**16BECE304A BUILDING MATERIALS & GEOLOGY**

**3 0 0 3 100**

**OBJECTIVE**:

At the end of this course the students should have learnt construction planning, Scheduling procedures and techniques, cost control monitoring and accounting, Quality control and safety during construction, Organization and use of project information

**UNIT I** **9**

Construction Planning: Basic concepts in the development of construction plans-choice of Technology and Construction method-Defining Work Tasks- Definition- Precedence relationships among activities-Estimating Activity Durations-Estimating Resource Requirements for work activities-coding systems

**UNIT II** **9**

Scheduling Procedures And Techniques: Relevance of construction schedules-Bar charts - The critical path method-Calculations for critical path scheduling-Activity float and schedules-Presenting project schedules-Critical path scheduling for Activity-on-node and with leads, Lags and Windows-Calculations for scheduling with leads, lags and windows-Resource oriented scheduling-Scheduling with resource constraints and precedences -Use of Advanced Scheduling Techniques-Scheduling with uncertain durations-Crashing and time/cost trade offs -Improving the Scheduling process – Introduction to application software

**UNIT III** **9**

Cost Control Monitoring And Accounting: The cost control problem-The project Budget-Forecasting for Activity cost control - financial accounting systems and cost accounts-Control of project cash flows-Schedule control-Schedule and Budget updates-Relating cost and schedule information

**UNIT II** **9**

Quality Control And Safety During Construction: Quality and safety Concerns in Construction-Organizing for Quality and Safety-Work and Material Specifications-Total Quality control-Quality control by statistical methods -Statistical Quality control with Sampling by Attributes-Statistical Quality control by Sampling and Variables-Safety.

**UNIT II** **9**

Organization And Use Of Project Information: Types of project information-Accuracy and Use of Information-Computerized organization and use of Information -Organizing information in databases-relational model of Data bases-Other conceptual Models of Databases-Centralized database Management systems-Databases and application programs-Information transfer and Flow **TOTAL HRS : 45**

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**TEXT BOOKS**

| Sl.No | Title of Book | Author of Book | Publisher | Year of Publishing |
|---|---|---|---|---|
| 1 | Construction Project Management Planning, Scheduling and Control | Chitkara, K.K | Tata McGraw-Hill Publishing Co., New Delhi | 2002 |
| 2 | Project Management for Construction–Fundamentals Concepts for Owners” | Chris Hendrickson and Tung Au | Prentice Hall, Pittsburgh | 2000 |

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| 2 | Financial and cost concepts for construction Management | Halpin, D.W | John Wiley and Sons New York | 2002 |

**KARPAGAM ACADEMY OF HIGHER EDUCATION**

*(Established Under Section 3 of UGC Act, 1956)*

COIMBATORE-641 021

**16BECE304A- BUILDING MATERIALS AND GEOLOGy**

**Lecture Plan**

**Staff Name : V. Venugopalan**
**Semester : 7 (2017 - 18 ODD)**
**Course Type : Core**
**Number of credits : 3**
**LTPC : 3 0 0 3**

| S.No | Lecture Duration (Hour) | Topics to be covered | Support Materials |
|---|---|---|---|
| **UNIT I Construction Planning:** | | | |
| 1. | 1 | Basic concepts in the development of construction plans | $T_1$/22,27,34 |
| 2. | 1 | choice of Technology and Construction method | $T_1$/22,27,34 |
| 3. | 1 | Defining Work Tasks- | $T_1$/50 |
| 4. | 1 | Definition- Precedence relationships among activities | $T_1$/22,27,34 |
| 5. | 1 | Estimating Activity Durations | $T_1$/22,27,34 |
| 6. | 1 | Estimating Activity Durations | $T_1$/235,251 |
| 7. | 1 | Estimating Resource Requirements for work activities | $T_1$/252,312, 325 |
| 8. | 1 | coding systems | $T_1$/306,312 |
| 9. | 1 | coding systems | $T_1$/306,433 |
| **UNIT II Scheduling Procedures And Techniques** | | | |
| 10. | 1 | Relevance of construction schedules-Bar charts - The critical path method | $T_1$/491 |
| 11. | 1 | Calculations for critical path scheduling-Activity float and schedules-Presenting project schedules- | $T_1$/494 |
| 12. | 1 | Critical path scheduling for Activity-on-node and with leads, Lags and Windows | $T_1$/212 |
| 13. | 1 | Calculations for scheduling with leads, lags and windows-Resource oriented scheduling | $T_1$/213 |
| 14. | 1 | Scheduling with resource constraints and precedences | $T_1$/218 |
| 15. | 1 | Use of Advanced Scheduling Techniques-Scheduling with uncertain durations | $T_1$/218 |
| 16. | 1 | Crashing and time/cost trade offs | T1/521,523 |
| 17. | 1 | Improving the Scheduling process | T1/523-532 |
| 18. | 1 | Introduction to application software | T1/553 |
| **UNIT III Cost Control Monitoring And Accounting** | | | |
| 19. | 1 | The cost control problem | T1/62 |
| 20. | 1 | The project Budget | T1/65,72 |

| | | | |
|---|---|---|---|
| 21. | 1 | Forecasting for Activity cost control | T1/86,146 |
| 22. | 1 | financial accounting systems and cost accounts | T1/53,15 |
| 23. | 1 | Control of project cash flows-Schedule control | T1/ 108 |
| 24. | 1 | Schedule and Budget updates | T1/ 68 |
| 25. | 1 | Schedule and Budget updates | T1/ 63 |
| 26. | 1 | Relating cost and schedule information | T1/ 72 |
| 27. | 1 | Relating cost and schedule information | T1/ 53 |
| **UNIT IV Quality Control And Safety During Construction:** | | | |
| 28. | 1 | Quality and safety Concerns in Construction | T1/ 98 |
| 29. | 1 | Organizing for Quality and Safety | T1/ 96 |
| 30. | 1 | Work and Material Specifications | T1/ 109 |
| 31. | 1 | Total Quality control | T1/ 89 |
| 32. | 1 | Quality control by statistical methods | T1/ 98 |
| 33. | 1 | -Statistical Quality control with Sampling by Attributes | T1/ 109 |
| 34. | 1 | Statistical Quality control by Sampling and Variables | T1/ 106 |
| 35. | 1 | Safety | T1/ 120 |
| 36. | 1 | Safety | T1/ 105 |
| **UNIT V Organization And Use Of Project Information:** | | | |
| 37. | 1 | Types of project information-Accuracy and Use of Information- | T1/ 90 |
| 38. | 1 | Computerized organization and use of Information - | T1/ 91 |
| 39. | 1 | Organizing information in databases-relational model of Data bases- | T1/ 98 |
| 40. | 1 | Other conceptual Models of Databases- | T1/ 96 |
| 41. | 1 | Centralized database Management systems- | T1/ 95 |
| 42. | 1 | Databases and application programs- | $T_1$/120 |
| 43. | 1 | Databases and application programs- | $T_1$/77 |
| 44. | 1 | Information transfer and Flow. | $T_1$/70 |
| 45. | 1 | Information transfer and Flow. | $T_1$/70 |

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| 1 | Construction Project Management Planning, Scheduling and Control | Chitkara, K.K | Tata McGraw-Hill Publishing Co., New Delhi | 2002 |
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**WEBSITES:**

- http://www.icivilengineer.com
- http://www.engineeringcivil.com/
- http://www.aboutcivil.com/
- http://www.engineersdaily.com
- http://www.asce.org/
- http://www.cif.org/
- http://icevirtuallibrary.com/
- http://www.ice.org.uk/
- http://www.engineering-software.com/ce/

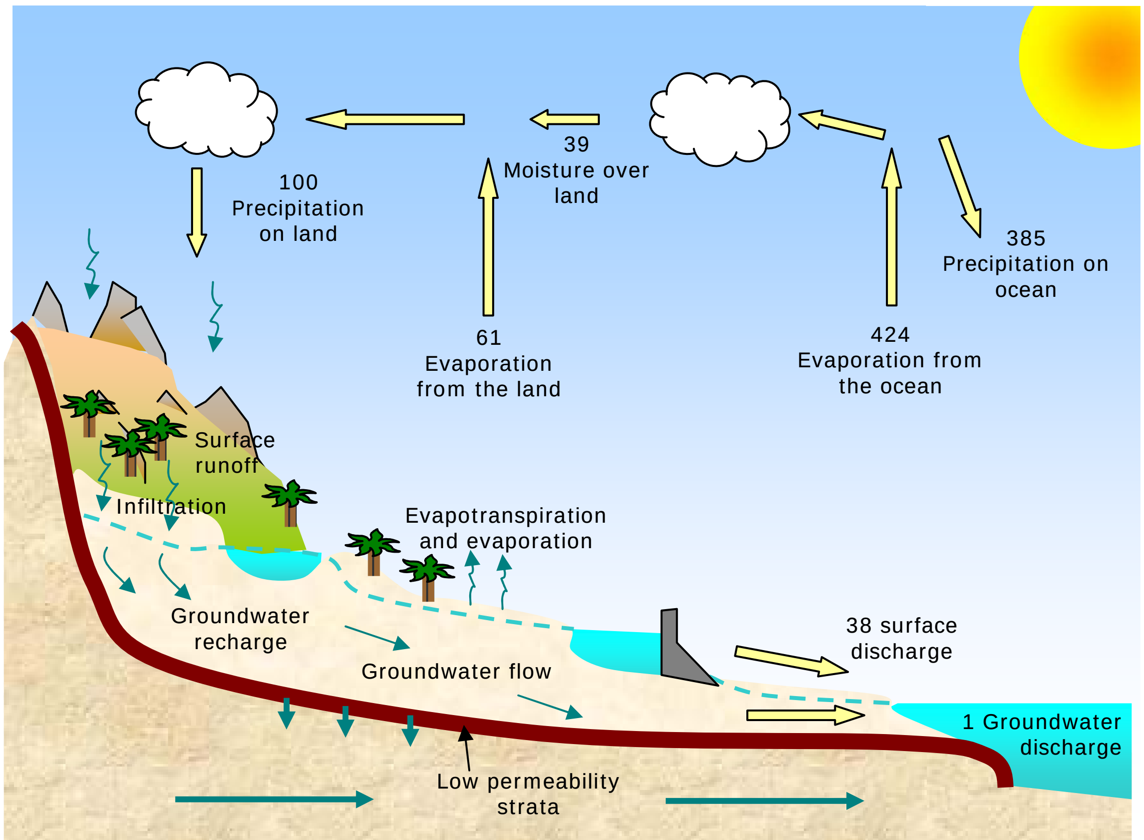
**COURSE CO ORDINATOR**

**HOD (CIVIL)** **DEAN/FOE**

**Lecture Packet #1: Course Introduction, Water Balance Equation**

**Where does water come from?**

It cycles. The total supply doesn't change much.



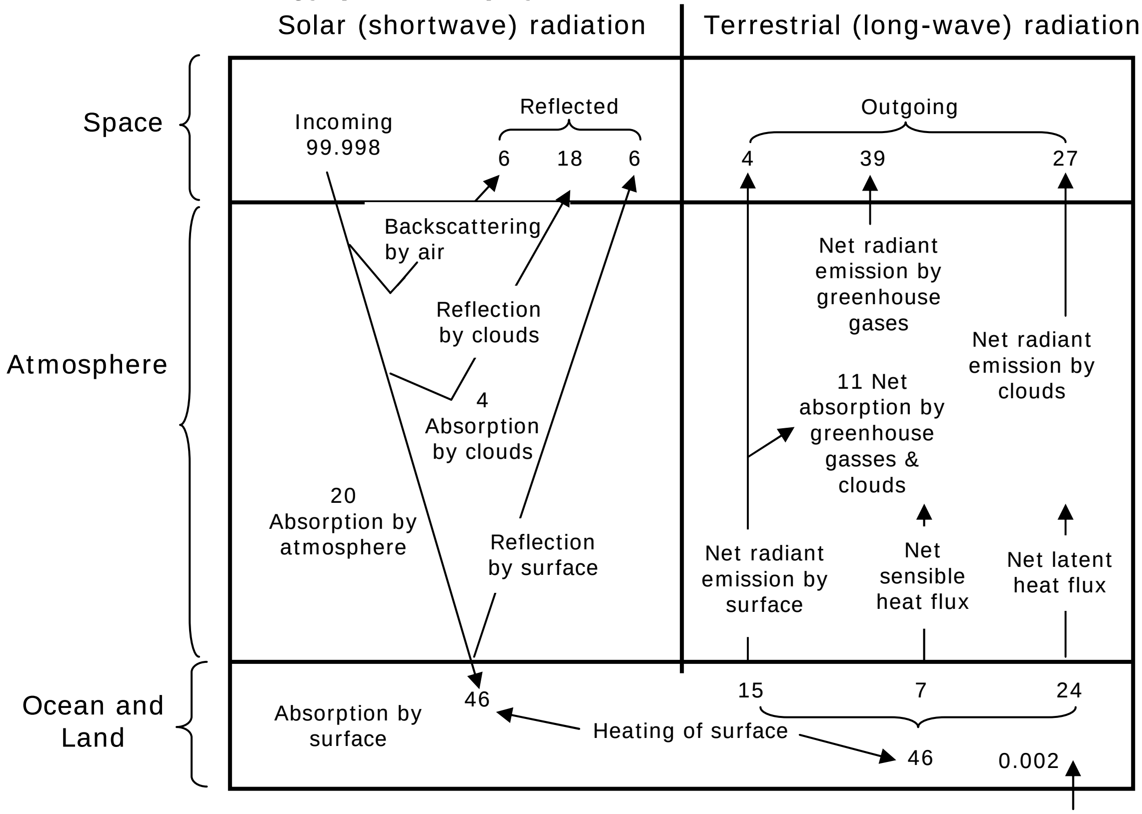

**Hydrologic cycle with yearly flow volumes based on annual surface precipitation on earth, ~119,000 $km^3$/year.**

<u>3000 BC – Ecclesiastes 1:7 (Solomon)</u>
"All the rivers run into the sea; yet the sea is not full; unto the place from whence the rivers come, thither they return again."

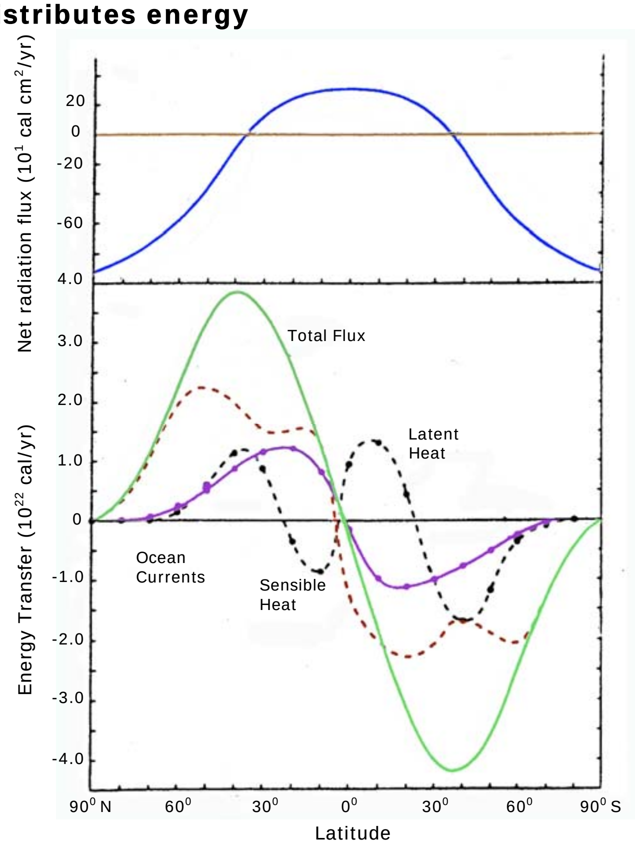
<u>Greek Philosophers (Plato, Aristotle)</u> embraced the concept, but mechanisms were not understood.

<u>17$^{th}$ Century</u> **–** Pierre Perrault showed that rainfall was sufficient to explain flow of the Seine.

**The earth's energy (radiation) cycle**

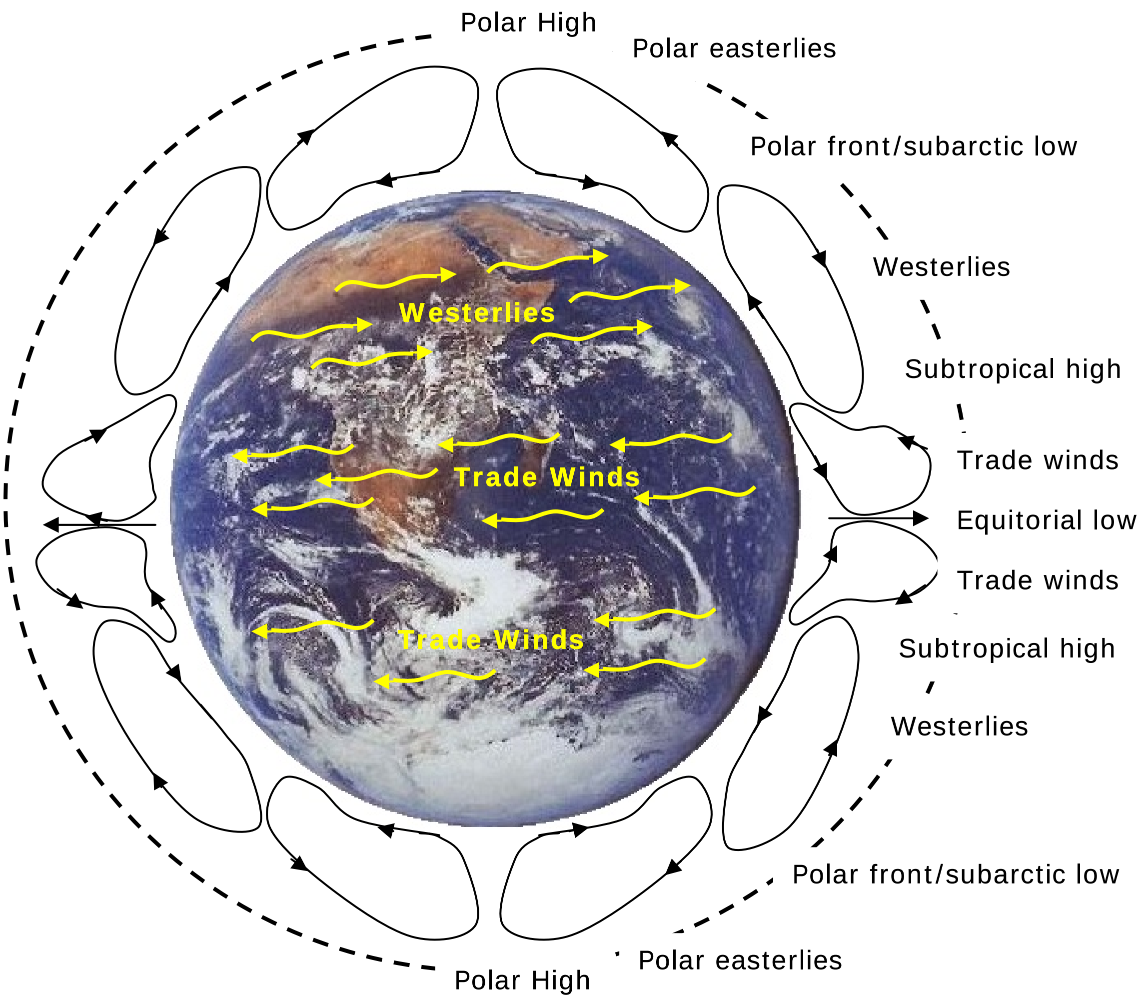



**Circulation redistributes energy**

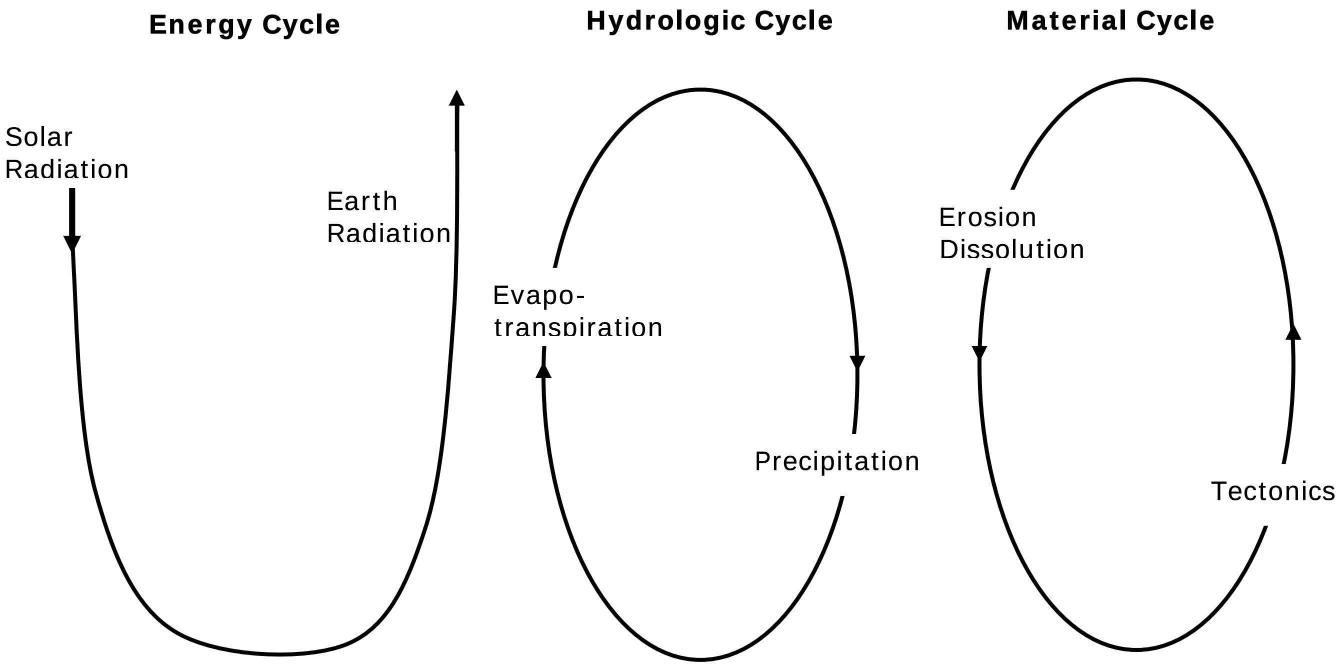


**The Earth's Energy (Radiation) Cycle:**

**Spatial distribution of energy and temperature drives circulation – both global and local:**



## Coupled Earth Cycles

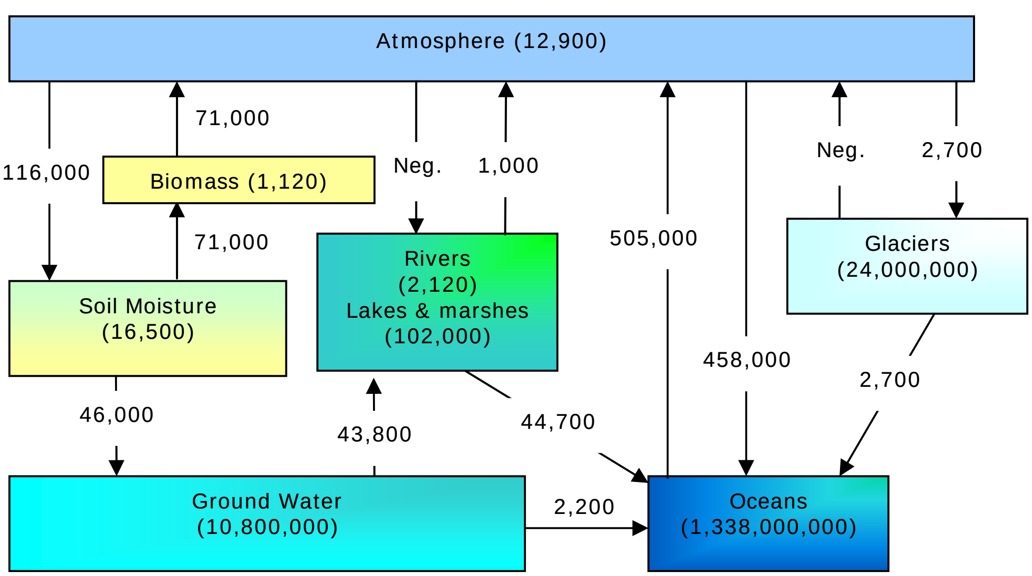


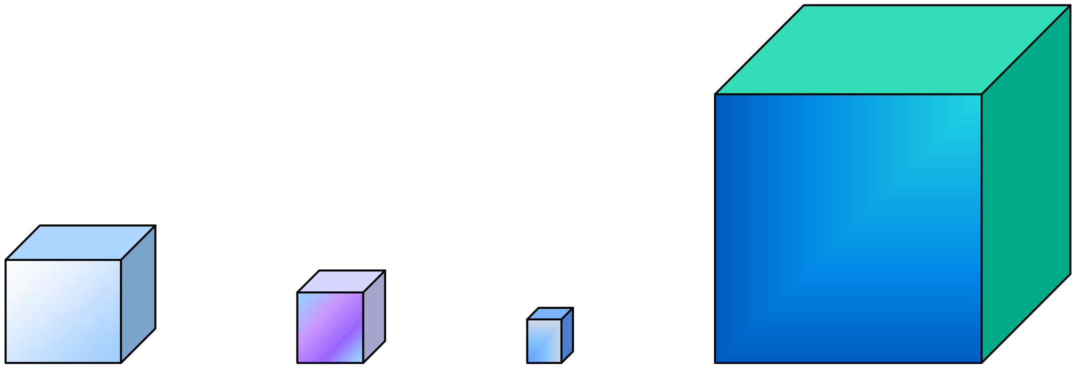

Estimates from river outflows indicate 17 x $10^9$ Tons/Year of material is transported into the Ocean. Another 2 or 3 x $10^9$ Tons/Year is trapped in reservoirs.

80% of material transported is particulate
20% is dissolved

0.05 mm/yr (may have been accelerated by man, and actual rates may be much larger)

Local rates depend on relief, precipitation, and rock type.






**Relative volumes of water in glaciers, fresh water, atmosphere and oceans.**

**Estimate of the World Water Balance**

| Parameter | Surface area ($km^2$) X $10^6$ | Volume (km3) X $10^6$ | Volume % | Equivalent depth (m) | Residence Time |
|---|---|---|---|---|---|
| Oceans and seas | 361 | 1370 | 94 | 2500 | ~4000 years |
| Lakes and reservoirs | 1.55 | 0.13 | <0.01 | 0.25 | ~10 years |
| Swamps | <0.1 | <0.01 | <0.01 | 0.007 | 1-10 years |
| River channels | <0.1 | <0.01 | <0.01 | 0.003 | ~2 weeks |
| Soil moisture | 130 | 0.07 | <0.01 | 0.13 | 2 weeks – 1 year |
| Groundwater | 130 | 60 | 4 | 120 | 2 weeks – 10,000 years |
| Icecaps and glaciers | 17.8 | 30 | 2 | 60 | 10-1000 years |
| Atmospheric water | 504 | 0.01 | <0.01 | 0.025 | ~10 days |
| Biospheric water | <0.1 | <0.01 | <0.01 | 0.001 | ~1 week |

Amazon is 6,000 $km^3$/yr (~5x more than Zaire-Congo)
0.0003% is potable and available.

What are water needs for humans?

Primitive conditions – 3 to 5 gallons/day
Urban use – 150 gallons/day
US Fresh Water Use – 1,340 gallons/day

Where does the water go? To make things…to clean things….

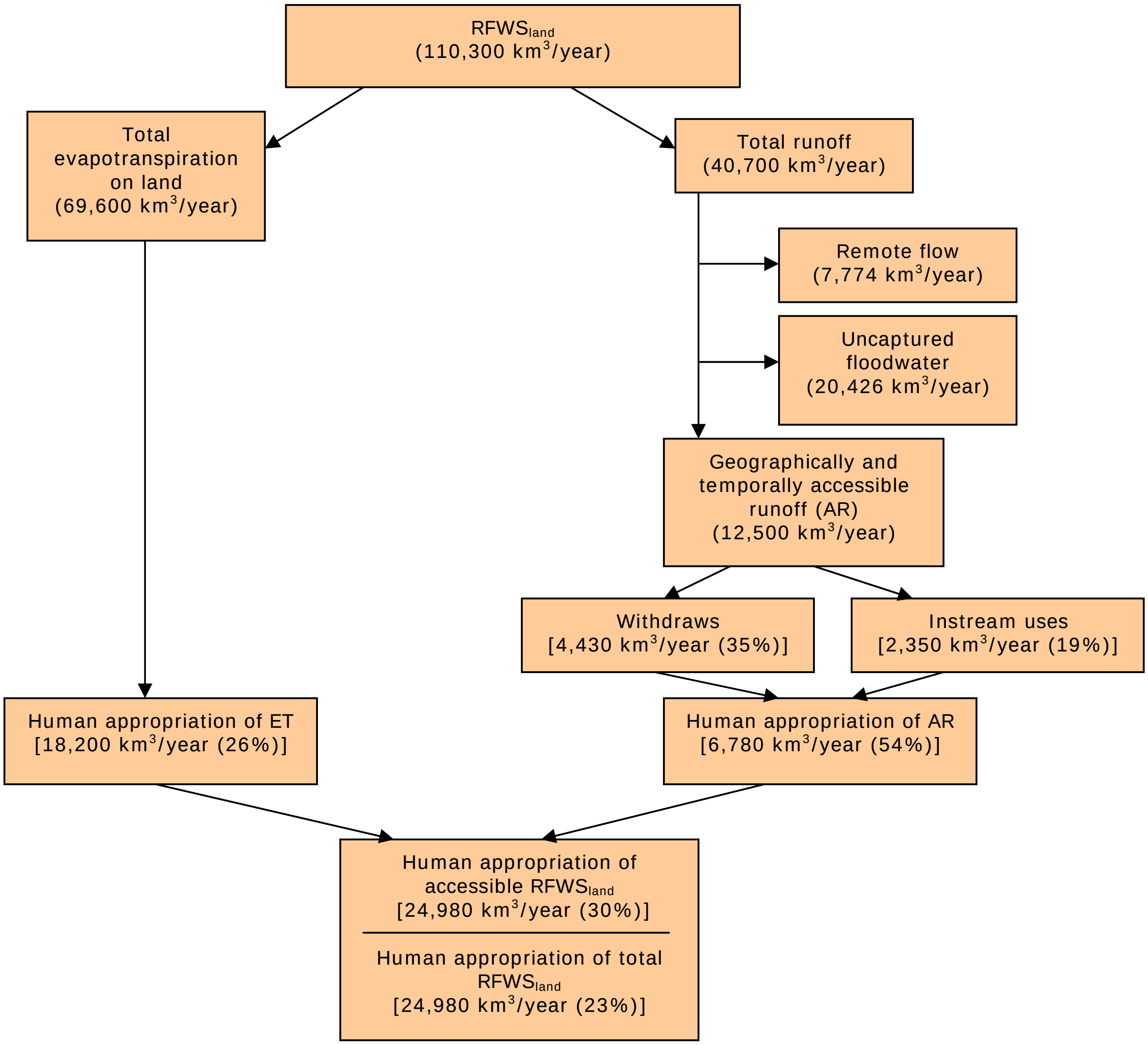
| Item | Gallons |
|---|---|
| 1 pound of cotton | 2,000 |
| 1 pound of grain-fed beef | 800 |
| 1 loaf of bread | 150 |
| 1 car | 100,000 |
| 1 kilowatt of electricity | 80 |
| 1 pound of rubber | 100 |
| 1 pound of steel | 25 |
| 1 gallon of gasoline | 10 |
| 1 load of laundry | 60 |
| 1 ten-minute shower | 25-50 |

**Some points:**

- A lot of water is used for agriculture
  - 56% (76 BGD) is consumptive use
  - 20% (28 BGD) is lost in conveyance
  - 24% (33 BGD) is return flow
- A lot of water is used for thermoelectric power generation
  - 87% of all industrial water use
- A small savings in either of these categories would free up significant quantities for public supplies

**Water Use in the US in 1990**

| Fresh Water Use 1990 Total 339 BGD | |
|---|---|
| California | 35.1 |
| Texas | 20.1 |
| Idaho | 19.7 |
| Illinois | 18.0 |
| Colorado | 12.7 |
| Louisiana | 11.7 |
| Michigan | 11.6 |
| New York | 10.5 |
| Pennsylvania | 9.8 |
| Indiana | 9.4 |
| Montana | 9.3 |
| Four states account for 27% | |
| 11 states account for 50% | |




Analysis of human appropriation of renewable freshwater supply (RFSW) on land. *Figure adapted from Postel et al., Science, (271) p. 758, Feb. 9, 1996*

## Postel et al.'s Calculations

1) Calculation of appropriated ET – indirect
    - 132 billion tons of biomass produced a year (Vitousek, 1992).
    - 30% used by people.
    - Approximate people's use of ET from the proportion of biomass – 30%.
    - Subtract agricultural irrigation (2,000 km$^3$/year) and assume half the water. from parks and lawns is irrigation (80 km$^3$/year).

2) Inaccessible runoff
    - 95 of the Amazon, Half of the Zaire-Congo and all of the Polar rivers 7774 .km$^3$/year (did not consider other northern rivers – conservative?).
    - Flood water. 11,100 km$^3$/year of runoff is base flow (6).
    - Capacity of man-made reservoirs is 5,500 km$^3$/year.
    - Accessible runoff = Baseflow + reservoir capacity = 12,500.

adjusted for accessible

3) Withdrawals (consumed)

- 12,000 $m^3$/ha average irrigation water application.
- 240,000,000 ha of world irrigated area.
- 2,880 $km^3$/year.
- 65% consumed = 1,870 $km^3$/year.

- Industrial use: 975 $km^3$/year, 9% consumed, 90 $km^3$/year.

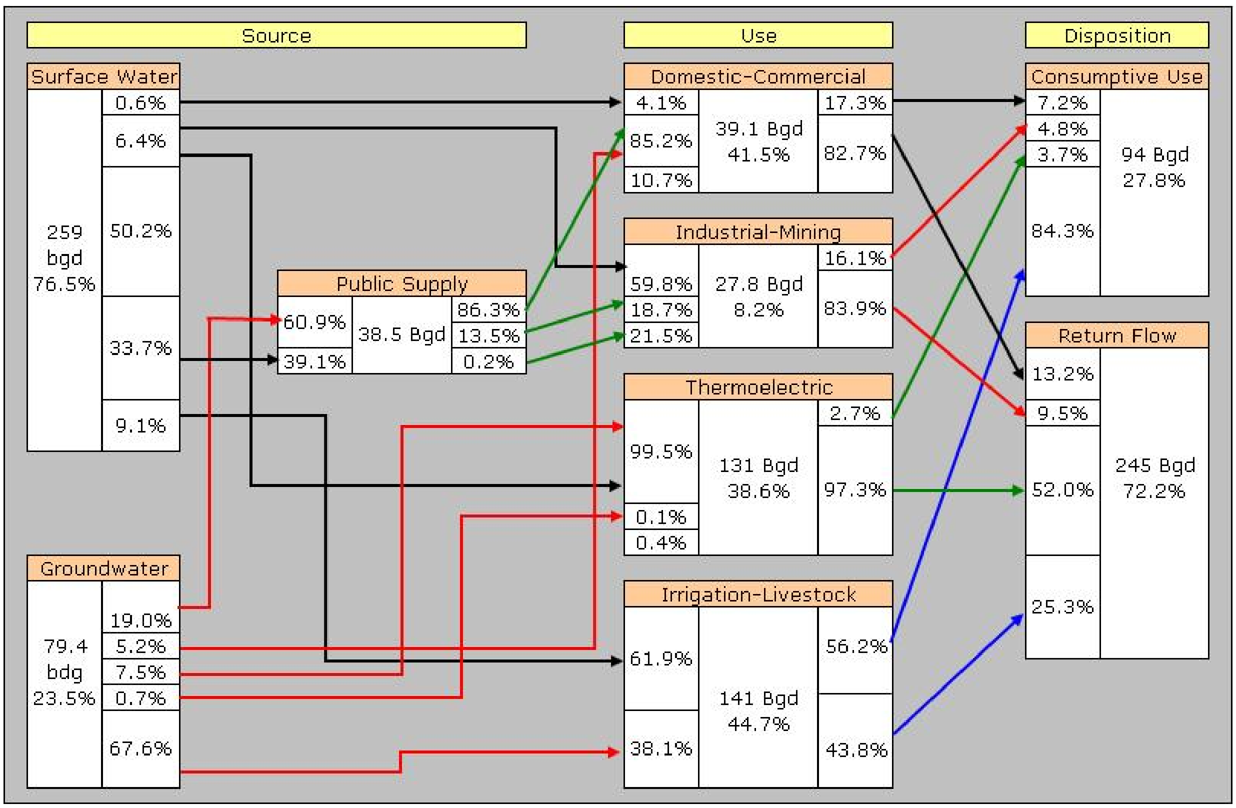
- Municipal use: 300 $km^3$/year, 17% consumed, 50 $km^3$/year.

- Reservoir losses – 5% loss, 275 $km^3$/year.

- Total consumed = 2,285 $km^3$/year.

4) Instream use

- 28.3 L/s per 1,000 population, applied to 1990 population, 4,700 $km^3$/year.
- Assuming 50% of waste gets treatment – 2,350 $km^3$/year.
- Neglects dispersed pollution (agricultural) and flood waters.




Estimated water use in the United States, billion gallons per day (bgd), for domestic and commercial purposes. *Adapted from Solley, Pierce, and Perlman, 1993.*

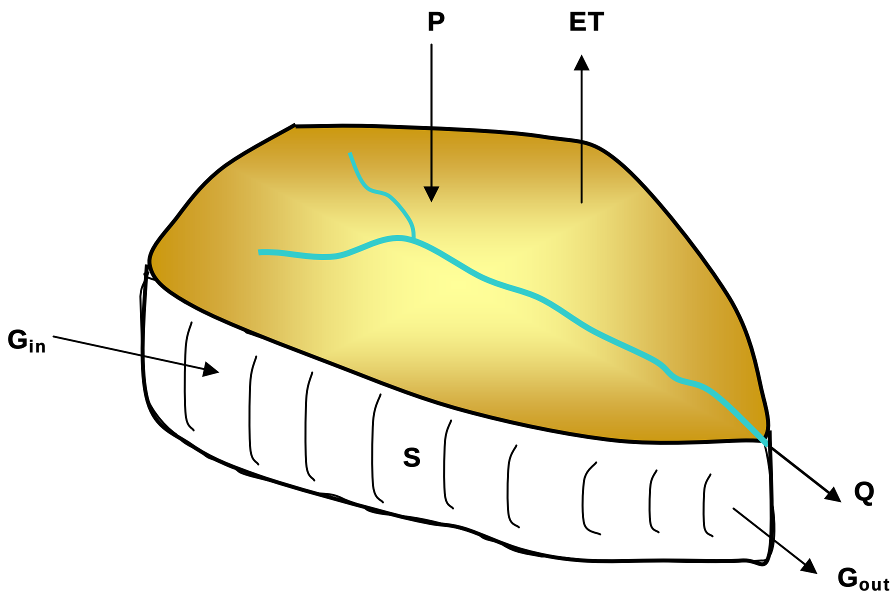
**Relative Merits of Surface and Subsurface Reservoirs**

| Surface Reservoirs | Subsurface Reservoirs |
|---|---|
| Disadvantages | Advantages |
| Few new sites free (in USA) | Many large-capacity sites available |
| High evaporative loss, even where humid climate prevails | Practically no evaporative loss |
| Need large areas of land | Need very small areas of land |
| May fail catastrophically | Practically no danger of failure |
| Varying water temperature | Water temperature uniform |
| Easily polluted | Usually high biological purity, although pollution can occur |
| Easily contaminated by radioactive fallout | Not rapidly contaminated by radioactive fallout |
| Water must be conveyed | Act as conveyance systems, thus obviating the need for pipes or canals |

## Watershed Hydrologic Budgets

Delineation of a watershed (drainage basin, river basin, catchment)

- Area that topographically appears to contribute all the water that flows through a given cross section of a stream. In other words, the area over which water flowing along the surface will eventually reach the stream, upstream of the cross-section.
- Horizontal projection of this area is the drainage area.
- The boundaries of a watershed are called a divide, and can be traced on a topographic map by starting at the location of the stream cross-section then drawing a line away from the stream that intersects all contour lines at right angles. If you do this right, the lines drawn from both sides of the stream should intersect. Moving to either side



**Water Balance Equation**

$$\frac{\partial S}{\partial t} = P + G_{in} - \left(Q + ET + G_{out}\right)$$

At steady-state: $\frac{\partial S}{\partial t} = 0$

$$P + G_{in} = \left(Q + ET + G_{out}\right)$$

Hydrologic Production

Runoff, or Hydrologic Circulation

Q and P are the only quantities that we can try to measure directly. If steady state is assumed, these measurements can be used to calibrate models of evapotranspiration and groundwater flow.

From and engineering point of view we are interested in understanding what controls Q.

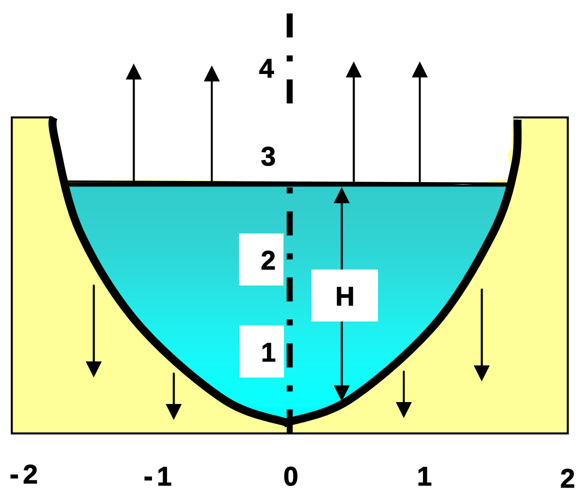
$$Q = P + (G_{in} - G_{out}) - ET$$

How much of Q is available to human use?

How can we increase Q?

**Lake Parabola**

(A simple quadratic model of the cross section of a circular lake)




$$H = cr^2$$

$$A = \pi r^2 = \frac{\pi}{c} H$$

$$V(H) = \int_0^H A(h)dh = \int_0^H \frac{\pi}{c} H dh = \frac{\pi}{2c} H^2$$

**Discharge into Lake Parabola**

If there is constant volume flux Q $m^3$/day into the lake, how does the depth depend on time? (At time 0, the depth is $H_o$ m)

Since there is no outflow, the charge in storage must equal the inflow

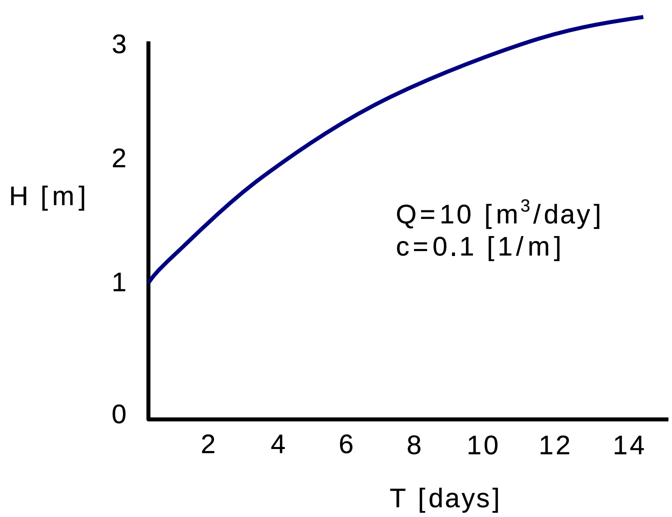
Mass-balance (water balance) equation

$$Q = \frac{dV(H)}{dt} = \frac{\pi}{2c}\frac{d(H^2)}{dt} = \frac{\pi}{c} H \frac{dH}{dt}$$

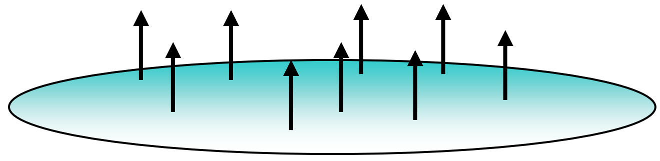
This differential equation can be solved by separation of variables

$$\int_0^T \frac{cQ}{\pi} dt = \int_{H_o}^{H} h dh$$

$$\frac{cQ}{\pi} T = \frac{H^2}{2} - \frac{H_o^2}{2} \qquad\qquad H = \sqrt{\frac{2cQ}{\pi} T + H_o^2}$$



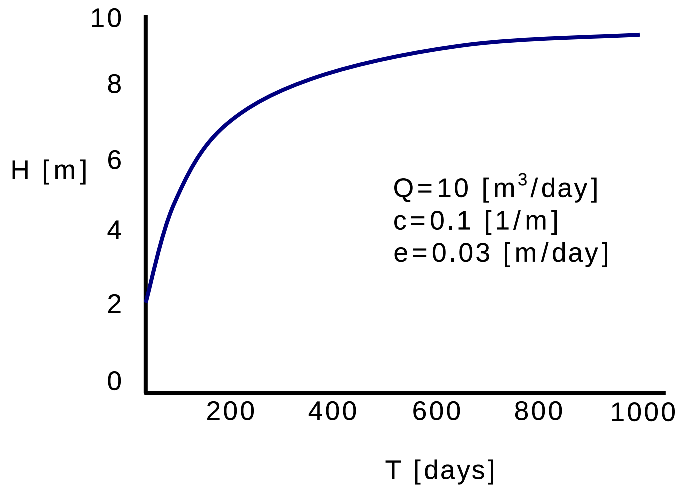

**Discharge into and Evaporation out of Lake Parabola**



e = rate of evaporation [L/T]

$$\frac{dV(H)}{dt} = Q - Ae = Q - \frac{\pi}{c} He$$

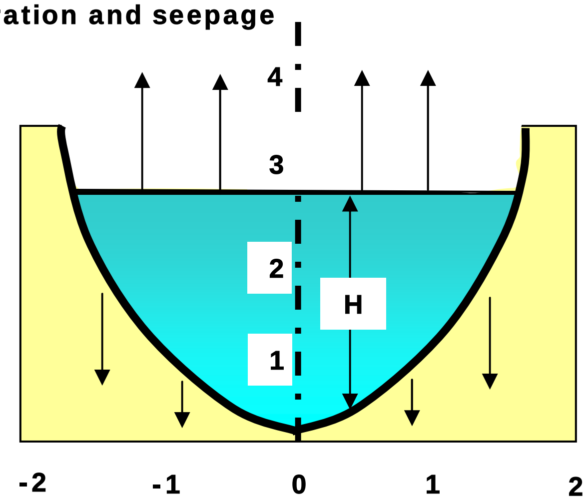
$$\frac{\pi}{c} H \frac{dH}{dt} = Q - \frac{\pi}{c} He \qquad\qquad \frac{dH}{dt} = \frac{Qc}{\pi H} - e$$




**Steady State Solution** $\cancel{\frac{dH}{dt}} = \frac{Qc}{\pi H} - e$

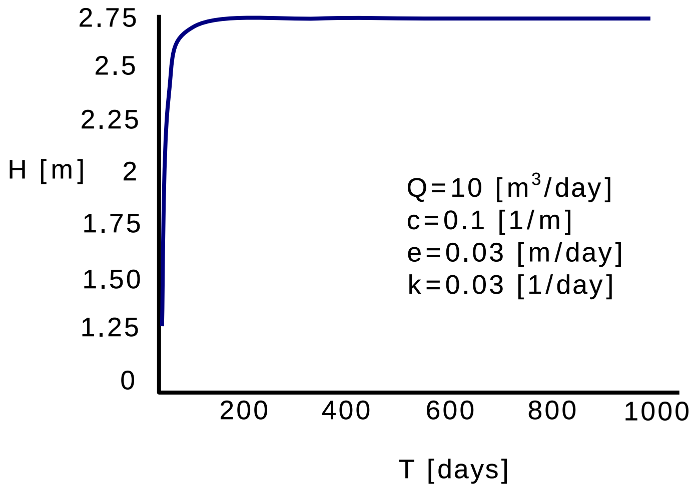
$$H = \frac{Qc}{\pi e} = \frac{(10)(0.1)}{\pi(0.03)} = 10.6 \ \boldsymbol{m}$$

**Discharge, evaporation and seepage**




$$\frac{dV(H)}{dt} = Q - eA - kHA = Q - \frac{e\pi}{c}H - \frac{k\pi}{c}H^2$$

$$\frac{\pi}{c}H\frac{dh}{dt} = Q - \frac{e\pi}{c}H - \frac{k\pi}{c}H^2 \qquad \frac{dh}{dt} = \frac{Qc}{\pi H} - e - kH$$




**Steady State Solution** $\cancel{\frac{dh}{dt}} = \frac{Qc}{\pi H} - e - kH$

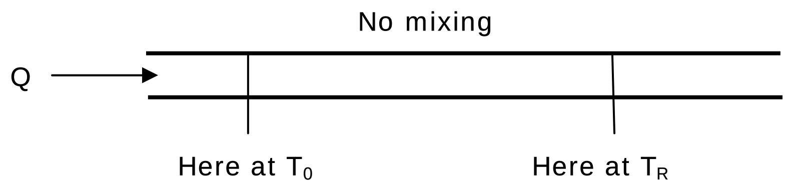
$$\frac{Qc}{\pi} - eH - kH^2 = 0$$

Quadratic formula:

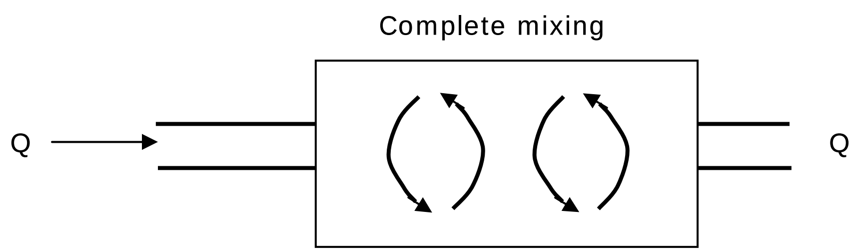
$$H = \frac{-e\pi \pm \sqrt{(e\pi)^2 - 4ck\pi Q}}{2k\pi} = -3.8\ m,\ 2.8\ m$$

## Residence Time (steady state, complete mixing)

Very useful concept for degradation and chemical reactions




$V = QT_R$ $\qquad$ $T_R = V/Q$ $\qquad$ Exactly




$T_R = V/Q$ $\qquad$ On Average

**For this problem:**

Consider discharge and evaporation out of Lake Parabola. Assume no seepage.

$Q$ = input = output = 10 [$m^3$/day]

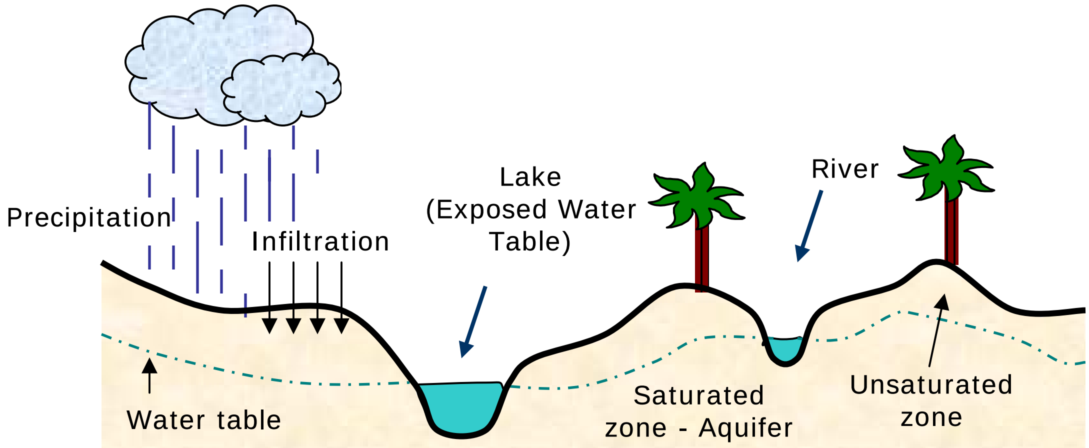
$$V(H) = \frac{\pi}{2c}H^2 = \frac{\pi}{2(0.1)}10.6^2 = 1{,}765m^3$$

$$T_R = V/Q = \frac{1{,}765m^3}{10m^3/day} = 176.5days$$

Consider yourself:

| Quantity | Men | Women |
|---|---|---|
| Water weight [Kg] | 60 | 50 |
| Average Intake [Kg/day] | 3 | 2.1 |
| Residence Time [day] | 14 | 14 |

**Lecture Packet #2: Aquifers, Porosity, and Darcy's Law**




Unsaturated Zone, Vadose Zone, Soil Moisture Zone, Zone of Aeration – rock, water and air

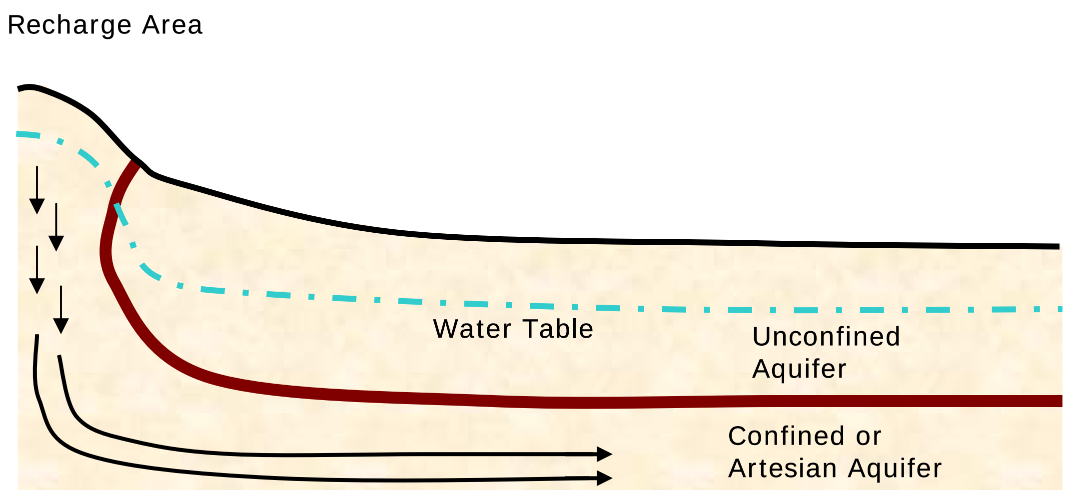
**Capillary Fringe** – region above water table where water rises due to capillary forces in the porous medium.

Saturated Zone, Phreatic Zone – rock and water

Water Table

- Top of saturated zone
- Depressed version of topography
- Surface waters are manifestations of the water table – exposed water table

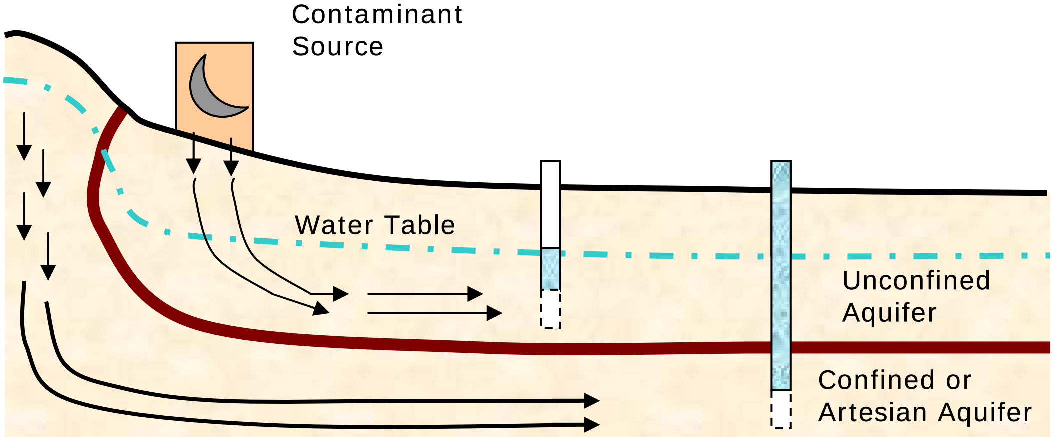
**Aquifer** - a geologic unit that stores and transmits water



**Unconfined Aquifer** – water is in contact with atmospheric pressure – drill and well hit the water table

**Confined Aquifer** – recharge upgradient forces water to flow down and get trapped under an aquiclude. Water is under pressure due to the weight of the upgradient water and the confinement of the water between "impermeable" layers. Water flows to surface under artesian pressure in an Artesian Well.

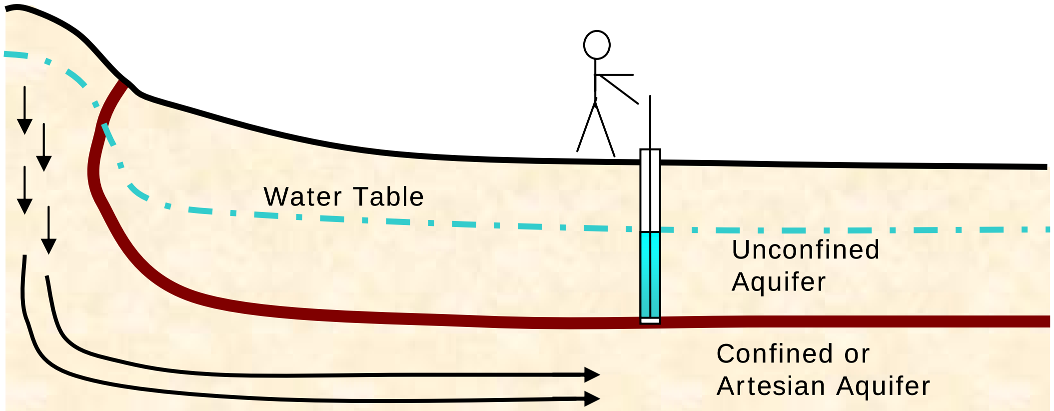
Aquifer contamination

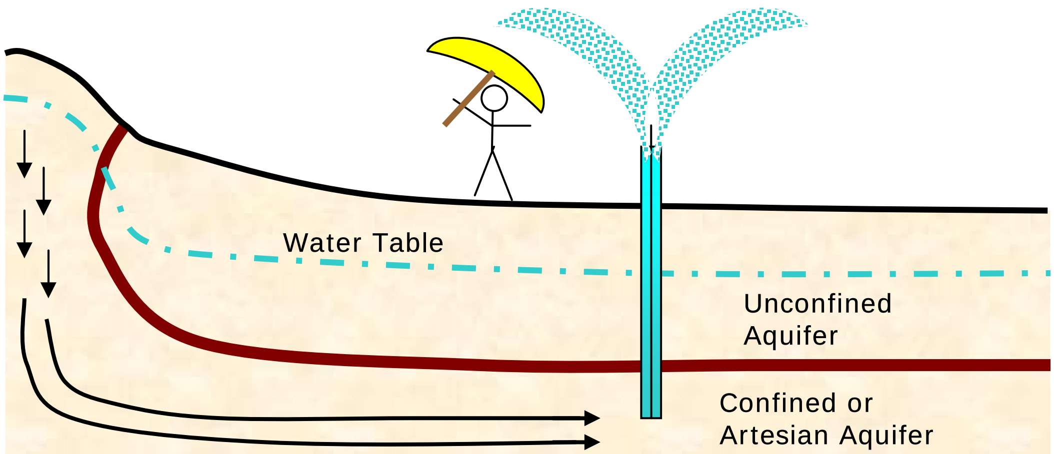



In confined vs. unconfined aquifers

- Although unconfined aquifers are used for water supply, they are often contaminated by wastes and chemicals at the surface.
- Confined aquifers are less likely to be contaminated and thereby provide supplies of good quality.
- Mechanisms of transport are advection and dispersion.
- There can be chemical interactions in aqueous phase or between the water and solid media
- Covered in Contaminant Hydrology course (1.72 is a prerequisite)

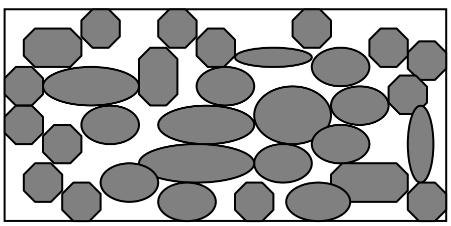
In A.D. 1126, in the province of Artois:




<u>Key Aquifer Properties</u>

**Porosity** – Percentage volume occupied by voids. Porosity is independent of scale. For example, a pile of marbles and a pile of beach balls have spherical shape and differing sizes; the porosities are identical due to the similar shaping.

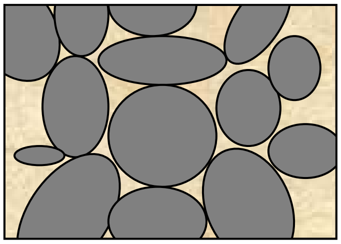


**Permeability** – Measures the transmission property of the media and the interconnection of the pores. Related to hydraulic conductivity and transmissivity. (more later)

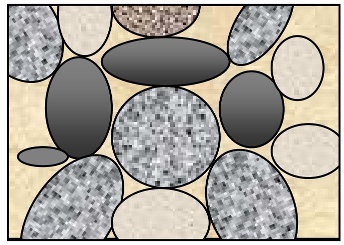
Good aquifer – High porosity + High permeability
- Sand and gravel, sandstone, limestone, fractured rock, basalt

Aqiuiclude, Confining bed, Aquitard – "impermeable" unit forming a barrier to groundwater flow.
- Granite, shale, clay

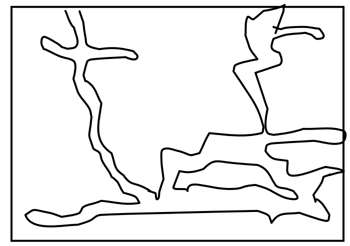
**Porosity**



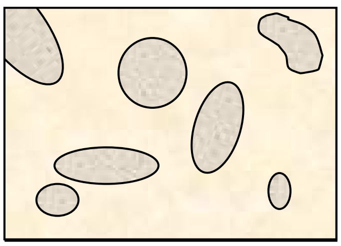
Well Sorted



Intragranular

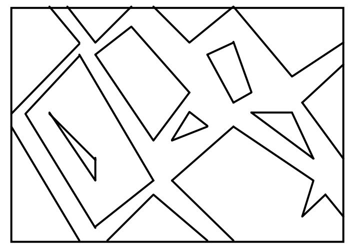


Dissolution



Poorly Sorted

Decreased Porosity
by Diogenesis



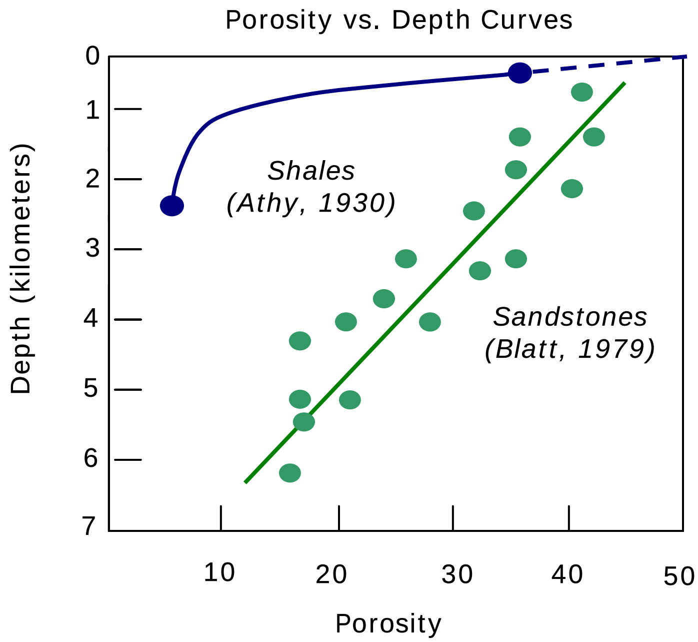
Fracture

**Diogenesis** – The formation of rock; pores fill up with precipitations of mineral and reduce porosity

<u>Two Origins of Porosity</u>

**Primary**

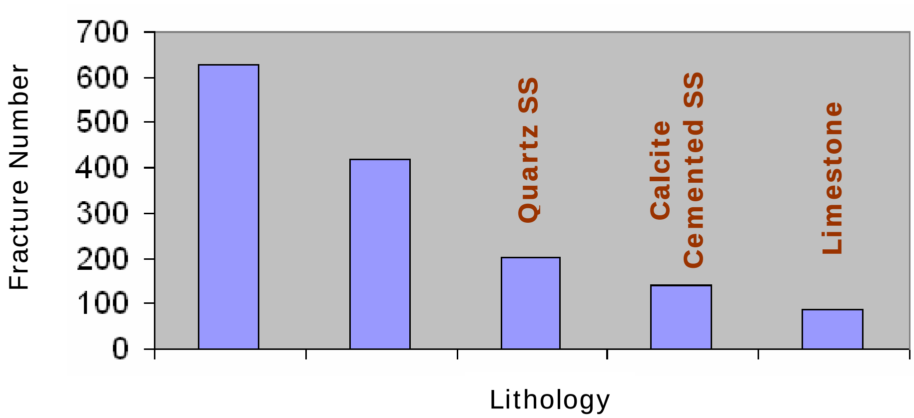
- A function of grain size distribution, also packing
- Decreases with depth – compaction and pressure solution

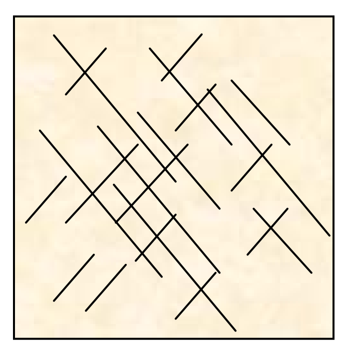



Porosity increases as depth decreases. This is on account of the weight on top of the deeper materials. Porosity also tends to increase with grainsize. Why?

**Secondary**

- Dissolution
- Fracture



Two types of Porosity

- Intergranular
  - Between grains, mostly part of the effect of porosity, but also dead-end pores
- Intragranular
  - Within grains
  - Usually not considered part of the effective porosity
  - Incredible wide range of widths and length scales
- Simple dichotomous model - dual porosity

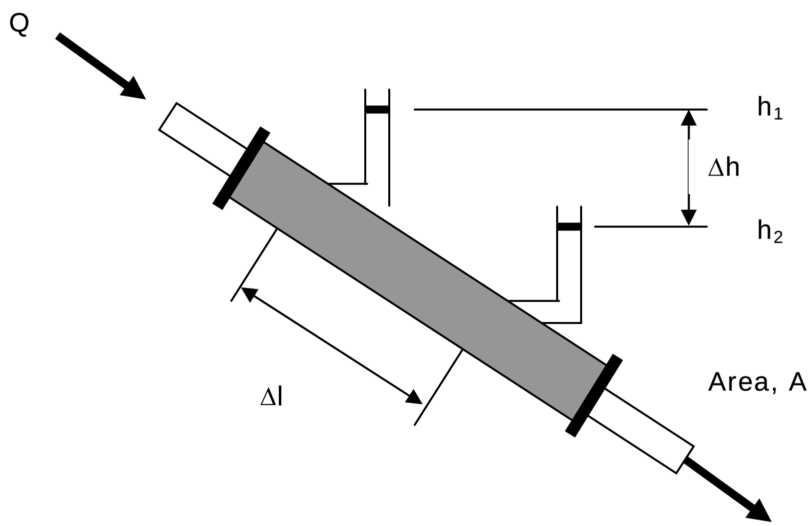
Darcy's Law

In 1856 in Dijon, France, Henry Darcy conducted his now famous experiment of pouring water through sediment-packed pipes to see how much would flow through them in a given amount of time [volume of flow per unit time].

Flow through column is Q in $L^3/T$ ⟵ most important quantity

The flow per unit area is **specific discharge**

$q = \frac{Q}{A}$ with units of velocity L/T – called **Darcian velocity** or **Darcian flux**, but not actual velocity of the fluid




Darcy showed that:

Q is in direction of decreasing head

q is proportional to $h_2 - h_1 = \Delta h$, given $\Delta l$ fixed, q $\alpha$ ($-\Delta h$)
q is inversely proportional to $\Delta l$, given $\Delta h$ fixed, q $\alpha$ ($1/\Delta l$)

The proportionality constant is K, and flow is from higher to lower hydraulic head.

$$q = -K\frac{\Delta h}{\Delta l} = -K\frac{dh}{dl} \longleftarrow \text{Hydraulic gradient}$$

*K* is hydraulic conductivity and has units of velocity (L/T). It is a function of both media and fluid.

Q is a flow per unit cross section and is **not** the actual velocity of groundwater flow.

$\Delta$h represents the frictional energy loss due to flow through media.

Darcy's law is a macroscopic law. It doesn't tell you about the flow through individual pores.

The discharge is Q in $L^3$/T

$$q = -K\frac{\Delta h}{\Delta l} = -KiA$$

*i* is commonly used for gradient.

Note the difference between Q and q!

<u>What is hydraulic conductivity?</u>

*K* is a property of both media and fluid.

Experiments show: $K = \frac{k\rho g}{\mu}$

- ***K*** is the intrinsic permeability ($L^2$), a property of media only
- $\rho$ is the mass density ($M/L^3$)
- $\mu$ is the dynamic viscosity (M/LT) and measures the resistance of fluid to shearing that is necessary for flow

<u>Range of Applicability of Darcy's Law</u>

At extreme gradients some have questioned the applicability of Darcy's Law (controversial)

Low Gradients
- Compacted clays and low gradients
- Threshold gradient to get flow
- Below a certain gradient – nonlinear

High Gradients

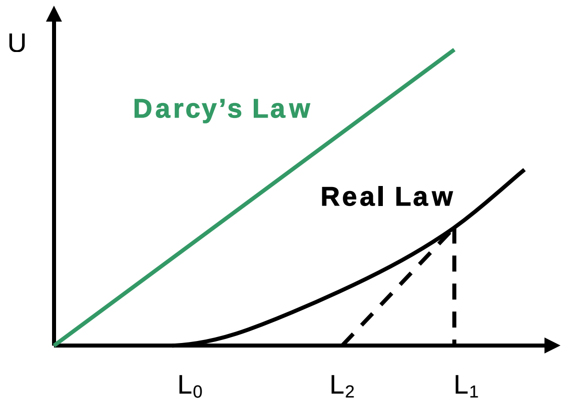
$$\frac{dh}{dl} = c_1 q + c_2 q^2$$

- Term 1 is loss – viscous friction against wall of solids and
- Term 2 is loss – dissipation of kinetic energy in pores – flow converges and diverges.

Must have laminar flow within pores.

$$R_e = \frac{\rho \bar{v} d}{\mu} \equiv \frac{Inertial\ forces}{Viscous\ forces}$$

Laminar in pipes < 2,000 in rocks < 10




Measures of hydraulic conductivity (L/T)
- Commonly cm/s, m/d, ft/d
- Older unit, gpd/ft$^2$, or meinzer

Measures of permeability, ($L^2$)

Often the **Darcy Unit** is used, recall

$$q = -\frac{k\rho g}{\mu}\frac{dh}{dl} \qquad Q = 1\ cm/s = -\frac{k\rho g}{cp}\frac{dh}{dl}$$

**1 darcy** is the permeability that gives a specific discharge of 1 cm/s for a fluid with a viscosity of 1 cp under a **hydraulic gradient times density times g of 1 atm/cm.**

- It equals about $10^{-8}$ cm$^2$
- About 0.831 m/d at 20$^o$

**Important** – A typical aquifer measure of the transmission property of media for the flow of water is given over a thickness, b

**Transmissivity** – T = Kb [$L^2$/T] $\longleftarrow$ 2D

- Very common quantity for site and regional studies
- Much more on this when we get to groundwater flow equation and well tests

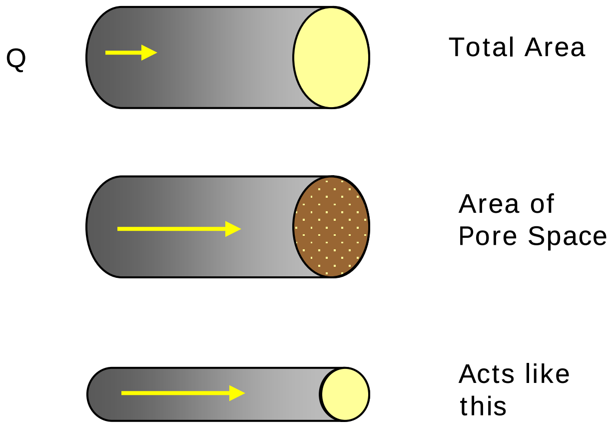
## Relations Between Grain Size and Hydraulic Conductivity

| Equation | Reference |
|---|---|
| $K = C(d_{10})^2$ | Hazen (1911) |
| $K = (9.66E-04)(760\, d_g^{\,2})EXP(-1.31\sigma_g)$ | Krumbein and Monk (1942) |
| $K = \left(\frac{\rho g}{\mu}\right)\frac{d^2\phi^3}{180(1-\phi)^2}$ | Kozeny-Carman (in Bear, 1972) |
| $K = \left(\frac{\rho g}{\mu}\right)\frac{\phi^3}{C_0 S_{sa}^{\,2}(1-\phi)^2}$ | Kozeny-Carman (in de Marsily, 1986) |
| $K = \left(\frac{\rho g}{\mu}\right)\frac{\phi^3}{C_T T(S_{sa})^2}$ | Kozeny Equation, modified by Collins (1961) |

**$C_0$** = factor reflecting pore shape and packing in the Kozeny-Carmen eqn. [-]
**$C_T$** = factor reflecting pore shape and packing in Kozeny eqn, mod. By Collins [-]
**C** = factor in the Hazen equation [T/L]
**$d_{10}$** = grain diameter for which 10% of particles are smaller [L]
**$d_g$** = geometric mean grain diameter [L]
**d** = representative grain diameter [L]
**g** = gravitational acceleration [$L/T^2$]
**K** = hydraulic conductivity [L/T]
$\phi$ = total porosity, accounting for compaction [-]
$\mu$ = dynamic viscosity [M/LT]
$\rho$ = density [$M/L^3$]
$\sigma_g$ = geometric mean standard deviation [L]
**$S_{sa}$** = surface area exposed to fluid per unit volume of solid medium [1/L]
**T** = tortuosity [-]

Remember Darcian Velocity is not an actual velocity; it is discharge per unit area (area is TOTAL cross section)

**Darcian Velocity**




Average Linear Velocity

**Primary porosity** – the original interstices

**Secondary porosity** – secondary dissolution or structural openings (fractures, faults, and openings along bedding planes).

**Computed porosity -** $n = 100[1-\rho_b/\rho_p]$

$\rho_b$ – bulk density, $M/L^3$ -> mass of dry sample/original volume

$\rho_p$ – particle density, $M/L^3$ -> mass of dry sample/volume of mineral matter from water-displacement test (2.65 g/cc)

**Effective porosity** – porosity available for flow, $n_e$

Can have isolated water as dead-end pores or trapped gas. IMPORTANT to transport!

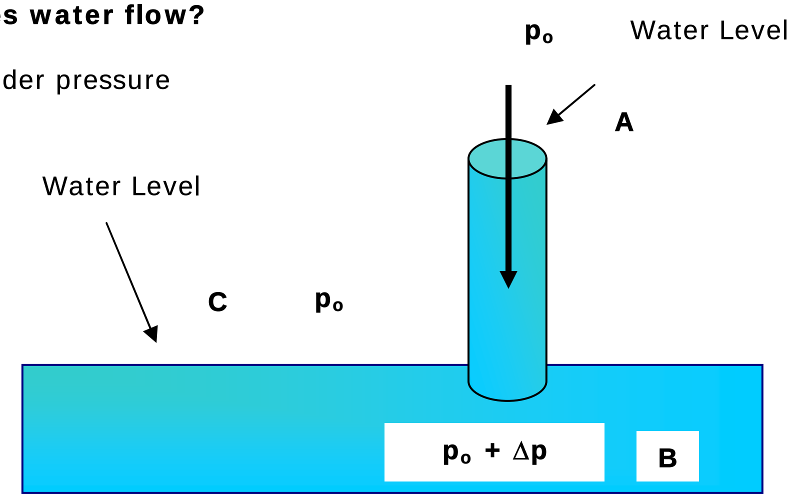
$$\overline{V} = \frac{-K}{n_e}\frac{\Delta h}{\Delta l} = \frac{-K}{n_e}\frac{dh}{dl}$$

$\overline{V}$ is the average linear pore water velocity. Measure of the rate of advection of a slug of water. $\overline{V}$ is larger than the Darcian Velocity. -> $q = n_e \overline{V}$

# Lecture Packet #3: Hydraulic Head and Fluid Potential

## What makes water flow?

Consider pressure




Pressure at A = atmospheric ($p_o$)
Pressure at B > atmospheric ($p_o + \Delta p$)
Pressure at C = atmospheric ($p_o$)

But flow is not from A to B to C.
Flow is not "down pressure gradient."

Hubbert (1940) – **Potential**

> A physical quantity capable of measurement at every point in a flow system, whose properties are such that flow always occurs from regions in which the quantity has less higher values to those in which it has lower values regardless of the direction in space.

Examples:
Heat conducts from high temperature to low temperature

- Temperature is a potential

Electricity flows from high voltage to low voltage

- Voltage is a potential

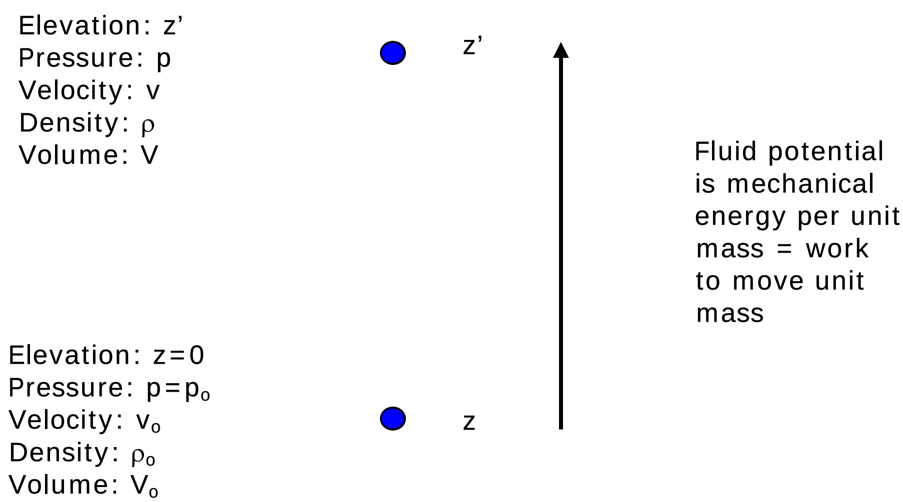
## Fluid potential and hydraulic head

Fluids flow from high to low fluid potential

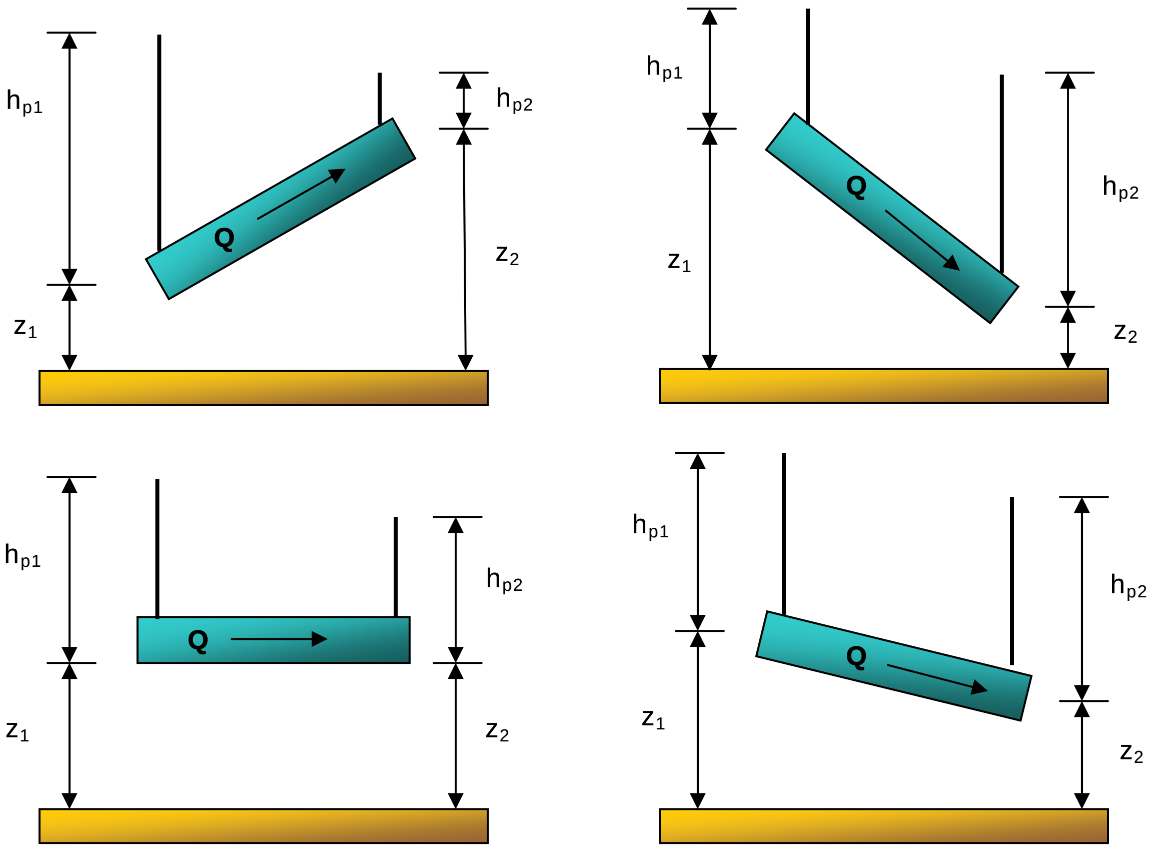
- Flow direction is away from location where mechanical energy per unit mass of fluid is high to where it is low.
- How does this relate to measurable quantity?

Groundwater flow is a **mechanical process** – forces driving fluid must overcome frictional forces between porous media and fluid. (generates thermal energy)

Work – mechanical energy per unit mass required to move a fluid from point z to point z'.



**Which way does water flow (function of Z and P) ?**




**Fluid potential** is the mechanical energy per unit mass of fluid potential at z' = fluid potential at datum + work from z to z'.

The work to move a unit mass of water has three components:

1) Work to lift the mass (where z = 0)

$$w_2 = mgz'$$

2) Work to accelerate fluid from v=0 to v'

$$w_2 = \frac{mv^2}{2}$$

3) Work to raise fluid pressure from $p=p_o$ to p

$$w_3 = \int_{p_o}^{p} Vdp = m\int_{p_o}^{p} \frac{V}{m}dp = m\int_{p_o}^{p} \frac{dp}{p}$$

Note that a unit mass of fluid occupies a volume V = 1/ρ

**The Fluid Potential (the mechanical energy per unit mass, m=1)**

$$\phi = gz + \frac{v^2}{2} + \int_{p_o}^{p} \frac{dp}{p} +$$

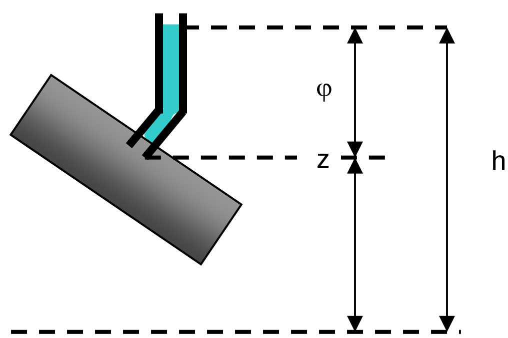
$$\phi = gz + \frac{v^2}{2} + \frac{p - p_0}{p}$$ for incompressible fluid (ρ is constant)

This term is almost always unimportant in groundwater flow, with the possible exception of where the flow is very fast, and Darcy's Law begins to break down.

How does potential relate to the level in a pipe?

At a measurement point pressure is described by:

P = ρg(depth) + $p_o$
p = ρgφ + $p_o$
p = ρg(h-z) + $p_o$




Return to fluid potential equation

$$\phi = gz + \frac{v^2}{2} + \frac{p - p_0}{p}$$

Neglect velocity (kinetic) term, and substitute for p

$$\phi = gz + \frac{\rho g(h - z) + p_0 - p_0}{\rho}$$

So,

$$\phi = gh \qquad \text{or} \qquad h = \phi / g$$

Thus, head h is a fluid potential.

- Flow is always from high h to low h.
- H is energy per unit weight.
- H is directly measurable, the height of water above some point.

$$h = z + \varphi$$

**Thermal Potential**

$$\varphi_t = \text{Thermal Potential}$$

Temperature can be an important driving force for groundwater and soil moisture.

- Volcanic regions
- Deep groundwater
- Nuclear waste disposal

Can cause heat flow and also drive water.

Chemical Potential
Adsorption Potential

Total Potential is the sum of these, but for saturated conditions for our initial cases we will have:

$$\varphi = \varphi_g + \varphi_p$$

$$q^* = -L_1 \frac{\partial h}{\partial l} - L_2 \frac{\partial T}{\partial l} - L_3 \frac{\partial c}{\partial l}$$

**Derivation of the Groundwater Flow Equation**

**Darcy's Law in 3D**

Homogeneous vs. Heterogeneous
Isotropic vs. Anisotropic

**Isotropy** – Having the same value in all directions. K is a scalar.

$$\longrightarrow \quad K \quad \longrightarrow$$

$$q_x = -K \frac{\partial h}{\partial x} \qquad q_y = -K \frac{\partial h}{\partial y} \qquad q_z = -K \frac{\partial h}{\partial z}$$

**Anisotropic** – having directional properties. **K** is really a tensor in 3D

$$\xrightarrow{\text{Gradient}} \quad K \quad \xrightarrow{\text{Flow}}$$

The value that converts one vector to another vector is a **tensor**.

$$q_x = -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} - K_{xz}\frac{\partial h}{\partial z}$$

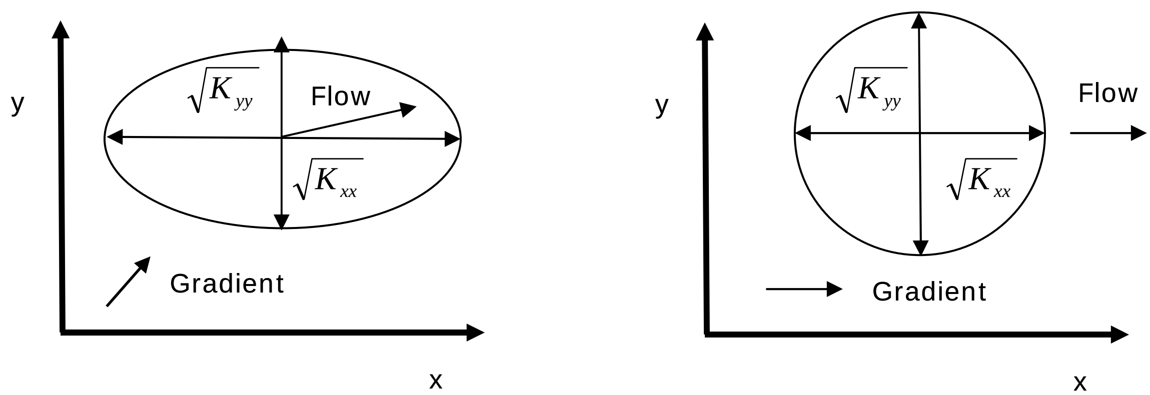
$$q_y = -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} - K_{yz}\frac{\partial h}{\partial z}$$

$$q_z = -K_{zx}\frac{\partial h}{\partial x} - K_{zy}\frac{\partial h}{\partial y} - K_{zz}\frac{\partial h}{\partial z}$$

- The first index is the direction of flow
- The second index is the gradient direction

**Interpretation** - $K_{xx}$ is a coefficient along the x-direction that contributes a component of flux along the x-axis due to the coefficient along the z-direction that contributes a component of flux along the z-axis due to the component of the gradient in the y-direction.

The conductivity ellipse (anisotropic vs. isotropic)




If $K_{yy}$ = $K_{xx}$ then the media is isotropic and ellipse is a circle.

It is convenient to describe Darcy's law as:

$$\vec{q} = -\overline{\overline{K}}\nabla h$$

Where $\nabla$ is called del and is a gradient operator, so $\nabla$h is the gradient in all three directions (in 3D). $\overline{\overline{K}}$ is a matrix.

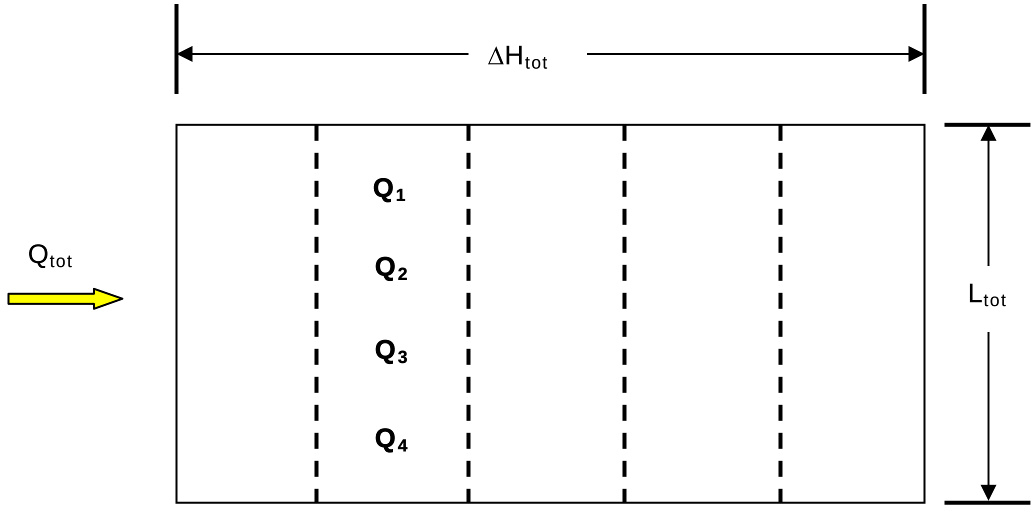
$$
\begin{aligned}
q_x &= \\
q_y &= -\begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix} \begin{matrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{matrix} \\
q_z &=
\end{aligned}
$$

The magnitudes of **K** in the principal directions are known as the principal conductivities.

If the coordinate axes are aligned with the principal directions of the conductivity tensor then the cross-terms drop out giving:

$$
\begin{aligned}
q_x &= \\
q_y &= -\begin{bmatrix} K_{xx} & & \\ & K_{yy} & \\ & & K_{zz} \end{bmatrix} \begin{matrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \\ \frac{\partial h}{\partial z} \end{matrix} \\
q_z &=
\end{aligned}
$$

**Effective Hydraulic Conductivity**



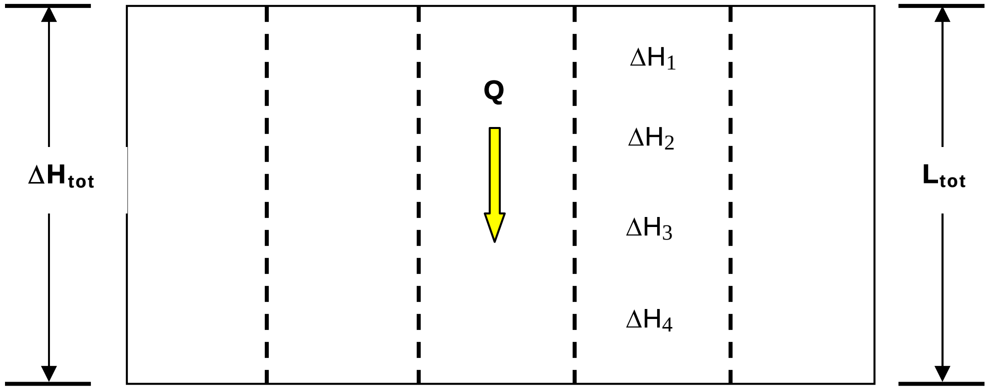

$$Q_{tot} = Q_1 + Q_2 + Q_3 + Q_4$$

$$= \frac{\Delta H}{\Delta x} L_1 K_1 + \frac{\Delta H}{\Delta x} L_2 K_2 + \frac{\Delta H}{\Delta x} L_3 K_3 + \frac{\Delta H}{\Delta x} L_4 K_4$$

$$= \frac{\Delta H}{\Delta x} \sum_i^n L_i K_i$$

$$= \frac{\Delta H}{\Delta x} L_{tot} K_{eff}$$

$$\boxed{K_{eff} = \frac{\sum_i^n L_i K_i}{\sum_i^n L_i}}$$




$\Delta H_{tot} = \Delta H_1 + \Delta H_2 + \Delta H_3 + \Delta H_4$

$$q = \frac{\Delta H_{tot}}{L_{tot}} K_{eff} = \frac{\Delta H_1}{L_1} K_1 = \frac{\Delta H_2}{L_2} K_2 = \frac{\Delta H_3}{L_3} K_3 = \frac{\Delta H_4}{L_4} K_4$$

$$\frac{qL_{tot}}{K_{eff}} = \sum_i^n \frac{qL_i}{K_i}$$

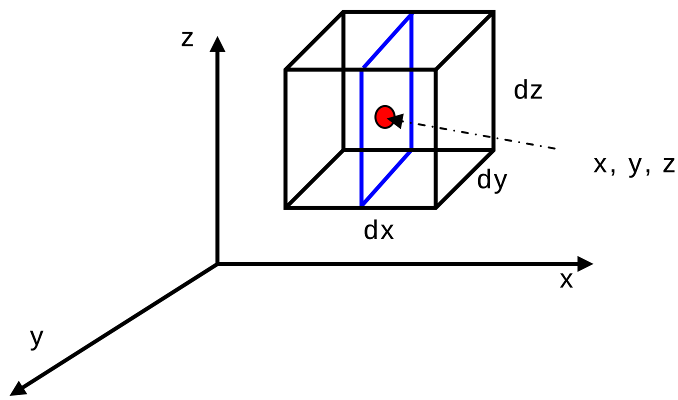
$$\boxed{K_{eff} = \frac{\sum_i^n L_i}{\sum_i^n \frac{L_i}{K_i}}}$$

**Lecture Packet #4: Continuity and Flow Nets**

## Equation of Continuity

- Our equations of hydrogeology are a combination of
  - o Conservation of mass
  - o Some empirical law (Darcy's Law, Fick's Law)

- Develop a control volume, rectangular parallelepiped, REV (Representative Elementary Volume)



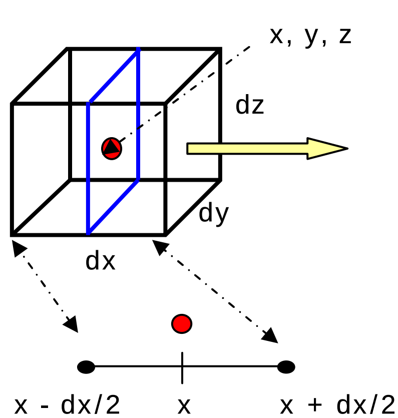

mass inflow rate – mass outflow rate = change in mass storage

$q_x$ = specific discharge in x-direction (volume flux per area)
at a point x,y,z $L^3/L^2$-T

Consider mass flow through plane y-z at (x,y,z)

$$q_x \, \rho \, dy \, dz \quad L/T \; M/L^3 \; L \; L = M/T$$

Rate of change of mass flux in the x-direction per unit time per cross-section is
$\frac{\partial}{\partial x}[\rho q_x]dydz$



mass flow into the entry plane y-z is

$$[\rho q_x]dydz - \frac{\partial}{\partial x}[\rho q_x]\frac{dx}{2}dydz$$

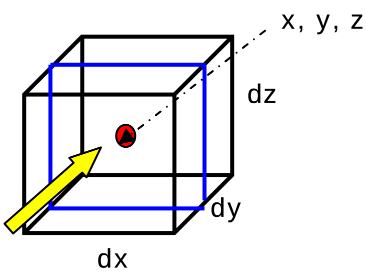
And mass flow out of the exit plane y-z is

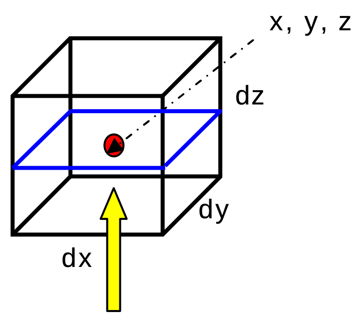
$$[\rho q_x]dydz + \frac{\partial}{\partial x}[\rho q_x]\frac{dx}{2}dydz$$

In the x-direction, the flow in minus the flow out is

$$-\frac{\partial}{\partial x}[\rho q_x]dxdydz$$

Similarly, the flow in the y-direction through the plane dxdz (figure on left)




$$\left[[\rho q_y]dxdz - \frac{\partial}{\partial y}[\rho q_y]\frac{dy}{2}dxdz\right] - \left[[\rho q_y]dxdz + \frac{\partial}{\partial y}[\rho q_y]\frac{dy}{2}dxdz\right] = -\frac{\partial}{\partial y}[\rho q_y]dxdydz$$

for the net y-mass flux.

Similarly, we get for the net mass flux in the z-direction:

$$-\frac{\partial}{\partial z}[\rho q_z]dxdydz$$

The **total mass flux** (flow out of the box) is

$$\left[-\frac{\partial[\rho q_x]}{\partial x} - \frac{\partial[\rho q_y]}{\partial y} - \frac{\partial[\rho q_z]}{\partial z}\right]dxdydz$$

Let's consider time derivative = 0 (Steady State System)

$$-\left[\frac{\partial[\rho q_x]}{\partial x}+\frac{\partial[\rho q_y]}{\partial y}+\frac{\partial[\rho q_z]}{\partial z}\right]dxdydz=\frac{\partial M}{\partial t}=0$$

How does Darcy's Law fit into this?

$$q_x=-K_{xx}\frac{\partial h}{\partial x} \qquad q_y=-K_{yy}\frac{\partial h}{\partial y} \qquad q_z=-K_{zz}\frac{\partial h}{\partial z}$$

- For anisotropy (with alignment of coordinate axes and tensor principal directions), or $\vec{q}=-K\nabla h$
- Assuming constant density for groundwater

$$0=-\rho\left[\frac{\partial[q_x]}{\partial x}+\frac{\partial[q_y]}{\partial y}+\frac{\partial[q_z]}{\partial z}\right]$$ substituting for q,

$$0=(-)(-)\rho\left[\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right)\right]$$

Steady-state flow equation for heterogeneous, anisotropic conditions:

$$0=\left[\frac{\partial}{\partial x}\left(K_x\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial h}{\partial z}\right)\right]$$

For <u>isotropic, homogeneous</u> conditions (K is not directional)

$$0=K\left[\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}\right]$$

This is the "diffusion equation" or "heat-flow equation"

For **Steady State** (K cancels) $0=\left[\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}\right]$

This is called the **Laplace equation.**
$\nabla^2$ is the "Laplacian" operator.

$$\nabla^2=\frac{\partial^2\cdot}{\partial x^2}+\frac{\partial^2\cdot}{\partial y^2}+\frac{\partial^2\cdot}{\partial z^2}$$ giving = $=\nabla^2\mathrm{h}$

Note that the full equation at this point can be written in summation notation as:

$$0 = \frac{\partial}{\partial x_i}\left(K_{ij}\right)\frac{\partial h}{\partial x_i}$$

$$\begin{matrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{matrix} \quad = \quad \begin{matrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{matrix}$$

The first Equation (flow in the x direction) is:

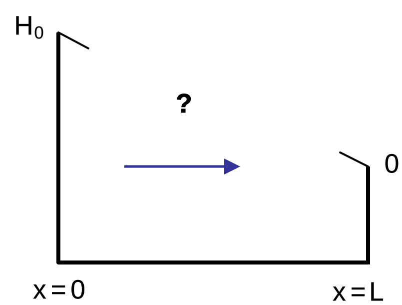
$$\frac{\partial}{\partial x}K_{xx}\frac{\partial h}{\partial x} + \frac{\partial}{\partial x}K_{xy}\frac{\partial h}{\partial y} + \frac{\partial}{\partial x}K_{xz}\frac{\partial h}{\partial z} \quad \text{or}$$

$$\frac{\partial}{\partial x_1}K_{11}\frac{\partial h}{\partial x_1} + \frac{\partial}{\partial x_1}K_{12}\frac{\partial h}{\partial x_2} + \frac{\partial}{\partial x_1}K_{13}\frac{\partial h}{\partial x_3}$$

What is the head distribution in a SS homogeneous system? Consider solution to steady-state problem (1-Dimensional) 1D Confined Aquifer – Head Distribution

$K(x=0) = H_0$ and $h(x=L) = 0$

1) $0 = \left[\frac{\partial^2 h}{\partial x^2}\right] \rightarrow 0 = \frac{\partial^2 h}{\partial x^2}$




Steady h distribution not f(K) – **h is independent of K.**

## Flow Nets

As we have seen, to work with the groundwater flow equation in any meaningful way, we have to find some kind of a *solution* to the equation. This solution is based on *boundary conditions*, and in the transient case, on *initial conditions*.

Let us look at the two-dimensional, steady-state case. In other words, let the following equation apply:

$$0 = \frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial x}\left(K_y \frac{\partial h}{\partial y}\right) \qquad \text{(Map View)}$$

A solution to this equation requires us to specify boundary condition. For our purposes with flow nets, let us consider

- No-flow boundaries ( $\frac{\partial h}{\partial n} = 0$, where *n* is the direction perpendicular to the boundary).
- Constant-head boundaries (*h* = constant)
- Water-table boundary (free surface, *h* is not a constant)

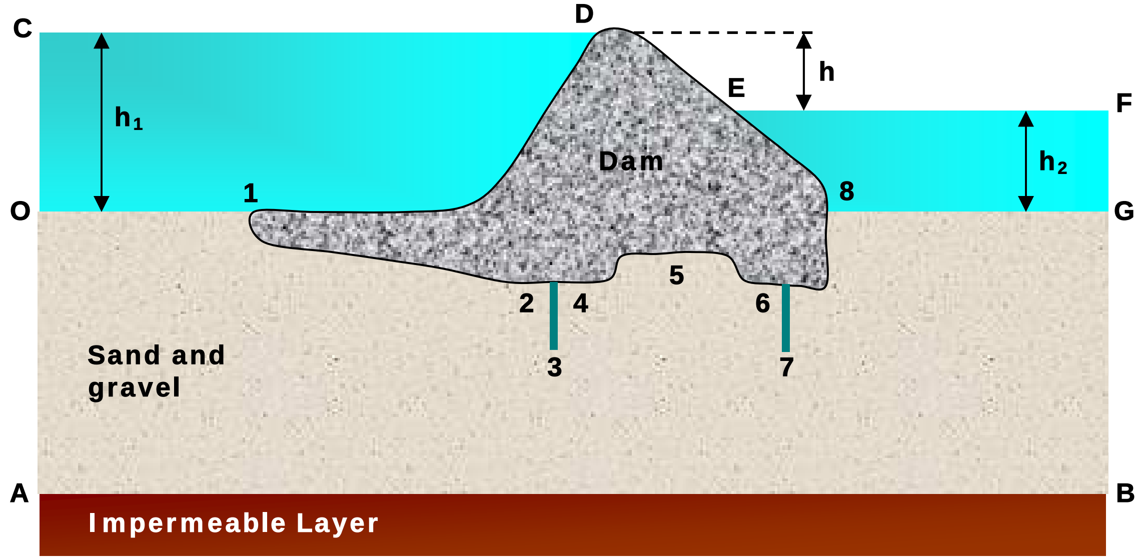
A relatively straightforward graphical technique can be used to find the solution to the GW flow equation for many such situations. This technique involves the construction of a **flow net**.

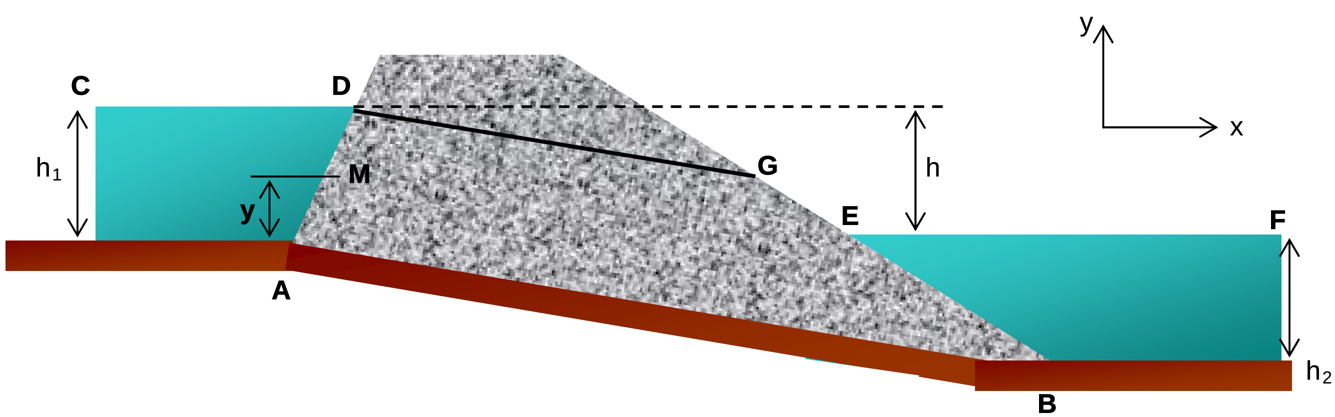
A flow net is the set of equipotential lines (constant head) and the associated flow lines (lines along which groundwater moves) for a particular set of boundary conditions.

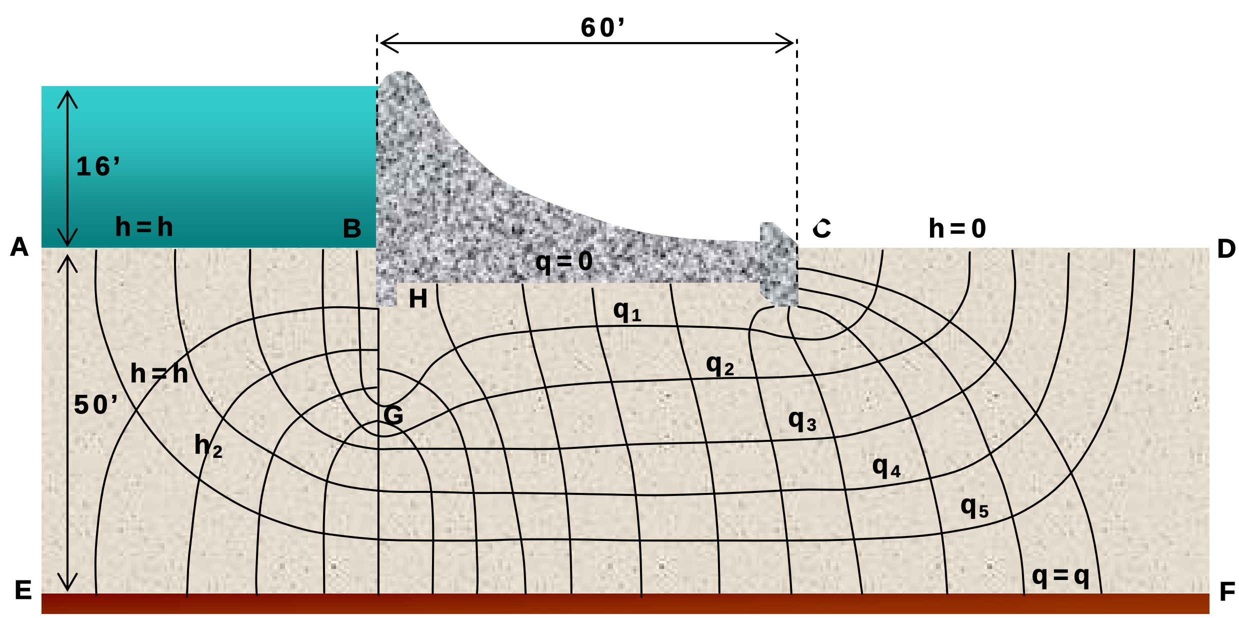
- For a given GW flow equation and a given value of K, the boundary conditions completely determine the solution, and therefore a flow net.

In addition, let us *first* consider only homogeneous, isotropic conditions:

$$0 = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} \qquad \text{(Cross-Section)}$$

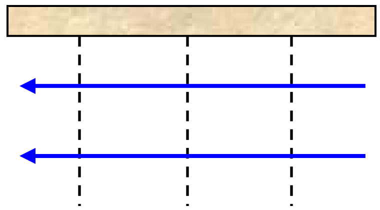

C
D
h
E
F
h1
Dam
8
h2
1
O
G
5
2
4
6
3
7
Sand and gravel
A
B
Impermeable Layer


y
x
C
D
h1
M
G
h
y
E
F
A
h2
B


60'
16'
h=h
B
C
h=0
A
D
q=0
H
q1
h=h
q2
50'
G
q3
h2
q4
q5
q=q
E
F

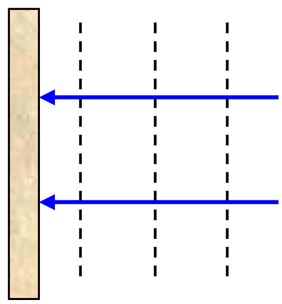
Let's look at flow in the vicinity of each of these boundaries. (Isotropic, homogeneous conditions).

**No-Flow Boundaries:** $\frac{\partial h}{\partial x} = 0$ or $\frac{\partial h}{\partial y} = 0$ or $\frac{\partial h}{\partial n} = 0$



- Flow is parallel to the boundary.
- Equipotentials are perpendicular to the boundary

**Constant-Head Boundaries:** *h* = constant



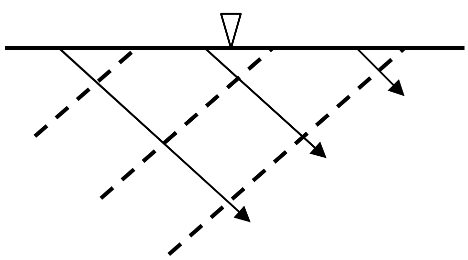
- Flow is perpendicular to the boundary.
- Equipotentials are parallel to the boundary.

**Water Table Boundaries:** $h = z$

Anywhere in an aquifer, total head is pressure head plus elevation head:

$$h = \psi + z$$

However, at the water table, $\psi = 0$. Therefore, $h = z$

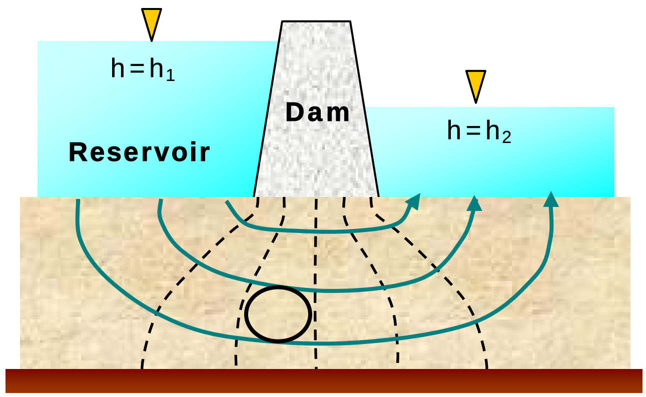


Neither flow nor equipotentials are necessarily perpendicular to the boundary.

**Rules for Flow Nets (Isotropic, Homogeneous System):**

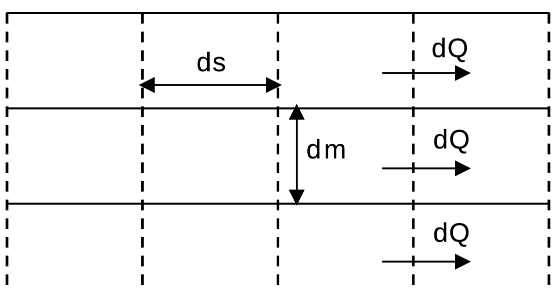
In addition to the boundary conditions the following rules must apply in a flow net:

1) Flow is perpendicular to equipotentials *everywhere*.
2) Flow lines never intersect.
3) The areas between flow lines and equipotentials are "curvilinear squares". In other words, the central dimensions of the "squares" are the same (but the flow lines or equipotentials can curve).
    - If you draw a circle inside the curvilinear square, it is tangential to all four sides at some point.




Why are these circles? It preserves dQ along any stream tube.

$dQ = K\ dm;\ dh/ds = K\ dh$




If $dm \neq ds$ (i.e. ellipse, not circle), then a constant factor is used.

**Other points:**
It is not necessary that flow nets have finite boundaries on all sides; regions of flow that extend to infinity in one or more directions are possible (e.g., see the figure above).

A flow net can have "partial" stream tubes along the edge. A flow net can have partial squares at the edges or ends of the flow system.

**Calculations from Flow Nets:**
It is possible to make many good, quantitative predictions from flow nets. In fact, at one time flow nest were the major tool used for solving the GW flow equation.

Probably the most important calculation is discharge from the system. For a system with one recharge area and one discharge area, we can calculate the discharge with the following expression:

$Q = n_f K\ dH \quad H = n_d\ dH$

Gives: $Q = n_f/n_d\ KH$

Where Q is the volume discharge rate *per unit thickness of section* perpendicular to the flow net; $n_f$ is the number of stream tubes (or flow channels); $n_d$ is the number of head drops; K is the uniform hydraulic conductivity; and H is the total head drop over the region of flow.

- Note that neither $n_f$ nor $n_d$ is necessarily an integer, but it is often helpful if you construct the flow net such that one of them is an integer.
- If you choose $n_f$ as an integer, it is unlikely that $n_d$ will be an integer.
- Note that to do this calculation, you do not need to know any lengths.

**Flow Nets in Anisotropic, Homogeneous Systems:**

Before construction of a flow net in an anisotropic system ($K_x \neq K_y$ or $K_x \neq K_z$ etc.), we have to *transform* the system.

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

For homogeneous K,

$$\frac{\partial^2 h}{\partial x^2} + \frac{K_z}{K_x}\frac{\partial^2 h}{\partial z^2} = 0$$

Introduce the transformed variable

$$Z = \sqrt{\frac{K_x}{K_z}}\, z$$

Applying this variable gives:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial Z^2} = 0$$

With this equation we can apply flownets exactly as we did before. We just have to remember how Z relates to the actual dimension z.

In an anisotropic medium, perform the following steps in constructing a flow net:

1. Transform the system (the area where a flow net is desired) by the following ratio:

$$Z = Z'\sqrt{\frac{K_z}{K_x}}$$

where *z* is the original vertical dimension of the system (on your page, in cm, inches, etc.) and Z' is the transformed vertical dimension.

$K_x$ is the hydraulic conductivity horizontally on your page, and $K_z$ is the hydraulic conductivity vertically on your page. This transformation is not specific to the x-dimension or the y-dimension.

2. On the transformed system, follow the exact same principles for flow nets as outlined for a homogeneous, isotropic system.
3. Perform the inverse transform on the system, i.e.

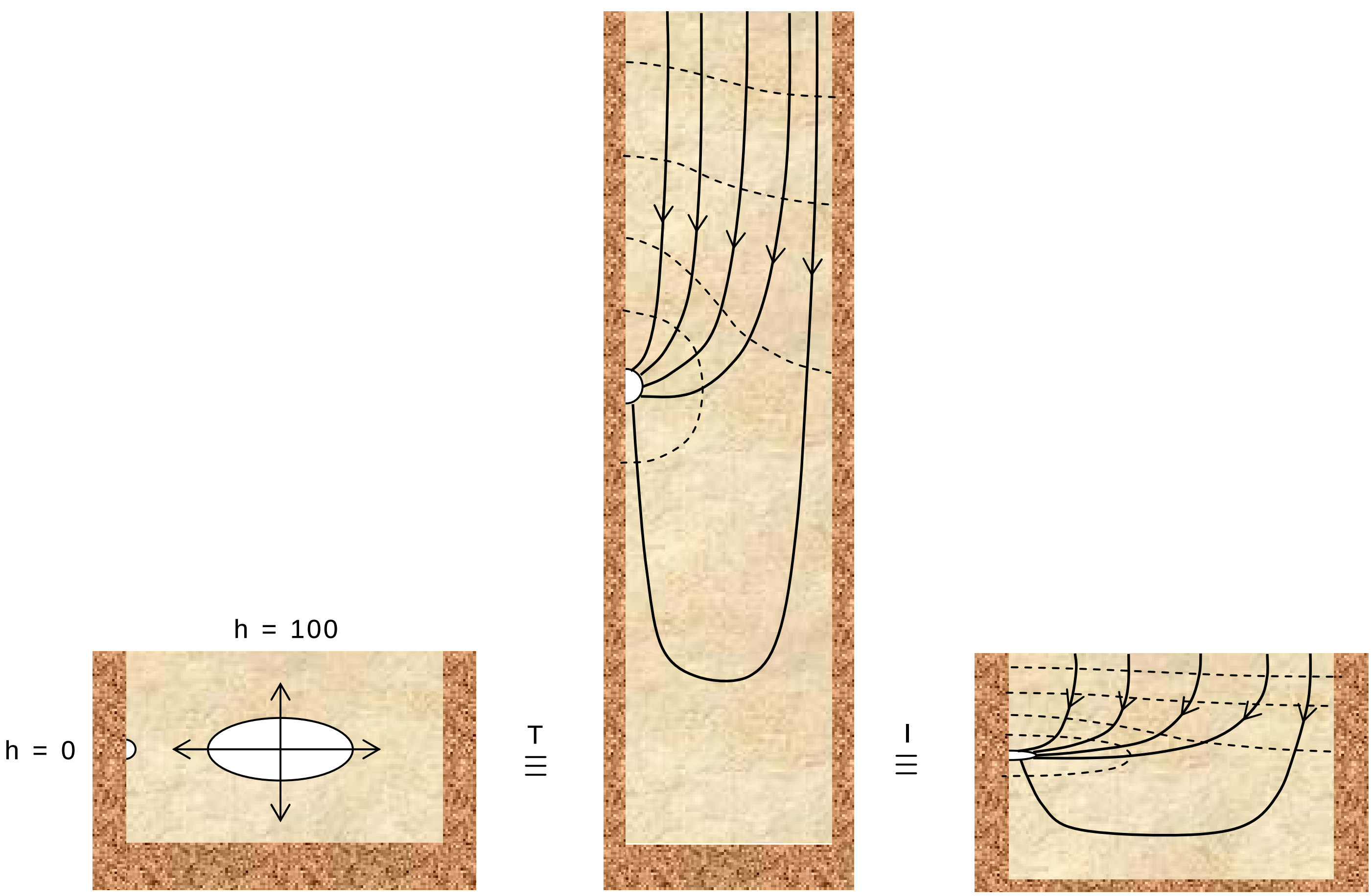
$$Z = Z'\sqrt{\frac{K_z}{K_x}}$$

4. If any flow calculations are needed, do these calculations on the homogeneous (step 2) section. Use the following for hydraulic conductivity:

$$K' = \sqrt{K_x K_z}$$

Where *K*' is the homogeneous hydraulic conductivity of the transformed section. (NOTE: This transformed K' is not real! It is only used for calculations on the transformed section.)

**Examples:**

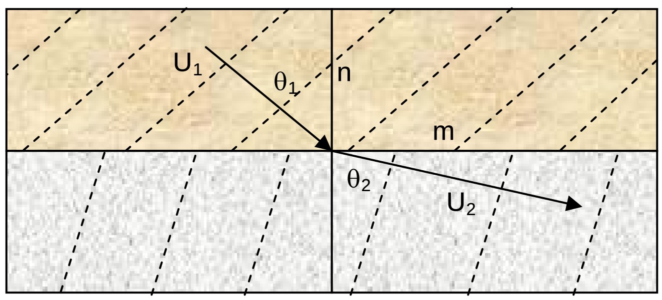


**Flow Nets in Heterogeneous Systems:**

We will only deal with construction of flow nets in the simplest types of heterogeneous systems. We will restrict ourselves to **layered heterogeneity**.

In a layered system, the same rules apply as in a homogeneous system, with the following important exceptions:

1. Curvilinear squares can only be drawn in ONE layer. In other words, in a two-layer system, you will only have curvilinear squares in one of the layers. Which layer to draw squares in is your choice: in general you should choose the thicker/larger layer.
2. At boundaries between layers, flow lines are refracted (in a similar way to the way light is refracted between two different media). The relationship between the angles in two layers is given by the "tangent law":




$h_1 = h_2 \longrightarrow \dfrac{\partial h_1}{\partial m} = \dfrac{\partial h_2}{\partial m}$ (1) No sudden head changes

$K_1 \dfrac{\partial h_1}{\partial n} = K_2 \dfrac{\partial h_2}{\partial n}$ (2) Conservation of Mass

| | Layer 1 | Layer 2 |
|---|---|---|
| $u_x$: | $U_1 \sin\theta_1 = -K_1 \dfrac{\partial h_1}{\partial m}$ | $U_2 \sin\theta_2 = -K_2 \dfrac{\partial h_2}{\partial m}$ |
| $u_y$: | $U_1 \cos\theta_1 = -K_1 \dfrac{\partial h_1}{\partial n}$ | $U_2 \cos\theta_2 = -K_2 \dfrac{\partial h_2}{\partial n}$ |

By (1): $\dfrac{U_1 \sin\theta_1}{K_1} = \dfrac{U_2 \sin\theta_2}{K_2}$

By (2): $U_1 \cos\theta_1 = U_2 \cos\theta_2$

$$\frac{K_1}{K_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

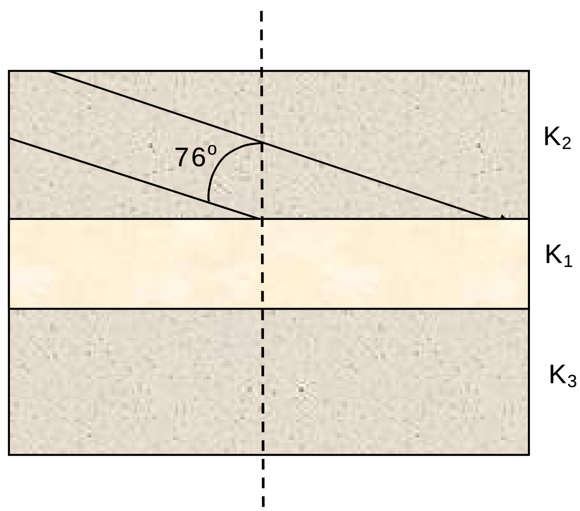
You can rearrange the tangent law in any way to determine one unknown quantity. For example, to determine the angle $\theta_2$:

$$\theta_2 = \tan^{-1}\left(\frac{K_2}{K_1}\tan\theta_1\right)$$

One important consequence for a medium with large contrasts in K: high-K layers will often have almost horizontal flow (in general), while low-K layers will often have almost vertical flow (in general).

**Example:**

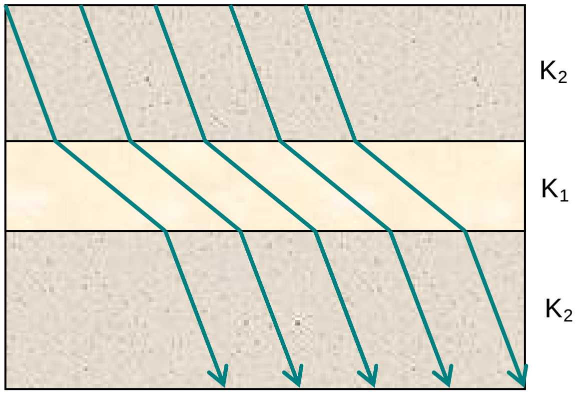
In a three-layer system, $K_1 = 1 \times 10^{-3}$ m/s and $K_2 = 1 \times 10^{-4}$ m/s. $K_3 = K_1$. Flow in the system is 14° below horizontal. What do flow in layers 2 and 3 look like?

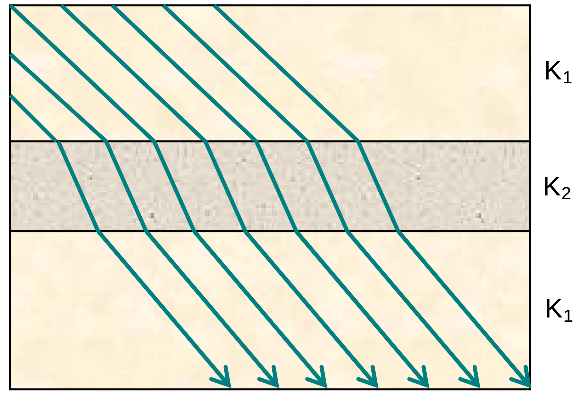



$$\theta_2 = \tan^{-1}\left(\frac{1\text{x}10^{-4}}{1\text{x}10^{-3}}\tan 76^\circ\right) = 22^\circ$$

What is the angle in layer 3? If you do the calculation, you will find it is 76° again.

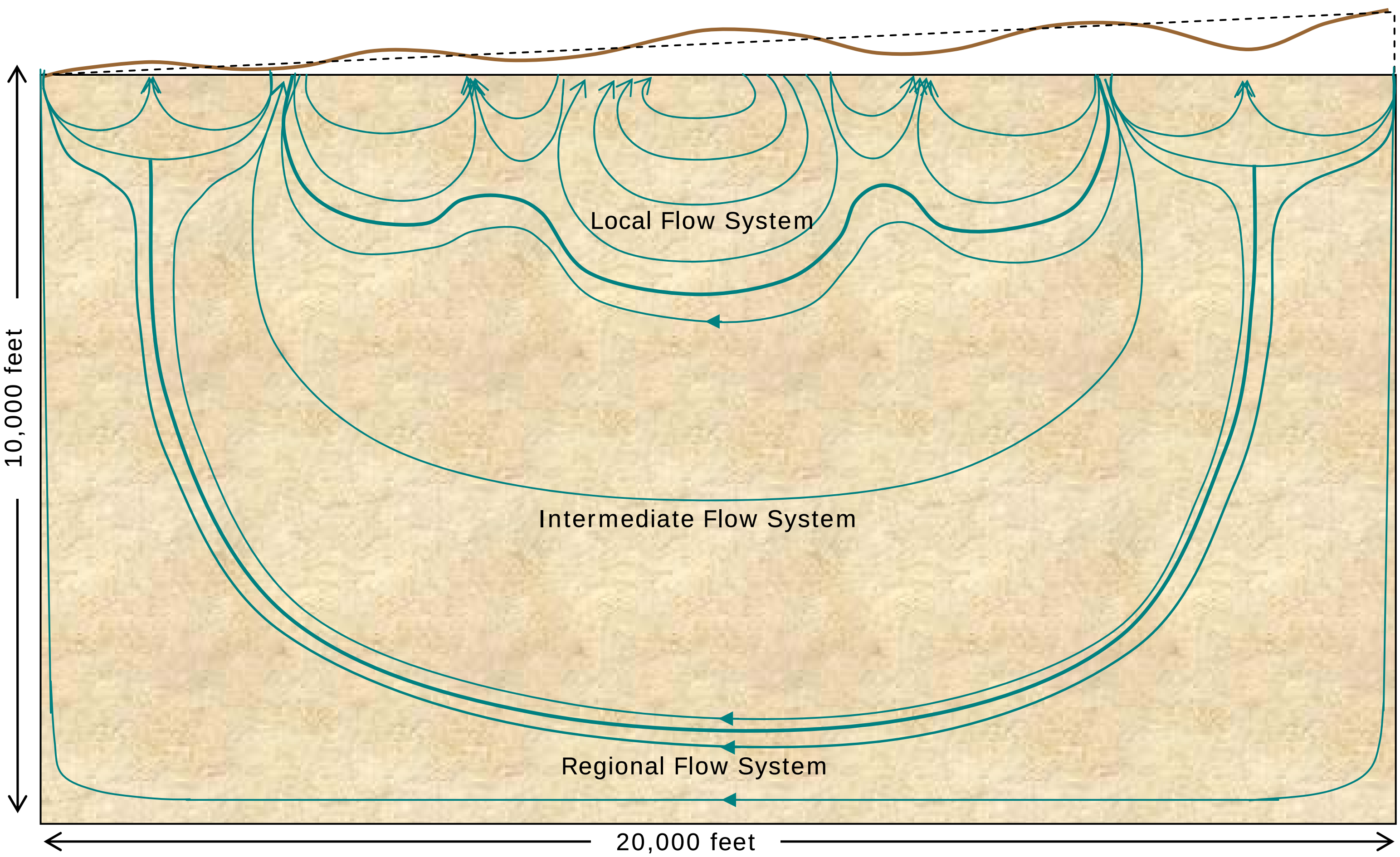
When drawing flow nets with different layers, a very helpful question to ask is "What layer allows water to go from the entrance point to the exit point the easiest?" Or, in other words, "What is the easiest (frictionally speaking) way for water to go from here to there?"




$$\frac{K_1}{K_2} = 10$$

**Lecture Packet #5: Groundwater Flow Patterns**




Hydrologic section showing local, intermediate and regional groundwater flow systems determined from an analytical solution to the groundwater flow equation

Conceptual picture based on Hubbert's potentials

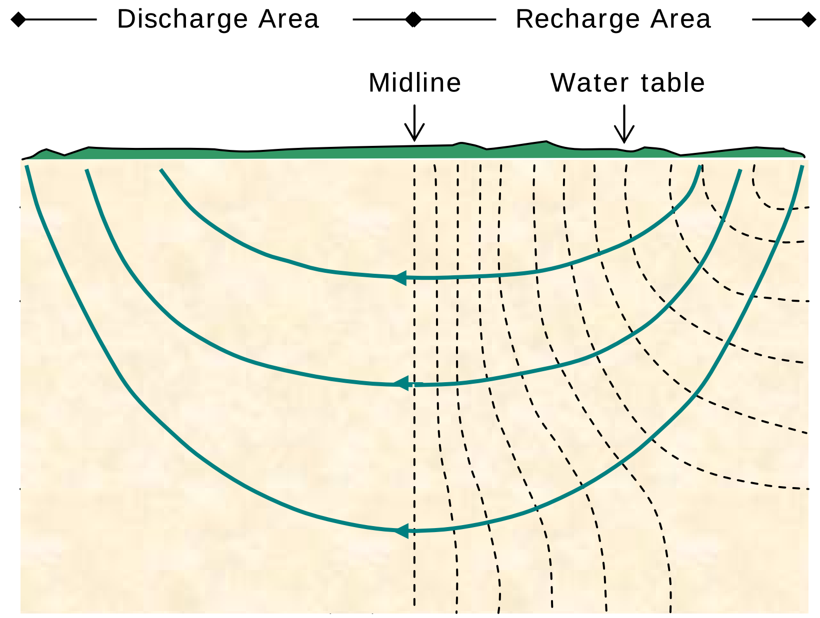
Recharge Zone between two streams.
Discharge Zone (generally 5% to 30% of a watershed)

Beneath ridge and valley one gets **Groundwater Divide**, an imaginary impermeable boundary.

- Recharge area – saturated flow is directed away from the water table.
- Discharge area – a component of flow upward and saturated flow is toward the water table.

Streamlines exist for steady-state systems. Is this realistic?

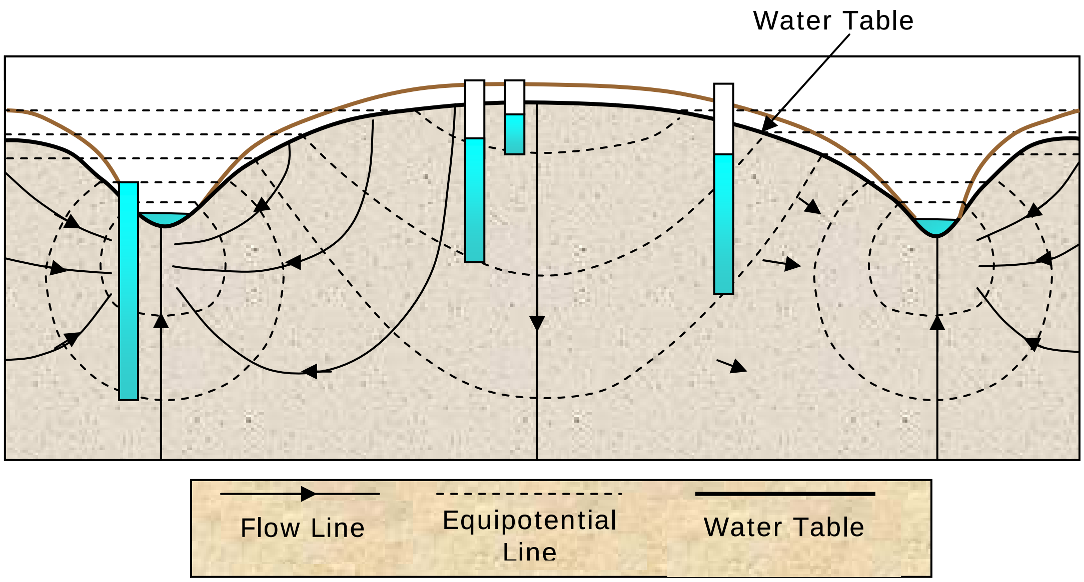
Pathlines can be constructed for transient systems. A **pathline** is the trajectory a particle would travel over time.



**Groundwater Flow Patterns in Homogeneous Aquifers**
From flow nets we know that:

- Equipotential lines corresponding to the water table reflect the elevation head.
- Equipotential lines beneath the water table are curvilinear and reflect both pressure and elevation.

Note: artesian conditions can be **geologically controlled** or **topographically controlled**.



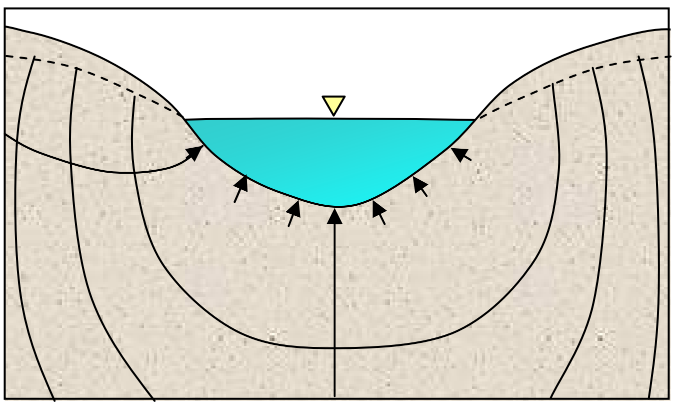

**Effects of Geology or Regional Flow (contrasts in K)**
Three most influential factors affected head distribution:
1. Ratio of basin depth to extent.
2. Configuration of the water table.
3. Variations in hydraulic conductivity.

Layered systems: lower layer 10 times the hydraulic conductivity.

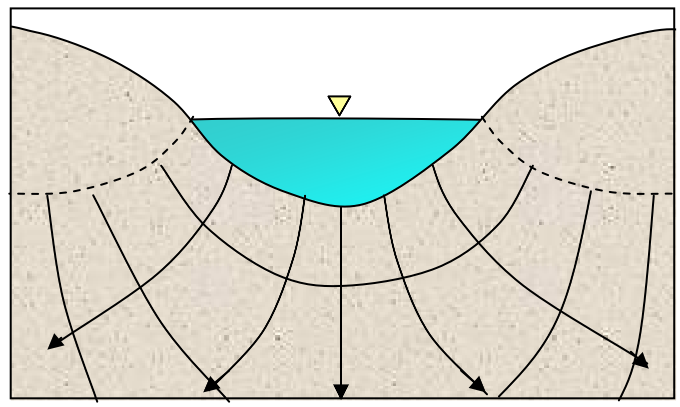
If the lower layer increased to 100 times the hydraulic conductivity:
- Lower gradient in lower aquifer.
- More flow through the system (less bottom impedance).
- Lower aquifer acts like a pirating agent for flow creating near vertical flow through upper aquifer.
- Large discharge area in upper aquifer for water to get out.

**Groundwater-Lake Interactions**

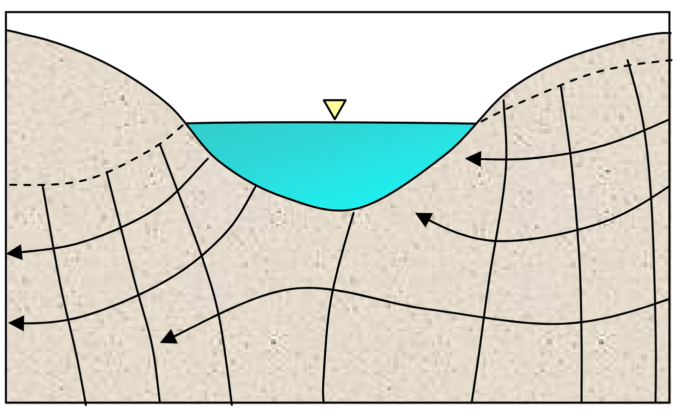
- Lake type
  - o Surface water dominated
    - Most of the lake water was originally surface water
    - Lake has an inlet and an outlet (rivers)
  - o **Groundwater dominated**
    - Most of the lake is groundwater
- Seepage Distribution
  - o Recharge Lake.
  - o Discharge Lake.
  - o Flow-through Lake.
  - o Combinations of the above.
- Seepage depends on groundwater-lake communication, which depends on the "Darcy parameters"
  - o Thickness and permeability of lake sediments.
  - o Hydraulic gradient across lake sediments.
  - o But lake is in the context of a regional flow system.



Discharge Lake



Recharge Lake



Flow-through Lake

## Numerical Modeling of Groundwater-Lake Interactions

- Winter (1976) presents numerical model of lake flow systems
- The focus of the study is the effect on the position and head value of stagnation point of
  - Height of water table relative to lake level
  - $K_h/K_v$
  - Position and size of aquifers
  - Lake depth

## The Model

- Boundary Conditions
  - Aquifer base is impermeable
  - Vertical no-flow boundaries on sides
    - Major topographic high on upslope side
    - Major drain (stream) on downslope side
  - Upper boundary is the water table
- Parameters chosen to be consistent with lakes and glacial terrain.
  - Silty till
  - Sand
  - Ratios of $K_h/K_v$ = 100 and 1000 based on field observations

## Stagnation Point

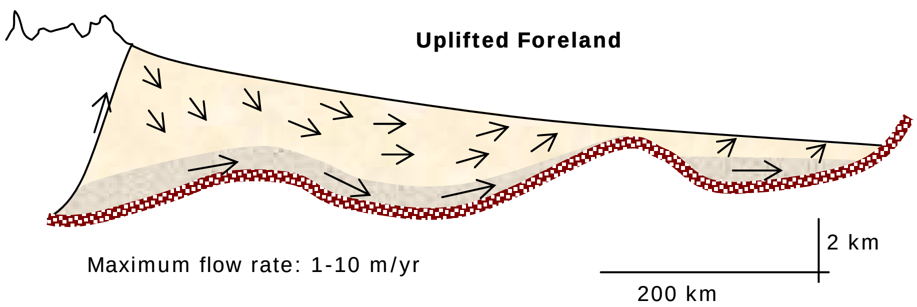
- A location where flow is zero (stagnant)
- Complex flow systems show one or more stagnation points
- The point of lowest head on dividing streamline is a stagnation point
  - There is a head value at the location of the stagnation point
  - For lake systems occurs under the shoreline on the downslope side of the lake
  - If the head at the stagnation point is higher than the head in a lake, flow is toward the lake (no water flows out the bottom of the lake)

## Groundwater-Lake Flow Systems

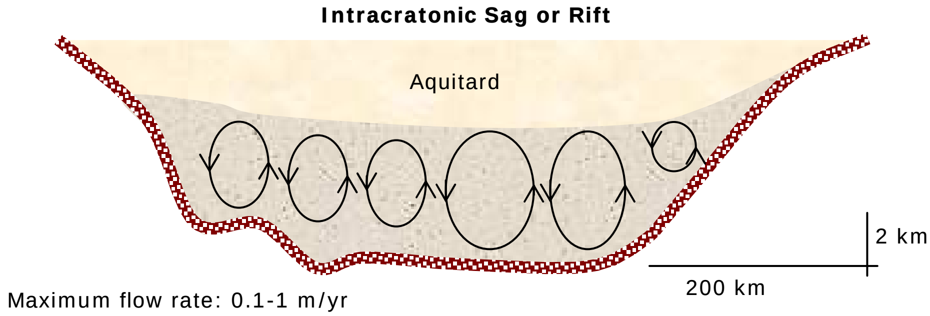
- Flow systems exist on different scales
  - Local
  - Intermediate
  - Regional
- Flow systems are separated by dividing streamlines
  - Separation of flow systems explains chemical differences between nearby lakes
  - More minerals dissolve the longer water is in flow system
  - Water in intermediate and regional flow systems has a higher mineral content

**Geologic Processes Controlling Regional Flow**

Flow controlled by single or coupled processes:

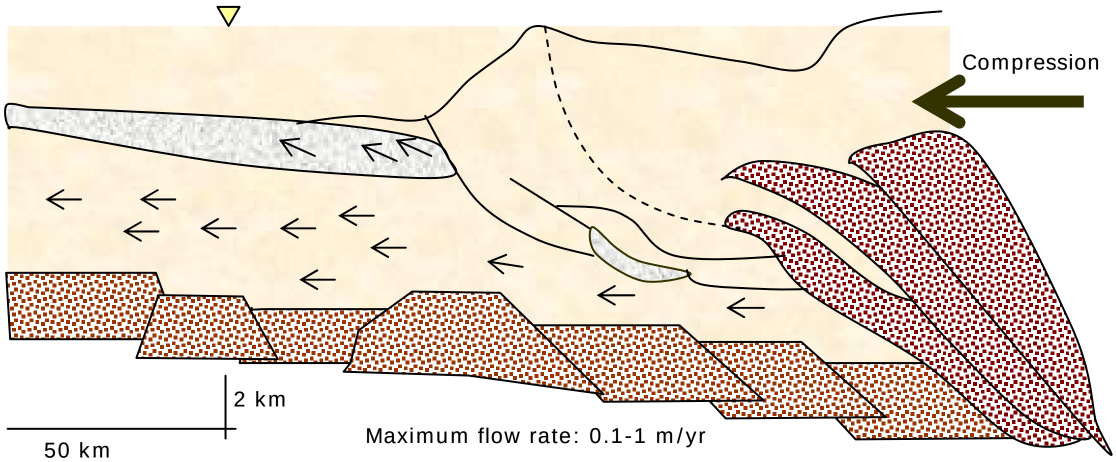



1) Topographically driven flow (gravity driven)
    - Dominant mechanism for shallow and deep groundwater flow systems
    - Water table subdued replica of landscape
    - Deep groundwater migrates 1 to 10 m/y in aquifers and much less in aquitards
    - Key factors: topography, conductivity, heterogeneity, anisotropy, and basin geometry



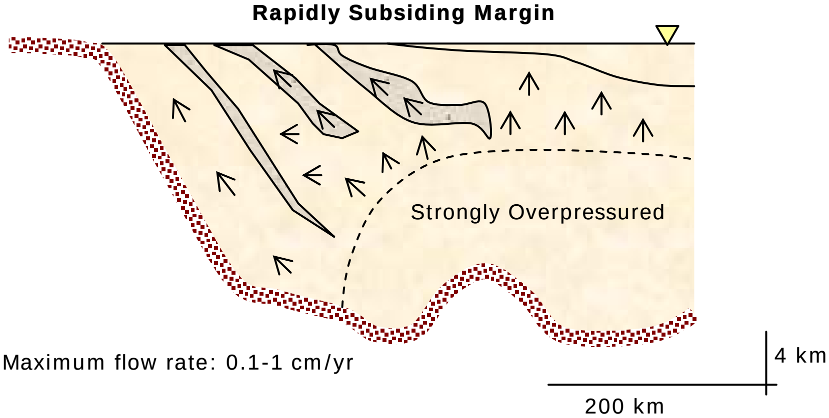

2) Free convection
    - Circulation cells develop with or without regional horizontal flow (can coexist with gravity-driven flow)
    - Driven by buoyancy forces associated with thermal and salinity fields
    - Flow rates are on the order of 1 m/y
    - Key factors: fluid-density gradient, aquifer thickness, and conductivity

**Thrust Terrane**



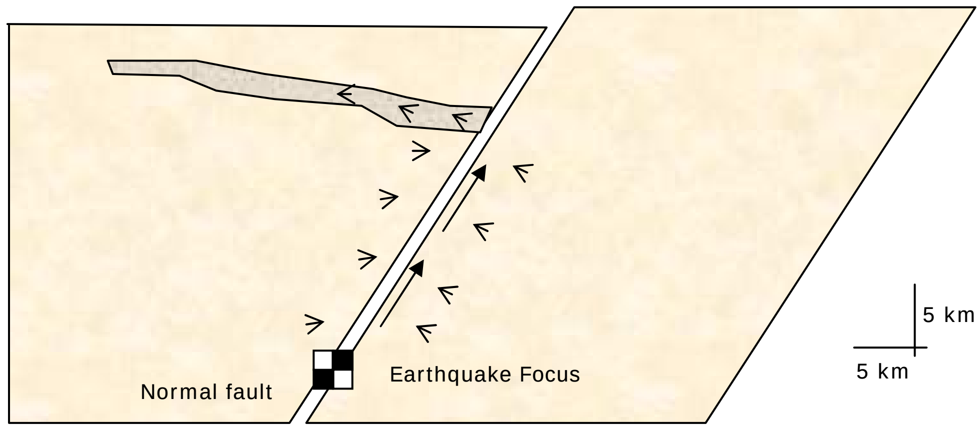

3) Tectonically-driven flow
   - Compression and thrusting during mountain building produces large overpressures in orogenic belts
   - Regime characteristic of accretionary wedges (subduction zones) with flow rate of cm/y
   - Theoretical flow (high) rates of 0.5 m/y

**Rapidly Subsiding Margin**



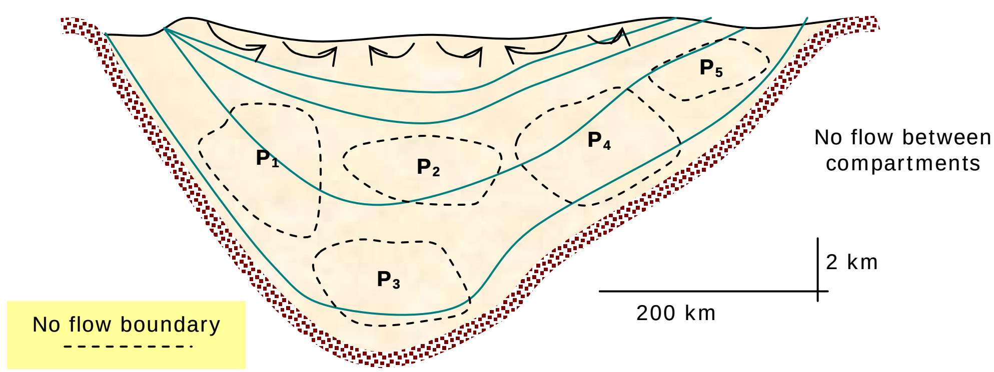

4) Overpressure buried continental margin
   - Fluid pressures can approach weight of overburden (lithostatic load)
   - Overpressuring typically occurs in young, low K basins
   - Also contributing to overpressure are: dehydration reactions and hydrocarbon generation
   - Flow rates generally < 1 cm/y (due to low K shales)

**Seismic Pumping in Rift**




5) Seismic pumping
   - Catastrophic faulting vents overpressure

**Pressure Compartments**

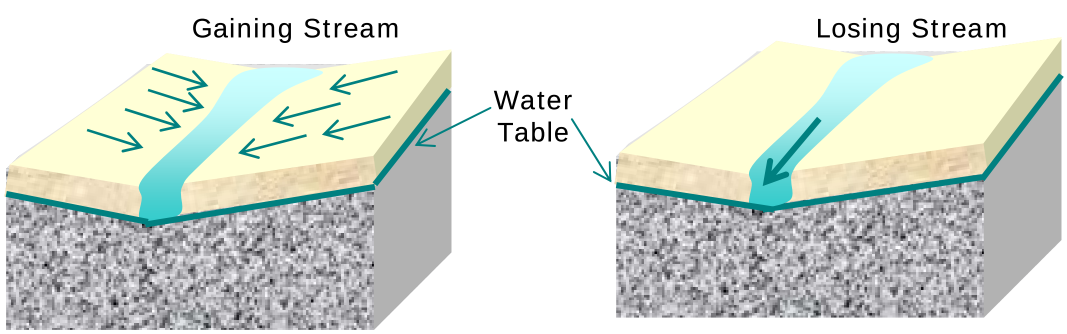



6) Pressure compartments
   - Permanent impermeable barrier maintains pressures forever; seals into isolated compartments
   - Not believed by most hydrogeologists who feel there is always some permeability and flow over geologic time

1.72, Groundwater Hydrology
Prof. Charles Harvey
**Lecture Packet #6: Groundwater-Surface Water Interactions**

**Streams**

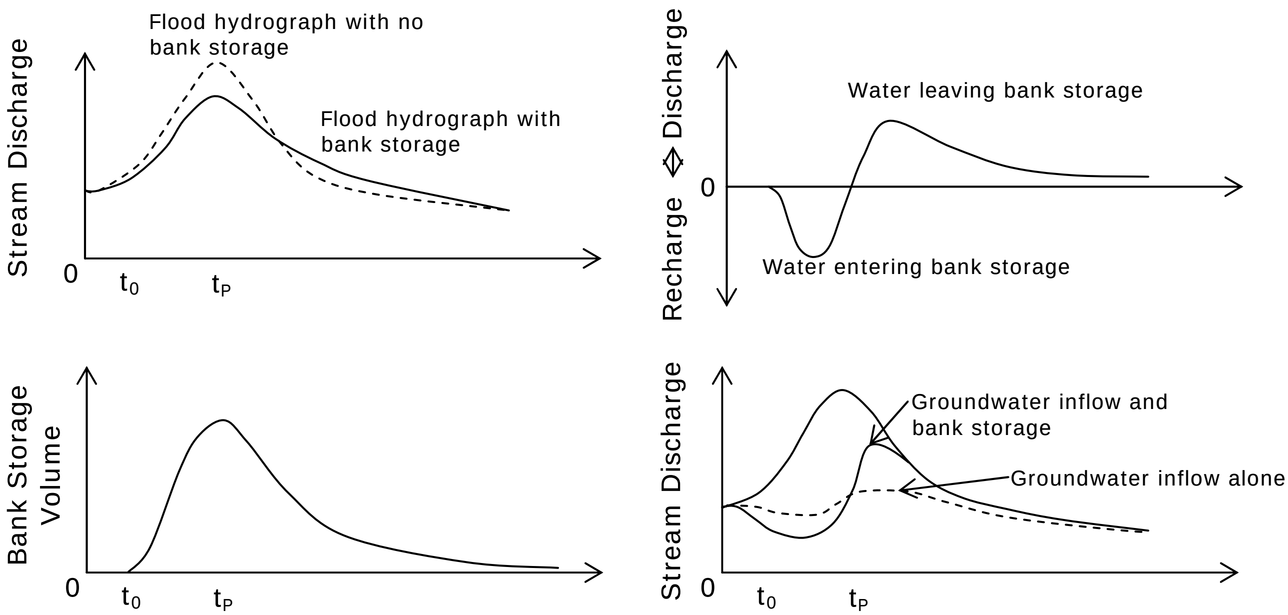
- Streamflow is made up of two components:
    - Surface water component
        - Surface Runoff
        - Direct Rainfall
    - Groundwater component (baseflow)
        - Seepage through the streambed or banks
- Definitions
    - Losing or influent stream → Stream feeds aquifer
    - Gaining or effluent stream → Aquifer discharges to stream



## Stream-Aquifer Interactions

**Base Flow** – Contribution to streamflow from groundwater

- Upper reaches provide subsurface contribution to streamflow (flood wave).
- Lower reaches provide bank storage which can moderate a flood wave.

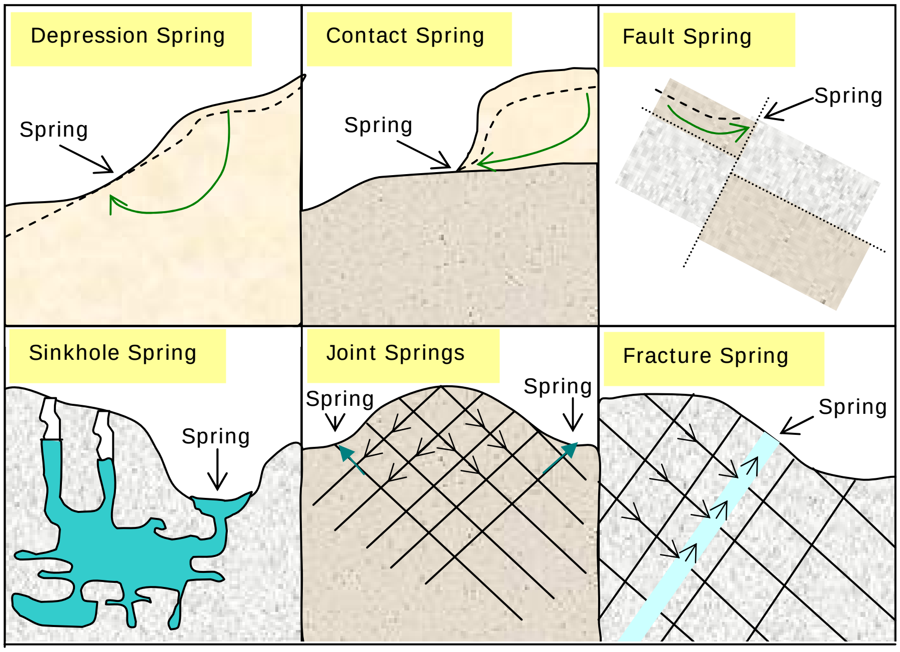



**Springs**

- A spring (or seep) is an area of natural discharge.
- Springs occur where the water table is very near or meets land surface.
    - o Where the water table does not actually reach land surface, capillary forces may still bring water to the surface.
- Discharge may be permanent or ephemeral
- The amount of discharge is related to height of the water table, which is affected by
    - o Seasonal changes in recharge
    - o Single storm events

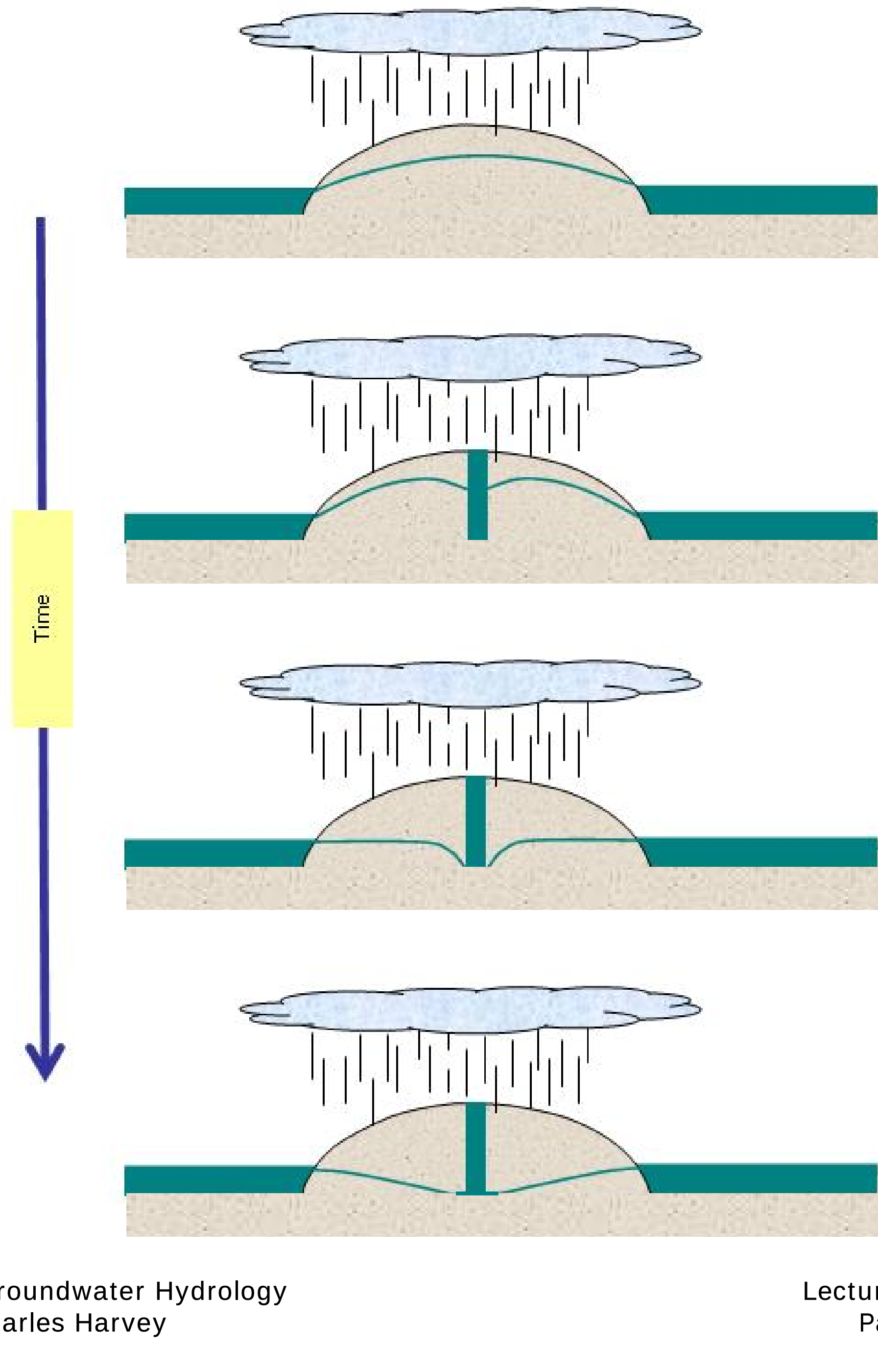
| Types of Springs | |
|---|---|
| • Depression spring | Land surface dips to intersect the water table |
| • Contact spring<br>• Fault spring | Water flows to the surface where a low permeability bed impedes flow |
| • Sinkhole spring<br>• Joint spring<br>• Fracture spring | Similar to a depression spring, but flow is limited to fracture zones, joints, or dissolution channels |

## Regional Groundwater Flow



**Where does water come from when pumping?**

- Initial rate of recharge balances initial rate of discharge.
- Water pumped comes from storage and recharge <u>within</u> cone of depression.
- Water pumping creates cone of depression reaches shoreline.
- Ultimate magnitude of pumpage (before well dries up – at the well) is dependent on hydraulic conductivity, thickness, available drawdown.
- Ultimate production of water depends upon how much rate of recharge can be changed and/or how much water can be captures. Steady-state production is not dependent on $S_y$.
- Although rate of recharge = discharge is interesting, it is almost irrelevant in determining the sustained yield of the aquifer. (Here, think of case where rainfall is small or does not exist – water source is ultimately the lake.)



**General Conclusions: Essential factors that determine response of the aquifer to well development**

- Distance to, and character of, recharge (precip vs. pond)
- Distance to, and character of, natural discharge
- Character of cone of depression (function T and S)

Prior to development aquifer is in equilibrium.
"All water discharged by wells is balanced by a loss of water from somewhere."
-Theis (1940)

When pumping occurs, water comes from storage <u>until</u> a new equilibrium is reached. Accomplished by:

- Increase in recharge → capture of a water source
- Decrease in discharge → reduction of gradient → outflow
- Both

Some water must always be **mined** (taken from storage) to create groundwater development.

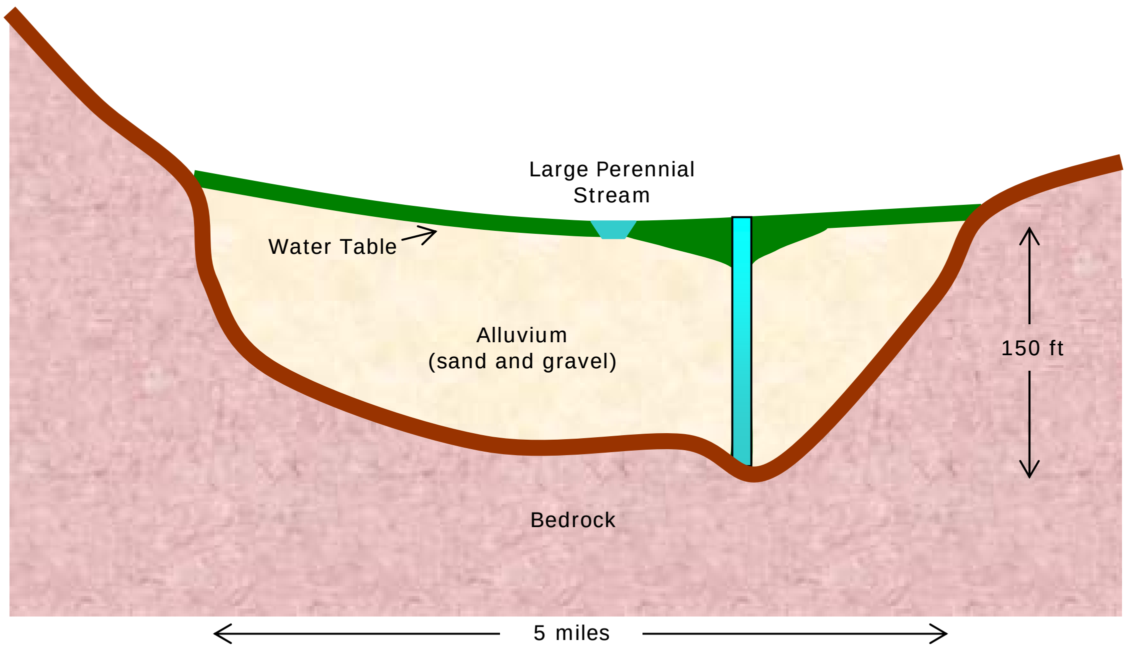
Mathematically the pumping water balance is:

$Q = (R + \Delta R) - (D + \Delta D) - S(\Delta h/\Delta t)$

If over the years R = D, and a new equilibrium (new steady state) is reached ($\Delta h/\Delta t = 0$)

$$Q = \Delta R - \Delta D$$

**Valley of Large Perennial Stream in Humid Region**




**Setting – East Coast**

- Thick, permeable alluvial valley cut into shale
- Large perennial stream
- Shallow water table with many phreatophytes (trees that can stick roots below water table and saturate their roots)
- Moderately heavy precipitation

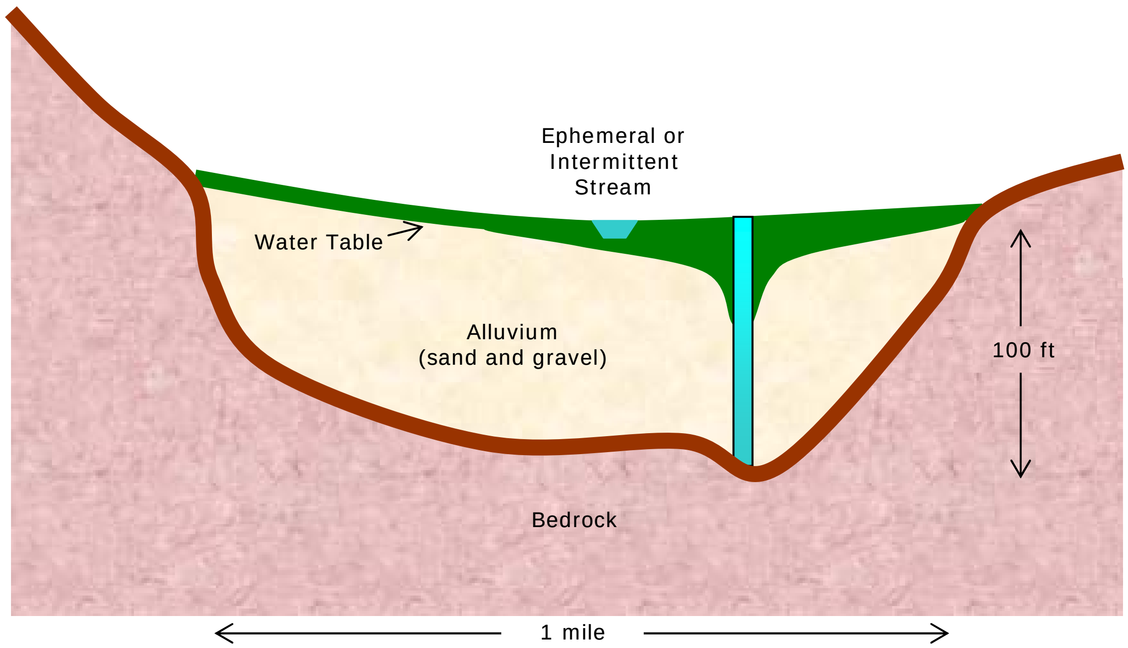
**Sources of Water**

- Withdrawal from storage (cone of depression)
- Salvaged rejected recharge (prevention of runoff to stream by making more room for recharge from precipitation – water goes to groundwater rather than stream – only some of stream water recharges.
- Salvaged natural discharge (natural discharge w/o pumping)
  - o Lowering water table beneath phreatophytes
  - o Decreasing gradient toward stream decreasing base flow (for low development rates) – river is a GW sing – a gaining stream under natural conditions
- Over long term at steady state

$$Q = \Delta R + \Delta D$$

Small developments $\Delta R$ source ← Room for precipitation
Large developments, stream capture, $\Delta D$, source.

**Valley of Ephemeral Stream in Semiarid Region**



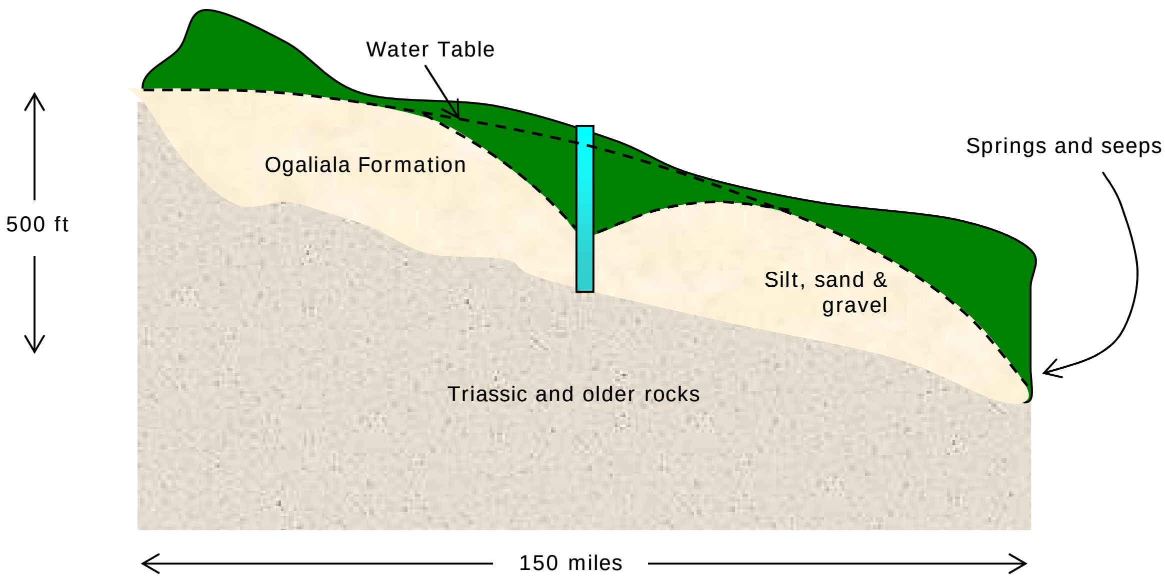

**Setting – West Coast (not northwest)**

- Moderately thick, permeable alluvial valley cut into shale
- Large ephemeral stream
- Water table beneath stream channel, below vegetation
- Precipitation like in Palo Alto, about 15 in/yr
- Stream dry most of year, floods in heavy rains

**Sources of Water**

- Withdrawal from storage (cone of depression)
- No salvaged rejected recharge (enough room for all recharge from low precipitation)
- Little salvaged natural discharge (no phreatophytes)
- Recharge directly from stream (water table low enough so that there is room for flood waters – evaporation-free-control reservoir) – can guarantee this with pumping
- $\Delta D = 0$
    - Capture floodwaters and get + $S\ \Delta h/\Delta t$
      When $\Delta R = 0$ loss from storage is only source
      $Q = -S\ \Delta h/\Delta t$ (water can be pumped seasonally for irrigation and later replenished)

## High Plains of Texas and New Mexico



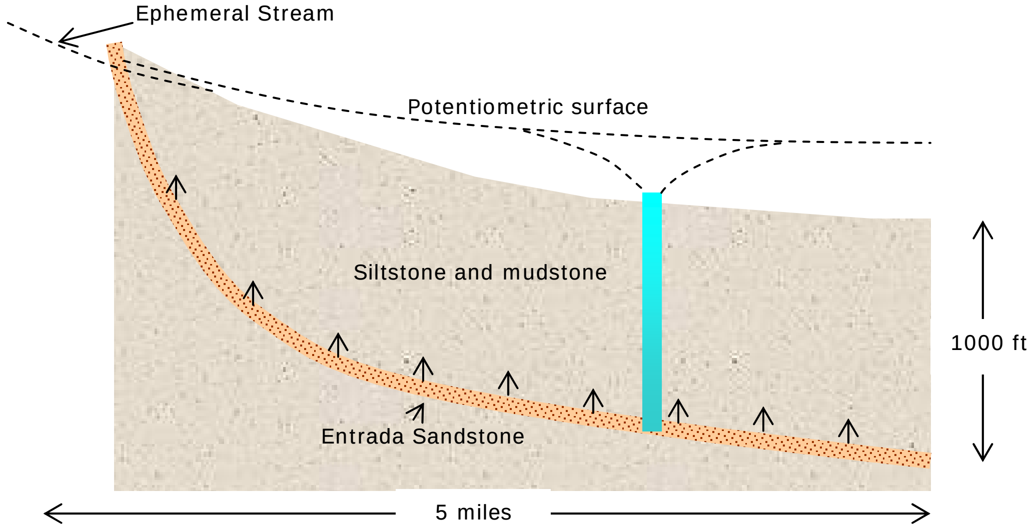

### Setting

- Remnant high plain sloping, cut off from external sources of water by escarpments both upgradient and downgradient
- Thick (300 ft to 600 ft) permeable rocks on impermeable rocks
- Recharge from precipitation is 1/20 to 1/2 in/yr
- Discharge from springs about the same
- Water table (>50 ft)

### Sources of Water

- Withdrawal from storage (cone of depression)
- No salvaged rejected recharge (ample space – 50 ft. unsaturated)
- Little salvaged natural discharge (gradient unchanged, but even if not, aquifer flow would only account for 1 to 2% of the withdrawal rate)
- $Q = -S\ \Delta h/\Delta t$ water from storage – mining only $S_y = 0.15$
- Big difference with Entrada Sandstone $S = 5 \times 10^{-5}$ per square foot for each foot of head decline

**Productive Artesian Aquifer System**



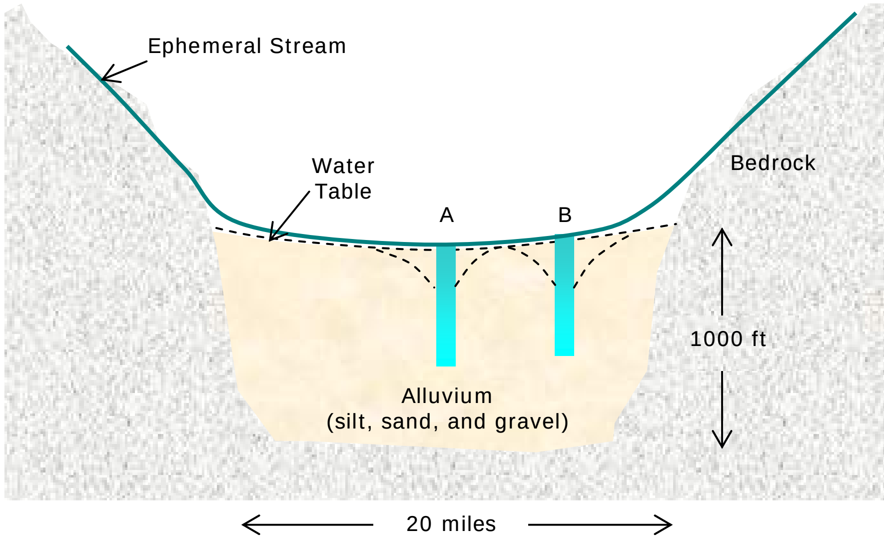

**Setting**

- Grand Junction Artesian Basin, Colorado
- Typical low conductivity artesian aquifer
- Fine-grained sandstone, party cemented with calc. carb.
- 150 ft. thick → T = 20 $ft^2/d$, S = 5 x $10^{-5}$
- Recharge from precipitation (7-8 in/yr) where outcrops are in contact with alluvium
- Discharge small and from upward leakage through relatively impermeable siltstone 500 to 1000 ft thick
- Artesian conditions, as much as 160 ft above land surface

**Sources of Water**

- Withdrawal from confined storage (large overlapping cones of depression)
- No salvaged rejected recharge – already room for recharge water; no extra would enter aquifer if water table in recharge area were lowered. (limiting unit is artesian aquifer)
- Little salvaged natural discharge (limited upward leakage)
- Acts like confined "bathtub" with little $\Delta$D due to pumping.
- Q = -S $\Delta h/\Delta t$ water from storage – mining only – mining artesian storage and not dewatering storage

## Closed Desert Basin




**Setting**

- Thick coalescing alluvial fans, gradational from mountains
- Basin receives precipitation of 3-5 in/yr, mountains 20-30 in/yr
- Very shallow water table near playa, deep near mountains
- Streams are ephemeral
- Phreatophytes near playa

**Sources of Water**

- Withdrawal from storage (create cone of depression)
- Salvaged rejected recharge (center none – precip in valley evaporates or transpires) (border some recharge from small ephemeral streams near surrounding mountains)
- Salvaged natural discharge
    - o Lowering water table near playa may reduce ET (roots)
    - o Near borders of basin discharge toward playa can be reduced (stop flow to center where ET occurs)
- **Operation** – increase rejected recharge and prevent existing discharge:
  $Q = \Delta R + \Delta D - (+/- \ S \ \Delta h/\Delta t)$
- Retention dams to capture flood waters for recharge

**Lecture Packet #7: Transient Systems and Groundwater Storage**

**How does the mass of water stored in an aquifer change?**

Confined Aquifer – Compression of both the material and the water

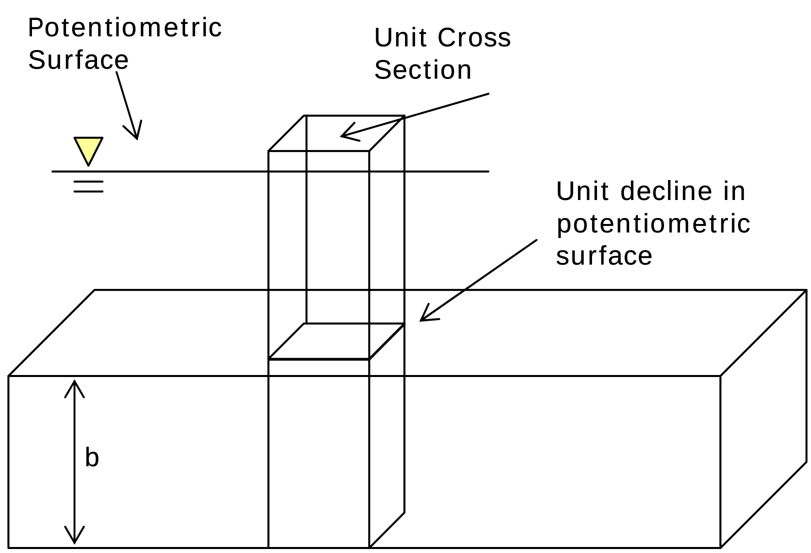
Unconfined Aquifer – Water drains out pores at the water table when the water table drops and fills pores when the water table rises. The change in store from compression is negligible.

Return to the time derivative or change in storage term, which may be written as

$$\frac{\partial \rho n}{\partial t} = \underbrace{n\frac{\partial \rho}{\partial t}}_{\text{Density changes (compressibility)}} + \underbrace{\rho\frac{\partial n}{\partial t}}_{\text{Porosity changes}}$$

Overall definition: **$S_s$, specific storage**, (1/L) of a saturated aquifer is the volume that a unit volume of aquifer released from storage for a unit decline in head. (Volume per Volume per head change).

**Confined Aquifers**




For a confined aquifer we must consider **compressibility**.

How can a reduction in aquifer volume occur?
- Compression of the individual grains or rock skeleton (assumed negligible – individual grains are incompressible)
- Rearrangement of the grains – more compact
- Compression of the water in the pores

- We will get $S_s = \rho g \alpha + \rho g \beta n$

Stress at any depth is due to:

**Total stress** acting downward on a plane

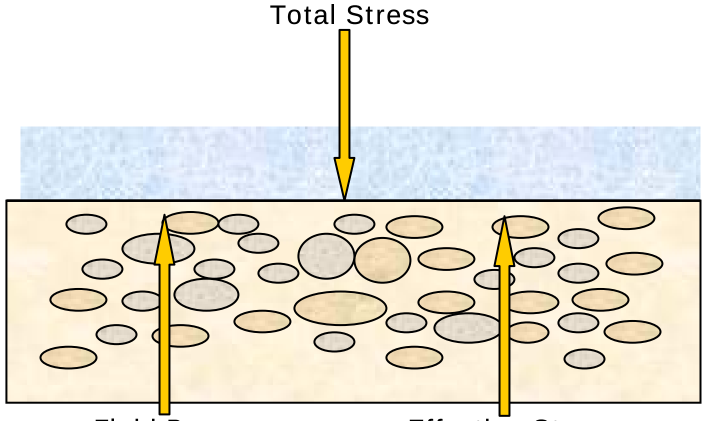
$\sigma_T$ = weight of rock and weight of water

Some of the stress is borne by rock skeleton; some by water.

$\sigma_T = \sigma_T + P \rightarrow$ effective stress (borne by rock) + fluid pressure

When pumping an aquifer the change in stress is:

$d\,\sigma_T = d\sigma_T + dP$




But the weight of overlying water and rock is essentially constant over time. So the change in total stress is zero.

$d\,\sigma_T = d\sigma_T + dP = 0$

$d\sigma_T = -dP$

Fluid pressure decreases, the stress on the grains becomes greater (imagine we took away the fluid).

Fluid pressure controls the volumetric deformation.

At a point, $P = \rho gh - \rho gz = \rho g(h - z)$ --- at a point, z is constant

$dP = d\rho gh$ and substituting ($d\rho gz = 0$ = derivative of a constant)

$d\sigma_e = -dP = -d\rho gh$

If pumping increases, head goes down, and the effective stress goes up. Consider what happens when $\sigma_e$ goes up.

### **Water Produced from Aquifer Compaction**

- Aquifer compressibility, $\alpha$, [$L^2/M$] is defined as follows (corresponds to shifting of grains and reduction in porosity.

$$\alpha = \frac{-(dV_t)/V_t}{d\sigma_e}$$

$V_t$ is the original volume (thickness)
$dV_t$ is the change in volume (thickness)
$d\sigma_e$ is the change in effective stress

Aquifer gets smaller with increase in effective stress.

Consider a unit volume $V_t = 1$.

$\alpha d\sigma_e = -dV_t$

Volume of water pumped that comes from aquifer compaction:

We also know that the change in volume of water produced by aquifer compaction must equal the volume of water and sand accounting for the reduction.

$dV_w = -dV_t$  substitution for the term on the right
$dV_w = \alpha d\sigma_e = -\alpha(d\rho gh) = -\alpha\rho g(dh)$

For a unit decrease in head due to pumping dh = -1

$$dV_w = \alpha\rho g$$

because this is for a unit decrease in head, this is a component of our $S_s$, specific storage.

### **Water Produced by the Expansion of Water**

We can also define fluid compressibility

$$\beta = \frac{-(dV_W)/V_W}{dP}$$

$\beta$ is the fluid compressibility -  compressibility of water
n is the porosity (volume of water is total volume times n)
so, for a unit total volume, $nV_t = n(1) = n$

$$dV_W = -\beta ndP = -\beta n(d\rho gh - d\rho gz) = -\beta n\rho gdh$$

For a unit decline in head, dh = -1,

$dV_w = +\beta n\rho g$       which is another component of $S_s$

Specific Storage is sum:

$$S_s = \rho g(\alpha + \beta n)$$

It has units [$L^{-1}$]. It is volume produced per aquifer volume per head decline.

- First term is from aquifer compressibility.
- Second term is from water expansion.

Let's return to flow equation (time derivative term):

$$\frac{\partial \rho n}{\partial t} = n\frac{\partial \rho}{\partial t} + \rho\frac{\partial n}{\partial t}$$

The total change in mass with time is due to

- A change in density with time term associated with water compressibility.
- A change in porosity with time term associated with the compressibility of the aquifer.

Our analysis of "where water from storage comes from" gives a term involving $S_s = \rho g(\alpha + \beta n)$ to replace the change in mass with time term above.

Going back to the continuity equation:

$$\frac{\partial \rho n}{\partial t} = -\left[\frac{\partial[\rho q_x]}{dx} + \frac{\partial[\rho q_y]}{dy} + \frac{\partial[\rho q_z]}{dz}\right]$$

Rate of Change of <u>Mass</u> stored:

$$\frac{\partial \rho n}{\partial t} = n\frac{\partial \rho}{\partial t} + \rho\frac{\partial n}{\partial t} = -\left[\frac{\partial[\rho q_x]}{dx} + \frac{\partial[\rho q_y]}{dy} + \frac{\partial[\rho q_z]}{dz}\right]$$

Substituting in for Darcy's law, pulling out K (homogeneous, optional), and substituting for the storage terms, we obtain:

$$\rho S_s\frac{\partial h}{\partial t} = \rho\frac{\partial h}{\partial t}\beta n\rho g = \rho K\left[\frac{\partial^2 h}{dx^2} + \frac{\partial^2 h}{dy^2} + \frac{\partial^2 h}{dz^2}\right]$$

Note storage term must convert a volume produced to a mass produced, so multiply by $\rho$.

Cancel first $\rho$, giving our final confined flow equation as:

$$S_s\frac{\partial h}{\partial t} = \rho g(\beta n + \alpha)\frac{\partial h}{\partial t} = K\left[\frac{\partial^2 h}{dx^2} + \frac{\partial^2 h}{dy^2} + \frac{\partial^2 h}{dz^2}\right]$$

**Summary of Storage Mechanisms – Confined Aquifer**

Water is released from storage during a decrease in h by two mechanisms:

- Compaction of aquifer caused by increasing effective stress and rearrangement of grains.
- Expansion of water caused by decreasing fluid pressure.

This gives rise to a specific storage coefficient with two terms and a function of $\alpha$ and $\beta$.

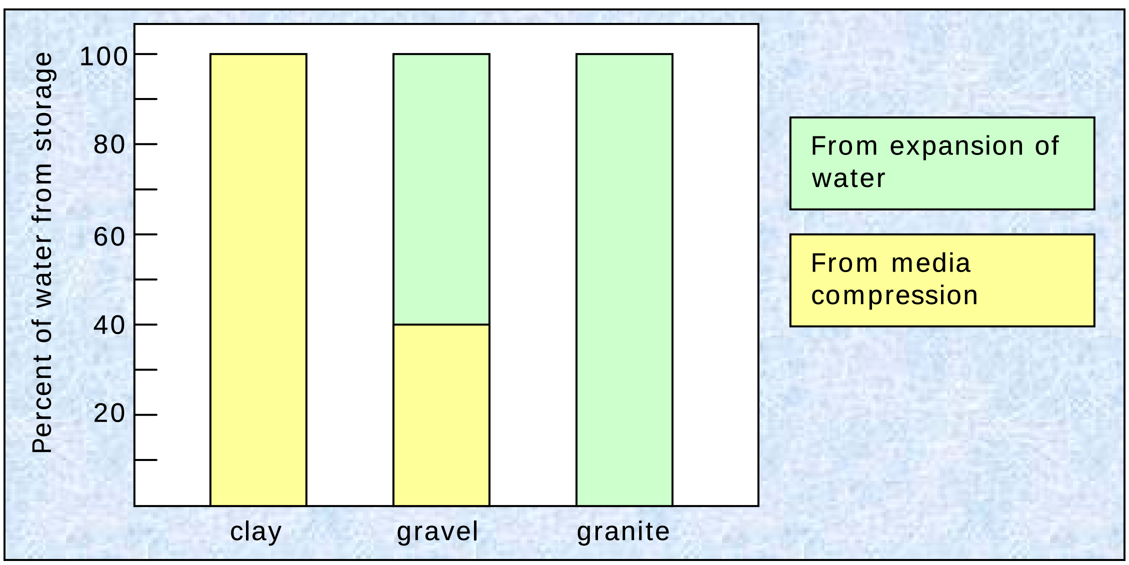
| Material | Compressibility [$m^2$/N or 1/Pa] |
|---|---|
| Clay | $10^{-6} - 10^{-8}$ |
| Sand | $10^{-7} - 10^{-9}$ |
| Gravel | $10^{-8} - 10^{-10}$ |
| Sound Rock | $10^{-9} - 10^{-11}$ |
| Water | $4.4 \times 10^{-10}$ |

Where does stored water come from in confined aquifers?

Assume: $S_s = \rho g(\alpha + \beta n)$

- 10 ft of drawdown (reduction in head)
- Porosity of 30%

| Compressibility ($m^2$/N) | | Water from Storage, $S_s \times \Delta H$ |
|---|---|---|
| Clay | $10^{-6}$ | 0.0303 |
| Sand | $10^{-8}$ | 0.000694 |
| Granite | $10^{-10}$ | 0.000398 |
| Water | $4.4 \times 10^{-10}$ | |




With more realistic porosities, results are similar

| | Porosity | Compression | Expansion | Total Water |
|---|---|---|---|---|
| Clay | 45% | 98.06% | 1.94% | 0.0305 |
| Gravel | 35% | 39.37% | 60.63% | 0.00759 |
| Granite | 25% | 0.90% | 99.10% | 0.00332 |

**Key Equations for a confined aquifer**

- 3D flow equation, Homogeneous Isotropic

$$S_s \frac{\partial h}{\partial t} = K\left[\frac{\partial^2 h}{dx^2} + \frac{\partial^2 h}{dy^2} + \frac{\partial^2 h}{dz^2}\right] \quad \text{or} \quad S_s \frac{\partial h}{\partial t} = K\nabla^2 h$$

- 3D Heterogeneous, Anisotropic

$$S_s \frac{\partial h}{\partial t} = \left[\frac{\partial}{dx} K_x \frac{\partial h}{dx} + \frac{\partial}{dy} K_y \frac{\partial h}{dy} + \frac{\partial}{dz} K_z \frac{\partial h}{dz}\right]$$

- 3D Heterogeneous, Isotropic

$$S_s \frac{\partial h}{\partial t} = \left[\frac{\partial}{dx}\left(K \frac{\partial h}{dx}\right) + \frac{\partial}{dy}\left(K \frac{\partial h}{dy}\right) + \frac{\partial}{dz}\left(K \frac{\partial h}{dz}\right)\right]$$

For **Steady State,** $S_s \frac{\partial h}{\partial t} = 0$ for any equation.

For steady state:
- No water from storage
- S values doesn't matter

## 2D Flow Equation

- Confined aquifer, homogeneous, isotropic.

$$S\frac{\partial h}{\partial t}=T\left[\frac{\partial^2 h}{dx^2}+\frac{\partial^2 h}{dy^2}\right]$$

S = $S_s$b = Storativity [$L^3$/ $L^3$] ← Values like $10^{-2}$ to $10^{-6}$
T = Kb = Transmissivity [$L^2$/T] Compare values to $S_y$ !
B = aquifer thickness [L]

- Confined aquifer, Heterogeneous, Anisotropic

$$S\frac{\partial h}{\partial t}=\left[\frac{\partial}{dx}T_x\frac{\partial h}{dx}+\frac{\partial}{dy}T_y\frac{\partial h}{dy}\right]$$

- Confined aquifer, Heterogeneous, Isotropic

$$S\frac{\partial h}{\partial t}=\left[\frac{\partial}{dx}\left(T\frac{\partial h}{dx}\right)+\frac{\partial}{dy}\left(T\frac{\partial h}{dy}\right)\right]$$

- Confined aquifer, Homogeneous, Anisotropic

$$S\frac{\partial h}{\partial t}=\left[T_x\frac{\partial^2 h}{dx^2}+T_y\frac{\partial^2 h}{dy^2}\right]$$

# Lecture Packet #8: Pump Test Analysis

The idea of a pump test is to stress the aquifer by pumping or injecting water and to note the drawdown over space and time.

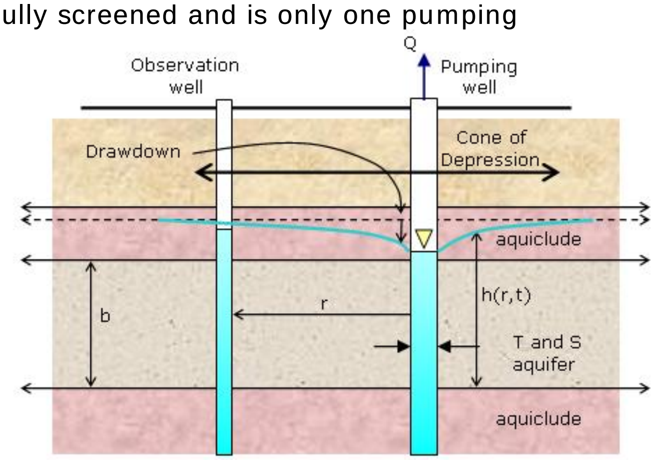
## History

- The earliest model for interpretation of pumping test data was developed by Thiem (1906)
  (Adolf and Gunther) for
  - Constant pumping rate
  - Equilibrium conditions
  - Confined and unconfined conditions
- Theis (1935) published the first analysis of transient pump test for
  - Constant pumping rate
  - Confined conditions
- Since then, many methods for analysis of transient well tests have been designed for increasingly complex conditions, including
  - Aquitard leakage (study of hydrogeology has become more a study of aquitards and less of aquifers)
  - Aquitard storage
  - Wellbore storage
  - Partial well penetration
  - Anisotropy
  - Slug tests
  - Recirculating well tests (water is not removed)

It is important to note the assumptions for a given analysis.

## Steady Radial Flow in a Confined Aquifer

Assume:
- Aquifer is confined (top and bottom)
- Well is pumped at a constant rate
- Equilibrium is reached (no drawdown change with time)
- Wells are fully screened and is only one pumping



Consider Darcy's law through a cylinder, radius r, with flow toward well.

$Q = K\frac{dh}{dr}2\pi rb$ and rearrange as $dh = \frac{Q}{2\pi Kb}\frac{dr}{r}$

Integrate from $r_1$, $h_1$ to $r_2$, $h_2$

$$\int_{h_1}^{h_2} dh = \frac{Q}{2\pi Kb}\int_{r_1}^{r_2}\frac{dr}{r}$$

$h_2 - h_1 = \frac{Q}{2\pi Kb}\frac{r_2}{r_1}$ or noting that T = Kb

$\boxed{T = \frac{Q}{2\pi(h_2 - h_1)}\ln\frac{r_2}{r_1}}$ this is the Thiem equation

Notes on the Thiem equation:

- Good with any self consistent units L and t
- If drawdown has stabilized can use any two observation wells
- Water is not coming from storage (S doesn't appear) cannot get S from this test
- Commonly used in USGS units and $\log_{10}$, T in gpd/ft (gallons per day per foot), Q in gpm (gallons per minute), r and h in ft.

$$T = \frac{527.7Q}{(h_2 - h_1)}\log\frac{r_2}{r_1}$$

**Specific Capacity of a Well** – Roughly estimating T

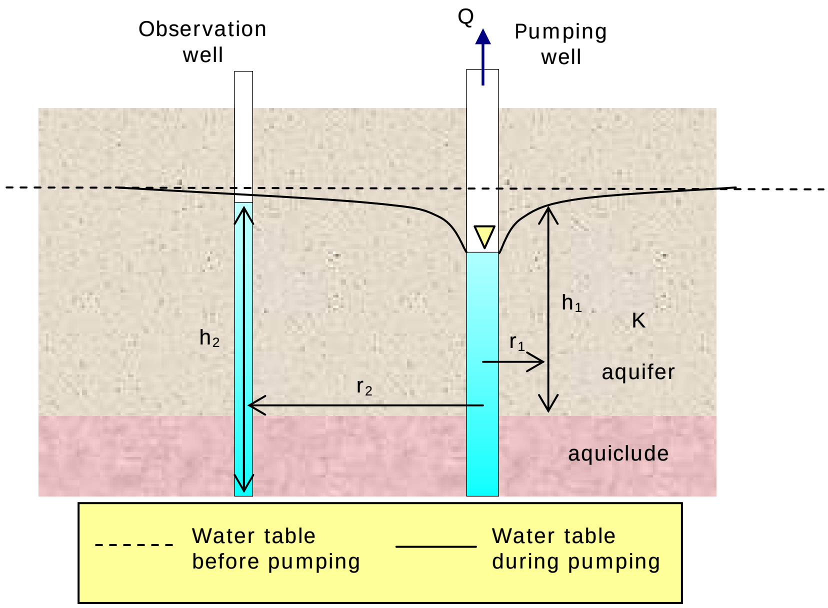
Specific Capacity = Discharge Rate/Drawdown in the well

1. A well is pumped to approximate equilibrium.
2. A good well would be 50 gpm per foot of drawdown, or 20 feet of drawdown at 1,000 gpm.
3. $h_e$ and $r_e$ are the head and corresponding distance from a well where drawdown is effectively zero.
4. Specific capacity = $T = \frac{Q}{(h_e - h_w)} = \ln\frac{T}{527.7\log\frac{r_e}{r_w}}$
5. Rule of Thumb – T ~ 1,800 x Specfic Capacity
6. What is $r_e$? It doesn't matter that much.
   $r_e = 1{,}000\ r_w$ $\quad\log r_e/r_w = 3$
   $r_e = 10{,}000\ r_w$ $\quad\log r_e/r_w = 4$
7. Case A → T = Specific Capacity [527 x 3] = 1,581 x SC
   Case B → T = Specific Capacity [527 x 4] = 2,108 x SC
8. If you use T ~ 1,800 x Specific Capacity you are not too far off. SC is gpm/ft and T is gpd/ft.

**Steady Radial Flow in an Unconfined Aquifer**

Assume:

- Aquifer is unconfined but underlain by an impermeable horizontal unit.
- Well is pumped at a constant rate
- Equilibrium is reached (no drawdown change with time)
- Wells are fully screened and
- There is only one pumping well




Radial flow in the unconfined aquifer is given by

$Q = K(2\pi rh)\frac{dh}{dr}$ and rearrange as $hdh = \frac{Q}{2\pi K}\frac{dr}{r}$

Integrate from $r_1$, $h_1$ to $r_2$, $h_2$

$$\int_{h_1}^{h_2} jdh = \frac{Q}{2\pi K}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$\frac{h_2^{\ 2} - h_1^{\ 2}}{2} = \frac{Q}{2\pi K}\ln\frac{r_2}{r_1}$$ or noting that T = Kb

$$\boxed{K = \frac{Q}{\pi(h_2^{\ 2} - h_1^{\ 2})}\ln\frac{r_2}{r_1}}$$

this is the Thiem equation for unconfined conditions (K not T, $h^2$ not h, no 2)

- If the greatest difference in head in the system is < 2%, then you can use the confined equation for an unconfined system.
- For r < $1.5h_{max}$ (the full saturated thickness) there will be errors using this equation because of vertical flow near the well.
- If difference in head is > 2% but < 25% use the confined equation with the following correction for drawdown

$(h_2 - h_1)_{new} = \Delta h - \dfrac{\Delta h^2}{2b}$ "measured" reduced compared to confined aquifer case

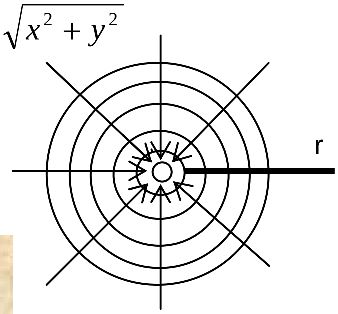
**Transient Pumping Tests**

$$S\frac{\partial h}{\partial t} = T\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right]$$

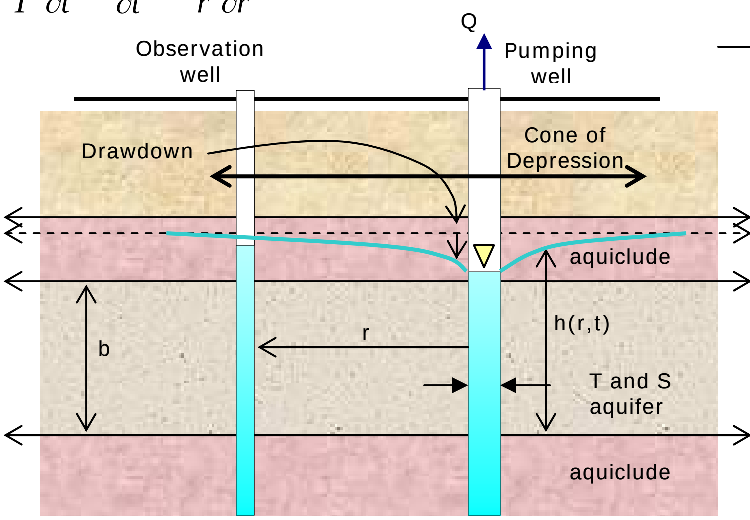
S = $S_s$b = Storativity [$L^3/L^3$]
T = Kb = Transmissivity [$L^2/T$]
b = aquifer thickness [L]
h is head [L]

Assume:

- Aquifer is horizontal, confined both top and bottom, infinite in horizontal extent, constant thickness, homogeneous, isotropic.
- Potentiometric surface is horizontal before pumping, is not changing with time before pumping, all changes due to one pumping well
- Darcy's law is valid and groundwater has constant properties
- Well is fully screened and 100% efficient
- Constant pumping from a well in such a situation is radial, and horizontal, where only 1 space dimension is needed $r = \sqrt{x^2 + y^2}$




$$\frac{S}{T}\frac{\partial h}{\partial t} = \frac{\partial^2 h}{\partial t^2} + \frac{1}{r}\frac{\partial h}{\partial r}$$



We want a solution that gives h(r,t) after we start pumping. To solve equation we need initial conditions (ICs) and boundary conditions (BCs)

IC: $h(r,0) = h_0$

BCs: $h(\infty,t)=h_0$ and $\lim_{r \to 0}\left( r\frac{\partial h}{\partial r}\right) = \frac{Q}{2\pi T}$ for t > 0
which is just the application of Darcy's law at the well

**Theis Equation – transient radial flow**

1935 – C.V. Theis solves this equation (with C.I. Lubin from heat conduction)

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}du}{u}$$

exponential integral in math tables but for our case it is "well function" ←

where $u = \frac{r^2 S}{4Tt}$

$h_0 - h(r,t) = \frac{Q}{4\pi T}W(u)$, well function is W(u)

Determining T and S from Pumping Test

Inverse method: Use solution to the PDE to identify the parameter values by matching simulated and observed heads (dependent variables); e.g., measure aquifer drawdown response given a known pumping rate and get T and S.

1. Identify pumping well and observation wells and their conditions (e.g., fully screened).
2. Determine aquifer type and make a quick estimate to predict what you think will happen during pumping test.
3. Theis Method: Arrange Theis equation as follows:

   $\Delta h = \left[\frac{114.6Q}{T}\right]W(u)$ (in USGS units) and

   $\frac{r^2}{t} = \left[\frac{T}{1.87S}\right]u$ → $t = \left[\frac{1.87\, Sr^2}{T}\right]\frac{1}{u}$

   $$\log \Delta h = \log\left[\frac{114.6Q}{T}\right] + \log W(u)$$

   $$\log t = \log\left[\frac{1.87Sr^2}{T}\right] + \log\frac{1}{u}$$
4. Plot the well function W(u) versus 1/u on log-log paper. (this is called a type curve)

5. Plot drawdown vs. time on log-log paper of same scale. (this is from data at a single observation well)
6. Superimpose the field curve on the type curve, keeping the axes parallel. Adjust the curves so that most of the data fall on the type curve. You trying to get the constants (bracketed terms) that make the type curve axes translate into your axes.
7. Select a match point (any convenient point will do like W(u) = 1.0 and 1/u = 1.0), and read off the values for W(u) and 1/u. Then read off the values for drawdown and t.
8. Compute the values of T and S from:

| Using USGS Units | Using Self-Consistent Units |
|---|---|
| $T = \dfrac{114.6QW(u)}{(h_0 - h)}$ | $T = \dfrac{QW(u)}{4\pi(h_0 - h)}$ |
| $S = \dfrac{Ttu}{1.87r^2}$ | $S = \dfrac{4Ttu}{r^2}$ |

If in USGS units of drawdown (ft), Q (gpm), T (gpd/ft), r (ft), t (days), S (decimal fraction).

$$h_0 - h(r,t) = \frac{Q}{4\pi T}W(u)$$

$$u = \frac{1.87r^2S}{Tt}$$

To predict drawdown – drawdown vs. distance or time

- Put in r, S, T, t and solve for u
- Find W(u) based on u and table
- Multiply Q/T and factor

The analytic solution describes:

- Geometric characteristics to the cone of depression, steepening toward the well
- For a given aquifer cone of depression increases in depth and extent with time
- Drawdown at a time and location increases linearly with pumping rate
- Drawdown at a time and location is greater for smaller T
- Drawdown at a time and location is greater for lower S

**Modified Nonequilibrium Solution – Jacob Method**

Recall from Theis solution:

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\int_u^\infty \frac{e^{-u}du}{u}$$

C.E. Jacob noted that the well function can be represented by a series.

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln u + u - \frac{u^2}{2\cdot 2!} + \frac{u^3}{3\cdot 3!} - \frac{u^4}{4\cdot 4!} + ....\right]$$

For small values of r and large values of t, u becomes small. (valid when u < 0.01).

Then most terms can be dropped leaving:

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\left[-0.5772 - \ln\frac{r^2 S}{4Tt}\right]$$

Noting that –ln u = ln 1/u and ln 1.78 = 0.5772

$$h_0 - h(r,t) = \frac{Q}{4\pi T}\left[\ln\frac{2.25Tt}{r^2 S}\right]$$

And ln u = 2.3 log u

$$h_0 - h(r,t) = \frac{2.30Q}{4\pi T}\left[\log_{10}\frac{2.25Tt}{r^2 S}\right]$$

Since Q, r, T, and S are constants, drawdown vs. log t should plot as a straight line.

In USGS units:

$$h_0 - h(r,t) = \frac{264Q}{4\pi T}\left[\log_{10}\frac{0.3Tt}{r^2 S}\right]$$

Over one log cycle you get change in drawdown

From $t_1$ to $t_2$ : $\Delta[h_0 - h] = \frac{264Q}{T}\left[\log_{10}\frac{t_2}{t_1}\right]$

$T = \frac{264Q}{\Delta[h_0 - h]}\left[\log_{10} 10\right]$ (USGS units)

$$T = \frac{2.3Q}{4\pi\Delta[h_0 - h]}$$ (self-consistent units)

**Procedure**

1. Plot on semi-log paper – t on log scale and drawdown on arithmetic scale.
2. Pick off two values of time and the corresponding values of drawdowns (over one log cycle make it easy).
3. Solve for T (just a function of Q and $\Delta$h)
4. Consider basic solution when drawdown is zero.

$$0 = h_0 - h = \frac{2.30Q}{4\pi T}\left[\log_{10}\frac{2.25Tt_0}{r^2S}\right]$$

$$0 = \left[\log_{10}\frac{2.25Tt_0}{r^2S}\right] \rightarrow \frac{2.25Tt_0}{r^2S} = 1 \text{ or}$$

$$S = \frac{2.25Tt_0}{r^2}$$ where $t_0$ is the time intercept at which drawdown is zero in USGS units.

$$S = \frac{2.25Tt_0}{r^2}$$ where $t_0$ is the time intercept in days and the straight line intersects the zero-drawdown axis

# Lecture Packet #9: Numerical Modeling of Groundwater Flow

Simulation: The prediction of quantities of interest (dependent variables) based upon an equation or series of equations that describe system behavior under a set of assumed simplifications.

## Groundwater Flow Simulation

- Predict hydraulic heads (1D, 2D, 3D)
- For particular conditions – confined, unconfined, isotropic, anisotropic, homogeneous, heterogeneous, infinite, finite, steady, transient.
- Varying levels of complexity:
  - Analystic solutions – Theim, Theis, etc.
    - Advantages: exact, simple, cheap, can provide sufficient insight
  - Analog simulation
  - Physical models – scale models of aquifers
  - Numerical Simulation

## Numerical Simulation

- Given a PDE and appropriate ICs and BCs
- Discretize the system
- Approximate the PDE corresponding to the discretization
- Solve the approximated PDE on a computer
- Commonly finite differences or finite elements for GW flow
- Advantages: can handle complex geometries, ICs, and C conditions; can be used for nonlinear systems.

## Beware of the term "Model" and how it is used

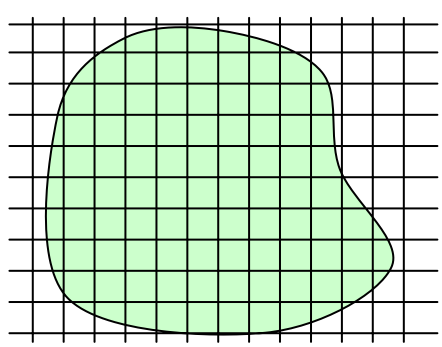
- "A model should be used as simple as possible, but not simpler"
- Mathematical "model" – a PDE
- Numerical "model" – a particular technique is applied
- Computer or simulation "model" – a code

## Finite Differences

A numerical method that approximates the governing PDE by replacing the derivatives in the equation with their respective difference representations.

Procedure involves: Grid (or Mesh) and Equation

Grid – a representation of the physical domain that enables one to account for the boundaries and internal features



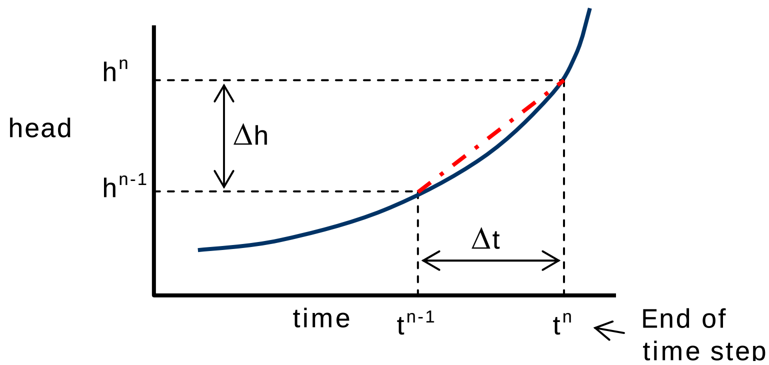
**Equation – Difference Approximation of Derivatives**

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$ 2D flow equation

Approximating the Time Derivative:

*Backward Difference:*

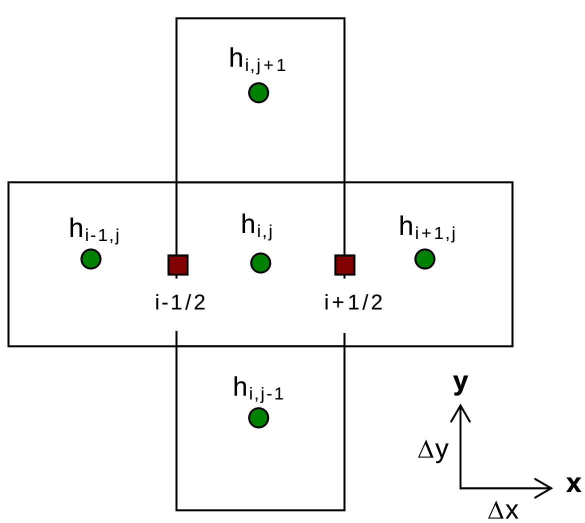
$$\left(\frac{\partial h_{i,j}}{\partial t}\right)_{n\Delta t} \approx \frac{\Delta h}{\Delta t} \approx \frac{h_{i,j,n} - h_{i,j,n-1}}{\Delta t}$$




where,
n = the current time step index
Δt = time step
i,j = x and y coordinate indices

Approximating the Space Derivatives:

Consider a 2D discretization, if we assume that the grid spacing in the x-direction and y-direction are the same, our discretized grid for an internal node will be:



*In the x-direction*:

Approximate derivative at location $h_{i,j}$:

$$\left(\frac{\partial^2 h}{\partial x^2}\right)$$

Approximate the **second** spatial derivative at **i,j** as follows:

$$\left(\frac{\partial^2 h}{\partial x^2}\right) = \frac{\partial}{\partial x}\left(\frac{\partial h}{\partial x}\right) \approx \frac{\left(\frac{\partial h_{i+1/2,j}}{\partial x} - \frac{\partial h_{i-1/2,j}}{\partial x}\right)}{\Delta x}$$

We can approximate the **first** spatial derivative at i-1/2,j and i+1/2,j as follows:

$$\left(\frac{\partial h_{i-1/2,j}}{\partial x}\right) \approx \left(\frac{h_{i,j} - h_{i-1,j}}{\Delta x}\right) \text{ and } \left(\frac{\partial h_{i+1/2,j}}{\partial x}\right) \approx \left(\frac{h_{i+1,j} - h_{i,j}}{\Delta x}\right)$$

Substitute the above equations to obtain:

$$\left(\frac{\partial^2 h_{i,j}}{\partial x^2}\right) \approx \frac{\left(\frac{h_{i+1,j} - h_{i,j}}{\Delta x} - \frac{h_{i,j} - h_{i-1,j}}{\Delta x}\right)}{\Delta x}$$

or

$$\left(\frac{\partial^2 h_{i,j}}{\partial x^2}\right) \approx \frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{\Delta x^2}$$

In the y-direction:

$$\left(\frac{\partial^2 h_{i,j}}{\partial y^2}\right) \approx \frac{h_{i,j-1} - 2h_{i,j} + h_{i,j+1}}{\Delta y^2}$$

Combining Flow Equation Terms:

$$\frac{\partial^2 h_{i,j}}{\partial x^2} + \frac{\partial^2 h_{i,j}}{\partial y^2} = \frac{S}{T}\left(\frac{\partial h_{i,j}}{\partial t}\right)_{\eta \Delta t}$$

For the backwards difference time derivative (note: all left hand side values of h are for time = n-1)

$$\frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{\Delta x^2} + \frac{h_{i,j-1} - 2h_{i,j} + h_{i,j+1}}{\Delta y^2} = \frac{S}{T}\frac{h_{i,j,n} - h_{i,j,n-1}}{\Delta t}$$

$\longleftarrow$ Linear diff. eq'n. This eq'n is not explicit for h at any particular time

If $\Delta$x and $\Delta$y are equal – this is the finite difference groundwater flow equation

$$\frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1} - 4h_{i,j}}{\Delta x^2} = \frac{S}{T}\frac{h_{i,j,n} - h_{i,j,n-1}}{\Delta t}$$

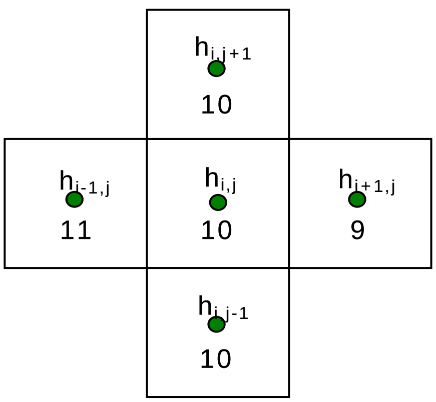
What does the finite-difference equation indicate for steady state conditions?

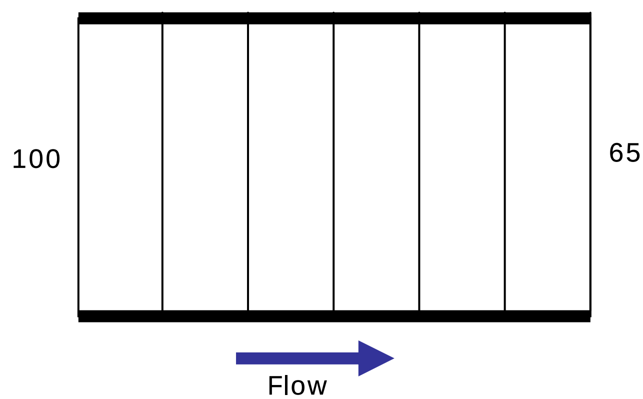
$$S\frac{\partial h}{\partial t} \approx S\left(\frac{h_{i,j,n} - h_{i,j,n-1}}{\Delta t}\right) = 0$$

$$\frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1} - 4h_{i,j}}{\Delta x^2} = 0$$

$$h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1} - 4h_{i,j} = 0$$

$$\frac{h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1}}{4} = h_{i,j}$$



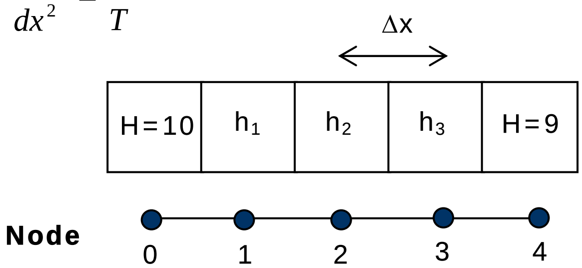




Value of head at node is the average of the surrounding nodes (for SS isotropic homogeneous case)

Consider the 1D steady-state flow equation with a sink is:

$$T\frac{d^2H}{dx^2} - w' = 0 \qquad \text{or}$$

$$\frac{d^2H}{dx^2} = \frac{w'}{T}$$



The nodal finite difference equation is:

$$\frac{h_{i,j-1} - 2h_{i,j} + h_{i,j+1}}{\Delta x^2} = \frac{w'}{T}$$

Or

$$h_{i,j-1} - 2h_{i,j} + h_{i,j+1} = \frac{(\Delta x)^2 w'}{T}$$

<u>Node 1:</u>

$1H_0 + -2h_1 + 1h_2 = 0$

$1(10) + -2h_1 + 1h_2 = 0$

$-2h_1 + 1h_2 = -10$

<u>Node 2:</u>

$1h_1 + -2h_2 + 1h_3 = 0$

<u>Node 3:</u>

Node 3 has a forcing term due to the pumping of the well. This translates into an initial righthand side of the equation:

Given $\Delta x = 0.1$, $\frac{(\Delta x)^2 w'}{T} = \frac{(0.1)^2 1}{.01} = 1$

$1h_2 + -2h_3 + 1H_4 = 1$

$1h_2 + -2h_3 + 1(9) = 1$

$1h_2 + -2h_3 = 1-9 = -8$

The <u>system</u> of finite-difference equations consists of 3 equations and 3 unknowns

$$\begin{array}{cccccc} -2h_1 & 1h_2 & & = & -10 \\ h_1 & -2h_2 & 1h_3 & = & 0 \\ & 1h_2 & -2h_3 & = & -8 \end{array}$$

**Unknowns** **Knowns**

Or in <u>coeffiecient matrix form</u>

$$\begin{pmatrix} -2 & 1 & \\ 1 & -2 & 1 \\ & 1 & -2 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} -10 \\ 0 \\ -8 \end{pmatrix}$$

FD Coeff. Matrix — Unknown Heads — RHS containing boundary conditions and known pumping

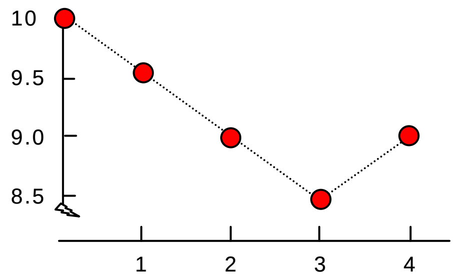
Or in matrix notation it can be written as

$Ah = b'$

A is the matrix of difference coefficients
H is the vector of unknown heads
b' is the RHS vector of known quantities

A computational linear solve yields the vector **h**:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 9.5 \\ 9 \\ 8.5 \end{pmatrix}$$




**Transient Simulation Using Finite Differences**

Procedure: March Through Time

- Start with initial conditions (these are known)
- Solve for heads at end of first time step $\Delta t$; this give the spatial distribution of head (a map) after a small time increment.
- Given known heads at end of first time step solve for heads at the end of the second time step.
- With known value at the end of time step solve for next time step – this is called marching through time

$Ah_n = b^*$ where $b^* = b' + h_{n-1}$

The right-hand side always contains knowns. The matrix A is the matrix of finite difference coefficients reflecting the system parameters and discretization.

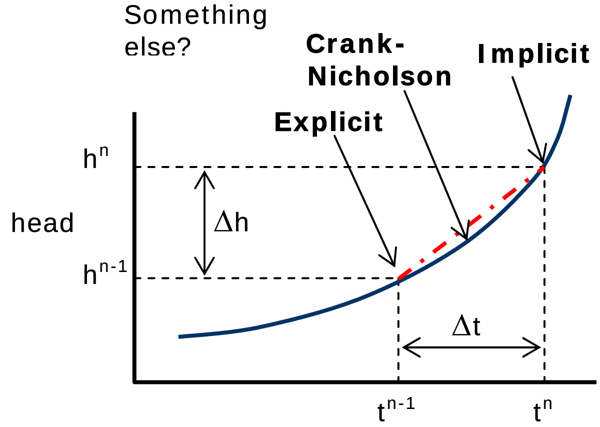
So if you can solve spatial equations for one time step. Then you can solve it for as many time steps as you like.

The time step must be small when changes in heads are rapid – such as, when you start to pump a well, $\Delta t$ must be seconds or minutes. It can be increased as changes in head become smaller.

**FD Simulation Models**: codes that solve the above system of linear algebraic equations – fairly robust.

**Time Stepping**

$$\left\{\frac{h_{i-1}-2h_i+h_{i+1}}{\Delta x^2}\right\} \quad \text{n? n-1? Something else?} \quad = \frac{S}{T}\frac{h_{i,n}-h_{i,n-1}}{\Delta t}$$




$$\frac{T\Delta t}{s\Delta x^2}\left(h_{i-1}-2h_i+h_{i+1}\right)=h_{i,n}-h_{i,n-1}$$

**Explicit:** $h_{i,n} = ch_{i-1,n-1} + (1-2C)h_{i,n-1} + ch_{i+1,n-1}$

Where: $c=\frac{T\Delta t}{s\Delta x^2}$

Easy to calculate – no linear algebra, but unstable if time-steps are too large.

**Implicit:** $ch_{i-1,n} - (1+2C)h_{i,n} + ch_{i+1,n} = -h_{i,n-1}$

Results in system of linear equations that must be solved simultaneously. Stable, but issues of accuracy.

**Crank Nicholson:** $\frac{c}{2}h_{i-1,n}-(1+c)h_{i,n}+\frac{c}{2}h_{i+1,n}=(c-1)h_{i-1,n}-\frac{c}{2}\left(h_{i-1,n-1}+h_{i-1,n-1}\right)$

Not much more numerical expense than fully implicit, but more accurate

**Boundary Conditions:**

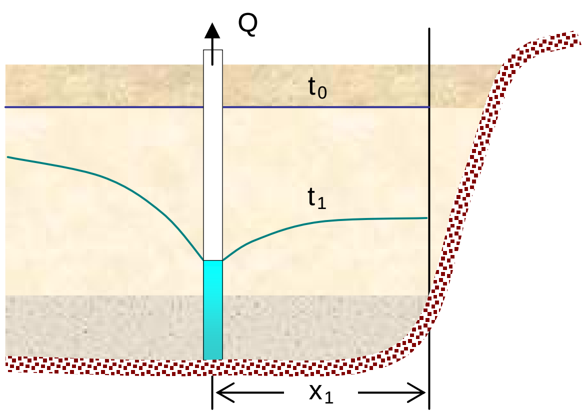
Constant Head – Don't write equation for constant head node. Use value of constant head in equations for neighboring nodes.

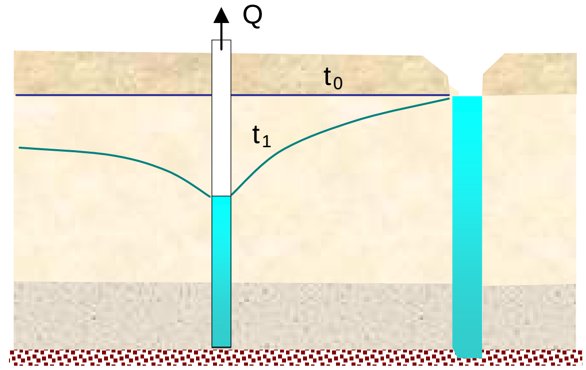
Flux Boundary – replace first derivative term with constant

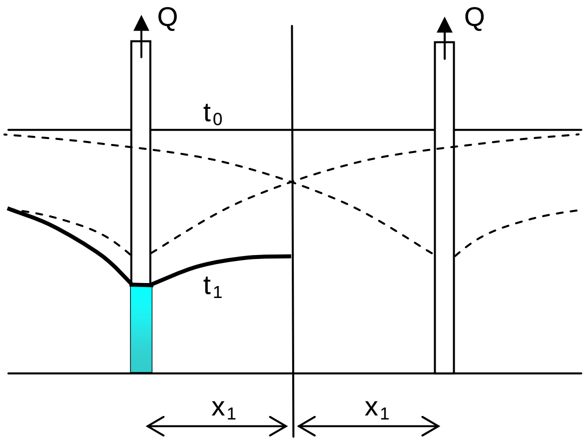
$$\left(\frac{\partial^2 h_{i,j}}{\partial x^2}\right)\approx\frac{\left(\frac{h_{i-1,j}-h_{i,j}}{\Delta x}-\frac{h_{i,j}-h_{i+1,j}}{\Delta x}\right)}{\Delta x}$$

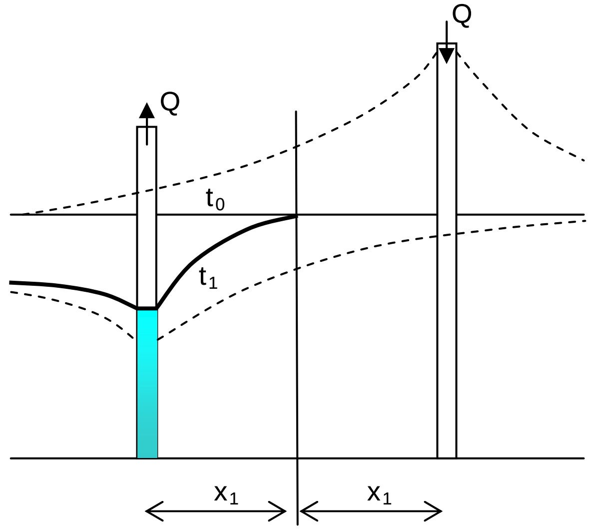
$$\frac{h_{i,j}-h_{i+1,j}}{\Delta x}=\frac{Q}{T}$$

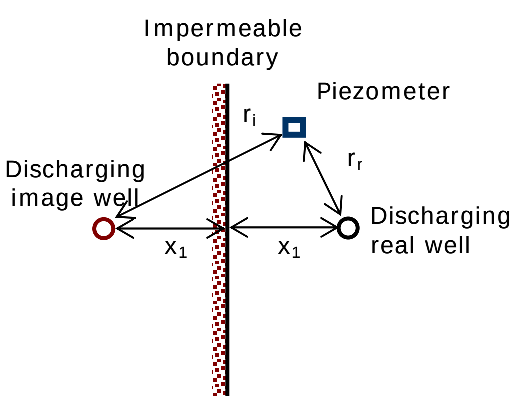
1.72, Groundwater Hydrology
Prof. Charles Harvey
**Lecture Packet #10: Superposition**

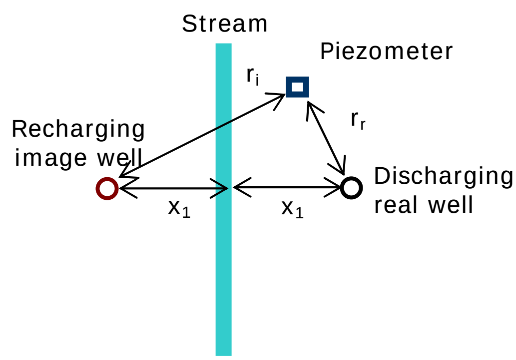


**Superposition**

Aquifer flow equation:

$$\frac{\partial T \partial H}{\partial x^2} + \frac{\partial T \partial H}{\partial y^2} + Q = S\frac{\partial H}{\partial t}$$

Subject to:

H = A, and/or $\frac{\partial H}{\partial \eta} = 0$ along some boundary

H = A at t = 0

Suppose the system has constant head boundaries (H = A, where A is uniform) and no-flow boundaries, then we can convert the GW equation to be in terms of drawdown rather than head.

d = A – H ⟹ H = A – d

$$\frac{\partial T \partial d}{\partial x^2} + \frac{\partial T \partial d}{\partial y^2} - Q = S\frac{\partial d}{\partial t}$$

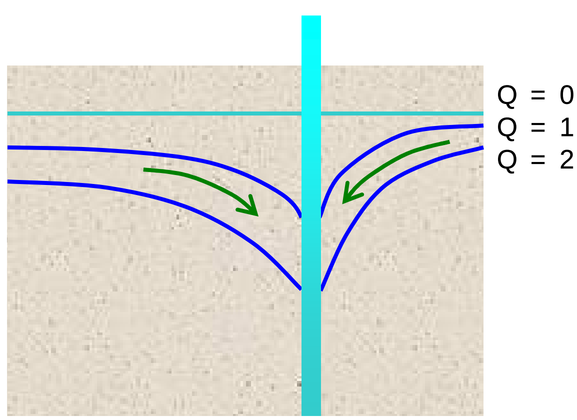
d = 0, and/or $\frac{\partial d}{\partial \eta} = 0$ along some boundary

Now we can investigate the effects on increasing the pumping rate, by multiplying the whole equation by a constant:

$$\frac{\partial T \partial 2d}{\partial x^2} + \frac{\partial T \partial 2d}{\partial y^2} - 2Q = S\frac{\partial 2d}{\partial t}$$

2d = 0, and/or $\frac{\partial 2d}{\partial \eta} = 0$ along some boundary

Thus the solution is the same, except all the drawdowns have been increased by the increase in the pumping rate. We did not make any assumptions about the geometry of the domain, the spatial pattern of T, or the location of the well.



Now we suppose we have two wells at different locations. We can add the two equations to get one equation:

$$\frac{\partial T \partial d_1}{\partial x^2} + \frac{\partial T \partial d_1}{\partial y^2} = Q_1$$

$$\frac{\partial T \partial d_2}{\partial x^2} + \frac{\partial T \partial d_2}{\partial y^2} = Q_2$$

$$\frac{\partial T \partial (d_1 + d_2)}{\partial x^2} + \frac{\partial T \partial (d_1 + d_2)}{\partial y^2} = Q_1 + Q_2$$

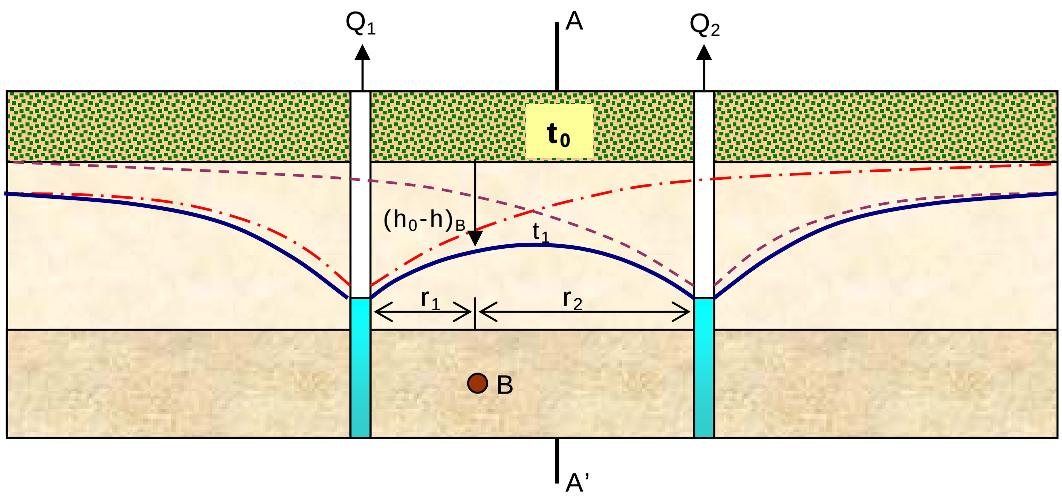
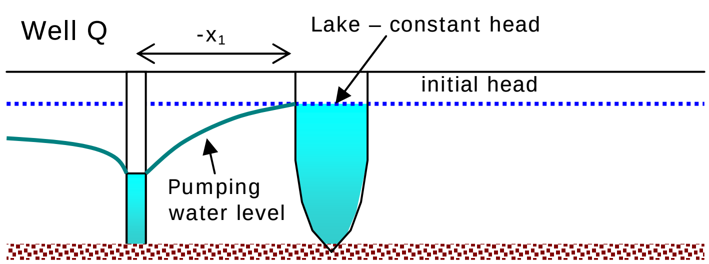
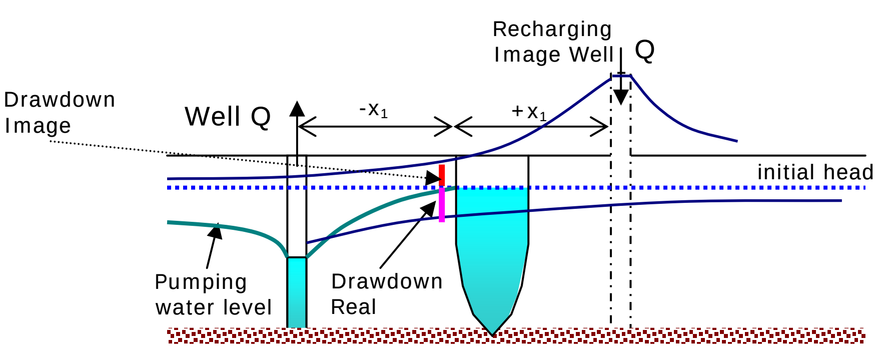



**Image Well Theory & Superposition**

Concept: To predict aquifer behavior in the presence of boundaries, introduce imaginary wells such that the response at the boundary is made true.



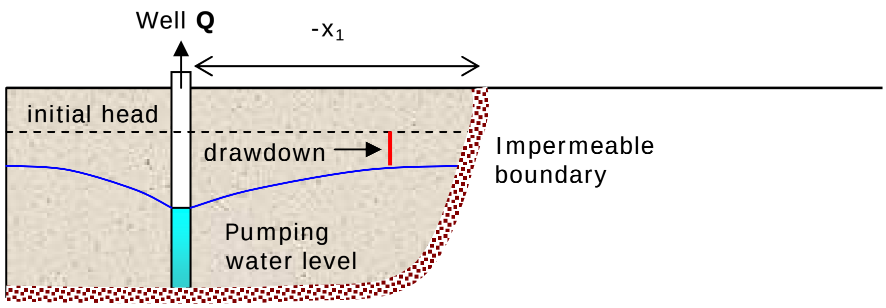



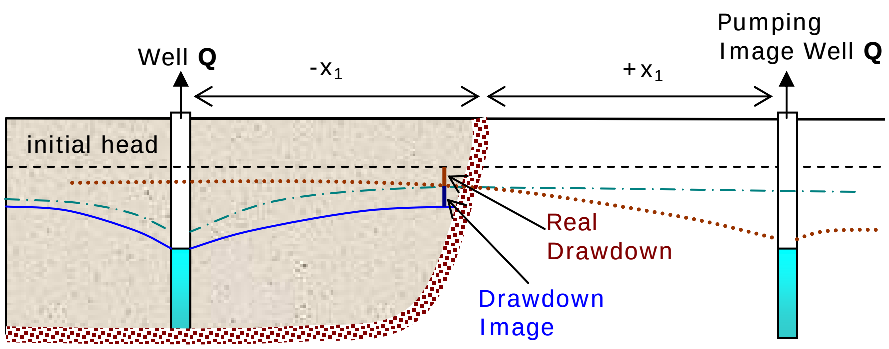
To maintain constant head at lake

- Introduce an image well
- It recharges (artificially)
- It creates a cone of impression
- Resultant cone is due to pumping well and recharge well

Consider an impermeable boundary

- Drawdown is enhanced due to reflection

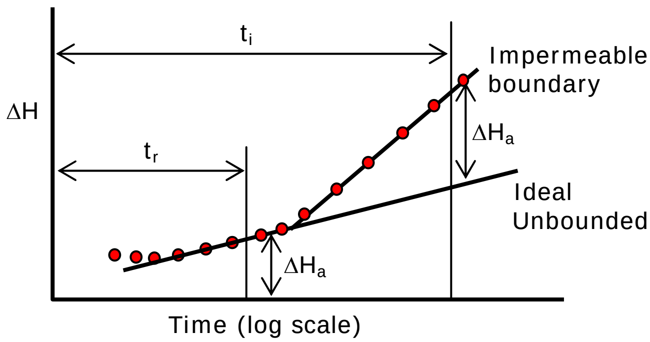


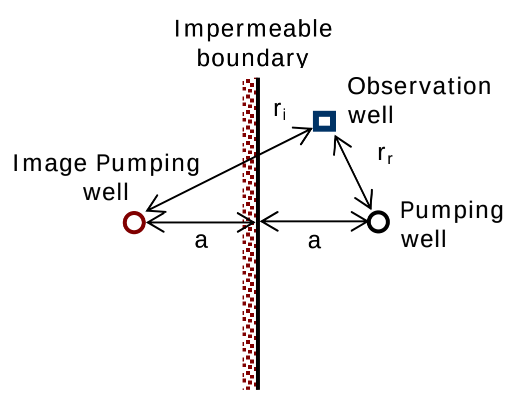




- A pumping image well creates the effect of the boundary

Two things you can do with superposition or image wells.

1) Find the hidden boundary
   For a semi-log plot one might obtain:




(applies to any linear system – can work with Theis equation as well)

$$[h_0 - h(r,t)]_r = \frac{2.30Q}{4\pi T}[\log_{10}]\frac{2.25Tt}{r^2 S}$$
$$[h_0 - h(r,t)]_i = \frac{2.30Q}{4\pi T}[\log_{10}]\frac{2.25Tt}{r_i^{\,2}}$$

Drawdown due to real well is set to drawdown due to image pumping well (at different times ← f(boundary))

$[h_0 - h(r,t)]_r = [h_0 - h(r,t)]_i$ which gives

$$\frac{r_i^{\,2}}{t_i} = \frac{r_r^{\,2}}{t_r} \rightarrow r_i = r_r\sqrt{\frac{t_i}{t_r}}$$

- Slope of line on semi-log plot depends on Q and T
- If an impermeable boundary is present, drawdown will double under the influence of the image well at some point in time
- Select an arbitrary drawdown $\Delta H_a$ and the time for this drawdown to occur under the real pumping well
- Find the same drawdown to be produced by the image well and its corresponding time
- Knowing drawdowns and times one can determine distance to boundary – uniqueness?
  - multiple observation wells are needed

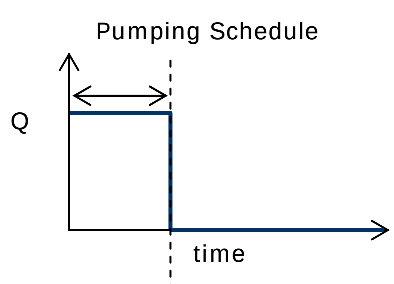
2) Analysis of water level recovery test data

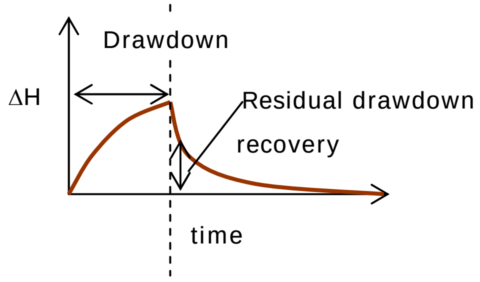
At end of pumping test, pump is stopped and water levels recover, this is called **recovery**.

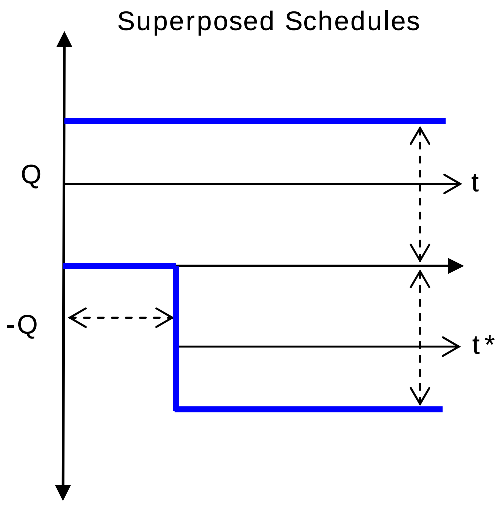
Water level below the original head during recover is called **residual drawdown**.

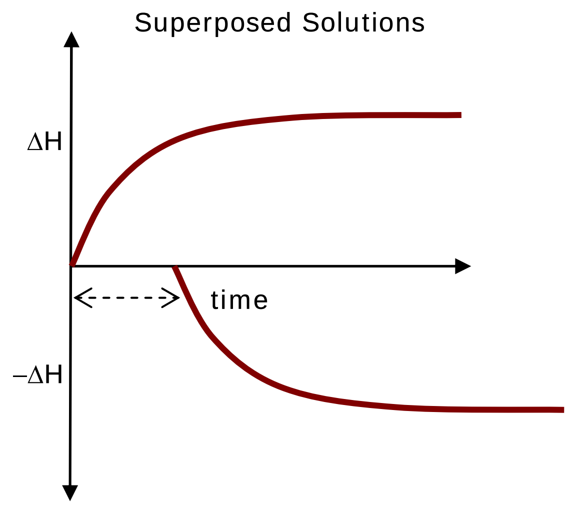
Advantages of recovery test – inexpensive and “pumping” rate is nearly constant.

What is drawdown from a pulse input?






residual drawdown

$$= \Delta h^* = \frac{Q}{4\pi T} W(u) - \frac{Q}{4\pi T} W(u^*) \quad \text{where}$$

$$u = \frac{r^2 S}{4Tt} \quad \text{and} \quad u^* = \frac{r^2 S}{4Tt^*}$$

For small r and large t*, we get $\Delta h^* = \frac{2.30Q}{4\pi T} \log \frac{t}{t^*}$

for a plot of $\Delta h$ vs. log t/t* over one log cycle, we get T as

$$T = \frac{2.30Q}{4\pi \Delta h^*}$$