SYLLABUS

SEMESTER VI
APPLIED OPTICS -16PHU601B

Objectives: Study of non linear optical properties of materials is very important as many of such materials are used in different instruments etc. This paper gives basic knowledge about different nonlinear optical properties and their theoretical aspects.

UNIT- I

Review of the concepts of polarizability and dielectric tensor of a medium. Frequency dependence of the dielectric tensor – wave vector dependence of the dielectric tensor – electromagnetic waves in an isotropic dielectrics.

UNIT- II

Introduction to non linear optics- Nonlinear dielectric response of matter – frequency variation of the nonlinear susceptibilities – properties of non linear susceptibilities- time domain descrption of optical non- linear susceptibilities- wave vector dependence of the nonlinear susceptibilities.

UNIT -III

Second harmonic generation – perturbation theory – phase matching evolution of SHW under phase matching conditions. Four wave mixing spectroscopy – optical phase conjugation – nonlinear materials.

UNIT- IV

Scattering of light – Raman scattering – Quantum theory of Raman scattering – Brillouin scattering. Interaction of atoms with nearly resonant fields – wave function under near resonant conditions. Bloch equations – self induced transparency.

UNIT- V

Fibre optics – normal modes of optical fibres – nonlinear Schrodinger equations – linear theory. Basic concepts of solutions and non-linear periodic structures. Effect of fibre loss – effect of waveguide property of a fibre – conditions of generation of a solutions in optical fibres.

TEXT BOOKS

1. D.L. Mills,(1998), Nonlinear Optics: Basic Concepts, 2nd Edition, Springer Science & Business Media
2. F.Zernike and J.E. Midwinter, Applied Nonlinear Optics , revised edition 2006, Dover books

REFERENCE BOOKS:

1. G.C. Baldwin (2011), An Introduction to Nonlinear Optics, Springer, US
2. AjoyGhatak & Tyagarajan 1st edition 2011,Introduction to Fibre Optics Tata McGraw Hills

KARPAGAM ACADEMY OF HIGHER EDUCATION

(Deemed to be University Established Under Section 3 of UGC Act 1956)

Coimbatore – 641 021.

LECTURE PLAN
DEPARTMENT OF PHYSICS

STAFF NAME: Dr. E. SIVASENTHIL
SUB.CODE: 16PHU601B
CLASS: III B.Sc., (PHYSICS)

SUBJECT NAME: APPLIED OPTICS
SEMESTER: VI

UNIT I

	Lecture Duration (hr.)	**Topics to be covered**	**Support materials**
1	1hr	Introduction	
2	1hr	Frequency dependence of the dielectric tensor	T1(11-27)
3	1hr	Wave vector dependence of the dielectric tensor	T1(27-30)
4	1hr	Electromagnetic waves in an isotropic dielectrics	T1(30-33)
5	1hr	Revision	
Total no. of hours planned for unit –I		5 hr	

UNIT II

S.No	**Lecture Duration (hr.)**	**Topics to be covered**	**Support Materials**
1	1hr	Nonlinear dielectric response of matter	T1(37-39)
2	1hr	Frequency variation of the nonlinear susceptibilities	T1(39-45)
3	1hr	Properties of nonlinear susceptibilities	T2(33-42)
4	1hr	Time domain description of optical non- linear susceptibilities-	T2(52-58)
5	1hr	Wave vector dependence of the nonlinear susceptibilities	T1(45- 47)
6	1hr	Revision	
Total no. of hours planned for unit –II		6 hr	

UNIT III

S.No	Lecture Duration (hr.)	Topics to be covered	Support Materials
1	1hr	Second harmonic generation	T2(96-104)
2	1hr	Perturbation theory, Phase matching evolution of SHW under phase matching conditions	T1(51-57), T1(59-63)
3	1hr	Four wave mixing spectroscopy	T1(63-69)
4	1hr	Optical phase conjugation; Nonlinear materials	T1(69-71)
5	1hr	Revision	
Total no. of hours planned for unit –III		5 hr	

UNIT IV

Si.No	Lecture Duration (hr.)	Topics to be covered	Support Materials
1	1hr	Raman Scattering; Quantum theory of Raman scattering	T1(77-82)
2	1hr	Brillouin scattering, Interaction of atoms with nearly resonant fields	T1(87-91), T1(94-101)
3	1hr	Wave function under near resonant conditions. Bloch equations	T1(101-107)
4	1hr	Self induced transparency	T1(107-111)
5	1hr	Revision	
Total no. of hours planned for unit –IV		5 hr	

UNIT V

Si.No	Lecture Duration (hr.)	Topics to be covered	Support Materials
1	1hr	Normal modes of optical fibres	T1(121-128)
2	1hr	Nonlinear Schrodinger equations	T1(128-132)
3	1hr	Linear theory	T1(132-135)
4	1hr	Basic concepts of solutions and non-linear periodic structures, Effect of fibre loss	T1(139-153), W1
5	1hr	Effect of waveguide property of a fibre	T1(154-155)
6	1hr	Conditions generation of a solutions in optical fibres	T1(156-160)
7	1hr	Revision and Old question paper discussion	
8	1hr	Old question Paper discussion	
8	1hr	Old question Paper discussion	
Total no. of hours planned for unit –v		9 hr	

Suggested Reading Books:

T1 : D.L. Mills,(1998), Nonlinear Optics: Basic Concepts, 2nd Edition, Springer Science & Business Media

T2 : Nonlinear Optics , Third edition by Robert W.Boyd

W1:http://www.physics.ttk.pte.hu/files/TAMOP/FJ_Nonlinear_Optics/3_the_nonlinear_polarization html

UNIT-I

SYLLABUS

Review of the concepts of polarizability and dielectric tensor of a medium. Frequency dependence of the dielectric tensor – wave vector dependence of the dielectric tensor – electromagnetic waves in an isotropic dielectrics.

WAVE VECTOR DEPENDENCE OF THE DIELECTRIC TENSOR:

The brief discussion of the wave vector dependence of the dielectric tensor, enters as a consequence of the nonlocal response of the system of interest, in coordinate space. In a microscopic theory of the wave vector and frequency dependent dielectric tensor of materials. The wave vector **k** enters the final expression in two distinctly different ways:

(i) In the energy denominator, the excitation energy ω_{no} is in fact a function of wave vector, $\omega_{no}(\mathbf{k})$. This is because, if the ground state wave function $|\Psi_o(0)\rangle$ is an eigenstate total momentum with eigenvalue zero, the eigenstate $|\Psi_n(k)\rangle$ which enters as the intermediate state has the total momentum $\hbar k$. One says the system has temporarily absorbed a photon with momentum $\hbar k$, and acquired its momentum in the process in this virtual transition. The energy denominator is thus $\omega_{no}(k)= [E_n(\mathbf{k}) - E_o(0)]/ \hbar$, and the resonance is shifted away from its zero wave vector value $[E_n(0) - E_o(0)] /\hbar$ by the recoil energy associated with the center of mass motion of the system.

(ii) In the numerator, of course $J_\alpha^p(\mathbf{k})$ depends on the wave vector **k**. If one lets $\mathbf{k} \rightarrow 0$ in the matrix elements which enter the numerator of, as we see from the discussion which leads, these matrix element become those associated with the electric dipole allowed transitions of the quantum theory of radiative transitions. There are selection rules that govern which of these matrix elements are nonzero. For instance, if the Hamiltonian is invariant under the parity operation, all eigenstates have even or odd parity. The ground state has even parity, so when the matrix elements are replaced by their limit as $\mathbf{k} \rightarrow 0$, as we have done to obtain , only odd parity states enter the sum on n. The even parity states are thus "silent," in this approximation; note that the wave vector dependence of the denominator remains, and can become important, as we shall see.

If the wave vector **k** is retained as nonzero in the matrix elements which remain in the numerator, the electric dipole selection rule breaks down, and it is possible for even parity intermediate states to contribute to the dielectric tensor, connected to the ground state wave function by the electric quadrupole operator, magnetic dipole operator, or a structure higher order in the multipole expansion of radiation theory. If the wavelength of the radiation is large compared to the size of the fundamental entity (atom, exciton, ...) in the system of interest, these matrix elements are small compared to their electric dipole counterparts. But if the frequency lies very dose to $\omega_{no}(\mathbf{k})$, a possibility that can be achieved readily in a variety of systems with tunable dye

lasers, then such dipole forbidden transitions can produce large contributions to $\varepsilon_{\alpha\beta}(\mathbf{k}, \omega)$. Spatial dispersion effects can thus be substantial, for frequencies very near resonance.
We may appreciate this point further by considering the behaviour of $\varepsilon_{\alpha\beta}(\mathbf{k}, \omega)$ near a dipole allowed transition, where the wave vector dependence of the denominator enters importantly. As a simple example, consider the contribution from electric dipole allowed transitions to $\varepsilon_{\alpha\beta}(\mathbf{k}, \omega)$ from an array of non-interacting atoms. If we define
$d_\alpha(no) = \langle\phi_n|\sum_{j=1}^{n} e_j r_\alpha^j|\phi_0\rangle$ the function $\chi_{\alpha\alpha}(\mathbf{k}, \omega)$ with n = N/V,

$$\chi_{\alpha\alpha}(\boldsymbol{k}, \omega) = \frac{2n}{\hbar} \sum_{n} \frac{\omega_{n0}(0)|d_\alpha(n0)|^2}{[\omega_{n0}^2(\boldsymbol{k}) - (\omega + i\eta)^2]} \quad \text{---------- (1)}$$

Suppose the frequency ω lies very close to one particular transition, so a single tenm on the right-hand side of equation (1) provides the dominant contribution to $\chi_{\alpha\alpha}(\mathbf{k}, \omega)$. To excellent approximation

$$\chi_{\alpha\alpha}(\boldsymbol{k}, \omega) = \frac{n}{\hbar} \frac{|d_\alpha(n0)|^2}{\omega_{n0}(k) - \omega - i\eta} \quad \text{---------- (2)}$$

Since the wave vector dependence of the excitation energy $\omega_{n0}(\mathbf{k})$arises from the recoil effect discussed above, we have

$$\omega_{n0}(\boldsymbol{k}) = \omega_{n0}(0) + \frac{\hbar k^2}{2M} \quad \text{---------- (3)}$$

with M the mass of the recoiling object. Then equation (2) may be rewritten to read

$$\chi_{\alpha\alpha}(\boldsymbol{k}, \omega) = \frac{2nM|d_\alpha(n0)|^2}{\hbar^2} \frac{1}{k^2 + \Gamma^2(\omega)} \quad \text{---------- (4)}$$

Where

$$\Gamma(\omega) = (2M/\hbar)^{1/2}[\omega_{n0}(0) - \omega - i\eta]^{1/2}$$

We shall always choose $\Gamma(\omega)$ so that $\mathrm{Re}\{\Gamma(\omega) > 0\}$ in the limit $\eta \to 0$.
We know that

$$\chi_{\alpha\alpha}(\boldsymbol{r} - \boldsymbol{r}', \omega) = \int \frac{d^3k}{(2\pi)^3} \chi_{\alpha\alpha}(\boldsymbol{k}, \omega) e^{i\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \quad \text{---------- (5)}$$

describes the spatial range of the response of the medium to a monochromatic

electric field of frequency ω. That is, if an electric field whose variation in space and time is given by

$$E_\alpha(r,t) = E_\alpha \delta(r - r_0)e^{-i\omega t} \quad \text{---------- (6)}$$

which is localized at the point r_0, then the α-th Cartesian of the dipole moment per unit volume induced by the field is

$$P_\alpha(r,t) = \chi_{\alpha\alpha}(r - r_0, \omega)E_\alpha e^{-i\omega t} \quad \text{---------- (7)}$$

With use of equation (4), a simple integration gives

$$\chi_{\alpha\alpha}(r - r';\omega) = \frac{nM|d_\alpha(n0)|^2}{2\pi\hbar^2}\frac{e^{-\Gamma(\omega)|r-r'|}}{|r-r'|} \quad \text{---------- (8)}$$

First consider frequencies ω which lie below $\omega_{n0}(0)$. We have

$$\lim_{\eta\to 0}\Gamma(\omega) = \left(\frac{2M}{\hbar}\right)^{1/2}[\omega_{n0}(0) - \omega]^{1/2}, \quad \omega < \omega_{n0}(0) \quad \text{---------- (9)}$$

Thus, $\chi_{\alpha\alpha}(r - r';\ \omega)$ has the Yukawa form, and falls off exponentially with increasing Ir - r'l. Notice, however, that as $\omega \to \omega_{n0}(0)$. from below, $\Gamma(\omega) \to 0$. Thus, for frequencies very close to resonance $\chi_{\alpha\alpha}(r - r';\ \omega)$ becomes very long ranged. It follows that close to resonance, one always has to be concerned about spatial dispersion effects, in that the spatial range of the response function becomes very long, and local dielectric theory breaks down.
Above the resonance, $\omega > \omega_{n0}(0)$. becomes pure imaginary,

$$\Gamma(\omega) = i\kappa(\omega), \quad \omega > \omega_{n0}(0) \quad \text{---------- (10)}$$

Where,

$$\kappa(\omega) = \left(\frac{2M}{\hbar}\right)^{1/2}[\omega - \omega_{n0}(0)]^{1/2} \quad \text{---------- (11)}$$

Thus,

$$\chi_{\alpha\alpha}(r - r', \omega) = \frac{nM|d_\alpha(n0)|^2}{2\pi\hbar^3}\frac{e^{-i\kappa(\omega)|r-r'|}}{|r-r'|} \quad \text{---------- (12)}$$

Just a bit above resonance, **k**(ω) is very small, and the wave length of the spatial oscillations in $\chi_{\alpha\alpha}(r - r';\ \omega)$ is very long. Spatial dispersion again asserts itself importantly. As ω increases, **k**(ω) becomes large, the spatial oscillations in $\chi_{\alpha\alpha}(r - r';\ \omega)$ are very rapid, and detailed

analysis shows that once again only the near vicinity of the point **r'** matters significantly. One recovers the local theory once again, though how this happens is not clear from what can be presented here.

The above discussion, based on the near resonant response of a noninteracting array of atoms or molecules is rather academic. The mass M is so very large that one has to be very close to resonance indeed for the spatial range of $\chi_{\alpha\alpha}(r - r';\ \omega)$ to be appreciable; on physical grounds one expects spatial dispersion effects to become important when the spatial range of $\chi_{\alpha\alpha}(r - r';\ \omega)$ becomes a non-negligible fraction of the wavelength of light in the medium.

However, discussions of the nature of the optical response of solids shows that for frequencies very close to the exciton absorption lines, a form identical to equation (2) and equation (4) describes the wave vector and frequency variation of the dielectric susceptibility tensor. The mass M which enters equation (3) is now the total mass of the bound electron-hole pair, which is the order of the free electron mass, rather than that of an atom or molecule. The latter is larger by a factor in the range of $10^4 – 10^5$. If the frequency ω lies within a few meV of the exciton resonance $\Gamma(\omega)$, lies in the range of 10^6 cm^{-1}. . The index of refraction of a typical semiconductor lies in the range of 3.5 or 4, so the wavelength of visible radiation in the medium is in the range of 10^{-5} cm. Thus $\Gamma(\omega)$ is an appreciable fraction of the wavelength. Theoretical and experimental studies show that the optical response of such materials is influenced in a rich and dramatic manner, for frequencies in the near vicinity of the exciton resonances.

While we shall not explore spatial dispersion effects further in the present text, the above discussion serves to acquaint us with the limitations of local dielectric theory, in which it is presumed the dipole moment per unit volume **P**(r, t) is proportional to the electric field **E**(r, t) at the same spatial point

ELECTROMAGNETIC WAVES IN AN ISOTROPIC DIELECTRICS:

With respect to the Maxwell's equation

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{H} = J + \frac{\partial D}{\partial t}$$
$$\nabla \times \mathbf{E} = \frac{-\partial B}{\partial t}$$

For isotropic medium,

$$J = \sigma E$$
$$B = \mu H$$
$$D = \varepsilon E$$

Here,

$$\sigma = 0$$
$$\rho = 0$$

Then the Maxwell's equation reduces to,

$$\nabla . E = 0$$

$$\nabla . H = 0$$
$$\nabla \times \mathrm{H} = \varepsilon \frac{\partial \mathrm{E}}{\partial \mathrm{t}}$$
$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

Taking curl for third and fourth equation, for third equation,

$$\nabla^2 H - \frac{1}{v^2}\frac{\partial^2 H}{\partial t^2} = 0$$

For fourth equation,

$$\nabla^2 E - \frac{1}{v^2}\frac{\partial^2 E}{\partial t^2} = 0$$

These two waves satisfies the wave equation,

$$\nabla^2 \Psi - \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} = 0$$

The solution for the wave equation is,

$$\Psi = \Psi_0 e^{-i(\omega t - k.r)}$$

The solution of equations are will be of the given in the form,

$$E = E_0 e^{-i(\omega t - kr)}$$
$$H = H_0 e^{-i(wt - kr)}$$

Where k is the wave vector

$$k = k_n = \frac{2\pi}{\lambda} n = \frac{2\pi f_n}{c} = \frac{\omega_n}{c}$$

With n as the unit vector in the direction of wave propagation
The equation can be written as

$$k . E = 0$$
$$k . H = 0$$
$$-k \times H = \omega \varepsilon_0 E$$
$$k \times E = \mu_0 H$$

Propagation of electromagnetic waves in dielectric:-

1. The waves E and H are orthogonal.
2. The electromagnetic wave is transverse in nature.
3. The electric and magnetic vectors are also mutually orthogonal.

POSSIBLE QUESTIONS
PART B

1. Define polarization.
2. Write a short note about electromagnetic waves?
3. Mention the three types of polarization.
4. Write any four characteristics of EM waves.
5. What is meant by wave vector (k)?

PART C

1. Briefly explain about frequency dependence of the dielectric tensor.
2. Discuss in detail about wave vector dependence of the dielectric tensor.
3. With the help of Maxwell's equations derive the expression for electromagnetic waves in an isotropic dielectrics.

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Question	**Choice 1**
UNIT 1	
The waves E and H are	parallel
The insulators whose behaviour gets modified in an electric field are c	semiconductor
Dielectric constant of any medium is always _____ than permitti	greater
Δ.D=	ρ
Δ.B=	W
ΔxH=	div J + ∂r/ ∂t =0
ΔxE=	ρ
In case of propagation of EMW in isotropic dielectric medium, the wa	longitudinal
The electric and magnetic vectors are also mutually_____.	parallel
In isotropic dielectric medium, k=kn, where k is	unit vector
Polarizability is a _____ of dipole moment to electric field.	product
Electric dipole moment per unit volume is called as	diffraction
Polarization vector can be defined as P=	N x p
Calculate the polarization vector of the material which has 100 dipoles	100
Which type of polarization depends on temperature?	Electronic
Calculate the polarization vector in air when the susceptibility is 5 and	3
The total polarization of a material is the ____	Product of all types of po
What is the process of producing electric dipoles inside the dielectric b	Polarization
When does the dielectric become a conductor?	In the presence of electric
In isotropic dielectric medium D =	εE
Maxwell first equation is	Δ.D=ρ
Maxwell second equation is	Δ.D=ρ
Maxwell third equation is	Δ.D=ρ
Maxwell fourth equation is	Δ.D=ρ
_____ is an insulating material in which all the electrons are tightly b	Dielectric
Example of dielectric is	iron
Example of non - polar molecules	H2
Example of polar molecules	H2O
p = α E, Where α is known as	molecular polarisability
The magnitude of the induced dipole moment p is directly proportiona	magnetic field
The alignment of the dipole moments of the permanent or induced dip	Polarization
The speed of electromagnetic wave in isotrophic dielectrics is _____	greater
Calculate the polarization vector of the material which has 100 dipoles	100
Calculate the polarization vector in air when the susceptibility is 5 and	50
Dielectric constant of any medium is always greater than _____.	strong skin effect
Electromagnetic waves propagates in free space with the velocity of _	light
The velocity of electromagnetic waves in free space is _____	30 X 10^8 m/s
The flow of energy in a electromagnetic wave in free space is in the di	electric field

The electromagnetic field vectors E and H are in _____.	out of phase
In an anisotrophic medium, the energy is _____ in the v	not propagated
The ratio of dipole moment to electric field is known as	field density
The insulators whose behaviour gets modified in an _____ are called	magnetic field
Electric dipole moment / unit volume =	diffraction
Orientational type polarisation depends on	external field
Example of non - polar molecules	O2
The wave vectors E and H are mutually _____	perpendicular
Identify a good dielectric.	Iron
A dielectric can be made a conductor by	Compression
Find the dielectric constant for a material with electric susceptibility o	3
What is the process of producing electric dipoles inside the dielectric b	Polarisation
How does ionic polarisation occur?	Splitting of ions
The resonance frequency wo=	(Ks/m)ˆ1/2
The creteria frequency for relaxation mechanis are in the range_____	KHz
_____describes the decay of excited states to ground state.	Relaxation
Electronic polarisation still works at optical ____ frequencies.	10ˆ3 Hz
For a dielectric which of the following properties hold good?	They are superconductors
Passive dielectrics restricts the flow of _____	electrical energy
Which of the following is the slowest polarisation method?	Ionic polarisation
When mobility increases, insulation resistance decreases and dielectric	positive
Calculate the polarization vector of the material which has 100 dipoles	300

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OF HIGHER EDUCATION, COIMBATORE – 21			
ARTMENT OF PHYSICS			
ASS: III B. Sc., PHYSI	**BATCH: 2016-2019**		
Sixth Semester			
IED OPTICS (16PHU601B)			
IPLE CHOICE QUESTIONS			
Choice 2	**Choice 3**	**Choice 4**	
orthogonal	rotational	irrotational	
superconductor	p-type semiconductor	dielectrics	
lesser	neither greater not lesse	neither lesser nor greater	
0	1	μ	
0	a	μ	
J + ∂D/ ∂t	J - ∂D/ ∂t	0	
π	J - ∂D/ ∂t	- ∂B/ ∂t	
parallel	transverse	none of the above	
orthogonal	rotational	irrotational	
tensor	gradient	scalar	
sum	difference	ratio	
interference	polarization	none of the above	
N-p	N+p	N/p	
50	200	0.02	
Orientational	Interfacial	Ionic	
2	2.4	60	
Sum of all types of pol	Total dipole moments i	Orientation directions of the dip	
Susceptibility	Magnetisation	Superconductivity	
At high temperature	At dielectric breakdow	In the presence of magnetic fiel	
sg	βe	Re	
Δ.B=0	ΔxH=J + ∂D/ ∂t	ΔxE=- ∂B/ ∂t	
Δ.B=0	ΔxH=J + ∂D/ ∂t	ΔxE=- ∂B/ ∂t	
Δ.B=0	ΔxH=J + ∂D/ ∂t	ΔxE=- ∂B/ ∂t	
Δ.B=0	ΔxH=J + ∂D/ ∂t	ΔxE=- ∂B/ ∂t	
Semiconductor	Conductor	none of the above	
mica	aluminium	mercury	
HNO3	H2So4	K2Cr2O7	
HCl	K2Cr2O7	H2So4	
polarisability vector	unit polarisation	propagation constant	
electric field	both a and b	none of the above	
dipole moment	non polar	polar	
lesser	absolute	none of the above	
300	700	50	
2.4	5.7	8.2	
charge density	permittivity	susceptibility	
sound	electron	proton	
356 X 10^8 m/s	330 m/s	3 X 10^8 m/s	
magnetic field	electrons	wave propagation	

phase	proportional	none of the above
propagated	orthogonal	parallel
dipole moment	Polarizability	charge density
electric field	both a and b	none of the above
interference	polarization	dipole moment
acceletator	modulator	temperature
HNO3	H2So4	K2Cr2O7
parallel	equal	greater
Magnesium	Copper	Ceramics
Heating	Doping	Freezing
10	5	32
Dipole moment	Susceptibility	Magnetisation
Passing magnetic field	Displacement of cation	Never occurs
(Ks/m)	2Ks/m	K/m
MHz	GHz	none of the above
Frequency	Time period	Oscillation
10^5 Hz	10^8 Hz	10^15 Hz
They are superconducto	They can never become	a superconductor
mechanical energy	wind energy	solar energy
Orientation polarisatior	Electronic polarisation	Space charge polarisation
negative	conducting	none of the above
400	200	500

Answer

orthogonal
dielectrics
greater
ρ
0
J + ∂D/ ∂t
- ∂B/ ∂t
transverse
orthogonal
unit vector
ratio
polarization
N x p
200
Orientational
60
Sum of all types of polarization
Polarization
At dielectric breakdown
εE
Δ.D=ρ
Δ.B=0
ΔxH=J + ∂D/ ∂t
ΔxE=- ∂B/ ∂t
Dielectric
mica
H2
H2O
molecular polarisability
electric field
Polarization
lesser
300
50
permittivity
light
3 X 10^8 m/s
wave propagation

phase
not propagated
Polarizability
electric field
polarization
temperature
O2
perpendicular
Ceramics
Heating
5
Polarisation
Displacement of cations and anions
(Ks/m)^1/2
GHz
Relaxation
10^15 Hz
They are superconductors at low temperatures
electrical energy
Space charge polarisation
conducting
500

UNIT-II

SYLLABUS

Introduction to nonlinear optics- Nonlinear dielectric response of matter – frequency variation of the nonlinear susceptibilities – properties of nonlinear susceptibilities- time domain description of optical non- linear susceptibilities- wave vector dependence of the nonlinear susceptibilities.

NONLINEAR DIELECTRIC RESPONSE OF MATTER:

In any real medium, the dipole moment per unit volume $P_\alpha(r,t)$ is not simply controlled by the instantaneous value of the electric field at the point r, but in fact we must realize that there is a time lag in the response of the medium. If we consider the extension that incorporates these features, then for a homogeneous medium, we may write

$$P_\alpha^{(NL)}(\boldsymbol{r},t) = \sum_{\beta\gamma}\int d^3r_1\,dt_1\,d^3r_2\,dt_2\,\chi^{(2)}_{\alpha\beta\gamma}(\boldsymbol{r}-\boldsymbol{r}_1,t-t_1;\boldsymbol{r}-\boldsymbol{r}_2,t-t_2)$$
$$\times E_\beta(\boldsymbol{r}_1,t_1)E_\gamma(\boldsymbol{r}_2,t_2) + \sum_{\beta\gamma\delta}\int d^3r_1\,dt_1\,d^3r_2\,dt_2\,d^3r_3\,dt_3$$
$$\times \chi^{(3)}_{\alpha\beta\gamma\delta}(\boldsymbol{r}-\boldsymbol{r}_1,t-t_1;\boldsymbol{r}-\boldsymbol{r}_2,t-t_2;\boldsymbol{r}-\boldsymbol{r}_3,t-t_3)$$
$$\times E_\beta(\boldsymbol{r}_1,t_1)E_\gamma(\boldsymbol{r}_2,t_2)E_\delta(\boldsymbol{r}_3,t_3) + \cdots . \qquad \text{---------- (1)}$$

We can proceed by introducing g a Fourier decomposition of the electric field. Consider for simplicity, just the second order contribution the nonlinear electric dipole moment proportional to $\chi^{(2)}_{\alpha\beta\gamma}$. The generalization to the case of the higher order terms will be clear after we examine the structure of this term. We introduce the wave vector and frequency dependent second order susceptibility $\chi^{(2)}_{\alpha\beta\gamma}(k_1\,\omega_1;\,k_2\omega_2)$ defined by

$$\chi^{(2)}_{\alpha\beta\gamma}(\boldsymbol{k}_1\omega_1;\boldsymbol{k}_2\omega_2) = \int \frac{d^3k_1d\omega_1d^3k_2d\omega_2}{(2\pi)^8}\,e^{-i\boldsymbol{k}_1\cdot(\boldsymbol{r}-\boldsymbol{r}_1)}e^{+i\omega_1(t-t_1)}e^{-i\boldsymbol{k}_2\cdot(\boldsymbol{r}-\boldsymbol{r}_2)}e^{+i\omega_2(t-t_2)}$$
$$\times\chi^{(2)}_{\alpha\beta\gamma}(\boldsymbol{r}-\boldsymbol{r}_1,t-t_1;\boldsymbol{r}-\boldsymbol{r}_2,t-t_2)\ . \qquad \text{---------- (2)}$$

One then finds that

$$P_\alpha^{(NL)}(\boldsymbol{r},t) = \int \frac{d^3k_1d\omega_1d^3k_2d\omega_2}{(2\pi)^3}\,e^{i(\boldsymbol{k}_1+\boldsymbol{k}_2)\cdot\boldsymbol{r}}e^{-i(\omega_1+\omega_2)t}$$
$$\times\sum_{\beta\gamma}\chi^{(2)}_{\alpha\beta\gamma}(\boldsymbol{k}_1\omega_1;\boldsymbol{k}_2\omega_2)E_\beta(\boldsymbol{k}_1\omega_1)E_\gamma(\boldsymbol{k}_2\omega_2) + \cdots . \qquad \text{---------- (3)}$$

The physical interpretation of the right-hand side of equation (3) is as follows: If we isolate the contribution from a particular wave vector and frequency (k_1 ω_1), combined with that form (k2ω2), we see that the two plane waves exp[i(k_1. r_1 - ω_1 t_1] and exp[i(k_2. r_2 - ω_2 t_2] "mix'' together in the nonlinear material, to produce an output wave at the frequency $\omega_1 + \omega_2$, and whose wave vector k is the sum of $k_1 + k_2$ of the wave vector of the two input waves. If we were to consider the third order terms generated by $\chi^{(3)}_{\alpha\beta\gamma\delta}$, then three waves with wave vector and frequency combinations (k_1 ω_1), ($k_2\omega_2$) and ($k_3\omega_3$) will generate output waves with wave vectors $k_1 + k_2 + k_3$ at frequency $\omega_1 + \omega_2 + \omega_3$.

It is evident from construction that $\chi^{(2)}_{\alpha\beta\gamma}(k_1\ \omega_1; k_2\omega_2)$ must be left invariant under simultaneous interchange of (β k_1 ω1) and (γ $k_2\omega_2$). A similar remark applies to $\chi^{(3)}_{\alpha\beta\gamma\delta}(k_1\ \omega_1; k_2\omega_2; k_3\omega_3)$, not displayed here in the interest of brevity.

If we consider just simple plane waves which propagate in a medium, then we must look a bit more closely to see the nature of the output. The point is that a real plane wave is not described by a single complex exponential exp[i(k. r - ωt], but by a superposition of this form with exp[-i(k. r - ωt]. For example, if we have the plane waves whose electric field has the form

$$E_\beta^{(a)}(\boldsymbol{r}, t) = E_\beta^{(a)} \cos(\boldsymbol{k}_a \cdot \boldsymbol{r} - \omega_a t + \phi_a)$$

---------- (4)

This can be written

$$E_\beta^{(a)}(\boldsymbol{r}, t) = \frac{1}{2} E_\beta^{(a)} e^{i\phi_a} e^{i(\boldsymbol{k}_a \cdot \boldsymbol{r} - \omega_a t)} + \frac{1}{2} E_\beta^{(a)} e^{-i\phi_a} e^{-i(\boldsymbol{k}_a \cdot \boldsymbol{r} - \omega_a t)}$$

---------- (5)

If we represent this wave may be in this form,

$$E_\beta(\boldsymbol{k}, \omega) = \frac{(2\pi)^4}{2} E_\beta^{(a)} e^{i\phi_a} \delta(\boldsymbol{k} - \boldsymbol{k}_a)\delta(\omega - \omega_a) + \frac{(2\pi)^4}{2} E_\beta^{(a)} e^{-i\phi_a} \delta(\boldsymbol{k} + \boldsymbol{k}_a)\delta(\omega + \omega_a)$$

---------- (6)

Suppose we have two plane waves which propagate in the medium, so the total 'input' electric field is given by

$$E_\beta(\boldsymbol{r}, t) = E_\beta^{(a)} \cos(\boldsymbol{k}_a \cdot \boldsymbol{r} - \omega_a t + \phi_a) + E_\beta^{(b)} \cos(\boldsymbol{k}_b \cdot \boldsymbol{r} - \omega_b t + \phi_b)$$

---------- (7)9

Then if we consider the contributions to the dipole moment per unit volume from the second order susceptibility $\chi^{(2)}_{\alpha\beta\gamma}$, there will be three classes of terms that we may describe as follows:

(i) Self interaction terms associated with wave (a):

These will be proportional to $E_\beta^{(a)} E_\gamma^{(a)}$ and will have Fourier components proportional to $k_a + k_a = 2\ k_a$ and $\omega_a + \omega_a = 2\ \omega_a$; this wave is called the second harmonic. Also, we have electric fields with Fourier components $k_a + (-k_a) = 0$ and $\omega_a + (-\omega_a) = 0$. Thus the self interactions within the wave also produce spatially uniform, dc electric fields.

(ii) Self interaction terms associated with wave (b):

The nature of these terms in identical to those associated wave (a). We thus get the second harmonic of wave (b) along with optical rectification from this wave.

(iii) Terms which describe the interaction between wave (a) and wave (b):

These terms are proportional to $E_\beta^{(a)}E_\gamma^{(b)}$ and $E_\beta^{(b)}E_\gamma^{(a)}$. We have contribution to the nonlinear dipole moment at the sum and differences $\pm$ ($k_a \pm k_b$) and $\pm$($\omega_a \pm \omega_b$). We thus find sum frequency generation and difference frequency generation, through the second order terms, in the presence of two input waves.

FREQUENCY VARIATION OF NONLINEAR SUSCEPTIBILITIES:

We know that $\chi^2_{\alpha\beta\gamma}$ depends on the frequencies ω_1 and ω_2 of the two interacting waves. Consider, for example, just the variation with ω_1,while the second frequency ω_2 is held fixed. For fixed ω_2 ,$\chi^{(2)}_{\alpha\beta\gamma}(k_1\ \omega_1; k_2\omega_2)$ considered a function of ω_1 obeys a Kramers-Kronig relation similar to that followed by the frequency dependent dielectric constant. That this is so follows simply from causality, which requires the real time representation of $\chi^{(2)}_{\alpha\beta\gamma}$ the functions $\chi^{(2)}_{\alpha\beta\gamma}$ ($r - r_1\ ;\ t - t_1\ ;\ r - r_2\ ;\ t - t_2$)$_{to}$ vanish when $t_1 > t$. Then, from the definitions of $\chi^{(2)}_{\alpha\beta\gamma}(k_1\ \omega_1; k_2\omega_2)$ one sees this function is an analytic function of ω_1, in the upper half ω_1 plane. On physical grounds, this object must vanish as $\omega_1 \to 0$. One may then write down the relevant Kramers-Kronig relation at once, following the discussion given by Landau and Lifshitz.

In the case of the dielectric we saw that the Kramers-Kronig relation proved most valuable indeed, in our effort to understand general aspects of the frequency variation of the complex dielectric constant $\varepsilon(\omega)$. We know that $\varepsilon_2(\omega)$ is directly related to the frequency dependent absorption constant of a material; from our understanding of the frequency dependence of the absorption constant, we can thus draw inferences about the behavior of $\varepsilon(\omega)$. To the experimentalist, the Kramers-Kronig relation for $\varepsilon(\omega)$ is particularly valuable. If the absorption rate, from which $\varepsilon_2(\omega)$ can be inferred, is measured over a wide spectral range, then $\varepsilon_1(\omega)$ can be calculated directly. It is possible also to measure both $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$ separately, in certain kinds of reflectivity studies. The Kramers-Kronig relations then serve as a consistency check on the results, if data is available over a sufficiently wide spectral range.

The Kramers-Kronig relations for the nonlinear susceptibilities are far less useful, and are encountered infrequently. The reason is that even at one pair of frequencies ω_1 and ω_2 the accurate measurement of a nonlinear susceptibility element like $\chi^{(2)}_{\alpha\beta\gamma}(k_1\ \omega_1; k_2\omega_2)$ proves rather difficult. If and when this can be done, say for fixed ω_2 , the measurement will cover a limited range of ω_1

The Kramers-Kronig integrals are then of little practical assistance in generating useful information. As we shall appreciate shortly, the dependence of the various nonlinear susceptibility elements on ω_1 and ω_2 is rather more complex than realized in the linear dielectric

response, where only a single frequency is involved, and one encounters singularities at the various excitation frequencies of the material.
simple model used frequently in the literature serves as a guide to the frequency variation of the nonlinear susceptibilities of a material.
Consider first a simple charged particle, with charge e, bound by a perfectly harmonic potential to an infinitely massive center. It is driven by an electric field E(t); we consider one dimensional motion only, and the electric field is applied parallel to the direction of the motion. We write the potential energy
$V_2(x)$ in the form

$$V_2(x) = \frac{1}{2} M\omega_0^2 x^2 \quad \text{---------- (1)}$$

where M is the mass of the particle, ω_0 its resonance frequency, and x the displacement from equilibrium. The equation of motion is of course

$$\ddot{x} + \Gamma\dot{x} + \omega_0^2 x = \frac{e}{M} E(t) \quad \text{---------- (2)}$$

with Γ a damping rate. For a field of frequency ω, E(t) = E(ω)exp(-i ω t), then
x(t) = x(ω) exp(-i ω t), where

$$x(\omega) = \frac{eE(\omega)}{M} \frac{1}{\omega_0^2 - i\omega\Gamma - \omega^2} \quad \text{---------- (3)}$$

The electric dipole moment is given by p(ω) = ex(ω). We write p(ω) = α(ω)E(ω),
where α(ω) is the polarizability, given by

$$\alpha(\omega) = \frac{e^2}{M} \frac{1}{\omega_0^2 - i\omega\Gamma - \omega^2} \quad \text{---------- (4)}$$

If there are n such charges per unit volume, and we can ignore interactions between them, then the frequency dependent dielectric constant is $\varepsilon(\omega) = 1 + 4\pi n\alpha(\omega)$. We arrive once again at a familiar frequency dependence.
The above simple model can be applied quite directly to a dilute array of linear diatomic molecules. Then M is the reduced mass of each molecule, ω_0 its vibrational frequency, and e the effective charge that describes its coupling to the external electric field.
If we now drive the above molecule with two superimposed electric fields, one with frequency ω_1 and one with frequency ω_2, we have for the total field felt by the molecule

$$E(t) = E(\omega_1)e^{-i\omega_1 t} + E(\omega_2)e^{-i\omega_2 t} + \text{complex conjugate} \quad \text{---------- (5)}$$

The fact that equation (2) is a linear differential equation means that the resulting displacement is just a simple superposition of that at frequency ω_1 and at frequency ω_2:

$$x(t) = x(\omega_1)e^{-i\omega_1 t} + x(\omega_2)e^{-i\omega_2 t} + \text{complex conjugate} \quad \text{---------- (6)}$$

We have no second harmonics, or interactions between the two input waves, for this very simple case.

For any actual physical system such as a linear diatomic molecule, the motion is not harmonic, but there are anharmonic terms in the total potential energy which supplement the principal terms $V_2(x) = M\frac{\omega_0^2 x^2}{2}$. For small amplitude displacements, we may expand the full potential V(x) in a power series, to obtain in addition to $V_2(x)$ the anharmonic terms that will be written

$$V_3(x) + V_4(x) + \cdots = \frac{1}{3}Max^3 + \frac{1}{4}Mbx^4 + \cdots \quad \text{---------- (7)}$$

Retaining the first two terms gives

$$\ddot{x} + \Gamma\dot{x} + \omega_0^2 x + ax^2 + bx^3 = \frac{e}{M}E(t) \quad \text{---------- (8)}$$

We now obtain a model description of the nonlinear interactions discussed earlier from a general perspective. Suppose we now drive the system with an electric field (necessarily real) that is a superposition of frequency ω_1 and ω_2.

$$E(t) = \frac{1}{2}(E_1 e^{-i\omega_1 t} + E_1^* e^{+i\omega_1 t}) + \frac{1}{2}(E_2 e^{-i\omega_2 t} + E_2^* e^{+i\omega_2 t}) \quad \text{---------- (9)}$$

We can analyse the response of the oscillator by expanding the displacement in powers of the electric field E(t). Thus, we shall write

$$x(t) = x^{(1)}(t) + x^{(2)}(t) + x^{(3)}(t) + \cdots \quad \text{---------- (10)}$$

where $x^{(i)}(t)$is proportional to the i-th power of the field E(t). If one inserts equation (10) into equation (8), and equates like power of E(t), one finds the hierarchy

$$\ddot{x}^{(1)} + \Gamma\dot{x}^{(1)} + \omega_0^2 x^{(1)} = \frac{e}{M}E(t) \quad \text{---------- (11a)}$$

$$\ddot{x}^{(2)} + \Gamma\dot{x}^{(2)} + \omega_0^2 x^{(2)} + a(x^{(1)})^2 = 0 \quad \text{---------- (11b)}$$

$$\ddot{x}^{(3)} + \Gamma\dot{x}^{(3)} + \omega_0^2 x^{(3)} + 2ax^{(1)}x^{(2)} + b(x^{(1)})^3 = 0 \quad \text{---------- (11c)}$$

where we ignore higher order terms.

The solution of equation (11a) can be written at once. We define

$$d(\omega) = \frac{1}{\omega_0^2 - i\omega\Gamma - \omega^2} \quad \text{---------- (12)}$$

and very much as above we have

$$x^{(1)}(t) = \frac{e}{2M}[E_1 d(\omega_1)e^{-i\omega_1 t} + E_1^* d^*(\omega_1)e^{+i\omega_1 t} + E_2 d(\omega_2)e^{-i\omega_2 t} + E_2^* d^*(\omega_2)e^{i\omega_2 t}]$$

---------- (13)

One proceeds by moving the term $a(x^{(1)})^2$ to the right-hand side of equation (11b), then treating it as an inhomogeneous driving term. The contributions to $x^{(2)}(t)$ are then the following:

(i) The second hannonic of the wave at frequency ω_1:

$$x^{(2)}(t) = -\frac{ae^2}{4M^2} E_1^2 d(2\omega_1) d^2(\omega_1) e^{-i2\omega_1 t} + \text{complex conjugate}$$

---------- (14a)

(ii) The second hannonic of the wave at frequency ω_2:

$$x^{(2)}(t) = -\frac{ae^2 E_2^2}{4M^2} d(2\omega_2) d^2(\omega_2) e^{-i2\omega_2 t} + \text{complex conjugate}$$

---------- (14b)

(iii) The time independent (dc) component of the displacement:

$$x^{(2)}(t) = -\frac{ae^2}{2M^2} (|d(\omega_1)|^2 |E_1|^2 + |d(\omega_2)|^2 |E_2|^2)$$

---------- (14c)

(iv) Generation of the sum frequency $\omega_1 + \omega_2$:

$$x^{(2)}(t) = -\frac{ae^2}{2M^2} d(\omega_1 + \omega_2) d(\omega_1) d(\omega_2) E_1 E_2 e^{-i(\omega_1 + \omega_2)t} + \text{complex conjugate ;}$$

---------- (14d)

(v) Generation of the difference frequency $\omega_1 - \omega_2$:

$$x^{(2)}(t) = -\frac{ae^2}{2M^2} d(\omega_1 - \omega_2) d(\omega_1) d^*(\omega_2) E_2 E_2^* e^{-i(\omega_1 - \omega_2)t} + \text{complex conjugate .}$$

---------- (14e)

The dipole moment p(t) of our model oscillator is just ex(t). Hence the various contributions to the second order nonlinear polarizability are obtained by multiplying the equations of equation (14) by e.

The third order terms are treated in a manner very similar to the second order terms. In equation(11c) one moves the terms $b(X^{(l)})^3 + 2ax^{(l)}X^{(2)}$ to the righth and side, and treats them as inhomogeneous driving terms, utilizing equation(13) and equation(14).The output $X^{(3)}(t)$ contains many terms, as one can appreciate from the expressions above. We shall quote only one contribution to the output that will prove useful later on, that at frequency $2\omega_1 - \omega_2$. At this frequency, we

$$x^{(3)}(t) = -\frac{e^2}{4M^3} d(2\omega_1 - \omega_2) d^2(\omega_1) d^*(\omega_2) \times \left[\frac{3}{2} b - 2a^2 d(\omega_1 - \omega_2) - a^2 d(2\omega_1)\right] E_1^2 E_2^* e^{-i(2\omega_1 - \omega_2)t} + \text{complex conjugate .}$$

---------- (15)

The various expressions given above have a common feature, which can be illustrated by examining (14d). We have the interaction between two input waves, one with frequency ω_1 and one with frequency ω_2 . Note that there is a factor d(ω), associated with each input wave which exhibits a resonance when the frequency ω is "tuned" to the frequency Wo of the oscillator. Thus, we obtain resonant enhancement of the output if either input wave lies close in frequency to ω_0, not surprisingly. The output is at the frequency $\omega_1 - \omega_2$, for the term displayed in equation (14e). An array of oscillators, each with a dipole moment with time dependence exp[-i($\omega_1 - \omega_2$)t], will generate an electromagnetic wave at frequency $\omega_1 - \omega_2$ as one may appreciate by inserting the nonlinear dipole moment density into the Maxwell equations; Notice that there appears a factor d($\omega_1 - \omega_2$) in equation (14e). Thus, we also get resonant enhancement if the frequency $\omega_1 - \omega_2$ lies close to the resonance frequency of the oscillator. Similarly, for the case of sum frequency generation, equation (14d) shows a resonance when ($\omega_1 - \omega_2$) lies near ω_0.

We have here an example of a three wave interaction: the wave of frequency ω_1 "mixes" with that of frequency ω_2 to, among other things, produce a third output wave at ($\omega_1 - \omega_2$). If anyone of the waves, including the output wave, is in resonance with the physical system (the oscillator in this case) that provides the nonlinearity responsible for their interaction, we obtain resonance enhancement. This is a general principle of nonlinear wave mixing phenomena. For the nonlinear susceptibilities we encounter in our study of nonlinear optics, one may appreciate this by examining the general structure of the quantum mechanical formulae for the nonlinear susceptibilities.

If we term to equation (15), we have a description of an interaction between four waves. There are two interactions between waves of frequency ω_1.a second with the wave at ω_2, and there is then an output wave at frequency $2\omega_1 - \omega_2$. First suppose the cubic term ax^3 vanished, while the quartic term bx^4 remains nonzero. We then have the direct interaction between four waves, as we see from the $b(X^{(I)})^3$ term in equation (11c) Upon noting $x^{(1)}$ is directly proportional to the total electric field E(t) in equation (9), we see the response of the oscillator has its origin in a driving term proportional to

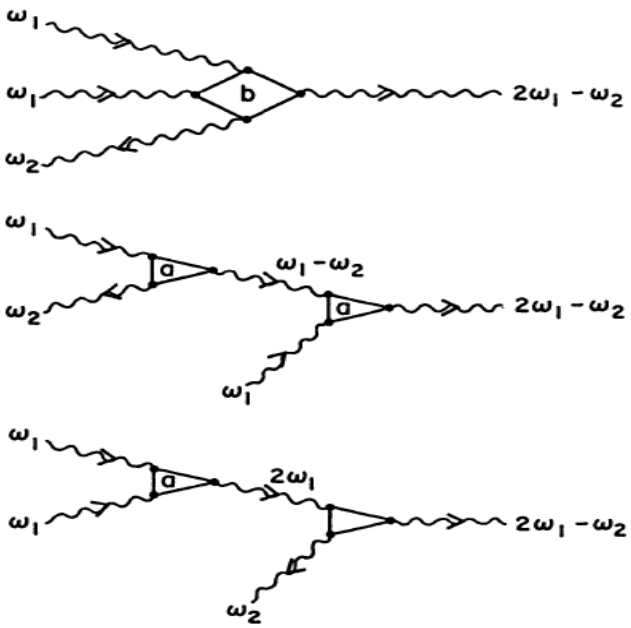
$$(E_1 e^{-i\omega_1 t})^2 (E_2^* e^{i\omega_2 t})$$

We thus have two enhancement factors associated with the frequency ω_1> $d(\omega_1)^2$, and a single resonant enhancement factor associated with ω_2 . There is finally an enhancement factor associated with the output wave at $2\omega_1 - \omega_2$. The term proportional to b describes a direct four wave interaction.

When a not equal to 0, so the cubic terms enter, we obtain additional contributions to the amplitude of the output at $2\omega_1 - \omega_2$ as we see in equation (15). These have the same overall enhancement factors as the direct four wave mixing promoted by the bx^4 term. However, there are additional resonant enhancement factors when a not equal to 0, inside the square bracket in equation (15).

We have seen that a, acting by itself, promotes the three wave interactions responsible for the "outputs" displayed in equation (14). An effective four wave interaction results from a succession of two three wave interactions, as illustrated in Figure, The second term in equation (15) is the consequence of the interaction displayed. We have an intermediate wave of frequency

$\omega_1 - \omega_2$ generated by mixing of the input waves at $\omega_1$and ω_2 which then immediately mixes with the ω_1 wave, to produce output at $(2\omega_1 - \omega_2)$. The presence of the intermediate wave, not part of the output, is responsible for the additional enhancement factor in equation (15). The third term comes from the process illustrated in Fig; the enhancement factor $d(2\omega_1)$ comes from the intermediate wave with frequency $2\omega_1$.



An illustration of the processes which contribute to the four wave mixing which leads to the output at frequency $2\omega_1 - \omega_2$, displayed in equation (15). The Fourier component of a wave with frequency dependence exp(-i ω t) is indicated by a left to right directed arrow, while the Fourier component of this wave which exhibits the frequency dependence exp(+ i ω t) is illustrated by a right to left directed arrow. We illustrate the interactions responsible for a the first, b the second, and e the third term of equation (15).

The anharmonic oscillator model discussed above can be applied in a literal manner to a simple molecule, where coupling to a single vibrational normal mode dominates the response of the above systems. Thus, the model applies in the infrared region of the spectrum. In fact, discrete, isolated electronic levels provide contributions to the nonlinear susceptibilities very similar in overall structure to those obtained for the anharmonic oscillator model. One may apply formulae for the nonlinear contributions to the dipole moment quite similar to those just derived to electronic excitations in gaseous media (the levels are quite sharp for such systems), and also to the exciton levels of semiconductors. Indeed, dye lasers can be tuned very close to exciton transitions in semiconducting crystals and films, with the consequence that the intensity of a desired nonlinear output can be increased by several orders of magnitude over those appropriate to off-resonance radiation. We have also seen that solids and other forms of dense matter contain absorption bands which extend over a broad frequency range. The nonlinear susceptibilities are enhanced substantially also for frequencies which lie close to the edges of such bands.

We can now appreciate that the frequency variations of the nonlinear susceptibilities are more complex than that of the linear dielectric constant. The higher order susceptibilities describe nonlinear interactions between several waves" and we can have resonant enhancement of the output if any of the participating waves has frequency close to an internal excitation energy of the system which promotes the mixing.

PROPERTIES OF NONLINEAR SUSCEPTIBILITIES:

Let us first see why it is important that we understand these symmetry properties. We consider the mutual interaction of three waves of frequencies ω_1, ω_2, and $\omega_3 = \omega_1 + \omega_2$, as illustrated in Fig. A complete description of the interaction of these waves requires that we know the nonlinear polarizations P(ωi) influencing each of them. Since these quantities are given in general by the expression

$$P_i(\omega_n + \omega_m) = \epsilon_0 \sum_{jk} \sum_{(nm)} \chi_{ijk}^{(2)}(\omega_n + \omega_m, \omega_n, \omega_m) E_j(\omega_n) E_k(\omega_m)$$

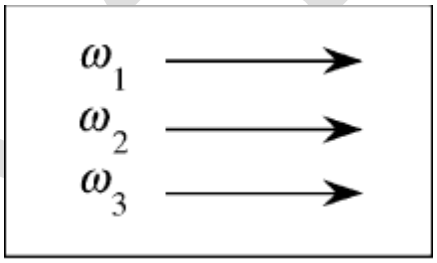
---------- (1)

we therefore need to determine the six tensors

$$\chi_{ijk}^{(2)}(\omega_1, \omega_3, -\omega_2), \quad \chi_{ijk}^{(2)}(\omega_1, -\omega_2, \omega_3), \quad \chi_{ijk}^{(2)}(\omega_2, \omega_3, -\omega_1),$$

$$\chi_{ijk}^{(2)}(\omega_2, -\omega_1, \omega_3), \quad \chi_{ijk}^{(2)}(\omega_3, \omega_1, \omega_2), \quad \text{and} \quad \chi_{ijk}^{(2)}(\omega_3, \omega_2, \omega_1)$$

and six additional tensors in which each frequency is replaced by its negative.



In these expressions, the indices i, j, and k can independently take on the values x, y, and z. Since each of these 12 tensors thus consists of 27 Cartesian components, as many as 324 different (complex) numbers need to be specified in order to describe the interaction.
Fortunately, there are a number of restrictions resulting from symmetries that relate the various components of $\chi^{(2)}$, and hence far fewer than 324 numbers are usually needed to describe the nonlinear coupling. In this section, we study some of these formal properties of the nonlinear susceptibility. The discussion will deal primarily with the second-order $\chi^{(2)}$ susceptibility, but can readily be extended to $\chi^{(3)}$ and higher-order susceptibilities.

(i) Reality of the fields:

Recall that the nonlinear polarization describing the sum-frequency response to input fields at frequencies ω_n and ω_m has been represented as

$$\tilde{P}_i(\mathbf{r},t) = P_i(\omega_n+\omega_m)e^{-i(\omega_n+\omega_m)t} + P_i(-\omega_n-\omega_m)e^{i(\omega_n+\omega_m)t} \quad \text{--------- (2)}$$

Since $\tilde{P}i(\mathbf{r}, t)$ is a physically measurable quantity, it must be purely real, and hence its positive- and negative-frequency components must be related by

$$P_i(-\omega_n-\omega_m) = P_i(\omega_n+\omega_m)^* \quad \text{---------- (3)}$$

The electric field must also be a real quantity, and its complex frequency components must obey the analogous conditions:

$$E_j(-\omega_n) = E_j(\omega_n)^* \quad \text{---------- (4A)}$$

$$E_k(-\omega_m) = E_k(\omega_m)^* \quad \text{---------- (4B)}$$

Since the fields and polarization are related to each other through the second order susceptibility of Eq. (1), we conclude that the positive- and negative frequency components of the susceptibility must be related according to

$$\chi^{(2)}_{ijk}(-\omega_n-\omega_m,-\omega_n,-\omega_m) = \chi^{(2)}_{ijk}(\omega_n+\omega_m,\omega_n,\omega_m)^* \quad \text{---------- (5)}$$

(ii) Intrinsic Permutation Symmetry:

The nonlinear susceptibility of a classical, anharmonic oscillator in the conventional form. In the present section, we treat the concept of intrinsic permutation symmetry from a more general point of view.

According to Eq.(1), one of the contributions to the nonlinear polarization $Pi(\omega_n + \omega_m)$ is the product $\chi^{(2)}_{ijk}$ $(\omega_n + \omega_m,\omega_n,\omega_m)$ E_j $(\omega_n)E_k(\omega_m)$. However, since j , k, n, and m are dummy indices, we could just as well have written this contribution with n interchanged with m and with j interchanged C with k, that is, as $\chi^{(2)}_{ikj}$ $(\omega_n + \omega_m,\omega_m,\omega_n)$ $E_k(\omega_m)E_j$ (ω_n). These two expressions are numerically equal if we require that the nonlinear susceptibility be unchanged by the simultaneous interchange of its last two frequency arguments and its last two Cartesian indices:

$$\chi^{(2)}_{ijk}(\omega_n+\omega_m,\omega_n,\omega_m) = \chi^{(2)}_{ikj}(\omega_n+\omega_m,\omega_m,\omega_n) \quad \text{---------- (6)}$$

This property is known as intrinsic permutation symmetry. More physically, this condition is simply a statement that it cannot matter which is the first field and which is the second field in products such as E_j (ω_n) $E_k(\omega_m)$. Note that this symmetry condition is introduced purely as a matter of convenience.

For example, we could set one member of the pair of elements shown in Eq. (6) equal to zero and double the value of the other member. Then, when the double summation of Eq. (1) was carried out, the result for the physically meaningful quantity P_j ($\omega_n + \omega_m$) would be left unchanged.

(iii) Symmetries for Lossless Media:

Two additional symmetries of the nonlinear susceptibility tensor occur for the case of a lossless nonlinear medium.

The first of these conditions states that for a lossless medium all of the components of $\chi^{(2)}_{ijk}$ ($\omega_n+\omega_m,\omega_n,\omega_m$) are real. This result is obeyed for the classical an harmonic oscillator as can be verified by evaluating the expression for $\chi^{(2)}$ in the limit in which all of the applied frequencies and their sums and differences are significantly different from the resonance frequency. The general proof that $\chi^{(2)}$ is real for a lossless medium is obtained by verifying that the quantum-mechanical expression for $\chi^{(2)}$ is also purely real in this limit.

The second of these new symmetries is full permutation symmetry. This condition states that all of the frequency arguments of the nonlinear susceptibility can be freely interchanged, as long as the corresponding Cartesian indices are interchanged simultaneously. In permuting the frequency arguments, it must be recalled that the first argument is always the sum of the latter two, and thus that the signs of the frequencies must be inverted when the first frequency is interchanged with either of the latter two. Full permutation symmetry implies, for instance, that

$$\chi^{(2)}_{ijk}(\omega_3 = \omega_1 + \omega_2) = \chi^{(2)}_{jki}(-\omega_1 = \omega_2 - \omega_3) \quad \text{---------- (7)}$$

However, according to Eq. (5), the right-hand side of this equation is equal

To $\chi^{(2)}_{jki}$ $(\omega 1 = -\omega 2 + \omega 3)^*$, which, due to the reality of $\chi^{(2)}$ for a lossless medium, is equal to $\chi^{(2)}_{jki}$ $(\omega 1 = -\omega 2 + \omega 3)$. We hence conclude that

$$\chi^{(2)}_{ijk}(\omega_3 = \omega_1 + \omega_2) = \chi^{(2)}_{jki}(\omega_1 = -\omega_2 + \omega_3) \quad \text{---------- (8)}$$

By an analogous procedure, one can show that

$$\chi^{(2)}_{ijk}(\omega_3 = \omega_1 + \omega_2) = \chi^{(2)}_{kij}(\omega_2 = \omega_3 - \omega_1) \quad \text{---------- (9)}$$

general proof of the validity of the condition of full permutation symmetry entails verifying that the quantum-mechanical expression for $\chi^{(2)}$ obeys this condition when all of the optical frequencies are detuned many linewidths from the resonance frequencies of the optical medium.

(iv) Kleinman's Symmetry:

Quite often nonlinear optical interactions involve optical waves whose frequencies ω_i are much smaller than the lowest resonance frequency of the material system. Under these conditions, the nonlinear susceptibility is essentially independent of frequency. For example, the expression for the second-order susceptibility of an anharmonic oscillator predicts a value of the susceptibility

that is essentially independent of the frequencies of the applied waves whenever these frequencies are much smaller than the resonance frequency ω_0. Furthermore, under conditions of low-frequency excitation the system responds essentially instantaneously to the applied field, and conditions the nonlinear polarization can be described in the time domain by the relation

$$\tilde{P}(t) = \epsilon_0 \chi^{(2)} \tilde{E}^2(t) \qquad \text{---------- (10)}$$

where $\chi^{(2)}$ can be taken to be a constant.

Since the medium is necessarily lossless whenever the applied field frequencies ω_i are very much smaller than the resonance frequency ω_0, the condition of full permutation symmetry Eq.(7) must be valid under these circumstances. This condition states that the indices can be permuted as long as the frequencies are permuted simultaneously, and it leads to the conclusion that

$$\begin{aligned}\chi^{(2)}_{ijk}(\omega_3 = \omega_1 + \omega_2) &= \chi^{(2)}_{jki}(\omega_1 = -\omega_2 + \omega_3) = \chi^{(2)}_{kij}(\omega_2 = \omega_3 - \omega_1) \\ &= \chi^{(2)}_{ikj}(\omega_3 = \omega_2 + \omega_1) = \chi^{(2)}_{kji}(\omega_2 = -\omega_1 + \omega_3) \\ &= \chi^{(2)}_{jik}(\omega_1 = \omega_3 - \omega_2).\end{aligned}$$

However, under the present conditions $\chi^{(2)}$ does not actually depend on the frequencies, and we can therefore permute the indices without permuting the frequencies, leading to the result

$$\begin{aligned}\chi^{(2)}_{ijk}(\omega_3 = \omega_1 + \omega_2) &= \chi^{(2)}_{jki}(\omega_3 = \omega_1 + \omega_2) = \chi^{(2)}_{kij}(\omega_3 = \omega_1 + \omega_2) \\ &= \chi^{(2)}_{ikj}(\omega_3 = \omega_1 + \omega_2) = \chi^{(2)}_{jik}(\omega_3 = \omega_1 + \omega_2) \\ &= \chi^{(2)}_{kji}(\omega_3 = \omega_1 + \omega_2)\end{aligned} \qquad \text{---------- (10)}$$

This result is known as the Kleinman symmetry condition. It is valid whenever dispersion of the susceptibility can be neglected.

(v) Contracted Notation:

We now introduce a notational device that is often used when the Kleinman symmetry condition is valid. We introduce the tensor

$$d_{ijk} = \frac{1}{2}\chi^{(2)}_{ijk} \qquad \text{---------- (11)}$$

and for simplicity suppress the frequency arguments. The factor of ½ is a consequence of historical convention. The nonlinear polarization can then be written as

$$P_i(\omega_n + \omega_m) = \epsilon_0 \sum_{jk} \sum_{(nm)} 2 d_{ijk} E_j(\omega_n) E_k(\omega_m). \qquad \text{---------- (12)}$$

We now assume that d_{ijk} is symmetric in its last two indices. This assumption is valid whenever Kleinman's symmetry condition is valid and in addition is valid in general for second-harmonic generation, since in this case ω_n and ωm are equal. We then simplify the notation by introducing a contracted matrix d_{il} according to the prescription

jk:	11	22	33	23, 32	31, 13	12, 21
l:	1	2	3	4	5	6

---------- (13)

The nonlinear susceptibility tensor can then be represented as the 3×6 matrix

$$d_{il} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix}$$

---------- (14)

If we now *explicitly* introduce the Kleinman symmetry condition—that is, we assert that the indices d_{ijk} can be freely permuted, we find that not all of the 18 elements of d_{il} are independent. For instance, we see that

$$d_{12} \equiv d_{122} = d_{212} \equiv d_{26} \quad \text{---------- (15A)}$$

and that

$$d_{14} \equiv d_{123} = d_{213} \equiv d_{25} \quad \text{---------- (15B)}$$

By applying this type of argument systematically, we find that d_{il} has only 10 independent elements when the Kleinman symmetry condition is valid; the form of d_{il} under these conditions is

$$d_{il} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{16} & d_{22} & d_{23} & d_{24} & d_{14} & d_{12} \\ d_{15} & d_{24} & d_{33} & d_{23} & d_{13} & d_{14} \end{bmatrix}$$

---------- (16)

We can describe the nonlinear polarization leading to second-harmonic generation in terms of d_{il} by the matrix equation

$$\begin{bmatrix} P_x(2\omega) \\ P_y(2\omega) \\ P_z(2\omega) \end{bmatrix} = 2\epsilon_0 \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix} \begin{bmatrix} E_x(\omega)^2 \\ E_y(\omega)^2 \\ E_z(\omega)^2 \\ 2E_y(\omega)E_z(\omega) \\ 2E_x(\omega)E_z(\omega) \\ 2E_x(\omega)E_y(\omega) \end{bmatrix}.$$

---------- (17)

When the Kleinman symmetry condition is valid, we can describe the nonlinear polarization leading to sum-frequency generation (with $\omega_3 = \omega_1 + \omega_2$) by the equation

$$\begin{bmatrix} P_x(\omega_3) \\ P_y(\omega_3) \\ P_z(\omega_3) \end{bmatrix} = 4\epsilon_0 \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix}$$

$$\times \begin{bmatrix} E_x(\omega_1)E_x(\omega_2) \\ E_y(\omega_1)E_y(\omega_2) \\ E_z(\omega_1)E_z(\omega_2) \\ E_y(\omega_1)E_z(\omega_2) + E_z(\omega_1)E_y(\omega_2) \\ E_x(\omega_1)E_z(\omega_2) + E_z(\omega_1)E_x(\omega_2) \\ E_x(\omega_1)E_y(\omega_2) + E_y(\omega_1)E_x(\omega_2) \end{bmatrix}. \quad \text{---------- (18)}$$

TIME DOMAIN DESCRIPTION OF OPTICAL NON-LINEAR SUSCEPTIBILITIES:

We found that the induced nonlinear polarization consists of a discrete summation of frequency components at the harmonics of and the sums and differences of the frequencies present in the applied field. In particular, we described the nonlinear response in the frequency domain by relating the frequency components P(ω) of the nonlinear polarization to those of the applied optical field, E(ω').

It is also possible to describe optical nonlinearities directly in the time domain by considering the polarization $\tilde{P}$(t) that is produced by some arbitrary applied field $\tilde{E}$ (t). These two methods of description are entirely equivalent, although description in the time domain is more convenient for certain types of problems, such as those involving applied fields in the form of short pulses; conversely, description in the frequency domain ismore convenient when each input field is nearly monochromatic.

Let us first consider the special case of a material that displays a purely linear response. We can describe the polarization induced in such a material by

$$\tilde{P}^{(1)}(t) = \epsilon_0 \int_0^\infty R^{(1)}(\tau)\tilde{E}(t-\tau)\,d\tau. \quad \text{---------- (1)}$$

Here $R^{(1)}(\tau)$ is the linear response function, which gives the contribution to the polarization produced at time t by an electric field applied at the earlier time $t-\tau$. The total polarization is obtained by integrating these contributions over all previous times τ. In writing Eq. (1) as shown, with the lower limit of integration set equal to zero and not to $-\infty$, we have assumed that $R^{(1)}(\tau)$ obeys the causality condition $R^{(1)}(\tau) = 0$ for $\tau < 0$. This condition expresses the fact that ˜ $\tilde{P}$ (t). (t) depends only on past and not on future values of $\tilde{E}$ (t). Equation (1) can be transformed to the frequency domain by introducing the Fourier transforms of the various quantities that appear in this equation. We adopt the following definition of the Fourier transform:

$$E(\omega) = \int_{-\infty}^{\infty} \tilde{E}(t)e^{i\omega t}\,dt \quad \text{---------- (2a)}$$

$$\tilde{E}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(\omega)e^{-i\omega t}\,d\omega$$ ---------- (2b)

with analogous definitions for other quantities. By introducing Eq. (2b) into Eq. (1), we obtain

$$\tilde{P}^{(1)}(t) = \epsilon_0 \int_0^{\infty} d\tau \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} R^{(1)}(\tau)E(\omega)e^{-i\omega(t-\tau)}$$

$$= \epsilon_0 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_0^{\infty} d\tau\, R^{(1)}(\tau)e^{i\omega\tau}E(\omega)e^{-i\omega t}$$ ---------- (3)

$$\tilde{P}^{(1)}(t) = \epsilon_0 \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \chi^{(1)}(\omega;\omega)E(\omega)e^{-i\omega t}$$ ---------- (4)

Or

where we have introduced an explicit expression for the linear susceptibility

$$\chi^{(1)}(\omega;\omega) = \int_0^{\infty} d\tau\, R^{(1)}(\tau)e^{i\omega\tau}$$ ---------- (5)

Equation (4) gives the time-varying polarization in terms of the frequency components of the applied field and the frequency dependent susceptibility.

By replacing the left-hand side of this equation with $\tilde{P}^{(1)}(\omega)\exp(-i\omega t)d\omega/2\pi$ and noting that the equality must be maintained for each frequency ω, we recover the usual frequency domain description of linear response:

$$P^{(1)}(\omega) = \epsilon_0\chi^{(1)}(\omega;\omega)E(\omega)$$ ---------- (6)

The nonlinear response can be described by analogous procedures. The contribution to the polarization second-order in the applied field strength is represented as

$$\tilde{P}^{(2)}(t) = \epsilon_0 \int_0^{\infty} d\tau_1 \int_0^{\infty} d\tau_2\, R^{(2)}(\tau_1,\tau_2)E(t-\tau_1)E(t-\tau_2)$$ ---------- (7)

where the causality condition requires that $R^{(2)}(\tau1, \tau2) = 0$ if either τ_1 or τ_2 is negative. As above, we write $E(t-\tau1)$ and $E(t-\tau2)$ in terms of their Fourier transforms using Eq. (2b) so that the expression for the second order polarization becomes

$$\tilde{P}^{(2)}(t) = \epsilon_0 \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} \int_0^{\infty} d\tau_1 \int_0^{\infty} d\tau_2 R^{(2)}(\tau_1,\tau_2)$$

$$\times E(\omega_1)e^{-i\omega_1(t-\tau_1)}E(\omega_2)e^{-i\omega_2(t-\tau_2)}$$

$$= \epsilon_0 \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} \chi^{(2)}(\omega_\sigma;\omega_1,\omega_2)E(\omega_1)E(\omega_2)e^{-i\omega_\sigma t},$$ ---------- (8)

where we have defined $\omega_\sigma = \omega_1 + \omega_2$ and have introduced the second-order susceptibility

$$\chi^{(2)}(\omega_\sigma;\omega_1,\omega_2) = \int_0^{\infty} d\tau_1 \int_0^{\infty} d\tau_2\, R^{(2)}(\tau_1,\tau_2)e^{i(\omega_1\tau_1+\omega_2\tau_2)}$$ ---------- (9)

This procedure can readily be generalized to higher-order susceptibilities. In particular, we can express the third-order polarization as

$$\tilde{P}^{(3)}(t) = \epsilon_0 \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_2}{2\pi} \int_{-\infty}^{\infty} \frac{d\omega_3}{2\pi} \chi^{(3)}(\omega_\sigma; \omega_1, \omega_2, \omega_3) \times E(\omega_1)E(\omega_2)E(\omega_3)e^{-i\omega_\sigma t}$$

---------- (10)

where $\omega_\sigma = \omega_1 + \omega_2 + \omega_3$ and where

$$\chi^{(3)}(\omega_\sigma; \omega_1, \omega_2, \omega_3) = \int_0^{\infty} d\tau_1 \int_0^{\infty} d\tau_2 \int_0^{\infty} d\tau_3 \times R^{(3)}(\tau_1, \tau_2, \tau_3)\, e^{i(\omega_1\tau_1+\omega_2\tau_2+\omega_3\tau_3)}$$

---------- (11)

WAVE VECTOR DEPENDENCE OF THE NONLINEAR SUSCEPTIBILITIES:

When the perturbation theoretic expressions for the nonlinear susceptibilities are examined, one sees wave vector dependencies that enter the formulae in a manner similar. The resonant denominators acquire excitation energies which depend on one or more of the wave vectors of the interacting waves. If one is operating at a frequency very close to that of an isolated resonance, this wave vector dependence can shift its apparent frequency. Also, the presence of the nonzero wave vector within the matrix elements in the numerator can "active" particular resonances, whose excitation matrix element vanishes in the long wavelength limit.

Finite wave vector effects can also modify selection rules deduced in the limit of very long wavelengths. Consider, for example the second order electric susceptibility tensor $\chi^{(2)}_{\alpha\beta\gamma}$(k₁ ω₁; k2ω2). We have seen that , in the limit $k_1, k_2 \rightarrow 0$ symmetry requires this tensor to vanish identically, for any material which possesses an inversion center. When $\chi^{(2)}_{\alpha\beta\gamma}$ vanishes, phenomena such as second harmonic generation, and the generation of the sum and difference frequencies by virtue of the interaction of a wave with frequency ω_2, will be absent.

However, as we know $\chi^{(2)}_{\alpha\beta\gamma}$ depends on the wave vectors k_1 and k_2. A Taylor series expression then yields, for a material with an inversion center,

$$\chi^{(2)}_{\alpha\beta\gamma}(\boldsymbol{k}_1\omega_1; \boldsymbol{k}_2\omega_2) = \sum_\delta \left(\frac{\partial \chi^{(2)}_{\alpha\beta\gamma}}{\partial k_1^\delta}\right)_0 k_1^\delta + \sum_\delta \left(\frac{\partial \chi^{(2)}_{\alpha\beta\gamma}}{\partial k_2^\delta}\right)_0 k_2^\delta + \cdots$$

---------- (1)

Under r ω₁otations and inversions of the coordinate system, derivatives such as $(\partial\chi^{(2)}_{\alpha\beta\gamma} / \partial k_1^\delta)_0$ are fourth rank tensors. There are nonzero elements of fourth rank tensors in any material, even if it is completely isotropic.

Thus, by exploiting the wave vector dependence of $\chi^{(2)}_{\alpha\beta\gamma}$, one may "activate" nonlinear processes forbidden by symmetry in the limit of long wavelengths. In the usual circumstance, where the wavelength of the radiation is long compared to the underlying microscopic lengths in

the system, the terms proportional to the wave vector will be quite small in magnitude. However, their magnitude can be enhanced substantially by operating near resonance.

Suppose we consider the second order term and endow $\chi^{(2)}_{\alpha\beta\gamma}$ with the wave vector dependence displayed in equation (1), we may define

$$E_\alpha(r_1, \omega_1) = \int \frac{d^3k_1}{(2\pi)^3} E_\alpha(k_1, \omega_1) e^{ik_1 \cdot r_1} \quad \text{---------- (2)}$$

Which describes the spatial variation of the component of the electric field with frequency ω_1. The first term in equation (1) can then be cast into the form

$$P_\alpha^{(NL)}(r,t) = \frac{1}{i}\sum_{\beta\gamma\delta} \int \frac{d\omega_1 d\omega_2}{(2\pi)^2} e^{-i\omega_1 t} e^{-i\omega_2 t} \left\{ \left(\frac{\partial \chi^{(2)}_{\alpha\beta\gamma}}{\partial k_1^\delta} \right)_0 \left[\frac{\partial E_\beta(r,\omega_1)}{\partial r^\delta} \right] E_\gamma(r,\omega_2) + \left(\frac{\partial \chi^{(2)}_{\alpha\beta\gamma}}{\delta k_2^\delta} \right)_0 \left[\frac{\partial E_\gamma}{\partial r^\delta}(r,\omega_2) \right] E_\beta(r,\omega_1) \right\} \quad \text{---------- (3)}$$

In equation (3), the nonlinear electric dipole moment per unit volume $P_\alpha^{(NL)}$(r,t) , is releted to quantities evaluated at the same point in space. We see, however, that is the spatial gradients of the various components of the electric field that enter, in addition to the electric field itself.

Before we encountered that the notation that there is time lag in the response of the system in the time domain, and also the response is nonlocal in space, we see that $\chi^{(2)}_{\alpha\beta\gamma}$ may be viewed as endowing the electric susceptibility of a medium with a dependence on electric field; in general, the dielectric susceptibility tensor is defined by writing $\chi_{\alpha\beta} = \partial P_\alpha / \partial E_\beta$, with the derivative evaluated at finite, nonzero applied field. When the nonlocal response of the medium is evaluated to the leading order in the wave-vector dependent corrections to a quantity such as $\chi^{(2)}_{\alpha\beta\gamma}(k_1\ \omega_1; k_2\omega_2)$, as in equation (1), the dipole moment per unit volume may be regarded to be a function of the gradient tensor $\partial E_\beta / \partial r_\gamma$. Electric field gradients in general interact with the quadrupolar components of the charge distribution of a system. One can refer to such contributions to the nonlinear dielectric response as quadrupole contributions to the nonlinear susceptibility.

POSSIBLE QUESTIONS
PART B

1. What is nonlinear optics?
2. Define the term 'susceptibility'.
3. Give some applications of nonlinear optics.
4. List out any four properties of nonlinear susceptibility.
5. Differentiate linear optics and nonlinear optics.

PART C

1. Discuss in detail about nonlinear dielectric response of matter.
2. Describe the frequency variation of the nonlinear susceptibilities.
3. State and explain the properties of nonlinear susceptibilities.
4. Briefly explain about the time domain description of optical non- linear susceptibilities.
5. Explain in detail about wave vector dependence of the nonlinear susceptibilities.

KARPAGAM ACADEMY C	
	DEP
	CL
	APPL
	MULT
Question	**Choice 1**
UNIT 2	
_____ is the study of phenomena that occur as a consequence of the n	opto electronics
Pi(r, t) is a physically measurable quantity, it must be purely and ____	positive
Which one of the following is the property of non-linear susceptibility'	time domain
In this more general case the nonlinear susceptibility becomes a _____	inverse
The nonlinear susceptibilities depend on the _____of the applied fiel	amplitude
The fields P(t) and E(t) to be _____ quantities.	scalar
The constant proportionality χ is known as	linear susceptibility
The demonstration of the first working laser by _____.	F.Zernike
The discovery of second-harmonic generation by Franken et al in the y	1876
The discovery of second-harmonic generation by	Newton
_____ is sufficiently intense to modify the optical properties of a ma	Laser light
Which one of the following is the property of non-linear susceptibility'	time reversal
The induced polarization depends linearly on the _____ strength.	electric field
which one of the following material cannot produce second-order nonl	liquids
The number density N of condensed matter is of the order of	a0
For condensed matter A381) is of the order of _____	N
Time-varying polarization can act as the source of new components of	nonlinear optical media
Which theorm states that accelerated charges generate electromagnetic	Gauss
The second-order polarization consists of a contribution at _____ frec	inverse
The second-order polarization consists of a contribution at _____ frec	2ω
A static _____ is created across the nonlinear crystal.	optical
Which one of the following is the property of non-linear susceptibility'	Intrinsic Permutation Syn
Nd:YAG laser operates in the near infrared at a wavelength of _____ μ	23.5
The conversion of the output of a fixed-frequency laser to a different s	second-harmonic generati
Second-harmonic generation is routinely used to convert the waveleng	1.55 μm
The dashed lines represent what are known as _____.	virtual levels
Non-linear is the study of phenomena that occur as a consequence of tl	field
Pi(r, t) is a physically _____ quantity.	measurable
The discovery of _____ by Franken et al.	first-harmonic generation
Laser light is sufficiently intense to modify the optical properties of a _	electrical
The complex field amplitudes E(ω) =	εe^ik
The complex field amplitudes E(-ω) =	εe^ik
Optical waves of frequencies ω1, ω2, and ω3 =	ω1xω2
The analogous condition in nonlinear susceptibility is	Ej (−ωn) = Ej (ωn)∗
The analogous condition in nonlinear susceptibilityis	Ek(−ωm) = Ek(-ωm)∗
The dispersion electrons are directly involved in the optical behavior; i	piezoelectricity
A plane-polarized fundamental can generate _____ which is polarized	fourth harmonic
Practically all SHG materials are _____crystals.	biaxial
Actually, second-order optical nonlinearity is forbidden in ____ media	conducting

Thus in mks units, the second-order susceptibility is in	meters
The _____ χijk applicable to their combined effect must be evaluated	rate of change of suscepti
Thus in cgs units, the second-order susceptibility is in	esu/dyne
The numerical values of the tensor coefficients is function of	frequency
The numerical values of the tensor coefficients is function of	temperature
The ____ is a dimensionless ratio, independent of the system.	Second-order linear susce
The ____ , which describes the nonlinearity independent of the system	first-order linear susceptil
_____ have dimensions inverse to those of electric field intensity.	Fourth-order linear susce
The frequency variations of the nonlinear susceptibilities are _____ th	less
_____model applies in the infrared region of the spectrum.	isolated electronic
The enhancement factor *d(2ω1)* comes from the intermediate wave wi	ω
p(ω)=a(ω)E(ω), Where a(ω) is	constant term
The frequency dependent dielectric constant is e(ω) =	1 + 4npa(ω)
Practically all _____materials are birefringent crystals.	THG

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OF HIGHER EDUCATION, COIMBATORE – 21			
ARTMENT OF PHYSICS			
ASS: III B. Sc., PHYSI	**BATCH: 2016-2019**		
Sixth Semester			
IED OPTICS (16PHU601B)			
IPLE CHOICE QUESTIONS			
Choice 2	**Choice 3**	**Choice 4**	
optics	material science	non-linear optics	
negative	imaginary	none of the above	
frequency domain	linearity	reality of the fields	
complex	tensor	integral	
time period	strength	frequency	
vector	tensor	none of the above	
Dielectric constant	Wave function	field strength	
G.C.Baldwin	Maiman	Einstein	
1874	1854	1961	
Franken et al	Einstein	Maxwell	
electromagnetic wave	digital signal	analog signal	
linearity	Symmetries for Lossles	causality	
optical	light	polarizer	
gases	amorphous solids	all the above	
(a0)^-3	n	a^3	
A^9	unity	none of the above	
inhomogeneous wave e	linear refractive index	electromagnetic field	
Stokes	Larmor's	Maxwell	
zero	ω^2	ω	
inverse	ω^2	ω	
electric field	light	filed	
linearity	time domain	periodicity	
1.06	35.4	65.3	
first-harmonic generati	third-harmonic generati	fourth-harmonic generation	
0.53 μm	0.99μm	0.563 μm	
energy levels	frequency level	quantum level	
electrical	material	optical	
non- measurable	imaginary	none of the above	
second-harmonic gener	third-harmonic generati	fourth-harmonic generation	
wave	material	sound	
εe^ik·r	1/2 εe^ik·r	none of the above	
εe^ik·r	1/2 εe^-ik·r	none of the above	
ω1-ω2	ω1/ω2	ω1+ω2	
Ej (−ωn) = Ej (-ωn)∗	Ej (ωn) = Ej (ωn)∗	Ej (ωn)* = Ej (ωn)∗	
Ek(ωm) = Ek(ωm)∗	Ek(−ωm)* = Ek(-ωm)∗	Ek(−ωm) = Ek(ωm)∗	
Photoelectric effect	Compton effect	Zeeman effect	
first harmonic	third harmonic	second harmonic	
uniaxial	birefringent	none of the above	
electrical	istropic	free space	

Volt	meters/volt	centimeter
susceptibility ratio	susceptibility tensor	susceptibility factor
Volt	meters	none of the above
amplitude	timeperiod	wavelength
amplitude	energy	none of the above
first-order linear suscep	Third-order linear susc	Fourth-order linear susceptibilit
Second-order linear sus	Fourth-order linear susc	Third-order linear susceptibility
Third-order linear susc	Second-order linear sus	first-order linear susceptibility
more complex	greater	equal
anharmonic oscillator	both a and b	none of the above
2ω	3ω	2ω + 2ω
normalization factor	polarizability	propagation constant
4npa(ω)	1 + 4a(ω)	4na(ω)
FHG	SHG	none of the above

Answer

non-linear optics
positive
reality of the fields
complex
frequency
scalar
linear susceptibility
Maiman
1961
Franken et al
Laser light
Symmetries for Lossless Media
electric field
all the above
(a0)^-3
unity
electromagnetic field
Larmor’s
zero
2ω
electric field
Intrinsic Permutation Symmetry
1.06
second-harmonic generation
0.53 μm
virtual levels
optical
measurable
second-harmonic generation
material
1/2 εe^ik·r
1/2 εe^-ik·r
ω1+ω2
Ej (−ωn) = Ej (ωn)∗
Ek(−ωm) = Ek(ωm)∗
piezoelectricity.
second harmonic
birefringent
istropic

meters/volt
susceptibility factor
esu/dyne
frequency
temperature
first-order linear susceptibility
Second-order linear susceptibility
Second-order linear susceptibility
more complex
anharmonic oscillator
2ω
polarizability
1 + 4npa(ω)
SHG

UNIT-III

SYLLABUS

Second harmonic generation – perturbation theory – phase matching evolution of SHW under phase matching conditions. Four wave mixing spectroscopy – optical phase conjugation – nonlinear materials.

SECOND HARMONIC GENERATION:

Consider, e.g., an experiment in which a monochromatic optical field at frequency ω is incident. In the second order, it can create $P^{(NLS)}$ at frequencies $2\omega \text{ and } 0$. The nonlinear polarization at the second harmonic frequency is given by

$$P_{\mu}^{(NLS)}(2\omega) = \varepsilon_0 \chi_{\mu\alpha\alpha}^{(2)}(-2\omega,\omega,\omega) E_{\alpha}(\omega) E_{\alpha}(\omega) \quad \text{---------- (1)}$$

where summation over repeated indices is implied. Now, as $\vec{E}(2\omega)$ becomes nonzero, it can also create a $\vec{P}^{(NLS)}$ at ω

$$P_{\mu}^{NLS}(\omega) = 2\varepsilon_0 \chi_{\mu\alpha\beta}^{(2)}(-\omega,2\omega,-\omega) E_{\alpha}(2\omega) E_{\beta}^{*}(\omega) \quad \text{---------- (2)}$$

The factor 2 accounts for the fact that there are two distinct frequencies. The two waves are therefore coupled and follow the wave equations

$$\nabla\times\nabla\times\vec{E}(2\omega) - \left(\frac{2\omega}{c}\right)^2 \vec{\varepsilon}(2\omega).\vec{E}(2\omega) = \varepsilon_0 \frac{(2\omega)^2}{c^2} \vec{\vec{\chi}}^{(2)}(-2\omega,\omega,\omega) : \vec{E}(\omega)\vec{E}(\omega) \quad \text{---------- (3)}$$

$$\nabla\times\nabla\times\vec{E}(\omega) - \frac{\omega^2}{c^2} \vec{\varepsilon}(\omega).\vec{E}(\omega) = 2\varepsilon_0 \frac{\omega^2}{c^2} \vec{\vec{\chi}}^{(2)}(-\omega,2\omega,-\omega) : \vec{E}(2\omega)\vec{E}(\omega) \quad \text{---------- (4)}$$

The coupling of the waves means that it is possible to exchange energy between them. If $\hat{z}$ is the direction of propagation then without the nonlinearity the solution would be

$$\vec{E}(\omega) = \hat{a} A_0 e^{ikz} \quad \text{---------- (5)}$$

where A_0 is the amplitude and $\hat{a}$ is the polarization. In the presence of nonlinear coupling the amplitude of the two waves can vary with z. So, we write,

$$\vec{E}(\omega) = \hat{a}_1 A_1(z) e^{ik_1 z} \quad \text{---------- (6a)}$$

And

$$\vec{E}(2\omega) = \hat{a}_2 A_2(z) e^{ik_1 z} \quad \text{---------- (6b)}$$

where $\hat{a}_1, \hat{a}_2$ are the polarization vectors such that

$$k_i^2\left(\hat{z}\times\hat{z}\times\hat{a}_i\right)+\frac{\omega_i^2}{c^2}\vec{\varepsilon}\left(\omega_i\right).\hat{a}_i=0$$ ---------- (7)

Thus, if $\chi^{(2)}=0$, A_1 and A_2 would be constants and no transfer of energy would be possible between the two waves. In the nonlinear case $\chi^{(2)}\neq 0$, we can assume that $A_i(z)$ are slowly varying functions of z. i.e.

$$\frac{\partial^2 A_i}{\partial z^2}<<k\frac{\partial A_i}{\partial z}<<k^2A_i$$ ---------- (8)

Substituting from eqs (5) and (6) in eqs (3) and (4) and using eq (7) and approximation (8), we get equations for $\partial A_1/\partial z$ and $\partial A_2/\partial z$.

Then taking scalar products of these equations with $\hat{a}_2$ and $\hat{a}_1$, respectively, we obtain

$$\frac{\partial A_2}{\partial z}=\frac{i\varepsilon_0 A_1^2 2\omega^2}{c^2k_2^2\cos^2\alpha_2}\vec{\chi}^{(2)}\left(-2\omega,\omega,\omega\right):\hat{a}_2\hat{a}_1\hat{a}_1e^{-i\Delta kz}$$ ---------- (9)

And

$$\frac{\partial A_1}{\partial z}=\frac{i\varepsilon_0 A_1^* A_2\omega^2}{c^2k_1^2\cos^2\alpha_1}\vec{\chi}^{(2)}\left(-\omega,2\omega,-\omega\right):\hat{a}_1\hat{a}_2\hat{a}_1e^{i\Delta kz}$$ ---------- (10)

where $\Delta k=k_2-2k_1$ is the wave vector mismatch and α_1 is the angle between the electric field $\vec{E}(\omega_1)$ and displacement vector $\vec{D}(\omega_1)$ at the fundamental frequency and similarly for α_2. The two susceptibility tensors involved here are not independent but are related to each other by the overall permutation symmetry. The susceptibility $\chi^{(2)}_{\mu\alpha\beta}(\omega,\omega_1,\omega_2)$ is invariant under the exchange of pairs $\mu\omega,\alpha\omega_1,\beta\omega_2$. This can be used to define a common coupling constant for the coupled wave propagation

$$K=\varepsilon_0\vec{\chi}^{(2)}\left(-2\omega,\omega,\omega\right):\hat{a}_2\hat{a}_1\hat{a}_1=\varepsilon_0\vec{\chi}^{(2)*}\left(-\omega,2\omega,-\omega\right):\hat{a}_1\hat{a}_2\hat{a}_1$$ ---------- (11)

If the conversion to the second harmonic is small, we may assume pump amplitude to remain constant. Then eq (9) gives:

$$\frac{\partial A_2}{\partial z}=K'e^{-i\Delta kz}$$ ---------- (12)

where the constant ***K'*** includes all factors independent of z. The solution of this is

$$A_2=zK'e^{-(i\Delta kz/2)}\frac{\sin(\Delta kz/2)}{(\Delta kz/2)}$$ ---------- (13)

where we have assumed that at z=0 the second harmonic wave amplitude is zero. This shows that the second harmonic intensity $I_2(z)$ is given by

$$I_2 \sim |A_2(z)|^2 = z^2 K'^2 \frac{\sin^2(\Delta k z/2)}{(\Delta k z/2)^2} \quad \text{---------- (14)}$$

As anticipated earlier this is an oscillatory function. The net conversion reaches a maximum at $z = \pi/\Delta k$ after which it starts to reduce. The length $l_{coh} = \pi/\Delta k$ is called the coherence length. This important limiting length would be infinite if there was no dispersion. It is finite only due to the small frequency dependence of the refractive index and that small dependence determines the small distance over which effective conversion takes place. For second harmonic generation of radiation of vacuum wavelength l, $\Delta k = 4\pi(\Delta n)/\lambda$ where Δn is the difference between the refractive indices of the medium at the fundamental and the harmonic frequencies. Typically for Δn as small as 0.01, $l_{coh} = \pi/\Delta k$ is 25 μm which is rather small. Oscillatory nature of the conversion efficiency is useful for obtaining Δk for a crystal by the "Maker Fringe" method. In this, we measure I_2 as a function of crystal length either by using a wedge shaped crystal or by rotating a parallel slab crystal and determine Δk from the period of oscillation.

For the phase matched case $(\Delta k = 0)$ the solutions are not oscillatory since the energy conversion to second harmonic proceeds without a phase mismatch between the field at 2ω and the nonlinear polarization at that frequency. In that case,

$$I_2 \sim z^2 I_1^2 \left(\chi_{eff}^{(2)}\right)^2 \quad \text{---------- (15)}$$

where the effective coupling coefficient is defined by

$$\chi_{eff}^{(2)} = \sum_{i,j,k} \chi_{ijk}^{(2)}(-2\omega, \omega, \omega) a_{2i} a_{1j} a_{1k} \quad \text{---------- (16)}$$

The condition $\Delta k = 0$ called the PHASE MATCHING condition can be achieved by compensating the effect of dispersion by that of birefringence (Birefringence Phase Matching or BPM) or by periodically changing the sign of the coupling coefficient $\chi_{eff}^{(2)}$. These techniques are described in the next two lectures.

To summarize, the second harmonic conversion efficiency in the phase matched case is

$$\eta_{SH} \sim (\text{crystal length})^2$$
$$\sim \text{Incident laser intensity}$$
$$\sim \left(\chi_{eff}^2\right)^2$$

Thus, to have efficient conversion to the second harmonic we need crystals which have large $\chi_{eff}^{(2)}$, have large damage threshold so that higher intensities can be used and be

available in relatively large sizes so that large lengths can be used. For efficient conversion of low power sources we need to focus the lasers to a relatively tight spot but that reduces the effective interaction length since the spot size increases rapidly due to diffraction. Exploiting guided wave propagation mitigates this problem to a great extent and this is particularly suitable for quasi phase matching. It is important to note that that the effective coupling coefficient depends on the polarization vectors of the two beams relative to the crystal axes. For example, if we rotate a crystal about the propagation directions $\chi_{eff}^{(2)}$ will change and effect η_{SH}.
It is remarkable that the ABDP formalism includes the effect of anisotropy of the crystal in an elegant way and also included the pump depletion effects fully. Recall that in birefringent crystals, for any given direction of propagation linear wave equation has two independent solutions. For a given $\vec{k}, \hat{a}$ has two possible solutions one corresponding to the ordinary ray and the other corresponding to the extra ordinary ray.

For the extraordinary wave $\vec{E}$ and $\vec{D}$ are not parallel and angle α between them depends on the direction of propagation. By α_1 we denote the angle between $\vec{E}(\omega)$ and $\vec{D}(\omega)$, and by α_2 that between $\vec{E}(2(\omega))$ and $\vec{D}(2(\omega))$. Writing $A_1 = \rho_1 e^{i\phi_1}$ and $A_2 = \rho_2 e^{i\phi_2}$ we can separate the equations for evolution of the amplitudes and phases. It is easy to show that

$2\rho_1^2 k_1 \cos^2\alpha_1 / \mu_0 \omega$ ---------- (17)

is the power flow per unit area in the ω wave while

$\rho_2^2 k_2 \cos^2\alpha_2 / (\mu_0 \omega)$ ---------- (18)

is that in the 2ω wave.

Equations of motion for ρ_1 and ρ_2 and the relative phase

$\theta = \Delta k z + \varphi_2(\varsigma) - 2\varphi_1(\varsigma)$ ---------- (19)

are obtained from those for A_1 and A_2,

$$\frac{d\rho_1}{dz} = -\frac{\omega^2 K}{k_1 \cos^2\alpha_1}\rho_1\rho_2 \sin\theta$$ ---------- (20)

$$\frac{d\rho_2}{dz} = +\frac{2\omega^2 K}{k_2 \cos^2\alpha_2}\rho_1^2 \sin\theta$$

---------- (21)

$$\frac{d\theta}{dz} = \Delta k - 2\omega^2 K \cos\theta\left[\frac{\rho_2}{k_1 \cos^2\alpha_1} - \frac{\rho_1^2}{\rho_2 k_2 \cos^2\alpha_2}\right]$$ ---------- (22)

It is important to note that the *energy transfer can occur from ω to 2ω or 2ω to ω, depending on the value of relative phase θ*. It does not matter, what the relative values

of ρ_1 and ρ_2 are, so long as both of them are non zero. ***This is the most important difference between coherent coupled waves and energy exchanges between incoherent systems governed by rate equations.*** If $\rho_2 = 0$, $\theta_{initial}$ is arbitrary because ϕ_2 is arbitrary. The value of ϕ_2 which gives $\sin\theta > 0$ will be adopted and ρ_2 can grow from a zero value. However, if $\rho_1 = 0$, the non-linear coupling between the two waves vanishes for all values of the phases and the fundamental wave cannot grow back. Thus second order nonlinearity cannot generate sub harmonic of a laser if none is present to begin with. However, the sub-harmonic can grow from an arbitrarily small amplitude but the growth rate depends on this amplitude itself.

To solve these coupled non-linear equations ABDP first found two constants of motion. The remaining one equation was then integrated out. We note that the total power flowing through any $z =$ constant surface must be same since no power is being absorbed by the medium. From eqs (29) and (30), one obtains

$$\rho_1 \frac{d\rho_1}{dz}\cdot\frac{k_1\cos^2\alpha_1}{\omega} + \rho_2\frac{d\rho_2}{dz}\cdot\frac{k_2\cos^2\alpha_2}{2\omega} = 0$$

or,

$$\left(\rho_1^2\frac{k_1\cos^2\alpha_1}{\omega} + \rho_2\frac{k_2\cos^2\alpha_2}{2\omega}\right) = W = \text{constant} \quad \text{---------- (23)}$$

where W is the total power flowing through a unit area of any ***z*** = constant plane.

While the relative phase determines the direction of energy flow, the rate of energy flow also depends on the total power in the system and the non-linear susceptibility $\chi^{(2)}_{eff}$ which in turn depends on the non-linear material and the polarization vectors $\hat{\alpha_1}$ and $\hat{\alpha_2}$. To understand the behavior of coupled waves in a wide variety of situations it is instructive to work in terms of scaled distance ζ and scaled amplitudes u and v. We put,

$$\zeta = \frac{\omega^2 Kz}{k_1\cos^2\alpha_1\sqrt{\frac{\omega W}{c^2 k_2\cos^2\alpha_2}}} \quad \text{---------- (24)}$$

$$u = \sqrt{\frac{k_1 c^2\cos^2\alpha_1}{\omega W}}\rho_1 \quad \text{---------- (25)}$$

and,

$$v = \sqrt{\frac{k_2 c^2\cos^2\alpha_2}{\omega W}}\rho_2 \quad \text{---------- (26)}$$

With these parameters eq(24) becomes

$$u^2 + v^2 = 1 \quad \text{---------- (27)}$$

and eqs (22) and (23) become

$$\frac{du}{d\zeta} = -uv\sin\theta$$ ---------- (28)

$$\frac{dv}{d\zeta} = u^2\sin\theta$$ ---------- (29)

And

$$\frac{d\theta}{d\zeta} = \Delta s - \cos\theta\left(2v - u^2/v\right)$$ ---------- (30)

With

$$\Delta s = \Delta kz/\zeta$$ ---------- (31)

Here, $\boldsymbol{u}^2$ and $\boldsymbol{v}^2$ are the fractional powers in the wave at the fundamental frequency and its second harmonic, respectively. Δs is the scaled phase mismatch. Having incorporated the conservation of power flow in the scaling, ABDP obtained the second constant of integration by observing that using eqs (28) and (29) in eq (30). We may write

$$\frac{d\theta}{d\zeta} = \Delta s + \frac{\cos\theta}{\sin\theta}\frac{d}{d\zeta}\left(ln\, u^2 v\right)$$

or,

$$u^2 v\sin\theta\frac{d\theta}{d\zeta} = \Delta s u^2 v\sin\theta + \cos\theta\frac{d}{d\zeta}\left(u^2 v\right)$$

$$= \Delta s v\frac{dv}{d\zeta} + \cos\theta\frac{d}{d\zeta}\left(u^2 v\right)$$

$$\frac{d}{d\zeta}\left(u^2 v\cos\theta + \frac{1}{2}\Delta s v^2\right) = 0$$

or,

$$u^2 v\cos\theta + \frac{1}{2}\Delta s v^2 = \text{constant} = \Gamma$$

Using (27) and (32) in Eq(28) we can obtain a non-linear equation for v^2

$$v\frac{dv}{d\zeta} = u^2 v\sin\theta = \pm\left(\left(u^2 v\right)^2 - \left(u^2 v\cos\theta\right)^2\right)^{1/2}$$

$$= \pm\left(\left(1-v^2\right)^2 v^2 - \left(\Gamma - \frac{1}{2}\Delta s\, v^2\right)^2\right)^{1/2}$$

or,

$$\frac{dv^2}{\zeta\left(v^2\left(1-v^2\right)^2 - \left(\Gamma - \frac{1}{2}\Delta s\, v^2\right)^2\right)^{1/2}} = \pm\, d\zeta$$

---------- (33)

The denominator is a cubic in v^2. If $v_a^{\,2} \le v_b^{\,2} \le v_c^{\,2}$ are the three roots of

$$v^2\left(1-v^2\right)^2 + \left(\Gamma - \frac{1}{2}\Delta s\, v^2\right)^2 = 0$$

---------- (34)

Then

$$\frac{dv^2}{\sqrt{\left(v^2 - v_a^{\,2}\right)\left(v^2 - v_b^{\,2}\right)\left(v^2 - v_c^{\,2}\right)}} = \pm\, 2d\zeta$$

The solution of this equation is a standard Jacobi elliptic function

$$v^2 = v_a^{\,2} + \left(v_b^{\,2} - v_a^{\,2}\right) sn^2\left(\sqrt{v_c^{\,2} - v_a^{\,2}}\,(\zeta - \zeta_0), \gamma\right)$$

---------- (35)

Where

$$\gamma^2 = \frac{v_b^{\,2} - v_a^{\,2}}{v_c^{\,2} - v_a^{\,2}}$$

---------- (36)

γ is called the modulus. The Jacobi elliptic functions sn and cn are defined as follows. If,

$$u = \int_0^{\varphi} \frac{d\psi}{\sqrt{1-\gamma^2\sin^2\psi}}$$

then $sn(u,\gamma) = \sin\phi$ and $cn(u,\gamma) = \cos\phi$. The Jocobi elliptic functions sn and cn are periodic function of u with period depending on g. They are easily evaluated using standard methods and are available directly in computing systems like mathematica. We note the two limiting forms of the function

Since $sn(u,\gamma)$ is bound by ± 1, maximum and minimum values for v^2 are given by the two roots of eq (34).

$(v^2)_{max} = v_b^2$

$(v^2)_{min} = v_a^2$

If initially $v = 0$ at $\zeta = z = 0$, $\Gamma = 0$, and eq (29) becomes

$$v^2(1-v^2)^2 - \left(\frac{1}{2}\Delta s\, v^2\right)^2 = 0$$

For this case the lowest root is $v_a^2 = 0$ and the other two roots are given by

$$v_{b,c}^2 = \left(1+\frac{1}{2}\left(\frac{\Delta s}{2}\right)^2\right) \pm \sqrt{\left(1+\frac{1}{2}\left(\frac{\Delta s}{2}\right)^2\right)^2 - 1} \quad \text{---------- (37)}$$

For, $\Delta s >> 1, v_b^2 = (2/\Delta s)^2$ and $v_c^2 = (\Delta s/2)^2$ and the maximum fractional power conversion to second harmonic is $(2/\Delta s)^2$.

Recalling that the scaled phase mismatch $\Delta s = \Delta k\,(z/\zeta)$ is the ratio between two lengths one characterizing the dispersion in the linear refractive index and the other characterizing the interaction between the two waves due to nonlinearity. The maximum power conversion is small if $\Delta s >> 1$ i.e. the dispersion occurs much faster than nonlinear coupling

In the phase matched case $\Delta s = 0$ eq (44) becomes $v_b^2 = v_c^2 = 1$ and $v_a^2 = 0,$ with $v_b^2 = v_c^2 = 1$. Then,

$$v^2 = sn^2(\zeta, \gamma = 1) = \tanh^2 \zeta$$

In this case full conversion is possible to second harmonic but it takes infinite interaction length since $\tanh \zeta \to 1$ only as $\zeta \to \infty$. However, $\tanh \zeta$ is close to unity for moderate values of ζ.

PERTURBUTION THEORY:

Suppose the material is illuminated with a laser beam, here taken to be a simple plane wave, of frequency ω1 and wave vector k1 • If we assume the intensity of the second harmonic radiation is very weak, then we can ignore the depletion of the primary wave due to a conversion of a portion of its energy into second harmonic. We may then calculate $P^{(N,L)}$ (r, ω) by simply inserting the expression for the amplitude of the initial wave into the appropriate terms in the power

series expansion of the dipole moment per unit volume in powers of electric field. Let the incident field be given by

$$\boldsymbol{E}(z,t) = \hat{\mathbf{e}}E(\omega_1)e^{ik_1 z}e^{-i\omega_1 t} + \hat{\mathbf{e}}E^*(\omega_1)e^{-ik_1 z}e^{+i\omega_1 t}$$

---------- (1)

where we orient the Z axis along the propagation direction of the beam. The components of the dipole moment with the frequency $2\omega_1$ are

$$P_\alpha^{(NL)} = E^2(\omega_1) \sum_{\beta\gamma} \chi^{(2)}_{\alpha\beta\gamma} \hat{e}_\beta \hat{e}_\gamma e^{i2k_1 z} e^{-i2\omega_1 t} + c.c. .$$

---------- (2)

If we suppose the dielectric is a simple isotropic dielectric, or if we suppose the propagation direction of the incident wave is aligned with a principal axis of the dielectric tensor in the more general case, the electric field of the incident wave will lie in the xy plane. The incident wave is then transverse. However, In general, even in such a simple case the nonlinear polarization in (2) may have a component parallel to the z axis, as weIl as in the xy plane, because of the tensor character of $\chi_{\alpha\beta\gamma}$ It is convenient to break $P^{(N,L)}$ into two pieces, one parallel to z and one in the xy plane

$$P^{(NL)} = \hat{z}P_\parallel^{(NL)} + P_\perp^{(NL)}$$ ---------- (3)

The amplitude of the second harmonic field will have amplitude dependent on only z, for an incident wave of plane wave character. We write, for the second harmonic field at the frequency c

$$E(z, \omega_2) = \hat{z}E_\parallel(z, \omega_2) + E_\perp(z, \omega_2)$$ ---------- (4)

One may show rather easily that V2E - V(V· E) = iPEl./az2. Furthermore, it will simplify our discussion to assume the z direction is a principal axis, and in fact that the dielectric material is isotropic in its response to electric fields in the xy plane. The discussion would then be applicable to second harmonic generation in a crystal such as quartz, for the case where the incident beam propagates parallel to the optic axis. We combine the decompositions described above with the model,

$$\frac{\partial^2}{\partial z^2} E_\perp + \omega_2^2 \frac{\varepsilon_\perp(\omega_2)}{c^2} E_\perp + \frac{4\pi\omega_2^2}{c^2} P_\perp^{(NL)} + \left[\frac{\omega_2^2 \varepsilon_\parallel(\omega_2)}{c^2} E_\parallel + \frac{4\pi\omega_2^2}{c^2} P_\parallel^{(NL)} \right] \hat{z} = 0$$

---------- (5a)

And

$$\frac{\partial}{\partial z} [\varepsilon_\parallel(\omega_2) E_\parallel + 4\pi P_\parallel^{(NL)}] = 0$$

---------- (5b)

where $\varepsilon_\perp$ and $\varepsilon_\parallel$ describe the dielectric response perpendicular and parallel to the optic axis, respectively.

Satisfaction of (5a) requires the two conditions:

$$E_\parallel(\omega_2, z) = -\frac{4\pi}{\varepsilon_\parallel(\omega_2)} P_\parallel^{(NL)}(\omega_2, z)$$ ---------- (6a)

And

$$\left(\frac{\partial}{\partial z^2} + k_2^2 \right) E_\perp(\omega_2, z) = -\frac{4\pi\omega_2^2}{c^2} P_\perp^{(NL)}(\omega_2, z)$$

---------- (6b)

Note that satisfaction of (6a) insures that (5b) is obeyed. We have introduced

$$k_2 = \frac{\omega_2}{c}\sqrt{\varepsilon_\perp(\omega_2)} \equiv \frac{2\omega_1}{c}\sqrt{\varepsilon_\perp(2\omega_1)}$$ ---------- (7)

which is the wave vector of a wave of frequency $\omega 2 = 2\omega 1$ as it propagates freely in the medium.

The component of the second harmonic wave parallel to the optic axis, and to the direction of propagation of the second harmonic wave, is given by the simple expression in (6a).

The analysis of $\varepsilon_\perp$ will prove of more interest. While it is not a difficult matter to solve (6b) as it stands, in fact we can turn to an approximate treatment based on a scheme used commonly in situations such as the present. As one progresses along the propagation direction, the amplitude of the second harmonic builds up very slowly, as a consequence of the smallness of $\chi_{\alpha\beta\gamma}$. The amplitude changes very little, if we move just one wavelength. Thus, for the amplitude of the second harmonic wave, we write

$$\boldsymbol{E}_\perp(\omega_2, z) = \boldsymbol{E}(\omega_2, z)e^{ik_2 z}$$ ---------- (8)

where the spatial variation of $\exp(ik_2 z)$ is assumed rapid compared to that of $E(\omega_2, z)$. Then we have

$$\frac{\partial^2 \boldsymbol{E}_\perp}{\partial z^2} = -\left(k_2^2\boldsymbol{E} - 2ik_2\frac{\partial \boldsymbol{E}}{\partial z} - \frac{\partial^2 \boldsymbol{E}}{\partial z^2}\right)e^{ik_2 z}$$

$$\cong -\left(k_2^2\boldsymbol{E} - 2ik_2\frac{\partial \boldsymbol{E}}{\partial z}\right)e^{ik_2 z}.$$ ---------- (9)

When this is inserted into (6b), and we write $E = e^{\wedge(2)}E(\omega_2, z)$ where $e^{\wedge(2)}$ is a unit vector parallel to $P_\perp^{(N,L)}$, we may write (6b) in the form

$$\frac{\partial E(\omega_2, z)}{\partial z} = \frac{2\pi i\omega_2^2}{c^2 k_2}\bar{\chi}^{(2)}E^2(\omega_1)e^{i(2k_1 - k_2)z}$$ ---------- (10)

Where

$$\bar{\chi}^{(2)} = \sum_{\alpha\beta\gamma}\chi^{(2)}_{\alpha\beta\gamma}\hat{e}^{(2)}_\alpha\hat{e}_\beta\hat{e}_\gamma$$ ---------- (11)

The scheme used to obtain (10) is called the slowly varying envelope approximation. It is an elementary matter to integrate (10). We shall assume the surface of the material is at z = O. The second harmonic field vanishes there, and builds in intensity as one moves into the material. We thus integrate (10)subject to the boundary condition $E(\omega_2, 0) = O$. The result may be arranged to read, with $\Delta k = 2k_1 - k_2$,

$$E(\omega_2, z) = \frac{4\pi i\omega_2^2\bar{\chi}^{(2)}E^2(\omega_1)}{c^2 k_2}e^{i\Delta kz/2}\frac{\sin(\Delta kz/2)}{\Delta k}$$ ---------- (12)

The energy per unit area per unit time carried by the second harmonic is found by evaluating the Pointing vector. In the slowly varying envelope approximation,

we have for the magnitude S of the Pointing vector $S = c^2k_2|E(\omega_2, z)|^2/2\pi\omega_2$, or

$$S = \frac{8\pi\omega_2^3|\bar{\chi}^{(2)}|^2}{c^3k_2}|E(\omega_1)|^4\frac{\sin^2(\Delta kz/2)}{(\Delta k)^2} \quad \text{---------- (13)}$$

The crucial parameter which controls the intensity of the second harmonic output is Δk. Recall that $k_1 = \omega_1\sqrt{\varepsilon(\omega_1)}/C$ is the wave vector of the primary wave of frequencyω_1, while $k_2 = 2\omega_1\sqrt{\varepsilon(2\omega_2)}/C$ is that of a freely propagating wave with frequency $2\omega_1$. If the dielectric constant were to be independent of frequency, then $\varepsilon(2\omega_1)=\varepsilon(\omega_1)$, and we have Δk = O.

$$\lim_{\Delta k\to 0}\frac{\sin(\Delta kz/2)}{\Delta k} = \frac{z}{2} \quad \text{---------- (14)}$$

Upon noting that in this limit the field envelope $E_.(\omega_2, z)$ grows linearly with z, and the power flow in the second harmonic increases as Z^2. Clearly, at large values of z our perturbation theory which ignores depletion of the first harmonic, breaks down, though it is quite c1ear that we wish to achieve the condition Δk = 0 to generate an intense second harmonic wave. As we have seen earlier, the dielectric constant of any medium depends on frequency, unfortunately. Thus, in general, the condition Δk = 0 is not realized. We shall discuss shortly how one arrange to satisfy this condition.

A direct experimental test of the predictions of (14) can be found in the work of Maker and his colleagues [1]. For example, as the path length z is varied, one expects oscillations with distance of travel in the second harmonic output, for$\Delta k\neq 0$. In [1], the path length is varied by using a thin film to generate the second harmonic, and rotating the film. The path length in the medium is then d/cosΘ, where d is the film thickness and Θ the angle between the pump beam (in the crystal) and the normal to the film. We show the comparison between theory (solid line) and experiment (dots), taken from [1]. When the condition Δk = 0 is obeyed, the interaction which leads to second harmonic generation is said to be phase matched. Physically, the reason why a phase matched interaction leads to intense output is the following: The nonlinear dipole moment exhibits the spatial variation $\exp(i2k_1z)$, as we have seen. If we consider two small regions of space separated by the distance d, the phase difference between the dipoles in each responsible for generating the second harmonic radiation is thus $2k_1d$. The radiation emitted by each set of dipoles has the frequeney$\omega_2=2\omega_1$, and will propagate through the medium with the wave vector k_2 . If $k_2 = 2k_1$, the radiation field emitted from the set of dipoles in the first volume is exactly in phase with that emitted by the dipoles in the second. The fields reinforce coherently. If we add up the fields radiated by all the dipoles in the strip which lies between 0 and z, since all the dipole fields reinforce coherently, the intensity of the second harmonic is just proportional to Z^2.

If $\Delta k\neq 0$, the length $l_c= 1/|\Delta k|$ has the following interpretation: A strip of "second harmonic" dipoles with width l_c radiate second harmonic fields which reinforce constructively. Thus, for $z<< l_c$, the second harmonic fields grow linearly with z. As z increases beyond l_c , we encounter destructive interference, the field no longer grows monotonically, and in fact the combined effects of constructive and destructive interference lead to the oscillatory behaviour for the field envelope displayed in (13). The length l_c is called the coherence length of the nonlinear interaction.

Some comments follow from the above remarks. First notice that if the process of second harmonic generation is phase matched, then no matter how small $\chi^{-(2)}$ is, our perturbation theoretic treatment of the phenomenon breaks down if the optical path length in the medium is sufficiently long. Validity of the perturbation treatment requires the path length L be shorter than $L_c = c^2k_2/[2\pi\omega_2{}^2|\chi^{-2}E(\omega_1)|]$. If $L > L_c$, the intensity of the second harmonic is comparable to that of the pump beam, even though $\chi^{-(2)}$ is small, and $\mathbf{P}^{(NL)}$ is everywhere small compared to the contribution to the dipole moment per unit volume linear in the external field.

It follows also from the above that if we have a medium whose thickness is small compared to the coherence length l_c, then we need not be concerned about the question of phase matching. The second harmonic intensity is then independent of l_c.

There is another means of understanding the phenomenon of phase matching. While our treatment here is based entirely on classical physics, we may also adopt a quantum mechanical view point, which treats the incident beam as a collection of photons, each of energy $\hbar\omega_1$. When $\chi^{(2)}{}_{\alpha\beta\gamma} \neq 0$, the photons may interact with each other. This may be appreciated by noting that the presence of $\chi^{(2)}{}_{\alpha\beta\gamma}$ leads to a term in the energy density of the medium proportional to $\sum_{\alpha\beta\gamma} \chi^{(2)}{}_{\alpha\beta\gamma} E_\alpha E_\beta E_\gamma$ (these remarks assume $\chi^{(2)}{}_{\alpha\beta\gamma}$ is real, and ignore complications which arise from its frequency dependence). When the electric field is expressed in terms of the photon annihilation and creation operators, one sees this term leads to three photon interactions. Two photons may "fuse" to form a third. Thus, two quanta in the pump beam may fuse to form a single quantum of energy $2\hbar\omega_1$. While such an interaction clearly conserves energy, a photon of wave vector k also carries momentum $\hbar k$. Thus, unless $k_2 = 2k_1$, momentum is not conserved in the interaction. The full classical treatment given below of the problem of phase matched second harmonic generation will show that under such conditions, all the pump beam is converted to second harmonic. Thus the photons of frequency ω_1 fuse until the supply is exhausted.

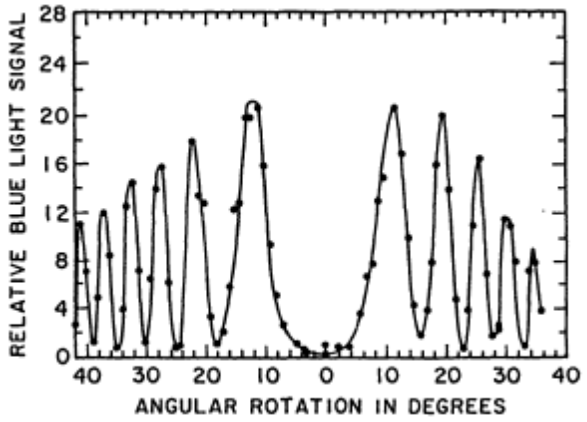

Fig. 4.1. The intensity of the second harmonic, as a function of rotation angle for a laser beam incident on a film of KDP. The experimental data (dots) is compared with theoretical prediction of (4.14) (solid line). This tests the dependence of the intensity of the second harmonic signal with path length in the medium; in (4.41), $z = d/\cos\theta$, with θ the angle between the pump beam inside the crystal, and the normal to the film. The figure has been reproduced from [4.1]

It is quite possible to give a theoretical treatment of second harmonic generation, with use of the photon annihilation and creation operators; of course the final answer must agree with that provided by our classical theory, when the occupation numbers of the states involved are large compared to unity. This is insured by the correspondence principle of quantum theory. Full quantum theoretical treatments are to be found in the literature. In the experience of the present

author, the classical approach to such problems is far more flexible and powerful. It is very tricky to incorporate the influence of absorption on the nonlinear interactions, within quantum theory, for example. Most such treatments ignore its role as a consequence, though in fact it is important in practice.

The treatment presented here, while very simple, in fact is valid in the presence of absorption as it stands, though in our discussion we regarded both $\varepsilon(\omega_1)$ and $\varepsilon(2\omega_1)$ as real. Also, the classical treatment is extended rather easily to incorporated boundary conditions and finite size effects (within the perturbation theoretic framework), while full quantum treatments of such influences are much more cumbersome, in the view of this writer. While we have encountered the notion of phase matching in the context of second harmonic generation, in fact the concept enters crucially into a diverse array of nonlinear interactions. A key ingredient critical to achieving an intense output is the realization of phase matching in a nonlinear interaction of interest. We next turn to a discussion of the means of achieving phase matching, in the specific case of second harmonic generation.

PHASE MATCHING EVOLUTION OF SHW UNDER PHASE MATCHING CONDITIONS:

In birefringent crystals for any given direction of propagation linear wave equation has two independent solutions.For a given $\vec{k}, \hat{a}$ has two possible solutions one corresponding to the ordinary ray and the other corresponding to the extra ordinary ray. In uniaxal crystals

$$\varepsilon = \begin{pmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \quad \text{---------- (1)}$$

in the frame of the principal axes of the crystal. If the direction of propagation makes an angle θ with the optic axis $\varepsilon_\parallel$, the o-ray has the refractive index $\eta_o = \sqrt{\varepsilon_\perp / \varepsilon_0}$ and its polarization vector $\hat{a}$ is perpendicular to the optics axis and the direction of propagation. For the e-ray, the polarization vector $\hat{a}$ lies in the plane containing the optic axis and the direction of propagation. It makes an angle $(\pi/2 - \alpha)$ with the direction of propagation such that

$$\frac{1}{\eta_e^2} = \frac{\varepsilon_0 \cos^2\theta}{\varepsilon_\perp} + \frac{\varepsilon_0 \sin^2\theta}{\varepsilon_\parallel} \quad \text{---------- (2)}$$

and the refractive index η_e is given by

$$\frac{1}{\eta_e^2} = \frac{\varepsilon_0 \cos^2\theta}{\varepsilon_\perp} + \frac{\varepsilon_0 \sin^2\theta}{\varepsilon_\parallel} \quad \text{---------- (3)}$$

In the normal dispersion region the refractive index varies like $\eta(\omega) = \eta(0) + A\omega^2$, so that the $\eta(2\omega) > \eta(\omega)$ for a given mode (or ordinary (o) or extraordinary (e)).

For a positive uniaxial crystal $\eta_e(\theta)$ lies between $\sqrt{\varepsilon_{\parallel}/\varepsilon_o} = \eta_{\parallel} > \eta_o \; and \; \eta_o$
Thus, if we chose the polarization such that the fundamental beam propagate as the e-wave and the second harmonic wave as o-wave, it is possible to achieve $\eta_e(\omega) = \eta_o(2\omega)$ for some value of θ, the angle between the optic axis and the deviation of the propagation, provides the birefringence $\eta_e(\omega) - \eta_o(\omega)$ is sufficiently large.

Other experimental issues:

1. Walk-off: As shown in Figure 1.4 the direction of the Poynting vector may differ from the direction of propagation which is normal to the planes of constant phase. Since practical light waves always have finite lateral size, this leads to reducing overlap between the fundamental frequency wave and the second harmonic wave because one of them is ordinary and the other is extra-ordinary wave. This effectively reduces the interaction length. The angle between $\vec{E}$ and $\vec{D}$ is called the walk off angle.

2. Noncritical Phase-Matching: If phase matching occurs at $\theta = \pi/2$, the $\vec{E}$ and $\vec{D}$ are again parallel and there is no walk off. This is called noncritical phase matching and is desirable for obtaining high conversion efficiency.

3. When birefringence is sufficient some times phase matching can be obtained for the process in which fundamental beam is split into an ordinary wave and an extraordinary wave. This is called type 2 phase matching. Since the two fundamental frequency waves now travel with different phase velocities they have to be treated as independent. So we now need to write 3 coupled equations –one for the second harmonic and two for the two polarizations of the fundamental wave this is a 3 wave mixing process.

4. For increasing the efficiency of second harmonic generation we need to increase the intensity. This can be done by focusing the fundamental wave but the tighter the focusing, faster the spread of the focal spot due to diffraction. This again limits the effective interaction length of the crystal. There is thus an optimum focusing condition.

5. Phase matching only determines the angle between the optic axis and the direction of propagation. The azimuthal angle is chosen to maximize $\chi^{(2)}_{eff}$, the effective coupling coefficient.

Quasi phase matching:

For isotropic crystals BPM is not possible. Since some crystals like semiconductors have large nonlinearities as well as other desirable properties like availability in good quality and relatively large sizes, alternate methods of compensation of phase mismatch were tried. We discuss these methods all originally proposed by ABDP. The most important of them is the quasi phase matching in which the sign of $\chi^{(2)}_{eff}$ is reversed after each coherence length. Since

$$\chi^{(2)}_{eff} = \sum_{i,j,k} \chi^{(2)}_{ijk}(-2\omega, \omega, \omega) a_{2i} a_{1j} a_{1k} \quad \text{---------- (4)}$$

it follows that the sign of $\chi^{(2)}_{eff}$ can be reversed if the crystal is rotated by 180° about the direction of propagation. This is because such a rotation reverses the direction of every vector normal to it

and all the polarization vectors are usually chosen to lie in a plane normal to the direction of propagation by cutting the crystal suitably.

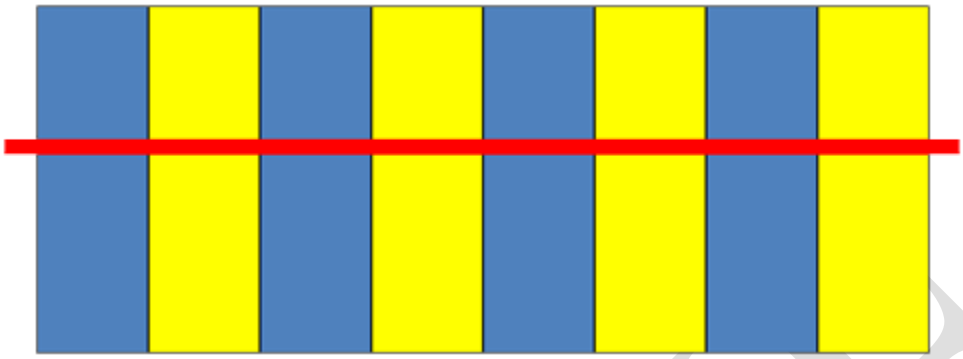

Figure 1 : $\chi_{eff}^{(2)}$ in a quasi-phase matched stack. Alternate segments are rotated by 180° about the direction of propagation shown by red line.

We can understand the QPM process in two ways.

First following the discussion we note that the rate at which the polarization radiates energy in the field at frequency Ω is $\sim \mathrm{Im}(P^{NLS}(\Omega).E^{*}(\Omega))$ which oscillates with a period $2\pi/\left|\vec{k}_{NLS}-\vec{k}\right|$ like where the denominator is the mismatch between the propagation vector for the nonlinear polarization and that for the field at the frequency Ω. The polarization travels with a wave vector determined by the fields that create this polarization. Thus the second harmonic grows till distance in the crystal becomes $l_{coh}=\pi/\left|\vec{k}_{NLS}-\vec{k}\right|$ when the direction of energy flow reverses. If now the sign $\chi_{eff}^{(2)}$ is reversed the second harmonic will continue growing for one more distance of $\pi/\left|\vec{k}_{NLS}-\vec{k}\right|$. The key idea, first proposed by ABDP is that phase mismatch can be compensated periodically. Next, let us see this more quantitatively in the small signal approximation.

In the first crystal of length $l_{coh}=\pi/\left|\vec{k}_{NLS}-\vec{k}\right|$, the amplitude of the second harmonic wave is given by

$$A_2 = zK'e^{-(i\Delta kz/2)}\frac{\sin(\Delta kz/2)}{(\Delta kz/2)} \quad \text{---------- (5)}$$

In the second crystal, the stating amplitude is $A_2(0)=-i(2/\pi)l_{coh}K'$ and the equation it follows is

$$\frac{\partial A_2}{\partial z} = -K'e^{-i\Delta kz} \quad \text{---------- (6)}$$

since the sign of $\chi_{eff}^{(2)}$ is reversed and the right hand side is proportional to it. Solving this gives

$A_2 = -i\,2(2/\pi) l_{coh} K'$ ---------- (7)

at the end od second crystal of length $z = l_{coh}$. Similarly at the end of the third crystal amplitude A2 is 3 time that in one coherence length.

This shows that the growth of the second harmonic in a QPM stack of crystals is like that in a BPM with $\chi^{(2)}_{eff}$ reduced by a factor 2/π.

This can be seen in yet another way which yields greater insight into this equivalence.

We again solve the equation of evolution of the second harmonic but assume now that the coupling constant K' is a periodic function of z. It can then be expanded into a Fourier series.

$$K'(z) = \sum_{n=-\infty}^{\infty} c_n e^{ik_n z}$$ ---------- (8)

with $k_n = 2\pi n / \Lambda$ where L is the period.

Substituting this in equation we get

$$\frac{\partial A_2}{\partial z} = K'(z) e^{-i\Delta k z} = \sum_{n=-\infty}^{\infty} c_n e^{i(k_n - \Delta k)z}$$ ---------- (9)

Integrating this we obtain

$$A_2 = \sum_{n=-\infty}^{\infty} c_n e^{i(k_n - \Delta k)z/2} \frac{\sin((k_n - \Delta k)z/2}{(k_n - \Delta k)/2}$$ ---------- (10)

If now the period of variation of $\chi^{(2)}_{eff}$ is chosen to be $\Lambda = 2l_{coh}$, $k_1 = \Delta k$ and only one term will dominate as z increases and we can write

$A_2 = c_1 z$ ---------- (11)

For the case when $\chi^{(2)}_{eff}$ is just changing sign in alternate segments $c_n = K'(2/n\pi)$ and so a QPM stack behaves exactly like a phase matched stack with $\chi^{(2)}_{eff}$ reduced by a factor of 2/π. This simple derivation also tells us more

1. Higher order quasi phase matching is possible in which the sign of $\chi^{(2)}_{eff}$ is changed after an odd number of coherence lengths but the $\chi^{(2)}_{eff}$ is reduced by 2/m π if mth order QPM is used. This is useful when the coherence length is too small for convenient fabrication. This is depicted in the Figure below.

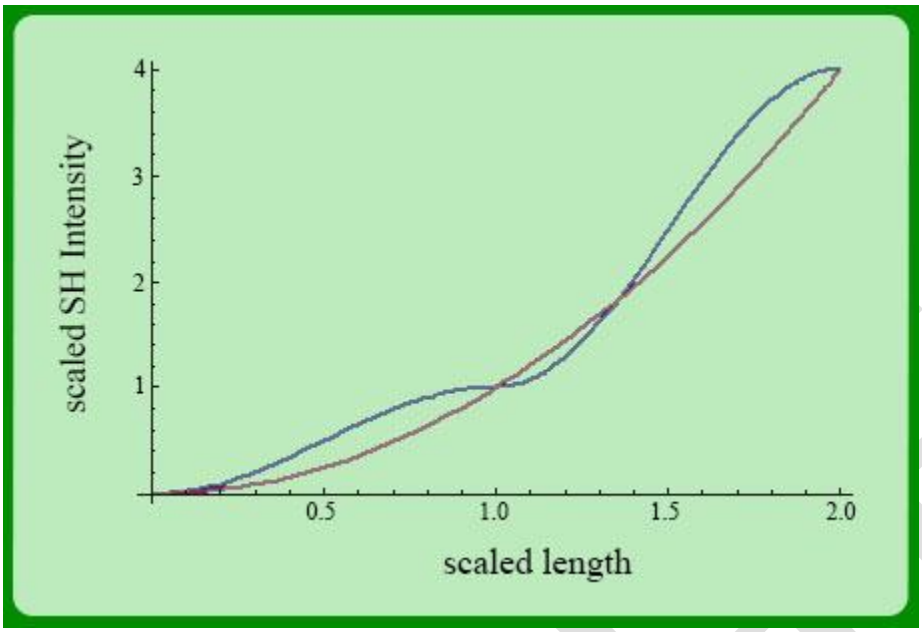

Figure 2: The second harmonic growth in the first two segments of the quasi-phase-matched stack. Length is scaled to the coherence length and the second harmonic intensity to that produced in the first segment. The blue curve corresponds to the solutions depicted by eqs (5) and (7). The magenta curve corresponds to the approximation given by eq (11). The difference between the two is due to neglect of higher Fourier components in eq (11).

2. The periodicity of $\chi^{(2)}$ is of importance and not its abrupt change of sign. Although at present it seems simpler to fabricate QPM structures with abrupt change of sign of $\chi^{(2)}$ with epitaxial techniques other structures may be possible.

3. when the period of $\chi^{(2)}$ does not match any odd multiple of $2l_{coh}$, other Fourier components become important.

FOUR WAVE MIXING SPECTROSCOPY:

There are a variety of other nonlinear interactions one may explore, as the remarks of the previous paragraph suggest. In this section, we comment on two that prove particularly interesting, in the view of the present author. Both of these are referred to as four wave interactions, and are mediated by the third order susceptibility $\chi^{(3)}_{\alpha\beta\gamma\delta}$.

The first is the process diagrammed in Fig 1. We have the interaction between two laser beams, one with frequency ω_1 and wave vector k_1 and one with frequency ω_2 and wave vector k_2. The output is at frequency $2\ \omega_1 - \omega_2$ and with wave vector $2k_l - k_2$.

A description of this process must recognize that the output can be generated in various steps, similar in character to the processes diagrammed in Fig 1. Three waves may mix directly through

$\chi^{(3)}_{\alpha\beta\gamma\delta}$, to produce a nonlinear dipole moment $P^{(NL)}_{\alpha}$ with the desired character. The input electric field is

$$E_\alpha(\boldsymbol{r}, t) = E_\alpha(\boldsymbol{k}_1\omega_1)e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)} + E_\alpha(\boldsymbol{k}_2\omega_2)e^{i(\boldsymbol{k}_2\cdot\boldsymbol{r}-\omega_2 t)} + \text{c.c.} \quad \text{---------- (1)}$$

and when due account is taken of the fact that $\chi^{(3)}_{\alpha\beta\gamma\delta}(k_1\omega_1, k_2\omega_2, k_3\omega_3)$ necessarily invariant under interchange of any pair of the collection $(k_1\omega_1\beta)$, $(k_2\omega_2\gamma)$, $(k_3\omega_3\delta)$, one has the contribution to the nonlinear moment given by

$$P^{(NL)}_\alpha(\boldsymbol{rt}) = 3\sum_{\beta\gamma\delta}\chi^{(3)}_{\alpha\beta\gamma\delta}E_\beta(\boldsymbol{k}_1\omega_1)E_\gamma(\boldsymbol{k}_1\omega_1)E^*_\delta(\boldsymbol{k}_2\omega_2) \times e^{i(2k_1-k_2)\cdot r}e^{-i(2\omega_1-\omega_2)t}. \quad \text{---------- (2)}$$

This may be inserted into the basic wave equation, to generate an expression for the output electric field. Within the framework of perturbation theory the analysis proceeds along lines very similar to our discussion of second harmonic generation.

However, in a medium which lacks an inversion center, by means of $\chi^{(2)}_{\alpha\beta\gamma}$ a wave of frequency ω_1 and wave vector k_1 may mix with a wave of frequency ω_2 and wave vector k_2 to produce a dipole moment per unit volume, and hence radiation at frequency ω_1 - ω_2, and wave vector k_1. The electric field so generated may then "mix" with the wave of frequency ω_1 and wave vector k_1 in the original beam again through action of $\chi^{(2)}_{\alpha\beta\gamma}$, to produce the output of interest. A second two step process involves the generation first of the second harmonic at $2\omega_1$, $2k_1$ of the input wave at frequency ω_1 then a second interaction with the "ω_2" wave to produce the output. We thus have a second contribution to the nonlinear dipole moment per unit volume that may be written as

$$P^{(NL)}_\alpha(\boldsymbol{r}, t) = 2\sum_{\beta\gamma}[\chi^{(2)}_{\alpha\beta\gamma}(\boldsymbol{k}_1 - \boldsymbol{k}_2, \omega_1 - \omega_2; \boldsymbol{k}_1\omega_1) \times E_\beta(\boldsymbol{k}_1 - \boldsymbol{k}_2, \omega_1 - \omega_2)E_\gamma(\boldsymbol{k}_1, \omega_1) + \chi^{(2)}_{\alpha\beta\gamma}(2\boldsymbol{k}_1, 2\omega_1; -\boldsymbol{k}_2 - \omega_2)E_\beta(2\boldsymbol{k}_1, 2\omega_1)E^*_\gamma(\boldsymbol{k}_2, \omega_2)] \times e^{i(2\boldsymbol{k}_1-\boldsymbol{k}_2)\cdot\boldsymbol{r}}e^{-i(2\omega_1-\omega_2)t}. \quad \text{---------- (3)}$$

The total output at the frequency $2\omega_1$ - ω_2 must be found by adding the dipole moment per unit volume in (3) to that in (2), then inserting the sum into the wave equation.

We can calculate the intermediate fields $E_\beta(k_1 - k_2, \omega_1 - \omega_2)$ and $E_\beta(2k_1, 2\omega_1)$, which, for simplicity we apply to an isotropic dielectric. Suppose on the right-hand side of this expression, we let $P^{(NL)}_\alpha(r, \omega)$ have the spatial variation exp(iQ· r). The solution of the equation will have $E_\alpha(r, \omega) = E_\alpha(Q, \omega)\exp(iQ\cdot r)$, and one finds $E_\alpha(Q, \omega)$ by inverting a 3 x 3 matrix:

$$\sum_\beta\left\{\delta_{\alpha\beta}\left[Q^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)\right] - Q_\alpha Q_\beta\right\}E_\beta(\boldsymbol{Q}, \omega) = \frac{4\pi\omega^2}{c^2}P^{(NL)}_\alpha(\boldsymbol{Q}, \omega) \quad \text{---------- (4)}$$

The result of the matrix inversion may be written, with $\hat{Q}$ a unit vector in the direction of Q,

$$E_\alpha(\boldsymbol{Q},\omega) = -\frac{4\pi}{\varepsilon(\omega)}\hat{Q}_\alpha[\hat{\mathbf{Q}}\cdot\boldsymbol{P}^{(\mathrm{NL})}(\boldsymbol{Q},\omega)] + \frac{4\pi\omega^2}{c^2}\frac{\{P_\alpha^{(\mathrm{NL})}(\boldsymbol{Q},\omega) - Q_\alpha[\boldsymbol{Q}\cdot\boldsymbol{P}^{(\mathrm{NL})}(\boldsymbol{Q},\omega)]\}}{Q^2 - \frac{\omega^2}{c^2}\varepsilon(\omega)}$$

---------- (5)

We may now choose the combination $Q = k_1 - k_2$, $\omega = \omega_1 - \omega_2$, and then the combination $Q = 2k_1$ and $\omega = 2\omega_1$ to generate the Fourier components $E_\beta(k_1 - k_2, \omega_1 - \omega_2)$ and $E_\beta(2k_1, 2\omega_1)$ which appear in (3). We are to take for this purpose, respectively,

$$P_\alpha^{(\mathrm{NL})}(\boldsymbol{k}_1 - \boldsymbol{k}_2, \omega_1 - \omega_2) = 2\sum_{\beta\gamma}\chi^{(2)}_{\alpha\beta\gamma}(\boldsymbol{k}_1\omega_1; -\boldsymbol{k}_2\omega_2)E_\beta(\boldsymbol{k}_1\omega_1)E^*_\gamma(\boldsymbol{k}_2\omega_2)$$

---------- (6a)

And

$$P_\alpha^{(\mathrm{NL})}(2\boldsymbol{k}_1, 2\omega_1) = 2\sum_{\beta\gamma}\chi^{(2)}_{\alpha\beta\gamma}(\boldsymbol{k}_1\omega_1; \boldsymbol{k}_1\omega_1)E_\beta(\boldsymbol{k}_1\omega_1)E_\gamma(\boldsymbol{k}_1\omega_1)$$

---------- (6b)

When these expressions are combined, in a notation that overlooks the wave vector dependence of the various elements of the nonlinear susceptibility tensor, we have a rather complex but rich expression for the nonlinear moment:

$$P_\alpha^{(\mathrm{NL})}(\boldsymbol{r},t) = 3e^{i(2\boldsymbol{k}_1-\boldsymbol{k}_2)\cdot\boldsymbol{r}}e^{-i(2\omega_1-\omega_2)t}\sum_{\beta\gamma\delta}\Bigg[\chi^{(3)}_{\alpha\beta\gamma\delta}(\omega_1;\omega_1;-\omega_2) + \frac{16\pi}{3}\sum_{\mu\nu}\chi^{(2)}_{\alpha\nu\beta}(\omega_1-\omega_2;\omega_1)\Bigg\{\frac{(\omega_1-\omega_2)^2[\delta_{\mu\nu}-\hat{n}_\nu(12)\hat{n}_\mu(12)]}{c^2|\boldsymbol{k}_1-\boldsymbol{k}_2|^2-(\omega_1-\omega_2)^2\varepsilon(\omega_1-\omega_2)} - \frac{\hat{n}_\nu(12)\hat{n}_\mu(12)}{\varepsilon(\omega_1-\omega_2)}\Bigg\}\chi^{(2)}_{\mu\gamma\delta}(\omega_1;-\omega_2) + \frac{16\pi}{3}\sum_{\mu\nu}\chi^{(2)}_{\alpha\nu\delta}(2\omega_1;-\omega_2)\Bigg\{\frac{(2\omega_1)^2(\delta_{\mu\nu}-\hat{k}_{1\nu}\hat{k}_{1\mu})}{c^2(2k_1)^2-(2\omega_1)^2\varepsilon(2\omega_1)} - \frac{\hat{k}_{1\nu}\hat{k}_{1\mu}}{\varepsilon(2\omega_1)}\Bigg\}\chi^{(2)}_{\mu\beta\gamma}(\omega_1;\omega_1)\Bigg]E_\beta(k_1\omega_1)E_\gamma(k_1\omega_1)E^*_\delta(k_2\omega_2)\ .$$

---------- (7)

In these expressions, $\hat{n}(12)$ is a unit vector in the direction of $k_1 - k_2$.

The output of the four wave mixing process is proportional to the square of the modulus of the magnitude of $P_\alpha^{(\mathrm{NL})}(r,t)$; this is controlled by the quantity in the large square brackets, which may be viewed as an effective third order susceptibility, in which the direct contribution $\chi^{(3)}_{\alpha\beta\gamma\delta}$ is supplemented by that

From $\chi^{(2)}_{\alpha\beta\gamma}$ taken to second order in perturbation theory.

Of particular interest are the energy denominators in the second order terms, which lead to resonances in the output of the four wave mixing experiment, under suitable conditions. In a typical experiment, ω_1 and ω_2 lie in the visible range, but the difference ω_1 - ω_2 may lie in the infrared. If one of the two laser sources is a dye laser, then one of the two frequencies may be "tuned," thus allowing the difference (ω_1 - ω_2) to be swept through resonances in the second and third term of (7). If ω_1 and ω_2 are fixed, the angle between the two beams may be varied, so one may scan the variable $|k_1 - k_2|$ and tune through resonance by this means.

The dispersion relation of an electromagnetic wave of frequency Ω and wave vector Q is given in an isotropic dielectric by

$$c^2Q^2 = \Omega^2\varepsilon(\Omega) \quad \text{---------- (8)}$$

Thus, the resonance in the second term of (7) occurs when $|k_1 - k_2|$ and (ω_1 - ω_2) are "tuned" to the wave vector and frequency of an electromagnetic normal mode of the material. One is driving the system at the difference frequency (ω_1 - ω_2) and wave vector ($k_1 - k_2$), through the contribution to the nonlinear dipole moment in (6 a). The response, which contains the resonance just described, is monitored in the visible frequency range at the frequency $2\omega_1$ - ω_2, through the electromagnetic wave generated by the third order dipole moment given in (3).

In the infrared frequency range, the lattice vibration modes of solids lead to sharp absorption lines in the infrared. Associated with each absorption line is a resonance in the dieletric constant. The four wave mixing process described here allows one to probe the dielectric response of the material in the infrared, though an experiment carried out at visible frequencies. This can be most useful, when applied to materials opaque in the infrared, but nominally transparent in the visible. One measures, through this technique, the dispersion curves of electromagnetic radiation in the near vicinity of ω_O; If the condition $C^2|k_1 - k_2|^2 \gg \varepsilon_\infty\omega_1 - \omega_2{}^2$ can be realized, then, one sees the resonance in the four wave mixing process lies very near the frequency ω_O of the vibration which produces the singularity in E(ω). One thus can use the method as a form of vibrational spectroscopy. More generally, one can probe features of the dispersion curves of electromagnetic waves near a vibrational resonance by this means;

We may refer to the resonance just described as the polariton resonance. For it to be excited, one must arrange for the nonlinear dipole moment $P_\alpha^{(NL)}$ in (6 a) to have a component transverse to the wave vector $k_1 - k_2$. One may appreciate this by noting the presence of the factor $[\delta_{\mu\nu} - \widehat{n_\mu}(12)\widehat{n_\nu}(12)]$

in the second term of (7). There is a second resonance in (7), as $\omega_1 - \omega_2$ is scanned through frequency regime where ε(ω) itself has a singularity This comes from the third term in (7), in which $\varepsilon(\omega_1 - \omega_2)$ appears in the denominator. Just above ω_O , there is a frequency where the real part of the dielectric constant vanishes, and $\varepsilon_2(\omega)$ will be very small if the absorption line is sharp. If we ignore absorption, then this frequency, denoted below as ω_L is found by setting ε(ω) to zero. One finds

$$\omega_L^2 = \omega_0^2 + \frac{\Omega_p^2}{\varepsilon_\infty} \equiv \left(\frac{\varepsilon_s}{\varepsilon_\infty}\right)\omega_0^2 \quad \text{---------- (9)}$$

where $\varepsilon_s = \varepsilon_\infty + \Omega_p^2/\varepsilon_\infty$ is the static dielectric constant. For this resonance to be excited, the nonlinear dipole moment at $(\omega_1 - \omega_2)$ must have a nonzero component parallel to $k_1 - k_2$ as one sees from the factors $\widehat{n_\mu}(12)\widehat{n_\nu}(12)$ in (7).

The resonance in the second term, excited by the transverse component of $P^{(NL)}$ at the frequency $\omega_1 - \omega_2$,had its origin in the coincidence between the intermediate state wave at frequency $(\omega_1 - \omega_2)$ and wave vector $(k_1 - k_2)$ and the electromagnetic normal mode of the medium subjected to this driving force. It is also the case that the resonance at ω_L has its origin in a true normal mode of the system excited by the $\omega_1 - \omega_2$ wave. This is a longitudinal normal mode, whose character is understood from the following:

The normal modes of such a dielectric medium requires us to explore the full set of Maxwell equations including the statement, for a disturbance of frequency ω.

$$\nabla \cdot \mathbf{D} = \varepsilon(\omega)\nabla \cdot \mathbf{E} = 0 \quad \text{---------- (10)}$$

For a transverse electromagnetic wave of wave vector k, the electric field E is perpendicular to k, so $\nabla \cdot E = ik \cdot E \equiv O$. Hence (10) is satisfied at any frequency.

Consider a longitudinal wave, in which k and E are parallel. Now $\nabla \cdot E \neq 0$, and in general such a disturbance fails to satisfy (10). However, at the special frequency ω_L where $\varepsilon(\omega_L) \equiv 0$, this equation is satisfied, and the remaining Maxwell equations are as well; one has k x E = 0, and this requires

B = O. Thus, no magnetic field is set up by the longitudinal wave.

Thus, a dielectric medium described by the dielectric constant ε(ω) supports not only transverse electromagnetic modes whose dispersion relation, electromagnetic modes whose dispersion relation is illustrated but in addition supports a longitudinal normal mode whose frequency ω_L is independent of wave vector (within our macroscopic theory, which is correct for wavelengths long compared to the underlying microscopic lengths). In ionic solids, such longitudinal modes are referred to as longitudinal

optical phonons; is referred to as the Lyddane-Sachs-Teller relation.

Four wave mixing, with attention to the resonances in the output induced by the second order process, has evolved into a powerful form of laser spectroscopy. The discussion presented here confirms its attention to the case where the second order susceptibility $\chi^{(2)}_{\alpha\beta\gamma}$ is responsible for the appearance of the

second order term. Thus, the phenomenon is confirmed to materials (crystals) which lack an inversion center. This term contributes to the optical response of materials which have inversion symmetry, including liquids and gasses. The Raman nonlinearity also provides a second order contribution to the effective third order susceptibility tensor, which exhibits a resonance rather similar to those in (10), as $(\omega_1 - \omega_2)$ is tuned through a vibrational resonance of the material. Thus, four wave mixing proves to be a versatile probe of matter, that can be applied to a wide class of materials. The technique is referred to by the acronym CARS, which is an abbreviation for Coherent Anti-Stokes Raman Spectroscopy. We shall appreciate the significance of the acronym when we tum our attention to four wave mixing spectroscopy.

Before we leave this topic, we call attention to the frequency dependence of the basic tensors $\chi^{(3)}_{\alpha\beta\gamma\delta}$ and $\chi^{(2)}_{\alpha\beta\gamma}$ in (10). If ω_1 and ω_2 lie in the visible frequency range, these response tensors are dominated by the contribution from the electronic degrees of freedom. As we have discussed earlier, there are resonances in these response functions when either input frequency lies close to that of an isolated electronic excited state, and also when the output frequency satisfies this condition. If infrared beams are used there can be contributions to the nonlinear susceptibility from nuclear motions excited by frequencies ω_1 and ω_2 and $2\omega_1$ - ω_2. We then encounter resonances in these quantities directly, when a wave frequency lies close to a vibrational resonance. We can see that the presence of anhannonicity leads to resonances in $\chi^{(3)}_{\alpha\beta\gamma\delta}$ when ω_1 - ω_2 or $2\omega_1$ lies near the vibrational frequency of the molecule. There have been few experimental studies of the resonances in nonlinear susceptibility tensors in the infrared, at the time of this writing.

The discussion so far has assumed the various interacting waves are simple plane waves, such as those emitted by a cw laser. The CARS technique proves most powerful when used in the pulsed mode of operation. One generates very short pulses with frequency and wave vector $(k_1\omega_{1)}$and $(k_2\omega_2)$, with the experimental geometry arranged so that one is in resonance with a vibrational mode or polariton of frequency $(\omega_1 - \omega_2)$, and wave vector $(k_1 - k_2)$. The pulses are arranged to physically overlap in the material. Assume the pulse duration T is short compared to the natural lifetime τ of the mode excited; this condition can be realized through use of pulses in the picosecond range, if we have the vibrational and polariton modes of dense matter in mind. The mode thus "rings" for a time τ after the pulses disappear. One then sends in a pulse $(\omega_1 k_1)$ into the excitation region a time Δt later. This mixes with the excitation $(\omega_1$ - $\omega_2, k_1 - k_2)$, to produce output a $2\omega_1$ - ω_2, $2k_1 - k_2$. The intensity of the signal is proportional to exp(- Δt / T), assuming exponential decay of the mode. By measuring the dependence of the relative intensity of the CARS signal on Δt, one has a direct, real time measurement of the lifetime of the mode. It can be the case that the decay is not a simple exponential. In this case, the method allows direct study of the real time decay profile.

This is an example of a class of experiments which have come to be known as "pulse-probe" experiments. Measurements can be devised to monitor the real time evolution of excited state populations in diverse systems. Current laser technology allows one to generate pulses as short as 10 femtoseconds. Through use of such pulses, one can obtain "snapshots" of the time profile of Phenomena that are characterized by time scales as short as 10^{-14} s.

OPTICAL PHASE CONJUGATION:

A brief discussion of a fascinating application of four wave mixing, referred to as the process of optical phase conjugation. This is once again a four wave mixing process; this time each of the waves involved has precisely the same frequency ω. One has two counter-propagating input waves, one with wave vector +k, and one wave vector -k. There is a signal wave to be processed, and this has frequency also, but need not be a simple plane wave. Thus, in the medium, within the interaction volume, we have an electric field we may write in real notation as

$$E_\alpha(\boldsymbol{r},t) = E_\alpha^{(+)}\cos(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \phi_+) + E_\alpha^{(-)}\cos(\boldsymbol{k}\cdot\boldsymbol{r} + \omega t - \phi_-)$$

$$+ \sum_i E_\alpha^{(s)}(i)\cos(\boldsymbol{k}_i\cdot\boldsymbol{r} - \omega t + \phi_i)\ .$$

---------- (1)

The last term is the signal wave of frequency ω, which may have a complex wave form represented by superimposing a range of wave vectors $\{k_i\}$. A plane wave which passes through a lens, or an in homogenous distorting medium could be represented by such a form.

The interaction between the three waves provided by the third order susceptibility $\chi^{(3)}_{\alpha\beta\gamma}$ is responsible for phase conjugation. We thus require the form of the function

$$F_\alpha(\boldsymbol{r},t) = \cos(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \phi_+)\cos(\boldsymbol{k}\cdot\boldsymbol{r} + \omega t - \phi_-)$$

$$\times \sum_i E_\alpha^{(s)}(i)\cos(\boldsymbol{k}_i\cdot\boldsymbol{r} - \omega t + \phi_i)$$

---------- (2)

to understand the output. Simple trigonometric identities give, with $\Delta t = (\Phi_+ + \Phi_-)/\omega$.

$$F_\alpha(\boldsymbol{r},t) = \frac{1}{4}\sum_i E_\alpha^{(s)}(i)\cos[\boldsymbol{k}_i\cdot\boldsymbol{r} + \omega(t-\Delta t) + \phi_i]$$

$$+ \frac{1}{4}\sum_i E_\alpha^{(s)}(i)\cos(\boldsymbol{k}_i\cdot\boldsymbol{r} - 3\omega t + \phi_i + \phi_- + \phi_+)$$

$$+ \frac{1}{4}\sum_i E_\alpha^{(s)}(i)\cos[(2\boldsymbol{k} + \boldsymbol{k}_i)\cdot\boldsymbol{r} - \omega t + \phi_- + \phi_+]$$

$$+ \frac{1}{4}\sum_i E_\alpha^{(s)}(i)\cos[(2\boldsymbol{k} - \boldsymbol{k}_i)\cdot\boldsymbol{r} + \omega t - \phi_i + \phi_- + \phi_+]$$

---------- (3)

The first term is the term of interest. Its form is precisely the same as the input signal in (1), save for the replacement $t \rightarrow -t$, and the introduction of an uninteresting time delay Δt. The four wave mixing process is thus a time reversal operation, which sends a signal back to the generating source, whose wave form is identical to that of the input signal.

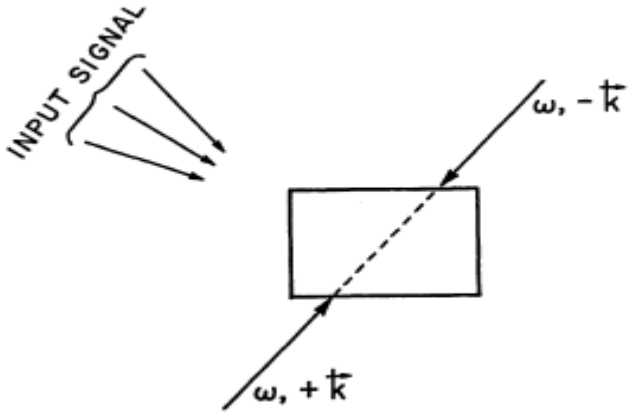

Fig. 4.4. The input waves in the optical phase conjugation process. We have two counter-propagating laser beams of frequency ω, and wave vectors $+k$ and $-k$. The input signal consists of an electromagnetic signal of frequency ω, and the nature of the output signal is discussed in the text. The rectangular region is the interaction volume

The process that leads to the time reversal can be described as a nonlinear mixing of three waves of frequency ω, one with wave vector +k, one with wave vector -k, and one with wave vector k_i , to produce output at frequency wand wave vector $–k_i$. This interaction is always phase matched, notice. Thus, if the interaction volume is large enough, or the counter-propagating beams sufficiently intense, one can actually amplify an incoming signal by this means. The remaining terms in (3) describe various nonlinear mixings between the input waves that in general are not phase matched, and which thus led to feeble output beams.

It is interesting to compare the effect of the phase conjugation process to that of an ordinary (passive) mirror, that also reflects a pulse back in the direction of the generating source, at least at normal incidence. We illustrate the two cases, we show the influence of a lens on the wave-fronts of a perfect plane wave. A reflector is placed behind the lens. If the reflector is a phase conjugate reflector, its effect is the time reversal operation illustrated. After the reflected pulse passes through the lens, a perfect plane wave emerges, as illustrated. The lens can be replaced by any inhomogeneous or non uniform medium, such as shower door glass, and the emergent beam will be identical in form to the incident beam, if phase conjugate reflection is employed. On the other hand, as we see, reflection from an ordinary mirror provides a distorted wave form which is modified further by its second pass through the in homogeneous medium. The property of phase conjugate reflection just described opens up very interesting possible applications .

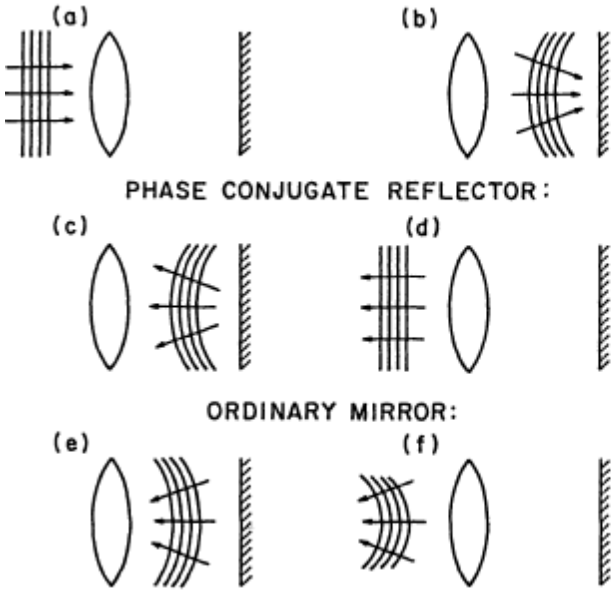

Fig. 4.5. A comparison between a phase conjugate reflector, and an ordinary mirror. In **a**, a plane wave is incident on a lens, which will distort its wave front. Behind the lens is a mirror. In **b**, the influence of the lens on the wave front is illustrated. If the mirror is a phase conjugate reflector, we show the emerging wave pattern **c** before and **d** after it has passed through the lens. In **e** and **f** we show the influence of an ordinary mirror

The two examples discussed above provide illustrations of the fascinating new applications of optical techniques opened up by non linear optics. We shall see further examples as we proceed.

NONLINEAR MATERIAL:

Right from the initial days it was realized that nonlinear optical effects provide a means of constructing new devices. The earliest nonlinear optical materials were inorganic crystals - usually those materials for which crystals in reasonably good optical quality and size (~ 1 cm^3) available. The earliest guide in the search for nonlinear optical materials was provided by Miller's rule discussed earlier in Lecture 7 which related the second order susceptibility $\chi^{(2)}$with $\chi^{(1)}$ through $\chi_{ijk}^{(2)}(\Omega,\omega_1,\omega_2) = \Delta_{ijk}\,\chi_{ii}^{(1)}(\Omega)\,\chi_{jj}^{(1)}(\omega_1)\,\chi_{kk}^{(1)}(\omega_2)$

The Miller's delta Δ_{ijk} was found to be nearly independent of material. The scope of search for new nonlinear optical materials broadened enormously- first, by the development of the powder second harmonic generation method by Kurtz and. Perry and later by the discovery of the optical Kerr shutter, degenerate four wave mixing (DFWM) and the Z-scan methods discussed in later lectures. The main application driving the search for third order nonlinear optical materials is "all optical logic" and switching while for second order materials it remains frequency conversion, specially of coherent sources based on semiconductor diode lasers.

Oxide nonlinear optical crystals:

Most of these crystals are ferroelectric although they are not always used in their ferroelectric phase. On the otherhand the ferroelectric property has been cruicial in poling them for quasi-phase-matching.

The nonlinear polarizable unit in many of these crystals such as $LiNbO_3$ and Li TaO_3 are the oxygen octahedra shown in Figure below.

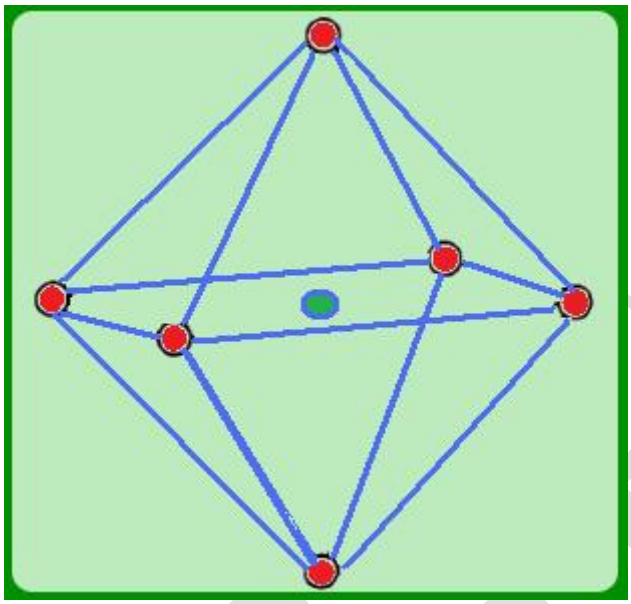

Figure : Oxygen octahedron –the basic building block of many nonlinear optical ferroelectric crystals (red circles: oxygen atoms, green circle cation)

The position of the central B cation atom becomes unstable at temperatures below the Curie Temperature. Once this atom moves of centre the above unit acquires a non-vanishing hyperpolarizability β. The polarizabilty is largely due partially delocalized p electrons of oxygen atoms. These are perturbed by the displacement of the B cation atoms. One can then relate βijk and hence χ(2) to the ionic displacement. This provided the first understanding of nonlinear response of these crystals. By now several first principle band structure based calculations have been reported for nonlinear susceptibility χ(2) based on sum over states expressions. It is fair to say that theory of nonlinear susceptibility for crystals has not played a crucial role in the development of new second order inorganic crystals. In part, this is due to the fact often other requirements like phase-matchability, transparency range and the potential to grow good crystals in the required size are often more crucial.

For example beta Barium Borate has a relatively small χ(2)) (d22~2.6pm/V compared to d33~27pm/V for LiNbO3). Yet, it is an excellent nonlinear optical crystal because it is transparent well into the ultraviolet region, has a high damage threshold and is phase-matchable over a wide spectral range and is a robust crystal. The hyperpolarizability β for each unit can be estimated by simpler LCAO models providing reasonably satisfactory results.

POSSIBLE QUESTIONS

PART B

1. Why do we stimulate nonlinear materials?
2. Define nonlinear materials.
3. What is meant by phase matching?
4. Write a short notes about second harmonic generation.
5. List out some applications of SHG.

PART C

1. What is meant by Second Harmonic Generation and explain about it.
2. Derive the expression for four wave mixing spectroscopy.
3. State Phase matching conditions and explain about it.
4. Write a short notes about nonlinear materials and phase matching.
5. Discuss about optical phase conjugation.
6. Explain about perturbation theory.

KARPAGAM ACADEMY C	
	DEP
	CL
	APPL
	MULT
Question	**Choice 1**
UNIT 3	
The nonlinear polarization present at the frequency can be viewed as ε	Optical medium
The incident wave is then _____.	Transverse
The electric field of the incident wave will lie in the ____ plane.	X
The electric field of the incident wave will lie in the XY plane because	Tensor character
The amplitude of the second harmonic field will have amplitude depen	x
e⊥ and ε II describe the dielectric response perpendicular and parallel t	Optic axis
The amplitude of SHM builds up _____	fastly
The energy per unit area per unit time carried by the second harmonic	pointing vector
The crucial parameter which controls the intensity of the second harm	Δk
If the dielectric constant were to be independent of frequency, then Δk	infinte
The power flow in the second harmonic increases as	Z
The dielectric constant of any medium depends on _____.	medium property
The condition Δk = 0 is obeyed, the interaction which leads to SHG is	frequency matched
The second harmonic intensity is independent of	wave vector mismatch
The second harmonic intensity is independent of	Δk
The second harmonic intensity is independent of	length lc
In the phenomenon of phase matching, a photon of wave vector k also	energy E
In the phenomenon of phase matching unless k2 = 2kl , momentum	conserved
There is the ordinary wave, with index ofrefraction no(θ,ω) =	√ e ⊥(ω)
There is the ordinary wave, with index of refraction no(θ,ω) independe	propagation direction
While second harmonic generation is not possible in	gases or liquids
In the case of optical phase Conjugation one has _____ counter-propa	three
The interaction between the three waves provided by the third order su	superimposing a range of
The four wave mixing process is thus a _____reversal operation.	amplitude
For a transverse electromagnetic wave of wave vector k, the electric fi	ik· E
Full form of CARS	Common Anti-Stokes Ra
In the case of _____ one has two counter-propagating input waves.	wavevector mismatch
There is the ordinary wave, with index of refraction no(θ,ω) _____ of	influenced
The second harmonic intensity is independent of	wave vector mismatch
The condition _____ is obeyed, the interaction which leads to SHG is s	Ñφ=1
The conversion of the input fields into the sum-frequency field is	small
_____ is known as the condition of perfect phase matching	Ñφ=1
The total power radiated by the ensemble of atomic dipoles thus scales	the number of atoms that
SHG also called as	general generation
SHG is a special case of	sum-frequency generatior
In some cases almost _____ percentage of the light energy can be conv	7
Generating the second harmonic is also a process in	signal processing
The Generation of the second harmonic is also a process in radio comn	12
The Generation of the second harmonic is also a process in radio comn	MHZ

The Generation of the second harmonic is also a process in radio comn	amplitude addition
Second harmonic generation was first demonstrated by	Peter Franken
Second harmonic generation was first demonstrated in the year	1876
The demonstration of SHG was made possible by the invention of the	micrometer
_____ created a high intensity coherent light in SHG.	micrometer
The formulation of SHG was intially described by	Sathya Prakash
The formulation of SHG was intially described in the year	1876
SHG occurs in _____ types.	10
The second lowest frequency at which a string could vibrated is knowr	second harmonic
_____ is a characteristic to the materials at the interface of study.	nonlinear susceptibility te
The 1064 nm light is fed through a bulk _____ crystal.	BCA
SHG provides	surface density
Birefringent materials have _____ refractive indices for different polar	same
Ordinary and extraordinary refractive indices can be different by up to	0.7
Application of SHG in materials	improvement of image qu
The ____ nm light is fed through a bulk KDP crystal.	1064

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OF HIGHER EDUCATION, COIMBATORE – 21			
ARTMENT OF PHYSICS			
ASS: III B. Sc., PHYSI	**BATCH: 2016-2019**		
Sixth Semester			
IED OPTICS (16PHU601B)			
IPLE CHOICE QUESTIONS			
Choice 2	**Choice 3**	**Choice 4**	
Radiation	electromagnetic radiati	None of these	
Longitudinal	Both a and b	None of these	
Y	YZ	XY	
Optical medium	Current	Extetnal field	
y	yz	z	
external field	amplitude	incident wave	
very fastly	moderetaly	Very slowly	
Gauss law	intensity	None of these	
Ej	γa	ϕr	
0	1	cannot be determined	
Z^2	Z^3	2Z	
external field	frequency	phase angle	
phase matched	intensity matching	None of these	
intensity variation	intensity distribution	none of the above	
ΔI	ΔA	ΔQ	
angle θ	intensity level	intensity variation	
momentum ħk	angle θ	intensity	
not conserved in the int	remains same	none of the above	
e (ω)	√ e (ω^2)	e (ω^2)	
propagation constant	propagation angle	propagation direction	
solids	Both a and b	none of the above	
five	ten	two	
rectangular region	phase Conjugation	complex wave	
time period	time	none of the above	
ik	ik· (E^2)	(-i)k· E	
Coherent Anti-Stokes F	Coherent Anti-Stokes R	Coherent Amplitude Raman Spe	
four wave mixing	optical phase Conjugati	orbital conjugation	
dependent	indepentent	none of the above	
Δk	length lc	All the above	
Δk = 0	Δθ=1	Δθ=0	
very small	vary	too large	
Δθ=1	Δk = 0	Δθ=0	
the cube of the number	the square of the numbe	the total number of atoms that p	
various harmonics	second frequency	frequency doubling	
general generation	various harmonics	second frequency	
8	9	100	
image processing	radio communication	none of the above	
13	14	20	
THZ	HZ	NHZ	

timeperiod inversion	frequency multiplicatio	none of the above
A.E.Hill	C.W.Peters	All the above
1961	1856	1879
ammeter	laser	voltmeter
solids Lamps	laser	none of the above
N.Bloembergen	Boyd	D.L.Mills
1786	1962	1876
3	20	30
third harmonic	fourth harmonic	first harmonic
optical susceptibility	nonlinear vector	optical tensor
KDP	GFE	ZYX
molecular orientations	electronic spectroscopy	All the above
minimum	different	0
0.6	0.1	0.4
used in probe magnetis	used in optical microsc	All the above
165	234	125

Answer

electromagnetic radiation
Transverse
XY
Tensor character
z
Optic axis
Very slowly
poiting vector
Δk
0
Z^2
frequency
phase matched
wavevector mismatch
Δk
length lc
momentum ħk
not conserved in the interaction
√ e (ω)
propagation angle
gases or liquids
two
phase Conjugation
time
ik· E
Coherent Anti-Stokes Raman Spectroscopy
optical phase Conjugation
indepentent
All the above
Δk = 0
too large
Δk = 0
the square of the number of atoms that participate
frequency doubling
sum-frequency generation
100
radio communication
20
MHZ

frequency multiplication
All the above
1961
laser
laser
N.Bloembergen
1962
3
second harmonic
nonlinear susceptibility tensor
KDP
All the above
different
0.1
All the above
1064

UNIT-IV

SYLLABUS

Scattering of light – Raman scattering – Quantum theory of Raman scattering – Brillouin scattering. Interaction of atoms with nearly resonant fields – wave function under near resonant conditions. Bloch equations – self induced transparency.

SCATTERING OF LIGHT:

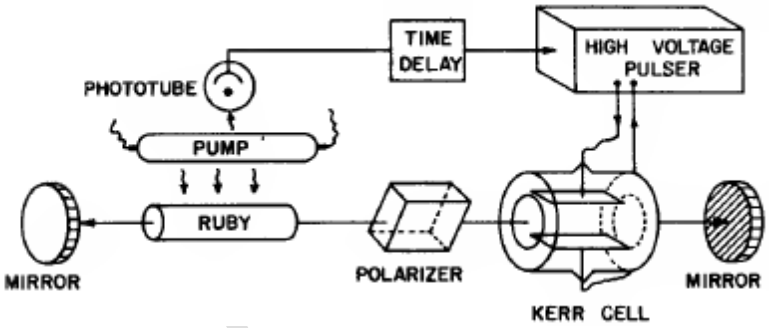
Lord Rayleigh was the first to deal with scattering of light by air molecules. The scattering of sunlight by the molecules of the gases in Earth's atmosphere is called Rayleigh scattering.
The basic process in scattering is absorption of light by the molecules followed by its re-radiation in different directions. The strength of scattering depends on the wavelength of the light and also the size of the particle which cause scattering.
The amount of scattering is inversely proportional to the fourth power of the wavelength. This is known as Rayleigh scattering law. Hence, the shorter wavelengths are scattered much more than the longer wavelengths. The blue appearance of sky is due to scattering of sunlight by the atmosphere. According to Rayleigh's scattering law, blue light is scattered to a greater extent than red light. This scattered radiation causes the sky to appear blue.
At sunrise and sunset the rays from the sun have to travel a larger part of the atmosphere than at noon. Therefore most of the blue light is scattered away and only the red light which is least scattered reaches the observer. Hence, sun appears reddish at sunrise and sunset.

RAMAN SCATTERING:

Although it was anticipated that laser beams would be especially useful in Raman spectroscopy, the discovery of stimulated Raman scattering was a surprise.
Raman scattering involves different selection rules than those applicable to emission and absorption processes. In particular, vibrational states which cannot combine directly with the ground state are accessible to Raman transitions. This has made Raman spectroscopy a valuable supplement to ordinary infrared and optical spectroscopy.
Because the anti-Stokes line appears in a normally-forbidden transition induced by incident light from an excited state to the ground state, the intensity of the incident light must be high, not only to maintain an adequate population in the excited state, but also to provide a sufficiently high flux of photons to induce the combination transition.
The ability of lasers to produce light of extremely high intensity makes them especially attractive sources for Raman spectroscopy of molecules by increasing the intensity of the anti-Stokes components in the Raman effect. Each resulting pair of lines, equally displaced with respect to the laser line, reveals a characteristic vibrational frequency of the molecule.
The need for high intensity in this and many other fields of optical research provided an incentive to the development of various techniques for the giant-pulsing of lasers. This in turn led to the unexpected discovery of Raman laser action. One of the earliest methods to accomplish this

employed a repetitively-pulsed Kerr cell as an elect optical shutter, or "Q-switch," enclosed within the laser "cavity," together with a polarizing prism (Figure). This allows laser action to occur only during the brief time intervals when the Kerr cell is transmitting; the laser avalanche then discharges energy stored over the much longer time interval since the preceding pulse on the Kerr cell. It was noticed that for sufficiently high laser-pulse intensity, the 6943 A°ruby-laser line is accompanied by a satellite line at 7670 A° which originates in the nitrobenzene of the Kerr cell. The satellite line increases markedly in intensity as the laser output is increased above a threshold level of the order of 1 MW/cm^2 , persists only while the laser output is above this threshold, shares the direction of the laser radiation, and becomes spectrally narrower at higher intensities. Its wavelength agrees with that of the known Raman-Stokes shift in nitrobenzene. The conclusion was therefore reached that the phenomenon is a stimulated Raman scattering process, pumped by the laser beam and resonated by the end-reflectors of the laser.

This interpretation has been sustained. Raman media are now widely used in conjunction with pulsed lasers to generate coherent radiation at frequencies other than those currently accessible to direct laser action. For this reason, the Raman scattering process has taken on a practical significance beyond its original one as a powerful spectroscopic technique.



QUANTUM THEORY OF RAMAN SCATTERING:

The quantum mechanical description of Raman scattering proves essential if the account is to be complete, as we discuss shortly. The picture of Raman scattering in quantum theory begins by noting that the vibrational levels of the molecule are quantized, given by $(n_v + \frac{1}{2})\hbar\omega_v$, with n_v an integer. An incoming photon of frequency ω_1 encounters the molecule in the quantum level n_v, and excites it to the level n_v + 1 as it scatters, thus leaving with energy $\hbar\omega_s = \hbar(\omega_1 - \omega_v)$. This is Stokes scattering. In the antiStokes event, the molecule is de-excited from n_v to n_v - 1, a process clearly impossible if the molecule is in the vibrational ground state n_v = 0, and the photon thus emerges with energy $\hbar\omega_s = \hbar(\omega_1 - \omega_v)$.

An account of these processes is obtained as follows: We have a quantization volume V, within which our molecule is placed. The molecule is located at the origin of the coordinate system. The Hamiltonian consists of that of the radiation field

$$H_R = \sum_{k\lambda} \hbar\omega_k \left(a^+_{k\lambda} a_{k\lambda} + \frac{1}{2} \right)$$ ---------- (1)

With $a_{k\lambda}$, $a^+_{k\lambda}$ the annihilation and creation operator of a photon of wave vector k and polarization λ. Then we have that of the vibrating molecule,

$$H_v = \sum_{v} \hbar\omega_v \left(b^+_v b_v + \frac{1}{2} \right)$$ ---------- (2)

With b_v, b^+_v the lowering and raising operators of the quantum theory of the harmonic oscillator. The Raman nonlinearity provides the coupling between the radiation field and the molecule, which one may write, using the classical expression for the energy of a polarized dipole in an electric field

$$H_I = -\int_0^E \boldsymbol{p} \cdot d\boldsymbol{E}' = -\frac{1}{2} \sum_{\alpha\beta} \sum_{v} a^{(R)}_{\alpha\beta} E_\alpha E_\beta q(v)$$ ---------- (3)

In (3), we use the quantum theoretic operator for the normal mode displacements q(v), and the electric field. For a quantum oscillator of frequency ω_v and effective mass M, we have

$$q(v) = \left(\frac{\hbar}{2M\omega_v} \right)^{1/2} (b_v + b^+_v)$$ ---------- (4a)

and the electric field at the origin, expressed in terms of photon annihilation and creation operators is

$$E_\alpha = \mathrm{i} \sum_{k\lambda} \hat{e}_\alpha(k\lambda) \left(\frac{2\pi\hbar\omega_k}{V} \right)^{1/2} (a_{k\lambda} - a^+_{k\lambda})$$ ---------- (4b)

Where $\hat{e}(k\lambda)$ is unit vector directed along the electric field associated with the mode kλ. When the expressions are inserted into (3), we have terms proportional to $a^+_{k'\lambda}, a_{k\lambda}$ that describe scattering of a photon from kλ to k'λ', accompanied by excitation or de-excitation of the oscillator. We consider scattering of a photon from a particular state I to a final state S, using these symbols everywhere rather than the more cumbersome notation kλ, k'λ'. When these terms are collected together, and those not of interest discarded, we have

$$H_I = \frac{2\pi\hbar}{V} \left(\frac{\hbar}{2M\omega_v} \right)^{1/2} (\omega_S \omega_I)^{1/2} \tilde{a}(IS; v) a^+_S a_I (b_v + b^+_v)$$ ---------- (5)

where we consider scattering with excitation or de-excitation of a particular vibrational mode v. We have defined

$$\tilde{a}(IS; v) = \sum_{\alpha\beta} a_{\alpha\beta}(v) \hat{e}_\alpha(I) \hat{e}_\beta(S)$$ ---------- (6)

We now calculate the transition rate using the Fermi golden rule, from a state with (n_1, n_s) photons in the initial and scattered photon states, to one with $(n_1 - 1, n_s + 1)$ photons in the two

states. The molecule is in state n_v initially, and is excited to n_v + I, so we are considering a Stokes event. The transition rate is also dn_s/dt, the increase in number of photons of frequency ω_s with time. We calculate the rate for one molecule, and multiply by N, the total number of molecules in V, to obtain a result appropriate to a sample with density n = N/V molecules per unit volume. Upon recalling the Dirac delta function identity $\delta(ax) = \delta(x)/|a|$, the result is

$$\frac{dn_S}{dt} = \frac{8\pi^3 n\omega_S\omega_I}{V}\left(\frac{\hbar}{2M\omega_v}\right)$$

$$|\bar{a}(IS;v)|^2 n_I(1+n_S)(1+n_v)\delta(\omega_s-\omega_I+\omega_v) \quad \text{---------- (7)}$$

The scattering rate for the anti-Stokes process is found by replacing I + n_v by n_v in (7), then changing the sign of ω_v in the energy conserving delta function.

It is useful to rearrange (7) a bit, for some purposes. It provides the rate for scattering into a single final photon state S. We can calculate the total number of photons scattered per unit time into solid angle $d\Omega_s$, $d^2N_s/\, d\Omega_s\, dt$. This is done by noting that the total number of photon states (of given polarization) in solid angle $d\Omega_s$ with frequency between ω_s and ω_s + $d\omega_s$ is $[V/(2\pi)^3](\omega_S^2 d\omega_s\, d\Omega_s\, /C^3)$ We multiple (7) by this density of states and integrate on ω_s to find

$$\frac{d^2N_s}{d\Omega_S dt} = \frac{n\omega_S^3\omega_I}{c^3}\left(\frac{\hbar}{2M\omega_v}\right)|\bar{a}(IS;v)|^2 n_I(1+n_S)(1+n_v) \quad \text{---------- (8)}$$

One speaks commonly in this field of the scattering efficiency per unit length, per unit solid angle. This is the fraction of photons scattered out of the beam, per unit length of travel in the medium. We find the scattering efficiency S by dividing the number of photons scattered per unit time by n_1. and convert to a scattering rate per unit length by dividing by the velocity of light, which we are taking to be c, in our gaseous medium. Hence, setting n_s = 0, appropriate for spectroscopic studies with rather weak exciting radiation,

$$\frac{d^2S}{d\Omega_s dz} = \frac{n\omega_s^3\omega_I}{c^4}\left(\frac{\hbar}{2M\omega_v}\right)|\bar{a}(Is;v)|^2(1+n_v) \quad \text{---------- (9)}$$

If the molecules are in thermal equilibrium, one averages over the quantum number n_v, to find it replaced by $n_v = [\exp\left(\frac{\hbar\omega_v}{K_B T}\right) -]^{-1}$ with k_B Boltzmann's constant and T the temperature. For anti-Stokes scattering, 1 + n_v is replaced by n_v itself. For visible frequency radiation, $\omega_S^3\omega_1 \cong \omega_S^4$. Then the ratio of the Stokes intensity to that of the anti-Stokes intensity in a Raman spectrum is $\exp\left(\frac{\hbar\omega_v}{K_B T}\right)$. The measurement of this ratio thus provides the sample temperature, a most useful result. For example, in a spectroscopic study, by monitoring the Stokes/anti-Stokes ratio, one may learn whether the sample has been heated by the laser beam, or one can infer the temperature of a remote object.

The frequency spectrum of the Raman radiation is a delta function, according to (7). This is, of course, an idealization. In actual matter, the excited vibration levels of the molecule are lifetime

broadened, and in this circumstance the delta function is replaced by a suitably normalized Lorentzian,
whose width is controlled by that of the excited vibrational levels. We use γ_v as a measure of this width, and replace (7) by

$$\frac{dn_S}{dt} = \frac{8\pi^2 n\omega_S\omega_I}{V}\left(\frac{\hbar}{2M\omega_v}\right)|\bar{a}(IS;v)|^2\frac{n_I(1+n_S)(1+n_v)\gamma_v}{\gamma_v^2+(\omega_S-\omega_I+\omega_v)^2} \quad \text{---------- (10)}$$

It is very useful to pause, and consider a semi-classical treatment of the Raman process.
In classical electrodynamics, one calculates the power radiated per unit solid angle by a point dipole to find, supposing the radiated power to have frequency ω_s.

$$\frac{dP}{d\Omega_s} = \frac{\omega_S^4}{8\pi c^3}|\hat{\mathbf{n}}_S\times(\hat{\mathbf{n}}_S\times\boldsymbol{p})|^2 \quad \text{---------- (11)}$$

Where $\widehat{n_s}$ is a unit vector in the emission direction. The right-hand side is to be time averaged. Noting
$\widehat{n_s}$ x ($\widehat{n_s}$ x p) = p - $\widehat{n_s}$ ($\widehat{n_s}$.p), one sees the intensity is proportional to the square of the projection of p onto the plane perpendicular to $\widehat{n_s}$. This projection is in fact parallel to the radiated electric field, which we suppose is parallel to the unit vector ê(S). Thus, (11) may also be written

$$\frac{dP}{d\Omega_S} = \frac{\omega_S^4}{8\pi c^3}|\hat{\mathbf{e}}(S)\cdot\boldsymbol{p}|^2 \quad \text{---------- (12)}$$

We apply this to the Raman process, using for p the prefactor of exp[-i($\omega_1 - \omega_v$)t]. We are thus examining the Stokes radiation. We then have, noting our earlier conventions $|\hat{e}(S).p|^2 \rightarrow 2|\bar{a}(I s;v)|^2 |E_1|^2|Q_V|^2$. Also, $(dp/d\Omega_s) = \hbar\omega_s[d^2N_s/(d\Omega_s\, dt)]$, where N_s is the number of scattered photons. Hence, the classical treatment gives, for the radiation from one molecule,

$$\frac{d^2N_s}{d\Omega_s dt} = \frac{\omega_S^3}{8\pi c^3}|\bar{a}(Is;v)|^2|E_I|^2|Q_v|^2 \quad \text{---------- (13)}$$

If there are n_1 photons in the incident beam, distributed over the volume V, then $n_1\hbar\omega_1 c/V$ is the energy per unit area per unit time in the incident beam. This is to be identified with the time average of the Poynting vector, $c|E_1|^2/2\pi$.
In classical theory, $|Q_V|^2$ should be replaced, for a molecule in contact with a thermal reservoir at temperature T, by the mean square displacement, or $K_BT/M\omega_v^2$. If we treat the vibrating molecule as a quantum system, we invoke the correspondence principle to replace $K_BT/\hbar\omega_v$ by $(1 + n_v)$ for a Stokes process.
When the comments of the two previous paragraphs are accounted for, (14) becomes, for N molecules in our volume V,

$$\frac{d^2N_s}{d\Omega_s dt} = \frac{n\omega_s^3\omega_I}{c^3}\left(\frac{\hbar}{2M\omega_v}\right)|\bar{a}(IS;v)|^2 n_I(1+n_v) \quad \text{---------- (14)}$$

This is the result for the number of photons per unit time per unit solid angle provided by a theory which treats the electromagnetic field classically, acknowledging the quantum character

of the molecular vibrations. The factor of $(I + n_s)$ which enters the full quantum treatment, however, is absent.

$$\bar{n}_I(1 + \bar{n}_S)(1 + \bar{n}_v) = \exp[\hbar(\omega_S - \omega_I + \omega_v)](1 + \bar{n}_I)\bar{n}_S\bar{n}_v \quad \text{---------- (15)}$$

With due account taken of energy conservation, we see the two processes balance exactly, and on the time average neither $\widehat{n_1}$, $\widehat{n_s}$, $\widehat{n_v}$ are changed by the presence of the Raman process. This is known as the principle of detailed balance in statistical mechanics.

The presence of the factor $(1 + n_s)$ is essential for detailed balancing to be achieved; the system of black-body radiation in contact with vibrating molecules could not remain in thermal equilibrium, if we accept the semi-classical formula in (14). The reader will recognize this argument follows closely that

given by Einstein, who was led to introduce a similar factor in the expressions for the rate of emissions of photos by an atom in its excited state.

The factor of $(1 + n_s)$ controls a most important phenomenon, the stimulated Raman effect. Initially, one may have a state for which $n_s \equiv 0$, and initiate Raman scattering which produces photons in the state S. The presence of the photons in the final state then increases the scattering rate, through the

factor $(1 + n_s)$. The process is unstable, and produces an avalanche of Raman photons, until the supply (the incident beam) is exhausted.

BRILLOUIN SCATTERING:

Brillouin scattering is describable entirely in classical terms, although, if we prefer, we can still describe it in terms of photons and phonons. This correspondence was presented in Section 3.4, where elementary relationships governing the scattering of light by acoustic waves. There it was shown how to relate the parameters of the incident and scattered light waves to those of the sound wave which by diffraction couples them, applying the Bragg law and the Dopplershift formula of classical wave theory. It was also shown how, by introducing Planck's constant $\hbar$, these relations are converted into statements of conservation of energy and momentum for two photons and a phonon in a replacement process. Ordinary Brillouin scattering, first observed as the Debye-Sears effect, is regarded as spontaneous scattering. Stimulated scattering becomes important when the creation of phonons in the scattering process is so copious that it overcomes their loss by decay. The very high population of phonons then stimulates the emission process, the laser beam acting as a pump, transferring energy from the light beam to the sound wave. Quantum-mechanical analysis is not required to predict the coefficients of interaction. We prefer to speak of e rather than of χ, since macroscopic, rather than microscopic, properties, are involved.

The classical description has a more appealing "physical" content to the non specialist. There, the light wave creates pressure through electrostriction; the resultant density change affects the susceptibility. Thus light pumps the sound wave which scatters it; the scattering creates a second, frequency-shifted light wave, which constitutes the idler wave of a parametric amplifier.

To describe the interaction quantitatively, therefore, one writes three coupled wave equations, just as for the other nonlinear processes. sections. Now, however, one of the three is an acoustical

wave equation. All three contain driving terms originating in the nonlinear interactions, which couple the various waves. The two electromagnetic waves have electric fields $E^{(\omega_1)}$, $E^{(\omega_2)}$ at, respectively, pump and idler frequencies. The sound wave is described by a Fourier component $\varrho^{(\omega_s)}$ of the density of the medium varying about the static value ϱ_0. The coupling arises from variation of the dielectric "constant" ε with density. This takes the form of electrostriction in the acoustical wave equation. A Fourier component of electrostrictively produced pressure exists, varying at the difference frequency of pump and idler. This drives the sound wave at frequency

$$\omega_s = \omega_1 - \omega_2$$

Simultaneously, a component of the charge polarization is made to vary at frequency ω_2 by the interaction of the laser light with the medium, in which the sound wave causes the dielectric susceptibility to vary at its frequency ω_s.

Similarly, the idler beam interacts with the acoustically-modulated susceptibility to provide the term in the fundamental wave equation which transfers its energy from the laser beam to sound and idler waves.

The Manley-Rowe conditions apply, as in other parametric amplifiers. The threshold of the stimulation process is that beam power density of the laser beam at which the amplifying action of the parametric process overcomes the attenuation losses, leading to growth of sound and idler waves.

INTERACTION OF ATOMS WITH NEARLY RESONANT FIELDS:

The discussion we have presented so far has supposed throughout that the primary quantity of interest, the electric dipole moment per unit volume P, can be expanded in powers of the electric field E. The justification for this procedure is the notion that the largest applied fields encountered in practice are very small compared to those encountered by an electron within an atom or a molecule. It follows that the electronic arrangement in the atom or molecule is perturbed only very slightly from that arrangement found in zero applied external field.

We now appreciate, however, that even though the nonlinearities are weak in the sense just described, their influence can be substantial, as illustrated by our model discussions of second harmonic generation, and the stimulated Raman effect. A bucket full of water may have only a small leak, but if one waits long enough, the bucket will be quite empty.

We also see that if the frequency ω of an oscillating electric field lies very close to that ω_{mo} of a transition between the ground state and an isolated excited state of the system, both the linear susceptibility and the various nonlinear susceptibilities can be enhanced enormously, over values appropriate to the nonresonant case. Indeed, the perturbation theoretic expressions produce a divergence as $\omega \rightarrow \omega_{mo}$, if no account is taken of the influence of the finite lifetime of the excited state. For frequencies very close to resonance, and for very intense applied fields, the perturbation theory breaks down. We then enter the regime of strong nonlinearities. This chapter is devoted to the analysis of the response of the system under these conditions, and the array of rather fascinating phenomena one encounters in this regime. For instance, when $\omega = \omega_{mo}$, elementary theory asserts that radiation will be absorbed by the system. When $\omega = \omega_{mo}$ and suitable conditions are satisfied, the

medium is quite transparent!
Our first task is to describe the response of the system to near resonant radiation. For simplicity, we shall confirm our attention to the response of an isolated atom, with a single nondegenerate excited level. We shall exploit the spherical symmetry of the atom in what follows:
The perturbation theoretic description of an atom subject to the perturbation described

$$|\psi\rangle = e^{-iE_0/\hbar t}\left[|\psi_0\rangle - \frac{1}{\hbar}\frac{\langle\psi_m|v|\psi_0\rangle}{\omega_{m0}-\omega-i\eta}|\psi_m\rangle e^{-i(\omega+i\eta)t}\right.$$

$$-\frac{1}{\hbar}\sum_{n\neq m}\frac{\langle\psi_n|v|\psi_0\rangle}{\omega_{n0}-\omega-i\eta}|\psi_n\rangle e^{-i(\omega+i\eta)t}$$

$$\left.-\frac{1}{\hbar}\sum_{n}\frac{\langle\psi_n|v^+|\psi_0\rangle}{\omega_{n0}+\omega+i\eta}|\psi_n\rangle e^{i(\omega-i\eta)t}+\cdots\right]. \quad \text{---------- (1)}$$

A weak perturbation is one for which matrix elements such as $\langle\Psi_n|v|\Psi_o\rangle$ are very small compared to the level splitting of the system. Our interest is clearly in this limit. Then the third and fourth terms on the right-hand side of (1) represent small corrections to the ground state wave function $|\Psi_o\rangle$. If ω is very close to ω_{mo} however, the second term can be very large. While perturbation theory clearly breaks down in this case, quite clearly in this circumstance the full wave function must have the form

$$|\psi(t)\rangle = a_0(t)|\psi_0\rangle + a_m(t)|\psi_m\rangle + \text{small terms} \quad \text{---------- (2)}$$

The treatment that follows proceeds by approximating the wave function by the first two terms only:

$$|\psi(t)\rangle = a_0(t)|\psi_0\rangle + a_m(t)|\psi_m\rangle \quad \text{---------- (3)}$$

This approximation, surely accurate when ω is close to ω_{mo}, allows one to obtain a simple and appealing description of the response of the atom.

SELF INDUCED TRANSPARENCY:
As remarked earlier, the Bloch equations apply to spin systems, and have been employed extensively in the description of both electron and nuclear resonance. Indeed, the formalism was developed many years ago with these applications in mind. Thus, the phenomena discussed above, such as the saturation of absorption lines at high power density, were well known and exploited by researchers in these fields, well before that advent of modem laser spectroscopy. Nuclear resonance involves application of radio frequency radiation, and electron spin resonance studies are carried out with microwaves. Under usual conditions,
the wavelength of the radiation is large compared to the sample size, and one can assume the driving field is spatially uniform, as we have done above. A unique aspect of the optical frequency probes is the fact that radiation wavelength is small compared to the sample size in nearly all experiments. Thus, we must combine our description of the response of a collection of atoms to a driving field, with the full Maxwell equations which describe propagation

of an electromagnetic disturbance in the medium. We do this in this section; we shall then encounter the remarkable phenomenon of self-induced transparency discovered some years ago by McCall and Hahn.

We have an electric field which we write as $E = \hat{n}E(z,t)$, and this has associated with it a dipole moment density $P(z, t) = \hat{n}P(z, t)$, where $P(z, t) = 2n[\gamma_1 S_1(z,t) + \gamma_2 S_2(z,t)]$. The Maxwell equation then reads

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{\bar{c}^2}\frac{\partial^2}{\partial t^2}\right)E(z,t) = \frac{8\pi n}{c^2}\frac{\partial^2}{\partial t^2}[\gamma_1 S_1(z,t) + \gamma_2 S_2(z,t)] \quad \text{---------- (1)}$$

Here $\bar{c} = c / \sqrt{\varepsilon}$, where ε is the dielectric constant of the medium in which the resonant atoms are embedded.

We- are to calculate S_1 and S_2 from the Bloch equation. We do this by describing the electric field as follows:

$$E(z,t) = e(z,t)\cos[\omega t - kz + \psi(z,t)] \quad \text{---------- (2)}$$

where the amplitude e(z, t) and the phase $\psi(z, t)$ are assumed to be slowly varying functions of both position z and the time t.

We now turn to the Bloch equations. We make a transformation to a local rotating frame, in a manner quite similar. We have

$$S_1' = S_1\cos(\omega t - kz + \psi) + S_2\sin(\omega t - kz + \psi) \quad \text{---------- (3a)}$$

$$S_2' = -S_1\sin(\omega t - kz + \psi) + S_2\cos(\omega t - kz + \psi) \quad \text{---------- (3b)}$$

And

$$S_3' = S_3 \quad \text{---------- (3c)}$$

One finds, upon ignoring terms in $(\partial\Psi/\partial t)$ in transforming from the laboratory to the rotating frame,

$$\frac{dS'}{dt} = \Lambda_{\text{eff}}' \, x \, S' - \frac{1}{T_1}\hat{x}_3'\left(S_3' - \frac{1}{2}\right) - \frac{1}{T_2}(\hat{x}_1' S_1' + \hat{x}_2' S_2') \quad \text{---------- (4)}$$

Where

$$\Lambda_{\text{eff}}' = (\omega_{m0} - \omega)\hat{x}_3' + \frac{1}{\hbar}e(z,t)(\gamma_1\hat{x}_1' + \gamma_2\hat{x}_2') \quad \text{---------- (5)}$$

is a function of position and time, because e(z, t) varies.

In what follows, we let both T_1 and T_2 become infinite. We also introduce new variables

$$S_{\parallel}' = S_1'\cos(\phi) + S_2'\sin(\phi) \quad \text{---------- (6a)}$$

And

$$S_{\perp}' = -S_1'\sin(\phi) + S_2'\cos(\phi) \quad \text{---------- (6b)}$$

where the phase angle ϕ.

One finds the following equations of motion, when these variables are introduced:

$$\frac{\partial}{\partial t} S'_{\parallel} = -(\omega_{m0} - \omega) S'_{\perp} \quad \text{---------- (7a)}$$

$$\frac{\partial}{\partial t} S'_{\perp} = (\omega_{m0} - \omega) S'_{\parallel} - \frac{\gamma}{\hbar} e(z,t) S'_3 \quad \text{---------- (7b)}$$

And

$$\frac{\partial}{\partial t} S'_3 = \frac{\gamma}{\hbar} e(z,t) S'_{\perp} \quad \text{---------- (7c)}$$

One has also

$$\gamma_1 S_1(z,t) + \gamma_2 S_2(z,t) = \gamma [S'_{\parallel}(z,t)\cos(\omega t - kz + \psi) - S'_{\perp}\sin(\omega t - kz + \psi)] . \quad \text{---------- (8)}$$

We next apply the slowly varying envelope approximation to the left-hand side of (1). After requiring that $k = \omega / \bar{c}$, one finds

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{\bar{c}^2}\frac{\partial^2}{\partial t^2}\right) E \cong \frac{2\omega}{\bar{c}}\left[\left(\frac{\partial e}{\partial z}\right) + \frac{1}{\bar{c}}\left(\frac{\partial e}{\partial t}\right)\right]\sin(\omega t - kz + \psi) + \frac{2\omega}{\bar{c}} e\left[\left(\frac{\partial \psi}{\partial z}\right) + \frac{1}{\bar{c}}\left(\frac{\partial \psi}{\partial t}\right)\right]\cos(\omega t - kz + \psi) \quad \text{---------- (9)}$$

When (9,8) are employed in (1), we obtain two first order differential equations which link the amplitude and phase of the electric field to the variables $S'_{\parallel}$ and $S'_{\perp}$. We have

$$\frac{\partial e}{\partial z} + \frac{1}{\bar{c}}\frac{\partial e}{\partial t} = \frac{4\pi n \omega \gamma}{c\sqrt{\varepsilon}} S'_{\perp} \quad \text{---------- (10a)}$$

And

$$e\left(\frac{\partial \psi}{\partial z} + \frac{1}{\bar{c}}\frac{\partial \psi}{\partial t}\right) = -\frac{4\pi n \omega \gamma}{c\sqrt{\varepsilon}} S'_{\parallel} . \quad \text{---------- (10b)}$$

The propagation of electromagnetic disturbances through the medium can be analyzed by solving (7) in combination with (10). e shall consider a pulse which encounters the atoms in the ground state, so as $t \rightarrow -\infty$, we have $S'_3 = +\frac{1}{2}$, $S'_{\parallel} = S'_{\perp} = 0$. We shall suppose also that the carrier frequency of the pulse is in precise resonance with the atomic transition. Thus, we set $\omega = \omega_{m0}$. Then (7a) in combination with the boundary condition requires that $S'_{\parallel}$ vanish at all times. It then

follows that ψ(z, t) vanishes identically at all times, from (10 b). Since the vector S' has constant length, we may write

$$S_3'(z,t) = \frac{1}{2}\cos[\theta(z,t)]$$ ---------- (11a)

And

$$S_\perp'(z,t) = \frac{1}{2}\sin[\theta(z,t)]$$ ---------- (11b)

Where as t → -∞, θ(z,t) → 0. Equations (7 b,c) then collapse into a single statement which describes the time evolution θ(z,t)

$$\frac{\partial\theta(z,t)}{\partial t} = -\frac{\gamma}{\hbar}e(z,t)$$ ---------- (12)

and we may then eliminate e from (10 a), to find

$$\frac{\partial^2\theta}{\partial t^2} + \bar{c}\frac{\partial^2\theta}{\partial t\partial z} = -\frac{2\pi n\omega\gamma^2}{\hbar\varepsilon}\sin(\theta)$$ ---------- (13)

As we shall see later, (13) is very close in form to a classical equation of nonlinear physics, the sine-Gordon equation. Indeed, an elementary transformation will convert this equation into precisely the sine-Gordon equation, as we shall see later. There are a rich and fascinating array of solutions to this equation. For the moment, we will be content to examine one special, but very important solution.

We seek solutions in the form of a pulse, which propagates through the system with some velocity v < $\bar{c}$. Thus, all quantities are assumed to be functions of the variable $\tau = t - z/v$. Equation (13) then reduces to

$$\frac{\partial^2\theta}{\partial\tau^2} = \Lambda^2\sin\theta$$ ---------- (14)

Where $\Lambda^2 = 2\pi n\omega\gamma^2 v/\hbar\varepsilon(\bar{c} - v)$

One may integrate (14) exactly, for our boundary conditions. Upon multiplying each side by $\partial\theta/\partial\tau$, we obtain the statement

$$\frac{\partial}{\partial\tau}\left[\frac{1}{2}\left(\frac{\partial\theta}{\partial\tau}\right)^2 + \Lambda^2\cos\theta\right] = 0$$ ---------- (14)

which, with the requirement θ → 0 as τ → -∞ requires

$$\left(\frac{\partial\theta}{\partial\tau}\right)^2 + 2\Lambda^2[\cos\theta - 1] = 0$$ ---------- (15)

Or

$$\frac{\partial\theta}{\partial\tau} = \pm 2\Lambda \sin\frac{\theta}{2}$$ ---------- (16)

One more integration yields two solutions:

$$\theta_{\pm}(\tau) = 4\tan^{-1}(e^{\pm\Lambda\tau})$$ ---------- (17)

Consider first the solution $\theta(\tau)$. For any fixed z, as $t \rightarrow -\infty$, $\theta(\tau) \rightarrow 0$. As $t \rightarrow +\infty$, $(\tau) \rightarrow 2\pi$, so after the pulse has passed over a given set of atoms, the atoms are returned to their ground state. Halfway through the pulse, when $\tau = 0$, $\theta = \pi$ all atoms reside in the excited state. One refers to such a pulse as a 2π pulse.

It is a straightforward matter to find the expression for the envelope function $e(\tau)$ that describes the electric field e(z, t):

$$e(\tau) = \pm\frac{2\hbar\Lambda}{\gamma}\frac{1}{\cosh(\Lambda\tau)}$$ ---------- (18)

There are two remarkable features of this pulse. First of all, the carrier frequency ω precisely matches the transition frequency ω_{mo} of our atoms. Under these circumstances, elementary considerations suggest the electromagnetic radiation should be absorbed, as we have discussed. In fact, the medium is perfectly transparent to this resonant radiation! The pulse propagates forever.

Also, for frequencies ω very close to the resonance frequency ω_{mo}, the dielectric constant $\varepsilon(\omega)$ of the medium exhibits the resonant behaviour. If we construct an electromagnetic wave packet localized in space, we must synthesize the wave packet from plane waves of different frequency and wave vector. The different wavelengths propagate at different phase velocities,
with the consequence that the wave packet distorts and spreads as it propagates through the medium, if its propagation is described by linear dielectric theory. The rate of spreading is particularly severe for frequencies near resonance. The wave packet described by not only has infinite mean free path, but in addition propagates through the medium perfectly undistorted in shape.

We have clearly encountered new solutions that describe propagation of radiation through the medium with characteristics that differ qualitatively from those encountered in the theory of wave propagation in the small amplitude, linear response limit (the limit of ordinary dielectric theory). This is a fascinating aspect of the physics of highly nonlinear systems. We have just encountered
an object referred to as a soliton.

One refers to the phenomenon just described as self-induced transparency. The atoms absorb energy from the leading edge of the pulse, but subsequently re-radiate it back into the tail in such a way as to produce a rigid, stable, propagating pulse. This remarkable phenomenon was discovered experimentally by McCall and Hahn, who also presented in their original papers a full and complete description of the theory.

Our soliton solution contains a parameter, the soliton propagation velocity v. The parameter Λ provides a measure of the width of the soliton, and we have from the definition of Λ,

$$\frac{1}{v} = \frac{1}{\bar{c}} + \frac{2\pi n \omega \gamma^2}{\hbar \varepsilon \Lambda^2 \bar{c}} \quad \text{---------- (19)}$$

The propagation velocity is thus always less than $\bar{c}$.

There remains the question of how the soliton pulse evolves from an input pulse whose shape may differ greatly from the soliton form. This is a complex question that in general must be addressed by a numerical solution of the full equations. However, a most important theorem proved by McCall and Hahn, the area theorem, tells one a great deal about this question.

POSSIBLE QUESTIONS
PART A

1. Why does the sun appear red at sunrise and sunset?
2. Write Bloch equations.
3. What is the quantum theory of Raman Effect?
4. Why the sky is blue?
5. Define Scattering.

PART B

1. Briefly explain about the wave function under near resonant conditions.
2. Describe about the Raman scattering.
3. Derive the expression for self-induced transparency.
4. What is meant by Scattering? Discuss about the Brillouin scattering.
5. Obtain Bloch equations.
6. Briefly explain about Interaction of atoms with nearly resonant fields.
7. Discuss about Quantum theory of Raman Effect.

KARPAGAM ACADEMY
DEP
CL
APPL
MULT

Question	Choice 1

UNIT 4

_____was the first to deal with scattering of light by air molecules.	Ramesh Babu
The scattering of sunlight by the molecules of the gases in Earth's atm	dispersion
The basic process in scattering is _____ of light.	evaporation
The strength of scattering depends on the _____ of the light.	wavelength
The strength of scattering depends on	light waves
The amount of scattering is _____ proportional to the _____ powe	inversely, third
The shorter wavelengths are scattered much _____ the longer wavele	smaller than
The ____ appearance of sky is due to scattering of sunlight by the atm	blue
The blue appearance of sky is due to scattering of _____by the atmosp	moon
Blue light is scattered to a _____ extent than red light.	greater
The amount of scattering is inversely proportional to the fourth power	T^3 law
The scattering of light by the colloidal particles is called	colloidal scattering
Raman effect was discovered in the year of _____	1765
The monochromatic light is scattered when it is allowed to pass throug	C.V.Raman
The lines whose frequencies have been modified in Raman effect are c	frequency lines
In Raman lines having frequencies lower than the incident frequency a	frequency lines
In Rama lines having frequencies higher than the incident frequency ar	incident lines
The intensity of Stoke's line is always _____ the corresponding An	lower than
The combination of a coherently scattered wave with the vacuum wave	refractive index
When a strong beam of light is passed through a colloidal solution, the	pass unchanged
The phenomenon of light associated with the appearance of blue colou	interference
Raman effect supports _____theory.	corpuscular
In Raman spectrum , if λ is the wavelength of incident radiation, then t	λ
Sun appears red at sun rise and sunset. This is due to scattering of	longer wavelengths
In Raman spectrum , if λ is the wavelength of incident radiation, then t	λ+Δλ
Tyndall effect is the scattering of the light by	air particles
Blue colour smoke emission by IC engine is an example of	total internal reflection
Very minute particles present in atmosphere scatter the light of	medium wavelength
The deflection of light by minute particles and molecules in all directi	dispersion
Which colour of light scatters maximum due to atmosphere?	blue
Bigger size particles present in atmosphere scatter the light of	smaller wavelength
Sometimes the smoke emitted by the combustion of oil appears blue du	dispersion
The time-dependent fluctuations in density produce _____ of light.	diffraction
The time-dependent modulations of the dipole moment per unit volum	internal frequency
Thermodynamic fluctuations in particle positions produce not only cha	force
Raman spectra explore vibrational frequencies roughly in the range of	0-90
_____ may be used to detect the frequency shift.	detectors
Frequency shift in the Brillouin event lies in the range of a few	Mehahertz
_____ is needed to scan the frequency spectrum of the scattered ligh	Michelson interferonmete

The Brillouin cross section for scattering from initial state I to a final s	(1 /ns)
The Brillouin scattering event from the vantage point of	quantum theory
The energy of the phonons is quantized in units of	Joule
Rayleigh scattering and Mie scattering are the types of	Linear scattering losses
Dominant intrinsic loss mechanism in low absorption window betwee	Rayleigh scattering
Rayleigh scattering is inversely proportional to the _____.	particle size
The scattering resulting from fiber imperfections like core-cladding RI	Rayleigh scattering
Mie scattering has in-homogeneities mainly in	Backward direction
In Mie scattering the scattering in-homogeneities size is greater than _	λ^3
Raman and Brillouin scattering are usually observed at _____ pow	Low optical
Raman and Brillouin scattering mechanism are _____.	Linear
Stimulated Brillouin scattering is mainly a _____ process.	forward
Stimulated Raman scattering occur in _____ direction.	forward
The nonlinear effects in optical fibers are _____.	intermediate
Which of the following is non releted to Kerr effect?	Self-phase modulation
Linear scattering effects are _____ in nature.	elastic

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OF HIGHER EDUCATION, COIMBATORE – 21			
ARTMENT OF PHYSICS			
ASS: III B. Sc., PHYSI	**BATCH: 2016-2019**		
Sixth Semester			
IED OPTICS (16PHU601B)			
IPLE CHOICE QUESTIONS			
Choice 2	**Choice 3**	**Choice 4**	

Raman	Lord Rayleigh	Sathya prakash
Rayleigh scattering	evaporation	molucular interference
change	energy transfer	absorption
momemtum	force	acceleration
size of the particle whi	energy of the source	scattering constant
inversely, fourth	directly, second	directly, third
less than	more than	equal to
rose	green	violet
sunlight	sky	particle
less than	small	none of the above
scattering constant	Raman scattering law	Rayleigh scattering law
Tyndal scattering	solution scattering	none of the above
1875	18745	1928
Boyd	Ramesh Babu	Sathya prakash
Raman lines	modified lines	none of the above
incident lines	Stoke's lines	none of the above
frequency lines	Anti−stokes lines	none of the above
greater than	equal to	less than
Cauchy constant	scattering constant	none of the above
absorbed	reflected	scattered
reflection	refraction	scattering
wave	quantum	electromagnetic
$\lambda+\Delta\lambda$	$\lambda-\Delta\lambda$	2λ
shorter wavelengths	lower frequencies	higher frequencies
λ	2λ	λ^2
solid particles	liquid particles	colloidal particles
Tyndall effect	refraction	dispersion
smaller wavelength	only blue colour	none of the above
interference	diffraction	scattering
yellow	pink	green
medium wavelength	larger wavelength	only blue colour
total internal reflection	Tyndall effect	diffraction
inelastic scattering	path difference	none of the above
constant dipole momen	internal vibrations	externel charge
density	momemtum	volume
100-2500	10-100000	30-800
sensors	Grating spectrometers	prism
Gigahertz	millihertz	Hertz
spectrometer	detectors	Fabry Perot interferometer

(1 + ns)	(1 -ns)	(2 + ns)
electromagnetic theory	solid state theory	wave theory
ħω(q)	h(q)	β(q)
fiber bends loss	splicing loss	none of the above
Mie scattering	Stimulated Raman scatt	Stimulated Brillouin scattering
wavelength	timeperiod	absorption
Mie scattering	Stimulated Raman scatt	Stimulated Brillouin scattering
All direction	Core-cladding interfere	Forward direction
λ/10	2λ	λ+d
Medium optical	High optical	Threshold
non-linear	transmitted	none of the above
downward	upward	backward
backward	upward	forward and backward
large	high	small
Cross-phase modulatio	Stimulated Raman scatt	Four-wave mixing
non-elastic	mechanical	electrical

Answer

Lord Rayleigh
Rayleigh scattering
absorption
wavelength
size of the particle which cause scattering
inversely, fourth
more than
blue
sunlight
greater
Rayleigh scattering law
Tyndal scattering
1928
C.V.Raman
Raman lines
Stoke's lines
Anti−stokes lines
greater than
refractive index
scattered
scattering
quantum
λ-Δλ
shorter wavelengths
λ+Δλ
colloidal particles
Tyndall effect
smaller wavelength
scattering
blue
larger wavelength
Tyndall effect
inelastic scattering
internal vibrations
density
100-2500
Grating spectrometers
Gigahertz
Fabry Perot interferometer

(1 + ns)
quantum theory
ħω(q)
Linear scattering losses
Rayleigh scattering
wavelength
Mie scattering
Forward direction
λ/10
High optical
non-linear
backward
forward and backward
small
Stimulated Raman scattering
non-elastic

UNIT-V

SYLLABUS

Fibre optics – normal modes of optical fibres – nonlinear Schrodinger equations – linear theory. Basic concepts of solutions and non-linear periodic structures. Effect of fibre loss – effect of waveguide property of a fibre – conditions of generation of a solutions in optical fibres.

INTRODUCTION TO FIBRE OPTICS:

Fiber optics as a research area exists since the 1950s. Although the main principle of fiber optics, namely total internal reflection (TIR), was discovered (but not explained) by Kepler in the 17th century, and uncladded ("bare") glass fibers based on this phenomenon were produced in the 1920s (J.L. Baird, C.W. Hansell), the core-and-cladding design that showed completely new characteristics was first used only in the 1950s by van Heel.

Since then many aspects of light propagation in fibers were discovered and used for various devices. The aim in the 1960s was to transmit images through a bunch of fibers, in which case the main limiting factor was the loss in the individual fibers. This problem was solved in the 1970s when loss was reduced below 0.2 dB/km using a wavelength of 1550 nm ; in that case Rayleigh-scattering was the limitation factor. These fibers could already be used for optical fiber communication.

The decade of 1970 was also the decade of recognizing the importance of nonlinear effects -like stimulated Raman- and Brillouin scattering (SRS, SBS), possibility of soliton-like pulses etc.- in fibers. Later, in the 1980s, nonlinear effects in fibers were used for pulse compression and optical switching. A change in the doping material of the fibers (using rare-earth elements) made it possible to create all-optical amplifiers, leading to a revolution in fiber-optic communication.

The new millennium saw the discovery of new types of fiber-optic amplifiers based on stimulated Raman scattering and four-wave mixing (FWM). The novelty of these amplifiers was that doping was unnecessary and these fibers could be used at any spectral region. With these new developments ultrafast signal processing became possible.

In the last decade new types of solitons such as dispersion-managed and dissipative solitons as well as photonic crystal fibers (PCF; also called holey fiber, hole-assisted fiber, microstructure fiber, or microstructured fiber) were developed. These fibers are characterized with a relatively narrow core and a cladding with air holes. This structure shows much larger nonlinear effects that can be observed in fibers of length in the cm range.

The above historical summary shows that the field of nonlinear fiber optics is a field of such diversity that a chapter can hardly cover this topic. Therefore in this book we only present the basic principles of the field. For further information the interested reader is referred to the books specialized on the topic.

NORMAL MODES OF OPTICAL FIBERS:

The simplest picture of an optical fiber is to regard it as a dielectric cylinder of radius R, with uniform dielectric constant E. The cylinder is of infinite length, and perfectly straight, with axis parallel to the z direction.

Such a structure can serve as a guide of electromagnetic waves, which propagate parallel to the axis of the cylinder, and are "bound" to the structure in a sense we shall appreciate shortly.

To see this, we examine the Maxwell equations for solutions in which all components of the electric and magnetic field are proportional to $\exp[i(k_z z - \omega t)]$. It is convenient to work in cylindrical coordinates in the xy plane. A full discussion of the modes bound to the cylinder is complex, so we confine attention to the simplest case where all field components are independent of ϕ, and thus depend on only ρ.

One condition that must be satisfied is $\nabla . E = 0$, and also $\nabla . B = 0$ everywhere.

In cylindrical coordinates, and in the absence of ϕ dependence of all field amplitudes, this gives

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho E_\rho) + ik_z E_z = 0 \quad \text{---------- (1)}$$

We have also the wave equations, which in the dielectric cylinder $0 < \rho < R$ read

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_\rho}{\partial\rho}\right) + \left(\frac{\omega^2}{c^2}\varepsilon - k_z^2 - \frac{1}{\rho^2}\right)E_\rho = 0 \quad \text{---------- (2a)}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_\phi}{\partial\rho}\right) + \left(\frac{\omega^2}{c^2}\varepsilon - k_z^2 - \frac{1}{\rho^2}\right)E_\phi = 0 \quad \text{---------- (2b)}$$

And

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right) + \left(\frac{\omega^2}{c^2}\varepsilon - k_z^2\right)E_z = 0 \quad \text{---------- (2c)}$$

In the vacuum, $R < \rho < \infty$, we use the same equations but with $\varepsilon = 1$. Given the various components of the electric field, one generates the magnetic field from Faraday's Law, $\nabla \times E = i\omega B/c$. Hence, in cylindrical coordinates,

$$B_\rho = -\frac{ck_z}{\omega}E_\phi \quad \text{---------- (3a)}$$

$$B_\phi = \frac{ck_z}{\omega}E_\rho + i\frac{c}{\omega}\frac{\partial E_z}{\partial\rho} \quad \text{---------- (3b)}$$

And

$$B_z = -\frac{ic}{\omega\rho}\frac{\partial}{\partial\rho}(\rho E_\phi) \quad \text{---------- (3c)}$$

There are two classes of solutions to the above set of equations. The first has Ep , E_z. nonzero, Then we see from (3) that B_ϕ. is the only nonzero component of the magnetic field. The magnetic field thus lies in the plane perpendicular to the direction of propagation. Such waves are referred to as transverse magnetic, or TM waves. We may also have $E_p = E_z == 0$, but $E_\phi \neq .0$. Then both B_p and Bz are nonzero, linked by the requirement $\nabla \cdot B = 0$. These are the transverse electric, or

TE waves. Notice there are no solutions in which both E and B lie in the xy plane. The physical reason for this will be apparent later.

In what follows, we confine our attention to the TM waves. Consider first the vacuum region $\rho > R$, where E. then obeys

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right)+\left(\frac{\omega^2}{c^2}-k_z^2\right)E_z=0 \quad \text{---------- (4)}$$

There are two distinct regimes which may be examined. The first is $k_z > \omega/C$, where (4) reads

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right)-\kappa_0^2E_z=0 \quad \text{---------- (5)}$$

Where $k_0^2 = k_z^2 - \omega^2/c^2 > 0$. This equation admits solutions where the electric field decays to zero exponentially with increasing distance from the cylinder. Most particularly, if $K_o(\xi)$ is the modified Bessel function of order zero we have,

$$E_z(\rho) = A^{>}K_0(\kappa_0\rho) \quad \text{---------- (6a)}$$

and from (1) upon noting the identity $\xi K_o(\xi) = - d(\xi k_1)/d\xi$, we have

$$E_\rho(\rho) = i\frac{k_z}{\kappa_0}A^{>}K_1(\kappa_0\rho) \quad \text{---------- (6b)}$$

Here $A^{>}$ is an arbitrary multiplicative constant.

The particular modified Bessel functions in (6) have the asymptotic behavior, for any order r,

$$\lim_{\rho\to\infty}K_\nu(\kappa_0\rho)=\left(\frac{\pi}{2\kappa_0\rho}\right)^{1/2}e^{-\kappa_0\rho} \quad \text{---------- (7)}$$

so, as remarked earlier, in the regime $k_z > \omega/C$ we have solutions in which the electromagnetic fields are "bound" to the dielectric cylinder. In the regime, $k_z > \omega/C$ (5) also admit solutions in which Ez is a linear combination of the regular Bessel functions J_O and N_o, while E_ρ is a linear combination of J I and N_1. If $k^2 = \omega^2/c^2 - k_z^2 > 0$, these have amplitude that falls off simply as $(k\rho)^{-½}$, multiplied by the oscillatory function $\cos(k\rho + \psi)$ where the phase angle ψ is controlled by the relative admixture of J_v and N_v in a given solution. These solutions describe cylindrically symmetric radiating waves which emanate from the cylinder, as opposed to waves "bound" to the cylinder. In the cylindrical geometry, radiation fields have amplitudes which falloff with distance from the source as $\rho^{-½}$

Our interest here is thus in the regime $k_z > \omega/C$. We need to consider the behavior of the fields within the cylinder $0 < \rho < R$. We suppose for the moment that we have

$$\frac{\omega}{c} < k_z < \frac{\omega}{c}\varepsilon^{1/2} \quad \text{---------- (8)}$$

Then inside the cylinder, one sees that the solutions regular at the origin are, with $k^2 = \omega^2\varepsilon/c^2 - k_z^2$

$$E_z(\rho) = A^{<}J_0(\kappa\rho) \quad \text{---------- (9a)}$$

And

$$E_\rho(\rho) = -i\frac{k_z}{\kappa}A^{<}J_1(\kappa\rho)$$ ---------- (9b)

To obtain a solution, the boundary conditions at the surface of the cylinder ρ = R must be satisfied. We use conservation of tangential components of E(E_z), and normal components of D(D_ρ) , to find

$$A^{>}K_0(\kappa_0 R) = A^{<}J_0(\kappa R)$$ ---------- (10a)

And

$$\frac{A^{>}}{\kappa_0}K_\phi(\kappa_0 R) = -\frac{\varepsilon}{\kappa}A^{<}J_1(\kappa R)$$ ---------- (10b)

Since these are two homogeneous equations, we have a nontrivial solution only if

$$\frac{J_1(\kappa R)}{J_0(\kappa R)} = -\frac{\kappa}{\varepsilon\kappa_0}\frac{K_1(\kappa_0 R)}{K_0(\kappa_0 R)}$$ ---------- (11)

The constraint in (11) may be viewed as an eigenvalue equation for the quantity $k^2 = \omega^2\varepsilon/c^2 - k_z^2$. Notice that we have $k_0^2 = \frac{\omega^2(\varepsilon-1)}{c^2} - k^2$, in fact k_o may be viewed as a function of k^2, which must lie in the range $0 < k < (\omega/c)(\varepsilon - 1)^{1/2}$.

Analysis of the consequences of (11) is assisted by letting KR = A sin θ where A = $(\omega/c)(\varepsilon - 1)^{1/2}$R. Then K_oR = A cos (J, so (11) becomes

$$\frac{J_1(\Lambda \sin\theta)}{J_0(\Lambda \sin\theta)} = -\frac{1}{\varepsilon}\tan\theta\frac{K_1(\Lambda\cos\theta)}{K_0(\Lambda\cos\theta)}$$ ---------- (12)

Here θ lies in the range $0 < \theta < \pi/2$. The ratio K_I/K_o is positive always, approaching unity for large values of its argument. One sees easily that the right-hand side diverges as $\theta \rightarrow \pi/2$, which is where K_O approaches zero, or where k_z approaches ω/c from above.

The behaviour of the left-hand side, and the number of solutions that emerge from (12), depend on the value of Λ. The first zero of $J_o(x)$ occurs at x = 2.4048, and that of $J_1(x)$ at x = 3.8317. Both Jo(x) and JI(x) are positive for 0< x < 2.4048. Hence, we have no solutions whatsoever if 0 < Λ < 2.4048. This means that, for fixed R, there are no solutions of the wave equations which describe modes bound to the cylinder, if the frequency ω lies below the cutoff frequency ω_c given by

$$\omega_c = 2.4048\frac{c}{R(\varepsilon - 1)^{1/2}}$$ ---------- (12)

The solutions of (11) are illustrated graphically, in Fig.

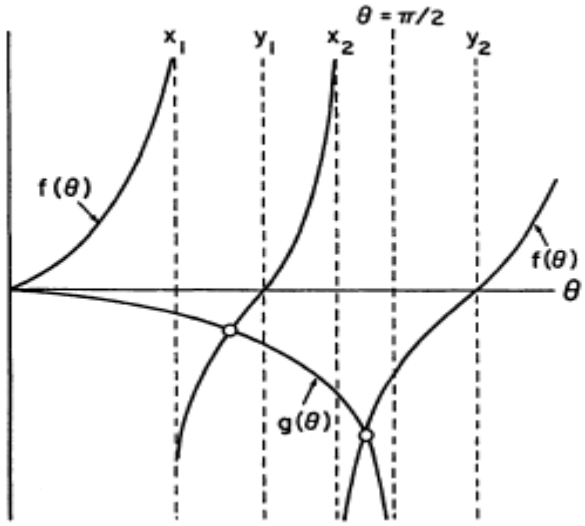


Fig. 7.2. A sketch of the graphical solution of (7.15). One has $g(\theta) = -\tan \theta K_1(\Lambda \cos \theta)/K_0(\Lambda \cos \theta)$, and $f(\theta) = J_1(\Lambda \sin \theta)/J_0(\Lambda \sin \theta)$. It is assumed the value of Λ is such that, as θ is scanned from 0 to $\pi/2$, two zeros of $J_0(\Lambda \sin \theta)$, and one zero of $J_1(\Lambda \sin \theta)$, are located in the interval of interest, as θ varies from 0 to $\pi/2$. The zeros of $J_0(x)$ are denoted by $x_1, x_2, \ldots$, and those of $J_1(x)$ by $y_1, y_2, \ldots$

If $J_0[A \sin \Lambda]$, in the interval $0 < 0 < \pi/2$, has n zeros, there will be n solutions of (11), and thus n distinct modes are guided by the cylinder, at the frequency ω. For the I th such solution, for which $\Lambda \sin \theta$ assumes the value $\Lambda \sin\theta i = \gamma_i \omega$, we have $K_i = \gamma_i \omega/R$. The dispersion relation of the i-th mode may then be written

$$\omega^2 = \frac{c^2}{\varepsilon}\left[\left(\frac{\gamma_i(\omega)}{R}\right)^2 + k_z^2\right] \quad \text{---------- (13)}$$

where the constraint $k_z > \omega/c$ must be recalled. As $k_z \rightarrow \omega/c$, $\gamma_i \omega$ approaches x_i;, the ith zero of the Bessel function (for then $K_o \rightarrow 0$, and the solution is such that $\theta_i \rightarrow \pi/2$). As $k_z \rightarrow \infty$, and we follow the i-th branch, $\omega \rightarrow \infty$, and also Λ becomes large. The right-hand side of (11) becomes independent of frequency, since K_1/K_o approaches unity, and γ_i approaches a frequency independent value, that zero of $J_1(x)$ which lies just above X_i (As $k_z \rightarrow \infty$, $w \rightarrow \infty$, and so does Λ . Use as the variable in (13) $y = \Lambda \sin\theta$, and note the righthand side vanishes for large Λ.) The dispersion relations are sketched in Fig.

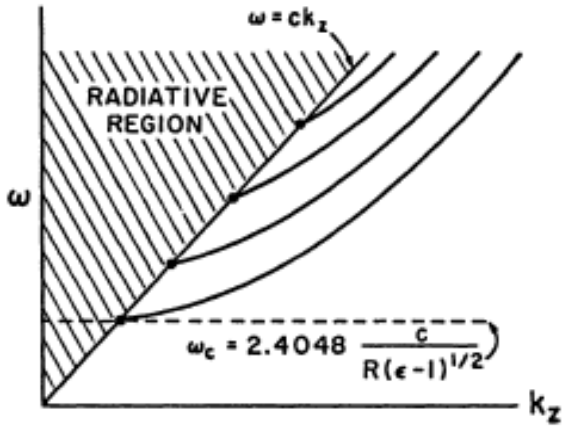


Fig. 7.3. The dispersion relation of the various normal modes of an optical fiber, with radius R and dielectric constant ε. There are no modes with frequency below the cutoff frequency ω_c

We have confined our attention to the region where $k_z < \omega\sqrt{\varepsilon}/c$. One may show that in the regime where where $k_z < \omega\sqrt{\varepsilon}/c$, there are no solutions which satisfy the boundary condition, are regular at the origin $\rho \approx 0$, and "bound" to the cylinder, in the sense described above. Also, one may construct a discussion of the TE modes, by following reasoning very similar to that given above. One finds dispersion relations of the various modes of the form given in (13); there is a cutoff frequency ω_c below which no propagating modes exist. The cutoff frequency for the TE modes is in fact identical to that for the TM modes, given in (12). One may also extend the discussion to the consideration of modes with azimuthal variation, where the various field components vary with the angle ϕ as $\exp(im\phi)$, When the azimuthal "quantum number" $m \neq 0$, the modes are no longer TE and TM in character, but rather have all components of E and B nonvanishing.

We can obtain a physical understanding of the origin of the condition $k_z > \omega/c$ required in our discussion of the guided modes, from the following argument: Consider a dielectric cylinder with radius R large compared to the wavelength of a light beam which is incident on one end of the cylinder, as illustrated in Fig. In this limit, we can use ray optics to trace out the trajectory of the beam. We assume the beam lies in the xz plane illustrated, with the z axis parallel to that of the cylinder. The beam will strike the lower surface of the cylinder, with θ the angle between the beam, and the local normal to its surface. It reflects off the lower surface, to propagate to the upper surface where it again strikes the surface, making the angle θ with respect to the nonnal. By a sequence of repeated reflections, the beam travels down the cylinder, as illustrated. The wave vector of the light in the cylinder is $k = \omega\sqrt{\varepsilon}/c$, and the z component of the wave vector is $k_z = (\omega\sqrt{\varepsilon}/c)\sin\theta$.

In general, such a beam will be attenuated after a relatively small number of reflections. The reason is that at each reflection, a portion of the radiation is transmitted through the surface, into the air outside, assumed here to have a dielectric constant of unity. By Snell's law, the angle $\theta_T = \sin\theta/\sqrt{\varepsilon}$. Now if $\theta > \theta_c$, where $\sin\theta_c = 1/\sqrt{\varepsilon}$, there is no transmitted beam. This is the condition that allows total internal reflection of a beam incident on an interface .The sequence of multiple reflections allow the light beam to propagate down the cylinder unattenuated. The light is "trapped" within the cylinder, and is guided by the cylinder. Notice that when $\theta > \theta_c$, we have $k_z > \omega/c$, precisely the condition that emerged from our analysis of Maxwell's equations, in our search

for waves bound to the cylinder. We saw also that when have $k_z > \omega/c$, the waves outside had radiative character. Discussions of the phenomenon of total internal reflection from a planar interface show that just outside the dielectric surface, there is an electromagnetic disturbance that decays to zero exponentially as one moves into the air above. In our full discussion of waves guided by the dielectric cylinder, outside the cylinder the wave fields exhibit a variation with radial distance ρ given by $K_O(k_o \rho)$ or $K_1(k_o \rho)$. Both functions decay to zero exponentially with ρ. We have, for the cylindrical geometry, the same behavior encountered in the theory of total internal reflection from the flat surface. In essence, the analysis we have given of the nonnal modes of the fiber based on Maxwell's equations is the full treatment of the multiple reflection-induced trapping of an optical wave illustrated in Fig.

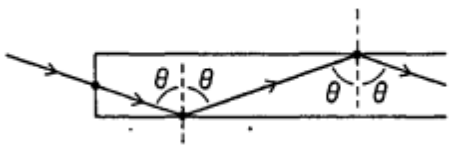


Fig. 7.4. The multiple reflections suffered by an optical beam injected into a dielectric cylinder

Notice also the light rays contain electric or magnetic fields which have components parallel to the axis of the cylinder. If the incident light has electric field parallel to the xz plane, $E_z \neq 0$ and we realize a TM mode in the ray tracing picture. Similarly, if B is parallel to the xz plane, we realize a TE wave. Both require the condition $k_z > \omega/c$ to be trapped in the fiber.

From the guided wave solutions described above, we can construct wave packets that describe the propagation of pulses down the optical fiber. We may do this by superimposing solutions associated with one of the branches of the dispersion relation illustrated in Fig. Let K; be the value of the parameter

$k = (\omega^2\varepsilon/c^2 - k_z^2)^{1/2}$ associated with the ith branch. The z component of the electric field associated with the solution of frequency ω can be written, from (9a), $E_oJ_o(k_ip)\exp[i(k_z - \omega t)]$ inside the cylinder, and the ρ component of the electric field is generated readily through use of (9b).

We may then synthesize a wave packet by superimposing such waves, by writing for the electric fields

$$E_z(\rho, z; t) = \int \frac{d\Omega}{2\pi} E(\Omega) J_0[\kappa_i(\Omega)\rho] e^{i[(k_z + Q_z)z - (\omega + \Omega)t]}$$

---------- (14a)

And

$$E_\rho(\rho, z; t) = -i \int \frac{d\Omega}{2\pi} \frac{(k_z + Q_z)}{\kappa_i(\omega)} E(\Omega) J_1[\kappa_i(\Omega)\rho] e^{i[(k_z + Q_z) - (\omega + \Omega)t]}$$

---------- (14b)

Here, given Ω, Ω_z is chosen from the dispersion relation in (13). That is, inthe dispersion relation ω is replaced by ω + Ω, k_z by $k_z + Q_z$ and then Q_z is found as a function of Ω. Then

$$\kappa_i(\Omega) = [\varepsilon(\omega + \Omega)^2/c^2 - (k_z + Q_z)^2]^{1/2}.$$

If we form an extended wave packet many wavelengths long, then E (Ω) is peaked around. Ω = 0, The Bessel functions and the prefactor may then be extracted from the integral over Ω , and we have

$$E_z(\rho, z; t) = J_0(\kappa_i \rho) E(z, t) e^{i(k_z z - \omega t)} \quad \text{---------- (15a)}$$

And

$$E_\rho(\rho, z; t) = -i \frac{k_z}{\kappa_i} J_1(\kappa_i \rho) E(z, t) e^{i(k_z z - \omega t)} \quad \text{---------- (15b)}$$

Where

$$E(z, t) = \int \frac{d\Omega}{2\pi} E(\Omega) e^{i(Q_z z - \Omega t)} \quad \text{---------- (16)}$$

The function E(z, t) is an envelope function that varies slowly in space and time, to produce a wave packet or pulse which propagates down the waveguide. The transverse variation of the field components, however, to excellent approximation is identical to that in the perfectly monochromatic normal mode solution. The boundary conditions force the transverse profile of the wave form to be "rigid," and the wave has only one degree of freedom, so to speak, its deformability in the z direction.

The self-interaction effects discussed qualitatively just after can thus lead to modifications of the profile of a pulse in the z direction, but will leave the radial or transverse variation unaffected. Thus, in the fiber, we may realize one-dimensional wave propagation in this sense, and self-focusing effects are suppressed by the "rigidity" of the transverse wave profile. This conclusion is

correct as long as the nonlinearity provided by $\chi^{-(3)}$ is weak in the following sense: When $\chi^{-(3)}$ is zero, of course the normal modes are the guided waves as discussed above. When $\chi^{-(3)}$ is "turned on," the full solution will contain admixtures from branches of the guided wave spectrum other than the ith branch discussed above. These admixtures will be small, if the splitting between adjacent branches is large compared to the effective matrix element proportional to $\chi^{-(3)}$ that is responsible for the mixing. At sufficiently low powers, such interbranch mixing can be expected to be small, and we can achieve the one-dimensional limit.

We now turn to a discussion of the self-interaction effects in one-dimensional wave propagation, with application to optical fibers in mind.

NONLINEAR SCHRODINGER EQUATION:

We start with the basic wave equation, for a wave propagating parallel to the z direction. This reads, with E(z, t) the electric field,

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 D}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2}{\partial t^2} P^{(NL)}(z, t) \quad \text{---------- (1)}$$

Assuming that the contribution to $p^{(NL)}$ (z, t) of interest is the self-interaction term displayed will be written

$$\frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2} = -\frac{12\pi\omega^2\bar{\chi}^{(3)}}{c^2}|E_\omega(z,t)|^2 E_\omega(z,t)e^{ikz}e^{-i\omega t}$$

---------- (2)

where in calculating the time derivatives of $p^{(NL)}(z, t)$, we ignore the influence of the slow time variation of the envelope function $E_\omega(z, t)$.

We wish to treat the left-hand side of (1) in the slowly varying envelope approximation, but we wish to retain terms second order in the space and time variation of the envelope function. Also, a central role in what follows will be played by the frequency variation of the dielectric function. A simple calculation gives

$$\frac{\partial^2 E}{dz^2} = -\left[k^2 E_\omega - 2ik\frac{\partial E_\omega}{\partial z} - \frac{\partial^2 E_\omega}{\partial z^2}\right]e^{ikz}e^{-i\omega t}$$

---------- (3)

We now need $\partial^2 D/\partial t^2$. We begin with the relation between D(z, t) and the electric field, taking the finite response time of the medium into account. We have, in the present notation,

$$D(z,t) = e^{ikz}\int dt'\,\varepsilon(t-t')E_\omega(z,t')e^{-i\omega t'}$$

---------- (4)

which may be written after a change of variable

$$D(z,t) = e^{ikz}e^{-i\omega t}\int_{-\infty}^{+\infty} d\tau\varepsilon(\tau)E_\omega(z,t-\tau)e^{+i\omega\tau}$$

---------- (5)

Since our envelope function varies in time slowly, we may expand $E_\omega(z,t - \tau)$ in powers of τ.

$$E_\omega(z,t-\tau) = E_\omega(z,t) - \tau\left(\frac{\partial E_\omega}{\partial t}\right) + \frac{1}{2}\tau^2\left(\frac{\partial^2 E_\omega}{\partial t^2}\right) + \cdots$$

---------- (6)

When this is inserted into (5), and we note the definition of the frequency dependent dielectric constant $e(\omega)$, we have the following relation between D(z, t), and the electric field:

$$D(z,t) = \left[\varepsilon(\omega)E_\omega(z,t) + i\left(\frac{\partial\varepsilon}{\partial\omega}\right)\left(\frac{\partial E_\omega}{\partial t}\right) - \frac{1}{2}\left(\frac{\partial^2\varepsilon}{\partial\omega^2}\right)\left(\frac{\partial^2 E_\omega}{\partial t^2}\right)\right]e^{ikz}e^{-i\omega t}.$$

---------- (7)

Upon taking the second derivative of (7) with respect to time, and ignoring all terms higher order than second in the differentiation of the envelope function $E_\omega(z, t)$ with respect to time, we have for the moment a rather complex expression:

$$\frac{\partial^2 D}{\partial t^2} = -\left[\omega^2 \varepsilon E_\omega + 2i\omega\left(\varepsilon + \frac{1}{2}\omega\frac{\partial \varepsilon}{\partial \omega}\right)\frac{\partial E_\omega}{\partial t} - \left(\varepsilon + 2\omega\frac{\partial \varepsilon}{\partial \omega} + \frac{1}{2}\omega^2\frac{\partial^2 \varepsilon}{\partial \omega^2}\right)\frac{\partial^2 E_\omega}{\partial t^2}\right] e^{ikz} e^{-i\omega t}$$

---------- (8)

One proceeds by substituting (3,8) into the left-hand side of the wave equation (1), then choosing the k to be that of an electromagnetic wave of frequency ω) in the linear dielectric. Thus, k, a function of frequency, is given by

$$k^2(\omega) = \frac{\omega^2}{c^2}\varepsilon(\omega)$$

---------- (9)

The wave equation then becomes

$$2i\left[k\frac{\partial E_\omega}{\partial z} + \frac{\omega}{c^2}\left(\varepsilon + \frac{1}{2}\omega\frac{\partial \varepsilon}{\partial \omega}\right)\frac{\partial E_\omega}{\partial t}\right] + \frac{\partial^2 E_\omega}{\partial z^2} \cdot$$

$$-\frac{1}{c^2}\left[\varepsilon + 2\omega\left(\frac{\partial \varepsilon}{\partial \omega}\right) + \frac{1}{2}\omega^2\frac{\partial^2 \varepsilon}{\partial \omega^2}\right]\frac{\partial^2 E_\omega}{\partial t^2} = -\frac{12\pi\omega^2\bar{\chi}^{(3)}}{c^2}|E_\omega|^2 E_\omega$$

---------- (10)

We can rewrite the left-hand side of (10) in terms of physically meaningful quantities. When the properties of electromagnetic waves in a medium with frequency dependent dielectric constant were discussed, we made mention of a characteristic velocity, the group velocity $v_g(\omega) = \frac{\partial \omega}{\partial k}$. Upon differentiating both sides of (9) with respect to w, one finds the identity

$$\frac{\omega}{c^2}\left(\varepsilon + \frac{1}{2}\omega\frac{\partial \varepsilon}{\partial \omega}\right) = \frac{k}{v_g}$$

---------- (11)

while differentiation a second time gives

$$\frac{1}{v_g^2} + k\frac{\partial}{\partial \omega}\left(\frac{1}{v_g}\right) = \frac{1}{c^2}\left(\varepsilon + 2\omega\frac{\partial \varepsilon}{\partial \omega} + \frac{1}{2}\omega^2\frac{\partial^2 \varepsilon}{\partial \omega^2}\right)$$

---------- (12)

With these two results, the left-hand side of (10) may be expressed in terms of physically meaningful quantities. We recall the definition of the phase velocity $v_p = \frac{\omega}{k}$. Then one has

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_g}\frac{\partial}{\partial t}\right)E_\omega-\frac{i}{2k}\left(\frac{\partial^2}{\partial z^2}-\frac{1}{v_g^2}\frac{\partial^2}{\partial t^2}\right)E_\omega+\frac{i}{2}\frac{\partial}{\partial\omega}\left(\frac{1}{v_g}\right)\frac{\partial^2 E_\omega}{\partial t^2}$$

$$=i\frac{6\pi\omega v_p\bar{\chi}^{(3)}}{c^2}|E_\omega|^2E_\omega\ .$$

---------- (13)

Note the identity

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_g}\frac{\partial}{\partial t}\right)E_\omega-\frac{i}{2k}\left(\frac{\partial^2}{\partial z^2}-\frac{1}{v_g^2}\frac{\partial^2}{\partial t^2}\right)E_\omega$$

$$=\left[1-\frac{i}{2k}\left(\frac{\partial}{\partial z}-\frac{1}{v_g}\frac{\partial}{\partial t}\right)\right]\left(\frac{\partial}{\partial z}+\frac{1}{v_g}\frac{\partial}{\partial t}\right)E_\omega$$

---------- (14)

Within the spirit of our slowly varying envelope approximation, the square bracket on the right-hand side of (14) may be replaced by unity, since the envelope function $E_\omega(z, t)$ varies little over a spatial region the size of a wavelength, and varies little over one cycle of the carrier wave at frequency ω. Hence, the term $(2k)^{-1}(\frac{\partial}{\partial z}-v_g^{-1}\partial/\partial t)$ inside the square bracket provides only a small correction to unity, and can be dropped.

We thus arrive at our primary equation,

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_g}\frac{\partial}{\partial t}\right)E_\omega(z,t)+\frac{i}{2}\frac{\partial}{\partial\omega}\left(\frac{1}{v_g}\right)\frac{\partial^2 E_\omega}{\partial t^2}=i\frac{6\pi\omega v_p\bar{\chi}^{(3)}}{c^2}|E_\omega|^2E_\omega$$

--------- (15)

which is the principal equation we shall analyse in this section.

If we set $\chi^{-(3)}$ to zero, and if furthermore we overlook the term which arises from the frequency variation of the group velocity, then (15) reduces to the elementary statement

$$\left(\frac{\partial}{\partial z}+\frac{1}{v_g}\frac{\partial}{\partial t}\right)E_\omega(z,t)=0$$

---------- (16)

Any function of the form

$$E_\omega(z,t)=E_\omega(z-v_gt)$$ ---------- (17)

will satisfy this equation. Thus, the envelope function propagates through the medium unchanged in from, with a speed equal to the group velocity vg • This is the basis of the assertion that a pulse propagates with a velocity equal to the group velocity, not the phase velocity.

Our next goal will be to study the influence of the effect of frequency variation of the group velocity, in combination with that of the nonlinearity. Before we do this, we rearrange (15). We define

$$\lambda = \frac{6\pi\omega v_p \bar{\chi}^{(3)}}{c^2}$$ ---------- (18)

a parameter that can be either positive or negative depending on the sign of $\chi^{-(3)}$, and also

$$\frac{\partial}{\partial\omega}\frac{1}{v_g} = -\frac{1}{v_g^2}\frac{\partial v_g}{\partial\omega} = -\sigma\mu$$ ---------- (19)

where μ is a positive number always, and $\sigma = \pm 1$. Thus, if $\partial v_g / \partial\omega$ is positive, we choose $\sigma = \pm 1$, while $\sigma = -1$. if this quantity is negative. Thus, (16) is written

$$-\frac{1}{2}\sigma\mu\frac{\partial^2 E_\omega}{\partial t^2} - \lambda|E_\omega|^2 E_\omega = i\left(\frac{\partial}{\partial z} + \frac{1}{v_g}\frac{\partial}{\partial t}\right)E_\omega$$ ---------- (20)

We now change variables, from (z, t) to a new set (ξ, τ), where

$$\tau = t - \frac{1}{v_g}z$$ ---------- (21a)

And

$$\xi = z$$ ---------- (21b)

Then (20) is transformed into

$$-\frac{1}{2}\sigma\mu\frac{\partial^2 E_\omega}{\partial\tau^2} - \lambda|E_\omega|^2 E_\omega = i\frac{\partial E_\omega}{\partial\xi}$$ ---------- (22)

which is a classical differential equation of nonlinear physics, the nonlinear Schrodinger equation.

If λ = 0, this has the form of the ordinary Schrodinger equation for a free particle, whose mass is inversely proportional to μ. The variable τ enters as an effective spatial coordinate, and ξ an effective time. When $\lambda \neq 0$, the combination $-\lambda|E_\omega|^2$ plays the role of an effective potential energy. The presence of the wave modifies the medium in a manner that may be represented as a potential energy that affects the wave form itself. If $\sigma = +1$ and $\lambda > 0$, the effective potential energy is attractive. The wave "digs a hole" in the medium, and can be trapped or confirmed to this self-induced hole. If $\sigma = -1$, and $\lambda < 0$, self-trapping can also occur. The solutions that describe such states will be the solitons of the nonlinear Schrodinger equation.

Before we examine the influence of the nonlinearity, we set λ = 0, and explore the influence of the frequency variation of the group velocity on wave packet propagation, in the limit where linear theory applies.

LINEAR THEORY:

If $\chi^{-(3)} = 0$ then the space and time evolution of the envelope function E_{ω}., is controlled, with $\lambda = 0$. Thus, we have

$$-\frac{\sigma\mu}{2}\frac{\partial^2 E_\omega}{\partial\tau^2} = \mathrm{i}\frac{\partial E_\omega}{\partial\xi}$$ ---------- (1)

Suppose we are supplied with the behaviour of the function $E_{\omega}(\xi, \tau)$ as a function of τ at $\xi = 0$.

We write $E_{\omega}(0, \tau)$ in the form

$$E_\omega(0,\tau) = \int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi} F(0,\Omega)\mathrm{e}^{-\mathrm{i}\Omega\tau}$$ ---------- (2)

The evolution of the envelope function with the variable ξ can then be written

$$E_\omega(\xi,t) = \int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi} F(\xi,\Omega)\mathrm{e}^{-\mathrm{i}\Omega\tau}$$ ---------- (3)

Where (1) implies

$$\frac{\partial F(\xi,\Omega)}{\partial\xi} = -\frac{\mathrm{i}}{2}\sigma\mu\Omega^2 F(\xi,\Omega),$$ ---------- (4)

$$F(\xi,\Omega) = F(0,\Omega)\exp\left(-\frac{\mathrm{i}}{2}\sigma\mu\Omega^2\xi\right)$$ ---------- (5)

Or

Hence, we have

$$E_\omega(\xi,\tau) = \int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi} F(0,\Omega)\mathrm{e}^{-\mathrm{i}\Omega\tau}\,\mathrm{e}^{-\mathrm{i}\sigma\mu\Omega^2\xi/2}$$ ---------- (6)

We now recognize $\tau = t - z/v_g$, and $\xi = z$, so in terms of the original variables in space and time,

$$E_\omega(z,t) = \int_{-\infty}^{+\infty} \frac{d\Omega}{2\pi} F(0,\Omega)\mathrm{e}^{-\mathrm{i}\Omega[t-(z/v_g)]}\mathrm{e}^{-\mathrm{i}\sigma\mu\Omega^2 z/2}$$ ---------- (7)

We see easily that if we are given the profile of the pulse in time at the point z = 0, then F(0,Ω) is the frequency Fourier transform of this pulse:

$$F(0,\Omega) = \int_{-\infty}^{+\infty} dt E_\omega(0,t) e^{+i\Omega t}$$

---------- (8)

The relations in (7,8) allow us to follow the subsequent evolution of any pulse whose time profile is known as z=0, Clearly, if the dielectric constant is independent of frequency,$\mu = 0$, and the pulse propagates through the medium unchanged in shape, with the group velocity v_g.

To illustrate the influence of dispersion, expressed here as the origin of the frequency dependence of the group velocity, one may consider the behaviour of a pulse of Gaussian profile, for which

$$E_\omega(0,t) = E_0 e^{-t^2/t_0^2}$$

---------- (9)

where t_o is a measure of its width. The various integrations may be carried out quite easily, to find

$$E_\omega(z,t) = \frac{t_0 E_0}{t_0^2 + 2i\sigma\mu z} \exp\left(-\frac{(t - z/v_g)^2}{t_0^2 + 2i\sigma\mu z}\right)$$

---------- (10)

The energy density in the pulse is proportional to $|E_\omega(z,t)|^2$, which has the form

$$|E_\omega(z,t)|^2 = \frac{t_0^2 E_0^2}{t_0(z)^2} \exp\left[-\frac{2}{v_g^2 t_0(z)^2}(z - v_g t)^2\right]$$

---------- (11)

where we have defined

$$t_0(z) = \left(t_0^2 + \frac{4\mu^2}{t_0^2} z^2\right)^{1/2}$$

---------- (12)

The expression in (11) describes a Gaussian pulse centered about the position $z= v_g t$. Thus, the energy density in the pulse still propagates with the group velocity, as before. However, as z increases, the pulse broadens, and its peak intensity decreases. The width of the pulse is $t_o z$ when the pulse has reached the position z. Notice that the pulse broadens, independently of the sign of the parameter μ.The various Fourier components from which the pulse was synthesized propagate at different phase velocities so as the pulse propagates down the material, necessarily it broadens in profile.

We can estimate the propagation distances that the pulse travels before the broadening becomes appreciable. We do this with optical fibers in mind. Before we turn to numerical estimates, a few comments on optical fibers will prove useful.

The optical fibers of interest are made from Si02 (quartz), and have diameters the order of a few microns. To realize very long propagation lengths in these transparent materials, one seeks a minimum in $\varepsilon_2(\omega)$. We discussed the general behaviour of $\varepsilon_2(\omega)$ in insulators. One has absorption in the far infrared from the interaction of radiation with lattice vibrations. As the frequency of the radiation increases above those characteristics of the lattice vibrations, we saw that $\varepsilon_2(\omega)$. decreases very rapidly with increasing frequency, in an exponential manner. This is the multiphoton regime. Then, as frequency is increased further, we encounter the "Urbach tail," which is precursor of the electronic absorption edge.

In the quartz fibers, the minimum in the absorption constant occurs near a wavelength of 1.5μm, which corresponds to a photon energy of roughly 0.8 eV. This lies in the near infrared. Most experiments are carried out in this wavelength regime, and in fact practical optical fibers utilize wavelengths in the one micron realm, since very long optical paths (kilometers) are desired.

The frequency variation of the group velocity can be inferred from the principles. Consider the dispersion curves of an electromagnetic wave in a medium with a sharp, well-defined resonance in $\varepsilon(\omega)$, at the frequency ω_o. For frequencies below ω_0 ,v_g decreases with increasing frequency, and frequencies above ω_0 ,v_g increases as frequency increases; the group velocity v_g is the slope of the dispersion curves displayed in Fig. In an optical fiber, the behaviour of $\varepsilon(\omega)$ is more complex, but the various lattice contributions to $\varepsilon(\omega)$ lie below the frequency ω, for ω in the near vicinity of the gap. The electronic transitions lie above. Somewhat below the minimum in $\varepsilon_2(\omega)$, the frequency of variation of $\varepsilon(\omega)$ is controlled importantly by the lattice contributions, and $\partial v_g/\partial\omega < 0$. As one moves through the minimum in $\varepsilon_2(\omega)$, one senses the higher frequency electronic contributions, and when these control the frequency variation of $\varepsilon(\omega)$, one has $\partial v_g/\partial\omega > 0$. It follows there is a point in the near vicinity of the absorption minimum where $\partial v_g/\partial\omega$ vanishes. In the quartz fibers, this occurs at the wavelength of 1.3μm Near this frequency, pulses can be propagated in the linear regime over appreciable distances, with minimal distortion.

Near, but not far from the zero in $\partial v_g/\partial\omega$, say at 1.5 μm, the value of $\partial v_g/\partial\omega$ is roughly 10-5 cm, which gives our parameter μ the value 2×10^{-26} S^3/cm, assuming $v_g \sim 2 \times 10^{10}$ cm/s in this spectral region. If the width of the pulse is t_o at z = 0, the distance the pulse must travel for its width to increase to $2t_o$ is $z = \sqrt{3}t_o^2/4\mu$. If t_o is 10 ps, a rather narrow pulse, then for the pulse to double in width, the propagation distance must be in the range of 2.5 km.

EFFECT OF FIBRE LOSS:

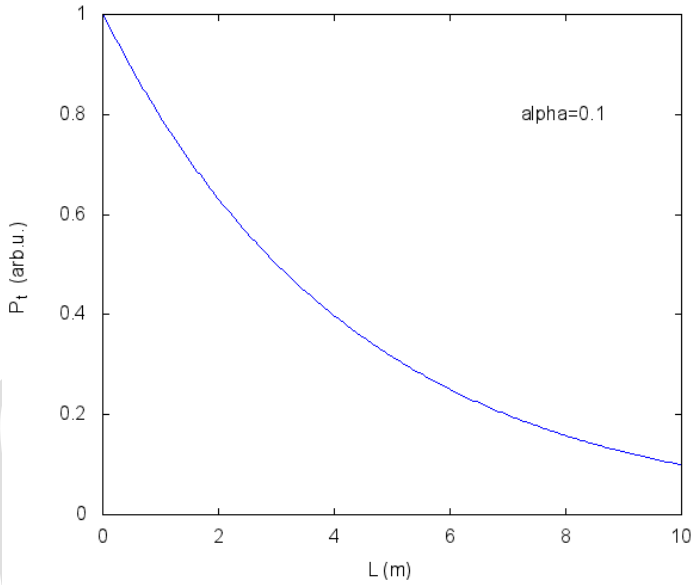
Attenuation, that is to say, intensity reduction with respect to distance traveled through a transmission medium, occurs in every fiber. To characterize it, first express the intensity ratio P_i/P_t as

$$\frac{P_i}{P_t} = 10^{\alpha L}, \qquad \text{---------- (1)}$$

where L is the fiber length, P_t is the transmitted intensity and P_i is the initial (input) intensity. The proportionality factor α is called attenuation coefficient; this is the quantity that describes the total fiber losses. It is generally measured in dB/km, and from (1) it can be expressed as follows (factor 10 causes the quantity to be measured in decibel):

$$\alpha = \frac{10}{L}\lg\left(\frac{P_t}{P_i}\right). \quad \text{---------- (2)}$$

Losses originate from various phenomena, such as material absorption, Rayleigh scattering, scattering on the core-cladding interface, or even bending of the fiber. From this list one may suspect that the loss should be wavelength dependent. Indeed, loss spectrum can be assigned to the individual fibers. As an example, the Rayleigh scattering loss, which is a fundamental loss mechanism, varies with the wavelength as λ^{-4}, therefore it is dominant at lower wavelengths.



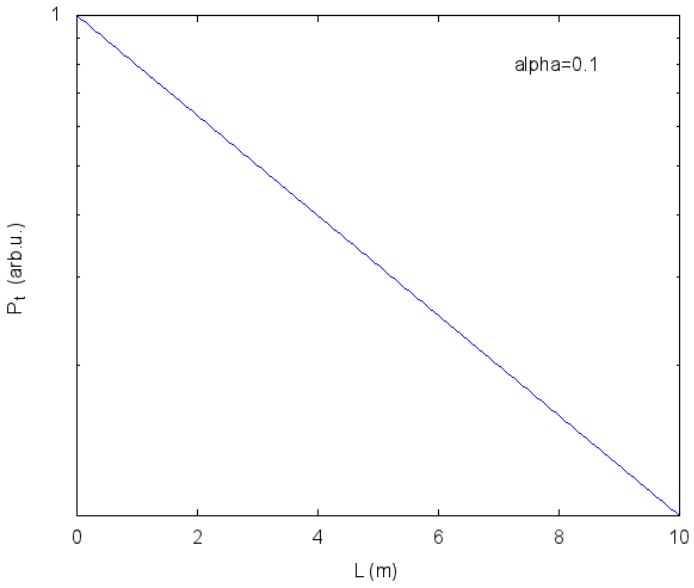


Figure: Transmitted power versus fiber length. (Left) Linear scale. (Right) Logarithmic scale.

POSSIBLE QUESTIONS

PART B

1. Mention any two advantages of fibre optics
2. Give short note on optical fibre.
3. Differentiate single-mode fibre and multi-mode fibre.
4. Define the term 'wave guides'.
5. What is fibre loss?

PART C

1. Discuss in detail about normal modes of optical fibres.
2. Derive the expression for nonlinear Schrodinger equations.
3. Briefly explain about the linear theory.
4. Explain about fibre optics.
5. Write a short notes about fibre loss.
6. Describe about conditions of generation of a solutions in optical fibres.

KARPAGAM ACADEMY C	
	DEP
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Question	**Choice 1**
UNIT 5	
The principle of fibre is same as that of _____	light
The core fibre is typically made of _____ doped with impurities	impurities
The _____ which surrounds the fibre core is made from pure sili	cladding
The first type of fibre optics put to use was called _____	step index fibre
The change in index also has the effect of _____ the light back towa	bending
The multimode step index fibre has a core of _____ in diameter	50-200 µm
Waveguides are used in _____ region.	MUF
Waveguides are used in _____ region.	microwave
Wave guide dispersion occurs only in fibers with a _____ mod	single
The most important application of optical fibres in the field of _____	communication
The fibre optical system is widely used in _____ services	defence
The refractive index of fibre _____ for parabolic	increases
The size of the step index fibre are _____ µm	125
Only fundamental mode is used to transmite _____ in fibre	energy
Optical fibre is made of _____	glass
Total internal reflection is the theory for _____	optical fiber coupler
Scattering of light in a fibre is also wavelength _____	dependent
Light is lost when it is first launched into any _____ along the f	splices
The variation in optical fibrre are known as _____	microwave
When data is sent in fibre as it cointains pulses in the intensity of ____	light
Modal dispersion occurs fibres that have morethan _____ mode	one
The loss of _____ in a fibre occurs because of mechanism	light
Fiber connectors are _____ joints	removable
_____ is a passive devices	optical fiber coupler
_____ is used as a joint in connecting 2 fibres	fiber slices
The loss of optical fibre is measured in terms of the _____	decibel
Fiber Optics is communication over_____	copper wires
Standard single mode fibers (SSMF) are utilized mainly for operation i	C-band
Single mode fibers allow single mode propagation; the cladding diame	Twice the core diameter
In single mode fibers, the most beneficial index profile is	Coaxial cable
The performance characteristics of multimode graded index fibers are	Better than multimode ste
Multimode graded index fibers are manufactured from materials with_	lower
Multimode step index fibers have a bandwidth of	2 to 30 KHz km
Multimode step index fiber has ____ core diameter.	large
Multimode step index fiber has ____ numerical aperture.	large
Which kind of dispersion phenomenon gives rise to pulse spreading in	group velocity
If a fiber operates at 1400nm with the diameter of about 10 µm, n1 = 1	6.125
Which rays exhibit the variation in the light acceptability ability of the	Meridional
Which is the transmission medium for VLF electromagnetic waves esp	Paired wires

In the structure of a fiber, which component provides additional streng	core
In the structure of fiber, the light is guided through the core due to tota	reflection
In an optical fiber, the concept of Numerical aperture is applicable in d	Light Collection
In single-mode fibers, how does the fraction of energy traveling throug	As a crescent wave
The principle of fibre optical communication	frequency modulation
What is the other name for maximum external incident angle?	optical angle
A single mode fibre has _____ intermodal dispersion than multimode.	very large
How does the refractive index vary in graded index fibre?	tangentially
The refractive index of the core is _____ along the fibre axis.	maximum
Which of the following loss occurs inside the fibre?	radiative loss
_____ used in fabrication of fibres.	mica
Optical fibre has ___ bandwidth.	large
Which of the following is known as fibre optic back bone?	telecommunication
_____are used for transmission of light.	Optical fibre
An optical fiber is a thin transparent rod, usually made of	metal
_____is lighter and less bulky than equivalent copper cable.	metal
Voice telephones is one of the application of	fiber optic communicatio
video phones is one of the application of	Analog communication
optical fiber coupler is _____device.	active
_____loss is low in fiber optic communication.	Power
In fiber optics the scale decibel is used to measure	fiber loss

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OF HIGHER EDUCATION, COIMBATORE – 21			
'ARTMENT OF PHYSICS			
ASS: III B. Sc., PHYSI	**BATCH: 2016-2019**		
Sixth Semester			
IED OPTICS (16PHU601B)			
IPLE CHOICE QUESTIONS			
Choice 2	**Choice 3**	**Choice 4**	

waves	particle	sound
silica	glass	copper
gratting	prism	none of these
Coaxial cable	tagging	core
tagging	covarient	invareint
20-50 cm	50-200 cm	20-50 cm
VHF	OHF	OWF
ultra-violet	infra-red	radio-wave
double	multi	tetra
transmission	modulation	propagation
computer	signal	none
decreases	remains constant	zero
250	500	350
particle	quantas	packets
mica	silica	iron
optical fibre	hologram	silica
independent	greater	smaller
slices	couplers	fibres
microbend	nanobend	millibend
sound	particle	wave
double	multi	none
sound	energy	particle
fixed	connecting	jointer
fibre splices	connectors	jointer
fiber splices	conncetors	coupler
ampere	hertz	intensity
speaker wire	thin stands of glass	thin wire
L-band	O-band	C-band and L-band
Thrice the core diamete	Five times the core diar	Ten times the core diameter
Graded index	fibre	core
Lesser than multimode	Same as multimode ste	Negligible
higher	negligible	none of the above
6 to 50 MHz km	10 to 40 KHz km	8 to 40 KHz km
small	infinte	none of the above
small	infinte	none of the above
phase velocity	Intramodal	extramodal
3.12	5.67	4.21
Skew	Leaky	all the above
copper wires	wireless	Coaxial cable

Fibre material	Buffer Coating	none of the above
refraction	diffraction	dispersion
Light Scattering	Light Dispersion	Light Polarization
As a gibbous wave	As an evanescent wave	All of the above
population inversion	total internal reflection	doppler effect
wave guide acceptance	refraction angle	diffraction angle
very small	low	infinite
longitudinally	transversely	radially
minimum	equal	none of the above
scattering	absorbtion	attenuation
glass	iron	aluminium
small	infinite	none of the above
cable television	delay lines	bus topology
modulator	demodulator	connecting wires
films	glass	chemicals
films	modulator	fibres
Digital communication	Analog communication	none of the above
fiber optic communicat	Digital communication	none of the above
passive	both a and b	none of the above
Data	Tranmission	Receiver
transmission rate	reflectance power	gain

Answer

light
silica
cladding
step index fibre
bending
50-200 μm
OHF
microwave
single
communication
defence
decreases
125
energy
glass
optical fibre
dependent
splices
microbend
light
one
light
removable
optical fiber coupler
fibre splices
decibel
thin stands of glass
O-band
Ten times the core diameter
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fiber optic communication
passive
Tranmission
fiber loss