KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE-21.

FACULTY OF ENGINEERING

DEPARTMENT OF SCIENCE AND HUMANITIES

13BECS601 NUMERICAL ANALYSIS 3 1 0 4 100

INTENDED OUTCOMES:

- To make the students acquainted with the basic concepts in numerical methods and their uses.
- To impart the procedure for solving different kinds of problems occur in engineering numerically.

UNIT- I TYPES OF ERRORS, SOLUTION OF ALGEBRAIC EQUATIONS
Different types of errors- Newton Raphson method, Modified Newton Raphson method, Method of false position.

UNIT -II SOLUTION OF ALGEBRAIC SIMULTANEOUS EQUATIONS
Gauss - Jordan elimination, Cholesky method, Crout's method, Gauss - Jacobi method, Gauss - Seidel method. Matrix Inverse by Gauss - Jordan method.

Eigenvalues and eigenvectors: Power method for finding dominant eigenvalue and inverse power method for finding smallest eigenvalue, Jacobi method for symmetric matrices.

UNIT- III FINITE DIFFERENCES AND INTERPOLATION
Finite difference operators $-E, \Delta, \nabla, \delta, \mu, D$ - Interpolation-Newton-Gregory forward and backward interpolation, Lagrange's interpolation formula, Newton divided difference interpolation formula.

UNIT- IV DIFFERENTIATION AND INTEGRATION
Numerical differentiation using Newton-Gregory forward and backward polynomials. Numerical Integration-Gaussian quadrature, Trapezoidal rule and Simpson's one third rule.

Ordinary differential equations:Taylor series method, Euler and Modified Euler method, (Heun's method). Runge-Kutta method, Milne's method, Adams-Moulton method, Solution of boundary value problems of second order by finite difference method.

UNIT- V PARTIAL DIFFERENTIAL EQUATIONS
Classification of partial differential equations of second order.Liebmann's method for Laplace equation and Poisson equation, Explicit method and Crank - Nicolson method for parabolic equations. Explicit method for hyperbolic equations.

MATLAB :Matlab – Toolkits – 2D Graph Plotting – 3D Graph Plotting.

TEXT BOOKS:

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Burden, R. L. and Faires,T. D	Numerical Analysis	Thomson Asia Pvt. Ltd., Singapore.	2002
2	Curtis F. Gerald and Patrick O. Wheatley	Applied Numerical Analysis	Pearson Education, South Asia	2009

REFERENCES:

S. No.	Author(s) Name	Title of the book	Publisher	Year of Publication
1	Steven C.Chapra and Raymond P.Canale	Numerical Methods for Engineers with Software and Programming Applications	Tata McGraw Hill, New Delhi	2004
2	Gerald,C.F. and Wheatley,P.O	Applied Numerical Analysis	Pearson Education Asia, New Delhi	2002
3	Balagurusamy.E	Numerical Methods	Tata McGraw Hill Pub.Co.Ltd, New Delhi.	2009

WEBSITES:

1.www.nr.com
2. www.numerical-methods.com
3. www.math.ucsb.edu
4. www.mathworks.com

KARPAGAM ACADEMY OF HIGHER EDUCATION

COIMBATORE-21.

FACULTY OF ENGINEERING
DEPARTMENT OF SCIENCE AND HUMANITIES

13BECS601 NUMERICAL ANALYSIS
LESSON PLAN

S.NO.	TOPICS TO BE COVERED	HOUR(S)
	UNIT I :TYPES OF ERRORS, SOLUTION OF ALGEBRAIC EQUATIONS	
	Different types of errors	1
	Different types of errors	1
	Newton Raphson method	1
	Newton Raphson method	1
Unit-I	Tutorial1Newton Raphson method	1
	Modified Newton Raphson method	1
	Modified Newton Raphson method	1
	Method of false position	1
	Method of false position	1
	Tutorial 2 Method of false position	1
	TOTAL	**10**
	UNIT II : SOLUTION OF ALGEBRAIC SIMULTANEOUS EQUATIONS	
	Gauss - Jordan elimination	1
	Cholesky method	1
	Crout's method	1
	Gauss - Jacobi method	1
	Gauss - Seidel method	1
	Tutorial 3Gauss - Jordan elimination, Cholesky method, Crout's method, Gauss - Jacobi method	1
	Matrix Inverse by Gauss - Jordan method	1
Unit-II	Matrix Inverse by Gauss - Jordan method	1
	Power method for finding dominant eigenvalue	1
	Inverse power method for finding smallest eigenvalue	1
	Power method for finding dominant eigenvalue and inverse power method for finding smallest eigenvalue	1
	Tutorial4Power method for finding dominant eigenvalue and inverse power method for finding smallest eigenvalue	1
	Jacobi method for symmetric matrices	1
	Jacobi method for symmetric matrices	1
	TOTAL	**14**
Unit-III	**UNIT III : FINITE DIFFERENCES AND**	

	INTERPOLATION ,	
	Finite difference operators $-E, \Delta, \nabla, \delta, \mu, D$	1
	Finite difference operators $-E, \Delta, \nabla, \delta, \mu, D$	1
	Interpolation	1
	Newton-Gregory forward and backward interpolation	1
	Newton-Gregory forward and backward interpolation	1
	Tutorial 5Finite difference operators – $E, \Delta, \nabla, \delta, \mu, D$ - Interpolation-Newton-Gregory forward and backward interpolation	1
	Lagrange's interpolation formula	1
	Lagrange's interpolation formula	1
	Newton divided difference interpolation formula	1
	Newton divided difference interpolation formula	1
	Tutorial 6 Lagrange's interpolation formula, Newton divided difference interpolation formula.	1
	TOTAL	**11**
Unit-IV	**UNIT IV : DIFFERENTIATION AND INTEGRATION**	
	Numerical differentiation using Newton-Gregory forward and backward polynomials	1
	Gaussian quadrature	1
	Trapezoidal rule	1
	Simpson's one third rule	1
	Tutorial 7Numerical differentiation using Newton-Gregory forward and backward polynomials. Numerical Integration-Gaussian quadrature, Trapezoidal rule and Simpson's one third rule.	1
	Taylor series method	1
	Euler and Modified Euler method	1
	Runge-Kutta method	1
	Runge-Kutta method	1
	Milne's method	1
	Adams-Moulton method	1
	Tutorial 8Taylor series method, Euler and Modified Euler method, (Heun's method). Runge-Kutta method, Milne's method, Adams-Moulton method	1
	Solution of boundary value problems of second order by finite difference method.	**1**
	Solution of boundary value problems of second	**1**

	order by finite difference method.	
	TOTAL	**14**
Unit-V	**UNIT V : PARTIAL DIFFERENTIAL EQUATIONS**	
	Classification of partial differential equations of second order	1
	Liebmann’s method for Laplace equation	1
	Liebmann’s method for Laplace equation	1
	Liebmann’s method for Poisson equation	1
	Liebmann’s method for Poisson equation	1
	Tutorial 9Liebmann’s method for Laplace equation and Poisson equation	1
	Explicit method for parabolic equations	1
	Crank - Nicolson method for parabolic equations	1
	Crank - Nicolson method for parabolic equations	1
	Explicit method for hyperbolic equations	1
	Tutorial 10 Explicit method and Crank - Nicolson method for parabolic equations. Explicit method for hyperbolic equations.	1
	TOTAL	**11**
	GRAND TOTAL	**50 + 10**

STAFF HOD

Unit - I

Solutions of Equations and Eigen Value Problems.

Iterative Method:

① Write the gn eqn $f(x) = 0$ into the form $x = \varphi(x)$

② Assume that $x = x_0$ be the root of the given eqn

③ The first approximation to the root is gn by $x_1 = \varphi(x_0)$

lly
$$x_2 = \varphi(x_1)$$
$$x_3 = \varphi(x_2)$$
$$\vdots$$
$$x_n = \varphi(x_{n-1})$$

x_n the n^{th} iteration & the value of
$\Rightarrow x_n$ is the root of the gn eqn.
x_n is the root

① Find the root of the equation $\cos x = 3x - 1$, using iteration Method.

Soln

$$f(x) = \cos x - 3x + 1$$
$$f(0) = \cos 0 - 3(0) + 1 = 2 \rightarrow +ve$$
$$f(1) = \cos 1 - 3(1) + 1 = 0 - 3(1) + 1 \rightarrow -ve$$

$\therefore$ The root lies between 0 and 1

The eqn can be written as

$$\cos x - 3x + 1 = 0$$
$$-3x = -\cos x - 1$$
$$3x = \cos x + 1$$
$$x = \frac{1}{3}[1 + \cos x]$$

Let $\varphi(x) = \frac{1}{3}[1 + \cos x]$

$$\varphi'(x) = -\frac{1}{3}\sin x$$
$$|\varphi'(x)| = \frac{1}{3}\sin x$$

$|\varphi'(0)| = 0 < 1$

$|\varphi'(1)| = \frac{1}{3}\sin 1 = 0.2804 < 1.$

Let $x_0 = 0$

$x_1 = \varphi(x_0) = \frac{1}{3}(1 + \cos x_0) = \frac{1}{3}(1 + \cos 0)$

$x_1 = 0.6667$

$x_2 = \varphi(x_1) = \frac{1}{3}(1 + \cos x_1) = \frac{1}{3}(1 + \cos 0.6667)$

$x_2 = 0.5953$

$x_3 = \varphi(x_2) = \frac{1}{3}(1 + \cos x_2) = \frac{1}{3}(1 + \cos 0.5953)$

$x_3 = 0.6093$

$x_4 = \varphi(x_3) = \frac{1}{3}(1 + \cos x_3) = \frac{1}{3}(1 + \cos 0.6093)$

$x_4 = 0.6067$

x_4

$x_5 = \varphi(x_4) = \frac{1}{3}(1+\cos x_4) = \frac{1}{3}(1+\cos 0.6067)$

$x_5 = 0.6072$

$x_6 = \varphi(x_5) = \frac{1}{3}(1+\cos x_5) = \frac{1}{3}(1+\cos 0.6072)$

$x_6 = 0.6071$

$x_7 = \varphi(x_6) = \frac{1}{3}(1+\cos x_6) = \frac{1}{3}(1+\cos 0.6071)$

$x_7 = 0.6071$

$\therefore$ The required root is 0.6071.

(2) Solve the equation $x^2 - 2x - 3 = 0$ for the +ve root by iteration method.

Soln

$$x^2 - 2x - 3 = 0$$

$$f(x) = x^2 - 2x - 3$$

$f(0) = 0 - 2(0) - 3 = -3 \rightarrow -ve$

$f(1) = -4 \rightarrow -ve$

$f(2) = -3 \rightarrow -ve$

$f(3) = 0 \rightarrow +ve$

$\therefore$ The root lies between 2 and 3

$$x^2 = 2x + 3$$

$$x = \sqrt{2x+3}$$

$$\varphi(x) = \sqrt{2x+3} = (2x+3)^{1/2}$$

$$\varphi'(x) = \frac{1}{\cancel{2}}(2x+3)^{-1/2} \cdot \cancel{2}$$

$$|\varphi'(x)| = |(2x+3)^{-1/2}|$$

$$|\varphi'(2)| < 1 \quad \& \quad |\varphi'(3)| < 1$$

Take $x_0 = 2.5$

$$x_1 = \varphi(x_0) = \sqrt{2x_0+3} = \sqrt{2(2.5)+3} = 2.8284$$

$$x_2 = \varphi(x_1) = \sqrt{2x_1+3} = \sqrt{2(2.8284)+3} = 2.9422$$

$$x_3 = \varphi(x_2) = \sqrt{2x_2+3} = \sqrt{2(2.94222)+3} = 2.9807$$

$$x_4 = \varphi(x_3) = \sqrt{2x_3+3} = \sqrt{2(2.9807)+3} = 2.9936$$

$$x_5 = \varphi(x_4) = \sqrt{2x_4+3} = \sqrt{2(2.9936)+3} = 2.9979$$

$$x_6 = \varphi(x_5) = \sqrt{2x_5+3} = \sqrt{2(2.979)+3} = 2.9993$$

$$x_7 = \varphi(x_6) = \sqrt{2x_6+3} = \sqrt{2(2.9993)+3} = 2.9998$$

$$x_8 = \varphi(x_7) = \sqrt{2x_7+3} = \sqrt{2(2.9998)+3} = 2.9999$$

$$x_9 = \varphi(x_8) = \sqrt{2x_8+3} = \sqrt{2(2.9999)+3} = 2.9999$$

$\therefore$ The required root is 2.9999

(3) Solve by iteration Method $2x - \log_{10} x = 7$

Soln

$$2x - \log_{10} x - 7 = 0$$

$$f(x) = 2x - \log_{10} x - 7$$

$f(1) = -5 \to -ve$

$f(2) = -3.3010 \to -ve$

$f(3) = -1.4771 \to -ve$

$f(4) = 0.3979 \to +ve$

$\therefore$ The root lies between 3 and 4

$$2x = 7 + \log_{10} x$$

$$x = \frac{1}{2}[7 + \log_{10} x]$$

$$\therefore \varphi(x) = \frac{1}{2}[7 + \log_{10} x]$$

$$\varphi'(x) = \frac{1}{2}\left[\frac{1}{x}\log_{10} e\right]$$

$$|\varphi'(x)| = \left|\frac{1}{2}\left[\frac{1}{x}\log_{10} e\right]\right| < 1 \text{ in } (3, 4)$$

Take $x_0 = 3.6$

$$x_1 = \varphi(x_0) = \frac{1}{2}[\log_{10} x_0 + 7]$$

$$= \frac{1}{2}[\log_{10} 3.6 + 7]$$

$$= 3.7782$$

$$x_2 = \varphi(x_1) = \frac{1}{2}\left[\log_{10} x_1 + 7\right]$$
$$= \frac{1}{2}\left[\log_{10} 3.7782 + 7\right]$$
$$x_2 = 3.7886$$
$$x_3 = \varphi(x_2) = \frac{1}{2}\left[\log_{10} x_2 + 7\right]$$
$$= \frac{1}{2}\left[\log_{10} 3.7886 + 7\right]$$
$$x_3 = 3.7892$$
$$x_4 = \varphi(x_3) = \frac{1}{2}\left[\log_{10} x_3 + 7\right]$$
$$= \frac{1}{2}\left[\log_{10} 3.7892 + 7\right]$$
$$x_4 = 3.7893$$
$$x_5 = \varphi(x_4) = \frac{1}{2}\left[\log_{10} x_4 + 7\right]$$
$$= \frac{1}{2}\left[\log_{10} 3.7893 + 7\right]$$
$$x_5 = 3.7893$$

$\therefore$ The required root is 3.7893

H.W 4) find the negative root of the eqn $x^3 - 2x + 5 = 0$

Gauss Jordan Method

(1) $2x - y + 6z = 22$
$x + 7y - 3z = -22$
$5x - 2y + 3z = 18$

Soln

$$[A, B] = \left[\begin{array}{ccc|c} 2 & -1 & 6 & 22 \\ 1 & 7 & -3 & -22 \\ 5 & -2 & 3 & 18 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{-1}{2} & 3 & 11 \\ 1 & 7 & -3 & -22 \\ 1 & \frac{-2}{5} & \frac{3}{5} & \frac{18}{5} \end{array}\right] \quad \begin{array}{l} R_1 \to \frac{R_1}{2} \\ R_3 \to \frac{R_3}{5} \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{-1}{2} & 3 & 11 \\ 0 & \frac{15}{2} & -6 & -33 \\ 0 & \frac{1}{16} & \frac{-12}{5} & \frac{-37}{5} \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} -2 & 1 & -6 & -22 \\ 0 & 1 & \frac{-4}{5} & \frac{-22}{5} \\ 0 & 1 & -24 & -74 \end{array}\right] \quad \begin{array}{l} R_1 \to -2R_1 \\ R_2 \to \frac{2}{15} R_2 \\ R_3 \to 10 R_3 \end{array}$$

$$= \left[\begin{array}{cccc} -2 & 0 & \frac{-26}{5} & \frac{-88}{5} \\ 0 & 1 & \frac{-4}{5} & \frac{-22}{5} \\ 0 & 0 & \frac{-116}{5} & \frac{-348}{5} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$= \left[\begin{array}{ccc|c} \frac{5}{13} & 0 & 1 & \frac{44}{13} \\ 0 & -\frac{5}{4} & 1 & \frac{11}{2} \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 \to R_1 \times \frac{-5}{26} \\ R_2 \to -\frac{5}{4} R_2 \\ R_3 \to -\frac{5}{116} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} \frac{5}{13} & 0 & 0 & \frac{5}{13} \\ 0 & -\frac{5}{4} & 0 & \frac{5}{2} \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \begin{array}{l} R_2 \to R_2 \times \frac{-4}{5} \\ R_1 \to R_1 \times \frac{13}{5} \end{array}$$

$x = 1$, $y = -2$, $z = 3$.

② Solve
$$\begin{aligned} x + 3y + 3z &= 16 \\ x + 4y + 3z &= 18 \\ x + 3y + 4z &= 19 \end{aligned}$$

$$[A,B] = \left[\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 1 & 4 & 3 & 18 \\ 1 & 3 & 4 & 19 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 3 & 3 & 16 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1/3 & 1 & 1 & 16/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \rightarrow \frac{R_1}{3}$$

$$= \left[\begin{array}{ccc|c} 1/3 & 0 & 1 & 10/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \rightarrow R_1 - R_2$$

$$= \left[\begin{array}{ccc|c} 1/3 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \rightarrow R_1 - R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad R_1 \rightarrow 3R_1$$

$$x = 1, \quad y = 2, \quad z = 3$$

(3) Solve

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

Soln

$$[A, B] = \left[\begin{array}{ccc|c} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1/10 & 1/10 & 6/5 \\ 1 & 5 & 1/2 & 13/2 \\ 1 & 1 & 5 & 7 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{10} \\ R_2 \to \frac{R_2}{2} \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & \frac{1}{10} & \frac{1}{10} & \frac{6}{5} \\ 0 & \frac{49}{10} & \frac{2}{5} & \frac{53}{10} \\ 0 & \frac{9}{10} & \frac{49}{10} & \frac{29}{5} \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{cccc} 10 & 1 & 1 & 12 \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & 1 & \frac{49}{9} & \frac{58}{9} \end{array}\right] \begin{array}{l} R_1 \to 10R_1 \\ R_2 \to \frac{10}{49} R_2 \\ R_3 \to \frac{10}{9} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 10 & 0 & \frac{45}{49} & \frac{535}{49} \\ 0 & 1 & \frac{4}{49} & \frac{53}{49} \\ 0 & 0 & \frac{2365}{441} & \frac{2365}{441} \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$= \left[\begin{array}{ccc|c} \frac{98}{9} & 0 & 0 & \frac{107}{9} \\ 0 & \frac{49}{4} & 1 & \frac{53}{4} \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_2 \to \frac{49}{45} R_1 \\ R_2 \to \frac{49}{4} R_2 \\ R_3 \to \frac{441}{2365} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} \frac{98}{9} & 0 & 0 & \frac{98}{9} \\ 0 & \frac{49}{4} & 0 & \frac{49}{4} \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_3 \\ R_1 \to R_1 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 \to R_1 \times \frac{9}{98} \\ R_2 \to R_2 \times \frac{4}{49} \end{array}$$

$x = 1 \quad y = 1 \quad z = 1$

Inverse of a Matrix

Gauss Jordan Method

(1) find the inverse of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$

using Gauss Jordan Method.

soln

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$$

$$(A/I) = \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{1} \\ R_2 \to \frac{R_2}{1} \\ R_3 \to \frac{R_3}{-2} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & -\frac{1}{2} \end{array}\right] R_2 \to$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 0 & -\frac{1}{2} \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -1 & -1 & 0 & -\frac{1}{2} \end{array}\right] \quad \begin{array}{l} R_1 \to \frac{R_1}{1} \\ R_2 \to \frac{R_2}{2} \\ R_3 \to \frac{R_3}{1} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} \frac{1}{6} & 0 & 1 & \frac{1}{4} & -\frac{1}{12} & 0 \\ 0 & -\frac{1}{3} & 1 & \frac{1}{6} & -\frac{1}{6} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1/6 \\ R_2 \to R_2/-3 \\ R_3 \to R_3/2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} \frac{1}{6} & 0 & 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{4} \\ 0 & -\frac{1}{3} & 0 & \frac{5}{12} & \frac{1}{12} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & \frac{3}{2} \\ 0 & 1 & 0 & -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 \times 6 \\ R_2 \to -3R_2 \end{array}$$

$$= [I / A]$$

$\therefore$ Inverse of A is $\begin{bmatrix} 3 & 1 & \frac{3}{2} \\ -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$

(2) find the inverse of the Matrix $\begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ using Gauss Jordan Method.

Soln

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ -1 & 4 & 10 \end{bmatrix}$$

$$[A/I] = \left[\begin{array}{ccc|ccc} 3 & -1 & 1 & 1 & 0 & 0 \\ -15 & 6 & -5 & 0 & 1 & 0 \\ -1 & 4 & 10 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 1 & -\frac{6}{15} & +\frac{1}{3} & 0 & -\frac{1}{15} & 0 \\ 1 & -4 & -10 & 0 & 0 & -1 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{3} \\ R_2 \to \frac{R_2}{-15} \\ R_3 \to \frac{R_3}{-1} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{15} & \frac{2}{3} & -\frac{1}{3} & -\frac{1}{15} & 0 \\ 0 & -\frac{11}{3} & -\frac{31}{3} & -\frac{1}{3} & 0 & -1 \end{array}\right] \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} -3 & 1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 10 & 5 & 1 & 0 \\ 0 & 1 & \frac{31}{11} & \frac{1}{11} & 0 & \frac{3}{11} \end{array}\right] \begin{array}{l} R_1 \to R_1(-3) \\ R_2 \to -15R_2 \\ R_3 \to \frac{-3}{11} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} -3 & 0 & -1 & -6 & -1 & 0 \\ 0 & 1 & 10 & 5 & 1 & 0 \\ 0 & 0 & -\frac{79}{11} & -\frac{54}{11} & -1 & \frac{3}{11} \end{array}\right] \begin{array}{l} R_3 \to R_3 - R_2 \\ R_1 \to R_1 - R_2 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} +3 & 0 & 1 & 6 & 1 & 0 \\ 0 & \frac{1}{10} & 1 & \frac{1}{2} & \frac{1}{10} & 0 \\ 0 & 0 & 1 & \frac{1}{31} & 0 & \frac{3}{31} \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{-1} \\ R_2 \to \frac{R_2}{10} \\ R_3 \to \frac{11}{31} R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 3 & 0 & 0 & \frac{185}{31} & 1 & -\frac{3}{31} \\ 0 & \frac{1}{10} & 0 & \frac{29}{62} & \frac{1}{10} & -\frac{3}{31} \\ 0 & 0 & 1 & \frac{1}{31} & 0 & \frac{3}{31} \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{185}{93} & \frac{1}{3} & -\frac{1}{31} \\ 0 & 1 & 0 & \frac{290}{62} & 1 & -\frac{30}{31} \\ 0 & 0 & 1 & \frac{1}{31} & 0 & \frac{3}{31} \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{3} \\ R_2 \to R_2 \times 10 \end{array}$$

$= [I \cancel{x} A]$

$\therefore$ inverse of A is $\begin{bmatrix} \frac{85}{93} & \frac{1}{3} & -\frac{1}{31} \\ \frac{290}{62} & 1 & -\frac{30}{31} \\ \frac{1}{31} & 0 & \frac{3}{31} \end{bmatrix}$

3) Using Gauss Jordan Method find the inverse of $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$

Soln

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$$

$$(A/I) = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 1 & \frac{4}{3} & \frac{5}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{1} \\ R_2 \to \frac{R_2}{3} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & \frac{4}{3} & \frac{8}{3} & -1 & \frac{1}{3} & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right] R_2 \to R_2 - R_1$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & \frac{2}{2} & \frac{-3}{4} & \frac{1}{4} & 0 \\ 0 & 1 & 7/6 & 0 & 0 & -\frac{1}{6} \end{array}\right] \begin{array}{l} R_2 \to \frac{3}{4}R_2 \\ R_3 \to \frac{R_3}{-6} \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & \cancel{2} & \frac{-3}{4} & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{5}{6} & 3/4 & -1/4 & \frac{-1}{6} \end{array}\right] \cdot R_3 \to R_3 - R_2$$

$$= \left[\begin{array}{ccc|ccc} -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & -3/8 & \frac{1}{8} & 0 \\ 0 & 0 & 1 & -\frac{9}{10} & \frac{3}{10} & \frac{1}{5} \end{array}\right] \begin{array}{l} R_1 \to \frac{R_1}{-1} \\ R_2 \to \frac{R_2}{2} \\ R_3 \to -\frac{6}{5}R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & -1/10 & -3/10 & -1/5 \\ 0 & 1/2 & 0 & 21/40 & -7/40 & -1/5 \\ 0 & 0 & 1 & -9/10 & 3/10 & 1/5 \end{array}\right] \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/10 & 3/10 & 1/5 \\ 0 & 1 & 0 & 21/20 & -7/20 & -2/5 \\ 0 & 0 & 1 & -9/10 & 3/10 & 1/5 \end{array}\right]$$

$$= [I / A]$$

Inverse of A is $\begin{bmatrix} 1/10 & 3/10 & 1/5 \\ 21/20 & -7/20 & -2/5 \\ -9/10 & 3/10 & 1/5 \end{bmatrix}$

Gauss Jacobi Method

(1) Solve the following eqns by Gauss Jacobi Method

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

$x = \frac{17 - y + 2z}{20}$	$y = \frac{-18 + z - 3x}{20}$	$z = \frac{25 - 2x + 3y}{20}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 0.85$	$y_1 = -0.9$	$z_1 = 1.25$
$x_2 = 1.02$	$y_2 = -0.965$	$z_2 = 1.03$
$x_3 = 1.0013$	$y_3 = -1.0015$	$z_3 = 1.0033$
$x_4 = 1.0004$	$y_4 = -1.0001$	$z_4 = 0.9996$
$x_5 = 0.9999$	$y_5 = -1.0001$	$z_5 = 0.9999$
$x_6 = 1$	$y_6 = -1$	$z_6 = 1$
$x_7 = 1$	$y_7 = -1$	$z_7 = 1$

$\therefore\ x = 1,\ y = -1,\ z = 1$

(2) Solve
$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

$x = \frac{32-4y+z}{28}$	$y = \frac{35-4x-2x}{17}$	$z = \frac{24-x-3y}{10}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 1.1429$	$y_1 = 2.0588$	$z_1 = 2.4$
$x_2 = 0.9345$	$y_2 = 1.3597$	$z_2 = 1.6681$
$x_3 = 1.0082$	$y_3 = 1.5564$	$z_3 = 1.898$
$x_4 = 0.9883$	$y_4 = 1.4935$	$z_4 = 1.8323$
$x_5 = 0.9949$	$y_5 = 1.514$	$z_5 = 1.8531$
$x_6 = 0.9931$	$y_6 = 1.5058$	$z_6 = 1.847$
$x_7 = 0.9937$	$y_7 = 1.5074$	$z_7 = 1.8490$
$x_8 = 0.9936$	$y_8 = 1.5069$	$z_8 = 1.8484$
$x_9 = 0.9936$	$y_9 = 1.5070$	$z_9 = 1.8486$
$x_{10} = 0.9936$	$y_{10} = 1.5070$	$z_{10} = 1.8485$
$x_{11} = 0.9936$	$y_{11} = 1.5070$	$z_{11} = 1.8485$

$\therefore$ The soln is

$x = 0.9936 \quad y = 1.5070 \quad z = 1.8485$

(3) Solve $27x + 6y - z = 85$

$x + y + 54z = 110$

$6x + 15y + 2z = 72$

$x = \frac{85 - 6y + z}{27}$	$y = \frac{72 - 6x - 2z}{15}$	$z = \frac{110 - x - y}{54}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 3.148$	$y_1 = 4.8$	$z_1 = 2.037$
$x_2 = 2.157$	$y_2 = 3.269$	$z_2 = 1.890$
$x_3 = 2.492$	$y_3 = 3.685$	$z_3 = 1.937$
$x_4 = 2.401$	$y_4 = 3.545$	$z_4 = 1.923$
$x_5 = 2.432$	$y_5 = 3.583$	$z_5 = 1.927$
$x_6 = 2.423$	$y_6 = 3.570$	$z_6 = 1.926$
$x_7 = 2.426$	$y_7 = 3.574$	$z_7 = 1.926$
$x_8 = 2.425$	$y_8 = 3.573$	$z_8 = 1.926$

$x = 2.425 \quad y = 3.573 \quad z = 1.926.$

Gauss Seidal Iteration Method

① Solve
$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Soln.

$x = \frac{17-y+2z}{20}$	$y = \frac{-18-3x+z}{20}$	$z = \frac{25-2x+3y}{20}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 0.82$	$y_1 = -1.0275$	$z_1 = 1.0109$
$x_2 = 1.0025$	$y_2 = -0.9998$	$z_2 = 0.9998$
$x_3 = 1.0000$	$y_3 = -1.000$	$z_3 = 1.0000$
$x_4 = 1.0000$	$y_4 = -1.0000$	$z_4 = 1.0000$

$x = 1 \quad y = -1 \quad z = 1$

② Solve
$$4x + 2y + z = 14$$
$$x + 5y - z = 10$$
$$x + y + 8z = 20$$

$x = \frac{85-6y+z}{27}$	$y = \frac{72-6x-2z}{15}$	$z = \frac{110-x-y}{54}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 3.148$	$y_1 = 3.541$	$z_1 = 1.913$
$x_2 = 2.432$	$y_2 = 3.572$	$z_2 = 1.926$
$x_3 = 2.426$	$y_3 = 3.573$	$z_3 = 1.926$
$x_4 = 2.426$	$y_4 = 3.573$	$z_4 = 1.926$

$\therefore$ $x = 2.426$

$y = 3.573$

$z = 1.926$.

$x = \frac{14-2y-z}{4}$	$y = \frac{10-x+z}{5}$	$z = \frac{20-x-y}{8}$
$x_0 = 0$	$y_0 = 0$	$z_0 = 0$
$x_1 = 3.5$	$y_1 = 1.3$	$z_1 = 1.9$
$x_2 = 2.375$	$y_2 = 1.905$	$z_2 = 1.965$
$x_3 = 2.056$	$y_3 = 1.982$	$z_3 = 1.995$
$x_4 = 2.010$	$y_4 = 1.997$	$z_4 = 1.999$
$x_5 = 2.002$	$y_5 = 1.999$	$z_5 = 2$
$x_6 = 2.001$	$y_6 = 2$	$z_6 = 2$
$x_7 = 2$	$y_7 = 2$	$z_7 = 2$
$x_8 = 2$	$y_8 = 2$	$z_8 = 2$

$\therefore \quad x = 2, \; y = 2, \; z = 2$

3 Solve
$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$

Eigen Values of a Matrix by Power Method

(1) Find the numerically largest eigen value of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and its corresponding eigen vector by power method, taking the initial eigen vector as $(1\ 0\ 0)^T$ (upto three decimal places.

Soln

(1) Given $X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$$A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$$

$$AX_1 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 25 \\ 1 \\ 2 \end{pmatrix} = 25\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = 25X_2$$

$$AX_2 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.04 \\ 0.08 \end{pmatrix} = \begin{pmatrix} 25.2 \\ 1.12 \\ 0.08 \end{pmatrix} = 25.2\begin{pmatrix} 1 \\ 0.0444 \\ 0.0667 \end{pmatrix}$$

$$= 25.2X_3$$

$$AX_3 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0444 \\ 0.0667 \end{pmatrix} = \begin{pmatrix} 25.1778 \\ 1.1332 \\ 1.7337 \end{pmatrix} = 25.1778\begin{pmatrix} 1 \\ 0.0450 \\ 0.0688 \end{pmatrix}$$

$$= 25.1778X_4$$

$$AX_4 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0450 \\ 0.06888 \end{pmatrix} = \begin{pmatrix} 25.1826 \\ 1.135 \\ 1.7248 \end{pmatrix}$$

$$= 25.1826\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = 25.1826 X_5$$

$$AX_5 = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = \begin{pmatrix} 25.1821 \\ 1.1353 \\ 1.7260 \end{pmatrix}$$

$$= 25.1821\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix} = 25.1821 X_6$$

Dominant eigen Value $\lambda = 25.1821$

corresponding eigen Vector is $\begin{pmatrix} 1 \\ 0.0451 \\ 0.0685 \end{pmatrix}$

(2) Determine by Power method the largest eigen value and the Corresponding eigen vector of the Matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$

Soln

$$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} = 3 \begin{bmatrix} 0.3333 \\ 1 \\ -0.333 \end{bmatrix} = 3X_2$$

$$AX_2 = \begin{bmatrix} 3.6666 \\ 1.6667 \\ 0.3337 \end{bmatrix} = 3.6666 \begin{bmatrix} 1 \\ 0.4546 \\ 0.0910 \end{bmatrix} = 3.6666\, X_3$$

$$AX_3 = \begin{bmatrix} 2.2728 \\ 4.2732 \\ 1.7284 \end{bmatrix} = 4.2732 \begin{bmatrix} 0.5319 \\ 1 \\ 0.4045 \end{bmatrix} = 4.2732\, X_4$$

$$AX_4 = \begin{bmatrix} 3.1274 \\ 5.2137 \\ 7.5131 \end{bmatrix} = 7.5131 \begin{bmatrix} 0.4163 \\ 0.6939 \\ 1 \end{bmatrix} = 7.5731\, X_5$$

$$AX_5 = \begin{bmatrix} 1.498 \\ 6.6367 \\ 12.3593 \end{bmatrix} = 12.3593 \begin{bmatrix} 0.1212 \\ 0.5370 \\ 1 \end{bmatrix} = 12.3596\, X_6$$

$$AX_6 = \begin{bmatrix} 0.7322 \\ 5.4376 \\ 12.0268 \end{bmatrix} = 12.0268 \begin{pmatrix} 0.0609 \\ 0.4521 \\ 1 \end{pmatrix} = 12.0268\, X_7$$

$$AX_7 = \begin{pmatrix} 0.4172 \\ 5.0869 \\ 11.7473 \end{pmatrix} = 11.7475 \begin{pmatrix} 0.0355 \\ 0.4330 \\ 1 \end{pmatrix} = 11.7475 X_8$$

$$AX_8 = \begin{pmatrix} 0.3345 \\ 4.9721 \\ 11.6965 \end{pmatrix} = 11.6965 \begin{pmatrix} 0.0286 \\ 0.4251 \\ 1 \end{pmatrix} = 11.6965 X_9$$

$$AX_9 = \begin{pmatrix} 0.3039 \\ 4.936 \\ 11.6718 \end{pmatrix} = 11.6718 \begin{pmatrix} 0.0260 \\ 0.4229 \\ 1 \end{pmatrix} = 11.6718 \begin{pmatrix} 0.0260 \\ 0.4229 \\ 1 \end{pmatrix}$$

$$AX_{10} = \begin{pmatrix} 0.2947 \\ 4.9238 \\ 11.6656 \end{pmatrix} = 11.6656 \begin{pmatrix} 0.0253 \\ 0.4221 \\ 1 \end{pmatrix} = 11.6656 X_{11}$$

$$AX_{11} = \begin{pmatrix} 0.2916 \\ 4.9201 \\ 11.6631 \end{pmatrix} = 11.6631 \begin{pmatrix} 0.025 \\ 0.4219 \\ 1 \end{pmatrix} = 11.6631 X_{12}$$

$$AX_{12} = \begin{pmatrix} 0.2907 \\ 4.9188 \\ 11.6626 \end{pmatrix} = 11.6626 \begin{pmatrix} 0.0249 \\ 0.4218 \\ 1 \end{pmatrix} = 0.6626 X_{13}$$

$$AX_{13} = \begin{pmatrix} 0.2903 \\ 4.9183 \\ 11.6623 \end{pmatrix} = 11.6623 \begin{pmatrix} 0.0249 \\ 0.4217 \\ 1 \end{pmatrix} = 11.6623 X_{14}$$

$$AX_{14} = \begin{pmatrix} 0.29 \\ 4.9181 \\ 11.6619 \end{pmatrix} = 11.6619 \begin{pmatrix} 0.0249 \\ 0.4217 \\ 1 \end{pmatrix} = 11.6619 X_{15}$$

$$AX_{15} = \begin{pmatrix} 0.29 \\ 4.9181 \\ 11.6619 \end{pmatrix} = 11.6619 \begin{pmatrix} 0.0249 \\ 0.4217 \\ 1 \end{pmatrix} = 11.6619 X_{16}$$

∴ The dominant eigen value is 11.6619

The corresponding eigen vector is

$$\begin{pmatrix} 0.0249 \\ 0.4217 \\ 1 \end{pmatrix}.$$

(3) Find the dominant eigen value and the corresponding eigen vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

Soln

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \qquad X_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$AX_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 1 \cdot X_2$$

$$AX_2 = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = 7\begin{pmatrix} 1 \\ 0.4286 \\ 0 \end{pmatrix} = 7 \cdot X_3$$

$$AX_3 = \begin{pmatrix} 3.5714 \\ 1.8572 \\ 0 \end{pmatrix} = 3.5714\begin{pmatrix} 1 \\ 0.52 \\ 0 \end{pmatrix} = 3.5714\, X_4$$

$$AX_4 = \begin{pmatrix} 4.12 \\ 2.04 \\ 0 \end{pmatrix} = 4.12\begin{pmatrix} 1 \\ 0.4951 \\ 0 \end{pmatrix} = 4.12\, X_5$$

$$AX_5 = \begin{pmatrix} +3.9706 \\ 1.9902 \\ 0 \end{pmatrix} = 3.9706 \begin{pmatrix} 1 \\ 0.5012 \\ 0 \end{pmatrix} = 3.9706\, X_6$$

$$AX_6 = \begin{pmatrix} 4.0072 \\ 2.0024 \\ 0 \end{pmatrix} = 4.0072 \begin{pmatrix} 1 \\ 0.4997 \\ 0 \end{pmatrix} = 4.0072\, X_7$$

$$AX_7 = \begin{pmatrix} 3.9982 \\ 1.9994 \\ 0 \end{pmatrix} = 3.9982 \begin{pmatrix} 1 \\ 0.5000 \\ 0 \end{pmatrix} = 3.9982\, X_8$$

$$AX_8 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix} = 4\, X_9$$

$$AX_9 = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix}$$

Dominant eigen value is $\lambda = 4$

Corresponding eigen vector is $\begin{pmatrix} 1 \\ 0.5 \\ 0 \end{pmatrix}$

Eigen Value of a Matrix by Jacobi Method for Symmetric Matrix

Let $P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2a_{ij}}{a_{ii}-a_{jj}}\right)$$

$$D = P^T A P$$

(1) Apply Jacobi process to evaluate the eigen values and eigen vectors of the Matrix $\begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$

Soln

$$A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$$

The largest non diagonal element is $a_{13} = a_{31} = 1$

$a_{11} = 5, \quad a_{33} = 5$

$$\tan 2\theta = \left[\frac{2a_{13}}{a_{11}-a_{33}}\right] = \frac{2}{5-5}$$

$$\tan 2\theta = \infty$$

$$2\theta = \tan^{-1}\infty$$

$$2\theta = \pi/2$$

$$\boxed{\theta = \pi/4}$$

$$P = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos \pi/2 & 0 & -\sin \pi/4 \\ 0 & 1 & 0 \\ \sin \pi/4 & 0 & \cos \pi/4 \end{bmatrix}$$

$$P = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

I[st] transformation

$$D = P^T A P$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$D = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

The eigen values are $6, -2, 4$

corresponding eigen vectors are

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(2) Find all the eigen values and eigen vectors of the Matrix

$$\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$ using Jacobi Method.

$$A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$$

Here the largest non diagonal element is $a_{13} = a_{31} = 2$.

$a_{11} = 1, a_{33} = 1$

$$S_1 = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\tan 2\theta = \frac{2a_{13}}{a_{11} - a_{33}} = \frac{4}{0}$$

$$\tan 2\theta = \infty$$

$$2\theta = \pi/2$$

$$\boxed{\theta = \pi/4}$$

$$S_1 = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$B_1 = S_1^{-1} A S_1 =$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

<u>II</u> <u>Transformation</u>

$$a_{12} = a_{21} = 2$$

$$a_{11} = 3 \quad a_{22} = 3$$

$$S_2 = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\tan 2\theta = \frac{2a_{12}}{a_{11} - a_{22}} = \frac{2 \times 2}{3 - 3} = \infty$$

$$2\theta = \tan^{-1} \infty$$

$$2\theta = \pi/2$$

$$\theta = \pi/4$$

$$S_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B_2 = S_1^{-1} B_1 S_2$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$\therefore$ A is reduced to the diagonal Matrix B_2.

Hence the eigen values of A is 5, 1, −1

$$S = S_1 S_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{-1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{-1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\therefore$ eigen vectors are $\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \begin{pmatrix} -1 \\ \sqrt{2} \\ 1 \end{pmatrix} \& \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

Questions	opt1	opt2	opt3	opt4
In Regula-Falsi method, to reduce the number of iterations we start with ________ interval	Small	large	equal	no
The rate of convergence in Newton-Raphson method is of order ________	1	2	3	4
The condition for convergence for Newton-Raphson method is	$\lvert f(x)\rvert<\lvert f'(x)\rvert^2$	$\lvert f(x)\rvert>\lvert f'(x)\rvert^2$	$\lvert f(x)f''(x)\rvert<\lvert f'(x)\rvert^2$	f(x) <1
Newton's method is useful when the graph of the function crosses the x-axis is nearly ______.	vertical	horizontal	close to zero	zero
If the initial approximation to the root is not given we can find any two values of x say a and b such that f (a) and f(b) are of _________signs.	opposite	same	positive	negative
$\lvert f(a)\rvert$___ $\lvert f(b)\rvert$ then 'a' can be taken as the first approximation to the root.	<	>	=	≥
The Newton – Raphson method is also known as method of __________	secant	tangent	iteration	interpolation
The Newton– Raphson method will fail if _________ in the neighborhood of the root	f'(x)=0	$\lvert f'(x)\rvert>0$	$\lvert f'(x)\rvert<0$	$\lvert f'(x)\rvert>1$
If f'(x)=0 _________ method should be used.	Newton – Raphson	Regula-Falsi	iteration	interpolation
The rate of convergence of Newton – Raphson method is _________	quadratic	cubic	4	5
If f (a) and f (b) are of opposite signs the actual root lies between _________	(a, b)	(0, a)	(0, b)	(0, 0)
The convergence of root in Regula-Falsi method is slower than ________	Gauss – Elimination	Gauss – Jordan	Newton – Raphson	Power method
Regula-Falsi method is known as method of _________	secant	tangent	chords	elimination
______________method converges faster than Regula-Falsi method.	Newton – Raphson	Power method	elimination	interpolation
f(x) is continuous in the interval (a, b) and if f (a) and f (b) are of opposite signs the equation f(x) = 0 has at least one _________ lying between a and b.	equation	function	root	polynomial
x^2 + 3x – 3 = 0 is a polynomial of order	2	3	1	0
x is a root of f(x)=0 with multiplicity p,then_________ method is used.	Generalized Newton – Raphson	Newton – Raphson	Regula-Falsi	Power
Errors which are already present in the statement of the problem are called____ errors.	Inherent	Rounding	Truncation	Absolute
Rounding errors arise during_________	Solving	Computation	Truncation	Absolute
The other name for truncation error is _________ error.	Absolute	Rounding	Inherent	Algorithm
Rounding errors arise from the process of _________ the numbers.	Truncating	Rounding off	Approximating	Solving
Absolute error is denoted by________	E_a	E_r	E_p	E_x
Truncation errors are caused by using _________ results.	Exact	True	Approximate	Real
Truncation errors are caused on replacing an infinite process by ________ one.	Approximate	True	Finite	Exact
Graffe's root squaring method is used for solving ___________ equation.	Polynomial	Algebraic	transcendental	wave
Bairstow's method is used for finding ____________ roots of a polynomial equation.	Complex	real	second order	first order
The actual root of the equation lies between a and b when f (a) and f (b) are of ____ signs.	Opposite	same	negative	positive
If a word length is 4 digits, then the truncation of 15.758 is	15.75	15.76	15.758	16
If a word length is 4 digits, then rounding off of 15.758 is	15.75	15.76	15.758	16

opt7
Small
2
|f(x)f''(x)|<|f'(x)|^2
vertical

opposite
<
tangent
f'(x)=0
Regula-Falsi
quadratic
(a, b)
Newton – Raphson
chords
Newton – Raphson

root
2
Generalized Newton – Raphson
Inherent
Computation
Algorithm
Rounding off
E_a
Approximate
Finite
Polynomial
Complex
Opposite
15.75
15.76

Numerical Methods

Unit - 2

Interpolation and Approximation

Lagrange's interpolation formula (unequal intervals)

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

---

① Using Lagrange's formula, find the polynomial to the given data

x	0	1	3
y	5	6	50

Soln

$$y = f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} \cdot y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} \cdot y_2$$

Here $x_0 = 0 \quad x_1 = 1 \quad x_2 = 3$

$y_0 = 5 \quad y_1 = 6 \quad y_2 = 50$

$$y = f(x) = \frac{(x-1)(x-3)}{(0-1)(0-3)}(5) + \frac{(x-0)(x-3)}{(1-0)(1-3)} \cdot (6)$$

$$+ \frac{(x-0)(x-1)}{} \cdot (50)$$

$$= \frac{(x-1)(x-3)}{3}(5) + \frac{x(x-3)}{-2}(6) + \frac{x(x-1)}{6}(50)$$

$$= \frac{5}{3}[x^2-4x+3] - 3[x^2-3x] + \frac{50}{6}[x^2-x]$$

$$= x^2\left[\frac{5}{3} - 3 + \frac{50}{6}\right] + x\left[-\frac{20}{3} + 9 - \frac{50}{6}\right] + \left[\frac{15}{3}\right]$$

$$= 7x^2 + (-6)x + 5$$

$$\boxed{y = f(x) = 7x^2 - 6x + 5}$$

---

(2) Using Lagranges interpolation find $y(2)$ from the following data

x	0	1	3	4	5
y	0	1	81	256	625

Soln

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 4. \quad x_4 = 5$

$y_0 = 0 \quad y_1 = 1 \quad y_2 = 81 \quad y_3 = 256 \quad y_4 = 625$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \cdot y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \cdot y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \cdot y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \cdot y_3$$

Put $x=2$

$$y(2) = \frac{(2-1)(2-3)(2-4)(2-5)}{(0-1)(0-3)(0-4)(0-5)}(0)$$
$$+ \frac{(2-0)(2-3)(2-4)(2-5)}{(1-0)(1-3)(1-4)(1-5)}(1)$$
$$+ \frac{(2-0)(2-1)(2-4)(2-5)}{(3-0)(3-1)(3-4)(3-5)}(81)$$
$$+ \frac{(2-0)(2-1)(2-3)(2-5)}{(4-0)(4-1)(4-3)(4-5)}(256)$$
$$+ \frac{(2-0)(2-1)(2-3)(2-4)}{(5-0)(5-1)(5-3)(5-4)}(625)$$

$$= \frac{(2)(-1)(-2)(-3)}{(1)(-2)(-3)(-4)} + \frac{(2)(1)(-2)(-3)}{(3)(2)(-1)(-2)}(81)$$
$$+ \frac{(2)(1)(-1)(-3)}{(4)(3)(1)(-1)}(256) + \frac{(2)(1)(-1)(-2)}{(5)(4)(2)(1)}(625)$$

$$= \frac{12}{24} + \frac{12}{12}(81) - \frac{6}{12}(256) + \frac{4}{40}(625)$$
$$= \frac{1}{2} + 81 - 128 + 62.5$$
$$= 0.5 + 81 - 128 + 62.5 = 16.$$

3) Use Lagranges Method to find $\log_{10} 656$, given that $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$ and $\log_{10} 661 = 2.8202$.

soln

x	654	658	659	661
$y=\log_{10} x$	2.8156	2.8182	2.8189	2.8202

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 656$

$$y = f(656) = \frac{(656-658)(656-659)(656-661)}{(654-658)(654-659)(654-661)} \cdot (2.8156) + \frac{(656-654)(656-659)(656-661)}{(658-654)(658-659)(658-661)} (2.8182) + \frac{(656-654)(656-658)(656-661)}{(659-654)(659-658)(659-661)} (2.8189) + \frac{(656-654)(656-659)(656-659)}{(661-654)(661-659)(661-659)} \cdot (2.8202)$$

$$= \frac{(-2)(-3)(-5)}{(-4)(-5)(-7)} (2.8156) + \frac{2(-3)(-5)}{4(-1)(-3)} (2.8182) + \frac{(2)(-2)(-5)}{(5)(1)(-2)} (2.8189) + \frac{(2)(-2)(-3)}{(7)(3)(2)} (2.8202)$$

$$= 0.6033 + 7.0455 - 5.6378 + 0.8058$$

$$= 2.8168.$$

4) Use Lagrange's formula to find the value of y at $x = 6$ from the following data

x :	3	7	9	10

Soln

$x_0 = 3 \quad x_1 = 7 \quad x_2 = 9 \quad x_3 = 10$

$y_0 = 168 \quad y_1 = 120 \quad y_2 = 72 \quad y_3 = 63.$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 6$

$$y = f(6) = \frac{(6-7)(6-9)(6-10)}{(3-7)(3-9)(3-10)}(168) + \frac{(6-7)(6-9)(6-10)}{(7-3)(7-9)(7-10)}(120) + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)}(72) + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)}(63)$$

$$= \frac{(-1)(-3)(-4)}{(-4)(-6)(-7)}(168) + \frac{(3)(-3)(-4)}{4(-2)(-3)}(120) + \frac{(3)(-1)(-4)}{(6)(2)(-1)}(72) + \frac{(3)(-1)(-3)}{(7)(3)(1)}(63)$$

$$= 12 + 180 - 72 + 27$$

$$= 147$$

5) Given the values

x	14	17	31	35
$f(x)$	68.7	64.0	44.0	39.1

find $f(27)$ by using Lagrange's interpolation formula.

Soln

$x_0 = 14 \quad x_1 = 17 \quad x_2 = 31 \quad x_3 = 35$

$y_0 = 68.7 \quad y_1 = 64 \quad y_2 = 44 \quad y_3 = 39.1$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3$$

Put $x = 27$

$$y = f(27) = \frac{(27-17)(27-31)(27-35)}{(14-17)(14-31)(14-35)} \cdot (68.7) + \frac{(27-14)(27-31)(27-35)}{(17-14)(17-31)(17-35)} (64.0) + \frac{(27-14)(27-17)(27-35)}{(31-14)(31-17)(31-35)} (44.0) + \frac{(27-14)(27-17)(27-31)}{(35-14)(35-17)(35-31)} (39.1)$$

$$= \frac{(10)(-4)(-8)}{(-3)(-17)(-21)} (68.7) + \frac{13(-4)(-8)}{(3)(-14)(-8)} (64) + \frac{(13)(10)(-8)}{(17)(14)(-4)} (44) + \frac{(13)(10)(-4)}{(21)(18)(4)} (39.1)$$

$$= -20.52 + 35.22 + 48.07 - 13.45$$

6) Find the Missing term in the following table using Lagranges interpolation

x	0	1	2	3	4
y	1	3	9	—	81

Soln

$x_0 = 0 \quad x_1 = 1 \quad x_2 = 3 \quad x_3 = 4$

$y_0 = 1 \quad y_1 = 3 \quad y_2 = 9 \quad y_3 = 81$

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3.$$

Put $x = 3$

$$y = f(3) = \frac{(3-1)(3-2)(3-4)}{(0-1)(0-2)(0-4)}(1) + \frac{(3-0)(3-2)(3-4)}{(1-0)(1-2)(1-4)}(3) + \frac{(3-0)(3-1)(3-4)}{(2-0)(2-1)(2-4)}(9) + \frac{(3-0)(3-1)(3-2)}{(4-0)(4-1)(4-2)}(81)$$

$$= \frac{-2}{-8}\{-3\} + \frac{27}{2} + \frac{81}{4}$$

$$= 31.$$

7) Using Lagranges formula prove

$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} + y_{-5})$

Soln

y_{-5}, y_{-3}, y_3, y_5 Occur in the answers.

So we can have the table

x	-5	-3	3	5
y	y_{-5}	y_{-3}	y_3	y_5

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \cdot y_{-5} + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_{-3} + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_3 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_5$$

put $x = 1$

$$y_1 = f(1) = \frac{(1+3)(1-3)(1-5)}{(-5+3)(-5-3)(-5-5)} \cdot y_{-5} + \frac{(1+5)(1-3)(1-5)}{(-3+5)(-3-3)(-3-5)} \cdot y_{-3} + \frac{(1+5)(1+3)(1-5)}{(3+5)(3+3)(3-5)} y_3 + \frac{(3+5)(3+3)(3-3)}{(5+5)(5+3)(5-3)} \cdot y_5$$

$$= \frac{(4)(-2)(-4)}{(-2)(-8)(-10)} \cdot y_{-5} + \frac{(6)(-2)(-4)}{(2)(-6)(-8)} \cdot y_{-3} + \frac{(6)(4)(-4)}{(8)(6)(-2)} \cdot y_3 + \frac{(6)(4)(-2)}{(10)(8)(2)} y_5$$

$= -0.2\,4 \quad +0.5\,4 \quad +4 \quad - 0.3\,4$

$$+\frac{(0+30)(0+13)(0-18)}{(3+30)(3+13)(3-18)}.(38)$$

$$+\frac{(0+30)(0+13)(0-3)}{(18+30)(18+13)(18-3)}(42)$$

$$= 37.23.$$

② Find the value of θ given $f(\theta) = 0.3887$ where $f(\theta) = \int_0^{\theta} \frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}}$ using the table

θ	$21°$	$23°$	$25°$
$f(\theta)$	0.3706	0.4068	0.4433

Soln

Let $\theta = x$

$f(\theta) = f(x) = y$

x	$21°$	$23°$	$25°$
y	0.3706	0.4068	0.4433

$$x = f(y) = \frac{(y-y_1)(y-y_2)}{(y_0-y_1)(y_0-y_2)}.x_0 + \frac{(y-y_0)(y-y_2)}{(y_1-y_0)(y_1-y_2)}.x_1 + \frac{(y-y_0)(y-y_1)}{(y_2-y_0)(y_2-y_1)}.x_2$$

Put $y = 0.3887$

$$x = f(0.3887) = \frac{(0.3887-0.4068)(0.3887-0.4433)}{(0.3706-0.4068)(0.3706-0.4433)}(21°)$$

$$+\frac{(0.3887-0.3706)(0.3887-0.4433)}{(0.4068-0.3706)(0.4068-0.4433)}(23°)$$

$$+(0.3887-0.3706)(0.3887-0.4068) \ldots$$

Newton's divided difference formula: (unequal)

$$y = f(x) = y_0 + (x-x_0)\,\Delta f(x_0) + (x-x_0)(x-x_1)\,\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\,\Delta^2 f(x_0) + \cdots$$

(1) Using Newton's divided difference formula find $f(x)$ and $f(6)$ from the following data.

x :	1 x_0	2 x_1	7 x_2	8 x_3
$f(x)$:	1	5	5	4

Soln

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	1			
		$\frac{5-1}{2-1} = 4$		
2	5		$\frac{0-4}{7-1} = -\frac{4}{6}$	
		$\frac{5-5}{7-2} = 0$		$\frac{-\frac{1}{6}+\frac{4}{6}}{8-1} = \frac{1/2}{7} = \frac{1}{14}$
7	5		$\frac{-1-0}{8-2} = \frac{-1}{6}$	
		$\frac{4-5}{8-7} = -1$		
8	4			

$$y = f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\,\Delta^2 f(x_0) + \cdots$$

$$= 1 + (x-1)(4) + (x-1)(x-2)\left(-\frac{4}{6}\right)$$

$$= x^3\left[\frac{1}{14}\right] + x^2\left[-\frac{4}{6}\right] - \frac{3}{14} - \frac{7}{14}\Big]$$

$$+ x\left[4 + \frac{12}{6} + \frac{2}{14} + \frac{21}{14}\right] + \left[1 - 4 - \frac{8}{6} - \frac{14}{14}\right]$$

$$f(x) = \frac{1}{14}x^3 - \frac{29}{21}x^2 + \frac{107}{14}x - \frac{16}{63}$$

Put $x = 6$

$$y = f(6) = \frac{1}{14}(6)^3 - \frac{29}{21}(6)^2 + \frac{107}{14}(6) - \frac{167}{36}$$

$$\cancel{= 54 - 114 + 40.4 - 7.833}$$

$$= 15.428 - 49.714 + 45.857 - 0.444$$

$$= 11.127$$

2) Find $f(x)$ as a polynomial in x for the following data by Newton's divided difference

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

Soln

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245				
		$\frac{33-1245}{-1+4} = -404$			
-1	33		$\frac{-28+404}{0+4} = 94$		
		$\frac{5-33}{0+1} = -28$		$\frac{10-94}{2+4} = -14$	
0	5		$\frac{2+28}{2+1} = 10$		$\frac{13+14}{5+4} = 3$
		$\frac{9-5}{2-0} = 2$		$\frac{88-10}{5+1} = 13$	
2	9		$\frac{442-2}{5-0} = 88$		
		$\frac{1335-9}{5-2} = 442$			
5	1335				

$$y = f(x) = f(x_0) + (x-x_0)\Delta f(x) + (x-x_0)(x-x_1)\Delta^2 f(x)$$
$$+ (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x)$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 f(x)$$

$$= 1245 + (x+4)(-404) + (x+4)(x+1)(94) + (x+4)(x+1)(x-0)(x-2)(3)$$
$$+ (x+4)(x+1)(x-0)(-14)$$

$$= 1245 - 404x - 1616 + (x^2+5x+4)\,94 + (x^2+5x+4)(3x^2-6x)$$
$$+ (x^2+5x+4)(-14x)$$

$$= 1245 - 404x - 1616 + 94x^2 + 470x + 376 + 3x^4 + 15x^3 + 12x^2$$
$$-14x^3 - 70x^2 - 56x$$
$$-6x^3 - 30x^2 - 24x$$

$$= \cancel{1245}\;\cancel{x(-404+470-16-24)} + \cancel{x^2[94-20+12-30]}$$
$$= x^4[3] + x^3[-14+15-6] + x^2[94-70+12-30] + [1245-1616+376]$$
$$+ x[-404+470-56-24]$$

$$= 3x^4 + 5x^3 + 6x^2 - 14x + 5$$

(3) Find the cubic polynomial from the following table using Newton's divided difference formula and hence find $f(4)$

x	0 x_0	1 x_1	2 x_2	5 x_3
y	2	3	12	147

Soln

x	$y=f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	2			
		$\frac{3-2}{1-0}=1$		
1	3		$\frac{9-1}{2-0}=4$	
		$\frac{12-3}{2-1}=9$		$\frac{9-4}{5-0}=1$
2	12		$\frac{45-9}{5-1}=9$	
		$\frac{147-12}{5-2}=45$		
5	147			

$$y=f(x) = y_0 + (x-x_0)\Delta f(x) + (x-x_0)(x-x_1)\Delta^2 f(x) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x).$$

$$= 2 + (x-0)(1) + (x-0)(x-1)(4) + (x-0)(x-1)(x-2)(1)$$

$$= 2 + x + 4x^2 - 4x + (x^2 - x)(x-2)$$

$$= 2 + x + 4x^2 - 4x + x^3 - x^2 - 2x^2 + 2x$$

$$= x^3 + x^2 - x + 2$$

Put $x=4$

$$y = f(4) = 4^3 + 4^2 - 4 + 2$$

$$= 78.$$

Cubic Spline Interpolation formula.

$$g(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right]$$
$$+ \frac{1}{h}(x_i - x)\left[y_{i-1} - \frac{h^2}{6}M_{i-1}\right]$$
$$- \frac{1}{h}(x_{i-1} - x)\left[y_i - \frac{h^2}{6}M_i\right]$$

where, $M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right]$

with $M_0 = M_n = 0$

---

① Obtain cubic spline polynomial which best fits with the following data, given that $y_0'' = y_3'' = 0$

x	-1 (x_0)	0 (x_1)	1 (x_2)	(x_3) 2
y	-1 (y_0)	1 (y_1)	3 (y_2)	35 (y_3)

Soln

Given $M_0 = M_3 = 0$, $h = 1$

WKT $M_{i-1} + 4M_i + M_{i+1} = 6\left[y_{i-1} - 2y_i + y_{i+1}\right]$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6\left[y_0 - 2y_1 + y_2\right]$$
$$4M_1 + M_2 = 6\left[-1 - 2 + 3\right]$$
$$4M_1 + M_2 = 0 \quad \text{———} \quad (1)$$

Put $i = 2$

$$M_1 + 4M_2 + M_3 = 6\left[y_1 - 2y_2 + y_3\right]$$
$$M_1 + 4M_2 = 6\left[1 - 6 + 35\right]$$

Solve ① & ②

$M_1 = -12 \qquad M_2 = 48$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i \right] + (x_i - x)\left[y_{i-1} - \frac{1}{6} M_{i-1} \right] - (x_{i-1} - x)\left[y_i - \frac{1}{6} M_i \right]$$

Case(i) $-1 < x < 0$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1 \right] + (x_1 - x)\left[y_0 - \frac{1}{6} M_0 \right] - (x_0 - x)\left[y_1 - \frac{1}{6} M_1 \right]$$

$$= \frac{1}{6}\left[-(-1-x)^3(-12) \right] + (0-x)(-1) - (-1-x)\left[1 + \frac{12}{6} \right]$$

$$= \frac{1}{6}\left[-12(1+x)^3 \right] + x + (1+x)(3)$$

$$= -2\left[1 + x^3 + 3x + 3x^2 \right] + x + 3 + 3x$$

$$= -2 - 2x^3 - 6x - 6x^2 + x + 3 + 3x$$

$$\boxed{S(x) = -2x^3 - 6x^2 - 2x + 1, \quad -1 < x < 0}$$

Case (ii) $0 < x < 1$

Put $i = 2$

$$S(x) = \frac{1}{6}\left[(x_2 - x_1)^3 M_1 - (x_1 - x)^3 M_2 \right.$$
$$+ (x_2 - x)\left[y_1 - \frac{1}{6} M_1 \right]$$
$$-(x_1 - x)\left[y_2 - \frac{1}{6} M_2 \right]$$
$$= \frac{1}{6}\left[(1-x)^3 (-12) - (0-x)^3 (48) \right]$$
$$+ (1-x)\left[1 - \frac{1}{6}(-12) \right] - (0-x)$$
$$\left[3 - \frac{1}{6} \times 48 \right]$$
$$= \frac{1}{6}\left[-12(1-x)^3 + 48x^3 \right] + 3(1-x) - 5x$$
$$= \frac{1}{6}\left[-12(1 - x^3 - 3x + 3x^2) + 48x^3 \right.$$
$$+ 3 - 3x - 5x$$
$$= \frac{1}{6}\left[-12 + 12x^3 + 36x - 36x^2 + 48x^3 \right]$$
$$+ 3 - 3x - 5x$$
$$= x^3[2 + 8] + x^2[-6] + x[6 - 8]$$
$$-2 + 3$$

$$\boxed{S(x) = 10x^3 - 6x^2 - 2x + 1, \quad 0 < x < 1}$$

Case(iii) $\quad 1 < x < 2$

Put $i = 3$

$$S(x) = \frac{1}{6}\left[(x_3 - x)^3 M_2 - (x_2 - x)^3 M_3 \right]$$
$$+ (x_3 - x)\left[y_2 - \frac{1}{6} M_2 \right] - (x_2 - x)$$
$$\left[y_3 - \frac{1}{6} M_3 \right]$$
$$= \frac{1}{6}\left[(2-x)^3 48 \right] + (2-x)\left[3 - \frac{1}{6} \times 48 \right]$$

$= 8(2-x)^3 + (2-x)(-5) - 35(1-x)$

$= 8[8 - x^3 - 12x + 6x^2] - 10 + 5x - 35 + 35x$

$= 64 - 8x^3 - 96x + 48x^2 - 10 + 5x - 35 + 35x$

$$\boxed{S(x) = -8x^3 + 48x^2 - 56x + 19, \quad 1 < x < 2}$$

The cubic Spline polynomial is

$$S(x) = \begin{cases} -2x^3 - 6x^2 - 2x + 1, & -1 < x < 0 \\ 10x^3 - 6x^2 - 2x + 1, & 0 < x < 1 \\ -8x^3 + 48x^2 - 56x + 19, & 1 < x < 2 \end{cases}$$

---

(2) From the following table

x	1 x_0	2 x_1	3 x_2
y	-8 y_0	-1 y_1	18 y_2

Compute $y(1.5)$ and $y'(1)$ using cubic Spline.

Soln

Take $M_0 = M_2 = 0.,\quad h = 1$

W.k.T $\quad M_{i-1} + 4M_i + M_{i+1} = 6[y_{i-1} - 2y_i + y_{i+1}]$

Put $i = 1$

$M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$

$4M_1 = 6[-8 + 2 + 18]$

$4M_1 = 72$

$\boxed{M_1 = 18}$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right] + (x_i - x)\left[y_{i-1} - \frac{1}{6}M_{i-1}\right] - (x_{i-1} - x)\left[y_i - \frac{1}{6}M_i\right]$$

Case (i) $1 < x < 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1\right] + (x_1 - x)\left[y_0 - \frac{1}{6}M_0\right] - (x_0 - x)\left[y_1 - \frac{1}{6}M_1\right]$$

$$= \frac{1}{6}\left[(2-x)^3(0) - (1-x)^3(18) + (2-x)\left[-8 - \frac{1}{6}(0)\right] - (1-x)\left[-1 - \frac{1}{6}(18)\right]\right]$$

$$= \frac{1}{6}\left[-(1-x)^3(18) + (2-x)(-8) - (1-x)\left[-1-3\right]\right]$$

$$= -18(1-x)^3 - 8(2-x) + 4(1-x)$$

$$= -18(1-x)^3 - 16 + 8x + 4 - 4x$$

$$= -18(1-x)^3 + 4x - 12, \quad 1 < x < 2$$

$$\boxed{S(x) = -18(1-x)^3 + 4x - 12, \quad 1 < x < 2}$$

Put $x = 1.5$

$$y(1.5) = S(1.5) = -18(1-1.5)^3 + 4(1.5) - 12$$

$$= -5.625$$

$y'(1) = 9(0) + 4 = 4$

$\boxed{y'(1) = 4}$ $\boxed{y(1.5) = -5.625}$

(3) Find the cubic spline interpolation

x :	1	2	3	4	5
f :	1 y_0	0 y_1	1 y_2	0 y_3	1 y_4

Soln

Take $M_0 = M_4 = 0$, $h = 1$

WKT

$$M_{i-1} + 4M_i + M_{i+1} = 6[y_{i-1} - 2y_i + y_{i+1}]$$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6[y_0 - 2y_1 + y_2]$$

$$4M_1 + M_2 = 6[1 - 0 + 1] = 12$$

$$4M_1 + M_2 = 12 \quad \text{——— (1)}$$

Put $i = 2$

$$M_1 + 4M_2 + M_3 = 6[y_1 - 2y_2 + y_3]$$

$$= 6[0 - 2 + 0]$$

$$M_1 + 4M_2 + M_3 = -12. \quad \text{——— (2)}$$

Put $i = 3$

$$M_2 + 4M_3 + M_4 = 6[y_2 - 2y_3 + y_4]$$

$$M_2 + 4M_3 = 6[1 - 0 + 1]$$

$$M_2 + 4M_3 = 12 \quad \text{——— (3)}$$

from (1) & (2)

$4 \times (1) \Rightarrow 16M_1 + 4M_2 = 48$

from ② & ③

$$M_1 + 4M_2 + M_3 = -12$$

$$② \Rightarrow M_1 + \cancel{4M_2} + M_3 = -12$$

$$4 \times ③ \Rightarrow \cancel{4M_2} + 16M_3 = 48$$

$$(-)\qquad(-)\qquad(-)$$

$$M_1 - 15M_3 = -60 \quad \text{—— ⑤}$$

Solve ④ & ⑤

$$\boxed{M_3 = \frac{30}{7}}$$

$$⑤ \Rightarrow M_1 = -60 + 15M_3$$

$$M_1 = -60 + \frac{450}{7}$$

$$\boxed{M_1 = \frac{30}{7}}$$

$$① \Rightarrow 4M_1 + M_2 = 12$$

$$M_2 = 12 - 4M_1$$

$$= 12 - 4\left(\frac{30}{7}\right)$$

$$\boxed{M_2 = -\frac{36}{7}}$$

The cubic Spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right]$$
$$+ (x_i - x)\left[y_{i-1} - \frac{1}{6}M_{i-1}\right]$$
$$- (x_{i-1} - x)\left[y_i - \frac{1}{6}M_i\right]$$

case (i) $-1 < x < 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1\right]$$
$$+ (x_1 - x)\left[y_0 - \frac{1}{6}M_0\right]$$
$$- (x_0 - x)\left[y_1 - \frac{1}{6}M_1\right]$$

$$= \frac{1}{6}\left[(2-x)^3(0) - (1-x)^3\left(\frac{30}{7}\right)\right.$$
$$+ (2-x)\left[1 - \frac{1}{6}(0)\right]$$

$(a-b)^3 = a^3-3$

$$= \frac{1}{6}\left[-(1-x)^3\left(\frac{30}{7}\right)\right] + (2-x)[1] - (1-x)\left[-\frac{1}{6}\frac{30}{7}\right]$$

$$= \frac{1}{6}\left[-\frac{30}{7}(1-x)^3 + (2-x) + \frac{5}{7}(1-x)\right]$$

$$= \frac{1}{6}\left[-\frac{30}{7}\left[1^3 - x^3 - 3x + 3x^2\right] + 2 - x + \frac{5}{7} - \frac{5}{7}x\right]$$

$$= -\cancel{\frac{5}{7}} + \frac{5}{7}x^3 + \frac{15}{7}x + 15x^2 + 2 - x + \cancel{\frac{5}{7}} - \frac{5}{7}x$$

$$= \frac{5}{7}x^3 + 15x^2 + x\left(\frac{15}{7} - 1 - \frac{5}{7}\right)$$

$$S(x) = \frac{5}{7}x^3 + 15x^2 + \frac{3}{7}x + 2, \quad 1 \le x \le 2$$

Case (ii) $2 < x < 3$.

Put $i = 2$.

$$S(x) = \frac{1}{6}\left[(x_2-x)^3 M_1 - (x_1-x)^3 M_2\right] + (x_2-x)\left[y_1 - \frac{1}{6}M_1\right] - (x_1-x)\left[y_2 - \frac{1}{6}M_2\right]$$

$$= \frac{1}{6}\left[(3-x)^3\frac{30}{7} - (2-x)\left(-\frac{36}{7}\right)\right] + (3-x)\left[0 - \frac{1}{6}\left(\frac{30}{7}\right)\right] - (2-x)\left[1 - \frac{1}{6}\left(-\frac{36}{7}\right)\right]$$

$$= \frac{1}{6}\left[\frac{30}{7}(3-x)^3 + \frac{36}{7}(2-x)\right] + (3-x)\left(-\frac{5}{7}\right)$$

$$-(2-x)\left[1+\frac{5}{7}\right]$$

$$= \frac{5}{7}\left[27-27x+9x^2-x^3\right] + \frac{6}{7}\left[4+x^2-4x\right]$$

$$-\frac{15}{7} + \frac{5x}{7} - \frac{26}{7} + \frac{13x}{7}$$

$$= x^3\left[-\frac{5}{7}\right] + x^2\left[\frac{45}{7} + \frac{6}{7} + \cancel{\frac{5}{7} + \frac{13}{7}}\right]$$

$$+ x\left[-135 - \frac{24}{7} + \frac{5}{7} + \frac{13}{7}\right] + \frac{135}{7} + \frac{24}{7} - \frac{15}{7}$$

$$-\frac{26}{7}$$

$$S(x) = -\frac{5}{7}x^3 + \frac{51}{7}x^2 - \frac{951}{7} + \frac{118}{7}, \quad 2<x<3$$

case (iii) $3<x<4$

Put $i=3$

$$S(x) = \frac{1}{6}\left[(x_3-x)^3 M_2 - (x_2-x) M_3\right]$$

$$+ (x_3-x)\left[y_2 - \frac{1}{6}M_2\right] - (x_2-x)\left[y_3 - \frac{1}{6}M_3\right]$$

$$= \frac{1}{6}\left[(4-x)^3\left(-\frac{36}{7}\right) - (3-x)^3\left(\frac{30}{7}\right)\right]$$

$$+ (4-x)\left[1 - \frac{1}{6}\left(-\frac{36}{7}\right)\right]$$

$$-(3-x)\left[0 - \frac{1}{6}\left(\frac{30}{7}\right)\right]$$

$$= \frac{1}{6}\Big[-\frac{36}{7}[64 - 48x + 12x^2 - x^3]$$

$$-\frac{30}{7}[27 - 27x + 9x^2 - x^3]\Big]$$

$$+(4-x)\left(1+\frac{6}{7}\right) - (3-x)\left(-\frac{5}{7}\right)$$

$$= \cancel{\Big[\frac{-384}{7}} \quad 13/7$$

$$= \frac{1}{7}\Big[-384 + 288x - 72x^2 + 6x^3 - 810$$

$$+ 810x - 270x^2 + 30x^3$$

$$+ 52 - 13x + 15 - 5x\Big]$$

$$= \frac{1}{7}\Big[x^3[30+6] + x^2[-72-270] +$$

$$+ x[288 + 810 - 13 - 5] +$$

$$[-384 - 810 + 52 + 15]\Big]$$

$$S(x) = \frac{1}{7}[36x^3 - 342x^2 + 1080x - 1127, \quad 3 \le x \le 4$$

case (iv) $4 < x < 5$

Put $i = 4$.

$$S(x) = \frac{1}{6}[(x_3 - x)^3 M_3 - (x_2 - x) M_4]$$

$$+ (x_4 - x)[y_3 - \frac{1}{6}M_3]$$

$$- (x_3 - x)[y_4 - \frac{1}{6}M_4]$$

$$= \frac{1}{6}[(5-x)^3\left(\frac{30}{7}\right) - 0] + (5-x)[0 - \frac{1}{6}(\frac{30}{7})]$$

$$+(x-4)[1-0]$$

4) Find the cubic spline for the data

x	1	2	3
y	-6	-1	16

Hence evaluate $y(1.5)$ given that $y_0'' = y_2'' = 0$.

Soln

Given $h = 1$ $\quad M_0 = M_2 = 0$

W.K.T

$$M_{i-1} + 4M_i + M_{i+1} = 6\left[y_{i-1} - 2y_i + y_{i+1}\right]$$

Put $i = 1$

$$M_0 + 4M_1 + M_2 = 6\left[y_0 - 2y_1 + y_2\right]$$

$$4M_1 = 6\left[-6 - 2(-1) + 16\right]$$

$$4M_1 = 72$$

$$\boxed{M_1 = 18}$$

The cubic spline polynomial is

$$S(x) = \frac{1}{6}\left[(x_i - x)^3 M_{i-1} - (x_{i-1} - x)^3 M_i\right]$$
$$+ (x_i - x)\left[y_{i-1} - \frac{1}{6}M_{i-1}\right]$$
$$- (x_{i-1} - x)\left[y_i - \frac{1}{6}M_i\right]$$

Case (i) $1 \le x \le 2$

Put $i = 1$

$$S(x) = \frac{1}{6}\left[(x_1 - x)^3 M_0 - (x_0 - x)^3 M_1\right]$$

$$= \frac{1}{6}\left[(2-x)^3(0) + (x-1)^3(18)\right]$$
$$+ (2-x)\left[-6 - \frac{1}{6}(0)\right]$$
$$+ (x-1)\left[-1 - \frac{1}{6}(18)\right]$$
$$= \frac{1}{6}\left[(x-1)^3(18)\right] + (2-x)(-6-0)$$
$$+ (x-1)(-1-3)$$
$$= 3(x^3 - 3x^2 + 3x - 1) - 12 + 6x - 4x + 4$$
$$S(x) = 3x^3 - 9x^2 + 11x - 11$$

Case (ii) $2 \le x \le 3$

Put $i = 2$.

$$S(x) = \frac{1}{6}\left[(x_2 - x)^3 M_1 - (x_1 - x)^3 M_2\right]$$
$$+ (x_2 - x)\left[y_1 - \frac{1}{6}M_1\right]$$
$$- (x_1 - x)\left[y_2 - \frac{1}{6}M_2\right]$$
$$= \frac{1}{6}\left[(3-x)^3\, 18 - (2-x)^3(0)\right]$$
$$+ (3-x)\left[-1 - \frac{1}{6}(18)\right]$$
$$- (x-2)\left[16 - \frac{1}{6}(0)\right]$$
$$= \frac{18}{6}\left[27 - 27x + 9x^2 - x^3\right]$$
$$- 12 + 4x + 16x - 32$$

$$S(x) = -3x^3 + 27x^2 - 61x + 37$$

$$y = S(x) = \begin{cases} 3x^3 - 9x^2 + 11x - 11, & 1 \le x \le 2 \\ -3x^3 + 27x^2 - 61x + 37, & 2 \le x \le 3 \end{cases}$$

To find $y(1.5)$

$$S(1.5) = 3(1.5)^3 - 9(1.5)^2 + 11(1.5) - 11$$
$$= -4.625$$

Newton's forward interpolation formula (equal intervals).

$$y = f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

where $u = \frac{x - x_0}{h}$

(1) Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence evaluate y at $x = 5$.

x	4	6	8	10
y	1	3	8	10

Soln

$u = \frac{x - x_0}{h}$, $h = 2$

$u = \frac{x-4}{2}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1			
		2		
6	3		3	
		5		-6
8	8		-3	
		2		

The Newton's forward interpolation form is

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 1 + \left(\frac{x-4}{2}\right)(2) + \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)}{2} \times 3$$

$$+ \frac{\left(\frac{x-4}{2}\right)\left(\frac{x-4}{2}-1\right)\left(\frac{x-4}{2}-2\right)}{3!} \times -6$$

$$= 1 + (x-4) + \frac{3(x-4)(x-6)}{8} - \frac{(x-4)(x-6)(x-8)}{8}$$

$$= \frac{1}{8}\left[8 + 8x - 32 + 3\left[x^2 - 10x + 24\right] - \left[x^3 - 18x^2 + 104x - 192\right]\right]$$

$$y = \frac{1}{8}\left[-x^3 + 21x^2 - 126x + 240\right]$$

Put $x = 5$

$$y(5) = \frac{1}{8}\left[-5^3 + 21 \times 5^2 - 126 \times 5 + 240\right]$$

$$\boxed{y(5) = 1.25}$$

② Fit a polynomial, by using Newtons forward interpolation formula to the data given below.

x	0 (x_0)	1 (x_1)	2 (x_2)	3 (x_3)
y	1 (y_0)	2 (y_1)	1 (y_2)	10 (y_3)

Soln

$$u = \frac{x - x_0}{h}, \quad h = 1$$

$$u = \frac{x - 0}{1} = x$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

$$= 1 + \frac{x}{1!}(2) + \frac{x(x-1)}{2!}(-1) + \frac{x(x-1)(x-2)}{3!}(10)$$

$$= -1 + 2x + \frac{(x^2 - x)}{2} + \frac{10}{6}\left[(x^2 - x)(x-2)\right]$$

$$= 1 + 2x + \frac{x^2}{2} - \frac{x}{2} + \frac{5}{3}\left[x^3 - 2x^2 - x^2 + 2x\right]$$

$$= \frac{5}{\;} x^3 + x^2\left[\frac{1}{2} - \frac{10}{3}\right] + x\left[2 - \frac{1}{2} + \frac{10}{3}\right] + 1$$

(3) From the data given below find the number of students whose weight is between 60 to 70.

Weight in kgs	0-40	40-60	60-80	80-100	100-120
No. of Students	250	120	100	70	50

Soln

$u = \frac{x - x_0}{h}$, $h = 20$

$u = \frac{x-40}{20}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
Below 60	370		-20		
		100		-10	
Below 80	470		-30		20
		70		10	
Below 100	540		-20		
		50			
Below 120	590				

The Newton's forward interpolation formula is

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

$$y = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \dots$$

$$= 250 + \left(\frac{x-40}{20}\right)120 + \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)}{2} \times -20$$

$$+ \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)\left(\frac{x-40}{20}-2\right)}{6} \times -10$$

$$+ \frac{\left(\frac{x-40}{20}\right)\left(\frac{x-40}{20}-1\right)\left(\frac{x-40}{20}-2\right)\left(\frac{x-40}{20}-3\right)}{24} \times 20$$

$$y = 250 + 6(x-40) - 10\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)$$

$$-\frac{5}{3}\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)\left(\frac{x-80}{20}\right)$$

$$+\frac{5}{6}\left(\frac{x-40}{20}\right)\left(\frac{x-60}{20}\right)\left(\frac{x-80}{20}\right)\left(\frac{x-100}{20}\right)$$

$$y(70) = 250 + 6(70-40) - 10\left(\frac{70-40}{20}\right)$$

$$\left(\frac{70-60}{20}\right) - \frac{5}{3}\left(\frac{70-40}{20}\right)\left(\frac{70-60}{20}\right)\left(\frac{70-80}{20}\right)$$

$$+\frac{5}{6}\left(\frac{70-40}{20}\right)\left(\frac{70-60}{20}\right)\left(\frac{70-80}{20}\right)\left(\frac{70-100}{20}\right)$$

$$= 250 + 180 - \frac{15}{2} + \frac{5}{8} + \frac{15}{32}$$

$$y(70) = 423.59 \simeq 424.$$

$$y(60) = 370.$$

No. of Students whose weight between 60–70 $\Big\} = y(70) - y(60)$

$= 424 - 370$

Newton's Backward Interpolation formula

$$y = y_0 + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n$$

Where $v = \frac{x - x_n}{h}$

(1) Use Newton's backward difference formula to construct an interpolating polynomial of degree 3 for the data

$f(-0.75) = -0.0718125$, $f(-0.5) = -0.024750$

$f(-0.25) = 0.33493750$, $f(0) = 1.10100$.

Hence find $f(-\frac{1}{3})$.

Soln.

$$v = \frac{x - x_n}{h} \quad \& \quad h = 0.25$$

$$v = \frac{x - 0}{0.25} = \frac{x}{0.25}$$

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
-0.75	-0.07181250			
		0.0470625		
-0.50	-0.024750		0.312625	
		0.3596875		0.09375
-0.25	0.33493750		0.40625	
		0.7660625		
0	1.10100			

The Newton's backward interpolation formula is

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

$$= 1.10100 + \left(\frac{x}{0.25}\right)(0.7660625)$$
$$+ \left(\frac{x}{0.25}\right)\left(\frac{x}{0.25}+1\right)(0.406375)$$
$$+ \frac{\left(\frac{x}{0.25}\right)\left(\frac{x}{0.25}+1\right)\left(\frac{x}{0.25}+2\right)}{3!}(0.09375)$$

$$= 1.10100 + (-1.33333)(0.7660625)$$
$$+ \frac{(-1.33333)(-0.33333)}{2}(0.406375)$$
$$+ \frac{(-1.33333)(-0.33333)(-0.66666)}{6}(0.09375)$$

$$= 1.10100 - 1.021414 + 0.090304426$$
$$+ 0.0046295$$

$$y(-1/3) = 0.165260.$$

(2) The amount A of a substance remaining in a reacting system after an interval of time t in a certain chemical experiment

T (min)	2	5	8	11
A (gm)	94.8	87.9	81.3	75.1

Obtain the value of A where $t = 9$ mins using Newton's interpolation formula.

Soln

T x	A y	∇y	$\nabla^2 y$	$\nabla^3 y$
2	94.8			
		-6.9		
5	87.9		0.3	
		-6.6		0.1
8	81.3		0.4	
		-6.2		
11	75.1			

$$v = \frac{x - x_n}{h}, \quad h = 3$$

The Newton's Backward interpolation formula is

$$y = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots$$

$$y = 75.1 + \left(\frac{x-11}{3}\right)(-6.2) + \frac{\left(\frac{x-11}{3}\right)\left(\frac{x-11}{3}+1\right)}{\ldots}(0.4)$$

$$y = 75.1 - 6.2\left(\frac{x-11}{3}\right) + \left(\frac{(x-11)(x-8)}{8}\right) \times 0.4$$

$$+ \frac{(x-11)(x-8)(x-5)}{162} \times 0.1$$

Put $x = 9$

$$y(9) = 75.1 - \frac{6.2(9-11)}{3} + \frac{(9-11)(9-8)}{18} \times 0.5$$

$$+ \frac{(9-11)(9-8)(9-5)}{162} \times 0.1$$

$$= 75.1 + \frac{62}{15} - \frac{2}{45} - \frac{2}{405}$$

$$y(9) = 79.1839.$$

Questions	opt1	opt2	opt3	opt4	opt7
The numerical method of solving linear equations is of two types one is direct, other is_________ method.	iterative	elimination	Newton	none	iterative
In Gauss –Jordan method the coefficient matrix is transformed into ________ matrix	scalar	unit	diagonal	column	unit
The convergence in Gauss –Jacobi method can be achieved only when coefficient of thematrix is __________ dominant	row wise	column wise	diagonally	none	diagonally
Gauss –Elimination and Gauss –Jordan are direct methods while Gauss –Jacobi and Gauss – Seidal are __________ methods	iterative	elimination	interpolation	none	iterative
The convergence of Gauss – Seidal method is _____times as fast as in Jacobi's method	1	2	3	4	3
The power method will work satisfactorily only if A has a__________ Eigen value	small	large	equal	dominant	dominant
In power method the element in vector in each iteration will become very large, to avoid this we divide each vector by its _________ component	smallest	largest	positive	negative	largest
Gauss – Jordan method is ________ method	direct	indirect	iteration	interpolation	direct
Gauss – Jacobi method is ________ method	direct	indirect	iteration	interpolation	indirect
Gauss – Jacobi method is ________ method	direct	indirect	iteration	interpolation	iteration
Gauss – Seidal method is ________ method	direct	indirect	iteration	interpolation	indirect
Gauss – Jordan method fails if the element in top of first column is ______	0	1	2	3	0
The successive approximations are called ________	interpolation	elimination	iterates	approximation	iterates
__________ method is a self - correcting method.	interpolation	elimination	iterates	approximation	iterates
In Gauss – Jacobi and Gauss – Seidal methods the co-efficient matrix must be ___dominant.	row wise	column wise	none	diagonally	diagonally
The matrix is ________ if the numerical value of the leading diagonal element in each row is greater than or equal to the sum of the numerical value of other element in that row.	orthogonal	symmetric	diagonally dominant	singular	diagonally dominant
The Gauss – Jordan method is the modification of ________ method.	Gauss –Elimination	Gauss – Jacobi	Gauss – Seidal	interpolation	Gauss –Elimination
The iterative procedure for finding the dominant Eigen value of the matrix is called ______ Power method.	Rayleigh's	Gaussian	Newton's	inverse	Rayleigh's
$x^2 + 5x + 4 = 0$ is a ________ equation.	algebraic	transcendental	wave	heat	algebraic
$a + b \log x + c \sin x + d = 0$ is a _________ equation.	algebraic	transcendental	wave	heat	transcendental
In Gauss – Jordan method, the augmented matrix is reduced into _____matrix	upper triangular	lower triangular	diagonal	scalar	diagonal
The 1st equation in Gauss – Jordan method, is called __________ equation.	pivotal	dominant	reduced	normal	pivotal
The element in Gauss – Jordan method is called _________ element.	Eigen value	Eigen vector	pivot	root	pivot
Power method generally gives the largest Eigen value of A provided the Eigen values are____.	equal	negative	positive	real and distinct	real and distinct
The system of simultaneous linear equation in n unknowns AX = B if A is diagonally Dominant then the system is said to be __________ system	dominant	diagonal	scalar	singular	diagonal
The convergence of Gauss – Seidal method is roughly _____ that of Gauss – Jacobi method	twice	thrice	once	4times	twice
In power method iterative process is repeated until _________ becomes negligibly small.	$X_r - X_{(r-1)}$	$X_{(r-1)} - X_r$	$X_r - X_{(r+1)}$	$X_{(r+1)} - X_r$	$X_r - X_{(r-1)}$
Cholesky's method is used for finding the _________ of a matrix.	determinant	value	inverse	rank	determinant
The smallest eigen value of A is the reciprocal of the dominant eigen value of___________	$A^{(-1)}$	det A	A^T	A	$A^{(-1)}$
Choleskey's method is used only when the matrix is ____	symmetric	skew-symmetric	singular	non-singular	symmetric
The Power method is used for finding ________ eigen value	dominant	least	central	positive	dominant
The Inverse Power method is used for finding ________ eigen value	dominant	least	central	positive	dominant
Jacobi's method is used only when the matrix is ____	symmetric	skew-symmetric	singular	non-singular	symmetric
Crout's method is a____ method to solve simultaneous linear equations.	Direct	Indirect	real	inverse	Direct
In Crout's method, if AX=B, then	LX=B	UX=B	L=B	LUX=B	LUX=B

8015290573

UNIT - 3

Numerical Differentiation and Integration

Numerical differentiation:

It is the Process of finding the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ & $\frac{d^3y}{dx^3}$, ... for some particular value of x.

① find the first derivatives of $f(x)$ at $x=2$ for the data $f(-1)=-21$, $f(1)=15$, $f(2)=12$, $f(3)=3$. using Newton's divided difference formula.

Soln

x	-1	1	2	3
y	-21	15	12	3

The Newton's divided difference formula is

$$y = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \cdots$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-1	-21			
		18		
1	15		-7	
		-3		1
2	12		-3	
		-9		
3	3			

$$y = -21 + (x+1)\,18 + (x+1)(x-1)(-7) + (x+1)(x-1)(x-2)\,(1)$$

$$= -21 + 18x + 18 - 7(x^2-1) + (x^2-1)(x-2)$$

$$= -21 + 18x + 18 - 7x^2 + 7 + x^3 - 2x^2 - x + 2$$

$$y = x^3 - 9x^2 + 17x + 6$$

$$y' = 3x^2 - 18x + 17$$

$$y'(2) = -7.$$

② find $f'(10)$ from the following data

x	3	5	11	27	34
$f(x)$	-13	23	899	17315	35606

The newtons divided difference formula is

$$y = f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + \cdots$$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
3	-13				
		18			
5	23		16		
		146		1	
11	899		40		0
		1026		1	
27	17315		69		
		2613			
34	35606				

$$y = f(x) = -13 + 18(x-3) + 16(x-3)(x-5) + (x-3)(x-5)(x-11)$$

$$= -13 + 18x - 54 + 16[x^2 - 8x + 15] + (x^2 - 8x + 15)(x - 11)$$

$$= -13 + 18x - 54 + 16x^2 - 128x + 240 + x^3 - 11x^2 - 8x^2 + 88x + 15x - 165$$

$$f(x) = x^3 - 3x^2 - 7x + 8$$

$$f'(x) = 3x^2 - 6x - 7$$

$$f'(10) = 233.$$

Newton's forward formula for derivatives

$$y = f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \cdots \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y''' = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

① Find the first three derivatives of $f(x)$ at $x = 1.5$ & at $x = 4.0$ using Newton's forward interpolation formula to the data given below.

x	1.5	2	2.5	3	3.5	4
y	3.375	7	13.625	24	38.875	59

Soln

$$f'(x) = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f'''(x) = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{x-1.5}{0.5}$$

When $x = 1.5$ $\quad \boxed{u=0}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375	3.625	3	0.75	0	0
2	7	6.625	3.75	0.75	0	
2.5	13.625	10.375	4.5	0.75		
3	24	14.875	5.25			
3.5	38.875	20.125				
4	59					

$$f'(1.5) = \frac{1}{0.5}\left[3.625 + (0-1)\cdot\frac{3}{2} + \frac{2}{6}(0.75)\right]$$

$$= \frac{1}{0.5}\left[3.625 - 1.5 + 0.25\right]$$

$$= 4.75$$

$$f''(1.5) = \frac{1}{0.5^2}\left[3 + (-6)\times\frac{0.75}{6}\right]$$

$$= \frac{1}{0.5^2}\left[3 - 0.75\right] = 9$$

$$f'''(1.5) = \frac{1}{0.5^3}\left[0.75\right] = 6$$

Newton's Backward Interpolation formula

$$y' = \frac{1}{h}\left[\nabla y_n + \frac{(2v+1)}{2!}\nabla^2 y_n + \frac{(3v^2+6v+2)}{3!}\nabla^3 y_n + \frac{(4v^3+18v^2+22v+6)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\nabla^2 y_n + (6v+6)\frac{\nabla^3 y_n}{3!} + \frac{(12v^2+36v+22)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y''' = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{(24v+36)}{4!}\nabla^4 y_n + \cdots\right]$$

$$v = \frac{x - x_n}{h} = \frac{x-4}{0.5}$$

When $x = 4 \Rightarrow \boxed{v = 0}$

$$f''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$f'''(x) = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{(24u-36)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{x-1.5}{0.5}$$

When $x = 1.5$ $\quad \boxed{u=0}$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.5	3.375					
		3.625				
			3			
2	7			0.75		
		6.625			0	
2.5	13.625		3.75			0
		10.375		0.75		
3	24		4.5		0	
		14.875		0.75		
3.5	38.875		5.25			
		20.125				
4	59					

$$y' = \frac{1}{0.5}\left[20.125 + \frac{1}{2}\times 5.25 + \frac{2}{6}\times 0.75\right]$$
$$= 46$$
$$y'' = \frac{1}{0.5^2}\left[5.25 + 6\times\frac{0.75}{6}\right] = 24$$
$$y''' = \frac{1}{0.5^3}\left[0.75\right] = 6.$$

(2) For the given data, find the first two derivatives at $x = 1.1$

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Soln

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{x-1.0}{0.1}$$

At $x = 1.1$ $\quad u = \frac{1.1-1.0}{0.1} = 1.$

$y' = \frac{1}{0.1}[0.4$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1.0	7.989					
		0.4140				
			−0.0360			
1.1	8.403					
		0.3780		0.0060		
1.2	8.781		−0.03		−0.0020	
		0.3480		0.0040		0.001
1.3	9.129		−0.0260		−0.0010	
		0.3220		0.003		0.003
1.4	9.451				0.002	
			−0.0230			
		0.2990		0.0050		
1.5	9.750		−0.0180			Δ^6
		0.2810				
1.6	10.031					0.00

$$y'(1.1) = \frac{1}{0.1}\left[0.414 + \frac{(2-1)}{2}(-0.0360) + \frac{(3-6+2)}{6}(0.006) + \frac{(4-18+22-6)}{24}(-0.002)\right]$$

$$= \frac{1}{0.1}\left[0.414 - 0.0180 - 0.0010 - 0.0002\right]$$

$$= 3.9480$$

$$y''(1.1) = \frac{1}{(0.1)^2}\left[(-0.0360) + \frac{(6-6)}{6}(0.0060) + \frac{(12-36+22)}{24}(-0.0020)\right]$$

$$= 100\left[-0.0360 + 0\right] + \frac{(-2)}{24}(-0.0020)\,]$$

$= -36 + 0.00016$

$= \cancel{-35.9998}\ -3.584$

(3) find the first two derivatives of $x^{1/3}$ at $x=50$ and $x=56$ for the given data

x	50	51	52	53	54	55	56
$y=x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Soln

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
50	3.6840						
		0.0244					
51	3.7084		-0.0003				
		0.0241		0			
52	3.7325		-0.0003		0		
		0.0238		0		0	
53	3.7563		-0.0003		0		0
		0.0235		0		0	
54	3.7798		-0.0003		0		
		0.0232		0			
55	3.8030		-0.0003				
		0.0229					
56	3.8259						

Newtons forward formula:

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$= -36 + 0.00016$

$= \cancel{-35.9998}\ -3.584$

③ find the first two derivatives of $x^{1/3}$ at $x = 50$ and $x = 56$ for the given data

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

Soln

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
50	3.6840						
		0.0244					
51	3.7084		-0.0003				
		0.0241		0			
52	3.7325		-0.0003		0		
		0.0238		0		0	
53	3.7563		-0.0003		0		0
		0.0235		0		0	
54	3.7798		-0.0003		0		
		0.0232		0			
55	3.8030		-0.0003				
		0.0229					
56	3.8259						

Newton's forward formula:

$$y' = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2!}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{3!}\Delta^3 y_0 + \frac{(4u^3-18u^2+22u-6)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{(6u-6)}{3!}\Delta^3 y_0 + \frac{(12u^2-36u+22)}{4!}\Delta^4 y_0 + \cdots\right]$$

$$u = \frac{x-x_0}{h} = \frac{50-50}{1} = 0$$

$$y' = \frac{1}{1}\left[0.02414 + \frac{(-1)}{2}(-0.0003)\right]$$

$$= 0.0244 + 0.0002$$

$$= 0.0246$$

$$y'' = \frac{1}{1}\left[-0.0003\right] = -0.0003$$

Newton's Backward Interpolation formula.

$$y' = \frac{1}{h}\left[\nabla y_n + \frac{(2v+1)}{2!}\nabla^2 y_n + \frac{(3v^2+6v+2)}{3!}\nabla^3 y_n + \frac{(4v^3+18v^2+22v+6)}{4!}\nabla^4 y_n + \cdots\right]$$

$$y'' = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{(6v+6)}{3!}\nabla^3 y_n + \frac{(12v^2+36v+22)}{4!}\nabla^4 y_n + \cdots\right]$$

$$v = \frac{x-x_n}{h} = \frac{x-56}{0.5}$$

$$v = \frac{56-56}{0.5} = 0$$

$$y' = \frac{1}{0.5}\left[0.0299 + \frac{(0+1)}{2!}(-0.0003) + \frac{2}{3!}(0) + 0\right]$$

$$= \frac{1}{0.5}\left[0.0299 - \frac{0.0003}{2}\right]$$

$$= 0.0595$$

$$y'' = \frac{1}{0.5^2}\left[-0.0003\right] = -0.0012$$

Numerical Integration

Trapezoidal rule

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\left[(\text{Sum of first and last ordinate}) + 2(\text{Sum of remaining ordinates})\right]$$

$$h = \frac{b-a}{n}$$

Simpson's 1/3 rule

$$I = \int_a^b f(x)\,dx = \frac{h}{3}\left[(\text{first} + \text{Last}) + 4(\text{Sum of odd ordinates}) + 2(\text{Sum of even ordinates})\right]$$

$$h = \frac{b-a}{n} - [\text{multiples of } 2]$$

Simpson's 3/8 rule

$$I = \frac{3h}{8}\left[(\text{first Last}) + 2(\text{Sum of multiples of } 3) + 3(\text{Sum of non-multiples of } 3)\right]$$

$$h = \frac{b-a}{n} \quad [\text{multiples of } 3]$$

(1) Using Trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

Soln

$$h = \frac{b-a}{n} = \frac{1+1}{8} = \frac{2}{8} = 0.25$$

x	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y	0.5	0.65	0.8	0.9412	1	0.9412	0.8	0.64	0.5

$$I = \frac{h}{2}\left[(y_0 + y_8) + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$$

$$= \frac{0.25}{2}\left[(0.5 + 0.5) + 2(0.65 + 0.8 + 0.9412 + 1 + 0.9412 + 0.8 + 0.64)\right]$$

$$= \frac{0.25}{2}\left[1 + 2(5.7624)\right]$$

$$= \frac{0.25}{2}\left[12.5248\right]$$

$$= 1.5656$$

2) Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ with $h = 1/6$ by Trapezoidal rule.

Soln

$$f(x) = \frac{1}{1+x^2} \qquad h = 1/6$$

x	0	$1/6$	$2/6$	$3/6$	$4/6$	$5/6$	1
y	1	$36/37$	$9/10$	$4/5$	$9/13$	$36/61$	$1/2$

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{(1/6)}{2}\left[(1+1/2) + 2\left(\frac{36}{37}+\frac{9}{10}+\frac{4}{5}+\frac{9}{13}+\frac{36}{61}\right)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 2(3.9554)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 7.9108\right]$$

$$= 0.7842$$

(3) Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ by Trapezoidal rule. Also check up the results by actual Integration

Soln

$f(x) = \frac{1}{1+x^2}$, $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
y	1.00	0.500	0.200	0.100	0.058824	0.038426	0.27026

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027027) + 2(0.5+0.2+0.1+0.058824+0.038462)\right]$$

$$= 1.41079950$$

By actual Integration

$$I = \int_0^6 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_0^6 = \tan^{-1}6 - \tan^{-1}0$$
$$= 1.40564765$$

(4) Evaluate $\int_{1.0}^{1.3} \sqrt{x}\,dx$ taking $h = 0.05$ by trapezoidal rule

Soln

$$f(x) = \sqrt{x}$$

$$h = \frac{b-a}{n} = 0.05$$

x	1.0	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0724	1.0954	1.1180	1.1402

$$I = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{0.05}{2}\left[(1 + 1.402) + 2(1.0247 + 1.0488 + 1.0724 + 1.0954 + 1.1180)\right]$$

$$= 0.1\left[2.1402 + 2(5.3593)\right]$$

$$= 0.1\left[2.1402 + 10.7186\right]$$

$$= \cancel{0.1}\ 0.025\left(12.8588\right)$$

$$= \cancel{1.28588}\ 0.3214$$

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{(1/6)}{2}\left[(1+1/2) + 2\left(\frac{36}{37}+\frac{9}{10}+\frac{4}{5}+\frac{9}{13}+\frac{36}{61}\right)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 2(3.9554)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 7.9108\right]$$

$$= 0.7842$$

(3) Evaluate $\int_0^6 \frac{1}{1+x^2}\,dx$ by Trapezoidal rule. Also check up the results by actual integration.

Soln

$f(x) = \frac{1}{1+x^2}$, $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
y	1.00	0.500	0.200	0.100	0.058824	0.038426	0.27026

$$I = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027027) + 2(0.5+0.2+0.1+0.058824+0.038462)\right]$$

$$= 1.41079950$$

By actual Integration

$$I = \int_0^6 \frac{1}{1+x^2}\,dx = \left[\tan^{-1}x\right]_0^6 = \tan^{-1}6 - \tan^{-1}0$$

$$= 1.40564765$$

(4) Evaluate $\int_{1.0}^{1.3} \sqrt{x}\,dx$ taking $h = 0.05$ by trapezoidal rule

Soln

$f(x) = \sqrt{x}$

$h = \frac{b-a}{n} = 0.05$

x	1.0	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0724	1.0954	1.1180	1.1402

$$I = \frac{h}{2}\left[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)\right]$$

$$= \frac{0.05}{2}\left[(1 + 1.402) + 2(1.0247 + 1.0488 + 1.0724 + 1.0954 + 1.1180)\right]$$

$$= 0.1\left[2.1402 + 2(5.3593)\right]$$

$$= 0.1\left[2.1402 + 10.7186\right]$$

$$= \cancel{0.1}\ 0.025\,(12.8588)$$

$$= \cancel{1.28588}\ 0.3214$$

(5) Dividing the range into 10 equal parts find the value of $\int_0^{\pi/2} \sin x \, dx$ by Simpson's $1/3$ rule.

Soln

$f(x) = \sin x \qquad h = \frac{b-a}{n} = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$

x	0	$\pi/20$	$2\pi/20$	$3\pi/20$	$4\pi/20$	$5\pi/20$	$6\pi/20$	$7\pi/20$	$8\pi/20$
$f(x)$	0	0.1564	0.3090	0.4540	0.5878	0.7071	0.8090	0.8910	0.9511

$$I = \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{\pi/20}{3}\left[(0+1) + 4(0.1564 + 0.4540 + 0.7071 + 0.8910) + 2(0.3090 + 0.5878 + 0.8090)\right]$$

$$= \frac{\pi}{60} * 19.0986 = 1$$

(6) The velocity v of a particle at a distance s from a point on its path is gn by the table below.

s	0	10	20	30	40	50	60
v	47	58	64	65	61	52	38

Estimate the time taken to travel 60 meters by Simpson's $1/3$ rule.

Soln

$$\text{Velocity} = \frac{\text{distance}}{\text{time}}$$

$$v = \frac{ds}{dt}$$

$$dt = \frac{1}{v}\,ds$$

$$t = \int_0^{60} \frac{1}{v}\,ds \;,\; h = 10$$

$$I = \int_0^{60} \frac{1}{v}\,ds = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

v	47	58	64	65	61	52	38
$1/v$	0.02127	0.01724	0.015625	0.01538	0.01639	0.01923	0.026316

$$I = \frac{10}{3}\left[(0.02127 + 0.026316) + 4(0.07124 + 0.01538 + 0.01923) + 2(0.015625 + 0.01639)\right]$$

$$I = 1.06338$$

(7) Compute $\int_0^{\pi/2} \sin x \, dx$ using Simpson's $3/8^{th}$ rule of numerical integration.

Soln

$$I = \int_0^{\pi/2} \sin x \, dx$$

$f(x) = \sin x \qquad h = \frac{\pi/2 - 0}{9} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$	$7\pi/18$	$8\pi/18$	$9\pi/18$
$f(x)$	0	0.1736	0.3420	0.50	0.6428	0.7660	0.8660	0.9397	0.9848	1

$$I = \frac{3h}{8}\left[(y_0 + y_9) + 3(y_1 + y_2 + y_4 + y_5 + y_7 + y_8) + 2(y_3 + y_6)\right]$$

$$= \frac{3\pi}{8 \times 18}\left[(0+1) + 3(0.1736 + 0.3428 + 0.6428 + 0.7660 + 0.9397 + 0.9848) + 2(0.5 + 0.8660)\right]$$

$I = 0.99998576$

$I \sim 1$

(7) The velocities of a car running on a straigh road at intervals of 2 minutes are gn below

Time (min)	0	2	4	6	8	10	12
Velocity (km/hr)	0	22	30	27	18	7	0

using simpsons 1/3 rule find the distance covered by the car.

Soln

$$\text{Velocity} = \frac{dx}{dt} \quad (\text{ie}) \quad v = \frac{dx}{dt}$$

$$dx = v\,dt$$

$$x = \int v\,dt$$

t	0	2	4	6	8	10	12
v	0	$\frac{22}{60}$	$\frac{30}{60}$	$\frac{27}{60}$	$\frac{18}{60}$	$\frac{7}{60}$	0

$$I = \frac{h}{3}\left[(y_0 + y_6) + 2(y_2 + y_4) + 4(y_1 + y_3 + y_5)\right]$$

$$= \frac{2}{3}\left[0 + 0 + 2\left(\frac{30}{60} + \frac{18}{60}\right) + 4\left(\frac{22}{60} + \frac{27}{60} + \frac{7}{60}\right)\right]$$

$$= 3.5556 \text{ Km}$$

Romberg Method

$$I = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

I_1 — Value of integral with $h = \frac{b-a}{2}$

I_2 — Value of integral with $h = \frac{b-a}{4}$

I_3 — " " " $h = \frac{b-a}{8}$.

① Compute $I = \int_0^{1/2} \frac{x}{\sin x}\,dx$, using Simpsons rule with $h = 1/4,\ 1/8,\ 1/16$ and then Rombergs Method.

Soln

$$I = \int_0^{1/2} \frac{x}{\sin x}\,dx$$

$$f(x) = \frac{x}{\sin x}$$

i) Take $h = 1/4$

x	0	$1/4$	$1/2$
$f(x)$	y_0 1	y_1 1.0105	y_2 1.0429

By Simpsons 1/3 rule,

$$I_1 = \frac{h}{3}\left[(y_0 + y_2) + 4(y_1) + 0\right]$$

$$= \frac{1}{12}\left[(1 + 1.0429) + 4(1.0105)\right]$$

$$I_1 = 0.507075$$

(ii) Take $h = \frac{1}{8}$

x	0	1/8	2/8	3/8	4/8
$f(x)$	1 (y_0)	1.0026 (y_1)	1.0105 (y_2)	1.0238 (y_3)	1.0429 (y_4)

$$I_2 = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$$

$$= \frac{1}{24}\left[(1+1.0429) + 4(1.0026+1.0236) + 2(1.0105)\right]$$

$$I_2 = 0.5070625$$

(iii) Take $h = \frac{1}{16}$

x	0	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{5}{16}$	$\frac{6}{16}$	$\frac{7}{16}$	$\frac{8}{16}$
$f(x)$	1 (y_0)	1.0007 (y_1)	1.0026 (y_2)	1.0059 (y_3)	1.0105 (y_4)	1.0165 (y_5)	1.0238 (y_6)	1.0326 (y_7)	1.0429 (y_8)

$$I_3 = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{1}{48}\left[(1+1.0429) + 4(1.0007+1.0059+1.0165+1.0326) + 2(1.0026+1.0105+1.0238)\right]$$

$$I_3 = 0.5070729$$

for I_1, I_2

Romberg formula is

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

$$= 0.5070625 + \left(\frac{0.5070625 - 0.507075}{3}\right)$$

$$I = 0.507058$$

for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3}\right)$$

$$= 0.5070729 + \left(\frac{0.5070729 - 0.5070625}{3}\right)$$

$$= 0.50707636 6.$$

Romberg for I_4 & I_5

$$I = I_5 + \left(\frac{I_5 - I_4}{3}\right)$$

(3) Evaluate $I = \int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method. Hence deduce an approximate value of π

Soln

$a = 0 \; ; \; b = 1$

$f(x) = \frac{1}{1+x^2}$

I $h = \frac{b-a}{2} = \frac{1-0}{2} = 0.5$

x	0	0.5	1
$f(x)$	1	0.8	0.5

$$I_1 = \frac{h}{2}\left[(y_0 + y_2) + 2(y_1)\right]$$

$$= \frac{0.5}{2}\left[(1+0.5) + 2 \times 0.8\right]$$

$$I_1 = 0.7750$$

II $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

(3) Evaluate $I = \int_0^1 \frac{dx}{1+x^2}$ by using Romberg's method. Hence deduce an approximate value of π

Soln

$a = 0$; $b = 1$

$f(x) = \frac{1}{1+x^2}$

I $h = \frac{b-a}{2} = \frac{1-0}{2} = 0.5$

x	0	0.5	1
$f(x)$	1	0.8	0.5

$I_1 = \frac{h}{2} [(y_0 + y_2) + 2(y_1)]$

$= \frac{0.5}{2} [(1 + 0.5) + 2 \times 0.8]$

$I_1 = 0.7750$

II $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.9412	0.8	0.64	0.5

$$I_2 = \frac{0.25}{2}\left[(1+0.5) + 2(0.9412 + 0.8 + 0.64)\right]$$

$$\boxed{I_2 = 0.7828}$$

(iii) $h = \frac{b-a}{8} = \frac{1-0}{8} = 0.125$

x	0	0.12	0.25	0.375	0.5
$f(x)$	1	0.9846	0.9412	0.8767	0.8

0.625	0.75	0.875	1
0.7191	0.64	0.5664	0.5

$$I_3 = \frac{0.5}{2}\left[(1+0.5) + 2(0.9846 + 0.9412 + 0.8767 + 0.8 + 0.7191 + 0.64 + 0.5664)\right]$$

$$\boxed{I_3 = 0.7848}$$

Romberg for I_1, I_2

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right) = 0.7854$$

Romberg for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3}\right) = 0.7855$$

Romberg for I_4, I_5

$$I = I_5 + \left(\frac{I_5 - I_4}{3}\right) = 0.7855$$

$$I = \int_0^1 \frac{dx}{1+x^2}$$

$$0.7855 = \left[\tan^{-1} x\right]_0^1$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$\frac{\pi}{4} = 0.7855$$

$$\pi = 3.1420.$$

(3) Using Romberg Integration, evaluate

$$\int_0^1 \frac{dx}{1+x}$$

Soln

I) Here $a=0$, $b=1$

x	0	0.5	1
$\frac{1}{1+x}$	1	0.6667	0.5

$$I_1 = \frac{h}{2}\left[(y_0 + y_2) + 2(y_1)\right]$$

$$= \frac{0.5}{2}\left[(1+0.5) + 2(0.6667)\right]$$

$$\boxed{I_1 = 0.7084}$$

II) $h = \frac{b-a}{4} = \frac{1-0}{4} = 0.25$

x	0	0.25	0.5	0.75	1
$f(x)$	1	0.8	0.6667	0.5714	0.5

$$I_2 = \frac{0.25}{2}\left[(1+0.5) + 2(0.8 + 0.6667 + 0.5714)\right]$$

$$\boxed{I_2 = 0.6970}$$

III) $h = \frac{b-a}{8} = \frac{1-0}{8} = 0.125$

x	0	0.125	0.25	0.375	0.5
$f(x)$	1	0.8889	0.8	0.7273	0.6667
		0.625	0.75	0.875	1
		0.6154	0.5714	0.5333	0.5

$$I_3 = \frac{0.125}{2}\left[(1+0.5) + 2(0.8889 + 0.8 + 0.7273 + 0.6667 + 0.6154 + 0.5714 + 0.5333)\right]$$

$$\boxed{I_3 = 0.6941.}$$

Romberg for I_1, I_2

$$I_4 = I_2 + \left(\frac{I_2 - I_1}{3}\right)$$

$$= 0.6970 + \left(\frac{0.6970 - 0.7084}{3}\right)$$

$$\boxed{I_4 = 0.6932}$$

Romberg for I_2, I_3

$$I_5 = I_3 + \left(\frac{I_3 - I_2}{3}\right)$$

$$= 0.6941 + \left(\frac{0.6941 - 0.6970}{3}\right).$$

$$\boxed{I_5 = 0.6931}$$

Romberg for I_4, I_5

$$I_6 = I_5 + \left(\frac{I_5 - I_4}{3}\right)$$

$$\boxed{I_6 = 0.6931}$$

Gauss Quadrature formula

Quadrature:

The process of finding a definite integral from a tabulated values of a function is known as Quadrature.

Gaussian two point Quadrature formula

Let $I = \int_a^b f(x)\,dx$.

Take $x = \left(\frac{a+b}{2}\right) + \left(\frac{b-a}{2}\right)t$

$$dx = \left(\frac{b-a}{2}\right)dt.$$

By using this transformation

$$I = \int_{-1}^{1} g(t)\, dt = g\left(\frac{-1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

① Evaluate $\int_{-1}^{1} e^{-x^2} \cos x \, dx$ by Gauss two Point Quadrature formula.

Soln

$$I = \int_{-1}^{1} e^{-x^2} \cos x \, dx$$

$$f(x) = e^{-x^2} \cos x$$

$$I = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-\left(\frac{1}{\sqrt{3}}\right)^2} \cos\left(\frac{-1}{\sqrt{3}}\right) + e^{-\left(\frac{1}{\sqrt{3}}\right)^2} \cos\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-1/3} \cos\left(\frac{-1}{\sqrt{3}}\right) + e^{-1/3} \cos\left(\frac{1}{\sqrt{3}}\right)$$

$$= e^{-1/3} \left[\cos(-1/\sqrt{3}) + \cos(1/\sqrt{3}) \right]$$

$$I = 1.2008.$$

② Apply Gauss two point formula to evaluate $\int_{-1}^{1} \frac{1}{1+x^2}\, dx$

Soln

$$I = \int_{-1}^{1} \frac{1}{1+x^2}\, dx$$

$$f(x) = \frac{1}{1+x^2}$$

$$I = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{1+\left(\frac{-1}{\sqrt{3}}\right)^2} + \frac{1}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{3}{4} + \frac{3}{4}$$

$$= \frac{6}{4}$$

$$= 1.5$$

③ Evaluate the integral $I = \int_1^2 \frac{2x}{1+x^4}\,dx$ using Gaussian two point formula

Soln

$$I = \int_1^2 \frac{2x}{1+x^4}\,dx$$

$$f(x) = \frac{2x}{1+x^4}, \quad a = 1, \quad b = 2$$

$$x = \frac{a+b}{2} + \left(\frac{b-a}{2}\right)t$$

$$x = \frac{3}{2} + \frac{1}{2}t$$

$$dx = \tfrac{1}{2}\,dt$$

$$I = \int_{-1}^{1} \frac{\cancel{2}\left(\frac{3}{2}+\frac{1}{2}t\right)}{1+\left(\frac{3}{2}+\frac{1}{2}t\right)^4} \cdot \frac{dt}{\cancel{2}}$$

$$= \int_{-1}^{1} \frac{\left(\frac{3+t}{2}\right)}{1+\left(\frac{3+t}{2}\right)^4}\, dt$$

$$g(t) = \frac{\frac{3+t}{2}}{1+\left(\frac{3+t}{2}\right)^4}$$

$$I = g\left(\frac{-1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)$$

$$= \frac{\frac{3-\frac{1}{\sqrt{3}}}{2}}{1-\left(\frac{3-\frac{1}{\sqrt{3}}}{2}\right)^4} + \frac{\frac{3+\frac{1}{\sqrt{3}}}{2}}{1+\left(\frac{3+\frac{1}{\sqrt{3}}}{2}\right)^4}$$

$$= \frac{1.2113}{3.1530} + \frac{1.7887}{11.2359}$$

$$= 0.3842 + 0.1592$$

$$= 0.5434.$$

$$= \frac{\pi}{4}\,[0.3259 + 0.9454]$$

$$= 0.9985$$

Gaussian Three Point Quadrature formula:

$$I = \int_a^b f(x)\,dx$$

Take $x = \left(\frac{a+b}{2}\right) + \left(\frac{b-a}{2}\right)t$

$$dx = \left(\frac{b-a}{2}\right)dt$$

$$I = \int_{-1}^{1} g(t)\,dt = \frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9}g(0)$$

① Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using 3 point Quadrature formula.

Soln

$$I = \int_0^1 \frac{dx}{1+x^2}$$

$$f(x) = \frac{1}{1+x^2}, \quad a = 0, \quad b = 1$$

Take $x = \left(\frac{a+b}{2}\right) + \left(\frac{b-a}{2}\right)t$

$$dx = \left(\frac{b-a}{2}\right) dt$$

$$\Rightarrow x = \frac{1}{2} + \frac{1}{2}t$$

$$dx = \frac{1}{2} dt$$

$$I = \int_{-1}^{1} \frac{\frac{1}{2} dt}{1 + \left(\frac{1+t}{2}\right)^2} = \frac{1}{2}\int_{-1}^{1} \frac{dt}{1 + \left(\frac{1+t}{2}\right)^2}$$

$$\therefore \quad g(t) = \frac{1}{1 + \left(\frac{1+t}{2}\right)^2}$$

$$I = \frac{1}{2}\left[\frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9} g(0)\right]$$

$$= \frac{1}{2}\left[\frac{5}{9}\left(\frac{1}{1+\left(\frac{1+(-\sqrt{3/5})}{2}\right)^2} + \frac{1}{1+\left(\frac{1+(\sqrt{\frac{3}{5}})}{2}\right)^2}\right) + \frac{8}{9}\left[\frac{1}{1+(\frac{1}{2})^2}\right]\right]$$

$$= \frac{1}{2}\left[\frac{5}{9}\left(0.9875 + 0.5595 + 0.7111\right]\right.$$

$$= 0.7853.$$

(2) Apply three point Gaussian Quadrature formula to evaluate $\int_0^1 \frac{\sin x}{x}\, dx$

Soln

$$I = \int_0^1 \frac{\sin x}{x}\, dx$$

$$f(x) = \frac{\sin x}{x}, \quad a = 0, \quad b = 1$$

$$x = \left(\frac{b+a}{2}\right) + \left(\frac{b-a}{2}\right) t$$

$$dx = \left(\frac{b-a}{2}\right) dt$$

$$\Rightarrow x = \tfrac{1}{2} + \tfrac{1}{2} t = \tfrac{1}{2}(1+t)$$

$$dx = \tfrac{1}{2}\, dt$$

$$I = \int_{-1}^{1} \frac{\sin \frac{1}{2}(1+t)}{\cancel{\frac{1}{2}}(1+t)} \cdot \cancel{\tfrac{1}{2}}\, dt$$

$$= \int_{-1}^{1} \frac{\sin \frac{1}{2}(1+t)}{(1+t)}\, dt$$

$$\therefore g(t) = \frac{\sin \frac{1+t}{2}}{1+t}$$

$$g(0) = \sin \tfrac{1}{2} = 0.47943$$

$$g\left(\sqrt{\tfrac{3}{5}}\right) = \sin\left[\frac{\sqrt{3/5}+1}{2}\right] \Big/ \sqrt{3/5}+1 = \frac{0.7754}{1.7746} = 0.437$$

$$g\left(-\sqrt{\tfrac{3}{5}}\right) = \frac{\sin\left[\frac{-\sqrt{\frac{3}{5}}+1}{2}\right]}{-\sqrt{\frac{3}{5}}+1} = \frac{0.1125}{0.2254} = 0.499$$

$$I = \frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9}g(0)$$

$$= \frac{5}{9}\left[0.499 + 0.437\right) + \frac{8}{9}(0.47943)$$

$$= 0.52 + 0.42616$$

$$= 0.94616.$$

③ Evaluate $\int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}\,dx$ by Gaussian three Point formula.

Soln

$$I = \int_0^2 \frac{x^2+2x+1}{1+(x+1)^4}$$

$$f(x) = \frac{x^2+2x+1}{1+(x+1)^4}, \quad a=0, \quad b=2$$

$$x = \frac{b+a}{2} + \left(\frac{b-a}{2}\right)t$$

$$dx = \left(\frac{b-a}{2}\right)dt$$

$\Rightarrow \quad x = 1 + t$

$dx = dt$

$$I = \int_{-1}^{1} \frac{(z+1)^2 + 2(z+1) + 1}{1 + [(z+1)+1]^4}\, dz$$

$$g(t) = \frac{(z+1)^2 + 2(z+1) + 1}{1 + [(z+1)+1]^4}$$

$$= \frac{z^2 + 2z + 1 + 2z + 2 + 1}{1 + (z+2)^4}$$

$$g(t) = \frac{(z+2)^2}{1 + (z+2)^4}$$

$$g(0) = \frac{4}{17}$$

$$g\left(-\sqrt{\tfrac{3}{5}}\right) = \frac{\left(-\sqrt{\tfrac{3}{5}} + 2\right)^2}{1 + \left(-\sqrt{\tfrac{3}{5}} + 2\right)^4} = \frac{1.50161}{3.2548} = 0.4614$$

$$g\left(\sqrt{\tfrac{3}{5}}\right) = \frac{\left(\sqrt{\tfrac{3}{5}} + 2\right)^2}{1 + \left(\sqrt{\tfrac{3}{5}} + 2\right)^4} = \frac{7.69839}{60.2652} = 0.12774$$

$$I = \frac{5}{9}\left[g\left(-\sqrt{\tfrac{3}{5}}\right) + g\left(\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9}\, g(0)$$

$$= \frac{5}{9}[0.4614 + 0.12774] + \frac{8}{9}\left(\frac{4}{17}\right)$$

$$= \frac{5}{9}[0.4614$$

$$= 0.5364$$

Double Integration

Trapezoidal rule:

$$I = \int_c^d \int_a^b f(x,y)\, dx\, dy$$

$$I = \frac{hk}{4}\Big[\text{Sum of four corners} + 2(\text{Sum of remaining boundary values}) + 4(\text{Sum of interior values})\Big]$$

Simpson's rule

$$I = \frac{hk}{9}\Big[\text{Sum of four corners} + \underbrace{2(\text{Sum of odd position values}) + 4(\text{Sum of even position values})}_{\text{Boundary}}$$

$$+ \underbrace{4(\text{Sum of odd position values}) + 8(\text{Sum of even position values})}_{\text{odd rows}}$$

$$+ \underbrace{8(\text{Sum of odd position values}) + 16(\text{Sum of even position values})}_{\text{even rows}}\Big]$$

$\cancel{I} = \frac{hk}{\cancel{4}}\Big\{$ ~~Sum of four corn~~

$$I = \frac{0.1 \times 0.1}{4}\left[0.5 + 0.4167 + 0.4545 + 0.3846 + 2(0.4762 + 0.4545 + 0.4348 + 0.4762 + 0.4 + 0.4348 + 0.4167 + 0.4) + 4(0.4545 + 0.4348 + 0.4167)\right]$$

$$= \frac{0.1 \times 0.1}{4}\left[1.7558 + 6.9864 + 5.2240\right]$$

$$= \frac{0.1 \times 0.1}{4} \times 13.9662 = 0.0349$$

(2) Evaluate $\int_1^2 \int_1^2 \frac{1}{x^2+y^2}\, dx\, dy$, numerically using $h = 0.2$, along x-direction and $k = 0.25$ along y-direction.

Soln

$$I = \int_1^2 \int_1^2 \frac{1}{x^2+y^2}\, dx\, dy$$

$$f(x, y) = \frac{1}{x^2+y^2}$$

By Trapezoidal

$$I = \frac{hk}{4}\left[\text{Sum of four corners} + 2(\text{Sum of remaining boundary}) + 4(\text{Sum of interiors})\right]$$

y \ x	1	1.2	1.4	1.6	1.8	2
1	0.5	0.4098	0.3378	0.2809	0.2359	0.2
1.25	0.3902	0.3331	0.2839	0.2426	0.2082	0.1798
1.5	0.3017	0.2710	0.2375	0.2079	0.1821	0.16
1.75	0.2462	0.2221	0.1991	0.1779	0.1587	0.1416
2	0.2	0.1838	0.1679	0.1524	0.1381	0.125

$$I = \frac{(0.2)(0.25)}{4}\Big[0.5 + 0.2 + 0.2 + 0.125$$
$$+ 2(0.4098 + 0.3378 + 0.2809 + 0.2359$$
$$+ 0.1798 + 0.16 + 0.1416$$
$$+ 0.1381 + 0.1524 + 0.1679 + 0.1838$$
$$+ 0.2462 + 0.2710 + 0.3331)$$
$$+ 4(0.3331 + 0.2839 + 0.2426$$
$$+ 0.2082 + 0.2710 + 0.2375$$
$$+ 0.2079 + 0.1821 + 0.2221$$
$$+ 0.1991 + 0.1779 + 0.1587)\Big]$$

$$= \frac{(0.2)(0.25)}{4}\Big[1.025 + 6.6642 + 10.8964)\Big]$$

$$= 0.2323.$$

3. Evaluate $I = \int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy}\, dx\, dy$ using Simpson's rule with $h = k = 1/4$

Soln

$$I = \int_0^{1/2}\int_0^{1/2} \frac{\sin(xy)}{1+xy}\, dx\, dy$$

$$f(x,y) = \frac{\sin xy}{1+xy}$$

By Simpsons $\frac{1}{3}$ rule,

$I = \frac{hk}{9}$ [Sum of four corners + 2 ($\underbrace{\text{Sum of odd position}}_{\text{Boundary}}$) + 4 (S E P) + 4 ($\underbrace{\text{SOP}) + 8(\text{SEP}}_{\text{odd rows}}$) + 8 ($\underbrace{\text{SOP}) + 16(\text{SEP}}_{\text{even rows}}$)]

	y \ x	0	1/4	1/2
I	0	0	0	0
II	1/4	0	0.0588	0.1108
III	1/2	0	0.1108	0.1979

$$I = \frac{0.1 \times 0.1}{9} \Big[0.5 + 0.4167 + 0.3571 + 0.2976$$
$$+ 2[0.4545 + 0.4167 + 0.3267 + 0.3472]$$
$$+ 4[0.4762 + 0.4348 + 0.3788 + 0.3205$$
$$+ 0.3401 + 0.3106 + 0.4545 + 0.3846]$$
$$+ 4(0.3788) + 8(0.3968 + 0.3623)$$
$$+ 8(0.4132 + 0.3497)$$
$$+ 16(0.4329 + 0.3953$$
$$+ 0.3663 + 0.3344) \Big]$$

$$= \frac{0.1 \times 0.1}{9} \Big[1.5714 + 3.0862 + 12.4004$$
$$+ 1.5152 + 6.0728 + 6.1032$$
$$+ 24.4624 \Big]$$

$I = 0.0613$

5 Evaluate $\int_0^2 \int_0^1 4xy \, dx \, dy$ using Simpsons rule by taking $h = 1/4$ & $k = 1/2$

Soln

$$I = \int_0^2 \int_0^1 4xy \, dx \, dy$$

Here $f(x, y) = 4xy$

$h = 0.25 \quad k = 0.5$

y \ x	0	0.25	0.5	0.75	1
0	0	0	0	0	0
0.5	0	0.5	1	1.5	2
1	0	1	2	3	4
1.5	0	1.5	3	4.5	6
2	0	2	4	6	8

$$I = \frac{0.25 \times 0.5}{9}\left[8 + 16 + 64 + 8 + 32 + 32 + 128\right]$$

$I = 4.$

Questions	opt1	opt2	opt3	opt4	opt7
The process of computing the value of the function inside the given range is called ________	Interpolation	extrapolation	reduction	expansion	Interpolation
If the point lies inside the domain [x_0, x_n], then the estimation of f(y) is called ________	Interpolation	extrapolation	reduction	expansion	Interpolation
The process of computing the value of the function outside the given range is called ________	Interpolation	extrapolation	reduction	expansion	extrapolation
If the point lies outside the domain [x_0, x_n], then the estimation of f(y) is called ________	Interpolation	extrapolation	reduction	expansion	extrapolation
Δ is called __________ difference operator.	forward	backward	central	none	forward
∇ is called __________ difference operator.	forward	backward	central	none	backward
In the forward difference table y_0 is called __________ element.	leading	ending	middle	positive	leading
In the forward difference table $\Delta y_0, \Delta^2 y_0, ...$ are called __________ difference.	leading	ending	middle	positive	leading
The difference of first forward difference is called ________.	divided difference	2nd forward difference	3rd forward difference	4th forward difference	2nd forward difference
Gregory – Newton forward interpolation formula is also called as Gregory – Newton forward_____________ formula.	Elimination	iteration	difference	distance	difference
Gregory – Newton backward interpolation formula is also called as Gregory – Newton backward_____________ formula	Elimination	iteration	difference	distance	difference
Gregory – Newton backward interpolation formula is also called as Gregory – Newton backward _____________ formula .	Elimination	iteration	difference	distance	difference
The divided differences are _________ in their arguments.	constant	symmetrical	varies	singular	symmetrical
In Gregory – Newton forward interpolation formula 1st two terms of this series give the result for the ________ interpolation.	Ordinary linear	ordinary differential	parabolic	central	Ordinary linear
Gregory – Newton forward interpolation formula 1st three terms of this series give the result for the ________ interpolation.	Ordinary linear	ordinary differential	parabolic	central	parabolic
Gregory – Newton forward interpolation formula is mainly used for interpolating the values of y near the _____________ of the set of tabular values.	beginning	end	centre	side	beginning
Gregory – Newton backward interpolation formula is mainly used for interpolating the values of y near the _______of the set of tabular values.	beginning	end	centre	side	end
From the definition of divided difference (u-u_0)/(x-x_0) we have __________ =	(y, y_0)	(x, y)	(x_0, y_0)	(x, x_0)	(x_0, y_0)
If f(x) =0, then the equation is called __________	Homogenous	non-homogenous	first order	second order	Homogenous
The order of y_(x+3) – 5 y_(x+2) + 7y_(x+1)+y_x= 10x is	2	0	1	3	3
A function which satisfies the difference equation is a ________of the difference equation.	Solution	general solution	complementary solution	particular solution	Solution
The degree of the difference equation is ________	The highest powers of y's	erence between the argum	The difference between the constant	The highest value of x	The highest powers of y's
The degree of (E2-5E +6)yx =e^x is	2	0	1	3	1
The order of y (x+3) – y (x+2) = 5x^2 is	3	2	1	0	1
The difference between the highest and lowest subscripts of y are called ______ of the difference equation	degree	order	power	value	order
E-1=	∇	Δ	μ	δ	Δ
Which of the following is the central difference operator?	∇	Δ	μ	δ	δ
1+Δ=	∇	E	μ	δ	E
μ is called the ______ operator	Central	average	backward	displacement	average
The other name of shifting operator is ______ operator	Central	average	backward	displacement	displacement
The difference of constant functions are___________	0	1	2	3	0
The nth order divided difference of xn will be a polynomial of degree ________.	0	1	2	3	2
The operator Δ is __________	homogenous	heterogeneous	linear	a variable	linear

Unit - IV

Initial Value Problem for ordinary differential Equation

Method - 1

Taylor Series:

The taylor series formula is

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + \cdots$$

1. Use taylor series method to find $y(0.1)$ and $y(0.2)$. Given that $\frac{dy}{dx} = 3e^x + 2y$; $y(0) = 0$;

Soln: Given $\frac{dy}{dx} = y' = 3e^x + 2y$; $y(0) = 0$;

The taylor series formula is,

$$y = y_0 + (x - x_0)\frac{y_0'}{1!} + (x - x_0)^2 \frac{y_0''}{2!} + (x - x_0)^3 \frac{y_0'''}{3!} + \frac{(x - x_0)^4}{}$$

x	0	x_0
y	0	y_0
$y' = 3e^x + 2y$	3	y_0'
$y'' = 3e^x + 2y'$	9	y_0''
$y''' = 3e^x + 2y''$	21	y_0'''
$y^{iv} = 3e^x + 2y'''$	45	y_0^{iv}

$$y = 0 + (x - 0)\cdot \frac{3}{1} + (x - 0)^2 \cdot \frac{9}{2} + (x - 0)^3 \frac{21}{6} + (x - 0)^4 \cdot \frac{45}{24};$$

$$y = 3x + \frac{9}{2}x^2 + \frac{7}{2}x^3 + \frac{15}{8}x^4.$$

$y(0.1) = 0.3487$.

$y(0.2) = 0.8110$.

2. use taylor series method, solve $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$ at $x = 0.1, 0.2, 0.3$.

Soln:

The taylor series formula is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!} + (x-x_0)^4\frac{y_0^{iv}}{4!}$$

$y' = x^2 - y$; $y(0) = 1$

x	0	x_0
y	1	y_0
$y' = x^2 - y$	-1	y_0'
$y'' = 2x - y'$	1	y_0''
$y''' = 2 - y''$	1	y_0'''
$y^{iv} = -y'''$	-1	y_0^{iv}

$$y = 1 + (x-0)\left(\frac{-1}{1!}\right) + (x-0)^2\frac{1}{2!} + (x-0)^3\times\frac{1}{3!} + (x-0)^4\left(\frac{-1}{4!}\right)$$

$$y = 1 - x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24}.$$

$y(0.1) = \cancel{0.9102}\ 0.9052$

$= \cancel{7/6\, x^4 + 4/3\, x^3}$

$y = 7/6\, x^4 + 4/3\, x^3 + x^2 + x + 1$

$y(0.1) = 1.1115$

$y(0.2) = 1.2525$

4. Obtain y by taylor series method given that $y' = xy + 1$; $y(0) = 1$; for $x = 0.1$; $x = 0.2$; Correct to four decimal places.

Soln: The formula is,

$$y = y_0 + (x - x_0)\frac{y_0'}{1!} + (x - x_0)^2 \frac{y_0''}{2!} + (x - x_0)^3 \frac{y_0'''}{3!} + (x - x_0)^4 \frac{y_0^{iv}}{4!} + \cdots$$

x	0	x_0
y	1	y_0
$y' = xy + 1$	1	y_0'
$y'' = y + xy'$	1	y_0''
$y''' = y' + y' + xy''$	2	y_0'''
$y^{iv} = y'' + y'' + y'' + xy'''$	3	y_0^{iv}

$$y = 1 + (x - 0)\frac{1}{1!} + (x - 0)^2 \frac{1}{2} + (x - 0)^3 \frac{2}{6} +$$

$$(x - 0)^4 \frac{3}{}$$

$$y = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{8}x^4.$$

$$y(0.1) = 1.1053$$

$$y(0.2) = 1.2229.$$

5. G.T $y'' + xy' + y = 0$; $y(0) = 1$; $y'(0) = 0$ obtain the value of y for $x = 0.1$ & $x = 0.2$; 0.3 by taylor series method.

Soln :

The taylor series formula is,

$$y = y_0 + (x - x_0)\frac{y_0'}{1!} + (x - x_0)^2\frac{y_0''}{2!} + (x - x_0)^3\frac{y_0'''}{3!} + (x - x_0)^4\frac{y_0^{iv}}{4!} + \cdots$$

x	0	x_0
y	1	y_0
y'	0	y_0'
$y'' = -xy' - y$	-1	y_0''
$y''' = -xy'' - y' - y'$	0	y_0'''
$y^{iv} = -xy''' - y'' - y'' - y''$	$+3$	y_0^{iv}

$$y = 1 + (x-0)\frac{0}{1!} + (x-0)^2\frac{1}{2} + (x-0)^3\frac{0}{6} + (x-0)^4\frac{+3}{24}$$

$$y = 1 - x^2/2 + x^4/8$$

$y(0.1) = 0.9950$

$y(0.2) = 0.9802$.

$y(0.3) = 0.9560$.

Method - II : Euler's method :

Consider $\frac{dy}{dx} = f(x, y)$

The Euler's formula is,

$$y_{n+1} = y_n + h f(x_n, y_n) \text{ (or)}$$

$$y_{n+1} = y_n + h y_n'$$

1. Solve $y' = \frac{y-x}{y+x}$, $y(0) = 1$ at $x = 0.1$ by taking $h = 0.02$; by using Euler's method.

Soln:

$$y' = \frac{y-x}{y+x} \; ; \; y(0) = 1$$

The Euler's formula is,

$$y_{n+1} = y_n + h f(x_n, y_n)$$

(or)

$$y_{n+1} = y_n + h \cdot y_n'$$

x	0	0.02	0.04	0.06	0.08	0.
y	1	1.02	1.0392	1.0577	1.0756	1.
$y' = \frac{y-x}{y+x}$	1	0.9615	0.9259	0.8926	0.8615	0

$n=0$;

$$y_1 = y_0 + hy_0' = 1 + 0.02 \times 1 = 1.02$$

$n=1$;

$$y_2 = y_1 + hy_1' = 1.02 + 0.02 \times 0.9615 = 1.0392$$

$n=2$;

$$y_3 = y_2 + h \cdot y_2' = 1.0892 + 0.02 \times 0.9259 = 1.0577$$

$n=3$;

$$y_4 = y_3 + hy_3' = \cancel{0.9} \; 1.0577 + 0.02 \times 0.8926$$

$$y_4 = 1.0756$$

$n=4$;

$$y_5 = y_4 + hy_4' = 1.0756 + 0.02 \times 0.8615$$

$$y_5 = 1.0928$$

~~n=5;~~

$$\cancel{y_6 = y_5 + hy_5' = 1.0928 + 0.}$$

2\. using Euler's method to find $y(0.4)$ G.

$\frac{dy}{dx} = x+y$, $y(0) = 1$. taking $h = 0.2$.

Soln:

Given $\frac{dy}{dx} = x+y$, $y(0)=1$.

The Euler's formula is $y_{n+1} = y_n + h y_n'$

x	0	0.2	0.4
y	1	1.2	1.48
$y' = x+y$	1	1.4	1.88

$n=0 \Rightarrow y_1 = y_0 + h y_0' = 1 + (0.2 \times 1) = 1.2$

$n=1 \Rightarrow y_2 = y_1 + h y_1' = 1.2 + (0.2 \times 1.4) = 1.48$

3. Using Euler's method find the solution of the initial value problem (IVP) $\frac{dy}{dx} = \log_e(x+y)$, $y(0)=2$ at $x=0.6$ by assuming $h=0.2$.

Soln:

Given $y' = \log_{10}(x+y)$; $y(0)=2$.

The Euler's formula is $y_{n+1} = y_n + h y_n'$

x	0	0.2	0.4	0.6
y	2	2.0602	2.1810	2.2117
$y' = \log_{10}(x+y)$	0.3010	0.3541	0.4033	0.4490

$n=0 \Rightarrow y_1 = y_0 + h y_0' = 2 + (0.2 \times 0.3010) = 2.0602$.

$n=1 \Rightarrow y_2 = y_1 + h y_1' = 2.0602 + (0.2 \times 0.3541) = 2.1810$.

$n=2 \Rightarrow y_3 = y_2 + h y_2' = 2.1810 + (0.2 \times 0.4033) = 2.2117$.

4. Using Euler's method, find $y(4.1)$ & $y(4.2)$ if $5x\frac{dy}{dx} + y^2 - 2 = 0$; $y(4)=1$

Soln :

Given $5\frac{dy}{dx}+y^2-2=0$; $y(4)=1$

$$\frac{dy}{dx}=\frac{-y^2+2}{5x}.$$

The Euler's formula is $y_{n+1}=y_n+hy_n'$

x	4	4.1	4.2
y	1	1.0050	1.0098
$y'=\frac{-y^2+2}{5x}$	0.05	0.0483	0.0467.

$n=0 \Rightarrow y_1=y_0+hy_0'=1+0.1(0.05)$

$=1.0050.$

$n=1 \Rightarrow y_2=y_1+hy_1'=1.005+0.1(0.0483)$

$=1.0098$ //.

5. Find $y(0.2)$ for $y'=y+e^x$, $y(0)=0$ by Euler's method. Take $h=0.1$

Soln :

Given $y'=y+e^x$, $y(0)=0$

The Euler's formula is $y_{n+1}=y_n+hy$

x	0	0.1	0.2
y	0	0.1	0.2205

$n=0 \Rightarrow$

$$y_1 = y_0 + hy_0' = 0+0.1(1) = 0.1$$

$n=1 \Rightarrow$

$$y_2 = y_1 + h y_1' = 0.1 + 0.1\times(1.2052) = 0.2205.$$

Fourth order Range - kutta method.

Consider, $g(x, y, y') = 0$.

$$y' = f(x,y)$$

$$k_1 = hf(x,y)$$

$$k_2 = hf(x+h/2, y+k_1/2)$$

$$k_3 = hf(x+h/2, y+k_2/2)$$

$$k_4 = hf(x+h, y+k_3)$$

$$y = y_0 + 1/6(k_1+2k_2+2k_3+k_4)$$

7\. using Runge - kutta method of order 4; find y value when $x=1.2$ in steps of 0.1 given that $y' = x^2+y^2$, $y(1)=1.5$.

soln:

The Runge - kutta formula is

$$K_1 = h\cdot f(x,y)$$

$$K_2 = h\cdot f(x+h/2, y+k_1/2)$$

$$k_3 = h\cdot f(x+h/2, y+k_2/2)$$

$k_4 = h \cdot f(x+h, y+k_3)$

Given $y' = x^2 + y^2$

here, $f(x,y) = x^2+y^2$; $h = 0.1$

x	1	1.1	1.2
y	1.5	y_1 1.8955	y_2 2.5044

To find y_1

$x=1$; $y=1.5$.

$k_1 = h f(x,y) = 0.1 \times f(1, 1.5)$.

$= 0.1 \times 3.25 = 0.325$.

$k_2 = h \cdot f(x+h/2, y+k_1/2) = 0.1 \times f(1.05, 1.662)$

$= 0.1 \times 3.8664 = 0.3866$.

$k_3 = h \cdot f(x+h/2, y+k_2/2) = 0.1 f(1.05, 1.6933)$

$= 0.1 \times 3.9698 = 0.3970$.

$k_4 = h \cdot f(x+h, y+k_3) = 0.1 \times f(1.1, 1.8970)$

$= 0.4809$.

$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1.5 + \frac{1}{6}[0.325 + 2 \times 0.3866 + 2 \times 0.3970 + 0.4809]$

$y_1 = 1.8955$

$f(x,y) = x^2 + y^2$

$k_1 = h \cdot f(x,y) = 0.1 \times f(1.2, 1.8955)$

$= 0.1 \times 4.8029 = 0.4803.$

$k_2 = h \, f\left(x + h/2,\ y + k_1/2\right) = \cancel{0.1\times}$

$= 0.1\, f(1.250,\ 2.1357)$

$= 0.1 \times 5.8837 = 0.5884.$

$k_3 = h\, f\left(x + h/2,\ y + k_2/2\right)$

$= 0.1 \times f(1.250,\ 2.1897)$

$= 0.1 \times 6.1173 = 0.6117.$

$k_4 = h\, f(x + h,\ y + k_3)$

$= 0.1 \times f(1.3,\ 2.5072)$

$= 0.7726.$

$y_2 = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1.8955 + \frac{1}{6}\left[0.4803 + 2\times 0.5884 + 2\times 0.6117 + 0.7726\right]$

2. Find $y(0.7)$ & $y(0.8)$ given that $y' = y - x^2$ by using RK method of $y(0.6) = 1.7379$ by using RK method of 4th order.

Soln:

$k_1 = h \cdot f(x, y)$

$k_2 = h \cdot f(x + h/2, y + k_1/2)$

$k_3 = h \cdot f(x + h/2, y + k_2/2)$

$k_4 = h \cdot f(x + h, y + k_3)$

Given $y' = y - x^2$.

Here $f(x, y) = y - x^2$; $h = 0.1$

	x_0	x_1	x_2
x	0.6	0.7	0.8
y	1.7379	1.8763	2.0145.

To find y_1:

$x = 0.6$; $y = 1.7379$.

$k_1 = h \cdot f(x, y) = 0.1 \times f(0.6, 1.7379)$

$= 0.1378$.

$k_2 = 0.1 \times f\left(0.6 + 0.1/2,\ 1.7379 + 0.1378/2\right)$

$k_2 = \cancel{0.0240} \; 0.1384$

$$k_3 = 0.1 \times f\left[0.6 + \frac{0.1}{2}, 1.7379 + 0.1384/2\right]$$

$$= 0.1 \times f(0.65, 1.8071)$$

$$= 0.1385.$$

$$k_4 = 0.1 \times f(0.7, 1.8764)$$

$$= 0.1386.$$

$$y_1 = \overset{1.7379}{\cancel{y_0}} + 1/6\,(0.1378 + 0.1384 \times 2 + 0.1385 \times 2 + 0.1386)$$

$$= 1.8763$$

To find y_2.

$x = 0.7 \; ; \; y = 1.8763.$

$k_1 = 0.1 \times f(0.7, 1.8763) = 0.1386$

$k_2 = 0.1 \times f(0.75, 1.9456) = 0.1383$

$k_3 = 0.1 \times f(0.75, 1.9455) = 0.1383.$

$k_4 = 0.1 \times f(0.8, 2.0146) = 0.1375$

$y_2 = y_1 + 1/6\,[k_1 + 2k_2 + 2k_3 + k_4].$

$$= 1.8763 + 1/6\,(0.1386 + 2 \times 0.1383 + 2 \times 0.1383 + \overset{0.1375}{\cancel{0.1402}})$$

8. using R.K method to find $y(0.2)$, $y(0.4)$. Given to $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$, $y(0)=1$

Soln:

$$y' = \frac{y^2-x^2}{y^2+x^2}$$

Here, $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$; $h=0.2$

x	0	0.2	0.4
y	1	1.1960	

To find y_1 :

$x=0$; $y=1$

$k_1 = h \cdot f(x,y) = 0.2 \times f(0,1)$

$= 0.2$

$k_2 = 0.2 \times f(0.1, 1.1) = 0.1967.$

$k_3 = 0.2 \times f(0.1, 1.0984) = 0.1967.$

$k_4 = 0.2 \times f(0.2, 1.1967) = 0.1891$

$y_1 = 1 + \frac{1}{6}(0.2 + 2 \times 0.1967 + 2 \times 0.1967 + 0.1891)$

$= 1.1960$

To find y_2 :-

$x = 0.2 ; \; y = 1.1960$

$k_1 = 0.2 \times f(0.2, 1.1960) = 0.1891$

$k_2 = 0.2 \times f(0.3, 1.2906) = 0.1795$

$k_3 = 0.2 \times f(0.3, 1.2842) = 0.1798$

$k_4 = 0.2 \times f(0.4, 1.3753) = 0.1688$

$$y_2 = 1.1960 + \frac{1}{6}\big(0.1891 + 2 \times 0.1763 + 0.1793 \times 2 + 0.1688\big)$$

$= 1.3753$

17/8/14. Using R.K method for solving simultaneous equation :

consider,

$$\frac{dy}{dx} = f(x,y,z) ; \frac{dz}{dx} = g(x,y,z)$$

$f(x,y,z)$	$g(x,y,z)$
$k_1 = h \cdot f(x,y,z)$	$l_1 = h \cdot g(x,y,z)$
$k_2 = h \cdot f(x+h/2, y+k_1/2, z+l_1/2)$	$l_2 = h \cdot g(x+h/2, y+k_1/2, z+l_1/2)$
$k_3 = h \cdot f(x+h/2, y+k_2/2, z+l_2/2)$	$l_3 = h \cdot g(x+h/2, y+k_2/2, z+l_2/2)$
$k_4 = h \cdot f(x+h, y+k_3, z+l_3)$	$l_4 = h \cdot g(x+h, y+k_3, z+l_3)$

$$y_1 = y_0 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4].$$

$$z_1 = z_0 + 1/6 [l_1 + 2l_2 + 2l_3 + l_4]$$

1. Solve for $y(0.1)$ and $z(0.1)$ from the simultaneous equation $\frac{dy}{dx} = 2y+z$; $\frac{dz}{dx} = y-3z$ $y(0) = 0$; $z(0) = 0.5$; using R.K method of order 4 !

Soln !

Given, $\frac{dy}{dx} = y-3z$; $g(x,y,z) = y-3z$

x	0	0.1
y	0	0.0481
z	0.5	0.3726

$h = 0.1$

$f(x,y,z) = 2y + z$	$g(x,y,z) = y - 3z$
$K_1 = h.f(x,y,z)$ $= 0.1 \times f(0,0,0.5)$ $K_1 = 0.05$	$l_1 = 0.1 \times g(0,0,0.5)$ $l_1 = -0.15$
$K_2 = h.f(x+h/2, y+k_1/2, z+l_1/2)$ $= 0.1 \times f(0.05, 0.025, 0.425)$ $K_2 = 0.0475$	$l_2 = 0.1 \times g(0.05, 0.025, 0.4$ $l_2 = -0.125$
$K_3 = h.f(x+h/2, y+k_2/2, z+l_2/2)$ $= 0.1 \times f(0.05, 0.0238, 0.4375)$ $k_3 = 0.0485$	$l_3 = 0.1 \times g(0.05 \times 0.238 \times 0.4375)$ $l_3 = -0.1289$
$K_4 = h.f(x+h, y+k_3, z+l_3)$ $= 0.1 \times f(0.1, 0.0485, 0.3711)$ $= 0.0468$	$l_4 = 0.1 \times g(0.1, 0.0485, 0.37$ $= -0.1065$

$$y_1 = y_0 + 1/6\,(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 0 + 1/6\,(0.05 + 2\times0.0475 + 2\times0.0485 + 0.0460)$$

$$= 0.0481$$

$$z_1 = 0.5 + 1/6\,(-0.15 - 2\times0.125 - 2\times0.1289 - 0.1065)$$

$$= 0.3726$$

R.K method for solving second order equation.

Consider, $\varphi(x, y, y', y'') = 0$ —— ①

take $y' = z$ —— ②

By using ② in ①, we get

$$z' = g(x, y, z)$$

1. Given $y'' + xy' + y = 0$; $y(0) = 1$; $y'(0) = 0$; find the value of $y(0.1)$ by using R.K method

Soln:

Given, $y'' + xy' + y = 0$. —— ①

Take $y' = z$;

$$z' + xz + y = 0.$$

$$z' = -xz - y.$$

x	0	0.1
y	1	0.9950
$z = y'$	0	-0.0995

$h = 0.1$

$f(x,y,z) = z$	$g(x,y,z) = -xz - y$
$k_1 = h \cdot f(x,y,z)$ $= 0.1 \times f(0,1,0)$ $k_1 = 0.$	$l_1 = 0.1 \times g(0,1,0)$ $= -0.1$
$k_2 = h \cdot f(x + h/2, y + k_1/2, z + l_1/2)$ $= 0.1 \times f(0.05, 1, -0.05)$ $= -0.005$	$l_2 = 0.1 \times g(0.05, 1, -0.05)$ $= -0.0998$
$k_3 = h \cdot f(x + h/2, y + k_2/2, z + l_2/2)$ $= 0.1 \times f(0.05, 0.9975, -0.0499)$ $= -0.005$	$l_3 = 0.1 \times g(0.05, 0.9975, -0.0499)$ $= -0.0995$
$k_4 = h \cdot f(x+h, y+k_3, z+l_3)$ $= 0.1 \times f(0.1, 0.9950, -0.0995)$ $= -0.0100$	$l_4 = 0.1 \times g(0.1, 0.9950, -0.0995)$ $= \cancel{0.1015} = -0.0985$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + \frac{1}{6}(0 - 2\times 0.005 - 2\times 0.005 - 0.01)$$

$$= 0.9950 //.$$

$$z_1 = 0 + \frac{1}{6}(-0.1 - 2\times 0.0998 - 2\times 0.0997 - 0.0985)$$

$$= -0.0995 //.$$

2. Consider the 2^{nd} Order initial value pbm $y'' - 2y' + 2y = e^{2t}\sin t$; $y(0) = -0.4$; $y'(0) = -0.6$ using ~~R~~ 4^{th} order R.K method find ~~y(0)~~ $y(0.2) = ?$

Soln:

Given $y'' - 2y' + 2y = e^{2x}\sin x$

Take $y' = z$.

$$f(x, y, z) = z.$$

$$z' - 2z + 2y = e^{2x}\sin x.$$

$$z' = e^{2x}\sin x - 2y + 2z.$$

$$g(x, y, z) = e^{2x}\sin x - 2y + 2z.$$

x	0	0.2
y	-0.4	
$z = y'$	-0.6	

$h = 0.2$

$f(x,y,z) = z$	$g(x,y,z) = e^{2x}\sin x - 2y + 2z$
$k_1 = h \cdot f(x,y,z)$ $= 0.2 \times f(0, -0.4, -0.6)$ $= -0.12$	$l_1 = 0.2 \times g(0, -0.4, -0.6)$ $l_1 = -0.08$
$k_2 = h \cdot f(x + h/2, y + k_1/2, z + l_1/2)$ $= 0.2 \times f(0.1, -0.46, -0.64)$ $= -0.1280$	$l_2 = 0.2 \times g(0.1, -0.46, -0.64)$ $= \cancel{-0.0592}$ -0.0476
$k_3 = h \cdot f(x + h/2, y + k_2/2, z + l_2/2)$ $= 0.2 \times f(0.1, \cancel{-0.9360}\ -0.4640, \cancel{-0.6288}\ -0.6238)$ $= \cancel{-0.1257}$ -0.1248	$l_3 = 0.2 \times g(0.1, -0.4640, -0.6286)$ $= \cancel{-0.0518}$ -0.0395
$k_4 = h \cdot f(x + h, y + k_3, z + l_3)$ $= 0.2 \times f(0.2, \cancel{-0.5257}\ -0.5248, \cancel{-0.6511}\ -0.6395)$ $= \cancel{-0.1302}$ -0.1279	$l_4 = 0.2 \times g(0.2, \cancel{-0.5257}\ -0.5248, \cancel{-0.6511}\ -0.6395)$ $= \cancel{+0.0086}$ -0.0136

$y_1 = y_0 + 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$

$= -0.47\ 1/6 (-0.12 - 2\times 0.1280 - 2\times \overset{0.1248}{\cancel{0.1257}} \overset{-0.1279}{\cancel{-0.1302}})$

$= \cancel{-0.5263} \quad -0.5256$

$z_1 = -0.6 + 1/6 (-0.08 - 2\times \overset{0.0476}{\cancel{0.0572}} - 2\times \overset{0.0395}{\cancel{0.0511}} + -0.0134 \cancel{0.0086})$

$= -0.6480$

$\simeq -0.6401$

18/8/14. Milne's Predictor - corrector Method.

Consider $\frac{dy}{dx} = f(x,y)$

$P: \quad y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$

$C: \quad y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$

1. By using Milne's predictor - corrector formula to find $y(0.4)$ & $y(0.5)$; G.T $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$, $y(0) = 1$; $y(0.1) = 1.06$; $y(0.2) = 1.12$; $y(0.3) = 1.21$

Soln: The Milne's predictor - corrector formula is,

$$P: \; y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right] \quad \text{--- (1)}$$

$$C: \; y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right] \quad \text{--- (2)}$$

x	0 (x_0)	0.1 (x_1)	0.2 (x_2)	0.3 (x_3)	0.4 (x_4)		0 (x_5)
y	1 (y_0)	1.06 (y_1)	1.12 (y_2)	1.21 (y_3)	1.2771 (y_4)	1.2797	$1.$
$y' = \frac{(1+x^2)y^2}{2}$	(y'_0) 0.5	(y'_1) 0.5674	(y'_2) 0.6523	(y'_3) 0.7979	0.9460	0.9478	$1.$

Put $n=3$ in (1).

$$P: y_4 = y_0 + \frac{4h}{3}\left(2y'_1 - y'_2 + 2y'_3\right)$$

$$= 1 + \frac{4 \times 0.1}{3}\left(2 \times 0.5674 - 0.6523 + 2 \times 0.7979\right)$$

$$P: y_4 = 1.2771.$$

put $n=3$ in eqn (2).

$$C: y_4 = y_2 + h/3\,(y'_2 + 4y'_3 + y'_4)$$

$$= 1.12 + \frac{0.1}{3}\left(0.6523 + 4 \times 0.7979 + 0.9460\right]$$

$$C: y_4 = 1.2797.$$

put n = 4 in ①,

$$P: y_5 = y_1 + \frac{4h}{3}\left[2y_2' - y_3' + 2y_4'\right]$$

$$= 1.06 + \frac{4 \times 0.1}{3}\left[2 \times 0.6523 - 0.7979 + 2 \times 0.9498\right]$$

$$P: y_5 = 1.3808.$$

put n = 4 in ②,

$$C: y_5 = y_3 + \frac{h}{3}(y_3' + 4y_4' + y_5')$$

$$= 1.21 + \frac{0.1}{3}(0.7979 + 4 \times 0.9498 + 1.1916.)$$

$$y_5 = 1.4030.$$

3808	1.4030
1916	1.2303

② Given $y' = \frac{1}{x+y}$; $y(0) = 2$; $y(0.2) = 2.0933$; $y(0.4) = 2.1755$, $y(0.6) = 2.2493$. Find $y(0.8)$ by using Milne's method.

Soln:

The Milne's formula is,

$$P: y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y_{n-2}' - y_{n-1}' + 2y_n'\right] \quad \text{①}$$

$$C: y_{n+1} = y_{n-1} + \frac{h}{3}\left[y_{n-1}' + 4y_n' + y_{n+1}'\right] \quad \text{②}$$

x	x_0 0	x_1 0.2	x_2 0.4	x_3 0.6	x_4 0.8
y	y_0 2	y_1 2.0933	y_2 2.1755	y_3 2.2493	2.3162
$y' = \frac{1}{x+y}$	y_0' 0.5	y_1' 0.4861	y_2' 0.3883	y_3' 0.3510	0.3209

put $n = 3$ in ①

$$P: y_4 = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

$$= 2 + \frac{4 \times 0.2}{3}\left[2 \times 0.4861 - 0.3883 + 2 \times 0.3510\right]$$

$$P: y_4 = 2.3162.$$

put $n = 3$ in ②

$$C: y_4 = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 2.1755 + \frac{0.2}{3}\left[0.3883 + 4 \times 0.3510 + 0.3209\right]$$

$$C: y_4 = 2.3164 //.$$

19|3|14.

3. Given $y' = xy + y^2$, $y(0) = 1$; $y(0.1) = 1.1169$; $y(0.2) = 1.2774$. using R.k method of 4th order, find $y(0.3)$. Continue the solution at $x = 0.4$ using milne's method.

soln:

x	0	0.1	0.2	0.3
y	1	1.1169	1.2774	1.5042.

Here, $h = 0.1$;

$y' = xy + y^2$

$f(x, y) = xy + y^2$

To find y_3;

$x = 0.2$; $y = 1.2774$.

$k_1 = h.f(x.y) = 0.1 \times f(0.2, 1.2774) = 0.1887$.

$k_2 = h \cdot f(x + h/2, y + \frac{k_1}{2}) = 0.1 \times f(0.25, 1.3718)$

$= 0.2225$.

$k_3 = h \cdot f(x + h/2, y + \frac{k_2}{2}) = 0.1 \times f(0.25, 1.3887)$

$= 0.2276$

$k_4 = h \cdot f(x + h, y + k_3) = 0.1 \times f(0.3, 1.5050) = 0.2717$.

$y_3 = y_2 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1.2774 + \frac{1}{6}[0.1887 + 2 \times 0.2225 + 2 \times 0.2276 + 0.2717]$

$= 1.5042$.

Milne's Formula is,

$$P: \ y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$C: \ y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

x	x_0 0	x_1 0.1	x_2 0.2	x_3 0.3	x_4 0.4	
y	y_0 1	y_1 1.1169	y_2 1.2774	y_3 1.5042	1.8345	1.8
$y' = xy + y^2$	y'_0 1	y'_1 1.3592	y'_2 1.8872	y'_3 2.7139	4.0992	4.1

Put $n = 3$ in (I)

$$P: \ y_4 = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$= 1 + \frac{4 \times 0.1}{3}\left[2 \times 1.3592 - 1.8872 + 2 \times 2.7139\right]$$

$$= 1.8345.$$

Put $n = 3$ in (II)

$$C: \ y_4 = y_2 + \frac{0.1}{3}\left[y'_2 + 4y'_3 + y'_4\right]$$

$$= 1.2774 + \frac{0.1}{3}\left[1.8872 + 4 \times 2.7139 + 4.09\right.$$

$$= 1.8388$$

4. Given that $y''+xy'+y=0$, $y(0)=1$; $y'(0)=0$ obtain y for $x=0.1, 0.2$ and 0.3 by Taylor series method and find the soln for $y(0.4)$ by milne's method.

soln:

The Taylor series is,

$$y = y_0 + (x-x_0)\frac{y_0'}{1!} + (x-x_0)^2\frac{y_0''}{2!} + (x-x_0)^3\frac{y_0'''}{3!}$$

$$+(x-x_0)^4\frac{y_0^{iv}}{4!} + \cdots$$

$y''+xy'+y=0.$

$y''=-xy'-y.$

x	0	x_0
y	1	y_0
y'	0	y_0'
$y''=-xy'-y$	-1	y_0''
$y'''=-xy''-y'-y'$	0	y_0'''
$y^{iv}=-xy'''-y'''-y''-y''$	3	y_0^{iv}

$$y = 1 + (x-0)\,0/1 + (x-0)^2 \times -1/2 + (x-0)^3\,0/6 +$$

$$y = 1 - \frac{x^2}{2} + \frac{x^4}{8}$$

$$y' = -\frac{\cancel{2}x}{\cancel{2}} + \frac{\cancel{4}x^3}{\cancel{8}_2} \Rightarrow y' = -x + x^3/2$$

$y(0.1) = 0.9950$

$y(0.2) = 0.9802$

$y(0.3) = 0.9560$

The Milne's formula is,

$$P: \quad y_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_{n}\right]$$

$$C: \quad y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

20/3/19

	x_0	x_1	x_2	x_3	x_4	
x	0	0.1	0.2	0.3	0.4	
y	y_0 1	y_1 0.9950	y_2 0.9802	y_3 0.9560	0.9232	0.9232
$y' = -x + x^3/2$	y'_0 0	y'_1 −0.0995	y'_2 −0.1960	y'_3 −0.2865	−0.3680	−0.3685

put $n = 3$;

$$P: \quad y_4 = y_0 + \frac{4 \times 0.1}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

$$= 1 + \frac{0.4}{3}\left[2 \times (-0.0995) + 0.1960 + 2 \times (-0.2\right.$$

$$= 0.9232.$$

C: put $n = 3$;

$$C: \quad y_4 = y_2 + h/3\left[y'_2 + 4y'_3 + y'_4\right]$$

$$= 0.9802 + \frac{0.1}{3}\left[-0.1960 - 4\times 0.2865 \cancel{+0.9232}\right] \quad -0.3680$$

$c: y_4 = 0.9232$

Adam's Bashforth predictor - corrector formula:

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

$$c: y_{n+1} = y_n + \frac{h}{24}\left[19y_n' - 5y_{n-1}' + y_{n-2}' + 9y_{n+1}'\right].$$

1. Using Adam's method find $y(1.4)$ given $y' = x^2(1+y)$, $y(1) = 1$; $y(1.1) = 1.233$; $y(1.2) = 1.548$ & $y(1.3) = 1.979$.

Soln:

The Adam's formula is,

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

$$c: y_{n+1} = y_n + \frac{h}{24}\left[19y_n' - 5y_{n-1}' + y_{n-2}' + 9y_{n+1}'\right]$$

x	x_0 1	x_1 1.1	x_2 1.2	x_3 1.3	x_4 1.4
y	y_0 1	y_1 1.233	y_2 1.548	y_3 1.979	y_4 2.5723 2.5749
$y' = x^2(1+y)$	y_0' 2	y_1' 2.7019	y_2' 3.6691	y_3' 5.0345	y_4' 7.0017 7.00..

put $n=3$;

$$P: y_4 = y_3 + \frac{0.1}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 1.979 + \frac{0.1}{24}\left[55\times 5.0345 - 59\times 3.6691 + 37\times 2.7019 - 9\times 2\right]$$

$$P: y_4 = 2.5723.$$

put $n=3$ in ①

$$C: y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$$

$$= 1.979 + \frac{0.1}{24}\left[19\times 5.0345 - 5\times 3.6691 + 2.70 + 9\times 7.0017\right]$$

$$C: y_4 = 2.5749.$$

2. Use Adam's method to find $y(2)$ if $y' = \frac{x+y}{2}$, $y(0) = 2$; $y(0.5) = 2.636$; $y(1) = 3$ and $y(1.5) = 4.968$.

Soln:

The Adam's formula is,

$$P: y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - \right.$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left[19y_n' - 5y_{n-1}' + y_{n-2}' + 9y_{n+1}'\right]$$

x	x_0 0	x_1 0.5	x_2 1	x_3 1.5	x_4 2
y	y_0 2	y_1 2.636	y_2 3.595	y_3 4.968	6.8708, 6.8731
$y' = \frac{x+y}{2}$	y_0' 1	y_1' 1.5680	y_2' 2.2975	y_3' 3.2340	4.4354, 4.4366

put $n=3$ in ①

$$P: \quad y_4 = y_3 + \frac{0.5}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 4.968 + \frac{0.5}{24}\left[55 \times 3.2340 - 59 \times 2.2975 + 37 \times 1.5680 - 9 \times 1\right]$$

$$= 6.8708$$

$$C: \quad y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$$

$$= 4.968 + \frac{0.5}{24}\left[19 \times 3.2340 - 5 \times 2.2975 + 1.5680 + 9 \times 4.4354\right]$$

$$= 6.8731.$$

21/3/14.

8. Using Adam's method find $y(0.4)$ given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$; $y(0.1) = 1.1169$; $y(0.2) = 1.2774$; and $y(0.3) = 1.5041$

Soln:

The Adam's formula is,

$$P: \quad y_{n+1} = y_n + \frac{h}{24}\left[55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}'\right]$$

$$C: y_{n+1} = y_n + \frac{h}{24}\left[19y_n' - 5y_{n-1}' + y_{n-2}' + 9y_{n+1}'\right]$$

	x_0	x_1	x_2	x_3	x_4
x	0	0.1	0.2	0.3	0.4
y	1	1.1169	1.2774	1.5041	1.8841
$\frac{dy}{dx} = xy + y^2$		1.3592	1.8872	2.7185	4.0976
		0.0895	0.1766		

put $n = 3$ in

$$P: y_4 = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$= 1.5041 + \frac{0.1}{24}\left[55 \times 2.7185 - 59 \times 1.8872 + 37 \times 1.3592 - 9 \times 1\right]$$

$$P: y_4 = 1.8841$$

put $n = 3$ in

$$C: y_4 = y_3 + \frac{h}{24}\left[19y_3' - 5y_2' + y_1' + 9y_4'\right]$$

$$= 1.5041 + \frac{0.1}{24}\left[19 \times 2.7185 - 5 \times 1.8872 + 1.3592 + 9 \times 4.0976\right]$$

$$= 1.8889.$$

unit-IV

Numerical differentiation and Integration

Questions	opt1	opt2	opt3	opt4	opt5	opt6	Answer
_________ Formula can be used for interpolating the value of f(x) near the end of the tabular values.	Newton's forward	Newton's backward	Lagrange	stirling			Newton's backward
_________ Formula can be used for interpolating the value of f(x) near the beginning of the tabular values.	Newton's forward	Newton's backward	Lagrange	stirling			Newton's forward
In Numerical integration, the length of all intervals is in ----------- distances.	Greater than the other	less than the other	equal	not equal			equal
When the function is given in the form of table values instead of giving analytical expression we use ___________.	numerical differentiation	numerical elimination	approximation	addition			numerical differentiation
_________ is the process of computing the value of the definite integral from the set of numerical values of the integrand.	numerical differentiation	numerical integration	Simpsons rule	Trapezoidal rule			numerical integration
Numerical integration is the process of computing the value of a ________ from a set of numerical values of the integrand.	indefinite integral	definite integral	expression	equation			definite integral
Numerical evaluation of a definite integral is called --------	integration	differentiation	interpolation	triangularisation			integration
What is the value of h if a=0,b=2 and n=2.	1	2	3	4			1
Integral (f(x) dx)=(h/2) [Sum of the first and last ordinates + 2(sum of the remaining ordinates)] is called _____	Constant rule	Simpsons rule	Trapezoidal rule	Rombergs rule			Trapezoidal rule
If the given integral is approximated by the sum of 'n' trapezoids, then the rule is called as ----------------.	Newton's method	Trapezoidal rule	simpson's rule	none			Trapezoidal rule
What is the formula for finding the length interval h in trapezoidal tule?	h=(b-a)/n	h=(b/a)/n	h=(b*a)/n	h=(b+a)/n			h=(b-a)/n

The accuracy of the result using the Trapezoidal rule can be improved by --------	Increasing the interval h	Decreasing the length of the interval h	Increasing the number of	altering the given function	Decreasing the length of the interval h
The order of error in Trapezoidal rule is -------------.	h	h^2	h^3	h^4	h^2
Simpson's rule is exact for a ----------------- even though it was derived for a Quadratic.	cubic	less than cubic	linear	quadratic	linear
The order of error in Simpson's rule is ----------	h	h^2	h^3	h^4	h^4
For what type of functions, Simpsons rule and direct integration will give the same result?	parabola	hyperbola	ellipse	cardiod	parabola
Simpson's rule gives exact result if the entire curve y=f(x) itself is a ___________.	parabola	hyperbola	ellipse	cardiod	parabola
To apply Simpsons one third rule the number of intervals must be_________.	odd	even	equally spaced	unequal	even
The end point coordinates y_0 and y_n are included in the Simpsons 1/3 rule, so it is called _________ formula.	Newton's	open	closed	Gauss	closed
Simpson's one-third rule on numerical integration is called a --------- formula.	closed	open	semi closed	semi opened	closed
The order of error in Simpson's formula is ________.	1	2	3	4	4
In two point Gaussian quadrature Formula n =	1	2	3	4	2
In Simpsons 1/3rd rule, the number of ordinates must be _________.	odd	even	0	3	odd
In three point Gaussian quadrature Formula n = ________.	1	2	3	4	3
Two point Gaussian quadrature Formula requires only _______ functional evaluations and gives a good estimate of the value of the integral.	1	2	3	4	2
_________ formula is based on the concept that the accuracy of numerical integration can be improved by choosing the sampling wisely , rather than on the basis	Newtons	elimination	Gauss quadrature	hermite	Gauss quadrature

Gauss Quadrature formula is also called as _________.	Newton's	Gauss-Legendre	Gauss-seidal	Gauss-Jordan	Gauss-Legendre
The 2 point Gauss-quadrature is exact for the polynomial up to degree _________.	1	2	3	4	3
The 3 point Gauss-quadrature is exact for the polynomial up to degree _________.	1	5	3	4	5
Integrating f(x)=5x^4 in the interval [-1,1] using Gaussion two point formula gives______.	1/2	9/5	10/9	5/9	10/9
The modified Eulers method is based on the ___________ of points	sum	multiplication	average	subratction	average
__________ prior values are required to predict the next value in Milne's method	1	2	3	4	4
__________ prior values are required to predict the next value in Adams method	1	2	3	4	3
The Eulers method is used only when the slope at point ____________ in computing is y(n+1)	(x(n), y)	(x, y(n))	(x(n), y(n))	(0, 0)	(x(n), y(n))
The Runge Kutta method agrees with Taylor series solution upto the ________ terms	h^2	h^3	h^4	h^r	h^r
Runge Kutta method agree with ________ solution upto the terms h^4	Taylor Series	Eulers	Milnes	Adams	Taylor Series
________ method is better than Taylor's series method	Runge Kutta	Milnes	Adams	Eulers	Runge Kutta
Taylors series method belongs to ____________ method	Single step	multi step	step by step	limination	Single step
If all the n conditions are specified at the initial point only then it is called a ______ problem	Initial value	final value	boundary value	semi defined	Initial value
The problem dy/dx = f(x,y) with the initial condition y(x(0)) = y(0)is ___________problem	initial value	final value	boundary value	multistep	initial value

The solution of an ODE means finding an explicit expression for y, in terms of a ______ number of elementary functions of x	finite	infinite	positive	negative	finite
The solution of an ODE is known as _____________ solution	infinite	open-form	closed-form	negative form	closed-form
The differential equation of the 2^{nd} order can be solved by reducing it to a ____ differential equation	lower order	higher-order	partial	simultaneous	lower order
The Eulers method is used only when the slope at point (x(n), y(n)) in computing is ___________	y(n+1)	y(n-1)	(dy/dx)(n+1)	(dy/dx)(n-1)	y(n+1)
The Eulers method is used only when the slope at point _____________ in computing is y(n+1)	(x(n),y)	(x, y(n))	(x(n), y(n))	(0, 0)	(x(n), y(n))
The modified Eulers method is a ____________ method of predictor-corrector type	Self-correcting	Self-starting	Self-evaluating	Self-predicting	Self-starting
The modified Eulers method has greater accuracy than ________ method	Taylor's	Picard's	Euler's	Adam's	Taylor's
The formula y(n+1) = y(n) + hf(x(n), y(n)) is ________formula	Euler's	modified Euler's	Picard's	Taylor's	Euler's
Modified Eulers method is the Runge-kutta method of________ order	1^{st}	2^{nd}	3^{rd}	4^{th}	2^{nd}
Modified Eulers method is same as the ________ method of 2^{nd} order	Eulers	Taylors	Picards	Runge Kutta	Runge Kutta
The process used in Eulers method is very slow and to obtain reasonable accuracy we need to take a ________ value of h	Smaller	Larger	negative	Positive	Smaller
The process used in Eulers method is very slow and to obtain reasonable accuracy we need to take a smaller value of ________	h	h^2	h^3	h^4	h
The ________ formula is given by y(i+1) = y(i) +hf (x(i), y(i))	Taylors	predictor	Corrector	Eulers	Eulers
The predictor formula and ________ formula are one and the same	Taylors	Eulers	Modified Eulers	Eulers	Eulers
The ________ formula is given by y(i+1) = y(i) + h/2[f(x(i), y(i)) + f(x(i+1), y(i+1))], i = 1,2,3…..	Taylors	predictor	Corrector	Picards	Corrector
The ________ formula is used to predict the value y(i+1) of y at x(i+1)	Predictor	Corrector	Corrector	Picards	Predictor

The ________ formula is used to improve the value of y(i+1)	Predictor	Corrector	Taylors	Picards	Corrector
In predictor corrector methods, ____ prior values of y are needed to evaluate the value of y at x(i+1)	1	2	3	4	4
In ________ methods, 4 prior values of y are needed to evaluate the value of y at x(i+1)	Taylor's	predictor	Predictor-corrector	Euler's	Predictor-corrector
In predictor corrector methods 4 prior values of ________ are needed to evaluate of values of are needed to evaluate of value of y at x(i+1)	y	y^2	y^3	y^4	y

UNIT - V

BOUNDARY VALUE PROBLEM IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATION.

Finite difference Method :

Replace x by x_k

y by y_k

y' by $\frac{y_{k+1} - y_k}{h}$

y'' by $\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2}$

where, $h = \frac{b-a}{n}$

1. Solve $y'' = x+y$ with the boundary condition $y(0) = y(1) = 0$.

Soln :

x	0	0.25	0.5	0.75	1
y	0	-0.0349	-0.0564	-0.05	0

$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$.

$y'' = x+y$.

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} = x_k + y_k.$$

$$y_{k-1} - 2y_k + y_{k+1} = h^2 x_k + h^2 y_k.$$

$$y_{k-1} - 2y_k + y_{k+1} - h^2 y_k = h^2 x_k$$

$$y_{k-1} + y_k(-2-h^2) + y_{k+1} = h^2 x_k$$

$$y_{k-1} - 2.0625\, y_k + y_{k+1} = 0.0625\, x_k$$

$k=1$;

$$y_0 - 2.0625\, y_1 + y_2 = 0.0625\, x_1$$

$$-2.0625 y_1 + y_2 = 0.0156 \quad \text{—} (1).$$

$k=2$;

$$y_1 - 2.0625 y_2 + y_3 = 0.0625 x_2$$

$$y_1 - 2.0625\, y_2 + y_3 = 0.0313 \quad \text{—} (2)$$

$k=3$;

$$y_2 - 2.0625\, y_3 + y_4 = 0.0625 x_3$$

$$y_2 - 2.0625\, y_3 = 0.0469 \quad \text{—} (3).$$

solve (1), (2) & (3)

$$y_1 = -0.0349;\quad y_2 = -0.0564;\quad y_3 = -0.0501;$$

2. using a finite difference method compute $y(0.5)$. Given $y'' - 64y + 10 = 0$; $y(0) = y(1) = 0$.

24/3/14.

Sub dividing the interval into i) 4 equal parts.

ii) 2 equal parts.

soln:

Given $y'' - 64y + 10 = 0$

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} - 64y_k + 10 = 0$$

$$\frac{y_{k-1} - 2y_k + y_{k+1} - 64y_k h^2 + 10h^2}{h^2} = 0$$

$$y_{k-1} + y_k(-2 - 64h^2) + y_{k+1} = -10h^2 \quad \text{--- (1)}$$

i) subdiving into 4parts.

$$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$$

	x_0	x_1	x_2	x_3	x_4
x	0	0.25	0.5	0.75	1
y	y_0 0	y_1 0.1287	y_2 0.1471	y_3 0.1287	y_4 0

for $h = 0.25$, (1) becomes,

$$y_{k-1} - 6y_k + y_{k+1} = -0.625 \quad \text{--- (2)}$$

put $k = 1$.

$$y_0 - 6y_1 + y_2 = -0.625$$

$$-6y_1 + y_2 = -0.625 \quad \text{--- (3)}$$

put $k=2$;

$$y_1 - 6y_2 + y_3 = -0.625 \quad \text{——} \; (4)$$

put $k=3$;

$$y_2 - 6y_3 + y_4 = -0.625$$

$$y_2 - 6y_3 = -0.625 \quad \text{——} \; (5)$$

Solving by (3) & (4) & (5)

$y_1 = 0.1287$; $\boxed{y_2 = 0.1471}$; $y_3 = 0.1287$.

ii) Sub dividing into 2 parts :

$$h = \frac{b-a}{n} = \frac{1-0}{2} = 0.5.$$

	x_1	x_2	x_3
x	0	0.5	1
	y_0	y_1	y_2
y	0	0.1389.	0.

for $h = 0.5$. Eqn (1) becomes

$$\cancel{y_{k-1} + y_k}$$

$$y_{k-1} - 18y_k + y_{k+1} = -2.5 \quad \text{——} \; (1)$$

$k=1$.

$$y_0 - 18y_1 + y_2 = -2.5.$$

$$\cancel{-}18y_1 = \cancel{-}2.5$$

$$\boxed{y_1 = 0.1389.}$$

3. Solve by finite difference method, the BVP $y'' - y = 0$ where $y(0)=0, y(1) = 1$; take $\Delta x = 0.25$.

Soln:

Given $y'' - y = 0$

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} - y_k = 0.$$

$$\frac{y_{k-1} - 2y_k + y_{k+1} - y_k h^2}{h^2} = 0.$$

$$y_{k-1} + y_k(-2-h^2) + y_{k+1} = 0 \quad \text{--- (1)}$$

for $h = 0.25$, eqn (1) becomes,

$$y_{k-1} - 2.0625\, y_k + y_{k+1} = 0 \quad \text{--- (2)}$$

x	x_0: 0	x_1: 0.25	x_2: 0.5	x_3: 0.75	x_4: 1
y	y_0: 0	0.2151	0.4437	y_3: 0.7	y_4: 1

$k = 1$;

$$y_0 - 2.0625\, y_1 + y_2 = 0.$$

$$-2.0625\, y_1 + y_2 = 0 \quad \text{--- (3)}$$

$k = 2$;

$$y_1 - 2.0625\, y_2 + y_3 = 0 \quad \text{--- (4)}$$

$k=3$;

$$y_2 - 2.0625 y_3 + y_4 = 0.$$

$$y_2 - 2.0625 y_3 + 1 = 0.$$

$$y_2 - 2.0625 y_3 = -1 \quad \text{---} (5).$$

Solve by (3), (4) & (5)

$$y_1 = 0.2181 \ ; \ y_2 = 0.4437 \ ; \ y_3 = 0.7000$$

24/3/14

Classification of partial differential Equation

Consider,

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = 0$$

$B^2 - 4ac < 0$ The P.D.E is elliptic

$B^2 - 4ac = 0$ The P.D.E is parapolic

$B^2 - 4ac > 0$ The P.D.E is hyperbolic

One dimensional heat equation :

The One dimensional heat eqn is

$$\frac{\partial^2 u}{\partial x^2} = a\frac{\partial u}{\partial t} \quad (\text{or}) \quad U_{xx} = a U_t$$

$$\frac{\partial^2 u}{\partial x^2} - a\frac{\partial u}{\partial t} = 0.$$

$A = 1$; $B = 0$; $C = 0$

$B^2 - 4AC = 0 - 4 \times 1 \times 0$

$= 0$

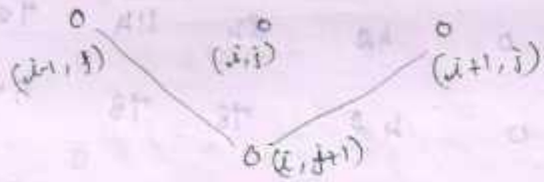
The one dimensional heat eqn is parabolic

There are two methods to solve one dimensional heat equations

i) Bender - Schmidt formula (Explicit)

ii) Crank - Nicolson method (Implicit)

Bender - Schmidt formula:



$$u_{i,j+1} = \frac{u_{i-1\,j} + u_{i+1\,j}}{2}$$

Here, $k = \frac{ah^2}{2}$

1. Solve $u_t = u_{xx}$ in $0 < x < 5$, $t > 0$ given that $u(0,t) = 0$, $u(5,t) = 0$, $u(x,0) = x^2(25 - x^2)$. Compute u upto 3sec. with $\Delta x = 1$ by using Bender Schmidt formula.

soln:

Given $u_t = u_{xx} \Rightarrow a = 1$

$h = \Delta x = 1$

$k = \frac{ah^2}{2} = \frac{1 \times 1}{2} = 0.5$

$$u_{i,j+1} = \frac{u_{i-1,j} + u_{i+1,j}}{2}$$

$t \backslash x$	0	1	2	3	4	5
0	0	24	84	144	144	0
0.5	0	42	84	114	72	0
1	0	42	78	78	57	0
1.5	0	39	60	67.5	39	0
2	0	30	53.25	49.5	33.75	0
2.5	0	26.625	39.75	43.5	24.75	0
3	0	19.875	35.0625	32.25	21.75	

2. Solve $u_{xx} = 32u_t$, $h = 0.25$ for $t \geq 0$, $0 < x < 1$, with $u(0,t) = 0$, $u(x,0) = 0$; $u(1,t) = t$.

Soln:

$u_{xx} = 32 u_t \qquad a = 32.$

$h = 0.25.$

$$k = \frac{ah^2}{2} = \frac{32 \times 0.25}{2} = 1$$

$$u_{i,j+1} = \frac{u_{i-1,j} + u_{i+1,j}}{2}$$

$t \backslash x$	0	0.25	0.5	0.75	1
0	0	0	0	0	0
1	0	0	0	0	1
2	0	0	0	0.5	2
3	0	0	0.25	1	3
4	0	0.125	0.5	1.625	4
5	0	0.25	0.875	2.25	5.

2. Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ subjected to $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin(\pi x)$ using Bender Schmidt method.

Soln:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

$U_{xx} = U_t \cdot \quad a=1$

$$h = \frac{B-A}{n} = \frac{1-0}{5} = 0.2.$$

$$k = \frac{ah^2}{2} = \frac{1\times 0.2^2}{2} = 0.02.$$

Bender Schmidth formula is,

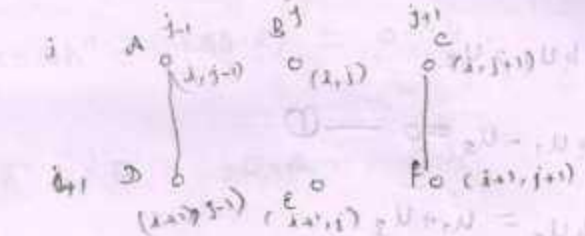
$$U_{i,j+1} = \frac{U_{i-1,j} + U_{i+1,j}}{2}$$

t \ x	0	0.2	0.4	0.6	0.8
0	0	0.5878	0.9511	0.9510	0.5878
0.02	0	0.4756	0.7695	0.7695	0.4756
0.04	0	0.3848	0.6226	0.6226	0.3848
0.06	0	0.3113	0.5037	0.5037	0.3115
0.08	0	0.2519	0.4075	0.4075	0.2519
1	0	0.2038	0.3297	0.3297	0.2038

25/3/14. Crank – Nicolson's Method (Implicit method):

Consider, $\frac{\partial^2 u}{\partial x^2} = a \frac{\partial u}{\partial t}$ (one dimensional heat eqn).

$k = ah^2$



$4u_E = u_A + u_C + u_D + u_F$

1. Using crank – Nicolson's scheme solve

$$16 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \ t > 0.$$

Subjected to $u(x, 0) = 0$; $u(0, t) = 0$; $u(1, t) = 100t$. Compute u for one step in t-direction. taking $h = 1/4$

Soln:

$$16 \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$a = 16$.

$h = 0.25$;

$k = ah^2 = 16 \times (0.25)^2 = 1$

t/x	0	0.25	0.5	0.75	
0	0	0	0	0	0
1	0	u_1	u_2	u_3	100

$4u_1 = u_2$

$4u_1 - u_2 = 0$ —— (1).

$4u_2 = u_1 + u_3$.

$u_1 - 4u_2 + u_3 = 0$ —— (2).

$4u_3 = u_2 + 100$

$-u_2 + 4u_3 = 100$ —— (3)

solve (1), (2) & (3).

$u_1 = 1.7857$.

$u_2 = 7.1429$.

$u_3 = 26.7857$.

2. find $u(x,t)$ for one time step the Equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ given $u(x,0) = \sin(\pi x)$; $u(0,t) = u(1,t) = 0$. Take $h = 0.2$ use implicit method.

Soln:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

$a = 1$

$h = 0.2$

$k = ah^2 = 1 \times (0.2)^2 = 0.04$

t\x	0	0.2	0.4	0.6	0.8	1
0	0	0.5878	0.9511	0.9511	0.5878	0
0.04	0	u_1	u_2	u_3	u_4	0

$4u_1 = u_2 + 0.9511$

$4u_1 - u_2 = 0.9511$ ——— ①

$4u_2 = u_1 + u_3 + 1.5389$

$\cancel{-u_1 + 4u_2}$ $u_1 - 4u_2 + u_3 = -1.5389$ ——— ②

$4u_3 = u_2 + u_4 + 1.5389$

$u_2 - 4u_3 + u_4 = -1.5389$ ——— ③

$4u_4 = 0.9511 + u_3$

$\cancel{u_3 + 4u_4 = 0.9511}$ ——— ④

$u_4 = \frac{u_3}{4} + 0.2378$ ——— ④

Sub ④ in ⑧

$$u_2 - 4u_3 + u_4 = -1.5389.$$

$$u_2 - 4u_3 + \frac{u_3}{4} + 0.2378 = -1.5389.$$

$$u_2 - \frac{15}{4}u_3 = -1.7767.$$

$$u_2 - 3.75u_3 = -1.7767 \quad \text{—— ⑤}$$

Solve ~~eqn~~ ①, ②, ⑤.

$u_1 = 0.3993$

$u_2 = 0.6461$

$u_3 = 0.6461$

④ $\Rightarrow$ $u_4 = \frac{0.6461}{4} + 0.2378 = 0.3993.$

$u_4 = 0.3993.$

27/3/14.

3. Solve by crank nicolson's method, eqn $u_{xx} = u_t$ subjected to $u(x,0) = 0$; $u(0,t) = 0$; $u(1,t) = t$ for two time step.

soln :

$u_{xx} = u_t$.

$a = 1$

$h = \frac{b-a}{n} = \frac{1-0}{4} = 0.25$.

$k = ah^2 = 1 \times 0.25^2 = 0.0625$.

$t \backslash x$	0	0.25	0.5	0.75	1
0	0	0	0	0	0
0.0625	0	0.0011	0.0045	0.0167	0.0625
0.125	0	0.0059	0.0191	0.0528	0.125

$4u_1 = u_2$.

$4u_1 - u_2 = 0$ — ①

$4u_2 = u_1 + u_3$

$u_1 - 4u_2 + u_3 = 0$ — ②

$4u_3 = u_2 + 0.0625$.

$-u_2 + 4u_3 = 0.0625$ — ③.

solve by ①, ②, ③

$u_1 = 0.0011$; $u_2 = 0.0045$; $u_3 = 0.0167$.

$4u_4 = u_5 + 0.0045$.

$4u_4 - u_5 = 0.0045$ — ④

$$4u_5 = u_4 + u_6 + 0.0178$$

$$u_4 - 4u_5 + u_6 = -0.0178 \quad \text{--- (5)}$$

$$4u_6 = u_5 + 0.1920$$

$$-u_5 + 4u_6 = 0.1920 \quad \text{--- (6)}$$

solve by (4), (5), & (6)

$$u_4 = 0.0059 \qquad u_5 = 0.0191 \qquad u_6 = 0.0598$$

One dimentional wave Equation:

The One dimentional wave Equation is, $\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial t^2}$; $k = ah$

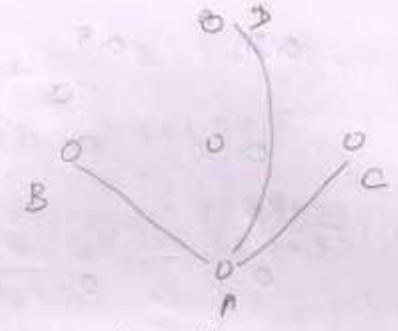
$$U_{xx} = a^2 U_{tt}$$

$$U_{xx} - a^2 U_{tt} = 0$$

$A = 1$; $B = 0$; $C = -a^2$

$$B^2 - 4AC = 0 + 4a^2 = 4a^2 > 0$$

The P.D.E is hyperbolic.



The formula is,

$$U_A = U_B + U_C - U_D$$

1. Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0<x<1$, $t>0$

Given $u(x,0)=0$; $\frac{\partial u}{\partial t}(x,0)=0$; $u(0,t)=0$; $u(1,t)=100\sin(\pi t)$. Compute $u(x,t)$ for 4 times steps with $h=0.25$

Soln:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

$a^2=1$

$a=1.$

$h=0.25$.

$k=ah=1\times 0.25=0.25.$

x: 0 0.25 0.5 0.75 1

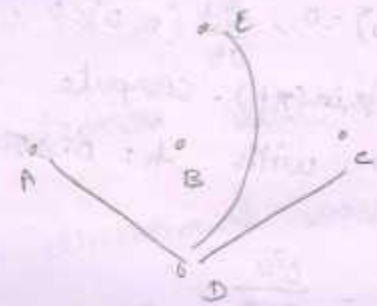
t: 0, 0.25, 0.5, 0.75

$u_1 = \frac{0+0}{2} + k.A$

$u_1 = \frac{\text{add side two values}}{2} + k.A$

$t \backslash x$	0	0.25	0.5	0.75	
0	0	0	0	0	
0.25	0	$\frac{0+0+\frac{1}{2}\cdot 0}{2}$ u_1 0	u_2 0	u_3 0	70
0.5	0	0	0	70.7107	10
0.75	0	0	70.7107	100	70.
1	0	70.7107	100	70.7107	0

22/3/14.



$$U_D = U_A + U_C - U_E$$

2. Solve the eqn. $\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with $u(0,t)=0$; $u(4,t)=0$; $u(x,0)=x(4-x)$; $\frac{\partial u}{\partial t}(x,0)=0$; by taking $h=1$; upto

soln:

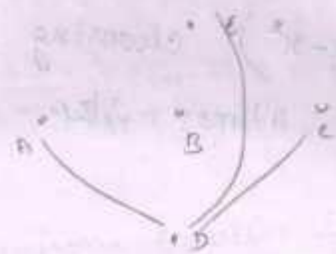
$$\frac{1}{4}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

$$a^2 = 1/4$$

$$a = 1/2.$$

$h = 1$

$k = ah = 0.5 \times 1 = 0.5$.



$$U_D = U_A + U_C - U_E$$

t \ x	0	1	2	3	4
0	0	$x(4-x)$ 3	$x(4-x)$ 4	$x(4-x)$ 3	0
0.5	0	2	3	2	0
1	0	0	0	0	0
1.5	0	−2	−3	−2	0
2	0	−3	−4	−3	0
2.5	0	−2	−3	−2	0
3	0	0	0	0	0
3.5	0	+2	3	2	0
4.	0	3	4	3.	0

8. Solve $u_{tt} = u_{xx}$. $0 < x < 2$; $t > 0$. subject to $u(x,0) = 0$; $u(0,t) = 0$; $u(2,t) = 0$; $u_t(x$

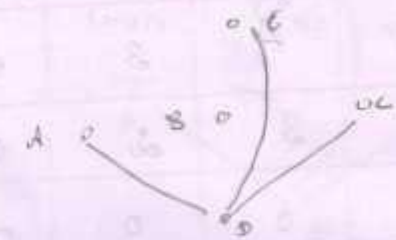
$u_t(x,0) = 100(2x - x^2)$ choosing $h = 1/2$ compute 'u' for 4 times step.

soln:

$u_{tt} = u_{xx}$.

$a^2 = 1 \Rightarrow a = 1$; $h = 0.5$.

$k = ah = 1 \times 0.5 = 0.5$.



$u_D = u_A + u_C - u_E$

$\frac{0+0}{2} + k \times 100(2x - x^2)$

$t \backslash x$	0	0.5	1	1.5	2
0	0	0	0	0	0
0.5	0	37.5	50	37.5	0
1	0	50	75	50	0
1.5	0	37.5	50	37.5	0
2	0	0	0	0	0

1/4/14. Laplace and poisson Equation

The Laplace Equation is $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.

$U_{xx}+U_{yy}=0$ (or) $\nabla^2 u=0$.

The poisons Equation is

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y)$$

(or)

$$U_{xx}+U_{yy}=f(x,y)$$

(or)

$$\nabla^2 u=f(x,y)$$

Here $A=1$; $B=0$; $C=1$

$B^2-4AC = 0-4\times1\times1$

$=-4<0$.

Hence, Laplace and poisson equation are elliptic

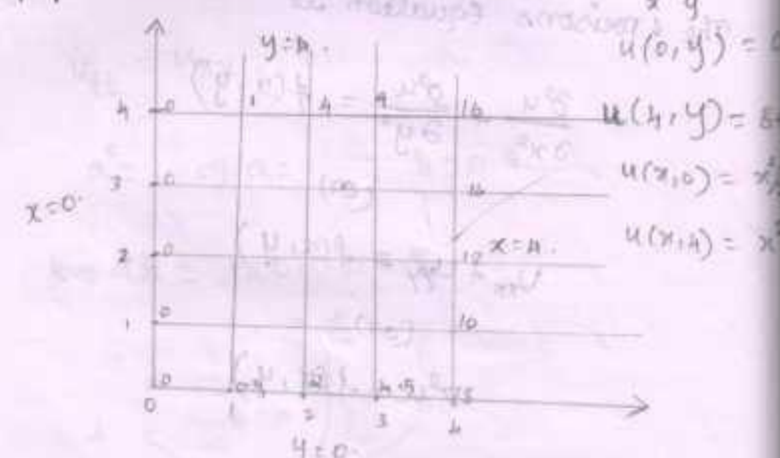
Standard Diagonal five point formula,

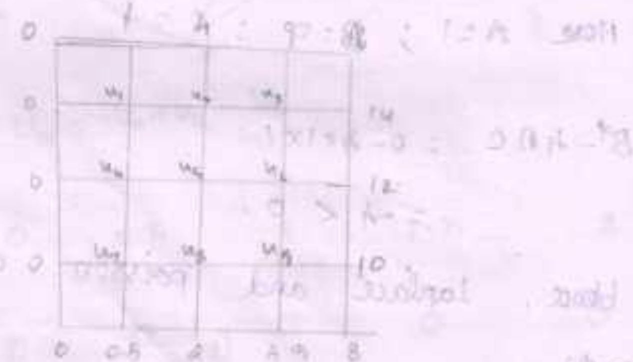
(+) SFPF : $U_E=\frac{U_B+U_D+U_F+U_H}{4}$.

(x) DFPF : $U_E=\frac{U_A+U_C+U_G+U_I}{4}$

1. By Liebmann iteration method solve $u_{xx}+u_{yy}$ over the ~~surface~~ square region of side 4 satisfying $u(0,y)=0$, $0 \le y \le 4$; $u(4,y)=8+2y$; $u(x,0)=x^2/2$, $0 \le x \le 4$, $u(x,4)=x^2$, $0 \le x \le 4$. Com the values at the interior points with $h=k=$

Soln:





Rough values:

SFPF : $u_5 = \frac{0+4+12+2}{4} = 4.5$

DFPF : $u_1 = \frac{0+4+0+u_5}{4} = 2.1$

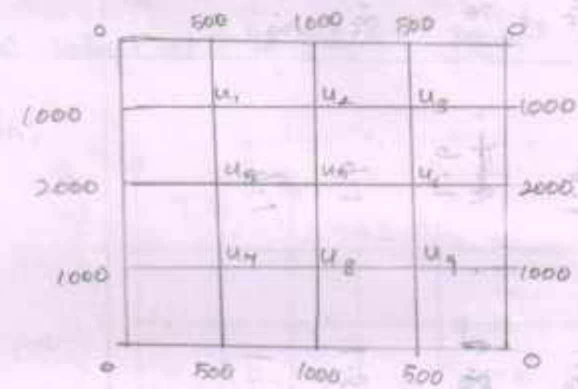
DFPF : $u_3 = \frac{4+16+12+u_5}{4} = 9.1$

DFPF : $u_7 = \frac{0+u_5+0+2}{4} = 1.6$

DFPF : $u_9 = \frac{u_5+12+2+8}{4} = 6.6$

$u_1 = \frac{1+u_2+u_4}{4}$	$u_2 = \frac{4+u_1+u_3+u_5}{4}$	$u_3 = \frac{38+u_2+u_6}{4}$	$u_4 = \frac{u_1+u_5+u_7}{4}$	$u_5 = \frac{u_2+u_4+u_6+u_8}{4}$	$u_6 = \frac{13+u_5+u_3+u_9}{4}$	$u_7 = \frac{0.5+u_4+u_8}{4}$	$u_8 = \frac{2+u_5+u_7+u_9}{4}$	$u_9 = \frac{14.5+u_6+u_8}{4}$
2.1	4.9.	9.1	2.1	4.5	8.1	1.6	3.7.	6.6
2.	4.9.	9.	2	~~2.8~~ 4.7.	8.1.	~~2.7~~. 1.6	3.7.	6.6.
2.	4.9.	9	2.1	4.7.	8.1	1.6	3.7.	6.6.
2	4.9	9	2.1	4.7	8.1	1.6	3.7	6.6.

2. Solve the Elliptic Eqn $U_{xx}+U_{yy}=0$ following square mesh with the bounda[ry] values are shown below



soln :

By symmetry

$u_1 = u_3 \qquad u_1 = u_7$

$u_4 = u_6 \quad \& \quad u_2 = u_8$

$u_7 = u_9 \qquad u_3 = u_9$

Hence,

$u_1 = u_3 = u_7 = u_9$

$u_2 = u_8$

$u_4 = u_6$

Now, we find only u_1, u_2, u_4, u

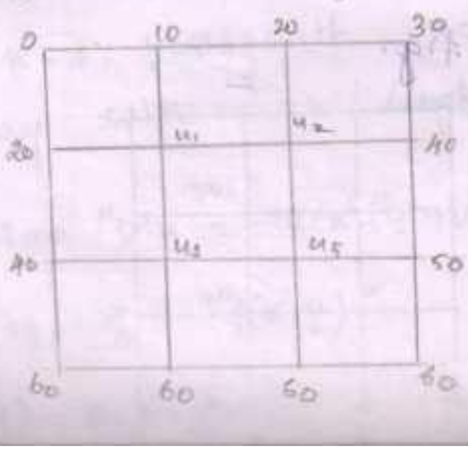
$u_1 = \frac{1500+u_2+u_4}{4}$	$u_2 = \frac{1000+2u_1+u_5}{4}$	$u_4 = \frac{2000+2u_1+u_5}{4}$	$u_5 = \frac{2u_2+2u_4+[illegible]}{4}$
1125	1187.5	1437.5	1500
~~1068.8~~ 1011.7	~~1190.9~~	~~1380.9~~	~~1234.4~~
1031.8	1140.7	1390.7	1265.4
1007.9	~~1099.0~~ 1070.3	1320.9	~~1254~~ 1195.4
992.7	1035.2	1285.2	1160.2
955.1	1017.6	1267.6	1142.6
946.3	1008.8	1258.8	1133.8
941.9	1004.4	1254.4	1129.4
939.7	1002.2	1252.2	1127.2
938.6	1001.1	1251.1	1126.1
938.1	1000.8	1250.6	1125.7
937.9	1000.4	1250.4	1125.4
937.9	1000.3	1250.3	1125.3

Rough value:

$$SFPF : u_5 = \frac{1000+2000+2000+1000}{4} = \frac{6000}{4} = 1500$$

$$DFPF : u_1 = \frac{0+1000+1500+2000}{4} = 1125$$

3. Solve $\nabla^2 u = 0$ over the square region given by the boundary condition as in the fig. below.



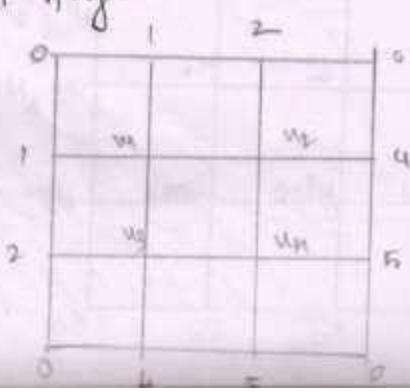
$u_1 = \frac{30+u_4+u_3}{4}$	$u_2 = \frac{60+u_1+u_4}{4}$	$u_3 = \frac{100+u_1+u_4}{4}$	$u_4 = 110+$
4	18.8	28.8	0
19.4	19.9	29.9.	40
20	30.	40	45
25	32.5	42.5	46.3
26.3	33.2	43.2.	46.6
26.6	33.3	43.3	46.7
26.7.	33.4	43.4	46.7.
26.7.	33.4	43.4	46.7.

~~soln~~: Rough :

$u_4 = 0.$

$DFPF: u_1 = \frac{0+20+40+0}{4} = 15.$

4. Solve $U_{xx} + U_{yy} = 0$ over the squar region given by the boundary condit shown in fig.



Soln:

By symmetry, $u_3 = u_2$.

Rough; Assume, $R_4 = 0$

DFPF : $u_1 = \frac{2+2+0+0}{4} = 1$

$u_1 = \frac{2+2u_2}{4}$	$u_2 = \frac{6+u_1+u_4}{4}$	$u_4 = \frac{10+2u_2}{4}$
1	1.6	0
1.4	1.9	3.5
1.5	2.8	3.9
1.9	3	4
2	3	4
2	3	4.

3/4/14. poisson equation problems. $U_{xx}+U_{yy} = -f(x,y)$

1. Solve the equation $\nabla^2 u = -10(x^2+y^2+10)$ over the square mesh with sides $x=0$; $y=0$; $x=3$; $y=3$, with $u=0$ on the boundary with mesh length 1 unit.

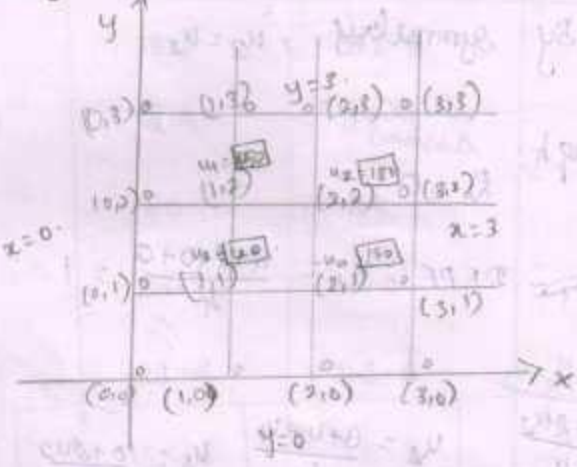
Soln: given $\nabla^2 u = -10(x^2+y^2+10)$

$$U_{xx}+U_{yy} = -f(x,y)$$

$f(x,y) = 10(x^2+y^2+10)$

$h^2 f(x,y) = f(x,y) = 10(x^2+y^2+10)$

$\therefore h =$ [illegible]



By symmetry,

$u_1 = u_4$.

$u_1 = \frac{u_2+u_3+150}{4}$	$u_2 = \frac{u_1+u_4+180}{3} = \frac{2u_1+180}{4}$	$u_3 = \frac{u_1+u_4 \ldots}{4} = \frac{2u_1+\ldots}{4}$
0	0	0
37.5	68.8	48.8
65.7	77.9	62.9
72.7	81.4	66.4
74.5	82.3	67.3
74.9	82.5	67.5
75	82.5	67.5
75	82.5	67.5

2. Solve $\nabla^2 u = 8x^2y^2$ over the square bounded by the lines $x=-2$; $x=2$, $y=-2$, $y=2$ with $u=0$ on the boundary and mesh length $=1$

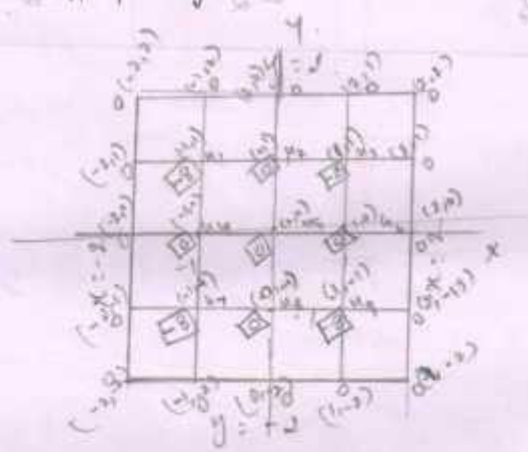
Soln :

Given $\nabla^2 u = 8x^2y^2$

w.k.t $\nabla^2 u = -f(x,y)$

$f(x,y) = -8x^2y^2$

$h^2 f(x,y) = -8x^2y^2$; $(\because h=1)$



By symmetry :

~~$u_1 = u_7$~~	~~$u_1 = u_3$~~	$u_2 = u_4$	~~$u_1 = u_9$~~
$u_2 = u_8$	$u_4 = u_6$	~~$u_3 = u_7$~~	$u_4 = u_8$
~~$u_3 = u_9$~~	~~$u_7 = u_9$~~	$u_6 = u_8$	$u_2 = u_6$

$u_1 = u_7 = u_3 = u_9$

$u_2 = u_8 = u_6 = u_4$.

$u_1 = \frac{u_2+u_4-8}{4} = \frac{2u_2-8}{4}$	$u_2 = \frac{u_1+u_3+u_5}{4} = \frac{2u_1+u_5}{4}$	$u_5 = \frac{u_2+u_4+u_6+u_8}{4}$, $u_5 = u_2$
0	0	0
-2	-1	-1
-2.5	-1.5	-1.5
-2.8	-1.8	-1.8
-2.9	-1.9	-1.9
-3	-2	-2
-3	-2	-2.

Questions	opt1	opt2	opt3	opt4	Answer
If B^2-4AC = 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	parabolic
If B^2-4AC > 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	hyperbolic
If B^2-4AC < 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	elliptic
(f(x+h)-f(x))/h is known as the _________	difference quotient	average	derivative	f(x)	difference quotient
The equation del^2(u) = 0 is _______ equation.	Laplace	Poisson	Heat	Wave	Laplace
One dimensional heat equation is the example of _______ equation.	Laplace	Poisson	Parabolic	Hyperbolic	Parabolic
One dimensional wave equation is the example of _______ equation.	Laplace	Poisson	Parabolic	Hyperbolic	Hyperbolic
The differential equation is said to be parabolic, if	B^2-4AC	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC =0	B^2-4AC
The differential equation is said to be elliptic, if	B^2-4AC	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC =0	B^2-4AC < 0
The differential equation is said to be hyperbolic, if	B^2-4AC	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC =0	B^2-4AC > 0
[x f(xx)+yf(yy)]=0 x>0, y>0 is ____ type of equation.	elliptic	Poisson	Parabolic	Hyperbolic	elliptic
[f(xx)-2f(xx)]=0, x>0, y>0 is ____ type of equation.	elliptic	Poisson	Parabolic	Hyperbolic	Hyperbolic
____________ process is used to solve two dimensional heat equations	Newtons	Gaussian	Laplace	Liebmanns iteration	Liebmanns iteration
The equation (Ñ^2) u = 0 is known as ____________ equation	Laplace	Poisson	heat	wave	heat
The ____________ formula is used to complete the improved value of u,	Newtons	elimination	Liebmanns iteratio	reduction	Liebmanns iteration
The value of u can be improved by ____________ process	Newtons	elimination	Liebmanns iteratio	reduction	Liebmanns iteration
The value of u is obtained at any ____________ lattice points which is the arithmetic mean of the values of u at 4 lattice points near to it	interior	exterior	positive	negative	interior
The value of $u_{i,j}$ in the difference equation are defined only at the _________points	equal	unequal	apex	lattice	lattice
The points of intersection of these families of lines are called ________ points	equal	unequal	apex	lattice	lattice
If B^2 – 4AC > 0 then the given equation is ____________	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	hyperbolic
The differential equation is said to be ____________ in a region R if B^2 - 4AC < 0 at all points of a region	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	elliptic
The differential equation is said to be ____________ in a region R if B^2- 4AC = 0 at all points of the region	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	Parabolic
If (ka)/h < 1, it is stable but the accuracy of the solution decrease with the increasing value of ____________	k	a	(ka)/h	k/h	(ka)/h
If (ka)/h < 1, it is stable but the accuracy of the solution decrease with the increasing value of ____________	k	a	k/h	(ka)/h	(ka)/h
The differential equation is said to be ____________ in a region R if B^2 - 4AC = 0 at all points of the region	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	Parabolic
The differential equation is said to be ____________ in a region R if B^2 - 4AC < 0 at all points of a region	Parabolic	elliptic	hyperbolic	rectangular hyperbolic	elliptic
The points of intersection of these families of lines are called ________ points	equal	unequal	apex	lattice	lattice
Schmidt method belongs to ____________ type	explicit	implicit	elliptic	hyperbolic	explicit
The Poisson's equation belongs to ____________ type	explicit	implicit	elliptic	hyperbolic	hyperbolic
One dimensional heat flow equation belongs to ____________ type	explicit	parabolic	elliptic	hyperbolic	parabolic
Laplace equation in two dimensions belongs to ____________ type	explicit	parabolic	elliptic	hyperbolic	explicit
The error in solving Poisson equation by ____________ methods is of order h^2	Difference	iteration	elimination	interpolation	Difference
The error in solving _________equation by difference method is of order h^2	Newton's	Jacobi's	Poisson	Gaussian	Poisson
The error in solving Poisson's equation by difference methods is of order_______	h	h^2	h^3	h^4	h^2
The equation del^ 2(u) = f(x, y) is known as_________ equation	Poisson	Newtons	Jacobis	Gaussian	Poisson
The value of $u_{i,j}$ is the average of its value at the ________ neighbouring diagonal mesh points	2	3	4	5	4
The value of u(i,j) is the __________of its values at the four neighbouring diagonal mesh points	sum	difference	average	product	average
The value of u(i,j) is the average of its values at the four neighbouring ____________ mesh points	Square	rectangle	diagonal	column	diagonal
The mesh points are also called ____________	grid point	starting point	Ending point	bisection	grid point
The points of intersection of the dividing lines are called ____________	bisection	mesh points	vertex	end point	mesh points
The differential equation is said to be hyperbolic, if	B^2-4AC = 0	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC <= 0	B^2-4AC > 0
The differential equation is said to be elliptic, if	B^2-4AC = 0	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC <=0	B^2-4AC < 0
The differential equation is said to be parabolic, if	B^2-4AC = 0	B^2-4AC > 0	B^2-4AC < 0	B^2-4AC <= 0	B^2-4AC = 0
One dimensional wave equation is the example of _______ equation.	Laplace	Poisson	Parabolic	Hyperbolic	Parabolic
One dimensional heat equation is the example of _______ equation.	Laplace	Poisson	Parabolic	Hyperbolic	Poisson
The equation del^2(u) = 0 is _______ equation	parabolic	elliptic	hyperbolic	equally spaced	parabolic
If B^2-4AC = 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	parabolic
If B^2-4AC > 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	hyperbolic
If B^2-4AC < 0, then the differential equation is said to be______	parabolic	elliptic	hyperbolic	equally spaced	elliptic