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2} = \frac{100-(-30)}{2}$$

$$= 65\ MPa.$$

3. Determine change in volume of an object as shown in figure below.



$\mu = 0.3$, $E = 200GPa$.

$$e_v = \frac{\cancel{200}}{75\times 100\cancel{0}} \cancel{/100}$$

$$\Delta V = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right)(1-2\mu) \times l \times b \times d$$

$$\sigma_x = \frac{200\times 10^3}{200\times 75} = 13.33\ MPa.$$

$$\sigma_y = -\frac{100\times 10^3}{100\times 75} = -13.33\ MPa.$$

$$\sigma_z = \frac{150 \times 10^3}{100 \times 200} = -7.5 \text{ MPa.}$$

$$\Delta V = \frac{(13.33 - 13.33 - 7.5)}{200 \times 10^3} \times (1 - 2 \times 0.3) \times 100 \times 200 \times 75$$

$$= -2225 \text{ mm}^3$$

$$\Delta V = -22.5 \text{ mm}^3$$

4. Determine change in length in x-direction if $\sigma_x = 100$ MPa tensile & $\sigma_y = 50$ MPa compressive. Take $\frac{1}{m} = 0.3$ & total length 700mm.

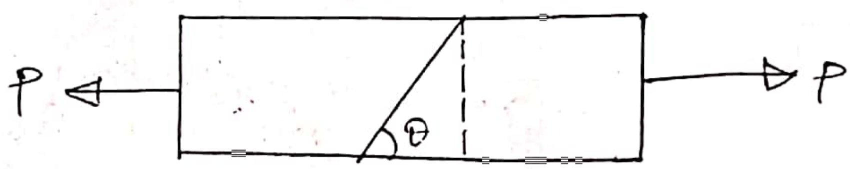
$$\epsilon_x = \frac{\overset{\text{tension}}{\sigma_x}}{E} - \frac{\overset{\text{compressive}}{\sigma_y}}{mE}$$

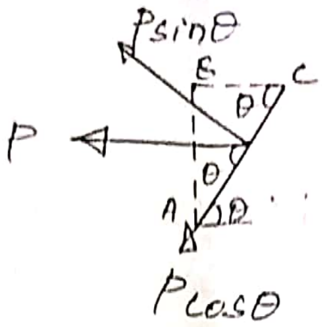
$$\frac{\delta l}{700} = \frac{100}{2 \times 10^5} + \frac{50}{2 \times 10^5} \times 0.3$$

$$\delta l = 0.4025 \text{ mm}$$

$$\epsilon_x = 5.75 \times 10^{-4}$$

5. Eg # Determine the Normal stress developed on a plane as shown in figure below.





$$F_n = P\sin\theta$$

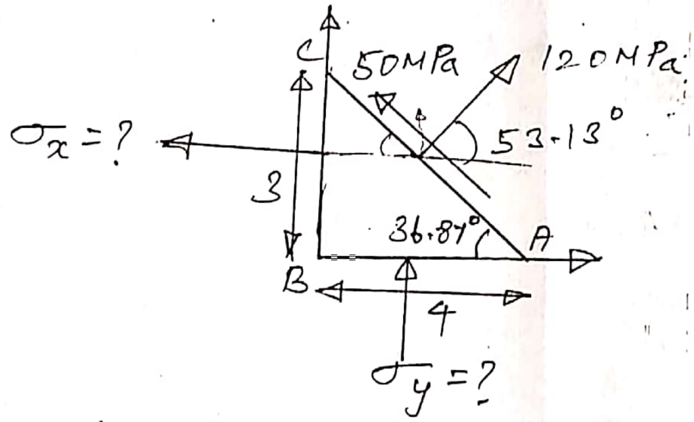
$$\sigma_n = \frac{F_n}{A} = \frac{P\sin\theta}{AC \times 1}$$ — unit thickness

$$= \frac{\sigma_x \times AB \times 1 \times \sin\theta}{AC}$$

$$= \sigma_x \sin\theta \times \sin\theta$$

$$\boxed{\sigma_n = \sigma_x . \sin^2\theta}$$

6. Determine the normal stresses on two ⊥r direction as shown in figure below.



$\Sigma H = 0.$

$$\sigma_x \times 3 \times 1 + 50 \times 5 \times 1 \cos 36.87^\circ - 120 \times 5 \times 1 \cos 53.13^\circ = 0$$

$$3\sigma_x = -199.99 + 360.00$$

$$\sigma_x = +53.34 \text{ MPa (Tensile)}$$

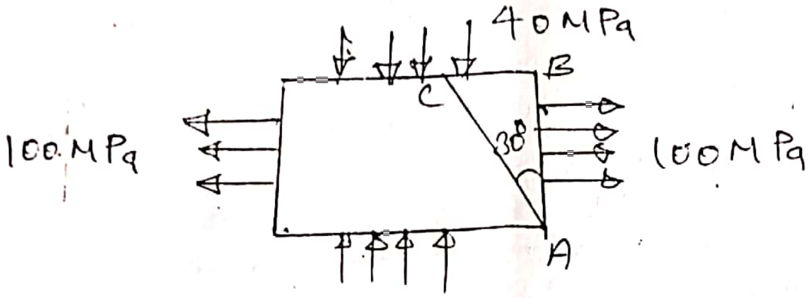
$\Sigma V = 0_y$

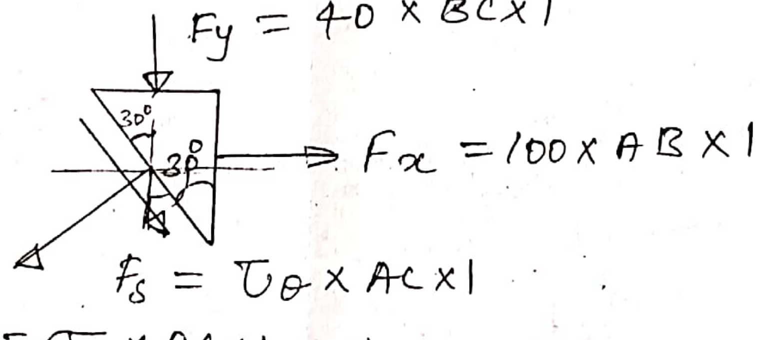
$\sigma_y \times 4 \times 1 + 50 \times 5 \times 1 \times \sin 36.87^0 + 120 \times 5 \times 1 \times \sin 53.13^0 = 0$

$4\sigma_y = -150.00 - 479.99$

$\sigma_y = -157.499$ MPa (Tensile)

7. Determine resultant stress on a given plane as shown in figure below.





ΣF along plane, $= 0$;

$\tau_\theta \times AC \times 1 + 40 \times BC \times 1 \cos 30^0 + 100 \times AB \times 1 \times \cos 60 = 0.$

$AC\, \tau_\theta = -40\, BC \cos 30^0 - 100\, AB \cos 60^0$

$\tau_\theta = -40 \cos 30 \sin 30 - 100 \cos 60 \cos 30$

$= -17.32 - 43.30$

$\tau_\theta = -60.62$ MPa (direction reversed)

$\Sigma F_{\perp r \text{ to plane}} = 0,$

$\sigma_n \times AC \times 1 = -40 \times BC \times 1 \times \sin 30^\circ$
$+ 100 \times AB \times 1 \times \sin 60$

$\sigma_n = -40 \frac{BC}{AC} \sin 30^\circ + 100 \frac{AB}{AC} \sin 60^\circ$

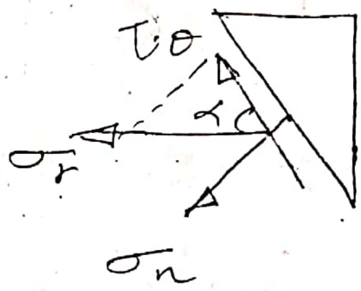
$= -40 \sin 30^\circ \times \sin 30^\circ + 100 \cos 30^\circ \sin 60^\circ$

$= -10 + 75$

$\sigma_n = 65 \text{ MPa}$

$\sigma_r = \sqrt{65^2 + 60.62^2}$

$\sigma_r = 88.88 \text{ MPa}$



$\alpha = \tan^{-1}\left(\frac{65}{60.62}\right)$

$= \cancel{66}.46.99^\circ$

* 8. A metallic bar having size ~~200~~ 300 mm in length & 40 mm square in cross section is subjected an axial load of 160 kN. The change in length is 0.12 mm & change in breadth is 0.005 mm. Determine,

i) Young's modulus
ii) poisson ratio
iii) Bulk modulus
iv) Modulus of rigidity.
v) Volumetric strain
vi) change in volume.

$$\mu = \frac{\delta b/b}{\delta l/l} = \frac{0.005/40}{0.12/\cancel{6000}\ 300} = \cancel{0.2}$$

$$= \frac{1.25\times10^{-4}}{6\times10^{-5}}$$

$$= \cancel{2.08}\ \cancel{0.625} = 0.3125$$

$$f = \frac{P}{A} = \frac{160\times10^{3}}{40\times40} = 100\,MPa$$

$$E = \frac{f}{e} = \frac{100}{0.12/\cancel{6000}\ 300} = \cancel{16.67}\ 2.5\times10^{5}\,MPa$$

$$E = 3k(1-2\mu)$$

$$2.5\times10^{5} = 3k(1-2\times0.3125)$$

$$k = 2.22\times10^{5}\,MPa$$

$$E = 2G(1+\mu)$$

$$2.5\times10^{5} = 2G(1+0.3125)$$

$$G = 0.95\times10^{5}\,MPa$$

$$\epsilon_v = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

$$= \frac{100 + 0 + 0}{2.5 \times 10^5}(1 - 2 \times 0.3125)$$

$$\epsilon_v = 1.5 \times 10^{-4}$$

$$\epsilon_v = \frac{\Delta v}{V}$$

$$\Delta v = \epsilon_v \times V$$

$$= 1.5 \times 10^{-4} \times 40 \times 40 \times 300$$

$$= 72 \text{ mm}^3$$ (increase in volume)

*****

III. PRINCIPAL STRESSES & THEORIES OF FA

1. The principal stress is the Normal Stress acting on any plane where shear stress is zero

2. The plane where only a Normal stress acts [& Tangential stress = 0] is called The principal plane.

3. There are two principal planes:—
   a) Major principal plane
   b) Minor principal plane

4. The principal planes always meet at Right angle to each other [90°]

** 5. The maximum shear stress is equal to

$$= \frac{\text{Major principal stress} - \text{Minor principal Stress}}{2}$$

$$\boxed{\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}}$$

* 6. The plane along which maximum shear stress [Tangential stress] acts makes an angle 45° from the principal plane

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7. To determine the Location of principal plane, the tangential stress [shear stress] [τ_θ] is made equal to zero. from where 'θ' can be determined.

$$\boxed{\tan(180-2\theta) = \frac{2\tau}{\sigma_x-\sigma_y}}$$

**

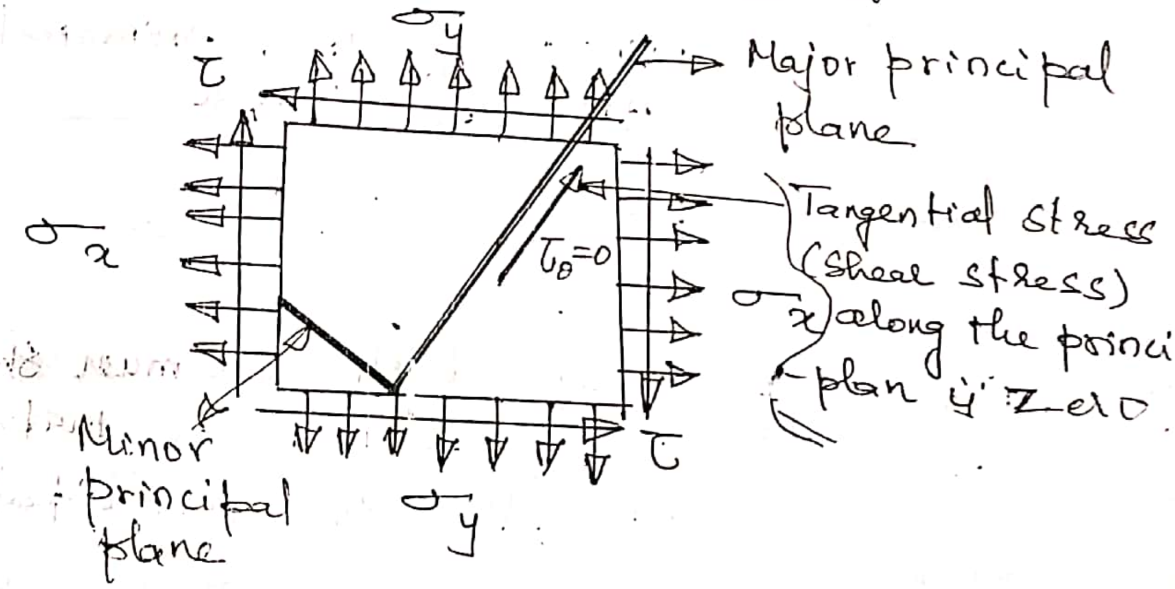
8. The major & minor principal stresses are given by formula:—

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \frac{1}{2}\sqrt{4\tau^2+(\sigma_x-\sigma_y)^2}$$

$\frac{1}{2}\sqrt{(\sigma_x-\sigma_y)^2+4\tau^2}$ — Radius of Mohr Circle.

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \frac{1}{2}\sqrt{4\tau^2+(\sigma_x-\sigma_y)^2}$$

9. Location of principal planes:→



$\tau_\theta = 0$

$$\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau \cos 2\theta = 0.$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{-\tau}{(\sigma_x - \sigma_y)/2}$$

$$\boxed{\tan 2\theta = -\frac{2\tau}{(\sigma_x - \sigma_y)}}$$

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}.$$

$$180 - 2\theta = \tan^{-1}\left(\frac{2\tau}{\sigma_x - \sigma_y}\right)$$

$$\boxed{\theta = 90 - \frac{1}{2}\tan^{-1}\left(\frac{2\tau}{\sigma_x - \sigma_y}\right)}$$

10. a) If $\sigma_x > \sigma_y$, $\theta_{p.p} = 45^\circ$ to 90° from σ_x-direction

b) If $\sigma_x = \sigma_y$, $\theta_{pp} = 45^\circ$ from σ_x direction

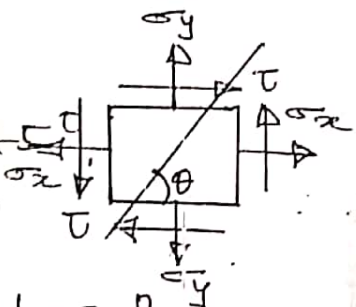
c) If $\sigma_x < \sigma_y$, $\theta_{pp} = 0$ to 45° from σ_x-direction

If τ - Reverse direction,

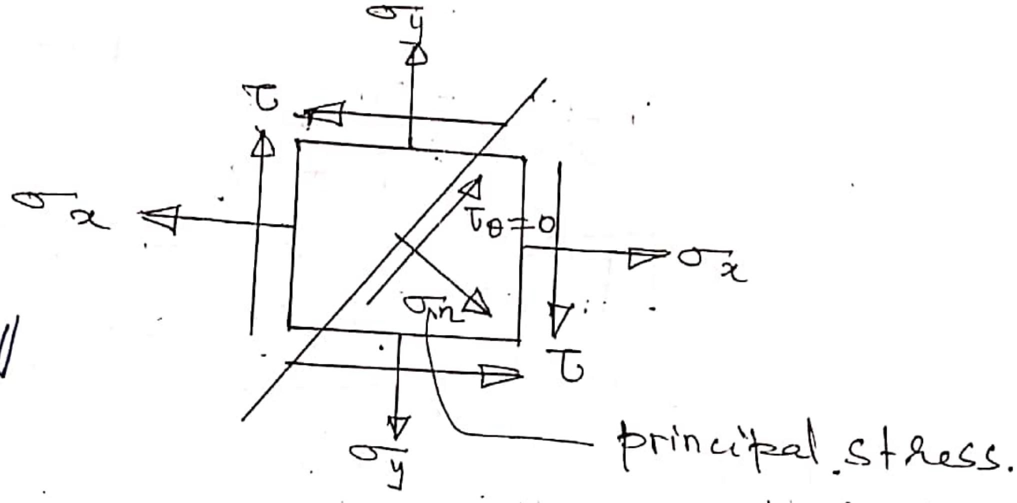
a) $\sigma_x > \sigma_y$, $\theta_{p.p} = 0$ to 45°

b) $\sigma_x = \sigma_y$, $\theta_{pp} = 45^\circ$

c) $\sigma_x < \sigma_y$, $\theta_{pp} = 45^\circ$ to 90° from σ_x-direction



10. The Magnitude of principal stress [Analytically]



$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta + \tau \sin 2\theta$$

$$= \sigma_x \left[\frac{1-\cos 2\theta}{2}\right] + \sigma_y \left[\frac{1+\cos 2\theta}{2}\right] + \tau \sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} - \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau \sin 2\theta$$

$$\tan 2\theta = \frac{-2\tau}{(\sigma_x - \sigma_y)} = \frac{\text{opposite}}{\text{adjacent}}$$

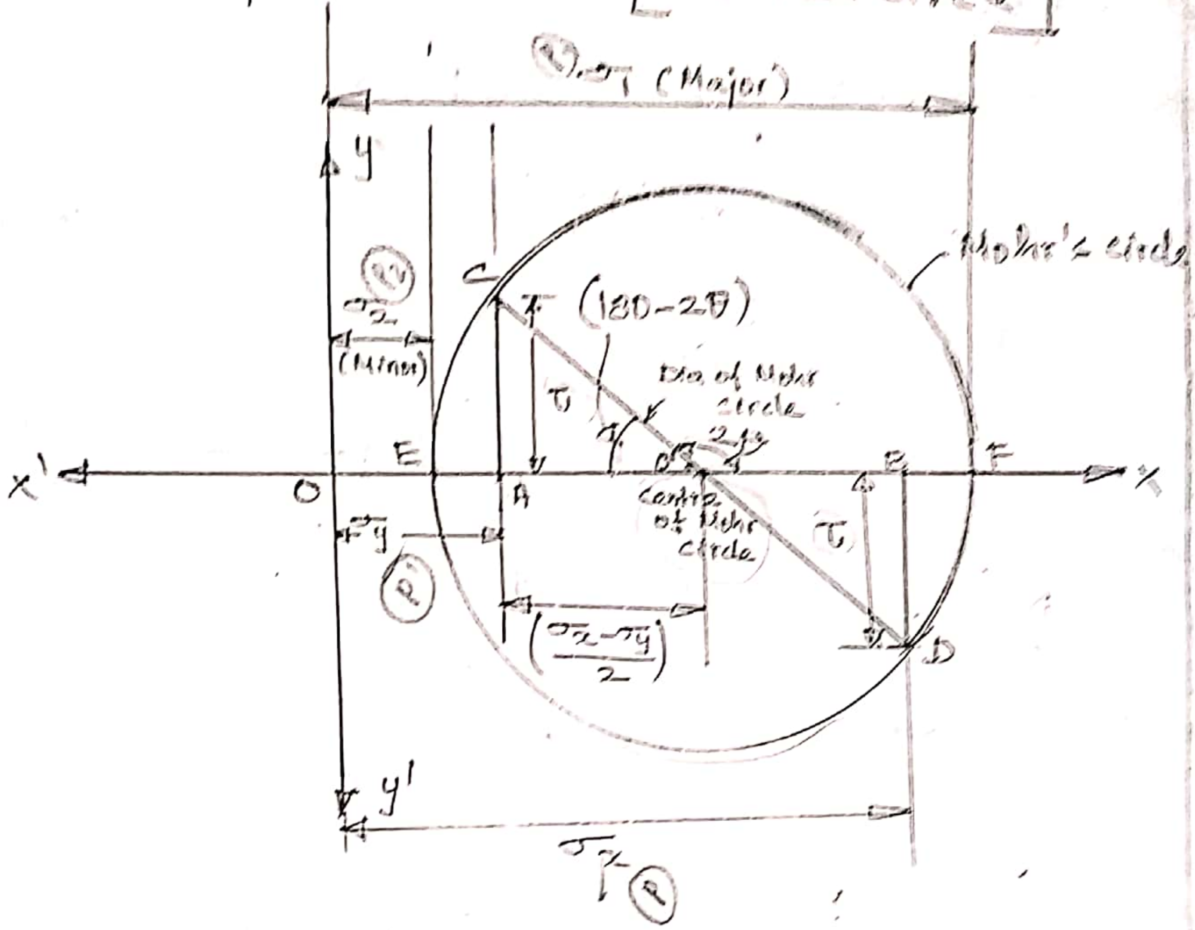
$$\sin 2\theta = \frac{-2\tau}{\pm\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

$$\left[hyp = \pm\sqrt{(-2\tau)^2 + (\sigma_x - \sigma_y)^2} = \pm\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2} \right]$$

$$\cos 2\theta = \frac{(\sigma_x - \sigma_y)}{\pm\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} - \frac{(\sigma_x-\sigma_y)}{2} \times \frac{\sigma_x-\sigma_y}{\pm\sqrt{4\tau^2+(\sigma_x-\sigma_y)^2}} + \tau \times \frac{-2\tau}{\pm\sqrt{4\tau^2+(\sigma_x-\sigma_y)^2}}$$

Graphical method [Mohr's Circle]



Minor principal stress,

$$OE = \sigma_2 = OO' - O'E$$

$$O'E = \sqrt{\tau^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$

$$O'C = \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

= Radius of Mohr circle.

$$OE = \sigma_2 = \sigma_y + \frac{\sigma_x - \sigma_y}{2} - \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\boxed{\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

Major principle stress,

$$OF = \sigma_1 = OO' + O'F$$

$$= \sigma_y + \frac{\sigma_x - \sigma_y}{2} + \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\boxed{\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}}$$

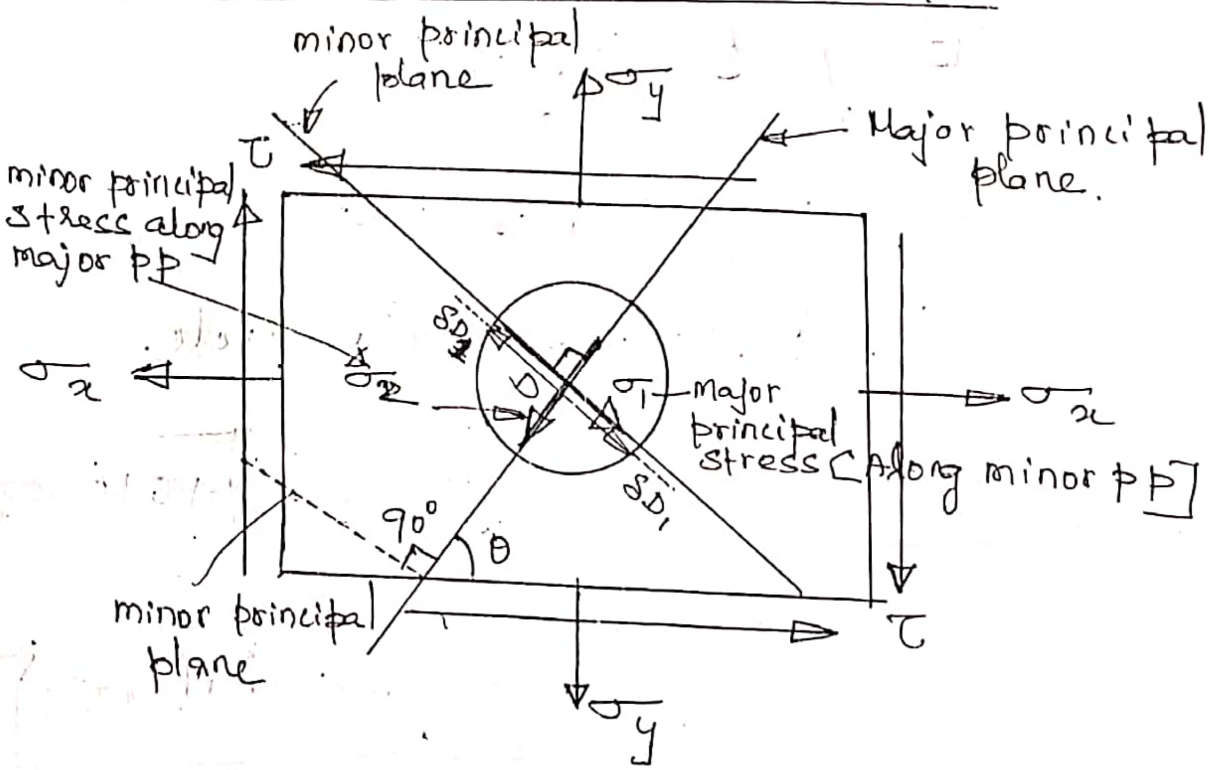
$\triangle^{le}$ O'CA,

$$\tan(180 - 2\theta) = \frac{\tau}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

$$\boxed{\tan(180 - 2\theta) = \frac{2\tau}{(\sigma_x - \sigma_y)}}$$

11-10-18

Circular Hole Become Ellipse :-



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Major principal strain,

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$\frac{\delta D_1}{D} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

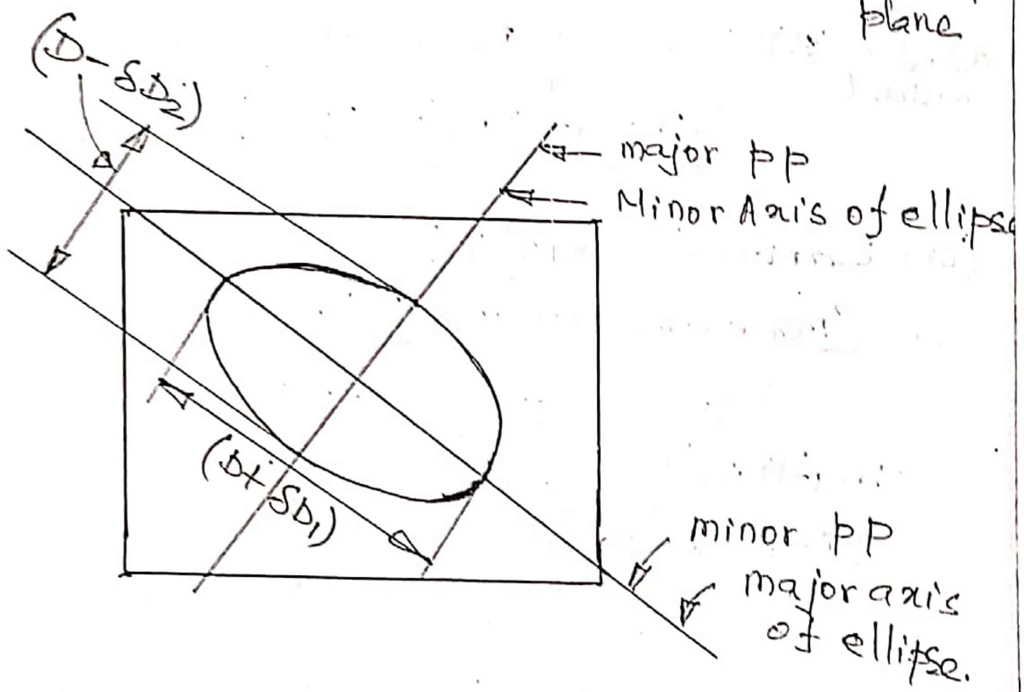
$$\boxed{\delta D_1 = D\left[\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}\right]} \Rightarrow$$ Along minor principal plane

Minor principal strain,

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$\frac{\delta D_2}{D} = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$\boxed{\delta D_2 = D\left[\frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}\right]} \Rightarrow$$ Along major principal plane



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THEORIES OF FAILURE :→

There are five theories of failure:-
[Based on Elastic condition of loading]
[Hooke's Law is obeyed]

Brittle material (a) Rankine's theory
[Maximum principal stress theory]

Brittle (b) Saint venent's theory
[Maximum principal strain theory]

Ductile material (c) Tresca or, Guest theory
[Maximum shear stress theory]

Ductile (d) Haig's theory
[Total strain energy theory]

General material (e) Von misses theory
[Shear-strain energy theory]

(a) Rankine's theory :→
[Maximum principal stress theory]

It is mainly applicable for Brittle material [$f_y = 0.002\epsilon$] [Do not have distinct yield point]

$$\sigma_1 > \sigma_0$$

$$\sigma_1 > \frac{f_y}{F.O.S} \rightarrow \text{for failure}$$

For no failure,

$$\sigma_1 \le \sigma_0$$

$$\boxed{\frac{\sigma_x+\sigma_y}{2}+\frac{1}{2}\sqrt{4\tau^2+(\sigma_x-\sigma_y)^2} \le \frac{f_y}{F.O.S}}$$

(b) Saint Vinent theory

[Maximum principal strain theory]

It is mainly applicable for General material [Neither Brittle nor Ductile]

$\epsilon_1 > \epsilon_0$, for failure

For No failure,

$$\epsilon_1 \le \epsilon_{0\,(\text{permissible})}$$

$$\frac{\sigma_1}{E}-\mu\frac{\sigma_2}{E} \le \frac{\sigma_0}{E} \rightarrow \text{As per Hooke's law.}$$

$$\sigma_1-\mu\sigma_2 \le \sigma_0$$

$$\boxed{\sigma_1-\mu\sigma_2 \le \frac{f_y}{F.O.S}}$$

(c) Tresca or, Guest theory :-

[Maximum shear stress theory]

It is mainly applicable for Ductile material.

$\tau_{max} > \tau_{0\,(perm)}$ — for failure for safety, No failure,

$$\tau_{max} \le \tau_{o\,(perm)}$$

$$\frac{\sigma_1 - \sigma_2}{\cancel{2}} \le \frac{\sigma_0}{\cancel{2}}$$

$$\sigma_1 - \sigma_2 \le \sigma_0.$$

$$\boxed{\sigma_1 - \sigma_2 \le \frac{f_y}{F.O.S}}$$

(d) Haig theory:-
[Total strain energy theory]
It is applicable for Ductile material & thick cylinder.

Equivalent stress;

$$\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{2}{m}(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)} \le \left[\sigma_0 = \frac{f_y}{FOS}\right]$$

For No failure

(e) Von mises theory:-
[Shear strain energy theory]
For General material

$$\sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}} \le \left[\sigma_0 = \frac{f_y}{FOS}\right]$$

For No failure

Numericals:—

1. A circular hole is drilled in a mild steel plate which is subjected to stresses $\sigma_x = 100$ MPa (tensile), $\sigma_y = 30$ MPa (tensile), $\tau = 40$ MPa. The dia of hole is 300mm. Determine,

(i) Major principal stress & Minor principal stress.

(ii) Maximum principal strain

(iii) principal plane location

(iv) Maximum shear stress

(v) Major axis of ellipse

take $\mu = 0.3$ & $E = 210$ GPa.

(i)

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$= \frac{100+30}{2} \pm \frac{1}{2}\sqrt{4 \times 40^2 + (100-30)^2}$$

$$= 65 \pm 53.15$$

$\sigma_1 = (+)118.15$ MPa (tensile)

$\sigma_2 = (+)11.85$ MPa (tensile)

ii)

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$= \frac{2 \times 40}{100 - 30}$$

$$\theta_1 = 65.59^\circ$$

$$\theta_2 = 90^\circ + \theta_1$$

$$\theta_2 = 155.59^\circ$$

iii)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{118.15 - 11.85}{2}$$

$$\tau_{max} = 53.15 \text{ MPa}$$

[Radius of Mohr circle]

(iv)

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$= \frac{1}{2.1 \times 10^5}(118.15 - 0.3 \times 11.85)$$

$$\epsilon_1 = 5.457 \times 10^{-4}$$

$$\frac{\delta D_1}{D} = 5.457 \times 10^{-4}$$

$$\delta D_1 = 5.457 \times 10^{-4} \times 300$$

$$\delta D_1 = 0.164 \text{ mm}$$

Major axis of Ellipse,

$$D_1 = D + \delta D_1$$

$$= 300 + 0.164$$

$$= 300.164 \text{ mm}.$$

Minor axis of ellipse,

$$\epsilon_2 = (11.85 - 0.3 \times 118.15)\frac{1}{2.1 \times 10^5}$$

$$\frac{\delta D_2}{D} = -1.124 \times 10^{-4}$$

$$\delta D_2 = -1.124 \times 10^{-4} \times 300$$

$$\delta D_2 = -0.0337 \text{ mm}.$$

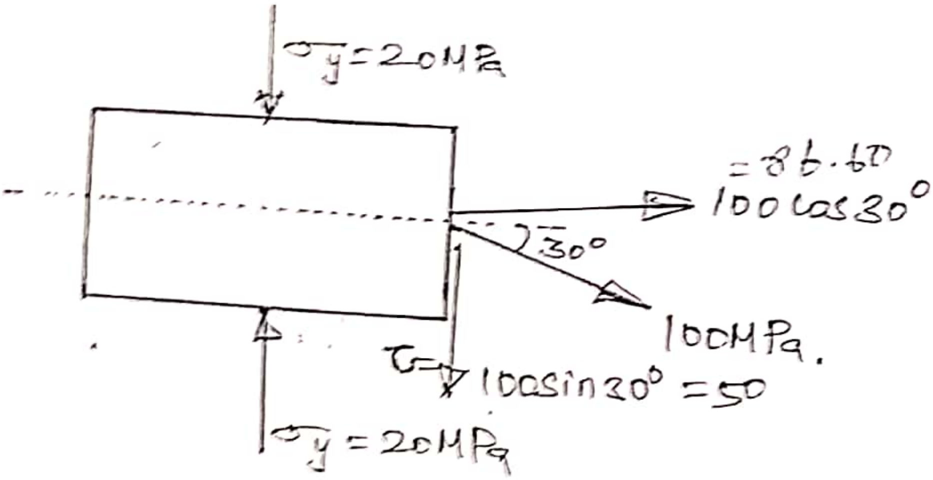
Minor axis of Ellipse,

$$D_2 = D + \delta D_2$$

$$= 300 + (-0.0337)$$

$$D_2 = 299.966 \text{ mm}$$

2. A steel material is subjected to direct stress at an angle 30° with horizontal having tensile nature of value 100MPa the material is subjected to compressive stress 20MPa in ⊥r direction. Determine principal stresses, maximum shear stress & principal planes.



$$\sigma_{1,2} = \frac{100\cos 30^\circ + (-20)}{2} \pm \frac{1}{2}\sqrt{4\times(100\sin 30)^2 + \left[100\cos 30^\circ - (-20)\right]^2}$$

$$= 33.30 \pm 73.08$$

$\sigma_1 = 106.38$ MPa (tensile)

$\sigma_2 = -39.78$ (Compressive)

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 73.08 \text{ MPa}$$

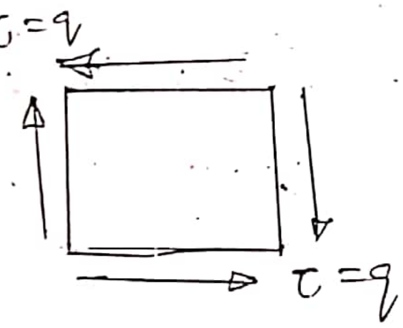
at 45° from PP

$$\tan(180 - 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y}$$

$$= \frac{2\times 50}{86.60 - (-20)}$$

$$\theta = 68.42^\circ$$

3. A steel material is subjected to pure shear [No Bending] 50MPa. Determine principal stresses, principal plane & Dia of Mohr's circle.



$$\sigma_{1,2} = \frac{0+0}{2} \pm \frac{1}{2}\sqrt{4\times 50^2 + (0-0)^2}$$

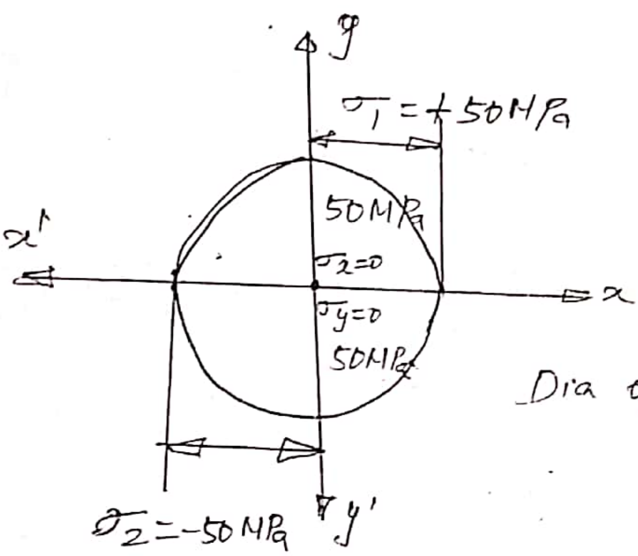
$$= 0 \pm 50$$

$$\sigma_1 = +50\,MPa$$

$$\sigma_2 = -50\,MPa$$

$$\tan(180 \cdot 2\theta) = \frac{2\tau}{\sigma_x - \sigma_y} = \frac{2\tau}{0} = \infty$$

$$\theta = 45^0$$



Dia of Mohr Circle $= 100\,MPa$

$= 2\tau_{max}$

$= 2 \times R_{mohr}$

4. Determine principal strains & Dia of Mohr circle of strain if $\epsilon_x = 2\times10^{-3}$, $\epsilon_y = 3\times10^{-4}$, $\phi_{xy} = 10^{-4}$.

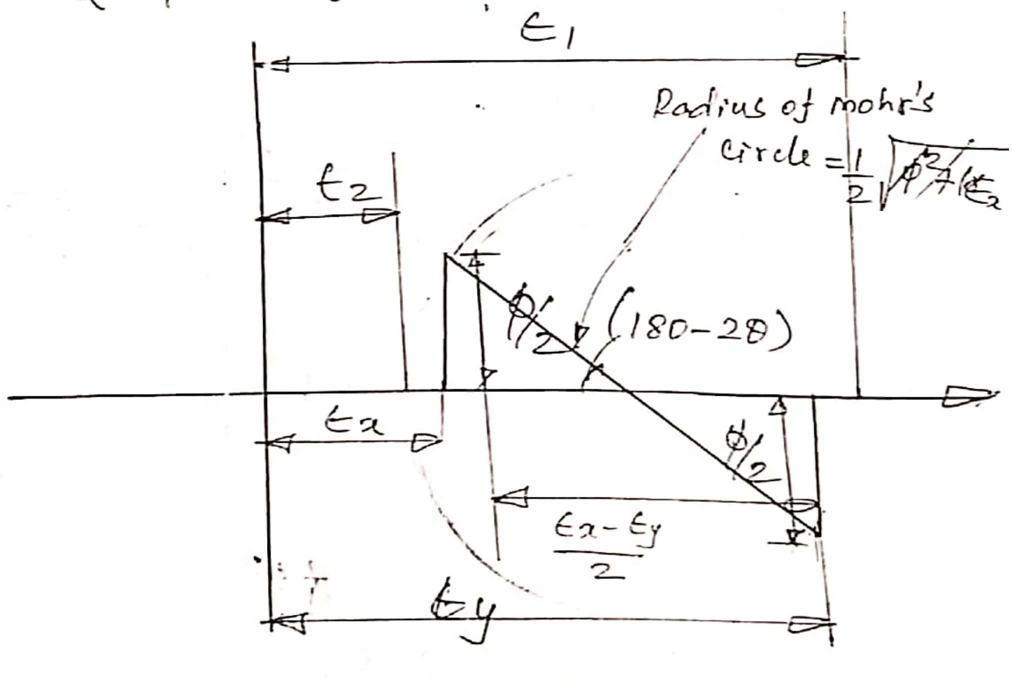
$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2}\sqrt{\phi_{xy}^2 + (\epsilon_x - \epsilon_y)^2}$$

$$= \frac{2\times10^{-3} + 3\times10^{-4}}{2} \pm \frac{1}{2}\sqrt{(10^{-4})^2 + (2\times10^{-3} - 3\times10^{-4})^2}$$

$$= 1.15\times10^{-3} \pm 8.515\times10^{-4}$$

$$\epsilon_1 = 2.00\times10^{-3}$$

$$\epsilon_2 = 2.985\times10^{-4}$$



Radius of Mohr's circle $= \frac{1}{2}\sqrt{\phi^2 + (\epsilon_x - \epsilon_y)^2}$

$$= 8.515\times10^{-4}$$

$$\text{Dia} = \sqrt{\phi^2 + (\epsilon_x - \epsilon_y)^2}$$
$$= 1.703\times10^{-3}$$

$$\tan(180-2\theta) = \frac{\phi/2}{(\epsilon_x-\epsilon_y)/2}$$

$$\tan(180-2\theta) = \frac{\phi}{(\epsilon_x-\epsilon_y)}$$

$$= \frac{10^{-4}}{(2\times10^{-3} - 3\times10^{-4})}$$

$$\theta = 88.32^\circ$$

5. A steel material is subjected to stresses, $\sigma_x = 90$ MPa, (tensile), $\sigma_y = 40$ MPa (compre) & $\tau = 30$ MPa. Determine the F.O.S if yield stress 270 MPa using Haig theory. take $\mu = 0.3$

$$\sigma_{1,2} = \frac{90+(-40)}{2} \pm \frac{1}{2}\sqrt{30^2\times4+(90-(-40))^2}$$

$$= 25 \pm 71.59$$

$$\sigma_1 = 96.59 \text{ MPa}.$$

$$\sigma_2 = -46.59 \text{ MPa}.$$

Haig theory,

$$\sqrt{96.59^2+(-46.59)^2 - \frac{2\times0.3}{\cancel{0.2}}(96.59\times(-46.59)) + 0 + 0} \leq \frac{\sigma_y}{F.os.}$$

$$\frac{119.17}{\cancel{208.72}} \leq \frac{270}{F.O.S}.$$

$$F.O.S \leq 2.27$$

6. A steel material is subjected to principal stresses 80MPa tensile & 30MPa compressive,
   (i) equivalent stress based on saint vinent theory & Tresca theory, also determine F.O.S based on Rankine theory & Von mises theory, take $\mu = 0.3$ & yield stress $= 270$ MPa.

Saint vinent,

$$\sigma_1 - \mu\sigma_2 \leq \cancel{\frac{f_y}{F.O.S}}\ \sigma_0$$

$$80 - 0.3 \times (-30) \leq \cancel{270}\ \sigma_0$$

$$\sigma_0 \geq 89 \text{ MPa}.$$

Tresca, $\sigma_1 - \sigma_2 \leq \sigma_0$

$$80 - (-30) \leq \sigma_0.$$

$$\sigma_0 \geq 110 \text{ MPa}.$$

Rankine,

$$\sigma_1 \leq \sigma_0$$

$$+80 \leq \frac{270}{F}$$

$$F \leq 270/80$$

$$F.O.S \leq 3.375$$

Von mises,

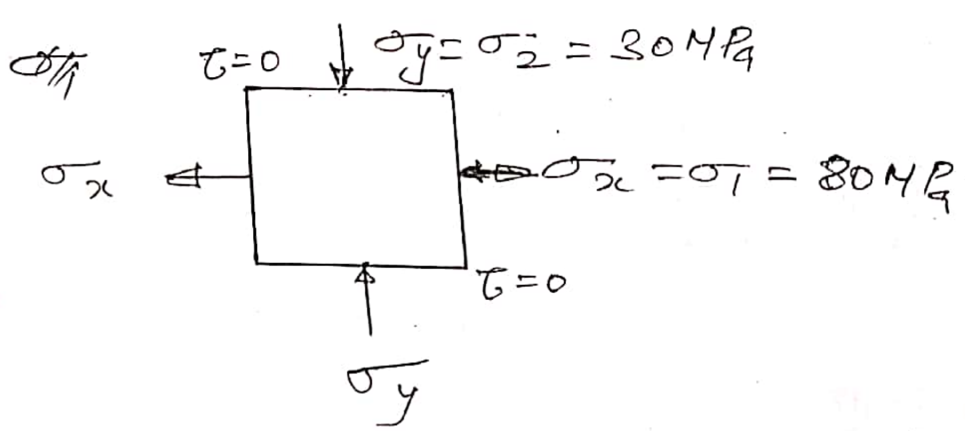
$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]} \leq \frac{f_y}{F.O.S}$$

$$\sqrt{\frac{1}{2}\left[(80-(-30))^2+(-30-0)^2+(0-80)^2\right]} \leq \frac{270}{F.O.S}$$

$$F.O.S \leq \frac{270}{98.49}$$

$$F.O.S \leq 2.741$$

7. If σ_2 is 30 MPa (compressive) & σ_1 is 80 MPa (tensile). Determine σ_x & σ_y, if $\tau = 0$.

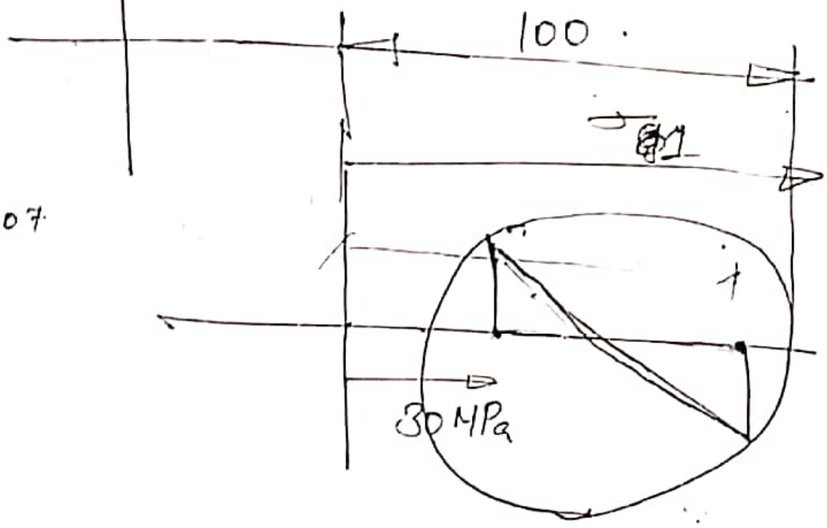


8. If $\sigma_1 = 100$ MPa (tensile), $\sigma_y = 30$ MPa (Comp), $\tau = 20$ MPa. Determine minor principal stress.

$$100 = \frac{\sigma_x + (-30)}{2} + \frac{1}{2}\sqrt{4\times20^2 + (\sigma_x - 30)^2}$$

$$= \frac{\sigma_x}{2} - 15 + \frac{1}{2}\sqrt{1600 + (\sigma_x - 30)^2}$$

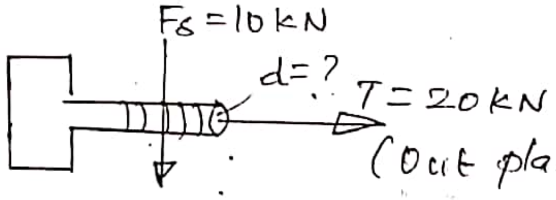
$\sigma_2^{(-)} = 33.07$



9. A mild bolt is subjected to transverse shear 10 kN & axial tension 20 kN. yield stress is 270 MPa & F.O.S = 3. Determine Dia of bolt by using,

1) Rankine theory
2) Vinent theory
3) Tresca theory
4) Haig theory
5) Von misses theory, take $\mu = 0.3$

$E = 210$ GPa.



(Out plane moment)

$$\tau = \frac{F_s}{A} = \frac{10 \times 10^3}{\pi/4 \times d^2} =$$

$$\sigma_x = \frac{T}{A} = \frac{20 \times 10^3}{\pi/4 \times d^2}$$

$\sigma_y = 0$

$$\left(\frac{\tau_{vt}}{\tau_d}\right)^2 + \left(\frac{\sigma_{at}}{\sigma_d}\right)^2 \leq 1$$

$$\sigma_{1,2} = \frac{20000}{\frac{\pi}{4} \times d^2} \times \frac{1}{2} \pm \frac{1}{2} \sqrt{4 \times \left(\frac{10000}{A}\right)^2 + \left(\frac{20000}{A}\right)^2}$$

$$= \frac{10000}{A} \pm \frac{28284.27}{2} / A$$

$\therefore \sigma_1 = 24142.14/A.$

$\sigma_2 = -4142/A$

1) Rankine,

$$\sigma_1 \leq \sigma_0.$$

$$\frac{24142}{\pi/4 d^2} \leq \frac{270}{F.O.S = 3}.$$

$$d \geq 18.48 \text{ mm}.$$

2) Viennent theory,

$$(\sigma_1 - \mu\sigma_2) \leq (\sigma_0 = f_y/FOS)$$

$$\left[\frac{24142}{\pi/4 d^2} - 0.3 \times \left(\frac{-4142}{A}\right)\right] \leq \frac{270}{3}.$$

$$d \geq 18.95 \text{mm}.$$

3) Tresca,

$$\sigma_1 - \sigma_2 \leq \sigma_0.$$

$$\frac{24142}{A} - \left(\frac{-4142}{A}\right) \leq \frac{270}{3}$$

$$d \geq 20.00 \text{mm}$$

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4) Haig theory,

$$\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2-2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)} \le \sigma_0$$

$$\sqrt{\left(\frac{24142}{A}\right)^2+\left(-\frac{4142}{A}\right)^2+0-2\times0.3\left(\frac{24142}{A}\right)\left(-\frac{4142}{A}\right)+0+0} \le \sigma_0$$

$$\frac{25690.27}{\pi/4\, d^2} \le \frac{270}{3}$$

$$d \ge 19.06\,mm$$

5) Vonmises theory,

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]} \le \frac{f_y}{F.}$$

$$\sqrt{\frac{1}{2}\left[\left[\frac{24142}{A}-\left(-\frac{4142}{A}\right)\right]^2+\left(-\frac{4142}{A}\right)^2+\left(-\frac{24142}{A}\right)^2\right]} \le \frac{f_y}{3}$$

$$\frac{26457.3}{\pi/4 \times d^2} \le \frac{270}{3}$$

$$d \ge 19.35\,mm.$$

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IV. TORSION & PRINCIPAL STRESSES & THEORIES OF FAILURE

1. Torque is the moment applied about the axis of the member [about z-axis] & this phenomenon is called **Torsion**.
2. The Torque develops shear stress.
3. The torque is represented by symbol 'T'

$$T = \text{Force} \times \text{radius}$$

$$\boxed{T = F \times r}$$

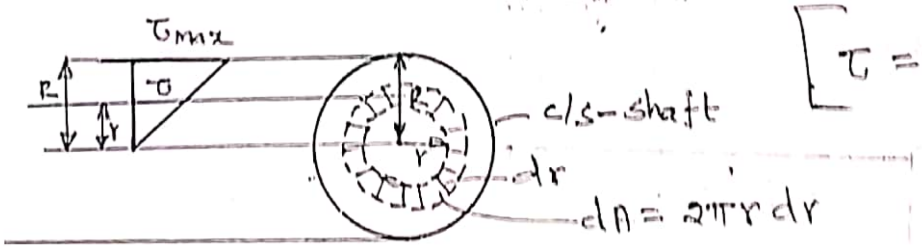
Unit – N-m ; Dimension: ML^2T^{-2}

4. Torsion is mainly used for screw-driver & to transmit power on the shaft.

5. **Expression for Torque:–**

From $\parallel^{r}$ Δ^{le}

$$\left[\tau = \frac{\tau_{max}}{R} \times r\right]$$



$$dT = dF \times r$$

$$= (\tau \times dA) \times r$$

$$= \tau \times 2\pi r \times dr \times r$$

$$dT = \tau_{max} \times \frac{r}{R} \times 2\pi r dr \times r.$$

$$T = \int_0^R \frac{\tau_{max}}{R} \times 2\pi r^3 dr$$

$$= \frac{2\pi \tau_{max}}{R} \left[\frac{r^4}{4}\right]_0^R = \frac{2\pi \tau_{max} \times R^{4^3}}{2R}$$

$$\boxed{T = \frac{\pi \tau_{max} R^3}{2}}$$

$$R = \frac{D}{2}$$

$$T = \frac{\pi \tau_{max}}{2} \left(\frac{D}{2}\right)^3$$

$$= \frac{\pi \tau_{max}}{2} \times \frac{D^3}{8} \times \frac{D}{D}$$

$$= \frac{\pi}{2} \times \frac{\tau_{max}}{8} \times \frac{D^4}{2R}$$

$$= \frac{\pi}{32} \times \tau_{max} \times D^4 \times \frac{1}{R}$$

$$= \frac{\pi}{32} \times D^4 \times \tau_{max} \times \frac{1}{R} \quad \left[\frac{\pi}{32} D^4 = J\right]$$

$$T = \tau_{max} \times J \times \frac{1}{R}$$

$$\boxed{\frac{T}{J} = \frac{\tau_{max}}{R}} \; ***$$

Polar M.I $= I_p = J =$ M.I about zz axis

$$= I_{xx} + I_{yy}$$

$$= \frac{\pi}{64} D^4 + \frac{\pi}{64} D^4$$

$$\boxed{J = I_p = \frac{\pi}{32} D^4} \text{ — solid shaft}$$

$$\boxed{J = I_p = \frac{\pi}{32} [D^4 - d^4]} \text{ — Hollow shaft}$$

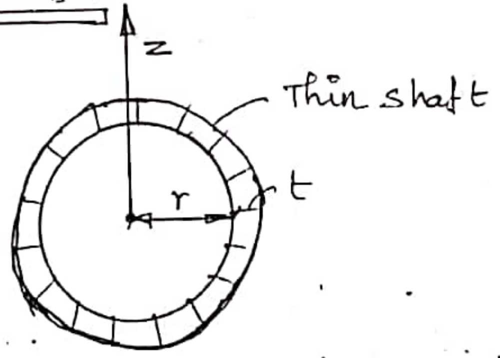
$$T = \frac{\tau_{max}}{R} \times J$$

$$= \frac{\tau_{max}}{D/2} \times \frac{\pi D^{4\,3}}{32\,16}$$

$$\boxed{T = \frac{\pi}{16} D^3 \tau_{max}}$$

$$\boxed{\tau_{max} = \frac{16T}{\pi D^3}}$$

For thin shaft:→



polar M.I = Moment of moment of Area

= Second moment of area

$= (2\pi r t) \times r \times r$

$= 2\pi t r^3$

$= 2\pi t \times \left(\frac{D}{2}\right)^3$

$= 2\pi t \frac{D^3}{8}$

$$\boxed{I_{polar} = \frac{\pi D^3 t}{4}}$$

For thin shaft:-

$$\frac{T}{J} = \frac{\tau_{max}}{R}$$

$$\frac{T}{\frac{\pi D^{\cancel{3}2} t}{\cancel{4}2}} = \frac{\tau_{max}}{\frac{\cancel{D}}{\cancel{2}}}$$

$$\boxed{\frac{T}{\frac{\pi}{4} D^3 t} = \frac{\tau_{max}}{\frac{D}{2}}}$$

Note:→

Incase of Hollow shaft, the shear stress distribution is non-uniform across the thickness. ~~But~~ therefore Law of integration is required. But incase of thin shaft the shear stress distribution is almost uniform. Therefore there is no need of applying integration.

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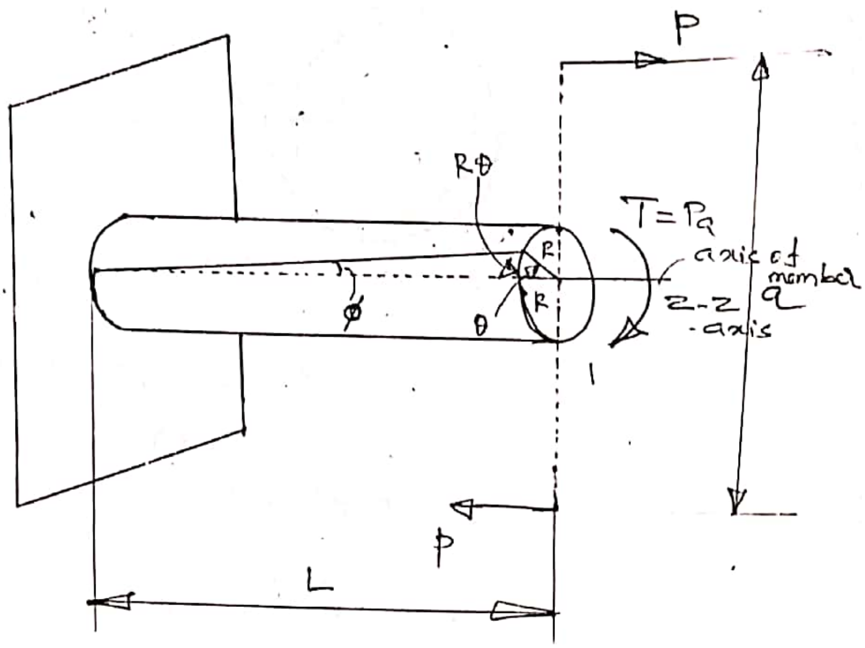
Expression for Angle of twist :-



$$\theta = \frac{l}{R}$$

$$l = R\theta$$

$$\tan\phi = \frac{R\theta}{L}$$

$$\boxed{\phi = \frac{R\theta}{L}} \quad \because \left[\begin{array}{l} \phi = \text{is very small} \\ \tan\phi \simeq \phi \end{array}\right]$$

ϕ = shear strain

shear stress = Shear strain × Modulus of Elasticity

$$\tau = \phi \times G$$

$$\phi = \frac{\tau}{G} \qquad [C = G = N]$$

$$\frac{R\theta}{L} = \frac{\tau}{G}$$

$$** \quad \boxed{\frac{G\theta}{L} = \frac{\tau}{R}}$$

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}}$$

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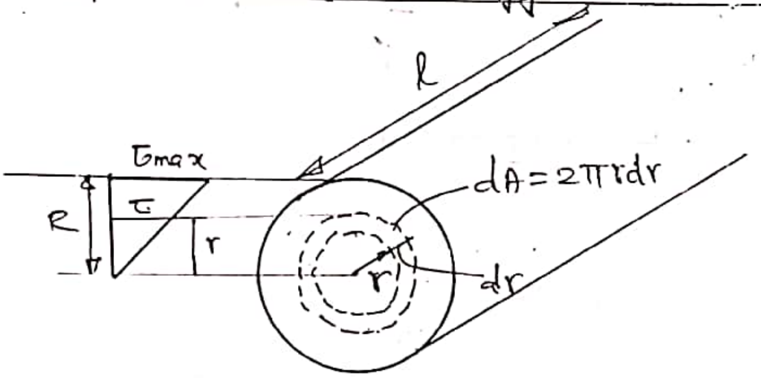
$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\boxed{\frac{TL}{\theta} = GJ} = \text{Tortional Rigidity.}$$

$$\boxed{\frac{T}{\frac{\pi}{32}D^4} = \frac{\tau_{max}}{D/2}} \quad \boxed{\frac{\tau_{max}}{D/2} = \frac{G\theta}{L}}$$

$D = ?$ Adopt greater value of 'D'.

Expression for Energy due to Torsion:-



$$U = \frac{f^2}{2E} \times \text{volume}$$

$$\partial U = \frac{\tau^2}{2G} \times dV$$

$$= \frac{\tau^2}{2G} \times (dA \times l)$$

$$= \frac{\tau^2}{2G} \times 2\pi r dr \times l$$

$$\partial U = \left(\tau_{max} \times \frac{r}{R}\right)^2 \times \frac{2\pi r dr \times l}{2G}$$

$$U = \int_0^R \frac{\tau_{max}^2}{\cancel{2}GR^2} \times \cancel{2}\pi r^3 l \, dr$$

$$= \frac{\tau_{max}^2 \times \pi l}{GR^2} \int_0^R r^3 dr$$

$$= \frac{\pi l \tau_{max}^2}{GR^2}\left[\frac{r^4}{4}\right]_0^R$$

$$U = \frac{\pi l \tau_{max}^2}{GR^2} \times \frac{R^4}{4}$$

$$= \frac{\tau_{max}^2}{4G} \times \pi R^2 \times l$$

$$= \frac{\tau_{max}^2}{4G} \times A \times l$$

$$\boxed{U = \frac{\tau_{max}^2}{4G} \times Volume}$$

$$U = \frac{\tau_{max}^2}{4G} \times Volume$$

Note:- Modulus of resilience

$U = \frac{f^2}{2E} \times volume$ — Axially applied load

$= \frac{\tau^2}{4G} \times Volume$ — Torsion effect ~~Axially loaded member~~

$= \frac{\tau^2}{2G} \times volume$ — ~~Torsion effe~~ Shear stress (SF)

$= \frac{p^2}{2k} \times volume$ — Volumetric change due to Hydrostatic force (Bulk mod)

$= \int \frac{M^2 ds}{2EI}$ — Bending (flextural)

$= \int \frac{V^2 ds}{2GA}$ — Shear force

EI - flextural rigidity.

GA - Shear rigidity.

GJ - Torsional rigidity.

AE - Axial rigidity.

$$U = \frac{\tau^2_{max}}{4G} \times volume$$

$$\left[\frac{T}{J} = \frac{\tau_{max}}{R}\right]$$

$$= \frac{\left[\frac{T}{J} \times R\right]^2}{4G} \times volume$$

$$= \frac{T^2 R^2}{J^2 \times 4G} \times \pi R^2 \times L$$

$$= \frac{T^2 L}{J^2 \times 4G} \times \pi R^4$$

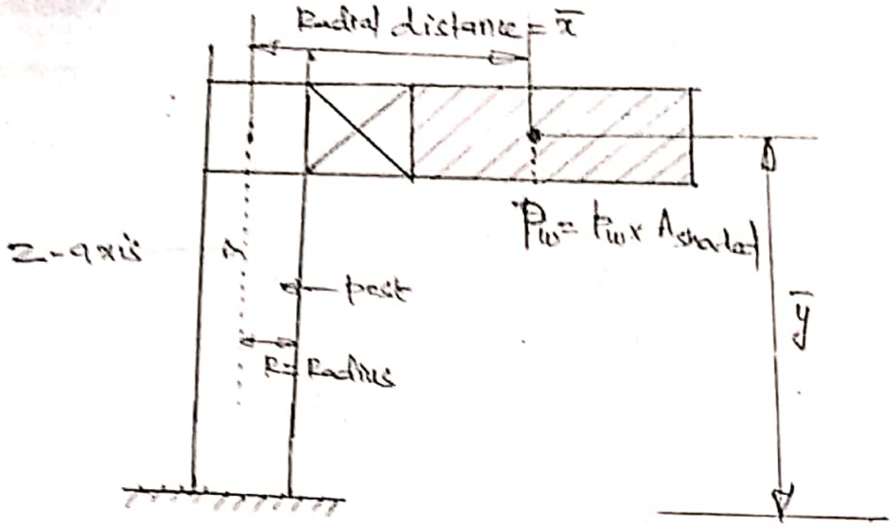
$$= \frac{T^2 L}{\left(\frac{\pi}{32} \times D^4\right)^2 \times 4G} \times \pi \times \left(\frac{D}{2}\right)^4$$

$$= \frac{T^2 L}{J^2 \times 2G} \times \frac{\pi D^4}{32}$$

$$= \frac{T^2 L}{J^2 \times 2G} \times J$$

$$\boxed{U = \frac{T^2 L}{2GJ}}$$

Expression for Equivalent Torque & equivalent Bending moment due to combined effect of Torsion & Bending moment :-



$P_w = p_w \times A_{shaded}$

$\text{Torque}, T = \left[P_w \times \bar{x} \right]$

$BM,\ M = P_w \times \bar{y}$

Due to torque, shear stress develops,

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \tau = \frac{T}{J} \times R$$

$$= \frac{T \times \frac{D}{2}}{\frac{\pi}{32} D^4}$$

✓ $$\boxed{\tau = \frac{16T}{\pi D^3}}$$

Due to BM, Bending stress develops,

$$\frac{M}{I} = \frac{f}{y}$$

$$f = \frac{M}{I} \times y$$

$$\sigma_x = \frac{M}{\frac{\pi D^4}{64}} \times \frac{D}{2}$$

✓ $$\boxed{\sigma_x = \frac{32M}{\pi D^3}} \ ; \ \sigma_y = 0$$

[other direction]
[fall down]

Principal stress,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$= \frac{\frac{32M}{\pi D^3} + 0}{2} \pm \frac{1}{2}\sqrt{\left(\frac{32M}{\pi D^3} - 0\right)^2 + 4 \times \left(\frac{16T}{\pi D^3}\right)^2}$$

$$= \frac{16M}{\pi D^3} \pm \frac{1}{2}\sqrt{\left(\frac{16}{\pi D^3}\right)^2 \left[(2M)^2 + 4 \times T^2\right.}$$

✓ $$\sigma_{1,2} = \frac{16}{\pi D^3}\left[M \pm \sqrt{M^2 + T^2}\right]$$

Major principal stress,

$$\sigma_1 = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

= Resultant Bending stress
[Equating bending stress].

$$\frac{M}{I} = \frac{f}{y}.$$

$$\frac{M_{equ}}{\frac{\pi}{64} D^4} = \frac{\sigma_1}{\frac{D}{2}}$$

$$\frac{32 M_{equi}}{\pi D^3} = \frac{16}{\pi D^3}\left[M + \sqrt{M^2 + T^2}\right]$$

✓ $$\boxed{M_{equi} = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \text{Radius of Mohr's circle}$$

$$= \left[\frac{16}{\pi D^3}\left[M + \sqrt{M^2+T^2}\right] - \frac{16}{\pi D^3}\left[M - \sqrt{M^2+T^2}\right]\right] \times \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{16}{\pi D^3}\left[2\sqrt{M^2+T^2}\right]$$

$$\tau_{max} = \frac{16}{\pi D^3}\left[\sqrt{M^2+T^2}\right] \quad \text{—} ①$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{T_{equi}}{\frac{\pi D^4}{32}} = \frac{\tau_{max}}{\frac{D}{2}}$$

$$\tau_{max} = \frac{16 T_{equ}}{\pi D^3} \quad \text{—} ②$$

$① = ② \Rightarrow$

$$\frac{16 T_{equi}}{\pi D^3} = \frac{16}{\pi D^3}\left[\sqrt{M^2+T^2}\right]$$

✓ $$\boxed{T_{equi} = \sqrt{M^2+T^2}}$$

power transmitted by shaft:-

$$\text{Power} = \frac{\text{W.D}}{\text{time}} = \frac{F \times \text{distance}}{t}$$

$$= F \times v_{(\text{Linear})}$$

$$= F \times r\omega$$

$$\text{power} = T\omega$$

$$\boxed{\text{power} = \frac{2\pi N T}{60}}$$

Theories of Failure :—

1. Rankine's theory :- [Maximum principal stress theory]
[Brittle material]

$$\sigma_1 \le \sigma_0$$

$$\frac{16}{\pi D^3}\left[M + \sqrt{M^2+T^2}\right] \le \left[\sigma_0 = \frac{f_y}{FOS}\right]$$

2. Saint vinent theory :- [Maximum principal strain theory]

$$\epsilon_1 \le \epsilon_0$$

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} \le \frac{\sigma_0}{E}$$

$$\sigma_1 - \mu\sigma_2 \le \frac{f_y}{FOS}$$

3. Tresca, or, Guest Theory :- [Ductile material]
[Maximum shear stress theory]

$$\frac{\sigma_1 - \sigma_2}{2} \le \frac{\sigma_0}{2}$$

$$\sigma_1 - \sigma_2 \le \frac{f_y}{FOS}$$

4. Haig theory :- [Total strain energy theory]
[Ductile material] [thin cylinder]

$$\sqrt{\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1\sigma_2)} \le \sigma_0 = \frac{f_y}{FOS}$$

5. Von misses theory :- [~~Max~~ shear strain energy]
[Distortion]

$$\sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]} \le \left[\sigma_0 = f_y\right.$$

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Numericals :→

1. A solid shaft has to transmit mean power of 1000kW at 300 rpm. The maximum shear stress is 60MPa. Determine the dia of shaft if the maximum torque is 25% more than the mean torque. Also determine % saving in material if Hollow shaft having inner dia 0.6 times of outer is used.

$$\frac{T}{J} = \frac{\tau}{R} \; ; \quad P = \frac{2\pi N T_{mean}}{60}$$

$$1000 \times 10^3 = \frac{2\pi \overset{300}{N} T_{mean}}{60}$$

$$T_{mean} = 31830.99 \, N\text{-}M$$

$$T_{max} = 25\% \times T_{mean} + T_{mean}$$

$$= 0.25 \times 31830.99 + 31830.99$$

$$= 7957.75 + 31830.99$$

$$= 39788.74 \, N\text{-}M$$

$$\frac{39788.74}{\frac{\pi}{16} \times D^3} = \frac{60 \times 10^6}{\frac{D}{2}}$$

$$D = 0.15 \, M = 150 \, MM.$$

Hollow shaft.

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{39788.74}{\frac{\pi}{32}(D^4 - d^4)} = \frac{60 \times 10^6}{\frac{D}{2}}$$

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$$\frac{39788.74}{\frac{\pi}{16}\left(D^4-(0.6D)^4\right)} = \frac{60\times10^6}{D}$$

$$D = 0.157\, m$$

$$D_H = 157\, MM$$

$$\%\ \text{Saving in material} = \frac{\text{wt. of Solid shaft} - \text{wt. of Hollow shaft}}{\text{wt. of Solid shaft.}}$$

$$= \frac{W_S - W_H}{W_S}\times 100$$

$$= \frac{\gamma_S A_S L_S - \gamma_H A_H L_H}{\gamma_S A_S L_S}$$

$$= \frac{A_S - A_H}{A_S}$$

$$= \frac{\pi/4\, D_S^2 - \pi/4\,(D^2-d^2)}{\pi/4\, D^2}$$

$$= \frac{\overset{150}{\cancel{D_S^2}} - \overset{157}{\cancel{D^2}} + (0.6D)^2\ 157}{D_S^2\ 150}$$

$$\approx \cancel{0.6}x$$

$$= 29.76\%$$

2. A thin cylinder having dia 300mm & thickness 10mm is subjected to external torque 100N-m & inner fluid pressure 1MPa. Determine,

i) ~~Maximum~~ shear stress due to torque

ii) Maximum principal stress

iii) Maximum shear stress

iv) Absolute maximum shear stress.

Soln:-

$t = 10\,mm$

$T = 100 \times 1000\ N\text{-}mm$

$R = D/2 = 300/2 = 150\,mm$

$\tau = ?$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{T}{\frac{\pi D^3 t}{4}} = \frac{\tau}{\frac{D}{2}}$$

$$\frac{100 \times 10^3}{\frac{\pi \times 300^2\ 10}{4}} = \frac{\tau}{\frac{300}{2}}$$

$$\tau = 0.071\ N/mm^2$$

$$\cancel{\sigma_1 = \frac{16}{\pi D^3}\left[M+\right]}$$

$$\sigma_x = \text{Hoop stress} = \frac{pD}{2t} = \frac{1 \times 300}{2 \times 10}$$

$$= 15\,MPa$$

$$\sigma_y = \text{Longitudinal stress} = \frac{pD}{4t} = 7.5\,MPa$$

$$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$= \frac{15+7.5}{2} \pm \frac{1}{2}\sqrt{4 \times 0.071^2 + (15-7.5)^2}$$

$$= 11.25 \pm 3.75$$

$$\sigma_1 = 15.00\ MPa$$

$$\sigma_2 = 7.50\ MPa$$

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$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = 3.75\ MPa$$

Absolute maximum shear $= \frac{\sigma_1}{2}$

$= 15/2$

$= 7.5\ MPa$

V. SFD & BMD

1. Beam is a structural member which is subject to load ⊥r to the axis of the member [Transverse load] & it transfers load to the support through Bending only. Beam is a bending member which is design on the basis of the maximum Bending moment & maximum shear force.

2. There are following types of load :-
   a) point load [concentrated load] [W] [KN]
   b) Uniformly distributed load (udl) (w) (KN/m) or Rectangular load.
   c) Gradually varying load [GVL] or Triangular load [0 to w] [KN/m]

3. The algebraic sum of all moments consider from extreme end to any section of the beam is called Bending moment at that section

BM

4. The graphical representation of Bending moment along with its nature (+ or -) called Bending moment diagram [BMD] which is very-very essential to find position of steel bars in RCC structures [Beams].

If the Bending moment is consider from left to right then clockwise moment is plus & Anticlockwise called minus.

But if the moment is consider from right to left then clockwise called minus & anti-clockwise called plus.

5. The algebraic sum of all forces consider from extreme end to any section of the beam is called shear force at that section. The graphical representation of shear force along with its nature is called shear force diagram (SFD) which is very very essential to determine the position of stirrups in RCC beams.

SF

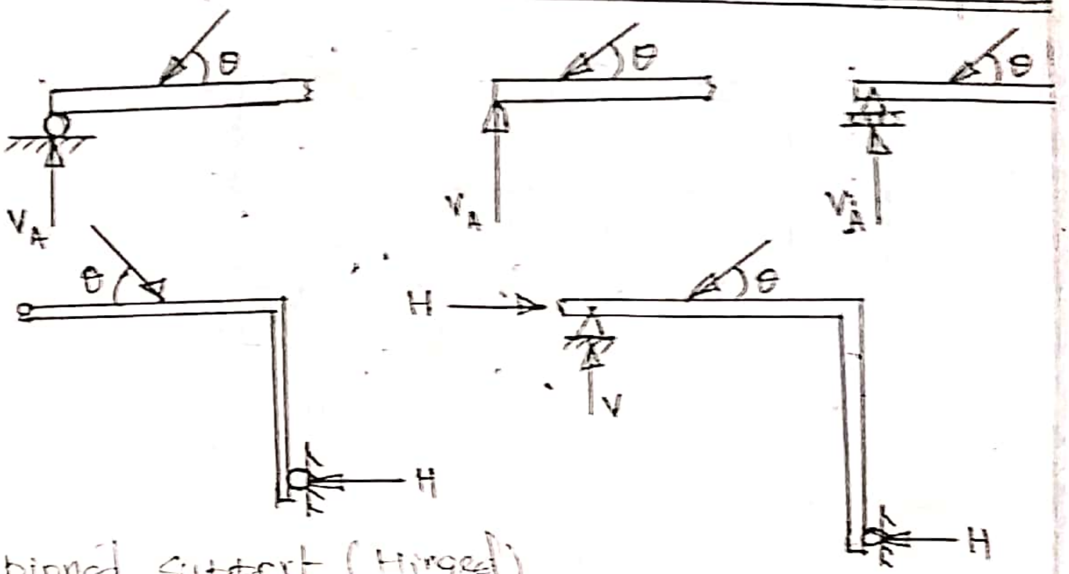
It the force is upward, consider from left to right, is considered as plus & downward force is taken as minus but if it is consider from right to left then upward force is taken as minus & downward force is taken as plus.

Any force developing clockwise moment from left to right then that force is consider as plus & if it develops anticlockwise moment [left to right] then that force is consider as negative.

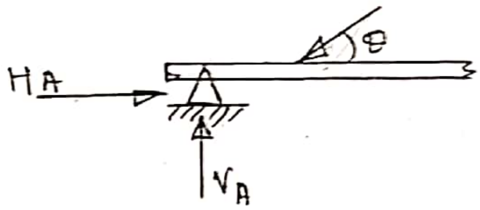
6.

6. Support reaction [Type of support]

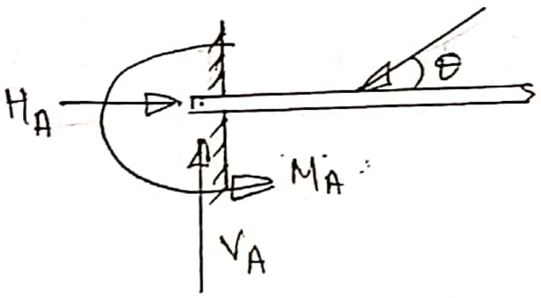
a) Roller support, simple support, Free support



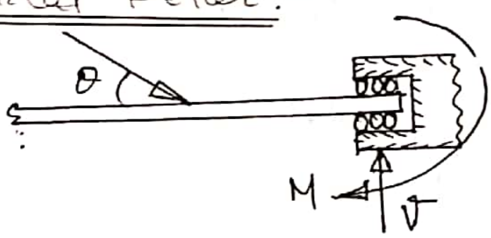
b) pinned support (Hinged)



c) Fixed support [Built-in] [Encastered]

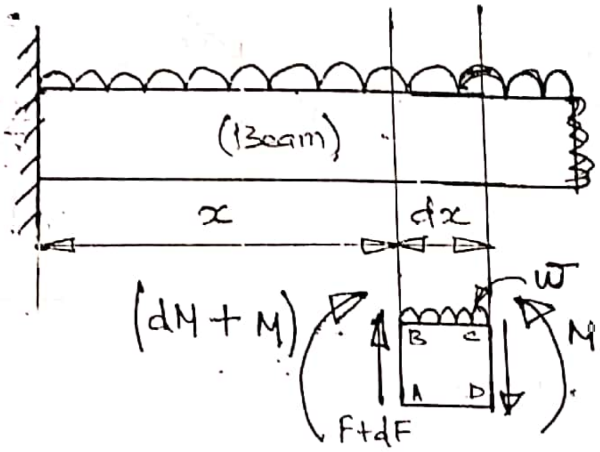


d) Guided Roller :-



7. Relation among shear force, Bending moment & Load :-

7. Relation among Shearforce, Bending moment & Load :-



– Free Body Diagram

Law of Equilibrium;

$$\Sigma V = 0$$

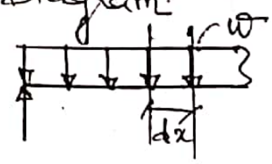
$$+(F+dF) - F - w\,dx = 0.$$

$$dF = w\,dx$$

$$\boxed{\frac{dF}{dx} = w}$$ – intensity of load

$$dF = w\,dx$$

$$\boxed{F = \int w\,dx}$$ – Area of loading Diagram



$$\Sigma M_A = 0;$$

$$-F\,dx + M - w\,dx \times \frac{dx}{2} - (dM+M) = 0$$

$$-F\,dx - \cancel{w\frac{(dx)^2}{2}} - dM = 0$$

neglected – dx – very small, $(dx)^2$ – very very small.

$$dM = -F\,dx$$

$$\boxed{\frac{dM}{dx} = -F}$$ → Rate of change of BM with respect to distance is called shearforce

$$\boxed{\frac{dM}{dx} = S.F}$$

For maximum Bending moment,

$$\frac{dM}{dx} = 0.$$

Shear force $= 0$.

$\boxed{V_{xx} = 0.}$ — for maximum BM

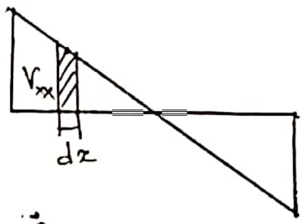
$$\frac{dM}{dx} = V_{xx}$$

$$dM = V_{xx}\,dx$$

$$\int dM = \int V_{xx}\,dx$$

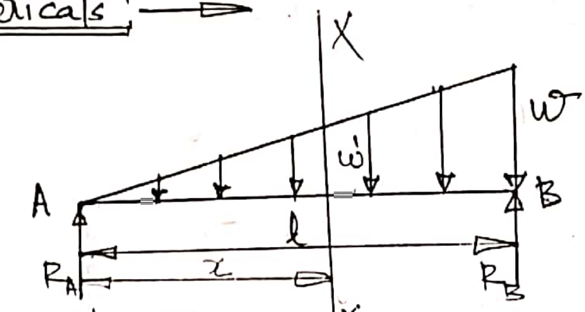
$$M = \int V_{xx}\,dx$$

$\boxed{\text{BM} = \text{Area of SFD}}$



Numericals :—

1.



(Triangular Load)

Total load, $= \frac{1}{2} w \times l$

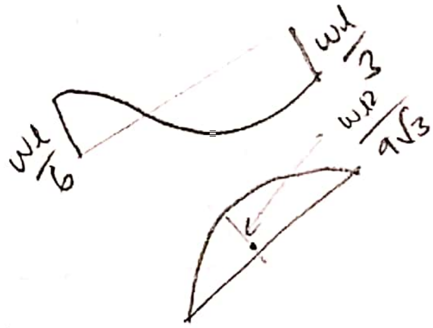
$\sum M_B = 0.$

$$R_A \times l = \frac{1}{2} \times w \times l \times \frac{l}{3}$$

$$R_A = \frac{wl}{6} \;;\; R_B = \frac{1}{2} \times w \times l - \frac{wl}{6} = \frac{wl}{3}$$

For Maximum BM,

$$V_{xx} = 0$$



$$\frac{wl}{6} - \frac{1}{2} \times w' \times x = 0.$$

$$\frac{wl}{6} - \frac{1}{2} \times \frac{w \times x \times x}{l} = 0.$$

parabolic SFD

$$\frac{\cancel{w} x^{2}}{\cancel{2} l} = \frac{\cancel{w} l}{\cancel{6}\,3}$$

$$x^2 = \frac{l^2}{3}$$

$$\boxed{x = l/\sqrt{3}} = 0.577\,l$$

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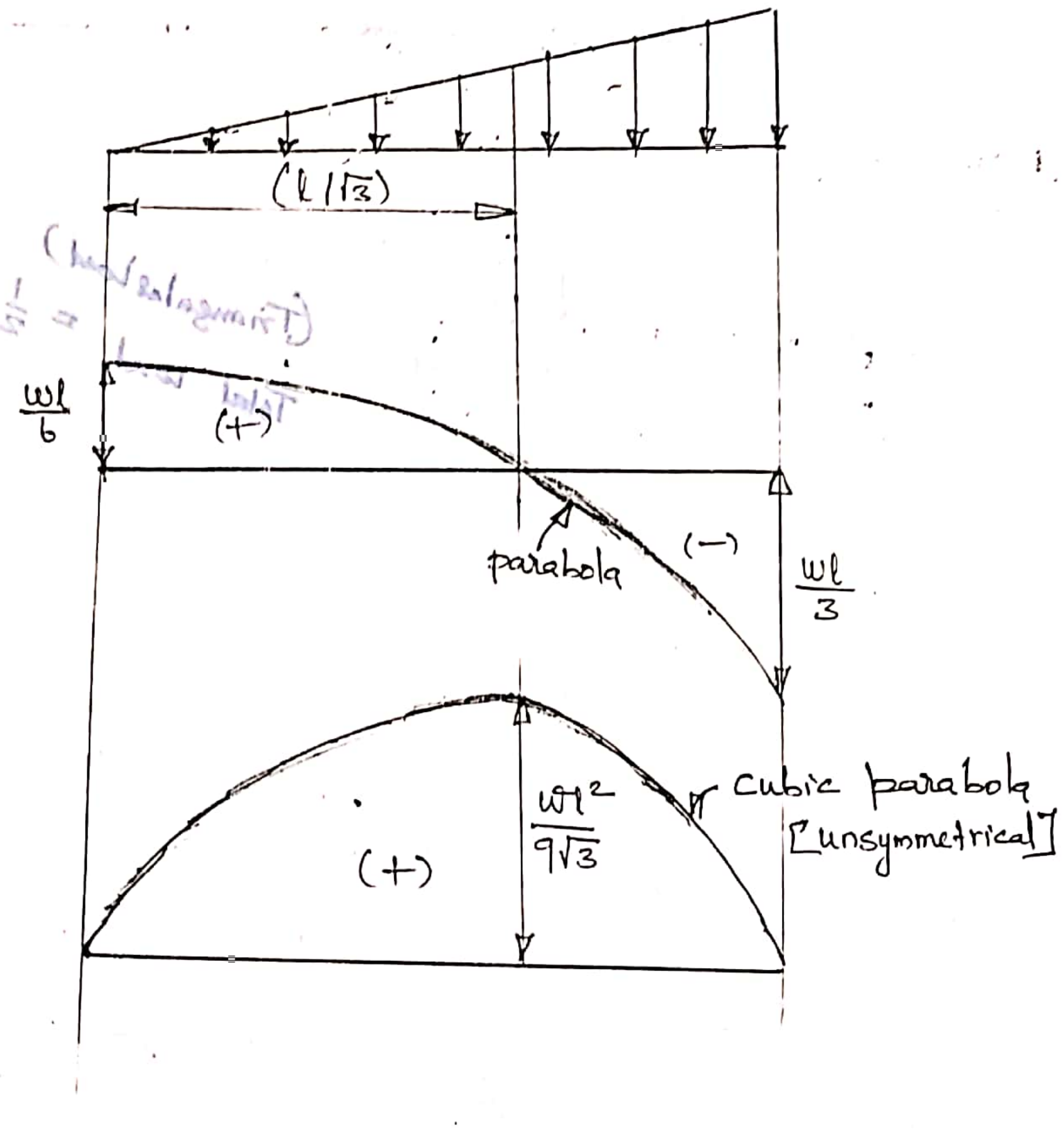
$$M_{xx} = M_{max} = \frac{wl}{6} \times x - \frac{1}{2} \times \frac{wx}{l} \times x \times \frac{x}{3}$$

$$= \frac{wlx}{6} - \frac{wx^{3}}{6l}$$ — Cubic parabolic BMD

$$= \frac{wl \times l}{\sqrt{3} \times 6} - \frac{w}{6l} \times \left(\frac{l}{\sqrt{3}}\right)^{3}$$

$$= \frac{wl^2}{6\sqrt{3}} - \frac{wl^2}{6 \times 3 \times \sqrt{3}}$$

$$\boxed{M_{max} = \frac{wl^2}{9\sqrt{3}}}$$



2. A beam is supported by a strut as shown in figure below. Determine,

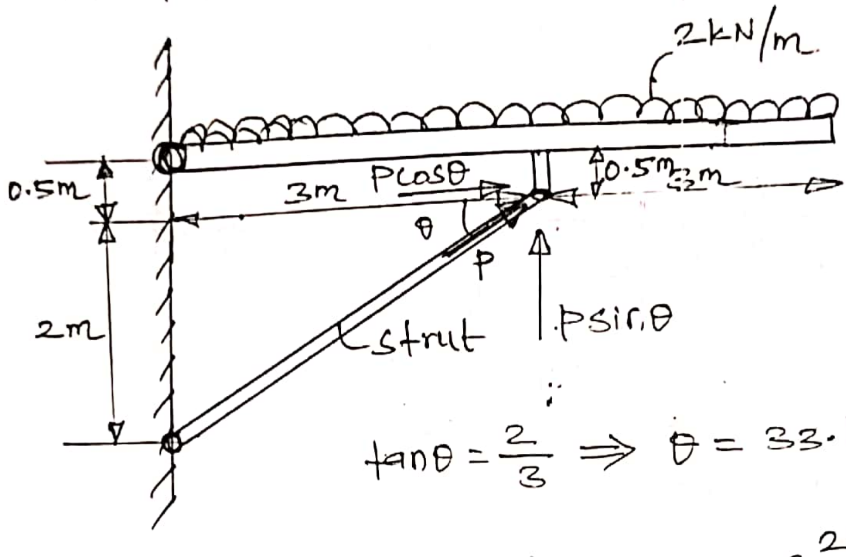
(i) Force in the strut

(ii) Thrust in the beam

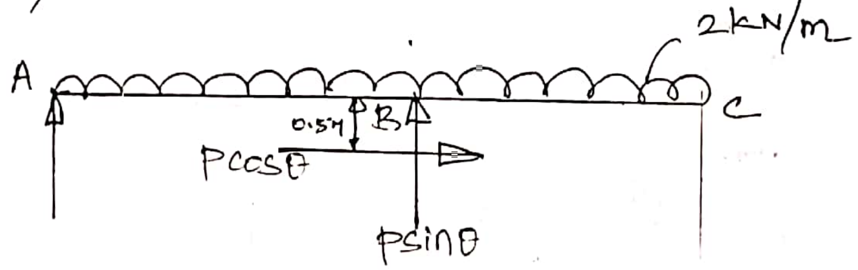
(iii) Maximum bending moment in the beam

(iv) Maximum shear force in the beam.

(v) SFD & BMD



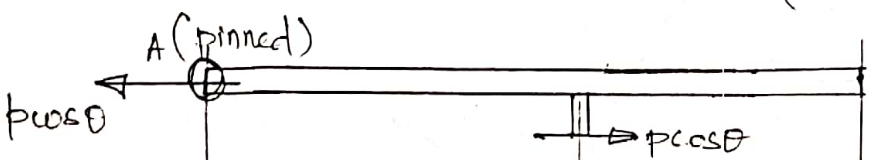
$\tan\theta = \frac{2}{3} \Rightarrow \theta = 33.69°$



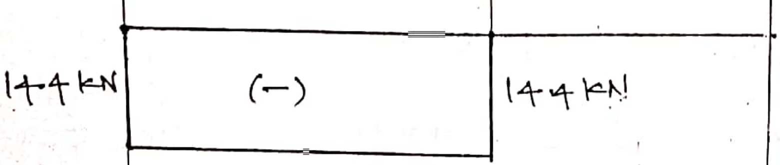
$\Sigma M_A = 0$;

$$-\frac{2 \times 6^2}{2} + P\cos\theta \times 0.5 + P\sin\theta \times 3 = 0$$

Force in the strut, $\boxed{P = +17.30\ kN}$ (compressive)

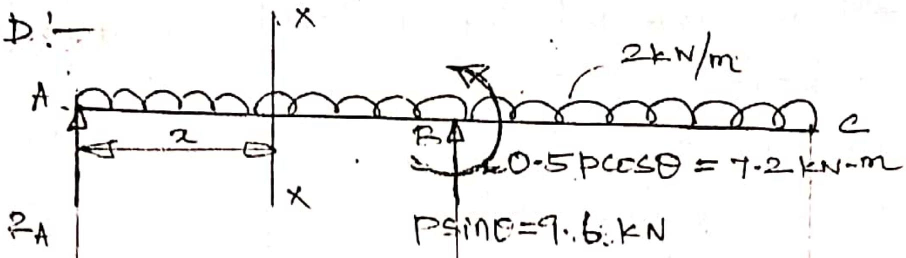


Axial thrust $= P\cos\theta = 17.3\cos\theta = 14.4$ kN (tensile)

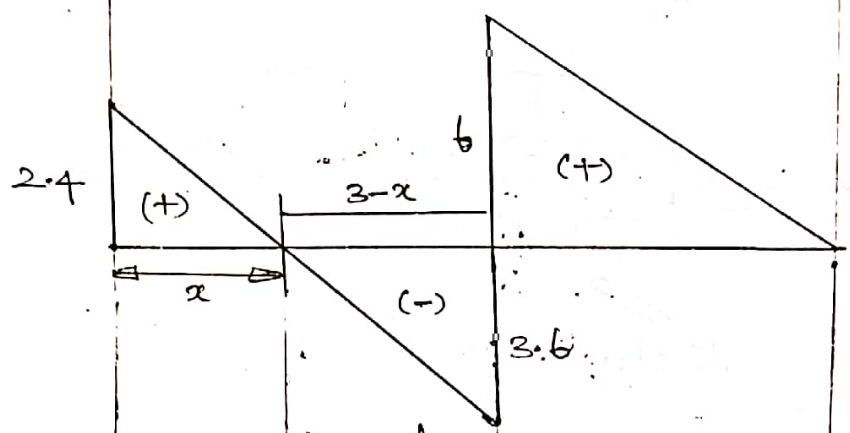


Thrust diagram.

SFD:-



$\Sigma V=0;\ R_A = 2\times 6 - P\sin\theta = 2.4\,kN$



Maximum shear force = 6kN

$$\frac{x}{2.4} = \frac{3-x}{3.6} \Rightarrow 3.6x = 7.2 - 2.4x$$

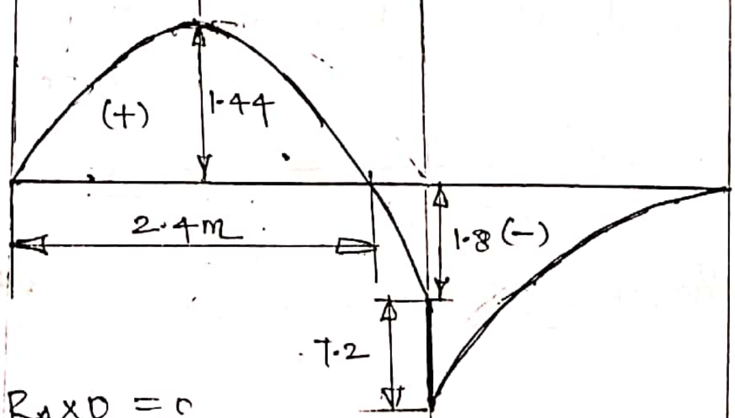
$$\boxed{x = 1.2\,m}$$

(or)

For max^m BM, $V_{xx}=0;\ R_A - 2x = 0$

$$\boxed{x = 1.2\,m}$$

BMD:-



$M_A = R_A \times 0 = 0$

$M_{xx} = R_A \times x - \frac{2x^2}{2} = 1.44\,kN\text{-}m$

$M_{A-B} = R_A \times 3 - \frac{2\times 3^2}{2} = -1.8\,kN\text{-}m$

$M_B = -1.8 - 7.2 = -9$ kN-m.

$M_C = 0$

For zero BM, $M_{xx} = 0$

$$R_A x - \frac{2x^2}{2} = 0$$

$$2.4x - x^2 = 0.$$

$$x = 0, 2.4\,m.$$

$\boxed{x = 2.4\,m}$ — point of inflection.

3. A structural member is shown in figure below. Determi

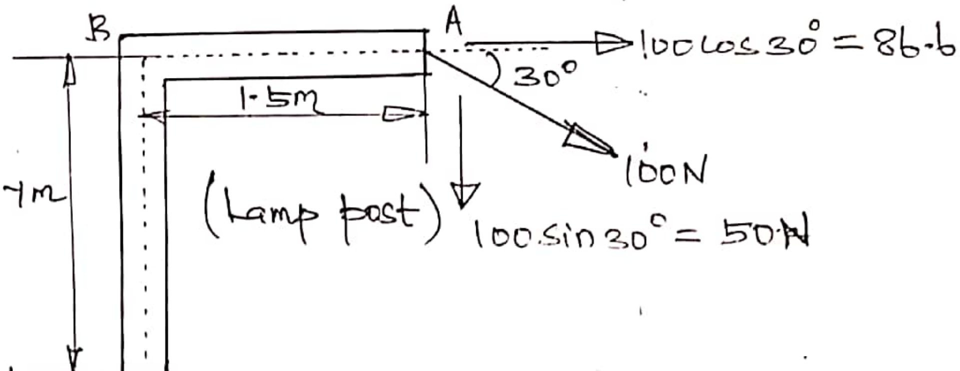
(i) Maximum shear force for the beam,

(ii) Maximum BM for the beam,

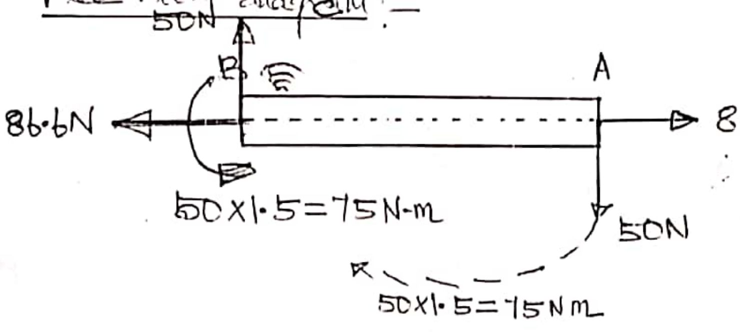
(iii) Maximum thrust in the beam,

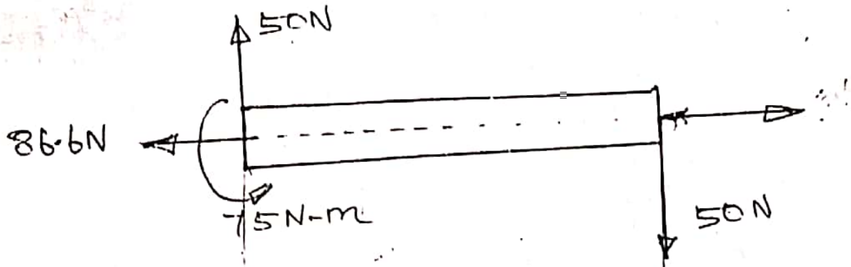
(iv) Maximum BM & thrust in the column,

(V) SFD & BMD & thrust diagram.

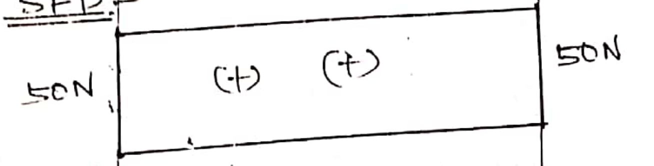


Free body diagram:

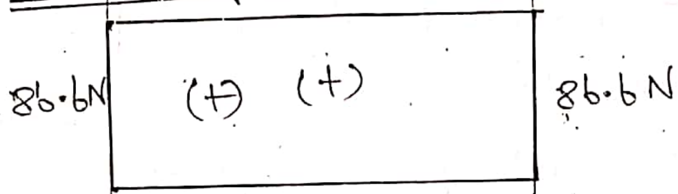




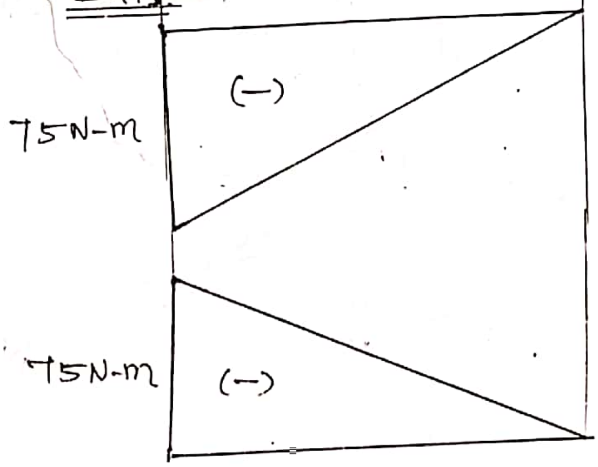
SFD:-

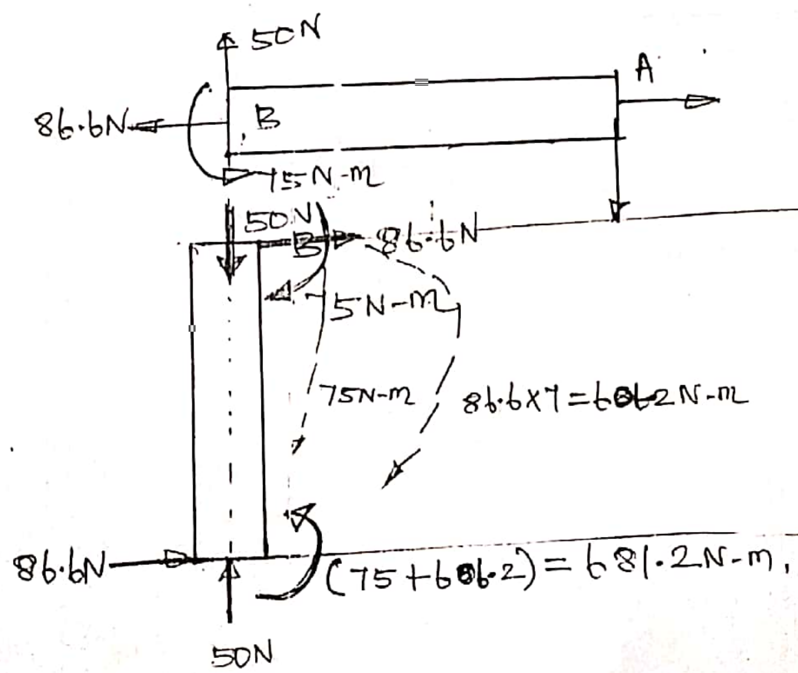


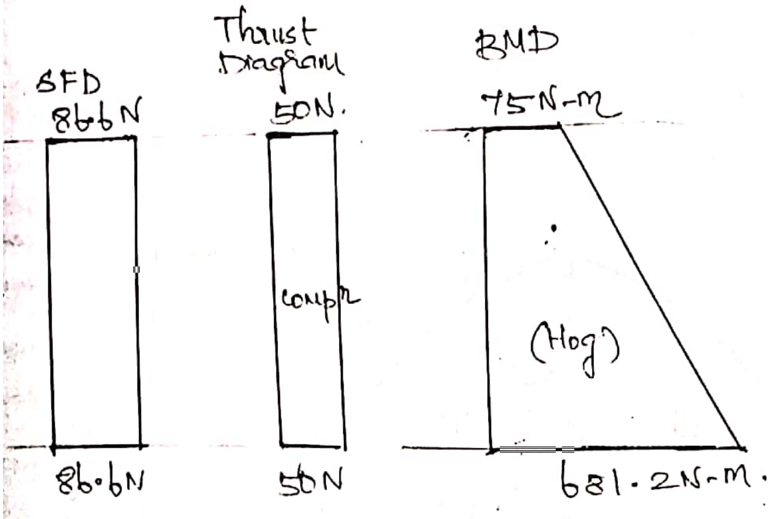
Thrust diagram:-



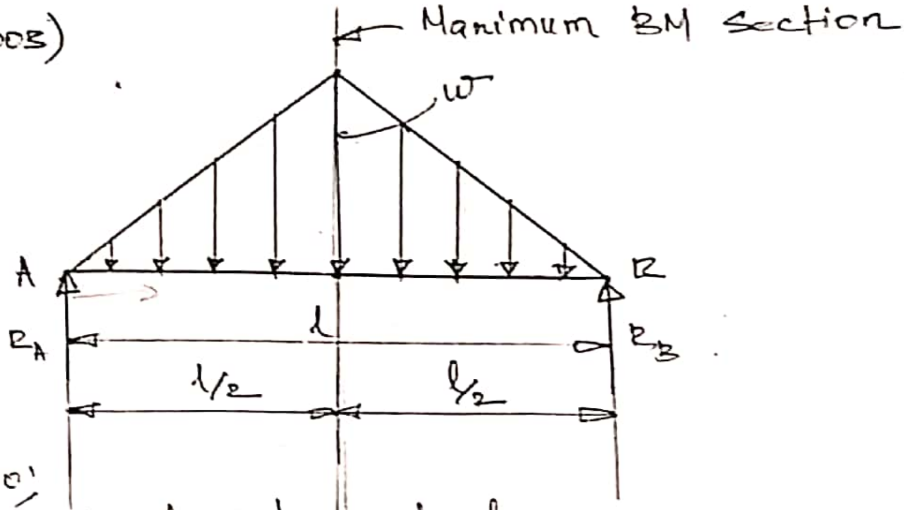
BMD:-






SFD
86.6 N
86.6N
Thrust Diagram
50N.
compr
50N
BMD
75 N-m
(Hog)
681.2N-m.

4. (IIT-M – 2003)

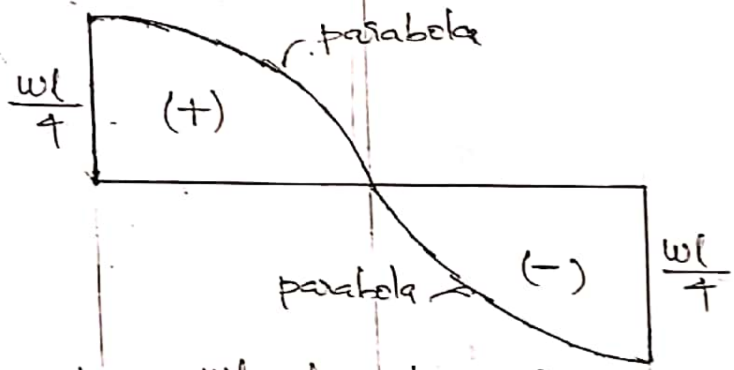


$\Sigma M_B = 0$;

$$R_A \times l = \frac{1}{2} \times w \times l \times \frac{l}{2}$$

$$R_A = \frac{wl}{4} ; \quad R_B = \frac{wl}{4}$$

SFD:-

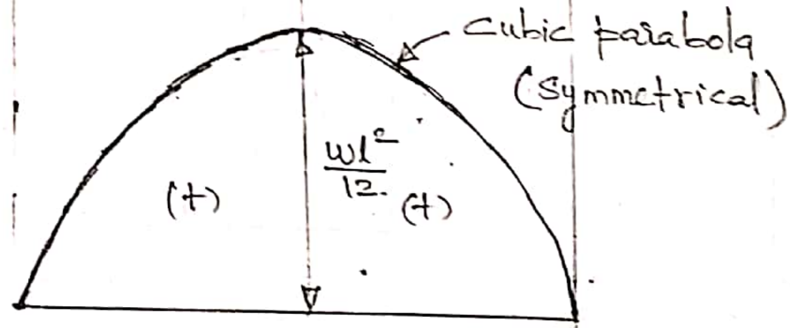


$$M_{max} = M_{mid} = \frac{wl}{4} \times \frac{l}{2} - \frac{1}{2} \times w \times \frac{l}{2} \times \frac{1}{3} \times \frac{l}{2}$$

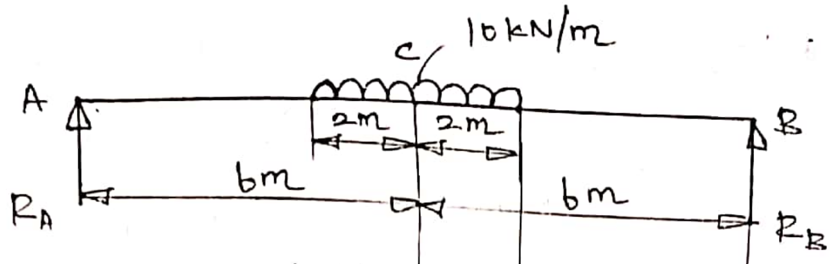
$$= \frac{wl^2}{8} - \frac{wl^2}{24}$$

$$M_{max} = \frac{wl^2}{12}$$

BMD:-



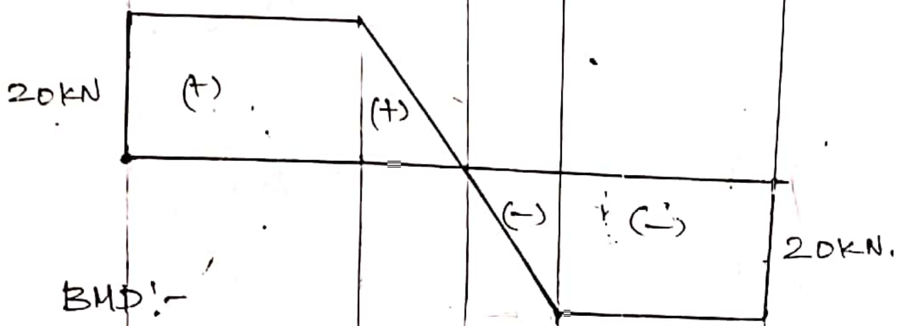
5.



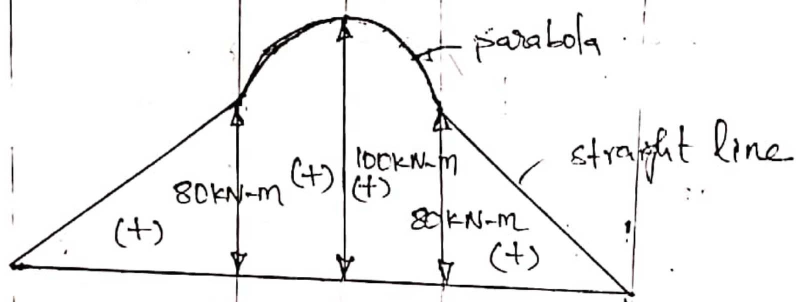
$$R_A = R_B = \frac{10 \times 4}{2} = 20\,kN$$

$$M_{max} = M_{mid} = 20 \times 6 - \frac{10 \times 2^2}{2} = 100\,kN\text{-}m$$

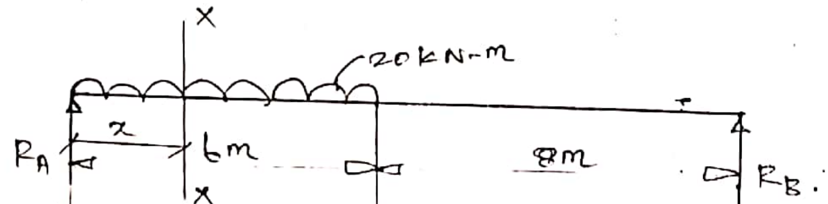
SFD:-



BMD:-



6.

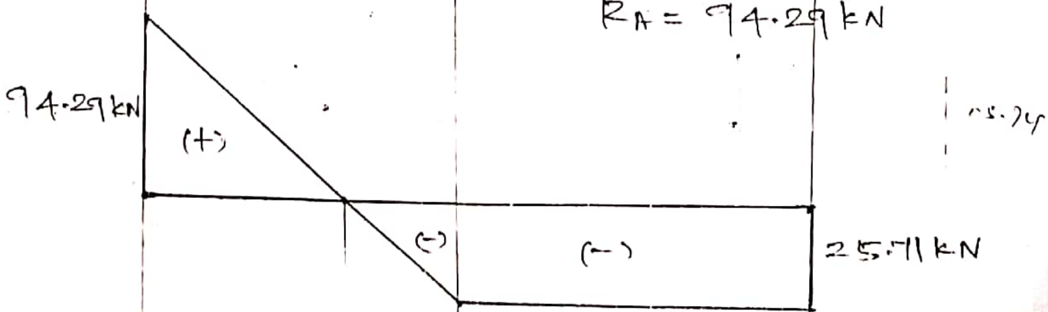


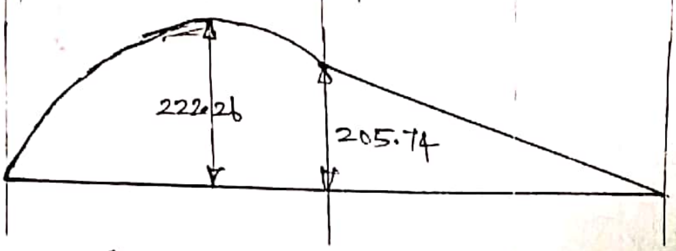
$\Sigma M_A = 0$

$$R_B \times 14 - \frac{20 \times 6^2}{2} = 0$$

$R_B = 25.71\,kN$

$R_A = 94.29\,kN$





$V_{xx} = 0;$ $\quad R_A - 20x = 0$

$$x = \frac{94.29}{20}$$

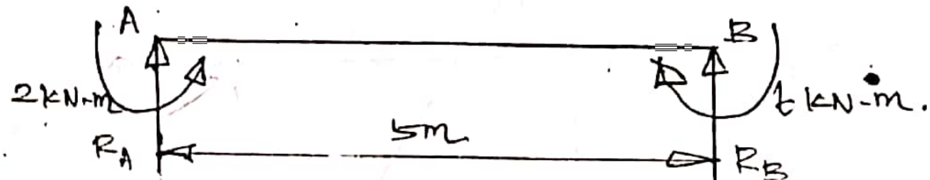
$$x = 4.71 m.$$

$$M_{max} = M_{x-x} = 94.29 \times x - 20 \times \frac{x^2}{2}$$

$$= 94.29 \times 4.71 - 20 \times \frac{4.71^2}{2}$$

$$= 222.26 kN\text{-}m$$

7. (HW)

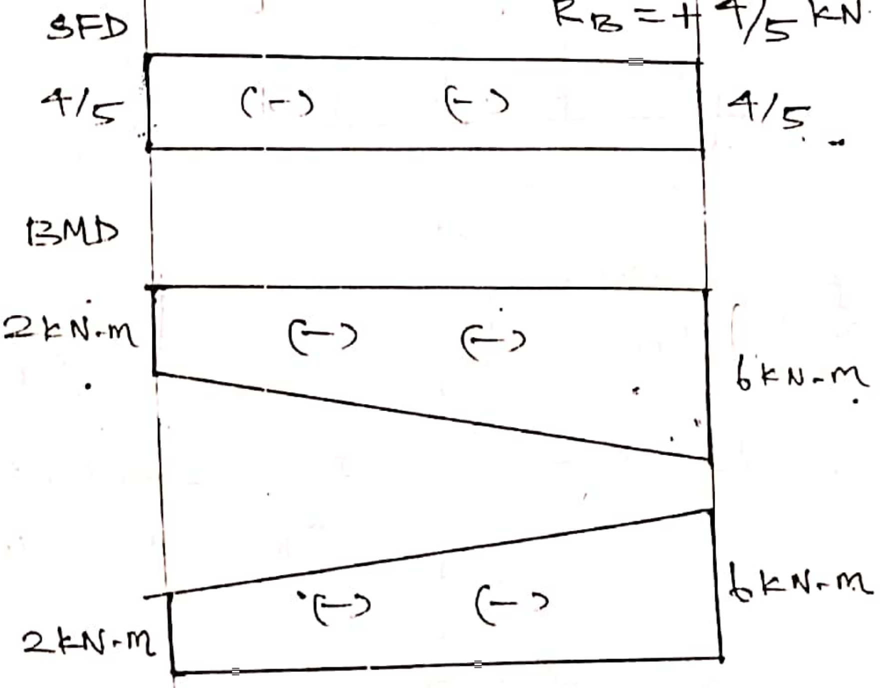


$R_A + R_B = 0;$

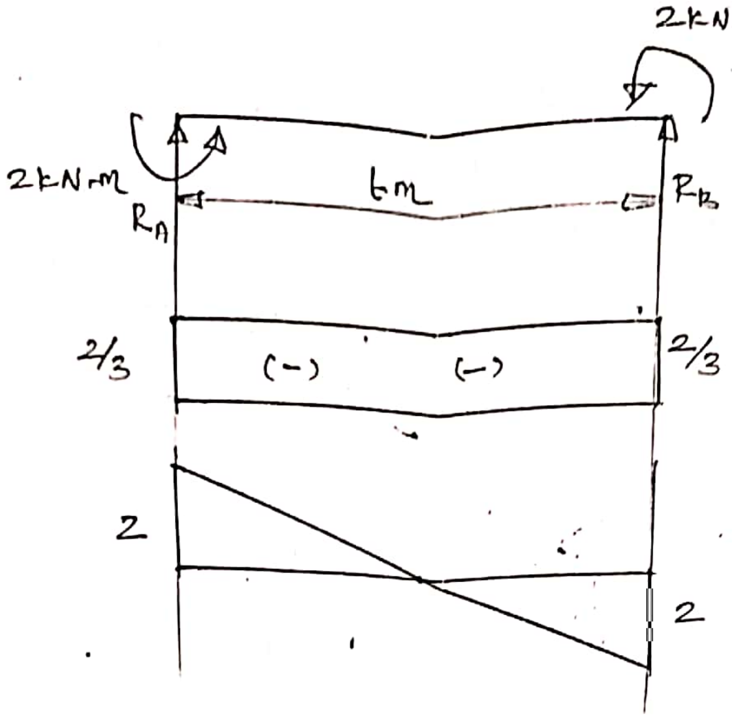
$\Sigma M_B = 0;$ $\quad R_A \times 5 - 2 + 6 = 0.$

$R_A = -4/5 \; kN$

$R_B = +4/5 \; kN$



8.



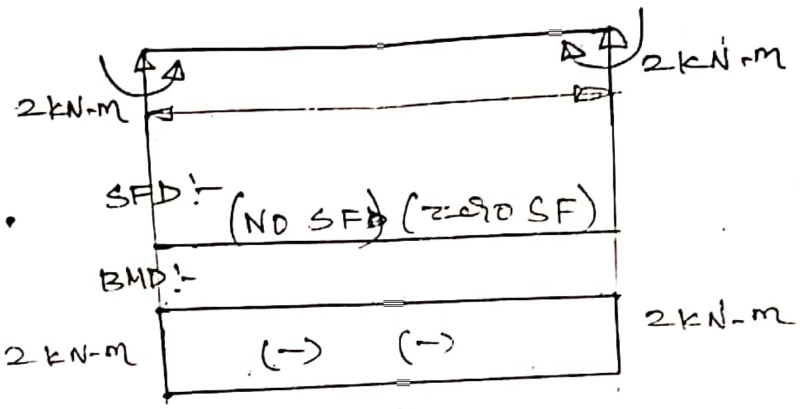
$R_A+R_B=0.$

$\Sigma M_B=0;$

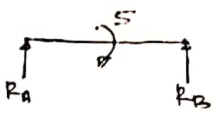
$R_A\times6=2+2$

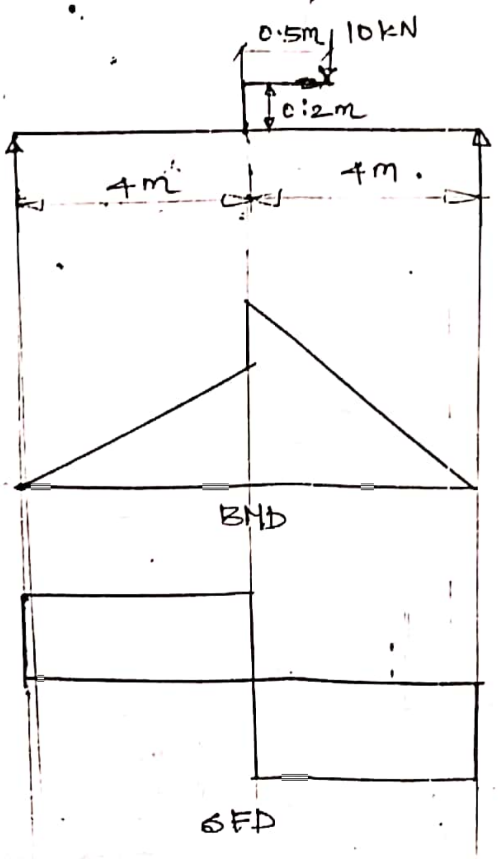
$R_A=2/3$

9.



10.





$R_A+R_B=10$

$R_A\times8=10\times3.5-10\times0.5$

$R_A=3.75$