KARAPAGAM ACADEMY OF HIGHER EDUCATION, COIMBATORE-21

SEMESTER IV

16PHU401 **WAVES AND OPTICS** **L T P C**
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Objective: This paper explains different properties of waves and their interaction with each other and with matter. Also it explains different aspects in optics using the theory of waves.

UNIT - I

Superposition of Two Collinear Harmonic oscillations: Simple harmonic motion (SHM). Linearity and Superposition Principle. (1) Oscillations having equal frequencies and (2) Oscillations having different frequencies (Beats). Superposition of Two Perpendicular Harmonic Oscillations: Graphical and Analytical Methods. Lissajous Figures (1:1 and 1:2) and their uses. Waves Motion- General: Transverse waves on a string. Travelling and standing waves on a string. Normal Modes of a string. Group velocity, Phase velocity. Plane waves. Spherical waves, Wave intensity.

UNIT - II

Sound: Sound waves, production and properties. Intensity and loudness of sound. Decibels. Intensity levels. musical notes. musical scale. Acoustics of buildings (General idea). Wave Optics: Electromagnetic nature of light. Definition and Properties of wave front. Huygens Principle.

UNIT – III

Interference: Interference: Division of amplitude and division of wavefront. Young's Double Slit experiment. Lloyd's Mirror & Fresnel's Biprism.Phase change on reflection: Stokes' treatment. Interference in Thin Films: parallel and wedge-shaped films. Fringes of equal inclination (Haidinger Fringes); Fringes of equal thickness (Fizeau Fringes). Newton's Rings: measurement of wavelength and refractive index. Michelson's Interferometer: Construction and working. Idea of form of fringes (no theory needed), Determination of wavelength, Wavelength difference, Refractive index, and Visibility of fringes.

UNIT – IV

Diffraction: Fraunhofer diffraction: Single slit; Double Slit. Multiple slits & Diffraction grating. Fresnel Diffraction: Half-period zones. Zone plate. Fresnel Diffraction pattern of a straight edge, a slit and a wire using half-period zone analysis.

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UNIT-V

Polarization: Transverse nature of light waves. Plane polarized light – production and analysis. Circular and elliptical polarization. Hygiene's explanation of double refraction, positive and negative uniaxial crystals, quarter and half wave plates, types of polarized light, production and analysis of plane, circularly and elliptically polarized light, optical activity (Sections 20.9,20.17-20.20,20.24 Brijlal, Subramaniyam, & Avadhanulu and Ajoy Ghatak)

Reference Books:

1. Fundamentals of Optics, F.A Jenkins and H.E White, 1976, McGraw-Hill Principles of Optics, B.K. Mathur, 1995, Gopal Printing
2. Fundamentals of Optics, A. Kumar, H.R. Gulati and D.R. Khanna, 2011, R. Chand Publications
3. University Physics. F.W. Sears, M.W. Zemansky and H.D. Young. 13/e, 1986. Addison-Wesley

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

LECTURE PLAN
DEPARTMENT OF PHYSICS

STAFF NAME: Mrs.A.SAHANA FATHIMA
SUBJECT NAME: WAVES AND OPTICS SUB.CODE:16PHU401
SEMESTER: IV CLASS: II B.Sc (PHY)

S.No	Lecture Duration Period	Topics to be Covered	Support Material/Page Nos
		UNIT-I	
1	1 hr	Simple harmonic motion (SHM)	T2(66-69)
2	1 hr	Linearity and superposition principe	T1(22-24)
3	1 hr	Oscillations having equal frequencies	T2(71-72)
4	1 hr	Oscillations having different frequencies	T2(73-75)
5	1 hr	Superposition of two perpendicular harmonic oscillations,graphical and analytical method	T1(99-101)
6	1 hr	Lissajous Figures andtheir uses	T2(76,81-84)
7	1 hr	Wave motion –General	T1(139-140)
8	1 hr	Transverse waves on a string	T1(140-141)
9	1 hr	Travelling and standing waves on a string	T1(214-218)
10	1 hr	Normal Modes of a string	T1(103-104)
11	1 hr	Group velocity, Phase velocity. Plane waves	T1(364-367)
12	1 hr	Spherical waves, Wave intensity	T2(107-108)
13	1 hr	Revision	
	Total No of Hours Planned For Unit 1=13		
		UNIT-II	

1	1 hr	Sound waves	T1(160-163)
2	1 hr	Production and properties	T2(142-144)
3	1 hr	Intensity and loudness of sound	T2(154-156)
4	1 hr	Decibels. Intensitylevels	T1(322-324)
5	1 hr	Musical notes. musical scale	T1(249-251)
6	1 hr	Acoustics of buildings	T1(335-336)
7	1 hr	Wave Optics:Electromagnetic nature of light.	T3(664-665)
8	1 hr	Definition	
9	1 hr	Properties of wave front	T3(275-277)
10	1 hr	Huygens Principle	T3(261-262)
11	1 hr	REVISION	
	Total No of Hours Planned For Unit II=11		
		UNIT-III	
1	1 hr	Interference: Division of amplitude and division of wavefront	T3(278-279)
2	1 hr	Young's Double Slit experiment	T3(280-281)
3	1 hr	Lloyd's Mirror & Fresnel's Biprism	T3(292-295)
4	1 hr	Phase change on reflection	T3(281-282)
5	1 hr	Stokes' treatmentInterference in Thin Films	T3(310)
6	1 hr	parallel and wedge-shaped films	T3(321-322)
7	1 hr	Fringes of equal inclination	T3(342-343)
8	1 hr	Fringes of equal thickness(Fizeau Fringes)	T3(344-345)
9	1 hr	Newton's Rings: measurement of wavelength and refractive index	T3(327-332)
10	1 hr	Michelson's Interferometer: Construction and working	T3(345-346)
11	1 hr	Idea of form of fringes (no theory needed)	T3(346-347)
12	1 hr	Determination of wavelength	T3(349)

13	1 hr	Wavelength difference	T3(350)
14	1 hr	Refractive index, and Visibility of fringes.	T3(347-349)
15	1 hr	REVISION	
	Total No of Hours Planned For Unit III=15		
		UNIT-IV	
1	1 hr	Fraunhofer diffraction	T3(448-451)
2	1 hr	Single slit	T3(451-455)
3	1 hr	Double Slit	T3(465-468)
4	1 hr	Multiple slits & Diffraction grating	T3(476-479)
5	1 hr	Fresnel Diffraction: Half-period zones	T3(404-405)
6	1 hr	Zone plate	T3(410-412)
7	1 hr	Fresnel Diffraction pattern of a straight edge	T3(444-447)
8	1 hr	A slit and a wire using half-period zone analysis.	T3(480-481)
9	1 hr	REVISION	
	Total No of Hours Planned For Unit IV=9		
		UNIT-V	
1	1 hr	Transverse nature of light waves	T2(58-59)
2	1 hr	Plane polarized light	T2(564-565)
3	1 hr	production and analysis	T2(574-576)
4	1 hr	Circular and elliptical polarization	T2(588-591)
5	1 hr	Hygiene's explanation of double refraction	T2(577-578)
6	1 hr	Positive and negative uniaxial crystals	T2(580-582)
7	1 hr	Quarter and half wave plates, types of polarized light, production	T2(591-592)
8	1 hr	Analysis of plane, circularly and elliptically polarized light, optical activity	T2(592-594)

9	1 hr	REVISION	
10	1 hr	OLD QUESTION PAPER	
11	1 hr	OLD QUESTION PAPER	
12	1 hr	OLD QUESTION PAPER	
	Total No of Hours Planned for unit V=12		
Total Planned Hours	**60**		

TEXT BOOK

1. T1 -Waves and oscillations by N .Subrahmanyam & Brij lal,1974
2. T2 - Fundamental of optics,1996,M.G.Raj
3. T3 - A Text books of optics by N. Subrahmanyam & Brij lal,1966

REFERENCES

1. Fundamentals of Optics, A. Kumar, H.R. Gulati and D.R. Khanna, 2011, R. Chand Publications.
2. George Arfken (2012), University Physics. Academic Press.

UNIT – I

SYLLABUS

Superposition of Two Collinear Harmonic oscillations: Simple harmonic motion (SHM). Linearity and Superposition Principle. (1) Oscillations having equal frequencies and (2) Oscillations having different frequencies (Beats). Superposition of Two Perpendicular Harmonic Oscillations: Graphical and Analytical Methods. Lissajous Figures (1:1 and 1:2) and their uses. Waves Motion- General: Transverse waves on a string. Travelling and standing waves on a string. Normal Modes of a string. Group velocity, Phase velocity. Plane waves. Spherical waves, Wave intensity.

SIMPLE HARMONIC MOTION:

Simple harmonic motion is a type of vibratory motion in which the acceleration is proportional to the displacement and is always directed toward the position of equilibrium. For such a motion to take place the force acting on the body should be directed towards the fixed point and should also be proportional to the displacement i.e, the displacement from the fixed point. The function of the force is to bring the body back to its equilibrium position and hence this force is often termed as restoring force.

The backward and forward swing of a pendulum, the up and down motion of a weight hanging on a spring, and the twisting and untwisting motion of a body suspended by a wire are the examples of motions which are nearly simple harmonic.

Suppose a particle of mass m is executing simple harmonic motion. If y represents the displacement of the particle from equilibrium position at any instant t, the restoring force F acting on the particle would be given by

or $F = -Sy$ ---------------- (1)

Where S denotes the force constant of proportionality or stiffness. In Equation (1) the negative sign is used to reveal that the direction of the force is opposite to the direction of increasing displacement.

If $\frac{d2y}{dt2}$ represents the acceleration of the particle at time then equation (1) becomes as follows:

$m\,\frac{d^2y}{dt^2} = -Sy$

or $\frac{d^2y}{dt^2} + \frac{S}{m}y = 0$

But $\frac{S}{m} = \omega^2$

$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0$ ----------- (2)

Equation (2) is the general differential equation of motion of a simple harmonic oscillator.

In order to find a solution of this equation, Equation (2) is multiplied by $2\frac{dy}{dt}$ and we get

$$2\frac{dy}{dt} \cdot \frac{d2y}{dt2} ++ \omega^2\, 2y\frac{dy}{dt} = 0 \qquad \text{------------ (3)}$$

On integrating equation (3), we get

$$(\frac{dy}{dt})^2 = -\omega^2 y^2 + C \qquad \text{------------(4)}$$

Where C is the constant of integration whose value can be calculated by using the initial conditions.

If the displacement is maximum i.e, at y=a where a refers to the amplitude of the oscillating particle, then

$$\frac{dy}{dt} = 0$$

i.e, the particle is momentarily at rest and starts its journey in the backward direction.

On substituting y = a and $\frac{dy}{dt} = 0$ in equation (4), we get

$$C = a^2\omega^2$$

On substituting this value of C in equation (4), we get

$(\frac{dy}{dt})^2 = \omega^2(a^2 - y^2)$

Or $\frac{dy}{dt} = \omega\sqrt{a^2 - y^2}$ --------------- (5)

Equation (5) gives the velocity of the particle if it is executing simple harmonic motion at a time t, when the displacement = y

$\frac{dy}{\sqrt{a^2-y^2}} = \omega\, dt$ ---------------- (6)

On integrating equation (6), we get

$$\sin^{-1}\frac{y}{a} = \omega t + \phi$$

Or $y = a \sin(\omega t + \phi)$ ----------------- (7)

Where φ denotes another constant of integration.

In equation (7) the term $(\omega t + \phi)$ represents the total phase of the particle at time t and ϕ is termed as the initial phase or phase constant. I f the time has been recorded from the instant then y = 0 and increasing then ϕ + o.

THE RESULTANT OF TWO SIMPLE HARMONIC MOTION VIBRATIONS OF THE SAME FREQUENCY ACTING ALONG THE SAME LINE BUT DIFFERING IN PHASE :

Suppose the two simple harmonic vibrations of angular velocity ω acting along X-axis and having initial phases ϕ_1 and ϕ_2 are given as follows:

$$x_1 = a_1 \sin(\omega t + \phi_1) \quad \text{------------- (1)}$$

$$x_2 = a_2 \sin(\omega t + \phi_2) \quad \text{------------- (2)}$$

As the two vibrations are assumed to be of the same frequency, it means that ω is the same for both. The resultant can be calculated as well as geometrically.

The resultant displacement due to the two simple harmonic vibrations may be given as follows.

$$x = x_1 + x_2 = a_1 \sin(\omega t + \phi_1) + a_2 \sin(\omega t + \phi_2)$$

$$= \sin \omega t\,(a_1 \cos \phi_1 + a_2 \cos \phi_2) + \cos \omega t\,(a_1 \sin \phi_1 + a_2 \sin \phi_2) \quad \text{-------------(3)}$$

As the amplitudes a_1 and a_2 and angles ϕ_1 and ϕ_2 are constant, the coefficients of sin ωt and cos ωt in equation (3) can be substituted by R cos θ and R sin θ i.e,

$$a_1 \cos \phi_1 + a_2 \cos \phi_2 = R \cos \theta \quad \text{---------- (4)}$$

$$a_1 \sin \phi_1 + a_2 \sin \phi_2 = R \sin \theta \quad \text{---------- (5)}$$

Where $x = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta = R \sin(\omega t + \theta)$ ----------- (6)

Equation (5) gives the equation of the resultant simple harmonic vibration of amplitude R and initial phase θ. The value of R is obtained by squaring equations (4) and (5) and then adding to yield.

$$R^2 = R^2 \cos^2 \theta + R^2 \sin^2 \theta = a_1^2 + a_2^2 + 2a_1a_2 (\sin \phi_1 \sin \phi_2 + \cos \phi_1 \cos \phi_2)$$

$$= a_1^2 + a_2^2 + 2a_1a_2 \cos(\phi_2 - \phi_1)$$

$$= a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \quad \text{----------- (7)}$$

And $\tan \theta = \frac{R \sin \theta}{R \cos \theta} = \frac{a_1 \sin \phi 1 + a_2 \sin \phi 2}{a_1 \cos \phi 1 + a_2 \cos \phi 2}$

LISSAJOUS FIGURES :

If a particle is acted upon simultaneously by two simple harmonic motions at right angles of each other, the resultant motion of particle traces a curve. This is called Lissajous figure. The nature or shape of curve traced out is depend upon:

(i) Time periods (or frequencies)

(ii) Amplitudes, and

(iii) Phase difference between two constituent vibrations.

Lissajous figures are useful for determining the ratio of the time periods of two vibrations and are also useful for comparing the frequencies of two turning forks.

THE RESULTANT OF TWO SIMPLE HARMONIC MOTIONS OF EQUAL PERIOD ACTING AT RIGHT ANGLES TO ONE ANOTHER :

If x is the displacement of the vibrating particle at any instant t, a the amplitude of vibration, ω the angular velocity and ϕ the initial phase, then the equation of a simple harmonic motion may be given as follows:

$$x = a \sin(\omega t + \phi)$$

Suppose the two simple harmonic vibrations having the same period are taking place along the X-axis and Y vibrations respectively and are represented by Or

$$x = a \sin(\omega t + \phi_1) \quad \text{----------- (1)}$$

$$y = b \sin(\omega t + \phi_2) \quad \text{----------- (2)}$$

Where b denotes the amplitude of the vibration along the Y-axis, ϕ_1 and ϕ_2 denote the initial phases of X and Y vibrations respectively. The phase difference between the two vibrations would be denotes as

$$\phi = \phi_2 - \phi_1$$

From equation (1), we get

$$\frac{x}{a} = \sin \omega t \cos \phi_1 + \cos \omega t \sin \phi_1 \quad \text{------------ (3)}$$

From equation (2), we get

$$\frac{y}{b} = \sin \omega t \cos \phi_2 + \cos \omega t \sin \phi_2 \quad \text{------------ (4)}$$

On multiplying equation (3) by $\sin \phi_2$ and equation (4) by $\sin \phi_1$ and subtracting, we get

$$(\frac{x}{a} \sin \phi_2 - \frac{y}{b} \sin \phi_1) = \sin \omega t \ (\cos \phi_1 \sin \phi_2 - \cos \phi_2 \sin \phi_1)$$

$$(\frac{x}{a} \sin \phi_2 - \frac{y}{b} \sin \phi_1) = \sin \omega t \sin (\phi_2 - \phi_1) \quad \text{------------- (5)}$$

Similarly on multiplying equation (4) by cos ϕ_1 and equation (3) by cos ϕ_2 and subtracting, we get

$(\frac{y}{b}\cos\phi_1 - \frac{x}{a}\cos\phi_2) = \cos\omega t\,(\sin\phi_2\cos\phi_1 - \cos\phi_2\sin\phi_1)$

$(\frac{y}{b}\cos\phi_1 - \frac{x}{a}\cos\phi_2) = \cos\omega t\sin(\phi_2 - \phi_1)$ ------------- (6)

On squaring equations (5) and (6) and then adding, we get

$(\frac{x}{a}\sin\phi_2 - \frac{y}{b}\sin\phi_1)^2 + (\frac{y}{b}\cos\phi_1 - \frac{x}{a}\cos\phi_2)^2 = \sin^2(\phi_2 - \phi_1)\,[\sin^2\omega t + \cos^2\omega t]$

$(\frac{x}{a}\sin\phi_2 - \frac{y}{b}\sin\phi_1)^2 + (\frac{y}{b}\cos\phi_1 - \frac{x}{a}\cos\phi_2)^2 = \sin^2(\phi_2 - \phi_1)$

Or $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}(\sin\phi_2\sin\phi_1 + \cos\phi_2\cos\phi_1) = \sin^2(\phi_2 - \phi_1)$

Or $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos(\phi_2 - \phi_1) = \sin^2(\phi_2 - \phi_1)$

On substituting $\phi_2 - \phi_1 = \phi$ in the above equation, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi = \sin^2 \quad \text{------------- (7)}$$

Equation (7) represents the equation of an ellipse whose major and minor axes get inclined to the co-ordinate axes. This ellipse can be inscribed in a rectangle whose sides are taken as 2a and 2b.

Important cases:

(i) If $\phi = 0$, $\cos\phi = 1$ and $\sin\phi = 0$.

Then equation (7) becomes as follows:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

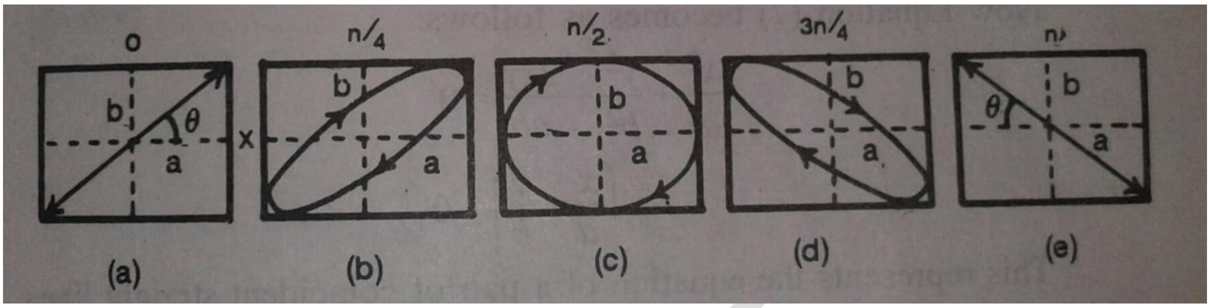
Or $$(\frac{x}{a} - \frac{y}{b})^2 = 0$$

This represents the equation of a pair of coincident straight lines which are lying in the first and third quadrant as shown in figure.

The straight line gets inclined to the X-axis at angle θ which is given by

$$\theta = \tan^{-1}(\frac{b}{a})$$

(ii) If $\phi = \frac{\pi}{4}$, then cos $\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and sin $\frac{\pi}{4}$ =



$\frac{1}{\sqrt{2}}$

Now equation (7) becomes as follows:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{\sqrt{2xy}}{ab} = \frac{1}{2}$$

This represents the equation of an oblique ellipse as shown in figure.

THE RESULTANT OF TWO RECTANGULAR SIMPLE HARMONIC MOTIONS HAVING AMPLITUDES AND PERIODS IN THE RATION 1:2 AND THE PHASE DIFFERENCE 90 ^{0} :

Suppose a particle is subjected to two simple harmonic motions of periods as well as amplitudes in ratio 1 : 2, and acting along the axes of x and y respectively. If the x-motion leads over the y-motion by 90^o i.e. $\pi/2$ radian in phase, then the equations of these motions would be

$$x = a \sin\left(1\ \omega t + \frac{\pi}{2}\right) \qquad \text{-------------- (1)}$$

and $$y = 2a \sin □□ \qquad \text{-------------- (2)}$$

Where a and 2a denote the amplitudes, and 2π/2□ and 2π/□ the periods respectively.

It is possible to get the equation of the resultant path of the particle by eliminating t between equations (1) and (2). From equation (1), we get

$$\frac{□}{□} = \sin\left(2\ □□ + \frac{□}{2}\right)$$

$$= \cos 2\ □□ = 1 - 2 \sin^2 □□$$

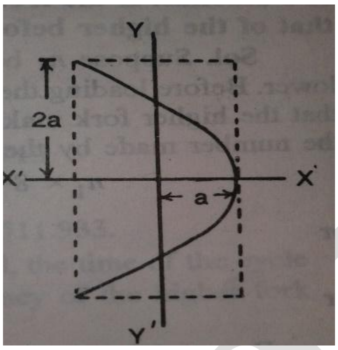
But from equation (2), sin □□ = y/2a

$$\therefore \frac{□}{□} = 1 - \frac{□^2}{2□^2}$$

Or $$\frac{□^2}{2□^2} = 1 - \frac{□}{□}$$

Or $y^2 = 2a\,(a-x)$

This has been the equation of a parabola as shown in figure.



We will now consider the case when the y-motion leads over the x-motion by π/2. Now the equations of motions will be

$$x = a \sin 2\,□□ \qquad \text{----------- (3)}$$

and $$y = 2a \sin \left(□□ + \frac{□}{2}\right) \qquad \text{----------- (4)}$$

From equation (3), we get

$$\frac{□}{□} = 2 \sin □□ \cos □□$$

$$= \sqrt{(1 - □□□^2\,□□)}\cos □□$$

But from equation (4), $\frac{□}{2□} = \sin\left(□□ + \frac{□}{2}\right) = \cos □□$

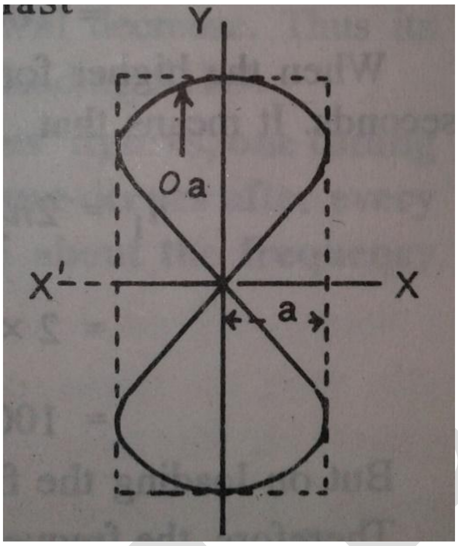
On substituting for cos □□ in the last equation we obtain

$$\frac{□}{□} = 2\sqrt{\left(1 - \frac{□}{4□^2}\right)}\,\frac{□}{2□}$$

Or $$\frac{□^2}{a^2} = \frac{□^2}{□^2}\left(1 - \frac{□^2}{4□^2}\right)$$

Or $$x^2 + y^2\left(\frac{□^2}{4□^2} - 1\right) = 0$$

This has been the equation of the figure of ‘8’ as shown in figure.



OSCILLATIONS HAVING DIFFERENT FREQUENCIES (BEATS) :

Consider two wave trains of frequencies n_1 and n_2 where $(n_1 - n_2)$ is small. Let a and b the amplitudes of the waves respectively. For the sake of simplicity, it is assumed that two waves are in phase at any point in the medium at t=0. The displacement y_1 and $y_2$due to each wave are given by

$$y_1 = a \sin \omega_1 t$$

$$y_2 = b \sin \omega_2 t$$

Here $\omega_1 = 2\pi\, n_1$

$$\omega_2 = 2\pi\, n_2$$

$\therefore$ $y_1 = a \sin 2\pi\, n_1\, t$ --------- (1)

And $y_2 = b \sin 2\pi\, n_2\, t$ ---------- (2)

The resultant displacement is given by

$y = y_1 + y_2$

$y = a \sin 2\pi\, n_1\, t + b \sin 2\pi\, n_2\, t$

$y = a \sin 2\pi\, n_1\, t + b \sin 2\pi\, [\, n_1 - (n_1 - n_2)\,]\, t$

$y = a \sin 2\pi\, n_1\, t + b\, [\, \sin 2\pi\, n_1\, t \cos 2\pi\, (n_1 - n_2)\, t - \cos 2\pi\, n_1\, t \sin 2\pi\, (n_1 - n_2)\, t\,]$

$y = a \sin 2\pi\, n_1\, t + b \sin 2\pi\, n_1\, t \cos 2\pi\, (n_1 - n_2)\, t - b \cos 2\pi\, n_1\, t \sin 2\pi\, (n_1 - n_2)\, t\,]$

$y = \sin 2\pi\, n_1\, t\, [\, a + b \cos 2\pi\, (n_1 - n_2)\, t\,] - \cos (2\pi\, n_1 t)\, [b \sin 2\pi\, (n_1 - n_2)\, t\,]$

Take $a + b \cos 2\pi\, (n_1 - n_2)\, t = A \cos \theta$

And $b \sin 2\pi\, (n_1 - n_2)\, t = A \sin \theta$

$$\therefore \qquad y = A \sin (2\pi n_1 t - \theta) \qquad \text{----------} (3)$$

Here
$$\tan\theta = \frac{\square \sin 2\square\,(\square_1 - \square_2)\,\square}{\square + \square \cos 2\square\,(\square_1 - \square_2)\square} \qquad \text{-----------} (4)$$

And
$$\square = \sqrt{\square^2 + \square^2 + 2\square\square \cos 2\square\,(\square_1 - \square_2)\square} \qquad \text{-----------} (5)$$

From equation (4), it is evident that the phase angle θ changes with respect to time. Similarly from equation (5) the amplitude of the resultant vibration also charges with time.

LINEARITY AND SUPERPOSITION PRINCIPL:

From a general simple harmonic oscillation wave equation,

$$\frac{d^2y}{dt^2} = -\,\square^2 y$$

We may add some constants,

$$\frac{\square^2 y}{d\square^2} = -\,\square^2 y + Ay^2 + By^2 + Cy^2 \ldots\ldots\ldots.. \qquad \text{------------} (1)$$

If A=B=C=0, then the equation (1) becomes,

$$\frac{\square^2 y}{d\square^2} = -\,\square^2 y \qquad \text{-----------} (2)$$

The above equation is known as linear homogeneous equation.

Let us consider, y_1 is the first solution of equation (2) at time t_1 and y_2 is the second solution of equation (2) at time t_2

$$\frac{\square^2 y}{d\square^2} = -\,\square^2 y_1 + Ay_1^2 + By_1^3 + Cy_1^4 \ldots\ldots\ldots.. \qquad \text{------------} (3)$$

$$\frac{\square^2 y}{d\square^2} = -\,\square^2 y_2 + Ay_2^2 + By_2^3 + Cy_2^4 \ldots\ldots\ldots.. \qquad \text{------------} (4)$$

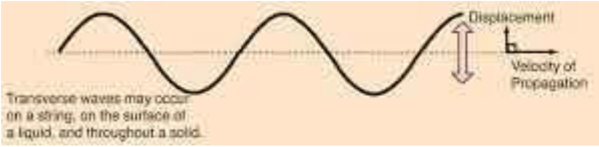
The equation for the resultant displacement

$$\frac{\square^2 y}{d\square^2} = \frac{\square^2}{d\square^2}(y_1 + y_2)$$

$$\frac{\square^2 y}{d\square^2} = \frac{\square^2}{d\square^2}(y_1 + y_2) = -\,\square^2 (y_1 + y_2) + A(y_1 + y_2)^2 + B(y_1 + y_2)^3 + C(y_1 + y_2)^4 \ldots\ldots.. \qquad \text{----------} (5)$$

TRANSVERSE WAVES:

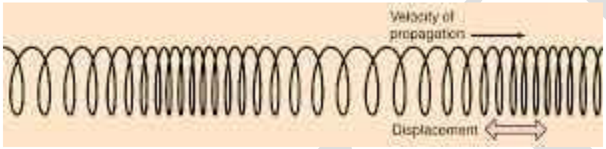
For transverse waves the displacement of the medium is perpendicular to the direction of propagation of the wave. A ripple on a pond and a wave on a string are easily visualized transverse waves.



Transverse waves cannot propagate in a gas or a liquid because there is no mechanism for driving motion perpendicular to the propagation of the wave.

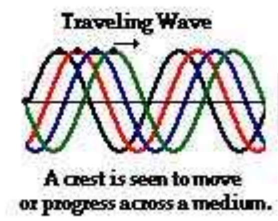
LONGITUDINAL WAVES:

In longitudinal waves the displacement of the medium is parallel to the propagation of the wave. A wave in a "slinky" is a good visualization. Sound waves in air are longitudinal waves.



TRAVELLING WAVES :

A mechanical wave is a disturbance that is created by a vibrating object and subsequently travels through a medium from one location to another, transporting energy as it moves. The mechanism by which a mechanical wave propagates itself through a medium involves particle interaction; one

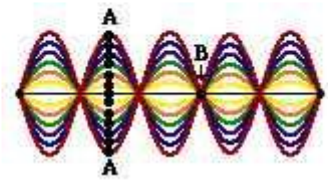


particle applies a push or pull on its adjacent neighbour, causing a displacement of that neighbour from the equilibrium or rest position. As a wave is observed traveling through a medium, a crest is seen moving along from particle to particle. This crest is followed by a trough that is in turn followed by the next crest. In fact, one would observe a distinct wave pattern (in the form of a sine wave) traveling through the medium. This sine wave pattern continues to move in uninterrupted fashion until it encounters another wave along the medium or until it encounters a boundary with another medium. This type of wave pattern that is seen traveling through a medium is sometimes referred to as a traveling wave.

Traveling waves are observed when a wave is not confined to a given space along the medium. The most commonly observed traveling wave is an ocean wave. If a wave is introduced into an elastic cord with its ends held 3 meters apart, it becomes confined in a small region. Such a wave has only 3 meters along which to travel. The wave will quickly reach the end of the cord, reflect and travel back in the opposite direction. Any reflected portion of the wave will then interfere with the portion of the wave incident towards the fixed end. This interference produces a new shape in the medium that seldom resembles the shape of a sine wave. Subsequently, a traveling wave (a repeating pattern that is observed to move through a medium in uninterrupted fashion) is not observed in the cord. Indeed there are traveling waves in the cord; it is just that they are not easily detectable because of their interference with each other. In such instances, rather than observing the pure shape of a sine wave pattern, a rather irregular and non-repeating pattern is produced in the cord that tends to change appearance over time. This irregular looking shape is the result of the interference of an incident sine wave pattern with a reflected sine wave pattern in a rather non-sequenced and untimely manner. Both the incident and reflected wave patterns continue their motion through the medium, meeting up with one another at different locations in different ways. For example, the middle of the cord might experience a crest meeting a half crest; then moments later, a crest meeting a quarter trough; then moments later, a three-quarters crest meeting a one-fifth trough, etc. This interference leads to a very irregular and non-repeating motion of the medium. The appearance of an actual wave pattern is difficult to detect amidst the irregular motions of the individual particles.

STANDING WAVES:

It is however possible to have a wave confined to a given space in a medium and still produce a regular wave pattern that is readily discernible amidst the motion of the medium. For instance, if an elastic rope is held end-to-end and vibrated at just the right frequency, a wave pattern would be produced that assumes the shape of a sine wave and is seen to change over time. The wave pattern is only produced when one end of the rope is vibrated at just the right frequency. When the proper frequency is used, the interference of the incident wave and the reflected wave occur in such a manner that there are specific points along the medium that appear to be standing still. Because the observed wave pattern is characterized by points that appear to be standing still, the pattern is often called a standing wave pattern. There are other points along the medium whose displacement changes over time, but in a regular manner. These points vibrate back and forth from a positive displacement to a negative displacement; the vibrations occur at regular time intervals such that the motion of the medium is regular and repeating. A pattern is readily observable.



The diagram at the right depicts a standing wave pattern in a medium. A snapshot of the medium over time is depicted using various colors. Note that point A on the medium moves from a maximum positive to a maximum negative displacement over time. The diagram only shows one-half cycle of the motion of the standing wave pattern. The motion would continue and persist, with point A returning to the same maximum positive displacement and then continuing its back-and-forth vibration between the up to the down position. Note that point B on the medium is a point that never moves. Point B is a point of no displacement. Such points are known as nodes and will be discussed in more detail later in this lesson. The standing wave pattern that is shown at the right is just one of many different patterns that could be produced within the rope.

PLANE WAVES:

In a plane wave disturbances travel in single direction.

<u>Plane waves examples :</u>

For example when a string is fixed at both ends and the string is plucked at one end ,then transverse waves are generated in the string in which particles of the medium vibrate in one direction.So the transverse waves are plane waves.It is not possible in practice to have a true plane wave.

A finite part of large spherical wave coming from the sun is considered a plane wave.

SPHERICAL WAVES:

A wave in which the disturbance a propagated outward in all directions from the source of wave is called a spherical wave.

<u>Spherical waves examples:</u>

The light waves are the example of spherical waves.

The light waves produced by a single light source ,are spherical waves.During the propagation of light waves ,the spherical wave fronts spread out in all directions.

PHASE VELOCITY:

Phase velocity is a concept discussed in propagation of waves. The phase velocity of a wave is the velocity of a "phase" which propagates. For clarification, assume a crest of a wave, which is travelling in the x direction of the axis. The phase velocity is the x component of the velocity of the selected point at the crest. This also can be obtained by dividing the wavelength by the time taken for a single wavelength to pass a selected point. This time is equal to the period of the oscillation, which is causing the wave. Now consider a standard sine wave A sin (wt – kx), where w is the angular velocity of the source, t is the time, k is the wave number (number of complete wavelengths per length of 2π), and x is the position on the x-axis. At the crest, wt – kx is equal to zero. Therefore, the phase velocity (x/t) is equal to w / k. mathematically, the value p=wt – kx is the phase of the wave.

GROUP VELOCITY:

Group velocity is discussed under superposition of waves. To understand group velocity one must first understand the concept of superposition. When two waves intercept each other in space, the resultant oscillation is somewhat complex than the sine behavior. Particle at a point oscillates with varying amplitudes. The maximum amplitude is the unison of the two amplitudes of the original waves. The minimum amplitude is the minimum difference between the two original amplitudes. If the two amplitudes are equal, the maximum is twice the amplitude and the minimum is zero. For the sake of clarity, let us assume that the two modulated waves are of the same amplitude and different frequencies. This causes the wave with the higher frequency to be enveloped in the wave with the lower frequency. This causes a group of waves packed in an envelope. The velocity of this envelope is the group velocity of the wave. It must be noted that, for a standing wave, the group velocity is zero. For the group velocity to be zero both of the waves have to be of the same frequency and they must have opposite directions of travel.

WAVE INTENSITY :

In general, an Intensity is a ratio. For example, pressure is the intensity of force as it is force/area. Also, density (symbol ρ) is the intensity of mass as it is mass/volume.

The Intensity of waves (called Irradiance in Optics) is defined as the power delivered per unit area.

The unit of Intensity will be W.m-2.

definition of intensity

$$\text{Intensity} = \frac{\text{power}}{\text{area}} \quad \text{in W.m}^{-2}$$
$$= \frac{\text{energy}}{\text{time} \times \text{area}}$$
$$= \frac{\text{energy} \times \text{length}}{\text{time} \times \text{volume}}$$
$$\text{Intensity} = \left(\frac{\text{energy}}{\text{volume}}\right) \times (\text{wave speed})$$

POSSIBLE QUESTIONS

PART B

1. Define Simple Harmonic Motion
2. Define frequency.
3. What is Lissajous Figures?
4. What is beats?
5. Define transverse wave.
6. Define longitudinal wave.
7. What is standing wave?
8. Define group velocity.
9. Define wave velocity.
10. Explain plain waves.
11. Explain spherical waves
12. Define wave intensity.

PART C

1. Explain in detail about the simple harmonic oscillations have same frequency.
2. Explain about the oscillations having different frequencies.
3. Discuss about the superposition of two perpendicular harmonic oscillations with Lissajous Figures of ratio 1:1
4. Discuss about the superposition of two perpendicular harmonic oscillations with Lissajous Figures of ratio 1:2
5. Write a note on (i) Transverse wave (ii) Longitudinal waves (iii) standing waves
6. Write a note on (i) Group velocity (ii) Phase velocity (iii) Wave intensity.

UNIT-II

SYLLABUS

Sound: Sound waves, production and properties. Intensity and loudness of sound. Decibels. Intensity levels. musical notes. musical scale. Acoustics of buildings (General idea). Wave Optics: Electromagnetic nature of light. Definition and Properties of wave front. Huygens Principle.

SOUND WAVES:

A sound wave is the pattern of disturbance caused by the movement of energy travelling through a medium (such as air, water, or any other liquid or solid matter) as it propagates away from the source of the sound. The source is some object that causes a vibration, such as a ringing telephone, or a person's vocal chords. The vibration disturbs the particles in the surrounding medium; those particles disturb those next to them, and so on. The pattern of the disturbance creates outward movement in a wave pattern, like waves of seawater on the ocean. The wave carries the sound energy through the medium, usually in all directions and less intensely as it moves farther from the source.

PRODUCTION OF SOUND WAVES:

Sound is a sensation or feeling that we hear. We produce sounds by doing something. The motion of materials or objects causes vibrations. A sound originates in the vibration of an object, which makes the air or another substance around the object vibrate. The vibration of the air moves outward in all directions in the form of a wave. The following are examples of how certain sounds are produced.

Human Voice:

The human voice is produced in the larynx, which is a part of the throat. There are two small pieces of tissue that stretch across the larynx with a small opening between them, these tissues are our vocal cords. As we speak, muscles in our larynx tighten the vocal cords making this small opening become narrower. When air from our lungs passes through the tightened cords a vibration is produced. This vibration produces vocal sounds. The tighter the vocal cords, the more rapidly the vocal cords vibrate and the higher the sounds that are produced. This is what causes the human voices to have different pitches.

Animal Sounds:

Animals also produce sounds. Almost all mammals, birds, and frogs have vocal cords or similar structures, which allow them to produce sounds in a similar way to humans. However, many other animals produce distinctly different sounds. For example, bees buzz as they fly because of the rapid movement of their wings. Their wings make the air vibrate producing a buzzing sound. A cricket produces a singing type sound as it scrapes parts of its front wings together. Some types of shellfish produce clicks by tapping their claws together.

Musical Sounds:

Musical instruments produce many different sounds in various ways. There are three categories of musical instruments, percussion, string, and wind. Some instruments need to be struck by an object in order to produce a sound, these are called percussion instruments. For example when the membrane of a drum is hit the membrane vibrates, producing a sound, or when a bar of a xylophone is struck, a sound is produced. Each bar of a xylophone produces a different note when struck. String instruments, such as a harp or violin, produce sounds when one or more of their strings are plucked, causing them to vibrate. This vibration causes parts of the body of the instrument to vibrate, creating sound waves in the air. The pitch of a stringed instrument depends upon the string's thickness, its length, the distance stretched, and the number of times it vibrates. Wind instruments, such as a flute or trumpet produce sound when a column of air inside the instrument vibrates. For example, with a trumpet it is the vibrating lips of the player which makes the air column vibrate.8 Sounds produced by musical instruments are usually pleasing for us to hear. "A musical sound is a regular vibration."

Noise:

Humans, animals, and instruments are not the only sounds we hear, many of us come across various other sounds or noise every day. For example thunder is caused when lightning heats the air, causing the air to vibrate. A car makes a rather loud noise, which is produced when the engine vibrates, causing the other parts of the car to vibrate. These types of noises are produced by irregular vibrations occurring at irregular intervals. This is what makes noise a rather unpleasant sound.

PROPERTIES OF SOUND:

Six Basic Properties of Sound are,

(i) Frequency/Pitch:

Frequency refers to how often something happens -- or in our case, the number of periodic, compression-rarefaction cycles that occur each second as a sound wave moves through a medium -- and is measured in Hertz (Hz) or cycles/second. The term pitch is used to describe our perception of frequencies within the range of human hearing.

(ii) Amplitude/Loudness:

Amplitude/Loudness refer to how loud or soft the sound.

(iii) Spectrum/Timbre:

Timbre (pronounced TAM-burr) refers to the characteristic sound or tone color of an instrument. A violin has a different timbre than a piano.

(iv)Duration:

Duration refers to how long a sound lasts.

(v) Envelope:

Envelope refers to the shape or contour of the sound as it evolves over time. A simple envelope consists of three parts: attack, sustain, and decay. An acoustic guitar has a sharp attack, little sustain and a rapid decay. A piano has a sharp attack, medium sustain, and medium decay. Voice, wind, and string instruments can shape the individual attack, sustain, and decay portions of the sound.

(vi) Location:

Location describes the sound placement relative to our listening position. Sound is perceived in three dimensional space based on the time difference it reaches our left and right eardrums.

SOUND INTENSITY:

Sound intensity is defined as the sound power per unit area. The usual context is the measurement of sound intensity in the air at a listener's location. The basic units are watts/m

^{2}or watts/cm^{2}. Many sound intensity measurements are made relative to a standard threshold of hearing intensity I_0 :

$$I_0 = 10^{-12} \text{ watts/m}^2 = 10^{-16} \text{ watts/cm}^2$$

The most common approach to sound intensity measurement is to use the decibel scale:

$$I(\text{dB}) = 10 \log_{10} \left[\frac{I_O}{I}\right] \quad \text{(intensity in decibles)}$$

Decibels measure the ratio of a given intensity I to the threshold of hearing intensity , so that this threshold takes the value 0 decibels (0 dB). To assess sound loudness, as distinct from an objective intensity measurement, the sensitivity of the ear must be factored in.

LOUDNESS OF SOUND:

Loudness of sound depends upon the intensity of sound. It is found that.

$$L \propto \text{Log } I$$

i.e. greater the amplitude, greater will be the intensity ($I \propto A2$) and so louder will be the sound.

The Unit of loudness is decibels (dB) and-

$$L = 10 \text{ Log10}(I/Io) \text{ in dB}$$

Here Io is constant i.e. minimum intensity (=10^{-12} W/m2) just audible at intermediate frequencies.

The loudness of normal talks is about 60 dB.

DECIBEL:

Decibel (dB), unit for expressing the ratio between two physical quantities, usually amounts of acoustic or electric power, or for measuring the relative loudness of sounds. One decibel (0.1 bel) equals 10 times the common logarithm of the power ratio. Expressed as a formula, the intensity of a sound in decibels is 10 log10 (S1/S2), where S1 and S2 are the intensity of the two sounds; i.e., doubling the intensity of a sound means an increase of a little more than 3 dB. In ordinary usage, specification of the intensity of a sound implies a comparison of the intensity of the sound with that of a sound just perceptible to the human ear. For example, a 60-dB, or 6-bel, sound, such as normal speech, is six powers of 10 (i.e., 106, or 1,000,000) times more intense than a barely detectable sound, such as a faint whisper, of 1 dB. Decibels are also used more generally to express the logarithmic ratio of two magnitudes of any unit, such as two electric voltages or currents (or analogous acoustic quantities). In cases where the ratio is of a squared quantity, 1 dB equals 20 times the common logarithm of the ratio.

The term bel is derived from the name of Alexander Graham Bell, inventor of the telephone. The unit decibel is used because a one-decibel difference in loudness between two sounds is the smallest difference detectable by human hearing.

INTENSITY LEVELS:

Another quality described by a decibel level is sound intensity, which is the rate of energy flow across a unit area. The reference for measuring sound intensity level is Io = 10^{-12} Watt/m2, and the sound intensity level (IL or Li,) is defined as:

$$L_i = 10 \log_{10} [\frac{I_O}{I}]$$

For a free progressive wave in air (e.g., a plane wave travelling down a tube or a spherical wave travelling outward from a source), sound pressure level and sound intensity level are nearly equal (L_P - L_i) This is not true in general, however, because sound waves from many directions contribute to sound pressure at a point. When we speak of simply sound level, we nearly always mean sound pressure level, L_P, since that is what is indicated by our sound-measuring instruments.

MUSICAL SCALE:

In music theory, a scale is any set of musical notes ordered by fundamental frequency or pitch. A scale ordered by increasing pitch is an ascending scale, and a scale ordered by decreasing pitch is a descending scale.

MUSICAL NOTE:

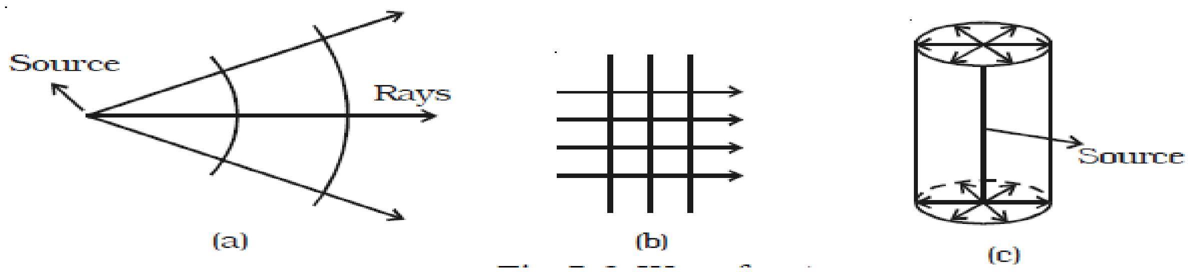
In music, a note is the pitch and duration of a sound, and also its representation in musical notation (♪, ♩). A note can also represent a pitch class.

WAVE FRONT:

(Fig.1)

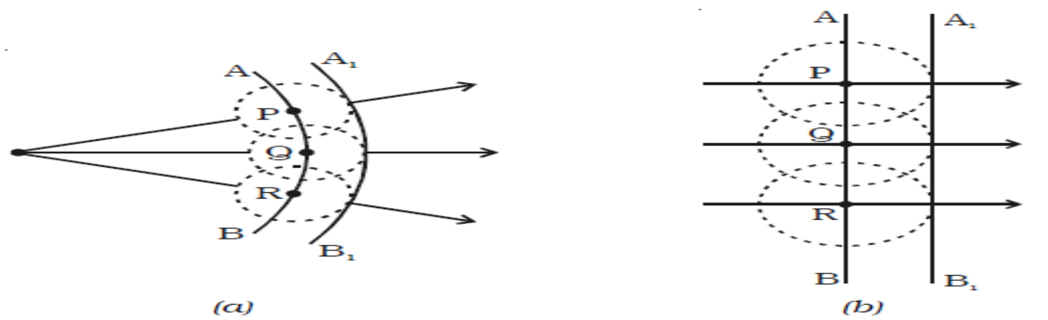
When a stone is dropped in a still water, waves spread out along the surface of water in all directions with same velocity. Every particle on the surface vibrates. At any instant, a photograph of the surface of water would show circular rings on which the disturbance is maximum (Fig.1). It is clear that all the particles on such a circle are vibrating in phase, because these particles are at the same distance from the source. Such a surface which envelopes the particles that are in the same state of vibration is known as a wave front. The wave front at any instant is defined as the locus of all the particles of the medium which are in the same state of vibration.

A point source of light at a finite distance in an isotropic medium emits a spherical wave front (Fig 2a). A point source of light in an isotropic medium at infinite distance will give rise to plane wavefront (Fig 2b). A linear source of light such as a slit illuminated by a lamp, will give rise to cylindrical wavefront (Fig 2c).



HUYGEN'S PRINCIPLE :

Huygen's principle helps us to locate the new position and shape of the wavefront at any instant, knowing its position and shape at any previous instant. In other words, it describes the progress of a wave front in a medium. Huygen's principle states that, (i) every point on a given wave front may be considered as a source of secondary wavelets which spread out with the speed of light in that medium and (ii) the new wavefront is the forward envelope of the secondary wavelets at that instant. Huygen's construction for a spherical and plane wavefront is shown in Fig.3a. Let AB represent a given wavefront at a time $t = 0$. According to Huygen's principle, every point on AB acts as a source of secondary wavelets which travel with the speed of light c. To find the position of the wave front after a time t, circles are drawn with points P, Q, R ... etc as centres on AB and radii equal to ct. These are the traces of secondary wavelets. The arc A1B1 drawn as a forward envelope of the small circles is the new wavefront at that instant. If the source of light is at a large distance, we obtain a plane wave front A1 B1 as shown in Fig.



POSSIBLE QUESTIONS

PART B

1. What is sound wave?
2. Define sound intensity.
3. Define decibels.
4. Write a note on musical note.
5. What is musical scale?
6. Define wave font.
7. Define frequency.
8. Write any two properties of waves.

PART C

1. Discuss about the sound waves , production and properties.
2. Explain about the electromagnetic nature of the light.
3. Explain about the Huygen's principle.
4. Write a note on (i) decibels (ii) loudness of sound (iii) musical note.
5. Write a note on (i) musical scale (ii) acoustics of building.

UNIT-III

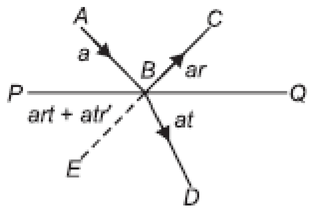
SYLLABUS

Interference: Interference: Division of amplitude and division of wavefront. Young's Double Slit experiment. Lloyd's Mirror & Fresnel's Biprism.Phase change on reflection: Stokes' treatment. Interference in Thin Films: parallel and wedge-shaped films. Fringes of equal inclination (Haidinger Fringes); Fringes of equal thickness (Fizeau Fringes). Newton's Rings: measurement of wavelength and refractive index. Michelson's Interferometer: Construction and working. Idea of form of fringes (no theory needed), Determination of wavelength, Wavelength difference, Refractive index, and Visibility of fringes.

PHASE CHANGE ON REFLECTION STOKE'S TREATMENT:

When a ray of light is reflected at the surface of a medium which is optically denser than the medium through which the ray is travelling, a change of phase equal to πor *a* path difference $\frac{\lambda}{2}$ is introduced.

When reflection takes place at the surface of a rarer medium, no change in phase or path-difference takes place.



Let *PQ* be the surface separating the denser medium below it from the rarer medium above it as shown in Fig. A ray of light *AB* of amplitude *a*incident on this surface is partly reflected along *BC* and partly refracted into the denser medium along *BD*. If *r* is the coefficient of reflection at the surface of a denser medium, i.e., the fraction of the incident light which is reflected, then

Amplitude of the ray *BC* = *ar*

If '*t*' is the coefficient of transmission from the rarer into the denser medium i.e., the fraction of the incident light transmitted, then

Amplitude of the refracted ray *BD* = *at* if there is no absorption of light, then

$$ar + at = a$$

or $r + t = 1$

If the reflected and the refracted rays are reversed the resultant should have the same amplitude '*a*' as that of the incident ray. When *CB* is reversed it is partly reflected along *BA* and partly refracted along *BE*.

The amplitude of the refracted ray along *BE* = *art* Similarly when the ray *DB* is reversed it is partly refracted along *BA* and partly reflectedalong *BE*. IF *r*' is the coefficient of reflection at the surface of a rarer medium, thenAmplitude of the reflected ray along *BE* = *atr*'

The two amplitudes along *BA* will combine together to produce the original amplitude, only if the total amplitude along *BE* is zero.

$$art + ar't = 0$$

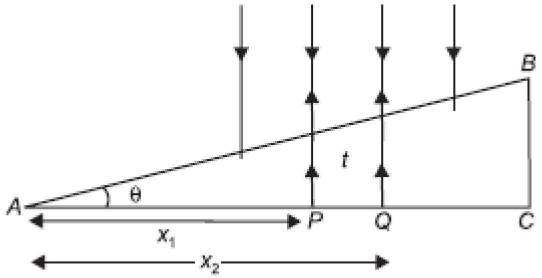
OR $r = -r'$

The negative sign shows that when one ray has a positive displacement the other has a negative displacement. Hence the two rays, one reflected on reaching a denser medium and the other reflected on reaching a rarer medium, differ in phase by πfrom each other. This explains the presence of a central dark spot in Newton's rings and is also responsible for the reversal of the condition of darkness and brightness produced in the reflected and transmitted systems in colours of thin films and in the fringes produced by Lloyd's single mirror.

INTERFERENCE IN THIN FILMS:

Newton and Hooke observed and developed the interference phenomenon due to multiple reflections from the surface of thin transparent materials. Everyone is familiar with the beautiful colours produced by a thin film of oil on the surface of water and also by the thin film of a soap bubble. Hooke observed such colours in thin films of mica and similar thin transparent plates. Newton was able to show the interference rings when a convex lens was placed on a plane glass-plate. Young's was able to explain the phenomenon on the basis of interference between light reflected from the top and the bottom surface of a thin film. It has been observed that interference in the case of thin films takes place due to (1) reflected light and (2) transmitted light.

FRINGES PRODUCED BY WEDGE SHAPED FILMS:



Let *ABC* be a wedge-shaped film of refractive index μ, having a very small angle at *A*, as shown in Fig. If a parallel beam of monochromatic light is allowed to fall on the upper surface and the surface is viewed by reflected light, then alternate dark and bright fringes become visible. Consider a point *P* at a distance *x*1 from *A*where the thickness of the film is *t*. When light isincident normally the total path-difference betweenthe light reflected at *R* from the upper face *AB* and that reflected at *P* from the lower face *AC* is 2μt + $\frac{\lambda}{2}$as an additional path-difference of $\frac{\lambda}{2}$is produced in the beam reflected from the upper face *AB* at *R* where reflection takes place at the surface of a denser medium. The point *P* will appear dark and a dark band will be observed across the wedge, if

$$2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad 2\mu t = n\lambda$$

The point *P* will appear bright and a bright band will be observed across the wedge, if

$$2\mu t + \frac{\lambda}{2} = n\lambda \quad \text{or} \quad 2\mu t = (2n-1)\frac{\lambda}{2}$$

If the *n*th dark fringe is formed at *P*, then

$$2\mu t = n\lambda$$

But

$$\frac{t}{x_1} = \theta$$

Or $$t = x_1\theta$$

Similarly for the (*n* + 1) the dark band, which is formed at *Q* at a distance *x*2 from *A*, we have

$$2\mu\theta x_2 = (n+1)\lambda$$

Subtracting (*i*) from (*ii*), we have

$$2\mu\theta\,(x2 - x1) = \lambda$$

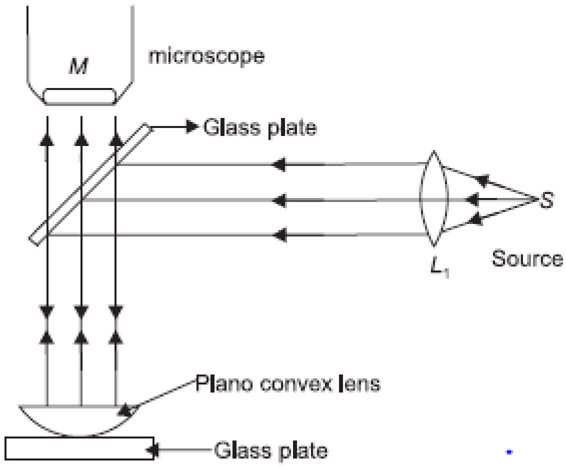
or Fringe-width $\beta = x_2 - x_1 = \dfrac{\lambda}{2\mu\theta}$

Similarly if we consider two consecutive bright fringes the fringe width β will be the same. A wedge-shaped air film can be obtained by inserting a thin piece of paper or hair between two plane parallel plates.

For air film μ= 1, and θ= *t/x*

Where *t*is the thickness of the hair and *x* its distance from the edge where the two plates touch each other.

NEWTON'S RING:



Circular interference fringes can be produced by enclosing a very thin film of air or any other transparent medium of varying thickness between a plane glass plate and a convex lens of a large radius of curvature. Such fringes were first obtained by Newton and are known as Newton's rings. When a plane-convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is very small at the point of contact and gradually increases from the centre outwards. The fringes produced with monochromatic light are circular. The fringes are concentric circles, uniform in thickness and with the point of contact as the centre. When viewed with white light, the fringes are coloured. With monochromatic light, bright and dark circular fringes are produced in the air film. *S* is a source of monochromatic light as shown in Fig. A horizontal beam of light falls on the glass plate *B* at 45°. The glass plate *B* reflects a part of the incident light towards the air film enclosed by the lens *L* and the plane glass plate *G*. The reflected beam from the air film is viewed with a microscope, Interference takes place and dark and bright circular fringes are produced. This is due to the interference between the light reflected from the lower surface of thelens and the upper surface of the glass plate G.

MEASUREMENT OF WAVELENGTH:

The arrangement used is shown earlier. In Figure *S* is a source of sodium light. A parallel beam of light from the lens *L*1 is reflected by the glass plate *B* inclined at an angle of 45° to the horizontal. *L* is a plano-convex lens of large focal length. Newton's rings are viewed through *B* by the travelling microscope *M* focussed on the air film. Circular bright and dark rings are seen with the centre dark. With the help of a travelling microscope, measure the diameter of the *n*th dark ring.

Suppose, the diameter of the nth ring = D_n

$$r_n^2 = n\lambda R$$

$$\text{But } r_n = \frac{D_n}{2}$$

$$\frac{(D_n)^2}{4} = n\lambda R$$

$$D_n^2 = 4n\lambda R \qquad \text{---------------- (1)}$$

Measure the diameter of the $(n + m)$th dark ring

Let it be

$$\frac{(D_{n+m})^2}{4} = (n + m)\,\lambda R$$

$$(D_{n+m})^2 = 4\,(n + m)\,\lambda R \qquad \text{---------------(2)}$$

Subtracting (1) from (2)

$$(D_{n+m})^2 - (D_n)^2 = 4m\lambda R$$

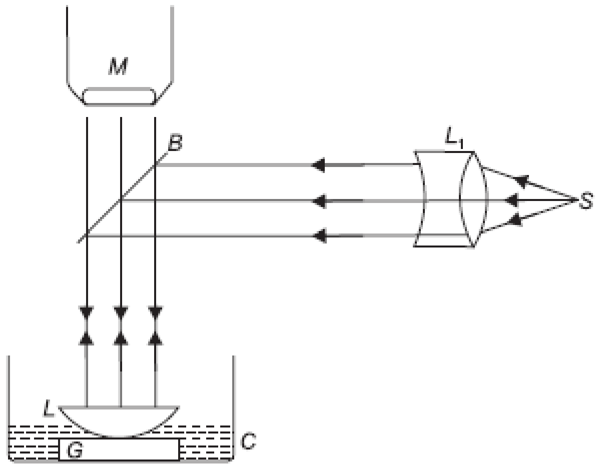
$$\lambda = \frac{(D_{n+m})^2 - (D_n)^2}{4mR}$$

Hence, λ can be calculated. Suppose the diameters of the 5th ring and the 15th ring are determined. Then $m = 15 - 5 = 10$.

$$\lambda = \frac{(D_{15})^2 - (D_5)^2}{4 \times 10\,R}$$

The radius of curvature of the lower surface of the lens is determined with the help of a spherometer but more accurately it is determined by Boy's method. Hence the wavelength of a given monochromatic source of light can be determined.

MEASUREMENT OF REFRACTIVE INDEX:



The experiment is performed when there is an air film between the plano-convex lens and the optically plane glass plate. These are kept in a metal container *C*. The diameter of the nth and the (*n*+*m*)th dark rings are determined with the help of a travelling microscope. For air,

$$(D_{n+m})^2 = 4\,(n + m)\,\lambda R, \quad D_n^{\,2} = 4n\lambda R$$

$$D_{n+m}^2 - D_n^{\,2} = 4\,m\lambda R \qquad \text{------------ (1)}$$

The liquid is poured in the container *C* without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. The diameters of the *n*th ring and the (*n*+*m*)th ring are determined.

$$2\mu t = n\lambda, \text{ But } t = \frac{r^2}{2R}$$

$$\frac{2\mu r^2}{2R} = n\lambda$$

$$r^2 = \frac{n\lambda R}{\mu} \quad \text{But } r = \frac{D}{2};$$

$$D^2 = \frac{4n\lambda R}{\mu}$$

For the liquid, $2\mu t \cos\theta = n\lambda$ for dark rings

If *D'*n is the diameter of the *n*th ring and *D'*n+m is the diameter of the (*n* + *m*)th ring then

$$(D'_{n+m})^2 = \frac{4(n+m)\lambda R}{\mu}; \quad (D'_n)^2 = \frac{4n\lambda R}{\mu} \qquad \text{---------------(1)}$$

$$(D'_{n+m})^2 - (D'_n)^2 = \frac{4m\lambda R}{\mu} \qquad \text{------------ (2)}$$

Or

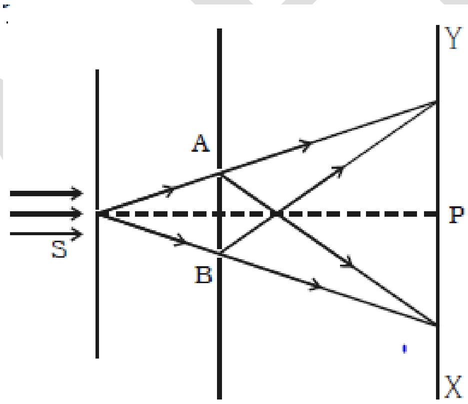
$$\mu = \frac{4m\lambda R}{(D^1_{n+m}) - (D^1_n)^2} \qquad \text{----------- (3)}$$

If m, λ, R, D'n +m and D'n are known μ can be calculated.

If $\lambda$is not known t hen divide (3) by (2) we get,

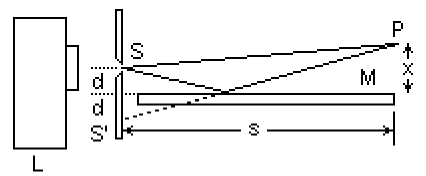
$$\mu = \frac{(D_{n+m})^2 - (D_n)^2}{(D'_{n+m})^2 - (D'_n)^2}$$

YOUNG'S DOUBLE SLIT EXPERIMENT:



The phenomenon of interference was first observed and demonstrated by Thomas Young in 1801. The experimental set up is shown in Fig. Light from a narrow slit S, illuminated by a monochromatic source, is allowed to fall on two narrow slits A and B placed very close to each other. The width of each slit is about 0.03 mm and they are about 0.3 mm apart. Since A and B are equidistant from S, light waves from S reach A and B in phase. So A and B act as coherent sources. According to Huygen's principle, wavelets from A and B spread out and overlapping takes place to the right side of AB. When a screen XY is placed at a distance of about 1 metre from the slits, equally spaced alternate bright and dark fringes appear on the screen. These are called interference fringes or bands. Using an eyepiece the fringes can be seen directly. At P on the screen, waves from A and B travel equal distances and arrive in phase. These two waves constructively interfere and bright fringe is observed at P. This is called central bright fringe. When one of the slits is covered, the fringes disappear and there is uniform illumination on the screen. This shows clearly that the bands are due to interference.

LOYD'S SINGLE MIRROR:



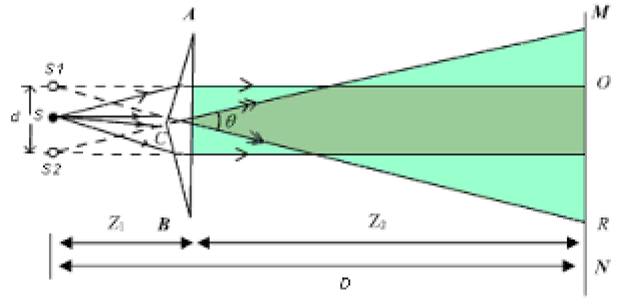
Lloyd's Mirror

This experiment for the production of interference fringes was performed by Lloyd in 1834. He used a plan mirror about 30 cm in length and 6 to 8 cm in breath. The mirror *M* is either a flat polished metal or a piece of black glass so that no reflection takes place from the back of the mirror. *S* is a source of monochromatic light and *B* is the image of *S* formed by reflection. *S* corresponds to one of the coherent source (*viz A)*. Hence *S* and *B* are very close and the rays from *S* are reflected almost at grazing incidence. Interference fringes are produced on the screen placed at a distance *D* from *S* in the shaded portion *EF*.

For complete theory refer to article 8.6. But there is one difference in the case. The central fringe, instead of being bright is dark. If the screen is brought at the end *R* of the mirror *M*, such that the point *C* of the screen touches the end of the mirror, *C* comes at the centre of the darkFringe. *C* is equidistant from *A* and *B.* according to the theory it should lie at the center of a bright fringe. Here, one of the two beams, producing interference fringes, has undergone a phase change of π. Due to this reason, the central fringe instead of being white is dark in this case. This experiment proves that a light beam after reflection form an optically denser medium undergoes a phase change of π.

One objection can be raised here, as to why the central fringe in the case of Frensnel's double mirror is bright and not dark. In the case undergo a phase change of π. Therefore, the path difference is not altered as in the case of Lloyd's single mirror.

FRESNEL'S BIPRISM:



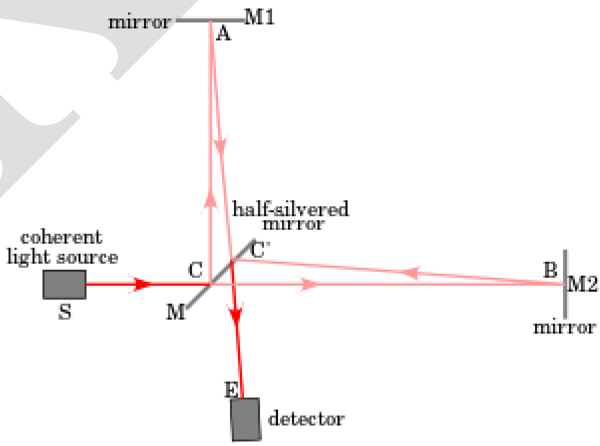
A Fresnel Biprism is a thin double prism placed base to base and have very small refracting angle (0.5o). This is equivalent to a single prism with one of its angle nearly 179° and other two of 0.5o each.

The interference is observed by the division of wave front. Monochromatic light through a narrow slit S falls on biprism , which divides it into two components. One of these component is refracted from upper portion of biprism and appears to come from S1 where the other one refracted through lower portion and appears to come from S2. Thus S1 and S2 act as two virtual coherent sources formed from the original source. Light waves arising from S1and S2 interfere in the shaded region and interference fringes are formed which can be observed on the screen .

Applications of Fresnel's Biprism:

Fresnel biprism can be used to determine the wavelength of a light source (monochromatic), thickness of a thin transparent sheet/ thin film, refractive index of medium etc.

MICHELSON'S INTERFEROMETER:



It is excellent device which is used to get interferometer fringes of various shapes which find a number of applications in optics.

Construction:

Its main optical part have been two plane mirrors M_1 and M_2 and two similar optically plane, parallel glass plates P_1 and P_2. The plane mirror M_1 and M_2 have been silvered on their front surfaces and get mounted vertically on two arms which are at right angles to each other.

It is possible to tilt their planes slightly about vertical and horizontal axes by adjusting screws at their backs. The mirror M_1 is mounted on a carriage provided with a very accurate and fine of a very uniform pitch and can be moved in the direction of the arrows.

The plates P_1 and P_2 are mounted vertically, exactly parallel to each other and inclined at 45^o to M_1 and M_2. The surface of P_1 towards P_2 are mounted has been partially silvered.

Working:

Light from an extended monochromatic source S, which is rendered nearly parallel by a lens L, falls on P_1. A ray of light incident on the partially-silvered surface of P_1 gets partly reflected and partly transmitted.

The reflected ray1, and the transmitted ray, travels to M_1 and M_2 respectively. After reflection at M_1 and M_2 ,the two ray re-combine at the partially-silvered surface and enter a short-focus telescope T. As the ray entering the telescope have been from derived from the same incident ray, they have been coherent and hence in apposition to interfere. The interference fringes can be seen in the telescope.

Function of the Plate P_2 :

After partial reflection and transmission at O, the ray 1 would travel through the glass plate P_1 twice, whereas ray 2 does not do so even once. Thus in the absence of P_2 , the path of rays 1 and 2 in glass would not be equal. To equalize these paths a glass plate P_2 , which is having the same thickness as P_1 , is kept parallel to P_1 . P_2 is termed as the 'compensating plate'.

Form of Fringes:

The form of the fringes has been found to depend on the inclination of M_1 and M_2 .Suppose M_2 is the image of M_2 formed by refection at the semi-silvered surface of P_1 is that $OM_2 = OM_2$.

The interference fringe may be considered to be formed by light reflected from the surfaces of M_1 and M_2 . Hence the arrangement would be equivalent to an air-film which is enclosed between the reflecting surfaces M_1 and M_2 .

DETERMINATION OF WAVELENGHT OF MONOCHROMATIC LIGHT:

First of all the interferometer is set for circular fringes and the position of the mirror M_1 is adjust to get a bright spot at the position of the mirror M_1 is adjust to get a bright spot at the centre of the field of view. If *d* denotes the thickness of the film and *n*the order of the spot obtained, we get

$$2d \cos r = n\,\lambda.$$

But at the centre r = 0, so that cos r = 1. Therefore, we get 2d = n λ.

If now the mirror M_1 is moved away from M_2 by a $\lambda/2$ then 2d increases by λ.

Therfore n + 1 would replace n in equation (1). Thus (n + 1)st bright spot now appears at the centre. Hence each time M_1 moves through a distance $\lambda/2$, next bright spot appears at the centre. Suppose during the movement of M1 through a distance x, N new fringes would appear at the center of the field. Then, we get

$X = N\frac{\lambda}{2}$

Or $$\lambda = \frac{2x}{N}$$

Hence, measuring the distance x on the micrometer screw and counting the number N, the value of λ would be obtained.

The determination of λ by this method this much accurate, as x can be measured to an accuracy of 10^{-4} mm, and the circular fringes can be obtained upto large path differences.

Determination of difference in Wavelengths:

If the source of light is having two wavelengths very close to each other (like sodium D-lines), each wavelength would produce its own system of rings.

If the movable mirror of the interferometer is made to move in one direction, the thickness of the air-film would increase and the rings would become alternately distinct and indistinct. This phenomenon can be used for determining the difference in the two wavelengths.

Suppose $\lambda 1$ and $\lambda 2$ denote the two wavelengths an dsuppose $\lambda 1 > \lambda 2$. If the thickness of the film is small, the rings due to $\lambda 1$ and $\lambda 2$ would almost coincides because $\lambda 1$ and $\lambda 2$ are nearly equal. The mirror M1 is moved away. Then, due different spacing between the rings $\lambda 1$ and $\lambda 2$, the rings of $\lambda 1$ would be slowly separated from those $\lambda 2$.

If the thickness of the air-film become such that a dark ring of $\lambda 1$ coincides with a bright ring of $\lambda 2$ (due to closeness of $\lambda 1$ and $\lambda 2$, he dark ring due to $\lambda 1$ would alomost coincide with bright rings due to $\lambda 2$ in the entire field of view), the rings are having maximum indistinctness.

The mirror 1 is moved further away through a distance x (say0 until the rings, after becoming most distinct, would once again become most indistinct.

Clearly, during this movement, n fringes of $\lambda 1$ and (n+1) fringes of $\lambda 2$ have appeared at the centre (because then the dark ring of $\lambda 1$ will again coincide with the bright ring of $\lambda 2$.) Now, as the movement of the mirror M1 by $\lambda/2$ gives rise the appearance of one new fringes at the centre, we get

X = $n\frac{\lambda 1}{2} = (n+1)\frac{\lambda 2}{2}$

Or $N = \frac{2x}{\lambda 1}$ and $(n+1) = \frac{2x}{\lambda 2}$

$\therefore \frac{2x}{\lambda 2} - \frac{2x}{\lambda 1} = 1$

Or $2x\frac{\lambda 1 - \lambda 2}{\lambda 1 \lambda 2} = 1$

Or $\lambda 1 - \lambda 2 = \frac{\lambda 1 \lambda 2}{2x}$

As $\lambda 1$ and $\lambda 2$ are close together, $\lambda 1 \lambda 2$ could be replace by $\lambda 2$ where $\lambda$is the mean of the mean of $\lambda 1$and $\lambda 2$. Thus

$\lambda - \lambda 2 = \frac{\lambda 2}{2x}$

Thus if one is able to measure the distance moved by 1 between two consecutive position of maximum indistinctness, and the mean wavelength is known one can determine the difference ($\lambda 1 - \lambda 2$).

Determination of refractive index of a thin plate:

This interferometer can be adjusted for producing straight white-light fringes and the cross wire is made to set on the achromatic fringe which is perfectly straight.

The given plate is now inserted in the path of one of the beam by (μ – 1) t would be introduced between the two interfering beams. The fringes are therefore shifted. The movable mirror M1 gets moved till the fringes are bought back to their initial position so that the achromatic fringes again gets coincide with the cross-wire. If the displacement of M1 is x, then we have

$$2x = 2(\mu - 1)\, t$$

Or

$$x = (\mu - 1)t.$$

Thus, by measuring x, t can be calculated if μ is known, or μ can be calculated if t is known.

POSSIBLE QUESTIONS

PART B

1. Define interference.
2. Define wavelength
3. Define amplitude.
4. What is wavelength difference?
5. Define refractive index.
6. Write a note on visibility of fringes.
7. What is meant by Fizeau fringes?
8. Define reflection.

PART C

1. Explain in detail about the Young's Double slit experiment.
2. Write a note on Fringes of equal thickness.
3. Discuss in detail about the determination of the wavelength of monochromatic source.
4. Determine the refractive index of the material by Newton's ring with neat diagram.
5. Explain in detail about the Michelson interferometer experiment and explain the determination of wavelength with neat diagram.
6. Explain in detail about the Michelson interferometer experiment and explain the determination of difference in wavelength with neat diagram.
7. Explain in detail about the Michelson interferometer experiment and explain the determination of refractive index of a thin plate with neat diagram.

UNIT-IV

SYLLABUS

Diffraction: Fraunhofer diffraction: Single slit; Double Slit. Multiple slits & Diffraction grating. Fresnel Diffraction: Half-period zones. Zone plate. Fresnel Diffraction pattern of a straight edge, a slit and a wire using half-period zone analysis.

FRAUNHOFER DIFFRACTION – SINGLE SLIT:

To obtain a Fraunhofer diffraction pattern, the incident wavefront must be plane and the diffracted light is collected on the screen with the help of lens. Thus, the source of light should either be at a large distance from the slit or a collimating lens must be used.

In figure, S is a narrow slit perpendicular to the plane of the paper and illuminated by monochromatic light. L_1 is the collimating lens and AB is a slit of width a. XY is the incident spherical wavefront. The light passing through the slit AB is incident on the lens L_2 and the final refracted beam is observed on the screen MN. the screen is perpendicular to the plane of the paper. The line SP is perpendicular to the screen. L_1 and L_2 are achromatic lenses.

A plane wavefront is incident on the slit AB and each point on this wavefront is a source of secondary disturbance. The secondary wave travelling in the direction parallel to OP viz. AQ and BV come to focus at P and a bright central image is observed. The secondary wave from points equidistant from O and situated in the upper and lower halves OA and OB of the wavefront travel the same distance is reaching P and hence the path difference is zero. The secondary waves reinforce one another and P will be a point of maximum intensity.

Now, consider the secondary waves travelling in the direction AR, inclined at an angel θ to the direction OP. All the secondary wave travelling in this direction reach the point P' on the screen. The point p' will be of maximum or minimum intensity depending on the path difference between the secondary waves orginating from the corresponding points of hte wavefront. Draw OC and BL perpendicular to AR.

Then, in the Δ ABL

$$\sin\theta = \frac{AL}{AB} = \frac{AL}{a}$$

or

$$AL = a\sin\theta$$

Where a is the width of the slit and AL is the path difference between the secondary waves orginating from A and B. If this path difference is equal to λ the wavelength of light used, then P' will be a point of minimum intensity. The whole wavefront can be considered to be of two halves OA and OB and if the path difference between the secondary waves from A and B is λ, then the path difference between the secondary waves from A and O will be $\frac{\lambda}{2}$. Similarly for every point in the upper half OA, there is a corresponding point in the lower half OB, and the path difference between the secondary waves from these points is $\frac{\lambda}{2}$. Thus, destructive interference takes place and point P' will be of minimum intensity. If the direction of the secondary waves is such that AL = 2 λ, then also the point they meet the screen will be minimum intensity. This is so because the secondary waves from the corresponding points of the lower half, differ in path by $\frac{\lambda}{2}$ and this again gives the position of minimum intensity. In general

$$a \sin \theta n = n \lambda$$

$$\sin \theta n = \frac{n\lambda}{a}$$

Where θn gives the direction of the n th minimum. Here n is an integer. If, however, the path difference is odd multiplies of $\frac{\lambda}{2}$, the directions of the secondary maxima can be obtained. In the case,

$$a \sin \theta n = (2n+1) \ \frac{\lambda}{2}$$

or $$\sin \theta n = \frac{(2n+1)\lambda}{2a}$$

Where n = 1,2,3 etc.

Thus, the diffraction pattern due to a single slit consisits of a central bright maximum at P followed by secondary maxima and minima on both the sides. The intensity distribution on the screen is given in figure. P corresponds to the position of the central bright maximum and the points on the screen for which the path difference between the points A and B is $\lambda, 2\lambda$ etc, correspond to the positions of secondary minima. The secondary maxima are of much less intensity. The intensity falls off rapidly from the point P outwards.

If the lens L_2 is very near the slit or the screen is far away from the lens L_2 , then

$$\sin \theta = \frac{x}{f} \qquad \text{----------(1)}$$

Where f is the focal length of the lens L_2, But $\sin\theta = \frac{\lambda}{a}$ -----------(2)

$$\frac{x}{f} = \frac{\lambda}{a}$$

or $x = \frac{f\lambda}{a}$

Where x is the distance of the secondary minimum from the point P. Thus, the width of the central maximum = 2x.

or $2x = \frac{2f\lambda}{a}$ -----------(3)

The width of the central maximum is proportional to λ, the wave-length of light. With red light (longer wavelength), the width of the central maximum is more than with violet light (shorter wavelength). With a narrow slit, the width of the central maximum is more. The diffraction pattern consists of alternate bright and dark bands with monochromatic light. With white light, the central maximum is white and the rest of the diffraction bands are coloured. From equation (2), if the width a of the slit is large, sin θ is small and hence θ is small. The maxima and minima are very close to the central maximum at P. But with a narrow slit , a is small and hence θ is large. This results a distinct diffraction maxima and minima on both sides of P.

FRAUNHOFER DIFFRACTION – DOUBLE SLIT:

In figure AB and CD are two rectangular slits parallel to one another and perpendicular to the plane of the paper. The width of each slit is a and the width of the opaque portion is b. L is a collecting lens and MN is a screen perpendicular to the plane of the paper. P is a point on the screen such that OP is perpendicular to the screen. Let a plane wavefront be incident on the surface of XY. All the secondary waves travelling a direction parallel to OP come to focus at P. Therefore, P coresponds to the position of the central bright maximum.

In the case, the diffraction pattern has to be considered in teo parts: (i) the interferene phenomenon due to the secondary waves emanating from th corresponding points of the two slits and (ii) the diffraction pattern due to the secondary waves from the two slits individually. For calculating the positions of interference maxima and minima, the diffracting angle is denoted as θ and for the diffraction maxima and minima it is denoted as φ. Both the angles θ and φ refer to the angle between the direction of the secondary waves and the initial direction of the incident light.

(i)Interference maxima and minima:

Consider the secondary waves travelling in a direction inclined at an angle θ with the initial direction.

In the ΔACN (Figure)

$$\sin\theta = \frac{CN}{AC} = \frac{CN}{a+b}$$

or $$c = (a+b)\sin\theta$$

If this path difference is equal to odd multiples of $\frac{\lambda}{2}$, θ gives the direction of minima due to interference of the secondary waves from the two slits .

$$CN=(a+b)\sin\theta_n =(2n+1)\frac{\lambda}{2} \quad \text{................(i)}$$

putting n=1,2,3,...etc , the values of $\theta_1\theta_2\theta_3$,etc. corresponding to the directions of minima can be obtained

from equation **(i)**

$$\sin\theta_n =\frac{(2n+1)\lambda}{2(a+b)} \quad \text{................(ii)}$$

On the other hand, if the secondary waves travel in a direction θ' such that the path difference is even multiple of $\frac{\lambda}{2}$, then θ' gives the direction of the maxima due to interference of light waves emanating from the two slits.

$$CN = (a+b)\sin\theta'_n = 2n.\frac{\lambda}{2}$$

or $$\sin\theta'_n = \frac{n\lambda}{(a+b)} \quad \text{-------------(iii)}$$

Putting n= 1,2,3 etc, the values of $\theta'_1, \theta'_2, \theta'_3$,etc. corresponding to the directions of minima can be obtained.

From equation (ii)

$$\sin\theta_1 = \frac{3\lambda}{2(a+b)}$$

and $$\sin\theta_2 = \frac{5\lambda}{2(a+b)}$$

$$\sin\theta_1 - \sin\theta_2 = \frac{\lambda}{a+b} \quad \text{------------(iv)}$$

Thus, the angular seperation between any two consective minima (or maxima) is equal to $\frac{\lambda}{a+b}$. The angular seperation is inversely proportional to (a+b), the distance between the two slits .

(ii)Diffraction maxima and minima:

Consider the secondary waves travelling in a direciton inclined at an angle φ with the initial direction of the incident light.

If the path difference BM is equal to λ the wavelength of light used, then φ will give the direction of diffraction minimum (figure). That is, the path difference between the secondary waves emanating from the extermities of a slit (i.e. points A and B) is equal to λ. Considering the wavefront on AB to be made up of two halves, the path difference between the corresponding points of the upper and the lower halves is equal to $\frac{\lambda}{2}$. The effect at P' due to the wavefront incident on AB is zero. Similarly for the same direction of the secondary waves, the effect at P' due to the wavefront incident on th slit CD is also zero. In general,

$$a \sin \varphi_n = n\lambda \qquad \text{------------(v)}$$

Putting n=1,2,3,etc, the values of $\theta_1\theta_2\theta_3$,etc. corresponding to the directions of diffraction minima can be obtained.

FRAUNHOFER DIFFRACTION – MULTIPLE SLIT:

Fraunhofer diffraction at two slits consists of diffraction maxima and minima governed by

$$\frac{sin^2 \alpha}{\alpha^2}$$

and sharp interference maxiam and minima, in each diffraction maximum governed by the $\cos^2 \beta$ term.

To derive an expression for the intensity distribution due to diffraction at N slits, the expression for dy has to be integrated for N slits.

For a single slit,

$$dy = k \int_{\frac{-a}{2}}^{\frac{+a}{2}} sin\ [\ 2\pi\ (\frac{t}{T} - \frac{r}{\lambda} + \frac{z \sin \Theta}{\lambda})\ ]\ dz$$

Let $\sin\ [\ 2\pi\ (\frac{t}{T} - \frac{r}{\lambda} + \frac{z \sin \Theta}{\lambda})\ ]\ dz$ be equal to φ (z) (i.e. function of z)

For N slits

$$dy = \int_{\frac{-a}{2}}^{\frac{+a}{2}} \Phi(z)dz + \int_{d\frac{-a}{2}}^{d\frac{+a}{2}} \Phi(z)dz + \int_{2d\frac{-a}{2}}^{2d\frac{+a}{2}} \Phi(z)dz + ...+ \int_{(N-1)d-\frac{a}{2}}^{(N+1)d+\frac{a}{2}} \phi(z)dz$$

On simplification

$$y = ka \frac{sin\alpha}{\alpha} [\ \sin 2\pi\ (\frac{t}{T} - \frac{r}{\lambda}) + \sin 2\pi\ (\frac{t}{T} - \frac{r}{\lambda} + \frac{d\ sin\theta}{\lambda}) + \sin 2\pi\ (\frac{t}{T} - \frac{r}{\lambda} + \frac{2d\ sin\theta}{\lambda}) + ...$$

.... + sin $2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{(N-1)d\,sin\theta}{\lambda}\right)$

Here $\alpha = \frac{\pi a\, sin\theta}{\lambda}$

For a general trigonometric summation

$$\sum_{p=0}^{p=n} \sin(x + pm) = \frac{\sin(x+\frac{nm}{2})\sin[(\frac{n+1}{2})m]}{\sin\frac{m}{2}}$$

Here x = $2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right)$

and m = $\frac{2\pi d\, sin\theta}{\lambda}$

= $2\left[\frac{\pi d\, sin\theta}{\lambda}\right] = 2\beta$

$\beta = \frac{\pi d\, sin\theta}{\lambda}$

n = (N-1)

Then y = ka $\left(\frac{\sin\alpha}{\alpha}\right)\frac{\sin N\beta}{\beta}$ [sin $2\pi\left(\frac{t}{T} - \frac{r}{\lambda} + \frac{(N-1)d\, sin\theta}{\lambda}\right)$] ------------(1)

The intensity at point P'

$$I = k^2a^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\frac{\sin^2 N\beta}{sin^2\beta} \quad \text{-----------(2)}$$

The maximum intensity, when $\alpha = k^2a^2$

$$I = I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right)\frac{\sin^2 N\beta}{sin^2\beta} \quad \text{------------(3)}$$

Since the expression $\frac{\sin^2\alpha}{\alpha^2}$ represents the diffraction pattern due to a single slit. The additional factor $\frac{\sin^2 N\beta}{sin^2\beta}$ represents the interference effects due to the secondary waves from the N slits.

The numerator will be zero when

N β = 0,π,2π,3π......etc. = kπ'

The denominator is also zero when

β = 0,π,2π,3π......etc

Since the quotient $\frac{0}{0}$ is indeterminate, therefore N β = kπ gives the condition for minimum intensity for all values of k other than

k = 0,N,2N,3N etc.

The directions of principal maxima correspond to the values of k=0,N,2N etc.

N$\beta = \frac{N\,\pi d\, sin\theta}{\lambda}$

or $k\pi = \frac{N\pi d \sin\theta}{\lambda}$

For the directions of principal maxima.

$k = 0, 1N, 2N, 3N$ etc. $= nN$

When $n = 0,1,2,3,....$etc.

$nN\pi = \frac{N\pi d \sin\theta}{\lambda}$

$d \sin\theta = n\lambda$

Here $n = 0,1,2,3$ etc.

If the width of the slit is a and the width of the opaque spacing is b.

$d = (a+b)$

$(a+b) \sin\theta = n\lambda$

Putting n = 1,2,3 etc., the directions of principal maxima $\theta_1\theta_2\theta_{3......}$etc can be determined.

For values of k in between 0 and N, between N and 2N etc, there arew (N-1) secondary minima and (N-2) secondary maxima.

The intensity distribution due to diffraction and N slits is shown in figure.

DIFFRACTION GRATING:

A diffraction grating is an extremely useful device and in one of it's forms it consists of a very large number of narrow slits are seprated by opaque spaces.when a wavefront is incident on a grating surface, light is transmitted through the slits and obstructed by the opaque portions. Such a grating is called a transmission grating. The secondary waves from the positions of the slits interfere with one another, similar to the interference of waves in Young's experiment. Joseph Fraunhofer used the first gratings which considered of a large number of parallel fine wires stretched on a frame. Now, gratings are prepared by ruling equidistant parallel lines on a glass surface. The lines are drawn with a fine diamond point. The space in between any two lines is transparent to light and the lined portion is opaque to light. Such surfaces act as transmission gratings. If on the other hand the lines are drawn on a silvered surface (plain or concave) then light is reflected from the positions of the mirror in between any two lines and such surfaces act as reflections gratings.

If the spacing between the lines is of the order of the wavelength of light, then the appreciable deviation of the light is produced. Gratings used for the study of the visible region of the spectrom contain 10,000 lines per centimeter. Gratings, width orginally ruled surfaces are only few. For practical purposes, replicas of the orginal grating is prepared. On the orginal grating surface a thin layer of collodion solution is poured and the solution is allowed to harden. Then, the film of collodion is removed from the grating surface and then fixed between two glass plates. This serves as a plane transmission grating. A large number of replicas are prepared in this way from a single orginal ruled surface.

POSSIBLE QUESTIONS

PART B

1. Define diffraction.
2. What is zone plate?
3. Explain about half-period zone.
4. Define wave font.
5. Constructive fringes – define.
6. Destructive fringes – define.

PART C

1. Write a note on Fraunhofer double slit experiment.
2. Write an expression of intensity distribution for multiple slit experiment.
3. Write a note on diffraction grating.
4. Write a note on (i) Half-period Zone (ii) Zone Plate.

UNIT – V

SYLLABUS

Polarization: Transverse nature of light waves. Plane polarized light – production and analysis. Circular and elliptical polarization. Hygiene's explanation of double refraction, positive and negative uniaxial crystals, quarter and half wave plates, types of polarized light, production and analysis of plane, circularly and elliptically polarized light, optical activity

TRANSVERSE NATURE OF ELECTROMAGNETIC WAVES:-

Consider an electromagnetic wave propagating in free space along z-direction. Then the E and H vary only in z-direction.

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0 \ and \ \frac{\partial}{\partial z} \neq 0$$

such wave is called as planar wave, since it's vector are functions of (z, t) only then we write,

$$E = E(z,t)$$
$$H = H(z,t)$$

from first Maxwell's equation,

$$\nabla . D = 0 \ or \ \varepsilon_0(\nabla . E) = 0$$
$$\nabla . E = 0 \ or \ \frac{\partial E_z}{\partial t} = 0$$
$$E_z = constant \ in \ time$$

from second Maxwell's equation,

$$\nabla . B = 0 \ or \ \mu_0(\nabla . H) = 0$$
$$\nabla . H = 0 \ or \ \frac{\partial H_z}{\partial z} = 0$$
$$H_z = constant \ in \ time$$

from third Maxwell's equation,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$(\nabla \times E)_z = -\mu_0 \frac{\partial H_z}{\partial t}$$
$$K\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) = -\mu_0 \frac{\partial H_z}{\partial t}$$

$$\frac{\partial H_z}{\partial t} = 0$$

$$H_z = constant\ in\ time$$

similarly by using fourth equation,

$$E_z = constant\ in\ time$$

Thus we have concluded that E_z and H_z are constant as regards for time and space. they represent the static components and consequently, no part of wave motion. we can therefore write,

$$E_z = H_z = 0$$

$$E = iE_x + jE_y$$

$$H = iH_x + jH_y$$

the electric E and magnetic H vector don't have any Z-component, the Z-direction being the direction of propagation ,both these vectors are perpendicular to the direction of propagation ,Maxwell's electromagnetic waves are purely transverse in nature.

DESCRIPTION OF PLANE, CIRCULAR AND ELLIPTICALLY POLARIZATION :

Light in the form of a plane wave in space is said to be linearly polarized. Light is a transverse electromagnetic wave, but natural light is generally unpolarized, all planes of propagation being equally probable. If light is composed of two plane waves of equal amplitude by differing in phase by 90°, then the light is said to be circularly polarized. If two plane waves of differing amplitude are related in phase by 90°, or if the relative phase is other than 90° then the light is said to be elliptically polarized.

Plane Polarization :

A plane electromagnetic wave is said to be linearly polarized. The transverse electric field wave is accompanied by a magnetic field wave as illustrated.

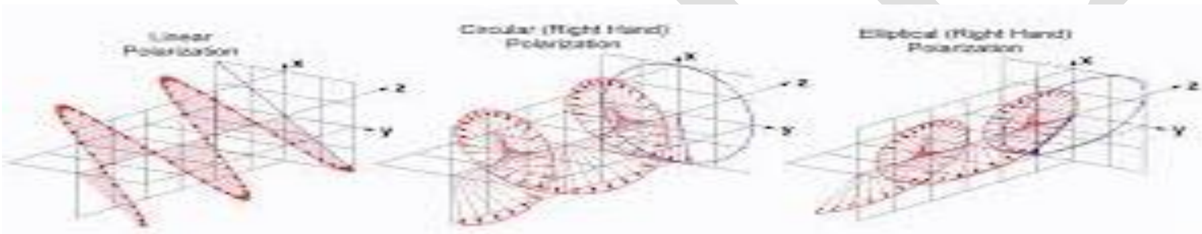
Circular Polarization :

Circularly polarized light consists of two perpendicular electromagnetic plane waves of equal amplitude and 90° difference in phase. The light illustrated is right- circularly polarized. If light is composed of two plane waves of equal amplitude but differing in phase by 90°, then the light is said to be circularly polarized. If you could see the tip of the electric field vector, it would appear to be moving in a circle as it approached you. If while looking at the source, the electric vector of the light coming toward you appears to be rotating counter clockwise, the light is said to be right-circularly polarized. If clockwise, then left-circularly polarized light. The electric field vector makes one complete revolution as the light advances one wavelength toward you. Another way of saying it is that if the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.

Circularly polarized light may be produced by passing linearly polarized light through a quarter-wave plate at an angle of 45° to the optic axis of the plate.

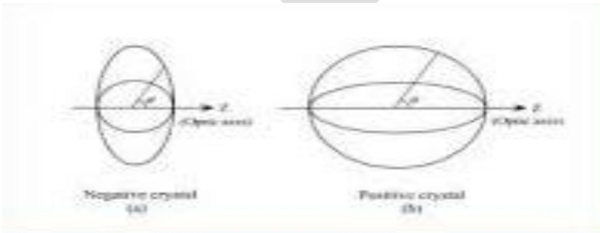
Elliptical Polarization :

Elliptically polarized light consists of two perpendicular waves of unequal amplitude which differ in phase by 90°. The illustration shows right- elliptically polarized light.
If the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.



HUYGENS EXPLANATION OF DOUBLE REFRACTION IN UNIAXIAL CRYSTALS :

Huygens explained the phenomenon of double refraction with the help of the principal of secondary wavelets. A point source of light in a double refracting medium is the origin of two wave fronts. For the ordinary ray, for which the velocity of light is same in all directions. The wave front is spherical. For extraordinary ray the velocity varies with the directions and wave front is ellipsoid. The velocities of ordinary and extraordinary rays are the same along the optic axis.



Consider a point source of light S in a calcite crystal, the sphere is the wave surface of the ordinary ray and ellipsoid is the wave surface of the extraordinary ray. The ordinary wave surface lies within the extraordinary wave surface. Such crystals are known as negative crystals. For crystals like quartz, which are known as positive crystals, The extraordinary wave surface lies within the ordinary wave surface

1) For the negative uniaxial crystals $\mu_0 > \mu_E$:

The velocity of the extraordinary ray varies as the radius vector of the ellipsoid. It is least and equal to the velocity of the ordinary ray along the optic axis but it is maximum at right angle to the direction of the optic axis.

2) For the positive uniaxial crystals $\mu_E > \mu_0$:

The velocity of the extraordinary ray is least in the direction at the right angles to the optic axis.I t is maximum and equal to the velocity of the ordinary ray along the optic axis. Hence from the Huygens theory, The wave fronts or surfaces in uniaxial crystals are a sphere and an ellipsoid and there are two points where these two wave fronts touch each other. The direction of the line joining these two point is the optic axis.

QUARTER WAVE PLATE:

It is a plate of doubly refracting uniaxial crystal of calcite (or) quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis. The incident plane –polarized light is perpendicular to its surface and the ordinary and the extraordinary rays travel along the same direction with different velocities. If the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays are μ_0 and μ_E respectively, Then the path difference introduced between the two rays is given by,

For negative crystals, path difference= $(\mu_0-\mu_E)t$

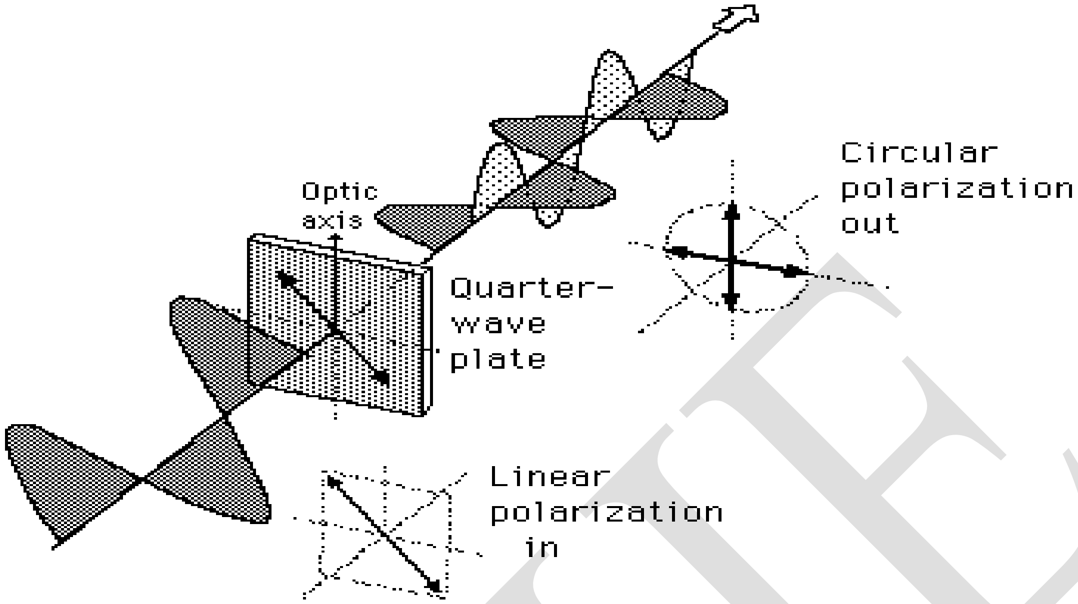
For positive crystals, path difference=$(\mu_E - \mu_0)t$

To produce a path difference of $\lambda/4$,in calcite

$$(\mu_0-\mu_E)t= \lambda/4$$

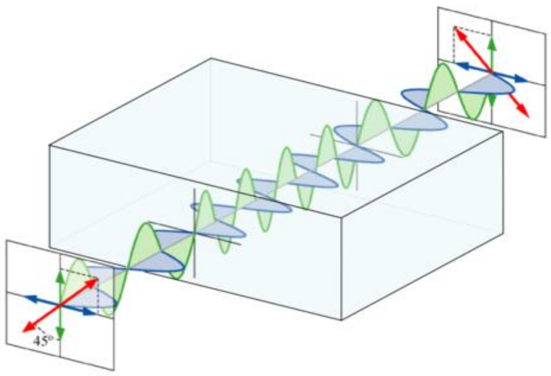
$$t= \lambda/4 (\mu_0-\mu_E) \qquad \text{---------------------} (1)$$

and in case of quartz, If the plane-polarized light, whose plane of vibration is inclined at an angle of 45^o to the optic axis, is incident on a quartz wave plate, the emergent light is circularly polarized.



HALF WAVE PLATE:

This plate is also made from a doubly refracting uniaxial crystal of calcite (or) quartz of suitable thickness whose refracting faces are cut parallel to the direction of the optic axis. the thickness of the plate is t and the refractive indices for the ordinary and the extraordinary rays have a path difference= λ/2 after passing through the crystals



For negative crystal, path difference =(μ_0-μ_E)t

For positive crystals, path difference=(μ_E - μ_0)t

To produce a path difference of λ/2,in calcite

$$(\mu_0-\mu_E)t= \lambda/2$$

$$t= \lambda/2\ (\mu_0-\mu_E) \quad \text{--------------------- (1)}$$

and in case of quartz,

$$t = \lambda/2\,(\mu_E - \mu_0) \qquad \text{-------------------- (2)}$$

When plane polarised light is incident on a half wave plate, such that it makes an angle of 45^o with the optic axis a path difference of $\lambda/2$ is introduced between the extraordinary and the ordinary rays. The emergent light is plane-polarized and the direction of polarization of the linear incident light is rotated through 90^o.Thus a half wave plate rotates the azimuth of a beam of plane-polarized light by 90^o ,provided the incident light makes an angle $45^o$with the optic axis of the half wave plate.

PRODUCTION OF PLANE, CIRCULAR AND ELLIPTICALLY POLARIZED LIGHT:

Production of plane polarized light:

A beam of monochromatic light is passed through a nicol prism. While passing through the nicol prism, the beam is spilt up into extraordinary ray and ordinary ray. The ordinary ray is totally internally reflected back at the Canada balsam layer, while the extra ordinary ray passes through the nicol prism. The emergent beam is plane polarized

Production of circularly polarized light:

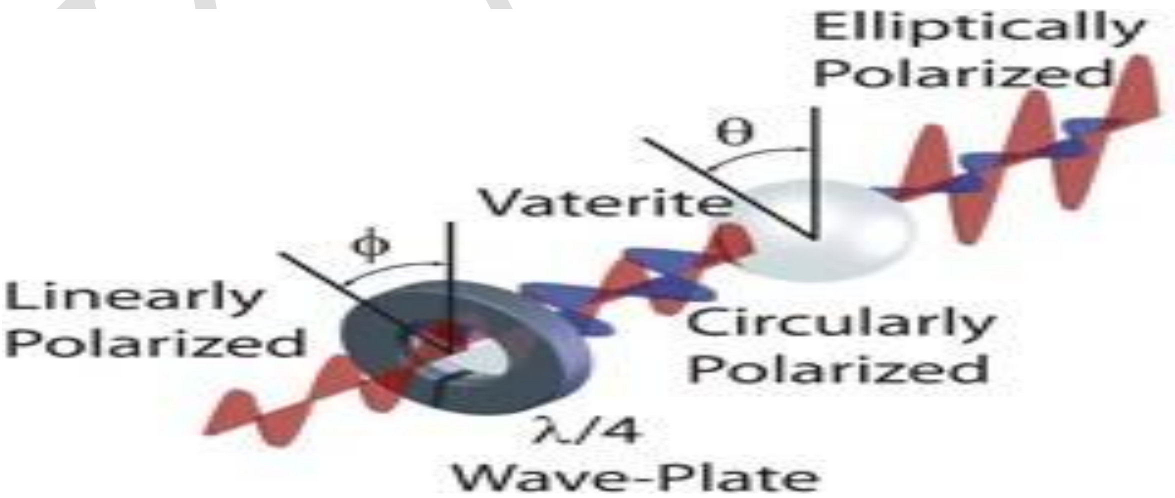
To produce circularly polarized light, the two waves vibrating at right angle to each other having the same amplitude and time period should have a phase difference of $\pi/2$ (or)a path difference of $\lambda/4$.for this purpose a parallel beam of monochromatic light is allowed to fall on a nicol prism N_1.The beam after passing through the prism N_1 is plane polarized.

The nicol prism N_2 is placed at some distance from N_1 so that N_1 and N_2 are crossed. The field of view will be dark as viewed by the eye in this position. A quarter wave plate p is mounted on a tube A .The tube A can rotate about the outer fixed tube B introduced between the nicol prism N_1 and N_2 .The plane polarised light from N_1 falls normally on P and the field of view may be bright. The quarter wave plate is rotated until the field of view may be dark keeping P fixed, A is rotated such that the mark S on P coincides with 0 mark on A. Afterwards, By rotating the quarter wave plate P, the mark S is made to coincide with the 45^o mark on A.

The quarter wave plate is in the desired position. In this case, the vibration of plane polarised light falling on the quarter wave plate makes an angle 45^o with the direction of optic axis of the quarter wave plate. The polarised light is split up into two rectangular components having equal amplitude and time period and on coming out of the quarter wave plate, the beam is circular polarised if the nicol prism N_2 is rotated at this stage, the field of view is uniform in intensity similar to the ordinary light passing through the nicol prism

Elliptically polarised light:

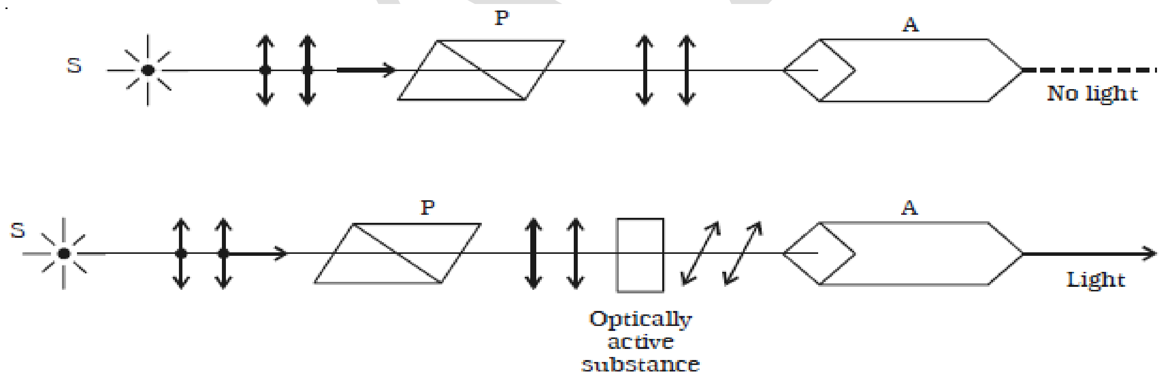
To produce elliptically polarised light, the two waves vibrating at right angle to each other having unequal amplitudes should have a phase difference of $\pi/2$ or a path difference of $\lambda/4$. The arrangement of figure can be used for this purpose. A parallel beam of monochromatic light is allowed to fall on the nicol prism N_1 .The prisms N_1 and N_2 are crossed and the field of view is dark. A quarter wave plate is introduced between N_1 and N_2.The plane polarised light from the nicol prism N_1 falls normally on the quarter wave plate. The field of view is illuminated and the light coming out of the quarter wave plate is elliptically polarised. The only precaution in the case is that the vibrations of the plane polarised light falling on the quarter plate should not make an angle of 45^o with the optic axis, in which case, the light will be circularly polarised. When the nicol prism N_2 is rotated, it is observed that the intensity of illumination of the field of view varies between a maximum and a minimum . This is just similar to the case when a beam consist of mixture of plane-polarised light and ordinary light is examined by a nicol prism.



OPTICAL ACTIVITY :

When a plane polarised light is made to pass through certain substances, the plane of polarisation of the emergent light is not the same as that of incident light, but it has been rotated through some angle. This phenomenon is known as optical activity. The substances which rotate the plane of polarisation are said to be optically active. Examples : quartz, sugar crystals, turpentine oil, sodium chloride etc. Optically active substances are of two types, (i) Dextro−rotatory (right handed) which rotate the plane of polarisation in the clock wise direction on looking towards the source. (ii) Laevo – rotatory (left handed) which rotate the plane of polarisation in the anti clockwisedirection on looking towards the source. Light from a monochromatic source S, is made to pass through a polariser P. The plane polarised light is then made to fall on an analyser A, which is in crossed position with P. No light comes out of A. When a quartz plate is inserted between the polariser and analyser some light emerges out of the analyzerA . The emerging light is cut off again, when the analyzer is rotated through a certain angle.

This implies that light emerging from quartz is still plane polarised, but its plane of polarisation has been rotated through certain angle.



The amount of optical rotation depends on :

(i) thickness of crystal

(ii) density of the crystal or concentration in the case of solutions.

(iii) wavelength of light used

(iv) the temperature of the solutions.

POSSIBLE QUESTIONS

PART B

1. Define Polarization.
2. What is plane polarized light?
3. What is circularly polarized light?
4. Write a note on elliptically polarized light.
5. What is positive uniaxial crystal?
6. What is negative uniaxial crystal?
7. What are the types of polarized light?

PART C

1. Discuss in detail about the transverse nature of light and polarization.
2. Write a note on production and analysis of plane polarized light.
3. Discuss in detail about quarter wave plate.
4. Discuss in detail about half wave plate.
5. Write a note on optical activity.
6. Explain about the circularly and elliptically polarized light.