SEMESTER IV

17PHU403 **DIGITAL SIGNAL PROCESSING** **L T P C**
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Objective: Digital signal processing has lot of applications in different fields of life. This objective of this paper is to give knowledge to students about the theory of signal processing and the different methods involved in it.

Unit I

Discrete-Time Signals and Systems: Classification of Signals, Transformations of the Independent Variable, Periodic and Aperiodic Signals, Energy and Power Signals, Even and Odd Signals, Discrete-Time Systems, System Properties. Impulse Response, Convolution Sum; Graphical Method; Analytical Method, Properties of Convolution; Commutative; Associative; Distributive; Shift; Sum Property System Response to Periodic Inputs, Relationship Between LTI System Properties and the Impulse Response; Causality; Stability; Invertibility, Unit Step Response.

Unit II

Discrete-Time Fourier Transform: Fourier Transform Representation of Aperiodic Discrete-Time Signals, Periodicity of DTFT, Properties; Linearity; Time Shifting; Frequency Shifting; Differencing in Time Domain; Differentiation in Frequency Domain; Convolution Property.

Unit III

The *z*-Transform: Bilateral (Two-Sided) *z*-Transform, Inverse *z*-Transform, Relationship Between *z*-Transform and Discrete-Time Fourier Transform, *z*-plane, Region-of-Convergence; Properties of ROC, Properties; Time Reversal; Differentiation in the *z*-Domain; Power Series Expansion Method (or Long Division Method); Analysis and Characterization of LTI Systems; Transfer Function and Difference-Equation System. Solving Difference Equations.

Unit IV

Filter Concepts: Phase Delay and Group delay, Zero-Phase Filter, Linear-Phase Filter, Simple FIR Digital Filters, Simple IIR Digital Filters, All pass Filters, Averaging Filters, Notch Filters.
Discrete Fourier Transform: Frequency Domain Sampling (Sampling of DTFT), The Discrete Fourier Transform (DFT) and its Inverse, DFT as a Linear transformation, Properties; Periodicity; Linearity; Circular Time Shifting; Circular Frequency Shifting.

Unit V

Fast Fourier Transform: Direct Computation of the DFT, Symmetry and Periodicity, Properties of the Twiddle factor (*WN*), Radix-2 FFT Algorithms; Decimation-In-Time (DIT) FFT Algorithm; Decimation-In-Frequency (DIF) FFT Algorithm, Inverse DFT Using FFT Algorithms. **Realization of Digital Filters:** Non Recursive and Recursive Structures, Canonic and Non Canonic Structures, Equivalent Structures (Transposed Structure), FIR Filter structures; Direct-Form; Cascade-Form; Basic structures for IIR systems; Direct-Form I.

SUGGESTED READINGS:

TEXT BOOKS

1. Digital Signal Processing, Tarun Kumar Rawat, 2015, Oxford University Press, India, Digital Signal Processing, S. K. Mitra, McGraw Hill, India.
2. Lathi, B.P.Zhi Ding, Modern Digital and Analog Communication Systems, 2009, 3rd Edn. Oxford University Press.

REFERENCE BOOKS:

1. Fundamentals of Digital Signal processing using MATLAB, R.J. Schilling and S.L. Harris, 2005, Cengage Learning.
2. Fundamentals of signals and systems, P.D. Cha and J.I. Molinder, 2007, Cambridge University Press, Digital Signal Processing Principles Algorithm & Applications, J.G. Proakis and D.G. Manolakis, 2007, 4th Edn., Prentice Hall.
3. Fundamentals of Digital Signal processing using MATLAB, R.J. Schilling and S.L. Harris, 2011, Cengage Learning, Digital Signal Processing , J.G. Proakis and D.G. Manolakis, 2013., Prentice.

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KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

LECTURE PLAN

DEPARTMENT OF PHYSICS

STAFF NAME:AMBILI VIPIN

SUBJECTNAME:DIGITAL SIGNAL PROCESSING **SUB.CODE:16PHU403**

SEMESTER:IV **CLASS :II B.Sc (PHYSICS)**

SNo	Lecture Duration (hr)	Topics to be covered	Support materials
	UNIT-I		
1	I Hr	**Discrete-Time Signals and Systems:** Classification of Signals, Transformations of the Independent Variable,	T1:1.3-1.9
2	I Hr	Periodic and Aperiodic Signals, Energy and Power Signals,	T1:1.36-1.41
3	I Hr	Even and Odd Signals,	T1:1.33-1.36
4	I Hr	Discrete-Time Systems, System Properties.	T1:1.36-1.42
5	I Hr	Impulse Response, Convolution Sum,Graphical method	T1:1.52-1.56
6	I Hr	Analytical Method, Properties of Convolution;	T1:1.58-1.60
7	I Hr	Commutative; Associative; Distributive;	T1:1.61
8	I Hr	Shift; Sum Property System Response to Periodic Inputs	T1:1.62
9	I Hr	Relationship Between LTI System Properties and the Impulse Response;	T1:1.63
10	I Hr	Causality; Stability	T1:1.71-1.75
11	I Hr	Invertibility, Unit Step Response	T1:1.61-1.63,1.80-1.99
12	Revision		X
		Total No of hours planned for Unit-I	12 hrs
	UNIT-II		
1	I Hr	Discrete-Time Fourier Transform: Fourier Transform Representation of Aperiodic	T1:1.107
2	I Hr	Discrete-Time Signals	T1:1.100-1.102

3	I Hr	Periodicity of DTFT,	T1:1.111
4	I Hr	Properties; Linearity;	T1:1.111
5	I Hr	Time Shifting;	T1:1.111
6	I Hr	Frequency Shifting	T1:1.112
7	I Hr	Differencing in Time Domain;	T1:1.112
8	I Hr	Differentiation in Frequency Domain;	T1:1.112-1.113
9	I Hr	Convolution Property	T1:1.114
10	I Hr	Revision	
		Total No of hours planned for Unit-II	**10 hrs**
		UNIT-III	
1	I Hr	The *z*-Transform: Bilateral (Two-Sided) *z*-Transform,	T1:2.1-2.2
2	I Hr	Inverse *z*-Transform, Relationship Between *z*-Transform and Discrete-Time Fourier Transform	T1:2.3-2.37
3	I Hr	Z Plane	T1:2.27
4	I Hr	Region-of-Convergence; Properties of ROC	T1:2.3-2.6
5	I Hr	Properties; Time Reversal; Differentiation in the *z*-Domain	T1:2.8-2.17
6	I Hr	Power Series Expansion Method (or Long Division Method)	T1:2.30
7	I Hr	Analysis and Characterization of LTI Systems	T1:2.3-2.45
8	I Hr	Transfer Function	T1:2.23
9	I Hr	Difference-Equation System.	T1:2.52
10	I Hr	Solving Difference Equation	T1:2.58
11	I Hr	Revision	
		Total No of hours planned for Unit-III	**11 hrs**
		UNIT-IV	
1	I Hr	**Filter Concepts:** Phase Delay and Group delay	T1:1.139-1.145
2	I Hr	Zero-Phase Filter, ,	T1:1.145
3	I Hr	Linear-Phase Filter	T1:1.145
4	I Hr	Simple FIR Digital Filters,	T1:1.147
5	I Hr	Simple IIR Digital Filters,	T1:1.153
6	I Hr	All pass Filters, Averaging Filters, Notch Filters.	T1:1.156-1.157
7	I Hr	**Discrete Fourier Transform:** Frequency Domain Sampling (Sampling of DTFT),	T1:3.00
8	I Hr	The Discrete Fourier Transform (DFT) and its Inverse, DFT as a Linear transformation,	T1:3.1-3.2
9	I Hr	Continuation	
10	I Hr	Properties; Periodicity; Linearity;	T1:3.2-3.5
11	I Hr	Circular Time Shifting; Circular Frequency Shifting.	T1:3.25-3.27

12	I Hr	Revision	
	Total No of hours planned for Unit IV		12 hrs
	UNIT-V		
1	I Hr	Fast Fourier Transform: Direct Computation of the DFT, Symmetry and Periodicity,	T1:4.1-4.3
2	I Hr	Properties of the Twiddle factor (*WN*),	T1:4.3
3	I Hr	Radix-2 FFT Algorithms;	T1:4.3-4.14
4	I Hr	Decimation-In-Time (DIT) FFT Algorithm;	T1:4.19-4.30
5	I Hr	Decimation-In-Frequency (DIF) FFT Algorithm,	T1:4.20
6	I Hr	Inverse DFT Using FFT Algorithms	T1:4.27
7	I Hr	Realization of Digital Filters: Non Recursive and Recursive Structures,	T1:5.63
8	I Hr	Canonic and Non Canonic Structures	T1:5.50
9	I Hr	Equivalent Structures (Transposed Structure)	T1:5.53
10	I Hr	FIR Filter structures; Direct-Form	T1:5.55
11	I Hr	Cascade-Form; Basic structures for IIR systems; Direct-Form I.	T1:5.54-5.58-5.65
12	I Hr	Revision	
13	I Hr	Old Question Paper discussion	
14	I Hr	Old Question Paper discussion	
15	I Hr	Old Question Paper discussion	
	Total No of hours planned for Unit-V		15 hr

TEXT BOOK:

T1:Digital Signal Processing , Fourth Edition ,P Ramesh Babu,2006,SCITE

REFERENCE BOOKS:

1. Fundamentals of Digital Signal processing using MATLAB, R.J. Schilling and S.L. Harris, 2005, Cengage Learning.
2. Fundamentals of signals and systems, P.D. Cha and J.I. Molinder, 2007, Cambridge University Press, Digital Signal Processing Principles Algorithm & Applications, J.G. Proakis and D.G. Manolakis, 2007, 4th Edn., Prentice Hall.
3. Fundamentals of Digital Signal processing using MATLAB, R.J. Schilling and S.L. Harris, 2011, Cengage Learning, Digital Signal Processing , J.G. Proakis and D.G. Manolakis, 2013., Prentice.

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UNIT-I

SYLLABUS

Discrete-Time Signals and Systems: Classification of Signals, Transformations of the Independent Variable, Periodic and Aperiodic Signals, Energy and Power Signals, Even and Odd Signals, Discrete-Time Systems, System Properties. Impulse Response, Convolution Sum; Graphical Method; Analytical Method, Properties of Convolution; Commutative; Associative; Distributive; Shift; Sum Property System Response to Periodic Inputs, Relationship Between

Definition

Anything that carries information can be called as signal. It can also be defined as a physical quantity that varies with time, temperature, pressure or with any independent variables such as speech signal or video signal.

The process of operation in which the characteristics of a signal (Amplitude, shape, phase, frequency, etc.) undergoes a change is known as signal processing.

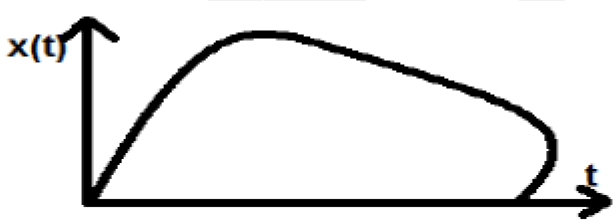
Note − Any unwanted signal interfering with the main signal is termed as noise. So, noise is also a signal but unwanted.

According to their representation and processing, signals can be classified into various categories details of which are discussed below.

Continuous Time Signals

Continuous-time signals are defined along a continuum of time and are thus, represented by a continuous independent variable. Continuous-time signals are often referred to as analog signals.

This type of signal shows continuity both in amplitude and time. These will have values at each instant of time. Sine and cosine functions are the best example of Continuous time signal.



Continuous Time signal

The signal shown above is an example of continuous time signal because we can get value of signal at each instant of time.

Discrete Time signals

The signals, which are defined at discrete times are known as discrete signals. Therefore, every independent variable has distinct value. Thus, they are represented as sequence of numbers.

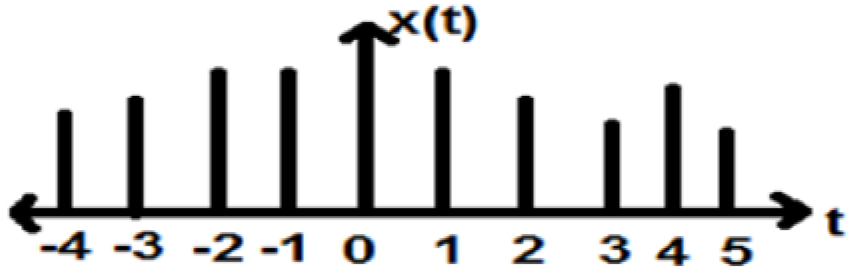
Although speech and video signals have the privilege to be represented in both continuous and discrete time format; under certain circumstances, they are identical. Amplitudes also show discrete characteristics. Perfect example of this is a digital signal; whose amplitude and time both are discrete.

The figure above depicts a discrete signal's discrete amplitude characteristic over a period of time. Mathematically, these types of signals can be formularized as;

$x=\{x[n]\},-\infty<n<\infty$

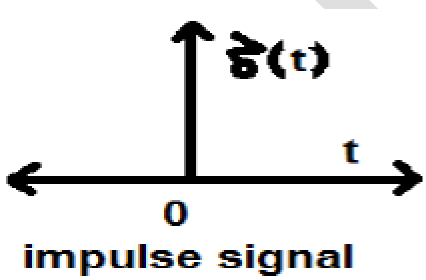
Where, *n* is an integer.

It is a sequence of numbers x, where n^{th} number in the sequence is represented as x[n].



Unit Impulse or Delta Function

A signal, which satisfies the condition, $\delta(t)=\lim\epsilon\to\infty x(t)\delta(t)=\lim\epsilon\to\infty x(t)$ is known as unit impulse signal. This signal tends to infinity when t = 0 and tends to zero when t ≠ 0 such that the area under its curve is always equals to one. The delta function has zero amplitude everywhere excunit_impulse.jpgept at t = 0.



Properties of Unit Impulse Signal

- $\delta(t)$ is an even signal.
- $\delta(t)$ is an example of neither energy nor power (NENP) signal.
- Area of unit impulse signal can be written as;

$A=\int_{-\infty}^{\infty}\delta(t)dt=\int_{-\infty}^{\infty}\lim_{\epsilon\to 0}x(t)dt=\lim_{\epsilon\to 0}\int_{-\infty}^{\infty}[x(t)dt]=1$

- Weight or strength of the signal can be written as;

  $y(t)=A\delta(t)$

  Area of the weighted impulse signal can be written as

  $y(t)=\int_{-\infty}^{\infty}y(t)dt=\int_{-\infty}^{\infty}A\delta(t)=A[\int_{-\infty}^{\infty}\delta(t)dt]=A=1=Wigthedimpulse$

Unit Step Signal

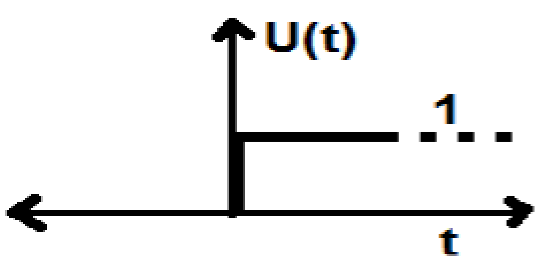
A signal, which satisfies the following two conditions

$U(t)=1(when t\geq 0) and$

$U(t)=0(when t<0)$

is known as a unit step signal.

It has the property of showing discontinuity at t = 0. At the point of discontinuity, the signal value is given by the average of signal value. This signal has been taken just before and after the point of discontinuity (according to Gibb's Phenomena).

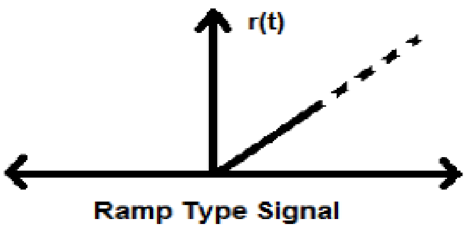


unit Step Signal

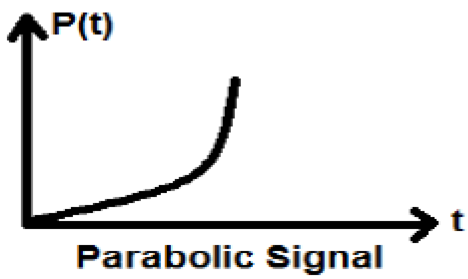
If we add a step signal to another step signal that is time scaled, then the result will be unity. It is a power type signal and the value of power is 0.5. The RMS (Root mean square) value is 0.707 and its average value is also 0.5

Ramp Signal

Integration of step signal results in a Ramp signal. It is represented by r(t). Ramp signal also satisfies the condition $r(t)=\int t-\infty U(t)dt=tU(t)r(t)=\int -\infty tU(t)dt=tU(t)$. It is neither energy nor power (NENP) type signal.



Parabolic Signal



Integration of Ramp signal leads to parabolic signal. It is represented by p(t). Parabolic signal also satisfies he condition $p(t)=\int t-\infty r(t)dt=(t2/2)U(t)p(t)=\int -\infty tr(t)dt=(t2/2)U(t)$. It is neither energy nor Power (NENP) type signal.

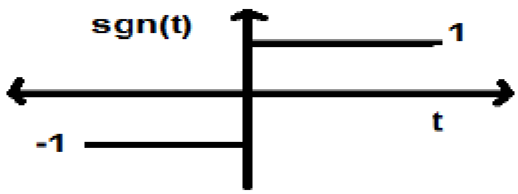
Signum Function

This function is represented as

$sgn(t)=\{1 for t>0$

$-1 for t<0$

It is a power type signal. Its power value and RMS (Root mean square) values, both are 1. Average value of signum function is zero.



Sinc Function

t is also a function of sine and is written as

SinC(t)=SinΠtΠT=Sa(Πt)

Properties of Sinc function

It is an energy type signal.

Sinc(0)=limt→0sinΠtΠt=1

Sinc(∞)=limt→∞sinΠ∞Π∞=0Sinc(∞)=limt→∞sinΠ∞Π∞=0(Range of sinπ∞ varies between -1 to +1 but anything divided by infinity is equal to zero)

If sinc(t)=0=>sinΠt=0

Πt=nΠ

t=n(n≠0)

Sinusoidal Signal

A signal, which is continuous in nature is known as continuous signal. General format of a sinusoidal signal is

x(t)=Asin(ωt+ϕ)

Here,

A = amplitude of the signal

ω = Angular frequency of the signal (Measured in radians)

φ = Phase angle of the signal (Measured in radians)

The tendency of this signal is to repeat itself after certain period of time, thus is called periodic signal. The time period of signal is given as;

T=2πωT=2πω

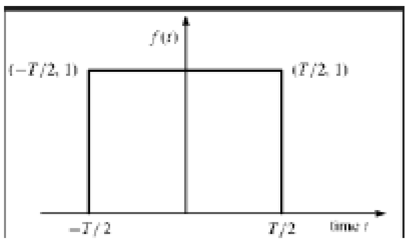
The diagrammatic view of sinusoidal signal is shown below.

Rectangular Function

A signal is said to be rectangular function type if it satisfies the following condition

π(t/τ)={1, fort≤τ/2

0, Otherwise



Being symmetrical about Y-axis, this signal is termed as even signal.

Triangular Pulse Signal

Any signal, which satisfies the following condition, is known as triangular signal.

Transformation of the Independent Variable

Signal Operation

Time Shifting

Time shifting is, as the name suggests, the shifting of a signal in time. This is done by adding or subtracting the amount of the shift to the time variable in the function. Subtracting a fixed amount from the time variable will shift the signal to the right (delay) that amount, while adding to the time variable will shift the signal to the left (advance).

$$y(t) = x(t - t_0)$$

Here, the original signal x(t) is shifted by an amount t_0.

Rule: set $t - t_0=0$ and move the origin of x(t) to t_0.

Example 1-2-1: Given x(t) = u(t+2) - u(t-2), find $x(t-t_0)$ and $x(t+t_0)$.

Time Scaling

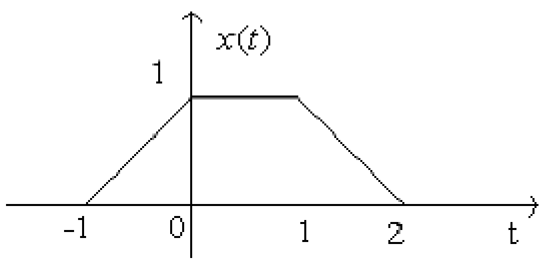
Time scaling compresses and dilates a signal by multiplying the time variable by some amount. If that amount is greater than one, the signal becomes narrower and the operation is called compression, while if the amount is less than one, the signal becomes wider and is called dilation. It often takes people quite a while to get comfortable with these operations, as people's intuition is often for the multiplication by an amount greater than one to dilate and less than one to compress.

The signal y(t) = x(at) is a time-scaled version of x(t).

If |a| > 1, we are SPEEDING UP x(t) by a factor of a.
If |a| < 1, we are SLOWING DOWN x(t) by a factor of a.

Combinations of Scale and Shift

Find x(2t+1) where x(t) is:



Method 1: Shift then scale: x(at+b)
(i) v(t)=x(t+b);
(ii) y(t) =v(at)= x(at+b).

v(t)=x(t+1)
y(t)=v(2t)

Time Reversal

A natural question to consider when learning about time scaling is: What happens when the time variable is multiplied by a negative number? The answer to this is time reversal. This operation is the reversal of the time axis, or flipping the signal over the y-axis.
We reverse a signal x(t) by flipping it over the vertical-axis to form a new signal y(t) = x(-t).
Signal Characteristics

Periodic Functions

How can we tell if a continuous- time signal x(t) is periodic? That is, given t and T, is there some period T >0 such that

$$x(t) = x(t + T).$$

If x(t) is periodic with period T, it is also periodic with period nT, that is:

$$x(t) = x(t + nT)$$

.
The minimum value of T that satisfies x(t) = x(t + T) is called the **fundamental period** of the signal and we denote it as T_0.

The fundamental frequency of the signal in hertz (cycles/second) is
and in radians/second, it is
If $x_1(t)$ is periodic with period T_1 and $x_2(t)$ is periodic with period T_2, then the sum of the two signals $x_1(t) + x_2(t)$ is periodic with period equal to the least common multiple(T_1, T_2) if the ratio of the two periods is a rational number, i.e.:
Let $T' = k_1T_1 = k_2T_2$, and $z(t) = x_1(t) + x_2(t)$,

$$z(t + T') = x_1(t + k_1T_1) + x_2(t + k_2T_2) = x_1(t) + x_2(t) = z(t)$$

Even and Odd Functions

Any continuous time signal can be expressed as the sum of an even signal and an odd signal:

$$x(t) = x_e(t) + x_o(t)$$

Even: $x_e(t)= x_e(-t)$
Odd: $x_o(t)=- x_o(-t)$

An even signal is symmetric across the vertical axis.
An odd signal is anti-symmetric across the vertical axis.

$$x_e(t)=(x(t)+x(-t))/2$$
$$x_o(t)=(x(t)-x(-t))/2$$

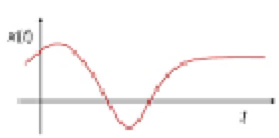
Example1-2-10: given the unit step function (a discontinuous continuous-time signal), find $u_e(t)$ and $u_o(t)$

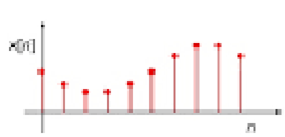
Signals are classified into the following categories:

- Continuous Time and Discrete Time Signals
- Deterministic and Non-deterministic Signals
- Even and Odd Signals
- Periodic and Aperiodic Signals
- Energy and Power Signals
- Real and Imaginary Signals

Continuous Time and Discrete Time Signals

A signal is said to be continuous when it is defined for all instants of time.





A signal is said to be discrete when it is defined at only discrete instants of time

Deterministic and Non-deterministic Signals

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or, signals which can be defined exactly by a mathematical formula are known as deterministic signals.

A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modelled in probabilistic terms.

Even and Odd Signals

A signal is said to be even when it satisfies the condition x(t) = x(-t)

Example 1: t2, t4… cost etc.

Let x(t) = t2

x(-t) = (-t)2 = t2 = x(t)

∴,∴, t2 is even function

Example: t, t3 ... And sin t

Let x(t) = sin t

x(-t) = sin(-t) = -sin t = -x(t)

∴,∴, sin t is odd function.

Any function $f(t)$ can be expressed as the sum of its even function $f_e(t)$ and odd function $f_o(t)$.

$f(t) = f_e(t) + f_0(t)$

where

$f_e(t) = ½[f(t) + f(-t)]$

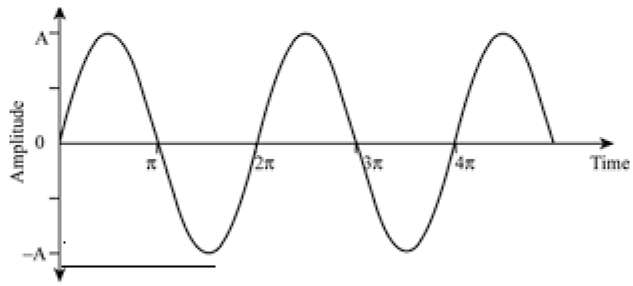
Periodic and Aperiodic Signals

A signal is said to be periodic if it satisfies the condition x(t) = x(t + T) or x(n) = x(n + N).

Where

T = fundamental time period,

1/T = f = fundamental frequency.



The above signal will repeat for every time interval T_0 hence it is periodic with period T_0.

Energy and Power Signals

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} x^2(t)dt$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

Power of energy signal = 0

Energy of power signal = ∞

Real and Imaginary Signals

A signal is said to be real when it satisfies the condition x(t) = x*(t)

A signal is said to be odd when it satisfies the condition x(t) = -x*(t)

Example:

If x(t)= 3 then x*(t)=3*=3 here x(t) is a real signal.

If x(t)= 3j then x*(t)=3j* = -3j = -x(t) hence x(t) is a odd signal.

Discrete-time systems

Discrete-time systems, "A set of connected parts or models which takes discrete-time signals as input, known as excitation, processes it under certain set of rules and algorithms to have a desired output of another discrete-time signal, known as response". In general, if a there is excitation x(n) and the response of the system is y(n), the we express the system as,

$$y(n) = T\,[x(n)]$$ or

$$x(n) \xrightarrow{T} y(n)$$

Where, ***T*** is the general rule or algorithm which is implemented on x(n) or the excitation to get the response y(n). For example, a few systems are represented as,

$$y(n) = -2x(n)$$

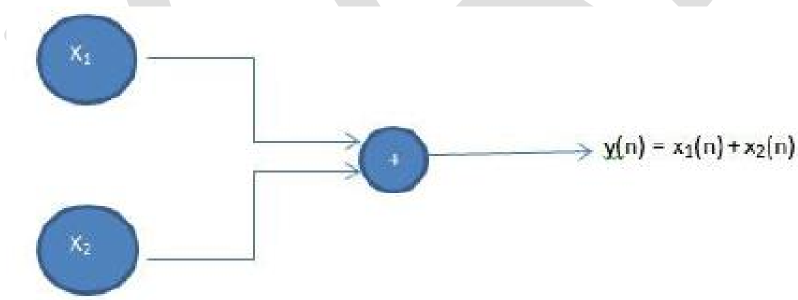
or, $$y(n) = x(n-1) + x(n) + x(n+1)$$

Block Diagram representation of Discrete-time systems

Digital Systems are represented with blocks of different elements or entities connected with arrows which also fulfills the purpose of showing the direction of signal flow,

Some common elements of Discrete-time systems are:-

Adder: It performs the addition or summation of two signals or excitation to have a response. An adder is represented as,



Constant Multiplier: This entity multiplies the signal with a constant integer or fraction. And is represented as, in this example the signal x(n) is multiplied with a constant "a" to have the response of the system as y(n).

Signal Multiplier: This element multiplies two signals to obtain one.

Unit-delay element: This element delays the signal by one sample i.e. the response of the system is the excitation of previous sample. This can element is said to have a memory which stores the excitation at time n-1 and recalls this excitation at the time n form the memory. This element is represented as,

Unit-advance element: This element advances the signal by one sample i.e. the response of the current excitation is the excitation of future sample. Although, as we can see this element is not physically realizable unless the response and the excitation are already in stored or recorded form.

Discrete-time systems are classified on different principles to have a better idea about a particular system, their behavior and ultimately to study the response of the system.

Relaxed system: If $y(n_o-1)$ is the initial condition of a system with response y(n) and $y(n_o-1)=0$, then the system is said to be initially relaxed i.e. if the system has no excitation prior to n_o .

Static and Dynamic systems: A system is said to be a Static discrete-time system if the response of the system depends **at most** on the current or present excitation and not on the past or future excitation. If there is any other scenario then the system is said to be a Dynamic discrete-time system. The static systems are also said to be memory-less systems and on the other hand dynamic systems have either finite or infinite memory depending on the nature of the system. Examples below will clear any arising doubts regarding static and dynamic systems.

Static System

$$y(n) = 2x(n) + nx^2(n)$$

$$y(n) = ax(n)$$

Dynamic system with finite memory

$$y(n) = ax(n) + bx(n-1) + cx(n+1)$$

$$y(n) = \sum_{k=0}^{n} x(n-k)$$

Dynamic system with in -finite memory

$$y(n) = \sum_{k=0}^{\infty} x(n-k)$$

Time-variant and Time-invariant system: A discrete-time system is said to be time invariant if the input-output characteristics do not change with time, i.e. if the excitation is delayed by k units then the response of the system is also delayed by k units. Let there be a system,

$x(n) \xrightarrow{T} y(n) \quad \forall\, x(n)$

Then the relaxed system ***T*** is time-invariant if and only if,

$x(n-k) \xrightarrow{T} y(n-k) \quad \forall\, x(n) \text{ and } k.$

Otherwise, the system is said to be time-variant system if it does not follows the above specified set of rules. For example,

$y(n) = ax(n)$

time-invariant }

$y(n) = x(n) + x(n-3)$

time-invariant }

Linear and non-Linear systems: A system is said to be a linear system if it follows the superposition principle i.e. the sum of responses (output) of weighted individual excitations (input) is equal to the response of sum of the weighted excitations. Pay attention to the above specified rule, according to the rule the following condition must be fulfilled by the system in order to be classified as a Linear system,

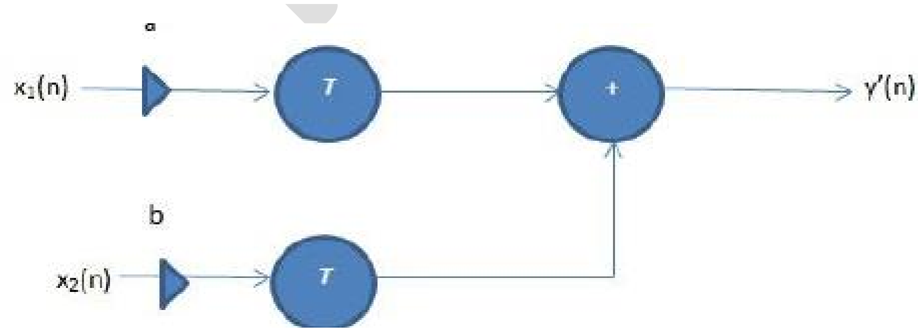
If, $y_1(n) = T[\, ax_1(n)\,]$

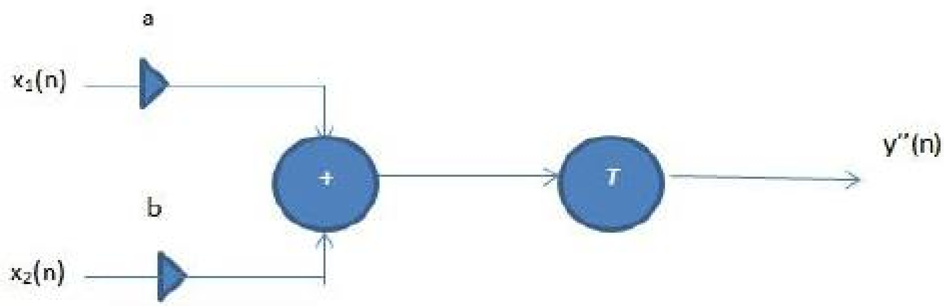
$y_2(n) = T[\, bx_2(n)\,]$

and, $y(n) = T[ax_1(n) + bx_2(n)]$

Then, the system is said to be linear if ,

$T[\, ax_1(n) + bx_2(n)] = T[\, ax_1(n)\,] + T[\, bx_2(n)\,]$





So, iff y'(n) = y''(n) then the system is said to be linear. I the system does not fulfills this property then the system is a non-Linear system

Causal and non-Causal systems: A discrete-time system is said to be a causal system if the response or the output of the system at any time depends only on the present or past excitation or input and not on the future inputs. If the system ***T*** follows the following relation then the system is said to be causal otherwise it is a non-causal system.

$$y(n) = \sum_{k=0}^{\infty} x(n-k) \qquad \{\text{ Causal }\}$$

$$y(n) = x(n) + x(n+1) \qquad \{\text{non-Causal }\}$$

Stable and Unstable systems: A system is said to be stable if the bounded input produces a bounded output i.e. the system is BIBO stable. If,

$$x(n) = M \quad \forall \quad -\infty < M < \infty$$

$$y(n) = N \quad \forall \quad -\infty < N < \infty$$

Then the system is said to be bounded system and if this is not the case then the system is unbounded or unstable.

The Basics of the Convolution Sum

Consider a DT LTI system, **L**.

$$x(n) \longrightarrow \boxed{\text{L}} \longrightarrow y(n)$$

DT convolution is based on an earlier result where we showed that any signal $x(n)$ can be expressed as a sum of impulses.

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

So let us consider $x(n)$ written in this form to be our input to the LTI system.

$$y(n) = \text{L}\left[x(n)\right] = \text{L}\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right]$$

This looks like our general linear form with a scalar $x(k)$ and a signal in n, $\delta(n-k)$. Recall that for an LTI system:

- Linearity (L): $ax_1(n) + bx_2(n) \longrightarrow \boxed{\text{L}} \longrightarrow ay_1(n) + by_2(n)$
- Time Invariance (TI): $x(n-n_o) \longrightarrow \boxed{\text{L}} \longrightarrow y(n-n_o)$

We can use the property of linearity to distribute the system **L** over our input.

$$y(n) = \text{L}\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right] = \sum_{k=-\infty}^{\infty} x(k)\text{L}\left[\delta(n-k)\right]$$

So now we wonder, what is $\text{L}\left[\delta(n-k)\right]$? Well, we can figure it out. Suppose we know how L acts on one impulse $\delta(n)$, and we call it

$$h(n) = \text{L}\left[\delta(n)\right]$$

then by time invariance we get our answer.

$$h(n-k) = \text{L}\left[\delta(n-k)\right]$$

$$\delta(n-k) \longrightarrow \boxed{\text{L}} \longrightarrow h(n-k)$$

This means that if we know *one* input-output pair for this system, namely

$$\delta(n) \longrightarrow \boxed{\text{L}} \longrightarrow h(n)$$

then we can infer

$$x(n) \longrightarrow \boxed{\text{L}} \longrightarrow y(n)$$

which gives us the following.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

This is the *convolution sum* for DT LTI systems.

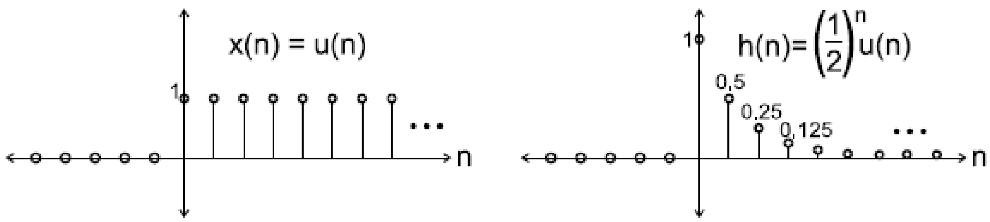
The convolution sum for $x(n)$ and $h(n)$ is usually written as shown here.

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Example 2.1: DT Convolution: Step Response

Say we are given the following signal $x(n)$ and system impulse response $h(n)$.

$$x(n) = u(n) \quad \text{and} \quad h(n) = \left(\frac{1}{2}\right)^n u(n)$$



We wish to find the step response $s(n)$ of the system (i.e. the response of the system to the unit step input $x(n) = u(n)$. This is shown below.

$$s(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

Thus the step response is as follows, found by substituting our actual signals into the general convolution sum.

$$s(n) = \sum_{k=-\infty}^{\infty} u(k) \left(\frac{1}{2}\right)^{n-k} u(n-k)$$

Let's look at this step response in smaller ranges to see what happens.

- First, consider the case where $n < 0$.

$$\begin{aligned}
s(n) &= \sum_{k=-\infty}^{\infty} u(k)\left(\frac{1}{2}\right)^{n-k} u(n-k) \\
&= \sum_{k=0}^{n} 1 \cdot \left(\frac{1}{2}\right)^{n-k} \cdot 1 \\
&\quad \text{We can pull out any terms only in } n \\
&\quad \text{since that is not the summation variable.} \\
&= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n} \left(\frac{1}{2}\right)^{-k} \\
&= \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{-k} \\
&= \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k}
\end{aligned}$$

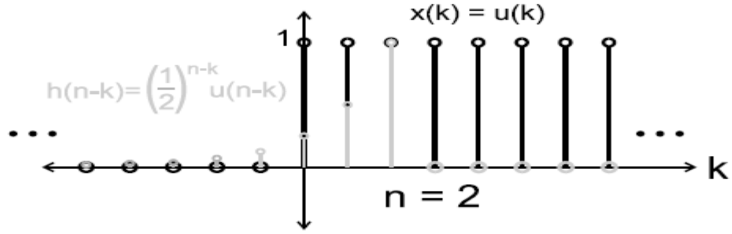
Now we have a form consistent with a geometric series. We can use that to solve.

$$\text{Recall} \quad \sum_{k=0}^{n} 2^k = \frac{1-2^{n+1}}{1-2} = 2^{n+1} - 1$$

So we have $s(n)$ as follows.

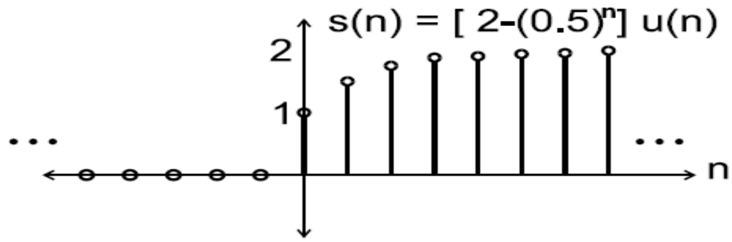
$$\begin{aligned}
s(n) &= \left(\frac{1}{2}\right)^{n} \left(2^{n+1} - 1\right) \\
&= \left(\frac{1}{2}\right)^{n} \left(2 \cdot 2^{n} - 1\right) \\
&= \left(\frac{1}{2}\right)^{n} \left(2 \cdot \left(\frac{1}{2}\right)^{-n} - 1\right) \\
&= 2 \cdot \left(\frac{1}{2}\right)^{-n} \left(\frac{1}{2}\right)^{n} - 1 \cdot \left(\frac{1}{2}\right)^{n} \\
s(n) &= 2 - \left(\frac{1}{2}\right)^{n}
\end{aligned}$$

We can visualize this, say for $n = 2$, as shown below. Note how the system output comes from the overlap of the input signal and the shifted and flipped impulse response.



So, overall, we have the following step response.

$$s(n) = \left[2 - \left(\frac{1}{2}\right)^n\right] u(n)$$



The $u(n)$ comes from our first case above since $s(n) = 0$ for $n < 0$, and obviously the other part comes from the expression found in the second case above.

3 Basic Properties of DT Convolution

Discrete-time convolution has several useful properties that allows us to solve systems more easily.

3.1 Commutativity

Convolution is a commutative operation, meaning signals can be convolved in any order.

$$x(n) * h(n) = h(n) * x(n)$$

This quite naturally is true of the convolution sums themselves, as well.

$$\sum_{k=-\infty}^{\infty} x(k)h(n-k) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

3.2 Associativity

Convolution is associative, meaning that convolution operations in series can be done in any order.

$$(x(n) * h(n)) * g(n) = x(n) * (h(n) * g(n))$$

This is significant because it means systems in series can be reordered.

Thus we have

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow \boxed{g(n)} \longrightarrow y(n)$$

is the same as

$$x(n) \longrightarrow \boxed{h(n) * g(n)} \longrightarrow y(n)$$

is the same as

$$x(n) \longrightarrow \boxed{g(n) * h(n)} \longrightarrow y(n)$$

is the same as

$$x(n) \longrightarrow \boxed{g(n)} \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

and so the systems in series can be reordered.

3.3 Distributivity

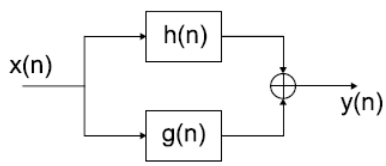
Convolution is distributive over addition.

$$x(n) * [h(n) + g(n)] = x(n) * h(n) + x(n) * g(n)$$

This is significant to all parallel connections because it means the following two arrangements are equivalent.

$$x(n) \longrightarrow \boxed{h(n) + g(n)} \longrightarrow y(n)$$

is the same as



3.4 Identity

We have previously established that $\delta(n)$ is the identity with respect to discrete-time convolution.

$$\text{Recall} \quad x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) = x(n) * \delta(n)$$

So $x(n) * \delta(n) = x(n)$.

This concept is quite easily extended, so $x(n) * \delta(n - n_o) = x(n - n_o)$ for $n_o \in \mathbb{Z}$ and $x(n - n_o) * \delta(n - n_1) = x(n - (n_o + n_1))$ for $n_o, n_1 \in \mathbb{Z}$.

Impulse Response of Discrete Time System:

Discrete Time System is an algorithm, which operates on a discrete time signal called as input signal according to some well-defined rules/operation. Impulse Response of a system is the reaction to any discrete time system in response to some external changes. Impulse Response is generally denoted as **h(t)** or **h[n]**. The output y[n] of any discrete LTI system is depended on the input (i.e. x(n)) and system's response to unit impulse (i.e. h[n]).

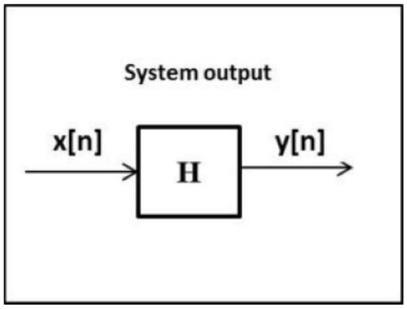
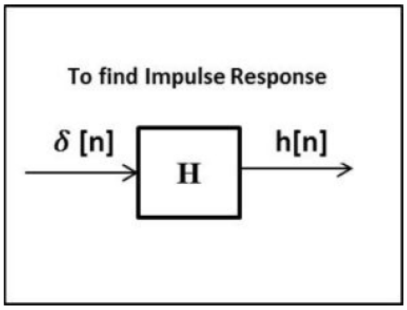


Figure 1



We can determine the systems output y[n], if we know system's impulse response, h[n], and the input, x[n]. To find the impulse response of the system we provide a **Unit impulse to the input** x[n].

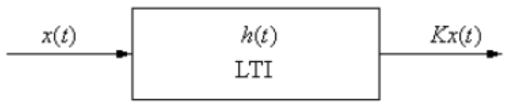
Systems with memory

In a memoryless system, the output $y(t)$ is a function of the input $x(t)$ at the time instant t alone. It does not depend on either past or future inputs.

An LTI system that is memoryless can only have this form:

$$y(t) = x(t) * h(t) = Kx(t)$$

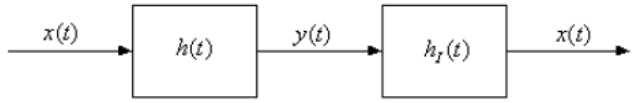
Here, K is the system gain and it must be constant or else the system would vary with time.



For $y(t) = Kx(t)$, the impulse response $h(t)$ must be of the form of a unit impulse weighted by a constant K:

$$h(t) = K\delta(t)$$

Invertible Systems



$$h(t) * h_I(t) = \delta(t)$$

A system is invertible if we can find $h_I(t)$ so that the original input $x(t)$ can be recovered from the output $y(t)$. For this to hold, the system must be *one-to-one*.

We will see how to do this when we study transforms.

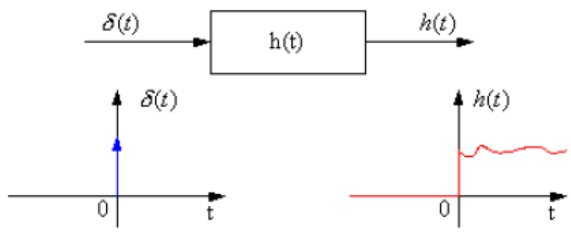
Causality

We know that for a causal system, the output depends only on past or present inputs and not on future inputs.

Equivalently, a causal system does not respond to an input until it occurs (the output is not based on the future).

In other words, a response to an input at $t = t_0$, would occur only for $t \geq t_0$ and not before t_0.

We know that $h(t)$ is the system response to $\delta(t)$, and that $\delta(t)$ occurs at $t = 0$.



A system is causal, if $h(t) = 0, \quad t < 0$

Another way to look at the causality condition: Let's examine the convolution equation, flipping $h(t)$ instead of $x(t)$:

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$

Causality: if $h(t)$ is causal then $h(t - \tau) = 0$, $t - \tau < 0$ or $t < \tau$.

So,

$$y(t) = \int_{-\infty}^{t} h(t-\tau)x(\tau)d\tau$$

which shows us that the output $y(t)$ depends only on values of the input $x(\tau)$ for $\tau \leq t$, i.e. it only depends on the past and present.

Stability

We can tell if an LTI system is BIBO stable from its impulse response.

$|x(t)| \leq B_1$, for all t, to determine if the system is BIBO stable, we need to determine if its output remains bounded for all time:

$$|y(t)| = \left|\int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau\right| \leq$$

$$\int_{-\infty}^{\infty} |x(t-\tau)h(\tau)|d\tau \text{ Why ?}$$

$$= \int_{-\infty}^{\infty} |x(t-\tau)||h(\tau)|d\tau \leq \int_{-\infty}^{\infty} B_1|h(\tau)|d\tau = B_1\int_{-\infty}^{\infty}|h(\tau)|d\tau$$

$$\text{Therefore, } |y(t)| \leq B_1\int_{-\infty}^{\infty}|h(\tau)|d\tau < \infty \text{ if } \int_{-\infty}^{\infty}|h(\tau)|d\tau < \infty$$

That is, the system is BIBO stable iff the impulse response $h(t)$ is absolutely integrable:

$$\int_{-\infty}^{\infty}|h(\tau)|d\tau = G < \infty$$

In this case, the output will be bounded by a second constant: $|y(t)| \leq B_1 G = B_2$ and thus, the system is BIBO stable.

POSSIBLE QUESTIONS

PART-B(2 MARKS)

1. Define signal.
2. Define periodic and aperiodic signal
3. Define energy and power signal
4. Define convolution
5. What are the properties of convolution
6. What do you mean by stability
7. What do you mean by invertibilty
8. Define even and odd signals
9. Define discrete time system
10. Define unit step response.

PART-C (6 MARKS)

1.Discuss about discrete time systems.

2. Find the convolution of the signal

$x(n)=1 \quad n=-2,0,1$

$=2 \quad n=-1$

$=0$ elsewhere

$h(n)=\delta(n) - \delta(n-1)+\delta(n-2)-\delta(n-3)$

3. Determine the following systems are linear or non-linear a)$Y(n)= n\, x(n)$ b)$y(n)= X^2(n)$

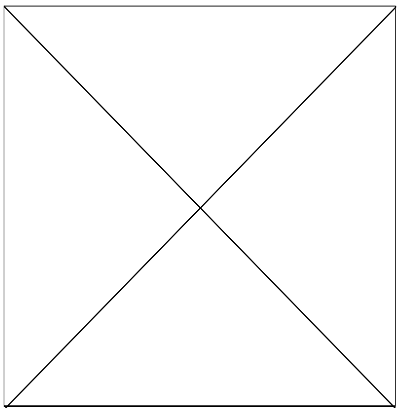
4. Determine the following systems are Causal or non causal a) $y(n)=x(n^2)$ b) $y(n)= a\, x(n)+b\, x(n-1)$

5. Derive expression for convolution operation and list out the properties of convolution.

6.Discuss about elementary discrete time signals.

7.Write short note on FIR and IIR systems ,Causal and non causal systems , Time variant and time in-varient system.

8.Write short note on Causality, Stability and Invertibility.



KARPAGAM ACADEMY OF HIGHER EDUCATION,COIMBATORE-21
DEPARTMENT OF PHYSICS
II B.Sc PHYSICS (2016-2019)
DIGITAL SIGNAL PROCESSING (16PHU403)

QUESTIONS	CHOICE1	CHOICE2	CHOICE3	CHOICE4	ANSWER
UNIT-I					
An LTI system is said to be causal if and only if	Impulse response is non-zero for positive values of n	Impulse response is zero for positive values of n	Impulse response is non-zero for negative values of n	Impulse response is zero for negative values of n	Impulse response is zero for negative values of n
The impulse response of a LTI system is h(n)={1,1,1}. What is the response of the signal to the input x(n)={1,2,3}?	{1,3,6,3,1}	{1,2,3,2,1}	{1,3,6,5,3}	{1,1,1,0,0}	{1,3,6,5,3}
The system described by the equation y(n)=ay(n-1)+b x(n) is	a recursive system.	causal	non-causal	superpostion	a recursive system.
$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$ Which of the following is a recursive form of a non-recursive system described by the equation	y(n)=y(n-1)+ 1/(M+1)[x(n)+x(n-1-M)].	) y(n)=y(n-1)+ 1/(M+1)[x(n)+x(n-1+M)].	y(n)=y(n-1)+ 1/(M+1)[x(n)-x(n-1+M)].	y(n)=y(n-1)+ 1/(M+1)[x(n)-x(n-1-M)].	y(n)=y(n-1)+ 1/(M+1)[x(n)-x(n-1-M)].
. If x(n) is a discrete-time signal, then the value of x(n) at non integer value of 'n' is:	0	positive	negative	not defined	not defined
The discrete time function defined as u(n)=n for n≥0;=0 for n<0 is an:	Unit sample signal	Unit step signal	Unit ramp signal	None of the mentioned	Unit ramp signal
The phase function of a discrete time signal $x(n)=a^n$, where $a=r.e^{j\theta}$ is:	tan(nθ)	nθ	$\tan^{-1}(n\theta)$	cosθ	$\tan^{-1}(n\theta)$
A real valued signal x(n) is called as anti-symmetric if:	x(n)=x(-n)	x(n)=-x(-n)	x(n)=-x(n)	x(n)=y(n)	x(n)=-x(-n)
The odd part of a signal x(t) is:	x(t)+x(-t)	x(t)-x(-t)	(1/2)*(x(t)+x(-t))	(1/2)*(x(t)-x(-t))	(1/2)*(x(t)-x(-t))
Time scaling operation is also known as:	Down-sampling	Up-sampling	Sampling	zer0 sampling	Down-sampling
What is the condition for a signal $x(n)=Br^n$ where $r=e^{\alpha T}$ to be called as an decaying exponential signal?	0<r<∞	0<r<1	r>1	r<0	0<r<1
The function given by the equation x(n)=1, for n=0;=0, for n≠0 is a:	Step function	Ramp function	Triangular function	Impulse function	Impulse function
The output signal when a signal x(n)=(0,1,2,3) is processed through an 'Identical' system is:	(3,2,1,0)	(1,2,3,0)	(0,1,2,3)	(0,2)	(1,2,3,0)
If a signal x(n) is passed through a system to get an output signal of y(n)=x(n+1), then the signal is said to be:	Delayed	Advanced	No operation	None of the mentioned	None of the mentioned
What is the output y(n) when a signal x(n)=n*u(n)is passed through a accumulator system under the conditions that it is initially relaxed?	$(n^2+n+1)/2$	(n(n+1))/2	(n+1)	(n+1)/2	(n(n+1))/2
The output signal when a signal x(n)=(0,1,2,3) is processed through an 'Delay' system is:	(3,2,1,0)	(1,2,3,0)	(2,3,0)	(3,2,1,3)	(3,2,1,3)
The system described by the input-output equation $y(n)=nx(n)+bx^3(n)$ is a:	Static system	Dynamic system	Identical system	ideal system	Static system
Whether the system described by the input-output equations y(n)=x(n)-x(n-1)	time varient	time in varient	delay	non-delay	time in varient

The system described by the input-output equations $y(n)=x^2(n)$ is	Linear	non linear	exponential	delay	non linear
If the output of the system of the system at any 'n' depends only the present or the past values of the inputs then the system is said to be:	Linear	Causal	Non-Linear	Non-causal	Causal
The system described by the input-output equations $y(n)=x(-n)$	Linear	Causal	Non-Linear	Non-causal	Non-causal
. If a system do not have a bounded output for bounded input, then the system is said to be:	Causal	Non-causal	Stable	Non-stable	Non-stable
The impulse response of a LTI system is h(n)={1,1,1}. What is the response of the signal to the input x(n)={1,2,3}?	{1,3,6,3,1}	{1,2,3,2,1}	{1,3,6,5,3}	{1,1,1,0,0}	{1,3,6,5,3}
Determine the output y(n) of a LTI system with impulse response $h(n)=a^n u(n)$,|a|<1with the input sequence x(n)=u(n).	$(1-a^{(n+1)})/(1-a)$	$(1-a^{(n-1)})/(1-a)$	$(1+a^{(n+1)})/(1+a)$	(1-a)	$(1-a^{(n+1)})/(1-a)$
Determine the impulse response for the cascade of two LTI systems having impulse responses $h1(n)=(1/2)^2 u(n)$ and $h2(n)= (1/4)^2 u(n)$.	$(1/2)^n[2-(1/2)^n]$, n<0	$(1/2)^n[2-(1/2)^n]$, n>0	$(1/2)^n[2+(1/2)^n]$, n<0	$(1/2)^n[2+(1/2)^n]$, n>0	$(1/2)^n[2-(1/2)^n]$, n>0
An LTI system is said to be causal if and only if	Impulse response is non-zero for positive values of n	Impulse response is zero for positive values of n	Impulse response is non-zero for negative values of n	Impulse response is zero for negative values of n	Impulse response is zero for negative values of n
$x(n)*\delta(n-n_0)=$	$x(n+n_0)$	$x(n-n_0)$	$x(-n-n_0)$	$x(-n+n_0)$	$x(n-n_0)$
The discrete impulse function is defined by	$\delta(n) = 1, n \geq 0 ,= 0, n \neq 1$	$\delta(n) = 1, n = 0,= 0, n \neq 1$	$\delta(n) = 1, n \leq 0,= 0, n \neq 1$	$\delta(n) = 1, n \leq 0,= 0, n \geq 1$	$\delta(n) = 1, n = 0,= 0, n \neq 1$
The computational procedure for Decimation in frequency algorithm takes	Log2 N stages	2Log2 N stages	Log2 N^2 stages	Log2 N/2 stages	Log2 N stages
The anti causal sequences have _____ components in the left hand sequences.	Positive	negative	not defined	0	Positive
The IIR filter designing involves	Designing of analog filter in analog domain and transforming into digital domain	Designing of digital filter in analog domain and transforming into digital domain	Designing of analog filter in digital domain and transforming into analog domain	Designing of digital filter in digital domain and transforming into analog domain	Designing of digital filter in analog domain and transforming into digital domain
Which among the following represent/s the characteristic/s of an ideal filter?	Constant gain in passband	Zero gain in stop band	Linear Phase Response	All of the above	All of the above
FIR filters _______	. are non-recursive	. are recursive	use feedback	linear	are non-recursive
In tapped delay line filter, the tapped line is also known as _______	Pick-on node	Pick-off node	Pick-up node	Pick-down node	Pick-off node
How is the sensitivity of filter coefficient quantization for FIR filters?	Low	Moderate	High	Unpredictable	Low
I I R digital filters are of the following nature	Recursive	Non Recursive	Reversive	Non Reversive	Recursive
In I I R digital filter the present output depends on	Present and previous Inputs only	Present input and previous outputs only	Present input only	Present Input, Previous input and output	Present Input, Previous input and output
Which of the following is best suited for I I R filter when compared with the FIR filter	Lower sidelobes in stopband	Higher Sidelobes in stopband	Lower sidelobes in Passband	No sidelobes in stopband	Lower sidelobes in stopband

In the case of I I R filter which of the following is true if the phase distortion is tolerable	. More parameters for design	More memory requirement	Lower computational Complexity	Higher computational complexity	Lower computational Complexity
A causal and stable I I R filter has	Linear phase	No Linear phase	Linear amplitude	No Amplitude	No Linear phase
Neither the Impulse response nor the phase response of the analog filter is Preserved in the digital filter in the following method	The method of mapping of differentials	Impulse invariant method	Bilinear transformation	Matched Z - transformation technique	Bilinear transformation
Out of the given I I R filters the following filter is the efficient one	Circular filter	Elliptical filter	Rectangular filter	Chebyshev filter	Elliptical filter
What is the disadvantage of impulse invariant method	Aliasing	one to one mapping	anti aliasing	d warping	Aliasing
. Which of the I I R Filter design method is antialiasing method?	a. The method of mapping of differentials	b. Impulse invariant method	c. Bilinear transformation	d. Matched Z - transformation technique	c. Bilinear transformation
The nonlinear relation between the analog and digital frequencies is called	a. aliasing	b. warping	c. prewarping	d. antialiasing	b. warping
The most common technique for the design of I I R Digital filter is	a. Direct Method	b. In direct method	c. Recursive method	d. non recursive method	b. In direct method
The I I R filter design method thatovercomes the limitation of applicability to only Lowpass filter and a limited class of bandpass filters is	a. Approximation of derivatives	b. Impulse Invariance	c. Bilinear Transformation	d. Frequency sampling	b. Impulse Invariance
The Fourier transform of a real valued time signal has	odd symmetry	conjugate symmetry	even symmetry	no symmetry	conjugate symmetry
A signal x(t) has a Fourier transform X(ω). If x(t) is a real and odd function of t, them X(ω) is	a real and even function of ω	an imaginary and odd function of ω	an imaginary and even function of ω	a real and odd function of ω	an imaginary and odd function of ω
The amplitude spectrum of a Gaussian pulse is	uniform	Gaussian	a sine function	An impulse function	Gaussian
If a signal f(t) has energy E, the energy of the signal f(2t) is equal to	E	2E	E/2	4E	E/2
The trigonometric Fourier series of an even function does not have the	dc term	cosine terms	sine terms	odd harmonic terms	sine terms
The Fourier series of an odd periodic function, contains only	odd harmonics	cosine terms	sine terms	even harmonic terms	sine terms
The trigonometric Fourier series of a periodic time function can have only	cosine terms	sine terms	dc term	even harmonic terms	cosine terms
The trigonometric Fourier series of an even function of time does not have	cosine terms	sine terms	dc term	even harmonic terms	cosine terms
A system with an input x(t) and output y(t) is described by the relation: y(t) = t. x(t). This system is	linear and time-invariant	linear and time-varying	non-linear & time-invariant	non-linear and time-varying	linear and time-varying
The input and output of a continuous time system are respectively denoted by x(t) and y(t). Which of the following descriptions correspond to a casual system?	( )= (−2)+ (+4)	( )=(−4) (+1)	( )=(+4) (−1)	( )=(+5) (+5)	( )=(+4) (−1)
A discrete-time signal [ ]=sin(2), being an integer, is	Periodic with period π	Periodic with period π/2	Periodic with period π2	Not periodic	Not periodic
Convolution of (+5) with impulse function (−7)is equal to	(−12)	(−2)	(+12)	(+2)	(−2)
Two systems with impulse responses h1(t) and h2(t) are connected in cascade. Then the overall impulse response of the cascade system is given by	product of h1(t) and h2(t)	sum of h1(t) and h2(t)	convolution of h1(t) and h2(t)	subtraction of h2(t) from h1(t)	convolution of h1(t) and h2(t)

Prepared by Ambili Vipin ,Assistnat Professor,Department of Physics ,KAHE

UNIT-II

SYLLABUS

Discrete-Time Fourier Transform: Fourier Transform Representation of Aperiodic Discrete-Time Signals, Periodicity of DTFT, Properties; Linearity; Time Shifting; Frequency Shifting; Differencing in Time Domain; Differentiation in Frequency Domain; Convolution Property

Discrete Time Fourier Transform

A discrete-time signal can be considered as a continuous signal $x(t)$ sampled at a rate $F = 1/t_0$ or $\Omega = 2\pi/t_0$, where t_0 is the sampling period (time interval between two consecutive samples). The corresponding sampling function (comb function) is:

$$comb(t) = \sum_{m=-\infty}^{\infty} \delta(t - mt_0)$$

The sampling process can be represented by

$$x_s(t) = x(t)\ comb(t) = x(t) \sum_{m=-\infty}^{\infty} \delta(t - mt_0) = \sum_{m=-\infty}^{\infty} x[m]\delta(t - mt_0)$$

where $x[m] = x(mt_0)$ is the value of $x(t)$ at $t = mt_0$. The Fourier transform of this discrete signal (treated as a special case of continuous signal) is:

$$X(j\omega) = \int_{\infty}^{\infty} x_s(t)e^{-j\omega t}dt = \int_{\infty}^{\infty} [\sum_{m=-\infty}^{\infty} x[m]\delta(t - mt_0)]\ e^{-j\omega t}dt$$

$$= \sum_{m=-\infty}^{\infty} x[m] \int_{\infty}^{\infty} \delta(t - mt_0)\ e^{-j\omega t}dt = \sum_{m=-\infty}^{\infty} x[m]\ e^{-j\omega m t_0}$$

This is the forward Fourier transform (analysis) of a discrete signal $x_s(t)$. The spectrum $X(j\omega)$ is periodic with period $\Omega = 2\pi F = 2\pi/t_0$:

$$X(j(\omega+\Omega)) = \sum_{m=-\infty}^{\infty} x[m]e^{-j(\omega+\Omega))mt_0} = \sum_{m=-\infty}^{\infty} x[m]e^{-j\omega mt_0}e^{-j\Omega mt_0} = X(j\omega)$$

as

$$e^{-j\Omega mt_0} = e^{-j2m\pi} = 1$$

To get back the time signal $x[m]$ from its spectrum:

$$X(j\omega) = \sum_{m=-\infty}^{\infty} x[m]\ e^{-j\omega mt_0}$$

we multiply the equation by $e^{j\omega nt_0}/\Omega$ and integrate both sides with respect to ω over the period $\Omega = 2\pi F = 2\pi/t_0$ to obtain the inverse Fourier transform (synthesis):

$$\frac{1}{\Omega}\int_\Omega X(j\omega)e^{j\omega nt_0}d\omega = \frac{1}{\Omega}\int_\Omega [\sum_{m=-\infty}^{\infty} x[m]e^{-j\omega mt_0}]e^{j\omega nt_0}d\omega$$

$$\sum_{m=-\infty}^{\infty} x[m]\frac{1}{\Omega}\int_\Omega e^{-j\omega(m-n)t_0}d\omega = \sum_{m=-\infty}^{\infty} x[m]\delta[m-n] = x[n]$$

Note that here we used

$$\frac{1}{\Omega}\int_\Omega e^{-j\omega(m-n)t_0}d\omega = \frac{1}{\Omega}\int_\Omega e^{-j(m-n)2\pi\omega/\Omega}d\omega = \delta[m-n] = \begin{cases} 1 & m=n \\ 0 & m\neq n \end{cases}$$

which can be compared this with

$$\frac{1}{T}\int_T e^{j(m-n)\omega_0 t}dt = \frac{1}{T}\int_T e^{j(m-n)2\pi t/T}dt = \delta[m-n] = \begin{cases} 1 & m=n \\ 0 & m\neq n \end{cases}$$

To summarize, the spectrum of a given discrete signal

$$x_s(t) = \sum_{m=-\infty}^{\infty} x[m]\delta(t - mt_0)$$

can be found by forward Fourier transform to be:

$$X_\Omega(j\omega) = \mathcal{F}[x[m]] = \sum_{m=-\infty}^{\infty} x[m]e^{-j\omega m t_0} = \sum_{m=-\infty}^{\infty} x[m]e^{-j2\pi m t_0}$$

and the signal can be expressed by inverse Fourier transform:

$$x[m] = \mathcal{F}^{-1}[X_\Omega(j\omega)] = \frac{1}{\Omega}\int_\Omega X_\Omega(j\omega)e^{j\omega m t_0}d\omega = \int_F X_F(f)e^{j2\pi m t_0}df$$

It is interesting to compare this discrete time Fourier transform pair with the Fourier series expansion - the Fourier transform of a periodic signal:

$$x_T(t) = \mathcal{F}^{-1}[X[n]] = \sum_{n=-\infty}^{\infty} X[n]e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} X[n]e^{j2\pi n f_0 t}$$

$$X[n] = \mathcal{F}[x_T(t)] = \frac{1}{T}\int_T x_T(t)e^{-jn\omega_0 t}dt = \frac{1}{T}\int_T x_T(t)e^{-j2\pi n f_0 t}dt$$

with discrete spectrum:

$$X(j\omega) = 2\pi \sum_{n=-\infty}^{\infty} X[n]\delta(\omega - n\omega_0) \quad \text{or} \quad X(f) = \sum_{n=-\infty}^{\infty} X[n]\delta(f - nf_0)$$

We see symmetry between these two different forms of Fourier transform. If the signal $x(t) = x(t+T)$ is periodic, its spectrum $X(j\omega)$ is discrete, the coefficients of the Fourier series with interval $\omega_0 = 2\pi/T$. On the other hand, if $x(t)$ is discrete with interval $t_0 = 2\pi/\Omega$, its spectrum $X(j\omega) = X(j\omega + \Omega)$ is periodic.

In particular, if the unit of time is so chosen that the sampling period is $t_0 = 1$, then $\Omega = 2\pi/t_0 = 2\pi$, and the forward Fourier transform of a discrete signal becomes:

$$X(j\omega) = \sum_{m=-\infty}^{\infty} x[m]e^{-jm\omega} = \sum_{m=-\infty}^{\infty} x[m]e^{-jm2\pi f}$$

The inverse transform becomes:

$$x_s[m] = \frac{1}{2\pi}\int_0^{2\pi} X(j\omega)e^{jm\omega}d\omega = \int_0^1 X(j\omega)e^{jm2\pi f}df$$

The spectrum $X(j\omega) = X(j\omega + 2\pi)$ is periodic.

he spectrum of a time signal (continuous or discrete) can be denoted by $X(j\omega)$ or $X(f)$ to emphasize the fact that the spectrum represents how the energy contained in the signal is distributed as a function of frequency ω or f. Moreover, if $X(f)$ is used, the factor $1/2\pi$ in front of the inverse transform is dropped so that the transform pair takes a more symmetric form. On the other hand, as Fourier transform of discrete signal can be considered as a special case of Z transform when the real part of $s = \sigma + j\omega$ is zero, i.e., $z = e^s = e^{j\omega}$:

$$X(z)|_{z=e^{j\omega}} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}|_{z=e^{j\omega}} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = X(e^{j\omega})$$

it is also natural to denote the spectrum of $x[n]$ by $X(e^{j\omega})$

DTFT Analysis of Discrete LTI Systems

The input-output relationship of an LTI system is governed by a convolution process: y[n] = x[n] * h[n] where h[n] is the discrete time impulse response of the system

Then the frequency-response is simply the DTFT of h[n]:

Properties of Discrete Fourier Transform

$$x(n) \xleftrightarrow[N]{\text{DFT}} x(k)$$

As a special case of general Fourier transform, the discrete time transform shares all properties (and their proofs) of the Fourier transform discussed above, except now some of these properties may take different forms. In the following, we always assume $\mathcal{F}[x[m]] = X(e^{j\omega})$ and $\mathcal{F}[y[m]] = Y(e^{j\omega})$.

Periodicity

Let x(n) and x(k) be the DFT pair then if

x(n+N) = x(n) for all n then

X(k+N) = X(k) for all k

Thus periodic sequence xp(n) can be given as

$$xp(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$

Linearity

$$\mathcal{F}[ax[m] + by[m]] = aX(e^{j\omega}) + bY(e^{j\omega})$$

The linearity property states that if

$$x1(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X1(k) \text{ And}$$

$$x2(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X2(k) \text{ Then}$$

Then

$$a1\, x1(n) + a2\, x2(n) \overset{DFT}{\longleftrightarrow} a1\, X1(k) + a2\, X2(k)\ N$$

DFT of linear combination of two or more signals is equal to the same linear combination of DFT of individual signals.

Time Shifting

$$\mathcal{F}[x[m-m_0]] = e^{-jm_0\omega}X(e^{j\omega})$$

Proof:

$$\mathcal{F}[x[m-m_0]] = \sum_{m=-\infty}^{\infty} x[m-m_0]e^{-j\omega m}$$

If we let $m' = m - m_0$, the above becomes

$$\mathcal{F}[x[m-m_0]] = \sum_{m=-\infty}^{\infty} x[m']e^{-j\omega(m'+m_0)} = e^{-j\omega m_0}X(e^{j\omega})$$

Time Reversal

$$\mathcal{F}[x[-m]] = X(e^{-j\omega})$$

Frequency Shifting

$$\mathcal{F}[x[m]e^{j\omega_0 m}] = X(e^{j(\omega-\omega_0)})$$

Differencing

Differencing is the discrete-time counterpart of differentiation.

$$\mathcal{F}[x[m]-x[m-1]]=(1-e^{-j\omega})X(e^{j\omega})$$

Proof:

$$\mathcal{F}[x[m]-x[m-1]]=\mathcal{F}[x[m]]-\mathcal{F}[x[m-1]]$$

$$X(e^{j\omega})-X(e^{j\omega})e^{-j\omega}=(1-e^{-j\omega})X(e^{j\omega})$$

Differentiation in frequency

$$\mathcal{F}^{-1}[j\frac{d}{d\omega}X(e^{j\omega})]=m\ x[m]$$

proof: Differentiating the definition of discrete Fourier transform with respect to ω, we get

$$\frac{d}{d\omega}X(e^{j\omega}) \quad \frac{d}{d\omega}\sum_{m=-\infty}^{\infty}x[m]e^{-j\omega m}=\sum_{m=-\infty}^{\infty}x[m]\frac{d}{d\omega}e^{-j\omega m}$$

$$=\sum_{m=-\infty}^{\infty}-jmx[m]e^{-j\omega m}$$

Convolution Theorems

The convolution theorem states that convolution in time domain corresponds to multiplication in frequency domain and vice versa:

$$\mathcal{F}[x[n]*y[n]]=X(e^{j\omega})\ Y(e^{j\omega}) \qquad (a)$$

$$\mathcal{F}[x[n]\ y[n]]=X(e^{j\omega})*Y(e^{j\omega}) \qquad (b)$$

Recall that the convolution of periodic signals $x_T(t)$ and $y_T(t)$ is

$$x_T(t) * y_T(t) \triangleq \frac{1}{T}\int_T x_T(\tau)y_T(t-\tau)d\tau$$

Here the convolution of periodic spectra $X(f)$ and $Y(f)$ is similarly defined as

$$X(e^{j\omega})*Y(e^{j\omega}) = \frac{1}{\Omega}\int_\Omega X(e^{j\omega'})Y(e^{j(\omega-\omega')})d\omega' = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega'})Y(e^{j(\omega-\omega')})d\omega'$$

Proof of (a):

$$\mathcal{F}[x[n]*y[n]] : \sum_{n=-\infty}^{\infty}[\sum_{m=-\infty}^{\infty} x[m]y[n-m]]e^{-jn\omega}$$

$$: \sum_{m=-\infty}^{\infty} x[m][\sum_{n=-\infty}^{\infty} y[n-m]e^{-j(n-m)\omega}]e^{-jm\omega}$$

$$: X(j\omega)\,Y(j\omega)$$

Proof of (b):

$$\mathcal{F}[x[n]y[n]] : \sum_{n=-\infty}^{\infty} x[n]y[n]e^{-jn\omega} = \sum_{n=-\infty}^{\infty}[\frac{1}{2\pi}\int_0^{2\pi} X(j\omega')e^{jn\omega'}d\omega']y[n]e^{-jn\omega}$$

$$: \frac{1}{2\pi}\int_0^{2\pi} X(j\omega')[\sum_{n=-\infty}^{\infty} e^{jn\omega'}y[n]e^{-jn\omega}]d\omega'$$

$$: \frac{1}{2\pi}\int_0^{2\pi} X(j\omega')\sum_{n=-\infty}^{\infty} y[n]e^{-jn(\omega-\omega')}d\omega'$$

$$\frac{1}{2\pi}\int_0^{2\pi} X(j\omega')Y(j(\omega-\omega'))d\omega' = X(j\omega) * Y(j\omega)$$

Parseval's Relation

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_0^{2\pi} |X(e^{j\omega})|^2 d\omega$$

Parseval'sTheorem

The Parseval s theorem states

$$\sum_{n=0}^{N-1} X(n)\, y^*(n) = 1/N \sum_{n=0}^{N-1} x(k)\; y^*(k)$$

This equation give energy of finite duration sequence in terms of its frequency components.

Example 1. The spectrum of

$$x[n] = a^n u[n] \qquad (|a| < 1)$$

is

$$X(e^{j\omega}) = \mathcal{F}[x[n]] = \sum_{n=-\infty}^{\infty} a^n u[n] e^{-jn\omega} = \sum_{n=0}^{\infty} (ae^{-j\omega})^n = \frac{1}{1-ae^{-j\omega}}$$

If the signal is two-sided:

$$x[n] = a^{|n|} = a^n u[n] + a^{-n} u[-n] - \delta[n], \qquad (|a| < 1)$$

Due to the time reversal property, its spectrum is

$$X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}} + \frac{1}{1-ae^{j\omega}} - 1 = \frac{1-a^2}{1-2a\ cos\omega + a^2}$$

Example 2. Consider an LTI system with impulse response

$$h[n] = a^n u[n], \qquad (|a| < 1)$$

and input

$$x[n] = b^n u[n], \qquad (|b| < 1)$$

The output $y[n]$ can be found in either time domain by convolution or in frequency domain by multiplication. In time domain, we have

$$y[n] = h[n] * x[n] = \sum_{m=-\infty}^{\infty} a^{n-m} u[n-m] b^m u[m] = a^n \sum_{m=0}^{n} a^{-m} b^m$$

$$= a^n \frac{1-(b/a)^{n+1}}{1-(b/a)} u[n] = \frac{1}{a-b}(a^{n+1} - b^{n+1}) u[n]$$

When $a = b$, we have

$$y[n] = a^n \sum_{m=0}^{n} a^{-m} b^m = (n+1) a^n u[n]$$

In frequency domain, we first find the spectra of both $x[n]$ and $h[n]$ to be:

$$X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}}; \qquad H(e^{j\omega}) = \frac{1}{1-be^{-j\omega}}$$

and the spectrum of the output is:

$$Y(e^{j\omega}) = H(e^{j\omega})\ X(e^{j\omega}) = \frac{1}{(1-be^{-j\omega})(1-ae^{-j\omega})}$$

To find $y(n)$ in time domain by inverse transform of $Y(e^{j\omega})$, we use partial fraction expansion to rewrite the above as

$$Y(e^{j\omega}) = \frac{A}{1-ae^{-j\omega}} + \frac{B}{1-be^{-j\omega}} = \frac{A - Abe^{-j\omega} + B - aBe^{-j\omega}}{(1-be^{-j\omega})(1-ae^{-j\omega})}$$

By equating the coefficients of $e^{-j\omega}$ and the constants, we get

$$A + B = 1, \quad aB + bA = 0$$

which can be solved to get

$$A = \frac{a}{a-b}, \qquad B = \frac{-b}{a-b}$$

In this form, $Y(j\omega)$ can be easily inverse transformed to yield

$$h[n] = [\frac{a}{a-b}a^n - \frac{b}{a-b}b^n]u[n] = \frac{1}{a-b}(a^{n+1} - b^{n+1})u[n]$$

same as the result from convolution. Again when $a = b$, we have

$$Y(e^{j\omega}) = \frac{1}{(1-ae^{-j\omega})^2} = \frac{e^{j\omega}}{a} j \frac{d}{d\omega}(\frac{1}{1-ae^{-j\omega}})$$

But since

$$\mathcal{F}^{-1}[\frac{1}{1-ae^{-j\omega}}]=a^n u[n]$$

by the frequency differentiation property, we have

$$\mathcal{F}^{-1}[j\frac{d}{d\omega}(\frac{1}{1-ae^{-j\omega}})]=na^n u[n]$$

and the output in time domain is obtained as:

$$y[n] \; : \; \mathcal{F}^{-1}[Y(e^{j\omega})]=\mathcal{F}^{-1}[\frac{e^{j\omega}}{a}j\frac{d}{d\omega}(\frac{1}{1-ae^{-j\omega}})]$$

$$: \; \frac{1}{a}(n+1)a^{n+1}u[n+1]=(n+1)a^n u[n+1]$$

$$: \; (n+1)a^n u[n]$$

Note that the time-shifting property is used due to the factor $e^{j\omega}$. Also note that $u[n+1]$ (starting at $n=-1$) is replaced by $u[n]$ (starting at $n=0$) as $n+1=0$ when $n=-1$.

Example 4. The impulse response of a discrete LTI system is

$$h[m]=a^m u[m]$$

where $|a|<1$ so that the system is stable. The output $y[m]$ of the system with an input

$$x[m] = cos(\frac{2\pi m}{N}) = \frac{e^{j2\pi m/N} + e^{-j2\pi m/N}}{2}$$

can be found in three different ways.

- **Time domain convolution:** The output is the convolution of $x[m]$ and $h[m]$:

$$y[m] \; : \; h[m] * x[m] = \sum_{k=0}^{\infty} a^k \frac{e^{j2\pi(m-k)/N} + e^{-j2\pi(m-k)/N}}{2}$$

$$: \; \frac{1}{2} e^{j2\pi m/N} \sum_{k=0}^{\infty} a^k e^{-j2\pi k/N} + \frac{1}{2} e^{-j2\pi m/N} \sum_{k=0}^{\infty} a^k e^{j2\pi k/N}$$

$$: \; \frac{1}{2} e^{j2\pi m/N} \frac{1}{1 - ae^{-j2\pi/N}} + \frac{1}{2} e^{-j2\pi m/N} \frac{1}{1 - ae^{j2\pi/N}}$$

-

- **The eigenequation method:** We first get the frequency response function from $h[m]$

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega k} = \sum_{k=0}^{\infty} (ae^{-j\omega})^k = \frac{1}{1 - ae^{-j\omega}}$$

which is the eigenvalue of the system when the input is a complex exponential $e^{jn\omega}$. Now the system's response to

$$x[m] = cos(\frac{2\pi m}{N}) = \frac{e^{j2\pi m/N} + e^{-j2\pi m/N}}{2}$$

can be found to be

$$y[m] : \frac{1}{2}[H(e^{j2\pi/N})e^{j2\pi m/N} + H(e^{-j2\pi/N})e^{-j2\pi m/N}]$$

$$: \frac{1}{2}e^{j2\pi m/N}\frac{1}{1-ae^{-j2\pi/N}} + \frac{1}{2}e^{-j2\pi m/N}\frac{1}{1-ae^{j2\pi/N}}$$

- **Frequency domain multiplication:** If we find the spectra of both $h[m]$ and $x[m]$ in the frequency domain, the spectrum of $y[m]$ can be found by multiplication. We already know

$$H(e^{j\omega}) = \mathcal{F}[h[m]] = \frac{1}{1-ae^{-j\omega}}$$

We next find the spectrum of $x[m]$

$$X(e^{j\omega}) : \mathcal{F}[x[m]] = \sum_{m=-\infty}^{\infty}\frac{e^{j2\pi m/N}+e^{-j2\pi m/N}}{2}e^{-jm\omega}$$

$$: \pi\sum_{k=-\infty}^{\infty}[\delta(\omega-2k\pi-2\pi/N)+\delta(\omega-2k\pi-2\pi/N)]$$

Now the spectrum of the output $y[m]$ can be found

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega})$$

$$= \frac{\pi}{1-ae^{-j\omega}} \sum_{k=-\infty}^{\infty} [\delta(\omega - 2k\pi - 2\pi/N) + \delta(\omega - 2k\pi - 2\pi/N)]$$

and the output $y[m]$ is obtained by inverse Fourier transform:

$$y[m] = \frac{1}{2\pi}\int_0^{2\pi} [\frac{\pi}{1-ae^{-j\omega}} \sum_{k=-\infty}^{\infty} [\delta(\omega - 2k\pi - \frac{2\pi}{N}) + \delta(\omega - 2k\pi - \frac{2\pi}{N})]]e^{jm\omega} d\omega$$

$$= \frac{1}{2}e^{j2\pi m/N}\frac{1}{1-ae^{-j2\pi/N}} + \frac{1}{2}e^{-j2\pi m/N}\frac{1}{1-ae^{j2\pi/N}}$$

The physical meaning of this result will be clear if we write $H(2\pi/N)$ in polar form:

$$H(e^{j2\pi/N}) = \frac{1}{1-ae^{j2\pi/N}} = re^{j\theta}$$

and the output becomes

$$y[m] = r\ cos(\frac{2\pi}{N}m + \theta)$$

That is, the output of the system is also a sinusoidal signal of the same frequency as the input, but with different magnitude r and a phase angle θ. For example, if $N=4$, we have

$$H(e^{j\pi/2}) = \frac{1}{1+ja} = \frac{1}{\sqrt{1+a^2}} e^{-j\, tan^{-1}(a)}$$

and the output is

$$y[m] = \frac{1}{\sqrt{1+a^2}} cos(\frac{\pi m}{2} - tan^{-1}(a))$$

POSSIBLE QUESTIONS

PART-B(2 MARKS)

1. Define Linearity
2. Define Time Shifting property
3. Define Frequency Shifting property
4. Define Differencing in Time Domain property
5. Define Differentiation in Frequency Domain property
6. Define Convolution Property
7. Define DTFT
8. Define periodicity of DTFT
9. State Parcevall's theorem.

PART-C (6 MARKS)

1. Derive expression for Fourier Transform Representation of Aperiodic Discrete-Time Signals.

2. Discuss the properties of DTFT.

3. Prove Differencing in Time Domain; Differentiation in Frequency Domain properties of DTFT.

4. Prove Convolution properties of DTFT.

KARPAGAM ACADEMY OF HIGHER EDUCATION,COIMBATORE-21
DEPARTMENT OF PHYSICS
II B.Sc PHYSICS (2016-2019)
DIGITAL SIGNAL PROCESSING (16PHU403)

QUESTIONS	**CHOICE1**	**CHOICE2**	**CHOICE3**	**CHOICE4**	**ANSWER**
UNIT-II					
DTFT is the representation of	Periodic Discrete time signals	Aperiodic Discrete time signals	Aperiodic continuous signals	Periodic continuous signals	Aperiodic Discrete time signals
The transforming relations performed by DTFT are	Linearity	nonlinearity	demodulation	periodicity	Linearity
The transforming relations performed by DTFT are	Modulation	nonlinearity	demodulation	periodicity	Modulation
The transforming relations performed by DTFT are	Shifting	nonlinearity	demodulation	periodicity	Shifting
The transforming relations performed by DTFT are	Convolution	nonlinearity	demodulation	periodicity	Convolution
DFT is preferred for	Removal of noise	Filter design	demodulation	periodicity	Filter design
The DFT is preferred for	Its ability to determine the frequency component of the signal	Quantization of signal	demodulation	periodicity	Quantization of signal
As compared to the analog systems, the digital processing of signals allow	Programmable operations	non progammable opertion	costly	not reliable	Programmable operations
As compared to the analog systems, the digital processing of signals allow	Flexibility in the system design	non progammable opertion	costly	not reliable	Flexibility in the system design
As compared to the analog systems, the digital processing of signals allow	Cheaper systems	non progammable opertion	costly	not reliable	Cheaper systems
As compared to the analog systems, the digital processing of signals allow	More reliability	non progammable opertion	costly	not reliable	More reliability
The Nyquist theorem for sampling	Relates the conditions in time domain and frequency domain	Helps in quantization	Gives the spectrum of the signal	calculate bndwidth	Relates the conditions in time domain and frequency domain
The Nyquist theorem for sampling	Gives the spectrum of the signal	Limits the bandwidth requirement	Helps in quantization	calculate bndwidth	Limits the bandwidth requirement

Roll-off factor is	The bandwidth occupied beyond the Nyquist Bandwidth of the filter	The performance of the filter or device	Aliasing effect	sampling	The bandwidth occupied beyond the Nyquist Bandwidth of the filter
Frequency selectivity characteristics of DFT refers to	Ability to resolve different frequency components from input signal	b. Ability to translate into frequency domain	c. Ability to convert into discrete time signal	Ability to convert continous time signal	Ability to resolve different frequency components from input signal
Which term applies to the maintaining of a given signal level until the next sampling?	Holding	Aliasing	Shannon frequency sampling	"Stair-stepping"	Holding
For a 4-bit DAC, the least significant bit (LSB) is	6.25% of full scale	0.625% of full scale	12% of full scale	1.2% of full scale	6.25% of full scale
The DTFT transforms an infinite-length discrete signal in the time domain into	an finite-length continuous signal in the frequency domain.	an finite-length discrete signal in the frequency domain.	an in finite-length continuous signal in the frequency domain.	an infinite-length discrete signal in the frequency domain.	an finite-length continuous signal in the frequency domain.
As with continuous-time, convolution is represented by the symbol *, and can be written as	y[n]=x[n]*∗h[n]	y[n]=x[n]*h[n]	y[n]=x[n]/h[n]	y[n]=x[n]h[n]	y[n]=x[n]*h[n]
Let f and g be two functions with convolution $f*g$.. Let F be the Fourier transform operator. Then	F(f*g)=F(f) .(g)	F(f*∗g)=F(f) .·F(g)	F(fg)=F(f) .·F(g)	F(f*∗g)=F(f)/·F(g)	F(f*g)=F(f) .(g)
Let f and g be two functions with convolution $f*g$.. Let F be the Fourier transform operator. Then	$F(f\cdot g)=F(f)*F(g)$	$F(f\cdot g)=F(f)F(g)$	$F(f\cdot * g)=F(f)*F(g)$	$F(f\cdot g)=F(f)/F(g)$	$F(f\cdot g)=F(f)*F(g)$
Inverse Fourier transform F−1, we can writ	f∗g=F−1(F(f)·F(g))	f∗g=F(F(f)·F(g))	fg=F−1(F(f)·F(g))	f∗g=F−1(F(f)/F(g))	f∗g=F−1(F(f)·F(g))
The Fourier transform of a convolution is the pointwise product of	Fouriertransform	Fourier serious	infinite series	FFT	Fouriertransform
convolution in one domain corresponds to point-wise ...... in the other domain (e.g., frequency domain).	multiplication	addition	subtraction	integration	multiplication
Symmetry property deals with the effect on the frequency-domain representation of a signal if the time variable is	altered	constant	added	subtracted	altered
a unit pulse with a very small duration, in time that becomes an infinite-length constant function in frequency.	delta function	impulse function	ramp function	step function	delta function
Time shifting shows that a shift in time is equivalent to a	linear phase shift in frequency	Non- linear phase shift in frequency	linear frequency shift in time	linear frequency shift	linear phase shift in frequency
frequency content depends only on the shape of a signal, which is . unchanged in a time shift, then	phasespectrum will be altered	amplitude spectrum will be altered	time spectrum will be altered	frequency spectrum will be altered	phasespectrum will be altered
convolution in time becomes……... in frequency	multiplication	addition	subtraction	integration	multiplication

convolution property is also another excellent example of ...... between time and frequency.	symmetry	antisymmetry	periodicity	aperiodicity	symmetry
Convolution property is also another excellent example of symmetry between	time andfrequency	time and phase	phase and frequency	phase and amplitude	time andfrequency
Parseval's relation tells us that the energy of a signal is equal to.	the energy of its Fourier transform	the power of its Fourier transform	the energy of its Z transform	the power of its Z transform	the energy of its Fourier transform
Continuous functions are sampled to form a	Fourier transform	fourierseries	Ztransform	digital image	digital image
2D Fourier transform and its inverse are infinitely	aperiodic	periodic	linear	nonlinear	periodic
Which property of delta function indicates the equality between the area under the product of function with shifted impulse and the value of function located at unit impulse instant ?	Replication	sampling	scaling	alising	sampling
Which among the below specified conditions/cases of discrete time in terms of real constant 'a' , represents the double-sided decaying exponential signal?	a>1	a<1	0	than a less than 1	than a less than 1
A system is said to be shift invariant only if_____	a shift in the input signal also results in the corresponding shift in the output	a shift in the input signal does not exhibit the corresponding shift in the output	c. a shifting level does not vary in an input as well as output	d. a shifting at input does not affect the output	a shift in the input signal also results in the corresponding shift in the output
Which condition determines the causality of the LTI system in terms of its impulse response ?	a. Only if the value of an impulse response is zero for all negative values of time	b. Only if the value of an impulse response is unity for all negative values of time	c. Only if the value of an impulse response is infinity for all negative values of time	d. Only if the value of an impulse response is negative for all negative values of time	a. Only if the value of an impulse response is zero for all negative values of time
An equalizer used to compensate the distortion in the communication system by faithful recovery of an original signal is nothing but an illustration of ________	static system	dynamic system	invertible system	non-invertible system	invertible system
Which block of the discrete time systems requires memory in order to store the previous input?	adder	multplier	unit delay	unit advance	unit delay
Which type/s of discrete-time system do/does not exhibit the necessity of any feedback ?	recursive system	nonrecursive system	linear	nonlinear	nonrecursive system
Which type of system response to its input represents the zero value of its initial condition?	Zero state response	b. Zero input response	c. Total response	d. Natural response	Zero state response
Which among the following operations is/are not involved /associated with the computation process of linear convolution?	Folding Operation	b. Shifting Operation	c. Multiplication Operation	d. Integration Operation	d. Integration Operation

A LTI system is said to be initially relaxed system only if ____	zero input produces zero output	b. zero input produces non-zero output	c. zero input produces an output equal to unity	d. none of the above	zero input produces zero output
What are the number of samples present in an impulse response called as?	string	b. array	c. length	d. element	c. length
Duality Theorem / Property of Fourier Transform states that ________	a. Shape of signal in time domain & shape of spectrum can be interchangeable	b. Shape of signal in frequency domain & shape of spectrum can be interchangeable	c. Shape of signal in time domain & shape of spectrum can never be interchangeable	d. Shape of signal in time domain & shape of spectrum can never be interchangeable	a. Shape of signal in time domain & shape of spectrum can be interchangeable
Which property of fourier transform gives rise to an additional phase shift of $-2\pi f t_d$ for the generated time delay in the communication system without affecting an amplitude spectrum ?	. Time Scaling	Linearity	Time Shifting	Duality	Time Shifting
What is/are the crucial purposes of using the Fourier Transform while analyzing any elementary signals at different frequencies?	Transformation from time domain to frequency domain	Plotting of amplitude & phase spectrum	Both a & b	Transformation from space domain to frequency domain	Both a & b
What is the possible range of frequency spectrum for discrete time fourier series (DTFS)?	0 to 2π	-π to +π	Both a & b	0	Both a & b
Which among the following assertions represents a necessary condition for the existence of Fourier Transform of discrete time signal (DTFT)?	Discrete Time Signal should be absolutely summable	Discrete Time Signal should be absolutely multipliable	Discrete Time Signal should be absolutely integrable	Discrete Time Signal should be absolutely differentiable	Discrete Time Signal should be absolutely summable
What is the nature of Fourier representation of a discrete & aperiodic signal?	Continuous & periodic	Discrete and aperiodic	Continuous & aperiodic	Discrete & periodic	Continuous & periodic
Which property of periodic signal in DTFS gets completely clarified / identified by the equation $x(n - n_0)$?	Conjugation	Time Shifting	Frequency Shifting	Time Reversal	Time Shifting
Which are the only waves that correspond/ support the measurement of phase angle in the line spectra?	Sine waves	Cosine waves	Triangular waves	Square wave	Cosine waves
What does the signalling rate in the digital communication system imply ?	Number of digital pulses transmitted per second	Number of digital pulses transmitted per minute	Number of digital pulses received per second	Number of digital pulses received per minute	Number of digital pulses transmitted per second
As the signalling rate increases, ______	Width of each pulse increases	Width of each pulse decreases	Width of each pulse remains unaffected	None of the above	Width of each pulse decreases
Which phenomenon occurs due to an increase in the channel bandwidth during the transmission of narrow pulses in order to avoid any intervention of signal distortion?	Compression in time domain	Expansion in frequency domain	Expansion in time domain	Compression in frequency domain	Expansion in frequency domain

What does the term y(-1) indicate especially in an equation that represents the behaviour of the causal system?	initial condition of the system	negative initial condition of the system	negative feedback condition of the system	response of the system to its initial input	initial condition of the system
Damped sinusoids are	sinusoid signals multiplied by growing exponentials	sinusoid signals divided by growing exponentials	sinusoid signals multiplied by decaying exponentials	sinusoid signals divided by decaying exponentials	sinusoid signals multiplied by decaying exponentials
Prepared by Ambili Vipin ,Assistnat Professor,Department of Physics ,KAHE					

UNIT-III

SYLLABUS

The *z***-Transform:** Bilateral (Two-Sided) *z*-Transform, Inverse *z*-Transform, Relationship Between *z*-Transform and Discrete-Time Fourier Transform, *z*-plane, Region-of-Convergence; Properties of ROC, Properties; Time Reversal; Differentiation in the *z*-Domain; Power Series Expansion Method (or Long Division Method); Analysis and Characterization of LTI Systems; Transfer Function and Difference-Equation System. Solving Difference Equations.

Analysis of continuous time LTI systems can be done using z-transforms. It is a powerful mathematical tool to convert differential equations into algebraic equations.

The bilateral (two sided) z-transform of a discrete time signal x(n) is given as

$$Z.T[x(n)] = X(Z) = \Sigma_{n=-\infty}^{\infty} x(n)z^{-n}$$

The unilateral (one sided) z-transform of a discrete time signal x(n) is given as

$$Z.T[x(n)] = X(Z) = \Sigma_{n=0}^{\infty} x(n)z^{-n}$$

Z-transform may exist for some signals for which Discrete Time Fourier Transform (DTFT) does not exist.

Concept of Z-Transform and Inverse Z-Transform

Z-transform of a discrete time signal x(n) can be represented with X(Z), and it is defined as

$X(Z) = \Sigma_{n=-\infty}^{\infty} x(n) z^{-n} \ldots\ldots (1)$

If $Z = re^{j\omega}$ then equation 1 becomes

$X(re^{j\omega}) = \Sigma_{n=-\infty}^{\infty} x(n)[re^{j\omega}]^{-n}$

$= \Sigma_{n=-\infty}^{\infty} x(n)[r^{-n}]e^{-j\omega n}$

$X(re^{j\omega}) = X(Z) = F.T[x(n)r^{-n}] \ldots\ldots (2)$

The above equation represents the relation between Fourier transform and Z-transform.

$X(Z)|_{z=e^{j\omega}} = F.T[x(n)]$.

INVERSE Z TRANSFORM

$$X(re^{j\omega}) = F.T[x(n)r^{-n}]$$

$$x(n)r^{-n} = F.T^{-1}[X(re^{j\omega}]$$

$$x(n) = r^n\, F.T^{-1}[X(re^{j\omega})]$$

$$= r^n \frac{1}{2\pi} \int X(re^j\omega)e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int X(re^j\omega)[re^{j\omega}]^n d\omega \ldots\ldots (3)$$

Substitute $re^{j\omega} = z$.

$$dz = jre^{j\omega} d\omega = jz d\omega$$

$$d\omega = \frac{1}{j} z^{-1} dz$$

Substitute in equation 3.

$$3 \rightarrow x(n) = \frac{1}{2\pi} \int X(z)z^n \frac{1}{j} z^{-1} dz = \frac{1}{2\pi j} \int X(z)z^{n-1} dz$$

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$x(n) = \frac{1}{2\pi j} \int X(z) z^{n-1} dz$$

Z-Transform has following properties:

Linearity Property

If $x(n) \overset{Z.T}{\longleftrightarrow} X(Z)$

and $y(n) \overset{Z.T}{\longleftrightarrow} Y(Z)$

Then linearity property states that

$$a\,x(n) + b\,y(n) \overset{Z.T}{\longleftrightarrow} a\,X(Z) + b\,Y(Z)$$

Time Shifting Property

If $x(n) \overset{Z.T}{\longleftrightarrow} X(Z)$

Then Time shifting property states that

$$x(n-m) \overset{Z.T}{\longleftrightarrow} z^{-m} X(Z)$$

Multiplication by Exponential Sequence Property

If $x(n) \stackrel{\text{Z.T}}{\longleftrightarrow} X(Z)$

Then multiplication by an exponential sequence property states that

$$a^n \,.\, x(n) \stackrel{\text{Z.T}}{\longleftrightarrow} X(Z/a)$$

Time Reversal Property

If $x(n) \stackrel{\text{Z.T}}{\longleftrightarrow} X(Z)$

Then time reversal property states that

$$x(-n) \stackrel{\text{Z.T}}{\longleftrightarrow} X(1/Z)$$

Differentiation in Z-Domain OR Multiplication by n Property

If $x(n) \stackrel{\text{Z.T}}{\longleftrightarrow} X(Z)$

Then multiplication by n or differentiation in z-domain property states that

$$n^k x(n) \stackrel{\text{Z.T}}{\longleftrightarrow} [-1]^k z^k \frac{d^k X(Z)}{dZ^K}$$

Convolution Property

If $x(n) \stackrel{\text{Z.T}}{\longleftrightarrow} X(Z)$

and $y(n) \stackrel{\text{Z.T}}{\longleftrightarrow} Y(Z)$

Then convolution property states that

$$x(n) * y(n) \stackrel{\text{Z.T}}{\longleftrightarrow} X(Z).\, Y(Z)$$

Correlation Property

If $x(n) \overset{Z.T}{\longleftrightarrow} X(Z)$

and $y(n) \overset{Z.T}{\longleftrightarrow} Y(Z)$

Then correlation property states that

$$x(n) \otimes y(n) \overset{Z.T}{\longleftrightarrow} X(Z).Y(Z^{-1})$$

Initial Value and Final Value Theorems

Initial value and final value theorems of z-transform are defined for causal signal.

Initial Value Theorem

For a causal signal x(n), the initial value theorem states that

$x(0) = \lim_{z \to \infty} X(z)$

This is used to find the initial value of the signal without taking inverse z-transform

Final Value Theorem

For a causal signal x(n), the final value theorem states that

$x(\infty) = \lim_{z \to 1} [z-1] X(z)$

This is used to find the final value of the signal without taking inverse z-transform.

Region of Convergence (ROC) of Z-Transform

The range of variation of z for which z-transform converges is called region of convergence of z-transform.

Properties of ROC of Z-Transforms

ROC of z-transform is indicated with circle in z-plane.

ROC does not contain any poles.

If x(n) is a finite duration causal sequence or right sided sequence, then the ROC is entire z-plane except at z = 0.

If x(n) is a finite duration anti-causal sequence or left sided sequence, then the ROC is entire z-plane except at z = ∞.

If x(n) is a infinite duration causal sequence, ROC is exterior of the circle with radius a. i.e. |z| > a.

If x(n) is a infinite duration anti-causal sequence, ROC is interior of the circle with radius a. i.e. |z| < a.

If x(n) is a finite duration two sided sequence, then the ROC is entire z-plane except at z = 0 & z = ∞.

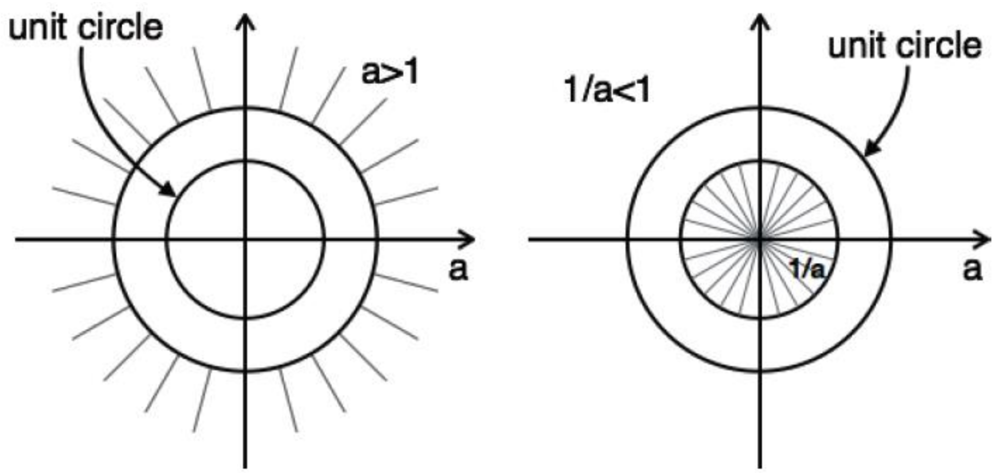
The concept of ROC can be explained by the following example:

Example 1: Find z-transform and ROC of $a^n u[n] + a^- nu[-n-1]$

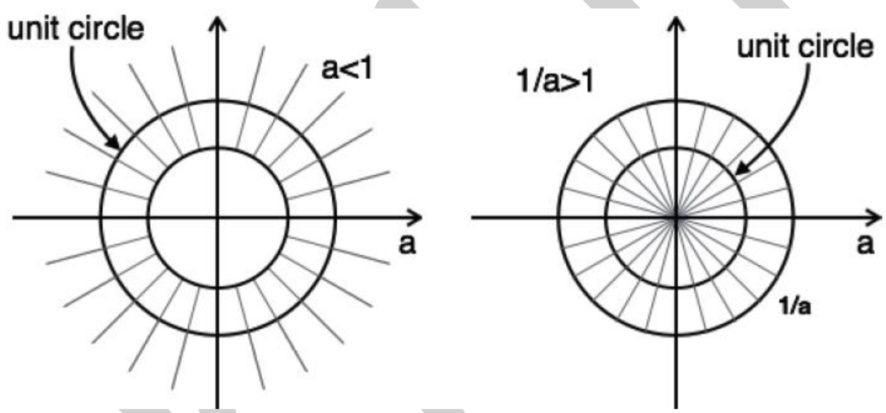
$$Z.T[a^n u[n]] + Z.T[a^{-n} u[-n-1]] = \frac{Z}{Z-a} + \frac{Z}{Z\frac{-1}{a}}$$

$$ROC : |z| > a \qquad ROC : |z| < \frac{1}{a}$$

The plot of ROC has two conditions as a > 1 and a < 1,



In this case, there is no combination ROC.



Here, the combination of ROC is from $a < |z| < \frac{1}{a}$

Hence for this problem, z-transform is possible when a < 1.

Causality and Stability

Causality condition for discrete time LTI systems is as follows:

A discrete time LTI system is causal when

ROC is outside the outermost pole.

In The transfer function H[Z], the order of numerator cannot be grater than the order of denominator.

Stability Condition for Discrete Time LTI Systems

A discrete time LTI system is stable when

its system function H[Z] include unit circle |z|=1.

all poles of the transfer function lay inside the unit circle |z|=1.

Power series expansion

If the z-transform is given as a power series in the form

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$
$$= \ldots + x[-2]z^2 + x[-1]z^1 + x[0] + x[1]z^{-1} + x[2]z^{-2} + \ldots,$$

then any value in the sequence can be found by identifying the coefficient of the appropriate power of z^{-1}.

Example: finite-length sequence

The z-transform

$$X(z) = z^2(1 - \frac{1}{2}z^{-1})(1 + z^{-1})(1 - z^{-1})$$

can be multiplied out to give

$$X(z) = z^2 - \frac{1}{2}z - 1 + \frac{1}{2}z^{-1}.$$

By inspection, the corresponding sequence is therefore

$$x[n] = \begin{cases} 1 & n = -2 \\ -\frac{1}{2} & n = -1 \\ -1 & n = 0 \\ \frac{1}{2} & n = 1 \\ 0 & \text{otherwise} \end{cases}$$

or equivalently

$$x[n] = 1\delta[n+2] - \frac{1}{2}\delta[n+1] - 1\delta[n] + \frac{1}{2}\delta[n-1].$$

Example: power series expansion by long division

Consider the transform

$$X(z) = \frac{1}{1 - az^{-1}}, \qquad |z| > |a|.$$

Since the ROC is the exterior of a circle, the sequence is right-sided. We therefore divide to get a power series in powers of z^{-1}:

$$\begin{array}{r} 1 + az^{-1} + a^2z^{-2} + \cdots \\ 1 - az^{-1}\overline{)\,1 \qquad\qquad\qquad\quad} \\ \underline{1 - az^{-1}} \qquad\qquad \\ az^{-1} \qquad\qquad \\ \underline{az^{-1} - a^2z^{-2}} \\ a^2z^{-2} + \cdots \end{array}$$

or

$$\frac{1}{1 - az^{-1}} = 1 + az^{-1} + a^2z^{-2} + \cdots.$$

Therefore $x[n] = a^n u[n]$.

Example: power series expansion for left-sided sequence
Consider instead the z-transform

$$X(z) = \frac{1}{1 - az^{-1}}, \qquad |z| < |a|.$$

Because of the ROC, the sequence is now a left-sided one. Thus we divide to obtain a series in powers of z:

$$\begin{array}{r} -a^{-1}z - a^{-2}z^2 - \cdots \\ -a + z \overline{\big) \; z \qquad\qquad\qquad} \\ \underline{z - a^{-1}z^2} \\ az^{-1} \end{array}$$

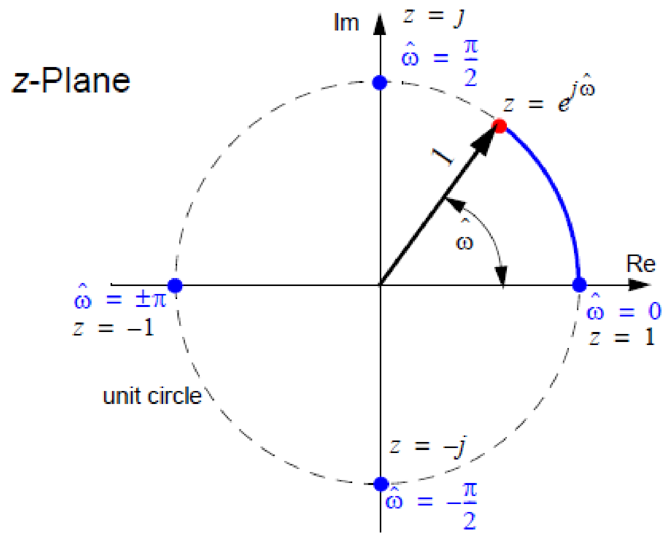
Thus $x[n] = -a^n u[-n-1]$.

Z-Transform of Basic Signals

x(t)	X[Z]
δ	1
$u(n)$	$\frac{Z}{Z-1}$
$u(-n-1)$	$-\frac{Z}{Z-1}$
$\delta(n-m)$	z^{-m}
$a^n u[n]$	$\frac{Z}{Z-a}$
$a^n u[-n-1]$	$-\frac{Z}{Z-a}$
$n\,a^n u[n]$	$\frac{aZ}{\|Z-a\|^2}$
$n\,a^n u[-n-1]$	$-\frac{aZ}{\|Z-a\|^2}$
$a^n \cos\omega n u[n]$	$\frac{Z^2-aZ\cos\omega}{Z^2-2aZ\cos\omega+a^2}$
$a^n \sin\omega n u[n]$	$\frac{aZ\sin\omega}{Z^2-2aZ\cos\omega+a^2}$

The *z*-Plane and the Unit Circle

- If we consider the z-plane, we see that $H(e^{j\hat{\omega}})$ corresponds to evaluating $H(z)$ on the unit circle



- From this interpretation we also can see why $H(e^{j\hat{\omega}})$ is periodic with period 2π
  - As $\hat{\omega}$ increases it continues to sweep around the unit circle over and over again

The Zeros and Poles of $H(z)$

- Consider

$$H(z) = 1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}$$

where we have assumed that $b_0 = 1$

- Factoring $H(z)$ results in

$$H(z) = (1 - z_1 z^{-1})(1 - z_2 z^{-1})(1 - z_3 z^{-1})$$

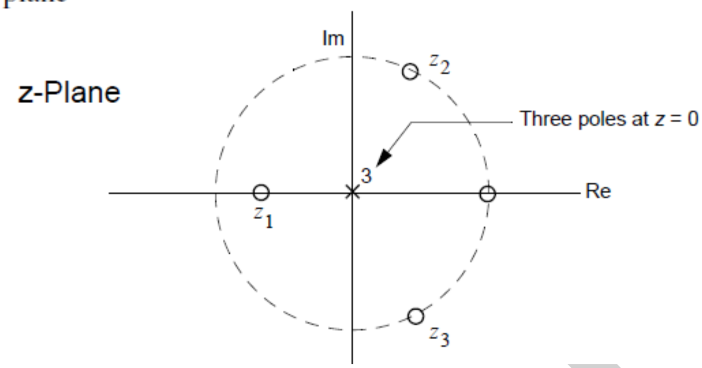
- Multiplying by z^3/z^3 allows to write $H(z)$ in terms of positive powers of z

$$H(z) = \frac{z^3 + b_1 z^2 + b_2 z^1 + b_3 z^0}{z^3}$$

$$= \frac{(z - z_1)(z - z_2)(z - z_3)}{z^3}$$

- The *zeros* are the locations where $H(z) = 0$, i.e., z_1, z_2, z_3
- The *poles* are where $H(z) \to \infty$, i.e., $z \to 0$

- A *pole-zero plot* displays the pole and zero locations in the z-plane



The Significance of the Zeros of *H*(*z*)

- The difference equation is the actual time domain means for calculating the filter output for a given filter input
- The difference equation coefficients are the polynomial coefficients in $H(z)$

For $x[n] = z_0^n$ we know that

$$y[n] = H(z_0)z_0^n,$$

so in particular if z_0 is one of the zeros of $H(z)$, $H(z_0) = 0$ and the output $y[n] = 0$

Differentation in Z –Domain

$$\mathcal{Z}[nx[n]] = -z\,\frac{d}{dz}X(z), \quad ROC = R_x$$

Proof:

$$\frac{d}{dz}X(z) = \sum_{n=-\infty}^{\infty} x[n]\frac{d}{dz}(z^{-n}) = \sum_{n=-\infty}^{\infty}(-n)x[n]z^{-n-1} = \frac{-1}{z}\sum_{n=-\infty}^{\infty} nx[n]z$$

i.e.,

$$\mathcal{Z}[nx[n]] = -z\frac{d}{dz}X(z)$$

Example: Taking derivative with respect to z of the right side of

$$\mathcal{Z}[a^n u[n]] = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

we get

$$\frac{d}{dz}\left[\frac{1}{1-az^{-1}}\right] = \frac{-az^{-2}}{(1-az^{-1})^2}$$

Due to the property of differentiation in z-domain, we have

$$\mathcal{Z}[na^n u[n]] = \frac{az^{-1}}{(1-az^{-1})^2} \quad |z| > |a|$$

Note that for a different ROC $|z| < |a|$, we have

$$\mathcal{Z}[-na^n u[-n-1]] = \frac{az^{-1}}{(1-az^{-1})^2} \quad |z| < |a|$$

Analysis and Characterization of LTI Systems Using z-Transform

The z-transform plays a particularly important role in the analysis and representation of discrete-time LTI systems. Many properties of a system can be tied directly to characteristics of the poles, zeros, and region of convergence of the system function.

Due to its convolution property, the z-transform is a powerful tool to analyze LTI systems

$$y[n] = h[n] * x[n] \xrightarrow{\mathcal{Z}} Y(z) = H(z)X(z)$$

when the input is the eigenfunction of all LTI system, i.e., $x[n] = e^{sn} = z^n$, the operation on this input by the system can be found by multiplying the system's eigenvalue $H(z)$ to the input:

$$y[n] = \mathcal{O}[z^n] = h[n] * z^n = H(z)z^n$$

Causality

A discrete-time LTI system is causal if and only if the ROC of its system function is the exterior of a circle, include infinity.

A discrete-time LTI system with rational system function H(z) is causal if and only if:
(a) the ROC is the exterior of a circle outside the outmost pole;
(b) with H(z) expressed as a ratio of polynomials in z, the order of the numerator cannot be greater than the order of the denominator.

Stability

An LTI system is stable if and only if the ROC of the system function H(z) includes the unit circle, |z|=1.

A causal LTI system with rational system function H(z) is stable if and only if all of the poles of H(z) lie inside the unit circle -i.e., they must all have magnitude smaller that 1.

The Transfer Function in the Z-domain

A LTI system is completely characterized by its impulse response $h[n]$ or equivalently the Z-transform of the impulse response $H(z)$ which is called the transfer function.

$$x[n] * h[n] \xrightarrow{Z} X(z)H(z).$$

In case the impulse response is given to define the LTI system we can simply calculate the Z-transform to obtain :math:`H(z).

In case the system is defined with a difference equation we could first calculate the impulse response and then calculating the Z-transform. But it is far easier to calculate the Z-transform of both sides of the difference equation.

As an example consider the following difference equation:

$$y[n] = 1.5y[n-1] - 0.5y[n-2] + 0.5x[n].$$

Remember that `x[n-n_0]ztarrow z^{-n_0}X(z)$ and knowing that the Z-transform is a linear transform we can apply the Z-transform to both sides of the above equation and obtain:

$$Y(z) = 1.5z^{-1}Y(z) - 0.5z^{-2}Y(z) + 0.5X(z)$$

This can be rewritten as:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.5}{1 - 1.5z^{-1} + 0.5z^{-2}} = \frac{z^2}{2z^2 - 3z + 1}$$

DIFFERENCE EQUATION

A difference equation is an equation which expresses a relation between an independent variable and the successive values of the dependent variable or the successive differences of the dependent variable.

Difference equations arise in the situations in which the discrete values of the independent variable
involve. Many practical phenomena are modelled with the help of difference equations.

Example

$$y_{x+3} + 2y_{x+2} - 3y_{x+1} + 5y_x = x^2$$

Order of a Difference Equation :

The difference between the largest and smallest arguments appearing in the difference equation is called its order.

Solution of a Difference Equation :

A solution of a difference equation is a relation between the independent variable and the dependent variable satisfying the equation.

e.g., The relation $y(x) = ca^x$ is a solution of the difference equation $y(x + 1) - ay(x) = 0$, $a \neq 1$ where c is an arbitrary constant.

The solution of a difference equation of order n shall generally contain n arbitrary constants.

A solution involving as many arbitrary constants as is the order of the equation, is called the **general solution**.

Any solution obtained from the general solution by assigning particular values to the arbitrary constants is called a **particular solution.**

In the above example, $y(x) = ca^x$ is the general solution and $y(x) = 3a^x$ is a particular solution.

A difference equation is formed by eliminating the arbitrary constants from a relation giving the order of the equation is equal to the number of arbitrary constants. The following examples illustrate the formation of difference equations :

Example: For the difference equation $y[n] - \frac{1}{2}y[n-1] = u[n]$ find $y[n]$ for $n \geq 0$.
Assume rest IC $y[-1] = 0$.
(Here $u[n]$ is the unit step function.)

<u>answer:</u> Rewrite the equation as $y[n] = u[n] + \frac{1}{2}y[n-1]$.

Make a table:

n	-1	0	1	2	3	4	…
$u[n]$	0	1	1	1	1	1	…
$y[n]$	0	1	3/2	7/4	15/8	31/16	…

We have already seen difference equations with Euler's formula. For example the IVP
$y' = ky$; $y(0) = 1$ becomes the difference equation
$y_{n+1} = y_n + khy_n = (1 + kh)y_n \Leftrightarrow y_{n+1} - (1 + kh)y_n = 0$.
Here instead of $y[n]$ we wrote y_n

Z-**transform** (analog of Laplace transform)

Let $x[n]$ be a sequence. Its z-transform is $X(z) = \sum_n x[n]z^{-n}$.

POSSIBLE QUESTIONS

PART-B(2 MARKS)

1.Define Z transform

2.Define inverse Z transform

3.Define Z plane

4.Define region of convergence

5.List out the properties of ROC

6.Define difference equation.

7.Define transfer function

8.Define stability in Z domain

9.Define causality in Z domain

10.What are the significance of poles and zeroes?

PART-C(6 MARKS)

1.Derive the expression for Z transform and inverse Z transform.

2. Explain and prove the properties of z transform

3.Explain region of convergence in Z transform

4.Explain the significance of poles and zeroes.

5.Writeshort note on transfer function and difference equation.

KARPAGAM ACADEMY OF HIGHER EDUCATION,COIMBATORE-21
DEPARTMENT OF PHYSICS
II B.Sc PHYSICS (2016-2019)
DIGITAL SIGNAL PROCESSING (16PHU403)

QUESTIONS	CHOICE1	CHOICE2	CHOICE3	CHOICE4	ANSWER
	/ –	/– –	/ +	Z/aT	/ –
The region of convergence of the z – transform of a unit step function is	| |>1	(Real part of z) > 0	| |<1	(Real part of z) < 0	| |>1
Two discrete time systems with impulse responses 1[ ] = [– 1] and 2[ ] = [– 2] are connected in cascade. The overall impulse response of the cascaded system is h n δ n h n δ n	[– 1] + [– 2]	[– 4]	[– 3]	[– 1] [– 2]	[– 3]
For a system function H(s) to be stable	The zeros lie in left half of the s plane	The zeros lie in right half of the s plane	The poles lie in left half of the s plane	The poles lie in right half of the s plane	The poles lie in left half of the s plane
The s plane and z plane are related as	$z = e^{sT}$	$z = e^{2sT}$	$z = 2e^{sT}$	$z = e^{sT}/2$	$z = e^{sT}$
The similarity between the Fourier transform and the z transform is that	Both convert frequency spectrum domain to discrete time domain	. Both convert discrete time domain to frequency spectrum domain	. Both convert analog signal to digital signal	Both convert digital signal to analog signal	Both convert discrete time domain to frequency spectrum domain
The ROC of a system is the	range of z for which the z transform converges	range of frequency for which the z transform exists	range of frequency for which the signal gets transmitted	range in which the signal is free of noise	range of z for which the z transform converges
For an expanded power series method, the coefficients represent	Inverse sequence values	Original sequence values	Negative values only	Positive values only	Inverse sequence values
Which of the following justifies the linearity property of z-transform?[x(n)↔X(z)].	x(n)+y(n) ↔X(z)Y(z)	x(n)+y(n) ↔X(z)+Y(z)	x(n)y(n) ↔X(z)+Y(z)	x(n)y(n) ↔X(z)Y(z)	x(n)+y(n) ↔X(z)+Y(z)
What is the z-transform of the signal x(n)=[3(2n)-4(3n)]u(n)?	3/(1-2z-1)-4/(1-3z-1)	3/(1+2z-1)-4/(1+3z-1)	3/(1-2z)-4/(1-3z)	None of the mentioned	3/(1-2z-1)-4/(1-3z-1)
According to Time shifting property of z-transform, if X(z) is the z-transform of x(n) then what is the z-transform of x(n-k)?	zkX(z)	z-kX(z)	X(z-k)	X(z+k)	z-kX(z)
If X(z) is the z-transform of the signal x(n) then what is the z-transform of anx(n)?	X(az)	X(az-1)	X(a-1z)	None of the mentioned	X(a-1z)
If the ROC of X(z) is r1<|z|<r2, then what is the ROC of X(a-1z)?	|a|r1<|z|<|a|r2	|a|r1>|z|>|a|r2	|a|r1<|z|>|a|r2	|a|r1>|z|<|a|r2	|a|r1<|z|<|a|r2
If X(z) is the z-transform of the signal x(n), then what is the z-transform of the signal x(-n)?	X(-z)	X(z-1)	X-1(z)	X(Z)	X(z-1)
X(z) is the z-transform of the signal x(n), then what is the z-transform of the signal nx(n)?	-z(dX(z))/dz	zdX(z)/dz	Z	d) z-1(dX(z))/dz	-z(dX(z))/dz
What is the set of all values of z for which X(z) attains a finite value?	Radius of convergence	Radius of divergence	Feasible solution	None of the mentioned	Radius of convergence
What is the ROC of the signal x(n)=δ(n-k),k>0?	z=0	z=∞	Entire z-plane, except at z=0	Entire z-plane, except at z=∞	Entire z-plane, except at z=0

What is the ROC of the z-transform of the signal x(n)= $a^n u(n)+b^n u(-n-1)$?	|a|<|z|<|b|	|a|>|z|>|b|	|a|>|z|<|b|	|a|<|z|>|b|	|a|<|z|<|b|
What is the ROC of z-transform of finite duration anti-causal sequence?	z=0	z=∞	Entire z-plane, except at z=0	Entire z-plane, except at z=∞	Entire z-plane, except at z=∞
What is the ROC of z-transform of an two sided infinite sequence?	|z|>r1	|z|<r1	r2<|z|<r1	z=1	|z|>r1
What is the ROC of the system function H(z) if the discrete time LTI system is BIBO stable?	Entire z-plane, except at z=0	Entire z-plane, except at z=∞	Contain unit circle	contain ellipse	Contain unit circle
The ROC of z-transform of any signal cannot contain	poles	zeros	ones	infinites	poles
What is the ROC of a causal infinite length sequence?	|z|<r1	|z|>r1	r2<|z|<r1	Z=0	r2<|z|<r1
If Z{x1(n)}=X1(z) and Z{x2(n)}=X2(z) then Z{x1(n)*x2(n)}=?	X1(z).X2(z)	X1(z)+X2(z)	X1(z)*X2(z)	X1(Z)-X2(Z)	X1(z).X2(z)
What is the convolution x(n) of the signals x1(n)={1,-2,1} and x2(n)={1,1,1,1,1,1}?	{1,1,0,0,0,0,1,1}	{-1,-1,0,0,0,0,-1,-1}	{-1,1,0,0,0,0,1,-1}	{1,-1,0,0,0,0,-1,1}	{1,-1,0,0,0,0,-1,1}
If Z{x1(n)}=X1(z) and Z{x2(n)}=X2(z) then what is the z-transform of correlation between the two signals?	X1(z).X2(z-1)	X1(z).X2(z-1)	X1(z).X2(z)	X1(z).X2(-z)	X1(z).X2(z-1)
If x(n) is causal, then lim┬(z→∞)X(z)=?	x(-1)	x(1)	x(0)	Cannot be determined	x(0)
What is the z-transform of the signal x(n)=δ(n-n0)?	zn0	z-n0	zn-n0	zn+n0	z-n0
If X(z) is the z-transform of the signal x(n), then what is the z-transform of x*(n)?	X(z*)	X*(z)	X*(-z)	X*(z*)	dX*(z*)
If x(n) is an imaginary sequence, then the z-transform of the real part of the sequence is:	1/2[X(z)+X*(z*)].	1/2[X(z)-X*(z*)].	1/2[X(-z)-X*(z*)].	1/2[X(-z)+X*(z*)].	1/2[X(z)+X*(z*)].
If x1(n)={1,2,3} and x2(n)={1,1,1}, then what is the convolution sequence of the given two signals?	{1,2,3,1,1}	{1,2,3,4,5}	{1,3,5,6,2}	{1,2,6,5,3}	{1,2,6,5,3}
What are the values of z for which the value of X(z)=0?	Poles	Zeros	Solutions	None of the mentioned	Zeros
What are the values of z for which the value of X(z)=∞?	Poles	Zeros	Solutions	None of the mentioned	Poles
If X(z) has M finite zeros and N finite poles, then which of the following condition is true?	|N-M| poles at origin(if N>M)	|N+M| zeros at origin(if N>M)	|N+M| poles at origin(if N>M)	|N-M| zeros at origin(if N>M)	|N-M| zeros at origin(if N>M)
If X(z) has M finite zeros and N finite poles, then which of the following condition is true?	|N-M| poles at origin(if N < M)	|N+M| zeros at origin(if N < M)	|N+M| poles at origin(if N < M)	|N-M| zeros at origin(if N < M)	|N-M| poles at origin(if N < M)
The z-transform X(z) of the signal x(n)=anu(n) has:	One pole at z=0 and one zero at z=a	One pole at z=0 and one zero at z=0	One pole at z=a and one zero at z=a	One pole at z=a and one zero at z=0	One pole at z=a and one zero at z=0
What are the values of z for which the value of X(z)=0?	Poles	Zeros	Solutions	None of the mentioned	Zeros
If Y(z) is the z-transform of the output function, X(z) is the z-transform of the input function and H(z) is the z-transform of system function of the LTI system, then H(z)=?	(Y(z))/(X(z))	(X(z))/(Y(z))	Y(z).X(z)	None of the mentioned	(Y(z))/(X(z))
What is the unit sample response of the system described by the difference equation y(n)=0.5y(n-1)+2x(n)?	0.5(2)nu(n)	2(0.5)nu(n)	0.5(2)nu(-n)	2(0.5)nu(-n)	2(0.5)nu(n)
Which of the following method is used to find the inverse z-transform of a signal?	Counter integration	Expansion into a series of terms	Partial fraction expansion	All of the mentioned	All of the mentioned
For what kind of signals one sided z-transform is unique?	All signals	Anti-causal signal	Causal signal	non-causal signal	Causal signal
What is the one sided z-transform X+(z) of the signal x(n)={1,2,5,7,0,1}?	z2+2z+5+7z-1+z-3	5+7z+z3	z-2+2z-1+5+7z+z3	5+7z-1+z-3	5+7z-1+z-3
What is the one sided z-transform of x(n)=δ(n-k)?	z-k	zk	0	1	z-k

What is the one sided z-transform of $x(n)=\delta(n+k)$?	z-k	zk	0	1	0
The impulse response of a relaxed LTI system is h(n)=anu(n),|a|<1. What is the value of the step response of the system as $n \to \infty$?	1/(1+a)	1/(1-a)	a/(1+a)	a/(1-a)	1/(1-a)
If all the poles of H(z) are outside the unit circle, then the system is said to be:	Only causal	Only BIBO stable	BIBO stable and causal	neither BIBO stable and neither causal	neither BIBO stable and neither causal
If all the poles have small magnitudes, then the rate of decay of signal is:	Slow	Rapid	Constant	0	Rapid
If one or more poles are located near the unit circle , then the rate of decay of signal is:	Slow	Rapid	Constant	0	Slow
If the ROC of the system function is the exterior of a circle of radius $r < \infty$, including the point $z = \infty$, then the system is said to be:	stable	Anti-causal signal	Causal signal	non-causal signal	Causal signal
A linear time invariant system is said to be BIBO stable if and only if the ROC of the system function:	Includes unit circle	Excludes unit circle	Is an unit circle	circle	Includes unit circle
If all the poles of H(z) are inside the unit circle, then the system is said to be:	Only causal	Only BIBO stable	BIBO stable and causal	BIBO stable and non causal	BIBO stable and causal
If x(n) is a discrete-time signal, then the value of x(n) at non integer value of 'n' is:	Zero	Positive	Negative	Not defined	Not defined
If the system is initially relaxed at time n=0 and memory equals to zero, then the response of such state is called as:	Zero-state response	Zero-input response	Zero-condition response	None of the mentioned	Zero-state response
Zero-state response is also known as:	Zero-state response	Forced response	Natural response	None of the mentioned	Forced response
The solution obtained by assuming the input x(n) of the system is zero is:	General solution	Particular solution	Homogenous solution	c) Complete solution	Homogenous solution
The total solution of the difference equation is given as:	yp(n)-yh(n)	yp(n)+yh(n)	yh(n)-yp(n)	y[n]=x[n]h[n]	yp(n)+yh(n)
What is the particular solution of the first order difference equation y(n)+ay(n-1)=x(n) where |a|<1, when the input of the system x(n)=u(n)?	1/(1+a) u(n)	1/(1+a)	1/(1-a) u(n)	1/(1-a)	1/(1+a) u(n)
The impulse response of a LTI system is h(n)={1,1,1}. What is the response of the signal to the input x(n)={1,2,3}?	{1,3,6,3,1}	{1,2,3,2,1}	{1,3,6,5,3}	{1,1,1,0,0}	{1,3,6,5,3}
The z – transform F(z) of the function $f(nT) = a^{nT}$ is					
Prepared by Ambili Vipin ,Assistnat Professor,Department of Physics ,KAHE					

UNIT IV

SYLLABUS

Filter Concepts: Phase Delay and Group delay, Zero-Phase Filter, Linear-Phase Filter, Simple FIR Digital Filters, Simple IIR Digital Filters, All pass Filters, Averaging Filters, Notch Filters.
Discrete Fourier Transform: Frequency Domain Sampling (Sampling of DTFT), The Discrete Fourier Transform (DFT) and its Inverse, DFT as a Linear transformation, Properties; Periodicity; Linearity; Circular Time Shifting; Circular Frequency Shifting.

DIGITAL FILTER

A digital filter is just a filter that operates on digital signals, such as sound represented inside a computer. It is a computation which takes one sequence of numbers (the input signal) and produces a new sequence of numbers (the filtered output signal). The filters mentioned in the previous paragraph are not digital only because they operate on signals that are not digital. It is important to realize that a digital filter can do anything that a real-world filter can do. That is, all the filters alluded to above can be simulated to an arbitrary degree of precision digitally. Thus, a digital filter is only a formula for going from one digital signal to another. Digital filters are defined by their impulse response, h[n], or the filter output given a unit sample impulse input signal. A discrete-time unit impulse signal is defined by:

- Digital filters are often best described in terms of their *frequency response*. That is, how is a sinusoidal signal of a given frequency affected by the filter.
- The frequency response of a filter consists of its *magnitude* and *phase* responses. The magnitude response indicates the ratio of a filtered sine wave's output amplitude to its input amplitude. The phase reponse describes the phase ``offset" or time delay experienced by a sine wave passing through a filter.

A *linear-phase filter* is typically used when a *causal* filter is needed to modify a signal'smagnitude-spectrum while preserving the signal's time-domain waveform as much as possible. Linear-phase filters have a *symmetric impulse response*, *e.g.*,

$$h(n) = h(N-1-n), \quad n = 0,1,2,\ldots,N-1.$$

The symmetric-impulse-response constraint means that *linear-phase filters must be FIR filters*, because a causal recursive filter cannot have a symmetric impulse response. Every real symmetric impulse response corresponds to a *real* frequency response times a *linear phase term* $e^{-j\alpha\omega T}$ where $\alpha = (N-1)/2$ is the *slope* of the linear phase. Linear phase is often ideal because a filter phase of the form $\Theta(\omega) = -\alpha\omega T$ corresponds to phase delay

$$P(\omega) \stackrel{\Delta}{=} -\frac{\Theta(\omega)}{\omega} = -\frac{-\alpha\omega T}{\omega} = \alpha T = \frac{(N-1)T}{2}$$

and group delay

$$D(\omega) \stackrel{\Delta}{=} -\frac{\partial}{\partial\omega}\Theta(\omega) = -\frac{\partial}{\partial\omega}(-\alpha\omega T) = \alpha T = \frac{(N-1)T}{2}.$$

That is, both the phase and group delay of a linear-phase filter are equal to $(N-1)/2$ samples of plain delay *at every frequency*.

ZERO-PHASE FILTERS

A *zero-phase filter* is a special case of a linear-phase filter in which the phase slope is $\alpha = 0$.`The real impulse response $h(n)$ of a zero-phase filter is *even.* That is, it satisfies

$$h(n) = h(-n), \quad n \in \mathbf{Z}$$

Every even signal is symmetric, but not every symmetric signal is even. To be even, it must be symmetric about time 0. A *zero-phase filter cannot be causal.*

PHASE DELAY

The phase response $\Theta(\omega)$ of an LTI filter gives the radian phase shift added to the phase of each sinusoidal component of the input signal. It is often more intuitive to consider instead the *phase delay*, defined as

$$\boxed{P(\omega) \triangleq -\frac{\Theta(\omega)}{\omega}.} \qquad \text{(Phase Delay)}$$

The phase delay gives the *time delay* in seconds experienced by each sinusoidal component of the input signal.

For example the phase response was $\Theta(\omega) = -\omega T/2$ which corresponds to a phase delay $P(\omega) = T/2$ or one-half sample. Thus, we can say precisely that the filter $y(n) = x(n) + x(n-1)$ exhibits half a sample of time delay at every frequency.

From a sinewave-analysis point of view, if the input to a filter with frequency response is

$$H(e^{j\omega T}) = G(\omega)e^{j\Theta(\omega)}$$

Is $x(n) = \cos(\omega nT)$

then the output is

$$\begin{aligned} y(n) &= G(\omega)\cos[\omega nT + \Theta(\omega)] \\ &= G(\omega)\cos\{\omega[nT - P(\omega)]\} \end{aligned}$$

and it can be clearly seen in this form that the phase delay expresses the phase response as a time delay in seconds.

GROUP DELAY

A more commonly encountered representation of filter phase response is called the *group delay*, defined by

$$\boxed{D(\omega) \triangleq -\frac{d}{d\omega}\Theta(\omega).} \qquad \text{(Group Delay)}$$

For linear phase responses, *i.e.,* $\Theta(\omega) = -\alpha\omega$ for some constant α he group delay and thephase delay are identical, and each may be interpreted as time delay. If the phase response is nonlinear, then the relative phases of thesinusoidal signal components are generally altered by the filter. A nonlinear phase response normally causes a ``smearing'' of attack transients such as in percussive sounds. Another term for this type of phase distortion is *phase dispersion*.

An example of a linear phase response is that of the simplest lowpass filter,

$$\Theta(\omega) = -\omega T/2 \;\Rightarrow\; P(\omega) = D(\omega) = T/2$$

Thus, both the phase delay and the group delay of the simplest lowpass filter are equal to half a sample at every frequency.

LINEAR-PHASE FILTER

Linear phase is a property of a filter, where the phase response of the filter is a linear function of frequency. The result is that all frequency components of the input signal are shifted in time (usually delayed) by the same constant amount (the slope of the linear function), which is referred to as the phase delay. And consequently, there is no phase distortion due to the time delay of frequencies relative to one another.

For discrete-time signals, perfect linear phase is easily achieved with a finite impulse response (FIR) filter. Approximations can be achieved with infinite impulse response (IIR) designs, which are more computationally efficient. Several techniques are**:**

- a Bessel transfer function which has a maximally flat group delay
- a maximally flat group delay approximation function
- a phase equalizer

If a discrete-time cosine signal

$$x_1(n) = \cos(\omega_1 n + \phi_1)$$

is processed through a discrete-time filter with frequency response

$$H^f(\omega) = A(\omega) \cdot e^{j\theta(\omega)}$$

then the output signal is given by

$$y_1(n) = A(\omega_1)\, \cos(\omega_1 n + \phi_1 + \theta(\omega_1))$$

or

$$y_1(n) = A(\omega_1)\, \cos\left(\omega_1 \left(n + \frac{\theta(\omega_1)}{\omega_1}\right) + \phi_1\right).$$

The LTI system has the effect of scaling the cosine signal and delaying it by $-\theta(\omega_1)/\omega_1$.

$$\Longrightarrow \quad \frac{\theta(\omega)}{\omega} = \text{constant}$$

$$\Longrightarrow \quad \theta(\omega) = K\,\omega$$

$$\Longrightarrow \quad \text{The phase is linear}$$

The function $\theta(\omega)/\omega$ is called the *phase delay*. A linear phase filter therefore has constant phase delay.

Linear-phase FIR filter can be divided into four basic types.

Type	impulse response	
I	symmetric	length is odd
II	symmetric	length is even
III	anti-symmetric	length is odd
IV	anti-symmetric	length is even

SIMPLE FIR DIGITAL FILTERS

SIMPLE IIR DIGITAL FILTERS

AVERAGING FILTERS,

ALL PASS FILTERS,

NOTCH FILTERS

DISCRETE FOURIER TRANSFORM-DFT

Like continuous time signal Fourier transform, discrete time Fourier Transform can be used to represent a discrete sequence into its equivalent frequency domain representation and LTI discrete time system and develop various computational algorithms.
X (jω) in continuous F.T, is a continuous function of x(n). However, DFT deals with representing x(n) with samples of its spectrum X(ω). Hence, this mathematical tool carries much importance computationally in convenient representation. Both, periodic and non-periodic sequences can be processed through this tool. The periodic sequences need to be sampled by extending the period to infinity.
Frequency Domain Sampling

From the introduction, it is clear that we need to know how to proceed through frequency domain sampling i.e. sampling X(ω). Hence, the relationship between sampled Fourier transform and DFT is established in the following manner. Similarly, periodic sequences can fit to this tool by extending the period N to infinity.
Let an Non periodic sequence be

$$X(n) = \lim_{N\to\infty} x_N(n)$$

Defining its Fourier transform

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-jwn} X(K\delta\omega)$$

Here, X(ω) is sampled periodically, at every δω radian interval.
As X(ω) is periodic in 2π radians, we require samples only in fundamental range. The samples are taken after equidistant intervals in the frequency range 0≤ω≤2π. Spacing between equivalent intervals is $\delta\omega = \frac{2\pi}{N}k$

Now evaluating, $\omega = \frac{2\pi}{N}k$

$$X(\frac{2\pi}{N}k) = \sum_{n=-\infty}^{\infty} x(n)e^{-j2\pi nk/N},$$

where k=0,1,......N-1

After subdividing the above, and interchanging the order of summation

$$X(\frac{2\pi}{N}k) = \sum_{n=0}^{N-1}[\sum_{l=-\infty}^{\infty} x(n-Nl)]e^{-j2\pi nk/N}$$

$\sum_{l=-\infty}^{\infty} x(n-Nl) = x_p(n) = a \quad periodic \quad function \quad of$
$period \quad N \quad and \quad its \quad fourier \quad series$
$= \sum_{k=0}^{N-1} C_k e^{j2\pi nk/N}$

where, n = 0,1,.....,N-1; 'p'- stands for periodic entity or function

The Fourier coefficients are,

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n)e^{-j2\pi nk/N} \qquad k=0,1,...,N-1$$

Comparing equations 3 and 4, we get ;

$$NC_k = X(\frac{2\pi}{N}k) \qquad k=0,1,...,N-1$$

$$NC_k = X(\frac{2\pi}{N}k) = X(e^{jw}) = \sum_{n=-\infty}^{\infty} x_p(n)e^{-j2\pi nk/N}$$

From Fourier series expansion,

$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} NC_k e^{j2\pi nk/N} = \frac{1}{N}\sum_{k=0}^{N-1} X(\frac{2\pi}{N}k)e^{j2\pi nk/N}$$

Where n=0,1,...,N-1

Here, we got the periodic signal from X(ω). $x(n)$ can be extracted from $x_p(n)$ only, if there is no aliasing in the time domain. $N \geq L$

N = period of $x_p(n)$ L= period of $x(n)$

$$x(n) = \begin{cases} x_p(n), & 0 \leq n \leq N-1 \\ 0, & Otherwise \end{cases}$$

The mapping is achieved in this manner.

The **inverse DFT** is given by:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi\frac{kn}{N}}.$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1}\left\{\sum_{m=0}^{N-1} x(m)e^{-j2\pi\frac{km}{N}}\right\}e^{j2\pi\frac{kn}{N}}$$
$$= \sum_{m=0}^{N-1} x(m)\underbrace{\left\{\frac{1}{N}\sum_{k=0}^{N-1} e^{-j2\pi\frac{k(m-n)}{N}}\right\}}_{\delta(m-n)} = x(n).$$

Properties of DFT

Linearity

It states that the DFT of a combination of signals is equal to the sum of DFT of individual signals. Let us take two signals $x_1(n)$ and $x_2(n)$, whose DFT s are $X_1(\omega)$ and $X_2(\omega)$ respectively. So, if

$x_1(n) \to X_1(\omega)$ and $x_2(n) \to X_2(\omega)$

Then $ax_1(n) + bx_2(n) \to aX_1(\omega) + bX_2(\omega)$

where **a** and **b** are constants.

Symmetry

The symmetry properties of DFT can be derived in a similar way as we derived DTFT symmetry properties. We know that DFT of sequence x(n) is denoted by X(K). Now, if x(n) and X(K) are complex valued sequence, then it can be represented as under

$$x(n) = x_R(n) + jx_1(n), 0 \leq n \leq N-1$$

And $X(K) = X_R(K) + jX_1(K), 0 \leq K \leq N-1$

Duality Property

Let us consider a signal x(n), whose DFT is given as X(K). Let the finite duration sequence be X(N). Then according to duality theorem,

If, $x(n) \longleftrightarrow X(K)$

Then, $X(N) \longleftrightarrow Nx[((-k))_N]$

So, by using this theorem if we know DFT, we can easily find the finite duration sequence.

Complex Conjugate Properties

Suppose, there is a signal x(n), whose DFT is also known to us as X(K). Now, if the complex conjugate of the signal is given as x*(n), then we can easily find the DFT without doing much calculation by using the theorem shown below.

If, $x(n) \longleftrightarrow X(K)$

Then, $x*(n) \longleftrightarrow X*((K))_N = X*(N-K)$

Circular Frequency Shift

The multiplication of the sequence x(n) with the complex exponential sequence $e^{j2\Pi kn/N}$ is equivalent to the circular shift of the DFT by L units in frequency. This is the dual to the circular time shifting property.

If, $x(n) \longleftrightarrow X(K)$

Then, $x(n)e^{j2\Pi Kn/N} \longleftrightarrow X((K-L))_N$

Multiplication of Two Sequence

If there are two signal $x_1(n)$ and $x_2(n)$ and their respective DFTs are $X_1(k)$ and $X_2(K)$, then multiplication of signals in time sequence corresponds to circular convolution of their DFTs.

If, $x_1(n) \longleftrightarrow X_1(K) \quad \& \quad x_2(n) \longleftrightarrow X_2(K)$

Then, $x_1(n) \times x_2(n) \longleftrightarrow X_1(K) © X_2(K)$

Parseval's Theorem

For complex valued sequences x(n) and y(n), in general

If, $x(n) \longleftrightarrow X(K) \quad \& \quad y(n) \longleftrightarrow Y(K)$

Then, $\sum_{n=0}^{N-1} x(n)y^*(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(K)Y^*(K)$

DFT Circular Convolution

Let us take two finite duration sequences $x_1(n)$ and $x_2(n)$, having integer length as N. Their DFTs are $X_1(K)$ and $X_2(K)$ respectively, which is shown below –

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n) e^{\frac{j2\Pi kn}{N}} \quad k = 0, 1, 2 \ldots N-1$$

$$X_2(K) = \sum_{n=0}^{N-1} x_2(n) e^{\frac{j2\Pi kn}{N}} \quad k = 0, 1, 2 \ldots N-1$$

Now, we will try to find the DFT of another sequence $x_3(n)$, which is given as $X_3(K)$

$$X_3(K) = X_1(K) \times X_2(K)$$

By taking the IDFT of the above we get

$$x_3(n) = \frac{1}{N} \sum_{n=0}^{N-1} X_3(K) e^{\frac{j2\Pi kn}{N}}$$

After solving the above equation, finally, we get

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m) x_2[((n-m))_N]$$
$$m = 0, 1, 2 \ldots N-1$$

Methods of Circular Convolution

Generally, there are two methods, which are adopted to perform circular convolution and they are –

- Concentric circle method,
- Matrix multiplication method.

Concentric Circle Method

Let $x_1(n)$ and $x_2(n)$ be two given sequences. The steps followed for circular convolution of $x_1(n)$ and $x_2(n)$ are

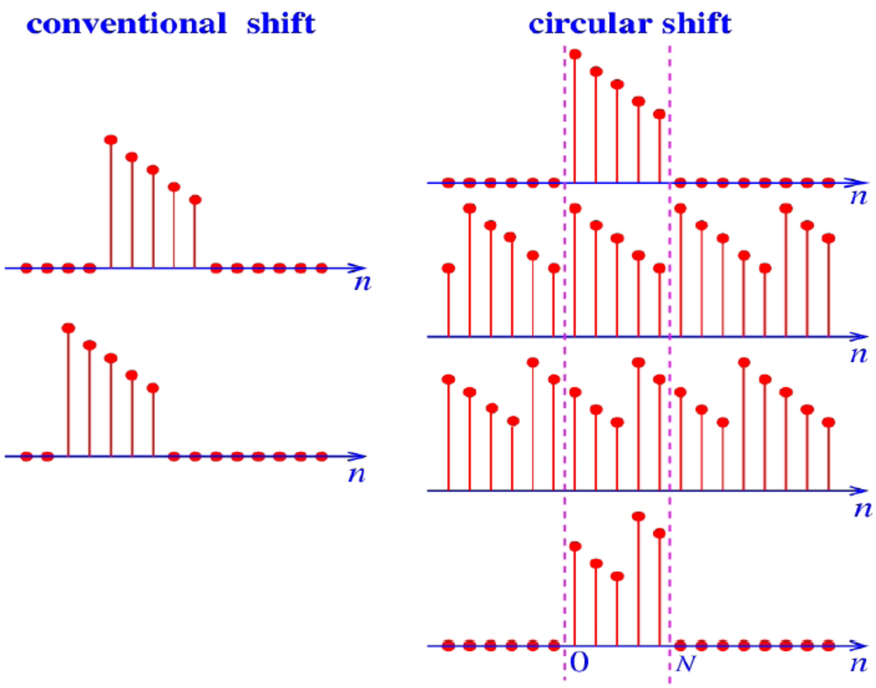
- Take two concentric circles. Plot N samples of $x_1(n)$ on the circumference of the outer circle (maintaining equal distance successive points) in anti-clockwise direction.
- For plotting $x_2(n)$, plot N samples of $x_2(n)$ in clockwise direction on the inner circle, starting sample placed at the same point as 0^{th} sample of $x_1(n)$
- Multiply corresponding samples on the two circles and add them to get output.
- Rotate the inner circle anti-clockwise with one sample at a time.

Matrix Multiplication Method

Matrix method represents the two given sequence $x_1(n)$ and $x_2(n)$ in matrix form.

- One of the given sequences is repeated via circular shift of one sample at a time to form a N X N matrix.
- The other sequence is represented as column matrix.
- The multiplication of two matrices give the result of circular convolution.

DFT: Circular Shift



$$\sum_{n=0}^{N-1} x((n-m)\mathrm{mod}N)W^{kn}$$

$$= W^{km}\sum_{n=0}^{N-1} x((n-m)\mathrm{mod}N)W^{k(n-m)}$$

$$
\begin{aligned}
&= W^{km} \sum_{n=0}^{N-1} x((n-m)\mathrm{mod}N) W^{k(n-m)\mathrm{mod}N} \\
&= W^{km} X(k),
\end{aligned}
$$

where we use the facts that $W^{k(l\mathrm{mod}N)} = W^{kl}$ and that the order of summation in DFT does not change its result.

Similarly, if $X(k) = \mathcal{DFT}\{x(n)\}$, then

$$X((k-m)\mathrm{mod}N) = \mathcal{DFT}\{x(n)e^{j2\pi\frac{mn}{N}}\}.$$

If

$$G[k] := W_N^{-mk} \cdot X[k]$$

then

$$g[n] = x[\langle n-m \rangle_N].$$

Derivation:

Begin with the Inverse DFT.

$$
\begin{aligned}
g[n] &= \frac{1}{N} \sum_{k=0}^{N-1} G[k]\, W_N^{nk} \\
&= \frac{1}{N} \sum_{k=0}^{N-1} W_N^{-mk}\, X[k]\, W_N^{nk} \\
&= \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, W_N^{k(n-m)} \\
&= x[n-m] \\
&= x[\langle n-m \rangle_N].
\end{aligned}
$$

Given an N-point signal $\{x[n],\ n \in \mathbb{Z}_N\}$, the signal

$$g[n] := x[\langle n - m \rangle_N]$$

represents a circular shift of $x[n]$ by m samples to the right. For example, if

$$g[n] := x[\langle n - 1 \rangle_N]$$

then

$$\begin{aligned} g[0] &= x[\langle -1 \rangle_N] = x[N-1] \\ g[1] &= x[\langle 0 \rangle_N] = x[0] \\ g[2] &= x[\langle 1 \rangle_N] = x[1] \\ &\vdots \\ g[N-1] &= x[\langle N-2 \rangle_N] = x[N-2] \end{aligned}$$

For example, if $x[n]$ is the 4-point signal

$$x[n] = (1, 3, 5, 2)$$

then

$$x[\langle n - 1 \rangle_N] = (2, 1, 3, 5).$$

$x[\langle n - m \rangle_N]$ represents a *circular* shift by m samples.

circular shift in frequency

If

$$g[n] := W_N^{mn} \cdot x[n]$$

then

$$G[k] = X[\langle k - m \rangle_N].$$

Derivation:

Begin with the DFT.

$$\begin{aligned} G[k] &= \sum_{n=0}^{N-1} g[n]\, W_N^{-nk} \\ &= \sum_{n=0}^{N-1} W_N^{mn}\, x[n]\, W_N^{-nk} \\ &= \sum_{n=0}^{N-1} x[n]\, W_N^{-n(k-m)} \\ &= X[k-m] \\ &= X[\langle k - m \rangle_N]. \end{aligned}$$

Verify Parseval's theorem of the sequence $x(n) = \frac{1^n}{4} u(n)$

Solution – $$\sum_{-\infty}^{\infty} |x_1(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X_1(e^{j\omega})|^2 d\omega$$

L.H.S $$\sum_{-\infty}^{\infty} |x_1(n)|^2$$

$$= \sum_{-\infty}^{\infty} x(n)x^*(n)$$

$$= \sum_{-\infty}^{\infty} (\frac{1}{4})^{2n} u(n) = \frac{1}{1-\frac{1}{16}} = \frac{16}{15}$$

R.H.S. $X(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e-j\omega} = \frac{1}{1-0.25\cos\omega + j0.25\sin\omega}$

$$\Longleftrightarrow X^*(e^{j\omega}) = \frac{1}{1-0.25\cos\omega - j0.25\sin\omega}$$

Calculating, $X(e^{j\omega}).X^*(e^{j\omega})$

$$= \frac{1}{(1-0.25\cos\omega)^2 + (0.25\sin\omega)^2} = \frac{1}{1.0625 - 0.5\cos\omega}$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{1.0625-0.5\cos\omega} d\omega$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{1.0625-0.5\cos\omega} d\omega = 16/15$$

We can see that, LHS = RHS. (Hence Proved)

Compute the N-point DFT of $x(n) = 3\delta(n)$

Solution – We know that,

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{j2\Pi kn}{N}}$$

$$= \sum_{n=0}^{N-1} 3\delta(n) e^{\frac{j2\Pi kn}{N}}$$

$$= 3\delta(0) \times e^{0} = 1$$

So, $x(k) = 3, 0 \leq k \leq N-1$ ... Ans.

Compute the N-point DFT of $x(n) = 7(n - n_0)$

Solution – We know that,

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{\frac{j2\Pi kn}{N}}$$

Substituting the value of x(n),

$$\sum_{n=0}^{N-1} 7\delta(n - n_0) e^{-\frac{j2\Pi kn}{N}}$$

$$= e^{-kj14\Pi k n_0 / N}$$

CIRCULAR TIME SHIFTING

If

$$g[n] := x[\langle -n \rangle_N]$$

then

$$G[k] = X[\langle -k \rangle_N].$$

Derivation:

$$\begin{aligned} G[k] &= \sum_{n=0}^{N-1} x[\langle -n \rangle_N] W_N^{-nk} \\ &= \sum_{m=0}^{N-1} x[m] W_N^{-\langle -m \rangle_N k} \\ &= \sum_{m=0}^{N-1} x[m] W_N^{mk} \\ &= X[-k] \\ &= X[\langle -k \rangle_N] \end{aligned}$$

where we used the change of variables $m = \langle -n \rangle_N$ (in which case $n = \langle -m \rangle_N$ for $0 \le n \le N-1$).

POSSIBLE QUESTIONS

PART-B(2 MARKS)

1.Define digital filters
2.Define phase delay
3. Define group delay
4.Define linear phase filter and notch filter.
5. Define FIR filter
6.Define IIR filter
7.Define DFT and inverse DFT.
8.Define circular time shifting
9. Define periodicity of DFT.
10. Define circular frequency shifting

PART-C (6 MARKS)

1.Derive the expression for Discrete fourier transform.

2.Explain time domain alising due to frequency sampling.

3.Write short note on IIR and FIR digital filters.

4. State and prove the properties of DFT.

5. Write short note on Zero-Phase Filter and Linear-Phase Filter digital filters.

6. Write short note on Averaging Filters and Notch Filters.

7. Explain the working principle of all pass filter.

KARPAGAM ACADEMY OF HIGHER EDUCATION,COIMBATORE-21
DEPARTMENT OF PHYSICS
II B.Sc PHYSICS (2016-2019)
DIGITAL SIGNAL PROCESSING (16PHU403)

QUESTIONS	CHOICE1	CHOICE2	CHOICE3	CHOICE4	ANSWER
UNIT-IV					
In the Frequency Transformations of the analog domain the transformation is	Low Pass to Lowpass	Lowpass to Highpass	Lowpass to Bandpass	Lowpass to Bandreject	Lowpass to Highpass
In the Frequency Transformations of the analog domain the transformation is	Low Pass to Lowpass	Lowpass to Highpass	Lowpass to Bandpass	Lowpass to Bandreject	Lowpass to Bandreject
The magnitude response of the following filter decreases monotonically as frequency increases	Butterworth Filter	Chebyshev type - 1	Chebyshev type - 2		Butterworth Filter
The transition band is more in	Butterworth Filter	Chebyshev type - 1	Chebyshev type - 2	FIR Filter	Butterworth Filter
. The poles of Butterworth filter lies on	sphere	circle	ellipse	parabola	circle
I I R digital filters are of the following nature	Recursive	Non Recursive	Reversive	Non Reversive	Recursive
In I I R digital filter the present output depends on	Present and previous Inputs only	Present input and previous outputs only	Present input only	Present Input, Previous input and output	Present Input, Previous input and output
Which of the following is best suited for I I R filter when compared with the FIR filter	Lower sidelobes in stopband	Higher Sidelobes in stopband	Lower sidelobes in Passband	No sidelobes in stopband	Lower sidelobes in stopband
In the case of I I R filter which of the following is true if the phase distortion is tolerable	More parameters for design	More memory requirement	Lower computational Complexity	Higher computational complexity	Lower computational Complexity
A causal and stable I I R filter has	Linear phase	No Linear phase	Linear amplitude	No Amplitude	No Linear phase
Neither the Impulse response nor the phase response of the analog filter is Preserved in the digital filter in the following method	The method of mapping of differentials	Impulse invariant method	Bilinear transformation	Matched Z - transformation technique	Bilinear transformation
Out of the given I I R filters the following filter is the efficient one	Circular filter	Elliptical filter	Rectangular filter	Chebyshev filter	Elliptical filter
What is the disadvantage of impulse invariant method	Aliasing	ne to one mapping	anti aliasing	warping	Aliasing
Which of the I I R Filter design method is antialiasing method?	The method of mapping of differentials	Impulse invariant method	Bilinear transformation	Matched Z - transformation technique	Bilinear transformation
The nonlinear relation between the analog and digital frequencies is called	aliasing	warping	prewarping	antialiasing	warping
The most common technique for the design of I I R Digital filter is	Direct Method	In direct method	Recursive method	non recursive method	In direct method
In the design a IIR Digital filter for the conversion of analog filter in to Digital domain the desirable property is	The axis in the s - plane should map outside the unit circle in the z - Plane	The Left Half Plane(LHP) of the s - plane should map in to the unit circle in the Z -plane	The Left Half Plane(LHP) of the s-plane should map outside the unit circle in the z-plane	The Right Half Plane(RHP) of the s-plane should map in to the unit circle in the Z -plane	The Left Half Plane(LHP) of the s - plane should map in to the unit circle in the Z -plane
The I I R filter design method thatovercomes the limitation of applicability to only lowpass filter and a limited class of bandpass filter is	Approximation of derivatives	Impulse Invariance	Bilinear Transformation	Frequency sampling	Impulse Invariance
The direct form II for realisation involves	The realisation of transfer function into two parts	Realisation fraction	division of two transfer functions	subtraction of two transfer functions	The realisation of transfer function into two parts

The direct form II for realisation involves	The realisation of transfer function into one part	Realisation after fraction	division of two transfer functions	subtraction of two transfer functions	Realisation after fraction
The direct form II for realisation involves	The realisation of transfer function into one part	Realisation after fraction	Product of two transfer functions	subtraction of two transfer functions	Product of two transfer functions
The direct form II for realisation involves	The realisation of transfer function into one part	Realisation after fraction	division of two transfer functions	sum of two transfer functions	sum of two transfer functions
The cascade realisation of IIR systems involves	The transfer function broken into product of transfer functions	The transfer function divided into multiplication of transfer functions	Factoring the numerator and denominator polynomials	integral of the transfer functions	The transfer function broken into product of transfer functions
The cascade realisation of IIR systems involves	The transfer function broken into product of transfer functions	The transfer function divided into addition of transfer functions	Factoring the numerator polynomials	integral of the transfer functions	The transfer function divided into addition of transfer functions
The advantage of using the cascade form of realisation is	It has same number of poles and zeros as that of individual components	The number of poles is the product of poles of individual components	The number of zeros is the product of poles of individual components	Over all transfer function cannot be determined	It has same number of poles and zeros as that of individual components
The advantage of using the cascade form of realisation is	It hasdifferent number of poles and zeros as that of individual components	The number of poles is the product of poles of individual components	The number of zeros is the product of poles of individual components	Over all transfer function may be determined	Over all transfer function may be determined
Which among the following represent/s the characteristic/s of an ideal filter?	Constant gain in passband	infinite gain in stop band	Non linear phase response	finite band width	Constant gain in passband
Which among the following represent/s the characteristic/s of an ideal filter?	zero gain in passband	zero gain in stop band	Non linear phase response	finite band width	zero gain in stop band
Which among the following represent/s the characteristic/s of an ideal filter?	zero gain in passband	constant gain in stop band	linear phase response	finite band width	linear phase response
FIR filters _______	are non-recursive	causal	are recursive	use feedback	are non-recursive
FIR filters _______	causal	do not adopt any feedback	use feedback	are recursive	do not adopt any feedback
In tapped delay line filter, the tapped line is also known as	Pick-on node	Pick-off node	Pick-up node	Pick-down node	Pick-off node
How is the sensitivity of filter coefficient quantization for FIR filters?	Low	Moderate	High	Unpredictable	Low
Decimation is a process in which the sampling rate is	enhanced	stable	reduced	unpredictable	reduced
Anti-imaging filter with cut-off frequency $\omega_c = \pi/ I$ is specifically used ______ upsampling process for the removal of unwanted images.	Before	At the time of	After	All of the above	After
The IIR filter designing involves	designing of analog filter in analog domain and transforming into digital domain	Designing of digital filter in analog domain and transforming into digital domain	Designing of analog filter in digital domain and transforming into analog domain	Designing of digital filter in digital domain and transforming into analog domain	Designing of digital filter in analog domain and transforming into digital domain
IIR filter design by approximation of derivatives has the limitations	Used only for transforming analog high pass filters	Used for band pass filters having smaller resonant frequencies			Used only for transforming analog high pass filters

IIR filter design by approximation of derivatives has the limitations	Used only for transforming analog low pass filters	Used for band pass filters having different resonant frequencies	Used only for transforming analog low pass filters	Used for band pass filters having high resonant frequencies	Used only for transforming analog low pass filters
The filter that may not be realized by approximation of derivatives techniques are	Band pass filters	#NAME?	Low pass filters	All pass filter	Band pass filters
The filter that may not be realized by approximation of derivatives techniques are	Band pass filters	Band reject filter	Low pass filters	All pass filter	Band reject filter
In direct form for realisation of IIR filters,	Denominator coefficients are the multipliers in the feed forward paths	Multipliers in the feedback paths are the positives of the denominator coefficients	Multipliers in the feedback paths are the negatives of the denominator coefficients	all the above	Multipliers in the feedback paths are the negatives of the denominator coefficients
In direct form for realisation of IIR filters,	Denominator coefficients are the multipliers in the feed forward paths	Multipliers in the feedback paths are the positives of the denominator coefficients	Numerator coefficients are the multipliers in the feed forward paths	all the above	Numerator coefficients are the multipliers in the feed forward paths
Roll-off factor is	The bandwidth occupied beyond the Nyquist Bandwidth of the filter	The performance of the filter or device	Aliasing effect	None of the above	The bandwidth occupied beyond the Nyquist Bandwidth of the filter
The DFT is preferred for	Its ability to determine the frequency component of the signal	Removal of noise	Quantization of signal	filter analysis	Its ability to determine the frequency component of the signal
The DFT is preferred for	Filter design	Removal of noise	Quantization of signal	sampling	Filter design
Frequency selectivity characteristics of DFT refers to	Ability to resolve different frequency components from input signal	Ability to translate into frequency domain	Ability to convert into discrete signal	None of the above	Ability to resolve different frequency components from input signal
DIT algorithm divides the sequence into	Positive and negative values	Even and odd samples	Upper higher and lower spectrum	Small and large samples	Even and odd samples
The transformations are required for	Analysis in time or frequency domain	Quantization	Modulation	sampling	Analysis in time or frequency domain
The transformations are required for	Easier operations	Quantization	Modulation	sampling	Easier operations
The computational procedure for Decimation in frequency algorithm takes	Log2 N stages	2Log2 N stages	Log2 N^2 stages	Log2 N/2 stages	Log2 N stages
Product of one even and one odd function is	even	odd	prime	aliasing	odd
If ƒ(x,y) is imaginary, then its Fourier transform is	conjugate symmetry	hermition	antihermition	symmetry	antihermition
ƒ(0,0) is sometimes called	ac component	dc component	jaggy	coordinate	dc component
Even functions are said to be	symmetric	antisymmetric	periodic	aperiodic	symmetric
Linear functions possesses property of	additivity	homogeneity	multiplication	Both A and B	Both A and B
Continuous functions are sampled to form a	Fourier series	Fourier transform	fast Fourier series	digital image	digital image
2D Fourier transform and its inverse are infinitely	aperiodic	periodic	linear	non linear	periodic
Odd functions are said to be	symmetric	antisymmetric	periodic	aperiodic	antisymmetric
Gradient computation equation is	|Gx|+|Gy|	|Gx|-|Gy|	|Gx|/|Gy|	|Gx|x|Gy|	|Gx|+|Gy|
Prepared by Ambili Vipin ,Assistnat Professor,Department of Physics ,KAHE					

UNIT V

SYLLABUS

Fast Fourier Transform: Direct Computation of the DFT, Symmetry and Periodicity, Properties of the Twiddle factor (*WN*), Radix-2 FFT Algorithms; Decimation-In-Time (DIT) FFT Algorithm; Decimation-In-Frequency (DIF) FFT Algorithm, Inverse DFT Using FFT Algorithms. **Realization of Digital Filters:** Non Recursive and Recursive Structures, Canonic and Non Canonic Structures, Equivalent Structures (Transposed Structure), FIR Filter structures; Direct-Form; Cascade-Form; Basic structures for IIR systems; Direct-Form I.

A fast Fourier transform (FFT) is any fast algorithm for computi the DFT. The development of FFT algorithms had a tremendc impact on computational aspects of signal processing and appli science. The DFT of an N-point signal

$$\{x[n], 0 \leq n \leq N-1\}$$

is defined as

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{-kn}, \qquad 0 \leq k \leq N-1$$

where

$$W_N = e^{j\frac{2\pi}{N}} = \cos\left(\frac{2\pi}{N}\right) + j\,\sin\left(\frac{2\pi}{N}\right)$$

is the principal N-th root of unity.

DIRECT DFT COMPUTATION

Direct computation of $X[k]$ for $0 \le k \le N-1$ requires

$$(N-1)^2 \text{ complex multiplications}$$

$$N(N-1) \text{ complex additions}$$

\

DFT as a Linear Transformation

- Matrix representation of DFT

  Definition of DFT:

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}, \qquad k = 0,1,\ldots,N-1$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-kn}, \quad n = 0,1,\ldots,N-1$$

where

Let

$$\mathbf{x}_N = \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}, \quad \mathbf{X}_N = \begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix},$$

and

$$\mathbf{W}_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & W_N^{(N-1)} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

Thus,

$$\mathbf{X}_N = \mathbf{W}_N \mathbf{x}_N \qquad N\text{ - point DFT}$$

$$\mathbf{x}_N = \mathbf{W}_N^{-1} \mathbf{X}_N \qquad N\text{ - point IDFT}$$

$$= \frac{1}{N}\mathbf{W}_N^{*}\mathbf{X}_N$$

Because the matrix (transformation) $\mathbf{W}_N$ has a specific structure and because W_N^k has particular values (for some k and n), we can reduce the number of arithmetic operations for computing this transform.

Example $x[n] = [0\ 1\ 2\ 3]$

$$\mathbf{W}_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Only additions are needed to compute this specific transform.
(This is a well-known *radix-4 FFT*)

Thus, the DFT of $x[n]$ is

$$\mathbf{X}_4 = \mathbf{W}_4\mathbf{x}_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

Fast Fourier Transform

-- Highly efficient algorithms for computing DFT

- General principle: *Divide-and-conquer*
- Specific properties of W_N^k
  - Complex conjugate symmetry: $W_N^{-kn} = (W_N^{kn})^*$
  - Symmetry: $W_N^{k+N/2} = -W_N^k$
  - Periodicity: $W_N^{k+N} = W_N^k$
  - Particular values of k and n: e.g., radix-4 FFT (no multiplications)
- Direct computation of DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{kn}, \quad k = 0,1,\ldots,N-1$$
$$= \sum_{n=0}^{N-1} \left\{ \begin{array}{l} \left[\mathrm{Re}(x[n]) \cdot \mathrm{Re}(W_N^{kn}) - \mathrm{Im}(x[n]) \cdot \mathrm{Im}(W_N^{kn})\right] + \\ j\left[\mathrm{Re}(x[n]) \cdot \mathrm{Im}(W_N^{kn}) + \mathrm{Im}(x[n])\mathrm{Re}(W_N^{kn})\right] \end{array} \right\}$$

For each k, we need N complex multiplications and N-1 complex additions. → 4N real multiplications and 4N-2 real additions.

We will show how to use the properties of W_N^k to reduce computations.

- Radix-2 algorithms: Decimation-in-time; Decimation-in-frequency
- Composite N algorithms: Cooley-Tukey; Prime factor
- Winograd algorithm
- Chirp transform algorithm

RADIX-2 FFT

The radix-2 FFT algorithms are used for data vectors of lengths $N = 2^K$. They proceed by dividing the DFT into two DFTs of length $N/2$ each, and iterating. There are several types of radix-2 FFT algorithms, the most common being the *decimation-in-time* (DIT) and the *decimation-in-frequency* (DIF).

The development of the FFT will call on two properties of W_N.

The first property is:

$$\boxed{W_N^2 = W_{N/2}}$$

which is derived as

$$\begin{aligned} W_N^2 &= e^{-j\frac{2\pi}{N}\cdot 2} \\ &= e^{-j\frac{2\pi}{N/2}} \\ &= W_{N/2}. \end{aligned}$$

More generally, we have

$$W_N^{2nk} = W_{N/2}^{nk}.$$

The second property is:

$$\boxed{W_N^{k+\frac{N}{2}} = -W_N^k}$$

which is derived as

$$\begin{aligned} W_N^{k+\frac{N}{2}} &= e^{j\frac{2\pi}{N}(k+\frac{N}{2})} \\ &= e^{j\frac{2\pi}{N}k} \cdot e^{j\frac{2\pi}{N}(\frac{N}{2})} \\ &= e^{j\frac{2\pi}{N}k} \cdot e^{j\pi} \\ &= -e^{j\frac{2\pi}{N}k} \\ &= -W_N^k \end{aligned}$$

Radix-2 Decimation-in-time Algorithms

-- Assume N-point DFT and $N = 2^{\nu}$

- Idea: N-point DFT → $N/2$-point DFT → $N/4$-point DFT

  $N/4$-point DFT

  $N/2$-point DFT → $N/4$-point DFT

  $N/4$-point DFT

- Sequence: $x[0]\ \ x[1]\ \ x[2]\ \ x[3]\ \ \cdots\ \ x[n/2]\ \ \cdots\ \ x[N-1]$

  Even index: $x[0]\ \ x[2]\ \ \cdots\ \ x[N-2]$

  Odd index: $x[1]\ \ x[3]\ \ \cdots\ \ x[N-1]$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad k = 0,1,\ldots,N-1$$

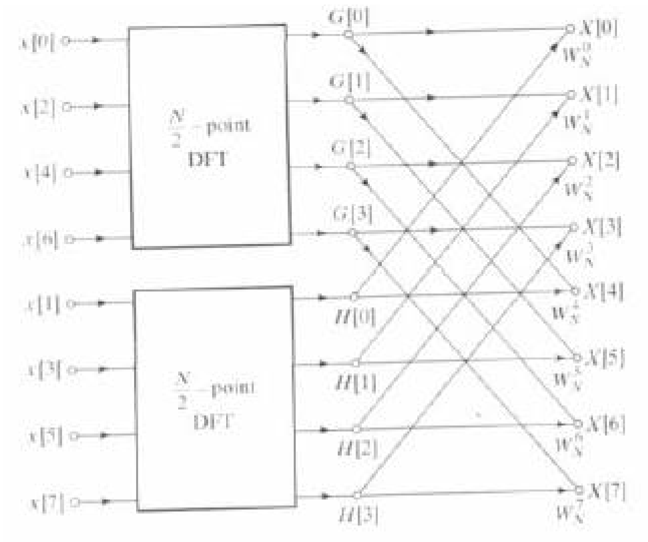
$$= \underbrace{\sum_{n\text{ even}} x[n] W_N^{kn}}_{n=2r} + \underbrace{\sum_{n\text{ odd}} x[n] W_N^{kn}}_{n=2r+1}$$

$$= \sum_{r=0}^{\frac{N}{2}-1} x[2r] W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_N^{(2r+1)k}$$

$$\because W_N^2 = e^{-2j\left(\frac{2\pi}{N}\right)} = e^{-2j\left(\frac{\pi}{N/2}\right)} = W_{N/2}$$

$$X[k] = \underbrace{\sum_{r=0}^{\frac{N}{2}-1} x[2r] W_{N/2}^{rk}}_{\frac{N}{2}\text{-point DFT}} + W_N^k \underbrace{\sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_{N/2}^{rk}}_{\frac{N}{2}\text{-point DFT}}$$

$$= G[k] + W_N^k H[k]$$



- Comparison:

  (a) Direct computation of N-point DFT (N frequency samples):

  $\sim N^2$ complex multiplications and N^2 complex adds

  (b) Direct computation of $N/2$-point DFT:

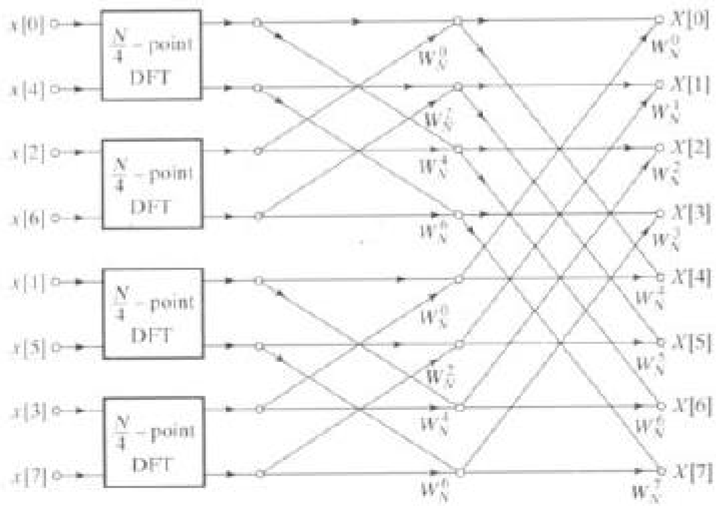
  $\sim \left(\frac{N}{2}\right)^2$ complex multiplications and $\left(\frac{N}{2}\right)^2$ complex adds

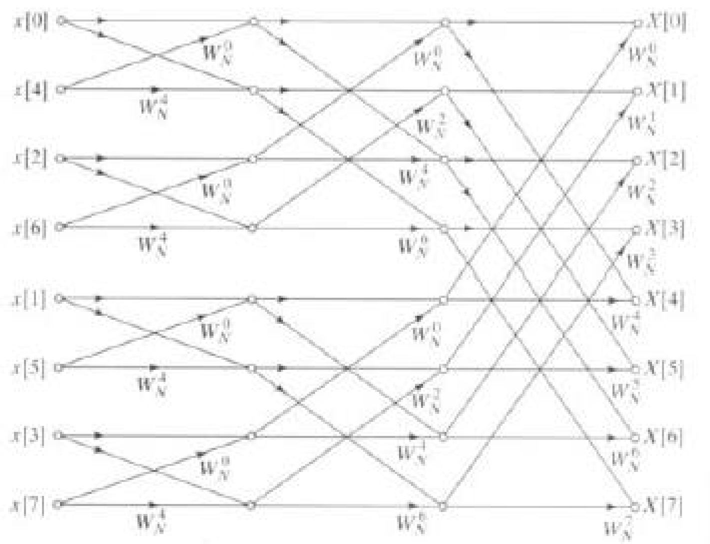
  $+$ additional N complex multis and N complex adds

  $\sim$ (Total:) $N+2\left(\frac{N}{2}\right)^2 = N+\frac{N^2}{2}$ complex multis and adds

  (c) $\log_2 N$ -stage FFT

  Since $N = 2^{\nu}$, we can further break $N/2$-point DFT into two $N/4$-point DFT and so on.



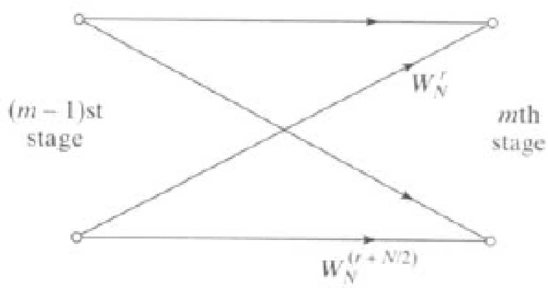


At each stage: ~ N complex multis and adds

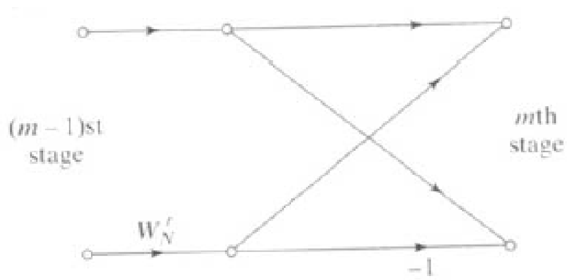
Total: ~ $N \log_2 N$ complex multis and adds (--> $\frac{N}{2}\log_2 N$)

Butterfly: Basic unit in FFT

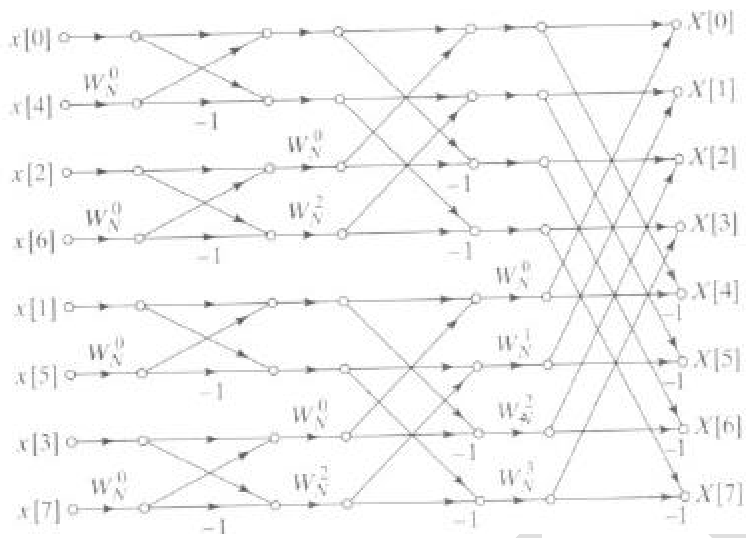
Two multiplications:



One multiplication:



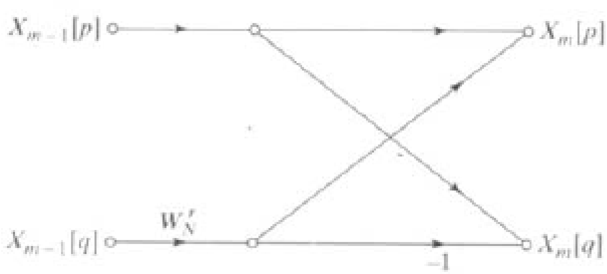
8-Point DFT



- **In-place** computations

  Only two registers are needed for computing a butterfly unit.

$$X_m[p] = X_{m-1}[p] + W_N^r X_{m-1}[q]$$
$$X_m[q] = X_{m-1}[p] - W_N^r X_{m-1}[q]$$



Radix-2 Decimation-in-frequency Algorithms

- Dividing the output sequence $X[k]$ into smaller pieces.

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}, \quad k = 0,1,\ldots,N-1$$

If k is even, $k = 2r$.

$$X[2r] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \qquad r = 0,1,\cdots,\frac{N}{2}-1$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{2nr} + \sum_{n=\frac{N}{2}}^{N-1} x[n] W_N^{2nr} \qquad n \leftarrow (n + N/2)$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{2nr} + \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] \cdot W_N^{2r\left(n+\frac{N}{2}\right)}$$

$$\because W_N^{2r[n+N/2]} = W_N^{2rn} W_N^{rN} = W_N^{2rn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + x\left[n+\frac{N}{2}\right] \right) \cdot W_N^{2nr}$$

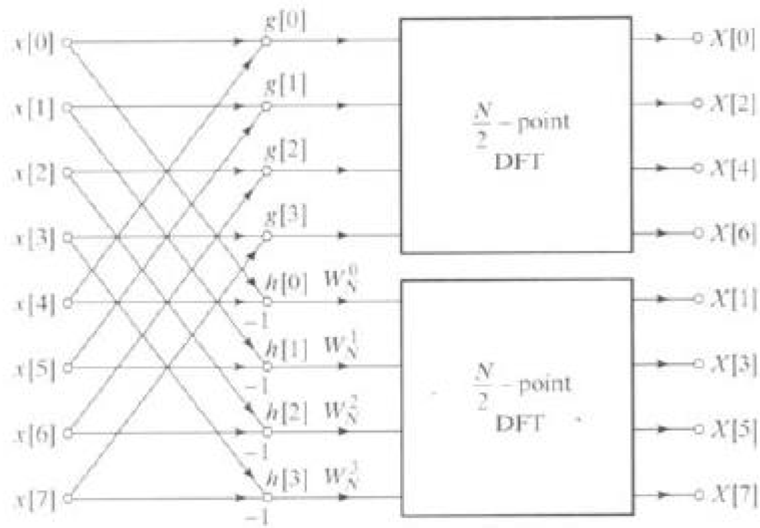
$$= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + x\left[n+\frac{N}{2}\right] \right) \cdot W_{N/2}^{nr}$$

Similarly, if k is odd, $k = 2r+1$.

$$X[2r+1] = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] - x\left[n+\frac{N}{2}\right] \right) \cdot W_N^{n} \cdot W_{N/2}^{nr}$$

$$\begin{cases} X[2r] = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + x\left[n+\frac{N}{2}\right] \right) \cdot W_{N/2}^{nr} \\ X[2r+1] = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] - x\left[n+\frac{N}{2}\right] \right) \cdot W_N^{n} \cdot W_{N/2}^{nr} \end{cases}$$

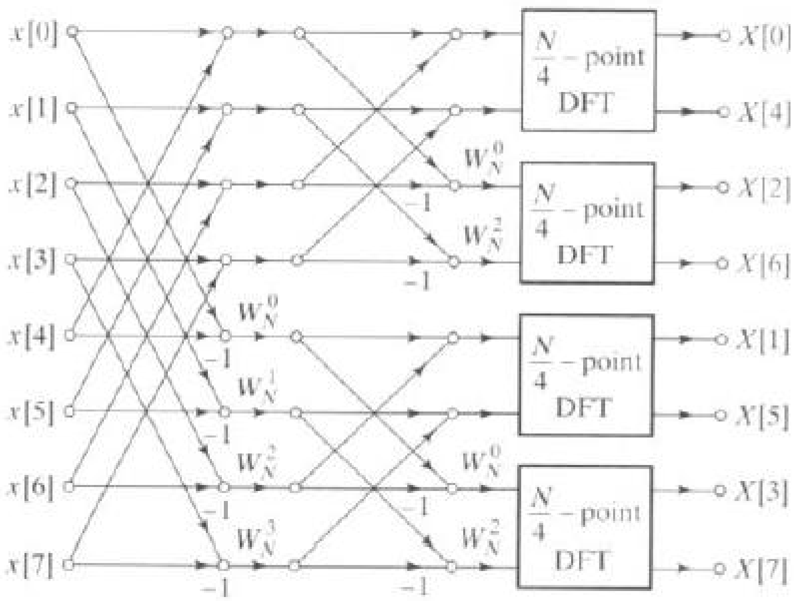
$$\text{Let} \quad \begin{cases} g[n] = x[n] + x\left[n+\frac{N}{2}\right] \\ h[n] = x[n] - x\left[n+\frac{N}{2}\right] \end{cases}$$

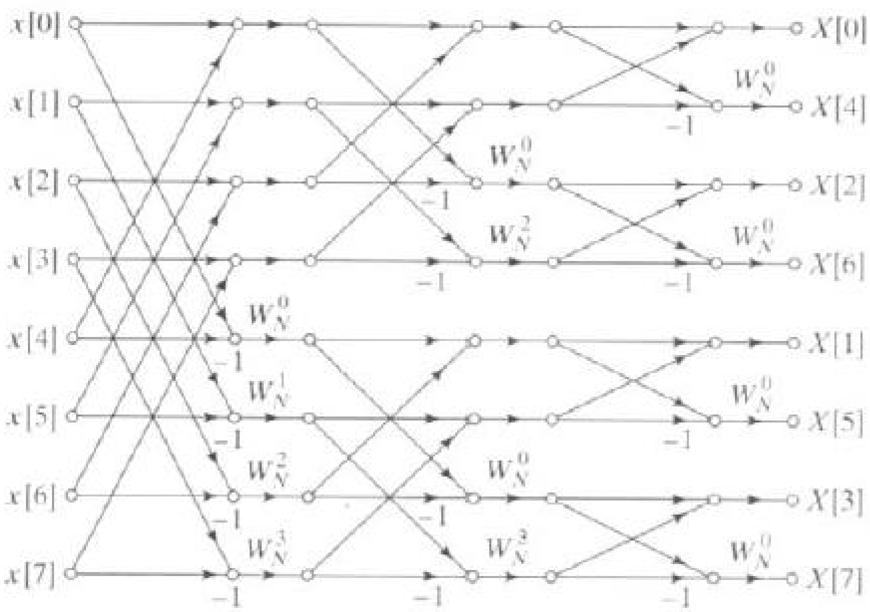


We can further break $X[2r]$ into even and odd groups …

Again, we can reduce the two-multiplication butterfly into one multiplication. Hence, the computational complexity is bout $\frac{N}{2}\log_2 N$. The in-place computation property holds if the outputs are in bit-reversed order (when inputs are in the normal order).

Flow chart of decimation –in-frequency decomposition of an 8 -point DFT in to four 2-point DFT computations





Flow graph of complete decimation –in- frequency decomposition of an 8 point DFT computation

Inverse FFT

- IDFT: $$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\cdot W_N^{-kn} \qquad (*)$$

  DFT: $$X[k] = \sum_{n=0}^{N-1} x[n]\cdot W_N^{nk}$$

  Hence, take the conjugate of $(*)$:

$$x^*[n] = \frac{1}{N}\left(\sum_{k=0}^{N-1} X[k]\cdot W_N^{-kn}\right)^*$$
$$= \frac{1}{N}\sum_{k=0}^{N-1}\left(X[k]\cdot W_N^{-kn}\right)^*$$
$$= \frac{1}{N}\sum_{k=0}^{N-1}\left(X^*[k]\cdot W_N^{kn}\right)$$
$$= \frac{1}{N}\,\mathrm{DFT}\left[X^*(k)\right]$$

  Take the conjugate of the above equation:

$$x[n] = \frac{1}{N}\left(\mathrm{DFT}\left[X^*(k)\right]\right)^*$$
$$= \frac{1}{N}\left(\mathrm{FFT}\left[X^*(k)\right]\right)^*$$

Thus, we can use the FFT algorithm to compute the inverse DFT.

POSSIBLE QUESTIONS

PART-B (2 MARKS)

1.Define FFT and inverse FFT.

2.Define Symmetry and Periodicity of FFT

3.List out the Properties of the Twiddle factor (WN).

4.Define Non Recursive and Recursive Structures

5.Define Canonic and Non Canonic Structures.

6.Define radix -2 FFT.

7.What is the basic operation of DIT algorithm?

8. What is the basic operation of DIF algorithm?

9.Draw the basic butterfly diagram for DIT algorithm

10. Draw the basic butterfly diagram for DIF algorithm

PART-C (6MARKS)

1. Draw the signal flow graph for 8-point DFT using DIT algorithm

2. Draw the signal flow graph for 8-point DFT using DIF algorithm

3.Compute an 8 point DFT for the sequence x(n)={ 1.-1,1,-1,0,0,0,0} using DIT algorithm

4. Compute an 8 point DFT for the sequence x(n)={ 1.-1,1,-1,0,0,0,0} using DIF algorithm

5.Explain the Properties of the Twiddle factor (WN).

6.Realize IIR filter structure using Direct form –I

7.Realize the second order digital filter $y(n) = 2 r \cos(\omega 0) y(n-1) - r^2 y(n-2) + x(n) - r\cos(\omega 0) x(n-1)$.

8.Explain cascade realization of FIR filter.

KARPAGAM ACADEMY OF HIGHER EDUCATION,COIMBATORE-21

DEPARTMENT OF PHYSICS
II B.Sc PHYSICS (2016-2019)
DIGITAL SIGNAL PROCESSING (16PHU403)

QUESTIONS	CHOICE1	CHOICE2	CHOICE3	CHOICE4	ANSWER
UNIT-V					
Which of the following is true regarding the number of computations required to compute an N-point DFT?	N2 complex multiplications and N(N-1) complex additions	N2 complex additions and N(N-1) complex multiplications	N2 complex multiplications and N(N+1) complex additions	N2 complex additions and N(N+1) complex multiplications	N2 complex multiplications and N(N-1) complex additions
Which of the following is true regarding the number of computations required to compute DFT at any one value of 'k'?	4N-2 real multiplications and 4N real additions	4N real multiplications and 4N-4 real additions	4N-2 real multiplications and 4N+2 real additions	4N real multiplications and 4N-2 real additions	4N real multiplications and 4N-2 real additions
WNk+N/2=	WNk	-WNk	WN-k	0	WNK
The computation of XR(k) for a complex valued x(n) of N points requires:	2N2 evaluations of trigonometric functions	4N2 real multiplications	4N(N-1) real additions	All of the mentioned	All of the mentioned
If the arrangement is of the form in which the first row consists of the first M elements of x(n), the second row consists of the next M elements of x(n), and so on, then which of the following mapping represents the above arrangement	n=l+mL	n=Ml+m	n=ML+l	n=0	n=Ml+m
If N=LM, then what is the value of WNmqL?	WMmq	WLmq	WNmq	W	WMmq
How many complex multiplications are performed in computing the N-point DFT of a sequence using divide-and-conquer method if N=LM?	N(L+M+2)	N(L+M-2)	N(L+M-1)	N(L+M+1)	N(L+M+1)
How many complex additions are performed in computing the N-point DFT of a sequence using divide-and-conquer method if N=LM?	N(L+M+2)	N(L+M-2)	N(L+M-1)	N(L+M+1)	N(L+M-2)
If we store the signal row wise and compute the L point DFT at each column, the resulting array must be multiplied by which of the following factors?	WNlq	WNpq	WNlq	WNpm	WNpm
If X(k) is the N/2 point DFT of the sequence x(n), then what is the value of X(k+N/2)?	F1(k)+F2(k)	F1(k)- WNk F2(k)	F1(k)+WNk F2(k)	F1(k)/WNk F2(k)	F1(k)- WNk F2(k)
How many complex multiplications are required to compute X(k)?	N(N+1)	N(N-1)/2	N2/2	N(N+1)/2	N(N+1)/2
The total number of complex multiplications required to compute N point DFT by radix-2 FFT is:	(N/2)log2N	Nlog2N	(N/2)logN	(N/2)lnN	(N/2)log2N
The total number of complex additions required to compute N point DFT by radix-2 FFT is:	(N/2)log2N	Nlog2N	(N/2)logN	(N/2)lnN	Nlog2N
For a decimation-in-time FFT algorithm, which of the following is true?	Both input and output are in order	Both input and output are shuffled	Input is shuffled and output is in order	Input is in order and output is shuffled	Input is shuffled and output is in order
. For a decimation-in-time FFT algorithm, which of the following is true?	Both input and output are in order	Both input and output are shuffled	Input is shuffled and output is in order	Input is in order and output is shuffled	Input is in order and output is shuffled

If x1(n) and x2(n) are two real valued sequences of length N, and let x(n) be a complex valued sequence defined as x(n)=x1(n)+jx2(n), 0≤ n≤ N-1, then what is the value of x2(n)?	(x(n)-x*(n))/2	(x(n)+x*(n))/2	(x(n)+x*(n))/2j	(x(n)-x*(n))/2j	(x(n)-x*(n))/2j
. If X(k) is the DFT of x(n) which is defined as x(n)=x1(n)+jx2(n), 0≤ n≤ N-1, then what is the DFT of x1(n)?	1/2 [X*(k)+X*(N-k)].	) 1/2 [X*(k)-X*(N-k)]	1/2j [X*(k)-X*(N-k)].	1/2j [X*(k)+X*(N-k)].	1/2 [X*(k)+X*(N-k)].
If X(k) is the DFT of x(n) which is defined as x(n)=x1(n)+jx2(n), 0≤ n≤ N-1, then what is the DFT of x1(n)?	(1/2) [X*(k)+X*(N-k)].	(1/2) [X*(k)-X*(N-k)].	(1/2j) [X*(k)-X*(N-k)].	(1/2j) [X*(k)+X*(N-k)].	(1/2j) [X*(k)-X*(N-k)].
If g(n) is a real valued sequence of 2N points and x1(n)=g(2n) and x2(n)=g(2n+1), then what is the value of G(k), k=0,1,2…N-1?	X1(k)-W2kNX2(k)	X1(k)+W2kNX2(k)	X1(k)+W2kX2(k)	X1(k)-W2kX2(k)	X1(k)+W2kNX2(k)
If g(n) is a real valued sequence of 2N points and x1(n)=g(2n) and x2(n)=g(2n+1), then what is the value of G(k), k=N,N-1,…2N-1?	X1(k)-W2kX2(k)	X1(k)+W2kNX2(k)	X1(k)+W2kX2(k)	X1(k)-W2kNX2(k)	X1(k)-W2kNX2(k)
How many complex multiplications are need to be performed for each FFT algorithm?	(N/2)logN	Nlog2N	(N/2)log2N	(N/2)ln2N	(N/2)log2N
How many complex additions are required to be performed in linear filtering of a sequence using FFT algorithm?	(N/2)logN	2Nlog2N	(N/2)log2N	Nlog2N	2Nlog2N
How many complex multiplication are required per output data point?	[(N/2)logN]/L	[Nlog22N]/L	[(N/2)log2N]/L	[(N/2)log2N]/L	[Nlog22N]/L
Which of the following is used in the realization of a system?	Delay elements	Multipliers	Adders	All of the mentioned	All of the mentioned
Computational complexity refers to the number of:	Additions	Arithmetic operations	Multiplications	division	Arithmetic operations
Which of the following refers the number of memory locations required to store the system parameters, past inputs, past outputs and any intermediate computed values?	Computational complexity	Finite world length effect	Memory requirements	bandwidth requirements	Memory requirements
Which of the following are called as finite word length effects?	Parameters of the system must be represented with finite precision	Computations are truncated to fit in the limited precision constraints	Whether the computations are performed in fixed point or floating point arithmetic	All of the mentioned	All of the mentioned
Which of the following is an method for implementing an FIR system?	Direct form	Cascade form	Lattice structure	All of the mentioned	All of the mentioned
How many memory locations are used for storage of the output point of a sequence of length M in direct form realization?	M+1	M	M-1	M/N	M-1
By combining two pairs of poles to form a fourth order filter section, by what factor we have reduced the number of multiplications?	25%	30%	40%	50%	50%
The desired frequency response is specified at a set of equally spaced frequencies defined by the equation:	π/2M(k+α)	π/M(k+α)	2π/M(k+α)	2π/M(k-α)	2π/M(k+α)
The zeros of the system function of comb filter are located:	Inside unit circle	On unit circle	Outside unit circle	circle	On unit circle
If M and N are the orders of numerator and denominator of rational system function respectively, then how many multiplications are required in direct form-I realization of that IIR filter?	M+N-1	M+N	M+N+1	M+N+2	M+N+1
If M and N are the orders of numerator and denominator of rational system function respectively, then how many additions are required in direct form-I realization of that IIR filter?	M+N-1	M+N	M+N+1	M+N+2	M+N

If M and N are the orders of numerator and denominator of rational system function respectively, then how many memory locations are required in direct form-I realization of that IIR filter?	M+N+1	M+N	M+N-1	M+N-2	M+N+1
If M and N are the orders of numerator and denominator of rational system function respectively, then how many memory locations are required in direct form-II realization of that IIR filter?	M+N+1	M+N	Min [M,N].	Max [M,N].	Max [M,N].
What are the nodes that replace the adders in the signal flow graphs?	Source node	Sink node	Branch node	Summing node	Summing node
If we reverse the directions of all branch transmittances and interchange the input and output in the flow graph, then the resulting structure is called as:	Direct form-I	Transposed form	Direct form-II	sampling	Transposed form
In IIR Filter design by the Bilinear Transformation, the Bilinear Transformation is a mapping from	Z-plane to S-plane	S-plane to Z-plane	S-plane to J-plane	J-plane to Z-plane	S-plane to Z-plane
The state space or the internal description of the system still involves a relationship between the input and output signals, what are the additional set of variables it also involves?	System variables	Location variables	State variables	variables	State variables
3. Which of the following gives the complete definition of the state of a system at time n0?	Amount of information at n0 determines output signal for n≥n0	Input signal x(n) for n≥n0 determines output signal for n≥n0	Input signal x(n) for n≥0 determines output signal for n≥n0	Amount of information at n0+input signal x(n) for n≥n0 determines output signal for n≥n0	Amount of information at n0+input signal x(n) for n≥n0 determines output signal for n≥n0
. If we interchange the rows and columns of the matrix F, then the system is called as:	Identity system	Transposed system	Diagonal system	system	Transposed system
A closed form solution of the state space equations is easily obtained when the system matrix F is:	Transpose	Symmetric	Identity	Diagonal	Diagonal
Which of the following is true regarding the number of computations required to compute an N-point DFT?	N^2 complex multiplications and N(N-1) complex additions	N^2 complex additions and N(N-1) complex multiplications	N^2 complex multiplications and N(N+1) complex additions	N^2 complex additions and N(N+1) complex multiplications	N^2 complex multiplications and N(N-1) complex additions
Which of the following is true regarding the number of computations required to compute DFT at any one value of 'k'?	4N-2 real multiplications and 4N real additions	4N real multiplications and 4N-4 real additions	4N-2 real multiplications and 4N+2 real additions	4N real multiplications and 4N-2 real additions	4N real multiplications and 4N-2 real additions
WNk+N/2=	WNk	-WNk	WN-k	W	-WNk
The computation of XR(k) for a complex valued x(n) of N points requires:	2N2 evaluations of trigonometric functions	4N2 real multiplications	4N(N-1) real additions	All of the mentioned	All of the mentioned
If N=LM, then what is the value of WNmqL?	WMmq	WLmq	WNmq	W	WMmq
What is the highest frequency that is contained in the sampled signal?	2Fs	Fs/2	Fs	F	Fs/2
If {x(n)} is the signal to be analyzed, limiting the duration of the sequence to L samples, in the interval 0≤ n≤ L-1, is equivalent to multiplying {x(n)} by:	Kaiser window	Hamming window	Hanning window	Rectangular window	Rectangular window
Which of the following is the advantage of Hanning window over rectangular window?	More side lobes	Less side lobes	More width of main lobe	width of main lobe	Less side lobes

Which of the following is the disadvantage of Hanning window over rectangular window?	More side lobes	Less side lobes	More width of main lobe	width of main lobe	More width of main lobe
If the input analog signal falls outside the range of the quantizer (clipping), e_q (n) becomes unbounded and results in ____________	Granular noise	Overload noise	Particulate noise	Heavy noise	Overload noise
What is the abbreviation of SQNR?	Signal-to-Quantization Net Ratio	Signal-to-Quantization Noise Ratio	Signal-to-Quantization Noise Region	Signal-to-Quantization Net Region	Signal-to-Quantization Noise Ratio
What is the scale used for the measurement of SQNR?	DB	db	dB	All of the mentioned	dB
In Overlap save method of long sequence filtering, what is the length of the input sequence block?	L+M+1	L+M	L+M-1	L	L+M-1
Which of the following is true in case of Overlap add method?	M zeros are appended at last of each data block	M zeros are appended at first of each data block	M-1 zeros are appended at last of each data block	M-1 zeros are appended at first of each data block	M-1 zeros are appended at last of each data block
What is the model that has been adopt for characterizing round of errors in multiplication?	Multiplicative white noise model	Subtractive white noise model	Additive white noise model	noise model	Additive white noise model
How many quantization errors are present in one complex valued multiplication?	One	Two	Three	Four	Four
What is the total number of quantization errors in the computation of single point DFT of a sequence of length N?	2N	4N	8N	12N	4N
How is the variance of the quantization error related to the size of the DFT?	Equal	Inversely proportional	Square proportional	Proportional	Proportional

Prepared by Ambili Vipin ,Assistnat Professor,Department of Physics ,KAHE

The z – transform F(z) of the function $f(nT) = a^{nT}$ is

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(16PHU403)

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE-21

DEPARTMENT OF PHYSICS

II B.Sc PHYSICS

Fourth Semester

I-Internal Examination (January2018)

DIGITAL SIGNAL PROCESSING

Time:2 hours **Maximum:50 marks**

PART-A(20x1=20Marks)

Answer all questions

1. If x(n) is a discrete-time signal, then the value of x(n) at non integer value of 'n' is:
   a).Zero b). Positive c). Negative d). Not defined
2. The discrete time function defined as u(n)=n for n≥0;=0 for n<0 is an:
   a). Unit sample signal b) Unit step signal c) Unit ramp signal d) None of the mentioned
3. .The phase function of a discrete time signal $x(n)=a^n$, where $a=r.e^{j\theta}$ is:
   a) tan (nθ) b) nθ c) $\tan^{-1}(n\theta)$ d) None of the mentioned
4. A real valued signal x(n) is called as anti-symmetric if:
   a) x(n)=x(-n) b) x(n)=-x(-n) c) x(n)=-x(n) d) None of the mentioned
5. The odd part of a signal x(t) is:
   a) x(t)+x(-t) b) x(t)-x(-t) c) (1/2)*(x(t)+x(-t)) d) (1/2)*(x(t)-x(-t))

The function given by the equation x(n)=1, for n=0;=0, for n≠0 is a:
a) Step function b) Ramp function c) Triangular function d) Impulse function

7. If a signal x(n) is passed through a system to get an output sig
of y(n)=x(n+1), then the signal is said to be:
   a) Delayed b) Advanced c) No operation d) None of the menti
8. The system described by the input-output equation $y(n)=nx(n)+bx^3(n)$ is a:
   a) Static system b) Dynamic system c) Identical system d) None of the mentioned
9. If the output of the system of the system at any 'n' depends c
the present or the past values of the inputs then the system is sa
be:
   a) Linear b) Non-Linear c) Causal d) Non-causal

10.If a system do not have a bounded output for bounded input
the system is said to be:
   a) Causal b) Non-causal c) Stable d) Non-stable

11.A discrete time signal may NOT be
   a) Samples of a continuous signal b) A time series which is a
domain of integers c) Time series of sequence of quantities
   d) Amplitude modulated wave

12. DTFT is the representation of
   a. Periodic Discrete time signals b. Aperiodic Discrete time signals c. Aperiodic continuous signals d. Periodic continuo
signals

13. The transforming relations performed by DTFT are Lineari
Modulation, Shifting, Convolution
   a)1, 2 and 3 are correct b) 1 and 2 are correct
   c) 1 and 3 are correct d) All the four are correct

14. DIT algorithm divides the sequence into
   a) Positive and negative values b) Even and odd samples
   c) Upper higher and lower spectrum d. Small and large sam

15.The signal given by the equation $\sum_{n=-\infty}^{\infty} |x(n)|^2$ is known as:
   a) Energy signal b) Power signal c) Work done signal
   d) None of the mentioned

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a) Down-sampling b) Up-sampling c) Sampling
d) None of the mentioned

17. The equation of average power of a periodic signal x(t) is given as:

a) $\sum_{k=0}^{\infty} |c_k|^2$ b) $\sum_{k=-\infty}^{\infty} |c_k|$ c) $\sum_{k=-\infty}^{0} |c_k|^2$ d) $\sum_{k=-\infty}^{\infty} |c_k|^2$

18. If $x(n)=Ae^{j\omega n}$ is the input of an LTI system and h(n) is the response of the system, then what is the output y(n) of the system?

a) $H(-\omega)x(n)$ b) $-H(\omega)x(n)$ c) $H(\omega)x(n)$ d) None of the mentioned

19. Which of the following represents the phase associated with the frequency component of discrete-time Fourier series (DTFS)?

a) $e^{j2\pi kn/N}$ b) $e^{-j2\pi kn/N}$ c) $e^{j2\pi knN}$ d) None of the mentioned

20. What is the average power of the discrete time periodic signal x(n) with period N ?

a) $\frac{1}{N}\sum_{n=0}^{N} |x(n)|$ b) $\frac{1}{N}\sum_{n=0}^{N-1} |x(n)|$ c) $\frac{1}{N}\sum_{n=0}^{N} |x(n)|^2$
d) $\frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$

PART-B (3x2=6 Marks)
Answer all the questions

21. Define time shifting.
22. Define time variant and time invariant system
23. Define even and odd signals

PART-C (8x3=24 Marks)
Answer all the questions

24.a. Discuss about discrete time systems.

OR

b. Find the convolution of the signal

x(n)=1 n=-2,0,1
=2 n=-1
=0 elsewhere

h(n)=δ(n) -δ(n-1)+δ(n-2)-δ(n-3)

25. a.i) Determine the following systems are linear or non-linear a) Y(n)= n x(n) b) $y(n)= X^2(n)$

(ii) Determine the following systems are Causal or non causal a) $y(n)=x(n^2)$ b) y(n)= a x(n)+b x(n-1)

OR

b. Derive expression for convolution operation and list out the properties of convolution.

26. a .Explain the properties of discrete – time Fourier transform.

OR

b.Derive the expression for discrete time Fourier transform pair and inverse discrete time Fourier transform