**15BTBT202** **MATHEMATICS II** **3 2 0 4**

**OBJECTIVES:**

- To impart analytical ability in solving mathematical problems as applied to the respective branches of Engineering.
- To understand the concepts ofMultiple integrals, Functions of several variables and fourier series
- To understand the concepts ofBoundary value problems and Statistics.

**INTENDED OUTCOMES:**

- The students will be able to understand mathematical tools needed in evaluating multiple integrals and their usage.
- The students will be able to familiarize functions of several variables which is used in many physical engineering problems.

**UNIT -I MULTIPLE INTEGRALS (12)**

Double integration in Cartesian – Change of order of integration – Area as a double integral – Triple integration in Cartesian coordinates.

**UNIT -II FUNCTIONS OF SEVERAL VARIABLES (12)**

Function of two variables – Taylor's expansion – maxima and minima – constrained maxima and minima by Lagrangian multiplier method – Jacobians.

**UNIT- III FOURIER SERIES (12)**

Dirchlet's conditions – statement of Fourier theorem – Fourier coefficients – change of scale – Half range series – Harmonic Analysis.

**UNIT-IV BOUNDARY VALUE PROBLEMS (12)**

Method of separation of variables – one dimensional wave equation – one dimensional heat equation – steady state conditions – zero and non – zero boundary conditions.

**UNIT – V STATISTICS (12)**

Measures of central tendency – Mean, Median, Mode, Standard deviation – moments – skewness and kurtosis-correlation – rank correlation.

**Total: 60**

**REFERENCES:**


| S. No. | AUTHOR(S) NAME | TITLE OF THE BOOK | PUBLISHER | YEAR OF PUBLICATIONS |
|---|---|---|---|---|
| 1 | Hemamalini. P.T | Engineering Mathematics I & II | McGraw-Hill Education Pvt.Ltd, New Delhi. | 2014 |
| 2 | Dr.P.Kandasamy , Dr.K.Thilagavathy, Dr.K.Gunavathy | Engineering Mathematics Volume III | S.Chand&Co., New Delhi. | 2013 |
| 3 | Veerarajan, T | Engineering Mathematics | Tata McGraw Hill Publishing Co., New Delh. | 2010 |
| 4 | Sundaram V., Balasubramanian R., Lakshminarayanan K.A. | Engineering Mathematics | Vikas publishing house Pvt. Ltd, New Delhi. | 2005 |
| 5 | Gupta S.C.,Kapoor V.K | Fundamentals of Mathematical Statistics | Sultan chand& Sons, New Delhi. | 2006 |


**WEBSITES:**

1. www.intmath.com
2. www.efunda.com
3. www.mathcentre.ac.uk

**KARPAGAM UNIVERSITY**
**KARPAGAM ACADEMY OF HIGHER EDUCATION**
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**COIMBATORE-641 021**
**DEPARTMENT OF SCIENCE AND HUMANITIES**
**FACULTY OF ENGINEERING**
**B.Tech - Biotechnology - (Regular) - II Semester**

# LESSON PLAN

**Name:P.KARPAGAM**
**Subject: MATHEMATICS-II** **Subject Code: 15BTBT202**

| S.NO | Topics covered | No. of hours |
|---|---|---|
| | **UNIT- I MULTIPLE INTEGRALS** | |
| 1. | **Introduction of integration and basic formulas** | 1 |
| 2. | Double integration in Cartesian coordinates | 1 |
| 3. | Problems based on double integration in Cartesian coordinates | 1 |
| 4. | Change of order of integration | 1 |
| 5. | Problems based on change of order of integration | 1 |
| 6. | Tutorial 1: Change of order of integration problems | 1 |
| 7. | Area as a double integral | 1 |
| 8. | Problems based on area as a double integral | 1 |
| 9. | Tutorial 2: Area as a double integral problems | 1 |
| 10. | Triple integration in Cartesian coordinates | 1 |
| 11. | Problems based on Triple integration in Cartesian coordinates | 1 |
| 12. | Tutorial 3: Triple integration in Cartesian coordinates. | 1 |
| | Total | 12 |
| | **UNIT II FUNCTIONS OF SEVERAL VARIABLES** | |
| 13. | Introduction of functions of several variables | **1** |
| 14. | Problems based on functions of several variables | 1 |
| 15. | Taylor's expansion | 1 |
| 16. | Problems based on Taylor's expansion | 1 |
| 17. | Tutorial 4: Problems based on Taylor's expansion | 1 |
| 18. | Concept of maxima and minima | 1 |
| 19. | Problems based on maxima and minima | 1 |
| 20. | Constrained maxima and minima by Lagrangian multiplier method | 1 |
| 21. | Problems based on maxima and minima by Lagrangian multiplier method | 1 |
| 22. | Tutorial 5: Maxima and minima by Lagrangian multiplier method problems | 1 |
| 23. | Introduction of Jacobians and problems | 1 |
| 24. | Tutorial 6: Problems on Jacobians | 1 |
| | Total | 12 |
| | **UNIT III FOURIER SERIES** | |
| 25. | Introduction to basic integration and Bernoulli's integration | 1 |
| 26. | Problems based on Bernoulli's integration | 1 |
| 27. | Periodic function - Dirchlet's conditions and Statement of Fourier theorem | 1 |
| 28. | Fourier coefficients and solving problems | 1 |

| | | |
|---|---|---|
| 29. | Full range series in the interval $(-\pi, \pi)$ and $(0,2\pi)$ | 1 |
| 30. | Problems based on full range series in the interval $(-\pi, \pi)$ and $(0,2\pi)$ | 1 |
| 31. | Tutorial:7 Problems based on full range series in the interval $(-\pi, \pi)$ and $(0,2\pi)$ | 1 |
| 32. | Concept of change of scale and Half range series | 1 |
| 33. | Problems based on half range series in the interval $(0, \pi)$ | 1 |
| 34. | Tutorial:8 Problems based on half range series in the interval $(0, \pi)$ | 1 |
| 35. | Harmonic Analysis | 1 |
| 36. | Tutorial: 9 Problems on Harmonic Analysis | 1 |
| | Total | 12 |
| | **UNIT IV BOUNDARY VALUE PROBLEMS** | |
| 37. | Introduction with application of partial differential equations | 1 |
| 38. | Classification of second order quasi linear PDE | 1 |
| 39. | Method of separation of variables | 1 |
| 40. | Tutorial : 10 Problems on method of separation of variables | 1 |
| 41. | Solution of One dimensional wave equation | 1 |
| 42. | Problems on One dimensional wave equation | 1 |
| 43. | Tutorial :11 Problems on One dimensional wave equation | 1 |
| 44. | Solution of One dimensional heat equation | 1 |
| 45. | Problems on One dimensional heat equation | 1 |
| 46. | Steady state solution of two dimensional heat equations | 1 |
| 47. | Problems based on zero boundary conditions | 1 |
| 48. | Tutorial :12 Problems based on zero boundary conditions | 1 |
| | Total | 12 |
| | **UNIT V STATISTICS** | |
| 49. | Introduction of Statistics | 1 |
| 50. | Concept of measures of central tendency | 1 |
| 51. | Concept of Mean, Median, Mode, Standard deviation | 1 |
| 52. | Problems on Mean, Median, Mode, Standard deviation | 1 |
| 53. | Problems on Mean, Median, Mode, Standard deviation | 1 |
| 54. | Tutorial:13 Problems on Mean, Median, Mode, Standard deviation | 1 |
| 55. | Moments – skewness and kurtosis | 1 |
| 56. | Tutorial: 14 Problems based on moments – skewness and kurtosis | 1 |
| 57. | Correlation – Types of correlation and formulas | 1 |
| 58. | Concept of Rank correlation | 1 |
| 59. | Problems based on rank correlation | 1 |
| 60. | Tutorial:15 Problems based on rank correlation | 1 |
| | Total | 12 |
| | **TOTAL** | **60** |

**Staff In charge** **HoD**

# Unit-1

# Matrices

## Definition of a Matrices.

A system of any $mn$ numbers arranged in a rectangular array of m-rows and n-columns is called Matrices of order $m \times n$ and is denoted by

$$A = (a_{ij})_{m\times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

where $a_{ij}$'s are called the entries or elements of the Matrices.

## Types of Matrices.

1) Row's Matrices:-

A Matrices having only one row is called as row Matrices

eg: $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$.

2) Column's matrices:-

A Matrices having only one column is called as column matrices.

eg: $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$.

3) Square Matrix.

A Matrix having equal number of rows and columns is called square Matrix

eg: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

4) Diagonal Matrix.

A Square Matrix having its entries along the leading diagonal and all other entries are zero is called diagonal matrices.

eg: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$

5) Scalar Matrix.

A diagonal Matrix whose leading diagonals are all same is called Scalar Matrix

eg: $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$

6) Unit Matrix:-

A Scalar Matrix whose diagonal elements are one is called a Unit Matrix

eg: $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$

7) Triangular Matrix.

Types: Upper triangular Matrix.

A Square Matrix in which all the elements below the leading diagonal are zero is called Upper triangular Matrix.

eg:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}_{3\times3}$

Lower triangular Matrix.

A Square Matrix in which all the elements above the leading diagonal are zero is called a lower triangular Matrix

eg: $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix}_{3\times3}$

8) Transpose of a Matrix.

The Matrix got from a given Matrix by interchanging its rows and columns is called Transpose of that Matrix.

ie:- if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

Then $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}_{3\times3}$

9) **Symmetric Matrix**

A Square Matrix "A" is said to be Symmetric if $A=A^T$ and Skew Symmetric if $A=-A^T$.

eg: Here

$$A=\begin{bmatrix}1 & 4 & 5\\ 4 & 2 & 6\\ 5 & 6 & 3\end{bmatrix}_{3\times3}.$$

Then

$$A^T=\begin{bmatrix}1 & 4 & 5\\ 4 & 2 & 6\\ 5 & 6 & 3\end{bmatrix}_{3\times3}.$$

$\therefore A=A^T$

$\therefore$ A is an Symmetric Matrix.

10) **Conjugate of a Matrix.**

A Matrix a obtained by replacing each element of A by its complex Conjugate is called Conjugate of A and is denoted by $\bar{A}$.

eg: if

$$A=\begin{bmatrix}1+i & 2 & 3-i\\ 4 & 5+i & i\\ 7 & 8+3i & 9\end{bmatrix}_{3\times3}.$$

Then

$$\bar{A}=\begin{bmatrix}1-i & 2 & 3+i\\ 4 & 5-i & -i\\ 7 & 8-3i & 9\end{bmatrix}_{3\times3}.$$

11) **Hermitian Matrices and Skew Hermitian Matrices.**

A Square Matrix A is said to be Hermitian if $A=\overline{A^T}$.

A Square Matrix A is said to be Skew Hermitian if $A=-\overline{A^T}$.

Eg: $$A=\begin{bmatrix}1 & 1-4i\\ 1+4i & 2\end{bmatrix}_{2\times2}.$$

Then $$A^T=\begin{bmatrix}1 & 1+4i\\ 1-4i & 2\end{bmatrix}_{2\times2} \quad \overline{A^T}=\begin{bmatrix}1 & 1-4i\\ 1+4i & 2\end{bmatrix}_{2\times2}$$

$A=\overline{A^T}$.

$\therefore$ A is Hermitian.

Eg: $$B=\begin{bmatrix}3i & 2+i\\ -2+i & i\end{bmatrix}_{2\times2}$$

Then. $$B^T=\begin{bmatrix}3i & -2+i\\ 2+i & i\end{bmatrix}_{2\times2}. \quad \overline{B^T}=\begin{bmatrix}-3i & -2-i\\ 2-i & -i\end{bmatrix}_{2\times2}$$

$$\overline{B^T}=-\begin{bmatrix}3i & 2+i\\ -2+i & i\end{bmatrix}_{2\times2}.$$

$B=-\overline{B^T}$.

$\therefore$ B is skew Hermitian.

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12) Trace of a Matrix

The sum of the main diagonal elements of a square matrix A is called trace of A and is denoted by trace (A) or tr(A).

if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

Then trace (A) = 1+5+9

tr(A) = 15.

13) determinant of a Matrix.

Determinant is calculating the numerical value of a matrix it is denoted by |A| or det(A) or $\Delta$(A).

eg: if

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}_{3\times3}$.

Then

$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{vmatrix}_{3\times3}$.

$|A| = 1[6-3] - 1[3+6] + 1[-1-4]$

$= 3-9-5 = -11$.

14) Singular and non Singular Matrices

Singular Matrices

If determinant of A is zero ie, |A|=0 then A is said to be a singular Matrix.

non singular Matrices.

If |A| ≠ 0 then A is said to be a non-singular Matrix.

15) Equal Matrix.

Two matrices A and B are said to be equal if

i) A & B have are of same order

ii) Each element of A is equal to the corresponding element of B.

eg: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$.

Then A=B.

16) Sub Matrix:

A Matrix are obtained from A by elementary transmation is called the sub matrix of A.

Properties of Determinants

Addition properties.

Addition of two matrices.

i) Two Matrices A and B can be added if and only if A and B are of same order.

ii) Each element of A is added with the corresponding element of B.

Eg: If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3}.$$

Then

$$A+B = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}_{3\times3}.$$

Types:

i) Commutative (ie) $A+B = B+A$.

ii) Associative (ie) $(A+B)+C = A+(B+C)$.

iii) Scalar Multiplication (ie) $(\alpha+\beta)A = \alpha A + \beta A$.

Multiplications of two matrices.

Two matrices A and B can be multiplied only if the number of columns of the first matrices is equal to the number of rows of the second matrices.

(ie) eg: If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times3}$

$$B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times2}.$$

16
25
36
77

Then.

$2\times\boxed{3\ 3}\times2$

$2\times2$

$$AB = \begin{bmatrix} 1+4+9 & 4+10+18 \\ 4+10+18 & 16+25+36 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix}_{2\times2}.$$

Properties:-

i) Commutative (ie) $AB \neq BA$.

ii) Associative (ie) $(AB)C = A(BC)$

iii) Scalar Multiplication (ie) $\alpha(A) = \alpha A$.

## Finding ~~aij~~ adjoint of a Matrix.

Given:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 3 \\ -2 & 1 & 3 \end{bmatrix}$$

The Co-factor of A is.

~~$A_{3\times3}$~~

$$A_{ij} = \begin{bmatrix} +\begin{vmatrix} -2 & 3 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -2 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} -6-3 & -(3+6) & +(1-4) \\ -(3-1) & +(3+2) & -(1+2) \\ +(3+2) & -(3-1) & +(-2-1) \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -9 & -3 \\ -2 & 5 & -3 \\ 5 & -2 & -3 \end{bmatrix}$$

$$\text{Adjoint } A\,(a_{ij})^{T} = \begin{bmatrix} -9 & -2 & 5 \\ -9 & 5 & -2 \\ -3 & -3 & -3 \end{bmatrix}$$

## Inverse of Matrix.

⇒ Inverse Matrix is also known as reciprocal of Matrix.

⇒ Inverse of a matrix can be obtained only for a square Matrix.

⇒ Inverse of a Matrix is denoted by $A^{-1}$ and defined as

$$A^{-1} = \frac{1}{|A|}(\text{adj } A).$$

Find the Problem. Inverse of the matrix

1) $A = \begin{bmatrix} 1 & 2 \\ -1 & -4 \end{bmatrix}.$

$$\text{Adj } A = \begin{bmatrix} -4 & -2 \\ 1 & 1 \end{bmatrix}.$$

$$|A| = (-4+2) = -2.$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-2}\begin{bmatrix} -4 & -2 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 1 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}.$$

ii) $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}.$

$$\text{Adj } A = \begin{bmatrix} -2 & -2 \\ -2 & 1 \end{bmatrix}.$$

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$|A| = -2-4$

$= -6.$

$A^{-1} = \frac{1}{|A|} adj(A)$

$= \frac{1}{-6}\begin{bmatrix} -2 & -2 \\ -2 & - \end{bmatrix}.$

$A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -1/6 \end{bmatrix}$

$(A^{-1}) \frac{(adj A)}{|A|} = A^{-1}$

2) $A = \begin{bmatrix} 8 & 3 \\ 2 & 3 \end{bmatrix}.$

$Adj\ A = \begin{bmatrix} 3 & -3 \\ -2 & 3 \end{bmatrix}$

$|A| = 9-6$

$= 3$

$A^{-1} = \frac{1}{|A|} adj(A)$

$A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & -3 \\ -2 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & -1 \\ -2/3 & 1 \end{bmatrix}$

3) $A = \begin{bmatrix} 3 & -2 \\ 7 & 8 \end{bmatrix}.$

$Adj\ A = \begin{bmatrix} 8 & +2 \\ -7 & 3 \end{bmatrix}$

$|A| = 24+14$

$= 38.$

$A^{-1} = \frac{1}{|A|} adj(A)$

$A^{-1} = \frac{1}{38}\begin{bmatrix} 8 & +2 \\ -7 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 4/19 & 1/19 \\ -7/38 & 3/38 \end{bmatrix}.$

$\frac{\cancel{8}\,4}{\cancel{38}\,19}$ $\frac{2^1}{\cancel{38}\,19}.$

$7/38$ $3/38$

4) $A = \begin{bmatrix} 1 & 0 & -4 \\ -2 & 2 & 5 \\ 3 & -1 & 2 \end{bmatrix}.$

$|A| = 1\begin{vmatrix} 2 & 5 \\ -1 & 2 \end{vmatrix} - 4\begin{vmatrix} -2 & 2 \\ 3 & -1 \end{vmatrix}$

$= 1(4+5) - 4(2-6)$

$= 1(9) - 4(-4)$

$= 9+16$

$= 25.$

$adj(A) = \begin{vmatrix} 1 & 0 & -4 \\ -2 & 2 & 5 \\ 3 & -1 & 2 \end{vmatrix}.$

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$$= \begin{bmatrix} +\begin{vmatrix} 2 & 5 \\ -1 & 2 \end{vmatrix} & -\begin{vmatrix} -2 & 5 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} -2 & 2 \\ 3 & -1 \end{vmatrix} \\ -\begin{vmatrix} 0 & -4 \\ -1 & 2 \end{vmatrix} & +\begin{vmatrix} 1 & -4 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 3 & -1 \end{vmatrix} \\ +\begin{vmatrix} 0 & -4 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 1 & -4 \\ -2 & 5 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ -2 & 2 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (4+5) & -(-4-15) & +(2-6) \\ -(-4) & +(2+12) & -(-1) \\ (8) & -(5-8) & +(2) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$$

$$adj(A) \Rightarrow (a_{ij})^T = \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$= \frac{1}{25} \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 9/25 & 4/25 & 8/25 \\ 19/25 & 14/25 & 3/25 \\ -4/25 & 1/25 & 2/25 \end{bmatrix}.$$

5) $A = \begin{bmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{bmatrix}.$

$$|A| = 5\begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} + 6\begin{vmatrix} 7 & -3 \\ 2 & 6 \end{vmatrix} + 4\begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix}$$

$= 5(24+3) + 6(42+6) + 4(7-8)$

$= 5(27) + 6(48) + 4(-1)$

$= 135 + 288 - 4$

$= 419.$

$288 + 135 = 423$

$27 \times 5 = 135$

$48 \times 6 = 288$

$$A_{ij} = \begin{bmatrix} +\begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} & -\begin{vmatrix} 7 & -3 \\ 2 & 6 \end{vmatrix} & +\begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} -6 & 4 \\ 1 & 6 \end{vmatrix} & +\begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix} & -\begin{vmatrix} 5 & -6 \\ 2 & 1 \end{vmatrix} \\ +\begin{vmatrix} -6 & 4 \\ 4 & -3 \end{vmatrix} & -\begin{vmatrix} 5 & 4 \\ 7 & -3 \end{vmatrix} & +\begin{vmatrix} 5 & -6 \\ 7 & 4 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 27 & -48 & -1 \\ +40 & 22 & -17 \\ +2 & 43 & 62 \end{bmatrix}$$

$$adj\ A = \begin{bmatrix} 27 & 40 & 2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}.$$

$24+3$

$42+6$

$7-8$

$-36-4$

$30-8$

$5+12$

$17$

$18-16$

$-15-28$

$28 + 15 = 43$

$20+42$

$$A^{-1} = \frac{1}{|A|}(adj(A))$$

$$= \frac{1}{419}\begin{bmatrix} 27 & 40 & 2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} \frac{27}{419} & 40/419 & 2/419 \\ -48/419 & 22/419 & 43/419 \\ -1/419 & -17/419 & 62/419 \end{bmatrix}$$

## Rank of the Matrix.

⇒ The Rank of the Matrix ~~Used~~ is the order ~~to find~~ of any highest ~~the degree of the~~ non-vanishing minor of the Matrix.

⇒ Eg: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$.

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1. \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} R_3 \to R_2 + R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$\rho(A) = 3$.

1) $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 1 & -4 \\ 1 & -7 & -1 \end{bmatrix}$.

$$\sim \begin{bmatrix} 1 & -7 & -1 \\ 2 & 1 & -4 \\ 4 & 3 & 2 \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & -7 & -1 \\ 0 & 15 & -2 \\ 0 & 1 & 6 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 2R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -7 & -1 \\ 0 & 15 & -2 \\ 0 & 0 & 88 \end{bmatrix} R_3 \to R_2 - 15R_3$$

$\rho(A) = 3$.

## Consistency and Inconsistency of the Matrix

| | | |
|---|---|---|
| Consistent | $\rho(A = \rho(A \cdot B) = n$ | Unique solution |
| | $\rho(A) = \rho(AB) \neq n$ | Infinite many soln |
| Inconsistent. | $\rho(A) \neq \rho(AB)$ | No solution. |

1) $x+y+z=6$

$x+2y+3z=10$

$x+2y+\lambda z=\mu$

$$A=\begin{bmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{bmatrix},\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}6\\10\\\mu\end{bmatrix}$$

$AX=B$

$$\begin{bmatrix}1&1&1\\1&2&3\\1&2&\lambda\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}6\\10\\\mu\end{bmatrix}$$

$$(A,B)=\begin{bmatrix}1&1&1&6\\1&2&3&10\\1&2&\lambda&\mu\end{bmatrix}$$

$$\sim\begin{bmatrix}1&1&1&6\\0&1&2&4\\0&0&\lambda-3&\mu-10\end{bmatrix}\begin{matrix}\\R_2\to R_2-R_1\\R_3\to R_3-R_2\end{matrix}$$

Case (i) = $\lambda=3$ $\mu\neq10$

$\rho(A)=2$ $\rho(A,B)=3$

$\rho(A)\neq\rho(A,B)$

inconsistent and has no solution.

Case (ii) = $\lambda\neq3$ $\mu=10$

$\rho(A)=\rho(AB)=3$ if $n=3$

Consistent & has unique soln.

Case (iii) $\lambda=3$, $\mu=10$

$\rho(A)=2$ $\rho(A,B)=2$ $\rho(A)=\rho(A,B)=2<\boxed{n=3}$

consistent and has infinitely many soln.

2) Test for consistency and Solve if consistent

1) $4x+3y+2z+7=0$ $\quad$ $4x+3y+2z=-7$

$2x+y-4z+1=0$ $\quad$ $2x+y-4z=-1$

$x-7y-z=0$ $\quad$ $x-7y-z=0$

$$A=\begin{bmatrix}4&3&2\\2&1&-4\\1&-7&-1\end{bmatrix},\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix},\quad B=\begin{bmatrix}-7\\-1\\0\end{bmatrix}$$

$AX=B$.

where,

$$\begin{bmatrix}4&3&2\\2&1&-4\\1&-7&-1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-7\\-1\\0\end{bmatrix}.$$

$$(A,B)=\begin{bmatrix}4&3&2&-7\\2&1&-4&-1\\1&-7&-1&0\end{bmatrix}$$

$$\sim\begin{bmatrix}1&-7&-1&0\\2&1&-4&-1\\4&3&2&-7\end{bmatrix}\quad R_1\leftrightarrow R_3$$

$$\sim\begin{bmatrix}1&-7&-1&0\\0&15&-2&-1\\0&31&6&-7\end{bmatrix}\begin{matrix}\\R_2\to R_2-2R_1\\R_3\to R_3-4R_1\end{matrix}$$

$$\sim\begin{bmatrix}1&-7&-1&0\\0&15&-2&-1\\0&1&10&-5\end{bmatrix}\begin{matrix}\\\\R_3\to R_3-2R_2\end{matrix}$$

$$\sim\begin{bmatrix}1&-7&-1&0\\0&15&-2&-1\\0&0&152&-74\end{bmatrix}\begin{matrix}\\\\R_3\to 15R_3-R_2\end{matrix}$$

$\rho(A) = 3$

$\rho(A,B) = 3$

$\rho(A) = \rho(A,B) = n$

The soln is consistent and it has Unique soln.

2) $4x+3y+2z+7=0$

$2x+y-4z+1=0$

$x-7y-z=0.$

$4x+3y+2z=-7$

$2x+y-4z=-1$

$x-7y-z=0$

$AX=B$

$$A=\begin{bmatrix}4&3&2\\2&1&-4\\1&-7&-1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}-7\\-1\\0\end{bmatrix}.$$

$$[A,B]=\begin{bmatrix}4&3&2&-7\\2&1&-4&-1\\1&-7&-1&0\end{bmatrix}$$

$$\sim\begin{bmatrix}4&3&2&-7\\0&-2&-20&10\\0&-31&-6&7\end{bmatrix} \begin{matrix}\\R_2\to 4R_2-2R_1\\R_3\to 4R_3-1R_1\end{matrix}$$

$$\sim\begin{bmatrix}4&3&2&-7\\0&-2&-20&+10\\0&0&-608&296\end{bmatrix} \begin{matrix}\\\\R_3\to -2R_3+31R_2\end{matrix}$$

$\rho(A)=3,$

$\rho(A,B)=3$

$\rho(A)=\rho(A,B)=n=3.$

The system of equation is consistent & has a unique soln.

$-608z=296$

$z=\frac{-296}{608}$

$-2y-20z=10$

$-2y-20\left(\frac{-37}{76}\right)=10$

$-2y+\frac{185}{19}=10$

$y=\left(10-\frac{185}{19}\right)\left(\frac{-1}{2}\right)$

$y=\frac{-5}{38}$

$4x+3y+2z=-7$

$4x+3\left(\frac{-5}{38}\right)+2\left(\frac{-37}{76}\right)=-7$

$x=\frac{-107}{76}.$

2) $x+y+z=3$

$x+y-z=1$

$3x+3y-5z=1$

Given

$x+y+z=3$

$x+y-z=1$

$3x+3y-5z=1$

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where $AX=B$

$$A=\begin{bmatrix}1&1&1\\1&1&-1\\3&3&-5\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix},\ B=\begin{bmatrix}3\\1\\1\end{bmatrix}$$

$$(A,B)=\begin{bmatrix}1&1&1&3\\1&1&-1&1\\3&3&-5&1\end{bmatrix}.$$

$$\sim\begin{bmatrix}1&1&1&3\\0&0&-2&-2\\0&0&-8&-8\end{bmatrix}\begin{matrix}\\R_2\to R_2-1R_1\\R_3\to R_3-3R_1\end{matrix}$$

$\rho(A)=3 \quad \rho(A,B)=3$

$\rho(A)=\rho(A,B)=n=3.$

The system is consistent and has unique solution.

$-8z=-8$

$\boxed{z=1}.$

$\cancel{y=0}$. Take $\boxed{y=k}$.

$x+y+z=3$

$x+0+1=3$

$x+1=3$

$x=3-1$

$\boxed{x=2}.$

$x+y+z=3$

$x+k+1=3$

$x+k=2$

$\boxed{x=2-k}$

$x=$

$\boxed{x=2-k,\ y=k,\ z=1}.$

3) $2x+3y-z=9$
$x+y+z=9$
$3x-y-z=-1$

$AX=B$

$$A=\begin{bmatrix}2&3&-1\\1&1&1\\3&-1&-1\end{bmatrix}\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix}\quad B=\begin{bmatrix}9\\9\\-1\end{bmatrix}.$$

$$[A,B]=\begin{bmatrix}2&3&-1&9\\1&1&1&9\\3&-1&-1&-1\end{bmatrix}$$

$$\sim\begin{bmatrix}2&3&-1&9\\0&-1&3&9\\0&-11&1&-29\end{bmatrix}\begin{matrix}\\R_2\to 2R_2-R_1\\R_3\to 2R_3-3R_1\end{matrix}$$

$$\sim\begin{bmatrix}2&3&-1&9\\0&-1&3&9\\0&0&-32&-128\end{bmatrix}\begin{matrix}\\\\R_3\to R_3-11R_2\end{matrix}$$

$\rho(A)=3$

$\rho(A,B)=3$

$\rho(A)=\rho(A,B)=n=3$

The System is Consistent and has Unique Soln.

$-32z=-128$

$z=\frac{+128}{+32}$

$\boxed{z=4}.$

$-1y + 3(z) = 9$

$-1y + 3(4) = 9$

$-1y + 12 = 9$

$-1y = 9 - 12$

$-1y = -3$

$\boxed{y = 3}$

$2x + 3y - 1z = 9$

$2x + 3(3) - 1(4) = 9$

$2x + 9 - 4 = 9$

$2x = 9 - 9 + 4$

$2x = 4$

$\boxed{x = 2}$

4) $x + y + z = 6$

$x + 2y - 2z = -3$

$2x + 3y + z = 11$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -2 \\ 2 & 3 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ -3 \\ 11 \end{bmatrix}$$

$Ax = B$

$$[A, B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & -2 & -3 \\ 2 & 3 & 1 & 11 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -3 & -9 \\ 0 & 1 & -1 & -1 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & -3 & -9 \\ 0 & 0 & -2 & 8 \end{bmatrix} R_3 \to R_3 - R_2.$$

$\rho(A) = 3$, $\rho(A, B) = 3$

$\rho(A) = \rho(A, B) = 3$

The System is Consistent and has Unique Solution

$-2z = 8$

$\boxed{z = -4}$

$1y - 3z = -9$

$1y - 3(-4) = -9$

$y + 12 = -9$

$y = -9 - 12$

$\boxed{y = -21}$

$x + y + z = 6$

$x - 21 - 4 = 6$

$x - 25 = 6$

$x = 6 + 25$

$\boxed{x = 31}$

Finding the Eigen Values and Eigen Vector

$$A = \begin{bmatrix} [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] \\ [illegible] & [illegible] & [illegible] \end{bmatrix}$$

The Characteristic eqn $|A - \lambda I| = 0$

In other words

For. 2×2 matrix

C.E $\lambda^2 - S_1\lambda + S_2 = 0$

where $S_1$ - Trace of the Matrix

$S_2 = |A|$.

For 3×3 Matrix

CE $\rightarrow \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

where

$S_1$ - Trace of the Matrix.

$S_2$ = sum of the minors of the main diagonals elements of A.

$S_3 = |A|$.

To Find the Eigen Values

Find the roots of the CE.

To Find Eigen vector:

$(A - \lambda I)X = 0$

where $X \neq 0$.

To find the roots:

eg: ①

$\lambda^2 + 4\lambda + 4 = 0$

$(\lambda+2)(\lambda+2) = 0$

$\lambda = -2, -2$.

eg: ②.

$2\lambda^2 + 4\lambda + 4 = 0$.

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{16 - 4(2)(4)}}{2 \times 2}$

$= \frac{-4 \pm \sqrt{-16}}{4}$

$= \frac{-4 \pm 4\sqrt{-1}}{4}$

$= \frac{-4 \pm 4i}{4}$

$= -1 \pm i$

$$\boxed{\begin{aligned} \lambda_1 &= -1 + i \\ \lambda_2 &= -1 - i \end{aligned}}$$

eg: $\lambda^3 + \lambda^2 + 2\lambda - 4 = 0$

$$\begin{array}{c|cccc} 1 & 0 & 1 & 2 & -4 \\ & 0 & 1 & 2 & 4 \\ \hline & 1 & 2 & 4 & \boxed{0} \end{array}$$

$\lambda^2 + 2\lambda + 4 = 0$.

$$\lambda = \frac{-2 \pm \sqrt{4-4(1)(4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-16}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}}{2}$$

$$\lambda = -1 \pm \sqrt{3}\, i$$

$$\boxed{\lambda_1 = -1 + \sqrt{3}\, i}$$

$$\boxed{\lambda_2 = -1 - \sqrt{3}\, i}$$

Find the Eigen values and Eigen vectors of the matrices ~~to~~ :

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

Soln.

Let $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

The characteristic eqn of $A$ is $|A - \lambda I| = 0$

i.e, $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

where

$S_1$ = Trace of $A$

$= 2+3+2$

$= 7$

$S_2$ = Sum of the minors of the main diagonals element of $A$.

$$= \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$$

$= (6-2) + (4-1) + (6-2)$

$= 4+3+4 = 11$.

$S_3 = |A|$

$$= \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$= 2(4) - 2(1) + 1(-1) = 5$

The characteristic eqn is

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

To find the Eigen values.

| | | | | |
|---|---|---|---|---|
| 1 | 1 | -7 | 11 | -5 |
| | 0 | 1 | -6 | 5 |
| | 1 | -6 | 5 | 0 |

$\boxed{\lambda_1 = 1}$.

$\lambda^2 - 6\lambda + 5 = 0$

$(\lambda - 1)(\lambda - 5) = 0$

$\lambda = 1, 5$.

$\boxed{\lambda_2 = 1}$

$\boxed{\lambda_3 = 5}$.

(or) $x = \frac{-b \pm \sqrt{b^2-}}{2a}$

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$\therefore$ The ~~The~~ Eigen Values are

$\boxed{\lambda_1 = 1}$, $\boxed{\lambda_2 = 1}$, $\boxed{\lambda_3 = 5}$ ...

To find the ~~vector~~ Eigen vectors.

$|A - \lambda I| x = 0$, where $x \neq 0$.

ie,

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Case (i) $\lambda = 1$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_1 + 2x_2 + x_3 = 0$

$x_1 + 2x_2 + x_3 = 0$

$x_1 + 2x_2 + x_3 = 0$

put $x_1 = 0$

$2x_2 + x_3 = 0$

$2x_2 = -x_3$

$\frac{x_2}{-1} = \frac{x_3}{2}$

$x_1 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$

Case (ii) $\lambda = 1$

put $x_2 = 0$

$x_1 + x_3 = 0$

$x_1 = -x_3$

$\frac{x_1}{-1} = \frac{x_3}{1}$

$x_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Case (iii) $\lambda = 5$

$$\begin{bmatrix} 2-5 & 2 & 1 \\ 1 & 3-5 & 1 \\ 1 & 2 & 2-5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left.\begin{aligned} -3x_1 + 2x_2 + x_3 &= 0 \\ x_1 - 2x_2 + x_3 &= 0 \\ x_1 + 2x_2 - 3x_3 &= 0 \end{aligned}\right\} \rightarrow (2)$$

$\frac{x_1}{6-2} = \frac{-x_2}{-3-1} = \frac{x_3}{2+2}$

$\frac{x_1}{4} = \frac{-x_2}{-4} = \frac{x_3}{4}$

$\div 4$ $\quad \frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}$

$x_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

The eigen vectors corresponding to the Eigen values $\lambda = 1, 1, 5$.

are $X_1 = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$, $X_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, $X_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

respectively.

Find the Eigen values and Eigen vectors of the Matrix :-

2. $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

Soln:-

Let

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

The characteristic eqn of A is $|A - \lambda I| = 0$

i,e, $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$.

where,

$S_1$ = Trace of A.

$= 3+2+3 = 8.$

$S_2$ = Sum of the minors of the main diagonals element of A.

$$= \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 2 \end{vmatrix}$$

$= (6-1) + (9-0) + (6-1)$

$= 5 + 9 + 5$

$= 19.$

$S_3 = |A|$

$$= \begin{vmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{vmatrix}$$

$= 3(6-1) + 1(-3+0) + 0$

$= 15 - 3$

$= 12.$

The characteristic eqn is

$\lambda^3 - 8\lambda^2 + 19\lambda - 12 = 0$

To find the Eigen value

$$\begin{array}{c|cccc} 1 & 1 & -8 & 19 & -12 \\ & 0 & 1 & -7 & 12 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$

$\boxed{\lambda_1 = 1}$

$\lambda^2 - 7\lambda + 12 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{7 \pm \sqrt{49 - 4(1)(12)}}{2(1)}$

$= \frac{7 \pm \sqrt{1 - 48}}{2}$

$= \frac{7 \pm \sqrt{-34}}{2}$

$= 7 \pm 1$

$(\lambda - 4)(\lambda - 3) = 0$

$\boxed{\lambda_2 = 4}$ $\boxed{\lambda_3 = 3}$.

The Eigen values are

$\lambda_1 = 1, \lambda_2 = 4, \lambda_3 = 3$

To find the Eigen vectors

$|A-\lambda I|\, x = 0$ where $x \neq 0$

ie $\begin{bmatrix} 3-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

Case (i) $\lambda = 1$

$\begin{bmatrix} 3-1 & -1 & 0 \\ -1 & 2-1 & -1 \\ 0 & -1 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$2x_1 - x_2 = 0$

$-x_1 + x_2 - x_3 = 0$

$-x_2 + 2x_3 = 0$

$\boxed{\begin{array}{l} 2x_1 - x_2 + 0 = 0 \\ 0 - x_2 + 2x_3 = 0 \end{array}}$

$\frac{-x_1}{-2-0} = \frac{x_2}{4-0} = \frac{-x_3}{-2-0} = 0$

$\frac{-x_1}{-2} = \frac{x_2}{4} = \frac{-x_3}{+2}$

$\frac{x_1}{2} = \frac{x_2}{4} = \frac{x_3}{2}$

÷ by 2

$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{1}$

$x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$\lambda_2 = 4$

$\begin{bmatrix} 3-4 & -1 & 0 \\ -1 & 2-4 & -1 \\ 0 & -1 & 3-4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$x_1 - x_2 = 0$

$-x_1 - 2x_2 - x_3 = 0$

$-x_2 - x_3 = 0$

$\frac{-x_1}{1-0} = \frac{-2x_2}{-1-0} = \frac{-x_3}{1-0}$

$\frac{-x_1}{1} = \frac{-2x_2}{-1} = \frac{-x_3}{-1}$

$x_1 = -x_2 = x_3$

$x_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

Case 3 $\lambda_3 = 3$

$\begin{bmatrix} 3-3 & -1 & 0 \\ -1 & 2-3 & -1 \\ 0 & -1 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$0 - x_2 + 0 = 0$

$-x_1 - x_2 - x_3 = 0$

$0 - x_2 + 0 = 0$ $\quad x_3 = 0$

$-x_1 - x_2 - 0 = 0$

$\frac{0}{0-1} = \frac{-x_2}{0-0} = \frac{0}{0}$

$-x_1 = x_2$

$\frac{x_1}{1} = \frac{x_2}{-1}$

$x_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

The eigen Vectors Corresponding to the Eigen value $\lambda = 1, 4, 3$ are

The Eigen $X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$,

$x_3 = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

1) $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

Soln:

$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

The characteristic eqn of A is $|A - \lambda I| = 0$

ie, $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1$ = Trace of A.

$= 2+1+2$

$= 5$

$S_2$ = Sum of the minors of the main diagonals element of A

$= \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$

$= (2-0) + (4-1) + (2-0)$

$= 2+3+2$

$= 7.$

$S_3 = |A|$

$= \begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix}$

$= 2(2-0) - 1(0-0) + 1(0-1)$

$= 4 - 1$

$= 3.$

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$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

To find Eigen:

$$\begin{array}{c|cccc} 1 & 1 & -5 & 7 & -3 \\ & 0 & 1 & -4 & 3 \\ \hline & 1 & -4 & 3 & 0 \end{array} \qquad \boxed{\lambda = 1}$$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\boxed{\lambda_2 = 3}$, $\boxed{\lambda_3 = 1}$.

The Eigen Values $\boxed{\lambda_1 = 1}$, $\boxed{\lambda_2 = 3}$, $\boxed{\lambda_3 = 1}$

To find the Eigen Vectors:-

Let

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2-\lambda & 1 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 1 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Case (i) $\lambda = 1$

$$\begin{bmatrix} 2-1 & 1 & 1 \\ 0 & 1-1 & 0 \\ 1 & 1 & 2-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_1 + x_2 + x_3 = 0$

$x_1 + x_2 + x_3 = 0$

$x_1 = 0$

$x_2 + x_3 = 0$

$x_2 = -x_3$

$\frac{x_2}{-1} = \frac{x_3}{1}$

$$X_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}.$$

Case (ii) $\lambda_2 = 3$

$$\begin{bmatrix} 2-3 & 1 & 1 \\ 0 & 1-3 & 0 \\ 1 & 1 & 2-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0.$$

$-x_1 + x_2 + x_3 - \ldots$

$-2x_2 = 0$

$x_1 + x_2 - x_3 = 0$

$$\frac{x_1}{-1-1} = \frac{-x_2}{1-1} = \frac{x_3}{-1-1}$$

$$\frac{x_1}{-2} = \frac{-x_2}{0} = \frac{x_3}{-2}$$

$$X_2 = \begin{bmatrix} -2 \\ 0 \\ -2 \end{bmatrix}$$

case: 3 $\lambda = 1$

consider $x_1 + x_2 + x_3 = 0$

sub: $x_2 = 0$.

$x_1 = -x_3$

$$\frac{x_1}{-1} = \frac{x_3}{1}$$

$$X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

The Eigen values and Eigen vectors are

$\lambda_1 = 1$, $\lambda_2 = 3$, $\lambda_3 = 1$ and Vectors

$$X_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, X_2 = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}, X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

2) Application of Matrix.

1. Physical related application:

i) Electrical Circuits, condom mechanical optics.

ii) Calculation of battery power outputs, Conversion of electrical energy to Useful energy of resistors

iii) solving problems of Kirchoff's law (voltage and current).

2) Computer related application:-

i) Projection of 3D image to 2D, Creating realistic Screening motion.

~~Sotastic mati~~

ii) Stochastic matrices and Eigen vectors Solvers are used in page rank algorithms, ie, used in ranking of web pages in Google search.

iii) Matrices Calculus:-

i) Generalization of anatical notions like ~~explants~~ exponationals to higher Dimensions.

ii) Coding are entrupting the message.

iv) Geology:-

i) Seismic servays.

ii) to plot graphs.

iii) statics and do scientific studies.
iv) robotic and automotions

i) Used in Graph theory, Quantum Mechanics, Computer Graphics, Solving linear equation, Cytrography.

## (2) Properties of Eigen values:

1) Sum of the Eigen Values is equal to Trace of the Matrix.

2) Product of the Eigen Values = |A|.

3) If $\lambda$ is the Eigen Values of A then $\frac{1}{\lambda}$ is the Eigen value of $A^{-1}$.

4) If $\lambda$ is of eigen value ~~the~~ of A the $\lambda^m$ is the eigen value of $A^m$.

5) If $\lambda$ is the eigen value of A then $k\lambda$ is the eigen value of $kA$.

6) A and $A^T$ have the same eigen values.

7) The eigen values of a real symmetric Matrix is all real numbers.

8) The eigen values of a triangular Matrix are its elements in the main diagonal.

9) Similar Matrices have the same Eigen values.

### Problems:

1) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and $\lambda_1 = 1$, $\lambda_2 = 3$ find. then $\lambda_3 = ?$

Soln:

W.K.T;

Sum of the Eigen Values = Trace of the Matrix A.

$$1+3+\lambda_3 = 2+1+2$$
$$4+\lambda_3 = 5$$
$$\lambda_3 = 5-4$$
$$\boxed{\lambda_3 = 1}$$

$$\therefore \boxed{\lambda_3 = 1}$$

2) If the Eigen values of a 3x3 matrix are 1, 3 and 1 find the determinant of |A| ~~without expanding~~.

Soln:

Product of the Eigen Values = |A|.

W.K.T: $1 \times 3 \times 1 = |A|$ ie, $\lambda_1\lambda_2\lambda_3 = |A|$

$$3 = |A|$$
$$\therefore \boxed{|A| = 3}$$

3) Similar Matrix

$A \sim B$

ie, $A = B^{-1}AB$.

Properties of Eigen vectors:-

1) Eigen vectors of a Matrix A is not Unique.

2) Two Eigen vectors $X_1, X_2$ are called Orthogonal if $X_1^T X_2 = 0$

3) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are distinct Eigen values of a $n \times n$ matrix then the Corresponding Eigen Vectors $X_1, X_2, \ldots, X_n$ form a linearly independent set.

30/6/17.

Unit-2:-

Theory of Equations:-

Relations between Co-efficient and roots.

i) Equations with rationals Co-efficient and irrationals roots.

⇒ Equations with rationals Co-efficient and irrational roots will occurs in pairs.

⇒ Let $f(x) = 0$ be a equation and Suppose that $a + \sqrt{b}$ is a root of the equation where $a$ & $b$ are rational and $\sqrt{b}$ is irrational, then $a - \sqrt{b}$ is also a root.

Problems:

Solve the equation :-

$x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$

Given that one of the root is

$1 - \sqrt{5}$.

Soln:-

Given the

we know that if $1 - \sqrt{5}$ is a root then $1 + \sqrt{5}$ is also an root.

Therefore, The factors are

$(x - (1 - \sqrt{5})) \cdot [x - (1 + \sqrt{5})]$

~~$[x + 2\sqrt{5}][x - \ldots] = 0$~~

$$x^2 - x(1+\sqrt{5}) - (1-\sqrt{5})x + (1-\sqrt{5})(1+\sqrt{5}) = 0$$
$$x^2 - x[1+\sqrt{5}+1-\sqrt{5}] + [1-5] = 0$$
$$x^2 - 2x - 4 = 0$$

$1-\sqrt{5}$ & $1+\sqrt{5}$

Sum of the roots $= 1-\sqrt{5}+1+\sqrt{5} = 2$

Product of the roots $= (1-\sqrt{5})(1+\sqrt{5}) = 1-5 = -4$.

$x^2 - [\text{Sum of the roots}]x + \text{product of the roots} = 0$

$$x^2 - 2x - 4 = 0.$$

$$x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$$

$$\begin{array}{r|l} & x^2 - 3x + 2 \\ \hline x^2-2x-4 & x^4 - 5x^3 + 4x^2 + 8x - 8 \\ & x^4 - 2x^3 - 4x^2 \\ \hline & -3x^3 + 8x^2 + 8x \\ & -3x^3 + 6x^2 + 12x \\ \hline & 2x^2 - 4x - 8 \\ & 2x^2 - 4x - 8 \\ \hline & 0 \end{array}$$

$\therefore x^2 - 3x + 2$ is also a factor.

$\therefore x^4 - 5x^3 + 4x^2 + 8x - 8 = (x^2 - 2x - 4)(x^2 - 3x + 2) = 0$

$[x - (1-\sqrt{5})][x - (1+\sqrt{5})]\,[(x-2)(x-1)] = 0$

$1-\sqrt{5}$, $1+\sqrt{5}$, $2$, $1$ are the roots of the eqn.

2) Solve the eqn $3x^3 - 23x^2 + 72x - 70 = 0$ having $(3+\sqrt{-5})$ as a root.

Soln:

W.K.T.

If $(3+\sqrt{-5})$ is a root then $(3-\sqrt{-5})$ is also a root.

Sum of the roots $= 3+\sqrt{-5}+3-\sqrt{-5}$
$= 6$

Product of the roots $= (3+\sqrt{-5})(3-\sqrt{-5}) = 9-(-5) = 14$

$x^2 - [\text{Sum of the roots}]x + \text{product of the roots} = 0$

$$x^2 - 6x + 14 = 0.$$

$$3x^3 - 23x^2 + 72x - 70 = 0$$

$$\begin{array}{r|l} & 3x - 5 \\ \hline x^2-6x+14 & 3x^3 - 23x^2 + 72x - 70 \\ & 3x^3 - 18x^2 + 42x \\ \hline & -5x^2 + 30x - 70 \\ & -5x^2 + 30x - 70 \\ \hline & 0 \end{array}$$

$3x^3 - 23x^2 + 72x - 70 = 0$

$\Rightarrow (x^2 - 6x + 14)(3x - 5) = 0$

$\Rightarrow (3+\sqrt{5}), (3-\sqrt{5}), 5/3$ are the roots of the above eqn:-

H/w

3) Find the eqn whose roots are.

$4\sqrt{3}$, $5+2\sqrt{-1}$.

Soln:-

The roots are $4\sqrt{3}, -4\sqrt{3}, 5+2\sqrt{-1}, 5-2\sqrt{-1}$

The factor is

$4\sqrt{3}, -4\sqrt{3}$.  $(4\sqrt{3})^2$  $16\times 3$ = 48

Sum of the roots $= 4\sqrt{3} - 4\sqrt{3} = 0$

product of the roots $= (4\sqrt{3})(-4\sqrt{3}) = -48$.

$x^2 - [\text{Sum of the roots}]x + \text{product of the roots} = 0$

$x^2 - 48 = 0. \rightarrow ①$

The factor is

$5+2\sqrt{-1}, 5-2\sqrt{-1}$.

Sum of the roots $= 5+2\sqrt{-1} + 5 - 2\sqrt{-1}$

$= 10$

Product of the roots $= (5+2\sqrt{-1})(5-2\sqrt{-1})$

$= 29$

$x^2 - [\text{Sum of the roots}]x + \text{product of the roots} = 0$

$x^2 - 10x + 29 = 0 \rightarrow ②$  $25 - 4$  $5^2 - 25 - 21$

The eqn is  $48\times 29$

$① \times ② \Rightarrow$

$(x^2 - 48)(x^2 - 10x + 29) = 0$  $(5-2\sqrt{-1})$  $25 + 4(-1)$  $25 + 4$

$x^4 - 10x^3 + 29x^2 - 48x^2 + 480x - 1392 = 0$

$x^4 - 10x^3 - 19x^2 + 480x - 1392 = 0$.

Find the eqn with rational co-efficients whose roots are $1+5\sqrt{-1}$ and $5-\sqrt{-1}$.

Soln:-

The roots are

$1+5\sqrt{-1}, 1-5\sqrt{-1}, 5-\sqrt{-1}, 5+\sqrt{-1}$.

The factor is  $(1-5\sqrt{-1})^2$

$1+5\sqrt{-1}, 1-5\sqrt{-1}$  $1 - 25(-1)$

Sum of the roots $= 1+5\sqrt{-1} + 1 - 5\sqrt{-1}$  $1+25$

$= 2$  $1^2 + 25$  $26$

Product of the roots $= (1+5\sqrt{-1})(1-5\sqrt{-1})$

$= 26$

$x^2 - [\text{Sum of the roots}]x + \text{Product of the roots} = 0$

$x^2 - 2x + 26 = 0. \rightarrow ①$

The factor is

$5-\sqrt{-1},\ 5+\sqrt{-1}$.

Sum of the root $= 5-\sqrt{-1}+5+\sqrt{-1}$.

$= 10$

Product of the roots $= (5-\sqrt{-1})(5+\sqrt{-1})$

$= 26$

$x^2 - [\text{Sum of the roots}]x + \text{Product of the roots} = 0$

$x^2 - 10x + 26 = 0 \longrightarrow (2)$

The eqn is,

(1) × (2).

$(x^2-2x+26)(x^2-10x+26)=0$

$x^4-10x^3+26x^2-2x^3+20x^2-52x+26x^2-260x+676=0$

$x^4-12x^3+72x^2-312x+676=0$.

Relations between roots and Co-efficients of eqn.

Let the eqn be.

$x^n+P_1x^{n-1}+P_2x^{n-2}+\dots+P_{n-1}x+P_n=0$.

If the eqn has roots

$\alpha_1, \alpha_2, \dots, \alpha_n$.

then

$\sum\alpha_1$ = Sum of the roots $= (-1)P_1$

$\sum\alpha_1\alpha_2$ = Sum of the roots taken 2 at a time $= (-1)^2P_2 = P_2$

$\sum\alpha_1\alpha_2\alpha_3$ = Sum of the roots taken 3 at a time $= (-1)^3P_3$

$\alpha_1\alpha_2\dots\alpha_n$ = Product of the roots $= (-1)^nP_n$.

$x^3+P_1x^2+P_2x+P_3=0$

$\alpha_1\ \alpha_2\ \alpha_3$

$\alpha_1\alpha_2+\alpha_1\alpha_3$

$\alpha\beta+\alpha\gamma+\beta\gamma = Q$.

$\sum\alpha_1 = \alpha_1+\alpha_2+\alpha_3 = 6$

$\sum\alpha_1\alpha_2 = \alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3 = 1\times2+1\times3+2\times3 = 11$

$\alpha_1\alpha_2\alpha_3 = 1\times2\times3 = 6$.

$x^3-6x^2+11x-6=0$

If $\alpha, \beta, \gamma$ are the roots of the eqn.

$x^3+px^2+qx+r=0$, express the Value

$\sum\alpha^2,\ \sum\frac{1}{\alpha},\ \sum\frac{1}{\alpha\beta},\ \sum\alpha^2\beta$

$\alpha+\beta+\gamma = (-1)P = -P$

$\alpha\beta + \quad = Q$

$\alpha\beta\gamma = R$

Soln:

The Given eqn.

$$x^3 + px^2 + Qx + R = 0$$

The roots are $\alpha, \beta, \gamma$.

$\Sigma\alpha = \alpha+\beta+\gamma = -P$.

$\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \alpha\gamma = Q$

$\alpha\beta\gamma = -R$

i) $\Sigma\alpha^2$.

$\Sigma\alpha^2 = \alpha^2+\beta^2+\gamma^2$.

$(\alpha+\beta+\gamma)^2 = \alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\alpha\gamma+2\beta\gamma$.

$\therefore = (\alpha+\beta+\gamma)^2 - 2\alpha\beta - 2\beta\gamma - 2\alpha\gamma$.

$= (-p)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$

$= p^2 - 2(Q)$.

$= p^2 - 2Q$.

ii) $\Sigma\frac{1}{\alpha}$

$\Sigma\frac{1}{\alpha} = \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$= \frac{\gamma\beta+\alpha\gamma+\beta\alpha}{\alpha\beta\gamma}$

$= -\frac{Q}{R}$.

iii) $\Sigma\frac{1}{\alpha\beta}$.

$\Sigma\frac{1}{\alpha\beta} = \frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$

$= \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma}$

$= \frac{-P}{-R}$

$= P/R$

iv) $\Sigma\alpha^2\beta$.

$\Sigma\alpha^2\beta = \alpha^2\beta+\alpha^2\gamma+\beta^2\alpha+\beta^2\gamma+\gamma^2\alpha+\gamma^2\beta$

$= \alpha\beta[\alpha+\beta]+\alpha\gamma[\alpha+\gamma]+\beta\gamma[\beta+\gamma]$

$= \alpha\beta[\alpha+\beta+\gamma-\gamma]+\alpha\gamma[\alpha+\gamma+\beta-\beta]+\beta\gamma[\beta+\gamma+\alpha-\alpha]$

$= \alpha\beta[\alpha+\beta+\gamma]-\alpha\beta\gamma+\alpha\gamma[\alpha+\beta+\gamma]-\alpha\beta\gamma+\beta\gamma[\alpha+\beta+\gamma]-\alpha\beta\gamma$

$= [\alpha+\beta+\gamma][\alpha\beta+\alpha\gamma+\beta\gamma]-3\alpha\beta\gamma$

$= -PQ+3R$.

2. If $\alpha, \beta, \gamma$ be the roots of the eqn $x^3 - px^2 + Qx - R = 0$. Then find.

i) $\Sigma\alpha^2$, $\Sigma\frac{1}{\alpha}$, $\Sigma\frac{1}{\alpha\beta}$, $\Sigma\alpha^2\beta^2$, $\Sigma\alpha^3$.

$\alpha+\beta+\gamma = (-1)(-P) = P$

$\alpha\beta+\beta\gamma+\gamma\alpha = (-1)^2 Q = Q$.

$\alpha\beta\gamma = (-1)^3(-R) = R$.

i) $\sum \alpha^2$

$= \alpha^2+\beta^2+\gamma^2$

$= (\alpha+\beta+\gamma)^2 - 2\alpha\beta - 2\alpha\gamma - 2\beta\gamma$

$= (\alpha+\beta+\gamma)^2 - 2[\alpha\beta+\alpha\gamma+\beta\gamma]$

$= P^2 - 2Q$.

ii) $\sum \frac{1}{\alpha}$.

$= \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}$

$= \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$

$= \frac{Q}{R}$.

iii) $\sum \frac{1}{\alpha\beta}$.

$= \frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$

$= \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma}$

$= P/R$

iv) $\sum \alpha^2\beta^2$

$= \alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2$

$= [\alpha\beta+\beta\gamma+\alpha\gamma]^2 - 2(\alpha\beta)(\beta\gamma) - 2(\alpha\beta)(\alpha\gamma) - 2(\beta\gamma)(\alpha\gamma)$

$= [\alpha\beta+\beta\gamma+\alpha\gamma]^2 - 2[\alpha\beta^2\gamma + \alpha^2\beta\gamma + \gamma^2\alpha\beta]$

$= [\alpha\beta+\beta\gamma+\alpha\gamma]^2 - 2[\beta(\alpha\beta\gamma)+\alpha(\alpha\beta\gamma)+\gamma(\alpha\beta\gamma)]$

$= Q^2 - 2[(\alpha\beta\gamma)[\alpha+\beta+\gamma]]$

$= Q^2 - 2RP$

v) $\sum \alpha^3$.

$= \alpha^3+\beta^3+\gamma^3$

$= (\alpha+\beta+\gamma)^3$

$(a+b+c)^3 = a^3+b^3+c^3 - 3[(a+b+c)(ab+bc+ca)] + 3abc$.

$(\alpha+\beta+\gamma)^3 = \alpha^3+\beta^3+\gamma^3 - 3\{[\alpha+\beta+\gamma][\alpha\beta+\gamma\beta+\alpha\gamma]\} + 3\alpha\beta\gamma$

$(\alpha+\beta+\gamma)^3 - 3[PQ] + 3R$

$= P^3 - 3PQ + 3R$

$= P^3 - 3PQ + 3R$.

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3) If $\alpha+\beta+\gamma$ be the roots of the eqn $x^3+px^2+Qx+R=0$ find the value of

$$\frac{(\beta^2+\gamma^2)}{\beta\gamma}+\frac{(\gamma^2+\alpha^2)}{\alpha\gamma}+\frac{(\alpha^2+\beta)^2}{\alpha\beta}.$$

$$\alpha+\beta+\gamma=(-1)p=-P$$
$$\alpha\beta+\beta\gamma+\gamma\alpha=(-1)^2Q=Q$$
$$\alpha\beta\gamma=(-1)^3(R)=-R.$$

$\alpha+\beta+\gamma=-P$
$\alpha\beta+\beta\gamma+\gamma\alpha=Q$
$\alpha\beta\gamma=-R.$

$$\frac{\beta^2+\gamma^2)}{\beta\gamma}+\frac{(\gamma^2+\alpha^2)}{\alpha\gamma}+\frac{(\alpha^2+\beta^2)}{\alpha\beta}$$

$$=\frac{(\alpha^2+\beta^2+\gamma^2-\alpha^2)}{\beta\gamma}+\frac{(\alpha^2+\beta^2+\gamma^2-\beta^2)}{\alpha\gamma}+\frac{(\alpha^2+\beta^2+\gamma^2-\gamma^2)}{\alpha\beta}$$

$$=\frac{(\alpha^2+\beta^2+\gamma^2)}{\beta\gamma}-\frac{\alpha^2}{\beta\gamma}+\frac{(\alpha^2+\beta^2+\gamma^2)}{\alpha\gamma}-\frac{\beta^2}{\alpha\gamma}+\frac{(\alpha^2+\beta^2+\gamma^2)}{\alpha\beta}-\frac{\gamma^2}{\alpha\beta}$$

$$=(\alpha^2+\beta^2+\gamma^2)\left[\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}+\frac{1}{\alpha\beta}\right]-\left[\frac{\alpha^2}{\beta\gamma}+\frac{\beta^2}{\alpha\gamma}+\frac{\gamma^2}{\alpha\beta}\right]$$

$$=\left[(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma)\right]\left[\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}\right]-\left[\frac{\alpha^3+\beta^3+\gamma^3}{\alpha\beta\gamma}\right]$$

$$=\left[(-P)^2-2Q\right]\left[\frac{-P}{-R}\right]-\left[\frac{1}{-R}(\alpha^3+\beta^3+\gamma^3)\right]$$

Consider:-

$$\alpha^3+\beta^3+\gamma^3=(\alpha+\beta+\gamma)^3+3\alpha\beta\gamma-3(\alpha+\beta+\gamma)(\alpha\beta+\beta\gamma+\gamma\alpha)$$
$$=(-P)^3+3(-R)-3(-P)(Q)$$
$$=(-P)^3-3R+3PQ.$$

Let.

$$=\left[P^2-2Q\right]\left[\frac{P}{R}\right]+\frac{(-P^3-3R+3PQ)}{R}$$

$$=\frac{P^3-2PQ-P^3-3R+3PQ}{R}$$

$$=\frac{PQ-3R}{R}$$

4) If $\alpha,\beta$ and $\gamma$ be the roots of the eqn $x^3+px^2+qx+R=0$. find the value of $\sum\alpha^2+1$.

Soln:

We know that.

$$\alpha+\beta+\gamma=-P$$
$$\alpha\beta+\beta\gamma+\gamma\alpha=Q$$
$$\alpha\beta\gamma=-R.$$

$$\sum\alpha^2+1=\alpha^2+1+\beta^2+1+\gamma^2+1$$
$$=\alpha^2+\beta^2+\gamma^2+3.$$
$$=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)+3$$
$$(-P)^2-2(Q)+3$$
$$P^2-2(Q)+3$$

Reciprocal Equation.

Conditions:

Let the eqn be.

$$x^n + p_1 x^{n-1} + p_2 x^{n-2} + \dots + p_{n-1} x + p_n = 0.$$

1) If the Co-efficients have all like Sign.

ii) Thn (-1) is a root.

If (-1) is a root then $(x+1)$ is a factor.

ii) If the Co-efficients of the term equi-distance from the first and the last have opposite sign. Thn (+1) is a root. thn $(x-1)$ is a factor.

(i) If the equation is of odd degree thn $(x-1)$ is a factor.

(ii) If the equation is of even degree thn (alternate + or -) thn $(x^2-1)$ is a factor.

Problems:

1) Find the roots of the equation.

$$x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0.$$

Soln:

All the Co-efficients are positive

$\therefore$ WKT

$(-1)$ is the root.

(i.e) $(x+1)$ is a factor.

$$x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0,$$

$$x^5 + x^4 + 3x^4 + 3x^3 + 3x^2 + 3x + x + 1 = 0$$

$$x^4(x+1) + 3x^3(x+1) + 3x(x+1) + (x+1) = 0$$

$$(x+1)(x^4 + 3x^3 + 3x + 1) = 0$$

$(x+1) \Rightarrow x = -1$ is a root.

$x+1 = 0$

$x = -1$

The eqn is divide by $x^2$.

$$x^2 \left[\frac{x^4}{x^2} + \frac{3x^3}{x^2} + \frac{3x}{x^2} + \frac{1}{x^2}\right] = 0$$

$$\left[x^2 + 3x + 3/x + \frac{1}{x^2}\right] = 0.$$

$$\left[x^2 + \frac{1}{x^2} + 3\left(x + \frac{1}{x}\right)\right] = 0 \rightarrow (1).$$

$(x+\frac{1}{x})$

$(x^2+\frac{1}{x^2})$

$(x+\frac{1}{x})$ $x^2+\frac{1}{x^2}$

Let put $\boxed{x + \frac{1}{x} = z}$

$x + \frac{1}{x} = z$

$$\left(x + \frac{1}{x}\right)^2 = z^2.$$

$\left(x+\frac{1}{x}\right)^2 = z^2$

$x^2+2x\frac{1}{x}+\frac{1}{x^2}=z^2$

$x^2+2+\frac{1}{x^2}=z^2$

$$\boxed{x^2+\frac{1}{x^2}=z^2-2}$$

① $\Rightarrow$ $z^2-2+3z=0$

$z^2+3z-2=0$

$z=\frac{-3\pm\sqrt{9-4(1)(-2)}}{2(1)}$

$=\frac{-3\pm\sqrt{9+8}}{2}$

$$\boxed{z=\frac{-3\pm\sqrt{17}}{2}}$$

We know.

$z=x+\frac{1}{x}$

$\frac{-3\pm\sqrt{17}}{2}=x+\frac{1}{x}$

$\frac{-3\pm\sqrt{17}}{2}=\frac{x^2+1}{x}$

$(-3\pm\sqrt{17})x=(x^2+1)2$

$2x^2-(-3\pm\sqrt{17})x+2=0$

$x=\frac{(-3\pm\sqrt{17})\pm\sqrt{(-3\pm\sqrt{17})^2-4(2)(2)}}{2(2)}$

$x=\frac{-3\pm\sqrt{17}\pm\sqrt{(-3\pm\sqrt{17})^2-16}}{4}$

$\& x=-1$ are the roots of the above equation.

2). Solve the eqn $6x^5-x^4-43x^3+43x^2+x-6=0$

Soln:

WKT:

$(x-1)$ is the factor.

Therefore $x=1$ is a root.

$6x^5-x^4-43x^3+43x^2+x-6=0$

$(6x^5-6x^4+5x^4-5x^3-38x^3+38x^2+5x^2-5x+6x-6$.

$6x^4(x-1)+5x^3(x-1)-38x^2(x-1)+5x(x-1)+6(x-1)=0$

$(x-1)(6x^4+5x^3-38x^2+5x+6)=0$

$(x-1)\Rightarrow x=1$ is a root.

The eqn is divide by $x^2$.

$\left[\frac{6x^4}{x^2}+\frac{5x^3}{x^2}-\frac{38x^2}{x^2}+\frac{5x}{x^2}+\frac{6}{x^2}\right]=0$

$$\left[6x^2+5x-38+\frac{5}{x}+\frac{6}{x^2}\right]=0$$

$$6\left(x^2+\frac{1}{x^2}\right)+5\left(x+\frac{1}{x}\right)-38=0 \rightarrow (1)$$

Let $z = x+\frac{1}{x}$

$$z^2-2 = x^2+\frac{1}{x^2}$$

Sub in (1).

$$6(z^2-2)+5z-38=0$$

$$6z^2+5z-12-38=0$$

$$6z^2+5z-50=0$$

$$z=\frac{-5\pm\sqrt{25-4(6)(-50)}}{2(6)}$$

$$=\frac{-5\pm\sqrt{25+1200}}{12}$$

$$=\frac{-5\pm\sqrt{1225}}{12}$$

$$=\frac{-5\pm 35}{12}$$

$$=\frac{30}{12}, \frac{-40}{12}$$

$$=\frac{5}{2}, \frac{-10}{3}$$

WKT:

$$z = x+\frac{1}{x}$$

$$xz = x^2+1$$

$$x^2-xz+1=0$$

Case (i) put $z=5/2$.

$$x^2-\frac{5}{2}x+1=0$$

$$x=\frac{+(5/2)\pm\sqrt{\frac{25}{4}-4(1)(1)}}{2(1)}$$

$$=\frac{5/2\pm 3/2}{2} = \frac{8}{2\times 2}, \frac{2}{2\times 2}$$

$$=2, \frac{1}{2}$$

Case (ii) put $z=-10/3$.

$$x^2+\frac{10}{3}x+1=0$$

$$x=\frac{\frac{-10}{3}\pm\sqrt{\frac{100}{9}-4(1)(1)}}{2}$$

$$=\frac{\frac{-10}{3}\pm\frac{8}{3}}{2} = \frac{-2}{6}, \frac{-18}{6}$$

$$=-\frac{1}{3}, -3.$$

The required roots are,

$$x = 1, 2, \frac{1}{2}, -\frac{1}{3}, -3.$$

3) Solve: $6x^6-35x^5+56x^4-56x^2+35x-6=0$.

Soln:

WKT: $(x^2-1)$ is the factor

$\therefore x=\pm 1$ is a root.

$6x^6 - 6 - 35x^5 + 35x + 56x^4 - 56x^2 = 0$

$6(x^6 - 1) - 35x(x^4 - 1) + 56x^2(x^2 - 1) = 0$

Consider.

$x^6 - 1 = (x^2)^3 - 1^3.$

$= (x^2 - 1)(x^4 + x^2 + 1).$

$x^4 - 1 = (x^2)^2 - 1^2$

$= (x^2 + 1)(x^2 - 1)$

$6[(x^2-1)[x^4 + x^2 + 1)] - 35x((x^2+1)(x^2-1) + 56x^2(x^2-1) = 0$

$(x^2-1)[6(x^4+x^2+1) - 35x(x^2+1) + 56x^2 = 0$

$x \pm 1$
is a root.

$6x^4 + 6x^2 + 6 - 35x^3 - 35x + 56x^2 = 0.$

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0.$

$\div x^2 \quad \frac{6x^4}{x^2} - \frac{35x^3}{x^2} + \frac{62x^2}{x^2} + \frac{6}{x^2} = 0$

$6x^2 - 35x + 62 + \frac{6}{x^2} = 0.$

$6\left(x^2 + \frac{1}{x^2}\right) - 35x + 62 = 0.$

Let $z = x + \frac{1}{x}$

$z^2 - 2 = x^2 + \frac{1}{x^2}.$

$\cancel{6(z^2 - 2) - 35x + 62 = 0.}$

$\cancel{6z^2 - 35x - 12 + 62 = 0}$

$6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0.$

$\div x^2 \quad \frac{6x^4}{x^2} - \frac{35x^3}{x^2} + \frac{62x^2}{x^2} - \frac{35x}{x^2} + \frac{6}{x^2} = 0$

$6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0$

$6\left(x + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0.$

Let $z = x + \frac{1}{x}.$

$z^2 - 2 = x^2 + \frac{1}{x^2}.$

$6(z^2 - 2) - 35(z) + 62 = 0$

$6z^2 - 12 - 35z + 62 = 0$

$6z^2 - 35z + 50 = 0.$

$z = \frac{35 \pm \sqrt{1225 - 4(6)(50)}}{2(6)}.$

$= \frac{35 \pm \sqrt{1225 - 1200}}{12}.$

$= \frac{35 \pm \sqrt{25}}{12}$

$= \frac{35 \pm 5}{12}$

$z = \frac{40}{12}, \frac{30}{12}.$

$\boxed{z = \frac{10}{3}, \frac{5}{2}}$

$35 \times 35$
$175$
$5$

$200 \times 6$
$1200$

$10$
$20$
$40$
$12$
$6$

$15$ $5$
$30$
$12$
$6$ $2$

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W.K.T.

$z = x + \frac{1}{x}$

$xz = x^2+1$

$x^2 - xz + 1 = 0$

Case(i) $z = \frac{10}{3}$

~~$\frac{100}{9} - \frac{10}{3}z + 1 = 0$~~

$x^2 - \frac{10}{3}z + 1 = 0$

$x = \frac{\frac{10}{3} \pm \sqrt{\frac{100}{9} - 4(1)(1)}}{2(1)}$

$= \frac{\frac{10}{3} \pm \sqrt{\frac{64}{9}}}{2}$

$= \frac{\frac{10}{3} \pm \frac{8}{3}}{2}$

$x = \frac{18}{3} \times \frac{1}{2} = \frac{18}{6}, \quad \frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$

$\boxed{x = 3, \frac{1}{3}}$

Case ii $z = \frac{5}{2}$.

$x^2 - \frac{5}{2}x + 1 = 0$

$x = \frac{\frac{5}{2} \pm \sqrt{\frac{25}{4} + (4)(1)(1)}}{2}$

$= \frac{5}{2} \pm \sqrt{\frac{9}{4}}$

$= \frac{\frac{5}{2} \pm \frac{3}{2}}{2}$

$x = \frac{8}{4}, \frac{2}{4}$.

$\boxed{x = 2, \frac{1}{2}}$.

The required roots $\pm 1, 2, 3, \frac{1}{3}, \frac{1}{2}$.

3) $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$.

$6(x^6-1) - 25x(x^4-1) + 31x^2(x^2-1) = 0$.

Consider.

$x^6 - 1 = (x^2)^3 - 1^3$.

$= (x^2-1)(x^4+x^2+1)$.

$x^4 - 1 = (x^2)^2 - 1$.

$= (x^2+1)(x^2-1)$.

$6(x^2-1)(x^4+x^2+1) - 25x(x^2+1)(x^2-1) + 31x^2(x^2-1) = 0$

$(x^2-1)\left[6(x^4+x^2+1) - 25x(x^2+1) + 31x^2\right] = 0$

$\therefore$ $x = \pm 1$ are the roots.

$6x^4 + 6x^2 + 6 - 25x^3 - 25x + 31x^2 = 0$

~~$\div x^2$~~

$\frac{6x^4}{x^2} + \frac{6x^2}{x^2} + \frac{6}{x}$

~~$6x^4+6x$~~

$6x^4 - 25x^3 + 37x^2 - 25x + 6 = 0$

$\div x^2$.

$\frac{6x^4}{x^2} - \frac{25x^3}{x^2} + \frac{37x^2}{x^2} - \frac{25x}{x^2} + \frac{6}{x^2} = 0$

$6x^2 - 25x + 37 - \frac{25}{x} + \frac{6}{x^2} = 0$

$6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x + \frac{1}{x}\right) + 37 = 0$

Let $z = x + \frac{1}{x}$.

$z^2 - 2 = x^2 + \frac{1}{x^2}$.

$6z^2 - 12 - 25z + 37 = 0$

$6z^2 - 25z + 25 = 0$.

$z = \frac{25 \pm \sqrt{225 - 4(6)(25)}}{2(6)}$

$= \frac{25 \pm \sqrt{625 - 600}}{12}$

$= \frac{25 \pm \sqrt{25}}{12}$

$z = \frac{25 \pm 5}{12}$

$z = \frac{30}{12}, \frac{20}{12}$

$\boxed{z = 5/3, 5/2}$.

37
12
25

100×6

Let $z = x + \frac{1}{x}$.

$zx = x^2 + 1$

$x^2 - zx + 1 = 0$

Case (i) $z = 5/3$.

$x^2 - 5/3\, x + 1 = 0$

$x = \frac{5/3 \pm \sqrt{\frac{25}{9} - 4(1)(1)}}{2}$

$= \frac{5/3 \pm \sqrt{\frac{-11}{9}}}{2}$

$= \frac{5/3 \pm \frac{1}{3}\sqrt{-11}}{2}$

$x = \frac{3}{2}, \frac{1}{6}$

$\boxed{x = 3/2, \frac{1}{6}}$

$\frac{36}{25}$
11

Case (ii) $z = 5/2$.

$x^2 - 5/2\, x + 1 = 0$

$x = \frac{5/2 \pm \sqrt{\frac{25}{4} - 4(1)(1)}}{2}$

$= \frac{5/2 \pm \sqrt{\frac{9}{4}}}{2}$

$= \frac{5/2 \pm 3/2}{2}$

$= \frac{8}{4}, 2/4$.

$\boxed{x = 2, \frac{1}{2}}$.

Case(ii) $\cancel{x} = \frac{5}{3}$

$x^2 - \frac{5}{3}x + 1 = 0.$

$x = \dfrac{\frac{5}{3} \pm \sqrt{\frac{25}{9} - 4(1)(1)}}{2}$

$= \dfrac{\frac{5}{3} \pm \sqrt{\frac{25-36}{9}}}{2}$

$= \dfrac{\frac{5}{3} \pm \sqrt{\frac{-11}{9}}}{2}$

$= \dfrac{\frac{5}{3} \pm \frac{1}{3}\sqrt{-11}}{2}$

$= \frac{1}{3} \times \frac{5 \pm \sqrt{-11}}{2}$

$= \frac{5 \pm \sqrt{-11}}{6}$

$x = \frac{5+\sqrt{-11}}{6}, \frac{5-\sqrt{-11}}{6}.$

∴ The required roots of the eqn are $\frac{5+\sqrt{-11}}{6}, \frac{5-\sqrt{-11}}{6}, 2, \frac{1}{2}, \pm 1.$

5) Solve $x^{10} - 3x^8 + 5x^6 - 5x^4 + 3x^2 - 1 = 0$

$x^{10} - 3x^8 + 5x^6 - 5x^4 + 3x^2 - 1 = 0$

$(x^2-1)$ is a factor.

$x^{10} - x^8 - 2x^8 + 2x^6 + 3x^6 - 3x^4 - 2x^4 + 2x^2 + x^2 - 1 = 0$

$x^8(x^2-1) - 2x^6(x^2-1) + 3x^4(x^2-1) - 2x^2(x^2-1) + (x^2-1) = 0$

$(x^2-1)(x^8 - 2x^6 + 3x^4 - 2x^2 + 1) = 0.$

$(x^2-1)$ is a factor.

$x^2 - 1 \Rightarrow x = \pm 1$ is a root.

The eqn divide by $x^4$.

$x^8 - 2x^6 + 3x^4 - 2x^2 + 1 = 0.$

$\frac{x^8}{x^4} - \frac{2x^6}{x^4} + \frac{3x}{x^4} - \frac{2x^2}{x^4} + \frac{1}{x^4} = 0$

$x^4 - 2x^2 + 3 - \frac{2}{x^2} + \frac{1}{x^4} = 0.$

$\left(x^4 + \frac{1}{x^4}\right) - 2\left(x^2 + \frac{1}{x^2}\right) + 3 = 0.$

$z = x^2 + \frac{1}{x^2}$ $\boxed{1 = z} = \frac{x^4+1}{x^2} =$

$\cancel{x^4 + \frac{1}{x^4} = z^2}$

$z =$

13/9/17

# Differential Calculus:

## And its Application:-

Differentiation and derivatives of Simple functions.

The rate of change in y with respect to x can be measured using the derivatives.

$\frac{dy}{dx}$ in physics velocity is equal to $\frac{dx}{dt}$.

The method of finding the derivative of a function is called differentiation.

Basic diffentiation formula:

1) $\frac{d}{dx} c = 0$,

2) $\frac{d}{dx} x^n = nx^{n-1}$

3) $\frac{d}{dx} e^{ax} = ae^{ax}$.

4) $\frac{d}{dx} \sin ax = a \cos ax$..

DSC

5) $\frac{d}{dx} \cos ax = -a \sin ax$

6) $\frac{d}{dx} \tan ax = a \sec^2 ax$.

7) $\frac{d}{dx} \sec ax = a \sec ax \tan ax$.

8) $\frac{d}{dx} \operatorname{cosec} ax = -a \operatorname{cosec} ax \cot ax$.

9) $\frac{d}{dx} \cot ax = -a \operatorname{cosec}^2 ax$.

10) $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$.

11) $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$.

12) $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$.

13) $\frac{d}{dx}(u \pm v) = \frac{d}{dx} u \pm \frac{dv}{dx}$.

14) $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

$= uv' + vu'$.

15) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$.

16) $\frac{d}{dx} cu = c\frac{d}{dx} u = cu'$.

$e^{x^2} = e$

$d(e^{x^2})$

put $u = x^2$

$\boxed{du = 2x\,dx}$.

$\frac{u}{v} = \frac{vu' - uv'}{v^2}$

I) Find the derivative of the following:-

i) $\frac{d}{dx}[4x^3]$

$= 4[3(x)^{3-1}]$

$= 12x^2$.

2) $\frac{d}{dx}[4x^3+5\sin x]$.

$\frac{d}{dx}[4x^3+5\sin x] = 4(3x^{3-1}) + 5\cos x$.

$= 12x^2 + 5\cos x$.

3) $\frac{d}{dx}[e^{7x}+8\cos 3x]$

$\frac{d}{dx}[e^{7x}+8\cos 3x] = 7e^{7x} - 8[3\sin 3x]$

$= 7e^{7x} - 24\sin 3x$.

4) $\frac{d}{dx}[5+x^6+\cos 5x+7e^{4x}]$

$\frac{d}{dx}[5+x^6+\cos 5x+7e^{4x}] = 0+6x^5 - 5\sin 5x + 7\cdot 4e^{4x}$.

$= 6x^5 - 5\sin 5x + 28e^{4x}$

5) $\frac{d}{dx}[5\cos 2x + 6\sin 2x + 7\sec 4x]$.

$\frac{d}{dx}[5\cos 2x + 6\sin 2x + 7\sec 4x] = 5(-2\sin 2x) + 6(2\cos 2x) + 7(4\sec 4x\tan 4x)$

$= -10\sin 2x + 12\cos 2x + 28\sec 4x\tan 4x$

II Find the derivative of the following:-

1). $\frac{d}{dx}[4\cot 5x + 6\tan 3x]$

$\frac{d}{dx}[4\cot 5x + 6\tan 3x] = -20\,\mathrm{cosec}^2 5x + 18\sec^2 3x$.

2) $\frac{d}{dx}[3+7\sec 8x + 4\,\mathrm{cosec}\,x] = 0 + 7[8\sec 8x\tan 8x] - [4\,\mathrm{cosec}\,x\cot x]$

$= 56\sec 8x\tan 8x - 4\,\mathrm{cosec}\,x\cot x$

3) $\frac{d}{dx}[8e^{4x}+9x^9+6e^{8x}] = 8(4e^{4x}) + 9(9x^{9-1}) + 6(8e^{8x})$

$= 32e^{4x} + 81x^8 + 48e^{8x}$.

4) $\frac{d}{dx}[8\cos^{-1}x + 4\sin^{-1}x] = 8\frac{-1}{\sqrt{1-x^2}} + 4\frac{1}{\sqrt{1-x^2}}$.

$= \frac{-8+4}{\sqrt{1-x^2}}$

$= \frac{-4}{\sqrt{1-x^2}}$.

$\sin^{-1} = \frac{1}{\sqrt{1-x^2}}$

$\cos^{-1} = \frac{-1}{\sqrt{1-x^2}}$

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$\frac{d}{dx}$ [...]

$= 4\cos 4x + \frac{1}{1+x^2}$.

$= \frac{\cancel{4\cos 4x}}{\cancel{1+x^2}} = 4\cos 4x + \frac{1}{1+x^2}$.

I)

$\log_a 2x$

$\frac{1}{2x} \cdot 2x$

1) $\frac{d}{dx} e^{\sin x^2}$.

$= \sin x^2 \, e^{\sin x^2} \cos x^2 (2x)$.

2) $\frac{d}{dx} (x^2+7x+2)(e^x - \log x)$.

$= (x^2+7x+2)\left(e^x - \frac{1}{x}\right) - (e^x - \log x)(2x+7)$

$=$

1) $y = x + \cos x$.

$\frac{dy}{dx} = 1 - \sin x$

$\frac{d^2y}{dx^2} = -\cos x$.

2) $y = xe^{-x} + be^x$.

$\frac{dy}{dx} = x(-e^{-x}) + e^{-x}(1) + b(e^x) + e^x$.

$= -xe^{-x} + e^{-x} + be^x$

$\frac{d^2y}{dx^2} = -x(-e^{-x}) + e^{-x}(-1) + (-e^{-x}) + b(e^x) + 0$

$= xe^{-x} - e^{-x} - e^{-x} + be^x$.

$= xe^{-x} - 2e^{-x} + be^x$.

I. If $y = x + 1/x$ show that $x^2y_2 + xy_1 - y = 0$

$y = x + 1/x$.

$y_1 = \frac{dy}{dx} = (1) - \frac{1}{x^2}$.

$y_2 = \frac{d^2y}{dx^2} = 2/x^3$.

Consisder

$x^2y_2 + xy_1 - y = x^2 \cdot 2/x^3 + x(1 - 1/x^2) - (x + 1/x)$

$= 2/x + x - 1/x - x - 1/x$

$= 2/x - 2/x = 0$.

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2) If $y = e^x \sin x$ PT $y_2 - 2y_1 + 2y = 0$

$y_1 \Rightarrow \frac{dy}{dx} = e^x \cos x + \sin x \, e^x$.

$y_2 \Rightarrow \frac{d^2y}{dx^2} = e^x(-\sin x) + \cos x \, e^x + \sin x \, e^x + \cos x \, e^x$

$= -\sin x \, e^x + \cos x \, e^x + \sin x \, e^x + \cos x \, e^x$

$= 2\cos x \, e^x$.

$y_2 - 2y_1 + 2y = 2\cos x \, e^x - 2(e^x \cos x + \sin x \, e^x) + 2e^x \sin x$

$= 2\cos x \, e^x - 2e^x \cos x - 2\sin x \, e^x + 2e^x \sin x$

$= 0$

$e^x \cos x + \sin x \, e^x$

3) If $y = e^{ax} \sin bx$. PT $y_2 - 2ay_1 + (a^2 + b^2)y = 0$

$y_1 = \frac{dy}{dx} = e^{ax}(b\cos bx) + \sin bx \, ae^{ax}$.

$= e^{ax} b\cos bx + \sin bx \, ae^{ax}$.

$y_2 = \frac{d^2y}{dx^2} = e^{ax}(-b^2 \sin bx) + b\cos bx \, ae^{ax} + \sin bx \, a^2 e^{ax}$
$+ ae^{ax} b\cos bx$

$= -b^2 e^{ax} \sin bx + b\cos bx \, ae^{ax} + \sin bx \, a^2 e^{ax}$
$+ ae^{ax} b\cos bx$

$y_2 - 2ay_1 + (a^2 + b^2)y = -b^2 e^{ax} \sin bx + b\cos bx \, ae^{ax} +$
$\sin bx \, a^2 e^{ax} + a e^{ax} b\cos bx$
$-2ae^{ax} b\cos bx - 2a \sin bx \, ae^{ax}$
$+ a^2 e^{ax} \sin bx + b^2 e^{ax} \sin bx$

$= 0$

4) $y = x^3 - 1$ PT $x^2 y_3 - 2xy_2 + 2y_1 = 0$

$y_1 = \frac{dy}{dx} = 3x^2 - 0 = 3x^2$

$y_2 = \frac{d^2y}{dx^2} = 3(2x) = 6x$

$y_3 = \frac{d^3y}{dx^3} = 6(1) = 6$

$x^2 y_3 - 2xy_2 + 2y_1 = x^2(6) - 2x(6x) + 2(3x^2)$

$= 6x^2 - 12x^2 + 6x^2$.

$= 0$.

$e^x \cos x + \sin x \, e^x$

7) $y = (x + \sqrt{x^2 - 1})^m$ PT $(x^2 - 1)y_2 + xy_1 - m^2 y = 0$.

$y_1 = \frac{dy}{dx} = m(x + \sqrt{x^2-1})^{m-1} \left(1 + \frac{1}{2}(x^2-1)^{-1/2}\right)(2x)$

$\cancel{= m(x + (x^2-1)^{1/2})^{m-1} \left(1 + \frac{1}{\sqrt{x^2-1}^{1/2}}\right)}$

$= m(x + (x^2-1)^{1/2})^{m-1} \left[1 + \frac{1}{(x^2-1)^{1/2}}\right]$

$y_2 = \frac{d^2y}{dx^2} = m(m-1)(x + (x^2-1)^{1/2})\left(1 + \frac{1}{2}(x^2-1)^{-1/2}\right)($

$\left[1 + \frac{1}{(x^2-1)^{1/2}}\right] + m(x + (x^2-1)^{1/2})^{m-1}$

$\left[\frac{(x^2-1)^{1/2}(1) - x \frac{1}{2}(x^2-1)^{-1/2} 2x}{x^2-1}\right]$

$(x^2-1)y_2 + xy_1 = m^2 y = 0$.

$x \, m(m-1)$

Application of differential.

~~finding~~ the Curve

radius of curvature.

radius of curvature $\rho = \frac{[1+y_1^2]^{3/2}}{y_2}$

where $y_1 = \frac{dy}{dx}$ and $y_2 = \frac{d^2y}{dx^2}$

1) Find the radius of curvature of the parabola $y_2 = 4ax$

Soln:

Given

$y^2 = 4ax$ at $(a,a)$

$2yy_1 = 4a$

$y_1 = \frac{4a}{2y} = \frac{2a}{y}$, $y_1(a,a) = \frac{2a}{a} = 2$

$y_2 = \frac{-2a}{y^2}\frac{dy}{dx}$

$y_2 = \frac{-2a}{y^2}y_1$

$y_2 = \frac{-2a}{y^2} \times \frac{2a}{y}$

$y_2 = -\frac{4a^2}{y^3}$

at $y_2(a,a) = \frac{-4a^2}{a^3} = \frac{-4}{a}$

Radius of curvature $\rho = \frac{[1+y_1^2]^{3/2}}{y_2}$

$= \frac{[1+4]^{3/2}}{-4/a}$

$= -\frac{5^{3/2}a}{4}$

Radius cannot be in negative

Radius of $\rho = \frac{a5^{3/2}}{4}$

2) Find the radius of curvature at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ on the curve $\sqrt{x}+\sqrt{y} = 1$.

Given $\sqrt{x}+\sqrt{y} = 1$ at $(1/4, 1/4)$ $\qquad \rho = \left[\frac{[1+y_1^2]^{3/2}}{y_2}\right]$

i.e $x^{1/2}+y^{1/2} = 1$

$\frac{1}{2}x^{-1/2}+\frac{1}{2}y^{-1/2}y_1 = 0$

$y_1 = -\frac{1}{2}x^{-1/2} \cdot 2y^{1/2} = \frac{-y^{1/2}}{x^{1/2}}$ $\qquad \rho = \frac{[1+y_1^2]^{3/2}}{y_2}$

$y_1(1/4, 1/4) = \frac{-(1/4)^{1/2}}{(1/4)^{1/2}} = -1$

$y_2 = -\frac{\left[x^{1/2}\left(-\frac{1}{2}y^{-1/2}\right)y_1 - y^{1/2}\left(-\frac{1}{2}\right)x^{-1/2}\right]}{(x^{1/2})^2}$

$= \frac{-\left[(1/4)^{1/2}\left(\frac{+1}{2(1/4)^{1/2}}\right)(-1) - (1/4)^{1/2}(-1/2)\frac{1}{(1/4)^{1/2}}\right]}{1/4}$

$= \frac{-[1/2 + 1/2]}{1/4} = \frac{+1}{1/4} = 4.$

$\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{[1+(-1)^2]^{3/2}}{+4} = \frac{2^{3/2}}{4} = \frac{\sqrt{2\times2\times2}}{4} = \frac{\sqrt{2}}{2}$

$= 1/\sqrt{2}$

radius of curvature $\rho = \frac{1}{\sqrt{2}}$

2) Find the radius of curvature $xy = c$ at $(1,1)$.

$xy = c$ at $(1,1)$.

$x\frac{dy}{dx} + y(1) = 0$

$xy_1 + y = 0$

$y_1 = -y/x$ at

$y_1(1,1) = \frac{-1}{1} = -1$

$y_2 = \frac{x \cdot (\frac{dy}{dx}) - y(1)}{x^2}$

$y_2 = \frac{-xy_1 + y}{x^2}$

$y_2 = \frac{-x \cdot \frac{-y}{x} + y}{x^2}$

$= \frac{y+y}{x^2} = \frac{2y}{x^2} = 2.$

$\rho = \frac{(1+(y_1)^2)^{3/2}}{y_2}$

$= \frac{(1+1)^{3/2}}{2} = \frac{(2)^{3/2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

radius of curvature $\rho = \sqrt{2}$.

1) Find the radius of curvature of the $xy^2 = a^3 - x^3$ at $(a,a)$.

$2yy_1 + y^2 = -3x^2$.

$y_1 = \frac{-3x^2 - y^2}{2xy}$.

$y_1(a,a) = \frac{-3(a^2) - (a)^2}{2(a)(a)}$

$= \frac{-4a^2}{2a^2}$

$y_1 = -2$.

$y_2 = \frac{2xy[-6x - 2yy_1] - [-3x^2 - y^2]2(xy_1 + y)}{4x^2y^2}$

$y_2(a,a) = \frac{2a^2[-6(a) - 2(a)(-2)] - [-3(a)^2 - a^2]\,2(a(-2) + a)}{4a^4}$

$= \frac{2a^2[-2a] + 4a^2[-2a]}{4a^4}$

$= \frac{2a^2[-2a] - 8a^3}{4a^4}$

$= \frac{-4a^3 - 8a^3}{4a^4} = \frac{-12a^3}{4a^4} = -3/a$

$\rho = \frac{[1+(y_1)^2]^{3/2}}{y_2}$

$= \frac{[1+4]^{3/2}}{-3/a} = -\frac{(5)^{3/2}a}{3} = -\frac{5\sqrt{5}a}{3}$

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Since radius cannot be in negative

So Radius of Curvature $\rho = \frac{5\sqrt{5}\,a}{3}$

5) Find the radius of curvature of the parabola $x = at^2$, $y = 2at$ at T.

$x = at^2$ | $y = 2at$

$\frac{dx}{dt} = 2at$ | $\frac{dy}{dt} = 2a$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$$

$$= \frac{d}{dt}\left(\frac{1}{t}\right)\frac{1}{2at}$$

$$= -\frac{1}{t^2}\,\frac{1}{2at}$$

$$y_2 = \frac{-1}{2at^3}$$

$$\rho = \frac{[1+(y_1)^2]^{3/2}}{y_2}$$

$$= \frac{\left[1+\frac{1}{t^2}\right]^{3/2}}{-\frac{1}{2at^3}}$$

$$= -\left[\frac{t^2+1}{t^2}\right]^{3/2} \cdot 2at^3$$

$$= \frac{-(t^2+1)^{3/2} \cdot 2at^3}{t^3}$$

$$= -(t^2+1)^{3/2} \cdot 2a$$

The radius cannot be in negative

So radius of curvature $\rho = (t^2+1)^{3/2} \cdot 2a$.

6) Find the radius of curvature at $(a,0)$ on the curve $xy^2 = a^3 - x^3$.

Given $xy^2 = a^3 - x^3$

$$y^2 = \frac{a^3 - x^3}{x}$$

$$2yy_1 = \frac{x(-3x^2) - (a^3 - x^3)}{x^2}$$

$$2yy_1 = \frac{-3x^3 - a^3 + x^3}{x^2}$$

$$2yy_1 = \frac{-2x^3 - a^3}{x^2} = \frac{-(2x^3 + a^3)}{x^2}$$

$$y_1 = \frac{-2x^3 + a^3}{2yx^2}$$

$$y_1(a,0) = \frac{-2a^3 + a^3}{2(0)x^3}$$

$$= \frac{a^3}{0} =$$

# Velocity and Acceleration:-

**Velocity:**

If $s$ is the distance travelled by the particle in time $t$ (sec), then the rate of ~~distance~~ displacement is given by $ds/dt$, it is denoted by V(velocity). ie,

$$V = \frac{ds}{dt}$$

**Acceleration:-**

The rate of change of velocity is the Acceleration and is given by

$$\text{acceleration} = \frac{dv}{dt} = \frac{d^2s}{dt^2} = a.$$

Note:

i) Initial Velocity $V = \frac{ds}{dt}$ at time $t=0$

ii) Initial acceleration $a = \frac{d^2s}{dt^2}$ at time $t=0$.

**Problems:-**

1) The distance-time formula of moving particles is $S = 2t^3 + 3t^2 - 72t + 1$ find

i) Velocity at $t = 3$ sec.

ii) Initial velocity.

iii) Initial acceleration.

iv) acceleration after 4 sec.

Soln:-

Given: $S = 2t^3 + 3t^2 - 72t + 1$.

$$V = \frac{ds}{dt} = 6t^2 + 6t - 72.$$

$$a = \frac{d^2s}{dt^2} = \frac{dv}{dt} = 12t + 6.$$

i) $V_{(t=3\,sec)} = 6(3)^2 + 6(3) - 72$

$= 0$

(ii) $V_{(t=0)} = 6(0) + 6(0) - 72 = -72$ units/sec.

iii) $a_{(t=0)} = 12(0) + 6 = 6$. units/sec$^2$

iv) $a_{(t=4)} = 12(4) + 6$

$= 54$. units/sec$^2$.

2) The distance-time formula of a moving particle is $S = \frac{t^3}{3} - \frac{7}{2}t^2 + 6t - 10$. find.

i) Velocity at $t = 3$ sec.

Soln:

Given $\Rightarrow S = \frac{t^3}{3} - \frac{7}{2}t^2 + 6t - 10$

$$V = \frac{ds}{dt} = \frac{3t^2}{3} - \frac{14t}{2} + 6$$

$$\frac{ds}{dt} = t^2 - 7t + 6.$$

$a = \frac{d^2s}{dt^2} = 2t - 7$

$= 2t - 7$

i) velocity at t=3 sec

$V_{(t=3sec)} = 3^2 - 7(3) + 6$

$= 9 - 21 + 6$

$= -6$ Units / sec.

ii) velocity Initial

$V_{(t=0)} = 0 - 0 + 6$

$= 6$. Units / sec.

iii) acceleration Initial.

$a_{(t=0)} = 0 - 7$

$= -7$ Units / sec$^2$.

iv) acceleration at t=4 sec.

$a_{(t=4sec)} = 8 - 7$

$= 1$ Units / sec$^2$.

8) The distance time formula of a moving particle is $s = \frac{t^3}{6} - 5t^2 + 6t - 10$. Find.

i) Initial Velocity.

$v = \frac{ds}{dt} = \frac{3t^2}{6_2} - 5(2t) + 6$

$= \frac{t^2}{2} - 10t + 6$.

$a = \frac{d^2s}{dt^2} = \frac{2t}{2} - 10$

$= t - 10$.

i) Initial Velocity.

$V_{(t=0)} = \frac{0^2}{2} - 0 + 6$

$= 6$ Units / sec.

ii) Velocity at t = 3 sec..

$V_{(t=3sec)} = \frac{9}{2} - 10(3) + 6$

$= \frac{9}{2} - 30 + 6$.

$= \frac{9}{2} - 24 = \frac{9-48}{2} = \frac{-39}{2}$ Units / sec

iii) Initial acceleration.

$a_{(t=0)} = 0 - 10 = -10$ Units / sec$^2$.

iv) acceleration at t = 4 sec.

$a_{(t=4)} = 4 - 10 = -6$ Unit / sec$^2$.

## Tangents and Normal:

**Tangent:**

The straight line which touches the curve at a point is called the tangent.

**Normal:**

Normal is the straight line which is perpendicular to the tangent and passing through the point at which the tangent touches the curve.

**Note:-**

i) slope of a tangent $= \frac{dy}{dx} = m$.

ii) slope of a Normal $= -\frac{1}{m}$.

iii) Eqn of a tangent $\Rightarrow y - y_1 = m(x - x_1)$

iv) Eqn of a normal $\Rightarrow y - y_1 = \frac{-1}{m}(x - x_1)$

## Problems:-

i) Find the eqn of tangent and normal to the curve $2y = 3 - x^2$ at $(1,1)$.

Given $2y = 3 - x^2$

$$2\frac{dy}{dx} = 0 - 2x.$$

$$\cancel{2}\frac{dy}{dx} = -\cancel{2}x$$

$$\frac{dy}{dx} = -x.$$

$$\left.\frac{dy}{dx}\right|_{(1,1)} = -1.$$

Slope of a tangent $m = -1$

Slope of a normal $\frac{-1}{m} = 1$.

Eqn of a tangent $\Rightarrow y - 1 = -1(x - 1)$

$$y - 1 = -x + 1$$

$$\boxed{x + y - 2 = 0}$$

Eqn of a normal $\Rightarrow y - 1 = 1(x - 1)$

$$x - y = 0$$

$$\boxed{x - y = 0}$$

2) Find the eqn of the Tangent and Normal to the Curve $y = 5 - 2x - 3x^2$ at $(2, -11)$

Given.

$y = 5 - 2x - 3x^2$.

$\frac{dy}{dx} = 0 - 2 - 3(2x)$

$= -2 - 6x$.

$\frac{dy}{dx} = -6x - 2$.

$\frac{dy}{dx}_{(2,-11)} = -6(2) - 2$

$= -12 - 2$

$= -14$.

Slope of the tangent $m = -14$.

Slope of the normal $\frac{-1}{m} = \frac{-1}{-14} = \frac{1}{14}$.

Eqn of the tangent $y - y_1 = m(x - x_1)$

$y + 11 = -14(x - 2)$

$y + 11 = -14x + 28$.

$14x + y + 11 - 28 = 0$

$14x + y - 17 = 0$.

Eqn of the Normal $y - y_1 = \frac{-1}{m}(x - x_1)$

$y + 11 = \frac{1}{14}(x - 2)$

$14y + 154 = x - 2$

$x - 14y - 156 = 0$

$x - 14y - 156 = 0$.

3) Find the eqn of the tangent and Normal to the Curve $y = \frac{5x^2}{1+x^2}$ at $(2, 4)$.

Given ⇒ $y = \frac{5x^2}{1+x^2}$.

$\frac{dy}{dx} = \frac{(1+x^2)(10x) - 5x^2(0+2x)}{(1+x^2)^2}$.

$= \frac{1+x^2(10x) - 5x^2(2x)}{(1+x^2)^2}$

$= \frac{10x + 10x^3 - 5 \times 10x^3}{(1+x^2)^2}$.

$\frac{dy}{dx} = \frac{10x}{(1+x^2)^2}$.

$\frac{dy}{dx} = \frac{10x}{(1+(2)^2)^2} = \frac{10(2)}{(5)^2} = \frac{20}{25} = \frac{4}{5}$.

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slope of the tangent $m = \frac{4}{5}$

slope of the normal $\frac{-1}{m} = -\frac{5}{4}$.

Eqn of the tangent $y - y_1 = m(x - x_1)$

$$y - 4 = \frac{4}{5}(x - 2)$$

$$5y - 20 = 4x - 8$$

$$4x - 5y + 12 = 0.$$

Eqn of the Normal $\Rightarrow y - y_1 = \frac{-1}{m}(x - x_1)$

$$y - 4 = -\frac{5}{4}(x - 2)$$

$$4y - 16 = -5x + 10$$

$$5x + 4y - 10 - 16 = 0$$

$$5x + 4y - 26 = 0$$

## Intergration:-

Intergration is reciprocal of differentiation

Basic formula:-

1) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

2) $\int e^{ax} dx = \frac{e^{ax}}{a} + c$

3) $\int \cos ax \, dx = \frac{\sin ax}{a} + c$

4) $\int \sin ax \, dx = \frac{-\cos ax}{a} + c$

5) $\int \frac{1}{x} dx = \log x + c$

6) $\int \tan ax \, dx = \frac{\log(\sec ax)}{a} + c$

7) $\int \sec ax \, dx = \frac{\log(\tan ax + \sec ax)}{a} + c$

8) $\int \sec^2 ax \, dz = \frac{\tan ax}{a} + c$

9) $\int \operatorname{cosec}^2 ax \, dx = \frac{-\cot ax}{a} + c$

10) $\int uv\,dx = v u_1 - u' v_2 + u'' v_3 - \ldots$

11) $\int u\,dv = uv - \int v\,du$.

12) $\int c\,dx = cx + c$

**definite integral:-**

Is an integral where the limits are given.

$(0,5) \to 0<x<5$

$[0,5] : 0 \le x \le 5$

$[0,5) : 0 \le x < 5$

$(0,5] : 0 < x \le 5$

**Indefinite integral:-**

Is an integral where the limits are not given.

1) Problems

1) $\int (5e^{3x} + \cos 4x + \frac{1}{x+3} + \sec^2 4x + 3)\,dx$

$= \frac{5e^{3x}}{3} + \frac{\sin 4x}{4} + \log(x+3) + \frac{\tan 4x}{4} + 3x + c.$

2) $\int x^2 e^{5x}\,dx$

$u = x^2$ $\quad v = e^{5x}$

$u' = 2x$ $\quad v_1 = \frac{e^{5x}}{5}$

$u'' = 2$ $\quad v_2 = \frac{e^{5x}}{25}$

$u''' = 0$ $\quad v_3 = \frac{e^{5x}}{125}$

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \cdots$

$\int x^2 e^{5x}\,dx = \frac{x^2 e^{5x}}{5} - \frac{2x e^{5x}}{25} + \frac{2e^{5x}}{125}.$

3) $\int x^2 \log x\,dx$

$\cancel{u = x^2}$ $\quad \cancel{v = \log x}$

$\cancel{u' = 2x}$ $\quad \cancel{v_1 =}$

$\cancel{u'' = 2}$

$\cancel{u''' = 0}$

$u = \log x$ $\quad v = x^2$

$u' = \frac{1}{x}$ $\quad v_1 = \frac{x^3}{3}$

$u'' = -\frac{1}{x^2}$ $\quad v_2 = \frac{x^4}{12}$

$u''' = +\frac{2}{x^3}$ $\quad v_3 = \frac{x^5}{60}$

$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \cdots$

$= \log x \frac{x^3}{3} - \frac{1}{x}\frac{x^4}{12} - \frac{1}{x^2}\frac{x^5}{60} \cdots + c$

$= \frac{\log x\, x^3}{3} - \frac{x^4}{12x} - \frac{x^5}{60x^2} \cdots + c$

$= \frac{\log x\, x^3}{3} - \frac{x^3}{12} - \frac{x^3}{60} \cdots + c.$

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3) $\int x \tan^2 x \, dx$

$\int x [\sec^2 x - 1] \, dx$

$\int [x \sec^2 x - x] \, dx$

$u = x$ $\quad v = \sec^2 x$

$u' = 1$ $\quad v_1 = \tan x$

$u'' = 0$ $\quad v_2 = \log \sec x$.

$\sin^2\theta + \cos^2\theta = 1$

$\sec^2\theta - \tan^2\theta = 1$

$\csc^2\theta - \cot^2\theta = 1$

$\sin^3\theta = \boxed{\frac{3\sin\theta - \sin 3\theta}{4}}$

$\cos^2\theta = \frac{1 \pm \cos 2\theta}{2}$

$\sin^2\theta = \frac{1 + \cos 2\theta}{2}$

$\int uv \, dx = uv_1 - u'v_2 + u''v_3 - \ldots$

$\int x \tan^2 x \, dx = x \tan x - \log \sec x - \frac{x^2}{2}$

H.W.

$\sin^n x = 0$

$\cos^n x =$

1) $\int e^{5x} x^5 \, dx$

2) $\int \sin 4x \, e^{3x} \, dx$

3) $\int x^3 \sin^3 x \, dx$

4) $\int x^4 \sin^2 x \, dx$

5) $\int x^3 \cos^2 4x \, dx$

Area.

Area bounded by a closed curve

The definite integral

$$\int_a^b y \, dx = \int_a^b x \, dy.$$

gives the area of the region which is bounded by the curve $y = f(x)$, the axis of $x$ and the two ordinates $x = a$ and $x = b$.

1) Find the area bounded by the curve $y = x^2 + x$ from $x = 1$ and $x = 3$.

$$\int_a^b y \, dx = \int_a^b x \, dy.$$

$a = 1$ ; $b = 3$.

$y = x^2 + x$.

$$\int_a^b y \, dx = \int_1^3 x^2 + x$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_1^3$$

$$= \left[\frac{27}{3} + \frac{9}{2}\right] - \left[\frac{1}{3} + \frac{1}{2}\right]$$

$$= \left[\frac{54+27}{6}\right] - \left[\frac{2+3}{6}\right]$$

$$= \left[\frac{81}{6}\right] - \left[\frac{5}{6}\right]$$

$$= \frac{67}{6} \cdot \frac{76}{6}$$

27
27
54

54
18
72

72
5
67

27
54
27
81

H/W)
1)

$$\int e^{5x} x^5\, dx$$

$u = x^5$ $\quad v = e^{5x}$

$u' = 5x$ $\quad v_1 = \frac{e^{5x}}{5}$

$u'' = 5$ $\quad v_2 = \frac{e^{5x}}{25}$

$u''' = 0$ $\quad v_3 = \frac{e^{5x}}{125}$

$$\int uv\,dx = uv_1 - u'v_2 + u''v_3 - \dots$$

$$\int e^{5x} x^5\, dx = \frac{x^5 e^{5x}}{5} - \frac{5x e^{5x}}{25} + \frac{5e^{5x}}{125}.$$

2) $\int \sin 4x\, e^{3x}\, dx$

$$\int \sin 4x\, e^{3x}\, dx = e^{3x} \sin 4x - \cos 3e^{3x} - \frac{\cos 4}{4} + 9e^{3x}\frac{\cos 4x}{16} + 27e^{3x} - \frac{\cos 4x}{64}$$

3) $\int x^3 \sin^3 x \, dx$

$= \int x^3 \left[\frac{3\sin\theta - \sin 3\theta}{4}\right] dx$

$= \frac{3}{4}\int x^3 \sin x dx - \frac{1}{4}\int x^3 \sin 3x \, dx$

$= \frac{3}{4}\int x^3(-\cos x - 3x^2(-\sin x) + 6x(\cos x) - 6\sin x)$

$= \frac{1}{4}\left(x^3\left(\frac{-\cos 3x}{3}\right) - 3x^2\left(\frac{-\sin 3x}{3^2}\right) - 6x\left(\frac{\cos^2 x}{3^2}\right) - 6\left(\frac{\sin 3x}{3^4}\right)\right)$

c) $\int x^3 \cos^2 4x \, dx$

$u = x^3$ | $v = \cos^2 4x$

$u' = 3x^2$ | $v_1 = \frac{1 - \cos^2 2x}{2} - \frac{x^4 \cos 2x}{16}$

$u'' = 6x$

$u''' = 6$ | $v_2 = \frac{\frac{x^5}{20} \cdot \frac{\sin 2x}{4}}{2}$

$u^{iv} = 0$

$\int x^3 \cos^2 4x \, dx = \frac{x^2}{2} \frac{x^3 \cos 2x}{2} + \frac{x^4}{8} - \frac{3x^4 \cos 2x}{16} + \frac{x^5}{10}\left(\frac{x^5}{20}\right) \cdot \frac{\sin 2x}{4 \times 2}$

4) $\int x^4 \sin^2 x \, dx$

$u = x^4$

$u' = 4x^3$

$u'' = 4(3x) = 12x$

$u''' = 12$

$u^{iv} = 0$

$v = \sin^2 x$

$v_1 = \frac{1 - \cos^2 2x}{2 \times 2}$

$v_1 = \frac{x - \sin 2x}{4 \times 2 \times 2}$

$v_2 = \frac{x^3 + \cos 2x}{16}$

$v_2 = x^3$

$v = \sin^2 x$

$v_1 = \frac{x - \sin 2x}{4}$

$= \frac{x^2 + \cos 2x}{4}$

$v_2 = x^3 + \frac{\cos 2x}{16}$

$= \frac{x^3 + \sin 2x}{96}$

$= x^4\left(\frac{1 - \cos 2x}{2}\right) - 4x^3\left(\frac{x - \sin 2x}{4}\right)$

$+ 12\left(\frac{x^3 + \sin 2x}{96}\right) + \ldots$

1) find the area bounded by the Curve $y=x^2+1$ from $x=1$, to 3.

$$\int_a^b y\,dx = \int_1^3 x^2+1$$

$$= \left[\frac{x^3}{3}+x\right]_1^3$$

$$= \left[\frac{x^3+3x}{3}\right]_1^3$$

$$= \left[\frac{27+9}{3}\right] - \left[\frac{1+3}{3}\right]$$

$$= \frac{36}{3} - \frac{4}{3}$$

$$= \frac{32}{3}$$

2) find the area bounded by the Curve $y=x^3$. $x=1$ to 4.

$$\int_a^b y\,dx = \int_1^4 x^3\,dx$$

$$= \left[\frac{x^4}{4}\right]_1^4$$

$$= \frac{4^4}{4} - \frac{1}{4}$$

$$= \frac{256}{4} - \frac{1}{4} = \frac{255}{4}$$

3) find the area bounded by the Curve, $y=x^2$ from $x=0$ to 1.

$$\int_a^b y\,dx = \int_0^1 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{3} - \frac{0}{3}$$

$$= \frac{1}{3}$$

$\sin 0 = 0$

$\sin 90 = 1$

$\sin \pi/2 = 1$

$\pi = 180$

4) find the area bounded by the Curve $y=2x^2$ from $x=0$ to 2

$$\int_a^b y\,dx = \int_0^2 2x^2\,dx.$$

$$= \left[\frac{2x^3}{3}\right]_0^2$$

$$= \frac{16}{3} - \frac{0}{3}$$

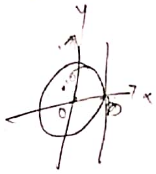
$$= \frac{16}{3}$$

9) Find the area of the circle of radius $b$ using integration method

The eqn of the circle with centre of the origin & radius '$b$' is

$x^2+y^2=b^2$

Here $x$ varies from $x=0$ to $x=b$ and



from $x^2+y^2=b^2$

$\Rightarrow \quad y^2=b^2-x^2$

$\Rightarrow \quad y=\pm\sqrt{b^2-x^2}$.

$\therefore$ Area of the circle $=4\int_0^b \sqrt{b^2-x^2}\,dx$.

$$=4\left[\frac{x}{b}\sqrt{b^2-x^2}+\frac{b^2}{2}\sin^{-1}\frac{x}{b}\right]_0^b.$$

$$=4\left[\frac{b^2}{2}\sin^{-1}1-0\right]$$

$$=4\,\frac{b^2}{2}\,\frac{\pi}{2}=\frac{4b^2\pi}{4}=b^2\pi.$$

## Volume:-

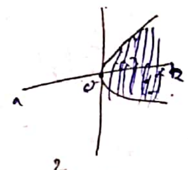
20

The volume of the solid obtained by rotating the area bounded by the curve $y=f(x)$ and $x$-axis between $x=a$ and $x=b$ about the $x$ axis is integral $\int_a^b \pi y^2\,dx$ is equal to $\int_a^b \pi(f(x))^2\,dx$.

The volume of when the area bounded by the curve $y=f(x)$ and $y$ axis between is revolved about the $y$ axis between $y=a$, $y=b$ is integral ... :

$\int_a^b \pi x^2\,dx$.

1) Area : Parabola $\rightarrow y^2=4ax$,



$y^2=4ax$ $\qquad x=\frac{y^2}{4a}$

$y=2\sqrt{a}\sqrt{x}$. $\qquad x=\frac{4ax}{4a}$

limit $x=0$, and $x=b$.

$y=2\sqrt{a}\sqrt{x}$

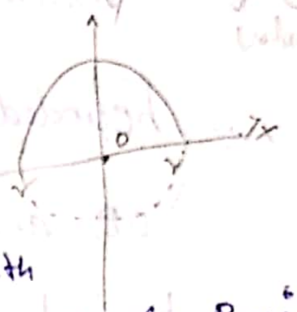
Parabola $=2\int_0^b 2\sqrt{a}\sqrt{x}\,dx$.

find the Volume of the Sphere of radius of r using integration.

Soln:

Volume of the Sphere is Obtained when the area bounded by the Semicircle

$x^2+y^2=r^2$ and the x axis when it is rotated about the x-axis




we known That $x^2+y^2=r^2$ is a Circle when center at the origin with radius r, when we Consisted a semicircle, and $y^2=r^2-x^2$.

x, varies from $-r$ to r

$\therefore$ Volume of the sphere is $\int_a^b \pi y^2 dx$

$$\int_a^b \pi y^2 dx = \int_{-r}^{r} \pi (r^2-x^2)\, dx$$

$$= 2\pi \int_0^r (r^2-x^2)\, dx$$

$$= 2\pi \left[ r^2 x - \frac{x^3}{3} \right]_0^r$$

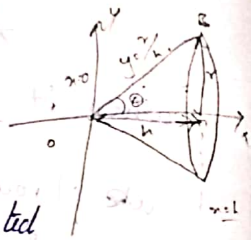
$= 2\pi \left[ r^3 - \frac{r^3}{3} \right] - 0$

$= \frac{4\pi r^3}{3}.$

2) Find the volume of the right circular cone of base radius 'r'.

Soln:

When the area of AOB bounded by the line OB and x-axis is rotated by about the x-axis, the solid cone obtained:



$\therefore$ Here $x$ varies from $0$ to $h$

and $y = \frac{r}{h}x$ , $y = mx$

$\Rightarrow y = \frac{r}{h}x$

where $m = \frac{r}{h}$.

$$\int_a^b \pi y^2 \, dx = \int_0^h \pi \frac{r^2}{h^2} x^2 \, dx.$$

$$= \frac{\pi r^2}{h^2} \int_0^h x^2 \, dx$$

$$= \frac{\pi r^2}{h^2} \left[ \frac{x^3}{3} \right]_0^h$$

$$= \frac{\pi r^2 h^3}{3h^2}$$

$$= \frac{\pi r^2 h}{3}$$

3) Reduction formula:

1) $\int \sin^m x \cos^n x = \frac{-\cos^{n+1} x \sin^{m-1} x}{m+n} + \frac{m-1}{m+n} \int \sin^{m-2} x \cos^n x$

$= \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \sin^m x \cos^{n-2} x$

2) $\int_0^{\pi/2} \sin^m x \cos^n x$.

Case(i) when m is odd ($n$ = odd or even)

$I_{m,n} = \frac{m-1}{m+n} \cdot \frac{m-3}{m+n-2} \cdots \frac{2}{3+n} \cdot \frac{1}{n+1}$.

Case(ii) when m is even & n is odd

$I_{m,n} = \frac{(m-1)}{m+n} \cdot \frac{(m-3)}{m+n-2} \cdots \frac{1}{2+n} \cdot \frac{(n-1)}{n} \cdot \frac{(n-3)}{n-2} \cdots$

Case (iii) :

when $m$ is even & $n$ is even.

$$I_{m,n} = \frac{m-1}{m+n} \cdot \frac{(m-3)}{m+n-2} \cdots \frac{1}{2+n} \cdot \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2}.$$

$$I_n = \int \sin^n x \, dx.$$

$$nI_n = -\sin^{n-1} x \cos x + (n-1) I_{n-2}.$$

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \text{ (n is odd)}$$

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2} \text{ (n is even)}$$

$$I_n = \int \cos^n x \, dx.$$

$$nI_n = \cos^{n-1} x \sin x + (n-1) I_{n-2}$$

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{2}{3} \text{ (n is odd)}$$

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2} \text{ (n is even)}$$

Evaluate Integral $\int \cos^3 x \, dx$ Using

i) Usual integration method

ii) Reduction formula.

iii) find $\int_0^{\pi/2} \cos^3 x \, dx$.

Soln

i) Usual integration method :

$$\int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx \quad \text{put } y = \sin x, \; dy = \cos x \, dx$$

$$= \int (1 - \sin^2 x) \cos x \, dx$$

$$= \int (1 - y^2) \, dy$$

$$= \left[ y - \frac{y^3}{3} \right] + c$$

$$= \left[ \sin x - \frac{\sin^3 x}{3} \right] + c$$

ii) Using Reduction formula

$$I_n = \int \cos^n x \, dx.$$

$$nI_n = \cos^{n-1} x \sin x + (n-1) I_{n-2}.$$

$n = 3$

$$\int \cos^3 x \, dx =$$

$$3I_3 = \cos^2 x \sin x + 2 I_{3-2}$$

$$3I_3 = \cos^2 x \sin x + 2I_1$$

$$I_3 = \frac{\cos^2 x \sin x + 2I_1}{3}$$

$$= \frac{1}{3} \left[ \cos^2 x \sin x + 2 \int \cos x \, dx \right]$$

$$= \frac{1}{3} \left[ \cos^2 x \sin x + 2 \sin x \right] + c$$

iii) $\int_0^{\pi/2} \cos^3 x \, dx$.

formula

$$\boxed{\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a}}$$

$\int_0^{\pi/2} \cos^3 x \, dx$.

Here $n=3$ (odd)

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{(n-1)(n-3)(n-5)}{n(n-2)(n-4)} \cdots \frac{2}{3}$$

$$\int_0^{\pi/2} \cos^3 x \, dx = \frac{2}{3}.$$

2) Evaluate ~~integral~~ $\int \sin^5 x \, dx$

i) Using Usual substutional method

2) Reduction formula.

3) $\int_0^{\pi/2} \sin^5 x \, dx$. find.

Soln:

i) Usual Substitution method.

$$\int \sin^5 x \, dx = \int \sin^4 x \sin x \, dx$$

Let $y = \cos x$ $\qquad = \int (\sin^2 x)^2 \sin x \, dx$

$dy = -\sin x dx$ $\qquad = \int (1-\cos^2 x) \sin x \, dx$

$-dy = \sin x dx$. $\qquad = -\int (1-y^2)^2 dy = -\int (1-2y^2+y^4) dy$

$$= -\left[y - \frac{2y^3}{3} + \frac{y^5}{5}\right] = -\cos x + \frac{2}{3}\cos^3 x - \frac{\cos^5 x}{5}$$

ii) Reduction method.

$$I_n = \int \sin^n x \, dx.$$

$$I_n = \frac{1}{n}\left[-\sin^{n-1} x \cos x + (n-1) I_{n-2}\right]$$

Here $n=5$

$$I_5 = \frac{1}{5}\left[-\sin^4 x \cos x + 4 I_3\right]$$

$$= \frac{1}{5}\left[-\sin^4 x \cos x + \frac{4}{3}\left[-\sin^2 x \cos x + 2 I_1\right)\right.$$

$$= \frac{1}{5}\left[-\sin^4 x \cos x + \frac{4}{3}\left[-\sin^2 x \cos x + 2 \int \sin x \, dx\right]\right.$$

$$= \frac{1}{2}\left[-\sin^4 x \cos x + \frac{4}{3}\left[-\sin^2 x \cos x - 2\cos x\right]\right] + C.$$

iii) $\int_0^{\pi/2} \sin^5 x \, dx$.

Here $n=5$ (odd)

$$\int_0^{\pi/2} \sin^5 x \, dx = \frac{(n-1)(n-3)(n-5)}{n \cdot (n-2)(n-4)} \cdots \frac{2}{3}$$

$$= \frac{4}{5} \cdot \frac{2}{3}$$

$$= \frac{8}{15}.$$

3) $\int \sin^4 x \, dx$.

i) Usual Integration Method.

$\int \sin^4 x \, dx =$

i) Using Reduction method.

$$\int \sin^4 x \, dx = \frac{1}{4}\left[-\sin^3 x \cos x + 3 I_2\right]$$

$$= \frac{1}{4}\left[-\sin^3 x \cos x + \frac{3}{2}\left[-\sin x \cos x + I_0\right]\right]$$

$$= \frac{1}{4}\left[-\sin^3 x \cos x + \frac{3}{8}\left[-\sin x \cos x + I_0 + c\right]\right]$$

(ii) $\int_0^{\pi/2} \sin^4 x \, dx$.

$$\int_0^{\pi/2} \sin^4 x \, dx = \frac{(n-1)(n-3)(n-5)}{n(n-2)(n-4)} \cdots \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \frac{3\pi}{16}.$$

4) (i) $\int \cos^6 x \, dx$

$$\int \cos^n x \, dx = \frac{1}{n}\left[\cos^{n-1} x \sin x + (n-1) I_{n-2}\right]$$

$$= \frac{1}{6}\left[\cos^5 x \sin x + 5 I_3\right]$$

$$= \frac{1}{6}\left[\cos^5 x \sin x + \frac{5}{3}\left[\cos^2 x \sin x + 2 I_1\right]\right]$$

$$= \frac{1}{6}\left[\cos^5 x \sin x + \frac{5}{3}\left[\cos^2 x \sin x + 2 \sin x\right] + C\right]$$

(ii) $\int_0^{\pi/2} \cos^6 x \, dx = \frac{(n-1)(n-3)(n-5)}{n \cdot (n-2)(n-4)} \cdots \frac{1}{2} \cdot \frac{\pi}{2}$

$$= \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}$$

$$= \frac{5\pi}{32}.$$

5) Evaluate:

$\int \sin^5 x \cos^7 x \, dx$

Using (i) Integration method

(ii) $\int_0^{\pi/4} \sin^5 x \cos^7 x \, dx$.

Soln!

1) Integral method.

$\int \sin^5 x \cos^7 x \, dx$.

$$\int \sin^m x \cos^n x \, dx = -\frac{\cos^{n+1} x \sin^{m-1} x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x \, dx$$

$$\int \sin^5 x \cos^7 x \, dx = -\cos^8 x \sin^4 x + \frac{4}{12}\int \sin^3 x \cos^7 x \, dx.$$

$$= -\cos^8 x \sin^4 x + \frac{1}{3}\left[-\cos^8 x \sin^2 x + \frac{2}{10}\int \sin x \cos^7 x \, dx\right.$$

$$= -\cos^8 x \sin^4 x + \frac{1}{3}\left[-\cos^8 x \sin^2 x + \frac{1}{5}\left[-\cos^8 x\right]\right] + C$$

(ii) $\int_0^{\pi/2} \sin^5 x \cos^7 x \, dx$.

$m = 5, \; n = 7$.

$$\int_0^{\pi/2} \sin^m x \cos^n x \, dx = \frac{(m-1)(m-3)\cdots(n-1)(n-3)\cdots 2}{(m+n)(m+n-2)(m+n-4)\cdots 2}$$

$$= \frac{4\cdot 2\cdot 6\cdot 4\cdot 2}{12\cdot 10\cdot 8\cdot 6\cdot 4\cdot 2}$$

$$= \frac{2}{12\cdot 5\cdot 3} = \frac{1}{30}$$

1) $\int \sin^n x \, dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{(n-1)}{n} I_{n-2}$

2) $\int \cos^n x \, dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{(n-1)}{n} I_{n-2}$.

Eg. $\int \sin^4 x \, dx = -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4} I_2 \; (n=4)$

$$= -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4}\left[-\frac{1}{2}\sin x \cos x + \frac{1}{2} I_0\right]$$

$$= -\frac{1}{4}\sin^3 x \cos x - \frac{3}{8}\sin x \cos x + C$$

Eg: $\int \cos^7 x \, dx = \frac{1}{7}\cos^6 x \sin x + \frac{6}{7} I_5 \; (n=7)$

$$= \frac{1}{7}\cos^6 x \sin x + \frac{6}{7}\left[\frac{1}{5}\cos^4 x \sin x + \frac{4}{5} I_3\right] \quad n=5$$

$$= \frac{1}{7}\cos^6 x \sin x + \frac{6}{7}\left[\frac{1}{5}\cos^4 x \sin x + \frac{4}{5}\left[\frac{1}{3}\cos^2 x \sin x + \frac{2}{3} I_1\right]\right] \quad n=3$$

$$= \frac{1}{7}\cos^6 x \sin x + \frac{6}{7}\left[\frac{1}{5}\cos^4 x \sin x + \frac{4}{5}\left[\frac{1}{3}\cos^2 x \sin x + \frac{2}{3}\sin x\right]\right] \quad (n=1).$$

$$\int_0^{\pi/2} \cos^n x \, dx = \int_0^{\pi/2} \sin^n x \, dx$$

$$= \frac{(n-1)(n-3)(n-5)\cdots}{n(n-2)(n-4)\cdots} \quad \text{(when } n \text{ is}$$

$$= \frac{(n-1)(n-3)(n-5)\cdots}{n(n-2)(n-4)\cdots}\cdot\frac{\pi}{2} \quad \text{(when } n \text{ is even}$$

Eg: $\int_0^{\pi/2} \sin^4 x\,dx = \frac{3\cdot 1}{4\cdot 2}\frac{\pi}{2} = \frac{3\pi}{16}$

Eg: $\int_0^{\pi/2} \cos^7 x\,dx = \frac{6\cdot 4\cdot 2}{7\cdot 5\cdot 3} = \frac{16}{35}$

① $\int \sin^m x \cos^n x\,dx = \frac{-1}{m+n}\cos^{n+1} x \sin^{m-1} x + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x\,dx$.

Eg: $\int \sin^5 x \cos^7 x\,dx = \frac{-1}{12}\cos^8 x \sin^4 x + \frac{4}{12}\int \sin^3 x \cos^7 x\,dx$.

$= \frac{-1}{12}\cos^8 x \sin^4 x + \frac{1}{3}\left[\frac{-1}{10}\cos^8 x \sin^2 x + \frac{2}{10}\int \sin x \cos^7 x\,dx\right)$

$= \frac{-1}{12}\cos^8 x \sin^4 x + \frac{1}{3}\left[\frac{-1}{10}\cos^8 x \sin^2 x + \frac{1}{5}\left[-\frac{1}{8}\cos^8 x\right]\right]$.

1) $\int_0^{\pi/2} \sin^m x \cos^n x\,dx$.

$= \frac{(m-1)(m-3)(m-5)\ldots\ldots(n-1)(n-3)(n-5)}{(m+n)(m+n-2)(m+n-4)\ldots\ldots}$

(when anyone value is odd).

$= \frac{(m-1)(m-3)(m-5)\ldots\ldots(n-1)(n-3)(n-5)\ldots}{(m+n)(m+n-2)(m+n-4)\ldots\ldots}\cdot\frac{\pi}{2}$

(when both m & n is even).

Eg $\int_0^{\pi/2} \sin^5 x \cos^7 x\,dx = \frac{4\cdot 2\cdot 6\cdot 4\cdot 2}{12\cdot 10\cdot 8\cdot 6\cdot 4\cdot 2} = \frac{1}{120}$

Eg $\int_0^{\pi/2} \sin^4 x \cos^3 x\,dx = \frac{3\cdot 1\cdot 2}{7\cdot 5\cdot 3\cdot 1} = \frac{2}{35}$

Eg $\int_0^{\pi/2} \sin^6 x \cos^8 x\,dx = \frac{5\cdot 3\cdot 1\cdot 7\cdot 5\cdot 3\cdot 1}{14\cdot 12\cdot 10\cdot 8\cdot 6\cdot 4\cdot 2}\left(\frac{\pi}{2}\right)$

$= \frac{5\pi}{2048}$.

$64\times 4 = 256$, $256\times 8 = 2048$

1) $\int_0^{\pi/2} \sin^7 x \cos^3 x\,dx$

2) $\int_0^{\pi/2} \sin^4 x \cos^4 x\,dx$

3) $\int_0^{\pi/2} \sin^6 x \cos^5 x\,dx$

4) $\int_0^{\pi/2} \sin^3 x \cos^3 x\,dx$.

Scanned with CamScanner

1) $\int 5x^4 dx = \frac{5x^5}{5} = x^5$

2) $\int \sin 8x \, dx = \frac{-\cos 8x}{8}$

3) $\int \cos x \, dx = \sin x$

4) $\int \frac{1}{x} dx = \log x$

5) $\int 0 \, dx = C$

6) $\int 5 \, dx = 5\int dx = 5x$

7) $\int \frac{1}{x+2} dx = \log(x+2)$

8) $\int \frac{x}{x^2+3} dx = \frac{2}{2}\int \frac{x}{x^2+3} dx = \frac{1}{2}\int \frac{2x}{x^2+3} dx = \frac{1}{2}\log(x^2+3)$.

$\int \sqrt{5^2-x^2}\, dx = \frac{x}{2}\sqrt{5^2-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}$

$\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\left(\sin^{-1}(x/a)\right)$

$\int \frac{dx}{2^2-x^2} = \frac{1}{2(2)}\log\left(\frac{2+x}{2+x}\right)+C$

$\int (5-2x)^5 dx = \frac{(5-2x)^6}{6(-2)} = \frac{(5-2x)^6}{-12}$.

$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+C$

$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)(a)}$



1) $\int x \log x \, dx$.

2) $\int x \tan^2 x \, dx$.

1) Area of the region bounded by $x=a$, $x=b$ and a curve, lies above the $x$ axis is $= \int_a^b y \, dx$.

2) Area of the region bounded by a curve, $x=a$, $x=b$ & lies below the $x$ axis $= \int_a^b -y \, dx$.

3) Area of the region bounded by a curve, $y=a$; $y=b$ & lies right to $y$-axis is $= \int_a^b x \, dy$.

4) Area of the region bounded by a curve $y=a$, $y=b$ & lies left to $y$-axis is $= \int_a^b -x \, dy$.

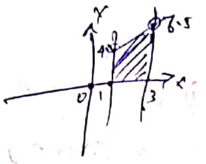
5) Area of the region bounded above and below the X-axis & $x=a$ & $x=b$ $= \int_a^c y\,dx + \int_c^b$

6) Area of the region bounded right and left of the $y$-axis & $y=a$ & $y=b$ $= \int_a^c x\,dy + \int_c^b -x$

1) Find the area of the region bounded by line $3x-2y+6=0$ and $x=1$, $x=3$ and $x$ axis.

Soln!

Given: $3x-2y+6=0$, $x=1$, $x=3$ and $x$-axis.




When $x=1$, $y=4.5$

when $x=3$, $y=7.5$

$\therefore y = \frac{3x+6}{2}$ ∴ Area $= \int_a^b y\,dx$

Area $= \int_1^3 \frac{3x+6}{2}\,dx$.

$$\int_1^3 \frac{3x+6}{2}\,dx = \frac{1}{2}\int_1^3 (3x+6)\,dx$$

$$= \frac{1}{2}\left[\frac{3x^2}{2}+6x\right]_1^3$$

$$= \frac{1}{2}\left[\left(\frac{3\times 9}{2}+6\times 3\right)-\left(\frac{3}{2}+6\right)\right]$$

$$= \frac{1}{2}\left[\frac{27}{2}+18-\frac{3}{2}-6\right]$$

$$= \frac{1}{2}\left[\frac{24}{2}+12\right]$$

$$= \frac{1}{2}\left[\frac{24+24}{2}\right]$$

$$= \frac{1}{2}\left[\frac{48}{2}\right]$$

$= 12$ sq. units.

Find the area of the region bounded by the Curve

2) Given:

Soln! $y=x^2-5x+4$, $x=2$, $x=3$ & $x$-axis.

when $x=2$, $y=-2$

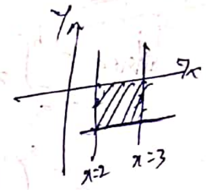
when $x=3$ $y=-2$

$4-10+4$

$8-10$

$9-15+4$

$13-15$




Soln:

Area $= \int_a^b -y\,dx$.

$$= \int_2^3 -(x^2-5x+4)\,dx.$$

$$= -\left[\frac{x^3}{3}-\frac{5x^2}{2}+4x\right]_2^3$$

$$= -\left[\left(\frac{27}{3}-\frac{45}{2}+12\right)-\left(\frac{8}{3}-\frac{20}{2}+8\right)\right]$$

$$= -\left[9-22.5+12-2.6+10-8\right]$$

$= 2.1$ sq. units.

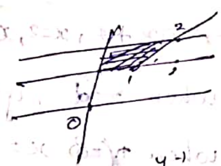
1) $y=2x+1$, $y=3$, $y=5$ & $y=$ axis. (right)

2) $y=2x+7$, $y=1$, $y=3$ & $y=$ axis (left).

3) $y=2x+1$

$y=3$, $x=1$

$y=5$, $x=2$.



$\frac{y-1}{2}$

Area $= \int_a^b x\,dy$

$= \int_3^5 \left(\frac{y-1}{2}\right) dy$

$= \frac{1}{2}\left[\frac{y^2}{2} - y\right]_3^5$

$= \frac{1}{2}\left[\frac{y^2-2y}{2}\right]_3^5$

$= \frac{1}{2}\left[\left[\frac{25-10}{2}\right] - \left[\frac{9-6}{2}\right]\right]$

$= \frac{1}{2}\left[\frac{+15}{2} - \left(\frac{3}{2}\right)\right]$

$= \frac{1}{2}\left[\frac{15-3}{2}\right]$

$= \frac{1}{2}\left[\frac{12}{2}\right]$

$= 3$ sq. units..

4). $y=2x+7$. $-\frac{1}{2}$

$y=1$, $x=-3/2$ $= x=-1.5$

$y=3$, $x=-1/2$ $= x=-0.5$

$\frac{y-7}{2}$

Area $= \int_a^b -x\,dy$

$= \int_1^3 -\left(\frac{y-7}{2}\right) dy$.

$= -\frac{1}{2}\left[\frac{y^2}{2} - 7y\right]_1^3$ $\quad \frac{-15}{2} = \left(\frac{-7}{2}\right)$ $\quad \frac{-15+7}{2}$

$= \frac{-1}{2}\left[\left[\frac{9}{2} - 12\right] - \left[\frac{1}{2} - 7\right]\right]$

$= -\frac{1}{2}\left[\left[\frac{9-21}{2}\right] - \left[\frac{1-8}{2}\right]\right]$

$= +\frac{1}{2}\left[+\frac{8}{2}\right]$

$= 2$ Sq. Units.

5) Find the area bounded by the Curve $y+3=x$, $x=1$, $x=5$.

$x=1$, $y=-2$ | $x=5$, $y=2$

$\int_1^3 -y\,dx + \int_3^5 y\,dx$.

$\Rightarrow -\int_1^3 (x-3)\,dx + \int_3^5 (x-3)\,dx$.

$= -\left[\frac{x^2}{2} - 3x\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^5$

$$-\left(\left(\frac{9-18}{2}\right)-\left(\frac{1-6}{2}\right)\right)+\left(\frac{25-30}{2}\right)-\left(\frac{9-18}{2}\right)$$

$$= -\left(\frac{-9}{2}+\frac{5}{2}+\frac{-5}{2}+\frac{9}{2}\right)$$

$$= \frac{9}{2} - \frac{5}{2} - \frac{5}{2} + \frac{9}{2}$$

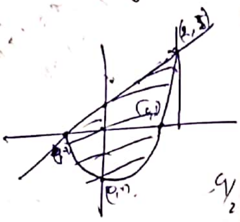
$$= \frac{18}{2} - \frac{10}{2} = \frac{8}{2}$$

$= 4$ Sq. Units.

Formula:

Area enclosed between two curves $R$ $\quad R = \int_a^b (f - g)\,dx$

6) Find the area between the lines $y = x+1$ and $y = x^2 - 1$





| $y = x+1$ | $y = x^2 - 1$ |
|---|---|
| $x = 0, y = 1$ | $x = 0, y = -1$ |
| $y = 0, x = -1$ | $y = 0, x = \pm 1$ |

From ① & ②

$x + 1 = x^2 - 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\boxed{x = 2, -1}$

when $x = 2$ then $y = 3$

$$= \int_a^b (f(x) - g(x))\,dx$$

$$= \int_{-1}^{2} \left[(x+1) - (x^2 - 1)\right] dx$$

$$= \left[\left[\frac{x^2}{2} + x\right] - \left[\frac{x^3}{3} - x\right]\right]_{-1}^{2}$$

$$= \left[\frac{x^2}{2} + x - \frac{x^3}{3} + x\right]_{-1}^{2}$$

$$\left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{-1}^{2}$$

$$= \left[\frac{4}{2} - \frac{8}{3} + 4\right] - \left[\frac{1}{2} + \frac{1}{3} - 2\right]$$

$$= \left[\frac{12 - 16 + 24}{6}\right] - \left[\frac{3 + 2 - 12}{6}\right]$$

$$= \frac{20}{6} + \frac{7}{6}$$

$$= \frac{27}{6}$$

15
12

36
16
20

1) Area of Circle with radius 'b' is

$x^2+y^2=b^2$


$y^2=b^2-x^2$

$y=\sqrt{b^2-x^2}$

$x=0$ to $b$.

$$\text{area} = 4\int_0^b y\,dx.$$

$$4\int_0^b y\,dx = 4\int_0^b \left(\sqrt{b^2-x^2}\right) dx$$

$$= 4\left[\frac{x}{2}\sqrt{b^2-x^2} + \frac{b^2}{2}\sin^{-1}\frac{x}{b}\right]_0^b.$$

$$= 4\left[\frac{b^2}{2}\sin^{-1}1 + 0\right]$$

$$= 4\left[\frac{b^2}{2}\cdot\frac{\pi}{2}\right]$$

$$= \cancel{4}\,\frac{b^2\pi}{\cancel{4}} \qquad = b^2\pi \text{ sq. Units.}$$

2) Area of ~~eppl~~ ellipse with radius 'b' is

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$

$y^2=b^2\left(1-\frac{x^2}{a^2}\right)$

$y=b\sqrt{1-\frac{x^2}{a^2}}.$ $\qquad x=0$ to $a$.

$$\text{Area} = 4\int_0^a y\,dx$$

$$= 4\int_0^a b\left(\sqrt{1-\frac{x^2}{a^2}}\right) dx$$

$$= \frac{4b}{a}\int_0^a \sqrt{a^2-x^2}\,dx.$$

$$= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{4b}{a}\left[\frac{a^2}{2}\sin^{-1}1\right]$$

$$= \frac{4b}{a}\cdot\frac{a^2}{2}\cdot\frac{\pi}{2}$$

$$= ba\pi \text{ Sq. Units}$$

Volume:

1) Volume generated about x-axis

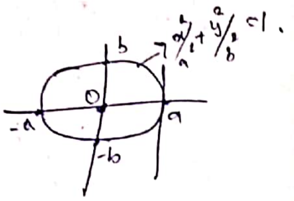
$$V=\int_a^b \pi y^2\,dx \text{ (cubic Units)}$$

2) Volume generated about y-axis

$$V=\int_a^b \pi x^2\,dy \text{ (cubic Units)}.$$

1) Find the volume of the solid generated by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolve about the x axis.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$




$$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$$

$$y^2=b^2\left(1-\frac{x^2}{a^2}\right).$$

limit $x = -a$ to $a$.

$$\text{volume} = \int_{-a}^{a}\pi\, y^2\, dx$$

$$= 2\int_0^a \pi\, b^2\left(1-\frac{x^2}{a^2}\right)dx.$$

$$= \frac{2\pi b^2}{a^2}\int_0^a (a^2-x^2)\,dx.$$

$$= \frac{2\pi b^2}{a^2}\left(a^2x-\frac{x^3}{3}\right)_0^a.$$

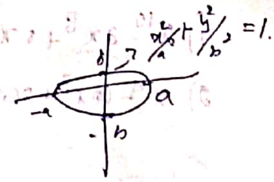
$$= \frac{2\pi b^2}{a^2}\left(a^3-\frac{a^3}{3}\right).$$

$$= \frac{2\pi b^2\, 2a^3}{a^2\;3}$$

$$= \frac{4\pi b^2 a}{3}\ \text{Cubic Units.}$$

2) Find the volume of the solid generated by the ellispe $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolve about the y-axis. (minor axis)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$




$$\frac{x^2}{a^2}=1-\frac{y^2}{b^2}$$

$$x^2=a^2\left(1-\frac{y^2}{b^2}\right).$$

limit $y = -b$ to $b$.

$$\text{volume } V = \int_{-b}^{b}\pi x^2\, dy$$

$$= 2\pi\int_0^b x^2\, dy.$$

$$= 2\pi\int_0^b a^2\left(1-\frac{y^2}{b^2}\right)dy.$$

$$= \frac{2\pi a^2}{b^2}\int_0^b (b^2-y^2)\,dy$$

$$= \frac{2\pi a^2}{b^2}\left[b^2 y-\frac{y^3}{3}\right]_0^b$$

$$= \frac{2\pi a^2}{b^2}\left[b^3-\frac{b^3}{3}\right]$$

$$= \frac{2\pi a^2}{b^2}\left[\frac{2b^3}{3}\right]$$

$$= \frac{4}{3}\pi b a^2\ \text{Cubic Units}$$

UNIT-I Multiple Integrals

| Objective type questions | Opt 1 | Opt2 | Opt3 | Opt4 | Answer |
|---|---|---|---|---|---|
| ∫ x^n dx=............. | x^(n+1)/ (n+1)+ C | x^(n-1)/ (n-1)+ C | nx^ (n-1)+ C | (n+1) x^ (n+1)+ C | x^(n+1)/ (n+1)+ C |
| ∫ cosx dx=............. | sinx + C | cosx + C | (-cosx)+C | (-sinx)+C | sinx + C |
| ∫ sinx dx=............. | sinx + C | cosx + C | (-cosx)+C | (-sinx)+C | (-cosx)+C |
| ∫ e^(x) dx=............. | (-e^x)+ C | e^(-x) + C | (-e^(-x))+C | e^x + C | e^x + C |
| ∫ e^(-x) dx=............. | (-e^x)+ C | e^(-x) + C | (-e^(-x))+C | e^x + C | (-e^(-x))+C |
| If u and v are differentiable functions then ∫ u dv =........... | uv+ ∫ v du | uv+ ∫ v du | (-uv)+ ∫ v du | (-uv)- ∫ v du | uv+ ∫ v du |
| ∫ cos^(4) x dx ( from 0 to π/2) =................. | 3π/16 | 5π/16 | 7π/16 | 9π/16 | 3π/16 |
| ∫ cos^(6) x dx ( from 0 to π/2) =................ | 3π/16 | 5π/16 | 7π/16 | 9π/16 | 5π/16 |
| ∫ cos^(9) x dx ( from 0 to π/2) =................... | 3π/16 | 5π/16 | 7π/16 | 9π/16 | 5π/16 |
| ∫ sin^(5) x dx ( from 0 to π/2) =................. | π/15 | π/15 | 8π/15 | 8π/15 | 8/15 |
| ∫ sin^(7) x dx ( from 0 to π/2) =..................... | π/15 | 1/15 | 16π/35 | 16/35 | 16/35 |
| ∫ cos2x dx=............. | (sin2x)/2 + C | (cos2x)/2 + C | (-cosx)/2+C | (-sinx)/2+C | (sin2x)/2 + C |
| ∫ sin3x dx=............. | (sin3x)/3 + C | (cos3x)/3 + C | (-cos3x)/3+C | (-sin3x)/3+C | (-cos3x)/3+C |
| ∫ (1/x) dx=............. | 1+ C | log x+C | (-1)+C | (-log x)+ C | log x+C |
| The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=a^2 about its diameter is.......... | (4/3)πa^3 | (2/3)πa^3 | (1/3)πa^3 | πa^3 | (4/3)πa^3 |
| The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=2^2 about its diameter is.......... | (32/3)π | (1/3)π | (2/3)π | π | (32/3)π |
| The volume of the solid of revolution generated by revolving the plane area bounded by the circle x^2+y^2=3^2 about its diameter is.......... | 16π | 9π | 36π | π | 36π |
| The Volume of a sphere of radius 'a' is................. | 2/3 π a^3 | 4/3 π a^3 | 1/3 π a^3 | π a^3 | 4/3 π a^3 |
| The surface are of the sphere of radius 'a' is................. | 4πa^2 | πa^2 | 3πa^2 | 2πa^2 | 4πa^2 |

| | | | | | |
|---|---|---|---|---|---|
| ∫ x e^(x) dx=............. | (-x)e^(x)-e^(x)+c | xe^(x)+e^(x)+ | (-x)e^(x)+e^ | xe^(x)-e^(x)+ | xe^(x)-e^(x)+c |
| ∫ cosmx dx=............. | (sinmx)/m + C | (cosmx)/m + C | (-cosmx)/m+ | (-sinmx)/m+C | (sinmx)/m+ C |
| ∫ sinnx dx=............. | (sinnx)/n + C | (cosnx)/n + C | (-cosnx)/n+C | (-sinnx)/n+C | (-cosnx)/n+C |
| ∫ dx=............. | x+C | 1 | 0 | x^2 | x+C |
| ∫ 5dx=............. | x+C | 5x+C | x^2+C | 5+C | 5x+C |
| ∫ 3x^(2) dx=............. | 3x^(2)+C | x+C | x^2+C | x^(3) +C | x^(3) +C |
| ∫ Sec ^ (2) x dx=............. | secx.tanx+C | tanx+C | tan^(2) x +C | Secx+C | tanx+C |
| ∫ Secx. tanx dx=............. | secx.tanx+C | tanx+C | tan^(2) x +C | Secx+C | Secx+C |
| ∫ e^(2x) dx=............. | (-e^2x)/2+ C | e^(-2x)/2 + C | (-e^(-2x))/2+ | e^2x/2+ C | e^2x/2 + C |
| ∫ e^(-2x) dx=............. | (-e^(-2x))/2+ C | e^(-2x)/2 + C | (-e^(-2x))/2+ | e^(-2x)/2+ C | e^2x/2 + C |
| | | | | | |
| The Volume of a sphere of radius '2' is................. | 16/3 π | 32/3 π | 8/3 π | 8 π | 32/3 π |
| The surface area of the sphere of radius '3' is................. | 36π | 9π | 27π | 18π | 36π |
| ∫ x^ (2) dx=............. | (x^(2)/2)+C | (x^(3)/3)+C | x+C | 2x+C | (x^(3)/3)+C |
| ∫ x logx dx=………. | 1-logx+C | logx+C | 0 | 1 | 1-logx+C |
| ∫cosec^(2) x dx=………. | cotx+C | tanx+C | (-tanx)+C | (-cotx)+C | (-cotx)+C |
| ∫sec^(2) x dx=………. | cotx+C | tanx+C | (-tanx)+C | (-cotx)+C | tanx+C |

**UNIT-II**

**Functions of several variables**

| **QUESTIONS** | **OPTION 1** | **OPTION 2** | **OPTION 3** | **OPTION 4** | **Answer** |
|---|---|---|---|---|---|
| The partial differentiation is a function of_________ or more variables . | two | zero | one | three | two |
| If z=f(x,y) where x and y are ______ function of another variable z | continuous | differential | two | one | continuous |
| If f(x,y)=0 then xand y are said to be an _________ function | implicit | extrem | explicit | differential | implicit |
| The concept of jacobian is used when we change the variables in________ | multiple integrals | single integrals | diffenential | function | multiple integrals |
| The extreme values of f(x,y,z) in such a situation are called________values | extreme | constrained extreme | boundry values | initial | constrained extreme |
| The ________series of f(x,y) at the point (0,0) is maclaurins series of f(x,y). | Maclaurins | Taylor | power | binomial | Taylor |
| The jacobian were introduced by_________ | C.G.Jacobi | johon | Gauess | | C.G.Jacobi |
| The Tayolr,s series of f(x,y) at the point (0,0) is ___________ series. | Maclaurins | Taylor | power | binomial | Maclaurins |
| f(a,b) is said to be exturemum value of f(x,y) if it is either a_______________ | maximum or minimum | zero | minimum | maximum | maximum or minimum |
| The expansion of f(x,y) by Taylor series is __________ | zero | unique | minimum | maximum | unique |
| If f(x,y)=e^x cosy at (0,1/2) then __________ | f=1 | $f^1$=0 | f=0 | f=4cos x | f=0 |
| The Lagrange multiplier is denoted by ________________ | a | b | l | d | a |
| Every extremum value is a stationary value but a stationary value need not be an ___________ value. | infimum | minimum | maximum | extremum | extremum |
| If u1,u2……un are functions of n variables x1,x2……..xn then the Jacobian of the transformation from x1,x2….xn to u1,u2...un is defined by__________ | 2 | 0 | 1 | -1 | 1 |
| F is differentiable and where not all of its first differential derivatives vanish simultaneously then the functions u1,u2…..un are said to be functionally_____ | independent | dependent | explicit | implicit | dependent |
| f(a,b) is a maximum value of f(x,y) if there exists some neighbourhood of the point (a,b) such that for every point (a+h,b+k) of the neighbourhood ________ | f(a,b)>f(a+h,b+k) | f(a,b)<f(a+h,b+k) | f(a,b)<0 | f(a,b)>0 | f(a,b)>f(a+h,b+k) |

| | | | | | |
|---|---|---|---|---|---|
| f(a,b) is a minimum value of f(x,y) if there exists some neighbourhood of the point (a,b) such that for every point (a+h,b+k) of the neighbourhood ________ | f(a,b)>f(a+h,b+k) | f(a,b)<f(a+h,b+k) | f(a,b)<0 | f(a,b)>0 | f(a,b)<f(a+h,b+k) |
| The necessary condition for maxima is __________ | ∂f/∂x (a,b)=0 | ∂f/∂x (a,b)= 1 | ∂f/∂y (a,b)=5 | ∂f/∂y (a,b)=1 | ∂f/∂x (a,b)=0 |
| The necessary condition for minimum is ____________ | ∂f/∂x (a,b)=0 | ∂f/∂y (a,b)=0 | ∂f/∂x (a,b)=1 | ∂f/∂y (a,b)=1 | ∂f/∂y (a,b)=0 |
| f(a,b) is said to be said to be a stationary value of f(x,y) if (x,y) is__________ | ∂f/∂x (a,b)=0 and ∂f/∂y (a,b)=0 | ∂f/∂x (a,b)=1 | ∂f/∂y (a,b)=0 | ∂f/∂y (a,b)=1 | ∂f/∂x (a,b)=0 and ∂f/∂y (a,b)=0 |
| f(x,y) =e^x siny at (1,π/2)then ___________ | f=0 | f=1 | f=2 | f=e | f=e |
| f(x,y) = e^xy at(1,1) then __________ | f=1 | f=e | f=0 | f=2 | f=e |
| The equation of the degree 1 is called _________ function | linear | homogenious | nonhomogenious | bilateral | linear |
| The expansion of f(x,y) by ________series is unique. | Maclaurins | Taylor | power | binomial | Taylor |
| If f(a,b) is said to be _________of f(x,y) if it is either maximum or minimum. | extremum value | boundary value | end | power | extremum value |
| The _______ differentiation is a function of two or more variables. | ODE | PDE | partial | total | partial |
| The ______ were introduced by C.G.Jacobi. | jacobian | millian | taylor | Gauss | jacobian |
| The______ series of f(x,y) at the point(0,0) is maclaurins series of f(x,y). | taylor | jacobian | gauss | maculaurin | taylor |
| The concept of ________ is used when we change the variables in multiple integrals | taylor | gauss | maculaurin | jacobian | jacobian |
| If the function u,v,w of three independent variables x,y,z are not independent then the Jacobian of u,v,w with respect to x,y,z is always equal to | 1 | 0 | Infinity | Jacobian of x,y,z with respect ro | 0 |
| The function f(x)=10+x^6 | is a decrasing function of x | has a minimum at x=0 | has neither a maximum nor a minimum at x=0 | saddle point | neither a maximum nor a minimum |

| | | | | | |
|---|---|---|---|---|---|
| The function f(x,y)=2x^2+2xy-y^3 has | only one stationary point at (0,0) | two stationary points at (0,0)and (1/6,1/3) | two stationary point at (0,0) and (1,-1) | not stationary points | two stationary points at (0,0)and (1/6,1/3) |
| If(a/3,a/3) is an extreme point on xy(a-x-y), the maxima is | a^3/27 | a/27 | a^3/9 | a/9 | a^3/27 |
| Any function of the type f(x,y)=c is called an _______function | Implicit | Explicit | Constant | composite | Implicit |
| If u=f(x,y) ,where x=pi(t),y=si(t) then u is a function of t and is called the ____ function | Implicit | Explicit | Constant | composite | composite |
| The point at which function f(x,y) is either maximum or minimum is known as ______ point | Stationary | Saddle point | extremum | implicit | Stationary |
| If rt-s^2>0 and r<0 at (a,b) the f(x,y) is maximum at (a,b) and the_______ value of the function(a,b) | Maximum | Minimum | maximum or minimum | zero | Maximum |
| If rt-s^2>0 and r>0 at (a,b) the f(x,y) is minimum at (a,b) and the_______ value of the function(a,b) | Maximum | Minimum | maximum or minimum | zero | Minimum |
| If rt-s^2>0 at (a,b) the f(x,y) is neither maximum nor minimum at (a,b) such point is known as _______ point | Stationary | Saddle point | extremum | implicit | Saddle point |
| If f(x,y) is a function of two variables x,y then _________ | lim f(x,y)=1 | lim f(x,y=0 | lim f(x,y)>0 | lim f(x,y)<0 | lim f(x,y)=1 |
| The concept of jacobian is used when we change the variables in ______ | multiple integrals | single integrals | diffenential | Function | multiple integrals |

**UNIT III**
**Foutier Series**

| Questions | opt 1 | opt 2 | opt 3 | opt 4 | Answer |
|---|---|---|---|---|---|
| Which of the following functions has the period 2π? | cos x | sin nx | tan nx | tan x | cos x |
| 1/π ∫ f(x) sinnx dx between the limits c to c+2π gives the Fourier coefficient_____ | a_0 | a_n | b_n | b_1 | b_n |
| If f(x) = -x for -π< x< 0 then its Fourier coefficient $a_0$ is_______- | (π^2)/2 | π/2 | π/3 | π | (π^2)/2 |
| If a function satisfies the condition f(-x) = f(x) then which is true? | $a_0$ = 0 | $a_n$ = 0 | $a_0$ = $a_n$ = 0 | $b_n$ = 0 | $b_n$ = 0 |
| If a function satisfies the condition f(-x) = -f(x) then which is true? | $a_0 = 0$ | $a_n = 0$ | $a_0$ = $a_n$ = 0 | $b_n$ = 0 | $a_0$ = $a_n$ = 0 |
| Which of the following is an odd function? | sin x | cos x | $x^2$ | x^4 | sin x |
| Which of the following is an even function? | $x^3$ | cos x | sin x | sin^3x | cos x |
| The function f(x) is said to be an odd function of x if | f(-x) = f( x) | f(x) = - f( x) | f(-x) = - f( x) | f(-x) = f(-x) | f(-x) = - f( x) |
| The function f(x) is said to be an even function of x if | f(-x) = f( x) | f(x) = - f( x) | f(-x) = - f( x) | f(-x) = f(-x) | f(-x) = f( x) |
| ∫f(x) dx = 2∫f(x) dx between the limits -a to a if f(x) is ------ | even | continuous | odd | discontinues | even |
| ∫f(x) dx = 0 between the limits -a to a if f(x) is ------ | even | continuous | odd | discontinues | odd |
| If a periodic function f(x) is odd, it's Fourier expansion contains no ------ terms. | coefficient $a_n$ | sine | coefficient $a_0$ | cosine | cosine |
| If a periodic function f(x) is even, it's Fourier expansion contains no ------ terms. | cosine | sine | coefficient $a_0$ | coefficient $a_n$ | sine |
| In dirichlet condition, the function f(x) has only a ----- number of maxima and minima. | uncountable | continuous | infinite | finite | finite |
| In Fourier series, the function f(x) has only a finite number of maxima and minima. This condition is known as ------- | Dirichlet | Kuhn Tucker | Laplace | Cauchy | Dirichlet |
| In dirichlet condition, the function f(x) has only a ----- number of discontinuities . | uncountable | continuous | infinite | finite | finite |
| The Fourier series of f(x) is given by ---- | $a_0$/2 + ∑ ($a_n$ cosnx+ bn sin nx ) | $a_0$/2 + ∑ ($a_n$ cos nx- bn sin nx ) | $a_n$/2 + ∑ ($a_n$ sin nx+ bn sin nx ) | $a_0$/2 + ∑ ($a_0$ sin nπx/ l ) | $a_0$/2 + ∑ ($a_n$ cosnx+ bn sin nx ) |

| | | | | | |
|---|---|---|---|---|---|
| In Fourier series, the expansion $f(x) = a_0/2 + \sum (a_n \cos nx + b_n \sin nx)$ is possible only if in the interval $c_1 \le x \le c_2$ the function f(x) satisfies --- condition. | kuhn- Tucker | Laplace | Dirichlet | Cauchy | Dirichlet |
| If the periodic function f(x) is even, then the Fourier expansion is of the form --- | $a_0/2 + \sum a_n \sin(n\pi x/l)$ | $a_0/2 + \sum a_n \cos(n\pi x/l)$ | $a_n/2 + \sum a_n \cos(n\pi x/l)$ | $a_0/2 + \sum a_0 \sin(n\pi x/l)$ | $a_0/2 + \sum a_n \cos(n\pi x/l)$ |
| If the periodic function f(x) is even, then it's Fourier co- efficient a_n is of the form --- | $2/l \int f(x) \sin(n\pi x/l)\, dx$ | $2/l \int f(x) \cos(n\pi x/l)\, dx$ | $1/l \int f(x)/l\, dx$ | $\int f(x)\, dx$ | $2/l \int f(x) \cos(n\pi x/l)\, dx$ |
| If the periodic function f(x) is even, then it's Fourier co- efficient $a_0$ is of the form --- | $2/l \int f(x)\, dx$ | $1/l \int f(x)\, dx$ | $2/l \int f(x)/l\, dx$ | $\int f(x)\, dx$ | $2/l \int f(x)\, dx$ |
| If the periodic function f(x) is odd, then it's Fourier co- efficient bn is of the form --- | $2/l \int f(x) \cos(n\pi x/l)\, dx$ | $2/l \int f(x) \sin(n\pi x/l)\, dx$ | $\int f(x)\, dx$ | $1/l \int f(x)/l\, dx$ | $2/l \int f(x) \sin(n\pi x/l)\, dx$ |
| If the periodic function f(x) is even, then it's Fourier co- efficient -------- is zero. | $a_0$ | $a_1$ | $b_n$ | $a_0$ & $a_n$ | $b_n$ |
| If the periodic function f(x) is odd, then it's Fourier co- efficient -------- is zero. | a_0 & a_n | a_1 | b_n | b_1 | a_0 & a_n |
| If the periodic function f(x) is even, then the Fourier expansion is of the form --- | ∑b_n sin nπx/ l | ∑b_n sin nπx/ l | ∑ b_n cos nπx/ l | a_0/2+∑ a_n cos (nπx/ l) | a_0/2+∑ a_n cos (nπx/ l) |
| If the periodic function f(x) is odd, then the Fourier expansion is of the form --- | $\sum b_n \sin n\pi x/l$ | $\sum a_n \sin n\pi x/l$ | $\sum b_n \cos n\pi x/l$ | $\sum a_n \cos n\pi x/l$ | $\sum b_n \sin n\pi x/l$ |
| 1/π∫f(x) cos nx dx gives the Fouier coefficient ----------- | a_0 | b_1 | b_n | a_n | a_n |
| 1/π∫f(x) dx gives the Fourier coefficient | a_0 | a_n | b_n | b_1 | a_0 |
| 1/π∫f(x)sin nx dx gives the Fouier coefficient ----------- | a_0 | a_n | b_n | b_1 | b_n |
| The period of cos nx where n is the positive integer is | 2π/n | π/2n | 2π | nπ | 2π/n |
| The Fourier co efficient $a_0$ for the function defined by f(x) = x for 0< x< π is | π | π/2 | 2π | 0 | π |
| If the function f(x) = x sin x, in –π< x< π then Fourier coefficient | $b_n = 0$ | $a_0 = 1$ | $a_0 = (\pi^2)/3$ | $a_0 = -1$ | $b_n = 0$ |
| For the cosine series, which of the Fourier coefficient will vanish? | a_n | b_n | a_1 | Both $a_0$ and $a_n$ | b_n |
| For the sine series, which of the Fourier coefficient variables will be vanish? | b_n | a_n | Both a_0 and a_n | a_0 | Both a_0 and a_n |
| For a function f(x) = x^3, in –π< x< π the Fourier coefficient | $b_n = 0$ | $a_n = 1$ | $a_0 = 1$ | $a_0 = a_n = 0$ | $a_0 = a_n = 0$ |

| | | | | | |
|---|---|---|---|---|---|
| F(x)=x cos x is an ------ function. | an odd function | even function | neither odd or even | both even and odd | an odd function |
| If f(x) = x, in $-\pi< x< \pi$ then Fourier co efficient | b_n = 0 | a_n = π | a_0 = a_n = 0 | a_n = 1 | a_0 = a_n = 0 |
| F(x)=e^x is in $-\pi< x< \pi$. | an odd function | even function | neither odd or even | Both even and odd | neither odd or even |
| Which of the coefficients in the Fourier series of the function $f(x) = x^2$ in $-\pi < x< \pi$ will vanish | $a_0$ | $a_0$ and $a_n$ | $b_n$ | $a_n$ | $b_n$ |
| If f(-x) = -f(x), then the function f(x) is said to be ----- | odd | Continuous | even | discontinuous | odd |
| If f(-x) = f(x), then the function f(x) is said to be ----- | odd | continuous | even | discontinuous | even |
| The function x sin x is a ------- function in $-\pi< x< \pi$. | even | odd | continuous | discontinuous | even |
| The function x cos x is a ------- function in $-\pi< x< \pi$. | even | odd | continuous | discontinuous | odd |
| The formula for finding the fourier coefficient a_0 in Harmonic analysis is ---- | (2/N)Σ y cos nx | ( 2/N)Σ y | (2/N)Σ y sin nx | a_0 /2 + ∑ (a_n cosnx+ b_n sin nx ) | ( 2/N)Σ y |
| The formula for finding the fourier coefficient a_n in Harmonic analysis is ---- | (2/N)Σ y cos nx | ( 2/N)Σ y | a_0 /2 + ∑ (a_n cosnx+ b_n sin nx ) | (2/N)Σ y sin nx | (2/N)Σ y cos nx |
| The formula for finding the fourier coefficient $b_n$ in Harmonic analysis is ---- | ( 2/N)Σ y | (2/N)ΣΣ y sin nx | (2/N)Σ y cos nx | $a_0/2 + \sum (a_n$ cosnx+ bn sin nx ) | (2/N)Σ y sin nx |
| The term $a_1\cos x+ b_1 \sin x$ is called the----- harmonic. | second | first | third | end | first |
| The term ------------- is called the first harmonic in Furier Series expansion. | $a_1$cosn x+ $b_1$ sin x | $a_1$cos2 x+ $b_1$ sin x | $a_1$cos x+ $b_1$ sin2 x | $a_1$cos x+ $b_1$ sin x | $a_1$cos x+ $b_1$ sin x |
| If f(x)= x in 0<x<2π and f(x)=f(x+2π) then the sum of the fourier series of f(x) at x=2π is------ | 2π | 2 | 0 | π | 2π |

| | | | | | |
|---|---|---|---|---|---|
| If f(x)= x^2 in 0<x<2π and f(x)=f(x+2π) then the sum of the fourier series of f(x) at x= 0 is------ | 2π^2 | 0 | 6π | 4π | 2π^2 |
| For any peroidic function f(x) in –π< x< π the point x=- π is a ----- point. | Continous | discontinous | intermediate | Continous and discontinous | discontinous |
| For any peroidic function f(x) in 0< x<2 π the point x= π is a ----- point. | Continous and discontinous | intermediate | Continous | discontinous | Continous |
| For any peroidic function f(x) in 0≤ x≤ π the point x= 0 is a ----- point. | discontinous | Continous and discontinous | Continous | intermediate | Continous |
| The process of finding the Fourier series for a function given by ------------ at equally spaced points is known as harmonic analysis. | initial value | numerical value | final value | fundamental value | numerical value |
| The process of finding the Fourier series for a function given by numerical values at ----------- points is known as harmonic analysis. | equally spaced | unequally spaced | intermediate | both equally and unequally spaced | equally spaced |
| The process of finding the Fourier series for a function given by numerical values at equally spaced points is known as ----------. | mathematical analysis | complex analysis | real analysis | harmonic analysis. | harmonic analysis. |

**UNIT -IV**

**Boundary value problems**

| Questions | opt 1 | opt 2 | opt 3 | opt 4 | Answer |
|---|---|---|---|---|---|
| Partial differential equation of second order is said to Elliptic at a point (x,y) in the plane if --------- | B^2-4AC<0 | B^2-4AC=0 | B^2-4AC>0 | B^2=4AC | B^2-4AC<0 |
| Partial differential equation of second order is said to Parabolic at a point (x,y) in the plane if --------- | B^2-4AC<0 | B^2-4AC=0 | B^2-4AC>0 | B^2=4AC | B^2-4AC=0 |
| Partial differential equation of second order is said to Hyperbolic at a point (x,y) in the plane if --------- | B^2-4AC<0 | B^2-4AC=0 | B^2-4AC>0 | B^2=4AC | B^2-4AC>0 |
| Two dimensional Laplace Equation is ---------- | u_xx+u_yy=1 | u_xx+u_yy=0 | u_x=u_y | u_x+u_y=0 | u_xx+u_yy=0 |
| One dimensional heat Equation is ---------- | u_xx=(1/α^2)u_t | u_xx=[(1/α^2) u_t]+10 | u_xx=u_tt | u_xx+u_tt=0 | u_xx=(1/α^2)u_t |
| One dimensional wave Equation is ---------- | u_xx=(1/α^2)u_t | u_xx+u_yy=0 | u_xx=(1/α^2)u_t^2 | u_xx=u_t | u_xx=(1/α^2)u_tt^2 |
| The D'Alembert's solution of the One dimensional wave Equation is----- | y(x,t)=φ(x-αt)+ψ(x+αt) | y(x,t)=0 | u_xx=(1/α^2)u_t | u_xx=(1/α^2)u_t^2 | y(x,t)=φ(x-αt)+ψ(x+αt) |
| The Possion equation is of the form ----- | y(x,t)=φ(x-αt)+ψ(x+αt) | u_xx=(1/α^2)u_t | u_xx=(1/α^2)u_tt | u_xx+u_yy=f(x,y) | u_xx+u_yy=f(x,y) |
| The steady state temperature of a rod of length l whose ends are kept at 30 and 40 is | u(x)= 10x/l + 30 | u(x)= 40x/l | u(x)= 30x/l | None | u(x)= 10x/l + 30 |
| Two dimensional heat Equation is known as ----------equation. | partial | Radio | laplace | Poisson | laplace |
| In one dimensional heat flow equation ,if the temperature function u is independent of time, then the solution is------ | u(x)= ax + b | u(x,t)= a(x,t) | u(t) = at + b | u(t) = at - b | u(x)= ax + b |
| f_xx+2f_xy+4f_yy=0 is a ______ | Elliptic | Hyperbolic | Parabolic | circle | Elliptic |
| f_xx=2f_yy is a --------- | Elliptic | Hyperbolic | Parabolic | circle | Hyperbolic |
| f_xx-2f_xy+f_yy=0 is a ----------- | Hyperbolic | Elliptic | Parabolic | circle | Parabolic |
| The diffusivity of substance is------ | k/pc | pc | k | pc/k | k/pc |
| Heat flows from a ------- temperature | higher to lower | lower to higher | normal | high | higher to lower |
| The Amount of heat required to produce a given temperature change in a bodies propostional to the ------- of the body and to the temperature change. | temperature | heat | mass | wave | mass |
| The rate at which heat flows through an area is------ to the area and to the temperature gradient normal to the area. | equal | not equal | lessthan | proportional | proportional |

| | | | | | |
|---|---|---|---|---|---|
| In steady state conditions the temperature at any particular point does not vary with --- | Time | temperature | mass | none | Time |
| The wave equation is a linear and ------ equation | non homogeneous | homogeneous | quadratic | none | homogeneous |
| In method of separation of variables we assume the solution in the form of ----- | u(x,y)=X(x) | u(x,t)=X(x)T(t) | u(x,0)=u(x,y) | u(x,y)=X(y)Y(x) | u(x,t)=X(x)T(t) |
| u(x,t)=(Acosλx+Bsinλx)Ce^(-(α^2))(λ^2)t) is the possible solution of ------ equation | heat | wave | laplace | none | heat |
| y=(Ax+B)(Ct+D) is the possible solution of ------ equation | heat | wave | laplace | none | wave |
| If the heat flow is one dimensional ,then the -------- is a function x and t only | heat | light | temperature | wave | temperature |
| The stream lines are parallel to the X-axis ,then the rate of change of the temperature in the direction of the Y-axis will be ---------. | one | two | zero | five | zero |
| To solve y_tt=(α^2)yxx, we need -------- boundary conditions. | y(0,t)=0 if t>=0; y(l,t)=0 if t>=0 | y(x,t)=0 if t>0; y(t)=0 if t=0 | y(x,t)=0 if t>0 | none | y(0,t)=0 if t>=0; y(l,t)=0 if t>=0 |
| The boundary condition with non zero value on the R.H.S of the wave equation should be taken as the -------------- boundary condition. | First | Second | Last | none | Last |
| In one dimensional heat equation u_t=(α^2)u_xx, What does α^2 stands for? | k/pc | pc | k | pc/k | k/pc |
| The possible solution of wave equation is ---------- | y=(Ax+B)(Ct+D) | u(x,t)=(Acosλx+Bsinλx)(Ce^(λy)+De^(-λy)) | u(x,t)=Acosλx+Bsinλx | u(x,t)=Acosλx-Bsinλx | y=(Ax+B)(Ct+D) |
| The possible solution of heat equation is ---------- | u(x,t)=(Acosλx+Bsinλx)Ce^(-(α^2))(λ^2)t) | u(x,t)=(Acosλx+Bsinλx)(Ce^(λy)+De^(-λy)) | u(x,t)=Acosλx+Bsinλx | u(x,t)=Acosλx-Bsinλx | u(x,t)=(Acosλx+Bsinλx)Ce^(-(α^2))(λ^2)t) |
| If B^2-4AC = 0, then the differential equation is said to be______ | parabolic | elliptic | hyperbolic | equally spaced | parabolic |
| If B^2-4AC > 0, then the differential equation is said to be______ | parabolic | elliptic | hyperbolic | equally spaced | hyperbolic |
| If B^2-4AC < 0, then the differential equation is said to be______ | parabolic | elliptic | hyperbolic | equally spaced | elliptic |

| questions | opt1 | opt2 | opt3 | opt4 | Answer |
|---|---|---|---|---|---|
| Which of the following is the most unstable average__________ | mode | median | geometric mean | harmonic mean | mode |
| The sum of deviations taken from arithmetic mean is_________ | minimum | zero | maximum | one | minimum |
| The sum of square deviations taken from arithmetic mean is______ | zero | maximum | minimum | one | minimum |
| When calculating the average growth of economy,the correct mean to use is_______________ | weighted mean | Geometric mean | arithmetic mean | median | geometric mean |
| When observation in the data is zero, then its geometric mean is__________________ | Negative | zero | positive | normal | zero |
| The best measure of central tendency is_______________ | arithmetic mean | Geometric mean | Harmonic mean | median | arithmetic mean |
| The point of inersection of the less than and more than corresponds to______________ | mean | median | geometric mean | mode | median |
| Median is same as__________quartile | first | second | third | four | second |
| Median is a ________average | first | second | positional | normal | positional |
| Median is dividing the series when arranged as an array into_______ parts | two | three | four | normal | two |
| Median and mode are called ___________average | first | second | positional | normal | positional |
| The geometric mean of a set of values lies between arithmetic mean and__________ | harmonic mean | Geometric mean | mean | median | harmonic mean |
| In a symmetrical distrbution mean__________median_________mode | is equal to,is equal to | is equal to,less than | less tnan or equal to | greater than or equal to | is equal to,is equal to |
| Harmonic mean is the ________ of the arithmetic mean of the values | positional | proposional | reciprocal | equal | reciprocal |
| The ___and _________ mark off the limits with in which the middle 50 % of the items lie | quartile one and three | deviation and one | median and the three | deviation | quartile one and three |
| ___________ can be calculated from a frequeny distribution with open end classes | median or mode | mode | mean or median | deviation | median or mode |
| In the calculation of _______ all the observations are taken into cosideraion | mean | mode | median | divation | mean |
| Median is the average suited for ________classes | open -end | middle | center | sub | open-end |
| When calculating the average rate of debt expansion for a company, the correct mean to use is the______ | arithmetic mean | weighted mean | geometric mean | either a (or) c | geometric mean |

| | | | | | |
|---|---|---|---|---|---|
| The mode has all the following disadvantages except________ | a data set may have no modal value | every value in a data set may be a mode | a multimodal data set is difficult to analvze | the mode is unduly affected byy extream values | the mode is unduly affected by extreme value |
| If one event is unaffected by the outcome of another event, the two events are said to be _____ | dependent | independent | mutually exclusive | event | independent |
| Calculate the mode of the data 13,13,14,15,13,15,14,15,13. | 14 | 13 | 13.5 | 15 | 13 |
| The empirical formula for Mode (m) = | 3 median – 2 mean | 2 median – 3 mean | 3 median + 2 mean | 3 mean – 2 median | 3 median – 2 mean |
| ___ is defined as the middle item of the given observations arranged in ascending order. | mean | mode | median | divation | median |
| The positional average is___ | mean | mode | median | divation | mean |
| Find the median 10, 15, 9, 25, 19. | 15 | 9 | 25 | 19 | 15 |
| Calculate the mode of the data 14,13,14,15,13,15,14,15,14. | 13 | 14 | 15 | 13.5 | 14 |
| What is the median of the numbers 4,12, 11, 6, 2? | 2 | 4 | 5 | 11 | 6 |
| What is the median of the numbers 3, 11, 6, 5, 4, 7, 12, 3 and 10? | 4 | 5 | 6 | 7 | 6 |
| What is the mean of the squares of the first ten natural numbers? | 30.25 | 31.67 | 38.5 | 50.5 | 38.5 |
| What is the mean of these numbers: 12, -1, 8, 2, -10, 0, -5, 3, 20, -2? | 6.3 | 5.3 | 3.7 | 2.7 | 2.7 |
| What is the mean of the numbers 8, 9, 13 and 18? | 10 | 11 | 12 | 16 | 12 |
| A booklet has 12 pages with the following numbers of words: 271, 354, 296, 301, 333, 326, 285, 298, 327, 316, 287 and 314. What is the mean number of words per page? | 307 | 309 | 311 | 313 | 309 |
| The coefficient of correlation is independent of change of_____ and ________ | scale,origin | vector,origin | variable, constant | interer, origin | scale,origin |
| If r is more than six times ________ it is called significant | scientific error | favourable Error | relative error | truncation error | favourable Error |
| The relationship between three or more variables is studied with the help of _________ correlation. | multiple | rank | perferct | spearman's rank | multiple |
| The coefficient of correlation___ | has no limits | can be less than 1 | can be more than 1 | varies between + or - one | varies between + or - one |
| which of the following is the highest range of r__________ | 0 and 1 | minus one and 0 | minus one and one | zero | minus one and one |

| | | | | | |
|---|---|---|---|---|---|
| The coefficient of correlation is independent of ________ | change of scale only | change of origin only | both change of scale and origin | change of variables | both change of scale and origin |
| The coefficient of correlation___ | cannot be positive | cannot be negative | can be either positive or negative | zero | can be either positive or negative |
| If X=Y , then correlation cofficient between them is _______ | 1 | zero | less than one | gerater than one | 1 |
| Correlation means relationship between ______ variables | two | one | two or more | three | two or more |
| The covariance of two independent random variable is ________ | Zero | two | three | two or more | Zero |
| Two random variables are said to be orthogonal if _______ | correlation is zero | rank is zero | covariance is zero | one | correlation is zero |
| Two random variables are said to be uncorrelated if correlation coefficient is____ | zero | one | two or more | orthogonal | zero |
| Perfect positive correlation is also called ___ correlation. | direct | indirect | inverse | partial | direct |
| If the variation of one variable has no relation with the variation on the other is is called ___ correlation. | positive | negative | partial | zero | zero |
| ------- is an positional average. | harmonic mean | Geometric mean | mean | median | median |
| The correlation between volume and pressure of a perfect gas is | positive | negative | partial | multiple | negative |
| If the value of y decreases as the value of x increases then there is ----- correlation between two variables. | positive | negative | partial | multiple | negative |
| The correlation between the income and expenditure is_____. | positive | negative | partial | multiple | positive |
| When the correlation coefficient is equal to___________ the correlation is perfect and positive. | 1 | 2 | 0 | 3 | 1 |
| If X & Y are independent then the correlation coefficient r = | 1 | 2 | 0 | 3 | 0 |