**SEMESTER – II**

**ELECTRICITY AND MAGNETISM** **L T P C**

**18PHU201** **5 - - 5**

**Objective:**

The aim of this course is to establish a grounding in electromagnetism in preparation for more advanced courses. The major concepts covered are: the abstraction from forces to fields using the examples of the gravitational, electric and magnetic fields, with some applications; the connection between conservative forces and potential energy; how charges move through electric circuits; the close connection between electricity and magnetism, leading to the discovery of electromagnetic waves.

**UNIT – I**

**Electrostatics:** Electrostatic Field, electric flux, Gauss's theorem of electrostatics. Applications of Gauss theorem- Electric field due to point charge, infinite line of charge, uniformly charged spherical shell and solid sphere, plane charged sheet, charged conductor.

**UNIT - II**

Electric potential as line integral of electric field, potential due to a point charge, electric dipole, uniformly charged spherical shell and solid sphere. Calculation of electric field from potential. Capacitance of an isolated spherical conductor. Parallel plate, spherical and cylindrical condenser. Energy per unit volume in electrostatic field. Dielectric medium, Polarisation, Displacement vector. Gauss's theorem in dielectrics. Parallel plate capacitor completely filled with dielectric.

**UNIT - III**

**Magnetostatics**: Biot-Savart's law and its applications- straight conductor, circular coil, solenoid carrying current. Divergence and curl of magnetic field. Magnetic vector potential. Ampere's circuital law.

**Magnetic properties of materials:** Magnetic intensity, magnetic induction, permeability, magnetic susceptibility. Brief introduction of dia-, para- and ferro-magnetic materials.

**UNIT – IV**

**Electromagnetic Induction**: Faraday's laws of electromagnetic induction, Lenz's law, self and mutual inductance, L of single coil, M of two coils. Energy stored in magnetic field.

**UNIT – V**

**Maxwell`s equations and Electromagnetic wave propagation**: Equation of continuity of current, Displacement current, Maxwell's equations, Poynting vector, energy density in electromagnetic field, electromagnetic wave propagation through vacuum and isotropic dielectric medium, transverse nature of EM waves, polarization.

**TEXT BOOKS**

1. Edward M. Purcell (2013), Electricity and Magnetism, Cambridge University Press
2. Textbook of electricity and magnetism-N Subrahmanyam, Brij Lal, Ratan Prakashan Ltd.
3. Electricity and Magnetism - D.L. Sehgal, K.L. Chopra, N.K.Sehgal, 2014, Sultan Chand & Co.
4. Electricity & Magnetism 7th Edition , R. Murugeshan, S Chand & Company Ltd

**REFERENCE BOOKS**

1. Griffiths D.J. (2013), Introduction to Electrodynamics, United States, Benjamin Cummings.
2. D Halliday, R Resnick and J Walker, Fundamentals of Physics (Extended) 6th ed., John Wiley, 2001.
3. Halliday D, Resnick R and Walker J (2013), Fundamentals of Physics (Extended) 10th ed., New Delhi, John Wiley.

# KARPAGAM ACADEMY OF HIGHER EDUCATION

***(Deemed to be University Established Under Section 3 of UGC Act 1956)***

**Coimbatore – 641 021.**

## LECTURE PLAN

## DEPARTMENT OF PHYSICS

**STAFF NAME**: Dr.A.SARANYA **SUB.CODE**:18PHU201

**SUBJECT NAME**: ELECTRICITY & MAGNETISM **CLASS**: I B.Sc (PHY)

**SEMESTER**: II

### UNIT-I

| Unit No. | No. of hours (8) | Topics to be covered | Support materials |
|---|---|---|---|
| | 1 | Electrostatic Field,electric flux | T2:47 |
| | 1 | Gauss's theorem of electrostatics,<br>Applications of Gauss theorem | T1:13-14 |
| | 2 | Electric field due to<br>point charge, infinite line of charge | T2:52 |
| | 1 | uniformly charged spherical shell and solid sphere | |
| | 1 | plane charged sheet | T1:21 |
| | 1 | charged conductor | T1:17-18 |
| | 1 | Revision | |

**TEXTBOOK:**

T1: Electricity and Magnetism by R.Murugesan, S.Chand & company, New Delhi.

T2: Electricity and Magnetism by D.C.Tayal, Himalaya publishing house.

T3: Electromagnetic theory by Chopra and Agarwal, S.Chand & company, New Delhi.

**UNTI-II**

| Unit No. | No. of hours (10) | Topics to be covered | Support materials |
|---|---|---|---|
| | 1 | Electric potential as line integral of electric field,potential due to a point charge,electric uniformly charged spherical shell and solid sphere | T1:35-42 |
| | 1 | Calculation of electric field from potential<br>Capacitance of an isolated spherical conductor | T1:43-44<br>T1:57-59 |
| | 1 | Parallel plate spherical and cylindrical condenser | T1:60 |
| | 1 | Energy per unit volume in electrostatic field | T1:67-68 |
| | 2 | Dielectric medium, Polarisation, | T1:279 |
| | 1 | Displacement vector | T1:280 |
| | 1 | Gauss's theorem in dielectrics | T1:281 |
| | 1 | Parallel plate capacitor completely filled with dielectric | T1:302-303 |
| | 1 | Revision | |

**TEXTBOOK:**

T1: Electricity and Magnetism by R.Murugesan, S.Chand & company, New Delhi.

T2: Electricity and Magnetism by D.C.Tayal, Himalaya publishing house.

T3: Electromagnetic theory by Chopra and Agarwal, S.Chand & company, New Delhi.

## UNIT-III

| Unit No. | No. of hours (10) | Topics to be covered | Support materials |
|---|---|---|---|
| III | 1 | Biot-Savart's law and its applications | T1:132-133 |
| | 1 | Straight conductor, circular coil, solenoid carrying current | T3:124, 125, 126 |
| | 1 | Divergence and curl of magnetic field | T1:416 |
| | 1 | Magnetic vector potential | T1:345-346 |
| | 1 | Ampere's circuital law | T1:155-156 |
| | 1 | Magnetic intensity, magnetic induction, | T1:247 |
| | 1 | permeability, magnetic susceptibility | T1:249-250 |
| | 1 | Brief introduction of dia-, para | T1:251 |
| | 1 | ferro-magnetic materials | T1:252 |
| | 1 | Revision | |

**TEXTBOOK:**

T1: Electricity and Magnetism by R.Murugesan, S.Chand & company, New Delhi.

T2: Electricity and Magnetism by D.C.Tayal, Himalaya publishing house.

T3: Electromagnetic theory by Chopra and Agarwal, S.Chand & company, New Delhi.

## UNIT-IV

| Unit No. | No. of hours (8) | Topics to be covered | Support materials |
|---|---|---|---|
| IV | 2 | Faraday's laws of electromagnetic | T1:162 |
| | 1 | Lenz's law | T1:163 |
| | 1 | Self and mutual inductance | T1:164-166 |
| | 1 | L of single coil | T1:166-170 |
| | 1 | M of two coils | T1:172 |
| | 1 | Energy stored in magnetic field | T2:428 |
| | 1 | Revision | |

**TEXTBOOK:**

T1: Electricity and Magnetism by R.Murugesan, S.Chand & company, New Delhi.

T2: Electricity and Magnetism by D.C.Tayal, Himalaya publishing house.

T3: Electromagnetic theory by Chopra and Agarwal, S.Chand & company, New Delhi.

## UNIT-V

| Unit No. | No. of hours (14) | Topics to be covered | Support materials |
|---|---|---|---|
| V | 1 | Equation of continuity of current | T3:171-172 |
| | 1 | Displacement current | T3:274-275 |
| | 2 | Maxwell's equations | T1:400-401 |
| | 2 | Poynting vector | T1:277 |
| | 1 | Energy density in electromagnetic field | T3:188-196 |
| | 2 | Electromagnetic wave propagation through vacuum isotropic dielectric medium | T3:230-234 |
| | 1 | Transverse nature of EM waves | T1:491 |
| | 1 | Polarization | T1: 492 |
| | 1 | Revision | |
| | 1 | Old question paper discussion | |
| | 1 | Old question paper discussion | |

**TEXTBOOK:**

T1: Electricity and Magnetism by R.Murugesan, S.Chand & company, New Delhi.

T2: Electricity and Magnetism by D.C.Tayal, Himalaya publishing house.

T3: Electromagnetic theory by Chopra and Agarwal, S.Chand & company, New Delhi.

**UNIT-I**

**SYLLABUS**

| |
|---|
| Electrostatics: Electrostatic Field, electric flux, Gauss's theorem of electrostatics. Applications of Gauss theorem- Electric field due to point charge, infinite line of charge, uniformly charged spherical shell and solid sphere, plane charged sheet, charged conductor. |

## Electrostatic Field:

To calculate the force exerted by some electric charges, $q_1$, $q_2$, $q_3$, ... (**the source charges**) on another charge $Q$ (**the test charge**) we can use the **principle of superposition**. This principle states that the interaction between any two charges is completely unaffected by the presence of other charges. The force exerted on $Q$by $q_1$, $q_2$, and $q_3$ (see Figure 2.1) is therefore equal to the vector sum of the force $\bar{F}_1$ exerted by $q_1$ on $Q$, the force $\bar{F}_2$ exerted by $q_2$ on $Q$, and the force $\bar{F}_3$ exerted by $q_3$ on $Q$.

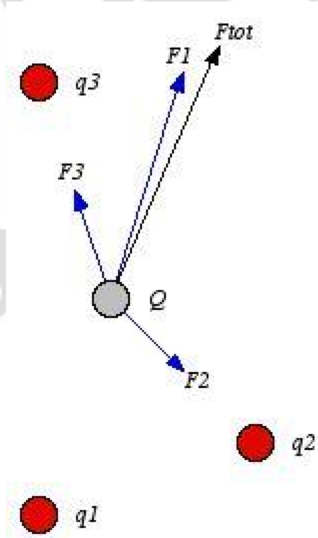



**Figure 1. Superposition of forces.**

The force exerted by a charged particle on another charged particle depends on their separation distance, on their velocities and on their accelerations. In this Chapter we will consider the special case in which the source charges are stationary.

The **electric field** produced by stationary source charges is called and **electrostatic field**. The electric field at a particular point is a vector whose magnitude is proportional to the total force acting on a test charge located at that point, and whose direction is equal to the direction of the force acting on a positive test charge. The electric field $\bar{E}$, generated by a collection of source charges, is defined as

$$\bar{E} = \frac{\bar{F}}{Q}$$

where $\bar{F}$ is the total electric force exerted by the source charges on the test charge $Q$. It is assumed that the test charge $Q$ is small and therefore does not change the distribution of the source charges. The total force exerted by the source charges on the test charge is equal to

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \dots = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{r_1^2}\hat{r}_1 + \frac{q_2 Q}{r_2^2}\hat{r}_2 + \frac{q_3 Q}{r_3^2}\hat{r}_3 + \dots\right) = \frac{Q}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

The electric field generated by the source charges is thus equal to

$$\bar{E} = \frac{\bar{F}}{Q} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

In most applications the source charges are not discrete, but are distributed continuously over some region. The following three different distributions will be used in this course:

1. **line charge** λ: the charge per unit length.

2. **surface charge** σ: the charge per unit area.

3. **volume charge** ρ: the charge per unit volume.

To calculate the electric field at a point $\bar{P}$ generated by these charge distributions we have to replace the summation over the discrete charges with an integration over the continuous charge distribution:

1. for a line charge: $\bar{E}(\bar{P}) = \frac{1}{4\pi\varepsilon_0}\int_{Line}\frac{\hat{r}}{r^2}\lambda dl$

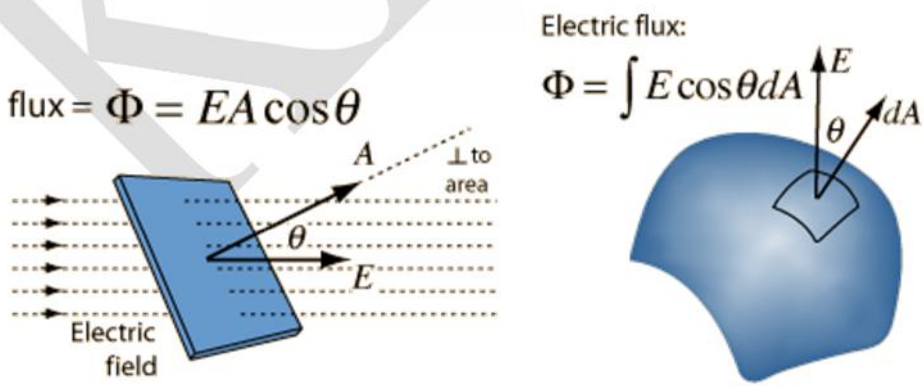
2. for a surface charge: $\bar{E}(\bar{P}) = \frac{1}{4\pi\varepsilon_0}\int_{Surface}\frac{\hat{r}}{r^2}\sigma da$

3. for a volume charge: $\bar{E}(\bar{P}) = \frac{1}{4\pi\varepsilon_0}\int_{Volume}\frac{\hat{r}}{r^2}\rho d\tau$

Here $\hat{r}$ is the unit vector from a segment of the charge distribution to the point $\bar{P}$ at which we are evaluating the electric field, and $r$ is the distance between this segment and point $\bar{P}$.

**Electric Flux:**

The concept of electric flux is useful in association with Gauss' law. The electric flux through a planar area is defined as the electric field times the component of the area perpendicular to the field. If the area is not planar, then the evaluation of the flux generally requires an area




integral since the angle will be continually changing.

When the area A is used in a vector operation like this, it is understood that the magnitude of the vector is equal to the area and the direction of the vector is perpendicular to the area.

**Gauss's theorem of electrostatics:**

**Statement :**

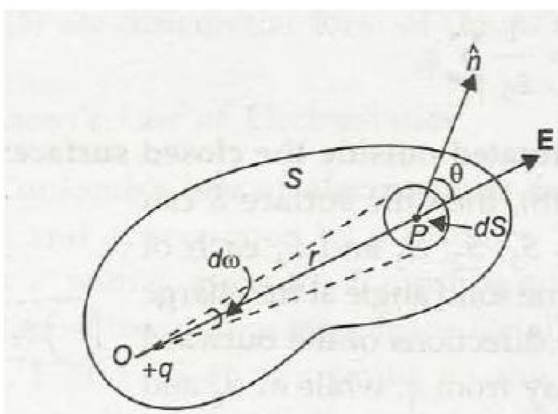
"The total normal electric flux over a closed surface in an electric field is equal to 1/$\varepsilon$0 times the total charge enclosed by that surface."

Mathematically it may be expressed as

$$\phi = \oint_S E \cos\theta \, dS = \oint_S \mathbf{E} . d\mathbf{S} .$$

**Proof.**

**(l) When the charge lies inside the closed surface:** Let us consider a source producing the field is a point charge + q situated at 0 inside the closed surface as shown in Figure 7.17.




Let dS be an infinitesimal element of the surface at point P and OP = r. As shown in Figure , the electric field strength vector E makes angle 8 with the unit vector n drawn normal to the surface element dS surrounding point P. The surface integral of the normal component of this electric field over the closed surface is given by ∫ $\int_s$ E. n dS.

The electric field strength E at the point P is given by

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^3} \mathbf{r}$$

$$\iint_S \mathbf{E} \cdot \hat{n}\, dS = \frac{1}{4\pi\varepsilon_0}\, q \iint_S \frac{r.\hat{n}}{r^3}\, dS$$

Now, it can be seen that the quantity ( r .n/r  dS) gives the projection of area dS on theplane perpendicular to r and therefore.

projected area $r^2$ =r. n dS /$r_3$= dw

where dw is the solid angle subtended by dS at 0.

From equation (1) and (2) we get

$\iint_S$ E .n ds =1/4π$\varepsilon_0$ q $\iint_s$dw

But $\iint_S$dw = 4π = solid angle subtended by entire closed surface at an internal point

$\iint_S$ E .n ds =1/4π$\varepsilon_0$ q .4 =q/$\varepsilon_0$

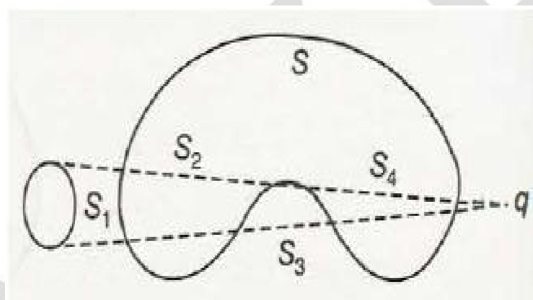
This result is known as Gauss's law for a single point charge enclosed by the surface.

If several point charges $q_1$, $q_2$, $q_3$ . . . . $q_n$ be enclosed by the surface 5, then total electric field is given by L qi (= $q_1$ + $q_2$ + $q_3$ + . ….. .. + qn). Each charge subtends a full solid angle 4n and equation (4) becomes ·

$$\iint_S \mathbf{E} \cdot \hat{n}\, dS = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i$$

**(ii) When the charge is situated outside the closed surface:**

If the charge q is outside the surface (Figure), then the surface 5 can be divided into areas $S_1$, $S_2$, $S_3$ and $S_4$each of which subtends the same solid angle at the charge q. But at $S_1$ and $S_3$the directions of the outward drawn normal are away from q, while at $S_2$ and $S_4$ they are toward q. Therefore the contributions of two pairs ($S_1$, $S_3$) and ($S_2$, $S_4$) to the surface integral are equal and opposite. Asa result the Net surface integral of the normal component of the electric field Evanishes, i.e.,




$\iint_S E .n\, ds = \int \int_S E .ds = 0$

equations (4) and (6) represents the integral form of the Gauss's law.

**Differential Form of Gauss's law:**

Let a charge q be distributed over a volume V of the closed surface 5 and p be the chargedensity; then the charge q may be given as

$q = \int \iint_V p dV = \int_V p dV$

Thus the total flux through the surface S $$\phi = \oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0} q = \frac{1}{\varepsilon_0}\int_V \rho\, dV$$

According to Gauss divergence theorem the surface integral may be converted into volume integral as

$$\oint_S \mathbf{E}\cdot d\mathbf{S} = \int_V (\text{div}.\ \mathbf{E})\, dV = \int_V (\nabla\cdot\mathbf{E})\, dV$$

Hence From equation (2) ,we obtain

$\int_v$ div E dv =1/$\varepsilon_0$ $\int_v$ p dv

div E =1/$\varepsilon_0$ P

Δ .E =1/$\varepsilon_0$ P

As the displacement vector D is defined as

D = $\varepsilon_0$ E, we have

Δ.D = div D = p

Equations (4) and (5) are differential form of Gauss's law of electrostatics

**Applications of Gauss theorem:**

(A) **Electric field due to point charge:**

We have considered Coulomb's law as fundamental equation of electrostatics and have derived Gauss's law from it. However Coulomb's law can also be derived from Gauss's law. This is done by using this law to obtain the expression for the electric field due to a point charge.

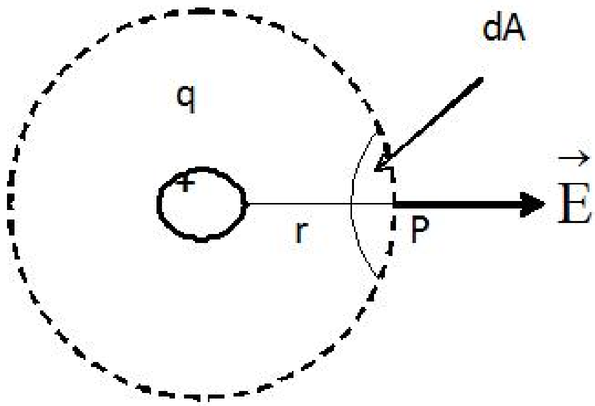



Figure : Electric field at a point on the spherical Gaussian surface surrounding a point charge.

Consider the electric field due to a single positive point charge q. By symmetry, the field is everywhere radial and its magnitude is the same at all points, that are at the same distance r from the charge as shown in figure 3B.1. Hence, if we select, as a Gaussian surface, a spherical surface of radius r, at all points on this surface and the field is radial. If we consider a small elementary area of this gaussian surface, the area vector is in the radial direction i.e. perpendicular to surface. Then.

$$\int_S \vec{E}_\perp . \vec{dA} = \int_S E_\perp dA = E_\perp \int_S dA = EA = E(4\pi r^2)$$

From Gauss's law

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

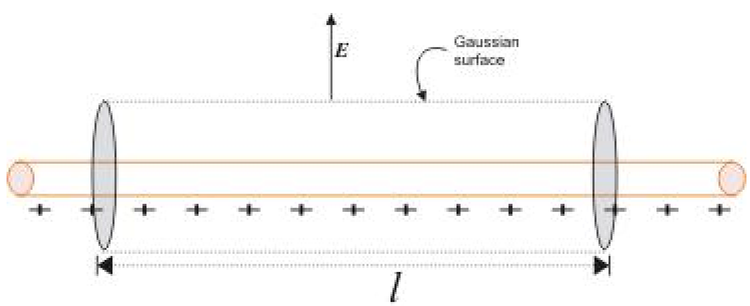
The force on a point charge q' at a distance r from the charge q is then

$$F = q'E = \frac{1}{4\pi\epsilon_0}\frac{qq'}{r^2}$$

which is Coulomb's law.

(B) **Electric field due to infinite line of charge:**

- Consider a long thin uniformly charged wire and we have to find the electric field intensity due to the wire at any point at perpandicular distance from the wire.
- If the wire is very long and we are at point far away from both its ends then field lines outside the wire are radial and would lie on a plane perpandicular to the wire.
- Electric field intensity have same magnitude at all points which are at same distance from the line charge.
- We can assume Gaussian surface to be a right circular cylinder of radius r and length l with its ends perpandicular to the wire as shown below in the figure.




- λ is the charge per unit length on the wire. Direction of **E** is perpendicular to the wire and components of **E** normal to end faces of cylinder makes no contribution to electric flux. Thus from Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$$

- Now consider left hand side of Gauss's law

$$\oint \mathbf{E} \cdot d\mathbf{a} = E\oint da$$

Since at all points on the curved surface **E** is constant. Surface area of cylinder of radius r and length l is A=2πrl therefore,

$$\oint \mathbf{E} \cdot d\mathbf{a} = E(2\pi r l)$$

- Charge enclosed in cylinder is q=linear charge density x length l of cylinder,
  or, $q=\lambda l$
  From Gauss's law

$$\oint E \cdot da = \frac{q}{\varepsilon_0}$$

$$or, E(2\pi r l) = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\lambda}{2\pi r \varepsilon_0}$$

$$\Rightarrow E \propto \frac{\lambda}{r}$$

Thus electric field intensity of a long positively charged wire does not depends on length of the wire but on the radial distance r of points from the wire.

(C) **Electric field due to uniformly charged spherical shell:**

**Electric Field due to a uniformly charged sphere**

A spherically symmetric distribution of charge means the distribution of charge where the charge density ρ depends only on the distance of the point from the center and not on the direction. Let the spherically symmetric charge distribution be characterised by a charge density function, ρ(r), which varies in a certain manner, with distance, from the center of the spherical surface. Consider first the case of a charge q that is uniformly distributed over a sphere of radius R. Let us calculate the electric field strength at any point distant r from the centre

(i) Electric field strength at an external point

$$\therefore\ E_o = \frac{1}{4\pi \epsilon_o} \frac{q}{r^2}$$

which is the same if the charge q were placed at the centre O.

Hence the electric field strength, at any point outside a spherical charge distribution, is the same as through the whole charge were concentrated at the centre.

(ii) Electric field strength at the surface of the spherical charge distribution

In this case, the point P lies on the surface of the spherical charge i.e. r=R. Hence the electric field strength, on the surface of the spherical charge distribution, is

$$E_s = \frac{1}{4\pi \epsilon_o} \frac{q}{R^2}$$

(iii) Electric field strength at an internal point

Let P be an internal point at a distance r (r < R)from the centre of the charge distribution. Consider a sphere of radius (OP=r) concentric with the spherical charge (figure 3B.3). Let $\rho$ be the volume charge density (charge per unit volume)

$$\therefore\ \rho = \frac{\text{charge}}{\text{volume}} = \frac{q}{\frac{4}{3}\pi R^3}$$

Let the whole surface be divided into thin spherical shells. The electric field strength $E_i$, at P, is the combined effect of shells outside the spherical surface of radius r, as well as those inside it. But the electric field strength contribution due to the outer spherical shells, is zero which may be seen as follows:

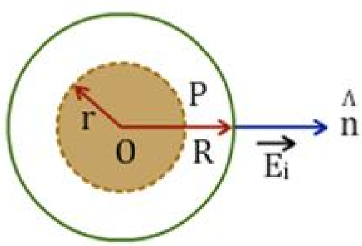



Figure 3B.3 Electric field at a point inside uniformly charged sphere.

Consider any point P, inside a charged thin shell of radius x. We have to find the electric field strength at P due to this shell. Consider a spherical surface of radius OP (= r) concentric with this spherical shell. By symmetry, the electric field strength, $\vec{E}$, at every point of this spherical surface, has the same magnitude and is directed along the outward drawn-normal to this surface. The electric flux through the whole surface $\int_s \vec{E}.\vec{da} = \int_s E da = E\int_s da = E(4\pi r^2)$. According to Gauss's theorem, $E 4\pi r^2 = \frac{1}{\epsilon_o} \times$ total charge enclosed by the surface. This equals zero since the net charge enclosed by this internal surface is zero. This implies that E=0.

Thus electric field strength, due to a charged spherical shell, at an internal point is zero. Hence the electric field strength, $E_i$, at P, is due to the inner shells only, which may be found as follows:

By symmetry, the electric field strength $E_i$ at every point of the spherical surface of radius r has the same magnitude and is directed along the outward drawn normal to the surface. The total electric flux through the whole surface

$$\int_S \vec{E_i} \cdot \vec{da} = \int_S E_i da = E_i \int_S da = E_i(4\pi r^2).$$

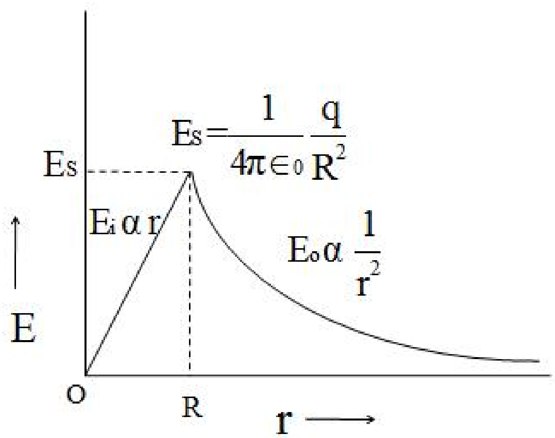



Figure 3B.4 Electric field due to a uniformly charged sbhere as a function of distance from the centre of sphere.

According to Gauss's law, $E_i 4\pi r^2 = \frac{1}{\epsilon_0} \times$ charge enclosed by the Gaussian surface

$$= \frac{1}{\epsilon_0}\int_0^r \rho(4\pi x^2)dx = \frac{1}{\epsilon_0}\rho\frac{4}{3}\pi r^3$$

$$\Rightarrow E_i = \frac{1}{4\pi\epsilon_0}\left(\frac{4}{3}\pi r\rho\right) = \frac{1}{4\pi\epsilon_0}\frac{4}{3}\pi r.\frac{q}{\frac{4}{3}\pi R^3} \qquad \left(\because \text{ since } \rho = \frac{q}{\frac{4}{3}\pi R^3}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{qr}{R^3}$$

Thus the electric field strength, at a point P inside a spherically symmetric charge distribution, is directly proportional to the distance of the point P from the centre of the spherical charge. The variation, of the magnitude of the electric field strength, with distance, from the centre of a spherically symmetric charge distribution, is, therefore, represented by the curve shown in figure 3B.4.

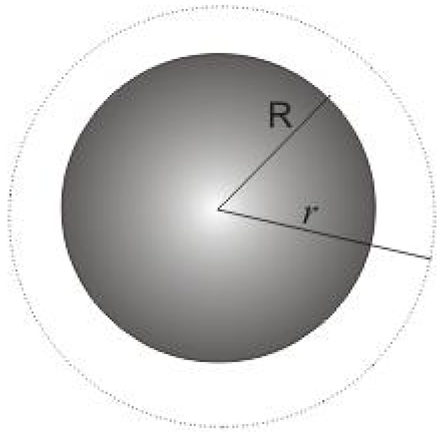
(D) **Electric field due to uniformly charged solid sphere:**

- We'll now apply Gauss's law to find the field outside uniformly charged solid sphere of radius R and total charge q.
- In this case Gaussian surface would be a sphere of radius r>R concentric with the charged solid sphere shown below in the figure.From Gauss's law

  $$\oint \mathbf{E}\cdot d\mathbf{a} = \frac{q_{enc}}{\varepsilon_0}$$

  where q is the charge enclosed.
- Charge is distributed uniformly over the surface of the sphere. Symmetry allows us to extract **E** out of the integral sign as magnitude of electric field intensity is same for all points at distance r>R.
- Since electric field points radially outwards we have

$$\oint \mathbf{E}\cdot d\mathbf{a} = E\oint da$$




also as discussed magnitude of **E** is constant over Gaussian surface so,

$$E\oint da = E(4\pi r^2)$$

where $4\pi r^2$ is the surface area of the sphere.

Again from Gauss's law we have

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$$

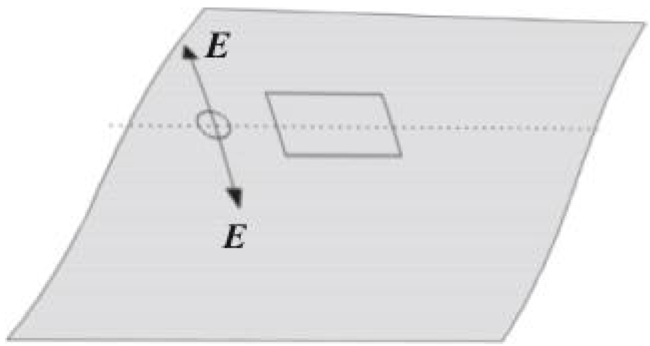
Thus we see that magnitude of field outside the sphere is exactly the same as it would have been as if all the charge were concentrated at its center.

(E) **Electric field due to plane charged sheet:**

**Electric field due to an infinite plane sheet of charge**

- Consider a thin infinite plane sheet of charge having surface charge density σ(charge per unit area).
- We have to find the electric field intensity due to this sheet at ant point which is distance r away from the sheet.

- We can draw a rectangular gaussian pillbox extending equal distance above and below the plane as shown below in the figure.




- By symmetry we find that **E**on either side of sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet.
- No field lines crosses the side walls of the Gaussian pillbox i.e., component of **E** normal to walls of pillbox is zero.
- We now apply Gauss's law to this surface

  $$\oint \boldsymbol{E} \cdot d\boldsymbol{a} = \frac{q_{enc}}{\varepsilon_0}$$

  in this case charge enclosed is

  $q=\sigma A$

  where A is the area of end face of Gaussian pillbox.
- **E** points in the direction away from the plane i.e., **E** points upwards for points above the plane and downwards for points below the plane. Thus for top and bottom surfaces,

  $$\oint \boldsymbol{E} \cdot d\boldsymbol{a} = 2A\,|\,\boldsymbol{E}\,|$$

  thus

  $2A|\mathbf{E}|=\sigma A/\varepsilon_0$

  or,

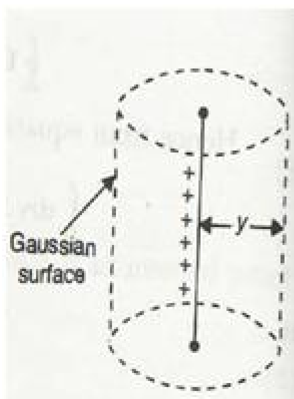
  $|\mathbf{E}|=\sigma/2\varepsilon_0$

Here one important thing to note is that magnitude of electric field at any point is independent of the sheet and does not decrease inversely with the square of the distance. Thus electric field due to an infinite plane sheet of charge does not falls of at all.

(F) **Electric field due to a charged conductor:**

**Field around a charged straight conductor (Line Charge):** Consider a linear charged conductor. Let the linear charge density be λ Imagine a Gaussian cylinderof radius 'y' and length 'I',closed at each end by plane caps normal to the axis. Bysymmetry, all the lines of force go radially outward.

As shown in Figure , no lines cross the flat ends of thecylinder and hence the ∫ E dS over these end surfaces are zero.




Over the remaining surface of cylinder, the field is uniform and outward.

. . Applying Gauss theorem

$\varphi = q/\varepsilon_0 = l\lambda/\varepsilon_0$

Net flux passing over the Gaussian surface

$$= \oint E\, dS \cos 0$$

$$= E \oint dS = E\,(2\pi y)\, l$$

From equations (1) and (2),= E (2ny) l or

But $q/\varepsilon 0 = \sigma\,(2\pi a l)/\varepsilon_0 = (\lambda/2\pi a)\,(2\pi a)/\varepsilon_0 = \lambda l$

From equations (1) and (2),

$\Lambda l/\varepsilon_0 = E\,(2\pi\varepsilon_0 r)$

**Electric field due to a charged conductor (Coulomb's theorem):**

**Statement :** The electric field at any point near a charged conductor is 1/c.0 times the surface density of charge on the surface.

Proof : Let 'AB' be a large conducting surface and σ be the surface charge density. To find the field at a point P,infinitely close to the charged surface, imagine a small cylinder of cross-section 'S' drawn with its faces parallel to the charged surface. Le one of the faces pass through P and the other through Q inside the conductor as shown in Figure 7.26. Normal component of E, through the sides of the cylinder, formed by faces enclosing P and Q i.e., Gaussian surface is zero.

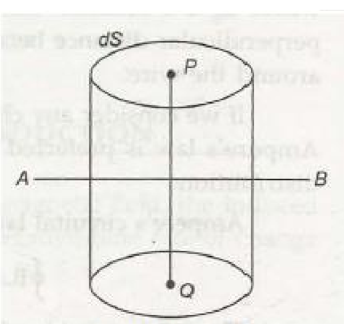


FIGURE 7.26

Since the charge resides on the surface of the conductor, there is no charge on the face enclosing 'Q' and therefore normal component of E through the area dS is also zero. Hence the normal component of 'E' is only through containing 'P'.

or

According to Gauss theorem,

$Eds = \sigma ds/\varepsilon_0$

$E = \sigma/\varepsilon_0$

**KARPAGAM UNIVERSITY**
**DEPARTMENT OF PHYSICS**
**I B.SC PHYSICS**
**ELECTRICITY AND MAGNETISM (17PHU201)**

| QUESTIONS | OPTION 1 | OPTION 2 |
|---|---|---|
| **UNIT-I** | | |
| If the distance between two charge is doubled the electrostat | fourtime more | four time less |
| The field due to a wire of uniform charge density at a perper | increases with in | decrease with increaa |
| Field due to a uniformly charged ring at an axial point at dis | independent of x | directly proportional |
| Electric charge enclosed by Gaussian surface is | 0 | 1 |
| distributed | arbitraril | sequentially |
| Gauss law is __________________ | $\epsilon_0 \int E.ds=q$ | $\Delta.D=\rho/\epsilon_0$ |
| The unit of Polarization is _________ | coulomb/$m^2$ | coulomb/m |
| Electric field intensity outside two charged parallel | $\sigma/2\varepsilon_0$ | $\sigma/\varepsilon_0$ |
| The total electric flux over any closed surface is | $\varepsilon_0$ | $\sigma/\varepsilon_0$ |
| Electric flux lines due to an infinite sheet of charge | convergi | radial |
| One electron volt is __________. | 1.6 x $10^{-19}$joule | 1.6 x 10-19volt |
| _________ law establishes a relationship between the electric flux and the electrostatic charge. | Lenz's | Keplers |
| The ratio $\epsilon/\epsilon 0$ is a dimensionless quantity known as ________ | relative permeability | relative permittivity |
| The electric field lines begin at the __________ charge and terminate at the _________ charge. | positive, negative | negative, positive |
| placed in air is ____________. | $q/\epsilon_0$ | $\epsilon_0$ q |
| Gauss's law due to different charge distribution is u | electric f | electric charg |
| The total flux across a closed surface enclosing cha | shape of the closed surface | |
| Electric field intensity two charged parallel plate is | $\sigma/2\varepsilon_0$ | #VALUE! |
| Electrostatic field is always ___________. | Solenoidal | Irrotational |
| The unit of Electric flux is __________. | Gauss's | Weber |
| .Mechanical pressure on the surface of a charged conductor having surface charge density σ is __________. | $\epsilon_0 \sigma^2$ | $\sigma^2 / \epsilon_0$ |
| Gauss's law in a dielectric medium takes the form ∫D.ds=q,where q is _________. | total free charge enclosed | polarization charges |

| | | |
|---|---|---|
| If the separation between two charges is increased the electric potential energy | always decreases | always increases |
| Electric intensity due to an infinitely long plane she | indepen | proportional |
| The total electric flux through a closed surface dep | location | the shape of |
| Electric field intensity due to an infinite plane shee | $\sigma/\varepsilon_0$ | $q/2\varepsilon_0$ |
| Law stated as flux is $^1/_{E_o}$ times total charge is | ohms la | newton's law |
| A Gaussian sphere closes an electric dipole within | half due | double due t |
| The Flux of electric field is____ | scalar | vector |
| Flux density is measured in | Tesla | Weber |
| Which of the following quantities are scalar? | dipole moment | electric force |
| A dipole is placed in a uniform electric field with its axis parallel to the field. It experiences | only a net force | only a torque |
| Electric potential energy U of two point charges is | $q_1q_2/4\varepsilon_0\pi r^2$ | $q_1q_2/4\varepsilon_0\pi r$ |
| If a point lies at a distance x from the midpoint of the dipole, the electric potential at the point is proportional to | $1/x^2$ | $1/x^3$ |
| The law that governs the force between electric charge is cal | Amperes | Coulomb law |
| The minimum value of the charge in any object cannot be le | $1.6 \times 10^{-19}$coulomb | $3.2 \times 10^{-19}$ Coulomb |
| An electric field can deflect | X rays | neutrons |
| Inside the hollow spherical conductor, the potential | is constant | varies directly as the |
| The intensity at a point due to a charge is inversely proportic | amount of the ch | size of the charge |
| The distance between two charge is douled then the force be | half | one-fourth |
| A surface enclosed an electric dipole, the flux through the su | infinite | positive |
| Electric potential is a | vector quantity | scalar quantity |
| The potential at any point inside a charged sphere is | zero | same as potential on t |
| Two smallspheres eaach carrying a charge q are placed r me | force between th | force between them x |
| State which of the following is correct? | J=Coulomb x vo | J=Coulomb / volt |
| A positively charged glass rod attracts an object. The object | negatively charg | either negative charg |
| A charge q islocated at the centre of a hypothetical cube. Th | $q/\epsilon_0$ | $q/2\epsilon_0$ |
| The force between two electrons seperated by a distance r va | r^2 | relative permittivity |
| The energy stored per unit volume of the medium of relative permittivity is ______________. | $\epsilon_r\epsilon_0E^2/2$ | $\epsilon_0E^2/2$ |
| All magnetic moments within a domain will point in the __________ direction. | Different | Same |
| The electrical energy consumed by a coil is stored in the form of: | magnetic field | force field |
| Electricity may be generated by a wire: | carrying current | wrapped as a coil |
| A magnetic field has: | lines of reluctan | polar fields |
| The polarity of induced voltage while a field is collapsing is | opposite to the fo | independent of the fo |

| | |
|---|---|
| What is magnetic flux? | the number of lin the number of lines o |

**Prepared by Dr.A.Saranya,Assistant Professor**

| OPTION 3 | OPTION 4 | ANSWER |
|---|---|---|
| will increase intc | will decrease into | four time less |
| remains constant | depends upon the l | decrease with increaase in y |
| directly proportic | inversely proportic | inversely proportional to x^2 |
| min | max | 0 |
| rational | in line | arbitrarily |
| $\Delta.D=q/\epsilon_0$ | $\int E.ds= \rho$ | $\epsilon_0 \int E.ds=q$ |
| coulomb.meter | coulomb.meter2 | coulomb/$m^2$ |
| infinity | 0 | 0 |
| $\varepsilon_0/ \sigma$ | $q/ \varepsilon_0$ | $q/ \varepsilon_0$ |
| uniform | uniform and paral | uniform and perpendicular to the sheet |
| 1.6 x 10-19joule | 1.6 x 10 -21joule | 1.6 x $10^{-19}$joule |
| Faraday | Gauss's | Gauss's |
| absolute permittivity | permeability | relative permittivity |
| both positive | both negative | positive, negative |
| q | $4\pi q$ | $q/\epsilon_0$ |
| electric i | electric fie | electric intensity |
| volume | actual spa | all |
| $\sigma/\varepsilon_0$ | 0 | $\sigma/\varepsilon_0$ |
| harmonic in character | rotational | Irrotational |
| $Nm^{-2}c^{-1}$ | $Nc^{-1}$ | $Nm^{-2}c^{-1}$ |
| $\sigma^2 / 2\epsilon_0$ | $\sigma /2\epsilon_0$ | $\sigma^2 / 2\epsilon_0$ |
| free and polarization | zero | charge enclosed |

| | | |
|---|---|---|
| remains the same | may increase or decrease | may increase or decrease |
| proporti | inversely | independent of r |
| the valu | both charg | the value of the net charge only |
| $\sigma/2\varepsilon_0$ | $q/\varepsilon_0$ | $\sigma/2\varepsilon_0$ |
| gauss's l | coulombs | gauss's law |
| zero | dependent | zero |
| zero | infinity | scalar |
| Ampere- turn | Maxwell | Tesla |
| electric field | electric potential | electric potential |
| both a net force and torque | neither a net force nor a torque | neither a net force nor a torque |
| $pE\sin\theta$ | $pE\cos\theta$ | $q_1q_2/4\varepsilon_0\pi r$ |
| $1/x^4$ | $1/x^{3/2}$ | $1/x^2$ |
| Faraday | Ohms | Coulomb law |
| $4.8 \times 10^{-19}$Coulomb | 1 coulomb | $1.6 \times 10^{-19}$coulomb |
| alpha particle | gamma rays | alpha particle |
| varies inversely a | varies inversely as | is constant |
| distance of the p | square of the dista | square of the distance from the charge |
| doubled | four times | one-fourth |
| negative | zero | zero |
| neither vector no | fictitious quantity | scalar quantity |
| smaller than the | greater than the po | same as potential on the surface |
| force between th | zero | zero |
| J= volt/ ampere | J= volt x ampere | J=Coulomb x volt |
| neutral | positively charged | either negative charged or neutral |
| $q/4\epsilon0$ | $q/4\epsilon0$ | $q/4\epsilon0$ |
| r^-1 | r^-2 | r^-2 |
| $\epsilon_r\epsilon_0E/2$ | $\epsilon_0E/2$ | $\epsilon_r\epsilon_0E^2/2$ |
| Positive | Negative | Same |
| electrostatic fielc | electrical field | magnetic field |
| passing through a | that has neutral do | passing through a flux field |
| lines of force | magnetomotive fo | lines of force |
| identical to the f | present only if the | opposite to the force creating it |

the number of lir the number of line the number of lines of force in webers

# UNIT-II

## SYLLABUS

Electric potential as line integral of electric field, potential due to a point charge, electric dipole, uniformly charged spherical shell and solid sphere. Calculation of electric field from potential. Capacitance of an isolated spherical conductor. Parallel plate, spherical and cylindrical condenser. Energy per unit volume in electrostatic field. Dielectric medium, Polarisation, Displacement vector. Gauss's theorem in dielectrics. Parallel plate capacitor completely filled with dielectric.

**Electric potential as line integral of electric field:**

The requirement that the curl of the electric field is equal to zero limits the number of vector functions that can describe the electric field. In addition, a theorem discussed in Chapter 1 states that any vector function whose curl is equal to zero is the gradient of a scalar function. The scalar function whose gradient is the electric field is called the **electric potential *V*** and it is defined as

$$\overline{E} = -\overline{\nabla} V$$

Taking the line integral of $\overline{\nabla} V$ between point *a* and point *b* we obtain

$$\int_a^b \overline{\nabla} V \bullet d\overline{l} = V(b) - V(a) = -\int_a^b \overline{E} \bullet d\overline{l}$$

Taking *a* to be the reference point and defining the potential to be zero there, we obtain for *V*(*b*)

$$V(b) = -\int_a^b \overline{E} \bullet d\overline{l}$$

The choice of the reference point *a* of the potential is arbitrary. Changing the reference point of the potential amounts to adding a constant to the potential:

$$V'(b) = -\int_{a'}^b \overline{E} \bullet d\overline{l} = -\int_{a'}^a \overline{E} \bullet d\overline{l} - \int_a^b \overline{E} \bullet d\overline{l} = K + V(b)$$

where *K* is a constant, independent of *b*, and equal to

$$K = -\int_{a'}^a \overline{E} \bullet d\overline{l}$$

However, since the gradient of a constant is equal to zero

$$E' = -\overline{\nabla} V' = -\overline{\nabla} V = E$$

Thus, the electric field generated by $V'$ is equal to the electric field generated by $V$. The physical behavior of a system will depend only on the difference in electric potential and is therefore independent of the choice of the reference point. The most common choice of the reference point in electrostatic problems is infinity and the corresponding value of the potential is usually taken to be equal to zero:

$$V(b) = -\int_{\infty}^{b} \overline{E} \bullet d\overline{l}$$

The unit of the electrical potential is the Volt (V, 1V = 1 Nm/C).

**Potential due to a point charge:**

- Consider a positive test charge +q is placed at point O shown below in the figure.


- We have to find the electric potential at point P at a distance r from point O.
- If we move a positive test charge q' from infinity to point P then change in electric potential energy would be

$$U_P - U_\infty = \frac{qq'}{4\pi\varepsilon_0 r}$$

- Electric potential at point P is

$$V_p = \frac{U_p - U_\infty}{q'} = \frac{q}{4\pi\varepsilon_0 r} \qquad (8)$$

- Potential V at any point due to arbitrary collection of point charges is given by

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad (9)$$

- here we see that like electric field potential at any point independent of test charge used to define it.

- For continuous charge distributions summation in above expressin will be replaced by the integration

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \qquad (10)$$

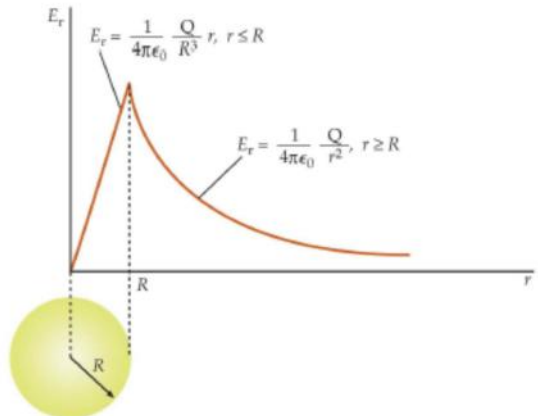
where dq is the differential element of charge distribution and r is its distance from the point at which V is to be calculated.

**Electric dipole:**

An electric dipole is two charged objects, with equal but opposite electric charges, that are separated by a distance. The electric field caused by a dipole falls off as the cube (third power) of the distance from the dipole, and has a directional variation that depends on whether you're moving along the line separating the two charges or perpendicular to it. A dipole can be created, for example, when you place a neutral atom in an electric field, because the positively-charged constituents of the atom will be pulled one way, and the negatively-charged constituents the other way, creating a separation of charge in the direction of the field.

**Electric field of Uniformly charged solid sphere:**

- Radius of charged solid sphere: $R$
- Electric charge on sphere: $Q = \rho V = \frac{4\pi}{3}\rho R^3$.
- Use a concentric Gaussian sphere of radius $r$.
- $r > R$: $E(4\pi r^2) = \frac{Q}{\epsilon_0}$
  $\Rightarrow \; E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$
- $r < R$: $E(4\pi r^2) = \frac{1}{\epsilon_0}\left(\frac{4\pi}{3}r^3\rho\right)$
  $\Rightarrow \; E(r) = \frac{\rho}{3\epsilon_0}\, r = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}\, r$

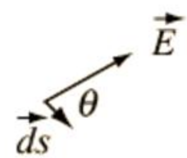



**Calculation of electric field from potential:**

One of the values of calculating the scalar electric potential (voltage) is that theelectric field can be calculated from it. The component of electric field in any direction is the negative of rate of change of the potential in that direction.

If the differential voltage change is calculated along a direction ds, then it is seen to be equal to the electric field component in that direction times the distance ds.

$$dV = -\vec{E}\cdot\vec{ds} = -E_s ds$$




Evaluate the voltage change dV along the direction of ds

The electric field can then be expressed as

$$E_s = -\frac{dV}{ds} \text{ along ds, or } E_s = -\frac{\partial V}{\partial s}$$

This is called a partial derivative.

For rectangular coordinates, the components of the electric field are

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

**Capacitance of an isolated spherical conductor:**

**Capacitance of an isolated conductor**

When a conductor is charged its potential increases. It is found that for an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.

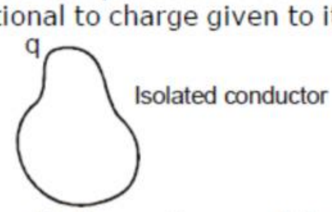
q = charge on conductor

V = potential of conductor

$q \propto V$

$\Rightarrow$ $q = CV$

Where C is proportionally constant called capacitance of the conductor.




Let there is charge Q on sphere.

$\therefore$ Potential $V = \frac{KQ}{R}$

Hence by formula : $Q = CV$

$$Q = \frac{CKQ}{R}$$

$$C = 4\pi\varepsilon_0 R$$

(i) If the medium around the conductor is vacuum or air.:

$C_{vacuum} = 4\pi\varepsilon_0 R$

R = Radius of spherical conductor. (may be solid or hollow)

(ii) If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity then

$C_{medium} = 4\pi\varepsilon_0 KR$

(iii) $\frac{C_{medium}}{C_{air / vaccum}}$ = K = dielectric constant.

**Capacitor:**

A capacitor or condenser consists of two coductors separated by an insulator or dielectric.

(i) When uncharged conductor is brought near to a charged conductor, the charge on conductors remains same but its potential dcreases resulting in the increase of capacitance.

(ii) In capacitor two conductors have equal but opposite charges.

(iii) The conductors are called the plates of the capacitor. The name of the capacitor depends on the shape of the capacitor.

(iv) Formulae related with capacitors:

(a) Q = CV

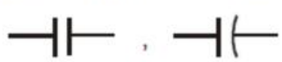
$$\Rightarrow \quad C = \frac{Q}{V} = \frac{Q_A}{V_A - V_B} = \frac{Q_B}{V_B - V_A}$$

Q = Charge of positive plate of capacitor.

V = Potential difference between positive and negative plates of capacitor

C = Capacitance of capacitor.

(v) The capacitor is represented as following :



(vi) Based on shape and arrangement of capacitor plates there are various types of capacitors:

(a) Parallel plate capacitor

(b) Spherical capacitor.

(c) Cylindrical capacitor

(v) Capacitance of a capacitor depends on

(a) Area of plates.

(b) Distance between the plates.

(c) Dielectric medium between the plates.

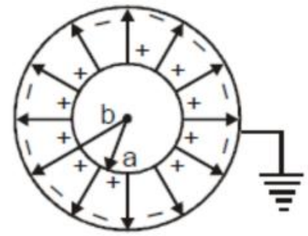
**Spherical Capacitor:**

A spherical capacitor consists of two concentric spheres of radii a and b as shown. The inner sphere is positively charged to potential V and outer sphere is at zero potential.

The inner surface of the outer sphere has an equal negative charge.

The potential difference between the spheres is

$$V = \frac{Q}{4\pi\varepsilon_0 a} - \frac{Q}{4\pi\varepsilon_0 b}$$



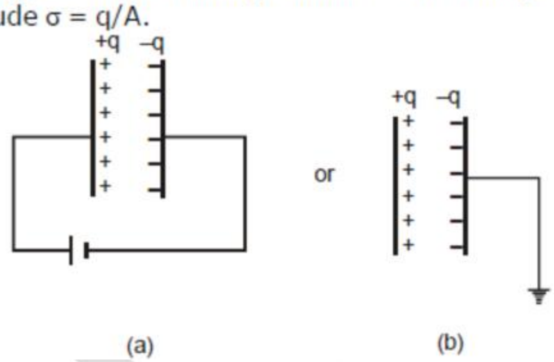

Hence, capacitance

$$C = \frac{Q}{V}\frac{4\pi\varepsilon_0 ab}{(b-a)}$$

**Parallel Plate Capacitor:**

Two metallic parallel plates of any shape but of same size and separated by small distance constitute parallel plate capacitor. Suppose the area of each plate is A and the separation between the two plates is d. Also assume that the space between the plates contains vacuum.

We put a charge q on one plate and a charge −q on the other. This can be done either by connecting one plate with the positive terminal and the other with negative plate of a battery (as shown in figure a ) or by connecting one plate to the earth and by giving a charge +q to the other plate only. This charge will induce a charge − q on the earthed plate. The charges will appear on the facing surfaces. The charges density on each of these surfaces has a magnitude σ = q/A.



If the plates are large as compard to the separation between them, then the electric field between the plates (at point B) is uniform and perpendicular to the plates except for a small region near the edge. The magnitude of this uniform field E may be calculated by using the fact that both positive and negative plates produce the electric field in the same direction (from positive plate towards negative plate) of magnitude $\sigma/2\varepsilon_0$ and therefore, the net electric field between the plates will be,

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Outside the plates (at point A and C) the field due to positive sheet of charge and negative sheet of charge are in opposite directions. Therefore, net field at these points is zero.

The potential difference between the plates is,

$$\therefore \quad V = E.d = \left(\frac{\sigma}{\varepsilon_0}\right)d = \frac{qd}{A\varepsilon_0}$$

$\therefore$ The capacitance of the parallel plate capacitor is,

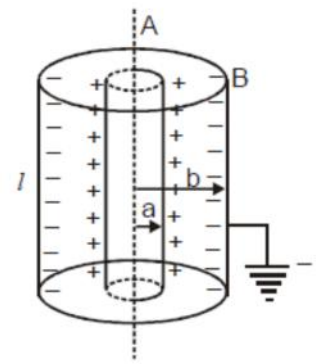
$$C = \frac{q}{V} = \frac{A\varepsilon_0}{d} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

**Cylindrical Capacitor:**

Cylindrical capacitor consists of two co-axial cylinders of radii a and b and length l. If a charge q is given to the inner cylinder, induced change −q will reach the inner surface of the outer cylinder. By symmetry, the electric field in region between the cylinders is radially outwards.

By Gauss's theorem, the electric field at a distance r from the axis of the cylinder is given by

$$E = \frac{1}{2\pi\varepsilon_0 l}\frac{q}{r}$$




The potential difference between the cylinders is given by

$$V = -\int_b^a \vec{E}\,\vec{dr} = -\frac{1}{2\pi\varepsilon_0 l} q \int_b^a \frac{dr}{r} = \frac{-q}{2\pi\varepsilon_0 l}\left(\ln\frac{a}{b}\right)$$

or,
$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\left(\ln\frac{a}{b}\right)}$$

**Energy per unit volume in electrostatic field:**

Let us consider charging an initially uncharged parallel plate capacitor by transferring a charge Q from one plate to the other, leaving the former plate with charge -Q and the later with charge +Q. Of course, once we have transferred some charge, an electric field is set up between the plates which opposes any further charge transfer. In order to fully charge the capacitor, we must do work against this field, and this work becomes energy stored in the capacitor. Let us calculate this energy.

Suppose that the capacitor plates carry a charge q and that the potential difference between the plates is $V$. The work we do in transferring an *infinitesimal* amount of charge dq from the negative to the positive plate is simply

$$dW = V\,dq. \qquad (1)$$

In order to evaluate the total work W(Q) done in transferring the total charge Q from one plate to the other, we can divide this charge into many small increments dq, find the incremental work $dW$ done in transferring this incremental charge, using the above formula, and then sum all of these works. The only complication is that the potential difference $V$ between the plates is a function of the total transferred charge. In fact, V(q)=q/C, so

$$dW = \frac{q\,dq}{C}. \qquad (2)$$

Integration yields

$$W(Q) = \int_0^Q \frac{q\,dq}{C} = \frac{Q^2}{2C}. \qquad (3)$$

Note, again, that the work $W$ done in charging the capacitor is the same as the energy stored in the capacitor. Since C=Q/V, we can write this stored energy in one of three equivalent forms:

$$W = \frac{Q^2}{2C} = \frac{C\,V^2}{2} = \frac{Q\,V}{2}. \qquad (4)$$

These formulae are valid for any type of capacitor, since the arguments that we used to derive them do not depend on any special property of parallel plate capacitors.

Where is the energy in a parallel plate capacitor actually stored? Well, if we think about it, the only place it could be stored is in the electric field generated between the plates. This insight allows us to calculate the energy (or, rather, the energy density) of an electric field.

Consider a vacuum-filled parallel plate capacitor whose plates are of cross sectional area $A$, and are spaced a distance $d$ apart. The electric field $E$ between the plates is approximately uniform, and of magnitude $\sigma/\epsilon_0$, where $\sigma = Q/A$, and Q is the charge stored on the plates. The electric field elsewhere is approximately zero. The potential difference between the plates is $V = E\,d$. Thus, the energy stored in the capacitor can be written

$$W = \frac{C\,V^2}{2} = \frac{\epsilon_0\,A\,E^2\,d^2}{2\,d} = \frac{\epsilon_0\,E^2\,Ad}{2}, \qquad (4)$$

Where, $A\,d$ is the volume of the field-filled region between the plates, so if the energy is stored in the electric field then the energy per unit volume, or *energy density*, of the field must be

$$w = \frac{\epsilon_0\,E^2}{2}. \qquad (5)$$

It turns out that this result is quite general. Thus, we can calculate the energy content of any electric field by dividing space into little cubes, applying the above formula to find the energy content of each cube, and then summing the energies thus obtained to obtain the total energy.

It is easily demonstrated that the energy density in a dielectric medium is

$$w = \frac{\epsilon\, E^2}{2}, \quad (6)$$

where $\epsilon = K\,\epsilon_0$ is the permittivity of the medium. This energy density consists of two elements: the energy density $\epsilon_0\, E^2/2$ held in the electric field, and the energy density $(K-1)\,\epsilon_0\, E^2/2$ held in the dielectric medium (this represents the work done on the constituent molecules of the dielectric in order to polarize them).

**Dielectric medium and Polarisation:**

Dielectrics are insulators, plain and simple. The two words refer to the same class of materials, but are of different origin and are used preferentially in different contexts.

- Since charges tend not to move easily in nonmetallic solids it's possible to have "islands" of charge in glass, ceramics, and plastics. The latin word for island is *insula*, which is the origin of the word *insulator*. In contrast, charges in metallic solids tend to move easily — as if someone or something was leading them. The latin prefix *con* or *com* means "with". A person you have bread with is a companion. (The latin word for bread is *panis*.) To take something with you on the road is to convey it. (The latin word for road is *via*.) The person you travel with who leads the way or provides safe passage is a conductor. (The latin word for leader is *ductor*.) A material that provides safe passage for electric charges is a *conductor*.
- Inserting a layer of nonmetallic solid between the plates of a capacitor increases its capacitance. The greek prefix *di* or *dia* means "across". A line across the angles of a

rectangle is a diagonal. (The greek word for angle is gonia — γωνία.) The measurement across a circle is a diameter. (The greek word for measure is metron — μέτρον.) The material placed across the plates of a capacitor like a little non-conducting bridge is a *dielectric*.

The plastic coating on an electrical cord is an insulator. The glass or ceramic plates used to support power lines and keep them from shorting out to the ground are insulators. Pretty much anytime a nonmetallic solid is used in an electrical device it's called an insulator. Perhaps the only time the word dielectric is used is in reference to the non-conducting layer of a capacitor. Dielectrics in capacitors serve three purposes:

1. to keep the conducting plates from coming in contact, allowing for smaller plate separations and therefore higher capacitances;
2. to increase the effective capacitance by reducing the electric field strength, which means you get the same charge at a lower voltage; and
3. to reduce the possibility of shorting out by sparking (more formally known as dielectric breakdown) during operation at high voltage.

**what's going on here:**

When a metal is placed in an electric field the free electrons flow against the field until they run out of conducting material. In no time at all, we'll have an excess electrons on one side and a deficit on the other. One side of the conductor has become negatively charged and the other positively charged. Release the field and the electrons on the negatively charged side now find themselves too close for comfort. Like charges repel and the electrons run away from each other as fast as they can until they're distributed uniformly throughout; one electron for every proton on average in the space surrounding every atom. A conducting electron in a metal is like a racing dog fenced in a pasture. They are free to roam around as much as they want and can run the entire length, width, and depth of the metal on a whim.

Life is much more restrictive for an electron in an insulator. By definition, charges in an insulator are not *free* to move. This is not the same thing as saying they *can't* move. An electron in an insulator is like a guard dog tied to a tree — free to move around, but within limits. Placing the electrons of an insulator in the presence of an electric field is like placing a tied dog in the presence of a mailman. The electrons will strain against the field as far as they can in much the same way that our hypothetical dog will strain against its leash as far as it can. Electrons on the atomic scale are more cloudlike than doglike, however. The electron is really spread out over the whole volume of an atom and isn't concentrated in any one location. A good atomic dog wouldn't be named Spot, I suppose.

When the atoms or molecules of a dielectric are placed in an external electric field, the nuclei are pushed with the field resulting in an increased positive charge on one side while the electron clouds are pulled against it resulting in an increased negative charge on the other side. This process is known as **polarization** and a dielectric material in such a state is said to be **polarized**. There are two principal methods by which a dielectric can be polarized: stretching and rotation.

Stretching an atom or molecule results in an induced dipole moment added to every atom or molecule.

**Displacement vector:**

The change in the position vector of an object is known as displacement vector. Suppose an object is at point A at time = 0 and at point B at time = t. The position vectors of the object at point A and at point B are given as:

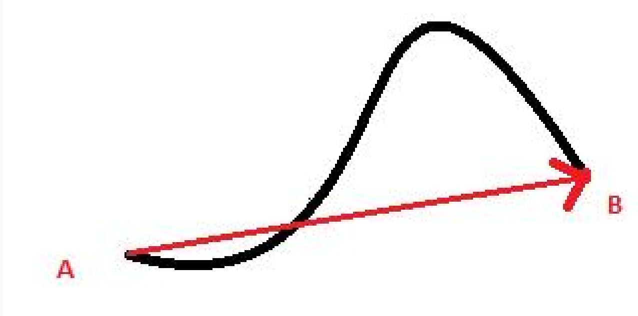
Position vector at point A $= \hat{r_A} = 5\hat{i}+3\hat{j}+4\hat{k}$

Position vector at point B $= \hat{r_B} = 2\hat{i}+2\hat{j}+1\hat{k}$

Now, the displacement vector of the object from time interval 0 to t will be:

$\hat{r_B}-\hat{r_B} = -3\hat{i}-\hat{j}-3\hat{k}$

The displacement of an object can also be defined as the vector distance between the initial point and the final point. Suppose an object travels from point A to point B in the path shown in the black curve:




The displacement of the particle would be the vector line AB, headed in the direction A to B. The direction of displacement vector is always headed from initial point to the final point.

**Gauss's theorem in dielectrics**:

Electrostatic field in the dielectric material is modified due to polarization and is not the same as in vacuum. Hence the Gauss law

$$\nabla E = \frac{\rho}{\varepsilon_0}$$

which is applicable in vacuum is reconsidered for dielectric media. It can be expressed in two forms- A) Integral 2) Differential - as follows

**A) Integral Form of Gauss Law**

(I) Consider two parallel-plate conductors having plane area S, separation d and vacuum between plates. Let charge +q and –q be the charges on the plates. Due to the charges, $\boldsymbol{E_0}$ is the uniform electric field directed from positive to negative plate (Fig. a).

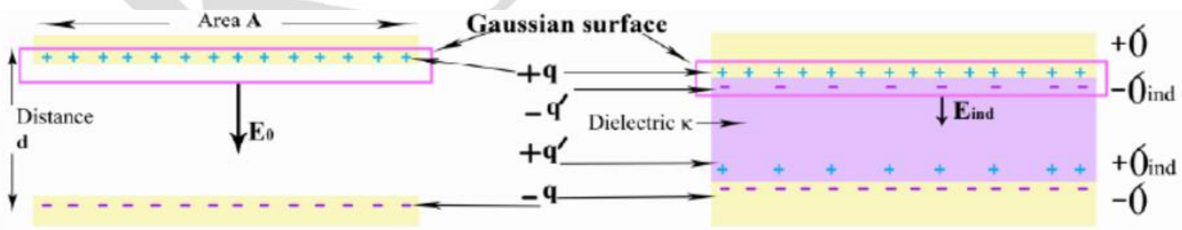



**Fig. parallel-plate conductors (a) without dielectric (b) with dielectric**

Consider the Gaussian surface around the upper conducting plate of positive charges. Applying Gauss's law the electric flux passing through the closed surface is given by

$\oint E\, ds = \frac{q}{\varepsilon_0}$ or $E_0 A = \frac{q}{\varepsilon_0}$

The field $E_0 = \frac{q}{A\varepsilon_0}$.

It is normal to the plate surfaces.

(II) Consider that a dielectric material of permittivity ε is filled completely between the plates (Fig. b). Charges –q' and +q' are induced on the surfaces of the dielectric that are in the proximity of the plates having charges q and -q respectively. The induced charges set up an electric field E in the dielectric. The dielectric is polarized. It remains as a whole electrically neutral as the positive induced surface charge must be equal to the negative induced surface charge.

If the dielectric is present, the surface encloses two types of charge:

Free charge on the upper conducting plate is q and

Induced charge on the top face of dielectric due to polarization is -q'

The net charge enclosed by the Gaussian surface around the (same upper conducting) plate (of positive charges +q) is q-q'.

According to Gauss's law

$$\oint E\, ds = \frac{q}{\varepsilon_0}(q - q')$$

$$E \cdot A = \frac{1}{\varepsilon_0}(q - q')$$

Or $E = \frac{q}{A \cdot \varepsilon_0} - \frac{q'}{A \cdot \varepsilon_0}$

Field **E** in the dielectric is in the opposite direction to that of the applied electric field $E_0$. The effect of the dielectric is to weaken the original field by the factor $k = \varepsilon/\varepsilon_0$.

$$\frac{E}{E_0} = \frac{1}{k}$$

Or

$$E = \frac{E_0}{k}$$

$$E = \frac{q}{k\varepsilon_0 A}$$

$$q' = q(1 - \frac{1}{k})$$

The magnitude of the net induced charge q` is always less than magnitude of the free charge q applied to the plates and is equal to zero if dielectric is absent.

$$\oint E\, ds = \frac{q}{\varepsilon_0 k}$$

Or $\oint E\, ds = \frac{q}{\varepsilon}$

i.e. $\oint D\, ds = q$ Where $D = \varepsilon E = \varepsilon_0 k E$.

**D** is called as the displacement vector. The induced surface charge is purposely ignored on the right side of this equation, since it is taken into account fully by introducing the dielectric constant k on the left side.

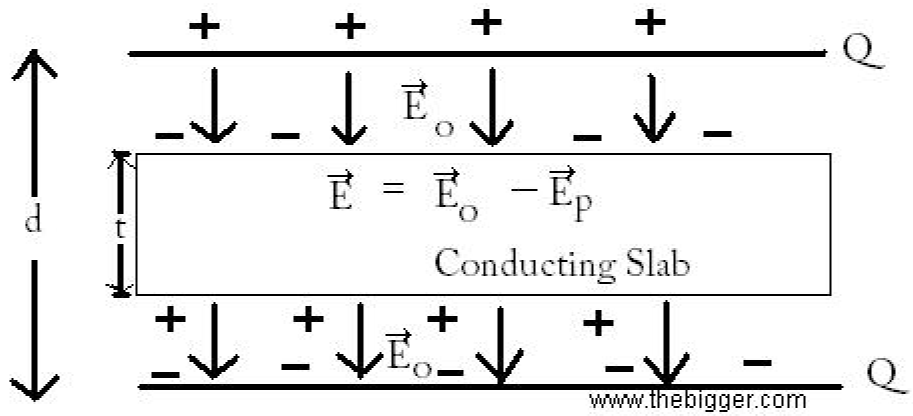
The equation states that "the surface integral of displacement vector 'D' over a closed surface is equal to the free charge enclosed within the surface" or "The outward flux of D over any closed surface S equals the algebraic sum of the free charges enclosed by S"

This important equation, although derived for parallel plate conductors, is true in general.It is the most general form of Gauss' law. The charge q enclosed by the Gaussian surface is the free charge only, which can be controlled and measured. Hence this form of Gauss law is very useful.

**Parallel plate capacitor completely filled with dielectric**.

Let us take a parallel plate capacitor. Suppose the separation distance between the plates is d. Use air or vacuum as a medium for this experiment.

Suppose +Q is the charge on one plate and –Q is charge on the second plate. Bring a rectangular slab made up of conducting material between the plates of the capacitor. The thickness of the slab must be less than the distance between the plates of the capacitor. When the electric field will be applied then polarization of molecules will be started. The polarization will take place in the direction same as that of electric field. Consider a vector that must be polarized, name it as P. The polarization vector must be in the direction of electric field $E_o$. Then this vector will start its functioning and will produce an electric field $E_p$ in the opposite direction to that of $E_o$. The net electric field in the circuit is shown by the figure.




$E= E_o – E_p$

The electric field $E_o$ in the outside region of the dielectric will be null. Now the equation of the potential difference between the plates will be :

$V=_o (d-t) + Et$

But $E_o= E_r$ or K

Therefore $E= E_o / k$

So

$V= E_o (d-t) + E_o t / k$

$V= E_o [d-t+t/k]$

As we know

$E_o = \sigma / \varepsilon_0$

$= Q / A \varepsilon_0$

$V = Q / A \varepsilon_0 [d-t+t/k]$

Capacitance of the capacitor is shown in the equation below:

$C = Q / V = A \varepsilon_0 / (d-t+t/k)$

$= \varepsilon_0 A / d-t (1-1/K)$

**I.e. C= $\varepsilon_0$ A/ d-t (1-1/k) ——- (a)**

So, $C > C_o$

Clearly, it is proved that if a dielectric slab is placed in the plates of a capacitor then its capacitance will increase by some amount.

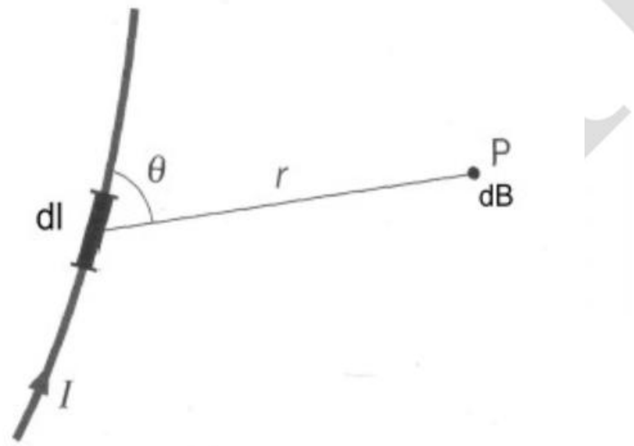
## UNIT-III

## SYLLABUS

Magnetostatics: Biot-Savart's law and its applications- straight conductor, circular coil, solenoid carrying current. Divergence and curl of magnetic field. Magnetic vector potential. Ampere's circuital law. Magnetic properties of materials: Magnetic intensity, magnetic induction, permeability, magnetic susceptibility. Brief introduction of dia-, para- and ferro-magnetic materials.

**Biot-Savart's law:**

Let a certain conductor be carrying a current 'I' in a direction on in the figure. Let 'P' be the point
where the magnetic field due to the wire is to be studied. Let a small portion be considered which is
of length 'dl'. Let the line joining 'dl' and point 'P' from an angle θ with the tangent to 'dl'.




Since the portion considered is very small, the magnetic field given by it at point P will also be small.
By experimental observations and empirically also, dB is found to depend on several factors.
i) Here dB is the measurement of magnetic energy which arises from the electrical energy represented by I which act respectively as output and input. Therefore they should have direct dependency. i.e.
$dB \propto I$
ii) In a certain length of a conductor, certain amount of charge is present at a moment and the magnetic effect it can produce depend on the total number of charges, which in turn depend on the length consider, i.e.
$dB \propto dl$
iii) Any force or phenomena which spread out spherically have inverse proportionality to the square of the distance between the source and point of observation.
dB r2
∝ 1
iv) Similarly the magnetic field is found to be least when the angle between r and dl is the

smallest (00) and it is largest when the angle is 90o. So,
dB∝sinθ
Therefore overall,

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

Or, $$dB = k\frac{Idl\sin\theta}{r^2}$$

In SI units, the value of $k = \frac{\mu_o}{4\pi} = 10^{-7}\ Hm^{-1}$

Or, $$dB = \frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2}$$

This expression is called as Biot Savart Law or Laplace Law. It is the basic formula to find the
magnetic field due to any structures for which all the dB's along the length of the structure have to be
added to find out the total B.

**Application of Biot Savart's Law:**

**(i)Magnetic field due to current carrying circular coil at its centre:**

Let a coil be considered which is bent in the form of almost complete circle. Let a current I be supplied in clockwise direction which will give the overall magnetic field away from the observer at the centre 'O' (according to Fleming's Right-Hand Thumb rule). Let a small portion 'dl' be considered somewhere and a radius be drawn from 'dl' to 'O'. Then according to Biot Savart Law, a small magnetic field 'dB' given by 'dl' can be expressed as:

$$dB = k\frac{Idl\sin\theta}{r^2}$$

, where θ is the angle between dl and r.
Here wherever 'dl' is considered, the angle between it and 'r' is always equal to 900.
Therefore,

$$dB = k\frac{Idl\sin 90^o}{r^2} = \frac{kIdl}{r^2}$$

The overall magnetic field will be equal to the sum of all these small magnetic fields.

i.e. $B = \int dB$ Or, $B = \int \frac{kIdl}{r^2}$

Or, $B = \frac{kI}{r^2}\int dl$

Here the variable is 'l'. if all the small "dl's" are added one by one, "l" will extend from l = 0 to l =
circumference (= 2 πr ).

$\therefore \quad B = \frac{kI}{r^2}\int_0^{2\pi r} dl$ Or, $B = \frac{kI}{r^2}[l]_0^{2\pi r}$ Or, $B = \frac{kI}{r^2}2\pi r$

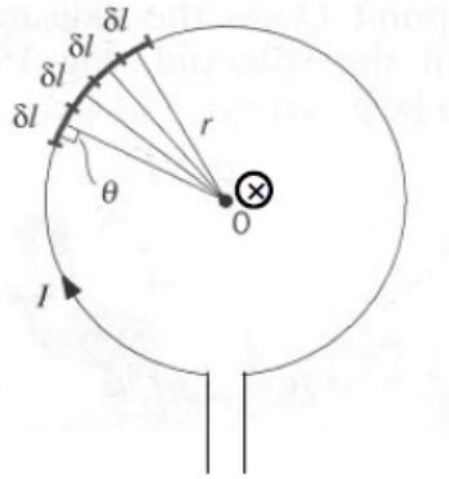
Or, $B = \frac{kI}{r}2\pi$ Or, $B = k\frac{2\pi I}{r}$ Or, $B = \frac{\mu_o}{4\pi}\frac{2\pi I}{r}$

Or, $B = \frac{\mu_o I}{2r}$,

(r is the radius of the coil)

If the number of coils is more than one, for example 'n', the magnetic field will be

$$B = \frac{\mu_o nI}{2r}$$




**(ii)Magnetic field due to a straight conductor:**

Let a straight wire be considered whose magnetic field is to be determined at a certain point P which is nearby the conductor at a distance ‘a’. Let a small portion of length dl be considered whose distance from P is ‘r’.Therefore the magnetic field at point P due to this small length is given by,

$$dB = \frac{kIdl\sin\theta}{r^2}$$

This magnetic field due to the whole wire is found by adding all the magnetic fields due to all these
dB’s of the whole wire for which the expression has to be changed to integrable form. For this, the point
‘P’ is joined to A, B & C. Similarly a perpendicular BD is drawn to AP at D.
Let ∠CPQ =α . Then ∠APB is the small variation in dα due to the consideration of the angles at the two ends of small length dl.
Since dl is very small, points A and C lie very close to each other.
Therefore ∠BAD = ∠BCP = θ.
So in triangle ABD,
sin∠BAD = AB/BD

$$\text{Or,}\quad \sin\theta = \frac{BD}{AB} \qquad \text{Or,}\quad AB\sin\theta = BD$$

$$\text{Or,}\quad dl\sin\theta = BD\ldots\ldots\ldots\ldots\text{(i)}$$

$$\text{Similarly in triangle BDP,}\quad \sin\angle BPD = \frac{BD}{BP}$$

$$\text{Or,}\quad \sin d\alpha = \frac{BD}{BP}$$

Since dl is very small, B & C also lie close together. So BP = CP = r. Similarly the angle dα is also
very small. So, sin d α = d α .
So, dα =BD/r
Therefore,
r dα = BD …………….. (ii)
Equations (i) and (ii) give dl sin θ = r d α

The expression for dB becomes, $dB = \frac{kIdl\sin\theta}{r^2}$ or, $dB = \frac{kIrd\alpha}{r^2}$

Or, $dB = \frac{kId\alpha}{r}$

In triangle CPQ, $\cos\angle CPQ = \frac{PQ}{CP}$ or, $\cos\alpha = \frac{PQ}{CP}$

or, $\cos\alpha = \frac{a}{r}$ or, $\frac{Cos\alpha}{a} = \frac{1}{r}$

which means $dB = \frac{kI\cos\alpha}{a}d\alpha$

The total magnetic field is given by summing up all these small dB's throughout the whole length of
the conductor.

$$\text{Total magnetic field (B)} = \int dBd\alpha = \int \frac{kI\cos\alpha}{a}d\alpha$$

Here the variable α varies within certain given values 1 α and 2 α , where 1 α is the angle formed by
the lower tip of the conductor at P and 2 α is by the upper tip. But when the angle goes below PQ, its
value becomes negative, since 1 α < 00.

$$B = \frac{kI}{a}\int_{\alpha_1}^{\alpha_2}\cos\alpha d\alpha = \frac{kI}{a}[\sin\alpha]_{\alpha_1}^{\alpha_2} \quad \text{Or, } B = \frac{kI}{a}[\sin\alpha_2 - \sin\alpha_1]$$

This is the expression for the magnetic field at a certain point due to a straight conductor of finite
length at a distance 'a' such that the angles formed at the two ends α 1 and α2 .

**Special case:**

In most cases the wires are very long compared to the distance of the point of observation from the
wire in such cases, the angles will be 90o at both sides.

$$B = \frac{kI}{a}\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right] = \frac{kI}{a}\left[1 + \sin\left(\frac{\pi}{2}\right)\right] = \frac{kI}{a}[1+1]$$

Or, $$B = \frac{2kI}{a}$$

Using the value of k as $\frac{\mu_o}{4\pi}$ gives $B = \frac{2\mu_o}{4\pi}\frac{I}{a}$

$$\therefore \quad B = \frac{\mu_o I}{2\pi a}$$

**(iii)Magnetic field inside a solenoid:**

Let the figure represent the linear cross section of a solenoid whose coiling is such that it has 'n'
number of coils per unit length. Let 'P' be a point where the magnetic field due to the whole solenoid
is to be determined. For this, let a small length 'dx' of the solenoid be considered first and the magnetic field at 'P' due to the coils within this length be determined first.

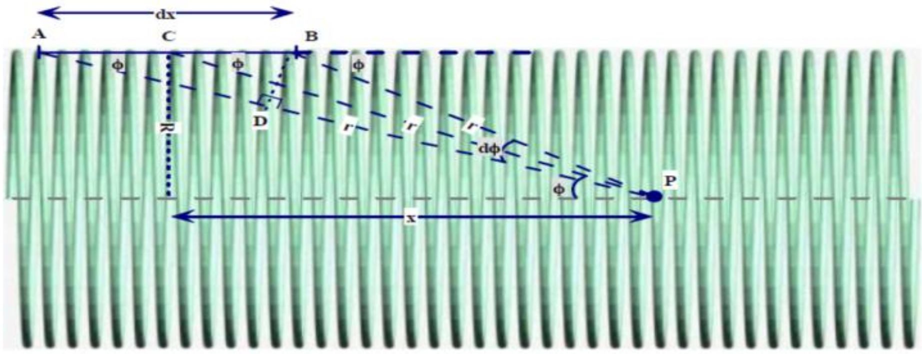
The magnetic field due to one circular coil at a certain distance 'x' from point 'P' is,

$$B = \frac{2\pi kIR^2}{(R^2 + x^2)^{3/2}}$$

Here the term R2 + x2 = r2 denotes the square of the distance of the point 'P' from each point of the
circumference of that single coil. The number of coils present in 'dx' length is equal to 'ndx'.

Therefore the magnetic field given by coils present in 'dx' length or 'ndx' number of coils is given by

$$B_{ndx} = \frac{2\pi kIR^2}{(R^2 + x^2)^{3/2}} \times (ndx)$$



However compared to the magnetic field exerted by the whole solenoid, this magnetic field is very small. So it is denoted by $dB_{ndx}$

$$\text{i.e.} \quad dB_{ndx} = \frac{2\pi k I R^2}{\left(R^2 + x^2\right)^{3/2}} \times (ndx)$$

Let P be connected to A, B as well as C, where C is the centre of AB. Let $\angle CPQ = \phi$, then $\angle BCP = \phi$.

Since AB is very short length, points A & C lie very close to each other. Therefore $\angle BQP = \phi$. Similarly $\angle BPA$ denotes the small variation in $\phi$, so, $\angle BPA = d\phi$

Let BD be drawn perpendicular to AP at P. In triangle BAD,

$$\sin\phi = \frac{BD}{AB} \qquad \text{or, } AB\sin\phi = BD$$

$$\text{or, } \sin\phi\, dx = BD \ldots\ldots \text{(i)}$$

In triangle BDP,

$$\sin d\phi = \frac{BD}{BP} \qquad \text{Or, } BP \sin d\phi = BD$$

Here, points B and c are very close to each other due to short length of AB. Therefore, BP = CP = r.
Similarly d φ is also a very small angle so, sin d φ ≈d φ
Therefore, rdφ = BD……. (ii)
Comparing (i) and (ii) gives,

$$\sin\phi\, dx = r\,d\phi \qquad \text{Or, } dx = \frac{r\,d\phi}{\sin\phi}$$

$$\text{Or, } dB_{ndx} = \frac{2\pi k I R^2 n}{\left(R^2 + x^2\right)^{3/2}} \frac{r\,d\phi}{\sin\phi}$$

$$\text{Or, } dB_{ndx} = \frac{2\pi k I R^2 n}{r^3} \frac{r\,d\phi}{\sin\phi} \quad (\text{since, } R^2 + x^2 = r^2)$$

$$\text{Or, } dB_{ndx} = 2\pi k n I \sin^2\phi \frac{d\phi}{\sin\phi}$$

$$\text{Or, } dB_{ndx} = 2\pi k n I \sin\phi\, d\phi$$

Therefore, the total magnetic field is given by, $B = \int dB_{ndx}$

Here from one end to the other end of the solenoid, $\phi$ varies from a minimum value $\phi_1$, to a maximum value $\phi_2$

$$B = 2\pi knI\int_{\phi_1}^{\phi_2}\sin\phi d\phi = 2\pi knI[-\cos\phi]_{\phi_1}^{\phi_2}$$

$$\text{i.e. } B = 2\pi knI(\cos\phi_1 - \cos\phi_2)$$

Generally solenoids are designed in such a way that the radius is very-very small compared to the
length. In such case angles φ 1 and φ 2 will range from the minimum 00 through the maximum 1800. In
such case,

$$B = 2\pi knI[\cos 0^0 - \cos 180^0]$$

$$\text{Or, } B = 2\pi knI[1-(-1)]$$

$$\text{Or, } B = 4\pi knI$$

Using $k = \frac{\mu_o}{4\pi}$ gives

$$B = \mu_o nI$$

**Divergence and curl of a magnetic field:**

**Divergence of B:**

According to Gauss law in electrostatics, divergence of the static electric field is equal to the total density of a stationary electric charge/s at a given point.

$$div.E = \nabla.E = \frac{\rho}{\epsilon_0}$$

However in magnetostatics a magnetic charge (i.e. monopole) is not found to exist. (The source of magnetic fields is moving electric charges, not the static ones). Due to the absence of magnetic charges, the magnetic field is divergenceless. **In Differential form**

*divB* = 0 or $\nabla.B = 0$ (where B is the magnetic field, $\nabla$. denotes divergence)

This is called as **Gauss's law for magnetism** (though this term is not universally adopted). It states that the magnetic field B has divergence equal to zero i.e. magnetic field is a solenoidal vector field. It is equivalent to the statement that magnetic monopole (isolated North or South magnetic pole) does not exist. The basic quantity for magnetism is the magnetic dipole, not the magnetic charge or monopole. Hence, the law is also called as "Absence of free magnetic poles".

The statement of Gauss's law for magnetism in integral form is given as

$$\oint_S B.dA = 0$$

Where S is any closed surface (the boundary enclosing a three-dimensional volume); dA is a vector, having magnitude equal to the infinitesimal area of the surface S and direction along the surface normal pointing outward.

The left-hand side of the equation in integral form denotes the net flux of the magnetic field out of the surface. The law implies that outward magnetic flux is always zero.

Thus Gauss's law for magnetism can be written in both- differential and integral- forms. These forms are equivalent due to the divergence theorem.

The magnetic field B, like any vector field, can be represented by field lines. Gauss's law for magnetism also implies that the field lines have neither a beginning nor an end. They either form a closed loop, or extend to infinity in both directions.

**CURL OF B:**

**Circulation** is the amount of pushing, twisting or turning force along a closed boundary / path when the path is shrunk down to a single point. Circulation is the integral of a vector field along a path. A vector field is usually the source of the circulation.

**Curl** is simply the circulation per unit area, circulation density, or rate of rotation (amount of twisting at a single point).

The curl of a force $FCurl(F) = \nabla \times F$ is calculated as follows.

Let the Force at position *r* = *F*(*r*) Direction at position *r* = *dr*

Total pushing force $= Circulation = \int F(r).dr$.

$$Curl = \frac{Circulation}{area} = \frac{\int F(r).dr}{\int S}$$

Curl is defined as the vector field having magnitude equal to the maximum "circulation" at each point and to be oriented perpendicularly to this plane of circulation for each point. The magnitude of $\nabla \times F$ is the limiting value of circulation per unit area.

If $\nabla \times F = 0$, then the field is said to be an irrotational field.

The physical significance of the curl of a vector field is the amount of "rotation" or angular momentum of the contents of given region of space. It arises in fluid mechanics and elasticity theory. It is also fundamental in the theory of electromagnetism.

In magnetostatics, it can be proved that the curl of magnetic field B is given by

$$\nabla \times B = \mu_0 j$$

Thus the curl of a magnetic B field at any point is equal to $\mu_0$ times the current density J at that point. This simple statement relates the magnetic field and moving charges. It is mathematically equivalent to the line integral equation given by Ampere’s law.

The equations in terms of Divergence and Curl of magnetic B-field are also called as the laws of Magnetostatics. They correspond to the curl and divergence of electric field E respectively in electrostatics as follows

| Electrostatics | Magnetostatics |
|---|---|
| $\nabla \times E = 0$, Field is without curl | $\nabla . B = 0$, Field is without divergence |
| $\nabla \times E = \frac{\rho}{\epsilon_0}$, Field E– Source ρ relation | $\nabla \times B = \mu_0 j$, Field B–Source j relation |

The equations for divergence and curl for vector fields are extremely powerful. Expressions for divergence and curl of a magnetic field describe uniquely any magnetic field from the current density j in the field in the same manner that the equations for the divergence and curl for the electric field describe an electric field from the electric charge density ρ in the electric field.

Above four equations are the versions of Maxwell’s equations for static electromagnetic fields. They describe mathematically the entire content of electrostatics and magnetostatics.

**Magnetic Vector Potential:**

The electric field E can always be expressed as the gradient of a scalar potential function

$$\vec{E} = -\vec{\nabla} V$$

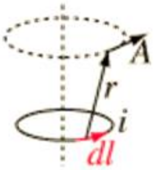
There is no general scalar potential for magnetic field B but it can be expressed as the curl of a vector function

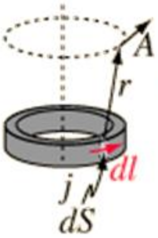
$$\vec{B} = \vec{\nabla} x \vec{A}$$

This function A is given the name "vector potential" but it is not directly associated with work the way that scalar potential is.

The vector potential is defined to be consistent with Ampere's Law and can be expressed in terms of either current i or current density j (the sources of magnetic field). In various texts this definition takes the forms

$$A = \frac{\mu_0 i}{4\pi} \oint \frac{\vec{dl}}{r}$$




$$A = \frac{\mu_0}{4\pi} \oint \frac{j d\vec{S} \vec{dl}}{r}$$

One rationale for the vector potential is that it may be easier to calculate the vector potential than to calculate the magnetic field directly from a given source current geometry. Its most common application is to antenna theory and the description of electromagnetic waves.

Since the magnetic field B is defined as the curl of A, and the by vector identity the curl of a gradient is identically zero, then any arbitrary function which can be expressed as the gradient of a scalar function may be added to A without changing the value of B obtained from it. That is, A' can be freely substituted for A where

$$\vec{A'} = \vec{A} + \vec{\nabla}\phi$$

Such transformations are called gauge transformations, and there have been a number of "gauges" that have been used to advantage is specific types of calculations in electromagnetic theory.

**Ampere’s circuital law:**

- Ampere's circuital law in magnetism is analogous to gauss's law in electrostatics
- This law is also used to calculate the magnetic field due to any given current distribution
- This law states that
  " The line integral of resultant magnetic field along a closed plane curve is equal to $\mu_0$ time the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant" Thus

  $$\oint \mathbf{B}.\mathbf{dl} = \mu_0 I_{enc}$$

  where $\mu_0$ is the permeability of free space and $I_{enc}$ is the net current enclosed by the loop as shown below in the figure

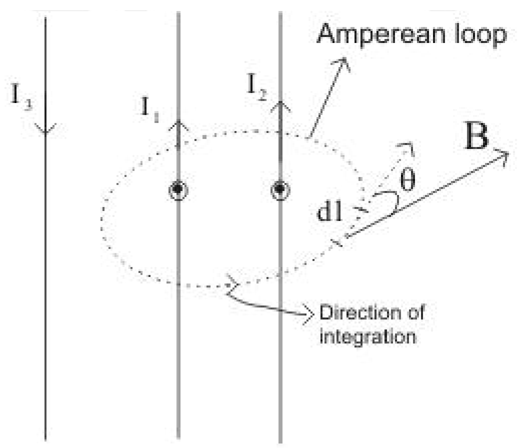



Figure 9. Ampere’s law applied to a loop containing two long straight wires.

- The circular sign in equation (21) means that scalar product **B.dl** is to be integrated around the closed loop known as Amperian loop whose beginning and end point are same
- Anticlockwise direction of integration as chosen in figure 9 is an arbitrary one we can also use clockwise direction of integration for our calculation depending on our convenience
- To apply the ampere's law we divide the loop into infinitesimal segments **dl** and for each segment, we then calculate the scalar product of **B** and **dl**
- **B** in general varies from point to point so we must use **B** at each location of **dl**
- Amperion Loop is usually an imaginary loop or curve ,which is constructed to permit the application of ampere's law to a specific situation

**Proof Of Ampere's Law**

- Consider a long straight conductor carrying current I perpendicular to the page in upward direction as shown below in the figure

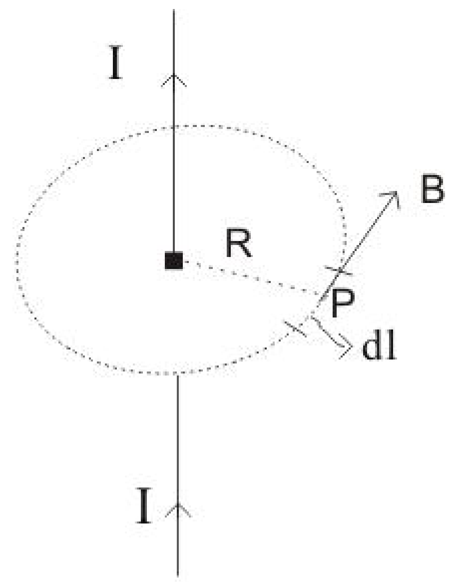



Fig: B is the magnetic field due to current carrying conductor at point P

- From Biot Savart law, the magnetic field at any point P which is at a distance R from the conductor is given by

$$B = \frac{\mu_0 I}{2\pi R}$$

- Direction of magnetic Field at point P is along the tangent to the circle of radius R withTh conductor at the center of the circle
- For every point on the circle magnetic field has same magnitude as given by

$$B = \frac{\mu_0 I}{2\pi R}$$

And field is tangent to the circle at each point

- The line integral of B around the circle is

$$\oint \mathbf{B.dl} = \oint \frac{\mu_0 I}{2\pi R} dl \quad = \frac{\mu_0 I}{2\pi R}\oint dl$$

since ∫dl=2πR ie, circumference of the circle so,

$$\oint \mathbf{B.dl} = \mu_0 I$$

This is the same result as stated by Ampere law
- This ampere's law is true for any assembly of currents and for any closed curve though we have proved the result using a circular Amperean loop
- If the wire lies outside the amperion loop, the line integral of the field of that wire will be zero

$$\oint \mathbf{B.dl} = 0$$

but does not necessarily mean that **B**=0 everywhere along the path ,but only that no current is linked by the path
- while choosing the path for integration ,we must keep in mind that point at which field is to be determined must lie on the path and the path must have enough symmetry so that the integral can be evaluated

## Classification of Magnetic Materials

**(1) Magnetising Field: -** The magnetic field that exists in vacuum and induces magnetism in a substance is called magnetizing field.
The magnetizing field setup inside a solenoid carrying current I and placed in vacuum, $B_0=\mu_0 nI$

**(2) Magnetising field intensity or Magnetising Force or Magnetic Intensity (H).**
The degree to which a magnetic field can magnetize a material is called magnetic field or magnetizing force or Magnetic Intensity and ***It is defined as the number of ampere-turns (=nI) per unit length of a solenoid***
Thus magnetizing force $H=nI$.

Also $B_0= \mu_0 nI= \mu_0 H$ for air core
$B= \mu H$ for iron core

Its SI unit is A-m

**(3) Intensity of Magnetization** ( ( *I* ) ):-

It is the extent to which a specimen is magnetized, when placed in a magnetic field and depends upon the nature of the material.

***It is defined as the magnetic moment developed per unit volume of the material.***

$$\left|\dot{I}\right| \quad \frac{\left|\dot{M}\right|}{V}$$

Where $M$ = magnetic moment developed in the material and
V = Volume of the material

If ***m*** is the pole strength and ***A*** is the area of cross section of material and ***2l*** is the length, then

$$I \quad \frac{m\;2l}{A\;2l} \quad \frac{m}{A}$$

Hence ***intensity of magnetization of a material may also be defined as the pole strength developed per unit area of cross section of the material.***
SI unit of it is A/m.

**(4) Magnetic permeability -**
The ratio of total magnetic flux density (B) & the magnetizing field intensity (H) is called absolute permeability

=B/H B= μ H

**(5) Relative Permeability-** ***The ratio of the absolute permeability of free space to the permeability of the medium is called relative permeability.***

μr = μ / μ0

**(6) Magnetic Flux density in Magnetic Materials-**The magnetic flux density inside a solenoid is directly proportional to the current. If we put a piece of ferromagnetic substance (e.g. iron), the magnetic flux density is greatly increased. This is due to the magnetisation of ferromagnetic substance by external field.

*The total magnetic flux density in the ferromagnetic substance is the sum of magnetic flux density ( $B_0$ )*

*due to the current in the wire and magnetic flux density ( $B_M$ )due to the magnetisation of ferromagnetic substance i.e.*

$$B \quad B_0 \quad B_M$$

(7) **Magnetic Susceptibility-**
This properly determines how easily a specimen can be magnetized. It is defined as the ratio of intensity of magnetization (I) to the applied magnetising force (H). It is represented by $_m$

$$_m \quad \frac{I}{H}$$

It has no unit

(8) **Relation between** $_r$ **&** $x_m$

$$H$$

$$B \quad _0 I$$

$$H \quad _0 H \quad I \qquad \frac{}{_0} \quad 1 \quad \frac{I}{H} \qquad _r \quad 1 \quad x_m$$

**Classification of magnetic materials**- On the basis of their behavior in external magnetic fields, Faraday classified the various substances into three categories:

**1. Diamagnetic substances-** Diamagnetic substances are those which develop feeble magnetisation in the opposite direction of the magnetising field. Such substances are feebly repelled by magnets and tend to move from stronger to weaker parts of a magnetic field.
e.g. - Bismuth, copper, lead, zinc, tin, gold, silicon, water, sodium chloride, etc.

**2. Paramagnetic substances-** Paramagnetic substances are those which develop feeble magnetisation in the direction of the magnetising field. Such substances are feebly attracted by magnets and tend to move from weaker to stronger parts of a magnetic field.
e.g. - Manganese, Aluminum, Chromium, Platinum, Sodium, Copper Chloride, Oxygen

**3. Ferromagnetic substances**- Ferromagnetic substances are those which develop strong magnetisation in the direction of the magnetising field. They are strongly attracted by magnets and tend to move from weaker to stronger parts of a magnetic field.
e.g. - Iron, cobalt, nickel, gadolinium and alloys like alnico.
Iron is a ferromagnetic substance. Its ferromagnetism decreases with the increase of temperature.

**Curie's law**- From experiments, it is found that the intensity of magnetisation (I) of a paramagnetic material is
(i) directly proportional to the magnetising field intensity H, because the latter tends to align the atomic dipole moments.
(ii) inversely proportional to the absolute temperature T, because the latter tends to oppose the alignment of the atomic dipole moments.

$$I \propto \frac{H}{T}$$

*or* $$I = C\frac{H}{T}$$

*or* $$\frac{I}{H} = \frac{C}{T}$$

*or* $$\chi_m = \frac{C}{T}$$

Where C is curie consant $\chi_m$ is the susceptibility
The above relation is called curie law. ***According to this law far away from the saturation region the magnetic susceptibility is inversely proportional to the absolute temperature***.

**Curie Temperature**- The temperature at which a ferromagnetic substance becomes paramagnetic is called ***Curie temperature or Curie point.***

**Modified Curie's law: Curie-Weiss law**- The magnetic susceptibility of a ferromagnetic substance above its curie temperature is inversely proportional to the excess of temperature above the Curie temperature.

$$_m \quad \frac{C}{T \quad T_C}$$

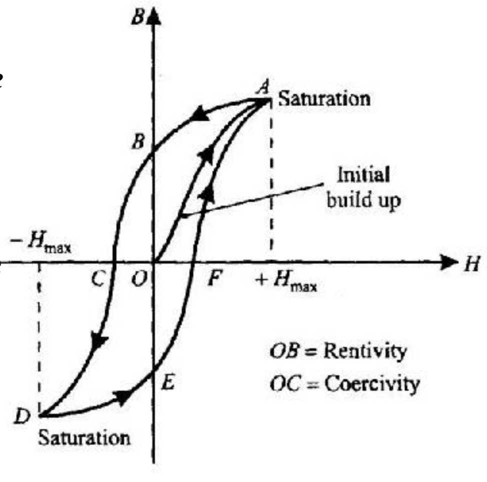
Where $T_C$ is Curie temperature

**Difference between Dia, Para and Ferro magnetic Materials-**

| **Property** | **Dia** | **Para`** | **Ferro** |
|---|---|---|---|
| Permeability ( ) | μ< $_0$ | > $_0$ | >> $_0$ |
| Relative Permeability ( $_r$ ) | 0< <1 | 1< $_r$ | $_r$ >>1 |
| Susceptibility ( $x_m$ ) | -1< $x_m$ <o | 0< $x_m$ <1 | $x_m$ >>1 |
| Temperature | independen t | $x_m$ 1/T | $x_m$ 1/T-TC |

**Hysteresis**- When a ferromagnetic sample is placed in a magnetising field, it gets magnetized by induction. Following figure shows the variation of magnetic induction B with magnetising field intensity H and is called as hysteresis curve.

As H increases, B first increases gradually and then attains a saturation value along the curve OA. Now we gradually decrease H to zero, B decreases but along a new path AB. At H =0, B=0. *The magnetic induction (BR =OB) left behind in the sample after the magnetising field has been removed is called*
***residual magnetism or retentivity or remanence***.
Now H is gradually increased in the reverse direction. B decreases and becomes zero at certain value of H (HC = OC). *The value of reverse magnetising field intensity H required for the residual magnetism of a sample to become zero is called* ***Coercivity*** *of the sample.*
The closed curve ABCDEFA which represents a cycle of magnetisation of the ferromagnetic sample is called its **hysteresis loop**. Throughout the cycle, the magnetic field B lags behind the magnetising field intensity H. ***The phenomenon of the lagging of magnetic induction behind the magnetising field is called hysteresis.***




**Significance of the area of hysteresis loop**- The area within a B— H loop represents the energy dissipated per unit volume in the material when it is carried through a cycle of magnetisation.
**Practical importance of hysteresis loops**- A study of hysteresis loop provides us information about retentivity, Coercivity and hysteresis loss of a magnetic material. This helps in proper selection of materials for designing cores of transformers and electromagnets and in making permanent magnets.

**KARPAGAM UNIVERSITY**
**DEPARTMENT OF PHYSICS**
**I B.SC PHYSICS**
**ELECTRICITY AND MAGNETISM (17PHU201)**

| QUESTIONS | OPTION 1 | OPTION 2 |
|---|---|---|
| **UNIT-II** | | |
| The energy stored in the charged capacitor is _________. | 1/2 $CV^2$ | 1/2qV |
| The arrangement in which one conductor is charged and other is earthed is named as ____________. | capacitor | condenser |
| __________ device is useful to reduce voltage fluctuations in electric power supplies. | capacitor | condenser |
| The capacitance of a capacitor C is___________ | q/v | qv |
| _______capacitors can bewidely used in the tuning circuits of radio receivers. | mica | electrolytic |
| _______capacitors are used widely in a radia-set as a smoothing capacitors | electrolytic | mica |
| ________capacitor is used in a.c bridges | electrolytic | variable air capacitor |
| ___________ device is used to measure electrostatic potentials | electrometers | magnetometers |
| A dielectric slab is introduced between the plates of an isolated charged parallel plate air capacitor.Which of the following quantities will remain unchanged? | charge on the capacitor | p.d across the capacitor |
| The p.d across a capacitor is kept constant.If a dielectric slab of dielectric constant K is introduced between the plates,the stored energy will be __________. | decreases by a factor K | increases by a factor K |
| Capacitance has the dimension ____________. | $M^{-1}L^{-2}T^{4}I^{2}$ | $ML^{2}T^{-4}I^{-2}$ |
| In gauss's law the electric flux E through a closed surface (s) depends on the value of net charge __________. | Inside the surface | outside the surface |
| The unit of capacitance is ____________. | Farad | coulomb/volt |
| The capacitance of the cylindrical capacitor is ____________. | $2\pi\epsilon_0 l/\log(b/a)$ | $4\pi\epsilon_r\epsilon_0 ab/(b-a)$ |
| The capacitance of the spherical capacitor (outer sphere earthed)is ____________. | $2\pi\epsilon_0 l/\log(b/a)$ | $4\pi\epsilon_r\epsilon_0 ab/(b-a)$ |
| The capacitance of the spherical capacitor (inner sphere is earthed) is __________. | $2\pi\epsilon_0 l/\log(b/a)$ | $4\pi\epsilon_0 b^2/(b-a)$ |
| The capacitance of a parallel plate capacitor is ___________. | $2\pi\epsilon_0 l/\log(b/a)$ | $4\pi\epsilon_0 b^2/(b-a)$ |

| | | |
|---|---|---|
| The energy stored in the capacitor is ___________. | $q^2/2C$ | $1/2\ CV^2$ |
| If a dielectric slab of dielectric constant K is introduced between the plates,when the charge remains the same then the stored energy will be ______________. | decreases by a factor $1/\epsilon_r$ | increases by a factor $1/\epsilon_r$ |
| Calculate the force between the plates of a parallel plate capacitor,when the area of the plate is 300cm2 each, the separation is 0.5cm and they are charged to p.d 1000volts. | $5.3x10^{-3}$ N | $4.2x10^{-3}$ N |
| _________ device is used to generate and detect electromagnetic oscillation of high frequency. | capacitor | voltmeter |
| The normal component of D are ________________ across the boundary by the surface charge density | continuous | Discontinuous |
| The tangential component of the electric field is __________ across the boundary. | continuous | Discontinuous |
| The potential difference between the conductors is proportional to the _______________ on the capacitor. | charge | voltage |
| The coaxial cable used in communication system is a common example for which type of capacitor __________. | spherical | cylindrical |
| Electric fields E1 and E2 on the two sides of an interface of two dielectric media having dielectric constants K1 and K2,make an angles Θ1 and Θ2 with the normal to the interface.Which of the following relations is true? | $K_1\sin \Theta_1= K_2\sin \Theta_2$ | $K_1\cos \Theta_1= K_2\cos \Theta_2$ |
| The unit of permittivity is _____ | $C^2N^{-1}M^{-2}$ | $C^2N^1M^2$ |
| The number of electric lines of force originating from a charge of 1C is_____ | $0.55x10^5$ | $0.055x10^{-5}$ |
| The value of ϵo is____ | $8.845x10^{12}$ | $8.854x10^{-12}$ |
| The net electrical charge in an isolated system remains constant. This is known as | Law of conservation of charge | Coulomb's first law |
| Variable air capacitor is used in _____________. | A.c bridges | D.c bridges |
| _________ capacitors can be used only in unidirectional power supplies. | mica | Electrolytic |
| An electron- volt (eV) is a unit of | Energy | Potential difference |
| The unit of electrical energy is | Joule | Watt- second |
| One electron volt (1 eV) is equivalent to _____ joules | $1.3 \times 10^{-19}$ | $1.4 \times 10^{-19}$ |
| Farad is the unit of | capacitance | self inductance |
| In a charged capacitor the energy is stored in | the field between | positive chaarge |

| | | |
|---|---|---|
| The capacitance of the parallel plate condenser does not depend on | area of the plates | medium between the plates |
| No current flows between two charged bodies connected together when they have the same | charge | potential |
| Two condensers of capcaitance C1 and C2 respectively are connected in parallel. The equivalent capcaitance of the system is | C1+C2 | C1+C2/(C1+C2) |
| Two condensers of capcaitance C1 and C2 respectively are connected in series. The equivalent capcaitance of the system is | C1+C2 | C1+C2/(C1+C2) |
| The radius of the earth is 6400km. Its capacitance is | 7.1 x 10^-4F | 6.4 x 10^-4F |
| When air in the capacitor is replaced by a medium of dielectric constant K,the capacity | decreases K times | increases K times |
| Materials which can store electrical energy are called | magnetic materials | semi conductors |
| The dielectric constant of air is practically taken as | more than unity | unity |
| Dielectric materials are | insulating materials | semiconducting materials |
| Dielectric constant of vacuum is | infinity | 100 |
| For making a capacitor it is better to select a dielectic having | low permittivity | high permitivitty |
| If three 15 micro F capacitors are connected in series, the ne | 5 micro Farad | 30 micro Farad |
| If three 10 micro F capacitors are connected in parallel, the r | 20 micro Farad | 30 micro Farad |
| A dielectric material must be a | resistor | insulator |
| The capacitance of the capacitor is not affected by | distance between the plates | area of the plates |
| The dissipation factor of a good dielectric is of the order | 0.0002 | 0.002 |
| Which of the following material has highest value of dielectric constant | glass | vacuum |
| Which of the following capacitor has relatively shortest shelf life? | mica | electrolytic |
| When a dielectric slab is introduced in a parallel plate capacitor, the potential difference between the plates will | remain unchanged | decrease |
| A capacitor consists of | two insulators seperated by a condenser | two conductors seperated by an insulator |
| A paper capacitor is usually availabble in the form of | tubes | rolled foil |

| | | |
|---|---|---|
| Which of the following material requires least magnetizing field to magnetize it? | Gold | Silver |

**Prepared by Dr.A.Saranya, Assistant Professor**

| OPTION 3 | OPTION 4 | ANSWER |
| --- | --- | --- |
| v/q | qV | 1/2 $CV^2$ |
| capacitor/ condneser | comparator | capacitor/ condneser |
| converter | comparator | capacitor |
| v/q | v/q | q/v |
| paper | variable air | variable air |
| both mica and electrolytic | variable | both mica and electrolytic |
| both mica and electrolytic | mica | variable air capacitor |
| potentiometer | galvanometer | electrometers |
| energy of the capacitor | electric field inside the capacitor | charge on the capacitor |
| remains constant | increases or decreases depending on the nature of the dielectric | increases by a factor K |
| $ML T^{-3} I^{-1}$ | $M^{-1}L^{-1}T^{3}I$ | $M^{-1}L^{-2}T^{4}I^{2}$ |
| on the surface | in the surface | Inside the surface |
| Farad and Coulomb/Volt | ohm | Farad and Coulomb/Volt |
| $\epsilon_r\epsilon_0A/d$ | $\epsilon_0A/d$ | $2\pi\epsilon_0 l/\log(b/a)$ |
| $\epsilon_r\epsilon_0A/d$ | $4\pi\epsilon_0b^2/(b-a)$ | $4\pi \epsilon_r\epsilon_0ab/(b-a)$ |
| $\epsilon_r\epsilon_0A/d$ | $4\pi \epsilon_0b^2/(b-a)$ | $4\pi \epsilon_0b^2/(b-a)$ |
| $\epsilon_0A/d$ | $4\pi\epsilon_0b^2/(b-a)$ | $\epsilon_0A/d$ |

| | | |
|---|---|---|
| both a and b | $q^2/C$ | both a and b |
| remains constant | increases or decreases depending on the nature of the dielectric | decreases by a factor $1/\epsilon_r$ |
| $5.0x10^{-3}$ N | $4.0x10^{-3}$ N | $5.3x10^{-3}$ N |
| Resistor | galvanometer | capacitor |
| Random | Discrete | Discontinuous |
| Random | Discrete | continuous |
| current | time | charge |
| Air capacitor | Parallel plate capacitor | Parallel plate capacitor |
| $K_1 \tan \Theta_1 = K_2 \tan \Theta_2$ | $K_1 \cot \Theta_1 = K_2 \cot \Theta_2$ | $K_1 \cot \Theta_1 = K_2 \cot \Theta_2$ |
| $C^{2} N^{1} M^{2}$ | $C^{-2} N^{-1} M^{-2}$ | $C^{-2} N^{-1} M^{-2}$ |
| $1.129x10^{11}$ | $1.921x10^{-11}$ | $0.055x10^{-5}$ |
| $8.888x10^{-12}$ | 5.845x1012 | $8.854x10^{-12}$ |
| Coulomb's second law | Law of conservation of energy | Law of conservation of charge |
| both a and b | tuning circuits | A.c bridges |
| paper | variable | Electrolytic |
| Charge | Momentum | Energy |
| Kilowatt- hour | All of these | All of these |
| $1.5 x 10^{-19}$ | $1.6 x 10^{-19}$ | $1.6 x 10^{-19}$ |
| mutual inductanc | conductance of an | capacitance |
| negative charge | neutral | the field between the plates |

| | | |
|---|---|---|
| distance between the plates | metal of the plates | metal of the plates |
| capacitance | resistance | potential |
| C1C2/(C1+C2) | C1-C2 | C1+C2 |
| C1C2/(C1+C2) | C1-C2 | C1C2/(C1+C2) |
| 0 | 6.4 x 10^6 F | 7.1 x 10^-4F |
| increases K^2 times | remains constant | increases K times |
| dielectric materials | super conductors | dielectric materials |
| less than unity | zero | unity |
| magnetic materials | ferroelectric materials | insulating materials |
| one | zero | one |
| permitivity same as that of air | permitivity slightly more thanair | high permitivitty |
| 40 micro Farad | 50 micro Farad | 5 micro Farad |
| 40 micro Farad | 50 micro Farad | 30 micro Farad |
| good conductor | semi conductor | insulator |
| thickness of the plates | atmosphere | thickness of the plates |
| 0.02 | 0.2 | 0.0002 |
| ceramics | oil | ceramics |
| ceramics | paper | electrolytic |
| increases | becomes zero | decrease |
| 2 insulator | 2 conductor | two conductors seperated by an insulator |
| disc | meshed plates | rolled foil |

| | | |
|---|---|---|
| Tungsten | Cobalt | Cobalt |

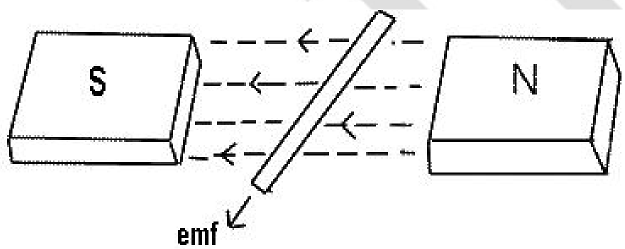
# UNIT-IV

## SYLLABUS

Electromagnetic Induction: Faraday's laws of electromagnetic induction, Lenz's law, self and mutual inductance, L of single coil, M of two coils. Energy stored in magnetic field.

**Faraday's laws of electromagnetic induction:**

**Faraday's laws of electromagnetic induction** explains the relationship between electric circuit and magnetic field. This law is the basic working principle of the most of the electrical motors, generators, transformers, inductors etc.




**Faraday's First Law:**

Whenever a conductor is placed in a varying magnetic field an EMF gets induced across the conductor (called as induced emf), and if the conductor is a closed circuit then induced current flows through it.
Magnetic field can be varied by various methods -
1. By moving magnet
2. By moving the coil
3. By rotating the coil relative to magnetic field

**Faraday's Second Law:**

**Faraday's second law of electromagnetic induction** states that, the magnitude of induced emf is equal to the rate of change of flux linkages with the coil. The flux linkages is the product of *number of turns* and *the flux associated with the coil.*

**Formula of Faraday's Law:**

Consider the conductor is moving in magnetic field, then
flux linkage with the coil at initial position of the conductor = $N\Phi_1$ (Wb) (N is speed of the

motor and Φ is flux)
flux linkage with the coil at final position of the conductor = $N\Phi_2$ (Wb)
change in the flux linkage from initial to final = $N(\Phi_1 - \Phi_2)$
let $\Phi_1 - \Phi_2 = \Phi$
therefore, change in the flux linkage = NΦ
and, rate of change in the flux linkage = NΦ/t
taking the derivative of RHS
rate of change of flux linkages = N (dΦ/dt)

According to **Faraday's law of electromagnetic induction**, rate of change of flux linkages is equal to the induced emf

So, E = N (dΦ/dt) (volts)

**Phenomenon Of Mutual Induction**

Alternating current flowing in a coil produces alternating magnetic field around it. When two or more coils are magnetically linked to each other, then an alternating current flowing through one coil causes an induced emf across the other linked coils. This phenomenon is called as mutual induction.

**Lenz's Law:**

**Lenz's law of electromagnetic induction** states that, when an emf is induced according to Faraday's law, the polarity (direction) of that induced emf is such that it opposes the cause of its production.

Thus, considering Lenz's law

E = -N (dΦ/dt) (volts)

The negative sign shows that, the direction of the induced emf and the direction of change in magnetic fields have opposite signs.

**self inductance L of single coil:**

- Consider the figure given below

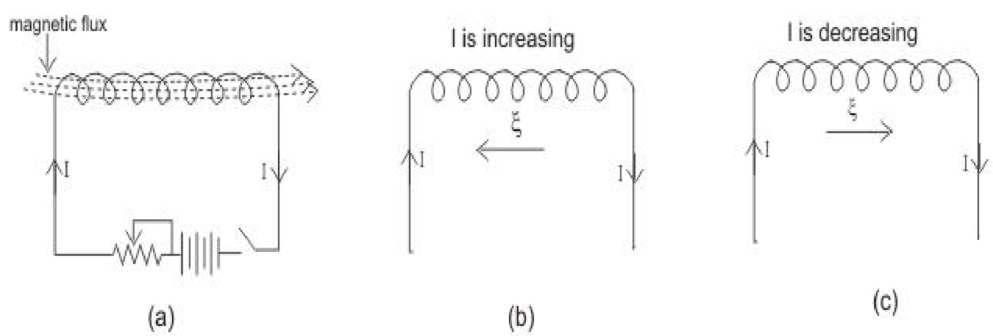



Figure 1. When current increases direction of induced emf is opposite to direction of current (b) and in case of decreasing current direction of induced emf is same as direction of current

- When we establish a current through an inductor or coil, it generates a magnetic field and this result in a magnetic flux passing through the coil as shown in figure 1(a).
- If we vary the amount of current flowing in the coil with time, the magnetic flux associated with the coil also changes and an emf $\xi$ is induced in the coil.
- According to the Lenz�s law, the direction of induced emf is such that it opposes its cause i.e. it opposes the change in current or magnetic flux.
- This phenomenon of production of opposing induced emf in inductor or coil itself due to time varying current in the coil is known as self induction.
- If I is the amount of current flowing in the coil at any instant then emf induced in the coil is directly proportional to the change in current i.e.

$$\xi \propto \left(\frac{-dI}{dt}\right)$$

or

$$\xi = -L\frac{dI}{dt} \qquad \text{--(1)}$$

where L is a constant known as coefficient of self induction.

- If (-dI/dt)=1 then $\xi$=L
  Hence the coefficient of self induction of a inductor or coil is numerically equal to the emf induced in the coil when rate of change of current in the coil is unity.

- Now from the faraday's and Lenz�s laws induced emf is

$$\xi = -\frac{d\phi}{dt} \qquad \text{---(2)}$$

comparing equation 1 and 2 we have,

$$L\frac{dI}{dt} = \frac{d\phi}{dt}$$

or $\Phi = LI$

- Again for I=1, $\Phi = L$
  hence the coefficient of self induction of coil is also numerically equal to the magnetic flux linked with the inductor carrying a current of one ampere
- If the coil has N number of turn�s then total flux through the coil is
  $\Phi_{tot} = N\Phi$
  where $\Phi$ is the flux through single turn of the coil .So we have,
  $\Phi_{tot} = LI$
  or $L = N\Phi/I$
  for a coil of N turns
- In the figure given below consider the inductor to be the part of a circuit and current flowing in the inductor from left to right

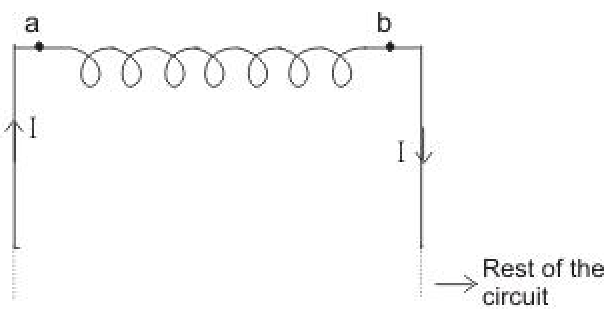



Figure 2. Inductor as the part of a circuit

- Now when a inductor is used in a circuit, we can use Kirchhoff�s loop rule and this emf(Self induced emf) can be treated as if it is a potential drop with point A at higher potential and B at lower potential when current flows from a to b as shown in the figure
- We thus have
  $V_{ab} = LdI/dt$

**Self induction of a long solenoid:**

- Consider a long solenoid of length l, area of cross-section A and having N closely wound turns.
- If I is the amount of current flowing through the solenoid them magnetic field **B** inside the solenoid is given by,

$$B = \frac{\mu_0 N I}{l}$$

- Magnetic flux through each turn of the solenoid is,

$$\phi = BA = \frac{\mu_0 N^2 A I}{l}$$

$$\text{but}, \phi = LI$$

$$\text{So}, LI = \frac{\mu_0 N^2 A I}{l}$$

or, coefficent of self induction

$$L = \frac{\mu_0 N^2 A}{l} \qquad (3)$$

**Energy in an inductor:**

- Changing current in an inductor gives rise to self induced emf which opposes changes in the current flowing through the inductor.
- This self inductance thus plays the role the inertia and it is electromagnetic analogue of mass in mechanics.
- So a certain amount of work is required to be done against this self induced emf for establishing the current in the circuit.
- In order to do so, the source supplying current in a circuit must maintain Potential difference between its terminals which is done by supplying energy to the inductor.
- Power supplied to the inductor is given by relation
  $P=\xi I$ ---(4)
  where

$$\xi = -L\frac{dI}{dt}$$

L is Self inductance and
dI/dt is rate of change of current I in the circuit.

- Energy dW supplied in time dt would be
  dW=Pdt
  =LI(dI/dt) dt
  =LIdI
  and total energy supplied while current I increases from o to a final value I is

$$W = L\int_0^I I dI = \frac{1}{2}LI^2 \qquad \text{----(5)}$$

- Once the current reaches its final value and becomes steady ,the power input becomes zero.
- The energy so far supplied to the inductor is stored in it as a form of potential energy as long as current is maintained.
- When current in circuit becomes zero, the energy is returned to the circuit which supplies it.

**Mutual Inductance**

- Consider two coils 1 and 2 placed near each other as shown below in the figure

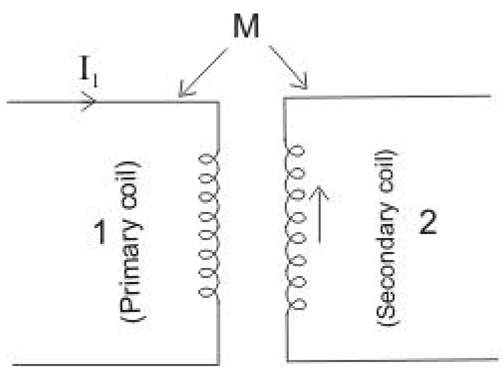



Fig: Two coils placed each other

- Let coil 1 be the primary coil and coil 2 be secondary coil
- When current is primary coil changes w.r.t time then the magnetic field produced in the coil also changes with time which causes a change in magnetic flux associated with secondary coil
- Due to this change of flux linked with secondary coil an emf is induced in it and this phenomenon is known as mutual induction
- Similarly change in current in secondary coil induces an emf in primary coild.This way as a result of mutual inductance emf is induced in both the coils
- If $I_1$ is the current in primary coil at any instant ,than the emf induced in secondary coil would be proportional to the rate of change of current in primary coil i.e.

$$\xi_2 \alpha \frac{dI_1}{dt}$$

Or

$$\xi_2 = -M \frac{dI_1}{dt}$$

Where M is a constant known as coefficient of mutual induction and minus sign indicates that direction of induced emf is such that it opposes the change of current in primary coil

- Unit of mutual inductance is Henry
- We know that a magnetic flux is produced in primary coil due to the flow of current $I_1$.If this is the magnetic flux associated with secondary coil then from faraday's law of EM induction ,emf induced in secondary coil would be

$$\xi_2 = \frac{-d\phi_{21}}{dt}$$

- comparing above equation we get

$$\phi_{21} = M_{21} I_1$$

Thus coefficient of mutual induction of secondary coil w.r.t primary coil is equal to magnetic flux linked with secondary coil when 1 Ampere of current flows in primary coil and vice-versa

- Similarly ,if $I_2$ is the current in secondary coil at any instant then flux linked with primary coil is

$$\phi_{12} = M_{12} I_2$$

where $M_{12}$ is coefficient of mutual induction of primary coil with respect to secondary coil

- EMF induced in primary coil due to change of this flux is

$$\xi_1 = -M_{12} \frac{dI_2}{dt}$$

- For any two circuits
  $M_{12}=M_{21}=M$
- In general mutual inductance of two coil depends on geometry of the coils (shape ,size, number of turns etc),distance between the coils and nature of material on which the coil is wound

**Mutual Inductance of two co-axial solenoids**

- Consider a long solenoid of length l and area of crossection A containing $N_p$ turns in its primary coil

Let a shorter secondary coil having $N_2$ number of turns be wounded closely over the central portion of primary coil as shown below in the figure.

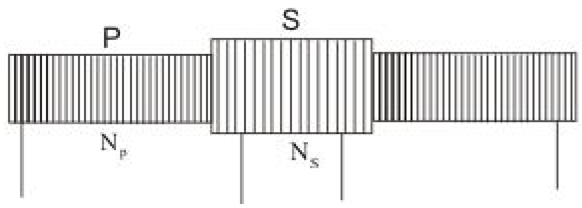



Fig: Two co-axial solenoids with secondary coil wounded closely over central portions of primary coil length l

- If $I_p$ is the current in the primary coil then magnetic field due to primary coil would be

$$B = \frac{\mu_0 N_P I_P}{l}$$

- So flux through each turn of secondary coil would be

$$\phi_s = \frac{\mu_0 N_P I_P A}{l}$$

where A is the area of crossection of primary coil

- Total magnetic flux through secondary coil is

$$\phi_{s(total)} = \frac{\mu_0 N_P N_s I_P A}{l}$$

- Emf induced in secondary coil is

$$\xi_s = \frac{-d\phi_{s(total)}}{dt} = \frac{-\mu_0 N_p N_S A}{l}\frac{dI_p}{dt}$$

Thus from equation 24

$$\xi_s = -M\frac{dI_p}{dt}$$

So

$$M = \frac{\mu_0 N_p N_S A}{l}$$

**Relation between Mutual inductance and self inductance:**

- Consider two coils of same length l and same area of cross-section placed near each other as shown below in the figure

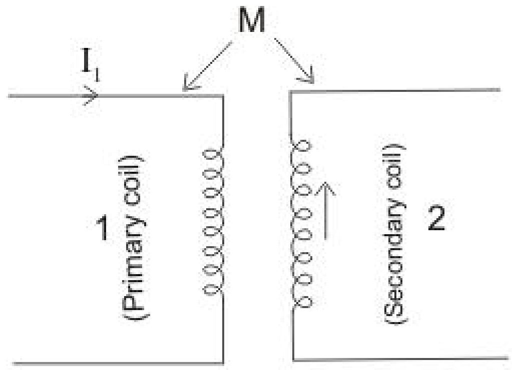



Fig: Two coils placed near each other

- Let there are $N_1$ number of turns in primary coil and $N_2$ number of turns in secondary coil
- A current $I_1$ in the primary coil produces a magnetic field

$$B = \frac{\mu_0 N_1 I_1}{l}$$

which in turns gives rise to flux?

$$\phi_{11} = BN_1A$$
$$= \frac{\mu_0 N_1^2 A I_1}{l}$$

in primary coil and

$$\phi_{21} = BN_2A$$
$$= \frac{\mu_0 N_1 N_2 A I_1}{l}$$

in the secondary coil due to current in primary coil.

- By the definition of self induction

$$\phi_{11} = L_1 I_1$$
So
$$L_1 = \frac{\mu_0 N_1^2 A}{l}$$

and by definition of mutual induction

$$\phi_{21} = M_{21} I_1$$
So
$$M_{21} = \frac{\mu_0 N_1 N_2 A}{l}$$

- Reversing the procedure if we first introduce the current $I_2$ in secondary coil then we get

$$L_2 = \frac{\mu_0 N_2^2 A}{l}$$

And

$$M_{12} = \frac{\mu_0 N_1 N_2 A}{l}$$

- So $L_1$ is the self inductance of primary coil,$L_2$ is the self induction of secondary coil and $M_{21}$=$M_{12}$=M is the mutual inductance between two coils

- Product of $L_1$ and $L_2$ is

$$L_1 L_2 = \frac{\mu_0^2 N_1^2 N_2^2 A^2}{l^2} = M^2$$

hence

$$M = \sqrt{L_1 L_2}$$

- In practice M is always less than eq due to leakage which gives

$$\frac{M}{\sqrt{L_1 L_2}} = k$$

Where K is called coefficient of coupling and K is always less then 1.

**Energy in an Inductor:**

When a electric current is flowing in an inductor, there is energy stored in themagnetic field. Considering a pure inductor L, the instantaneous power which must be supplied to initiate the current in the inductor is

$$P = iv = Li\frac{di}{dt}$$

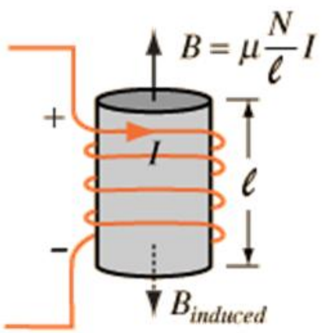
so the energy input to build to a final current i is given by the integral

$$\text{Energy stored} = \int_0^I P dt = \int_0^I Li' di' = \frac{1}{2} LI^2$$

**Energy in Magnetic Field:**

From analysis of the energy stored in an inductor,

$$\text{Energy stored} = \frac{1}{2}LI^2$$




the energy density (energy/volume) is

$$\frac{\frac{1}{2}LI^2}{A\ell} = \frac{\frac{1}{2}\frac{\mu N^2 A}{\ell}\frac{B^2\ell^2}{\mu^2 N^2}}{A\ell}$$

so the energy density stored in the magnetic field is

$$\eta_B = \frac{energy}{volume} = \frac{1}{2}\frac{B^2}{\mu}$$

**KARPAGAM UNIVERSITY**
**DEPARTMENT OF PHYSICS**
**I B.SC PHYSICS**
**ELECTRICITY AND MAGNETISM (18PHU201)**

| QUESTIONS | OPTION 1 | OPTION 2 |
|---|---|---|
| **UNIT-IV** | | |
| Corresponding to Maxwell the unit in SI system is | weber | Henry |
| Lenz's law is a consequence of the law of conservation of | energy | momentum |
| Lenz's law does not violate the principle of | conservation of c | conservation of energ |
| In SI system of units, Henry is the unit of | inductance | self inductance |
| Magnitude of induced emf is proportional to | of current. | voltage. |
| Alternative current generator is basically based upon | amperes law | Lenz's law |
| In system international, unit of mutual inductance is | henry | $VsA^{-1}$ |
| Change of current of 1 $As^{-1}$ causes e.m.f of 1 V to be equal t | 1 henry | 1 volt/m |
| The direction of the induced emf during electromagnetic ind | Lenz's law | Amperes law |
| The magnitude of induced emf during electromagnetic induc | electric field | electric flux |
| The knowledge of electromagnetic induction has been used i | electric motor | generator |
| In alternative current generator, AC current reverses its direc | T times per secoı | f times per second |
| Induced current depends upon speed with which conductor r | resistance of galv | voltage of loop |
| Moving a coil in and out of magnetic field induces | force | potential difference |
| Induced current in coil by a magnet turns it into an | straight wire | magnet |
| Changing current in one coil induces e.m.f in another this ph | emf | induced emf |
| Negative sign of equation of self induction shows that | deduce e.m.f. | it maintains change |
| Changing current in a coil produces e.m.f in same coil is kno | emf | induced emf |
| Energy that is stored in an inductor can be represented by | $^1/_2(L^2I)$ | $^1/_2(LI^2)$ |
| Which of the following circuital elementsstore energy in ele | inductance | condenser |
| The emf which produces induced circuit is called as | induced emf | emf |
| When a magnet is moved towards the coil, galvanometer sho | one direction | two direction |
| When a magnet is moving away from the coil, galvanometer | same direction | all direction |
| When the magnet is stationary, the galvanometer shows | no deflection | opposite side deflecti |
| Coefficeint of self inductance is represented by | M | Z |
| Self inductance is also called as | coefficient of sel | emf |
| Induced emf is called | coefficient of sel | emf |
| ___________ of a coil is numerically equal to the induced e | inductance | emf |
| ___________ of a coil is numerically equal to the emf when | inductance | back emf |
| will result in what type of filtering? | blocking of a cer | passing of the lower f |
| Higher self induction of a coil | lesser its weber t | lower the emf induce |
| Faraday's law states that the: | irection of the in | emf is related to the d |
| What does Faraday's law concern? | a magnetic field | a magnetic field cutti |
| What is hysteresis? | lead between vol | lag between cause an |
| What type of device consists of a coil with a moveable iron core? | solenoid | armature |

| | | |
|---|---|---|
| Which two values are plotted on a B-H curve graph? | reluctance and fl | permeability and relu |
| coil are doubled. Its self-inductance will be | unaffected | doubled |
| Lenz'a law self-induced voltage will | aid the increasing | tend to decrease the a |
| on the | flux density of th | amount of flux cut |
| coil depends on | permeability of th | number of their turns |
| Which of the following is unit of inductance? | ohm | Henry |
| induced emf is equal to | magnerising forc | magnetising flux |
| When a coil is rotated in a magnetic field with steady speed | no emf is induced | a periodic emf is proc |
| 1 Tesla = | 1 wb- $m^2$. | 1 wb/ $m^2$. |
| A conductor of length 1 metre moves at right angles to a ma | 25 V | 50 V |
| A magnetic material has a total flux B of 80 micro Wb with | $2 \times 10^{-6}$. | $0.2 \times 10^{-6}$. |
| An emf of 16 volts is induced in a coil of inductance 4 H. Th | 64 A/s | 32 A/s |
| The emf induced in a conductor rotating in bipolar field is | DC | AC |
| If current in one coil becomes steady then magnetic field line | zero | constant |
| Production of induced current in one coil due to production o | electromagnetism | induction |
| Both the number of turns and the core length of an inductive | unaffected | doubled |
| A conductor of length 100 cm moves at right angles to a ma | 150 V | 50 V |
| Self inductance of magnetic coil is proportional to | N | 1/N |
| 1 Maxwell is the same as | $10^{-8}$ weber | $10^{8}$ weber |

**Prepared by Dr.A.Saranya,Assistant Professor**

| OPTION 3 | OPTION 4 | ANSWER |
|---|---|---|
| Tesla | Gauss | weber |
| mass | charge | energy |
| conservation of mass | conservation c | conservation of energy |
| mutual inductanc | both self and mutu | inductance |
| rate of change of | rate of change of r | rate of change of magnetic flux linkage |
| faradays law | coulombs law | faradays law |
| Wb | both henry and Vs. | both henry and VsA-1 |
| 1 ampere | 1 joule | 1 henry |
| Maxwell law | Faaradays law | Lenz's law |
| magnetic field | magnetic flux | magnetic flux |
| voltmeter | galvanometer | generator |
| once/sec | per sec | f times per second |
| current of loop | resistance of loop | resistance of loop |
| emf | voltage | emf |
| ammeter | electromagnet | electromagnet |
| self induction | mutual induction | mutual induction |
| it opposes chang | induced e.m.f | it opposes change |
| self induction | mutual induction | self induction |
| $L(^I/_T)$ | LI | $^1/_2(LI^2)$ |
| variable resistor | resistance | inductance |
| potential differen | magnetic flux | induced emf |
| all direction | rest | one direction |
| rest | opposite direction | opposite direction |
| at rest | slowly moving | no deflection |
| H | L | L |
| back emf | induced e.m.f | coefficient of self inductance |
| back emf | self induction | back emf |
| back emf | self induction | self induction |
| self induction | mutual induction | mutual induction |
| passing of the hi | blocking of the hig | passing of the higher frequencies |
| greater the flux p | longer the dlay in e | longer the dlay in estabilishing steady current through it |
| emf depends on t | direction of an inc | emf depends on the rate of cutting flux |
| a magnetic field | a magnetic field hy | a magnetic field cutting a conductor |
| lag between volt | lead between caus | lag between cause and effect |
| read switch | relay | solenoid |

| | | |
|---|---|---|
| magnetizing for | flux density and m | flux density and magnetizing force |
| halved | quadrupled | doubled |
| produce current ( | aid the applied vol | produce current opposite to the increasing current |
| amount of flux li | rate of changes of | rate of changes of flux linkages |
| cross sectional a | all | all |
| ampere turns | webers/meter | Henry |
| rate of change of | intensity | rate of change of magnetic flux linked with the coil |
| unidirectional en | bidirectional | a periodic emf is produced |
| 1 wb | 1 wb/m | 1 wb/ $m^2$. |
| 75 V | 100 V | 25 V |
| $2 \times 10^6$. | $20 \times 10^{-6}$. | $2 \times 10^{-6}$. |
| 16 A/s | 4 A/s | 4 A/s |
| AC and DC both | zero | AC |
| less than before | more than before | zero |
| mutual induction | steady current | mutual induction |
| halved | quadrupled | doubled |
| 75 V | 37.5 V | 75 V |
| N^2 | 1/N^2 | N^2 |
| $10^4$ weber | $10^{-4}$ weber | $10^{-8}$ weber |

## UNIT-V

## SYLLABUS

Maxwell`s equations and Electromagnetic wave propagation: Equation of continuity of current, Displacement current, Maxwell's equations, Poynting vector, energy density in electromagnetic field, electromagnetic wave propagation through vacuum and isotropic dielectric medium, transverse nature of EM waves, polarization.

**Equation of continuity of current:**

If we do some simple mathematical tricks to Maxwell's Equations, we can derive some new equations. On this page, we'll look at the continuity equation, which can be derived from Gauss' Law andAmpere's Law.

To start, I'll write out a vector identity that is always true, which states that the divergence of the curl of any vector field is always zero:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0$$

If we apply the divergence to both sides of Ampere's Law, then we obtain:

$$\nabla \cdot \left( \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \right) = \nabla \cdot (\nabla \times \mathbf{H}) = 0$$

$$\frac{\partial (\nabla \cdot \mathbf{D})}{\partial t} = -\nabla \cdot \mathbf{J}$$

If we apply Gauss' Law to rewrite the divergence of the Electric Flux Density (D), we have derived the continuity equation:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_V}{\partial t}$$

Equation [3] looks nice, but what does it mean? The left side of the equation is the divergence of theElectric Current Density (J). This is a measure of whether current is flowing into a volume (i.e. the divergence of J is positive if more current leaves the volume than enters).

Recall that current is the flow of electric charge. So if the divergence of J is positive, then more charge is exiting than entering the specified volume. If charge is exiting, then the amount of charge within the volume must be decreasing. This is exactly what the right side is a measure of - how much electric charge is accumulating or leaving in a volume. Hence, the continuity equation is about continuity - if there is a net electric current is flowing out of a region, then the charge in that region must be decreasing. If there is more electric current flowing into a given volume than exiting, than the amount of electric charge must be increasing.

**Displacement current:**

Displacement current, in electromagnetism, a phenomenonanalogous to an ordinary electric current, posited to explain magnetic fields that are produced by changing electric fields. Ordinary electric currents, called conduction currents, whether steady or varying, produce an accompanying magnetic field in the vicinity of the current. The British physicist James Clerk Maxwell in the 19th century predicted that a magnetic field also must be associated with a changing electric field even in the absence of a conduction current, a theory that was subsequently verified experimentally. As magnetic fields had long been associated with currents, the predicted magnetic field also was thought of as stemming from another kind of current. Maxwell gave it the name displacement current, which was proportional to the rate of change of the electric field that kept cropping up naturally in his theoretical formulations.

As electric charges do not flow through the insulation from one plate of a capacitor to the other, there is no conduction current; instead, a displacement current is said to be present to account for the continuity of the magnetic effects. In fact, the calculated size of the displacement current between the plates of a capacitor being charged and discharged in an alternating-current circuit is equal to the size of the conduction current in the wires leading to and from the capacitor. Displacement currents play a central role in the propagation of electromagnetic radiation, such as light and radio waves, through empty space. A traveling, varying magnetic field is everywhere associated with a periodically changing electric field that may be conceived in terms of a displacement current. Maxwell's insight on displacement current, therefore, made it possible to understand electromagnetic waves as being propagated through space completely detached from electric currents in conductors.

**Maxwell's Equations:**

**Maxwell's equations** represent one of the most elegant and concise ways to state the fundamentals of electricity and magnetism. From them one can develop most of the working relationships in the field. Because of their concise statement, they embody a high level of mathematical sophistication and are therefore not generally introduced in an introductory treatment of the subject, except perhaps as summary relationships.

These basic equations of electricity and magnetism can be used as a starting point for advanced courses, but are usually first encountered as unifying equations after the study of electrical and magnetic phenomena.

| Symbols Used | | |
|---|---|---|
| E = Electric field | ρ = charge density | i = electric current |
| B = Magnetic field | $\varepsilon_0$ = permittivity | J = current density |
| D = Electric displacement | $\mu_0$ = permeability | c = speed of light |
| H = Magnetic field strength | M = Magnetization | P = Polarization |

**Integral form in the absence of magnetic or polarizable media:**

I. Gauss' law for electricity $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$

II. Gauss' law for magnetism $\oint \vec{B} \cdot d\vec{A} = 0$

III. Faraday's law of induction $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

IV. Ampere's law $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2}\frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{A}$

**Differential form in the absence of magnetic or polarizable media:**

I. Gauss' law for electricity $\nabla \cdot E = \frac{\rho}{\varepsilon_0} = 4\pi k\rho$

II. Gauss' law for magnetism $\nabla \cdot B = 0$

III. Faraday's law of induction $\nabla x\, E = -\frac{\partial B}{\partial t}$

IV. Ampere's law

$$\nabla\, x\;\, B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$= \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$k = \frac{1}{4\pi\varepsilon_0} = \text{Coulomb's constant} \qquad c^2 = \frac{1}{\mu_0\varepsilon_0}$$

Note: $\nabla \cdot E$ and $\nabla x E$ here represent the vector operations divergence and curl, respectively.

**Differential form with magnetic and/or polarizable media:**

I. Gauss' law for electricity $\nabla \cdot D = \rho$

$D = \varepsilon_0 E + P$ *General case*

$D = \varepsilon_0 E$ *Free space*

$D = \varepsilon E$ *Isotropic linear dielectric*

II. Gauss' law for magnetism $\nabla \cdot B = 0$

III. Faraday's law of induction $\nabla x\, E = -\frac{\partial B}{\partial t}$

IV. Ampere's law $\nabla\, x\, H = J + \frac{\partial D}{\partial t}$

$B = \mu_0 (H + M)$ *General case*

$B = \mu_0 H$ *Free space*

$B = \mu H$ *Isotropic linear magnetic medium*

Note: $\nabla \cdot E$ and $\nabla x E$ here represent the vector operations divergence and curl, respectively.

**Poynting vector:**

The Poynting Theorem is in the nature of a statement of the conservation of energy for a configuration consisting of electric and magnetic fields acting on charges. Consider a volume V with a surface S. Then

*the time rate of change of electromagnetic energy within V plus the net energy flowing out of V through S per unit time is equal to the negative of the total work done on the charges within V.*

Consider first a single particle of charge q traveling with a velocity vector v. Let E and B be electric and magnetic fields external to the particle; i.e., E and B do not include the electric and magnetic fields generated by the moving charged particle. The force on the particle is given by the Lorentz formula

$$F = q(E + v\times B)$$

The work done by the electric field on that particle is equal to $qv\cdot E$. The work done by the magnetic field on the particle is zero because the force due to the magnetic field is perpendicular to the velocity vector v.

For a vector field of current density J the work done on the charges within a volume V is

$$\int_V J\cdot E\,dV$$

For a single particle of charge q traveling with velocity v the above quantity reduces to $qv\cdot E$.

One form of the Ampere-Maxwell's Law says that

$$J = (c/4\pi)\nabla\times H - (1/4\pi)(\partial D/\partial t)$$

When the RHS of the above is substituted for J the work done by the external fields on the charges within a volume V is

$$(1/4\pi)\int_V[cE\cdot(\nabla\times H) - E\cdot(\partial D/\partial t)]dV$$

There is a vector identity

$$\nabla\cdot(A\times B) = B\cdot(\nabla\times A) - A\cdot(\nabla\times B)$$

*which can be rewritten as*

$$A\cdot(\nabla\times B) = -[\nabla\cdot(A\times B)] + B\cdot(\nabla\times A)$$

This means that

$$E\cdot(\nabla\times H) = -\nabla\cdot(E\times H) + H\cdot(\nabla\times E)$$

When this expression is substituted into the expression for the rate at which work is being done the result is

$$\int_V J\cdot E dV = (1/4\pi)\int_V[-c\nabla\cdot(E\times H) - E\cdot(\partial D/\partial t) + cH\cdot(\nabla\times E)]dV$$

Faraday's law states that

$$\nabla\times E = -(1/c)(\partial B/\partial t)$$

When Faraday's law is taken into account the previous equation can be expressed as:

$$\int_V J\cdot E dV = (-1/4\pi)\int_V[c\,\nabla\cdot(E\times H) + E\cdot(\partial D/\partial t) + H\cdot(\partial B/\partial t)]dV$$

The total energy density U of the fields at a point is

$$U = (1/8\pi)(E\cdot D + B\cdot H)$$

where D=εE and H=(1/μ)B and ε and μ, called the dielectric and permabiity, respectively, are properties of the material in which the fields are located. The dielectric and permability are independent of the location.

This means that

$$U = (1/8\pi)(\varepsilon E\cdot E + (1/\mu)B\cdot B)$$

*and thus*

$$(\partial U/\partial t) = (1/4\pi)(\varepsilon E\cdot(\partial E/\partial t) + (1/\mu)B\cdot(\partial B/\partial t))$$

*which is equivalent to*

$$(\partial U/\partial t) = (1/4\pi)(E\cdot(\partial D/\partial t) + B\cdot(\partial H/\partial t))$$

The RHS of this latter expression occurs in a previous expression so that

$$-\int_V J\cdot E dV = \int_V [(\partial U/\partial t) + (c/4\pi)\nabla\cdot(E\times H)] dV$$

It is convenient to define a vector P, known as the Poynting vector for the electrical and magnetic fields, such that

$$P = (c/4\pi)(E\times H)$$

The previous equation then becomes

$$-\int_V J\cdot E dV = \int_V [(\partial U/\partial t) + \nabla\cdot P] dV$$

By Gauss' Divergence Theorem

$$\int_V (\nabla\cdot P) dV = \int_S n\cdot P dS$$

where S is the surface of the volume V and n is the unit normal to the surface element dS. The vector P has the dimensions of energy×time per unit area. Thus $\int_S n\cdot P dS$ is the net flow of energy out of the volume V.

The above means that work done by the electric and magnetic fields on the charges within a volume must match the rate of decrease of the energy of the fields within that volume and the net flow of energy into the volume. The big question is what does the net flow of energy into the volume correspond to physically. One possibility is that it might correspond to electromagnetic radiation. The above equation can also be stated as the negative of the work done on the charges within a volume must be equal to the increase in the energy of the electric and magnetic fields within the volume plus the net flow of energy out of the volume.

There is a major problem with the Poynting vector P; it is independent of the charges involved. It is the same whether there is one charge or one hundred million charges, or for that matter, zero charges. It can change with time but only as a result of the changes in the electric and magnetic fields.

Usually any difference between the change in energy and the work done is the energy of radiation. This is what is universally presumed in the case of the Poynting theorem, but the empirical evidence is that this cannot be so. If the Poynting vector corresponded to radiation then if a permanent magnet was placed in the vicinity of a body charged with static electricity the combination should glow and is that is not the case.

The Poynting vector is completely independent of the charges and their velocities in the volume being considered. In a word it is *exogenous*. The charges and their velocities are also exogenous. It is the rate of change of the energy stored in the fields that is *endogenous*. The Poynting theorem should read rate of change of energy in the fields = negative of work done by the fields on the charged particles minus the Poynting vector term.

However in the case of a permanent magnet and static electric charge the fields cannot change. Charged particles impinging upon an electric and magnetic field would experience work of them. The compensating change in momentum and energy would occur in the bodies holding the electric and magnetic fields. The charged particles hitting the electric and magnetic fields would induce a reaction as though they hit the magnet and charged body which creates the fields.

The dimensions of the Poynting vector term are energy per unit area per unit time. This is what would be expected if there were radiation generated in the volume. But the fact that the Poynting vector is exogenous means that without any charged particles at all being involved there would be radiation generated. The amount of radiation generated is fixed and no matter how many charged particles are injected into the volume at whatever velocities the same amount of radiation would be generated.

So the Poynting vector term apparently does not correspond to radiation. It is a puzzle as to what it does correspond to but there is no possibility that it corresponds to radiation.

The Differential Form of the Poynting Theorem:

Since the volume element is arbitrary the above equation implies that

$$(\partial U/\partial t) + \nabla \cdot P = -E \cdot J$$

The interpretation of the term $\nabla \cdot P$ is also problematical. It has a sign but it does not have a direction. It also is independent of the charge distribution, in this case J. In another study the case will be made that $\nabla \cdot P$ is the time rate of change of the energy resulting from the interaction of the electrical and magnet field

.

**Energy density in electromagnetic field:**

Let us calculate the energy density of electromagnetic field and see how the energy contained in such field can change in a closed volume.

we have seen that the energy of a collection of charges can be written as follows:

$$W_{el} = \frac{1}{2}\int \rho(\vec{r})\varphi(\vec{r})d^3r$$
$$= \frac{1}{2}\int (\nabla \cdot \vec{D})\varphi(\vec{r})d^3r$$
$$= -\frac{1}{2}\int (\vec{D} \cdot \nabla\varphi)d^3r$$
$$= \frac{1}{2}\int \vec{E} \cdot \vec{D}d^3r$$

where in the penultimate step, we have used the relation

$$\varphi\nabla \cdot \vec{D} = \nabla \cdot (\varphi\vec{D}) - \vec{D} \cdot \nabla\varphi$$

and converted the integral of the first term on the right to a surface integral by the divergence theorem and discarded the surface integral by taking the surface to infinity so that the remaining integral is all over space. The electric energy density can be thus written as

$$u_E = \frac{\vec{E} \cdot \vec{D}}{2} \rightarrow \frac{\epsilon E^2}{2}$$

the last relation being true for a linear electric medium.

Likewise, the magnetic energy can be written as

$$W_{mag} = \frac{1}{2}\int \vec{A} \cdot \vec{J}\, d^3r$$
$$= \frac{1}{2}\int \vec{A} \cdot (\nabla \times \vec{H})d^3r$$
$$= \frac{1}{2}\int \vec{H} \cdot (\nabla \times \vec{A})d^3r$$
$$= \frac{1}{2}\int \vec{H} \cdot \vec{B}d^3r$$

The magnetic energy density is given by

$$u_m = \frac{\vec{H}\cdot\vec{B}}{2} \to \frac{B^2}{2\mu}$$

the last relation is valid for a linear magnetic medium.

The total energy density in electromagnetic field is thus given by

$$u = \frac{\vec{E}\cdot\vec{D}}{2} + \frac{\vec{H}\cdot\vec{B}}{2}$$

**Electromagnetic wave propagation through vacuum and isotropic dielectric medium:**

In regions of space where there are no charges and currents, Maxwell equations read

$$\nabla\cdot\mathbf{E} = 0 \tag{3.1}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{3.2}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \tag{3.3}$$

$$\nabla\times\mathbf{B} = \mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t} \tag{3.4}$$

They are a set of coupled, first order, partial differential equations for **E** and **B**. They can be decoupled by applying curl to eqs. (3.3) and (3.4):

$$\nabla\times(\nabla\times\mathbf{E}) = \nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E} = -\nabla\times\frac{\partial\mathbf{B}}{\partial t} = -\frac{\partial}{\partial t}(\nabla\times\mathbf{B}) = -\mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} \tag{3.5}$$

$$\nabla\times(\nabla\times\mathbf{B}) = \nabla(\nabla\cdot\mathbf{B}) - \nabla^2\mathbf{B} = \nabla\times\left(\mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right) = \mu_0\varepsilon_0\frac{\partial}{\partial t}(\nabla\times\mathbf{E}) = -\mu_0\varepsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2} \tag{3.6}$$

Since $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$ we have

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \tag{3.7}$$

$$\nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \tag{3.8}$$

We now have *separate* equations for **E** and **B**, but they are of *second* order; that's the price you pay for decoupling them. In vacuum, then, each Cartesian component of **E** and **B** satisfies the three-dimensional wave equation

$$\nabla^2 f = \mu_0 \varepsilon_0 \frac{\partial^2 f}{\partial t^2} \tag{3.9}$$

The solution of this equation is a wave. So Maxwell's equations imply that empty space supports the propagation of electromagnetic waves, traveling at a speed

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.00 \cdot 10^8 m/s \tag{3.10}$$

which happens to be precisely the velocity of light, *c*. The implication is astounding: light is an electromagnetic wave. Of course, this conclusion does not surprise anyone today, but imagine what a revelation it was in Maxwell's time! Remember how $\varepsilon_0$ and $\mu_0$ came into the theory in the first place: they were constants in Coulomb's law and the Biot-Savart law, respectively. You measure them in experiments involving charged pith balls, batteries, and wires—experiments having nothing whatever to do with light. And yet, according to Maxwell's theory you can calculate *c* from these two numbers. Notice the crucial role played by Maxwell's contribution to Ampere's law; without it, the wave equation would not emerge, and there would be no electromagnetic theory of light.

**Transverse nature of EM waves:**

We will now consider the other field components that must accompany the solution we have found for the electric field. First, we note that with the assumptions we have made so far, Maxwell's equations can be rewritten in the following time-independent form:

$$\text{div}(\varepsilon \underline{E}) = 0 \quad (1)$$
$$\text{div}(\mu_0 \underline{H}) = 0 \quad (2)$$
$$\text{curl}(\underline{E}) = -j\omega\mu_0 \underline{H} \quad (3)$$
$$\text{curl}(\underline{H}) = j\omega\varepsilon \underline{E} \quad (4)$$

2.4-13

Now, our solution has so far contained only an x-component of the electric field. In this case:

$$\text{curl}(\underline{E}) = \underline{j}\ \partial E_x/\partial z - \underline{k}\ \partial E_x/\partial y \qquad 2.4\text{-}14$$

Given that $E_x = E_{x+} \exp(-jkz)$, we find:

$$\text{curl}(\underline{E}) = -jk\ E_{x+} \exp(-jkz)\ \underline{j} \qquad 2.4\text{-}15$$

Now, from Equation (3) in 2.4-13, we must have $\text{curl}(\underline{E}) = -j\omega\mu_0 \underline{H}$ . Hence, the magnetic field accompanying our solution only has a component in the y-direction. Writing this as:

$$\underline{H} = H_{y+} \exp(-jkz)\ \underline{j}$$

we can obtain the following constant relation between the electric and magnetic field amplitudes:

$$H_{y+}/E_{x+} = k/\omega\mu_0 \qquad 2.4\text{-}17$$

The solution therefore really consists of two travelling waves - an electric and a magnetic component. Both are in-phase, but the field directions are at right angles to each other. We can represent the complete solution at any given instant in time as in Figure 2.4-2, which shows the real parts of the two components together. Both exhibit similar cosinusoidal variations with distance.

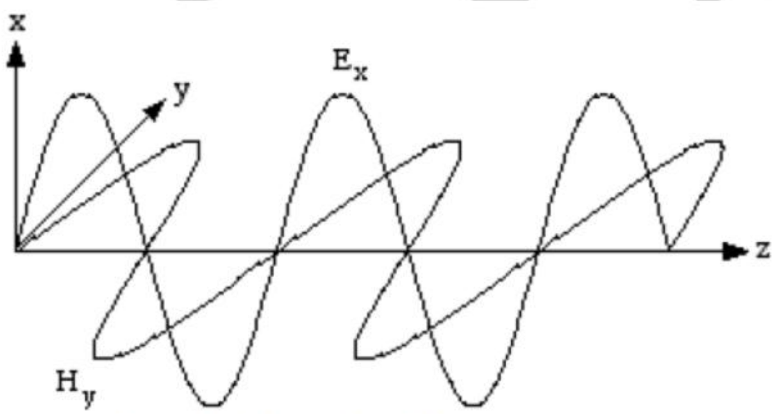



Figure 2.4-2 A plane electromagnetic wave.

Is this solution the only one possible? It would seem reasonable to repeat the analysis, starting with the assumption that the electric field only has a component in the y-direction. In this case, we find that if $\underline{E} = E_{y+} \exp(-jkz)\ \underline{j}$, then $\underline{H} = H_{x+} \exp(-jkz)\ \underline{i}$, so the magnetic field now only has a component in the x-direction. As before, we can find a relation between the two field amplitudes. This time, we get:

$$H_{x+}/E_{y+} = -k/\omega\mu_0 \qquad 2.4\text{-}18$$

Apart from the minus sign, the amplitude ratio is as before.

What happens if we assume instead that the electric field only has a component in the z-direction? Well, in an isotropic medium, div($\varepsilon\underline{E}$) = $\varepsilon$ div($\underline{E}$), so Equation (1) in 2.4-13 must reduce to div($\underline{E}$) = 0. Remember that we can expand this as:

$$\partial E_x/\partial x + \partial E_y/\partial y + \partial E_z/\partial z = 0 \qquad 2.4\text{-}19$$

However, since we have already assumed that $\partial\underline{E}/\partial x$ and $\partial\underline{E}/\partial y = \underline{0}$, it follows that $\partial E_x/\partial x = \partial E_y/\partial y = 0$. Hence, $\partial E_z/\partial z$ must be zero, so $E_z$ must be a constant independent of z. We therefore do not find travelling-wave solutions for $E_z$, and a similar argument can be used to show that there are no wave solutions for $H_z$. Plane electromagnetic waves are therefore strictly transverse. They are therefore often described as **TEM** (standing for Transverse ElectroMagnetic) waves.

**Polarization:**

We now consider some of the wider properties of the solutions found so far, beginning with the important feature of optical polarization. We start by noting that the two independent travelling wave solutions discussed above can be combined into a more general solution, in the form:

$$\underline{E} = E_{x+} \exp[j(\omega t - kz + \phi_x)]\, \underline{i} + E_{y+} \exp[j(\omega t - kz + \phi_y)]\, \underline{j}$$

where $\phi_x$ and $\phi_y$ are arbitrary (but constant) phase factors. The nature of the resulting wave then depends on the values of $E_{x+}$, $E_{y+}$, $\phi_x$ and $\phi_y$. Several combinations are particularly important.

(i) If $\phi_x = \phi_y$, the solution can be written as:

$$\begin{aligned}\underline{E} &= [E_{x+}\, \underline{i} + E_{y+}\, \underline{j}] \exp[j(\omega t - kz + \phi)] \\ &= \underline{E}_0 \exp[j(\omega t - kz + \phi)]\end{aligned}$$

In this solution, the direction of the electric field vector is independent of time and space, and is defined by a new vector $\underline{E}_0$, which is the vectorial sum of $E_{x+}\,\underline{i}$ and $E_{y+}\,\underline{j}$ as shown in Figure 2.4-3. This type of wave is known as a **linearly polarized** wave, and the direction of the electric field vector $\underline{E}_0$ represents the **direction of polarization**. Linearly polarized light is particularly important in engineering optics. It can be produced from natural light (which has random polarization) by passing it through a **polarizer**. More importantly, it is emitted directly by many types of laser.

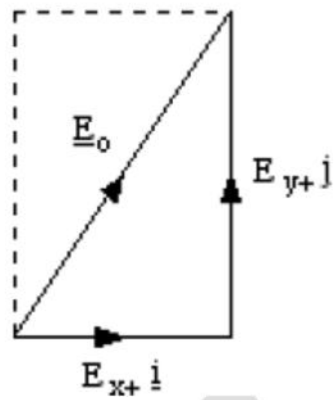



Fig: Construction of polarization vector

(ii) If $E_{x+} = E_{y+}$, and $\phi_y = \phi_x \pm \pi/2$, the solution can be written as:

$$\underline{\mathbf{E}} = E_0 \exp[j(\omega t - kz + \phi)]\, \underline{i} + E_0 \exp[j(\omega t - kz + \phi \pm \pi/2)]\, \underline{j}$$

Or, alternatively, as:

$$\underline{\mathbf{E}} = E_0\, (\underline{i} \pm j\, \underline{j}) \exp[j(\omega t - kz + \phi)]$$

Now, ultimately, we are interested in the real part of $\underline{\mathbf{E}}$. This is given by:

$$\mathrm{Re}\{\, \underline{\mathbf{E}}\, \} = E_0\, \{\cos(\omega t - kz + \phi)\, \underline{i} \pm \sin(\omega t - kz + \phi)\, \underline{j}\}$$

In this case, the amplitude of the electric field vector is still constant (and equal to $E_0$), but the direction of polarization is not. Instead, it rotates as a function of space and time. This solution is known as a **circularly polarized** wave, because the locus traced of the electric field vector as a function of time (at a given point) is a circle, as shown in Figure 2.4-4a. Right- and left-hand circular polarizations are both possible, depending on the sign of the $\pi/2$ phase-shift. If $E_{x+} \neq E_{y+}$, the locus becomes an ellipse, and the wave is described as being **elliptically polarized** (Figure 2.4-4b).

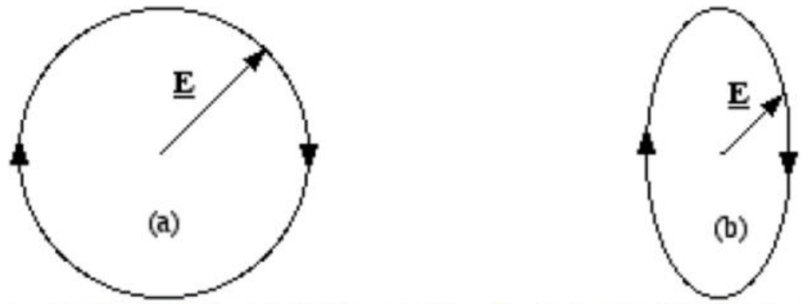



Figure 2.4-4 Loci of the electric field vector for a) circular and b) elliptic polarization.

**KARPAGAM UNIVERSITY**
**DEPARTMENT OF PHYSICS**
**I B.SC PHYSICS**
**ELECTRICITY AND MAGNETISM (18PHU201)**

| QUESTIONS | OPTION 1 | OPTION 2 |
|---|---|---|
| **UNIT-V** | | |
| All electromagnetic waves travel through a vacuum at | same | speeds |
| Electromagnetic waves are | longitudinal | transverse |
| The **E** and **B** fields in electromagnetic waves are oriented | parallel to the wa | parallel to the waves |
| An electromagnetic wave is radiated by a straight wire anten | vertically. | horizontally and in a |
| An electromagnetic wave is traveling to the east. At one ins | north | down |
| Which of the following correctly lists electromagnetic wave | gamma rays, ult | microwaves, ultravio |
| What is the wavelength of light waves if their frequency is 5 | 0.60 m | 6 m |
| How long does it take light to travel 1.0 m? | 3.3 ns | 3.3 micro s |
| What is the wavelength of a 92.9 MHz radio wave? | 32 mm | 32 cm |
| What frequency are 20 mm microwaves? | 100 MHz | 400 MHz |
| Electromagnetic waves travel | without medium | with medium |
| Which one of following have lowest frequency? | radio | infrared |
| Electromagnetic waves carry | positive charge | negative charge |
| Which one of following have highest wavelength? | radio waves | infrared |
| The coupled fields produce ___________ waves called elect | longitudinal | transverse |
| Ups and downs in longitudinal waves are termed as | compression and | crests and rarefactio |
| A pendulum bob is a good example of | vibration | oscillation |
| If we increase wavelength frequency would | increase | decrease |
| An electro magnetic wave consists of __________ | both electric and | an electric field only |
| Who propounded electro magnetic radiation theory? | Sir Edward App | James Clerk Maxwel |
| Electromagnetism is the | magnetic field ca | action between a perr |
| In electromagnetic waves, polarization __________. | is caused by refl | is due to the transvers |
| Electromagnetic Waves are refracted when they _________ | pass into a mediu | are polarized at right |
| In a vacuum, the speed of an electromagnetic wave | depends on its cc | depends on its wavel |
| Most of the effects an electro magnetic wave produces when | magnetic field | speed |
| When the electric field is perpendicular in the surface of the | elliptical | vertical |
| When the magnetic field is perpendicular to the surface of th | elliptical | vertical |
| When the magnetic field is parallel to the surface of the eartl | elliptical | vertical |
| What are the two interrelated fields considered to make up a | an electric field a | an electric field and v |
| A changing magnetic field gives rise to | sound field | magnetic field |
| At what speed do electromagnetic waves travel in free space | approximately 4( | approximately 18630 |
| Electric field that lies in a plane perpendicular to the earths | vertical polarizat | horizontal polarizatio |
| Electric field that lies in a plane parallel to the earths surfac | vertical polarizat | horizontal polarizatio |
| in electromagnetic waves, polarization means | the physical orie | the physical orientati |
| an electromagnetic waves travel in free space, only one of th | absorption | attenuation |
| In an electromagnetic wave the electric field is | Parallel to both n | Perpendicular to both |

| | | |
|---|---|---|
| Electromagnetic waves transport | Wavelength | charge |

**Prepared by Dr.A.Saranya,Assistant Professor**

| OPTION 3 | OPTION 4 | ANSWER |
| --- | --- | --- |
| | | sam |
| both longitudinal | any form | transverse |
| perpendicular to | perpendicular to th | perpendicular to the wave's direction of travel, and also to eacl |
| horizontally and | in a direction perpe | vertically. |
| east | south | south |
| radio waves, infr | radio waves, infrar | radio waves, infrared, gamma rays, ultraviolet |
| 0.06 mm | 0.6o micro m | 0.6o micro m |
| 3.3 ms | 3.3 s | 3.3 ns |
| 3.2 m | 32 m | 3.2 m |
| 15 GHz | 73 GHz | 15 GHz |
| with medium anc | in a disturbed path | with medium and without medium |
| ultraviolet | gamma rays | radio |
| no charge | both positive and r | no charge |
| ultraviolet | gamma rays | radio waves |
| travelling | sine | travelling |
| compressions an | crests and troughs | compression and rarefaction |
| ventillation | periodic motion | periodic motion |
| remain same | may increase or de | decrease |
| a magnetic field | non-magnetic field | both electric and magnetic fields |
| Christian Huyger | Sir Isaac Newton | James Clerk Maxwell |
| magnetic field a | current in the coil | magnetic field action with a current-carrying wire |
| results from the l | is always vertical | is due to the transverse nature of the waves |
| encounter a perfe | pass through a sm | pass through a small slot in a conducting plane |
| depends on its el | is a universal cons | is a universal constant |
| frequency | electric field | electric field |
| horizontal | circular | vertical |
| horizontal | circular | horizontal |
| horizontal | circular | elliptical |
| an electric field a | a voltage and curre | an electric field and a magnetic field |
| electric field | nothing in particul | electric field |
| approximately 3( | approximately 300 | approximately 300 million m/s |
| circular polarizat | elliptical polarizati | vertical polarization |
| circular polarizat | elliptical polarizati | horizontal polarization |
| ionization | the presence of po | the physical orientation of electric field in space |
| refraction | reflection | attenuation |
| Parallel to the ma | Perpendicular to th | Perpendicular to both the magnetic field and the wave directio |

frequency energy energy

ı other

n