**KARPAGAM ACADEMY OF HIGHER EDUCATION**

**(Deemed to be University)**

**(Established Under Section 3 of UGC Act 1956)**

Coimbatore- 641 021

(For the candidates admitted from 2016 onwards)

**DEPARTMENT OF CIVIL ENGINEERING**

**SUBJECT CODE: 16BECE303** **SUBJECT: MECHANICS OF FLUIDS**

**SEMESTER:III** **CLASS: II Civil Engineering** **L T P C = 3 0 0 3**

## Course Objectives

- To understand the properties of fluids and fluid statics.
- To solve kinematic problems such as finding particle paths and stream lines.
- To use important concepts of continuity equation, Bernoulli's equation and turbulence, and apply the same to problems.
- To study about specific speed and performance characteristics of different types of turbines.
- To study types of centrifugal Pumps, work done and efficiency of the different types centrifugal pumps and also study about performance of pumps & characteristic curves.

**UNIT I** **9**

**TYPES AND PROPERTIES OF FLUIDS:** Introduction– Types of fluids- Basic properties – calculation of Viscosity, compressibility, surface tension.

**Fluid statics**: Fluid pressure-various methods of measurement. Total pressure and centre of pressure – determination on plane surface only – Equilibrium of floating bodies – conditions and analysis.

**UNIT II** **9**

**KINEMATICS OF FLUID FLOW:** Classification of fluid flow – stream function and velocity potential – (Reynolds number and its application) - Linear acceleration and constant rotation of fluids in a container – application and simple problems.

**UNIT III** **9**

**DYNAMICS OF FLUID FLOW:** Euler's equation of motion – Bernoulli's theorem – Limitation of Bernoulli's theorem – Application – simple problems. Venturimeter – Flow nozzle meter – Bend meter – Pitot tube – current meter.

**UNIT IV** **9**

**FLOW THROUGH PIPES:** Laminar and Turbulent flow – friction and minor losses (Study of Moody's diagram).Transmission of power through pipes – flow between reservoirs – parallel, series and siphon pipes – water hammer.

**UNIT V** **9**

**DIMENSIONAL AND MODEL ANALYSIS**

Dimensional Homogeneity – Need – Rayleigh's method &Buckingham'sPitheorem – Significance of dimensionless numbers-Reynolds number, Froude number, Euler's number, Mach number and Weber number – Distorted models – Scale effect

**TOTAL: 45HRS**

**TEXT BOOKS:**

| S.No | Title of the Book | Author of the Book | Publisher | Year of Publishing |
|---|---|---|---|---|
| 1 | Text book of Fluid Mechanics and Hydraulic Machines | Bansal. R.K | Lakshmi Publications, Madras | 2005 |

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|---|---|---|---|---|
| 1 | Fluid Mechanics & Hydraulic Machines | R K Rajput | M/s.S.Chand Co., Madras | 2008 |
| 2 | Fluid Mechanics, Hydraulics & Fluid Machinery | Ramamrutham.S | M/s.Dhanpatrai& Sons, New Delhi | 2006 |
| 3 | Fluid Mechanics, Hydraulics and Hydraulic machines | Arora K.R | Standard Publishers Distributors, New Delhi | 2011 |

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**DEPARTMENT OF CIVIL ENGINEERING**

**LECTURE PLAN**

**MECHANICS OF FLUIDS (16BECE303)**

**LECTURER : Mr.S. Velmurugan**
**SEMESTER : III (2017-2018)ODD**
**NUMBER OF CREDITS : 3**
**COURSE TYPE : Regular Course**

| S.No | Hours | Topics to be Covered | Text Book | Page No. |
|---|---|---|---|---|
| **UNIT I- TYPES AND PROPERTIES OF FLUIDS** | | | | |
| 1. | 1 | Introduction- Types of fluids | T1 | 1 |
| 2. | 1 | Basic properties – calculation of Viscosity, | T1 | 1,3 |
| 3. | 1 | Compressibility, surface tension. | T1 | 2 |
| 4. | 1 | Fluid pressure-various methods of measurement | T1 | 35 |
| 5. | 1 | Total pressure and centre of pressure | T1 | 37 |
| 6. | 1 | Total pressure and centre of pressure | T1 | 37 |
| 7. | 1 | Total pressure and centre of pressure | T1 | 37 |
| 8. | 1 | determination on plane surface only | T1 | 39 |
| 9. | 1 | Equilibrium of floating bodies – conditions and analysis. | T1 | 52 |
| | | | | **Total 09 hours** |
| **UNIT II- KINEMATICS OF FLUID FLOW** | | | | |
| 10. | 1 | Classification of fluid flow | T1 | 65 |
| 11. | 1 | Stream function and velocity potential | T1 | 68 |
| 12. | 1 | Reynolds number and its application | T1 | 73 |
| 13. | 1 | Linear acceleration and constant rotation of fluids in a container | T1 | 85 |
| 14. | 1 | application and simple problems | T1 | 88 |
| 15. | 1 | Problems | T1 | 92 |
| 16. | 1 | Problems | T1 | 93 |
| 17. | 1 | Problems | T1 | 93 |
| 18. | 1 | Problems | T1 | 93 |
| | | | | **Total 09 hours** |
| **UNIT III-DYNAMICS OF FLUID FLOW** | | | | |
| 19. | 1 | Euler’s equation of motion | T1 | 174 |
| 20. | 1 | Bernoulli’s theorem | T1 | 183 |
| 21. | 1 | Limitation of Bernoulli’s theorem | T1 | 177 |
| 22. | 1 | Application of Bernoulli’s theorem | T1 | 189 |
| 23. | 1 | simple problems | T1 | 193 |
| 24. | 1 | Venturimeter | T1 | 189 |
| 25. | 1 | Flow nozzle meter | T1 | 199 |
| 26. | 1 | Bend meter | T1 | 210 |
| 27. | 1 | Pitot tube – current meter | T1 | 212 |
| | | | | **Total 09 hours** |
| **UNIT IV- FLOW THROUGH PIPES** | | | | |

| | | | | |
|---|---|---|---|---|
| 28. | 1 | Laminar and Turbulent flow | T1 | 354 |
| 29. | 1 | Friction and minor losses | T1 | 367 |
| 30. | 1 | Friction and minor losses | T1 | 398 |
| 31. | 1 | Transmission of power through pipes | T1 | 376 |
| 32. | 1 | Flow between reservoirs | T1 | 379 |
| 33. | 1 | Flow between reservoirs | T1 | 389 |
| 34. | 1 | Parallel, series and siphon pipes | T1 | 397 |
| 35. | 1 | Parallel, series and siphon pipes | T1 | 397 |
| 36. | 1 | Water hammer. | T1 | 402 |
| | | | | **Total 09 hours** |
| | | **UNIT V- DIMENSIONAL AND MODEL ANALYSIS** | | |
| 37. | 1 | Dimensional Homogeneity | T1 | 599-610 |
| 38. | 1 | Rayleigh's method | T1 | 599-610 |
| 39. | 1 | Buckingham'sPitheorem | T1 | 599-610 |
| 40. | 1 | Significance of dimensionless numbers | T1 | 599-610 |
| 41. | 1 | Reynolds number | T1 | 599-610 |
| 42. | 1 | Froude number | T1 | 599-610 |
| 43. | 1 | Euler's number | T1 | 599-610 |
| 44. | 1 | Mach number and Weber number | T1 | 599-610 |
| 45. | 1 | Distorted models and Scale effect | T1 | 599-610 |
| | | | | **Total 09 hours** |
| | | | | **Total 45 hours** |

**TEXT BOOKS:**

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|---|---|---|---|---|
| 1 | Text book of Fluid Mechanics and Hydraulic Machines | Bansal. R.K | Lakshmi Publications, Madras | 2005 |

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# CE 6303 MECHANICS OF FLUIDS

## UNIT I FLUID PROPERTIES AND FLUID STATICS

Fluid – definition, distinction between solid and fluid - Units and dimensions - Properties of fluids -density, specific weight, specific volume, specific gravity, temperature, viscosity, compressibility, vapour pressure, capillarity and surface tension - Fluid statics: concept of fluid static pressure, absolute and gauge pressures - pressure measurements by manometers and pressure gauges- forces on planes – centre of pressure – bouncy and floatation.

## INTRODUCTION TO FLUIDS

***Definition***

There are three states of matter: solids, liquids and gases.

Both liquids and gases are classified as fluids.

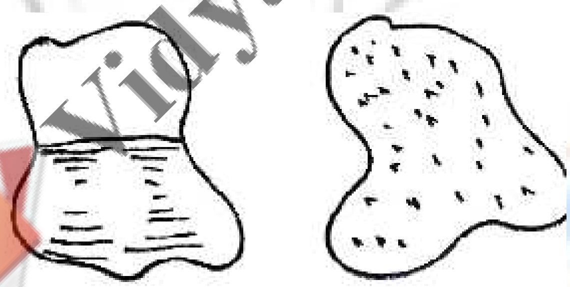
Fluids do not resist a change in shape. Therefore fluids assume the shape of the container they occupy.

Liquids may be considered to have a fixed volume and therefore can have a free surface.

Liquids are almost incompressible.

Conversely, gases are easily compressed and will expand to fill a container they occupy.

We will usually be interested in liquids, either at rest or in motion.

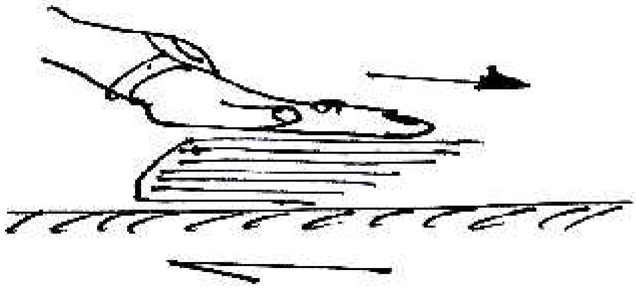


Liquid showing free surface Gas filling volume

**Definition**

The strict definition of a fluid is: *A fluid is a substance which conforms continuously under the action of shearing forces.*

To understand this, remind ourselves of what a shear force is:



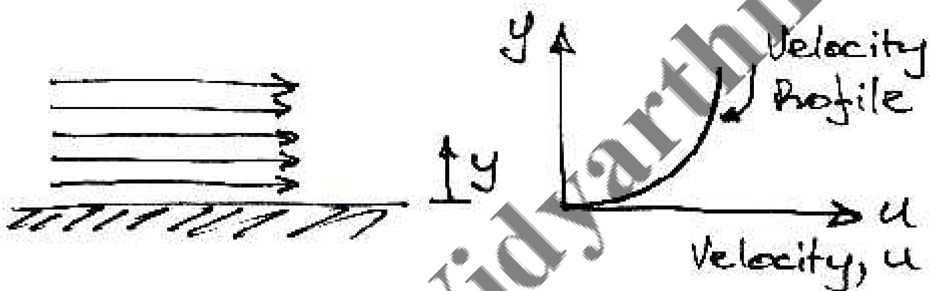
**Application and effect of shear force on a book**

**Definition Applied to Static Fluids**

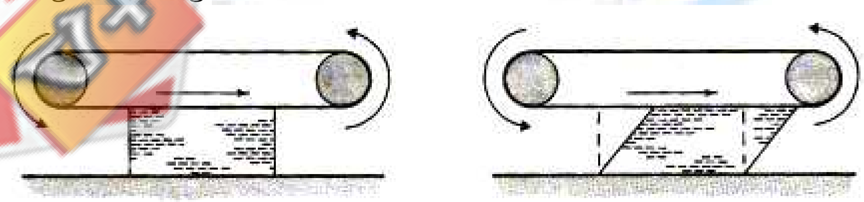
According to this definition, if we apply a shear force to a fluid it will deform and take up a state in which no shear force exists. Therefore, we can say: *If a fluid is at rest there can be no shearing forces acting and therefore all forces in the fluid must be perpendicular to the planes in which they act.* Note here that we specify that the fluid must be at rest. This is because, it is found experimentally that fluids in motion can have slight resistance to shear force. This is the source of *viscosity*.

**Definition Applied to Fluids in Motion**

For example, consider the fluid shown flowing along a fixed surface. At the surfacethere will be little movement of the fluid (it will 'stick' to the surface), whilst furtheraway from the surface the fluid flows faster (has greater velocity):




If one layer of is moving faster than another layer of fluid, there must be shear forcesacting between them. For example, if we have fluid in contact with a conveyor beltthat is moving we will get the behaviour shown:



**Ideal fluid** **Real (Viscous) Fluid**

When fluid is in motion, any difference in velocity between adjacent layers has the same effect as the conveyor belt does.
Therefore, to represent real fluids in motion we must consider the action of shear forces
Consider the small element of fluid shown, which is subject to shear force and has a dimension $s$into the page. The force $F$ acts over an area $A$ = BC×$s$. Hence we have a *shear stress* applied:

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

$$\tau = \frac{F}{A}$$

Any stress causes a deformation, or strain, and a shear stress causes a *shear strain*. This shear strain is measured by the angle φ . Remember that a fluid *continuously* deforms when under the action of shear. This is different to a solid: a solid has a single value of φ for each value of τ . So the longer a shear stress is applied to a fluid, the more shear strain occurs. However, what is known from experiments is that the rate of shear strain (shear strain per unit time) is related to the shear stress:

$$\text{Shear stress} \propto \text{Rate of shear strain}$$

$$\text{Shear stress} = \text{Constant} \times \text{Rate of shear strain}$$

We need to know the rate of shear strain. From the diagram, the shear strain is:

$$\phi = \frac{x}{y}$$

If we suppose that the particle of fluid at *E* moves a distance *x* in time *t*, then, using $S = R\theta$ for small angles, the rate of shear strain is:

$$\frac{\Delta\phi}{\Delta t} = \left(\frac{x}{y}\right)\Big/ t = \frac{x}{t} \cdot \frac{1}{y}$$

$$= \frac{u}{y}$$

Where *u* is the velocity of the fluid. This term is also the change in velocity with height. When we consider infinitesimally small changes in height we can write this in differential form, *du/ dy* . Therefore we have:

$$\tau = \text{constant} \times \frac{du}{dy}$$

*Newton's Law of Viscosity*:

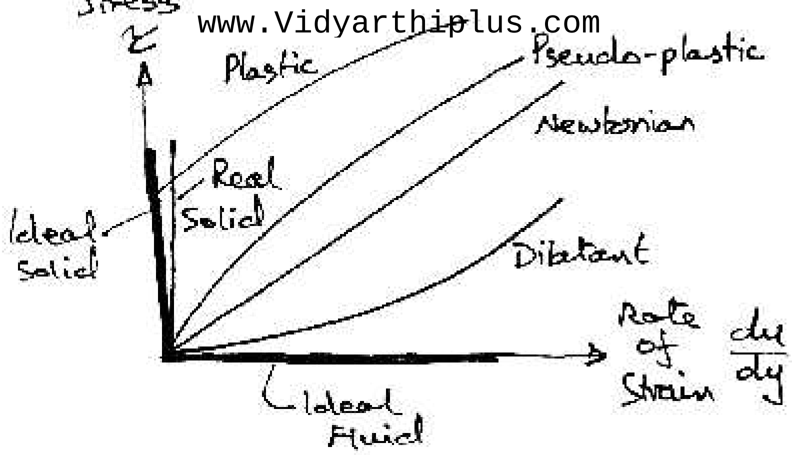
$$\tau = \mu \frac{du}{dy}$$

**Generalized Laws of Viscosit**

We have derived a law for the behaviour of fluids – that of *Newtonian fluids*. However, experiments show that there are *non-Newtonian* fluids that follow a *generalized law of viscosity*:

$$\tau = A + B\left(\frac{du}{dy}\right)^n$$

Where *A*, *B* and *n* are constants found experimentally. When plotted these fluids show much different behaviour to a Newtonian fluid:




**Behaviour of Fluids and Solids**

In this graph the Newtonian fluid is represent by a straight line, the slope of which is μ . Some of the other fluids are:

*Plastic:* Shear stress must reach a certain minimum before flow commences.

*Pseudo-plastic:* No minimum shear stress necessary and the viscosity decreases with rate of shear, e.g. substances like clay, milk and cement.

*Dilatant substances;* Viscosity increases with rate of shear, e.g. quicksand.

*Viscoelastic materials:* Similar to Newtonian but if there is a sudden large change in shear they behave like plastic.

*Solids:* Real solids do have a slight change of shear strain with time, whereas ideal solids (those we idealise for our theories) do not. Lastly, we also consider the *ideal fluid*. This is a fluid which is assumed to have no viscosity and is very useful for developing theoretical solutions. It helps achieve some practically useful solutions.

***Properties***

**Further Reading**

Here we consider only the relevant properties of fluids for our purposes. Find out about surface tension and capillary action elsewhere. Note that capillary action only features in pipes of ≤ 10 mm diameter.

**FLUID PROPERTIES:**

**1. Density or Mass density:** Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a is called density.

$$\textit{Mass density} = \frac{\textit{Mass of fluid}}{\textit{Density of fluid}}$$

The unit of density in S.I. unit is kg/m$^3$. The value of density for water is 1000kg/m$^3$.

**2.Specific weight or weight density:** Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. The weight per unit volume of a fluid is called weight density.

$$\text{Weight density} \quad \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$w \quad \frac{\text{Mass of fluid x } g}{\text{Volume of fluid}}$$

$$w \quad x \quad g$$

The unit of specific weight in S.I. units is $N/m^3$. The value of specific weight or weight density of water is
$9810 N/m^3$.

**3.)Specific Volume:** Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid.

$$\text{Specific volume} \quad \frac{\text{Volume of a fluid}}{\text{Mass of fluid}} \quad \frac{1}{}$$

Thus specific volume is the reciprocal of mass density. It is expressed as $m^3/kg$. It is commonly applied to gases.

**4.)Specific Gravity:** Specific gravity is defined as the ratio of the weight density of a fluid to the weight density of a standard fluid.

$$\text{Specific gravity} \quad \frac{\text{Weight density of liquid}}{\text{Weight density of water}}$$

**VISCOSITY:**

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over adjacent layer of the fluid. When two layers of a fluid, a distance 'dy' apart, move one over the other at different velocities, say u and u+du as shown in figure. The viscosity together with relative velocity causes a shear stress acting between the fluid layers.

**COMPRESSIBILITY:**

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Consider a cylinder fitted with a piston as shown in figure. Let V= Volume of a gas enclosed in the cylinder

P= Pressure of gas when volume is V

Let the pressure is increased to p+dp, the volume of gas decreases from V to V-dV. Then increase in pressure =dp kgf/m$^2$

Decrease in volume= dV

Volumetric Strain $= -\frac{dV}{V}$

- ve sign means the volume decreases with increase of pressure.

$$\text{Bulk modulus } K = \frac{\text{Increase of pressure}}{\text{Volumetric Strain}}$$

$$= -\frac{dp}{\frac{dV}{V}}$$

$$\text{Compressibility is given by} = \frac{1}{K}$$

**Relationship between K and pressure (p) for a Gas:**

The relationship between bulk modulus of elasticity (K) and pressure for a gas for two different processes of comparison are as:

(i) **For Isothermal Process:** The relationship between pressure (p) and density (ρ) of a gas as

$\frac{p}{}$ = Constant

$V$ = Constant

Differentiating this equation, we get (p and V are variables)

PdV +Vdp = 0 or pdV= - Vdp or p= $\frac{\tilde{} Vdp}{dV}$

Substituting this value K =p

**(ii) For adiabatic process**. For adiabatic process

$\frac{p}{}$ Constant or $pV^k$ = Constant

**SURFACE TENSION:**

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two two immiscible liquids such that the contact surface behaves like a membrane under tension

**CAPILLARITY :**

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

**Problem 1.**

**Calculate the capillary effect in millimeters a glass tube of 4mm diameter, when immersed in (a) water (b) mercury. The temperature of the liquid is $20^0$ C and the values of the surface tension of water and mercury at $20^0$ C in contact with air are 0.073575 and 0.51 N/m respectively. The angle of contact for water is zero that for mercury $130^0$. Take specific weight of water as 9790 N / $m^3$**

Given:

Diameter of tube $\Rightarrow$ d = 4 mm = $4 \times 10^{-3} m$

Capillary effect (rise or depression) $\Rightarrow$ $h = \frac{4\pm \cos^{-}}{p \times g \times d}$

$\pm$ = Surface tension in kg f/m

$\theta$ = Angle of contact and p = density

### i. Capillary effect for water

$\sigma = 0.073575\ N/m, \quad \theta = 0^0$

$p = 998\ kg/m^3\ @\ 20^0 c$

$$h = \frac{4\times 0.73575\times Cos0^0}{998\times 9.81\times 4\times 10^{-3}} = 7.51\times 10^{-3} m$$

= 7.51 mm.

**Capillary effect for mercury:**

$\sigma = 0.51\,N/m, \qquad \theta = 130^0$

$p = sp\ gr\times 1000 = 13.6\times 1000 = 13600\,kg/m^3$

$$h = \frac{4\times 0.51\times Cos130^0}{13600\times 9.81\times 4\times 10^{-3}}$$

$= -2.46\times 10^{-3}$ m

= - 2.46 mm.

-Ve indicates capillary depression.



**Problem 2.**

**A cylinder of 0.6 m$^3$ in volume contains air at 50$^0$C and 0.3 N/ mm$^2$ absolute pressure. The air is compressed to 0.3 m$^3$. Find (i) pressure inside the cylinder assuming isothermal process (ii) pressure and temperature assuming adiabatic process. Take K = 1.4**

Given:

Initial volume $\forall_1 = 0.36\,m^3$

Pressure $P_1$ = 0.3 N/mm$^2$

$$= 0.3\times 10^6\ N/m^2$$

Temperature, $t_1$ = 50$^0$ C

$T_1 = 273 + 50 = 323\,^0 K$

Final volume, $\forall_2 = 0.3 m^3$

$K = 1.4$

**i. Isothermal Process:**

$$\frac{P}{p} = Cons\tan t \quad (or)\ p\forall = Cons\tan t$$

$$p_1\forall_1 = p_2\forall_2$$

$$p_2 = \frac{p_1\forall_1}{\forall_2} = \frac{30\times 10^4 \times 0.6}{0.3} = 0.6\times 10^6\ N/m^2$$

$$= 0.6\ N/mm^2$$

**ii. Adiabatic Process:**

$$\frac{p}{p^K} = Cons\tan t \quad or$$

$$\boxed{p\forall^K = cons\tan t}$$

$$p_1.\forall_1^K = p_2\forall_2^K$$

$$p_2 = p_1\frac{\forall_1 K}{\forall_2 K} = 30\times 10^4 \times \left(\frac{0.6}{0.3}\right)^{1.4} = 30\times 10^4 \times 2^{1.4}$$

$$= 0.791\times 10^6\ N/m^2 = 0.791 N/mm^2$$

**For temperature,** $p\forall = RT$, $p\forall^k = cons\tan t$

$$p = \frac{RT}{\forall} \text{ and } \frac{RT}{\forall}\times\forall^k = cons\tan t$$

$$RT\forall^{k-1} = Cons\tan t$$

$$T\forall^{k-1} = Cons\tan t \qquad (\because R \text{ is also } cons\tan t)$$

$$T_1V_1^{k-1} = T_2V_2^{k-1}$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{k-1} = 323 \quad \left(\frac{0.6}{0.3}\right)^{1.4-1.0}$$

$$= 323 \times 2^{0.4} = 426.2^0\,K$$

$$t_2 = 426.2 - 273 = 153.2^0\,C$$

**<u>Problem 3</u>**

**If the velocity profile of a fluid over a plate is a parabolic with the vertex 202 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stress at a distance of 0,10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.**

Given,

Distance of vertex from plate = 20 cm.

Velocity at vertex, u = 120 cm / sec.

Viscosity, $« = 8.5\,poise = \frac{8.5}{10}\frac{Ns}{m^2} = 0.85$

Parabolic velocity profile equation, $u = ay^2 + by + C$ -------------- (1)

Where, a, b and c constants. Their values are determined from boundary conditions.

i) At y = 0, u = 0

ii) At y = 20cm, u = 120 cm/se.

iii) At y = 20 cm, $\frac{du}{dy} = 0$

Substituting (i) in equation (1), C = 0

Substituting (ii) in equation (1), $120 = a(20)^2 + b(2) = 400a + 20b$ -----------(2)

Substituting (iii) in equation (1), $\frac{du}{dy} = 2ay + b$

$$0 = 2 \times a \times 20 + b = 40a + b$$ -----------(3)

solving 1 and 2, we get,

$$400\,a + 20\,b = 0$$

(−)

$$800\,a + 20\,b = 0$$

40 a + b = 0

b = - 40 a

$$120 = 400\,a + 20\,b(-40\,a) = 400\,a - 800\,a = -400\,a$$

$$a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$b = -40 \times (-0.3) = 1.2$$

Substituting a, b and c in equation (i) $u = -0.3y^2 + 12y$

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

**Velocity gradient**

at y = 0, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 = 12/s.$

at y =10 cm, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 = 6/s.$

at y = 20 cm, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 = 0$

**Shear Stresses:**

Shear stresses is given by, $2 = « \frac{du}{dy}$

i. Shear stress at y = 0 , $2 = « \left(\frac{du}{dy}\right)_{y=0} = 0.85 \times 12.0 = 10.2 N/m^2$

ii. Shear stress at y = 10 , $2 = « \left(\frac{du}{dy}\right)_{y=10} = 0.85 \times 6.0 = 5.1 N/m^2$

iii. Shear stress at y = 20 , $2 = « \left(\frac{du}{dy}\right)_{y=20} = 0.85 \times 0 = 0$



**Problem 4**

**A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. Both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 rpm determine the viscosity of the fluid.**

Solution:

Diameter of cylinder = 15 cm = 0.15 m

Diameter of outer cylinder = 15.10 cm = 0.151 m

Length of cylinder ⇒ L = 25 cm = 0.25 m

Torque T= 12 Nm ; N = 100 rpm.

Viscosity = μ

Tangential velocity of cylinder $u = \frac{® \; DN}{60} = \frac{® \times 0.15 \times 100}{60} = 0.7854 \quad m/s$

Surface area of cylinder $A = ®D \times L = ® \times 0.15 \times 0.25$

= $0.1178 \; m^2$

$2 = « \frac{du}{dy}$ $\qquad du = u - 0 = u = 0.7854 \; m/s$

$$dy = \frac{0.151 - 0.150}{2} = 0.0005 \; m$$

$$2 = \frac{« \times 0.7854}{0.0005}$$

Shear force, $F = Shear \; Stress \times Area = \frac{« \times 0.7854}{0.0005} \times 0.1178$

Torque $T = F \times \frac{D}{2}$

$$12.0 = \frac{« \times 0.7854}{0.0005} \times 0.1178 \times \frac{0.15}{2}$$

$$« = \frac{12.0 \times 0.0005 \times 2}{0.7854 \times 0.1178 \times 0.15} = 0.864 Ns/m^2$$

$$« = 0.864 \times 10 = 8.64 \; poise.$$

**Problem 5**

**The dynamic viscosity of oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.**

Given, $\mu = 6\,poise = \frac{6}{10}\frac{Ns}{m^2} = 0.6\frac{Ns}{m^2}$

D = 0.4 m $L = 90mm = 90 \times 10^{-3}\, m$

N = 190 rpm. $t = 1.5mm = 1.5 \times 10^{-3}\, m$

$$Power = \frac{2\pi\, NT}{60} W$$

$$T = force \times \frac{D}{2} Nm.$$

$$F = Shear\ stress \times Area \quad = \tau \times \pi DL$$

$$\tau = \mu \frac{du}{dy} N/m^2$$

$$u = \frac{\pi\, DN}{60} m/s.$$

Tangential Velocity of shaft, $u = \frac{\pi\, DN}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98\, m/s.$

du = change of velocity = u – 0 = u = 3.98 m/s.

$dy = t = 1.5 \times 10^{-3}\, m.$

$$\tau = \mu \frac{du}{dy} \Rightarrow \quad \tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 N/m^2$$

Shear force on the shaft F = Shear stress x Area

$$F = 1592 \times \pi D \times L = 1592 \times \pi \times 0.4 \times 90 \times 10^{-3} = 180.05N$$

Torque on the shaft, $T = Force \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01 \quad Ns.$

$$\text{Power lost} = \frac{2\pi NT}{60} = \frac{2\pi \times 190 \times 36.01}{60} = 716.48W$$

## Problem 6

**If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which U is the velocity in m/s at a distance y meter above the plate, determine the shear stress at y = 0 and y = 0.15 m. Take dynamic viscosity of fluid as 8.63 poise.**

Given:

$$u = \frac{2}{3}y - y^2$$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3}$$

$$\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2\times(0.17) = 0.667 - 0.30$$

$$\mu = 8.63\, poise = \frac{8.63}{10} \text{ SI units} = 0.863 \text{ Ns/m}^2$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

i. Shear stress at y = 0 is given by

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = 0.5756\ N/m^2$$

ii. Shear stress at y = 0.15 m is given by

$$(\tau)_{y=0.15} = \mu\left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = 0.3167\ N/m^2$$

**<u>Problem 7</u>**

**The diameters of a small piston and a large piston of a hydraulic jack at3cm and 10 cm respectively. A force of 80 N is applied on the small piston Find the load lifted by the large piston when:**

- **The pistons are at the same level**
- **Small piston in 40 cm above the large piston.**

**The density of the liquid in the jack in given as 1000 kg/m$^3$**

Given: Dia of small piston d = 3 cm.

∴ Area of small piston , $a = \frac{\text{®}}{4} d^2 = \frac{\text{®}}{4} \times (3)^2 = 7.068 cm^2$

Dia of large piston, D = 10 cm

∴ Area of larger piston, $A = \frac{P}{4} \times (10)^2 = 78.54 cm^2$

Force on small piston, F = 80 N

Let the load lifted = W

a. When the pistons are at the same level

Pressure intensity on small piston

$$P = \frac{F}{a} = \frac{80}{7.068} N/cm^2$$

This is transmitted equally on the large piston.

∴ Pressure intensity on the large piston $= \frac{80}{7.068}$

∴ Force on the large piston = Pressure x area

$$= = \frac{80}{7.068} \text{ x } 78.54 \text{ N} = 888.96 \text{ N.}$$

b. when the small piston is 40 cm above the large piston

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} N/cm^2$$

∴ Pressure intensity of section A – A

$= \frac{F}{a} +$ pressure intensity due of height of 40 cm of liquid. P = pgh.

But pressure intensity due to 40cm. of liquid

$$= p \times g \times h = 1000 \times 9.81 \times 0.4 N/m^2$$

$$= \frac{1000 \times 9.81 \times 0.4}{10^4} N/cm^2 = 0.3924 N/cm^2$$

∴ Pressure intensity at section

$$A - A = \frac{80}{7.068} + 0.3924$$

= 11.32 + 0.3924 = 11.71 $N/cm^2$

Pressure intensity transmitted to the large piston = 11.71 $N/cm^2$

Force on the large piston = Pressure x Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54$$

= 919. 7 N.

**FLUID STATICS**

**Pressure**

In fluids we use the term pressure to mean:

*The perpendicular force exerted by a fluid per unit area.*

This is equivalent to stress in solids, but we shall keep the term pressure.

Mathematically, because pressure may vary from place to place, we have:

$$p = \lim_{\Delta \to 0} \frac{\Delta F}{\Delta A}$$

As we saw, force per unit area is measured in N/m2 which is the same as a pascal (Pa). The units used in practice vary:

1 kPa = 1000 Pa = 1000 N/m2

1 MPa = 1000 kPa = 1 × 106 N/m2

1 bar = 105 Pa = 100 kPa = 0.1 MPa

1 atm = 101,325 Pa = 101.325 kPa = 1.01325 bars = 1013.25 millibars

For reference to pressures encountered on the street which are often imperial:

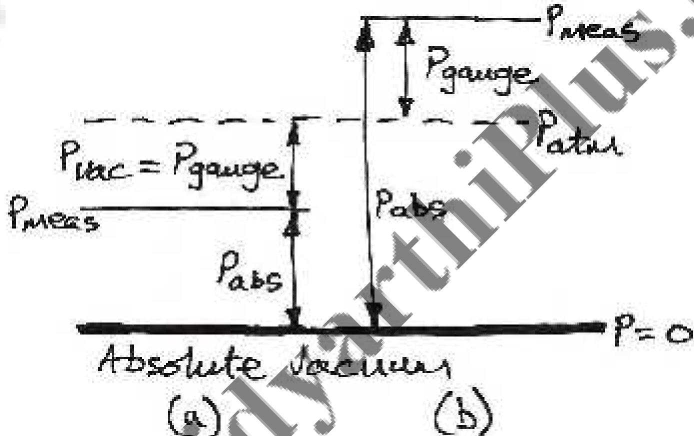
1 atm = 14.696 psi (i.e. pounds per square inch)

1 psi = 6894.7 Pa ≈ 6.89 kPa ≈ 0.007 MPa

**Pressure Reference Levels**

The pressure that exists anywhere in the universe is called the *absolute pressure*, *abs P*

.This then is the amount of pressure greater than a pure vacuum. The atmosphere onearth exerts *atmospheric pressure*, *atm P* , on everything in it. Often when measuringpressures we will calibrate the instrument to read zero in the open air. Any measuredpressure, *meas P* , is then a positive or negative deviation from atmospheric pressure.We call such deviations a *gauge pressure*, *gauge P* . Sometimes when a gauge pressureis negative it is termed a *vacuum pressure, vac P* .

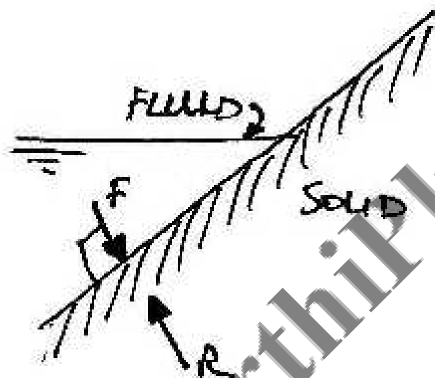


The above diagram shows:
(a) the case when the measured pressure is below atmospheric pressure and so is a negative gauge pressure or a vacuum pressure;
(b) the more usual case when the measured pressure is greater than atmospheric pressure by the gauge pressure.

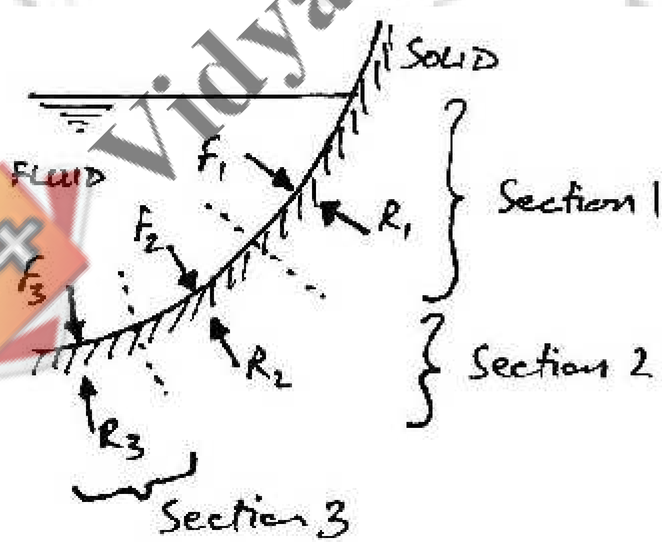
***Pressure in a Fluid***

**Statics of Definition**

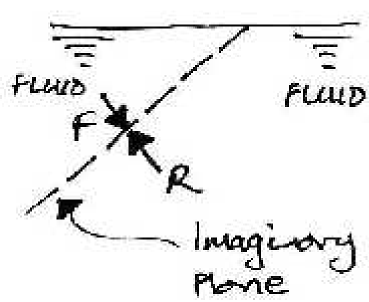
We applied the definition of a fluid to the static case previously and determined that there must
be no shear forces acting and thus only forces normal to a surface act in a
fluid. For a flat surface at arbitrary angle we have:



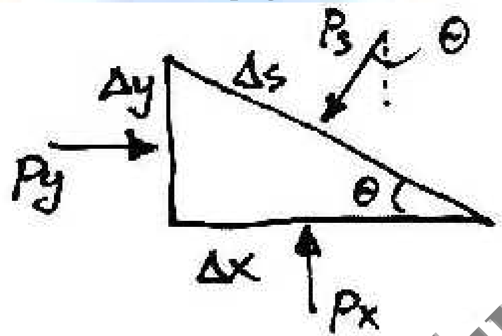

A curved surface can be examined in sections:




And we are not restricted to actual solid-fluid interfaces. We can consider imaginary planes through a fluid:



**Pascal's Law**
This law states:
*The pressure at a point in a fluid at rest is the same in all directions.*
To show this, we will consider a very small wedge of fluid surrounding the point. This wedge is unit thickness into the page:




As with all static objects the forces in the *x* and *y* directions should balance. Hence:

$$\sum F_x = 0: \qquad p_y \cdot \Delta y - p_s \cdot \Delta s \cdot \sin\theta = 0$$

But $\sin\theta = \frac{\Delta y}{\Delta s}$, therefore:

$$p_y \cdot \Delta y - p_s \cdot \Delta s \cdot \frac{\Delta y}{\Delta s} = 0$$
$$p_y \cdot \Delta y = p_s \cdot \Delta y$$
$$p_y = p_s$$

$$\sum F_y = 0: \qquad p_x \cdot \Delta x - p_s \cdot \Delta s \cdot \cos\theta = 0$$

But $\cos\theta = \frac{\Delta x}{\Delta s}$, therefore:

$$p_x \cdot \Delta x - p_s \cdot \Delta s \cdot \frac{\Delta x}{\Delta s} = 0$$
$$p_x \cdot \Delta x = p_s \cdot \Delta x$$
$$p_x = p_s$$

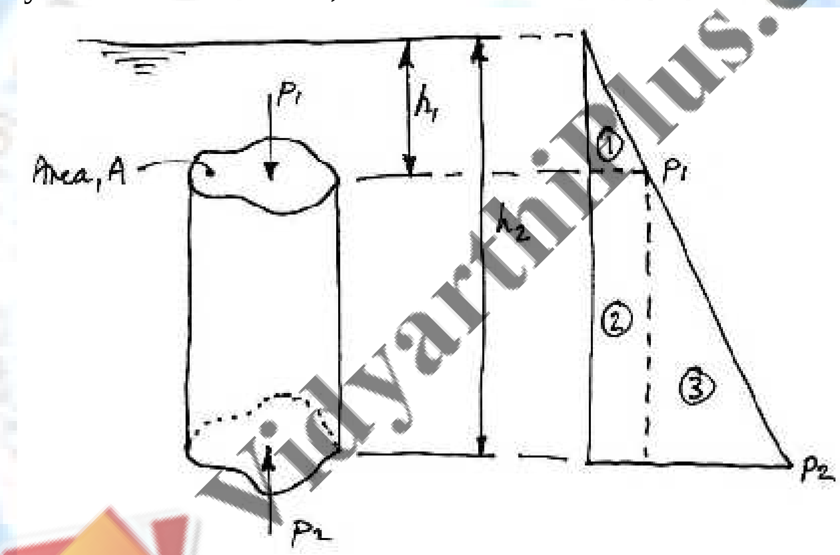
Hence for any angle:

$$p_y = p_x = p_s$$

And so the pressure at a point is the same in any direction. Note that we neglected the weight of the small wedge of fluid because it is infinitesimally small. This is why Pascal's Law is restricted to the pressure at a point.

**Pressure Variation with Depth**

Pressure in a static fluid does not change in the horizontal direction as the horizontal forces balance each other out. However, pressure in a static fluid does change with depth, due to the extra weight of fluid on top of a layer as we move down wards. Consider a column of fluid of arbitrary cross section of area, *A*:




**Column of Fluid** **Pressure Diagram**

Considering the weight of the column of water, we have:

$$\sum F_y = 0: \qquad p_1 A + \gamma A(h_2 - h_1) - p_2 A = 0$$

Obviously the area of the column cancels out: we can just consider pressures. If we say the height of the column is $h = h2 - h1$ and substitute in for the specific weight, we see the difference in pressure from the bottom to the top of the column is:

$$p_2 - p_1 - \rho g h$$

This difference in pressure varies linearly in *h*, as shown by the Area 3 of the pressure diagram. If we let $h1 = 0$ and consider a gauge pressure, then $p1 = 0$ and we have:

$$p_2 = \rho g h$$

Where *h* remains the height of the column. For the fluid on top of the column, this isthe source of

1 *p* and is shown as Area 1 of the pressure diagram. Area 2 of thepressure diagram is this same pressure carried downwards, to which is added more pressure due to the extra

fluid.
*The gauge pressure at any depth from the surface of a fluid is:*

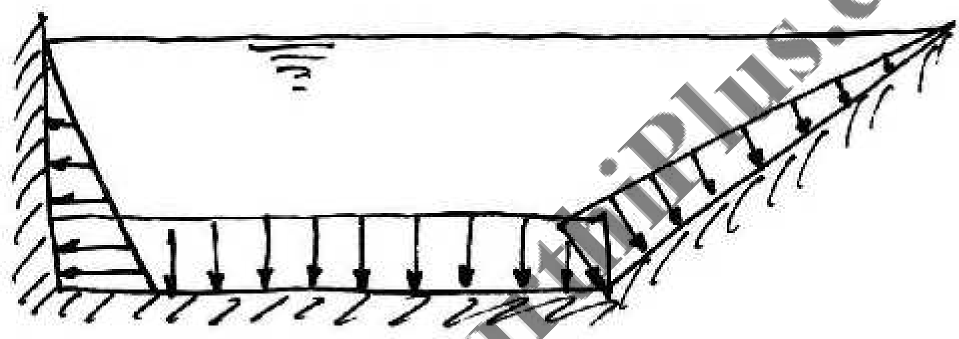
$$p = \rho g h$$

**Summary**

1. Pressure acts normal to any surface in a static fluid;
2. Pressure is the same at a point in a fluid and acts in all directions;

3. Pressure varies linearly with depth in a fluid.

By applying these rules to a simple swimming pool, the pressure distribution around the edges is as shown:

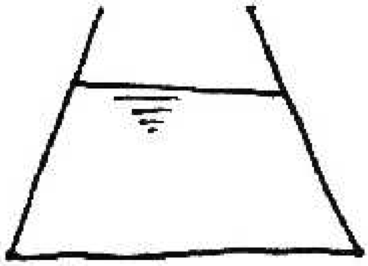


Note:

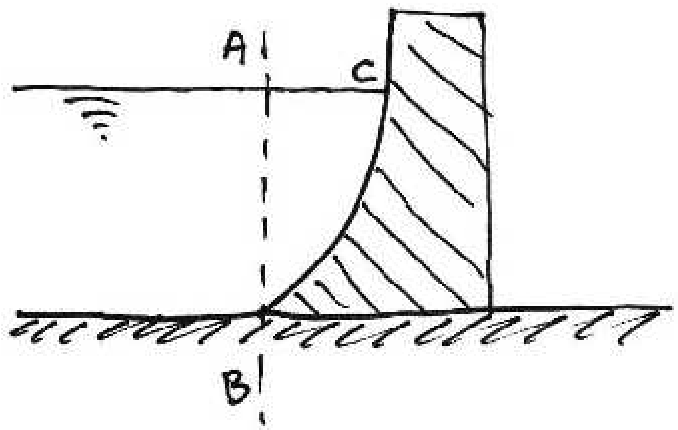
1. Along the bottom the pressure is constant due to a constant depth;
2. Along the vertical wall the pressure varies linearly with depth and acts in the horizontal direction;
3. Along the sloped wall the pressure again varies linearly with depth but also acts normal to the surface;
4. At the junctions of the walls and the bottom the pressure is the same.

**Problems - Pressure**

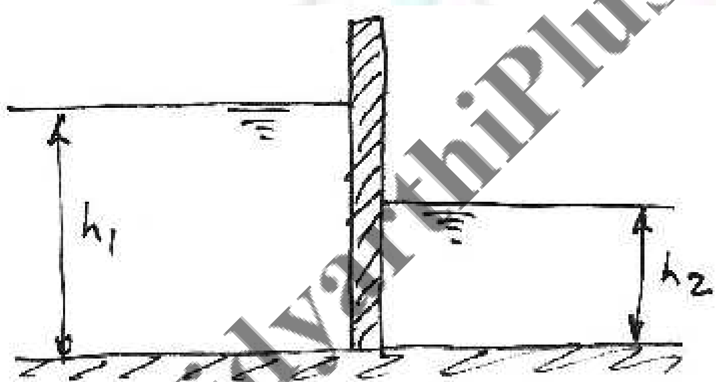
1. Sketch the pressure distribution applied to the container by the fluid:



2. For the dam shown, sketch the pressure distribution on line *AB* and on the surface of the dam, *BC*. Sketch the resultant force on the dam.




3. For the canal gate shown, sketch the pressure distributions applied to it. Sketch the resultant force on the gate? If $h1 = 6.0$ m and $h2 = 4.0$ m, sketch the pressure distribution to the gate. Also, what is the value of the resultant force on the gate and at what height above the bottom of the gate is it applied




**Pressure Head**

Pressure in fluids may arise from many sources, for example pumps, gravity, momentum etc.

Since $p = \rho gh$, a height of liquid column can be associated with thepressure $p$ arising from such sources. This height, $h$, is known as the pressure head.

**Example:**

The gauge pressure in a water mains is 50 kN/m$^2$, what is the pressure head?

The pressure head equivalent to the pressure in the pipe is just:

$$p = \rho g h$$

$$h = \frac{p}{\rho g}$$

$$= \frac{50 \times 10^3}{1000 \times 9.81}$$

$$\approx 5.1 \text{ m}$$

So the pressure at the bottom of a 5.1 m deep swimming pool is the same as the pressure in this pipe.

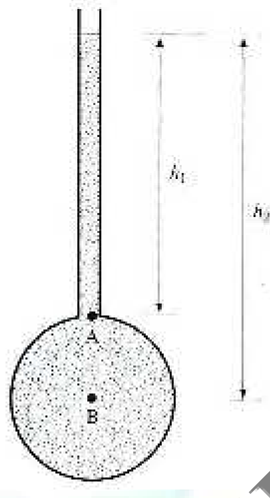
**Manometers**

A manometer (or liquid gauge) is a pressure measurement device which uses the

relationship
between pressure and head to give readings.
In the following, we wish to measure the pressure of a fluid in a pipe.

**Piezometer**

This is the simplest gauge. A small vertical tube is connected to the pipe and its top is left open
to the atmosphere, as shown.




The pressure at *A* is equal to the pressure due to the column of liquid of

$$p_A = \rho g h_1$$

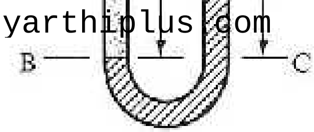
height $h_1$ : Similarly,

$$p_B = \rho g h_2$$

The problem with this type of gauge is that for usual civil engineering applications the pressure is large (e.g. 100 kN/m2) and so the height of the column is impractical(e.g.10 m). Also, obviously, such a gauge is useless for measuring gas pressures.

**U-tube Manometer**

To overcome the problems with the piezometer, the U-tube manometer seals the fluid by using a
measuring (manometric)
liquid:




Choosing the line *BC* as the interface between the measuring liquid and the fluid, we

$$\text{Pressure at } B,\ p_B = \text{Pressure at } C,\ p_C$$

know: For the left-hand side of the U-tube:

$$p_B = p_A + \rho g h_1$$

For the right hand side:

$$p_C = \rho_{man} g h_2$$

Where we have ignored atmospheric pressure and are thus dealing with gauge pressures. Thus:

$$p_B = p_C$$

$$p_A + \rho g h_1 = \rho_{man} g h_2$$

And so:

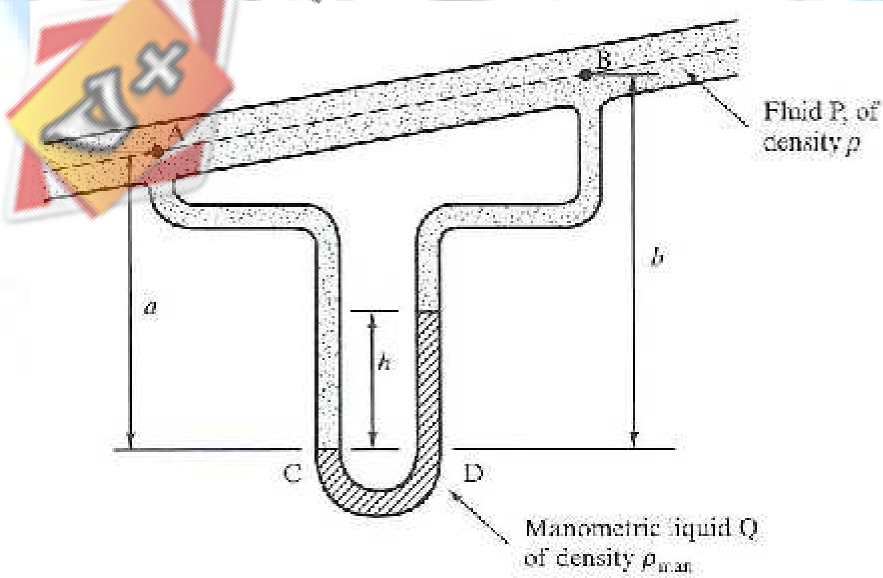
$$p_A = \rho_{man} g h_2 - \rho g h_1$$

Notice that we have used the fact that in any continuous fluid, the pressure is the same at any horizontal level.



**Differential Manometer**

To measure the pressure difference between two points we use a u-tube as shown:




Using the same approach as before:

Pressure at $C$, $p_C$ = Pressure at $D$, $p_D$

$$p_A + \rho g a = p_B + \rho g (b - h) + \rho_{man} g h$$

Hence the pressure difference is:

$$p_A - p_B = \rho g (b - a) + h g (\rho_{man} - \rho)$$

**PROBLEM 8**

**A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs U. U tube is 10 cm and the free surface of mercury is in level with over the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m$^2$, Calculate the new difference in the level of mercury. Sketch the arrangement in both cases.**

Given,

Difference of mercury = 10 cm = 0.1 m.

Let $P_A$ = pr of water in pipe line (ie, at point A)

The point B and C lie on the same horizontal line. Hence pressure at B should be equal to pressure at C.

But pressure at B = Pressure at A and Pressure due to 10 cm (or) 0.1m of water.

$$= PA + p \times g \times h$$

where , P = 1000kg/m$^3$ and h = 0.1 m

$$= P_A + 1000 \times 9.81 \times 0.1$$

$$= P_A + 981 N / m^2 \qquad \text{(i)}$$

Pressure at C = Pressure at D + pressure due to 10 cm of mercury

$$0 + P_0 \times g \times h_0$$

where $p_o$ for mercury $= 13.6 \times 1000 kg / m^3$

$h_0$ = 10 cm = 0.1 m

Pressure at $C = 0 + (13.6 \times 1000) \times 9.81 \times 0.1$

$$= 13341.6 \text{ N} \qquad \text{(ii)}$$

But pressure at B is = to pr @ c. Hence, equating (i) and (ii)

$$P_A + 981 = 13341.6$$

$$p_A = 13341.6 - 981 = 12360.6 N/m^2$$

II part: Given $p_A = 9810 N/m^2$

In this case the pressure at A is 9810 $N/m^2$ which is less than the 12360.6 $N/m^2$. Hence the mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let, x = Ries of mercury in left limb in cm

Then fall of mercury in right limb = x cm.

The points B, C and D show the initial condition.

Whereas points B*, C*, and D* show the final conditions.

The pressure at B * = pressure at C*

Pressure at A + pressure due to (10-x) cm of water.

= pressure at D* + pressure due to (10-2x) cm of mercury.

(or)

$$P_A + p_1 g \times h_1 = pD^* + p_2 \times g \times h_2$$

(or)

$$\frac{9810}{9.8} + \frac{1000 \times 9.81}{9.81}\left(\frac{10-x}{100}\right)$$

$$= 0 + \frac{(13.6 \times 1000) \times 9.81}{9.81} \times \left(\frac{10-2x}{100}\right)$$

Dividing by 9.81 , we get,

$$1000 + 100 - 10x = 1360 - 272x$$

$$272x - 10x = 1360 - 1100$$

262 x = 260

$$x = \frac{260}{262} = 0.992cm$$

∴ New difference of mercury = 10 - 2 x cm

$$= 10 - 2 \times 0.992$$

= 8.016 cm.

**PROBLEM 9**

**A differential manometer is connected at the two points A and B of two pipes a shown in figure. The pipe A contains a liquid of sp. Gr = 1.5 while pipe b contains a liquid of sp . gr = 0.9 . The pressures at A and B are 1 kgf / cm$^2$ respectively. Find the difference in mercury level in the differential manometer.**

**Given,**

**Sp. Gr of liquid at A,** $S_1 = 1.5$ y $p_1 = 1500$

**Sp. Gr of liquid at B,** $S_2 = 0.9$ y $p_2 = 900$

**Pr at A,** $P_A = 1 kgf / cm^2 = 1 \times 10^4 kgf / m^2$

$$= 10^4 \times 9.81 N / m^2$$ **(1 kgf= 9.81 N)**

**Pr at B,** $P_B = 1.8$ **kgf / cm$^2$**

$$= 1.8 \times 10^4 \times 9.81 \quad N / m^2$$

**Density of mercury** $= 13.6 \times 1000 kg / m^3$

**Pr above X – X in left limb =** $13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2+3) + P_A$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 \times 10^4$$

**Pr above X – X in the right limb** $= 900 \times 9.81 \times (h+2) + P_B$

$$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$$

**Equating two pressure, we get,**

$$13.6\times1000\times9.81h+7500\times93.81+9.81\times10^{4}$$

$$=900\times9.81\times(h+2)+1.8\times10^{4}\times9.81$$

**Dividing by** $1000\times9.81,$ **we get**

$$13.6h+7.5+10=(h+2.0)\times0.9+18$$

$$13.6h+17.5=0.9h+1.8+18=0.9h+19.8$$

$(13.6-0.9)h=19.8-17.5$ **or 12.7 h = 2.3**

$$h=\frac{2.3}{12.7}=0.181m=18.1cm$$

# *CE 6303 MECHANICS OF FLUIDS*

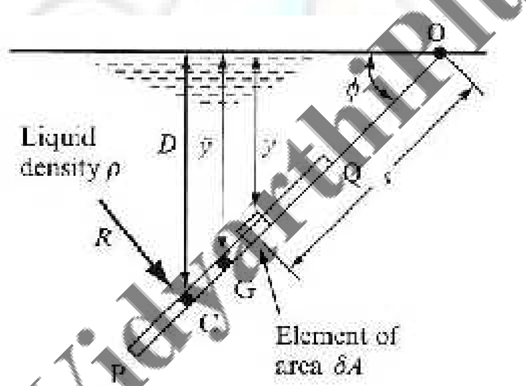
## UNIT II FLIUD KINEMATICS AND DYNAMICS

Fluid Kinematics - Flow visualization - lines of flow - types of flow - velocity field and acceleration - continuity equation (one and three dimensional differential forms)- Equation of streamline - stream function - velocity potential function - circulation - flow net. Fluid dynamics - equations of motion - Euler's equation along streamline - Bernoulli's equation – applications - Venturi meter, Orifice meter and Pitot tube. Linear momentum equation and its application.

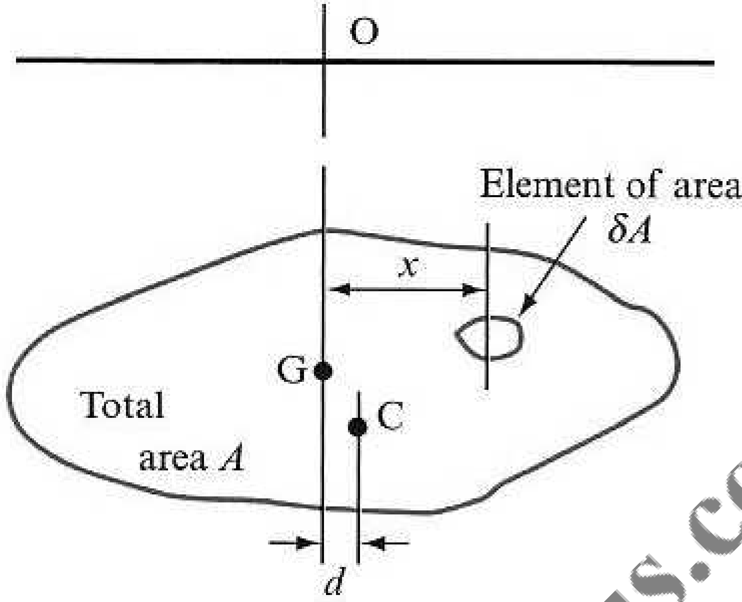
### *Fluid Action on Surfaces*

**Plane Surfaces**

We consider a plane surface, *PQ*, of area *A*, totally immersed in a liquid of density ρ and inclined at an angle φ to the free surface:



**Side Elevation**




**Front Elevation**

If the plane area is symmetrical about the vertical axis *OG*, then $d = 0$ . We will assume that this is normally the case.

**Find Resultant Force:**

The force acting on the small element of area, $\delta A$, is:

$$\delta R = p \cdot \delta A = \rho g y \cdot \delta A$$

The total force acting on the surface is the sum of all such small forces. We can integrate to get the force on the entire area, but remember that *y* is not constant:

$$R = \int \rho g y \cdot \delta A$$
$$= \rho g \int y \cdot \delta A$$

But $\int y \cdot \delta A$ is just the first moment of area about the surface. Hence:

$$R = \rho g A \bar{y}$$

Where *y* is the distance to the centroid of the area (point *G*) from the surface.

**Vertical Point Where Resultant Acts:**

The resultant force acts perpendicular to the plane and so makes an angle $90° - \varphi$ to the horizontal. It also acts through point *C*, the centre of pressure, a distance *D* below the free surface. To determine the location of this point we know:

$$\text{Moment of } R \text{ about } O = \begin{array}{l}\text{Sum of moments of forces} \\ \text{on all elements about } O\end{array}$$

Examining a small element first, and since $y = s\sin\varphi$ , the moment is:

$$\text{Moment of } \delta R \text{ about } O = \left[\rho g \left(s \sin\phi\right) \cdot \delta A\right] s$$
$$= \rho g \sin\phi \left(s^2 \cdot \delta A\right)$$

In which the constants are taken outside the bracket. The total moment is thus:

$$\text{Moment of } R \text{ about } O = \rho g \sin\phi \cdot \int s^2 \cdot \delta A$$

But ∫s2 .δ.*A* is the second moment of area about point *O* or just *O I* . Hence we have:

$$\text{Moment of } R \text{ about } O = \rho g \sin\phi \cdot I_O$$

$$\rho g A\bar{y} \times OC = \rho g \sin\phi \cdot I_O$$

$$A\bar{y} \times \frac{D}{\sin\phi} = \sin\phi \cdot I_O$$

$$D = \frac{I_O}{A\bar{y}} \cdot \sin^2\phi$$

If we introduce the parallel axis theorem:

$$I_O = I_G + A \times (OC)^2$$

$$= I_G + A \cdot \left(\frac{\bar{y}}{\sin\phi}\right)^2$$

Hence we have:

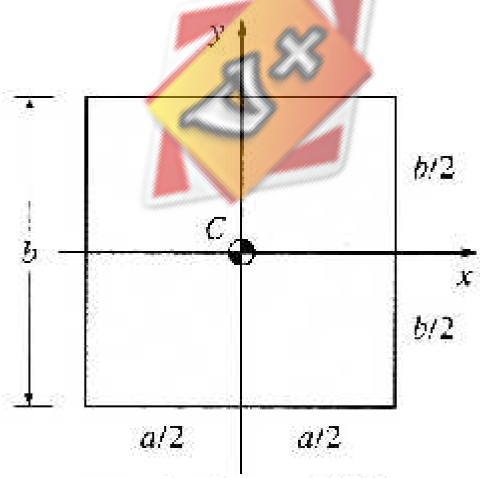
$$D = \frac{I_G + A\bar{y}^2}{A\bar{y}} \cdot \frac{\sin^2\phi}{\sin^2\phi}$$

$$= \bar{y} + \frac{I_G}{A\bar{y}}$$

Hence, the centre of pressure, point *C*, always lies below the centroid of the area, *G*.

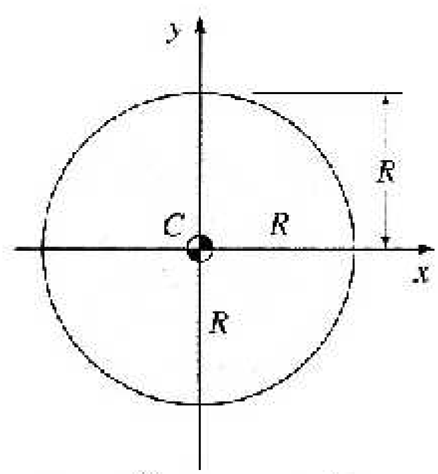


**Plane Surface Properties**



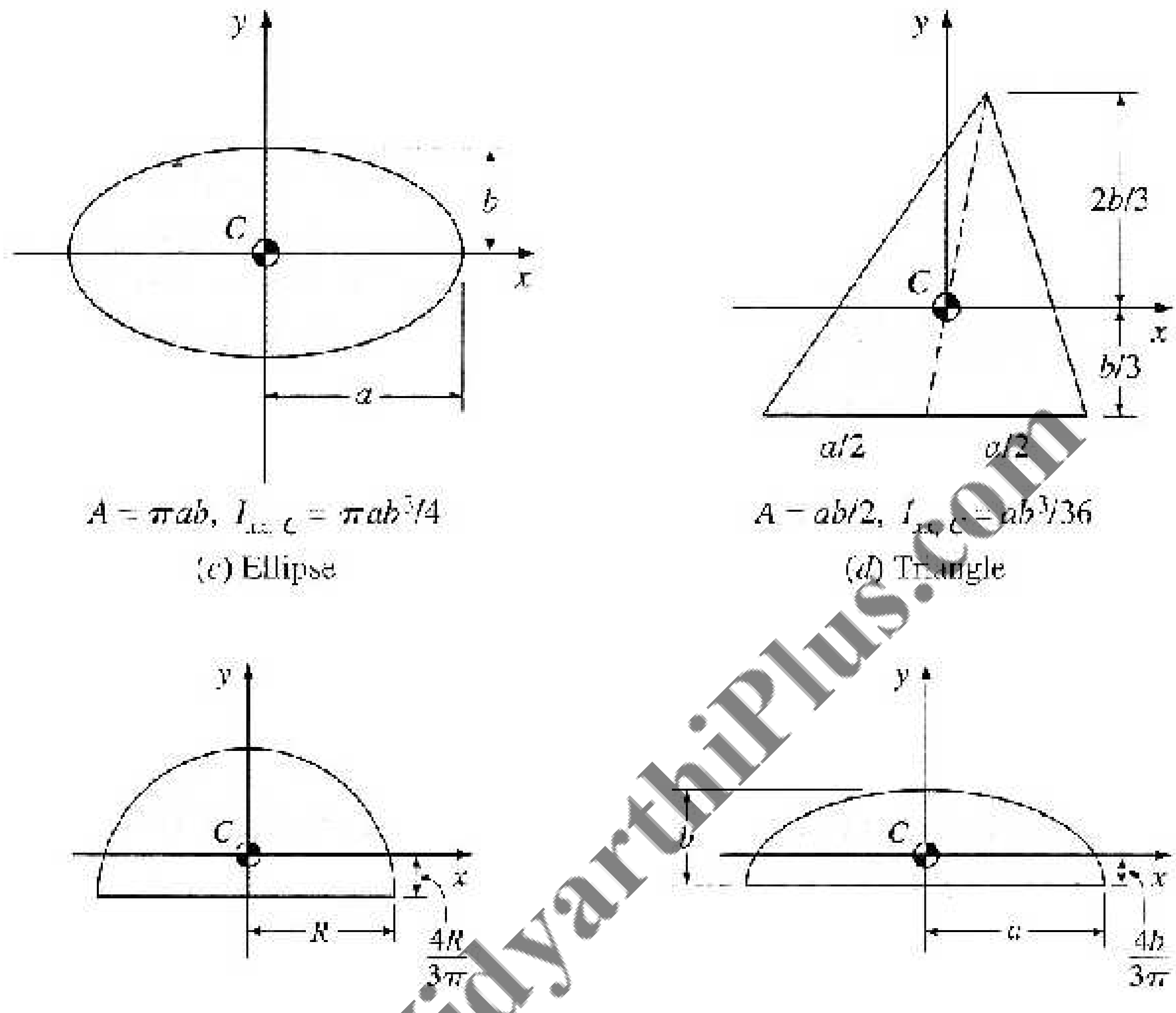

$A = ab,\ I_{xx,C} = ab^3/12$

(*a*) Rectangle




$A = \pi R^2,\ I_{xx,C} = \pi R^4/4$

(*b*) Circle




$A = \pi ab,\ I_{xx,C} = \pi ab^3/4$
(c) Ellipse

$A = ab/2,\ I_{xx,C} = ab^3/36$
(d) Triangle

$A = \pi R^2/2,\ I_{xx,C} = 0.109757R^4$
(e) Semicircle

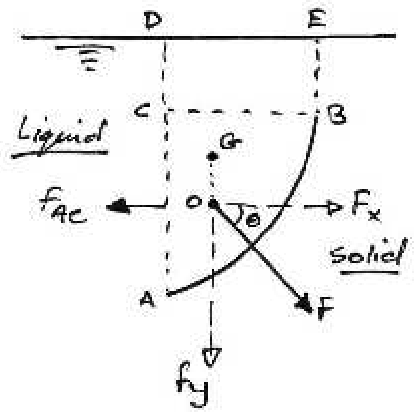
$A = \pi ab/2,\ I_{xx,C} = 0.109757ab^3$
(f) Semiellipse

**Curved Surfaces**

For curved surfaces the fluid pressure on the infinitesimal areas are not parallel and so must be combined vectorially. It is usual to consider the total horizontal and vertical force components of the resultant.

**Surface Containing Liquid**

Consider the surface AB which contains liquid as shown below:




**Horizontal Component**
Using the imaginary plane *ACD* we can immediately see that the horizontal component of force on the surface must balance with the horizontal force *AC F* .
Hence:

$$F_x = \text{Force on projection of surface onto a vertical plane}$$

*F* must also act at the same level as *F AC* and so it acts through the centre of pressure of the projected surface.
**Vertical Component**
The vertical component of force on the surface must balance the weight of liquid above the surface. Hence:

$$F_y = \text{Weight of liquid directly above the surface}$$

Also, this component must act through the centre of gravity of the area *ABED*, shown as *G* on the diagram. Resultant
The resultant force is thus:

$$F = \sqrt{F_x^2 + F_y^2}$$

This force acts through the point *O* when the surface is uniform into the page, at an angle of:

$$\theta = \tan^{-1} \frac{F_y}{F_x}$$

to the horizontal. Depending on whether the surface contains or displaces water the angle is measured clockwise (contains) or anticlockwise (displaces) from the horizontal.

## KINEMATICS OF FLUIDS

Fluid motion observed in nature, such as the flow of waters in rivers is usually rather chaotic. However, the motion of fluid must conform to the general principles of mechanics. Basic concepts of mechanics are the tools in the study of fluid motion.

Fluid, unlike solids, is composed of particles whose relative motions are not fixed from time to time. Each fluid particle has its own velocity and acceleration at any instant of time. They change both respects to time and space. For a complete description of fluid motion it is necessary to observe the motion of fluid particles at various points in space and at successive instants of time.

Two methods are generally used in describing fluid motion for mathematical analysis, the *Lagrangian* method and the *Eulerian* method.

The Lagrangian method describes the behavior of the individual fluid during its course of motion through space. In rectangular Cartesian coordinate system, Lagrange adopted a, b, c, and t as independent variables. The motion of fluid particle is completely specified if the following equations of motion in three rectangular coordinates are determined:

$$x = F_1(a.b.c.t)$$
$$y = F_2(a.b.c.t)$$
$$z = F_3(a.b.c.t)$$

Eqs. (3.1) describe the exact spatial position (x, y,z) of any fluid particle at different times in terms of its *initial position* (x0 = a, y0 = b, z0 = c) at the given initial time t = t0. They are usually referred to as parametric equations of the path of fluid particles. The attention here is focused on the paths of different fluid particles as time goes on. After the equations describing the paths of fluid particles are determined, the instantaneous velocity components and acceleration components at any instant of time can be determined in the usual manner by taking derivatives with respect to time.

$$u = \frac{dx}{dt} \quad , a_x = \frac{du}{dt} = \frac{d^2x}{dt^t}$$

$$v = \frac{dy}{dt} \quad , a_y = \frac{dv}{dt} = \frac{d^2y}{dt^2}$$

$$w = \frac{dz}{dt} \quad , a_z = \frac{dw}{dt} = \frac{d^2z}{dt^2}$$

In which u, v, and w, and ax, ay, and az are respectively the x, y, and z components of velocity and acceleration.

In the Eulerian method, the individual fluid particles are not identified. Instead, a fixed position in space is chosen, and the velocity of particles at this position as a function of time is sought. Mathematically, the velocity of particles at any point in the space can be written,

$$u = f_1(x, y, z, t)$$
$$v = f_2(x, y, z, t)$$
$$w = f_3(x, y, z, t)$$

Euler chose x, y, z, and t as independent variables in his method.

The relationship between Eulerian and Lagrangian methods can be shown. According to the Lagrangian method, we have a set of Eqs. (3.2) for each particle which can be combined with Eqs. (3.3) as follows:

$$\frac{dx}{dt} = u(x, y, z, t)$$
$$\frac{dy}{dt} = v(x, y, z, t)$$
$$\frac{dz}{dt} = w(x, y, z, t)$$

The integration of Eqs. (3.4) leads to three constants of integration, which can be considered as initial coordinates a, b, c of the fluid particle. Hence the solutions of Eqs. (3.4) give the equations of Lagrange (Eqs. 3.1). Although the solution of Lagrangian equations yields the complete description of paths of fluid particles, the mathematical difficulty encountered in solving these equations
makes the Lagrangian method impractical. In most fluid mechanics problems, knowledge of the behavior of each particle is not essential. Rather the general state of motion expressed in terms of velocity components of flow and the change of velocity with respect to time at various points in the flow field are of greater practical significance. Therefore the Eulerian method is generally adopted in fluid mechanics. With the Eulerian concept of describing fluid motion, Eqs. (3.3) give a specific velocity field in which the velocity at every point is known. In using the velocity field, and noting that x, y, z are functions of time, we may establish the acceleration components ax, ay, and az by employing the chain rule of partial differentiation,

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt} + \frac{\partial u}{\partial t}\frac{dt}{dt}$$

$$u = f_1(x, y, z, t) \quad , \quad a_x = \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) + \left(\frac{\partial u}{\partial t}\right)$$

$$v = f_2(x, y, z, t) \quad , \quad a_y = \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) + \left(\frac{\partial v}{\partial t}\right) \qquad (3.5)$$

$$w = f_3(x, y, z, t) \quad , \quad a_z = \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) + \left(\frac{\partial w}{\partial t}\right)$$

The acceleration of fluid particles in a flow field may be imagined as the superposition of two effects:

1) At a given time t, the field is assumed to become and remain steady. The particle, under such circumstances, is in the process of changing position in this steady field. It is thus undergoing a change in velocity because the velocity at various positions in this field will be different at any time t. This time rate of change of velocity due to changing position in the field is called *convective acceleration*, and is given the first parentheses in the preceding acceleration equations.

2) The term within the second parentheses in the acceleration equations does not arise from the change of particle, but rather from the rate of change of the velocity field itself at the position occupied by the particle at time t. It is called *local acceleration*.

**UNIFORM FLOW AND STEADY FLOW**

Conditions in a body of fluid can vary from point to point and, at any given point, can vary from one moment of time to the next. Flow is described as *uniform* if the velocity at a given instant is the same in magnitude and direction at every point in the fluid. If, at the given instant, the velocity changes from point to point, the flow is described as *non-uniform*.

A *steady* flow is one in which the velocity and pressure may vary from point to point but do not change with time. If, at a given point, conditions do change with time, the flow is described as *unsteady*.

For example, in the pipe of Fig. 3.1 leading from an infinite reservoir of fixed surfaceelevation, unsteady flow exits while the valve A is being opened or closed; with the valve opening fixed, steady flow occurs under the former condition, pressures, velocities, and the like, vary with time and location; under the latter they may vary only with location.

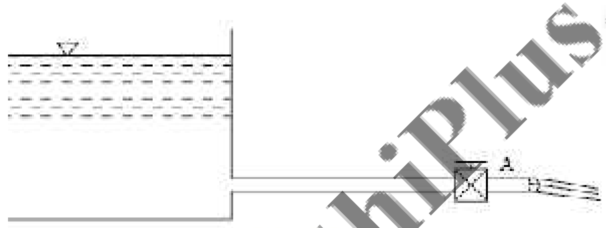



**Fig. 3.1**

There are, therefore, four possible types of flow.

1) *Steady uniform flow*. Conditions do not change with position or time. The velocity of fluid is the same at each cross-section; e.g. flow of a liquid through a pipe of constant diameter running completely full at constant velocity.

*2) Steady non-uniform flow*. Conditions change from point to point but not with time. The velocity and cross-sectional area of the stream may vary from crosssection to cross-section, they will not vary with time; e.g. flow of a liquid at a constant rate through a conical pipe running completely full.

*3) Unsteady uniform flow*. At a given instant of time the velocity at every point is the same, but this velocity will change with time; e.g. accelerating flow of a liquid through a pipe of uniform diameter running full, such as would occur when a pump is started up.

*4) Unsteady non-uniform flow*. The cross-sectional area and velocity vary from point to point and also change with time; a wave travelling along a channel.

**STREAMLINES AND STREAM TUBES**

If curves are drawn in a steady flow in such a way that the tangent at any point is in the direction of the velocity vector at that point, such curves are called *streamlines*. Individual fluid particles must travel on paths whose tangent is always in the direction of the fluid velocity at any point. Thus, path lines are the same as streamlines in steady flows.

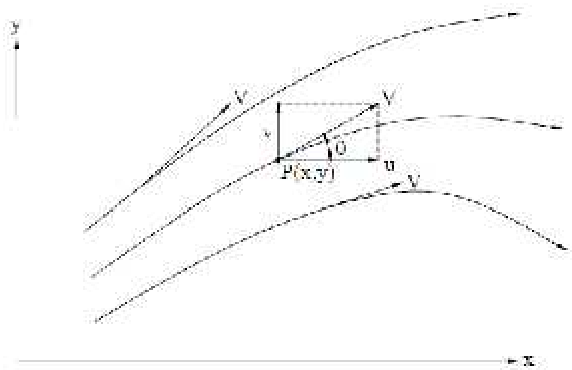



Fig. 3.2

Streamlines for a flow pattern in the xy-plane are shown in Fig. 3.2, in which a streamline passing through the point P (x, y) is tangential to the velocity vector $\vec{V}$ at P. If u and v are the x and y components of $\vec{V}$

$$\frac{v}{u} = Tan\theta = \frac{dy}{dx}$$

Where dy and dx are the y and x components of the differential displacement ds along the streamline in the immediate vicinity of P. Therefore, the differential equation for streamlines in the xy-plane may be written as

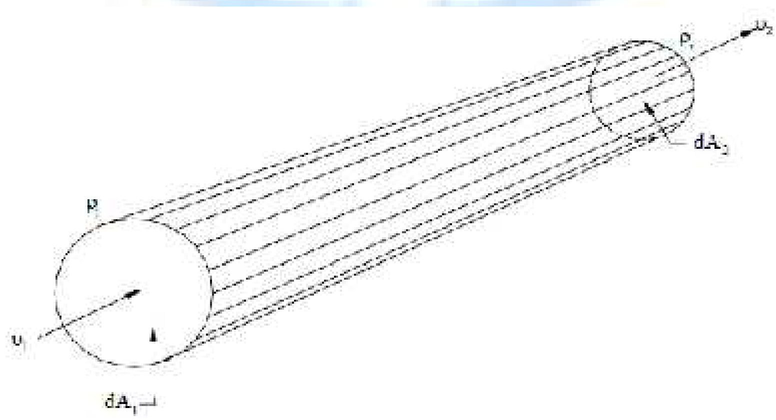
$$\frac{dx}{u} = \frac{dy}{v} \quad \text{or} \quad udy - vdx = 0$$

The differential equation for streamlines in space is,

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

Obviously, a streamline is everywhere tangent to the velocity vector; *there can be no flow occurring across a streamline*. In steady flow the pattern of streamlines remains invariant with time.

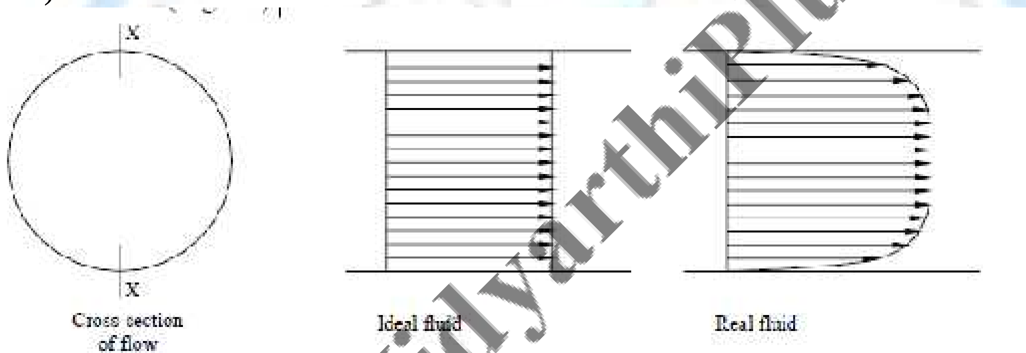
A *stream tube* such as that shown in Fig. 3.3 may be visualized as formed by a bundle of streamlines in a steady flow field. *No flow crosses the wall of a stream tube*. Often times in simpler flow problems, such as fluid flow in conduits, the solid boundaries may serve as the periphery of a stream tube since they satisfy the condition of having no flow crossing the wall of the tube.



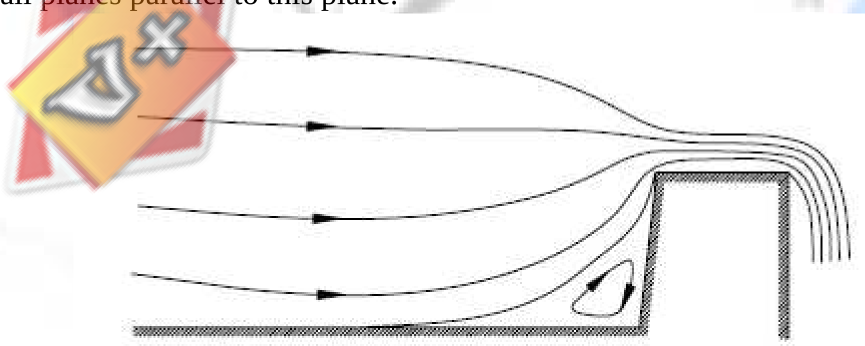
In general, the cross-sectional area may vary along a stream tube since streamlines are generally curvilinear. Only in the steady flow field with uniform velocity will streamlines be straight and parallel. By definition, the velocities of all fluid particles in a uniform flow are the same in both magnitude and direction. If either the magnitude or direction of the velocity changes along any one streamline, the flow is then considered *non-uniform*.

**ONE, TWO AND THREE-DIMENSIONAL FLOW**

Although, in general, all fluid flow occurs in three dimensions, so that, velocity, pressure and other factors vary with reference to three orthogonal axes, in some problems the major changes occur in two directions or even in only one direction. Changes along the other axis or axes can, in such cases, be ignored without introducing major errors, thus simplifying the analysis. Flow is described as *one-dimensional* if the factors, or parameters, such as velocity,pressure and elevation, describing the flow at a given instant, vary only along the direction of flow and not across the cross-section at any point. If the flow is unsteady, these parameters may vary with time. The one dimension is taken as the distance along the streamline of the flow, even though this may be a curve in space, and the values of velocity, pressure and elevation at each point along this streamline will be the average values across a section normal to the streamline (Fig.3.4).




In *two-dimensional* flow it is assumed that the flow parameters may vary in the direction of flow and in one direction at right angles, so that the streamlines are curves lying in a plane and identical in all planes parallel to this plane.



Thus, the flow over a weir of constant cross-section (Fig.3.5) and infinite width perpendicular to the plane of the diagram can be treated as two-dimensional. In *three-dimensional* flow it is assumed that the flow parameters may vary in space, x in the direction of motion, y and z in the plane of the cross-section.

\

## ENERGY EQUATION:

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as: Consider a stream-line in which flow is taking place in S-direction as shown in figure. Consider a cylindrical element of cross-section dA and length dS. The forces acting on the cylindrical element are:

1.Pressure force pdA in the direction of flow.

2.Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.

3.Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of S must be equal to the mass of fluid element x acceleration in the S direction.

$$pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA dS \cos\theta = \rho dA dS \times a_s \quad \text{-------------(1)}$$

Where $a_s$ is the acceleration in the direction of S.

Now $a_s = \frac{dv}{dt}$, where v is a function of s and t.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \qquad \left\{ \frac{ds}{dt} = v \right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of $a_s$ in equation (1) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g dA ds \cos\theta = \rho dA ds \times \frac{v\partial v}{\partial s}$$

Dividing by $\rho dsdA$, $$-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$$

$$\frac{\partial p}{\rho \partial s} + g\cos\theta + \frac{v\partial v}{\partial s} = 0$$

But from the figure $\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = 0 \quad \text{or} \quad \frac{\partial p}{\rho} + gdz + vdv = 0$$

$$\frac{\partial p}{\rho} + gdz + vdv = 0 \qquad \text{-----------------(2)}$$

The above equation is known as Euler's equation of motion.

Bernoulli's equation is obtained by integrating the above Euler's equation of motion.

$$\int \frac{\partial p}{\rho} + \int gdz + \int vdv = \text{constant}$$

If the flow is incompressible, $\rho$ is a constant and

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \text{------------(3)}$$

The above equation is known as Bernoulli's equation.

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure Head

$\frac{v^2}{2g}$ = kinetic energy per unit weight or kinetic Head

z= potential energy per unit weight or potential Head

**ASSUMPTIONS:**

The following are the assumptions made in the derivation of Bernoulli's equation:

(i)The fluid is ideal, i.e. viscosity is zero (ii)The flow is steady

(iii)The flow is incompressible (iv)The flow is irrotational

**StatementofBernoulli'sTheorem:**

It states in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are:

Pressure energy $= \frac{p}{\rho g}$ Kinetic energy $= \frac{v^2}{2g}$ Datum energy $= z$

Thus mathematically, Bernoulli's theorem is written as $\frac{p}{w} + \frac{v^2}{2g} + z = \text{cons tan t}$

**RATE OF FLOW OR DISCHARGE (Q):**

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (or liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(i) For liquids the units of Q are $m^3/s$ or liters/s (ii)For gases the units of Q are kgf/s or Newton/s Consider a fluid flowing flowing through a pipe in which

A= Cross-sectional area of pipe.

V= Average area of fluid across the section

Then discharge $Q=A \times v$

**CONTINUITY EQUATION:**

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in figure.

Let $V_1$=Average velocity at cross-section at 1-1

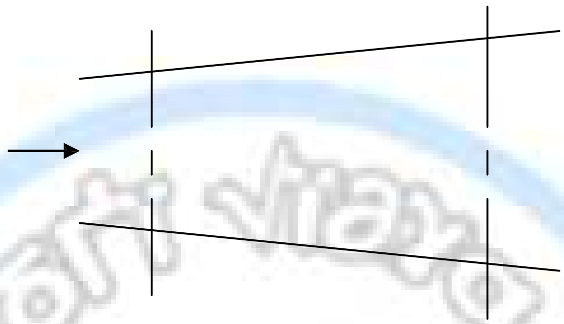
$\rho_1$ =Density at section 1-1

$A_1$=Area of pipe at section 1-1

And $V_2$, $\rho_2$, $A_2$ are corresponding values at section 2-2

Then rate of flow at section 1-1 = $V_1 \rho_1 A_1$

Rate of flow at section 2-2 = $V_2 \rho_2 A_2$



According to law of conservation of mass

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \ldots\ldots\ldots\ldots\ldots(1)$$

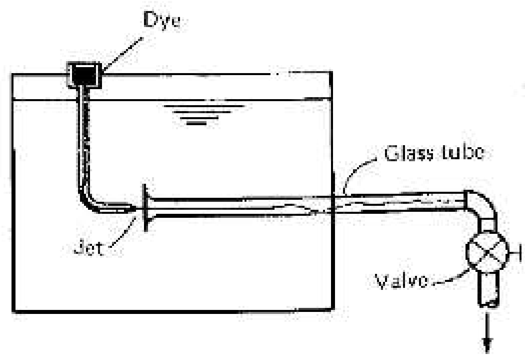
The above equation is applicable to the compressible as well as incompressible fluids is called Continuity Equation. If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (1) reduces to

$$A_1 V_1 = A_2 V_2$$

The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 5m/s. Determine the velocity at section 2.

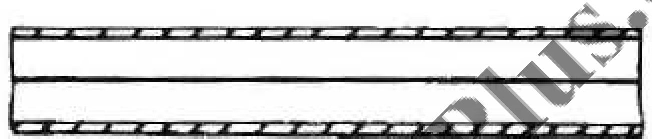
***General Concepts***

The real behaviour of fluids flowing is well described by an experiment carried out by Reynolds in 1883. He set up the following apparatus:




The discharge is controlled by the valve and the small ‘filament’ of dye (practically astreamline) indicates the behaviour of the flow. By changing the flow Reynolds noticed:

• At low flows/velocities the filament remained intact and almost straight. This type of flow is known as **laminar flow**, and the experiment looks like this:



• At higher flows the filament began to oscillate. This is called **transitional flow** and the experiment looks like:

Lastly, for even higher flows again, the filament is found to break up completely and gets diffused over the full cross-section. This is known as **turbulent flow**:

Reynolds experimented with different fluids, pipes and velocities. Eventually he

found that the following expression predicted which type of flow was found:

$$\mathrm{Re} \quad \frac{\rho \overline{vl}}{\mu}$$

In which Re is called the Reynolds Number; ρ is the fluid density; *v* is the averagevelocity; *l* is the characteristic length of the system (just the diameter for pipes), and;μ is the fluid viscosity. The Reynolds Number is a ration of forces and hence has nounits.

Flows in pipes normally conform to the following:

• Re < 2000: gives laminar flow;

• 2000 < Re < 4000: transitional flow;

• Re > 4000: turbulent flow.

These values are only a rough guide however. Laminar flows have been found at

Reynolds Numbers far beyond even 4000.

For example, if we consider a garden hose of 15 mm diameter then the limiting average velocity for laminar flow is:

$$\mathrm{Re} = \frac{\rho \bar{v} l}{\mu}$$

$$2000 = \frac{(10^3)\bar{v}(0.015)}{0.55 \times 10^{-3}}$$

$$\bar{v} = 0.073 \text{ m/s}$$

This is a very low flow and hence we can see that in most applications we deal with turbulent flow. The velocity below which there is no turbulence is called the **critical velocity**.

**Characteristics of Flow Types**

**For laminar flow:**

• Re < 2000;

• ‘low’ velocity;

• Dye does not mix with water;

• Fluid particles move in straight lines;

• Simple mathematical analysis possible;

• Rare in practical water systems.

**Transitional flow**

• 2000 < Re < 4000

• ‘medium’ velocity

• Filament oscillates and mixes slightly.

**Turbulent flow**

• Re > 4000;

• ‘high’ velocity;

• Dye mixes rapidly and completely;

• Particle paths completely irregular;

• Average motion is in the direction of the flow;

• Mathematical analysis very difficult - experimental measures are used;

• Most common type of flow.

**Background to Pipe Flow Theory**

To explain the various pipe flow theories we will follow the historical development of the subject:

| Date | Name | Contribution |
|---|---|---|
| ~1840 | Hagen and Poiseuille | Laminar flow equation |
| 1850 | Darcy and Weisbach | Turbulent flow equation |
| 1883 | Reynolds | Distinction between laminar and turbulent flow |
| 1913 | Blasius | Friction factor equation for smooth pipes |
| 1914 | Stanton and Pannell | Experimental values of friction factor for smooth pipes |
| 1930 | Nikuradse | Experimental values of friction factor for artificially rough pipes |
| 1930s | Prandtl and von Karman | Equations for rough and smooth friction factors |
| 1937 | Colebrook and White | Experimental values of the friction factor for commercial pipes and the transition formula |
| 1944 | Moody | The Moody diagram for commercial pipes |
| 1958 | Ackers | Hydraulics Research Station charts and tables for the design of pipes and channels |
| 1975 | Barr | Solution of the Colebrook-White equation |

***Laminar Flow***

**Steady Uniform Flow in a Pipe: Momentum Equation**

The development that follows forms the basis of the flow theories applied to laminar flows. We remember from before that at the boundary of the pipe, the fluid velocity is zero, and the maximum velocity occurs at the centre of the pipe. This is because of the effect of viscosity. Therefore, at a given radius from the centre of the pipe the velocity is the same and so we consider an elemental annulus of fluid:

**FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE:**

For the flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of pressure for a given length is to be determined. The flow through circular pipe will be viscous or laminar, if the Reynold's number is less than 2000. The expression for Reynold's number is given by

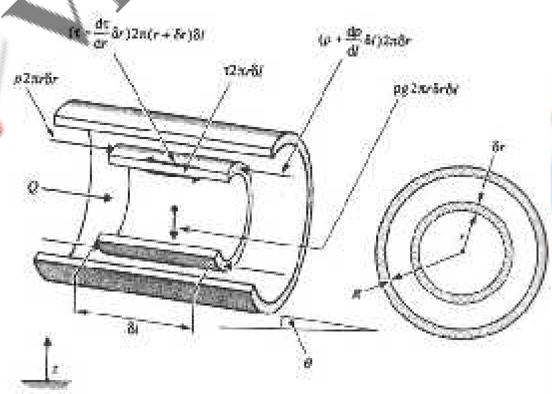
$$R_e = \frac{\rho v d}{\mu}$$

Where ρ= Density of fluid flowing through pipe,

V = Average velocity of fluid,

D = Diameter of pipe and,

μ = Viscosity of fluid




Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in figure. Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius (r+dr). Let the length of fluid element be $\Delta x$. If ‘p’ is the intensity of pressure on the face AB, then the intensity of pressure on the face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$. The the forces acting on the fluid element are:

1. The pressure force, p x $\pi r^2$ on face AB

2. The pressure force $\left(p + \frac{\partial p}{\partial x}\Delta x\right).\pi r^2$ on face CD

3. The shear force, $\tau \times 2\pi r \Delta x$ on the surface of fluid element. As there is no acceleration, hence the summation of all forces in the direction of flow must be zero.

$$p\ \pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right).\pi r^2 - \tau \times 2\pi r \Delta x = 0$$

$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \Delta x = 0$$

$$-\frac{\partial p}{\partial x} r - 2\tau = 0$$

$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{-------------------(1)}$$

The shear stress $\tau$ across a section varies with ‘r’ as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress across a section is linear as shown in figure.

**(i) Velocity Distribution**: To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \frac{\partial u}{\partial y}$ is substituted in equation (1)

But in the relation $\tau = \mu \frac{\partial u}{\partial y}$, y is measured from the pipe wall. Hence

$$y = R - r \qquad \text{and} \quad dy = -dr$$

$$\tau = \mu \frac{\partial u}{-\partial r} = -\mu \frac{du}{dr}$$

Substituting this value in equation (1)

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x} r$$

Integrating the equation w.r.t 'r' we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C \text{ ------------- (2)}$$

Where C is the constant of integration and its value is obtained from the boundary condition that at r=R, u=0

$$0 = \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 + C$$

$$C = -\frac{1}{4\mu}\frac{\partial p}{\partial x} r^2$$

Substituting this value of C in equation (2), we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] \text{ ------------------------ (3)}$$

In equation (3) values of $\mu$, $\frac{\partial p}{\partial x}$ and r are constant, which means the velocity u, varies with the square of r. Thus the equation (3) is a equation of parabola. This shows that the velocity distribution across the section of a pipe is parabolic. This velocity distribution is shown in fig.

**(ii) Ratio of Maximum velocity to average velocity:**

The velocity is maximum, when r =0 in equation (3). Thus maximum velocity, $U_{max}$ is obtained as

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2 \text{ ------------------- (4)}$$

The average velocity, $\bar{u}$, is obtained by dividing the discharge of the fluid across the section by the area of the pipe $(\pi R^2)$. The discharge (Q) across the section is obtained by considering the through a ring element of radius r and thickness the as shown in fig (b). The fluid flowing per second through the elementary ring

dQ= velocity at a radius r x area of ring element

$$= u \times 2\pi r dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] \times 2\pi r dr$$

$$Q = \int_o^R dQ = \int_0^R = \frac{1}{4\mu}\frac{\partial p}{\partial x}\left[R^2 - r^2\right] \times 2\pi r dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \int_0^R \left(R^2 - r^2\right) r dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \int_0^R \left(R^2 r - r^3\right) dr$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{4}\right] = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right) \times 2\pi R^4$$

Average velocity, $\bar{u} = \frac{Q}{Area} = \frac{\frac{\pi}{8\mu =}\left(\frac{-\partial p}{\partial x}\right)R^4}{\pi R^2}$

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2 \text{ --------------- (5)}$$

Dividing equation (4) by equation (5)

$$\frac{U_{max}}{\bar{u}} = \frac{\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2} = 2.0$$

Ratio of maximum velocity to average velocity = 2.0

**(iii) Drop of pressure for a given length (L) of a pipe:**

From equation (5), we have

$$\bar{u} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2 \quad \text{or} \quad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

Integrating the above equation w.r.t . x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}dx$$

$$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_1]$$

$$[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2}L \qquad \{ x_2 - x_1 = L \text{ from equation (3)}\}$$

$$= \frac{8\mu\bar{u}L}{\left(D/2\right)^2} \qquad \{R = \frac{D}{2}\}$$

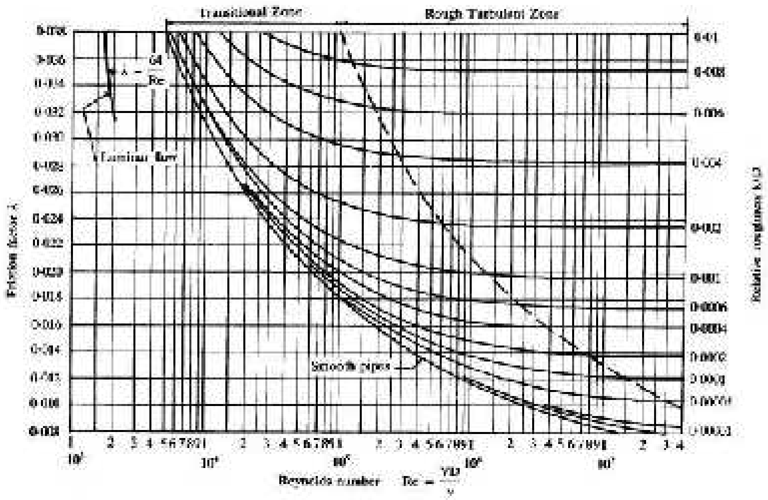
$$[p_1 - p_2] = \frac{32\mu\bar{u}L}{D^2},$$ Where $p_1 - p_2$ is the drop of pressure

Loss of pressure head = $\frac{p_1 - p_2}{\rho g}$

$$\frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2} \quad \text{---------------------- (6)}$$

Equation (6) is called Hagen Poiseuille Formula.

- **DARCY – WEISBACH EQUATION (Derivation refer class notes)**
- **Moody diagram for friction factor:**




***The Momentum Equation***

**Development**

We consider again a general streamtube:


In a given time interval, $\delta t$ , we have:

$$\text{momentum entering} = \rho Q_1 \delta t \, v_1$$
$$\text{momentum leaving} = \rho Q_2 \delta t \, v_2$$

From continuity we know $Q_1 = Q_2 = Q$ . Thus the force required giving the change in momentum between the entry and exit is, from Newton's Second Law:

$$F \quad \frac{d(mv)}{dt}$$

$$F \quad \frac{\rho Q \delta t (v_2 \quad v_1)}{\delta t}$$

$$\rho Q (v_2 \quad v_1)$$

This is the force acting on a fluid element in the direction of motion. The fluid exerts an equal but opposite reaction to its surroundings.

**Application – Fluid Striking a Flat Surface**

Consider the jet of fluid striking the surface as shown:


The velocity of the fluid normal to the surface is:

$$v_{normal} = v\cos\theta$$

This must be zero since there is no relative motion at the surface. This then is also the change in velocity that occurs normal to the surface. Also, the mass flow entering the control volume is:

$$\rho Q = \rho Av$$

Hence:

$$F = \frac{d(mv)}{dt}$$

$$= (\rho Av)(v\cos\theta)$$

$$= \rho Av^2 \cos\theta$$

And if the plate is perpendicular to the flow then:

$$F \quad \rho Av^2$$

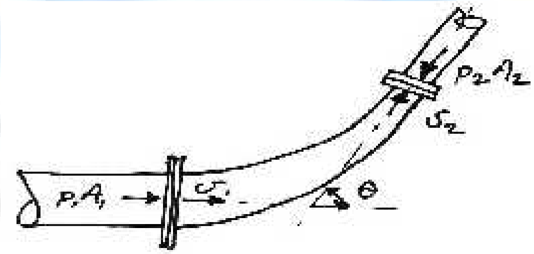
Notice that the force exerted by the fluid on the surface is proportional to the velocity squared. This is important for wind loading on buildings. For example, the old wind loading code

$$q = 0.613 v_s^2 \qquad (\text{N/m}^2)$$

In which *s v* is the design wind speed read from maps and modified to take account of relevant factors such as location and surroundings.

**Application – Flow around a bend in a pipe**

Consider the flow around the bend shown below. We neglect changes in elevation and consider the control volume as the fluid between the two pipe joins.



The net external force on the control volume fluid in the *x*-direction is:

$$p_1A_1 - p_2A_2\cos\theta + F_x$$

In which *x F* is the force on the fluid by the pipe bend (making it 'go around the corner'). The above net force must be equal to the change in momentum, which is:

$$\rho Q(v_2\cos\theta - v_1)$$

Hence:

$$p_1A_1 - p_2A_2\cos\theta + F_x = \rho Q(v_2\cos\theta - v_1)$$

$$F_x = \rho Q(v_2\cos\theta - v_1) - p_1A_1 + p_2A_2\cos\theta$$

$$= (\rho Q v_2 + p_2A_2)\cos\theta - (\rho Q v_1 + p_1A_1)$$

Similarly, for the *y*-direction we have:

$$-p_2A_2\sin\theta + F_y = \rho Q(v_2\sin\theta - 0)$$

$$F_y = \rho Q(v_2\sin\theta - 0) + p_2A_2\sin\theta$$

$$= (\rho Q v_2 + p_2A_2)\sin\theta$$

The resultant is:

$$F = \sqrt{F_x^2 + F_y^2}$$

And which acts at an angle of:

$$\theta = \tan^{-1}\frac{F_y}{F_x}$$

This is the force and direction of the bend on the fluid. The bend itself must then be supported for this force. In practice a manhole is built at a bend, or else a thrust block is used to support the pipe bend.

***PROBLEM -1***

**The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 lit/sec. the section 1 is 6m above datum. If the pressure at section 2 is 4m above the datum. If the pressure at section 1 is 39.24 N/cm$^2$, find the intensity of pressure at section 2.**

**Given:**

**At section 1,** $D_1 = 20$ cm = 0.2m

$A_1 = \frac{\Pi}{4}(0.2)^2 = 0.314m^2$.

$P_1 = 39.24$ N/cm$^2$ = 39.24 X10$^4$ N/m$^2$.

$Z_1 = 6.0$m

**At section 2,** $D_2 = 0.10$m

$A_2 = \frac{\Pi}{4}(0.1)^2 = 0.0785m^2$.

$P_2 = ?$

$Z_1 = 4.0$m

Rate of flow $Q = 35$ lit/sec = 35/1000 = 0.035m$^3$/s

$Q = A_1V_1 = A_2V_2$

$V_1 = Q / A_1 = 0.035 / 0.0314 = 1.114$ m/s

$V_2 = Q / A_2 = 0.035 / 0.0785 = 4.456$ m/s.

Applying Bernoulli's Equations at sections at 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Or (39.24 X10$^4$ / 1000X9.81) + ((1.114)$^2$/2X9.81) + 6.0

$= (p_2 / 1000 X 9.81) + ((4.456)^2 / 2 X 9.81) + 4.0$

$40 + 0.063 + 6.0 = (p_2/9810) + 1.012 + 4.0$

$46.063 = (p_2/9810) + 5.012$

$(p_2/9810) = 46.063 – 5.012 = 41.051$

$p_2 = (41.051 X 9810/10^4) = 40.27\ N/cm^2$

***PROBLEM -2***

**In avertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16 cm and 8 cm respectively. A is 2 m above B. the pressure gauge readings have shown that the pressure at B is greater than at A by 0.981 N/cm$^2$. Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U – tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.**

**Given:**

Sp.gr.. of oil, $S_o = 0.8$

Density, $\rho = 0.8 X 1000 = 800\ kg/m^3$.

Dia at A, $D_A = 16\ cm = 0.16m$

Area at A, $A_1 = \frac{\Pi}{4} X (0.16)^2 = 0.0201 m^2$.

Dia. At B $D_B = 8\ cm = 0.08m$

Area at B, $A_B = = \frac{\Pi}{4} X (0.08)^2 = 0.005026\ m^2$

(i). Difference of pressures, $p_B – p_A = 0.981\ N/cm^2$.

$= 0.981 X 10^4\ N/m^2. = 9810\ N/m^2$.

Difference of pressure head $(p_B – p_A)/ \rho g = (9810 / (800 X 9.81)) = 1.25$

Applying Bernoulli's theorem at A and B and taking reference line passing through section B, we get

$$\frac{p_A}{^\circ g} + \frac{V_A^2}{2g} + Z_A = \frac{p_B}{^\circ g} + \frac{V_B^2}{2g} + Z_B$$

$$\frac{p_A}{{}^\circ g}-\frac{p_B}{{}^\circ g}+Z_A-Z_B=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}$$

$$\frac{p_A-p_B}{{}^\circ g}+2.0-0.0=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}$$

$$-1.25+2.0=\frac{V_B^2}{2g}-\frac{V_A^2}{2g} \qquad \frac{p_B-p_A}{{}^\circ g}=1.25$$

$$0.75=\frac{V_B^2}{2g}-\frac{V_A^2}{2g} \text{ --------------------- (i)}$$

Now applying continuity equation at A and B, we get

$$V_A X A_1 = V_B X A_2$$

$$V_B=\frac{V_A X A_1}{A_2}=\frac{V_A X \frac{\text{®}}{4}(.16)^2}{\frac{\text{®}}{4}(.08)^2}=4V_A$$

Substituting the Value of $V_B$ in equation (i), we get

$$0.75=\frac{16V_A^2}{2g}-\frac{V_A^2}{2g}=\frac{15V_A^2}{2g}$$

$$V_A=\sqrt{\frac{0.75 X 2 X 9.81}{15}}=0.99 m/s.$$

Rate of flow, $Q = V_A X A_1$

$= 0.99 \text{ X } 0.0201 = 0.01989 \text{ m}^3/\text{s}.$

(ii). Difference of mercury in the U –tube.

Let h = difference of mercury level.

Then $$h = x\left(\frac{S_g}{S_o}-1\right)$$

Where $$h=\left(\frac{p_A}{{}^\circ g}+Z_A\right)-\left(\frac{p_B}{{}^\circ g}+Z_B\right)=\frac{p_A-p_B}{{}^\circ g}+Z_A-Z_B$$

$= -1.25+2.0-0 = 0.75.$

$$\therefore 0.75 = x\left[\frac{13.6}{0.8} - 1\right] = x \text{ X } 16$$

x = (0.75 / 16 ) = 0.04687 cm.

## EXPRESSION FOR RATE OF FLOW THROUGH VENTURIMETER.

Venturi meter is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three pats (i). A short converging part (ii) Throat and (iii). Diverging part

Let $d_1$ = diameter at inlet or at section 1

Let $P_1$ = pressure at section 1

Let $V_1$ = velocity of fluid at section 1

Let $a_1$ = area of section 1 = $\frac{®}{4} d_1^2$

And $d_2$, $P_2$, $V_2$, $a_2$ are the corresponding values at section 2.

Applying the Bernoulli's equation at section 1 & 2

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2$

since the pipe is horizontal $Z_1 = Z_2$

$(P_1/\rho g) + (V_1^2 / 2g) = (P_2/\rho g) + (V_2^2 / 2g)$

$$\frac{P_1 - P_2}{\circ g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

We know that $\frac{P_1 - P_2}{\circ g}$ is the difference or pressure head and is equal to h.

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \text{ ------------- (1).}$$

Now applying, continuity equation at 1 & 2

$a_1V_1 = a_2V_2$ or $V_1 = ( a_2V_2/ a_1)$ --------------- (2).

Sub (2) in equation (1) we get

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2V_2}{a_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$V_2^2 = 2gh ( a_1^2 / ( a_1^2 - a_2^2))$

$$V_2 = \sqrt{2gh}.\frac{a_1}{\sqrt{a_1^2 - a_2^2}}$$

Discharge, $Q = a_2V_2$

$$Q = \frac{a_2 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$ theoretical discharge

Actual discharge

$$Q_{act} = C_d \text{ X } \frac{a_2 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Where $C_d$ = co – efficient of venturi meter.

***PROBLEM 3***

**Water flows through a pipe AB 1.2m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter at C, the pipe branches. Branch CD is 0.8m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.**

**Solution. Given:**

Diameter of Pipe AB, $D_{AB} = 1.2$ m.

Velocity of flow through AB $V_{AB} = 3.0$ m/s.

Dia. of Pipe BC, $D_{BC} = 1.5$m.

Dia. of Branched pipe CD, $D_{CD} = 0.8$m.

Velocity of flow in pipe CE, $V_{CE} = 2.5$ m/s.

Let the rate of flow in pipe AB = Q $m^3$/s.

Velocity of flow in pipe BC = $V_{BC}$ $m^3$/s.

Velocity of flow in pipe CD = $V_{CD}$ $m^3$/s.



Diameter of pipe CE = $D_{CE}$

Then flow rate through CD = Q / 3

And flow rate through CE = Q – Q/3 = 2Q/3

(i). Now the flow rate through AB = Q = $V_{AB}$ X Area of AB

$= 3 \times (\pi / 4) \times (D_{AB})^2 = 3 \times (\pi / 4) \times (1.2)^2$

$= 3.393 \; m^3/s.$

(ii). Applying the continuity equation to pipe AB and pipe BC,

$V_{AB}$ X Area of pipe AB = $V_{BC}$ x Area of Pipe BC

$3 \times (\pi / 4) \times (D_{AB})^2 = V_{BC} \times (\pi / 4) \times (D_{BC})^2$

$3 \times (1.2)^2 = V_{BC} \times (1.5)^2$

$V_{BC} = (3 \times 1.2^2) / 1.5^2 = 1.92 \; m/s.$

(iii). The flow rate through pipe

$CD = Q_1 = Q/3 = 3.393 / 3 = 1.131 \; m^3/s.$

$Q_1 = V_{CD}$ x Area of pipe $C_D \times (\pi / 4) (C_{CD})^2$

$1.131 = V_{CD} \times (\pi / 4) \times (0.8)^2$

$V_{CD} = 1.131 / 0.5026 = 2.25 \; m/s.$

(iv). Flow through CE,

$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262 \; m^3/s$

$Q_2 = V_{CE}$ X Area of pipe CE $= V_{CE} \times (\pi / 4) (D_{CE})^2$

$2.263 = 2.5 \times (\pi / 4) (D_{CE})^2$

$D_{CE} = \sqrt{(2.263 \times 4) / (2.5 \times \pi)} = 1.0735 \; m$

Diameter of pipe CE = 1.0735m.

***PROBLEM 4***

**A horizontal Venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d$ = 0.98.**

**Given:**

$d_1 = 30$ cm

$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2$

$= 706.85 \text{ cm}^2$

$d_2 = 15$ cm

$a_2 = \frac{\pi}{4} d_2^{\,2} = \frac{\pi}{4}(15)^2$

$= 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer = x = 20 cm of mercury.

Difference of pressure head, $h = x\left(\frac{S_h}{S_o} - 1\right)$

$= 20\,[\,(13.6 / 1) - 1] = 252.0$ cm of mercury.

$$Q_{act} = C_d \times \frac{a_2 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= 0.98 \times \frac{706.85 x 176.7 \sqrt{2 x 9.81 x 252}}{\sqrt{706.85^2 - 176.7^2}}$$

$= 125756 \text{ cm}^3 / \text{s}$

**= 125.756 lit / s.**

# *CE 6303 MECHANICS OF FLUIDS*

**UNIT III FLOW THROUGH PIPES**

Viscous flow - Shear stress, pressure gradient relationship - laminar flow between parallel plates - Laminar flow through circular tubes (Hagen poiseulle's) – Hydraulic and energy gradient - flow through pipes - Darcy -Weisbach's equation - pipe roughness -friction factor- Moody's diagram- Major and minor losses of flow in pipes - Pipes in series and in parallel.

## Fluid Flow in Pipes

We will be looking here at the flow of real fluid in pipes – *real* meaning a fluid that possesses viscosity hence looses energy due to friction as fluid particles interact with one another and the pipe wall.

Recall from Level 1 that the shear stress induced in a fluid flowing near a boundary is given by Newton's law of viscosity:

$$\tau \propto \frac{du}{dy}$$

This tells us that the shear stress, τ, in a fluid is proportional to the velocity gradient - the rate of change of velocity across the fluid path. For a "Newtonian" fluid we can write:

$$\tau = \mu \frac{du}{dy}$$

where the constant of proportionality, μ, is known as the coefficient of viscosity (or simply viscosity).

Recall also that flow can be classified into one of two types, **laminar** or **turbulent** flow (with a small transitional region between these two). The non-dimensional number, the Reynolds number, Re, is used to determine which type of flow occurs:

$$Re = \frac{\rho u d}{\mu}$$

For a pipe

| | |
|---|---|
| Laminar flow: | Re < 2000 |
| Transitional flow: | 2000 < Re < 4000 |
| Turbulent flow: | Re > 4000 |

It is important to determine the flow type as this governs how the amount of energy lost to friction relates to the velocity of the flow. And hence how much energy must be used to move the fluid.

## HYDRAULIC GRADIENT AND TOTAL ENERGY LINE:

This concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as

**1.Hydraulic Gradient Line:** It is defined as the line which gives the sum of pressure head (p/w) and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L (Hydraulic Gradient Line).

**2.Total Energy Line:** It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L (Total Energy Line)

## EXPRESSION FOR LOSS OF HEAD DUE TO FRICTION IN PIPES OR DARCY – WEISBACH EQUATION

Consider a uniform horizontal pipe, having steady flow as shown figure. Let 1 -1 and 2-2 is two sections of pipe.

Let $P_1$ = pressure intensity at section 1-1.
Let $P_2$ = Velocity of flow at section 1-1.
L = length of the pipe between the section 1-1 and 2-2
d = diameter off pipe.
$f^1$ = Frictional resistance per unit wetted area per unit velocity.
$h_f$ = loss of head due to friction.

And $P_2$,$V_2$ = are the values of pressure intensity and velocity at section 2-2.

Applying Bernoulli's equation between sections 1-1 & 2-2

Total head 1-1 = total head at 2-2 + loss of head due to friction between 1-1&2-2

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2+h_f$ ------------(1)

but $Z_1 = Z_1$ [ pipe is horizontal ]

$V_1= V_2$ [ diameter of pipe is same at 1-1 & 2-2]

(1) becomes,

$(P_1/ \rho g) = (P_2/\rho g)+h_f$

$h_f = (P_1/ \rho g) - (P_2/\rho g)$

frictional resistance = frictional resistance per unit wetted area per unit velocity X wetted area X velocity $^2$.

$F = f^1 \times \pi d_1 \times V^2$ [ Wetted area = $\pi$ d x L, and Velocity $V = V_1 = V_2$]

$F_1 = f^1 x P x L x V^2$ ----------- (2). [$\pi$ d = wetted perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are,

1) Pressure force at section 1-1 = $P_1 X A$

2) Pressure force at section 2-2 = $P_2 X A$

3). Frictional force $F_1$

Resolving all forces in the horizontal direction.,

$P_1 A - P_2 A - F_1 = 0$

$(P_1-P_2)A = F_1 = f^1 x P x L x V^2$

$(P_1-P_2) = (f^1 x P x L x V^2 / A)$.

But from (1) we get

$P_1 - P_2 = \rho g h_f$

Equating the values of $(P_1 - P_2)$ we get

$\rho g h_f = (f^1 x P x L x V^2 / A)$.

$h_f = (f^1 / \rho g) X (P/A) X L X V^2$

$(P/A) = (\pi d / (\pi d^2/4)) = (4/d)$

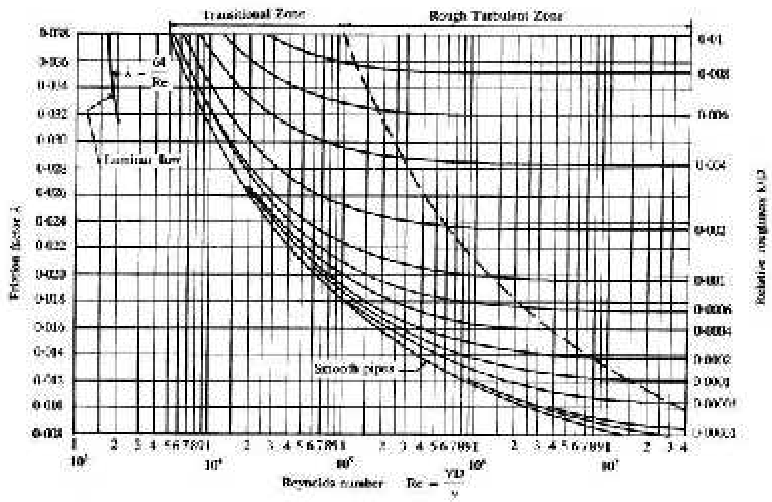
Hence, $h_f = ( f^1 / \rho g) x ( 4/d) x L x V^2$.

Putting $(f^1 / \rho) = (f / 2)$, where f is the co – efficient of friction

$$h_f = \frac{4fLV^2}{2gd}$$

This equation is known as Darcy – Weisbach equation. This equation is commonly used to find loss of head due to friction in pipes.

- **Moody diagram for friction factor:**




**Minor losses: (Derivation and formulas refer class notes)**

- **Sudden enlargement**
- **Sudden contraction**
- **Sudden obstruction**
- **Entrance in pipe**
- **Exit in pipe**
- **Losses by bend**
- **Losses by using fittings**

**FLOW THROUGH PIPES IN SERIES OR FLOW THROUGH COMPOUND PIPES:**

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

**FLOW THROUGH PARALLEL PIPES:**

Loss of head for branch pipe 1= Loss of head for branch pipe 2 or $$\frac{4f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4f_2 L_2 V_2^2}{d_2 \times 2g}$$

If $f_1 = f_2$, then $$\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}$$

***PROBLEM 1***

**The rate of flow through a horizontal pipe is 0.25 $m^3$/s. the diameter of the pipe which is 200mm is suddenly enlarged to 400mm. the pressure intensity in the smaller pipe is 11.772 N/$cm^2$. Determine (i). Loss of head due to sudden enlargement (ii). Pressure intensity in large pipe. (iii). Power lost due to enlargement.**

**Given:**

| | |
|---|---|
| Discharge | $Q = 0.25\ m^3/s$. |
| Dia. Of smaller pipe | $D_1 = 200mm = 0.2m$ |
| Area | $A_1 = \frac{®}{4}(0.2)^2 = 0.03141\ m^2$. |
| Dia of large pipe | $D_2 = 400mm = 0.4m$ |
| Area | $A_2 = \frac{®}{4}(0.4)^2 = 0.12566\ m^2$. |
| Pressure in smaller pipe | $p_1 = 11.772\ N/cm^2 = 11.772\ X10^4\ N/m^2$. |
| Now velocity | $V_1 = Q / A_1 = 0.25 / 0.03414 = 7.96$ m/s. |
| Velocity | $V_2 = Q / A_2 = 0.25 / 0.12566 = 1.99$ m/s. |

(i). Loss of head due to sudden enlargement,

$h_e = (V_1 - V_2)^2 / 2g = (7.96 - 1.99)^2 / 2X\ 9.81 =$ **1.816 m**.

(ii). Let the pressure intensity in large pipe = $p_2$.

Then applying Bernoulli's equation before and after the sudden enlargement,

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2 + h_e$

But $Z_1 = Z_2$

$(P_1/\rho g) + (V_1^2 / 2g) = (P_2/\rho g) + (V_2^2 / 2g) + h_e$

Or $(P_1/\rho g) + (V_1^2 / 2g) = (P_2/\rho g) + (V_2^2 / 2g) + Z_2 + h_f$

$(P_2/\rho g) = (P_1/\rho g) + (V_1^2 / 2g) - (V_2^2 / 2g) - h_e$

$= \frac{11.772X10^4}{1000X9.81} + \frac{7.96^2}{2X9.81} - \frac{1.99^2}{2X9.81} - 1.816$

$= 12.0+3.229-0.2018-1.8160$

$= 15.229 - 20.178 = 13.21$m of water

$p_2 \quad = 13.21\ X\ \rho g = 13.21\ X\ 100X\ 9.81\ N/m^2$

$= 13.21X1000X9.81X10^{-4}\ N/cm^2$. = **12.96N/$cm^2$.**

(iii). Power lost due to sudden enlargement,

P = ($\rho$g **Q $h_e$**) / 1000 = (1000X9.81X0.25X1.816)/1000 = **4.453kW.**

***PROBLEM 2***

**A horizontal pipeline 40m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25m of its length from the tank, the pipe is 150mm diameter is suddenly enlarged to 300mm. the height of water level in the tank is 8m above the centre of the pipe. Considering all losses of head, which occur. Determine the rate of flow. Take f = 0.01 for both sections of the pipe.**

**Given:**

Total length of pipe, $L = 40m$

Length of 1 st pipe, $L_1 = 25m$

Dia of 1st pipe $d_1 = 150mm = 0.15m$

Length of 2nd pipe $L_2 = 40 – 25 = 15m$

Dia of 2nd pipe $d_2 = 300mm = 0.3m$

Height of water $H = 8m$

Co-effi. Of friction $f = 0.01$

Applying the Bernoulli's theorem to the surface of water in the tank and outlet of pipe as shown in fig. and taking reference line passing through the center of the pipe.

$0+0+8 = (P_2/\rho g) + (V_2^2 / 2g) +0+\text{all losses}$

$8.0 = 0+(V_2^2 / 2g)+h_i+ h_{f1}+ h_e+ h_{f2}$

Where, $h_i$ = loss of head at entrance = $0.5 V_1^2/ 2g$

$h_{f1}$ = head lost due to friction in pipe 1 = $\dfrac{4XfXL_1XV_1^2}{d_1X2g}$

$h_e$ = loss of head due to sudden enlargement = $(V_1- V_2)^2/2g$

$h_{f2}$ = head lost due to friction in pipe 2 = $\dfrac{4XfXL_2XV_2^2}{d_2X2g}$

But from continuity equation, we have

$A_1V_1 = A_2V_2$

$$V_1 = (A_2V_2/A_1) = \frac{\frac{®}{4}d_2^2XV_2}{\frac{®}{4}d_1^2} = \left(\frac{d_2}{d_1}\right)^2 XV_2 = \left(\frac{0.3}{0.15}\right)^2 XV_2 = 4V_2$$

Substituting the value of $V_1$ in different head losses, we have

$h_i = 0.5 V_i^2/ 2g = (0.5 X (4V_2)^2 )/ 2g = 8V_2^2/2g$

$$h_{f1} = \frac{4X0.01X25X(4V_2^2)}{0.15X2g} = \frac{4X0.01X25X16}{0.15}X\frac{V_2^2}{2g} = 106.67\frac{V_2^2}{2g}$$

$he = (V_1 – V_2)^2/ 2g = (4V_2 – V_2)^2/2g = 9V_2^2/2g$

$$h_{f2} = \frac{4X0.01X15X(V_2^2)}{0.3X2g} = \frac{4X0.01X15}{0.3}X\frac{V_2^2}{2g} = 2.0\frac{V_2^2}{2g}$$

Substituting the values of these losses in equation (i), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2X\frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}[1+8+106.67+9+2] = 126.67\frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\frac{8.0x2xg}{126.67}} = \sqrt{\frac{8.0X2X9.81}{126.67}} = 1.113\text{ m/s}$$

Rate of flow $Q = A_2XV_2 = \frac{®}{4}(0.3)^2 X 1.113 = 0.07867\ m^3/s$ = **78.67 litres/sec.**

***PROBLEM 3***

**A pipe line, 300mm in diameter amd 3200m long is used to pump up 50kg per second of an oil whose density is 950n kg/m³.and whose Kinematic viscosity is 2.1 stokes. The center of the pipe at upper end is 40m above than at the lower end. The discharge at the upper end is atmospheric. Find the pressure at the lower end and draw the hydraulic gradient and the total energy line**.

**Given:**

Dia of pipe d = 300mm = 0.3m

Length of pipe L = 3200m

Mass M = 50kg/s = ρ. Q

Discharge Q = 50/ ρ = 50/950 = 0.0526 $m^3/s$

Density ρ = 950 $kg/m^3$

Kinematic viscosity ν = 2.1 stokes = 2.1 $cm^2/s$ = 2.1 $X10^{-4}\ m^2/s$

Height of upper end = 40m

Pressure at upper end = atmospheric = 0

Renolds number, $R_e$ = VXD/ν, where V = Discharge / Area

$= 0.0526/(\frac{®}{4}(0.3)^2) = 0.744$ m/s

$R_e = (0.744X0.30)/(2.1X10^{-4}) = 1062.8$

Co – efficient of friction, $f = 16/R_e = 16/1062.8 = 0.015$

Head lost due to friction, $h_f$

$$= \frac{4XfXL\ XV^2}{d\ X2g} = \frac{4X0.015X3200X(0.744)^2}{0.3X2X9.81} = 18.05m \text{ of oil.}$$

Applying the Bernoulli's equation at the lower and upper end of the pipe and taking datum line passing through the lower end, we have

$$(P_1/\rho g) + (V_1^2/2g) + Z_1 = (P_2/\rho g) + (V_2^2/2g) + Z_2 + h_f$$

but $Z_1 = 0$, $Z_2 = 40m.$, $V_1 = V_2$ as diameter is same.

$P_2 = 0$, $h_f = 18.05m$

Substituting these values, we have

$= 5400997\ N/m^2 = 54.099\ N/cm^2$.

**H.G.L. AND T.E.L.**

$V^2/2g = (0.744)^2/2X9.81 = 0.0282$ m

$p_1/\rho g = 58.05$ m of oil

$p_2/\rho g = 0$

Draw a horizontal line AX as shown in fig. From A draw the centerline of the pipe in such way that point C is a distance of 40m above the horizontal line. Draw a vertical line AB through A such that AB = 58.05m. Join B with C. then BC is the hydraulic gradient line.

Draw a line DE parallel to BC at a height of 0.0282m above the hydraulic gradient line. Then DE is the total energy line.

***PROBLEM 4***

**A main pipe divides into two parallel pipes, which again forms one pipe as shown. The length and diameter for the first parallel pipe are 2000m and 1.0m respectively, while the length and diameter of 2$^{nd}$ parallel pipe are 2000m and 0.8m. Find the rate of flow in each parallel pipe, if total flow in main is 3.0 $m^3$/s. the co-efficient of friction for each parallel pipe is same and equal to 0.005.**

**Given:**

Length of Pipe 1 $L_1 = 2000$m

Dia of pipe1 $d_1 = 1.0$m

Length of pipe 2 $L_2 = 2000$m

Dia of pipe 2 $d_2 = 0.8$m

Total flow $Q = 3.0 m^3/s$

$f_1 = f_2 = f = 0.005$

let $Q_1$ = discharge in pipe 1

let $Q_2$ = discharge in pipe 2

from equation, $Q = Q_1 + Q_2 = 3.0$ ---------------(i)

using the equation we have

$$\frac{4Xf_1XL_1XV_1^2}{d_1X2g} = \frac{4Xf_2XL_2XV_2^2}{d_2X2g}$$

$$\frac{4X0.005X2000XV_1^2}{1.0X2X9.81} = \frac{4X0.005X2000XV_2^2}{0.8X2X9.81}$$

$$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8} or V_1^2 = \frac{V_2^2}{0.8}$$

$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{0.894}$$

Now, $Q_1 = \frac{\pi}{4} d_1^2 X V_1 = \frac{\pi}{4} (1)^2 X (V_2/0.894)$

And $Q_2 = \frac{\pi}{4} d_2^2 X V_2 = \frac{\pi}{4} (0.8)^{2} X (V_2) = \frac{\pi}{4} (0.64) X (V_2)$

Substituting the value of $Q_1$ and $Q_2$ in equation (i) we get

$\frac{\text{®}}{4}$ $(1)^2X(V_2 / 0.894)+$ $\frac{\text{®}}{4}$ $( 0.64)X(V_2) = 3.0$ or $0.8785\ V_2 + 0.5026\ V_2 = 3.0$

$V\ _2[0.8785+0.5026] = 3.0$ or $V = 3.0 / 1.3811 = 2.17$ m/s.

Substituting this value in equation (ii),

$V_1\ = V_2 / 0.894 = 2.17 / 0.894$ m/s

Hence $Q_1 = \frac{\text{®}}{4}\ d_1^{\,2}XV_1 = \frac{\text{®}}{4}\ 1^2X2.427 =$ **$1.096\ m^3/s$**

$Q_2 = Q - Q_1 = 3.0 - 1.906 =$ **$1.904\ m^3/s$**.

***PROBLEM 5***

**Three reservoirs A, B, C are connected by a pipe system shown in fig. Find the discharge into or from the reservoirs B and C if the rate of flow from reservoirs A is 60 litres / s. find the height of water level in the reservoir C. take f = 0.006 for all pipes.**

**Given:**

Length of pipe AD, $L_1 = 1200$m

Dia of pipe AD, $d_1 = 30$cm $= 0.3$m

Discharge through AD, $Q_1 = 60$lit/s $= 0.06\ m^3/s$

Height of water level in A from reference line , $Z_A= 40$m

For pipe DB, length $L_2 = 600$mm, Dia., $d_2 = 20$cm $= 0.20$m, $Z_B= 38.0$

For pipe DC, length $L_3 = 800$mm, Dia., $d_3= 30$cm $= 0.30$m,

Applying the Bernoulli's equations to point E and, $Z_A = Z_D+ \frac{p_D}{{}^\circ g} +h_f$

Where $h_f = \frac{4Xf_1XL_1XV_1^2}{d_1X2g}$, where $V_1 = Q_1$ / Area $= 0.006 / (\frac{\text{®}}{4}\ ( 0.3)^2) = 0.848$ m/s.

$$h_f = \frac{4X0.006X1200X0.848^2}{0.3X2X9.81} = 3.518 \text{ m.}$$

$$\{Z_D+ \frac{p_1}{{}^\circ g}\} = 40.0 - 3.518 = 36.482 \text{ m}$$

Hence piezometric head at D = 36.482m. Hence water flows from B to D.

Applying Bernoulli's equation to point B and D

$$Z_B = \{Z_D+ \frac{p_D}{{}^\circ g}\}+h_{f2} \text{ or } 38 = 36.482 + h_{f2}$$

$$h_{f2} = 38 - 36.482 = 1.518\text{m}$$

But $h_{f2} = \frac{4XfXL_2XV_2^2}{d_2X2g} = \frac{4X0.006X600XV_2^2}{0.2X2X9.81}$

$$1.518 = \frac{4X0.006X600XV_2^2}{0.2X2X9.81}$$

$$V_2 = \sqrt{\frac{1.518X0.2X2X9.81}{4X0.006X600}} = 0.643m/s$$

Discharge $Q_2 = V_2X\frac{\text{®}}{4}\ (d_2)^2 = 0.643\ X\ \frac{\text{®}}{2}X(0.2)^2 =$ **$0.0202m^3/s$ = 20.2lit/s.**

Applying Bernoulli's equation to D and C

$\{Z_D + \frac{p_D}{^\circ g}\} = Z_C + h_{f3}$

$36.482 = Z_C + \frac{4 X f X L_3 X V_3^2}{d_3 X 2g}$ $where, V_3 = \frac{Q_3}{\frac{®}{4} d_3^2}$

but from continuity $Q_1 + Q_2 = Q_3$

$Q_3 = Q_1 + Q_2 = 0.006 + 0.0202 = 0.0802\ m^3/s$

$V_3 = \frac{Q_3}{\frac{®}{4} d_3^2} = \frac{Q_3}{\frac{®}{4}(0.9)^2} = 1.134 m/s$

$36.482 = Z_C + \frac{4 X 0.006 X 800 X 1.134^2}{0.X 2 X 9.81} = Z_C + 4.194$

$Z_C = 36.482 - 4.194 =$ **32.288m**

***PROBLEM 6***

**A Pipe line of length 2000 m is used for power transmission. If 110.365 kW power is to be transmitted through the pipe in which water having pressure of 490.5 N/cm$^2$ at inlet is flowing. Find the diameter of the pipe and efficiency of transmission if the pressure drop over the length of pipe is 98.1 N/cm$^2$. Take f = 0.0065.**

**Given:**

Length of pipe L = 2000m.

H.P transmitted = 150

Pressure at inlet, $p = 490.5\ N/cm^2 = 490.5 X 10^4\ N/m^2$.

Pressure head at inlet, $H = p / \rho g$

Pressure drop $= 98.1\ N/cm^2 = 98.1 X 10^4/m^2$

Loss of head $h_f = 98.1 X 10^4 / \rho g = 98.1 X 10^4 / (1000 X 9.81) = 100m$

Co – efficient of friction f = 0.0065

Head available at the end of the pipe = $H - h_f = 500 - 100 = 400m$

Let the diameter of the pipe = d

Now power transmitted is given by,

$P = [\rho g X Q X (H - h_f)] / 1000$ kW.

$110.3625 = [1000 X 9.81 X Q X 400] / 1000$

$Q = [110.3625 X 1000 / (1000 X 9.81 X 400)] = 0.02812 m^3/s$

But discharge $Q = AXV = \frac{®}{4} d^2 X V$

$\frac{®}{4} d^2 X V = 0.02812$

$V = (0.02812 X 4) / 3.14 X d^2 = 0.0358 / d^2$----------(1)

Total head lost due to friction,

$h_f = \frac{4 f X L X V^2}{d X 2g}$

but, $h_f = 100m$

$$100 = h_f = \frac{4XfXL \;\; XV^2}{d \;\; X2g} = \frac{4X0.0065X2000XV^2}{dX2X9.81} = \frac{2.65XV^2}{d}$$

$$= (2.65 / d) \text{ X } (0.358/d^2)^2 = 0.003396 / d^5$$

from equation (1),

$$V = 0.0358 / d^2$$

$$100 = 0.003396 / d^5$$

$$d = (0.00396 / 100)^{1/5} = 0.1277m = 127.7mm.$$

Efficiency of power transmission is given by equation

$$¦ = \frac{H - h_f}{H} = \frac{500 - 100}{500} = 0.80 = 80\%$$

# *CE 6303 MECHANICS OF FLUIDS*

## UNIT V DIMENSIONAL ANALYSIS AND MODEL STUDIES

### Units

Fluid mechanics deals with the measurement of many variables of many different types of units. Hence we need to be very careful to be consistent.

### Dimensions and Base Units

The *dimension* of a measure is independent of any particular system of units. For example, velocity may be in metres per second or miles per hour, but dimensionally, it is always length per time, or $L/T = LT^{-1}$ . The dimensions of the relevant base units of the Système International (SI) system are:

### Derived Units

From these we have some relevant derived units (shown on the next page). Checking the dimensions or units of an equation is very useful to minimize errors. For example, if when calculating a force and you find a pressure then you know you've made a mistake.

| Quantity | Dimension | SI Unit | |
|---|---|---|---|
| | | Derived | Base |
| Velocity | $LT^{-1}$ | m/s | $m\,s^{-1}$ |
| Acceleration | $LT^{-2}$ | $m/s^2$ | $m\,s^{-2}$ |
| Force | $MLT^{-2}$ | Newton, N | $kg\,m\,s^{-2}$ |
| Pressure<br>Stress | $ML^{-1}T^{-2}$ | Pascal, Pa<br>$N/m^2$ | $kg\,m^{-1}\,s^{-2}$ |
| Density | $ML^{-3}$ | $kg/m^3$ | $kg\,m^{-3}$ |
| Specific weight | $ML^{-2}T^{-2}$ | $N/m^3$ | $kg\,m^{-2}\,s^{-2}$ |
| Relative density | Ratio | Ratio | Ratio |
| Viscosity | $ML^{-1}T^{-1}$ | $Ns/m^2$ | $kg\,m^{-1}\,s^{-1}$ |
| Energy (work) | $ML^2T^{-2}$ | Joule, J<br>Nm | $kg\,m^2\,s^{-2}$ |
| Power | $ML^2T^{-3}$ | Watt, W<br>Nm/s | $kg\,m^2\,s^{-3}$ |

*Note:* The acceleration due to gravity will always be taken as $9.81\ m/s^2$.

### SI Prefixes

SI units use prefixes to reduce the number of digits required to display a quantity. The prefixes and multiples

Be very particular about units and prefixes. For example:

- kN means kilo-Newton, 1000 Newtons;
- Kn is the symbol for knots – an imperial measure of speed;
- KN has no meaning;

**Dimensional Homogeneity:**

Dimensional homogeneity means the dimensions of each terms in an equation on both sides equal. Thus if the dimensions of each term on both sides of an equation are the same the equation is known as dimensionally homogeneous equation. The powers of fundamental dimensions (i.e., L, M, T) on both sides of the equation will be identical for a dimensionally homogeneous equation

Let us consider the equation $v = \sqrt{2gh}$

Dimension of L.H.S $= V = \frac{L}{T} = LT^{-1}$

Dimension of R.H.S $= \sqrt{2gH} = \sqrt{\frac{L}{T^2} x\, L} = \sqrt{\frac{L^2}{T^2}}$

$= \frac{L}{T} = LT^{-1}$

Dimension of L.H.S = Dimension of R.H.S $= LT^{-1}$

Equation $v = \sqrt{2gh}$ is dimensionally homogeneous.

**METHODS OF DIMENSIONAL ANALYSIS:**

If the number of variables involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods.

1.Rayleigh's method, and

2. Buckingham's π theorem

**1.Rayleigh's method:**

This method is used for determining the expression for a variable which depends upon maximum three or four variables only. If the number of independent variables becomes more than four then it is very difficult to find the expression for the dependent variable.

Let X is a variable, which depends on $X_1$, $X_2$ and $X_3$ variables. Then according to Rayleigh's method, X is function of $X_1$, $X_2$ and $X_3$ and mathematically it is written as

$$X = f\,[X_1, X_2, X_3]$$

This can also be written as $X = K\, X_1^a.\, X_2^b.\, X_3^c$

Where K is constant and a, b and c are arbitrary powers.

The values of a, b and c are obtained by comparing the powers of the fundamental dimension on both sides. Thus the expression is obtained for dependent variable.

**2.Buckingham's π theorem:**

If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into (n-m) dimensionless numbers. Each term is called π term.

Let $X_1$, $X_2$, $X_3$,……… $X_n$ are the variables involved in a physical problem. Let $X_1$ be the dependent variable and $X_2$, $X_3$,……… $X_n$ are the independent variables on which $X_1$

depends. Then $X_1$ is a function of $X_2$, $X_3$,……… $X_n$ and mathematically it is expressed as

$$X_1 = f\,(X_2, X_3, \ldots\ldots\ldots X_n) \quad \text{--------------------(1)}$$

The above equation can also be written as

$$f_1\,(X_1, X_2, X_3, \ldots\ldots\ldots X_n) = 0 \quad \text{-------------------(2)}$$

The above (2) is a dimensionally homogeneous equation. It contains n variables. If there are m fundamental dimensions then according to Buckingham's π theorem, equation (2) can be written on terms of dimensionless groups or π- terms is equal to (n-m). Hence equation (2) becomes as

$f_1(\pi_1, \pi_2, \pi_3, \dots \pi_{n-m})=0.$ -------------(3)

Each π- term is dimensionless and is independent of the system. Division or multiplication by a constant does not change the character of the π- term. Each π- term contains m+1 variables, where m is the number of fundamental dimensions and is also called repeating variables. Let in the above case $X_2$, $X_3$, and $X_n$ are repeating variables if the fundamental dimension m (M, L,T)=3. Then each π- term is written as

$\pi_1 = X_2^{a1}.X_3^{b1}.X_4^{c1}.X_1$

$\pi_2 = X_2^{a2}.X_3^{b2}.X_4^{c2}.X_5$

.

.

$\pi_{n-m} = X_2^{an-m}.X_3^{b\,n-m}.X_4^{cm}.X_n$ ----------------------(4)

Each equation is solved by the principle of dimensional homogeneity and values of $a_1$, $b_1$, $c_1$ etc. are obtained. These values are substituted in equation (4) and values of $\pi_1, \pi_2, \pi_3, \dots \pi_{n-m}$ are obtained. These values of π's are substituted in equation (3). The final equation for the phenomenon is obtained by expressing any one of the π- terms as a function of others as

$\pi_1 = \Phi[\pi_2, \pi_3, \dots \pi_{n-m}]$

$\pi_2 = \Phi[\pi_1, \pi_3, \dots \pi_{n-m}]$ ---------------------(5)

**Method of selecting Repeating variables:** The number of repeating variables are equal to the number of fundamental dimensions of the problem. The choice of repeating variables if governed by the following considerations.

1. As far as possible, the dependent variable should not be selected as repeating variable.

2. The repeating variables should be choosen in such a way that one variable contains geometric property, other variable contains flow property and third variable contains fluid property.

Variables with geometric property are (i) Length, l (ii) d (iii) Height H etc.

Variables with flow property are (i) Velocity, V (ii) Acceleration etc.

Variables with fluid property are (i) μ (ii) ρ (iii) w etc.

3. The repeating variables selected should not form a dimensionless group.

4. The repeating variables together must have the same number of fundamental dimensions.

5. No two repeating variables should have the same dimensions.

In most of fluid mechanics problems, the choice of repeating variables may be (i) d, v, ρ (ii) I, v, ρ or (iii) I, v, μ or (iv) d, v, μ.

**MODEL ANALYSIS:**

For predicting the performance of the hydraulic structures (such as dams, spill ways etc.) or hydraulic machines (such as turbines, pumps etc.), before actually constructing or manufacturing, models of the structures or machines are made and tests are performed on them to obtain the desired information.

The model is the small scale replica of the actual structure or machine. The actual structure or machine is called prototype. It is not necessary that the models should be smaller than the prototypes (though in most of cases it is), they may be larger than the prototype. The study of models of actual machines is called model analysis. Model analysis is actually an experimental method of finding solutions of complex flow problems.

The followings are the advantages of the dimensional and model analysis.

1.The performance of the hydraulic structure or hydraulic machine can be easily predicted, in advance, from its model.

2. With the help of dimensional analysis, a relationship between the variables influencing a flow problem in terms of dimensionless parameters is obtained. This relationship helps in conducting tests on the model.

3. The merits of alternative designs can be predicted with the help of model testing. The most economical and safe design may be, finally, adopted.

4.The tests performed on the models can be utilized for obtaining, in advance, useful information about the performance of the prototypes only if a complete similarity exists between the model and the prototype.

**SMILITUDE –TYPES OF SIMILARITIES:**

Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype are completely similar. Three types of similarities must exit between the model and prototype. They are

1. Geometric Similarity 2. Kinematic Similarity 3. Dynamic Similarity

1.Geometric Similarity:

The geometric similarity is said to exist between the model and the prototype if the ratio of all corresponding linear dimension in the model and prototype are equal.

$L_m$= Length of model , $b_m$= Breadth of model

$D_m$= Dismeter of model $A_m$= area of model

$\not{V}_m$= Volume of model

and $L_p$, $B_p$, $D_p$, $A_p$, $\not{V}_p$ =Corresponding values of the prototype.

For geometric similarity between model and prototype, we must have the relation,

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r$$

$L_r$ is called the scale ratio.

For area's ratio and volume's ratio the relation should be as given below.

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2$$

$$\frac{\not{V}_p}{\not{V}_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3$$

**2. Kinematic Similarity :**

Kinematic similarity means the similarity of motion between model and prototype. Thus kinematic similarity is said to exist between the model and the prototype if the ratios of the velocity and acceleration at the corresponding points in the model and at the corresponding points in the prototype are the same. Since the velocity and acceleration are vector quantities, hence not only the ratio of magnitude of velocity and acceleration at the corresponding points in the model and prototype should be same, but the directions of velocity and accelerations at the corresponding points in the model and prototype also should be parallel.

$V_{p1}$ = velocity of fluid at point 1 in prototype,

$V_{p2}$= velocity of fluid at point 2 in prototype,

$a_{p1}$= Acceleration of fluid at point 1 in prototype,

$a_{p2}$= Acceleration of fluid at point 2 in prototype,

$V_{m1}$, $V_{m2}$, $a_{m1}$, $a_{m2}$ = Corresponding values at the corresponding points of fluid velocity and acceleration in the model.

For kinematic similarity, we have

$$\frac{V_{p1}}{V_{m1}} = \frac{V_{p2}}{V_{m2}} = V_r$$

where $V_r$ is the velocity ratio.

For acceleration, we have $\frac{a_{p1}}{a_{m1}} = \frac{a_{p2}}{a_{m2}} = a_r$

where $a_r$ is the acceleration ratio.

Also the directions of the velocities in the model and prototype should be same.

**3. Dynamic Similarity:**

Dynamic similarity means the similarity of forces between the model and prototype. Thus dynamic similarity is said to exist between the model and prototype if the ratios of the corresponding forces acting at the corresponding points are equal. Also the directions of the corresponding forces at the corresponding points should be same.

$(F_i)_p$= Inertia force at a point in prototype,

$(F_v)_p$= Viscous force at the point in prototype,

$(F_g)_p$= Gravity force at the point in prototype,

$F_i)_p$, $(F_v)_p$, $(F_g)_p$ = Corresponding values of forces at the corresponding point in model.

Then for dynamic similarity, we have

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

where $F_r$ is the force ratio.

Also the directions of the corresponding forces at the corresponding points in the model and prototype should be same.

<u>**DIMENSIONLESS NUMBERS:**</u>

Dimensionless numbers are those numbers which are obtained by dividing the inertia force

by viscous force or pressure force or surface tension force or elastic force. As this is a ratio of one force to the other force, it will be a dimensionless number. These dimensionless numbers are also called non-dimensional parameters. The following are the important dimensionless numbers:

1. Reynold's number
2. Froud's number
3. Euler's number
4. Weber's number

5. Mach's number

**1.)Reynold's number:** It is defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid. The expression for Reynold's number is obtained a

$$R_e = \frac{V \times d}{\nu} \quad \text{or} \quad \frac{\rho V d}{\mu}$$

**2.Froud's Number ($F_e$) :** The Froud's Number is defined as the square root of the ratio of inertia force of a flowing fluid to the gravitational force. Mathematically, it is expressed as

$$F_e = \sqrt{\frac{F_i}{F_g}}$$

$$= \sqrt{\frac{\rho A V^2}{\rho A L g}} = \sqrt{\frac{V^2}{Lg}} \; \frac{V}{\sqrt{Lg}}$$

**3. Euler's number ($E_u$):** It is defined as the square root of the ratio of inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

Euler's number $E_u = \sqrt{\dfrac{F_i}{F_p}}$

**4. Weber's number ($W_e$):** It is defined as the square root of the ratio of inertia force of a flowing fluid to the surface tension force. Mathematically, it is expressed as

Weber's number $W_e = \sqrt{\dfrac{F_i}{F_g}}$

**5. Mach number (M):** Mach number is defined as the square root of the ratio of inertia force of a flowing fluid to the elastic force. Mathematically, it is expressed as

Mach number $M = \sqrt{\dfrac{\text{Inertia force}}{\text{Elastic force}}} = \sqrt{\dfrac{F_i}{F_e}}$

$$M = \frac{V}{C}.$$

**MODEL LAWS OR SIMILARITY LAWS:**

1. Reynold's model law
2. Froud's model law

3. Euler's model law
4. Weber's model law
5. Mach's model law

**1.Reynold's model law:** Reynold's model law is the law in which models are based on Reynold's number. Model based on Reynold's number includes:

$$= \rho_r A_r V_r = \rho_r L_r^{2} V_r$$

**2.Froude Model law:** Froude Model law is the law in which the models are based on Froude number which means for dynamic similarity between the model and prototype, the Froude number for both of them should be equal. Froude Model law is applicable when the gravity force is only predominant force which controls the flow in addition to the force of inertia.

$$\frac{V_p}{V_m} = V_r = \sqrt{L_r}$$

**3. Weber's Model law:** Weber's Model law is the law in which models are based on Weber's number which is the ratio of the square root of inertia force to surface tension force. Hence where surface tension effects predominant in addition to inertia force, the dynamic similarity between the model and prototype is obtained by equating the Weber number of the model and its prototype. Hence according to this law:

$(W_e)_{model} = (W_e)_{prototype}$ where $W_e$ is Weber number = $\dfrac{V}{\sqrt{\sigma/\rho L}}$

**4.Mach Model law:** Mach Model law is the law in which models are based on Mach number which is the ratio of the square root of inertia force to elastic force of a fluid. Hence where force due to elastic compression predominant in addition to inertia force, the dynamic similarity between the model and prototype is obtained by equating the Weber number of the model and its prototype.

Hence according to this law:

$(M)_{model} = (M)_{prototype}$

***PROBLEM:1***

**The resisting force of (R) of a supersonic flight can be considered as dependent upon the length of aircraft "l", velocity 'V', air viscosity 'μ', air density 'ρ', and bulk modulus of air 'k'. Express the functional relationship between these variables and the resisting force.**

**(Nov 2005.)**

**Solution:**

Step 1. $R = f(l, V, \mu, \rho, k)$

$\Phi$ ( R, l, V, $\mu$, $\rho$, k) = 0

Number of variables, n = 6

Number of primary variable, m = 3

Number of **$\pi$** terms = n – m = 6 – 3 = 3

f ($\pi_1, \pi_2, \pi_3$) = 0

Step 2. Assume l, V and $\rho$ to be the repeating variables.

$\pi_1 = l^x, V^y, \mu^z, R$

**$M^O L^O T^O = [L]^X [LT^{-1}]^y [ML^{-3}]^z [MLT^{-2}]$**

**$z_1+1 = 0;$ $x_1+y_1-3z_1+1 = 0$ $-y_1-2 = 0$**

**$z_1 = -1$ $x_1-2+3+1 = 0$ $y_1 = -2.$**

**$x_2 = -2.$**

**$\pi_1 = l^{-2}V^{-2}\rho^{-1}R = \frac{R}{l^2V^2\rho}$**

**Step 3:** **$\pi_2 = l^x V^y \rho z \mu$**

**$M^O L^O T^O = [L]^X [LT^{-1}]^y [ML^{-3}]^z [ML^{-1}T^{-1}]$**

**$z_1+1 = 0;$ $x_2+y_2-3z_2 - 1 = 0$ $-y_2 - 1 = 0$**

**$z_1 = -1$ $x_2 - 1+3 - 1 = 0$ $y_2 = -1.$**

**$x_2 = -1.$**

**$\pi_2 = l^{-1}V^{-1}\rho^{-1}\mu = \frac{\mu}{lV\rho}$**

**Step 4.** **$\pi_2 = l^x V^y \rho z K$**

**$M^O L^O T^O = [L]^X [LT^{-1}]^y [ML^{-3}]^z [ML^{-1}T^{-2}]$**

**$Z_3+1 = 0;$ $x_3+y_3-3z_3 - 1 = 0$ $-y_3 - 2 = 0$**

**$Z_3 = -1$ $x_3 - 2+3 - 1 = 0$ $y_2 = -2.$**

**$x_3 = 0.$**

**$\pi_2 = l^{-0}V^{-2}\rho^{-1} K = \frac{K}{\rho V^2}$**

**Step 5.**

$$\Phi\,(\pi_1, \pi_2, \pi_3) = 0 \qquad \Phi\left(\frac{R}{l^2V^2\circ}, \frac{«}{lV\circ}, \frac{K}{\circ V^2}\right) = 0$$

$$R = l^2\,V^2\,\boldsymbol{\rho}\,\Phi\left(\frac{«}{lV\circ}, \frac{K}{\circ V^2}\right) = 0$$

***PROBLEM:2***

**A Ship is 300m long moves in seawater, whose density is 1030 kg/m$^3$, A 1:100 model of this to be tested in a wind tunnel. The velocity of air in the wind tunnel around the model is 30m/s and the resistance of the model is 60N. Determine the velocity of ship in seawater and also the resistance of the ship in sea water. The density of air is given as 1.24 kg/m$^3$. Take the Kinematic viscosity of seawater and air as 0.012 stokes and 0.018 stokes respectively.**

**Given:**

For prototype,

Length $L_p$ = 300m

Fluid = Sea water

Density of water = 1030 kg/m$^3$

Kineamtic viscosity $v_p$ = 0.018 stokes = 0.018 X10$^{-4}$ m$^2$/s

Let velocity of ship = $V_p$

Resitance = $F_p$

**For Model**

Length $L_m$ = (1/100)X300 = 3m

Velocity $V_m$ = 30m/s

Resistance $F_m$ = 60N

Density of air $\rho_m$ = 1.24 kg/m$^3$

Kinematic viscosity of air $v_m$ = 0.018 stokes = 0.018 X10$^{-4}$ m$^2$/s

For dynamic similarity between the prototype and its model, the Reynolds's number for both of them should be equal.

$(V_p X L_p / v_p) = (V_m X L_m / v_m)$ or $(v_p / v_m) X (L_m/L_p) X V_m$

= (0.012X10$^{-4}$ / 0.018X10$^{-4}$) X (3/300) X30 = 0.2 m/s.

Resistance = Mass X Acceleration

$= \rho\, L^3 X (V/t) = \rho\, L^2 X (V/1) X (L / t) = \rho\, L^2V^2$

Then $F_p / F_m = (\rho\, L^2V^2)_p / (\rho\, L^2V^2)_m = (\rho_p/\rho_m) X (L_p/Lm)^2 X (V_p/V_m)$

$(\rho_p/\rho_m)$ = 1030 / 1.24

$F_p / F_m$ = (1030 /1.24) X (300/3)$^2$X (0.2/30) = 369.17

$F_p = 369.17 \times F_m = 369.17 \times 60 = 22150.2$ N.

***PROBLEM:3***

**A 7.2 m height and 15m long spill way discharges 94 $m^3/s$ discharge under a head of 2.0m. If a 1:9 scale model of this spillway is to be constructed, determine model dimensions, head over spillway model and the model discharge. If model experience a force of 7500N (764.53Kgf), determine force on the prototype.**

**Given:**

For prototype: height $h_p = 7.2m$

Length, $L_p = 15m$

Discharge $Q_p = 94\ m^3/s$

Head, $H_p = 2.0m$

Size of model = 1/9. of the size of prototype

Linear scale ratio, $L_r = 9$

Force experienced by model $F_p = 7500N$

Find : (i) Model dimensions i.e., height and length of model ($h_m$ and $L_m$)

(ii). Head over model i.e., $H_m$

(iii). Discharge through model i.e., $Q_m$

(iv). Force on prototype (i.e., $F_p$)

(i). Model dimensions ( $h_m$ and $L_m$ )

$$h_p/ h_m = L_p/ L_m = L_r = 9$$

$$h_m = h_p / 9 = 7.2 / 9 = 0.8\ m$$

$$L_m = L_p / 9 = 15 / 9 = 1.67\ m.$$

(ii). Head over model ($H_m$)

$$h_p/ H_m = L_r = 9$$

$$H_m = H_p / 9 = 2/9 = 0.222\ m.$$

(iii). Discharge through model ($Q_m$)

Using equation we get, $Q_p / Q_m = L_r^{2.5}$

$$Q_m = (Q_p / L_r^{2.5}) = 94 / 9^{2.5} = 0.387\ m^3/s.$$

(iv). Force on the prototype($F_p$)

Using $Fr = F_p / F_m = L_r^3$

$$F_p = F_m \times L_r^3\ 7500 \times 9^3 = 5467500N.$$

***PROBLEM:4***

**A quarter scale turbine models is tested under a head of 12m. The full-scale turbine is to work under a head of 30m and to run at 428 rpm. Find N for model. If model develops 100 kW and uses 1100 l/s at this speed, what power will be obtained from full scale turbine assuming its n is 3% better than that of model.**

**Solution :**

$D_m / D_p = 1/4$ ; $H_m = 30$ m; $N_p = 428$ rpm.

w.k.t.

$(DN/\sqrt{H})_m = (DN/\sqrt{H})_p$

$(D^2N^2)_m / (D^2N^2)_P = H_m / H_p)$

$$N_m = \sqrt{\left(\frac{D_p}{D_m}\right)^2 X \frac{H_m}{H_p} X (N_p)^2}$$

$$= \sqrt{16 X \frac{12}{30} X 428^2}$$

= 1082.7638 rpm.

(ii). $P_m = 100$ kW; $Q_m = 1.1\ m^3/s$.

$(\eta_0)_m$ = ( P/ ρg Q H)$_m$ = $100X10^3$ / (9810X1.1X12) = 0.772248 or 77.22%.

It is given in this problem that the efficiency of the actual turbine is 3% better than the model.

$(\eta_0)_p$ = 79.54159%

We know that,

$(Q / D^2\sqrt{H})_m = (Q / D^2\sqrt{H})_p$

$Q_p = (Q / \sqrt{H})_m (\sqrt{H})_p (D_p/D_m)^2$.

$Q_p = (1.1 / \sqrt{12}) X (\sqrt{30}) X 16$

$= 27.82\ m^3/s$

$P_p = (\eta_0)_p X (\rho g Q H)_p$

= 0.7954159 X 9810 X 27.82 X 30

= 6514.2917 kW.