NUMERICAL METHODS 3 1 0 4 100

INTENDED OUTCOMES:

- The roots of algebraic or transcendental equations, solutions of large system of linear equations and Eigen value problem of a matrix can be obtained numerically.
- When huge amounts of experimental data are involved, the methods discussed on interpolation will be useful in constructing approximate polynomial to represent the data and to find the intermediate values.
- The numerical differentiation and integration find application when the function in the analytical form is too complicated or the huge amounts of data are given such as series of measurements, observations or some other empirical information.

UNIT -I SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS

Method of false position – Newton's method – Statement of fixed point theorem – Fixed point iteration: $x = g(x)$ method – Solution of linear system by Gaussian elimination and Gauss-Jordon methods - Iterative methods: Gauss Jacobi and Gauss-Seidel methods - Inverse of a matrix by Gauss Jordon method – Eigen value of a matrix by power method.

UNIT- II INTERPOLATION AND APPROXIMATION

Lagrangian Polynomials – Divided differences Interpolation formula – Newton's forward and backward difference formulas.

UNIT- III NUMERICAL DIFFERENTIATION AND INTEGRATION

Derivatives from difference tables –Derivatives using interpolation formula–Numerical integration by trapezoidal and Simpson's 1/3 and 3/8 rules – Romberg's method – Two and Three point Gaussian quadrature formulas – Double integrals using trapezoidal and Simpson's rules.

UNIT -IV INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

Single step methods: Taylor series method – Euler and modified Euler methods (Heun's method) – Fourth order Runge – Kutta method for solving first and second order equations – Multistep methods: Milne's and Adam's predictor and corrector methods.

UNIT- V BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Finite difference solution of second order ordinary differential equation – Finite difference solution of one dimensional heat equation by explicit and implicit methods – One dimensional wave equation and two dimensional Laplace and Poisson equations.
MATLAB : Matlab – Toolkits – 2D Graph Plotting.

TEXT BOOKS:

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Burden, R. L. and Faires,T. D	Numerical Analysis	Thomson Asia Pvt. Ltd., Singapore.	2002
2	Curtis F. Gerald and Patrick O. Wheatley	Applied Numerical Analysis	Pearson Education, South Asia	2009

REFERENCES:

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication

1	Sankar Rao. G	Numerical Methods	Prentice Hall of India Pvt. Ltd, New Delhi.	2003
2	Gerald. C. F. and Wheatley. P. O	Applied Numerical Analysis	Pearson Education Asia, New Delhi.	2002
3	Balagurusamy. E	Numerical Methods	Tata McGraw Hill Pub. Co. Ltd., New Delhi.	2009
4	Kandaswamy, P., Thilagavathy, K. and Gunavathi, K.	Numerical Methods	S. Chand Publishing, New Delhi.	2010

WEBSITES:

1. www.nr.com
2. www.numerical-methods.com
3. www.math.ucsb.edu
4. www.mathworks.com

KARPAGAM UNIVERSITY, COIMBATORE-21.
FACULTY OF ENGINEERING
B.E Aeronautical Engineering- Fourth Semester
NUMERICAL METHODS-- LECTURE PLAN

S.No	Topic covered	No. of hours
	UNIT I : SOLUTION OF EQUATIONS AND EIGENVALUE PROBLEMS	
1	Basics – Scientific Calculator usage, Equation, Simultaneous equation, algebraic equation, ODE, PDE, Trigonometric function	5
2	Method of false position – Formula, methodology, Problems	1
3	Method of false position –Problems	1
4	Newton's method - Formula, methodology, Problems	1
5	Modified Newton's method - Formula, methodology, Problems	1
6	Fixed point iteration - Formula, methodology, Problems	1
7	Tutorial 1 – Regular Falsi and Newton's Method	1
8	Gauss elimination method - Methodology, Problems	1
9	Gauss-Jordan method - Methodology, Problems	1
10	Gauss-Jacobi method - Methodology, Problems	1
11	Gauss-Seidal method - Methodology, Problems	1
12	Tutorial 2 – Gauss Jordan, Jacobi, Seidal	1
13	Inverse of the matrix by Gauss-Jordan method	1
14	Inverse of the matrix by Gauss-Jordan method	1
15	Eigenvalue of the matrix by power method	1
16	Tutorial 3 – Eigen value by Power method	1
17	MATLAB – Toolkits	1
	Total	21
	UNIT II : INTERPOLATION AND APPROXIMATION	
1	Introduction – Interpolation, equal interval and Unequal interval	1
2	Lagrange's polynomial - Formula, methodology, Problems	1
3	Lagrange's polynomial – Problems	1
4	Newton's divided difference formula - Formula, methodology, Problems	1
5	Newton's divided difference formula – Problems	1
6	Newton's forward difference formula - Formula, methodology, Problems	1
7	Newton's backward difference formula- Formula, methodology, Problems	1
8	Newton's forward and backward difference formula	1
9	Tutorial 4-Newton's Forward and Backward difference formula	1
	Total	9
	UNIT III : NUMERICAL DIFFERENTIATION AND INTERGRATION	
1	Derivatives from difference table - Formula, methodology, Problems	1
2	Derivatives from difference table - Problems	1
3	Derivatives at initial final values	1
4	Tutorial 5 – Derivatives from difference table	1
5	Numerical integration by trapezoidal rule	1
6	Numerical integration by Simpson's $1/3^{rd}$ and $3/8^{th}$ rules	1
7	Romberg's method	1
8	Romberg's method	1
9	Two point Gaussian quadrature formula	1
10	Three point Gaussian quadrature formula	1
11	Double integral's using trapezoidal rule	1
12	Double integral's using Simpson's rule	1
13	Double integral – Problems	1
14	Tutorial 6 – Trapezoidal and Simpson's methods	1
	Total	14
	UNIT IV: INITIAL VALUE PROBLEM FOR ORDINARY DIFFERENTIAL EQUATIONS	
1	Taylor's series method - Formula, methodology, Problems	1
2	Taylor's series method – Problems	1
3	Taylor's series method – Second order Problems	1
4	Euler Finite difference solution	1
5	Modified Euler Finite difference solution	1
6	Euler's method – Problems	1
7	Tutorial 7 – Taylor's and Euler's methods	1

8	Fourth order Runge-Kutta method for solving 1st order equations	1
9	Fourth order Runge-Kutta method for solving 2nd order equations	1
10	Runge- Kutta – Problems	1
11	Milne's predictor method	1
12	Milne's corrector method	1
13	Adam's predictor method	1
14	Adam's corrector method	1
15	Tutorial 8 – Runge-Kutta method	1
	Total	15
UNIT V:BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS		
1	Finite difference solution of 2nd order ODE	1
2	Finite difference method – Problems	1
3	Finite difference solution of one dimensional heat equation by explicit method	1
4	Finite difference solution of one dimensional heat equation by implicit method	1
5	Tutorial 9 – Finite difference Explicit and implicit methods	1
6	One dimensional wave equation and two dimensional Laplace equation	1
7	Two dimensional Laplace equation	1
8	Two dimensional Laplace equation – Problems	1
9	Two dimensional Poisson equation	1
10	Two dimensional Poisson equation – Problems	1
11	Tutorial 10 – Laplace equation	1
		11
	TOTAL	**60 +10**

Reference Books:

1. Gerald,C.F, and wheatly,P.O,2002.Applied Numerical Analysis, Sixth Edition, Pearson Education Asia, New Delhi
2. Balagurusamy.E.,1999; Numerical Methods, TataMcGraw, Hill Pub.Co.Ltd.,New Delhi.
3. Kandasamy.P, Thilagavathy, K.Gunavathy,K.,1999,Numerical Methods; S.Chand Co. New Delhi.
4. Burden,R.L and T.D., 2002 numerical Analyis, 7th Edition,Thomson asia Pvt. Ltd., Singapore.
5. Venkataraman M.K.1991.Numerical Methods National Pub.Co, Chennai.
6. Sankara Rao., 2004. Numerical Methods for Scientists and engineers, 2nd Ed.Prentice Hall.India

Unit - I

Solutions of Equations and Eigen Value Problems.

Iterative Method:

① Write the gn eqn $f(x) = 0$ into the form $x = \varphi(x)$

② Assume that $x = x_0$ be the root of the given eqn

③ The first approximation to the root is gn by $x_1 = \varphi(x_0)$

IIIly $x_2 = \varphi(x_1)$

$x_3 = \varphi(x_2)$

$\vdots$

$x_n = \varphi(x_{n-1})$

x_n the n^{th} iteration → the value of

=> x_n is the root of the gn eqn.

x_n is the root

---

① Find the root of the equation $\cos x = 3x - 1$, using iteration Method.

Soln

$f(x) = \cos x - 3x + 1$

$f(0) = \cos 0 - 3(0) + 1 = 2 \rightarrow +ve$

$f(1) = \cos 1 - 3(1) + 1 = 0 - 3(1) + 1 \rightarrow -ve$

∴ The root lies between 0 and 1

The eqn can be written as

$$\cos x - 3x + 1 = 0$$

$$-3x = -\cos x - 1$$

$$3x = \cos x + 1$$

$$x = \frac{1}{3}[1 + \cos x]$$

Let $\varphi(x) = \frac{1}{3}[1 + \cos x]$

$$\varphi'(x) = -\frac{1}{3}\sin x$$

$$|\varphi'(x)| = \frac{1}{3}\sin x$$

$$|\varphi'(0)| = 0 < 1$$

$$|\varphi'(1)| = \frac{1}{3}\sin 1 = 0.2804 < 1.$$

Let $x_0 = 0$

$x_1 = \varphi(x_0) = \frac{1}{3}(1 + \cos x_0) = \frac{1}{3}(1 + \cos 0)$

$x_1 = 0.6667$

$x_2 = \varphi(x_1) = \frac{1}{3}(1 + \cos x_1) = \frac{1}{3}(1 + \cos 0.6667)$

$x_2 = 0.5953$

$x_3 = \varphi(x_2) = \frac{1}{3}(1 + \cos x_2) = \frac{1}{3}(1 + \cos 0.5953)$

$x_3 = 0.6093$

$x_4 = \varphi(x_3) = \frac{1}{3}(1 + \cos x_3) = \frac{1}{3}(1 + \cos 0.6093)$

$x_4 = 0.6067$

x_4

$x_5 = \varphi(x_4) = \frac{1}{3}(1+\cos x_4) = \frac{1}{3}(1+\cos 0.6067)$

$x_5 = 0.6072$

$x_6 = \varphi(x_5) = \frac{1}{3}(1+\cos x_5) = \frac{1}{3}(1+\cos 0.6072)$

$x_6 = 0.6071$

$x_7 = \varphi(x_6) = \frac{1}{3}(1+\cos x_6) = \frac{1}{3}(1+\cos 0.6071)$

$x_7 = 0.6071$

$\therefore$ The required root is 0.6071.

(2) Solve the equation $x^2 - 2x - 3 = 0$ for the +ve root by iteration method.

Soln

$x^2 - 2x - 3 = 0$

$f(x) = x^2 - 2x - 3$

$f(0) = 0 - 2(0) - 3 = -3 \rightarrow -ve$

$f(1) = -4 \rightarrow -ve$

$f(2) = -3 \rightarrow -ve$

$f(3) = 0 \rightarrow +ve$

$\therefore$ The root lies between 2 and 3

$x^2 = 2x + 3$

$x = \sqrt{2x+3}$

$$\varphi(x) = \sqrt{2x+3} = (2x+3)^{1/2}$$

$$\varphi'(x) = \frac{1}{2}(2x+3)^{-1/2} \cdot 2$$

$$|\varphi'(x)| = |(2x+3)^{-1/2}|$$

$$|\varphi'(2)| < 1 \quad \& \quad |\varphi'(3)| < 1$$

Take $x_0 = 2.5$

$x_1 = \varphi(x_0) = \sqrt{2x_0+3} = \sqrt{2(2.5)+3} = 2.8284$

$x_2 = \varphi(x_1) = \sqrt{2x_1+3} = \sqrt{2(2.8284)+3} = 2.9422$

$x_3 = \varphi(x_2) = \sqrt{2x_2+3} = \sqrt{2(2.9422)+3} = 2.9807$

$x_4 = \varphi(x_3) = \sqrt{2x_3+3} = \sqrt{2(2.9807)+3} = 2.9936$

$x_5 = \varphi(x_4) = \sqrt{2x_4+3} = \sqrt{2(2.9936)+3} = 2.9979$

$x_6 = \varphi(x_5) = \sqrt{2x_5+3} = \sqrt{2(2.979)+3} = 2.9993$

$x_7 = \varphi(x_6) = \sqrt{2x_6+3} = \sqrt{2(2.9993)+3} = 2.9998$

$x_8 = \varphi(x_7) = \sqrt{2x_7+3} = \sqrt{2(2.9998)+3} = 2.9999$

$x_9 = \varphi(x_8) = \sqrt{2x_8+3} = \sqrt{2(2.9999)+3} = 2.9999$

$\therefore$ The required root is 2.9999

(3) Solve by iteration Method $2x - \log_{10} x = 7$

Soln

$$2x - \log_{10} x - 7 = 0$$

$$f(x) = 2x - \log_{10} x - 7$$

$f(1) = -5 \to -ve$

$f(2) = -3.3010 \to -ve$

$f(3) = -1.4771 \to -ve$

$f(4) = 0.3979 \to +ve$

$\therefore$ The root lies between 3 and 4

$$2x = 7 + \log_{10} x$$

$$x = \frac{1}{2}\left[7 + \log_{10} x\right]$$

$$\therefore \varphi(x) = \frac{1}{2}\left[7 + \log_{10} x\right]$$

$$\varphi'(x) = \frac{1}{2}\left[\frac{1}{x}\log_{10} e\right]$$

$$|\varphi'(x)| = \left|\frac{1}{2}\left[\frac{1}{x}\log_{10} e\right]\right| < 1 \text{ in } (3, 4)$$

Take $x_0 = 3.6$

$$x_1 = \varphi(x_0) = \frac{1}{2}\left[\log_{10} x_0 + 7\right]$$

$$= \frac{1}{2}\left[\log_{10} 3.6 + 7\right]$$

$$= 3.7782$$

$x_2 = \varphi(x_1) = \frac{1}{2}[\log_{10} x_1 + 7]$

$= \frac{1}{2}[\log_{10} 3.7782 + 7]$

$x_2 = 3.7886$

$x_3 = \varphi(x_2) = \frac{1}{2}[\log_{10} x_2 + 7]$

$= \frac{1}{2}[\log_{10} 3.7886 + 7]$

$x_3 = 3.7892$

$x_4 = \varphi(x_3) = \frac{1}{2}[\log_{10} x_3 + 7]$

$= \frac{1}{2}[\log_{10} 3.7892 + 7]$

$x_4 = 3.7893$

$x_5 = \varphi(x_4) = \frac{1}{2}[\log_{10} x_4 + 7]$

$= \frac{1}{2}[\log_{10} 3.7893 + 7]$

$x_5 = 3.7893$

∴ The required root is 3.7893

H.W 4) find the negative root of the eqn $x^3 - 2x + 5 = 0$

Gauss Jordan Method

(1)
$$2x - y + 6z = 22$$
$$x + 7y - 3z = -22$$
$$5x - 2y + 3z = 18$$

Soln

$$[A, B] = \left[\begin{array}{ccc|c} 2 & -1 & 6 & 22 \\ 1 & 7 & -3 & -22 \\ 5 & -2 & 3 & 18 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{-1}{2} & 3 & 11 \\ 1 & 7 & -3 & -22 \\ 1 & \frac{-2}{5} & \frac{3}{5} & \frac{18}{5} \end{array}\right] \quad R_1 \to \frac{R_1}{2}, \quad R_3 \to \frac{R_3}{5}$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{-1}{2} & 3 & 11 \\ 0 & \frac{15}{2} & -6 & -33 \\ 0 & \frac{1}{16} & \frac{-12}{5} & \frac{-37}{5} \end{array}\right] \quad R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$$

$$= \left[\begin{array}{ccc|c} -2 & 1 & -6 & -22 \\ 0 & 1 & \frac{-4}{5} & \frac{-22}{5} \\ 0 & 1 & -24 & -74 \end{array}\right] \quad R_1 \to -2R_1, \quad R_2 \to \frac{2}{15} R_2, \quad R_3 \to 10 R_3$$

$$= \left[\begin{array}{ccc|c} -2 & 0 & \frac{-26}{5} & \frac{-88}{5} \\ 0 & 1 & \frac{-4}{5} & \frac{-22}{5} \\ 0 & 0 & \frac{-116}{5} & \frac{-348}{5} \end{array}\right] \quad R_1 \to R_1 - R_2, \quad R_3 \to R_3 - R_2$$

$$= \left[\begin{array}{ccc|c} \frac{5}{13} & 0 & 1 & \frac{44}{13} \\ 0 & -\frac{5}{4} & 1 & \frac{11}{2} \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 \to R_1 \times \frac{-5}{26} \\ R_2 \to -\frac{5}{4} R_2 \\ R_3 \to -\frac{5}{116} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} \frac{5}{13} & 0 & 0 & \frac{5}{13} \\ 0 & -\frac{5}{4} & 0 & \frac{5}{2} \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_2 \to R_2 \times \frac{-4}{5} \\ R_1 \to R_1 \times \frac{13}{5} \end{array}$$

$x = 1$, $y = -2$, $z = 3$.

(2) Solve

$$x + 3y + 3z = 16$$
$$x + 4y + 3z = 18$$
$$x + 3y + 4z = 19$$

$$[A,B] = \left[\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_2 \to R_2 - R_1, \; R_3 \to R_3 - R_1$$

$$= \left[\begin{array}{ccc|c} 1/3 & 1 & 1 & 16/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to \frac{R_1}{3}$$

$$= \left[\begin{array}{ccc|c} 1/3 & 0 & 1 & 10/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to R_1 - R_2$$

$$= \left[\begin{array}{ccc|c} 1/3 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to R_1 - R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \to 3R_1$$

$x = 1, \; y = 2, \; z = 3$

(3) Solve

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

Soln

$$[A, B] = \left[\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1/10 & 1/10 & 6/5 \\ 1 & 5 & 1/2 & 13/2 \\ 1 & 1 & 5 & 7 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{10} \\ R_2 \to \frac{R_2}{2} \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{1}{10} & \frac{1}{10} & \frac{6}{5} \\ 0 & \frac{49}{10} & \frac{2}{5} & \frac{53}{10} \\ 0 & \frac{9}{10} & \frac{49}{10} & \frac{29}{5} \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{cccc} 10 & 1 & 1 & 12 \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & 1 & \frac{49}{9} & \frac{58}{9} \end{array}\right] \begin{array}{l} R_1 \to 10 R_1 \\ R_2 \to \frac{10}{49} R_2 \\ R_3 \to \frac{10}{9} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 10 & 0 & \frac{45}{49} & \frac{535}{49} \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & 0 & \frac{2365}{441} & \frac{2365}{441} \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$= \left[\begin{array}{ccc|c} \frac{98}{9} & 0 & 1 & \frac{107}{9} \\ 0 & \frac{49}{4} & 1 & \frac{53}{4} \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_2 \to \frac{49}{45} R_1 \\ R_2 \to \frac{49}{4} R_2 \\ R_3 \to \frac{441}{2365} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} \frac{98}{9} & 0 & 0 & \frac{98}{9} \\ 0 & \frac{49}{4} & 0 & \frac{49}{4} \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_3 \\ R_1 \to R_1 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 \times \frac{9}{98} \\ R_2 \to R_2 \times \frac{4}{49} \end{array}$$

$x = 1 \quad y = 1 \quad z = 1$

Inverse of a Matrix

Gauss Jordan Method

(1) find the inverse of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ using Gauss Jordan Method.

Soln

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

$$(A/I) = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{1} \\ R_2 \to \frac{R_2}{1} \\ R_3 \to \frac{R_3}{-2} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & -\frac{1}{2} \end{array}\right] R_2 \to$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & -\frac{1}{2} \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -1 & -1 & 0 & -\frac{1}{2} \end{array}\right] \quad \begin{array}{l} R_1 \to \frac{R_1}{1} \\ R_2 \to \frac{R_2}{2} \\ R_3 \to \frac{R_3}{1} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} \frac{1}{6} & 0 & 1 & \frac{1}{4} & -\frac{1}{12} & 0 \\ 0 & -\frac{1}{3} & 1 & \frac{1}{6} & -\frac{1}{6} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1/6 \\ R_2 \to R_2/-3 \\ R_3 \to R_3/2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{4} \\ 0 & -\frac{1}{3} & 0 & \frac{5}{12} & \frac{1}{12} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & \frac{3}{2} \\ 0 & 1 & 0 & -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 \times 6 \\ R_2 \to -3R_2 \end{array}$$

$$= [\, I / A \,]$$

$\therefore$ Inverse of A is $\begin{bmatrix} 3 & 1 & \frac{3}{2} \\ -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$

(2) find the inverse of the Matrix $\begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ using Gauss Jordan Method.

Soln

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ -1 & 4 & 10 \end{bmatrix}$$

$$[A/I] = \left[\begin{array}{ccc|ccc} 3 & -1 & 1 & 1 & 0 & 0 \\ -15 & 6 & -5 & 0 & 1 & 0 \\ -1 & 4 & 10 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 1 & -\frac{6}{15} & +\frac{1}{3} & 0 & -\frac{1}{15} & 0 \\ 1 & -4 & -10 & 0 & 0 & -1 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{3} \\ R_2 \to \frac{R_2}{-15} \\ R_3 \to \frac{R_3}{-1} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{15} & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{15} & 0 \\ 0 & -\frac{11}{3} & -\frac{31}{3} & -\frac{1}{3} & 0 & -1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} -3 & 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 10 & 5 & 1 & 0 \\ 0 & 1 & \frac{31}{11} & \frac{1}{11} & 0 & \frac{3}{11} \end{array}\right] \begin{array}{l} R_1 \to R_1(-3) \\ R_2 \to -15 R_2 \\ R_3 \to \frac{-3}{11} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} -3 & 0 & -1 & -6 & -1 & 0 \\ 0 & 1 & 10 & 5 & 1 & 0 \\ 0 & 0 & -\frac{79}{11} & -\frac{54}{11} & -1 & \frac{3}{11} \end{array}\right] \begin{array}{l} R_3 \to R_3 - R_2 \\ R_1 \to R_1 - R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} +3 & 0 & 1 & 6 & 1 & 0 \\ 0 & \frac{1}{10} & 1 & \frac{1}{2} & \frac{1}{10} & 0 \\ 0 & 0 & 1 & \frac{1}{31} & 0 & \frac{3}{31} \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{-1} \\ R_2 \to \frac{R_2}{10} \\ R_3 \to \frac{11}{31} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 3 & 0 & 0 & \frac{185}{31} & 1 & -\frac{3}{31} \\ 0 & \frac{1}{10} & 0 & \frac{29}{62} & \frac{1}{10} & -\frac{3}{31} \\ 0 & 0 & 1 & \frac{1}{31} & 0 & \frac{3}{31} \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{185}{93} & \frac{1}{3} & -\frac{1}{31} \\ 0 & 1 & 0 & \frac{290}{62} & 1 & -\frac{30}{31} \\ 0 & 0 & 1 & \frac{1}{31} & 0 & \frac{3}{31} \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{3} \\ R_2 \to R_2 \times 10 \end{array}$$

$= [I \; A]$

$\therefore$ inverse of A is $\begin{bmatrix} \frac{85}{93} & \frac{1}{3} & -\frac{1}{31} \\ \frac{290}{62} & 1 & -\frac{30}{31} \\ \frac{1}{31} & 0 & \frac{3}{31} \end{bmatrix}$

3) Using Gauss Jordan Method find the inverse of $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$

Soln

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$$

$$(A/I) = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & \frac{4}{3} & \frac{5}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{1} \\ R_2 \to \frac{R_2}{3} \\ R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & \frac{4}{3} & \frac{8}{3} & -1 & \frac{1}{3} & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right] R_2 \to R_2 - R_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & \frac{2}{2} & -\frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 1 & \frac{7}{6} & 0 & 0 & -\frac{1}{6} \end{array}\right] \begin{array}{l} R_2 \to \frac{3}{4} R_2 \\ R_3 \to \frac{R_3}{-6} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -\frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{5}{6} & \frac{3}{4} & -\frac{1}{4} & \frac{-1}{6} \end{array}\right] R_3 \to R_3 - R_2$$

$$= \left[\begin{array}{ccc|ccc} -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & -\frac{3}{8} & \frac{1}{8} & 0 \\ 0 & 0 & 1 & -\frac{9}{10} & \frac{3}{10} & \frac{1}{5} \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{-1} \\ R_2 \to \frac{R_2}{2} \\ R_3 \to -\frac{6}{5} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & -\frac{1}{10} & -\frac{3}{10} & -\frac{1}{5} \\ 0 & \frac{1}{2} & 0 & \frac{21}{40} & -\frac{7}{40} & -\frac{1}{5} \\ 0 & 0 & 1 & -\frac{9}{10} & \frac{3}{10} & \frac{1}{5} \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{10} & \frac{3}{10} & \frac{1}{5} \\ 0 & 1 & 0 & \frac{21}{20} & -\frac{7}{20} & -\frac{2}{5} \\ 0 & 0 & 1 & -\frac{9}{10} & \frac{3}{10} & \frac{1}{5} \end{array}\right]$$

$$= [\,I / A\,]$$

Inverse of A is $\begin{bmatrix} \frac{1}{10} & \frac{3}{10} & \frac{1}{5} \\ \frac{21}{20} & -\frac{7}{20} & -\frac{2}{5} \\ -\frac{9}{10} & \frac{3}{10} & \frac{1}{5} \end{bmatrix}$

Gauss Jacobi Method

① Solve the following eqns by Gauss Jacobi Method

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

$x = \frac{17 - y + 2z}{20}$	$y = \frac{-18 + z - 3x}{20}$	$z = \frac{25 - 2x + 3y}{20}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 0.85$	$y_1 = -0.9$	$z_1 = 1.25$
$x_2 = 1.02$	$y_2 = -0.965$	$z_2 = 1.03$
$x_3 = 1.0013$	$y_3 = -1.0015$	$z_3 = 1.0033$
$x_4 = 1.0004$	$y_4 = -1.0001$	$z_4 = 0.9996$
$x_5 = 0.9999$	$y_5 = -1.0001$	$z_5 = 0.9999$
$x_6 = 1$	$y_6 = -1$	$z_6 = 1$
$x_7 = 1$	$y_7 = -1$	$z_7 = 1$

$\therefore\ x = 1,\ y = -1,\ z = 1$

② Solve
$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

$x = \frac{32-4y+z}{28}$	$y = \frac{35-4x-2x}{17}$	$z = \frac{24-x-3y}{10}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 1.1429$	$y_1 = 2.0588$	$z_1 = 2.4$
$x_2 = 0.9345$	$y_2 = 1.3597$	$z_2 = 1.6681$
$x_3 = 1.0082$	$y_3 = 1.5564$	$z_3 = 1.898$
$x_4 = 0.9883$	$y_4 = 1.4935$	$z_4 = 1.8323$
$x_5 = 0.9949$	$y_5 = 1.514$	$z_5 = 1.8531$
$x_6 = 0.9931$	$y_6 = 1.5058$	$z_6 = 1.847$
$x_7 = 0.9937$	$y_7 = 1.5074$	$z_7 = 1.8490$
$x_8 = 0.9936$	$y_8 = 1.5069$	$z_8 = 1.8484$
$x_9 = 0.9936$	$y_9 = 1.5070$	$z_9 = 1.8486$
$x_{10} = 0.9936$	$y_{10} = 1.5070$	$z_{10} = 1.8485$
$x_{11} = 0.9936$	$y_{11} = 1.5070$	$z_{11} = 1.8485$

∴ The soln is

$x = 0.9936 \quad y = 1.5070 \quad z = 1.8485$

(3) Solve
$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

$x = \frac{85 - 6y + z}{27}$	$y = \frac{72 - 6x - 2z}{15}$	$z = \frac{110 - x - y}{54}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 3.148$	$y_1 = 4.8$	$z_1 = 2.037$
$x_2 = 2.157$	$y_2 = 3.269$	$z_2 = 1.890$
$x_3 = 2.492$	$y_3 = 3.685$	$z_3 = 1.937$
$x_4 = 2.401$	$y_4 = 3.545$	$z_4 = 1.923$
$x_5 = 2.432$	$y_5 = 3.583$	$z_5 = 1.927$
$x_6 = 2.423$	$y_6 = 3.570$	$z_6 = 1.926$
$x_7 = 2.426$	$y_7 = 3.574$	$z_7 = 1.926$
$x_8 = 2.425$	$y_8 = 3.573$	$z_8 = 1.926$

$x = 2.425 \quad y = 3.573 \quad z = 1.926.$

Gauss Seidal Iteration Method

(1) Solve

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Soln.

$x = \frac{17 - y + 2z}{20}$	$y = \frac{-18 - 3x + z}{20}$	$z = \frac{25 - 2x + 3y}{20}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 0.82$	$y_1 = -1.0275$	$z_1 = 1.0109$
$x_2 = 1.0025$	$y_2 = -0.9998$	$z_2 = 0.9998$
$x_3 = 1.0000$	$y_3 = -1.000$	$z_3 = 1.0000$
$x_4 = 1.0000$	$y_4 = -1.0000$	$z_4 = 1.0000$

$x = 1$ $y = -1$ $z = 1$

(2) Solve

$$4x + 2y + z = 14$$
$$x + 5y - z = 10$$
$$x + y + 8z = 20$$

$x = \frac{85-6y+z}{27}$	$y = \frac{72-6x-2z}{15}$	$z = \frac{110-x-y}{54}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 3.148$	$y_1 = 3.541$	$z_1 = 1.913$
$x_2 = 2.432$	$y_2 = 3.572$	$z_2 = 1.926$
$x_3 = 2.426$	$y_3 = 3.573$	$z_3 = 1.926$
$x_4 = 2.426$	$y_4 = 3.573$	$z_4 = 1.926$

$\therefore \quad x = 2.426$

$y = 3.573$

$z = 1.926.$

$x = \frac{14-2y-z}{4}$	$y = \frac{10-x+z}{5}$	$z = \frac{20-x-y}{8}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 3.5$	$y_1 = 1.3$	$z_1 = 1.9$
$x_2 = 2.375$	$y_2 = 1.905$	$z_2 = 1.965$
$x_3 = 2.056$	$y_3 = 1.982$	$z_3 = 1.995$
$x_4 = 2.010$	$y_4 = 1.997$	$z_4 = 1.999$
$x_5 = 2.002$	$y_5 = 1.999$	$z_5 = 2$
$x_6 = 2.001$	$y_6 = 2$	$z_6 = 2$
$x_7 = 2$	$y_7 = 2$	$z_7 = 2$
$x_8 = 2$	$y_8 = 2$	$z_8 = 2$

$\therefore \quad x = 2, \; y = 2, \; z = 2$

(3) Solve
$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

Eigen Values of a Matrix by Power Method

(1) Find the numerically largest eigen value of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and its corresponding eigen vector by power method, taking the initial eigen vector as $(1\ 0\ 0)^T$ (upto three decimal places.

Soln

(1) Given $X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

$$AX_1 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 25 \\ 1 \\ 2 \end{pmatrix} = 25\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = 25X_2$$

$$AX_2 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = \begin{pmatrix} 25.2 \\ 1.12 \\ 0.08 \end{pmatrix} = 25.2\begin{pmatrix} 1 \\ 0.0444 \\ 0.0667 \end{pmatrix}$$

$$= 25.2X_3$$

$$AX_3 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0444 \\ 0.0667 \end{pmatrix} = \begin{pmatrix} 25.1778 \\ 1.1332 \\ 1.7337 \end{pmatrix} = 25.1778\begin{pmatrix} 1 \\ 0.0450 \\ 0.0688 \end{pmatrix}$$

$$= 25.1778X_4$$

$$AX_4 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0450 \\ 0.06888 \end{pmatrix} = \begin{pmatrix} 25.1826 \\ 1.135 \\ 1.7248 \end{pmatrix}$$

$$= 25.1826\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = 25.1826 X_5$$

$$AX_5 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \begin{pmatrix} 25.1821 \\ 1.1353 \\ 1.7260 \end{pmatrix}$$

$$= 25.1821\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = 25.1821 X_6$$

Dominant eigen value $\lambda = 25.1821$

corresponding eigen vector is $\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$

(2) Determine by Power method the largest eigen value and the corresponding eigen vector of the Matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$

Soln

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = 3 \begin{bmatrix} 0.3333 \\ 1 \\ -0.333 \end{bmatrix} = 3X_2$$

$$AX_2 = \begin{bmatrix} 3.6666 \\ 1.6667 \\ 0.3337 \end{bmatrix} = 3.6666 \begin{bmatrix} 1 \\ 0.4546 \\ 0.0910 \end{bmatrix} = 3.6666\, X_3$$

$$AX_3 = \begin{bmatrix} 2.2728 \\ 4.2732 \\ 1.7284 \end{bmatrix} = 4.2732 \begin{bmatrix} 0.5319 \\ 1 \\ 0.4045 \end{bmatrix} = 4.2732\, X_4$$

$$AX_4 = \begin{bmatrix} 3.1274 \\ 5.2137 \\ 7.5131 \end{bmatrix} = 7.5131 \begin{bmatrix} 0.4163 \\ 0.6939 \\ 1 \end{bmatrix} = 7.5731\, X_5$$

$$AX_5 = \begin{bmatrix} 1.498 \\ 6.6367 \\ 12.3593 \end{bmatrix} = 12.3593 \begin{bmatrix} 0.1212 \\ 0.5370 \\ 1 \end{bmatrix} = 12.3596\, X_6$$

$$AX_6 = \begin{bmatrix} 0.7322 \\ 5.4376 \\ 12.0268 \end{bmatrix} = 12.0268 \begin{pmatrix} 0.0609 \\ 0.4521 \\ 1 \end{pmatrix} = 12.0268\, X_7$$

$$AX_7 = \begin{pmatrix} 0.4172 \\ 5.0869 \\ 11.7473 \end{pmatrix} = 11.7475 \begin{pmatrix} 0.0355 \\ 0.4330 \\ 1 \end{pmatrix} = 11.7475 X_8$$

$$AX_8 = \begin{pmatrix} 0.3345 \\ 4.9721 \\ 11.6965 \end{pmatrix} = 11.6965 \begin{pmatrix} 0.0286 \\ 0.4251 \\ 1 \end{pmatrix} = 11.6965 X_9$$

$$AX_9 = \begin{pmatrix} 0.3039 \\ 4.936 \\ 11.6718 \end{pmatrix} = 11.6718 \begin{pmatrix} 0.0260 \\ 0.4229 \\ 1 \end{pmatrix} = 11.6718 \begin{pmatrix} 0.0260 \\ 0.4229 \\ 1 \end{pmatrix}$$

$$AX_{10} = \begin{pmatrix} 0.2967 \\ 4.9238 \\ 11.6656 \end{pmatrix} = 11.6656 \begin{pmatrix} 0.0253 \\ 0.4221 \\ 1 \end{pmatrix} = 11.6656 X_{11}$$

$$AX_{11} = \begin{pmatrix} 0.2916 \\ 4.9201 \\ 11.6631 \end{pmatrix} = 11.6631 \begin{pmatrix} 0.025 \\ 0.4219 \\ 1 \end{pmatrix} = 11.6631 X_{12}$$

$$AX_{12} = \begin{pmatrix} 0.2907 \\ 4.9188 \\ 11.6626 \end{pmatrix} = 11.6626 \begin{pmatrix} 0.0249 \\ 0.4218 \\ 1 \end{pmatrix} = 0.6626 X_{13}$$

$$AX_{13} = \begin{pmatrix} 0.2903 \\ 4.9183 \\ 11.6623 \end{pmatrix} = 11.6623 \begin{pmatrix} 0.0249 \\ 0.4217 \\ 1 \end{pmatrix} = 11.6623 X_{14}$$

$$AX_{14} = \begin{pmatrix} 0.29 \\ 4.9181 \\ 11.6619 \end{pmatrix} = 11.6619 \begin{pmatrix} 0.0249 \\ 0.4217 \\ 1 \end{pmatrix} = 11.6619 X_{15}$$

$$AX_{15} = \begin{pmatrix} 0.29 \\ 4.9181 \\ 11.6619 \end{pmatrix} = 11.6619 \begin{pmatrix} 0.0249 \\ 0.4217 \\ 1 \end{pmatrix} = 11.6619 X_{16}$$

∴ The dominant eigen value is 11.6619

The corresponding eigen vector is

$$\begin{pmatrix} 0.0249 \\ 0.4217 \\ 1 \end{pmatrix}.$$

(3) Find the dominant eigen value and the corresponding eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Soln

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$AX_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1 \cdot X_2$$

$$AX_2 = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = 7\begin{pmatrix} 1 \\ 0.4286 \\ 0 \end{pmatrix} = 7 \cdot X_3$$

$$AX_3 = \begin{pmatrix} 3.5714 \\ 1.8572 \\ 0 \end{pmatrix} = 3.5714\begin{pmatrix} 1 \\ 0.52 \\ 0 \end{pmatrix} = 3.5714\, X_4$$

$$AX_4 = \begin{pmatrix} 4.12 \\ 2.04 \\ 0 \end{pmatrix} = 4.12\begin{pmatrix} 1 \\ 0.4951 \\ 0 \end{pmatrix} = 4.12\, X_5$$

$$AX_5 = \begin{pmatrix} +3.9706 \\ 1.9902 \\ 0 \end{pmatrix} = 3.9706 \begin{pmatrix} 1 \\ 0.5012 \\ 0 \end{pmatrix} = 3.9706\, X_6$$

$$AX_6 = \begin{pmatrix} 4.0072 \\ 2.0024 \\ 0 \end{pmatrix} = 4.0072 \begin{pmatrix} 1 \\ 0.4997 \\ 0 \end{pmatrix} = 4.0072\, X_7$$

$$AX_7 = \begin{pmatrix} 3.9982 \\ 1.9994 \\ 0 \end{pmatrix} = 3.9982 \begin{pmatrix} 1 \\ 0.5000 \\ 0 \end{pmatrix} = 3.9982\, X_8$$

$$AX_8 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} = 4\, X_9$$

$$AX_9 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix}$$

Dominant eigen value is $\lambda = 4$

Corresponding eigen vector is $\begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix}$

Eigen Value of a Matrix by Jacobi Method for Symmetric Matrix

Let $P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$\theta = \frac{1}{2} \tan^{-1}\left(\frac{2a_{ij}}{a_{ii} - a_{jj}}\right)$$

$$D = P^T A P$$

(1) Apply Jacobi process to evaluate the eigen values and eigen vectors of the Matrix $\begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$

Soln

$$A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$$

The largest non diagonal element is $a_{13} = a_{31} = 1$

$a_{11} = 5$, $a_{33} = 5$

$$\tan 2\theta = \left[\frac{2 a_{13}}{a_{11} - a_{33}}\right] = \frac{2}{5-5}$$

$$\tan 2\theta = \infty$$

$$2\theta = \tan^{-1}\infty$$

$$2\theta = \pi/2$$

$$\boxed{\theta = \pi/4}$$

$$P = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\pi/2 & 0 & -\sin\pi/4 \\ 0 & 1 & 0 \\ \sin\pi/4 & 0 & \cos\pi/4 \end{bmatrix}$$

$$P = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

I^{st} transformation

$$D = P^T A P$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$D = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

The eigen values are $6, -2, 4$

Corresponding eigen vectors are

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(2) Find all the eigen values and eigen vectors of the Matrix

$$\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$ using Jacobi Method.

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

Here the largest non diagonal element is $a_{13} = a_{31} = 2$.

$a_{11} = 1, a_{33} = 1$

$$S_1 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\tan 2\theta = \frac{2a_{13}}{a_{11} - a_{33}} = \frac{4}{0}$$

$$\tan 2\theta = \infty$$

$$2\theta = \pi/2$$

$$\boxed{\theta = \pi/4}$$

$$S_1 = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$B_1 = S_1^{-1} A S_1$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

II Transformation

$$a_{12} = a_{21} = 2$$

$$a_{11} = 3 \quad a_{22} = 3$$

$$S_2 = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\tan 2\theta = \frac{2a_{12}}{a_{11}-a_{22}} = \frac{2\times 2}{3-3} = \infty$$

$$2\theta = \tan^{-1}\infty$$

$$2\theta = \pi/2$$

$$\theta = \pi/4$$

$$S_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_2 = S_1^{-1} B_1 S_2$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$\therefore$ A is reduced to the diagonal Matrix B_2.

Hence the eigen values of A is 5, 1, −1

$$S = S_1 S_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{-1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\therefore$ eigen vectors are $\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix} \& \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

Questions	opt1	opt2	opt3	opt4	Answer
In Regula Falsi method, to reduce the number of iterations we start with ________ interval	Small	large	equal	no	Small
The rate of convergence in Newton Raphson method is of order ________	1	2	3	4	2
The condition for convergence for Newton Raphson method is	$\lvert f(x)\rvert<\lvert f'(x)\rvert^2$	$\lvert f(x)\rvert>\lvert f'(x)\rvert^2$	$\lvert f(x)f''(x)\rvert<\lvert f'(x)\rvert^2$	$f(x) <1$	$\lvert f(x)f''(x)\rvert<\lvert f'(x)\rvert^2$
Newtons method is useful when the graph of the function crosses the x-axis is nearly ______.	vertical	horizontal	close to zero	zero	vertical
If the initial approximation to the root is not given we can find any two values of x say a and b such that f (a) and f(b) are of _________signs.	opposite	same	positive	negative	opposite
$\lvert f(a)\rvert$___ $\lvert f(b)\rvert$ then a can be taken as the first approximation to the root.	<	>	=	greaterthan or equal	<
The Newton Raphson method is also known as method of __________	secant	tangent	iteration	interpolation	tangent
The Newton Raphson method will fail if _________ in the neighborhood of the root	f'(x)=0	$\lvert f'(x)\rvert>0$	$\lvert f'(x)\rvert<0$	$\lvert f'(x)\rvert>1$	f'(x)=0
If f'(x)=0 _________ method should be used.	Newton Raphson	Regula Falsi	iteration	interpolation	Regula Falsi
The rate of convergence of Newton – Raphson method is _________	quadratic	cubic	4	5	quadratic
If f (a) and f (b) are of opposite signs the actual root lies between _________	(a,b)	(0,a)	(0,b)	(0,0)	(a,b)
The convergence of root in Regula Falsi method is slower than ________	Gauss Elimination	Gauss Jordan	Newton Raphson	Power method	Newton Raphson
Regula Falsi method is known as method of _________	secant	tangent	chords	elimination	chords
______________method converges faster than Regula Falsi method.	Newton – Raphson	Power method	elimination	interpolation	Newton – Raphson
f(x) is continuous in the interval (a, b) and if f (a) and f (b) are of opposite signs the equation f(x) = 0 has at least one _________ lying between a and b.	equation	function	root	polynomial	root
x^2 + 3x -3 = 0 is a polynomial of order	2	3	1	0	2
x is a root of f(x)=0 with multiplicity p,then_________ method is used.	Generalized Newton	Newton Raphson	Regula-Falsi	Power	Generalized Newton Raphson
Errors which are already present in the statement of the problem are called____ errors.	Inherent	Rounding	Truncation	Absolute	Inherent
Rounding errors arise during_________	Solving	Computation	Truncation	Absolute	Computation
The other name for truncation error is _________ error.	Absolute	Rounding	Inherent	Algorithm	Algorithm
Rounding errors arise from the process of _________ the numbers.	Truncating	Rounding off	Approximating	Solving	Rounding off
Absolute error is denoted by________	E_a	E_r	E_p	E_x	E_a
Truncation errors are caused by using _________ results.	Exact	True	Approximate	Real	Approximate
Truncation errors are caused on replacing an infinite process by ________ one.	Approximate	True	Finite	Exact	Finite
Graffes root squaring method is used for solving ___________ equation.	Polynomial	Algebraic	transcendental	wave	Polynomial
Bairstows method is used for finding ____________ roots of a polynomial equation.	Complex	real	second order	first order	Complex
The actual root of the equation lies between a and b when f (a) and f (b) are of ____ signs.	Opposite	same	negative	positive	Opposite
If a word length is 4 digits, then the truncation of 15.758 is	15.75	15.76	15.758	16	15.75
If a word length is 4 digits, then rounding off of 15.758 is	15.75	15.76	15.758	16	15.76

Numerical Methods

Unit - 2

Interpolation and Approximation

Lagrange's interpolation formula (unequal intervals)

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

---

① Using Lagrange's formula, find the polynomial to the given data

x :	0	1	3
y :	5	6	50

Soln

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \cdot y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \cdot y_2$$

Here $x_0 = 0 \quad x_1 = 1 \quad x_2 = 3$

$y_0 = 5 \quad y_1 = 6 \quad y_2 = 50$

$$y = f(x) = \frac{(x-1)(x-3)}{(0-1)(0-3)}(5) + \frac{(x-0)(x-3)}{(1-0)(1-3)} \cdot (6)$$

$$+ \frac{(x-0)(x-1)}{\ldots} \cdot (50)$$

$$= \frac{(x-1)(x-3)}{3}(5) + \frac{x(x-3)}{-2}(6) + \frac{x(x-1)}{6}(50)$$

$$= \frac{5}{3}[x^2-4x+3] - 3[x^2-3x] + \frac{50}{6}[x^2-x]$$

$$= x^2\left[\frac{5}{3} - 3 + \frac{50}{6}\right] + x\left[-\frac{20}{3} + 9 - \frac{50}{6}\right] + \left[\frac{15}{3}\right]$$

$$= 7x^2 + (-6)x + 5$$

$$\boxed{y = f(x) = 7x^2 - 6x + 5}$$

---

② Using Lagrange's interpolation find $y(2)$ from the following data

x	0	1	3	4	5
y	0	1	81	256	625

Soln

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 4. \quad x_4 = 5$

$y_0 = 0 \quad y_1 = 1 \quad y_2 = 81 \quad y_3 = 256 \quad y_4 = 625$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \cdot y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \cdot y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \cdot y_3$$

Put $x = 2$

$$y(2) = \frac{(2-1)(2-3)(2-4)(2-5)}{(0-1)(0-3)(0-4)(0-5)}(0) + \frac{(2-0)(2-3)(2-4)(2-5)}{(1-0)(1-3)(1-4)(1-5)}(1) + \frac{(2-0)(2-1)(2-4)(2-5)}{(3-0)(3-1)(3-4)(3-5)}(81) + \frac{(2-0)(2-1)(2-3)(2-5)}{(4-0)(4-1)(4-3)(4-5)}(256) + \frac{(2-0)(2-1)(2-3)(2-4)}{(5-0)(5-1)(5-3)(5-4)}(625)$$

$$= \frac{(2)(-1)(-2)(-3)}{(1)(-2)(-3)(-4)} + \frac{(2)(1)(-2)(-3)}{(3)(2)(-1)(-2)}(81) + \frac{(2)(1)(-1)(-3)}{(4)(3)(1)(-1)}(256) + \frac{(2)(1)(-1)(-2)}{(5)(4)(2)(1)}(625)$$

$$= \frac{12}{24} + \frac{12}{12}(81) - \frac{6}{12}(256) + \frac{4}{40}(625)$$

$$= \frac{1}{2} + 81 - 128 + 62.5$$

$$= 0.5 + 81 - 128 + 62.5 = 16.$$

3) Use Lagranges Method to find $\log_{10} 656$, given that $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$ and $\log_{10} 661 = 2.8202$.

Soln

x	654	658	659	661
$y = \log_{10} x$	2.8156	2.8182	2.8189	2.8202

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 656$

$$y = f(656) = \frac{(656-658)(656-659)(656-661)}{(654-658)(654-659)(654-661)} \cdot (2.8156) + \frac{(656-654)(656-659)(656-661)}{(658-654)(658-659)(658-661)} (2.8182) + \frac{(656-654)(656-658)(656-661)}{(659-654)(659-658)(659-661)} (2.8189) + \frac{(656-654)(656-659)(656-659)}{(661-654)(661-659)(661-659)} \cdot (2.8202)$$

$$= \frac{(-2)(-3)(-5)}{(-4)(-5)(-7)} (2.8156) + \frac{2(-3)(-5)}{4(-1)(-3)} (2.8182) + \frac{(2)(-2)(-5)}{(5)(1)(-2)} (2.8189) + \frac{(2)(-2)(-3)}{(7)(3)(2)} (2.8202)$$

$= 0.6033 + 7.0455 - 5.6378 + 0.8058$

$= 2.8168$.

4) Use Lagrange's formula to find the value of y at $x = 6$ from the following data

x :	3	7	9	10

Soln

$x_0 = 3 \quad x_1 = 7 \quad x_2 = 9 \quad x_3 = 10$

$y_0 = 168 \quad y_1 = 120 \quad y_2 = 72 \quad y_3 = 63.$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 6$

$$y = f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)}(168) + \frac{(6-7)(6-9)(6-10)}{(7-3)(7-9)(7-10)}(120) + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)}(72) + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)}(63)$$

$$= \frac{(-1)(-3)(-4)}{(-4)(-6)(-7)}(168) + \frac{(3)(-3)(-4)}{4(-2)(-3)}(120) + \frac{(3)(-1)(-4)}{(6)(2)(-1)}(72) + \frac{(3)(-1)(-3)}{(7)(3)(1)}(63)$$

$$= 12 + 180 - 72 + 27$$

$$= 147$$

5) Given the values

x	14	17	31	35
$f(x)$	68.7	64.0	44.0	39.1

find $f(27)$ by using Lagrange's interpolation formula.

Soln

$x_0 = 14 \quad x_1 = 17 \quad x_2 = 31 \quad x_3 = 35$

$y_0 = 68.7 \quad y_1 = 64 \quad y_2 = 44 \quad y_3 = 39.1$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 27$

$$y = f(27) = \frac{(27-17)(27-31)(27-35)}{(14-17)(14-31)(14-35)} \cdot (68.7)$$

$$+ \frac{(27-14)(27-31)(27-35)}{(17-14)(17-31)(17-35)} (64.0)$$

$$+ \frac{(27-14)(27-17)(27-35)}{(31-14)(31-17)(31-35)} (44.0)$$

$$+ \frac{(27-14)(27-17)(27-31)}{(35-14)(35-17)(35-31)} (39.1)$$

$$= \frac{(10)(-4)(-8)}{(-3)(-17)(-21)} (68.7) + \frac{13(-4)(-8)}{(3)(-14)(-8)} (64)$$

$$+ \frac{(13)(10)(-8)}{(17)(14)(-4)} (44) + \frac{(13)(10)(-4)}{(21)(18)(4)} (39.1)$$

$$= -20.52 + 35.22 + 48.07 - 13.45$$

6) Find the Missing term in the following table using Lagranges interpolation

x	0	1	2	3	4
y	1	3	9	—	81

Soln

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 4$

$y_0 = 1 \quad y_1 = 3 \quad y_2 = 9 \quad y_3 = 81$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3.$$

Put $x = 3$

$$y = f(3) = \frac{(3-1)(3-2)(3-4)}{(0-1)(0-2)(0-4)}(1) + \frac{(3-0)(3-2)(3-4)}{(1-0)(1-2)(1-4)}(3) + \frac{(3-0)(3-1)(3-4)}{(2-0)(2-1)(2-4)}(9) + \frac{(3-0)(3-1)(3-2)}{(4-0)(4-1)(4-2)}(81)$$

$$= \frac{-2}{-8}\{-3\} + \frac{27}{2} + \frac{81}{4}$$

$$= 31.$$

7) Using Lagranges formula prove

$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} + y_{-5})$

Soln

y_{-5}, y_{-3}, y_3, y_5 Occur in the answers.

So we can have the table

x	-5	-3	3	5
y	y_{-5}	y_{-3}	y_3	y_5

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_{-5} + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_{-3} + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_3 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_5$$

put $x = 1$

$$y_1 = f(1) = \frac{(1+3)(1-3)(1-5)}{(-5+3)(-5-3)(-5-5)} \cdot y_{-5} + \frac{(1+5)(1-3)(1-5)}{(-3+5)(-3-3)(-3-5)} \cdot y_{-3} + \frac{(1+5)(1+3)(1-5)}{(3+5)(3+3)(3-5)} y_3 + \frac{(3+5)(3+3)(3-3)}{(5+5)(5+3)(5-3)} \cdot y_5$$

$$= \frac{(4)(-2)(-4)}{(-2)(-8)(-10)} \cdot y_{-5} + \frac{(6)(-2)(-4)}{(2)(-6)(-8)} \cdot y_{-3} + \frac{(6)(4)(-4)}{(8)(6)(-2)} \cdot y_3 + \frac{(6)(4)(-2)}{(10)(8)(2)} y_5$$

$= -0.2y \; +0.5y \; + y - 0.3y$

$$+\frac{(0+30)(0+13)(0-18)}{(3+30)(3+13)(3-18)}.(38)$$

$$+\frac{(0+30)(0+13)(0-3)}{(18+30)(18+13)(18-3)}(42)$$

$$=37\cdot 23.$$

(2) Find the value of θ given $f(\theta) = 0.3887$ where $f(\theta) = \int_0^\theta \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}}$ using the table

θ	$21°$	$23°$	$25°$
$f(\theta)$	0.3706	0.4068	0.4433.

Soln

Let $\theta = x$

$f(\theta) = f(x) = y$

x	$21°$	$23°$	$25°$
y	0.3706	0.4068	0.4433

$$x = f(y) = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)}\cdot x_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)}\cdot x_1 + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)}\cdot x_2$$

Put $y = 0.3887$

$$x = f(0.3887) = \frac{(0.3887-0.4068)(0.3887-0.4433)}{(0.3706-0.4068)(0.3706-0.4433)}(21°)$$

$$+\frac{(0.3887-0.3706)(0.3887-0.4433)}{(0.4068-0.3706)(0.4068-0.4433)}(23°)$$

$$+\frac{(0.3887-0.3706)(0.3887-0.4068)}{}$$

Newton's divided difference formula : (unequal)

$$y = f(x) = y_0 + (x-x_0)\,\Delta f(x_0) + (x-x_0)(x-x_1)\,\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\,\Delta^3 f(x_0) + \cdots$$

(1) Using Newton's divided difference formula find $f(x)$ and $f(6)$ from the following data.

x :	1 (x_0)	2 (x_1)	7 (x_2)	8 (x_3)
$f(x)$:	1	5	5	4

Soln

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	1			
		$\frac{5-1}{2-1} = 4$		
2	5		$\frac{0-4}{7-1} = -\frac{4}{6}$	
		$\frac{5-5}{7-2} = 0$		$\frac{-\frac{1}{6}+\frac{4}{6}}{8-1} = \frac{1/2}{7} = \frac{1}{14}$
7	5		$\frac{-1-0}{8-2} = \frac{-1}{6}$	
		$\frac{4-5}{8-7} = -1$		
8	4			

$$y = f(x) = f(x_0) + (x-x_0)\,\Delta f(x_0) + (x-x_0)(x-x_1)\,\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\,\Delta^3 f(x_0) + \cdots$$

$$= 1 + (x-1)(4) + (x-1)(x-2)\left(-\frac{4}{6}\right)$$

$$= x^3\left[\frac{1}{14}\right] + x^2\left[-\frac{4}{6}\right] - \frac{3}{14} - \frac{7}{14}\Big]$$

$$+ x\left[4 + \frac{12}{6} + \frac{2}{14} + \frac{21}{14}\right] + \left[1 - 4 - \frac{8}{6} - \frac{14}{14}\right]$$

$$f(x) = \frac{1}{14}x^3 - \frac{29}{21}x^2 + \frac{107}{14}x - \frac{16}{63}$$

Put $x = 6$

$$y = f(6) = \frac{1}{14}(6)^3 - \frac{29}{21}(6)^2 + \frac{107}{14}(6) - \frac{16}{36}$$

~~$= 54 - 114 + 40.4 - 7.833$~~

$$= 15.428 - 49.714 + 45.857 - 0.444$$

$$= 11.127$$

2) Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

Soln

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		$\frac{33-1245}{-1+4} = -404$			
-1	33		$\frac{-28+404}{0+4} = 94$		
		$\frac{5-33}{0+1} = -28$		$\frac{10-94}{2+4} = -14$	
0	5		$\frac{2+28}{2+1} = 10$		$\frac{13+14}{5+4} = 3$
		$\frac{9-5}{2-0} = 2$		$\frac{88-10}{5+1} = 13$	
2	9		$\frac{442-2}{5-0} = 88$		
		$\frac{1335-9}{5-2} = 442$			
5	1335				

$$y = f(x) = f(x_0) + (x-x_0)\,\Delta f(x) + (x-x_0)(x-x_1)\,\Delta^2 f(x)$$
$$+ (x-x_0)(x-x_1)(x-x_2)\,\Delta^3 f(x)$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)\,\Delta^4 f(x)$$

$$= 1245 + (x+4)(-404) + (x+4)(x+1)(94) + (x+4)(x+1)(x-0)(x-2)(3)$$
$$+ (x+4)(x+1)(x-0)(-14)$$

$$= 1245 - 404x - 1616 + (x^2+5x+4)\,94 + (x^2+5x+4)(3x^2-6x)$$
$$+ (x^2+5x+4)(-14x)$$

$$= 1245 - 404x - 1616 + 94x^2 + 470x + 376 + 3x^4 + 15x^3 + 12x^2$$
$$-14x^3 - 70x^2 - 56x$$
$$-6x^3 - 30x^2 - 24x$$

$$= x(-404 + 470 - 16 - 24) + x^2[94 - 70 + 12 - 30]$$
$$= x^4[3] + x^3[-14 + 15 - 6] + x^2[94 - 70 + 12 - 30] + [1245 - 1616 + 376]$$
$$+ x[-404 + 470 - 56 - 24]$$

$$= 3x^4 + 5x^3 + 6x^2 - 14x + 5$$

(3) Find the cubic polynomial from the following table using Newton's divided difference formula and hence find $f(4)$

x	0 x_0	1 x_1	2 x_2	5 x_3
y	2	3	12	147

Soln

x	$y=f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	2			
		$\frac{3-2}{1-0}=1$		
1	3		$\frac{9-1}{2-0}=4$	
		$\frac{12-3}{2-1}=9$		$\frac{9-4}{5-0}=1$
2	12		$\frac{45-9}{5-1}=9$	
		$\frac{147-12}{5-2}=45$		
5	147			

$$y=f(x)=y_0+(x-x_0)\Delta f(x)+(x-x_0)(x-x_1)\Delta^2 f(x)+(x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x).$$

$$=2+(x-0)(1)+(x-0)(x-1)(4)+(x-0)(x-1)(x-2)(1)$$

$$=2+x+4x^2-4x+(x^2-x)(x-2)$$

$$=2+x+4x^2-4x+x^3-x^2-2x^2+2x$$

$$=x^3+x^2-x+2$$

Put $x=4$

$$y=f(4)=4^3+4^2-4+2$$

$$=78.$$

Cubic Spline Interpolation formula.

$$g(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right]$$
$$+ \frac{1}{h}(x_i - x)\left[y_{i-1} - \frac{h^2}{6} M_{i-1}\right]$$
$$- \frac{1}{h}(x_{i-1} - x)\left[y_i - \frac{h^2}{6} M_i\right]$$

where, $M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$

with $M_0 = M_n = 0$

---

① Obtain cubic spline polynomial which best fits with the following data, given that $y_0'' = y_3'' = 0$

x	-1 x_0	0 x_1	1 x_2	x_3 2
y	-1 y_0	1 y_1	3 y_2	35 y_3

Soln

Given $M_0 = M_3 = 0$, $h = 1$

WKT $M_{i-1} + 4M_i + M_{i+1} = 6\left[y_{i-1} - 2y_i + y_{i+1}\right]$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6\left[y_0 - 2y_1 + y_2\right]$$
$$4M_1 + M_2 = 6\left[-1 - 2 + 3\right]$$
$$4M_1 + M_2 = 0 \quad \text{———} \quad (1)$$

Put $i = 2$

$$M_1 + 4M_2 + M_3 = 6\left[y_1 - 2y_2 + y_3\right]$$
$$M_1 + 4M_2 = 6\left[1 - 6 + 35\right]$$

Solve ① & ②

$M_1 = -12 \quad M_2 = 48$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i \right] + (x_i - x)\left[y_{i-1} - \frac{1}{6} M_{i-1} \right] - (x_{i-1} - x)\left[y_i - \frac{1}{6} M_i \right]$$

Case(i) $-1 < x < 0$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1 \right] + (x_1 - x)\left[y_0 - \frac{1}{6} M_0 \right] - (x_0 - x)\left[y_1 - \frac{1}{6} M_1 \right]$$

$$= \frac{1}{6}\left[-(-1-x)^3(-12) \right] + (0 - x)(-1) - (-1-x)\left[1 + \frac{12}{6} \right]$$

$$= \frac{1}{6}\left[-12(1+x)^3 \right] + x + (1+x)(3)$$

$$= -2\left[1 + x^3 + 3x + 3x^2 \right] + x + 3 + 3x$$

$$= -2 - 2x^3 - 6x - 6x^2 + x + 3 + 3x$$

$$\boxed{S(x) = -2x^3 - 6x^2 - 2x + 1, \quad -1 < x < 0}$$

Case (ii) $0 < x < 1$

Put $i = 2$

$$S(x) = \frac{1}{6}\left[(x_2 - x_1)^3 M_1 - (x_1 - x)^3 M_2 \right.$$
$$+ (x_2 - x)\left[y_1 - \frac{1}{6} M_1 \right]$$
$$\left. - (x_1 - x)\left[y_2 - \frac{1}{6} M_2 \right] \right]$$

$$= \frac{1}{6}\left[(1-x)^3 (-12) - (0-x)^3 (48) \right]$$
$$+ (1-x)\left[1 - \frac{1}{6}(-12) \right] - (0-x)\left[3 - \frac{1}{6} \times 48 \right]$$

$$= \frac{1}{6}\left[-12(1-x)^3 + 48x^3 \right] + 3(1-x) - 5x$$

$$= \frac{1}{6}\left[-12(1 - x^3 - 3x + 3x^2) + 48x^3 \right] + 3 - 3x - 5x$$

$$= \frac{1}{6}\left[-12 + 12x^3 + 36x - 36x^2 + 48x^3 \right] + 3 - 3x - 5x$$

$$= x^3[2+8] + x^2[-6] + x[6-8] - 2 + 3$$

$$\boxed{S(x) = 10x^3 - 6x^2 - 2x + 1, \quad 0 < x < 1}$$

Case (iii) $1 < x < 2$

Put $i = 3$

$$S(x) = \frac{1}{6}\left[(x_3 - x)^3 M_2 - (x_2 - x)^3 M_3 \right]$$
$$+ (x_3 - x)\left[y_2 - \frac{1}{6} M_2 \right] - (x_2 - x)\left[y_3 - \frac{1}{6} M_3 \right]$$

$$= \frac{1}{6}\left[(2-x)^3 48 \right] + (2-x)\left[3 - \frac{1}{6} \times 48 \right]$$

$$= 8(2-x)^3 + (2-x)(-5) - 35(1-x)$$

$$= 8[8 - x^3 - 12x + 6x^2] - 10 + 5x - 35 + 35x$$

$$= 64 - 8x^3 - 96x + 48x^2 - 10 + 5x - 35 + 35x$$

$$\boxed{S(x) = -8x^3 + 48x^2 - 56x + 19, \quad 1 < x < 2}$$

The cubic Spline Polynomial is

$$S(x) = \begin{cases} -2x^3 - 6x^2 - 2x + 1, & -1 < x < 0 \\ 10x^3 - 6x^2 - 2x + 1, & 0 < x < 1 \\ -8x^3 + 48x^2 - 56x + 19, & 1 < x < 2 \end{cases}$$

---

(2) From the following table

x	1 x_0	2 x_1	3 x_2
y	-8 y_0	-1 y_1	18 y_2

Compute $y(1.5)$ and $y'(1)$ using cubic Spline.

Soln

Take $M_0 = M_2 = 0.$, $h = 1$

W.k.T $M_{i-1} + 4M_i + M_{i+1} = 6[y_{i-1} - 2y_i + y_{i+1}]$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$$

$$4M_1 = 6[-8 + 2 + 18]$$

$$4M_1 = 72$$

$$\boxed{M_1 = 18}$$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i \right] + (x_i - x)\left[y_{i-1} - \frac{1}{6} M_{i-1} \right] - (x_{i-1} - x)\left[y_i - \frac{1}{6} M_i \right]$$

Case (i) $1 < x < 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1 \right] + (x_1 - x)\left[y_0 - \frac{1}{6} M_0 \right] - (x_0 - x)\left[y_1 - \frac{1}{6} M_1 \right]$$

$$= \frac{1}{6}\left[(2 - x)^3 (0) - (1 - x)^3 (18) \right] + (2 - x)\left[-8 - \frac{1}{6}(0) \right] - (1 - x)\left[-1 - \frac{1}{6}(18) \right]$$

$$= \frac{1}{6}\left[-(1-x)^3 (18) \right] + (2 - x)(-8) - (1-x)\left[-1 - 3 \right]$$

$$= -18(1-x)^3 - 8(2-x) + 4(1-x)$$

$$= -18(1-x)^3 - 16 + 8x + 4 - 4x$$

$$= -18(1-x)^3 + 4x - 12, \quad 1 < x < 2$$

$$\boxed{S(x) = -18(1-x)^3 + 4x - 12, \quad 1 < x < 2}$$

Put $x = 1.5$

$$y(1.5) = S(1.5) = -18(1 - 1.5)^3 + 4(1.5) - 12$$

$$= -5.625$$

$y'(1) = 9(0) + 4 = 4$

$\boxed{y'(1) = 4}$ $\boxed{y(1.5) = -5.625}$

(3) Find the cubic spline interpolation

x :	1	2	3	4	5
f :	1 y_0	0 y_1	1 y_2	0 y_3	1 y_4

Soln

Take $M_0 = M_4 = 0$, $h = 1$

WKT $M_{i-1} + 4M_i + M_{i+1} = 6[y_{i-1} - 2y_i + y_{i+1}]$

Put $i=1$ $M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$

$4M_1 + M_2 = 6[1 - 0 + 1] = 12$

$4M_1 + M_2 = 12$ ——— (1)

Put $i=2$ $M_1 + 4M_2 + M_3 = 6[y_1 - 2y_2 + y_3]$

$= 6[0 - 2 + 0]$

$M_1 + 4M_2 + M_3 = -12$ ——— (2)

Put $i=3$ $M_2 + 4M_3 + M_4 = 6[y_2 - 2y_3 + y_4]$

$M_2 + 4M_3 = 6[1 - 0 + 1]$

$M_2 + 4M_3 = 12$ ——— (3)

from (1) & (2)

$4 \times (1) \Rightarrow 16M_1 + 4M_2 = 48$

from ② & ③

$M_1 + 4M_2 + M_3 = -12$

② $\Rightarrow M_1 + 4M_2 + M_3 = -12$

$4 \times$ ③ $\Rightarrow 4M_2 + 16M_3 = 48$

$M_1 - 15M_3 = -60$ —— ⑤

Solve ④ & ⑤

$$M_3 = \frac{30}{7}$$

① $\Rightarrow 4M_1 + M_2 = 12$

⑤ $\Rightarrow M_1 = -60 + 15M_3$

$M_1 = -60 + \frac{450}{7}$

$$M_1 = \frac{30}{7}$$

$M_2 = 12 - 4M_1$

$= 12 - 4\left(\frac{30}{7}\right)$

$$M_2 = -\frac{36}{7}$$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right]$$
$$+ (x_i - x)\left[y_{i-1} - \frac{1}{6}M_{i-1}\right]$$
$$- (x_{i-1} - x)\left[y_i - \frac{1}{6}M_i\right]$$

case (i) $-1 < x < 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1\right]$$
$$+ (x_1 - x)\left[y_0 - \frac{1}{6}M_0\right]$$
$$- (x_0 - x)\left[y_1 - \frac{1}{6}M_1\right]$$
$$= \frac{1}{6}\left[(2-x)^3(0) - (1-x)^3\left(\frac{30}{7}\right)\right.$$
$$+ (2-x)\left[1 - \frac{1}{6}(0)\right]$$

$(a-b)^3 = a^3 - 3$

$$= \frac{1}{6}\left[-(1-x)^3\left(\frac{30}{7}\right)\right] + (2-x)[1] - (1-x)\left[-\frac{1}{6}\frac{30}{7}\right]$$

$$= \frac{1}{6}\left[-\frac{30}{7}(1-x)^3 + (2-x) + \frac{5}{7}(1-x)\right]$$

$$= \frac{1}{6}\left[-\frac{30}{7}\left[1^3 - x^3 - 3x + 3x^2\right] + 2 - x + \frac{5}{7} - \frac{5}{7}x\right]$$

$$= -\frac{5}{7} + \frac{5}{7}x^3 + \frac{15}{7}x + 15x^2 + 2 - x + \frac{5}{7} - \frac{5}{7}x$$

$$= \frac{5}{7}x^3 + 15x^2 + x\left(\frac{15}{7} - 1 - \frac{5}{7}\right)$$

$$S(x) = \frac{5}{7}x^3 + 15x^2 + \frac{3}{7}x + 2, \quad 1 \le x \le 2$$

Case (ii) $2 < x < 3$.

Put $i = 2$.

$$S(x) = \frac{1}{6}\left[(x_2 - x)^3 M_1 - (x_1 - x)^3 M_2\right] + (x_2 - x)\left[y_1 - \frac{1}{6}M_1\right] - (x_1 - x)\left[y_2 - \frac{1}{6}M_2\right]$$

$$= \frac{1}{6}\left[(3-x)^3\frac{30}{7} - (2-x)\left(-\frac{36}{7}\right)\right] + (3-x)\left[0 - \frac{1}{6}\left(\frac{30}{7}\right)\right] - (2-x)\left[1 - \frac{1}{6}\left(-\frac{36}{7}\right)\right]$$

$$= \frac{1}{6}\left[\frac{30}{7}(3-x)^3 + \frac{36}{7}(2-x)\right] + (3-x)\left(-\frac{5}{7}\right)$$

$$-(2-x)\left[1+\frac{5}{7}\right]$$

$$= \frac{5}{7}\left[27 - 27x + 9x^2 - x^3\right] + \frac{6}{7}\left[4 + x^2 - 4x\right]$$

$$-\frac{15}{7} + \frac{5x}{7} - \frac{26}{7} + \frac{13x}{7}$$

$$= x^3\left[-\frac{5}{7}\right] + x^2\left[\frac{45}{7} + \frac{6}{7} + \frac{5}{7} + \frac{13}{7}\right]$$

$$+ x\left[-135 - \frac{24}{7} + \frac{5}{7} + \frac{13}{7}\right] + \frac{135}{7} + \frac{24}{7} - \frac{15}{7}$$

$$-\frac{26}{7}$$

$$S(x) = -\frac{5}{7}x^3 + \frac{51}{7}x^2 - \frac{951}{7} + \frac{118}{7}, \quad 2 < x < 3$$

case (iii) $3 < x < 4$

Put $i = 3$

$$S(x) = \frac{1}{6}\left[(x_3 - x)^3 M_2 - (x_2 - x) M_3\right]$$

$$+ (x_3 - x)\left[y_2 - \frac{1}{6}M_2\right] - (x_2 - x)\left[y_3 - \frac{1}{6}M_3\right]$$

$$= \frac{1}{6}\left[(4-x)^3\left(-\frac{36}{7}\right) - (3-x)^3\left(\frac{30}{7}\right)\right]$$

$$+ (4-x)\left[1 - \frac{1}{6}\left(-\frac{36}{7}\right)\right]$$

$$-(3-x)\left[0 - \frac{1}{6}\left(\frac{30}{7}\right)\right]$$

$$= \frac{1}{6}\Big[-\frac{36}{7}[64 - 48x + 12x^2 - x^3]$$

$$-\frac{30}{7}[27 - 27x + 9x^2 - x^3]\Big]$$

$$+(4-x)\left(1+\frac{6}{7}\right) - (3-x)\left(-\frac{5}{7}\right)$$

~~$= [\frac{-384}{7}$~~ $\quad 13/7$

$$= \frac{1}{7}\Big[-384 + 288x - 72x^2 + 6x^3 - 810$$

$$+ 810x - 270x^2 + 30x^3$$

$$+ 52 - 13x + 15 - 5x \Big]$$

$$= \frac{1}{7}\Big[x^3[30+6] + x^2[-72-270] +$$

$$+ x[288 + 810 - 13 - 5] +$$

$$[-384 - 810 + 52 + 15]\Big]$$

$$S(x) = \frac{1}{7}[36x^3 - 342x^2 + 1080x - 1127, \quad 3 \le x \le 4$$

case (iv) $4 < x < 5$

Put $i = 4$.

$$S(x) = \frac{1}{6}[(x_3 - x)^3 M_3 - (x_2 - x) M_4]$$

$$+ (x_4 - x)\left[y_3 - \frac{1}{6}M_3\right]$$

$$-(x_3 - x)\left[y_4 - \frac{1}{6}M_4\right]$$

$$= \frac{1}{6}\left[(5-x)^3\left(\frac{30}{7}\right) - 0\right] + (5-x)\left[0 - \frac{1}{6}\left(\frac{30}{7}\right)\right]$$

$$+(x-4)[1-0]$$

4) Find the cubic spline for the data

x	1	2	3
y	-6	-1	16

Hence evaluate $y(1.5)$ given that $y_0'' = y_2'' = 0$.

Soln

Given $h = 1$ $\quad M_0 = M_2 = 0$

W.K.T

$$M_{i-1} + 4M_i + M_{i+1} = 6\left[y_{i-1} - 2y_i + y_{i+1}\right]$$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6\left[y_0 - 2y_1 + y_2\right]$$

$$4M_1 = 6\left[-6 - 2(-1) + 16\right]$$

$$4M_1 = 72$$

$$\boxed{M_1 = 18}$$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right]$$
$$+ (x_i - x)\left[y_{i-1} - \frac{1}{6} M_{i-1}\right]$$
$$- (x_{i-1} - x)\left[y_i - \frac{1}{6} M_i\right]$$

Case (i) $1 \le x \le 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1\right]$$

$$= \frac{1}{6}\left[(2-x)^3(0) + (x-1)^3(18)\right]$$
$$+ (2-x)\left[-6 - \frac{1}{6}(0)\right]$$
$$+ (x-1)\left[-1 - \frac{1}{6}(18)\right]$$
$$= \frac{1}{6}\left[(x-1)^3(18)\right] + (2-x)(-6-0)$$
$$+ (x-1)(-1-3)$$
$$= 3(x^3 - 3x^2 + 3x - 1) - 12 + 6x - 4x + 4$$
$$S(x) = 3x^3 - 9x^2 + 11x - 11$$

Case (ii) $2 \le x \le 3$

Put $i = 2$.

$$S(x) = \frac{1}{6}\left[(x_2 - x)^3 M_1 - (x_1 - x)^3 M_2\right]$$
$$+ (x_2 - x)\left[y_1 - \frac{1}{6}M_1\right]$$
$$- (x_1 - x)\left[y_2 - \frac{1}{6}M_2\right]$$
$$= \frac{1}{6}\left[(3-x)^3 18 - (2-x)^3(0)\right]$$
$$+ (3-x)\left[-1 - \frac{1}{6}(18)\right]$$
$$- (x-2)\left[16 - \frac{1}{6}(0)\right]$$
$$= \frac{18}{6}\left[27 - 27x + 9x^2 - x^3\right]$$
$$-12 + 4x + 16x - 32$$

$$S(x) = -3x^3 + 27x^2 - 61x + 37$$

$$y = S(x) = \begin{cases} 3x^3 - 9x^2 + 11x - 11, & 1 \le x \le 2 \\ -3x^3 + 27x^2 - 61x + 37, & 2 \le x \le 3 \end{cases}$$

To find $y(1\cdot 5)$

$$S(1\cdot 5) = 3(1\cdot 5)^3 - 9(1\cdot 5)^2 + 11(1\cdot 5) - 11$$

$$= -4\cdot 625$$

Newton's forward interpolation formula (equal intervals).

$$y = f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

where $u = \frac{x-x_0}{h}$

(1) Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence evaluate y at $x=5$.

x	4	6	8	10
y	1	3	8	10

Soln

$u = \frac{x-x_0}{h}$, $h=2$

$u = \frac{x-4}{2}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	(1)			
		(2)		
6	3		(3)	
		5		(−6)
8	8		−3	
		2		

The Newton's forward interpolation form is

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 1 + \left(\frac{x-4}{2}\right)(2) + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)}{2} \times 3 + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)\left(\frac{x-4}{2}-2\right)}{3!} \times -6$$

$$= 1 + (x-4) + \frac{3(x-4)(x-6)}{8} - \frac{(x-4)(x-6)(x-8)}{8}$$

$$= \frac{1}{8}\left[8 + 8x - 32 + 3\left[x^2 - 10x + 24\right] - \left[x^3 - 18x^2 + 104x - 192\right]\right]$$

$$y = \frac{1}{8}\left[-x^3 + 21x^2 - 126x + 240\right]$$

Put $x = 5$

$$y(5) = \frac{1}{8}\left[-5^3 + 21 \times 5^2 - 126 \times 5 + 240\right]$$

$$\boxed{y(5) = 1.25}$$

(2) Fit a polynomial, by using Newtons forward interpolation formula to the data given below.

x	0 (x_0)	1 (x_1)	2 (x_2)	3 (x_3)
y	1 (y_0)	2 (y_1)	1 (y_2)	10 (y_3)

Soln

$$u = \frac{x - x_0}{h}, \quad h = 1$$

$$u = \frac{x - 0}{1} = x$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	(1)			
		(1)		
			(-2)	
1	2			(12)
		-1		
			10	
2	1			
		9		
3	10			

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 1 + \frac{x}{1!}(2) + \frac{x(x-1)}{2!}(-1) + \frac{x(x-1)(x-2)}{3!}(10)$$

$$= -1 + 2x + \frac{(x^2-x)}{2} + \frac{10}{6}\left[(x^2-x)(x-2)\right]$$

$$= 1 + 2x + \frac{x^2}{2} - \frac{x}{2} + \frac{5}{3}\left[x^3 - 2x^2 - x^2 + 2x\right]$$

$$= \frac{5}{[illegible]}x^3 + x^2\left[\frac{1}{2} - \frac{10}{3}\right] + x\left[2 - \frac{1}{2} + \frac{10}{3}\right] + 1$$

(3) From the data given below find the number of students whose weight is between 60 to 70.

Weight in kgs	0-40	40-60	60-80	80-100	100-120
No. of Students	250	120	100	70	50

Soln

$$u = \frac{x - x_0}{h}, \quad h = 20$$

$$u = \frac{x - 40}{20}$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
Below 60	370		-20		
		100		-10	
Below 80	470		-30		20
		70		10	
Below 100	540		-20		
		50			
Below 120	590				

The Newton's forward interpolation formula is

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \cdots$$

$$= 250 + \left(\frac{x-40}{20}\right)120 + \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)}{2}\times -20$$

$$+ \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)\left(\frac{x-40}{20}-2\right)}{6}\times -10$$

$$+ \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)\left(\frac{x-40}{20}-2\right)\left(\frac{x-40}{20}-3\right)}{24}\times 20$$

$$y = 250 + 6(x-40) - 10\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)$$

$$-\frac{5}{3}\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)\left(\frac{x-80}{20}\right)$$

$$+\frac{5}{6}\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)\left(\frac{x-80}{20}\right)\left(\frac{x-100}{20}\right)$$

$$y(70) = 250 + 6(70-40) - 10\left(\frac{70-40}{20}\right)\left(\frac{70-60}{20}\right) - \frac{5}{3}\left(\frac{70-40}{20}\right)\left(\frac{70-60}{20}\right)\left(\frac{70-80}{20}\right)$$

$$+\frac{5}{6}\left(\frac{70-40}{20}\right)\left(\frac{70-60}{20}\right)\left(\frac{70-80}{20}\right)\left(\frac{70-100}{20}\right)$$

$$= 250 + 180 - \frac{15}{2} + \frac{5}{8} + \frac{15}{32}$$

$$y(70) = 423.59 \simeq 424.$$

$$y(60) = 370.$$

No. of Students whose weight between 60–70 $= y(70) - y(60)$

$= 424 - 370$

Newton's Backward Interpolation formula

$$y = y_0 + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n$$

Where $v = \frac{x - x_n}{h}$

(1) Use Newton's backward difference formula to construct an interpolating polynomial of degree 3 for the data

$f(-0.75) = -0.07181250$, $f(-0.5) = -0.024750$

$f(-0.25) = 0.33493750$, $f(0) = 1.10100$.

Hence find $f(-\frac{1}{3})$.

Soln.

$v = \frac{x - x_n}{h}$ & $h = 0.25$

$v = \frac{x - 0}{0.25} = \frac{x}{0.25}$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
-0.75	-0.07181250			
		0.0470625		
-0.50	-0.024750		0.312625	
		0.3596875		0.09375
-0.25	0.33493750		0.40625	
		0.7660625		
0	1.10100			

The Newton's backward interpolation formule is

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

$$= 1.10100 + \left(\frac{x}{0.25}\right)(0.7660625)$$
$$+ \left(\frac{x}{0.25}\right)\left(\frac{x}{0.25}+1\right)(0.406375)$$
$$+ \frac{\left(\frac{x}{0.25}\right)\left(\frac{x}{0.25}+1\right)\left(\frac{x}{0.25}+2\right)}{3!}(0.09375)$$

$$= 1.10100 + (-1.33333)(0.7660625)$$
$$+ \frac{(-1.33333)(-0.33333)}{2}(0.406375)$$
$$+ \frac{(-1.33333)(-0.33333)(-0.66666)}{6}(0.09375)$$

$$= 1.10100 - 1.021414 + 0.090304426$$
$$+ 0.0046295$$

$$y(-1/3) = 0.165260.$$

② The amount A of a substance remaining in a reacting system after an interval of time t in a certain chemical experiment

T (min)	2	5	8	11
A (gm)	94.8	87.9	81.3	75.1

Obtain the value of A where t = 9 mins using Newton's interpolation formula.

Soln

T x	A y	∇y	$\nabla^2 y$	$\nabla^3 y$
2	94.8			
		-6.9		
5	87.9		0.3	
		-6.6		0.1
8	81.3		0.4	
		-6.2		
11	75.1			

$$v = \frac{x - x_n}{h}, \quad h = 3$$

The Newton's Backward interpolation formula is

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

$$y = 75.1 + \left(\frac{x-11}{3}\right)(-6.2) + \frac{\left(\frac{x-11}{3}\right)\left(\frac{x-11}{3}+1\right)}{2}(0.4)$$

$$y = 75.1 - 6.2\left(\frac{x-11}{3}\right) + \left(\frac{(x-11)(x-8)}{8}\right) \times 0.4$$

$$+ \frac{(x-11)(x-8)(x-5)}{162} \times 0.1$$

Put $x = 9$

$$y(9) = 75.1 - \frac{6.2(9-11)}{3} + \frac{(9-11)(9-8)}{18} \times 0.5$$

$$+ \frac{(9-11)(9-8)(9-5)}{162} \times 0.1$$

$$= 75.1 + \frac{62}{15} - \frac{2}{45} - \frac{2}{405}$$

$$y(9) = 79.1839.$$

UNIT- II Solution of Simultinous Linear equations

Questions	opt1	opt2	opt3	opt4	opt 5	opt 6	Answer
The numerical method of solving linear equations is of two types one is direct, other is_________ method.	iterative	elimination	Newton	exact			iterative
The direct method fails if any one of the pivot elements become ----.	Zero	one	two	negative			Zero
The given system of equations can be taken as in the form of -----------	A = B	BX= A	AX= B	AB = X			AX= B
------------- Method produces the exact solution after a finite number of steps.	Gauss Siedal	Gauss Jacobbi	Iterative method	Direct			Direct
Gauss elimination method is a -----------------.	Direct method	InDirect method	Iterative method	convergent			Direct method
Gauss Elimination and Gauss Jordan are direct methods while Gauss Jacobi and Gauss Seidal are __________ methods	iterative	elimination	interpolation	none			iterative
The modification of Gauss – Elimination method is called ---------------------	Gauss Jordan	Gauss Siedal	Gauss Jacobbi	Gauss Elimination			Gauss Jordan
When Gauss Jordan method is used to solve AX = B, A is transformed into -----	Scalar matrix	diagonal matrix	Upper triangular matrix	lower triangularmatrix			diagonal matrix
In Gauss Jordan method the coefficient matrix is transformed into ________ matrix	upper triangular	lower triangular	diagonal	column			diagonal
Gauss Jordan method is ________ method	direct	indirect	iteration	interpolation			direct
The first equation in Gauss – Jordan method, is called __________ equation.	pivotal	dominant	normal	reduced			pivotal
The element a_11not equal to zero in Gauss – Jordan method is called _________ element.	Eigen value	root	Eigen vector	pivot			pivot
The Gauss Jordan method is the modification of ________ method.	Gauss Elimination	Gauss Jacobi	Gauss Seidal	interpolation			Gauss Elimination
In Crouts method, if AX=B, then	LX=B	UX=B	L=B	LUX=B			LUX=B
Crouts method is a____ method to solve simultaneous linear equations.	Direct	Indirect	real	inverse			Direct
Choleskeys method is used only when the matrix is ____	symmetric	skew-symmetric	singular	non-singular			symmetric
Choleskys method is used for finding the _________ of a matrix.	determinant	value	inverse	rank			determinant
In the absence of any better estimates, the -------------of the function are taken as x = 0, y = 0, z = 0.	initialapproximations	roots	points	final value			initialapproximations
In the absence of any better estimates, the initial approximations are taken as---	x = 0, y = 0, z = 0	x = 1, y = 1, z = 1	x = 2, y = 2, z = 2	x = 3, y = 3, z = 3			x = 0, y = 0, z = 0
Gauss Jordan method fails if the element in top of first column is ______	0	1	2	3			0
Gauss Jacobi method is ________ method	direct	indirect	elimination	interpolation			indirect
Gauss Jacobi method is ________ method	direct	elimination	iteration	interpolation			iteration
Gauss Seidal method is ________ method	direct	indirect	elimination	interpolation			indirect
The successive approximations are called ________	interpolation	elimination	iterates	approximation			iterates
__________ method is a self correcting method.	interpolation	elimination	Iteration	approximation			Iteration
The convergence in Gauss Jacobi method can be achieved only when coefficient of the matrix is __________ dominant	row wise	column wise	diagonally	none			diagonally
The convergence of Gauss Seidal method is _____times as fast as in Jacobis method	1	2	3	4			2
The convergence of Gauss Seidal method is roughly _____ that of Gauss Jacobi method	twice	thrice	once	4times			twice

The convergence in Gauss Seidal method can be achieved only when coefficient of the matrix is __________ dominant	row wise	column wise	diagonally	none	diagonally
The matrix is ________ if the numerical value of the leading diagonal element in each row is greater than or equal to the sum of the numerical value of other element in that row.	orthogonal	symmetric	diagonally dominant	singular	diagonally dominant
The system of simultaneous linear equation in n unknowns AX = B if A is diagonally Dominant then the system is said to be system	dominant	diagonal	scalar	singular	diagonal
In Gauss Jacobi and Gauss Seidal methods the co-efficient matrix must be ___dominant.	row wise	column wise	none	diagonally	diagonally
In finding the inverse of the matrix using Gauss Jordan method the condition for convergence is achieved by changing the given matrix into a _______ matrix.	upper triangular	lower triangular	diagonal	unit	unit
The iterative procedure for finding the dominant Eigen value of the matrix is called ______ Power method.	Rayleighs	Gaussian	Newtons	inverse	Rayleighs
The power method will work satisfactorily only if A has a__________ Eigen value	small	unequal	equal	dominant	dominant
In power method the element in vector in each iteration will become very large, to avoid this we divide each vector by its _________ component	smallest	largest	positive	negative	largest
Power method generally gives the largest Eigen value of A provided the Eigen values are____.	equal	negative	positive	real and distinct	real and distinct
In power method iterative process is repeated until _________ becomes negligibly small.	$X_r – X_{(r-1)}$	$X_{(r-1)} – X_r$	$X_r – X_{(r+1)}$	$X_{(r+1)} – X_r$	$X_r – X_{(r-1)}$
If the eigen values of A are -3,3,1 then the dominant eigen value of A is________.	3	1	-3	No dominant eigen value	No dominant eigen value
The smallest eigen value of A is the reciprocal of the dominant eigen value of___________	$A^{(-1)}$	det A	A^T	A	$A^{(-1)}$
If the Eigen values of A are -6, 2, 4 then ______ is dominant.	2	4	-6	-2	-6
If the eigen values of A are 4,3,1 then the dominant eigen value of A is________.	3	1	4	none	4
The Power method is used for finding ________ eigen value	dominant	least	central	positive	dominant
The Inverse Power method is used for finding ________ eigen value	dominant	least	central	positive	dominant
Jacobis method is used only when the matrix is ____	symmetric	skew-symmetric	singular	non-singular	symmetric

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UNIT - 3

Numerical Differentiation and Integration

Numerical differentiation:

It is the Process of finding the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ & $\frac{d^3y}{dx^3}$, ... for some particular value of x.

① find the first derivatives of $f(x)$ at $x=2$ for the data $f(-1)=-21$, $f(1)=15$, $f(2)=12$, $f(3)=3$. using Newtons divided difference formula.

Soln

x	-1	1	2	3
y	-21	15	12	3

The Newton's divided difference formula is

$$y = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \cdots$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	-21			
		18		
1	15		-7	
		-3		1
2	12		-3	
		-9		
3	3			

$$y = -21 + (x+1)\,18 + (x+1)(x-1)(-7) + (x+1)(x-1)(x-2)\,(1)$$

$$= -21 + 18x + 18 - 7(x^2-1) + (x^2-1)(x-2)$$

$$= -21 + 18x + 18 - 7x^2 + 7 + x^3 - 2x^2 - x + 2$$

$$y = x^3 - 9x^2 + 17x + 6$$

$$y' = 3x^2 - 18x + 17$$

$$y'(2) = -7.$$

② find $f'(10)$ from the following data

x	3	5	11	27	34
$f(x)$	-13	23	899	17315	35606

The newtons divided difference formula is

$$y = f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \cdots$$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
3	-13				
		18			
5	23		16		
		146		1	
11	899		40		0
		1026		1	
27	17315		69		
		2613			
34	35606				

$$y = f(x) = -13 + 18(x-3) + 16(x-3)(x-5) + (x-3)(x-5)(x-11)$$

$$= -13 + 18x - 54 + 16[x^2 - 8x + 15] + (x^2 - 8x + 15)(x-11)$$

$$= -13 + 18x - 54 + 16x^2 - 128x + 240 + x^3 - 11x^2 - 8x^2 + 88x + 15x - 165$$

$$f(x) = x^3 - 3x^2 - 7x + 8$$

$$f'(x) = 3x^2 - 6x - 7$$

$$f'(10) = 233.$$

Newton's forward formula for derivatives

$$y = f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \cdots \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y''' = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

① Find the first three derivatives of $f(x)$ at $x = 1.5$ & at $x = 4.0$ using Newton's forward interpolation formula to the data given below.

x	1.5	2	2.5	3	3.5	4
y	3.375	7	13.625	24	38.875	59

Soln

$$f'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f'''(x) = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x - x_0}{h} = \frac{x - 1.5}{0.5}$$

When $x = 1.5$ $\boxed{u = 0}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375					
		3.625				
			3			
2	7			0.75		
		6.625			0	
2.5	13.625		3.75			0
		10.375		0.75		
3	24		4.5		0	
		14.875		0.75		
3.5	38.875		5.25			
		20.125				
4	59					

$$f'(1.5) = \frac{1}{0.5}\left[3.625 + (0-1)\cdot\frac{3}{2} + \frac{2}{6}(0.75)\right]$$

$$= \frac{1}{0.5}\left[3.625 - 1.5 + 0.25\right]$$

$$= 4.75$$

$$f''(1.5) = \frac{1}{0.5^2}\left[3 + (-6)\times\frac{0.75}{6}\right]$$

$$= \frac{1}{0.5^2}\left[3 - 0.75\right] = 9$$

$$f'''(1.5) = \frac{1}{0.5^3}\left[0.75\right] = 6$$

Newton's Backward Interpolation formula

$$y' = \frac{1}{h}\left[\nabla y_n + \frac{(2v+1)}{2!}\nabla^2 y_n + \frac{(3v^2+6v+2)}{3!}\nabla^3 y_n + \frac{(4v^3+18v^2+22v+6)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\nabla^2 y_n + (6v+6)\frac{\nabla^3 y_n}{3!} + \frac{(12v^2+36v+22)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y''' = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{(24v+36)}{4!}\nabla^4 y_n + \cdots\right]$$

$$v = \frac{x - x_n}{h} = \frac{x-4}{0.5}$$

When $x = 4 \Rightarrow \boxed{v = 0}$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f'''(x) = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{x-1.5}{0.5}$$

When $x = 1.5$ $\boxed{u=0}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375	3.625	3	0.75	0	0
2	7	6.625	3.75	0.75	0	
2.5	13.625	10.375	4.5	0.75		
3	24	14.875	5.25			
3.5	38.875	20.125				
4	59					

$$y' = \frac{1}{0.5}\left[20.125 + \frac{1}{2}\times 5.25 + \frac{2}{6}\times 0.75\right]$$

$$= 46$$

$$y'' = \frac{1}{0.5^2}\left[5.25 + 6\times\frac{0.75}{6}\right] = 24$$

$$y''' = \frac{1}{0.5^3}\left[0.75\right] = 6.$$

(2) For the given data, find the first two derivatives at $x = 1.1$

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Soln

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{x-1.0}{0.1}$$

At $x = 1.1$ $\quad u = \frac{1.1-1.0}{0.1} = 1.$

$y' = \frac{1}{0.1}[0.4$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	7.989					
		0.4140				
1.1	8.403		-0.0360			
		0.3780		0.0060		
1.2	8.781		-0.03		-0.0020	
		0.3480		0.0040		0.001
1.3	9.129		-0.0260		-0.0010	
		0.3220		0.003		0.003
1.4	9.451		-0.0230		0.002	
		0.2990		0.0050		
1.5	9.750		-0.0180			Δ^6
		0.2810				
1.6	10.031					0.00

$$y'(1.1) = \frac{1}{0.1}\left[0.414 + \frac{(2-1)}{2}(-0.0360) + \frac{(3-6+2)}{6}(0.006) + \frac{(4-18+22-6)}{24}(-0.002)\right]$$

$$= \frac{1}{0.1}\left[0.414 - 0.0180 - 0.0010 - 0.0002\right]$$

$$= 3.9480$$

$$y''(1.1) = \frac{1}{(0.1)^2}\left[(-0.0360) + \frac{(6-6)}{6}(0.0060) + \frac{(12-36+22)}{24}(-0.0020)\right]$$

$$= 100\left[-0.0360 + 0\right] + \frac{(-2)}{24}(-0.0020)\Big]$$

$= -36 + 0.00016$

$= ~~-35.9998~~ \; -3.584$

③ find the first two derivatives of $x^{1/3}$ at $x = 50$ and $x = 56$ for the given data

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Soln

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
50	3.6840						
		0.0244					
51	3.7084		-0.0003				
		0.0241		0			
52	3.7325		-0.0003		0		
		0.0238		0		0	
53	3.7563		-0.0003		0		0
		0.0235		0		0	
54	3.7798		-0.0003		0		
		0.0232		0			
55	3.8030		-0.0003				
		0.0229					
56	3.8259						

Newton's forward formula:

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$= -36 + 0.00016$

$= ~~35.9998~~ \; -3.584$

(3) find the first two derivatives of $x^{1/3}$ at $x = 50$ and $x = 56$ for the given data

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Soln

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
50	3.6840						
		0.0244					
51	3.7084		−0.0003				
		0.0241		0			
52	3.7325		−0.0003		0		
		0.0238		0		0	
53	3.7563		−0.0003		0		0
		0.0235		0		0	
54	3.7798		−0.0003		0		
		0.0232		0			
55	3.8030		−0.0003				
		0.0229					
56	3.8259						

Newton's forward formula:

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{50-50}{1} = 0$$

$$y' = \frac{1}{1}\left[0.02414 + \frac{(-1)}{2}(-0.0003)\right]$$

$$= 0.0244 + 0.0002$$

$$= 0.0246$$

$$y'' = \frac{1}{1}[-0.0003] = -0.0003$$

Newton's Backward Interpolation formula.

$$y' = \frac{1}{h}\left[\nabla y_n + \frac{(2v+1)}{2!}\nabla^2 y_n + \frac{(3v^2+6v+2)}{3!}\nabla^3 y_n + \frac{(4v^3+18v^2+22v+6)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{(6v+6)}{3!}\nabla^3 y_n + \frac{(12v^2+36v+22)}{4!}\nabla^4 y_n + \cdots\right]$$

$$v = \frac{x-x_n}{h} = \frac{x-56}{0.5}$$

$$v = \frac{56-56}{0.5} = 0$$

$$y' = \frac{1}{0.5}\left[0.0299 + \frac{(0+1)}{2!}(-0.0003) + \frac{2}{3!}(0) + 0\right]$$

$$= \frac{1}{0.5}\left[0.0299 - \frac{0.0003}{2}\right]$$

$$= 0.0595$$

$$y'' = \frac{1}{0.5^2}[-0.0003] = -0.0012$$

Numerical Integration

Trapezoidal rule

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\left[(\text{Sum of first and last ordinate}) + 2(\text{Sum of remaining ordinates})\right]$$

$$h = \frac{b-a}{n}$$

Simpson's 1/3 rule

$$I = \int_a^b f(x)\,dx = \frac{h}{3}\left[(\text{first} + \text{Last}) + 4(\text{Sum of odd ordinates}) + 2(\text{sum of even ordinates})\right]$$

$$h = \frac{b-a}{n} - [\text{multiples of } 2]$$

Simpson's 3/8 rule

$$I = \frac{3h}{8}\left[(\text{first Last}) + 2(\text{Sum of multiples of } 3) + 3(\text{Sum of non-multiples of } 3)\right]$$

$$h = \frac{b-a}{n} \quad [\text{multiples of } 3]$$

① Using Trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

Soln

$$h = \frac{b-a}{n} = \frac{1+1}{8} = \frac{2}{8} = 0.25$$

x	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y	0.5	0.65	0.8	0.9412	1	0.9412	0.8	0.64	0.5

$$I = \frac{h}{2}\left[(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$$

$$= \frac{0.25}{2}\left[(0.5 + 0.5) + 2(0.65 + 0.8 + 0.9412 + 1 + 0.9412 + 0.8 + 0.64)\right]$$

$$= \frac{0.25}{2}\left[1 + 2(5.7624)\right]$$

$$= \frac{0.25}{2}\left[12.5248\right]$$

$$= 1.5656$$

2) Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ with $h = 1/6$ by Trapezoidal rule.

Soln

$$f(x) = \frac{1}{1+x^2} \qquad h = 1/6$$

x	0	1/6	2/6	3/6	4/6	5/6	1
y	1	36/37	9/10	4/5	9/13	36/61	1/2

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{(1/6)}{2}\left[(1+1/2) + 2\left(\frac{36}{37}+\frac{9}{10}+\frac{4}{5}+\frac{9}{13}+\frac{36}{61}\right)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 2(3.9554)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 7.9108\right]$$

$$= 0.7842$$

③ Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ by Trapezoidal rule. Also check up the results by actual Integration

Soln

$f(x) = \frac{1}{1+x^2}$, $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
y	1.00	0.500	0.200	0.100	0.058824	0.038426	0.27026

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027027) + 2(0.5+0.2+0.1+0.058824+0.038462)\right]$$

$$= 1.41079950$$

By actual Integration

$$I = \int_0^6 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_0^6 = \tan^{-1}6 - \tan^{-1}0$$

$$= 1.40564765$$

(4) Evaluate $\int_{1.0}^{1.3} \sqrt{x}\,dx$ taking $h = 0.05$ by trapezoidal rule

Soln

$f(x) = \sqrt{x}$

$h = \frac{b-a}{n} = 0.05$

x	1.0	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0724	1.0954	1.1180	1.1402

$$I = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{0.05}{2}\left[(1 + 1.402) + 2(1.0247 + 1.0488 + 1.0724 + 1.0954 + 1.1180)\right]$$

$$= 0.1\left[2.1402 + 2(5.3593)\right]$$

$$= 0.1\left[2.1402 + 10.7186\right]$$

$$= ~~0.1~~ \; 0.025\,(12.8588)$$

$$= ~~1.28588~~ \; 0.3214$$

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{(1/6)}{2}\left[(1+1/2) + 2\left(\frac{36}{37}+\frac{9}{10}+\frac{4}{5}+\frac{9}{13}+\frac{36}{61}\right)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 2(3.9554)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 7.9108\right]$$

$$= 0.7842$$

(3) Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ by Trapezoidal rule. Also check up the results by actual Integration

Soln

$f(x) = \frac{1}{1+x^2}$, $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
y	1.00	0.500	0.200	0.100	0.058824	0.038426	0.27026

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027027) + 2(0.5+0.2+0.1+0.05824+0.038462)\right]$$

$$= 1.4107950$$

By actual Integration

$$I = \int_0^6 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_0^6 = \tan^{-1}6 - \tan^{-1}0$$

$$= 1.40564765$$

(4) Evaluate $\int_{1.0}^{1.3} \sqrt{x}\,dx$ taking $h = 0.05$ by trapezoidal rule

Soln

$f(x) = \sqrt{x}$

$h = \frac{b-a}{n} = 0.05$

x	1.0	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0724	1.0954	1.1180	1.1402

$$I = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{0.05}{2}\left[(1 + 1.402) + 2(1.0247 + 1.0488 + 1.0724 + 1.0954 + 1.1180)\right]$$

$$= 0.1\left[2.1402 + 2(5.3593)\right]$$

$$= 0.1\left[2.1402 + 10.7186\right]$$

$$= \sout{0.1}\ 0.025\ (12.8588)$$

$$= \sout{1.28588}\ 0.3214$$

(5) Dividing the range into 10 equal parts find the value of $\int_0^{\pi/2} \sin x \, dx$ by Simpsons 1/3 rule.

Soln

$f(x) = \sin x \qquad h = \frac{b-a}{n} = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$

x	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$	$6\pi/20$	$7\pi/20$	$8\pi/20$
$f(x)$	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511

$$I = \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{\pi/20}{3}\left[(0+1) + 4(0.1564 + 0.4540 + 0.7071 + 0.8910) + 2(0.3090 + 0.5878 + 0.8090)\right]$$

$$= \frac{\pi}{60} * 19.0986 = 1$$

(6) The velocity v of a particle at a distance s from a point on its path is gn by the table below.

s	0	10	20	30	40	50	60
v	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by Simpsons 1/3 rule.

Soln

$$\text{Velocity} = \frac{\text{distance}}{\text{time}}$$

$$v = \frac{ds}{dt}$$

$$dt = \frac{1}{v}\, ds$$

$$t = \int_0^{60} \frac{1}{v}\, ds \;,\; h = 10$$

$$I = \int_0^{60} \frac{1}{v}\, ds = \frac{h}{3}\Big[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4) \Big]$$

v	47	58	64	65	61	52	38
$1/v$	0.02127	0.01724	0.015625	0.01538	0.01639	0.01923	0.026316

$$I = \frac{10}{3}\Big[(0.02127 + 0.026316) + 4(0.07124 + 0.01538 + 0.0923) + 2(0.015625 + 0.01639) \Big]$$

$$I = 1.06338$$

(7) Compute $\int_0^{\pi/2} \sin x\, dx$ using Simpson's $3/8^{th}$ rule of numerical integration.

Soln

$$I = \int_0^{\pi/2} \sin x \, dx$$

$f(x) = \sin x \qquad h = \frac{\pi/2 - 0}{9} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$	$7\pi/18$	$8\pi/18$	$9\pi/18$
$f(x)$	0	0.1736	0.3420	0.50	0.6428	0.7660	0.8660	0.9397	0.9848	1

$$I = \frac{3h}{8}\left[(y_0 + y_9) + 3(y_1 + y_2 + y_4 + y_5 + y_7 + y_8) + 2(y_3 + y_6)\right]$$

$$= \frac{3\pi}{8 \times 18}\left[(0 + 1) + 3(0.1736 + 0.3428 + 0.6428 + 0.7660 + 0.9397 + 0.9848) + 2(0.5 + 0.8660)\right]$$

$I = 0.99998576$

$I \sim 1$

(7) The velocities of a car running on a straigh road at intervals of 2 minutes are gn below

Time (min)	0	2	4	6	8	10	12
Velocity (km/hr)	0	22	30	27	18	7	0

using Simpsons 1/3 rule find the distance covered by the car.

Soln

$$\text{Velocity} = \frac{dx}{dt} \quad (\text{ie}) \quad v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$x = \int v\,dt$$

t	0	2	4	6	8	10	12
v	0	$\frac{22}{60}$	$\frac{30}{60}$	$\frac{27}{60}$	$\frac{18}{60}$	$\frac{7}{60}$	0

$$I = \frac{h}{3}\left[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)\right]$$

$$= \frac{2}{3}\left[0 + 0 + 2\left(\frac{30}{60} + \frac{18}{60}\right) + 4\left(\frac{22}{60} + \frac{27}{60} + \frac{7}{60}\right)\right]$$

$$= 3.5556 \text{ Km}$$

Romberg Method

$$I = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

I_1 — Value of integral with $h = \frac{b-a}{2}$

I_2 — Value of integral with $h = \frac{b-a}{4}$

I_3 — " " " $h = \frac{b-a}{8}$.

① Compute $I = \int_0^{1/2} \frac{x}{\sin x}\, dx$, using Simpsons rule with $h = 1/4,\ 1/8,\ 1/16$ and then Rombergs Method.

Soln

$$I = \int_0^{1/2} \frac{x}{\sin x}\, dx$$

$$f(x) = \frac{x}{\sin x}$$

i) Take $h = 1/4$

x	0	1/4	1/2
$f(x)$	y_0 1	y_1 1.0105	y_2 1.0429

By Simpsons 1/3 rule,

$$I_1 = \frac{h}{3}\left[(y_0 + y_2) + 4(y_1) + 0\right]$$

$$= \frac{1}{12}\left[(1 + 1.0429) + 4(1.0105)\right]$$

$$I_1 = 0.507075$$

(ii) Take $h = 1/8$

x	0	1/8	2/8	3/8	4/8
$f(x)$	1 (y_0)	1.0026 (y_1)	1.0105 (y_2)	1.0238 (y_3)	1.0429 (y_4)

$$I_2 = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$$

$$= \frac{1}{24}\left[(1 + 1.0429) + 4(1.0026 + 1.0236) + 2(1.01051)\right]$$

$$I_2 = 0.5070625$$

(iii) Take $h = \frac{1}{16}$

x	0	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$	$\frac{8}{16}$
$f(x)$	1 (y_0)	1.0007 (y_1)	1.0026 (y_2)	1.0059 (y_3)	1.0105 (y_4)	1.0165 (y_5)	1.0238 (y_6)	1.0326 (y_7)	1.0429 (y_8)

$$I_3 = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{1}{48}\left[(1+1.0429) + 4(1.0007+1.0059+1.0165+1.0326) + 2(1.0026+1.0105+1.0238)\right]$$

$$I_3 = 0.5070729$$

for I_1, I_2

Romberg formula is

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

$$= 0.5070625 + \left(\frac{0.5070625 - 0.507075}{3}\right)$$

$$I = 0.507058$$

for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3}\right)$$

$$= 0.5070729 + \left(\frac{0.5070729 - 0.5070625}{3}\right)$$

$$= 0.50707636 6.$$

Romberg for I_4 & I_5

$$I = I_5 + \left(\frac{I_5 - I_4}{3}\right)$$

(3) Evaluate $I = \int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method. Hence deduce an approximate value of π.

Soln

$a = 0 \; ; \; b = 1$

$f(x) = \frac{1}{1+x^2}$

I $h = \frac{b-a}{2} = \frac{1-0}{2} = 0.5$

x	0	0.5	1
$f(x)$	1	0.8	0.5

$I_1 = \frac{h}{2}\left[(y_0 + y_2) + 2(y_1) \right]$

$= \frac{0.5}{2}\left[(1 + 0.5) + 2 \times 0.8 \right]$

$I_1 = 0.7750$

II $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

(3) Evaluate $I = \int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method. Hence deduce an approximate value of π

Soln

$a = 0 \quad ; \quad b = 1$

$f(x) = \frac{1}{1+x^2}$

I $h = \frac{b-a}{2} = \frac{1-0}{2} = 0.5$

x	0	0.5	1
$f(x)$	1	0.8	0.5

$I_1 = \frac{h}{2}\left[(y_0 + y_2) + 2(y_1)\right]$

$= \frac{0.5}{2}\left[(1 + 0.5) + 2 \times 0.8\right]$

$I_1 = 0.7750$

II $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

$$I_2 = \frac{0.25}{2}\left[(1+0.5) + 2(0.9412 + 0.8 + 0.64)\right]$$

$$\boxed{I_2 = 0.7828}$$

(iii) $h = \frac{b-a}{8} = \frac{1-0}{8} = 0.125$

x	0	0.12	0.25	0.375	0.5	0.625	0.75	0.875	1
f(x)	1	0.9846	0.9412	0.8767	0.8	0.7191	0.64	0.5664	0.5

$$I_3 = \frac{0.5}{2}\left[(1+0.5) + 2(0.9846 + 0.9412 + 0.8767 + 0.8 + 0.7191 + 0.64 + 0.5664)\right]$$

$$\boxed{I_3 = 0.7848}$$

Romberg for I_1, I_2

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right) = 0.7854$$

Romberg for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3} \right) = 0.7855$$

Romberg for I_4, I_5

$$I = I_5 + \left(\frac{I_5 - I_4}{3} \right) = 0.7855$$

$$I = \int_0^1 \frac{dx}{1+x^2}$$

$$0.7855 = \left[\tan^{-1} x \right]_0^1$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$\frac{\pi}{4} = 0.7855$$

$$\pi = 3.1420.$$

③ Using Romberg Integration, evaluate

$$\int_0^1 \frac{dx}{1+x}$$

Soln

I) Here $a=0,\ b=1$

x	0	0.5	1
$\frac{1}{1+x}$	1	0.6667	0.5

$$I_1 = \frac{h}{2}\left[(y_0 + y_2) + 2(y_1)\right]$$

$$= \frac{0.5}{2}\left[(1+0.5) + 2(0.6667)\right]$$

$$I_1 = 0.7084$$

II) $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.8	0.6667	0.5714	0.5

$$I_2 = \frac{0.25}{2}\left[(1+0.5) + 2(0.8 + 0.6667 + 0.5714)\right]$$

$$I_2 = 0.6970$$

III $h = \frac{b-a}{8} = \frac{1-0}{8} = 0.125$

x	0	0.125	0.25	0.375	0.5
$f(x)$	1	0.8889	0.8	0.7273	0.6667
		0.625	0.75	0.875	1
		0.6154	0.5714	0.5333	0.5

$$I_3 = \frac{0.125}{2}\left[(1+0.5) + 2(0.8889+0.8+0.7273+0.6667+0.6154+0.5714+0.5333)\right]$$

$$\boxed{I_3 = 0.6941.}$$

Romberg for I_1, I_2

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

$$= 0.6970 + \left(\frac{0.6970 - 0.7084}{3}\right)$$

$$\boxed{I_4 = 0.6932}$$

Romberg for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3}\right)$$

$$= 0.6941 + \left(\frac{0.6941 - 0.6970}{3}\right).$$

$$\boxed{I_5 = 0.6931}$$

Romberg for I_4, I_5

$$I_6 = I_5 + \left(\frac{I_5 - I_4}{3}\right)$$

$$\boxed{I_6 = 0.6931}$$

Gauss Quadrature formula

Quadrature:

The Process of finding a definite integral from a tabulated values of a function is known as Quadrature.

Gaussian two point Quadrature formula

Let $I = \int_a^b f(x)\,dx$.

Take $x = \left(\frac{a+b}{2}\right) + \left(\frac{b-a}{2}\right)t$

$$dx = \left(\frac{b-a}{2}\right)dt.$$

By using this transformation

$$I = \int_{-1}^{1} g(t)\,dt = g\left(\frac{-1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

① Evaluate $\int_{-1}^{1} e^{-x^2} \cos x \, dx$ by Gauss two Point Quadrature formula.

Soln

$$I = \int_{-1}^{1} e^{-x^2} \cos x \, dx$$

$$f(x) = e^{-x^2} \cos x$$

$$I = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-\left(\frac{-1}{\sqrt{3}}\right)^2} \cos\left(\frac{-1}{\sqrt{3}}\right) + e^{-\left(\frac{1}{\sqrt{3}}\right)^2} \cos\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-1/3} \cos\left(\frac{-1}{\sqrt{3}}\right) + e^{-1/3} \cos\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-1/3} \left[\cos\left(-1/\sqrt{3}\right) + \cos\left(1/\sqrt{3}\right) \right]$$

$$I = 1.2008.$$

② Apply Gauss two point formula to evaluate $\int_{-1}^{1} \frac{1}{1+x^2} dx$

Soln

$$I = \int_{-1}^{1} \frac{1}{1+x^2} dx$$

$$f(x) = \frac{1}{1+x^2}$$

$$I = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{1+\left(\frac{-1}{\sqrt{3}}\right)^2} + \frac{1}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{3}{4} + \frac{3}{4}$$

$$= \frac{6}{4}$$

$$= 1.5$$

③ Evaluate the integral $I = \int_1^2 \frac{2x}{1+x^4}\,dx$ using Gaussian two point formula

Soln

$$I = \int_1^2 \frac{2x}{1+x^4}\,dx$$

$$f(x) = \frac{2x}{1+x^4}, \quad a = 1, \quad b = 2$$

$$x = \frac{a+b}{2} + \left(\frac{b-a}{2}\right)t$$

$$x = \frac{3}{2} + \frac{1}{2}t$$

$$dx = \frac{1}{2}\,dt$$

$$I = \int_{-1}^{1} \frac{2\left(\frac{3}{2}+\frac{1}{2}t\right)}{1+\left(\frac{3}{2}+\frac{1}{2}t\right)^4} \cdot \frac{dt}{2}$$

$$= \int_{-1}^{1} \frac{\left(\frac{3+t}{2}\right)}{1+\left(\frac{3+t}{2}\right)^4}\, dt$$

$$g(t) = \frac{\frac{3+t}{2}}{1+\left(\frac{3+t}{2}\right)^4}$$

$$I = g\left(\frac{-1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\frac{3-\frac{1}{\sqrt{3}}}{2}}{1-\left(\frac{3-\frac{1}{\sqrt{3}}}{2}\right)^4} + \frac{\frac{3+\frac{1}{\sqrt{3}}}{2}}{1+\left(\frac{3+\frac{1}{\sqrt{3}}}{2}\right)^4}$$

$$= \frac{1.2113}{3.1530} + \frac{1.7887}{11.2359}$$

$$= 0.3842 + 0.1592$$

$$= 0.5434.$$

$= \frac{\pi}{4}[0.3259 + 0.9454]$

$= 0.9985$

Gaussian Three Point Quadrature formula:

$$I = \int_a^b f(x)\,dx$$

Take $x = \left(\frac{a+b}{2}\right) + \left(\frac{b-a}{2}\right)t$

$$dx = \left(\frac{b-a}{2}\right)dt$$

$$I = \int_{-1}^{1} g(t)\,dt = \frac{5}{9}\left[g\left(-\sqrt{\frac{3}{5}}\right) + g\left(\sqrt{\frac{3}{5}}\right)\right] + \frac{8}{9}g(0)$$

① Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using 3 point Quadrature formula.

Soln

$$I = \int_0^1 \frac{dx}{1+x^2}$$

$f(x) = \frac{1}{1+x^2}$, $a = 0$, $b = 1$

Take $x = \left(\frac{a+b}{2}\right) + \left(\frac{b-a}{2}\right)t$

$$dx = \left(\frac{b-a}{2}\right) dt$$

$$\Rightarrow x = \frac{1}{2} + \frac{1}{2}t$$

$$dx = \frac{1}{2} dt$$

$$I = \int_{-1}^{1} \frac{\frac{1}{2} dt}{1 + \left(\frac{1+t}{2}\right)^2} = \frac{1}{2}\int_{-1}^{1} \frac{dt}{1 + \left(\frac{1+t}{2}\right)^2}$$

$$\therefore \quad g(t) = \frac{1}{1 + \left(\frac{1+t}{2}\right)^2}$$

$$I = \frac{1}{2}\left[\frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9}g(0)\right]$$

$$= \frac{1}{2}\left[\frac{5}{9}\left(\frac{1}{1+\left(\frac{1+(-\sqrt{3/5})}{2}\right)^2} + \frac{1}{1+\left(\frac{1+(\sqrt{\frac{3}{5}})}{2}\right)^2}\right) + \frac{8}{9}\left[\frac{1}{1+(\frac{1}{2})^2}\right]\right]$$

$$= \frac{1}{2}\left[\frac{5}{9}(0.9875 + 0.5595 + 0.7111\right]$$

$$= 0.7853.$$

(2) Apply three point Gaussian Quadrature formula to evaluate $\int_0^1 \frac{\sin x}{x}\, dx$

Soln

$$I = \int_0^1 \frac{\sin x}{x}\, dx$$

$$f(x) = \frac{\sin x}{x}, \quad a = 0, \; b = 1$$

$$x = \left(\frac{b+a}{2}\right) + \left(\frac{b-a}{2}\right) t$$

$$dx = \left(\frac{b-a}{2}\right) dt$$

$$\Rightarrow x = \tfrac{1}{2} + \tfrac{1}{2} t = \tfrac{1}{2}(1+t)$$

$$dx = \tfrac{1}{2}\, dt$$

$$I = \int_{-1}^{1} \frac{\sin \frac{1}{2}(1+t)}{\frac{1}{2}(1+t)} \cdot \frac{1}{2}\, dt$$

$$= \int_{-1}^{1} \frac{\sin \frac{1}{2}(1+t)}{(1+t)}\, dt$$

$$\therefore g(t) = \frac{\sin \frac{1+t}{2}}{1+t}$$

$$g(0) = \sin \tfrac{1}{2} = 0.47943$$

$$g\left(\sqrt{\tfrac{3}{5}}\right) = \sin\left[\frac{\sqrt{3/5}+1}{2}\right] \Big/ \sqrt{3/5}+1 = \frac{0.7754}{1.7746} = 0.437$$

$$g\left(-\sqrt{\tfrac{3}{5}}\right) = \frac{\sin\left[\frac{-\sqrt{\frac{3}{5}}+1}{2}\right]}{-\sqrt{\frac{3}{5}}+1} = \frac{0.1125}{0.2254} = 0.499$$

$$I = \frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9}g(0)$$

$$= \frac{5}{9}\left[0.499 + 0.437\right) + \frac{8}{9}(0.47943)$$

$$= 0.52 + 0.42616$$

$$= 0.94616$$

③ Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}\,dx$ by Gaussian three Point formula.

Soln

$$I = \int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}$$

$$f(x) = \frac{x^2+2x+1}{1+(x+1)^4}, \quad a=0, \quad b=2$$

$$x = \frac{b+a}{2} + \left(\frac{b-a}{2}\right)t$$

$$dx = \left(\frac{b-a}{2}\right)dt$$

$\Rightarrow \quad x = 1 + t$

$dx = dt$

$$I = \int_{-1}^{1} \frac{(z+1)^2 + 2(z+1) + 1}{1 + [(z+1)+1]^4} dz$$

$$g(t) = \frac{(z+1)^2 + 2(z+1) + 1}{1 + [(z+1)+1]^4}$$

$$= \frac{z^2 + 2z + 1 + 2z + 2 + 1}{1 + (z+2)^4}$$

$$g(t) = \frac{(z+2)^2}{1 + (z+2)^4}$$

$$g(0) = \frac{4}{17}$$

$$g\left(-\sqrt{\tfrac{3}{5}}\right) = \frac{\left(-\sqrt{\tfrac{3}{5}} + 2\right)^2}{1 + \left(-\sqrt{\tfrac{3}{5}} + 2\right)^4} = \frac{1.50161}{3.2548} = 0.4614$$

$$g\left(\sqrt{\tfrac{3}{5}}\right) = \frac{\left(\sqrt{\tfrac{3}{5}} + 2\right)^2}{1 + \left(\sqrt{\tfrac{3}{5}} + 2\right)^4} = \frac{7.69839}{60.2652} = 0.12774$$

$$I = \frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9} g(0).$$

$$= \frac{5}{9}[0.4614 + 0.12774] + \frac{8}{9}\left(\frac{4}{17}\right)$$

$$= 0.5364$$

Double Integration

Trapezoidal rule:

$$I = \int_c^d \int_a^b f(x,y)\, dx\, dy$$

$I = \frac{hk}{4}$ [Sum of four corners + 2 (Sum of remaining boundary values) + 4 (Sum of interior Values)]

Simpson's rule

$I = \frac{hk}{9}$ [Sum of four corners +

2 (Sum of odd position values) + 4 (Sum of even position values)

Boundary

+ 4 (Sum of odd position values) + 8 (Sum of even position values)

odd rows

+ 8 (Sum of odd position values) + 16 (Sum of even position values)]

even rows

~~$I = \frac{hk}{4}$ { Sum of four corn~~

$$I = \frac{0.1 \times 0.1}{4}\Big[0.5 + 0.4167 + 0.4545 + 0.3846$$
$$+ 2(0.4762 + 0.4545 + 0.4348 + 0.4762$$
$$+ 0.4 + 0.4348 + 0.4167 + 0.4)$$
$$+ 4(0.4545 + 0.4348 + 0.4167)\Big]$$

$$= \frac{0.1 \times 0.1}{4}\left[1.7558 + 6.9864 + 5.2240\right]$$

$$= \frac{0.1 \times 0.1}{4} \times 13.9662 = 0.0349.$$

② Evaluate $\int_1^2 \int_1^2 \frac{1}{x^2+y^2}\,dx\,dy$, numerically using $h = 0.2$, along x-direction and $k = 0.25$ along y-direction.

<u>Soln</u>

$$I = \int_1^2 \int_1^2 \frac{1}{x^2+y^2}\,dx\,dy$$

$$f(x, y) = \frac{1}{x^2+y^2}$$

By Trapezoidal

$$I = \frac{hk}{4}\Big[\text{Sum of four corners} + 2(\text{Sum of remaining boundary}) + 4(\text{Sum of interiors})\Big]$$

y \ x	1	1.2	1.4	1.6	1.8	2
1	0.5	0.4098	0.3378	0.2809	0.2359	0.2
1.25	0.3902	0.3331	0.2839	0.2426	0.2082	0.1798
1.5	0.3077	0.2710	0.2375	0.2079	0.1821	0.16
1.75	0.2462	0.2221	0.1991	0.1779	0.1587	0.1416
2	0.2	0.1838	0.1679	0.1524	0.1381	0.125

$$I = \frac{(0.2)(0.25)}{4}\Big[0.5 + 0.2 + 0.2 + 0.125$$
$$+ 2(0.4098 + 0.3378 + 0.2809 + 0.2359$$
$$+ 0.1798 + 0.16 + 0.1416$$
$$+ 0.1381 + 0.1524 + 0.1679 + 0.1838$$
$$+ 0.2462 + 0.2710 + 0.3331)$$
$$+ 4(0.3331 + 0.2839 + 0.2426$$
$$+ 0.2082 + 0.2710 + 0.2375$$
$$+ 0.2079 + 0.1821 + 0.2221$$
$$+ 0.1991 + 0.1779 + 0.1587)\Big]$$

$$= \frac{(0.2)(0.25)}{4}\Big[1.025 + 6.6642 + 10.8964)\Big]$$

$$= 0.2323.$$

3. Evaluate $I = \int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy}\,dx\,dy$ using Simpson's rule with $h = k = 1/4$

Soln

$$I = \int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy}\,dx\,dy$$

$$f(x,y) = \frac{\sin xy}{1+xy}$$

By Simpsons 1/3 rule,

$I = \frac{hk}{9}$ [Sum of four corners +

2 (Sum of odd position) + 4(S E P) (Boundary)

+ 4 (S O P) + 8 (S E P) (odd rows)

+ 8 (S O P) + 16 (S E P)] (even rows)

y \ x	0	1/4	1/2
I. 0	0	0	0
II. 1/4	0	0.0588	0.1108
III. 1/2	0	0.1108	0.1979

$$I = \frac{0.1 \times 0.1}{9} \Big[0.5 + 0.4167 + 0.3571 + 0.2976$$
$$+ 2[0.4545 + 0.4167 + 0.3247 + 0.3472)$$
$$+ 4[0.4762 + 0.4348 + 0.3788 + 0.3205$$
$$+ 0.3401 + 0.3106 + 0.4545 + 0.3846)$$
$$+ 4(0.3788) + 8(0.3968 + 0.3623)$$
$$+ 8(0.4132 + 0.3497)$$
$$+ 16(0.4329 + 0.3953$$
$$+ 0.3663 + 0.3344) \Big]$$

$$= \frac{0.1 \times 0.1}{9} \Big[1.5714 + 3.0862 + 12.4004$$
$$+ 1.5152 + 6.0728 + 6.1032$$
$$+ 24.4624 \Big]$$

$I = 0.0613$

5 Evaluate $\int_0^2 \int_0^1 4xy \, dx \, dy$ using Simpsons rule by taking $h = 1/4$ & $k = 1/2$

Soln

$$I = \int_0^2 \int_0^1 4xy \, dx \, dy$$

Here $f(x, y) = 4xy$

$h = 0.25 \quad k = 0.5$

y \ x	0	0.25	0.5	0.75	1
0	0	0	0	0	0
0.5	0	0.5	1	1.5	2
1	0	1	2	3	4
1.5	0	1.5	3	4.5	6
2	0	2	4	6	8

$$I = \frac{0.25 \times 0.5}{9}[8 + 16 + 64 + 8 + 32 + 32 + 128]$$

$$I = 4.$$

Unit-III **Interpolation**

Questions	opt1	opt2	opt3	opt4	opt5	opt6	Answer
The process of computing the value of the function inside the given range is called ________	Interpolation	extrapolation	reduction	expansion			Interpolation
If the point lies inside the domain [x_0, x_n], then the estimation of f(y) is called _________	Interpolation	extrapolation	reduction	expansion			Interpolation
The process of computing the value of the function outside the given range is called ________	Interpolation	extrapolation	reduction	expansion			extrapolation
If the point lies outside the domain [x_0, x_n], then the estimation of f(y) is called _________	Interpolation	extrapolation	reduction	expansion			extrapolation
Interpolation is the process of computing ________ values of a function from a given set of tabular values of a function	positive	negative	constant	intermediate			intermediate
The estimation of values between well-known discrete points are called ________.	Interpolation	extrapolation	reduction	expansion			Interpolation
______ is the process of finding the most appropriate estimate for missing data.							
For making the most probable estimate the changes in the series are must be ______ within a period.	uniform	Normal	Exponentially	periodic			uniform
For making the most probable estimate the frequency distribution must be ______.	Normal	uniform	periodic	Exponentially			Normal
Lagrange's interpolation formula can be used when the values of independent variable x are ____	equally – spaced	unequally – spaced	both equally andunequally – spaced	positive			both equally andunequally – spaced
To find the unknown value of x for some y, which lies at the unequal intervals we use ------------------- formula.	Newton's forward	Newton's backward	Newtons divided difference	inverse interpolation			Newtons divided difference
If the values of the variable y are given, then the method of finding the unknown variable x is called ----------------	Newton's forward	Newton's backward	interpolation	inverse interpolation			inverse interpolation
In Newton's backward difference formula, the value of n is calculated by -------.	n = (x–x_n) / h	n = (x_n–x) / h	$n = (x–x_0) / h$	n = (x_0–x) / h			n = (x–x_n) / h
In Newton's forward difference formula, the value of n is calculated by -------.	n = (x–x_n) / h	n = (x_n–x) / h	$n = (x–x_0) / h$	n = (x_0–x) / h			$n = (x–x_0) / h$
In the forward difference table y_0 is called ___________ element.	leading	ending	middle	positive			leading
In the forward difference table forward symbol ((y_0)), forward symbol(^2(y_0)) , ... are called ___________ difference.	leading	ending	middle	positive			leading
The difference of first forward difference is called ________.	divided difference	2nd forward difference	3rd forward difference	4th forward difference			2nd forward difference
_________ Formula can be used for interpolating the value of f(x) near the end of the tabular values.	Newton's forward	Newton's backward	Lagrange	stirling			Newton's backward
Gregory Newton forward interpolation formula is also called as Gregory Newton forward_____________ formula.	Elimination	iteration	difference	distance			difference
Gregory Newton backward interpolation formula is also called as Gregory Newton backward_____________ formula	Elimination	iteration	difference	distance			difference
Gregory Newton backward interpolation formula is also called as Gregory Newton backward _____________ formula .	Elimination	iteration	difference	distance			difference

The divided differences are _________ in their arguments.	constant	symmetrical	varies	singular	symmetrical
In Gregory Newton forward interpolation formula 1st two terms of this series give the result for the _________ interpolation.	Ordinary linear	ordinary differential	parabolic	central	Ordinary linear
Gregory Newton forward interpolation formula 1st three terms of this series give the result for the _________ interpolation.	Ordinary linear	ordinary differential	parabolic	central	parabolic
Gregory Newton forward interpolation formula is mainly used for interpolating the values of y near the _____________ of the set of tabular values.	beginning	end	centre	side	beginning
Gregory Newton backward interpolation formula is mainly used for interpolating the values of y near the _______of the set of tabular values.	beginning	end	centre	side	end
From the definition of divided difference (u-u_0)/(x-x_0) we have __________ =	(y,y_0)	(x,y)	(x_0, y_0)	(x,x_0)	(x_0, y_0)
If f(x) =0, then the equation is called __________	Homogenous	non-homogenous	first order	second order	Homogenous
If the values $x_0 = 0$, $y_0 = 0$ and h = 1 are given for Newton's forward method, then the value of x is ------------.	0	1	n	X	n
The n th order difference of a polynomial of n th degree is ------------.	constant	zero	polynomial in first degree	polynomial in n-1 degree	constant
What will be the first difference of a polynomial of degree four?	Polynomial of degree one	Polynomial of degree two	Polynomial of degree three	Polynomial of degree four	Polynomial of degree three
A function which satisfies the difference equation is a ________of the difference equation.	Solution	general solution	complementary solution	particular solution	Solution
The degree of the difference equation is ________	The highest powers of y's	The difference between the arguments of y	The difference between the constant	The highest value of x	The highest powers of y's
The degree of the difference equation is ________	2	0	1	3	1
The difference between the highest and lowest subscripts of y are called ______ of the difference equation	degree	order	power	value	order
E-1=	backward difference operator	forward difference operator	μ	δ	forwarddifference operator
Which of the following is the central difference operator?	E		μ	δ	δ
1+(forward difference operator)=	backward difference symbol	E	μ	δ	E
μ is called the ______ operator	Central	average	backward	displacement	average
The other name of shifting operator is ______ operator	Central	average	backward	displacement	displacement
The difference of constant functions are___________	0	1	2	3	0

The nth order divided difference of x_n will be a polynomial of degree ________.	0	1	2	3	2
The operator forward symbol is __________	homogenous	heterogeneous	linear	a variable	linear

Unit - IV

Initial Value Problem for ordinary differential Equation

Method - 1

Taylor Series:

The taylor series formula is

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + \cdots$$

1. Use Taylor series method to find $y(0.1)$ and $y(0.2)$. Given that $\frac{dy}{dx} = 3e^x + 2y$; $y(0) = 0$;

Soln: Given $\frac{dy}{dx} = y' = 3e^x + 2y$; $y(0) = 0$;

The taylor series formula is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + \frac{(x-x_0)^4}{}$$

x	0	x_0
y	0	y_0
$y' = 3e^x + 2y$	3	y_0'
$y'' = 3e^x + 2y'$	9	y_0''
$y''' = 3e^x + 2y''$	21	y_0'''
$y^{iv} = 3e^x + 2y'''$	45	y_0^{iv}

$$y = 0 + (x-0)\cdot 3/1 + (x-0)^2 \cdot 9/2 + (x-0)^3 \frac{21}{6} + (x-0)^4 \cdot \frac{45}{24};$$

$$y = 3x + 9/2\, x^2 + 7/2\, x^3 + 15/8\, x^4.$$

$y(0.1) = 0.3487.$

$y(0.2) = 0.8110.$

2. use taylor series method, solve $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ at $x = 0.1, 0.2, 0.3$.

Soln:

The taylor series formula is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + (x-x_0)^4\frac{y_0^{iv}}{4!}$$

$y' = x^2 - y$; & $y(0) = 1$

x	0	x_0
y	1	y_0
$y' = x^2 - y$	-1	y_0'
$y'' = 2x - y'$	1	y_0''
$y''' = 2 - y''$	1	y_0'''
$y^{iv} = -y'''$	-1	y_0^{iv}

$$y = 1 + (x-0)\left(\frac{-1}{1!}\right) + (x-0)^2\frac{1}{2!} + (x-0)^3\frac{\times 1}{3!} + (x-0)^4\left(\frac{-1}{4!}\right)$$

$$y = 1 - x + x^2/2 + x^3/6 - \frac{x^4}{24}.$$

$y(0.1) =$ ~~0.9102~~. 0.9052

$= \cancel{7/6\, x^4 + 4/3\, x^3}$

$$y = 7/6\, x^4 + 4/3\, x^3 + x^2 + x + 1$$

$y(0.1) = 1.1115$

$y(0.2) = 1.2525$

4. Obtain y by <u>taylor series</u> method given that $y' = xy + 1$; $y(0) = 1$; for $x = 0.1$; $x = 0.2$; Correct to four decimal places.

Soln: The formula is,

$$y = y_0 + (x - x_0)\frac{y_0'}{1!} + (x - x_0)^2 \frac{y_0''}{2!} + (x - x_0)^3 \frac{y_0'''}{3!} + (x - x_0)^4 \frac{y_0^{iv}}{4!} + \cdots$$

x	0	x_0
y	1	y_0
$y' = xy + 1$	1	y_0'
$y'' = y + xy'$	1	y_0''
$y''' = y' + y' + xy''$	2	y_0'''
$y^{iv} = y'' + y'' + y'' + xy'''$	3	y_0^{iv}

$$y = 1 + (x-0)\frac{1}{1!} + (x-0)^2\frac{1}{2} + (x-0)^3\frac{2}{6} +$$

$(x-0)^4\ 3$

$$y = 1+x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{8}x^4.$$

$y(0.1) = 1.1053$

$y(0.2) = 1.2229.$

5. G.T $y''+xy'+y=0$; $y(0)=1$; $y'(0)=0$ obtain the value of y for $x=0.1$ & $x=0.2$; 0.3 by taylor series method.

Soln :

The taylor series formula is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + (x-x_0)^4\frac{y_0^{iv}}{4!} + \cdots$$

x	0	x_0
y	1	y_0
y'	0	y_0'
$y''=-xy'-y.$	-1	y_0''
$y'''=-xy''-y'-y'$	0	y_0'''
$y^{iv}=-xy'''-y''-y''-y''$	+3	y_0^{iv}

$$y = 1 + (x-0)\frac{0}{1!} + (x-0)^2\frac{1}{2} + (x-0)^3\frac{0}{6} + (x-0)^4\frac{+3}{24}$$

$$y = 1 - x^2/2 + x^4/8$$

$y(0.1) = 0.9950$

$y(0.2) = 0.9802$

$y(0.3) = 0.9560$

Method-II: Euler's method:

Consider $\frac{dy}{dx} = f(x,y)$

The Euler's formula is,

$$y_{n+1} = y_n + h f(x_n, y_n) \text{ (or)}$$

$$y_{n+1} = y_n + h y_n'$$

1. Solve $y' = \frac{y-x}{y+x}$, $y(0) = 1$ at $x = 0.1$ by taking $h = 0.02$; by using Euler's method.

Soln:

$$y' = \frac{y-x}{y+x} ; y(0) = 1$$

The Euler's formula is,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

(or)

$$y_{n+1} = y_n + h \cdot y_n'$$

x	0	0.02	0.04	0.06	0.08	0.
y	1	1.02	1.0392	1.0577	1.0756	1.
$y' = \frac{y-x}{y+x}$	1	0.9615	0.9259	0.8926	0.8615	0

$n=0$;

$y_1 = y_0 + hy_0' = 1 + 0.02 \times 1 = 1.02$.

$n=1$;

$y_2 = y_1 + hy_1' = 1.02 + 0.02 \times 0.9615 = 1.0392$.

$n=2$;

$y_3 = y_2 + h \cdot y_2' = 1.0892 + 0.02 \times 0.9259 = 1.0577$.

$n=3$;

$y_4 = y_3 + hy_3' = ~~0.9~~ \cdot 1.0577 + 0.02 \times 0.8926$

$y_4 = 1.0756$

$n=4$;

$y_5 = y_4 + hy_4' = 1.0756 + 0.02 \times 0.8615$

$y_5 = 1.0928$.

~~$n=5$;~~

~~$y_6 = y_5 + hy_5' = 1.0928 + 0.$~~

2\. using Euler's method to find $y(0.4)$ G.

$\frac{dy}{dx} = x+y$, $y(0) = 1$. taking $h = 0.2$.

Soln:

Given $\frac{dy}{dx} = x+y$, $y(0)=1$.

The Euler's formula is $y_{n+1} = y_n + h y_n'$

x	0	0.2	0.4
y	1	1.2	1.48
$y'=x+y$	1	1.4	1.88

$n=0 \Rightarrow y_1 = y_0 + h y_0' = 1 + (0.2\times 1) = 1.2$

$n=1 \Rightarrow y_2 = y_1 + h y_1' = 1.2 + (0.2\times 1.4) = 1.48$

2. Using Euler's method find the solution of the initial value problem (IVP) $\frac{dy}{dx} = \log_e(x+y)$ $y(0)=2$ at $x=0.6$ by assuming $h=0.2$.

Soln:

Given $y' = \log_{10}(x+y)$; $y(0)=2$.

The Euler's formula is $y_{n+1} = y_n + h y_n'$

x	0	0.2	0.4	0.6
y	2	2.0602	2.1810	2.2117
$y' = \log_{10}(x+y)$	0.3010	0.3541	0.4033	0.4490

$n=0 \Rightarrow y_1 = y_0 + h y_0' = 2 + (0.2\times 0.3010) = 2.0602$.

$n=1 \Rightarrow y_2 = y_1 + h y_1' = 2.0602 + (0.2\times 0.3541) = 2.1810$.

$n=2 \Rightarrow y_3 = y_2 + h y_2' = 2.1810 + (0.2\times 0.4033) = 2.2117$.

3. Using Euler's method, find $y(4.1)$ & $y(4.2)$ if $5x\frac{dy}{dx} + y^2 - 2 = 0$; $y(4)=1$

Soln:

Given $5\frac{dy}{dx} + y^2 - 2 = 0$; $y(4) = 1$

$$\frac{dy}{dx} = \frac{-y^2+2}{5x}.$$

The Euler's formula is $y_{n+1} = y_n + h y_n'$

x	4	4.1	4.2
y	1	1.0050	1.0098
$y' = \frac{-y^2+2}{5x}$	0.05	0.0483	0.0467.

$n=0 \Rightarrow y_1 = y_0 + h y_0' = 1 + 0.1(0.05)$

$= 1.0050.$

$n=1 \Rightarrow y_2 = y_1 + h y_1' = 1.005 + 0.1(0.0483)$

$= 1.0098$ //.

5. Find $y(0.2)$ for $y' = y + e^x$, $y(0) = 0$ by Euler's method. Take $h = 0.1$

Soln:

Given $y' = y + e^x$, $y(0) = 0$

The Euler's formula is $y_{n+1} = y_n + hy$

x	0	0.1	0.2
y	0	0.1	0.2205

$n=0 \Rightarrow$

$y_1 = y_0 + hy_0' = 0 + 0.1(1) = 0.1$

$n=1 \Rightarrow$

$y_2 = y_1 + hy_1' = 0.1 + 0.1 \times (1.2052) = 0.2205.$

Fourth order Range - kutta method.

Consider, $g(x, y, y') = 0$.

$y' = f(x, y)$

$k_1 = hf(x, y)$

$k_2 = hf(x + h/2, y + k_1/2)$

$k_3 = hf(x + h/2, y + k_2/2)$

$k_4 = hf(x+h, y+k_3)$

$y = y_0 + 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

7. using Runge - kutta method of order 4; find y value when $x = 1.2$ in steps of 0.1 given that $y' = x^2 + y^2$, $y(1) = 1.5$.

Soln:

The Runge - kutta formula is

$K_1 = h \cdot f(x, y)$

$K_2 = h \cdot f(x + h/2, y + k_1/2)$

$K_3 = h \cdot f(x + h/2, y + k_2/2)$

$k_4 = h \cdot f(x+h, y+k_3)$

Given $y' = x^2+y^2$

here, $f(x,y) = x^2+y^2$; $h = 0.1$

x	1	1.1	1.2
y	1.5	y_1 1.8955	y_2 2.5044

To find y_1

$x = 1$; $y = 1.5$.

$k_1 = h\, f(x,y) = 0.1 \times f(1, 1.5)$.

$= 0.1 \times 3.25 = 0.325$.

$k_2 = h \cdot f(x+h/2, y+k_1/2) = 0.1 \times f(1.05, 1.662)$

$= 0.1 \times 3.8664 = 0.3866$.

$k_3 = h \cdot f(x+h/2, y+k_2/2) = 0.1\, f(1.05, 1.6933)$

$= 0.1 \times 3.9698 = 0.3970$.

$k_4 = h \cdot f(x+h, y+k_3) = 0.1 \times f(1.1, 1.8970)$

$= 0.4809$.

$y_1 = y_0 + \frac{1}{6}\left[k_1+2k_2+2k_3+k_4\right]$

$= 1.5 + \frac{1}{6}\left[0.325 + 2 \times 0.3866 + 2 \times 0.3970 + 0.4809\right]$

$y_1 = 1.8955$

$f(x,y) = x^2 + y^2$

$k_1 = h \cdot f(x,y) = 0.1 \times f(1.2, 1.8955)$

$= 0.1 \times 4.8029 = 0.4803$

$k_2 = h\, f(x + h/2,\; y + k_1/2)$ ~~$= 0.1\times$~~

$= 0.1\, f(1.250,\; 2.1357)$

$= 0.1 \times 5.8837 = 0.5884$

$k_3 = h\, f(x + h/2,\; y + k_2/2)$

$= 0.1 \times f(1.150,\; 2.1897)$

$= 0.1 \times 6.1173 = 0.6117$

$k_4 = h\, f(x + h,\; y + k_3)$

$= 0.1 \times f(1.2,\; 2.5072)$

$= 0.7726$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1.8955 + \frac{1}{6}\left[0.4803 + 2 \times 0.5884 + 2 \times 0.6117 + 0.7726\right]$

2. Find $y(0.7)$ & $y(0.8)$ given that $y' = y - x^2$ by using RK method of $y(0.6) = 1.7379$ 4th order.

Soln:

$k_1 = h \cdot f(x, y)$

$k_2 = h \cdot f(x + h/2, y + k_1/2)$

$k_3 = h \cdot f(x + h/2, y + k_2/2)$

$k_4 = h \cdot f(x + h, y + k_3)$

Given $y' = y - x^2$.

Here $f(x, y) = y - x^2$; $h = 0.1$

	x_0	x_1	x_2
x	0.6	0.7	0.8
y	1.7379	1.8763	2.0145.

To find y_1 :

$x = 0.6$; $y = 1.7379$.

$k_1 = h \cdot f(x, y) = 0.1 \times f(0.6, 1.7379)$

$= 0.1378$.

$k_2 = 0.1 \times f(0.6 + 0.1/2, 1.7379 + 0.1378/2)$

$K_2 = ~~0.0240~~ \; 0.1384$

$$K_3 = 0.1 \times f\left[0.6 + \frac{0.1}{2},\ 1.7379 + 0.1384/2\right]$$

$$= 0.1 \times f(0.65,\ 1.8071)$$

$$= 0.1385.$$

$$K_4 = 0.1 \times f(0.7,\ 1.8764)$$

$$= 0.1386.$$

$$y_1 = 1.7379 + 1/6\ (0.1378 + 0.1384 \times 2 + 0.1385 \times 2 + 0.1386)$$

$$= 1.8763$$

To find y_2.

$x = 0.7$; $y = 1.8763$.

$k_1 = 0.1 \times f(0.7,\ 1.8763) = 0.1386$

$k_2 = 0.1 \times f(0.75,\ 1.9456) = 0.1383$

$k_3 = 0.1 \times f(0.75,\ 1.9455) = 0.1383$.

$k_4 = 0.1 \times f(0.8,\ 2.0146) = 0.1375$

$$y_2 = y_1 + 1/6\ [k_1 + 2k_2 + 2k_3 + k_4).$$

$$= 1.8763 + 1/6\ (0.1386 + 2 \times 0.1383 + 2 \times 0.1383 + ~~0.1402~~\ 0.1375)$$

8. using R.K method to find $y(0.2)$, $y(0.4)$. Given to $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y(0)=1$

Soln:

$$y' = \frac{y^2 - x^2}{y^2+x^2}$$

Here, $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$; $h = 0.2$

x	0	0.2	0.4
y	1	1.1960	

To find y_1:

$x=0$; $y=1$

$k_1 = h \cdot f(x,y) = 0.2 \times f(0,1)$

$= 0.2$

$k_2 = 0.2 \times f(0.1, 1.1) = 0.1967$.

$k_3 = 0.2 \times f(0.1, 1.0984) = 0.1967$.

$k_4 = 0.2 \times f(0.2, 1.1967) = 0.1891$

$y_1 = 1 + \frac{1}{6}(0.2 + 2\times 0.1967 + 2 \times 0.1967 + 0.1891)$

$= 1.1960$

To find y_2:

$x = 0.2$; $y = 1.1960$

$k_1 = 0.2 \times f(0.2, 1.1960) = 0.1891$

$k_2 = 0.2 \times f(0.3, 1.2906) = 0.1795$

$k_3 = 0.2 \times f(0.3, 1.2842) = 0.1798$

$k_4 = 0.2 \times f(0.4, 1.3753) = 0.1688$

$y_2 = 1.1960 + \frac{1}{6}(0.1891 + 2 \times 0.1763 + 0.1793 \times 2 + 0.1688)$

$= 1.3753$

14/8/14. Using R.K method for solving simultaneous Equation:

consider,

$$\frac{dy}{dx} = f(x,y,z) ; \frac{dz}{dx} = g(x,y,z)$$

$f(x,y,z)$	$g(x,y,z)$
$k_1 = h \cdot f(x,y,z)$	$l_1 = h \cdot g(x,y,z)$
$k_2 = h \cdot f(x+h/2, y+k_1/2, z+l_1/2)$	$l_2 = h \cdot g(x+h/2, y+k_1/2, z+l_1/2)$
$k_3 = h \cdot f(x+h/2, y+k_2/2, z+l_2/2)$	$l_3 = h \cdot g(x+h/2, y+k_2/2, z+l_2/2)$
$k_4 = h \cdot f(x+h, y+k_3, z+l_3)$	$l_4 = h \cdot g(x+h, y+k_3, z+l_3)$

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4].$$

$$z_1 = z_0 + 1/6 [l_1 + 2l_2 + 2l_3 + l_4]$$

1. Solve for $y(0.1)$ and $z(0.1)$ from the simultaneous equation $\frac{dy}{dx} = 2y + z ; \frac{dz}{dx} = y - 3z$ $y(0) = 0 ; z(0) = 0.5$; using R.K method of order 4 !

Soln !

Given, $\frac{dy}{dx} = y - 3z ; g(x,y,z) = y - 3z$

x	0	0.1
y	0	0.0481
z	0.5	0.3726

$h = 0.1$

$f(x,y,z) = 2y + x$	$g(x,y,z) = y - 3z$.
$k_1 = h.\,f(x, y, z)$ $= 0.1 \times f(0, 0, 0.5)$ $k_1 = 0.05$.	$l_1 = 0.1 \times g(0, 0, 0.5)$ $l_1 = -0.15$.
$k_2 = h.\,f(x+h/2, y+k_1/2, z+l_1/2)$ $= 0.1 \times f(0.05, 0.025, 0.425)$ $k_2 = 0.0475$.	$l_2 = 0.1 \times g(0.05, 0.025, 0.4$ $l_2 = -0.125$.
$k_3 = h.\,f(x+h/2, y+k_2/2, z+l_2/2)$ $= 0.1 \times f(0.05, 0.0238, 0.4375)$ $k_3 = 0.0485$.	$l_3 = 0.1 \times g(0.05 \times 0.238 \times 0.4375)$ $l_3 = -0.1289$.
$k_4 = h.\,f(x+h, y+k_3, z+l_3)$ $= 0.1 \times f(0.1, 0.0485, 0.3711)$ $= 0.0468$	$l_4 = 0.1 \times g(0.1, 0.0485, 0.37$ $= -0.1065$.

$y_1 = y_0 + 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

$= 0 + 1/6 (0.05 + 2 \times 0.0475 + 2 \times 0.0485 + 0.0460)$

$= 0.0481$

$z_1 = 0.5 + 1/6 (-0.15 - 2 \times 0.125 - 2 \times 0.1289 - 0.1065)$

$= 0.3726$

R.K method for solving second order equation.

Consider, $\varphi(x, y, y', y'') = 0$ —— ①

take $y' = z$ —— ②

By using ② in ①, we get

$z' = g(x, y, z)$

1. Given $y'' + xy' + y = 0$; $y(0) = 1$; $y'(0) = 0$; find the value of $y(0.1)$ by using R.K method

Soln:

Given, $y'' + xy' + y = 0$ —— ①

Take $y' = z$;

$z' + xz + y = 0$.

$z' = -xz - y.$

x	0	0.1
y	1	0.9950
$z = y'$	0	-0.0995

$h = 0.1$

$f(x,y,z) = z$	$g(x,y,z) = -xz - y$
$k_1 = h \cdot f(x,y,z)$ $= 0.1 \times f(0,1,0)$ $k_1 = 0.$	$l_1 = 0.1 \times g(0,1,0)$ $= -0.1$
$k_2 = h \cdot f(x + h/2, y + k_1/2, z + l_1/2)$ $= 0.1 \times f(0.05, 1, -0.05)$ $= -0.005$	$l_2 = 0.1 \times g(0.05, 1, -0.05)$ $= -0.0998$
$k_3 = h \cdot f(x + h/2, y + k_2/2, z + l_2/2)$ $= 0.1 \times f(0.05, 0.9975, -0.0499)$ $= -0.005$	$l_3 = 0.1 \times g(0.05, 0.9975, -0.0499)$ $= -0.0995$
$k_4 = h \cdot f(x+h, y+k_3, z+l_3)$ $= 0.1 \times f(0.1, 0.9950, -0.0995)$ $= -0.0100$	$l_4 = 0.1 \times g(0.1, 0.9950, -0.0995)$ $= ~~0.1015~~ -0.0985$

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1 + 1/6 (0 - 2 \times 0.005 - 2 \times 0.005 - 0.01)$

$= 0.9950$ //.

$z_1 = 0 + 1/6 (-0.1 - 2 \times 0.0998 - 2 \times 0.0997 - 0.0985)$

$= -0.0995$ //.

2. Consider the 2^{nd} order initial value pbm $y'' - 2y' + 2y = e^{2t} \sin t$; $y(0) = -0.4$; $y'(0) = -0.6$ using 4th order R.K method find ~~y(0)~~ $y(0.2) = ?$

Soln:

Given $y'' - 2y' + 2y = e^{2x} \sin x$

Take $y' = z$.

$f(x, y, z) = z$.

$z' - 2z + 2y = e^{2x} \sin x$.

$z' = e^{2x} \sin x - 2y + 2z$.

$g(x, y, z) = e^{2x} \sin x - 2y + 2z$.

x	0	0.2
y	-0.4	
$z = y'$	-0.6	

$h = 0.2$

$f(x,y,z) = z$	$g(x,y,z) = e^{2x}\sin x - 2y + 2z$
$k_1 = h \cdot f(x,y,z)$ $= 0.2 \times f(0, -0.4, -0.6)$ $= -0.12$	$l_1 = 0.2 \times g(0, -0.4, -0.6)$ $l_1 = -0.08$
$k_2 = h \cdot f(x + h/2, y + k_1/2, z + l_1/2)$ $= 0.2 \times f(0.1, -0.46, -0.64)$ $= -0.1280$	$l_2 = 0.2 \times g(0.1, -0.46, -0.64)$ $= \sout{-0.0592}$ -0.0476
$k_3 = h \cdot f(x + h/2, y + k_2/2, z + l_2/2)$ $= 0.2 \times f(0.1, \sout{-0.4360}\ -0.4640, \sout{-0.6296}\ -0.6238)$ $= \sout{-0.1247}$ -0.1248	$l_3 = 0.2 \times g(0.1, -0.4640, -0.6286)$ $= \sout{-0.0518}$ -0.0395
$k_4 = h \cdot f(x + h, y + k_3, z + l_3)$ $= 0.2 \times f(0.2, \sout{-0.5247}\ -0.5248, \sout{-0.6511}\ -0.6395)$ $= \sout{-0.1302}$ -0.1279	$l_4 = 0.2 \times g(0.2, \sout{-0.5247}\ -0.5248, \sout{-0.6511}\ -0.6395)$ $= \sout{+0.0086}$ -0.0136

$y_1 = y_0 + 1/6 \left(k_1 + 2k_2 + 2k_3 + k_4\right)$

$= -0.47\ 1/6\ (-0.12 - 2\times0.1280 - 2\times \overset{0.1248}{\cancel{0.1277}} \overset{-0.1279}{\cancel{-0.1802}})$

$= \cancel{-0.5263} \quad -0.5256$ //

$z_1 = -0.6 + 1/6\ (-0.08 - 2\times \overset{0.0476}{\cancel{0.0572}} - 2\times \overset{0.0395}{\cancel{0.0511}} + -0.0134\cancel{0.0086})$

$= -0.6480$ //

$= -0.6401$ //

18/3/14. Milne's Predictor - corrector Method.

Consider $\frac{dy}{dx} = f(x, y)$

$P: \quad y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$

$C: \quad y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$

1. By using Milne's predictor - corrector formula to find $y(0.4)$ & $y(0.5)$; G.T $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$, $y(0) = 1$; $y(0.1) = 1.06$; $y(0.2) = 1.12$; $y(0.3) = 1.21$

Soln: The Milne's predictor - corrector formula is,

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right] \quad \text{—(1)}$$

$$C: y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right] \quad \text{—(2)}$$

x	0 x_0	0.1 x_1	0.2 x_2	0.3 x_3	0.4 x_4	$0.$[illegible] x_5
y	1 y_0	1.06 y_1	1.12 y_2	1.21 y_3	1.2771 y_4	1.2797
$y' = \frac{(1+x^2)y^2}{2}$	y'_0 0.5	y'_1 0.5674	y'_2 0.6523	y'_3 0.7979	0.9460	0.9978

Put $n=3$ in (1).

$$P: y_4 = y_0 + \frac{4h}{3}\left(2y'_1 - y'_2 + 2y'_3\right)$$

$$= 1 + \frac{4\times 0.1}{3}\left(2\times 0.5674 - 0.6523 + 2\times 0.7979\right)$$

$$P: y_4 = 1.2771.$$

put $n=3$ in eqn (2).

$$C: y_4 = y_2 + h/3\left(y'_2 + 4y'_3 + y'_4\right)$$

$$= 1.12 + \frac{0.1}{3}\left(0.6523 + 4\times 0.7979 + 0.9460\right]$$

$$C: y_4 = 1.2797.$$

put n = 4 in ①,

$$P: y_5 = y_1 + \frac{4h}{3}\left[2y_2' - y_3' + 2y_4'\right]$$

$$= 1.06 + \frac{4\times 0.1}{3}\left[2\times 0.6523 - 0.7979 + 2\times 0.9498\right]$$

$$P: y_5 = 1.3808.$$

put n = 4 in ②,

$$C: y_5 = y_3 + \frac{h}{3}\left(y_3' + 4y_4' + y_5'\right)$$

$$= 1.21 + \frac{0.1}{3}\left(0.7979 + 4\times 0.9498 + 1.1916\right).$$

$$y_5 = 1.4030.$$

② Given $y' = \frac{1}{x+y}$; $y(0) = 2$; $y(0.2) = 2.0933$; $y(0.4) = 2.1755$, $y(0.6) = 2.2493$. Find $y(0.8)$ by using Milne's method.

Soln:

The Milne's formula is,

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y_{n-2}' - y_{n-1}' + 2y_n'\right] \quad — ①$$

$$C: y_{n+1} = y_{n-1} + \frac{h}{3}\left[y_{n-1}' + 4y_n' + y_{n+1}'\right] \quad — ②$$

x	0	0.2	0.4	0.6	0.8
y	y_0 2	y_1 2.0933	y_2 2.1755	y_3 2.2493	2.3162
$y' = \frac{1}{x+y}$	y_0' 0.5	y_1' 0.4861	y_2' 0.3883	y_3' 0.3510	0.3209

put $n=3$ in ①

$$P: \; y_4 = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 2 + \frac{4\times 0.2}{3}\left[2\times 0.4861 - 0.3883 + 2\times 0.3510\right]$$

$$P: y_4 = 2.3162.$$

put $n=3$ in ②

$$C: \; y_4 = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 2.1755 + \frac{0.2}{3}\left[0.3883 + 4\times 0.3510 + 0.3209\right]$$

$$C: y_4 = 2.3164 \,//.$$

19/3/14.

3. Given $y' = xy + y^2$, $y(0) = 1$; $y(0.1) = 1.1169$; $y(0.2) = 1.2774$. using R.k method of 4th order, find $y(0.3)$. Continue the solution at $x = 0.4$ using Milne's method.

Soln:

x	0	0.1	0.2	0.3
y	1	1.1169	1.2774	1.5042.

Here, $h = 0.1$;

$y' = xy + y^2$

$f(x, y) = xy + y^2$

To find y_3;

$x = 0.2$; $y = 1.2774$.

$k_1 = h.f(x.y) = 0.1 \times f(0.2, 1.2774) = 0.1887$.

$k_2 = h \cdot f(x + h/2, y + \frac{k_1}{2}) = 0.1 \times f(0.25, 1.3718)$

$= 0.2225$.

$k_3 = h \cdot f(x + h/2, y + \frac{k_2}{2}) = 0.1 \times f(0.25, 1.3887)$

$= 0.2276$

$k_4 = h \cdot f(x + h, y + k_3) = 0.1 \times f(0.3, 1.5050) = 0.2717$.

$y_3 = y_2 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1.2774 + \frac{1}{6}[0.1887 + 2 \times 0.2225 + 2 \times 0.2276 + 0.2717]$

$= 1.5042$.

Milne's formula is,

$$P: \; y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$C: \; y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

x	x_0 0	x_1 0.1	x_2 0.2	x_3 0.3	x_4 0.4	
y	y_0 1	y_1 1.1169	y_2 1.2774	y_3 1.5042	1.8345	1.[illegible]
$y' = xy + y^2$	y'_0 1	y'_1 1.3592	y'_2 1.8872	y'_3 2.7139	4.0992	4.[illegible]

Put $n = 3$ in (1)

$$P: \; y_4 = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$= 1 + \frac{4 \times 0.1}{3}\left[2 \times 1.3592 - 1.8872 + 2 \times 2.7139\right]$$

$$= 1.8345.$$

Put $n = 3$ in (2)

$$C: \; y_4 = y_2 + \frac{0.1}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$= 1.2774 + \frac{0.1}{3}\left[1.8872 + 4 \times 2.7139 + 4.099\right]$$

$$= 1.8388$$

4. Given that $y'' + xy' + y = 0$, $y(0) = 1$; $y'(0) = 0$ obtain y for $x = 0.1, 0.2$ and 0.3 by Taylor series method and find the soln for $y(0.4)$ by Milne's method.

soln:

The Taylor series is,

$$y = y_0 + (x - x_0)\frac{y_0'}{1!} + (x - x_0)^2 \frac{y_0''}{2!} + (x - x_0)^3 \frac{y_0'''}{3!}$$

$$+ (x - x_0)^4 \frac{y_0^{iv}}{4!} + \cdots$$

$y'' + xy' + y = 0.$

$y'' = -xy' - y.$

x	0	x_0
y	1	y_0
y'	0	y_0'
$y'' = -xy' - y$	-1	y_0''
$y''' = -xy'' - y' - y'$	0	y_0'''
$y^{iv} = -xy''' - y''' - y'' - y''$	3	y_0^{iv}

$$y = 1 + (x - 0)\,0/1 + (x - 0)^2 \times -1/2 + (x - 0)^3\,0/6 +$$

$$y = 1 - \frac{x^2}{2} + \frac{x^4}{8}$$

$$y' = -\frac{2x}{2} + \frac{4x^3}{8} \Rightarrow y' = -x + \frac{x^3}{2}$$

$y(0.1) = 0.9950$

$y(0.2) = 0.9802$

$y(0.3) = 0.9560$

The Milne's formula is,

$$P: \quad y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

$$C: \quad y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_{n} + y'_{n+1}\right]$$

x	x_0 0	x_1 0.1	x_2 0.2	x_3 0.3	x_4 0.4	
y	y_0 1	y_1 0.9950	y_2 0.9802	y_3 0.9560	0.9232	0.9232
$y' = -x + \frac{x^3}{2}$	y'_0 0	y'_1 −0.0995	y'_2 −0.1960	y'_3 −0.2865	−0.3680	−0.3685

put $n = 3$;

$$P: \quad y_4 = y_0 + \frac{4 \times 0.1}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$= 1 + \frac{0.4}{3}\left[2 \times (-0.0995) + 0.1960 + 2 \times (-0.2\ldots\right.$$

$$= 0.9232.$$

C: put $n = 3$;

$$C: \quad y_4 = y_2 + \frac{h}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$= 0.9802 + \frac{0.1}{3}\left[-0.1960 - 4\times 0.2865 \; \overset{-0.3680}{\cancel{+0.9232}}\right]$$

$$C: y_4 = 0.9232$$

Adam's Bashforth predictor - corrector formula:

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55 y_n' - 59 y_{n-1}' + 37 y_{n-2}' - 9 y_{n-3}'\right]$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left[19 y_n' - 5 y_{n-1}' + y_{n-2}' + 9 y_{n+1}'\right].$$

1. Using Adam's method find ~~y(1.4)~~ y(1.4)
given $y' = x^2(1+y)$, $y(1) = 1$; $y(1.1) = 1.233$;
$y(1.2) = 1.548$ & $y(1.3) = 1.979$.

Soln:

The Adam's formula is,

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55 y_n' - 59 y_{n-1}' + 37 y_{n-2}' - 9 y_{n-3}'\right]$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left[19 y_n' - 5 y_{n-1}' + y_{n-2}' + 9 y_{n+1}'\right]$$

x	x_0 1	x_1 1.1	x_2 1.2	x_3 1.3	x_4 1.4
y	y_0 1	y_1 1.233	y_2 1.548	y_3 1.979	y_4 2.5723 2.5749
$y' = x^2(1+y)$	y_0' 2	2.7019	3.6691	5.0345	y_4' 7.0017 7.0068

put n=3;

$$P: y_4 = y_3 + \frac{0.1}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 1.979 + \frac{0.1}{24}\left[55\times 5.0345 - 59\times 3.6691 + 37\times 2.7019 - 9\times 2\right]$$

$$P: y_4 = 2.5723.$$

put n=3 in ①

$$C: y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$$

$$= 1.979 + \frac{0.1}{24}\left[19\times 5.0345 - 5\times 3.6691 + 2.70 + 9\times 7.0017\right]$$

$$C: y_4 = 2.5749.$$

2. Use Adam's method to find $y(2)$ if $y' = \frac{x+y}{2}$, $y(0) = 2$; $y(0.5) = 2.636$; $y(1) = $ and $y(1.5) = 4.968$.

Soln:

The Adam's formula is,

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' -\right.$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left[19y_n' - 5y_{n-1}' + y_{n-2}' + 9y_{n+1}'\right]$$

x	x_0 0	x_1 0.5	x_2 1	x_3 1.5	x_4 2	
y	y_0 2	y_1 2.636	y_2 3.595	y_3 4.968	6.8708	6.8731
$y' = \frac{x+y}{2}$	y_0' 1	y_1' 1.5680	y_2' 2.2975	y_3' 3.2340	4.4354	4.4366

put n = 3 in ①

$$P: \ y_4 = y_3 + \frac{0.5}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 4.968 + \frac{0.5}{24}\left[55 \times 3.2340 - 59 \times 2.2975 + 37 \times 1.5680 - 9 \times 1\right]$$

$$= 6.8708$$

$$C: \ y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$$

$$= 4.968 + \frac{0.5}{24}\left[19 \times 3.2340 - 5 \times 2.2975 + 1.5680 + 9 \times 4.4354\right]$$

$$= 6.8731.$$

21/3/14.

3. Using Adam's method find y(0.4) given $\frac{dy}{dx} = xy + y^2$, y(0) = 1; y(0.1) = 1.1169; y(0.2) = 1.2774; and y(0.3) = 1.5041

Soln: The Adam's formula is;

$$P: \ y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

$C: y_{n+1} = y_n + \frac{h}{24}\left[19y_n' - 5y_{n-1}' + y_{n-2}' + 9y_{n+1}'\right]$

	x_0	x_1	x_2	x_3	x_4
x	0	0.1	0.2	0.3	0.4
y	y_0 = 1	y_1 = 1.1169	y_2 = 1.2774	y_3 = 1.5041	1.8841
$\frac{dy}{dx} = xy + y^2$	y_0' = 1	y_1' = 1.3592	y_2' = 1.8872	y_3' = 2.7185	4.0976

put n = 3 in

$P: y_4 = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$

$= 1.5041 + \frac{0.1}{24}\left[55 \times 2.7185 - 59 \times 1.8872 + 37 \times 1.3592 - 9 \times 1\right]$

$P: y_4 = 1.8841$

put n = 3 in

$C: y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$

$= 1.5041 + \frac{0.1}{24}\left[19 \times 2.7185 - 5 \times 1.8872 + 1.3592 + 9 \times 4.0976\right]$

$= 1.8889.$

unit-IV

Numerical differentiation and Integration

Questions	opt1	opt2	opt3	opt4	opt5	opt6	Answer
_________ Formula can be used for interpolating the value of f(x) near the end of the tabular values.	Newton's forward	Newton's backward	Lagrange	stirling			Newton's backward
_________ Formula can be used for interpolating the value of f(x) near the beginning of the tabular values.	Newton's forward	Newton's backward	Lagrange	stirling			Newton's forward
In Numerical integration, the length of all intervals is in ----------- distances.	Greater than the other	less than the other	equal	not equal			equal
When the function is given in the form of table values instead of giving analytical expression we use ___________.	numerical differentiation	numerical elimination	approximation	addition			numerical differentiation
_________ is the process of computing the value of the definite integral from the set of numerical values of the integrand.	numerical differentiation	numerical integration	Simpsons rule	Trapezoidal rule			numerical integration
Numerical integration is the process of computing the value of a ________ from a set of numerical values of the integrand.	indefinite integral	definite integral	expression	equation			definite integral
Numerical evaluation of a definite integral is called --------	integration	differentiation	interpolation	triangularisation			integration
What is the value of h if a=0,b=2 and n=2.	1	2	3	4			1
Integral (f(x) dx)=(h/2) [Sum of the first and last ordinates + 2(sum of the remaining ordinates)] is called _____	Constant rule	Simpsons rule	Trapezoidal rule	Rombergs rule			Trapezoidal rule
If the given integral is approximated by the sum of 'n' trapezoids, then the rule is called as ----------------.	Newton's method	Trapezoidal rule	simpson's rule	none			Trapezoidal rule
What is the formula for finding the length interval h in trapezoidal tule?	h=(b-a)/n	h=(b/a)/n	h=(b*a)/n	h=(b+a)/n			h=(b-a)/n

The accuracy of the result using the Trapezoidal rule can be improved by --------	Increasing the interval h	Decreasing the length of the interval h	Increasing the number of	altering the given function	Decreasing the length of the interval h
The order of error in Trapezoidal rule is -------------.	h	h^2	h^3	h^4	h^2
Simpson's rule is exact for a ----------------- even though it was derived for a Quadratic.	cubic	less than cubic	linear	quadratic	linear
The order of error in Simpson's rule is ----------	h	h^2	h^3	h^4	h^4
For what type of functions, Simpsons rule and direct integration will give the same result?	parabola	hyperbola	ellipse	cardiod	parabola
Simpson's rule gives exact result if the entire curve y=f(x) itself is a ___________.	parabola	hyperbola	ellipse	cardiod	parabola
To apply Simpsons one third rule the number of intervals must be_________.	odd	even	equally spaced	unequal	even
The end point coordinates y_0 and y_n are included in the Simpsons 1/3 rule, so it is called _________ formula.	Newton's	open	closed	Gauss	closed
Simpson's one-third rule on numerical integration is called a --------- formula.	closed	open	semi closed	semi opened	closed
The order of error in Simpson's formula is ________.	1	2	3	4	4
In two point Gaussian quadrature Formula n =	1	2	3	4	2
In Simpsons $1/3^{rd}$ rule, the number of ordinates must be _________.	odd	even	0	3	odd
In three point Gaussian quadrature Formula n = ________.	1	2	3	4	3
Two point Gaussian quadrature Formula requires only _______ functional evaluations and gives a good estimate of the value of the integral.	1	2	3	4	2
_________ formula is based on the concept that the accuracy of numerical integration can be improved by choosing the sampling wisely , rather than on the basis	Newtons	elimination	Gauss quadrature	hermite	Gauss quadrature

Gauss Quadrature formula is also called as _________.	Newton's	Gauss-Legendre	Gauss-seidal	Gauss-Jordan	Gauss-Legendre
The 2 point Gauss-quadrature is exact for the polynomial up to degree _________.	1	2	3	4	3
The 3 point Gauss-quadrature is exact for the polynomial up to degree _________.	1	5	3	4	5
Integrating f(x)=5x^4 in the interval [-1,1] using Gaussion two point formula gives______.	1/2	9/5	10/9	5/9	10/9
The modified Eulers method is based on the ___________ of points	sum	multiplication	average	subratction	average
__________ prior values are required to predict the next value in Milne's method	1	2	3	4	4
__________ prior values are required to predict the next value in Adams method	1	2	3	4	3
The Eulers method is used only when the slope at point ____________ in computing is y(n+1)	(x(n), y)	(x, y(n))	(x(n), y(n))	(0, 0)	(x(n), y(n))
The Runge Kutta method agrees with Taylor series solution upto the ________ terms	h^2	h^3	h^4	h^r	h^r
Runge Kutta method agree with ________ solution upto the terms h^4	Taylor Series	Eulers	Milnes	Adams	Taylor Series
________ method is better than Taylor's series method	Runge Kutta	Milnes	Adams	Eulers	Runge Kutta
Taylors series method belongs to ____________ method	Single step	multi step	step by step	limination	Single step
If all the n conditions are specified at the initial point only then it is called a ______ problem	Initial value	final value	boundary value	semi defined	Initial value
The problem dy/dx = f(x,y) with the initial condition y(x(0)) = y(0)is ___________problem	initial value	final value	boundary value	multistep	initial value

The solution of an ODE means finding an explicit expression for y, in terms of a ______ number of elementary functions of y	finite	infinite	positive	negative	finite
The solution of an ODE is known as _____________ solution	infinite	open-form	closed-form	negative form	closed-form
The differential equation of the 2^{nd} order can be solved by reducing it to a ____ differential equation	lower order	higher-order	partial	simultaneous	lower order
The Eulers method is used only when the slope at point (x(n), y(n)) in computing is ____________	y(n+1)	y(n-1)	(dy/dx)(n+1)	(dy/dx)(n-1)	y(n+1)
The Eulers method is used only when the slope at point _____________ in computing is y(n+1)	(x(n),y)	(x, y(n))	(x(n), y(n))	(0, 0)	(x(n), y(n))
The modified Eulers method is a ____________ method of predictor-corrector type	Self-correcting	Self-starting	Self-evaluating	Self-predicting	Self-starting
The modified Eulers method has greater accuracy than ________ method	Taylor's	Picard's	Euler's	Adam's	Taylor's
The formula y(n+1) = y(n) + hf(x(n), y(n)) is ________formula	Euler's	modified Euler's	Picard's	Taylor's	Euler's
Modified Eulers method is the Runge-kutta method of________ order	1^{st}	2^{nd}	3^{rd}	4^{th}	2^{nd}
Modified Eulers method is same as the ________ method of 2^{nd} order	Eulers	Taylors	Picards	Runge Kutta	Runge Kutta
The process used in Eulers method is very slow and to obtain reasonable accuracy we need to take a ________ value of h	Smaller	Larger	negative	Positive	Smaller
The process used in Eulers method is very slow and to obtain reasonable accuracy we need to take a smaller value of ________	h	h^2	h^3	h^4	h
The ________ formula is given by y(i+1) = y(i) +hf (x(i), y(i))	Taylors	predictor	Corrector	Eulers	Eulers
The predictor formula and ________ formula are one and the same	Taylors	Eulers	Modified Eulers	Eulers	Eulers
The ________ formula is given by y(i+1) = y(i) + h/2[f(x(i), y(i)) + f(x(i+1), y(i+1))], i = 1,2,3.....	Taylors	predictor	Corrector	Picards	Corrector
The ________ formula is used to predict the value y(i+1) of y at x(i+1)	Predictor	Corrector	Corrector	Picards	Predictor

The ________ formula is used to improve the value of y(i+1)	Predictor	Corrector	Taylors	Picards	Corrector
In predictor corrector methods, ____ prior values of y are needed to evaluate the value of y at x(i+1)	1	2	3	4	4
In ________ methods, 4 prior values of y are needed to evaluate the value of y at x(i+1)	Taylor’s	predictor	Predictor-corrector	Euler’s	Predictor-corrector
In predictor corrector methods 4 prior values of ________ are needed to evaluate of values of are needed to evaluate of value of y at x(i+1)	y	y^2	y^3	y^4	y

UNIT - V

BOUNDARY VALUE PROBLEM IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATION.

Finite difference Method :

Replace x by x_k

y by y_k

y' by $\frac{y_{k+1} - y_k}{h}$

y'' by $\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2}$

where, $h = \frac{b-a}{n}$

1. Solve $y'' = x+y$ with the boundary condition $y(0) = y(1) = 0$.

Soln :

x	0	0.25	0.5	0.75	1
y	0	-0.0349	-0.0564	-0.05	0

$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$.

$y'' = x+y$.

$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} = x_k + y_k$.

$$y_{k-1} - 2y_k + y_{k+1} = h^2 x_k + h^2 y_k$$

$$y_{k-1} - 2y_k + y_{k+1} - h^2 y_k = h^2 x_k$$

$$y_{k-1} + y_k(-2-h^2) + y_{k+1} = h^2 x_k$$

$$y_{k-1} - 2.0625\, y_k + y_{k+1} = 0.0625\, x_k$$

$k=1$;

$$y_0 - 2.0625\, y_1 + y_2 = 0.0625\, x_1$$

$$-2.0625 y_1 + y_2 = 0.0156 \quad \text{---} (1)$$

$k=2$;

$$y_1 - 2.0625 y_2 + y_3 = 0.0625 x_2$$

$$y_1 - 2.0625 y_2 + y_3 = 0.0313 \quad \text{---} (2)$$

$k=3$;

$$y_2 - 2.0625 y_3 + y_4 = 0.0625 x_3$$

$$y_2 - 2.0625 y_3 = 0.0469 \quad \text{---} (3)$$

solve (1), (2) & (3)

$$y_1 = -0.0349;\quad y_2 = -0.0564;\quad y_3 = -0.0501;$$

2. using a finite difference method compute

24/3/14. $y(0.5)$. Given $y'' - 64y + 10 = 0$; $y(0) = y(1) = 0$.

Sub dividing the interval into i) 4 equal parts.

ii) 2 equal parts.

soln:

Given $y'' - 64y + 10 = 0$

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} - 64y_k + 10 = 0$$

$$\frac{y_{k-1} - 2y_k + y_{k+1} - 64y_k h^2 + 10h^2}{h^2} = 0$$

$$y_{k-1} + y_k(-2 - 64h^2) + y_{k+1} = -10h^2 \quad \text{—— (1)}$$

i) subdiving into 4parts.

$$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

	x_0	x_1	x_2	x_3	x_4
x	0	0.25	0.5	0.75	1
	y_0	y_1	y_2	y_3	y_4
y	0	0.1287	0.1471	0.1287	0

for $h = 0.25$, (1) becomes,

$$y_{k-1} - 6y_k + y_{k+1} = -0.625 \quad \text{—— (2)}$$

put $k = 1$.

$$y_0 - 6y_1 + y_2 = -0.625$$

$$-6y_1 + y_2 = -0.625 \quad \text{—— (3)}$$

put $k=2$;

$$y_1 - 6y_2 + y_3 = -0.625 \quad \text{——} (4)$$

put $k=3$;

$$y_2 - 6y_3 + y_4 = -0.625$$

$$y_2 - 6y_3 = -0.625 \quad \text{——} (5)$$

Solving by (3), (4) & (5)

$y_1 = 0.1287$; $\boxed{y_2 = 0.1471}$; $y_3 = 0.1287$.

ii) Sub dividing into 2 parts :

$$h = \frac{b-a}{n} = \frac{1-0}{2} = 0.5.$$

	x_1	x_2	x_3
x	0	0.5	1
	y_0	y_1	y_2
y	0	0.1389.	0.

for $h = 0.5$. eqn (1) becomes

$$y_{k-1} - 18y_k + y_{k+1} = -2.5 \quad \text{——} (1)$$

$k=1$.

$$y_0 - 18y_1 + y_2 = -2.5.$$

$$-18y_1 = -2.5$$

$$\boxed{y_1 = 0.1389.}$$

3. Solve by finite difference method, the BVP $y'' - y = 0$ where $y(0)=0, y(1)=1$; take $\Delta x = 0.25$.

Soln:

Given $y'' - y = 0$

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} - y_k = 0.$$

$$\frac{y_{k-1} - 2y_k + y_{k+1} - y_k h^2}{h^2} = 0.$$

$$y_{k-1} + y_k(-2-h^2) + y_{k+1} = 0 \quad \text{——} (1)$$

for $h = 0.25$ eqn (1) becomes,

$$y_{k-1} - 2.0625\, y_k + y_{k+1} = 0. \quad \text{——} (2)$$

put

x	x_0 0	x_1 0.25	x_2 0.5	x_3 0.75	x_4 1
y	y_0 0	0.2151	0.4437	y_3 0.7	y_4 1

$k = 1$;

$$y_0 - 2.0625\, y_1 + y_2 = 0.$$

$$-2.0625\, y_1 + y_2 = 0. \quad \text{——} (3)$$

$k = 2$;

$$y_1 - 2.0625\, y_2 + y_3 = 0. \quad \text{——} (4)$$

$k = 3$;

$$y_2 - 2.0625 y_3 + y_4 = 0$$

$$y_2 - 2.0625 y_3 + 1 = 0$$

$$y_2 - 2.0625 y_3 = -1 \quad \text{--- (5)}$$

Solve by (3), (4) & (5)

$$y_1 = 0.2181 \; ; \; y_2 = 0.4437 \; ; \; y_3 = 0.7000$$

24/3/14 Classification of partial differential Equation

Consider,

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = 0$$

$B^2 - 4ac < 0$ The P.D.E is elliptic

$B^2 - 4ac = 0$ The P.D.E is parabolic

$B^2 - 4ac > 0$ The P.D.E is hyperbolic.

One dimensional heat equation :

The One dimensional heat eqn is

$$\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t} \quad \text{(or)} \quad u_{xx} = a u_t$$

$$\frac{\partial^2 u}{\partial x^2} - a\frac{\partial u}{\partial t} = 0.$$

$A = 1$; $B = 0$; $C = 0$

$B^2 - 4AC = 0 - 4 \times 1 \times 0$

$= 0$

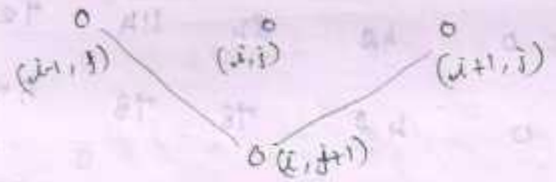
The one dimensional heat eqn is parabolic

There are two methods to solve one dimensional heat equations

i) Bender-Schmidt formula (Explicit)

ii) Crank-Nicolson method (Implicit)

Bender-Schmidt formula:



$$u_{i,j+1} = \frac{u_{i-1\,j} + u_{i+1\,j}}{2}$$

Here, $k = \frac{ah^2}{2}$

1. Solve $u_t = u_{xx}$ in $0<x<5$, $t>0$ given that $u(0,t)=0$, $u(5,t)=0$, $u(x,0) = x^2(25-x^2)$. Compute u upto 3sec with $\Delta x = 1$ by using Bender Schmidt formula.

soln:

Given $u_t = u_{xx} \Rightarrow a = 1$

$h = \Delta x = 1$

$k = \frac{ah^2}{2} = \frac{1 \times 1}{2} = 0.5$

$u_{i,j+1} = \frac{u_{i-1,j} + u_{i+1,j}}{2}$

t \ x	0	1	2	3	4	5
0	0	24	84	144	144	0
0.5	0	42	84	114	72	0
1	0	42	78	78	57	0
1.5	0	39	60	67.5	39	0
2	0	30	53.25	49.5	33.75	0
2.5	0	26.625	39.75	43.5	24.75	0
3	0	19.875	35.0625	32.25	21.75	0

2. Solve $u_{xx} = 32u_t$, $h = 0.25$ for $t \geq 0$, $0 < x < 1$, with $u(0,t) = 0$, $u(x,0) = 0$; $u(1,t) = t$

Soln:

$u_{xx} = 32 u_t \qquad a = 32.$

$h = 0.25.$

$$k = \frac{ah^2}{2} = \frac{32 \times 0.25}{2} = 1$$

$$u_{i,j+1} = \frac{u_{i-1,j} + u_{i+1,j}}{2}$$

t \ x	0	0.25	0.5	0.75	1
0	0	0	0	0	0
1	0	0	0	0	1
2	0	0	0	0.5	2
3	0	0	0.25	1	3
4	0	0.125	0.5	1.625	4
5	0	0.25	0.875	2.25	5.

2. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subjected to $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin(\pi x)$ using Bender Schmidt method.

Soln:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

$u_{xx} = u_t$. $a = 1$

$$h = \frac{B-A}{n} = \frac{1-0}{5} = 0.2.$$

$$k = \frac{ah^2}{2} = \frac{1 \times 0.2^2}{2} = 0.02.$$

Bender Schmidth formula is,

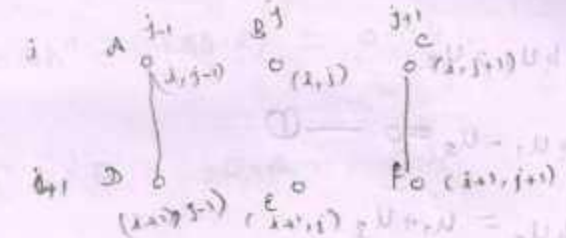
$$u_{i,j+1} = \frac{u_{i-1,j} + u_{i+1,j}}{2}$$

t \ x	0	0.2	0.4	0.6	0.8
0	0	0.5878	0.9511	0.9510	0.5878
0.02	0	0.4756	0.7695	0.7695	0.4756
0.04	0	0.3848	0.6226	0.6226	0.3848
0.06	0	0.3113	0.5037	0.5037	0.3115
0.08	0	0.2519	0.4075	0.4075	0.2519
1	0	0.2038	0.3297	0.3297	0.2038

~~5~~/3/14. Crank – Nicolson's Method (Implicit method):

Consider, $\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t}$ (one dimensional heat eqn).

$k = ah^2$



$4u_E = u_A + u_C + u_D + u_F$

1. Using crank – Nicolson's scheme solve

$$16\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0<x<1,\ t>0.$$

Subjected to $u(x,0)=0$; $u(0,t)=0$; $u(1,t)=100t$. Compute u for one step in t-direction. taking $h = 1/4$

Sln:

$$16\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$a = 16.$

$h = 0.25$,

$k = ah^2 = 16 \times (0.25)^2 = 1$

t/x	0	0.25	0.5	0.75	
0	0	0	0	0	0
1	0	u_1	u_2	u_3	100

$4u_1 = u_2$

$4u_1 - u_2 = 0$ —— (1).

$4u_2 = u_1 + u_3$.

$u_1 - 4u_2 + u_3 = 0$ —— (2).

$4u_3 = u_2 + 100$

$-u_2 + 4u_3 = 100$ —— (3)

solve (1), (2) & (3).

$u_1 = 1.7857$.

$u_2 = 7.1429$.

$u_3 = 26.7857$.

2. find $u(x,t)$ for one time step the Equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given $u(x,0) = \sin(\pi x)$; $u(0,t) = u(1,t) = 0$ Take $h = 0.2$ use implicit method.

Soln:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

$a = 1$

$h = 0.2$.

$k = ah^2 = 1 \times (0.2)^2 = 0.04$.

t\x	0	0.2	0.4	0.6	0.8	1
0	0	0.5878	0.9511	0.9511	0.5878	0
0.04	0	u_1	u_2	u_3	u_4	0

$4u_1 = u_2 + 0.9511$

$4u_1 - u_2 = 0.9511$ — (1)

$4u_2 = u_1 + u_3 + 1.5389$.

~~$-u_1 + 4u_2$~~ $u_1 - 4u_2 + u_3 = -1.5389$ — (2)

$4u_3 = u_2 + u_4 + 1.5389$.

$u_2 - 4u_3 + u_4 = -1.5389$ — (3)

$4u_4 = 0.9511 + u_3$.

~~$-u_3 + 4u_4 = 0.9511$ — (4)~~

$u_4 = \frac{u_3}{4} + 0.2378$ — (4).

Sub ④ in ⑧

$u_2 - 4u_3 + u_4 = -1.5389.$

$u_2 - 4u_3 + \frac{u_3}{4} + 0.2378 = -1.5389.$

$u_2 - \frac{15}{4}u_3 = -1.7767.$

$u_2 - 3.75u_3 = -1.7767$ —— ⑤

Solve ~~eqn~~ ①, ②, ⑤.

$u_1 = 0.3993$

$u_2 = 0.6461$

$u_3 = 0.6461$

④ ⇒ $u_4 = \frac{0.6461}{4} + 0.2378 = 0.3993.$

$u_4 = 0.3993.$

27/3/14.

3. Solve by crank nicolson's method, eqn $u_{xx} = u_t$ subjected to $u(x,0) = 0$; $u(0,t) = 0$; $u(1,t) = t$ for two time step.

soln :

$u_{xx} = u_t$.

$a=1$

$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$.

$k = ah^2 = 1 \times 0.25^2 = 0.0625$.

t \ x	0	0.25	0.5	0.75	1
0	0	0	0	0	0
0.0625	0	0.0011	0.0045	0.0167	0.0625
0.125	0	0.0059	0.0191	0.0528	0.125

$4u_1 = u_2$.

$4u_1 - u_2 = 0$ —— ①

$4u_2 = u_1 + u_3$

$u_1 - 4u_2 + u_3 = 0$ —— ②

$4u_3 = u_2 + 0.0625$.

$-u_2 + 4u_3 = 0.0625$ —— ③ .

solve by ①, ②, ③

$u_1 = 0.0011$; $u_2 = 0.0045$; $u_3 = 0.0167$.

$4u_4 = u_5 + 0.0045$.

$4u_4 - u_5 = 0.0045$ —— ④

$4u_5 = u_4 + u_6 + 0.0178$.

$u_4 - 4u_5 + u_6 = -0.0178$ — (5)

$4u_6 = u_5 + 0.1920$.

$-u_5 + 4u_6 = 0.1920$ — (6)

solve by (4), (5), & (6)

$u_4 = 0.0059 \quad u_5 = 0.0191 \quad u_6 = 0.0598$

One dimentional wave Equation:

The One dimentional wave Equation is,

$$\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial t^2} \quad ; \quad k = ah$$

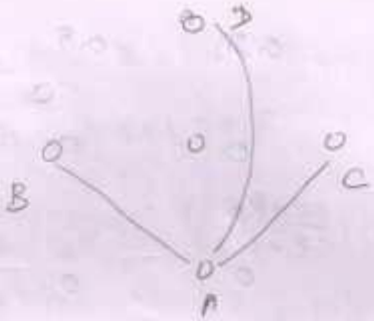
$$U_{xx} = a^2 U_{tt}$$

$$U_{xx} - a^2 U_{tt} = 0$$

$A = 1 \; ; \; B = 0 \; ; \; C = -a^2$

$B^2 - 4AC = 0 + 4a^2 = 4a^2 > 0$

The P.D.E is hyperbolic.



The formula is,

$$U_A = U_B + U_C - U_D$$

1. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0<x<1$, $t>0$

Given $u(x,0)=0$; $\frac{\partial u}{\partial t}(x,0)=0$; $u(0,t)=0$; $u(1,t)=100\sin(\pi t)$. compute $u(x,t)$ for 4 times steps with $h=0.25$

soln:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

$a^2=1$

$a=1$.

$h=0.25$.

$k = ah = 1 \times 0.25 = 0.25$.

t \ x	0	0.25	0.5	0.75	1
0					
0.25					
0.5					
0.75					

$u_1 = \frac{0+0}{2} + k \cdot A$

$u_1 = \frac{\text{add side two values}}{2} + k \cdot A$

22/5/14.

t \ x	0	0.25	0.5	0.75	
0	0	0	0	0	
0.25	0	$\frac{0+0+k.0}{2}$ u_1 0	u_2 0	u_3 0	70
0.5	0	0	0	70.7107	10
0.75	0	0	70.7107	100	70.
1	0	70.7107	100	70.7107	0

$$U_D = U_A + U_C - U_E$$

2. Solve the Eqn. $\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with $u(0,t)=0$; $u(4,t)=0$; $u(x,0) = x(4-x)$; $\frac{\partial u}{\partial t}(x,0)=0$; by taking $h=1$; upto

Soln:

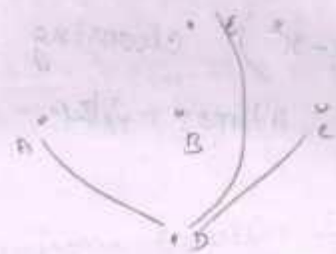
$$\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

$$a^2 = 1/4$$

$$a = 1/2.$$

$h = 1$

$k = ah = 0.5 \times 1 = 0.5$



$U_D = U_A + U_C - U_E$

t \ x	0	1	2	3	4
0	0	x(4−x) 3	x(4−x) 4	x(4−x) 3	0
0.5	0	2	3	2	0
1	0	0	0	0	0
1.5	0	−2	−3	−2	0
2	0	−3	−4	−3	0
2.5	0	−2	−3	−2	0
3	0	0	0	0	0
3.5	0	+2	3	2	0
4.	0	3	4	3.	0

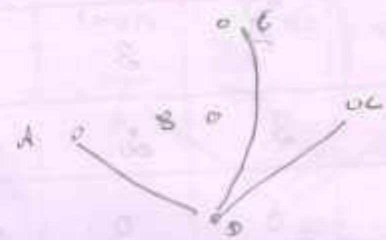
8. Solve $u_{tt} = u_{xx}$, $0 < x < 2$; $t > 0$ subject to $u(x,0) = 0$; $u(0,t) = 0$; $u(2,t) = 0$; $u_t(x,0) = 100(2x - x^2)$ choosing $h = 1/2$ compute 'u' for 4 times step.

soln:

$u_{tt} = u_{xx}$.

$a^2 = 1 \Rightarrow a = 1$; $h = 0.5$.

$k = ah = 1 \times 0.5 = 0.5$.



$U_D = U_A + U_C - U_E$

t \ x	0	0.5	1	1.5	2
0	0	0	0	0	0
0.5	0	37.5	50	37.5	0
1	0	50	75	50	0
1.5	0	37.5	50	37.5	0
2	0	0	0	0	0

$\frac{0+0}{2} + k \times 100(2x - x^2)$

1/4/14. Laplace and poisson Equation.

The Laplace Equation is $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

$U_{xx} + U_{yy} = 0$. (or) $\nabla^2 u = 0$.

The poisons Equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

(or)

$$U_{xx} + U_{yy} = f(x, y)$$

(or)

$$\nabla^2 u = f(x, y)$$

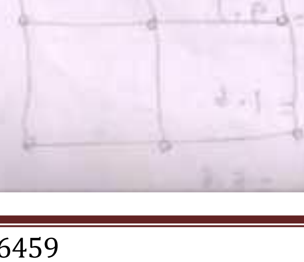
Here $A = 1$; $B = 0$; $C = 1$

$B^2 - 4AC = 0 - 4 \times 1 \times 1$

$= -4 < 0$.

Hence, Laplace and poisson equation are elliptic

Standard Diagonal five point formula,

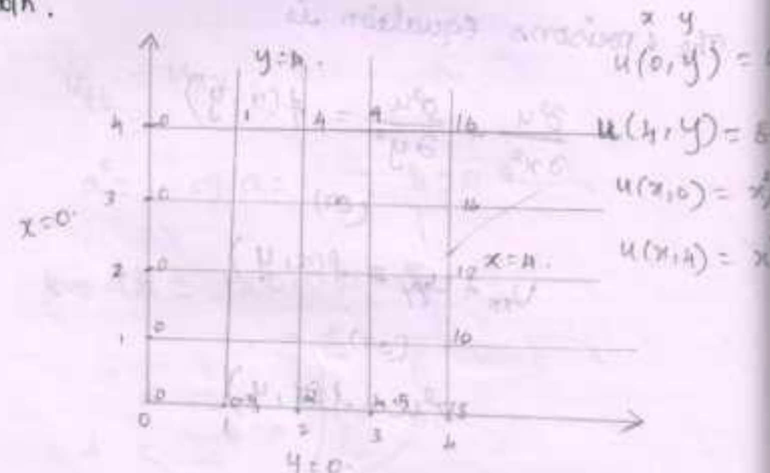


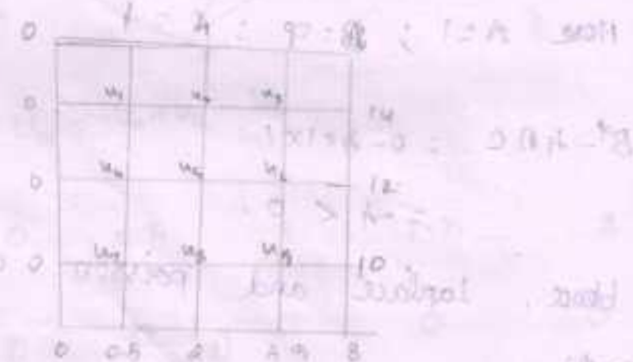
(+) SFPF : $U_E = \frac{U_B + U_D + U_F + U_H}{4}$.

(x) DFPF : $U_E = \frac{U_A + U_C + U_G + U_I}{4}$

1\. By Liebmann iteration method solve $u_{xx}+u_{yy}$ over the ~~surface~~ square region of side 4 satisfying $u(0,y)=0$ $0\le y\le 4$; $u(4,y)=8+2y$ $u(x,0)=x^2/2$ $0\le x\le 4$, $u(x,4)=x^2$, $0\le x\le 4$. Com[pute] the values at the interior points with $h=k=$

Soln:





Rough values:

SFPF : $u_5 = \frac{0+4+12+2}{4} = 4.5$

DFPF : $u_1 = \frac{0+4+0+u_5}{4} = 2.1$

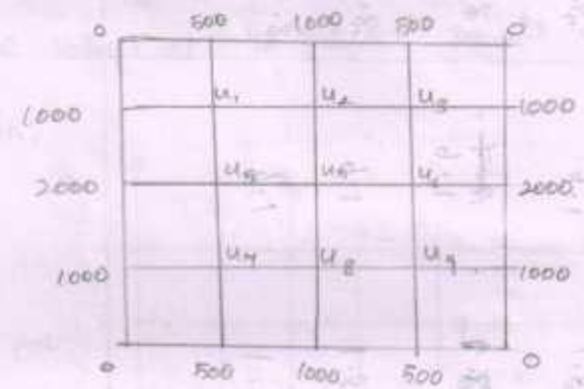
DFPF : $u_3 = \frac{4+16+12+u_5}{4} = 9.1$

DFPF : $u_7 = \frac{0+u_5+0+2}{4} = 1.6$

DFPF : $u_9 = \frac{u_5+12+2+8}{} = 6.6$

$u_1 = \frac{1+u_2+u_4}{4}$	$u_2 = \frac{4+u_1+u_3+u_5}{4}$	$u_3 = \frac{38+u_2+u_6}{4}$	$u_4 = \frac{u_1+u_5+u_7}{4}$	$u_5 = \frac{u_2+u_4+u_6+u_8}{4}$	$u_6 = \frac{18+u_5+u_3+u_9}{14}$	$u_7 = \frac{0.5+u_4+u_8}{4}$	$u_8 = \frac{2+u_5+u_7+u_9}{4}$	$u_9 = \frac{14.5+u_6+u_8}{4}$
2.1	4.9.	9.1	2.1	4.5	8.1	1.6	3.7.	6.6
2.	4.9.	9.	2	~~2.8~~ 4.7.	8.1.	~~2.7~~. 1.6	3.7.	6.6.
2.	4.9.	9	2.1	4.7.	8.1	1.6	3.7	6.6.
2	4.9	9	2.1	4.7	8.1	1.6	3.7	6.6.

2. Solve the Elliptic Eqn $U_{xx}+U_{yy}=0$ following square mesh with the bounda values are shown below



Soln :

By symmetry

$u_1 = u_3$ $\quad\quad$ $u_1 = u_7$

$u_4 = u_6$ & $u_2 = u_8$

$u_7 = u_9$ $\quad\quad$ $u_3 = u_9$

Hence,

$u_1 = u_3 = u_7 = u_9$

$u_2 = u_8$

$u_4 = u_6$

Now, we find only u_1, u_2, u_4, u

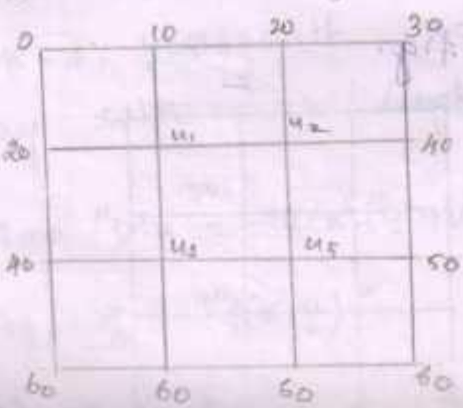
$u_1 = \frac{1500+u_2+u_4}{4}$	$u_2 = \frac{1000+2u_1+u_5}{4}$	$u_4 = \frac{2000+2u_1+u_5}{4}$	$u_5 = \frac{2u_2+2u_4+\ldots}{4}$
1125	1187.5	1437.5	1500
~~1068.8~~ ~~1011.7~~	~~1190.9~~	~~1380.9~~	~~1234.4~~
1031.3	1140.7	1390.7	1265.4
1007.9	~~1089.0~~ 1070.3	1320.4	~~1251~~ 1195.4
992.7	1035.2	1285.2	1160.2
955.1	1017.6	1267.6	1142.6
946.3	1008.8	1258.8	1133.8
941.9	1004.4	1254.4	1129.4
939.7	1002.2	1252.2	1127.2
938.6	1001.1	1251.1	1126.1
938.1	1000.8	1250.6	1125.7
937.9	1000.4	1250.4	1125.4
937.9	1000.3	1250.3	1125.3

Rough value :

SFPF : $u_5 = \frac{1000+2000+2000+1000}{4} = \frac{6000}{4} = 1500$

DFPF : $u_1 = \frac{0+1000+1500+2000}{4} = 1125$

3. Solve $\nabla^2 u = 0$ over the square region given by the boundary condition as in the fig. below.



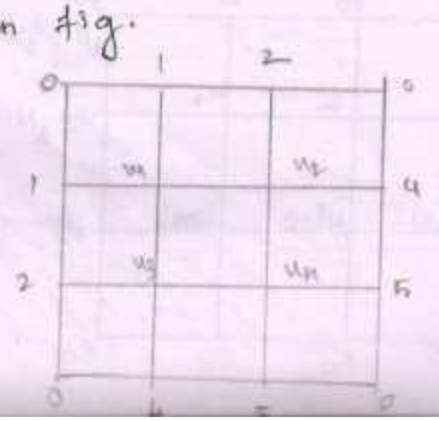
$u_1 = \frac{20+u_2+u_3}{4}$	$u_2 = \frac{60+u_1+u_4}{4}$	$u_3 = \frac{100+u_1+u_4}{4}$	$u_4 = \frac{110+}{}$
4	18.8	28.8	0
19.4	19.9	29.9.	40
20	30	40	45
25	32.5	42.5	46.3
26.3	33.2	43.2.	46.6
26.6	33.3	43.3	46.7
26.7.	33.4	43.4	46.7.
26.7.	33.4	43.4	46.7.

~~Soln:~~ Rough :

$u_4 = 0$.

DFPF: $u_1 = \frac{0+20+40+0}{4} = 15$.

4. Solve $V_{xx} + V_{yy} = 0$ over the squa[re] region given by the boundary condi[tions] shown in fig.



Soln:

By symmetry, $u_3 = u_2$.

Rough; Assume, $R_4 = 0$

DFPF: $u_1 = \frac{2+2+0+0}{4} = 1$

$u_1 = \frac{2+2u_2}{4}$	$u_2 = \frac{6+u_1+u_4}{4}$	$u_4 = \frac{10+2u_2}{4}$
1	1.8	0
1.4	1.9	3.5
1.5	2.8	3.9
1.9	3	4
2	3	4
2	3	4.

3/4/14. poisson equation problems. $U_{xx}+U_{yy} = -f(x,y)$

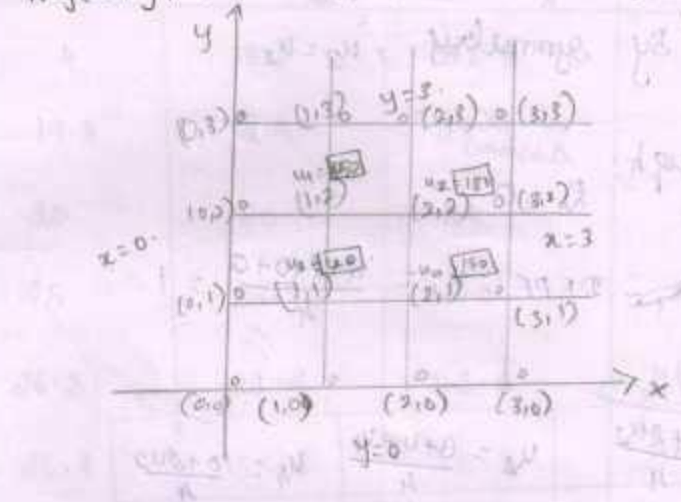
1. Solve the equation $\nabla^2 u = -10(x^2+y^2+10)$ over the square mesh with sides $x=0$; $y=0$; $x=3$; $y=3$, with $u=0$ on the boundary with mesh length 1 unit.

Soln: given $\nabla^2 u = -10(x^2+y^2+10)$

$$U_{xx}+U_{yy} = -f(x,y)$$

$f(x,y) = 10(x^2+y^2+10)$

$h^2 f(x,y) = f(x,y) = 10(x^2+y^2+10)$ $\therefore h =$



By symmetry,

$u_1 = u_4$.

$u_1 = \frac{u_2+u_3+150}{4}$	$u_2 = \frac{u_1+u_4+180}{3} = \frac{2u_1+180}{4}$	$u_3 = \frac{u_1+u_4 \ldots}{4} = \frac{2u_1 \ldots}{4}$
0	0	0
37.5	68.8	48.8
65.7	77.9	62.9
72.7	81.4	66.4
74.5	82.3	67.3
74.9	82.5	67.5
75	82.5	67.5
75	82.5	67.5

2. Solve $\nabla^2 u = 8x^2y^2$ over the square bounded by the lines $x = -2$; $x = 2$, $y = -2$, $y = 2$ with $u = 0$ on the boundary and mesh length $= 1$

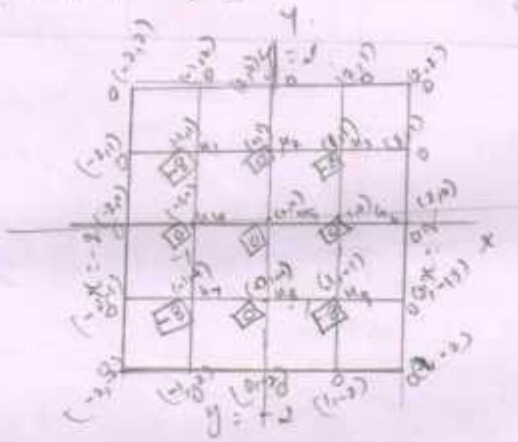
Soln :

Given $\nabla^2 u = 8x^2y^2$

w.k.t $\nabla^2 u = -f(x,y)$

$f(x,y) = -8x^2y^2$

$h^2 f(x,y) = -8x^2y^2$; $(\because h = 1)$



By symmetry :

~~$u_1 = u_7$~~	~~$u_1 = u_3$~~	$u_2 = u_4$	~~$u_1 = u_9$~~
$u_2 = u_8$	$u_4 = u_6$	~~$u_3 = u_7$~~	$u_4 = u_8$
~~$u_3 = u_9$~~	~~$u_7 = u_9$~~	$u_6 = u_8$	$u_2 = u_6$

$u_1 = u_7 = u_3 = u_9$

$u_2 = u_8 = u_6 = u_4$.

$u_1 = \frac{u_2+u_4-8}{4} = \frac{2u_2-8}{4}$	$u_2 = \frac{u_1+u_3+u_5}{4} = \frac{2u_1+u_5}{4}$	$u_5 = \frac{u_2+u_4+u_6+u_8}{4}$, $u_5 = u_2$
0	0	0
-2	-1	-1
-2.5	-1.5	-1.5
-2.8	-1.8	-1.8
-2.9	-1.9	-1.9
-3	-2	-2
-3	-2	-2.

Questions	opt1	opt2	opt3	opt4	Answer
If B^2-4AC = 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	parabolic
If B^2-4AC > 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	hyperbolic
If B^2-4AC < 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	elliptic
(f(x+h)-f(x))/h is known as the _________	difference quotient	average	derivative	f(x)	difference quotient
The equation del^2(u) = 0 is _______ equation.	Laplace	Poisson	Heat	Wave	Laplace
One dimensional heat equation is the example of _______ equation.	Laplace	Poisson	Parabolic	Hyperbolic	Parabolic
One dimensional wave equation is the example of _______ equation.	Laplace	Poisson	Parabolic	Hyperbolic	Hyperbolic
The differential equation is said to be parabolic, if	B^2-4AC	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC =0	B^2-4AC
The differential equation is said to be elliptic, if	B^2-4AC	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC =0	B^2-4AC < 0
The differential equation is said to be hyperbolic, if	B^2-4AC	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC =0	B^2-4AC > 0
[x f(xx)+yf(yy)]=0 x>0, y>0 is ____ type of equation.	elliptic	Poisson	Parabolic	Hyperbolic	elliptic
[f(xx)-2f(xx)]=0, x>0, y>0 is ____ type of equation.	elliptic	Poisson	Parabolic	Hyperbolic	Hyperbolic
____________ process is used to solve two dimensional heat equations	Newtons	Gaussian	Laplace	Liebmanns iteration	Liebmanns iteration
The equation (Ñ^2) u = 0 is known as ____________ equation	Laplace	Poisson	heat	wave	heat
The ____________ formula is used to complete the improved value of u,	Newtons	elimination	Liebmanns iteratio	reduction	Liebmanns iteration
The value of u can be improved by ____________ process	Newtons	elimination	Liebmanns iteratio	reduction	Liebmanns iteration
The value of u is obtained at any ____________ lattice points which is the arithmetic mean of the values of u at 4 lattice points near to it	interior	exterior	positive	negative	interior
The value of $u_{i,j}$ in the difference equation are defined only at the _________points	equal	unequal	apex	lattice	lattice
The points of intersection of these families of lines are called ________ points	equal	unequal	apex	lattice	lattice
If B^2 – 4AC > 0 then the given equation is ____________	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	hyperbolic
The differential equation is said to be ____________ in a region R if B^2 - 4AC < 0 at all points of a region	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	elliptic
The differential equation is said to be ____________ in a region R if B^2- 4AC = 0 at all points of the region	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	Parabolic
If (ka)/h < 1, it is stable but the accuracy of the solution decrease with the increasing value of ____________	k	a	(ka)/h	k/h	(ka)/h
If (ka)/h < 1, it is stable but the accuracy of the solution decrease with the increasing value of ____________	k	a	k/h	(ka)/h	(ka)/h
The differential equation is said to be ____________ in a region R if B^2 - 4AC = 0 at all points of the region	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	Parabolic
The differential equation is said to be ____________ in a region R if B^2 - 4AC < 0 at all points of a region	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	elliptic
The points of intersection of these families of lines are called ________ points	equal	unequal	apex	lattice	lattice
Schmidt method belongs to ____________ type	explicit	implicit	elliptic	hyperbolic	explicit
The Poisson's equation belongs to ____________ type	explicit	implicit	elliptic	hyperbolic	hyperbolic
One dimensional heat flow equation belongs to ____________ type	explicit	parabolic	elliptic	hyperbolic	parabolic
Laplace equation in two dimensions belongs to ____________ type	explicit	parabolic	elliptic	hyperbolic	explicit
The error in solving Poisson equation by ____________ methods is of order h^2	Difference	iteration	elimination	interpolation	Difference
The error in solving _________equation by difference method is of order h^2	Newton's	Jacobi's	Poisson	Gaussian	Poisson
The error in solving Poisson's equation by difference methods is of order_______	h	h^2	h^3	h^4	h^2
The equation del^ 2(u) = f(x, y) is known as_________ equation	Poisson	Newtons	Jacobis	Gaussian	Poisson
The value of $u_{i,j}$ is the average of its value at the ________ neighbouring diagonal mesh points	2	3	4	5	4
The value of u(i,j) is the __________of its values at the four neighbouring diagonal mesh points	sum	difference	average	product	average
The value of u(i,j) is the average of its values at the four neighbouring ____________ mesh points	Square	rectangle	diagonal	column	diagonal
The mesh points are also called ____________	grid point	starting point	Ending point	bisection	grid point
The points of intersection of the dividing lines are called ____________	bisection	mesh points	vertex	end point	mesh points
The differential equation is said to be hyperbolic, if	B^2-4AC = 0	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC <= 0	B^2-4AC > 0
The differential equation is said to be elliptic, if	B^2-4AC = 0	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC <=0	B^2-4AC < 0
The differential equation is said to be parabolic, if	B^2-4AC = 0	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC <= 0	B^2-4AC = 0
One dimensional wave equation is the example of _______ equation.	Laplace	Poisson	Parabolic	Hyperbolic	Parabolic
One dimensional heat equation is the example of _______ equation.	Laplace	Poisson	Parabolic	Hyperbolic	Poisson
The equation del^2(u) = 0 is _______ equation	parabolic	elliptic	hyperbolic	equally spaced	parabolic
If B^2-4AC = 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	parabolic
If B^2-4AC > 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	hyperbolic
If B^2-4AC < 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	elliptic