		Semester-VI
		LTPC
15PHU604A	ELECTROMAGNETIC THEORY	5 5
Scope: Electromagnet	ic theory is a very important branch of physics, and is very m	uch useful in the study

of communication theory. It is important to know the theory for the design of antennas which is essential for any types of communication.

**Objectives:** This paper aims at giving a theoretical treatment of the electromagnetic theory.

### UNIT- I

**Electrostatics**: Electric intensity – Electric potential – Gauss Law - Dielectric and its polarization - Electric displacement D – Dielectric constant  $\epsilon$ r – Polarisibility $\alpha$  - Clausius-Mossotti relation (Non-polar molecules) – The Langevin equation (Polar molecules) – Electrostatic energy

### UNIT II

**Magnetostatics:** Current density J – Ampere's law of force – Biot-Savart law – Ampere's circuital law – Magnetic scalar potential  $\phi$ m (no applications) – Magnetic vector potential A – Magnetisation and magnetization current – Magnetic intensity – Magnetic susceptibility and Permeability.

#### UNIT III

#### Maxwell's Equations

Maxwell Equations-Displacement Current-Vector and Scalar Potentials. Gauge Transformations: Lorentz and Coulomb Gauge. Boundary Conditions at Interface between Different Media. Wave Equations. Plane Waves in Dielectric Media. Poynting Theorem and Poynting Vector. Electromagnetic Energy Density. Physical Concept of Electromagnetic Field Energy Density, Momentum Density and Angular Momentum Density.

#### UNIT IV

Interaction of E.M.Waves with matter (Macroscopic): Boundary conditions at interfaces - Reflection and refraction – Frenel's laws-Brewster's law and degree of polarization - Total internal reflection and critical angle.

### UNIT V

Interaction of E.M.Waves with matter (Microscopic): Scattering and Scattering parameters - Scattering by a free electron (Thomson Scattering) - Scattering by a Bound electron (Rayleigh scattering) – Dispersion Normal and Anomalous – Dispersion in gases (Lorentz theory)

#### **Books for study**

1. Chopra & Agarwal 2004, Electromagnetic theory, 6th Edition, Nath& Co, Meerut.

#### REFERENCES

- 1. Jacson. J.D., 1998, Classical Electro dynamics, 3rd Edition, Willey Eastern, New Delhi.
- 2. Schwaritz. M. 1972, Revised edition, Principles of Electro dynamics, McGraw Hill, Auckland.
- 3. Jordon and Balmain 2<sup>nd</sup> edition 2002, EMW radiating systems, Prentice Hall of India Pvt Ltd, New Delhi.
- 4. Gupta, Kumar and Singh, 2007, Electro dynamics, 19th Edition, PragatiPrakasan, Meerut, New Delhi.
- 5. SatyaPrakash10<sup>th</sup> edition 2003, Electromagnetic theory and Electro dynamics, KedarNath Ram Nath& Co, Meerut.
- 6. Griffiths D., 1998, Introduction to Electrodynamics, 3rd Edition, Printice Hall of India, New Delhi.



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## LECTURE PLAN DEPARTMENT OF PHYSICS

## STAFF NAME: Mrs.S.SHARMILA SUBJECT NAME: PHYSICS SEMESTER: VI

## SUB.CODE:15PHU604A CLASS: III B.Sc (PHY)

S.No	Lecture Duration Period	Topics to be Covered	Support Material/Page Nos
		UNIT-I	
1	1	Electric intensity	T1-4-5
2	1	Electric potential	T1-5-7
3	1	Gauss Law	T1-10-12
4	1	Dielectric and its polarization	T1-20-21
5	1	Continuation	
6	1	Electric displacement D	T1- 30-31
7	1	Dielectric constant $\varepsilon_r$	
8	1	Polarisibiltiy $\alpha$	T1-31-34
9	1	Continuation	
10	1	Clausius-Mossotti relation (Non- polar molecules)	T1-38-40
11	1	Continuation	
12	1	The Langevin equation (Polar molecules)	T1-40-43,
13	1	Continuation	
14	1	Electrostatic energy	T1-45-47
15	1	Revision	
	Total No of Ho	urs Planned For Unit 1=15	
		UNIT-II	

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1	1	Current density J	T1-120-121
2	1	Ampere's law of force	T1-122
3	1	Biot-Savart law	T1-123-124
4	1	Continuation	
5	1	Ampere's circuital law	T1-128-131
6	1	Magnetic scalar potential $\phi_m$ (no applications)	T1-136-138
7	1	Continuation	
8	1	Continuation	
9	1	Magnetic vector potential A	T1-141-142
10	1	Continuation	
11	1	Continuation	
12	1	Magnetisation and magnetization current	T1-151-152
13	1	Magnetic intensity	T1-153-154
14	1	Magnetic susceptibility and Permeability	T1-155
15	1	Revision	
	Total No of Ho	urs Planned For Unit II=15	
		UNIT-III	
1	1	Maxwell's Equation	T1-180-185
2	1	Continuation	
3	1	Displacement currents, Vector and scalar potentials	T1-174-176
4	1	Gauge transformations, Lorentz gauge	T1-213-126
5	1	Coulomb gauge	T1-217-220
6	1	Continuation	
7	1	Boundary Conditions at Interface between Different Media	R1-40-45
8	1	Wave Equations	

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9	1	Plane Waves in Dielectric Media	R1-53-62
10	1	Pointing theorem	T1-188-193
11	1	Poynting vector	T1-196-200
12	1	Electromagnetic Energy Density	
13	1	Physical Concept of Electromagnetic Field Energy Density	
14	1	Momentum Density and Angular Momentum Density	
15	1	Revision	
	Total No of Hou	urs Planned For Unit III=15	
		UNIT-IV	
1	1	Boundary conditions at interfaces	T1-265-268
2	1	Continuation	
3	1	Continuation	
4	1	Reflection and refraction	T1-269-270
5	1	Continuation	
6	1	Fresnel's laws	T1-270-276
7	1	Continuation	
8	1	Continuation	
9	1	Brewster's law and degree of polarization	T1-278-280
10	1	Continuation	
11	1	Continuation	
12	1	Total internal reflection and critical angle	T1-281-284
13	1	Continuation	
14	1	Revision	
	Total No of Hou	irs Planned For Unit IV=14	
		UNIT-V	
1	1	Scattering and Scattering	T1-318-319

		parameters	
2	1	Continuation	
3	1	Scattering by a free electron (Thomson Scattering)	T1-319-124
4	1	Continuation	
5	1	Continuation	
6	1	Scattering by a Bound electron (Rayleigh scattering)	T1-325-329
7	1	Continuation	
8	1	Continuation	
9	1	Dispersion Normal and Anomalous	T1-334
10	1	Continuation	
11	1	Dispersion in gases (Lorentz theory)	T1-334-338
12	1	Continuation	
13	1	Revision	
14	1	Old question paper discussion	
15	1	Old question paper discussion	
16	1	Old question paper discussion	
	Total no.	of of hours for Unit-V =16	
Total Planned Hours	75		

## **TEXT BOOK**

T1- Electromagnetic theory by Chopra and Agarwal, KedarNath RamNath & Co.

## REFERENCES

R1-Electromagnetic theory & Electrodynamics by Sathyaprakash, KedarNath RamNath & Co.

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## <u>UNIT-I</u>

## **SYLLABUS**

**Electrostatics:** Electric intensity – Electric potential – Gauss Law - Dielectric and its polarization - Electric displacement D – Dielectric constant  $\varepsilon_r$  – Polarisibility  $\alpha$  - Clausius-Mossotti relation (Non-polar molecules) – The Langevin equation (Polar molecules) – Electrostatic energy

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## **ELECTRIC INTENSITY**

The space surrounding any charge or charge distribution in which the effects of its presence can be felt is called the field of the charge or charge distribution. The strength of the field called electric intensity at a point is defined by a vector function of position in space which determines the force on a unit positive charge at rest at that point of space. The unit of electric intensity in M.K.S system is volts/meter.

If a test charge  $q_0$  is placed at the position r in space at to determine electric field and the test charge experience a force F there, then electric field at the point will be E = F/q0 ------ (1)

Thus in turn implies that the electric field of a point charge at a point  $r_{j0}$  will be

 $E_{j0} = F_{j0}/q_0 = (1/4\pi\epsilon_0) (q_j/r_{j0}^3) r_{j0}$ Or E = (qr/4\pi \epsilon\_0 r^3) ------ (2)

Where r is the vector distance from q to the point at which E is evaluated.

The total field due to a set of charges i.e., discrete distribution of charges will be

 $E = 1/4\pi\epsilon_0 \sum_{j=1}^{n} (q_j/r_{jo}^3) r_{jo}$ ------ (3)

In continuous distribution of charge, the summation can be changed to integration,

where r represents the vector distance from the element of integration dq to the point at which E is evaluated.

## **ELECTRIC POTENTIAL**

The electric field at a point due to a point charge q is given by  $E = 1/4\pi\epsilon_0 \frac{r}{r^3}$ 

But as grad 
$$(1/r) = -(\frac{r}{r^3})$$
  
 $E = \frac{q}{4\pi\varepsilon_0} \left[-grad\left(\frac{1}{r}\right)\right]$   
 $E = -grad\left(\frac{q}{4\pi\varepsilon_0 r}\right)$   
 $E = -grad V$  with  $V = \frac{q}{4\pi\varepsilon_0 r}$  ------ (1)

The electric field can be described in terms of a scalar function of position V. The scalr function of position whose negative gradient gives the electric field is called electrostatic potential.

In the light of eqn. (1) it is apparent that  $\int_{ref}^{R} grad(V) dr = -\int_{ref}^{R} E dr - - - - (2)$ 

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The limit ref stands for reference point for which V is known.

But grad V = 
$$i \frac{\partial v}{\partial x} + j \frac{\partial v}{\partial y} + k \frac{\partial v}{\partial z}$$
  
and dr =  $i dx + j dy + k dz$   
grad V. dr =  $\frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz$   
i.e. (grad V). dr = dV  
eqn., (2) reduces to  $\int_{ref.}^{V} dV = -\int_{ref.}^{R} E. dr$  ----- (3)

This eqn shows that the potential at any point may physically interpreted as the work done in moving a unit positive charge from some reference point to the point at which it is to be evaluated against the direction of the field. The unit of electric potential in M.K.S system is Joule/ coul which is called volt.

If the reference point is taken at infinity where the potential is assumed to be zero and the electric field is assumed to be due to a point charge then eqn (3) reduces to

$$\int^{V} dV = -\int_{\infty}^{R} \left( \frac{q}{4\pi\varepsilon_{0}} \frac{r}{r^{3}} \right) dr$$

$$V = -\int_{\infty}^{R} \left( \frac{q}{4\pi\varepsilon_{0}} \frac{dr}{r^{2}} \right)$$

$$V = \frac{1}{4\pi\varepsilon_{0}} \frac{q}{R} - \dots \dots (4)$$

In case of a discrete distribution of charges the potential at a point by principle of superposition will be  $V = \sum_{j=1}^{n} V_j = \frac{1}{4\pi\varepsilon_0} \sum_{R_j}^{q_j} - - - -(5)$ 

Where  $R_i$  is the distance of the point from the charge  $q_i$ .

If there is a continuous distribution of charge within a finite volume V =  $\frac{1}{4\pi\epsilon_0}\int \frac{dq}{R}$ 

$$V_{(r)} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')dr}{R} - - - -(6)$$

## GAUSS' LAW FOR ELECTRICITY

The electric flux out of any closed surface is proportional to the total charge enclosed within the surface.

The integral form of Gauss' Law finds application in calculating electric fields around charged objects.

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In applying Gauss' law to the electric field of a point charge, one can show that it is consistent with Coulomb's law.

While the area integral of the electric field gives a measure of the net charge enclosed, the divergence of the electric field gives a measure of the density of sources. It also has implications for the conservation of charge.

**Integral Form** 

$$\oint \vec{E}.\vec{dA} = \frac{q}{\varepsilon_0} = 4\pi kq$$

**Differential form** 

$$\nabla . E = \frac{\rho}{\varepsilon_0} = 4\pi k\rho$$

## GAUSS' LAW FOR MAGNETISM

The net magnetic flux out of any closed surface is zero. This amounts to a statement about the sources of magnetic field. For a magnetic dipole, any closed surface the magnetic flux directed inward toward the south pole will equal the flux outward from the north pole. The net flux will always be zero for dipole sources. If there were a magnetic monopole source, this would give a non-zero area integral. The divergence of a vector field is proportional to the point source density, so the form of Gauss' law for magnetic fields is then a statement that there are no magnetic monopoles

> Integral form,  $\oint \vec{B} \cdot \vec{dA} = 0$ Differential form,  $\Delta \cdot B = 0$

## POLARIZABILITY

Dielectric polarization occurs when a dipole moment is formed in an insulating material because of an externally applied electric field. When a current interacts with a dielectric (insulating) material, the dielectric material will respond with a shift in charge distribution with the positive charges aligning with the electric field and the negative charges aligning against it. By taking advantage of this response, important circuit elements such as capacitors can be made.

Dielectric polarization is the term given to describe the behavior of a material when an external electric field is applied on it. A simple picture can be made using a capacitor as an

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example. The charges in the material will have a response to the electric field caused by the plates.



Using the capacitor model, it is possible to define the relative permittivity or the dielectric constant of the material by setting its relative permittivity equivalent to the ratio of the measured capacitance and the capacitance of a test capacitor, which is also equal to the absolute permittivity of the material divided by the permittivity of a vacuum.

$$\epsilon_r = Q/Q_0 = C/C_0 = \epsilon/\epsilon_0$$
 (1)

The dielectric constant is an important term, because another term known as the electronic polarizability or  $\alpha_e$  can be related to the dielectric constant. The electronic polarizability is a microscopic polarization phenomena that occurs in all materials and is one of the main mechanisms that drives dielectric polarization.

To explain how the dielectric constant relates to the electronic polarizability of a material, the polarization or P of a material should be determined. The polarization of a material is defined as the total dipole moment per unit volume, and its equation is,

 $P=N\alpha e E=\chi_e \epsilon_0 E - \dots (2)$ 

where the  $\chi$  term is known as the electric susceptibility of the material given by the equation  $\chi = \epsilon_r - 1$ . Then, from substituting  $\epsilon_r - 1$  for  $\chi$ , an equation relating the relative permittivity and the electronic polarizability is determined.

## $\epsilon_r = 1 + N \epsilon_e / \epsilon_0$

Where N is the number of molecules per unit volume.

While this equation does relate the dielectric constant with the electronic polarizability, it only represents the material as a whole, and does not take into effect the local field, or the field experienced by a molecule in a dielectric. This field is known as the Lorentz field, and the equation to define this is given as,

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 $E_{\text{loc}} = E + 1/3\epsilon_0 P \dots (3)$ 

and by substituting this value back for the field used in the previous method, the following equation is determined

## $\epsilon_r - 1/\epsilon_r + 2 = N\alpha_e/3\epsilon_0 \dots (4)$

This equation is known as the **Clausius-Mossotti equation** and is the way to interchange between the microscopic property of electronic permittivity and the dielectric constant. In addition to knowing the electronic polarizability of a material, there are also other sub-factors, such as chemical composition and bond type that determine the total dielectric behavior of a material. However, electronic polarization is always inherent in a dielectric material.

## **Ionic Polarization**

Ionic polarization is a mechanism that contributes to the relative permittivity of a material. This type of polarization typically occurs in ionic crystal elements such as NaCl, KCl, and LiBr. There is no net polarization inside these materials in the absence of an external electric field because the dipole moments of the negative ions are canceled out with the positive ions. However, when an external field is applied, the ions become displaced, which leads to an induced polarization.



The equation to describe this effect is given by,

 $P_a v {=} \alpha_i E_{loc} \ {\dots} \ {(5)}$ 

Where  $P_{av}$  is the induced average dipole moment per ion pair,  $\alpha i \alpha i$  is the ionic polarizability, and E is the local electric field experienced by the pair of ions. Usually the ionic polarizability is greater than the electronic polarizability by a factor of 10 which leads to ionic substances having high dielectric constants. Similar to electronic polarization, ionic polarization also has a total polarization associated with it. The equation is given by

 $P=N_iP_av=N_i\alpha_iE_{loc}$  (6)

which will also lead to a Clausius-Mossotti equation for ionic polarization,

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 $\epsilon_r - 1/\epsilon_r + 2 = 1/3\epsilon_0 N_i \alpha_i \dots (7)$ 

Note that these equations assume that there is a charge balance inside the ionic material (eg. NaCl) whereas if a charge imbalance is present, such as in materials like  $CaF_2$ , a different set of equations must be used.

## **Orientational Polarization**

Orientational polarization arises when there is a permanent dipole moment in the material. Materials such as HCl and  $H_2O$  will have a net permanent dipole moment because the charge distributions of these molecules are skewed. For example, in a HCl molecule, the chlorine atom will be negatively charged and the hydrogen atoms will be positively charged causing the molecule to be dipolar. The dipolar nature of the molecule should cause a dipole moment in the material, however, in the absence of an electric field, the dipole moment is canceled out by thermal agitation resulting in a net zero dipole moment per molecule. When an electric field is applied, however, the molecule will begin to rotate to align the molecule with the field, causing a net average dipole moment per molecule.



To determine if the induced average dipole moment along the electric field, the average potential energy of the dipole must be calculated and compared to the average thermal energy 1/5kT, as determined by thermodynamics for five degrees of freedom. To accomplish this, Force F torque  $\tau$  acting on the rigid body of the dipolar molecule. Using this model, the equation for the torque is given by

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#### $\tau = (Fsin\theta)a = EP_0 sin(\theta) \dots (8)$

where  $P_0$  is given by  $P_0=aQ$ . From this equation the maximum potential energy can be calculated by taking the integral at the instant of maximum torque. This maximum potential energy is calculated out to be  $2P_0E$  which then means that the average dipole potential energy is  $1/2E_{max}$  or  $P_0E$ . Once knowing the average dipole potential energy, it is then possible to calculate the average dipole moment  $P_{av}$  through Boltzmann's statistics, which would lead to the answer,

$$P_a v = P_0^2 E/3kT \dots (9)$$

This, leads to the dipolar orientational polarizability,  $\alpha_d$  per molecule, which is shown by the equation,

## $\alpha_d = P_0^2/3kt$

The equation for orientational polarizability shows, that unlike electronic polarizability and ionic polarizability, orientational polarizability is temperature dependent. This is an important factor to consider when choosing a dielectric material for electronic and optical applications.

### **Interfacial Polarization**

Interfacial or space charge polarization occurs when there is an accumulation of charge at an interface between two materials or between two regions within a material because of an external field. This can occur when there is a compound dielectric, or when there are two electrodes connected to a dielectric material. This type of electric polarization is different from orientation and ionic polarization because instead of affecting bound positive and negative charges i.e. ionic and covalent bonded structures, interfacial polarization also affects free charges as well. As a result interfacial polarization is usually observed in amorphous or polycrystalline solids. The electric field will cause a charge imbalance because of the dielectric material's insulating properties. However, the mobile charges in the dielectric will migrate over maintain charge neutrality. This then causes interfacial polarization.



The equation to show the space charge polarizability constant is

$$\alpha_c = \alpha - \alpha_\infty - \alpha_0$$

Electric Field (E)

where  $\alpha_c$  is the space charge polarizability and  $\alpha$ ,  $\alpha_{\infty}$ , and  $\alpha_0$  refer to the total, electronic, and orientational polarizations respectively. It is important to note that because the charges are free charges, defects such as grain boundaries or other interfaces can serve as a medium for interfacial polarizability to form.

From the equation of space charge polarization, it is then determined that the total amount of dielectric polarization in a material is the sum of the electronic, orientations, and interracial polarizabilities, or  $\alpha = \alpha_c + \alpha_\infty + \alpha_0$ .

## **CLAUSIUS-MOSSOTTI RELATION**

According to Lorentz eqn., the electric field on single molecule of non-polar isotropic dielectric when in a macroscopic field E is given by  $E_m = E + P/3\epsilon_0$  ------(1)

Under the action of this field, if the induced dipole moment in the molecule is  $p_m$  then by definition of polarisibility,

If there are n molecule per unit volume of the dielectric, then polarization will be

$$P=n p_m = n \alpha (E + P/3\epsilon_0)$$

$$P\left[\frac{1}{n\alpha} - 1/3\epsilon_0\right] = E \dots (4)$$

By definition

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 $P = \epsilon_0 \chi_e E = \epsilon_0 (\epsilon_r - 1) E \dots (5)$ 

So eliminating P between eqn. (4) and (5)

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{n\alpha}{3\epsilon_0} \qquad \dots \dots (A)$$

Now if M is molecular weight of the material  $\rho$  its density and N Avagadro's no. The no. of molecules per unit volume n= N  $\rho$  /M. So eqn. A reduces to

$$\frac{M}{\rho} = \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{n\alpha}{3\epsilon_0} \qquad \dots \dots \dots (B)$$

Eq. A or B is the desired result and it is called Clausius- Mossotti relation. This eqn. relates the microscopic property, polarisability,  $\alpha$  to the macroscopic property, dielectric constant and it is applicable to non polar liquids. From eqn b it is clear that  $\frac{M}{\rho} = \frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{n\alpha}{3\epsilon_0}$  is constant if then polarization is valid, this eqn of Claussius Mossotti eqn has been satisfactorily for a number of gases

## **Discussion:**

i) In cases of transparent dielectric as according to electromagnetic theory the refractive index

So Clausius-mossotti en reduces to

 $\frac{M}{\rho}\left(\frac{n^2-1}{n^2+2}\right)$  =constant this law is known as Lorentz- Lorentz formula

ii) In case of gases as the behavior of each molecule is completely independent of the action of all others  $\epsilon r \rightarrow 1$ 

## Eqn A reduces to $\alpha = \epsilon_0 \chi_e/n$

iii) For most o the gases and liquids due to due to small polarisability  $n\alpha/3\epsilon_0$  is less than one. However in few crystalline solid such as barium titanate  $n\alpha/3\epsilon_0$  tends to one so that  $\epsilon_r$  tends to infinity such dielectrics materials are called Ferro electric sand exhibit the properties of spontaneous polarization analog to Ferro magnetic materials

iv) According to Clausius- mossotti eqn for mono atomic gas

$$\mathbf{R}^3 = \varepsilon_r - 1/4\pi \mathbf{n}.$$

## DIELECTRIC CONSTANT

Electric displacement vector is defined by

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And also  $P = \epsilon_0 E \chi_e$  ----- (2)

By eliminating P between the above eqn.,  $D = \epsilon_0 (1 + \chi_e) E$ 

With  $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_0 (1 + \chi_e)$  ------ (4)

 $D = \epsilon E$  ----- (3)

The parameter  $\varepsilon$  is called permittivity of the dielectric. So the permittivity of dielectric relative to free space will be given by  $\epsilon_r = \epsilon/\epsilon_0 = (1 + \chi_e)$  (5)

And is called relative permittivity, dielectric constant or specific inductive capacity and also represented by k. Regarding dielectric constant it must be noted that

- (i) For isotropic dielectrics it is a scalar having no units and dimensions.
- (ii) It is always greater than 1 as  $\epsilon > \epsilon_0$
- (iii) In terms of it the displacement vector D is given by  $D = \epsilon E = \epsilon_r \epsilon_0 E$ .
- (iv) It depends on the nature of the material.
- In case of polar molecules it depends on frequency of applied field and decreases with increase in frequency.

## ELECTRIC DISPLACEMENT D

According to Gauss law in a region of charge density  $\rho$  div  $E = \rho / \epsilon_0$ 

In a dielectric of infinite extent having free charge density  $\rho$  and bound charge density  $\rho'$ , by the principle of superposition, div  $E = (\rho + \rho')/\epsilon_0$ 

Or div (
$$\epsilon_0 E + P$$
) =  $\rho$  ------ (1)

If define a vector D by  $D = \epsilon_0 E + P$  ------ (2)

Eqn., (1) reduces to div  $D = \rho$  ------ (3)

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Eqn., (3) is called differential form of Gauss's law for dielectrics. The vector D is defined by Eqn., (2) is called electric displacement. It is note that

(i) It has the same unit as P (coulomb/ $m^2$ ).

(ii) Its divergence is equal to the free charge density div  $D = \rho$ .

(iii)In free space i.e. vacuum as P = 0,  $D = \epsilon_0 E$ .

## LANGEVIN EQUATION

Langevin modified the Clausius –Mossotti relation for polar molecules. The atom has a permanent dipole moment p0 and the only force acting on it due to the field is Em. The couple acting on the dipole whose axis subtends an angle  $\theta$  with the field.

 $C=q \ 2l \sin \theta E_m = p_0 E_m \sin \theta$ 

So the work done on the dipole in a small rotation  $d_{\theta}$ ,  $dW = Cd\theta = p_0 E_m \sin \theta \ d\theta$ 

W= -  $p_0 E_m \cos \theta$  ------ (1)

This work is store by the dipole as the potential energy.

On the basis of the classical statistical mechanics, the number of molecules per unit volume whose axes makes an angle  $\theta$  with the field is proportional to  $e^{-W/kT}$ , where k is the Boltzmann constant and T is the temperature in degree Kelvin. Hence the number of dipoles per unit volume whose axes makes an angle  $\theta$  and  $d\theta$  within the solid angle will be

 $dn = Ae^{-W/kT} d\omega$  $n = 2\pi A \int_0^{\pi} e^{-W/kT} \sin \theta \ d\theta \dots (2)$ 

dn particles contribute a component of electric moment  $p_0 \cos \theta$  partallel to the field

while by symmetry the component perpendicular to the field neutralize one another. Hence the polarization of the atoms  $P = 2\pi A$  ------ (3)  $\int_{0}^{\pi} (p0\cos\theta) \ e^{-W/kT}\sin\theta \ d\theta$ 

From equation (2) and (3)  $p/n = \frac{p0 \int_0^{\pi} (\cos\theta) e^{-W/kT} \sin\theta \, d\theta}{\int_0^{\pi} e^{-W/kT} \sin\theta \, d\theta}$  ------ (4)

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if we take,  $p_0 E_m / kT = u$  and  $u \cos \theta = t$ 

or P = Ps [ coth u - 1/u ] = PsL(u) ------(A)

where  $Ps = nP_0$  is the saturation value of polarization. Eqn., (A) is known as Langevin equation.

## **ELECTROSTATIC ENERGY:**

The work done in assembling the charge from infinity to establish the distribution is called the electrostatic energy of the field. Assume that the density of the initial charge distribution is 0 and the charge is brought uniformly from infinity so that at any time

 $\int$  the charge density is  $\propto P$  where  $\propto$ 

is a parameter lying in the range from 0 to 1.When the density from the  $\propto$  P the potential

$$V' = \propto V$$
 (as  $v \propto p$ ) (1)

If we increases the charge density at every point in the distribution from  $\propto P$  to  $(\propto +d \propto)P$  the charge in volume element  $d\tau$  increases by

And the energy supplied to the system in adding charge dq to the particular volume element is

Sub. The values of v'and dq from eqn (1) and (2) respect in (3) if repeat the contribution for all the elements in  $v_0/\tau$ 

$$du = (\propto v) (d \propto \rho d\tau) \qquad \dots \dots (4)$$

If repeat the orientation for all the elements in  $v_0/\tau$ 

As

 $du=\int \propto v\rho d \propto d\tau$ 

To increases the charge density to 1 from 0 everywhere

$$U=\int_0^1 \propto d \propto \int_\tau \rho v d\tau$$

- - Or,  $U=1/2\int_{\tau} VDiv Dd\tau$

div (SA) = S div A + A. grad S

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Div (VD) = V div D + D grad V

Therefore

 $u=1/2 \oint V.D ds+1/2 \int E.D d\tau$ 

As  $\tau$  can be any volume which includes all the charge in the system. So if the volume  $\tau$  extends to infinity i.e. all space, the surface will vanish and the surface contribution. This in turn means

 $U=1/2\int_{all \ space} E.Dd\tau$ 

1. The interpretation of eqn (B) is certainly possible throughout the field with an energy density

$$U_E = du/d\tau = 1/2 E.D$$

2. In case of free spaces as

U= 
$$1/2 \int [E0E.D] d\tau = 1/2 \varepsilon_0 E^2 d\tau$$

And electrostatic energy will be,

 $U_E = 1/2\epsilon_0 E^2$ 

3. If the distribution is discrete

$$\int_{all \ space} E^2 \mathrm{d}\tau = \sum_{j=1}^n E j^{\wedge} 2 \sum_{j=1}^n E j \sum_{j=1}^n E j$$

For the first term the position of other charges is immaterial for any point i.e. this term represents the work done in the creation of charges and is called self-energy  $u_0$ 

 $U=u_0+E_0/2(\sum_{j=1}^n \sum_{j=1}^n E_J.E_J)$  with  $u_0=E_0/2(\sum_{j=1}^n E_J2)$ 

Second terms represents the work done by the charge in bringing then from infinity to the space to constitute the given distribution of charges

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## **Possible Questions**

## 8 marks

- 1. Explain electric potential and intensity.
- 2. Derive Clausius Mossotti equation.
- 3. State and explain Gauss law.
- 4. What is called electric displacement? Discuss it.
- 5. Obtain Langevin equation for polar molecules.
- 6. What is called dielectric? Explain dielectric and its polarization.
- 7. Explain different types of polarization.
- 8. Explain electrostatic energy.
- 9. Obtain Lorentz-Lorentz equation for non polar molecules.

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#### DEPARTMENT OF PHYSICS

III BSc PHYSICS

BATCH: 2015-2018

#### ELECTROMAGNETIC THEORY (15PHU604A)

#### MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	Answer
UNIT-I					
charge enclosed by it.	ε <sub>0</sub>	$\mu_0$	$1/\epsilon_0$	$1/\mu_0$	$1/\epsilon_0$
The differential form of Gauss's law is	div $E = \rho/\epsilon_0$	-div $E = \rho/\epsilon_0$	$\operatorname{div} \mathbf{E} = 0$	div $E = \rho$	div $E = \rho/\epsilon_0$
The insulators whose behaviour gets modified in an electric field are called					
	semiconductor	superconductor	p-type semiconductor	dielectrics	dielectrics
of the applied field, the dielectric is called	anisotrophic	isotrophic	heterogeneous	homogeneous	isotrophic
applied field, the dielectric is called	anisotrophic	isotrophic	heterogeneous	homogeneous	anisotrophic
acquires an	moment	electric polarization	electronic polarization	induced dipole moment	induced dipole moment
The induced dipole moment per unit volume is called	orientation polarization	electronic polarization	polarization	none of the above	polarization
Dielectric polarization is proportional to	applied electric field	applied magnetic field	field	applied electrostatic field	applied electric field
The unit of polarization is	coul/m	coul/m <sup>2</sup>	coul/m <sup>3</sup>	coul <sup>2</sup> /m <sup>2</sup>	coul/m <sup>2</sup>
Charge appear on the dielectric surface is	bound charge density	volume charge density	free charge density	surface charge density	surface charge density
The charge appear throughout the volume of a dielectric is	bound charge density	volume charge density	free charge density	surface charge density	volume charge density
The total bound charge on a dielectric is	1	-1	0	Same	0
E =	–grad V	Div V	Curl V	Grad V	–grad V
The total space-time average electric field acting on a single molecule of a	electric field	magnetic field	molecular field	electrostatic field	molecular field
field E.	lesser	greater	equal	neither lesser nor greater	greater
E <sub>m</sub> =	$-E + P/3\varepsilon_0$	-Ε - Ρ/3ε <sub>0</sub>	E - P/3ε <sub>0</sub>	$E + P/3\epsilon_0$	$E + P/3\varepsilon_0$
Gauss's law for dielectric is	div $D = \rho$	div D = -ρ	$\operatorname{div} \mathbf{D} = 0$	div D = B	div D = $\rho$
In free space i.e., vacuum, P =	1	-1	2	0	0
The unit for electric displacement D is	coul <sup>2</sup> /m	coul/m <sup>2</sup>	coul/m <sup>3</sup>	coul <sup>2</sup> /m <sup>2</sup>	coul/m <sup>2</sup>
In case of transparent dielectric, the refractive index n =	√ε <sub>ρ</sub>	ε <sub>r</sub>	-E <sub>r</sub>	$\epsilon_r^2$	√ε <sub>ρ</sub>
Dielectric materials are called	ferromagnetic	paramagnetic	ferroelectric	diamagnetic	ferroelectric
For isotrophic dielectrics, the dielectric constant has	no value	constant value	infinite value	no dimensions	no dimensions
frequency of applied field.	increases	decreases	is proportional	is same	decreases
Dipole moment acquired by a molecule per unit polarizing field is called	displacement current	current density	polarizability	charge density	polarizability
Clausius-Mossotti relation is	$(e_r - 1/e_r + 2) = (na/3e_0)$	$-(e_r - 1/e_r + 2) = (na/3e_0)$	$(e_r - 1/e_r - 2) = (na/3e_0)$	$(e_r + 1/e_r + 2) = (na/3e_0)$	$(e_r - 1/e_r + 2) = (na/3e_0)$
In case of gases, Clausius-Mossotti relation is	$-(e_r - 1/3) = (na/3e_0)$	$-(e_r + 1/3) = (na/3e_0)$	$-(e_r - 1/4) = (na/3e_0)$	$(e_r - 1/3) = (na/3e_0)$	$(e_r - 1/3) = (na/3e_0)$
Langevin equation is applicable to	non-polar molecules	polar molecules	complex molecules	simple molecules	polar molecules
polarisability of polar molecules is proportional to absolute	inversely	directly	not	infinitely	inversely
Debye relation is used to study the structure of	atoms	molecules	bond	ionic bond	molecules
In H <sub>2</sub> O, the center of +ive and -ive charge coincide.	is supposed to	changed and again	always	do not	do not
distribution is called energy.	electrostatic	electromagnetic	electric	magnetic	electrostatic
The divergence of electric displacement D is equal to	surface charge density	free charge density	volume charge density	bound charge density	free charge density
Dielectric constant of any medium is always than permittivity.	greater	lesser	neither greater not lesser	neither lesser nor greater	greater
Clausius-Mossotti relation is applicable to	polar liquids	solids	gases	rigid solids	gases

Clausius-Mossotti relation is applicable to	polar liquids	non-polar liquids	solids	rigid solids	non-polar liquids
Clausius-Mossotti relation relates microscopic property polarisability to the	dielectric constant	electric displacement D	permittivity	permeability	dielectric constant
CCl <sub>4</sub> is	complex molecule	simple molecule	polar	non-polar	non-polar
CO <sub>2</sub> is	non-polar	complex	simple	polar	non-polar
polarisation is	elliptical	circular	linear	perpendicular	linear
polarisation?	elliptical	circular	linear	perpendicular	circular
be	elliptical	circular	linear	parallel	elliptical
For a non-zero Ex component and zero Ey component, the polarisation is	elliptical	circular	linear	parallel	parallel
Polarization occurs due to	magnesium in dilute acid	magnesium in conc. Acid	covered with bubbles of	covered with bubbles of	covered with bubbles of
The best definition of polarisation is	random direction	per unit volume	moments	every dipole	per unit volume
Polarizability is defined as the	and electric field	electric field	dipole moment	constant and dipole	electric field
Identify which type of polarisation depends on temperature.	Electronic	ionic	orientational	interfacial	orientational
In the given types of polarisation, which type exists in the semiconductor?	Electronic	ionic	orientational	interfacial	interfacial
Solids do not have which type of polarisation?	Electronic	ionic	orientational	interfacial	interfacial
Many ionic compounds have some covalent ability due to	Electronic	ionic	proton	interfacial	ionic
A dielectric can be made a conductor by	Compression	Heating	Doping	Freezing	Heating
The magnetic field which destroys the superconductivity is called	Diamagnetic field	Ferromagnetic field	Ferrimagnetic field	Critical field	Critical field
Magnetic materials which store energy are called	magnetic materials	semi conductors	dielectric	super conductors	dielectric
The dielectric constant of air is practically taken as	more than unity	less than unity	unity	zero	unity
dielectric materials are	insulating materials	semiconducting materials	magnetic material	ferromagnetic materials	insulating materials
Dielectric constant of vacuum is	infinity	100	one	zero	one
On placing the dielectric in an electric field, the field strength	increases	decreases	remains constant	reduces to zero	decreases
Mica is a	dielectric material	insulating material	magnetic material	dielectric	dielectric
How does ionic polarisation occur?	Splitting of ions	Passing magnetic field	and anions	Never occurs	and anions

Physics, KAHE, Coimbatore.

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## UNIT-II

## SYLLABUS

Magnetostatics: Current density J – Ampere's law of force – Biot-Savart law – Ampere's circuital

law – Magnetic scalar potential  $\phi$ m (no applications) – Magnetic vector potential A – Magnetisation

and magnetization current – Magnetic intensity – Magnetic susceptibility and Permeability.

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## **CURRENT DENSITY**

In the field of electromagnetism, Current Density is the measurement of electric current (charge flow in amperes) per unit area of cross-section  $(m^2)$ . This is a vector quantity, with both a magnitude (scalar) and a direction.

J = I/A

 $J = current density in amperes/m^2$ 

I = current through a conductor, in amperes

A = cross-sectional area of the conductor,  $m^2$ 

## AMPERE'S LAW OF FORCE

In magnetostatics, the force of attraction or repulsion between two current-carrying wires (see first figure below) is often called **Ampere's force law**. The physical origin of this force is that each wire generates a magnetic field, as defined by the Biot-Savart law, and the other wire experiences a magnetic force as a consequence, as defined by the Lorentz force.

### Equation

The best-known and simplest example of Ampère's force law, which underlies the definition of the ampere, the SI unit of current, states that the force per unit length between two straight parallel conductors is

$$F_m = 2k_A rac{I_1 I_2}{r},$$

where  $k_A$  is the magnetic force constant, r is the separation of the wires, and  $I_1$ ,  $I_2$  are the direct currents carried by the wires. This is a good approximation for finite lengths if the distance between the wires is small compared to their lengths, but large compared to their diameters. The value of  $k_A$  depends upon the system of units chosen, and the value of  $k_A$  decides how large the unit of current will be.



In the SI system,



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with  $\mu_0$  the magnetic constant, *defined* in SI units as  $\mu_0 \stackrel{\text{def}}{=} 4\pi \times 10^{-7}$  newtons / (ampere)<sup>2</sup>.

Thus, in vacuum, the force per meter of length between two parallel conductors – spaced apart by 1 m and each carrying a current of 1 A is exactly  $2 \times 10^{-7}$  N/m.

The general formulation of the magnetic force for arbitrary geometries is based on line integrals and combines the Biot-Savart law and Lorentz force in one equation as shown below.

$$ec{F_{12}} = rac{\mu_0}{4\pi} \int_{L_1} \int_{L_2} rac{I_1 dec{\ell}_1 imes (I_2 dec{\ell}_2 imes \hat{\mathbf{r}}_{21})}{|r|^2}$$

where

- $\vec{F}_{12}$  is the total force felt by wire 1 due to wire 2 (usually measured in newtons),
- $I_1$  and  $I_2$  are the currents running through wires 1 and 2, respectively (usually measured in amperes),
- The double line integration sums the force upon each element of wire 1 due to the magnetic field of each element of wire 2,
- $\overrightarrow{dl_1}$  and  $\overrightarrow{dl_2}$  are infinitesimal vectors associated with wire 1 and wire 2 respectively (usually measured in metre); see line integral for a detailed definition,

The vector  $\hat{\mathbf{r}}_{21}$  is the unit vector pointing from the differential element on wire 2 towards the differential element on wire 1, and /r/ is the distance separating these elements.

The multiplication  $\times$  is a vector cross product,

• The sign of  $I_n$  is relative to the orientation  $\overline{dl_n}$  (for example, if  $\overline{dl_1}$  points in the direction of conventional current, then  $I_1>0$ ).

To determine the force between wires in a material medium, the magnetic constant is replaced by the actual permeability of the medium.

## **BIOT-SAVART LAW**

The Biot-Savart Law relates magnetic fields to the currents which are their sources. In a similar manner, Coulomb's law relates electric fields to the point charges which are their sources. Finding the magnetic field resulting from a current distribution involves the vector product, and is inherently a calculus problem when the distance from the current to the field point is continuously changing.

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 $\vec{dL}$  –Infinitesimal length of conductor carrying electric current I

 $\vec{l_r}$  –Unit vector tospecify the direction of the vector distance r from the current to the field point.

## **AMPERE'S CIRCUITAL LAW**

Ampere's Circuital Law states the relationship between the current and the magnetic field created by it. This law says, the integral of magnetic field density (B) along an imaginary closed path is equal to the product of current enclosed by the path and permeability of the medium.

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 \ I$$

It alternatively says, the integral of magnetic field intensity (H) along an imaginary closed path is equal to the current enclosed by the path.

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 I$$

$$\Rightarrow \oint \frac{\overrightarrow{B}}{\mu_0} \cdot \overrightarrow{dl} = I$$

$$\Rightarrow \oint \overrightarrow{H} \cdot \overrightarrow{dl} = I$$

$$\left[ \because \overrightarrow{H} = \frac{\overrightarrow{B}}{\mu_0} \right]$$

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Let us consider an electrical conductor, carrying a current of I ampere. Let us take an imaginary loop around the conductor. This loop is called as Amperian loop, also imagine the radius of the loop is r and the flux density created at any point on the loop due to current through the conductor is B. Consider an infinitesimal length dl of the Amperian loop at the same point. At each point on the Amperian loop, the value of B is constant since the perpendicular distance of that point from the axis of conductor is fixed, but the direction will be along the tangent on the loop at that point. The close integral of the magnetic field density B along the amperian loop, will be,

A B. dl

$$\int dl = B \cdot (2\pi r)$$

Now, according to Ampere's Circuital Law

$$\oint B \cdot dl = \mu_0 \cdot I$$

Therefore,

$$2\pi r B = \mu_0 I$$
$$\Rightarrow \frac{B}{\mu_0} = \frac{I}{2\pi r}$$
$$\Rightarrow H = \frac{I}{2\pi r}$$

Instead of one current carrying conductor, there are N number of conductors carrying same current I, enclosed by the path, then

COURSE CODE. 151 HO004A	$H = \frac{NI}{2\pi r}$	DATCH-2013-2016
	$H = rac{NI}{2\pi r}$	
	$H = \frac{111}{2\pi r}$	
	2	
MAGNETIC SCALAR AND VE	CTOR POTENTIALS	
Let us relate the magnetic f	ield intensity to a scalar magnetic	<b>c potential</b> and write:
-		
H =	$= -\nabla V_{m}$ (1)	
Fro	m Ampere's law, we know that	
V×1	$H = J \qquad (2)$	
Therefore,	$(\vee \times (-\vee V_m) = J$	(3)
		-
But using vector identity, V	$7 \ge (\nabla V) = 0$ we find that $\vec{H} = -\nabla V$	$V_{\rm m}$ is valid only where $J = 0$
		_
Thus the scalar magnetic potentia	l is defined only in the region w	here $\dot{J} = 0$ . Moreover, $V_m$ in
general is not a single valued funct	ion of position.	
Let us consider the cross se	ection of a coaxial line. In the	
$\vec{H} = \frac{I}{I}$	$-\hat{a}_{\phi}$	
region $a < \rho < b$ , $\vec{j} = 0$ and $2\pi$	o '	
	A y	

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If  $V_m$  is the magnetic potential then,

$$-\nabla V_m = -\frac{1}{\rho} \frac{\partial V_m}{\partial \phi}$$
$$= \frac{I}{2\pi\rho}$$

$$\therefore V_m = -\frac{I}{2\pi}\phi + c$$

If we set  $V_m = 0$  at  $\phi = 0$  then c=0 and  $V_m = -\frac{1}{2}$ 

$$\therefore \text{At } \phi = \phi_0 \qquad V_m = -\frac{I}{2\pi}\phi_0$$

The lap around the current carrying conductor is complete, reach  $\overset{\text{Ph}}{\to}$  again but  $V_m$  becomes

$$V_m = -\frac{I}{2\pi}(\phi_0 + 2\pi)$$

The value of  $V_m$  keeps changing as additional laps to the same point.  $V_m$  analogous to electostatic potential V. But for static electric fields,  $\nabla \times \vec{E} = 0$  and  $\oint \vec{E} \cdot d\vec{l} = 0$ , whereas for steady magnetic field  $\nabla \times \vec{H} = 0$  wherever  $\vec{J} = 0$  but  $\oint \vec{H} \cdot d\vec{l} = I$  even if  $\vec{J} = 0$  along the path of integration.

Thee **vector magnetic potential** which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.

Since  $\nabla \cdot \vec{B} = 0$  and we have the vector identity that for any vector  $\vec{A}$ ,  $\nabla \cdot (\nabla \times \vec{A}) = 0$ , we can write  $\vec{B} = \nabla \times \vec{A}$ .

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Here, the vector field  $\vec{A}$  is called the vector magnetic potential. Its SI unit is Wb/m. Thus if can find  $\vec{A}$  of a given current distribution,  $\vec{B}$  can be found from  $\vec{A}$  through a curl operation. The vector function  $\vec{A}$  and related its curl to  $\vec{B}$ . A vector function is defined fully in terms of its curl as well as divergence. The choice of  $\nabla \cdot \vec{A}$  is made as follows.

$$\nabla \times \nabla \times \vec{A} = \mu \nabla \times \vec{H} = \mu \vec{J} \tag{4}$$

By using vector identity,  $\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$  .....(5)

 $\nabla(\nabla \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J} \qquad (6)$ 

if we choose  $\nabla \cdot \vec{A} = 0$ .

substituting  $\nabla \vec{A} = 0$ , we get  $\nabla^2 \vec{A} = -\mu \vec{J}$  which is vector poisson equation. In Cartesian coordinates, the above equation can be written in terms of the components as

 $\nabla^2 A_x = -\mu J_x \dots (7a)$   $\nabla^2 A_y = -\mu J_y \dots (7b)$   $\nabla^2 A_z = -\mu J_z \dots (7c)$ 

The form of all the above equation is same as that of

for which the solution is

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In case of time varying fields we shall see that  $\nabla \vec{A} = \mu \varepsilon \frac{\partial V}{\partial t}$ , which is known as Lorentz condition, *V* being the electric potential. Here we are dealing with static magnetic field, so  $\nabla \vec{A} = 0$ .

By comparison, we can write the solution for  $A_x$  as

Computing similar solutions for other two components of the vector potential, the vector potential can be written as

This equation enables to find the vector potential at a given point because of a volume current density  $\vec{J}$ . Similarly for line or surface current density we can write

$$\vec{A} = \frac{\mu}{4\pi} \int_{\vec{k}'} \vec{R} d\vec{l}' \qquad .....(12)$$
$$\vec{A} = \frac{\mu}{4\pi} \int_{\vec{k}'} \vec{K} ds' \qquad respectively. .....(13)$$

The magnetic flux  $\psi$  through a given area S is given by

Substituting  $\vec{B} = \nabla \times \vec{A}$ 

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Vector potential thus have the physical significance that its integral around any closed path is equal to the magnetic flux passing through that path.

## MAGNETIC VECTOR POTENTIAL

Electric fields generated by stationary charges obey  $\nabla \times \mathbf{E} = \mathbf{0}$ . .....(1)

since the curl of a gradient is automatically zero. In fact, whenever we come across an irrotational vector field in physics we can always write it as the gradient of some scalar field. This is clearly a useful thing to do, since it enables us to replace a vector field by a much simpler scalar field. The quantity  $\phi$  in the above equation is known as the *electric scalar potential*.

Magnetic fields generated by steady currents (and unsteady currents, for that matter)

satisfy 
$$\nabla \cdot \mathbf{B} = 0$$
.....(3)

This immediately allows us to write  $\mathbf{B} = \nabla \times \mathbf{A}$ , ......(4)

since the divergence of a curl is automatically zero. In fact, whenever we come across a solenoidal vector field in physics we can always write it as the curl of some other vector field. This is not an obviously useful thing to do, however, since it only allows us to replace one vector field by another. Nevertheless, Eq. (4) is one of the most useful equations we shall come across in this lecture course. The quantity  $\mathbf{A}$  is known as the *magnetic vector potential*.

The curl of the vector potential gives us the magnetic field via Eq. (4). However, the divergence of  $\mathbf{A}$  has no physical significance. the magnetic field is invariant under the

transformation  $\mathbf{A} \to \mathbf{A} - \nabla \psi$ .\_\_\_\_(5)

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In other words, the vector potential is undetermined to the gradient of a scalar field. This is just another way of saying that we are free to choose  $\nabla$ .A. The electric scalar potential is undetermined to an arbitrary additive constant, since the transformation  $\phi \rightarrow \phi + c$  ...... (6)

leaves the electric field invariant. The transformations are examples call gauge transformations.

The choice of a particular function  $\psi$  or a particular constant c is referred to as a choice of the gauge. We are free to fix the gauge to be whatever we like. The most sensible choice is the one which makes our equations as simple as possible. The usual gauge for the scalar potential  $\phi$  is such that  $\phi \rightarrow 0$  at infinity. The usual gauge for **A** is such that  $\nabla$ .A=0 ----- (7). This particular choice is known as the *Coulomb gauge*.

It is obvious to add a constant to  $\phi$  so as to make it zero at infinity. Suppose that we have found some vector field A whose curl gives the magnetic field but whose divergence in non-zero. Let  $\nabla A = v(r)$  ----- (8). The question is, can we find a scalar field  $\psi$  such that after we perform

the gauge transformation we are left with  $\nabla$ .A. Taking the divergence of Eq. to find a

function  $\psi$  which satisfies  $\nabla^2 \psi = v$ ......(9)

But this is just Poisson's equation, we can always find a unique solution of this equation. This proves that, in practice, the divergence of  $\mathbf{A}$  equal to zero.

Let us again consider an infinite straight wire directed along the z-axis and carrying a current I. The magnetic field generated by such a wire is written

$$\mathbf{B} = \frac{\mu_0 I}{2\pi} \left( \frac{-y}{r^2}, \frac{x}{r^2}, 0 \right) .$$
(10)

To find a vector potential  $\mathbf{A}$  whose curl is equal to the above magnetic field, and whose divergence is zero. It is not difficult to see that

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$$\mathbf{A} = -\frac{\mu_0 I}{4\pi} \left( 0, 0, \ln[x^2 + y^2] \right)_{------} (11)$$

fits the bill. Note that the vector potential is parallel to the direction of the current. This would seem to suggest that there is a more direct relationship between the vector potential and the current than there is between the magnetic field and the current. The potential is not very well-behaved on the z-axis, but this is just because we are dealing with an infinitely thin current.

Let us take the curl of Eq. (11).

$$\nabla \times \mathbf{B} = \nabla \times \nabla \times \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = -\nabla^2 \mathbf{A},$$
(12)

where use has been made of the Coulomb gauge condition. We can combine the above relation

with the field equation to give  $\nabla^2 \mathbf{A} = -\mu_0 \mathbf{j}$ . (13)

By writing this in component form, we obtain  $\nabla^2 A_x = -\mu_0 j_x$ , ...... (14)

This is just Poisson's equation three times over. The unique solutions to the above equations:

$$A_{\mathbf{x}}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{j_{\mathbf{x}}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 \mathbf{r}',$$
......(17)

$$A_y(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{j_y(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 \mathbf{r}',$$
(18)

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$$A_{z}(\mathbf{r}) = \frac{\mu_{0}}{4\pi} \int \frac{j_{z}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^{3}\mathbf{r}'.$$
(19)

These solutions can be recombined to form a single vector solution

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{j}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 \mathbf{r}'.$$
(20)

$$\phi(\mathbf{r}) = \frac{1}{4\pi \epsilon_0} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3 \mathbf{r}'.$$
(21)

Equations (20) and (21) are the unique solutions (given the arbitrary choice of gauge) to the field equations. They specify the magnetic vector and electric scalar potentials generated by a set of stationary charges, of charge density  $\rho(\mathbf{r})$ , and a set of steady currents, of current density j(r). Incidentally, Eq. (20) satisfies the gauge condition  $\nabla \cdot \mathbf{A} = \mathbf{0}$  by repeating the  $\mathbf{C} \rightarrow \mu_0 \mathbf{j}$  analysis of (with W $\rightarrow$ J and ), and using the fact that  $\nabla \cdot \mathbf{j} = 0$  for steady currents.

## MAGNETISATION AND MAGNETIC INTENSITY

The Magnetic behavior of a magnet is characterized by the alignment of the atoms inside a substance. When a ferromagnetic substance is brought under the application of a strong external magnetic field, then they **experience a torque** wherein the substance **aligns themselves in the direction of the magnetic field applied** and hence gets strongly magnetized in the direction of the magnetic field.

### MAGNETIC INTENSITY OR INTENSITY OF MAGNETIC FIELD

The magnetic intensity at a point is defined as the force that unit north - Pole experiences when it is placed in that field.

The intensity of magnetic field at P due to single pole is given by:





### **INTENSITY OF MAGNETIZATION**

The Magnetic moment of a magnet undergoes a change when it is placed in a magnetic field. This change that is, the magnetic moment change per unit volume is known as in **Intensity of Magnetization**.

Formula of Intensity of Magnetization

$$I = \frac{\text{Magnetic Moment}}{\text{Volume}}$$
$$= \frac{M}{V}$$
$$= \frac{mX2}{AX2} \quad [\text{since } M = mX2| \text{ and } V = AX2]$$
$$= m/A$$

Where, m – Pole strength

A - Area of cross section

Intensity of Magnetisation Unit

The S.I unit of intensity of magnetisation is Ampere/metre or A/m

#### MAGNETIC SUSCEPTIBILITY AND PERMEABILITY

In a large class of materials, there exists an approximately linear relationship

between **M** and **H**. If the material is isotropic then  $\mathbf{M} = \chi_m \mathbf{H}$ , ..... (1)
where  $\chi_m$  is called the *magnetic susceptibility*. If  $\chi_m$  is positive then the material is called *paramagnetic*, and the magnetic field is strengthened by the presence of the material. On

the other hand, if  $\chi_m$  is negative then the material is *diamagnetic*, and the magnetic field is weakened in the presence of the material. The magnetic susceptibilities of paramagnetic and diamagnetic materials are generally extremely small.

A linear relationship between  $\mathbf{M}$  and  $\mathbf{H}$  also implies a linear relationship between  $\mathbf{B}$  and  $\mathbf{H}$ . In fact, we can write  $B=\mu H$  ------ (1)

where  $\mu = \mu_0(1+\chi_m)$  ----- (2)

is termed the *magnetic permeability* of the material in question. (Likewise,  $\mu_0$  is termed the *permeability of free space*.) Note that  $\mu$  has the same units as  $\mu_0$ . The permeabilities of common diamagnetic and paramagnetic materials do not differ substantially from the permeability of free space.

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#### **Possible Questions**

#### 6 marks

- 1. State Magnetic vector potential. Explain it briefly.
- 2. State Magnetic scalar potential. Explain it briefly.
- 3. State and derive Ampere circuital law.
- 4. Explain Ampere law of force.
- 5. State and derive Biot-Savart law.
- 6. Define Magnetization, magnetic intensity, Susceptibility, permeability and relativity.

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#### DEPARTMENT OF PHYSICS

III BSc PHYSICS

BATCH: 2015-2018

#### ELECTROMAGNETIC THEORY (15PHU604A)

#### MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	Answer
UNIT-II					
According to Ampere's law of force, the force between current					
carrying conductors varies as the produce of					
magnitudes of current.	inversely	independently	directly	infinitely	independently
According to Ampere's law of force, the force between current					
carrying conductors depends on of the medium.	colour	nature	property	length	nature
According to Ampere's law of force, the force is if					
the current flows in the same direction.	repulsive	infinite	attractive	finite	attractive
According to ampere's law of force, the force is if the					
current flows in the opposite direction.	attractive	infinite	finite	repulsive	4
The vector B is called vector.	magnetic flux	magnetic intensity	magnetic induction	magnetic force	magnetic induction
The unit of B is	Tesla	Web	Web/m	Web <sup>2</sup> /m	Tesla
The Biot-Savart law for B otherwise called as	laplace formula	poisson formula	Debye formula	Langevin formula	laplace formula
The line integral of magnetic induction vector B around a closed					
path is equal to times the total current crossing					
any surface bounded by the line integral path.	ε <sub>0</sub>	$\mu_0$	$1/\epsilon_0$	$1/\mu_0$	μ <sub>0</sub>
Differential form of Ampere's circuital law is	$\nabla X B = e_0 J$	$\nabla X B = -m_0 J$	$\nabla X B = m_0 J$	$\nabla X B = 0$	$\nabla X B = m_0 J$
Ampere's circuital law signifies that magnetic field is					
	finite	infinite	irrotational	rotational	rotational
B can be expressed in terms of magnetic scalar potential as					
·	$B = grad \phi_m$	$B = -grad \phi_m$	$B = curl \phi_m$	$B = div \phi_m$	$B = -grad \phi_m$
Magnetic scalar potential satisfies equation.	poisson	Debye	Laplace	Langevin	Laplace
$\nabla^2 \mathbf{f}_m = $	0	1	-1	2	0
Magnetic vector potential satisfies equation.	laplace	poisson	debye	Langevin	poisson
The line integral of magnetic vector A round a closed path gives the				Ť	
flux linked with the area enclosed by the closed					
path.	electrostatic flux	magnetostatic	electric flux	magnetic flux	magnetic flux
The divergence of magnetic vector potential A is	-1	0	1	2	0
B =	curl A	–curl A	Div A	-div A	curl A
$\nabla X B = $	-μ <sub>0</sub> J	ε <sub>0</sub> J	$\mu_0 J$	-ε <sub>0</sub> J	$\mu_0 J$
$\nabla^2 A = $	$\mu_0 J$	-µ₀J	ε <sub>0</sub> J	-ε <sub>0</sub> J	-μ <sub>0</sub> J
The magnetic field is	rotational	non-solenoidal	solenoidal	irrotational	solenoidal

According to Ampere's law of force, the force between current					
carrying conductors depends upon of the two current					
elements.	length	diameter	cross-section	area	length
According to Ampere's law of force, the force between current					
carrying conductors varies as the square of the					
distance between the two current elements	infinitely	finitely	directly	inversely	inversely
	initially		directly	inversery	inversery
Biot Savart law in magnetism is analogous to which law in electricity?	Gauss law	Faraday law	Coulomb's law	Ampere law	Coulomb's law
Which of the following cannot be computed using the Biot Savart law?	Magnetic field intensity	magnetic flux density	Electric field intensity	permeability	Electric field intensity
Find the magnetic field of a finite current element with 2A current and height $1/2\pi$ is	1	2	0.5	0.25	1
The current element of the solenoid of turns 100, length 2m and current					
0.5A is given by	100 dx	200 dx	25 dx	50 dx	200 dx
Find the magnetic field intensity at the centre O of a square of the sides					
equal to 5m and carrying 10A of current.	1.2	1	1.6	1.8	1.8
The point form of Ampere law is given by	Curl(B) = I	$\operatorname{Curl}(\mathrm{D}) = \mathrm{J}$	Curl(V) = I	Curl(H) = J	Curl(H) = J
	~	~	~		~
The Ampere law is based on which theorem?	Green's theorem	Gauss divergence theorem	Stoke's theorem	Maxwell theorem	Stoke's theorem
The value of J H.dL will be	J	I	В	Н	1
		Determination of			
In Ampere's circuital law, what is the purpose of an 'Amperian	Computation of magnetic	differential element of	Estimation of electric flux	Detection of loop in a	Computation of magnetic
Path'?	field intensity	path length	density	constant plane	field intensity
What is the direction of magnetic field intensity vector due to					
infinite long straight filament?	Radial	Elliptical	Parabolic	Circumferential	Circumferential
What is the value of cross product for two similar unit vectors?	zero	infinity	third unit vector	negative vector	zero
			Sine of angle between	Square of the distance	Square of the distance
According to Biot-Sayart law, which parameter/s exhibit/s an/the		Magnitude of differential	filament & line connecting	from differential element	from differential element
inverse relationship to the differential magnetic field intensity (dH)?	Current	length	differential length to point	to point	to point
In magnetism Biot Savart law is well known as law of	Current	length	differential length to point	to point	to point
aurrent element	Amporo's	Coulomb's	Joule's	Ohm's	A mnoro's
	Amperes	Couloillos	Joules	Ohillis	Amperes
what does the constant $\mu$ indicate, while specifying the relation		-	~		-
between magnetic flux density (B) and magnetic field intensity (H)?	Persistivity	Permittivity	Permissibility	Permeability	Permeability
In a magnetic flux density, the total magnetic lines of force crossing					
a unit area in plane to the direction of flux are at	Acute angles	Obtuse angles	Right angles	None of the above	Right angles
The H quantity is analogous to which component in the following?	В	D	E	V	E
The expression for magnetization is given by(I-current, A-area, V-volume)	M = IAV	M = IA/V	M = V/IA	M = 1/IAV	M = IA/V
Find the permeability of a medium whose susceptibility is 100.	-100	99	-99	101	101
Very small and positive susceptibility is found in	Ferromagnetic	Diamagnetic	Paramagnetic	Antiferromagnetic	Paramagnetic
Which of the following materials is ferrimagnetic?	Fe	Sn	Fe2O3	FeCl	Fe2O3
Identify the diamagnetic material.	Silicon	Germanium	Silver	Cobalt	Germanium
Which of the following is vector quantity?	relative permeability	magnetic field intensity	flux density	magnetic potential	magnetic field intensity
The ratio of intensity of magnetisation to the magnetisation force is known	•	•			•
as	flux density	susceptibility	relative permeability	none	susceptibility
The unit of relative permeability is	henry/meter	henry	henry/sq.m	dimensionless	dimensionless
Strength of magnetic field is known as	flux	density	magnetic strength	magnetic flux density	magnetic flux density
Weakest force in nature is	electric force	gravitational force	weak force	magnetic force	gravitational force
Which of the following is a vector quantity ?	Relative permeability	Magnetic field intensity	Flux density	Magnetic potential	Magnetic potential

Magnetising steel is normals difficult because	it corrodes easily	it has high permeability	it has high specific gravity	it has low permeability	it has low permeability
The Biot-savart's law is a general modification of	Kirchhoffs law	Lenz's law	Ampere's law	Faraday's laws	Ampere's law
Magnetic moment is a	pole strength	universal constant	scalar quantity	vector quantity	vector quantity
For which of the following materials the saturation value is the					
highest ?	Ferromagnetic materials	Paramagnetic materials	Diamagnetic materials	Ferrites	Ferrites
The torque expression of a current carrying conductor is	$T = BIA \cos \theta$	$T = BA \cos \theta$	$T = BIA \sin \theta$	$T = BA \sin \theta$	$T = BIA \sin \theta$
Which of the following laws of electromagnetic theory is associated with the force experienced by two loops of a wire carrying currents?	Maxwell's law	Coulomb's law	Ampere's law	Laplace's law	Ampere's law
The force in a magnetic field is given by $F = qv B$	F and $q$ are perpendicular and $v$ and B are perpendicular	F and q only are perpendicular to each other	F and $v$ , F and B are mutually perpendicular to each other and $v$ and B at any angle between them	All the four components are perpendicular to each other	F and $q$ are perpendicular and $v$ and B are perpendicular
The value of susceptibility in case of vacuum is taken as	zero	0.:	5 unity	infinity	zero
will have the least value of relative permeability	Palladium	Silver	Cobalt	Iron	Silver
The relative permeability is taken as unity for which of the					
following?	Air	Cobalt	Vacuum	Bismuth	Air

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# <u>UNIT-III</u>

# **SYLLABUS**

#### Maxwell's Equations

Maxwell Equations-Displacement Current-Vector and Scalar Potentials. Gauge Transformations: Lorentz and Coulomb Gauge. Boundary Conditions at Interface between Different Media. Wave Equations. Plane Waves in Dielectric Media. Poynting Theorem and Poynting Vector. Electromagnetic Energy Density. Physical Concept of Electromagnetic Field Energy Density, Momentum Density and Angular Momentum Density.

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#### MAXWELL EQUATIONS

The four fundamental equation of electromagnetism and corresponds to a generalization of certain experimental observations-regarding electricity and magnetism. The following four laws of electricity and magnetism constitutes the so called differential form of Maxwell's equation.

1. Guass law for the electric field of charge yields



D – electric displacement in coulombs / m  $^2$ 

 $\rho$  – free charge density in coulombs / m <sup>3</sup>

2. Guass law for magnetic field yields

**div B** = **∇**.**B** =0 -----(B)

B – magnetic induction in web / m  $^3$ 

3. Ampere's Law in circuital form for the magnetic field accompanying a current when modified by Maxwell yields

**Curl H = 
$$\nabla$$
 x H = J +  $\partial$ D /  $\partial$ t ------(C)**

H – magnetic field intensity in amperes / m

I – current density amperes / m  $^2$ 

4. Faradays law in circuital form for the induced electromotive force produced by the rate of change of magnetic flux linked with the path yields.

**Curl E** = 
$$\nabla$$
 **X E** = -  $\partial$ **B** /  $\partial$ **t** -----(D)

E – electric field intensity in Volts / m

#### **DERIVATIONS**

#### 1. div $\mathbf{D} = \nabla \cdot \mathbf{D} = \rho$

Let us consider a surface S bounding a volume  $\tau$  with in a dielectric. The volume  $\tau$  contains no net charge but we allow the dielectric to be polarised say by placing it is an electric field. Some charge on the dielectric body are placed. Thus we have two types charges

a) real charge of density p

b) bound charge density  $\rho^{\prime}$  , Guass law then can be written as,

$$\oint_{s} E \cdot ds = \frac{1}{\varepsilon \circ} \int (\rho + \rho') d\tau$$

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 $\varepsilon_0 \oint_s E.\mathrm{ds} = \int_\tau \rho d + \tau \int \rho' d\tau - \dots (1)$ 

But as the bound charge density  $\rho'$  is defined as

 $\rho' = - div P$ 

$$\oint_{s} E ds = \int_{\tau} div E d\tau$$

Equation (1) can be written as,

$$\varepsilon_0 \int_{\tau} div \ E \ d\tau = \int_{\tau} \rho \ \tau - \int_{\tau} div \ P \ d\tau$$
$$\int_{\tau} div \ ( \ \varepsilon 0 \ E + P) \ d\tau = \int_{\tau} \rho \ d\tau \qquad [\varepsilon_0 E + P = D]$$

$$\int_{\tau} (div D - \rho) d\tau = 0$$

This equation is true for all volumes, the integration must vanish.

div  $D = \nabla D = \rho$ 

#### 2. div B = **∇**.B =0

Experiments to data have shown that magnetic poles do not exist. This in turn implies that the magnetic lines of force are either closed group or go off to infinity. Hence the no of magnetic lines of force entering any arbitrary closed surface is exactly the same as leaving it. The flux of magnetic induction B across any closed surface is always zero.

$$\oint_{s} B.\,ds = 0$$

Transforming this surface integral to volume integral by Guass theorem, we get,

$$\int_{\tau} div \ B \ d\tau = 0$$

But as the surface bounding the volume is quite arbitrary the above equation will be true only when the integrated vanishes.

div  $B = \nabla B = 0$ 

#### 3. Curl H = $\nabla$ x H = J + $\partial$ D / $\partial$ t

From Ampere's circuital law the work done in carrying unit magnetic pole once round a closed arbitrary path linked with the current I is expressed by,

 $\oint_C H.\,d\ I = I$ 

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 $\oint_{C} H.dI = \int J.ds$  [I= $\int I.ds$ ]

Where S is the surface bounded by the closed path C.

Now changing the line integral into surface integral by Stokes theorem, we get

 $\int_{s} curl \ H.ds = \int_{s} J.ds$ 

Curl H = J -----(2)

But Maxwell found it to be incomplete for changing electric fields and assumed that a quantity,

 $J_d = \partial D / \partial t$  is called displacement current must also be included in it so that it may satisfy the continuity equation, J must be replaced by J + J<sub>d</sub>, so the law becomes,

$$Curl H = J + J_d$$
$$Curl H = J + \partial D / \partial t$$

#### 4. Curl E = $\nabla$ X E = - $\partial$ B / $\partial$ t

According to Faradays law of electromagnetic induction, we know that the induced e.m.f is proportional to the rate of change flux

$$\boldsymbol{\epsilon} = -\boldsymbol{d} \, \Phi_{\rm B} \, / \, \mathrm{dt} - \dots \quad (3)$$

Now if E be the electric intensity at a point the work done in moving a unit charge through a small distance d I is E. d I. So the work done in moving the unit charge once round the circuit is  $\oint_c E. dI$ . Now as e.m.f is defined as the amount of work done in moving a unit charge once round the electric circuit.

$$\boldsymbol{\epsilon} = \oint_{\boldsymbol{c}} \boldsymbol{E}.\,\boldsymbol{d}\boldsymbol{l} \qquad \dots \qquad (4)$$

Comparing equation (3) and (4), we get,

 $\oint_{c} E. dI = d \Phi_{B} / dt \qquad \dots \qquad (5)$   $\Phi_{B} = \int_{s} B. ds$ So  $\oint_{c} E. dI = -\frac{d}{dt} \int B. ds$ 

Now changing the line integral into surface integral by Stokes theorem, we get

$$\int_{s} curl \ E.\, ds = -\frac{d}{dt} \int B.\, ds$$

The surface S is fixed in space and only B changes with time, above equation yields,

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$$\int_{s} (curl + \frac{\partial B}{\partial T}) . ds = 0$$

Integrated vanish is the integral is true for arbitrary,

 $Curl E = - \partial B / \partial t$ 

Special Cases :

- 1. In a conducting medium of relative permittivity  $\epsilon_r$  and permeability  $\mu_r$  as
  - $D = \varepsilon E = \epsilon_r \epsilon_0 E$  $B = \mu H = \mu_r \mu_0 H \dots (A)$

And Maxwell equation reduced to

(i) 
$$\nabla . E = \rho / \epsilon_r \epsilon_0$$
  
(ii)  $\nabla . H = 0$   
(iii)  $\nabla X H = J + \epsilon_r \epsilon 0 \frac{\partial E}{\partial t}$   
(iv)  $\nabla X E = -\mu_r \mu_0 \frac{\partial H}{\partial t}$ 

2. In a non-conducting media of relative permittivity  $\epsilon_r$  and permeability  $\mu_r$  as

 $\rho = \sigma = 0$ 

$$J = \sigma E = 0$$
 ------ (B)

And Maxwell equation reduced to

(i) 
$$\nabla . E = 0$$
  
(ii)  $\nabla . H = 0$   
(iii)  $\nabla X H = \epsilon_r \epsilon \ 0 \frac{\partial E}{\partial t}$   
(iv)  $\nabla x E = -\mu_r \mu_0 \frac{\partial H}{\partial t}$  .....(C)

3. In free space as

 $\epsilon_r = \mu_r = 1$ 

 $\rho = \sigma = 0$ 

And Maxwell equation reduced to

(i)  $\nabla . E = 0$ 

- (ii) **∇**. H = 0
- (iii)  $\nabla X H = \epsilon \ 0 \frac{\partial E}{\partial t}$ (iv)  $\nabla X E = -\mu_0 \frac{\partial H}{\partial t}$

Discussion :

(i) The equation are based on experimental observations the equation (A) and (C) correspond to electricity and (B) and (D) to magnetism.

(ii) These equations are general and apply to all electromagnetic phenomena in media, which are at rest with to respect to the co-ordinate system.

(iii) These equation are not independent of each other as form equation (D) we can derive (B) and from (C), (A). The equation (B) and (D) are called first pair of Maxwell's equation while (A) and (C) are called the second pair.

(iv) The equation A represents coulombs law while C the law of conservation of charge (i.e.) continuity equation.

# DISPLACEMENT CURRENT

Ampere's circuital law in its most general form is given by

 $\oint_{C} H.d I = \int_{S} J.ds$  J – Current density

 $\int_{s} curl \ H.ds = \int_{s} J.ds$ 

Curl H = J ------ (1)

Let us now examine the validity of this equation in the event that the fields are allowed to vary with time. If we take the divergence of both sides of equation (1) then,

Now as div of curl of any vector is zero, we get from equation (2),

div J = 0 ----- (3)

Now the continuity equation in general state

div J = 
$$-\frac{\partial \rho}{\partial t}$$
 ----- (4)

and will therefore vanish only in the special case that the charge density is static. We must conclude that Ampere's law as stated in equation (1) is valid only for steady state conditions and

is insufficient for the case of time-dependent fields. Because of this Maxwell assumed that equation (1) is not complete but should have something be denoted be  $J_d$ , then equation (1) can be written as

 $\operatorname{curl} H = J + J_d$  ----- (5)

In order to identify  $J_d$ , we calculate the divergence of equation (2) again and get

div curl H = div  $(J + J_d)$ 

[ div curl H = 0 ]

 $div (J + J_d) = 0$  $div J + div J_d = 0$ 

 $divJ_d = - div J$ 

 $divJ_d = \frac{\partial \rho}{\partial t}$ 

{ from a equation (4) }

$$\operatorname{div} J_d = \frac{\partial}{\partial t} \operatorname{div} D$$

$$\operatorname{div} J_{d^{-}} \frac{\partial}{\partial t} \operatorname{div} D = 0 \qquad \{ \operatorname{div} D = \rho \}$$

div 
$$(J_d - \frac{\partial D}{\partial t}) = 0$$
 -----(6)

As equation (6) is true for any arbitrary volume  $J_d = \frac{\partial D}{\partial t}$ 

And so the modified form of Ampere's circuital law becomes,

$$\operatorname{Curl} \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t}$$

 $\partial D / \partial t$  called the displacement current to distinguish it from J, the conduction current. By adding this term to Ampere's law, Maxwell assumed that the time rate of change of displacement produce a magnetic field just as a conduction current.

# ELECTROMAGNETIC POTENTIALS A AND **\$**

The analysis of an electromagnetic field is often facilitated by the use of auxiliary functions know as electromagnetic potentials. At every point of space the field vectors satisfy the equations,

div D $\rho = \rho$	(A)
div B =0	(B)
$\operatorname{Curl} \mathbf{H} = \mathbf{J} + \partial \mathbf{D} / \partial \mathbf{t}$	(C)

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 $Curl = -\partial B / \partial t \qquad \dots \qquad (D)$ 

According to equation (B) , the field of vector B is always solenoidal, B can be represented as the curl of another vector say A .

 $B = \operatorname{curl} A \qquad ------(1)$ 

Where A is a vector which is function of space (x, y, z) and time (t) both. Now sub the value B in equation (1) we get,

$$\operatorname{Curl} \mathbf{E} = -\frac{\partial}{\partial t} \operatorname{curl} \mathbf{A}$$
$$\operatorname{Curl} \left( \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \qquad -----(2)$$

(i.e.)  $E + \frac{\partial A}{\partial t}$  is a irrotational and must be equal to the gradient of some scalar function.

Thus we have introduced a vector A and a scalar  $\varphi$  both being functions of position and time.

These are called electromagnetic potentials. The scalar  $\phi$  is called the scalar potential and vector

A, vector potential.

Properties of scalar and vector potential :

(i) These are mathematical function, which are not physically measurable.

(ii) They are not independent of each other.

(iii) They play an important role in relativistic electrodynamics.

NON-UNIQUENESS OF ELECTROMAGNETIC POTENTIALS AND CONCEPT OF GAUGE

In terms of electromagnetic potentials field vectors are given by,

$$B = \operatorname{curl} A \qquad -----(1)$$

$$E = -\operatorname{grad} \varphi - \frac{\partial A}{\partial t} \qquad -----(2)$$

And

From equations (1) and (2) it is clear that for a given A and  $\varphi$ , each of the field vectors B and E has only value i.e. A and  $\varphi$  determine B and E uniquely. However the converse is not true i.e. field vectors do not determine the potentials A and  $\varphi$  completely. This in turn implies that for a given A and  $\varphi$  there will be only one E and B while for a given E and B there can be infinite

number of A' S and  $\phi$ ' S. This is because the curl of the gradient of any scalar vanishes identically and hence we may add to A the gradient of a scalar  $\Lambda$  without affecting B. That is A may be replaced by,

$$A' = A + \text{grad } \Lambda$$
-----(3)

But if this is done equation (2) becomes,

$$E = -\operatorname{grad} \varphi - \frac{\partial}{\partial t} (A' - \operatorname{grad} \Lambda)$$
$$E = -\operatorname{grad} (\varphi - \frac{\partial A'}{\partial t}) - \frac{\partial A'}{\partial t}$$

So if we make the transformation given by equation (3). We must also replace  $\varphi$  by

$$\Phi' = \varphi - \frac{\partial A}{\partial t} - \dots$$
 (4)

 $B = curl A = curl (A' - grad \Lambda) = curl A'$ 

The expressions for field vectors E and B remain unchanged under transformations equations (3) and (4).

And

$$E = -\operatorname{grad} \varphi - \frac{\partial A}{\partial t} = -\operatorname{grad} \left( \varphi' + \frac{\partial A}{\partial t} \right) - \frac{\partial}{\partial t} \left( A' - \operatorname{grad} \Lambda \right)$$
$$E = \operatorname{grad} \varphi' - \frac{\partial A'}{\partial t}$$

We get the same field vectors whether we use the set  $(A, \phi)$  or  $(A', \phi')$ . So electromagnetic potentials define the field vectors uniquely through they themselves are non-unique.

The transformations given by equations (3) and (4) are called gauge transformations and the arbitrary scalar  $\Lambda$  gauge function. From the above it is also clear that even though we add the gradient of a scalar function, the field vectors remain unchanged. Now it is the field quantities and not the potentials that possess physical meaningfulness. We therefore Say that the field vectors are invariant to gauge transformations i.e. they are gauge invariant.

#### LORENTZ GAUGE

The Maxwell's field equations in terms of electromagnetic potentials are,

A casual glance at equations (1) and (2) reveals that these equations will be much more simplified (i.e. will become identical and uncoupled) if we choose

div A + 
$$\mu \varepsilon \frac{\partial \phi}{\partial t} = 0$$
 ------ (3)

This requirement is called the Lorentz condition when the vector and scalar potential satisfy it, the gauge is called is known as Lorentz gauge.

So with Lorentz condition field equation reduce to

But as  $\mu \varepsilon = 1/v^2$ 

So equations (4) and (5) can be written as

Equations (6) and (7) are inhomogeneous wave equations and are known as D' Alembertian equations and can be solved in general by a method similar to that we use to solve Poisson's equation. The potentials obtained by solving these equations are called retarted potentials.

In order to determine the requirement that Lorentz condition places on  $\Lambda$ , we substitute A' and  $\phi$ ' from equations (3) and (4).

$$div (A' - grad \Lambda) + \mu \varepsilon \frac{\partial}{\partial t} (\phi' + \frac{\partial \Lambda}{\partial t}) = 0$$
$$div A' + \mu \varepsilon \frac{\partial \phi'}{\partial t} = \nabla^2 \Lambda - \mu \varepsilon \frac{\partial 2\Lambda}{\partial t^2}$$

So A' and  $\varphi$ ' will also satisfy equation (3) i.e. Lorentz condition provided that

$$\nabla^2 \Lambda - \mu \varepsilon \frac{\partial^2 \Lambda}{\partial \tau^2} = 0$$
  
i.e. 
$$\boxdot^2 \Lambda = 0 \qquad ----- (8)$$

Lorentz condition is invariant under those gauge transformations for which the gauge functions are solutions of the homogeneous wave equations.

The advantages of this particular gauge are :

(i) It makes the equations for A and  $\phi$  independent of each other.

(ii) It leads to the wave equations which treat  $\varphi$  and A on equivalent footings.

(iii) It is a concept which is independent of the co-ordinate system chosen and so fits naturally

into the considerations of special theory of relativity.

#### **COULOMB GAUGE**

An inspection of field equations in terms of electromagnetic potentials,

$$\nabla^{2} A - \mu \varepsilon \frac{\partial 2A}{\partial t^{2}} - \operatorname{grad} \left( \operatorname{div} A + \mu \varepsilon \frac{\partial \varphi}{\partial t} \right) = -\mu J \qquad (1)$$

$$\nabla^{2} \varphi - \mu \varepsilon \frac{\partial 2\Box}{\partial t^{2}} + \frac{\partial}{\partial t} (\operatorname{div} A + \mu \varepsilon \frac{\partial \Box}{\partial t}) = -\frac{\rho}{\varepsilon}$$
i.e.
$$\nabla^{2} \varphi + \frac{\partial}{\partial t} (\operatorname{div} A) = -\frac{\rho}{\varepsilon} \qquad (2)$$

Shows that if we assume,

div A = 0 ----- (3)

equation (2) reduces to Poisson's equation

$$\nabla^2 \varphi_{(\mathbf{r},t)} = - \frac{\rho}{\varepsilon} (\mathbf{r}', t) - \dots (4)$$

Whose solution is,

$$\varphi_{(r,t)} = \frac{1}{4\pi\varepsilon} \int \frac{\rho(r',t)}{R} d\tau' \qquad (5)$$

i.e. the scalar potential is just the instantaneous Coulombian potential due to charge  $\rho$  (x', y', z',

t). This is the origin of the name Coulomb gauge. Equation (1) in the light of (3) reduced to

$$\nabla^2 \mathbf{A} - \frac{1}{\mathbf{v}_2} \frac{\partial^2 \mathbf{A}}{\partial \mathbf{t}_2} = -\mu J + \mu \varepsilon \nabla \frac{\partial \Box}{\partial t} \qquad (6)$$

Now to express equation (6) in more convenient way we use Poisson's equation (4) which with the help of (5) can be written as

Now as Poisson's equation holds good for scalar and vectors both, replacing  $\rho(r^{\prime}\,,\,t)$  by J' we get,

$$\nabla^2 \left\{ \frac{1}{4\pi\varepsilon} \int \frac{J'}{R} \, \mathrm{d}\tau' \right\} = -\frac{J'}{\varepsilon} \tag{8}$$

Now from the vector identity

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	$\nabla \mathbf{x} \nabla \mathbf{x} \mathbf{G} = \nabla (\nabla . \mathbf{G}) - \nabla^2 \mathbf{G}$					
	$\nabla^2 \mathbf{G} = \nabla \ (\nabla \cdot \mathbf{G}) - \nabla \mathbf{X} \nabla \mathbf{X} \mathbf{G}$					
	Taking G = $\int \left(\frac{J}{R}\right) d\tau'$ , we get					
$\nabla^2 \int \left( \int_{\mathbf{F}} \right)^2$	$\nabla^2 \int \left(\frac{J}{R}\right) d\tau' = \nabla \left(\nabla \cdot \int \frac{J'}{R} d\tau'\right) - \nabla x \nabla x \int \frac{J'}{R} d\tau'$					
Which in the light of equation (8) reduces to						
$-4\pi J' = \nabla \left(\nabla \int \frac{J'}{R} d\tau'\right) - \nabla x \nabla x \int \frac{J'}{R} d\tau'$						
<i>i.e.</i> $J' = -\frac{1}{4\pi}\nabla$	$\left(\nabla \cdot \int \frac{J'}{R} d\tau'\right) + \frac{1}{4\pi} \nabla x \nabla x \int \frac{J'}{R} d\tau'  \dots \dots$	(9)				
Now as $\nabla . \int \frac{J'}{R} d\tau'$						
$= \int \left[\frac{1}{R} \nabla J' + J' \cdot \nabla \left(\frac{1}{R}\right)\right] \{a\}$	$s\nabla(sv) = s\nabla \cdot v + v \cdot \nabla s \}$					
$=\int J'.\nabla \frac{1}{R} d\tau' \{ as J' is not$	t a function x, y and z }					
$= - \int \mathbf{J}' \cdot \nabla' \frac{1}{R} d\tau' \{ as \nabla \left( \frac{1}{R} \right) \}$	$= -\nabla'\left(\frac{1}{R}\right)$					
$= \int \left[ \nabla' \cdot \frac{J'}{R} - \nabla' \cdot \left( \frac{J'}{R} \right) \right] d\tau'$	$I'\{ as \nabla' \cdot \frac{J'}{R} = \left(\frac{1}{R}\right) \nabla' \cdot J' + J' \cdot \nabla' \left(\frac{1}{R}\right) \}$					
$= \int \nabla' \cdot \frac{\mathbf{J}'}{\mathbf{R}}  d\tau' - \oint_{\mathbf{s}} \left( \frac{\mathbf{J}'}{\mathbf{R}} \right) \cdot d\mathbf{s}$	s { as $\int \nabla' \left( \frac{J'}{R} \right) d\tau' = \oint_{s} \left( \frac{J'}{R} \right) . ds }$					
As J' is confined to the volt', the	ne surface contribution will vanish so					
$\nabla \cdot \int \left(\frac{J'}{R}\right)$	$\int d\tau' = \nabla' \cdot \frac{J'}{R} d\tau' \qquad \dots $					
$A = d \nabla = \left( \begin{pmatrix} j' \end{pmatrix} d z' \right)$						

And  $\nabla x \int \left(\frac{I}{R}\right) d\tau'$  $= \int \left[ \nabla x \frac{J'}{R} - J' X \nabla \left(\frac{1}{R}\right) \right] d\tau' \{ \text{ as curl } SV = S \text{ curl } V - V X \text{ grad } S \}$   $= -\int J' X \nabla \left(\frac{1}{R}\right) d\tau' \{ \text{ as } J' \text{ is not a function of } x, y, \text{ and } z \}$   $= \int J' X \nabla ' \left(\frac{1}{R}\right) d\tau' \{ \text{ as } \nabla \left(\frac{1}{R}\right) = -\nabla' \left(\frac{I}{\Box}\right) \}$   $= \int \left[ \nabla' X \frac{J'}{R} - \nabla' X \left(\frac{J'}{R}\right) \right] d\tau' \{ \text{ as } \nabla x \left(J'/R\right) = (1/R) \nabla' X J' - J' X \nabla' (1/R) \}$   $= \int \nabla' X \frac{J'}{R} d\tau' + \oint \frac{J'}{R} X ds \{ \text{ as } \int \nabla X V d\tau' = -\oint_s V X ds \}$ 

As J' is confined to vol', surface contribution will vanish so

So equation (9) becomes

$$J' = -\frac{1}{4\pi} \nabla \int \nabla' \cdot \frac{J'}{R} d\tau' + \frac{1}{4\pi} \nabla X \int \nabla' X \frac{J}{R} d\tau'$$
  
i.e.  $J' = J'_1 + J'_T$  (12)  
With  $J'_1 = -\frac{1}{4\pi} \nabla \int \nabla' \cdot \frac{J'}{R} d\tau'$  and  $J'_T = \frac{1}{4\pi} \nabla X \int \nabla' X \frac{J'}{R} d\tau'$  (13)

Now as

The first term on R.H.S of equation (12) is irrotational and second is solenoid. The first term is called longitudinal current and the other transverse current.

So in the light of equation (12),(6) can be written as

$$\nabla^{2}A \quad \frac{1}{V2} \frac{\partial^{2}A}{\partial t_{2}} = -\mu (J_{1} + J_{T}) + \mu \varepsilon \nabla \frac{\partial \phi}{\partial t}$$

$$\nabla^{2}A \quad \frac{1}{V2} \frac{\partial^{2}A}{\partial t_{2}} = -\mu J_{T} - \mu J_{1} + \mu \varepsilon \nabla \frac{\partial}{\partial t} \left[ \frac{1}{4\pi\varepsilon} \int \frac{\rho(r',t)}{R} d\tau' \right] \quad \{ \text{ Substituting } \phi \text{ from equation } (5) \}$$

$$\nabla^{2}A \quad \frac{1}{V2} \frac{\partial^{2}A}{\partial t_{2}} = -\mu J_{T} - \mu J_{1} + \mu \frac{1}{4\pi} \nabla \int -\frac{-\nabla J}{R} d\tau' \quad \{ \text{ as from continuity equation } \partial \frac{\rho(r',t)}{R} = -\nabla J \}$$

$$Or \nabla^{2}A \quad \frac{1}{V2} \frac{\partial^{2}A}{\partial t_{2}} = -\mu J_{T} - \mu J_{1} + \mu J_{1} \quad \{ \text{ from equation } (13) \}$$

$$\nabla^{2}A \quad \frac{1}{V2} \frac{\partial^{2}A}{\partial t_{2}} = -\mu J_{T} \quad (16)$$

The equation for A can be expressed entirely in terms of the transverse current. So this gauge sometimes is also called as transverse gauge.

The Coulomb gauge has a entire advantage. In it the scalar potential is exactly the electrostatic potential and electric field,

$$E = - \operatorname{grad} \varphi - \frac{\partial A}{\partial t}$$

Is separable into an electrostatic field V =  $\varphi$  and a wave field given by  $-(\partial A/\partial t)$ .

This gauge is often used when no sources are present. Then according to equation 5,  $\varphi = 0$  and A satisfies the homogeneous wave equation 16. The fields are given by,

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial \mathbf{t}}$$
 and  $\mathbf{B} = \nabla \mathbf{X} \mathbf{A}$ 

# POYNTING THEOREM (OR) ENERGY IN ELECTROMAGNETIC FIELDS :

"The rate of decrease of energy in the electrodynamic fields in a specific region in equal to the sum of rate of work done on charges and rate of escape of energy through the surface in the form of electromagnetic radiation."

According to Lorentz law, the force acting in a electromagnetic field is given by

$$\mathbf{F} = \begin{bmatrix} \mathbf{E} + (\mathbf{v} + \mathbf{B}) \end{bmatrix} \qquad ------(1)$$

For an elementary volume  $d\tau'$ , the force experienced in an electromagnetic field is given by

 $\vec{\mathbf{F}} = \oint_{\mathbf{v}} [\vec{\mathbf{E}} + (\vec{\mathbf{v}} + \vec{\mathbf{B}})] \rho d\tau [q = \oint_{\mathbf{v}} \rho d\tau]$ 

The work done in causing a displacement dl in the electromagnetic field is given by

$$W = \oint_{v} \left[ \vec{E} + (\vec{v} + \vec{B}) \right] \rho d\tau. \vec{dl} \qquad (2)$$
$$W = \oint_{v} \left[ \vec{E} + (\vec{v} + \vec{B}) \right] \rho d\tau. \vec{v} dt$$

Rate of work done in an electromagnetic field is given by

$$\frac{\mathrm{dW}}{\mathrm{dt}} = \oint_{\mathbf{v}} \left[ \vec{\mathbf{E}} + (\vec{\mathbf{v}} + \vec{\mathbf{B}}) \right] \rho \mathrm{d\tau} \cdot \vec{\mathbf{v}} \qquad -----(3)$$

Assuming the rate of work done in the electric field only, we get

$$\frac{dW}{dt} = \oint_{V} \vec{E} \cdot \rho d\tau \cdot \vec{v} \{ \vec{v} \times \vec{B} = 0 \}$$
$$= \oint_{V} \vec{E} \cdot \rho \vec{v} d\tau$$
$$P = \frac{dW}{dt} = \oint_{V} (\vec{E} \cdot \vec{J}) d\tau \qquad ------(4)$$

We know that the modified Ampere's law is applicable to electrodynamics.

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Putting the value of J from equation (5) in equation (4), we get

$$\oint_{v} (\vec{E} \cdot \vec{J}) d\tau = \oint_{v} \vec{E} \cdot [\frac{\nabla X \vec{E}}{\mu 0} - \epsilon 0 \frac{\partial \vec{E}}{\partial t}] d\tau$$

$$= \oint_{v} [\vec{E} \cdot \frac{\nabla X \vec{E}}{\mu 0}] d\tau - \oint_{v} \epsilon 0 \vec{E} \cdot \frac{d\vec{E}}{dt} d\tau \qquad (6)$$
We know that  $\nabla \cdot [\vec{E} \cdot X \frac{\vec{E}}{\mu 0}] = [\frac{\vec{E}}{\mu 0} \cdot (\nabla X \vec{E}) - \vec{E} \cdot \frac{\nabla X \vec{E}}{\mu 0}]$ 
Now
$$\oint_{v} \vec{E} \cdot \vec{J} \cdot d\tau = \oint_{v} \frac{\vec{E}}{\mu 0} (\nabla X \vec{E}) d\tau - \oint_{v} \nabla \cdot [\vec{E} \cdot X \frac{\vec{E}}{\mu 0}] d\tau - \oint_{v} \epsilon 0 \vec{E} \cdot \frac{d\vec{E}}{dt} d\tau$$

$$= \oint_{v} \frac{\vec{E}}{\mu 0} \cdot (\nabla X \vec{E}) d\tau - \oint_{v} \nabla \cdot [\vec{E} \cdot X \frac{\vec{E}}{\mu 0}] d\tau - \frac{\epsilon 0}{2} \oint_{v} \frac{dE \cdot 2}{dt} d\tau$$

According to Maxwell's third equation in the differential form,

$$\nabla \mathbf{x} \, \vec{\mathbf{E}} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint_{\mathbf{v}} \vec{\mathbf{E}} \cdot \vec{\mathbf{j}} \cdot d\tau = \frac{1}{\mu 0} \oint_{\mathbf{v}} [\vec{\mathbf{B}} \cdot (-\frac{\partial \vec{\mathbf{E}}}{\partial t})] \, d\tau - \oint_{\mathbf{v}} \nabla \cdot (\vec{\mathbf{E}} \cdot \vec{\mathbf{H}}) \, d\tau - \frac{\varepsilon_0}{2} \oint_{\mathbf{v}} \frac{d\mathbf{E} \, 2}{dt} \, d\tau$$

$$\oint_{\mathbf{v}} \vec{\mathbf{E}} \cdot \vec{\mathbf{j}} \cdot d\tau = -\frac{1}{2\mu 0} \oint \frac{d\mathbf{B} \, 2}{dt} \, d\tau - \frac{\varepsilon_0}{2} \oint_{\mathbf{v}} \frac{d\mathbf{E} \, 2}{dt} \, d\tau - \oint_{\mathbf{v}} \nabla \cdot (\vec{\mathbf{E}} \cdot \vec{\mathbf{H}}) \, d\tau$$

$$= -\frac{\partial}{\partial t} \oint_{\mathbf{v}} [\frac{\mathbf{B} \, 2}{2\mu 0} + \frac{1}{2} \varepsilon_0 \mathbf{E}^2] \, d\tau - \oint_{\mathbf{v}} \nabla \cdot (\vec{\mathbf{E}} \cdot \vec{\mathbf{H}}) \, d\tau$$

$$= -\frac{\partial}{\partial t} \oint_{\mathbf{v}} [\frac{\mathbf{B} \, 2}{2\mu 0} + \frac{1}{2} \varepsilon_0 \mathbf{E}^2] \, d\tau - \oint_{\mathbf{S}} (\vec{\mathbf{E}} \cdot \vec{\mathbf{H}}) \, d\vec{\mathbf{a}} \qquad (8)$$

$$-\frac{\partial}{\partial t} \oint_{\mathbf{v}} [\frac{\mathbf{B} \, 2}{2\mu 0} + \frac{1}{2} \varepsilon_0 \mathbf{E}^2] \, d\tau = \oint_{\mathbf{v}} (\vec{\mathbf{E}} \cdot \vec{\mathbf{j}}) \, d\tau + \oint_{\mathbf{S}} (\vec{\mathbf{E}} \cdot \mathbf{X} \, \vec{\mathbf{H}}) \, d\vec{\mathbf{a}} \qquad (9)$$

 $\vec{E}X \vec{H}$  is called the pointing vector or power density. It is denoted by symbol S.

$$\vec{S} = \vec{E} X \vec{H}$$

The unit of pointing vector is Watts/m<sup>2</sup>. The pointing theorem,

 $-\frac{\partial}{\partial t}\oint_{v}\left[\frac{B2}{2\mu0}+\frac{1}{2}\epsilon_{0}E^{2}\right]d\tau=\oint_{v}\left(\vec{E}\cdot\vec{j}\right)\,d\tau+\oint_{S}\left(\vec{E}\cdot X\,\vec{H}\right)\,.\,d\vec{a}\text{ is integral form.}$ 

# POYNTING VECTOR (OR) POWER DENSITY

According to the law of conservation of energy is an electromagnetic fields,

S = E X H

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The emount of the field energy	y passing through unit area of the su	urface in a direction

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### **Possible Questions**

#### 8 marks

- 1. Obtain Maxwell equations.
- 2. Derive Poynting vector.
- 3. Explain the non uniqueness of electromagnetic potential and Lorentz Gauge.
- Obtain an equation for electromagnetic potential (A and φ) and Maxwell equation in terms of electromagnetic potential.
- 5. Discuss about displacement current.
- 6. What is the concept of gauge? Explain Lorentz gauge.

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DEPARTMENT OF PHYSICS III BSc PHYSICS ELECTROMAGNETIC THEORY (15PHU604A) MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	Answer
UNIT-III					
The div of curl of any vector is	1	-1	Grad	0	0
The magnitude of displacement current is equal to the time rate of change	electric displacement				electric displacement
of	vector D	current density J	charge density r	none of the above	vector D
The displacement current in a good conductor is	constant	proportional to time	current density	negligible	negligible
Displacement current have a value in perfect vacuum.	finite	infinite	proportional value	no	finite
J <sub>d</sub> =	$-\partial D/\partial t$	$J + \partial D /  \partial t$	$\partial D / \partial t$	J - $\partial r / \partial t$	$\partial D / \partial t$
Displacement current density J <sub>d</sub> makes conduction current					
across a discontinuity.	discontinuous	continuous	finite	infinite	continuous
The unit for free charge density is	coul/m <sup>2</sup>	coul <sup>2</sup> /m	coul/m <sup>3</sup>	$coul^2/m^2$	coul/m <sup>3</sup>
Maxwell's first equation signifies that the total flux of electric displacement	coul/m		coul/m		coul/m
linked with a closed surface is the total have of electric displacement					
enclosed by the closed surface	equal to	lesser than	greater than	inversely proportional to	equal to
Maxwell's second equation signifies that the total flux of magnetic	equal to	iesser than	greater than	inversely proportional to	equal to
induction linked with a closed surface is	unity	same	zero	constant	zero
Maxwell's third equation signifies that force around a	unity	Sume	Lero	constant	2010
closed path is equal to the conducting current plus displacement current					
linked with the path	magnetomotive	electromotive	restoring	none of the above	magnetomotive
Maxwell's fourth equation signifies that is equal to the	magnetomotive	cicculomotive	restoring		magnetomotive
negative rate of change of magnetic flux linked with the path	magnetomotive	electromotive	electric	magnetic	electromotive
is the amount of field energy passing through unit area of	magnetomotive	cicculomotive	ciccure	magnetie	cicculomotive
the surface in a direction perpendicular to the plane containing F and H per					
unit time	electric energy	magnetic energy	novnting vector	mechanical energy	novnting vector
Povnting vector at any arbitrary point in the field varies	creetine energy	Inaghetic chergy	poynting vector	incentation energy	poynting vector
as the square of the distance from the point source of radiation	inversely	directly	sinusoidally	abnormally	inversely
The definition of a poynting vector is not a	vector	scalar	mandatory	none of the above	mandatory
If the poynting vector is then no electromagnetic	Vector	Scalar	Indicatory		mandatory
energy flows across a closed surface	unity	finite	infinite	zero	7610
In case of time varying fields $S = E \times H$ gives the value of	unity	lillite	minine	2010	2010
the poynting vector	instantaneous	total	random	half the	instantaneous
The time rate of change of the sum of particle momentum and field	Instantaneous	lotai	Tandom		Instantaneous
momentum is equal to the					
everted by the Maxwell stresses on the region considered	magnetic force	electric force	electromotive force	total force	total force
Electromagnetic potentials define the field vectors	uniquely	non-uniquely	sinusoidally	randomly	uniquely
Field vectors are to gauge transformation	variant	proportional	equal	invariant	invariant
Lorentz condition is div A + me $\partial f/\partial t =$	1	_1	2	0	0
	1	1	-	0	0
Gauge functions are solutions of the wave equation	non-homogeneous	Homogeneous	Linear	Symmetric	Homogeneous
Lorentz gauge is of the co-ordinate system	Dependent	Independent	Linear function	Exponential function	Independent
gauge is offen used when no sources are present	lorentz	screw	coloumb	none of the above	coloumb
Maxwell's equation are under Lorentz transformation	variant	proportional	invariant	none of the above	invariant
ander Estenz traisformation.	at right angles to each	proportional		anti-perpendicular to	at right angles to each
F = BIL can only be used if magnetic field and electric current are	other	in same direction	anti-narallel to each other	each other	other
Maxwell's equations, were arrived at mostly through various experiments of	James Clerk Maxwell	Thomas Clerk Maxwell	Charles Drawin	Stephen Hawking	James Clerk Maxwell
interest in the anti-ter at mostly in ough surfous experiments of					

				either in integral or	either in integral or
Maxwell's equations can be written in	integral form	differential form	logical form	differential form	differential form
Relationship of electric and magnetic field is governed by physical laws, whi	Kirchhoff's Equations	Millman's Equations	Maxwell's Equations	Arithmetic Equations	Maxwell's Equations
In Maxwell's equation, $\nabla \cdot D = \rho_{ev}$ ; D is	electric flux density	magnetic flux density	magnetic field intensity	electric field intensity	electric field intensity
According to Maxwell's first equation in integral form	. = v	$\int = \int (.D) dv$	$\int = \int (XH) ds$	$\nabla XH = Jc + Jd$	$\int = \int (.D) dv$
According to Maxwell's first equation in differential form gives	. = v	$\int = \int (.D) dv$	$\int = \int (XH) ds$	$\nabla XH = Jc + Jd$	= v
According to Maxwell's second equation in differential form gives	. = v	$\int = \int (.D) dv$	$\int = \int (XH) ds$	$\nabla XH = Jc + Jd$	$\nabla XH = Jc + Jd$
According to Maxwell's second equation in integral form gives	. = v	$\int = \int (.D) dv$	$\int = \int (XH) ds$	$\nabla XH = Jc + Jd$	$\int = \int (XH) ds$
Maxwell's third equation in differential form gives	. = v	$\int = \int (XE) ds$	$\int = \int (XH) ds$	$\nabla XE = -\partial B/\partial t$	$\nabla XE = -\partial B/\partial t$
Maxwell's third equation in integal form gives	. = v	$\int = \int (XE) ds$	$\int = \int (XH) ds$	$\nabla XE = -\partial B/\partial t$	$\int = \int (XE) ds$
Maxwell's fourth equation in differential form gives	. = 0	$\int = 0$	$\int = \int (XH) ds$	$\nabla XE = -\partial B/\partial t$	. = 0
Maxwell's fourth equation in integral form gives	. = 0	$\int = 0$	$\int = \int (XH) ds$	$\nabla XE = -\partial B/\partial t$	$\int = 0$
Maxwell's equations involve	Charge density	Current density	Magnetic intensity	All of these	All of these
Maxwell's equations are based on law(s)	Faraday's	Gauss's	Ampere's	All of these	All of these
Maxwell's equations in form give information at points of disco	Differential	Integral	Algebraic	none	Differential
For a lossy dielectric medium,	$\sigma = 0$	$\sigma \neq 0$	None	Cannot say	None
Poynting Vector is obtained as	$= \times H$	$= \times H$	$= \times H$	$= \times P$	$= \times H$
Curl of magnetic field intensity is	Current density	Magnetic field intensity	Current	None	None
Any two shortest points in a wave that are in phase are termed as	wave distance	wavelength	phase length	amplitude	wavelength
Waves which require medium for propagation are	electromagnetic	mechanical	transverse	longitudinal	mechanical
Energy can be transferred from one place to another through	linear motion	circular motion	force	waves	waves
Momentum possessed by spinning objects is called	linear momentum	angular momentum	normal momentum	degrees' momentum	angular momentum
Poynting vector gives the	direction of propagation	rate of energy flow	intensity of electric field	intensity of magnteic field	rate of energy flow
When the phase velocity of an EM wave depends on frequency in any mediu	scattering	polarization	absorption	dispersion	polarization
The distance between maxima and minima of a standing wave is	λ/4	$\lambda/2$	λ	2λ	2λ
Poynting theorem relates the electric intensity E, magnetic intensity H to the	P = E/H	P = H/E	$E = 1/E \times H$	$P = E \times H$	$P = E \times H$
The absorption of EM wave by the atmosphere depends on	polarization of waves	frequency in use	dist. from $T_y$	height of antenna	dist. from $T_y$
Poynting vector is the rate of energy flow having	W/m <sup>2</sup>	W-m	kWh/s	W/m	W/m <sup>2</sup>
Refraction of electromagnetic waves occurs when they		pass into a medium of	are polarized at right		
	pass through a small slot	different dielectric	angles to the direction of	encounter a perfectly	pass through a small slot
	in a conducting plane	constant	propagation	conduction surface	in a conducting plane
If a wave is propagated in free space, it will be in nature of	TE	TM	TEM	Any one of above	TEM
In electromagnetic wave polarization is caused by			longitudinal nature of	transverse nature of	transverse nature of
	refraction	reflection	electromagnetic waves	electromagnetic waves	electromagnetic waves
	Power density vector	Power density vector	Current density vector	Current density vector	Power density vector
	producing electrostatic	producing	producing electrostatic	producing	producing
Poynting vector signifies which of the following?	field	electromagnetic field	field	electromagnetic field	electromagnetic field
	the conductivity of the			the conductance of the	
When a magnetic circuit is compared with an electric circuit, the equivalent	lines	the voltage of the source	the current in the lines	load	the current in the lines

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# UNIT-IV

# **SYLLABUS**

**Interaction of E.M.Waves with matter (Macroscopic):** Boundary conditions at interfaces - Reflection and refraction – Frenel's laws-Brewster's law and degree of polarization - Total internal reflection and critical angle.

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#### **BOUNDARY CONDITION**

The electromagnetic field vectors changes as one moves across the boundary.

- (i) The normal component of the electric displacement is discontinuous by an amount equal to the free surface density of charge at the boundary  $D_{1n}$ - $D_{2n}$ = $\sigma$
- (ii) The normal component of the magnetic induction is continuous across s surface of discontinuity  $B_{1n}-B_{2n}=0$
- (iii) The tangential component of magnetic intensity is discontinuous by an amount equal to the free surface current density  $H_{1t}$ - $H_{2t}$ = $J_s$
- (iv) The tangential component of E is continuous across surface of discontinuity  $E_{1t}-E_{2t}=0$

# **REFLECTION AND REFRACTION OF ELECTROMAGNETIC WAVE**

Consider that when plane electromagnetic waves which are travelling in one medium are incident upon an infinite plane surface separation this medium from another, with different electromagnetic properties. When an electric wave is travelling through space there is an exact balance between the electric and magnetic field. Half of the energy of wave as a matter of fact is the electric field and half in the magnetic. If the wave enters some different medium, there must be a new distribution of energy, whether the new medium is a dielectric a magnetic a conducting or an ionised region, there will have to be a readjustment of energy related as the wave reaches its surface. Since no energy can be added to the wave as it only way that a new balance can be achieved is for some of the incident energy to be reflected.

The transmitted energy constitutes the reflected wave and the reflected one the reflected wave. The reflection and refraction of light at a plane surface between two media of different dielectric properties is a familiar, example of reflection and refraction of electromagnetic waves. The various aspects of the phenomenon divide themselves into two classes.

#### **Kinematic properties :**

Following are the kinematic properties of reflection and refraction.

#### (i) Law of frequency :

The frequency of the wave remains unchanged by reflection or refraction.

(ii) The reflected and refracted waves are in the same plane as the incident wave and the normal to the boundary surface.

(iii) Law of reflection :

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In case of reflection the angle of reflection is equal to the angel of incident.

 $\theta_{\rm I} = \theta_{\rm R}$ 

(IV) Snell's Law :

In case of reflection the ratio of the sin of the angle of refraction to the sin of angle of incident is equal to the ratio of the refractive indices of two media.

 $n_1 \sin \theta_i = n_2 \sin \theta_R$ 

# **Dynamic properties**

These properties are concerned with the

- (i) intensities of reflected and refracted waves
- (ii) Phase changes and polarisation of waves

The kinematic properties follow immediately from the wave nature of phenomenon and the fact that these are boundary condition to be satisfied. But they do not depends on the nature of the wave or the boundary conditions.

### FRESNEL FORMULAE

The formulae relating the amplitude of the reflected and transmitted electromagnetic waves with that of incident one when the boundary is between two dielectrics are called Fresnel formulae. These are contained in the boundary condition.

$(D_i)_n + (D_R)_n = (D_T)_n$	(1)
$(B_i)_n + (B_R)_n = (B_T)_n$	(2)
$(E_i)_t + (E_R)_t = (E_T)_t$	(3)
$(H_i)_t + (H_R)_t = (H_T)_t$	(4)

The condition (1) and (2) when coupled with Snell's law yield no information not included in the (3) and (4) conditions. So it is necessary to consider only condition (3) and (4). Now to derive the desired formulae we consider a plane EMW in x-z plane incident on a plane boundary and consider it as a superposition of two waves one with the electric vector perpendicular to the plane of incidence. Therefore it is sufficient to consider these two cases separately. The general result may be obtained from the appropriate linear combination of the two cases.

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#### **CASE I : E parallel to the plane of incidence**

The situation is shown in figure. The electric and propagation vectors in two media are indicated. The directions of H vector are chosen as to give a positive flow of energy in the direction of wave vectors. In this situation the magnetic vectors are all parallel to the boundary surface.

#### $(H_i)_t = H_i$

 $(H_R)_t = H_R$  $(H_T)_t = H_T$ 

And

 $(E_I)_t = E_i \cos \theta_i$ 

 $(E_R)_t = -E_R \cos \theta_R$ 

 $(E_T)_t = E_T \cos \theta_T$ 

So the boundary condition (3) and (4) reduce to,

$$E_{i} \cos \theta_{i} - E_{R} \cos \theta_{R} = E_{T} \cos \theta_{T} \quad ------(5)$$
  
$$H_{i} - H_{p} = H_{T} \quad ------(6)$$

In equation (5) and (6) we have emitted the zero subcript on E and H, it being understood that the phases now cancel and equation are relations between amplitudes.

$$\theta_{i\,=}\,\theta_{R}$$
 and  $H=(E/Z)=(n/\mu_{r}\,Z_{0})=E$ 

$$\mathbf{H} = (\mathbf{n}/\mathbf{Z}_0)\mathbf{E}$$

So equation (5) and (6) reduce to

$$E_{i} \cos \theta_{i} - E_{R} \cos \theta_{R} = E_{r} \cos \theta_{r} - \dots (7)$$

$$n_{1} E_{i} + n_{2} E_{R} = n_{2} E_{r} - \dots (8)$$

The interest lies in the fraction of incident amplitudes which are reflected and transmitted. So eliminating E  $_{\rm r}$  from equation (7) with the help of (8) we get



#### CASE II : E perpendicular to the plane of incidence

The situation is shown the magnetic field vectors and the propagation vectors are indicated. The electric vectors all directed into the plane of the figure. Since the electric vectors are all parallel to the boundary surface,

$$(E_{I})_{t} = E_{i}$$
$$(E_{R})_{t} = E_{R}$$
$$(E_{T})_{t} = E_{T}$$

And

 $(H_i)_t = -H_I \cos \theta_i$  $(H_R)_t = H_R \cos \theta_p$  $(H_T)_t = -H_T \cos \theta_T$ 

So boundary condition (3) and (4) reduce to

 $E_I - E_R = E_T$ 

 $H_{i} \cos \theta_{i} - H_{R} \cos \theta_{R} = H_{T} \cos \theta_{T} \qquad -----(9)$ 

 $\theta_{i=} \theta_{R}$  and  $H = (E/Z) = (n\epsilon / Z_{0})$  ------(10)

So equation (10) reduce to

 $n_1 E_1 \cos \theta_1 - n_2 E_R \cos \theta_R = n_2 E_T \cos \theta_T$  ------(11)

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# KARPAGAM ACADEMY OF HIGHER EDUCATION **CLASS: III BSc Physics** COURSE NAME: ELECTROMAGNTEIC THEORY COURSE CODE: 15PHU604A UNIT: IV(Interaction of E.M.Waves with matter (Macroscopic)) BATCH-2015-2018 Now eliminating E $_{R}$ from equation (11) with help of (9) we get, $(E_i - E_R) n \log \theta_I = n_2 \cos \theta_T (E_i + E_R)$ $\left(\frac{ER}{Ei}\right)_{*} = \frac{\cos\theta \ i - \frac{n^2}{n^1}\cos\theta \ T}{\cos\theta \ i + \frac{n^2}{n^1}\cos\theta \ T}$ $\left(\frac{E R}{E i}\right)_{+} = \frac{\cos \theta i - \frac{\sin \theta i}{\sin \theta T} \cos \theta T}{\cos \theta i + \frac{\sin \theta i}{\sin \theta T} \cos \theta T}$ $\left(\frac{E\ R}{E\ i}\right) = \frac{\sin\ \theta\ r\ \cos\ \theta\ i - \cos\ \theta\ T\ \sin\ \theta\ i}{\sin\ \theta\ r\ \cos\ \theta\ i + \cos\ \theta\ T\ \sin\ \theta\ i}$ $\left(\frac{E R}{E i}\right) = \frac{\sin\left(\theta \ i - \theta \ T\right)}{\sin\left(\theta \ i - \theta \ T\right)} \qquad ----- (C)$ Similarly eliminating $E_R$ from equation (11) with the help of (9) we get, $n_1 E_1 \cos \theta_1 - n_1 (E_T - E_I) \cos \theta_I = n_2 E_T \cos \theta_T$ $\left(\frac{E T}{E i}\right)_{\perp} = \frac{2\cos\theta i}{\cos\theta i + \frac{n2}{n1}\cos\theta T}$ $\left(\frac{E T}{E i}\right)_{*} = \frac{2\cos\theta i}{\cos\theta i + \frac{\sin\theta i}{\sin\theta T}\cos\theta T}$

$$\left(\frac{ET}{Ei}\right)_{*} = \frac{2\cos\theta \, i\sin\theta \, T}{\cos\theta \, i\sin\theta \, T + \sin\theta \, i\cos\theta \, T}$$
$$\left(\frac{ET}{Ei}\right)_{*} = \frac{2\cos\theta \, i\sin\theta \, T}{\sin(\theta i + \theta \, T)} \qquad -----(D)$$

Equation (A), (B), (C), (D) are the desired result known as Fresnel formulae. BREWSTER'S LAW AND POLARIZATION OF E.M.W.

From Fresnel's formula (A) i.e

$$\left(\frac{E_R}{E_i}\right)_{\mathrm{I}} = \frac{\tan\left(\theta\mathrm{I} - \theta\mathrm{r}\right)}{\tan\left(\theta\mathrm{I} + \theta\mathrm{r}\right)}$$

It is evident that  $(E_R/E_i)_1=0$  for

$$\tan(\theta_{\rm I} - \theta_{\rm r}) = 0 \quad \text{i.e} \quad \theta_{\rm I} - \theta_{\rm r} = 0$$
$$\tan(\theta_{\rm I} + \theta_{\rm r}) = \infty \quad \text{i.e} \quad \theta_{\rm I} + \theta_{\rm r} = \pi/2$$

Or

The first result is trivial since it implies that the two media are optically identical\*. But the second result shows that when the reflected and refracted rays are perpendicular there is no

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energy carried by the reflected ray. The angle of incidence for which this occurs is called Brewster's angle $\theta_{B}$ .

Now, as from snell's law

 $n_1 \sin \theta_i = n_2 \sin \theta_r$ 

$$\frac{\sin \theta I}{\sin \theta r} = \frac{n_2}{n_1}$$

i.e

i.e

 $\sin \theta = n_1$   $\sin \frac{\sin \theta}{\sin \frac{\theta}{2} - \theta} = \frac{n_2}{n_1} \quad [\text{as } \theta_B + \theta_r] = \frac{n_2}{2}$   $\tan \theta = \frac{n_2}{n_1}$   $\theta_B = \tan^{-1}[\frac{n_2}{n_1}]$  $\theta_B = \tan^{-1}(n_2) \quad \dots (a)$ 

 $\theta_{\rm B}$  =tan (1n2) .....(a) Thus if an unpolarised wave is incident on the boundary surface with  $\theta_i = \theta_{\rm B}$  only the portion of the wave with electric vector perpendicular to thr plan of incidence will be reflected. That is the reflected wave is linearly polarised with vibration to the plan of incidence. Brewster's angle is therefore some times called the polarising angle. For light incident on glass with n2=1.5  $\theta_{\rm B}$ =56° and so  $\theta_r$ =34°. So if light is incident on a glass plate at 56°, the reflected light will be plane polarized with vibrations to the plane of incidence and transmitted light will also be plane polarized with vibration parallel to the plane incidence. For radio wave incident on water as n2=9,  $\theta_{\rm B}$ =84° and so  $\theta_r$ =6° so water cannot reflect radio wave which are polarised with vibration in the plane of incident and are incident on it at 84°,



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Even if the unpolarised incident wave is reflected at angles other than the Brewster's angle, there is a tendency for the reflected wave to be predominantly polarized with vibration perpendicular to the plane of incident. The success of dark glasses which selectively transmits only one direction of vibration depends on this fact. In such cases the degree of polarisation is defined as

$$P(\theta_i) = \frac{R1 - R1}{R1 + R1} \qquad \dots (b)$$

Where  $R_1$  and  $R_1$  are the reflection coefficient for the <u>I</u> and <u>II</u> components of reflected light and are given by

$$R_1 = \frac{\tan^2(\theta i - \theta r)}{\tan^2(\theta i + \theta r)} \text{ and } R_1 = \frac{\sin^2(\theta i - \theta r)}{\sin^2(\theta i - \theta r)}$$

The curves for  $R_I$  and  $R_{\underline{I}}$  for glass are shown in fig.

From these it is clear that

For 
$$\theta_{I}=0$$
  $R_{i}=R_{\underline{I}}=04$  so  $P_{(0)}=0$   
For  $\theta_{I}=\theta_{B}$   $R_{I}=0$  and  $R_{\underline{I}}=16$  so  $P_{(\theta_{B})}=1$   
For  $\theta_{I}=90$   $R_{I}=R_{\underline{I}}=1$  so  $P_{(90)}^{o}=0$ 

*i.e* the reflection wave is partially polarized with vibrations perpendicular to the plane of incidence except near  $0^{\circ}$  and  $90^{\circ}$ .



Further the curves it is clear that the reflecting power at normal incidence is 4% only and it falls to zero at polarising angle for the light with vibration in the plane of incidence and for light with its vibration to the plane of incidence it is 15%. All the light is reflected as the angle of incidence approaches  $90^{\circ}i.e$  grazing angle.

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#### TOTAL INTERNAL REFLECTION AND CRITICAL ANGLE

If  $n_1$  is greater than  $n_2$  then in the light of snell's law we

$$\sin \theta_i = \frac{n_2}{n_1} \sin \theta_r \quad \dots (1)$$
$$\theta_i < \theta_r \qquad (as \frac{n_2}{n_1}) < 1)$$

So for a particular value of angle of incidence,  $\theta_r$  will become 90°. The value of angle of incidence for which  $\theta_r$  become 90° is called critical angle and is represented by  $\theta_c$ . so for

$$\sin \theta_i = \frac{n_2}{n_1} \sin 90^\circ = \frac{n_2}{n_1}$$
 .....(2)

The refracted wave is propagated parallel to the boundary surface

Now it is certainly possible to have  $\theta_i > \theta_c$  in this situation from equation (1) in the light of (2) we get

$$\sin \theta_i = \frac{\sin \theta_i}{(\frac{n_2}{n_1})} = \frac{\sin \theta_i}{\sin \theta_c} > 1 \quad [as \ \theta_i > \theta_c] \quad \dots (3)$$

Equation (3) means that  $\theta_r$  is imaginary (as the sine of any real angle can never be greater than

1). To find the meaning of imaginary  $\theta_r$  we consider the transmitted electromagnetic wave

$$E_r = E_c^{e^{-i(\omega 1 - K.r)}}$$

$$E_r = E_c e^{-i[\omega t - Kr(x \sin \theta_r + z \cos c)]}$$
(4)

but from equation (3)

$$\cos \theta_r = \sqrt{(1-\sin^2 \theta_r)} = I \sqrt{\left\{\left(\frac{\sin \theta_r}{\sin \theta_c}\right)^2 - 1\right\}} = ib \qquad (5)$$

So equation (4) reduces to

Equation (6) shows that  $\theta_i > \theta_c$  the transmitted wave is propagated only parallel to the surface and is attenuated exponentially beyond the interface.\* regarding the surface wave it is worthy to note that

(i) The phase velocity of the wave is a function of angle of incidence i.e

$$\gamma_r = \frac{\omega}{Kr\left(\frac{\sin \theta_i}{\sin \theta_c}\right)} = \frac{\omega}{Kr} \frac{\sin \theta_i}{\sin \theta_c} = \frac{\omega}{Kr} \frac{n_2}{n_1 \sin \theta_i}$$
$$\gamma_r = \frac{\gamma_{xc}/\gamma}{n_1 \sin \theta_i} = \frac{c}{n_1 \sin \theta_i} \text{ [as } K_{r=\omega}/\gamma \text{ and } n=c/\gamma$$

Or

i.e

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And in the event the first medium is air it become equal to c for grazing incidence as expected

(ii) These waves do not carry any energy inyo the second medium This is because the time average normal component of the pointing vector just inside the surface

$$*n=\cos\theta_{r}$$

$$*n=\frac{1}{2} \operatorname{Re} (E_{r} * 11 *)\cos\theta_{r}$$

$$*n=\frac{1}{2} \operatorname{Re} [E_{r} * \frac{n_{1}(u*E_{r}*)}{Z_{o}}]\cos\theta_{r}$$

$$*n=\frac{1}{2} \operatorname{Re} \frac{n_{1}}{Z_{o}}[E_{r}. E_{r} *]\cos\theta_{r} n \text{ (as n and } E_{r} \text{ are perpendicular)}$$

i.e

i.e 
$$< S_r > *n = \frac{n_1}{2Z_o} \operatorname{Re}[E_o^{e-bkrZ}]^2$$
 (ib) n

i.e 
$$\langle S_r \rangle^* n=0$$
 (as it has no real part)

such waves are called evanescent waves and are set by the transient effect of the incident waves which strike the surface first.

It is also interesting to see what Fresnel's equations tells us about the field when we use the value of  $\sin\theta_r$  and  $\cos\theta_r$  given by equation (3) and (5) we consider first the case in which E is perpendicular to the plane of incidence i.e

Similarly when E is parallel to the plane of incidence

$$\binom{E_R}{E_r} = \frac{\cos \theta_i - \frac{n_2}{n_1} \cos \theta_r}{\cos \theta_i + \frac{n_2}{n_1} \cos \theta_r} = \frac{a - ib}{a + ib} \text{ with } \{a = \cos \theta_r \ b = \frac{n_2}{n_1} \cos \theta_r \}$$

$$\binom{E_R}{E_r} = e^{-\emptyset 1} \text{ with } \tan \frac{\emptyset 1}{2} = \frac{b*}{a} \dots \dots \dots (8)$$

Or

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So from equation 7 and 8 it is clear that the amplitudes and the intensity of the reflected wave is equal to that of incident wave i.e the wave is totally reflected and that the phases of the perpendicular and parallel reflected wave are  $\emptyset 1$  and  $\emptyset 1$  and depend on the angle of incidence. The general consequence of this is that if a linearly polarised wave is reflected from the boundary at an incident angle greater than the critical angle the reflected wave will be elliptically polarised.

Assume that the incident wave is linearly polarized by this we mean that the oscillating electric field always lies along the same direction. If two write the field in their real forms. The component of the linearly polarised field can be written.

 $(E_R)_{\rm I} = (E_o)_{\rm I} \cos \omega t$  and  $(E_R)_{\rm I} = (E_o)_{\rm I} \cos \omega t$ 

After reflection each of these component will have its phase changed by different amount and can be written as

 $(E_R)_{\mathrm{I}} = (E_o)_{\mathrm{I}} \cos (\omega t \cdot \phi_1) \text{ and } (E_R)_{\mathrm{I}} = (E_R)_{\mathrm{I}} = (E_o)_{\mathrm{I}} \cos (\omega t \cdot \phi_1)$ 

So now the components no longer vanish nor reach their maxima simultaneously and when we add the two we find the resultant E trace an ellipse.

The phenomenon of total internal reflection is used to produce elliptically or circularly polarised light. It is also exploited in many applications where it is required to transmit light without loss in intensity. In nuclear physics Lucite or 'light pipes' are used to carry light emitted from a scintillation crystal because of the passage of an ionizing particle to a photo-multiplier . where it is converted into a useful electric signal.

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### **Possible Questions**

#### 8 marks

- 1.Explain briefly about boundary conditions.
- 2.Briefly discuss about Rayleigh scattering.
- 3.Explain Frenel's law.
- 4.Discuss the phenomenon of total internal reflection and critical angle.
- 5.Discuss about dynamic properties.
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DEPARTMENT OF PHYSICS III BSc PHYSICS

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ELECTROMAGNETIC THEORY (15PHU604A) MULTIPLE CHOICE QUESTIONS

Questions	opt1	opt2	opt3	opt4	Answer
UNIT-IV					
The factor is called degree of polarization.	$-1/2 (1 + \cos^2 f)$	$1/2 (1 - \cos^2 f)$	$-1/2 (1 - \cos^2 f)$	$1/2 (1 + \cos^2 f)$	$1/2 (1 + \cos^2 f)$
direction.	forward	backward	random	none of the above	forward
An oscillating charge behave like an oscillating dipole with dipole					
moment p =	-qx	$qx^2$	qx <sup>2</sup>	$-qx^2$	qx <sup>2</sup>
The relation between the wavelength of the scattered radiation at an angle					
f and the incident radiation is	$l_s = l_i + (h/mc) (1 - \cos f)$	$l_s = l_i - (h/mc) (1 - \cos f)$	$l_s = l_i + (h/mc) (1 + \cos f)$	$l_s = -l_i + (h/mc) (1 + \cos f)$	$l_s = l_i + (h/mc) (1 - \cos f)$
light has longest wavelength in the visible region.	blue	violet	red	green	red
medium is said to be	rarer	dispersive	denser	none of the above	dispersive
	dispersion	refreactive index	dispersive power	none of the above	dispersive power
The index of refractionas the frequency increases.	equals	decreases	varies	increases	increases
in the gas molecules.	electrostatic energy	dipole moment	magnetostatic energy	none of the above	dipole moment
	covalent bond	ionic bond	linear restoring force	none of the above	linear restoring force
In polarization the positions of the electrons are altered from their	nuclei	neutron	proton	muon	nuclei
The classical radius of the electron $r_0 = $	$-q^2/4pe_0mc^2$	$q^2/4pe_0mc$	$q^2/4pe_0mc^2$	$q/4pe_0mc^2$	$q^2/4pe_0mc^2$
In a plane wave B =	- (n X E)	(n X E)	-(n X E)/c	(n X E)/c	(n X E)/c
The dipole moment results from the displacement of the electron is p=					
· · · ·	er	-er	er <sup>2</sup>	-er <sup>2</sup>	er
comparison to the natural frequency of the electron.	very large	very small	zero	none of the above	very small
moment $p = qx$ .	electric dipole	magnetic dipole	electromagnetic dipole	none of the above	electric dipole
The restoring force is	$mw_0^2 x$	$-mw_0^2 x$	-mw <sub>0</sub> x	$-m^2w_0x$	$-mw_0^2 x$
Example of resonance scattering is	neon lamp	mercury vapour lamp	fluorescent lamp	sodium vapour lamp	sodium vapour lamp
The color of the sky during sunset or sunrise is	red	blue	yellow	yellowish red	red
light has shorter wavelength in visible region.	red	blue	violet	green	violet
For a given medium light has the lowest index of					
refraction in the optical range of frequencies.	blue	green	yellow	red	red
For a given medium light has the largest index of refraction					
in the optical range of frequency.	red	violet	blue	green	violet
For anomalous dispersion, there is natural frequency.	one	two	four	three	one
The index of refraction is a function of frequency of	linear	logarithmic	complex	exponential	complex
unity.	lesser than	greater than	equal to	proportional to	greater than
The imaginary part of index of refraction corresponds to the	emission	reflection	absorption	refraction	absorption
Brewster angle is valid for which type of polarisation?	Perpendicular	Parallel	S polarised	P polarised	Parallel
The Brewster angle is expressed as	Tan-1(n)	Tan-1(n1/n2)	Tan-1(n2/n1)	Tan (n)	Tan-1(n2/n1)
The Brewster angle is the angle of	Incidence	Reflection	Refraction	Transmission	Incidence
The transmission coefficient of a wave propagating in the Brewster angle					
is	0	1	-1	infinity	1
When medium of electromagnetic waves changes from air to water, their					
speed	increases	remains same	decreases	may increase or decrease	decreases
Faster among waves and rays is	waves	both have no speed	rays	both have equal speed	rays
Speed of electromagnetic waves in vacuum is	$30 \times 10^8 \text{ m s}^{-1}$	$4 \times 10^8 \text{ m s}^{-1}$	$3.0 \times 10^8 \text{ m s}^{-1}$	$6.0 \times 10^8 \mathrm{m  s^{-1}}$	$3.0 \times 10^8 \text{ m s}^{-1}$

As speed decreases if we change medium of electromagnetic waves from					
air to water, frequency	also decreases	also increases	remains same	may increase or decrease	remains same
Electromagnetic waves that include light belong to	light spectrum	electronic spectrum	electromagnet group	electromagnetic spectrum	electromagnetic spectrum
Electromagnetic waves	transfer energy	cannot transfer energy	may transfer energy	are longitudinal	transfer energy
Electromagnetic waves are	transverse waves	longitudinal waves	latitudinal waves	reversal waves	transverse waves
	travels at the same speed	frequency is greater in air	frequency is greater in	travels slower in glass	travels slower in glass
Light refracts when traveling from air into glass because light	in air and in glass	than in glass	glass than in air	than in air	than in air
Diffraction is a result of	refraction.	reflection	interference	polarization	interference
Refraction results from differences in light's	frequency	amplitude	speed	none	speed
Which of these electromagnetic waves has the shortest wavelength?	radio waves	infrared waves	X rays	ultraviolet waves	X rays
Interference occurs due to of light	Wave nature	particle nature	none	both wave and particle	Wave nature
Light will not pass through a pair of polarizing lenses when their axes are	parallel.	perpendicular.	45 degrees to each other	two of these	perpendicular.
	light passes from a denser	light comes into air from	light goes to vacuum from	light passes from denser to	light passes from denser to
Total internal reflection occurs when	to a lighter medium	vacuum	air	less dense medium	less dense medium
Ratio equal to constant (in refraction) is	sin r/sin i	sin r (sin i)	sin i/sin r	sin i²/sin r	sin i/sin r
Difference in speed of light in air and in denser medium is termed as	reflective index	refractive index	transactional index	density index	refractive index
What principle is responsible for the fact that certain sunglasses can					
reduce glare from reflected surfaces?	refraction	polarization	diffraction	total internal reflection	polarization
The principle which allows a rainbow to form is	refraction	polarization	diffraction	total internal reflection	diffraction
When unpolarised beam of light is incident on a glass slab, the rflected					
beam is found to be completely plane polarised. The angle between the					
reflected beam and the transmitted beam is	45°	25°	60°	90°	60°
Critical angle of water when refracted angle is 90° and refractive index for					
water and air is 1.33 and 1 is	48.8°	49.1°	50°	51°	48.8°
Critical angle for glass is	50°	42°	45°	30°	42°
Angle which is formed between normal and ray entering a medium is					
known as	critical angle	angle of incidence	angle of reflection	angle of refraction	angle of incidence

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### UNIT-V

### **SYLLABUS**

**Interaction of E.M.Waves with matter (Microscopic):** Scattering and Scattering parameters - Scattering by a free electron (Thomson Scattering) - Scattering by a Bound electron (Rayleigh scattering) – Dispersion Normal and Anomalous – Dispersion in gases (Lorentz theory).

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### SCATTERING AND SCATTERING PARAMETERS

If an electromagnetic wave is incident on a system of charged particles, the electric and magnetic components of the wave will Exert Lorentz force on the charge and they will be set into motion. Since the electromagnetic wave is periodic in time, so will be the motion of the particles Thus there will be changes in the directions of motion and hence there will be acceleration. The system will therefore radiate ; that is energy will be absorbed from the incident wave by the particles and will be re-emitted into space in all directions. We describe such a process as scattering of the electromagnetic wave by the system of charge particles. If the energies of the incident and scattered radiations are equal the scattering is called elastic otherwise inelastic.

Scattering is most conveniently characterized by following parameters-

(A) **Differential Scattering Cross-Section :** It is defined as the ratio of the amount of energy scattered by the system per unit time per unit solid angle to the energy flux density or intensity (i.e energy per unit area per unit time in a normal direction) of the incident radiations. So if a solid angle d $\Omega$  is substandard at the system of an area ds. The mean power (i.e energy per unit time) scattered by the system will be given by  $dP_{sr}=S_{sr}ds$ 

Where  $S_{sr}$  is the intensity of scattered radiation. The mean energy scattered per unit time per unit solid angle will therefore be

$$\frac{dPsr}{d\Omega} = \frac{Ssrds}{d\Omega}$$

 $\frac{dPsr}{sr} = S_{sr}r^2$ 

Or

If the incident energy flux density i.e intensity (pointing vector is  $S_{tr}$ , the differential scattering cross-section will be

$$\frac{do}{d\Omega} = \frac{dPsr}{d\Omega} / S_{tr} \qquad \dots \dots (A)$$

 $(as d\Omega = \frac{ds}{r^2})$  .....1

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$$\frac{do}{d\Omega} = \frac{Ssr.r2}{SIr}....(1A)$$

(Substituting the value of  $\frac{dPsr}{d\Omega}$  from eqn .1)

From the above it is clear that the differential scattering cross section has the dimension of the area.

(B) Total Scattering Cross-section : we have defined the differential scattering cross-section as

$$\frac{do}{d\Omega} = \frac{Ssr.r2}{SIr}$$

$$d\sigma = \frac{Ssr.r2d\Omega}{SIr}$$

$$\sigma = \int \frac{Ssr}{SIr} ds \qquad (as \ d\Omega = \frac{ds}{r2})$$

$$\sigma = \frac{p_{sr}}{SIr} \qquad [as \ P_{sr} = \int Ssrds]...(B)$$

 $\sigma$  is called the total scattering cross-section and is defined as the ratio of the power scattered (total energy scattered per sec) to the intensity (energy per unit area per unit time) of the incident radiations.

### SCATTERING BY A FREE ELECTRON (THOMSON SCATTERING)

Let there be an electron of mass m and change q in the path of a plane polarized monochromatic wave in vacuum. Both the electric and magnetic wave vector E and B will exert a force on a q given by the Lorentz formula

$$F=q(E+v \times B)$$
 .....(1)

Where v is the velocity of the particle its self-produced by wave, and assumed though out this treatment to be non-relativistic so that  $v \ll c$ . because in a plane wave.

$$B=n \times \frac{E}{c}$$

So expression (1) become

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 $F=q[E+v/c\times (n\times E)]$ -----(2)

F=qE

Thus only action of the electricity field on the charge needs to be considered. And so the equation of motion.

 $F=ms=m\frac{d2r}{dt^2}$ ----->(3)

In the light of (2) becomes

And becomes in a plane wave

 $E = E_0 e^{-i(\omega t - k.r)}$ 

Equation (4) becomes

$$\frac{d2r}{dt^2} = \frac{qEo}{m} e^{-i\omega t - k.t} \qquad (5)$$

Equation (5) implies that the acceleration, velocity and displacement of the particle are all in the same direction as  $E_0$  which itself is constant and that the charge is oscillating sinusoid ally. Now if the incident electromagnetic wave in which the electric vector is along x-axis is moving along z-axis as shown in fig 7.2 (a) then the acceleration in the x- direction will be given by

So that the displacement x at time t will be given by

$$x = \frac{qE0}{m\omega^2} e - i(^{\omega t - kz})....(7)$$

Now as an oscillating charge behaves like an oscillating dipole with dipole moment

P=qx.

It follows from equation (7) that

$$p0 = q2.\frac{E0}{m\omega^2}\dots\dots(8)$$

But as the average energy radiated per sec. per unit area in a normal direction by an oscillating dipole is given by

$$Ssr = \frac{1}{4\pi\varepsilon 0} \frac{q4E02}{8\pi m^2 c^3 r^4} sin 2\theta \dots (9)$$

Further as for a plane wave

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 $S1r=E\times H$ 

So the average value of  $S_1r$  will be given by

 $S1r = \frac{1}{2}\varepsilon 0cE02\dots(10).$ 

So the differential scattering cross-section  $\frac{d\sigma}{d\theta} = ro2 s\theta = rc2 cos2\Phi$  .....(A)

the incident radiation here has been taken to be plane polarized. For unpolarised an average must be taken over all orientation of the plane of AB is the direction of E in another wave of incident on the particle of fig (a) containing field point is  $\psi$ . It is now preferable to express the scattering in terms of the angle  $\boldsymbol{\Phi}$  which common to all azimuth . in fig (b) the plane POB is drawn perpendicular to the plane containing AB so that the length BQ is given both by r cos $\theta$  and by r sin $\boldsymbol{\Phi} \cos \psi$ .

averaging equation (11) over all  $\psi$ , we get

 $\sin^2\theta = 1/2(1 + \cos^2 \phi)$  -----(12)

substituting the value of  $\sin^2\theta$  from eqn (12) in (A), we get

$$\frac{d\sigma}{d\Omega} = r02\frac{1}{2}(1+\cos 2\Phi) \dots (B)$$

This is called Thomson formula for scattering of radiation and is appropriate for the scattering of x-ray by electrons or gamma rays by photons. In it angle is called the scattering angle and the factor  $1/2(1+\cos^2 \boldsymbol{\phi})$  is called degree of depolarization. From expression (B) it is clear that: i) Scattering of electromagnetic waves is independent of the nature of incident wave,

(ii) Scattering occurs in all direction and is maximum when  $\varphi=0$  or  $\pi$ , while when is minimum

$$\varphi = \frac{\pi}{2} or 3\pi/2,$$

(iii) Scattering depends on the nature of the charge particle i.e. scattered and is symmetrical about the line given by  $\varphi = \pi/2$ .

For s plane polarized light as  $\psi=0$ 

 $\sin^2\theta = 1 - (1 - \cos^2\varphi) = \cos^2\varphi$  which is also evident from fig (a) in which  $\theta = (\frac{\pi}{2} - \varphi)$ 

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The total scattering cross-section will be

$$\sigma = \frac{\int d\sigma}{dQ} \ dQ$$

$$= \int r 02 \frac{1}{2} (1 + \cos 2\varphi) dQ$$
$$\Omega = 2\pi (1 - \cos \varphi)$$
$$\sigma r = \frac{8\pi}{3} r 02....(c)$$

Result (c) was first of all derived by Thomson and so after his name it is called Thomson scattering cross-section.

A quantum mechanical calculation carried out by Klein and Mishna shows that derivations from Thomson result become significant for incident photon energy  $h\gamma$  whoch is comperable with or larger than the rest energies of the scattering electron mc<sup>2</sup>.according to them

$$\sigma KN = r02 \left\langle \frac{8\pi}{3} \left( 1 - 2h\frac{\gamma}{mc^2} + \cdots \right) \right\rangle \quad \text{for } h\gamma \ll mc2.$$
  
$$\sigma KN = r02 \left\langle \frac{\pi mc^2}{h\gamma} \left( loge(2h\frac{\gamma}{mc^2} + 1/2) \right) \right) \text{for } h\gamma \gg mc2$$

From these curves it is clear that;

(i) The scattering depends on the nature of incident radiations.

Quantum mechanical result approaches the classical one on the long wavelength side as the frequency  $\gamma = \frac{\omega}{2\pi}$  goes to zero.

(ii) The scattering is not symmetrical. In general the scattered radiation is more concentrated in the forward direction.

Apart from these is another feature to Thomson scattering which is modified by quantum considerations. The relation between the wavelength of the scattered radiation at an angle  $\phi$  and the incident radiation is

$$\lambda s = \lambda 1 + \frac{h}{mc} (1 - cos\phi)$$

### SCATTERING BY A BOUND ELECTRON (RAYLEIGH SCATTERING)

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Considering a charge whose restoring force is  $m\omega_0^2 x$  is displaced. Assume that there is a small amount of damping proportional to dx/dt which may be produced in particle by collisions or radiation. The quation of motion becomes,

 $m(d^2x/dt^2) = qE - (m\gamma) dx/dt - m\omega_0^2 x$ 

where  $\gamma$  is the damping constant per unit mass. Thus

$$(d^{2}x/dt^{2}) + (\gamma) dx/dt + \omega_{0}^{2}x = (q/m)Eoe^{-i(\omega t-kz)} - \dots (1)$$

The solution of this differential equation consists of two parts:

(a) The complementary function: It is obtained by solving equation

 $(d^{2}x/dt^{2}) + (\gamma) dx/dt + \omega_{0}^{2}x = 0 - \dots (2)$ 

Let the solution of equation (2) be

X=A<sup>αt</sup> ----- (3)

So that  $dx/dt = A\alpha e^{\alpha t}$ 

Substituting the values of x, dx/dt and  $d^2x/dt^2$  in equation (2) we get

$$\alpha^{2} + \gamma \alpha + \omega^{2}_{0} = 0$$
  
i.e. 
$$\alpha = -\frac{\gamma}{2} \pm i \sqrt{\omega_{0}^{2} - \frac{\gamma^{2}}{4}}$$

so that  $x = A_1 e^{\left[-(\gamma/2) + i\left(\sqrt{(\omega_0^2 - \gamma^2/4)t}\right) + A_2 e^{\left[-(\frac{\gamma}{2}) - i\left(\sqrt{(\omega_0^2 - \gamma^2/4)t}\right) + A_2 e^{(-(\gamma/2) + i\left(\sqrt{(\omega_0^2 - \gamma^2/4)t}\right) + A_$ 

i.e.  $x = e^{-(\gamma/2)t} [A_1 e^{i\beta t} + A_2 e^{-i\beta t}]$  ------ (a)

The constant  $A_1$  and  $A_2$  can be determined by applying initial conditions.

(b) Perpendicular integral:

It is obtained by solving the equation

$$(d^{2}x/dt^{2}) + (\gamma) dx/dt + \omega_{0}^{2}x = (q/m)Eoe^{-i(\omega t - kz)} - \dots - (4)$$
  
So that  $dx/dt = -i\omega Be^{-i(\omega t - kz)}$   
 $d^{2}x/dt^{2} = \omega^{2}Be^{-i(\omega t - kz)}$ 

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substituting these values of x, dx/dt and  $d^2x/dt^2$  in eqn., (4) we obtain

$$-\omega^{2}B + \gamma(-2\omega B) + \omega^{2}_{0}B = q/m (E_{0})$$

Or B = 
$$\frac{qE_0}{m[\omega_0^2 - \omega^2 - i\gamma\omega]}$$

So that 
$$x = \frac{qE_0}{m[\omega_0^2 - \omega^2 - i\gamma\omega]} e^{-i(\omega t + kz)}$$

with 
$$\delta = \tan^{-1}(\gamma \omega / [\omega_0^2 - \omega^2)]$$

So from expression (a) and (b) conclude that the general solution of equation (1) will be

$$x = e - (\gamma/2)t[A_1e^{i\beta t} + A_2e^{-i\beta t}] + \frac{qE_0 e^{-i(\omega t - kz - \delta)}}{m[(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2]^{\frac{1}{2}}} - \dots - (c)$$

In this solution first term on RHS represents free damped vibrations of the charge. These vibrations die out soon on account of the factor of the charge  $e(\gamma/2)t$  and the first term can be neglected in considering the final motion. So eqn., (c) reduces to

$$x = \frac{qE_0 e^{-i(\omega t - kz - \delta)}}{m[(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2]^{\frac{1}{2}}} - \dots - (d)$$

An oscillating charge is equivalent to an induced electric dipole of moment p=qx

It follows from eqn., (d)

$$p = \frac{q^{2}E_{0}e^{-i(\omega t - kz - \delta)}}{m[(\omega_{0}^{2} - \omega^{2})^{2} + \gamma^{2}\omega^{2}]^{\frac{1}{2}}}$$
  
or  $q = \frac{qE_{0}}{m[(\omega_{0}^{2} - \omega^{2})^{2} + \gamma^{2}\omega^{2}]^{\frac{1}{2}}}$  ------ (6)

But as average energy radiated per sec per unit area in a normal direction by an oscillating dipole is given by

S=(1/4πε<sub>0</sub>) (
$$\omega^4 p_0^2 / 8\pi c^3 r^2$$
) sin<sup>2</sup>θ

For the present situation 
$$d\sigma/d\Omega = r_0^2 \frac{\omega^4 \sin^2 \theta}{m[(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2]^{\frac{1}{2}}}$$
----- (A)

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Now if  $\phi$  is the angle of scattering and incident radiations are unpolarised then  $\sin^2\theta = \frac{1}{2}$ (1+cos<sup>2</sup> $\phi$ ) ------(7)

So eqn. (A) in the light of (7) becomes,  $d\sigma/d\Omega = \frac{\frac{1}{2}(1+\cos^2\phi)r_0^2\omega^4}{m[(\omega_0^2-\omega^2)^2+\gamma^2\omega^2]^{\frac{1}{2}}}$ ----- (B)

This is the required result. It is clear that

- (i) Scattering depends on the nature of the incident radiations,  $\omega$
- (ii) Scattering depends on the angle of scattering  $\phi$
- (iii) Scattering depends on the nature of the scatter  $\omega_0$  and  $\gamma$ .



The total scattering cross-section will be  $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$ 

The expression c gives the total scattering cross-section for an elastically bound electron. The total scattering cross-section is an evident of a function of the frequency of incident radiations.

- (i) If  $\omega \gg \omega_0$  then  $\sigma \rightarrow \sigma_T$
- (ii) If  $\omega \sim \omega_0$ , then  $\sigma = (8\pi/3) r_0^2 (\omega_0/\gamma)^2$  which is very large compared to Thomson scattering cross-section. This is known as resonance scattering.
- (iii) If  $\omega \ll \omega_0$  the  $\sigma \rightarrow K/\lambda^4$  i.e., amount of scattered light is proportional to  $1/\lambda^4$  where  $\lambda$  is the wavelength of the incident radiation. This scattering is known as Rayleigh

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scattering. This will occur when  $\omega$  corresponds to the frequencies of visible light and  $\omega$ o to ultraviolet.

### DISPERSION NORMAL AND ANOMALOUS

If in the medium the index of refraction various with frequency then the medium is said to be dispersive the phenomena its self is called dispersion and the rate of change refractive index with wavelength i.e.  $dn/d\lambda$  is known as dispersive power

Generally the variation of  $\eta$  is such that

- (i) The index of refraction increase as the frequency increase
- (ii) The rate of increases dn/d $\omega$ i.e. the slop of the  $\eta \omega$  curve is greater at the high frequency

However it is also found that over small frequency range there is often decrease of index of refraction with increase in frequency.



In these is narrow spectral region due to it is abnormal behavior the dispersion is called anomalous **DISPERSION IN GASES (LORENTZ THEORY)** 

In order to investigate the frequency dependence of refractive index n or dielectric constant

*er* and to discuss dispersive Lorentz assumed that in cases of gases

- (i) There is no appreciable interaction between the atom cases of atomic gases or between the molecules in case of molecules gases.
- (ii) As a electromagnetic wave passes through a gas electric field induces dipole moment in the gas molecule.
- (iii)In polarization the position of the electrons are altered from their equilibrium value while nuclei remain stationary.
- (iv)The electrons are bound to the nucleus is an atom by linear restoring force.
- (v) There is a damping proportional to the velocity of the electron.

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(vi)Over an atom or a molecule E is constant in space i.e

 $E=E_0e^{-i(\omega t-k.r)} \approx E0e-i\omega t$ 

In the light of above assumption the equation of motion of an electron will be

$$m\frac{d2r}{dt^2} + m\gamma 0\frac{dr}{dt} + m\omega o^2 r = eE$$
$$\frac{d2r}{dt^2} + \gamma 0\frac{dr}{dt} + \omega o^2 r = \frac{e}{M}E.....(1)$$

This equation is fully discussed in 7.3 and its solutions was found to be

So that dipole moment which results from the displacement of the electron under consideration

$$p = er = \left(\frac{e^2}{m}\right) EOe - i\omega t / \omega o^2 - \omega^2 - i\gamma o\omega \qquad (3)$$

Now if there are N electrons per unit volume in the gas, the polarization vector

P=Np

In the light of equation (3) is given by

$$P = N\left(\frac{\epsilon^2}{m}\right)E/\omega o^2 - \omega^2 - i\gamma o\omega \qquad \dots \qquad (4)$$

In deriving the above equation we have assumed that there is only one type of charge which is characterized by the constants  $\omega g$  and  $\gamma g$ . It is quite reasonable to expect that the electrons are not all in identical situations within molecules and that there should be different pair of characteristic frequency  $\omega_{oj}$  and associated damping factors  $\gamma_{oj}$ , pair reflecting that particular environment in which the given type of electron is found. So if we define  $f_j$  as the probability that an electron has characteristics frequency  $\gamma 0f$  and damping coefficient then the generalization of equation yields This is the required result which expresses the frequency dependence of  $\epsilon r$  or n. to study what this equation implies we consider the following situations;

(A) Static case : if  $\omega \to 0$  i.e the frequency of the incident wave is very small in comparison to

the natural frequency of the electrons, equation (A) reduces to

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$$n2 = 1 + \frac{1}{4\pi\epsilon o} \frac{4\pi Ne2}{m} \frac{\Sigma f 1}{\omega o j 2} \dots \dots \dots (a)$$

Equation (a) clearly shows that the index of refraction is a constant greater than unity and depends on the nature of the medium

(B) **normal dispersion:** If  $\gamma_{0j}$  and  $\omega < \omega o j$  i.e the region is remote from the natural frequency is of the electrons equation a reduce to

$$n2 = \left[1 + \frac{1}{4\pi\epsilon o} \frac{4\pi Ne2}{m} \frac{\Sigma f1}{\omega o j2 - 2\omega} \dots \right]$$
(b)

From equation B is clear that the refractive index in real and increases with frequency of the incident wave

(c) Anomalous dispersion If  $\omega \simeq \omega o j$  i.e in the extremely narrow spectral region in which the impressed frequencies include one of the so many natural frequencies of the electron for simplicity we assume that there is one natural frequency i.e.,  $\omega 0 = \omega o j$  so that equation(A)becomes Now as the index of refraction of gases under normal conditions is a approximately unity so that expression  $(1+y)^{1/2}=1+1/2y$  for y < 1 may be employed to obtain Multiplying numerator and denominator of the second term on R.H.S by its complex conjugated equation shows that the index of refraction is a complex function of frequency of the electromagnetic waves propagating through the gas.

The real part of equation i.e n is plotted as a function of frequency  $\omega$ . At very low frequency n is slightly greater than unity, n increase with increasing  $\omega$ , reaching a maximum at  $\omega 1 \simeq \left(\omega 0 - \frac{\gamma 0}{2}\right)$  falling to unity  $\left(\omega 0 - \frac{\gamma 0}{2}\right)$  where apart it increase again and approache unity asymptotically for large values of  $\omega$ 

The imaginary part of n2 is corresponding to absorption of the electromagnetic waves propagating through the gas. The imaginary of n\* gas a typical resonance shape. X is maximum at  $\omega = \omega 0$  where n is unity and a width at half maximum approximately equal to  $\gamma 0$  .therefore in region where n changes rapidly, the gas is relatively high absorbing.

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For any real gas exist many imaginary resonant frequency  $\omega 0j$  and corresponding damping coefficient  $\gamma 0_i$  so that

$$n * 1 + \frac{1}{4\pi\varepsilon o} \cdot \frac{2\pi N e^2}{m} \Sigma fr / (\omega 0 j 2 - \omega 2 + \gamma 0 j 2 \omega 4) \dots (d)$$

the behavior of the real and imaginary parts of the equation (c) is illustrated.

It is worthy to note here that this classical theory cannot of course predict the values of the resonance  $\omega 0j$ ; only a correlation of observation relating to optical properties of matter can be attempted. Quantum theory must be used for complete description and even though a quantum calculation can in principle yield value for the resonance frequency, the computation can be carried out exactly only for the most simple cases.

CLASS: III BSc Physics COURSE CODE: 15PHU604A COURSE NAME: ELECTROMAGNTEIC THEORY UNIT: V(Interaction of E.M.Waves with matter (Microscopic))

BATCH-2015-2018

### **Possible Questions**

### 6 marks

- 1. Explain the dispersion in gases.
- 2. Explain scattering and scattering parameters.
- 3. Explain Rayleigh Scattering.
- 4. Explain Thomson scattering.
- 5. Briefly discuss about Lorentz theory.

#### KARPAGAM ACADEMY OF HIGHER EDUCATION, COIMBATORE - 641 021

**DEPARTMENT OF PHYSICS** 

III BSc PHYSICS

BATCH : 2015-2018

ELECTROMAGNETIC THEORY (15PHU604A) MULTIPLE CHOICE QUESTIONS

opt2 opt3 Questions opt1 opt4 Answer UNIT-V If the energies of the incident and scattered radiations are equal the scattering is called inelastic coherent elastic incoherent elastic If the energies of the incident and scattered radiations are not equal. the scattering is called inelastic coherent elastic incoherent inelastic is defined as the ratio of the energy scattered by the system per unit time per unit solid angle to the energy flux density scattering differential scattering differential of the incident radiation. surface cross-section area cross-section cross-section cross-section cross-section is defined as the ratio of the power scattered to the total scattering crossdifferential scattering total scattering crossintensity of the incident radiation. cross section area cross section section cross section section Scattering depends on the nature of the particles. charged elementary none of the above uncharged charged Thomson formula for scattering is appropriate for the scattering for alpha electrons electrons neutrons cosmic Scattering occurs in all directions and is maximum when f 0° 270° 90° 45° 0° Scattering occurs in all directions and is minimum when f = 180° 90° or 270° 360° 90° or 270° 0 The total scattering cross-section according to Thomson's scattering is  $-8\pi/3 \rho_0^2$  $8\pi/3 \rho_0^2$  $8\pi/3 \rho_0^2$  $S_T =$  $8\pi/3 \rho_0$  $-8\pi/3 \rho_0$ Scattering of electromagnetic waves is of the nature of the incident wave. dependent independent infinite independent finite If the amount of scattered light is proportional to  $1/l^4$  where 1 is the wavelength of the incident radiation, then scattering is known as scattering. Thomson Inelastic Rayleigh Rayleigh Compton The blue color of the sky is due to Rayleigh None of the above Ravleigh scattering. Compton Thomson If the index of refraction decreases with the increase in frequency over small frequency range, then it is called dispersion. normal abnormal finite anomalous anomalous In dispersion in gases, there is a damping proportional to the velocity of the electron neutron muon electron proton In case of gases,  $\varepsilon_r \rightarrow$ -1 Thomson scattering is known as scattering. abnormal normal anomalous resonance resonance Thomson result become significant for incident photon energy hu which is comparable with or larger than rest energies of the scattering mc<sup>2</sup> mc<sup>2</sup>  $mv^2$ electron. mc mv According to quantum mechanical calculations, the frequency of the scattered radiation is that of the incoming waves. greater than lesser than equal to none of the above lesser than According to quantum mechanical calculations, the frequency of the scattered radiation depends on of the scattering. angle nature energy momentum angle

According to normal dispersion, the refractive index is					
	Real	imaginary	complex	rational	Real
According to normal dispersion, the refractive index					
with frequency of the incident waves.	proportional	equals	increases	decreases	increases
For any real gas, there exists resonant frequencies.	many	two	three	four	many
By which optical phenomenon does the splitting of white light into					
seven constituent colours occur?	Refraction	Reflection	Dispersion	Interference	Dispersion
	Angle of incidence is	Angle of incidence is	Angle of incidence is	Angle of incidence is	Angle of incidence is
For which of the following cases will the total internal reflection of	less than the critical	equal to the critical	greater than the critical	equal to the angle of	greater than the critical
light be possible?	angle.	angle	angle	refraction	angle
Which of the following is not due to total internal reflection?	Sparkling of diamond	Mirage	Looming	Twinkling of stars	Twinkling of stars
Which colour of light deviates minimum in the dispersion of white					
light by prism?	Violet	Blue	Green	Red	Red
Which optical phenomenon is responsible for twinkling of stars?	Atmospheric refraction	Atmospheric reflection	Total internal reflection	Dispersion	Atmospheric refraction
The sparkling of diamond and the signal communication through					
optical fibres is due to	refraction	scattering	total internal reflection	dispersion	total internal reflection
In London dispersion forces dipole which is created is	temporary	permanent	weak	strong	temporary
London dispersion forces are present in	HC1	oxygen	He	NaCl	He
Lorentz electric force has direction	similar to electric field	opposite of electric field	scalar quantity	None	similar to electric field
Lorentz extended drude model and thus extended classical model is		Drude-Sommerfeld			
called	Drude-Hendrik model	model	Drude-Lorentz model	Lorentz model	Drude-Lorentz model
Simplest analysis of Drude model assumes that electric field E is	non uniform	uniform	infinite	random	uniform
				both negative and	
Drude model considers metal to be formed of a mass of ions which are	negatively charged	positively charge	not charged	positive charged	positively charge
In Maxwell's equation, $\nabla \cdot B=0$ , which term is basically zero	$\rho_{ev}$ (electrical)	$\rho_{mv}$ (magnetic)	$\sigma_{ev}$ (electrical)	$\sigma_{mv}$ (magnetic)	$\rho_{mv}$ (magnetic)
If the EMW incident on a system of charged particles, the electric and					
magnetic fields of the wave exert a force on the					
charges.	Lorentz	Mechanical	Electrical	No	Lorentz
Lorentz formula is F=	-q(E + vXB)	q(E + vXB)	q (E - v X B)	-q (E - v X B)	q (E + v X B)
The velocity of electromagnetic radiation in a material medium varies					
with	wavelength	interference	diffraction	scattering	wavelength
If the energies of the incident and scattered radiations are unequal the					
scattering is called	elastic	inelastic	flux density	none	elastic

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