KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Established Under Section 3 of UGC Act 1956)
Pollachi Main Road, Eachanari (Po),
Coimbatore –641 021.
DEPARTMENT OF MATHEMATICS
SYLLABUS

Semester – V

		L	T	P	C
15MMU505A	ELECTIVE-I DISCRETE MATHEMATICS	5	0	0	5

Course Objective: On successful completion of this course the learner gain a complete knowledge about the Formal languages, Automata Theory, Lattices & Boolean Algebra and Graph Theory which plays a crucial role in the field of computers.

Course Outcomes: To enable the students to learn about the interesting branches of Mathematics such as Mathematical logic , Formal languages and Automata, Lattices and Boolean algebra, Directed and undirected graphs etc .

UNIT-I
Mathematical logic: Connections well formed formulas, Tautology, Equivalence of formulas, Tautological implications, Duality law, Normal forms, Predicates, Variables, Quantifiers, Free and bound Variables. Theory of inference for predicate calculus.

UNIT-II
Relations and functions: Composition of relations, Composition of functions, Inverse functions, one-to- one, onto, one-to-one & onto, onto functions, Hashing functions, Permutation function.

UNIT-III
Formal languages and Automata: Grammars: Phrase–structure grammar, context-sensitive grammar, context-free grammar, regular grammar. Finite state automata- Deterministic finite automata and Non deterministic finite automata-conversion of non deterministic finite automata to deterministic finite automata.

UNIT-IV
Lattices and Boolean algebra: Partial ordering, Poset, Lattices, Boolean algebra, Boolean functions, Theorems, Minimization of Boolean functions.

UNIT-V
Graph Theory: Directed and undirected graphs, Paths, Reachability, Connectedness, Matric representation, Eular paths, Hamiltonean paths, Trees, Binary trees simple theorems, and applications.

TEXT BOOK
1.Tremblay J.P., and R.P Manohar., 1975 . Discrete Mathematical Structures with applications to computer science, Tata Mc.Graw Hill, New Delhi .

REFERENCES

1. Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002. Discrete Mathematics, A.R. Publications, Nagapatinam.

2. Veerarajan T.,2007.Discrete Mathematics with graph theory and combinatorics,Tata Mc-graw hill companies,New Delhi.

3. Sharma.J.K, 2005.Discrete Mathematics, Second Edition , Macmillan India Ltd, New Delhi .

4. Discrete mathematics by Neeru Sharma, Publisher: New Delhi, India : University Science Press (An imprint of Laxmi Pubications Limited, Pvt. Ltd.), [2016] ©2011

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DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics **Subject Code: 15MMU505A**

Class: III-B.Sc.Mathematics **Semester: V**

LESSON PLAN

UNIT I

S.NO	DURATION HOURS	TOPICS TO BE COVERED	SUPPORT MATERIAL
1	1	Connectives	T1: ch 1 Pg.No: 7 -13
2	1	Well formed formulas	T1: ch 1 Pg.No: 23-26
3	1	Tautology	T1: ch 1 Pg.No: 26-28
4	1	Equivalence formulas	T1: ch 1 Pg.No: 28-31
5	1	Tautology Implication	R1: ch 2 Pg.No: 2.5-2.6
6	1	Continuation of Tautology Implication	T1:ch 1 pg.No30-32
7	1	Duality Law	T1:ch 1 pg.No30-32
8	1	Normal forms	T1: ch 1 Pg.No: 50-53 R1:ch 2 pg No: 2.7 – 2.9
9	1	Definitions – Predicates , variables	R1:ch 2 pg.No:2.14
10	1	Quantifiers	R1:ch 2.14 pg.No:2.18
11	1	Free bounded variables	T1:ch 1 pg No:86-87
12	1	Theory of Inference for predicate calculus	R1:ch 2 :pg No:2.20-2.22
13	1	Recapitulation and discussion of possible questions	
Total	**13 Hours**		

TEXT BOOK

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

REFERENCES

1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

UNIT II

S.NO	DURATION HOURS	TOPICS TO BE COVERED	SUPPORT MATERIAL
1	1	Relations	T1: ch 2 Pg.No: 149-151
2	1	Continuation of Relations	T1: ch 2 Pg.No: 151-153
3	1	Properties of binary relations in a set	T1: ch 2 Pg.No: 154 -155
4	1	Composition of Relations	T1: ch 2 Pg.No: 17 6-179
5	1	Continuation Composition of Relations	T1: ch 2 Pg.No: 17 9-182
6	1	Functions – Definition and introduction	T1: ch 2 Pg.No: 192 -194
7	1	Continuation of Functions theorems	T1: ch 2 Pg.No: 194 -197
8	1	Composition of Functions	T1: ch 2 Pg.No: 198-201
9	1	Inverse Function	T1: ch 2 Pg.No: 201-203
10	1	Continuation of Inverse Function	T1: ch 2 Pg.No: 203-206
11	1	Classification of Function	R2: ch 4 Pg.No: 184 – 186
12	1	Hashing Function	T1: ch 2 Pg.No: 212-215
13	1	Permutations	R1: ch 3 Pg.No: 3.24-3.26
14	1	Recapitulation and discussion of possible questions.	
Total	**14 Hours**		

TEXT BOOK

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

REFERENCES

1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

2. Veerarajan T., 2007.Discrete mathematics with graph theory and combinatorics, Tata Mc.Graw Hill,New Delhi.

UNIT III

S.NO	DURATION HOURS	TOPICS TO BE COVERED	SUPPORT MATERIAL
1	1	Introduction to Formal Language and Automata	R1: ch 7 Pg.No: 7.1 – 7.2
2	1	Grammar : Phrase – structure grammar	R1: ch 7 Pg.No: 7.2 – 7.4
3	1	Types of grammar	R1: ch 7 Pg.No: 7.5 – 7.6
4	1	Context – free grammar	T1: ch 3 Pg.No: 302- 303
5	1	Context sensitive grammar	T1: ch 3 Pg.No: 303- 306
6	1	Regular grammar and Examples	R1: ch 7 Pg.No: 7.5 – 7.7
7	1	Finite State Automata – Definitions	R1: ch 7 Pg.No: 7.20
8	1	Deterministic Finite Automata(DFA)- definitions and Examples	R1: ch 7 Pg.No: 7.20 – 7.26
9	1	Non- Deterministic Finite Automata(NFA) : definitions and Examples	R1: ch 7 Pg.No: 7.1 – 7.2
10	1	Conversion of NFA to DFA- Procedure	R1: ch 7 Pg.No: 7.29 – 7.32
11	1	Theorems for Finite state Automata	R1: ch 7 Pg.No: 7.32 – 7.34
12	1	Recapitulation and discussion of possible Questions.	
Total	**12 Hours**		

TEXT BOOK

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

REFERENCES

1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

UNIT IV

S.NO	DURATION HOURS	TOPICS TO BE COVERED	SUPPORT MATERIAL
1	1	Lattices as partially ordered sets	T1: ch 4 Pg.No 379 - 380
2	1	Posets – Definitions	R1: ch 6 Pg.No: 6.1 – 6.3
3	1	Least upper bound and Greatest lower bound	R1: ch 6 Pg.No: 6.3– 6.5
4	1	Lattices and properties of Lattices	R1: ch 6 Pg.No: 6.6 – 6.8
5	1	Theorems for Lattices	R1: ch 6 Pg.No: 6.8 – 6.11
6	1	Continuation of Theorems for Lattices	R1: ch 6 Pg.No: 6.11– 6.14
7	1	Boolean Algebra	R1: ch 6 Pg.No: 6.19 – 6.23
8	1	Continuation of Boolean Algebra	R1: ch 6 Pg.No: 6.24 – 6.26
9	1	Boolean Expression and Boolean Functions	R1: ch 6 Pg.No: 6.28 – 6.29
10	1	Examples –Boolean Functions	R1: ch 6 Pg.No: 6.29 – 6.33
11	1	Minimization of Boolean Functions	T1: ch 4 Pg.No 424- 426
12	1	Simplification of Boolean Functions by map method and examples	R1: ch 6 Pg.No: 6.36 – 6.39
13	1	Continuation the problems of Simplification Boolean Functions by map method	R1: ch 6 Pg.No: 6.40 – 6.42
14	1	Recapitulation and discussion of possible Questions.	
Total	**14 Hours**		

TEXT BOOK

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

REFERENCES

1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

UNIT V

S.NO	DURATION HOURS	TOPICS TO BE COVERED	SUPPORT MATERIAL
1	1	Basic concepts and Basic definitions of Graph Theory	T1: ch 4 Pg.No 469 – 470
2	1	Directed graphs	R3: ch 4 Pg.No: 243 – 248
3	1	Undirected graphs	R3: ch 4 Pg.No: 259 – 262
4		Continuation of Undirected graphs	R3: ch 4 Pg.No: 263 – 265
5	1	Walks, Paths and circuits	R1: ch 5 Pg.No: 5.5 – 5.7
6	1	Continuation of Walks, Paths and circuits	R3: ch 9 Pg.No: 263-266
7	1	Basic theorems	R1: ch 5 Pg.No: 5.7 – 5.8
8	1	Reachability	R1: ch 5 Pg.No: 5.40 – 5.41
9	1	Connectedness	T1: ch 4 Pg.No 480 – 482
10	1	Matrix representation of graphs Undirected graphs and their Matrices	R1: ch 5 Pg.No: 5.70 – 5.74
11	1	Continuation of Matrix representation of graphs Undirected graphs and their Matrices	R1: ch 5 Pg.No: 5.74 – 5.77
12	1	Directed graphs and their Matrices	R1: ch 5 Pg.No: 5.77 – 5.82
13	1	Continuation of Directed graphs and their Matrices	R1: ch 5 Pg.No: 5.82 – 5.86
14	1	Euler path	R1: ch 5 Pg.No: 5.10 – 5.11
15	1	Hamiltonian path	R3: ch 9 Pg.No: 292-295
16	1	Trees	R1: ch 5 Pg.No: 5.43 – 5.45
17	1	Continuation of the topic of Trees	R1: ch 5 Pg.No: 5.45 – 5.48
18	1	Binary trees	R1: ch 5 Pg.No: 5.56 – 5.58
19	1	Recapitulation and discussion of possible Questions.	
20	1	Discussion of previous year ESE question papers	
21	1	Discussion of previous year ESE question papers	
22	1	Discussion of previous year ESE question papers	
Total	**22 Hours**		

TEXT BOOK

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

REFERENCES

1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

3.Sharma .J.K,2005.Discrete Mathematics ,Second Edition, Macmillan India Ltd,New Delhi.

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DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics	**Semester :V**	**L T P C**
Subject Code: 15MMU505A	**Class : III- B.Sc Mathematics**	**5 0 0 5**

UNIT-I

Mathematical logic: Connections well formed formulas, Tautology, Equivalence of formulas, Tautological implications, Duality law, Normal forms, Predicates, Variables, Quantifiers, Free and bound Variables. Theory of inference for predicate calculus.

Text Book

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

References

1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

2. Veerarajan T., 2007.Discrete mathematics with graph theory and combinatorics, Tata Mc.Graw Hill,New Delhi.

3.Sharma .J.K,2005.Discrete Mathematics ,Second Edition, Macmillan India Ltd,New Delhi.

4. Discrete mathematics by Neeru Sharma, Publisher: New Delhi, India : University Science Press (An imprint of Laxmi Pubications Limited, Pvt. Ltd.), [2016] ©2011

UNIT – I

Mathematical Logic

Prepositional Logic – Definition

A proposition is a collection of declarative statements that has either a truth value "true" or a truth value "false". A propositional consists of propositional variables and connectives. We denote the propositional variables by capital letters (A, B, etc). The connectives connect the propositional variables.

Some examples of Propositions are given below:

- "Man is Mortal", it returns truth value "TRUE"
- "12 + 9 = 3 – 2", it returns truth value "FALSE"

Connectives

In propositional logic generally we use five connectives which are:

- OR (V)
- AND (Λ)
- Negation/ NOT (¬)
- Implication / if-then (→)
- If and only if (⇔).

OR (V): The OR operation of two propositions A and B (written as A V B) is true if at least any of the propositional variable A or B is true.

The truth table is as follows:

A	B	A V B
True	True	True
True	False	True
False	True	True
False	False	False

AND (∧): The AND operation of two propositions A and B (written as A ∧ B) is true if both the propositional variable A and B is true.

The truth table is as follows:

A	**B**	**A ∧ B**
True	True	True
True	False	False
False	True	False
False	False	False

Negation (¬): The negation of a proposition A (written as ¬A) is false when A is true and is true when A is false.

The truth table is as follows:

A	**¬A**
True	False
False	True

Implication / if-then (→): An implication A →B is the proposition "if A, then B". It is false if A is true and B is false. The rest cases are true.

The truth table is as follows:

A	**B**	**A → B**
True	True	True
True	False	False
False	True	True
False	False	True

If and only if (⇔): A ⇔B is bi-conditional logical connective which is true when p and q are same, i.e. both are false or both are true.

A	**B**	**A ⇔ B**
True	True	True
True	False	False
False	True	False
False	False	True

Tautologies

A Tautology is a formula which is always true for every value of its propositional variables.

Example: Prove [(A → B) ∧ A] →B is a tautology

The truth table is as follows:

A	**B**	**A → B**	**(A → B) ∧ A**	**[(A → B) ∧ A] →B**
True	True	True	True	True
True	False	False	False	True
False	True	True	False	True
False	False	True	False	True

As we can see every value of [(A → B) ∧ A] →B is "True", it is a tautology.

Contradictions

A Contradiction is a formula which is always false for every value of its propositional variables.

Example: Prove (A ∨ B) ∧ [(¬A) ∧ (¬B)] is a contradiction

The truth table is as follows:

A	B	A V B	¬A	¬B	(¬A) Λ (¬B)	(A V B) Λ [(¬A) Λ (¬B)]
True	True	True	False	False	False	False
True	False	True	False	True	False	False
False	True	True	True	False	False	False
False	False	False	True	True	True	False

As we can see every value of (A V B) Λ [(¬A) Λ (¬B)] is "False", it is a contradiction.

Contingency

A Contingency is a formula which has both some true and some false values for every value of its propositional variables.

Example: Prove (A V B) Λ (¬A) a contingency

The truth table is as follows:

A	B	A V B	¬A	(A V B) Λ (¬A)
True	True	True	False	False
True	False	True	False	False
False	True	True	True	True
False	False	False	True	False

As we can see every value of (A V B) Λ (¬A) has both "True" and "False", it is a contingency.

Propositional Equivalences

Two statements X and Y are logically equivalent if any of the following two conditions hold:

- The truth tables of each statement have the same truth values.
- The bi-conditional statement X ⇔ Y is a tautology.

Example: Prove ¬ (A V B) and [(¬A) Λ (¬B)] are equivalent

Testing by 1st method (Matching truth table):

A	B	A V B	¬ (A V B)	¬A	¬B	[(¬A) Λ (¬B)]
True	True	True	False	False	False	False
True	False	True	False	False	True	False
False	True	True	False	True	False	False
False	False	False	True	True	True	True

Here, we can see the truth values of ¬ (A V B) and [(¬A) Λ (¬B)] are same, hence the statements are equivalent.

Testing by 2nd method (Bi-conditionality):

A	B	¬ (A V B)	[(¬A) Λ (¬B)]	[¬ (A V B)] ⇔[(¬A) Λ (¬B)]
True	True	False	False	True
True	False	False	False	True
False	True	False	False	True
False	False	True	True	True

As [¬ (A V B)] ⇔ [(¬A) Λ (¬B)] is a tautology, the statements are equivalent.

Inverse, Converse, and Contra-positive

Implication / if-then (→) is also called a conditional statement. It has two parts-

- Hypothesis , p
- Conclusion , q

As mentioned earlier, it is denoted as p → q.

Inverse: An inverse of the conditional statement is the negation of both the hypothesis and the conclusion. If the statement is "If p, then q", the inverse will be "If not *p*, then not *q*". Thus the inverse of $p \to q$ is $\neg p \to \neg q$.

Example : The inverse of "If you do your homework, you will not be punished" is "If you do not do your homework, you will be punished."

Converse: The converse of the conditional statement is computed by interchanging the hypothesis and the conclusion. If the statement is "If p, then q", the converse will be "If *q*, then *p*". The converse of $p \to q$ is $q \to p$.

Example : The converse of "If you do your homework, you will not be punished" is "If you will not be punished, you do not do your homework".

Contra-positive: The contra-positive of the conditional is computed by interchanging the hypothesis and the conclusion of the inverse statement. If the statement is "If p, then q", the contra-positive will be "If not *q*, then not *p*". The contra-positive of $p \to q$ is $\neg q \to \neg p$.

Example : The Contra-positive of " If you do your homework, you will not be punished" is "If you are not punished, then you do not do your homework".

Duality Principle

Duality principle states that for any true statement, the dual statement obtained by interchanging unions into intersections (and vice versa) and interchanging Universal set into Null set (and vice versa) is also true. If dual of any statement is the statement itself, it is said **self-dual** statement.

Example: The dual of $(A \cap B) \cup C$ is $(A \cup B) \cap C$

Normal Forms

We can convert any proposition in two normal forms:

- Conjunctive normal form

Conjunctive Normal Form

A compound statement is in conjunctive normal form if it is obtained by operating AND among variables (negation of variables included) connected with ORs. In terms of set operations, it is a compound statement obtained by Intersection among variables connected with Unions.

Examples

- $(A \vee B) \wedge (A \vee C) \wedge (B \vee C \vee D)$
- $(P \cup Q) \cap (Q \cup R)$

Disjunctive Normal Form

A compound statement is in conjunctive normal form if it is obtained by operating OR among variables (negation of variables included) connected with ANDs. In terms of set operations, it is a compound statement obtained by Union among variables connected with Intersections.

Examples

- $(A \wedge B) \vee (A \wedge C) \vee (B \wedge C \wedge D)$
- $(P \cap Q) \cup (Q \cap R)$

Predicate Logic – Definition

A predicate is an expression of one or more variables defined on some specific domain. A predicate with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.

The following are some examples of predicates:

- Let E(x, y) denote "x = y"
- Let X(a , b, c) denote "a + b + c = 0"
- Let M(x, y) denote "x is married to y"

Well Formed Formula

Well Formed Formula (wff) is a predicate holding any of the following -

- All propositional constants and propositional variables are wffs
- If x is a variable and Y is a wff, $\forall x\, Y$ and $\exists x\, Y$ are also wff
- Truth value and false values are wffs
- Each atomic formula is a wff
- All connectives connecting wffs are wffs

Quantifiers

The variable of predicates is quantified by quantifiers. There are two types of quantifier in predicate logic: Universal Quantifier and Existential Quantifier.

Universal Quantifier

Universal quantifier states that the statements within its scope are true for every value of the specific variable. It is denoted by the symbol $\forall$.

$\forall x\ P(x)$ is read as for every value of x, P(x) is true.

Example: "Man is mortal" can be transformed into the propositional form $\forall x\ P(x)$ where P(x) is the predicate which denotes x is mortal and the universe of discourse is all men.

Existential Quantifier

Existential quantifier states that the statements within its scope are true for some values of the specific variable. It is denoted by the symbol $\exists$.

$\exists x\ P(x)$ is read as for some values of x, P(x) is true.

Example: "Some people are dishonest" can be transformed into the propositional form $\exists x\ P(x)$ where P(x) is the predicate which denotes x is dishonest and the universe of discourse is some people.

Nested Quantifiers

If we use a quantifier that appears within the scope of another quantifier, it is called nested quantifier.

Examples

- $\forall a\ \exists b\ P(x, y)$ where $P(a, b)$ denotes $a + b=0$
- $\forall a\ \forall b\ \forall c\ P(a, b, c)$ where $P(a, b)$ denotes $a + (b+c) = (a+b) +c$

Note: $\forall a\ \exists b\ P(x, y) \neq \exists a\ \forall b\ P(x, y)$

Inference

Table of Rules of Inference

Rule of Inference	Name	Rule of Inference	Name
P ---------- ∴ P ∨ Q	Addition	P ∨ Q ¬P ---------- ∴ Q	Disjunctive Syllogism
P Q ---------- ∴ P ∧ Q	Conjunction	P → Q Q → R ---------- ∴ P → R	Hypothetical Syllogism
P ∧ Q ---------- ∴ P	Simplification	(P → Q) ∧ (R → S) P ∨ R ---------- ∴ Q ∨ S	Constructive Dilemma
P→Q P ---------- ∴ Q	Modus ponens	(P → Q) ∧ (R → S) ¬Q ∨ ¬S ---------- ∴ ¬P ∨ ¬R	Destructive Dilemma

Part -B (5x8=40 Marks)

Possible Questions:

1. i) Verify that a proposition $P \vee \rceil(P \wedge Q)$ is a tautology.
   ii) Prove that $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

2. Show that the following premises are Inconsistent.
   i) If Jack misses many classes through illness, he fails in school.
   ii) If jack fails in school, then he is uneducated.
   iii) If jack reads a lot of books, then he is not uneducated.
   iv) Jack misses many classes through illness and reads a lot of books.

3. i) Prove that $(\rceil Q \wedge P) \wedge Q$ is contradiction.
   ii) Show that the following implication without constructing truth table
   $\rceil Q \wedge (P \rightarrow Q) \Rightarrow \rceil P$

4. Find the min term normal form of $(\rceil((P \vee Q) \wedge R)) \wedge (P \vee R)$

5. i) Construct the truth table for $(P \leftrightarrow R) \wedge (\rceil Q \rightarrow S)$
   ii) Obtain PDNF of $(\rceil((P \vee Q) \wedge R)) \wedge (P \vee R))$

6. Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \rightarrow \rceil H$, $\rceil H \rightarrow (A \wedge \rceil B)$ and $(A \wedge \rceil B) \rightarrow (R \vee S)$.

7. Define disjunctive normal form and conjunctive normal form. Also obtain disjunctive normal form of $\rceil(P \vee Q) \leftrightarrow (P \wedge Q)$

8. Prove that $(P \vee Q) \wedge \rceil(\rceil P \wedge (\rceil Q \vee \rceil R) \vee (\rceil P \wedge \rceil Q) \vee (\rceil P \wedge \rceil R))$ is a tautology.

9. Obtain PCNF and PDNF of $(P \wedge Q) \vee (\rceil P \wedge Q \wedge R)$

10. i) Prove that $R \vee S$ follows logically from the premises
    $C \vee D$, $(C \vee D) \rightarrow \rceil H$, $\rceil H \rightarrow (A \wedge \rceil B)$ and $(A \wedge \rceil B) \rightarrow (R \vee S)$.
    ii) Show that (x) M(x) follows logically from the premises (x)(H(x) $\rightarrow$ M(x)) and (x)H(x).

s

UNIT I

Part A (20x1=20 Marks)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
UNIT I					
Let p be “He is tall” and let q “He is handsome”. Then the statement “It is false that he is short or handsome” is:	p^q	~(~pv q)	~pv q	p v q	~(~pv q)
The proposition p^ (~ p v q) is……………………….	A tautulogy	a contradiction	Logically equivalent to p ^q	an assumption	Logically equivalent to p ^ q
Which of the following is/are tautology?	a v b → b ^ c	a ^ b → b v c	a v b → (b → c)	a v b → b v c	a ^ b → b v c
Identify the valid conclusion from the premises Pv Q, Q → R, P → M, ꞁM	P ^ (R v R)	P ^ (P ^ R)	R ^ (P v Q)	Q ^ (P v R)	Q ^ (P v R)
Let a, b, c, d be propositions. Assume that the equivalence a ↔ (b v ꞁb) and b ↔ c hold. Then truth value of the formula (a ^ b) → ((a ^ c) v d) is always	TRUE	FALSE	Same as the truth value of a	Same as the truth value of b	TRUE
Which of the following is a declarative statement?	It's right	He says	Two may not be an even integer	I love you	He says
P → (Q → R) is equivalent to	(P ^ Q) → R	(P v Q) → R	(P v Q) → ꞁR	(P v Q) → P	(P ^ Q) → R
If F1, F2 and F3 are propositional formulae such that F1 ^ F2 → F3 and F1 ^ F2→F3 are both tautologies, then which of the following is TRUE?	Both F1 and F2 are tautologies	The conjuction F1 ^ F2 is not satisfiable	Neither is tautologies	F1v F2 is tautology	Both F1 and F2 are tautologies
Consider two well-formed formulas in propositional logic F1 : P →ꞁP F2 : (P →ꞁP) v (ꞁP →), then	F1 is satisfiable, F2 is unsatisfiable	F1 is unsatisfiable, F2 is satisfiable	F1 is unsatisfiable, F2 is valid	F1 & F2 are both satisfiable	F1 is unsatisfiable, F2 is valid
What can we correctly say about proposition P1 : (p v ꞁq) ^ (q →r) v (r v p)	P1 is tautology	P1 is satisfiable	If p is true and q is false and r is false, the P1 is true	If p as true and q is true and r is false, then P1 is true	If p is true and q is false and r is false, the P1 is true
(P v Q) ^ (P → R)^ (Q →S) is equivalent to	S ^ R	S → R	S v R	S U R	S v R
In propositional logic , which of the following is equivalent to p → q?	~p → q	~p v q	~p v~ q	p →q	~p v q
ꞁ(P → Q) is equivalent to	P ^ ꞁQ	P ^ Q	ꞁP v Q	ꞁP ^ Q	P ^ ꞁQ
(P v Q) ^ (P→R) ^ (Q → R) is equivalent to	P	Q	R	True=T	R
How ∧ many ∨ rows would be in the truth table for the following compound proposition: (p q) ¬(q t) (r → s)	32	34	27	25	32
Which of the following statement is the negation of the statement,“2 is even and –3 is negative”?	2 is even and –3 is not negative.	2 is odd and –3 is not negative.	2 is even or –3 is not negative.	2 is odd or –3 is not negative.	2 is odd or –3 is not negative.
p→q is logically equivalent to	~ q→p	~ p→q	~ p ^ q	~ p v q	~ p v q
Which of the following is not a well formed formula?	"for all x [P(x)→f (x)^ x]	for all x1,x2,x3 { x1 = x2 ^ x2 = x3 Þ x1 = x3}	~ (p →q)→q	[T v P(a, b)]→zQ(z)	for all x1,x2,x3 { x1 = x2 ^ x2 = x3 Þ x1 = x3}
[~ q ^ (p→q)]→~ p is,	Satisfiable	Unsatisfiable	Tautology	Invalid	Tautology
An and statement is true if, and only if, both components are ……………..	TRUE	FALSE	not true	neither true nor false	TRUE
If P : It is hot & Q : It is humid,then what does P ^ (~ Q):mean?	It is not hot and it is not humid	It is hot and it is humid	It is hot and it is not humid	It is not hot and it is not humid	It is hot and it is not humid
An or statement is false if, and only if, both components are ………………..	TRUE	FALSE	not true	neither true nor false	FALSE
Two statement forms are logically equivalent if, and only if they always have………………………….	not same truth values	the same truth values	the different truth values	the same false values	the same truth values
A tautology is a statement that is always ………………….	TRUE	FALSE	not true	neither true nor false	TRUE
A contradiction is a statement that is always ………………….	FALSE	TRUE	not true	neither true nor false	FALSE
The statement (p^q) Þ p is a…………………	Satisfiable	Unsatisfiable	Tautology	Invalid	Tautology
In propositional logic which one of the following is equivalent to p→q ?	p→q	p→q	p v q	p v q̶	p̶ v q
Which of the following proposition is a tautology?	(p v q)→p	p v (q→p)	p v(p→q)	(p v q)→q	p v(p→q)
Which one is the contrapositive of q → p ?	~p → ~ q	p → ~ q	~p → q	p → q	~p → ~ q
The statement form pv(~p) is a……………………..	Satisfiable	Unsatisfiable	Tautology	Invalid	Tautology

Let p and q be statements given by “p →q". Then q is called	hypothesis	conclusion	TRUE	FALSE	conclusion
The statement form p^(~p) is a……………………..	contradiction	Unsatisfiable	Tautology	Invalid	contradiction
If p and q are statement variables, the conditional of q by p is given by ……………..	~p → ~ q	p → ~ q	~p → q	p → q	p → q
Let p and q be statements given by “p →q". Then p is called………………	hypothesis	conclusion	TRUE	FALSE	hypothesis
The statement (p → r) (q → r) is equivalent to…………	p q →~ r	p q → r	p ~ q → r	~p q → r	p q → r
The Negation of a Conditional Statement p →q is given by ………………..	p q	~p q	p V q	p q	p q
Given statement variables p and q, the biconditional of p and q is given by …………	p«~q	p→q	~p«q	p«q	p«q
The inverse of “if p then q” is …………………………	if p then q	if p then q	if p then q	if p then q	if p then q
“R is a………………………. condition for S” means “if R then S .”	valid	inevitable	sufficient	necessary	sufficient
A conditional statement and its contrapositive are …………………	A tautulogy	a contradiction	Logically equivalent	an assumption	Logically equivalent
A rule of inference is a form of argument that is …………….	valid	a contradiction	an assumption	A tautulogy	valid

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DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics	**Semester :V**	**L T P C**
Subject Code: 15MMU505A	**Class : III- B.Sc Mathematics**	**5 0 0**

UNIT-II

Relations and functions: Composition of relations, Composition of functions, Inverse functions, one-to- one, onto, one-to-one & onto, onto functions, Hashing functions, Permutation function.

Text Book

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

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1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

2. Veerarajan T., 2007.Discrete mathematics with graph theory and combinatorics, Tata Mc.Graw Hill,New Delhi.

3.Sharma .J.K,2005.Discrete Mathematics ,Second Edition, Macmillan India Ltd,New Delhi.

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UNIT II

RELATIONS AND FUNCTIONS

Definition and Properties

A binary relation R from set x to y (written as xRy or R(x,y)) is a subset of the Cartesian product x × y. If the ordered pair of G is reversed, the relation also changes.

Domain and Range

If there are two sets A and B, and relation R have order pair (x, y), then:

- The **domain** of R, Dom(R), is the set { x | (x, y) ∈ R for some y in B }
- The **range** of R, Ran(R), is the set { y | (x, y) ∈ R for some x in A }

Examples

Let, A = {1,2,9} and B = {1,3,7}

- Case 1: If relation R is 'equal to' then R = {(1, 1), (3, 3)}
  Dom(R) = { 1, 3}, Ran(R) = { 1, 3}
- Case 2: If relation R is 'less than' then R = {(1, 3), (1, 7), (2, 3), (2, 7)}
  Dom(R) = { 1, 2}, Ran(R) = { 3, 7}
- Case 3: If relation R is 'greater than' then R = {(2, 1), (9, 1), (9, 3), (9, 7)}
  Dom(R) = { 2, 9}, Ran(R) = { 1, 3, 7}

Types of Relations

1. The **Empty Relation** between sets X and Y, or on E, is the empty set ∅
2. The **Full Relation** between sets X and Y is the set X×Y
3. The **Identity Relation** on set X is the set {(x,x) | x ∈ X}
4. The Inverse Relation R' of a relation R is defined as: R'= {(b,a) | (a,b) ∈R}

   Example: If R = {(1, 2), (2,3)} then R' will be {(2,1), (3,2)}

5. A relation R on set A is called **Reflexive** if ∀a∈A is related to a (aRa holds).

   Example: The relation R = {(a,a), (b,b)} on set X={a,b} is reflexive

6. A relation R on set A is called **Irreflexive** if no a∈A is related to a (aRa does not hold).

   Example: The relation R = {(a,b), (b,a)} on set X={a,b} is irreflexive

7. A relation R on set A is called **Symmetric** if xRy implies yRx, ∀x∈A and ∀y∈A.

   Example: The relation R = {(1, 2), (2, 1), (3, 2), (2, 3)} on set A={1, 2, 3} is symmetric.

8. A relation R on set A is called **Anti-Symmetric** if xRy and yRx implies x=y ∀x ∈ A and ∀y ∈ A.

   Example: The relation R = { (x,y) ∈ N | x ≤ y } is anti-symmetric since x ≤ y and y ≤ x implies x = y.

9. A relation R on set A is called **Transitive** if xRy and yRz implies xRz, ∀x,y,z ∈ A.

   Example: The relation R = {(1, 2), (2, 3), (1, 3)} on set A= {1, 2, 3} is transitive.

10. A relation is an **Equivalence Relation** if it is reflexive, symmetric, and transitive.

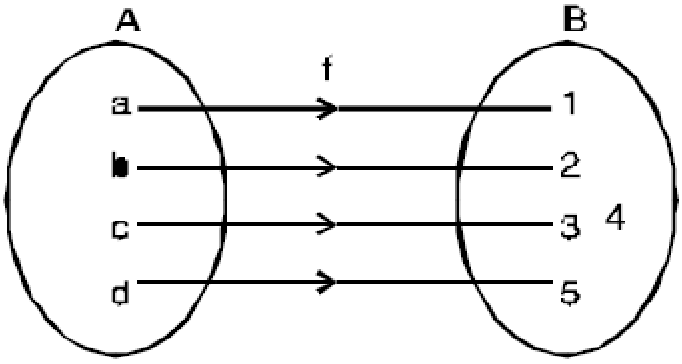
Example: The relation R = {(1, 1), (2, 2), (3, 3), (1, 2),(2,1), (2,3), (3,2), (1,3), (3,1)} on set A= {1, 2, 3} is an equivalence relation since it is reflexive, symmetric, and transitive.

Function – Definition

A function or mapping (Defined as f: X→Y) is a relationship from elements of one set X to elements of another set Y (X and Y are non-empty sets). X is called Domain and Y is called Codomain of function 'f'.

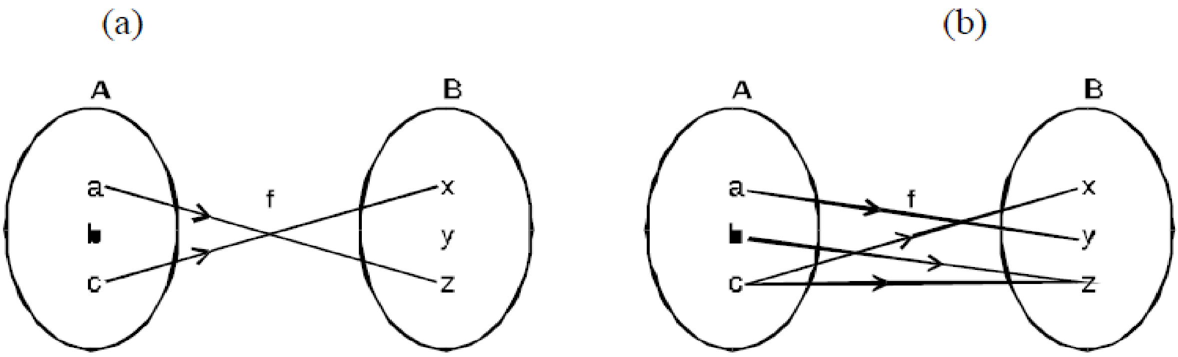
Function 'f' is a relation on X and Y such that for each x ∈ X, there exists a unique y ∈ Y such that (x,y) ∈ R. 'x' is called pre-image and 'y' is called image of function f.

A function can be one to one or many to one but not one to many.

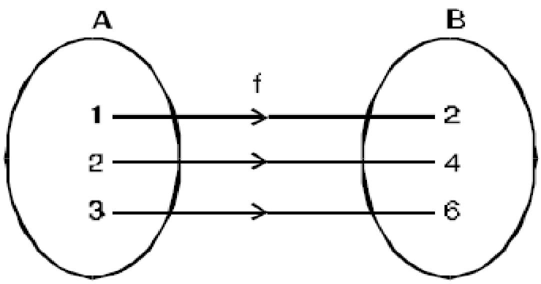


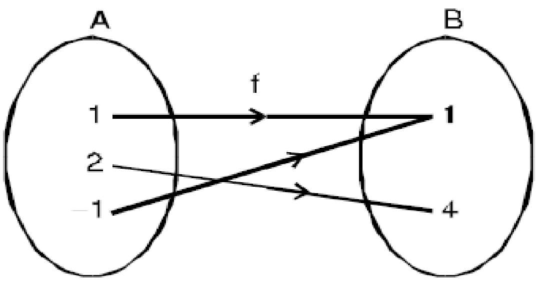
Example

State whether each of the following relations represent a function or not.



(c) (d)





Solution :

(a) *f* is not a function because the element *b* of *A* does not have an image in *B*.

(b) *f* is not a function because the element *c* of *A* does not have a unique image in *B*.

(c) *f* is a function because every element of *A* has a unique image in *B*.

(d) *f* is a function because every element in *A* has a unique image in *B*.

Injective / One-to-one function

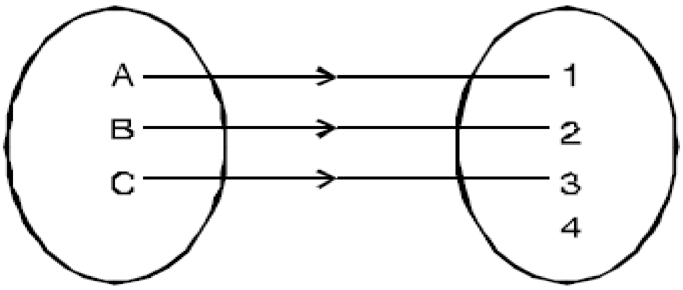
A function f: A→B is injective or one-to-one function if for every b ∈ B, there exists at most one a ∈ A such that f(s) = t.

This means a function **f** is injective if $a_1 \neq a_2$ implies $f(a_1) \neq f(a_2)$.

Example

1. f: N →N, f(x) = 5x is injective.
2. f: N→N, f(x) = x^2 is injective.
3. f: R→R, f(x) = x^2 is not injective as $(-x)^2 = x^2$

One-to-one function



The domain is { A , B , C}

The co-domain is {1, 2 , 3 , 4}

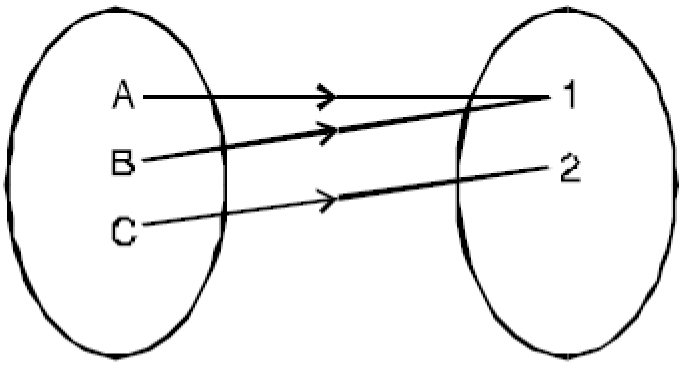
The range is {1 , 2 , 3}

Surjective / Onto function

A function f: A →B is surjective (onto) if the image of f equals its range. Equivalently, for every b ∈ B, there exists some a ∈ A such that f(a) = b. This means that for any y in B, there exists some x in A such that y = f(x).

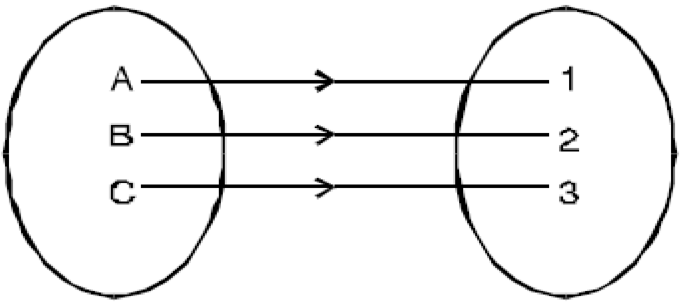
Example

1. f : N→N, f(x) = x + 2 is surjective.
2. f : R→R, $f(x) = x^2$ is not surjective since we cannot find a real number whose square is negative.



Bijective / One-to-one Correspondent

A function f: A →B is bijective or one-to-one correspondent if and only if **f** is both injective and surjective.



Problem:

Prove that a function f: R→R defined by f(x) = 2x – 3 is a bijective function.

Explanation: We have to prove this function is both injective and surjective.

If $f(x_1) = f(x_2)$, then $2x_1 - 3 = 2x_2 - 3$ and it implies that $x_1 = x_2$.

Hence, f is **injective**.

Here, 2x – 3= y

So, x = (y+5)/3 which belongs to R and f(x) = y.

Hence, f is **surjective**.

Since **f** is both **surjective** and **injective**, we can say **f** is **bijective**.

Inverse of a Function

The **inverse** of a one-to-one corresponding function f : A → B, is the function g : B → A, holding the following property:

f(x) = y ⇔ g(y) = x

The function f is called **invertible**, if its inverse function g exists.

Example:

- A function f : Z → Z, f(x) = x + 5, is invertible since it has the inverse function g : Z → Z, g(x) = x – 5
- A function f : Z→Z, $f(x) = x^2$ is not invertible since this is not one-to-one as $(-x)^2 = x^2$.

Composition of Functions

Two functions f: A→B and g: B→C can be composed to give a composition g o f. This is a function from A to C defined by (gof)(x) = g(f(x))

Example

Let f(x) = x + 2 and g(x) = 2x + 1, find (f o g)(x) and (g o f)(x)

Solution

(f o g)(x) = f (g(x)) = f(2x + 1) = 2x + 1 + 2 = 2x + 3

(g o f)(x) = g (f(x)) = g(x + 2) = 2 (x+2) + 1 = 2x + 5

Hence, (f o g)(x) ≠ (g o f)(x)

Some Facts about Composition

- If f and g are one-to-one then the function (g o f) is also one-to-one.
- If f and g are onto then the function (g o f) is also onto.

Monotonic Function

Let $F : A \to B$ be a function then F is said to be monotonic on an interval (a,b) if it is either increasing or decreasing on that interval.

For function to be increasing on an interval (a,b)

$$x_1 < x_2 \quad \Rightarrow F(x_1) < F(x_2) \quad \forall\, x_1\, x_2 \in (a, b)$$

and for function to be decreasing on (a,b)

$$x_1 < x_2 \quad \Rightarrow F(x_1) > F(x_2) \quad \forall\, x_1\, x_2 \in (a, b)$$

A function may not be monotonic on the whole domain but it can be on different intervals of the domain.

Consider the function $F : R \to R$ defined by $f(x) = x^2$.

Now $\forall \; x_1, x_2 \in [0, \infty]$

$$x_1 < x_2 \Rightarrow F(x_1) < F(x_2)$$

$\Rightarrow$ F is a ***Monotonic Function*** on $[0, \infty]$.

($\because$ It is only increasing function on this interval)

But $\forall\ x_1, x_2 \in (-\infty, 0)$

$x_1 < x_2 \Rightarrow F(x_1) > F(x_2)$

$\Rightarrow$ F is a ***Monotonic Function*** on $[-\infty, 0]$

($\because$ It is only a decreasing function on this interval)

Therefore if we talk of the whole domain given function is not monotonic on R but it is monotonic on $(-\infty, 0)$ and $(0, \infty)$.

Again consider the function $F : R \to R$ defined by $f(x) = x^3$.

Clearly $\forall\ x_1\ x_2 \in$ domain

$x_1 < x_2 \Rightarrow F(x_1) < F(x_2)$

$\therefore$ Given function is ***monotonic*** on R i.e. on the whole domain.

EVEN FUNCTION

A function is said to be an even function if for each x of domain

$F(-x) = F(x)$

For example, each of the following is an ***even function***.

(i) If $F(x) = x^2$ then $F(-x) = (-x)^2 = x^2 = F(x)$

ODD FUNCTION

A function is said to be an odd function if for each x

$f(-x) = -f(x)$

For example,

(i) If $f(x) = x^3$

then $f(-x) = (-x)^3 = -x^3 = -f(x)$

Polynomial Function

Any function defined in the form of a polynomial is called a polynomial function.

For example,

(i) $f(x) = 3x^2 - 4x - 2$

(ii) $f(x) = x^3 - 5x^2 - x + 5$

Rational Function

Function of the type $f(x) = \frac{g(x)}{h(x)}$, where $h(x) \neq 0$ and $g(x)$ and $h(x)$ are polynomial functions are called rational functions.

For example, $f(x) = \frac{x^2 - 4}{x + 1}, x \neq -1$

is a rational function.

Reciprocal Function

Functions of the type $y = \frac{1}{x}, x \neq 0$ is called a reciprocal function.

Exponential Functions

A swiss mathematician Leonhard Euler introduced a number e in the form of an infinite series. In fact

$$e = 1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \ldots + \frac{1}{\lfloor n} + \ldots \qquad \ldots\ldots(1)$$

It is well known that the sum of its infinite series tends to a finite limit (i.e., this series is convergent) and hence it is a positive real number denoted by e. This number e is a transcendental irrational number and its value lies between 2 an 3.

Consider now the infinite series

$$1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots + \frac{x^n}{\lfloor n} + \ldots$$

It can be shown that the sum of its infinite series also tends to a finite limit, which we denote by e^x. Thus,

$$e^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \ldots + \frac{x^n}{\lfloor n} + \ldots \qquad \ldots\ldots(2)$$

This is called the **Exponential Theorem** and the infinite series is called the **exponential series**.

We easily see that we would get (1) by putting x = 1 in (2).

The function $f(x) = e^x$, where x is any real number is called an **Exponential Function**.

The graph of the exponential function

$$y = e^x$$

Logarithmic Functions

Consider now the function

$$y = e^x$$

We write it equivalently as

$$x = \log_e y$$

Thus, $y = \log_e x$

is the inverse function of $y = e^x$

The base of the logarithm is not written if it is e and so $\log_e x$ is usually written as log x.

As $y = e^x$ and $y = \log x$ are inverse functions, their graphs are also symmetric w.r.t. the line $y = x$

The graph of the function $y = \log x$ can be obtained from that of $y = e^x$ by reflecting it in the line y = x.

Part -B (5x8=40 Marks)

Possible Questions:

1. Explain about types of relation with examples.

2. Let A={1,2,3} and f,g,h and s be functions from A to A given by
   f ={ (1,2), (2,3),(3,1) } ; g = { (1,2), (2,1), (3,3) } ;
   h = { (1,1), (2,2), (3,1) } and s = { (1,1), (2,2), (3,3) }.
   Find $f_o g$, $g_o f$, $f_o h_o g$, $g_o s$, $s_o s$, $f_o s$.

3. Write about the types of function with example.

4. If f : A→B and g : B→C are one – one function then prove that $g_o f$: A→C is also 1-1.

5. Let R denotes a relation on the set of all ordered pairs of positive integers by (x, y) R (u, v) iff xv=yu. Show that R is a equivalence relations.

6. Let S={1,2,3,4,5} and T={1,2,3,8,9} and define the functions f : S→ T and g: S → S by f={(1,8), (3,9),(4,3),(2,1),(5,2)} and g={(1,2),(3,1),(2,2),(4,3),(5,2)} ,then find the values of the following $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$.

7. For integers a,b define aRb if and only if a – b is divisible by m. Show that R defines an equivalence relationon Z.

8. Let A be the set A={xϵR \ x>0} and define f ,g, h :A→ R by $f(x)=\frac{x}{x+1}$,$g(x)=\frac{1}{x}$,h(x)=x+1 find $g \circ f$,$f \circ g$,$h \circ g \circ f$ and $f \circ g \circ h$.

9. If R and S are equivalence relations defined on a set S,Prove that R∩S is an equivalence relation.

10. Show that the following functions are 1-1
    i) f : R → R given by $f(x)= 5x^2 - 1$
    ii) f: Z → Egiven by $f(n)=3x^3 – x$

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DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics **Subject Code: 15MMU505A**
Class : III-B.Sc Mathematics **Semester : V**

UNIT II

Part A (20x1=20 Marks)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
If R= {(1,2),(3,4),(2,2)} and S = {(4,2),(2,5),(3,1),(1,3)} are relations then RoS = ------	{(4,2),(3,2),(1,4)}	{(1,5),(3,2),(2,5)}	{(1,2),(2,2)}	{(4,5),(3,3),(1,1)}	{(1,5),(3,2),(2,5)}
If f(x) = x+2 and g(x) = x^2 –1 then(gof)(x) = ------	x^2 +4x+4	x^2 +4x-3	x^2 -4x+4	x^2 +4x+3	x^2 +4x+3
A relation R in a set X is ---------- if for every $x\in X,(x,x)\notin R$	transitive	symmetric	irreflexive	reflexive	irreflexive
Suppose in RxR, the ordered pairs (x-2, 2y+1) and (y-1, x+2) are equal. The values of x and y are	2,3	3,2	2,-3	3,-2	3,2
A relation R on a set is said to be an equivalence relation if it is ------	Reflexive	Symmetric	Reflexive,Symmetric, Transitive	Transitive	Reflexive,Symmetric, Transitive
Let f : R→R where R is a set of real numbers.Then f(x) = -2x is a -----	One-to-one	Onto	into	bijection	bijection
A mapping f : x→y is called ------ if distinct elements of x are mapped into distinct elements	one-to-one	Onto	into	many to one	one-to-one
If the relation R and S are both reflexive then R ∨ S is ---------	symmetric	reflexive	transitive	not reflexive	reflexive
A One – to –one function is also known as ---------	injective	surjective	bijective	objective	injective
A On to function is also known as ---------	injective	surjective	bijective	objective	surjective
A One – to –one and onto function is also known as ---------	injective	surjective	bijective	objective	bijective
Let f : x→y , g : y→x be the functions then g is equal to f^{-1} only if --------	fog = Iy	gof = I_x	gof=I_y	fog=I_x	gof = I_x
In N, define aRb if a+b = 7. This is symmetric when --------	b+a =7	a+a =7	b+c =7	a + c = 7	b+a =7
If the relation is ---------- relation if aRb,bRa →a = b -------	symmetric	reflexive	Antisymmetric	not reflexive	Antisymmetric
f : R→R, g : R→R defined by f(x) = 4x-1 and g(x) = cos x..The value of fog is ------	4cosx –1	4cosx	4cosx +1	1/4cosx	4cosx –1
Let f : N→N be a function such that f(x) = 5 ,x∈N then the f(x) is called---------function	identity	inverse	equal	constant	constant
A binary relation R in a set X is said to be symmetric if ------	aRa	aRb⇒bRa	aRb,bRc⇒aRc	aRb,bRa⇒a=b	aRb⇒bRa
A binary relation R in a set X is said to be reflexive if ------	aRa	aRb⇒bRa	aRb,bRc⇒aRc	aRb,bRa⇒a=b	aRa
A binary relation R in a set X is said to be antisymmetric if ------	aRa	aRb⇒bRa	aRb,bRc⇒aRc	aRb,bRa⇒a=b	aRb,bRa⇒a=b
A binary relation R in a set X is said to be transitive if ----	aRa	aRb⇒bRa	aRb,bRc⇒aRc	aRb,bRa⇒a=b	aRb,bRc⇒aRc
If R= {(1,2),(3,4),(2,2)} and S = {(4,2),(2,5),(3,1),(1,3)} are relations then SoS = ------	{(4,2),(3,2),(1,4)}	{(1,5),(3,2),(2,5)}	{(1,2),(2,2)}	{(4,5),(3,3),(1,1)}	{(4,5),(3,3),(1,1)}
Let x = {1,2,3,4}, R = {(2,3),(4,1)} then the domain of R = ----------	{1,3}	{2,3}	{2,4}	{1,4}	{2,4}
Let x = {1,2,3,4}, R = {(2,3),(4,1)} then the range of R = ---------	{1,3}	{3,1}	{2,4}	{1,4}	{3,1}
In a relation matrix all the diagonal elements are one then it satisfies -------	symmetric	antisymmetric	transitive	reflexive	reflexive
In a relation matrix A=(aij) a_{ij} =a_{ji} then it satisfies ------- relation	symmetric	reflexive	transitive	antisymmetric	symmetric
An ordered arrangement of r - element of a set containning n - distinct element is called an ---------	r permutation of n elements	r - combination of n elements	n permutation of r elements	n combination of r elements	r permutation of n elements
The r - permutation of n elements is denoted by -----------	P (r, n)	P(n,r)	c(r, n)	c(n, r)	P(n,r)
The r - permutation of n elements is denoted by P (n, r) where -------	r ≤ n	r = n	r ≥ n	r > n	r ≤ n
An unordered pair of r elements of a set containing n distinct elements is called an-------	r permutation of n elements	r - combination of n elements	n permutation of r elements	n combination of r elements	r - combination of n elements
The number of different permutations of the word BANANA is -------	720	60	120	360	60
The number of way a person roundtrip by bus from A to C by way of B will be ------	12	48	144	264	144
How many 10 digits numbers can be written by using the digits 1 and 2 ?	C (10, 9) + C (9, 2)	1024	C(10, 2)	10!	1024
The number of ways to arrange th a letters of the word CHEESE are ------	120	240	720	6	120

r - combination of n elements is denoted by ------	P (r, n)	P(n,r)	C(r, n)	C(n, r)	C(n, r)
The value of C(n,n) is ----------	0	1	n	n-1	1
C (n, n-r) = ------	C(n, r)	C(n-1, r)	C(n-1, r-1)	C(n, r-1)	C(n, r)
C (n, r) + C (n, r-1) = ------	C(n, r)	C (n+1, r-1)	C (n+1, r)	C(n, r+1)	C (n+1, r)
The number of arranging different crcular arrangement of n objects = ----------	n!	(n+1)!	(n -1)!	0!	(n -1)!
The number of ways of arranging n beads in the form of a necklace = ----------	(n-1)!	(n-1)!/2	n!	n!/2	(n-1)!/2
The value of C(10, 6) + C(9, 5) + C(8, 4) + C(8, 3) is ----------	C(10, 7)	C(9,7)	C(8, 5)	C(11, 5)	C(11, 5)
The value of C(10, 8) + C(10,7) is -------------	990	165	45	120	165
The number of different words can be formed out of the letters of the word VARANASI, is---------	64	120	40320	720	720
The number of ways can a party of 7 persons arrange themselves around a circular table-----	6!	7!	5!	7	6!
The sum of entries in the fourth row of Pascal's triangle is ----------	8	4	10	16	8
The number of wors can be formed out of the letters of the word PECULIAR beginning with P and ending with R is ------	100	120	720	150	720
The value of P(n,n) = -------------	1	0	n	n-1	n
The value of P(10, 3) is -----------	120	720	60	45	720
If P (10, r) is 720, then the value of r is --------	2	3	4	5	3
In how many ways 5 children out of a class of 20 line for a picture?	P (20, 4)	P(20, 5)	P (5, 20)	P(5, 5)	P(20, 5)
The value of C(n, r) is ----------	an integer	a fraction	an integer or a fraction	a rational number less than 1	an integer
The value of P(n, r) / r! is ---------	r	C(n, r)	n /r	nr	c(n,r)

KARPAGAM ACADEMY OF HIGHER EDUCATION
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DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics	**Semester :V**	**L T P C**
Subject Code: 15MMU505A	**Class : III- B.Sc Mathematics**	**5 0 0 5**

UNIT-III

Formal languages and Automata: Grammars: Phrase–structure grammar, context-sensitive grammar, context-free grammar, regular grammar. Finite state automata-Deterministic finite automata and Non deterministic finite automata-conversion of non deterministic finite automata to deterministic finite automata.

Text Book

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

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UNIT – III

Formal Language and and Automata:

Finite Automata

Definition: A finite state automaton (.F S A) or simply an **automaton** M or **finite state acceptor** consists of

(1) a finite set I, called the input alphabet of input symbols

(2) a finite set S of states

(3) a subset A of S of accepting states

(4) an initial state s_0 in S

(5) a next state function f from $S \times I \to S$.

Such an automaton is denoted by $M = (I, S, A, s_0, f)$. Thus, finite automaton does not have an output alphabet, instead it has a set of acceptance state. The plural of automaton is **automata.**

Example: 1. Let us take

$$I = \{a, b\}$$

$$S = \{s_0, s_1, s_2)$$

$$O = \{x, y, z\}$$

Initial State is s_0

Next state function $f : S \times I \to S$ defined by

$$f(s_0, a) = s_1, \quad f(s_1, a) = s_2, \quad f(s_2, a) = s_0$$

$$f(s_0, b) = s_2, \quad f(s_1, b) = s_{1,} \quad f(s_2, b) = s_1$$

Output function $g : S \times I \to O$ defined by

$$g(s_0, a) = x, \quad g(s_1, a) = x, \quad g(s_2, a) = z$$

$$g(s_0, b) = y, \quad g(s_1, b) = z, \quad g(s_2, b) = y/$$

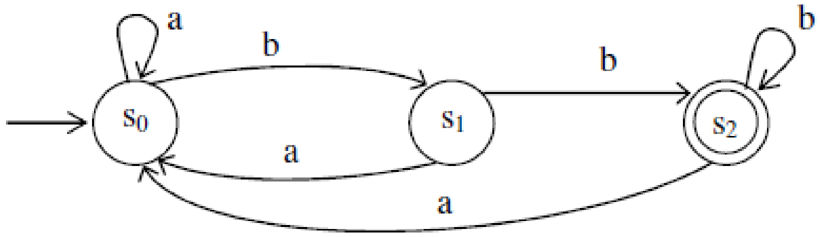
Then $M = M(I, S, O, s_0, f, g)$ in a finite state machine.

Example : Let

I = {a, b}, S = {s_0, s_1, s_2}, A = {s_2}, $s_0 \in$ S, the initial state and f is given by the table

S \ I	f: a	f: b
s_0	S_0	s_1
s_1	s_0	s_2
s_2	s_0	s_2

The transition diagram of a finite – state automation is usually drawn with accepting states in double circles. Thus transition diagram for the example in question is



Example: Let

I = {a, b}, input symbols

S = {s_0, s_1, s_2}, internal states

A = {s_0, s_1}, yes states (accepting states)

s_0, initial state

Next state function f : S × I → S defined by

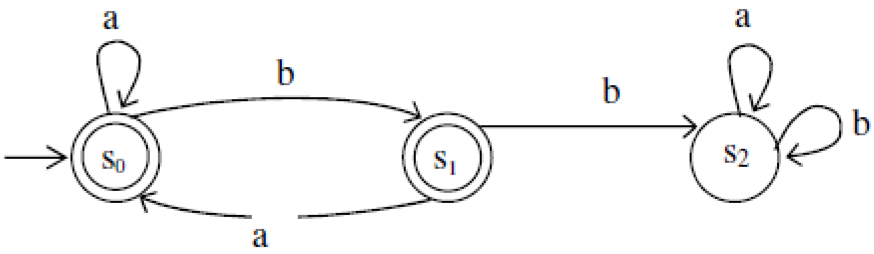
$$f(s_0, a) = s_0,\ f(s_1, a) = s_0,\ f(s_2,a) = s_2$$

$$f(s_0, b) = s_1,\ f(s_1, b) = s_2,\ f(s_2,b) = s_2$$

Then M = (I, S, A, s_0, f) is a finite state automaton. Its transition table is

S \ I	f: a	f: b
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_2	s_2

and the transition diagram is



If a string is input to a finite state automaton, we will end at either an accepting or a non-accepting state. The status of this final state determines whether the string is accepted by the finite state automaton.

Definition: Let $M = (I, S, A, f, s_0)$ be a finite state automaton. Let $x_1 \ldots x_n$ be a string over I. If there exist states $s_0, s_1, \ldots, s_n$ such that

$$f(s_{i-1}, x_i) = s_i \text{ for } i = 1, 2, \ldots, n$$

and

$$s_i \in A,$$

then we say that the string $x_1 \ldots x_n$ is accepted by A.

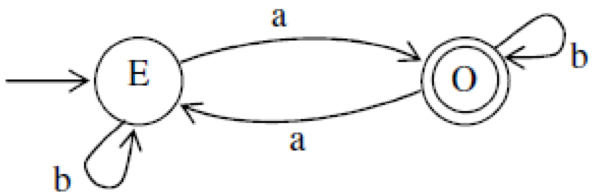
We call the directed path P $(s_0, \ldots, s_n)$ the path representing $x_1, \ldots, x_n$ in M. Thus M accepts $x_1 \ldots x_n$ if and only if path P ends at an accepting state.

Example: Design a finite – state – automaton that accepts precisely those strings over {a, b} that contains an odd number of a's.

Solution: There shall be two states:

E : An even number of a's was found
O : An odd number of a's was found

The initial state is E and the accepting state is O.



If f is next – state function, then we have

$$f(E, a) = O$$

$$f(E, b) = E$$

$$f(O, a) = E$$

$$f(O, b) = O$$

Example : Let M = {I, S, A, s_0, f) be a finite state automaton with

$$I = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$S = \{s_0, s_1, s_2\}$$

$$A = \{s_0\}$$

$$a \in \{0, 3, 6, 9\}, b \in \{1, 4, 7\}, c \in \{2, 5, 8\}.$$

Next – state function f defined by

$$f(s_0, a) = s_0, \quad f(s_0, b) = s_1, \quad f(s_0, c) = s_2$$

$$f(s_1, a) = s_1, \quad f(s_1, b) = s_2, \quad f(s_1, c) = s_0$$

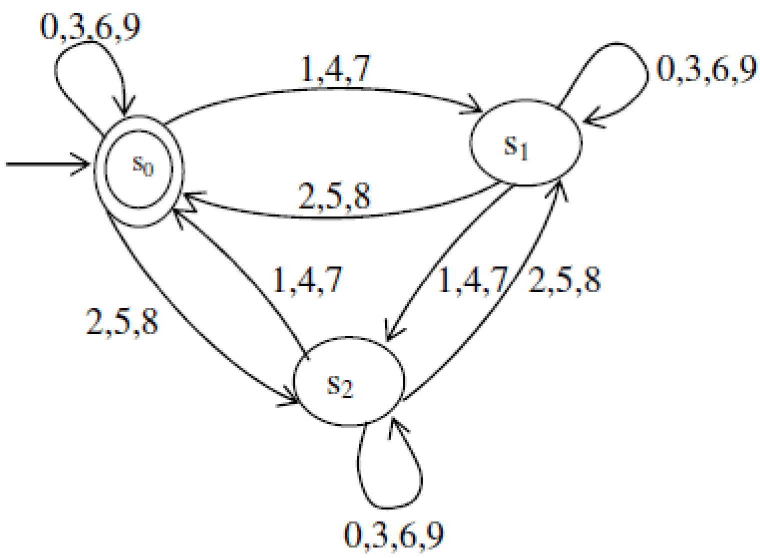
$$f(s_2, a) = s_2, \quad f(s_2, b) = s_0, \quad f(s_2, c) = s_1$$

Draw transition table and transition diagram for this F.S.A. Does this automaton accept 258 and 142 ?

Solution: The transition table for F.S.A. is

I \ S	f: a	b	c
s_0	s_0	s_1	s_2
s_1	s_1	s_2	s_0
s_2	s_2	s_0	s_1

The transition diagram for this F.S.A. is



Here A = {s_0} is the initial stage and also is an acceptor. Further, we note that

$$
\begin{aligned}
f(s_0, 258) &= f(f(s_0, 25), 8) \\
&= f(f(f(s_0, 2), 5), 8) \\
&= f(f(s_2,5), 8) \\
&= f(s_1, 8) = s_0 \in A
\end{aligned}
$$

Thus, the string 258 determines the path

$$s_0 \xrightarrow{2} s_2 \xrightarrow{5} s_1 \xrightarrow{8} s_0 \in A$$

Hence 258 is accepted by the given Finite State Automation.

On the other hand,

$$f(s_0, 142) = f(f(s_0, 14), 2)$$
$$= f(f(f(s_0, 1), 4), 2)$$
$$= f(f(s_1, 4), 2)$$
$$= f(s_2, 2)$$
$$= s_1 \notin A$$

$$s_0 \to s_1 \to s_2 \xrightarrow{2} s_1 \notin A.$$

Hence 142 is not accepted by the given Finite State Automaton.

Non – Deterministic Finite State Automaton

Definition: A non – deterministic finite – state automaton is a 5 – tuple M = (I, S, A, s_0, f) consisting of
(1) A finite set I of input symbols
(2) A finite set S of states
(3) A subset A of S of accepting states
(4) An initial state function $s_0 \in S$
(5) A next state function f from $S \times I$ into P(S)

Thus, in a non – deterministic finite state automaton, the next state function leads us to a set of states, whereas in a finite state automaton, the next state function takes us to a uniquely defined state.

Example: Find the transition diagram for N D F S A

$$M = (I, S, A, s_0, f),$$

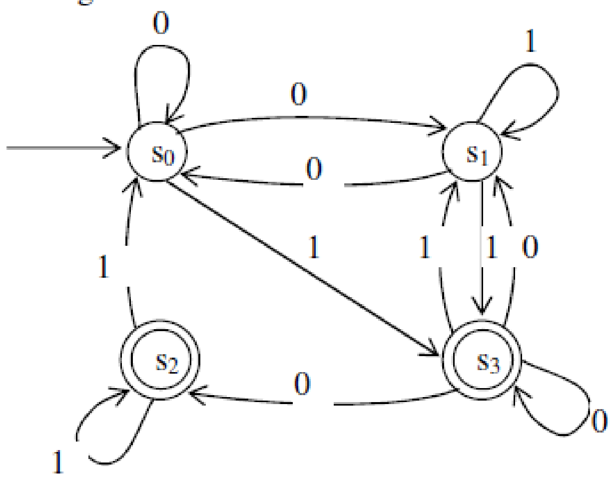
where

$$I = \{0, 1\}, S = \{s_0, s_1, s_2, s_3\}, A = \{s_2, s_3\}$$

and the next state function f is given by

S \ I	f: 0	1
s_0	$\{s_0, s_1\}$	$\{s_3\}$
s_1	$\{s_0\}$	$\{s_1, s_3\}$
s_2	ϕ	$\{s_0, s_2\}$
s_3	$\{s_1, s_2, s_3\}$	$\{s_1\}$

Solution: Here the initial state is s_0 and the accepting states are s_2 and s_3. The transitional diagram of this N D F S A is



Definition: Let M = (I, S, A, s_0, f) be a non – deterministic finite state automaton. The null string is **accepted** by M if and only if $s_0 \in$ A. **If w = a_1 a_2…..a_n is a non – null string over I and there exists states s_0, s_1, ….., s_n such that**

(1) **s_0 is the initial state**

(2) $s_i = f(s_{i-1}, a_i)$

(3) $s_n \in A$,

then we say that w is accepted by M.

We denote by AC(M), the set of strings accepted by M and say that M accept AC(M).

Definition: Two non – deterministic finite state automata M and M′ are said to be **equivalent** if

$$AC(M) = AC(M').$$

Example: Let

$$M = (I, S, A, s_0, f)$$

be a N D F S A with

$$I = \{0, 1\},\ S = \{s_0, s_1, s_2, s_3, s_4\},\ A = \{s_2, s_4\},$$

so as the initial state and the next state function defined by the transition table given below:

S \ I	f: 0	f: 1
s_0	$\{s_0, s_3\}$	$\{s_0, s_1\}$
s_1	φ	$\{s_2\}$
s_2	$\{s_2\}$	$\{s_2\}$
s_3	$\{s_4\}$	ϕ
s_4	$\{s_4\}$	$\{s_4\}$

Determine whether M accept the words (i) w = 010 and (ii) w = 01001.

Solution: (i) The word w = 010 determines the path $s_0 \xrightarrow{0} \{s_0, s_3\} \xrightarrow{1} f(s_0, 1) \cup f(s_3, 1) = \{s_0, s_1\}$

$\cup\ \varphi = \{s_0, s_1\} \xrightarrow{0} f(s_0, 0) \cup f(s_1, 0) = \{s_0, s_3\} \cup \varphi = \{s_0, s_3\}$

But $A \cap \{s_0, s_3\} = \{s_2, s_4\} \cap \{s_0, s_3\} = \phi$. Hence the word w = 010 is not acceptable to the given non – deterministic finite state automaton.

(ii) We have seen above that

$$s_0 \xrightarrow{0} \{s_0, s_3\} \xrightarrow{1} \{s_0, s_1\} \xrightarrow{0} \{s_0, s_3\}$$

Therefore the word w = 01001 determines the path

$$s_0 \xrightarrow{0} \{s_0, s_3\} \xrightarrow{1} \{s_0, s_1\} \xrightarrow{0} \{s_0, s_3\} \xrightarrow{0} f(s_0, 0) \cup f(s_3, 0)$$

$$= \{s_0, s_3\} \cup \{s_n\}$$

$$= \{s_0, s_3, s_4\} \xrightarrow{1} f(s_0, 1) \cup f(s_3, 1\} \cup f(s_4, 1\}$$

$$= \{s_0, s_1\} \cup \varphi \cup \{s_4\}$$

$$= \{s_0, s_1, s_4\}$$

so that

But $A \cap \{s_0, s_3\} = \{s_2, s_4\} \cap \{s_0, s_3\} = \phi$. Hence the word w = 010 is not acceptable to the given non – deterministic finite state automaton.

(ii) We have seen above that

$$s_0 \xrightarrow{0} \{s_0, s_3\} \xrightarrow{1} \{s_0, s_1\} \xrightarrow{0} \{s_0, s_3\}$$

Therefore the word w = 01001 determines the path

$$s_0 \xrightarrow{0} \{s_0, s_3\} \xrightarrow{1} \{s_0, s_1\} \xrightarrow{0} \{s_0, s_3\} \xrightarrow{0} f(s_0, 0) \cup f(s_3, 0)$$

$$= \{s_0, s_3\} \cup \{s_n\}$$

$$= \{s_0, s_3, s_4\} \xrightarrow{1} f(s_0, 1) \cup f(s_3, 1\} \cup f(s_4, 1\}$$

$$= \{s_0, s_1\} \cup \varphi \cup \{s_4\}$$

$$= \{s_0, s_1, s_4\}$$

so that

$$A \cap \{s_0, s_1, s_4\} = \{s_2, s_4\} \cap \{s_0, s_1, s_4\} = \{s_4\} \neq \varphi.$$

Hence the string 01001 is acceptable to the given N D F S A.

REGULAR GRAMMARS

Regular Expansions

One way of describing regular languages is via the notation of regular expressions. This notation involves a combination of strings of symbols from some alphabet Σ, parentheses, and the operators +, ., and *. The simplest case is the language {a}, which will be denoted by the regular expression a Slightly more complicated is the language {a, b, c}, for which, using the + to denote union, we have the regular expression a+b+c. We use · for concatenation and * for star-closure in a similar way. The expression (a + (b·c))* stands for the star-closure of {a} ∪; {b}, that is, the language {λ, *a*, *bc*, *aa*, *abc*, *bca*, *bcbc*, *aaa*, *aabc*,…}.

Definition:

Let Σ be a given alphabet. Then

1. Ø,λ and a ∈ Σ are all regular expressions. These are called **primitive regular expressions.**

2 If r_1 and r_2 are regular expressions, so are r_1+ r_2,r_1.r_2,r_1^*, and (r_1).

3. A string is a regular expression if and only if it can be derived from the primitive regular expressions by a finite number of applications of the rules in (2).

Definition:

The language $L(r)$ denoted by any regular expression r is defined by the following rules.

1. Ø is a regular expression denoting the empty set,
2. λ is a regular expression denoting {λ}.
3. For every $a \in \Sigma$, a is a regular expression denoting $\{a\}$.

If r_1 and r_2 are regular expressions, then

4. $L(r_1 + r_2) = L(r_1) \cup L(r_2)$,

5. $L(r_1 \cdot r_2) = L(r_1) \cup L(r_2)$;

6 $L((r_1)) = L(r_1)$,

7. $L(r_1^*) = (L(r_1))^*$.

Example:

For $\Sigma = \{0, 1\}$, give a regular expression r such that

$L(r) = \{w \in \Sigma^*: w$ has at least one pair of consecutive zeros$\}$.

One can arrive at an answer by reasoning something like this: Every string in $L(r)$ must contain 00 somewhere, but what comes before and what goes after is completely arbitrary. An arbitrary string on $\{0,1\}$ can be denoted by $(0+1)^*$. Putting these observations together, we arrive at the solution

$$r = (0 + 1)^* 00(0 + 1)^*.$$

Definition:

A grammar $G = (V, T, S, P)$ is said to be **right-linear** if all productions are of the form

$$A \to xB,$$

$$A \to x,$$

where $A, B \in V$, and $x \in T^*$. A grammar is said to be **left-linear** if all productions are of the form

$$A \to Bx,$$

or

$$A \to x.$$

Example:

The grammar $G_1 = (\{S\}, \{a, b\}, S, P_1)$, with P_1 given as

$$S \to abS|a$$

is right-linear. The grammar $G_2 = (\{S, S_1, S_2\}, \{a, b\}, S, P_2)$, with productions

$$S \to S_1ab,$$

$$S_1 \to S_1ab|S_2,$$

$$S_2 \to a,$$

is left-linear. Both *G1* and *G2* are regular grammars.

The sequence

S implus **abS** implus **ababS** implus **ababa**

Example:

The grammar $G = (\{S, A, B\}, \{a, b\}, S, P)$ with productions

$$S \to A$$

$$A \to aB|\lambda,$$

$$B \to Ab,$$

is not regular. Although every production is either in right-linear or left-linear form, the grammar itself is neither right-linear nor left-linear, and therefore is not regular. The grammar is an example of a **linear grammar**.

Context Free Grammar:

A grammar $G = (V, T, S, P)$ is said to be **context-free** if all productions in P have the form

$$A \to x,$$

where $A \in V$ and $x \in (V \cup T)^*$.

A language L is said to be context-free if and only if there is a context-free grammar G such that $L = L(G)$.

Example:

The grammar $G = (\{S\}, \{a, b\}, S, P)$, with productions

$$S \to aSa,$$
$$S \to bSb,$$
$$S \to \lambda,$$

is context-free. A typical derivation in this grammar is

$$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabbaa.$$

This, and similar derivations, make it clear that

$$L(G) = \{ww^R : w \in \{a, b\}^*\}.$$

Example:

The language

$$L = \{a^n b^m : n \neq m\}$$

is context-free.

To show this, we need to produce a context-free grammar for the language. The case of $n = m$ is solved in Example 1.11 and we can build on that solution. Take the case $n > m$. We first generate a string with an equal number of a's and b's, then add extra a's on the left. This is done with

$$\begin{aligned} S &\rightarrow AS_1, \\ S_1 &\rightarrow aS_1b|\lambda, \\ A &\rightarrow aA|a. \end{aligned}$$

We can use similar reasoning for the case $n < m$, and we get the answer

$$\begin{aligned} S &\rightarrow AS_1|S_1B, \\ S_1 &\rightarrow aS_1b|\lambda, \\ A &\rightarrow aA|a, \\ B &\rightarrow bB|b. \end{aligned}$$

The resulting grammar is context-free, hence L is a context-free language. However, the grammar is not linear.

Example: Construct deterministic finite state automaton equivalent to the following non – deterministic finite state automaton :

$$M = (\{0, 1\}, \{s_0, s_1\}, s_0, \{s_1\}, f),$$

where f is given by the table

	f	
I \ S	0	1
s_0	$\{s_0, s_1\}$	$\{s_1\}$
s_1	φ	$\{s_0, s_1\}$

Solution: Let

$$M' = \{\{0, 1\}, \{\varphi, \{s_0\}, \{s_1\}, \{s_0, s_1\}, s_0' = \{s_0\}, A', f'\}$$

be the D F S A, where

$$A' = \{s \in \{\varphi, (s_0, \{s_1\}, \{s_0, s_1\} : s \cap \{s_1\} \neq \varphi$$

and $= \{s_1\}$ and $\{s_0, s_1\}$ (**Accepting states**)

$$f'(s, a) = \bigcup_{\sigma \in s} f(\sigma, a) \text{ for } s \in \{\varphi, \{s_0\}, \{s_1\}, \{s_0, s_1\}\}$$

We have

$\{s_0\}$ as the initial state

The finite set of states is $\{\varphi, \{s_0\}, \{s_1\}, \{s_0, s_1\}\}$
The finite set of inputs is $\{0, 1\}$
The accepting states are $[s_1]$ and $[s_0, s_1]$.
Now

$$f'(\varphi, 0) = \phi \text{ and } f'(\varphi, 1) = \varphi$$

$$f'([s_0], 0) = f(s_0, 0) = [s_0, s_1]$$

$$f'([s_0], 1) = f(s_0, 1) = [s_1]$$

$$f'([s_1], 0) = f(s_1, 0) = \phi$$

$$f'([s_1], 1) = f(s_1, 1) = [s_0, s_1]$$

$$f'([s_0, s_1], 0) = f(s_0, 0) \cup f\{s_1, 0\}$$

$$= \{s_0, s_1\} \cup \{s_1\}$$

$$= [s_0, s_1]$$

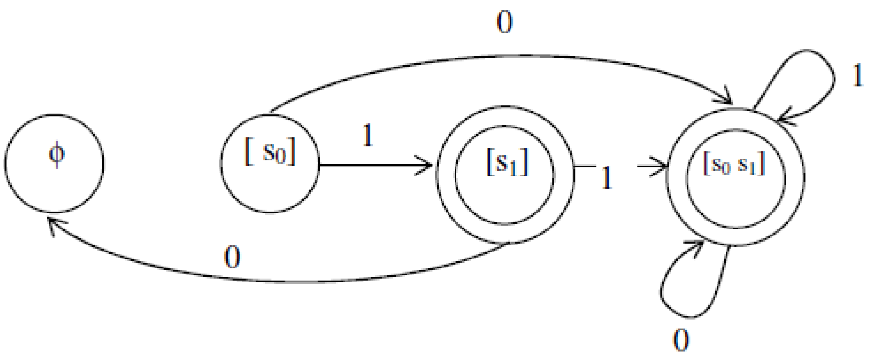
$$f'(\{s_0, s_1\}, 1) = f(s_0, 1) \cup f(s_1, 1)$$

$$= \{s_1\} \cup \{s_0, s_1\}$$

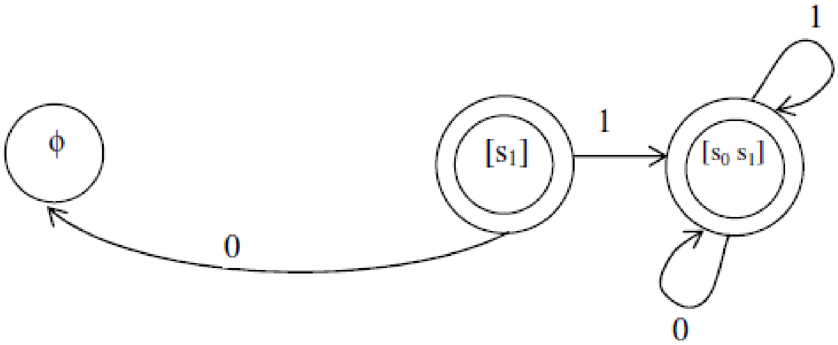
$$= [s_0, s_1]$$

Hence the next state function and the transition diagram for D F S A are as given below :

S \ I	f′ 0	f′ 1
φ	φ	φ
$[s_0]$	$[s_0, s_1]$	$[s_1]$
$[s_1]$	ϕ	$[s_0, s_1]$
$[s_0, s_1]$	$[s_0, s_1]$	$[s_0, s_1]$



It may be mentioned here that a **state which is never entered may be deleted from the transition diagram**. In view of this, the above transition diagram becomes



Thus, we note that **if N D F S A has n states, then D F S A will have 2^n states.**

Part -B (5x8=40 Marks)

Possible Questions:

1. Explain the types of grammars with examples.

2. i) Obtain the Context sensitive grammar for the language (a^{m^2} / m ≥ 1)
   ii) Let G = { (S,B),(a),S, φ} Define production function φ as
   (i) S→ aS ,(ii) S→ aB (iii) B→ aS (iv) S→ a
   iii) Define transition diagram. Draw a transition diagram which will accept those words from A, which have an even number of a's.

3. i) Prove that L(G) = { $a^n b^n c^n$ / n ≥ 1} where G = ({S,B,c},{a,b,c},S, φ) and
   Q = { S→ aSBc, S→ aBc, cB→ Bc, aB→ ab, bC→ bc, cC→ cc}
   ii) Explain with an example for conversion of non-deterministic finite automata to finite state automata .

4. Show that the language L(G_5) = {a^nbc^m/ m,n ≥ 1} is generated by the following grammar: G_5 = ({S,A,B,C}, {a,b}, S, φ), where the set φ consists of production is S → aS, S → aB, B → bC, C → ac, C → a.

5. Show that the language L(G_4) = {a^nba^n/ n ≥ 1} is generated by the following grammar: G_4 = ({S,C}, {a,b}, S, φ), where φ consists of productions {S → aCa, C → aCa, C → b}.

6. i)Consider the grammar G = ({ S,A,B,C} , {a,b},S,φ) where φ is the set of productions S→ aAab , A→ aAa , A→ bB , Ba→ aB, Bb→ Cbb ,aC→ Ca, A→ b. Find L(G).
   ii). Construct the grammar for the language L(G) = { $a^n b^{2n}$ / n ≥ 1}

7. Construct the equivalent DFSA for the following NDFSA

   M=({0,1},{q_0,q_1},δ,q_0,{q_1}) here δ is given by

δ	0	1
q_0	{q_0,q_1}	{q_1}
q_1	ϕ	{q_0,q_1}

8. i)Construct the grammar for the language L(G)={a^nb^{2n}/ n ≥ 1}.
   ii) Construct the grammar for the language L(G) = {a^nba^m/ n, m ≥ 1}.

9. Let M = ({a,b},{ q_0,q_1,q_2 },q_0 ,δ ,{q_2}) be a non-deterministic finite –state automata. where δ is given as follows: δ(q_0 ,a) = { q_0,q_1}, δ(q_1 ,a) = {q_1}, δ(q_2,a)={ q_0}, δ(q_0 ,b) = { q_2}, δ(q_1 ,b) = { q_0}, δ(q_2 ,b) = { q_1,q_2}. Construct an equivalent deterministic finite-state automata.

10. Prove that every regular set is accepted by a finite state state automata.

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DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics **Subject Code: 15MMU505A**
Class : III-B.Sc Mathematics (A) **Semester : V**

UNIT III

Part A (20x1=20 Marks)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
A finite non- empty set E of sympols called ----------	string	word	letters	alphabet	alphabet
The ------------ of the word is the number of letters in it.	degree	weight	length	height	length
A---------- over E is sequence of symbols of E with possible repetitions	alphabet	letters	word	length	word
The specification of profer construction of sentences is called the ------------of the language	alphabet	monoid	syntax	semantics	syntax
A ------------- is any derivative of the unique non-terminal symbol S.	sentential form	language	type 0 grammer	type -1 grammer	sentential form
A grammar G is said to be -------------- if there is some word in L(G) has atleast two derivation trees	un ambiguous	ambiguous	language	syntex	ambiguous
a derivation in which the right most non terminal symbol is replaced at each step is said to be -----------	word	sentential form	left most derivation	right most derivation	rightmost derivation
The pictorial method of specifying the finite state machine is called --------------	state diagram	sequential diagram	digrapgh	right most derivation	state diagram
Every regular language is -----------------	ambiguous	unambiguous	inherently	language	unambiguous
Any subset L of A* is called a -------------over A.`	language	letters	alphabet	sensitive	language
The specification of the meaning of sentences is called the ------------of the language	syntax	semantics	E*	empty set	semantics
A pharse structure grammer with no restrictions is called a ---------------	Type -0 grammer	Type - 1 grammer	type- 2 grammer	type-3 grammer	Type -0 grammer
A grammer G is said to be --------- if every production is of the form A→α.	context - sensitive	context-free	regular	type -1 grammer	context- free
A grammer G is said to be --------- if every production is of the form A→a, A→aB	context - sensitive	context-free	type-1 grammer	regular	regular
A language for which there exists a recongnition algorithm is said to be ------------	recursive	relation	syntax	semantics	recursive
A language generated by type -0 grammer is called a --------------	Type -0 grammer	Type - 1 grammer	type 2 grammar	type-3 grammer	Type-0 language
grammar of type - 1 are often called --------	context - sensitive	context-free	regular	syntex	context -sensitive grammar
A language generated by type - 1 grammer is called a --------------	Type -0 language	Type - 1 language	type -2 language	type -3 language	type -1 language
The length of a word W is the ------------- in W	number of letters	number of alphabets	number of words	number of strings	number of letters
DFA and NFA represent the -------------- language .	context - sensitive	context-free	regular	type-4	regular
Every grammar generating a context - free language is ------------	ambiguous	unambiguous	string	word	ambiguous
Every finite state machine has a -------------- associated with it.	monoid	unambiguous	regular	semantics	monoid
If L accepted by a NFA, then there exists a DFA , that accepts ----------------.	L	E	E*	M	L
If a language L is accepted by a multitape TM , it is accepted by a single tape ------------.	TM	MT	E*	L	TM
Every regular language is accepted by a ---------------.	finite state automata	infinte state automata	Regular automata	Irregular automata	finite state automata
If L is N(M) for some PDA M, then L is a -------------- language.	context - sensitive	context-free	regular	empty set	context - free
Push - down automata is denoted by -----------	PDA	PAD	DAP	DAA	PAD
If the syntex is correct then it produces............ code	Verb	sentence	sentence	object	object

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Subject Code: 15MMU505A	**Class : III- B.Sc Mathematics**	**5 0 0 5**

UNIT-IV

Lattices and Boolean algebra: Partial ordering, Poset, Lattices, Boolean algebra, Boolean functions, Theorems, Minimization of Boolean functions.

Text Book

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

References

1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

2. Veerarajan T., 2007.Discrete mathematics with graph theory and combinatorics, Tata Mc.Graw Hill,New Delhi.

3.Sharma .J.K,2005.Discrete Mathematics ,Second Edition, Macmillan India Ltd,New Delhi.

4. Discrete mathematics by Neeru Sharma, Publisher: New Delhi, India : University Science Press (An imprint of Laxmi Pubications Limited, Pvt. Ltd.), [2016] ©2011

UNIT -IV

LATTICES

Definitions and Examples

Definition: A **lattice** is a partially ordered set (L, $\leq$) in which every subset {a, b} consisting of **two element** has a **least upper bound** and a **greatest lower bound.**

We denote lub({a, b}) by a $\vee$ b and call it **join** or **sum of a and b.**

Similarly,

we denote GLB({a, b}) by a $\wedge$ b and call it **meet** or **product of a and b.**
Other symbol used are:

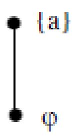
LUB : $\oplus$, +, $\cup$
GLB : *, **.** , $\cap$

Thus **Lattice is** a mathematical structure with **two binary operations, join and meet.** Lattice structures often appear in computing and mathematical applications.

A totally ordered set is obviously a lattice but not all partially ordered sets are lattices.

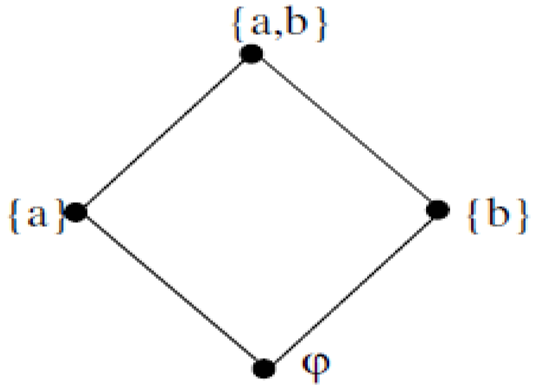
Example 1. Let A be any set and P(A) be its power set. The partially ordered set (P(A), $\subseteq$) is a lattice in which the meet and join are the same as the operations $\cap$ and $\cup$ respectively. If A has single element, say a, then P(A) = {φ, {a}} and
LUB({ φ, {a}) = {a}
GLB({φ, {a}) = φ

The Hasse diagram of (P(A), $\subseteq$) is a chain containing two elements φ and {a} as shown below:



If A has two elements, say a and b. Then P(A) = {φ, {a}, {b}, {a, b}}. The

Hasse diagram of {P(A), $\subseteq$) is then as shown below :



We note that

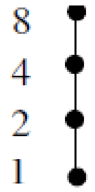
1. LUB exists for every two subsets and is $L \cup M$

2. GLB exists for every two subsets and is in $L \cap M$ for $L, M \in P(A)$.

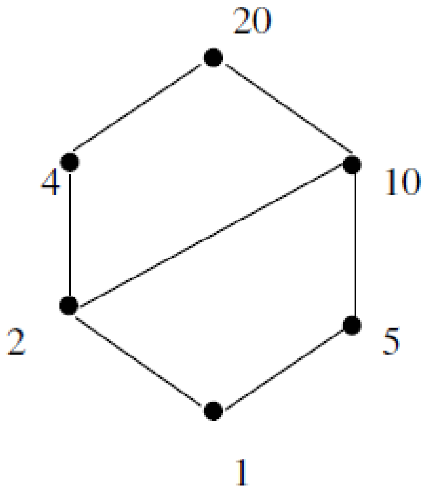
Hence P(A) in a lattice.

Example 2. Consider the poset (**N**, $\leq$), where $\leq$ is relation of divisibility. Then **N** is a lattice in which
join of a and b = $a \vee b$ = L C M(a, b)
meet of a and b = $a \wedge b$ = G C D (a, b) for a, b $\in$ **N**.

Example 3. Let n be a positive integer and let D_n be the set of all positive divisors of n. Then D_n is a lattice under the relation of divisibility**.** The Hasse diagram of the lattices D_8, D_{20} and D_{30} are respectively.

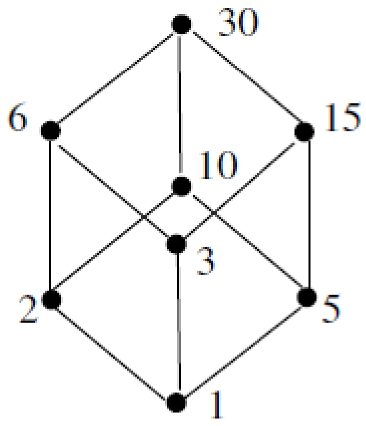


$D_8 = \{1, 2, 4, 8\}$



$D_{20} = \{1, 2, 4, 5, 10, 20\}$

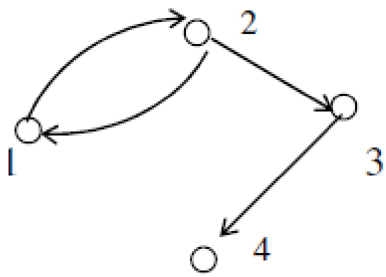
and



$D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$.

The TransiDefinition: The **Transitive closure** of a relation R is the smallest transitive relation containing R. It is denoted by R_∞.

Example: Let A = {1, 2, 3, 4} and R = [(1, 2), (2, 3), (3, 4), (2, 1)] Find the transitive closure of R.
Solution: The digraph of R is



We note that from vertex 1, we have paths to the vertices 2, 3, 4 and 1. Note that path from 1 to 1proceeds from 1 to 2 to 1. Thus we see that the ordered pairs (1, 1), (1, 2), (1, 3) and (1, 4) are in R_∞. Starting from vertex 2, we have paths to vertices 2, 1, 3 and 4 so the ordered pairs (2, 1), (2, 2), (2, 3) and (2, 4)
are in R_∞. The only other path is from vertex 3 to 4, so we have
R_∞ = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3,4)}

Example: Let R be the set of all equivalence relations on a set A. As such R consists of subsets of A $\times$ A and so R is a partially ordered set under the partial order of set inclusion. If R and S are equivalence relations on A, the same property may be expressed in relational notations as follows:
R $\subseteq$ S if and only if x R y _ x S y for all x y $\in$ A.

Then **(**R, $\subseteq$) is a poset. R is a lattice**,** where the meet of the equivalence relations R and S is their intersection R $\cap$ S and their join is $(R \cup S)_\infty$, the transitive closure of their union.

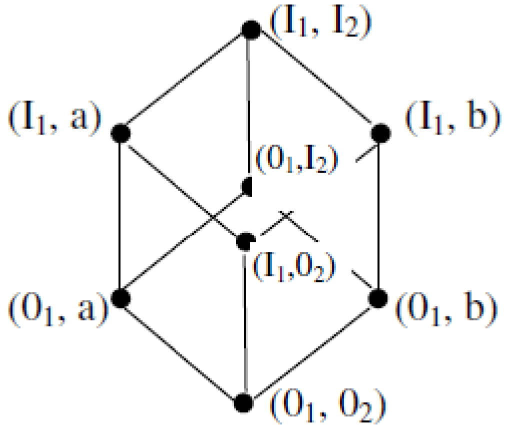
Definition: Let (L, $\leq$) be a poset and let (L, $\geq$) be the dual poset. If (L, $\leq$) is a lattice, we can show that (L, $\geq$) is also a lattice. In fact, for any a and b in L, the
L U B of a and b in (L, $\leq$) is equal to the GLB of a and b in (L, $\geq$). Similarly, the GLB of a and b in (L, $\leq$) is equal to L U B in (L, $\geq$).
The operation $\vee$ and $\wedge$ are called **dual of each other**.

Example: Let S be a set and L = P(S). Then (L, $\subseteq$) is a lattice and its **dual lattice** is (L, $\supseteq$), where $\supseteq$ represents "contains". We note that in the poset (L, $\supseteq$), the join A $\vee$ B is the set A $\cap$ B and the meet A $\wedge$ B is the set A $\cup$ B.

Cartesian Product of Lattices

Theorem: If (L_1, ≤) and (L_2, ≤) are lattices, then (L, ≤) is a lattice, where $L = L_1 \times L_2$ and the partial order ≤ of L is the product partial order.

Proof: We denote the join and meet in L_1 by $\vee_1$, and $\wedge_1$ and the join and meet in L_2 by $\vee_2$ and $\wedge_2$ respectively.
We know that Cartesian product of two posets is a poset.
Therefore $L = L_1 \times L_2$ is a poset. Thus all we need to show is that if
(a_1, b_1) and $(a_2, b_2) \in L$,
Then $(a_1, b_1) \vee (a_2, b_2)$ and $(a_1, b_1) \wedge (a_2, b_2)$ exist in L.
Further, we know that
$(a_1, b_1) \vee (a_2, b_2) = (a_1 \vee a_2 , b_1 \vee b_2)$ and
and
$(a_1, b_1) \wedge (a_2, b_2) = (a_1 \wedge a_2 , b_1 \wedge b_2)$
Since L_1 is lattice, $a_1 \vee_1 a_2$ and $a_1 \wedge_1 a_2$ exist. Similarly, since L_2 is a lattice, $b_1 \vee b_2$ and $b_1 \wedge b_2$ exist. Hence $(a_1, b_1) \vee (a_2, b_2)$ and $(a_1, b_1) \wedge (a_2, b_2)$ both exist and therefore (L, ≤) is a lattice, called **the direct product of (L_1, ≤) and (L_2, ≤).**



$L = L_1 \times L_2$

Properties of Lattices:

Let (L, ≤) be a lattice and let a, b , c ∈ L. Then, from the definition of ∨ (join) and ∧ (meet)

we have

(i) $a \le a \vee b$ and $b \le a \vee b$; $a \vee b$ is an upper bound of a and b.

(ii) if $a \leq c$ and $b \leq c$, then $a \vee b \leq c$; $a \vee b$ is the least bound of a and b.

(iii) $a \wedge b \leq a$ and $a \wedge b \leq b$; $a \wedge b$ is a lower bound of a and b.

(iv) if $c \leq a$ and $c \leq b$, then $c \leq a \wedge b$; $a \wedge b$ is the greatest lower bound of a and b

Theorem:
Let L be a lattice. Then for every a and b in L,
(i) $a \vee b = b$ if and only if $a \leq b$
(ii) $a \wedge b = a$ if and only if $a \leq b$
(iii) $a \wedge b = a$ if and only if $a \vee b = b$
Proof:
(i) Let $a \vee b = b$. Since $a \leq a \vee b$, we have $a \leq b$.
Conversely, if $a \leq b$, then since $b \leq b$, it follows that b is an upper bound of a and b. Therefore, by the definition of least upper bound, $a \vee b \leq b$. Also $a \vee b$ being an upper bound, $b \leq a \vee b$. Hence $a \vee b = b$.

(ii) Let $a \wedge b = a$. Since $a \wedge b \leq b$, we have $a \leq b$. Conversely, if $a \leq b$ and since $a \leq a$, a is a lower bound of a and b and so, by the definition of greatest lower bound, we have

$$a \leq a \wedge b$$

Since $a \wedge b$ is lower bound,

$$a \wedge b \leq a$$

Hence

$$a \wedge b = a.$$

(iii) From (ii)

$$a \wedge b = a \Leftrightarrow a \leq b \text{.......(iv)}$$

From (i)

$$a \leq b \Leftrightarrow a \vee b = b \text{..........(v)}$$

Hence, combining (iv) and (v),

we have

$$a \wedge b = a \Leftrightarrow a \vee b = b.$$

Example: Let L be a linearly (total) ordered set. Therefore $a, b \in L$ imply either $a \leq b$ or $b \leq a$. Therefore, the above theorem implies that
$a \vee b = a$

$a \wedge b = a$

Thus for every pair of elements a, b in L, $a \vee b$ and $a \wedge b$ exist. Hence **a linearly ordered set is a lattice.**

Theorem :

Let $(L, \leq)$ be a lattice and let $a, b, c \in L$. Then we have

L_1 : **Idempotent property**

(i) $a \vee a = a$

(ii) $a \wedge a = a$

L_2 : **Commutative property**

(i) $a \vee b = b \vee a$

(ii) $a \wedge b = b \wedge a$

L_3 : **Associative property**

(i) $a \vee (b \vee c) = (a \vee b) \vee c$

(ii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

L_4 : **Absorption property**

(i) $a \vee (a \wedge b) = a$

(ii) $a \wedge (a \vee b) = a$

Proof: L_1 : The idempotent property follows from the definition of LUB and GLB.

L_2 : Commutativity follows from the symmetry of a and b in the definition of LUB and GLB.

L_3 : (i) From the definition of LUB, we have

$$a \leq a \vee (b \vee c) \text{ ....(1)}$$

$$b \vee c \leq a \vee (b \vee c) \text{ ....(2)}$$

Also $b \leq b \vee c$ and $c \leq b \vee c$ and so transitivity implies

$$b \leq a \vee (b \vee c)\text{................. (3)}$$

and

$$c \leq a \vee (b \vee c) \text{ ....(4)}$$

Now, (1) and (3) imply that $a \vee (b \vee c)$ is an upper bound of a and b and hence by the definition of least upper bound, we have

$$a \vee b \leq a \vee (b \vee c) \text{ ....(5)}$$

Also by (4) and (5), $a \vee (b \vee c)$ is an upper bound of c and $a \vee b$. Therefore

$$(a \vee b) \vee c \leq a \vee (b \vee c) \text{ ....(6)}$$

Similarly

$$a \vee (b \vee c) \leq (a \vee b) \vee c\text{.......... (7)}$$

Hence, by antisymmetry of the relation ≤, (6) and (7) yield

$a \vee (b \vee c) = (a \vee b) \vee c$

The proof of (ii) is analogous to the proof of part (i).

L_4 : (i) Since $a \wedge b \leq a$ and $a \leq a$, it follows that a is an upper bound of $a \wedge b$ and a. Therefore, by the definition of least upper bound

$$a \vee (a \wedge b) \leq a \text{ ....(8)}$$

On the other hand, by the definition of LUB, we have

$$a \leq a \vee (a \wedge b) \text{ ....(9)}$$

The expression (8) and (9) yields

$$a \vee (a \wedge b) = a.$$

(ii) Since $a \leq a \vee b$ and $a \leq a$, it follows that a is a lower bound of $a \vee b$ and a.

Therefore, by the definition of GLB,

$$a \leq a \wedge (a \vee b) \text{ ....(10)}$$

Also, by the definition of GLB, we have

$$a \wedge (a \vee b) \leq a \text{ ....(11)}$$

Then (10) and (11) imply

$$a \wedge (a \vee b) = a$$

and the proof is completed.

In view of L_3, we can write $a \vee (b \vee c)$ and $(a \vee b) \vee c$ as $a \vee b \vee c$. Thus, we can express

LUB ({$a_1, a_2, \ldots a_n$}) as $a_1 \vee a_2 \vee \ldots \vee a_n$

GLB ({$a_1, a_2, \ldots a_n$}) as $a_1 \wedge a_2 \wedge \ldots \wedge a_n$

Remark:

Using commutativity and absorption property, part (ii) of previous Theorem can be proved as follows :

$$\text{Let } a \wedge b = a.$$

We note that

$$b \vee (a \wedge b) = b \vee a$$

$$= a \vee b \text{ (Commutativity)}$$

But

$$b \vee (a \wedge b) = b \text{ (Absorption property)}$$

Hence

$$a \vee b = b$$

and so by part (i), $a \leq b$. Hence $a \wedge b = a$ if and only if $a \leq b$.

Theorem: Let $(L, \leq)$ be a lattice. Then for any $a, b, c \in L$, the following properties hold :

1. (**Isotonicity) :** If $a \leq b$, then
   (i) $a \vee c \leq b \vee c$
   (ii) $a \wedge c \leq b \wedge c$

This property is called "Isotonicity".

2. $a \leq c$ and $b \leq c$ if and only if $a \vee b \leq c$

3. $c \leq a$ and $c \leq b$ if and only if $c \leq a \wedge b$

4. If $a \leq b$ and $c \leq d$, then
(i) $a \vee c \leq b \vee d$
(ii) $a \wedge c \leq b \wedge d$.

Proof : 1 (i). We know that
$a \vee b = b$ if and only if $a \leq b$.

Therefore, to show that $a \vee c \leq b \vee c$, we shall show that

$$(a \vee c) \vee (b \vee c) = b \vee c.$$

We note that

$$(a \vee c) \vee (b \vee c) = [(a \vee c) \vee b] \vee c = a \vee (c \vee b) \vee c$$
$$= a \vee (b \vee c) \vee c$$
$$= (a \vee b) \vee (b \vee c)$$
$$= b \vee c \ (\Theta a \vee b = b \text{ and } c \vee c = c)$$

The part 1 (ii) can be proved similarly.

2. If $a \leq c$, then 1(i) implies

$$a \vee b \leq c \vee b$$

But

$$b \leq c \Leftrightarrow b \vee c = c$$
$$\Leftrightarrow c \vee b = c \text{ (commutativity)}$$

Hence $a \leq c$ and $b \leq c$ if and only if $a \vee b \leq c$

3. If $c \leq a$, then 1(ii) implies $c \wedge b \leq a \wedge b$

But

$$c \leq b \Leftrightarrow c \wedge b = c$$

Hence

$$c \leq a \text{ and } c \leq b \text{ if and only if } c \leq a \wedge b.$$

4 (i) We note that 1(i) implies that if $a \leq b$, then $a \vee c \leq b \vee c = c \vee b$

if $c \leq d$, then $c \vee b \leq d \vee b = b \vee d$

Hence, by transitivity

$$a \vee c \leq b \vee d$$

(ii) We note that 1(ii) implies that

if $a \leq b$, then $a \wedge c \leq b \wedge c = c \wedge b$

if $c \leq d$, then $c \wedge b \leq d \wedge b = b \wedge d$.

Therefore transitivity implies

$$a \wedge c \leq b \wedge d.$$

Theorem:

Let $(L, \leq)$ be a lattice. If $a, b, c \in L$, then

(1) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

(2) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

These inequalities are called "Distributive Inequalities".

Proof: We have

$a \leq a \vee b$ and $a \leq a \vee c$ (i)

Also, by the above theorem, if $x \leq y$ and $x \leq z$ in a lattice, then $x \leq y \wedge z$.

Therefore (i) yields

$$a \leq (a \vee b) \wedge (a \vee c) \text{................ (ii)}$$

Also

$$b \wedge c \leq b \leq a \vee b$$

and

$$b \wedge c \leq c \leq a \vee c,$$

that is, $b \wedge c \leq a \vee b$ and $b \wedge c \leq a \vee c$ and so, by the above argument, we have

$$b \wedge c \leq (a \vee b) \wedge (a \vee c) \text{ (iii)}$$

Also, again by the above theorem if $x \leq z$ and $y \leq z$ in a lattice, then

$x \vee y \leq z$

Hence, (ii) and (iii) yield

$a\ c\ (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

This proves (1).

The second distributive inequality follows by using the **principle of duality.**

Theorem: (Modular Inequality) : Let (L, ≤) be a lattice. If a, b, c ∈ L, then

$a \leq c$ if and only if $a \vee (b \wedge c) \leq (a \vee b) \wedge c$

Proof: We know that $a \leq c \Leftrightarrow a \vee c = c$ ....(1)

Also, by distributive inequality,

$a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

Therefore using (1) $a \leq c$ if and only if

$a \vee (b \wedge c) \leq (a \vee c) \wedge c$,

which proves the result.

The modular inequalities can be expressed in the following way also:

$(a \wedge b) \vee (a \wedge c) \leq a \wedge [b \vee (a \wedge c)]$

$(a \vee b) \wedge (a \vee c) \geq a \vee [b \wedge (a \vee c)]$

Example: Let (L, ≤) be a lattice and a, b, c ∈ L. If $a \leq b \leq c$, then

(i) $a \vee b = b \wedge c$, (ii) $(a \wedge b) \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Solution: (i) We know that

$a \leq b \Leftrightarrow a \vee b = b$

and

$b \leq c \Leftrightarrow b \wedge c = b$

Hence $a \leq b \leq c$ implies $a \vee b = b \wedge c$.

(ii) Since $a \leq b$ and $b \leq c$, we have

$a \wedge b = a$ and $b \wedge c = b$

Thus

$(a \wedge b) \vee (b \wedge c) = a \vee b$

$= b$,

since $a \leq b \Leftrightarrow a \vee b = b$.
Also, $a \leq b \leq c$ _ $a \leq c$ by transitivity. Then
$a \leq b$ and $a \leq c$ _ $a \vee b = b$, $a \vee c = c$
and so
$(a \vee b) \wedge (a \vee c) = b \wedge c$
$= b$ since $b \leq c \Leftrightarrow b \wedge c = b$.
Hence
$(a \wedge b) \vee (b \wedge c) = b = (a \vee b) \wedge (a \vee c)$,
which proves (ii).

1.21. Lattices as Algebraic System

Definition. A **Lattice** is an algebraic system $(L, \vee, \wedge)$ with two binary operations $\vee$ and $\wedge$, called **join** and **meet** respectively, on a non-empty set L
which satisfy the following axioms for $a, b, c \in L$:

1. Commutative Law :
$a \vee b = b \vee a$ and $a \wedge b = b \wedge a$.
2. Associative Law :

$(a \vee b) \vee c = a \vee (b \vee c)$
and
$(a \wedge b) \wedge c = a \wedge (b \wedge c)$

3. Absorption Law :
(i) $a \vee (a \wedge b) = a$
(ii) $a \wedge (a \vee b) = a$
We note that Idempotent Law follows from axiom 3 above. In fact,
$a \vee a = a \vee [a \wedge (a \vee b)]$ using ....3(ii)
$= a$ using ....3(i)
The proof of $a \wedge a = a$ follows by principle of duality.

1.22 Partial Order Relations on a Lattice

A partial order relation on a lattice (L) follows as a consequence of the axioms for the binary operations $\vee$ and $\wedge$.
We define a relation $\leq$ on L such that for a, b $\in$ L ,

$$a \leq b \Leftrightarrow a \vee b = b$$

or analogously,

$$a \leq b \Leftrightarrow a \wedge b = a .$$

We note that

(i) For any a $\in$ L
$a \vee a = a$ (idempotent law),
therefore $a \leq a$ showing that $\leq$ is **reflexive**.

(ii) Let $a \leq b$ and $b \leq a$. Therefore

$$a \vee b = b$$
$$b \vee a = a$$

But

$$a \vee b = b \vee a \text{ (Commutative Law in lattice)}$$

Hence

$$a = b ,$$

showing that $\leq$ is **antisymmetric.**

(iii) Suppose that $a \leq b$ and $b \leq c$. Therefore $a \vee b = b$ and $b \vee c = c$. Then

$$\begin{aligned} a \vee c &= a \vee (b \vee c) \\ &= (a \vee b) \vee c \text{ (Associativity in lattice)} \\ &= b \vee c \\ &= c , \end{aligned}$$

showing that $a \leq c$ and hence $\leq$ is transitive.

This shows that a **lattice is a partially ordered set**

1.23 Least Upper Bounds and Latest Lower Bounds in a Lattice

Let (L, $\vee$, $\wedge$) be a lattice and let a, b $\in$ L. We now show that LUB of {a, b} $\subseteq$ L with respect to the partial order introduced above is a $\vee$ b and GLB of {a, b} is a $\wedge$ b.

From absorption law
a $\wedge$ (a $\vee$ b) = a
b $\wedge$ (a $\vee$ b) = b

Therefore a $\leq$ a $\vee$ b and b $\leq$ a $\vee$ b, showing that a $\vee$ b is upper bound for {a,b}. Suppose that there exists c $\in$ L such that a $\leq$ c, b $\leq$ c. Thus we have a $\vee$ c = c and b $\vee$ c = c

and then

$$(a \vee b) \vee c = a \vee (b \vee c) = a \vee c = c$$

implying that a $\vee$ b $\leq$ c.
Hence a $\vee$ b is the least upper bound of a and b.
Similarly, we can show that a $\wedge$ b is GLB of a and b.
The above discussion shows that the two definitions of lattice given so far are equivalent.

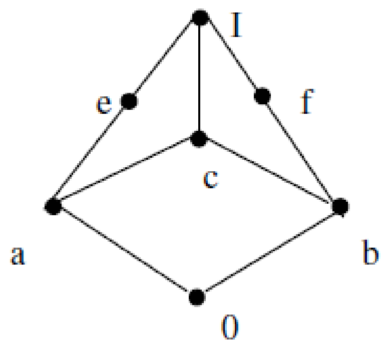
Sublattices

Definition: Let (L, $\leq$) be a lattice. A non-empty subset S of L is called a **sublattice** of L if a $\vee$ b $\in$ S and a $\wedge$ b $\in$ S whenever a $\in$ S, b $\in$ S.

(Or)

Let (L, $\vee$, $\wedge$) be a lattice and let S $\subseteq$ L be a subset of L. Then (S, $\vee$, $\wedge$) is
called a sublattice of (L, $\vee$, $\wedge$) if and only if S is closed under both operations of join($\vee$) and meet($\wedge$).

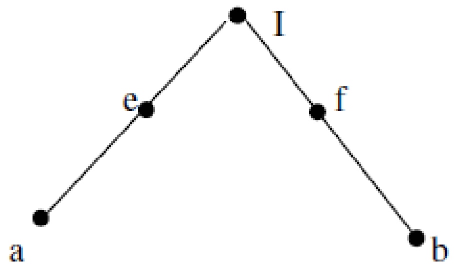
From the definition it is clear that **sublattice itself is a lattice.** However, **any subset of L which is a lattice need not be a sublattice.** For example, consider the lattice shown in the diagram:



L

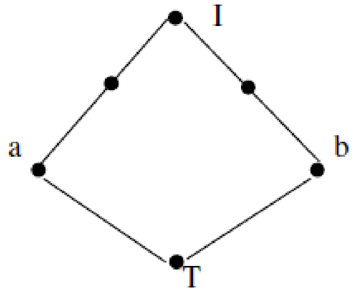
We note that

(i) the subset S shown by the diagram below is not a sublattice of L, since $a \wedge b \notin S$ and $a \vee b \notin S$.



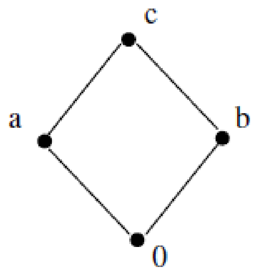
S

(ii) the set T shown below is not a sublattice of L since $a \vee b \notin T$.



However, T is a lattice when considered as a poset by itself.

(iii) the subset ∪ of L shown below is a sublattice of L:



U

Example: Let A be any set and P(A) its power set. Then (P(A), ∨ , ∧) is a
lattice in which join and meet are union of sets and intersection of sets respectively.

A family _ of subsets of A such that S ∪ T and S ∩ T are in _ for S,
T ∈ _ is a sublattice of (P(A), ∨ , ∧). **Such a family _ is called a ring of**
subsets of A and is denoted by (R(A), ∨ , ∧) (This is not a ring in the sense of algebra). **Some author call it lattice of subsets.**

Definition:

A lattice (L, ∨ **,** ∧ **)** is called a **distributive lattice** if for any elements a, b and c in L,

(1) a ∧ (b ∨ c) = (a ∧ b) ∨ (a ∧ c)

(2) a ∨ (b ∧ c) = (a ∨ b) ∧ (a ∨ c)

Properties (1) and (2) are called **distributive properties.**

Thus, in a distributive lattice, the operations ∧ and ∨ are distributive over each other.

We further note that, by the principle of duality, the condition (1) holds if and only if (2) holds. Therefore it is sufficient to verify any one of these two equalities for all possible combinations of the elements of a lattice.

If a lattice L is not distributive, we say that L is **non-distributive.**

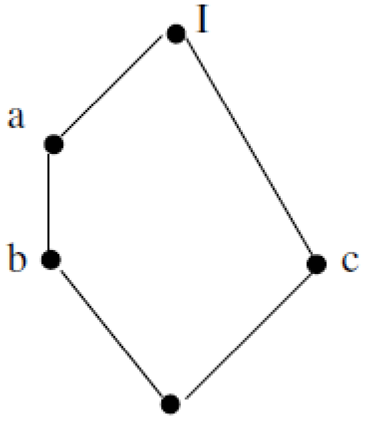
Example: For a set S, the lattice (P(S), $\subseteq$) is distributive. The meet and join operation in P(S) are $\cap$ and $\cup$ respectively. Also we know, by set

theory, that for A, B, C $\in$ P(S),

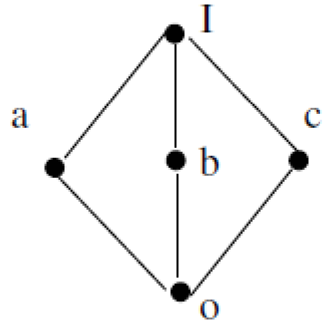
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Example:
The **five elements** lattices given in the following diagrams are **non distributive.**



(i)



(ii)

In fact for the lattice (i), we note that $a \wedge (b \vee c) = a \wedge I = a$,
while

$(a \wedge b) \vee (a \wedge c) = b \vee 0 = b$

Hence

$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$,

showing that (i) is non-distributive.

For the lattice (ii) ,

we have

$a \wedge (b \vee c) = a \wedge I = a$,

while

$(a \wedge b) \vee (a \wedge c) = 0 \vee 0 = 0$.

Hence

$a \wedge (b \vee c) \neq (a \wedge b) \vee (a \wedge c)$,

showing that (ii) is also non-distributive

Boolean Algebra

Definitions and Examples

Definition: A non-empty set B with two binary operations $\vee$ and $\wedge$, a unary operation $'$, and two distinct elements 0 and I is called a **Boolean Algebra** if the following axioms holds for any elements a, b, c $\in$ B:

[B_1]: **Commutative Laws:**

$$a \vee b = b \vee a \quad \text{and} \quad a \wedge b = b \wedge a$$

[B_2]: **Distributive Law:**

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \text{ and } a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

[B_3]: **Identity Laws:**

$$a \vee 0 = a \quad \text{and} \quad a \wedge I = a$$

[B_4]: **Complement Laws:**

$$a \vee a' = I \quad \text{and} \quad a \wedge a' = 0$$

We shall call 0 as zero element, 1 as unit element and a' the complement of a.

We denote a Boolean Algebra by (B, $\vee$, $\wedge$, ~, 0, I).

Example 1. Let A be a non-empty set and P(A) be its power set. Then the set algebra (P(A), $\cup$, $\cap$, $-$, ϕ, A) is a Boolean algebra.

Example 2 : Let B = {0, 1} be the set of bits (binary digits) with the binary operations $\vee$ and $\wedge$ and the unary operation $'$ defined by the following tables:

$\vee$	1	0
1	1	1
0	1	0

,

$\wedge$	1	0
1	1	0
0	0	0

$'$	1	0
	0	1

Here the operations $\vee$ and $\wedge$ are logical operations and complement of 1 is 0 whereas complement of 0 is 1. Then (B, $\vee$, $\wedge$, $'$, 0, 1) is a Boolean Algebra. It is the simplest example of a two-element algebra.

Further, a two element Boolean algebra is the only Boolean algebra whose diagram is a chain.

Example 3 : Let B_n be the set of n tuples whose members are either 0 or 1. Let $a = (a_1, a_2, \ldots, a_n)$ and $b = (b_1, b_2, \ldots, b_n)$ be any two members of B_n. Then we define

$$a \vee_1 b = (a_1 \vee b_1, a_2 \vee b_2, \ldots, a_n \vee b_n)$$

$$a \wedge_1 b = (a_1 \wedge b_1, a_2 \wedge b_2, \ldots, a_n \wedge b_n),$$

where $\vee$ and $\wedge$ are logical operations on {0, 1}, and

$$a' = (\sim a_1, \sim a_2, \ldots, \sim a_n),$$

where $\sim 0 = 1$ and $\sim 1 = 0$.

If 0_n represents (0, 0,.....,0) and 1_n = (1, 1,......,1), then $(B_n, \vee_1, \wedge_1, ', 0_n, 1_n)$ is a Boolean algebra.

This algebra is known as Switching Algebra and represents a switching network with n inputs and one output.

Example 4. The poset D_{30} = {1, 2, 3, 5, 6, 10, 15, 30} has eight element. Define $\vee$, $\wedge$ and $'$ on D_{30} by

$$a \vee b = \mathrm{lcm}(a, b), \quad a \wedge b = \gcd(a, b) \qquad \text{and} \qquad a' = \frac{30}{a}.$$

Then D_{30} is a Boolean Algebra with 1 as the zero element and 30 as the unit element.

Example 5: Let S be the set of statement formulas involving n statement variables. The algebraic system $(S, \wedge, \vee, \sim, F, T)$ is a Boolean algebra in which $\wedge, \vee, \sim$ denotes the operations of conjunction, disjunction and negation respectively. The element F and T denotes the formulas which are contradictions and Tautologies respectively. The partial ordering corresponding to $\wedge, \vee$ is implication $\Rightarrow$.

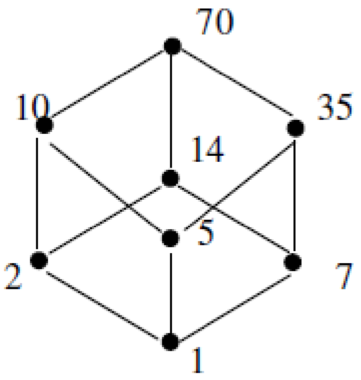
We have seen that B_n is a Boolean algebra. Using this fact, we can also define Boolean algebra as follows:

Definition: A finite lattice is called a **Boolean Algebra** if it is isomorphic with B_n for some non-negative integer n.

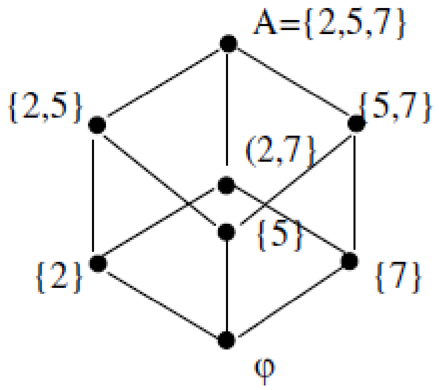
Definition: Let (B, ∨, ∧, ′, 0, 1) be a Boolean algebra and S ⊆ B. If S contains the elements 0 and 1 and is closed under the operation ∨, ∧ and 1, then (S, ∧, ∨, ′, 0, 1) is called **Sub-Boolean Algebra**.

Example: Consider the Boolean algebra

$D_{70} = \{1, 2, 5, 7, 10, 14, 35, 70\}$



D_{70}



P(A)

We note that the diagram for D_{70} and P(A) are structurally the same.

Then the set of atoms of D_{70} is

$$A = \{2, 5, 7\}$$

The unique representation of each non-atom by atoms is

$$10 = 2 \vee 5$$

$$14 = 2 \vee 7$$

$$35 = 5 \vee 7$$

$$70 = 2 \vee 5 \vee 7$$

The diagram of the Boolean algebra of the power set e(A) of the set A of atoms is given below :

Boolean Function

Definition: Let (B, . , +, ′, 0, 1) be a Boolean algebra. A function $f : B_n \to B$ which is associated with a Boolean expression (polynomial) is n variables is called a **Boolean function.**

Thus a Boolean function is completely determined by the Boolean expression $\alpha\ (x_1,\ x_2,\ldots,x_n)$ because it is nothing but the evaluation function of the expression. It may be mentioned here that every function $g : B_n \to B$ needs not be a Boolean function.

If we assume that the Boolean algebra B is of order 2^m for $m \geq 1$, then the number of function from B_n to B is greater than 2^{2n} showing that there are functions from B_n to B which are not Boolean functions. On the other hand, for m = 1, that is, for a two element Boolean algebra, the number of function from B_n to B is 2^{2n} which is same as the number of distinct Boolean expressions in n variable. Hence every function from B_n to B in this case is a Boolean function.

Representation of Boolean Functions using Karnaugh Map

Karnaugh Map is a graphical procedure to represent Boolean function as an "or" combination of minterms where minterms are represented by squares. This procedure is easy to use with functions f: $B_n \to B$, if n is not greater than 6. We shall discuss this procedure for n = 2, 3, and 4.

A Karnaugh map structure is an area which is subdivided into 2^n cells, one for each possible input combination for a Boolean function of n variables. Half of the cells are associated with an input value of 1 for one of the variables and the other half are associated with an input value of 0 for the same variable. This association of cell is done for each variable, with the splitting of the 2^n cells yielding a different pair of halves for each distinct variable.

Case of 1 variable: In this case, the Karnaugh map consists of $2^1 = 2$ squares.

0	1
x′	x

The variable x is represented by the right square and its complement x′ by the left square.

Case of 2 variables: For n = 2, the Boolean function is of two variable, say x and y. We have $2^2 = 4$ squares, that is, a 2×2 matrix of squares. Each squares contains one possible input from B_2.

The variable x appears in the first row of the matrix as x′ whereas x appears in the second row as x. Similarly y appears in the first column as y′ and as y in the second column.

	0	1
0	00	01
1	10	11

	y′	y
x′	x′y′	x′y
x	xy′	xy

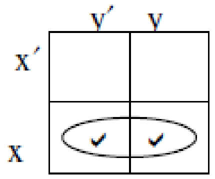
(2 variable Karnaugh Map)

Example : Find the prime implicants and a minimal sum-of-products form from each of the following complete sum-of-products Boolean expression:

(a) $E_1 = x\,y + x\,y'$ (b) $E_2 = x\,y + x'\,y + x'\,y'$

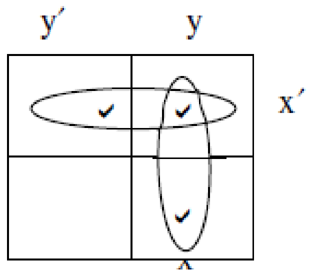
(c) $E_3 = x\,y + x'\,y'$.

Solution: (a) The Karnaugh map for E_1 is



Check the squares corresponding to $x\,y$ and $x\,y'$. We note that E_1 consists of one prime implicant, the two adjacent square designated by the loop. The pair of adjacent square represents the variable x. So x is the only prime implicant of E_1. Consequently $E_1 = x$ is its minimal sum.

(**b**) The Karnaugh map for E_2 is



Check the squares corresponding to $x\,y$, $x'\,y$, $x'\,y'$. The expression E_2 contains two pairs of adjacent squares (designated by two loops) which include all the squares of E_2. The vertical pair represents y and the horizontal pair x'. Hence y and x' are the prime implicants of E_2. Thus

$$E_2\,(x, y) = x' + y$$

is minimal sum.

(**c**) The Karnaugh map for E_3 is

	y'	y
x'	$x'y'$ ✓	
x		xy ✓

Check (tick) the squares corresponding to x y and x′ y′. The expression E_3 consists of two isolated squares which represent x y and x′ y′. Hence and x y and x′ y′ are the prime implicants of E_3 and so $E_3 = x\,y + x'\,y'$ is its minimal sum.

Case of 3 variables: We now turn to the case of a function f: $B_3 \to B$ which is function of x, y and z. The Karnaugh map corresponding to Boolean expression E(x, y, z) is shown in the diagram below:

y' y

	00	01	11	10
0	000	001	011	010
1	100	101	111	110

	$y'z'$	$y'z$	yz	yz'
x'	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
x	$xy'z'$	$xy'z$	xyz	xyz'

z

z′

Here x, y, z are respectively represented by lower half, right half and middle two quarters of the map.

Similarly, x′, y′, z′ are respectively represented by upper half, left half and left and right quarter of the map.

Part -B (5x8=40 Marks)

Possible Questions:

1. Let (L,≤) be a lattice. For any a ,b ϵ L, a ≤ b⇔ $a \wedge b = a \Leftrightarrow a \vee b = b$.
2. State and prove Demorgan's Law.
3. Prove that algebraically $a\bar{b} + b\bar{c} + c\bar{a} = \bar{a}b + \bar{b}c + \bar{c}a$.
4. Simplify the following Boolean functions to a minimum number of literals
   (i) $x \vee (x' \wedge y)$ (ii)$(x' \wedge y' \wedge z) \vee (x' \wedge y \wedge z) \vee (x \wedge y')$
5. Express the Boolean function F=$A \vee (B' \wedge C)$ in a sum of min terms.
6. Simplify the Boolean function $F(x, y, z) = \sum(0,1,2,4,5,6,8,9,12,13,14,)$.
7. Explain the basic laws of Boolean Algebra.
8. Show that a lattice is distributive iff
   $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$.
9. a)Let D24={1,2,3,4,6,8,12,24}and let the relation / be a partial ordering on D24.
   i) draw the Hasse diagram for D24 with /.
   ii) Find all the lower bounds of 8 and 12.
   iii) Find the GLB of 8 and 12.
   iv)Find all the upper bounds of 8 and 12
   v) Find the LUB of 8 and 12.
10. Express the Boolean function F=$(x \wedge y) \vee (x' \wedge z)$ in a product of max term form.

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DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics **Subject Code: 15MMU505A**
Class : III-B.Sc Mathematics **Semester : V**

UNIT IV

Part A (20x1=20 Marks)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
A setL on which apartial ordering ≤ is called a…………set	partially ordered	maximum ordered	quarterly ordered	ordered	partially ordered
The least member or Greatest member,if it exists, is……………………	finite	infinite	unique	zero	unique
Distinct minimal members are………………	comparable	incomparable	finite	in - finite	incomparable
By Idempotent Law (aΛa) =………	0	1	1	2a	a
Every pair of elements has LUB and GLB,the given poset is a……………	Lattice	duality	supremum	infimum	Lattice
In any Boolean algebra , the immediate successors of the O-element are called……	join	meet	atoms	dual	atoms
Every ………………..Boolean algebra is atomic	finite	infinite	unique	lesser	finite
Orered set (or) poset denoted by …….	(L,>)	(L,≥)	(L,<)	(L,≤)	(L,≤)
The LUB {d,b}=……..	b	d	b,d	b,d	b
Finite Boolean Algebra as n - tuples of …….	0's and 1's	1's only	0's only	n< o,n<1	0's and 1's
Every finite Boolean algebra has ……..	1^n elements	o elements	2^n elements	n elements	2^n elements
Every finite boolean algebra of order 2^n elements are …….	Endomorphic	Homomorphic	Atomic	Isomorphic	Isomorphic
The GLB {a,b} = …….	b,a	b	a,b	a	b
By Commutative Law (aΛb) =……………	b≥a	bΛa	b=a	bva	bΛa
In Boolean Algebra the value of (a+b)(a'+c)=………	ac+a'b+bc	ab+a'b+bc	ac+a'b'+bc	ac+ab'+b'c'	ac+a'b+bc
A……. Is a variable or the complement of a variable.	complementary	literal	biliteral	unilateral	literal
If x y z → 0 0 0 the min terms =...........	x'Λy'Λz'	x'Λy'vz'	xΛy'Λz'	xΛyΛz	x'Λy'Λz'
Boolean Function expressed as a product of maxterms is said to be ……..	canonical form	maxi terms	mini terms	maxi mini terms	canonical form
For n variables , we will have ……..different minterms and maxterms.	2^n	2/n	2n	(n+1)	2^n
Every finite Boolean algebra has ----elements for some positive integer n.	2^n+1	2^n-2	2^n-1	2^n	2^n
A Lineral is a variable or the ------------of a variable.	complement	commutative	distributive	associative	complement
Boolean function expressed as a ………..of mix terms.	sum	difference	product	equal	sum
Boolean function expressed as a ………..of max terms.	difference	sum	product	equal	product
In involution law , if a lattice be a complemented ……..lattice	commutative	associative	distributive	identity	Isomorphic
In the complement axioms aΛa'=………..	0	1	-1	a	1
Aboolean algebra is alattice which contains a least element and a greatest element and which is both..	commutative and distributive	associative and commutative	complemented and associative		complemented and distributive
A walk with no repeated vertices is called as ………..	cycle	path	circuit	trail	trail
The complemente og any function is same as the complement of each literal in the …of that function	lattice	boolean	dual	canonical	dual
The number of edges in a path is called …..	length of the path	size of the path	degree of the path	order of path	length of the path
If some edges are directed and some edges are undirected in a graph,then the graph is called….	digraph	weighted graph	isomorphic	mixed	mixed
An elementary cycle is a cycle if its path is …..	simple path	elementary path	simple trail	elementary trail	trail
The dual of an (bLC) = ….	a	b	c	0	a

KARPAGAM ACADEMY OF HIGHER EDUCATION
(Deemed to be University Established Under Section 3 of UGC Act 1956)
Pollachi Main Road, Eachanari (Po),
Coimbatore –641 021
DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics	**Semester :V**	**L T P C**
Subject Code: 15MMU505A	**Class : III- B.Sc Mathematics**	**5 0 0 5**

UNIT-V

Graph Theory: Directed and undirected graphs, Paths, Reachability, Connectedness, Matric representation, Eular paths, Hamiltonean paths, Trees, Binary trees simple theorems, and applications.

Text Book

1.Trembly J.P., and R.p Mahohar., 1975.Discrete Mathematical structures with applications to computer science,Tata Mc.Graw Hill,New Delhi.

References

1.Sundaresan V., Ganapathy Subramanian K.S., and Ganesan K., 2002.Discrete Mathematics Publications , Nagapattinam.

2. Veerarajan T., 2007.Discrete mathematics with graph theory and combinatorics, Tata Mc.Graw Hill,New Delhi.

3.Sharma .J.K,2005.Discrete Mathematics ,Second Edition, Macmillan India Ltd,New Delhi.

4. Discrete mathematics by Neeru Sharma, Publisher: New Delhi, India : University Science Press (An imprint of Laxmi Pubications Limited, Pvt. Ltd.), [2016] ©2011

UNIT -V

GRAPH THEORY

What is a Graph?

Definition: A graph (denoted as G = (V, E)) consists of a non-empty set of vertices or nodes V and a set of edges E.

Example: Let us consider, a Graph is G = (V, E) where V = {a, b, c, d} and E = {{a, b}, {a, c}, {b, c},{c, d}}

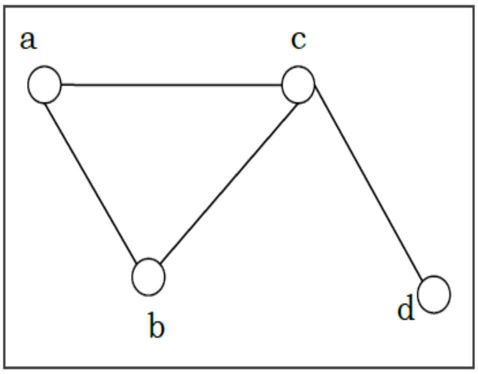


Figure: A graph with four vertices and four edges

Degree of a Vertex: The degree of a vertex V of a graph G (denoted by deg (V)) is the number of edges incident with the vertex V.

Vertex	Degree	Even / Odd
a	2	even
b	2	even
c	3	odd
d	1	odd

Even and Odd Vertex: If the degree of a vertex is even, the vertex is called an even vertex and if the degree of a vertex is odd, the vertex is called an odd vertex.

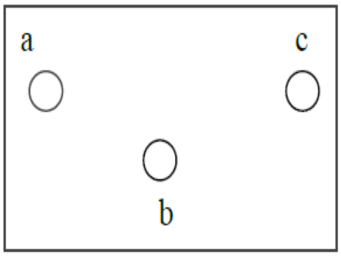
Degree of a Graph: The degree of a graph is the largest vertex degree of that graph. For the above graph the degree of the graph is 3.

Types of Graphs

There are different types of graphs, which we will learn in the following section.

Null Graph

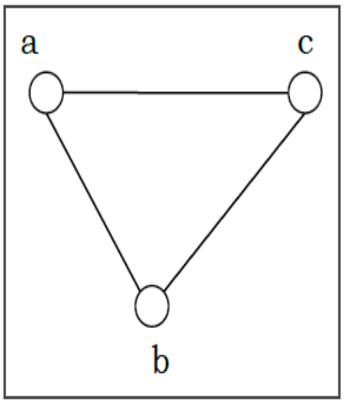
A null graph has no edges. The null graph of n vertices is denoted by N_n



Null graph of 3 vertices

Simple Graph

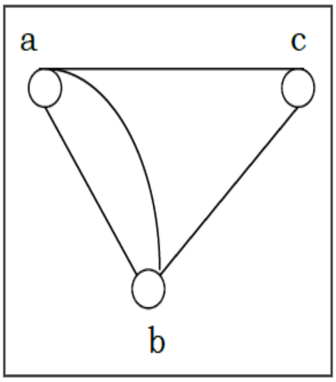
A graph is called simple graph/strict graph if the graph is undirected and does not contain any loops or multiple edges.



Simple graph

Multi-Graph

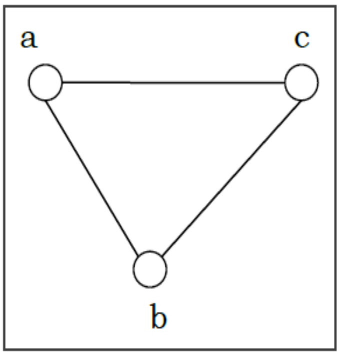
If in a graph multiple edges between the same set of vertices are allowed, it is called Multi-graph. In other words, it is a graph having at least one loop or multiple edges.



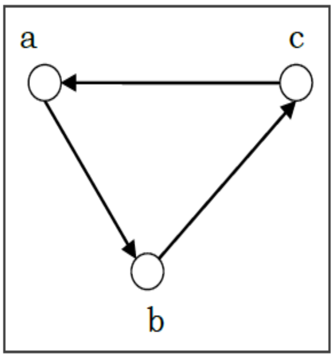
Multi-graph

Directed and Undirected Graph

A graph G = (V, E) is called a directed graph if the edge set is made of ordered vertex pair and a graph is called undirected if the edge set is made of unordered vertex pair.



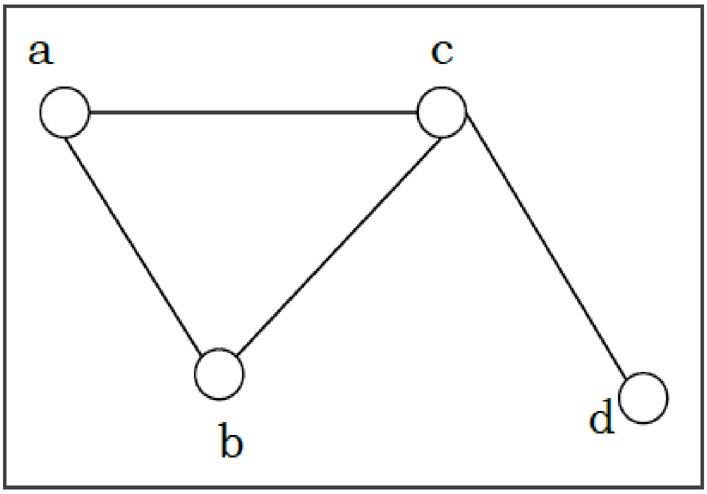
Undirected graph



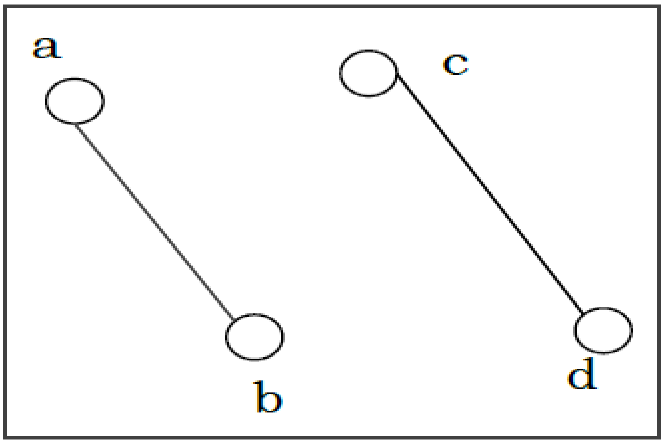
Directed graph

Connected and Disconnected Graph

A graph is connected if any two vertices of the graph are connected by a path; while a graph is disconnected if at least two vertices of the graph are not connected by a path. If a graph G is disconnected, then every maximal connected subgraph of G is called a connected component of the graph G.



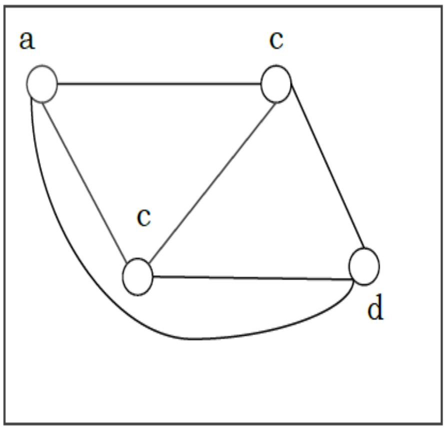
Connected graph



Unconnected graph

Regular Graph

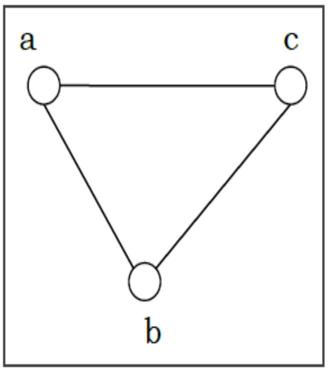
A graph is regular if all the vertices of the graph have the same degree. In a regular graph G of degree r, the degree of each vertex of G is r.



Regular graph of degree 3

Complete Graph

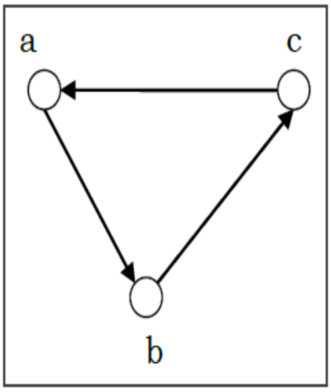
A graph is called complete graph if every two vertices pair are joined by exactly one edge. The complete graph with n vertices is denoted by K_n



Complete graph K_3

Cycle Graph

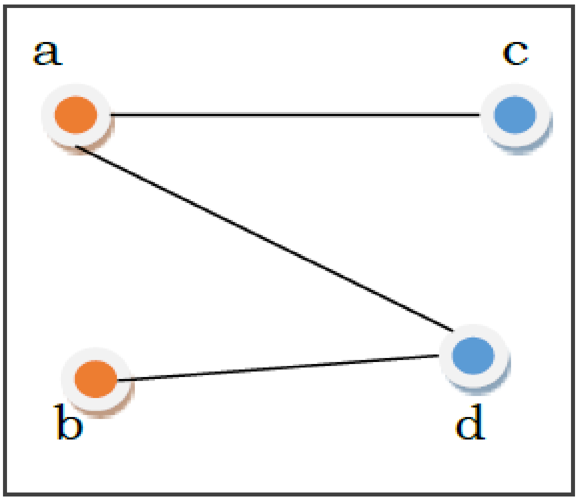
If a graph consists of a single cycle, it is called cycle graph. The cycle graph with n vertices is denoted by C_n



Cyclic graph C_3

Bipartite Graph

If the vertex-set of a graph G can be split into two disjoint sets, V_1 and V_2, in such a way that each edge in the graph joins a vertex in V_1 to a vertex in V_2, and there are no edges in G that connect two vertices in V_1 or two vertices in V_2, then the graph G is called a bipartite graph.



Bipartite graph

Representation of Graphs

There are mainly two ways to represent a graph:

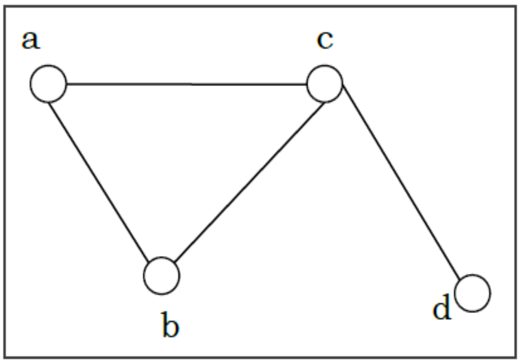
- Adjacency Matrix
- Adjacency List

Adjacency Matrix

An Adjacency Matrix A[V][V] is a 2D array of size V×V where V is the number of vertices in a undirected graph. If there is an edge between V_x to V_y then the value of $A[V_x][V_y]=1$ and $A[V_y][V_x]=1$, otherwise the value will be zero. And for a directed graph, if there is an edge between V_x to V_y, then the value of $A[V_x][V_y]=1$, otherwise the value will be zero.

Adjacency Matrix of an Undirected Graph

Let us consider the following undirected graph and construct the adjacency matrix:



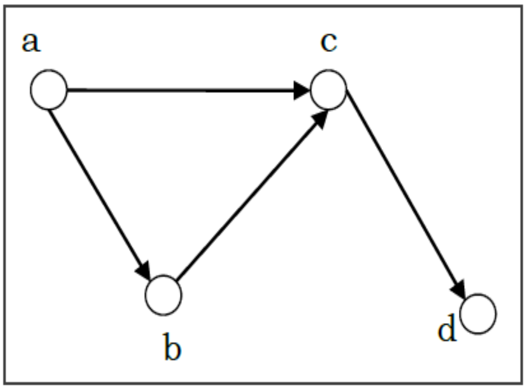
An undirected graph

Adjacency matrix of the above undirected graph will be:

	a	b	c	d
a	0	1	1	0
b	1	0	1	0
c	1	1	0	1
d	0	0	1	0

Adjacency Matrix of a Directed Graph

Let us consider the following directed graph and construct its adjacency matrix:



A directed graph

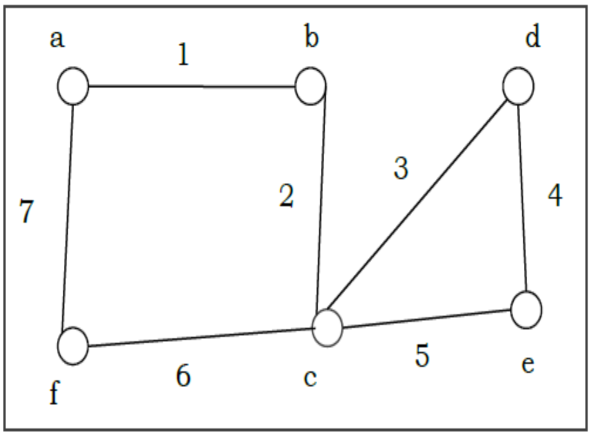
Adjacency matrix of the above directed graph will be:

	a	b	c	d
a	0	1	1	0
b	0	0	1	0
c	0	0	0	1
d	0	0	0	0

Euler Graphs

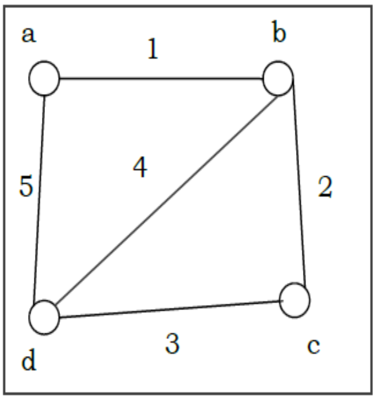
A connected graph G is called an Euler graph, if there is a closed trail which includes every edge of the graph G. An Euler path is a path that uses every edge of a graph exactly once. An Euler path starts and ends at different vertices.

An Euler circuit is a circuit that uses every edge of a graph exactly once. An Euler circuit always starts and ends at the same vertex. A connected graph G is an Euler graph if and only if all vertices of G are of even degree, and a connected graph G is Eulerian if and only if its edge set can be decomposed into cycles.



Euler graph

The above graph is an Euler graph as "a 1 b 2 c 3 d 4 e 5 c 6 f 7 g" covers all the edges of the graph.



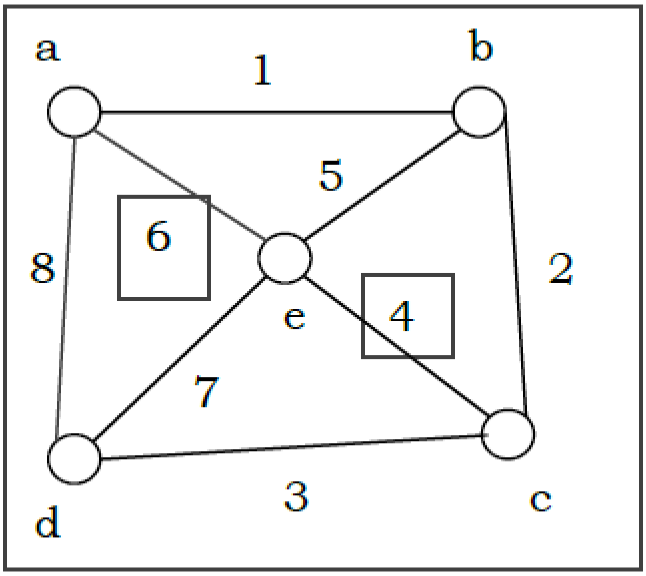
Non-Euler graph

Hamiltonian Graphs

A connected graph G is called Hamiltonian graph if there is a cycle which includes every vertex of G and the cycle is called Hamiltonian cycle. Hamiltonian walk in graph G is a walk that passes through each vertex exactly once.

If G is a simple graph with n vertices, where $n \geq 3$ If $deg(v) \geq n/2$ for each vertex v, then the graph G is Hamiltonian graph. This is called **Dirac's Theorem**.

If G is a simple graph with n vertices, where $n \geq 2$ if $deg(x) + deg(y) \geq n$ for each pair of non-adjacent vertices x and y, then the graph G is Hamiltonian graph. This is called **Ore's theorem**.



Hamiltonian graph

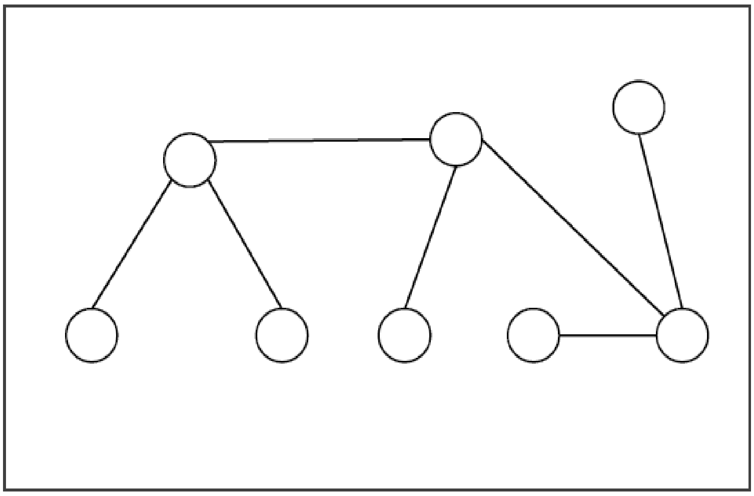
Tree

Tree is a discrete structure that represents hierarchical relationships between individual elements or nodes. A tree in which a parent has no more than two children is called a binary tree.

Tree and its Properties

Definition: A Tree is a connected acyclic undirected graph. There is a unique path between every pair of vertices in G. A tree with N number of vertices contains (N-1) number of edges. The vertex which is of 0 degree is called root of the tree. The vertex which is of 1 degree is called leaf node of the tree and the degree of an internal node is at least 2.

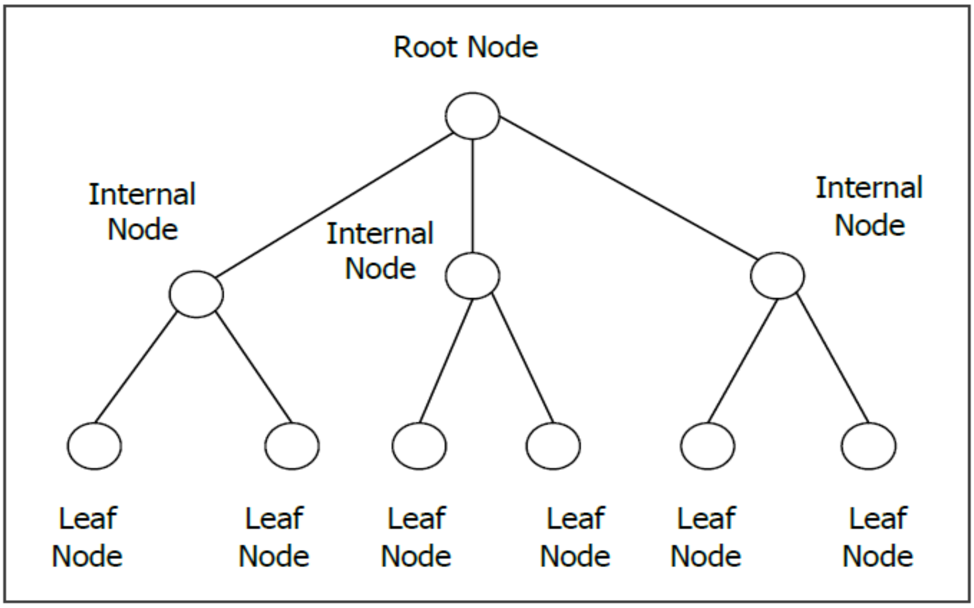
Example: The following is an example of a tree:



A tree

Rooted Tree

A rooted tree G is a connected acyclic graph with a special node that is called the root of the tree and every edge directly or indirectly originates from the root. An ordered rooted tree is a rooted tree where the children of each internal vertex are ordered. If every internal vertex of a rooted tree has not more than m children, it is called an m-ary tree. If every internal vertex of a rooted tree has exactly m children, it is called a full m-ary tree. If m = 2, the rooted tree is called a binary tree.



A Rooted Tree

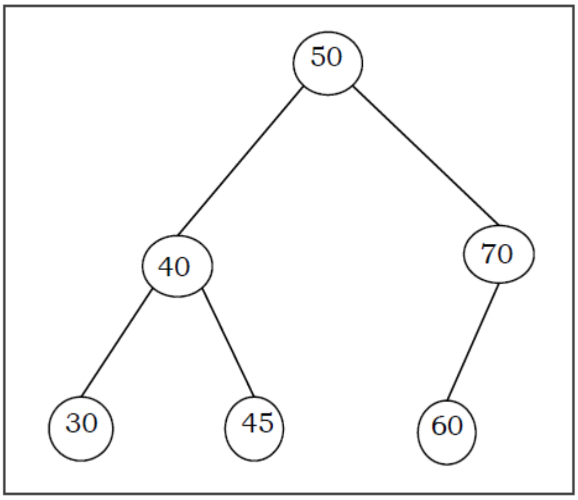
Binary Search Tree

Binary Search tree is a binary tree which satisfies the following property:

- X in left sub-tree of vertex V, Value(X) ≤ Value (V)
- Y in right sub-tree of vertex V, Value(Y) ≥ Value (V)

So, the value of all the vertices of the left sub-tree of an internal node V are less than or equal to V and the value of all the vertices of the right sub-tree of the internal node V are greater than or equal to V. The number of links from the root node to the deepest node is the height of the Binary Search Tree.

Example



A Binary Search Tree

Part -B (5x8=40 Marks)

Possible Questions:

1. Define the following terms by giving with examples:

i)Adjacency matrix
ii)Incidence matrix
iii)Path matrix
iv)Circuit matrix

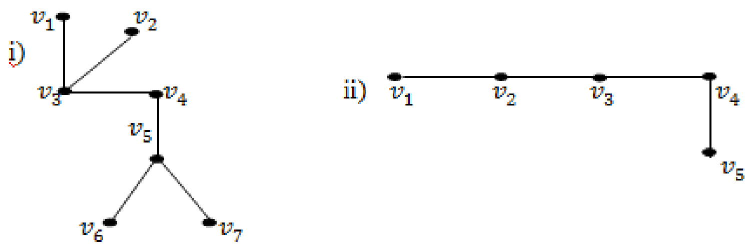
2. Define a tree and path length of a vertex with example.

3. State and prove handshaking lemma

4. Show that if a fully binary tree has i internal vertices then it has i+1 terminal vertices and (2i+1)total vertices.

5. Describe about konigsberg bridge problem.

6. Find the eccentricity of all vertices, center, radius and diameter of the following graph.



7. Prove that the number of vertices of odd degree in a graph is always even.

8. Prove that the number of pendent vertices of a tree is equal to $\frac{n+1}{2}$

9. Define graph. Explain the various types of graph with an example.

10. State and prove polyhedron formula.

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DEPARTMENT OF MATHEMATICS

Subject: Discrete Mathematics | **Subject Code: 15MMU505A**
Class : III-B.Sc Mathematics | **Semester : V**

UNIT V

Part A (20x1=20 Marks)

Possible Questions

Question	Choice 1	Choice 2	Choice 3	Choice 4	Answer
A graph is said to be ……if there exists atleast one path between every pair of veticesin G.	connected	disconnected	complete	regular	connected
A tree with ……vertices has atleast two vertices of ……1	n-1, degree	n-2 , order	n, degree	n-1, size	n, degree
The chromatic number of the chess board is …….	2	5	64	60	2
A tree with n vertices has ……..edges.	n-2	n-3	n-1	n	n-1
kruskal's algorithm is used to find …………… in a graph G	shortest spanning tree	longest spanning tree	binary tree	diameter spanning tree	shortest spanning tree
The number of internal vertices in a binary tree with n vertices is	n-1/2	n-2/2	n/2	n/3	n-2/2
A tree has atleast …………..pendant vertices	three	two	four	ten	two
An acyclic graph is called as ……….	forest	cycle	tree	trail	tree
Any vertex having degree one is called ………… vertex	pendant	loop	parallel	isolated	pendant
Any vertex having degree zero is called ……………vertex	pendant	loop	parallel	isolated	isolated
Any graph with edge set is empty is called as …………	complete	connected	disconnected	null	null
vertices with which a walk begins or ends are called its …………….	terminal vertices	terminal edges	pendant vertices	pendant edges	terminal vertices
A walk with no repeated vertices is called as………	cycle	path	circuit	trail	trail
The number of edges in a path is called ……….	length of the path	size of the path	degree of the path	order of the path	length of the path
If some edges are directed and some edges are undirected in a graph , then the graph is called……….	digraph	weighted graph	isomorphic	mixed	mixed
An elementary cycle is cycle if its path is ……………..	simple path	elementary path	simple trail	elementary trail	elementary path
A tree can have more than ……… centre.	one	two	three	four	one
Every edge of a weekly connected digraph ties exactly in one …………component.	weak	weakly connected	strong	strongly connected	weakly connected
A graph G=(V,E) in which every edge is directed is called as………..	digraph	undirected	connected	disconnected	digraph
A tree is a ………….. Graph without any cycle.	connected	disconnected	directed	undirected	connected
Two edges are said to be ------------ if they are incident on a common vertex.	adjacent	incident	pendant	isolated	adjacent
A graph has neither loops nor parallel edges is called a ----------------	digraph	simple	undirected	shell	digraph
A graph in which every vertex has the same degree is called --------------	digraph	undirected	simple	regular	regular
A walk is also called ----------------	chain	trail	cycle	path	chain
A graph having a Hamiltonian circuit is called------------	hamiltonian graph	digraph	euler graph	regular graph	hamiltonian graph
A graph in which weights are assigned to each edge is called a --------------graph	weighted	isomorphic	directed	undirected	weighted
A --------tree is rooted tree in which every vertex has either or no children	binary tree	ordered tree	rooted tree	rooted binary tree	binary tree
A __________ is a graph whose components are all trees.	tree	graph	forest	walk	forest
. A ________________ consists of set of vertices and edges such that each edge is incident with vertices.	graph	path	forest	walk	graph
A vertex having no edge incident on it is called_______________.	end vertex	pendant vertex	isolated vertex	null graph	isolated vertex
A graph is said to be ___________ if there exists at least one path between every pair of vertices in G.	connected	disconnected	null graph	hamiltanion	connected
A tree with n vertices has _____ edges	n	n-1	n-2	n+1	n-1
A graph in which all nodes are of equal degrees is known as…………………	regular graph	complete graph	simple graph	null graph	regular graph
A ___________ is connected graph without circuit	graph	directed graph	undirected graph	tree	tree
The sum of the degrees of all vertices of a graph is equal to __________ the number of edges.	twice	thrice	same	any	twice
A node with no children is called____________.	siblings	node	leaf	tree	leaf
A graph is ___________ if it has no parallel edges or self-loops	simple	directed	adjacent	self-loop	simple
A graph in which some edges are directed and some are undirected is called________________.	mixed graph	regular graph	complete graph	simple graph	mixed graph

A graph is a collection of.... ?	Row and columns	Vertices and edges	Equations	lines	Vertices and edges
In a tree between every pair of vertices there is ?	Exactly one path	A self loop	Two circuits	*n* number of paths	Exactly one path

Extra

Reg. No -----------

(15MMU505A)

KARPAGAM ACADEMY OF HIGHER EDUCATION

Karpagam University

COIMBATORE-21

DEPARTMENT OF MATHEMATICS

Fifth Semester

I INTERNAL TEST- JUL '17

DISCRETE MATHEMATICS

Date: . 07.17 ( ) **Time:2 hours**

Class: IIIB.Sc(Mathematics) **Maximum: 50 Marks**

PART-A (20x 1 =20 Marks)

ANSWER ALL THE QUESTIONS

1. $\neg(P\vee Q)$ = -----------
   a) $\neg P\wedge\neg Q$ b) $\neg P\wedge Q$ c) $\neg P\vee\neg Q$ d) $P\wedge Q$
2. $P\vee P$ is equivalent to -----------
   a)P b) $\neg P$ c)T d)F
3. $\neg(P\to Q)\Leftrightarrow$----------
   a) $Q\to\neg P$ b) $\neg Q\to P$ c) $\neg Q\wedge P$ d) $\neg Q\wedge\neg P$
4. $A\Leftrightarrow B$ states that --------- is a tautology
   a) $A\leftrightarrow B$ b) $A\to B$ c) $A\downarrow B$ d) $A\leftarrow B$
5. P has truth value T ,Q has truth value F then $P\to Q$ has truth value----------
   a)T b)F c)P d)Q
6. From (x) A(x) one can conclude A(y)
   a)Rule US b)Rule ES c)Rule EG d)Rule UG
7. $P\downarrow Q$ is equivalent to -------
   a) $P\vee Q$ b) $\neg(P\vee Q)$ c) $P\wedge\neg Q$ d) $Q\vee P$
8. A formula which is equivalent to a given formula and which consists of product of elementary sums is called a --------------
   a)PCNF b)DNF c)CNF d)PDNF
9. A statement that is always false is called --------------
   a)Contradiction b)Tautology c)Tautology implications d)none of these
10. $P\wedge P\Leftrightarrow P$ is called ----------- law.
    a) Idem potent b)Associative
    c)Commutative d)distributive
11. The dual of $(P\wedge Q)\vee T$ is ----------
    a) $(P\wedge Q)\wedge F$ b) $(P\vee Q)\wedge F$ c) $(P\vee Q)\wedge T$ d) $(P\wedge Q)\vee F$
12. $\neg(P\to Q)\Leftrightarrow$----------
    a) $Q\to\neg P$ b) $\neg Q\to P$ c) $\neg Q\wedge P$ d) $\neg Q\wedge\neg P$
13. From (x) A(x) one can conclude A(y)
    a)Rule US b)Rule ES c)Rule EG d)Rule UG
14. For three variables P,Q and R there are ------ maxterms
    a)2 b)4 c)6 d)8
15. If P then Q is called a -------------- statement.
    a) Conjunction b) Disjunction
    c) Conditional d)Biconditional
16. If R= {(1,2),(3,4),(2,2)} and S = {(4,2),(2,5),(3,1),(1,3)} are relations then SoS = ------
    a) {(4,2),(3,2),(1,4)} b){(1,5),(3,2),(2,5)}
    c){(1,2),(2,2)} d){(4,5),(3,3),(1,1)}
17. If f(x) = x+2 and g(x) = x^2 –2 for $x\in R$ then fog is ---------
    a) x^2-2 b) x^2+2 c) x^2 d) x^2-1
18. A One – to –one function is also known as ---------
    a)injective b)surjective c)bijective d)disjunctive
19. If f(x) = x+2 and g(x) = x^2 –1 then(gof)(x) = ------
    a) x^2+4x+4 b) x^2+4x-3 c) x^2-4x+4 d) x^2+4x+3
20. A binary relation R in a set X is said to be reflexive if
    a)aRa b) $aRb\Rightarrow bRa$
    c) $aRb, bRc\Rightarrow aRc$ d) $aRb, bRa\Rightarrow a=b$

PART-B (3x 10 = 30 Marks)

ALL THE QUESTIONS CARRY EQUAL MARKS

21.(a) Prove that $(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R))$ is a tautology.

(OR)

(b) Show that the following premises are Inconsistent.

i) If Jack misses many classes through illness, he fails in school.

ii) If jack fails in school, then he is uneducated.

iii)If jack reads a lot of books, then he is not uneducated.

iv) Jack misses many classes through illness and reads a lot of books.

22.(a) Find the PDNF and PCNF of the formula

$P \vee (\neg P \rightarrow (Q \vee (\neg Q \rightarrow R)))$

(OR)

(b) Construct the truth table for $\neg [P \vee (Q \wedge R)] \leftrightarrow [(P \vee Q) \wedge (Q \vee R)]$

23.(a) Explain the properties of relations with examples.

(OR)

(b) Let {1,2,3} , f , g, h and s be functions from X to X given by f={(1,2) , (2,3),(3,1)}, g={(1,2) , (2,1),(3,3)} , h ={(1,1), (2,2),(3,1)} and s={(1,1) , (2,2),(3,3)} find $f \circ g$, $g \circ f$, $h \circ g$, $g \circ s$, $s \circ s$, $f \circ s$, $f \circ h \circ g$.

Reg no-----------------

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Karpagam University

Coimbatore-21

DEPARTMENT OF MATHEMATICS

Fifth Semester

II Internal Test - AUG'2017

Elective-1 Discrete Mathematics

Date: 11-08-2017 **Time: 2 Hours**

Class: III B.Sc Mathematics **Maximum Marks:50**

PART-A (20X1=20 Marks)

Answer all the Questions:

1. If f(x) = x+2 and g(x) = x2 –1 then(gof)(x) = ________
   a) x^2+4x+4 b) x^2+4x-3 c) x^2-4x+4 d) x^2+4x+3
2. If the relation R and S are both reflexive then R Ú S is ________
   a) Symmetric b) reflexive c) transitive d) not reflexive
3. Let $f: x \to y$, $g: y \to x$ be the functions then g is equal to $f-1$ only if ______
   a) $fog = I_y$ b) $gof = I_x$ c) $gof=I_y$ d) $fog=I_x$
4. A binary relation R in a set X is said to be transitive if ______
   a) aRa b) aRb $\Rightarrow$ bRa c) aRb,bRc $\Rightarrow$ aRc d) aRb,bRa $\Rightarrow$ a=b
5. The function fog is called the ____________ function.
   a) Inverse b) identity c) composition d) bijective
6. A One – to –one and onto function is also known as ________
   a)injective b) surjective c)bijective d)objective
7. In N, define aRb if a+b = 7. This is symmetric when ________
   a) a+a =7 b) b+a =7 c) b+c =7 d)a + c = 7
8. Suppose in RxR, the ordered pairs (x-2, 2y+1) and (y-1, x+2) are equal. then values of x and y are_____
   a) 2,3 b) 3,2 c) 2,-3 d)3,-2
9. A mapping $f: X \to Y$ is called ________ if distinct elements of x are mapped into distinct elements.
   a) one-to-one b) Onto c) into d) many to one
10. Let $f: N \to N$ be a function such that $f(x) = 5, x \in N$ then the f(x) is called________function.
    a) identity b) inverse c) equal d) constant
11. Let x = {1,2,3,4}, R = {(2,3),(4,1)} then the range of R = ______
    a) {1, 2,3, 4} b) {3,1} c) {2,4} d) {1,4}
12. From the below, the identity function is__________
    a) F(x) = 2x b) $F(x) = x^2$ c)f(x) = x d) F(x) = G(x)
13. A string containing no symbol the __________
    a) Empty word b) two word c) single word d)Simple sentence
14. The elements of a vocabulary are called__________
    a) Letter b) Number c) Numeric d) Verb
15. The syntax of a small subset of the English language can be described by using________
    a) Symbol b) Numeric c) Number d) small letters
16. The sentence has two subdivision __________
    a)Article and Verb phrase b) Subject and verb phrase
    c) Verb and subject d) Noun and verb
17. A system or language which describe another language is called ________
    a) Meta language b) Beta language c) verb d) Beta verb
18. If the syntax is correct then it produces __________ code
    a) Verb b) sentence c) language d)object
19. The another name of phrase structure is ______
    a) Sentence b) Noun c) n-tuples d) grammar
20. A context-sensitive grammar contains only productions of the form $\alpha \to \beta$ where________
    a) $|\alpha| \le |\beta|$ b) $|\alpha| \ne |\beta|$ c) $|\alpha| \ge |\beta|$ d) $|\alpha| = |\beta|$

PART-B(3X10=30 Marks)

Answer all the Questions:

21. (a) Define equivalence relation. Let X = {1, 2, 3…7} and R = {(x, y) |x – y is divisible by 3}. Show that R is an equivalence relation.

(OR)

(b) Explain the types of grammars with examples.

22. (a) i) Let the functions f and g on the real numbers be defined by $f(x) = x^2 + 2x - 3$, $g(x) = 3x - 4$. Find the formulas which define the product $g \circ f$ and $f \circ g$.

ii) Find $f \circ g$ and $g \circ f$ when $f: R \to R$ and $g: R \to R$ defined by $f(x) = 2x - 1$, $g(x) = x^2 - 2$.

(OR)

(b) Prove that $L(G) = \{ a^n b^n c^n / n \geq 1\}$ where $G = (\{S,B,c\},\{a,b,c\},S,\varphi)$ and $Q = \{ S \to aSBc, S \to aBc, cB \to Bc, aB \to ab, bC \to bc, cC \to cc\}$.

23. (a) Explain the properties of relations with examples.

(OR)

(b) Show that the language $L(G_4) = \{a^n b a^n / n \geq 1\}$ is generated by the following grammar:

$G_4 = (\{S,C\}, \{a,b\}, S, \varphi)$, where φ consists of productions $\{S \to aCa, C \to aCa, C \to b\}$.

Reg.No.....................

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KARPAGAM UNIVERSITY

Karpagam Academy of Higher Education

Coimbatore – 21

DEPARTMENT OF MATHEMATICS

Fifth Semester

Model Examination – September'2017

Discrete Mathematics

Date : .09.2017( ) **Time:3Hours**

Class: III B.Sc Mathematics **Maximum :60 Marks**

PART – A (20X1=20 Marks)

Answer all the questions

1. A ....is a sentence that is true or false but not both.
a) proposition b) logic
c)sentence d) empty

2. Given a statement p, the sentence "~ p" is read "not p" or "It is not the case that p" and is called the.............
a) negation of p b) conjunction of p and q
c)sentence d) logic

3. Suppose x is a real number. Let q, and r symbolize "$x < 3$," and "$x = 3$," respectively, then $x \leq 3$ is given by.......
a) ~q v r b) q ^ r
c) q v r d) ~r v q

4. Two statement forms are called ....if and only if, they have identical truth values for each possible substitution of statements for their statement variables.
a)logically in equivalent b) isomorphic
c) invalid d)logically equivalent

5. The .... of a function as the image of its domain
a) domain b) range
c) co domain d) image

6. In one-one mappings an element in B has only................pre image in A
a) zero b)two
c) one d) three

7. If f:A→B in this set B is called the ....of the function f.
a) domain b) co domain
c) set d) element

8. The element a may be referred to as the ....of f(a)
a) f-image b) pre-image
c) domain d) co domain

9. FAS have only......... amount of memory and recognition of a CFL may require storing an unbounded amount of information.
a) finite b) infinite
c) bounded d) unbounded

10. A PDA provides ....Memory.
a)finite b) infinite
c) unlimited d)un bounded

11. A pushdown automaton is said to be .... with two conditions.
a) Deterministic b) Non deterministic
c) well known d) unknown

12. If L is a Context free language, then there exists a.........that accepts L.
a) FSA b)CFL
c) PDA d) ID

13. In Involution law ,if a lattice be complemented,.............lattic
a) commutative b) associative
c) distributive d) identity

14. In the complement axioms ana'=..............
a) 0 b) 1
c) -1 d) a

15. A Boolean algebra is a lattice which contains a least element and a greatest element and which is both ....
a) commutative and distributive
b) associative and commutative
c) complemented and distributive
d) complemented and associative

16. The complement of any function is same as the complement of each literal in the ....of that function
a) lattice b) boolean
c) dual d) canonical

17. A tree is a .... Graph without any cycle.
a) connected b) disconnected
c) directed d) undirected

18. A tree can have more than .... centre.
a) one b) two
c) three d) four

19. Every edge of a weekly connected digraph ties exactly in one ....component.
a) weak b) weakly connected
c) strong d) strongly connected

20. A graph G=(V,E) in which every edge is directed is called as..........
a) digraph b) undirected
c) connected d) disconnected

PART – B (5X8=40 Marks)

Answer all the questions:

21. a) Prove that:
$(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology.

(OR)

b) Show that the following premises are Inconsistent.
i) If Jack misses many classes through illness, he fails in school.
ii) If jack fails in school, then he is uneducated.
iii) If jack reads a lot of books, then he is not uneducated.
iv) Jack misses many classes through illness and reads a lot of books.

22. a) For integers a,b define aRb iff a – b is divisible by m. Show that R defines an equivalence relation on Z.

(OR)

b) Let S={1,2,3,4,5} and T={1,2,3,8,9} and define the functions $f: S \rightarrow T$ and $g: S \rightarrow S$ by f={(1,8), (3,9),(4,3),(2,1),(5,2)} and g={(1,2),(3,1),(2,2),(4,3),(5,2)} ,then find the values of the following $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$.

23. a) Show that the language $L(G_4) = \{a^n b a^n / n \geq 1\}$ is generated by the following grammar:
$G_4 = (\{S,C\}, \{a,b\}, S, \varphi)$, where φ consists of productions $\{S \rightarrow aCa, C \rightarrow aCa, C \rightarrow b\}$.

(OR)

b) Show that the language $L(G_5) = \{a^n b c^m / m,n \geq 1\}$ is generated by the following grammar:
$G_5 = (\{S,A,B,C\}, \{a,b\}, S, \varphi)$, where the set φ consists of production is $S \rightarrow aS$, $S \rightarrow aB$, $B \rightarrow bC$, $C \rightarrow ac$, $C \rightarrow a$.

24. a) Prove that algebraically $a\bar{b} + b\bar{c} + c\bar{a} = \bar{a}b + \bar{b}c + \bar{c}a$.

(OR)

b) State and prove Demorgan's Law.

25. a) Prove that the number of vertices of odd degree in a graph is always even.

(OR)

b) Prove that the number of pendent vertices of a tree is equal to $\frac{n+1}{2}$.