SEMESTER – I

MECHANICS **L T P C**

18PHU101 **5 - - 5**

SCOPE:

The main objective of the course is to know how to use Newton's laws of motion to solve advanced problems involving the dynamic motion of mechanical systems and the use of differential equations and other advanced mathematics in the solution of the problems. Also to find the use of conservation of energy and linear and angular momentum to solve dynamics problems.

OBJECTIVE:

- To explain the applications of Newton's law of motion.
- To solve advanced problems involving the dynamic motion of mechanical systems

UNIT - I

Vectors: Vector algebra. Scalar and vector products. Derivatives of a vector with respect to a parameter. Ordinary Differential Equations: 1st order homogeneous differential equations. 2nd order homogeneous differential equations with constant coefficients.

UNIT - II

Laws of Motion: Frames of reference. Newton's Laws of motion. Dynamics of a system of particles. Centre of Mass. Momentum and Energy: Conservation of momentum. Work and energy. Conservation of energy. Motion of rockets. Rotational Motion: Angular velocity and angular momentum. Torque. Conservation of angular momentum.

UNIT - III

Gravitation: Newton's Law of Gravitation. Motion of a particle in a central force field (motion is in a plane, angular momentum is conserved, areal velocity is constant). Kepler's Laws (statement only). Satellite in circular orbit and applications. Special Theory of Relativity: Constancy of speed of light. Postulates of special theory of Relativity. Length contraction. Time dilation. Relativistic addition of velocities.

UNIT - IV

Oscillations: Simple harmonic motion. Differential equation of SHM and its solutions. Kinetic and Potential Energy, Total Energy and their time averages. Damped oscillations.

UNIT - V

Motion of rigid body: Moment of inertia of a rod, disc, spherical shell, solid and hollow spheres - Theory of compound pendulum and Kater‘s pendulum - Determinat ion of ‘g’. Derivation of expressions for angular momentum and kinetic energy of a system of N particles.

Friction-Static Friction - Laws of Friction-Angle and cone of Friction - Motion up and down on a rough inclined plane.

Suggested Reading Books

1. Mathur D.S. (2014), *Mechanics*, New Delhi, S. Chand & Co.
2. Physics – Resnick, Halliday & Walker 9/e, 2010, Wiley
3. Engineering mechanics by D.P. Sharma, 2010, Pearson edition, Delhi, ISBN 978-81-317-3222-9.
4. D. S. Mathur "Elements of Properties of Matter" S. Chand & Co.
5. Mechanics, Second edition, H. S. Hans, S. P. Puri, 2006, Tata McGraw Hill Publishing Company Limited, ISBN 0-07-047360-9.
6. Mechanics Berkeley Physics course, v.1:Charles Kittel, et.al. 2007, Tata McGraw Hill
7. Engineering Mechanics, Basudeb Bhattacharya, 2nd edn., 2015, Oxford University Press
8. Engineering mechanics and Statistics by N. H. Dubey, 2013, Tata McGraw Hill
9. Education Private Limited, ISBN: 978-0-07-107259-5.

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Established Under Section 3 of UGC Act 1956)
Coimbatore – 641 021.

LECTURE PLAN
DEPARTMENT OF PHYSICS

STAFF NAME : **Dr. S. KARUPPUSAMY** SUBJECT NAME: **MECHANICS**
SUB.CODE:**18PHU101** SEMESTER: **I**
CLASS: **I B.Sc., (PHYSICS)**

Sl.No.	Lecture Duration Period	Topics to be covered	Support Material/Page Nos.
UNIT I			
1.	1 hr	**Vectors:** Vector algebra	T1 (2-3)
2.	1 hr	Scalar and vector products	T1(11-20)
3.	1 hr	Derivatives of a vector with respect to a parameter.	T1(38-39)
4.	1 hr	Ordinary Differential Equations	R2 (397-395)
5.	1 hr	1st order homogeneous differential equations	R2(395-396)
6.	1 hr	2nd order homogeneous differential equations with constant coefficients.	R2(396-398)
7.	1 hr	Continuation	R2(396-398)
8.	1 hr	Revision	
Total Number of Hours Planned For Unit I = 8			
UNIT II			
9.	1 hr	**Laws of Motion:** Frames of reference	T1-(67-69)
10.	1 hr	Newton's Laws of motion	T1-(70)
11.	1 hr	Dynamics of a system of particles	T1-(252)
12.	1 hr	Centre of Mass	T1-(259-264)

13.	1 hr	Continuation	T1-(259-264)
14.	1 hr	Momentum and Energy: Conservation of momentum	T1-(257-258)
15.	1 hr	Work and energy. Conservation of energy	T1-(230-233)
16.	1 hr	Continuation	T1-(230-233)
17.	1 hr	Motion of rockets	T1-(275-277)
18.	1 hr	Rotational Motion	T1-(278)
19.	1 hr	Angular velocity and angular momentum	T1-(278)
20.	1 hr	Torque.	T1-(278)
21.	1 hr	Conservation of angular momentum.	T1-(280-281)
22.	1 hr	Revision	
Total Number of Hours Planned For Unit II = 14			
UNIT III			
23.	1 hr	**Gravitation:** Newton's Law of Gravitation.	T1-(626-627)
24.	1 hr	Motion of a particle in a central force field	T1-(627-629)
25.	1 hr	Kepler's Laws	T1-(676)
26.	1 hr	Satellite in circular orbit and applications	T1-(671-672)
27.	1 hr	Special Theory of Relativity	T1-(114-116)
28.	1 hr	Constancy of speed of light	T2-(1022-1023)
29.	1 hr	Postulates of special theory of Relativity	T2-(1022-1023)
30.	1 hr	Length contraction, Time dilation	T2-(1027-1029)
31.	1 hr	Relativistic addition of velocities.	T2-(1039-1042)
32.	1 hr	Continuation	T2-(1039-1042)
33.	1 hr	Revision	
Total Number of Hours Planned For Unit III = 11			
UNIT IV			
34.	1 hr	**Oscillations**: Simple harmonic motion	T1-(322)
35.	1 hr	Differential equation of SHM and its solutions.	T1-(322-327)

36.	1 hr	Continuation	T1-(322-327)
37.	1 hr	Kinetic and Potential Energy	T1-(327-330)
38.	1 hr	Continuation	T1-(327-330)
39.	1 hr	Total Energy and their time averages	T1-(327-330)
40.	1 hr	Continuation	T1-(327-330)
41.	1 hr	Damped oscillations	T1-(390-393)
42.	1 hr	Continuation	T1-(390-393)
43.	1 hr	Revision	
Total Number of Hours Planned For Unit IV = 10			
UNIT V			
44.	1 hr	**Motion of rigid body:** Moment of inertia of a rod	T1-(574-575)
45.	1 hr	Continuation	T1-(577)
46.	1 hr	Moment of inertia of a disc, spherical shell	T1-(581)
47.	1 hr	Continuation	T1-(581)
48.	1 hr	Moment of inertia of a solid and hollow Spheres	T1-(582-583)
49.	1 hr	Continuation	T1-(582-583)
50.	1 hr	Theory of compound pendulum and Kater‘s pendulum	W1
51.	1 hr	Determination of ‘g’	W2
52.	1 hr	Derivation of expressions for angular momentum and kinetic energy of a system of N particles	W2
53.	1 hr	Friction - Static Friction - Laws of Friction	T2 (116)
54.	1 hr	Angle and cone of Friction	T2 (119)
55.	1 hr	Motion up and down on a rough inclined plane	T2 (120)
56.	1 hr	Continuation	T2 (120)
57.	1 hr	Revision	
58.	1 hr	Old Question Paper Revision	

59.	1 hr	Old Question Paper Revision	
60.	1 hr	Old Question Paper Revision	
Total Number of Hours Planned For Unit V = 17			

Suggested Reading Books

1. Mathur D.S. (2014), Mechanics, New Delhi, S. Chand & Co.
2. Physics – Resnick, Halliday & Walker 9/e, 2010, Wiley
3. Engineering mechanics by D.P. Sharma, 2010, Pearson edition, Delhi, ISBN 978-81-317-3222-9.
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8. Engineering mechanics and Statistics by N. H. Dubey, 2013, Tata McGraw Hill
9. Education Private Limited, ISBN: 978-0-07-107259-5.
10. W1 - https://nptel.ac.in/courses/122103010/3
11. W2 - https://people.emich.edu/jthomsen/p332/expt1.pdf

SYLLABUS

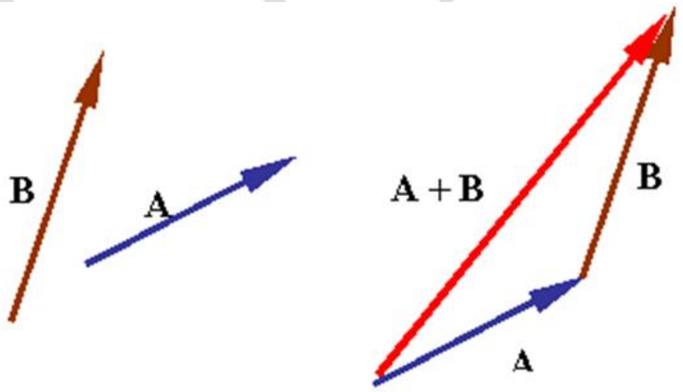
Vectors: Vector algebra. Scalar and vector products. Derivatives of a vector with respect to a parameter. Ordinary Differential Equations: 1st order homogeneous differential equations. 2nd order homogeneous differential equations with constant coefficients.

VECTOR ALGEBRA

Vectors and vector addition:

A scalar is a quantity like mass or temperature that only has a magnitude. On the other had, a vector is a mathematical object that has magnitude and direction. A line of given length and pointing along a given direction, such as an arrow, is the typical representation of a vector. Typical notation to designate a vector is a boldfaced character, a character with and arrow on it, or a character with a line under it (i.e., $\mathbf{A}, \vec{A}, \text{or } \underline{A}$). The magnitude of a vector is its length and is normally denoted by $|\mathbf{A}|$ or *A*.

Addition of two vectors is accomplished by laying the vectors head to tail in sequence to create a triangle such as is shown in the figure.



The following rules apply in vector algebra.

$$a\mathbf{P} = \mathbf{P}a$$

$$a(\mathbf{P} + \mathbf{Q}) = a\mathbf{P} + a\mathbf{Q}$$

$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P}$$

where **P** and **Q** are vectors and *a* is a scalar.

Unit vectors:

A unit vector is a vector of unit length. A unit vector is sometimes denoted by replacing the arrow on a vector with a "^" or just adding a "^" on a boldfaced character (i.e., $\hat{e}$, or $\hat{\mathbf{e}}$). Therefore,

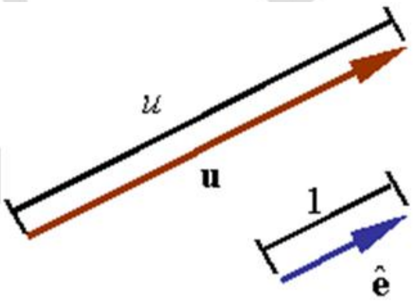
$$|\hat{\mathbf{e}}| = 1$$

Any vector can be made into a unit vector by dividing it by its length.

$$\hat{\mathbf{e}} = \frac{\mathbf{u}}{|\mathbf{u}|}$$

Any vector can be fully represented by providing its magnitude and a unit vector along its direction.

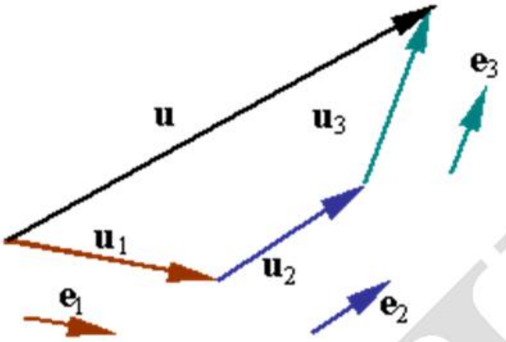
$$\mathbf{u} = u\hat{\mathbf{e}}$$



Base vectors and vector components:

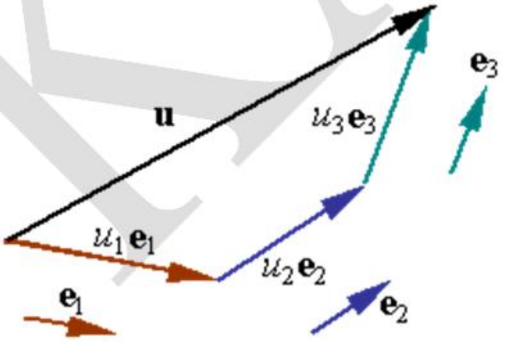
Base vectors are a set of vectors selected as a base to represent all other vectors. The idea is to construct each vector from the addition of vectors along the base directions. For example, the vector in the figure can be written as the sum of the three vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$, each along the direction of one of the base vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$, so that

$$\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3$$



Each one of the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ is parallel to one of the base vectors and can be written as scalar multiple of that base. Let u_1, u_2, and u_3 denote these scalar multipliers such that one has

$$\mathbf{u}_1 = u_1\mathbf{e}_1$$
$$\mathbf{u}_2 = u_2\mathbf{e}_2$$
$$\mathbf{u}_3 = u_3\mathbf{e}_3$$

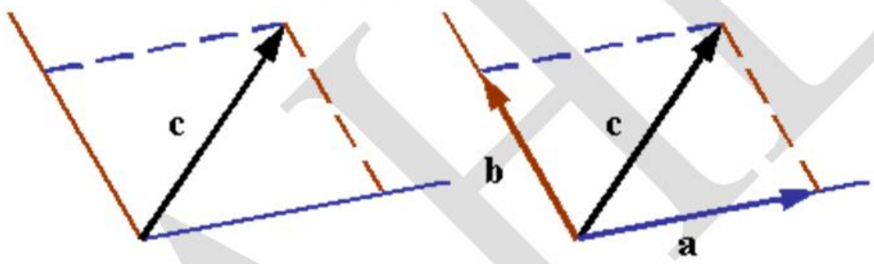


The original vector $\mathbf{u}$ can now be written as

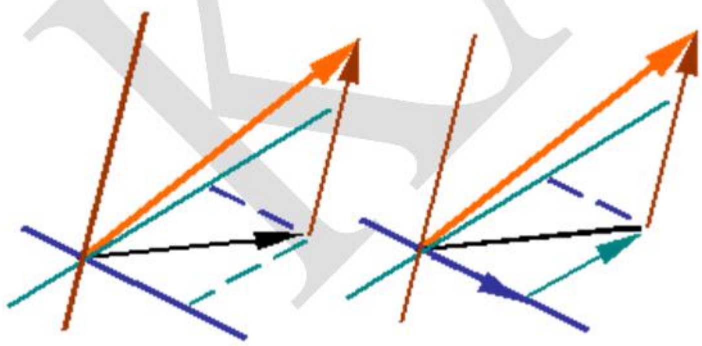
$$\mathbf{u} = u_1\mathbf{e}_1 + u_2\mathbf{e}_2 + u_3\mathbf{e}_3$$

The scalar multipliers u_1, u_2, and u_3 are known as the components of **u** in the base described by the base vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$. If the base vectors are unit vectors, then the components represent the lengths, respectively, of the three vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$. If the base vectors are unit vectors and are mutually orthogonal, then the base is known as an orthonormal, Euclidean, or Cartesian base.

A vector can be resolved along any two directions in a plane containing it. The figure shows how the parallelogram rule is used to construct vectors **a** and **b** that add up to **c**.



In three dimensions, a vector can be resolved along any three non-coplanar lines. The figure shows how a vector can be resolved along the three directions by first finding a vector in the plane of two of the directions and then resolving this new vector along the two directions in the plane.



When vectors are represented in terms of base vectors and components, addition of two vectors results in the addition of the components of the vectors. Therefore, if the two vectors **A** and **B** are represented by

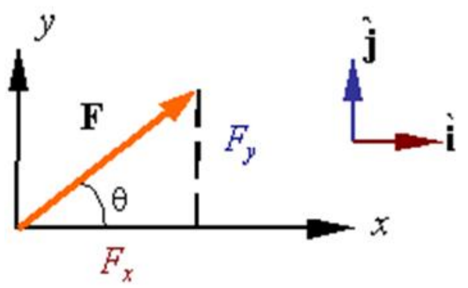
$$\mathbf{A} = A_1\mathbf{e}_1 + A_2\mathbf{e}_2 + A_3\mathbf{e}_3$$
$$\mathbf{B} = B_1\mathbf{e}_1 + B_2\mathbf{e}_2 + B_3\mathbf{e}_3$$

then,

$$\mathbf{A} + \mathbf{B} = (A_1 + B_1)\mathbf{e}_1 + (A_2 + B_2)\mathbf{e}_2 + (A_3 + B_3)\mathbf{e}_3$$

Rectangular components in 2-D:

The base vectors of a rectangular *x*-*y* coordinate system are given by the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ along the *x* and *y* directions, respectively.



Using the base vectors, one can represent any vector **F** as

$$\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}}$$

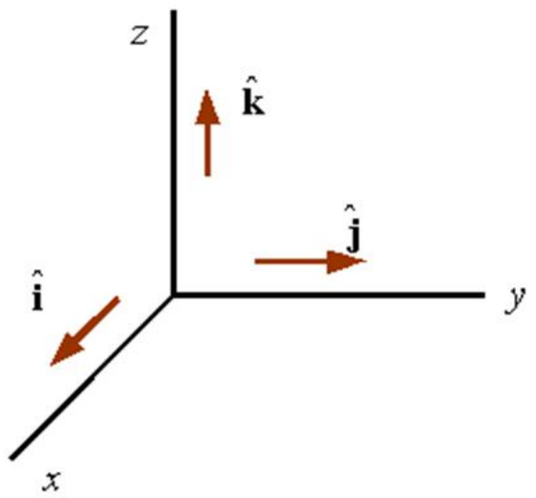
Due to the orthogonality of the bases, one has the following relations.

$$F = \sqrt{F_x^2 + F_y^2}$$
$$F_x = F\cos(\theta)$$
$$F_y = F\sin(\theta)$$

$$\tan(\theta) = \frac{F_y}{F_x}$$

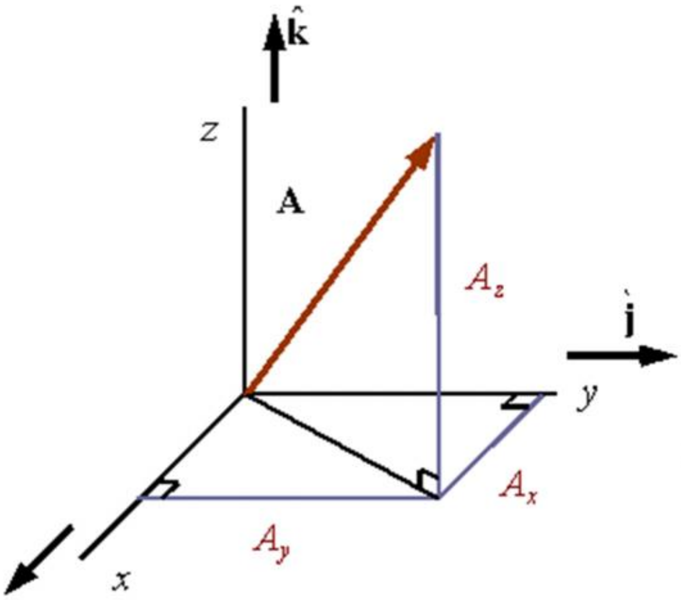
Rectangular coordinates in 3-D:

The base vectors of a rectangular coordinate system are given by a set of three mutually orthogonal unit vectors denoted by $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ that are along the *x*, *y*, and *z* coordinate directions, respectively, as shown in the figure.



The system shown is a right-handed system since the thumb of the right hand points in the direction of *z* if the fingers are such that they represent a rotation around the *z*-axis from *x* to *y*. This system can be changed into a left-handed system by reversing the direction of any one of the coordinate lines and its associated base vector.

In a rectangular coordinate system the components of the vector are the projections of the vector along the *x*, *y*, and *z* directions. For example, in the figure the projections of vector **A** along the *x*, *y*, and zdirections are given by A_x, A_y, and A_z, respectively.



$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

As a result of the Pythagorean theorem, and the orthogonality of the base vectors, the magnitude of a vector in a rectangular coordinate system can be calculated by

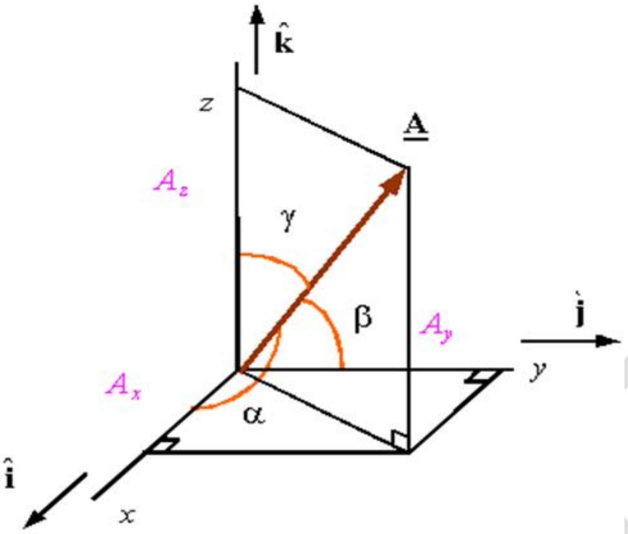
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Direction cosines:

Direction cosines are defined as

$$l = \cos(\alpha)$$
$$m = \cos(\beta)$$
$$n = \cos(\gamma)$$

where the angles α , β, and γ are the angles shown in the figure. As shown in the figure, the direction cosines represent the cosines of the angles made between the vector and the three coordinate directions.



The direction cosines can be calculated from the components of the vector and its magnitude through the relations

$$l = \cos(\alpha) = \frac{A_x}{A}, \quad m = \cos(\beta) = \frac{A_y}{A}, \quad n = \cos(\gamma) = \frac{A_z}{A}$$

The three direction cosines are not independent and must satisfy the relation

$$l^2 + m^2 + n^2 = 1$$

This results form the fact that

$$\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = \frac{A_x^2}{A^2} + \frac{A_y^2}{A^2} + \frac{A_z^2}{A^2} = 1$$

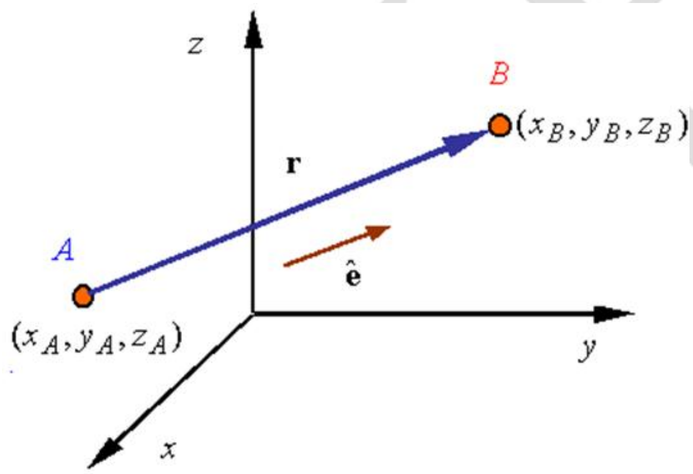
A unit vector can be constructed along a vector using the direction cosines as its components along the *x*, *y*, and *z* directions. For example, the unit-vector $\hat{\mathbf{e}}$ along the vector **A** is obtained from

$$\hat{\mathbf{e}} = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\hat{\mathbf{i}} + \frac{A_y}{A}\hat{\mathbf{j}} + \frac{A_z}{A}\hat{\mathbf{k}}$$
$$= \cos(\alpha)\hat{\mathbf{i}} + \cos(\beta)\hat{\mathbf{j}} + \cos(\gamma)\hat{\mathbf{k}}$$
$$= l\hat{\mathbf{i}} + m\hat{\mathbf{j}} + n\hat{\mathbf{k}}$$

Therefore,

$$\mathbf{A} = A\hat{\mathbf{e}} = A\cos(\alpha)\underline{\mathrm{i}} + A\cos(\beta)\underline{\mathrm{j}} + A\cos(\gamma)\underline{\mathrm{k}}$$

A vector connecting two points:



The vector connecting point *A* to point *B* is given by

$$\mathbf{r} = (x_B - x_A)\hat{\mathbf{i}} + (y_B - y_A)\hat{\mathbf{j}} + (z_B - z_A)\hat{\mathbf{k}}$$

A unit vector along the line *A*-*B* can be obtained from

$$\hat{\mathbf{e}} = \frac{\mathbf{r}}{r}$$

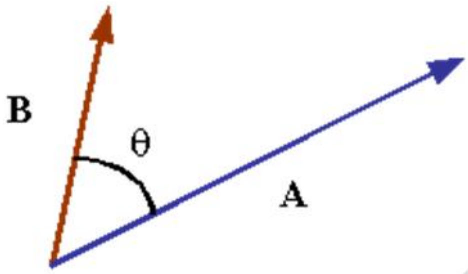
A vector **F** along the line *A*-*B* and of magnitude *F* can thus be obtained from the relation

$$\mathbf{F} = F\hat{\mathbf{e}} = F\frac{\mathbf{r}}{r}$$

Dot product:

The dot product is denoted by " ∘ " between two vectors. The dot product of vectors **A** and **B** results in a scalar given by the relation

$$\mathbf{A} \circ \mathbf{B} = AB\cos(\theta)$$



where θ is the angle between the two vectors. Order is not important in the dot product as can be seen by the dot products definition. As a result one gets

$$\mathbf{a} \circ \mathbf{b} = \mathbf{b} \circ \mathbf{a}$$

The dot product has the following properties.

$$a(\mathbf{b} \circ \mathbf{c}) = (a\mathbf{b}) \circ \mathbf{c} = \mathbf{b} \circ (a\mathbf{c})$$
$$\mathbf{a} \circ (\mathbf{b} + \mathbf{c}) = \mathbf{a} \circ \mathbf{b} + \mathbf{a} \circ \mathbf{c}$$

Since the cosine of 90^o is zero, the dot product of two orthogonal vectors will result in zero.

Since the angle between a vector and itself is zero, and the cosine of zero is one, the magnitude of a vector can be written in terms of the dot product using the rule

$$\mathbf{A} \circ \mathbf{A} = A^2$$

Rectangular coordinates:

When working with vectors represented in a rectangular coordinate system by the components

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$
$$\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$$

then the dot product can be evaluated from the relation

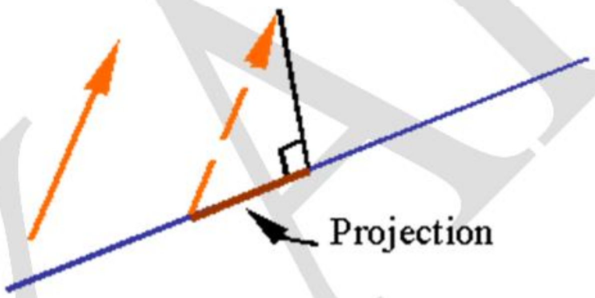
$$\mathbf{A} \circ \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

This can be verified by direct multiplication of the vectors and noting that due to the orthogonality of the base vectors of a rectangular system one has

$$\hat{\mathbf{i}} \circ \hat{\mathbf{j}} = 0$$
$$\hat{\mathbf{i}} \circ \hat{\mathbf{k}} = 0$$
$$\hat{\mathbf{j}} \circ \hat{\mathbf{k}} = 0$$

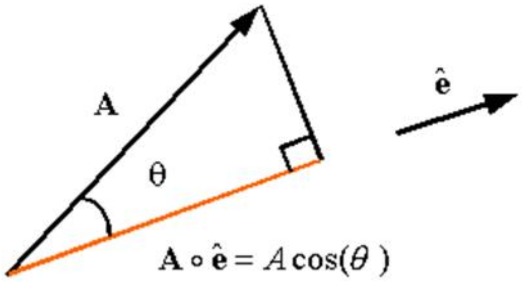
Projection of a vector onto a line:

The orthogonal projection of a vector along a line is obtained by moving one end of the vector onto the line and dropping a perpendicular onto the line from the other end of the vector. The resulting segment on the line is the vector's orthogonal projection or simply its projection.



The scalar projection of vector **A** along the unit vector $\hat{\mathbf{e}}$ is the length of the orthogonal projection **A** along a line parallel to $\hat{\mathbf{e}}$, and can be evaluated using the dot product. The relation for the projection is

$$\text{Scalar projection of } \mathbf{A} \text{ along } \hat{\mathbf{e}} = \mathbf{A} \circ \hat{\mathbf{e}}$$

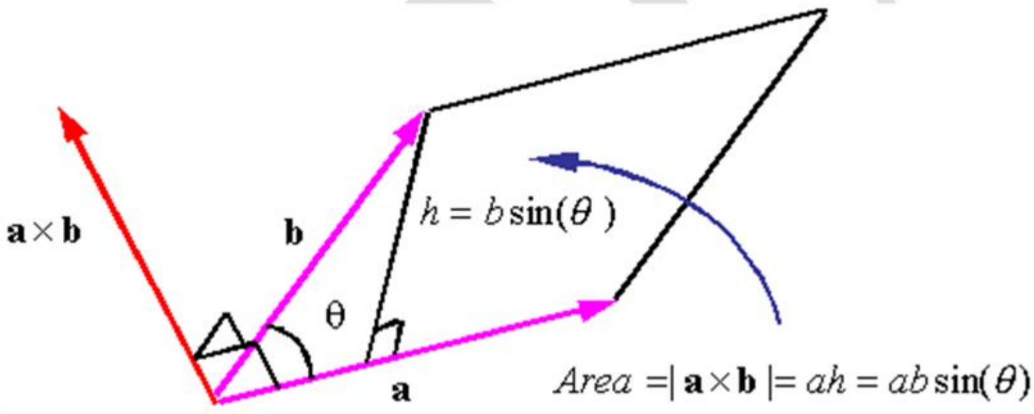


The vector projection of **A** along the unit vector $\hat{\mathbf{e}}$ simply multiplies the scalar projection by the unit vector $\hat{\mathbf{e}}$ to get a vector along $\hat{\mathbf{e}}$. This gives the relation

$$\text{Vector projection of } \mathbf{A} \text{ along } \hat{\mathbf{e}} = (\mathbf{A} \circ \hat{\mathbf{e}})\hat{\mathbf{e}}$$

The cross product:

The cross product of vectors **a** and **b** is a vector perpendicular to both **a** and **b** and has a magnitude equal to the area of the parallelogram generated from **a** and **b**. The direction of the cross product is given by the right-hand rule . The cross product is denoted by a "$\times$" between the vectors



Order is important in the cross product. If the order of operations changes in a cross product the direction of the resulting vector is reversed. That is,

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$$

The cross product has the following properties.

$$a(\mathbf{b} \times \mathbf{c}) = (a\mathbf{b}) \times \mathbf{c} = \mathbf{b} \times (a\mathbf{c})$$
$$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$$
$$(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$$

Rectangular coordinates:

When working in rectangular coordinate systems, the cross product of vectors **a** and **b** given by

$$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$
$$\mathbf{b} = b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}} + b_z\hat{\mathbf{k}}$$

can be evaluated using the rule

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$
$$= (a_yb_z - a_zb_y)\hat{\mathbf{i}} - (a_xb_z - a_zb_x)\hat{\mathbf{j}} + (a_xb_y - a_yb_x)\hat{\mathbf{k}}$$

One can also use direct multiplication of the base vectors using the relations

$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}} \quad \hat{\mathbf{i}}\times\hat{\mathbf{i}} = \mathbf{0}$$
$$\hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \mathbf{0}$$
$$\hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}} \quad \hat{\mathbf{k}}\times\hat{\mathbf{k}} = \mathbf{0}$$

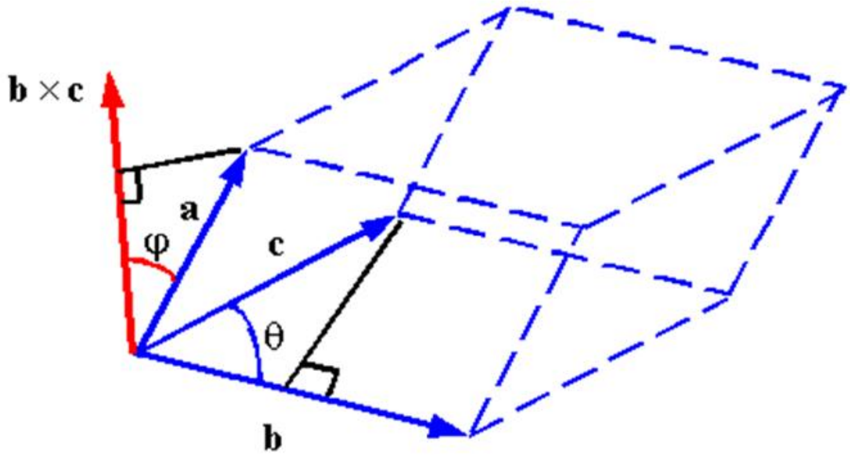
The triple product:

The triple product of vectors **a**, **b**, and **c** is given by

$$\mathbf{a}\circ(\mathbf{b}\times\mathbf{c})$$

The value of the triple product is equal to the volume of the parallelepiped constructed from the vectors. This can be seen from the figure since

$$volume = abc\sin(\theta)\cos(\varphi)$$
$$\mathbf{a}\circ(\mathbf{b}\times\mathbf{c}) = a\,|\mathbf{b}\times\mathbf{c}|\cos(\varphi) = abc\sin(\theta)\cos(\varphi)$$



The triple product has the following properties

$$\mathbf{a} \circ (\mathbf{b} \times \mathbf{c}) = (\mathbf{b} \times \mathbf{c}) \circ \mathbf{a}$$
$$\mathbf{a} \circ (\mathbf{b} \times \mathbf{c}) = \mathbf{c} \circ (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \circ (\mathbf{c} \times \mathbf{a})$$
$$\mathbf{a} \circ (\mathbf{b} \times \mathbf{c}) = -\mathbf{a} \circ (\mathbf{c} \times \mathbf{b})$$

Rectangular coordinates:

Consider vectors described in a rectangular coordinate system as

$$\mathbf{a} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$
$$\mathbf{b} = b_x\hat{\mathbf{i}} + b_y\hat{\mathbf{j}} + b_z\hat{\mathbf{k}}$$
$$\mathbf{c} = c_x\hat{\mathbf{i}} + c_y\hat{\mathbf{j}} + c_z\hat{\mathbf{k}}$$

The triple product can be evaluated using the relation

$$\mathbf{a} \circ (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$
$$= (b_y c_z - b_z c_y)a_x - (b_x c_z - b_z c_x)a_y + (b_x c_y - b_y c_x)a_z$$

SCALAR PRODUCTS

The scalar product of two vectors A and B is denoted by A.B .It is also known as the dot product of the two vectors.

It is definied as the product of the magnitudes of the two vectors. A and B and the cosine of their included angle θ irrespective of the co-ordinate system used.

$$A.B=AB\cos\theta$$

The scalar product is clearly commutative and

$$A.B=B.A$$

The order of the factors may be reversed without in anyway affecting the value of the product.

SOME IMPORTANT POINTS ABOUT SCALAR PRODUCT:

I. If the two vectors have the same direction,θ=0

$$A.B=AB\ Cos0°$$
$$A.B=AB$$

so that the scalar product is equal is to the product of the magnitude of two vectors.

II. If the two vectors have the opposite direction ,θ=π

$$A.B=AB\ \cos\pi$$
$$A.B=AB(-1)$$
$$A.B=-AB$$

so that the scalar product is equal to the negative of their magnitudes

III. If A.B=0,it means that either A or B=0 or A and B are perpendicular to each other

IV. The scalar product obeys the distributive law

$$A(B+C)=AB+AC$$

VECTOR PRODUCT

The vector product of two vectors is denotd by A*B. It is also called the cross product of the two vectors.

It is definied as the vector R whose magnitude is equal to the product of the magnitudes of the two vectors A and B and the sine of their included angle θ.

$$R=A*B=AB\ Sin\ \theta$$

The direction R is perpendicular to the plain containing the vectors A and B.It is to be noted that if the order of vectors is reveresd the sign of the vector product changes

A*B=-B*A

SOME IMPORTANT POINTS ABOUT VECTOR PRODUCT:

I. if the two vector are perpendicular to each other ,then θ=90°

A*B=AB Sin 90°

A*B=AB

II. If the two vectors be parellel then θ=0

A*B=AB Sin θ

A*B=0

Therefore, the vector product of two parellel (or) equal vectors is zero.

III. The distributive law holds good for the cross product of vectors.

DEREVATIVES OF A VECTOR WITH RESPECT TO A PARAMETER:

VECTOR DERIVATIVES-VELOCITY *ACCELERATION

Let r be a single –valued function of a scalar variable t such that for every value of t there exists only one value of r. then as t varies continuously r also changes.

In this case , t reprecents the time variable and r represents the position vector of a moving particle with respect to a fixed origin o.then as t varies continuously ,the point moves along a continuous curve in space so that ,if r and r+δ be the position vectors of the point in positions P and Ṕ relative to origin 0 for the values t and t+δt of the scalar variable, we have

change in the value of r=δr

as δt→0, point Ṕ approaches P and the chord PṔ tends to coincide with the tangent to the curve at P

The time derivative of r with respect to t can be written as

$$\frac{dr}{dt} = \lim_{\delta t \to 0} \delta r \frac{\delta r}{\delta t}$$

the second and third derivatives of r are respectively d^2r/dt^2 and d^3r/dt^3clearly ,δr represents the displacement of the particle in time inrerval δt and δr/δt gives its average velocity cluring interval δt. the limiting value of this average velocity ,as δt→ 0,is the instantaneous velocity v of the particle .thus we have ,

v^2=dr/dt

along the tangent to the path of the particle in the same manner ,if δv be the increase in the velocity v of the particle during the time-interval δt,the rate of change of velocity (or) the average acceleration during the interval =δv/δt and therefore ,instantaneous accleration a of the particle is the limiting value dv/dt of δv/δt as δt→ 0 thus ,

$$a=dv/dt=d^2r/dt^2$$

now since,r=xi+yi+zk and since x,y and z are functions of time ,we also have

$$v=dr/dt=dx/dt \quad i+dy/dt \; j+dz/dt \; k \quad \text{and}$$

$$a=d^2r/dt^2=d^2x/dt^2 \; i+d^2y/dt^2 \; j+d^2z/dt^2 \; k$$

ORDINARY DIFFERENTAL EQUATIONS:

1^{ST} ORDER HOMOGENEOUS DIFFERENTIAL EQUATIONS

The general linear differential equation of first order is

$$dy/dx+p(x)y=f(x) \quad \rightarrow (1)$$

to solve this equation let us substitute

$$y=u(x)v(x) \quad \rightarrow(2)$$

where u and v are function of x to be determined substituting y from (2) in (1).we get,

$$u\,dv/dx+v\;du/dx+puv=f(x)$$

this may be expressed in the form

$$v[du/dx+pu]+u\,dv/dx=f(X) \quad \rightarrow(3)$$

Since u and v are orbitary functions of x ,we may choose u such that

$$du/dx+pu=0 \quad \rightarrow(4)$$

then equation (3) would reduce to

$$u\,dv/dx=f(x) \quad \rightarrow(5)$$

equation (4) may be put in the form

$$du/u=-p(x)dx$$

integrating we get,

$$\log_e u=-\int p(x)dx+\log k \quad \rightarrow(6)$$

where log k is constant of integration

from equations (6) we have

$$\log_e \frac{u}{k}=-\int p(x)dx \text{ or}$$

$$u=ke^{-\int p(x)dx} \quad \rightarrow (7)$$

now from equation (5) we have

$$dv= f(x)dx/u=1/k \quad e^{\int p(x)dx)} \; f(x)\,dx$$

integrating both sides with respect to x, we get

$$v=1/k\int \quad e^{+\int p(x)dx}\ f(x)\ dx+c$$

C being a constant of integration

substituting these values of u and v in equation (2)

we get,

$$y=k\ e^{-\int p(x)dx}[1/k\int e\int p(x)dx f(x)\ dx+c]$$

$$=A\ e^{-\int p(x)dx}+e^{\int p(x)dx}\ e^{\int p(x)}f(x)dx \quad \rightarrow(8)$$

where A=KC is an orbitary constant equation (8) represents the required solution of differentrial equation (1)

SECOND ORDER DIFFERENTIAL EQUATIONS WITH CONSTANT CO-EFFICIENTS

Consider the differential equation

$$d^2y/dx^2+a1dy/dx+a2\ y=f(x) \quad \rightarrow(1)$$

where a1 and a2 are constants and f (x) is a known function of x

let us introduce the symbol of operation

$$D^r=d^r/dx^r \text{ i.e } d/dx=D \text{ and } d^{2/dx^2}=D^2 \quad \rightarrow (2)$$

then equation (1) may be expressed in the form

$$(D^2+a1\ D+a2)y=f(x) \quad \rightarrow (3)$$

Equation (3) can be written as ,

$$L(D)y=f(x)\rightarrow (4)$$

If f(x)=0,then equation (4) reduces to

$$L(D)y=0\rightarrow (5)$$

this is called the veduced equation and its solution is called the complementary function denoted by yc then we may specity

$$L(D)yc=o\rightarrow (6)$$

the general solution of equation (a) consists of the sum of two parts :

the complementary function yc and the particular integral yp which may be seen as follows

the particular integral Yp satisfies the equation (4)

$$L(D)yp=f(x) \rightarrow(7)$$

Adding (6) and (7) we get ,

$$L(D)yc+L(D)yp=f(x)$$

$$L(D)[y_c+y_p]=f(x) \rightarrow (8)$$

If we substitute ,

$$y=y_c+y_p \rightarrow (9)$$

we obtain

$$L(D)y=f(x) \rightarrow (10)$$

this proves the proposition that the general solution of a linear differntial equation with constant co-efficient is the sum of a particular integral yp and the complementary function yc.

If α and β are the roots of auxiliary equation

$$(D^2+a_1 D+a_2)y=0$$

then we may write equation (3) in the form

$$(D-\alpha)(D-\beta)y=f(x) \rightarrow (11)$$

substituting $(D-\beta)y=u \rightarrow (12)$

equation (11) becomes ,

$$(D-\alpha)u=f(x) \text{ (or) } du/dx - \alpha u=f(x) \rightarrow (13)$$

this is a first order linear equation with $p(x)=-\alpha L$ $y(x)=u(x)$

$$u=A1e^{\alpha x}+e^{\alpha x}+\int e^{-\alpha x} f(x)\, dx$$

$$=e^{\alpha x}[A1+\varphi(x)] \rightarrow (14)$$

where

$$d(x) = \int_0^x e^{\alpha x} f(x)dx \rightarrow (15)$$

if we substitute this value of u in equation (12) we get,

$$(D-\beta)y = e^{\alpha x}[A_1 + \varphi(x)] \rightarrow (16)$$

$$(D-\beta)y = F(x) \rightarrow (17)$$

$$F(x) = e^{\alpha x}[A_1 + \varphi(x)] \rightarrow (18)$$

Equation (17) is again first order liner differential equation, hence its solution is,

$$Y= A_2 e^{\beta x} + e^{\beta x} \int e^{-\beta x} F(x)dx \rightarrow (19)$$

Substitiuting value of F(x) from (18) in (19)

we get,

$$Y= A_2 e^{\beta x} + e^{\beta x} \int e^{-\beta x} e^{\alpha x}[A_1 + \varphi(x)]\, dx$$

$$Y= A_2 e^{\beta x} + e^{\beta x} \int e^{(\alpha-\beta)x} [A_1 + \varphi(x)]\, dx$$

$$Y= A_2 e^{\beta x}+ e^{\beta x}A_1 \int e^{(\alpha-\beta)x}dx + e^{\beta x} \int e^{(\alpha-\beta)x} \varphi(x)\, dx$$

$$Y = A_2\, e^{\beta x} + \frac{e^{\beta x} A_1}{\alpha-\beta}\, e^{(\alpha-\beta)x} + e^{\beta x} \int e^{(\alpha-\beta)x} \;\; \varphi(x)\, dx \rightarrow (20)$$

On changing the meaning of constant A_1, the solution of equation (1) may be written as

$$Y = A_1\, e^{\alpha x} + A_2\, e^{\beta x} + e^{\beta x} \int e^{(\alpha-\beta)x} \;\; \varphi(x)\, dx \rightarrow (21)$$

where $\varphi(x)$ is given by equation (15)

In this solution the first two terms represent the complementary function while the remaining last term represents the particular integral.

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POSSIBLE QUESTIONS

2 MARKS

1. Write an example for scalar quantity.
2. Write an example for vector quantity.
3. Define scalar.
4. Solve the dot product vector $(\vec{A}\times\vec{B}).(\vec{C}\times\vec{D})$
5. Solve the cross product vector $(\vec{A}\times\vec{B}) \times (\vec{C}\times\vec{D})$
6. Define vector.

8 MARKS

1. Explain the scalar products with suitable examples.
2. Obtain the derivatives of a vector with respect to a parameter.
3. Explain the vector product with suitable examples
4. Define vector algebra and give some applications of the vector algebra.
5. Explain in detail about the differential equations.
6. Derive the expression for first order homogenous differential equations.
7. Give the final solution for second order homogeneous differential equations.
8. Obtain the differentiation of scalar and vector products $\vec{A}.\vec{B}$ and $\vec{A}\times\vec{B}$.
9. Find the solution for the equation : $X-Y+X\frac{dy}{dx} = 0$
10. Find the solution for the equation : $X^2 = Y^2\frac{dy}{dx}$
11. Prove the relations

    (i) A×(B×C)+B× (C×A)+C×(A×B) = 0

    (ii) A×[B×{C×(D×E)}] = {(C.E)(A.D) – (C.D)(A.E)}B + (C.D)(A.B)E-(C.E)(A.B)D

KARPAGAM ACADEMY OF HIGHER EDUCATION, COIMBATORE-21
DEPARTMENT OF PHYSICS
CLASS : I B.SC PHYSICS
BATCH: 2018-2021
PART A : MULTIPLE CHOICE QUESTIONS (ONLINE EXAMINATIONS)
SUBJECT : MECHANICS
SUBJECT CODE : 18PHU101
UNIT I

Sl. No.	Questions	opt1	opt2	opt3	opt4	Answer
1.	A physical quantity which posses only magnitude are called --------------	scalar	tensor	vector	none	scalar
2.	A physical quantity which posses magnitude and direction are called ------------	scalar	tensor	vector	none	vector
3.	Force is a ------------ quantity	tensor	vector	scalar	both a and b	vector
4.	Mass is a -------------- quantity	vector	tensor	scalar	none	scalar
5.	Physical quantities can be divided into ----------- types.	2	3	4	5	2
6.	The commutation law of addition can be written as ------------	A+B=B+A	A+B+C=(A+B)C	A+(B+C)=(A+C)+B	none	A+B=B+A
7.	The association law of addition can be written as ------------	A+B=B+A	A+B+C=(A+B)C	A+(B+C)=(A+B)+C	none	A+(B+C)=(A+B)+C
8.	The distributive law can be written as ------------	AX(B+C)=(AXB)+(AXC)	A+(B+C)=(A+B)+C	A+B+C=(A+B)C	none	AX(B+C)=(AXB)+(AXC)
9.	The commutation law of multiplication can be written as ------------	AXB=BXA	AX(BXC)=(AXB)XC	A+B+C=(A+B)C	AX(B+C)=(AXB)+(AXC)	AXB=BXA
10.	The association law of multiplication can be written as ------------	AXB=BXA	AX(BXC)=(AXB)XC	A+B+C=(A+B)C	AX(B+C)=(AXB)+(AXC)	AX(BXC)=(AXB)XC
11.	A vector of unit magnitude is called a --------------	zero vector	null vector	unit vector	proper vectors	unit vector
12.	Vectors other than null vectors are referred as -----------	zero vector	unit vector	proper vectors	none	proper vectors
13.	Mechanics can be divided into ------------- types	2	3	4	5	2
14.	The scalar product of two vectors A and B is denoted as -------------	A.B	A+B	AXB	A-B	A.B
15.	If the two vectors A and B are equal and have same direction then the product can be written as -----------	A.A=0	A.A=A	A.A=1	$A.A=A^2$	$A.A=A^2$
16.	The rate of doing work is known as -----------	energy	acceleration	momentum	power	power
17.	The rate of change of velocity is --------------	momentum	impulse	acceleration	power	acceleration
18.	velocity is a ------------ quantity	scalar	vector	tensor	none	scalar
19.	If the lines of action of all forces lying in one plane, then the force system is sai to be -------------	concurrent forces	coplanar forces	equal forces	none	coplanar forces
20.	The couple can be written as ------------	aXf	aXp	axr	none	aXf
21.	The product of mass and volume can be written as ------------	acceleration	density	momentum	energy	density
22.	The vector product of two vectors A and B can be written as -------------	A.B	A+B	AXB	A-B	AXB
23.	The velocity of a particle of zero rest mass is always	c	1	0	>1	c
24.	The energy equivalent of the mass loss E=mC2 is called	gravity	center of mass	momentum	binding energy	binding energy
25.	The relativistic law of conservation of momentum is	p=rm0v	p=mv	p=m0	none	p=rm0v
26.	The first experimental confirmation of the relativity of mass came from	bohr	thomson	bucherer	none	bucherer
27.	The vectors associated with a linear or directional effect are called	unit vectros	polar vectors	scalar	position vectors	polar vectors
28.	A null vector is denoted by	1	-1	0	none	0
29.	The process by which we obtain a product is called multiplication and each of the two vectors is called	factor of product	factor of addition	factor of subtraction	none	factor of product

30.	The interacting forces after the cllision became effectively	1	-1	0	none	0
31.	The external torque applied determines the rotation of the system about	gravity	center of mass	momentum	none	center of mass
32.	In physics, ------------ is the motion of study of particles.	dynamics	mechanics	statics	kinetics	mechanics
33.	In physics, ------------ is the study of rest state of particles.	dynamics	mechanics	statics	kinetics	statics
34.	Inertia is a property of a body by virtue of which the body is	unable to change by itself the state of rest	unable to change by motion in a straight line	unable to change by itself the direction of motion	none	unable to change by itself the state of rest
35.	When an object undergoes acceleration	its speed always increases	its velocity always increases	it always falls towards the earth	a force always acts on it	a force always acts on it
36.	The spin angular momentum of an electron is also referred to as its	angular momentum	linear momentum	momentum	intrinsic angular momentum	intrinsic angular momentum
37.	The magnitude of the angular velocity and hence also that of the angular momentum of the body remains	changed	increases	decreases	unaltered	unaltered
38.	The property momentum was introduced by --------------	galileo	newton	einstein	bohr	newton
39.	Newton's law's remain unchanged or invariant	Under Galilean transformation	Under Lorentz transformation	both a and b	none	Under Galilean transformation
40.	The SI unit of momentum is --------------	kg ms^{-1}	kg ms^{-2}	kg ms^{-3}	kg s^{-2}	kg ms^{-1}
41.	The couple can be written as ------------	aXF	a+F	0	r+F	aXF
42.	The region of space concerned is known as	scalar field	vector field	both a and b	none	both a and b
43.	Density can be defined as	mxv	m/v	ma	none	m/v
44.	Of the following units, the one that is a unit of energy is	Joule	Meter	Newton	none	Joule
45.	The term inertia means which of the following? The tendency of an object to	maintain its mass	remain in motion	remain in rest or motion	stop the motion of other objects	remain in rest or motion
46.	Which pair of variables defines motion?	speed and distance	time and momentum	change of position and passage of time	speed and passage of time	change of position and passage of time
47.	Which two fundamental properties are used to describe motion?	mass and distance	length and time	speed and time	distance and speed	length and time
48.	What is a difference between an object's speed and velocity?	Speed includes direction as well as the rate of travel	Velocity includes time during which travel occurred.	Velocity includes the direction of travel whereas speed does not.	none	Velocity includes the direction of travel whereas speed does not.
49.	What feature of motion is described by acceleration?	The rate at which speed changes	How quickly final velocity is reached	The rate at which velocity changes.	Whether motion is speeding up or slowing down	The rate at which velocity changes.
50.	If the forces on an object are balanced, the object will	remain at rest if initially at rest.	continue moving in a straight line if initially moving in a straight line	both A and B	none	both A and B
51.	According to Newton's second law of motion, acceleration is proportional to force. That means a larger force	produces a smaller acceleration	doesn't affect acceleration	produces a smaller mass	produces a larger acceleration	produces a larger acceleration
52.	Which of the following statements are true of both weight and mass?	Weight is a force, mass is a measure of inertia	Mass depends on gravity, weight does not	Heavier objects weigh more than light objects	Gravity is necessary to measure both weight and mass	Weight is a force, mass is a measure of inertia
53.	What is the unit of weight in the metric system?	kilogram	newton	pound	meters per second squared	newton
54.	According to Newton's second law of motion, what causes a change in the motion of an object? A (An)	decrease in inertia	change in velocity	net force	acceleration	net force
55.	Which of the following indicate that an object has been subjected to an unbalanced force?	The object speeds up	The object slows down	The object changes direction	Any of the above	Any of the above
56.	Which of the following correctly states Newton's third law of motion? Forces occur in matched pairs that are	equal in magnitude and equal in direction	opposite in magnitude and equal in direction	equal in magnitude and opposite in direction	opposite in magnitude and opposite in direction	equal in magnitude and opposite in direction

57.	A person walking on a level surface moves forward because the forces of	his feet pushing forward on the ground	his feet pushing backward on the ground	the ground pushing forward on his feet	the ground pushing backward on his feet	the ground pushing forward on his feet
58.	What are the metric units for momentum?	kg • m/s^2	newton	kg • m/s	newton • m	kg • m/s
59.	In Newton's second law of motion, what is the relationship between acceleratio and mass? Acceleration	is directly proportional tc mass	is inversely proportiona to mass	does not depend on mass	none	is inversely proportional to mass

SYLLABUS

Laws of Motion: Frames of reference. Newton's Laws of motion. Dynamics of a system of particles. Centre of Mass. Momentum and Energy: Conservation of momentum. Work and energy. Conservation of energy. Motion of rockets. Rotational Motion: Angular velocity and angular momentum. Torque. Conservation of angular momentum.

FRAMES OF REFERENCE

Inertia frames of reference are those reference frames in which Newton's laws are valid. They are non-accelerating frames (constant velocity frames).

If a person performed an experiment in a train, he will not observe any difference between the readings obtained when the train is moving at constant velocity or when the train is at rest.

Consider an airplane in flight, moving with a constant velocity. If a passenger in the airplane throws a ball straight up in the air, the passenger observes that the ball moves in a vertical path. The motion of the ball is precisely the same as it would be if the ball were thrown while at rest on Earth. The law of gravity and the equations of motion under constant acceleration are obeyed whether the airplane is at rest or in uniform motion.

Now consider the same experiment when viewed by another observer at rest on Earth. This stationary observer views the path of the ball in the plane to be a parabola and the ball has a velocity to the right equal to the velocity of the plane. Although the two observers disagree on the shape of the ball's path, both agree that the motion of the ball obeys the law of gravity and Newton's laws of motion and even agree on how long the ball is in the air.

Hence, we can say: There is no preferred frame of reference of describing the laws of mechanics.

NEWTON'S LAWS OF MOTION

First Law: A body must continue in its state of rest or of uniform motion along a straight line unless acted upon by an external force.

$$\vec{F} = 0$$

Second Law:

The rate of change of momentum is proportional to the impressed force and takes place in the direction of force.

$$\vec{F} = m\vec{a}$$

Third Law:

For every action, there is an equal and opposite reaction, Thus if F_{12} and F_{21} be the forces exerted

on each other by two interacting bodies respectively, we have $F_{12}= -F_{21}$

The first law implies that, if there be F_{10} force applied, there will be no change in velocity and therefore no acceleration or that,

$$\text{If} \quad F = 0 \text{ we have } a = 0$$

The second law gives a measure of force as the rate of change of momentum, implying that

If P be the momentum of a particle = mv ,

Where v is its velocity,

we have $F = \frac{dp}{dt} = \frac{(mv)}{dt}$

$$F = \frac{dp}{dt} = \frac{d\,(mv)}{dt}$$

If therefore, m be constant and independent of the velocity of the particle, we have

$$F = m\,\frac{d\,(v)}{dt} = ma$$

So that, again if F = 0, a= 0 which is the first law.

Thus, the first law is simply a special case of the second law when F=0 and is thus contained in it. Hence it is that the second law is taken be the law of motion.

This relation F=ma would not hold good in case m does not remain constant.

CENTER OF MASS OF A SYSTEM:

Center of mass is defined as a point in which the entire mass of the system is concentrated.

For simple case, Let us consider a system containing two particles of masses m1 and m2 at points p1 and p2 and r_1, r_2 are the corresponding distances from the orgin O. Then the velocity and acceleration of the particles are:

For first particle:

$$v1 = \frac{dr1}{dt} \qquad a1 = \frac{dv1}{dt}$$

The particle at p1 experiences two forces,

(i) a force F12 due to the particle at p2

(ii) a force Fe1 the external force due to some particles external to the system.

So F1 can be written as,

$$F_1 = F_{21} + F_{e1} \qquad \text{-----------------} \quad (1)$$

By using Newton's second law of motion

$$F1 = m1a1 \qquad \text{----------------------} \ (2)$$

Similarly for second particle:

$$v2 = \frac{dr2}{dt} \qquad a2 = \frac{dv2}{dt}$$

The particle at P2 experiences two forces:

(i) a force F_{21} due to the particle at p1

(ii) a force Fe2 the external forces due to some particles external to the system.

So F2 can be written as,

$$F_2 = F_{21} + F_{e2} \ \text{----------------} \ (3)$$

By using Newton's second law of motion

$$F_2 = m_2a_2 \qquad \text{----------------------} \ (4)$$

Adding equation (2) and (4) we get,

$$F_1 + F_2 = m_1a_1 + m_2a_2$$

$$F = (m_1a_1 + m_2a_2) \ \text{-----------------}(5)$$

Sub $F_1$and F_2 values from equation (1) and (3)

$$F_{12} + F_{e1} + F_{21} + F_{e2} = m_1a_1 + m_2a_2$$

From Newton's third law

$$F_{12} = - F_{21}$$

$$F = F_{e1} + F_{e2} \ \text{-----------}(7)$$

where F is the net force acting on the system.

Let the external force F acting on the system and produces acceleration a_{CM} called acceleration of center of mass of the system.

By Newton's second law,

$$F = Ma_{CM} \text{-------}(8)$$

$$Ma_{CM} = m_1a_1 + m_2a_2$$

Let R be the position vector of center of mass,

$$a_{CM} = d^2R_{CM}/dt^2$$

$$R_{CM} = 1/M\,(m_1r_1 + m_2r_2) \text{---------------------}(9)$$

This equation gives the position of center of mass of a system for two particles.

CONSERVATION OF LINEAR AND ANGULAR MOMENTUM:

From Newton's second law of motion

$$F = dp/dt$$

$$F = d(mv)/dt$$

If the total force F is zero then dp/dt = 0 and the linear momentum is conserved.

CONSERVATION OF ANGULAR MOMENTUM:

The angular momentum (L) of the parrticle can be written as,

$$L = I\omega$$

where I is the moment of inertia and ω is the angular velocity. we can write the moment of inertia as $I = mr^2$ and the angular velocity $\omega = v/r$.

$$L = mrv$$

$$L = r \times p \text{--------}(1)$$

The torque can be written as,

$$\tau = I\alpha$$

$$\tau = r \times F \text{-----}(2)$$

Let us form the vector product of r with both sides of equation,

$$F = dP/dt \text{------}(3)$$

Subsitituted (3) in (2)

$$\tau = r \times dP/dt \text{-------}(4)$$

By using the vector product rule,

$$= \frac{d}{dt}(r \times p) - \frac{dr}{dt} \times p$$

If both the vectors are parllel then the second term is Zero,

$$\tau = \frac{dL}{dt} \text{-------}(7)$$

Thus time rate of change of the vector angular momentum of a particle equal to the vector torque acting on it. Equation (7) is the analogue newton's second law of motion in the case of rotational motion.

If the total torque T=0 , then dL/dt = 0 and therefore the angular momentum is conserved in the absence of an external torque.

WORK AND ENERGY

In classical mechanics, we are concerned with the concepts of work and energy.

Work-energy concepts are based on Newton's laws and do not involve any new physical principles.

The work-energy concept can therefore be applied to the dynamics of a mechanical system without resorting to Newton's laws. This is useful in complex situation (e.g. variable forces) and applicable to a wide range of phenomena.

Energy

- Energy is present in the Universe in various forms.
- Energy can be transformed from one form to another.

Conservation of energy states that when energy is changed from one form to another, the total amount in the system remains the same.

Work Done By A Constant Force

When force is parallel to displacement, Work done by a force = force ×× displacement

$$W=FsW=Fs$$

The SI unit is N mN m or joule (J).

In general, the work done by an agent exerting a constant force is the product of the component of the force in the direction of the displacement and the magnitude of the displacement of the force.

When the force is an an angle ϕ to the displacement,

$$W=Fs\cos\phi=\vec{F}.\vec{s}$$

A force does no work if the object does not move.
Work done by a force is zero if the force is perpendicular to the displacement.
Work can be positive or negative. E.g. When an object is lifted from the ground, the work done by the applied force is positive, while the work done by the gravitational force is negative.

CONSERVATION OF ENERGY:

The energy of a particle is its capacity to do work and is measured by the amount of work it is capable of doing in virtue of (i) its motion (ii) position. The first one is called as the Kinetic energy and the second one is called as the potential energy of the particle represented by the symbols T and U respectively.

Since kinetic energy (T) of a particle is due to its motion, let us obtain a proper expression for it for a particle moving with velocity v. Conservation of energy principle implies that the total energy of the particle (kinetic + potential) is conserved.

Suppose, under the action of force, the particle moves from position1 to position 2. Then workdone by the particle will be ,

$$W_{12} = \int_1^2 F.\mathrm{dr} \text{ ----------(1)}$$

$$= \int_1^2 \frac{dp}{dt}\ .dr \text{--------------------(2)}$$

The momentum can be written as,

$$p=mv$$

$$p = m(dr/dt) \text{ ------(3)}$$

take this $dr/dt = \dot{r}$ and $dr = \dot{r}\ dt$ -------(4)

Sub. equations (3) and (4) in (2)

$$W_{12} = \int_1^2 \frac{dm\dot{r}}{dt}\ .\dot{r}dt$$

$$= \int_1^2 \frac{d}{dt}(mv^2)\ dt$$

$$W_{12} = T_2 - T_1 \text{ ------------(5)}$$

Force can be written in terms of potential energy,

$$F = -\ \Delta\ v \text{ ----(6)}$$

Sub. equation (6) in eqn (1),

$$W_{12} = v_1 - v_2 \text{------(7)}$$

By equating (5) and (7) we get

$$T2 + V2 = T1 + V1$$

$$T + V = \text{a constant} \text{ ---------------(8)}$$

From equation (8) we can say that the sum of kinetic and potential energies of a total system is conserved (or) unchanged.

MOTION OF ROCKETS

A rocket is based on the principle of conservation of momentum. It has a combution chamber in which the fuel (solid or liquid) is burnt. The burnt gas escaping in a vertically downward direction gives an equal vertical upward momentum to the rocket.

Suppose the intial mass of the rocket is m and the velocity is v. Let in a short time t, the mass of the gas esacaping be m.

The mass of escaping gas = m

The velocity of escaping gas relative to the rocket = v_r

The velocity of the gas with respect to the earth = $v_1 = v - v_r$ ---------(1)

The momentum of the escaping gas = $mv_1 = m (v - v_r)$ [downward] -------(2)

The momentum of the rocket = $m(v- v_r)$ [upwards] -------(3)

Intial momentum of the rocket = MV -----(4)

If the increase in velocity of the rocket is v,

Then the momentum of the rocket = (M+m) (V+v) ---------(5)

Final momentum of the rocket = sum of mometum of rocket in upward and the momentum of the rocket with velocity v

Add equations (3) and (5)

Final momentum of the rocket = $(M-m) (V+v) + m (v - v_r)$ ------(6)

From equations (4) and (6),

Total change in momentum of the system = $(M-m) (V+v) + m (v - v_r) – MV$ ---------(7)

But In general

Change in momentum = F x t ----------(8)

Equating (7) and (8),

$$F \times t = (M-m) (V+v) + m (v - v_r) – MV$$

We can write force interms of gravity

$$F = -Mg \text{ -------(9)}$$

(Here the negative sign showns that the weight acts in the downward direction)

$$-Mgt = (M-m)(V+v) + m(v - v_r) - MV$$

$$-Mgt = Mv - mv - mv_r$$

By neglecting mv, we get,

$$-Mgt = Mv - mv_r$$

$$mv_r - Mgt = Mv$$

Divide both sides by t,

$$(mv_r/t) - (Mgt/t) = (Mv/t)$$

$$(mv_r/t) - Mg = (Mv/t)$$

$$M(v/t) = (m/t)\, v_r - Mg$$

$$Ma = v_r(m/t) - Mg$$

$$a = [(v_r/M) \times (m/t)] - g$$

The term $[(v_r/M) \times (m/t)]$ is the upward acceleration of the rocket and g is the downward acceleration due to gravity.

The rocket ascends higher, the value of g decreases. Also due to the increase in the value of M, the acceleration of the rockets increases. Thus the acceleration of the rocket continuously increases till the whole of the fuel is burnt.

ANGULAR MOMENTUM:

Consider a particle of mass m moving with a velocity v about an axis at a distance r.

The momentum of the particle = mv

moment of momentum of the particle = mvr

Moment of momentum is also called angular momentum and is denoted by L.

Angular momentum $L = mvr$

$$L = m(r\omega)r$$

$$L = mr^2\omega$$

The moment of inertia of the particle about the axis of rotation,

$$I = mr^2$$

$$L = I\,\omega$$

Angular momentum is a vector quantity. Its dimensions are $[ML^2T^{-1}]$.

ANGULAR VELOCITY:

The rate of change of angular displacement is the angular velocity of the particle.

Let dθ be the angulr displacement made by the particle in time dt, then the angular velocity of the particle

$$\omega = d\theta/dt$$

Unit ---rad S^{-1}

Dimension -----T^{-1}

TORQUE:

Consider a particleof mass m moving about an axis in a circular path of radius r. Let an external force F act on the particle along the tangent to the circular path. The moment of the force = Fr. This moment of force is also called torque represent by the symbol τ.

$$\tau = Fr.$$

But

$$F = ma = mr\alpha$$

$$\tau = (mr\alpha)r$$

$$\tau = (mr^2 \alpha)$$

$$I = mr^2$$

$$\tau = I\alpha$$

Hence torque is equal to the product of moment of inertia and angular acceleration. Torque is also defined as the rate of change of angular momentum.

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POSSIBLE QUESTIONS

2 MARKS

1. State Newton's first law and give an example.
2. State Newton's second law and give an example.
3. Define Work and give a unit for work.
4. What is the difference between angular momentum and linear momentum?
5. Briefly explain about energy.
6. Define Torque and give an expression for Torque.

8 MARKS

1. Explain Newton 'three laws of motion in detail.
2. Write short notes on dynamic of a system of particles.
3. Define centre of mass and obtain the expression for position of centre of mass.
4. Explain the conservation of momentum with suitable examples.
5. Describe the motion of rockets.
6. Explain the conservation of energy with examples.
7. Write a short note on conservation of angular momentum with examples.
8. Define (i) torque (ii) couple (iii) angular velocity (iv)work.
9. What are the difference between mass and weight

KARPAGAM ACADEMY OF HIGHER EDUCATION, COIMBATORE-21
DEPARTMENT OF PHYSICS
CLASS : I B.SC PHYSICS
BATCH: 2018-2021
PART A : MULTIPLE CHOICE QUESTIONS (ONLINE EXAMINATIONS)
SUBJECT : MECHANICS
SUBJECT CODE : 18PHU101
UNIT II

Sl. No.	Questions	opt1	opt2	opt3	opt4	Answer
1.	The acceleration in a body is due to ------------	unbalanced forces	gravitational forces	balanced forces	electromagnetic forces	unbalanced forces
2.	The Newton's first law of motion can also be called as -----------	momentum	inertia	acceleration	impulse	inertia
3.	The moment of inertial of an object is independent of --------------	rotation axis	mass	center of mass	angular velocity	angular velocity
4.	The inertia of an object can be measured by its --------------	velocity	mass	density	energy	mass
5.	The momentum of an object can be defined as the product of mass and -------------	force	volume	velocity	energy	velocity
6.	The property momentum was introduced by --------------	galileo	newton	einstein	bohr	newton
7.	Angular momentum is a vector product of --------------	linear momentum and r	velocity and mass	torque and r	force and acceleration	linear momentum and r
8.	The inability of a body to change its state of rest or motion by itself is called ------------	velocity	momentum	inertia	impulse	inertia
9.	Momentum is a ----------- quantity	scalar	vector	tensor	none	vector
10.	Force is the product of mass and ---------------	acceleration	velocity	density	none	acceleration
11.	The SI unit of momentum is --------------	kg ms^{-1}	kg ms^{-2}	kg ms^{-3}	kg s^{-2}	kg ms^{-1}
12.	The unit of moment of the force is -------------	N	Nm	Nsm	none	Nm
13.	Two equal and opposite forces whose lines of action do not coincide are said to constitute a -------------	torque	couple	moment of inertia	moment of force	couple
14.	The product of force and displacement of the body is known as --------------	work	impulse	power	momentum	work
15.	A single force which gives the effect of all other forces acting together is called -------------	concurrent forces	coplanar forces	resultant	none	resultant
16.	A system of co-ordinate axes which defines the position of a particle in a three dimensional space is called -------------	reference axis	axial co-ordinates	frame of reference	none	frame of reference
17.	The rate of change momentum is directly proportional to the applied force is called Newton's ------------ law	first	second	third	none	second
18.	The relation F=ma holds good for only -------------- frames	inertial	non-inertial	both a and b	none	inertial
19.	In physics, ------------ is the motion of study of particles.	dynamics	mechanics	statics	kinetics	mechanics
20.	In physics, ------------ is the study of rest state of particles.	dynamics	mechanics	statics	kinetics	statics
21.	The word conservation means ------------	co-efficient	variable	constant	none	constant
22.	The angular momentum L can be written as ------------	rxF	rXp	FXp	ma	rXp
23.	The twisting force that will cause rotation in a body is known as -----------	torque	couple	coplanar forces	none	torque
24.	The torque T can be written as ------------	rxF	rXp	FXp	ma	rxF
25.	A body must continue its state of rest or in motion unless acted upon by an external force is called Newton's ---------------- law.	first	second	third	none	first
26.	The moment of the couple is the rotating effect of the -----------	mass	couple	torque	acceleration	couple
27.	The change in angular momentum is equal to	moment of inertia	moment of force	force	angular acceleration	moment of force
28.	The angular momentum of an object	always unchanged	always changed	will changes in the absence of external force	will changes in the presence of external force	will changes in the absence of external force
29.	Rocket is based on the principle of Newton's ----------- law.	first	second	third	all the above	third
30.	When the external torue applied is zero, then the total -------------- is conserved	energy	linear momentum	angular momentum	none	angular momentum
31.	Among the following which one is a vector quantity	displacement	speed	mass	velocity	velocity

32.	By changing the planes of rotation to center is about the disturbing effect due to	mass	energy	position	gravity	gravity
33.	The first law was proposed by Newton in the year of	1985	1876	1764	1687	1687
34.	The couple is provided by the clamp in support and that is called	momen of inertia	center of mass	gravity	fixed moment	fixed moment
35.	No two electrons in an atom can exist in the same state.this was stated by	newton	pauli	bohr	fermi	pauli
36.	If a bus starts suddenly, the passengers in the bus will tend to fall	in the direction opposite to the direction of motion of bus	in the direction same to the direction of motion of bus	sideways	none	in the direction opposite to the direction of motion of bus
37.	An athelete runs some distance before taking a long jump because	he gains to take him through long distance	it helps him to apply large force	by running action and reaction forces increase	by running the athelete gives himself larger inertia of motion	by running the athelete gives himself larger inertia of motion
38.	Inertia is a property of a body by virtue of which the body is	unable to change by itself the state of rest	unable to change by motion in a straight line	unable to change by itself the direction of motion	none	unable to change by itself the state of rest
39.	Qualitative definition of force is given by	newton's first law	newton's second law	newton's third law	newton's gravitational law	newton's second law
40.	SI unit of force is	kg ms-1	Newton	Dyne	none	Newton
41.	The action and reaction forces referred to in the third law	must act on the same object	may act on different objects	must act on different objects	none	must act on different objects
42.	When an object undergoes acceleration	its speed always increases	its velocity always increases	it always falls towards the earth	a force always acts on it	a force always acts on it
43.	When a net force acts on an object, the object will be accelerated in the direction of the force with acceleration proportional to	the force on the object	the velocity of the object	the mass of the object	the inertia of the object	the force on the object
44.	Rate of change of momentum is equal to	acceleration	work	force	impulse	force
45.	kg m/s is the unit of	momentum	speed	acceleration	force	force
46.	A rocket or jet engine works on the principle of	conservation of energy	conservation of mass	conservation of momentum	none	conservation of mass
47.	A canon after firing recoils due to	conservation of energy	backward thrust of gases	newton's third law	newton's gravitational law	newton's third law
48.	An object will continue to accelerate untill	the resultant force begins to decrease	the resultant force on it is zero	the velocity changes direction	the resultant force on it is increased continuously	the resultant force on it is zero
49.	A rider on a horseback falls back when horse starts running all of a sudden because	rider is taken back	rider is suddenly afraid of falling	inertia of rest keeps the upper part of body at rest	none	inertia of rest keeps the upper part of body at rest

SYLLABUS

Gravitation: Newton's Law of Gravitation. Motion of a particle in a central force field (motion is in a plane, angular momentum is conserved, areal velocity is constant). Kepler's Laws (statement only). Satellite in circular orbit and applications. Special Theory of Relativity: Constancy of speed of light. Postulates of special theory of Relativity. Length contraction. Time dilation. Relativistic addition of velocities.

NEWTON'S LAW OF GRAVITION:

Everybody in the universe attracts every other body with a force which is directly propotional to the product of their masses and inversely propotional to the square of the distance between their centers.

Consider two bodies A and B of masses M and m respectively.

The distance between their centers = R

$$F \propto Mm \rightarrow (1)$$

$$F \propto \frac{1}{R^2} \rightarrow (2)$$

Combining (1) and (2) we get,

$$F \propto \frac{Mm}{R^2}$$

$$F = \frac{GMm}{R^2}$$

Here G is the universal constant of gravitation.

If M = m = 1 , R = 1, F = G

The constant of gravitation is numerically equal to the force of attraction between two masses at a unit distance apart.

Therefore $G = 6.670 \times 10^{-11}$ Nm^2Kg^{-2}

MOTION OF A PARTICLE IN A CENTRAL FORCE FIELD:

Centeral force is that force which is always directed away or towards a fixed center and the magnitude of which is a function of the distance from the fixed center (origin).

Let us consider a particle of a mass m moving in a plane and attracted towards the origin of co-ordinates with a force that is inversely proportional to the square of the distance from it. If (r, θ) be its plane polar co-ordinates, the kinetic energy will be

$$T = ½ m(\dot{r}^2 + r^2 \dot{\theta}^2)$$

and the potential energy

$$V = -\int_{\alpha}^{r} F.dr$$
$$= -\int_{\alpha}^{r} -k/r^2.dr$$
$$= -k/r$$

because force F is attractive in nature and varies inversely as the square of the distance from the origin (i.e) $F = \frac{-K}{r^2}$, where K is the constant of proportionality. The Lagrangian will become

L = T – V

$$= ½\, m(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{K}{r^2} \text{----------(1)}$$

Here

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r}, \frac{\partial L}{\partial r} = mr\dot{\theta}^2 - \frac{K}{r^2} \quad \text{--------(2)}$$
$$\frac{\partial L}{\partial \dot{\theta}} = m\dot{\theta}r^2 ., \frac{\partial L}{\partial \theta} = 0 \text{ --------(2)}$$

The equations of motion for such a case will be

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0 \text{ --------(3)}$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \text{--------(3)}$$

applying equation (2) to equation (3) we find that,

$$\frac{d}{dt}(m\dot{r}) - mr\dot{\theta}^2 + \frac{K}{r^2} = 0$$
$$\frac{d}{dt}(mr\dot{\theta}^2) = 0$$

Solving further, we get

$$(m\ddot{r}) - mr\dot{\theta}^2 + \frac{K}{r^2} = 0 \quad \text{------- (4)}$$
$$2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta} = 0 \text{ ------ (4)}$$

Equation (4) are the desired equation of motion for the particle moving in the influence of a centeral force.

KEPLER'S LAWS:

(i) SHAPE OF THE ORBIT:

Every planet moves in an elliptical orbit with the sun being at one of its foci.

(ii) VELOCITY IN THE ORBIT : [Laws of areas]

The radius vector drawn from the sun to the planet sweeps out equal areas in equal intervals of time.

Consider a planet moving in an elliptical orbit with the sun at the focus S. Let the radius vector of the planet describe a small angle dθ in time dt while moving from the position A to B. Arc AB = R dθ

Area swept in time dt

$$= \text{Area ABS} = ½ R \times R\, d\theta$$

$$= ½ R^2\, d\theta$$

$$\text{Area Velocity} = \frac{½ R^2\, d\theta}{dt}$$

$$= ½ R^2 \left(\frac{d\theta}{dt}\right)$$

$$= ½ R^2 \omega = \text{Constant}$$

In this Case ½ R^2 ω represents the areal velocity of the radius when R decreases, ω increases but ½ R^2 ω is always constant.

As areal velocity ½ R^2 ω is constant, therefore ½ MR^2 ω is also a constant. Therefore MR^2 ω remains constant throughout the motion of a planet in its orbit. The term MR^2 ω represents the angular momentum of the planet.

Area of the ellipse = π ab

Here a and b are the semi-major and semi-minor axes of the ellipse respectively. time period of revolution of the planet around the sun,

$$T = \frac{Area}{Areal\ Velocity}$$

$$T = \frac{\pi\, ab}{½\, R2\, \omega} = \frac{2\pi\, ab}{R2\, \omega}$$

(iii) TIME PERIOD OF PLANETS:

The square of the time period of revolution of a planet around the sun is propotional to the cube of the semi-major axis of the orbit. Thus if T_1 and T_2 are the semi-major axes of the planets respectively.

$$(T^{1}{}_{2})/(a^{3}{}_{1}) = (T^{2}{}_{2})/(a^{3}{}_{2}) = \text{Constant}$$

$$T^2 / a^3 = \text{constant}$$

$$T^2 \; \alpha \; a^3$$

SATELLITES IN CIRULAR ORBITS:

In the solar system, different planets revolve round the sun. The radii of the orbits and their time periods of revolution are different for different planets. In the these cases, the force of gravition between the sun and the planet provides the necessary centripetal force. similarly the moon revolves around the earth and the force of gravition between the earth and the moon provides the necessary centripetal force for the moon to be in the orbit. Here moon the satellite of the earth.

ORBITAL VELOCITY:

Suppose a satellite of mass m is moving in an orbit of radius r around the earth. If v is the velocity of the satellite in its orbit, the necessary centripetal force (mv^2/ r) is provided by the gravitional force of attraction between the earth and the satellite.

$$\text{Gravitional Force} = GMm / r^2 \text{ ------ (1)}$$

$$\text{Centripetal Force} = mv^2 / r \text{ ------ (2)}$$

(diagram)

Equating (1) and (2)

$$GMm / r^2 = mv^2 / r$$

$$v = \sqrt{\frac{GM}{r}} \text{ -------(3)}$$

If g is the acceleration due to gravity at the position of the satellite.

$$mg = GMm / r^2$$

$$GM/ r = gr \text{ -----------(4)}$$

$$V = \sqrt{gr}$$

For a satellite, revolving very near the earth's surface

$$v = \sqrt{gR} \text{------(5)}$$

Taking the value of

$$g = 9.80 \text{ m/s}^2$$

$$R = 64 \times 10^5 \text{ m}$$

$$v = \sqrt{9.8} \times 10^5$$

$$v = 7.920 \text{ km/s}$$

TIME PERIOD:

$$v = \sqrt{GM/r}$$

$$v = r\omega$$

$$r\omega = \sqrt{GM/r}$$

$$\omega = \sqrt{GM/r^3} \text{------- (6)}$$

Substituted the ω value in time period expression,

$$T = 2\pi/\omega$$

$$T = 2\pi\sqrt{r^3/GM}$$

$$r = R+h$$

$$T = 2\pi\sqrt{(R+h)^3/GM}$$

$$T = 2\pi\sqrt{(R+h)/g} \text{ ------------------(7)}$$

ESCAPE VELOCITY:

Escape velocity is defined as the velocity with which a body has to be projected vertically upwards from the earth's surface so that is escapes the earth's gravitional pull.

If v is the esape velocity, then the intial kinetic energy of projection ½ mv^2 must be equal to the workdone in moving the body from the surface of the earth to infinity.

Mass of the earth = M

Radius of the earth = R

Mass of the body = m

The gravitional force $F = GMm/r^2$------------(1)

Workdone in moving the body from the surface of the earthto the infinity

$$W = \int F \, . dr$$

$$= \int_R^r \frac{GMm}{r^2} . dr$$

$$W = GMr/R \text{ ------- (2)}$$

In general, the kinetic energy can be written as,

$$\text{Kinetic Energy} = \tfrac{1}{2} mv^2$$

Equating (2) and (3)

$$\frac{1}{2} m v_e^2 = GMm/R$$

$$v_e^2 = 2GM/R$$

$$v_e = \sqrt{(2GM)}/R$$

The object is at the surface of the earth

$$mg = GMm/R^2$$

$$GM = gR^2 \text{--------} (5)$$

Sub equation (5) in (4)

$$V_e = \sqrt{2gR} \text{---------}(6)$$

The value of g at the earth's surface = 9.8 m/s^2

$$R = 64 \times 10^5 \text{ m}$$

$$V = \sqrt{2} \times 9.8 \times 64 \times 10^5$$

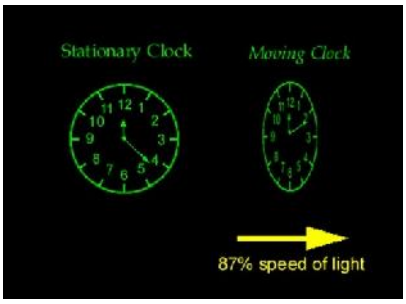
$$V = 11.2 \text{ km/s.}$$

SPECIAL THEORY OF RELATIVITY:

In the Special Theory of Relativity, published in his so-called "miraculous year" of 1905, Einstein had the audacity to turn the question around and ask: what must happen to our common notions of space and time so that when the distance light travels in a given time is measured, the answer is always 300,000 km/s? For example, if a spaceship fires a laser beam at a piece of space debris flying towards it at half the speed of light, the laser beam still travels at exactly the speed of light, not at one-and-a-half times the speed of light. He began to realize that either the measurement of the distance must be smaller than expected, or the time taken must be greater than expected, or both.

In fact, Einstein realized, the answer is both: space "contracts" and time "dilates" (or slows). Some of the motion through space can be thought of as being "diverted" into motion through time (and vice versa), in much the same way as a car travelling north-west diverts some of its northwards motion towards the west. Thus, the dimensions of space and time affect each other, and both space and time are therefore relative concepts, with only the unvarying speed of light providing the bedrock on which the universe is built. This revolutionary idea flew in the face of the long-held notion of simultaneity (the idea that events that appear to happen at the same time for one person should appear to happen at the same time for everyone in the universe) and suggested that it was impossible to say in an absolute sense whether two events occurred at the same time if those events were separated in space.

In a nutshell, the Special Theory of Relativity tells us that a moving object measures shorter in its direction of motion as its velocity increases until, at the speed of light, it disappears. It also tells us that moving clocks run more slowly as their velocity increases until, at the speed of light, they stop running altogether. In fact, it also tells us (as we will see in subsequent sections) that the mass of a moving object measures more as its velocity increases until, at the speed of light, it becomes infinite.



Thus, one person's interval of space is not the same as another person's, and time runs at different rates for different observers travelling at different speeds. To some extent, the faster you go, the slower you age and the slimmer you are! The reason this is not obvious in everyday situations is that the differences at everyday speeds are infinitesimally small, and only really become apparent at speeds approaching that of light itself ("relativistic" speeds). The closer the speed of an objects approaches to the speed of light, the more warped lengths and time intervals become.

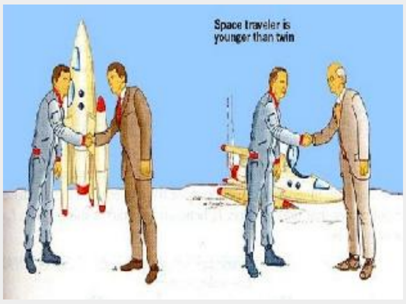
The amount of length contraction and time dilation is given by the Lorentz factor, named after the Dutch physicist Hendrik Lorentz, who had been exploring such transformation equations since as early as 1895, long before Einstein began his work (indeed some would claim that Lorentz and Henri Poincaré between them anticipated almost everything in Einstein's Special Theory of Relativity). The Lorentz factor, γ (gamma) is given by the equation $\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, so that the effect increases exponentially as the object's velocity *v* approaches the speed of light *c*. Thus, the calculations show that at 25% of the speed of light, the effect is just 1.03 (a mere 3% slowing of time or contraction of length); at 50% of the speed of light, it is just 1.15; at 99% of the speed of light, time is slowed by a factor of about 7; and at 99.999, the factor is 224. So, if it were possible to travel in a spaceship at, say, 99.5% of the speed of light, a hypothetical observer looking in would see the clock moving about 10 times slower than normal and the astronaut inside moving in slow-motion, as though through treacle.

A couple of real-life examples may help to make the effects of special relativity clearer. Experiments have been carried out where two identical super-accurate atomic clocks were synchronized, and then one was flown around the world on an airplane while the other stayed at home. The clock which travelled recorded marginally less passage of time than the other (as predicted by the theory), although the difference was of course minimal due to the relatively slow speeds involved. Our fastest military airplanes can only travel at about 1/300,000 of the of the speed of light, so the time dilation effect γ is only about a ten-thousandth of 1%.

At very high speeds, however, the effect is much more noticeable. Experiments have demonstrated that an ultra-short-lived muon particle, which habitually travels at 99.92% of the speed of light, actually lives about 25 times longer and travels about 25 times further than it theoretically should. Particles travelling at speeds up to 99.99% the speed of light in the CERN particle accelerator in Switzerland experience the same kind of relativity-induced time travel, experiencing a γ factor of around 5,000, allowing the artificial persistence of even shorter-lived particles such as phi mesons.

So, travelling at close to the speed of light would theoretically allow time travel into the future, as time slows down for the speeding object in order to "protect" the cosmic speed limit of the speed of light. A corollary of all this is that, if it were possible to exceed the speed of light, then it would also be possible to go back in time, which raises the possibility of time-travel paradoxes (where a person goes back in time and interferes in their own past or kills their own grandparents, etc), although some scientists believe that some as yet undiscovered law of physics may intervene to prevent such paradoxes. Actually, special relativity does not specifically forbid the existence of particles that travel faster than light, and there is a hypothetical sub-atomic particle called a tachyon, which would indeed spend its entire life travelling faster than the speed of light, but it is currently still hypothetical.

Another phenomenon associated with the dilation of time is the so-called “twins effect” (sometimes referred to as the “twins paradox”), where an astronaut returns from a near-light speed voyage in space to find his stay-at-home twin many years older than him (as travelling at relativistic high speeds has allowed him to experience only one year of time while ten years have elapsed on Earth). This is sometimes considered a paradox in that each twin sees the other twin as travelling, and so, it is argued, each should see the other aging more slowly. But in fact this is based on a misunderstanding of relativity, because in reality only one twin experiences acceleration and deceleration, and so only one twin ages less.



(Click for a larger version) In the "twins effect" (or paradox), a space traveller returns to Earth younger than his twin (Source: The Reference Frame:http://motls.blogspot.com/2007/02/resolving-einsteins-twin-paradox.html)

An equivalent paradox concerning the related phenomenon of length contraction is often referred to as the "tunnel paradox", whereby a hypothetical train approaching a tunnel at near-light speed sees the tunnel as much shorter than it really is, whereas someone in the tunnel sees the approaching train as short.

Essentially, then, the Special Theory of Relativity can be boiled down to its two main postulates: firstly, that physical laws have the same mathematical form when expressed in any inertial system(so that all motion, and the forces that result from it, is relative); and secondly that the speed of light is independent of the motion of its source and of the observer, and so it is NOT relative to anything else and will always have the same value when measured by observers moving with constant velocity with respect to each other. Not such a scary proposition at first glance, perhaps, but it does lead to some rather interesting implications, which we will begin to consider in subsequent sections.

CONSTANCY OF SPEED OF LIGHT

Logically, one would expect the ultimate cosmic speed limit to be infinity, which after all is defined as the biggest number imaginable. However, in our universe, the relatively modest speed of 300,000 kilometres per second, the speed of light, is the de facto maximum speed, and in practice one can never catch up with a beam of light. It was the 16 year old Albert Einstein who first gave serious consideration to why this might be the case, in the final years of the 19th Century.

The speed of light had been measured often and very accurately, going back to the Danish astronomer Ole Rømer (or Roemer) who had shown in 1675 that light travels at a finite (although very high) speed. Rømer's observations of the moons of Jupiter yielded a speed of light of about 225,000 km/s, although subsequent, more accurate, experiments have actually shown it to be 299,792,458 metres per second (about 300,000 km/s).

In 1868, the equations of the Scottish mathematician and physicist James Clerk Maxwell, building on the earlier work of Ampère, Coulomb and Faraday, noted that all electromagnetic waves travelled at exactly the same speed as light in empty space, and that light itself was a kind of wave rippling through the invisible magnetic and electric fields. Maxwell concluded that light and other electromagnetic waves should travel at a certain fixed speed relative to some unconfirmed ambient medium he called "aether".

The famous Michelson-Morley experiments of 1887, in a failed attempt to prove that light travels through a medium known as aether, had unexpectedly demonstrated that light travels at the same speed regardless of whether it was measured in the direction of the Earth's motion or at right angles to it. At least this is the case when light travels through a vacuum: when light moves from medium to medium (like from air to glass, for example), its speed can of course change depending on the new medium's index of refraction, and this "bending" of light is essentially how lenses work, as had long been understood.

Thus, whether a source of light is moving towards you or away from you, the light still travels at a steady 300,000 km/s, completely contrary to classical physics and common sense. It was the young Einstein's genius to explain just WHY the speed of light is constant and does not depend on the speed of its source or its observer. In 1905, Einstein (and also the French mathematician Henri Poincaré, who was coming to similar conclusions at around the same time, although from a more mathematical point of view) realized that the whole idea of aether as a medium for light to travel in was totally unnecessary, providing, as we will see, that one was willing to abandon the idea of absolute time.

Einstein also realized that that Maxwell's equations led to an apparent paradox or inconsistency in the laws of physics, because it suggested that if one could catch up to a beam of light one would see a stationary electromagnetic wave, which is an impossibility. Einstein hypothesized, therefore, that the speed of light actually plays the role of infinite speed in our universe, and that in fact nothing can ever travel faster than light (and certainly that nothing in the universe could ever travel at anything like infinite speed). The constant speed of light was to become one of the two main planks of his Special Theory of Relativity, which we will examine in more detail in the next section. The other main plank was the "principle of relativity" (or "principle of invariance"), an idea first stated by the great Italian physicist Galileo Galilei as early as 1632. Galileo argued that the mechanical laws of physics are the same for every inertial observer (those moving uniformly with constant speed in a straight line), and therefore that, purely by observing the outcome of mechanical experiments, one cannot distinguish a state of rest from a state of constant velocity.

Galileo used the example of a ship travelling at constant speed, without rocking, on a smooth sea, and he noted that any observer doing experiments in a dark room below deck would not be able to tell whether the ship was moving or stationary. As a slightly updated example, a ball thrown in an airplane flying at 800 kilometres per hour 12,000 metres above the Earth follows the same path, and is indistinguishable from, one thrown in the airplane at rest on the ground.

When he combined the principle of relativity with the constant speed of light, it became clear to Einstein that the speed of light was also independent of the speed of the observer (as well as of the speed of the source of the light), and that everyone in the universe, no matter how fast they were moving, would always measure the speed of light at exactly the same 300,000 km/s.

POSTULATES OF SPECIAL THEORY OF RELATIVITY

Einstein discerned two fundamental propositions that seemed to be the most assured, regardless of the exact validity of the (then) known laws of either mechanics or electrodynamics. These propositions were the constancy of the speed of light and the independence of physical laws (especially the constancy of the speed of light) from the choice of inertial system. In his initial presentation of special relativity in 1905 he expressed these postulates as:[1]

- The Principle of Relativity – The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems in uniform translatory motion relative to each other.[1]

- The Principle of Invariant Light Speed – "... light is always propagated in empty space with a definite velocity [speed] *c* which is independent of the state of motion of the emitting body" (from the preface).[1] That is, light in vacuum propagates with the speed *c* (a fixed constant, independent of direction) in at least one system of inertial coordinates (the "stationary system"), regardless of the state of motion of the light source.

The derivation of special relativity depends not only on these two explicit postulates, but also on several tacit assumptions (made in almost all theories of physics), including the isotropy and homogeneity of space and the independence of measuring rods and clocks from their past history.

Following Einstein's original presentation of special relativity in 1905, many different sets of postulates have been proposed in various alternative derivations. However, the most common set of postulates remains those employed by Einstein in his original paper. A more mathematical statement of the Principle of Relativity made later by Einstein, which introduces the concept of simplicity not mentioned above is:

Special principle of relativity: If a system of coordinates K is chosen so that, in relation to it, physical laws hold good in their simplest form, the *same* laws hold good in relation to any other system of coordinates K' moving in uniform translation relatively to K.

Henri Poincaré provided the mathematical framework for relativity theory by proving that Lorentz transformations are a subset of his Poincaré group of symmetry transformations. Einstein later derived these transformations from his axioms.

Many of Einstein's papers present derivations of the Lorentz transformation based upon these two principles.

Einstein consistently based the derivation of Lorentz invariance (the essential core of special relativity) on just the two basic principles of relativity and light-speed invariance. He wrote:

The insight fundamental for the special theory of relativity is this: The assumptions relativity and light speed invariance are compatible if relations of a new type ("Lorentz transformation") are postulated for the conversion of coordinates and times of events... The universal principle of the special theory of relativity is contained in the postulate: The laws of physics are invariant with respect to Lorentz transformations (for the transition from one inertial system to any other arbitrarily chosen inertial system). This is a restricting principle for natural laws.

Thus many modern treatments of special relativity base it on the single postulate of universal Lorentz covariance, or, equivalently, on the single postulate of Minkowski spacetime.

The constancy of the speed of light was motivated by Maxwell's theory of electromagnetism and the lack of evidence for the luminiferous ether. There is conflicting evidence on the extent to which Einstein was influenced by the null result of the Michelson–Morley experiment. In any case, the null result of the Michelson–Morley experiment helped the notion of the constancy of the speed of light gain widespread and rapid acceptance.

LENGTH CONTRACTION

The length of an object measured in a reference frame that is moving with respect to the object is always less than the proper length. This effect is known as **length contraction**.

- Length contraction only takes place along the direction of motion.

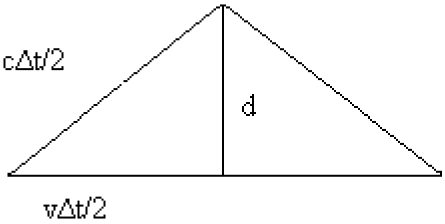
The proper length, Lp, of an object is the length of the object measured by someone at rest relative to the object.

Derivation:

Consider a spaceship traveling with a speed v from one star to another. There are two observers: one on Earth and the other in the spaceship. The observer at rest on Earth and also assumed to be at rest with respect to the two stars measures the distance between the stars to be L_p. According to this observer, the time it takes the spaceship to complete the voyage is $\Delta t = L_p/v$. Because of time dilation, the space traveler using the spaceship clock measures a smaller time of travel: $\Delta t_p = \Delta t/\gamma$. The space traveler claims to be at rest and sees the destination star moving towards the spaceship with speed v. Because the spaceship reaches the star in the time Δt_p, the traveler concludes that the distance L between the stars is shorter than L_p. The distance measured by the space traveler is $L = v\Delta t_p = v(\Delta t/\gamma)$.

Because $L_p = v\Delta t$, we see that $\mathbf{L_p = \gamma L}$

TIME DILATION



Consider an observer O' on a train moving at velocity v relative to the ground. A mirror is fixed on the ceiling of the train carriage. An observer O is outside the train and stationary relative to the ground.

A pulse of light directed toward the mirror is emitted, and at some time later after reflecting from the mirror, the pulse arrives back at the laser. Observer O' measure the time interval ΔtpΔtp. Because the light pulse has speed c, the time it takes to travel from point A to the mirror and back to point A is
Δtp=2d/c.
Now consider the same set of events as viewed by O in a second frame. According to this observer, the mirror and laser are moving to the right with a speed v, and as a result the sequence of events appears different. The observer O will see the light leaving the source at an angle with respect to the vertical direction. According to second postulate of special theory of relativity, both observers must measure c for the speed of light.

$$\left(\frac{c\Delta t}{2}\right)^2 - \left(\frac{v\Delta t}{2}\right)^2 + d^2$$

$$\Delta t = \frac{2d}{c\sqrt{1-\frac{v^2}{c^2}}}$$

Because $\Delta t_p = \frac{2d}{c}$, we can express the result as:

$$\Delta t = \frac{\Delta t_p}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma \Delta t_p$$

A clock moving past an observer at a speed v runs more slowly than an identical clock at rest with respect to the observer by a factor of 1/γ.

Proper time, ΔtpΔtp is the time interval between two events as measured by an observer who sees two events occur at the same position.

RELATIVISTIC ADDITION OF VELOCITIES

$$U_{x'} = \frac{U_x - v}{1 - \frac{U_x V}{c^2}}$$

where

U_x will be velocity of object in S frame. S' frame will be moving with velocity v wrt S. $U_{x'}$ will be velocity of object in S' frame.

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POSSIBLE QUESTIONS

2 MARKS

1. Define gravitation.
2. Explain briefly the Newton's law of gravitation.
3. State Kepler's law.
4. What are the Postulates of special theory of Relativity?
5. Write a short note on theory of relativity.

6 MARKS

1. Explain the motion of particle in a central force field.
2. Describe the Kepler's three laws in detail.
3. Explain the length contraction and time dilation in relativity.
4. Obtain the expression for escape velocity of a satellite.
5. Give the applications of satellites in various fields.
6. Derive the expression for orbital velocity of a satellite.
7. Explain the special theory of relativity and give its postulates.

KARPAGAM ACADEMY OF HIGHER EDUCATION, COIMBATORE-21
DEPARTMENT OF PHYSICS
CLASS : I B.SC PHYSICS
BATCH: 2018-2021
PART A : MULTIPLE CHOICE QUESTIONS (ONLINE EXAMINATIONS)
SUBJECT : MECHANICS
SUBJECT CODE : 18PHU101
UNIT III

sl. No.	Questions	opt1	opt2	opt3	opt4	Answer
1.	Total energy of body is sum of	kinetic energies	potential energies	forces	both a and b	both a and b
2.	Energy can neither be created nor be destroyed, but it can be changed from one form to another This law is known as	kinetic energies	potential energies	conservation of energy	both a and b	conservation of energy
3.	Which one of the following choices is an example of a conservative force?	normal force	tension	static frictional force	elastic spring force	elastic spring force
4.	According to the Law of Conservation of Energy, the total amount of energy in the universe----------	increases	decreases	remains constant	none	remains constant
5.	________ happens when a force causes an object to move in the same direction that the force is applied.	heat	density	work	none	work
6.	Special theory of relativity treats problems involving	inertial frame of reference	non-inertial frame of reference	non-accelerated frame of reference	accelerated frame of reference	inertial frame of reference
7.	According to special theory of relativity which one is not an absolute quantity	time	mass	height	both a and b	both a and b
8.	Length contraction happens only	perpendicular to direction of motion	along direction of motion	parallel to direction of motion	both a and b	along direction of motion
9.	Relativity mechanics is applicable for a particle which is moving with a velocity	Greater than that of light	Less than that of light	Comparable to that of light	none	Comparable to that of light
10.	According to special theory of relativity which one is not an absolute quantity	The state of motion of the observer as well as upon the quality	The state of motion of the observer only	The quantity that is being measured.	none	The quantity that is being measured.
11.	A frame which is moving with zero acceleration is called	Non-inertial frame	Inertial frame	both a and b	none	Inertial frame
12.	Newton's law's remain unchanged or invariant	Under Galilean transformation	Under Lorentz transformation	both a and b	none	Under Galilean transformation
13.	The laws of mechanics in all initial frame of reference are	Same	Different	constant	none	Same
14.	The acceleration of a particle under Galilean transformation is	Invariant	Non-invariant	variant	none	Invariant
15.	The special theory of relativity was proposed by	Newton	Madam Currie	Einstein	bohr	Einstein
16.	The mass energy relation was proposed by	Newton	Madam Currie	Einstein	bohr	Einstein
17.	The Lorentz transformation will converted to Galilean transformation when the relative velocity v between two inertial frames will satisfy the condition	v>>c	v<<c	v=c	none	v<<c
18.	The length of an object is maximum in a reference frame in which it is	at rest	in motion	moving in a circular path	none	at rest
19.	The length contraction phenomenon is known is known as	Galilean contraction	Lorentz Fitzgerald contraction	Lorentz contraction	none	Lorentz Fitzgerald contraction
20.	The time interval between two event in a reference in a reference frame which is in motion is	Maximum	minimum	unchanged	none	Maximum
21.	If the velocity of a moving particle is comparable to velocity of light then the mass of the movi object is	Greater than when it is rest	Smaller than when it is at rest	Equal	both a and b	Greater than when it is rest
22.	Einistein's mass energy equation E=mc2 implies that	Energy disappears to reappear as mass	Mass disappears to reappear as energy	All the above statements are correct	All the above statements are wrong	All the above statements are correct
23.	Relative velocity of two particles moving with velocity of light of light in opposite direction is	0	1	c	2c	c

24.	The force which appears to be acting on a body due to the acceleration non-inertial frame is known as	Pseudo force	Coriolis force	balanced force	unbalanced force	Pseudo force
25.	What is a difference between an object's speed and velocity?	Speed includes direction as well as the rate of travel.	Velocity includes time during which travel occurred.	Velocity includes the direction of travel whereas speed does not.	There is no difference	Velocity includes the direction of travel whereas speed does not.
26.	Of the following units, the one that is a unit of energy is	Joule	Meter	Newton	none	Joule
27.	A stationary object may have	potential energy	kinetic energy	velocity	none	potential energy
28.	Which is the best example that something has kinetic energy?	a car parked on a steep hill	a tennis ball rolling across the court	a picture hanging on the wall	a piece of coal before it's burned	a tennis ball rolling across the court
29.	Conservation of energy means that	energy can be created but not destroyed	energy can be destroyed but not created	energy can both be created and destroyed	energy can neither be created nor destroyed	energy can neither be created nor destroyed
30.	An inertial frame is	Accelerated	Decelerated	Moving with uniform velocity or at rest	May be accelerated, decelerated or moving with constant velocity	Moving with uniform velocity or at rest
31.	"All the inertial frames are equivalent" this statement is called the principle of -----------	relative motion	equivalence	inertia	Correspondence.	relative motion
32.	Special theory of relativity deals with the events in the frames of reference which move with constant-----------------	speed	velocity	acceleration	momentum.	velocity
33.	According to relativity, the length of a rod in motion	is same as its rest length	is more than its rest length	is less than its rest length	may be more or less than or equal to rest length depending on the speed of rod.	is less than its rest length
34.	According to special theory of relativity	speed of light is relative	speed of light is same in all inertial frames	time is relative	mass is relative.	speed of light is same in all inertial frames
35.	In any collision	total momentum is not conserved.	total kinetic energy is conserved.	total momentum is conserved.	total momentum is not conserved but total kinetic energy is conserved	total momentum is conserved.
36.	Two objects collide and stick together. Which of the following is false?	momentum is conserved	kinetic energy is lost	kinetic energy is conserved	momentum is lost	kinetic energy is conserved
37.	The momentum for any object is calculated by	dividing mass by velocity.	multiplying mass by velocity.	both a and b	none	multiplying mass by velocity.
38.	An impulse is equivalent to	the change in mass of an object.	a force applied to an object for a period of time.	both a and b	none	a force applied to an object for a period of time.
39.	The force of gravity on a body varies slightly from place to place on the earth for two reason. (i shape of earth and (ii) ?	the circumference of the earth	the mass of the earth	the rotation of the earth	All of these	the rotation of the earth
40.	If the radius of the earth were to shrink and its mass were to remain the same, the acceleration due to gravity on the surface of the earth with?	increase	remain the same	zero	decrease	increase
41.	'g' decrease with?	horizontal displacement	amplitude	weight	altitude	altitude
42.	Newton apply Huygens's formula to calculate?	Centripetal acceleration of the earth.	Centrifugal acceleration of the Sun	Centripetal acceleration of the moon	Centripetal acceleration of the Satellite.	Centripetal acceleration of the moon
43.	The magnitude of the weight is expressed in the units of?	Displacement (meter)	Force (Newton)	Mass (Kilogram)	None of these	Force (Newton)

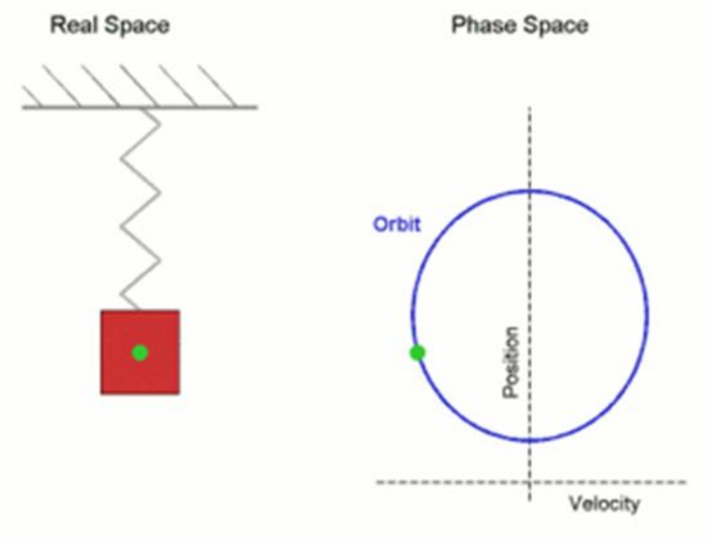
44.	The motion of falling bodies towards earth is due to the?	Acceleration due to gravity.	Gravitational rotation.	Weightlessness	Gravitational force	Gravitational force
45.	The weight of an object in a satellite orbiting around the earth is?	Actual weight	Greater than the actual weight	Zero	Less than the actual weight	Zero
46.	The acceleration of the moon is because of the?	Gravitational force exerted by the Sun.	Gravitational force exerted on the moon by the earth.	Gravitational force exerted on the earth by the moon	Gravitational force exerted by the planets	Gravitational force exerted on the moon by the earth.
47.	The Universal law of gravitation must apply to?	The earth and the apple.	The earth and the moon	Any pair of bodies.	The planets around the Sun	Any pair of bodies.
48.	In order to derive the law of gravitation, Newton assume that the moon's orbit is?	Straight	Uniform	Circular	Parabolic	Circular
49.	Gravitational force is responsible for?	Keeping planets on theft axes.	Keeping planets in their radii.	For the motion of planets around the Sun.	None of these.	For the motion of planets around the Sun.
50.	The weight of a body in a frame of reference is equal and opposite to the force required to prevent it?	from increasing its momentum.	from accelerating from rest in that frame of reference.	from providing a velocity equal to its instantaneous velocity	from an elastic collision	from accelerating from rest in that frame of reference.
51.	What is Areal Velocity?	Velocity of Air	Velocity of Aeroplane in air	Area per unit time	Unreal velocity	Area per unit time
52.	Kepler's First Law: Every planet revolves around the Sun in _______ orbit	circular	rectangular	elliptical	none	elliptical
53.	A heavy body which revolves round a planet in a _____ orbit is known as "Satellite"	Circular	Elliptical	Stable	Synchronized	Stable
54.	The horizontal velocity given to a satellite so as to put it into a circular orbit round the Earth is called "Critical Velocity". This is also called as _________	Projection Velocity	Linear Velocity	Orbital Velocity	Angular Velocity	Orbital Velocity
55.	Every body in the universe attracts every other body. This attraction is called _______	gravitational force	electrostatic force	electromagnetic force	nuclear forces	gravitational force

SYLLABUS

Oscillations: Simple harmonic motion. Differential equation of SHM and its solutions. Kinetic and Potential Energy, Total Energy and their time averages. Damped oscillations.

SIMPLE HARMONIC MOTION

The motion of a particle moving along a straight line with an acceleration which is always towards a fixed point on the line and whose magnitude is proportional to the distance from the fixed point is called simple harmonic motion [SHM].



In the diagram, a simple harmonic oscillator, consisting of a weight attached to one end of a spring, is shown. The other end of the spring is connected to a rigid support such as a wall. If the system is left at rest at the equilibrium position then there is no net force acting on the mass. However, if the mass is displaced from the equilibrium position, the spring exerts a restoring elastic force that obeys Hooke's law.

Mathematically, the restoring force **F** is given by

$$F = -kx$$

where **F** is the restoring elastic force exerted by the spring (in SI units: N), *k* is the spring constant ($N \cdot m^{-1}$), and **x** is the displacement from the equilibrium position (m).

For any simple mechanical harmonic oscillator:

When the system is displaced from its equilibrium position, a restoring force that obeys Hooke's law tends to restore the system to equilibrium.

Once the mass is displaced from its equilibrium position, it experiences a net restoring force. As a result, it accelerates and starts going back to the equilibrium position. When the mass moves closer to the equilibrium position, the restoring force decreases. At the equilibrium position, the net restoring force vanishes. However, at $x = 0$, the mass has momentum because of the acceleration that the restoring force has imparted. Therefore, the mass continues past the equilibrium position, compressing the spring. A net restoring force then slows it down until its velocity reaches zero, whereupon it is accelerated back to the equilibrium position again.

As long as the system has no energy loss, the mass continues to oscillate. Thus simple harmonic motion is a type of periodic motion.

DIFFERNTIAL EQUATIONS OF SIMPLE HARMONIC MOTION:

A particle may be said to execute a simple harmonic motion if its acceleration is directed towards a fixed point and is proportional to the displacementnof the vibrating particle.

Consider a particle at P moving on the circumference of a circle of radius **a** with a uniform velocity v. The vertical displacement of particle along Y axis is OM (y). If the particle moves from X to P in time t, then

$$\angle POX = \angle MPO = \theta = \omega t$$

From the Δ MPO,

$$\sin\theta = OM / OP$$

$$OM = OP \sin\theta$$

$$y = a \sin\omega t \text{ -------(1)}$$

Here y is the displacement and a is the amplitude. The maximum displacement of a vibrating particle is called its amplitude. The rate of change of displacement is called the velocity of a vibrating particle.

Differentiating equation (1) we get,

$$v = dy/dt = a\omega \cos \omega t \text{ -------- (2)}$$

Differentiating equation (2) with respect to time, we get the acceleration

$$a = d^2y / dt^2 = -a\omega^2 \sin \omega t$$

$$a = - \omega^2 t \text{ -----------(3)}$$

From Acceleration,

$$(d^2y / dt^2) + \omega^2 t = 0$$

The time period can be calculated from the above expression,

$$\omega^2 = \frac{Acceleration}{Displacement}$$

$$\omega = \sqrt{\frac{Acceleration}{Displacement}}$$

If ω = 2πn = 2π/T , then

$$2\pi/T = \sqrt{\frac{Acceleration}{Displacement}}$$

$$1/T = 1/2\pi \sqrt{\frac{Acceleration}{Displacement}}$$

$$T = 2\pi \sqrt{\frac{Displacement}{Acceleration}}$$

$$T = 2\pi \sqrt{k}$$

Where K is the displacement per unit acceleration.

If the particle revolvesround the circle n times per second, then the angular velocity ω is given by,

$$\omega = 2\pi n = 2\pi/T$$

therefore y = a sin 2πt/T.

AVERAGE KINETIC ENERGY OF A VIBRATING PARTICLE:

The displacement of a vibrating particle is given by

$$y = a \sin(\omega t + \alpha)$$

$$v = dy/dt = a\,\omega \sin(\omega t + \alpha)$$

If m is the mass of the vibrating particle, the kinetic energy at any instant

$$K.E = ½\, mv^2$$

$$K.E = ½\, m\, (a^2\, \omega^2 \cos^2(\omega t + \alpha))$$

The average kinetic energy of the vibrating particle in one complete vibration

$$K.E = 1/T \int_o^T \frac{1}{2}\, m\, a^2\, \omega^2 \cos^2(\omega t + \alpha)\, dt$$

$$K.E = 1/T\, (m\, a^2\, \omega^2 \cos^2)/2 \int_o^T \omega 2\cos 2(\omega t + \alpha)\, dt$$

By using cosine half angle formula,

$$K.E = (m\, a^2\, \omega^2)/4T \int_o^T 1 + \cos 2(\omega t + \alpha)\, dt$$

$$K.E = (m a^2 \omega^2)/4T \left[\int_o^T dt + \int_o^T \cos 2(\omega t + \alpha)\, dt\right.$$

$$K.E = ((m a^2 \omega^2)/4T) . T +0$$

$$K.E = (m a^2 \omega^2)/4$$

$$K.E = ma^2 \pi^2 n^2$$

Where m is the mass of the vibrating particle and a is the amplitude of vibration. In the above equation the average kinetic energy of a vibrating particle is directly propotional to the square of the amplitude.

TOTAL ENERGY OF A VIBRATING PARTICLE:

Total energy is the sum of kinetic energy and potential energy of a vibrating particle.

The general equation for kinetic energy is = ½ mv^2

In simple harmonic motion, the velocity can be represented as

$v = a \omega \cos (\omega t + \alpha)$ ------(2)

To determine $\cos (\omega t + \alpha)$, we are taking the displacement equation

$$y = a \sin (\omega t + \alpha)$$

$$y/a = \sin (\omega t + \alpha)$$

By using trignometric formula,

$\cos (\omega t + \alpha) = \sqrt{1-y^2/a^2} = \sqrt{\frac{a2-y2}{a2}}$

$\cos (\omega t + \alpha) = \sqrt{(a^2-y^2)}/ a$ ----------(3)

Sub equation (3) in equation (2) we get,

$v = a \omega\sqrt{(a^2-y^2)}/ a$

$v = \omega\sqrt{(a^2-y^2)}$ ----------(4)

sub eqnation (4) in equation (1)

$K.E = ½ m \omega^2 (a^2-y^2)$ --------(5)

we already know that,

Potential energy of the vibrating particle is the amount of workdone. so we can write the potential energy as,

$P.E = \int_0^y m \omega^2 y. dy$ --------(6)

P.E = $m\omega^2 \int_0^y y\, dy$

P.E = $m\omega^2 - y^2/2$

P.E = $\frac{1}{2} m\omega^2 y^2$ ---------(7)

By adding equation (5) and (6) we get,

Total energy = $\frac{1}{2} m\, \omega^2 (a^2 - y^2) + \frac{1}{2} m\omega^2 y^2$

Total energy = $\frac{1}{2} m\, \omega^2 a^2$

The value of ω is 2πn,

T.E = $\frac{1}{2} m\, (2\pi n)^2 a^2$

T.E = $\frac{1}{2} m\, 4\, \pi^2 n^2 a^2$

T.E = $2m\, \pi^2 n^2 a^2$---------(8)

Equation (8) represents the total energy of a vibrating particle.

SIMPLE HARMONIC OSCILLATION OF A LOADED SPRING:

Consider a spring S whose upper end is fixed to a right support and the lower end is attached to a mass M. In the equillibrium position, the mass is at A. When the mass is displaced downwards and left, it oscillates simple harmonically in the vertical direction.

Suppose at any instant the mass is at B. The distance AB = y. Let the tension per unit displacement of the spring be 'k'

Force extered by the spring = ky

According to Newton's second law,

Force $Md^2y/ dt^2 = -ky$

The negative sign indicates that the force is directed upwards.

$M(d^2y/dt^2) + ky = 0$

$(d^2y/dt^2) + (k/M)y = 0$ ------------(1)

This equation is simliar to the equation of simple harmonic motion,

$(d^2y/dt^2) + \omega^2 y = 0$ ------------(2)

complaring equations (1) and (2),

$\omega^2 = k/M$

$\omega = \sqrt{k/M}$ ------(3)

Time Period $T = 2\pi/\omega$ -------(4)

$\omega = 2\pi/T$ -----------(5)

Comparing equations (3) and (5),

$2\pi/T = \sqrt{k/M}$

$1/T = 1/2\pi \sqrt{(k/M)}$

$T = 2\pi \sqrt{(M/k)}$-------------(6)

Knowing the values of M and k, the value of T can be calculated.

DETERMINATION OF k:

To determine the value of 'k' , a small mass m is attached to the free end of the spring. The increase in length of a spring is noted. Let it be x

Then, $k = (mg/x)$ -------(7)

Subsititue the equation (7) in equation (6)

$T = 2\pi \sqrt{(M/mg)}$-------------(8)

It is to be noted mg/x is constant for a given spring.

GRAPHICAL REPRESENTATION OF SHM:

Let p be a partical moving on the circumference of a circle of radius a. The displacement equation can be written as

$y = a \sin \omega t$

The changes in the displacement, velocity and acceleration of a vibrating particle in one complete vibration are given in the following table.

Time (t)	Angle (ωt)	Displacement (y) a sin ωt	Velocity (v) a ω cos ωt	Acceleration (a) $-a\,\omega^2 \sin \omega t$
0	0	0	+ a ω	0
T/4	π/2	+a	0	$-a\,\omega^2$
T/2	Π	0	-a ω	0
3T/4	3π/2	-a	0	$+a\,\omega^2$
T	2π	0	+ a ω	0

The velocity of a particle moving with simple harmonic motion is $v = dy/dt = + a\,\omega \cos \omega t$

The acceleration of a particle moving with simple harmonic motion is $d^2y/dt^2 = - a\,\omega^2 \sin \omega t$

TYPES OF OSCILLATIONS

There are three types of oscillations.

- Free oscillations
- **DAMPED OSCILLATIONS**
- Undamped oscillations

(i) Free Oscillations:

When a body vibrates with its own natural frequency is said to be free oscillations.

The frequency of oscillations depends upon the spring fractor.

$n = 1/2\pi \sqrt{(k/M)}$

Eg: Vibrations of a tunning fork

Vibrations in a stretched string

(ii) Damped Oscillations:

In this case during each oscillations, some energy is lost. So the amplitude of the oscillation will be reduced to Zero. There is no compensating arragements in provided. Here the only parameters that will remain unchanged are the frequency (or) time period.

Here the energy loss (or) dissipation component denoted as R. If the R value is positive, then the amplitude of particle will increases with t. If the R value is negative, then the amplitude will decreases with t.

Eg: Oscillations of a pendulum

Electromagnetic oscillations in a tank circuit.

(iii) Undamped Oscillations:

The undamped oscillations have constant amplitude oscillations. In this case this dissipation component R will be Zero. The correct amount of energy is supplied to overcome the losses at the right time in each cycle.

These oscillations are also called as sustained oscillations. An oscillator is mainly required to produce undamped oscillations for utilizing in various electronics equipment.

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POSSIBLE QUESTIONS

2 MARKS

1. Define simple harmonic motion.
2. State the principle of Kater's pendulum.
3. Give some applications of compound pendulum.
4. What are the types of oscillations?
5. What is the difference between compound and Kater's pendulum?
6. Briefly explain about oscillations.

6 MARKS

1. Explain simple harmonic motion detail.
2. Describe the graphical representations of SHM with neat diagram.
3. Obtain the expression for the average kinetic energy of a vibrating particle.
4. Explain the working of simple pendulum in detail.
5. Explain the types of oscillations with suitable examples.
6. Derive the expression for the total kinetic energy of a vibrating particle.
7. Explain the simple harmonic oscillations of a loaded spring.
8. Explain the working of a compound pendulum in detail.

KARPAGAM ACADEMY OF HIGHER EDUCATION, COIMBATORE-21
DEPARTMENT OF PHYSICS
CLASS : I B.SC PHYSICS
BATCH: 2018-2021
PART A : MULTIPLE CHOICE QUESTIONS (ONLINE EXAMINATIONS)
SUBJECT : MECHANICS
SUBJECT CODE : 18PHU101
UNIT IV

sl. No.	Questions	opt1	opt2	opt3	opt4	Answer
1.	A particle moves in x-y plane according to the equation the motion of the particle is	on a straight line	on an ellipse	periodic	simple harmonic	on a straight line
2.	Which of the following quantities are always positive in a simple harmonic motion ?	F–> . a–>	v–> . r–>	a–> . r–>	F–> . r–>	F–> . a–>
3.	The magnitude of average acceleration in half time period in a simple harmonic motion is	2 ω2A /π	ω2A /2 π	ω2A /√2 π	Zero	2 ω2A /π
4.	A small block oscillates back and forth on a smooth concave surface of radius R. The time period of small oscillation is	T = 2π √(R/g)	T = 2π √(2R/g)	T = 2π √(R/2g)	None of these	T = 2π √(R/g)
5.	A particle of mass 10 gm lies in a potential field v = 50 x2 + 100. The value of frequency of oscillations in Hz is	5 Hz	5/π Hz	10π/3 Hz	none of these.	5/π Hz
6.	A particle executes SHM with a frequency f. The frequency with which it's KE oscillates is	f/2	f	2f	4f	2f
7.	A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitudes is decreased by 5 % ?	6%	3%	1.50%	0%	0%
8.	The work done by the string of a simple pendulum during one complete oscillation is equal to	total energy of the pendulum	KE of the pendulum	PE of the pendulum	Zero	Zero
9.	A particle oscillates with undamped simple harmonic motion. Which one of the following statements about the acceleration of the oscillating particle is true?	It is least when the speed is greatest.	It is always in the opposite direction to its velocity.	It is proportional to the frequency.	It decreases as the potential energy increases.	It is least when the speed is greatest.
10.	Which one of the following statements is true when an object performs simple harmonic motion about a central point O?	The acceleration is always away from O.	The acceleration and velocity are always in opposite directions.	The acceleration and the displacement from O are always in the same direction.	The graph of acceleration against displacement is a straight line.	The graph of acceleration against displacement is a straight line.
11.	The frequency of a body moving with simple harmonic motion is doubled. If the amplitude remains the same, which one of the following is also doubled?	the time period	the total energy	the maximum velocity	the maximum acceleration	the maximum velocity
12.	Which one of the following statements always applies to a damping force acting on a vibrating system?	It is in the same direction as the acceleration.	It is in the same direction as the displacement.	It is in the opposite direction to the velocity.	It is proportional to the displacement.	It is in the opposite direction to the velocity.
13.	Which one of the following statements concerning the acceleration of an object moving with simple harmonic motion is correct?	It is constant.	It is at a maximum when the object moves through the centre of the oscillation.	It is zero when the object moves through the centre of the oscillation.	It is zero when the object is at the extremity of the oscillation.	It is zero when the object moves through the centre of the oscillation.
14.	When the length of a simple pendulum is decreased by 600 mm, the period of oscillation is halved. What was the original length of the pendulum?	800 mm	1000 mm	1200 mm	1400 mm	800 mm
15.	A mass on a spring undergoes SHM. The maximum displacement from the equilibrium is called?	Period	Frequency	Amplitude	Wavelength	Amplitude
16.	In a periodic process, the number of cycles per unit of time is called?	Period	Frequency	Amplitude	Wavelength	Frequency
17.	In a periodic process, the time required to complete one cycle is called?	Period	Frequency	Amplitude	Wavelength	Period
18.	When the mass reaches point x = +A its instantaneous velocity is?	Maximum and positive	Maximum and negative	Zero	Less than maximum and positive	Zero
19.	When the mass reaches point x = 0 its instantaneous velocity is?	Maximum and can be positive or negative	Constant and doesn't depend on the location	Zero	Slightly less than maximum and positive	Maximum and can be positive or negative
20.	When the mass reaches point x = +A its instantaneous acceleration is?	Maximum and positive	Maximum and negative	Zero	Slightly less than maximum and positive	Maximum and negative
21.	A mass-spring oscillating system undergoes SHM with maximum amplitude A. If the amplitude is doubled what effect will it produce on the mechanical energy of the system?	The energy is increased by factor two	The energy is increased by factor four	The energy is decreased by factor two	The energy is decreased by factor four	The energy is increased by factor four
22.	A mass-spring oscillating system undergoes SHM with maximum amplitude A. If the spring constant is doubled what effect will it produce on the mechanical energy of the system?	The energy is increased by factor two	The energy is increased by factor four	The energy is decreased by factor two	The energy is decreased by factor four	The energy is increased by factor two
23.	A mass M suspended from a string L undergoes SHM. Which of the following is true about the period of oscillations?	The period increases with increasing amplitude	The period increases with increasing mass	The period increases with decreasing length	The period increases with increasing length	The period increases with increasing length
24.	A simple pendulum is moved from the Earth to the Moon. How does it change the period of oscillations? Acceleration due to gravity on moon= 1.6 m/s2	The period is increased by factor √6	The period is increased by factor four	The period is decreased by factor √6	The period is decreased by factor four	The period is increased by factor √6

25.	The length of a simple pendulum oscillating with a period T is quadrupled, what is the new period of oscillations in terms of T?	2T	T	3T	4T	2T
26.	A simple pendulum has a period of 1 s. What is the length of the string?	1m	2m	1/2m	1/4m	1/4m
27.	Mass doesn't stop at mean position due to	inertia	friction	torque	force	inertia
28.	In SHM of a simple pendulum, component of weight which is directed towards mean position is	mg cosθ	mg sinθ	0	none	mg sinθ
29.	Expression for Hooke's law is	F = ma	F = -kx	F = vl	F = m1m2	F = -kx
30.	If time period of simple pendulum is 2 s then it's length is	1.02m	1.03m	1.04m	1.05m	1.02m
31.	For an object undergoing simple harmonic motion	the amplitudes are usually regarded as being large	the acceleration is greatest when the displacement is greatest	the acceleration is greatest when the speed is greatest.	the displacement is greatest when the speed is greatest.	the acceleration is greatest when the displacement is greatest
32.	If both the mass m of a simple pendulum and its length L are doubled, the period will	increase by a factor of 1.4	increase by a factor of 2.	increase by a factor of 0.71.	be unchanged.	increase by a factor of 1.4
33.	Sound travels at 340 m/s in air and 1500 m/s in water. A sound of 256 Hz is made under water. In the air,	the frequency remains the same but the wavelength is shorter.	the frequency is higher but the wavelength stays the same.	the frequency is lower but the wavelength is longer.	the frequency is lower and the wavelength is shorter.	the frequency remains the same but the wavelength is shorter.
34.	We can hear sounds that are produced around a corner but cannot see light that is produced around a corner because	sound has more energy than light.	sound has shorter wavelengths than light	sound has longer wavelengths than light.	none	sound has longer wavelengths than light.
35.	Length of a Second's Pendulum is constant, equal to	$g/2\pi$	$g/2\pi^2$	$g/4\pi$	g/π^2	g/π^2
36.	Angular speed (ω) of a Second's Pendulum is	equal to 2π	greater than π	less than π	equal to π	equal to π
37.	Total energy in a Damped Oscillations ______ with time.	Increases Linearly	Decreases Linearly	Increases Exponentially	Decreases Exponentially	Decreases Exponentially
38.	The velocity of a spring-mass system at mid-position (when x=a/2) is _____ aω	(√3/4)	(√3/8)	(√5/8)	(√3/2)	(√3/2)
39.	The kinetic energy of a spring-mass system at mid-position (when x=a/2) is _____ ka^2	(1/2)	(3/8)	(5/3)	(2/3)	(3/8)
40.	The equation describing Linear Simple Harmonic Motion (Linear SHM) is	$F = k x$	$F = -k x^2$	$F = -k \sqrt{x}$	$F = -k x$	$F = -k x$
41.	Law of Mass: The period of simple pendulum __________ its mass.	directly proportional to	inversely proportional to	does not depend upon	directly proportional to square-root of	does not depend upon
42.	The expression {asin(ωt + α)} represents the physical quantity _____ in Linear SHM	Velocity	Speed	Displacement	Acceleration	Displacement
43.	Oscillations are damped due to presence of	restoring force	frictional force	mechanical force	none	frictional force
44.	As amplitude of resonant vibrations decreases, degree of damping	increases	remains constant	varies	Decreases	Decreases
45.	Oscillations become damped due to	normal force	friction	parallel force	none	friction
46.	In s.h.m, object's acceleration depends upon	displacement from equilibrium position	magnitude of restoring force	both A and B	force exerted on it	displacement from equilibrium position
47.	Angular frequency of s.h.m is equal to	2πf	1f	2f	none	2πf
48.	For a resonating system it should oscillate	bound	only for some time	freely	none	freely
49.	An oscillator differs from an amplifier because it ………	Has more gain	Requires no input signal	Requires no d.c. supply	Always has the same input	Requires no input signal
50.	Quartz crystal is most commonly used in crystal oscillators because …………	It has superior electrical properties	It is easily available	It is quite inexpensive	None of the above	It has superior electrical properties
51.	When an object is oscillating in simple harmonic motion in the vertical direction, its maximum speed occurs when the object	is at its highest point	is at its lowest point.	is at the equilibrium point.	has the maximum net force exerted on it.	is at the equilibrium point.
52.	Any body moving with simple harmonic motion is being acted by a force that is	constant.	proportional to a sine or cosine function of the displacement	proportional to the inverse square of the displacemen	directly proportional to the displacement.	directly proportional to the displacement.
53.	A system consists of a mass vibrating on the end of a spring. The total mechanical energy of this system	varies as a sine or cosine function	is constant only when the mass is at maximum displacement.	is a maximum when the mass is at its equilibrium position only.	is always equal to the square of the amplitude	is constant only when the mass is at maximum displacement.
54.	A mass m hanging on a string with a spring constant k has simple harmonic motion with a period T. If the mass is doubled to 2m, the period of oscillation is	2T	T /2	T / √2	√2 T	√2 T

SYLLABUS

Motion of rigid body: Moment of inertia of a rod, disc, spherical shell, solid and hollow spheres - Theory of compound pendulum and Kater's pendulum - Determination of 'g' - Derivation of expressions for angular momentum and kinetic energy of a system of N particles.
Friction-Static Friction - Laws of Friction-Angle and cone of Friction - Motion up and down on a rough inclined plane.

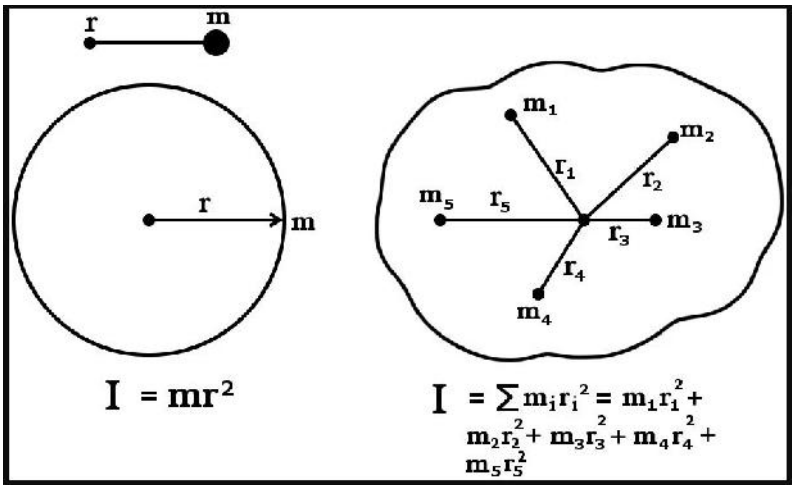
MOMENT OF INERTIA OF A ROD

Inertia is the measure of resistance that a body of a certain mass offers when plunged into motion or, on the contrary, bought to a halt by an external force. Inertia, or the tendency of objects to resist change, varies with mass. Heavier objects are difficult to accelerate when at rest and equally difficult to stop when in motion, as compared to lighter objects.

The prefix 'moment of' in physics is used to depict the rotational counterpart of a linear quantity. Thus, the 'moment of inertia' is the rotational equivalent of mass for linear motion. It is denoted by **'I'**. Similarly, the 'moment of force' is the rotational equivalent of linear force, also known as *torque*.

How do we calculate moment of inertia?

The moment of inertia 'I' of a rotating object with respect to its axis of rotation is given by the product of its mass and the square of its distance from the axis of rotation. However, this is only true for uniform or ordinary objects, such as an orb attached to a string whirling around at a certain angular velocity.



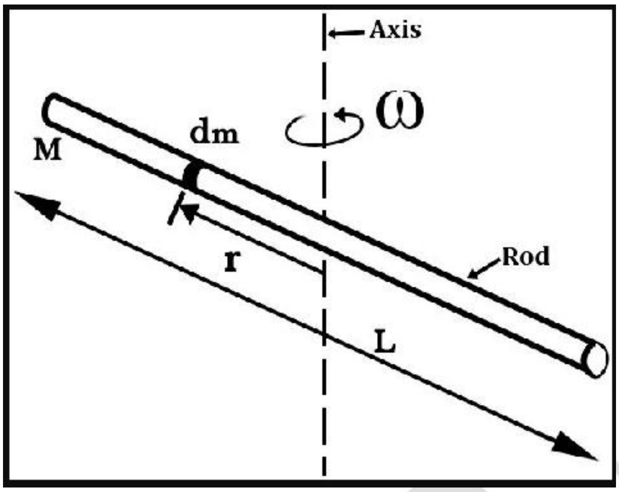
For non-uniform objects, moment of inertia is calculated by the sum of the products of individual *point* masses and their corresponding distance from the axis of rotation. This generalized relationship can be used to calculate the moment of inertia of any system, since any object can be constituted as an aggregation of similar *point* masses.

To calculate the moment of inertia of such a continuous distribution of mass at various distances, we use calculus, due to its dexterity with continuous variables.
We use a *differential* element of mass, an infinitesimal chunk of mass **dm**. The differential moment of inertia is then, $\mathbf{dI = r^2 dm}$. To calculate the moment of inertia **'I'** of the whole of mass **'M',** we sum the differential moment of inertia **dI** contributed by **dm** throughout the surface. Or simply, we integrate.

$$I = \int dI = \int_0^M r^2 dm$$

DERIVATION OF THE MOMENT OF INERTIA OF A ROD

Consider a rod of mass 'M' and length 'L' such that its linear density **λ** is *M/L*. Depending on the position of the axis of rotation, the rod illustrates two moments: one, when the axis cuts perpendicular through the center of mass of the rod, exactly through the middle; and two, when the axis is situated perpendicular through one of its two ends.



Axis through the center of mass

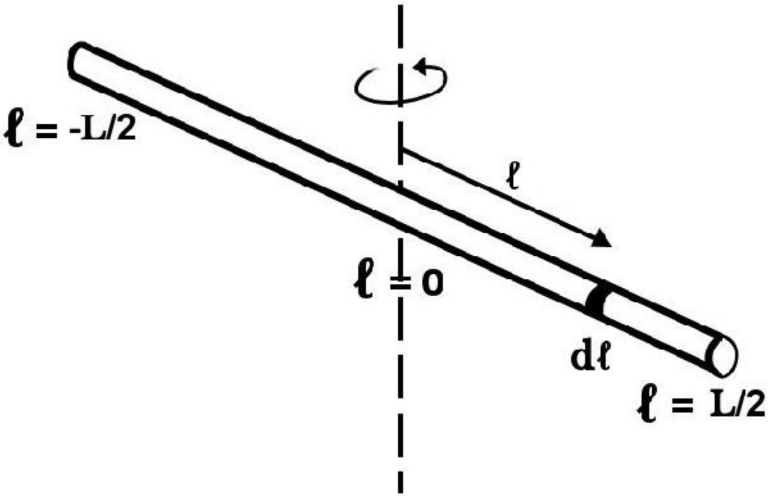
Similar to the infinitesimal element of mass **dm,** consider an infinitesimal element of length **dl** corresponding to it**.** Drawing the origin at the center of mass resting on the line of the axis, we realize that the distance of the rod to the left from the origin to its end is *-L/2,* while the distance from the origin to the other end to its right is *+L/2.*

Assuming that the rod is uniform, the linear density remains a constant such that:

$$\lambda = M / L = dm / d\ell$$

$$\text{or, } dm = \frac{M}{L} d\ell$$

Substituting the value of **dm** in our expression to calculate moment of inertia, we get:



$$I = \int_0^M r^2 dm = \int_{-L/2}^{L/2} \ell^2 \frac{M}{L} d\ell$$

Because the variable of integration is now length **(dl),** the limits have changed from the previously depicted M to a required fraction of L.

$$I_{com} = \frac{M}{L} \int_{-L/2}^{L/2} \ell^2 d\ell$$

$$= \frac{M}{L} \left[\frac{\ell^3}{3} \right]_{-L/2}^{L/2}$$

$$= \frac{M}{3L} \left[\left(\frac{L}{2}\right)^3 - \left(\frac{-L}{2}\right)^3 \right]$$

$$= \frac{M}{3L} \left[\frac{L^3}{8} + \frac{L^3}{8} \right]$$

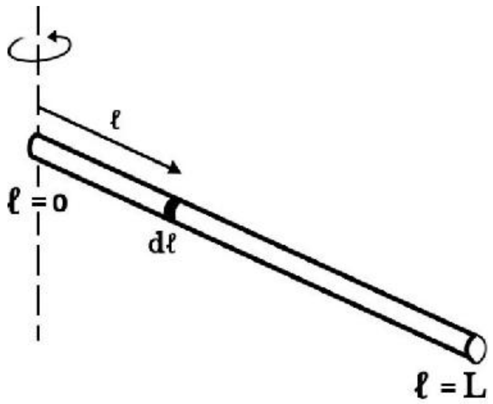
$$= \frac{M}{3L} \cdot \frac{2L^3}{8}$$

$$\text{or,} \boxed{I_{com} = \frac{1}{12} ML^2}$$

Axis through an end

In order to calculate the moment of inertia of a rod when the axis is at one of its ends, we draw the origin at this end.

We are required to use the same expression, however, with a different limit now. Because the axis rests at the end, the limit over which we integrate is now *zero (the origin) to L (the opposite end).*



$$I = \int_0^M r^2 dm = \int_0^L \ell^2 \frac{M}{L} d\ell$$

After integrating, we get:

$$I_{end} = \frac{M}{L} \int_0^L \ell^2 d\ell$$

$$= \frac{M}{L} \left[\frac{\ell^3}{3} \right]_0^L$$

$$= \frac{M}{3L} \left[L^3 - 0^3 \right]$$

$$\boxed{I_{end} = \frac{1}{3} ML^2}$$

We can also arrive at the same result for the moment of inertia about the end by using the *parallel axis theorem,* according to which:

$$I_{end} = I_{com} + M.d^2_{(com, end)}$$

Here $d_{(com, end)}$ is the distance from the center of mass (com) to the rod's end.

As L (com,end) is *L/2,* we find that:

$$I_{end} = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12}ML^2 + \frac{1}{4}ML^2$$

$$\boxed{I_{end} = \frac{1}{3}ML^2}$$

MOMENT OF INERTIA OF A DISK

Derivation of the moment of inertia of a disk

In physics, the rotational equivalent of mass is something called the *moment of inertia*. The definition of the moment of inertia of a volume element dV which has a mass dm is given by

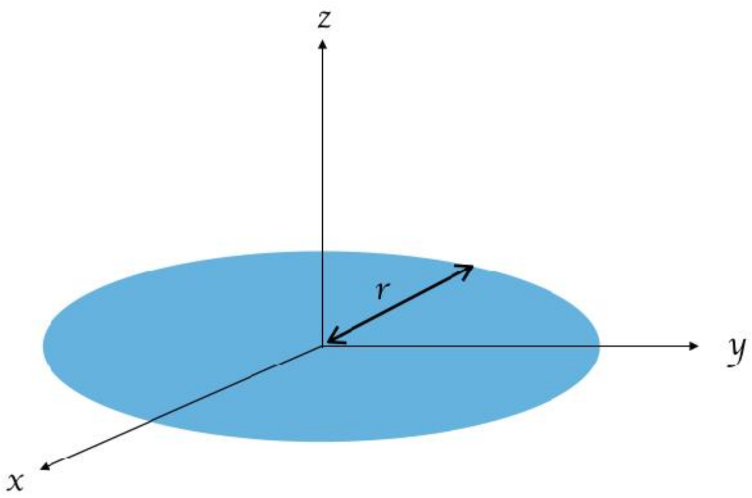
$$dI = r_{\perp}^2 dm$$

where $r_{\perp}$ is the perpendicular distance from the axis of rotation to the volume element. To find the total moment of inertia of an object, we need to sum the moment of inertia of all the volume elements in the object over all values of distance from the axis of rotation. Normally we consider the moment of inertia about the vertical (z-axis), and we tend to denote this by I_{zz}. We can write

$$I_{zz} = \int_{r_1}^{r_2} r_{\perp}^2 dm$$

The moment of inertia about the other two cardinal axes are denoted by I_{xx}, and I_{yy}, but we can consider the moment of inertia about any convenient axis.

For our purposes, a disk is a solid circle with a *small* thickness t ($t \ll r$, small in comparison to the radius of the disk). If it has a thickness which is comparable to its radius, it becomes a cylinder, which we will discuss in a future blog. So, our disk looks something like this.



To calculate the moment of inertia of this disk about the z-axis, we sum the moment of inertia of a volume element dV from the centre (where $r = 0$) to the outer radius r.

$$I_{zz} = \int_{r=0}^{r=r} r_{\perp}^{2}\, dm \quad (\text{Equ. } 1)$$

The mass element dm is related to the volume element dV via the equation $dm = \rho dV$ (where ρ is the density of the volume element). We will assume in this example that the density $\rho(r)$ of the disk is uniform; but in principle if we know its dependence on r, $\rho(r) = f(r)$, this would not be a problem.

The volume element dV can be calculated by considering a ring at a radius r with a width dr and a thickness t. The volume of this ring is just this rings circumference multiplied by its width multiplied by its thickness.

$$dV = (2\pi r dr)t$$

so we can write

$$dm = \rho(2\pi r dr)t$$

and hence we can write equation (1) as

$$I_{zz} = \int_{r=0}^{r=r} r_{\perp}^{2} \rho(2\pi r dr)t = 2\pi\rho t \int_{r=0}^{r=r} r_{\perp}^{3}\, dr$$

Integrating between a radius of $r = 0$ and r, we get

$$I_{zz} = 2\pi\rho t[\frac{r^4}{4} - 0] = \frac{1}{2}\pi\rho t r^4 \text{ (Equ. 2)}$$

If we now define the *total mass* of the disk as M,

Where $M = \rho V$ and V is the *total volume* of the disk.

The total volume of the disk is just its area multiplied by its thickness,$V = \pi r^2 t$

so the total mass is $M = \rho\pi r^2 t$

Using this, we can re-write equation (2) as

$$\boxed{I_{zz} = \frac{1}{2}\pi\rho t r^4 = \frac{1}{2}Mr^2}$$

What are the moments of inertia about the x and y-axes?

To find the moment of inertia about the x or the y-axis we use the *perpendicular axis theorem*. This states that, for objects which lie within a plane, the moment of inertia about the axis parallel to this plane is given by $I_{zz} = I_{xx} + I_{yy}$ where I_{xx} and I_{yy} are the two moments of inertia in the plane and perpendicular to each other.

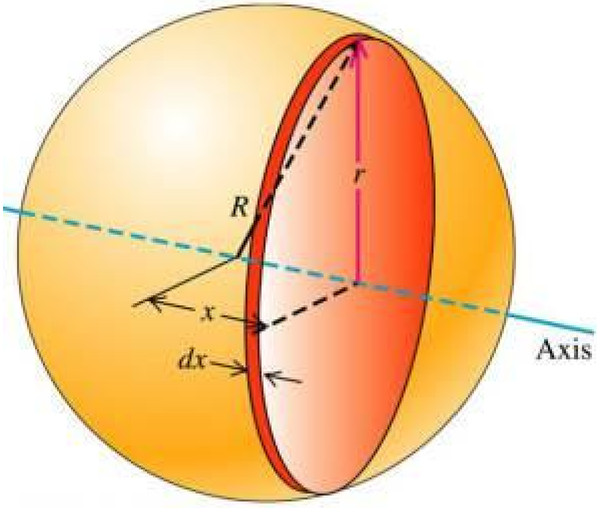
We can see from the symmetry of the disk that the moment of inertia about the x and y-axes will be the same, so $I_{zz} = 2I_{xx}$.

Therefore we can write

$$\boxed{I_{xx} = I_{yy} = \frac{1}{2}I_{zz} = \frac{1}{4}Mr^2}$$

DERIVATION OF MOMENT OF INERTIA OF A THIN SPHERICAL SHELL

A thin uniform spherical shell has a radius of R and mass M. Calculate its moment of inertia about any axis through its centre.



Recall: Moment of inertia for a hoop: $I = r^2\,dm$

Hence,

$$dI=r^2dm$$

Finding dm,

$$dm=M/AdA$$

Where A is the total surface area of the shell: $4\pi R^2$

Now, dA is the area of the ring.

$$dA=Rd\theta\times 2\pi r$$

Note: $2\pi r$ is the circumference of the hoop while $R\,d\theta$ is the "thickness" of the hoop (its dx in the above picture). The $R\,d\theta$ comes from the equation for arc length: $S = R\theta$.

Now, we have to find a way to relate r with θ. Consider the above picture, notice that there is a right-angle triangle with angle θ at the centre of the circle. Hence,

$$\sin\theta=rR$$

$$r=R\sin\theta$$

Hence, dA becomes:

$$dA = 2\pi R^2 \sin\theta\, d\theta$$

Substituting the equation for dA into the equation for dm, we have:

$$dm = \frac{M\sin\theta}{2}d\theta$$

Substituting the equation above and the equation for r into the equation for dI, we have:

$$dI = \frac{MR^2}{2}\sin^3\theta\, d\theta$$

Integrating with the proper limits, (from one end to the other)

$$I = \frac{MR^2}{2}\int_0^{\pi}\sin^3\theta\, d\theta$$

For those who knows how to integrate $\sin^3\theta$, you're done with this post. For those who needs a little bit more help, read on.

Now, we split the $\sin^3\theta$ into two,

$$I = \frac{MR^2}{2}\int_0^{\pi} sin^2\theta\, sin\,\theta\, d\theta$$

$$I = \frac{MR^2}{2}\int_0^{\pi}(1 - cos^2\theta)\, sin\,\theta\, d\theta$$

Now, at this point, we will use the substitution: u = cos θ. Hence,

$$I = \frac{MR^2}{2}\int_1^{-1} u^2 - 1\, du$$

I'm pretty sure you can handle this simple integration by yourself. Hence, we have:

$$I = \frac{2}{3}MR^2$$

DERIVATION OF MOMENT OF INERTIA OF AN UNIFORM SOLID SPHERE

An uniform solid sphere has a radius R and mass M. calculate its moment of inertia about any axis through its centre.

First, we set up the problem.

1. Slice up the solid sphere into infinitesimally thin solid cylinders
2. Sum from the left to the right

Recall the moment of inertia for a solid cylinder:

$$I = \frac{1}{2}MR^2$$

Hence, for this problem,

$$dI = \frac{1}{2}r^2\ dm$$

Now, we have to find dm,

$$dm = \rho\, dV$$

Finding dV,

$$dV = \pi r^2\ dx$$

Substitute dV into dm,

$$dm = \rho\pi r^2\ dx$$

Substitute dm into dI,

$$dI = \frac{1}{2}\rho\pi r^4\ dx$$

Now, we have to force x into the equation. Notice that x, r and R makes a triangle above. Hence, using Pythagoras' theorem,

$$r^2 = R^2 - x^2$$

Substituting,

$$dI = \frac{1}{2}\rho\pi(R^2 - x^2)^2\,dx$$

Hence,

$$I = \frac{1}{2}\rho\pi\int_{-R}^{R}(R^2 - x^2)^2\,dx$$

After expanding out and integrating, you'll get

$$I = \frac{1}{2}\rho\pi\frac{16}{15}R^5$$

Now, we have to find what is the density of the sphere:

$$\rho = \frac{M}{V}$$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

Substituting, we will have:

$$I = \frac{2}{5}MR^2$$

COMPOUND PENDULUM

Consider an extended body of mass M with a hole drilled though it. Suppose that the body is suspended from a fixed peg, which passes through the hole, such that it is free to swing from side to side, as shown in Fig. This setup is known as a *compound pendulum*.

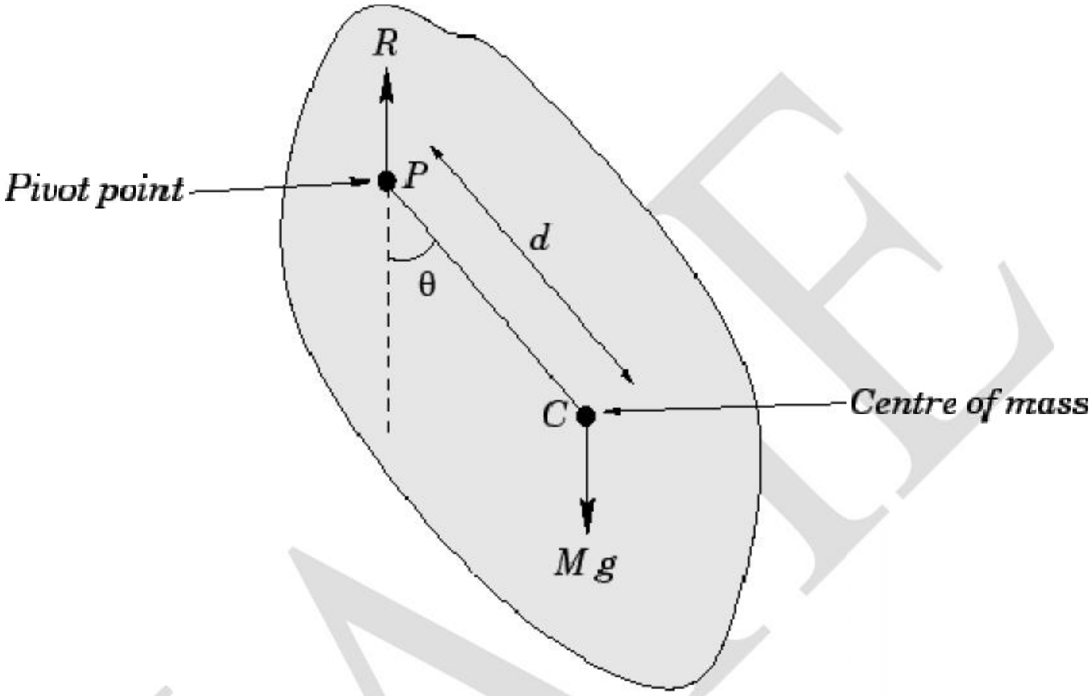


Fig. *A compound pendulum.*

Let P be the pivot point, and let C be the body's centre of mass, which is located a distance d from the pivot. Let θ be the angle subtended between the downward vertical (which passes through point P) and the line PC. The equilibrium state of the compound pendulum corresponds to the case in which the centre of mass lies vertically below the pivot point: *i.e.*, $\theta = 0$. The angular equation of motion of the pendulum is simply

$$I\,\ddot{\theta} = \tau, \qquad (1)$$

where I is the moment of inertia of the body about the pivot point, and τ is the torque. Using similar arguments to those employed for the case of the simple pendulum (recalling that all the weight of the pendulum acts at its centre of mass), we can write

$$\tau = -M\,g\,d\,\sin\theta. \qquad (2)$$

Note that the reaction, R, at the peg does not contribute to the torque, since its line of action passes through the pivot point. Combining the previous two equations, we obtain the following angular equation of motion of the pendulum:

$$I\ddot{\theta} = -M\,g\,d\,\sin\theta. \qquad (3)$$

Finally, adopting the small angle approximation, $\sin\theta \simeq \theta$, we arrive at the simple harmonic equation:

$$I\ddot{\theta} = -M\,g\,d\,\theta. \qquad (4)$$

It is clear, by analogy with our previous solutions of such equations, that the angular frequency of small amplitude oscillations of a compound pendulum is given by

$$\omega = \sqrt{\frac{M\,g\,d}{I}}. \qquad (5)$$

It is helpful to define the length

$$L = \frac{I}{M\,d}. \qquad (6)$$

Equation (5) reduces to

$$\omega = \sqrt{\frac{g}{L}}, \qquad (7)$$

which is identical in form to the corresponding expression for a simple pendulum.
We conclude that a compound pendulum behaves like a simple pendulum with *effective length* L.

KATER'S PENDULUM

Kater's pendulum, shown in Fig. 1, is a physical pendulum composed of a metal rod 1.20 m in length, upon which are mounted a sliding metal weight W_1, a sliding wooden weight W_2, a small sliding metal cylinder w, and two sliding knife edges K_1 and K_2that face each other. Each of the sliding objects can be clamped in place on the rod. The pendulum can suspended and set swinging by resting either knife edge on a flat, level surface. The wooden weight W_2 is the same size and shape as the metal weight W_1. Its function is to provide as near equal air resistance to swinging as possible in either suspension, which happens if W_1 and W_2, and separately K_1 and K_2, are constrained to be equidistant from the ends of the metal rod. The centre of mass G can be located by balancing the pendulum on an external knife edge. Due to the difference in mass between the metal and wooden weights W_1 and W_2, G is not at the centre of the rod, and the distances h_1 and h_2 from G to the suspension points $O_1$and O_2 at the knife edges K_1 and K_2 are not equal. Fine adjustments in the position of G, and thus in h_1 and h_2, can be made by moving the small metal cylinder w.

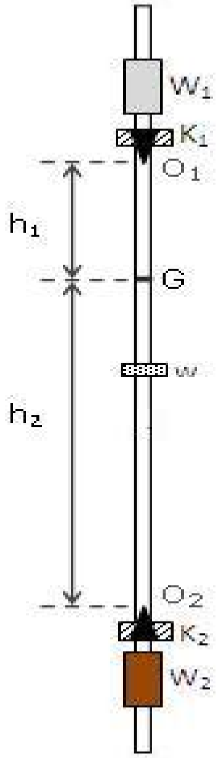


Figure 1

In Fig. 1, we consider the force of gravity to be acting at G. If h_i is the distance to G from the suspension point O_i at the knife edge K_i, the equation of motion of the pendulum is

$$I_i\ddot{\theta} = -Mgh_i \sin\theta$$

where I_i is the moment of inertia of the pendulum about the suspension point O_i, and i can be 1 or 2. Comparing to the equation of motion for a simple pendulum

$$Ml_i^2\ddot{\theta} = -Mgl_i \sin\theta$$

we see that the two equations of motion are the same if we take

$$Mgh_i / I_i = g / l_i \qquad (1)$$

It is convenient to define the radius of gyration of a compound pendulum such that if all its mass M were at a distance from O_i, the moment of inertia about O_i would be I_i , which we do by writing

$$I_i = Mk_i^2$$

Inserting this definition into equation (1) shows that

$$k_i^2 = h_i l_i \qquad (2)$$

If I_G is the moment of inertia of the pendulum about its centre of mass G, we can also define the radius of gyration about the centre of mass by writing

$$I_G = Mk_G^2$$

The parallel axis theorem gives us

$$k_i^2 = h_i^2 + k_G^2$$

so that, using (2), we have

$$l_i = \frac{h_i^2 + k_G^2}{h_i}$$

The period of the pendulum from either suspension point is then

$$T_i = 2\pi\sqrt{\frac{l_i}{g}} = 2\pi\sqrt{\frac{h_i^2 + k_G^2}{gh_i}} \qquad (3)$$

Squaring (3), one can show that

$$h_1T_1^2 - h_2T_2^2 = \frac{4\pi^2}{g}\left(h_1^2 - h_2^2\right)$$

and in turn,

$$\frac{4\pi^2}{g} = \frac{h_1T_1^2 - h_2T_2^2}{(h_1+h_2)(h_1-h_2)} = \frac{T_1^2 + T_2^2}{2(h_1+h_2)} + \frac{T_1^2 - T_2^2}{2(h_1-h_2)}$$

which allows us to calculate g,

$$g = 8\pi^2\left[\frac{T_1^2 + T_2^2}{h_1+h_2} + \frac{T_1^2 - T_2^2}{h_1-h_2}\right]^{-1} \qquad (4)$$

Applications

Pendulums are used to regulate pendulum clocks, and are used in scientific instruments such as accelerometers and seismometers. Historically they were used as gravimeters to measure the acceleration of gravity in geophysical surveys, and even as a standard of length. The problem with using pendulums proved to be in measuring their length.

A fine wire suspending a metal sphere approximates a simple pendulum, but the wire changes length, due to the varying tension needed to support the swinging pendulum. In addition, small amounts of angular momentum tend to creep in, and the centre of mass of the sphere can be hard to locate unless the sphere has highly uniform density. With a compound pendulum, there is a point called the centre of oscillation, a distance l from the suspension point along a line through the centre of mass, where l is the length of a simple pendulum with the same period. When suspended from the centre of oscillation, the compound pendulum will have the same period as when suspended from the original suspension point. The centre of oscillation can be located by suspending from various points and measuring the periods, but it is difficult to get an exact match in the period, so again there is uncertainty in the value of l.

With equation (4), derived by Friedrich Bessel in 1826, the situation is improved. $h_1 + h_2$, being the distance between the knife edges, can be measured accurately. $h_1 – h_2$ is more difficult to measure accurately, because accurate location of the centre of mass G is difficult. However, if T_1 and T_2 are very nearly equal, the second term in (4) is quite small compared to the first, and $h_1 – h_2$ does not have to be known as accurately as $h_1 + h_2$.

Kater's pendulum was used as a gravimeter to measure the local acceleration of gravity with greater accuracy than an ordinary pendulum, because it avoids having to measure l. It was popular from its invention in 1817 until the 1950’s, when began to be possible to directly measure the acceleration of gravity during free fall using a Michelson interferometer. Such an absolute gravimeter is not particularly portable, but it can be used to accurately calibrate a relative gravimeter consisting of a mass hanging from a spring adjacent to an accurate length scale. The relative gravimeter can then be carried to any location where it is desired to measure the acceleration of gravity.

FRICTION

Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other. There are several types of friction:

- **Dry friction** is a force that opposes the relative lateral motion of two solid surfaces in contact. Dry friction is subdivided into *static friction* ("stiction") between non-moving surfaces, and *kinetic friction* between moving surfaces. With the exception of atomic or molecular friction, dry friction generally arises from the interaction of surface features, known as asperities
- **Fluid friction** describes the friction between layers of a viscous fluid that are moving relative to each other.
- **Lubricated friction** is a case of fluid friction where a lubricant fluid separates two solid surfaces.
- **Skin friction** is a component of drag, the force resisting the motion of a fluid across the surface of a body.
- **Internal friction** is the force resisting motion between the elements making up a solid material while it undergoes deformation.

What is Static Friction?

The force that has to be overcome in order to get something to move is called static friction. This is the force that prevents an object, placed on a sloped surface, from sliding.

At solid surfaces, the static friction occurs as a consequence of the surface roughness of the objects in contact. Its value depends on the type of the contacting surfaces. It is higher for rough and dry surfaces and lower for wet and smooth ones.

The force necessary to induce motion (i.e. to overcome the static friction), is bigger than the one necessary to continue the motion (i.e. to overcome kinetic friction). So the coefficient of static friction (μ_s), exceeds the one of kinetic friction (μ_k).

The coefficient of static friction has a constant value for each pair of contacting surfaces (materials). For example it is 0.74 for steel / steel contact, 0.61 for steel / aluminum contact, etc.

In order to make a stationary object move, we have to overcome the static friction force by an applied force. When a small force is applied to a nonmoving object, the static friction is of equal magnitude, but in the opposite direction to the applied force. When the force is being increased, at a certain point it reaches the maximum static friction value. At that point, the static friction is overcome and the object starts to move.

The maximum static friction (fs max) equals to:

$$f_{s\,max} = \mu_s n,$$

where μs is the coefficient of static friction, and n – the size of the normal contact force between the surfaces.

Difference Between Kinetic and Static Friction

1) Definition of Kinetic and Static Friction

Kinetic Friction: The retarding force between two objects in contact that are moving against each other is called kinetic friction.
Static Friction: The force that has to be overcome in order to get something to move is called static friction.

2) Formula for Kinetic and Static Friction

Kinetic friction: The kinetic friction (fk) equals to $f_k = \mu_k n$, where μk is the coefficient of kinetic friction, and n – the size of the normal contact force between the surfaces.
Static friction: The maximum static friction (fs max) equals to $f_{s\,max} = \mu_s n$, where μs is the coefficient of static friction, and n – the size of the normal contact force between the surfaces.

3) Magnitude of Kinetic and Static Friction

Kinetic Friction: The force necessary to induce motion is always bigger than the one necessary to continue the motion. So the kinetic friction coefficient is smaller than the static friction one.

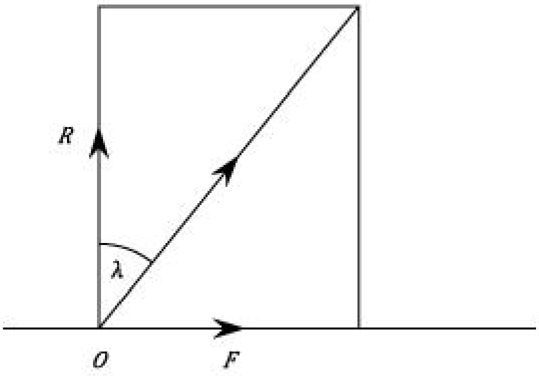
Static Friction: The coefficient of static friction exceeds the one of kinetic friction.

Summary of Kinetic and Static Friction:

- The tangential component of the force of interaction between two surfaces in contact is called friction. It leads to resistance against movement between the surfaces and can cause mechanical deformation and heating.
- The retarding force between two objects that are moving against each other is called kinetic friction. The force that has to be overcome in order to get something to move is called static friction.
- Friction depends on the type of the contacting surfaces. It is high for rough and dry surfaces and low for wet and smooth ones.
- The force necessary to induce motion (i.e. to overcome the static friction), is bigger than the one necessary to continue the motion (i.e. to overcome kinetic friction). So the coefficient of static friction (μs), exceeds the one of kinetic friction (μk).
- The kinetic friction (fk) equals to fk = μkn, where μk is the coefficient of kinetic friction, and n – the size of the normal contact force between the surfaces in contact. The maximum static friction (fs max) equals to fsmax = μsn, where μs is the coefficient of static friction, and n – the size of the normal contact force between the surfaces in contact.

ANGLE AND CONE OF FRICTION

Engineers often refer to either the coefficient of friction or the angle of friction. This section describes their relationship and goes on to explain the Cone of friction:



Let *R* be the normal reaction at a point of contact O and *F* the frictional force acting in a direction perpendicular to *R*. Then the total force at O is given by

$$\sqrt{R^2 + F^2}$$

(1)

acting in a direction making an angle

$$tan^{-1}\left(\frac{F}{R}\right)$$

(2)

with the normal reaction.

If friction is limiting, $F = \mu\, R$ and the action at O makes an angle of $tan^{-1}\mu$ with the normal reaction. This angle is denoted by λ.

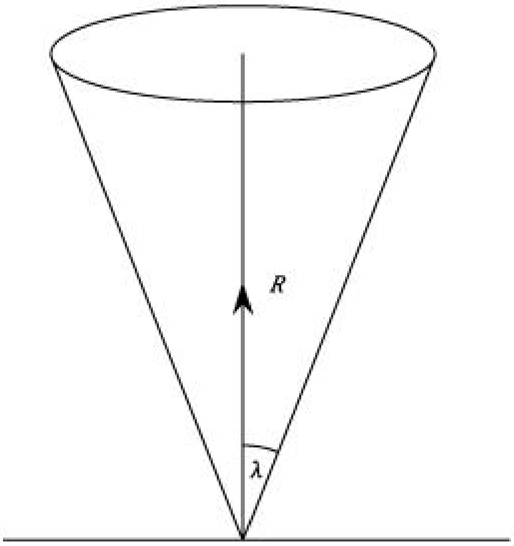
Thus

$$\mu = tan\ \lambda$$

(3)

and the magnitude of the limiting friction can be found if either μ or λ is known.

If the direction in which the body tends to move is varied the the force of limiting friction will always lie in the plane through O perpendicular to the normal reaction and the direction of the total action at O will always lie 0n a cone with it's vertex at O and axis along the line of the normal reaction. The semi vertical angle will be λ.



This cone is called the cone of friction. If friction is not limiting, the angle made by the total action at O will be less than λ. Hence whether the friction be limiting or not, the direction of the total action at O must be inside or on the cone of friction. It follows that if *P* is the resultant of the other forces acting on the body which is in equilibrium, the direction of *P* must lie inside or on the cone o friction, since *P* must balance the total action at O.

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POSSIBLE QUESTIONS

2 MARKS

1. State friction.
2. Write the types of friction.
3. Give a brief note on compound pendulum.
4. Define moment of inertia.
5. Explain briefly about static friction.
6. Give any two practical example for moment of inertia.

8 MARKS

1. How to calculate the moment of inertia of a rod?
2. Derive an expression of moment of inertia of a given rod.
3. Derive an expression of moment of inertia of a given disc.
4. Derive an expression of moment of inertia of a given spherical cell.
5. Derive an expression of moment of inertia of a given hollow sphere.
6. Derive an expression of moment of inertia of a given solid sphere.
7. Derivation of expressions for angular momentum and kinetic energy of a system of N particles.
8. Derive an expression for Angle and cone of Friction.

KARPAGAM ACADEMY OF HIGHER EDUCATION, COIMBATORE-21
DEPARTMENT OF PHYSICS
CLASS : I B.SC PHYSICS
BATCH: 2018-2021
PART A : MULTIPLE CHOICE QUESTIONS (ONLINE EXAMINATIONS)
SUBJECT : MECHANICS
SUBJECT CODE : 18PHU101
UNIT V

Sl. No.	Questions	opt1	opt2	opt3	opt4	Answer
1.	__________ law in simple says that strain is directly proportional to stress.	Kepler's	Hooke's	Newton's	Stoke's	Hooke's
2.	_________ is defined as the restoring force per unit area.	strain	stress	slip	dislocation	stress
3.	The ratio of change in length to original length is called _________.	shearing strain	volume strain	tensile strain	longitudinal strain	longitudinal strain
4.	The property of a body to regain its original state on removal of the applied forces is called _________.	plasticity	elasticity	Pseudoelasticity	viscoelasticity	elasticity
5.	The ratio of the change in any dimension to its original value is called _________.	strain	stress	slip	dislocation	strain
6.	The ratio of change in angle to original angle is called _________.	longitudinal strain	shearing strain	tensile strain	volume strain	shearing strain
7.	The ratio of change in volume to original volume is called ________.	longitudinal strain	shearing strain	tensile strain	volume strain	volume strain
8.	________ is defined as the ratio of longitudinal stress to longitudinal strain within elastic limits.	Young's modulus	rigidity modulus	bulk modulus	dynamic modulus	Young's modulus
9.	________ is defined as the ratio of tangential stress to shearing strain.	Young's modulus	rigidity modulus	bulk modulus	dynamic modulus	Rigidity modulus
10.	________ is defined as the ratio of volume stress to volume strain.	Young's modulus	rigidity modulus	bulk modulus	dynamic modulus	Bulk modulus
11.	Mathematically, Hooke's law states that F = _____________	- kx	kx	k/x	k/x	- kx
12.	___________ exhibits linear-elastic behavior in most engineering applications.	Steel	Iron	carbon	chromium	Steel
13.	_________ is generally regarded as a "non-hookean" material because its elasticity is stress dependent and sensitive to temperature and loading rate.	Steel	rubber	iron	carbon	rubber
14.	Most materials have Poisson's ratio values ranging between ___________.	0.0 and 0.5	0.0 and 0.1	0.0 and 0.05	0.5 and 1.0	0.0 and 0.5
15.	Rubber has a Poisson ratio of nearly ___________.	0.5	0.1	0.2	0.3	0.5
16.	A perfectly incompressible material deformed elastically at small strains would have a Poisson's ratio of exactly _________.	0.2	0.3	0.4	0.5	0.5
17.	A beam is a ___________ that is capable of withstanding load primarily by resisting bending.	structural element	inline element	interface element	linear element	structural element
18.	A ___________ is a beam supported on only one end.	cantilever	pile	pillar	stay	cantilever
19.	_____________ are the most ubiquitous structures in the field of microelectromechanical systems (MEMS).	Cantilevered beams	piles	pillars	stays	Cantilevered beams
20.	An early example of a MEMS cantilever is the ________, an electromechanical monolithic resonator.	Resonistor	Sensor	Actuator	Accelerometers	Resonistor
21.	__________ cantilevers are commonly fabricated from silicon (Si), silicon nitride (SiN), or polymers.	MEMS	AFM	deck	roof	MEMS
22.	Without cantilever transducers, _______________ would not be possible.	atomic force microscopy	scanning electron microscopy	transmission electron microscopy	scanning tunneling microscopy	atomic force microscopy
23.	______________ cantilevers are also finding application as radio frequency filters and resonators.	MEMS	AFM	deck	roof	MEMS
24.	The ____________ cantilevers are commonly made as unimorphs or bimorphs.	MEMS	AFM	deck	roof	MEMS
25.	In solid mechanics, __________ is the twisting of an object due to an applied torque.	torsion	stress	strain	shear	torsion
26.	_________ modulus describes the material's response to linear strain.	Young's	shear	bulk	dynamic	Young's
27.	The _________ modulus describes the material's response to uniform pressure.	Young's	shear	bulk	dynamic	bulk
28.	The __________ modulus describes the material's response to shearing strains.	Young's	shear	bulk	dynamic	shear
29.	_____________ materials such as wood and paper exhibit differing material response to stress or strain when tested in different directions.	Anisotropic	isotropic	bi isotropic	quasi isotropic	Anisotropic
30.	The _________ modulus of metals measures the resistance to glide over atomic planes in crystals of the metal.	Young's	shear	bulk	dynamic	shear
31.	In solid mechanics, ___________ modulus is also known as the tensile modulus.	Young's	shear	bulk	dynamic	Young's
32.	The __________ modulus calculates the change in the dimension of a bar made of an isotropic elastic material under tensile or compressive loads.	Young's	shear	bulk	dynamic	Young's
33.	_____________ modulus is not always the same in all orientations of a material.	Young's	shear	bulk	dynamic	Young's
34.	The _________ modulus of a substance measures the substance's resistance to uniform compression.	Young's	shear	bulk	dynamic	bulk
35.	It is possible to measure the __________ modulus using powder diffraction under applied pressure.	Young's	shear	bulk	dynamic	bulk
36.	The inverse of the bulk modulus gives a substance's __________.	compressibility	Susceptibility	permeability	Permittivity	compressibility

37.	The _________ modulus of an object is defined as the slope of its stress-strain curve in the elastic deformation region.	elastic	Young's	shear	bulk	elastic
38.	_________ is the force causing the deformation divided by the area to which the force is applied.	stress	strain	deformation	bending	stress
39.	__________ is the ratio of the change caused by the stress to the original state of the object.	stress	strain	deformation	bending	strain
40.	The ___________ modulus is an extension of Young's modulus to three dimensions.	Young's	shear	bulk	dynamic	bulk
41.	Unit of moment of inertia__________	Kgm^{-2}	Kgm^{-1}	Kgm	Kgm^{2}	Kgm^{2}
42.	Dimensional formula for I _____________	ML^{2}	MLT^{2}	$ML^{2}T$	$M^{2}LT$	ML^{2}
43.	Area of a ring in circular disc___________	2πxdx	2xdx	πxdx	2πx	2πxdx
44.	Moment of inertia of a solid sphere __________	2/5 MR^{2}	1/5 MR^{2}	2/3MR^{2}	5/2 MR^{2}	2/5 MR^{2}
45.	Mass of solid sphere ___________	4/3 $\pi\rho R^{3}$	$4\pi\rho R^{3}$	2/3 $\pi\rho R^{3}$	1/3 $\pi\rho R^{3}$	4/3 $\pi\rho R^{3}$
46.	Volume of hollow sphere _____________	(4/3)(R32-R31)	(4)π(R32-R31)	(1/3)π(R32-R31)	(4/3)π(R31-R32)	(4/3)π(R32-R31)
47.	Unit of g ____________	ms^{-2}	ms^{2}	s^{-2}	m^{-2}	ms^{-2}
48.	Value of g_______	9.8	8.9	98.8	0.98	9.8
49.	Compound pendulum is used to determine ______________	L	m	t or g	v	t or g
50.	___________ is the force resisting the relative motion of solid surfaces, fluid layers, or material elements sliding against each other.	Friction	Drag	Wear	Tire	Friction
51.	ratio of the lateral contraction to longitudinal elongation	poissions ratio	Young's modulus	rigidity modulus	bulk modulus	poissions ratio
52.	a rod or bar of uniform cross section whose length is grater than thickness	beam	disc	rod	sphere	beam
53.	which method is used to find out rigidity modulus	jaegars method	stoke's method	millikan method	searl's method	searl's method
54.	which method is used to find out rigidity modulus	jaegars method	stoke's method	millikan method	torsion pendulum	torsion pendulum
55.	which method is used to find out rigidity modulus using static torsion	jaegars method	stoke's method	millikan method	searl's method	searl's method
56.	torsion involves	longitudinal strain	shearing strain	tensile strain	volume strain	shearing strain
57.	which method is used to determin young's modulus	pin & microscope	jaegars method	stoke's method	millikan method	pin & microscope
58.	which method is used to determin young's modulus	Scale & telescope	jaegars method	stoke's method	millikan method	Scale & telescope
59.	which method is used to determin young's modulus	optic lever	jaegars method	stoke's method	millikan method	optic lever
60.	which method is used to determin young's modulus	Cantilever	jaegars method	stoke's method	millikan method	Cantilever
61.	which method is used to determin young's modulus	Konig's method	jaegars method	stoke's method	millikan method	Konig's method
62.	which method is used to determin young's modulus	Searl's method	jaegars method	stoke's method	millikan method	Searl's method