## UNIT-I

Planck's quantum, Planck's constant and light as a collection of photons; Blackbody Radiation: Quantum theory of Light; Photo-electric effect and Compton scattering. DeBroglie wavelength and matter waves; Davisson-Germer experiment. Wave description of particles by wave packets. Group and Phase velocities and relation between them.Two-Slit experiment with electrons. Probability. Wave amplitude and wave functions.

**PLANCK HYPOTHESIS**

Planck suggested that the correct results could be obtained if the energy of oscillating electrons is taken as discrete rather than continuous. He suggested quantum theory of radiation. Planck suggested in deriving the formula, which agrees extremely well with experimental results. He derived the radiation law by using the following assumptions.

a. A black body chamber is filled up not only with radiation, but also with simple harmonic oscillators or harmonic oscillators or resonators of molecular dimensions. They can vibrate with all possible frequencies.

b. The frequency of radiation emitted by an oscillator is the same as the frequency of its vibration.

c. An oscillator cannot emit energy in a continuous manner. It can emit energy in the multiples of a small unit called Quantum (Photon).

If an oscillator is vibrating with a frequency , it can radiate in quantas of magnitude h .
The oscillator can have only discrete energy En given by

$$E_n = n\,h\gamma = n\varepsilon \text{ where } h\,\gamma = \varepsilon$$

Here n is an integer and h is the Planck's constant

The energy of the single photon of the frequency $\gamma$ is

$$\varepsilon = h\,\gamma$$

d. The oscillators can emit or absorb radiation energy in packets of h .

This implies that the exchange of energy between the radiation and matter cannot take place continuously but are limited to discrete set of values 0, $h\gamma$, $2h\gamma$, $3h\gamma$, $nh\gamma$

The emission of radiation corresponds to a decrease and absorption to an increase in the energy and amplitude of an oscillator

**Planck's quantum theory :**

According to Planck's quantum theory,

Different atoms and molecules can emit or absorb energy in discreet quantities only. The smallest amount of energy that can be emitted or absorbed in the form of electromagnetic radiation is known as quantum.

The energy of the radiation absorbed or emitted is directly proportional to the frequency of the radiation. The energy of radiation is expressed in terms of frequency as,

$$E = h\nu$$

Where,

**E** = energy of the radiation

**h** = Planck's constant ($6.626\times10^{-34}$ J.s)

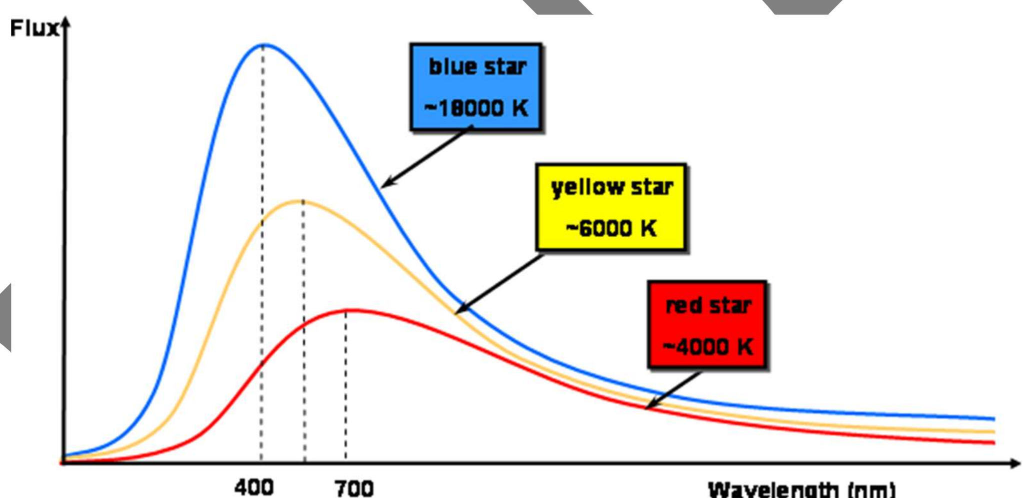
**ν**= Frequency of radiation

**BLACKBODY RADIATION**

All objects with a temperature above absolute zero (0 K, -273.15 ºC) emit energy in the form of **electromagnetic radiation**.

A **blackbody** is a theoretical or model body which absorbs all radiation falling on it, reflecting or transmitting none. It is a hypothetical object which is a "perfect" absorber and a "perfect" emitter of radiation over all **wavelengths**.

The spectral distribution of the thermal energy radiated by a blackbody (i.e. the pattern of the intensity of the radiation over a range of wavelengths or frequencies) depends ***only on its temperature***.




The characteristics of blackbody radiation can be described in terms of several laws:

1. **Planck's Law** of blackbody radiation, a formula to determine the spectral energy **density** of the emission at each **wavelength** *($E_\lambda$)* at a particular absolute temperature (T).

$$E_\lambda = \frac{8\pi hc}{\lambda^5(e^{(hc/\lambda\kappa T)}-1)}$$

2. **Wien's Displacement Law**, which states that the **frequency** of the peak of the emission ($f_{max}$) increases linearly with absolute temperature (T). Conversely, as the temperature of the body *increases*, the wavelength at the emission peak *decreases*.

$$f_{max} \alpha T$$

3. **Stefan–Boltzmann Law**, which relates the *total* energy emitted *(E)* to the absolute temperature (T).

$$E \alpha T^4$$

The blackbody radiation curves have quite a complex shape (described by Planck's Law). The spectral profile (or curve) at a specific temperature corresponds to a specific peak wavelength, and vice versa.

As the temperature of the blackbody increases, the peak wavelength decreases (Wien's Law).

The intensity (or **flux**) at all wavelengths increases as the temperature of the blackbody increases.

The total energy being radiated (the **area** under the curve) increases rapidly as the temperature increases (Stefan–Boltzmann Law).

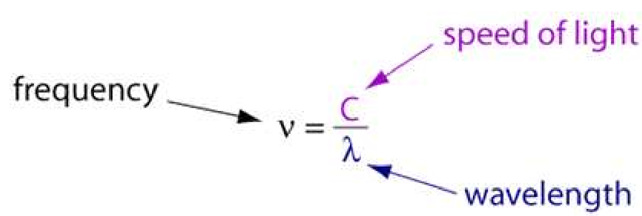
Although the intensity may be very low at very short or long wavelengths, at any temperature above absolute zero energy is theoretically emitted at *all* wavelengths (the blackbody radiation curves never reach zero).

**QUANTUM THEORY OF LIGHT**

Quantum theory describes that matter, and light consists of minute particles that have properties of waves that are associated with them. Light consists of particles known as photons and matter are made up of particles known as protons, electrons, and neutrons. Let's understand how the light behaves as a particle and as a wave.

**Wave Theory of Light**

Diffraction is one of the behaviours of waves. Interference is the other behaviour of waves. James Clerk Maxwell showed that light is an electromagnetic wave that travels at the speed of light through space. The light frequency is relevant to its wavelength according to the following relation.



**Particle Behavior of Light**

The major feature of the photoelectric experiment is the electron is emitted by the metal with a particular kinetic energy. For instance, the bigger is the wave at oceans; higher is the energy associated with it. As the light gets brighter, some electrons are emitted while the kinetic energy remains same.

There exists a critical frequency for every metal, ν0 lower than which no electrons are not emitted. This describes that the kinetic energy equals to the light frequency times a constant, known as Planck's Constant by the symbol h.

$$\mathbf{h = 6.63 \times 10\text{-}34\ J \cdot s \leftarrow \text{Planck's Constant}}$$

The equation for the kinetic energy of an emitted electron is written as follows.

$$E_{kin} = hu\text{-}hu_o$$

A consistent explanation would be when the picture of light comes in discrete packages known as photons, and every photon should have sufficient energy to eject a single electron. Hence, the energy of a single photon is given by,

$$\mathbf{Ephoton = h\ \nu}$$

Hence, when all the phenomenons are put together, it can be concluded that light is a particle with wave behaviour.

**Dual Nature of Light :**

There are some experimental phenomena of light like reflection, refraction, interference, diffraction etc., which can be explained only on the basis of wave theory of light, i.e. these phenomena verify the wave nature of light. There are some experimental phenomena of light itself like photoelectric effect, compton effect raman effect etc….which can be explained only on the basis of the particle nature of light (i.e. quantum theory) i.e. these phenomenon verify the particle nature of light on the basis of the above experimental phenomena it was inferred that light does not have any definite nature, rather its nature depends on its experimental phenomenon.

In some experimental phenomena it behaves like particles (i.e. photons). This is known as the dual theory of light. The wave nature and particles nature both can be possible simultaneously.

**De Broglie Hypothesis**

de Broglie imagined that as light (i.e. energy in general) possesses both nature (i.e. wave and particle) similarly matter must also posses both nature particle as well as wave. As matter consists of minute particles, hence its nature is particles nature. de Broglie imagined that despite particle nature of matter waves must also be associated with material particles. These imaginary waves presumed to be associated with material particles, are defined as matter waves.

**Matter Waves:**

The waves presumed to be associated with moving material particles on the imagination of de Broglie are defined as matter waves.

**Characteristics of Matter Waves:**

The wavelength of matter waves is inversely proportional to the momentum of the particle.

Matter waves travels even in vacuum, hence these are not mechanical waves.

Matter waves are produced due to the motion of material particle. These waves are associated with every moving particles.

Actually matter waves are probabilistic waves because these waves represent the probability of fining a particle in space.

Practical observation of matter waves is possible only when the wave length of matter wave is greater than the size of the particle (i.e. $\lambda >> a$).

These waves are also associated with electrically natural particle hence these cannot be the electromagnetic waves even.

Matter waves propagate in the form of wave packet with group velocity. The phase velocity of matter waves can be greater than the light.

The wavelength of matter waves does not depend on the nature and charge of the particle.

According to Plank's quantum theory the energy of photon is given by

$$E = h\nu, \text{ where } h = \text{Plank's constant}$$

$$\text{since },\ c = \nu\lambda$$

$$E = hc/\lambda$$

$\lambda$ = wavelength of photon

$\nu$ = frequency of photon

**EFFECTIVE MASS OF PHOTON**

According to Einstein's theory, if the energy (hn) of photon is converted into matter then the mass of matter created or the mass of photon in moving state is defined as the effective mass of photon. If the effective mass of the photon is m then according to Einstein's mass – energy relation its energy is

$$E = mc^2 = h\nu = hc/\lambda$$

Effective mass of photon , $m = E/c^2 = h\nu/c^2$

The momentum of photon , $P = mc = E/c = h\nu/c = h/\lambda$

The wavelength associated with photon , $\lambda = h/P = h/mc$

Photon is an uncharged particle, its rest mass is zero, spin is h and its velocity is equal to that of light.

***de Broglie Wavelength Associated with Moving Particles :***

Energy of a particle of mass m and moving with velocity v .

$$E = \frac{1}{2}mv^2 = \frac{P^2}{2m}$$

where, P = momentum of particle

momentum of particle $P = mv = \sqrt{(2mE)}$

According to de Broglie theory the wavelength associated with the particles.

$$\lambda = \frac{h}{P} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

The order of magnitude of wave lengths associated with microscope particles is $10^{-24}$Å. Whereas the smallest wavelength whose measurement is possible is that of g-rays (g ~ $10^{-5}$ Å – 1Å). This is the reason why the wave nature of microscopic particles is not observable.

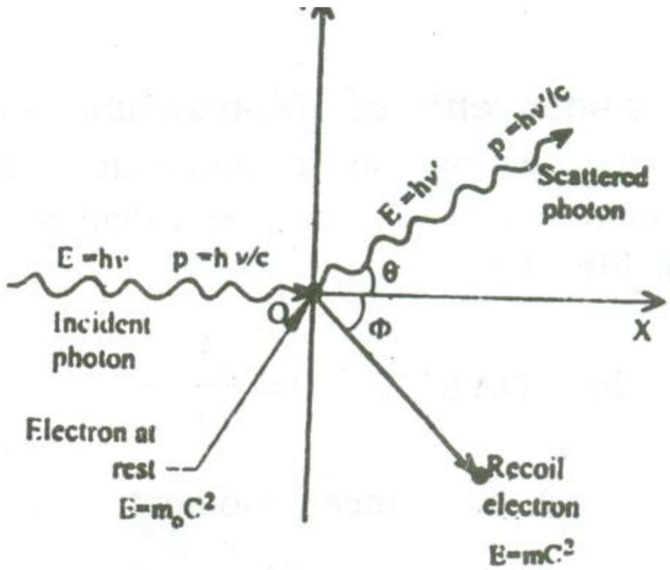
**COMPTON EFFECT**

When a beam of monochromatic radiation such as x- rays, gamma rays etc of high frequency is allowed to fall on the scatterer, the beam is scattered into two components

i. One component having the same frequency or wavelength as that of the incident photon, so called unmodified radiation

ii. The other component having lower frequency or higher wavelength compared to incident radiation, so called modified radiation

This effect is called Compton Effect.




When a photon of energy 'hγ' collides with an electron of a scatterer at rest; the photon gives its energy to the electron. Therefore the scattered photon will have lesser energy or lower frequency or higher wavelength compared to the wavelength of incident photon. Since the electron gains energy, it recoils with velocity's'. This effect is called Compton Effect and the shift in wavelength is called Compton shift.

Thus as a result of Compton scattering, we get (i) unmodified radiations (ii) modified radiations and (iii) a recoil electron.

**Principle:**

In Compton scattering the collision between a photon and an electron is considered. Then by applying the laws if conservation of energy and momentum, the expression for Compton wavelength is derived.

Assumptions:

1. The collision occurs between the photon and an electron in the scattering material
2. The electron is free and is at rest before collision with the incident photon.

With these assumptions, let us consider a photon energy hγ colliding within electron at rest.

During the collision process, a part of kinetic energy is given to the electron, which in turn increases the kinetic energy of the electron and hence it recoils at an angle of φ as shown.

The scattered photon moves with an energy h ( lesser than hγ) , at an angle θ with respect to the original direction.

Let us find the energy and momentum components before and after collision.

**ENERGY BEFORE COLLISION.**

i. Energy of the incident photon = hν

ii. Energy of the electron at rest = $c^2m_0$

Where $m_0$ is the rest mass of the electron

Total energy before collision = $h\nu+m_0c^2$ ----------- (1)

**ENERGY AFTER COLLISION:**

i. Energy of the scattered photon = hν'

ii. Energy of the recoil electron = $m_0c^2$

Where m is the mass of the electron moving with velocity 'v'

Total energy after collision= $h\nu' + mc^2$ ---------- (2)

We know according to law of conservation of energy

Total energy before collision = Total energy after collision

$h\nu+m_0c^2 = h\nu' + mc^2$ ---------- (3)

**X-component of Momentum before collision**:

i. X-component momentum of the incident photon = hν/c

ii. X-component momentum of the electron at rest = 0

**Total X-component of Momentum before collision** = hν/c --------- (4)

**X-component of Momentum after collision:**

i. X-component of the scattered photon can be calculated from the figure.

In ΔOAB cos θ = Mx/ (hν'/c)

X-component of Momentum of the scattered photon = cosθ( hν'/c )

ii X-component Momentum of recoil electron an be calculated as follows

in ΔOBC cosϕ = Mx/mv

X-component Momentum of recoil electron = cosϕ mv

**Total X-component of Momentum after collision** = cos θ (hν'/c ) + cosϕ mv ------ 5

We know according to the law of conservation of momentum

Total Momentum before collision = Total Momentum after collision

$h\nu/c = \cos\theta\ (h\nu'/c) + \cos\phi\ mv$ ---------- (6)

**Y-component of Momentum before collision**

I. Y – component momentum of the incident photon = 0

II. Y – component of the electron at rest = 0

Total Y _component of momentum before collision = 0 -------- (7)

**Y-component of Momentum after collision**

I. From figure in $\Delta OAE$ $\sin\theta = My/(h\nu'/c)$,

Y-component Momentum of the scattered Photon = $\sin\theta\ h\nu'/c$

from figure in $\Delta$ OCD $\sin\phi = -My/mv$ ,

Y-component Momentum of the recoil electron = $-\sin\phi\ mv$

**Total Y-component of Momentum after collision** = $mv\ h\nu'/c - mv\sin\phi$ ------ (8)

According to the law of conservation of momentum

$0 = h\nu'/c\ \sin 0 - mv\sin\phi$ -------- 9

From equation (6), we can write

$mcv\cos\phi = h(-v'\cos\theta)$ -------- 10

From equation 9 we can write

$mcv\sin\phi = h\nu'\sin\theta$ ---------- (11)

Squaring and adding both sides of equation 10 and 11, we have,

$m^2c^2v^2 = h^2 - 2\nu\nu'\cos\theta + \nu'^2$ ------------- (12)

change in wavelength $\Delta\lambda = h/m_0c\ (-\cos\theta)$ ------- A

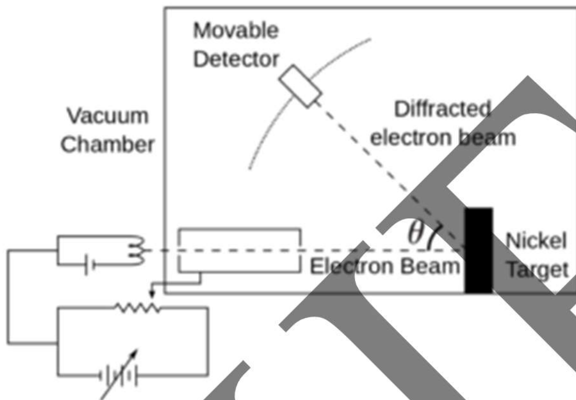
Equation A represents the shift in wavelength, i.e., Compton shift which is independent of the incident radiation as well as the nature of the scattering substance.

Thus the shift in wavelength or Compton shift purely depends upon the angle of scattering.

**DAVISSON AND GERMER EXPERIMENT**

Davisson and Germer Experiment, for the first time, proved the wave nature of electrons and verified the de Broglie equation. de Broglie argued the dual nature of matter back in 1924, but it was only later that Davisson and Germer experiment verified the results. The results established the first experimental proof of quantum mechanics. In this experiment, we will study the scattering of electrons by a Ni crystal.

The experimental setup for the Davisson and Germer experiment is enclosed within a vacuum chamber. Thus the deflection and scattering of electrons by the medium are prevented.




Electron gun: An electron gun is a Tungsten filament that emits electrons via thermionic emission i.e. it emits electrons when heated to a particular temperature.

Electrostatic particle accelerator: Two opposite charged plates (positive and negative plate) are used to accelerate the electrons at a known potential.

Collimator: The accelerator is enclosed within a cylinder that has a narrow passage for the electrons along its axis. Its function is to render a narrow and straight (collimated) beam of electrons ready for acceleration.

Target: The target is a Nickel crystal. The electron beam is fired normally on the Nickel crystal. The crystal is placed such that it can be rotated about a fixed axis.

Detector: A detector is used to capture the scattered electrons from the Ni crystal. The detector can be moved in a semicircular arc as shown in the diagram above.

**The Thought Behind the Experimental Setup**

The basic thought behind the Davisson and Germer experiment was that the waves reflected from two different atomic layers of a Ni crystal will have a fixed phase difference. After reflection, these waves will interfere either constructively or destructively. Hence producing a diffraction pattern.

In the Davisson and Germer experiment waves were used in place of electrons. These electrons formed a diffraction pattern. The dual nature of matter was thus verified. We can relate the de Broglie equation and the Bragg's law as shown below:

From the de Broglie equation, we have:

$$\lambda = h/p$$
$$= h/\sqrt{2mE}$$
$$= h/\sqrt{2meV} \quad \ldots (1)$$

where, m is the mass of an electron, e is the charge on an electron and h is the Plank's constant.

Therefore for a given V, an electron will have a wavelength given by equation (1).

The following equation gives Bragg's Law:

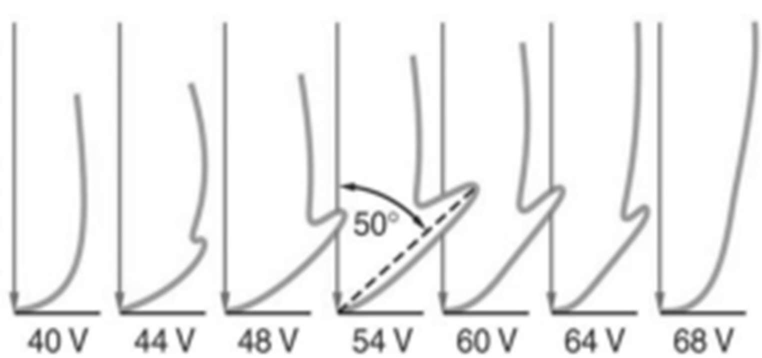
$$n\lambda = 2d \sin(90^o - \theta/2) \quad \ldots(2)$$

Since the value of d was already known from the X-ray diffraction experiments. Hence for various values of θ, we can find the wavelength of the waves producing a diffraction pattern from equation (2).

**Observations of the Davisson and Germer Experiment**

The detector used here can only detect the presence of an electron in the form of a particle. As a result, the detector receives the electrons in the form of an electronic current. The intensity (strength) of this electronic current received by the detector and the scattering angle is studied. We call this current as the electron intensity.

The intensity of the scattered electrons is not continuous. It shows a maximum and a minimum value corresponding to the maxima and the minima of a diffraction pattern produced by X-rays. It is studied from various angles of scattering and potential difference. For a particular voltage (54V, say) the maximum scattering happens at a fixed angle only ( 50° ) as shown below:



**Results of the Davisson and Germer Experiment**

From the Davisson and Germer experiment, we get a value for the scattering angle θ and a corresponding value of the potential difference V at which the scattering of electrons is maximum. Thus these two values from the data collected by Davisson and Germer, when used in equation (1) and (2) give the same values for λ. Therefore, this establishes the de Broglie's wave-particle duality and verifies his equation as shown below:

From (1), we have:

$$\lambda = h/\sqrt{2meV}$$

For V = 54 V, we have

$$\lambda = 12.27/\sqrt{54} = 0.167 \text{ nm} \quad \ldots. (3)$$

Now the value of 'd' from X-ray scattering is 0.092 nm. Therefore for V = 54 V, the angle of scattering is 500, using this in equation (2), we have:

$$n\lambda = 2\,(0.092 \text{ nm})\sin(\,900-50°/2)$$

For n = 1, we have:

$$\lambda = 0.165 \text{ nm} \quad \ldots.. (4)$$

Therefore the experimental results are in a close agreement with the theoretical values got from the de Broglie equation. The equations (3) and (4) verify the de Broglie equation.

**WAVE VELOCITY AND GROUP VELOCITY**

The phase difference between the vibrations is continually changing, the specification of some initial nonzero phase difference is in general not of major significance in this case.

So we can suppose that the individual vibrations have an initial phase of 0, and hence can be written as:

$$E_1 = a\cos(\omega_1 t - k_1 z)$$

$$E_2 = a\cos(\omega_2 t - k_2 z)$$

Then the sum of these two waves is:

$$E = E_1 + E_2 = a[\cos(\omega_1 t - k_1 z) + \cos(\omega_2 t - k_2 z)]$$

Using the following triangular formula

$$\cos(\alpha)+\cos(\beta)=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

We get

$$E=2a\cos\frac{1}{2}[(\omega_1-\omega_2)t-(k_1-k_2)z]\cos\frac{1}{2}[(\omega_1+\omega_2)t-(k_1+k_2)z]$$

then introduce the notation of average angular frequency $\bar{\omega}$ and average wave number $\bar{k}$

$$\bar{\omega}=\frac{1}{2}(\omega_1+\omega_2)$$

$$\bar{k}=\frac{1}{2}(k_1+k_2)$$

And modulation frequency $\boldsymbol{\omega_m}$ and modulation wave number $\mathbf{k_m}$

$$\omega_m=\frac{1}{2}(\omega_1-\omega_2)$$

$$k_m=\frac{1}{2}(k_1-k_2)$$

$$E=2a\cos(\omega_m t-k_m z)\cos(\bar{\omega}t-\bar{k}z)$$

We can make

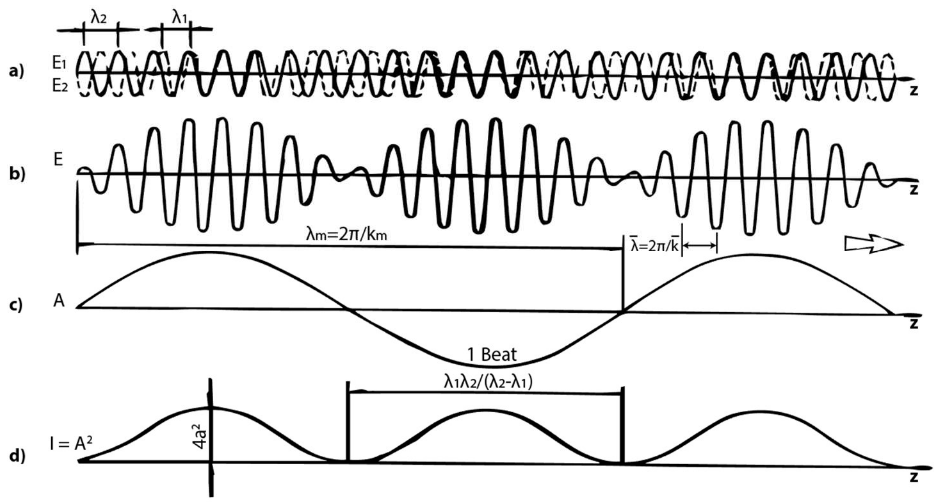
$$A=2a\cos(\omega_m t-k_m z)$$

Then we get

$$E=A\cos(\bar{\omega}t-\bar{k}z)$$

This means that the resultant superposed wave has an angular frequency $\bar{\omega}$, and its amplitude varies between **0** and **2a** with time **t** and position **z**.

The following picture shows the superposition result. Since light waves have very high frequency, if $\omega_1 \approx \omega_2$, then $\bar{\omega} >> \omega_m$, which means that **A** varies slowly but **E** varies extremely fast.




Intensity **I** of the superposed wave is proportional to $\mathbf{A^2}$, we have

$$I = A^2 = 4a^2 \cos^2(\omega_m t - k_m z)$$

or

$$I = A^2 = 2a^2[1 + \cos 2(\omega_m t - k_m z)]$$

So intensity **I** varies between **0** and $\mathbf{4a^2}$ with time **t** and position **z**. This phenomenon is called "**beat**". From the last formula we can see that the beat frequency is 2 times of modulation frequency $\omega_m$, from $\omega_m$'s definition $\omega_m = (\omega_1 - \omega_2)/2$, we can see that the beat frequency equals to $\omega_1 - \omega_2$.

This process, as a purely mathematical result, can be carried out for any values of $\omega_1$ and $\omega_2$. But its description as a "beat" phenomenon is physically meaningful only if $|\omega_1 - \omega_2| << \omega_1 + \omega_2$.

**PHASE VELOCITY AND GROUP VELOCITY**

However, in the case of a superposed wave, we need to carefully define its propagation velocity.

Let's continue using the superposed wave equation from above:

$$E = 2a\cos(\omega_m t - k_m z)\cos(\bar{\omega} t - \bar{k} z)$$

The superposed wave has two propagation velocities: equiphase surface propagation velocity (called Phase Velocity **Vp**), and equiamplitude surface propagation velocity (called Group Velocity **Vg** as defined by Rayleigh).

**PHASE VELOCITY OF THE SUPERPOSED WAVE:**

Phase velocity Vp can be concluded by keeping the phase a constant:

$$\bar{\omega} t - \bar{k} z = constant$$

$$z = \frac{\bar{\omega} t}{\bar{k}} - \frac{constant}{\bar{k}}$$

Then by doing derivative of z we get the Phase Velocity **Vp** of the superposed wave:

$$V_p = \frac{dz}{dt} = \frac{\bar{\omega}}{\bar{k}}$$

**GROUP VELOCITY OF THE SUPERPOSED WAVE:**

Similarly we can get the Group Velocity **Vg** by keeping the amplitude a constant:

$$\omega_m t - k_m z = constant$$

Following the same steps, we get the Group Velocity of the superposed wave:

$$V_g = \frac{dz}{dt} = \frac{\omega_m}{k_m} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\Delta\omega}{\Delta k}$$

when **Δω** is very small, we then get:

$$V_g = \frac{\partial \omega}{\partial k}$$

So **Vg** is the partial derivative of **ω**.

**RELATIONSHIP BETWEEN GROUP VELOCITY VG AND PHASE VELOCITY VP**

Based on the definition **Vp = ω/k**, we can replace **ω** with **k*Vp**, then we get

$$V_g = \frac{\partial \omega}{\partial k} = \frac{\partial (k V_p)}{\partial k} = V_p + k \frac{dV_p}{dk}$$

Since

$$k=\frac{2\pi}{\lambda}$$

and

$$dk=-\frac{2\pi}{\lambda^2}d\lambda$$

then we get

$$V_g=V_p-\lambda\frac{dV_p}{d\lambda}$$

**Possible Questions**

**2 marks**

1. What is called black body radiation?
2. Define group velocity.
3. What is called photo electric effect?
4. State Compton Effect.
5. State Planck hypothesis.
6. What is called wave functions?

**8 marks**

1. Briefly explain Davison and Germer's experiment.
2. Find the relationship between particle and group velocity for de Broglie wavelength.
3. Obtain an expression for group velocity.
4. Explain wave and Group velocity. Obtain an expression for group velocity.

KARPAGAM ACADEMY OF HIGHER EDUCATION
Coimbatore-641021.
(For the candidates admitted from 2017 onwards)

DEPARTMENT OF PHYSICS
ELEMENTS OF MODERN PHYSICS (17PHU502A)

Unit - I

| Question | Option 1 |
|---|---|
| The phenomena of interference, diffraction and polarization can only be explained based on _________. | wave theory of light |
| Which is not characteristics of Planck's quantum theory of radiation? | Radiation is associated with energy |
| Einstein's theory of photoelectric effect is based on | Newtons corpuscular theory of light |
| The equation E= hν was deduced by: | Heisenberg |
| De Broglie wavelength (λ) associated with moving particles, mass, m, and velocity v is | h/mv |
| The wavelength associated with a 54eV is | 1.61Å |
| The propagation constant (k) = | λ |
| Based on quantum theory of light, the bundles of energy = | hν |
| De Broglie wavelength (λ) associated with moving particles of K.E is | h/mv |
| Wave nature is not observed in daily life because we are using ___________. | Microscopic particles |
| Group velocity (u) = | dω |
| _______ year de Broglie proposed that the idea of dual nature. | 1921 |
| de Broglie wavelength (λ) associated with charge q and potential difference of V volts is | h/mv |
| The interplanar distance of gold crystal is _______ Å. | 4.02 |
| In relativistic particle, the group velocity (G) is equal to | v |

| | |
|---|---|
| The wave velocity (v) = | ω/k |
| What is the interplanar distance of gold crystal (Å)? | 4.02 |
| In non-relativistic particle, the group velocity (G) is equal to | v/4 |
| Classical physics could not explain the behavior of a black body radiator at very short wavelengths. What was this problem called? | Absorption failure |
| The photoelectric effect was explained by Albert Einstein by assuming that: | light is a wave. |
| The Compton Effect supports which of the following theories? | Special Theory of Relativity. |
| How does the energy of a photon change if the wavelength is doubled? | Doubles |
| How does the momentum of a photon change if the wavelength is halved? | Doubles |
| Which one of the following objects, moving at the same speed, has the greatest de Broglie wavelength? | Neutron |
| Which of the following formulas can be used to determine the de Broglie wavelength? | λ = hmv |
| The value of Plank's constant is | 6.62 X 10-34 JS2 |
| The idea of dual nature of light was proposed by | Plank |
| Which of the following terms refers to the molecular modelling computational method that uses equations obeying the laws of classical physics? | Quantum mechanics |
| Which of the following terms refers to the molecular modelling computational method that uses quantum physics? | Quantum mechanics |
| According to the de Broglie's hypothesis of matter waves, the concepts of energy, momentum and wavelength are applicable to | moving particles but not to radiation (photon) |

| | |
|---|---|
| Experimental verification of de Broglie's matter waves was obtained in | Einstein‟s Photoelectric experiment |
| A pattern of alternate dark and bright bands is obtained in the double slit experiments on | Single photon at a time |
| Probabilistic interpretation of matter waves (as in the double slit experiment) was given by | Einstein |
| Phase velocity Vp of a wave is expressed as | Vp = ω / k where ω = Angular frequency, k = propagation constant of the wave |
| The quantum theory of radiation was proposed by ____ | Einstein |
| The wave nature of electron was experimentally verified by | Einstein |
| Classical mechanics could not explain the stability of ________ | atoms |
| Classical mechanics correctly explain the motion | plantes |
| Classical mechanics could not explain the variation of specific heat of metals and ________ | solids |
| The first experimental evidence for matter waves was given by _______ | Einstein |
| The acclerated potential difference for Davisson and Germer experiment was ________ | 30 to 1000 V |
| The type of crystal used in Davisson and Germer experiment was _______ | Ni |
| The wavelength of bullet of mass 1 g moving with a velocity of 1000 m/ s is given by ________ | $6.625 \times 10^{-34}$ nm |
| In davisson and germer experiment the angle of incidence relative to the family of Bragg plane is ________ | 65 |
| Calculate the de Broglie wavelength associated with a proton moving with a velocity equal to 1/20 th of the velocity of light | $2.62 \times 10^{-14}$ m |
| The wave property for momentum is _____ | energy |

| | |
|---|---|
| The wave property for energy is _____ | momentum |
| The particle property for waqvelength is | momentum |
| The particle property for frequency is | momentum |
| In Wave velocity the cosine factor represents a slowly varying function of ______ | w |
| In the transmission process light (radiation) behaves as a | Wave |
| ‘The moving particles behave like waves’ was first theoretical established by | Einstein |
| ‘The material particles behave like waves’ was first experimentally extablished by | Einstein |
| The de Broglie wavelength a particle of a particle of mass m and charge e subjected to a potential difference V volt is | λ = h/(2eV)1/2 |
| The de Broglie wavelength wave length of a moving electron subjected to a potential V is | 1.26/V1/2 |
| Compute the de Broglie wavelength of an electron that has been accelerated through a potential difference of 9.0 kV. Ignore relativists effects. | 1.3 x 10-11 m |

| option 2 | option 3 | option 4 | option 5 | option 6 | Answer |
|---|---|---|---|---|---|
| photoelectric effect. | compton effect. | quantum theory of light. | | | wave theory of light |
| Energy is not absorbed or emitted in whole number or multiples of quantum | The magnitude of energy associated with a quantum is proportional to the frequency | Radiation energy is neither emitted nor absorbed continuously but in small packets called quanta | | | Radiation is associated with energy |
| Huygen's wave theory of light | Maxwell's electromagnetic theory of light | Planck's quantum theory of light | | | Planck's quantum theory of light |
| de Broglie | Einstein | Planck | | | Einstein |
| h/√2mEk | h/√2mqV | h/√2mkT | | | h/mv |
| 1.63Å | 1.67Å | 1.69Å | | | 1.67Å |
| 2π/λ | 2πλ | λ /2π | | | 2π/λ |
| hλ | h/ν | h/λ | | | hν |
| h/√2mEk | h/√2mqV | h/√2mkT | | | h/mv |
| macroscopic particles | molecules | atoms | | | macroscopic particles |
| dk | dωdk | dω/dk | | | dω/dk |
| 1922 | 1923 | 1925 | | | 1923 |
| h/√2mEk | h/√2mqV | h/√2mkT | | | h/mv |
| 4.04 | 4.06 | 4 | | | 4.06 |
| u | 1/u | 1/v | | | u |

| | | | |
|---|---|---|---|
| ωk | k/ω | ω | ω/k |
| 4.04 | 4.06 | 4 | 4.04 |
| v/2 | v | 2v | v/2 |
| Ultraviolet Explosion | Wavelength decrease | Photoelectric Effect | Ultraviolet Explosion |
| light is a particle. | an electron behaves as a wave. | an electron behaves as a particle. | light is a particle |
| Light is a wave. | Thomson model of the atom. | Light is a particle. | Light is a particle. |
| Quadruples | Stays the same | Is cut to one-half | Is cut to one-half |
| Quadruples | Stays the same | Is cut to one-half | doubles |
| Electron | Tennis ball | Bowling ball | electron |
| λ = h/mv | λ = mv/h | λ = hm/c | λ = h/mv |
| 6.62 X 10-31 JS | 6.62 X 10-34 JS | 6.62 X 10-31 JS2 | $6.62 \times 10^{-34}$ JS |
| De Broglie | Einstein | Maxwell | De Broglie |
| Molecular calculations | Molecular mechanics | Quantum theory | Molecular mechanics |
| Molecular calculations | Molecular mechanics | Quantum theory | Quantum mechanics |
| moving particles as well as to radiation (photon) | radiation (photon) but not to moving particles | neither to moving particles nor to radiation (photon). | moving particles as well as to radiation (photon) |

| | | | | |
|---|---|---|---|---|
| Davisson and Germer Experiment | Compton‟s Experiment | Plank | | Davisson and Germer Experiment |
| Single electron at a time | Single bullet at a time | Electron Beam | | Electron Beam |
| De Broglie | Max Planck | Davisson | | Davisson |
| where λ = wavelength and T = period of the wave | Vp = E/p where E = Energy, p= Momentum of the particle | No relation between Phase velocity and Group velocity | | Vp = ω / k where ω = Angular frequency, k = propagation constant of the wave |
| De Broglie | Max Planck | Davisson | | Max Planck |
| De Broglie | Max Planck | Davisson | | Davisson |
| proton | neutron | electron | | atoms |
| stars | atoms | both a and b | | both a and b |
| gases | liquids | inert gas | | gases |
| de Broglie | Plancks | Davisson and Germer | | Davisson and Germer |
| 30 to 100 eV | 30 to 100 V | 3 to 100 V | | 30 to 100 V |
| Al | Cu | Fe | | Ni |
| 6.625 x $10^{-34}$m | 6.625 x $10^{34}$m | 6.625 x $10^{-34\ c}$m | | 6.625 x $10^{-34}$m |
| 56 | 54 | 48 | | 65 |
| 6.62 x $10^{-14}$ m | 26.2 x $10^{-14}$ m | 0.262 x $10^{-14}$ m | | 2.62 x $10^{-14}$ m |
| frequency | velocity | wavelength | | wavelength |

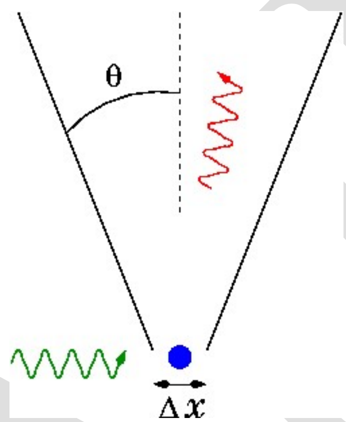
| | | | |
|---|---|---|---|
| frequency | velocity | wavelength | frequency |
| frequency | velocity | energy | momentum |
| wavelength | velocity | energy | energy |
| k | x | x and t | x and t |
| Particle | Wave-particle | matter | Wave |
| De Broglie | Davison & Germer | Plancks | De Broglie |
| De Broglie | Davison & Germer | Plancks | Davison & Germer |
| λ = h/(2meV)1/2 | λ = h/(2V)1/2 | λ = h/(meV)1/2 | λ = h/(2meV)1/2 |
| 12.26/V1/2 | 12.26/V | 2.26/V1/2 | 12.26/V1/2 |
| $1.7 \times 10^{-22}$ m | $1.2 \times 10^{-26}$ m | 1.7 x 10 3 m | $1.7 \times 10^{-22}$ m |

## UNIT-II

Position measurement- gamma ray microscope thought experiment; Wave-particle duality, Heisenberg uncertainty principle (Uncertainty relations involving Canonical pair of variables): Derivation from Wave Packets impossibility of a particle following a trajectory; Estimating minimum energy of a confined particle using uncertainty principle; Energy-time uncertainty principle- application to virtual particles and range of an interaction.

**Position measurement- Gamma ray microscope thought experiment**

Heisenberg pictured a microscope that obtains very high resolution by using high-energy gamma rays for illumination. No such microscope exists at present, but it could be constructed in principle. Heisenberg imagined using this microscope to see an electron and to measure its position. He found that the electron's position and momentum did indeed obey the uncertainty relation he had derived mathematically.




In the corrected version of the thought experiment, a free electron sits directly beneath the center of the microscope's lens (see the picture above). The circular lens forms a cone of angle 2A from the electron. The electron is then illuminated from the left by gamma rays--high energy light which has the shortest wavelength. These yield the highest resolution, for according to a principle of wave optics, the microscope can resolve (that is, "see" or distinguish) objects to a size ofDx, which is related to and to the wavelength L of the gamma ray, by the expression:

Dx = L / (2sinA)

However, in quantum mechanics, where a light wave can act like a particle, a gamma ray striking an electron gives it a kick. At the moment the light is diffracted by the electron into the microscope lens, the electron is thrust to the right. To be observed by the microscope, the gamma ray must be scattered into any angle within the cone of angle 2A. In quantum mechanics, the gamma ray carries momentum, as if it were a particle. The total momentum p is related to the wavelength by the formula

$$p = h / L$$, where h is Planck's constant.

In the extreme case of diffraction of the gamma ray to the right edge of the lens, the total momentum in the x direction would be the sum of the electron's momentum $p'_x$ in the x direction and the gamma ray's momentum in the x direction:

$$p'_x + (h \sin A) / L'$$, where L' is the wavelength of the deflected gamma ray.

In the other extreme, the observed gamma ray recoils backward, just hitting the left edge of the lens. In this case, the total momentum in the x direction is:

$$p''_x - (h \sin A) / L''.$$

The final x momentum in each case must equal the initial x momentum, since momentum is never lost (it is *conserved*). Therefore, the final x momenta are equal to each other:

$$p'_x + (h \sin A) / L' = p''_x - (h \sin A) / L''$$

If A is small, then the wavelengths are approximately the same, $L' \sim L'' \sim L$. So we have

$$p''_x - p'_x = Dp_x \sim 2h \sin A / L$$

Since $Dx = L / (2 \sin A)$, we obtain a reciprocal relationship between the minimum uncertainty in the measured position, Dx, of the electron along the x axis and the uncertainty in its momentum, $Dp_x$, in the x direction:

$$Dp_x \sim h / Dx \quad \text{or} \quad Dp_x\, Dx \sim h.$$

For more than minimum uncertainty, the "greater than" sign may be added.

Except for the factor of 4p and an equal sign, this is Heisenberg's uncertainty relation for the simultaneous measurement of the position and momentum of an object.

**Heisenberg uncertainty principle**

The position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision. There is a minimum for the product of the uncertainties of these two measurements. There is likewise a minimum for the product of the uncertainties of the energy and time.

$$\Delta x \Delta p \geq \hbar/2$$

$$\Delta t \Delta E \geq \hbar/2$$

This is not a statement about the inaccuracy of measurement instruments, nor a reflection on the quality of experimental methods; it arises from the wave properties inherent in the quantum mechanical description of nature. Even with perfect instruments and technique, the uncertainty is inherent in the nature of things.

**Energy-time uncertainty principle**

The energy-time form of the Heisenberg uncertainty principle is interpreted in a very different way than the position-momentum form.

First, the generalized uncertainty principle for two physical observables A and B can be written as

$$\Delta A\Delta B\geq 1/2|\langle[A^{\wedge},B^{\wedge}]\rangle|$$

where ΔA and ΔB are the uncertainties in A and B, || is the absolute value, ⟨⟩ is the expectation value, [,] is the commutator and A^ and B^ are the quantum mechanical operators corresponding to A and B. If you insert A=x (position), B=p (momentum) and the canonical commutation relation, $[x^{\wedge},p^{\wedge}]=i\hbar$, you get the position-momentum uncertainty relation,

$\Delta x\Delta p\geq\hbar^2$

If there were a time "operator" t^ which obeyed a similar relation with the Hamiltonian H^ (which is the energy operator), $[t^{\wedge},H^{\wedge}]=i\hbar$, then

$$\Delta t\Delta E\geq\hbar/2$$

But there is no time operator. That is because in non-relativistic quantum mechanics, time is not an observation that can take on a multitude of values in an experiment. Time is an independent variable that invariably increases with absolute certainty. So, Δt cannot be interpreted as a statistical uncertainty.

However, Δt as some characteristic timescale. According to Ehrenfest's theorem, the rate of change of an expectation value of an observable is given by,

$$\partial\langle A^{\wedge}\rangle\partial t=1/i\hbar\langle[A^{\wedge},H^{\wedge}]\rangle$$

Inserting this into the generalized uncertainty relation above, with B=E (B^=H^), we get

$$\Delta A|\partial\langle A^{\wedge}\rangle/\partial t|\Delta E\geq\hbar/2$$

The fraction on the left can be interpreted as the time Δt it takes for the quantum state to change significantly, with respect to the observable A (sort of like the time it takes your car to travel a distance Δx is Δx/v, where v=∂x/∂t is the speed of your car). With this we have

$$\Delta t \Delta E \geq \hbar/2$$

This says that ***the time it takes for the system to change significantly times the uncertainty in the energy is always greater than*** ℏ/2.

The probability amplitude for a free particle with momentum $\vec{p}$ and energy

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$

is the complex wave function

$$\psi_{\text{free particle}}(\vec{x}, t) = e^{i(\vec{p}\cdot\vec{x} - Et)/\hbar}.$$

Note that $|\psi|^2 = 1$ everywhere so this does not represent a localized particle. In fact we recognize the wave property that, to have exactly one frequency, a wave must be spread out over space. can build up localized wave packets that represent single particles by adding up these free particle wave functions (with some coefficients).

$$\psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \phi(p) e^{i(px - Et)/\hbar} dp$$

Similarly we can compute the coefficient for each momentum.

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x) e^{-ipx/\hbar} dx.$$

These coefficients, $\phi(p)$, are actually the state function of the particle in momentum space. We can describe the state of a particle either in position space with $\psi(x)$ or in momentum space with $\phi(p)$. We can use $\phi(p)$ to compute the probability distribution function for momentum.

$$P(p) = |\phi(p)|^2$$

We will show that wave packets like these behave correctly in the classical limit, vindicating the choice we made for $\psi_{free\ particle}(\vec{x}, t)$.

The Heisenberg Uncertainty Principle is a property of waves that we can deduce from our study of localized wave packets.

$$\Delta p \Delta x \geq \frac{\hbar}{2}$$

It shows that due to the wave nature of particles, we cannot localize a particle into a small volume without increasing its energy. For example, we can estimate the ground state energy (and the size of) a Hydrogen atom very well from the uncertainty principle.

**Possible Questions**

**2 marks**

State uncertainty principle.

What is called wave packets?

What is the application of uncertainty principle?

**8 marks**

Explain gamma ray microscope.

Explain uncertainty principle.

Explain the applications of uncertainity principle.

KARPAGAM ACADEMY OF HIGHER EDUCATION

Coimbatore-641021.

(For the candidates admitted from 2017 onwards)

DEPARTMENT OF PHYSICS

ELEMENTS OF MODERN PHYSICS (17PHU502A)

**Unit - II**

| Question | Option 1 |
|---|---|
| Heisenberg's uncertainty principle states for the energy and time is | ΔEΔt = h |
| Based on optical theory, the limits of distance between two points (Δx) is | λ/2sinɵ |
| The angular frequency (ω) = | √k/m |
| In which of the following is the radius of the first orbit minimum? | hydrogen atom |
| The Kinetic energy of electron of mass (m) is given by (T) | p/2m |
| Heisenberg's uncertainty principle states for the angular momentum and angle is | ΔJΔɵ = h |
| The radius of the nucleus of any atom is of the order of ____ m | 10-8 m |
| The minimum energy of harmonic oscillator (Emin) = | ½hω |
| Which of following formula satisfy the diffraction pattern? | nλ= 2dsinɵ |

| | |
|---|---|
| Heisenberg's Uncertainty Principle states: | The more precise a particle's energy can be measured, the less precise its position can be measured. (b) |
| Heisenberg's uncertainty principle states for the position and momentum is | ΔpΔq = h |
| The product of the uncertainties in determine the angular momentum and angle of the particle can never be smaller that the number of order | #NAME? |
| The uncertainty in the total energy (ΔE) is | ΔT + ΔV |
| Constructive interference happens when two waves are | out of phase |
| Based on the uncertainty principle, the minimum momentum (Pmin) = | h/I |
| Who proposed the uncertainty principle? | Bohr |
| The kinetic energy of electron in the atoms is | 4 Mev |

| | |
|---|---|
| A particle has position (x, y, z) and corresponding momenta (px, py, pz). According to Heisenberg‟s Uncertainty principle, following observables cannot be measured simultaneously. | The shorter the lifetime of an excited state of an atom, the less accurately can its energy be measured. ( |
| What is the atom life time in the excited states? | $10^{-8}$ sec |
| Planck's constant has the same units as | angular momentum |
| Which of the following is NOT a correct consequence of the Heisenberg uncertainty principle: | x and px |
| According to Heisenberg‟s Uncertainty principle, Indeterminism in the measurement of canonically conjugate variables is due to | imperfection in measuring instruments |
| Potential energy of Hydrogen atom in the ground state is | negative |
| The value of ħ is | 6.625 x $10^{-34}$ nm |
| The mass of an electron is | 9 x $10^{-34}$ nm |
| If we measure the position of a particle accurately then the uncertainty in measurement of momentum at the same instant becomes | 0 |
| If we measure the energy of a particle accurately then the uncertainty in measurement of the time becomes | 0 |
| If a photon and the electron have the same wavelength, then the energy of an electron is | Smaller than that of a photon |

| | |
|---|---|
| For a photon and an electron with equal energy, the Broglie wavelength of the electron is | Much smaller than that of a photon |
| For destructive interference, the path difference is | odd number of half wavelengths |
| Two waves with phase difference 180° have resultant of amplitude | one |
| Extra distance travelled by one of waves compared with other is called | path |
| Bohr stated that electron is | material particle |
| Uncertainty principle is applicable to | macroscopic particles |
| Uncertainty principle can be easily understandable with help of | Dalton's effect |
| Which of the following is *false* about the uncertainty principle? | The uncertainty principle is an important relationship between position and momentum |
| In quantum mechanical theory, it is possible for a particle confined to a region surrounded by a high potential barrier to escape by | tunneling |

| | |
|---|---|
| Wolfgang Pauli concluded that | in any single atom, no more than three electrons can occupy a particular orbit |
| Heisenberg gave his concept in | 1923 |
| Delta x is related to delta p | directly |
| The de Broglie wavelength of an object | is equal to Planck's constant divided by the momentum of the object |
| Heisenberg uncertainty principle is used for | data processing |
| Duality principle is used when SE is | square |
| The Heisenberg uncertainty principle is concerned with what two properties? | mass and velocity |
| Wavelength of slow moving neutrons is about | $10^{-34}$ m |
| Energy of photon is directly related to the | wavelength |

| option 2 | option 3 | option 4 | option 5 | option 6 | Answer |
|---|---|---|---|---|---|
| ΔEΔt = h/2π | ΔEΔt = 2πh | ΔEΔt = 2 π /h | | | ΔEΔt = h/2π |
| λ/sinө | $λ^2$sinө | λsinө | | | λ/2sinө |
| √m/k | √k | √m | | | √k/m |
| A tritium atom | Triply ionized beryllium | Doubly ionized helium | | | hydrogen atom |
| $p^2$/2m | 2mp | 2m$p^2$ | | | $p^2$/2m |
| ΔJΔө = h/2π | ΔJΔө = 2πh | ΔJΔө = 2 π /h | | | ΔJΔө = h/2π |
| 10 -14 cm | 10-14m | 10-10 m | | | 10-14m |
| hω | -hω | ω | | | ½hω |
| nλ= 2sinө/d | nλ= sinө/2d | nλ= 2d/sinө | | | nλ= 2dsinө |

| | | | | |
|---|---|---|---|---|
| A particle's position can be measured exactly | A particle's energy can be measured exactly | The more precise a particle's momentum can be measured, the less precise its position can be measured. | | The more precise a particle's momentum can be measured, the less precise its position can be measured. |
| $\Delta p\Delta q = h/2\pi$ | $\Delta p\Delta q = 2\pi h$ | $\Delta p\Delta q = 2\ \pi /h$ | | $\Delta p\Delta q = h/2\pi$ |
| $\leq \frac{1}{2}\hbar$ | $\geq \frac{1}{2}\hbar$ | $\neq \frac{1}{2}\hbar$ | | #NAME? |
| $\Delta T - \Delta V$ | $\Delta T$ | $\Delta V$ | | $\Delta T + \Delta V$ |
| zero amplitude | in phase | in front | | in phase |
| $\hbar$ | $\hbar l$ | $l/\hbar$ | | $\hbar$ |
| De Broglie | Heisenberg | Schroedinger | | Heisenberg |
| 6 Mev | 8 MeV | 97 Mev | | 97 Mev |

| | | | |
|---|---|---|---|
| An electron in an atom cannot be described by a well-defined orbit | The momentum of an electron cannot be measured exactly | Measurement of one variable in an atomic system can affect subsequent measurements of other variables. | Measurement of one variable in an atomic system can affect subsequent measurements of other variables. |
| $10^{-8}$ min | $10^{-10}$ sec | $10^{-10}$ min | $10^{-8}$ sec |
| The Hamiltonian | quantum number | frequency | angular momentum |
| x and py | py and pz | x and z | x and z |
| imperfection in measurement methods | the interminisim inherent in the auantum world itself | | imperfection in measuring instruments |
| zero | imfinity | cannot be determined | zero |
| 5 x $10^{-34}$ nm | 1.055 x $10^{34}$nm | 1.0555 x $10^{-34}$ nm | 1.055 x $10^{34}$nm |
| 9x $10^{-31}$ m | 6 x $10^{-34}$ nm | 6.625 x $10^{-30}$ nm | 9x $10^{-31}$ m |
| Infinity | 1 | constant | Infinity |
| Infinity | 1 | constant | Infinity |
| Greater than that of a photon | 0 | Equal | Greater than that of a photon |

| | | | |
|---|---|---|---|
| Much greater than of a proton | 0 | Equal | Much smaller than that of a photon |
| even number of half wavelengths | whole number of wavelengths | even whole number of wavelengths | odd number of half wavelengths |
| zero | same as the single wave | doubles the single wave | zero |
| displacement | phase difference | path difference | path difference |
| wave | energy | none | material particle |
| microscopic particles | gases | liquids | microscopic particles |
| Compton's effect | electron effect | photoelectric effect | Compton's effect |
| The uncertainty principle is an important relationship between energy and time | The uncertainty principle is an important relationship between energy and position | The uncertainty principle helped overthrow the idea of strict determinism in physics | The uncertainty principle is an important relationship between energy and position |
| barrier climbing | energy relocation | conversion of mass to energy | tunneling |

| | | | | | |
|---|---|---|---|---|---|
| the quantum numbers for a particular electron in an atom can never be changed | only one electron in an atom can exist in a given quantum state | there is a unique set of quantum numbers for every single atom in the universe | | | only one electron in an atom can exist in a given quantum state |
| 1927 | 1957 | 1933 | | | 1927 |
| inversely | no relation | none | | | inversely |
| is significant only if the object is moving at 1% of the speed of light or faster | cannot be determined accurately for any subatomic particles | increases as the velocity of the particle increases | | | is equal to Planck's constant divided by the momentum of the object |
| information processing | data processin erosion | dilation | | | information processing |
| symmetric | asymmetric | translated | | | symmetric |
| momentum and position | position and velocity | momentum and mass | | | momentum and position |
| $10^{-20}$ m | $10^{-19}$ m | $10^{-10}$ m | | | $10^{-10}$ m |
| wave number | frequency | amplitude | | | frequency |

## UNIT-III

Two slit interference experiment with photons, atoms and particles; linear superposition principle as a consequence; Matter waves and wave amplitude; Schrodinger equation for non-relativistic particles; Momentum and Energy operators; stationary states; physical interpretation of a wave function, probabilities and normalization; Probability and probability current densities in one dimension

**Matter Waves:**

The waves presumed to be associated with moving material particles on the imagination of de Broglie are defined as matter waves.

**Characteristics of Matter Waves:**

The wavelength of matter waves is inversely proportional to the momentum of the particle.

Matter waves travels even in vacuum, hence these are not mechanical waves.

Matter waves are produced due to the motion of material particle. These waves are associated with every moving particles.

Actually matter waves are probabilistic waves because these waves represent the probability of fining a particle in space.

Practical observation of matter waves is possible only when the wave length of matter wave is greater than the size of the particle (i.e. $\lambda >> a$).

These waves are also associated with electrically natural particle hence these cannot be the electromagnetic waves even.

Matter waves propagate in the form of wave packet with group velocity. The phase velocity of matter waves can be greater than the light.

The wavelength of matter waves does not depend on the nature and charge of the particle.

According to Plank's quantum theory the energy of photon is given by

$$E = h\nu, \quad \text{where} \quad h = \text{Plank's constant}$$

$$\text{since}, \ c = \nu\lambda$$

$$E = hc/\lambda$$

$\lambda$ = wavelength of photon

$\nu$ = frequency of photon

**EFFECTIVE MASS OF PHOTON**

According to Einstein's theory, if the energy (hn) of photon is converted into matter then the mass of matter created or the mass of photon in moving state is defined as the effective mass of photon. If the effective mass of the photon is m then according to Einstein's mass – energy relation its energy is

$$E = mc^2 = h\nu = hc/\lambda$$

Effective mass of photon , $m = E/c^2 = h\nu/c^2$

The momentum of photon , $P = mc = E/c = h\nu/c = h/\lambda$

The wavelength associated with photon , $\lambda = h/P = h/mc$

Photon is an uncharged particle, its rest mass is zero, spin is h and its velocity is equal to that of light.

***de Broglie Wavelength Associated with Moving Particles :***

Energy of a particle of mass m and moving with velocity v .

$$E = \frac{1}{2}mv^2 = \frac{P^2}{2m}$$

where, P = momentum of particle

momentum of particle $P = mv = \sqrt{(2mE)}$

According to de Broglie theory the wavelength associated with the particles.

$$\lambda = \frac{h}{P} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

The order of magnitude of wave lengths associated with microscope particles is $10^{-24}$Å. Whereas the smallest wavelength whose measurement is possible is that of g-rays (g ~ $10^{-5}$ Å – 1Å). This is the reason why the wave nature of microscopic particles is not observable.

**Momentum Energy Operator**

The numerical quantities that the old Newtonian physics uses, (position, momentum, energy, ...), are just shadows of what really describes nature: operators. The operators described in this section are the key to quantum mechanics.

As the first example, while a mathematically precise value of the position $x$ of a particle never exists, instead there is an $x$-position *operator* $\widehat{x}$. It turns the wave function $\Psi$ into $x\Psi$:

$$\Psi(x,y,z,t) \xrightarrow{\quad \widehat{x} \quad} x\Psi(x,y,z,t) \tag{1}$$

The operators $\widehat{y}$ and $\widehat{z}$ are defined similarly as $\widehat{x}$.

Instead of a linear momentum $p_x = mu$, there is an $x$-momentum *operator*

$$\boxed{\widehat{p}_x = \frac{\hbar}{i}\frac{\partial}{\partial x}} \tag{2}$$

that turns $\Psi$ into its $x$-derivative:

$$\Psi(x,y,z,t) \xrightarrow{\widehat{p}_x = \frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial x}} \frac{\hbar}{\mathrm{i}}\Psi_x(x,y,z,t) \tag{3}$$

The constant $\hbar$ is called "Planck's constant." (Or rather, it is Planck's original constant $h$ divided by $2\pi$.) If it would have been zero, all these troubles with quantum mechanics would not occur. The blobs would become points. Unfortunately, $\hbar$ is very small, but nonzero. It is about $10^{-34}$ kg m$^2$/s.

The factor $\mathrm{i}$ in $\widehat{p}_x$ makes it a Hermitian operator (a proof of that is in derivation {D.9}). All operators reflecting macroscopic physical quantities are Hermitian.

The operators $\widehat{p}_y$ and $\widehat{p}_z$ are defined similarly as $\widehat{p}_x$:

$$\boxed{\widehat{p}_y = \frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial y} \qquad \widehat{p}_z = \frac{\hbar}{\mathrm{i}}\frac{\partial}{\partial z}} \tag{4}$$

The *kinetic energy operator* $\widehat{T}$ is:

$$\widehat{T} = \frac{\widehat{p}_x^2 + \widehat{p}_y^2 + \widehat{p}_z^2}{2m} \tag{5}$$

Its shadow is the Newtonian notion that the kinetic energy equals:

$$T = \frac{1}{2}m\left(u^2+v^2+w^2\right) = \frac{(mu)^2+(mv)^2+(mw)^2}{2m}$$

This is an example of the "Newtonian analogy": the relationships between the different operators in quantum mechanics are in general the same as those between the corresponding numerical values in Newtonian physics. But since the momentum *operators* are gradients, the actual kinetic energy operator is, from the momentum operators above:

$$\widehat{T} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right). \tag{6}$$

Mathematicians call the set of second order derivative operators in the kinetic energy operator the Laplacian, and indicate it by $\nabla^2$:

$$\boxed{\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}} \tag{7}$$

In those terms, the kinetic energy operator can be written more concisely as:

$$\boxed{\widehat{T} = -\frac{\hbar^2}{2m}\nabla^2} \tag{8}$$

Following the Newtonian analogy once more, the total energy operator, indicated by $H$, is the the sum of the kinetic energy operator above and the potential energy operator $V(x, y, z, t)$:

$$\boxed{H = -\frac{\hbar^2}{2m}\nabla^2 + V} \tag{9}$$

This total energy operator $H$ is called the *Hamiltonian* and it is very important. Its eigenvalues are indicated by $E$ (for energy), for example $E_1$, $E_2$, $E_3$,...with:

$$H\psi_n = E_n\psi_n \quad \text{for } n = 1, 2, 3, \ldots \tag{10}$$

where $\psi_n$ is eigenfunction number $n$ of the Hamiltonian.

It is seen later that in many cases a more elaborate numbering of the eigenvalues and eigenvectors of the Hamiltonian is desirable instead of using a single counter $n$. For example, for the electron of the hydrogen atom, there is more than one eigenfunction for each different eigenvalue $E_n$, and additional counters $l$ and $m$ are used to distinguish them.

**The Time-Dependent Schrödinger Equation**

To derive the single-particle time-independent Schrödinger equation starting from the classical wave equation and the de Broglie relation, the time-dependent Schrödinger equation cannot be derived using elementary methods and is generally given as a postulate of quantum mechanics. It is possible to show that the time-dependent equation is at least *reasonable* if not derivable, but the arguments are rather involved

The single-particle three-dimensional time-dependent Schrödinger equation is

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) \quad (1)$$

where $V$ is assumed to be a real function and represents the potential energy of the system *Wave Mechanics* is the branch of quantum mechanics with equation (1) as its dynamical law. Note that equation (1) does not yet account for spin or relativistic effects.

Of course the time-dependent equation can be used to derive the time-independent equation. If we write the wavefunction as a product of spatial and temporal terms, $\psi(\mathbf{r},t) = \psi(\mathbf{r})f(t)$, then equation (1) becomes

$$\psi(\mathbf{r})i\hbar\frac{df(t)}{dt} = f(t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r}) \quad (2)$$

or

$$\frac{i\hbar}{f(t)}\frac{df}{dt} = \frac{1}{\psi(\mathbf{r})}\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r}) \quad (3)$$

Since the left-hand side is a function of $t$ only and the right hand side is a function of $\mathbf{r}$ only, the two sides must equal a constant. If we tentatively designate this constant $E$ (since the right-hand side clearly must have the dimensions of energy), then we extract two ordinary differential equations, namely

$$\frac{1}{f(t)}\frac{df(t)}{dt} = -\frac{iE}{\hbar} \quad (4)$$

and

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad (5)$$

The latter equation is once again the time-independent Schrödinger equation. The former equation is easily solved to yield

$$f(t) = e^{-iEt/\hbar} \quad (6)$$

The Hamiltonian in equation (5) is a Hermitian operator, and the eigenvalues of a Hermitian operator must be real, so $E$ is real. This means that the solutions $f(t)$ are purely oscillatory, since $f(t)$ never changes in magnitude (recall Euler's formula $e^{\pm i\theta} = cos\theta \pm i\ sin\theta$). Thus if

$$\psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-iEt/\hbar} \quad (7)$$

then the total wave function $\psi(\mathbf{r},t)$ differs from $\psi(\mathbf{r})$ only by a phase factor of constant magnitude. There are some interesting consequences of this. First of all, the quantity $|\psi(\mathbf{r},t)|^2$ is time independent, as we can easily show:

$$|\psi(\mathbf{r},t)|^2 = \psi^*(\mathbf{r},t)\psi(\mathbf{r},t) = e^{iEt/\hbar}\psi^*(\mathbf{r})e^{-iEt/\hbar}\psi(\mathbf{r}) = \psi^*(\mathbf{r})\psi(\mathbf{r}) \quad (8)$$

Secondly, the expectation value for any time-independent operator is also time-independent, if $\psi(\mathbf{r},t)$ satisfies equation (27). By the same reasoning applied above,

$$< A >= \int \psi^*(\mathbf{r},t)\hat{A}\psi(\mathbf{r},t) = \int \psi^*(\mathbf{r})\hat{A}\psi(\mathbf{r}) \quad (9)$$

For these reasons, wave functions of the form (7) are called *stationary states*. The state $\psi(\mathbf{r},t)$ is ``stationary," but the particle it describes is not!

Of course equation (7) represents a particular solution to equation (1). The general solution to equation (1) will be a linear combination of these particular solutions, i.e.

$$\psi(\mathbf{r},t) = \sum_i c_i e^{-iE_i t/\hbar}\psi_i(\mathbf{r})$$

# The Time-Independent Schrödinger Equation

To start with the one-dimensional classical wave equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} \quad (1)$$

By introducing the separation of variables

$$u(x,t) = \psi(x) f(t) \quad (2)$$

we obtain

$$f(t)\frac{d^2\psi(x)}{dx^2} = \frac{1}{v^2}\psi(x)\frac{d^2 f(t)}{dt^2} \quad (3)$$

If we introduce one of the standard wave equation solutions for $f(t)$ such as $e^{i\omega t}$ (the constant can be taken care of later in the normalization), we obtain

$$\frac{d^2\psi(x)}{dx^2} = \frac{-\omega^2}{v^2}\psi(x) \quad (4)$$

Now we have an ordinary differential equation describing the spatial amplitude of the matter wave as a function of position. The energy of a particle is the sum of kinetic and potential parts

$$E = \frac{p^2}{2m} + V(x) \quad (5)$$

which can be solved for the momentum, $p$, to obtain

$$p = \{2m[E - V(x)]\}^{1/2} \quad (6)$$

Now we can use the de Broglie formula to get an expression for the wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\{2m[E - V(x)]\}^{1/2}} \quad (7)$$

The term $\omega^2/v^2$ in equation can be rewritten in terms of $\lambda$ if we recall that $\omega = 2\pi\nu$ and $\nu\lambda = v$.

$$\frac{\omega^2}{v^2} = \frac{4\pi^2\nu^2}{v^2} = \frac{4\pi^2}{\lambda^2} = \frac{2m[E - V(x)]}{\hbar^2} \quad (8)$$

When this result is substituted into equation 2 we obtain the famous *time-independent Schrödinger equation*

$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0 \quad (9)$$

which is almost always written in the form

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x) \quad (10)$$

This single-particle one-dimensional equation can easily be extended to the case of three dimensions, where it becomes

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad (11)$$

A two-body problem can also be treated by this equation if the mass $m$ is replaced with a reduced mass $\mu$.

**PHYSICAL SIGNIFICANCE OF WAVE FUNCTION**

1. The variable quantity that characterizes d-Broglie wave is called wave function .

2. The wave function represents the variations in the matter waves and it connects the particle nature and its associated wave nature statistically.

3. The wave function associated with a moving particle at a particular instant of time and at a particular point in space is related to the probability of finding the particle at that instant and at that point.

4. The probability 0 corresponds to the certainty of not finding the particle and probability 1 corresponds to the certainty of finding the particle.

$\iiint \psi\psi d\tau = 1$ if particle is present

$\iiint \psi\psi d\tau = 0$ if particle is not present

5. The wave function is a complex quantity that cannot be measured.

6. The probability of finding a particle at particular region must be real and positive, but the wave function is in general a complex quantity.

KARPAGAM ACADEMY OF HIGHER EDUCATION

Coimbatore-641021.

(For the candidates admitted from 2017 onwards)

DEPARTMENT OF PHYSICS

ELEMENTS OF MODERN PHYSICS (17PHU502A)

**Unit - III**

| Question | Option 1 | option 2 | option 3 |
|---|---|---|---|
| ____ forms of Schroedinger's equation describe the motion of non-relativistic material particle. | Hψ = Eψ | Hψ ≠ Eψ | Hψ < Eψ |
| If $\psi_1$ and $\psi_2$ are two different wave functions, both being satisfactory solution of wave equation for a given system, then these functions will be normalized, if | $\psi_j^*\psi_j d\tau = 1$ | $\psi_j^*\psi_j d\tau \neq 1$ | $\psi_j^*\psi_j d\tau > 1$ |
| Schroedinger suggested seeking solutions of the waves equation which represents ___ waves. | non-progressive | progressive | non-standing |
| Newton's law may be written as | (dp/dt) > -gradV | (dp/dt) < -gradV | (dp/dt) ≠ -gradV |
| Kinetic energy operator is | $(-\hbar^2/2m)^2$ | $(-2m/\hbar^2)^2$ | $(-2m\hbar^2)^2$ |
| Momentum operator in Schroedinger equation (Pop) is | ħ/i | ħi | i/ħ |
| The minimum energy of a particle in a box (E) is | $\hbar^2/ml^2$ | $\hbar^2/2ml^2$ | $ml^2/\hbar^2$ |
| The Schroedinger time-dependent wave equation is | Hψ = Eψ | Hψ ≠ Eψ | Hψ < Eψ |

| | 1 | 2 | 3 |
|---|---|---|---|
| The time-dependent Schroedinger equation is partial differential equation having ___ variables. | | | |
| The Schroedinger equation for a free particle is | $\Delta^2\psi + (2m/\hbar^2)(E)\psi = 0$ | $\Delta^2\psi + (2m/\hbar^2)(E)\psi \neq 0$ | $\Delta^2\psi + (2m/\hbar^2)(E)\psi < 0$ |
| The time independent form of Eop is | H | V | U |
| Wave function Ψ of a particle is | real quantity | a complex quantity | an imaginary quantity |
| Which of the following quantities are complex quantities? | Wave function Ψ of a particle | Probability of a particle having Wave function Ψ | Probability Density of a particle having Wave function Ψ |
| The wave function Ψ of the particle is | solution to the wave equation | not a variable quantity | goes through repeating, periodic maxima and minima or oscillations |
| The probability current of a particle is | dependent on time | number of particles per unit volume per unit time | not a real quantity |

| | | | |
|---|---|---|---|
| The time-independent Schrödinger equation | is a partial differential equation | involves only one independent variable r | can be derived from time-dependent Schrödinger equation |
| In the Stationary states | probability distribution of finding the particle is time independent | measurements of total energy yield different values | the expectation values of time-independent operators are dependent on time |
| Time dependent Schrödinger equation | intrinsically includes the unit of imaginary numbers, hence cannot describe the physical reality of the micro-world | equates first order space derivative with second time derivative | is a more general and fundamental postulate of quantum physics |

| | | | |
|---|---|---|---|
| Operators in quantum physics | are used to represent physical observables in classical physics | are used to translate equations in classical physics into equations of quantum physics | corresponding to canonically conjugate variables commute |
| The continuity equation in quantum physics implies | conservation of energy | equation of the continuous functions | the conservation of wavefunction |
| The time evolution equation of the expectation values of position and momentum of a quantum mechanical particle is given by | Continuity Equation | Ehrenfest's Theorem | Divergence theorem (Green's Second Theorem) |
| Wave function is represented by ________ | $\psi$ | E | H |
| Schroedinger attempt the physical interpretation of $\psi$ in terms of _______ | volume density | current density | density |
| In wave function, energy per unit volume is equal to ___ | $A^2$ | $E^2$ | H2 |
| Photon density is ____ | hν | $A^2/h$ | $A^2/\nu$ |
| Photon density is proportional to ______ | hν | $A^2$ | h |
| Particle density is proportional to ______ | hν | $\psi^2$ | h |
| Complex conjugate of wave function is ______ | $E^*$ | $H^*$ | $\psi$ |
| To remove the above discrepancy another physical interpretation of wave function generally accepted at present was suggested by Max Born in the year | 1923 | 1927 | 1926 |
| The total probability of finding the particle in the entire space is _____ | unity | 0 | ∞ |
| At $x = \pm\infty$ then $\psi^*\psi =$ | 1 | 0 | ∞ |
| Normalising factor is | $\sqrt{N}$ | $1/\sqrt{N}$ | $2/\sqrt{N}$ |
| Normalised wavefunction is | $\psi$ | $\sqrt{N}$ | $1/\sqrt{N}$ |
| If probability distribution of finding the particle is time independent the it is said to be | orthogonal | noramalised | stationary state |

| | | | |
|---|---|---|---|
| Characteristic function is also called as ________ | wave function | eigen value | noramalised |
| _________ of a dynamical quantity is the mathematical expectation for the result of a single measurements | expectation value | eigen value | noramalised |
| In electromagnetic wave system if A is amplitude,then energy density is | $A^2$ | $E^2$ | $H^2$ |
| $\|\psi^2\|$ is the measure of $\|\psi^2\|$ $\|\psi^2\|$ ____ | volume density | current density | particle density |
| ________ is the measure of particle density | $\|E^2\|$ | $\|H^2\|$ | $\|\psi\|$ |
| Probability density of the particle in the state of $\phi$ | $\psi\psi^*$ | $\psi$ | $\psi^*$ |
| The operator for momentum is | $(\hbar/i)\,\Delta$ | $(\hbar)\,\Delta$ | $p\,\Delta$ |
| The probability amplitude for the position of the particle is represented by | P | H | E |
| The probability density is the | square root of the wave function | absolute value of the wave function | inverse of the wave function |
| All of the following statements about Schrodinger's cat are true EXCEPT | it was conducted in 1935 | the cat is sealed inside of a box | the cat is both alive and dead |
| Any two operators that do not commute have what property associated with their observables? | It is impossible to specify simultaneously the values of both observables without uncertainty | The values of their observables are coupled and decay exponentially in time | Their observables cannot be simultaneously non-zero |
| The eigen value of the even function of the parity operator is | $\lambda = 0$ | $\lambda = 1$ | $\lambda = \pm 1$ |
| The eigen value of the odd function of the parity operator is | $\lambda = 0$ | $\lambda = 1$ | $\lambda = \pm 1$ |

| | | | |
|---|---|---|---|
| The wave equation for a moving particle is represented by | $\tilde{N}^2\psi + (1/v^2)\,\partial^2\psi/\partial t^2 = 0$ | $\tilde{N}^2\psi - (1/v^2)\,\partial^2\psi/\partial t^2 = 0$ | $\tilde{N}^2\psi + (v^2)\,\partial^2\psi/\partial t^2 = 0$ |
| The quantum concept was introduced by ____ | Schrodinger | Bohr | Planck |
| Old quantum theory explains _____ | particle in a box | Pauli’s exclusion principle | Spectral lines of hydrogen molecule |
| If A and B are unitary operators, then the product is | Hermitian | Unitary | Hamiltonian |
| The quantum mechanical operator for momentum is | $P = - i\hbar\tilde{N}$ | $P = i\hbar\tilde{N}$ | $P = \hbar\tilde{N}$ |
| The operator for kinetic energy is | $(\hbar^2 \tilde{N}^2)/2m$ | $-(\hbar^2 \tilde{N}^2)/2m$ | $(h^2\tilde{N}^2)/2m$ |
| The operator for velocity is | $i\,\hbar\tilde{N}/m$ | $-i\,\hbar m\tilde{N}$ | $-i\,\hbar\tilde{N}/m$ |
| In Schrodinger picture, the state vector and operator are respectively | time dependent and time independent | time independent and time dependent | both time independent |
| The operator for energy is ____ | $i\hbar\partial/\partial t$ | $-i\hbar\partial/\partial t$ | $i\hbar\partial/\partial x$ |
| Schrodinger's equation described the | wave function | complement of the wave function | behavior of "matter" waves |
| Solutions to Schrodinger's equation are labeled with | psi | phi | mu |

| option 4 | option 5 | option 6 | Answer |
|---|---|---|---|
| $H\psi > E\psi$ | | | $H\psi = E\psi$ |
| $\psi_j{}^*\psi_j d\tau < 1$ | | | $\psi_j{}^*\psi_j d\tau = 1$ |
| standing | | | standing |
| (dp/dt) = -gradV | | | (dp/dt) = -gradV |
| $(-2\hbar^2)^2$ | | | $(-\hbar^2/2m)^2$ |
| Ħ | | | ħ/i |
| $2ml^2/\hbar^2$ | | | $\hbar^2/2ml^2$ |
| $H\psi > E\psi$ | | | $H\psi = E\psi$ |

| | |
|---|---|
| 4 | 3 |
| $\Delta^2\psi + (2m/\hbar^2)(E)\psi > 0$ T | $\Delta^2\psi + (2m/\hbar^2)(E)\psi = 0$ H |
| any one of these | real quantity |
| Probability Current of a particle having Wave function Ψ | Wave function Ψ of a particle |
| variable quantity | solution to the wave equation |
| always positive | number of particles per unit volume per unit time |

| | |
|---|---|
| has solutions which are the stationary states | involves only one independent variable r |
| the general solution is a linear combination of separable solutions | probability distribution of finding the particle is time independent |
| is the Eigenvalue equation for the energy operator (Hamiltonian operator | is a more general and fundamental postulate of quantum physics |

| | |
|---|---|
| are nonlinear, hermitian corresponding to classical dynamical variables | are used to translate equations in classical physics into equations of quantum physics |
| the conservation of momentum of the particle | the conservation of wavefunction |
| Schrödinger equation | Schrödinger equation |
| W | $\psi$ |
| charge density | charge density |
| $\psi^2$ | $A^2$ |
| $A^2/h\nu$ | $A^2/h\nu$ |
| $\nu$ | $A^2$ |
| $\nu$ | $\psi^2$ |
| $\psi^*$ | $\psi^*$ |
| 1929 | 1926 |
| vary | unity |
| vary | 1 |
| N | $1/\sqrt{N}$ |
| $\psi/\sqrt{N}$ | $\psi/\sqrt{N}$ |
| orthonormal | stationary state |

| | |
|---|---|
| stationary state stationary state | wave function expectation value |
| $\psi^2$ | $A^2$ |
| density | particle density |
| / | / |
| 0 | $\psi\psi^*$ |
| $i\ \Delta$ | $(\hbar/i)\ \Delta$ |
| $\psi$ | $\psi$ |
| absolute square of the wave function | absolute square of the wave function |
| there is a 50 percent chance a vial of poison is released inside the box | it was conducted in 1935 |
| Their observables correspond to imaginary eigenvalues | Their observables cannot be simultaneously non-zero |
| $\lambda = -1$<br>$\lambda = -1$ | $\lambda = -1$<br>$\lambda = 0$ |

| | |
|---|---|
| $\tilde{N}^2\psi - (v^2) \partial^2\psi/\partial t^2 = 0$ | $\tilde{N}^2\psi - (1/v^2) \partial^2\psi/\partial t^2 = 0$ |
| Einstein | Planck |
| Spin of the electrons | Spectral lines of hydrogen molecule |
| Inverse | Hermitian |
| $P = - \hbar\tilde{N}$ | $P = i\hbar\tilde{N}$ |
| $-(h^2\tilde{N}^2)/2m$ | $(h^2\tilde{N}^2)/2m$ |
| $i \hbar m\tilde{N}$ | $-i \hbar m\tilde{N}$ |
| both time dependent | time independent and time dependent |
| $-i\hbar v$ | $i\hbar\partial/\partial t$ |
| motion of light | behavior of "matter" waves |
| pi | psi |

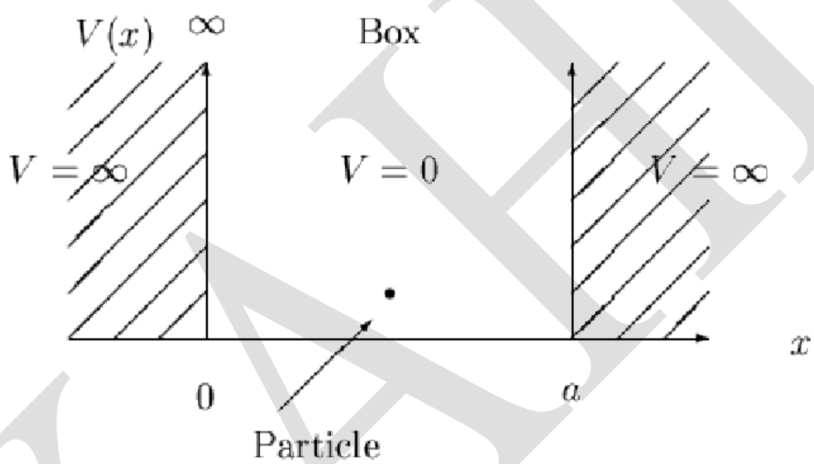
## UNIT-IV

One dimensional infinitely rigid box- energy eigenvalues and eigenfunctions, normalization; Quantum dot as example; Quantum mechanical scattering and tunneling in one dimension-across a step potential & rectangular potential barrier. Size and structure of atomic nucleus and its relation with atomic weight; Impossibility of an electron being in the nucleus as a consequence of the uncertainty principle. Nature of nuclear force, NZ graph, Liquid Drop model: semi-empirical mass formula and binding energy, Nuclear Shell Model and magic numbers.

## ONE DIMENSIONAL INFINITELY RIGID BOX

A particle in a 1-dimensional box is a fundamental quantum mechanical approximation describing the translational motion of a single particle confined inside an infinitely deep well from which it *cannot*escape.

The particle in a box problem is a common application of a quantum mechanical model to a simplified system consisting of a particle moving horizontally within an infinitely deep well from which it cannot escape. The solutions to the problem give possible values of E and ψ that the particle can possess. E represents allowed energy values and ψ(x) is a wavefunction, which when squared gives us the probability of locating the particle at a certain position within the box at a given energy level.




*A particle in a 1D infinite potential well of dimension* L.

The potential energy is *0 inside the box* (V=0 for 0<x<L) and *goes to infinity at the walls of the box* (V=∞ for x<0 or x>L). We assume the walls have infinite potential energy to ensure that the particle has zero probability of being at the walls or outside the box. Doing so significantly simplifies our later mathematical calculations as we employ these **boundary conditions** when solving the Schrödinger Equation.

The time-independent Schrödinger equation for a particle of mass *m* moving in one direction with energy *E* is

$$(-\hbar^2/2m)d^2\psi(x)/dx^2+V(x)\psi(x)=E\psi(x) \quad ---------- (1)$$

with

ℏ is the reduced Planck Constant where ℏ=h/2π

m is the mass of the particle

ψ(x) is the stationary time-independent wavefunction

V(x) is the potential energy as a function of position

EE is the energy, a real number

This equation can be modified for a particle of mass *m* free to move parallel to the x-axis with zero potential energy (V = 0 everywhere) resulting in the quantum mechanical description of free motion in one dimension:

$$(-\hbar^2/2m)d^2\psi(x)/dx^2\ E\psi(x) \quad \text{----------- (2)}$$

This equation has been well studied and gives a general solution of:

$$\psi(x)=A\sin(kx)+B\cos(kx) \quad \text{-----------(3)}$$

where A, B, and k are constants.

The solution to the Schrödinger equation we found above is the general solution for a 1-dimensional system. We now need to apply our **boundary conditions** to find the solution to our particular system. According to our boundary conditions, the probability of finding the particle at x=0 or x=L is zero. When x=0x=0, sin(0)=0sin (0)=0, and cos(0)=1cos (0)=1; therefore, *B must equal 0* to fulfill this boundary condition giving:

$$\psi(x)=A\sin(kx) \quad \text{---------(4)}$$

We can now solve for our constants (A and k) systematically to define the wavefunction.

**Solving for k**

Differentiate the wavefunction with respect to x:

$$d\psi/dx=kA\cos(kx) \quad \text{---------- (5)}$$

$$d^2\psi/dx^2=-k^2A\sin(kx) \quad \text{--------(6)}$$

Since ψ(x)=Asin(kx)ψ(x)=Asin(kx), then

$$d^2\psi/dx^2=-k^2\psi \quad \text{----------- (7)}$$

$$k=(8\pi^2mE/\ h^2)^{1/2} \quad \text{------(8)}$$

Now we plug k into our wavefunction:

$$\psi=A\sin(8\pi^2mE/h^2)^{1/2}x \quad \text{------(9)}$$

**Solving for A**

To determine A, we have to apply the boundary conditions again. Recall that the *probability of finding a particle at x = 0 or x = L is zero.*

When x=L

$$0 = A\sin(8\pi^2 mE/h^2)^{1/2}L$$

This is only true when

$$(8\pi^2 mE/h^2)^{1/2}L = n\pi$$

where n = 1,2,3…

Plugging this back in gives us:

$$\psi = A\sin n\pi/Lx$$

To determine A, recall that the total probability of finding the particle inside the box is 1, meaning there is no probability of it being outside the box. When we find the probability and set it equal to 1, we are *normalizing* the wavefunction.

$$\int L0\psi^2 dx = 1$$

For our system, the normalization looks like:

$$A^2\int L0\sin^2(n\pi/L)xdx = 1$$

Using the solution for this integral from an integral table, we find our normalization constant, A:

$$A = \sqrt{2}/L$$

Which results in the normalized wavefunction for a particle in a 1-dimensional box:

$$\psi = \sqrt{2}/L\ \sin n\pi/L\ x$$

Solving for E results in the allowed energies for a particle in a box:

$$E_n = n^2h^2/8mL^2$$

This is an important result that tells us:

The energy of a particle is quantized and

The lowest possible energy of a particle is **NOT** zero. This is called the **zero-point energy** and means the particle can never be at rest because it always has some kinetic energy.

This is also consistent with the Heisenberg Uncertainty Principle: if the particle had zero energy.

**ENERGY EIGENVALUES AND EIGENFUNCTIONS**

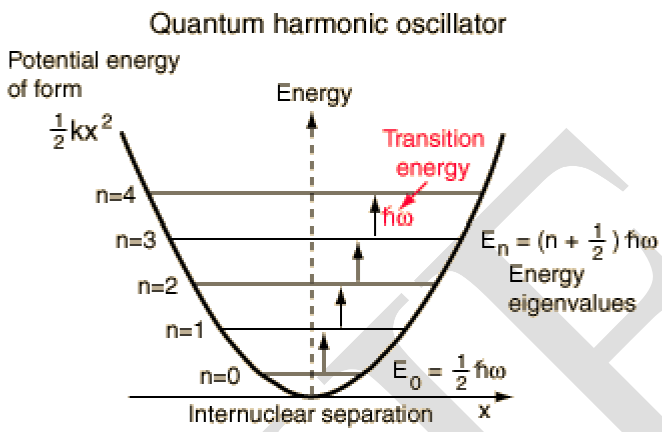
The wavefunction with the quantum mechanical operator associated with energy, which is called the Hamiltonian. The operation of the Hamiltonian on the wavefunction is the Schrodinger equation. Solutions exist for the time-independent Schrodinger equation only for certain values of energy, and these values are called "eigenvalues" of energy.

For example, the energy eigenvalues of the quantum harmonic oscillator are given by

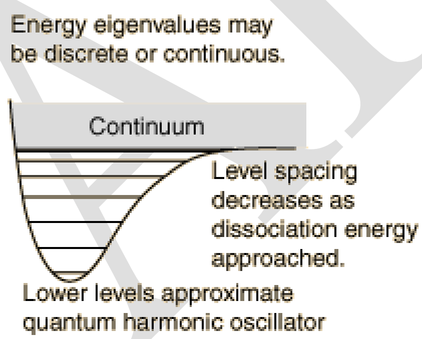
$$E_n = (n + \tfrac{1}{2})\hbar\omega \qquad n = 0,1,2,3 \ldots$$

$$\omega = 2\pi(\text{frequency})$$

$$\hbar = \text{Planck's constant}/2\pi$$




The lower vibrational states of diatomic molecules often fit the quantum harmonic oscillator model with sufficient accuracy to permit the determination of bond force constants for the molecules.




The energy eigenvalues may be discrete for small values of energy, they usually become continuous at high enough energies because the system can no longer exist as a bound state. For a more realistic harmonic oscillator potential (perhaps representing a diatomic molecule), the energy eigenvalues get closer and closer together as it approaches the dissociation energy. The energy levels after dissociation can take the continuous values associated with free particles.

Corresponding to each eigenvalue is an "eigenfunction*". The solution to the Schrodinger equation for a given energy $E_i$ involves also finding the specific function $\psi_i$ which describes that energy state. The solution of the time independent Schrodinger equation takes the form

$$H_{op}\psi_i = E_i\psi_i$$

The eigenvalue concept is not limited to energy. When applied to a general operator Q, it can take the form

$$\underbrace{Q_{op}}_{\text{operator}}\psi_i = \underbrace{q_i}_{\text{eigenvalue}}\overbrace{\psi_i}^{\text{eigenfunction}}$$

if the function $\psi_i$ is an eigenfunction for that operator. The eigenvalues qi may be discrete, and in such cases we can say that the physical variable is "quantized" and that the index i plays the role of a "quantum number" which characterizes that state.

**NORMALIZATION OF WAVE FUNCTION**

An outcome of a measurement which has a probability 0 is an impossible outcome, whereas an outcome which has a probability 1 is a certain outcome. According to Eq., the probability of a measurement of $x$ yielding a result between $-\infty$ and $+\infty$ is

$$P_{x\in-\infty:\infty}(t) = \int_{-\infty}^{\infty}|\psi(x,t)|^2\,dx. \tag{1}$$

However, a measurement of $x$ *must* yield a value between $-\infty$ and $+\infty$, since the particle has to be located somewhere. It follows that $P_{x\in-\infty:\infty} = 1$, or

$$\int_{-\infty}^{\infty}|\psi(x,t)|^2\,dx = 1, \tag{2}$$

which is generally known as the *normalization condition* for the wavefunction.

For example, suppose that we wish to normalize the wavefunction of a Gaussian wave packet, centered on $x = x_0$, and of characteristic width $\sigma$ *i.e.*,

$$\psi(x) = \psi_0\,\mathrm{e}^{-(x-x_0)^2/(4\sigma^2)}. \tag{3}$$

In order to determine the normalization constant $\psi_0$, we simply substitute Eq. (3) into Eq. (2), to obtain

$$|\psi_0|^2\int_{-\infty}^{\infty}\mathrm{e}^{-(x-x_0)^2/(2\sigma^2)}\,dx = 1. \tag{4}$$

Changing the variable of integration to $y = (x - x_0)/(\sqrt{2}\,\sigma)$, we get

$$|\psi_0|^2 \sqrt{2}\,\sigma \int_{-\infty}^{\infty} e^{-y^2}\,dy = 1. \tag{5}$$

However,

$$\int_{-\infty}^{\infty} e^{-y^2}\,dy = \sqrt{\pi}, \tag{6}$$

which implies that

$$|\psi_0|^2 = \frac{1}{(2\pi\,\sigma^2)^{1/2}}. \tag{7}$$

Hence, a general normalized Gaussian wavefunction takes the form

$$\psi(x) = \frac{e^{i\,\varphi}}{(2\pi\,\sigma^2)^{1/4}}\,e^{-(x-x_0)^2/(4\sigma^2)}, \tag{8}$$

where $\varphi$ is an arbitrary real phase-angle.

Now, it is important to demonstrate that if a wavefunction is initially normalized then it stays normalized as it evolves in time according to Schrödinger's equation. If this is not the case then the probability interpretation of the wavefunction is untenable, since it does not make sense for the probability that a measurement of $x$ yields *any* possible outcome (which is, manifestly, unity) to change in time. Hence, we require that

$$\frac{d}{dt}\int_{-\infty}^{\infty} |\psi(x,t)|^2\,dx = 0, \tag{9}$$

for wavefunctions satisfying Schrödinger's equation. The above equation gives

$$\frac{d}{dt}\int_{-\infty}^{\infty} \psi^*\,\psi\,dx = \int_{-\infty}^{\infty}\left(\frac{\partial \psi^*}{\partial t}\,\psi + \psi^*\,\frac{\partial \psi}{\partial t}\right)dx = 0. \tag{10}$$

Now, multiplying Schrödinger's equation by $\psi^*/(i\,\hbar)$, we obtain

$$\psi^* \frac{\partial\psi}{\partial t} = \frac{i\hbar}{2m}\psi^*\frac{\partial^2\psi}{\partial x^2} - \frac{i}{\hbar}V|\psi|^2. \tag{11}$$

The complex conjugate of this expression yields

$$\psi\frac{\partial\psi^*}{\partial t} = -\frac{i\hbar}{2m}\psi\frac{\partial^2\psi^*}{\partial x^2} + \frac{i}{\hbar}V|\psi|^2 \tag{12}$$

[since $(A\,B)^* = A^*\,B^*$, $A^{**} = A$, and $i^* = -i$]. Summing the previous two equations, we get

$$\frac{\partial\psi^*}{\partial t}\psi + \psi^*\frac{\partial\psi}{\partial t} = \frac{i\hbar}{2m}\left(\psi^*\frac{\partial^2\psi}{\partial x^2} - \psi\frac{\partial^2\psi^*}{\partial x^2}\right) = \frac{i\hbar}{2m}\frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right). \tag{13}$$

Equations (10) and (13) can be combined to produce

$$\frac{d}{dt}\int_{-\infty}^{\infty}|\psi|^2\,dx = \frac{i\hbar}{2m}\left[\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right]_{-\infty}^{\infty} = 0. \tag{14}$$

The above equation is satisfied provided

$$|\psi| \to 0 \quad \text{as} \quad |x| \to \infty. \tag{15}$$

However, this is a necessary condition for the integral on the left-hand side of Eq. (2) to converge. Hence, we conclude that all wavefunctions which are *square-integrable* [*i.e.*, are such that the integral in Eq. (2) converges] have the property that if the normalization condition (2) is satisfied at one instant in time then it is satisfied at all subsequent times.

It is also possible to demonstrate, via very similar analysis to the above, that

$$\frac{dP_{x\in a:b}}{dt} + j(b,t) - j(a,t) = 0, \tag{16}$$

where $P_{x\in a:b}$ is defined in Eq. , and

$$j(x,t) = \frac{i\hbar}{2m}\left(\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}\right) \tag{17}$$

is known as the *probability current*. Note that $j$ is real. Equation (15) is a *probability conservation equation*. According to this equation, the probability of a measurement of $x$ lying in the interval $a$ to $b$evolves in time due to the difference between the flux of probability into the interval [*i.e.*, j(a,t)], and that out of the interval [*i.e.*, j(b,t)]. Here, we are interpreting j(x,t) as the *flux* of probability in the +x -direction at position x and time t.

Note, finally, that not all wavefunctions can be normalized according to the scheme set out in Eq. (2). For instance, a plane wave wavefunction

$$\psi(x,t) = \psi_0 \, e^{i(kx-\omega t)} \tag{18}$$

is not square-integrable, and, thus, cannot be normalized. For such wavefunctions, the best we can say is that

$$P_{x \in a:b}(t) \propto \int_a^b |\psi(x,t)|^2 \, dx.$$

**NUCLEAR SIZE**

Nuclear size is calculated from the formula R=$r_0 A^{1/3}$, where A is the mass number of the nucleus. For example $_6C^{12}$,R=1.3*$10^{-15}$*$(12)^{1/3}$, R=2.976*$10^{-15}$

Practical verification for nuclear size was done by electric and nuclear method. Electrical method such as Mesonic X-ray, Electronic scattering and Coulomb-energy of mirror nuclei. Nuclear method like neutron scattering, α-decay and α-scattering isotopic shift in line spectra.

**NUCLEAR MASS**

Nuclear consists of protons and neutrons the mass of the nuclei can be assumed nuclear mass=$Zm_p+Nm_n$, where $m_p$ and $m_n$ is the mass of the proton and neutrons respectively, the nuclear mass can be obtained experimentally using a mass spectrometer. It shows that real nuclear mass < $Zm_p+Nm_n$ the difference in mass is

$Zm_p+Nm_n$-real nuclear mass-$\Delta m$

$$\Delta m - mass\ defect$$

**NUCLEAR DENSITY**

Nuclear density= $\frac{nuclear\ mass}{nuclear\ volume}$

$$= \frac{Am_N}{\frac{4}{3}\pi(A^{1/3}r_0)^3}$$

$$= \frac{m_N}{\frac{4}{3}\pi r_0{}^3}$$

nuclear density=1.816*$10^{17}$ kg/$m^3$

This shows that nuclear matter is in extremely compressed state certain starts like white dwarf composed of atoms whose electron shell have collapsed owing enormous pressure and density such states apporaches pure nucleus matter.

**Impossibility of an electron being in the nucleus as a consequence of the uncertainty principle**

The diameter of nucleus of any atom is of the order of $10^{-14}$m. If any electron is confined within the nucleus then the uncertainty in its position (Δx) must not be greater than $10^{-14}$m. According to Heisenberg's uncertainty principle, equation (1.27)

$\Delta x\ \Delta p \geq h / 2\pi$

The uncertainty in momentum is

$\Delta p \geq h / 2\pi\Delta x$ , where $\Delta x = 10^{-14}$m

$\Delta p \geq (6.63X10^{-34}) / (2X3.14X10^{-14})$

i.e. $\Delta p \geq 1.055X10^{-20}$ kg.m /s ————-(1.30)

This is the uncertainty in the momentum of electron and then the momentum of the electron must be in the same order of magnitude. The energy of the electron can be found in two ways one is by non relativistic method and the other is by relativistic method.

Nuclear Force

The nucleus is held by the forces which protect them from the enormous repulsion forces of the positive protons. It is a force with short range and not similar to the electromagnetic force.

We know that the nucleus is made up with its fundamental particles that are the protons and neutrons. These are formed with quarks which are held together with strong force. This strong force is residual color force. The basic exchange particle is called gluon which works as mediator forces between quarks. Both the particles; gluons and quarks are present in protons and neutrons.

The range of force between particles is not determined by the mass of particles. Thus, the force which balanced the repulsion force between the positively charged particles protons are is referred as nuclear attraction which overcomes the electric repulsion force in a short range of order. It is quite stronger than the Columbic force of atomic nuclei and short range force for larger nuclei. Now we discuss about the nuclear force and some examples of weak and strong range nuclear forces.

**Nuclear Force Definition**

Nuclear Force is defined as the force exerted between numbers of nucleons. This force is attractive in nature which binds protons and neutrons in the nucleus together. Since the protons are of same positive charge they exert a repulsive force among them. This can result in bursting of the nucleus. Hence to hold them together nuclear force is responsible.

Because of this attractive Nuclear Force, the total mass of the nucleus is less than the summation of masses of nucleons that is protons and neutrons. This force is highly attractive between nucleons at a distance of $10^{-15}$ m or 1 femtometer (fm) approximately from their centers.

Nuclear forces are of two types namely,

Strong Nuclear Force

Weak Nuclear Force.

Nuclear forces are independent of the charge of protons and neutrons. This property of nuclear force is called charge independence. It depends on the spins of the nucleons that is whether they are parallel or no and also on the non central or tensor component of nucleons.

**Strong Nuclear Force**

**Strong nuclear force is about 100 times stronger than electromagnetism. These forces is also known as strong interactions.**

Strong nuclear forces can be applied in two aspects: One is on Larger Scale and other on Lower Scale.

On larger scale of about 1 to 3 fm distance, it is the force that binds the nucleons together to form nuclei.

On smaller scale of abut less than 0.8 fm distance, it is the force that binds quarks together to form nucleons that is protons and neutrons and also other particles like hadrons.

Strong interactions bring into account the concept of Color charge. It is completely different from the visual sense of color. Color charge is similar to electric charge. As the electromagnetic force is due to electric charge, in the similar way strong nuclear forces are due to color charge. Other particles which do not have color charge are not responsible for strong forces. Fundamentally quarks are colored charges which feel strong forces.

Nucleons (protons and neutrons) are considered to be a part of baryon class that contains 3 types of quarks, each having a color charge among 3 fundamental colors Red, Blue and Green. They are decided according to the rule of Quantum Chromo Dynamics.

To carry strong nuclear force between quarks or anti quarks, gluons act as mediators. Gluons in turn carry color charge for interaction between quarks and gluons. Strong force acts directly on quarks and gluons only. These particles interact with each other through strong force.

**Strong Nuclear Force Examples**

Strong nuclear forces help in holding sub atomic particles of protons together and also the nucleons together at larger scale.

Strong nuclear force leads to release of energy when heat is generated in Nuclear Power Plant to generate steam for generating electricity.

Energy is released when a Nuclear Weapon detonates which is due to strong nuclear forces.

**Weak Nuclear Force**

Weak Nuclear Force is one of the four fundamental force. Electromagnetic force, gravitational force and strong nuclear force are the other forces. Weak Nuclear Force is caused by the emission or exchange of W and Z bosons. Weak nuclear forces are very short range because of the heaviness of the W and Z particles. Weak nuclear force results in the change of one type of quark to another type. This is also known as change of flavor / flavor change of quarks.

Weak Nuclear Force can transform a neutron into proton or proton into neutron. Weak nuclear forces act between quarks and leptons both. Weak interaction is responsible for the flavor change of quarks and leptons. The significance of weak nuclear force in flavor change of quarks makes it the interaction indulged in many decay phenomenons of nuclear particles which need a change of quark from one type to another.

Weak nuclear force is of two types:

- Charged current Nuclear Force and
- Neutral current Nuclear Force.

Charged current nuclear force is so called because this force is carried by electric charge carriers i.e. W+ and W- boson particles.

Neutral current nuclear force is carried by neutral particles i.e. Z boson particles. These forces occurs in many reactions namely,

- Radioactive decay
- Beta decay
- Burning of sun
- Initiating the process of hydrogen fusion in stars.
- In production of deuterium
- Radiocarbon dating and
- Radio luminescence.

**BINDING ENERGY**

The theoretical explanation of mass defect is based on Einstein equation E=m$c^2$.When Z protons and N neutrons combine to make the nucleus. Some of the mass $\Delta m$ disappear because it is converted into an amount of energy $\Delta E = \Delta mc^2$. This energy is called binding energy. To disrupt a stable nucleus into its constituent neutrons and protons.The energy required is binding energy. The magnitude of the binding energy of a nucleus determines its stability against disintegration. If the binding energy is large the nucleus is stable. The nucleus having least possible energy as binding energy it is said to be in the ground state. If the nucleus has an energy

$E < E_{minimum}$ is said to existed state. The case E=0 corresponds to disassociation of the nucleus into constitutional nucleons.

If m is experimentally determined mass of nuclei having Z proton and N neutron then binding energy={(Z$m_p$+N$m_n$)-M} $c^2$

If binding energy>0 the nucleus is stable and the energy is must be supplied from outside to disrupt. The binding energy <0 the nucleus is unstable and it will disintegrate by itself.

Example

Let us illustrate the calculation of binding energy by taking example as deuterons is formed by a proton and neutron.

mass of proton =1.007276a.m.u

mass of neutron=1.008665a.m.u

therefore,

mass of proton + mass of neutron in free state =2.015941a.m.u

mass of deuteron nucleus= 2.013553a.m.u

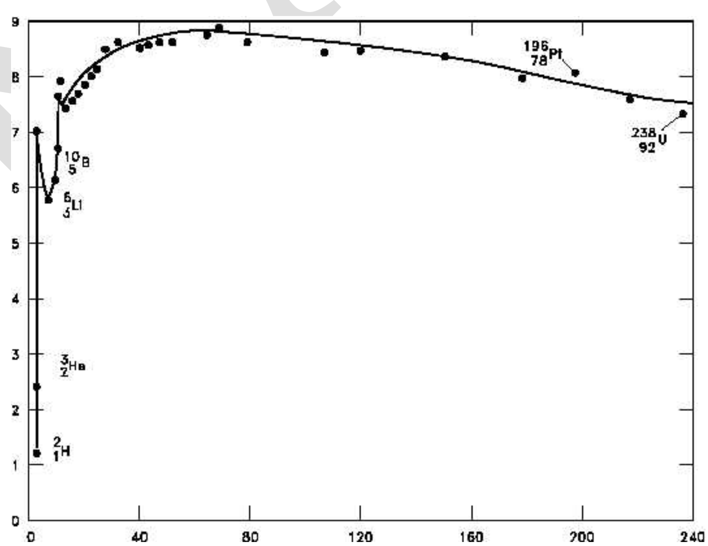
mass defect $\Delta m = 2.015941 - 2.013553 = 0.002388 \text{a.m.u}$

binding energy=$\Delta m * 931 = 0.002388 * 931 = 2.23 Mev$

1a.m.u=931Mev

***STABILITY OF NUCLEUS AND BINDING ENERGY***

Binding energy per nucleon=$\frac{\text{total binding energy of nucleus}}{\text{the number of nucleon it contains}}$



The binding energy per nucleon is plotted as the function of mass number (A) the curve rises steeply at first, then more gradually until it reaches a maximum of 8.75Mev at A=56 corresponding to the iron nucleus $Fe_{26}^{56}$ . The curve then drops slowly to 7.6Mev at the highest mass number. Evidently, nuclei of intermediate masses or the most stable since the greatest amount of energy must be supplied to liberate each of their nucleons. This fact suggests that a larger amount of energy will be liberated if heavier nuclei can be split into lighter one or if lighter nuclei be joined to form heavier one.

**LIQUID-DROP MODEL**

In the liquid-drop model the forces acting in the nucleus are assumed to the molecular forces in a droplet of some liquid. This model was proposed by Neils Bohr, who observed that there are certain marked similarities between an atomic nucleus and a liquid drop.

1. The nucleus is supposed to be spherical in shape in the stable state just as a liquid drop is spherical due to the symmetrical surface tension forces.
2. The forces of surface tension acts on the surface of the liquid drop, similarly there is a potential barrier at the surface of the nucleus.
3. The density of a liquid drop is independent of its volume similarly the density of the nucleus is independent of its volume.
4. The intermolecular forces in a liquid are short ranges the molecules in a liquid drop interact only with their immediate neighbors, similarly the nucleus also interacts only with their immediate neighbors. This leads to the saturation in the nuclear forces and a constant B.E per nucleons
5. The molecules evaporates from a liquid drop on raising the temperature of the liquid due to the their increased energy of thermal agitation. Similarly when energy is given to a nucleus by bombarding it with nuclear projectiles, a compound nucleus is formed which emits nuclear radiation almost immediately.
6. When a small drop of liquid is allowed to oscillate it breaks up into two smaller nuclei.

**Merits**

The liquid drop model accounts for many of the silent features of nucleus matters such as the observed binding energy of nuclei and the stability against the $\alpha, \beta$ disintegration as well as nuclear fusion.Calculation of atomic nucleus and binding energy can be done good with accuracy in liquid drop model however this model fails to explain the other properties in particular magic numbers and spin and magnetic moment of the nuclei.

**Semi-empirical mass formula**

It helps to obtain an expression for binding energy of the nuclear Weizacker proposed semi-empirical nuclear of mass number A containing Z protons and N neutrons

$$B.E = aA - bA^{2/3} - c\frac{z(z-1)}{A^{1/3}} - \frac{d(n-z)}{A} \pm \frac{\delta}{A^{3/4}}$$

where a, b ,c ,d are constant

The first term is the equation is called as volume energy of the nucleus the larger the total number of nucleon A the more difficult to remove it will be to remove the individual protons and neutrons the binding energy is directly proportional to total number of nucleons A.

The nucleon at the surface of the nucleus are not completely surrounded by other nucleons depends upon the surface area of the nucleus. A nucleus of radius R has an area of $4\pi R^2=4\mu r_0^2A^{2/3}$. Hence the surface effect reduces the binding energy by $e_x=bA^{2/3}$. The negative energy it is most significant for the lighter nuclei since a greater fraction of nucleons on the surface.

The electrostatic repulsion between each pair of protons in a nucleus also constitutes towards decrease its BE. The coulomb energy $E_e$ of a nucleus the work that must be done to bring together Z protons from infinity into a volume equal to that a nucleus. Hence $E_c$ is proportional to Z (Z-1)/2 and $E_c$ is inversely proportional to the nuclear radius $R=r_0A^{1/3}$. $E_c$ is negative because it arises from a force that opposes nuclear stability.

The final correction term $\delta$ allows for the fact that even-even nuclei are more stable than odd nuclei. Z is positive fir even-even pair and negative for odd-odd pair, zero for odd A.

**SHELL MODEL**

The shell model of the nucleus assumes that the energy structure of the nucleus is similar to that of an electron shell in an atom. According to this model the protons and the neutrons are grouped in shells in the nucleus extra -nuclear electrons in various shells outside the nucleus. The shells are regarded as filled. When they contain a specific number of protons and neutrons are both the no. of nucleons in each shell is limited by Pauli exclusion principle. The shell model Referred to independent particle model because it assumes that each nucleons moves independently of all the other nucleons and acted by an average nuclear field produced by the action of all other nucleus.

*Evidence for shell model*

Nucleus is stable if it has a certain definite number of a the protons and neutrons, these number are known as magic numbers the magic numbers are 2 8 20 50 82 126.Thus nuclei containing 2 8 20 50 82 126 nucleons of same kind from sort of closed nuclear shell structure. the main points are:

1.The inert Gases with the closest electron shells exhibit a high degree of chemical stability. Similarly nucleus whose nuclei containing a magic number of nucleons of same kind exhibit more than average stability.

2. Isotopes of elements having an isotope abundance greater than 60 % belongs to the magic number category

3.Tin has 10 stable isotopes while calcium has 6 stable isotopes so elements with Z=50, 20 are more than usually stable

4.The three Radioactive series uranium actinium Thorium decay to $Pb_{82}^{208}$ with Z=82 N=126 thus lead is the most stable isotope this again shows that 82 and 126 indicates stability

5.Nuclei having no.of neutrons equal to magic number cannot capture a neutrons because the shells are closed and they cannot contain an extra neutron.

6. It is found that some isotopes are spontaneous neutron emitters when excited about the nuclear binding energy by a preceding beta decay. These are $_8O^{17}$ $_{36}Kr^{87}$ and $_{54}Xe^{137}$ for which N= 9,51 and 83 which can be written as 8+1, 50+1 and 82+1 if We interpret this loosely bound neutron as a valance neutron, the neutron numbers 8, 50, 82 to the represent greatest stability than other neutron number.

Nuclear behavior is often determined by the are excess or deficiency of nucleons with respect to closed shells of nucleons corresponding to magic numbers. The nucleons revolve inside the nucleus just as electrons revolve outside in specific permitted orbits.

The neutrons and protons move in two separate systems of orbits around the centre of mass of all nucleons. It moves in Orbit around a common centre of constituents of all nucleus. each nucleon shelll has a specific maximum capacity. They give rise to particular number of characteristics of unusual stability.

The shell model is able to account for several nuclear phenomena in addition to magic numbers.

1. It is observed that even - even nuclei are more stable then odd-odd nuclei. This is obvious from the shell model. According to Pauli principle, a single energy sub level can have a maximum of two nucleons. Therefore in an even-even nucleus only completed sub level are present, which means greater stability on the other hand odd-odd nucleus contain incomplete sub levels which means lesser stability .

2. The shell model Able to predict the total angular momentum of nuclei. In even-even nuclei the protons and neutrons should pair off so as to cancel out one another spin and orbital angular momentum. Thus even-even nuclei have zero nuclear angular momenta. In even odd & odd-even nuclei, the half integral spin off the single extra nucleon should be combined with the integral angular momentum of the rest of nucleons for a half integral total angular momentum. Odd odd nuclei each have an extra neutron and an extra proton whose half integral spin should yield integral total angular momenta. These are experimentally confirmed.

**Possible Questions**

**2 marks**

What is called eigen value and eigen function?
What is called strong nuclear force?
What is called weak nuclear force?
Define semi-empirical formula.
Give a note on liquid drop model.
What is called magic numbers?
Give any four similarities between liquid drop and nucleus.

**8 marks**

Discuss about 1D rigid box.
Explain normalization of wave function.
Compare liquid drop model and shell model.
What is the significance of terms in the liquid drop model?
Give a note on magic numbers.
What are the basic assumptions of shell model?
Briefly discusses about the liquid drop model.
Discuss in detail about shell model of the nucleus.

KARPAGAM ACADEMY OF HIGHER EDUCATION

Coimbatore-641021.

(For the candidates admitted from 2017 onwards)

DEPARTMENT OF PHYSICS

ELEMENTS OF MODERN PHYSICS (17PHU502A)

**Unit - IV**

**Question**

Quantum mechanical tunneling can be explained only with ____

The potential involved outside the nucleus is ____

The eigenvalues of the Hamiltonian of a system refer to what property of the system?

The atomic mass is almost equal to _____

The nuclear radius is proportional to

The nucleon density at the centre of any nucleus is

The force which holds the nucleons together in a nucleus is

The non-central part of the nuclear force is called

Nuclear exchange forces arise due to

Nucleus is

Proton has the charge

Neutrons has the charge

The difference between the total mass of the individual nucleons and the mass of the nucleus is known as

The mass of the nucleus is normally -------- the total mass of the nucleons

The hypothesis that nuclear forces possess an exchange character was put forward by

Instrument used to measure nuclear masses and their other properties is called

The existence of mesons were first observed in

The density of nucleus is approximately

The number of nucleons per unit unit volume is approximately

BE/A curve shows that iron nucleus is

The constant nucleon density inside the nucleus supports

The nuclear wave functions and particle motions support

The constant binding energy per nucleon supports

In which of the following model of nucleus, the protons and neutrons are considered as gas particles?

In the Fermi gas model of the nucleus, the gas is characterised by the kinetic energy of the highest filled state called

In the Fermi gas model, the neutron gas is contained in a potential energy well of depth

The depth of the potential well for proton gas in a Fermi model is

The degenerate gas model was suggested by

The liquid drop model was suggested by

In the liquid drop model, the restoring force after deformation is supplied by

The surface energy is proportional to ---- where A is the mass number

The liquid drop model could not explain satisfactorily ----

According to alpha particle model, a nucleus can be considered as

Alpha particle model could not describe the ground and excited states of

It is seen that nuclei with ----- nucleons are most stable, where n=1,2,3,….

The nuclei with Z = ----- and ------ are found to be more than usually stable

In ------ model, the nucleus is assumed to be containing a gas of protons and neutrons

The resemblance of the nucleus with a drop of liquid led to the suggestion of ----- model.

In Fermi Gas model, the neutron is in a potential well of depth

Fermi gas model is not useful for explaining

In the liquid drop model, the nuclear force is

In the liquid drop model, the nuclei consist of

Which of the following statements is correct?

For certain numbers of neutrons and protons, called ------, nuclei exhibit specral characteristics of stability

The nuclear fission can be best explained using

Bohr-Wheeler theory of nuclear fission is based on

As per liquid drop model, if the energy of the incident neutron is less than the critical energy, ----- takes place.

Standing waves will occur whenever the radius of the body is an odd multiple of the wavelength divided by

Which model is the combination of liquid drop and shell model

The unified model was developed by

Which is the hybrid of liquid drop model and distorted shell model

In which model the shell model potential is assumed non-spherical and the nucleons move independently

The mathematical theory of unified model was developed by

The optical model of the nucleus is developed from an analogy of nuclear scattering with that of-

The collective motion of the nucleons in a deformed nucleus may be ……………………..in character

The nuclear isomerism has been successfully explained by

Nuclei with N or Z near the end of a shell are found in …………. Distinct groups, known as islands of isomerism

The mechanism of nuclear fission was first explained by Bohr and Wheeler on the basis of

Angular momenta and parity for $N^{16}$ is

The expected shell model spin and parity assignment for the ground state of $^{11}B$ is

| Option 1 | option 2 | option 3 | option 4 | option 5 | option 6 | Answer |
|---|---|---|---|---|---|---|
| particle nature of matter | wave nature of matter | mass of the particle | volume of the particle. | | | mass of the particle |
| gravitational | electromagnetic | nuclear | Coulombic | | | Coulombic |
| kinetic energy | potential energy | total energy | total entropy | | | total entropy |
| the mass of the electron | the mass of the nucleus | the mass of the protons | the mass of the neutrons | | | the mass of the nucleus |
| A^2/3 | A | A^1/3 | A^2 | | | A^1/3 |
| proportional to A | proportional A^2 | proportional Z | almost the same | | | almost the same |
| elelctromagnetic force | gravitational force | strong nuclear force | weak interaction | | | strong nuclear force |
| elelctromagnetic force | tensor force | magnetic force | static force | | | tensor force |

| | | | | | |
|---|---|---|---|---|---|
| exchange of mesons | exchange of charge | exchange moments | exxchange of strangeness | | exchange of mesons |
| positively charged | negatively charged | neutral | charge keeps on changing | | positively charged |
| 1637 times of an electron | 1737 times of an electron | 1837 times of an electron | 1937 times of an electron | | 1837 times of an electron |
| 1639 times of an electron | 1739 times of an electron | 1839 times of an electron | 1939 times of an electron | | 1839 times of an electron |
| mass defect | binding energy | packing fraction | mass excess | | mass defect |
| greater than | equal to | less than | can be anything | | less than |
| Pauli | Rutherford | Heisenberg | Max Plank | | Heisenberg |
| Mass spectrograph | nuclear spectrometer | NMR spectrometer | magnetic spectrometer | | Mass spectrograph |
| particle accelerators | cosmic rays | mass spectrometers | none of the above | | cosmic rays |
| 10^17 g/m^3 | 10^44 kg/m^3 | 10^20 kg/m^3 | 10^17 kg/m^3 | | 10^17 kg/m^3 |
| 10^17 /m^3 | 10^44 /m^3 | 10^20 /m^3 | 10^17 /m^3 | | 10^44 /m^3 |
| most stable | unstable | radio active | heavy | | most stable |
| liquid drop model of the nucleus | shell model | collective model | unified model | | liquid drop model of the nucleus |
| Fermi gas model | unified model | collective model | liquid drop model | | Fermi gas model |

| | | | | | |
|---|---|---|---|---|---|
| shell model | collective model | liquid drop model | unified model | | liquid drop model |
| shell model | Fermi gas model | unified model | liquid drop model | | Fermi gas model |
| ionisation energy | binding energy | Fermi energy | packing fraction | | Fermi energy |
| 38 MeV | 83 MeV | 3.8 MeV | 38 keV | | 38 MeV |
| equal to the depth of potential well of neutron gas | less than the depth of the potential well of the neutron gas | more than the depth of the potential well of the neutron gas | can be less or more than that of neutron gas | | less than the depth of the potential well of the neutron gas |
| Rutherford | Niel Bohr | Fermi | Prout | | Fermi |
| Bohr and Kalcker | Fermi | Rutherford | Fermi | | Bohr and Kalcker |
| internal force | gravitational attraction | surface tension | repulsion | | surface tension |
| A | A^1/3 | A^2/3 | A^2 | | A^2/3 |
| surface vibration of the nuclei | surface energy of the nuclei | all the above | low lying discrete energy levels of nuclei | | low lying discrete energy levels of nuclei |
| a sphere of individual nucleons | poly-atomic molecule of alpha particles | alpha and beta particles | poly-atomic molecule of beta particles | | poly-atomic molecule of alpha particles |
| nuclei other than even-even nuclides | even-even nuclides | even-odd nuclides | odd-even nuclides | | nuclei other than even-even nuclides |
| 2n-1 | 4n-2 | 4n | 2n | | 4n |

| | | | | | |
|---|---|---|---|---|---|
| 50, 20 | 50,40 | 20, 40 | 30, 40 | | 50, 20 |
| liquid drop model | alpha particle model | collective model | Fermi gas model | | Fermi gas model |
| Fermi gas model | collective model | liquid drop model | Shell model | | liquid drop model |
| 8 MeV | 16 MeV | 38 MeV | 38 keV | | 38 MeV |
| higher level energy levels | low lying energy states | medium level energy states | all the three | | low lying energy states |
| identical for every nucleon | different for every nucleon | higher for inner nucleons and lower for surface nucleuons | lower for inner nucleons and higher for surface nucleons | | identical for every nucleon |
| compressible matter | incompressible matter | liquid matter | solid matter | | incompressible matter |

| | | | | |
|---|---|---|---|---|
| Liquid drop model could nto give atomic masses and binding energy accurately | Liquid drop model could not predict anpha and beta emission properties | Liquid drop model could give atomic masses and binding energy accurately, but could not predict alpha and beta emission properties | Liquid drop model not could give atomic masses and binding energy accurately, but also could predict alpha and beta emission properties | Liquid drop model not could give atomic masses and binding energy accurately, but also could predict alpha and beta emission properties |
| nuclear quantum numbers | isospin | magic numbers | isomers | magic numbers |
| shell model | liquid drop model | Fermi gas model | collective model | liquid drop model |
| shell model | Fermi gas model | collective model | Liquid drop model | Liquid drop model |
| radiative capture | fusion | gamma ray emission | fission | radiative capture |
| 4 | 3 | 2 | 1 | 4 |
| Collective model | Unified model | optical model | Super-conductivity model | Collective model |
| Bohr | Mottelson | Bohr and Mottelson | Rainwater | Bohr and Mottelson |

| | | | | | |
|---|---|---|---|---|---|
| Collective model | optical model | Unified model | Fermi gas model | | Unified model |
| Collective model | Liquid drop model | Optical model | unified model | | unified model |
| Nilsson | Rainwater | Davydov and Chaban | Bohr and Kalcker | | Nilsson |
| Scattering of light | Reflection | Diffraction | refraction | | Scattering of light |
| rotational | vibrational | rotational or vibrational | electronic | | rotational or vibrational |
| Liquid drop model | unified model | single particle model | Fermi gas model | | single particle model |
| three | two | seven | four | | four |
| liquid drop model of the nucleus | Shell model | Optical model | Unified model | | liquid drop model of the nucleus |
| $½^-$ | $5/2^+$ | $2^-$ | $3^-$ | | $2^-$ |
| $3/2^+$ | $3/2^-$ | $5/2^+$ | $½^-$ | | $3/2^-$ |

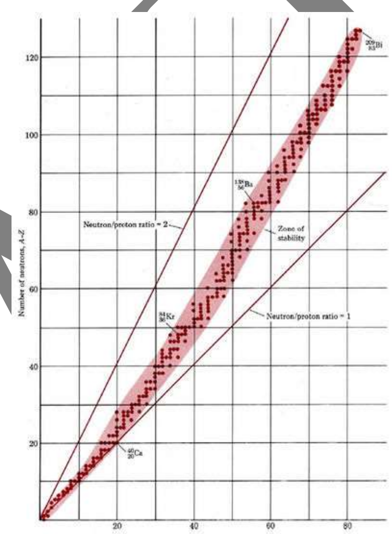
## UNIT-V

Radioactivity: stability of the nucleus; Law of radioactive decay; Mean life and half-life; Alpha decay; Beta decay- energy released, spectrum and Pauli's prediction of neutrino; Gamma ray emission, energy-momentum conservation: electron-positron pair creation by gamma photons in the vicinity of a nucleus

## STABILITY OF THE NUCLEUS

The exact nature of the forces holding the nucleons together is still only partially understood. Several factors which affect nuclear stability. The most obvious is the neutron/proton ratio.

All the stable nuclei lie within a definite area called the zone of stability. For low atomic numbers most stable nuclei have a neutron/proton ratio which is very close to 1. As the atomic number increases, the zone of stability corresponds to a gradually increasing neutron/proton ratio. In the case of the heaviest stable isotope, $Bi_{83}^{209}$ for instance, the neutron/proton ratio is 1.518. If an unstable isotope lies to the left of the zone of stability, it is neutron rich and decays by β emission. If it lies to the right of the zone, it is proton rich and decays by positron emission or electron capture.




Another factor affecting the stability of a nucleus is whether the number of protons and neutrons is even or odd. Among the 354 known stable isotopes, 157 (almost half) have an even number of protons and an even number of neutrons. Only five have an odd number of both kinds of nucleons. In the universe as a whole (with the exception of hydrogen) we find that the even-numbered elements are almost always much more abundant than the odd-numbered elements close to them in the periodic table.

Finally, there is a particular stability associated with nuclei in which either the number of protons or the number of neutrons is equal to one of the so-called "magic" numbers 2, 8, 20, 28, 50, 82, and 126. These numbers correspond to the filling of shells in the structure of the nucleus.

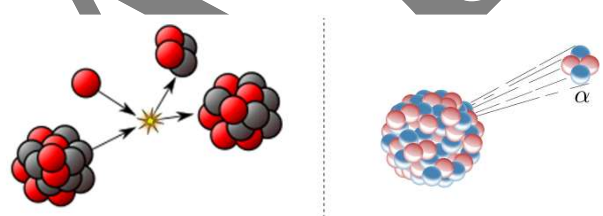
These shells are similar in principle but different in detail to those found in electronic structure. Examples are ${}^{4}_{2}He$, ${}^{16}_{8}O$, ${}^{40}_{20}Ca$, and ${}^{208}_{82}Pb$.

**RADIOACTIVITY DECAY**

Radioactivity is the phenomenon exhibited by the nuclei of an atom as a result of nuclear instability. It is a process by which the nucleus of an unstable atom loses energy by emitting radiation. Radioactivity was discovered by Henry Becquerel completely by accident. He wrapped a sample of a Uranium compound in a black paper and put it in a drawer that contained photographic plates, he found that they had already been exposed. This phenomenon was termed as Radioactive Decay. The element or isotope which emits radiation and undergoes the process of radioactivity is called Radioactive Element.

The radioisotope of an element has unstable nuclei and thus do not have sufficient binding energy to hold all the particles of an atom. To stabilize, these isotopes constantly decay. In this entire process of radioactive decay, they release a lot of energy in the form of radiations and often transform into a new element.

This process of transformation of an isotope into an element with a stable nucleus is known as Transmutation. Transmutation can occur naturally or can be done artificially.




**RADIOACTIVE DECAY LAW**

In a radioactive material, it is found that the radioactive decays per unit time are directly proportional to the total number of nuclei of radioactive compounds in the sample. Through this, we can mathematically quantify the rate of radioactive decay.

If the number of nuclei in a sample is N and the number of radioactive decays per unit time $\Delta t$ is $\Delta N$ then,

$$\Delta N \ \Delta t \propto N$$

$$\text{or } \Delta N \ \Delta t = \lambda N,$$

Where, λ is the constant of proportionality called the radioactive decay constant or disintegration constant. Also, the number of radioactive decays ΔN is reducing the total number present in the sample. Convention tells us that this should be termed negative.

$$dN/dt=-\lambda N$$

Rearranging this,

$$dN/N=-\lambda dt$$

Integration of both sides then results in,

$$\int^{N}_{N0} dN\ N=-\lambda\int^{t}_{t0} dt$$

$$\ln N-\ln N_0=-\lambda(t-t_0)$$

Here, $N_0$ represents the original number of nuclei in the sample at a time t0, i.e. t=0. Applying that in the equation results in;

$$\ln N/N_0=-\lambda t$$

This further leads to,

$$N_t=N_0^{e-\lambda t}$$

**RATE OF DECAY**

Rate here is the change per time. Calculating the rate of decay,

$$R=-dN/dt$$

Substituting N(t) in the equation and differentiating it,

$$N(t)=N_0e^{-\lambda t}$$

Differentiation result is'

$$R=-dN/dt=\lambda N_0e^{-\lambda t}$$

$$R=R_0e^{-\lambda t}$$

$R_0$ here represents the Radioactive decay rate at time, t=0.
Substituting the original equation back here,

$$\Delta N/\Delta t=\lambda N$$

We get,

$$R=\lambda N$$

The total decay rate R of a radioactive sample is called the activity of that sample. SI unit of the activity is Becquerel, in the honor of Radioactivity's discoverer, Henry Becquerel.

1 becquerel = 1 Bq = 1 decay per second

Another unit is the curie.

$$1 \text{ curie} = 1 \text{ Ci} = 3.7\times1010\text{Bq}$$

**TYPES OF RADIOACTIVE DECAY**

There are four kinds of radioactive decays namely: Alpha decay, Beta decay, and Gamma decay.

1. Alpha decay:

The process of alpha decay involves the emission of a nucleus from an alpha particle.The alpha particle is usually a nucleus of helium which is very stable. It is a group of two neutrons and two protons. The example of alpha decay involves the uranium-238 as shown below-

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$$

This process of transformation of one element into another element is known as transmutation.

2. Beta Decay:

An electron is often a beta particle, but it can also be a positron, which is a positively-charged particle. If the electrons are involved in the reaction, then the number of neutrons decreases in the nucleus one by one. Also, the proton number increases one by one. The example for the beta decay process is as shown below:

$$^{234}_{90}Th \rightarrow ^{234}_{91}Pa + ^{0}_{-1}e$$

3. Gamma Decay:

The electrons orbiting around the nucleus have energy levels, and that each time an electron moves from a high energy level to a low energy level, it emits a photon. The same thing happens in the nucleus: when it rearranges into a lower energy state, it shoots out a high-energy photon known as a gamma ray.

**Half-life**

In radioactivity, the life of a sample is measured using two different measurements.

Half-Life, T 1/2 : Half-life is the time period in which both the number of nuclei, N and the rate of decay, R have been reduced to a half of the original value. If we start with 100 nuclei, after one half-life the number left will be 50. Second, 25. Third, 12.5. Fourth, 6.25 and so on.

$$R = R_0 e^{-\lambda t}$$

At the first half-life, $R = 1/2 \times R_0$ and t = T 1/2. Substituting and solving for T 1/2.

$$T\ 1/2=\ln 2/\lambda=0.693/\lambda$$

Average or Mean life, tav: Average life refers to the average number of radioactive decays in a given span of time. Assuming that the time period is Δt then the rate R(t) is;

$$R(t)\Delta t=(\lambda N_0 e^{-\lambda t}\Delta t)$$

Though some nuclei decay in a short while, some live much longer. In order to take these decays into consideration, we integrate these from zero to infinity.

$\tau=1/\lambda$ which can then be summarized as;

$$T^{1/2}=\ln 2/\lambda=\tau \ln 2$$

Half-life doesn't mean that if there are 15 nuclei, then after one half-life there will 7.5 atoms left. Half-life just tells us the probability of the atoms decaying. The probability of a radioactive atom decaying within its half-life is 50%. But since the graph is exponential, it never really reaches zero. It approaches zero asymptotically. It just reduces to small number of atoms without ever becoming zero.

**MEAN LIFE**

**Mean life**, in radioactivity, average lifetime of all the nuclei of a particular unstable atomic species. This time interval may be thought of as the sum of the lifetimes of all the individual unstable nuclei in a sample, divided by the total number of unstable nuclei present. The mean life of a particular species of unstable nucleus is always 1.443 times longer than its half-life (time interval required for half the unstable nuclei to decay). Lead-209, for example, decays to bismuth-209 with a mean life of 4.69 hours and a half-life of 3.25 hours.

Formulas of Calculating Radioactivity Mean Life

The mean life of an element equals the half-life of the substance divided by the natural logarithm of 2 which is about 0.693. In fact, the mean life turns out to equal the number *τ which appears in the exponential term* $e^{-t/\tau}$ *involved in the description of decay or growth. It is termed as the time constant.*

Mean lifetime is a very significant quantity that can be measured directly for small number of atoms. If there are 'n' active nuclei, (atoms) (of the same type, of course), the mean life is

$$\tau = \tau$$

$$_1 + \tau_3 + \ldots + \tau_2 / n$$

Where $\tau_1$, $\tau_2$,....... $\tau_n$ represent the observed lifetime of the individual nuclei and n is a very large number. It can also be calculated as a weighted average:

$$\tau = \tau_1 N_1 + \tau_3 N_2 + ... + \tau_2 N_n / N_1 + ... + N_n$$

Where $N_1$ nuclei live for time $\tau_1$,

$N_2$ nuclei live for time $\tau_2$....... and so on.

This quantity may be related with γ. Using calculus we may rewrite it as:

$$\bar{\tau} = \frac{\int t\,|dN|}{\int |dN|}$$

Where |dN| is the number of nuclei decaying between t, t + dt; the modulus sign is required to ensure that it is positive.

$$dN = -\lambda N_0 e^{-\lambda t} dt$$

$$\text{and } |dN| = \lambda N_0 e^{-\lambda t} dt$$

$$\bar{\tau} = \frac{\int_0^\infty t \lambda N_0 e^{-\lambda t} dt}{\int_0^\infty \lambda N_0 e^{-\lambda t} dt}$$

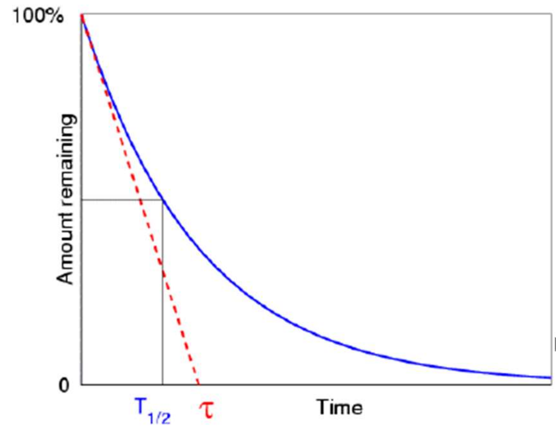
$$= \frac{\int_0^\infty t e^{-\lambda t} dt}{\int_0^\infty e^{-\lambda t} dt} = \frac{\left(\frac{1}{\lambda^2}\right) t}{\left(\frac{1}{\lambda}\right) t}$$

$$\text{or } \quad \tau = 1/\lambda.$$

The half-life and the mean life of substances are related to each other by the formula

$$T = \tau \ln 2 \approx 0.693\,\tau$$

The two parameters vary drastically for different substances. For example the half-life of Polonium-212 is less than 1 microseconds, while for Thorium-232, the half-life crosses even 1 billion years.




## GAMMA RAY EMISSION

The gamma ray is a photon of high energy, short wavelength electromagnetic nuclear radiation that is not made up matter at all. It is pure energy.

Having no mass and no charge its ionizing power is very low. As its ionizing power is so low it penetrates very deeply into matter before its energy has been used up. Its penetrating power is therefore very high (about 99.9% is absorbed by 1 km of air or 10 cm lead).

If a barrier is thick enough it will absorb most of the gamma rays that fall on it. Very few of the gamma rays emitted from the Sun reach the Earth's surface because the atmosphere is thick enough to absorb virtually all of them.

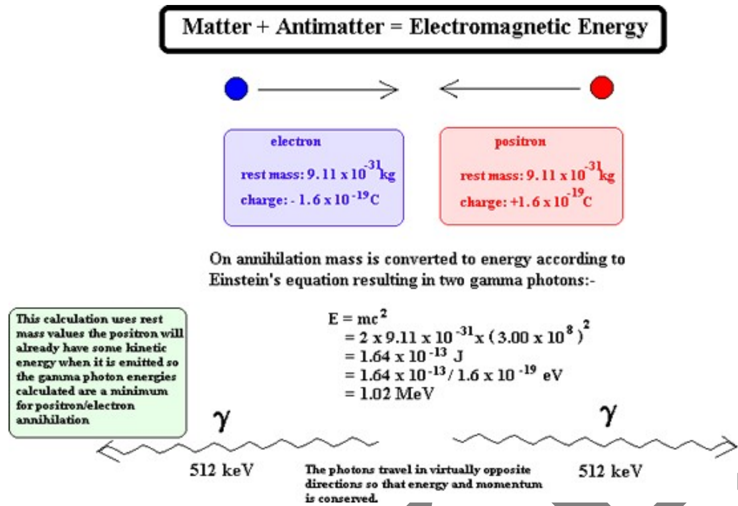
**Production of Gamma rays**

For gamma ray emission to occur the nucleus must be in an **excited state** after emitting an alpha, beta or positron particle. Sometimes it stays like that for quite a while before the gamma ray is emitted; sometimes the emission of a gamma ray is instantaneous.

Technicium 99m is a metastable form of technicium that is used widely in medical establishments.

When gamma emission occurs there is no emission of matter particles therefore the nucleon number and the proton number remain the same. The remaining nucleus is of the **same isotope** but at a **lower energy state**.

When antimatter meets with its matter counterpart annihilation occurs. Both particles disappear and two gamma rays are produced instead.




Once emitted a positron does not get very far through matter before it reaches a 'matter electron' therefore its ionizing power is great. As its ionizing power is great it cannot penetrate very far at all into matter before it is annihilated. Its penetrating power is therefore very low indeed. However on interaction with matter it produces gamma rays and these have great penetration power.

## ENERGY-MOMENTUM CONSERVATION

The investigation of the fundamental constituents of matter and their interactions comes from the experimental and theoretical analysis of reactions. These reactions can be scattering experiments with or without production of particles, and decays of the unstable particles produced in these reactions.

Various fundamental conservation laws govern nuclear reactions. The laws allow the identification of particles, i.e. the determination of their masses, spins, energies, momenta etc. The most important laws are energy-momentum conservation, angular momentum conservation and electric charge conservation. In nuclear physics, other laws play an important role such as lepton number, baryon number and isospin conservation.

### Energy-momentum conservation

By far the most important conservation law is that for Energy-momentum.

For example, in nuclear β-decay

$$(A,Z) \rightarrow (A,Z+1)e^- \bar{\nu}_e \text{ --------- } (1)$$

we require

$$E_{A,Z} = E_{A,Z+1} + E_e + E_{\bar{\nu}e}, \text{ ---------- } (2)$$

and

$$\mathbf{p}_{A,Z} = \mathbf{p}_{A,Z+1} + \mathbf{p}_e + \mathbf{p}_{\bar{\nu}e} \text{ ------------ } (3)$$

These two laws are only constraints. The way that momentum and energy are distributed between the decay products depends on the details of the interaction responsible for the reaction.

When one applies energy-momentum conservation, it is of course necessary to take into account the masses of initial and final particles by using the relativistic expression for the energy

$$E = (p^2c^2 + m^2c^4)^{1/2} \text{ -------- } (4)$$

for a free particle of mass m. The square root in this formula often makes calculations very difficult. However, in nuclear physics, nuclei and nucleons are usually non-relativistic, $v = pc^2/E << c$, and one can use the non-relativistic approximation :

$$E = \sqrt{(p^2c^2 + m^2c^4} \sim mc^2 + p^2/2m \text{ , ---------- } (5)$$

i.e. the energy is the sum of the rest energy $mc^2$ and the non-relativistic kinetic energy $p^2/2m$. On the other hand, photons and neutrinos are relativistic:

$$E = \sqrt{(p^2c^2 + m^2c^4} \sim pc + m^2c^4/2pc \text{ ------------ } (6)$$

where the mass term $m^2c^4/2pc$ can usually be neglected for neutrinos and always for the massless photon E = pc. The presence of non-relativistic and relativistic particles in a given reaction results in the very useful fact that, viewed in the center-of-mass, the momentum is shared democratically between all final state particles whereas the kinetic energy is carried mostly by the relativistic particles. This is most easily seen in the decay of an excited nucleus:

$$(A,Z)^* \rightarrow (A,Z)\ \gamma \text{ -------- } (7)$$

Energy conservation in the initial rest frame implies

$$m^*c^2 = mc^2 + p^2/2m + pc \text{ ---------- } (8)$$

where m* and m are the masses of the excited and unexcited nuclei and p is the common momentum of the final nucleus and photon. (Momentum conservation requires that these two momenta be equal.) It is clear that the photon energy pc is much greater than the nuclear kinetic energy:

$$p^2/2m = pv/2 << pc \text{ for } v << c \text{ -------- } (9)$$

Neglecting $p^2/2m$ in (8), we see that the photon energy is then to good approximation proportional to the mass difference $pc * (m^* - m)c^2$ ------------ (10)

Using this value for the momentum, we find that the ratio between the nuclear kinetic energy and the photon energy is

$$(p^2/2m)/pc = (1/2)\,(m^* - m)/m \quad \text{---------- (11)}$$

This is at most of order $10^{-3}$ in transitions between nuclear states. In more complicated reactions like three-body decays, one generally finds that the momentum is evenly distributed on average among the final-state particles. Once again, this implies that the kinetic energy is taken by the lightest particles.

In nuclear physics, one often mentions explicitly the energy balance in writing reactions

$$A + B \rightarrow a_1 + a_2 + a_n + Q \quad \text{----------- (12)}$$

where

$$Q = (\Sigma mi - \Sigma mf\,)c^2 \quad \text{-------- (13)}$$

If the reaction can take place when A and B are at rest, Q is the total kinetic energy of the particles in the final state. If Q is negative, the reaction is endothermic and it can only take place if the energy in the center-of-mass is above the energy threshold.

An important example in producing heavy elements is neutron capture accompanied by the production of k photons:

$$n + (A, Z) \rightarrow (A + 1, Z) + k\gamma + Q \quad \text{----------- (14)}$$

The fact that binding energies per nucleon are ~ 8MeV means that Q is positive and of order 8MeV (near the bottom of the stability valley). Since the final state photons are the only relativistic particles, we can expect that they take all the energy, $E\gamma \sim Q$.

Of course, some reactions involve no relativistic particles, for example

$$d\ t \rightarrow n\ {}^4He + 17.58MeV \quad \text{---------- (15)}$$

**Electron-Positron pair creation by gamma photons in the vicinity of a nucleus**

Pair production is the phenomena Here a photon is incident on the atomic nucleus, electron-positron pair is produced.It is the process to get mass from energy.

**Pair Production Process**

Pair Production is a method where energy can be converted into mass. In this process a pair of elementary particle and its antiparticle is formed by high energy photon incident on a heavy nucleus. It explains the concepts that in what way our internal world is converted into physical world that we see.

This process can be viewed in two ways :

First is as a particle and an anti particle

Second as a particle and a hole.

The basic process of pair production includes the interaction of a packet or wave of energy with a nucleus which forms an electron positron pair. The process of pair production occurs naturally a photon of energy greater than 1.02 million electron volt passes nearby the electric field of a heavy atom which has large number of protons with atomic number of about 80 to 90. If the initial energy photon has its energy greater than 1.02 eV than energy is divided into the kinetic energy of motion of the two particles.

Pair production is represented by following equation:

$$E = 2(m_o c^2) + KE(p) + KE(p')$$

This equation obeys conservation of energy.

Here,

E is the initial energy of incident photon.

$m_o c^2$ is rest energy of elementary particle which is equal to the rest energy of its anti particle.

KE (p) is the kinetic energy of elementary particle

KE (p)’ is the kinetic energy of anti particle.

**Electron Positron Pair Production**

Electron Positron Pair Production includes the formation of electron and its anti particle i.e. positron. This is formed by the interaction of high energy photon with a heavy nucleus. Photon splits into electron and positron. Both electron and its anti particle have rest mass energy equal to 0.511 million electron volts.

As mass and energy are similar or we can say equivalent mass energy is the amount of energy that it needs to convert into mass. The equivalent energy for an electron is 0.511 million electron volts (i.e. for mass of electron which is $9.11 \times\times 10^{-24}$ gms).

For electron positron pair production energy conservation equation can be written as:

$E = 2(m_o c^2) + KE(-e) + KE(+e)$

Here,

E is the initial energy of incident photon.

$m_oc^2$ is rest energy of the electron which is equal to the rest energy of its anti particle i.e. positron. Its value is 0.511 million electron volts.

KE (-e) is the kinetic energy of electron

KE (+e) is the kinetic energy of anti particle of electron.

Pair Production Cross Section

Pair production cross section is measured as Z2, Here Z is atomic number of the medium. The cross section increases highly from threshold.

**Schwinger Pair Production**

Schwinger pair production means instantaneous production of pair of charged fermion and its anti particle or we can say instantaneous pair production. This is done in the presence of strong electric field. Schwinger effect is also called vacuum pair production as it is the dielectric breakdown of vacuum.

The rate of production of pairs per unit volume is dependent on the strength of electric field applied.

$$\Gamma_s = (eE)^2/4\pi^3 \sum (n=1)\ (n-2)\ e^-\ n\pi m^2/eE$$

In above formula change in E with space and time is slow in comparison to other scales. Grapheme is used to probe the dynamics of Schwinger pair production.

Pair Production Effect

When a low energy photon interacts with a metal surface, it is completely absorbed with the emission of electron which is called Photoelectric effect. We also know that when a high energy photon such as that of X rays is scattered by an atomic electron transferring a part of its energy to the electron which is called Compton effect.

A third kind of interaction in which a very high energy photon such as that of gamma rays interact with. This is called pair production in which photon energy is changed into an electron positron pair or some another elementary particle and its anti particle pair. A positron is an elementary particle with mass and charge equal to that of electron but have a positive charge.

The creation of two particles with equal and opposite charges is essential for charge conservation in the universe. The interaction usually takes place in the electric field in the vicinity of heavy nucleus. In this case momentum and energy remains conserved.

**Muon Pair Production**

Muon Pair Production is the formation of muon and its anti particle. Muon is similar to electrons. These are also elementary particles and have a mass greater than electrons and their mass is 200 times greater. Muon with energy larger than a few giga electron volts escapes from the detector but they leave some tracks through which they can be identified. In scattering of electron and proton, muon pairs are produced. This is done by two photon interactions where the incident photons are radiated from beam particles.

When the momentum transfer to the scattering particles is less and the photons are quasi-real then the pair production cross section becomes large.

In the field of the nucleus, muon pair production on atomic electrons,

$$\gamma + e \rightarrow e + \mu^+ + \mu^-$$ Where,

e is the electron and

γ is the gamma radiation.

It has a threshold of $2m_\mu(m_\mu+m_e)/m_e \approx 43.9$ GeV. Up to some hundred GeV this process of muon pair production has a much lower cross section than the corresponding process on the nucleus. But at higher energies, the cross section on electrons represents a correctness of approximately 1Z (Z is atomic number of the nucleus) to the total cross section.

For elastic scattering as considered in case of muon pair production, Change in momentum occurs but no energy is transferred to the nucleon. The energy of photon is totally transferred to the two muons according to the following relation:

$$E_\gamma = E_{\mu+} + E_{\mu-}$$

Where,

$E_\gamma$ = Energy of the Photon,

$E_{\mu+}$ = Muon as particle,

$E_{\mu-}$ = Muon as antiparticle.

**Possible Questions**

**2 marks**

1. What is called radioactivity?
2. State law of radioactive decay.
3. Give the properties of alpha decay.
4. What is called gamma ray emission?
5. Give the properties of beta decay.
6. What is called mean life and half life?

**8 marks**

1. Give the properties of alpha and beta decay.
2. Explain gamma ray emission.
3. Briefly discuss about electron positron pair production.

KARPAGAM ACADEMY OF HIGHER EDUCATION

Coimbatore-641021.

(For the candidates admitted from 2017 onwards)

DEPARTMENT OF PHYSICS

ELEMENTS OF MODERN PHYSICS (17PHU502A)

**Unit - V**
**Question**

1 MBq is equal to:

The Half Life of 99m-Tc is 6 hours. After how much time will one eighth of the radioactivity in a sample remain?

When the Half Life increases:

The Radioactive Decay Law is expressed by:

When the Decay Constant increases:

Activity is proportional to number of

Energy given to nucleus to dismantle it increases the

Radioactivity is confined almost entirely to the elements __ to __ in the periodic table

The difference in the mass of the resultant nucleus and the sum of the masses of two parent nuclear particle is known as

When the nuclei of U235 is splitted into approximately two equal nuclei, the amount of energy released per nucleon is

As per radioactive decay law, the small amount of disintegration of the isotope in a small period is equal to

The International system of units (SI) of radioactivity activity is

The half life of radioactive nuclei is

The average (mean) life for particle decay is

The average (mean) life for particle decay is

The materials used to decelerate fast moving neutrons is called

A radioactive isotope undergoes decay with respect to time following __________ law

The half-life period of a radioactive element is 100 days. After 400 days, one gm of the element will be reduced to __________ gm.

Alpha particles have relatively

Type of rays that affect the nucleus are

Isotope A has a half-life measured in minutes, whereas isotope B has a half-life of millions of years. Which is more radioactive?

The decay rate of a radioactive isotope can be increased by increasing the

A measure of radiation that takes into account the possible biological damage produced by different types of radiation is called a

When an isotope releases gamma radiation the atomic number

The emission of a beta particle from a nucleus results in

The nucleus of the greatest stability is found in the isotope of the element

This type of radiation is released when Rn-224 decays to Po-220

In Alpha Decay

An alpha-particle consists of

What is emitted durring Beta Radiation?

Alpha particle is emitted from

The spin of an alpha particle is

Alpha particle is of ----- parity

The penetrating power of alpha particle is

Range of alpha particle

There are ---- types of beta emission
The spin of the beta particle is

When a beta particle is emitted

The alpha particles emitted by a nuclide are of

Beta decay includes

When an alpha particle is emitted, the atomic mass

When an alpha particle is emitted, its atomic number

The minimum energy required by the photon for pair production is

The intensity of the gamma-ray beam passed through thin sheet of thickness t reduces

In pair production by a photon, the particles produced are

The process in which the excited daughter nucleus, after emisison of an alpha particle, gives its energy to an orbital electron, is called

When a particle is incident on a nucleus, it can cause

The charge to mass ratio (specific charge) of an alpha particle is ------

What is the most penetrating radiation?

Which types of radiation is the most dangerous?

| Option 1 | option 2 | option 3 | option 4 | option 5 | option 6 | Answer |
|---|---|---|---|---|---|---|
| 1 billion decays per second | 37 thousand, million curies | 1 decay per second | 1 million decays per second | | | 1 million decays per second |
| 6 hours | 12 hours | 18 hours | 24 hours | | | 18 hours |
| the Decay Constant increases | the Decay Constant decreases | the Decay Constant remains unchanged | none | | | the Decay Constant decreases |
| an exponential function | a logarithmic function | a sinusoidal function | a linear function | | | an exponential function |

| | | | | |
|---|---|---|---|---|
| the Half Life decreases | the Half Life increases | the Half Life remains unchanged | none | the Half Life decreases |
| daughter nuclei | decayed nuclei | undecayed nuclei | father nuclei | undecayed nuclei |
| kinetic energy of individual nucleons | mechanical energy of individual nucleons | potential energy of individual nucleons | chemical energy of individual nucleons | potential energy of individual nucleons |
| 60, 92 | 83, 106 | 92, 118 | none | 83, 106 |
| mass defect | solid defect | weight defect | nucleus defect | mass defect |
| 0.45 MeV | 0.9 MeV | 1.35 MeV | 1.7 MeV | 0.9 MeV |
| $-\lambda N$ | $\lambda N$ | $-2\lambda N$ | $2\lambda N$ | $-\lambda N$ |
| Becquerel | Curie | Fermi | Moles | Becquerel |
| $0.693 / \lambda$ | $0.793 / \lambda$ | $0.693\lambda$ | $0.793\lambda$ | $0.693 / \lambda$ |
| 1.145 times greater than half life | 1.245 times greater than half life | 1.345 times greater than half life | 1.445 times greater than half life | 1.445 times greater than half life |
| 1.145 times greater than half life | 1.245 times greater than half life | 1.345 times greater than half life | 1.445 times greater than half life | 1.445 times greater than half life |
| coolant | moderator | controller | reactor | moderator |
| logarithmic | exponential | inverse square | linear | exponential |
| 01-Feb | 01-Apr | 0.15 | Jan-16 | Jan-16 |
| low kinetic energies | high potential energy | high mechanical energy | high kinetic energy | high kinetic energy |
| alpha | beta | gamma | EM | alpha |

| | | | | | |
|---|---|---|---|---|---|
| isotope A | isotope B | both are equally dangerous | it depends on the sample size | | isotope A |
| temperature. | pressure | sample size | none | | sample size |
| rem | rad | roentgen | curie | | rem |
| decreases by two and the mass number decreases by four | increases by one and the mass number remains the same | and the mass number decrease by one | and the mass number remain the same | | and the mass number remain the same |
| a decrease in the atomic number | an increase in the atomic number | no change in the atomic number | none | | an increase in the atomic number |
| aluminium | iron | hydrogen | lead | | iron |
| alpha | beta | gamma | all | | alpha |
| Z and A are unchanged | Z decreases by 4 and A decreases by 2 | Z decreases by 2 and A decreases by 4 | Z decreases by 4 and A decreases by 4 | | Z decreases by 2 and A decreases by 4 |
| one proton and two neutrons | two protons and one neutron | two protons and two neutrons | one proton and one neutron | | two protons and two neutrons |
| A high-energy Electron | protons | neutrons | Jimmy Nutrin | | A high-energy Electron |
| outside the nucleus | inside the nucleus | from the external orbits | inside a proton | | inside the nucleus |
| 1 | 01-Feb | 03-Feb | 0 | | 0 |
| no parity | odd | even | odd or even | | even |
| large | small | medium | zero | | small |

| | | | | |
|---|---|---|---|---|
| $R = kE^{1.5}$ | $R = kE$ | $R = kE^{2}$ | $R = kE^{0.5}$ | $R = kE^{1.5}$ |
| 2 | 1 | 3 | 4 | 3 |
| 01-Feb | 03-Feb | 1 | 0 | 01-Feb |
| proton is converted to a neutron | neutron is converted to proton | no conversion takes place | gamma ray is emitted | neutron is converted to proton |
| same energy | continuous energies | any discrete value of energy | groups of discrete energies | groups of discrete energies |
| emission of electrons only | emission of positrons only | emission of electrons and positrons | emission of electrons, emission of positrons and electron capture | emission of electrons and positrons |
| reduces by 4 | reduces by 2 | increases by 2 | remains the same | reduces by 4 |
| reduces by 4 | reduces by 2 | increases by 4 | increases by 2 | reduces by 2 |
| 100 MeV | 5 MeV | 0.51 MeV | 1.02 MeV | 1.02 MeV |
| linearly with t | linearly with t^2 | exponentially with t | exponentially with t^2 | exponentially with t |
| an electron | a positron | a proton | a pair of electron and a positron | a pair of electron and a positron |
| electron emission | internal conversion | beta decay | de-excitation | internal conversion |
| either scattering or reaction | scattering only | reaction only | both scattering and reaction | either scattering or reaction |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4.826 x 10^7 Coulomb /kg | 4.826 x 10^17 Coulomb /kg | 2.826 x 10^7 Coulomb /kg | 1.6 x 10^-17 Coulomb /kg | | | | 4.826 x 10^7 Coulomb /kg |
| gamma | alpha | beta | positron | | | | gamma |
| gamma | alpha | beta | they are equally dangerous | | | | gamma |